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ESTIMATION STRATEGY USES IN LENGTH AND AREA MEASUREMENT TASKS BY FIFTH AND SEVENTH GRADE STUDENTS

The Ohio State University

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ESTIMATION STRATEGY USES IN LENGTH AND AREA
MEASUREMENT TASKS BY FIFTH AND SEVENTH GRADE STUDENTS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By
David John Hildreth, B.S., M.S.

* * * * *

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1.1 Introduction

The NCTM's Agenda For Action (NCTM, 1980) makes a strong brief for increasing the curricular emphasis on estimation. The rationale was partly in recognition of evidence of poor performance and partly in recognition of needs of learners. However, examination of research literature, available instructional materials, and methods literature reveals little guidance for curricular or instructional design other than simply providing estimation experience. The purpose of this study was to identify and describe estimation processes that are used by individuals and to make a preliminary exploration of instruction focused by these processes.

In an agrarian society measurement is an important element in the education of its citizens. During the nineteenth century, the school mathematics curricula in the United States reflected this need (Burton, 1980). Estimation of measurements was also deemed necessary. In 1893, the Committee of Ten recommended:

The child should learn to estimate by the eye and to measure with some degree of accuracy the length of lines, the magnitude of angles, and the areas of simple figures; and to make accurate plans and maps from his own actual measurements and estimates (Henderson, 1973, p. 16).
As society became more industrialized, precision and accuracy became a very important topic in measurement learning. The Twelfth Yearbook of the National Council of Teachers of Mathematics, Approximate Computation, dealt with this topic (Bakst, 1937). The Twenty-fourth Yearbook, The Growth of Mathematical Ideas, Grades K-12, contained a chapter entitled "Measurement and approximation" (Payne and Seber, 1959). Measurement and approximation was one of seven concepts that the authors of the yearbook thought to be among "the most basic mathematical themes which should be central to the entirety of a modern mathematics curriculum and...the similarly key concepts of modern teaching techniques" (p. vi).

Recently, there has been a call for more emphasis upon measurement estimation. It has been proposed that the curriculum for learning the SI measurement system include estimation of measurement as a method for meaningful learning. The National Council of Teachers of Mathematics Metric Implementation Committee suggested that teachers "Develop meaning and feeling for units through experiences centering around estimating, and checking of the estimates" (Henry, Choate, & Firl, 1974, p. 75). The metric system is listed under the NACOME's "New Curricular Emphasis" (National Advisory Committee on Mathematical Education, 1975). This advisory committee apparently saw new hope for measurement learning:
The move to metric might well give a fresh boost to the teaching about measurement, particularly emphasizing the importance of physical experience, estimation and laboratory style investigations (p. 44).

Both computational and measurement estimation have been listed as basic skills by the National Council of Supervisors of Mathematics (1978). The PRISM project survey (PRISM Project Staff, 1980) found that teachers, supervisors, mathematicians, and mathematics educators all placed relatively high importance on both types of estimation. Measurement estimation skills are perceived as important because of many practical problems that do not permit the use of measurement tools or because estimates provide a check on the results of measurement. The Agenda For Action reiterated the idea that estimation is a basic skill and recommended increased measurement estimation activities.

Teachers should incorporate estimation activities into all areas of the program on a regular and sustaining basis, in particular encouraging the use of estimation skills to pose and select alternatives and to assess what a reasonable answer may be (NCTM, 1980, p. 7).

More may be at stake than practical skills. Estimation has been recommended as an activity that will promote measurement learning. According to Bright (1976), estimation practice helps students develop a mental frame of reference for the sizes of units of measure relative to each other and to the real world and it provides students with activities that illustrate basic properties of measurement. Bright
suggested that students should guess the measurement of objects before measuring and that they should keep a record of their guesses.

The processes used by students while estimating the measurement of objects is of interest. Appropriate estimation strategies are related to basic properties of measurement. For example, to estimate the length of a line in inches, the length could be subdivided mentally into four equal segments (chunks) and then the length of one of the chunks could be estimated. The estimator that uses this chunking strategy exhibits an understanding of the additivity, congruency and Archimedean properties of the length measure function. The mathematical properties of measure functions and estimation strategies are discussed in more detail later.

Studying the estimation strategies that students use could reveal how measurement is understood. Teaching estimation strategies could add to the students' understanding of measurement concepts as well as add to their estimation ability. Estimation strategies appear a productive direction for research. No research has been found that explored the use of estimation strategies or the teaching of estimation strategies. If estimation is as important as NCSM, NCTM Agenda For Action, and Bright suggest, then it is important to provide a better research base for the design of
curricula and instruction.

1.2 Statement of the Problem

The purpose of this study was to investigate the use of estimation strategies and estimation ability. The particular questions of interest were:

1. What are the strategies that children and adults use when estimating length and area measurements?
2. What is the relationship between strategy use and estimation ability?
3. Is mathematical ability or perceptual ability related to strategy use or estimation ability?
4. Does strategy use or estimation ability vary with grade level or sex?
5. Is there a difference in strategy use or in estimation ability between children who have been taught to use strategies and those who have been taught to use a guess-and-check method?
6. Do either of the teaching methods (emphasis on strategy use or the guess-and-check method) have an effect on estimation ability or strategy use?

A fifth-grade class, a seventh-grade class, and a class of college students were used as subjects in this study. The fifth- and seventh-graders were taught to estimate length and area SI system measurements using two different teaching
methods. Half of the students in each class were taught to use a guess-and-check method while the other half were taught to use estimation strategies. The college students and half of each of the treatment groups were individually pretested on estimation ability and strategy use. Mathematical and perceptual ability scores were obtained for all students. All of the fifth- and seventh-graders were individually post-tested on estimation ability and strategy use. The data were analyzed using appropriate statistical procedures.

This problem was significant for a number of different reasons. First, discovering the estimation strategies used by adults and children can reveal misconceptions about length and area measurement. It is important that teachers recognize these misconceptions in their students. Second, if the estimation strategies that children use reveal errors, teachers could use this technique to uncover misconceptions about measurement. Third, analysis of the area estimation strategies could shed light on the conflict between the information integration theory (Anderson & Cuneo, 1978) and the centration theory of Piaget as a means of explaining children's errors in area measurement tasks. Fourth, if estimation strategies can be taught and their use is related to estimation ability or measurement concept learning, then this would have important implications for the school mathematics curriculum. Finally, if the guess-and-check method
suggested by Bright has an effect upon estimation ability or measurement concept learning, then this would provide needed research to support a suggested practice.

1.3 Definitions

For the purpose of this study the following definitions were used.

ESTIMATION: The process of comparing an attribute of an object to some unit which is selected to quantify that attribute. In estimation problems the unit is not usually contiguous to the target object. The object or the unit may be out of sight; or if the unit is contiguous to the object, the iteration of the unit is not readily apparent as it would be if a measuring device were being used.

ESTIMATION ABILITY: The ability to use the estimation process. For the purpose of the study, estimation ability was operationally defined to be the score that is obtained on the 24-item, length and area, Estimation Ability Test or the alternate form.

APPROPRIATE ESTIMATION STRATEGY: A systematic approach to an estimation problem. For this study, it was expected that the estimation strategies would include those listed in chapter 4. Wild guessing is not an appropriate strategy.

MEASURING: The process of comparing an attribute of an object to some unit which is selected to quantify that
attribute. In measurement, it is possible to move the unit so that it is contiguous to the attribute, as in length measurement; or a tool is used to measure the attribute, as in time or temperature measurement; or the measurement is derived by measuring other attributes and using an algorithm to compute the measurement of the desired attribute, as in velocity.

PERCEPTUAL ABILITY: An individual's ability to select relevant stimuli from an embedding context and to resist the interfering effect of the contextual stimuli. For the purpose of the study, perceptual ability was operationally defined to be the score that is obtained on the Group Embedded-Figures Test.

WAG: Wild Guessing, no systematic approach is used to answer an estimation problem.

Several concepts are important in measurement and in measurement estimation.

ATTRIBUTES: All physical objects have attributes that are perceived, directly or indirectly, by the senses. Such concepts as numerosity, length, area, mass, volume, velocity, and temperature, are attributes.

COMPARISON: Before children learn to measure, they learn to make comparisons of attributes. The ability to compare two objects on some attribute (e.g. which of two sticks is longer) is prerequisite for learning to measure the
attribute. Without this ability, no comparison can be made between units and target objects.

UNIT: The concepts of unit and unit iteration are also important in learning to measure. It is possible that measurement learning is enhanced when children have experience with both standard and non-standard units and when they have experience with both unit iteration (measuring objects that are longer than the unit) and with subunits (measuring objects that are shorter than the unit).

Measuring is an observational process of comparing an attribute of an object to some unit which has been selected to quantify that attribute. A measurement is the result of the quantification of the attribute using the chosen unit. A distinction is made between measurement which is the result of an inexact measuring process and measure which is a mathematical idea independent of the observational process. Function is the unifying idea in the mathematical concept of measure.

Blakers (1967) and Osborne (1975) describe the use of functions in measurement. A measure function, which is determined by the attribute, assigns to the attribute of an object a unique real number. Measure functions have particular properties that children need to understand on an intuitive level. For example, the distance function could be defined to be the map from $L$, the set of all line segments,
to the set of all nonnegative real numbers.

\[ d : L \rightarrow R^+ \]

If A and B are the endpoints of a line segment of length \( p \), then \( d(A,B) = p \). This function has the following properties:

i) If A and B are endpoints of any line segment, then
\( d(A,B) = d(B,A) \).

ii) A = B if and only if \( d(A,B) = 0 \).

iii) If B is between A and C on a line, then
\( d(A,B) + d(B,C) = d(A,C) \). (Additivity Property)

iv) If line segment AB is congruent to line segment CD
then \( d(AB) = d(CD) \). (Congruency Property)

v) If B is between A and C on a line, then there
exists a positive integer \( n \) such that
\( n[d(A,B)] > d(A,C) \). (Archimedean Property)

The scientific process of assigning a positive real number using units, subunits, the additivity property, and the Archimedean property has imprecision built into it. When measuring, one can not be sure when to stop dividing units into subunits in order to make a "perfect match" between the iterated units and subunits and the target length. One can only go as far as the measurement tools and the perceptual judgement of equality will allow. A range of numbers (e.g. 1.52 ± .005 cm) is then used to indicate the possible values of the actual measure and the last subunit that
was used for comparison.

