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AN EXPERIMENTAL AND THEORETICAL STUDY OF THE INTERACTION OF AN ELECTROSTATIC FIELD WITH A TWO-DIMENSIONAL JET FLOW

The Ohio State University

Ph.D. 1981

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AN EXPERIMENTAL AND THEORETICAL STUDY
OF THE INTERACTION OF AN
ELECTROSTATIC FIELD WITH A TWO-DIMENSIONAL
JET FLOW

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By
Farrokh Kaveh, B.S., M.S.

* * * * *

The Ohio State University
1981

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LIST OF SYMBOLS

\( b \) = Mobility of the air, \( m^2/kV \cdot \text{sec} \)

\( \mathbf{E} \) = Electric field strength, kV/m

\( \mathbf{J} \) = Current density, \( \mu A/m^2 \)

\( n \) = Charge number density, \( m^{-3} \)

\( e \) = Elementary electric charge, C

\( E_x \) = Electric field strength in x-direction, kV/m

\( E_y \) = Electric field strength in y-direction, kV/m

\( \mathbf{D} \) = Electric field flux, \( F \cdot kV/m^2 \)

\( J_{\omega_0} \) = Maximum current density in the ground grid, \( \mu A/m^2 \)

\( D \) = Distance between wire and the ground grid, m

\( n_o \) = Characteristic charge number density = \( D J_{\omega_0} / e b \phi_o \), \( m^{-3} \)

\( \bar{n} \) = Normalized charge number density = \( \frac{n}{n_o} \)

\( \bar{x} \) = Normalized distance in the x-direction = \( \frac{x}{D} \) or \( \frac{x}{h} \)

\( \bar{y} \) = Normalized distance in the y-direction = \( \frac{y}{D} \) or \( \frac{y}{h} \)

\( Y \) = Constant = \( \frac{D^3 J_{\omega_0}}{e b \phi_o} \)

\( i, j \) = Grid indices

\( \text{Diff}_{i,j} \) = Finite difference equivalent of the diffusion term

\( u \) = Velocity in the x-direction, m/sec

\( v \) = Velocity in the y-direction, m/sec

\( P \) = Pressure, Pa.
\( t \) = Time, sec
\( h \) = Half-width of the 2-D jet, m
\( U_0 \) = Centerline velocity of the jet at the exit, m/sec
\( \bar{t} \) = Normalized time = \( t \cdot \frac{2h}{U_0} \) or \( t \cdot \frac{U_E}{D} \)
\( Re \) = Reynolds number = \( \frac{2U_0 h}{v} \)
\( \bar{u} \) = Normalized velocity in the x-direction \( \frac{u}{U_0} \) or \( \frac{u}{U_E} \)
\( \bar{v} \) = Normalized velocity in the y-direction \( \frac{v}{U_0} \) or \( \frac{v}{U_E} \)
\( \Delta \bar{t} \) = Normalized time increment
\( Conv_{i,j} \) = Finite difference equivalent of the convective term
\( S_{i,j} \) = \( \Delta^2 \cdot \bar{\omega}_{i,j} \)
\( j_{\text{max}} \) = Maximum number of grid points in the y-direction
\( U_E \) = Characteristic EHD velocity = \( \sqrt{DJ \frac{\omega}{\rho b}} \), m/sec
\( R_{\text{ehd}} \) = EHD Reynolds number = \( \frac{U_E \cdot D}{\nu} \)
\( Elec_{i,j} \) = Finite difference equivalent of the electric body force term
\( EN \) = EHD number = \( \frac{U_E}{U_0} = \frac{2h}{D} \cdot \frac{R_{\text{ehd}}}{Re} \)
\( U \) = Jet average velocity in the x-direction, m/sec
\( U_c \) = Jet centerline average velocity, m/sec
\( F(x,y) \) = Arbitrary body force term
\( u' \) = Velocity fluctuation in the x-direction, m/sec
\( v' \) = Velocity fluctuation in the y-direction, m/sec
\( c \) = Complex wave velocity
\( c_r \) = Wave propagation velocity
\( c_i \) = Rate of amplification of the small disturbances

Greek Symbols
\( \nabla \) = Gradient operator
\( \mu \) = Permeability = F/m
\( \phi \) = Electrical potential, kV
\( \phi_0 \) = Electrical potential between wire and ground grid, kV
\( \bar{\phi} \) = Normalized electrical potential = \( \frac{\phi}{\phi_0} \)
\( \Delta x \) = Normalized mesh size of numerical calculation for x-coordinate
\( \Delta y \) = Normalized mesh size of numerical calculation for y-coordinate
\( \Delta \) = \( \Delta x = \Delta y \)
\( \Omega \) = Relaxation factor
\( \rho \) = Air density, kg/m\(^3\)
\( \nu \) = Kinematic viscosity of air, m\(^2\)/sec
\( \psi \) = Stream function, m\(^2\)/sec
\( \omega \) = Vorticity, sec\(^{-1}\)
\( \bar{\psi} \) = Normalized Stream function = \( \frac{\psi}{2U_0h} \) or \( \frac{\psi}{U_ED} \)
\( \bar{\omega} \) = Normalized vorticity = \( \frac{\omega}{U_0} \cdot 2h \) or \( \frac{\omega}{U_E} \cdot D \)
\( \bar{\psi}_0 \) = Normalized value of the streamfunction at the centerline
\( \bar{\psi}_{rb} \) = Normalized value of the stream function at the right boundary in the numerical calculation

\( \tau' \) = Amplitude of the velocity fluctuations in the x-direction

\( \alpha \) = Wave-number of the velocity fluctuations

\( \delta \) = Half-width of the jet, m

\( \eta \) = Normalized distance in the y-direction = \( y/\delta \)
CHAPTER I
INTRODUCTION

A. Background

The problem of unbounded flows in general and free jets in particular have attracted the attention of many investigators in the past four decades. The initial work done in this field was concerned with the laminar mode of the flow, the solution of which was obtained by Schlichting (1).

In the majority of practical cases the jets were observed to be turbulent, and it was thus predictable that the stability of jets and its transition into turbulence became the focal point of the later investigations in this field. Attempts were made to explain the instability of free boundary layer type flows using the stability theories developed by Rayleigh (2), Tollmein (3), and Schlichting (1). These theories had already been verified experimentally in case of flow over a plate by Schubauer & Skrumstad (4).

There was a major difference though between wall and free boundary layer flows. It was found by Tollmain that viscosity was instrumental in the onset of instability in case of bounded flows. Neglecting viscosity led to completely stable flows which was in conflict with experimental results. This was in direct contrast with free boundary layer type flows where the instability was an inviscid effect, with...
viscosity having only a damping influence. This significant difference that existed between the two types of flow was seemingly due to the different mean velocity distributions associated with each flow field. The free boundary layer flows possessed at least an inflexion point, and based on the theories of Lord Rayleigh and Tollmein this resulted in the growth of all small disturbances for inviscid flows. The mentioned theories are known as the necessary and sufficient conditions for instability in the absence of viscosity.

It was stated by Helmholtz (5) that a disturbed free boundary layer would roll up into vortices. Now free boundary layers have always one or more inflexion points. Thus for large Reynolds numbers, amplified disturbances were expected to exist, and moreover it was expected after Helmholtz that the free boundary layer should roll up into vortices. Now these vortices would not have been certainly of the well-known potential type. The vortices were real vortices and could be defined by the existence of a local concentration of vorticity as, for instance, was found in a Hamel-Oseen vortex (5).

The above observations were verified by the theoretical and experimental works of Lessen (6), Esch (7), Tatsumi & Kakvtani (8), Sato (9), and Michalke & Wille (10).

Concerning the rolling up of free boundary layers Rosenhead (11), Birkhoff & Fisher (12) calculated the rolling up of a vortex sheet, which was the simplest case of a free-boundary layer profile, approximately. More recent theoretical studies of the phenomena has been reported by Beaver & Wilson (13) who made an inviscid representation of the flow by discrete arrays of point vortices, and Moore
who followed basically the same procedure. Another type of theoretical work was done based on the classical linearized theory, of which the works of Michalke (15) and Batcheler & Grill (16), were major contributions. Michalke showed by means of the stability theory that disturbed hyperbolictangent velocity profiles gave rise to areas of concentrated vorticity. Sato (9) also concluded that the solution of the linearized stability theory was very much dependent on the assumed average velocity profile, and that for slow varying profiles the rate of amplification of small disturbances was smaller than that of "top-hat" profiles. The same type of observation was made by Grant (17) when he failed to exhibit instability through the numerical solution of the complete Navier Stokes equations for axisymmetric jets using shallow profiles. On the other hand his numerical result for "top-hat" type velocity profiles indicated that the flow field was dominated by large-scale vortices which drifted in the down-stream direction with a Strouhal number of around 0.55.

The disturbances that grew and evolved into vortices have been found to be of periodic nature, exhibiting both symmetric and antisymmetric characteristics depending on the initial velocity profiles, Sato (9). The disturbances grew exponentially at first, in the so called linear region, where the results of linear stability theory applied. These observations were made in experiments at sufficiently high Reynolds numbers based upon the theoretical studies of the inviscid case. However, it was known, as stated before, that viscosity had a damping effect on the growth of the periodic fluctuations. In order for the disturbances to grow it was expected a
critical Reynolds number would exist beyond which the damping effect of viscosity would be insignificant, and the flow would behave as if it were inviscid. No definite value for the critical Reynolds number has been reported in the literature. Values ranging from 3, Tatsumi & Kakutani (8), up to 1000, Crow & Champayne (18) have been observed. It has been noted that these wide ranges of the observed critical Reynolds number were due to the differences that existed in the conditions under which the experiments have been carried out, Grant (17).

Beyond the linearity range the theories stated so far were unable to predict the characteristics of the flow. This happened because the flow was not laminar anymore, and thus could not be described by any of the established theoretical methods described before. Because of these complications most of the work done in the transition and turbulent regions of the flow have been of an experimental nature. The experimental work has been mostly concerned with the utilization of the flow visualization and hot-wire anemometry techniques to try to explain the flow characteristics. It has to be mentioned though that in the initial stages of such studies, due to the scarcity of hot-wire anemometers, pitot tubes were employed for the study of the jets, an example of such work would be that of Miller & Comings (19), who mapped the pressure distribution in the jet. They concluded that the flow was far from being isobaric due to the existence of turbulent stresses. Their results though generally true were subject to doubt because of the uncertainty of the direction of flow in the turbulent region, Sato (9).
With the development of highly sensitive and accurate constant temperature type hot-wire anemometers significant advancement in the study of turbulent flows in general and jets in particular occurred. Also the development of high speed motion picture cameras, along with fast developing films enabled researcher to visualize and record the mentioned flow fields, and to make measurements of velocities and frequencies based on these.

The first reported experimental study of two-dimensional jet was done by Andrade (20) using water. He basically repeated Reynolds famous experiment in the case of jet issuing from a narrow slit, and found that above certain values of Reynolds number the jet became turbulent. Brown (21) published photographs showing the periodic motions of smoke-laden two-dimensional air jets subjected to external acoustic excitation. Extensive studies of vortex formation was done by Wille and co-workers in Berlin. Wehrmann & Wille (22) clearly illustrated the breakdown of the vortices at a Reynolds number of 10,000. Examples of studies of a more recent nature are those of Beaver & Wilson (13) who observed a regular periodic pattern when the Reynolds number of the water jet was between 500 and 3000. The organized structure was observed to persist a limited distance in the down-stream direction. Beyond that, the break-down of vortices occurred, and the flow seemed to become completely chaotic, lacking any kind of regularity. Yet it was noticed that acoustically exciting the jet resulted in an extension of the organized structure in the down-stream direction. Similar observations was made by Crow & Champagne (18) who observed the orderly structure up to a Reynolds
number of $10^4$, while forming at an average Strouhal number of about 0.3 based on frequency, exit speed, and diameter.

The phenomena of vortex breakdown observed in the experiments stated above, has been a major concern of the ongoing studies in the area of free boundary layer type flows. Timme (23) suggested that a critical Reynolds number $Re = \frac{\Gamma}{\nu}$ of a single vortex was relevant for the breakdown, and Fabian (24) applied this hypothesis to the mixing region of a free jet. $\Gamma$ was a critical circulation of a single vortex and $\nu$ the viscosity. It was assumed that if a vortex had reached the stated Reynolds number during its growth, it would breakdown. Obviously viscosity played an important role in this hypothesis. Freymuth (25) showed by changing a single dimensionless flow parameter, while keeping the others constant, that Reynolds number and thus viscosity was not a deciding parameter in the turbulent region of the flow. This would tend to reject the hypothesis mentioned above for which viscosity was the corner stone. Using hot-wire signals, Freymuth also showed that the sole parameter which affected the flow was the Strouhal number. Regarding the breakdown of vortices Donn (26) proposed that centrifugal instability could be relevant for the observed phenomenas. There has been no proof of this hypothesis. Another suggestion has been forwarded by Lykoudis & Papailiou (27). They concluded that vortices moved in the down-stream direction lost their strength owing to the diffusion of vorticity which caused their gradual deceleration. Moreover the vortices increased their size as they drifted in the down-stream direction by entraining outside fluid.
These combined effects caused the vortices to reach a state of instability leading to their breakdown.

The more recent explanations of the phenomena of vortex breakdown, have been in concord with the observation of Freymuth (25). Through the visualization of the flow field of an axisymmetric jet it was concluded that the vortex breakdown was presumably caused by the interaction of the vortices, i.e. by induction. The vortices through mutual induction coalesce, forming bigger circulatory cells which consequently breakdown into complete turbulence. Similar observation has been reported by Winant & Browand (28), who found that turbulent vortices in the mixing layer of two streams having different velocities interact by rolling around each other forming a single vortical structure with approximately twice the spacing of the former vortices. Roshko (29) has also verified the mentioned phenomena through high speed photography of a free boundary layer type flow at very high Reynolds numbers. Reynolds & Hussain (30) have observed the vortex pairing process in axisymmetric jets, and Reynolds (31) has suggested that vortex pairing is instrumental in the entrainment of outside fluid into the jet.

The observed pairing process was believed to be the result of the mutual interaction of neighbouring vortices and thus, in turn, depended upon slight imperfections in the vortex spacing and strength.

A model of turbulent shear-layer growth based on vortex pairing, as opposed to the "nibbling & straining" action theory of Townsend (32), has been developed by Winant (33). The mixing layer was considered to be a double row of Stuart (34) vortices swept downstream at
some mean velocity. The rows were slightly vertically offset, and vortices were distance of a wave-length apart. Calculation showed that adjoining pairs of vortices, one from each row, did rotate around one another and drew closer.

The calculations also revealed that the growth of the shear layer was dependent on two factors, the offset distance of vortices and their strength. The larger the offset distance was (corresponding in a sense to a more irregular initial distribution of vortices) the faster the growth of the shear layer. Also large values of vortex strength, corresponding to more concentrated vorticity distributions led to larger shear-layer growth rates.

In recent years considerable interest has been shown in the area of electrohydrodynamics. Various fluid phenomena have been found to be affected by electrostatic fields or charges. Examples of such cases would be: boiling and condensation, boundary layer type flows, flow attachment, fluid diagnostics and convective heat transfer Velkoff (35).

Utilization of the phenomena of electric or corona wind, has been one of the more active areas in EHD in the past two decades. The basic concept of a corona wind is very old, but it has been only in recent years that it has been put into practical use. A corona wind is a flow of air induced by the action of an intense electrical field between a highly charged sharp point and another ground electrode. The air breaks down in the vicinity of the point, ions are created which move away from the point under the action of the electrical
field, colliding with the neutral gas molecules and result in a corona or electrical wind.

In a series of earlier studies in this area Velkoff (36), (37), (38) investigated the effects of an electric wind on heat transfer and boundary layer analysis. He demonstrated that a corona discharge increased the free convection heat transfer rates, and doubled the pressure drop in a channel at low Reynolds numbers. Godfrey & Velkoff (39) reported similar results for a corona discharge in forced convection, and Chuang & Velkoff (40) reported increased frost formation in a region close to a corona discharge.

Industrial use of the phenomena include gas or liquid pumping Melcher (41), electrostatic type precipitators, Yamamoto & Velkoff (42), bread baking, Kulacki (43), and enhancement of combustion processes.

In the early studies in this area no complete analytical or numerical solutions of the electrical field and fluid flow problems associated with the corona wind could be found. The reason was the intractability of the governing equations, along with the lack of advanced computers. The solutions on hand were of analytical nature and limited to either zero charge density or constant charge density. Ramadan and Soo (44) and later Hughes and Stephans (45) presented analytical solutions of the corona wind for a wire-plate geometry. In their analysis it was assumed that the space charge density distribution was proportional to the square of the electric field, and the pressure and viscous terms were neglected.
A complete solution of the governing equations of electric field was reported by Leutert & Bohlen (46) for an electrostatic precipitator. The current continuity equation along with Poisson's equation was solved and the significant effect of the space charge on the electrical potential distribution was demonstrated. No attempt for solving the associated fluid problem was made.

Along this line Adachi, Masuda, and Akutsu (47) investigated the negative electric wind for a needle to plane geometry. Robinson (48) studied the electric wind velocity by using ion diffusion models, and experimentally obtained helium concentration profiles for a wire plate in a duct geometry.

It was only recently that Yabe, Mori, and Hijikata (49) were able to carry out a comprehensive numerical and experimental analysis of the electric wind. The geometry considered was that of a single-wire plate which corresponded to a semi-infinite region. In order to solve the problem numerically the domain was made finite by a bi-polar transformation of the cartesian coordinate system. The current continuity equation along with the Poisson's equation were solved and the potential and space charge distributions were evaluated. Consequently the Navier-Stokes and continuity equations with the inclusion of the known electrical body forces were solved by backward differencing and successive over-relaxation methods, successively. The numerical results were in good agreement with the initial experimental data.

Yamamoto (50) applied refined versions of the described methods in solving the problem of electrohydrodynamic secondary flow interaction in an electrostatic precipitator. The numerically produced
fluid field configuration agreed well with the experimental results
obtained by the use of Schlieren visualization technique. Strong
interaction between the electric wind and the primary flow was
observed at low Reynolds numbers.

An experimental work by Velkoff & Carpenter (51) studied the
effects of a corona discharge on the stability of a two-dimensional
jet. The hot-wire signals obtained showed a definite reduction in
the level of turbulence in the flow, Figure 1. No further detailed
investigation of the phenomena was attempted. Nevertheless it was
hypothesized by Velkoff that the electric body forces could be
thought of as a pseudo-pressure gradient imposed on the flow field,
and thus it was possible to get electrical equivalents of positive or
negative pressure gradients in the flow.

In order to put this hypothesis on firm ground the present study
was initiated into the nature of the action of the electric body
force within the fluid.

B. Present Study

It is known that an incompressible potential flow is determined
at any instant in time by the existing conditions on the boundary. In
effect any incompressible potential flow could be controlled by a
suitable change in the conditions at the boundary, and it is not
required to reach within the boundary to affect the character of the
flow.

The situation changes though if vorticity is shed from the
boundary into the flow, as in the case of a two-dimensional jet flow.
Figure 1. Turbulence Level in the 2-D Jet. (a) Re = 375, $\phi_0 = 0$, (b) Re = 375 , $\phi_0 = 15.0$ kV, (c) Re = 375, $\phi_0 = 17.5$ kV
The flow no longer would be dependent upon the conditions at the boundary, and one would be unable to control the rotational region of the velocity field by changing the boundary conditions. Thus the control over the flow would be lost. The flow would depend not solely on instantaneous surface conditions, but on the entire history of vortex shedding from the boundary in all detail, Crow & Champagne (18).

In order to restore control, one must either control the vorticity field directly by means of body forces, or control the entire history of the boundary conditions. Usually neither approach is taken, and the flow gives way to chaos.

Liepmann (52) has noted that the possibility of turbulence control by direct interference with the observed vortical structures would lead to very significant technological advances. Controlled excitation of jets have been observed to result in prolongment of the orderly structures, Beaver & Wilson (13).

Use of body forces, specially in the case of jet flows, as an alternative approach of controlling the flow field has been rare. In the present study this technique was attempted, based on the promising results of Velkoff & Carpenter. The geometry considered included a two-dimensional nozzle, a corona wire placed above the nozzle, and finally a ground grid system placed further down-stream. By maintaining a high electrical potential between the corona wire and ground grid system, an electrical body force field was created. The mentioned force field was expected to interact with the initial two-dimensional jet flow enhancing its stability characteristics. Consequently the course of the study was directed towards the
experimental and numerical investigation of the effects of the coupling of an electrostatic field with a two-dimensional jet flow.

The experimental objectives were:

1. Measurement of the current density distribution at the ground grid. This was required in the numerical solution of the electric field and space charge density distributions.
3. Use of hot-wire anemometry techniques to evaluate the unsteady characteristics of the flow as well as the average properties.
4. Visualization of the two-dimensional jet with and without the presence of an electric wind, by means of the Schlieren optical method, and smoke-laser techniques.

