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DOYLE, WILLIAM HOWARD

USING AN ADVANCE ORGANIZER TO ANCHOR A SUBSUMING FUNCTION CONCEPT TO FACILITATE LEARNING, TRANSFER, AND RETENTION IN REMEDIAL COLLEGE MATHEMATICS

The Ohio State University

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USING AN ADVANCE ORGANIZER TO ANCHOR A SUBSUMING FUNCTION
CONCEPT TO FACILITATE LEARNING, TRANSFER, AND
RETENTION IN REMEDIAL COLLEGE MATHEMATICS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

William Howard Doyle, B.S., M.S.

* * * * *

The Ohio State University

1981

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Education
To my wife and children who have lovingly and patiently waited for me during my captivity to a long doctoral program.
ACKNOWLEDGEMENTS

I would like to thank my committee, Dr. Joseph Ferrar, Dr. Alan Osborne, and Dr. Arthur White, for their help in preparing this paper. In particular I extend my appreciation to Dr. Osborne, my major advisor, for the time, advice, and encouragement freely given throughout the course of my degree program. I am appreciative of the help and facilities provided by Dr. Franklin Demana, Dr. John Riner, and others in the Department of Mathematics.

It is most important to me to thank the good Lord Who continues to make my life meaningful and worthwhile. In the completion of this program, I have found that the promise given to Joshua in the fourteenth century B.C. has been fulfilled for me today:

"This book of the law shall not depart out of your mouth, but you shall meditate on it day and night, that you may be careful to do according to all that is written in it; for then you shall make your way prosperous, and then you shall have good success. Have I not commanded you? Be strong and of good courage; be not frightened, neither be dismayed; for the Lord your God is with you wherever you go." Joshua 1:8,9; the Bible.
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CHAPTER I

INTRODUCTION

Overview of the Study

Ausubel's theory of meaningful learning is perceived to be useful as an explanation of many events in the learning of mathematics. Moreover, in the nearly twenty years since its initial description, the extensions and refinements of the theory have proved productive in identifying questions that need answers in order to improve the robustness and understanding of the theory. The problem that this research addressed was the exploration and further refinement of the pivotal relationship between two key constructs of the theory, subsumption and advance organizers, as they affect the learning of mathematics.

An advance organizer was developed for instruction concerning lines and slope. Presented in the advance organizer was the subsuming concept functions and their graphical representations. The primary purpose of the research was to determine if the advance organizer can anchor this subsuming concept in the cognitive structures of the learners. To achieve this purpose, a measurement process that included the use of pencil and paper transfer items along with interviews was used. The secondary purpose was to determine if the ability to learn, retain, and transfer knowledge about lines and slope is facilitated for those subjects who have the subsuming concept in cognitive structure.
The study utilized 36 sections of Mathematics 100 during Autumn quarter, 1980. All sections studied learning materials on lines and slope as presented in the text used in the course. The students and topics were selected for this study because students typically experience much difficulty in comprehending the concepts presented in the unit on lines and slope. The learning materials were adopted as found in the text in order to enhance the practical significance of the study for instruction.

The Psychological Theory

Ausubel's subsumption theory of meaningful learning and retention has generated much interest in mathematics education. According to this theory cognitive structure is hierarchically arranged. At the top of the hierarchy are highly inclusive, general concepts. Branching down are concepts that are less inclusive, less general, and more differentiated from each other. A simple but illustrative model might be the relationship between a composite integer and its prime factors organized via a factor tree. The primes represent the detailed facts within a given sphere of knowledge. The intermediate and original composite numbers represent concepts that subsume facts and other less inclusive concepts below them in the tree. The cognitive structure of an individual consists of a number of such hierarchies that vary in degree of complexity. The individual concepts (composites) belonging to the structure are called subsumers. The process by which the meanings of the less inclusive concepts are incorporated into the more inclusive concepts above them in the structure is called subsumption.

Subsumers are the basis of Ausubel's theory. When new material enters cognitive structure, it is linked with subsumers already existing in the structure. The linking process facilitates organization of the new material in order that it can become an integrated part of cognitive structure.
(Ausubel, 1962). If no subsumers of any degree of appropriateness for the content of the learning materials are present in cognitive structure, learning will be rote. Discrete chunks of knowledge are added to memory, but these do not integrate well into cognitive structure. The result is less stable learning (Ausubel, 1962).

The theory predicts that the quality of learning and retention is a function of the quality of the integration and thus a quality of the subsumers (Ausubel, 1962; Mayer, 1979). The quality of a subsumer is related in part to its clarity and stability in cognitive structure. Ausubel does not explicitly define these terms. For purposes of this study the clarity of a subsumer shall define a state of understanding that is free of ambiguity and confusion. The stability of a subsumer shall define a state in which the subsumer can resist the tendency to become a non-differentiated part of a more inclusive subsumer. These two aspects of the quality of a subsumer are not independent. Each plays an important part in Ausubel's theory. Repetition, relative age, use of exemplars, and multi-contextual exposure are variables which affect the clarity and stability of subsumers (Ausubel, 1963, p. 29).

When subsumers of an appropriate degree of clarity and stability for particular learning materials are not available in cognitive structure, subsumers of lesser clarity and stability are pressed into use. The discrimination between new material and existing cognitive structure will be poor, and new concepts will not be well integrated into existing cognitive structure. Consequently, unclear, ambiguous, misconceived meanings with little retentive value will be added to cognitive structure. (Ausubel, 1963, p. 84).

To remedy these problems (absence of relevant subsumers; potential use of unclear or unstable ones), Ausubel suggested the use of advance
organizers. "An advance organizer is a set of introductory instructional materials designed to establish a concept in cognitive structure which can serve as a subsumer for the detailed learning materials to follow" (Kozlow, 1978, p. 18). Advance organizers should be written at a higher level of generality and inclusiveness than the learning materials to follow. Mayer (1979) has presented a version of subsumption theory (he calls it "assimilation coding") which also calls for the use of advance organizers.

Advance organizers can serve both to establish subsumers in cognitive structure if none are present (expository advance organizers) and to add clarity and stability to subsumers already present (comparative advance organizers). If either of these potential results occurs, meaningful learning and retention should be improved.

Ausubel (1963, p. 83) gave more insight into the role of comparative advance organizers. He said they should relate the similarities of learning materials to knowledge already in cognitive structure. They should do this by explicitly drawing upon and mobilizing the available subsuming concepts already in cognitive structure for use as subsumers. They should also increase the discriminability of genuine differences between new learning materials and ideas already in cognitive structure.

The use of advance organizers should allow the more detailed information in the learning materials to be incorporated into cognitive structure with a minimum of ambiguity, competing meanings, and misconceptions caused by the learner's prior knowledge of related concepts. "... as these clearer, less confused new meanings interact with analogous established meanings during the retention interval, they would be more likely to retain their identity" (Ausubel, 1963, p. 83).
Ausubel and associates performed several experiments during the early sixties (1960, 1961, 1962) which demonstrated facilitative effects for advance organizers on learning and retention. Since that time there has been a host of studies yielding inconclusive results.

In 1975 Barnes and Clawson reviewed 32 studies and concluded that given the then current state of advance organizer preparation, they did not facilitate learning (p. 651). The conclusion was drawn based on counting those studies that found significance and comparing this number, 12, with the number, 20, of remaining studies. (Included in the remaining studies were those with non-significant differences favoring the organizer group or the control group, and those with significant differences in favor of the control group.)

This study sparked a renewed interest in advance organizer research. Kozlow (1978), Luiten et al. (1980), and Mayer (1979) each took a fresh look at the status of this research. Each found fault with the methods and resulting conclusion of the Barnes and Clawson study.

Mayer (1979, p. 373) pointed out three limitations of the study:
"... they did not specify the conditions under which advance organizers should have an effect, they did not present data that adequately analyzed the nature of the learning outcomes, and they did not adequately discuss the problem of experimental control in the studies they cited." He suggested future research should test the cognitive process theories which underlie the advance organizers.

Luiten et al. (1980, p. 3) suggested Barnes' and Clawson's counting technique was flawed in that it failed to consider positive treatment effects which are not statistically significant. They concluded that the results of such an analysis are biased against a favorable finding for
advance organizers.

Kozlow (1978) found two major limitations with the Barnes and Clawson study. First, they did not perform any statistical analysis of their data. Because of the large number of variables involved in advance organizer studies (such as grade level, ability level, mode of organizer presentation, subject area, and length of treatment), "... simple frequency counts probably will not provide a sufficient basis for the formation of valid generalizations" (p. 2). Second, they did not consider the content and organizational properties of the materials used as advance organizers.

Kozlow (1978), Luiten et al. (1980), and Mayer (1979) each suggested that advance organizers have positive effects on learning and retention. Kozlow and Luiten et al., using meta-analysis techniques, attempted to identify those variables which have had the most effect in advance organizer research.

Kozlow found that advance organizers may be more likely to show facilitative effects when the reading level is appropriate to the grade level, when the rate of introduction of new ideas is slower, when students are given more time to process the advance organizer information, and when comparative advance organizers are used (1978, p. 139). Both Kozlow and Luiten et al. (1980, p. 16) found that advance organizers may be more effective at higher grade levels.

Kozlow suggested the need to control research design in future advance organizer research. Procedures should be included both to determine if information in the advance organizer alone can help answer test questions over the learning materials and if extra study time for the organizer group compared to the control group can improve treatment group achievement. Further, he suggested that the experimenter should determine if subjects have the desired subsumer in cognitive structure before the
experiment begins. Finally, he suggested the need to make the level of interest of the control introduction similar to that of the advance organizer introduction.

Each of these variables is a significant factor that may affect outcomes in any advance organizer study. This study carefully controlled or collected data concerning effects of many of these variables. The following model may help summarize that part of Ausubel's theory which forms the foundation for this study.

![Diagram](image)

1. Advance organizers produce subsumers.
2. Subsumers facilitate learning.
3. Potential influence (direct) of advance organizers on learning the content of the learning materials to follow.

Figure 1. A model for the roles of subsumers, advance organizers, and learning materials.

This model shows the basic role of subsumers in facilitating the integration of new material into cognitive structure (2). It suggests that advance organizers may be able to plant appropriate subsumers in cognitive structure (1). Finally, it suggests that some advance organizers may have a direct effect on the desired learning outcomes (3). This can
happen if the advance organizer contains information that will help sub-
jects answer questions on the learning materials achievement test without studying the learning materials.

Need for the Study

Looking at advance organizer literature in terms of the model in Figure 1 gives some indication as to the need for this study. Most of the studies with advance organizers have considered the question: Do advance organizers facilitate learning and/or retention? In terms of the model, the studies have made no attempt to distinguish between its three stages. Perhaps this lack of refinement has contributed to the non-significance syndrome.

Mayer (1979) inferred that the failure to consider the separate stages of his "assimilation coding" model has caused much confounding of results in advance organizer studies. Examination of the individual stages may help clarify the role of advance organizers in the meaningful learning theory.

Objectives of the Study

The study consisted of two parts. Part I looked specifically at stage (1) of the model. Kozlow (1978, p. 144) suggested such an approach. The question investigated was the following: Can an advance organizer establish a desired subsumer in cognitive structure? Part II looked specifically at stage (2) of the model. If an advance organizer can establish a subsumer in cognitive structure, will the subsumer facilitate understanding of the learning materials to follow? If the answer to both questions is "yes" and if Ausubel's theory holds, then advance organizers
should help facilitate meaningful learning, retention, and transfer. If advance organizers do not establish subsumers, then a new means of generating subsumers in cognitive structure should be found or advance organizer theory should be modified or rejected.

The literature does not provide any clear direction as to how to answer the major question of part I. Shumway (note 1) suggested that a transfer task might be used along with an achievement test over the advance organizer to verify the existence of a subsumer in cognitive structure. On the basis of this suggestion, it was decided to administer transfer items in an interview setting. The rationale for using interviews is twofold: (1) Direct interaction with students gives more confidence that what they really know is being assessed; and (2) Subjects are required to deal with the subsuming concept in a second (interview) context. Success on the transfer items in the interview setting will be additional evidence that the students understood the concept.

Why use transfer items: (1) To determine if a proposition is true, one often decides what ought to happen in light of that proposition. According to Ausubel, subsumptive learning occurs if a person extends or modifies previously learned propositions. If subjects have the subsuming concept in cognitive structure, they ought to be capable of success in a task which requires an extension of that concept. Appropriately written transfer items require an extension of the ability to identify functions and to justify the choice. Therefore, if subjects are successful on a transfer task, it is an indication or bit of evidence that the subsuming concept was present in cognitive structure.

(2) Success on transfer items indicates the person has a more global perspective as to what a function is. The importance of a global
perspective is evidenced by Thomas' (1975) statement that a person must be able to identify functions in several contexts before the person has mastered the function concept at the classification level.

To test these ideas, a pilot study was designed in which all subjects were given an achievement test over the concepts presented in the advance organizer entitled "Extending Number Systems." Then a transfer task was administered verbally to selected subjects in an interview setting. The Pearson correlation between achievement test results and transfer task results was 0.80. This result indicates that students having the desired subsumer in cognitive structure can be identified by using an appropriate achievement test, validated by a transfer task in an interview setting. Joseph Novak (note 2) concurred that this approach may be a valid one to answer the desired question.

Part II considered two other subquestions. First, which students do advance organizers help? Many studies (Ausubel & Fitzgerald, 1961, 1962; Caponecchi, 1973; Christie & Shumacher, 1975; Kuhn, 1967; Allen, 1970) indicate lower ability students benefit more from advance organizers. Perhaps they are not able to organize the concepts in the learning materials by themselves. However, low ability students will have greater difficulty understanding the more inclusive, subsuming concept presented by the organizer materials. This implies that it may be more difficult to establish a subsumer in the cognitive structure of a low ability student. In contrast, Grotelueschen and Sjogren (1968), Kozlow (1978), and Luiten et al. (1980) found that advance organizers may help high ability students more. This result is possible especially when the content is mathematics, when high level questions are asked on the learning materials achievement test (Kozlow, 1978), and when the learning materials are more complex
(Grotelueschen & Sjogren, 1968). No consistent approach to grouping students by ability has been used.

Second, what learning outcome(s) are facilitated by an advance organizer? Mayer (1979) found evidence that advance organizers should facilitate performance on transfer items. Ausubel (1978, p. 225) stated the following:

Tests of application of concepts to novel problems, especially when administered six weeks or more after instruction, are much more likely to show positive facilitation of meaningful learning that should result from appropriately designed advance organizers.

Groteluschen and Sjogren (1968) found a similar result. They stated that an advance organizer for the study of a mathematical unit on measurement facilitated transfer.

The formal reasoning ability of the subjects was also measured. The subsuming concept in the advance organizer for this study is a formal level concept (Thomas, 1975). Thomas found that concrete level subjects were able to work with functions in finding images, preimages, domain, and range. He further found that subjects were not able to discriminate instances and non-instances of function and to formulate a correct criterion for making such discriminations until the post-concrete stage. Lawson and Renner (1975) found that subjects were unable to answer formal concept questions over chemistry, physics, and biology correctly until the post-concrete stage. A strong correlation between students' formal reasoning ability and ability to understand the advance organizer material was expected.
**Problem Statement**

The major problem was to determine if a comparative advance organizer could establish "Functions and Their Graphical Representations" as a subsuming concept in cognitive structure. Part (1) of the model was studied. Two potentially confounding factors were controlled. The first was the preexistence of the desired subsumer in cognitive structure. Any attempt to measure the effectiveness of an advance organizer in generating the subsumer would be confounded to the extent that the subsumer already existed in cognitive structure.

To control this factor, learning materials were identified for which it was agreed the chosen subsuming concept would probably be unfamiliar to the identified population. This agreement was based on both the researcher's and others' teaching experiences with subjects at the level identified for this study. Both pretesting of advance organizer and control groups and interviewing of control subjects confirmed this opinion.

The second confounding factor was the relative levels of interest of the control and advance organizer groups. The effect on the subjects' state of arousal should be similar for the two groups. A short semantic differential was administered between part I and part II of the study. The semantic differential measured the subjects' attitudinal readiness to study the materials on lines and slope.

The secondary problem was to determine if the advance organizer (AO) group would outperform the control group in achievement over the learning materials on lines and slope. Part (2) of the model was studied. This problem was broken into two sub-questions as follows;
1) Will the total AO group outperform the total control group on any of the following three measures?
   a) an achievement test over the learning materials;
   b) specially written transfer items as suggested by Mayer (1979);
   c) a short-term (seven day) retention test.

2) On the same three measures listed in sub-question 1), will the smaller sample of subjects who were judged to have the subsumer in cognitive structure outperform a sample of control subjects matched by ability as measured by average performance on Midterms I and II?

A third potentially confounding factor, stage (3) of the model, was controlled. If information in the advance organizer made possible an improvement in the performance on an achievement test over the learning materials for students who did not read the learning materials, then it would be impossible to determine the ability of a subsumer to facilitate learning (stage (2) of the model). This factor was controlled by forming two groups, matched by performance on the first two midterms, which were not part of the main study. One group was given only the advance organizer and an achievement test over the learning materials. The other group was given only the achievement test over the learning materials. Any significant difference in achievement between the two groups should be attributable to the direct effect of reading the advance organizer described in stage (2) of the model.

**Method of the Study**

The mathematics course used in this study was designed to review elementary concepts of algebra and geometry. The advance organizer and control groups were formed by matching sections by ability and then
randomly assigning each member of a matched pair to one of the treatments.

The advance organizer presented the subsuming concept "Functions and Their Graphical Representations." Five major tests were administered including a test of formal reasoning ability, a pretest, and three achievement tests. Forty-one subjects completed transfer questions in an interview setting.

Assumptions and Limitations

The population for this study typically has been average or below average in high school mathematics. Most earned a minimal number of credits in secondary mathematics. However, it was assumed these observations were indicative of mathematical aptitudes rather than general intelligence. In particular, it was assumed the full spread of Piagetian reasoning abilities was present in the classes.

The criteria for placement into Mathematics 100 was a score roughly in the lowest quartile on the mathematics placement test taken by all freshmen entering the university. Compared to the total freshman population, the mathematical ability of these students was very low. The generalizability of the study is limited. However, within this context, it was assumed the students' mathematical abilities would vary widely. Student performance in the past has been distributed throughout the full grade range. This assumption made feasible the use of blocking on ability.

It was further assumed that the instructors for the advance organizer sections, mostly graduate teaching associates, used the concept maps and made use of the advance organizer for instruction during the study of the learning materials. Even though the organization of this mathematics program requires a high degree of standardization as to schedule, topics, examinations, and teaching approaches, experience has shown that past
instructors have remained highly individualistic, in many instances following their own rather than the suggested procedures.

The effects of instructor and absenteeism were equally distributed by the random assignment of sections to the advance organizer and control groups.

**Definition of Terms**

1. **Achievement/Learning**: The raw scores attained by students on the achievement tests over the advance organizer (ATAO) and learning materials are used as measures of achievement. These two examinations assess understanding of the content presented during this study.

2. **Advance Organizer**: "An advance organizer is a set of introductory instructional materials designed to establish a concept in cognitive structure which can serve as a subsumer for the detailed learning materials to follow" (Kozlow, 1978, p. 18). In this study the advance organizer was read by the subjects.


4. **Comparative Organizer**: An advance organizer which both explicitly discriminates and describes the similarities between concepts relatively familiar to the learner and the new ideas to be learned.

5. **Concrete Level Reasoner**: An individual who is able to think in a logically coherent manner about objects that do exist and have real properties and about actions that are possible; who can perform the mental operations involved both when asked purely verbal questions and when manipulating objects (Sinclair, 1971). A TOLT
(Tobin and Capie, 1980) score of zero operationally defines a concrete level reasoner.

6. **Formal Level Reasoner:** An individual who has fully mastered the operations described for the concrete stage and, in addition, has the capability of reasoning hypothetically, possesses a system of combinatorical operations which give rise to the hypothetical thought, and has the ability to use proportional reasoning schemes (Sinclair, 1971). A TOLT score of four or greater operationally defines a formal level reasoner.

7. **Function Classification:** "... the ability to discriminate instances and non-instances of function and to formulate a correct criterion for making such discriminations." (Thomas, 1975, p. 163).

8. **Function Process:** "... the ability to work with various representations and names of functions in finding images, preimages, domain, range, and sets of images." (Thomas, 1975, p. 163).

9. **Kinetic Structure Analysis:** Developed by O. R. Anderson (1971), this procedure concerns the serial ordering of information in a verbal communication and the relatedness of adjacent segments of the communication (O. R. Anderson, 1971, p. 7). The first two of the following four terms are used in the coding procedures for obtaining the data from the communication for the analysis. The last two define properties of the communication which are computed from the data.

   a. **Discourse Unit:** A complete thought in a verbal communication. It is a sentence or portion of a sentence containing a subject, predicate, and modifying subordinate clauses (Anderson, 1971, p. 29).
   
   b. **Verbal Element:** A word or phrase in a verbal communication which
contributes to the substantive meaning of the communication
(Anderson, 1971, p. 31).

c. Commonality: A measure of the extent to which adjacent discourse
units in a verbal communication contain common elements

d. Progression Density: A measure of the rate at which new elements
are introduced in a verbal communication (Anderson, 1971, p. 17).

10. Learning Materials: Those materials, activities, lectures, and films
designed to teach the concepts, facts, and generalizations which the
subjects are to learn. They are distinguished from the advance
organizer by a lesser degree of abstraction, generality, and
inclusiveness.

11. Levels of the Function Concept: The function concept was pre­
sented in the advance organizer at two levels which for understanding
require increasing ability to think formally (Thomas, 1975). They
are function process and function classification.

12. Short Term Retention: The raw scores attained by students on a
subtest of the third midterm were used as measures of retention. As
such, the time span for the memory task was approximately seven days.
The subtest consisted of five weighted items testing understanding
of the unit on lines and slope.

13. Subsumer: A general, inclusive concept in cognitive structure to
which the information in the detailed learning materials can be
linked for integration into cognitive structure (Ausubel, 1963, p. 28).

14. Subsumption: The use of a subsumer for integration of the information
contained in detailed learning materials into cognitive structure.
15. TOLT: Test of Logical Thinking (Tobin and Capie, 1980). A paper-pencil test comprising ten items designed to measure students' ability to reason formally. Eight items are multiple choice, requiring the student to choose both the correct answer and a best reason for the correct answer. Two items require written answers.

16. Transfer items: Items 16-18 from the learning materials achievement test, they are designed to measure students' ability to apply concepts from the learning materials to new situations.

17. Transfer task: Written and verbal questions designed for use during the interview to test students' ability to apply the subsuming concept "Functions and Their Graphical Representations" to new situations.

Hypotheses

Part I

Hypothesis 1: There will be a significant correlation between subjects' scores on TOLT and their scores on the achievement test over the advance organizer.

Hypothesis 2: The advance organizer group will perform significantly better than the control group on the ATAO process and classification subtests.

Hypothesis 3: This hypothesis was considered in two parts which were tested separately:

3a: Subjects at all levels of TOLT performance will score significantly higher than their control counterparts on the ATAO process subtest.

3b: Subjects at all levels of TOLT performance will score significantly higher than their control counterparts on the ATAO classification subtest.
Hypothesis 4: This hypothesis was considered in two parts which were tested separately:

4a: Formal reasoners will perform significantly better than non-formal reasoners on the ATAO classification subtest.

4b: There will be no significant difference between formal and non-formal reasoners on the ATAO process subtest.

Hypothesis 5: There will be a significant correlation between interview scores and combined scores on the classification and transfer subtests of the ATAO.