Estimation has a structure that parallels measurement. Estimation is the process of comparing an attribute of an object to some unit which is selected to quantify that attribute. The estimate is a number that is a result of applying the appropriate property(ies) of a measure function. Estimation is imprecise for the same reasons that measuring is. The difference between length estimation and length measurement is that in measurement the units are contiguous to the attribute that is being measured. In estimation the units may be present or out of sight and the object may be present or out of sight. If both are in sight, then the problem situation will not allow the estimator to move the unit next to the object or to use the unit in an iterative process. The estimator must use perceptual abilities in order to compare the unit, subunits, and iterations of each with the object. The perceptual problem is that of translating the unit from one place to another and making comparisons mentally.

Summary

In this study the strategies that children and adults use while estimating length and area were investigated. The relationships between estimation ability, strategy use, mathematical ability, perceptual ability, grade level, and sex
were examined. A comparison of two methods of teaching estimation in the SI measurement system was made. In the next chapter some of the research that is related to this problem is discussed. In chapter 3 a more detailed description of the research design and methods is given. The strategies that were found in pilot study work and in the current study are described in chapter 4. The statistical results of testing each of the hypotheses are given in chapter 5. In the last chapter an attempt is made to relate the results found in the current study with previous research; some suggestions for the classroom teacher and for further research are given.
II. RELATED RESEARCH

The purpose of this chapter is two-fold; to review relevant research and to outline a theory about the use of estimation in measurement learning. Emphasis is placed on pre-measurement and measurement; estimation; teaching of measurement and estimation of measurement; and variables which might have an effect on measurement and estimation learning (sex, spatial ability, mathematical ability, age, and strategies used).

2.1 Pre-measurement and Measurement

Pre-measurement could be defined as "establishing empirical procedures for directly comparing, ordering, and combining elements of some domain of elements that possess a given attribute" (Carpenter, 1976, p. 47). Most of the research that has been done on measurement has been on what could be called pre-measurement skills; and much of this research has been done in an effort to confirm, extend, or disconfirm the research of Piaget and his associates.

Measurement is one aspect of the development of the child's concept of space. There are three themes that characterize the Genevan research on space (Smock, 1976). First,
acquisition of concepts of the spatial world is a product of general intellectual development. Second, spatial representations are built up through the process of organization of actions and/or logico-mathematical experience. Third, the critical characterization of spatial concept acquisitions is according to the type of geometric concepts involved: topological, projective, or Euclidean; and the order of acquisition is this order (Piaget & Inhelder, 1967, chap. 15). Measurement is a Euclidean function.

Underlying the Piagentian notion of measurement are the concepts of conservation of size, subdivision, change of position, and a coordinate system (Smock, 1976). The use of a unit of measure requires conservation of size and a coordination between change of position and subdivision. An understanding of transitivity is also a precondition of measurement since subdivision involves transitive relations.

Table 1 summarizes the Piagetian research on conservation and measurement. Children use transitivity intuitively when the comparison has favorable conditions such as when objects are contiguous to each other. Operational transitivity is not restricted to situational factors and depends solely on logical inference. Measurement is operational when area and volumes are measured using linear or linear and two-dimensional units.
Identifying the sequence of development of conservation and transitive operations seems to depend upon how transitivity is defined. Piaget proposed that conservation and transitivity develop simultaneously. Most of the replications of Piaget's studies found that conservation develops before transitivity (Carpenter, 1976). Brainerd (1973) found that transitivity preceded conservation in development.

Table 1

Summary of Piagetian Research on Conservation and Measurement.

<table>
<thead>
<tr>
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<td>II B</td>
<td>Intermediate Responses</td>
<td>Intuitive transitivity</td>
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<tr>
<td>III A</td>
<td>Conservation</td>
<td>Operational transitivity</td>
</tr>
<tr>
<td>III B</td>
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<td>Unit iteration</td>
</tr>
<tr>
<td>IV</td>
<td></td>
<td>Mathematical multiplication</td>
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</table>

(Smock, 1976, p. 13)

Bailey (1971) found that children had more difficulty identifying nontransitive situations than in making transitive judgements. In further study, (Bailey (1974) found a
hierarchical development of recognition of transitivity, substitution, then recognition of incompatible situations. Recognition of incompatible situations has been found to be a critical variable in developing a need for a standard unit of measure in children (Steffe, 1971).

The Piagetian studies (1967) did not include numbers in the assessment of logical reasoning. The importance of the inclusion of number in the assessment has been pointed out by Osborne (1975). In a study by Lamb (1978) it was concluded that "Introduction of 'measurement' (numerical information) aids the logical reasoning of children at second grade level or higher" (p. 556).

There has been a number of studies that have investigated the errors that children make in measurement situations. Piaget, Inhelder, and Szeminska (1960) used the term centration to describe the tendency of children (particularly preoperational children) to center their attention on a limited aspect of the visual stimuli. When children were asked to draw a square that had twice the area of a given square, the preoperational children tended to center their attention on the length of the side. These children doubled the length of both sides of the square. Centering was also found by Carpenter (1975), but the order of cues within the problem seemed to be the more salient variable.
Other researchers (Anderson & Cuneo, 1978; Wilkening, 1979; Cuneo, 1980) found that children did not center their attention on only one cue in area and volume measurement tasks, but used a height + width rule. Anderson and Cuneo (1978) criticized the logical-operational approach of the Piagetians for ignoring perceptual judgement. The information integration theory was suggested as an alternative to the Piagetian theory. Information integration theory places emphasis on the problems of stimulus integration and multiple causation. Multiple causation has two complementary aspects: analysis and synthesis. Analysis aims to dissect the observed response into its causal components; synthesis aims at determining the integration function that combines the causal components into a unitary response. Anderson and Cuneo found that many children up to about 11 years of age used a height plus width integration rule for finding the area of a rectangle. This could be the result of the curricular treatment of measurement in the curriculum.

The National Assessment of Educational Progress (Carpenter, Coburn, Reys, & Wilson, 1978, p. 98) reports that 44% of the 9-year olds chose a 5 by 3 rectangle as having the same area as a 4 by 4 rectangle while only 38% chose the correct response, a 2 by 8 rectangle. This seems to support the Anderson and Cuneo findings. However, there are some other plausible explanations for the results on this N.A.E.P.
question. The children who responded by choosing the 5 by 3 rectangle may be using a one more and one less primitive compensation method or they may be confusing perimeter with area.

Another possible explanation for the results on this N.A.E.P. question is that the 5 by 3 rectangle was the only response choice in which the longer dimension was horizontal (i.e. the responders were centering on the spatial axis). Verge and Bogartz (1978) using the information integration theory found evidence to support this hypothesis. They found that the vertical dimension was not the salient dimension for single dimension centers, but that the spatial axis on which the compared objects were placed determined the displacement of attention. When the spatial axis effect was controlled, the longer of the two dimensions was the salient dimension for centering subjects.

There are many other plausible explanations for the errors that are made in area measurement. Hirstein, Lamb, and Osborne (1978) interviewed 106 children in grades three through six and identified five misconceptions about area; (1) centering on one dimension, (2) using primitive compensation methods, (3) point-counting area, (4) counting around the corner, and (5) point-counting linear units.
Summary

The literature on pre-measurement and measurement is primarily centered on the Genevans. Measurement is understood by children within the more complex notion of space. Measurement has several component elements which are inter-related but not well understood. These components include conservation of size, subdivision, change of position, transitivity, unit iteration, and coordinate system. Children perceive area in a number of ways. Centering of attention and the length plus width rule are two theories that have been posed to explain how children perceive area.

2.2 Estimation

Estimation of measurement is an important area of study for two reasons. First, asking children to respond to estimation type tasks can reveal how they think about measurement. Many of the Piagetian studies actually used estimation of measurement tasks to study the child's conception of geometry (Piaget, Inhelder, & Szeminska, 1960). Their studies have shown that a great deal can be learned about how children think about measurement from observing them perform estimation tasks. In one of the tasks children were asked to build a tower that was the same height as a model tower using blocks of various sizes. Visual comparisons were used to accomplish this task by 4- and 5-year olds (level I); 5-7 year
olds (level II) used manual transfer to bring the towers closer together or they used body comparisons; and children over 8-years of age (level III) used a rod for comparisons. It was reported that, "in some ways visual comparison is more accurate in young children than it is in older children and even in adults" (p. 29).

This last finding was disputed by Pinard and Lavoie (1974). In a cross cultural comparative study it was found that the precision of the estimates was more pronounced for the operational children than for children at the preoperational level. Corle (1960) studied the ability of fifth- and sixth-grade children to estimate weight, length, temperature, and time. He found that the fifth-graders had an average error of 52% on linear measurement estimation and the sixth-graders had an average error of 40%. In a similar study with elementary teachers he found an average error of 34% (Corle, 1963).

The second reason for studying estimation of measurement is that children can practice measurement skills while doing estimation. O'Daffer (1979, p. 47) claims that, when asked to estimate a distance or length, for example, a child often goes through the measurement process mentally and thinks repeatedly about the size of the unit used. Thus the act of estimation helps develop better understanding of the measurement process and familiarity with the size of the basic unit.
It is for this reason that Bright (1976) suggested a number of activities to coincide with different types of estimation. Bright uses the scheme of Figure 1 in categorizing eight basic types of estimation. The four types of estimation in class A can be used to illustrate and emphasize the mathematical properties of a measure function. Those in class B can be used to illustrate the inverse relationships.

There is empirical data to support Bright's suggestion. Love (1977) compared two methods of teaching the metric measurement system to middle school students. One of the methods was to teach within the metric system and the alternative was to compare the metric units of measure with the American units of measure. The posttest achievement scores had two components. Cognitive metric measurement estimation was defined as responses based on remembering and imagery. Visual perceptual metric measurement estimation was defined as responses based on the way things look and the impressions these objects make upon the senses. The results indicated that the group that compared the metric units with the American units had significantly higher achievement on cognitive metric estimation achievement while no significant difference was found on visual perceptual metric estimation.

Love's result is not so surprising if it is considered within the context of Ausubel's learning theory. "The most important single factor influencing learning is what the
Fig. 1. Types of Estimation (from Bright, 1976, p. 90)
learner already knows (Ausubel, 1978, p. iv)." The finding that there was no significant difference on visual perceptual estimation seems to support Bright's types of estimation.

