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TARIFFS AND BALANCE-OF-PAYMENTS ADJUSTMENT: A PORTFOLIO APPROACH

The Ohio State University

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TARIFFS AND BALANCE-OF-PAYMENTS ADJUSTMENT:

A PORTFOLIO APPROACH

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

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The Ohio State University
1981

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To my mother and father
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The monetary approach, being the dominant approach to balance-of-payments (BOP) theory for the past ten years,\(^1\) has been subjected to various criticisms. First, there is no real financial sector in the typical monetary model. With no alternative asset to money, only the adjustment of the trade account and its implications for the money supply is examined. The capital account merely accommodates the trade account. The response of the demand for money to changes in the interest rate is thereby assumed away.

The monetary approach is severely limited in terms of its usefulness for policy analysis. For example, in an empirical study, Kouri and Porter (1974) found that monetary policy will only have a minor effect on the goods market whenever capital flows are given their appropriate role in the model. Also, Miles (1979) demonstrates, in an empirical study, the relative insignificance of the real balance effect compared to the portfolio composition adjustment. Miles shows that a devaluation will cause a balance-of-payments improvement mainly through a capital account instead of a trade account surplus.
Second, except in a growth context, in which the effect of economic growth on the balance-of-payments is considered, the monetary approach usually "assumes" that full employment exists in the economy, without defending this typically unrealistic assumption in any manner. Also, because of this assumption, which is made solely for the sake of simplicity rather than the sake of analysis, the advocates of the monetary approach neglect the traditional role assigned to the change in the "level of income" in studying mechanisms which bring the BOP to zero: (i.e. the multiplier approach).

Third, the type of economy usually postulated by the monetary approach is the so-called "small" open economy, in the sense that the prices of traded goods are treated as externally fixed. By virtue of this assumption, when analyzing the standard question of the effect of devaluation on the balance-of-payments, the monetary approach therefore neglects the other traditionally cited adjustment mechanism: the effect of changes in the "terms of trade", or the relative price effect. This is the general practice of the monetary approach, with the exception of Dornbusch (1973(a)) in which he considers this particular adjustment mechanism in a transfer problem framework. Therefore, the only way the monetary approach is able to restore the effect of devaluation on the real side of the economy via relative price changes is to incorporate nontraded goods into the model for a small country.
Finally, the monetary approach, while concentrating on the long-run steady state equilibrium situation following an exogenous disturbance, tends to overlook the interim dynamics of the adjustment processes and their possible influence on the final outcome.

As pointed out by both Bilson (1978) and Blejer (1979), it is not sufficient to restrict attention to either the short-run or the long-run as is generally done in static models. This is because the effect of a devaluation in the intermediate run may be quite different from either short-run or long-run, and also because when the economy adjusts to the exchange rate variations, the period of disequilibrium is likely to be very long (according to Blejer's empirical study (1977)). So, the nature of the transition process becomes important for evaluating the implications of the devaluation, and for any empirical implementation of a model.

In short, according to Hahn (1977): "... In particular, one is continuously diverted from the main question by naive monetarism, ..., too simple-minded and somewhat loose theorizing."

Therefore, BOP theorists should approach the problem from a new direction. One such potential development\(^2\) is the so-called "portfolio-balance" approach, which is underdeveloped in its application to the analysis of international economics. This approach, with a sector incorporating international capital flows, is essentially a more rigorous version
of the monetary model. (Note that this mainly overcomes the first limitation, as listed in the beginning, for the typical monetary approach).

The purpose of my dissertation, therefore, is fourfold. First, the dissertation will develop a portfolio-balance model for an open economy which modifies and remedies major drawbacks associated with the conventional portfolio approach. For one thing, the assumption of an instantaneous portfolio adjustment, which is usually assumed by a so-called "stock" approach, is questioned as to its empirical applicability. Also, the role of price expectation in affecting the selection of the portfolio mix, when price is freely flexible, will be discussed in detail in my analysis.

The price expectational factor in the demand function for assets, which has been a newly developed research topic in the finance literature (e.g. see Andrew H. Chen (1978)), has been generally found to have a significant effect on the portfolio selection of an investor or on the financing and investment policies of a firm. However, it has not been incorporated properly in the international economics literature. Therefore, in my dissertation, although the common practice of explicit incorporation of a utility maximization model will not be employed to derive the effect of inflation on the optimal portfolio decision of wealth holders, I will incorporate their basic findings in the construction of my model.
Second, the dissertation will provide a specific example of the application of my portfolio-balance model by analyzing the effects of an imposition of a tariff on the balance-of-payments, for reasons stated later in this Chapter.

Third, the emphasis of my dissertation is on the interim dynamics of the BOP adjustment process between short-run and long-run equilibrium. By focusing on this period, we can investigate rigorously the effect of possible dynamic adjustments on the final outcome. Specifically, I will experiment with different specifications of my model as well as different hypothetical values for some key parameters to see if the resulting dynamics exhibit any fundamental changes, such that the comparative statics are affected.

Finally, I will incorporate in my analysis some particular features besides the presence of the second asset, in order to relax some other restrictive assumptions inherent in the traditional monetary approach. For example, I will release the full employment assumption by investigating the situation of unemployment as well as full employment. Furthermore, non-traded goods are incorporated into my model. Also, to make my analysis more policy oriented, I apply my dynamic analysis to the case in which the monetary authorities may pursue an active policy designed to offset the effect of the balance-of-payments on the money supply, such as open market operations.

Although there is an existing literature that is concerned, in the framework of a portfolio-balance model, with the
more typical question of the effects of a devaluation on the BOP, I choose to examine the effects of an imposition of a tariff for the following reasons:

First, under the assumption of a small country with all goods traded, a devaluation leaves the relative price of commodities unchanged and thus has no effect on the real variables of the economy (except the supply of real cash balances). A tariff on the importable, on the other hand, will always change the domestic relative price between goods and thus affect the pattern of production and consumption. By virtue of this basic difference in terms of impact on the economy, the case with a tariff change offers a better example of how we can eliminate the dichotomy between real (barter) and monetary theories which is often cited as a striking limitation in the international economic theory. 3

Second, in a portfolio balance framework in which the portfolio adjustment has the main role in the BOP adjustment process, the type of disturbances which are caused by the imposition of a tariff are quite different from those caused by a devaluation. Following a devaluation, which raises the domestic price of foreign currency, the domestic prices of all traded things (commodities as well as assets) will increase equi-proportionately by the rate of devaluation, and therefore so will the general price level. Consequently, the only asset whose real value has been reduced will be money, while the traded non-monetary asset maintains its
initial real value after the devaluation. An excess demand for real cash balances relative to the other asset is therefore created as a natural result of the portfolio balance requirements. However, in case of an imposition of a tariff, the general price level rises only as a result of the higher prices of importables (and nontraded goods, if there are any) faced by domestic economy, and the real value of the non-monetary asset as well as the cash balances will be reduced equipropotionately. In other words, disturbances occur in the markets for both assets. Consequently, whether the portfolio composition equilibrium between the two assets has been disturbed is not as obvious as in the case of a devaluation. For this reason, we can therefore infer possible different portfolio equilibrium adjustments and resulting BOP adjustments.

Third, in terms of the value of the total portfolio stock, there might be a capital gain which occurs in the case of a devaluation if domestic residents hold some assets dominated in foreign currency. By augmenting the value of wealth of a devaluing country, this makes the net effect of a devaluation on real wealth ambiguous. However, in the tariff case, there is no such complication, and therefore, it is easier for us to divert our attention from the traditionally emphasized "real cash balance" to the portfolio "composition" adjustment aspect of the BOP which is presently so underdeveloped.
On the other hand, all the existing portfolio models that discuss the adjustment of the balance-of-payments are more or less partial equilibrium and incomplete. In some cases, only the financial sector of the economy is formulated, such as Branson (1974), and Girton and Henderson (1976). In other cases, although a seemingly general equilibrium framework that includes various sectors of the economy is developed, there is seldom a clear explanation of the interaction across sectors. In particular, the interrelationship between the determinants of the composition of the portfolio and that of the size of portfolio itself are never explained.

In other words, few of the conventional models recognize the fact that changes in the factors that entail a reallocation of an existing portfolio among different assets will also in general induce a new desired scale of portfolio. Furthermore, the role of international capital mobility has not been sufficiently discussed in most of these analyses.

Moreover, in the conventional portfolio approach in which a static approach to the wealth effect is employed, an instantaneous portfolio adjustment is usually assumed. That is, under the convention of the so-called stock approach, transaction costs associated with trade in assets are assumed to be negligible so that it is feasible that assets are always held in desired proportions at any instant. As pointed out by Kouri (1976), whether this is a realistic assumption is in fact an empirical matter. This may be a reasonable
simplified assumption for a large segment of the international short-term money market, as Kouri asserts. One might wonder, however, under certain circumstances, whether this assumption will still be equally applicable. For example, the country with which we are concerned is a small, non-reserve currency country in which most of its capital flows are likely to be "autonomous" rather than "accommodating" (i.e. capital flows for financing the trade account balance), with longer-term maturities, and there exist high costs associated with a more rapid transaction of these capital flows. Consequently, it is probably more realistic to assume a relatively sluggish rather than instantaneous type of portfolio adjustment in composition as well as in scale in this situation.

In this study, I attempt to remedy these shortcomings associated with conventional portfolio balance models. In order to accomplish this goal, each factor which affects the composition of a given portfolio as well as its scale will be carefully examined. One of the major characteristics of my model is that the respective effects of the real rate of interest and the expected rate of inflation on the desired portfolio holdings are distinguished. This has seldom been clearly expressed in the existing literature. Capital mobility is also explicitly considered throughout the qualitative analysis. Also, I will take into account the appropriate assumption as to the speed of portfolio adjustment, rather
than follow the conventional practice of instantaneous adjustment.

A literature review comprises Chapter II, and my basic model is then developed in Chapter III. In Chapter IV, by applying my model, an analysis of the full employment case is presented in the first section; the case in which unemployment exists is then discussed in the second section. In Chapter V, a dynamic model is developed and utilized to prove rigorously (quantitatively) all the qualitative inferences made in Chapter IV. Particularly, we can derive precisely the characteristics of the adjustment paths and the resulting magnitude of the BOP improvement under different hypothetical situations. The case with unemployment is also mathematically analyzed in this chapter. Furthermore, some alternative specifications of the model are also experimented with in order to investigate whether the adjustment paths will exhibit different characteristics under different basic assumptions. Finally, in Chapter VI, an extension is made to cover the case of open market operations aimed to eliminate the effect of the external disturbances (as reflected in the balance-of-payments balance) on the money supply. A summary, conclusions, and suggestions for further research are contained in Chapter VII.
CHAPTER II

LITERATURE REVIEW

The most relevant work which I can find in the literature on this topic is Michael Mussa (1974). Mussa takes the issue in a pure real trade model first. He finds that with a neutral rebate of tariff proceeds, an increase in the tariff rate not only reduces the value of imports, but also reduces that of exports by the same amount, such that the trade balance, measured in foreign currency, will always be balanced. Therefore, Mussa concludes: "From these facts, it follows that, within the context of the exclusively real theory model, an increase in the tariff rate cannot be seen to have any effect on the balance of payments. For this reason, the real theory model, by itself, cannot provide the basis for an adequate analysis of the BOP effects of tariff changes".

Mussa proposes that the effect of a tariff on the arguments of the money demand function is the determinant of the long-run effect of a tariff on the level of foreign exchange reserves (i.e. the cumulative effect on the balance of payments). Also, he claims that a divergence between income and expenditure which gives rise to the needed BOP
surplus will not only be created by a real balance effect working directly on expenditure under the monetary approach. Expenditure may also be affected by changes in interest rates or changes in the value of non-monetary assets. For example, in the world of capital mobility, the requirements of portfolio balance may lead to purchases or sales of assets to the rest of the world. In a variable employment model, the divergence between income and expenditure may come as a result of changes in income rather than changes in expenditure. All of these mechanisms can play a role in the process of adjustment of the balance of payments. Mussa therefore employed a "semi-portfolio-balance" model in which there are two assets: money and an internationally non-traded domestic security so that the demand function for money is a function of the interest rate as well as real income.

The main arguments implied in his mathematical derivation of the model are as follows. Because real cash balances decline due to the higher general price level following the imposition of a given tariff, the interest rate increases to maintain money market equilibrium. This higher interest rate will also reduce expenditure and brings forth the needed money stock through a cumulative trade balance surplus. Over time, as the nominal money balance increases towards its initial real value, the interest rate gradually falls and so a smaller trade account surplus is generated. Eventually, the trade surplus reduces to zero when the stock of money
has risen by the same proportion as the price level, such that money market equilibrium is restored.

There are two serious shortcomings in his analysis. First, it is not clear how the interest rate is (if ever) affected by portfolio balance considerations. It is not clear that he ever gives explicit consideration to the equilibrium of the portfolio balance between assets. After a rise in the general price level, he implies that the money market is the only market in which equilibrium has been disturbed (with the real cash balance reduced by the extent of the increase in the general price level). And thus, with excess demand for real cash balances relative to bonds, the interest rate must be bid up to restore equilibrium in both asset markets. The only justification for such an argument, it seems to me, is the following unrealistic assumption: wealth holders, while holding money free of any money illusion, hold bonds with full illusion, and thus, bonds are held in terms of their nominal value.

Second, it excludes completely the possibility of capital mobility because the non-monetary asset is nontraded, and hence there is no capital account in the balance of payments. Therefore, the BOP adjustment following an imposition or an increase of tariff is essentially a trade balance adjustment through a variant of the conventional real balance effect (Although the expenditure cutting arises from the higher interest rate, rather than being a direct
result of the real balance effect). And so, it is not at all obvious that Mussa really moves beyond the traditional monetary approach, since he incorporates merely the trade account in his analysis.

In short, because of the assumption of non-tradedness of the non-monetary asset across countries (and thus, no capital mobility) as well as the implicit inconsistency between demand functions for different assets, Mussa obtained his peculiar BOP adjustment process which relies entirely on the expenditure effect of a higher interest rate. As a result, the consideration for portfolio balance is not clearly seen in his analysis at all. Instead, by simply applying the Walras Law and thus conducting his entire analysis in terms of only commodity and money markets, it is not obvious as to what difference the introduction of the second asset ever made in Mussa's BOP adjustment process compared to a conventional money-only model. In a word, the incorporation of another asset besides money in his model does not exert the proper function which it should have served.

However, in spite of all these limitations, Mussa made an important contribution by pointing out that the possible influence of any policy change or exogenous disturbances on the arguments of the money demand function will, in turn, affect the long-run stock equilibrium and thus the BOP adjustment. It is his work which provides much of the
motivation for this study.

Next, we turn to a paper written by Frenkel and Rodriguez (1975), which has been cited as the most complex model using the portfolio balance approach. With two assets in their model, money and an internationally traded equity (which represents ownership of capital goods or claims for the income streams yielded by capital), Frenkel and Rodriguez analyze the process by which a small fully-employed economy adjusts in response to monetary changes (creation of outside and inside money and devaluation respectively). With the explicit consideration of the role of portfolio balance, they distinguish between short- and long-run effects and explore the exact channels through which adjustments take place.

Their process of adjustment of the balance-of-payments resulting from devaluation is illustrated graphically by Figure 1. Where, m = real cash balance.

\[ k_d = \text{real capital stock owned by domestic residents.} \]

\[ i = \text{interest rate; pegged at the world level because of the perfect capital mobility and small country assumptions.} \]

\[ a = \text{the total real value of assets, } a = m + P_k k_d. \]

\[ P_k = \text{the price of capital goods.} \]

\[ P = \text{the domestic general price level.} \]

\[ a^* = \text{the steady state value of real assets.} \]
e = the exchange rate.

The desired ratio of real cash balances to the real value of capital (equities) is assumed to be a decreasing function of the rate of interest, i.e. \( m = \xi(i)P_kk_d \), and \( \xi'(i) < 0 \). So, the ray \( \xi(i) \) reflects the specific desired ratio of \( m \) to \( k_d \) with an externally determined interest rate \( i^* \), and any portfolio composition which lies along the locus \( \xi(i^*) \) implies portfolio balance.

Consider an initial long-run equilibrium at point A. With all goods traded, following a devaluation, the general price level rises proportionately to the devaluation. So, the real value of cash balances declines while that of internationally traded bonds remains constant. (We are now at point C). An instantaneous adjustment of the portfolio balance motivates the private sector to sell securities
In exchange for cash balances and to arrive at point B, (where the portfolio composition equilibrium is restored), and also to build up immediately part of the depreciated money stock. On the other hand, the "scale" requirement to maintain the steady state value of assets sets in motion the "real wealth effect", as the community reduces expenditure relative to income and thereby runs a balance of trade surplus. The accumulation of assets along the path between point B and A is reflected in a deficit in the capital account as the community stops selling securities and instead, buys back securities in order to maintain equilibrium between the gradually recovered cash balance and equities. (Note: the same point is made in Connolly and Taylor's paper (1974)). The process of adjustment is completed once the nominal stock of money has risen equi-proportionally to the depreciation of the currency.

Finally, Frankel and Rodriguez propose a set of empirically testable hypotheses which evolve from the main results of their paper: that is, a capital account surplus should be observed in the earlier phase and then a deficit in the later stage of the adjustment following a devaluation.

Although they capture most of the essence of a portfolio balance model, and thus remedy the major flaws of Mussa's work as discussed above, Frenkel and Rodriguez do not explain how their "steady state" value for the
asset has been established. This is partly a result of their restrictive assumption of perfect capital mobility, which means that the rate of interest is viewed by the small country as pegged externally. On the other hand, this is also caused by their failure to make a clear distinction between actual and desired real wealth which is, in turn, a result of peculiar assumption that planned saving is always fulfilled. (Refer to their eq.(13) and footnote 7).

Furthermore, by simply assuming an instantaneous portfolio composition adjustment, the adjustment path will not deviate from $l(i^*)$ once point B is reached, but they give no explanation as to why this must be so.

Another important step forward in the portfolio balance literature has been made by Floyd (1978). In his attempt to reconcile the monetary approach and the more traditional approach (income-expenditure or IS-LM models), Floyd examines the respective roles of real and monetary factors in balance-of-payments adjustment under alternative assumptions about international capital mobility. His main contribution lies in his finding that when the degree of capital mobility is explicitly considered, the smaller the degree of capital mobility, the less ascendant is the role of monetary factors in the balance of payments adjustment process.

However, while having the same shortcomings as those of Frenkel and Rodriguez, there is also a vital flaw involved in Floyd's portfolio analysis. He apparently organizes his
core of analysis solely on the composition equilibrium of the portfolio, while ignoring the other traditional type of portfolio equilibrium condition, i.e. the scale requirement of the stock of wealth. This leads to his conclusion that: "... the traditional sort of real balance effect will only exist when the model incorporates only one asset (money) (p. 27), ... and excess money holdings (due to external money creation) is a portfolio problem and, in the absence of restrictions on transactions in bonds and equities, will be eliminated by a one-shot conversion into the non-monetary asset, and not by increased consumption flow".

To investigate Floyd's spirit of balance of payments adjustment more closely, we can use Figure 1 to illustrate a case with devaluation in his model. With perfect capital mobility, he starts from an initial equilibrium position like point A. "... The devaluation switches expenditure from traded to non-traded goods, 'driving up non-traded goods prices and the general price level'. The higher level of prices implies an increased level of desired money holdings (as a result of reduced real cash balances) (point C). "Domestic residents acquire these additional nominal balances by selling assets in the international market. To maintain the exchange rate at its new higher level (after devaluation), the authorities have to provide these additional money balances by purchasing foreign exchange reserves.
This causes the nominal money balance to increase until the desired cash balance is fulfilled (and therefore, portfolio balance between assets) (point B). The devaluation thus brings about a temporary balance of payments surplus\textsuperscript{1}. (p.13)

However, since money stock is accumulated completely through selling the non-monetary asset abroad, the size of the total wealth will not be increased. So, by suggesting the above adjustment process, Floyd apparently fails to recognize the community's desire to restore the initial level of real wealth, which will derive the community to cut their expenditure on goods. That is, Floyd obviously has left out the remaining part of the adjustment path from point B to point A, the long-run equilibrium point.

Therefore, in his final conclusion (p.27), he states: "Those which rigorously incorporate such real balance effects contain only one asset, money, and no marketable real capital. The direct expenditure effect of excess money stock (or excess demand for money) on commodities makes sense—indeed, is necessary— in these models because portfolio adjustments are ruled out". In other words, Floyd tends to assert that once there is another asset besides money, the composition adjustment of portfolio will instead become the sole driving force behind the balance of payments adjustment while the traditional real balance effect, on the other hand, is rendered completely useless and unnecessary.
Although there is the foregoing drawback in Floyd's analysis, his giving proper consideration to real factors in the BOP adjustment when capital mobility is less than perfect is correctly done (compared to Mussa), and probably pioneering. I will try to incorporate his findings in establishing my model in the next chapter.

The other important work which is relevant to my model is Blejer (1979). Although he uses a single asset (money-only) model which has no bearing on the portfolio balance consideration which I incorporate as the major feature of my model, Blejer formulates explicitly the short-run dynamics of prices following an once-and-for-all devaluation.

With the distinction between traded and non-traded goods, which has come increasingly to occupy the attention of the trade theorists, Blejer follows the convention of Dornbusch (1973) and Krueger (1974), to integrate the substitution effect as well as the aggregate spending effect of a devaluation on the balance-of-payments through changes of the relative price between traded and non-traded goods and reduction of the real balance of money stock.

However, instead of focusing on the analysis of long-run equilibrium and the demonstration of the purely transitional effects of an exchange rate adjustment on the balance-of-payments, Blejer formulates mathematically the short-run adjustment path of the domestic inflation as well as the
balance-of-payments in returning to the final equilibrium.

Blejer sets up the relative price of the non-traded good with respect to the traded goods as a function of the monetary disequilibrium which results from either a adoption of domestic monetary policy or a devaluation. Specifically, he formulates the relative price of the non-traded goods as a simple exponential function of the monetary disequilibrium with a coefficient which is in turn a function of the elasticities of substitution between traded and non-traded goods in production and consumption as well as the income elasticity of non-traded goods (i.e. the real balance effect on expenditure on non-traded good). In fact, in one of his earlier papers (1977), he already proved empirically the appropriateness of this specification of the function.

Moreover, Blejer also mathematically derives the result that the price of the non-traded good tends to increase by the same proportion as the devaluation (and thus as the price of the traded goods) in the long-run in his appendix, by aggregating the changes of the price of the non-traded good over all time periods following the once-and-for-all devaluation. Although the long-run neutrality of a devaluation on the relative price of the non-traded goods with respect to traded goods has been indicated verbally or illustrated graphically by some predecessors (e.g. Dornbusch (1973), Krueger (1974), and Wolf (1978)), Blejer gives us a rigorous mathematical proof.
Therefore, in my Chapter III in which I derive the comparative static change of price level, I will follow suit to take into account the relative price effect (real side) as well as the real wealth effect (asset side) which a tariff has on the price level and thus the balance of payments.