Part II

Hypothesis 6: There will be no significant difference between advance organizer and control groups on the semantic differential.

Hypothesis 7: There will be no significant difference on the learning materials achievement test between the two matched groups, one of which studied the advance organizer, neither of which studied the learning materials.

Hypothesis 8: The total advance organizer group will perform significantly better than the total control group on the achievement test over the learning materials, items 1-15 (called LMAT).

Hypothesis 9: The total advance organizer group will perform significantly better than the total control group on the transfer items (questions 16-18) from the learning materials achievement test.

Hypothesis 10: The total advance organizer group will perform significantly better than the total control group on the retention test.

Hypothesis 11: The advance organizer subjects who are judged to have the subsumer in cognitive structure will perform significantly better on the LMAT than their control counterparts matched by ability as measured by
average performance on Midterms I and II.

**Hypothesis 12**: The advance organizer subjects who are judged to have the subsumer in cognitive structure will perform significantly better on the transfer items from the learning materials achievement test than their control counterparts matched by ability.

**Hypothesis 13**: The advance organizer subjects who are judged to have the subsumer in cognitive structure will perform significantly better on the retention test than their control counterparts matched by ability.

**Plan of the Report**

The report is presented in five chapters. The intention of Chapter I was to familiarize the reader with the problem being investigated and to state the research hypotheses. Chapter II presents a review of related literature and some findings of pertinent studies. Chapter III describes the design, research procedures, instrumentation, the treatment, and statistical procedures. Chapter IV presents the results. Chapter V presents discussion of results, final conclusions, and recommendations.
CHAPTER II

REVIEW OF LITERATURE

Theories of Meaningful Verbal Learning

Ausubel has set forth a subsumption theory of meaningful verbal learning (1962). The theory deals only with problems of cognitive organization and interaction: that is, "(a) Systematic changes in the availability and identifiability of presented ideational materials as they interact with and are incorporated into existing cognitive structure, and (b) variables increasing or decreasing the incorporability of these materials as well as their subsequent availability" (Ausubel, 1962, p. 213-214).

Meaningful learning is characterized by the interaction of new material with existing cognitive structure. Rote learning is contrasted as it adds discrete chunks of knowledge to memory. These chunks do not interact with existing cognitive structure and hence are lost easily. During the subsumption process, new material enters the cognitive field where it interacts with subsumers. This interaction makes possible insightful relationships. If the material is not subsumable, rote learning occurs. In time the separate concepts subsumed under more inclusive concepts in cognitive structure are subjected to the erosive influence of a conceptualizing trend in cognitive structure. The memory retains the more inclusive concept, losing recall of the more specific facts and concepts. This process is called "obliterative subsumption" (Ausubel, 1962, p. 217).
Ausubel identifies three variables which influence meaningful learning. First is the availability of appropriate subsuming concepts in cognitive structure at an appropriate level of inclusiveness. He suggests that appropriate subsumers are usually not present for most learning materials. Therefore, learners tend to utilize the most relevant and proximate ones available. On the basis of this theory, it seems desirable to present to the learner appropriate subsumers which can be made part of cognitive structure before learning materials are introduced. The introduced subsumers would thus constitute efficient advance organizers.

Second, the extent to which new material is retainable is also a function of its discriminability from established conceptual systems which subsume it. Only discriminable, categorical variants of more inclusive concepts have long term retentive value. The discriminability of new materials could be enhanced by repetition or by explicitly pointing out similarities and differences between them and their presumed subsumers in cognitive structure. Third, the longevity of retaining newly-learned materials is a function of the clarity and stability of its subsumers. Repetition, relative age of subsumers, use of exemplars, and multi-contextual exposure are variables which affect the clarity and stability of subsumers (Ausubel, 1963, p. 29).

Ausubel (1968, p. 50-57) postulated a hierarchically organized cognitive structure. Within a given sphere of knowledge, the hierarchy rises from facts at the base to ever more inclusive, complex concepts toward the top. Novak (1977, p. 75) used tree diagrams to represent these organizational properties. The tree trunk represents a broad concept (top of a hierarchy) and the branches and twigs represent the less inclusive concepts and facts which are interrelated through the broad concept. A person's
cognitive structure consists of several of such hierarchies with varying degrees of complexity. The "trees" grow as new information is linked to existing structures and as two or more structures are joined through their relationships with concepts higher in the hierarchy. "Subsumer" is the name given to a concept having a number of less inclusive facts and concepts linked to it. Mayer (1979) adds to this theory under the name "assimilation coding" theory. In this theory a subsumer is called "anchoring knowledge."

![Image](image)

(a) Reception: Is information received into working memory?
(b) Availability: Is there anchoring knowledge in Long Term Memory?
(c) Activation: Is anchoring knowledge transferred from Long Term Memory to Working Memory so that it can be actively integrated with received information?

ENCODING: Integrate anchoring knowledge and received information; then transfer new cognitive structure to Long Term Memory.

Figure 2: Assimilation coding model (Mayer, 1979, p. 374)

In this theory of meaningful learning, new knowledge enters the Working Memory at (a). If anchors (subsumers) for the new knowledge already exist in Long Term Memory (b), they are transferred to Working Memory where the new knowledge is actively integrated with the anchors (c) during learning. The new cognitive structure is then returned to Long Term Memory.

Mayer made several predictions from this theory. First, increasing the number of relevant anchors (b) should improve learning. The use of appropriate advance organizers should be a valid means of accomplishing
this goal. Second, facilitating the transfer of anchors from Long Term Memory to Working Memory should improve learning. The use of advance organizers should also accomplish this goal. Third, students who have developed the ability to provide their own structure to learning materials ((b) and (c)) may be more successful in learning than other students. The use of advance organizers may not be particularly beneficial to such students.

**Definition and Construction of Advance Organizers**

Advance organizers are introductory instructional materials designed to establish a concept (anchor) in cognitive structure which can serve as a subsumer for the detailed learning materials to follow (Kozlow, p. 18). They should be more inclusive and more highly generalized than the learning materials that follow. The ideas presented in the advance organizer should be selected so as to maximize their potential for explaining and integrating the ideas and facts in the learning materials. Furthermore, advance organizers should use terms which are familiar to the learner. The specific relevance of the ideas presented in the advance organizer to the content of the learning materials should be specified (Ausubel, 1970, p. 2-6).

According to both Ausubel's subsumption theory and Mayer's "assimilation coding" theory, advance organizers can facilitate meaningful verbal learning. Advance organizers either provide subsumers to cognitive structure when none exist (expository organizers) or provide a context in which learners can recall relevant subsumers preexisting in long term memory (comparative advance organizers). Ausubel and Fitzgerald (1962, 1961) performed two definitive experiments that illustrated the use of these two types of advance organizers.
Ausubel (1963, p. 81) stated the difference between advance organizers and summaries/overviews. Advance organizer content is selected on the basis of its ability to explain, integrate, and interrelate the concepts in the learning materials to existing cognitive structure. Summaries and overviews are ordinarily presented at the same level of abstraction, generality, and inclusiveness as the learning materials. They emphasize the major points of the material, omit less important information, and largely achieve their result by repetition. Ausubel gave no operational definitions.

There is no known foolproof way to produce effective advance organizers, but Mayer (1979), Kozlow (1978), and Jones (1979) have made suggestions. Mayer offered a checklist for advance organizer construction (p. 382): "(1) Does the advance organizer allow one to generate all or some of the logical relationships in the to-be-learned material? (2) Does the advance organizer provide a means of relating unfamiliar material to familiar, existing knowledge? (3) Is the advance organizer learnable, i.e., is it easy for the particular learner to acquire and use it? (4) Would the learner fail to normally use an organizing assimilative set for this material, e.g., due to stress or inexperience?" He suggested that the answer should be "yes" to each of these questions.

Kozlow (1978, p. 148) suggested that the identification of the concept which is to subsume the learning materials is probably the most important decision to be made in constructing an advance organizer. To accomplish this, he suggested creating a cognitive map for the concepts to be taught in the learning materials. This could be a tree-type diagram which shows the relationship among concepts and which concepts subsume others. A broad, more inclusive concept should be identifiable in this procedure. Kozlow found that an advance organizer has a lesser tendency to show facilitative
effects as one increases the number of concepts it contains. Therefore, an attempt should be made to present only the main concept in an advance organizer.

Jones (1979) suggested that advance organizers which utilize declarative statements, specific questions and specific examples instead of general concepts may not be advance organizers at all. Furthermore, if the level of abstraction is the same as that of the learning materials, the advance organizer may not be valid.

Ausubel (1963, p. 81) suggested an appropriate level of inclusiveness for an advance organizer would be as close as possible to the degree of conceptualization required by the learning task. The more unfamiliar the learning material, the more inclusive or highly generalized the advance organizer must be in order to be as near as possible to the degree of conceptualization of the learning task.

Research Supporting Advance Organizers

Ausubel and Fitzgerald (1962) assigned teacher education students to an advance organizer or control group. The advance organizer group received a 500 word expository advance organizer to provide anchorage for a 1400 word passage on the endocrinology of pubescence. Pretesting verified the subjects were unfamiliar with the material to be presented. The researchers concluded that no constructs which could serve as subsumers preexisted in the subjects' cognitive structure. The control group received a 500 word introductory passage that had no organizational value.

On the first day the groups studied their respective 500 word passages for eight minutes. Two days later both groups restudied these passages for six minutes followed by 25 minutes on the 1400 word learning materials.
The test on the endocrinology passage came 48 hours later. A second learning passage of 1600 words was studied for 27 minutes three days after the first examination. A test over the second passage was given four days later. The learning and retention difference of the two groups on the first test was not quite significant at the .05 level. However, students in the lower third level of the experimental group on verbal ability (SAT scores) scored significantly better than the lower third of the control group.

The authors claimed that evidence exists "... for believing that advance organizers, by providing ideational anchorage, facilitate the learning and retention of totally unfamiliar material for those subjects who have relatively little verbal ability. Subjects of average and better ability are evidently capable of spontaneously organizing new material around relevant, more inclusive concepts, and hence derive little or no benefit from introduced advance organizers" (p. 247).

The authors made two other relevant observations. First, their advance organizer proved to be difficult for subjects to understand. The authors suggested that an effort should be made to create the advance organizer in terms of concepts that are already familiar to subjects and already established in the learners' cognitive structures.

Second, those subjects who performed best on the test over the first passage tended to learn the second passage better. The clarity and stability of knowledge of the first passage constituted a significant limiting condition with respect to learning and retaining material in the second passage.

Ausubel and Fitzgerald (1961), using university undergraduates, compared the effectiveness of three types of introductory passages: a
comparative advance organizer, an expository advance organizer, and a historical introduction. The comparative organizer was designed to increase discriminability between concepts in learning materials on Buddhism and concepts about Christianity already familiar to the learners. The comparative organizer pointed out explicitly the differences and similarities between the two sets of concepts. The expository organizer presented the principal Buddhist doctrines at a higher level of abstraction, generality, and inclusiveness without making reference to Christianity. The historical introduction, a control treatment, presented historical and human interest information. It contained no comparisons between religions.

A three-day treatment was followed by a posttest and then by a retention test ten days after treatment. On the posttest the comparative organizer was found to be significant ($p < .05$) when compared with the expository and historical introductions. On the retention test both the expository and comparative organizers were significantly more facilitative ($p < .05$, $p < .02$) than the historical introduction.

Ausubel (1960) compared the effects of an expository advance organizer and a historical passage, both 500 words in length, on the learning of concepts about metallurgy. A pretest showed the 120 college seniors in educational psychology were unfamiliar with the topic. The historical passage was written to create interest among the students but did not relate directly to the concepts to be tested. The advance organizer was written at a higher level of abstraction, generality, and inclusiveness than the learning materials. The treatment was administered in one 35 minute period, and the posttest was given three days later. The difference between means was almost significant at the .01 level in favor of the group receiving the expository advance organizer.
Mayer (1979) listed nine series of experiments testing "assimilation encoding" theory. All of the tests involved advance organizers, college underclassmen, and computer programming materials. All showed facilitative effects for the organizer. In the first series, results indicated that the advance organizer group (EG) excelled on far-transfer items from a posttest. The control group (CG) excelled on near-transfer items. Mayer suggested the EG "... assimilated the material to a broader set of past experiences; this allowed superior transfer to new situations but may have caused some loss of the original organization" (p. 377).

Mayer replicated this test with a different instructional text and test. Again he found superior far-transfer for the EG, while there was no difference between the groups on near-transfer items. He defined far-transfer test items as those which were very different from the text. Near-transfer items were very similar to information presented in the text.

In a second series of studies Mayer and Bromage gave students an advance organizer either before the learning passage (EG) or after (CG). Then students were requested to recall selected portions of the text. The test measured recall of technical ideas, recall of conceptual ideas, references to the model (presented in the organizer), vague summaries, and inappropriate intrusions (relating material to new ideas). A discriminant analysis of the results of two separate studies showed the EG excelled on recall of conceptual ideas and on relating the material to new ideas. Mayer suggested this type of outcome would seem most capable of generating far-transfer as noted above. The CG retained technical ideas, as would be most useful for near-transfer.

A third finding of Mayer's was that the facilitative effect of the advance organizer was primarily among subjects scoring below average on
SAT-Mathematics. He suggested that this is reasonable in that high ability students might already possess meaningful contexts in which new knowledge can be assimilated.

Finally, a fourth test looked for interaction between advance organizer treatment and text organization (in random or logical order). Results showed that the organizer facilitated achievement only for those subjects who studied the randomly organized text. These results suggested that the organizer helped subjects to reorganize the incoming materials.

Jones (1979) studied the effects of level-specific advance organizers on A- and C-track non-science majors from grades ten and eleven in a large, southern, metropolitan school district. The learning materials concerned the concept of feedback. The organizers differed in level of abstraction. One was prepared specifically for A-track students (ATO), the other for C-track (CTO). The results of an achievement test showed a significant track by organizer interaction. A-track students who received the ATO performed significantly higher than those who received the CTO or no organizer. C-track students who received the CTO performed significantly better than those who received the ATO, but not better than those who received no organizer. Results on a retention test also showed significant interaction. However, no significant differences were found among A-track students or among C-track students on levels of advance organizer. Jones concluded that it may not be possible to prepare an advance organizer for all students. Students with different cognitive abilities may benefit from level-specific advance organizers.
Analyses of Advance Organizer Research

Mayer (1977, p. 374) has pointed out that though Ausubel's and his colleagues' early work was promising, their studies "generally found only small advantages in recall due to advance organizers." Barnes and Clawson (1975) raised the question as to whether there is any empirical evidence for the claim that advance organizers influence the learning of conceptual materials. They reviewed 32 studies of which 20 failed to produce significant results. They were forced to conclude: "Advance organizers, as presently constructed, do not facilitate learning" (1975, p. 651).

Mayer (1979, p. 372) pointed out three limitations of the Barnes and Clawson review: "(1) inadequate statement of the to-be-tested theory, (2) inadequate analysis of the learning outcomes, (3) inadequate experimental control." The first limitation was stated because Ausubel (1978) did not claim advance organizers would be facilitative in all situations. Their greatest promise holds when subjects do not have relevant subsumers for the learning materials and when the subjects would not otherwise have used prior knowledge subsumers.

The second limitation was suggested because many of the 32 studies which were reviewed tested for the amount of overall knowledge of learning materials retained. Mayer suggested organizers should enhance retention of conceptual ideas and far transfer. Thus, dependent measures of amount of overall knowledge retained may not be sensitive to the effects of advance organizers.

The third limitation was claimed because many of the studies reviewed by Barnes and Clawson did not adequately control the amount and kind of information about the learning materials presented to the subjects. Control and experimental subjects did not receive the same information. Therefore,
any differences in examination performance could have been due to information in the advance organizers.

Mayer (1979, p. 373) suggested that advance organizer studies might do well to test the cognitive process theories which underlie the organizers. The nine studies by Mayer and colleagues reviewed earlier in this chapter were an attempt to overcome these shortcomings and to test his underlying "assimilation encoding" theory.

Using a meta-analysis of 99 selected advance organizer studies, Kozlow (1978, p. 1) attempted "... to determine whether or not there were any characteristics of the experimental settings or materials that had consistent relationships with the treatment effects obtained through the use of advance organizers." He undertook the study because the literature reviews of advance organizer research had failed to reach any definitive conclusions.

Kozlow identified two dependent variables and 63 independent variables. The dependent variables were measures of the magnitude of the difference between the achievement means for the advance organizer and control groups. "The first variable is the t-statistic for testing for significant differences between two means and the second is based on the probability level associated with the obtained t-statistic" (1978, p. 53-54). Independent variables described characteristics of the sample, treatment administration conditions, type of subject matter, quality of research procedures, and characteristics of the advance organizers and learning materials (p. 2).

He found ten independent variables had significant correlations with both dependent variables, while seven had significant correlations with only one dependent variable. Included in the former group were grade level, gain score, progression density, number of concepts in the advance
organizer, type of advance organizer, and mode of the advance organizer. Included in the latter group was the percent of achievement test questions answerable from the advance organizer alone (p. 102-103).

Carefully considering some of the independent variables, Kozlow found there was a greater tendency for the advance organizers to show facilitative effects as the grade level of the subjects increased. From this he did not suggest that young children cannot benefit from the use of advance organizers, but rather that younger students may have had difficulty understanding the advance organizers which typically contained a high concentration of information.

Kozlow did not find significant correlations for the learning ability of the students. Possible causes cited were the variety of criteria such as IQ, basic skills, and reading ability used to identify the ability levels in various experiments; and the separation between high and low ability levels being inconsistent across experiments. Investigating the effect of ability level in more detail, he found there was a greater tendency for advance organizers to be more facilitative for high ability groups than for low ability groups under the following conditions (seven were listed): when the percentage of higher level questions in the achievement test was increased, when mathematics content was used, when the presentation mode for the advance organizer and learning materials was other than reading, and when the length of the organizer was increased (p. 106-108). These findings may be related to the age and grade level of the subjects involved.

Kozlow found there was a lesser tendency for the advance organizer to show facilitative effects as the number of concepts it presented was increased. A general concept should be emphasized since the function of
the advance organizer is to generate an inclusive subsumer in cognitive structure (p. 113-114).

Next he found comparative advance organizers showed a greater facilitative effect than expository ones. He suggested this relationship might be expected since comparative organizers tend to utilize existing concepts in cognitive structure to a greater extent (p. 115). Kozlow found that some of the significant findings in favor of advance organizers may have been due to the contributions of advance organizers alone to answering test questions or to an inequality of study time for the treatment groups.

He further stated (p. 3): "Student inability to understand the advance organizer information may account for some of the non-significant results. Advance organizers are probably more likely to show facilitative effects when the reading level is appropriate for the grade level, when the rate of introduction of new material is slower, when the content is less complex, and when students are given more time to process the advance organizer information."

Luiten et al. (1980) examined approximately 170 published and unpublished studies from 1960 to 1979 of which 135 were included in a meta-analysis. The goal of this technique, proposed by Glass (1978), was to quantify, standardize, and compare treatment effects across studies. The fundamental statistic in this technique is the "effect size" (ES):

\[
E.S. = \frac{X_t - X_c}{S.D.} = \frac{X_t}{S.D.} - \frac{X_c}{S.D.}
\]

where \( X_t \) = treatment group mean,
\( X_c \) = control group mean,
\( S.D. \) = control group standard deviation

Although Glass' technique offered no statistical test for the significance of the E.S., Luiten et al. suggested that "All effect sizes,
regardless of magnitude, are worthy in their own right. In this way the bias of the voting technique is avoided and the detection of small, but consistent treatment effects is made possible across studies" (p. 3,4).

Luiten et al. suggested that Barnes' and Clawson's technique was flawed in that it failed to consider positive treatment effects which were not statistically significant. The results of such an analysis were biased against a favorable finding for advance organizers (p. 3).

As did Kozlow (1978), Luiten et al. made several observations about variables which affect advance organizer research. They computed 110 effect sizes for learning and 50 for retention. Subsets of the studies generating these effect sizes were selected and their effect sizes averaged to consider such variables as grade level, subject area studied, organizer presentation mode, and subject ability level.

With the help of a standard normal distribution table one can interpret an effect size as the percent of subjects receiving treatment who performed better on learning and retention than the control subjects. A positive effect size will always yield a percent greater than 50.

The mean of 110 effect sizes on learning was .21, standard error = .04. On retention the average effect size increased consistently from a low of .19 for 2-6 day retention to a high of .38 for 22 day retention (p. 15).

When studies were classified by grade level (college, secondary (grades 9-12), primary (grades 3-8)), the mean effect sizes for learning were .26, .17, .17 respectively. The values for retention were .21, .26, .33 respectively (p. 16).

When studies were classified by subject area (mathematics; physical, biological, and social sciences) the mean effect sizes for learning were
.10, .15, .11, and .34 respectively. For retention they were .17, .50, .18, and .26 respectively (p. 17). When classified by ability level (high, middle, low), mean effect sizes for learning were .23, .08, and .13 respectively (p. 18). When classified by presentation mode (written, aural), the mean effect sizes for learning were .17 and .37 respectively (p. 19). When those studies with aural presentation mode were classified by grade level, the mean effect sizes were .68, .11, and .34 respectively (p. 20).

Having posited these findings, the authors concluded that advance organizers showed a small but facilitative effect on learning and retention. They suggested this is not surprising considering the short duration of the treatment (often 1-2 periods) in the typical study. Furthermore, they stated that advance organizers facilitated learning in all content areas examined, and with individuals of all grade and ability levels (p. 11).

**Recommendations for Future Advance Organizer Research**

Mayer (1979) suggested that future research test the cognitive process theories which underlie the use of advance organizers. He presented nine series of studies in which he tested his "assimilation encoding" theory. Jones (1979) suggested that future research use advance organizers prepared specifically for the cognitive ability level of the subjects.

Kozlow (1978) recommended that research be designed to answer two questions: (1) Do subsumers already in cognitive structure facilitate learning? If research would show that the answer is "no," then the validity of subsumption theory would be damaged since the role of subsumers is crucial in both Ausubel's and Mayer's meaningful learning theory.
(2) Are advance organizers an effective means of generating subsumers? If research would again show the answer is "no," a new means of generating subsumers would then be crucial to the theory's application. Advance organizers are an application of subsumption theory. If they do not anchor subsumers, the theory's validity is not affected. Tests should be included to determine if information in the advance organizer alone helped answer test questions over the learning materials and if extra study time for the advance organizer group compared to the control group could have improved treatment group achievement. If these alternative explanations of significant results are controlled, the researcher will have a stronger basis upon which to claim the advance organizer is facilitative.

Third, he suggested controls to enhance generalizability of non-significant results. It should be determined whether or not subjects have the subsumer in cognitive structure before the experiment begins. Also, it should be determined if students understand the organizer information. If the desired subsumer preexists in cognitive structure, the organizer would exhibit minimal effect. If the subjects do not understand the organizer, it most likely will not produce the desired subsumer in cognitive structure. Kozlow suggested these tests be done on equivalent groups of students before the experiment is begun. Pre- and posttests on the organizer could be used.

Another possible confounding factor is the potential affective effects of the control introduction (non-advance organizer). If it is more interesting or more boring than the organizer, the effect on the subjects' state of arousal could produce significant results. Perhaps this effect could be tested in a pilot study by obtaining an indication of student interest in the two introductions, control and organizer.
Kozlow suggested that judges should be utilized to rule on the appropriateness of the advance organizer. What is the main concept in the organizer? Does this concept subsume the learning materials? Will students be able to understand the advance organizer content? Is the level of abstraction too difficult for the subjects? Judges could rule on questions such as these. Finally, his study suggested a presentation mode other than verbal text be utilized when possible. Concrete experience or audio-visual aids in the advance organizer have been helpful.