Moyer and Dumais (1978) used the terms "perceptual judgments" and "memory judgments" in their study of these two types of estimation. Moyer and Dumais proposed two models to explain the difference that was found between subjects who because of the experiment used memory to make size judgments and subjects who used perception. In the perceptual group the test objects were out of sight but the subjects were asked to project an image of the object onto a screen. One kind of model maintains that memory judgments are more compressive than perceptual judgments because people adopt a more conservative strategy when making memory judgments; this restricts the range of magnitude estimates. The other model maintains that perceptual and memorial judgments differ because the objects judged in memory are not the referents, or actual objects, but instead are the stored representations of the perceptual transformation. Consequently, the remembered size is simply a function of the perceptual transform applied twice to the original object. The second of these two models is supported by the research of Kerst and Howard (1978).
Summary

Estimation of measurement is important because it reveals how children think about measurement and because estimation practice can help children learn measure concepts. It was reported that fifth- and sixth-graders had average errors of 52% and 40% respectively on linear measurement estimation tasks. Bright's classification of types of estimation was discussed. Love's results seem to support Bright's classification, but is not entirely supportive of the "think metric" movement. The research of Moyer and Dumais supports Bright's distinctions. They proposed a model to explain why estimations of the size of objects which are out of sight tend to be more conservative. The model indicates a variable that needs to be controlled in research and evaluations concerning estimations.

2.3 Teaching Measurement and Estimation

In this section several articles that reveal the "state of the art" in teaching measurement and estimation will be discussed first. Secondly, the empirical research on the teaching of measurement and the teaching of estimation will be reviewed followed by a discussion of the "think metric" literature.
2.31 State of the Art

The data from the National Assessment of Educational Progress can be used to gauge how the schools are doing in the teaching of measurement. A comparison of the results from the 1972-73 assessment with the results from the 1977-78 assessment shows some gains in knowledge of common metric units but that a gap still exists between familiarity of metric units and common customary units (Carpenter et al. 1980a; 1980b). The 1977-78 results reveal that 9- and 13-year-olds are generally successful in making simple linear measurements, but they have difficulty with measuring the distance around a triangle, figuring out the correct length of a line that is not aligned with the ruler, perimeter, and area. Table 2 lists these results along with the results on area and estimation questions for 9-, 13- and 17-year-olds.

There are several possible reasons for the low level of achievement indicated by the N.A.E.P. results: 1) teachers may not be or feel competent in measurement or estimation of measurement; 2) the textbooks may not adequately cover measurement and estimation; 3) measurement equipment may not be available for classroom use or if available it is not used. There is some data to support each of these three conjectures.

First, Rowsey and Henry (1978) found that a group of science and mathematics teacher education majors lacked the
<table>
<thead>
<tr>
<th>Problem</th>
<th>Age 9</th>
<th>Age 13</th>
<th>Age 17</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple linear measurement</td>
<td>81</td>
<td>91</td>
<td>*</td>
</tr>
<tr>
<td>Distance around a triangle</td>
<td>38</td>
<td>65</td>
<td>70</td>
</tr>
<tr>
<td>Line segment length with misaligned ruler</td>
<td>19</td>
<td>59</td>
<td>*</td>
</tr>
<tr>
<td>Length of a pencil to the nearest quarter of an inch</td>
<td>*</td>
<td>53</td>
<td>81</td>
</tr>
<tr>
<td>&quot;Distance around&quot; rectangle</td>
<td>40</td>
<td>69</td>
<td>*</td>
</tr>
<tr>
<td>&quot;Perimeter&quot; of rectangle</td>
<td>8</td>
<td>49</td>
<td>*</td>
</tr>
<tr>
<td>Area of a rectangle partitioned into square units</td>
<td>28</td>
<td>71</td>
<td>*</td>
</tr>
<tr>
<td>Area of a rectangle</td>
<td>4</td>
<td>51</td>
<td>74</td>
</tr>
<tr>
<td>Area of a square</td>
<td>*</td>
<td>12</td>
<td>42</td>
</tr>
<tr>
<td>Area of a parallelogram</td>
<td>*</td>
<td>*</td>
<td>19</td>
</tr>
<tr>
<td>Area of a right triangle</td>
<td>*</td>
<td>4</td>
<td>18</td>
</tr>
<tr>
<td>Reasonable estimates of length and weight in metric units</td>
<td>20</td>
<td>37</td>
<td>50</td>
</tr>
</tbody>
</table>

*Missing entries indicate the question was not given to these age groups.
skill to estimate metric measurements. This conclusion was based upon the results of giving a 20-item estimation test for which an acceptable range of responses was predetermined. Based upon these acceptable responses, the 32 subjects had a mean score of only 8.7. The research concluded:

It appears that all the Ss (science and mathematics) have not been able to transfer their somewhat limited knowledge about relationships of metric units of measure to the description of everyday objects in terms of these units. Because of Ss' poor performance in applying metric units of measure, science and mathematics educators should provide instruction in applying metric units of measure (p. 89).

The effect of practice on estimating linear measurements by experienced teachers of grades 6-8 was studied by Bright (1979). It was found that the skills of experienced teachers in estimating lengths seemed to improve with practice.

The NCTM In-Service Surveys (Osborne, Bowling, 1977, p. 44) indicated that 92% of the elementary teachers and 86% of the secondary teachers felt that metrication ought to be a topic for in-service education in their schools. Fortunately, 45% of the elementary teachers and 39% of the secondary teachers indicated that metrication was a topic for in-service education in their schools. This was the highest reported topic for both groups of teachers.

Second, the PRIMES analysis of textbook content (PRIMES, 1979) found that approximately 1.7% of the 180
lessons a year is on computational or measurement estimation for grades 1-7.

Finally, in a survey of elementary mathematics teachers (Fey, 1979) it was found that 40% of the teachers of grades K-3 reported that metric tools were either not needed or needed but not available. Another 23% of these teachers reported using metric tools less than ten days a year. Of the teachers of grades 4-6, 36% reported that metric tools were either not needed or needed but not available and another 20% used metric tools less than ten days a year. On the other hand, 48% of the teachers of grades K-3 used nonmetric measurement tools more than ten days a year and 55% of the teachers of grades 4-6 used nonmetric tools more than ten days a year.

2.32 Empirical Research

A few studies have been done on the teaching of measurement and the teaching of estimation which are pertinent to this study. Classes of students who used the School Mathematics Study Group materials were compared with classes that used traditional materials on measurement and estimation (Friebel, 1967). A block of 185 seventh-grade students were randomly assigned to experimental and control groups. It was found that the SMSG classes scored significantly better on a group inventory of knowledge of
approximate nature of measurement, selection of units of measure, vocabulary, accuracy, precision of measurement, conversion of units, and computation process. An estimation test was administered individually and the methods that the students used were categorized. The two groups were equally adept in the process of estimation except on area and volume. The "modern mathematics" students showed more knowledge of processes on area and volume estimation.

The use of concrete materials could have an effect upon learning to measure. Johnson (1970) used three levels of concreteness in teaching a unit on perimeter, area, and volume to students in grades four, five, and six. The posttest results on perimeter and area significantly favored the highest level of concreteness. However, no treatment effect was found for the volume questions.

The formalization of the generalization of formulas for area measure was studied by Urbach (1972). In four 5th-grade classes area formulas were introduced (verbalization group) and in another four 5th-grade classes the area formulas were not introduced (non-verbalization group). The non-verbalization treatment was more effective for the high ability students on the posttest than was the verbalization treatment. This method was also more effective for low ability students on the retention test.
Taloumis (1973) studied the relationship between the scores on area conservation tasks and area measurement tasks, as affected by two sequences of presentation of the tasks in grades one through three. It was found that conservation scores were higher if the measurement tasks were given first. This result raises the question whether training in area measurement concepts would enhance area conservation.

In an aptitude-treatment interaction study by Montgomery (1973), the treatments differed in their emphasis on the unit of area, and the aptitude was two levels of the child's ability to learn mathematical concepts about a unit-of-length measure. In one of the treatment groups the children measured regions using congruent units and compared regions covered with the same units. In the other group, the children measured regions using noncongruent units and compared regions covered with different units. No aptitude-treatment interaction was found. However, the noncongruent unit treatment group scored significantly higher on achievement than the congruent unit treatment group for both levels of aptitude.

Ibe (1971) investigated the effects of teaching a unit on angular measurement using a guess-then-measure approach. The subjects estimated the measure of angles before measuring them. A comparison was made with a no-estimation group in terms of the effects on simple recall, transfer,
retention, achievement and estimating ability. No significant difference was found on simple recall and retention scores. Significance was found in favor of the estimation treatment group on achievement and estimation ability. Ibe also found that estimation ability correlated with general intellectual ability, number facility, and flexibility of perceptual closure. No difference was found between the groups on propensity for guessing.

Pattison (1972) did a non-statistical study of teaching concepts of estimation and measurement to first-graders. She concluded that the children acquired skills in measurement and in estimation and demonstrated growth in the ability to hypothesize reasonably about discrepant measures.

2.33 Think Metric Literature

Many articles and books have been published advocating a think metric approach to the teaching of the metric system (Henry et al., 1974; Shumway et al., 1974; Leffin, 1975; Potteringer, 1975; Kurtz, 1978; Zalewski, 1978). All of these authors advocate using estimation as a means to "think metric".

The research of Love (1977) suggested that children and adults who have had prior experience with the English system of measure be taught the metric system by comparing the metric units with the English system. For example, many adults
have a good conception of meter because they have remembered that a meter is a little longer than a yard. McFee (1967) found no significant difference between classes of seventh-graders that were taught the metric system by comparing with the English system and classes that were taught without comparing with the English system. Those who suggest throwing away all yard sticks may be wrong. The customarily used units are still predominantly the English system units. Most children have a foot ruler or yard stick at home. The schools can ignore the problems of operating in a society with two systems of measure only when the United States has completely adopted the SI system.

2.34 Summary

The N.A.E.P. data reveals that the school children do not have a good understanding of measurement concepts. All age groups seem to have difficulty with the concepts of perimeter and area. Although most children could not "reasonably" estimate length and weight in metric units, there has been improvement in familiarity with common metric units from the 1972-73 assessment to the 1977-78 assessment. Evidence has been cited to show that teachers may not be or feel competent in teaching measurement. Metric measurement tools are used extensively by some teachers but most teachers in grades K-6 use metric measurement tools less than
ten days a year. The textbooks contain very few lessons involving estimation of measurement. It is most likely that the lessons on measurement that are in the textbooks are either ignored or used as extra materials by teachers.

There is empirical evidence to indicate that: using concrete materials has an effect upon measurement learning; area formulas should not be given to students who are learning about area measurement; a variety of units should be used in teaching about area measurement; using the guess-then-measure teaching strategy can have an effect on measurement achievement; and practice with area measurement tasks can have an effect on area conservation.

2.4 Variables Which May Affect Estimation Ability

In this section the literature on spatial ability, age, mathematical ability, sex, and strategy use will be reviewed. Of special interest is the possible effects these variables may have on estimation and measurement ability.

2.41 Perceptual Ability

There does not seem to be a formally accepted definition of perception in the literature. Silverman (1971, p. 276) uses a very broad definition: "Perception is an individual's awareness of and reaction to the stimuli that reach him." Gagne's information processing model is useful for
focussing on the aspects of perception that are most pertinent to measurement and estimation.