The theoretical objectives were:

1. Numerical solutions of the electric field potential and space charge distribution, using the method of Yabe (49).
2. Numerical solutions of the Navier-Stokes equation after being written in terms of vorticity and stream functions covered the following cases: (a) two-dimensional jet flow, (b) electric wind flow, (c) two-dimensional jet flow in the presence of an electrostatic field. In part c the electric body forces were assumed to be unaffected by the jet flow, and thus were known from the calculations described in part 1.
3. Stability analysis based on the linear stability theory, in order to get an insight into the factors relevant to the stability of the described flows. The analysis was limited
to the neutral case, and the average velocity distributions used were those found from the experiment.
A. **Experimental Arrangement**

The objectives of the experimental investigation were three-fold. The first objective was to measure the current density and the electrical potential. The experimentally determined current density values were needed as boundary conditions for the determination of the electrical potential and space charge density distributions when the numerical techniques were employed. The second objective was to measure and analyze the flow fluctuations in the jet, the electric wind, and the combined flow field. The third objective was to visualize the mentioned flow fields.

A schematic diagram of the test apparatus that was designed to meet the specific needs of this study is illustrated in Figure 2. In the initial stages of the study a two-dimensional nozzle (0.5 cm in width and 12.5 cm in length) made of plexiglass was used as the jet source. Later on another type of nozzle which had two side plates was used, in order to make the experiment comparable with the theory that was employed in numerical solution of the problem.

The surge tank that was directly connected to the nozzle established a uniform flow of air. Electrically grounded steel gauze packed in the tank further secured the uniformity of the flow,
Figure 2. Schematic Diagram of Experimental Arrangement
minimized the existing velocity fluctuations in the supply air, and simultaneously discharged any electrical charge that happened to exist in the shop air. A Dwyer instruments rotameter type flowmeter with an accuracy of 1.0% of full range was employed to measure the flow rate through the nozzle. Heated air had to be used when the Schlieren technique was used. In order to accomplish this a cylindrical tank with a built-in electrical heater was placed before the surge tank.

To impose an electrical field on the jet flow a corona wire-collector grid system was used. A corona wire 89 μm in diameter and 12.5 cm in length placed parallel to the nozzle slot, was fixed at both ends to two pins, which were placed at either end of the nozzle. The pins were thin enough, 1.0 mm in diameter, so as to minimize interference of the passage of light through the test section when the Schlieren system was employed for flow visualization.

In most of the tests the high voltage was applied to the corona wire, while the collector grid was grounded. A Peschel electronics PS-10Y high voltage supply with maximum of 50 kV, coupled to two filters was used as the high voltage source. The collector grid consisted of 14 brass rods, 1.6 mm in diameter, placed 12.7 mm apart. This grid system was used instead of a plate electrode because it allowed the jet flow to move in a relatively unrestricted fashion, served as a ground plate, and also provided a means to measure the current density of the electric wind. The current density was simply measured by connecting each rod, in turn, to a Westinghouse DC microammeter. More accurate measurement of the current density was accomplished by use of the set-up shown in Figure 3. In this
Figure 3. Test Set Up for Current Density Measurements
Figure 3. Test Set Up for Current Density Measurements
design the ground plate consisted of 62 copper plates 1.6 mm wide and 10 cm long. The plates were connected together through teflon insulation tape. In this manner the plates were electrically isolated from one another and by connecting each plate, in turn, to the microammeter accurate measurements of the current densities was possible.

The outline of the experimental set-up for the electrical potential measurement is shown in Figure 4. As an electrostatic Longmuir probe, a fine wire of 89 µm in diameter and 1.5 cm in length was placed in parallel to the corona wire, and was covered with a tapered glass tube. The electrostatic probe was placed in the region of interest, while being mounted on a traverse mechanism. The probe location had 0.5 mm accuracy in the vertical direction and 0.02 mm accuracy in the horizontal direction. The positive lead of a second high voltage source was connected to the probe to nullify the electrical current reading of the microammeter (2) shown in Figure 4.

In order to find the frequency content, and the average properties of the 2-D jet, the electric wind, and the modified jet flow fields hot wire anemometry technique was employed. The hot wire that was used was a Data-metrics model that was calibrated by the use of the calibration tank. The frequency response of the hot wire was also determined using the direct technique, as proposed by Velkoff [53]. The anemometer used was found to have a flat frequency response up to 500 Hz, which was sufficient for our purposes. The frequency spectrum analyser that was employed was a Federal Scientific Ubiquitous Spectrum Analyser model UA-500.
Figure 4. Test Set Up for Electrical Potential Measurements
The defining parameters of turbulence were also measured for the various flow fields described in above. A Thermo-Systems Inc. Model 1050 anemometer, along with a Tsi 1242-5 V-type hot wire probe was employed to measure the r.m.s. value of the velocity fluctuations in the x and y directions. The frequency response of the system was measured by the use of a built-in square wave testing system, and was found to be flat up to 16000 Hz for a wire diameter of 0.02 mm.

The Reynolds stresses were also evaluated by combining a Tsi Model 1015c Correlator with the system described above.

The Schlieren optical technique was employed to visualize the flow fields of the two-dimensional jet, the electric wind and the two-dimensional jet in the presence of an electric field. Figure 5 and 6 show the schematic diagram, and experimental set-up of the system employed. A General Radio type 1531-AB strobotal electronic stroboscope was used as the light source. The frequency of the strobe light was adjustable and the lamp was covered so that only a slit of light could be emitted from it. Light from the mentioned line source was collimated by the first mirror, and then passed through the test section. It was then brought to focus by a second mirror. At the focal point where there existed an image of the source, a knife-edge was introduced. The light rays that passed through the knife-edge were projected on a screen or camera. Detailed information concerning the Schlieren technique is discussed by Shapiro (54). The camera that was used to photograph the produced images was a Polaroid Speed Graphic. The needed density gradient for the Schlieren method to work was obtained by heating the inlet air to the nozzle.
Figure 5. Schematic Diagram of Test Arrangement for the Schlieren System.
Figure 6. Overall Test Arrangement
In the case where the visualization of the entrainment process was of concern, the set-up shown in Figure 7 was used. The Nichrome wire shown was 0.2 mm in diameter, and 13 cm long. It was used to heat the fluid particles in the entrainment region of the jet. The input power to the wire was furnished by a 12 volt DC battery, which was floated electrically to avoid any electrical discharges through the Nichrome wire when corona discharge testing was taking place. The wire was stretched parallel to the nozzle slot, and was mounted on a traversing mechanism, so that it could be positioned in different locations.

Smoke was also used to visualize the different flow fields. Smoke from a cigar smoke generator was premixed with the air flow, and a laser was used as the light source. The cigar smoke generator consisted of two glass bottles in one of which a cigar that was soaked in oil was placed. The other bottle was connected to the first one through a piece of tubing and was used as storage for the smoke. After lighting the cigar one would wait for enough smoke to be collected in the storage bottle. Shop air was then passed through one end of the system forcing the smoke out of the other side. The arrangement is shown in Figure 8.

B. Experimental Procedure

1. Current Density Measurement

To measure the current density of the electrical current associated with the corona wind the ground rods were connected in turns, to
Figure 7. Test Set Up for Observation of the Entrainment Process
Figure 8. Laser-Smoke Arrangement
the microammeter, while the other rods were collectively grounded. The same procedure was followed in the case where copper plates were used for the precise measurements of the current densities. The mentioned procedure was followed, in two general cases. One in which side plates were placed below the corona wire and when no plates were present. The results were then compared and the effect of the insulation plate below the corona wire on the current density distribution was analyzed. Current densities were obtained for various two-dimensional jet flow velocities, to find out if in fact the electrical field distribution was independent of the fluid flow characteristics. Because of the dependence of the corona discharge process on factors such as temperature, pressure, and relative humidity, their values were recorded in each test.

2. Electrical Potential Measurement

A Langmuir electrostatic probe was used in the measurement of the electrical potential distribution. The probe was placed parallel to the corona wire in the region of interest. By use of a traverse mechanism the exact position of the electrostatic probe was known at all times. The probe was connected through a microammeter to a high voltage power supply. If there existed a difference between the applied potential to the probe and the electrical potential in its vicinity, a current would flow in the positive or negative directions, depending on whether the probe potential was higher or lower than that of the surroundings. The potential was varied until the current was reduced to zero. The minimum value of the potential applied to the probe was equal to the electrical potential of the point where
the probe was located. By placing the probe at various locations in
the region, the electrical potential distribution was determined. To
assure that accurate data was taken care had to be taken so that no
electrical discharges through the probe system existed. To accomplish
this, the microammeter had to be floated.

3. Hot Wire Measurements

The average velocity distributions of the two-dimensional jet, the
electric wind, and the modified jet were measured at different
down-stream positions. The data obtained was also used in the evalu­
ation of the diffusion and decay rates of the jets. The measurements
were taken above the ground grid to avoid any damage to the hot wire,
and to prevent any distortion of the hot wire readings due to dis­
charge through the probe.

The frequency spectrum of the different jets was measured above
the ground grid system. By traversing the hot wire probe along
the mixing layer of the two-dimensional jet, different stages of
breakdown were also investigated. The data were used to find the
drift velocity of the observed vortices. This velocity was also
measured by placing two hot wire probes, 6.6 mm apart, in the shear
layer. Using the time difference between the peaks of the hot wire
signals and the known distance between the vortices the drift vel­
was calculated.

The fluctuating components of the velocity field along with the
Reynolds stresses were measured above the ground grid at $x/2h = 10$
where $x$ was distance from the nozzle and $h$ was the half width of the
jet. The procedure followed is illustrated in the Instruction
Manual for the 1015c Tsi correlator given in Appendix B.

4. Flow Visualization

The Schlieren system was employed to visualize and record the various flow fields under study. In order to improve the intensity of the images produced, air was preheated to 160°F in the cylindrical chamber. The temperature used was high enough to create the needed density variation for the Schlieren technique to work; however the subsequent buoyancy force field was not strong enough to considerably affect the flow field of the jet.

In order to visualize the entrainment process of the free air into the two-dimensional jet flow, the pathlines of the entrained fluid had to be made visible to the Schlieren system. For this purpose the Nichrome wire indicated in Figure 7 was used as a heat source to heat the fluid particles in the entrainment region of the jet. As mentioned before the input power to the wire was furnished by a floating battery, so that not to distort the existing electric field. The wire system was mounted on a traversing mechanism. The system could be positioned in different locations within the entrainment region of the jet. The same procedure was followed to visualize the entrainment process in the case of the electric wind, and the modified jet flows.

When the objective was to visualize the electric wind by itself the high voltage was connected to the collector grid system, while the corona wire was grounded. The DC battery was connected to the corona wire and thus was at corona wire potential.
Smoke was also used to visualize the flow fields of the two-dimensional jet, and the jet in the presence of the electric wind. Smoke from the cigar smoke generator was premixed with the air flow, and the laser was used as the light source. By placing a glass tube of small diameter in front of the laser beam, the beam was converted into a vertical plane of light which by cutting through the smoke produced an image of the two-dimensional jet flow. This was studied and photographed.

The electrode geometries considered other than the one described before included the following: a) the collector grid rods shown in Figure 2 were reduced to two central rods only, b) a rod was added on each side of these two central rods, c) the two central rods were removed, while leaving all other rods in place, d) two corona wires were placed in symmetry with respect to the centerline in different distances from each other, while the collector grid was similar to case c. The resulting electrically-induced flow fields and the coupled effect on the two-dimensional jet were visualized and studied.

5. Jet Impingement and Bouyancy

A brief study was made of the effect of an electric field on the case of the air jet impinging on a flat plate. In this case the collector grid system was replaced by a nonporous aluminum plate electrode, and the Schlieren technique was used to visualize and photograph the flow. It should be noted that in this case a positive pressure gradient was imposed on the jet, because of the presence of the solid plate.
The effect of the bouyant force on the stability of the jet was also briefly investigated. The study was done to insure that the bouyancy force which arose in the case of the heated jet, did not have a significant effect on the stability of the jet. By mounting the nozzle upside down, the jet was forced to flow in the downward direction. The bouyant force caused by the heated jet then was in the opposite direction to the flow and acted as a retarding force. The resulting action was observed and studied with the use of the Schlieren technique.
CHAPTER III
THEORETICAL STUDY

Numerical Solution Method

1. Electrical Potential and Space Charge Calculations

To be able to solve the equations of motion in the presence of an electric field, the electric potential and space charge density distributions had to be evaluated in the region of interest. The geometry that was considered is shown in Figure 9. It consisted of a corona wire, a ground plate placed above the wire, and an electrically insulated plate located below the wire.

In order to effect a solution the following assumptions had to be made:

1) A steady two-dimensional field was considered, and the analysis was limited to constant discharge current without fluctuations.

2) There were only positive ions and neutral molecules present in the region. This was justified by the fact that the electrons were confined to a very narrow zone in the vicinity of the wire.

3) The diffusion of the ions was neglected, since the average ion velocity was ten times higher than the typical jet velocities used.
Figure 9. Geometry for Electric Potential, and Space Charge Density Calculations
4) The drift velocity of ions was expressed as $b \vec{E}$, where $b$ was the mobility and $\vec{E}$ the electric field strength.

Based on these assumptions a modified version of the current continuity and Poisson's equations were derived from the basic equations as follows:

Current continuity equation for steady state condition is expressed as:

$$\nabla \cdot J = 0$$

(1)

Now Ohm's law states that:

$$J = \rho_c \cdot (bE + q)$$

(2)

where $q$ is the fluid velocity, and its contribution to ionic current will be neglected based on assumption 3. So one gets that:

$$J = n e b E$$

(3)

where $n$ is the charge number density and $e$ the electron charge.

The value of $J$ was substituted in equation (1), which resulted in the modified version of the current continuity equation:

$$E_x \frac{\partial n}{\partial x} + E_y \frac{\partial n}{\partial y} + n \left( \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} \right) = 0$$

(4)

where constant positive ion mobility was assumed.

The next step involved the derivation of the Poisson's equation.

The Maxwell equation is given as:

$$\nabla \cdot D = \rho_c = ne$$

(5)

and

$$D = \varepsilon E$$

(6)

$\varepsilon$ being the permeability of air. Also we have that:
\[ \vec{E} = \nabla \phi \quad (7) \]

Combining equations (5), (6) and (7) we get:

\[ \nabla \cdot \nabla \phi = \frac{ne}{\varepsilon} \quad (8) \]

or

\[ \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \frac{ne}{\varepsilon} \quad (9) \]

Equations (4) and (9) were to be solved numerically. But prior to this, both were non-dimensionalized to simplify the procedure. Each variable was divided by its characteristic value, and the following equations were obtained:

\[ \frac{\partial \tilde{n}}{\partial x} \cdot \frac{\partial \phi}{\partial x} + \frac{\partial \tilde{n}}{\partial x} \cdot \frac{\partial \phi}{\partial y} + \tilde{n} \left( \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right) = 0 \quad (10) \]

\[ \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \gamma \cdot \tilde{n} \quad (11) \]

where

\[ \tilde{n} = \frac{n}{n_0}, \quad \bar{\phi} = \frac{\phi}{\phi_0}, \quad \bar{x} = \frac{x}{D}, \quad \bar{y} = \frac{y}{D}, \quad \text{and} \quad \gamma = \frac{D^3 J w_0}{\varepsilon b \phi_0^2} \]

The boundary conditions associated with the problem were:

\[ \bar{\phi} = 0 \quad \text{at the ground plate,} \quad \bar{x} = 0 \quad (12) \]

\[ \frac{\partial \bar{\phi}}{\partial \bar{y}} = 0 \quad \text{at the right boundary,} \quad \bar{y} = 0 \quad (13) \]

\[ \frac{\partial \bar{\phi}}{\partial \bar{y}} = 0 \quad \text{at the symmetry line,} \quad \bar{y} = 3.0 \quad (14) \]

\[ \bar{\phi} = 1.0 \quad \text{at the wire,} \quad \bar{x} = 1 \text{ and } \bar{y} = 3.0 \quad (15) \]
\[ \frac{\partial \Phi}{\partial x} = 0 \] at the insulated plate, \( x = 1.5 \) \hfill (16)

Known values of charge number density, \( n \), at the ground plate constituted the remaining boundary condition. These were evaluated from the experimentally obtained current density distribution.

Boundary condition (13) was based on the fact that the right hand boundary was far from the corona wire, and thus zero field strength in the \( y \) direction was expected.

In order to solve equations (10) and (11) with the given set of boundary conditions, the finite difference method was employed. The required network was obtained by dividing the region into a series of equal size squares, as depicted in Figure 10.

The finite difference approximation of equation (10) becomes:

\[
\frac{n_{i,j} - n_{i,j-1}}{\Delta x} + \frac{\phi_{i,j} - \phi_{i,j-1}}{\Delta x} + \frac{n_{i,j+1} - n_{i-1,j}}{\Delta y} - \frac{\phi_{i+1,j} - \phi_{i-1,j}}{\Delta y} + n_{i,j} \\
\quad \frac{\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}}{\Delta x^2} + \frac{\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}}{\Delta y^2} = 0 \hfill (17)
\]

Solving for \( n_{i,j} \) we get that:

\[
n_{i,j} = \frac{\bar{E}_{x,i,j} + \bar{E}_{y,i,j} + \Delta \cdot \text{Diff}_{i,j}}{\bar{E}_{x,i,j} + \bar{E}_{y,i,j} + \Delta} \hfill (18)
\]

where

\[
\bar{E}_{x,i,j} = \frac{\phi_{i,j} - \phi_{i,j-1}}{\Delta x} \hfill (19)
\]

\[
\Delta \frac{\partial}{\partial x} = \Delta y \hfill (20)
\]
Figure 10. Computational Network for Electrical Potential Calculation
The finite difference equivalent of equation (11) is:

\[
E_{y_{i,j}} = \frac{\phi_{i,j} - \phi_{i-1,j}}{\Delta y} \tag{21}
\]

The finite difference equivalent of equation (11) is:

\[
\text{Diff}_{i,j} = \frac{\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1} + \phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}}{\Delta x^2} + \frac{\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}}{\Delta y^2} \tag{22}
\]

The finite difference equivalent of equation (11) is:

\[
\frac{\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1} + \phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}}{\Delta x^2} + \frac{\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}}{\Delta y^2} = \gamma \cdot n_{i,j} \tag{23}
\]

Solving for \( \phi_{i,j} \), we get that:

\[
\phi_{i,j} = \frac{1}{4} \cdot (\phi_{i,j+1} + \phi_{i,j-1} + \phi_{i+1,j} + \phi_{i-1,j} + \Delta x^2 \cdot \gamma \cdot n_{i,j}) \tag{24}
\]

Equation (24) was solved by the successive-over-relaxation method (SOR). In order to do so, a modified version of (24) was used, as given below:

\[
\phi_{i,j}^{(m+1)} = \frac{1}{4} \cdot (\phi_{i,j+1}^{(m)} + \phi_{i,j-1}^{(m)} + \phi_{i+1,j}^{(m)} + \phi_{i-1,j}^{(m)} + \Delta x^2 \cdot \gamma \cdot n_{i,j}^{(m)}) \Omega \\
+ (1.0 - \Omega) \cdot \phi_{i,j}^{(m)} \tag{25}
\]

where \( \Omega \) was the relaxation factor, satisfying the condition \( 1 < \Omega < 2 \). Superscript \( m \) denoted the number of iterations performed.

Equations (18) and (25) were solved using the following scheme:

1. Initial guessed values for potential were obtained by solving Laplace's equation, \( \nabla^2 \phi = 0 \), corresponding to the charge free case.

2. Using the estimated values for potential, the field strength at the ground plate was determined. With the now known field strength values and the experimentally determined values of current density at the ground plate, the charge number density at the plate was evaluated from Ohm's law, Eq. (3). Once the charge number density
was obtained Eq. (18) was solved by the backward difference method, from the ground plate down, until values of $n$ were determined in the entire region.

3. Using values of $n$ and the estimated values for $\phi$'s, Eq. (25) was solved by the SOR method until convergence was gained.

4. Steps 2 and 3 were repeated until overall convergence based on the criterion that $\frac{\phi(m+1) - \phi(m)}{\phi(m)} \leq 0.001$ was achieved. For $\phi_0$'s above 8kV the outlined procedure could not be followed exactly, since numerical instability was encountered due to the nonlinear convective terms in Eq. (18), and the specific geometry considered. After several trials the way stability and convergence was gained in case of high $\phi_0$, was by using potential values calculated for a lower $\phi_0$, instead of the solution to the Laplace's equation, as the initial guess.

The flow chart depicting the solution procedure is shown in Figure 11.

2. 2-D Jet Flow Calculations

To predict fluid flow behavior and manifestation of instability in a 2-D jet the geometry in Figure 12 was assumed. The 2-D jet flanked by two plates issued into an "infinite" medium.