The literature cited above is representative of most recent and critical developments in applying the portfolio approach to balance of payments adjustment. Of course, there has been much written since the 1960s with regard to the portfolio balance model for an open economy. Some of the important works, such as Mundell (1963), McKinnon and Oates (1966), Floyd (1969), and more recently Branson (1970), Allen (1973), Dornbusch (1975), and Lapan and Enders (1978) which have applied the so-called "new view" of international capital movements (which distinguish capital movements resulting from stock vs. flow adjustments), provide more or less some useful general ideas for forming my model. Because nothing specific was taken from any one of them, however, I do not intend to discuss each of these papers separately here in my review of the literature.
CHAPTER III
THE MODEL

Introduction:

Defining the overall balance-of-payments as being equal to the divergence between the stock demand for and supply of money, the monetary approach often cites the so-called "real balance effect" as the main adjusting force which brings the balance-of-payments (BOP) back to equilibrium following any exogenous disturbance. (The one most often seen in existing literature is the discussion of the effect of a devaluation on the BOP). The basic reasoning behind the real balance effect is a stable money demand function and thus any divergence between actual and desired money balance will cause adjustment in expenditure on commodities flows through the trade balance such that the money stock equilibrium is restored.

However, by focusing on the role of the real balance effect in the process of balance-of-payments adjustment, the monetary approach tends to ignore other feasible adjustment mechanisms through changes of real factors which impinge directly upon the demand for money function following any exogenous disturbances. In particular, if the interest rate
and employment level, which are the main determinants of money demand, vary after the occurrence of disturbances, then the adjustment of BOP as a whole can be described as a combination of real as well as monetary phenomena. 8

Therefore, in the following Chapter, with a portfolio balance model, I will attempt to demonstrate that after a tariff is imposed:

(1) The fact that not only the composition of a given BOP improvement (in terms of trade balance or capital account balance) but also the size of that BOP change may be affected by the preference pattern between different assets (money and non-monetary assets) at the margin, and

(2) With the degree of capital mobility explicitly considered, how the relative significance of real factors (interest rate and employment level) vs. monetary factors in the adjustment of the balance-of-payments will be affected.

(3) Under what kind of situation different degrees of capital mobility will result in different degrees of improvement of the balance-of-payments.

(4) With variable rather than constant employment, i.e. with the more realistic assumption that there may exist some unemployed resources, there may be a significant shift in the final outcomes in terms of the change of the BOP.

The basic assumptions used in my model are set forth below.
First, I consider a small open economy, with three commodities (importables, exportables, and nontraded goods), and two assets (money and an internationally traded non-monetary asset, a composite bond-equity bearing interest rate \( i \)). (In the rest of paper, I will simply call this asset a bond). Assume that there exists perfect international commodity arbitrage such that the prices of the traded goods always converge to their world levels; and assume that this small country imposes a tariff to protect its "infant industries" for the long-run goal of import substitution. (Notice that by assuming a small country in the international commodity market, the optimal tariff argument under which a country gains from the imposition of tariff due to the resulting favorable terms of trade is no longer applicable).

Second, the bond could be perfectly mobile across countries such that the asset price, and thus the interest rate in the home country, would be the same as in the rest of the world by virtue of asset arbitrage. Alternatively, the portfolio-asset could be imperfectly mobile such that the domestic interest rate can deviate from the foreign interest rate, at least in the short-run. (An example would be the case in which there are capital flow controls or in situations in which the exclusively domestic nature of the asset itself precludes mobility\(^9\)). If capital is perfectly mobile,\(^{10}\) given that the domestic interest rate will be fixed at the world level for a small country in the
international asset market, the interest rate can be simply treated as externally determined, and will not be affected by any economic policies in the home country. 11

Third, labor is the only variable input in the production process, and the stock of capital is assumed fixed in both sectors in the short-run time framework to which our analysis applies. Each good can be used either for consumption or as a new addition to the capital stock. Investment in each industry proceeds to the point at which the marginal efficiency of investment is equal to the current market interest rate. However, the short-run constancy of the capital stock requires that current investment flows be small in relation to the stocks of capital employed.

Fourth, the bond in our model is assumed to be a variable-return and fixed-price variety for which the issuer always pays a coupon equal to the prevailing market rate of interest such that the market value of a bond equals its face value and is unaffected by the market rate of interest. I choose this special type of bond for two reasons. First, it simplifies the expression for the value of the bond such that the interest rate does not appear in the expression. Secondly, during inflation, this makes the nominal rate of return to the "bond" commensurate with the rising price level. Examples for this type of earning assets are some floating rate securities whose yield is indexed in some way with inflation.
Fifth, the government sector is not explicitly formalized in our model because of the assumption that a balanced budget is maintained over time with the proceeds of tariff being rebated to the private sector in a non-distorting way. Besides, there is assumed to be no domestic credit creation, no new debt issuance, and no sterilization operation during the process of BOP adjustments with which we are concerned. (We will relax this assumption in Chapter VI). Nevertheless, the government does play the role of maintaining a fixed exchange rate regime.

The features of the model which distinguish it from existing portfolio balance models are the presence of a price expectational factor in the asset demand function, and the less than instantaneous adjustment of the portfolio balance. The model incorporates a price expectation variable in the asset demand functions to act as a shift parameter in the desired ratio of real cash balances relative to real bond holdings. Consequently, this will enable us to capture the possibility of an asymmetrical response on the part of wealth holders between different assets in face of the general price level increase brought about by the imposition of a tariff.

In our analysis in Chapter IV, we will see that the utilization of this specification will make possible a diversity of dynamics. However, it is noteworthy that the resulting increase in the general price level following
the imposition of a given tariff will be an once-and-for-all increase if all goods are traded. This is so because of the instantaneous operation of international commodity arbitrage, as we have assumed. Therefore, the different price expectation schemes used in the analysis of Chapter IV will be theoretically more plausible if there is a nontraded good in addition to the traded goods. Because only then will the general price level change along the entire path of the adjustment due to the demand and supply pressure as well as the aggregate spending effect created by the imposition of tariff upon the nontraded goods sector. This arises from the distorted relative price among goods and the depressed stock of the real wealth. Since the general price level changes continuously, this lends credibility to the proposition that people can form their expectation regarding future inflation during the transitional period of adjustment. Consequently, in the absence of employment variation or interest rate changes (in other words, making the assumptions of full employment or perfect capital mobility), the different possibilities of price expectations will result in different possible changes in the desired ratio between assets, which in turn leads to all the interesting feasible adjusting patterns.

However, notice here even if all goods are traded, there could still be transitory price expectations during the adjustment period, an issue that is more empirical than
it is theoretical. Nevertheless, in our analysis, we will consider both cases with and without nontraded goods in order to investigate the difference in comparative statics.

Before we proceed to construct our model, note that the analysis in Chapter IV mainly contains qualitative inferences which are presented to provide some intuitive explanations or economic rationales for the resulting BOP adjustments. We do not use the techniques of a typical dynamic analysis, such as solving for the time path of the endogenous variables from differential equations or illustrating the movements of the system by using phase diagrams, until Chapter V.
The Model:

The model contains two asset demand functions:

(1) $M^d = PL(y, i_m) a = PL(y, i_r, \pi_e) a$ (demand for nominal cash balances)

(2) $B^d = PG(y, i_m) a = PG(y, i_r, \pi_e) a$ (demand for bond-equity)

where, $L+G = 1$ and thus $L/\partial x + G/\partial x = 0$, where $x$ is any independent variable. And, where

$P$ = the general price level, which is a weighted average index of individual commodity prices.

$a$ = the real value of total existing financial assets.

$y$ = real income for a certain time period.

$i_m$ = nominal rate of interest.

$i_r$ = real rate of interest, and,

$\pi_e$ = expected rate of inflation; where $i_m = i_r + \pi_e$.

The fraction of real wealth ($a = M^s/P + B^s/P$, where subscript "s" refers to "the amount supplied") which people want to hold in form of real cash balances and real bonds is a function of $y$ and $i_m$. The signs attached to each argument indicate the signs of partial derivatives of $M^d$ and $B^d$ with respect to each variable.

If an individual's real wealth increases, he will usually want to hold part of the increase in form of money as well as non-monetary assets because each of them will bring him different kinds of return, i.e. the high liquidity of cash balance versus the money return yielded by the
illiquid assets. As \( y \) increases, an individual tends to substitute cash balances for part of his illiquid assets, for now he may need a larger balance to provide conveniently for his expenditures in the periods between income payments (i.e. the so-called transaction demand for money) and partly because he can more readily afford to forego the earnings received from holding illiquid assets. Therefore, other things being equal, \( M^d \) is assumed to increase while \( B^d \) declines as \( y \) increases.

As the nominal rate of interest on non-monetary assets increases, we assume that the share of desired real cash balances will decline. And in fact, we can decompose the effect of an increase in the nominal rate of interest into two different effects. On one hand, when the real rate of interest rises, the non-monetary assets become more attractive to hold. On the other hand, if the rate of inflation is expected to increase over time, on account of the depreciated purchasing power of a given nominal cash balance, money will become inferior in portfolio selection relative to other earning assets because these assets are assumed to have a rate of return commensurate with inflation due to the higher market interest rates which prevail during inflation. Therefore, the higher the rate of expected inflation, the lower the share of desired real cash balances relative to bonds in the portfolio holdings.\(^{12}\)
(3) \( \bar{a} = F(y, i_r) = F(i_r)y \) (demand for financial assets as a whole), (Note: \( i_r = i_m - \pi^e \))

And,

(4) \( S = \frac{dA}{dt} \) (Saving in nominal terms), where \( A \) is the nominal stock of total wealth.

From eq. (3), the desired wealth in real terms, \( \bar{a} \), is a function of \( y \), and \( i_r \). More specifically, the fraction of real income which people want to hold in the form of financial assets rather than physical commodities (and thus, the planned rate of saving, according to eq. (4), which reflects the desired rate of asset accumulation), will increase as the real interest rate rises. Note that: it is the income constraint which is relevant when people make decision regarding the rate of wealth accumulation, rather than the wealth constraint by which a constant real wealth is implied. And we will discuss later in this Chapter the specific form which the accumulation of wealth could have.

Therefore, upon investigation of equations (1), (2), and (3), we can see that the following implicit assumptions lie behind these demand functions:

(5) \( \frac{\partial L}{\partial y} > \left| \frac{\partial G}{\partial y} \right| > 0 \),

(6) \( \frac{\partial G}{\partial i_r} > \left| \frac{\partial L}{\partial i_r} \right| > 0 \), and

(7) \( \frac{\partial L}{\partial \pi^e} = \frac{\partial G}{\partial \pi^e} > 0 \)
The notion that the desired level of real wealth is positively related to real income implies that the increased transaction demand for money is more than offsetting the negative effect on the demand for bonds of a given increase in real income. And for a similar reason, an increase in the real interest rate culminates in a higher $\bar{a}$. However, when $\pi^e$ changes, this causes the desired ratio between different assets in the portfolio to shift, (refer to eqs. (1) and (2)), yet we assume that the net effect on the desired level of total wealth is nil.15

One final point about the variable $\pi^e$ needs strong emphasis. We assume that it is in turn a function of the current inflation, i.e.

(8) $\pi^e = f(P)$, ("." denotes the change of price over time, i.e., $\dot{P} = dP/dt$), with $f' = 3\pi^e/\partial P > 0$,

and we assume that the formation of expectations could be either positively related, independent, or negatively related to current inflation (i.e., $f'$ could be either positive, zero, or negative).

Should people believe that the current inflation will last for long and will increase over time, we assume that the expected rate of inflation will be revised upward. At the other extreme, if the belief is that there is a normal level of inflation and thus, the future inflation will always revert to its normal level, then the expected rate of inflation will adjust downward if the current inflation is higher
than normal. Finally, in the case in which the current inflation is regarded as an once-and-for-all movement, then \( f' = 0 \), i.e. there is no linkage between the current inflation and future expected rate of inflation.

Also, based on the two individual asset demand functions (equations (1) and (2)) we can derive the desired ratio of real cash balances with respect to bonds, a key relationship which we will utilize later in our analysis:

\[
(9) \quad \ell = \frac{M^d}{B^d} = \frac{L(y, i_r, \pi^e) + - \frac{\pi^e}{G(y, i_r, \pi^e)} + - + - + + \pi^e}{y, i_r, \pi^e, \pi^e} = (y, i_r, \pi^e)
\]

The signs of the partial derivatives of \( \ell \) with respect to \( y \) and \( i_r \) are obvious, while the sign attached to the current inflation is contingent upon the way that expectations are formed.

If the expectation of future rate of inflation is related to current inflation in a positive fashion, the sign corresponding to \( \pi^e \) will be negative, indicating a lower desired ratio for real cash balances during an explosive inflationary period. Or, if the expectation is regressive, then the current inflation will simply lead to a relatively higher transaction demand for cash balances because it is expected that money will not permanently be degraded as an inferior type of asset to hold.\(^{16}\) For a given set of \( y, i_r, \) and \( \pi^e \), there is a specific desired portfolio composition between money and bonds.
Next, we turn to the commodity sector of the economy:

(10) \( D_X = D_X(\bar{a}, a, q, q') \) (demand function for the exportable)

where, \( q = \frac{P_M}{P_X} = e(1+\tau)\frac{P^\#_M}{P^\#_X} \) (the relative domestic price of the importable to the exportable)

\( q' = \frac{P_{NT}}{P_X} \) (the relative price of the nontraded good to the exportable)

\( P_M = \) the domestic price of the importable
\( P_X = \) the domestic price of the exportable
and "\( \# \)" denotes foreign currency price.
\( e = \) the exchange rate.
\( \tau = \) the ad valorem rate of tariff.

(11) \( D_M = D_M(\bar{a}, a, q, q'') \) (demand function for the importable)

where, \( q'' = \frac{P_M}{P_{NT}} \) (the relative domestic price of the importable to the nontraded good)

(12) \( D_{NT} = D_{NT}(\bar{a}, a, q', q'') \) (demand function for the nontraded good).

(13) \( z = D_X + D_M + D_{NT} = z(\bar{a}) \) (total real expenditure on goods).

And so, the total expenditure in nominal terms can be written as:

(14) \( Z = P_XD_X + P_MD_M + P_{NT}D_{NT} = Z(A^d - A^s) \)
Note that the relative prices of commodities, \( q, q', \)
and \( q'' \), do not occur in the expenditure function, by the
assumption that \( |\partial D_M/\partial q| = \partial D_X/\partial q, \]
\( |\partial D_M/\partial q'| = \partial D_X/\partial q', \)
and \( |\partial D_M/\partial q''| = \partial D_N/\partial q''. \)
Note that if we choose the unit of goods such that \( P_X = P_M = P_N \),
this assumption is simply the condition for consumer equilibrium at
which the marginal rate of substitution between two different goods has
to be equal to the relative price to maximize the total utility
of a consumer subject to his fixed budget constraint. For
example, here we have the marginal rate of substitution
between \( X \) and \( M \) as \( \partial D_X/\partial D_M = \partial D_X/\partial q \)/(\partial D_M/\partial q) = P_M/P_X = 1. \)

The divergence between desired and actual real wealth,
\( \bar{a} - a \), is incorporated in these demand functions for commodities
to capture the so-called "real balance effect" (in fact, the "real wealth" effect here, since money is not the only asset),
and it reflects a trade-off relationship between consumption
and savings, given the income constraint. Therefore, any
disturbance which causes a wider gap between \( \bar{a} \) and \( a \) will
reduce \( z \), as people attempt to rebuild their desired stock
of real wealth, holding other things constant. For example,
the effect of an increase in the real interest rate is to
depress the expenditure on commodity flows through increasing
the desired level of assets holding.

There are three supply functions:

\[
(15) \quad S_X = S_X(q, q'), \quad S_M = S_M(q, q''), \quad \text{and} \quad S_N = S_N(q', q'').
\]
When there is full employment, supply functions for commodities are functions of relative prices alone, since the production point lies along the production possibility frontier of the economy under full employment.

However, with unemployment:

\[(16) \, S_i = S_i(N_i) = S_i(W/P, P_i), \, i = X, M, \text{ and NT.} \]

where \( S_i \) is the output of each sector respectively, and is related directly to the employment level \( (N_i) \) via the production function. \( N_i \) is in turn negatively related to the real wage rate \( (W/P) \) and positively related to the commodity price in each sector, i.e.

\[(17) \, N_i = N_i(W/P, P_i) \]

In this case, because of unemployed resources, the resources constraint is no longer binding. Consequently, the trade-off among production of the three goods no longer exists, and the relative prices do not play a role in the supply functions. However, a higher nominal commodity price, on the one hand, by lowering the real wage rate, will lead to a higher employment level and thus higher output in each sector. On the other hand, by signaling a larger market demand for the specific goods, it will stimulate production in that sector.\(^{18}\)

\[(18) \, M^s = R + D \, (\text{money supply}) \]

where \( R \) is the country's foreign exchange reserve, and \( D \) is the domestic credit component of the monetary base, (e.g. the
stock of domestic securities held by the central bank or the fiat issue of money by the central bank). By our assumption, there is no domestic credit creation for the time period concerned, so the only way the money stock can change is by changes in foreign reserves, which in turn comes from an imbalance of the balance of payments.

In the following, we set up the different accounts of the balance of payments:

\[
B_T = P_X(S_X - D_X) - P_M(D_M - S_M)
\]

(19) $B_T$ is defined as the net domestic excess supply of commodities. Notice that if there is no tariff, which causes a wedge between the domestic price and the foreign price of traded goods, the domestic prices of importables and exportables are simply the domestic currency equivalent of the respective foreign prices under our perfect commodity arbitrage assumption. Therefore, $B_T$ is also the trade balance expressed in home currency. (i.e. $B_T = B_T^e$). However, if there is a tariff, $P_M = P_M^e(l+\tau)$, and the domestic currency trade balance, or, the domestic currency equivalent of the foreign currency trade balance, is no longer the $B_T$ as defined in eq. (19). Instead, the trade balance is now:

\[
B_T^e = P_X^e(S_X - D_X) - P_M^e(D_M - S_M)
\]

(20) $B_T^e = B_T + T$

where $T$ is the tariff collected by the government on imports.
In an open economy, $B_T$ represents the "hoarding" of the economy (i.e. the accumulation of wealth). We can demonstrate this as follows:

\[(21) \quad Z + B_T - Y^d = 0\]

where $Z$ is total domestic absorption of both home and foreign-produced goods, and $Y^d$, disposable income, is the sum of factor income ($Y^f$) and the tariff proceeds, $T$.\(^{19}\) (Recall that we assume the government redistributes tariff proceeds to the private sector as a lump-sum rebate). Notice that eq.(21) can be reduced to $Z + B_T - Y^f = 0$, because $B_T = B_T + T$, and $Y^d = Y^f + T$. And, because $Z + B_T$ represents the total demand (home and abroad) for domestically produced commodities, the above expression ($Z + B_T - Y^f = 0$) states that the excess demand for domestically produced commodities is zero. In fact, if we substitute eq.(14) for $Z$, eq.(20) for $B_T$, and $Y^f + T = P_X S_X + P_M S_M + P_N T S_N + T$ for $Y^d$ into eq.(21), we find that eq.(21) is in fact the condition for market clearance of the nontraded good sector which by definition has to be maintained in equilibrium. (Short-run equilibrium of the economy, Dornbusch (1973(a)).

Furthermore, because each economic agent can spend his income either on commodities or on financial assets, for the economy as a whole the aggregate budget constraint is as follows:

\[(22) \quad (Z - Y^d) + dA/dt = 0\]

where $dA/dt$ represents the time derivative of the financial
asset stock, i.e. the rate of wealth accumulation over time.

Therefore, from eq. (21) and (22), we get:

(23) \frac{dA}{dt} = \frac{dA}{dt}

We now derive an asset accumulation function (or the hoarding function in Dornbusch (1973(a)), that is, the function describes the change in an asset, through the so-called "stock adjustment approach". This is a standard approach employed in the conventional monetary macroeconomics, e.g. Niehans (1978), ..., or Dornbusch. Essentially, this approach postulates that the accumulation of a specific asset will in general depend on the stock excess demand(s) for all asset(s). Specifically, it is an increasing function of the excess demand for the asset itself. And, if there are any alternative assets in the portfolio which are potential substitutes for this asset, the accumulation of the asset will be negatively related to the excess demands for the other assets. According to Niehans: "If an individual's house burned down, this will not only cause him to replace it and thus to accumulate capital goods, but also to draw down his cash balance and liquidate some of his securities". In a multi-asset model, this cross effect between the demand for different assets reflects the particular "composition" consideration of the portfolio adjustment, given a certain wealth constraint in a certain time period, although this does not necessarily imply the extreme case in which the desired ratio of portfolio mix is always held.
Consider first the simplest case in which there is only one asset, money, in the economy. According to the stock adjustment approach, the accumulation of nominal cash balance is proportional to the excess demand for money:

\[(24) \frac{dM^S}{dt} = \lambda(M^d - M^s)\]

This equation expresses the asset effect (real balance effect) in dynamic terms, where \(\lambda\) is the speed of adjustment for each unit of time in the process of adjustment. Generally, \(\lambda < \infty\) due to the fact that a more rapid adjustment of asset may be more expensive than a slow adjustment, so individuals find it efficient to spread the adjustment over time. Notice that we are now dealing with a continuous time model in which all variables \((M^d\text{ and } M^s\) here) presumably are changing continuously. So, at any instant, the gap \(M^d-M^s\) also varies and the adjustment of the money stock is instantaneous when \(\lambda\) equals infinity. However, if this were a discrete time model, then a fractional adjustment would be reflected as \(\lambda < 1\). And, if \(\lambda=1\) in a discrete time analysis, it will imply a complete adjustment of the actual to the desired level of nominal cash balance within one time period.

Therefore, in this "money-only" open economy, from eq. (23),

\[(25) B^T = \frac{dA}{dt} = \frac{dM^S}{dt} = \lambda(M^d - M^s)\]

Now, we turn to our two-asset model (money and bonds). Again, the trade balance is equal to the asset accumulation:

\[(26) B^T = \frac{dA}{dt} = \frac{dM^S}{dt} + \frac{dB^S}{dt}\]
According to the generalized stock adjustment approach postulated above, the accumulation function for each asset can be written in the following general form:

(27) \( \frac{dM^g}{dt} = m_1(M^d - M^g) - m_2(B^d - B^g) \), where \( m_1 > 0, m_2 > 0 \).

And,

(28) \( \frac{dB^g}{dt} = b_1(B^d - B^g) - b_2(M^d - M^g) \), where \( b_1 > 0, b_2 > 0 \).

Notice that the subscript "1" in the coefficients denotes the direct adjustment effect, while "2" refers to the cross effect.

Intuitively, the cross-effect between the demands for alternative assets in a portfolio is in some way similar to the substitution effect between different commodities in a consumer's consumption basket. Therefore, in this model which describes the most general case, when an individual has an excess demand for a specific asset, running down the holding of the other assets in his portfolio is certainly one of the ways to replenish the particular asset.

Also notice here again if it is a discrete model, such constraints on the coefficients of adjustment as \( m_1, b_1 \leq 1 \) would be necessary for our stock adjustment analysis. However, these restrictions would have no special meaning for our continuous time model.