Gagne (1977) reports that visual information is registered in the sensory register in a more or less complete form for a few hundredths of a second. The details of this complex representation which persist for a longer period must be the object of the process of attention. This selective perception process depends upon the individual's ability to attend to certain features while ignoring others and is one of the functions of the executive control. Attention is also influenced by the expectancies of the individual. According to Gagne, the short term memory can be used for the storage of visual imagery.

The organization of the environment affects the measurement and estimation response in several ways. Color contrast, illumination of the stimulus, and stimulus intensity directly affect the receptors. The literature on these three factors and on illusions will not be reviewed here.

Gogel and Graham suggested factors related to the stimulus context which are more complex in their effects. Gogel (1978, p. 126) posits the adjacency principle: "The weight the visual system gives to a relative cue is inversely related to the apparent separation of the test object from the induction object in 3-dimensional space." Relative cues are the factors that change the perception when other objects
are present. The test object is the object whose perceived characteristic is being measured and the induction object is an additional object whose presence modifies the perception.

Graham (1966, p. 50) lists several visual cues that elicit spatial discriminations.

1. Relative size: Discrimination of distances is dependent on the size of the retinal image provided by an object and by past and present experience with objects of the same size.

2. Interposition: Overlapping objects appear to be nearer than an overlapped object.

3. Linear perspective: A constant distance between points subtends a smaller angle at the eye as the points recede from the subject.

4. Stereoscopic vision: When a subject views an object in space, the retinal image in the right eye is different from the image in the left eye.

Form perception and contour discrimination (the ability to discriminate the difference between the boundary of a figure) are important in length and area estimation tasks.

The "motor copy" perceptual learning theory developed through research by the Russians Leontiev and Zaporozhets (Gibson, 1969) is useful in explaining how children learn to perceive the shapes and relative sizes of objects or figures. Manipulation of the object with the hands is the first stage
in motor copying, followed by tracing of the object with the eyes. In the next stage, the process progressively loses the character of an external action toward the object and becomes a mental tracing of the object. In the final stage a copy of the object is formed within the mind. Of course, individuals vary in this copying ability.

In the tower building experiment of Piaget (Piaget, Inhelder and Szeminska, 1960) it was found that children in level I made visual comparisons; in level II the towers were moved closer together or the children used body comparisons; and in level III children used independent objects for comparison. This progression from visual comparison to measurement and the Russians' motor copy theory have been criticized by Daehler (1972). Daehler found that the use of the body or hands was considerably below that predicted by Piaget.

It could be that the use of hands simply helps the child draw closer attention to the task. The use of hands then is symptomatic of the child reacting differentially to the stimuli. Within this context Gibson's (1969) definition of perceptual learning makes sense:

Perceptual learning (is) an increase in the ability of the organism to get information from its environment ... There are potential variables of stimuli which are not differentiated within the mass of impinging stimulation ... As they are differentiated, the resulting perceptions become more specific. ... There is a change in what the organism can respond to ...
not acquisition or substitution of a new response (p. 77).

The Embedded-figures Test (EFT) test the ability of the subject to trace a given figure which is embedded within a more complex figure (Witkin, Oltman, Raskin, & Karp, 1971). A group form of this test (GEFT) has been developed for subjects ten years and older (Oltman, Raskin Witkin, & Karp, 1971). A reliability estimate of .82 is reported for the GEFT (Buros, 1978).

The EFT has been used widely for studying the field dependence-independence cognitive style. Field dependence-independence is currently treated by Witkin as the perceptual aspect of a more pervasive global-analytic cognitive style. There is evidence (Witkin, Moore, Goodenough, & Cox, 1977) that subjects who are field dependent or global may ignore nonsalient cues in problem solving. In contrast, field independent or analytic subjects use a wider variety of cues both salient and nonsalient. There is a possibility that estimation ability is related to this cognitive style.

Eisner (1972) studied the relationship between line length and angle size judgements and Embedded-figures Test performance in adult males. Six black line drawings of two- and three-dimensional figures were presented to twenty subjects who were asked to draw them in the same size, shape, and proportion. The absolute errors in line length and angle measurement were found to correlate (r = .38) with the
time scores on the EFT.

It is concluded that the Group Embedded-figures Test is a valid and reliable measure of an aspect of perception which is important in studying estimation ability.

2.42 Age

The age of the learner may have an effect upon the ability to learn about measurement and estimation. The purpose of this section is to review some of the literature pertaining to age and the sequencing of measurement and estimation topics in the school curriculum.

The research of Piaget et al. (1960) provides some guidelines about the ages at which children are hypothesized to be able to perform measurement tasks. No measurement is accomplished in stages I and IIA (ages: approximately 4-7). Substage IIB (ages: 6-7) is a transitional stage in which conservation, transitivity, and the role of a measuring unit are beginning to appear. In substage IIIA (ages 7-9) conservation of length and area are achieved, transitivity comparison can be made, and measuring units are used; however, the need for using a common unit for comparing is not understood. In substage IIIB (ages: 9-12) measurement of length in one, two or three dimensions and measurement of area by using unit iteration is accomplished. The logical operations of multiplying length measurements to obtain area and
volume do not appear until stage IV (ages: 12-13). Piaget et al. maintain that metric measurement of length and area are achieved simultaneously during stage IIIB. This conclusion was supported by Kosanovich (1971).

The conclusion that length and area measurement are acquired simultaneously was questioned by Beilin and Franklin (1962). They found that length, area, and volume are achieved in that order. They also found that the Piaget testing situation acts to facilitate acquisition of the operation. Furthermore, third graders (many in stage IIIA) were influenced by measurement instruction while first graders (many in stage IIA) did not achieve operational area measurement even with instruction.

Further support for the developmental stages of Piaget was given by Kamps' (1970) research. Second grade children (mean age: 8.2 years) were given Piagetian length conservation and measurement tasks. Only 15 percent of the subjects demonstrated unit iteration while 95 percent demonstrated transitivity achievement.

Bargmann (1973) studied pupil achievement in learning the metric system in grades three through six. Some recommendations for the placement of metric system topics were given. Bargmann found no differences by grade level in achievement of the following: (a) understanding the meaning and approximate sizes of various metric units, (b) use of
metric measurement equipment, (c) understanding the multiples of tens organization of the metric system, and (d) measuring length in the metric system using whole numbers. It was suggested that the above topics be introduced in grade three. These topics plus determination of area and cubic volume may be taught in grade four. Grade five may include all of these plus measuring length, liquid volume, and weight using decimals. In grade six, all of the topics mentioned may be taught.

Age-level achievements in area measurement postulate use by 8-, 10-, and 11-year-olds was studied by Wagman (1975). At least one task was designed to test the stage of attainment of four School Mathematics Study Group postulates (SMSG, 1963, p. 989). The postulates used were:

Area axiom: If R is any given polygonal region, there is a correspondence which associates to each polygonal region in space a unique positive number such that the number assigned to the given polygonal region R is one.

Additivity postulate: Suppose that the polygonal region R is the union of two polygonal regions R₁ and R₂ such that the intersection of R₁ and R₂ is contained in a union of a finite number of segments. Then relative to a given unit area, the area of R is the sum of the areas of R₁ and R₂.

Congruence postulate: If two triangles are congruent, then the respective triangular regions consisting of the triangles and their interiors have the same area relative to any given unit area.

Unit postulate: Given a unit pair for measuring distance, the area of a rectangle relative to a unit square is the product of the measures (relative to the given unit pair) of any two consecutive
sides of the rectangle.

An area conservation task was also given. The findings included:

1. About 10 percent of the 11-year-old group did not conserve area.

2. Twenty percent of the 8-year-olds, 28 percent of the 10-year-olds, and 42 percent of the 11-year-olds were successful with the unit area task. The results on the 8-year-olds were in close agreement with the Beilin and Franklin results. The results for the 11-year-olds is lower than would be expected from the Piagetian studies.

3. On the additivity axiom task with decomposition (subjects were asked to cover polygonal regions with subregions) over 70 percent of the 8-year-olds were successful and about 10 percent of the 11-year-olds were not successful.

4. On the additivity axiom task without decomposition (parallelograms which could be rearranged into rectangles were used) 35 percent of the 11-year-olds were successful. This is in contrast to the expected 75 percent results from the Piagetian results.

5. On the unit postulate task most of the 8-year-olds were in the lowest levels of achievement while over half of the 11-year-olds were using the multiplicative rule for finding area or were in transition to this stage.
6. No difference on the unit postulate task was found between the fifth-grade subjects who received no instruction and the sixth-grade subjects who received intruction related to this task.

7. Approximately a third of the subjects in the study confused area and perimeter.

Wagman suggests that children in transitional stages are most likely to benefit from instruction.

Age is also a factor in perceptual ability (Witkin et al., 1971, p. 5). A marked continuous increase in field independence occurs between 8- and 15-years. After age 15, a plateau is reached and is maintained until somewhere in the late 30's.

2.43 Mathematical Ability

One study was found that used mathematical ability as a variable in a measurement or estimation study. Paull (1971) studied the abilities of eleventh-graders to estimate length, area, and arithmetical computation as well as the ability to solve problems by trial and error. Subjects were not found to be consistent across tasks in their ability to estimate answers to problems. The estimation ability scores were found to be correlated with both the PSAT Mathematical and Verbal scores. The abilities to estimate length and area did not correlate with the ability to do mathematical
computations rapidly. Estimation abilities were not found to be related to the tendency to categorize quantities in broad, medium or narrow band-widths. No significant difference was found between the sexes on estimation ability.

A possible direction for further study using mathematical ability as a variable is suggested by Beilin and Franklin (1962) who used IQ as a variable in their study of the effects of training on area and length measurement. "The notion that children with higher IQs may profit from such training to a greater extent than children with lower IQs is barely suggested by the data (p. 618)."

A number of studies have questioned the relationship between mathematical ability and field-independence (Witkin et al., 1977, p. 46). In the 11 studies which used women as subjects, all found a significant relation: the mean of the correlations in the nine studies that used correlations was .44. In 11 of 16 studies that used men, a significant relation was found between mathematical ability and field-independence; the mean correlation on 13 of these studies was .29.

2.44 Sex

Males and females have been found to differ on visual-spatial ability and mathematical ability (Maccoby and Jacklin, 1974). Both of these differences have been attributed
to sex roles, practice, expectations and problem context. The spatial skills difference is one of the most consistent sex differences found among older children and adults. It is less easy to attribute this difference to sex roles, expectations, or interest. However, the different life experiences of boys and girls may be an underlying factor causing these differences.