**Kinetic Structure**

Kinetic structure refers to the serial ordering of information in a communication according to some organizational principle. Kinetic structure coefficients are measures of the extent to which the order of the terms in a communication corresponds to the order defined by an organizational principle (Anderson, 1971, p. 7). The organizational principle of potential interest for advance organizer research is a derived dimension which defines kinetic structure in terms of the relatedness of the adjacent discourse units in the advance organizer. The discourse unit is the simple sentence consisting of subject, predicate, and subordinate clauses.

Two measures of this derived dimension may prove useful. One is the commonality coefficient, the other the progression density coefficient. The list of verbal elements, words, or phrases which contribute to the substantive meaning of the communication in each discourse unit provide the data for the calculation of the two coefficients. Commonality is a measure of the extent to which adjacent discourse units contain common verbal elements (Anderson, 1979, p. 13). Progression density is a measure of the rate at which new elements are introduced in a verbal communication.

Kozlow (1978, p. 26) felt it worthwhile to consider kinetic structure in that it can be a measure of the learning difficulty of the advance organizer. Anderson (1971, p. 19) felt learning materials should be most easily understood when the commonality coefficient is .50; that is, when there is a balance between commonality and progression. Kozlow (1978, p. 27) cited three research studies which all concluded that "students exposed to materials with higher commonality coefficients obtain higher achievement acores."

Kozlow found average commonality and progression density coefficients of .26 and 1.94 respectively in the advance organizers he analyzed. He stated (1978, p. 117) that Anderson feels progression density is the more important variable to be considered in the construction of advance organizers. If the introduction of new material is too rapid, students may have difficulty in forming the generalizations necessary for establishing the desired subsumer. Kozlow found (1978, p. 117) that "there was a lesser tendency for advance organizers to show facilitative effects as the rate of introduction of new materials increased." There is no indication as to an acceptable range for the progression density coefficient. The only data available is the average Kozlow found for the advance organizers he analyzed.

Level of Reasoning

Piaget hypothesized four stages of cognitive development (Piaget, 1971). The behaviors that subjects use to achieve cognitive knowledge of their
world characterize the structures that define each stage. The four stages are the sensory motor (ages 0-2); pre-operational (ages 2-6); concrete operations (ages 6-11); and formal operations (ages 11-16). Only the last two stages and a possible transitional stage between them is of interest in this study.

Sinclair (1971) summarized the differences in the concrete and formal stages: "Concrete operational means that the student can think in a logically coherent manner about objects that do exist and have real properties and about actions that are possible; he can perform the mental operations involved both when asked purely verbal questions and when manipulating objects. For formal operations, the main condition is the same as for concrete except for an absence of objects; the formal operational student can solve problems dealing only with propositions or hypotheses." Much research attention is presently being given to Piaget's theory. That minute portion of the research directly relevant to this study follows.

Not all students attain formal reasoning on all concepts at ages 11-16. Chiappetta (1975) and Parete (1978) suggested that at least 50% of American college freshmen and senior high school students are at or below the concrete stage. Lawson and Renner (1975) found that roughly 65% of a sample of 134 chemistry, physics, and biology students were beginning formal thinkers and above. Chiappetta and Parate suggested that such findings might have serious implications for the teaching of beginning college and senior high school mathematics since a large portion of the subject matter deals with formal level concepts. Lawson and Renner (1975) found that most of the concepts taught in their three subject matters were formal level, and also suggested teaching implications. They found that the percentage of formal level questions answered correctly increased as
the subjects' reasoning level increased. Virtually no formal level questions were answered correctly until a stage they identified as post-concrete.

Lawson et al. (1975) suggested that to develop meaningful understandings students at the concrete level need courses which involve concrete objects, events, and situations rather than formal lectures. Their curriculum can include abstractions, but the material should proceed from the concrete to the abstract.

Lawson and Renner (1975) gave definitions to formal and concrete concepts. They defined formal concepts as those whose meanings are derived through position in the systems of which they are a part. Meaning is given to these concepts not through the senses but through their logical relationships within the system. Concrete models can represent such concepts; therefore, concrete level students should be able to verbalize about them. But a full comprehension of the concepts will not be attainable for concrete level students. In evaluating understanding of such concepts, evaluation cannot be made solely in terms of the concrete model. Such a plan would fail to test for complete understanding.

Concrete-operational concepts were defined as ones whose meaning can be developed from first hand experience with objects or events. These concepts may arise through direct experience in which the entire meaning of the concepts (such as color) is given through the senses. Concrete objects may also be used by postulation. If so, part of the meaning of the concept must be sensed or immediately apprehended. Examples would be chair, table, and person.

Collis (1973) found that subjects have a difficult time dealing with abstract mathematical operations until formal operations are available.
Rosskopf (1971) included in his volume studies that show non-formal students cannot fully comprehend the concepts of function and limit. Some aspects of the concepts could be attained by concrete students if the material was introduced in a very concrete manner. But students who were able to tackle such tasks as domain and range from a graph and ordered pairs were in the early stages of the formal level.

Grady (1976), Lawson and Renner (1975), and Parete (1978) found that Piagetian level of reasoning is strongly correlated with mathematical ability. Lawson et al. (1975) found Piagetian task scores correlated, after correction for attenuation, .40 with the College Entrance Examination Board for Mathematics, .47 with SAT-Mathematics, and .63 with SAT-verbal, \( p < .001 \). Subjects who were formal reasoners performed significantly higher than subjects who were concrete reasoners, \( p < .05 \) on both mathematics aptitude tests, \( p < .01 \) on the SAT-verbal. Seventy-one elementary education majors were subjects.

**Summary**

The literature provides evidence that advance organizers have generally been facilitative. Kozlow (1978) and Luiten et al. (1980) have used the statistical technique of meta-analysis to provide evidence for this claim. Many variables which are important in advance organizer studies have been identified. Progression density, type of advance organizer, grade level, and student ability are examples.

Both Kozlow (1978) and Mayer (1979) raised questions for exploration in future studies. Kozlow identified several variables which should be controlled in order to make interpretation of results more robust. Finally, some evidence was presented that suggested a relationship between level of reasoning and ability to understand advance organizers.
CHAPTER III

RESEARCH PROCEDURES

Design

Population

The study was conducted with a population of 36 sections of Mathematics 100 during Autumn Quarter, 1980, at the Ohio State University. Mathematics 100 is a five quarter-hour course designed as a remedial review of secondary school algebra and geometry skills and concepts. Students receive no credit toward degree requirements for the course. Most students who enroll are freshmen.

Mathematics 100 was taught in small sections of approximately 28 students by either a professor or a graduate student. Graduate students were assigned to professors who served as resource persons and/or supervisors. A course coordinator supervised the entire program. Common midterm and final examinations were written by the course coordinator and given in the evenings. The course coordinator also provided a daily syllabus for content and exercise coverage. The calculator was a central instructional tool. Its use made possible the assignment of interesting and realistic problems such as the effect of inflation on prices.
Selection of Sections

There were approximately 60 sections of Mathematics 100 during Autumn Quarter. At a meeting prior to the beginning of the quarter, instructors representing 40 sections volunteered to participate. For technical reasons only 36 sections began the experiment. Previous experience with this program indicated that performance on examinations varied widely among sections. To control for differences in ability among sections, section means on the first two midterms were converted to standard scores (Z) and averaged. Then 18 pairs of sections were formed, matched by average midterm performance. Each of the 36 sections had at least 20 students still attending on a regular basis after the second midterm.

Nader (1971) found that the first midterm score has a high correlation with success in Mathematics 116, another introductory mathematics course at the Ohio State University. This course also covers the content of intermediate high school algebra. It was predicted that the use of the average of Midterms I and II for matching into equivalent groups would prove valid for this study. Later analyses confirmed this prediction.

Description

Figure 3 presents the design which was used:

\[ R_{11} \quad 0_1 \quad X_1 \quad 0_2 \quad X_2 \quad 0_3 \quad 0_4 \]
\[ R_{12} \quad X_1 \quad 0_2 \quad X_2 \quad 0_3 \quad 0_4 \]
\[ R_{21} \quad 0_1 \quad X_1 \quad 0_2 \quad X_2 \quad 0_3 \quad 0_4 \]
\[ R_{22} \quad 0_2 \quad X_2 \quad 0_3 \quad 0_4 \]
\[ \underline{R_3} \quad X_1 \quad 0_3 \]
\[ \underline{R_4} \quad 0_3 \]

Figure 3. The research design.
$R_1$ and $R_2$ represent the random assignment of sections to the AO and C groups respectively to test hypotheses 1-6 and 8-13. $R_{11}$ and $R_{12}$, $R_{21}$ and $R_{22}$ represent the random assignment of AO and C sections to pretest and no pretest conditions. $R_3$ and $R_4$ represent the random assignment of sections to groups designated to test hypothesis 7. The remaining symbols were defined as follows:

- $O_1$: Pretest over the advance organizer materials.
- $O_2$: Achievement test over the advance organizer materials.
- $O_3$: Achievement test over the learning materials.
- $O_4$: Retention test over the learning materials.
- $X_1$: Administration of the advance organizer.
- $X_2$: Study of the learning materials.

The primary independent variables were formal reasoning ability and treatment group membership. Tests $O_1$ through $O_4$ were the primary dependent variables.

**Assignment of Sections to Treatments**

The sections of each matched pair were randomly assigned to either the advance organizer or the control group. Two matched pairs were designated as the special sections needed to control for the direct effect of the advance organizer on performance over the learning materials achievement test.

There were nine instructors who taught two sections each. In eight cases, after one of these instructor's two sections was randomly assigned to a treatment group, the other section was assigned to the same group. This procedure minimized the potential for control sections to come into contact with treatment materials.
Eight matched pairs were assigned the pretest condition. The sixteen matched pairs were ranked by ability (on Midterms I, II) and marked off in groups of two matched pairs each. From each of the eight groups of two pairs, one pair was randomly assigned the pretest condition.

Table 1 presents the results of the assignment of sections to treatment and pretest conditions. The higher the section in the table, the higher its average on Midterms I and II.

Procedure

Part I

TOLT was administered to all sections on 6 November. Then all sections began a five-day unit on graphing. Polynomial, rational, logarithmic, and trigonometric equations were carefully graphed. The advance organizer was administered on 13 November, the day preceding the beginning of study of lines. On 12 November, control classes were assigned as homework the graphing of \( y = -3x + 1, \ -5 \leq x \leq 5 \), with increments of 0.5 in \( x \). On 13 November, instructors of control groups discussed the graphs for 25 minutes and then administered the ATAO. The advance organizer sections thus spent four days on graphing and one day on the advance organizer. The control group spent five days on graphing including the special assignment on the linear equation.

The ATAO was part of the treatment packet for the advance organizer group. Subjects moved through the advance organizer and ATAO materials at their own pace. They had access to the advance organizer throughout the test. The ATAO was handed to control subjects as a separate entity.
Table 1

Time and Random Assignment of Pairs of Sections to Treatment and Pretest Conditions

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</table>

*Indicates instructor with two sections
Conducting Interviews

Interviews had to be completed between consecutive class periods on 13 and 14 November. Furthermore, experience indicated that finding volunteers for interviews was difficult in the constraints of the experimental environment. To help circumvent these problems, six instructors were asked to provide interview subjects. In a one-on-one basis the six instructors approached their students and asked for volunteers. The instructors chosen were ones for whom it was felt students would respond positively to the request. Of 49 students who volunteered, 41 came at the appointed time. Results on four of these were discarded due to a confounding of results. Of the remaining interview subjects, 29 were from AO sections and 8 from control sections.

Only higher ability control subjects were interviewed. Results of an exploratory study indicated that control Mathematics 100 students do poorly on both the ATAO and the interview task. In general this indicated Mathematics 100 students do not initially have the desired subsumer in cognitive structure. Therefore, it was considered probable that only a higher ability control subject might have the subsumer in cognitive structure.

The interview task required students to respond first in written form and then verbally to transfer items specially designed to determine if the students could classify positive and negative instances of functions and justify their choices. The interview sessions were recorded. The consistency in scoring the interviews was validated by listening to the tapes and looking at the written records. Two cases were discarded as a result. Interviewers were not told the treatment group membership of their interview subjects.
Part II

All Mathematics 100 sections began the three-day study of lines and slope on 14 November, the day after treatment. The AO and C sections began the period by filling out a short semantic differential. Students in the advance organizer sections were provided a concept map to which they were to refer during the three-day study. Instructors were given directions suggesting how and when to refer to the advance organizer materials during the study. At the beginning of the period after the study of lines and slope (19 November), an achievement test was given. This test was to be counted as a part of the 100 points assigned by instructors on the basis of daily work. Midterms and the final examination comprised the remaining 500 points. Grades were based upon 600 points. A subtest of Midterm III was used as the short-term retention test. It would have been preferable to use a subtest of the final examination as a retention measure for a longer time period, but this was not practical. Similar logistical problems prevented the incorporation of transfer items on the retention measure.

Special Sections

The instructors of the four special sections were asked to carry out their assigned task on 13 November. Two sections were to give the advance organizer treatment and the achievement test over the learning materials while the other two sections were to give the achievement test over the learning materials only. Two sections failed to complete the assignment. Fortunately, of the two sections which did complete the assignment, one was in each treatment condition.
Instrumentation

Six instruments were used in this study: TOLT, a pretest and post-test over the advance organizer, a short semantic differential, an achievement test over the learning materials, and a retention test.

TOLT

This test is a paper-pencil measure of Piagetian level of reasoning. It includes two-item subscales for five modes of formal thinking: proportional reasoning, controlling variables, probabilistic reasoning, correlational reasoning, and combinatorial reasoning. The authors (Tobin and Capie, 1980) report an internal consistency reliability of .85 for 682 students in grades six through college. The internal consistency reliability of the five subscales ranged from .56 to .82.

Construct validity estimates were obtained by a factor analysis of the measures on each of the five modes. A one-factor solution accounted for 43% of the variance. Each of the five modes of reasoning was highly correlated with the factor, with the factor structure loadings ranging from .60 to .72. A strong criterion-related validity estimate of .82 between scores for 25 students on clinical interviews and TOLT was obtained (Tobin and Capie, 1980). There is some evidence of the predictive validity of TOLT for mathematical tasks. A Pearson correlation of .58 between TOLT scores and an achievement test over the formal level concept "extending number systems" was found in the pilot study. Fifty-two algebra II students were the sample. Tobin and Capie report the results of two studies with college students in which Pearson correlations of .64 and .66 between TOLT and SAT quantitative scores were obtained.
Pretest and Posttest over the Advance Organizer

The 18 items on the pretest were parallel to 18 of the 26 items on the posttest. Designed by the researcher, they included the following subtests:

1) Questions written to measure a subject's ability to classify correctly examples and non-examples of functions and their graphical representations (pre- and posttests).

2) Questions written to measure a subject's ability to understand the function concept at what Thomas (1975) called the "process" level (pre- and posttests).

3) Transfer questions written to measure a subject's ability to classify an instance as a positive or negative example of a function. Subjects were required to justify their choices (posttest only).

Thomas included in an understanding of "process" the ability to work with various representations and names of functions in finding images, pre-images, the domain, the range, and sets of images. Questions in the "process" subtest were written to measure these ideas.

Wagner (1977, p. 3) suggested that to understand a concept is to know which attributes define the concept and which attributes are irrelevant. A measure of a person's understanding of a concept of abstract structure is to test the person's ability to conserve structure under a transformation of some irrelevant attributes. For this study the number of ordered pairs, the type of functional representation, the continuity of the curve, and the open or closed state of an interval of the domain were all irrelevant attributes. Whether or not there existed two or more different range elements for any domain element was the relevant attribute. Questions were designed to measure students' abilities to conserve the concept of function.
Semantic Differential

The semantic differential, developed by Leitzel et al. (1979) as an instrument to aid in the evaluation of a program to train elementary education majors in mathematics, reported an average Cronbach's alpha of .95 for multiple testings. The original instrument contained nine adjective pairs, one of which was changed for this study. Its purpose in this research was to determine the attitudinal readiness of the subjects to begin a study of lines and slope.

Achievement Test over the Learning Materials

This 18 item, multiple choice test, was designed to measure the subject's understanding of the unit on lines and slope. Questions were written at the recognition, understanding, and applications-analysis levels of Bloom's taxonomy.

In addition, the test included three items (16-18) which required the students to transfer their understanding of lines to situations different from those encountered in the reading. The researcher did not judge these items entirely satisfactory, but after searching available resources accepted the items as the best available. Except for the transfer ones, most items were similar in content to items normally included on course examinations by the faculty-supervisor of the Mathematics 100 program.

Items 1-15 are called the LMAT. Items 15-18 are called the transfer items.

Retention Test

This test was a subtest of the normal third examination for the course. The items were written by the faculty supervisor of Mathematics 100 and graded by the individual instructors. Instructors were required to use a grading guide prepared by the faculty supervisor of the course.
Interview Items

Three written items were presented to each interview subject. The three items were taken with some modification from Thomas (1975). The first item required the students both to classify a graph as an example or a non-example of a graphical representation of a function and to justify their choices. Subjects who classified the instance correctly were then asked to change the graph so that it would represent a function.

The second item required the students to compute and then locate on a grid range elements for a given function rule and domain set. The final item required the students to examine a relation presented in the context of a correspondence (a representation not presented in the advance organizer) and classify it as an example or non-example of a function.

During the four class sessions before the experiment began, all students studied graphing of functions. By utilizing the process skills developed during the study of graphing, most students were expected to perform well on process questions during this study. Thus, it was anticipated that most students would correctly answer the second transfer item.

Thomas (1975) presented evidence that process skills require a lower reasoning ability than do classification skills. The subsuming concept was a classification concept requiring formal level reasoning for understanding. Therefore, it was reasonable that success on the two classification transfer items would be the best criterion for measuring who understood the subsumer. The process item was included in the event the logic in these last two paragraphs was incorrect. Results on this item were not to be included in analyses of interview results if most students answered the item correctly.
Summary

A summary listing of the measurements made with dates of administration and internal consistency estimates of the instruments is provided in Table 2. Content validity of all instruments was judged adequate by professors familiar with the Mathematics 100 program and the purpose of this research.

Table 2
Summary of Dependent Measures with Dates and Reliabilities

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<th>ATAO Attitude</th>
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Treatments

Comparative Advance Organizer

This set of introductory instructional materials was designed to establish the subsuming concept of functions and their graphical representations in the cognitive structures of the subjects. It was a programmed learning device with 23 questions to be answered included in the narrative. Subjects were instructed to answer each question in order and then, before progressing on, to check their answers immediately on a detachable answer
The advance organizer was prepared by the researcher in an evolutionary process. First, the learning materials on lines and slope were examined for concept relationships (see the concept map in Appendix A). Then the subsumer "Functions and Their Graphical Representations" was chosen after discussion with both mathematicians and mathematics educators.

The present form of the materials represents the fourth version. Changes were made after suggestions by educators familiar with advance organizer research and after a pre-experimental testing of the materials. The suggestions of Kozlow (1978) were followed as carefully as possible throughout the process. The reading level of the advance organizer was determined to be grade nine by use of a procedure developed by Fry (cited in Kozlow, 1978). The progression density coefficient was approximately 1.4, two-thirds of a standard deviation below the mean found by Kozlow for 72 advance organizers.

To teach the relevant and irrelevant attributes of the function concept as presented, matched examples and non-examples as well as divergent examples were utilized in a concept-learning format as suggested by Tennyson et al. (1972). Research by Stout (note 3) provided the universe from which examples and non-examples were chosen.

Learning Materials

These materials were the two sections on lines and slope, 9.3-9.4, from the text Basic College Mathematics by Ash and Robinson (1981) which was used in the Mathematics 100 program during Autumn quarter, 1980.
Statistical Procedures

Verification of Equality of AO and C Sections

Because some of the instructors failed to administer required instruments, the number of sections retained for testing hypotheses varied from hypothesis to hypothesis. Measurements to test for any ability differences between the AO and C groups were made by use of one- and two-factor analyses of variance. Average scores on Midterms I and II were included in the analyses for those subjects who completed the instruments involved in the hypotheses.

Part I

To investigate the relationships between formal reasoning ability (as measured by TOLT), treatment group membership, and performance on the two major subtests of the ATA0 (function process and classification), a two-way multivariate analysis of variance was performed. Only students who completed both TOLT and the ATA0 were included in the analyses. Students from each section were placed into one of four blocks depending upon their performance on TOLT. The resulting micro groups (four per section) were the units of analysis. Dependent measures were the two subscores on the ATA0. The number of students by TOLT block by treatment condition for these analyses is included in Table 3. A complete breakdown of the number of students in each micro group is presented in Table 32, Appendix E.

To find the best subscale from the ATA0 for prediction of who had the subsumer anchored in cognitive structure, factor analysis, Pearson correlation, and multiple regression were used on the ATA0 and/or interview results.
Table 3

Number of Students by Treatment by TOLT Block for Analyses in Part I

<table>
<thead>
<tr>
<th>Students Per TOLT Block</th>
<th>Treatment</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>AO 0-3</td>
<td>4-5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>26</td>
<td>108</td>
</tr>
<tr>
<td></td>
<td></td>
<td>22</td>
<td>94</td>
</tr>
<tr>
<td></td>
<td></td>
<td>255</td>
<td>215</td>
</tr>
</tbody>
</table>

Part II

Step 1. Three dependent measures were made in Part II: the LMAT, the three transfer items, and a retention test. Since retention test results were missing for seven sections, three separate analyses were performed. Independent variables were treatment group membership and level of performance (high, average, low) on the average of Midterms I and II. Micro groups (three per section) formed by the ability blocks were the units of analysis. Midterm scores were converted to T-scores. Criterion levels used for formation of the ability groups were T-scores of 45 and 55.

The three analyses were as follows:

1) A multivariate analysis of variance with LMAT and transfer scores as dependent measures.

2) A one-way analysis of variance on retention results (seven sections missing).
3) A multivariate analysis of variance with LMAT, transfer, and retention scores as dependent measures.

For the third analysis only those sections were retained for which scores were available on all three measures.

The number of students by ability block by treatment condition for the analyses in step 1 is included in Table 4. A complete breakdown of the number of students in each micro group is presented in Table 33, Appendix E.

Table 4

Number of Students by Treatment by Ability Block for Two Step Analyses in Part II

<table>
<thead>
<tr>
<th>Step</th>
<th>Treatment</th>
<th>Ability Block</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AO</td>
<td>Lower Middle Upper</td>
</tr>
<tr>
<td>I</td>
<td></td>
<td>77 105 91</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>62 91 67</td>
</tr>
<tr>
<td>II</td>
<td></td>
<td>46 48</td>
</tr>
<tr>
<td></td>
<td></td>
<td>49 45</td>
</tr>
</tbody>
</table>
Step 2. The three analyses listed above were repeated only for those subjects judged to have the subsumer in cognitive structure (score on the ATAO classification subtest ≥ 9 and question 24 correct) and their matched control counterparts. Since there were no students predicted to have the subsumer in the lower ability block of several sections, the number of blocks was reduced to two to facilitate analysis. Results of one advance organizer section with two subjects predicted to have the subsumer anchored were discarded to further reduce the number of cells with no observations.

After the students predicted to have the subsumer anchored were identified, their individual average T-score means on Midterms I and II were computed. Then, for each AO section, students from the originally matched control section were matched to the AO students by T-score average. A control student was chosen within 0.5 T-score units when possible. When impossible, two criteria were satisfied by the choice of a match: 1) keep the number of advance organizer and control students per section in each ability block the same; 2) keep the average mean T-score the same for the AO and matched control groups.

A T-score of 54 was chosen as the criterion level to nearly equalize in the two ability blocks the number of students predicted to have the subsumer. Consequently, the upper ability block for the analyses in step 2 consisted of students roughly in the upper third of the overall ability range. The T-score means were 59.9 (SD=3.8) and 58.6 (SD=3.1) for the AO and matched C subjects respectively. The lower ability block consisted of students from the lower two-thirds of the ability range. The T-score means were 46.1 (SD=6.8) and 46.9 (SD=5.2) for the AO and C subjects respectively.

The number of students by ability block by treatment condition for the analyses in step 2 is included in Table 4. A complete breakdown of the number of students in each micro group is presented in Table 35, Appendix E.
CHAPTER IV

ANALYSIS OF RESULTS

The empirical data are presented and analyzed in this chapter. Checks on the equality of the matched groups where necessary, analyses for Part I data on only those subjects experiencing full treatment, and analyses for Part II on both those subjects experiencing full and partial treatment are presented. For these analyses TOLT scores, ATAO pre- and post-scores, LMAT, scores, retention scores, attitude scores, and interview scores are reported. A .05 level of significance was used for all statistical tests unless otherwise reported. All hypotheses are stated in null form in this chapter.

Analyses of Part I Data

Treatment Group Equality

Equivalent treatment groups were originally formed by matching on the basis of average performance on Midterms I and II and random assignment to treatments. Section averages were based upon scores for all students who completed the two midterms. Some of these students did not complete all of the tests administered in Part I. Thus new section averages on Midterms I and II were computed based upon only the midterm scores of those students who were to be included in Part I analyses.

Table 5 presents the results of a two-way analysis of variance of average midterm scores. Factors A and B were treatment group membership and pretest condition respectively. The results indicate that based on
average performance on Midterms I and II, differences in ability among
the partitioned groups of importance in Part I were negligible. The
results suggest the procedure used to create groups equal in ability was
effective.

Table 5
Analysis of Variance of Differences Among Treatment
and Pretest Groups as Measured by the Average of Midterms I and II

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>p&lt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment(A)</td>
<td>9.21</td>
<td>1</td>
<td>9.21</td>
<td>.410</td>
<td>.527</td>
</tr>
<tr>
<td>Pretest(B)</td>
<td>0.01</td>
<td>1</td>
<td>0.01</td>
<td>.000</td>
<td>.987</td>
</tr>
<tr>
<td>AB</td>
<td>18.93</td>
<td>1</td>
<td>18.93</td>
<td>.884</td>
<td>.367</td>
</tr>
<tr>
<td>Within</td>
<td>583.37</td>
<td>26</td>
<td>22.47</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

ATAO-TOLT Correlation

A Pearson correlation was computed to test the relationship between
performance of students on TOLT and the ATAO. The following hypothesis was
tested:

H₁: There will be no significant correlation between subjects' scores
on TOLT and their scores on the ATAO.

A statistically significant correlation of r = .23, p < .0005, n = 470,
was obtained. When only those subjects who studied the advance organizer
were included, a statistically significant correlation of r = .34, p < .0005,
n = 255, was obtained. H₁ was rejected.
Analysis of ATAO Pretest Effect

A randomly selected half of both A0 and C sections was pretested. Results on 14 sections were retained. Descriptive statistics for all pretested students are presented in Table 6. Subtest S1 contained eight multiple choice questions, each with four distractors, measuring function process skills, while subtest S2 contained 10 items, each with a yes/no format, measuring function classification skills.

Table 6

Descriptive Statistics by Treatment For ATAO Pretest Subscales Process (S1) and Classification (S2)

<table>
<thead>
<tr>
<th>Treatment</th>
<th>S1 Mean</th>
<th>SD</th>
<th>S2 Mean</th>
<th>SD</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>A0</td>
<td>3.21</td>
<td>1.46</td>
<td>5.58</td>
<td>1.81</td>
<td>163</td>
</tr>
<tr>
<td>C</td>
<td>3.38</td>
<td>1.43</td>
<td>5.49</td>
<td>1.64</td>
<td>164</td>
</tr>
<tr>
<td>Totals</td>
<td>3.30</td>
<td>1.45</td>
<td>5.54</td>
<td>1.75</td>
<td>327</td>
</tr>
</tbody>
</table>

Table 6 indicates students performed slightly above the random guess level on process questions, but on classification questions performed well within the random guess range predicted by the binomial distribution. Since later analyses will be based upon only the classification subtest of the ATAO, the naivity of the subjects on concepts measured by this subtest was verified.

A two-way analysis of variance indicated the effect of the pre-test on ATAO performance. Independent variables were treatment condition (A) and pretest condition (B). The dependent variable was ATAO scores. Only
those students who took both tests are included in this analysis. Cell means and standard deviations are presented in Table 7 while ANOVA results are presented in Table 8.

Table 7

Cell Means and Standard Deviations on ATAO Scores by Treatment and Pretest Conditions

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Pretest Condition</th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>16.92</td>
<td>14.75</td>
</tr>
<tr>
<td>AO</td>
<td></td>
<td>0.72</td>
<td>1.31</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11.48</td>
<td>12.00</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>0.82</td>
<td>2.14</td>
</tr>
</tbody>
</table>

Table 8

Analysis of Variance of ATAO Pretest Effect on ATAO Performance Among Treatment Groups

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>p&lt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment(A)</td>
<td>100.62</td>
<td>1</td>
<td>100.62</td>
<td>53.809</td>
<td>.001</td>
</tr>
<tr>
<td>Pretest(B)</td>
<td>4.05</td>
<td>1</td>
<td>4.05</td>
<td>2.168</td>
<td>.156</td>
</tr>
<tr>
<td>AB</td>
<td>10.86</td>
<td>1</td>
<td>10.86</td>
<td>5.808</td>
<td>.026</td>
</tr>
<tr>
<td>Within</td>
<td>37.40</td>
<td>20</td>
<td>1.87</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Inspection of Table 7 indicates a much larger difference between the means of the pretested AO and C sections than between the means of the non-pretested AO and C sections. The statistically significant interaction term ($F(1,20) = 5.808, p < .026$) reported in Table 8 confirms this interpretation of the cell mean pattern. Table 8 also shows that the mean (11.48) of the pretested C sections was the same as the mean (12.00) of C sections not pretested. The pretest did not enhance C group performance on the ATAO. Figure 4 shows the interaction between pretest condition and treatment.

Figure 4. Pretest condition by treatment interaction with ATAO scores as dependent variable

Using Tukey's test to perform a post hoc followup of the interaction showed that AO groups at both pretest conditions outperformed their C counterparts, $p < .05$. The comparison between means of AO sections pretested and AO sections not pretested was not statistically significant at the .05 level. However, if the less conservative Dunn test is used for making only
the two comparisons between means of pretested sections and sections not
pretested, the AO group pretested did outperform the AO group not pre-
tested at the .05 level. Perhaps the pretest exerted a sensitizing effect
on subjects who read the advance organizer and/or had a negative effect
on the morale of control subjects.

Analysis of Achievement on the ATAO

A two-way multivariate analysis of variance was performed to look at
achievement on the ATAO with treatment condition and TOLT blocks as inde-
dependent variables. The two dependent variables were the two ATAO subtests,
function process and function classification. The function process subtest
(S1) contained 12 multiple choice items, each with four distractors, while
the classification subtest (S2) contained 14 items, four of which were mul-
tiple choice (four distractors) and 10 of which were of the yes/no format.
All results are presented as percents, and only those students who took
both TOLT and the ATAO are included in the analysis. Due to the nature of
the substantive interest in the hypotheses, follow-up of significant mul-
tivariate F ratios will emphasize univariate interpretations.

Test of H2. A two-way multivariate analysis of variance was conducted
to test hypothesis two:

H2: There will be no significant difference between the two treatment
groups on the ATAO process and classification subtests.

The test in null form was H0:

\[
\begin{bmatrix}
u_{11} \\
u_{12}
\end{bmatrix} = \begin{bmatrix}
u_{21} \\
u_{22}
\end{bmatrix}.
\]

Descriptive statistics for those students used in the analysis of ATAO
achievement are presented in Table 9. Table 32 in Appendix E presented num-
bbers of students by section and treatment for each TOLT block. Results of
the two-way multivariate analysis of variance are presented in Table 10.
<table>
<thead>
<tr>
<th>TOLT Block</th>
<th>Treatment</th>
<th>N</th>
<th>Subtest</th>
<th>S1</th>
<th>S2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>AO</td>
<td>14</td>
<td></td>
<td>70.22</td>
<td>68.18</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7.58</td>
<td>11.62</td>
</tr>
<tr>
<td>6-10</td>
<td>C</td>
<td>15</td>
<td></td>
<td>47.31</td>
<td>52.68</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>13.15</td>
<td>7.02</td>
</tr>
<tr>
<td></td>
<td>AO</td>
<td>14</td>
<td></td>
<td>62.35</td>
<td>65.46</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>9.23</td>
<td>10.82</td>
</tr>
<tr>
<td>4-5</td>
<td>C</td>
<td>15</td>
<td></td>
<td>40.32</td>
<td>49.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10.94</td>
<td>14.89</td>
</tr>
<tr>
<td></td>
<td>AO</td>
<td>15</td>
<td></td>
<td>54.26</td>
<td>56.69</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7.63</td>
<td>10.58</td>
</tr>
<tr>
<td>1-3</td>
<td>C</td>
<td>15</td>
<td></td>
<td>42.80</td>
<td>46.70</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>9.41</td>
<td>8.16</td>
</tr>
<tr>
<td></td>
<td>AO</td>
<td>13</td>
<td></td>
<td>43.59</td>
<td>54.40</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>14.40</td>
<td>15.98</td>
</tr>
<tr>
<td>0</td>
<td>C</td>
<td>12</td>
<td></td>
<td>43.52</td>
<td>47.62</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>15.01</td>
<td>7.67</td>
</tr>
</tbody>
</table>
Table 10
Multivariate Analysis of Variance of Function Process (S1) and Classification (S2) scores by Treatment and TOLT Block

<table>
<thead>
<tr>
<th>Source</th>
<th>F (Wilkes Lambda)</th>
<th>DF</th>
<th>p &lt;</th>
<th>Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Std Dis Function</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>S1</td>
</tr>
<tr>
<td>TOLT Blocks(A)</td>
<td>4.725</td>
<td>6,208</td>
<td>.001</td>
<td>.82</td>
</tr>
<tr>
<td>Treatment(B)</td>
<td>31.030</td>
<td>2,104</td>
<td>.001</td>
<td>.72</td>
</tr>
<tr>
<td>AB</td>
<td>2.990</td>
<td>6,208</td>
<td>.008</td>
<td>.96</td>
</tr>
</tbody>
</table>

The statistically significant multivariate interaction, $F(6,208) = 2.990, p < .008$, indicates there is significant interaction between TOLT blocks and treatment condition on the dependent measures. The relative magnitudes of the standardized discriminate function coefficients indicates that scores on the process subtest may be primarily responsible for the interaction. Tables 11 and 12 present univariate results for the process and classification subtests respectively.

Table 11
Univariate Analysis of Variance of Function Process Scores by Treatment Condition and TOLT Block

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>p &lt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>TOLT Blocks(A)</td>
<td>3121.85</td>
<td>3</td>
<td>1040.62</td>
<td>8.427</td>
<td>.001</td>
</tr>
<tr>
<td>Treatment(B)</td>
<td>6007.14</td>
<td>1</td>
<td>6007.14</td>
<td>48.644</td>
<td>.001</td>
</tr>
<tr>
<td>AB</td>
<td>2290.59</td>
<td>3</td>
<td>763.53</td>
<td>6.183</td>
<td>.001</td>
</tr>
<tr>
<td>Within</td>
<td>12966.54</td>
<td>105</td>
<td>123.49</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 12
Univariate Analysis of Variance of Function Classification Scores by Treatment Condition and TOLT Block

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>p &lt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>TOLT Blocks(A)</td>
<td>1575.22</td>
<td>3</td>
<td>525.07</td>
<td>4.151</td>
<td>.008</td>
</tr>
<tr>
<td>Treatment(B)</td>
<td>4312.04</td>
<td>1</td>
<td>4312.04</td>
<td>34.092</td>
<td>.0001</td>
</tr>
<tr>
<td>AB</td>
<td>430.93</td>
<td>3</td>
<td>143.64</td>
<td>1.136</td>
<td>.338</td>
</tr>
<tr>
<td>Within</td>
<td>13280.50</td>
<td>105</td>
<td>126.48</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Inspection of the interaction terms in Tables 11 and 12 supports the interpretation that scores on the process subtest are primarily responsible for the significant multivariate interaction. The significant multivariate root for the main effects of treatment $F(2,104) = 31.030, p < .001$, indicates that one treatment group outperformed the other on the subtests. $H_2$ can be rejected at the .001 level. Cell means clearly favor the AO group.

Following rejection of $H_2$, further interpretation of multivariate and univariate results was performed to provide additional insight concerning $H_2$. Inspection of the relatively large structure coefficients, .80 and .86 for the process and classification subtests respectively, indicates the differential performance may hold on both subtests. The highly significant univariate $F$ ratios for treatment main effects on both subtests confirm this indication.

Test of $H_3$. Hypothesis three was considered in two parts which were tested separately.
\[ H_{3a}: \] There will be no significant difference between the two treatment groups on the ATAO process subtest at each of the four levels of TOLT performance.

\[ H_{3b}: \] There will be no significant difference between the two treatment groups on the ATAO classification subtest at each of the four levels of TOLT performance.

Figure 5 shows the significant univariate interaction \( F(3,105) = 6.183, p < .001 \) between TOLT blocks and treatment for the process subtest of the ATAO.

Follow-up of the interaction was performed by use of the generalized Dunn procedure. This procedure permits pairwise and compound comparisons with unequal \( n \). Kennedy (1978) suggested that when less than one-fourth of the total number of pairwise comparisons is desired, Dunn's procedure is more powerful than other hypothesiswise techniques. Since five of
twenty-eight possible comparisons were desired, Dunn's procedure was used. Results of the follow-up showed that the A0 sections significantly outperformed the C sections at all TOLT levels except the concrete (TOLT score = 0), $p < .05$ at the 1-3 block, $p < .01$ at the two formal level blocks. $H_{3a}$ cannot be rejected at the .05 level of significance.

The univariate interaction term for the classification subtest did not attain significance, $F(3,105) = 1.136, p < .338$. $H_{3b}$ cannot be rejected at the .05 level. However, it is of interest to note that at both formal level TOLT blocks, 4-5 and 6-10, the difference between A0 and C group means was large enough to yield a significant Dunn test (unequal n) at the .01 level.

Test of $H_4$. A test of $H_4$ is of interest only for those subjects who studied the advance organizer. The hypothesis will be considered in two parts.

$H_{4a}$: There will be no significant difference on the ATAO process subtest between formal and non-formal reasoners.

$H_{4b}$: There will be no significant difference on the ATAO classification subtest between formal and non-formal reasoners.

Figure 6 shows the mean (percent) performance on both ATAO subtests of each treatment group by TOLT block. Inspection of Figure 6 shows a relatively large variation among the means of the A0 group by TOLT block and a relatively small variation for the C group. This suggests that differences among A0 group means were the major contributor to the significant main effect of TOLT blocks reported in Tables 12 and 13 for the process and classification subtests respectively.
A compound Dunn comparison (unequal n) was used to test $H_{4a}$. This test for differences among TOLT blocks on the process subtest was a continuation of the follow-up of the significant univariate interaction on process scores reported in Table 12. The Dunn test was significant, $p < .01$. Formal reasoners in the AO group performed significantly better than non-formal reasoners. $H_{4a}$ was rejected at the .01 level.

$H_{4b}$ could be tested by a one-way analysis of variance (four levels of TOLT blocks). This approach would cause an inflation of $\alpha$ by requiring a second analysis of the same set of data. A second approach would be to follow-up the significant main effect for TOLT blocks reported in Table 13. This would require using cell means computed by collapsing over treatment
groups. Since these means would be based on data from both treatment groups, they are not useful. Only AO group means are of interest.

Having rejected the other approaches, the following one was used: The data were necessarily submitted to a second test. To minimize any inflation of $\alpha$, a compound Sheffe' comparison (unequal n) was used. The test was significant at the .01 level. Formal reasoners in the AO group performed significantly better than non-formal reasoners on the classification subtest. $H_{4b}$ was rejected at the .01 level of significance.

In summary of the analyses of achievement on the ATAO, a significant multivariate interaction was caused primarily by a univariate ordinal interaction on the process subtest. Follow-up tests showed the AO group performed significantly better than the C group at all TOLT levels except the concrete level. Although the univariate interaction on the classification subtest was not statistically significant, the differences between AO and C group means at the two formal level TOLT blocks were large enough to yield significant Dunn tests. Finally, as might be expected, there were significant main effects for the AO group on both subtests, and formal level AO subjects performed significantly better than non-formal AO subjects on both subtests.

Analyses of Interview Results

In terms of the definitions in this study, process skills are developed prior to classification skills. This claim was based on evidence reported by Thomas (1975) that classification skills require a higher formal level of reasoning ability. Support for Thomas' finding derives from a significant Pearson correlation of $r = .37$, $p < .0005$, $n = 470$, between process and classification subscores on the ATAO.
Because of Thomas' findings and because they spent at least four
days graphing functions immediately before the experiment began,
students were expected to perform well on the process question included
in the interview task. Of the 37 interview cases retained, 30 received
at least one of two possible points on the process question. This result
confirmed the expectation stated above. Therefore, only the two
classification questions were included in the following analyses. Each
question was assigned a score of two points.

The two retained interview questions tested the subjects' ability
to transfer their understanding of the subsuming concept, functions and
their graphical representations, which was presented by the advance
organizer. A breakdown of the number of interview subjects by treatment
and interview score is presented in Table 13.

<table>
<thead>
<tr>
<th>Score</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>AO Subjects</td>
<td>10</td>
<td>1</td>
<td>4</td>
<td>2</td>
<td>12</td>
<td>29</td>
</tr>
<tr>
<td>Control Subjects</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>Totals</td>
<td>15</td>
<td>2</td>
<td>5</td>
<td>2</td>
<td>13</td>
<td>37</td>
</tr>
</tbody>
</table>

Hypothesis five was tested by performing a Pearson correlation
between interview scores and combined scores on the classification and
transfer subtests of the ATAO. The resulting correlation,
\( r = .62, p < .0005 \) was highly significant. \( H_5 \) can be rejected at the .0005 level of significance.

\( H_5: \) There is no significant correlation between interview scores and combined scores on the classification and transfer subtests of the ATAO.

To further investigate the relationship between interview scores and ATAO subtest scores, a stepwise multiple regression procedure was run. Six of the classification questions accounted for 76\% of the variance in interview scores. Entering the remaining eight questions increased \( R^2 \) slightly to 79\%. However, due to the small sample (\( n = 37 \)) upon which the regression was based and to a desire to maximize the reliability of the predictor set, all 14 classification and transfer questions were included in the predictor set. A Hoyt reliability estimate of .68 was obtained for the 14 item predictor set which shall be called the classification subtest for the remainder of this report. Scores on this subtest were part of the criteria established to predict which subjects had the subsumer in cognitive structure.

Among the 12 control subjects interviewed (four of whom were not included in the analyses due to missing ATAO data), only one demonstrated a good understanding of the subsuming concept. This student had four years of high school mathematics including precalculus. Her background was unusual for a Mathematics 100 student. Generally, Mathematics 100 students who did not read the advance organizer did not understand the subsuming concept. This conclusion is supported both by results of the interviews of 12 control subjects and by control group performance on the classification subtest of the ATAO.
Control students seemed to relate the term "function" with the verb "process." One student responded:

"A function is something you have to go through, a process" (comma added).

Control subjects also tended to connect the irrelevant attributes of continuity and "normal" appearance to graphical representations of relations. In response to interview question 1 a student said:

"A function is one line not two separate lines as above."

"How would you change the graph so that it would represent a function?"

"I'd fix um [sic] so the lines are connected."

Upon which the student drew a vertical line connecting the endpoints.

Of the 10 ATAO questions (items 13-22; see Appendix C) students were asked to classify as examples or non-examples of functions, only items 15 and 18 were correctly classified by over 70% of the control respondents. Possible explanations for the high rate of correct classification of these items could be a misunderstanding of the irrelevant attributes of continuity and "normal" appearance. Item 15, an example of a function, is a continuous curve normally encountered by Mathematics 100 students. Item 18, a non-example of a function, is not continuous and is the least "normal" appearing of all the graphs.

Students who read the advance organizer had difficulty in resolving the relationship between the informal definition of function and the visual representation of the definition by graphical examples and non-examples of functions. This result is unexpected since many examples
with explanations were presented in the advance organizer along with the use of matched and divergent examples and non-examples in the concept learning sequences.

One student stated:

"Something has to be the same to be a function."

When probed, the student was unable to explain what the statement meant.

Students who read the advance organizer also attached significance to the irrelevant attribute of continuity. A student gave the following justification for a "no" response to interview item 1.

"The flow of the graph isn't even--It doesn't have a way to be equal."

Further probing indicated he felt the curve had to be continuous.

Probably the concept misconceived most often was the idea that an unbroken line segment contains an infinite number of points, not just possible endpoints and/or plotted points. Students misconceived this concept in a preliminary study, and even though attempts were made to clarify this concept in the advance organizer, misconception still occurred. An illustration might be helpful at this point.

As a follow up to interview question 1, students were asked to classify the following graph of a relation as an example or non-example of a function.
Since the end point on the lower line was no longer included in the domain of the relation, many students who correctly answered and justified their answer to question 1, misclassified this non-example. They failed to recognize that points A and B have the same first values. In particular, they failed to understand that B is a point. This interpretation is supported by student response to ATAO item 24. Thirty percent of the AO subjects chose incorrect distractor b, indicating the same lack of understanding.

This misconception typically persists through the study of lines and slope. Indeed, after plotting two points and drawing a line through them, students fail to comprehend the significance of the line. An objective of the advance organizer was to alleviate this lack of understanding. This objective may not have been met.

Two interview students who read the advance organizer reversed the roles of the domain and range sets. They appeared to understand the other relevant and irrelevant attributes of the function concept but misconceived the role of the two sets.

From those interview students who were judged to have the subsumer in cognitive structure came some well stated responses. In response to
the question of whether the correspondence in interview task 3 represented a function, one student replied:

"No, because there are two C's and two D's, and the graph would have two points on the same vertical line."

At no time was the vertical line test for functions explained to the students. Yet this student discovered the concept behind the test as was evidenced by his mental ability to transform an instance presented in mapping notation to an instance in graphical notation.

In response to interview task 1, another student responded:

"No, because -1 is repeated in the domain set."

To interview task 3, the same student responded:

"It is not a function, because the letters C and D are repeated in the domain. There can be no repeating in the domain unlike the range."

**Predicting Anchoring of the Subsumer**

The primary purpose of the interview process was to validate the use of the ATA0 (or a subtest of the ATA0) for identifying those subjects who had the subsuming concept anchored in their cognitive structures. Performance on the classification subtest (items 13-26) of the ATA0 was used for this purpose.

Interview subjects who correctly responded to at least one and one-half of the two transfer items (score ≥ 3) were considered to have attained the function concept at Thomas' level three. Thus, they were predicted to have the subsumer anchored. The objective was to determine the magnitude of ATA0 subtest score which could be used to determine which uninterviewed students had the subsumer anchored. By requiring them both to score ≥ 9 on the classification subtest and to respond to item 24 correctly, 30 of the 37 interview students were correctly classified.
(compared to their classification based on interview results) as having or not having the subsumer anchored. Question 24 was required because it was a transfer item which entered the regression equation second (after question 21).

Based on this rate of success at classification, the conjunctive requirement, score $\geq 9$ and item 24 correct, was the criterion chosen for categorizing students as having the subsumer in cognitive structure. Table 14 presents the number of students by TOLT block by treatment condition predicted to have the subsumer in cognitive structure by the specified criteria.

Table 14
Number of Students by TOLT Block by Treatment Categorized as Having the Subsumer Anchored

<table>
<thead>
<tr>
<th>Treatment</th>
<th>TOLT BLOCK</th>
<th></th>
<th></th>
<th></th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1-3</td>
<td>4-5</td>
<td>6-10</td>
<td></td>
</tr>
<tr>
<td>AO</td>
<td>4</td>
<td>25</td>
<td>23</td>
<td>26</td>
<td>78</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>Totals</td>
<td>5</td>
<td>27</td>
<td>25</td>
<td>30</td>
<td>87</td>
</tr>
</tbody>
</table>

Few control subjects (9 of 215) met the specified criteria. By contrast 78 of 255 (30.59%) subjects who read the advance organizer met the criteria. Fifty-five subjects judged to have the subsumer anchored were formal reasoners (TOLT scores $\geq 4$) while 32 were non-formal. Of the 32 non-formal reasoners, 27 were in the transition state (TOLT scores of 1-3).
Analyses of Part II Data

Part II of the study was considered a pilot attempt to determine if exposure to the advance organizer differentially affected learning, transfer, and short-term retention. Two problems occurred that limit the potential substantive value of this part of the study: (1) The retention measure had to be taken only seven days after the achievement test over the learning materials; and (2) seven instructors failed to provide a record of their students' performances on the retention test (a five item subtest of Midterm III).

More precisely the analyses of Part II attempted to determine if those students in whose cognitive structures the subsumer was anchored performed significantly better on the three achievement measures than their control counterparts matched by ability. No evidence was found in the literature that this type of measurement has been attempted before. The advance organizer should not be expected to facilitate learning for those subjects who are unable to understand it.

Analysis of Attitudes

A two-factor analysis of variance with six section means per cell was conducted to test hypothesis six. Treatment group membership and pretest condition were the factors:

$$H_6:$$ There will be no significant difference between advance organizer and control groups on results of the semantic differential.

Results of the analysis are presented in Table 15. The F ratios indicate there were no significant differences in attitude among the
partitioned groups. $H_6$ cannot be rejected at the .05 level of significance.

Table 15

Two-Way Analysis of Variance of Attitude Scores by Treatment and Pretest Conditions

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>TREATMENT (A)</td>
<td>0.28</td>
<td>1</td>
<td>0.28</td>
<td>0.085</td>
<td>.773</td>
</tr>
<tr>
<td>PRETEST CONDITION (B)</td>
<td>1.31</td>
<td>1</td>
<td>1.31</td>
<td>0.350</td>
<td>.561</td>
</tr>
<tr>
<td>AB</td>
<td>0.70</td>
<td>1</td>
<td>0.70</td>
<td>0.216</td>
<td>.647</td>
</tr>
<tr>
<td>WITHIN</td>
<td>64.60</td>
<td>20</td>
<td>3.23</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Analysis of Special Control Group Performance

Four sections were assigned to a special control design to determine if reading the advance organizer alone could help students on the learning materials achievement test without their having studied the learning materials on lines and slope (stage 3 of the model).

Hypothesis seven was tested:

$H_7$: There will be no significant difference on the learning materials achievement test between the two matched groups, one of which studied the advance organizer, neither of which studied the learning materials.

Data were available for only two sections, one section per treatment. Since the two sections were not equal in ability as measured by average performance on Midterms I and II, students in each section
were ranked by average score on Midterms I and II and then matched by this ability measure. Eleven closely matched student pairs were formed. Group 1 both studied the advance organizer and took the learning materials achievement test in a single class period. On the same day group 2 took the learning materials achievement test only. The means of both groups were identical: 3.909. The resulting one-way analysis of variance yielded an $F(1,20) = 0.000$. $H_7$ cannot be rejected at the .05 level of significance.

Analyses of Total Treatment Versus Total Control Group Performance

In this section comparisons will be made between the AO and C groups on the three dependent measures of Part II: LMAT, a set of three transfer items, and a seven day retention test. The LMAT and the transfer items formed an 18-item multiple choice test with the last three questions being the transfer items. Questions 15 and 16 had five distractors, while all other items had four. The retention test contained five weighted items worth 5,5,8,8, and 8 points respectively. The retention test, a subtest of Midterm III, was prepared by the supervisor of the Mathematics 100 program and graded by the individual instructors according to a predetermined grading guide.

Due to missing retention data for seven class sections, the analyses were performed in three steps: 1) a two-way multivariate analysis of variance for all sections on both the LMAT and the transfer items; 2) a two-way univariate analysis of variance on the retention test results; and 3) a two-way multivariate analysis of variance of all three dependent measures for the sections which completed all three measures. If any differences found in steps 1) and 2) persisted through step 3),
some evidence would have been found that these differences were not merely characteristic of the particular sample used.

**Equality of treatment groups.** Twenty-nine sections were included in the analyses for steps 1) and 2). To test the equality of the two groups by average performance on Midterms I and II (all student raw scores converted to T scores), a one-way analysis of variance was performed. The resulting $F(1,27) = .437$ was not significant. The AO group mean was 50.40 while the C group mean was 49.69.

When only those matched samples retained for step 3) were included, the advance organizer group mean was 51.77 while the control group mean was 50.30. A second one-way analysis of variance was performed on the total of 16 section means. The resulting $F(1,14) = 1.262$ was not statistically significant, $p < .280$. Thus, there was no evidence that the groups compared in Part II of the study were significantly different in mathematical ability.

Raw score descriptive statistics for AO and C sections on the three dependent measures and on the average performance on Midterms I and II are in Table 35 in Appendix E.

**Analysis of LMAT and transfer scores.** To test hypotheses eight and nine, a two-way multivariate analysis of variance was conducted. Ability blocks and treatment group membership were the factors.

- $H_8$: There will be no significant difference between total AO and total C groups on the LMAT.

- $H_9$: There will be no significant difference between total AO and total C groups on the transfer items.
Ability blocks were formed on the basis of average performance on Midterms I and II. Criterion levels for formation of the three blocks were T-scores of 45 and 55. Table 16 presents cell means (percents) and standard deviations for both dependent measures by treatment and ability block. Inspection of Table 16 shows the AO section means higher than the C means on both tests at all ability levels.

Table 16

Cell Means and Standard Deviations for LMAT and Transfer Scores (Percent) by Ability (T-Score) Blocks and Treatment Condition

<table>
<thead>
<tr>
<th>Ability Blocks</th>
<th>Treatment</th>
<th>N</th>
<th>Test (Percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>LMAT</td>
</tr>
<tr>
<td>Upper</td>
<td>AO</td>
<td>15</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>SD</td>
</tr>
<tr>
<td>Third</td>
<td>C</td>
<td>14</td>
<td>x</td>
</tr>
<tr>
<td>(T ≥ 55)</td>
<td></td>
<td></td>
<td>SD</td>
</tr>
<tr>
<td>Middle</td>
<td>AO</td>
<td>15</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>SD</td>
</tr>
<tr>
<td>Third</td>
<td>C</td>
<td>14</td>
<td>x</td>
</tr>
<tr>
<td>(45 ≤ T &lt; 55)</td>
<td></td>
<td></td>
<td>SD</td>
</tr>
<tr>
<td>Lower</td>
<td>AO</td>
<td>15</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>SD</td>
</tr>
<tr>
<td>Third</td>
<td>C</td>
<td>14</td>
<td>x</td>
</tr>
<tr>
<td>(T &lt; 45)</td>
<td></td>
<td></td>
<td>SD</td>
</tr>
</tbody>
</table>
Table 17 presents results of the multivariate analysis.

### Table 17

Multivariate Analysis of Variance of LMAT and Transfer Scores by Ability Block and Treatment Condition

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>F (WILKS LAMBDA)</th>
<th>DF</th>
<th>P&lt;</th>
<th>COEFFICIENTS</th>
<th>STRUCTURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABILITY BLOCKS</td>
<td>13.687</td>
<td>4,160</td>
<td>.001</td>
<td>.99</td>
<td>.99</td>
</tr>
<tr>
<td>TREATMENT</td>
<td>3.189</td>
<td>2,80</td>
<td>.047</td>
<td>.77</td>
<td>.81</td>
</tr>
<tr>
<td>AB</td>
<td>.158</td>
<td>4,160</td>
<td>.959</td>
<td>-.32</td>
<td>1.01</td>
</tr>
</tbody>
</table>

The significant multivariate roots for both treatment condition ($F(2,80) = 3.189, p < .047$) and ability block ($F(4,160) = 13.687, p < .001$) indicate there were significant differences among the respective groups on the dependent measures. Inspection of the structure coefficients indicates the differences were caused primarily by LMAT results among the ability blocks and by both dependent measures between treatment conditions.

Tables 18 and 19 present univariate results for the LMAT and transfer scores respectively.
<table>
<thead>
<tr>
<th>SOURCE</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABILITY BLOCKS (A)</td>
<td>5698.12</td>
<td>2</td>
<td>2849.06</td>
<td>32.327</td>
<td>.001</td>
</tr>
<tr>
<td>TREATMENT (B)</td>
<td>431.90</td>
<td>1</td>
<td>431.90</td>
<td>4.901</td>
<td>.030</td>
</tr>
<tr>
<td>AB</td>
<td>3.23</td>
<td>2</td>
<td>1.61</td>
<td>0.018</td>
<td>.982</td>
</tr>
<tr>
<td>WITHIN</td>
<td>7138.75</td>
<td>81</td>
<td>88.13</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 19

Univariate Analysis of Variance of Transfer Scores by Treatment Condition and Ability Block

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABILITY BLOCKS (A)</td>
<td>883.99</td>
<td>2</td>
<td>442.00</td>
<td>2.137</td>
<td>.125</td>
</tr>
<tr>
<td>TREATMENT (B)</td>
<td>567.09</td>
<td>1</td>
<td>567.09</td>
<td>2.741</td>
<td>.102</td>
</tr>
<tr>
<td>AB</td>
<td>115.21</td>
<td>2</td>
<td>57.61</td>
<td>0.278</td>
<td>.758</td>
</tr>
<tr>
<td>WITHIN</td>
<td>16755.54</td>
<td>81</td>
<td>206.86</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Inspection of Tables 18 and 19 indicates significant differences between treatment groups and among ability blocks on LMAT scores while there are no significant univariate differences on transfer scores. Cell means indicate the AO group significantly outperformed the C group on the LMAT. Hypothesis 8 can be rejected at the .05 level of significance. While hypothesis 9 cannot be rejected at the .05 level of significance, the AO group mean was higher than the C group mean.

Follow up of the significant difference among ability blocks on LMAT scores is of interest since the T-score criterion levels used to form the blocks divided the total number of students equally into the three ability groups. Using the Sheffe' formula for pairwise and compound comparisons and $\alpha = .05$ level of significance, the upper block outperformed both the middle and lower blocks and the middle block outperformed the lower block. As expected ability significantly influenced performance.

**Analysis of retention scores.** For this analysis eight AO and eight C sections were retained. Table 20 presents cell means (percents) and standard deviations for the retention measure by ability block and treatment condition. Table 21 presents results of the two-way univariate analysis of variance used to test hypothesis 10.

$H_{10}$: There will be no significant difference between total AO and total C groups on the retention test.
Table 20

Cell Means and Standard Deviations for Retention Scores by Ability Block (T-Scores) and Treatment Condition

<table>
<thead>
<tr>
<th>Ability Blocks</th>
<th>Treatment</th>
<th>N</th>
<th>( \bar{x} )</th>
<th>SD</th>
<th>Retention Scores (Percents)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper Third ((T \geq 55))</td>
<td>AO</td>
<td>8</td>
<td>81.11</td>
<td>9.10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>8</td>
<td>85.70</td>
<td>6.03</td>
<td></td>
</tr>
<tr>
<td>Middle Third ((45 \leq T &lt; 55))</td>
<td>AO</td>
<td>8</td>
<td>69.21</td>
<td>10.81</td>
<td></td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>8</td>
<td>71.47</td>
<td>10.82</td>
<td></td>
</tr>
<tr>
<td>Lower Third ((T &lt; 45))</td>
<td>AO</td>
<td>8</td>
<td>56.24</td>
<td>18.40</td>
<td></td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>8</td>
<td>48.57</td>
<td>17.26</td>
<td></td>
</tr>
</tbody>
</table>

Table 21

Univariate Analysis of Variance of Retention Scores by Ability Block and Treatment Condition

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>p&lt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABILITY BLOCKS (A)</td>
<td>7750.57</td>
<td>2</td>
<td>3875.29</td>
<td>23.499</td>
<td>.001</td>
</tr>
<tr>
<td>TREATMENT (B)</td>
<td>0.93</td>
<td>1</td>
<td>0.93</td>
<td>0.006</td>
<td>.941</td>
</tr>
<tr>
<td>AB</td>
<td>339.04</td>
<td>2</td>
<td>169.52</td>
<td>1.028</td>
<td>.367</td>
</tr>
<tr>
<td>WITHIN</td>
<td>6926.19</td>
<td>42</td>
<td>164.91</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Inspection of Table 21 indicates $H_{10}$ cannot be rejected at the .05 level of significance:

The Sheffe test was used to follow-up the significant effect of the ability blocks. Results indicated that the upper ability group outperformed both the middle and lower ability groups and that the middle group outperformed the lower group at the .05 level of significance.

Analysis of LMAT, transfer, and retention scores for the restricted number of sections. Only those 16 sections utilized in the analysis of retention results were included in this analysis. The objective was to determine if the differences found in the previous two analyses would persist with a modified sample. Table 22 presents cell means (percents) and standard deviations for the reduced sample for all three dependent measures by ability block and treatment condition.
Table 22

Cell Means and Standard Deviations of Restricted Sections for LMAT, Transfer, and Retention Scores (Percents) by Ability Blocks (T-Score) and Treatment Condition

<table>
<thead>
<tr>
<th>Ability Blocks</th>
<th>Treatment</th>
<th>N</th>
<th>Test (Percents)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>LMAT</td>
</tr>
<tr>
<td>Upper Third</td>
<td>A O</td>
<td>8</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>SD</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>8</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>SD</td>
</tr>
<tr>
<td>Middle Third</td>
<td>A O</td>
<td>8</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>SD</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>8</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>SD</td>
</tr>
<tr>
<td>Lower Third</td>
<td>A O</td>
<td>8</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>SD</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>8</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>SD</td>
</tr>
</tbody>
</table>

Table 23 presents results of the two-way multivariate analysis of variance on LMAT, transfer, and retention scores with ability blocks and treatment condition as independent variables.
Table 23
Multivariate Analysis of Variance for Restricted Sections of LMAT, Transfer, and Retention Scores by Ability Block and Treatment Condition

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>F (WILKS LAMBDA)</th>
<th>DF</th>
<th>P&lt;</th>
<th>COEFFICIENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>STD DIS FUNCTION</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>LMAT TRANSFER</td>
</tr>
<tr>
<td>ABILITY BLOCKS (A)</td>
<td>8.448</td>
<td>6,80</td>
<td>.001</td>
<td>.59</td>
</tr>
<tr>
<td>TREATMENT (B)</td>
<td>.625</td>
<td>3,40</td>
<td>.213</td>
<td>1.16</td>
</tr>
<tr>
<td>AxB</td>
<td>.433</td>
<td>6,80</td>
<td>.855</td>
<td>.43</td>
</tr>
</tbody>
</table>

The multivariate root for treatment condition indicates there were no significant multivariate differences between the two treatments on the three dependent measures for this restricted sample. However, a relatively large structure coefficient (.69) indicates that standardized achievement differences on the LMAT were greater than on the other dependent measures.

The significant root for the effect of ability indicates significant differences among the three blocks on the dependent measures. Examination of structure coefficients indicates that scores on the retention test (coefficient = .91) made the most important contribution to ability differences on the discriminant function scores. LMAT scores (coefficient = .67) also made an important contribution but to a lesser degree.
Examination of univariate results supports the interpretation of the structure coefficients. Significant F ratios for both the LMAT ($F(2,42) = 25.045, p < .001$) and retention tests ($F(2,42) = 23.499, p < .001$) were found. The $F(2,42) = 2.376, p < .105$, for the transfer test was not significant. Pairwise and compound follow-up tests using the Sheffe' procedure indicated that the upper ability group significantly outperformed both the middle and lower ability groups and that the middle ability group significantly outperformed the lower group on both the LMAT and retention measures. Summaries of the three univariate analyses of variance are in Tables 36-38, Appendix E.

**Summary for Part II, all subjects retained.** In this section in which achievement differences between the treatment groups on the three dependent measures were reported, the following results were found: A significant difference between treatment groups on results of the first LMAT analysis, a difference which did not persist when the sample was necessarily restricted. All analyses showed significant differences among ability groups on the LMAT and retention tests. The upper ability group consistently outperformed both lower groups and the middle group consistently outperformed the lower group. Results of the transfer test made little contribution to the differences among the three ability groups.

**Analyses of Performances of Those AO Subjects Predicted to Have the Subsumer Anchored and Their Control Counterparts**

The reporting of analyses in this section follow the same pattern used above. Only those AO subjects predicted to have the subsuming concept in cognitive structure (score on ATAO classification subtest
and question 24 correct) and their control counterparts matched by ability were included. If the advance organizer benefited any students, they should be those students who understood the concepts presented. Descriptive statistics for the dependent measures of this section and for the ability measure (average T-score mean of Midterms I and II) by treatment by section are in Table 39 in Appendix E.

Table 24 shows the number of students by treatment condition and ability block predicted to have the subsumer anchored. The original number of AO and C students were 273 and 220 respectively.

Table 24

<table>
<thead>
<tr>
<th>Ability Block</th>
<th>Treatment Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AO</td>
</tr>
<tr>
<td>Upper (T-Score ≥ 54)</td>
<td>49</td>
</tr>
<tr>
<td>Lower (T-Score &lt; 54)</td>
<td>47</td>
</tr>
</tbody>
</table>

Equality of treatment groups. To test the validity of the procedure for producing treatment groups of equal ability as measured by average performance on the T-score average of Midterms I and II, a one-way analysis of variance was run on section T-score means. The results of this analysis are presented in Table 25:
Table 25

One-Way Analysis of Variance of Section Midterm Means (T-Scores) by Treatment Group Membership

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>TREATMENT GROUP</td>
<td>4.01</td>
<td>1</td>
<td>4.01</td>
<td>0.251</td>
<td>.621</td>
</tr>
<tr>
<td>WITHIN</td>
<td>415.57</td>
<td>26</td>
<td>15.98</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The results indicate that the two groups were evenly matched on the designated ability measure. The means were 53.29% (standard deviation = 4.33) and 52.53% (standard deviation = 3.64) for the AO and control groups respectively.

Analysis of LMAT and transfer scores. To test hypotheses 11 and 12, a two-way multivariate analysis of variance was conducted. Ability blocks (two levels) and treatment condition were the independent variables.

**H_{11}**: There will be no significant difference on the LMAT between AO subjects who are judged to have the subsumer in cognitive structure and their control counterparts matched by average performance on Midterms I and II.

**H_{12}**: There will be no significant difference on the transfer items between AO subjects who are judged to have the subsumer in cognitive structure and their control counterparts matched by average performance on Midterms I and II.
Table 26 presents cell means and standard deviations while Table 27 presents results of the multivariate analysis of variance.

Table 26

Cell Means and Standard Deviations for LMAT and Transfer Scores (Percent) by Ability Level (T-Score) and Treatment Condition for AO Subjects with Subsumer and Their Matched Control Counterparts

<table>
<thead>
<tr>
<th>Ability Blocks</th>
<th>Treatment</th>
<th>N</th>
<th>Test (Percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>LMAT</td>
</tr>
<tr>
<td>Upper</td>
<td>AO</td>
<td>13</td>
<td>x</td>
</tr>
<tr>
<td>Half (T ≥ 54)</td>
<td>C</td>
<td>13</td>
<td>x</td>
</tr>
<tr>
<td>Lower</td>
<td>AO</td>
<td>14</td>
<td>x</td>
</tr>
<tr>
<td>Half (T ≤ 54)</td>
<td>C</td>
<td>14</td>
<td>x</td>
</tr>
</tbody>
</table>

Table 27

Multivariate Analysis of Variance of LMAT and Transfer Scores by Ability Block and Treatment Condition for AO Subjects with Subsumer and Their Matched Control Counterparts

<table>
<thead>
<tr>
<th>Source</th>
<th>F (Wilks Lambda)</th>
<th>DF</th>
<th>P&lt;</th>
<th>Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Std. Dis. Function</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>LMAT</td>
</tr>
<tr>
<td>Ability Blocks (A)</td>
<td>8.23</td>
<td>2,49</td>
<td>.001</td>
<td>.93</td>
</tr>
<tr>
<td>Treatment (B)</td>
<td>10.65</td>
<td>2,49</td>
<td>.001</td>
<td>-.41</td>
</tr>
<tr>
<td>AB</td>
<td>.15</td>
<td>2,49</td>
<td>.862</td>
<td>-.76</td>
</tr>
</tbody>
</table>
Inspection of Table 26 shows AO group means were higher than C group means in all cells, while inspection of Table 27 shows significant multivariate roots for both treatment condition ($F(2,49) = 10.65, \ p < .001$) and ability level ($F(2,49) = 8.23, \ p < .001$). The structure coefficients indicate that scores on the transfer items (coefficient = .96) made the more important contribution to the discriminant function's ability to separate the treatment groups. LMAT scores (coefficient = .47) made a smaller contribution. Between the ability groups, the achievement tests were important in an inverse fashion. Scores on the LMAT (coefficient = .99) made the more important contribution to the separation of the groups while transfer scores (coefficient = .37) made a smaller contribution.

Tables 28 and 29 present univariate results for the LMAT and transfer scores respectively.

Table 28

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>P &lt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABILITY BLOCKS(A)</td>
<td>2471.26</td>
<td>1</td>
<td>2471.26</td>
<td>15.57</td>
<td>.001</td>
</tr>
<tr>
<td>TREATMENT (B)</td>
<td>860.51</td>
<td>1</td>
<td>860.51</td>
<td>5.42</td>
<td>.024</td>
</tr>
<tr>
<td>AB</td>
<td>21.99</td>
<td>1</td>
<td>21.99</td>
<td>0.14</td>
<td>.711</td>
</tr>
<tr>
<td>WITHIN</td>
<td>7936.96</td>
<td>50</td>
<td>158.74</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 29

Univariate Analysis of Variance of Transfer Scores by Ability Level and Treatment Condition for AO Subjects with Subsumer and Their Matched Control Counterparts

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>p  &lt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABILITY BLOCKS (A)</td>
<td>566.71</td>
<td>1</td>
<td>566.57</td>
<td>2.32</td>
<td>.134</td>
</tr>
<tr>
<td>TREATMENT (B)</td>
<td>4437.64</td>
<td>1</td>
<td>4437.64</td>
<td>18.19</td>
<td>.001</td>
</tr>
<tr>
<td>AB</td>
<td>32.39</td>
<td>1</td>
<td>32.39</td>
<td>0.13</td>
<td>.717</td>
</tr>
<tr>
<td>WITHIN</td>
<td>12195.35</td>
<td>50</td>
<td>343.91</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The significant univariate F ratios for treatment condition, \( F(1,50) = 5.42, \ p < .024 \), on the LMAT; and \( F(1,50) = 18.19, \ p < .001 \), on the transfer items, indicates those AO subjects who had the subsuming concept established outperformed their control counterparts matched by ability. Hypotheses 11 and 12 can be rejected at the .05 level of significance.

Table 28 also shows that upper ability students outperformed lower ability students on the LMAT, \( F(1,50) = 15.57, \ p < .001 \). Table 29 shows there was no significant difference in performance of the two ability groups on the transfer items, \( F(1,50) = 2.32, \ p < .134 \). However, the mean (34.41%) of the upper ability group was higher than the mean (27.93%) of the lower ability group.
Analysis of retention scores. A two-way univariate analysis of variance was conducted to test hypothesis 13. Ability blocks (two levels) and treatment condition were the independent variables.

H_{13}: There will be no significant difference on the retention test between AO subjects who are judged to have the subsumer in cognitive structure and their control counterparts matched by average performance on Midterms I and II.

Table 30 presents cell means and standard deviations while Table 31 presents results of the analysis of variance.

Table 30

Cell Means and Standard Deviations for Retention Scores (Percents) by Ability Level (T-Score) and Treatment Condition for AO Subjects with Subsumer and Their Matched Control Counterparts

<table>
<thead>
<tr>
<th>Ability Blocks</th>
<th>Treatment</th>
<th>N</th>
<th>Mean ± SD</th>
<th>Retention Test (Percents)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper Half \ (T ≥ 54)</td>
<td>AO</td>
<td>8</td>
<td>91.16 ± 4.10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>8</td>
<td>87.03 ± 4.82</td>
<td></td>
</tr>
<tr>
<td>Lower Half \ (T ≤ 54)</td>
<td>AO</td>
<td>8</td>
<td>69.36 ± 14.69</td>
<td></td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>8</td>
<td>55.27 ± 14.10</td>
<td></td>
</tr>
</tbody>
</table>
Table 31

Univariate Analysis of Variance of Retention Scores
by Ability Level and Treatment Condition for Subjects
with Subsumer and Their Matched Control Counterparts

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABILITY BLOCKS (A)</td>
<td>5736.67</td>
<td>1</td>
<td>5736.67</td>
<td>50.49</td>
<td>.001</td>
</tr>
<tr>
<td>TREATMENT (B)</td>
<td>664.82</td>
<td>1</td>
<td>664.82</td>
<td>5.58</td>
<td>.022</td>
</tr>
<tr>
<td>AB</td>
<td>198.17</td>
<td>1</td>
<td>198.17</td>
<td>1.74</td>
<td>.197</td>
</tr>
<tr>
<td>WITHIN</td>
<td>3182.05</td>
<td>28</td>
<td>113.65</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Inspection of Table 31 shows that the AO subjects who had the subsumer anchored significantly outperformed their matched control counterparts, $F(1,28) = 5.58, p < .022$. The upper ability group also significantly outperformed the lower ability group, $F(1,28) = 50.47, p < .001$.

Hypothesis 13 can be rejected at the .05 level.

Analysis of LMAT, transfer, and retention scores for the restricted number of sections. Only those 16 sections utilized in the analysis of retention results were included in this analysis. The differences found with this restricted number of sections were closely parallel to the differences found in the first two analyses. The probability levels varied slightly. Further interpretation of these results will not be
made since they closely parallel the interpretation of the previous two analyses. These results lend some support to a conclusion that the differences in performance between those AO students who have the subsumer and their matched control counterparts persist regardless of the analysis sample drawn.

Descriptive statistics and results of the two-way multivariate analysis of variance on LMAT, transfer, and retention scores with ability blocks and treatment condition as independent variables can be found respectively in Tables 40 and 41 in Appendix E.

Summary for Part II, subjects with subsumer and their matched control counterparts. In this section in which achievement differences were investigated between AO subjects predicted to have the subsumer in cognitive structure and their matched control counterparts, the following results were found: Significant differences in favor of the AO group on all three dependent measures, the LMAT, transfer, and retention tests. These differences persisted when the number of sections was restricted due to missing data on the retention test. The upper ability block significantly outperformed the lower ability block on all measures except the transfer test. The differences between ability blocks also persisted when the number of sections was restricted due to missing data on the retention test.
CHAPTER V

CONCLUSION

Summary

The primary purpose of this study was to determine if an appropriate advance organizer can establish a desired mathematical subsuming concept in students' cognitive structure. A procedure was developed and used to validate the use of an achievement test over the advance organizer to predict whether the subsumer was anchored in a particular student's cognitive structure. The procedure utilized results on transfer items administered in an interview setting.

Multiple regression analysis showed that the 14 item classification subtest of the ATAO accounted for 78.67% of the variance in interview scores. By requiring students to have both a minimum of nine items correct on the classification subtest and transfer item 24 correct, 30 of 37 interview subjects were correctly classified (compared to their classifications on interview results) as having or not having the subsumer anchored. Evidence was found that a short, adequately prepared advance organizer can establish a subsuming concept in cognitive structure, and that students' inability to think formally severely limits their ability to understand a formal level concept.

The secondary purpose was to determine if the advance organizer used in this study facilitated learning, transfer, and retention of the concepts.
Two variables, the potential direct effect of the advance organizer on the learning materials achievement test, stage 3 of the model, and the affective state of arousal of the students, were successfully controlled as Kozlow (1978) suggested.

Results of Part II were used to predict that by reading the advance organizer the subsumer was established in the cognitive structures of 96 of 273 students in the AO group. The performances of these 96 students were compared to their control counterparts matched by ability as measured by average performance on Midterms I and II. The AO group performed significantly better on all three dependent measures: LMAT, transfer, and retention tests. Thus, the anchoring of the subsuming concept "Functions and Their Graphical Representations" helped students learn, transfer, and retain the concepts in the learning materials over lines and slope. Structure coefficients indicated the transfer items and the LMAT made the most important contributions to the significant discriminant function.

When all students in both advance organizer and control sections were included in the analyses, a significant difference was found only for results of the LMAT. This difference did not persist when the number of sections included in the analysis was restricted due to missing sectional scores on the retention measure.

The results presented in this summary provide an explanation for the lack of consistent results in previous advance organizer studies: Measurements were not made to determine if and for whom the desired subsumer was anchored. If the number of students who understood the subsumer was relatively small, better performance could have been "diluted" by the larger number of scores of students who did not understand the subsumer. Students'
ability to understand abstract materials presented in short lessons may be the critical factor in advance organizer theory.

Part I of the Study

Discussion

The advance organizer was able to anchor the subsuming concept in the cognitive structures of some of the students. The subsuming concept, functions and their graphical representations, is a formal level concept (Thomas, 1975). Perhaps formal reasoning ability is a prerequisite for the anchoring of this concept.

Of those students who were judged to have the subsumer anchored, 55 of 87 were formal reasoners as measured by TOLT. Furthermore, on the classification subtest of the ATAO, only formal reasoners from the AO group benefited from studying the advance organizer. Formal reasoners performed significantly better than their control counterparts, while non-formal reasoners had higher means but not significantly so.

The validity of the conclusion that formal reasoning is a prerequisite for anchoring of the subsumer is further strengthened by appealing to the results on the process subtest. Here students who did not exhibit formal level reasoning (TOLT scores of 1-3) were able to perform significantly better than their control counterparts. All students except those with TOLT scores of zero benefited from studying the advance organizer. Therefore, the evidence indicated that non-formal reasoners were unable to understand the formal classification concepts, not the advance organizer in general.

This finding supports two conclusions of Kozlow (1978) and Luiten et al. (1980): 1) that care must be taken not to make the advance
organizer so abstract it cannot be understood; and 2) that in mathematics
higher ability students (probably with higher formal reasoning abilities)
may be differentially benefited by exposure to an advance organizer.
Conclusion two may be true because non-formal reasoners are not able to
understand those advance organizers which present inclusive, formal level
conds which subsume the learning materials to follow.

Interview results showed that control subjects generally had no under­
standing of the subsuming concept. They tended to relate the term "func­
tion" with the verb "process" and to relate the irrelevant attributes of
continuity and "normal" appearance to graphical representations of
functions.

Students who read the advance organizer had difficulty in resolving
the relationship between the informal definition of function and the visual
representation of the definition by graphical examples and non-examples of
functions. This finding was unexpected. Concept learning research by
Stout (note 3) indicated that the presentation in a concept learning format
of matched and divergent examples and non-examples of functions was effec­
tive (see the advance organizer in Appendix D for an example of the format).
The use of different populations may account for this finding. The
freshman sample used by Stout was taken from the final introductory course
for the standard calculus sequence. This sample had a stronger average
mathematical aptitude. Consequently, a larger percentage of formal level
reasoners was represented. The ability to reason formally is an important
correlate of success in understanding the subsuming concept in this study.

Results of this study indicate that the procedure developed to deter­
mine which subjects had the subsumer anchored in cognitive structure is
effective. It is the firm conclusion of this researcher that those
subjects who were successful on the two interview transfer tasks understood the function concept as presented in the advance organizer. A strong argument was presented that success on appropriate transfer tasks indicates the existence in cognitive structure of the concept which is being extended by the transfer task.

Student performance on the classification subtest of the ATAO accounted for 78% of the variance in interview scores. This $R^2$ value is evidence that success on the subtest may be a valid indication not only of understanding of the advance organizer but also of anchoring of the subsuming concept. ATAO item 24, a transfer item, entered the regression equation second. This position offers support for the inclusion of transfer items on the ATAO.

**Limitations**

At least two uncontrolled factors influenced how well students understood the advance organizer. The first was not taking the task seriously. The researcher visited one classroom where most students completed the full treatment, reading the advance organizer and writing the ATAO, in 25 minutes. Students on task normally required 50 minutes to complete the task. A superficial reading of materials presenting a formal level mathematical concept would be fruitless for most students.

The second factor was an apparent lack of experience in acquiring new mathematical ideas by reading. The advance organizer was a programmed learning device at a grade 9 reading level requiring students to respond to 23 questions for which there was available immediate feedback. Nevertheless, several students stated that they had difficulty understanding what they read.
Both Kozlow (1978) and Luiten et al. (1980) found evidence that presentation modes other than verbal text may be more effective. However, verbal text may be the only practical method for a large scale presentation of an advance organizer. If advance organizers were to be purposely included in textbooks, teacher-led discussion could alleviate the dependence on verbal text to communicate ideas. But dependence upon such interaction with instructors would present potentially large control problems for experimental research.

**Part II of the Study**

**Discussion**

A univariate interpretation suggests that those AO students in whose cognitive structures the subsumer was anchored significantly outperformed their control counterparts matched by ability (as measured by average performance on Midterms I and II) on learning, transfer, and retention measures. A multivariate interpretation is also of interest. The large structure coefficients, .96 for the analysis of LMAT and transfer scores and .82 for the restricted analysis of all three dependent measures, indicated that differential performance on the transfer items made the greatest contribution to the significant multivariate differences between treatment groups.

When all students were included in the analyses, there was no significant difference between the treatment groups in performance on the transfer items. However, if considered from the multivariate viewpoint, the structure coefficient for the transfer measure was large, .79, although not as large as for the LMAT, .81.

These results support the position stated by Mayer (1979) that having the subsumer anchored should make the greatest achievement difference in
transfer tasks. Transfer tasks require an extension of concept(s) to contexts that are different from those previously experienced. This extension process should be facilitated by having the concept(s) to be extended in a proper hierarchical arrangement with respect to related concepts. An objective of the subsumer is to provide a framework around which such structure can be imposed on the concepts presented in the learning materials. This finding regarding the discriminatory power of the transfer items lends further support to the possible conclusion that the advance organizer was effective.

It is puzzling that there were no significant univariate differences between ability groups on transfer items, and that the structure coefficients for the transfer items were so low. The small coefficients indicated that performance on the transfer items made relatively little contribution to the significant multivariate differences among ability groups. The transfer items, as mentioned before, were not entirely satisfactory. The low percentage of students answering them correctly indicated they were very difficult. Perhaps there were no differences among ability groups for those who did not have the subsumer because these students were basically arriving at their answers by guessing. The average performance of those students not predicted to have the subsumer was 24.0%. The random guess rate was 21.7%. These figures support the possibility that the students who did not have the subsumer were randomly guessing.

The students predicted to have the subsumer, on the other hand, performed at the rate of 39.8%, nearly twice the random guess rate. An explanation for the non-significant difference in performance between the two ability groups in this analysis is that an anchored subsumer helps students at any ability level equally. No evidence can be supplied for this
hypothesis. It must remain a conjecture for further study.

Some would argue that the extra day on the advance organizer, material related to lines and slope, provided a distinct advantage for the advance organizer sections. Therefore, the extra day may be responsible for any differential performance. In response to this implication, the following issues should be considered: 1) Control sections were also provided an experience with lines and slope. They carefully graphed the linear function $y = -3x + 1$, $-5 \leq x \leq 5$, with increments in x of 0.5. They also took the ATA0. 2) Neither the AO nor the C treatment introduced from the unit on lines and slope any of the concepts upon which the students were tested on the learning materials achievement test. 3) It is generally agreed among those familiar with the Mathematics 100 program that formally spending a fourth day on the textual material on lines and slope would be counterproductive. 4) The advance organizer was shown to have no direct effect on the learning materials achievement test performance. 5) Advance organizers are supposed to establish in cognitive structure a subsumer to which the concepts in the learning materials can be related for better learning, retention, and transfer.

Careful consideration of these five issues lead to the rejection of this alternative explanation of differential performance. However, the issue of using additional instructional days to present advance organizers is a concern that will be addressed in the recommendation section at the conclusion of this chapter.

In previous studies the advance organizers proved to be differentially effective for different ability groups. No interaction terms of any analyses were significant in this study. Therefore, there is no evidence of differential performance by treatment subjects at different ability levels.
The more important question may be the following: Who can understand the advance organizer? Before students who had the subsumer were collapsed from three to two ability blocks, a count of students at the original three levels was made. There were 20, 34, and 49 students predicted to have the subsumer at the lower, middle, and upper ability levels respectively. There is a greater chance that an upper ability student can understand the subsuming concept. But once the subsumer was anchored, performance on transfer items did not significantly differ for the ability levels.

Limitations

Any interpretation of these results must consider that the sample used was not representative of the general university population in freshman mathematics courses. Since Ohio State University has an open enrollment policy, large numbers of students are enrolled in remedial mathematics courses. The beginning remedial course was the source of the sample. Several assumptions were made regarding this population, assumptions which proved to be true: There would be both a (1) full range of formal reasoning abilities; and (2) a large range of mathematical abilities (see p. 14). But care must be exercised in any attempt to generalize the results.

Test reliabilities were not as high as desired. Although this result may be accounted for by the suspected failure of a portion of the students to take the test seriously, the relatively low reliabilities are a factor to be considered in any interpretation of results. Of particular concern is the low internal consistency of the three-item transfer test used in Part II. It is desirable that this reliability be higher in light of the interpretations made on the differential treatment group performance for this test. However, a low internal consistency was expected for two reasons: 1) The difficulty level of the three items was very high; and
2) there were only three items.

The proper unit of analysis for this experiment was the section. However, to answer the questions asked, use of the micro group as the unit of analysis was the best compromise. Since each section contributed one mean to each ability block, the independence of means among ability blocks can be assumed. Since each section contributed two, three, or four means, depending on the analysis, to the same treatment condition, independence of error effects within treatment conditions cannot be assumed. Positively biased $F$ statistics for treatment group differences may have resulted. This positive bias is minimal since the micro group was the unit of analysis rather than the student. The only significant $F$ ratios reported which may represent type I errors are those associated with treatment effect in Tables 18, 19, 29, and 32. Since none of these ratios is of substantive importance in the interpretation of results, adjustments to the ratios were not made.

**Final Conclusions**

Evidence has been presented that an appropriate advance organizer can establish a subsuming concept in students' cognitive structures. The trade-off of decreasing the level of abstraction and inclusiveness at a loss of subsumptive power to increase the percentage of students who can attain the concept is a problem. As Novak (note 2) inferred in an informal presentation at the Ohio State University, the better response may be to decrease the level of inclusiveness. Novak and associates are pursuing this course in their work at Cornell with concept mapping (Novak, 1980). They are emphasizing the presentation of subsuming concepts over long periods of time, from several weeks to a year, via concept maps.
The relatively short advance organizer can be useful. This study has presented evidence that it can anchor a subsuming concept. Continued use of the procedure developed for this study can shed further light on this ability with different concepts and different, perhaps more typical, populations.

The ability to reason formally appears to play a significant role in the attainment of formal level mathematical concepts. This evidence supports the need to present new concepts initially in terms as concrete as possible, starting with what is already known and working toward what is not known. Frequent use of and referral to concrete referents may be very important. The objective is to provide a handle or reference point for the non-formal reasoner to use in an attempt to master the new ideas. Advance organizers should be constructed in this fashion.

The procedure used in this study to predict those subjects who have the subsumer in cognitive structure is a valid one for future use. Approximately 78% of the variance in interview results was accounted for by the classification subtest of the ATAO. The sample of subjects predicted to have the subsumer significantly outperformed on all three dependent measures their control counterparts matched by mathematical ability. This significantly better performance validates the prediction that these subjects had the subsumer.

Teaching for transfer is a difficult, elusive goal in mathematics education. Among the three dependent measures, the transfer items used on the learning materials achievement test made the largest contribution to the significant multivariate difference between the two treatment groups. Mayer (1979) suggested that appropriate advance organizers should facilitate transfer. When considering those AO students predicted to have the
subsumer and their matched control counterparts, the results of this study support Mayer's claim. Perhaps the conscientious use of advance organizers where appropriate can assist mathematics educators to accomplish this elusive goal of teaching for transfer.

**Recommendations**

Additional studies should be planned in which measurements are made to determine if the advance organizer is anchored. In particular, the robustness of the transfer process as indicative of anchoring the subsumer needs further study. Populations, subsuming concepts, and learning materials should be varied to increase the generalizability of results. A goal of future research should be to develop a list of subsuming concepts helpful for both specific learning materials and populations. Such a list should be helpful in curriculum design. A textbook with several effective advance organizers for specific, perhaps typically difficult, topics may be an effective tool to increase transfer in mathematics.

A second suggestion involves instructor-led discussion of advance organizer material. All instructors of advance organizer sections were encouraged to refer during study of the learning materials to both the concepts in the advance organizer and to the concept map showing the relationships among the concepts presented in the learning materials. Many instructors followed these instructions. However, additional well-planned interaction for the students with the instructor (or perhaps ideally with a computer) may be helpful. Such interaction may partially offset the effect caused by students not normally acquiring new ideas by reading. Interaction of this type may also assist non-formal reasoners in understanding the subsuming concept.
Potential control and design problems accompany the use of instructor-student interaction. The control problem can be minimized by careful planning of the content of the interaction. A potential approach to the design problem is to use a hierarchical design in which the instructor effects are built into the design as a nested variable.

A practical question which must be faced concerns the time required to study advance organizers. Where and when in an already crowded curriculum should advance organizers be used? At this point trade offs must be considered. If an advance organizer can help students understand, transfer, and retain concepts on lines and slope, then one day devoted to study of an effective advance organizer may be worthwhile. Substitution of advance organizers for regular curricular topics should occur only when the facilitative effect of the advance organizer has been demonstrated. It is true in some cases that the sequencing of topics and daily assignments can be changed to utilize existing curricular materials as effective advance organizers. For example, a section(s) on functions often precedes sections on lines and slope in secondary algebra texts. Some modification of the content of the section(s) on functions may be necessary to create an effective advance organizer, but no additional time in the curriculum would be required.

After the experience gained from this study, it is the firm conclusion of the researcher that some incentive must be provided for participating students. The use of test results in the determination of student grades would probably be sufficient. But, as was true in this study, this approach is often not available to the researcher. Some other external motivator such as extra credit may be necessary. Otherwise absenteeism can be very
high on days during which data is to be collected. High absenteeism on multiple testing days severely limits the size of the sample available for analysis.
APPENDIX A

CONCEPT MAP
FUNCTIONS AND THEIR GRAPHICAL REPRESENTATIONS

Linear Equations
- Visual relationship between x, y variables, graphs
- Ordered pairs
- Rectangular coordinate systems
- Ratio
- Points
- Axes
- Quadrants
- Intercepts
- Rising, falling, horizontal, vertical
- Positive, negative, zero, undefined
- Slope, change, rise, run
- Numerator, denominator
- Chart, chart
- Change
APPENDIX B

ACHIEVEMENT TEST OBJECTIVES
Learning Objectives for the Advance Organizer, Functions and Their Graphical Representations

<table>
<thead>
<tr>
<th>Test Items</th>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>21, 22</td>
<td>1. Recognition of functions using ordered pairs in table format.</td>
</tr>
<tr>
<td>13-20</td>
<td>2. Classification of functions by their graphical representations.</td>
</tr>
<tr>
<td>1, 5</td>
<td>3. Understanding the notions set, ordered pair, and Cartesian Coordinate System.</td>
</tr>
<tr>
<td>2, 3, 6</td>
<td>4. Recognition of the domain and range sets.</td>
</tr>
<tr>
<td>4, 11</td>
<td>5. The need for representations of functions in addition to a listing of ordered pairs.</td>
</tr>
<tr>
<td>7, 8, 10</td>
<td>6. Generation of ordered pairs from the function rule.</td>
</tr>
<tr>
<td>9, 12</td>
<td>7. An understanding that an unbroken line segment contains an infinite number of points.</td>
</tr>
<tr>
<td>23-26</td>
<td>8. Transfer knowledge to situations not seen in the advance organizer presentation.</td>
</tr>
<tr>
<td>Question #</td>
<td>Objectives of the Unit on Lines</td>
</tr>
<tr>
<td>------------</td>
<td>--------------------------------</td>
</tr>
<tr>
<td>13</td>
<td>1. Be able to transform from 1 form of linear equation in 2 variables to a different form.</td>
</tr>
<tr>
<td></td>
<td>2. Be able to write a linear equation in 2 variables given</td>
</tr>
<tr>
<td></td>
<td>a. two points</td>
</tr>
<tr>
<td></td>
<td>b. one point and a slope.</td>
</tr>
<tr>
<td>16, 6</td>
<td>3. Be able to determine the slope of a line from</td>
</tr>
<tr>
<td></td>
<td>a. two points</td>
</tr>
<tr>
<td></td>
<td>b. a linear equation.</td>
</tr>
<tr>
<td>5, 7, 16</td>
<td>4. Understand the concept of slope.</td>
</tr>
<tr>
<td>2, 3</td>
<td>5. Be able to graph a linear equation.</td>
</tr>
<tr>
<td>8, 9, 11, 12, 14</td>
<td>6. Be able to understand and determine the x- and y-intercepts given a linear equation.</td>
</tr>
<tr>
<td>15</td>
<td>7. Be able to transfer an understanding of linear equations in two variables to new situations not previously encountered.</td>
</tr>
<tr>
<td>17, 18</td>
<td></td>
</tr>
</tbody>
</table>

n implies understanding at the recall-understanding levels of Bloom's taxonomy.

n implies understanding at the applications-analysis levels of Bloom's taxonomy.
APPENDIX C

INSTRUMENTS
Test of Logical Reasoning

Item 1 Orange Juice #1

Four large oranges are squeezed to make six glasses of juice. How much juice can be made from six oranges?

a. 7 glasses  
b. 8 glasses  
c. 9 glasses  
d. 10 glasses  
e. other

Reason

1. The number of glasses compared to the number of oranges will always be in the ratio 3 to 2.
2. With more oranges, the difference will be less.
3. The difference in the numbers will always be two.
4. With four oranges the difference was 2. With six oranges the difference would be two more.
5. There is no way of predicting.
Item 2 Orange Juice #2

How many oranges are needed to make 13 glasses of juice?

a. 6½ oranges
b. 8 2/3 oranges
c. 9 oranges
d. 11 oranges
e. other

Reasons

1. The number of oranges compared to the number of glasses will always be in the ratio 2 to 3.
2. If there are seven more glasses, then five more oranges are needed.
3. The difference in the numbers will always be two.
4. The number of oranges will be half the number of glasses.
5. There is no way of predicting the number of oranges.
Suppose you wanted to do an experiment to find out if changing the length of a pendulum changed the amount of time it takes to swing back and forth. Which pendulums would you use for the experiment?

a. 1 and 4  
b. 2 and 4  
c. 1 and 3  
d. 2 and 5  
e. all

Reason

1. The longest pendulum should be tested against the shortest pendulum.
2. All pendulums need to be tested against one another.
3. As the length is increased the number of washers should be decreased.
4. The pendulums should be the same length but the number of washers should be different.
5. The pendulums should be different lengths but the number of washers should be the same.
Suppose you wanted to do an experiment to find out if changing the weight on the end of the string changed the amount of time the pendulum takes to swing back and forth. Which pendulums would you use for the experiment?

a. 1 and 4
b. 2 and 4
c. 1 and 3
d. 2 and 5
e. all

Reason

1. The heaviest weight should be compared to the lightest weight.
2. All pendulums need to be tested against one another.
3. As the number of washers is increased the pendulum should be shortened.
4. The number of washers should be different but the pendulums should be the same length.
5. The number of washers should be the same but the pendulums should be different lengths.
A gardener bought a package containing 3 squash seeds and 3 bean seeds. If just one seed is selected from the package, what are the chances that it is a bean seed?

a. 1 out of 2  
b. 1 out of 3  
c. 1 out of 4  
d. 1 out of 6  
e. 4 out of 6

Reasons

1. Four selections are needed because the three squash seeds could have been chosen in a row.

2. There are six seeds from which one bean seed must be chosen.

3. One bean seed needs to be selected from a total of three.

4. One half of the seeds are bean seeds.

5. In addition to a bean seed, three squash seeds could be selected from a total of six.
Item 6

The Flower Seeds

A gardener bought a package of 21 mixed seeds. The package contents listed:

- 3 short red flowers
- 4 short yellow flowers
- 5 short orange flowers
- 4 tall red flowers
- 2 tall yellow flowers
- 3 tall orange flowers.

If just one seed is planted, what are the chances that the plant that grows will have red flowers?

- a. 1 out of 2
- b. 1 out of 3
- c. 1 out of 7
- d. 1 out of 21
- e. other

Reason

1. One seed has to be chosen from among those that grow red, yellow or orange flowers.

2. 1/4 of the short and 4/9 of the tall are red.

3. It does not matter whether a tall or a short is picked. One red seed needs to be picked from a total of seven red seeds.

4. One red seed must be selected from a total of 21 seeds.

5. Seven of the twenty one seeds will produce red flowers.
Item 7  The Mice

The mice shown represent a sample of mice captured from a part of a field. Are fat mice more likely to have black tails and thin mice more likely to have white tails?

a. Yes  
b. No

Reason

1. 8/11 of the fat mice have black tails and 3/4 of the thin mice have white tails.
2. Some of the fat mice have white tails and some of the thin mice have white tails.
3. 10 mice out of thirty have black tails and 12 have white tails.
4. Not all of the fat mice have black tails and not all of the thin mice have white tails.
5. 6/12 of the white tailed mice are fat.
Item 8

The Fish

Are fat fish more likely to have broad stripes than thin fish?

a. Yes
b. No

Reason

1. Some fat fish have broad stripes and some have narrow stripes.
2. 3/7 of the fat fish have broad stripes.
3. 12/28 are broad striped and 16/28 are narrow striped.
4. 3/7 of the fat fish have broad stripes and 9/21 of the thin fish have broad stripes.
5. Some fish with broad striped are thin and some are fat.
Item 9  
The Student Council

Three students from grades 10, 11, 12 were elected to the student council. A three member committee is to be formed with one person from each grade. All possible combinations must be considered before a decision can be made. Two possible combinations are Tom, Jerry and Dan (TJD) and Sally, Anne and Martha (SAM). List all other possible combinations in the spaces provided.

More spaces are provided on the Answer Sheet than you will need.

<table>
<thead>
<tr>
<th>STUDENT COUNCIL</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade 10</td>
<td>Grade 11</td>
<td>Grade 12</td>
</tr>
<tr>
<td>Tom (T)</td>
<td>Jerry (J)</td>
<td>Dan (D)</td>
</tr>
<tr>
<td>Sally (S)</td>
<td>Anne (A)</td>
<td>Martha (M)</td>
</tr>
<tr>
<td>Bill (B)</td>
<td>Connie (C)</td>
<td>Gwen (G)</td>
</tr>
</tbody>
</table>
In a new Shopping Center, 4 store locations are going to be opened on the ground level.

A BARBER SHOP (B), a DISCOUNT STORE (D), a GROCERY STORE (G), and a COFFEE SHOP (C) want to move in there. Each one of the stores can choose any one of four locations. One way that the stores could occupy the 4 locations is BDGC. List all other possible ways that the stores can occupy the 4 locations.

More spaces are provided on the Answer Sheet than you will need.
INSTRUCTIONS: Pretest on "Functions and Their Graphical Representations"

Tomorrow you will study some materials designed to help you understand sections 9.3 and 9.4 of the text. The purpose of this pretest is to measure how well you understand tomorrow's materials before you actually study them. Do not be concerned if you know only a very few answers. Simply make your best guess at each question. The results of this pretest will not count as part of your grade in Math 100. Please do your best, and good luck.

For questions 1-8, darken in the correct response on the answer sheet. For each question choose the one best answer.

1. Which of the following are sets? Choose letter a, b, c, or d for your answer.

   I  \[\{7, 9, 11, 13\}\]
   II  \[-1, 10, 9\]
   III  \[\{(-1, 1), (10, 9), (9, 11), (10, 13)\}\]
   IV  \[\{\square, 3, -1\}\]


2. A person drives from Columbus to Indianapolis at 60 mph. Suppose the trip requires 4.5 hours. Which ordered pair of the form \((t, d)\) would not belong in a table determined by the function rule \(d = 60t\), \(0 \leq t \leq 4.5\)?

   a. (1, 60)  b. (1.5, 90)  c. (3, 180)  d. (5, 300)

3. A function is specified by the rule \(y = 50x\). If the domain set is \(\{0, 1, 2, 3, \ldots\}\), which is the range set?

   a. \(\{50, 100, 150, \ldots\}\)  b. \(\{50, 100, 150, \ldots\}\)  c. \(\{0, 50, 100, 150, \ldots\}\)  d. \(\{0, 50, 100, 150, \ldots\}\)

Question 4 refers to the graph below.

4. Which domain value is paired with "0" of the range?

   a. 1  b. 2  c. -1  d. 0

5. Which ordered pair of the form \((t, d)\) does not belong to a table determined by the function rule \(d = 60t\), \(0 \leq t \leq 9\)?

   a. (0, 0)  b. (1, 60)  c. (4.68, 280.8)  d. (9, 540)

6. Consider the function rule and domain set, \(d = 45t\), \(0 \leq t \leq 7\). Which number completes the ordered pair \((\_, 148.5)\) which is of the form \((t, d)\)?

   a. 3.30  b. 4.50  c. 270  d. none of the above

7. Consider the function rule and domain set, \(d = 45t - 5\), \(0 \leq t \leq 10\). Which number completes the ordered pair \((\_, 198.5)\) which is of the form \((t, d)\)?

   a. 4.08  b. 430  c. 8482.5  d. none of the above

8. Consider the following sets of ordered pairs in the form \((x, y)\) which can be generated from \(x = y^2\). Which set would you use to convince your instructor that \(x = y^2\) is not a function rule?

   a. \[\{(9, 3), (16, 4)\}\]  b. \[\{(9, 3), (9, -3)\}\]  c. \[\{(0, 0), (9, -3)\}\]  d. \[\{(9, 3), (16, -4)\}\]
For questions 9-18, darken in the first circle "a" on the answer sheet if the instance represents a function, darken in the second circle "b" if the instance does not represent a function.
INSTRUCTIONS: Exam on "Functions and Their Graphical Representations"

Read each question on the following exam carefully. The first time through the exam answer only those questions for which you know the answer. Then go through the exam a second time to answer all remaining questions.

Please darken in heavily in pencil all answers on the answer sheet provided. Erase thoroughly if you need to change an answer.

You may refer back to the reading materials as you take the exam.
For questions 1 - 12, darken in heavily the correct response on the answer sheet. For each question choose the one best answer.

1. Which of the following are sets? Choose letter a, b, c, or d for your answer.

<table>
<thead>
<tr>
<th>I</th>
<th>II</th>
</tr>
</thead>
<tbody>
<tr>
<td>{4, 5, 7, 11}</td>
<td>{-3, 6, 5}</td>
</tr>
<tr>
<td>III</td>
<td>IV</td>
</tr>
<tr>
<td>{(-3,4), (6,5), (5,7), (6,11)}</td>
<td>{6, Δ, 7}</td>
</tr>
</tbody>
</table>


Question 2 refers to the graph below.

2. Which domain value is paired with "1" of the range?
   a. 1   b. 2   c. -1   d. 0

3. A function is specified by the rule \( y = 40x \). If the domain set is \( \{0, 1, 2, 3, \ldots\} \), which is the range set?
   a. \( \{0, 40, 80, 120, \ldots\} \)   b. \( \{40, 80, 120, \ldots\} \)   c. \( \{0, 40, 80, 120\} \)   d. \( \{0, 40, 80, 120, \ldots\} \)

4. Which reason best explains why one might want to represent a function by a rule and domain set rather than a listing of ordered pairs?
   a. The rule and domain set representation is more mathematically correct.
   b. The rule and domain set provide more information than the listing of ordered pairs.
   c. If the domain set is infinite, one cannot list all possible ordered pairs.
   d. The definition of function is given in terms of a rule.
5. Which statement best explains how the concept of ordered pair is helpful in a study of functions?
   a. Ordered pairs provide a good format which helps one to remember the numbers.
   b. Ordered pairs are very easy to fit into tables which help in organizing information.
   c. Ordered pairs are a way to represent the relation between domain values and range values.
   d. Ordered pairs help one remember that domain values are written first.

6. A person drives from Columbus to Indianapolis at 50 mph. Suppose the trip requires $4\frac{1}{2}$ hours. Which ordered pair of the form $(t,d)$ would not belong in a table determined by the function rule $d = 50t$, $0 \leq t \leq 4\frac{1}{2}$?
   a. $(1,50)$ b. $(1.5,75)$ c. $(3,150)$ d. $(5,250)$

7. Which ordered pair of the form $(t,d)$ does not belong to the set of ordered pairs determined by the function rule $d = 50t$, $0 \leq t \leq 10$?
   a. $(0, 0)$ b. $(1.21, 59.5)$ c. $(4.65, 232.5)$ d. $(10, 500)$

8. Consider the function rule and domain set, $d = 55t$, $0 \leq t \leq 6$. Which number best completes the ordered pair $(\_, 280.50)$ which is of the form $(t,d)$?
   a. 5.10 b. 1.54 c. 225.50 d. none of the above

9. A function rule is given by $d = 45t$, $0 \leq t \leq 2$. Choose the statement which best describes how many ordered pairs of the form $(t,d)$ can be generated from the rule.
   a. The number of ordered pairs is infinite.
   b. Many ordered pairs, but the set is finite.
   c. At most two ordered pairs can be generated.
   d. No meaningful description is possible.
10. Consider the function rule and domain set, \( d = 55t - 3, 0 \leq t \leq 5 \).
Which number correctly completes the ordered pair \((\_, 189.5)\) which is of the form \((t,d)\)?

- a. 3.39
- b. 3.50
- c. 10,419.50
- d. none of the above

11. Which one of these diagrams pictures a graph of a function which can be more conveniently represented by a graph than by a set of ordered pairs?

12. Consider the following graph:

Which statement best justifies why a line segment connects the points?

- a. \(t\) values of 1.1, 1.2, 1.3, ..., 1.8, 1.9 are included in the domain set.
- b. The domain set contains all the numbers between 1 and 2.
- c. The line shows how to get from \(t = 1\) to \(t = 2\).
- d. The domain set is an infinite set of numbers.
For questions 13 - 22, darken in the first circle "A" on the answer sheet if the instance represents a function, darken in the second circle "B" if the instance does not represent a function.
23. Consider the following sets of ordered pairs in the form \((x,y)\) which can be generated from \(x = y^2\). Which set would you use to convince your instructor that \(x = y^2\) is not a function rule?
   a. \((4,2),(4,-2)\)  b. \((4,2),(9,-3)\)  c. \((4,2),(9,3)\)  d. \((4,-2),(0,0)\)

24.

Which diagram below indicates how the above diagram can be changed to represent the graph of a function?

- [Diagram a]
- [Diagram b]
- [Diagram c]
- [Diagram d]

25. Which set shows all the ordered pairs generated by this correspondence?
   a. \(\{(0,1),(1,2),(2,3),(3,4),(4,5)\}\)
   b. \(\{(0,1),(1,0),(1,2),(2,3),(3,2),(4,3),(5,4)\}\)
   c. \(\{(1,0),(2,1),(3,2),(4,3),(5,4)\}\)
   d. \(\{(1,0),(0,1),(2,1),(3,2),(4,3),(5,4)\}\)

26. Is the correspondence a function?
   a. Yes, because there is exactly one element of the range paired with each element of the domain.
   b. Yes, because the domain and range elements are the same.
   c. No, because some elements of the domain correspond with more than one element of the range.
   d. No, because both elements "1" and "3" of the domain correspond to element "2" of the range.
Interview Task

1. Does the following graph represent a function?

Justify your answer to question 1.

2. Consider the function rule and domain set \( d = t^2 + 1, \ -4 \leq t \leq 4 \). On the given lattice with its indicated scale, circle each point which belongs to the graph of the function.
To assign one locker to each student, a gym teacher put the following chart on the board:

<table>
<thead>
<tr>
<th>Locker</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Steve</td>
</tr>
<tr>
<td>B</td>
<td>Jim</td>
</tr>
<tr>
<td>C</td>
<td>Hal</td>
</tr>
<tr>
<td>C</td>
<td>Ed</td>
</tr>
<tr>
<td>D</td>
<td>Ralph</td>
</tr>
<tr>
<td>D</td>
<td>George</td>
</tr>
<tr>
<td>E</td>
<td>Hank</td>
</tr>
</tbody>
</table>

a. List all the ordered pairs of the form (locker, student) generated by this assignment.

b. Is the correspondence a function? Why or why not?
Example: For each pair of words below place an X on the blank that best tells how you feel about--

SNOW

like: X: ---: ---: ---: ---: hate
cold: X: ---: ---: ---: ---: hot
work: ---: ---: X: ---: ---: play

These responses would indicate that the person likes snow but is not crazy about it. The person thinks snow is very cold and that snow means some work and some play.

Directions: For each pair of words below place an X on the blank that tells how you feel about

MATH

confident: ---: ---: ---: ---: ---: insecure
thinking: ---: ---: ---: ---: ---: memorizing
mysterious: ---: ---: ---: ---: ---: understandable
uncomfortable: ---: ---: ---: ---: ---: at ease
easy: ---: ---: ---: ---: ---: hard
exciting: ---: ---: ---: ---: ---: dull
practical: ---: ---: ---: ---: ---: useless
boring: ---: ---: ---: ---: ---: challenging
rigid: ---: ---: ---: ---: ---: flexible
INSTRUCTIONS: Quiz on Lines and Slope

Read each question on the following quiz carefully. The first time through the quiz answer only those questions for which you know the answer. Then go through the quiz a second time to answer all remaining questions.

Please darken in heavily in pencil all answers on the answer sheet provided. Erase thoroughly if you need to change an answer. Good luck!

1. What is the x-intercept of the line $3x - y = -6$?
   a. 3  b. -6  c. -2  d. -\frac{1}{3}

2. What is the slope of the line $-2x + y + 3 = 0$?
   a. -3  b. 3  c. -2  d. 2

3. What is the slope of the line $y = 7x - 20$?
   a. 1  b. 7  c. 20  d. -20

4. What is the y-intercept of the line $y = 15 + 4x$?
   a. 1  b. 15  c. 4  d. 2

5. The slope of the line passing through the points $(1,6)$ and $(2,3)$ is
   a. 3  b. -3  c. 1/3  d. -1/3

6. Find an equation of the line through $(1,-6)$ with slope 2.
   a. $y - 6 = 2(x - 1)$  b. $y - 6 = 2(x + 1)$
   c. $y + 6 = 2(x + 1)$  d. $y + 6 = 2(x - 1)$

7. To compute the slope of the line through the points $(5,-3)$ and $(12,300)$, which is the best choice?
   a. $\frac{300 - (-3)}{12 - 5}$  b. $\frac{12 - (-3)}{300 - 3}$
   c. $\frac{12 - 5}{300 - 3}$  d. $\frac{300 + 3}{12 - 5}$

8. If a line contains the points $(0,3)$ and $(2,0)$, which one of the following is true?
   a. The line has x-intercept 3.
   b. The line has y-intercept 0.
   c. The slope of the line is positive.
   d. The slope of the line is negative.
9. If the slope of a line is \(-\frac{2}{3}\), then a point moving along the line
   a. goes up 2 units for each 3 units it moves to the left.
   b. goes up 3 units for each 2 units it moves to the left.
   c. goes up 2 units for each 3 units it moves to the right.
   d. goes up 3 units for each 2 units it moves to the right.

10. If a line crosses the x axis at \(x = 1\) and the y axis at \(y = -2\), which point is on the line?
    a. \((1, -2)\)    b. \((0, 1)\)    c. \((-2, 0)\)    d. \((1, 0)\)

Questions 11 and 12 refer to the equation \(y = mx + b\), for any real numbers \(x\) and \(y\).

11. Which characteristic of a linear equation is associated with the following graph?
    ![Graph](image)
    a. \(m > 0\)    b. \(m = 0\)    c. \(b < 0\)    d. \(b = 0\)

12. Which characteristic of a linear equation is associated with a graph that intersects the negative x-axis and the positive y-axis?
    a. \(m > 0\)    b. \(m = 0\)    c. \(b < 0\)    d. \(b = 0\)

13. If \(3x + y - 2 = 0\) were written in the form \(y = mx + b\), which equation would result?
    a. \(y = -3x - 2\)    b. \(y = -3x + 2\)    c. \(y = 3x - 2\)    d. \(y = 3x + 2\)

14. Which graph represents a line with positive slope and positive x-intercept?
    ![Graphs](image)
    a. ![Graph A](image)    b. ![Graph B](image)    c. ![Graph C](image)    d. ![Graph D](image)

15. Which graph represents the line \(2x - 3y = 6\)?
    ![Graphs](image)
    a. ![Graph A](image)    b. ![Graph B](image)    c. ![Graph C](image)    d. ![Graph D](image)
16. The four vertices of the above rectangle are A(1,3), B(5,3), C(5,-3), and D(1,-3). Find the equation of the diagonal through vertex A.

   a. \( y - 3 = -\frac{6}{4}(x - 5) \)  
   b. \( y - 3 = -\frac{4}{6}(x + 3) \)  
   c. \( y + 3 = -\frac{4}{6}(x - 5) \)  
   d. \( y - 3 = -\frac{6}{4}(x - 1) \)  
   e. \( y - 3 = -\frac{4}{6}(x - 1) \)

Questions 17 and 18 refer to the following situation which assumes a linear relationship between the variables. A manufacturing company is interested in introducing a new power mower. Its market research department gave management the demand-price forecast listed in the table below:

<table>
<thead>
<tr>
<th>Price (x)</th>
<th>Estimated Demand (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$70</td>
<td>7,800 mowers</td>
</tr>
<tr>
<td>$120</td>
<td>4,800 mowers</td>
</tr>
<tr>
<td>$160</td>
<td>2,400 mowers</td>
</tr>
<tr>
<td>$200</td>
<td>0 mowers</td>
</tr>
</tbody>
</table>

17. Which equation shows the relationship between Price (x) and Demand (y) for the mowers?

   a. \( y = 7800 - x - 70 \)  
   b. \( y = 7800 = -\frac{60}{(x - 70)} \)  
   c. \( y = x - 200 \)  
   d. \( y = 7800 = 200(x - 70) \)  
   e. \( y = 200 = x \)

18. If $70 is the minimum price and $200 the maximum price under consideration, which graph best represents this situation?

   a.  
   b.  
   c.  
   d.  

   [Graphs not visible in text]
Retention Test - A Subtest of Midterm III

1. Write $5x + 6 = 3y$ in standard form, $Ax + By = C$, and identify $A$, $B$, and $C$.

2. Draw the line through $(-2,1)$ with slope $3/4$.

3. Write the equation of the line passing through the points $(-2,-1)$ and $(3,3)$.

4. Find the slope, $x$-intercept, $y$-intercept, and sketch the graph of $3y + 2x = 6$.

5. Determine whether the triangle determined by the points $(0,2)$, $(2,0)$, and $(5,5)$ is an isosceles triangle.
APPENDIX D

THE ADVANCE ORGANIZER
Please read the following material on "Functions and Their Graphical Representations" carefully. Placed throughout the study materials are 23 questions. Answer each question carefully in the provided space and then immediately check the answer sheet on pages 10 and 11 for the correct answer. You may detach the answer sheet for easier reference. The object of the "read and answer questions" approach is to help you understand the materials. The purpose of this exercise is to help you understand the material on lines and slope which you begin to study tomorrow.

After you have completed the reading and the exercises, take the achievement test at the end. You may refer back to the materials as you take the test. You have 48 minutes for the entire exercise, reading and exam. Use the time wisely.
FUNCTIONS AND THEIR GRAPHICAL REPRESENTATIONS

This is an introduction to a unit on lines which you will be studying shortly. Hopefully, the ideas you encounter in this introduction will help you understand the ideas about lines better. Look at examples 1) - 3):

1) \( \{21, 1.1, 3x, .05, \sqrt{y}, -16\} \)
2) \( \{1, 2, 3, 4, 5, 6, \ldots\} \)
3) \( \{(1,2), (6,6), (1,4), (2,-8), (.21,-.21), (x,y)\} \)

The three examples are all sets, or collections of things. What things and how many things there are in a set does not matter. The third set is of particular interest to us. It is a set of ordered pairs.

ORDERED: Which member comes first makes a difference.

PAIR: There are two members.

The ordered pairs \((1,2)\) and \((2,1)\) represent different points on a graph. In the last few days you have plotted from 40 - 120 such ordered pairs as part of making graphs.

1. PLOT \((1,2)\) and \((2,1)\).

In graphing you worked with rectangular coordinate systems as illustrated in question 1 above. Note the \(x\) and \(y\) axes. Recall they are number lines.

2. AT WHAT ORDERED PAIR DO THE TWO AXES INTERSECT?

Having reviewed the above ideas, let us consider a concept which can help you in your work with lines. Consider the situation on the next page:
If your goal is Florida, it might be of interest to you to associate with each amount of time travelled a corresponding distance travelled at the constant speed of 50 mph. One way to represent such a correspondence is by a table.

<table>
<thead>
<tr>
<th>t(hrs)</th>
<th>0</th>
<th>½</th>
<th>1</th>
<th>1½</th>
<th>2</th>
<th>2.7</th>
<th>12</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>d(miles)</td>
<td>0</td>
<td>25</td>
<td>50</td>
<td>75</td>
<td>100</td>
<td>135</td>
<td>600</td>
<td>1000</td>
</tr>
</tbody>
</table>

Another way to represent this correspondence between time and distance is by a set of ordered pairs of the form \((t,d)\):

\[
S = \{(0,0),(½,25),(1,50),(1½,75),(2,100),(2.7,135),(12,600),(20,1000)\}
\]

where \(t\) represents a time and \(d\) a corresponding distance travelled in the specified time. The set can be expanded to include more ordered pairs of the form \((t,d)\).

3. Give another ordered pair of the form \((t,d)\).

4a. How many different time values are there between 0 and 20 hours?

b. Referring to your answer for part (a), how many different ordered pairs should be included in set \(S\) to completely describe the correspondence?

All the first elements of a set of ordered pairs are called the domain set. The domain set = \(\{0, ½, 1, 1½, 2, 2.7, 12, 20\}\) for set \(S\) above. The set of all second elements of a set of ordered pairs is called the range set.

5. Give the range set for \(S\).
Representing a correspondence between time and distance by a table of ordered pairs (such as on page two) is often not very satisfactory. The table is inconvenient when the set of ordered pairs is large. A rule and domain set are often a better way to represent the correspondence. The rule can be used to generate any ordered pairs which are desired. In the previous example, a rule which indicates how to find all ordered pairs of the form \((t, d)\) is \(d = 50t\). We must specify what the domain set is for this rule. A trip of 1000 miles at 50 mph would require 20 hours. Time values larger than 20 hours would not be necessary. Therefore, \(0 \leq t \leq 20\) represents the domain set. Remember: \(t\) values such as \(\frac{1}{2}\), 1.615, .001, \(2\sqrt{2}\), and any other real numbers between 0 and 20 are included as possible time values.

This rule has a special property: With each time value, it associates one distance value. That is, for any time travelled, say two hours, there can be only one distance associated, in this case 100 miles. These ideas introduce the important concept of function. A function can be thought of as a rule which assigns to each element of a set of numbers (the domain set) exactly one element of another set (the range set).

On the next page are four tables, two which represent functions and two which do not represent functions. Below each table is its corresponding graph:
FUNCTION

<table>
<thead>
<tr>
<th>Domain</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<td>2</td>
<td>2</td>
<td>2</td>
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</tbody>
</table>

NOT A FUNCTION

<table>
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<tr>
<th>Domain</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>3</th>
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<tbody>
<tr>
<td>Range</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

6. WHAT DISTINGUISHES THE EXAMPLES OF FUNCTIONS FROM THE NON-EXAMPLES?

You should now be able to decide whether some examples are functions or not by looking at either the graphs or the tables resulting from the rules. Try your skill on this exercise:

7. Circle a point which must be removed in order to create a function.

Many useful functions can be represented by smooth curves instead of sets of isolated points as in the examples on page four. You graphed functions such
as \( y = x^2 + 3x, -3 \leq x \leq 3 \). You used the rule to find as many as 40 points, plotted these points, and then smoothed in a curve.

A graphical representation of the function specified by \( d = 50t, \ 0 \leq t \leq 20 \), is a straight line beginning at the origin and ending at \((20,1000)\).

Why do we connect points with a smooth curve? We do so in this example because every time value between 0 and 20 hours (there are an infinite number of them) has a corresponding distance value associated with it. It is impossible to compute all the corresponding distance values and plot the resulting ordered pairs. Therefore, we compute a few values, plot the corresponding points (heavy dots), and by smoothing between these points predict what the rest of the graph should be.

8. How do the following graphs differ?

9. Is the point \((1,1)\) included in the following graph?

In your study of lines the domain set will usually include all real numbers. If the domain set in the example at the top of the page included all real numbers, we could indicate this on the graph as follows:

The arrows indicate the line continues in both directions indefinitely. This means every real number represents a \(t\) value which has a corresponding
10. **WHY IS THE FOLLOWING GRAPH NOT A VALID REPRESENTATION OF THE FUNCTION \( d = 50t, \ 0 \leq t \leq 20? \)**

![Graph](image)

11. **TWO CARS TRAVEL TO NEW YORK CITY. YOU ARE IN ONE AND YOUR FRIENDS ARE IN THE OTHER. YOU AGREE TO TRAVEL ACCORDING TO THE FOLLOWING RULE: \( d = 55t, \ 0 \leq t \leq 16. \) IN 12\( \frac{1}{2} \) HOURS, YOU TRAVELLED 687.5 MILES WHILE YOUR FRIENDS SAID THEY TRAVELLED A DIFFERENT DISTANCE (700 MILES).**

   a. **WHICH GRAPH REPRESENTS THIS SITUATION?**

   ![Graphs](image)

   b. **WHAT DO WE KNOW ABOUT FUNCTIONS THAT MAKES THIS SITUATION NOT POSSIBLE?**

12. **SPECIFY THE DOMAIN SET FOR THE FOLLOWING GRAPH:**

![Graph](image)

13. **BELOW IS A GRAPH OF A FUNCTION. IS THERE A POINT ON THE GRAPH DIRECTLY ABOVE THE POINT 3? IF SO, WHAT IS ITS COORDINATES?**

![Graph](image)
Study the following examples and non-examples of graphical representations of functions. Try to understand the important factor which distinguishes between examples and non-examples.

FUNCTION

\[ f(x) = x^2 \]

NOT A FUNCTION

\[ f(x) = x \]

FUNCTION

\[ f(x) = \sqrt{x} \]

NOT A FUNCTION

\[ f(x) = \sqrt{-x} \]
Based on your study of the above examples, classify each instance below as an example or a non-example of a function.

14. 

15. 

16. 

17. 

18. 

19. 

20. 

21. 

22. 

23. 
You will begin a study of lines tomorrow. The new material will present ideas such as graphing which are similar to what you have already learned. In addition you will be asked to learn some new ideas such as how to distinguish one line from another. Since all lines (except vertical lines) represent functions, you may find the ideas presented in this introduction helpful to you. Try to keep the following ideas in mind as you begin your study:

1. A function can be thought of as a rule which assigns to each element of the domain set one element of the range set.
2. Each function rule with its domain set can be represented by one set of ordered pairs.
3. A graph can be used to represent a function pictorially.
4. In many cases one can easily determine if a graph represents a function or not.
ANSWERS FOR THE 23 QUESTIONS IN THE STUDY MATERIALS.

You may detach this sheet from the packet for easier reference.

1. \[ \begin{align*}
A(1,2) \\
B(2,1)
\end{align*} \]

2. At the ordered pair \((0,0)\), the origin.

3. \((3,150)\) and \((4,400)\) are examples which are easy to find.

4. a. You can pick as many as you want, which means an infinite number of time values.

   b. Since each time value can be a first element in an ordered pair, an infinite number of ordered pairs could be included.

5. \[ \{0, 25, 50, 75, 100, 135, 600, 1000\} \]

6. For the function, there is associated only one range value with a given domain value. Graphically, this means two different points cannot lie on a vertical line, for two such points would have the same domain value but different range values:

   Notice that listing the same ordered pair more than once does not affect whether or not the example is a function.

7. \[ \begin{align*}
A & \in \mathbb{R} \\
B & \in \mathbb{R}
\end{align*} \]

   Either point can be removed. Then, for each domain value there would be exactly one range value. Also no two points will be on the same vertical line.

8. They are the same graph. They represent the same rule and domain set, \(d = 50t, \ 0 \leq t \leq 20\). The only difference is the choice of points plotted. In (a) only two points are emphasized. In (b) four points are emphasized. Heavy dots are usually found at the ends of smooth graphs to emphasize the starting and ending points of the domain. Remember: heavy dots are only representative of the infinite number of points which make up a curve.

9. Yes; the line goes through the point on the paper representing \((1,1)\). Therefore, \((1,1)\) is a point on the graph. That there is no heavy dot at \((1,1)\) is irrelevant.

10. The open circle at \((20,1000)\) indicates this point is not included in the graphical representation. The correct domain specification for the graph as drawn is \(0 \leq t \leq 20\). The domain specification, \(0 \leq t \leq 20\) says all time values from 0 to and including 20 hours should be included. Therefore, the open circle should be darkened in to indicate that the point \((20,1000)\) is included in the graph.
11. a. 2) because this graph shows two different distances travelled (range values) for one time (domain) value.

b. A function rule assigns exactly one element of the range set to one element of the domain set. Since \( d = 55t \) is a function rule, two different distances travelled cannot be assigned to the same time value.

12. The domain set is all the positive real numbers since the arrowhead on the slanted line indicates the line continues indefinitely. Zero is not included in the domain as indicated by the open circle at the (0,0) point.

13. Yes. Its coordinates are the ordered pair (3,5).

14. example

15. non-example; there are two range values for all but two domain values. Note how the vertical line intersects the graph in more than one point, A and B.

16. example; for a given domain value, there is only one range value.

17. example; for each domain value there is only one range value.

18. non-example; there are two range values for all but one domain value. For instance, points A and B have the same domain value but different range values.

19. non-example; the one domain value has many range values.

20. non-example; notice that points A and B have the same domain value but different range values. Points C and L also have the same domain value but different range values.

21. example; there is only one range value associated with a domain value. A vertical line could intersect the graph in at most one point.

22. example; there is only one range value associated with each domain value.

23. non-example; the vertical line shows points A and B have the same domain value but different range values.
APPENDIX E

DESCRIPTIVE STATISTICS, MANOVA AND ANOVA
SUMMARY TABLES FOR SELECTED VARIABLES
Table 32
Number of Students in Each Micro Group for Analyses of Part I

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<tr>
<td>14</td>
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<td>10</td>
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<td>2</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Totals</td>
<td></td>
<td>26</td>
<td>108</td>
<td>72</td>
<td>49</td>
<td>22</td>
<td>94</td>
<td>55</td>
<td>44</td>
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<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
Table 33

Students Per Micro Group for the Three Analyses of Part II in Which All Students Were Retained

<table>
<thead>
<tr>
<th>AO SECTION</th>
<th>LOWER</th>
<th>MIDDLE</th>
<th>UPPER</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ability Block</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lower</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>77</td>
</tr>
<tr>
<td>Middle</td>
<td>7</td>
<td>7</td>
<td>3</td>
<td>105</td>
</tr>
<tr>
<td>Upper</td>
<td>4</td>
<td>6</td>
<td>5</td>
<td>91</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CONTROL SECTION</th>
<th>1 2 3 4 5 6 7 8 9 10 11 12 13 14 15</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ability Block</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lower</td>
<td>1 9 4 4 8 7 3 3 7 3 2 3 5 3</td>
<td>62</td>
</tr>
<tr>
<td>Middle</td>
<td>4 5 7 4 4 6 6 3 11 11 7 5 4 14</td>
<td>91</td>
</tr>
<tr>
<td>Upper</td>
<td>8 2 5 6 6 4 3 5 5 6 4 5 3 5</td>
<td>67</td>
</tr>
</tbody>
</table>

\(^a\) A micro-group was the set of students in one ability block in one section.

\(^b\) Underlining indicates those sections used in analyses II and III.
Table 34

Students Per Micro Group\textsuperscript{a} for the Three Analyses of Part II in Which AO
Students with the Subsumer and Their Matched Control Counterparts Were Included\textsuperscript{b}

<table>
<thead>
<tr>
<th>AO SECTION</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ability</td>
<td>Lower</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>7</td>
<td>5</td>
<td>8</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Block</td>
<td>Upper</td>
<td>0</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>8</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>1</td>
<td>6</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>CONTROL SECTION</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
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<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>TOTAL</td>
<td></td>
</tr>
<tr>
<td>Ability</td>
<td>Lower</td>
<td>3</td>
<td>5</td>
<td>1</td>
<td>8</td>
<td>3</td>
<td>2</td>
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<td>1</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td>6</td>
<td>49</td>
</tr>
<tr>
<td>Block</td>
<td>Upper</td>
<td>8</td>
<td>2</td>
<td>7</td>
<td>3</td>
<td>3</td>
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<td>3</td>
<td>5</td>
<td>0</td>
<td>2</td>
<td>45</td>
</tr>
</tbody>
</table>

\textsuperscript{a} A micro-group was the set of students in one ability block in one section.

\textsuperscript{b} Underlining indicates those sections used in analyses II and III.

\textsuperscript{c} Data for this section were discarded.
Table 35
Means and Standard Deviations on LMAT, Transfer Items, Retention Test, and Midterms I and II Average (T-Score) by Treatment and Section

<table>
<thead>
<tr>
<th>AO Section</th>
<th>N</th>
<th>Test</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>LMAT&lt;sup&gt;a&lt;/sup&gt;</td>
<td>Transfer&lt;sup&gt;b&lt;/sup&gt;</td>
<td>Retention&lt;sup&gt;c&lt;/sup&gt;</td>
<td>Ability</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\bar{X}$</td>
<td>SD</td>
<td>$\bar{X}$</td>
<td>SD</td>
<td>$\bar{X}$</td>
</tr>
<tr>
<td>1</td>
<td>13</td>
<td>7.77</td>
<td>2.20</td>
<td>.69</td>
<td>.75</td>
<td>25.29</td>
</tr>
<tr>
<td>2</td>
<td>17</td>
<td>7.47</td>
<td>2.87</td>
<td>1.00</td>
<td>.87</td>
<td>20.20</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>6.50</td>
<td>2.17</td>
<td>.60</td>
<td>.97</td>
<td>22.87</td>
</tr>
<tr>
<td>4</td>
<td>18</td>
<td>7.44</td>
<td>2.77</td>
<td>.94</td>
<td>.87</td>
<td>25.43</td>
</tr>
<tr>
<td>5</td>
<td>21</td>
<td>7.00</td>
<td>2.72</td>
<td>.67</td>
<td>.80</td>
<td>24.18</td>
</tr>
<tr>
<td>6</td>
<td>23</td>
<td>6.57</td>
<td>2.33</td>
<td>.70</td>
<td>.76</td>
<td>21.39</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
<td>8.05</td>
<td>2.46</td>
<td>1.45</td>
<td>1.00</td>
<td>28.04</td>
</tr>
<tr>
<td>8</td>
<td>21</td>
<td>8.33</td>
<td>2.92</td>
<td>1.43</td>
<td>.93</td>
<td>25.08</td>
</tr>
<tr>
<td>9</td>
<td>22</td>
<td>6.82</td>
<td>2.61</td>
<td>1.00</td>
<td>.82</td>
<td>25.43</td>
</tr>
<tr>
<td>10</td>
<td>23</td>
<td>8.61</td>
<td>2.41</td>
<td>.61</td>
<td>.78</td>
<td>28.61</td>
</tr>
<tr>
<td>11</td>
<td>21</td>
<td>9.67</td>
<td>2.67</td>
<td>.90</td>
<td>.83</td>
<td>25.08</td>
</tr>
<tr>
<td>12</td>
<td>13</td>
<td>7.62</td>
<td>2.79</td>
<td>.46</td>
<td>.66</td>
<td>21.39</td>
</tr>
<tr>
<td>13</td>
<td>18</td>
<td>6.44</td>
<td>2.68</td>
<td>.83</td>
<td>.71</td>
<td>25.08</td>
</tr>
<tr>
<td>14</td>
<td>21</td>
<td>7.38</td>
<td>2.27</td>
<td>.95</td>
<td>.92</td>
<td>21.39</td>
</tr>
<tr>
<td>15</td>
<td>12</td>
<td>5.92</td>
<td>2.78</td>
<td>.58</td>
<td>.51</td>
<td>25.08</td>
</tr>
</tbody>
</table>

Control Section

|   |   | LMAT<sup>a</sup> | Transfer<sup>b</sup> | Retention<sup>c</sup> | Ability |
|---|---|------|---|---|---|---|
|   |   | $\bar{X}$  | SD | $\bar{X}$  | SD | $\bar{X}$  | SD | $\bar{X}$  | SD |
| 1 | 13| 8.00  | 3.51| 1.31 | .95| 24.77  | 9.17| 57.4  | 7.8 |
| 2 | 16| 5.81  | 1.97| 1.00 | .73| 24.77  | 9.17| 43.4  | 10.8|
| 3 | 16| 4.44  | 2.71| .88  | .62| 24.77  | 9.17| 49.5  | 7.8 |
| 4 | 14| 3.07  | 2.76| .29  | .47| 24.77  | 9.17| 50.0  | 7.9 |
| 5 | 18| 6.33  | 2.99| .56  | .70| 24.77  | 9.17| 49.8  | 7.7 |
| 6 | 17| 5.76  | 2.49| .53  | .51| 24.77  | 9.17| 47.8  | 9.4 |
| 7 | 12| 7.17  | 2.66| .92  | .67| 24.77  | 9.17| 48.4  | 8.7 |
| 8 | 11| 7.27  | 2.61| .55  | .52| 24.77  | 9.17| 49.6  | 10.2|
| 9 | 23| 6.70  | 2.30| .70  | .76| 24.77  | 9.17| 49.7  | 6.5 |
| 10| 20| 8.50  | 1.85| 1.05 | .76| 24.77  | 9.17| 50.3  | 8.7 |
| 11| 13| 7.62  | 2.53| .38  | .51| 24.77  | 9.17| 51.0  | 6.8 |
| 12| 13| 7.62  | 2.99| .46  | .52| 24.77  | 9.17| 51.6  | 8.4 |
| 13| 12| 6.25  | 2.77| .58  | 1.00| 24.77  | 9.17| 47.8  | 8.7 |
| 14| 22| 8.00  | 2.85| .82  | .73| 24.77  | 9.17| 49.4  | 8.1 |

<sup>a</sup>15 items  
<sup>b</sup>3 items  
<sup>c</sup>34 points
## Table 36

### Univariate Analysis of Variance for Restricted Sections of LMAT Scores by Ability Block and Treatment Condition

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABILITY BLOCK(A)</td>
<td>4104.66</td>
<td>2</td>
<td>2052.33</td>
<td>25.045</td>
<td>.001</td>
</tr>
<tr>
<td>TREATMENT(B)</td>
<td>131.05</td>
<td>1</td>
<td>131.05</td>
<td>1.599</td>
<td>.213</td>
</tr>
<tr>
<td>AB</td>
<td>133.63</td>
<td>2</td>
<td>66.81</td>
<td>.815</td>
<td>.449</td>
</tr>
<tr>
<td>WITHIN</td>
<td>3441.75</td>
<td>42</td>
<td>81.95</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 37

Univariate Analysis of Variance for Restricted Sections
Of Transfer Scores by Ability Block and Treatment Condition

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABILITY</td>
<td>1009.27</td>
<td>2</td>
<td>504.64</td>
<td>2.367</td>
<td>.105</td>
</tr>
<tr>
<td>BLOCK(A)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TREATMENT(B)</td>
<td>8.67</td>
<td>1</td>
<td>8.67</td>
<td>.041</td>
<td>.841</td>
</tr>
<tr>
<td>AB</td>
<td>54.92</td>
<td>2</td>
<td>27.46</td>
<td>.129</td>
<td>.879</td>
</tr>
<tr>
<td>WITHIN</td>
<td>8921.88</td>
<td>42</td>
<td>212.43</td>
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</tr>
</tbody>
</table>
Table 38

Univariate Analysis of Variance for Restricted Sections (Part II)
Of Retention Scores by Ability Block and Treatment Condition

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABILITY BLOCK(A)</td>
<td>7750.57</td>
<td>2</td>
<td>3875.29</td>
<td>23.499</td>
<td>.001</td>
</tr>
<tr>
<td>TREATMENT(B)</td>
<td>0.92</td>
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<td>0.92</td>
<td>.010</td>
<td>.941</td>
</tr>
<tr>
<td>AB</td>
<td>339.04</td>
<td>2</td>
<td>169.52</td>
<td>1.028</td>
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<tr>
<td>WITHIN</td>
<td>6926.19</td>
<td>42</td>
<td>164.91</td>
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Table 39
Means and Standard Deviations on LMAT, Transfer Items, Retention Test, and Average Midterm Means (T-Score) by Treatment and Section for AO Subjects with Subsumer and Their Matched Control Counterparts

<table>
<thead>
<tr>
<th>Section</th>
<th>Test</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>LMAT&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td>X</td>
</tr>
<tr>
<td>AO</td>
<td>N</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
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<td>8</td>
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<td>5</td>
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<tr>
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<tr>
<td>13</td>
<td>4</td>
</tr>
<tr>
<td>14</td>
<td>8</td>
</tr>
</tbody>
</table>

<sup>a</sup>15 items
<sup>b</sup>3 items
<sup>c</sup>34 points
Table 40

Cell Means and Standard Deviations of Restricted Sections for LMAT, Transfer, and Retention Scores (Percents) for AO Subjects with Subsumer and Their Matched Control Counterparts by Ability Blocks (T-Score) and Treatment Condition

<table>
<thead>
<tr>
<th>ABILITY BLOCKS</th>
<th>TREATMENT</th>
<th>N</th>
<th>TEST (PERCENTS)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>LMAT</td>
</tr>
<tr>
<td>UPPER</td>
<td>AO</td>
<td>8</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>SD</td>
</tr>
<tr>
<td>HALF</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(T ≥ 54)</td>
<td>C</td>
<td>8</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>SD</td>
</tr>
<tr>
<td>LOWER</td>
<td>AO</td>
<td>8</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>SD</td>
</tr>
<tr>
<td>HALF</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(T &lt; 54)</td>
<td>C</td>
<td>8</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>SD</td>
</tr>
</tbody>
</table>
Table 41

Multivariate Analysis of Variance for Restricted Sections of LMAT, Transfer, and Retention Scores for AO Subjects with Subsumer and Their Matched Control Counterparts by Ability Block and Treatment Condition

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>F (WILKS LAMBDA)</th>
<th>DF</th>
<th>p</th>
<th>COEFFICIENTS</th>
<th>Std DIS FUNCTION</th>
<th>STRUCTURE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>LMAT_Transfer Retention</td>
<td>LMAT_Transfer Retention</td>
<td></td>
</tr>
<tr>
<td>ABILITY BLOCKS(A)</td>
<td>18.49 3,26 .001</td>
<td></td>
<td></td>
<td>.32 .18 .91 .51 .10 .96</td>
<td>.39 .63 .65 .69 .82 .52</td>
<td></td>
</tr>
<tr>
<td>TREATMENT(B)</td>
<td>4.88 3,26 .008</td>
<td></td>
<td></td>
<td>.24 -.82 -.75</td>
<td>.24 -.82 -.75</td>
<td></td>
</tr>
<tr>
<td>AB</td>
<td>1.52 3,26 .233</td>
<td></td>
<td></td>
<td>.24 -.82 -.75</td>
<td>.24 -.82 -.75</td>
<td></td>
</tr>
</tbody>
</table>
REFERENCE NOTES


LIST OF REFERENCES

Allen, Donald J. Effects on the learning and retention of written social studies material of the use of advance organizers with memory level or higher order questions. (Doctoral Dissertation, University of California, Berkeley, 1969). Dissertation Abstracts International. 1970, 30, 4267A. (University Microfilms No. 70-6036)


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