Therefore, upon substituting eqs. (27) and (28) into (26), we obtain:

(29) \( B_T' = (m_1 - b_2)(M^d - M^g) + (b_1 - m_2)(B^d - B^g) \)
Note here the coefficients \((m_1-b_2)\) and \((b_1-m_2)\) are in fact respectively the sum of the direct and cross effect which each single asset has on the accumulation of the total wealth.

Furthermore, it is reasonable to assume that excess demand for any asset does not result in a reduction in total wealth. That is, the direct effects dominates the cross effects in eq.(29):

\[(30) \, m_1 > b_2 \quad \text{and} \quad b_1 > m_2\]

(We will see later in the dynamic analysis of Chapter V that this is the only constraint on the adjustment speeds needed for the stability requirement of our system).

The above derivation is for the trade balance account. As for the capital account, on the other hand,

\[(31) \, B_K = -dB^S/dt = -b_1(B^d-B^s) + b_2(M^d-M^s) \] from eq.(28).

That is, the capital account surplus (or the net capital inflow) comes from the decumulation of bonds when individuals sell bonds abroad in our open economy. 20

Finally, the overall balance of payments is, from eqs. (29) and (31):

\[(32) \, BOP = B_T + B_K = dR/dt = dM^S/dt = m_1(M^d-M^s) - m_2(B^d-B^s)\]

The overall balance of payments is the sum of the trade balance and capital account balance, 21 and the resulting foreign reserve accumulation constitutes the sole source of changes in the money stock. 22 When there is an excess stock demand for either money or bonds, both will entail
a change in the holding of money stock in this generalized stock adjustment model.

The above general model is rather complicated, particularly to derive dynamic properties of the adjustment path from such a model. Therefore, in our next Chapter and the first part of Chapter V, we make some special assumptions about the adjustment coefficients to simplify our mathematical task while preserving the basic features of the stock adjustment approach and thus the fundamental insights. Then, in the second part of Chapter V, we investigate also the dynamic features of the more general models.

The special case which we are going to explore is set up by giving the specific form for eq. (4), the saving function, as follows:

\[(33) \ S = \frac{dA}{dt} = k(A^d - A^s)\]

I.e. the accumulation of wealth is in proportion to the excess stock demand for total asset.

And, if we further assume that to the above accumulation of total wealth, each single asset makes equal proportionate contribution:

\[(34) \ B_T^j = \frac{dA}{dt} = k(M^d - M^s) + k(B^d - B^s)\]

Comparing eq. (34) to the general form of \(B_T^j\) in eq.(29), we find that the special restrictions imposed on the coefficients of adjustment here are:

\[(35) \ (m_1 - b_2) = (b_1 - m_2) = k\]
That is, we are in fact assuming that the excess stock demand for each asset has the same effect on the degree of accumulation of total wealth. I.e., whenever there is an excess demand for any asset, the induced saving will take place at the same speed.

Also, we further simplify the $B_k$ equation as follows:

\[ B_k = \frac{dB^s}{dt} = k(M^d - M^s) - k(B^d - B^s) \]

That is, we assume the direct effect ($b_1$) and the cross effect ($b_2$) on the accumulation of bonds are the same. Notice that this is analogous to saying that the own price effect is equal to the cross price effect on the demand for a certain good. Also, notice that when $b_1 = b_2$, and if the excess demand for bonds is equal to the excess demand for money (i.e. $B^d - B^s = M^d - M^s$), there will be no adjustment of the bonds in this particular specification. This assumption is crucial to our later analysis, and we will discuss its implication in detail after we derive the overall BOP equation. From eq. (36), the accumulation function of the stock of bonds is:

\[ \frac{dB^s}{dt} = k(B^d - B^s) + k(M^s - M^d) \]

Notice that by setting $b_1 = k$, we have also implicitly assumed that upon the occurrence of an excess demand for either asset, the stock of bonds will vary monotonically with the change of total wealth such that the speed of bond stock accumulation is the same as the speed of total wealth restoration, $k$. (I.e. we have assumed that $b_1 = b_1 - m_2$,
and \( b_2 = m_1 - b_2 \). For instance, the assumption that \( b_2 = m_1 - b_2 \) means that when there is an excess demand for money, expenditure on goods will be reduced at a speed of \( m_1 - b_2 \) (\( = k \)), and bonds will be sold abroad at the same speed, \( b_2 \).

Therefore, the overall balance of payments becomes:

\[
(38) \quad \text{BOP} = B_T' + B_K = dM^g/dt = 2k(M^d - M^g)
\]

In other words, in this special case, we have \( m_1 = 2k \) while \( m_2 = 0 \). I.e., a certain asymmetry in the asset accumulations is implied. More specifically, while people will sell bonds if there is an excess demand for money, an excess demand for bonds has no such cross effect on the accumulation of money. This is so because we have assumed that when there is an excess demand for bonds, wealth holders will either cut their spending on goods, which leads to a \( B_T \) surplus and increases the money stock at a speed \( k \) (refer to eq. (34) in which \( (b_1 - m_2) = k \)); or they will buy bonds from abroad, which results in a \( B_K \) deficit and reduces the money stock at the same speed (refer to eq. (36) in which \( b_1 = k \)). I.e., we have assumed equal \( k \) associated with \( (B^d - B^g) \) in eqs. (34) and (36). Therefore, the excess demand for bonds has no net effect on the overall BOP (and thus the money stock). On the other hand, if there is an excess demand for money, people will either cut expenditure on goods, which leads to a \( B_T \) surplus at a speed \( k \) (see eq. (34)), or they will sell bonds abroad and create a \( B_K \) surplus at the same speed (see eq. (36)). Consequently, the total effect of excess demand
for money on the overall BOP will be an improvement at a speed of 2k.

Furthermore, if we investigate eqs. (34), (36), and (38), we find that in this special model, money adjusts faster than bonds and thus a "creeping" portfolio adjustment is implied. This can be seen from the fact that if there is an equal amount of excess demand for the two assets, because in eq. (36) we have $b_1 = b_2 = k$, the stock of bonds will not accumulate, while the money stock and the total wealth will accumulate immediately as indicated by eqs. (38) and (34). Notice that this special case reflects mainly our assumption that there exists a relative high cost for the international transaction of bonds such that changing the holding of bonds is not as convenient as adjusting money. This is a fairly realistic assumption for a small, developing country characterized by a huge trade sector and in which the financial institution for securities transactions has not been sufficiently developed (e.g. Taiwan or Korea).

However, despite of all these special assumptions, it is noteworthy that the basic assumption about the dominance of the direct effect over the cross effect in asset accumulation is still maintained in our special model. (I.e., $m_1 = 2k > b_2 = k$; and $b_1 = k > m_2 = 0$).

In Chapter V, we will investigate the sensitivity of the dynamic stability of our models to this particular assumption under different hypotheses about the coefficients of adjustment.
Now we will derive the comparative statics steady state price level of our model.

First of all, we consider the case in which there is no nontraded good, that is, if there are only importables and exportables in the model. Because the price of the exportable is fixed at the international level under the assumption of commodity arbitrage, the imposition of a given tariff on the importable leads only to change in the price of the importable by the change in the tariff. So, the change of the general price index, \( P \), which is a weighted average of the prices of the exportable and the importable (i.e., \( P = \alpha_1 P_X + (1-\alpha_1) P_M \), where \( \alpha_1 \) is the expenditure share of the exportable) will equal:

\[
(39) \quad dP = \alpha_1 dP_X + (1-\alpha_1) dP_M = (1-\alpha_1) d\tau,
\]

because \( dP_X = 0 \) and \( dP_M = d\tau \) \((P_M = P_{M_0}(1+\tau))\), if we assume \( P_{M_0} = 1 \) by choosing the unit of goods, and assume a constant \( \alpha_1 \).

On the other hand, if there is a nontraded good besides the exportable and the importable, the imposition of the tariff on the importable will lead to changes of the price of the nontraded good as well as the price of the importable. This is because when the relative price of the importable rises, there is a transmission of demand pressure from the importable sector to the nontraded good sector due to the effects on consumption and production substitution. Furthermore, there occurs a depressed effect on the level of aggregate spending as a result of the higher general price level.
Consequently, the price of the nontraded good will change in response to such pressures in order to restore the equilibrium of the nontraded good sector.

Notice that the incorporation of a nontraded good into our model not only adds to the plausibility of the occurrence of price expectation during the transitional period of adjustment as we have already mentioned, but also will change the comparative statics price level relative to the all-traded good case as we will demonstrate below:

From equations (10), (11), (12), and (15), we can derive the real excess demand \( = D_{i} - S_{i}, \ i = X,M,NT \) for each good as a function of relative prices and any discrepancy between actual and desired wealth. These relationships can be expressed in nominal terms as in the following specific forms:

\[
\begin{align*}
(40) \ P_{X}^{*}XD_{X} &= P_{X}(D_{X} - S_{X}) = P_{X}(aq + bq') - i(A^{d} - A^{S}), \\
(41) \ P_{M}^{*}XD_{M} &= P_{M}(D_{M} - S_{M}) = P_{M}(-cq - rq'') - J(A^{d} - A^{S}), \text{ and} \\
(42) \ P_{NT}^{*}XD_{NT} &= P_{NT}(D_{NT} - S_{NT}) = P_{NT}(-fq' + gq'') - o(A^{d} - A^{S})
\end{align*}
\]

Where \( XD \) stands for the real excess demand and all the coefficients are positive.

Notice that in these specifications of the excess demand functions for goods, we capture the aggregate spending as well as the relative price (substitution) effect in the spirit of Blejer (1979) and his predecessors. And, from these excess demand functions, we will derive the change of the price of the nontraded good.
When the nontraded good sector is in equilibrium, according to eq. (23), the trade balance, \( B^i_T \), equals the hoarding of an economy. Moreover, because in the long-run equilibrium, hoarding (or the trade balance) must equal zero such that the stock of wealth remains constant, we get from eq. (20) that, assume for simplicity that \( e=1 \):

\[
(43) \quad B^i_T = P_X^*(S_X - D_X) - P_M^*(D_M - S_M) = -P_X^*X^* - P_M^*X^* = 0
\]

That is, when income equals expenditure and so hoarding equals zero, the excess supply of the exportable equals the excess demand for the importable.

By substituting eqs. (40) and (41) into eq. (43), and keeping in mind that \( A^d \), the desired level of wealth, equals the actual wealth, \( A^s \), in the steady state equilibrium, we get the following long-run equilibrium condition:

\[
(44) \quad P_X^*(a + bq') + P_M^*(-cq - rq'') = 0
\]

Recall that \( q = P_M/P_X, q' = P_{NT}/P_X, \) and \( q'' = P_M/P_{NT} \). Because both \( P_i^* (i=X, M) \) are fixed at their initial world levels, thus, \( P_X = P_{X0} \), and \( P_M = P_{M0} (1+\tau) \) after the imposition of the tariff, \( \tau \). Therefore, in eq. (44), \( P_{NT} \) is the only endogenous variable and we can derive the equilibrium change of \( P_{NT} \) following the imposition of the tariff by differentiating eq. (44).

Upon substitution of \( P_X^* (=P_{X0}), P_M^* (=P_{M0}), q, q', \) and \( q'' \) into eq. (44), and manipulation (multiplication by \( P_{NT} \)), we obtain:

\[
(45) \quad bP_{NT}^2 + (1+\tau)aP_{M0}P_{NT} - (1+\tau)P_{M0}^2(cP_{NT}/P_{X0} + r) = 0
\]
Differentiating eq. (45) with respect to $\tau$, we obtain:

$$\frac{dP_{NT}}{d\tau} = \frac{(cP_{NT}P_{Mo}^2 + rP_{Xo}P_{Mo}^2) - aP_{Xo}P_{Mo}P_{NT}}{2bP_{Xo}P_{NT} + aP_{Xo}P_{Mo}(1+\tau) - cP_{Mo}^2(1+\tau)}$$

If we choose the appropriate units of goods such that $P_X = P_M = P_{NT} = 1$, then eq. (46) becomes:

$$\frac{dP_{NT}}{d\tau} = \frac{(c + r) - a}{2b + (1+\tau)(a-c)}$$

Recall from eqs. (40) and (41), the coefficients in eq. (47) represents respectively $a = \frac{\alpha(D_X-S_X)}{\partial q} = \partial XD_X/\partial q$, $b = \partial XD_X/\partial q'$, $c = \frac{\alpha(S_M-D_M)}{\partial q} = \partial(-XD_M)/\partial q$, and $r = \partial(-XD_M)/\partial q''$. By investigating eq. (46), we find that the increase of the price of the nontraded good following the imposition of a given tariff, $\tau$, will be greater, the larger the parameters $c$ and $r$, or the smaller the $a$ and $b$. Intuitively, we can interpret these results in terms of the underlying economics as follows.

First, when parameter $c$ is relatively large, the resulting decline of excess demand for the importable following the tariff imposition is relatively great. Therefore, the demand pressure imposed on the nontraded good sector is now larger and the price of the nontraded good must increase by a greater amount.

Second, when the parameter $r$ is relatively large, upon the imposition of the tariff, the resulting initial increase of $q''$ ($=P_{M}/P_{NT}$) leads to a greater loss of demand for the
importable because the own price effect of the excess demand for the importable is larger. Subsequently, a greater rise of $P_{NT}$ is needed to restore equilibrium in the nontraded good sector.

Third, when parameter $a$ is relatively large, the resulting excess demand for the exportable following the tariff imposition is relatively great. This implies that most of the demand pressure released from the importable sector shifts to the exportable sector with less "spillover" effect on the nontraded good sector. As a result, the rise in price of the nontraded good will be relatively small.

Finally, because $P_X$ is fixed by the assumption of commodity arbitrage, the relative price of the nontraded good relative to the exportable will rise gradually after the imposition of the tariff. If the parameter $b$ is relatively large, the subsequent switching of the excess demand from the nontraded good sector to the exportable sector will be relatively large. Consequently, the rise of the price of the nontraded good will be relatively small because the cross price effect of the demand for the exportables with respect to the price of the nontraded good is relatively strong.

In short, the extent to which the price of the nontraded good (and thus the general price level) will rise in equilibrium following the imposition of a given tariff on the importable depends on these underlying parameters.
In particular, if the ensuing reduction in excess demand for the importable is relatively great, or the induced excess demand for the exportable is relatively small, the price of the nontraded good will increase even more. This is because a greater loss in demand for the importable implies that a stronger demand pressure has been created to affect the remaining sectors of the economy. (Notice that the aggregate loss of demand for the importable is reflected as \((c+r)\) in the numerator of eq. \((47)\)). Furthermore, given this demand pressure, if little of it is shifted to the exportable sector (reflected as a relatively small \(a\) in eq. \((47)\)), the other alternative good, the nontraded good, must bear most of the burden (reflected as a relatively large numerator in eq. \((47)\)). Therefore, a larger increase in the price of the nontraded good is necessary in order to restore equilibrium.

Furthermore, notice that \(P_{NT}\) will not increase in equal proportion to \(P_M\) unless the numerator in eq. \((47)\) equals the denominator, which generally will not hold.\(^{24}\)

This is in contrast to the case of a devaluation in which the relative price between importable and exportable remains fixed upon the devaluation. In the short-run, a devaluation will push up the relative price of traded goods with respect to nontraded goods in order to generate the needed \(B_T\) surplus to replenish the lost real cash balances following the higher general price level caused by the devaluation. (See Dornbusch (1973b), Krueger (1974), Wolf (1978),
and Blejer (1979)). However, in the long-run equilibrium, $P_{NT}$ will rise equiproportionately with the price of traded goods such that the relative price between them returns to its initial level and thus the trade account balance becomes zero. ($B_T = 0$ assures steady state because only then the financial wealth stops accumulating as we discussed before).

The basic reason for this difference lies in the fact that in our analysis, because $P_M$ rises permanently with respect to $P_X$ (which is assumed to be fixed by commodity arbitrage) following the imposition of the tariff, the nontraded goods, the other alternative goods in the economy whose price is free to change, have to adjust their prices to absorb the residual demand pressure after what has been absorbed by the quantity adjustment of the exportables. Therefore, there is no longer an assurance for proportionate change of the price of nontraded goods with respect to traded goods as in the case of a devaluation.

In the following, we want to compare the magnitude of change in the general price level when there is a nontraded good to the situation when there are only importables and exportables. This issue is important quantitatively because if the change of the price level is different, the resulting magnitude of BOP improvement might also be different as we will analyze later in Chapter V. However, unfortunately, by investigating eq. (47), we get the conclusion that it is ambiguous whether the incorporation of the nontraded good
will lead to a larger price change. This is formally demonstrated as follows:

If there is a nontraded good, with \( \alpha_2 \) and \( \beta_2 \) as the expenditure shares of the exportable and importable, and if we assume for simplicity that \( dP_{NT} = d\tau = dP_M \), the change of the general price level will be:

\[
(48) \quad dP = \beta_2 d\tau + (1-\alpha_2-\beta_2) d\tau = (1-\alpha_2) d\tau
\]

Upon comparing eq. (48) with eq. (39), and if we further assume that \( \alpha_1 = \alpha_2 \), i.e. if the spending share of the exportable remains the same as in the all-traded good case, the general price is the same under both cases with and without the nontraded good.

However, in reality, when there exists a nontraded good, according to our derivation, the change of the general price level is:

\[
(49) \quad dP = \beta_2 d\tau + (1-\alpha_2-\beta_2) \frac{(c+r)-a}{2b+(1+r)(a-c)} d\tau
\]

by substituting eq. (47) for \( dP_{NT} \) and recalling that \( dP_x = 0 \), and \( dP_M = d\tau \).

Therefore, even if we assume that \( \alpha_1 = \alpha_2 \), the change of the general price level with nontraded good will only equal to that without the nontraded good if the following equality holds:

\[
(50) \quad \frac{(c+r)-a}{2b+(1+r)(a-c)} = 1
\]
And, as we can infer from eq. (50), whether this will be the case depends on the relative magnitude of c, r, a, and b. The larger are c and r with respect to a and b, the greater the magnitude of the left-hand expression in eq. (50), and, subsequently, it is more likely that the change in the general price level will be larger with the inclusion of the nontraded good.

On the other hand, given that eq. (50) holds, then we have in eq. (49) that dP=(1-\alpha_2)dt. And, if the exportable share rises (\alpha_2>\alpha_1), the change in the general price level will be smaller. This is because the price of the exportable is fixed in our model due to the assumption of commodity arbitrage. Thus, if its share increases in consumer's spending, the resulting increase in the general price level will be smaller upon the imposition of the tariff.

Therefore, the incorporation of the nontraded good is very likely to lead to a different degree of price change and thus a difference in the resulting size of BOP improvement in the comparative statics. However, it is noteworthy that the dynamic properties of the adjustment path of our model which we are mainly concerned with should remain unchanged. Because in our "portfolio-balance" model in which all adjustments originate from disequilibrium of the asset markets, as long as the general price level does change as a function of the tariff, the resulting asset market imbalance and thus the adjustment path should have the same qualitative characteristics.
CHAPTER IV

QUALITATIVE ANALYSIS

Section 1 - Full Employment Case:

With the Portfolio approach model developed in Chapter III, we can proceed to analyze the balance-of-payments adjustment following any exogenous shocks or policy change. Suppose a tariff of rate $\tau$ is imposed upon the importable in a small country for reasons of import substitution. This causes the domestic price of the importable to increase by the rate of the tariff, and also increases the price of the nontraded good, as we discussed in Chapter III. But before the price of the nontraded goods increases enough to restore the equilibrium of the nontraded goods sector, the relative price of the importable will have increased along with the general price level. In the following, we investigate this situation under the assumption of full employment.

With resources in the economy fully employed, the resources constraint is binding, so a substitution effect take place on the supply side as well as on the demand side as the result of a higher relative price for the importable. The supply of the exportable and the nontraded -58-
good decreases while the demand for them increases. However, due to the fact that the nominal price of the exportable is fixed at the world level by commodity arbitrage, the excess demand for this good will be eliminated through real trade flows (i.e. by reducing the amount available for export), while the excess demand for the nontraded good will cause an increase in the price of that good. On the other hand, there is a "real balance effect" or "real wealth effect" which arises from the fact that the rising general price level deprives people of part of their real wealth. Therefore, consumers will cut their expenditure on goods in order to rebuild their real financial asset stock such that it maintains a constant relationship with real income (refer to eq. (3)). To the extent that spending on imports is reduced, there will be an improvement in the trade balance and this will cause an increase in the money supply which is needed to restore the asset position.

In our specific example, when the general price level rises, total wealth as well as each single asset in the portfolio decline in their real value by an equal proportion. According to eq. (33), the decline of real wealth or equivalently the increase in the demand for the nominal wealth results in an accumulation of wealth at a speed $k$ for each unit of time. More specifically, we have assumed that excess demands for each single asset will culminate in an equal speed of restoration of total wealth through $B_m$ surplus
which causes accumulation of money stock (i.e., we have assumed that \( m_1 - b_2 = b_1 - m_2 = k \) in eq. (34)).

The adjustment mechanism described above is basically that of the traditional real balance effect of the monetary approach. However, with the presence of the second asset, there is another type of portfolio adjustment besides the conventional one which arises solely from the scale requirement for assets. We could name this second channel of adjustment a portfolio "composition" adjustment, which comes from the fact that wealth holders tend to hold their assets in an optimal ratio, given the values of those determinants for asset holdings, e.g. real income, real interest rate and expected rate of inflation. We now discuss this new channel of adjustment in more detail as follows.

According to eq. (9), changes in the current general price level (\( P \)) may alter the desired relative ratio of real cash balances in individual's portfolios if the expected rate of inflation is affected by these changes. When the general price level rises following the imposition of a tariff, cash balances and the bond stock decline in real value by an equal proportion. Wealth holders, in this event, can be expected to have three possible kinds of responses in their formation of price expectations and thus preference pattern for holding assets between money and bonds:
(A) A symmetrical response for both assets such that an equal magnitude excess demand for money and for bonds is created, with neither of the excess demands dominating the other. In other words, in terms of eqs. (9) and (8), we have $\lambda \eta / \eta P = 0$ due to $f' (\eta \pi^e / \eta P) = 0$. That is, the expected inflation rate is independent of the current inflation. Therefore, from eq. (36), this implies no adjustment of bonds will ever occur in the early stage of the adjustment because no argument in the $M^d$ and $B^d$ has changed except the rising general price level which leads to an equal increase of $M^d$ and $B^d$. (We can see that this particular result comes from our very special assumption that $b_1 = b_2 = k$, so that when the excess demand for the two assets are equal, there will be no net effect on the accumulation of bonds).

(B) An asymmetrical response in favor of money such that the desired ratio of real cash balances with respect to bonds increases as a result of rising price level due to the negative relationship between price expectation and current inflation. (I.e., in terms of eq. (9), $\lambda \eta / \eta P > 0$ due to a negative $f'$). In other words, because $\pi^e$ declines (assume $\pi^e$ equals zero initially), the demand for the nominal cash balance increases by a larger amount than the demand for bonds given the increase of the general price level (refer to eqs. (1) and (2)). So, in terms of our eq. (36), there will be an immediate decumulation of bonds.
(C) The last possible response is asymmetrical in favor of bonds due to the explosive type of price expectation formation (i.e. \( f' > 0 \)) such that the cash balance is regarded as inferior to other earning assets in portfolio selection. (I.e. \( \delta \lambda / \delta \bar{P} < 0 \)). In terms of eq. (36), because the positive \( \pi^e \) upon the rising general price level leads to a larger increase in \( B^d \) relative to \( M^d \), so there will be an immediate accumulation of bonds and an accompanying capital account deficit.

Now we look at each case separately:

Under case (A), because wealth holders view the equal proportionate loss in real value of money and bonds equally, there will be no tendency for a typical individual (and thus, wealth holders as a whole) to adjust his portfolio composition, and the initial equilibrium composition between money and bonds remains optimal. Therefore, the only automatic force generated is the portfolio stock adjustment, which occurs when people attempt to restore the overall value of wealth stock while maintaining the initial portfolio composition, by cutting expenditure on commodity flows.

However, due to the underlying assumption implied in our model specification that the adjustment of money is faster than that of bonds, or equivalently, that the stock of bonds will not change when there are equal excess demands for the two assets from eq. (36), (refer to the discussion on p. 48 and 61), when the real wealth effect discussed above restores the real cash balance to its initial level by creating a \( B_T \)
surplus and thus a foreign reserve inflow, the initially non-existing "portfolio composition problem" comes into the picture. Notice here, for illustrative purpose, we have assumed that only after the nominal cash balance has accumulated to its initial level will the bonds start to increase. This is an extreme case under our assumption that money adjusts faster than bonds. Specifically, there will now be an excess demand for bonds in asset holdings. We can illustrate the process graphically by Figure 2. The initial level of total real wealth is $a_o$ which is determined by a consumption-saving decision regarding income allocation. $l_o$ stands for the original optimal ratio between real cash balances and bonds, given values for $i_r$, $\pi^e$, and $y$; and point A is the initial equilibrium determined by the wealth constraint and the specific preference between different assets. After the general price level rises as a result of the imposition of a tariff, total real wealth declines from $a_o$ to $a_1$, and due to the equal proportionate losses in the real value of both assets, we are now moving from point A to point B which lies along the initial portfolio composition line. $l_o$ remains optimal since $\pi^e$ is unaffected by current inflation, and so $i_m$ as well as $y$ are constants due to the assumptions of perfect capital mobility and full employment. So, at point B, we have maintained portfolio composition equilibrium.

In terms of our model built in Chapter III, in this case the nominal $M^d$ and $B^d$ rise by the same amount when the general
2. Comparative Statics and Dynamics of Case A in which Money and Bonds are Equally Preferred at the Margin.
price level increases. This is because no other argument in the demand functions for money and bonds has changed. Therefore, from eq. (36), this implies that no capital flows will ever occur in the early stage of the adjustment. However, because of the attempt to restore wealth size, we have a movement towards point C. At point C, initial real cash balances are restored, but the corresponding portfolio ratio ξ is higher than optimal. Therefore, another attempt by wealth holders to restore composition equilibrium for their portfolio is brought about. More specifically, because an excess demand for bonds relative to money occurs, a capital outflow will take place when wealth holders try to buy bonds from abroad (and so, B^3 starts to increase). However, a deficit in the capital account (B_T=-dB^3/dt<0) will offset the initial B_T surplus and thus a further B_T surplus is needed to maintain the real cash balance at its initial level. As shown in the diagram, the adjustment path thus moves from point C back to point A, the original as well as the ultimate equilibrium position for the economy.26,27

Notice that we have slightly simplified the matter for illustrative purposes by implicitly assuming that real wealth rises monotonically from the beginning of the adjustment. However, it is feasible that the continuous increase in the price of the nontraded good and thus the increase in the general price level may outweigh the accumulation of nominal wealth, especially in the early stage of the adjustment.
Thus, real wealth might decline at these points rather than rising monotonically.

Also, note that with full employment and perfect capital mobility and thus constancy of real income as well as the nominal interest rate, (and thus $i_r$ since $\pi^e$ remains constant in this case), the desired level of real wealth ($\bar{a}$) is left unchanged after the price level rises, while the actual real wealth ($a$) immediately declines by the extent of general price level increase. And, due to the fact that every capital flow involves an equal amount of counter money flow, the changes in $B_K$ account change only the composition but not the scale of portfolio. Therefore, in the steady state, the size of the cumulative $B_T$ surplus has to be equal to the initial loss in real wealth, and cumulative BOP surplus has to equal to the initial loss of real cash balance in order to restore stock equilibrium for asset markets.

In other words, as a final result, the cumulative BOP $= \int_{t}^{\infty} (dM^S/dt) dt = M^d - M^s = BC$ in terms of our eq. (32); and the cumulative $B_T = \int_{t}^{\infty} (dA/dt) dt = A^d - A^s = BD$ from eq. (29).28

If capital is perfectly immobile, then as portfolio composition adjustment takes place, under which wealth holders as a whole try to purchase bonds, the domestic real interest rate will be forced down as the result. This interest rate adjustment will help to bring the desired portfolio mix in line with the actual one by lowering demand
for bonds and inducing people to hold the now existing
cash balances which have been restored to their initial
level by the real wealth effect, as indicated by point C
in Figure 2. Also, according to eq. (6) and footnote 14,
there is a "spillover effect" onto expenditure on commodi­
dities because of the incomplete portfolio mix adjusting
function of a given interest rate change. Thus, expenditure
on goods will increase as the interest rate falls as a
result of a new consumption-saving decision which gives rise
to a lower planned level of wealth holdings. In other words,
the lower interest rate arising from portfolio composition
adjustment entails a smaller desired size of real wealth.
So, for stock equilibrium, the needed $B_T$ improvement (Here,
in fact, $BOP=B_T$, since $B_K=0$ if capital is immobile) will be
smaller than that required when capital is mobile. The final
position in this case is at point C where the new desired
ratio locus ($\ell_1$) intersects with the new wealth constraint
$\bar{a}_2$ which is smaller than $a_0$.

In short, when capital is increasingly immobile, the
role of real factors (we refer to the interest rate here)
become more and more important in the BOP adjustment process
relative to the conventional real wealth effect. Furthermore,
the size of the resulting cumulative $B_T$ improvement
from imposing a given tariff is larger when there are fewer
barriers for international assets trade. However, this result
is not common to all the cases, as we will see from case (B)
and (C) below.

Case (B): When wealth holders as a whole have an asymmetrical preference changing toward money because of a negative price expectation, then there will be a higher desired ratio for holding real cash balances relative to real bond holdings for any given value of \( y \) and \( r \).

Consider the circumstance in which capital is perfectly mobile. In this case, the portfolio composition imbalance mentioned above will cause a capital account surplus when people attempt to sell bonds to the foreign country. In other words, because \( \pi^e \) declines (assume \( \pi^e \) equals zero initially), the demand for the nominal cash balance increases by a larger amount than the demand for bonds given the increase of the general price level (refer to eqs. (1) and (2)). So, in terms of our eq. (36), there will be an immediate \( B_K \) surplus via decumulation of bonds.

On the other hand, the portfolio stock adjustment brings about a \( B_T \) surplus through the real wealth effect. Both adjustment mechanisms work to accumulate money balances and to raise the ratio of money to bonds toward the new desired level. As time passes, an excess demand for bonds will evolve (relative to money, since bonds are sold abroad while the money stock accumulates continuously). Particularly, once the new desired ratio of money to bonds has been achieved, any further accumulation of money stock due to the real wealth effect will cause a portfolio composition
imbalance in the opposite direction (i.e. the initial excess demand for money becomes an excess demand for bonds), people stop selling bonds or even try to purchase bonds from abroad. Consequently, cumulative $B_k$ surplus becomes smaller (but not all the way back to nil because the new desired ratio of cash balances to bonds is higher than before), while the $B_T$ surplus keeps on growing as long as stock requirement for real wealth has not been fulfilled.

In short, the adjustment process can be described as follows: In the short-run, a capital account surplus shifts part of the adjustment burden away from the trade account, thereby serving as a cushion in the adjustment process. However, in the long-run, the adjustment is taken over by the trade account as the real wealth effect exerts its full power gradually, while the capital account balance dwindles because of the portfolio composition requirement.\(^{29}\)

We can illustrate this adjustment process in Figure 3. After the imposition of the tariff and the resulting higher general price level, the position of the economy moves from the initial equilibrium point, A, to point B due to the equal proportionate loss in real value of both assets. Also, due to the lower expected rate of inflation as a result of regressive expectation formation, the corresponding lowered $i_m$ will increase the desired ratio of real cash balance to bonds from $l_0$ to $l_1$. Therefore, the short-run equilibrium position is at point F. (Notice that at point F, the initially
lost real wealth has been recovered in part. This will be explained in more detail later).

3. Comparative Statics and Dynamics of Case B in which Money is More Preferred to Bonds at the Margin.
When the process of interest arbitrage proceeds, however, under which capital is exported due to the lower domestic $i_m$ relative to the rest of the world, the domestic $i_m$ will be restored to its world level in the case of perfect capital mobility. This implies that the outflow of capital will drive the domestic real interest rate upward by exactly the same percentage as the expected rate of deflation which occurs in the beginning. So, the desired level of real wealth ($\bar{a}$) must become higher since it is a positive function of $i_r$ (from eq. (3)). On the other hand, the desired ratio between assets remains at the initial level, because it's a function of the unchanged $i_m$, and the long-run semi-equilibrium position will be at point $F'$, the intersection of $l_o$ and $\bar{a}_2$, the latter being larger than $\bar{a}_0$.

However, according to our price analysis in Chapter III, the general price level as well as the price of the nontraded goods will eventually settle down to a certain level following the impulse result from the imposition of the tariff, given the underlying parameters of the economy such as elasticities of substitution in consumptions and productions. Therefore, the negative price expectation here must reverse itself if people are assumed to be rational in forming their expectation. That is, when people learn that the general price level will not completely return to its initial level, but, instead, increases to a certain level, they will modify their expected rate of inflation from
negative to positive and it will eventually level off at zero.

So $F'$ cannot be our steady state equilibrium. Instead, the final equilibrium will only be achieved at point A, at which the real interest rate as well as the nominal interest rate return to their initial levels. This is because with no expected inflation, in the long-run there is no necessity for a change in the level of the real interest rate to restore the domestic nominal interest rate to its international level as required by the equilibrium of international capital market.

The initial stage of adjustment is characterized by a composition adjustment which induces people to sell bonds for cash balances as well as a scale adjustment which leads to an accumulation of money stock by incurring a $B_T$ surplus. So, we move from point B toward point F where the new desired ratio between money and bonds is fulfilled and the initially lost real wealth has been recovered in part.

Notice that in eqs. (36) and (38), because the magnitude of decumulation of bonds will be smaller than that of cumulation of money stock, the size of total real wealth must have increased from $\bar{a}_l$ when the adjustment proceeds. Therefore, when we move from point B to F, the adjustment path must locate to the northeast of the $\bar{a}_l$ line.

However, any further $B_T$ surplus, driven by the remaining strength of the real wealth effect, will cause a new
composition imbalance in the opposite direction. As shown at point E, the corresponding portfolio composition is $\ell_2$ which is higher than what is desired ($\ell_1$). Therefore, wealth holders, in turn, start to buy bonds from abroad and the economy finally reaches the point of semi-equilibrium $F'$, by a combination of composition adjustment through international capital flows and scale adjustment. The backward bending of the adjustment path from point F indicates the fact that an initial $B_K$ surplus, which decreases bond holdings, is followed by a $B_K$ deficit as people restore their bond stock. From then on, the reversing of $\pi^e$ from negative to positive, by increasing the nominal interest rate, will lead to a counterbalancing decreasing of real interest rate, due to the resulting asset arbitration (inflow of capital because of the higher domestic $i_m$). This change will restore $i_r$ as well as $i_m$ to their initial levels as indicated by point A. And, as we have seen, A is also the long-run equilibrium point for case (A).

Because the capital inflow which decreases bonds and the remaining real wealth effect which increases money stock, the system will move from point $F'$ to point A from below.

Notice that we have divided the adjustment process as if it occurs discretely in three different successive stages. This is done just for illustrative purpose. Realistically, the reversing of $\pi^e$ may occur concurrently with the changes of $i_r$ which take place along the process of
the international asset arbitration.

So, compared to case (A) in which the size of the cumulative BOP surplus is equal to BC (the amount of replenished cash balances), here the resulting cumulative BOP improvement from the imposition of a given tariff again equals BC.

If capital is immobile, then the impact effect of portfolio composition adjustment will be to drive the domestic real interest rate upward when wealth holders as a whole try to sell their bonds. This increase of the real interest rate serves to restore the initial level of $i_m$ and thus make people content with their existing portfolio ($\ell_0$). It also induces an increased desired level of real wealth (i.e. a lowered expenditure on goods)\textsuperscript{30} equal to that under perfect capital mobility. Because given a fall in $\pi^e$, the restoration of $i_m$ to its initial level in the long-run equilibrium under both situations implies an equal rise of $i_r$. $i_m$ will return to its initial level due to the operation of asset arbitrage in the case in which capital is mobile, and because of the need to induce a desired portfolio ratio equal to the initially existing $\ell_0$ in the case when capital is immobile. Then, as we consider, as above, the factor of $\pi^e$ reversal (from negative to positive) due to the long-run stability of price, $i_m$ increases, the resulting excess demand for bonds will push $i_r$ down to its initial level such that the desired level of real wealth is restored to its original
level. So, a symmetrical pattern of changes of $\bar{i}_r$ (first an increase and then a decline in response to the negative and then positive price expectations, in order to maintain the constancy of the domestic $i_m$) is implied, which in turn lead to an equal level of desired wealth regardless of the degree of capital mobility.

Therefore, in this case, the size of $B_T$ improvement which is needed to close the gap between the desired and the actual stock of wealth when capital is immobile will be the same as when capital is mobile. This result is different from what we obtained for case (A) in which the resulting $B_T$ improvement is larger when capital is mobile than that if capital is immobile. The key factor which causes this difference lies in the fact that in case (B), when capital is immobile, because there is a portfolio imbalance immediately upon imposition of the tariff, $i_m$ has to be restored to its initial level (just as what happens when capital is mobile) to induce people to be content with the initial portfolio mix, $\bar{\xi}_0$. In case (A), however, because a portfolio imbalance will not occur until the real cash balance has been restored relative to bonds, when capital is immobile, $i_m$ has to adjust only to a lower level which is consistent with the then prevailing portfolio composition characterized by a higher ratio of real cash balance with respect to bonds. And, with a relative low $i_m$, because $\pi^e = 0$ in case (A), a relative low $i_r$ and thus a relative low desired level of
wealth are implied. Consequently, a smaller cumulative $B_T$ surplus is called for to achieve stock equilibrium in case (A).

Next, we consider the last feasible situation, case (C). When wealth holders in the aggregate have an asymmetrical preference changing toward bonds because of a positive price expectation, then there will be a lower desired ratio for real cash balances relative to real stock of bonds for any given values of $y$ and $i_r$.

By virtue of the attempt of wealth holders to keep an optimal portfolio mix, bonds will be purchased from abroad and thus a $B_K$ deficit generated. In terms of our eq. (36), because the positive $\pi^s$ upon the rising general price level leads to a larger increase in $B^d$ relative to $M^d$, there will be an immediate accumulation of bonds and thus the accompanying $B_K$ deficit. This deficit of $B_K$ will offset part of the $B_T$ surplus which arises from the gradual real wealth effect. One particular feature which is unique in this case compared to case (B) is that the portfolio composition adjusting function of the capital account and the stock adjustment function of the trade account work against one another. (In case (B), the two accounts are complementary in achieving the main targets of each). Here, while $B_T$ improves as individuals attempt to restore their initial real cash balances, the $B_K$ deficit tends to offset its effect. On the other hand, while the $B_K$ deficit serves to drive the desired ratio
of money to bonds toward the new lower equilibrium level, the surplus of $B_T$ works in opposite direction by increasing the money supply. As a consequence, both the composition and the stock requirements of portfolio adjustment may take longer to be accomplished in this case than in previous cases. Except for this, there will be a similar adjustment process, however, characterized by the interaction of stock and composition effects as in previous cases. Should the new desired ratio of real cash balance to bonds be achieved before the scale equilibrium is reestablished, then any further increases in $B_T$ surplus driven by the unsatisfied scale requirement will cause another composition problem and lead to another composition adjustment.

In Figure 4, $l_1$ is the new desired ratio which is lower than the initial ratio, $l_o$, indicating a now stronger preference to hold bonds relative to money. Thus, the short-run equilibrium point is at point G. However, again due to perfect capital mobility, $i_m$ will be fixed at the world level while $i_r$ declines by the same amount as the increase of the expected rate of inflation in the long-run equilibrium. So, the semi-long-run equilibrium is now at point $G'$, which is determined by the fulfillment of both the long-run preference pattern $l_o$ and the now desired level of wealth $\bar{\alpha}_2$ which corresponds to the reduced level of $i_r$.

Because of the counter effects of $B_T$ and $B_K$, the initial stage of the adjustment will be characterized by a movement
Comparative Statics and Dynamics of Case C in which Bond is More Preferred to Money at the Margin.
as shown from point B to point G where the new desired ratio between the two assets is satisfied. Then, a further BT surplus creates a second-generation composition imbalance implied by point I at which the corresponding ratio £2 is higher than £1. And thus, another composition adjustment is set in motion in which people buy more bonds in order to keep pace with the continuously growing money stock, and the economy finally reaches the semi-equilibrium point G'.

Note that the initial and the second composition imbalances here are of the same direction with excess demand for bonds, while they are in opposite direction in case (B). Also, notice that from point B to point G, because there are accumulation of bonds as well as money (reflected in eqs. (36) and (38), both dB^s/dt and dM^s/dt are positive), total wealth must have increased during the adjustment. Therefore, the movement from point B to point G must be located to the northeast of the a^1 line.

This would be the whole story if we did not consider the long-run absence of π^e. With π^e returning to zero, the long-run equilibrium will still be the same as in the previous two cases in which both i_m and i_r and thus l and ā remain unchanged. Specifically, in this case, after some point, π^e becomes negative as people learned that the price level is leveling off. So, i_m declines correspondingly which leads to capital outflow until i_r have been bid up to restore i_m to its initial level. With a fixed i_m, and since π^e is reduced
to zero in the long-run, \( i_r \) converges to its initial level, too. Therefore, we conclude that the comparative statics will be the same for all three cases despite the difference in the dynamics of the adjustments due to the initial difference of changes in \( \pi^e \). In other words, the final equilibria for all three cases are at point A. This stage of the adjustment is reflected in Figure 4 as the movement from point \( G' \) to point A. In particular, during this segment of the adjustment path, because domestic \( i_m \) is lower than its equilibrium level (the world level) and the accompanying outflow of capital, the holding ratio of real cash balances relative to bonds will be lower than its equilibrium level \( (\lambda_o) \). Therefore, the system moves from point \( G' \) to point A from below as shown in Figure 4.

Finally, if capital is immobile, a stronger marginal preference in favor of bonds will bring a downward pressure on the real interest rate in order to eliminate the portfolio imbalance. In other words, a lowered \( i_r \) will restore the initial level of \( i_m \) which has been driven up by the positive \( \pi^e \) such that \( \lambda_o \) is maintained. However, this lower \( i_r \) will also mean a smaller desired level of real wealth at the same level as in the long-run equilibrium of perfect capital mobility case, because the restoration of the initial \( i_m \) under both cases implies \( di_r = d\pi^e \). And again, if \( \pi^e \) converges to zero, this implies \( i_r \) and thus \( \bar{\alpha} \) will return to their initial levels. Specifically, when \( \pi^e \) becomes negative in
the later stage of the adjustment, the resulting decline of $i_m$ leads to an excess demand for money. In the absence of capital mobility, this will bid up the domestic $i_r$ until the initial level of $i_m$ is restored such that the initial $\pi_0$ is contended. With a zero $\pi^e$ in the long-run, the restoration of the $i_m$ to its initial level also indicates the returning of $i_r$ to its original level.

Therefore, similar to case (B), the same cumulative $B_T$ surplus is needed for the return to full stock equilibrium regardless of the extent of capital mobility.

The results of this section can be summarized as follows. First, the role of real factors relative to monetary factors (real balance effect) in the process of balance-of-payments adjustment becomes more and more ascendant the smaller the degree of capital mobility. (Here, we refer especially to the real rate of interest, because real income is constant with the full employment assumption). As capital becomes more immobile, any portfolio disequilibrium can only be eliminated through changes in the interest rate. This, in turn, affects the level of expenditure on goods and brings forth the needed $B_T$ improvement. Alternatively, in the case where capital retains mobility, it can always flow to fulfill portfolio demand and leaves the interest rate unchanged.

Second, the size of the ultimate cumulative BOP improvement following the imposition of a given tariff will be the same regardless of the alternative type of expectation
formations and thus different marginal responses for portfolio selection. However, the short-run dynamics do differ substantially across cases, as reflected in the different configuration of $B_T$ and $B_K$ changes.

On the other hand, the degree of capital mobility will lead to different resulting $B_T$ improvement only for case (A). Since $i_m$ will be restored to its initial level when there is perfect capital mobility for all the three cases, and since under cases (B) and (C) $i_m$ will also be restored to its initial level in order to eliminate the beginning portfolio imbalance upon the imposition of the tariff when capital is immobile, there will be no difference in the size of $B_T$ improvement for these two cases when capital mobility different in degree. Specifically, since in both cases (B) and (C), facing a given change in the expected rate of inflation upon the imposition of the tariff, $i_m$ has to be restored to its initial level when capital is immobile to make people contend with whatever there initial is the portfolio mix, the implied change in $i_r$, and thus the corresponding change of the desired level of real wealth, will be the same as in the situation of perfect capital mobility. And thus, the resulting $B_T$ improvement which is needed to close the gap between the desired and actual level of real wealth will be the same. However, for case (A), although $i_m$ is restored to its initial level by asset arbitrage when capital is perfectly mobile, it will adjust
to a different level (in fact, a lower level) when capital is immobile. The certain level of \( i_m \) which it will adjust to is compatible with a different portfolio mix (characterized by a higher ratio of real cash balance with respect to bonds than the initial ratio) which exists when the imbalance of portfolio occurs somewhere along the adjustment path of case (A). (Recall that in case (A), the portfolio composition problem will only occur after the real cash balance has been accumulated relative to the bonds; while in the latter two cases, there are portfolio mix disequilibria right upon the imposition of the tariff, with case (B) featured by an initial relative excess demand for money, and case (C) characterized by an initial relative excess demand for bonds).

Finally, the adjustments of the balance-of-payments described above characterize a portfolio stock-composition interactive process which incorporates a combination of real as well as monetary phenomena.

In the next section, we return to the situation in which there is unemployment.
Section 2 - Unemployment Case:

With the labor force partially unemployed, the supply functions for commodities are as follows: From eq. (16),

\[ S_i = S_i(N_i) = S_d(N_i^d) = S_i(W/P, P_i), \text{ for } i = X, M, \text{ and } NT. \]

Where, \( N \) = employment level, \( N_i^d \) = demand for labor by firms in each sector, and \( W \) = nominal wage rate.

In other words, production functions are determined by the level of employment in each sector which, in turn, is determined by the labor demanded by firms.\(^31\)

It should be noted that the relative prices of the importable with respect to the exportable and nontraded goods do not appear in these functions. This is so by virtue of the existence of unemployed resources (labor, here) and therefore the trade-off relationship in the supply of the three goods no longer exists.

Barro and Grossman (1971) argue that the demand for labor is not uniquely determined by the real wage rate. Rather, because of the nature of derived demand for inputs, one critical determinant is the demand for output. Therefore, we incorporate the absolute commodity price level in these functions in order to capture the effect of demand for output on demand for labor.

Observe that this specification implicitly assumes that product suppliers have interpreted the higher absolute price as a higher relative price and thus a symbol of higher
relative profitability for their commodities. This will be rational in our analysis with the assumption of a small country if all goods are traded. Because the price of the exportable is determined externally, an increase in the domestic absolute price of the importable due to the tariff is also a rise in the relative price of the importable. However, if there exists a nontraded good, we have seen in Chapter III that the cumulative rising of the general price level might be more than proportionate to the increase of the price of the importable (i.e. the rate of the tariff). Therefore, the increase in \( P_M \) can only be regarded by their producers as a signal of relative profitability of their products if there exists money illusion among the producers.

Now, suppose, that a tariff is imposed to promote import substitution in response to a persisting balance-of-payments deficit caused by a prevailing excess demand for commodities. The domestic price of the importable increases by the amount of the tariff, and thus raises the "effective" demand for labor in the importable sector and thus employment and output level in that sector, while leaving supplies unchanged in the other two sectors. (The "effective" demand for labor is defined as the actual amount of labor demanded by a profit-maximizing firm through the production function relationship in order to produce the actual demand-determined level of sales). However, when consumers substitute commodities X and NT for importables in their consumption, this leads to higher
demand for goods X and NT and a upward pressure on their prices, and thus $S_X$ and $S_{NT}$ will also increase in this variable employment case.

Consider the other explanatory variable, the real wage rate, which is traditional in these functions. Due to the rigidity of the money wage rate, which causes unemployment in the first place, firms will pay a lower real wage rate in the face of rising general price level following the imposition of the tariff. This, again, constitutes another source for the higher demand for labor and thus a larger output level produced in all sectors.

Compared to the full employment case, in which $S_M$ must expand at the expense of $S_X$ and $S_{NT}$ due to the resource constraint, we now have a net increase in the value of final output in the economy upon the imposition of a given tariff. This implies an increase in the long-run equilibrium value of real cash balances demanded (since $\partial L/\partial y > 0$). Therefore, a larger cumulative BOP surplus is needed in order to restore stock equilibrium in this "variable" employment case than in the full employment model. Specifically, since the desired level of real wealth increases (i.e. $\partial a/\partial y > 0$ by eq. (3)), there is now a stronger incentive to cut expenditure on goods (to save more), and thus lead to a larger cumulative $B_T$ surplus. On the other hand, there will also be a larger $B_K$ surplus because the desired level of bonds declines when $y$ increases and so more bonds will be sold abroad. Therefore,
the imposition of a tariff tends to improve the BOP by more than in a comparable full-employment model through both accounts.

In other words, because $3L/3y > -3G/3y > 0$ by expression (5), a larger output level will lead to an increase in the desired ratio for holding real cash balances with respect to bonds. As far as the portfolio composition adjustment is concerned, it is therefore more likely that wealth holders will have an asymmetrical response in favor of money here in this case than in the full employment case.

Recall that in the last section in which the economy was initially assumed to be at full employment, changes of $\bar{a}$ and $\ell$ are only temporary because of the change of $\pi^e$ is only illusory and eventually reverses itself. In terms of our Figure 3, the long-run equilibrium position is now located at point H at which the new desired ratio $\bar{l}_1$ and the new desired level of real wealth $\bar{a}_2$, both higher than before, are fulfilled. As a result, the cumulative improvement of BOP is $B'H$ which is obviously larger than $BC$, the size of BOP improvement for all the three cases when capital is perfectly mobile in the last section.
CHAPTER V

DYNAMIC ANALYSIS

Section 1 - Basic Model:

In this Chapter, I develop the dynamic aspect of my analysis explicitly with the model which I set up in Chapter III.

Assume that there is perfect capital mobility such that the nominal interest rate is determined internationally due to covered interest arbitrage. Also, assume full employment prevails in the economy such that real output remains constant. Then, demand functions for money and bonds from eqs. (1) and (2) become as follows:

\begin{align}
M^d &= PL(\gamma, i_m)a = P(\tau)La \\
B^d &= PG(\gamma, i_m)a = P(\tau)Ga, \text{ where } L+G = 1.
\end{align}

Recall that \(a\) is defined as the real value of actual stock of wealth, \(\tau\) is the rate of tariff, and \(P\) is the domestic price index, which is in turn a weighted average of the prices of the importable, exportable, and the nontraded good. \(P\) is a function of \(\tau\) here, because after the imposition of a given tariff, the domestic price of the importable will rise...
by the same percentage as the tariff rate. Besides, the
price of the nontraded good will also rise to a certain
degree in comparative statics as demonstrated in eq. (47).
(Recall that the price of exportable is fixed at the world
level due to the commodity arbitrage).

In the following, for the sake of simplifying our math-
ematical derivation, we will treat the change in the price
of the nontraded good and thus the change in the general
price level as if they occur instananeously. The more real-
istic consideration of a continuous, and a possible oscilla-
tory change in the price of the nontraded goods due to the
interaction of the positive effect of the substitution among
goods and the negative effect of the depressed aggregate
spending might lead to much more complicated dynamics.
Nevertheless, the simplified assumption should leave the
basic dynamics as well as the comparative statics of our
model unchanged. (And, the more realistic case is a feasible
extension of this work in further research).

Precisely, from the price equation, eq. (49), if we
assume that all elasticities of excess demands, a, b, c, r,
and the expenditure shares of goods,α, and β, are constants
for the time period concerned,^33 then we could obtain the
change of the general price level as:
\[
(54) \quad dP = Adt + B(\tau)d\tau = F(\tau)
\]
where, A and B are two constants determined by the parameters
a, b, c, r, α₂, and β₂, and so is the function F.
Therefore, the general price level as a function of the tariff could be written as:

\[(55) \ P = P(\tau) = P_0 + F(\tau), \] because \(dP = P-P_0; \) and,

\[\approx P_0 + C\tau, \] by linear approximation.

Where, \(C\) is also a function of the underlying parameters.

Therefore, eqs. (52) and (53) become:

\[(56) \ M^d = (P_0+C\tau)L_1 = P_0L_1 + C\tau L_1 = M^d_0 + \phi_1\tau\]

\[(57) \ B^d = (P_0+C\tau)G_0 = P_0G_0 + C\tau G_0 = B_0^d + \phi_2\tau\]

Where, \(\phi_1\tau = M^d_0 - M_0 - C\lambda_1, \) i.e., the level of excess demand for money; and, \(\phi_2\tau = B^d_0 - B_0 - C\gamma_1, \) i.e., the level of excess demand for bonds. Notice that \(\phi_1 + \phi_2 = \gamma_1\).

The change in the stock of bonds is then:

\[(58) \ dB^S/dt = -k(M^d_0 + \phi_1\tau - M^S) + k(B^d_0 + \phi_2\tau - B^S)\]

from eq. (37) and upon substitution of eqs. (56) and (57).

From eq. (38), the change in the stock of money is:

\[(59) \ dM^S/dt = 2k(M^d_0 + \phi_1\tau - M^S)\]

Recall that we have assumed that (refer to p.45-46):

(a) the speed of portfolio adjustment, \(k, \) is the same as that of real wealth effect on expenditure, and also that

(b) disequilibria in different asset markets have the same effect on the economy in specifying our eqs. (37) and (38).

We will investigate later in Section 2 whether these
assumptions have any serious effect on our results.

Upon imposition of a given tariff, $\tau$, because the general price level increases, there is a proportionate loss in the real value of both assets. And in response to this portfolio disturbance, there will be subsequent compensating accumulation or decumulation of both assets. Denote for any variable, $x$, the deviation of its current from its initial level by $\bar{x}$, i.e. $\bar{x} = x - x_0$. (Note, therefore, $d\bar{x}/dt = dx/dt$).

Assume that initially each asset market is in equilibrium, i.e. $M_0^d = M_0^s$, and $B_0^d = B_0^s$. So, $M^d - M^s = M^d - M^s$ and $B^d - B^s = B^d - B^s$. Also, because there is no tariff in the beginning, so $\bar{\tau}$ is simply the newly imposed tariff in rate $\tau$.

Then, we can rewrite eqs. (59) and (58) in deviation form as follows:

(60) $dM^S/dt = 2k(\phi_1 \bar{\tau} - M^S)$

(61) $dB^S/dt = -k(\phi_1 \bar{\tau} - M^S) + k(\phi_2 \bar{\tau} - B^S)$

Now, we can derive the time path of the change in the money stock (that is, equivalently, the accumulation of international reserves here) and that of the stock of bonds by solving the system equations (60) and (61). (See Appendix). We get the following solutions: (Here, $e$ denotes exponential function).

(62) $M^S(t) = -\phi_1 \bar{\tau} e^{-2kt} + \phi_1 \bar{\tau}$

(63) $B^S(t) = \phi_2 \bar{\tau} + \frac{\phi_2 \rho_2 - (\phi_2 - \phi_1)k\bar{\tau}}{\rho_1 - \rho_2} e^{-\rho_1 t} - \frac{\phi_2 \rho_1 - (\phi_2 - \phi_1)k\bar{\tau}}{\rho_1 - \rho_2} e^{-\rho_2 t}$
Where, $-\rho_1$ and $-\rho_2$ are the two roots of the characteristic equation of our system as derived in the Appendix and they satisfy $\rho_1\rho_2=2k^2$, and $\rho_1+\rho_2=3k$. (In fact, the two roots are $-k$ and $-2k$ respectively).

Upon substitution of $\rho_1$ and $\rho_2$ into eq. (63) where $\rho_1=k$, and $\rho_2=2k$:

$$B^S(t) = \phi_2e^{-kt} - (\phi_1+\phi_2)e^{-kt} + \phi_1e^{-2kt}$$

To analyze the effect of the imposition of a tariff on the different accounts of the balance of payments, we derive the following equations by differentiating eqs. (62) and (64):

$$\frac{d\bar{M}^S}{dt} = 2k\phi_1e^{-2kt}$$

$$\frac{d\bar{B}^S}{dt} = \bar{T}(k(\phi_1+\phi_2)e^{-kt} - 2k\phi_1e^{-2kt})$$

From eq. (65), it is obvious that $\frac{d\bar{M}^S}{dt}>0$ for all values of $t$. If we differentiate eq. (65), we get $\frac{d^2\bar{M}^S}{dt^2} = -4k^2\phi_1e^{-2kt}$, which is, in contrast to the first derivative, negative in sign. So, we can conclude that the stock of money, following the imposition of a tariff, will increase at a decreasing rate over time. Furthermore, we can derive the steady state value of the money stock which the system will converge to at the intertemporal equilibrium by letting $t$ go to infinity in eq. (62). We obtain:

$$\bar{M}^S^* = \phi_1$$

(the asterisk "*" denotes the equilibrium value)
Since the intertemporal equilibrium level of money stock is a finite value when a given tariff is imposed, we can therefore infer that the time adjustment path of the money stock will approach the steady state value asymptotically as time goes to infinity. The time path of the money stock adjustment is depicted in Figure 5. What the path describes is the following process: As a result of the increased nominal demand for money in face of the higher general price level, people will cut their expenditure on goods or sell bonds abroad such that money stock is gradually accumulated. However, the closer the money market moves toward equilibrium, the weaker the adjusting forces, and the accumulation of money stock will completely cease when the money market reaches its intertemporal equilibrium. Therefore, the total accumulated stock of money is a finite figure which is just enough to restore the equilibrium of the money market. Since the balance-of-payments is equal to the incremental money stock, the corresponding time path for the BOP is also depicted in Figure 5.

As for the time path of adjustment of the bond stock, from eq. (66), we get the following expression:

\[ dB^B/dt = k\bar{T}(\phi_2 - \phi_1), \text{ as } t \text{ approaches } 0. \]

The sign of this expression could be positive, zero, or negative, depending on whether \( \phi_2 > \phi_1 \) right after the tariff is imposed. Moreover, we can get the intertemporal equilibrium value of the stock of bond from eq. (64):
5. The Effect of Imposition of a Tariff on the BOP, Money and Reserves
(69) \( B_s^* = \phi_2 t \)
And, again, this is a finite number.

Also, we can derive further the second derivative of the time path of bond stock adjustment by differentiating eq. (66):

(70) \( \frac{d^2 B_s}{dt^2} = -k^2 (\phi_1 + \phi_2) e^{-kt} + 4k^2 \phi_1 e^{-2kt} \)
And again, the sign of eq. (70), just like eq. (66), is contingent upon the relative values of \( \phi_1 \) to \( \phi_2 \).

Note that in eq. (66) for \( \frac{dB_s}{dt} \), the first term on the right hand side is a positive term while the second term is negative. Also, the first term in eq. (70) \( \frac{d^2 B_s}{dt^2} \) is negative while the second term is positive.

In order to investigate the adjustment path of the bond stock, we need, therefore, to look at each case separately.

Case (A): If \( \phi_1 = \phi_2 \), in other words, if upon the imposition of the tariff, people have an equal amount of excess demand for both assets, then \( \frac{dB_s}{dt} \) (eq. (66)) will be always positive because the first, positive term will always outweigh the second, negative term for all possible values of \( t \). However, \( \frac{d^2 B_s}{dt^2} \) (eq. (70)) could be positive for small values of \( t \) and becomes negative when \( t \) increases. This is due to the fact that \( e^{-2kt} \) declines faster than the \( e^{-kt} \) as \( t \) rises. Furthermore, we observe in this case that in eq. (68), \( \frac{dB_s}{dt} \) equals zero as \( t \) approaches 0. The adjustment path of the stock of bonds following the imposition of a given tariff is drawn in Figure 6. In sum, in the initial stage after the
imposition of the tariff, there will be no accumulation of bonds (refer eq. (36)). As time passes, however, the stock of bonds will start to increase first at an increasing rate and then at a decreasing rate until the intertemporal value of the bond stock $B^s_t$ as stated in eq. (69) is reached. Note that the shape of the time path of adjustment is completely in conformity with my qualitative inference in the static analysis chapter (Chapter IV, see p. 62-65 in which the economic implication of this particular adjustment path has been explained).

Case (B): If $\phi_1 > \phi_2$, i.e. if individuals have a larger demand for money relative to bonds, then there will be an initial decumulation of the bond stock as reflected by the negative sign of eq. (68). However, since the steady state change of bond stock is a positive finite number, as shown by eq. (69), there must be a reverse capital flow along the path of adjustment as time passes. Upon investigating eq. (66), we find that although the negative term might outweigh the positive term when $t$ is small because $\phi_1 > \phi_2$, the positive term will become the dominating force in the later stage of the adjustment. In this case, therefore, we will observe an adjustment path of the bond stock which shows a decrease in the earlier stage of the adjustment but an increase in the later stage of the time path. Again, the characteristics of this time path are consistent with my predictions. (See p. 68 and 69 for an economic interpretation). This adjustment
is depicted in Figure 7.

Case (C): Finally, we might have $\phi_1 < \phi_2$, i.e. people have a greater excess demand for bonds than for money. If this is the case, the $dB^B/dt$ of eq. (66) as well as eq. (68) (when $t$ approaches $0$) will both be positive. That is, the stock of bonds will accumulate right from the beginning (upon the imposition of the tariff). And moreover, the rate of accumulation will accelerate for the earlier stage of the adjustment and then slow down. This is so because the second derivative as shown by eq. (70) will go from positive to negative when $t$ increases. Furthermore, if we compare case (C) with cases (A) and (B), we see that the time interval for the bond stock to change at an increasing rate is the shortest for case (C). In other words, in case (C), the adjustment path will turn from convex to concave at an earlier point of time than under cases (A) and (B). This is so because the larger the $\phi_2$, the more quickly the negative term in the second derivative becomes dominant, and this also explains why it might take longer for the system to reach the final equilibrium under cases (C) than in the other two cases. (See p. 76-77 for the relevant discussion regarding this point). This adjustment path is depicted in Figure 8.
The Effect of a Tariff on the Holding of International Traded Bonds.
In the following, I investigate the stability condition and the possibility of oscillation of our dynamic adjustment.

Since the two roots of the characteristic equation of the system are respectively \(-k\) and \(-2k\), the system is dynamically stable because the real part of both roots is negative. In fact, we have already shown that the steady state values for both assets are finite when time goes to infinity. (See eqs. (67) and (69)). In other words, since \(M^S(t)\) and \(B^S(t)\) diminish as \(t\) increases due to the negative exponent of the growth factor, \(e\) (or \(\exp\), the exponential function), the system is dynamically stable. (See eqs. (62) and (63)).

On the other hand, since both roots are real numbers, the adjustment path will be monotonic rather than cyclical.

In order to ascertain the qualitative properties of the adjustment paths - primarily, whether \(M^S(t)\) and \(B^S(t)\) converge - we employ the technique of the phase diagram illustration as follows:

From eqs. (60) and (61), when we set both equations equal to zero (i.e. when \(dM^S/dt=0\) and also \(dB^S/dt=0\)), we can obtain the following expressions:

\[
\begin{align*}
(71) & \quad dM^S/dt = 2k\phi_1T - 2kM^S = 0, \text{ and} \\
(72) & \quad dB^S/dt = -k\phi_1T + kM^S + k\phi_2T - kB^S = 0
\end{align*}
\]

Therefore, the intercept and slope of eq. (71) are \(M^S=\phi_1T\) and \(dM^S/dB^S=0\); while for eq. (72), they are \(M^S=(\phi_1-\phi_2)T+B^S\) and \(dM^S/dB^S=1\). Also, we can get \(\delta(dM^S/dt)/\delta M^S<0\) from eq. (71) and \(\delta(dB^S/dt)/\delta B^S<0\) from eq. (72). Now, we
can plot the two curves $dM^S/dt = 0$ and $dB^S/dt = 0$ in Figure 9.

As we can see from the phase diagram (Figure 9), the system is dynamically stable such that it will always move toward the long-run equilibrium (the intersection point, E, at which the accumulation of both assets ceases and both assets have obtained their steady state values), no matter where the system starts.\footnote{35}

9. Phase Diagram for Basic Model in which Excess Demand for Each Single Asset Culminates in an Equal Speed of Restoration of Total Wealth and the Stock of Bonds Adjusts Monotonically with Total Wealth.
We will now go on to investigate the comparative statics of our dynamic model:

Assume that the system is initially at the equilibrium point, \( E \), and a given tariff is then imposed. As a result of the imposition of the tariff and the resulting equal proportionate loss in the real value of both assets, wealth holders might have three possible types of reaction: they might now have a stronger preference for money (corresponds to case (B)), or for bonds (case (C)), or regard the two different assets equally (case (A)) in their portfolio selection decision. Assume that wealth holders have now a stronger preference towards money after the imposition of the tariff, i.e., an increase of \( \phi_1 \) relative to \( \phi_2 \) (case (B)). This could happen under the following circumstances: (1) if people have a negative expectation for the future rate of inflation in face of the rising general price level caused by the imposition of the tariff; or, (2) if there is an increase in the level of national output in the event of initial unemployment. Because under these situations, if other things remain constant, these changes will induce people to have a relatively stronger preference for holding money than for bonds. (Refer to eqs. (52) and (53), the demand functions for money and bonds respectively).

Given that \( \phi_1 \) increases (equivalently, \( \phi_2 \) drops since the sum of these two parameters is a constant, see p. 90), the two curves \( dH^S/dt = 0 \) and \( dB^S/dt = 0 \) will both shift upward
in our phase diagram. Consequently, the initial equilibrium point, E, now becomes a disequilibrium point characterized by an excess demand for cash balance and an excess supply of bonds. As we can see from Figure 10, point E is now located to the south of the new equilibrium point, E'. Therefore, according to our dynamic analysis, the system will move toward point E' in the direction indicated in our Figure 10. That is, the bond stock will be reduced and money will accumulate in the early stage of the adjustment, but after a certain length of time, there will be a restoration of bond stock to some extent while the money stock keeps on increasing. Notice that this is exactly the adjustment path which we have described in our previous discussion in Chapter IV (p. 68-70).

In other words, in response to the excess demand for money, individuals begin to sell bonds abroad as well as cutting back their expenditure on goods, and consequently the stock of bonds dwindles while stock of money is built up. Ultimately, however, an excess demand for bonds eventually develops which leads to another stage of the adjustment characterized by buying bonds from abroad while cutting expenditure on goods continues. Although we have made these qualitative predictions, we are only now able to show them rigorously through our dynamic analysis developed in this chapter. (ϕ_1 in Figure 10 denotes the new ϕ_1).
10. Comparative Statics for the Case in which Money Becomes More Preferred to Bonds.
Furthermore, the dynamic analysis also enable us to analyze quantitatively the effect of the imposition of a given tariff on the balance-of-payments under different hypothetical situations. More specifically, because the total cumulative improvement in the balance-of-payments is here identical to the replenished amount of money stock needed to bring the money stock to its intertemporal equilibrium level (see eq. (38)), and also, since the intertemporal equilibrium value of money balance will be higher, the higher is the \( \phi_1 \) (see eq. (67)), we thus conclude that the total cumulative balance-of-payments improvement will be greater under case (B) than in cases (A) and (C) if there is any change of any real variable which raises the demand for money permanently.\(^{36}\)

As a example, as already discussed in the section of Chapter IV dealing with unemployment, if we compare the situation in which unemployment exists to the case in which full employment prevails, there will be a net increase of real output and thus a stronger relative demand for cash balance in the former case. This will lead, according to our dynamic analysis, to a larger cumulative BOP improvement in the less than full employment case, given the imposition of a tariff.

We will now prove the foregoing assertion for the unemployment case.
From eqs. (52) and (53), the demand functions for money and bonds become:

(73) \[ M^d = P(\tau)L(y)a, \]

(74) \[ B^d = P(\tau)G(y)a, \]

where \( L + G = 1 \), and notice that \( m \) is suppressed from these functions due to the assumption of perfect capital mobility.

By linear approximation, eqs. (73) and (74) can be written as:

(75) \[ M^d = PL'ya + LP'^\tau a, \]

(76) \[ B^d = PG'ya + GP'^\tau a \]

Then, if written in deviation form:

(77) \[ \Delta M^d = PL'y\bar{a} + LP'^\tau \bar{a} = h_1\bar{y} + \phi_1\bar{\tau}, \]

where \( h_1 = PL'a > 0 \), and \( \phi_1 = LP'a > 0 \) (note here because \( \phi_1 = L\alpha C \) from eq. (56) and \( P'^\gamma a\alpha P/\alpha a = C \) from eq. (55)). And,

(78) \[ \Delta B^d = PG'y\bar{a} + GP'^\tau \bar{a} = h_2\bar{y} + \phi_2\bar{\tau}, \]

where \( h_2 = PG'a < 0 \), and \( \phi_2 = GP'a > 0 \).

Also, notice that we have assumed \( L'(y) > |G'(y)| > 0 \) such that \( h_1 > |h_2| \) because of our specification that an income increase has a net positive effect on the desired level of real wealth as a whole (see eq. (3)).

Now, substitute eqs. (77) and (78) into the deviation form of eqs. (59) and (58):

(79) \[ \frac{dM^S}{dt} = 2k(h_1\bar{y} + \phi_1\bar{\tau} - M^S), \]

(80) \[ \frac{dB^S}{dt} = -k(h_1\bar{y} + \phi_1\bar{\tau} - M^S) + k(h_2\bar{y} + \phi_2\bar{\tau} - B^S) \]
Solving the above two differential equations, we get the time paths of the adjustments of the money stock and the stock of bonds (see Appendix):

\[(81) \quad M^s(t) = -(h_1\overline{y}+\phi_1\overline{r})e^{-2kt} + (h_1\overline{y}+\phi_1\overline{r}), \text{ and} \]
\[(82) \quad B^s(t) = -\{(h_1+h_2)\overline{y}+(\phi_1+\phi_2)\overline{r}\}e^{-kt} + (h_1\overline{y}+\phi_1\overline{r})e^{-2kt} + (h_2\overline{y}+\phi_2\overline{r}) \]

Therefore, the steady state values for the two assets when time goes into infinity are respectively:

\[(83) \quad M^s* = h_1\overline{y} + \phi_1\overline{r} > \phi_1\overline{r}, \text{ because } h_1>0 \text{ (and } \overline{y}>0 \text{ here since } y \text{ rises), and} \]
\[(84) \quad B^s* = h_2\overline{y} + \phi_2\overline{r} < \phi_2\overline{r}, \text{ because } h_2<0. \]

These results confirm the above assertion (also that made in Chapter IV, see p. 86-87) that when there is unemployment, a larger BOP improvement will result from an imposition of a given tariff than if full employment prevails. The larger BOP surplus comes from a larger \(B_k\) surplus as reflected by the lower desired level of the bond stock of eq. (84), as well as a larger \(B_T\) surplus as represented by the now larger desired level for the wealth as a whole. (Because \(h_1>|h_2|>0\), if we add up eqs. (83) and (84), the long-run incremental desired level of wealth as a whole will be larger than that under full employment).

However, the speed of adjustment to the final equilibrium is now slower. This is reflected by the fact that in
eqs. (81) and (82), given the same roots of adjustment, \(-k\) and \(-2k\) (the same as in eqs. (62) and (64) of the full employment case), the coefficients of both \(e^{-k}\) and \(e^{-2k}\) are now larger in absolute value than those under full employment. In other words, comparing eqs. (81) and (82) to eqs. (62) and (64), we now have \((h_1\bar{y} + \phi_1) > \phi_1\bar{r}\), and, \((h_1 + h_2)\bar{y} + (\phi_1 + \phi_2)\bar{r} > (\phi_1 + \phi_2)\bar{r}\). The latter occurs because \((h_1 + h_2) > 0\) when we assume that \(h_1 > |h_2| > 0\):

Economically, this is so because there is now an output effect besides the price effect. Both affect the desired level of wealth holding, so it takes longer to achieve the stock equilibrium.

If we compare an all-traded goods model with the case in which a nontraded good exists, we could again, by virtue of the dynamic analysis here, quantitatively distinguish these two situations in terms of the magnitude of the cumulative improvement of the balance-of-payments. Specifically, if all goods are tradeable, following the imposition of the tariff, the general price level, \(P\), will only increase to the extent of the domestic price of the importable increases, because the price of the exportable is fixed at the world level by international commodity arbitrage. In other words, according to our expression (eq. (39)), \(dP/d\tau = -a_1 = \beta_1\), where \(\beta_1\) is the expenditure share of the importable. However, if there exist nontraded goods, \(P\) might rise by a different percentage. According to our derivation (see eq. (49)),
now we have:

\[ \frac{dP}{dt} = a_2 + (1-\alpha_2-\beta_2) \frac{(c+r) - a}{2b + (1+\tau)(a-c)} \]

This could be either greater or smaller than \((1-a_1)\), depending on elasticities \(a, b, c, r\) and expenditure shares \(\alpha\) and \(\beta\) as demonstrated in Chapter III.

Therefore, imposition of a tariff in the case with a nontraded good may generate a different cumulative improvement in the balance-of-payments as the system moves to its intertemporal equilibrium position. This is because a different price level, given everything else constant, will cause a different excess demand for nominal cash balances which is turn must be eliminated through a balance-of-payments surplus.
Section 2 - Alternative Models:

In this section, I will investigate the issue of whether the dynamic nature (e.g. stability and monotonicity) of our adjustment path is only a result of the specific assumptions which have been employed to simplify our model. In other words, instead of using a model which is built upon equations (37) and (38), if we now use a more general model for our dynamic analysis, will our resulting time path of adjustments exhibit instability or oscillation?

We now examine two alternative models:

Model 2:

(86) \( B_T = k(M^d - M^s) + k(B^d - B^s), \quad k > 0, \) and

(87) \( B_K = v(M^d - M^s) + v(B^s - B^d), \quad v > 0 \)

What is implied in this specification of the balance of trade and capital account is that the speed of bond stock adjustment, \( v \), is different from the speed of real wealth restoration, \( k \). However, we still retain the assumptions that excess stock demand for each asset has equal effect on the accumulation of total wealth as well as on the bonds.

Therefore, we have the overall balance-of-payments or the change in money stock as:

(88) \( \text{BOP} = \frac{dM^s}{dt} = (k+v)(\phi_1^r-M^s) + (k-v)(\phi_2^r-B^s) \)

And, the change in bond stock is:

(89) \( \frac{dB^s}{dt} = -B_K = -v(\phi_1^r-M^s) + v(\phi_2^r-B^s) \)
Deriving a phase diagram by setting both eqs. (88) and (89) equal to zero, we get the following:

The slope of \( \frac{dM^S}{dt} = 0 \) is:

\[
\frac{dM^S}{dB^S} = -\frac{(k-v)}{(k+v)} \geq 0, \text{ if and only if } k \leq v.
\]

And the slope of \( \frac{dB^S}{dt} = 0 \) is:

\[
\frac{dM^S}{dB^S} = 1
\]

The partial derivatives are:

\[
\frac{\partial (dM^S/dt)}{\partial M^S} = -(k+v) < 0, \text{ and}
\]

\[
\frac{\partial (dB^S/dt)}{\partial B^S} = -v < 0.
\]

The phase diagram associated with this model is depicted in Figure 11 for the case in which we assume \( k < v \) such that the \( \frac{dM^S}{dt} = 0 \) curve has a positive slope but is flatter than the \( \frac{dB^S}{dt} = 0 \) curve. The system is found to be dynamically stable upon investigating the two roots of the characteristic equation of this model (see Appendix). We also find that the adjustment path of money stock as well as bond stock are, again, monotonic in this model (because both roots are real with no imaginary component), despite the modified assumptions behind the model.

As a matter of fact, comparing eqs. (88) and (89) to our general model as shown in eqs. (27) and (28), we found that the feature of the dominance of the direct effect over the cross effect of asset accumulations is still preserved in this model. That is, in Model 2, that \( m_1 = k+v > b_2 = v \), and, \( b_1 = v > m_2 = v-k \). Also, as we can see from the discussion of the
11. Phase Diagram for Model 2 in which Excess Demand for Each Single Asset Culminates in an Equal Speed of Restoration of Total Wealth but the Stock of Bonds Varies at Different Pace from the Total Wealth.
condition for stability in our Appendix, this particular assumption fulfills the sufficient condition for dynamic stability. Recall from our discussion in Chapter III that this assumption implies that excess demand for any one asset does not result in a reduction in total wealth.

Intuitively, this constitutes the source of stability for our system. Specifically, since the imposition of the tariff culminates in declines in the real values of total wealth as well as each single asset in the portfolio, if the accumulation of either asset during the process of the adjustment leads to a decumulation of total wealth, the scale requirement of the total wealth may never be attained. Therefore, the system becomes dynamically restless.

Let us try another alternative model which is even more general than the last one:

Model 3:

(94) \( B_T = k(M^d - M^s) + \delta(B^d - B^s), k, \delta > 0; \)

(95) \( B_K = v(M^d - M^s) + U(B^s - B^d), v, U > 0. \)

We can see from this model that not only the speed of bond stock adjustment is different from that of the real wealth restoration, but also that different asset market's imbalances generate different speeds of adjustment for portfolios as well as expenditure on goods.

Similarly, we can get the following equations for the phase diagram:
\( dM_s^S/dt = B_T + B_K = BOP \)

\[ = (k + v)(\phi_1^T - M^S) + (\delta - U)(\phi_2^T - B^S), \] and

\( dB_s^S/dt = -B_K = -v(\phi_1^T - M^S) + U(\phi_2^T - B^S) \)

The slopes of \( dM_s^S/dt = 0 \) and \( dB_s^S/dt = 0 \) are:

\( dM_s^S/dB_s^S = (U - \delta)/(k + v) < 0, \) if and only if \( U < \delta, \)

\( dM_s^S/dB_s^S = U/v > 0 \)

Also, the partial derivatives for each asset are:

\( (dM_s^S/dt)/M_s^S = -(k + v) < 0, \) and

\( (dB_s^S/dt)/B_s^S = -U < 0 \)

The phase diagram of this model is depicted in Figure 12 in which we assume that \( U > \delta, \) i.e., the portfolio adjustment resulting from disequilibrium in the bond market is faster than that of expenditure adjustment to bond market disequilibrium such that \( dM_s^S/dt = 0 \) has a positive slope. Also, \( dM_s^S/dt = 0 \) is flatter than \( dB_s^S/dt = 0 \) because eq. (98) is smaller in absolute value than eq. (99).

This system, as shown in the diagram, is dynamically stable just like the previous cases. Upon comparison of eqs. (96) and (97) to our general model, we found that stability once again arises from the dominance of the direct effect over the cross effect in asset accumulations.

That is, in Model 3, \( m_1 = k + v > b_2 = v, \) and, \( b_1 = U > m_2 = U - \delta. \)

However, upon investigating the characteristic equation of this model (see Appendix), we find that there is now a
possibility that the adjustment path will be cyclical rather than monotonic. The condition under which oscillation might occur is when the following relationship holds:

\[ \delta > \frac{(k+U)}{2} \]

In other words, if the disequilibrium of the bond market has a significant expenditure effect on commodity flows, the system could move towards the final equilibrium via a fluctuating adjustment path rather than monotonically. However, because the system remains dynamically stable, the fluctuation of the adjustment will be damped as the system moves closer to its intertemporal equilibrium.

Since Model 3 is the only model under which the adjustment path might fluctuate, we conclude that the system will move back toward equilibrium with oscillation only when imbalances in different asset markets cause different speeds of portfolio as well as expenditure adjustments. Intuitively, that a larger \( \delta \) constitutes the source of oscillation can be easily explained. In all three cases the stock of bonds begins to accumulate at some points in the time path of adjustment (see Figures 6-8). In case (C), for instance, bonds are accumulated immediately upon imposition of the tariff. If the increase in bonds has a large expansionary effect on expenditure upon goods (i.e. a significant negative effect on \( B_T \)), this will obviously counteract the real wealth effect which works to reduce expenditure on goods. Therefore, there is a process characterized by fluctuations in which the
positive and negative forces in restoring the desired level of wealth are interacting with each other.

12. Phase Diagram for Model 3 in which There is No Uniformity in Asset Accumulation.
In this chapter, we apply our dynamic model to analyze the situation in which the government sector actively uses its most popular monetary policy instrument, open market operations, to affect the performance of the economy. The purpose of adopting the open market operation of the monetary authority (the central bank) in these circumstances is to sterilize the effects of external monetary disturbances on the stock of money through the balance-of-payments.\footnote{38}

A typical open market operation is a one to one exchange of assets from a liquid to a non-liquid type (or vice versa) initiated by the central bank with the private sector (individuals or commercial banks). The central bank's decision about the direction and degree of the open market operation is a reaction to the extent of deviation of the prevailing economic performance from desired goals. For example, during an inflationary period, the central bank usually sells government securities in the open market in order to reduce the stock of money held by the public and thereby curb inflation.
The following equation is the reaction function for open market operations in this analysis:

\[ \text{(103)} \, \frac{dM^G}{dt} = -\frac{dB^G}{dt} = \theta \text{BOP} = \theta B_T + \theta B_K \]

where \( M^G \) is the stock of money held by the central bank and \( B^G \) is the stock of bonds owned by the central bank.

In the face of a balance-of-payments surplus, the central bank will sell bonds for money in order to reduce the domestic credit component of the monetary base such that the total size of the monetary base remains stable.

In our notation, \( M^s (= R+D) \), the total money supply, consists of the foreign reserve asset, \( R \), which increases due to the balance-of-payments surplus, and the domestic credit component, \( D \), which declines by the execution of the open market operation of the central bank.

We assume that the degree of sterilization would range from none to complete \((0 \leq \theta \leq 1)\). We will also discuss over-sterilization \((\theta > 1)\) and negative sterilization \((\theta < 0)\).

By definition, the money supply and supply of bonds are the amount in circulation (held by the public - i.e. non-government sector, individuals or commercial banks), so the accumulation of each asset will have a modified functional form as follows:

\[ \text{(104)} \, \frac{dM^s}{dt} = \text{BOP} - \frac{dM^G}{dt}, \text{ and} \]

\[ \text{(105)} \, \frac{dB^s}{dt} = -B_K + \frac{dM^G}{dt} \]

That is, if the government sells bonds for money, the
public-held $M^s$ will decline while the $B^s$ held by the public increases by the same amount.

Note that these two equations differ from the previous ones in that there is no open market operation represented by an extra additive term in each equation.

Recall from Chapter III that $BOP=2k(M^d-M^s)$, $B_t=k(M^d-M^s)$ + $k(B^d-B^s)$, $B_K=k(M^d-M^s)+k(B^s-B^d)$, and $M^d=M^o+\phi_1t$ while $B^d=B^o+\phi_2t$. So, upon substitution of respective terms in equations (103) to (105) and rewrite eqs. (104) and (105) in their deviation form, we get the following differential equations for each asset:

\begin{align}
(106) \frac{dM^s}{dt} &= 2k(1-\theta)(\phi_1^T - M^s) \\
(107) \frac{dB^s}{dt} &= k(2\theta-1)(\phi_1^T - M^s) + k(\phi_2^T - B^s)
\end{align}

We can now derive the time path of the adjustment of each asset in order to investigate the dynamic nature of the new system. By solving eqs. (106) and (107)(see Appendix).

\begin{align}
(108) M^s(t) &= \{M^s(0)-\phi_1^T\}e^{-2(1-\theta)kt} + \phi_1^T \\
(109) B^s(t) &= \{\phi_2 + \frac{\rho_2\phi_2-(\phi_2+(2\theta-1)\phi_1)k}{\rho_1 - \rho_2} e^{-\rho_1 t} + \frac{\rho_1\phi_2-(\phi_2+(2\theta-1)\phi_1)k}{\rho_2 - \rho_1} e^{-\rho_2 t}\} \tau
\end{align}

where $-\rho_1$ and $-\rho_2$ are the two characteristic roots of the system, with $\rho_1=k$ and $\rho_2=2k(1-\theta)$. Upon substitution of $\rho_1$ and $\rho_2$, we get:
(110) $B^S(t) = \{\phi_2 - (\phi_1 + \phi_2)e^{-kt} + \phi_1 e^{-2k(1-\theta)t}\}$

Notice that when $\theta = 0$, we simply degenerate to the previous results of Chapter V. On the other hand, if $\theta = 1$, i.e. with complete sterilization, then the equations (108) and (110) become:

(111) $M^S(t) = M^S(0) = 0$, i.e. $M^S(t) = M^S(0)$, the initial level of the stock of money. And,

(112) $B^S(t) = \{(\phi_1 + \phi_2)(1-e^{-kt})\}$

Eq. (111) implies that with perfect sterilization, the stock of money will simply be maintained at its initial level throughout the entire adjustment period. From equation (112) we can derive the first and the second derivatives of the change of the stock of bonds:

(113) $dB^S/dt = kT(\phi_1 + \phi_2)e^{-kt} > 0$, while
(114) $d^2B^S/dt^2 = -k^2T(\phi_1 + \phi_2)e^{-kt} < 0$

Notice that the signs of these derivatives are no longer contingent on the relative magnitudes of $\phi_1$ and $\phi_2$ as in the previous case in which there is no sterilization. In particular, now the stock of bonds increases at a decreasing rate during the adjustment period following the imposition of a given tariff with complete sterilization.

This is due to the fact that a perfect sterilization which aims to offset the natural adjustment pattern of the money stock (recall from Chapter V that the money stock will
increase at a decreasing rate without sterilization), will lead to a counteracting adjustment of the bond stock in exactly the same configuration. I.e., the Fed has to sell bonds to the public in whatever pattern the fluctuation of the money stock originally has in order to maintain the constancy of the money stock. This explains why the bond stock will exhibit such an adjusting path regardless its own original feasible paths determined by the relative magnitude of $\phi_1$ and $\phi_2$ when there is no sterilization.

For the normal situation in which a partial sterilization is imposed, i.e. $0<\theta<1$, the speed of adjustment toward the steady state equilibrium of the system is slower than before. This is reflected by the facts that now we have a smaller absolute value for one of the characteristic roots ($-p_2$) while the other remains the same (before, $-p_1=k$ and $-p_2=2k$), and the coefficients associated with the two exponential factor are also the same. This is simply because the intervention of the central bank's sterilization policy retards the motion of the automatic adjustment process of the money market towards its equilibrium, and thus the time period needed for adjustment is prolonged. However, with sterilization, the system still remains dynamically stable with no oscillation since the two roots are again negative and real.

However, we observe that $0<1$ is the necessary condition for the stability. If $\theta>1$, the system will be unstable, with
the possibility for both assets to accumulate explosively. This is so because an over-sterilization will in fact not only retard the natural adjustment toward equilibrium but also by itself "creates" the fundamental disequilibrium which calls for an endless adjustment. Notice that the possibility of unstability is precluded in the no sterilization case. Also, should \( \theta \) be negative, then the central bank's open market operation is in fact a complementary policy which serves to reinforce the pace of the adjustment toward the stock equilibrium by speeding up the accumulation of the money stock toward its desired steady state level.

Notice that the sign of \( \frac{dB^8}{dt} \) and \( \frac{d^2B^8}{dt^2} \) as derived below are now dependent on not only the relative magnitude of \( \phi_1 \) and \( \phi_2 \), but also on the value of \( \theta \). By differentiating eq. (110):

\[
(115) \frac{dB^8}{dt} = \{k(\phi_1 + \phi_2)e^{-kt} - 2(1-\theta)k\phi_1 e^{-2(1-\theta)kt}\}T, \quad \text{and}
\]

\[
(116) \frac{d^2B^8}{dt^2} = \{-k^2(\phi_1 + \phi_2)e^{-kt} + 4(1-\theta)^2k^2\phi_1 e^{-2(1-\theta)kt}\}T.
\]

Upon investigating these two equations, we found that the larger the value of \( \theta \) (the closer it is to 1), the more likely the \( \frac{dB^8}{dt} \) will have a positive sign and the \( \frac{d^2B^8}{dt^2} \) will be negative. This is particularly in contrast to case (B) in Chapter V in which \( \phi_1 > \phi_2 \) and \( \frac{dB^8}{dt} \) is negative when \( t \) approaches 0. Now, we will derive the critical value of \( \theta \) which makes the sign of \( \frac{dB^8}{dt} \) become positive. When \( t \) is close to 0, from eq. (115):
\[
\frac{dB^S}{dt} = \{k(\phi_1 + \phi_2) - 2(1-\theta)k\phi_1\} \overline{\tau} \\
= \{(2\theta-1)k\phi_1 + k\phi_2\} \overline{\tau} > 0, \text{ if and only if} \\
(118) \theta > \frac{1-\phi_2/\phi_1}{2}.
\]

We can see from this expression that the higher the relative magnitude of \(\phi_1\) with respect to \(\phi_2\), the larger the critical value of \(\theta\) has to be. All of these results come from the fact that a larger sterilization will make the corresponding adjustment path of the bonds more closely resemble the natural fluctuation of the money stock.

Therefore, in a case in which there is an initial excess demand for money (i.e., with \(\phi_1 > \phi_2\)) so that bonds will be sold abroad to accumulate the desired money stock, it takes a larger sterilization effort to change this initial natural adjustment of the stock of bonds.

To derive the phase diagram, set both equations (106) and (107) equal to zero, we get the following:

\[
\begin{align*}
(119) \frac{dM^S}{dt} &= 2k(1-\theta)\phi_1 \overline{\tau} - 2k(1-\theta)M^S = 0, \text{ and} \\
(120) \frac{dB^S}{dt} &= k(2\theta-1)\phi_1 \overline{\tau} - k(2\theta-1)M^S + k\phi_2 \overline{\tau} - kB^S = 0
\end{align*}
\]

Therefore, the intercept of the \(dM^S/dt=0\) is \(M^S=\phi_1 \overline{\tau}\), and the slope is \(dM^S/dB^S=0\). (Notice that they are exactly the same as in the no sterilization case). On the other hand, the intercept of the \(dB^S/dt=0\) is as follows: Set \(B^S=0\) in eq. (120):

\[
(121) M^S = \overline{\tau}(\phi_1 + \frac{1}{2\theta-1}\phi_2) > (\phi_1-\phi_2) \overline{\tau} \text{ as } \theta \leq 1/2
\]
Also notice that eq. (121) will be greater than or smaller than \((\phi_1-\phi_2)^{-1}\) (the intercept of the \(dB^S/dt=0\), when there is no sterilization) whenever \(\theta \geq 1/2\). On the other hand, the slope of the \(dB^S/dt=0\) is as follows:

\[(122) \frac{dM^S/dB^S}{dB^S} = -\frac{1}{2\theta - 1} \geq 0 \text{ if and only if } \theta \leq 1/2\]

If we compare with the no-sterilization slope, which is of value 1 from the derivation of Chapter V, we can see that the slope now will be greater in absolute value for all feasible values of \(\theta\) between 0 and 1. Especially, when \(\theta > 1/2\), the slope is now negative, but has absolute value greater than 1, and in fact, when \(\theta = 1/2\), the slope is infinite.

Therefore, from the comparison we just made between this open market operation model and the no-sterilization model in the last chapter, we can see the \(dB^S/dt=0\) locus will now has a steeper slope with the possibility of a negative slope when \(\theta\) gets larger in its value from 0 to 1. In other words, the \(dB^S/dt=0\) locus now tends to rotate counterclockwise, and the degree of rotation gets larger, the larger the degree of sterilization.

As shown by Figure 13 (for \(0 < \theta < 1/2\)), and Figure 14 (for \(1/2 < \theta < 1\)), we now can proceed to explain what the steeper slope of the \(dB^S/dt=0\) means and what it implies for our dynamic analysis of the counterclockwise rotation of the locus.

First, the steeper slope of \(dB^S/dt=0\), i.e. the larger absolute value of \(dM^S/dB^S\) can be interpreted as follows:
Suppose both asset markets are in equilibrium initially. Assume that there is a BOP deficit which leads to a given drop in the money stock. (Recall that $M^S$ can only change in our analysis through the BOP imbalance, aside from the sterilization policy of the government). Because there is now active government intervention in the form of buying bonds in the open market to offset to some extent the drop of $M^S$, there will be a smaller need for private sector to sell bonds abroad in order to rebuild their desired stock of money. Therefore, with government's open market operation which takes part of the burden of compensating the fluctuation of $M^S$, a given shortage of $M^S$ now needs only a smaller capital inflow from abroad to restore equilibrium in the domestic asset market. This is exactly what is reflected by the steeper slope $(dM^S/dB^S)$ of the $dB^S/dt=0$ locus.

Before we proceed to analyze the implications of the counterclockwise rotation of the $dB^S/dt=0$ locus for our comparative statics, we notice that the equilibrium values of $M^S$ and $B^S$ remain the same as before. That is, from eqs. (108) and (110) we can derive the results that $M^S(t=\infty)=\phi_1\overline{T}$ and $B^S(t=\infty)=\phi_2\overline{T}$, which are the same as before. So, the locus $dB^S/dt=0$ must still intersect $dM^S/dt=0$ at the same equilibrium point $E$ when it is rotating in a counterclockwise direction due to the open market operation.

The counterclockwise rotation of the $dB^S/dt=0$ locus will lead to an enlarged excess-demand-for-bonds zone.
Given the new locus (shown as B'B' in Figure 13) instead of the original one (BB) (i.e. the one when there is no sterilization), an initial position like that depicted by point A, at which an excess demand for money and excess supply of bonds were implied before, now becomes a position at which there is excess demand for bonds as well as money. Therefore, the resulting adjustment path will move in a quite different direction from point A to the equilibrium point E. In particular, the stock of bonds, instead of being reduced, will now be accumulated in the early phase of the adjustment.

Moreover, the region of excess-demand-for-bonds-zone will be larger, the larger the value of $\theta$, because the degree of rotation will be greater. This confirms our previous analysis that in order to maintain the constancy of the money stock which would otherwise accumulate through the BOP surplus following the imposition of a given tariff, the more intensively the central bank practice the open market operation, the more likely and more prolonged the stock of bonds which circulates in the private sector will be continuously increased by the government over the span of the entire adjustment process. As we can see from Figure 13, the adjustment is quite likely to undergo a longer journey to reach the final equilibrium point $E$, with the possibility that the path exhibits a quasi-cyclical pattern. (As indicated by the dotted locus from A to E).
For example, in the extreme case in which $\theta > 1$, the system may be possibly unstable. In this case, the slope of the $dB^S/dt = 0$ (see eq. (122)) will not only be negative but also have an absolute value smaller than 1, which is the value of the slope before. Furthermore, we now can obtain from eq. (119) the derivative $2(dM^S/dt)/2M^S (-2k(1-\theta)) > 0$. From this, we can illustrate the instability caused by a oversterilization policy by Figure 15 in which an explosive adjustment path is implied.

Finally, when $\theta < 0$, because $1/(2\theta - 1) < 0$, and its absolute value is smaller than 1 in eq. (121), the intercept of $dB^S/dt = 0$ will be greater than before. On the other hand, from eq. (122), the slope will now be positive while having a absolute value smaller than before, so that $dB^S/dt = 0$ will rotate clockwise compared to the case of no sterilization.

As depicted in Figure 16, we see that a position like point A will now be characterized by an excess supply of bonds rather than an excess demand. Therefore, the resulting change in the bond stock in the early phase of the adjustment will be a reduction instead of an increase. This arises from the fact that now government, under its negative sterilization policy, will reinforce the accumulative effect on $M^S$ of a BOP surplus through buying bonds in the open market. According to our mathematical derivation (refer to eqs. (108) and (110), where the exponent increases when $\theta < 0$), this will speed up the pace of adjustment toward the final equilibrium $E$. 
13. Phase Diagram for the Case with Intensive Open Market Operation ($0 < \theta < 1/2$)
14. Phase Diagram for the Case with Weak Open Market Operation \((1/2 < \theta < 1)\)
15. Phase Diagram for Oversterilization ($\theta > 1$)
16. Phase Diagram for Negative Sterilization ($\theta < 0$)
CHAPTER VII
SUMMARY AND CONCLUSIONS

This study analyzes the dynamics of the adjustment of the balance-of-payments within a two-asset model. Specifically, because of the typical neglect of the capital account in the traditional balance-of-payments approaches, we stress the respective changes of each of the two major accounts of the balance-of-payments, the capital account as well as the trade account, during the process of the adjustment.

This is accomplished by setting our analysis in the context of "portfolio approach" to the balance-of-payments. This approach emphasizes the portfolio composition adjustment as well as the portfolio scale adjustment (which is the sole driving force in a money-only model) following any disturbance in the framework of a multi-asset model.

We are not concerned here with the traditional real-trade analysis of the effect of a tariff, i.e. the impact of a tariff on the changes of the production and consumption structure of the economy arising from the establishment of price distortions. Instead, we focus on the monetary effect which a tariff has on the economy, that is, the effect on
the balance-of-payments. We first found this issue interesting because of the striking fact that the effect of a tariff on the balance-of-payments itself can not be analyzed in a pure real-trade context by using our conventional analysis of trade theories (Refer to Mussa (1974)). Besides, the imposition of a tariff is a good expository tool with which to analyze the theory of the balance-of-payments adjustments. This is because: (a) The imposition of a tariff has real as well as monetary implications, that is, it will change real variables as well as monetary variables which have influences on the balance-of-payments; (b) The portfolio imbalance caused by the imposition of a tariff could be of any form, and, as a consequence, the resulting dynamics may be quite different.

Therefore, in this dissertation, I approach the issue by focusing on how a tariff changes individual's preference regarding the composition of their asset holdings as well as the scale of these holdings. Both effects are important in determining the resulting changes of the balance-of-payments. The exposition of the subject is made more interesting by considering the effects of inflationary factors of short-run duration as well as the effects of changes of any real variables on the marginal portfolio decisions of wealth holders following the imposition of the tariff. Furthermore, we employ a unconventional variant of "the" portfolio approach in which the speed of portfolio
adjustment is sluggish in achieving the optimal ratio as well as the desired scale rather than instantaneous as traditionally assumed by the "stock" approach. In order to derive the resulting steady state stock equilibrium, the comparative static change of the price level following the imposition of the tariff is also carefully drawn.

By establishing a mathematical model for the dynamic analysis, the time path of adjustments of our system is analyzed quantitatively as well as qualitatively. In particular, techniques of Laplace Transform for solving differential equations and phase diagrams for illustrating the dynamics are employed in Chapter V. As a result, not only the qualitative predictions for the properties of the adjustment path made in Chapter IV are proved rigorously, but we also gain some insights which would not be feasible without establishing this more rigorous dynamic analysis.

The main conclusions of this work are as follows: First, upon the imposition of a tariff and the resulting higher general price level, regardless of how wealth holders respond in their portfolio decision, the resulting time path of money stock adjustment will rise at a decreasing rate. This is because the money stock is accumulated through the so-called "real wealth effect" as wealth holders cut back on their expenditure on goods, in order to satisfy their increased stock demand for nominal cash balances. This effect becomes weaker, however, as the stock of money accumulates
closer to its desired level. (Notice that this is also the typical result in a conventional monetary approach). We have also assumed a second asset (internationally traded bonds), which involves another type of portfolio adjustment caused by the requirement of composition equilibrium. Thus, we have also assumed the existence of a capital account in which each bonds transaction implies a counter money flow. Consequently, the real wealth effect as described above remains as the dominating force in replenishing the nominal cash balances to its desired level.

However, the adjustment paths of bonds will be different as wealth holders exhibit different marginal changes of portfolio preference. If there is initially no portfolio composition disequilibrium, under our assumption of "sluggish" portfolio adjustment in which money adjusts relatively faster than bonds, the stock of bonds will not start to increase until some point in the later stage of the adjustment (refer to case (A) in the context). However, if people prefer bonds after the disturbance, the stock of bonds will be accumulated right from the beginning of the adjustment (case (C)). Finally, if money is preferred to bonds at the margin, bonds will be sold abroad in the early stage of the adjustment (case (B)). Nevertheless, the stock of bonds will eventually show an increase in all three cases to a certain level in order to maintain balance with the money stock which has been accumulated continuously
throughout the entire adjustment process.

Second, as far as the comparative statics is concerned, if the imposition of the tariff leads to only a temporary change in the demand for assets in terms of composition and scale through a variable like inflationary expectation of a short-run nature, then, if everything else remains constant, the resulting size of the balance-of-payments improvement will be the same in the steady state for all cases with different short-run expectation schemes. However, the interim dynamics do differ substantially as reflected in the different configurations of the two accounts of the balance-of-payments as discussed above. This is because in long-run equilibrium, the effect of the tariff on the general price level will be the same regardless of how people form their price expectation in the short run. The ultimate change in the general price index will be determined by the underlying parameters of the economy, for instances, the elasticities of substitution among different goods in production and consumption as well as the income elasticities, and also the given rate of the tariff. This was demonstrated in the comparative static price change section of our Chapter III. Also, when we compare the comparative statics between an all-traded good model and a model with nontraded goods, although we could expect a different price change and thus the resulting changes of the balance-of-payments, it is not obvious which
case will have the larger balance-of-payments improvement unless we make specific quantitative assumptions regarding the parameters.

If the imposition of the tariff leads to a change in certain real variables (e.g. real income, or interest rate), then not only the composition (trade account vs. capital account) of a given cumulative change of the balance-of-payments will be different, but also the comparative static size of the improvement of the balance-of-payments itself will also be different. In particular, in Chapter V, we derive the dynamics of the balance-of-payments adjustment for the case of unemployment, in which the level of income rises as a result of the increase in the realtive price of the importable caused by the tariff. We find that the resulting cumulative balance-of-payments improvement will be greater in this case than in the full-employment case because of a larger capital account surplus as well as a larger trade account surplus. This is because when real income rises, the desired holding ratio of money to bonds as well as the desired level of total wealth will both increase so that people have a stronger incentive to cut spending and sell bonds abroad. However, we also find that the speed of adjustment when employment is variable is slower than that with full employment. This occurs because we now have in the adjustment an output effect besides the price effect on the desired level of wealth holding, so it
takes longer to achieve the final stock equilibrium.

Third, if capital is immobile, the comparative statics for both cases (B) and (C) in which wealth holders exhibit a stronger preference toward either one of the two assets will remain the same as in the perfect capital mobility case. This is because the nominal interest rate, rather than capital flows, is the adjustment force which restores the portfolio equilibrium when capital is immobile. If portfolio imbalance occurs immediately upon imposition of the tariff, if wealth holders are to remain content with the initial portfolio ratio, the nominal interest rate has to return to its initial level in equilibrium. Without a long-run price expectation factor, this also implies a real interest rate at its initial level. Therefore, the desired level as well as the desired ratio of portfolio holdings will return to their original values just as in the perfect capital mobility case in which the nominal interest rate is fixed at its original (world) level by asset arbitrage. And consequently, the resulting cumulative balance-of-payments improvement will be the same regardless of capital mobility in these two cases. However, if wealth holders exhibit no preference upon the imposition of the tariff on the two assets (i.e. refer to case (A)), there will be no composition problem until the money stock accumulates relative to bonds in our "creeping" portfolio adjustment model. Thus, when capital is immobile, in this case the nominal interest rate
will only adjust to a lower level which is compatible with the then prevailing ratio of real cash balances to bonds (which is higher than the initial ratio) to achieve portfolio equilibrium. On the other hand, the nominal interest rate still has to return to its initial level under perfect capital mobility case. Therefore, only in this specific case will the resulting comparative statics be different as a result of capital mobility. More specifically, according to our analysis, a smaller cumulative balance-of-payments improvement will occur because the equilibrium level of the nominal interest rate (and thus the real interest rate and the implied desired level of real wealth) will be lower than that under perfect capital mobility.

Fourth, for alternative models in which we either drop the assumption of a monotonic change of the bonds with total wealth, while maintaining the uniformity of contribution of each single asset to the total wealth (Model 2), or relax even further the latter assumption (Model 3), the system will be dynamically stable and without any oscillation in all models except that in which the bond market disequilibrium has a particularly strong effect on individual's spending. (Chapter V, p.118-119). In this situation, the system, although remaining stable, will move toward the final equilibrium with fluctuations which are damped over time. The inherent dynamic stability of the system is built upon the crucial assumption that the direct effect (that is, the effect
of an excess demand for a specific asset on its own accumulation) dominates the cross effect (that is, the effect of an excess demand for a specific asset on the accumulation of the other asset) in asset accumulation functions as discussed in Chapters III and V. (Recall that this assumption implies that an excess demand for any single asset does not lead to a reduction of total wealth). Therefore, all the models, as long as they preserve this basic feature, are dynamically stable. (For rationales, see p. 116).

Fifth, if the government sterilizes the effect of the balance-of-payments on the money supply through open market operations, we obtain the following results.

First, in the normal situation in which the degree of sterilization is greater than zero but less than perfect, the steady state value of the cumulative balance-of-payments improvement remains the same as that without the open market operation, and the adjustment also remains dynamically stable with no oscillation. However, the speed of adjustment toward the final equilibrium is slower than that without government intervention. This is, in fact, an expected result because it is intended that the open market operation will retard the natural operation of the portfolio adjustment which ultimately brings the system to its final equilibrium.

Second, if there is complete sterilization such that the stock of money is always maintained at its initial level, the other asset, bonds, which is used as the tool of the open
market operation, will have a adjustment path which simply duplicates the shape of the time path of the money stock when there is no sterilization. More specifically, with complete sterilization, the stock of bonds will always increase at a decreasing rate during the adjustment period regardless of its own original paths under different short-run expectation schemes.

Third, should there be an oversterilization, that is, if the central bank overacts by engaging in open market operations which more than offset the effect of a balance-of-payments disequilibrium on the money supply, then the system will be dynamically unstable, with the possibility that both assets could accumulate explosively. This is so because the oversterilization itself will create a fundamental disequilibrium which calls for an endless adjustment. For example, if the central bank, in response to a balance-of-payments surplus, sells bonds to the public to such a large extent that the money supply actually declines, the desired level of nominal cash balance will never be achieved, and a basic instability is thus created.

Finally, should the sterilization be negative, that is, if the central bank is in fact using open market operations to reinforce the effect of the balance-of-payments on the money supply, the speed of adjustment toward the final equilibrium will be faster than in the event of perfect or less than perfect sterilization.
In the following, I list some of the feasible extensions and elaborations which I think would be interesting and worthwhile to pursue in the near future.

The first promising extension that I want to make is to remove the conventional "small country" assumption, and to consider, instead, a country which is "large" in the world commodity and/or asset markets.

Consider a country which is large in the world commodity market, such that changes in the demand and supply for commodities will affect the world prices of these goods. An imposition of a tariff on the importable will thus make the world price level of the importable decline because of the reduced demand of the home country for this commodity. This, implies an improvement of this country's terms of trade and therefore, an improvement in real income.

As a result, the desired level of real wealth increases (since $\delta y/\delta y > 0$ by eq. (3)) and there is a stronger incentive to cut expenditure on goods (to save more). This will lead to a larger cumulative trade account surplus, and therefore, the cumulative balance-of-payments improvement is magnified.

Therefore, we can assert that the traditional argument for "optimum tariff" which applies mainly to the domestic objective to raise the welfare level for the economy as a whole will also help to achieve an external target by bringing forth a larger balance-of-payments improvement.
On the other hand, if the country is large in the international asset market, with everything else remaining unchanged as before, we have an additional effect on the balance-of-payments adjustments similar to that described above following the imposition of a tariff on imports. Suppose there is now also an interest differential tax on foreign securities imposed by the government in order to discourage capital outflow. This is, in essence, a tariff on imported assets, and will serve to reduce domestic demand for foreign bonds by distorting the relative rate of return of foreign assets. Because the country is important in the world asset market, its reduced demand will lead to a lower asset price offered by other countries. Recall that in the case of a small country, the asset price in foreign currency remains constant and thus the nominal value of assets measured in domestic currency after the imposition of the tariff will also remain constant, and the real value of bonds declines by exactly the degree of the rise in the general price level. Consequently, the real cash balance and the real value of bonds would decrease in proportion. However, with the decline in the world price of bonds, the real value of bonds will decrease by a greater proportion than the rise in the general price level. Thus, we are more likely to observe an excess demand for bonds relative to money in this case than that in the case of a fixed asset price (i.e., refer to case (C) discussed in Chapter IV). And from Chapter IV,
we can infer that additional factor of "imperfection" of asset market tends to weaken the balance-of-payments improving effect of a given tariff, while the monopolistic elements in the commodity market contribute to the achievement of a better external position for a country.

I might also want to incorporate a nontraded asset into my model, i.e., to extend my model to a three-asset context: money, a traded bond, and a home bond. The existence of a nontraded bond is one of the factors which could make capital mobility less than perfect, and so, represents a more general case than those of zero or perfect capital mobility. In this case, in which capital mobility is less than perfect, the most obvious result will be a slower speed of the balance-of-payments adjustment toward the final portfolio equilibrium. On the other hand, with the existence of the home bonds, whose market has to be cleared all the time, following the imposition of the tariff and thus the higher general price level and the excess demand for the home bonds, the interest rate of the home bonds has to fall to restore the equilibrium of the home bonds market. If we assume the supply of the home bonds is fixed within the time period concerned (with no new issuance) and also perfect capital mobility for the traded bond such that its rate of return is fixed at the world level, the induced falling of the domestic bonds' interest rate indicated above will lead to a larger equilibrium desired level for the other two assets when people substitute
the other two assets for home bonds in their portfolio. Therefore, compared to the all‐traded assets case in which there is no such interest rate effect, the equilibrium desired levels for money and the world bonds will now be greater following the imposition of the given tariff. Therefore, the resulting balance‐of‐payments improvement as well as the trade balance surplus will be greater. (Since a greater demand for the world bonds in equilibrium reflects a larger capital account deficit, so the improved balance‐of‐payments must come from an improved trade balance).

Finally, I am wondering whether it is feasible to carry out an empirical study such as that suggested by Frenkel and Rodriguez (1975) in the last section of their paper. On the one hand, what we need in our model are data for a small, non key‐currency country for which there are no speculative capital flows. (Since all the capital movements occur as a result of portfolio adjustments). On the other hand, we may need to collect very short‐run data, such as quarterly or even monthly data, in order to capture and contrast between the effects on capital flows which follow immediately after the imposition of the tariff and those which occur in the later phase of the adjustment. Because of all these data problems, I am skeptical about the feasibility of empirical work based on this kind of dynamic portfolio balance model.
For Chapter V:

Expressed in matrix notation, the eqs. (60) and (61) become as follows:

\[
\begin{bmatrix}
2k+D & 0 \\
-k & k+D
\end{bmatrix}
\begin{bmatrix}
\bar{M}^S \\
\bar{B}^S
\end{bmatrix} = 
\begin{bmatrix}
2k\phi_1\bar{\tau} \\
k(\phi_2-\phi_1)\bar{\tau}
\end{bmatrix}
\]  

(A.1)

where \(D\) is the differential operator \(d/dt\).

The system has the characteristic equation:

\[(k+D)(2k+D) = 0\]  

(A.2)

So, the two roots of this equation are \(-k\) and \(-2k\) respectively.

Using Cramer's rule, the differential equation for \(\bar{M}^S\) is:

\[(2k+D)\bar{M}^S = 2k\phi_1\bar{\tau}, \text{ i.e. } d\bar{M}^S/dt = 2k\phi_1\bar{\tau} - 2k\bar{M}^S\]  

(A.3)

Solving (A.3), we obtain:

\[
\bar{M}^S(t) = (\bar{M}^S(0) - \phi_1\bar{\tau})e^{-2kt} + \phi_1\bar{\tau} \\
= -\phi_1\bar{\tau} e^{-2kt} + \phi_1\bar{\tau}
\]  

(A.4)

since \(\bar{M}^S(0) = 0\). This equation gives eq. (62) for the change in money supply following the imposition of a given tariff \(\tau\).

On the other hand, the differential equation for \(\bar{B}^S\) is:

\[(2k+D)(k+D)\bar{B}^S = 2k^2\phi_2\bar{\tau} + (\phi_2-\phi_1)Dk\bar{\tau}\]  

(A.5)
If we let the two roots of the characteristic equation be \(-\rho_1\) and \(-\rho_2\), then eq. (A.5) becomes:

\[(\rho_1 + D)(\rho_2 + D)\bar{B}^s = \rho_1 \rho_2 \phi_2 \tau + (\phi_2 - \phi_1)Dk\tau\]  
(A.6)

A convenient method for solving equations such as (A.6) with step changes such as an imposition of a given tariff is that of Laplace transforms as outlined in Allen (1967, p. 357-362). As discussed there, we need only specify the initial conditions just before the imposition of the tariff, then apply the technique of Laplace transforms. Prior to the tariff, assume the initial conditions are: \(B^s = DB^s = D^2 B^s = 0\).

Then, the solution of \(B^s\) with an imposition of a tariff of rate of \(\tau\) has Laplace transform \(b(r) = \int_0^\infty e^{-rt}B^s(t)dt\) given by:

\[b(r) = \frac{(\rho_1 \rho_2 \phi_2 \tau + (\phi_2 - \phi_1)kr\tau)}{r(r + \rho_1)(r + \rho_2)} = \frac{F(r)}{G(r)} \]  
(A.7)

where \(F(r)\) is of lower degree of \(r\) than \(G(r)\).

Eq. (A.7) can be written in partial fractions as:

\[b(r) = \frac{F(r)}{G(r)} = \sum_{i=1}^{2} \frac{F(r_i)}{G'(r_i)(r_i)} \frac{1}{r-r_i} \]  
(A.8)

where \(r_i\) is the \(i\)th root of \(r\), and \(G'(r_i)\) is the partial derivative of \(G\) with respect to \(r\) when \(r=r_i\). (Here, \(r_1^* = 0\), \(r_2^* = -\rho_1\), and \(r_3^* = -\rho_2\).

Therefore, we get the expression of:

\[b(r) = \left\{\frac{1}{r} \phi_2 + \frac{\phi_2 \rho_2 - (\phi_2 - \phi_1)k}{\rho_1 - \rho_2} \cdot \frac{1}{r + \rho_1} - \frac{\phi_2 \rho_1 - (\phi_2 - \phi_1)k}{\rho_1 - \rho_2} \cdot \frac{1}{r + \rho_2}\right\} \tau \]  
(A.9)
From (A.9), since the Laplace transforms of $1$ is $1/r$ and of $e^{-\rho t}$ is $1/(r+\rho)$, therefore, the inverse Laplace transform for (A.9) is:

$$B^s(t) = \frac{\phi_2 \rho_2 - (\phi_2 - \phi_1)k}{\rho_1 - \rho_2} e^{-\rho_1 t} - \frac{\phi_2 \rho_1 - (\phi_2 - \phi_1)k}{\rho_1 - \rho_2} e^{-\rho_2 t}$$

(A.10)

Equation (A.10) gives eq. (63) for the change in the international asset following the imposition of the tariff, $\tau$.

As far as the dynamic stability is concerned, because the real part of the two characteristic roots, $-k$ and $-2k$ are both negative, the system is stable.

An alternative way to investigate stability is as follows: for a system,

$$\frac{dX}{dt} = F(X,Y) = a_{11}X + a_{12}Y$$

$$\frac{dY}{dt} = G(X,Y) = a_{21}X + a_{22}Y$$

(A.11)

the condition for stability is as follows: the trace of the coefficient matrix $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ less than or equal to zero, while the determinant greater than zero. That is,

$$a_{11} + a_{22} \leq 0, \text{ and } a_{11}a_{22} - a_{21}a_{12} > 0$$

(A.12)

In our general model, from eqs. (31) and (32), we have:

$$\frac{dM}{dt} = m_1(M^d - M^s) - m_2(B^d - B^s) = -m_1M^s + m_2B^s + (m_1\phi_1 - m_2\phi_2)\tau$$

$$\frac{dB}{dt} = b_1(B^d - B^s) - b_2(M^d - M^s) = b_2M^s - b_1B^s + (-b_2\phi_1 + b_1\phi_2)\tau$$

(A.13)
Therefore, the coefficient matrix is \[
\begin{bmatrix}
-m_1 & m_2 \\
b_2 & -b_1
\end{bmatrix},
\]
and the stability condition is:
\[-m_1b_1 - b_1 < 0, \text{ and } (m_1b_1 - m_2b_2) > 0 \quad (A.14)\]

And, we can see that this will be the case if \( m_1 > b_2 \), and \( b_1 > m_2 \), given that \( m_1, b_1 > 0 \), \( i = 1, 2 \). (However, notice that this is just a sufficient condition, but not the necessary condition, and recall that we made this particular assumption for its economic meaning).

From our eqs. (37) and (38), (and thus (60) and (61) of Chapter V), we have \( m_1 = 2k > b_2 = k \), and \( b_1 = k > m_2 = 0 \). Therefore, the system is dynamically stable under this specific model.

On the other hand, the condition for monotonic solution is that the characteristic roots are real numbers with no imaginary part. With \(-k\) and \(-2k\) as the roots, the adjustment is therefore monotonic for our case. Or, alternatively, if \((a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{21}a_{12}) > 0\) for the system postulated in eq. (A.11), the solution is monotonic. By investigating our model, again, this condition is fulfilled.

In the following, we derive the time path for the unemployment case:

From eqs. (79) and (80), expressed in matrix notation:

\[
\begin{bmatrix}
2k+D \\
-k
\end{bmatrix}
\begin{bmatrix}
\Phi
\end{bmatrix} = \begin{bmatrix}
2k(h_1\bar{y} + \phi_1\bar{r}) \\
k((h_2-h_1)\bar{y} + (\phi_2-\phi_1)\bar{r})
\end{bmatrix}
\]

(A.15)

The characteristic equation is again, \((k+D)(2k+D) = 0\), so, the two roots are again, \(-k\) and \(-2k\).
The differential equation for $M^S$ is: 
\[(2k+D)M^S = 2k(h_1\bar{y} + \phi_1 \bar{r})\]
i.e.,
\[
dM^S/dt = 2k(h_1\bar{y} + \phi_1 \bar{r}) - 2kM^S \quad (A.16)
\]
By solving this, we get the time path of $M^S$ as:
\[
M^S(t) = -(h_1\bar{y} + \phi_1 \bar{r})e^{-2kt} + (h_1\bar{y} + \phi_1 \bar{r}) \quad (A.17)
\]
This gives eq. (81).

On the other hand, the differential equation for $B^S$ is:
\[(2k+D)(k+D)B^S = 2k^2(h_2\bar{y} + \phi_2 \bar{r}) \quad (A.18)
\]
Let the two roots of the characteristic equation be $-\rho_1$ and $-\rho_2$, then eq. (A.18) becomes:
\[(\rho_1+D)(\rho_2+D)B^S = 2k(h_2\bar{y} + \phi_2 \bar{r}) + \rho_1 \rho_2 (h_2\bar{y} + \phi_2 \bar{r}) \quad (A.19)
\]
By applying the technique of Laplace transforms as described before:
\[
b(r) = \frac{r k((h_2-h_1)\bar{y} + (\phi_2-\phi_1) \bar{r}) + \rho_1 \rho_2 (h_2\bar{y} + \phi_2 \bar{r})}{r(r+\rho_1)(r+\rho_2)} = \frac{F(r)}{\sigma(r)} \quad (A.20)
\]
Written in partial fractions (i.e., $b(r) = \frac{3}{r_1} \frac{F(r_1)}{d(r_1)} \frac{1}{r-r_1}$) substitute $k$ and $2k$ for $\rho_1$ and $\rho_2$, and rearrange:
\[
b(r) = \frac{1}{r}(h_2\bar{y} + \phi_2 \bar{r}) + \frac{1}{r+k}(-(h_1+h_2)\bar{y} - (\phi_1 + \phi_2) \bar{r}) + \frac{1}{r+2k}(h_1\bar{y} + \phi_1 \bar{r}) \quad (A.21)
\]
Therefore, the solution for the time path of the stock of bonds adjustment is:
\[
B^S(t) = (h_2\bar{y} + \phi_2 \bar{r}) - (h_1+h_2)\bar{y} + (\phi_1 + \phi_2) \bar{r})e^{-kt} + (h_1\bar{y} + \phi_1 \bar{r})e^{-2kt} \quad (A.22)
\]
This gives equation (82).
In the following, we derive the characteristic equation and roots for model 2 and 3:

For model 2, the system (equations (88) and (89)) expressed in matrix notation is as follows:

$$
\begin{bmatrix}
  k+v+D & k-v \\
  -v & v+D
\end{bmatrix}
\begin{bmatrix}
  M^S \\
  B^S
\end{bmatrix}
= 
\begin{bmatrix}
  (k+v)\phi_1 + (k-v)\phi_2 \tau \\
  v(\phi_2-\phi_1)\tau
\end{bmatrix}
$$

(A.23)

So, the characteristic equation is:

$$(k+D)(v+D) + v(k+D) = 0, \text{ i.e., } D^2 + (k+2v)D + 2kv = 0$$

(A.24)

Therefore, the two roots are:

$$s_i = -(k+2v) \pm \sqrt{(k+2v)^2 - 8kv}/2, \text{ i.e. } s_1 = -2v, s_2 = -k.$$  

(A.25)

Since both roots are real, we conclude that the time path of adjustment is monotonic under this model.

For model 3, the system (equations (96) and (97)) expressed in matrix notation is:

$$
\begin{bmatrix}
  k+v+D & \delta-U \\
  -v & U+D
\end{bmatrix}
\begin{bmatrix}
  M^S \\
  B^S
\end{bmatrix}
= 
\begin{bmatrix}
  ((k+v)\phi_1 + (\delta-U)\phi_2)\tau \\
  (U\phi_2-v\phi_1)\tau
\end{bmatrix}
$$

(A.26)

So, the characteristic equation is:

$$D^2 + (k+U+v)D + (kU+v\delta) = 0$$

(A.27)

And, the two roots are:

$$s_j = -(k+U+v) \pm \sqrt{(k+U+v)^2 - 4(kU+v\delta)}/2, \text{ where } j=1, 2.$$  

(A.28)

For the roots to be complex (involving imaginary element) such that the adjustment path is oscillatory rather than
monotonic, the condition below has to be satisfied.
\[(k+U+v)^2 < 4(kU+v\delta), \text{ i.e., } (k-U)^2+v^2+2v(k+U) < 4v\delta \text{ (A.29)}\]

And, this could only be possible if \(2\delta > (k+U)\). This condition gives rise to eq. (102). (Note that this is only a necessary but not a sufficient condition).

Suppose eq. (A.29) holds, then the roots can be written as: \(s_j = h^+ci\), where \(h\), the real part of the roots, equals to \(-{(k+U+v)/2}\). Since \(h<0\), the system, although fluctuating, remains dynamically stable. Specifically, the solutions for the time path of changes of money stock and bond stock will have the following general forms:

\[M^s(t) = e^{ht}(pcos.ct+qsin.ct) - A, \text{ and} \]
\[B^s(t) = e^{ht}(pcos.ct+qsin.ct) - B, \text{ where } p \text{ and } q \text{ are two arbitrary constants, and } A \text{ and } B \text{ are the equilibrium values of each asset. When } h<0, \text{ e}^{ht} \text{ decreases as } t \text{ increases, the amplitude of the oscillation declines. (I.e., the deviation from equilibrium becomes smaller and smaller in each successive cycle). In other words, the time path of the system is characterized by a damped fluctuation. This is depicted in our Figure 12. (Here, cosine and sine are the so-called circular functions which have a period } 2\pi \text{ - i.e. the graphs of the functions will repeat their own configuration for every } 2\pi, \text{ and have a periodic fluctuation between +1 and -1. Therefore, it is due to the presence of the circular functions in our solutions, the resulting time paths may exhibit a fluctuating pattern).} \]
For Chapter VI:

In the following, we derive solutions for Chapter VI.

With open market operations, the model (equations (106) and (107)), expressed in matrix notation, becomes as:

\[
\begin{bmatrix}
2(1-\theta)k+D & 0 \\
k(2\theta-1) & k+D
\end{bmatrix}
\begin{bmatrix}
M^s \\
B^s
\end{bmatrix}
= 
\begin{bmatrix}
2(1-\theta)k\phi_1\tau \\
(\phi_2+(2\theta-1)\phi_1)k\tau
\end{bmatrix}
\]  

(A.31)

where \(D\) is the differential operator \(d/dt\).

The characteristic equation is:

\[(k+D)(2(1-\theta)k + D) = 0 \]  

(A.32)

So, the two characteristic roots are \(-k\) and \(-2k(1-\theta)\).

Using Cramer's rule, the differential equation for \(M^s\) is as follows:

\[(2k(1-\theta)+D)\dot{R}_s = 2k(1-\theta)\phi_1\tau, \text{ i.e.,} \]

\[
\frac{dM^s}{dt} = 2k(1-\theta)\phi_1\tau - 2k(1-\theta)M^s
\]  

(A.33)

Solving eq. (A.33), we obtain the time path for money stock:

\[
M^s(t) = (M^s(0)-\phi_1\tau)e^{-2k(1-\theta)t} + \phi_1\tau
\]  

(A.34)

This gives eq. (108).

On the other hand, the differential equation for \(B^s\) is:

\[(2k(1-\theta)+D)(k+D)\dot{B}_s = 2(1-\theta)k^2\phi_2\tau + (\phi_2+(2\theta-1)\phi_1)Dk\tau \]  

(A.35)

Let the two characteristic roots be \(-\rho_1\) and \(-\rho_2\), then eq. (A.35) becomes:

\[(\rho_1+D)(\rho_2+D)\dot{B}_s = \rho_1\rho_2\phi_2\tau + (\phi_2+(2\theta-1)\phi_1)Dk\tau \]  

(A.36)

To solve equation (A.36), again, we apply the Laplace transforms, by assuming the initial conditions \(B^s=DB^s=D^2B^s=0\).
Then, the solution of $B^s$ with an imposition of a tariff of rate $\tau$ has Laplace transform $b(r) = \int_0^\infty e^{-rt}B^s(t)dt$ given by:

$$b(r) = \left(\rho_1\rho_2\phi_2 + (\phi_2 + (2\theta - 1)\phi_1)kr\tau\right)/r(r+\rho_1)(r+\rho_2)$$  \hspace{1cm} (A.37)

Write eq. (A.37) in form of partial fraction, we can get our solution from the inverse Laplace transform:

$$B^s(t) = \left(\phi_2 + \frac{\rho_2\phi_2 - (\phi_2 + (2\theta - 1)\phi_1)k}{\rho_1 - \rho_2} \cdot e^{-\rho_1 t} \right) + \frac{\rho_1\phi_2 - (\phi_2 + (2\theta - 1)\phi_1)k}{\rho_2 - \rho_1} \cdot e^{-\rho_2 t} \tau$$  \hspace{1cm} (A.38)

This gives equation (109).


FOOTNOTES

1. Actually, the basic idea of the monetary approach in terms of its emphasizing the role of monetary factor in balance-of-payments adjustment could be attributed to Hume's "price-specie-flow mechanism" which prevailed under the gold standard. The credit for its recent revival, however, must go to Mundell, R.A. (1968), Johnson, H.G. (1951 and 1971) and their students at the University of Chicago.


3. This statement can be found in Corden, W.M., The Theory of Protection, Chapter 4, p. 72.


5. As has been pointed out by Leamer, E.E. and Stern, R.M. (1972), the static concept of wealth is the major flaw associated with the conventional portfolio approach.


7. This terminology is seen from Branson (1970) and is designed mainly to replace the traditional view (like that in Mundell (1963)) in which the capital flows arising from one-shot stock adjustment is viewed as permanent.

8. Mussa mentions these points (1974), and develops the idea in a semi-portfolio model in which the non-monetary asset is internationally non-traded. The major limitations of his work are discussed in Chapter II.

9. Instances of home assets or nontraded assets such as consumer loans, mortgages, equities of local interest, and particular government bonds with limited marketability, are cited by Boyer (1975).
10. Strictly speaking, the interest rates established on these assets will be the same inside and outside the country only when these assets, whether they are denominated in foreign or domestic currency, are perfect substitutes from the point of view of wealth holders. This requires, of course, perfect capital mobility with no exchange risk and significant transaction cost involved.

11. Later, we will make a distinction between the nominal and real rates of interest when there is a non-zero anticipated rate of inflation. Under that situation, although the nominal rate is in equilibrium externally determined in the world asset market due to asset arbitration, a positive expected domestic inflationary rate will lead to a domestic nominal rate higher than the world rate by exactly the differential rate of expected inflation in the short-run.

12. See Chen, A.H. (1978) in which he discusses the effect of inflation on the portfolio policies of an individual by utilizing an explicit utility maximization model. He concludes that the effect of inflation is as important as an interest rate change in altering investor's portfolio choices between risky and risk-free assets.

13. For the distinction between income vs. wealth constraints, see Hellwig (1975) and May (1970).

14. As pointed out by McKinnon (1969), when \( \frac{\partial G/\partial \pi_r}{\partial \pi_r} > |\frac{\partial L/\partial \pi_r}{\partial \pi_r}| \) \( > 0 \), there will be a "spillover" effect onto expenditure on commodities of any portfolio adjustment such that \( \frac{\partial z/\partial \pi_r}{\partial \pi_r} < 0 \). As shown in our model, this is consistent with our specification of an expenditure function in which \( \frac{\partial a/\partial \pi_r}{\partial \pi_r} + \frac{\partial z/\partial \pi_r}{\partial \pi_r} = 0 \), where \( \frac{\partial a/\partial \pi_r}{\partial \pi_r} = \frac{\partial L/\partial \pi_r}{\partial \pi_r} + \frac{\partial G/\partial \pi_r}{\partial \pi_r} \) \( > 0 \), due to the income constraint. Also notice that McKinnon's model is different in this aspect from that of Floyd (1978), in which there will never be such a spillover effect on the goods market. (See Chapter II).

15. Note that it is the real interest rate, (rather than the nominal rate) which is relevant here in the determination of income allocation between durable goods and financial assets. Because we have assumed that the expected rate of inflation is fully taken into account in forming the nominal interest rate, a rise in \( \pi \) causes no change of real interest rate and thus has no net effect on the holding of financial assets. However, this is a somewhat simplified specification in order to avoid the complexity of the need to distinguish between ex-ante
and ex-post real interest rate. More realistically, because the adjustment of the expected inflation and thus the nominal interest rate generally lags behind that of the actual inflation, the ex-post real rate tends to decline during an inflationary period. And so, with a rising $\pi_e$, people generally switch away from financial assets to non-perishable consumer goods.

16. The argument here is based for the most on Cagan (1956) in which he presents empirical evidence to support his hypothesis that a lower real cash balance is desired during periods of the so-called "hyperinflation". I adopt his argument here in my model because when inflation is explosive, it could have the basic feature of a hyperinflation.

17. Note that if we do not make this assumption here, we certainly will have the total expenditure as a function of relative prices among different goods. Then, in order to sign these relative prices, we will have to make some special assumptions which are redundant to our analysis.

18. For further details on this point, see my discussion in Chapter IV, Section 2.

19. If we define $Y$ as the factor income, with a positive tariff, total expenditure will then exceed factor income by the lump-sum tariff rebate paid to consumers. So, $B_T^T$ will be negative in equilibrium. (Recall that $B_T^T = Y - Z + T$, where $T$ is the tariff rebate). Therefore, $Y$ should be defined as disposable income (i.e., the sum of factor income and the tariff rebate) such that $B_T^T$ equals 0 in equilibrium.

20. In our model, capital flows occur mainly for the purpose of portfolio adjustments. Therefore, any disequilibrium in either one of the two asset markets will cause adjusting capital flows for the purpose of restoring portfolio balance equilibrium. Also, note that the specification for the net capital flow here is only justified for a small country which faces a perfectly elastic foreign supply and demand for the asset such that the net export of the asset can be treated as the residual of domestic supply and demand.

21. We abstract from the debt service account by the assumption that the interest yields from net asset holdings are sufficiently small to be ignored or that the government sterilizes interest payments by imposing lump sum taxes (subsidies) equal in magnitude to the net interest
receipts of domestic residents from abroad. Therefore, factor income is equal to the value of domestic production and the change in wealth is equal to the trade balance as there is no debt service account.

22. Here, we assume there is no sterilization policy and the reserve requirement facing the banking system is 100%. Therefore, the monetary expansion multiplier equals 1.

23. Aileen (1973) simply assumes the equality of k in $B_m$ and $B_k$ equations without any explanation, while Connolly and Taylor (1974) use different coefficients, but again, no rationale is stated for their specification.

24. If $a=c$, i.e., if the loss of excess demand for M caused by a rising relative price of M relative to X is completely absorbed by the X sector (as an increase of excess demand for X), eq. (47) reduces to $dP_{MN}/dt=r/2b$, which will equal to 1 if and only if $r=2b$. That is, $dP_{MN}$ will equal $dP_M$ if and only if the part of lost demand for M caused by the rising relative price of M with respect to NT is twice as much as that of demand shifted from NT sector to X sector during the adjustment.

25. The contrast is the case of devaluation and balance-of-payments as discussed by Connolly and Taylor (1974). By assuming a small country with perfect capital mobility, the price of the internationally traded asset in terms of foreign currency is fixed. Thus, a devaluation will cause an equal proportionate rise in the home currency price of the asset by the "law of one price". So, after devaluation, the real value of the internationally traded asset will not be disturbed, while real cash balances decline by the percentage rate of price increase. The same situation is also discussed in Frenkel and Rodriguez (1975).

26. In fact, if the adjustment coefficient of portfolio, k, has a value equal to $\alpha$, (i.e. instantaneous portfolio adjustment), the path will move from point B immediately toward point A without any deviation. In general, the smaller the k is, the slower the adjustment would be, and so, the more diviation there could be. It is more realistic to have a k smaller than $\alpha$ because of the income constraint which suggests that each individual can only partially adjust his portfolio to a certain extent within a certain time period, and also because the existence of transaction cost associated with portfolio adjustments.
27. Point B is the so-called short-run equilibrium at which portfolio composition is at its desired level; while point A is the long-run equilibrium point at which not only composition but also scale of wealth is at its desired level.

28. The discussion here focuses solely on the interim dynamics of the adjustment process, rather than the long-run situation. In the long-run, the BOP surplus will dwindle over time and become nil in the steady state. For this kind of analysis, see Mussa's model surveyed in Chapter II.

29. A similar adjustment process could be seen in Connolly and Taylor (1974). But their capital account moves all the way back to equilibrium finally because of the requirement for the bond stock to be restored to its initial level in order to maintain portfolio mix equilibrium with respect to money. This is similar to our case (A) in which the initial portfolio ratio remains optimal after the imposition of the tariff.

30. Notice that when capital is immobile, case (B) corresponds to Mussa's analysis (1974).

31. When there is unemployment in the labor market, the level of employment is determined by the demand for labor. This is in contrast with the situation discussed in Brito and Richardson (1975) in which the employment level is determined by the suppliers of labor. This happens because there is an initial excess demand for labor in the market such that the supplier of labor, rather than the buyer, is the one who determines the level of employment. Therefore, with a devaluation and the subsequent drop in the real wage rate, the supplier-determined level of employment declines even further in their model.

32. The nominal wage rate is rigid either because a widespread "wage illusion" exists among workers so that they do not recognize the declining real value of their income in face of rising general price level, or it arises from the constraint of institutional factors, such as long-term contracts and/or lack of collective bargaining power.

33. This assumption is made mainly for simplicity. If we make an alternative assumption, i.e., if these coefficients are assumed to be variables rather than constants, the mathematical task would become much more complicated. More precisely, we will have these coefficients as determinants of our resulting dynamics.
34. In fact, the point of inflection in the time path of change of bonds can be found by setting $d^2B_\delta/dt^2=0$. By doing so, we find that the resulting $t$, the specific point in time at which the path turns from convex to concave, will be smaller as $\phi_2$ is larger.

35. For example, suppose we start with a position such as point A, at which there exists excess money stock as well as bond stock. Subsequently, due to the properties of $3(dM^s/dt)/3M^s<0$, and $3(db^s/dt)/3B^s<0$, there must follow a process of decumulation of both assets until the equilibrium position, point E, is restored. This adjustment process is reflected in Figure 9 as a southwestern movement from point A towards point E as indicated by the arrow.

36. That is, because people have a higher demand for money at the margin under case (B), i.e. a larger value of $L$ in eq. (52), the value of $\phi_1$ will also be correspondingly higher. (Recall that $\phi_1$ = LaC from eq. (52) and (55). Therefore, the resulting steady state value of money stock, $M_s^\#$, which equals to $\phi_1\tau$, will also be greater (from eq. (67)).

37. Notice that this constitutes only the sufficient but not the necessary condition for stability. However, we made this assumption for its economic rationale, rather than to satisfy the stability condition.

38. It is noteworthy that if capital is perfectly mobile, the domestic interest rate will be fixed for our small country. Therefore, any monetary policy which aims to achieve any internal goal by changing the interest rate will be rendered impotent, and thus the practice of sterilization can not sustain long. However, here we assume that the stability of the money supply itself rather than any other goal is the very concern of the monetary authority. For example, the Federal Reserve recently announced to shift the focus of monetary policy from interest rate to the amount of money supply (October, 1979).
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