Small but consistent differences have been found between males and females on the Embedded-figures Test and other measures of field-independence (Witkin et al., 1971). These differences have been found in the United States, western Europe, Africa, Japan, Hong Kong and Israel. Males tend to be more field-independent than females.

No definite conclusions can be stated at this time, for the research on sexual differences is continuing. However, two directions in research can been seen.

Three cognitive variables (mathematical ability, verbal ability and spatial visualization) and eight affective variables were used in a study of 1320 sixth through eighth grade students by Fennema and Sherman (1978). No sex-related differences were found in verbal ability or spatial visualization. When sex-related differences in mathematics favored the males, sex-related differences in favor of males were also found in six affective variables including: mathematics confidence, stereotyping mathematics as a male
domain, attitude toward success, and usefulness of mathematics.

A number of researchers are studying apparent sex-related differences in brain hemisphere specialization and lateralization (Goleman, 1978). The theory is that right-handed boys develop their right hemispheres and hence their spatial skills more rapidly than girls. However, it could be that boys and girls spend more time practicing the skills that they are expected to have, and therefore develop the associated hemisphere.

2.45 Strategies

The Friebel (1967) study of the SMSG materials, cited earlier, is the only research that has been found that used strategies as a variable in a study of measurement or estimation of measurement. However, strategies or heuristics are frequently mentioned as important variables in studies of problem solving. This raises the question about the possibility of studying the strategies that are used in estimation and measurement problems.

Summary

Estimation is a basic skill, like many others, that cannot be measured in absolute terms. Its importance varies with the particular application. However, the studies by Corle, and Rowsey and Henry seem to indicate a low
achievement. Estimation of areas is more difficult than estimation of lengths. Estimation ability seems to be correlated with mathematical ability (Ibe, 1971; Pauli, 1971), estimation practice (Ibe, 1971), Embedded-figures Test scores (Eisner, 1972), and age (Corle, 1960; Piaget et al., 1960).

The argument could be made that the lack of strategies is due to low achievement in measurement. The data from the 1972 National Assessment of Educational Progress and the preliminary data from 1978 indicate that a significant number of children and adults have difficulty in finding the area of a rectangle. These persons would probably guess or use inappropriate strategies in attempting to solve an area estimation problem. If the lack of available strategies to the estimator can account for a significant amount of the variance in estimation ability, then the question arises as to whether or not strategies can be taught. If strategies can be taught, then the curricular efforts in measurement ought to reflect this. If estimating is a skill which can not be taught, then the current emphasis on estimation may be about right.
III METHODS

3.1 Restatement of the Problem

The purpose of this study was to investigate the use of estimation strategies and estimation ability. The particular questions of interest were:

1. What are the strategies that children and adults use when estimating length and area measurements?
2. What is the relationship between strategy use and estimation ability?
3. Is mathematical ability or perceptual ability related to strategy use or estimation ability?
4. Does strategy use or estimation ability vary with grade level or sex?
5. Is there a difference in strategy use or in estimation ability between children who have been taught to use strategies and those who have been taught to use a guess-and-check method?
6. Do either of the teaching methods (emphasis on strategy use or the guess-and-check method) have an effect on estimation ability or strategy use?

In order to investigate the questions listed above, the following null hypotheses were posed:
1. Estimation ability is not correlated with use of estimation strategies.
2. Estimation ability is not correlated with mathematical ability.
3. Estimation ability is not correlated with perceptual ability.
4. Strategy use is not correlated with perceptual ability.
5. Strategy use is not correlated with mathematical ability.
6. There is no difference between males and females on estimation ability.
7. There is no difference between males and females on the use of strategies.
8. There is no difference on estimation ability between grade levels (grade five, seven, and thirteen).
9. There is no difference in the use of estimation strategies between grade levels (grades five, seven and thirteen).
10. There is no difference on estimation ability between groups of fifth- and seventh-graders who have been taught to use estimation strategies and those who have been taught to use a guess-and-check method for estimating measurement.
11. There is no difference on the use of estimation strategies between groups of fifth- and seventh-graders who have been taught to use estimation strategies and those who have been taught to use a guess-and-check method for estimating measurements.

12. There is no interaction effect on estimation ability by treatment and grade level (grades five and seven).

13. There is no interaction effect on estimation strategy use by treatment and grade level (grade five and seven).

3.2 Research Design

In the first part of the study, 24 college students were given the Group Embedded-figures Test and mathematical ability scores were obtained for each subject. During individual interviews, the subjects were given a 24-item estimation ability test and the strategies used on each item were recorded.

In the second part of the study, two intact public school classes, one fifth-grade and one seventh-grade were used. In order to control for pretest practice effect, if any, half of the students in each class were pretested on estimation ability and estimation strategy use and half were not. All of the students were given a perceptual ability test and mathematical ability test scores were collected for
all subjects. Half of the students in each class were given an estimation treatment that emphasized estimation strategies and the other half were given an estimation treatment that utilized a guess-and-check method. The subjects were post-tested on estimation ability and strategy use. The tests and the treatments were given during the first five months of 1980 (See Appendix A). The data were analyzed to test all 13 hypotheses.

Students were randomly assigned to the two treatment groups. Further, one-half of the subjects from each of the two treatment groups were randomly assigned to the posttest only group (See Table 3).

3.3 Subjects

Three groups of subjects, a fifth-grade class, a seventh-grade class, and a group of college students were used in this study. The fifth- and seventh-grade classes were chosen at random from the four fifth-grade and four seventh-grade classes at Delaware Academy Central School. The school was chosen because it was near the two-year college from which the college student sample was drawn. The Delaware Academy Central School with a total K-12 student population of 1280 in 1979-80 is situated in the village of Delhi in upstate New York. The school district encompasses many farms and includes many professional families because of the college
and because the village is the county seat.

Table 3

<table>
<thead>
<tr>
<th>Grade Level</th>
<th>Fifth</th>
<th>Seventh</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Strategy Treatment</td>
<td>Guess &amp; Check Treatment</td>
</tr>
<tr>
<td>Pretest and Posttest</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Posttest only</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

The fifth- and seventh-grade classes were chosen because the mean ages would be a little less and a little more than 12 years, the expected age for transition to formal operations on length and area measurement. The fifth-grade class that was chosen had 24 students when the pretesting began. A new student was added to the roll before the treatments began. He was randomly assigned to the strategy treatment group and he participated in all of the class activities; however, his scores were not included in the data analyses. The mean age of the 24 fifth-graders was 10.6 years.
at the beginning of the testing with an age range of 10.0-12.0.

The seventh-grade class had 25 students when the pre-testing began. One student whose attendance at school had been very erratic was expected by the classroom teacher to be dropping out of school on his sixteenth birthday. Because of his absence and the possibility that he might drop out of school in the middle of the study, he was randomly assigned as the seventh subject in one of the cells (Table 3). Since the boy did drop out, there were 24 students who participated in all phases of the study. The mean age of these 24 seventh-graders was 12.8 years with a range of 12.1-13.7.

For comparison purposes, a group of 24 college freshmen was chosen. These subjects were all members of a General Psychology class chosen at random from four such classes at the State University of New York Agricultural and Technical College at Delhi.

Delhi College is one of six two-year agricultural and technical colleges in the SUNY system. The enrollment for the fall semester 1979 was 2586 with 434 students enrolled in vocational education certificate programs; 179 in continuing education; 107 in liberal arts; and the remainder in two-year technical degree programs including: business management, building construction, agriculture, and
laboratory technology. The means of the SAT Mathematics and Verbal scores for the 1979 freshmen class were 451 and 399, respectively. In a survey of 1078 graduates in 1978, 19% of those who responded (668) said they were enrolled in further education.

The General Psychology class was chosen because it was thought to be representative of the two-year degree students at Delhi. The majors of the students in the class were representative of the various majors at the college. The mean of the SAT Mathematics and Verbal scores of the 17 students who had taken the test was 427 and 446, respectively. The original class had 24 students. Two of the students dropped the class after the group perceptual ability test was given. These two students were contacted and agreed to continue with the study by taking the individually administered estimation ability test.

3.4 Instruments

In this section the testing materials are described. The two forms of the estimation ability test and the strategy use form were developed by the researcher. The standardized mathematical ability tests and the Group Embedded-figures Test that were used in the study are discussed.
3.41 Estimation Ability Tests

Two 24-item length and area estimation ability tests were developed. Both the pretest form (Appendix B) and the posttest form (Appendix C) were individually administered. The tests consisted of 12 length and 12 area questions. Three length and three area questions involved the use of customary units of measure (inches, feet, square inches and square feet). Three length and three area questions involved the use of SI units of measure (centimeters, meters, square centimeters, and square meters). Six length and six area questions involved the use of non-standard units of measure. The object to be estimated was exhibited for all items. The unit was not exhibited for the items that involved standard units. The non-standard units were exhibited. The items on the posttest form were developed so that that the ratios of unit to target objects were within 10% of the ratios on the pretest form. The items on the two tests have very similar contexts. A complete description of each item is given in the appendices.

The pretest and posttest were both given to 10 college freshmen with a two-week time lapse between the two administrations. Each test was scored using two different methods. In method A, the relative error was first calculated for each item; the total score was the average of the absolute value of these relative error scores. In method B, each
item was counted correct if the absolute relative error was less than 1/3; the total score was the total number of "correct" items. This second method was used in the research of Crawford and Zylstra (1952) and by Rowsey and Henry (1978). The correlations between the pretest and posttest were found to be significant using scoring methods A and B, \( r = 0.65, p < 0.05 \) and \( r = 0.82, p < 0.01 \), respectively. Scoring method A is probably less reliable because of the possible influence that one or two bad over estimates can have on the total score. Scoring method B was used in this study.

The items on the pretest form were piloted with seven children in grades three through six. It was found that the children had difficulty with the term "perimeter"; hence, the term "distance around" was used on item 3 instead of the word "perimeter". It was found that because of the dimensions of some of the objects, some of the children that used inappropriate methods had estimates that were very near the measurements. For example, consider the problem of estimating the area of a rectangle which is 5 units by 4 units. If a subject had used the inappropriate method of counting around the edge and responded by saying that the area was 18 square units, then their relative error on the item would only be -.10. Although it is difficult to anticipate all of the inappropriate responses, the dimensions of the objects and units were adjusted to anticipate some of the obvious
errors.

The estimation ability test was given during individual interviews. The interviews for the college students were conducted in a conference room on the Delhi college campus. The interviews for the fifth- and seventh-graders were conducted in an office in the Delaware Academy High School building. Both the fifth- and seventh-grade classes met in the same building and the office was equally accessible to both groups. A table, two chairs, and an easel were used in each setting. The easel was used for holding some of the test items. The placement of the chairs, table, and easel was the same in each setting.

The college students were tested during two, two-week periods (see Schedule of Testing and Treatments, Appendix A). The fifth- and seventh-grade students were scheduled so that the grade level and treatment groups were spread equally over the testing period. The seventh-grade students were tested during study periods and the fifth-grade students were tested during the times scheduled for social studies, individual work, or reading.

Before each test began, the following dialogue took place between the experimenter and the subject:

E: I am going to ask you some questions about the size of several objects. There are no right or wrong answers to these questions. I will also be asking you to tell me what you were thinking when you got the answers you did. For example, if I asked you how long this desk is in feet,
what would you say?

S: Five feet.

E: What were you thinking when you got five feet?

If the subject responded with a WAG (Wild Guess) statement (e.g. "I just guessed", "It just looks like it", or "I don't know what I was thinking") during this practice problem, the experimenter carefully probed to make sure that a strategy was not used. The experimenter was careful not to verbalize value judgements about guessing. After each test question, the experimenter only probed if it were not clear what method the subject used. Every response was given positive encouragement with a smile, nodding, or one of the following verbal reenforcers: "O.K.", "you are doing just fine", or "we are coming along quite nicely".

3.42 Estimation Strategy Form

The purpose of the estimation strategy form (see Appendix E) was to provide a place to record information about the strategies that were used in answering each of the questions on the estimation ability test. The 17 strategies and inappropriate methods that appear on the estimation strategy form were gathered from the literature (Anderson and Cuneo, 1978; Friebel, 1967; Herstein et al., 1978; Piaget et al., 1960) and from pilot studies. These strategies and inappropriate methods are discussed in chapter 4. The length of the interview to the nearest minute and the subject's name,
grade level, and birthdate were recorded on the estimation strategy form.

During the study, each interview was tape recorded for the purpose of checking the reliability of the experimenter's use of the estimation strategy form. A trained rater listened to 10 randomly selected interviews and filled out an estimation strategy form. The experimenter's and rater's forms were checked for agreement on the 240 items. There was a 90% agreement between the experimenter and the trained independent rater on the use of the estimation strategy form. There was 100% agreement on whether a strategy or an inappropriate method was used. The disagreements occurred over which of the strategies were being used or over which of the inappropriate methods were being used. On 2% of the items it was impossible for the independent rater to make a judgment about which strategy or inappropriate method was being used because of the poor quality of the tape recording or because the strategy used was not communicated orally.

3.43 Estimation-olympics Team Competition Test

The estimation-olympics team competition test (Appendix D) was designed to be given during the final day of the treatments in the regular classroom. One of the events, the javelin throw, provides an example of the procedures used in scoring and test development.
Javelin Throw  You will be given a javelin (plastic straw) at the starting line. You should throw the javelin as far as you can without crossing the starting line. You will then estimate how far it is from the starting line to the nearest point of the javelin in centimeters. Write down this estimate. Write down the number that the official gives you for your measurement.

Measurements were given in feet and hundred parts of a foot so that the measurement on one event was difficult to use for estimating in subsequent events.

The events were trial tested with members of the Delhi College staff. The units on some of the items were adjusted to reflect the actual distances thrown. For example, in the shot put event the contestants were expected to estimate how far they had thrown a balloon. Originally the unit for this event was a half meter. During the trial testing, the furthest that anyone threw a balloon was 2.5 m so the unit for the shot put event was changed to a centimeter.

3.44 Mathematical Ability Tests

For the college students in the study, the scores on the Minimum Competency Test were used as a measure of mathematical ability. The 46-item Minimum Competency Test (Callas, Note 1) was developed by the Mathematics Department of the State University of New York Agricultural and Technical College at Delhi. The test has been used for three years for the purpose of screening students who are weak in basic
mathematical skills and for placement of students in proper courses. The number of students taking the test, means, standard deviations, Kuder-Richardson reliability estimate, and Pearson-product-moment correlation with the Scholastic Aptitude Test mathematics subscore are given for the Minimum Competency Test for 1978 and 1979 (Table 4).

Table 4

<table>
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<th>Statistic</th>
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</thead>
<tbody>
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</tr>
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<td>Mean</td>
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<td>27.8</td>
</tr>
<tr>
<td>Standard Deviation</td>
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</tr>
<tr>
<td>Reliability Coefficient</td>
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<td>.87</td>
</tr>
<tr>
<td>Correlation with SAT(^a)</td>
<td>.70</td>
<td>.79</td>
</tr>
</tbody>
</table>

\(^a\)This correlation is based on the 953 students in 1978 and 908 students in 1979 who had SAT scores reported.

For the fifth- and seventh-grade students, scores on the Stanford Achievement Test were used as measures of mathematical ability. The total mathematics scores from the Intermediate Level I-Form A and Intermediate Level II-Form B were used for the respective grades. The raw scores were
converted to scaled scores using the Norms Booklet (Madden, R., Gardner, E., Rudman, H., Karesen, B., & Merwin, J., 1973). According to Madden et al., scaled scores are directly comparable across grade levels, batteries and forms. The tests were given in June 1979 when the students were in the fourth- and sixth-grades, respectively. Test scores were not available for two students, one in each grade.

3.45 Group Embedded-figures Test

The Group Embedded-figures Test (Oltman et al., 1971) was given to all subjects before the estimation ability pretesting was begun. The test was given during the regular class periods in the classrooms. The classroom teachers assisted in the administering of the GEFT by checking to see that the students understood the instructions during the practice section of the test. The test consists of three sections with nine problems in each section. Only the last two sections are scored, hence the possible range of scores is 0 to 18. Witkin et al. (1971, p. 28) reported the means and standard deviations for college students, males: \( N = 155, \overline{M} = 12.0, \overline{SD} = 4.1 \); females: \( N = 242, \overline{M} = 10.8, \overline{SD} = 4.2 \).
3.5 Treatments

Two different length and area estimation treatments were given in this study. The guess-and-check treatment essentially followed the suggestions of Bright (1976). The students were taught how to use metric measurement tools and they were given a list of objects for which they estimated the length and area, recorded their estimates, measured, and then recorded their measurements. Help was given in measuring and in reading and following the instructions. Strategies were not discussed. The materials used during the first day are in Appendix F.

A primary characteristic of the strategy treatment was that all discussion focused on the use of appropriate estimation strategies. The students in the strategy treatment groups were taught how to use metric measurement tools; length and area estimation strategies were discussed; and the students were given a list of objects for which they estimated the length and area using several different strategies, recorded their estimates, measured the objects, and then recorded their measurements. Help was given in measuring, in following the instructions, and in using estimation strategies. The lists of objects for the guess-and-check groups were a little longer to ensure that the time on tasks for both groups would be the same; however, all of the objects on the strategy group's lists were also on the guess-
and-check group's lists. Appendix F contains the materials that were used on the first day.

The materials for the second, third and fourth days were similar to the materials used during the first day. The topics for the four days were: (1) estimations of the length of objects in centimeters, decimeters, and meters (meter sticks were used for measuring); (2) estimation of the area of rectangles in square centimeters, square decimeters, and square meters (square decimeter grids and meter sticks were used for measuring); (3) estimation of the area of triangles, parallelograms, and pentagons in square centimeters and square decimeters (square decimeter grids and meter sticks were used for measuring); and (4) estimation of the length, area, and perimeter of objects (the units and measuring devices from the first three days were used).

On the fifth and final day of the treatment each group participated in an Estimation-olympic Competition (see Appendix D). In order to improve motivation the children in each of the two original classes were told that they would be split into two equal olympic teams, that some of the team members would be given individual pre-training trials, that two different training methods would be used, that there would be a team competition, and that everyone would participate in the individual competition. The children were told that it was important that they not share the training
methods that they received with the other team. The purpose of the team competition on the last day was threefold. First, it was thought that this would improve motivation. Second, the estimation competition as envisioned would provide an additional setting for the practice of length and area estimation skills. Finally, the results from such a competition could be used for comparing treatment and grade level differences.

Each of the treatment groups within the grade levels were taught by the researcher while the classroom teacher taught other topics to the other half of the class in another room. Since the junior high school class periods were 41 minutes long, it was decided that all treatment periods would be 41 minutes. Hence, each group of 12 students was given one of the two treatments for five, 41 minute sessions.

Summary

The scores on the estimation ability pretest, pretest strategy use, mathematical ability test, and perceptual ability test were subjected to Pearson-product-moment correlations. An analysis of variance was conducted on the pretest data to test for sex and grade level differences. The reliability of the estimation strategy form was checked. The posttest data were subjected to an analysis of variance to check for a pretest practice effect. An analysis of
variance was conducted on the posttest data to assess the effects of the two treatments and possible interaction effects with the grade levels. The strategies that were observed being used are discussed in chapter 4. The estimation ability results and the effects of the use of estimation strategies on estimation ability are discussed in chapter 5.
IV ESTIMATION STRATEGIES

The estimation strategies that are listed on the estimation strategy form have origins in the literature and in pilot study work conducted by the researcher. Since estimation strategies permeate the current study, the results of two pilot studies are discussed in this chapter. Descriptions of strategies and inappropriate methods displayed by learners in estimating lengths and areas are given. Finally, the estimation strategies and inappropriate strategies found in the current study are discussed. The statistical analysis of the relationships between the use of estimation strategies, estimation ability and other variables are discussed in chapter five.

4.1 Pilot Studies

During the spring of 1979, a pilot study was conducted to determine what estimation strategies are employed by college students in making estimations of length and area. Nineteen students were individually given a 16-item length and area estimation test. An interview format was used to determine what, if any, strategies were being utilized by young adults. It was found in the study that the poorest
estimators did not use an explicit strategy on 40% of the responses while the best estimators had no strategy on only 4% of the responses. Five of the students used strategies on the area estimation tasks that revealed that they did not fully understand the concept of area. The pilot study did not systematically control for possible intervening variables such as mathematical or perceptual abilities.

During the summer of 1979, a 12-item estimation test was given to 5 third- and fourth-grade students. The primary question in this study was: Could the strategies that young children use in length and area estimation tasks be found by interviewing them? It was concluded in the study that the estimation strategies of young children could be ascertained by using an interviewing technique. Section 4.2 and 4.3 contain definitions of the length and area estimation strategies that were observed in the two pilot studies. The relationship between these strategies and measurement concepts are also discussed.

4.2 Length Estimation Strategies

USE OF SUBDIVISION CLUES: Subdividing the length using information that is available within the problem. For example, to estimate the length of a hallway in meters one could use the distances between doorways and the doors as subdivisions. These subdivisions have lengths which are closer to
the unit lengths, thus making the iteration process (Archimedean Property) easier to use.

UNIT ITERATION: Successively applying the unit perceptually to the object and determining a count of the unit. It should be noted that the use of unit iteration is perceptually difficult when the ratio of the unit size compared to the object size is small.

PRIOR KNOWLEDGE: Using information that the estimator has about the target object or the unit. For example, the prior knowledge that inside doors are about 80 inches high could be used for estimating ceiling height. In this case, the door becomes an intermediate unit which is contiguous to the target object attribute. The knowledge that ceiling tiles are one square foot could be used to get a very accurate estimate, almost a measurement, of the length of the room in feet.

COMPARISON: Comparing the target object with another object, either in sight or out of sight, about which the estimator has prior knowledge. For example, in a pilot study, a 10-year-old boy compared the length of the room to a twenty-foot crocodile that he had seen. Comparing the target object with objects that are in sight may be a better strategy than comparing with objects that are out of sight, since the attribute and referrent are in closer proximity. However, this depends upon the prior knowledge that the
estimator has about the referrents. Using the comparison strategy could help in the application of the congruency, additivity and Archimedean properties.

CHUNKING: Subdividing a length into equal segments so that when comparing the unit to one of the segments, the use of the Archimedean property involves a smaller n. For example, to estimate the length of a line in inches, the length could be subdivided mentally into four equal segments (chunks) and then the length of one of the segments could be estimated.

SQUEEZING: Making estimations that are a little less and a little more than the object, the subject attempts to narrow in on the measurement. For example, to estimate the length of a 70 cm line, the estimator might first estimate the length to be between a half of a meter and a meter. This strategy involves the use of the Archimedean property to make gross estimates; it should be followed by the use of a subunit comparison in order to obtain a better estimate.

4.3 Area Estimation Strategies

Each of the length estimation strategies were used in area estimation, sometimes with minor modifications to accommodate for the multiplication rule. The use of unit iteration in an area problem may indicate that the estimator is at the concrete operational level on area measurement but
not at the formal operational level of using a length times width rule. The term "repeated addition" was used to distinguish between unit iteration on a length item and unit iteration on an area item. Three other unique area strategies were observed. Before listing these three strategies for area estimation, area measure functions are briefly described. For rectangular regions, the area function is tied directly to the distance function in the usual way, length times width. For some polygons, the polygonal region is decomposed into disjointed triangular regions which are then tied to the distance function through the formula $A = \frac{bh}{2}$. The area function has analogous properties to the congruency, additivity and Archimedean properties of the distance function. Regions other than polygons were not considered in this study.

**LENGTH X WIDTH:** Using length estimation strategies to estimate the dimensions of a polygonal plane figure and then using an algorithm to obtain the area of the region. This strategy becomes more difficult to use as the number of sides of the polygon increases. The algorithms for calculating the area of polygonal regions are forgotten or incorrectly applied even for rectangular regions.

**REARRANGEMENT:** Rearranging part of the target figure or the unit in order to obtain a more easily estimated area. For example, to estimate the area of an isosceles trapezoid,
a triangle at one end could be pictured as being moved to the other end to make a rectangle.

**REPEATED ADDITION:** Using unit iteration for estimating the area of a rectangle. The use of unit iteration for estimating the area of a rectangle is neither an inappropriate method nor the best strategy. Repeated addition is used to describe the estimation method used by subjects who seem to have acquired a concrete concept of area but who are not at the formal operations level of using multiplication to find area.

4.4 Inappropriate Strategies

A number of inappropriate strategies were used particularly for estimating area. The following inappropriate strategies were observed during pilot study work:

**LENGTH PLUS WIDTH:** Finding the area of a rectangle by adding estimates of the length and width. It could be that the use of this method indicates that the estimator recognizes the need for integrating the two dimensions but does not understand how. It is also possible that the estimator uses addition rather than multiplication because it is easier.

**COUNT AROUND:** Estimating the number of units it would take to go around the outside of a polygon and reporting this number as the area. Use of this method indicates that
there is confusion about the concepts of area and perimeter.

UNIT TURNED: Estimating the area of a rectangle using a nonstandard rectangular unit by estimating the length of the rectangle with the unit turned in one direction and the width of the rectangle with the unit turned 90 degrees (see Figure 2). A length times width rule is usually applied to the two estimates to get an answer.

Actual Area = 12 units

Turned unit area estimate = 8 units

Figure 2. An example of the unit being turned in an area estimation problem.
CENTERING: Reporting the area of an object as being the estimated length of one of its dimensions. This inappropriate method was used by children in the primary grades in studies done by Piaget et al. (1960). This method was not used by any of the subjects that were observed in the pilot study.

WAG: Wild guessing, no systematic approach is used to answer a length or area estimation problem.

The following three categories were also used in the pilot study to record what the subjects did in estimating length and area. When a subject was observed making an arithmetic or unit size error, the error and the appropriate or inappropriate strategy were both noted.

ARITHMETIC ERROR: Errors in arithmetic that were detected in the methods used for estimating length or area were noted.

UNIT SIZE ERROR: Obvious errors in judgement of unit size were noted. For example, one subject thought that a centimeter was about twice as long as an inch.

OTHER METHOD: A systematic method that was not previously identified as appropriate or an inappropriate strategy. Descriptions of other methods that were used were noted on the estimation strategy form.

In the current study, the nine length and area strategies, the five inappropriate strategies, and the three other
categories were included on the estimation strategy form (Appendix E). For each of the 24 items on the estimation ability pretest and posttest exactly one of the following was marked: one of the nine appropriate strategies, one of the inappropriate strategies, or Other Method. On the items in which arithmetic or unit size errors were made the errors were also noted (i.e. some items were marked twice).

4.5 Strategies and Inappropriate Methods

In this section the strategies and inappropriate methods that college, fifth- and seventh-graders were observed using while estimating lengths and areas are described. One estimation strategy, squeezing, and one inappropriate strategy, centering, were not observed in this study; hence, they were dropped from further analysis. All of the strategies noted under Other Methods were found to be inappropriate; in subsequent analyses they were included in the Inappropriate Strategies category. A strategy use score (STRUSE) was computed for each student by subtracting the number of items in which an inappropriate strategy was used from 24.

Table 5 contains the total number of times that each of the appropriate strategies and each of the inappropriate strategies were observed for the college population and for the combined fifth- and seventh-grade populations by test, treatment and two treatments combined. Except for iterate,
Table 5

Frequencies of Strategies Used by Treatment Groups on the Pretest and Posttest and for College Students.

<table>
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<tr>
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<th>Guess &amp; Check</th>
<th>Strategy</th>
<th>Pre-Test</th>
<th>Post-Test</th>
<th>Pre-Test</th>
<th>Post-Test</th>
<th>Pre-Test</th>
<th>Post-Test</th>
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<th>Post-Test</th>
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<td></td>
</tr>
<tr>
<td>L X W</td>
<td>35</td>
<td>41</td>
<td>33</td>
<td>52</td>
<td>68</td>
<td>93</td>
<td>125</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Rearrange</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td>12</td>
<td>9</td>
<td>15</td>
<td>15</td>
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</tr>
<tr>
<td>Repeated Addition</td>
<td>20</td>
<td>9</td>
<td>25</td>
<td>8</td>
<td>45</td>
<td>17</td>
<td>13</td>
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<tr>
<td>Total</td>
<td>207</td>
<td>210</td>
<td>199</td>
<td>234</td>
<td>406</td>
<td>444</td>
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<td></td>
<td>Inappropriate</td>
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<tr>
<td></td>
<td>Strategies</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Count Around</td>
<td>17</td>
<td>13</td>
<td>15</td>
<td>8</td>
<td>32</td>
<td>21</td>
<td>16</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L X W</td>
<td>10</td>
<td>2</td>
<td>7</td>
<td>4</td>
<td>17</td>
<td>6</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unit Turned</td>
<td>6</td>
<td>14</td>
<td>5</td>
<td>10</td>
<td>11</td>
<td>24</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
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<td>20</td>
<td>22</td>
<td>28</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wag</td>
<td>36</td>
<td>35</td>
<td>54</td>
<td>24</td>
<td>90</td>
<td>59</td>
<td>87</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>81</td>
<td>78</td>
<td>89</td>
<td>54</td>
<td>170</td>
<td>132</td>
<td>137</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[a_n = 12; \text{Column Total = 288}\]

\[b_n = 24; \text{Column Total = 576}\]
unit turned and other, the changes favored an increase in STRUSE on the posttest. A decrease in inappropriate strategies indicates an increase in STRUSE. The decreases in Repeated Addition, Count Around, and Length + Width augmented by the increases in Length × Width and Rearranging imply learning about area concepts. It is interesting to note that the increase of 33 in the use of clues is attributable to both treatment groups; however, the decrease of 31 in Wagging seems to be attributable to the strategy treatment groups. The college students seemed to use more appropriate area strategies than the fifth- and seventh-graders did on either test. The 87 Wags by the college group indicates that they averaged about 3.6 wild guesses per person. Unit iteration was the most popular strategy for all grade levels for both treatment groups, and on both tests with the fifth- and seventh-graders using the strategy more than the college group.

The decrease in the number of unit size errors (Table 6) should be expected because of the practice with the metric units. Both the college students and the fifth- and seventh-grade pretest subjects averaged about one unit size error per person. Typical of these errors were the following:

Steve (college, pretest) insisted that 12 in. = 1 1/2 cm and he used this proportion on all questions involving cm.
Peggie (grade 7, pretest) had no idea what a cm or a square cm was. She wagged on all centimeter items, however, she did know that a meter is a little longer than a yard.

Laura (grade 5, posttest) confused cm and m. When asked to show with her hands about how long a cm would be, she placed her hands approximately a m apart.

Many students displayed good unit size sense especially on the posttest. Although none of the items used dm as the
unit, several of the subjects judiciously utilized the dm or dm$^2$ as an intermediate size unit.

Roger (grade 7, posttest) estimated the length of the line in item 4 (29 cm) of the posttest to be "3 dm or 30 cm".

Matt (grade 5, posttest) estimated the area of the rectangle (130 cm$^2$) in item 13 of the posttest by comparing the rectangle with a dm$^2$, "It looks like a square decimeter, it must be about 100 cm$^2$".

The following two methods were included in the Other Methods column. These methods were not previously observed during any of the pilot studies.

Susan (grade 5, pretest, item 18) estimated the area of a large sheet of paper (4 m$^2$) by estimating the number of meter sticks it would take to cover the paper.

Another fifth-grade student and one seventh-grade student estimated the number of rulers it would take to cover the poster on item 14 of the posttest. On the posttest Susan and two other fifth-graders and one seventh-grader revealed a confusion of perimeter and area.

Susan (grade 5, posttest, item 14) used a square that was 3 inches on each side as the unit instead of a square foot.

Stephanie (grade 7, posttest, item 17) used a square that was 25 cm on each side as the unit instead of a m$^2$.

A progression of errors was observed in problems requiring an estimate of the area of a rectangle (pretest and posttest items 13, 14, 16, 18, 20, 22, and 23). Six different methods were observed ranging from Length + Width to
Length X Width. Each of these methods is described:

Length + Width

Mike B. (grade 5, pretest, item 13) estimated the length of the rectangle to be 12 inches and the width to be 7 inches. His estimate for the area was 19 square inches. On item 17 Mike seemed to estimate the area by using only the length measurement; however, this was the only item upon which he did this and it was not entirely clear that he was centering only on the length.

Count Around

Marty (college, pretest, item 16) estimated the length of the rectangle to be 9 cm, the width 7 cm, and the area 32 cm² (i.e. 9 + 9 + 7 + 7). On item 13, Marty estimated the area to be 60 cm² in this fashion:

\[20 \text{ cm} + 20 \text{ cm} + 20 \text{ cm} = 60 \text{ cm}^2\]

He did problem 22 in a similar manor.

Karen (grade 7, posttest, item 20) estimated the number of units for the width to be 5, the length to be 8 units and the area to be 26 square units (i.e. 5 + 5 + 8 + 8 = 26). Karen used the count around method on six items on the posttest.

Count Around Plus Some for Middle

Pamela (grade 5, pretest, item 16) first estimated the perimeter as if she were using the Count Around Method and then added some more for the middle. "It looks like about 20 cm around the edge plus 16 more for the middle. I guess it's about 36 altogether." Pamela used this same method on five other items. Pamela recognized that perimeter and area are different concepts but she did not know how to measure interior
area. This method was used by three college students on the pretest and by four seventh-graders and five fifth-graders on the posttest.

Repeated Addition

Pamela (grade 5, posttest, item 20) counted four units across the top of the rectangular book, moved her eyes down a little and counted four more, she continued this until she got to the bottom of the book. Her use of unit iteration was exact, she estimated the area to be 40 units.

Turned Unit

The subjects that turned the unit after counting across the width of a rectangle and before counting the length used a multiplication rule, but failed to recognize that the unit must be kept in the same relative position.

Mike (college, pretest, item 23) did this problem as shown:

\[
\begin{array}{c}
\text{3 across top} \\
\hline
\text{2 down the side}
\end{array}
\]

Area = 3 x 2 = 6

Four other college students did this problem in the same way. On the posttest, 9 seventh-graders and 10 fifth-graders turned the unit on a very similar item.

Length X Width

As would be expected, the use of the Length X Width strategy involved multiplying the estimates of the lengths
of the two sides of the rectangles.

Bill (grade 7, posttest, item 8) estimated that two of the unit rectangles would fit across the top of the target rectangle and four would fit down the side. He estimated that it would take eight of the smaller rectangles to cover the larger one.

The subjects were asked to estimate the area of a triangle in square inches on item 15 of the pretest and posttest and the area of a triangle in cm$^2$ on items 17 of the pretest and 18 on the posttest. In item 19 of both tests the subjects were asked to estimate the number of small rectangles it would take to cover a five-sided figure. With some modifications the six methods that were observed being used for estimating the area of a rectangle were also used for estimating the areas of the two triangles and the pentagon. In addition, the Rearranging estimation strategy was also observed.

Length + Width

Jennifer (grade 5, pretest, item 15) estimated the length of one side of the triangle to be 11 inches and the base side to be 10. She estimated the area to be 21 square inches. Jennifer used the same method on item 17.

Count Around

Mike (grade 5, pretest, item 15) estimated the length of all three sides of the triangle to be 10 inches and the area to be 30 square inches. Mike used the same method on items 17 and 19. On the pretest, he did not use the Length X Width strategy on any of the problems. On the posttest, Mike used the Length X Width strategy correctly on three items. On item 15 of the posttest, he multiplied his estimates of the lengths of the 3 sides (i.e. $10 \times 10 \times 10 = 1000$ square inches).
Count Around Plus Some for the Middle

This method was popular with the college students that could not recall a formula for finding the area of a pentagon.

Cindi (college, pretest, item 19) estimated the perimeter of the pentagon to be 12 and added 6 more to get her estimate of 18. On item 17, Cindi estimated the lengths of two sides of the triangle to be 8 cm and 12 cm. She added the lengths of these two sides and "10 more for the middle" to get her estimate of 30 cm².

Repeated Addition

The students used two different patterns in estimating the area of the triangles and the pentagons using unit iteration.

Ed (grade 7, pretest, item 15) started at the base and counted around the outside and when he got back to where he started he moved toward the center of the triangle and counted around again. His counting pattern was spiraling toward the center.

Ladd (grade 7, pretest, item 17) used the pattern in the triangle to estimate the area to be 56 cm². Ladd also had an arithmetic error since 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 should be 41.

Turned Unit

Christy (grade 5, pretest, item 7) used a Rearranging method but she estimated the base with the unit turned in one direction and the altitude with the unit turned 90 degrees.
Rearranging

Geoff (college, pretest, item 19) pictured a rectangle superimposed on the pentagon so that the rectangular area was about the same as the pentagon. He used a Length X Width strategy to estimate the area of the rectangle to be 30 units.

Length X Width

Using the Length X Width strategy for estimating the area of a triangle involves the use of the formula $A = \frac{1}{2} bh$. Only three college students were observed using this strategy. No fifth- or seventh-graders used this strategy on either the pretest or posttest.

Summary

In this chapter the estimation strategies that were observed in the pilot studies and in the current study were discussed. The strategies that various subpopulations used were enumerated and described. Differences between the college students' and the school children's use of strategies were noted. Apparent differences were found between the strategies that were used in the pretest and in the posttest and between the treatment groups. In chapter 5 the results of the tests for significant differences in strategy use and in estimation ability with respect to grade level, sex, treatment, and pretest practice will be given. The relationship between estimation ability, strategy use and other variables will be described.
V ESTIMATION ABILITY

The pretest and posttest results on estimation ability and strategy use are given in this chapter. The hypotheses about the relationships between estimation ability, strategy use, mathematical ability, and perceptual ability are tested. The differences on estimation ability and in strategy use between males and females and between grade levels are discussed. Finally, the results of the treatments on estimation ability and on strategy use are given.

5.1 Estimation Ability, Strategy Use, Perceptual Ability, and Mathematical Ability

The means and standard deviations of the average absolute relative error scores by grade level and test are given in Table 7. Since average absolute relative error scores tend to be less reliable (see Section 3.41), estimation ability scores were calculated for each student by counting each item of the estimation ability test as correct if the absolute relative error on the item was less than 1/3. Estimation ability is the total number of items, out of 24, that were "correct".

84
Table 7

Average Absolute Relative Error Score Means and Standard Deviations by Grade Level and Test.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Pretest</th>
<th></th>
<th></th>
<th>Posttest</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>n</td>
<td>M</td>
<td>SD</td>
<td>n</td>
<td>M</td>
<td>SD</td>
</tr>
<tr>
<td>College</td>
<td>24</td>
<td>.632</td>
<td>.395</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grade Five</td>
<td>12</td>
<td>.792</td>
<td>.509</td>
<td>24</td>
<td>.625</td>
<td>.363</td>
</tr>
<tr>
<td>Grade Seven</td>
<td>12</td>
<td>.612</td>
<td>.251</td>
<td>24</td>
<td>.458</td>
<td>.184</td>
</tr>
</tbody>
</table>

Table 8 contains the pretest means and standard deviations on estimation ability, strategy use, perceptual ability, and mathematical ability by grade level and treatment group. Pearson product-moment correlations were calculated to test the following null hypotheses:

1. Estimation ability is not correlated with use of estimation strategies.

2. Estimation ability is not correlated with mathematical ability.

3. Estimation ability is not correlated with perceptual ability.

4. Strategy use is not correlated with perceptual ability.
Table 8

Pretreatment Means and Standard Deviations
By Grade Level and Treatment Group

<table>
<thead>
<tr>
<th>GRADE</th>
<th>Estimation Ability</th>
<th>Strategy Use</th>
<th>Math. a</th>
<th>GEFT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>SD</td>
<td>M</td>
<td>SD</td>
</tr>
<tr>
<td>College b</td>
<td>12.83</td>
<td>2.90</td>
<td>18.23</td>
<td>3.08</td>
</tr>
<tr>
<td>Five</td>
<td>11.00</td>
<td>2.52</td>
<td>16.58</td>
<td>3.75</td>
</tr>
<tr>
<td>Guess &amp; Check c</td>
<td>12.33</td>
<td>2.25</td>
<td>17.67</td>
<td>4.84</td>
</tr>
<tr>
<td>Strategy c</td>
<td>9.67</td>
<td>2.16</td>
<td>15.50</td>
<td>2.17</td>
</tr>
<tr>
<td>Seven</td>
<td>11.50</td>
<td>3.09</td>
<td>17.25</td>
<td>3.67</td>
</tr>
<tr>
<td>Guess &amp; Check c</td>
<td>10.83</td>
<td>3.71</td>
<td>16.83</td>
<td>2.32</td>
</tr>
<tr>
<td>Strategy c</td>
<td>12.17</td>
<td>2.93</td>
<td>17.67</td>
<td>4.89</td>
</tr>
</tbody>
</table>

The Math data is based on two different tests:

College-Delhi Minimum Competency Test;
Five and seven Stanford Achievement, scaled-total mathematics.

\[ b_n = 24 \]

\[ c_n = 6 \]
5. Strategy use is not correlated with mathematical ability.

Since the mathematical ability scores for the college students are from the Minimum Competency Test and from the Stanford Achievement Tests for the fifth- and seventh-graders, the correlations with mathematical ability are given separately for the two groups. The correlations between estimation ability and the GEFT score and between strategy use and the GEFT score are broken down similarly (Table 9). The results indicated that null hypotheses 1, 3 and 4 were rejected and supported the following:

1. Estimation ability is positively correlated with strategy use and perceptual ability.

2. Strategy use is positively correlated with perceptual ability.

The results on the correlations with mathematical ability were mixed. The correlations between mathematical ability and estimation ability ($r = .484$) and between mathematical ability and strategy use ($r = .408$) were significant for the college students. For the fifth- and seventh-graders neither of the two correlations ($r = .0972$ and $r = .052$) were significant.

On the items in which a strategy was used on the pretest the college students had 65% correct (65% of the absolute relative errors were less than $1/3$). On the items in
which an inappropriate method was used the college students had 16% correct. The corresponding figures for the school children were: grade seven, 58% when a strategy was used and 21% when an inappropriate method was used; grade five, 60% when a strategy was used and 17% when an inappropriate method was used.

Table 9

Correlations Between Estimation Ability, