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TURBULENT THERMAL COUNTERFLOW OF HELIUM II IN LARGE CIRCULAR CHANNELS

The Ohio State University

Ph.D. 1982

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Turbulent Thermal Counterflow of HeII in Large Circular Channels

Dissertation

Presented in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in the Graduate School of the Ohio State University

by

Kevin Paul Martin, B.S., M.S.

The Ohio State University

1982

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Chapter I

Introduction

1.1 Introduction

Since the liquefaction of $\text{He}^4$ in 1908 by Kamerlingh-Onnes\textsuperscript{1} and the discovery that the HeII phase is a superfluid\textsuperscript{2,3}, HeII has entertained much theoretical and experimental interest as a quantum liquid. In the last 40 years considerable attention has been paid to flow properties of superfluid helium. Experiments to determine the viscosity of HeII produced two entirely different results. The results of experiments which measured the viscous resistance to flow\textsuperscript{4} showed a viscosity of essentially zero; HeII can flow without dissipation. Other types of experiments\textsuperscript{5,6,7} measured the drag on a body moving through the fluid. These findings indicated a viscous drag nearly the size of that from He\textsuperscript{4} gas. These contradictory discoveries were explained in terms of the two-fluid model suggested by Tisza\textsuperscript{3}. In this model HeII behaves as if it were composed of two distinct non-interacting fluids totally immiscible in one another. These components are a superfluid portion which flows without friction and a normal fluid component which has an ordinary viscosity. The
entropy of HeII is only carried by the normal fluid fraction. The
density of HeII is the sum of normal and superfluid densities

\[ \rho = \rho_s + \rho_n \]

The flow of HeII at low velocities is laminar and is accurately
described by the Landau two-fluid equations.

\[ \rho_n \frac{\partial \mathbf{v}_n}{\partial t} + \rho_n (\mathbf{v}_n \cdot \nabla) \mathbf{v}_n = \eta \nabla^2 \mathbf{v}_n - \frac{n}{\rho} \mathbf{v} \cdot \nabla \mathbf{v} + \rho_s SVT \quad (1.1) \]

\[ \rho_s \frac{\partial \mathbf{v}_s}{\partial t} + \rho_s (\mathbf{v}_s \cdot \nabla) \mathbf{v}_s = - \frac{\rho_s}{\rho} \mathbf{v} \cdot \nabla \mathbf{v} + \rho_s SVT \quad (1.2) \]

where \( \mathbf{v}_n \) and \( \mathbf{v}_s \) are velocities of the normal and superfluid components. These equations describe the motion of HeII in terms of \( \rho_s \) and \( \rho_n \) flowing independently. There is no interaction between \( \rho_s \) and \( \rho_n \). The only dissipation is from the normal fluid viscosity.

1.2 Mutual Friction Force

At higher velocities the flow of HeII is no longer laminar but instead becomes turbulent (for a recent and comprehensive review of superfluid turbulence see reference 10). When this occurs a dissipation in addition to that from normal fluid viscosity appears.
This was explained phenomenologically in terms of a mutual friction force $F_{sn}$ by Gorter and Mellink\textsuperscript{11}. The independent motion of $\rho_s$ and $\rho_n$ breaks down and their interaction via $F_{sn}$ produces an excess dissipation. Since $F_{sn}$ is between the normal and superfluid components it enters both two-fluid equations with opposite signs. For turbulent flow of HeII $F_{sn}$ is included in equations 1.1 and 1.2 and they are re-written as:

$$\rho_n \frac{\partial \mathbf{v}_n}{\partial t} + \rho_n (\mathbf{v}_n \cdot \nabla) \mathbf{v}_n = -\eta \nu^2 \mathbf{v}_n - \frac{\rho_n}{\nu} \mathbf{v}_n - \rho_s \mathbf{v} \tau + F_{sn}$$

(1.3)

$$\rho_s \frac{\partial \mathbf{v}_s}{\partial t} + \rho_s (\mathbf{v}_s \cdot \nabla) \mathbf{v}_s = -\frac{\rho_s}{\nu} \mathbf{v}_s + \rho_s \mathbf{v} \tau - F_{sn}$$

(1.4)

A force similar to $F_{sn}$ was observed by Hall and Vinen\textsuperscript{12} in their experiment measuring attenuation of second sound in rotating helium. They determined the force to be from scattering of normal fluid excitations off quantized vortex lines introduced by the rotation. Following a suggestion by Feynman\textsuperscript{13} that turbulence in HeII could be a distribution of quantized vortex lines and using the results of the rotating HeII experiment, Vinen\textsuperscript{14} proposed that $F_{sn}$ arose from a homogeneous tangle of vortex lines. When the vortex line distribution is in a steady state $F_{sn}$ has the form
The coefficient $B$ is determined from experiments on the attenuation of second sound in rotating HeII.$^{12,15}$ $K$ is the quantum of circulation (h/m), $L_0$ is the steady state vortex line density (the length of line per unit volume), and $V$ is the relative velocity between the normal and superfluid components.

$$V=V_n-V_s$$ (1.6)

In many cases experimental probes of superfluid turbulence are limited to measuring temporal and spatial averages of the system in the steady state. For flow through a channel then equations 1.4 and 1.3 may be re-written as

$$0 = \eta V^2 <V_n> - \frac{\rho_n}{\rho} <\nabla p> - \rho_s S <\nabla T> + F_{sn}$$ (1.7)

$$0 = -\frac{\rho_s}{\rho} <\nabla p> + \rho_s S <\nabla T> - F_{sn}$$ (1.8)

using the mutual friction approximation$^{11}$. The bracketed terms are time averages of the axial components. Experimental results$^{16}$
indicate that $\langle \nabla P \rangle$ in turbulent flow is not much different than the laminar value (the "Allen and Reekie" rule). For a channel of length $l$ this leads to a uniform temperature gradient given very accurately as:

$$\nabla T = \frac{\Delta T_L}{L} + \frac{F_{sn}}{\rho_s S}$$

The dissipation from $F_{sn}$ appears as a temperature difference in excess of that from laminar flow ($\Delta T_L$).

1.3 Theory

A phenomenological description of the length of vortex line per unit volume ($L$) was developed by Vinen. The model pictured the vortex tangle as being homogeneous, isotropic and drifting with the superfluid component. The so-called modified Vinen equation describes the rate of change of line density ($dL/dt$) as being determined by two independent competing processes of line production and annihilation. A steady state line density ($L_0$) is calculated by setting $dL/dt=0$. Vinen's model also incorporates the experimentally observed fact that there is a critical relative velocity ($V_c$) below which $L_0$ is zero. The critical condition for $L_0$ is
For $1.2V$ to $4V$ in a flow channel with small dimension $d$, $L_0$ is accurately described by the approximate form

$$L_0^{1/2}(T,V)d = 2$$  \hspace{1cm} (1.10)

The dependence of $L_0$ on $V$ is given directly by this model. The phenomenological parameters $\gamma(T)$ and $\alpha$ are determined experimentally. The parameter $\gamma(T)$ is temperature dependent, while $\alpha$ is a temperature independent number of order unity, included to account for the critical velocity. The model constructed by Vinen has successfully explained experiments dealing with turbulent flow in HeII, but the model's origins should be kept in mind. Vinen's assumptions of homogeneity and isotropy of the vortex tangle are for the most part not justified in most experimental systems. In addition to this, the model is purely phenomenological.

A modern approach for describing superfluid turbulence has been produced by Schwarz. The theory considers a vortex tangle in terms of a microscopic distribution function for the lines. It assumes a homogeneous distribution of vortex lines in the presence of a uniform relative velocity. Using the localized induction approximation and the self induced motion of a piece of vortex line he determines the vortex line dynamics locally. The microscopic structure of the line
distribution $\lambda(V_x,t)$ is written as a function of the local velocity of line ($V_x$) and time. The integral of $\lambda(V_x,t)$ over all $V_x$ is $L$. Using $\lambda(V_x,t)$ and the local line dynamics he determines an equation for $(d\lambda/dt)$ which has no adjustable parameters. Starting from a particular $V_x$ and $T$ and an arbitrary line distribution, $d\lambda/dt$ is numerically integrated forward in time and integrated over all $V_x$ space to determine $L$. The steady state line density which results has a form remarkably similar to that found from Vinen's model. This agreement is important in that it allows one to adopt previous analyses done in terms of Vinen's phenomenological model to Schwarz's theory of homogeneous turbulence.

1.4 Experiment

Many experiments have been performed to clarify the properties of superfluid turbulence. These experiments utilized a variety of probes for the investigation. They include measuring temperature, pressure, chemical potential differences, second sound attenuation, ion entrainment on vortex lines, and local temperature gradient measurement. The channel's small dimension $d$ often determined what sort of probes could be used. The range of $d$ studied extends from 1 cm to $3 \times 10^{-4}$ cm. Channel geometry is another experimental factor which varies among experiments. Different cross sections that were used include circular, square, and rectangular. The results show that many aspects
of superfluid turbulence are determined by $T$, channel characteristics, and the size and nature of $V$ (i.e., pure superflow or a particular combination of $V_n$ and $V_s$).

Several forms of turbulence have been identified and associated with different flow configurations. A major influence on turbulence arises from channel geometry. Thermal counterflow in circular and square tubes has two different turbulent states (identified as states $T_1$ and $T_3$). In high aspect ratio rectangular channels there is only one turbulent counterflow state (called $T_3$) present. Experimental results in different channel geometries have shown that Vinen's critical condition in equation 1.10 is actually closer to $L d=2.5^{36}$. The form of turbulent flow not only depends on geometry but also depends on what particular $V_n$ and $V_s$ combination is present. Pure superflow exhibits only one turbulent state ($T_4$) for both circular and rectangular tubes$^{37,38}$. Dutch workers at Leiden$^{39,40}$ have pioneered methods of varying $V_n$ and $V_s$ independently. Their results have revealed a complex texture of flow behavior as a function of different $V_n$ and $V_s$ combinations.

Many experiments (including this one) have investigated superfluid turbulence in thermal counterflow. A schematic of a simple thermal counterflow apparatus is seen in figure 1.1. The helium bath is connected to a sample cell by a flow channel with cross-sectional area $A$. A heat current $Q$ is emitted by the cell heater. The cell temperature and pressure have values of $P+\Delta P$ and $T+\Delta T$. If $P+\Delta P$ and
Figure 1.1

A schematic illustration of a typical thermal counterflow apparatus
Figure 1.1
\[ V_n = \frac{Q}{(\rho_{\text{STA}})} \] (1.12)

The superfluid fraction carries no entropy and does not leave the cell. Instead, it satisfies the condition that the net mass current be zero by flowing into the cell. This constraint determines the average velocity of \( \rho_n \) to be:

\[ V_s = \frac{\rho_n}{\rho_s} V_n \] (1.13)

The relative velocity between \( \rho_n \) and \( \rho_s \) is

\[ V = V_n - V_s = \frac{A}{\rho_s \text{STA}} \] (1.14)

1.5 Motivation of Research

The preceding introduction has shown that turbulent flow of HeII has been characterized in detail. However, at the beginning of this thesis work, the wide variety of accumulated research had not answered some important questions. The experimental evidence for strong geometry dependence of the turbulent flow state was unexpected and unexplained. Uncertainty arose regarding the validity of homogeneity.
assumptions in theoretical descriptions of the vortex line tangle.

Turbulent counterflow in small channels was typically probed via the measurement of temperature differences between channel ends. Results of these experiments were used to determine the Vinen parameters $\gamma(T)$ and $\alpha$ (equations 1.10 and 1.11). In large channels temperature differences are very small and difficult to detect. Other aspects of superfluid turbulence were investigated in large channels using different methods such as second sound or ions. Size limitations prevent using these probes in small channels. A comparison of data from small and large channels is not useful unless one can establish some similarity between the flow states observed in them.

This experiment was undertaken to provide a comparison between large and small channels. A small channel type of experiment was performed in a large (1 mm diameter) channel. The aim of this experiment was to determine what features of superfluid turbulence present in small channels also appear in large channels. By determining the Vinen parameters in a large channel one can observe how they scale with size. If state TII is observed in a large channel an investigation of TII could add to understanding of its origin. To accomplish these goals an apparatus for measuring temperature changes smaller than 0.5 uK had to be constructed. This method was developed and first used by a previous student C. Piotrowski. The following is a description of the present apparatus and the results of the experiment.
for which it was used. The experiment must be considered as very successful since all of the original goals were met.
Chapter II
Apparatus

2.1 Introduction

A schematic diagram of the apparatus may be seen in figure 2.1. This apparatus is similar in design and construction to that used by a previous student. The counterflow channel connected the sample cell to the helium bath. The counterflow channel and cell were enclosed in a vacuum can so that the cell's only thermal link to the bath was by counterflow of HeII through the flow tube. The vacuum can made a superleak tight seal to the probe flange with an indium O-ring. The cell contained two heaters for the production of counterflow and the superconducting magnet and bolometer (used in conjunction with the SQUID magnetometer) for measuring small temperature changes in helium filling the cell. The leads for the magnet, secondary inductance coil \(L_2\), and heaters exited the can by a Stycast 1266 epoxy feedthrough on the probe flange. The heater and magnet leads were attached to a terminal board on the probe flange. The secondary inductance coil was connected to the pickup coil \(L_p\) of the SHE Multi Function SQUID Probe. The leads for the bolometer and primary inductance coil \(L_1\)
Figure 2.1

A schematic diagram of the experimental apparatus used for the investigation of thermal counterflow in large circular channels.
Figure 2.1
left the can by running up a hollow support post and out a feedthrough at the top of the probe. All leads were inductively wound to minimize RF pickup and were connected across RF feedthrough capacitors to the leads that ran to the top of the probe. The details of these parts will be discussed in the following sections.

2.2 Flow Channel

The flow channel was a circular glass tube 10±.05 cm. long with a 1±.01 mm inside diameter and a wall thickness of .25 mm. The tube was much longer than the entrance length \(^{43}\) for channels of this diameter (about 5mm for laminar flow) so that the counterflow observed was not characterized by end effects. The tube was microscopically inspected to make sure it had no flaws or dirt on the inner walls and that the inside diameter was the same at both ends. After being selected the tube was flushed with acetone and blown out with dry nitrogen. The tube was reinforced by covering it with a fiberglass sleeve \(^{44}\) which was painted with 1266 epoxy to make the covering rigid and bond it to the channel walls. One end of the tube was glued into a hole in the cell cap (figure 2.2) and the other end into the stainless steel adapter. Great care was taken in the gluing operations so as not to block the tube ends. After cell assembly was completed the adapter was soldered into the stainless steel tube on the flange at the end of the probe.
A diagram of the cell, flow channel and stainless steel adapter assembled together and soldered to the vacuum can flange.
Figure 2.2
2.3 Cell

The cell and cap (figure 2.2) were machined from blocks of 1266 epoxy. The cell contained heaters used to generate counterflow and the magnet and bolometer used to detect temperature changes of helium in the cell. The leads for these left the cell by small holes drilled in the sides of the cell that were sealed with stycast. The cell and cap were also glued together with 1266 epoxy. The cell held 5.03 cm of helium with the heaters, bolometer and magnet in place. During the experiment the heat capacity of cell and its components were negligible compared with that of the helium in the cell.

2.4 Heaters

The heaters had resistances of 531 and 1054 ohms, and were made of Evanohm wire\textsuperscript{45} that had a resistance of 200 ohms/foot. The wire was inductively wound to eliminate self inductance and RF pickup. Then the wire was wrapped around thin wooden dowels about 1.5 cm long and lightly coated with GE varnish\textsuperscript{46} to prevent unwinding. The leads that connected the heaters to terminal boards outside of the can were of the same superconducting wire as in the magnet. The leads that ran from the terminal boards to the top of the probe were manganin wire\textsuperscript{47} with a resistance of 11 ohms/ft.
2.5 Bolometer

The bolometer was a superconducting thin film made from a 95% Sn and 5% In alloy evaporated onto a glass substrate. The details of the bolometers construction are in Appendix A. A film of this alloy in zero magnetic field has a sharp resistive transition at about 3.5 K (figure 2.3) but as the schematic shows the transition is broadened and shifted to a lower temperature when the field is nonzero. At a chosen temperature $T$, and by picking the appropriate field the resistance can be set to $R = \frac{R_0}{2}$ where $R_0$ is the normal resistance of the film before its superconducting transition. One can see from figure 2.3 that for small changes in temperature ($dT$) around the centering temperature the corresponding resistance changes ($dR$) are:

$$dR \leq dT$$

The temperature changes produced in the experiment are well within the region where the resistance has this dependence.

The bolometer had a resistance of 146 ohms at room temperature, 33 ohms at 77K, and 8.9 ohms at 4.2 K. The experiment was run with the bolometer approximately centered at about 4.3 ohms. This small resistance produced a negligible amount of heating, less than 10 microwatts. The thermal response time of the bolometer was less than 1 ms. For the time scale of the experiment, the bolometer could
Figure 2.3

A typical plot of the resistance as a function of temperature for a thin superconducting film in a magnetic field. Note that $B_1$ is less than $B_2$. 
Figure 2.3

Graph showing the relationship between $R/R_0$ and $T$ (K) with two curves labeled $B_1$ and $B_2$, where $B_1 < B_2$. The graph also indicates $B=0$ at the rightmost point.
always be considered as being at the temperature of helium in the
cell. The bolometer's sensitivity was determined at 1.6 K (figure
2.4) and the line fit through the data corresponds to a \( \frac{1}{R} \frac{dR}{dT} \) of
6.9.

2.6 Inductance Loops

The bolometer was coupled to the squid pickup \( (L_a) \) coil by two
inductance loops (figure 2.1). The first coil \( (L_1) \) was in series with
the bolometer and consisted of 10 turns around a .8'' diameter epoxy
former. The wire used was 12 mil in diameter and composed of NbTi
filaments embedded in a copper matrix. The second inductance loop
\( (L_2) \), was 6 turns of 5 mil Nb wire wrapped concentrically around the
first coil. A lead foil blanket was wrapped around the loops in order
to shield them from any stray magnetic fields. The calculated self
inductances of the loops were \( L_1=3.52 \ \mu H \) and \( L_2=1.86 \ \mu H \). Their
calculated mutual inductance was \( 2 \ \mu H \). Since the self inductance of
\( L_2 \) was very nearly the same as \( L_a \), the expected mutual inductance \( (M) \),
between \( L_1 \) and \( L_2 \) would also be \( 2 \ \mu H \). These values of \( L_1 \) and \( L_2 \) were
chosen in order to maximize the signal coupled into the SQUID. The
second coil was connected to \( L_a \) using pressed contacts on the SQUID
terminal board similar to those used with the persistent current
switch (see section 2.10). This type of connection was used because
the second inductance loop and the squid pickup coil had to form a
Figure 2.4

Resistance versus temperature at T=1.6 for the InSn thin film superconducting bolometer used in this experiment. The solid line represents a \((1/R)(dR/dT)\) graph.
Figure 2.4
superconducting loop to prevent decay of signal flux coupled into the SQUID. The leads of $L_2$ were threaded through a length of solder tube that ran from the epoxy former out the feedthrough and into the squid case. The solder tube was grounded to the probe to serve as a shield for the leads. The ends of the solder tube were filled with stycaast to keep the vacuum can leak tight.

The effective mutual inductance ($M_{\text{eff}}$) between $L_1$ and $L_2$ was experimentally determined by passing an AC current through the first coil and measuring the output voltage of the SQUID. The output of the SQUID VSC is 20 mV per flux quanta (one flux quanta $\phi_0$ is $2.07 \times 10^{-15}$ Wb/m$^2$) coupled into $L_B$. The measured output voltage was 30 mV for an injected current of 1.7μA, which yielded an effective mutual inductance of 0.62 μH. This was less than the expected mutual inductance of 2 μH. The difference arose because the calculation of $M$ was done for an isolated pair of concentrically wound loops without considering the boundary conditions due to the lead blanket wrapped around $L_1$ and $L_2$.

2.7 RF Shunt

The RF noise that coupled to the SQUID was reduced by placing an RF shunt across the input to the SQUID pickup coil (figure 2.1). The shunt was a 3.5mm piece of manganin wire with a resistance of 0.12 ohms. This resistance was chosen so that in conjunction with $L_B$ the
3db rolloff would occur at approximately 20 kHz. This cutoff frequency was much higher than any component frequencies of measurements made in this experiment.

2.8 Magnet

The magnetic field used to shift the resistive transition of the SnIn film was generated by a superconducting persistent current solenoid. The bolometer was centered along the bore of the magnet. The magnet was made by wrapping superconducting wire around a tubular stycast former .234" in diameter and .79" long. The wire had a NbTi core .0024" in diameter, a layer of copper cladding .0009" thick, and a layer of formvar insulation .0015" thick. There were 12 layers of coils with an average of 165 coils per layer. The bolometer was then inserted into the former. The calculated field for the magnet was 1115 Gauss per ampere at the center of the solenoid. Deviations from this value were less than 7% over the length of the bolometer. The fields required to shift the bolometer's resistance to 4.3 ohms in the temperature range of the experiment were about 600 Gauss.

2.9 Persistent Current Switch

The magnet was built to run in a persistent current mode to insure stability of the field. This also reduced heating of the bath
Figure 2.5

A schematic diagram of the magnet and persistent current switch used together to operate the magnet in the persistent current mode.
Figure 2.5
Figure 2.6

The current persistent switch encased in a Stycast 1266 cylinder.
Figure 2.6
from the magnet leads which otherwise would be carrying approximately .6 A. The persistent current operation required a switch and superconducting contacts between the magnet and switch (figure 2.5). The persistent current switch is shown in figure 2.6 and consisted of a piece of superconducting wire like that used in the magnet, attached to a 300 ohm heater resistor. These were encased in a hollow 1266 epoxy cylinder. The superconducting wire was about 10" long and had its insulation and copper cladding completely stripped off. The middle of the wire was attached to the 1/4 watt resistor by tying the wire on with a piece of cotton thread and gluing it down with GE varnish. The separate holes by which the resistor leads and superconducting wire passed outside the tube were sealed shut with 1266 epoxy. When the apparatus was at experimental temperatures the air inside condensed out creating a vacuum space in the tube. The resistor and superconducting wire were the thermally isolated from the bath. The removal of copper cladding from the superconducting wire also decreased the heat leak from the switch out to the bath. A current of 5mA through the resistor produced enough heat needed to drive a section of the wire normal and open the switch.

2.10 Superconducting Contacts

The magnet and persistent switch had to form a continuous superconducting loop when the magnet was run in persistent current
Figure 2.7

The superconducting contact between the magnet and persistent current switch. The contact includes both spot welding the wire and a pressed contact of it to the NbTi flat.
Figure 2.7
mode. The current would quickly decay to zero if there was a resistive section in the loop. The superconducting contacts that joined the switch and magnet together were made from both pressed contacts and spotwelding the magnet leads and persistent current wire to flat pieces of NbTi (figure 2.7). The lead-tin washer was made from a piece of solder tube wrapped once around the brass screw. The pressed contact was formed when the wire from either the switch or magnet was wrapped once around the screw, and then it was screwed tight to the NbTi flat. This contact was further supplemented by spot welding a length of wire to the flat. The wires were stripped of their copper cladding and lightly sanded with emery cloth to insure that a good NbTi to NbTi metallic contact was made in the spot welds and press contacts. There was no evidence of any field decay when the experiment was run in persistent current mode, which suggested that the contacts were adequate.

The apparatus discussed was used for the investigation of thermal counterflow. The various techniques for this investigation are explained in the following chapter.
III Experimental Procedure

3.1 Introduction

The experimental procedure for the investigation of turbulent counterflow involved probing the turbulence by measuring attenuation of second sound and the thermal resistance of the helium in the flow channel. These measurements were used to determine the vortex line density as a function of temperature and relative velocity. The investigation was conducted at 1.5, 1.6 and 1.7 K. The second sound method was able to probe counterflow at small velocities and the heat pulsing technique at higher velocities. The flow regions over which these techniques were used overlapped slightly yielding complementary measurements on the counterflow system. More will be said about these methods later. Some general aspects of experimental procedure which were common to both of these techniques will be discussed.

3.2 Bath Temperature: Regulation and Measurement

The bath temperature was electronically regulated and the temperature fluctuations were kept to less than 40 uK. The
fluctuations were monitored by measuring the resistance of an Allen-Bradley carbon resistor in the helium bath with a SHE Model 120 resistance bridge. The long term stability of the regulation was observed by tracing the off balance signal of the resistance bridge with a chart recorder. The vapor pressure of the bath was determined by measuring the fluid levels in a differential oil manometer with a Wild Haerbrug Cathetometer. The manometers were filled with butyl pthyalate oil. The bath temperature was determined from this pressure and the "1958 Helium Scale of Temperature" providing a temperature resolution of .5 mK. The experiment was run so that the bath was within 1 mK of the temperature required.

3.3 Detection of Small Temperature Changes

Both methods of investigating turbulent counterflow required measuring small changes in temperature of helium in the cell. These temperature variations were detected with the bolometer SQUID system discussed in chapter II. By creating a magnetic field with the persistent current magnet the SnIn film would have a temperature dependence as seen in figure 2.3. Any small change in temperature (dT) from the centering temperature changed the bolometer resistance by an amount (dR_b) proportional to dT. A fixed voltage (figure 3.1) was placed across the bolometer so that the current changed by dI where:
Figure 3.1

A schematic of the circuit used to maintain a constant voltage \( V_b \) across the bolometer.
$R_0 =$ Voltage Supply Output Impedance

$R_L =$ Lead Resistance

$R_B =$ Bolometer Resistance

Figure 3.1
\[ dI = -(V_b/R^2)(dR_b/dT)dT \]  
\[ R = R_o + R_L + R_b \]

The magnetic flux coupled into the SQUID is changed by an amount:

\[ d\Phi_s = M_{\text{eff}} dI \]  

\( M_{\text{eff}} (.62 \text{ uH}) \) is the measured effective mutual inductance between the \( L_1 \) and the SQUID. The output voltage \( (V_s) \) of the SQUID changes in proportion to \( dT \) by:

\[ dV_s = \frac{-20 \text{ mV}}{\phi_0} M_{\text{eff}} \frac{V_b}{R^2} \frac{dR_b}{dT} dT \]

3.4 Bolometer Voltage Supply

The bolometer voltage supply had to have both long term stability and low output noise to ensure that the real signal output of the SQUID closely followed temperature changes in the cell. A circuit diagram of the constant voltage supply built for the bolometer is seen in figure 3.2. The first stage was a voltage follower using 1.42 volt mercury battery as the reference voltage. This section was followed by a voltage divider so the bolometer bias voltage which was typically 150 to 175 mV could be set. The selected voltage went to the input of
the gain 1 amplifier the output of which was connected across the bolometer (figure 3.1).

The voltage follower stage was used so that no appreciable current load was put on the reference battery thus causing its output voltage to decay. The gain 1 amplifier stage was used to improve sensitivity because it minimized the output impedance $R_o$ of the voltage supply. This can be seen from equation 3.1 which indicates that $dI$ goes as $1/(R_b+R_l+R_o)^2$. Battery power for the supply was provided by two 9V transistor radio batteries. Batteries were used so that the circuit could be decoupled from 60 Hz and RF pickup which plagued line powered voltage sources. The components listed in table 3.1 were chosen for temperature stability and low levels of inherent noise. Despite this, changes in room temperature caused $V$ to drift noticeably. The supply's temperature was stabilized by enclosing it in a waterproof camera bag and immersing them in an ice water bath. The supply was removed for short intervals to change batteries or adjust the bolometer voltage. The circuit was installed in a metal box and connected to the bolometer leads with shielded cable. The supply provided less than 1uV of noise from dc to 20 Hz. The voltage stability was better than .01 mV for several hours running with an output of 150 to 175 mV at 1.5 to 1.75 mA.
Figure 3.2

A circuit diagram of the bolometer voltage supply shown in figure 3.1
Figure 3.2
Table 3.1

Components:

- $C_1 = 10 \text{ pfd tantulum capacitor}$
- $C_2 = 175 \text{ ufd electrolytic capacitor}$
- $R_1 = 330 \text{ ohm metal film resistor}$
- $R_2 = 0 \text{ to } 50 \text{ ohm pot}$
- $R_3 = 684 \text{ ohm metal resistor}$
- $R_4 = 684 \text{ ohm metal film resistor}$
- $OA1 = \text{Signetics NE5533N low noise op-amp}$
- $OA2 = \text{Signetics NE5533N low noise op-amp}$
3.5 Tuning the SQUID

The SQUID was tuned before each set of data runs according to the procedure outlined in the SHE SQUID manual. Tuning was done after the bath was regulated at the temperature that data was to be taken. All instrumentation was disconnected from the probe except for the electronic temperature regulation, otherwise the noise coupled into the SQUID obscured the triangles making tuning difficult. The SQUID Variable Slew Control (VSC) was run with the gain dial set at 5 and in the slow response mode. The faster modes could not be used because the SQUID was too slow to be able to track flux changes from noise coupled into the pickup coil. This caused the SQUID to unlock.

3.6 Setting the Bolometer Resistance

The procedure for setting the bolometer's resistance to $R_0/2$ began by passing a 5mA current through the persistent current heater. A section of superconducting wire was driven normal, opening the switch (figure 2.5). The magnet current supply was connected to the magnet leads at the top of the probe. The magnet current was increased from zero while monitoring $R_b$. When the bolometer had the desired resistance the superconducting switch was closed by turning off the heater current. This put the magnet in its persistent current mode. The magnet current supply was turned off and disconnected. To de-energize the magnet the current supply was reconnected, and the
current roughly set to its value when the magnet was energized. The
switch heater was turned on and then the magnet current turned off.
This de-energized the magnet and the switch heater could be turned
off.

3.7 Mechanical Vibration Isolation

The cryostat was mounted on springs and weighed down with 2000
pounds of sand to isolate the probe from floor vibrations. Spring
mounting the dewar stand did not completely isolate the experiment
from vibrations. Chairs could not be dragged across the floor or
drawers slammed shut. Even someone standing at the opposite end of
the room could disturb the experiment by holding a metal trash can off
the floor and banging it sharply with a hammer. The acoustic impulse
cause the VSC output to jump, indicating that the SQUID had unlocked.
Other precautions for avoiding mechanical disturbances included
holding down cables running from the top of the probe to the
instrument racks with clamps or bags of lead shot and taking care that
none of the cables were touched while the experiment was running.

After the basic procedures of regulating and measuring bath
temperature, SQUID tuning, and setting the bolometer resistance were
completed, data taking could proceed.
3.8 Data Taking: Second Sound Helmholtz Oscillations

The cell and counterflow channel formed a damped second sound Helmholtz resonator. One method used to investigate turbulent counterflow was to measure attenuation of the second sound Helmholtz oscillations as a function of DC heat current. When the flow was laminar, damping of second sound was from normal fluid viscosity. When the counterflow became turbulent and vortex lines were present, attenuation increased because of an additional dissipation due to the mutual friction force $F_{ss}$. A derivation of the resonance curve $dT(\ )$ and damping mechanisms for the second sound appears in Appendix B.

A block diagram of the experimental configuration used for creating and detecting second sound is shown in figure 3.3. A 30 uW rms sinusoidal heat current with a frequency $\omega$ was produced by connecting the 531 ohm heater to a Wavetek Model 147 signal generator. The Wavetek’s output was a sine wave of frequency $\omega/2$. A DC heat current was maintained by sending a fixed current from a Hewlett Packard 6177b current source through the 1087 ohm heater. The second sound generated by the AC heat current produced temperature oscillations in the cell that were detected by the bolometer SQUID system. The largest amplitudes of these oscillations were about 1.5 uK. They were resolved by sending the output of the SQUID VSC to an Ithaco 393 lock-in amplifier. The amplifier was set up in the maximum noise rejection mode with a .1mV sensitivity and a time constant of 12
Figure 3.3

A block diagram of the instrumentation configuration used for measuring second sound Helmholtz oscillations.
Figure 3.3
seconds. The lock-in had the capability of simultaneously detecting the in and out of phase components of the signal and their vector sum. This eliminated the need to readjust the reference phase at each new second sound frequency.

The oscillations were further enhanced by signal averaging the A, Asin, and Acos outputs of the lock-in with the analog to digital converter (ADC) module of the Minc-11 computer. The averaging was usually done with 100 samples, and a sampling rate of four samples taken per second. The results were stored on magnetic disk for analysis later. The program which measured the lock-in's output, signal averaged it, and stored it on magnetic discs is listed in Appendix C. The second sound amplitude was recorded at frequencies from 1 to 10 hz around the resonance peak to experimentally determine $dT(\omega)$. Each resonance curve was taken with a fixed AC heat current amplitude and DC heat current. Some resulting $dT(\omega)$'s are shown in figures 3.4 to 3.12. These show that as the DC heat current was increased the second sound damping remained constant until the counterflow became turbulent. At this point attenuation increased as $Q_{dc}$ was set to higher values, indicating larger vortex line densities.

This method of determining vortex line densities by measuring attenuation of second sound was used at low DC heat currents. When $Q_{dc}$ became large enough the Helmholtz oscillations were very highly damped and barely discernible. At this point and at higher heat currents the turbulence was investigated using another technique.
Figure 3.4

Second sound resonance data taken at T=1.6 K, $Q_{dc} = 1$ mW and $dQ=30$ uW rms.
Figure 3.4
Figure 3.5

Second sound resonance data taken at $T=1.6$ K, $Q_{dc}=.1$ mW and $\dot{Q}=30$ uW rms.
Amplitude (Arbitrary Units)

Figure 3.5

Frequency (Hz)
Figure 3.6

Second Sound resonance data taken at T=1.6 K, $Q_{dc}$ = 0.5 mW and $dQ$=30 uW rms.
Figure 3.6

Amplitude (Arbitrary Units)

Frequency (Hz)
Figure 3.7

Second sound resonance data taken at $T=1.6$ K, $Q_{dc}=7$ mW and $\Delta Q=30uW$ rms.
Figure 3.7

Amplitude (Arbitrary Units)

Frequency (Hz)

0 2 4 6 8 10

0 2 4 6 8 10
Figure 3.8

Second sound resonance data taken at $T=1.6\,\text{K}$, $Q_{dc}=8\,\text{mW}$ and $dQ=30\mu\text{W}$ rms.
Figure 3.8

Amplitude (Arbitrary Units)

Frequency (Hz)
Figure 3.9

Second Sound resonance data taken at $T=1.6$ K, $Q_{dc}=9$ mW and $dQ=30uW$ rms.
Figure 3.9

Amplitude (Arbitrary Units)

Frequency (Hz)
Figure 3.10

Second Sound resonance data taken at $T=1.6$ K, $Q_{dc}=1.0$ mW and $dQ=30$ uW rms.
Figure 3.10

Amplitude (Arbitrary Units)

Frequency (Hz)

0

2

4

6

8

10
Figure 3.11

Second sound resonance data taken at $T=1.6$ K, $Q_{dc}=1.1mW$ and $dQ=30uW$
Figure 3.11

Amplitude (Arbitrary Units)

Frequency (Hz)

0  2  4  6  8  10

0  2  4  6  8  10
Figure 3.12

Second Sound resonance data taken at T=1.6 K, $Q_\text{dc}=1.2 \text{ mW}$ and $dQ=30\text{uW}$ rms.
3.9 Data Taking: Heat Pulsing

When there is a fixed heat current $Q$ along the flow tube there will also be a fixed temperature difference ($\Delta T$) between the channel ends.

$$\Delta T = \Delta T(Q)$$  \hspace{1cm} (3.5)

For small variations $dQ$ from $Q$ the steady state temperature difference will be:

$$\Delta T(Q) + \Delta T(Q + dQ)$$

$$\Delta T(Q + dQ) = \Delta T(Q) + dT$$

$$\Delta T(Q) + \frac{\delta(\Delta T(Q))}{\delta Q} dQ$$  \hspace{1cm} (3.6)

The dynamic thermal resistance $R(Q)$ will be defined as:

$$R(Q) = \frac{\delta(\Delta T)}{\delta Q}$$  \hspace{1cm} (3.7)

The second method of investigating turbulent thermal counterflow measured $R(Q)$ as a function of DC heat current. This method was developed and used by a previous student. A block diagram of the experimental configuration is shown in figure 3.13. The DC temperature difference $\Delta T$ was established with a constant heat current from the DC heater (1087 ohms). The AC heater (531 ohms) emitted small heat pulses $dQ$ which were accompanied by small temperature
Figure 3.13

A block diagram of the instrumentation configuration used for measuring the thermal resistance of HeII in the flow channel as a function of DC heat current and temperature.
perturbations $dT(t)$. These heat pulses were generated by square voltage pulses from a Hewlett Packard 8005b pulser across the 531 ohm heater. The pulses were chosen to be from 50 to 100 uW. Their width and spacing were long enough so that the temperature perturbations reached steady state values on both the rise and fall.

The resulting perturbations appeared in two different forms. For DC heat currents comparable to those at which the Helmholtz oscillation data was taken, the temperature variations typically took the form seen in figure 3.14. At the start and finish of the heat pulse a second sound ringing appeared. Their frequency was approximately equal to the resonant frequency of the Helmholtz oscillations. The ringing was attenuated by mechanisms discussed in Appendix B. They became critically damped at the same heat current as the driven second sound Helmholtz oscillations. The heat pulses used were typically 100uW and one to two seconds long. The signal averager sweep time was from two to four seconds. At heat currents larger than those where the ringing was critically damped, $dT(t)$ abruptly took on another form. The perturbations rose and fell exponentially as is seen from the data point shown in figure 3.15. The AC heat pulse that produced the exponential was chosen to be small enough (50 uW) so that the amplitude of $dT(t)$ is

$$dT_0 = R(Q)dQ \quad (3.8)$$
A typical second sound ringing type data point, taken at $T=1.7$ K, $Q_{dc} = 0.9$ mW, $dQ=100\mu$W. This point was recorded by the signal averager and stored on magnetic disk. The temperature perturbation is plotted as digital counts versus address register. The dwell time per address was 4 ms.
Figure 3.15

A typical exponential type data point, taken at $T=1.7$, $Q_{dc}=2.0$ mW, $dQ=50$ uW. This point was recorded by the signal averager and stored on magnetic disk. The temperature perturbation is plotted as digital counts versus address register. The dwell time per address is 8 ms.
Figure 3.15
The time constant of the exponential was just $R(Q)C$ where $C$ is the heat capacity of helium in the cell. The pulses were from 1.5 to 10 seconds long and sweep times were from three to twenty seconds. The cell's heat capacity and the thermal conductivity of the flow tube walls were much smaller than that of HeII so that the time constant and the thermal resistance are determined solely by characteristics of the helium.

Both types of temperature perturbations detected were small, so to resolve them out of experimental noise the heat pulses were repeated many times at each DC heat current. The output of the SQUID VSC was put into a PAR 113 pre-amp, noise filtered with a Frequency Devices Model 901F 8 pole Butterworth filter and then finally was signal averaged with a Tracor Northern NS 570-A signal averager (figure 3.13). A schematic of the sequence of triggering, pulsing and data taking is shown in figure 3.16. A trigger signal from the pulser preceded each voltage pulse and started the averager on a signal sweep. The sweep time was set so that a complete temperature perturbation and an adequate baseline could be recorded in one sweep. The averaging required as few as 75 sweeps for data taken at the largest DC heat currents (where $dT(t)$ had the largest values) to several hundred sweeps for data taken at the smallest heat currents.

The pre-amp had a bandwidth of DC to 30 hz and a gain of either 200 or 500. The Butterworth filter was set with a gain of one and a bandwidth of DC to 15 hz. The high frequency cutoff of the filter
Figure 3.16

The sequence of triggering and heat pulsing and the corresponding temperature perturbations that were generated.
Figure 3.16

Trigger

A.C. Heat Pulses

Temperature Perturbations
was large enough so as not to distort the signal. This was checked by filtering and averaging the voltage measured across a charging and discharging capacitor in an RC circuit. The time constant was comparable to the largest thermal time constants $R(Q)C$. No distortion of the capacitor voltage was evident for the filter settings used. The settings of the signal averager were input filter 1 mS, coarse gain 1 V, and fine gain at 7.5. In most cases the averager's internal time base was used to set the dwell time per address. For exponentials with the longest time constants the Wavetek 147 oscillator along with a Hewlett Packard 5326A Timer-Counter provided an external time base for the averager.

Figures 3.14 and 3.15 indicate that besides the expected temperature variation the signal has a baseline offset. The signal averager could only digitize positive voltages within a 0 to 1 volt window. A voltage offset was subtracted from the pre-amp output so the signal arriving at the averager was within this window. Both drift and bolometer voltage drift caused the signal to shift so the compensating voltage was reset before each sweep. The amount of offset sent to the B input of the Butterworth filter was determined by averaging the output of the pre-amp during one sweep with the ADC of the Hinc computer (figure 3.13). The Hinc then calculated the offset voltage necessary for keeping the signal in a 0-1V window, and fed the voltage back to the filter during the next sweep. A listing of the offset voltage correction program is found in Appendix C. The
remaining baseline offset in the signal did not effect $dT(t)$ and was subtracted out when a data point was analyzed.

The digitized data points recorded by the signal averager were transferred to the Minc via the RS 232 serial port and stored on magnetic disks for analysis later. The program which transferred and stored the data is listed in Appendix C.

The pulsing method was complementary to the second sound technique. Pulsing data not only probed turbulent counterflow at heat currents equal to the largest at which the second sound data was recorded, but also was able to investigate counterflow at heat currents much higher than where second sound Helmholtz oscillations are damped out. Together both methods provided the means to investigate turbulent counterflow over a wide range of heat currents in the same flow tube.
Chapter IV

Data Analysis

4.1 Introduction

The experimental data was divided into two sets, one being resonance curves of second sound Helmholtz oscillations, and the other being temperature perturbation data from the heat pulsing method. The methods of analysis used on these two sets of data will explained in this chapter. The purpose of the analysis was to determine the vortex line density, \( L_0 \) as a function of temperature and relative velocity \( V \) where \( V \) is determined from the DC heat current and equation 1.14. This information was useful for characterizing several aspects of turbulent counterflow.

4.2 Second Sound

The vortex line density was obtained by fitting the absolute value of equation B.15 (which is the functional form for the Helmholtz oscillations derived in Appendix B) to the second sound data (figures 3.4 to 3.12). The fitting procedure was done with a Minc-11 computer.
on second sound data stored on magnetic disks. The first step was to determine the optimum values for the resonant frequency \( \omega_0 \) and the overall amplitude \( A_0 \) of the signal detected by the lock-in (\( A_0 \) is proportional to \( dT_0 \) defined in equation B.16). Resonant frequencies were obtained by fitting equation B.15 to data taken at zero or very small DC heat currents where counterflow was laminar and the attenuation was due to normal fluid viscosity only. The resonant frequency was not an entirely free parameter. It could be calculated from equation B.17 which predicted a resonant frequency of 3.87 Hz. When fitting was done \( \omega_0 \) was allowed to vary slightly from its calculated value to allow for error in measuring the cell and tube dimensions and the second sound driving frequencies. The resulting optimum frequency was found to be as much as 8% smaller than the calculated value. The systematic deviation is most likely due to the tube diameter being smaller than what was measured. The overall amplitude of the detected signal depended on the size of the AC heat current and the bolometer SQUID detection system's sensitivity. These factors were fixed for data taken on the same day. This meant \( A_0 \) was a constant for all resonance data taken in that day's run, and the amplitude of B.15 did not have to be known in terms of temperature. The optimum resonant frequency and amplitude were found to change very little from day to day, however they were only used for fitting data taken on the same day.

Once values of these two parameters were established resonance
The fit of the absolute value of equation B.15 to the resonance data seen in figure 3.4. The resulting values of the fitting parameters are $L_0^{1/2} = 0$, $f_0 = 3.76$ Hz.
Figure 4.1

Amplitude (Arbitrary Units)

Frequency (Hz)

0 2 4 6 8 10
0 2 4 6 8 10
Figure 4.2

The fit of the absolute value of equation B.15 to the resonance data seen in figure 3.5. The resulting values of the fitting parameters are $L_0^{1/2} = 0$ and $f_0 = 3.76$ Hz.
Figure 4.3

The fit of the absolute value of equation B.15 to resonance data seen in figure 3.6. The resulting values of the fitting parameters are \( L_0^{1/2} d = 0 \) and \( f_0 = 3.78 \) Hz.
The fit of the absolute value of equation B.15 to resonance data seen in figure 3.7. The resulting values of the fitting parameters are $L_0^{1/2} = 2.5$ and $f_0 = 3.78$ Hz.
Amplitude (Arbitrary Units)

Frequency (Hz)

Figure 4.4
The fit of the absolute value of equation B.15 to resonance data seen in figure 3.8. The resulting values of the fitting parameters are $L_{0}^{1/2} = 4.6$ and $f_0 = 3.76$ Hz.
The fit of the absolute value of equation B.15 to resonance data seen in figure 3.9. The resulting values of the fitting parameters are \( L^{1/2}_0 \approx 6.1 \) and \( f \approx 3.74 \) Hz.
Figure 4.6

Amplitude (Arbitrary Units)

Frequency (Hz)

0  2  4  6  8  10

0  2  4  6  8  10
The fit of the absolute value of equation B.15 to resonance data seen in figure 3.10. The resulting values of the fitting parameters are $L_0^{1/2}d=9.3$ and $f=3.66$ Hz.
Figure 4.7

Amplitude (Arbitrary Units)

Frequency (Hz)

10

0 2 4 6 8 10
Figure 4.8

The fit of the absolute value of equation B.15 to resonance data seen in figure 3.11. The resulting values of the fitting parameters are $L_{0d}^{1/2}=10.5$ and $f=3.77$ Hz.
Amplitude (Arbitrary Units)

Figure 4.8

Frequency (Hz)
The fit of the absolute value of equation B.15 to resonance data seen in figure 3.12. The resulting values of the fitting parameters are $L_0^{1/2} = 24$ and $f_0 = 3.78$ Hz.
A plot of $L_0^{1/2}$ obtained from second sound data versus $V$. These values were from data taken at $T=1.5$. 
Figure 4.10
Figure 4.11

A plot of $L_{0}^{1/2}$ obtained from second sound data versus $V$. These values were from data taken at $T=1.6$ K.
A plot of $L_{0}^{1/2}$ obtained from second data versus $v$. These values were from data taken at $T=1.7$ K.
Figure 4.12
data taken at DC heat currents where counterflow was turbulent could be analyzed to determine vortex line density. Using values of $A_0$ and $\omega_0$ previously determined, $L_0$ was determined by selecting the value of line density that gave the best fit of equation B.15 to the resonance data.

Figures 4.1 to 4.9 show examples of fitting by plotting equation B.15 over some second sound data. The vortex line densities resulting from analysis of all the resonance data are seen in figure 4.10 to 4.12 where the dimensionless number $L_0^{1/2}$ is plotted against $\nu$.

4.3 Thermal Resistance

The data produced by the heat pulsing method were either ringing Helmholtz oscillations (figure 3.14) or an exponential rise and fall in temperature (figure 3.15). The size of the former were small enough to be at the limits of the detection system. An analysis of different data taken with the same size heat pulse and DC heat current produced scattered and inclusive results which could not be effectively used in quantitatively determining the vortex line density. The second type of data were analyzed by fitting $dT(t)$ on both the rise and fall portion to an exponential with a time constant

$$\tau = R(Q)C$$  \hspace{1cm} (4.1)
and amplitude

\[ dT_0 = R(Q)dQ \]  

(4.2)

where \( R(Q) \) is the thermal resistance (equation 3.7), \( C \) is the heat capacity of HeII in the cell, and \( dQ \) is the heat pulse. Either of these measurements could then be used to determine \( R(Q) \). It was thought that the values of \( R(Q) \) resulting from \( t \) would be the same as those determined from measurements of \( dT_0 \). A comparison showed an important discrepancy between them that will be mentioned briefly in this section and discussed in chapter V.

The amplitude of \( dT(t) \) obtained initially in the fitting, was not in units of temperature. Instead, \( dT_0 \) was measured in units of digital counts/sweep, the result of the signal being digitized by the signal averager. The size of \( dT_0 \) in counts/sweep (\( dT_0(c/s) \)) is proportional to \( dT_0 \) in units of temperature (\( dT_0(K) \)). A proportionality constant \( A \) may be defined by:

\[ A = \frac{dT_0(c/s)}{dT_0(K)} \]  

(4.3)

Using \( R(Q) \) already determined from the time constant

\[ A = \frac{dT_0(c/s)}{(R(Q)dQ)} \]  

(4.4)
The size of A is controlled not only by R(Q) and dQ in the denominator but also by the size of dT_0(c/s) which is determined by the temperature detection system's sensitivity and the size of the heat pulse dQ. The detection system's sensitivity is controlled by gain settings of the pre-amp and signal averager and by the bolometer bias voltage. All values of dT_0(c/s) were rescaled for one standard sensitivity and dQ size. Then an A determined from one data point could be used for converting dT_0(c/s) to dT_0(K) for any other data point. The amplitudes were normalized for a heat pulse size of 50 uW and a sensitivity determined by a pre-amp gain of 500, Butterworth filter gain of 1, signal averager coarse gain of 1 volt and fine gain of 7.5, and bolometer bias voltage of 175mV. The conversion factor for each temperature was determined by finding the averages of the values of dT_0(c/s) and thermal resistances from τ for data that had the least amount of scatter at a given DC heat current. These averages were used to calculate A where

\[ A = \frac{dT_0(c/s)}{<R(Q)dQ>} \]  

(4.5)

Once A was calculated and the values of dT_0(c/s) were converted to dT_0(K) R(Q) could be determined using equation 4.3. The resulting thermal resistances from measurement of τ and dT_0 are shown in figures 4.13 to 4.18 and are plotted separately as functions of DC heat current.
The thermal resistance measured from the time constant of $dT(t)$ versus $Q_{dc}$, for $T=1.5$ K.
The thermal resistance measured from the time constant of $dT(t)$ versus $Q_{dc}$, for $T=1.6\ K$. 
Figure 4.14
Figure 4.15

The thermal resistance measured from the time constant of $dT(t)$ versus $Q_{dc}$, for $T=1.7$ K.
Figure 4.15
The thermal resistance from the amplitude of $dT(t)$ versus $Q_{dc}$ for $T=1.5$ K.
Figure 4.16
Figure 4.17

The thermal resistance measured from the amplitude of $dT(t)$ versus $Q_{dc}$, for $T=1.6$ K.
Figure 4.17
The thermal resistance measured from the amplitude of $dT(t)$ versus $Q_{dc}$, for $T=1.7$ K.
Figure 4.18
Figure 4.19

An exponential temperature perturbation with a skewed baseline taken at $T=1.7$ K, $Q_{dc}=2.6$ mW, $dQ=50$ uW and plotted as digital counts versus address register. The dwell time per address is 10 ms.
Figure 4.19
The initial form of the data (figure 3.15) was not appropriate for fitting to an exponential. It was necessary to subtract the signal offset to reduce the baseline of $dT(t)$ to zero. As mentioned in chapter III the offset baseline was an artifact of data taking and not part of the temperature perturbation, so it could be removed without distorting $dT(t)$. The amount removed was determined by averaging the contents of the signal averager address registers which preceded the start of the heat pulse along with the contents of the last 50 addresses. These regions were chosen since the counterflow was in a steady state and $dT(t)$ at its zero value. Any non-zero signal was excess baseline that was to be removed. This average offset was subtracted from the contents of all address registers.

In some cases in addition to an offset the averaged signal would have a linearly slanted baseline (figure 4.19). This occurred if the bath temperature was drifting or was undergoing long term oscillations or that the bolometer voltage ($V_b$) was slowing changing. One can easily see that a drifting bath temperature or $V_b$ would produce a skewed baseline. Temperature oscillations are responsible for slanted baselines by the fact that signal averaging eliminates any rise in bath temperature ($T_b$) with a following decline in $T_b$ unless the averaging were to occur over a time interval that ended on an uncompensated rise or fall in $T_b$. The slant was removed from the baseline by noting that when the heat pulse was turned on $dT(t)$ looked like:
\( d_{\text{rise}}(t) = dT_0(1 - e^{-t/\tau}) \) \hspace{1cm} (4.6)

for \( t=0 \) at the heat pulse start

and when the heat pulse was turned off it looked like:

\( d_{\text{fall}}(t) = dT_0 e^{-t/\tau} \) \hspace{1cm} (4.7)

for \( t=0 \) at the heat pulse end

If in addition to this behavior there was a linear component with a slope \( m \) added to the signal so that

\[ dT(t) \rightarrow dT(t) + mt \] \hspace{1cm} (4.8)

\( mt \) could separated from the exponential part by adding equation 4.7 to equation 4.6 producing

\[ d_{\text{rise}}T(t) + d_{\text{fall}}T(t) = 2mt + dT_0 \] \hspace{1cm} (4.9)

The linear slope was eliminated by adding the contents of the address register at the beginning of the pulse to the contents of the register at the end of pulse. Then the contents of the register one address past the pulse start was added to the contents of the register one step past the end of the pulse. This process of pairing and adding
the contents of corresponding registers was repeated until the address register just preceding the end of the pulse was reached. The folding back process yielded a slanted line like equation 4.9 whose slope was determined by fitting it to a straight line. A line with half this fitted slope was removed from dT(t). Following removal of the slant the baseline offset was subtracted by the procedure explained above. When the baseline adjustment was complete the data was ready to be fitted to an exponential.

The important discrepancy noted earlier in this section can be immediately seen by looking at figures 4.20 to 4.22. These figures show the average thermal resistances from time constant measurements compared to the average thermal resistance from dT_0 measurements as functions of DC heat current. The averages were calculated using thermal resistance data shown in figures 4.13 to 4.18. The values of R(Q) resulting from the two different measurements are consistent at higher heat currents as expected. But at the lowest heat currents their values are systematically different. In this region the results from measurement of time constants are unexpectedly larger than thermal resistances arrived at using dT_0. Possible explanations of the difference in R(Q) will be discussed in the next chapter.

The thermal resistance cannot be directly converted into a vortex line density, instead R(Q) must be integrated to obtain T(Q) by the relation:
Figure 4.20

A plot of the average thermal resistance from the time constant and the average thermal resistance from the amplitude of $dT(t)$ as functions of $Q_{dc}$, for $T=1.5$ K.

- Data from the measurement of $dT_0$

- Data from the measurement of $\tau$
Figure 4.20
Figure 4.21

A plot of the average thermal resistance from the time constant and the average thermal resistance from the amplitude of $dT(t)$ as functions of $Q_{dc}$, for $T=1.6$ K.

- Data from the measurement of $dT_0$

- Data from the measurement of $\tau$
Figure 4.21
Figure 4.22

A plot of the average thermal resistance from the time constant and the average thermal resistance from the amplitude of $dT(t)$ as functions of $Q_{dc}$, for $T=1.7$ K.

* Data from the measurement of $dT_0$

■ Data from the measurement of $\tau$
Figure 4.22

$R(\Omega)$ (K/Watt) vs. $Q$ (mWatts)
\[ \Delta T(Q) = \Delta T(Q_0) + \int_{Q_0}^{Q} R(Q) dQ \]

The line density may then be determined from \( \Delta T(Q) \) via equation 1.9. The constant of integration \( \Delta T(Q_0) \), is determined by the line density obtained from second sound data at the heat current corresponding to the point where the integration of \( R(Q) \) begins. This starting point was chosen to be at \( Q_0 \). The thermal resistance data determined from \( dT_0 \) was chosen for integration for two reasons. First, \( dT_0 \) data was determined by the steady state properties of \( \Delta T(Q) \) according to the definition of \( R(Q) \) in equation 3.7. Secondly the inconsistency of these results and \( R(Q) \) from measurements of \( \tau \) (figures 4.20 to 4.22) revealed that there was an additional relaxation time in the thermal counterflow system in the region of the lowest heat currents. The time constant from that region was not determined by just \( R(Q)C \).

Results of integrating \( R(Q) \) and then reducing \( \Delta T(Q) \) to \( L_0 \) are seen in figure 4.23 along with the line densities measured from attenuation of second sound. The dimensionless number \( L_{0}^{1/2}d \) is plotted as a function of the relative velocity.
A plot of $L_{0}^{1/2}$ as a function of $V$ from the second sound and heat pulsing data for all temperatures.

Second Sound Data

- T=1.5 K
- T=1.6 K
+ T=1.7 K

Pulsing Data

+ T=1.5 K
- T=1.6 K
* T=1.7 K
Figure 4.23
5.1 Different Turbulent States

The experimental results show that counterflow produces two distinct superfluid turbulent states in this 1 mm channel. These are flow states II and III which have been identified in small circular and low aspect ratio channels\(^{10}\). The existence of two turbulent flow states is confirmed from several aspects of the data. The most apparent is a plot of \(L_{1/2}^d\) as a function of \(V\) (figure 5.1 to 5.3). The absence of vortex lines at low velocities indicates laminar thermal counterflow. The onset of turbulence is marked by the appearance of vortex lines at \(V = V_{c1}\). The line density increases smoothly with increasing velocity until \(V = V_{c2}\). At this point the turbulence undergoes a dramatic change. Beyond \(V_{c2}\), \(L_{1/2}^d\) enters a different turbulent state III.

An inspection of the data before it was reduced to \(L_{1/2}^d\) also shows that there are two turbulent flow states. Figures 4.13 to 4.18 show that the local peak in the thermal resistance at \(Q = Q_{c2}\) marks the
A plot of $\frac{1}{2}L_0 d$ as a function of $V$ from the second sound and heat pulsing data at taken at $T=1.5$ K. The solid line represent the fit of equation 5.2 to state TII data.

- Second Sound Data

* Pulsing Data

Figure 5.1

139
Figure 5.1
Figure 5.2

A plot of $L_0^{1/2}$ as a function of $V$ from the second sound and heat pulsing data taken at $T=1.6$ K. The solid line represents the fit of equation 5.2 to state TII data.

- Second sound Data

* Pulsing Data
Figure 5.2
Figure 5.3

A plot of $L_{0}^{1/2}$ as a function of $V$ from the second sound and heat pulsing data taken at $T=1.7$ K. The solid line represents the fit of equation 5.2 to state TII data.

- Second Sound Data
- Pulsing Data
Figure 5.3
point where \( R(Q) \) rises sharply from its small value below \( Q_{c2} \) to much larger values above \( Q_{c2} \). In addition the ringing of \( dT(t) \) is damped out within a narrow range of \( Q \) just below \( Q_{c2} \). These features indicate a major change occurs in the flow near \( Q_{c2} \). This is not the initial appearance of turbulence in the channel as proposed by some. The increased attenuation of second sound seen in the resonance data (figures 3.4 to 3.12) indicate vortex lines are present for heat currents less than \( Q_{c2} \). The radical change is a transition from one turbulent state to another.

5.2 Turbulent State TI

The turbulent state at low velocities is identified as state TI. The Vinen parameters \( \gamma(T) \) and \( \alpha \) were determined by fitting equation 1.11 to second sound data shown in figures 4.10 to 4.12. The region where \( L^{1/2}d_0 \) deviates from linear behavior is regarded as a precursor to the TI-TIII transition. Data points in that region were not used for determining \( \gamma(T) \) and \( \alpha \). The values of \( \gamma(T) \) listed in figure 5.4 are in good agreement with results obtained in small channel experiments. The parameter \( \alpha \) is approximately 4.5 at all three temperatures. This value is larger than those determined in small channel experiments (where \( \alpha \) is approximately unity) and concurs with the weak \( d \) dependence for \( \alpha \) inferred by Ladner. The temperature independence of \( \alpha \) is consistent with previous experimental findings and Vinen's phenomenological model.
Figure 5.4

The values of $Y(T)$ obtained from fitting equation 1.11 to the $L^{1/2}_0$ data shown in figures 4.10 to 4.12.
Figure 5.4

The values of $\gamma(T)$ obtained from fitting equation 1.11 to the $L_0^{1/2}$ data shown in figures 4.10 to 4.12.

<table>
<thead>
<tr>
<th>Temperature (K)</th>
<th>$\gamma(T)$ (s/cm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>77</td>
</tr>
<tr>
<td>1.6</td>
<td>86</td>
</tr>
<tr>
<td>1.7</td>
<td>92</td>
</tr>
</tbody>
</table>
The transition between laminar flow and state TI has several features worth noting. Figures 4.10 to 4.12 show the transition from TI to laminar flow to be "first order like". In some cases it was possible for the laminar state to persist metastably above $V_{c1}$. The line densities obey the condition that $L^{1/2} a^{2.5}$ at the critical velocity $V_{c1}$. All these results are consistent with features observed in the transition between laminar flow and TI in small circular and low aspect ratio channels.

5.3 Turbulent State TII

The second turbulent state is observed from the temperature perturbation measurements. The TI-TII transition region begins with a local peak in the thermal resistance $R(Q)$ at $Q_{c2}$ (figures 4.13 to 4.18). This peak has also been observed in a small channel thermal relaxation experiment. The present data show no evidence for saturation of the transition. This is in agreement with previous experimental findings. The excess relaxation time observed at heat currents slightly larger than $Q_{c2}$ is consistent with results obtained using the "Vinen waiting time technique". The presence of a local peak in $R(Q)$ at $Q=Q_{c2}$, an excess relaxation time close to $Q_{c2}$ and the inability to establish state TI metastably beyond $Q_{c2}$ show that the TI-TII transition appears "second order like" as opposed to the "first order like" behavior of the laminar-TI transition. There is no evidence of an extra time at heat currents smaller than $Q_{c2}$. 
The critical velocity $V_{c2}$ is determined from $Q_{c2}$ via equation 1.14. The behavior of $V_{c2}$ as a function of tube size and temperature is of primary interest. Previous experiments performed over a wide range of channel sizes have shown $V_{c2}$ to be a strong function of $d$. No consistent temperature dependence has been determined. Most of the size dependence of $V_{c2}$ can be scaled out by plotting $V_{c2}d$ instead of $V_{c2}$ (figure 5.5). The present values were checked for consistency with other experimental results by including $V_{c2}d$ from different experiments in figure 5.5. Not all available experimental results were suitable for use. In some cases it is not apparent whether the critical velocities given from different experiments of the same size channel were $V_{c1}$ or $V_{c2}$. Different experiments performed in metal channels of the same size have produced inconsistent results. The irregularity is most likely due to wall roughness. In addition not all data from experiments whose results are unambiguous are included in figure 5.5. If all of them were, the values of $V_{c2}d$ would appear to be widely spread. This would be misleading as the apparent scatter arises from the dependence of $V_{c2}d$ on $T$ and $d$. To avoid possible confusion and to make clear the essential features of the behavior of $V_{c2}d$ as a function of $T$ and $d$, we plot the results from this experiment and another performed in a channel of similar size along with $V_{c2}d$ from experiments performed in channels an order of magnitude smaller in figure 5.5. The resulting plot indicates that $V_{c2}d$ is weakly dependent on $d$ and there is a consistent temperature behavior for a particular channel size. In addition the temperature
Figure 5.5

A plot of $V_{c2d}$ as a function of temperature.
Figure 5.5

$V_{c2d}$ (cm$^2$/s)

T(K)

Figure 5.5
Table 5.1

Sources of the experimental values of $V_{c2d}$ shown in figure 5.5.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Reference</th>
<th>Channel Size (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>■</td>
<td>Brewer and Edwards</td>
<td>0.0108</td>
</tr>
<tr>
<td>○</td>
<td>Ladner et al</td>
<td>0.0131</td>
</tr>
<tr>
<td>●</td>
<td>Chase</td>
<td>0.08</td>
</tr>
<tr>
<td>▲</td>
<td>This work</td>
<td>0.1</td>
</tr>
</tbody>
</table>
dependence seems to be somewhat different for different size channels. The behavior of $V_{c2d}$ is not as "irregular" as previously thought but must considered carefully in light of the dependence of $V_{c2d}$ on $T$ and $d$.

The inability to understand the TI-TII transition in terms of a simple a critical velocity suggests considering a critical condition similar to that for state TI. One problem with this approach concerns the nature of the critical condition. The question is does the transition occur because $L_{0}^{1/2}d$ reaches a critical value while in state TI or does the turbulence undergo a transition because the value of $L_{0}^{1/2}d$ it can have in state TII is somehow unique. If the critical condition is chosen to be $L_{0}^{1/2}d$ constant while the counterflow is in state TI one gets a corresponding expression for $V_{c2d}$ from equation 1.11.

\[
\frac{(L_{0}^{1/2})_{\text{critical}} + 1.44a}{\gamma(T)} = V_{c2d} \tag{5.1}
\]

Equation 5.1 cannot account for the experimental values of $V_{c2d}$. Although the weak $d$ dependence may arise from $a$'s $d$ dependence the experimentally determined values of $\gamma(T)$ provide the wrong $T$ dependence. It is worth noting that the results from this experiment do indicate that $L_{0}^{1/2}d=12$ at $V=V_{c2}$, approximately the same as values of $(L_{0}^{1/2}d)_{c}$ from other experiments\textsuperscript{10}. 


The most common type of theoretical treatment of $V_{c2}$ has been in terms of various modified critical Reynolds numbers $R_{c}^{10}$. This sort of picture has been unsuccessful because of its inability to correctly predict the $T$ dependence of $V_{c2}$ even for a particular value of $d$. The results of this experiment along with the others shown in figure 5.5 indicate that the temperature and $d$ dependence of $V_{c2}d$ are linked. The idea of a critical Reynolds number will fail unless one of the parameters used in $R_{c}$ has an "effective $d$" ($d_{eff}$) dependence where $d_{eff}$ is a function of temperature.

Figure 4.23 indicates that state TII does not become fully developed at velocities immediately above $V_{c2}$. The long approach for TII to reach its asymptotic form is also observed in other experiments. In these cases $L_{0}^{1/2}d$ was measured out to velocities at least equal to $3V_{c2}$. In some experimental results this aspect of the transition is overlooked because the line densities were not measured to high enough velocities. A function of the following form

$$L_{0}^{1/2}d = \sqrt{(B(T)V_{d})^{2} + C^{2}} - D$$

was fit to the state TII data shown in figure 4.23. The data and accompanying fits are seen in figure 5.1 to 5.3. One can see equation 5.2 has the curvature of $L_{0}^{1/2}d$ close to the transition and it also develops the asymptotic linear behavior of the data at higher velocities. The values of the parameters obtained from these fits are
shown in figure 5.6. The results of different experiments performed in circular channels which measured L^{1/2} well into state TII were also fit to equation 5.2 with the resulting values of the parameters agreeing well with the present results. Asymptotically the linear slope of L^{1/2} is B(T). There is remarkable agreement between the values of B(T) determined from the present results, the other data fitted to equation 5.2 and those slopes of L^{1/2} determined from pure superflow experiments, and slopes calculated from Schwarz's theory of homogenous turbulence. This suggests that state TII is closer to homogeneous turbulence than state TI.

5.4 Conclusion

The primary goal of this experiment was to investigate thermal counterflow in large channels in terms of a small tube experiment. This was achieved in the apparatus described. The temperature resolution was better than .1 uK for the second sound measurements and .5 uK for the pulsing measurements. The sensitivity would be improved primarily by decreasing the ambient noise in the system and maintaining the fixed bolometer bias voltage across the thin film only instead of across the film and its manganin leads.

The experimental observations of thermal counterflow performed in this 1 mm channel indicates two turbulent states. These can be
Values of the parameters $B(T)$, $C$ and $D$ after fitting equation 5.2 to state TII data plotted in figures 5.1 to 5.3.
<table>
<thead>
<tr>
<th>Temperature (K)</th>
<th>B(T) (a/cm²)</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>145</td>
<td>31</td>
<td>19</td>
</tr>
<tr>
<td>1.6</td>
<td>163</td>
<td>33</td>
<td>20</td>
</tr>
<tr>
<td>1.7</td>
<td>190</td>
<td>33</td>
<td>20</td>
</tr>
</tbody>
</table>
identified as states TI and TII that are seen in small circular and low aspect ratio channels. The features of TI and TII that were observed are consistent with those seen in other experiments. One new aspect that was observed was the excess relaxation time in the region of the TI-TII transition. This experiment is currently the only one performed in any size channel which has probed the laminar flow state, TI, the TI-TII transition, and the full development of TII. These results suggest that state TII is closer to homogeneous turbulence than state TI. The second critical velocity exhibits the same temperature dependence in channels of the same size. At this point the behavior of $V_c^d$ is not explained in terms of any theoretical model, and remains an outstanding problem in the field of superfluid turbulence.
Appendix A

Bolometer Construction

A.1 Construction

The bolometer was a thin metal film sputter evaporated onto a pyrex substrate. The film was made from an alloy which was 95% indium and 5% tin. The substrate had dimensions of 1/8"x1/16"x5/8". The substrate was cut from a larger plate and the cleaned using the process outlined in the following steps.

1. Scrub the substrate with a cotton bud and a solution of Alconox detergent and hot water.
2. Flush the substrate in the following order with this series of rinses:
   A. Hot tap water
   B. Distilled water
   C. Methanol
   D. Acetone
   E. Freon

The substrates were handled with tweezers throughout these steps.
During cleaning the washing and rinsing solutions were not allowed to dry on the substrate.

At this point a gold coating was fired onto each end of the substrate (figure A.1). Inspection of the substrate after firing revealed a crust-like deposit around the perimeter of the gold film. This was polished off by rubbing a thin paste of aluminum oxide powder and water onto the gold with a wood stick. The crust was removed because otherwise it caused a break in the electrical conductivity of the InSn film evaporated over the gold pad.

After the gold pads were prepared the substrate was again cleaned using the previously listed steps. The substrates were mounted in a jig which held several of them during the evaporation of InSn. The substrates were held secure to the jig by metal fingers at either end. A lead from each finger was connected to an electrical feedthrough in the evaporator. This connection permitted monitoring film resistances externally during evaporation.

A molybdenum boat was burned clean by running a 100A current through it for 1 minute. The substrate jig was mounted about 4" above the boat. The charge was heated for 2.5 minutes at 130A with the shutter closed. The shutter was opened until the film resistances were from 7.5 to 28 ohms, which was approximately one minute. These are the times and currents for the particular evaporation batch that produced the bolometer used in the experiment.
A schematic diagram of the InSn thin film superconducting bolometer used for the detection of small temperature changes.
Superconducting Lead

Scratches

Gold Pad

InSn Film

Silver Epoxy

Pyrex Substrate

Figure A.1
Superconducting leads (using wire like that in the superconducting solenoid magnet) were glued with silver conducting epoxy\textsuperscript{59} to sections of the gold pads not covered by InSn (figure A.1). Bolometer resistances were increased by making alternating scratches part way across the film (figure A.1) to increase the path length. The bolometers would have their resistance raised by a factor of 10 to 20.

A.2 Testing

The bolometers were cycled from room temperature to 4.2 K several times. This procedure screened out unreliable bolometers by causing them to fail. Either leads broke off the substrate or a break in the film occurred. The survivors were honored at a special banquet and inducted into the Bolometer Hall of Fame.

Their sensitivities were determined by placing them inside magnets identical to that mentioned in section 2.8 and measuring \((1/R)(dR/dT)\) at experimental temperatures. The bolometer used had a \((1/R)(dR/dT)\) of 6.9 per K at 1.6 K.

A.3 Comments

The construction process outlined above is not a guaranteed method for making reliable bolometers. On the contrary, it was found
that one batch of bolometers could be wildly different from another even though they were constructed using the same procedure. It appears that successful results depended as much on black magic as on carefully following the steps outlined in this appendix.
Appendix B

Second Sound Helmholtz Oscillations

As mentioned in chapter 3 the cell and counterflow channel form a second sound Helmholtz oscillator. In this appendix the frequency response of the resonance curve and primary damping mechanisms of the temperature oscillations are calculated. Consider the system shown in figure B.1 and let there be a DC heat current $Q$ from one heater and a small AC heat current $dQ(t)$

$$dQ(t) = dQe^{-i\omega t}$$

from the other. These two heat currents generate thermal counterflow through the tube. The normal and superfluid velocities in the channel are written as:

$$V_n(r) = V_{no}(r) + dV_{n}(r)e^{-i\omega t} \quad (B.1)$$

$$V_s(r) = V_{so}(r) + dV_{s}(r)e^{-i\omega t} \quad (B.2)$$
Figure B.1

A schematic illustration of a second sound Helmholtz oscillator
Figure B.1
Where $V_{\text{no}}(r)$ and $V_{\text{so}}(r)$ are the time independent velocities for steady state counterflow. The AC velocity terms are produced by $dQ(t)$.

Entrance length effects appear in a small fraction of channel length and so will be neglected. The wavelengths of second sound at these frequencies are much larger than the dimensions of the cell and flow tube. This allows the $\exp(ikz)$ dependence to be eliminated from equations B.1 and B.2. Temperature oscillations may be assumed uniform throughout the cell and equal to the spatial average of temperature across the cell end of the flow channel.

After inserting B.1 and B.2 into equations 1.3 and 1.4 and keeping only terms linear in $dV_n(r)$ and $dV_s(r)$, the axial components of 1.3 and 1.4 are averaged over the channel's cross section. The result is:

\[-i\omega dV_n e^{-i\omega t} = \frac{1}{A} \int dA \left( \frac{\rho}{\rho_n} v_n^2 - \frac{1}{\rho} \frac{\partial P}{\partial z} - \frac{\rho}{\rho_n} S \frac{\partial T}{\partial z} + \frac{F}{\rho_n} \right) \]  \hspace{1cm} (B.3)

\[-i\omega dV_s e^{-i\omega t} = \frac{1}{A} \int dA \left( -\frac{1}{\rho} \frac{\partial P}{\partial z} + S \frac{\partial T}{\partial z} - \frac{F}{\rho_s} \right) \]  \hspace{1cm} (B.4)

The terms $dV_n$ and $dV_s$ are averages of the AC velocities across the
Equation B.4 is subtracted from B.3 resulting in

\[-i\omega (dV_n - dV_s) e^{-i\omega t} = \]

\[\frac{1}{A} \int dA \left( \frac{n}{\rho_n} \frac{\rho S}{\rho_n} \frac{\partial T}{\partial z} + \frac{\rho}{\rho_n} F_{sn} \right) \quad (B.5)\]

The temperature gradient

\[\frac{\partial T}{\partial z} = \frac{\Delta T_o}{L} + \frac{dT_{e^{-i\omega t}}}{L} \quad (B.6)\]

is assumed uniform through the tube. The gradient has a DC part \(\nabla T_o/L\) from steady state counterflow and an AC part, \(dT(t)/L\) from second sound oscillations.

The relative velocity also has an AC and DC part

\[V = V_o + dV e^{-i\omega t} \]

\[V_o = \text{steady state relative velocity} \]

This may be used to re-write \(F_{sn}\) from equation 1.5 as

\[F_{sn} = F_{sno} + dF_{sn} \quad (B.7)\]

\[F_{sn} = \frac{K B \rho_n}{3}\rho_s (V_o + dV e^{-i\omega t}) L_o (T, V) \quad (B.8)\]
Equations B.6 and B.7 are inserted into B.5. The laminar mean flow assumption allows the steady state counterflow portion to be separated from equation B.5 leaving

\[ 0 = \frac{1}{A} \int \frac{n}{\rho_n} v^2 v_{no}(r) - \frac{\rho_S}{\rho_n} \frac{T_o}{\rho} dT + \frac{\rho}{\rho_S \rho_n} F_{sn} \]

For thermal counterflow there is no net mass flow through the channel. The continuity equation provides the following relation between \( dV_n \) and \( dV_s \):

\[ dV_s = -\frac{\rho_n}{\rho_s} dV_n \]
Substituting for $dV_g$ and $dF_{sn}$ in equation B.9 and solving for $dT$ gives:

$$dT = \frac{\rho \ell}{\rho_s} [(i\omega + \frac{B K L}{3}) \frac{p}{\rho_s} dV_n + \frac{n}{\rho_n A} \int dA (v^2 dV_n(r))]$$

(B.10)

The AC relative velocity appearing in $dF_{sn}$ has been replaced with $dV$ via equation 1.12. The viscosity term is partially reduced to

$$\frac{1}{A} \frac{n}{\rho_n} \int dA v^2 dV_n(r) = \frac{n}{\rho_n} \frac{2}{a} \frac{\partial (dV_n(r))}{\partial r} \bigg|_{r=a}$$

This is evaluated arbitrarily close to the flow tube wall, so a suitable approximation for $dV_n(r)$ near the wall is $^{60}$:

$$dV_n(r) = dV_n(1-e^{-(1+i)(a-r)/\delta})$$

$$\delta = \sqrt{\frac{2n}{\rho_n \omega}} \quad \text{viscous penetration depth}$$

The resulting viscosity term is:

$$= -\omega^2 \frac{(i+1)}{a} \delta dV_n$$
Substituting this into equation B.10 results in:

\[ dT = \frac{\rho_n l}{S} \left[ (i\omega + \frac{BLK}{3}) - \frac{\omega (1+i) \delta}{a} \right] dV_n \]  

(B.11)

Some portion of AC heat current is carried down the flow tube by AC counterflow. The rest of the AC heat current changes the temperature of helium in the cell. The amount of heat flowing out the tube or cell walls or used in changing the temperature of the cell walls is small and is ignored. The AC heat balance equation is written as

\[ dQe^{-i\omega t} = dQ_t e^{-i\omega t} + dQ_c e^{-i\omega t} \]  

(B.12)

\[ dQ_c = -i\omega C dT \]  

(B.13)

\[ C = \text{heat capacity of HeII in the cell} \]

Where \( dQ_t \) is the heat current that changes the temperature of helium in the cell. Heat carried out of the cell by counterflow \( dQ_t \) is related to \( dV_n \) by the relation

\[ dQ_t = STa^2 dV_n \]  

(B.14)
Substituting equations B.11, B.13 and B.14 into equation B.12 and solving for results in

\[
\frac{dT(\omega)}{d\omega} = \frac{T_o}{\rho_n} \left( \frac{\rho_s}{\rho_n} \right)^2 \frac{(x_1 + \omega y_1) x_2 + i(x_1 y_1 - \omega^2 x_2^2)}{x_1^2 + \omega^2 x_2^2}.
\]  
(B.15)

\[
\chi_1 = [\omega_0^2 - \omega^2 (1 - \frac{\rho_s \delta}{\rho_a})]
\]

\[
\chi_2 = -\left( \frac{\rho_s \omega \delta}{\rho_a} + \frac{KBL}{3} \right)
\]

\[
y_1 = \omega \rho_n \left( \frac{1}{\rho_s} - \frac{\delta}{\rho_a} \right)
\]

\[
\frac{dT_o}{dQ/C} = \frac{dQ/C}{C}
\]  
(B.16)

The absolute value of equation B.15 is used for analysis of second sound as discussed in chapter 4. The attenuation of the Helmholtz oscillations for laminar counterflow is from transverse viscous effects of the normal fluid. In turbulent counterflow there will be an additional damping due to the dissipative presence of vortex lines.
Appendix C

Computer Programs

These programs were used for controlling the experiment, measuring and storing data, and analysis of data. They were written for a MINC-11 microcomputer in the 1.0 version BASIC language.

The following program is called LKIN2. This program recorded the output of the Ithaco lock-in when it was detecting the second sound Helmholtz oscillations and stored the data on magnetic disks. The data stored included the amplitude of the oscillations as well as the in and out of phase components.

```
10 DISPLAYCLEAR
20 GETCURSOR(R1,C1)
30 MOVECURSOR(5,1)
40 PRINT "THIS PROGRAM RECORDS THE ACOS, ASIN AND A OUTPUTS OF THE LOCKIN"
50 PRINT "IT STORES THE DATA IN A VIRTUAL ARRAY, Y(3,24)"
60 PRINT "CONNECT A TO CHANNEL 0, ACOS TO 1 AND ASIN TO 2"
70 PRINT "ENTER FILE NAME, DD0R" ; INPUT F$
80 PRINT "ENTER D.C. HEAT CURRENT IN MILLIWARTS" ; INPUT Q1
90 PRINT "ENTER A.C. HEAT CURRENT IN MICROWARTS" ; INPUT Q2
100 PRINT "ENTER FULL SCALE SENSITIVITY OF LOCKIN IN MILLVOLTS" ;
    INPUT V
110 DIM D(2) PRINT "ENTER TIME PER POINT" ; INPUT T
120 PRINT "ENTER NUMBER OF POINTS" ; INPUT N
130 DISPLAYCLEAR
140 MOVECURSOR(1,1) PRINT "FILE NAME F$
150 PRINT " Q(dc)="Q1'MILLIWARTS" Q(ac)="Q2'MICROWARTS"
160 PRINT "LOCKIN SENSITIVITY="V"mV"
170 MOVECURSOR(5,35) PRINT "F A ACOS ASIN"
180 DIM Y(3,24) FOR J=0 TO 3 FOR I=0 TO 24 Y(J,I)=0 NEXT I NEXT J
190 Y(0,21)=N
200 FOR I=1 TO 20
210 MOVECURSOR(10,1) PRINT "ENTER F'I" ; INPUT P Y(0,I)=2*F P
220 HTEXT(11,1, 'PRESS RETURN WHEN')
230 HTEXT(12,1, 'LOCKIN IS IN EQUILIBRIUM')
240 MOVECURSOR(13,1) INPUT S$
```

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This program is called OFFSET and was used to keep the signal going into the signal averager within the range of 0 to 1 volt.

10 DIM V(49)
20 PRINT "ENTER GAIN ON PAR 113"; INPUT G
30 PRINT "ENTER TIME PER ADDRESS"; INPUT T1
40 T=(1024*T1-.4)/50
50 PRINT "WAIT 1 SWEEP TIME BEFORE STARTING AVERAGER"
60 AOUT("ST2",V,1,0,2,1)
70 D=0
80 AIN(,V(),50,T,0,1)
90 FOR I=0 TO 49 D=D+V(I) NEXT I
100 V1=D/50 V=2047*(V1-.3/G)/5
The name of this next program is VRTRAY. It was used to transfer data from the signal averager to the MINC-11 and then store it on magnetic disk.

10 DIM Y$(127)
20 PRINT "THIS TRANSfers DATA FROM AVERAGEr TO THE MINC"
30 PRINT "D.C. HEAT CURRENT Q1 IN MILLIWATTS"; INPUT Q1
40 PRINT "A.C. HEAT CURRENT Q2 IN MICROWATTS"; INPUT Q2
50 PRINT "TIME PER ADDRESS T1"; INPUT T1
60 PRINT "START AND FINISH ADDRESSES T2,T3 FOR PULSE"; INPUT T2,T3
70 PRINT "PRESS START READOUT ON AVERAGER"
80 CIN(,A$,1)
90 FOR I=0 TO 126
  100 CIN(‘RETRIEVE’,Y$(I),,1)
  110 NEXT I
120 CIN(‘RETRIEVE’,Y$(127),57,1)
130 PRINT ‘DATA TRANSFER COMPLETE’
140 PRINT ‘INPUT FILE NAME-MODDR’; INPUT F$
150 OPEN ‘SY1:’+F$ FOR OUTPUT AS FILE 1
160 DIM 1,J(1024,1)
170 FOR I=0 TO 127
  180 FOR K=0 TO 7
  190 J(K+(I*8),1)=VAL(SEC$(Y$(I),3+K*7,8+K*7))
  200 J(K+(I*8),0)=0
  210 NEXT K
  220 NEXT I
230 J(1024,1)=0
240 J(1,0)=Q1 J(2,0)=Q2 J(3,0)=T1 J(4,0)=T2 J(5,0)=T3
250 CLOSE 1
260 PRINT ‘FILE HAS BEEN RECORDED’
270 END

This program is called NUSOND and was used to analyze the second sound data. It could analyze the amplitude of the oscillations as well as the in and out of phase components.
10 DIM X(500)
20 DISPLAYCLEAR
30 PRINT "THIS WILL ANALYZE SECOND SOUND CORRECTLY"
40 PRINT "IT WILL RETRIEVE FILES OF THE FORM Y(3,24)"
50 PRINT "ENTER FILE NAME, DDMO \"; INPUT F$
60 DIM Y(3,24)
70 OPEN "SY lt'+F$ FOR INPUT AS FILE 1
80 DIM 1,Q(3,24)
90 FOR I=0 TO 24
100 FOR J=0 TO 3
110 Y(J,I)=Q(J,I)
120 NEXT J
130 NEXT I
140 CLOSE 1
150 PRINT "FILE NAME \"F$
160 PRINT "Q(dc)=\"Y(0,0)\"MILLIWARTS"
170 PRINT "Q(ac)=\"Y(1,0)\"MICROWARTS"
180 WINDOW(\"EXACT\",0,0,10,10,)
190 GRAPH(,24,Y(0,1),Y(1,1),)
200 PRINT "PUSH RETURN TO CONTINUE\"; INPUT P$
210 DISPLAYCLEAR
220 PRINT "ENTER Y OR N IF NEW CONSTANTS ARE NEEDED\"; INPUT A$
230 IF A$='Y' GO TO 240 IF A$='N' GO TO 290
240 PRINT "ENTER Ps,Pn,P\"; INPUT P1,P2,P
250 P1=P1*P2 P=P2/P
260 PRINT "ENTER N,U2\"; INPUT N,U2
270 PRINT "ENTER C,B,K\"; INPUT C,B,K
280 PRINT "ENTER ST\"; INPUT S1
290 A=Y(1,0)/(P*5.3)
300 PRINT "ENTER SQR(Lb),Wo\"; INPUT L,W1
310 L=L2 DIM T(501)
320 PRINT "ENTER AMPLITUDE SCALER, S\"; INPUT S
330 A=Y(1,0)/(P*1.5)
340 V=Y(0,0)/(P*0.52*P1*S1)
350 FOR I=100 TO 400 X(I)=I NEXT I
360 FOR I=125 TO 375
370 W=(2*PI*1*7)/500
380 D=SQR((1.000000E-03*2*N)/(W*P2))
390 X1=(P2/P1)*(W12-(W2)*(1-(P1*D)/(P*.05)-V*(P2-P1)/(P1*U2)))/((P2*W*D)/(P*.05)+(1.00000E-03*W2*N)/(P*U22)+(B*L*K*P2)/(3*P1))
400 X2=((P2*W*D)/(P*.05)+(1.00000E-03*W2*N)/(P*U22)+(B*L*K*P2)/(3*P1))
410 Y1=Y*X2
420 Y2=Y*X2*((1-V*(P2-P1)/(P1*U2))/P1-D/(P*.05))
430 T(I)=S*A*SQR((X1*X2-Y1*Y2)+((X2*Y1+X1*Y2)*2)/((X12+Y12)
440 NEXT I
450 WINDOW(\"EXACT\",0,0,500,10,1)
460 GRAPH(,300,X(125),T(125),,1
470 FOR I=1 TO 24
480 POINT(,INT(Y(0,1)*500/7),Y(1,1),1)
490 NEXT I
This program is called MANFI1 and was used to fit exponentials to the pulsing data. It also adjusted the non-zero baseline artifact that was part of the raw data.
10 DIM Y(1024)
20 PRINT "ENTER FILE DESIGNATION"; INPUT M1$
30 OPEN "ST1:"+M1$ FOR INPUT AS FILE 1
40 DIM 1, J(1024,1)
50 FOR I=0 TO 1024 Y(I)=J(I,1) NEXT I
60 Q1=J(1,0) Q2=J(2,0) T1=J(3,0) T2=J(4,0) T3=J(5,0)
70 CLOSE 1
80 Y(1024)=Y(1023)
90 PRINT Q1=" MILLIWATTS"; Q2=" MICROWATTS"
100 PRINT " OF SWEEPS"; Y(0)
110 DIM S(1024)
120 IF (T3-T2)<(1024-T3) GO TO 170
130 IF (T3-T2)>(1024-T3) GO TO 140
140 FOR I=T3+1 TO 1024 S(I+T2-T3)=Y(I)+Y(I-T3+T2) NEXT I
150 I1=T3+1 I2=T2+1024-T3+1 GOSUB 220
160 GO TO 350
170 FOR I=T2+1 TO T3
180 S(I)=Y(I)+Y(I-T2+T3)
190 NEXT I
200 I1=T2+1 I2=T3 GOSUB 220
210 GO TO 350
220 FOR I=I1 TO I2 L=L+S(I) NEXT I
230 L=L/(I2+1-I1)
240 S1=0 S2=0 S3=0 S4=0 S5=0 M=0
250 FOR I=I1 TO I2
260 IF S(I)<.9*L THEN 300
270 IF S(I)>1.1*L THEN 300
280 M=M+1
290 S1=S1+I S2=S2+I*I S3=S3+S(I) S4=S4+S(I)*S(I) S5=S5+I*S(I)
300 NEXT I
310 D=M*S2-S1*S1
320 A4=(S2*S3-S1*S5)/D
330 B3=(M*S5-S1*S3)/D
340 RETURN
350 B3=B3/2 PRINT " M="B3
360 FOR I=1 TO 1024
370 S(I)=Y(I)-B3*I
380 NEXT I
390 C=0
400 FOR I=1 TO T2 C=C+S(I) NEXT I
410 C=C/(T2)
420 FOR I=1 TO 1024 S(I)=S(I)-C NEXT I
430 GRAPH('EXACT',1,-1.2*ABS(S(500)),1024,1.2*S(500),0)
440 FOR I=1 TO 50 POINT(,4*I,0,2) NEXT I
450 FOR I=200 TO 250 POINT(,4*I,0,2) NEXT I
460 PRINT 'Y TO CONT. N TO CHANGE SLOPE'; INPUT W$
470 IF W$="Y" GO TO 510 IF W$="N" GO TO 490
480 IF W$="Y" GO TO 510 IF W$="N" GO TO 490
490 PRINT "INPUT NEW SLOPE M"; INPUT B3
500 GO TO 360
510 PRINT 'INPUT C1'; INPUT C1 PRINT 'INPUT C2'; INPUT C2  
520 PRINT 'INPUT A1'; INPUT A1 PRINT 'INPUT A2'; INPUT A2  
530 PRINT 'INPUT T1'; INPUT B1 PRINT 'INPUT T2'; INPUT B2  
540 FOR I=1 TO T2 D=C1 POINT(I,D,2) NEXT I  
550 FOR I=T2+1 TO T3 D=C1+A1*(1-EXP(-(I-T2-1)*T1/B1)) POINT(I,D,2) NEXT I  
560 FOR I=T3+1 TO 1024 D=C2+A2*EXP(-(I-T3-1)*T1/B2) POINT(I,D,2) NEXT I  
570 GEICURSOR(R1,G1)  
580 MOVECURSOR(3,40)  
590 PRINT 'C1='C1'A1='A1'T1='B1'SEC'  
600 MOVECURSOR(4,40)  
610 PRINT 'C2='C2'A2='A2'T2='B2'SEC'  
620 MOVECURSOR(5,40)  
630 PRINT 'R1='B1/U,'R2='B2/U  
640 MOVECURSOR(R1,G1)  
650 PRINT 'Y TO CONTINUE N FOR ANOTHER POINT. ; INPUT A$  
660 IF A$='Y' GO TO 670 IF A$='N' GO TO 10  
670 PRINT 'T2='T2'T3='T3 INPUT T2 INPUT T3  
680 GO TO 510  
690 DISPLAYCLEAR END
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