The Navier-Stokes equations in terms of primary variables in a two-dimension geometry, along with the continuity equation could be written as (1):

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$ (26)
Figure 11. Flow Chart for Solving Electrical Potential and Space Charge Density Distributions
Figure 12. Geometry for 2-D Jet Flow Calculations
Simplified versions of equations (26) and (27) were determined in terms of stream function $\psi$ and vorticity $w$. To accomplish this the pressure terms in equations (26) and (27) were eliminated by differentiating eq. (26) with respect to $y$, (27) with respect to $x$, and subtracting one from another. The following equation resulted:

$$\frac{\partial w}{\partial t} + \mathbf{u} \cdot \nabla w + v \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)$$

(29)

where

$$w = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

(30)

and

$$u = -\frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \psi}{\partial x}$$

(31)

The resulting vorticity equation is nonlinear due to the presence of convective terms. The equation is parabolic in time, and the time dependent term is required since unsteady solutions were expected above certain values of the Reynolds number. Even for subcritical Reynolds numbers the time dependent term was retained, since it enhanced numerical stability (55).

The defining equation of vorticity, eq. (30), was also written in terms of $\psi$ and $w$.

$$\nabla^2 \psi = w$$

(32)
which is a Poisson's equation and falls into the category of an elliptic type equation.

Before proceeding with the solution of equations (29) and (32), they were put into non-dimensional form using the following definitions:

\[ \bar{x} = \frac{x}{2h}, \quad \bar{y} = \frac{y}{2h}, \quad \bar{t} = t \cdot \frac{U_0}{2h} \]

\[ \bar{w} = \frac{w}{U_0} \cdot 2h, \quad \bar{\psi} = \frac{\psi}{2U_0 h}, \quad \text{Re} = \frac{2U_0 h}{\nu} \quad (33) \]

where \( h \) was the half width of the jet and \( U_0 \) the centerline velocity at the jet exit.

In non-dimensional form the vorticity equation became:

\[ \frac{\partial \bar{w}}{\partial \bar{t}} + \frac{\partial \bar{\psi}}{\partial \bar{y}} \cdot \frac{\partial \bar{w}}{\partial \bar{x}} - \frac{\partial \bar{\psi}}{\partial \bar{x}} \cdot \frac{\partial \bar{w}}{\partial \bar{y}} = -\frac{1}{\text{Re}} \cdot \left( \frac{\partial^2 \bar{w}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{w}}{\partial \bar{y}^2} \right) \quad (34) \]

and the Poisson's equation appeared as:

\[ \nabla^2 \bar{\psi} = \bar{w} \quad (35) \]

The boundary conditions were:

\[ \bar{\psi} = \text{const}, \quad \frac{\partial \bar{\psi}}{\partial \bar{x}} = 0, \quad \bar{w} = 0, \quad \text{at } \bar{y} = 0 \quad (36) \]

because of symmetry at centerline.

\[ \frac{\partial \bar{\psi}}{\partial \bar{x}} = 0 \quad \text{and} \quad \frac{\partial \bar{\psi}}{\partial \bar{y}} = 0 \quad \text{along the wall} \quad (37) \]

due to the no-slip condition.

\[ \frac{\partial^2 \bar{\psi}}{\partial \bar{x}^2} \quad \text{and} \quad \frac{\partial^2 \bar{\psi}}{\partial \bar{x}^2} = 0 \quad \text{at } \bar{x} = 10 \quad (38) \]

since it was far from the jet exit.
because it was far from the jet exit.

Boundary conditions (38) and (39) were required because the second derivative could not be evaluated at the boundaries using space-centered differences, since no mesh point existed beyond the boundaries. Setting the second derivatives equal to zero was justifiable based on the fact that these were small everywhere compared to other terms, specially away from the jet exit. This approximation should not be interpreted as a rigorous boundary condition imposed on the problem, which may have excluded certain solutions, but rather as an approximation to the values of the appropriate terms in the equations of motion, Grant (17).

Boundary condition (37) had to be manipulated to be useable in the solution of the problem. The stream function was expanded in its Taylor series around \( x = 0 \), which corresponded to points along the wall:

\[
\psi \bigg|_{x=0} = \psi \bigg|_{x=0} - \frac{\partial \psi}{\partial x} \bigg|_{x=0} \cdot (\Delta x) + \frac{1}{2} \cdot \frac{\partial^2 \psi}{\partial x^2} \bigg|_{x=0} \cdot (\Delta x)^2 + O(\Delta x)^3
\]

\( (40) \)

now

\[
\frac{\partial \psi}{\partial x} \bigg|_{x=0} = v \bigg|_{x=0} = 0 \quad \text{based on B.C. 37.}
\]

Since along the wall \( u = 0 \) so \( \frac{\partial u}{\partial y} \) was also zero since \( y \) was measured along the wall. This indicated that along the wall:
Combining equations (40) and (41) yielded the following equation for vorticity along the wall:

\[ \bar{w} = \frac{\partial v}{\partial x} = \frac{\partial^2 \bar{\psi}}{\partial x^2} \]  

(41)

Combining equations (40) and (41) yielded the following equation for vorticity along the wall:

\[ \bar{w}|_{x=0} = -\frac{2}{(\Delta y)^2} \cdot (\bar{\psi}|_{x=0} - \bar{\psi}|_{x=\Delta x}) + O(\Delta x) \]  

(42)

Several other formulations of the above equations such as Lagrange's interpolation formula, Roache (55), were used in the present study. No noticeable advantages over the formulation given in (42) was observed in this very slow-flow region close to the wall.

To complete the set of given boundary conditions the velocity profile at the jet exit had to be defined, from which values of stream-function and vorticity were determined.

In order to solve the problem numerically, the finite difference method was used. In writing the finite difference equivalent of equation (34) the nonlinear convective terms were of prime concern as far as the numerical stability of the calculation were concerned. In the initial stage of the study central differencing was used in the evaluation of the convective terms. Numerical instability was observed irrespective of time step chosen. Subsequently the upwind differential method (Torrence 56) was tried for the convective terms. The method has been used extensively because of its superior stability and convergence characteristics. In the present study though, the method did not prove useful, because of its poor accuracy at high Reynolds numbers, which was required for physical instability to occur. This occurred because the magnitude of the first order
truncation error was equal to or greater than the diffusion term. Because of this, all solutions tended asymptotically towards a steady state, up to a Reynolds number of 6000, for all initial velocity profiles assumed.

In order to get desired results, other methods were tested with no success, until it was found that the method of Arakawa (57), which conserved vorticity, square of vorticity, and kinetic energy appeared to be satisfactory. As far as the other terms were concerned, forward differencing in time was employed for the local rate of change of vorticity, while a central difference scheme was used to express the diffusion terms.

The local time step was chosen as large as possible under the condition of satisfying the stability criterion as given by Mori (49):

$$\Delta t < \frac{0.9}{2/Re \cdot \left( \frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} \right) + \frac{\bar{u}}{\Delta x} + \frac{\bar{v}}{\Delta y}}$$

(43)

where

$$\Delta t = \Delta t \cdot \frac{U_0}{2h} \quad , \quad \bar{x} = \frac{x}{2h} \quad , \quad \bar{y} = \frac{y}{2h} \quad , \quad \bar{u} = \frac{u}{U_0} \quad , \quad \text{and} \quad \bar{v} = \frac{v}{U_0}$$

(44)

$\Delta t$ being the half width of the jet, and $U_0$ the centerline velocity at the jet exit. The above criterion assumes that the convective terms are evaluated by the upwind differential method, rather than Arakawa's method which has a higher order of accuracy. As a result, a more stringent condition is placed on the chosen time step (58). This was decided on purpose, in order to avoid time step related numerical instabilities.
In order to obtain an approximate solution via the finite difference method, the region of interest was divided into a network of squares as shown in Figure 13. After doing so, the equivalent of equations (34) and (35), according to the arguments presented above, were written. Equation (34) becomes:

$$\frac{w_{i,j}^{(m+1)} - w_{i,j}^{(m)}}{\Delta t} = \text{Conv}_{i,j}^{(m)} + \frac{1}{Re} \cdot \text{Diff}_{i,j}^{(m)}$$

(45)

where \(\text{Conv}_{i,j}^{(m)}\) represented the convective terms after \(m\) iterations and was equal to:

$$\text{Conv}_{i,j}^{(m)} = -\frac{1}{12} \cdot (\Delta) \cdot \left( \left( \psi_{i-1,j}^{(m)} + \psi_{i,j+1}^{(m)} - \psi_{i+1,j}^{(m)} - \psi_{i,j-1}^{(m)} \right) \cdot \left( w_{i,j}^{(m)} + w_{i,j}^{(m+1)} \right) \right)$$

$$- \left( \psi_{i,j}^{(m)} \cdot \left( w_{i,j}^{(m)} + w_{i,j}^{(m+1)} \right) \right)$$

$$+ \left( \psi_{i+1,j}^{(m)} - \psi_{i,j}^{(m)} \right) \cdot \left( w_{i,j}^{(m)} + w_{i,j}^{(m+1)} \right)$$

$$+ \left( \psi_{i,j}^{(m)} \cdot \left( w_{i,j}^{(m)} + w_{i,j}^{(m+1)} \right) \right)$$

$$- \left( \psi_{i,j+1}^{(m)} \cdot \left( w_{i,j}^{(m)} + w_{i,j}^{(m+1)} \right) \right)$$

$$+ \left( \psi_{i,j}^{(m)} \cdot \left( w_{i,j}^{(m)} + w_{i,j}^{(m+1)} \right) \right)$$

$$- \left( \psi_{i,j}^{(m)} \cdot \left( w_{i,j}^{(m)} + w_{i,j}^{(m+1)} \right) \right)$$

$$+ \left( \psi_{i,j}^{(m)} \cdot \left( w_{i,j}^{(m)} + w_{i,j}^{(m+1)} \right) \right)$$

$$- \left( \psi_{i,j}^{(m)} \cdot \left( w_{i,j}^{(m)} + w_{i,j}^{(m+1)} \right) \right)$$

$$- \left( \psi_{i,j}^{(m)} \cdot \left( w_{i,j}^{(m)} + w_{i,j}^{(m+1)} \right) \right)$$

(46)

\(\text{Diff}_{i,j}^{(m)}\) was the diffusion term, after \(m\) iterations, and is given as:

$$\text{Diff}_{i,j}^{(m)} = \frac{1}{2} \cdot \left( \left( w_{i+1,j}^{(m)} + w_{i,j}^{(m)} \right) \cdot \left( w_{i+1,j}^{(m)} + w_{i+1,j}^{(m)} \right) \right)$$

$$+ \left( \left( w_{i,j+1}^{(m)} + w_{i,j+1}^{(m)} \right) \cdot \left( w_{i,j+1}^{(m)} + w_{i,j+1}^{(m)} \right) \right)$$

$$+ \left( \left( w_{i+1,j}^{(m)} + w_{i+1,j}^{(m)} \right) \cdot \left( w_{i+1,j}^{(m)} + w_{i+1,j}^{(m)} \right) \right)$$

$$- 4 \cdot \left( w_{i,j}^{(m)} \right)$$

(47)

\(w_{i,j}^{(m)}\) was solved for from equation (45), and is equal to:

$$w_{i,j}^{(m+1)} = w_{i,j}^{(m)} + \Delta t \cdot \left( \text{Conv}_{i,j}^{(m)} + \frac{1}{Re} \cdot \text{Diff}_{i,j}^{(m)} \right)$$

(48)

Now equation (35) in finite difference form becomes:
Figure 13. Computational Network for 2-D Jet Flow Calculations
The speed of convergence as well as the associated cost was strongly dependent on the computational method employed for solving the Poisson's equation, (49). The successive-over-relaxation method proved to be quite inefficient and thus costly in this particular case, because of the enormous amount of iterations involved. In view of this the Fast Direct Solution (FDS) which was a non-iterative computational method was employed. A brief explanation of the procedure is given below, while the details could be found in Nakamura (59), Equation (49) for all i's and for $1 < j < j_{\text{max}}$ could be written in the following form:

\[
\begin{bmatrix}
\psi_{1,j-1} \\
\psi_{2,j-1} \\
\cdot \\
\cdot \\
\cdot \\
\psi_{\text{imax},j-1}
\end{bmatrix}
- 
\begin{bmatrix}
(\psi_{1,j}^{(m+1)})^{-1} & \cdots \\
\cdot & \cdot \\
0 & \cdots
\end{bmatrix}
+ 
\begin{bmatrix}
\psi_{1,j} \\
\psi_{2,j} \\
\cdot \\
\cdot \\
\cdot \\
\psi_{\text{imax},j}
\end{bmatrix}
= 
\begin{bmatrix}
\psi_{1,j+1} \\
\psi_{2,j+1} \\
\cdot \\
\cdot \\
\cdot \\
\psi_{\text{imax},j+1}
\end{bmatrix}
- 
\begin{bmatrix}
\psi_{1,j} \\
\psi_{2,j} \\
\cdot \\
\cdot \\
\cdot \\
\psi_{\text{imax},j}
\end{bmatrix}
= 
\begin{bmatrix}
\psi_{1,j+1} \\
\psi_{2,j+1} \\
\cdot \\
\cdot \\
\cdot \\
\psi_{\text{imax},j+1}
\end{bmatrix}
\]
Where $\bar{\psi}_o$ represented the stream function value at the center line and $\bar{\psi}_{rb}$ was the stream function at the right boundary. For $j=1$ values of stream function at the jet exit had to be added, similar to the above, to the right hand side of the equation, since the program was written based on a homogeneous boundary condition. The equivalent form of equation (50) was:

$$- \bar{\psi}_{j-1} + A\bar{\psi}_j - \bar{\psi}_{j+1} = \bar{s}_j$$

where $A$ was the tridiagonal matrix given as:

$$A = \begin{bmatrix} 4,-1,0,...,0 \\ -1,4,-1,...,0 \\ 0,-1,4,-1,0 \\ \vdots \\ 0,...,-1,4,-1 \\ 0,...,0,-1,4 \end{bmatrix}$$

and

$$\bar{\psi}_j = \text{Col}[\bar{\psi}_{1,j}, \bar{\psi}_{2,j}, ..., \bar{\psi}_{\text{max},j}]$$

Equation (51) was written for all $j$'s in matrix form as:

$$M\vec{\psi} = \vec{0}$$

where $M$ was a block-tridiagonal matrix given as:

$$M = \begin{bmatrix} A,-I,0 \ldots 0 \\ -I,A,-I,0,0 \\ \vdots \\ 0,...,-I,A,-I \\ 0,...,-I,A \end{bmatrix}$$
and $Q$ was a matrix having $\psi_{i,j-1} + \psi_{i,j+1} + s_{i,j}$ as its elements.

The following procedure had to be followed to reach a solution:

1. All values of stream function and vorticity were set to zero, except at the jet exit, where a velocity profile was assumed.

2. Equation (48) was used to evaluate new values for vorticity at all grid points, by advancing in time.

3. Using these new values for vorticity distribution equation (49) was solved by the FDS method.

4. New values for vorticity and stream function were evaluated at the boundaries using equations (36)-(39).

5. Using equation (31) new values for the velocity components $\bar{u}$ and $\bar{v}$ were evaluated. These were used in equation (43) to calculate the local time step at each grid point.

6. Steps 2 through 5 were repeated until the desired solution, being physically stable or not, was reached.

The non-dimensional grid size used in this part of the study was equal to $\frac{1}{2}$ which corresponded to a physical size of $\frac{1}{24}$ cm. Number of grid points in the $x$ direction was 128, while the number of grid points in the $y$ direction was only 36. The appropriate number of grid points were evaluated based on the following considerations:

(a) the number of grids had to be so chosen as to justify the boundary conditions assumed at the upper and right boundaries, (b) the minimum number of grid points which satisfied the first criterion had to be chosen so as to reduce the cost of computation. Appendix D includes a detailed discussion concerning the mentioned considerations.
3. Electric Wind Flow Calculations

The electric wind was the flow induced by the action of the electrical body forces. In view of this when considering the Navier-Stokes equations, terms representing the body forces had to be included too. The value of body forces were known values, being calculated in part (a), with the assumption that the electric problem was not affected by the fluid problem. This was justified on the basis that the ion velocities were 3 orders of magnitude greater than the highest velocities present in the flow.

In arriving at the appropriate equations of motion the following assumptions had to be made:

1. Incompressible flow
2. Newtonian flow
3. Absence of magnetic fields
4. Constant temperature field
5. All current was due to ion motion

Based on these assumptions the Navier-Stokes equations with the inclusion of electrical body forces in terms of primary variables $u$ and $v$ could be written as:

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial x} + v \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + neE_x \]  

(57)

\[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial y} + v \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + neE_y \]  

(58)

where $neE_x$ and $neE_y$ represented the electric body forces per unit volume.
By following a similar procedure as given in part (b) the two above equations were combined into a single equation in terms of the stream and vorticity functions, as given below:

$$\frac{\partial \psi}{\partial t} + \frac{\partial \psi}{\partial y} - \frac{\partial \psi}{\partial x} - \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} = 1 - \frac{\partial n E_x}{\partial y} - \frac{\partial n E_y}{\partial x} + \frac{n^2 w}{a^2 x^2} + \frac{n^2}{a^2 y^2} \quad (59)$$

now $E_x = \frac{\partial \psi}{\partial x}$ and $E_y = \frac{\partial \psi}{\partial y}$ where $\phi$ was the electrical potential. Substituting for $E_x$ and $E_y$ from above in equation (59), the following equation resulted:

$$\frac{\partial w}{\partial t} + \frac{\partial \psi}{\partial y} - \frac{\partial \psi}{\partial x} - \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} = \frac{\partial n}{\partial y} - \frac{\partial n}{\partial x} - \frac{\partial \psi}{\partial x} - \frac{\partial \psi}{\partial y} + \frac{\partial \psi}{\partial x} - \frac{\partial \psi}{\partial y} + \frac{n^2 w}{a^2 x^2} + \frac{n^2}{a^2 y^2} \quad (60)$$

Before proceeding with the solution the obtained equation was put into non-dimensional form. In order to do so the following non-dimensional quantities were adopted:

$$\bar{x} = x \quad D \quad \bar{y} = y \quad D \quad \bar{n} = n \quad n_0 \quad \bar{\psi} = \frac{\psi}{\psi_0}$$

$$\bar{w} = \frac{D}{U_E} \quad w \quad \bar{\psi} = \frac{\psi}{U_E D} \quad \bar{t} = t \quad \frac{U_E}{D} \quad (61)$$

where $D$ was the distance between the corona wire and ground plate, $n_0$ the characteristic charge number density, $\phi_0$ the potential applied to the wire, and $U_E$ the characteristic CHD velocity. In doing so the following equation resulted:

$$\bar{w} + \bar{\psi} - \bar{\psi} - \bar{\psi} - \bar{\psi} - \bar{\psi} = \bar{\psi} - \bar{\psi} - \bar{\psi} - \bar{\psi} - \bar{\psi} + \frac{1}{R_{ehd}} \left( \frac{\bar{\psi}^2}{a^2 x^2} + \frac{\bar{\psi}^2}{a^2 y^2} \right) \quad (62)$$

Similar to equation (35) the defining equation of vorticity was:

$$\frac{\partial \bar{\psi}}{\partial x^2} + \frac{\partial \bar{\psi}}{\partial y^2} = \bar{w} \quad (63)$$
The parameter $R_{ehd}$ in equation (62) was expressed as follows:

$$R_{ehd} = \frac{D \cdot U_E}{\nu} = \frac{D \cdot \sqrt{\frac{\nu}{\rho}}}{\nu} = \frac{D \cdot \sqrt{\rho \cdot J \cdot \rho}}{\nu}$$

(64)

which in effect was the Reynolds number based on the EHD characteristic velocity $U_E$.

The equations (62) and (63) were solved for the geometry shown in Figure 14. The specific geometry considered resulted in the following boundary conditions:

$$\psi = 0, \quad \frac{\partial \psi}{\partial x} = 0, \quad w = 0 \quad \text{at } y = 0$$

(65)

because of symmetry at centerline.

$$\psi = 0, \quad \frac{\partial \psi}{\partial x}, \quad \text{and } \frac{\partial \psi}{\partial y} = 0 \quad \text{at } x = 0$$

(66)

due to the no-slip condition.

$$\frac{\partial^2 \psi}{\partial x^2}, \quad \text{and } \frac{\partial^2 w}{\partial x^2} = 0 \quad \text{at } x = 6$$

(67)

since it was far from the jet exit.

$$\frac{\partial^2 \psi}{\partial y^2}, \quad \text{and } \frac{\partial^2 w}{\partial y^2} = 0 \quad \text{at } y = 3$$

(68)

because it was considered far from the jet exit.

The boundary conditions were similar to those presented in part (b), except for condition (66). In this case the wall was extended to the centerline, since no 2-D jet was present, and thus no initial velocity profiles had to be assumed.
Figure 14. Geometry for Electric Wind Flow Calculations
Based on the grid network shown in Figure 15, the finite difference approximates of the governing equations (62) and (63) was written. Equation (62) became:

\[
\bar{w}_{i,j}^{(m+1)} = \bar{w}_{i,j}^{(m)} + \Delta t \cdot \left( \text{Conv}_{i,j}^{(m)} + \frac{1}{Re_{hd}} \cdot \text{Diff}_{i,j}^{(m)} + \text{Elec}_{i,j} \right) \tag{69}
\]

where \( \text{Conv}_{i,j}^{(m)} \) and \( \text{Diff}_{i,j}^{(m)} \) were defined in equations (69), (46) and (47). \( \text{Elec}_{i,j} \) was given as below:

\[
\text{Elec}_{i,j} = \frac{1}{\Delta^2} \cdot \left[ (\bar{n}_{i,j+1} - \bar{n}_{i,j}) (\bar{\phi}_{i+1,j} - \bar{\phi}_{i,j}) - (\bar{n}_{i+1,j} - \bar{n}_{i,j}) \cdot (\bar{\phi}_{i,j+1} - \bar{\phi}_{i,j}) \right] \tag{70}
\]

where in deriving it a forward differencing scheme was employed.

Equation (63) in finite difference form was similar to equation (49), and for its solution the FDS method was used. The resulting matrices for all \( i \)'s and a fixed \( j < j_{\text{max}} \) were of the form:

\[
\begin{bmatrix}
\bar{\psi}_{1,j-1} \\
\bar{\psi}_{2,j-1} \\
\vdots \\
\bar{\psi}_{i_{\text{max}},j-1}
\end{bmatrix} + \begin{bmatrix}
4, -1, \ldots, 0 \\
-1, 4, -1, \ldots, 0 \\
\vdots \\
\hat{0}, -1, 4, -1
\end{bmatrix} \begin{bmatrix}
\bar{\psi}_{1,j} \\
\bar{\psi}_{2,j} \\
\vdots \\
\bar{\psi}_{i_{\text{max}},j}
\end{bmatrix} - \begin{bmatrix}
\bar{\psi}_{1,j+1} \\
\bar{\psi}_{2,j+1} \\
\vdots \\
\bar{\psi}_{i_{\text{max}},j+1}
\end{bmatrix} = \begin{bmatrix}
s_{1,j} \\
s_{2,j} \\
\vdots \\
s_{i_{\text{max}},j+\bar{\psi}_{rb}}
\end{bmatrix}
\tag{71}
\]

The remaining formulation of the problem was similar to the one described in part (b). The solution procedure was as follows:

1. All values of stream function and vorticity were set to zero at all grid points.
Figure 15. Computational Network for Electric Wind Flow Calculations
2. With the known values of electrical potential and space charge in the region, the term associated with the electrical body forces, \( \text{Elec}_{i,j} \), was evaluated at each grid point.

3. Equation (69) was used to evaluate new values for vorticity at all grid points, by advancing in time.

4. Using these new values for vorticity, equation (49) was solved by the FDS method.

5. New values for vorticity and stream function were evaluated at the boundaries using equations (65)-(68).

6. Using equation (31) new values for the velocity components \( \bar{u} \) and \( \bar{v} \) were found. These were used in equation (43) to calculate the local time step at each grid point.

7. Steps 3 through 6 were repeated until convergence was reached.

The non-dimensional grid size used in this part of the study was equal to \( 1/12 \) which corresponded to a physical size of \( 1/6 \) of a cm. Number of grid points in the \( x \) direction was equal to 128, which in the physical sense corresponded to a region extending about 18 cm above the ground grid. The number of points in the lateral direction was 36 points which translated into a distance equal to 6 cm. Number of grids in the \( x \) direction had the most significant effect on the solution of the problem. Smaller values than the one stated above resulted in the propagation of instabilities from the upper boundary in the upstream direction, before a steady state solution was realized. It was of course realized that any increase in the number of grids, or reductions in the grid size meant increased costs of computation. The
numbers mentioned above were found to be satisfactory resulting in stable and correct solutions as well as relatively low computational expenses.

4. Modified Jet Calculations

The vorticity equation associated with the problem was similar to equation (60). In non-dimensionalizing the equation, the characteristic velocity used was $U_0$ the jet exit velocity, rather than $U_E$ the electric wind characteristic velocity. In view of this and the use of other non-dimensional parameters shown below:

$$\bar{x} = \frac{x}{2h}, \quad \bar{y} = \frac{y}{2h}, \quad \bar{n} = \frac{n}{n_0}, \quad \bar{\phi} = \frac{\phi}{\phi_0},$$

$$\bar{w} = \frac{2h}{U_0} \cdot w, \quad \bar{\psi} = \frac{\psi}{2U_0 h}, \quad \bar{t} = t \cdot \frac{U_0}{2h}$$

the vorticity equation became:

$$\frac{\partial w}{\partial t} + \frac{\partial \psi}{\partial y} \cdot \frac{\partial w}{\partial x} - \frac{\partial \psi}{\partial x} \cdot \frac{\partial w}{\partial y} = (EN) \cdot \left( \frac{\partial n}{\partial y} \frac{\partial \phi}{\partial x} - \frac{\partial n}{\partial x} \frac{\partial \phi}{\partial y} \right) + \frac{1}{Re} \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)$$

in which $EN$ and $Re$ were expressed as follows:

$$EN = \frac{U_E}{U_0} = \frac{R_{ehd}}{Re} = \frac{Djw_0}{\rho b U_0^2}, \quad \text{and} \quad Re = \frac{2U_0 h}{v}$$

$EN$ represented the ratio of electric body forces over the inertia forces, while $Re$ was the Reynolds number based on 2-D jet exit velocity and width.

The set of boundary conditions resulting from the geometry considered in Figure 16 was exactly the same as the ones given in part b.
Figure 16. Geometry for Modified Jet Flow Calculations
The finite difference form of equation (73) was given as:

\[
\frac{w_i, j (m+1)}{w_i, j (m)} = w_i, j (m) + \Delta t \cdot \left( \text{Conv}_{i, j} (m) + \frac{1}{Re} \cdot \text{Diff}_{i, j} (m) + (EN) \cdot \text{Elec}, j \right) \tag{75}
\]

the individual terms in the above being the same as those defined in parts (b) and (c). As can be seen two parameters \(Re\) and \(EN\) were relevant in the solution of the problem, unlike previous cases which were dependent on a single parameter, namely the Reynolds number.

The defining equation of vorticity was the same as before, and the resulting matrices for use in the FDS method were similar to those described in part (b).

The solution procedure is given below:

1. All values of stream function and vorticity were set equal to zero at all grid points, except at the jet exit where a velocity profile was assumed.

2. With the known values of electrical potential and space charge in the region between the jet and the ground grid the term \(\text{Elec}, j\) present in equation 75 was calculated. The remaining steps that had to be taken to effect a solution were similar to the ones outlined in part (c).

The non-dimensional grid size used in this part of the study was equal to 1/12 or 1/24 of cm. Number of grid points in the \(x\) direction was 256 points. Greater number of grid points, as compared to part (c) had to be used since the grid size was smaller in this case. Number of grid points in the lateral direction, \(y\), was 60.
CHAPTER IV
PRESENTATION OF RESULTS

A. Experimental Results

1. Current Distribution

The electric wind current density distribution was measured under various conditions. For the case where no insulated plates were present, it was found that the normalized current distributions were similar regardless of the electrical potential or the distance between the corona wire and the ground grid, Figure 17. The normalized current density distribution was expressable by the following formula, which was consistent with Velkoff (35).

\[
\frac{J}{J_w} = (1 + 0.4n^2)^{-2}
\]

\(\eta \) was \(y/\delta\), where \(\delta\) was the half width of the distribution.

It was also noted that the distribution, and the magnitude of the total current was unaffected by the 2-D Jet flow, for velocities below 10 m/sec.

The electric wind distribution in the presence of the insulated plate was measured. It was found that for distances more than 3.0 cm the effect of the plate on the distribution was insignificant. Below 3.0 cm the effect of the plate began to show. A narrowing of the
Figure 17. Normalized Current Density Distribution
Figure 18. Current Density Distribution in the Presence and Absence of an Insulated Plate
current distribution occurred, accompanied with a reduction in the total current. These effects are best illustrated in Figure 18.

2. Current-Voltage Relationship

The observed corona current-voltage relation, in the absence of the insulated plate, is shown in Figure 19. The established empirical formula $I = KV(V - V_0)$ agreed well with the data. In the formula $V_0$ represented the onset potential, above which current started to flow. The constant $K$ was dependent on the distance between the wire and ground grid, and the electrode geometry. In the case when the insulated plate was placed 1.0 cm below the corona wire the same relationship as above was observed. It should be noted though that the total current was lower due to the presence of the insulated plate, as apparent from the data included in Figure 19. When the distance between the insulated plate and the wire was more than 3.0 cm, the effect of the plate became insignificant.

Doubling the number of ground rods did not affect the total current. In the case where segmented plate electrode was used instead of the ground rods also the same total current for given wire to ground grid distance and electrical potential was observed. The current density distribution calculated based on the center to center spacing of the ground rods, also did not deviate more than 2% from the values found from those measured from the segmented plate electrode.

3. Electrical Potential Distribution

The electrical potential distribution in the region between the insulated plate and the ground grid was evaluated using the Langmuir electrostatic probe. An example of the current-voltage relationship
Figure 19. Current-Voltage Relationship of the Corona Wind
of the probe is given in Figure 20. Above certain values of applied potential to the probe, depending on its location, the wire potential, and the wire to ground electrode distance, the probe current was reduced to zero. This indicated that no electron current existed. The relationship shown in Figure 20 was observed throughout the region as the probe was traversed, except when the probe was adjacent to the corona wire.

The lowest potential for which no current through the probe was observed, was considered the electrical potential at that point. Figure 21 shows the electrical equipotential lines for the case when no insulation plate was present. In Figure 22 the equipotential lines are shown for the case when the insulated plate was placed 1.0 cm below the corona wire.

Figure 23 shows the electrical potential distribution along three vertical lines at various distances from the centerline.

4. Average Velocity Distributions

4a. 2-D Jet

The average velocity distribution of the 2-D Jet flow was measured for three different Reynolds number, for varying downstream positions. Figures 24-27 are results of such measurements. The velocities have been normalized with respect to the velocity of the Jet at centerline, \( U_c \). The distance \( \bar{y} \) represented the nondimensional lateral distance from the centerline \( y/h \), where \( h \) was the half width of the nozzle. As can be seen from the figures, the change in the distribution was
Figure 20. Current-Voltage Relationship for the Langmuir Probe
Figure 21. Potential Distribution in the Absence of the Insulated Plate, at 7.5 kV
Figure 22. Potential Distribution in the Presence of the Insulated Plate, at $\phi_0 = 10 \, \text{kV}$
Figure 23. Potential Distribution at Different y Positions, for $\phi_0 = 10$ kV
gradual, up to $x/2h = 10$ except at the outer region of the Jet where the velocity increased significantly.

In order to further define the behavior of the 2-D Jet with respect to different Reynolds numbers, $500 < R_e < 1312.5$, and downstream locations, two other characteristic parameters were evaluated. Figure 28 shows the diffusion of the 2-D Jet versus downstream location. The parameter representing the diffusion of the Jet, being the normalized half width of the Jet. The half width was defined as the distance from the centerline at which the velocity became half of the centerline velocity. As can be seen the rate of diffusion of the jet was insignificant up to $x/2h = 8$, after which a sharp increase in its value occurred. From that point on a linear relationship was observed. The effect of the Reynolds number could be deduced from the plot. The second parameter that was evaluated was the square of the inverse of the normalized central velocity, $U_c$. The normalization factor was the central velocity of the Jet at the exit, $U_0$. The change of this parameter in the downstream direction, was indicative of the decay of the 2-D Jet. This particular parameter was used to exhibit the decay of the central velocity of the jet since based on the previously known theoretical (1) and experimental (2) results was expected to have a linear relationship with the downstream position. Figure 29 represents the decay of the Jet as exemplified by the reduction of the Jet central velocity, due to viscous diffusion and the dissipating action of turbulence.

In Figure 30 the normalized velocity distribution of the 2-D Jet for $\frac{x}{2h} > 10$ are shown. The difference between this plot and Figures
$U_0 = 1.31 \text{ m/sec (Re = 500)}$

$U_0 = 2.46 \text{ m/sec (Re = 937.5)}$

$U_0 = 3.44 \text{ m/sec (Re = 1312.5)}$

Figure 24. 2-D Jet Average Velocity Distribution at the Jet Exit
Figure 25. 2-D Jet Average Velocity Distribution at $\frac{x}{2h} = 4$
Figure 26. 2-D Jet Average Velocity Distribution at $\frac{x}{2h} = 10$

- $U_0 = 1.31$ m/sec (Re = 500)
- $U_0 = 2.46$ m/sec (Re = 937.5)
- $U_0 = 3.44$ m/sec (Re = 1312.5)
Figure 27. 2-D Jet Average Velocity Distribution at $\frac{x}{2h} = 15$
\( U_0 = 1.31 \text{ m/sec (Re = 500)} \)

\( U_0 = 2.46 \text{ m/sec (Re = 937.5)} \)

\( U_0 = 3.44 \text{ m/sec (Re = 1312.5)} \)

Figure 28. 2-D Jet Diffusion Rate
Figure 29. 2-D Jet Decay Rate

- $U_0 = 1.31 \text{ m/sec (Re = 500)}$
- $U_0 = 2.46 \text{ m/sec (Re = 937.5)}$
- $U_0 = 3.44 \text{ m/sec (Re = 1312.5)}$
Figure 30. 2-D Jet Average Velocity Distribution in the Turbulent Region
24-27 being that the lateral distance was normalized with respect to \( \delta \) the half breadth of the Jet. As can be seen the data points seemed to from a single describing curve. The analytical function which fitted best to the produced data was

\[
\frac{U}{U_c} = \exp(-0.67n^2(1 + 0.565n^4))
\]

The equation was similar to that used by Townsend (60) to describe the velocity distribution in a turbulent wake. Same type of equation was reported by Bradbury (61) to fit his data in the turbulent region of a 2-D Jet.

4b. The Electric Wind

The mean velocity distribution of the electric wind was measured at different down-stream location, for varying electrical potentials.

The normalized velocity distribution, measured above the ground grid system, agreed well with the result of Velkoff (35). The observed profile, shown in Figure 31 was similar to that of the electrical current distribution, and could be expressed by the following formula:

\[
\frac{U}{U_c} = (1 + 0.4n^2)^{-2}
\]

where \( U_c \) was the centerline velocity, and \( n = y/\delta \), \( \delta \) being the half breadth of the electric wind.

The velocity distribution was not affected by a change in the collector grid size from 1.6 mm to 0.5 mm.

The velocity profile was still expressable by the above formula, in the case when an insulated plate was placed 1.0 cm below the corona wire.
Figure 31. Electric Wind Average Velocity Distribution at $\frac{x}{2h} = 10$
The diffusion of the electric wind, measured above the ground rods and defined in a manner similar to that of the 2-D Jet, is shown in Figure 32. Insignificant diffusion of the flow in contrast with that of the 2-D Jet (Figure 28), in the region $10 < \frac{x}{2h} < 15$ was observed. It was also noticed that the half breadth of the electric wind ($\frac{\delta}{h} = 2.3$ at $\frac{x}{2h} = 10$), was greater than that of the 2-D Jet ($\frac{\delta}{h} = 1.3$ at $\frac{x}{2h} = 10$ & $Re = 500$). This was indicative of the fact that the electric wind possessed a wider velocity profile than that of the 2-D Jet in the laminar region.

The decay rate of the electric wind, defined similar to that of the 2-D Jet, is also shown in Figure 32. The velocities were normalized by the electric wind velocity measured at $\frac{x}{2h} = 10$ above the ground rods. It was observed that the decay rate was insignificant, lacking the sharp rise observed in case of the 2-D Jet (Figure 29).

4c. Modified Jet

The average velocity profiles of the modified jet were measured above the ground grid system, and the results are shown in Figures 33-35. The profiles exhibit the effects of three parameters which were varied in turns while the others were kept constant. These were: (a) Reynolds number, (b) down-stream position, (c) electrical potential.

The diffusion rate of the modified jet, defined similar to that of the plain jet, was measured above the ground grid system and the results are illustrated in Figures 36 & 37. The modified jet generally exhibited a smaller diffusion rate as compared to the 2-D Jet flowing at the same Reynolds number. The reduction in the slope of the
Figure 32. Electric Wind Diffusion and Decay Rates, $\phi_0 = 10$ kV
Figure 33. Modified Jet ($\phi_o = 10$ kV) Normalized Average Velocity Distributions at $x/2h = 10$
Figure 34. Modified Jet ($\phi_0 = 10$ kV) Normalized Average Velocity Distribution at $\frac{x}{2h} = 15$
Figure 35. Normalized Average Velocity Distribution of the Modified Jet, $\phi_0 = 10 \text{kV}$
Figure 36. Modified Jet Diffusion Rate at $\phi_0 = 10$ kV
- $U_0 = 1.31 \text{ m/sec (Re = 500, EN = 1.0)}$
- $U_0 = 2.46 \text{ m/sec (Re = 937.5, EN = 0.54)}$
- $U_0 = 3.44 \text{ m/sec (Re = 1312.5, EN = 0.4)}$

Figure 37. Modified Jet Diffusion Rate at $\phi_0 = 12.5 \text{ kV}$
diffusion curves was most significant, for the modified Det at $Re = 500$ and $\phi_0 = 10$ kV. In this case the slope was reduced by a factor of 2.5 to 1.

The decay rate of the modified jet defined as before is shown in Figure 38. The decay curves exhibited substantial decrease in the decay rate of the modified jet, as compared to that of the 2-D Jet. Moreover it was observed that with increased electrical potentials, the curves were shifted downwards with their slopes being reduced. 

5. **Velocity Fluctuations**

The fluctuating components of the velocity in the $x$ and $y$ directions were measured. The measurements for the $x$ component $u'$, were performed at the center of the Jet for different down-stream locations, Reynolds numbers, and electrical potentials. Figure 39 shows the results of such measurements, $u'$ being normalized by the mean velocity at the center. It was observed that the magnitude of $\bar{u}$ was reduced at all downstream locations under the action of the electrical field. The reduction being the greatest at low Reynolds number, when a 1/6 reduction was seen.

The distribution of $\bar{u}$ and $\bar{v}$ across the flow field was measured, under the conditions described in above. Figure 40 shows the distribution of $\bar{u}$ at $\frac{x}{2h} = 10$. For plain 2-D Jet the distribution showed a maximum at about $y/h = 0.5$, while $\bar{u}$ was reduced to zero away from the center of the Jet, at about $y/h = 2$. For the modified Jet a definite reduction in $\bar{u}$ occurred; the change being the greatest at low Reynolds number. For the modified Jet at high $Re$ the shape of the distribution
Figure 38: Modified Jet Decay Rate
Figure 39. Variation of $\hat{u}$ with Downstream Position
- $Re = 500, E_N = 0$
- $Re = 500, E_N = 0.75$
- $Re = 500, E_N = 1.0$
- $Re = 937.5, E_N = 0$
- $Re = 937.5, E_N = 0.4$
- $Re = 937.5, E_N = 0.54$
- $Re = 1312.5, E_N = 0$
- $Re = 1312.5, E_N = 0.28$
- $Re = 1312.5, E_N = 0.4$
- Electric Wind, $R_{ehd} = 1500$
- Electric Wind, $R_{ehd} = 2000$

Figure 40. $\tilde{u}$ Distribution at $\frac{x}{2H} = 10$
remained similar to that of the 2-D Jet. For low \( R_e \) and the case of the electric wind a rather monotonic profile was observed.

Figure 41 shows the distribution of \( \tilde{v} \), the normalized fluctuating component of velocity in the \( y \) direction. For the 2-D Jet the maximum of \( \tilde{v} \) occurred at the centerline, and was equal to \( \tilde{u} \) at that point. For some cases of the modified jet (e.g. \( R_e = 1312.5, \phi_0 = 10 \text{kV} \)) it was seen that differences existed in the magnitude of \( \tilde{v} \) compared to \( \tilde{u} \). In case of the electric wind \( \tilde{u} \) was similar to \( \tilde{v} \), and reduced magnitude of \( \tilde{v} \) was observed with increased electrical potentials.

6. Reynolds Stresses

The normalized Reynolds stresses \( \langle \frac{u'v'}{2} \rangle \), where \( U_0 \) was Jet exit velocity, were measured at \( \frac{x}{2h} = 10 \) for the 2-D Jet flow, electric wind, and the modified jet flows. The results are given in Figure 42. Significant changes were observed in the value of Reynolds stresses after the application of the electric field. Insignificant values of Reynolds stresses were observed in case of the electric wind.

The maximum value of Reynolds stresses occurred close to \( n = 1 \), and its value was reduced to zero at about \( n = 2 \). The values for higher Reynolds numbers were close, while for \( R_e = 500 \), lower Reynolds stresses as compared to the other two were observed.
\[ \bar{v} = \frac{\sqrt{\nu}}{U_c} \]

Figure 41. \( \bar{v} \) Distribution at \( \frac{x}{2h} = 10 \)
Figure 42. Reynolds Stresses Distribution at $\frac{x}{2h} = 10$
7. **Frequency Spectrum Analysis**

7a. Electrical Current

The frequency content of the electrical current of the corona discharge, for various electrical potentials was measured. Figure 43 shows an example of such measurements. The frequency content seemed to be limited to the low frequency portion of the spectrum, less than 20 Hz. The significant peaks shown were due to the 60 Hz noise and its harmonics.

7b. Electric Wind

The frequency spectrum of the electric wind for different electrical potentials is shown in Figure 44. Similar to the electrical current spectrum, the frequency content was limited to the low frequencies, with the peaks being rather small. For higher voltages a very small peak appeared in the high frequency side of the spectrum. The observed frequencies were believed to be due to the vortex shedding from the collector rods.

7c. 2-D Jet

The frequency spectrum of the 2-D Jet was measured above the ground grid system at $\frac{x}{2h} = 10$ for three different Reynolds numbers. Figure 45 shows the result of such measurements. The observed peaks in the spectrum were believed to be the symmetric and antisymmetric type sinusoidal fluctuation, that could have existed in the flow field of the jet.

The frequency spectrum of the 2-D jet was measured along the mixing layer at different down-stream positions. The results, shown in Figure 46, exhibited a peak at about 285 Hz, for $\text{Re} = 1240$ at $\frac{x}{2h} = 2$. 
Figure 43. Frequency Spectra of the Electric Wind Measured Electrical Current
Figure 44. Frequency Spectra of the Electric Wind Measured Velocity
Figure 45. Frequency Spectra of 2-D Jet Measured Velocity
Figure 46. Frequency Spectra of 2-D Jet at Different Downstream Positions at Re = 1240
The frequency of the observed peak was reduced to half at down-stream positions.

7d. Modified Jet

The frequency spectrum of the modified jet, for various electrical potentials and Reynolds numbers were measured. Figures 47-52 show the results of such measurements for the electrical currents as well as the fluid flow velocities associated with the modified jet. For lower Reynolds numbers of the jet flow, an example of which is shown in Figure 47, increased electrical potentials caused the peaks in the low frequency side of the spectrum to disappear, while another distinct peak at a higher frequency was created. The frequency associated with this peak increased linearly with increased voltages. At higher Reynolds numbers (Figures 48 & 49) the same behavior was observed, with the difference that the value of the peak remained almost constant.

In the case of the current spectrum no other significant peaks were observed except for the one mentioned above, as could be seen in Figures 51&52. The observed peak that was present in both the velocity and current spectrums was believed to be the vortex shedding frequency from the collector rods.

8. Visualization of the Flow Fields

The flow fields associated with the 2-D jet, electric wind and the modified jet were visualized, with the results appearing in the following sections.
Figure 47. Frequency Spectra of the Modified Jet Measured Velocity at $U_0 = 0.33$ m/sec ($Re = 125$)
Figure 48. Frequency Spectra of the Modified Jet Measured Velocity at $U_0 = 0.91$ m/sec (Re = 375)
Figure 49. Frequency Spectra of the Modified Jet Measured Velocity at $U_0 = 1.31$ m/sec (Re = 500)
Figure 50. Frequency Spectra of the Electrical Current of Modified Jet at $U_0 = 0.33$ m/sec ($Re = 125$)
Figure 51. Frequency Spectra of the Electrical Current of Modified Jet at $U_o = 0.91$ m/sec ($Re = 375$)
Figure 52. Frequency Spectra of the Electrical Current of the Modified Jet at $U_o = 1.3$ m/sec, $Re = 500$
8a. 2-D Jet

An example of a 2-D jet flow exhibiting different stages of breakdown into turbulence is shown in Figure 53. Three distinct modes of breakdown were detected upon careful examination of the flow. The initial stage of instability manifested itself as a wave-like motion of the fluid at the mixing layer of the jet, accompanied by small size ripples. The second stage was the formation of large eddy structure in the form of vortices. The distance between vortices, for sufficiently high Reynolds numbers $R_e > 1000$ was found to be independent of the jet velocity, and the relationship $d_v = 2c_v h$ was established. The value of $c_v$ ranged from 1.0 to 2.0.

The effect of side plates on the characteristics of the 2-D jet was examined. Figure A1 (in Appendix A) shows the results for different Reynolds numbers. It appeared that the plates did not have a significant effect on the structure of the 2-D jet flow.

8b. Electric Wind

The electric wind flow configuration is shown in Figure 54. No side plates were present in the case shown. From the photographs, which show both the core of the flow and the entrainment of outside fluid into the wind, no trace of any structure similar to that of 2-D jet could be seen.

Figure 55 shows the electric wind flow for the case where side plates were present. No changes in the structure of flow as compared to the previous case were observed.
Figure 53. 2-D jet flow at $U_0 = 5.0$ m/sec ($Re = 1925$)
Figure 54. Electric Wind Flow in the Absence of the Insulated Plate at $\phi_0 = 10$ kV, $D = 5$ cm
Figure 55. Electric Wind Flow in the Presence of the Insulated Plates: (a) $R_{ehd} = 1500$, (b) $R_{ehd} = 2000$
8c. Modified Jet

The modified jet for different Reynolds numbers, electric field strengths, and corona wire to ground grid distances was visualized and photographed. Figure 56 shows a comparison of 2-D jet flow and modified jet at the same Reynolds numbers. The accompanying pictures are the hot wire signals corresponding to either case. Significant changes existed between the two flow fields. Under the action of the field, the final stage of breakdown into complete turbulence did not occur. The vortices persisted in the down-stream direction, maintaining their initial size. The distance between vortices remained constant in the down-stream direction, in contrast to the 2-D jet flow where at times an increase in the wavelength associated with the vortices was observed.

Figure 57 shows the case where the original 2-D jet was at a lower Reynolds number, as compared to the one in Figure 56. It is seen that under the action of the field the wavelike instability at the mixing layer disappeared, and the modified jet behaved as a laminar jet.

An example of smoke-laser visualization tests is shown in Figure 58. For the 2-D jet the flow seemed to diffuse rapidly. By increasing the electrical field strength, progressive reduction in the diffusion of the jet was observed.

Figure 59 compares the phenomena of entrainment of outside fluid into the 2-D jet, and the modified jet. In case of the 2-D jet, the entrained fluid was drawn into the jet by the action of vortices. The outer region of the modified jet appeared totally different than that
Figure 56. 2-D jet flow and corresponding hot wire signals (D = 5 cm), (a) $U_0 = 3.67$ m/sec, $\phi_0 = 0$ kV ($Re = 1400$, $EN = 0$), (b) $U_0 = 3.67$ m/sec, $\phi_0 = 20$ kV ($Re = 1400$, $EN = 0.72$)
Figure 57. 2-D jet flow (D = 5 cm) at: (a) $U_0 = 1.97$ m/sec, $\phi = 0$ kV ($Re = 750$, $EN = 0$), (b) $U_0 = 1.97$ m/sec, $\phi = 20$ kV ($Re = 750$, $EN = 1.34$)
Figure 58. Laser-smoke pictures of the 2-D jet (D=5 cm)

(a) $U_0 = 0.98$ m/sec, $\phi_0 = 0$ kV ($Re = 375$, $EN = 0$),
(b) $U_0 = 0.98$ m/sec, $\phi_0 = 15$ kV ($Re = 375$, $EN = 1.57$)
(c) $U_0 = 0.98$ m/sec, $\phi_0 = 20$ kV ($Re = 375$, $EN = 2.68$)
Figure 59. Entrainment process in the jet (D = 5 cm), (a) $U_0 = 3.67$ m/sec, $\phi_0 = 0$ kV ($Re = 1400$, $EN = 0$), (b) $U_0 = 3.67$ m/sec, $\phi_0 = 15$ kV ($Re = 1400$, $EN = 1.5$)
of 2-D jet. The entrained streamlines behaved as if the modified jet was a 2-D laminar jet.

The modified jet in the presence of side plates, placed below the corona wire, is shown in Figure 60-62. The photographs exhibit the flow structure of the modified jet at three different Reynolds numbers and two levels of electrical potentials. The corresponding 2-D jet flow is also included for comparison. The distance between the corona wire to the ground plate was 2.0 cm and the wire to side plate distance was 1.0 cm. For \( R_e = 500 \) under the action of the field the wave-like instability that was present in the mixing layer of the jet disappeared. With an increase in the applied electrical potential progressive reduction in the initial width of the jet was observed. For higher Reynolds numbers the vortices that had formed were reduced in size, or even their formation delayed with increased electrical potentials.

The outer flow region of the modified jet was compared to that of 2-D jet at the same Reynolds number. The results are shown in Figures 63-65. For \( R_e = 500 \) not much change occurred, although it seemed that the entrained outside air was drawn more to the center of the jet under the action of the electrical field. For \( R_e = 937.5 \) with increased electric field strengths, a definite alteration of the outer region occurred. The wave-like disturbance observed in the case of 2-D jet disappeared, and the flow field looked like that of the jet at lower Reynolds numbers. For \( R_e = 1312.5 \) the entrained streamline was markedly distored in case of the 2-D jet. The distortion was progressively reduced by increasing the electrical potential.
Figure 60. Jet flow (D = 2.0 cm) at (a) $U_o = 1.31$ m/sec, $\phi_0 = 0$ kV ($Re = 500$, $EN = 0$), (b) $U_o = 1.31$ m/sec, $\phi_0 = 10$ kV ($Re = 500$, $EN = 0.75$), (c) $U_o = 1.31$ m/sec, $\phi_0 = 12.5$ kV ($Re = 500$, $EN = 1.0$)
Figure 61. Jet flow (D = 2.0 cm) at (a) \( U_0 = 2.46 \text{ m/sec}, \phi_0 = 0 \text{ kV} \) (Re = 937.5, EN = 0),
(b) \( U_0 = 2.46 \text{ m/sec}, \phi_0 = 10 \text{ kV} \) (Re = 937.5, EN = 0.4), (c) \( U_0 = 2.46 \text{ m/sec}, \phi_0 = 12.5 \text{ kV} \) (Re = 937.5, EN = 0.5)
Figure 62. Jet flow (D = 2.0 cm) at: (a) $U_0 = 3.44$ m/sec, $\phi_0 = 0$ kV (Re = 1312.5, EN = 0),
(b) $U_0 = 3.44$ m/sec, $\phi_0 = 10$ kV (Re = 1312.5, EN = 0.28), (c) $U_0 = 3.44$ m/sec,
$\phi_0 = 12.5$ kV (Re = 1312.5, EN = 0.4)
Figure 63. Entrainment into the jet (D = 2.0 cm) at: (a) $U_0 = 1.31$ m/sec, $\phi_0 = 0$ kV ($Re = 500$, $EN = 0$), (b) $U_0 = 1.31$ m/sec, $\phi_0 = 10$ kV ($Re = 500$, $EN = 0.75$), (c) $U_0 = 1.31$ m/sec, $\phi_0 = 12.5$ kV ($Re = 500$, $EN = 1.0$)
Figure 64. Entrainment into the jet (D = 2.0 cm) at: (a) \( U_0 = 2.46 \text{ m/sec}, \varphi_0 = 0 \text{kV} \) (Re = 937.5, EN = 0.0), (b) \( U_0 = 2.46 \text{ m/sec}, \varphi_0 = 10 \text{kV} \) (Re = 937.5, EN = 0.4), (c) \( U_0 = 2.46 \text{ m/sec}, \varphi_0 = 12.5 \text{kV} \) (Re = 937.5, EN = 0.54)
Figure 65. Entrainment into the (D = 2.0 cm) jet at: (a) $U_0 = 3.44$ m/sec, $\zeta_0 = 0$ kV 
(Re = 1312.5, EN = 0), (b) $U_0 = 3.44$ m/sec, $\zeta_0 = 10$ kV (Re = 1312.5, EN = 0.28),
(c) $U_0 = 3.44$ m/sec, $\zeta_0 = 12.5$ kV (Re = 1312.5, EN = 0.4)
Figure 66. Jet Flow (D = 5.0 cm) at: (a) $U_o = 1.31 \text{ m/sec, } \zeta = 0 \text{ kV (Re = 500, EN = 0)}$, (b) $U_o = 1.31 \text{ m/sec, } \zeta = 14 \text{ kV (Re = 500, EN = 1.2)}$
Figures 66-68 exhibit the modified jet flow, when the distance between the corona wire and the ground grid was increased to 5.0 cm. The modified jet was compared to 2-D jet flow at the same Reynolds number. The electric field seemed to have a significant effect only at the lower Re's.

The case when two corona wires were used was investigated. Figure 69 shows the results. Apparently the electrical field in this case did not have a significant effect on the 2-D jet flow, although some narrowing of the jet was observed.

The number of ground rods were reduced to two central rods only. A comparison of this case with the case when all rods were present is shown in Figure 70. It seemed that for the same total current the effect of electric wind on the 2-D jet was reduced, as compared to the case when all rods were present, in the sense that the final breakdown of the jet occurred in the region of study. The entrainment of outside fluid into the jet is also shown in Figure 71. The entrainment process, unlike the case of the electric wind exhibited a cross flow nature.

The impingement of the 2-D jet on a flat plate, with and without the field is shown in Figure 72. The flow fields in the region between the nozzle and the ground electrode resembled that of the case when the ground grid was used. The vortices under the action of the electrical field maintained their characteristics up to the ground plate, where the final breakdown occurred.
Figure 66. Jet Flow (D = 5.0 cm) at: (a) $U_0 = 1.31$ m/sec, $\phi_0 = 0$ kV ($Re = 500, EN = 0$), (b) $U_0 = 1.31$ m/sec, $\phi_0 = 14$ kV ($Re = 500, EN = 1.2$)
Figure 67. Jet Flow (D = 5.0 cmP at: (a) $U_0 = 2.46$ m/sec, $\phi_0 = 0$ kV ($Re = 937.5$, $EN = 0$), (b) $U_0 = 2.46$ m/sec, $\phi_0 = 14$ kV ($Re = 937.5$, $EN = 0.64$)
Figure 68. Jet flow (D = 5.0 cm) at: (a) $U_0 = 3.44 \text{ m/sec}, \varphi_0 = 0 \text{ kV}$ ($Re = 1312.5, EN = 0$), (b) $U_0 = 3.44 \text{ m/sec}, \varphi_0 = 14 \text{ kV}$ ($Re = 1312.5, EN = 0.45$)
Figure 69. Jet flow (D = 5 cm) at: (a) \( U_0 = 5.19 \text{ m/sec}, \phi_0 = 0 \text{ kV} \) (Re = 2000, EN = 0), (b) \( U_0 = 5.19 \text{ m/sec}, \phi_0 = 17.5 \text{ kV} \) (Re = 2000, EN = 0.48), two corona wires each 5 mm from centerline, (c) \( U_0 = 5.19 \text{ m/sec}, \phi_0 = 17.5 \text{ kV} \) (Re = 2000, EN = 0.48), two corona wires each 18 mm from centerline
Figure 70. Jet flow (D=5 cm) at: (a) $U_0 = 3.67$ m/sec, $\phi_0 = 0$ kV ($Re = 1400, EN = 0$), (b) $U_0 = 3.67$ m/sec, $\phi_0 = 20$ kV ($Re = 1400, EN = 0.72$), all rods in, (c) $U_0 = 3.67$ m/sec, $\phi_0 = 24$ kV ($Re = 1400, EN = 0.72$), two central rods in
Figure 71. Entrainment into the electric wind when central rods were present \((D = 5 \text{ cm}, \phi_0 = 15.5 \text{ kV})\)
Figure 72. Jet impinging on a flat plate (D = 7 cm), (a) $U_0 = 3.67 \text{ m/sec}, \phi_0 = 0 \text{ kV} (Re = 1400, EN = 0)$, (b) $U_0 = 3.67 \text{ m/sec}, \phi_0 = 20 \text{ kV} (Re = 1400, EN = 0.76)$
B. Theoretical Results

1. Electrical Potential Distribution

The electrical potential distribution for the geometry shown in Figure 9 was evaluated for different electric potentials. Example of such calculations is shown in Figures 73 & 74. To exhibit the effect of space charge on the field strength, the potential distribution at the centerline for different applied electrical potentials is plotted in Figure 75, along with the solution to the Laplace's equation corresponding to the charge free case. Also included is the experimentally evaluated data for $\phi_0 = 10$ kV, exhibiting the close agreement between the theoretical and experimental values.

2. 2-D Jet

The 2-D jet flow was simulated on the computer by use of the finite difference scheme outlined in Chapter III. The calculations were performed for Reynolds numbers in the range of 500 to 1500, in order to observe both stable and unstable flows. The specific range of Reynolds numbers considered was based on the previously obtained experimental values. It has to be emphasized that the equations of motion considered were limited to that of laminar flows. As a result far field characteristics of the flow, which corresponded to that of fully turbulent flows, could not be accounted for. Nevertheless the near field properties of the flow such as the onset of instability was demonstrated upon the solution of the governing equations of motion. The validity of the presented results were limited to the region $0 < \frac{x}{2h} < 10$, the linear region, since beyond that it was observed in
Figure 73. Equi-potential Lines for $\phi_0 = 10$ kV
Figure 74. Equi-potential lines for $\phi_0 = 12.5$ kV
Figure 75. Electrical Potential Distribution at Centerline

- \( \phi_0 = 10 \text{ kV} \)
- \( \phi_0 = 12.5 \text{ kV} \)
- Experimental values at \( \phi_0 = 10 \text{ kV} \)
- No space charge
the experimental phase of the study that breakdown of the jet into turbulence occurred.

As mentioned in Chapter III, in order to effect a solution, the initial velocity profile at the jet exit had to be known. In the present study three profiles were considered from which the values of stream function and vorticity were calculated at the jet exit. The profiles are shown in Figure 76 and are denoted as profiles I, II, III and IV. Profile I was the experimentally evaluated profile corresponding to Re = 500, profile II was measured at Re = 937.5, profile III was measured at Re = 1312.5, and IV was an assumed distribution resembling a "top-hat" profile.

For Re < 750-800 the solution was stable irrespective of the Reynolds number, and the initial velocity profile considered, as shown in Figures 77-80. The streamlines were similar to that of a stable laminar jet and the isodines, or lines of constant vorticity, stretched, without separation, in the down-stream direction.

The critical Reynolds number was found to be around 750 to 800, since a step increase in the Reynolds number from 750 to 800 produced instabilities in the solution. This occurred irrespective of the initial velocity profiles used, as can be seen in Figures 81-84, unlike the case of axisymmetric jets, Grant (17).

The solution for Reynolds number ranging from 500 to 1312.5, for comparison with the experimental results reported before, was obtained, and the results are shown in Figures 85-87. Significant distortion of streamlines, and existence of circulatory cells in the form of vortices were clearly observed at Re = 1312.5 (Figure 87). This agreed in form
I - \( U_0 = 1.31 \text{ m/sec} \) (Re = 500)
II - \( U_0 = 2.46 \text{ m/sec} \) (Re = 937.5)
III - \( U_0 = 3.44 \text{ m/sec} \) (Re = 1312.5)
IV - Assumed profile

Figure 76. 2-D Jet Velocity Distributions Used in the Numerical Calculations
Figure 77. 2-D Jet Flow Characteristics at Re = 650 (Profile I): (a) Streamlines, (b) Isodines
Figure 78. 2-D Jet Flow Characteristics at Re = 650 (Profile II): (a) Streamlines, (b) Isodines
Figure 79. 2-D Jet Flow Characteristics at $Re = 650$ (Profile III): (a) Streamlines, (b) Isodines
Figure 80. 2-D Jet Flow Characteristics at Re = 650 (Profile IV): (a) Streamlines, (b) Isodines
Figure 81. 2-D Jet Flow Characteristics at $Re = 800$ (Profile I), (a) Streamlines, (b) Isodines
Figure 82. 2-D Jet Flow Characteristics at Re = 800 (Profile II), (a) Streamlines, (b) Isodines
Figure 83. 2-D Jet Flow Characteristics at $Re = 800$ (Profile III), (a) Streamlines, (b) Isodines
Figure 84. 2-D Jet Flow Characteristics at Re = 800 (Profile IV), (a) Streamlines, (b) Isodines
Figure 85. 2-D Jet Flow Characteristics at Re = 500 (Profile I), (a) Streamlines, (b) Isodines
Figure 86. 2-D Jet Flow Characteristics at $Re = 937.5$ (Profile II), (a) Streamlines, (b) Isodines
Figure 87. 2-D Jet Flow Characteristics at Re = 1312.5 (Profile III), (a) Streamlines, (b) Isodines
with the flow visualization results. The isodine plot also demonstrated the existence of vortices as islands of concentrated vorticity were detached from the initial isodines initiating at the nozzle exit. These areas of constant vorticity moved in the down-stream direction as time progressed, Figures 88 & 89. The circulatory cell that was formed (Figure 88a & 89a), increased its strength as it became more concentrated while forcing a bigger vortical cell around it as it moved in the down-stream direction, Figure 88b & 89b. Further down-stream the initial cell began to expand and its vorticity appeared to be diffusing into the surrounding fluid, Figure 88c & 89c.

The movement of the mentioned cells was essential in the creation of unstable jet flows, since existence of mere areas of concentrated vorticity is not representative of such flows (15).

3. Electric Wind

The electric wind flow was evaluated for three different Reynolds numbers based on the EHD characteristic velocity. The results are shown in Figures 90 & 91, which illustrate both the streamline and the isodine plots. The solutions were stable in the range of Reynolds numbers considered, in accordance with the experimental results. The isodine plot exhibited areas of concentrated vorticity close to the centerline, unlike the 2-D jet where the islands existed at the shear-layer. Also contrary to 2-D jet flow the islands were stationary, and did not move in the down-stream direction upon increasing the number of iterations.

The average velocity distributions at $\frac{x}{2h} = 10$ is shown in Figure 92, along with the experimental results. It is seen that good
Figure 88. 2-D Jet Flow Streamline Plot at Different Instances in Time (Re = 1312.5, Profile III)
(a) $t_1$ (b) $t_1 + 2$ msec (c) $t_1 + 4.5$ msec
Figure 89. 2-D Jet Flow Isodine Plot at Different Instances in Time (Re = 1312.5, Profile III)
(a) $t_1$ (b) $t_1 + 2$ msec (c) $t_1 + 4.5$ msec
Figure 90. Electric Wind Flow Characteristics at $R_{ehd} = 1500$
($\phi_0 = 10$ kV, $D = 2.0$ cm): (a) Streamlines, (b) Isodines
Figure 91. Electric Wind Flow Characteristics at $R_{ehd} = 2000$
($\phi_0 = 12.5$ kV, $D = 2.0$ cm): (a) Streamlines,
(b) Isodines
Figure 92. Electric Wind Average Velocity Distribution at $\frac{x}{2h} = 10$
($\phi_0 = 10$ kV, $D = 2.0$ cm)
agreement existed between the two, especially close to the centerline where the difference was not more than 2%. The centerline velocity Reh = 1500 at x/2h = 10 was 1.23 m/sec comparable to the experimental result of 1.3 m/sec.

4. Modified Jet

The modified jet was simulated on the computer. The results of which is shown in Figures 93-100. The computation covered Reynolds numbers ranging from 500 to 1312.5, and EHD numbers (EN) extending from 0 to 1.0. Depending on the respective values of Reynolds and EHD numbers, both stable and unstable flows were observed, in accordance with the previously established results. The produced flow fields, especially at lower Reynolds numbers, exhibited unique characteristics as the electric body force field was coupled with the 2-D jet flow. The changes were most significant above the corona wire where a narrowing of the initial jet occurred under the accelerating action of the electric field. The observed phenomena is best illustrated in Figures 101 & 102 where the diffusion and decay curves of the modified jet between the nozzle and the ground grid system are presented. The results exhibit characteristics unlike that of any ordinary jet since negative slopes of the decay and diffusion curves were observed.

The average velocity distribution at $\frac{x}{2h} = 10$ for the case Re = 500, $\phi_0 = 10$ kV is shown in Figure 103, along with the experimentally evaluated results. It is obvious that reasonable agreement exists between the two, as the maximum deviation was not more than 5%.
Figure 93. Modified Jet Flow Characteristics at Re = 500, and EN = 0.75 ($U_0 = 1.31$ m/sec, $\phi_0 = 10$ kV, $D = 2.0$ cm):
(a) Streamlines, (b) Isodines
Figure 94. Modified Jet Flow Characteristics at Re = 500, and EN = 1.0 ($U_0 = 1.31$ m/sec, $\phi_0 = 12.5$ kV, $D = 2.0$ cm):
(a) Streamlines, (b) Isodines
Figure 95. Modified Jet Flow Characteristics at $Re = 800$, and $En = 0.56$ ($U_0 = 2.09 \text{ m/sec}, \phi_0 = 10 \text{ kV}, D = 2.0 \text{ cm}$): (a) Streamlines, (b) Isodines
Figure 96. Modified Jet Flow Characteristics at Re = 800 and EN = 0.81 ($U_0 = 2.09$ m/sec, $\phi_0 = 12.5$ kV, D = 2.0 cm):

(a) Streamlines, (b) Isodines
Figure 97. Modified Jet Flow Characteristics at $Re = 937.5$ and $EN = 0.4 \ (U_0 = 2.46 \text{ m/sec}, \phi_0 = 10 \text{ kV}, D = 2.0 \text{ cm})$:
(a) Streamlines, (b) Isodines
Figure 98. Modified Jet Flow Characteristics at $Re = 937.5$ and $EN = 0.54$ ($U_0 = 2.46$ m/sec, $\phi_0 = 12.5$ kV, $D = 2.0$ cm):
(a) Streamlines, (b) Isodines
Figure 99. Modified Jet Flow Characteristics at \( Re = 1312.5 \) and \( EN = 0.28 \) (\( U_0 = 3.44 \) m/sec, \( \phi_0 = 10 \) kV, \( D = 2.0 \) cm):
(a) Streamlines, (b) Isodines
Figure 100. Modified Jet Flow Characteristics at $Re = 1312.5$ and $EN = 0.4$ ($U_0 = 3.44$ m/sec, $\phi_0 = 12.5$ kV, $D = 2.0$ cm):
(a) Streamlines, (b) Isodines
Figure 101. Modified Jet Interpolated Diffusion Rate
Figure 102. Modified Jet Interpolated Diffusion Rate
Figure 103. Modified Jet Average Velocity Distribution
Re = 500, E = 0.75, and $\frac{x}{2h} = 10$
Figures 104 & 105 show average velocity and vorticity distributions of the modified jet at $\frac{x}{2h} = 4$. 
Figure 104. Modified Jet Average Velocity Distribution at $\frac{x}{2h} = 4$
Figure 105. Modified Jet Vorticity Distribution at $\frac{x}{2h} = 4$
CHAPTER V
DISCUSSION OF RESULTS

A. Current Distribution

The current distribution of the electric wind was measured. The observed distribution, shown in Figure 17, was in agreement with the results of Velkoff (35). The distribution was unaffected by the 2-D jet flow, for velocities below 10 m/sec. This was expected because the ion velocities for even moderate electrical potentials, were orders of magnitude higher than the typical jet velocities used in the present study.

The electric wind distribution was unaffected by the presence of the insulated plate, as long as the plate was at least 3.0 cm away from the corona wire. This occurred inspite of the plate forcing a zero field strength, in the x direction, below the wire. The reason being that the majority of ions created existed above the corona wire, in the region between the wire and the collector grid system. So if the insulated plate was sufficiently far, in this case 3.0 cm, from the corona wire its effect on the field lines and the space charge was insignificant. The idea of the ions being present mostly above the wire comes from the observation that the field lines below the wire move away from the corona wire then turn upwards towards the collector.
rods. The field lines closest to the centerline would tend towards "infinity" before finding their way back to the ground rods. At infinity no space charge existed so if a field line was to be traced back to the area below the wire essentially no space charge would be encountered along the way. So it is only fair to say that the field below the wire did not contribute significantly to the electrical current characteristics. The same argument holds in the presence of the insulated plate. The field lines close to the plate were forced to go to infinity, reducing their current carrying capacity. Based upon these arguments, when the insulated plate was placed close to the corona wire (less than 3.0 cm away), an increasing number of field lines were forced to behave as mentioned above. Thus reduction in the total current and narrowing of the current distribution was to be expected. These results were confirmed by the numerical solution of the problem, as presented before.

B. Current-Voltage Relationship

The observed corona current-voltage relationship, in the absence of the insulated plate agreed well with the established empirical formula, \( I = KV(V-V_0) \) as indicated by Velkoff (35), and Mori (49). So the onset potential was strongly dependent on the distance between the corona wire and the ground grid, and less significantly on the wire diameter, temperature, pressure (density), and the relative humidity of the room. The effect of the insulated plate on the total current, became apparent for distances below 3.0 cm. Reduced total currents were observed, in accordance to the argument stated before.
C. Electrical Potential Distribution

The relationship between probe voltage and current in Figure 20 was observed throughout the region as the probe was traversed, except where the probe was adjacent to the corona wire. Therefore it could be said that there were few electrons in the region except for the limited space close to the wire. This indicated that positive ions and neutral molecules were the dominant charge forms in the entire region.

The equipotential lines shown in Figure 21 showed good agreement with the results of Mori & Hijikata (49). For the case when the insulated plate was used, again the same distribution of potential in the region was observed, except close to the insulated plate, when the equipotential lines became perpendicular to the plate to satisfy the zero field strength in the x direction.

Figure 73 shows the potential distribution at the centerline above the corona wire. Pronounced difference existed between the charge free case and the one when current was flowing. This indicated the important effect of the existing ions in the region on the electrical potential distribution. Thus it would be erroneous to use the electrical potential without space charge, i.e. the solution of the Laplaces equation for an analysis of corona discharge. The result of the numerical analysis of the phenomena was also supportive of the above statement, as seen in the present study and those of Velkoff, Mori, and Yamamoto.
D. Average Velocity Distribution

1. 2-D Jet

Velocity distribution of the 2-D jet for 3 different Reynolds numbers was shown in Figure 24. The shape of the velocity profiles at the jet exit was determined by the velocity distribution in the two-dimensional nozzle which was equivalent to a short two-dimensional channel. Based on this, Schlichting (62) has shown that the shape of the average velocity distribution was dependent on a non-dimensional parameter given as \( \frac{\nu L}{h^2 U_0} \). The magnitude of this number then would indicate the degree of development of the flow through the nozzle. For small values of \( \frac{\nu L}{h^2 U_0} \) the profile would correspond to that of non-fully-developed channel flow, characterized by an existing potential core. According to this it was observed that by increasing the Reynolds number, or reducing \( \frac{\nu L}{h^2 U_0} \) the profiles resembled more and more that of a top hat type velocity distribution. The observed results were in accord with that of Sato (9).

The variation of average velocity distribution with respect to the distance down-stream was found to be in compliance with the results of Sato (9), and Krothapalli, et al (63). Before transition into turbulence happened, the change in the distribution was gradual except at the outer region of the jet where the velocity increased significantly. This occurred due to the change from wall to free boundary condition at the jet exit. At about a distance 4 to 8 times the width
of the slit in the down-stream direction, depending on the initial condition (initial profile, and level of turbulence) the change in the velocity profiles became rather significant. A large reduction in the magnitude of the velocity at the center, and broadening of the jet took place due to the action of turbulence. Now this behavior was best illustrated in the curves representing the diffusion and decay of the 2-D jet. The transition into turbulence occurred in the region where a sudden increase in the diffusion and decay took place. As shown in Figures 28 & 29 after transition into turbulence took place both the diffusion and decay curves showed a linear dependence in the down-stream position. This was an expected behavior based on the results of Schlichting (62), Bradbury (61), Gutmark and Wygnanski(64), and Krothapalli (63). The present result showed good agreement with the results of Sato, while it exhibited higher diffusion rates as compared to the results of others. The reason being most probably due to the differences in the conditions under which the experiments were conducted. Example of which would be the jet exit profile, initial level of turbulence, room turbulence, and draughts. In the present experiment no attempt was made to isolate the experiment from the room condition, such as the use of vertical side plates (Gutmark & Wygnanski), or calming streams at the outer boundaries of the jet (Bradbury). The result of the diffusion of the 2-D jet was clearly in accord with the Schlieren photographs taken under the same condition.

The effect of Reynolds number on jet profile in the turbulent region was found to be insignificant. This was in accordance with the results of Gutmark & Wygnanski(64), and Freymuth (25). It has to be
mentioned though that the range of Reynolds numbers reported in the experiments of Freymuth and others was much higher Re > 50,000 that that of the present study. This fact still did not cause the present results to deviate from their observations, that in the turbulent region of a jet the physical viscous forces have an insignificant effect upon the overall behavior of the flow.

Figure 30 showed the normalized velocity profiles of the jet in the turbulent region. It was stated that the distribution was expressable by the formula, \( f(n) = \exp(-0.67 n^2(1 + 0.565n^4)) \) in accordance with the results of Townsend (60), and Bradbury (61). The results tend to indicate partial self preservation of the jet. The flow could not be said to be completely self preserved because as will be seen later velocity fluctuations did not satisfy the condition under which self preservation was achieved.

The results also indicated that the jet became turbulent before the similarity distribution for laminar flow, as calculated by Schlicting (62), was realized. The results further, in compliance with the remarks of Michalke (15), showed that similarity solutions of the form tanhn were not suitable choices for stability analysis of the 2-D jet flow.

The results obtained for the average properties of the 2-D jet flow, were obtained with the ground rods in place. The presence of the ground rods did not noticeably change the average characteristics of the 2-D jet flow, after the results were compared to that of the case when no ground rods were present. The following reasons explain why: (a) low velocities of the jet discouraged any significant shedding
of vorticity from the collector rods, (b) at higher velocities the level of turbulence in the 2-D jet was sufficiently high that the rods did not act as effective trip wires. The Schlieren photographs at higher velocities clearly support this observation, while at Re = 500 some distortion of the flow due to the action of rods were observed.

The effect of side plates on the velocity field was insignificant, based on both the hot wire results and Schlieren photograph, Figure A1. This was in compliance with the observations of Hussain (65).

2. Electric Wind

The electric wind velocity distribution agreed well with the results of Velkoff (35). The distribution was given as:

$$\frac{U}{U_c} = \frac{1}{(1 + 0.4n^2)^2}$$

The change in the normalized profile with downstream position in the range considered was less visible than that of the 2-D jet. The reason for this was the inherent stability of the electric wind flow. It should be noted though that beyond the ground grid the electric body forces did not exist. So it was expected that the flow would become unstable some distance downstream from the ground grid, \( \frac{x}{2h} > 15 \). This in fact happened, and the break down of the jet occurred.

As in the case of electrical current distribution addition of the side plates did not have any noticeable effect on the mean velocity distribution of the electric wind, when the side plates were at least 3.0 cm away from the corona wire.
3. **Modified Jet**

Depending on the 2-D jet flow exit velocity and the applied electric field, the 2-D jet mean velocity distribution was to a greater or lesser degree affected by the action of the electrostatic field. As can be seen in Figure 104 higher electrical potentials resulted in sharper average velocity profiles, at the same Reynolds number, in the ionized region. It was noted that the sharpest profiles corresponded to the lowest Reynolds number of 500, resulting in a greater maximum value of vorticity associated with each profile, as shown in Figure 105. Figure 105 also revealed that the observed maximum in the vorticity distribution progressively moved towards the centerline of the jet with an increase in the value of the EHD number, EN. The mentioned phenomena clearly demonstrated the effect of the electric field on the 2-D jet, manifested in terms of a shift of the point of maximum vorticity from the shear layer towards the centerline, resulting in reduced vorticity at the shear layer.

By comparing Figures 25 & 104 it was concluded that the modified jet possessed a wider velocity profile than that of the 2-D jet. This conclusion was supported by the Schlieren photographs (Figure 60), and the hot wire measurements.

It is seen from Figure 35 that the normalized velocity distribution of the modified jet for \( R_e = 937.5 \) and \( R_e = 1312.5 \), were identical above ground grid. This again emphasized the insignificant effect of the Reynolds number in the turbulent region, as was equally observed in the case of 2-D jet flow. By examining Figure 35 further, it was noticed
that partial self-preservation observed for 2-D jet was not realized in this case up to $x = 15$.

Based on the numerical results presented in Figure 101, the modified jet exhibited negative slopes in the diffusion curve, within the ionized region. This behavior was unique to the modified jet and in contrast with all plain jet flows. This occurred as sharper velocity profiles of the 2-D jet were encountered with an application of an electric field.

Above the ground rods reduced diffusion rates of the modified jet, as compared to that of the 2-D jet, for $\phi_0 = 10$ kV (Figure 36) was observed. Even lower diffusion rates, as in the case of $R_e = 500$, could have been possible if the ground rods did not influence the flow in the downstream direction (see Appendix A for full discussion). Increased electrical potentials from 10 to 12.5 kV resulted in reduced half breadth of the modified jet (Figure 101), while slightly higher diffusion rates were observed above the ground rods (Figure 37).

The decay rate of the modified jet, in the ionized region, is shown in Figure 102. Once again the uniqueness of the 2-D jet flow in the presence of an electric field was demonstrated as decay curves with negative slopes were observed. Figure 102 was also indicative of the fact that the velocity of the jet actually increased some distance away from the nozzle exit, contrary to plain jet flows. The mentioned phenomena was also observed in the streamline plot shown in Figure 93, as streamlines converged under the action of the electric field. Now the fact that a narrowing of the core occurred, along with the observation that in the laminar region no flow could ever cross the streamlines
would indicate a rise in the velocities. With increased electrical potentials, the mentioned narrowing of the core became more pronounced, resulting in higher core velocities. The conclusions drawn above were supported by the Schlieren photographs of the modified jet, Figure 60, as a narrowing of the jet was observed due to the accelerating action of the electric field.

The decay rates of the modified jet above the ground grid system was shown in Figure 38. Significantly reduced decay rates of the modified jet as compared to that of the 2-D jet flow were observed. This occurred due to the action of the field which maintained or at times reduced the initial size of the jet; an effect which propagated beyond the ground rods. The observed effect resulted in the preservation of the central velocity of the jet, and thus reduced decay rates. It was further observed that an increase in the electrical potential was translated into further reduction in the decay rates.

The fact that increased potentials caused reduced decay rates and at the same time increased diffusion rates above the ground grid may have been somewhat surprising. This seemingly paradox could be explained based on the fact that it was the small scale turbulence which was responsible for the decay of the velocities rather than the diffusion of the jet, Hussain & Clark (66). Increased electrical potentials reduced the initial level of turbulence, thus lower decay rates were observed.

E. Velocity Fluctuations

The variation of $\bar{u}$ with respect to the downstream position was shown in Figure 39. For the 2-D jet the results agreed well inform with the results of Miller and Comings (19), Wygnanski (67), and Krothapalli (63). The measurements indicated in the range considered,
That \( \frac{x}{2h} \leq 25 \) that \( u \) was not a constant. This would point to the fact that self-preservation was not achieved up to \( \frac{x}{2h} = 25 \). Self-preservation of the 2-D jet seemed to be dependent upon several factors such as the initial conditions and the Reynolds number. Comings reported the distance required for self-preservation to be \( \frac{x}{2h} = 30 \), while others have reported distances up to \( \frac{x}{2h} = 70 \). In the present study the result seemed to comply with that of Comings, although no definite statement could be made since the measurements were not extended further down-stream.

Under the action of the electrical field the magnitude of \( u \) was reduced at all down-stream locations. This happened inspite of the induced disturbances by the ground rods. The reduction of \( u \) was greatest in the case of \( Re = 500 \), using a smaller central rod \((d = 5 \text{ mm})\) where a 1/6 change was observed, Figure 39. It was also observed that further down-stream the effect of the applied electrical field began to diminish, and a rise in the value of \( u \) was observed. The electric wind showed the smallest fluctuation as compared to the 2-D jet and the modified jet flows.

The distribution of \( u \) and \( v \) across the jets was measured, at \( \frac{x}{2h} = 10 \). The distribution for the 2-D jet was in fair agreement with the results of Sato (9), Wygnanski (67), and Krothapalli (63). The distribution showed a maximum at about \( \frac{y}{h} = 1.0 \), corresponding to the edge of the jet at the exit. Then \( u \) was reduced to zero at about \( \frac{y}{h} = 2 \). The effect of Reynolds number was not significant on the distribution for the two higher Re's. For \( Re = 500 \) the distribution was shifted downward, indicating that the flow was far from being self-preserved.
After the electrical field was applied a definite reduction in the magnitude of $\bar{u}$ occurred, the change being the greatest for lower Reynolds numbers. For the modified jet at high Re's the shape of the distribution remained similar to that of the plain jet, exhibiting a maximum at about the same point, but having a lower value. For low Re and the case of electric wind, a rather monotonic profile was observed, indicating a change in the structure of the flows compared to the 2-D jet at the point where measurements were taken. The reason for this to happen could be the extension of the linear range in the down-stream direction and delay of complete turbulence due to the stabilizing effect of the electric field. It was observed that with increased electric field strengths further reduction in the magnitude of $\bar{u}$ occurred, indicating increased stability under higher EHD numbers.

The distribution of $\bar{v}$, the normalized r.m.s. value of the fluctuating component of velocity in the y direction, was shown in Figure 41. The results were in good agreement with the results of Miller and Comings (19), and Gutmark & Wygnanski (64). For higher Reynolds numbers the distributions were similar, exhibiting the lack of effect of Reynolds number on the characteristics of the turbulent jet. This was in compliance with the observations of Freymuth (19), and Michalke (68). The centerline value of $\bar{v}$ was found to be equal to the corresponding $\bar{u}$. This suggested isotropy of turbulence at the centerline in accordance with the remarks of Townsend (32), and the data of Krothpali (63).

After the application of the electric field, for some cases, an example of which was $R_e = 1312, EN = 0.28$ the magnitude of $\bar{v}$ deviated
from that of \( \bar{u} \) at the centerline. This suggested anisotropy, as opposed to the plain 2-D jet case. The reason again could be the lack of complete turbulence, at the point where measurements were taken, due to the stabilizing action of the field.

F. Reynolds Stresses

The Reynolds stresses distributions for different cases are illustrated in Figure 42. The measured values of Reynolds stresses were higher than those reported in literature (61). This happened most probably because all the data reported in literature has been those measured in the self-preserved region of the jet (\( \frac{x}{2h} > \) at least 50), whereas the measurements reported herein were performed at \( \frac{x}{2h} = 10 \), that is in the near field region of the jet.

After the application of the electric field significant reduction in the values of Reynolds stresses occurred, similar to those observed in the multiple free jet flow (63). The electric wind also exhibited insignificant values of Reynolds stresses, which was believed to be due to the observed stable characteristics of the flow.

G. Stability of the Jets

1. 2-D Jet

   Below some "critical" Reynolds number the 2-D jet flow was expected to be stable, that is damping of all small disturbances had to occur, based on the results of the numerous literature that was concerned with such flows. This behavior was observed in both the
numerical and experimental phases of the present study. Experiment exhibited that the 2-D jet behaved as a laminar jet in the region of study, up to \( R_e \approx 750 \). The numerical results shown in Figure 77 also exhibited the same behavior. The streamlines extended in the down-stream direction, while transferring momentum with the outer regions of the jet. The corresponding isodines also shown in Figure 77, exhibited a stretching of the lines of constant vorticity in the down-stream direction, with no signs of eventual shedding of vortices. Generally speaking the observed results were in excellent agreement with the results of Grant (17), taking into consideration that his study was concerned with the flow of axisymmetric jets.

Different stages of breakdown of a 2-D jet were exhibited in Figure 53. The observed characteristics were present when the Reynolds number was sufficiently high, that is above a "critical" Reynolds number. This critical value has been noted by many investigators to be dependent on the conditions that the experiments have been carried out. Reynolds numbers in the range of 3 (Tatsumi & Katakami,8) to 1000 (Crow,18) have been reported as the critical Reynolds number. Factors such as the initial boundary layer thickness, i.e. vorticity distribution, level of turbulence in the nozzle and room turbulence and draught have been known to have significant effect on the onset of instability. In the present study the experimental critical Reynolds number was found to be around 750. These values were in fair agreement with the experimental results of Beavers & Wilson (13), who found small irregularities in the flow in the range of Reynolds numbers between 500 and 600. In the present study the
observed numerical value for the critical Reynolds number was around 750 to 800 for all the four profiles considered. This value was higher than the experimental values. The reason could have been the existing initial turbulence in the flow which was around 0.3%, the existing turbulence in the room, and the vibration of the test setup. Sato has warned against outside disturbances, and their strong effect on the onset of turbulence.

Above the "critical" Reynolds number, the initial disturbances in the flow grew in the down-stream direction, forming the wave-like instability shown in Figure 51. This region of small disturbance growth, has been termed the linear range, inside of which the linear stability theory applies, as described by Sato (9), Michalke (15), etc. The same behavior was seen in the numerical solution of the problem for all four profiles considered. This was in direct contrast with the results of Grant (17) in case of axisymmetric jets. He found that thick boundary layer type profiles were unable to amplify the disturbances, and solutions were stable up to a Reynolds number of 2500. It was thus suggested by Grant that only "top hat" profiles had the potential to amplify two-dimensional disturbances, the kind of disturbances that could be introduced in the numerical scheme considered.

As mentioned before, in the present study all the four profiles considered did amplify the small disturbances when the Reynolds number was above 750-800. The "critical" Reynolds number was found to be unaffected by the shape of the initial profile considered. Nevertheless the rate of amplification of small disturbances was strongly dependent on the initial profiles, as shown in Figures 81-84. The
sharper the distribution the higher was the amplification rate, in the sense that for the same Reynolds number the distortion of the streamlines became more pronounced in the down-stream direction. This was in compliance with the stability analysis of Sato (9), based on the linear stability theory, and the results of the present study reported in section e, Chapter IV.

The results were supportive of the stability analysis reported in literature, e.g. Sato (9), Michalke (15), who have taken into account only two-dimensional effects. It further emphasized the differences that existed between 2-D and axisymmetric jet flows, in which three dimensional effects such as helical disturbances (Crow & Champagne) were instrumental in the onset of instability.

The observed wave-like instability evolved into large eddy structures in the form of vortices. Figure 53 distinctly exhibits the existence of such vortices, and was in compliance with the results of the numerous investigators concerned with the stability of jets, such as Beavers & Wilson, Crow, Freymuth(25), and Roshko(29). Vortices were representative of regions of concentrated vorticity, their formation the result of the roll up of the vortex sheet present at the mixing layer, Michalke (15). The existence of vortices in the numerical results were clearly manifested through the observation of the isodine and streamline plots, or the lines of constant vorticity, Figure 89. For the case shown, in the isodine plot, a regular train of islands of constant vorticity appeared at the mixing layer, which drifted in the down-stream direction. Based on the remarks of Michalke, these travelling regions of concentrated vorticity were
indicative of the existence of vortices in the flow.

The distance between vortices, in the numerical results for profile III, was found to be about $3h$. This distance was close to the shorter wave lengths observed in the experimental results. The resultant average Strouhal number was around 0.47. The result could be compared to 0.25-0.5 the experimental values obtained at Reynolds numbers of 1200. The values obtained in the present study, for the range of Reynolds numbers considered, seemed to be higher than those reported by Michalke & Wille (10), Sato (9), and Beaver & Wilson (13). Sato reported a constant Strouhal number of 0.23 for Reynolds numbers between about 1500 and 8000 and an increasing value for higher Reynolds numbers, $S = 0.013\sqrt{Re}$. Nevertheless he observed that the important factor which distinguished the observed wave length and corresponding Strouhal number, was not the Reynolds number but rather the thickness of the boundary layer at the exit of the jet, or the parameter $\frac{\nu L}{2hU_0}$. Moreover he reported a constant value of 0.015 for the Strouhal number, $N_j$, based on the momentum thickness, $\theta$. In the present study the non-dimensionalized momentum thickness for profile III, $\frac{\theta}{2h}$, was found to be equal to 0.037. Now $N_j$ and $S$ were related in the following fashion: $S = \left(\frac{2h}{\theta}\right)N_j$. Using the values of $N_j$ and $\frac{\theta}{2h}$ it was found that the Strouhal number, $S$ (based on nozzle width), was equal to 0.4. This was in close agreement with the results of the numerical solution of the problem.

The experimental Strouhal number of 0.25 reported above was based on the frequency of vortices having twice the wave-length of the
initial disturbances. The observed doubling of the wave-length was believed to be due to the coalition of the initial vortices into a single vortical structure. This was in accord with the results of Winant & Browand (28), Freymuth (9), and Roshko (29), Reynolds (31). The frequency spectrum analysis of the present study were supportive of the visualization results too, Figure 46. The observed peak frequency was reduced to half its initial value as the probe was traversed in the down-stream direction. This was indicative of a doubling of the wave-length associated with the disturbances since the drift velocity could be considered constant in that region, $\frac{x}{2h} < 8$.

The role of vortices in shaping the overall characteristics of jets is significant as documented by Rosko (29), Winant & Browand (28), Reynolds (31), etc. In the present study similar observations was made, as the role of vortices on the entrainment process and the growth of the jet was investigated. As was shown in Figure 59 bulk of the entrainment of the stagnant fluid into the jet took place mainly due to the presence of vortices. Due to the viscous action of the vortices, and their subsequent coalition, outside fluid was drawn into the jet and was trapped between the vortices, leading to their growth. Simultaneously some of the high velocity fluid from the core of the jet was ejected outwardly. Hot wire signals obtained from traversing the probe across the jet pointed out to the above idea, as first suggested by Lau et al (69), in the case of axisymmetric jets. Signals from the hot wire placed in the entrainment region showed upward spikes, suggesting a rise in the velocity whenever a vortex passed by, Figure 106a. When the probe was placed in the core region
Figure 106. Hot Wire Spikes at Different Locations Across the 2-D Jet (Re=750),
(a) $y/2h = 1.0$, (b) $y/2h = 0.3$, (c) $y/2h = 0.0$
of the mixing layer the reverse was found, as can be seen in Figure 106b. Also it was noted that the level of turbulence in the potential core region, Figure 109c, was smaller than the level of turbulence in the shear layer, suggesting that the effect of vortices was mostly confined to the shear layer.

The entrainment process described above not only was responsible for the partial growth of the vortices, but for the existence of the Reynolds stresses in the jet. The "periodic" cross flow of entrained fluid into and out of the jet, resulted in non-zero value of $u\overline{v'}$, as suggested by Lau et al. (69).

2. Electric Wind

The electric wind was visualized with the help of the heated-wire, as shown in Figure 52. The wind itself did not exhibit any large eddy type structure contrary to the 2-D jet flow above the critical Reynolds number. The flow field was similar to that of a laminar jet, with the major difference that converging rather than diffusing streamlines were observed. This was due to the accelerating effect of the existing electrical body forces, which were mostly present in the region between the wire and the ground grid system. This behavior was propagated beyond the ground grid system until the eventual breakdown of the jet occurred. The entrained streamlines were not noticeably affected by an increase in electrical potential from 10 KV to 12.5 KV as shown in Figure 55. The same behavior was observed in the numerical solution of the problem as shown in Figures 90 & 91. The solution was steady in the sense that the streamlines did not change by advancing in time. The closed contours in the vorticity plot were not indicative of any
vortices, since they did not drift in the down-stream direction with progression in time. No travelling wave type solutions, indicative of any instabilities in the flow were observed.

The results of the stability analysis of the electric wind based on the linear stability theory were supportive of the remarks made in above. For the electric wind velocity profile, it was found that the velocity fluctuations for the neutral case had a single sign only, for the case when antisymmetric disturbances were assumed. Based on the result of Sato (9) & Michalke (15) this indicated reduced rates of amplification, $c_i$, in the inviscid case. So it should have been of no surprise that due to reduced rate of amplification, coupled with the damping effect of viscosity the electric wind, behaved as a stable flow. Since the velocity distribution of the electric wind, based on experimental as well as numerical results, was independent of the applied potential it was expected that the flow remained stable immaterial of the applied electrical potential. This was clearly observed in the numerical and experimental results obtained at different potentials. This behavior was in contrast with that of the 2-D jet flow, and indicative of the fundamental differences that existed between the two. It has to be mentioned that these differences were expected since the corona wind unlike any so-called free jet did not initiate as a bounded flow. The constant shedding of vorticity from the boundary, i.e. the nozzle, was not present in the electric wind flow which would lead to its inherent stability.

The frequency spectrum analysis of the electric wind flow also was supportive of the arguments put forward in above. No significant
frequency peaks were observed in the spectrum, in the wide range of electrical potentials considered. This was indicative of the stable nature of the electric wind. Initially, it was expected though that a peak should have appeared in the high frequency side due to shedding of vorticity from the collector grid rods. The absence of a significant peak was indicative of the special nature of the electric wind flow, in the sense that it appeared that the wind maintained attachment to the rods, possibly due to the accelerating action of the electric field around the rod.

3. Modified Jet

The experimental as well as theoretical study of the modified jet exhibited the profound effect of the electric field on the characteristics of the 2-D jet flow. The action of the electric field was mostly felt, at Reynolds numbers below 1500, its influence diminishing at higher values.

The presence of the electric body forces induced higher velocities in the core of the jet, while at the same time creating a flow field in the outer-flow region of the initial 2-D jet which was otherwise stationary. This resulted in wider average velocity profile of the modified jet as compared to the 2-D jet flow, as evident from the hot-wire measurements presented in Chapter III. The velocity measurements as well as the visualization of the modified jet showed that the outer-flow region was dominated by the flow induced by the electric field, at all Reynolds numbers up to 2000. The effect of the electric field on the 2-D jet in the core region was expectedly dependent on the two defining variables of the flow, namely the Reynolds and the
EHD numbers. High EHD numbers, at low Reynolds numbers (e.g., \( \text{Re} = 500, \text{EN} = 1.0 \)) resulted in significant modifications of the initial 2-D jet flow. Increased core velocities, as much as twice with subsequent reduction in the width of the initial jet was observed based on both experimental and theoretical results. The observed streamlines were converging towards the centerline of the jet similar to the electric wind case, and in contrast with the 2-D jet flow where the flow diffused into the surrounding still air.

With reduced EHD numbers, which represented the ratio of the electric field body forces over inertia forces, the effect of the electric field on the two-dimensional jet was reduced. This fact was clearly exhibited in both the experimental and theoretical phases of the study. Figures 62 & 100.

It was stated before that the "critical" Reynolds number was close to 750, according to both experimental and theoretical results. The Schlieren photographs of the flow at the mentioned Reynolds number exhibited the existence of the formation of wave-like instabilities at the shear-layer. The same behavior was observed in the numerical solution of the problem. Upon the application of the electric field, above \( \text{EN} = 0.7 \), the mentioned instabilities disappeared and the modified jet exhibited characteristics similar to a stable laminar jet. It was thus concluded that complete stabilization of the 2-D jet was possible under certain conditions (low \text{Re}, and moderate \text{EN}'s). It is believed that this happened due to the alteration of the average velocity profile, resulting in reduced amplification rates of small disturbances, according to the findings of the present study, and Sato
The solution of the Rayleigh's equation for the modified jet profile exhibited the fundamental changes that occurred, as far as the stability characteristics of the jet was concerned due to the action of the field. This was similar to the stabilization effect of favorable pressure-gradients on fluid flows, which result in average profiles producing stable flows. In fact upon the review of the Navier-Stokes equation, it was seen that the electric body force and the pressure terms were interchangeable, and thus the same stabilization influence as favorable pressure gradient could be realized upon the use of electric fields.

The alteration of the average velocity profile due to the action of the electric field is best illustrated by comparing the vorticity distributions of the 2-D jet and that of the modified jet at the same Reynolds number as shown in Figure 107. It was observed that the maximum value of vorticity was greater in case of the modified jet indicating a sharper velocity profile. Also it was noticed that the point of maximum vorticity of the 2-D jet was shifted towards the centerline under the action of the field, accompanied with a reduction in the value of vorticity at the shear layer.

In looking at the numerical results, specially at lower Reynolds numbers, Figure 93, the isodine plot showed an interesting characteristic, as both 2-D jet and electric wind flow influenced the produced contours. Similar to the 2-D jet flow, series of isodines initiated at the nozzle exit, through the shear-layer, extending in the down-stream direction. Also areas of concentrated vorticity were
Re = 1312.5, EN = 0

Re = 1312.5, EN = 0.28

Re = 1312.5, EN = 0.4

Figure 107. A Comparison of the 2-D Jet and the Modified Jet Vorticity Distributions at $\frac{x}{2h} = 4$
observed close to the centerline of the jet resembling the vorticity distribution in the electric wind flow.

At higher Reynolds numbers (Re > 800) the observed instabilities evolved into vertical structure in the region between the nozzle and the ground grid for the geometry shown in Figure 16. With the application of an electric field, progressive stabilization of the jet occurred as the formation of vortices was delayed or eliminated as apparent from the Schlieren photographs presented before. It was also observed that the distortion in the entrained stream-lines were progressively reduced upon an increase in the electric field strength. The same behavior was observed in the numerical results as a definite reduction in the distortion of the streamlines occurred resulting in delayed vortex formation. The observed stability was believed to be related to the modification of the average velocity profiles as mentioned before.

The flow field of the modified jet was not only dependent on the Reynolds and EHD numbers, but on the electrode geometry considered as well. The behavior mentioned above was not realized when the distance between the corona wire and the ground grid was increased. An example of such a case was presented in Figure 56. It was seen that upon the application of the electric field, and although the EHD number was high (EN = 1.5), the formation of vortices was not prevented. Nevertheless it was observed that the vortices retained their initial size, unlike the 2-D jet flow, as they drifted in the downstream direction and their final breakdown was not observed. The distance between the vortices remained constant, as possible coalition of vortices was
eliminated. As stated by Winant & Browand (28), the growth of the shear-layer was strongly dependent on the action of vortices, and higher growth rates were realized for strong vortices. Based on the results presented in Figure 107, it was believed that due to the action of the electric field the vorticity and circulation (strength) associated with the vortices was reduced, and thus according to the argument stated above a reduction in the growth rate of the layer was expected. It was also believed that with weaker vortices and the dominant effect of the electric field in the outer flow region of the jet, reduced rates of entrainment into the jet occurred. This further discouraged the growth of vortices as some of the entrained fluid would have otherwise remained in the vortex, leading to its growth, and subsequent breakdown.
A. Summary & Conclusions

Experimental and theoretical studies of the flow fields associated with a two-dimensional jet, an electric wind, and a two-dimensional jet in the presence of an applied electric field were performed. The following were the important aspects of the study:

1) Inclusion of an insulated plate less than about 3.0 cm below the corona wire, resulted in narrower current density distributions and lower total currents, as compared to the case where no plate was present. Nevertheless the normalized current density distribution and the current-voltage relationship was found to be similar to previously established empirical formulas.

2) Based on the experimental results the critical Reynolds number, beyond which the 2-D jet became turbulent, was found to be $R_e = 750$. Above this value the small disturbances grew into vortical structures which drifted in the downstream direction. The observed vortices eventually broke down into complete turbulence. The Strouhal number corresponding to the vortices was found to be equal to 0.5, and their drift velocity equal to $0.5 U_0$. The distance between the vortices was about $2h$, although at times a doubling of the wave-length
and a subsequent reduction in the Strouhal number to 0.25 occurred. It is believed that the observed phenomena was due to the coalition of vortices.

Computer simulation of the problem indicated a critical Reynolds number in the range of 750-800. The Strouhal number at high Reynolds number (Re > 800) was found to be equal to 0.47, and the observed wave-length was about 1.6h. In the numerical results coalition of vortices was not realized, similar to the behavior encountered by Grant (17) in case of axisymmetric jets. But unlike Grant the effect of the initial velocity distribution at the nozzle exit was not as critical to the onset of instability in the two-dimensional jet. This illustrated the fundamental differences that existed between the two mentioned jet flows, axisymmetric and 2-D, as far as the more unstable modes of disturbances were concerned.

3) Insignificant diffusion and decay rates were observed up to \( \frac{x}{2h} = 6 \) to 8, for the 2-D jet flow. Beyond this the jet became turbulent, and a linear relationship between the mentioned parameters and the downstream location was observed. The role of the Reynolds number was rather insignificant as tripling its value did not greatly affect the slope of the lines.

4) Normalized component of the velocity fluctuation in the x direction \( \bar{u} \), exhibited its maximum value at the shear layer, while its counterpart in the y direction \( \bar{v} \), had its maximum value at the center-line. This value was equal to \( \bar{u} \) at centerline suggesting isotropy of turbulence at the centerline.
Variation in $\bar{u}$ was observed in the downstream direction. This indicated that self-preservation was not achieved, in the range covered in the present study, $\frac{X}{2h} \leq 25$.

5) The electric wind flow was defined solely by the parameter $R_{ehd}$, the Reynolds number based on the EHD characteristic velocity $U_E$, for a specific geometry considered. In the range of $R_{ehd}$'s considered, $R_{ehd} < 2000$, the flow was found to be stable, exhibiting converging streamlines due to the accelerating action of the electric body forces. The rates of decay and diffusion were insignificant as compared to the 2-D jet flow. Consequently low $\bar{u}$ and $\bar{v}$ were observed, along with values of Reynolds stress close to zero at $\frac{X}{2h} \approx 10$.

6) Modification of the 2-D jet flow was realized upon the application of an electrostatic field. The modified flow field could be defined in terms of two parameters $Re$ and $EN$, for a specific geometry that was considered. Significant changes in the structure of the initial 2-D jet flow occurred when $EN$ was higher than 0.4. The effect of the electric field was most significant when the 2-D jet flow exit velocity was lower or the same order of magnitude as the EHD characteristic velocity.

At low Reynolds numbers the flow was stable, and a narrowing of the jet occurred due to the accelerating action of the electric field. The isodine plots exhibited characteristics observed in both the electric wind and 2-D jet flows, as areas of concentrated vorticity was observed both at the shear layer and close to the centerline.

At higher Reynolds numbers, when $EN > 0.4$, enhanced stability of the jet was observed, as formation of vortices were either delayed or
completely eliminated. In the case where vortices formed their growth was prevented, and their coalition became non-existent. This occurred presumably due to the modification of average velocity profiles, with the consequent reduction in the strength of vortices.

Due to the reduced strength of vortices and the action of the electric field in the outer flow region of the jet, significant reduction in the entrainment of outside fluid into the jet occurred.

7) The stabilizing effect of the electric field is believed to be the result of the alteration of the average velocity profile, or the basic flow. The electric body forces were independent of the flow field, and thus did not directly enter the governing equation of stability. Their effect was implicit as the base flow was affected due to the presence of the mentioned body forces. The resulting average velocity profiles produced smaller growth rates of the small disturbances as compared to those associated with the 2-D jet flow.

8) Significant reduced diffusion and decay rates of the 2-D jet was observed under the stabilizing action of the electric field, specifically in the region bounded by the nozzle and the ground grid. Progressive reduction in the mentioned rates occurred as the $E_N$ value was increased.

Pronounced reduction in the values of $\tilde{u}$ and $\tilde{v}$ occurred due to the action of the electric field. The change was most significant at low Reynolds numbers and high EHD numbers.

The Reynolds stresses were affected upon the application of the electric field. The reduction in the value of Reynolds stresses was reminiscent of the similar occurrence observed in case of multiple
free jets. This happened due to reduced entrainment into the jet, and the observed stability of the jet as the linear range was extended in the downstream direction.

9) In closing, possible applications of the phenomenon in drying or cooling processes can be pointed out. First, because the level of turbulence in the jet was very much reduced after the application of the electric field, one could expect that the jet would be able to dry a given surface more uniformly, and with a controllable rate. Secondly because of the reduction of the entrainment of surrounding air which may be at a different temperature than the jet itself, the change in the temperature of the jet in the downstream direction would be less, and so the efficiency of the jet would increase. Thirdly, the observed reduction in the decay and diffusion rates, would enhance the heat transfer potential of the jet, leading to even higher efficiencies.

B. Recommendations for Further Study

The following are recommended for the purpose of further studies, as continuation of the present study:

1) Other electrode geometries, in addition to ones investigated in the present study, could be considered, and their effect on the corona discharge characteristics analyzed.

2) Different corona wire-ground grid systems should be integrated into the 2-D jet flow, in order to maximize the stabilizing effect of the electric field on the 2-D jet flow.
3) Extensive study of the different average velocity profiles, based on the linear stability theory, should be performed. This would result in the clear definition of stable and unstable regions in the \( \alpha R_e - c_r \) plane.

4) The heat transfer problem associated with the impingement of a 2-D jet, with or without the presence of an electric field, on a heated flat plate could be considered. The study could include both experiments and theoretical considerations.

5) A similar procedure as followed in the present study could be applied in the case of axisymmetric jets, to investigate the effect of an electric field on the stability characteristics of the jet.
APPENDIX A

a. Effect of Side Plates on the Onset of Instability

A brief visual study was initiated in order to observe the 2-D jet flow characteristics in the presence and absence of side plates, and to compare the results. Figures 108 & 109 are the results of such a study.

It was observed that the presence of the side plates did not have any significant effects on the onset of instability and the structure of the 2-D jet. The above observation was in compliance with the results of Hussain & Hussain (65) in case of axisymmetric jets.

b. Effect of Ground Rods on the Up-Stream Characteristics of the Jet Flow

The change in the flow structure upstream of the ground grid proved to be significant, specially when effective stabilization of the jet had occurred. The Schlieren photographs shown in Figure 110 demonstrated the fact that beyond the ground grid the flow became turbulent, even though significant stabilization of the jet was observed in the ionized region, between the corona wire and the ground grid system. Initially this was considered to be totally due to the fact that the stabilizing electrical body forces die not exist beyond the ground rods. This idea was soon to be discarded since use of a streamlined smaller size (0.5 mm in diameter) central rod resulted in the continuation of the stable flow above the ground grid system, Figure 111. The observed behavior was limited though to low Reynolds numbers and moderate electrical potentials, e.g. Re = 500 & $\phi_0 = 10$ kV.
Figure 108. 2-D Jet Flow in the Presence and Absence of Side Plates, Re = 750
Figure 109. 2-D Jet Flow in the Presence and Absence of Side Plates, Re = 937.5
Figure 110. Jet Flow at:

(a) $U_0 = 2.46 \text{ m/sec}, \phi_0 = 0 \text{ kV} \ (Re = 937.5, EN = 0)$

(b) $U_0 = 2.46 \text{ m/sec}, \phi_0 = 10 \text{ kV} \ (Re = 937.5, EN = 0.54)$
Figure 111. Jet Flow at:
(a) $U_o = 1.31$ m/sec, $\phi_o = 10$ kV ($Re = 500$, $EN = 0.75$), Rod Diameter = 1.6 mm
(b) $U_o = 1.31$ m/sec, $\phi_o = 10$ kV ($Re = 500$, $EN = 0.75$), Rod Diameter = 0.5 mm
For higher Reynolds numbers or higher electrical potentials no significant changes were observed above the ground grid by using the smaller size rod.

Nevertheless the observation depicted in Figure 111 exhibited the fact that it was mostly the tripping action of the ground rods that caused the flow to become turbulent immediately above the ground grid. It was of course expected the flow become turbulent some distance away from the ground grid system due to the absence of the stabilizing electrical body forces.
APPENDIX B

a. Procedure to Measure the Velocity Fluctuations

The following is a step by step operating procedure that was followed for measuring the rms value of the components of the velocity fluctuations in the x and y directions.

1) The anemometer signals were connected to inputs A and B on the Model 1015c Correlator.

2) The sensitivities of the input signals were equalized by adjusting the variable potentiometer on channel A or channel B (but not both) as required. The sensitivities were equalized when either the true rms or the DC reading of both channels were the same.

3) The V type hot wire probe was aligned so that each sensor was at approximately 45° to the mean flow.

4) The fixed gains were adjusted (equally) so that the true rms value of A and B did not exceed 10 volts.

5) With the output jacks connected to a true rms meter the rms value of the x and y components of the velocity fluctuations were read respectively on the A+B and A-B settings.

b. Procedure to Measure the Reynolds Stresses

The following is a step by step operating procedure that was followed for measuring the Reynolds stresses.
1) The anemometer signals were connected to inputs A and B on the Model 1015c Correlator.

2) The sensitivities of the input signals were equalized by adjusting the variable potentiometer on channel A or channel B (but not both) as required. The sensitivities were equalized when either the true rms or the DC reading of both channels were the same.

3) The V type hot wire probe was aligned so that each sensor was at approximately 45° to the mean velocity.

4) The fixed gains were adjusted (equally) so that the true rms value of A and B did not exceed 10 volts.

5) The true rms meter was connected to the auxiliary output of the correlator. A+B and A-B were adjusted so that they both read 0.5 on the 0-1 meter scale of the true rms (same range position for both but any range position was satisfactory).

6) The "uv correlation" knob was switched to u+v, the function to uv correlation, and the true rms meter to the mean square position. The number read on the rms meter was \( \frac{u'v'}{\sqrt{u'^2} \sqrt{v'^2}} \).
APPENDIX C

Stability Analysis

In this section the procedure for the stability analysis of two velocity profiles is outlined. The procedure of calculation presented here was one commonly employed, and was based on the linear stability theory. The basic flow was assumed to be two-dimensional and function of the lateral distance \( y \) only. Parallel flow was assumed as justified by the experimental results (7). Moreover it was assumed that there existed a known body force field which was function of \( x \), and \( y \) only and independent of the velocity field. The disturbance velocities \( u' \) and \( v' \) were assumed to be functions only of \( x \), \( y \), and time. The equations of motion and continuity were made non-dimensional by using half-breadth of jet \( \delta \), and central velocity \( U_c \) as the characteristic values, with the assumption that these did not vary with \( x \). This again was justifiable based on the experimental results, which exhibited insignificant decay and diffusion rates in the linear region.

Assuming that the disturbance velocities \( u' \) and \( v' \) were small the momentum equations were linearized and combined to give the Orr-Sommerfeld equation (9), with the inclusion of a body force term \( F(x,y) \) (e.g., \( F(x,y) = \frac{\partial \Psi}{\partial y} - \frac{\partial \Psi}{\partial x} \)).

\[
\left( \frac{U}{U_c} - c \right) (\tau'' - \alpha^2 \tau) - \frac{U'}{U_c} \cdot \tau = \frac{1}{4 \alpha R_e} \left( \tau v' - 2 \alpha^2 \tau'' + \alpha^4 \tau \right) + F(x,y)
\]  

(76)
in which primes meant differentiation with respect to \( y \). The function \( \tau \) shown was defined as follows: if \( u' \) and \( v' \) were expressed in terms of a stream function \( \Psi \) as

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then \( \psi \) would be expressed as
\[
\psi = \tau(y) \exp[i\alpha(x - ct)]
\]
where \( \alpha \) was the wave-number, and \( c \) was complex in the form \( c = c_r + ic_i \), in which \( c_r \) was the propagation velocity and \( c_i \) was a measure of the rate of amplification of the small disturbances.

Now since it was known that the instability of jets was an inviscid one (15), corresponding to \( Re \to \infty \), equation (76) was simplified to yield the Rayleigh equation with the inclusion of a body force term
\[
\left( \frac{U}{U_c} - c \right) (\tau'' - \alpha^2 \tau) - \frac{U''}{U_c} \tau = F(x,y)
\]
(79)

Now \( F(x,y) \) could be considered as a forcing function, and thus irrelevant in the stability analysis, since only solution of the homogeneous problem had to be considered (9). Consequently the inclusion of a body force field, such as above (independent of the velocity field) did not affect the governing equation (Rayleigh's equation). Nevertheless its effect was included implicitly in the specific average velocity, \( U(y) \), that was considered.

Based on the argument set forth in above the following equation had to be solved, irrespective of the fact that there existed a body force field or not
\[
\left( \frac{U}{U_c} - c \right) (\tau'' - \alpha^2 \tau) - \frac{U''}{U_c} \tau = 0
\]
(80)

It has to be emphasized again that different conditions, i.e. presence or absence of a body force field, was accounted for in the
velocity profile assumed for the basic flow, with profiles being evaluated from experiment.

Solution to equation (80) was found for the neutral case, that is when \( c_i = 0 \), so that the disturbances were neither damped nor amplified. It was known that when the basic flow has an inflexion point in the velocity distribution, the propagation velocity \( c_r \) was equal to the mean velocity at the inflexion point (15). In view of this \( c_r \) was easily determined for the mean velocity distributions considered.

For the neutral case \( c = c_r \) and thus equation (80) was written as:

\[
\left( \frac{U}{U_c} - c_r \right) \left( \tau'' - \alpha^2 \tau \right) - \frac{U''}{U_c} \tau = 0
\]

which was a Sturm-Liouville type equation, having \( \alpha^2 \) as its eigenvalue, and \( \tau \) as the eigenvector. The solution of equation (81) was obtained by numerical integration using the Runge-Kutta procedure (10). In evaluating the boundary conditions only anti-symmetrical disturbances were considered. In view of this and because of the symmetry of the basic flow the following boundary conditions resulted:

\[
\tau, \tau' \text{ were bounded as } y \to \infty
\]

\[
\tau = \text{const}, \text{ and } \tau' = 0 \text{ at } y = 0
\]

Having equation (81) with the boundary conditions given above the numerical integration was started at \( y = 0 \) and proceeded along the \( y \) axis. Procedure was repeated until the boundary condition at large \( y \) was satisfied.
The stability characteristics of two velocity profiles were investigated using the mentioned procedure. The two profiles that were considered were those of the two-dimensional jet flow at Re = 937.5, and the electric wind flow. Each distribution was expressed in terms of an analytical function which best fitted the data obtained in the experimental part of the study, Figure 112.

The values of $c_r$, the propagation velocity, and $\alpha$, the wave-number of neutral oscillations is shown in Figure 113 for the cases considered. The amplitude function of neutral oscillations for the three profiles considered is shown in Figure 114, the functions being evaluated for the anti-symmetrical case.

In accordance with the results of Sato(9) and Michalke(15), the results demonstrated the significant effect of the average velocity profile on the propagation velocity, $c_r$, and the amplitude function, $\tau$. In addition, based on the results of Sato, for the case when $c_i \neq 0$, the amplification rate of the small disturbances in the case of the electric wind (having an amplitude function as shown in Figure 114) is expected to be smaller than that of the two-dimensional jet flow.
2-D Jet
\[ \Pi - \frac{U}{U_c} = (1 + 0.138n^2 \left( \frac{2 - n^2 - \frac{4}{n}}{1 + 0.1977n} \right) \text{Sech}^2 0.8836n \]

Electric Wind
\[ \Pi - \frac{U}{U_c} = \frac{1}{(1 + 0.4n^2)^2} \]

Figure 112. Profiles Used in Stability Analysis
<table>
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<tr>
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<th>Profile I</th>
<th>Profile II</th>
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</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.21</td>
<td>0.2</td>
</tr>
<tr>
<td>$c_r$</td>
<td>0.46</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Figure 113. Calculated Values of $\alpha$ & $c_r$.
Figure 114. $\tau'$ Distribution for the Two Assumed Profiles
APPENDIX D
Numerical Stability & Accuracy

a. Truncation Errors

Equation 45 is the vorticity equation in its finite difference form, the accuracy of which is heavily dependent on the order of truncation errors involved.

In the process of developing the mentioned equation special attention was given to the convective terms. The objective was to minimize the truncation error associated with these non-linear terms. In view of this the Arakawa's method(57) having a truncation error of order $(\Delta x)^4$ was employed. As far as the other terms were concerned, a central differencing scheme, having truncation error of order $(\Delta x)^2$, was used for the diffusion terms, while a forward differencing scheme ($O(\Delta t)$) was employed to express the local rate of change of vorticity.

b. Grid Size

In choosing the appropriate grid size three factors were taken into account. These were: a) the grid size be sufficiently small so that the velocity profile at the jet exit could be adequately represented, b) the grid size be of a reasonable size to contain the computational cost within an acceptable budgetary limit, c) to simulate the general characteristics of the 2-D jet flow, and the effect of the electric field on it, for comparison with the flow visualization results.
The chosen grid size of 1/12 in the present study was found to be satisfactory so far as the mentioned criterions were concerned. The numerical results agreed reasonably well, in form, with the experimental results. In spite of this the order of accuracy of the results were of concern. In order to investigate this the grid size was changed to 1/24, while the number of grids in the x and y directions were also doubled to maintain the same distances, from nozzle to boundaries, as before.

An example calculation was performed in case of 2-D jet flow at $Re = 650$, with the new grid size. It was observed that although a maximum of 15% difference existed between the new flow parameters (vorticity, velocity, etc.) and the previous ones, the overall characteristics of the flow (Figure 115) remained similar to that observed in case of the larger grid size of 1/12. Based on this it could be concluded that the use of a smaller grid size is expected to result in flow fields similar in nature to that of the larger grid. Nevertheless the details of the flow such as the value of the critical Reynolds number may be affected, to a certain degree, by the specific choice of the grid size.

At this point it is appropriate to mention that the difference observed in the results, due to a change in the grid size, is believed to be partly because of the differences in the representation of the velocity profile at the jet exit. This happens because the values of vorticity and stream function at the jet exit, which were used as a boundary condition, were evaluated directly from experimentally obtained velocity distributions. As a result the values of vorticity
Figure 115. 2-D Jet flow Characteristics (Re = 650, Profile 1, Δ = 1/24):
(a) Streamlines, (b) Isodines
and stream function were to a degree affected by the specific choice of grid size.

c) Extent of the Computational region

The extent of the computational region in the x and y directions was determined based on both experimental and computational considerations. Based upon the experimental results, it was known that the diffusion effect of the 2-D jet in the laminar region was limited to not more than 0.75 cm from the centerline. In view of this the chosen distance of 1.5 cm from centerline to the right hand boundary (Figure 12), was considered appropriate for the assumed boundary condition to be applicable. In order to verify the above statement, the mentioned distance was doubled. An example calculation was performed. The results indicated that the flow parameters, as compared to the initial study remained unchanged up to two significant figures in the region of interest (0 < y < 1.5), Figure 116.

The number of grid points in the x direction was determined by the formula $2^n - 1$, as required by the FDS method (n is an integer). The number of grid points in case of 2-D jet flow was 128. As a result the downstream boundary was located at $x/2h = 10$, where h was the half width of the nozzle. Based on the experimental results the 2-D jet became fully turbulent in the region $x/2h > 6$ to 8. Because of this the validity of the numerical results was practically limited to $x/2h < 6$.

Now the objective of the study was to investigate the stabili-
Figure 116. 2-D Jet Flow Characteristics (Re = 650, Profile I, no. grids = 72x128):
   (a) Streamlines, (b) Isodines
lity characteristics of the 2-D jet. The transition region of the jet, which was of prime concern was known to be, based on the experimental results, to be in the region, $0 < x/2h < 6$, beyond which the flow became turbulent. In view of this, and the limitations on the choice of number of grids, because of FDS scheme, the chosen number of grids ($128, n = 7$) was found to be the minimum value which resulted in successful simulation of the transition region of the jet. It is appropriate to mention that the number of grid points that was smaller than 128, and could be used in FDS was 64 ($n = 6$). This then would result in the upper boundary to be well inside the transition region.

In order to investigate the validity of the arguments put forward in above, the number of grids in the $x$ direction was increased to 256 ($n = 8$), and an example calculation was performed. The results indicated that the flow parameters remained unchanged up to two significant figures in the region of interest, $0 < x/2h < 6$, as shown in Figure 117.
Figure 117. 2-D Jet Flow Characteristics (Re = 650, Profile I, no. grids = 36x256):
(a) Streamlines, (b) Isodines
APPENDIX E

Computer Program Used in the Numerical Study of the Problem
NUMERICAL SOLUTION OF A 2-D JET FLOW

DIMENSION U(100,140),V(50,140),NW(100,140),L(100,140),T(100,140)

C SOLVING THE VORTICITY EQUATION (N-S EQ)

DO 110 J=1,NJ
DO 10 I=1,NI
SF(I,J)=0.0
10 CONTINUE
110 CONTINUE
INPUT DATA TO FDS FOR SOLVING POISSON'S EQUATION

DO 112 I=1,IMAX
   A(I)=1
   C(I)=1
   Q(I,1)=W(I,1)*DEL*SF(I,1)
   W(I,1)=W(I,1)+D(I,1)*DEL*SF(I,1)
   Q(I,1)=W(I,1)*DEL*SF(I,1)
DO 112 I=1,IMAX
   Q(I,J)=W(I,J)*DEL*SF(I,J)
   W(I,J)=W(I,J)+D(I,J)*DEL*SF(I,J)
   Q(I,J)=W(I,J)*DEL*SF(I,J)
CONTINUE

TRANSFORMING OUTPUT OF FDS TO THE ASSUMED GRID SYSTEM

DO 113 J=1,JMAX
   SF(J,1)=W(J,1)*DEL*SF(N1,J+1)
   SF(J,J)=W(J,J)*DEL*SF(N1,J+1)
CONTINUE

IF (J.LT.IMAX) GO TO 6

WRITE(*,*) T
220 CONTINUE
T1=U+1:T0=U+1
T3=U+1:
X1=E(1)/T1:Y1=E(2)/T1:Z1=E(3)/T1:
E(1)=T1:E(2)=T1:E(3)=T1:
CONTINUE

250 CONTINUE
!
DO 140 L=2,140

-REDUCTION TO THE SECOND LEVEL-

CONTINUE
!
-REDUCTION TO THE THIRD LEVEL-

-REDUCTION TO THE SECOND LEVEL-

-REDUCTION TO THE THIRD LEVEL-

-REDUCTION TO THE SECOND LEVEL-

-REDUCTION TO THE THIRD LEVEL-

END
CALL INVL(N-1,TO)
K=K+1
IF(K.GT.NMAX) GO TO 350

300 CONTINUE

BACKWARD SUBSTITUTION
J=J+1,NMAX
C
CONTINUE

310 CONTINUE

SOLUTION FOR THE HIGHEST LEVEL
IF(N.GT.NMAX) GO TO 315
CALL INVL(N,TO)
JX1=LIND(N-1,J-M)
JX2=LIND(N-1,J+M)
DO 340 T=1,NMAX
340 CONTINUE

SOLUTION FOR 2ND HIGHEST THRU LOWEST LEVEL
350 CONTINUE

J=J+1
IF(N.EQ.1) GO TO 320

320 CONTINUE

J=LIND(N,1)
GO TO 340

330 CONTINUE

J=J+2
IF(K.EQ.1) GO TO 350
J=J+1
IF(K.EQ.2) GO TO 330
J=J+1

340 CONTINUE

J=J+1
IF(K.EQ.2) GO TO 330
J=J+1

350 CONTINUE

J1=J1+1
IF(J1.EQ.J) GO TO 330

330 CONTINUE

J=J+1
IF(J.GT.NMAX) GO TO 320
J=J+1
IF(J.EQ.J1) GO TO 340

340 CONTINUE

J=J+1
IF(J.EQ.J1) GO TO 340

350 CONTINUE
340 CONTINUE
C CALL INVI(T,UI)
C
345 CONTINUE
IND J = 1
GO TO 350
320 DO 325 J = 1, IMAX
IF (Kuğu, T(I)) T(I) = T(I) + X(I,J-1)
IF (K > KMAX) T(I) = T(I) + X(I,J+1)
325 CONTINUE
CALL INVI(T,UI)
GO TO 320
327 CONTINUE
350 CONTINUE
C
360 CONTINUE
C
RETURN
END
FUNCTION IF 1 NO ('It KX')
COMMON /I/ Xi(10U+1), XI(10U+2), XI(10U+6), XI(10U+9), XI(10U+10), XI(10U+5)
X(11U+1), XI(11U+6), XI(11U+9), XI(11U+10)
END
SUBROUTINE INV(N,H,1)

C THIS SUBROUTINE INVERTS THE n x n DIAGONAL SOLUTION NMAX TIMES

COMMON /Y/ Y(100,100),U(100,100),T(100,100),LA(100),A(100),P(100)
C
L.C(10),MAX1
DIMENSION H(100),P(100),T(100),U(100),A(100)

DO 20 I=1,NMAX

20 IF (I.EQ.1) THEN

DO 10 I=1,MAX1

10 H(I)=C(I)-KAN

CONTINUE

IF (I.EQ.NMAX) THEN

DO 50 J=1,MAX1

50 H(I)=C(I)*S

CONTINUE

END
BIBLIOGRAPHY


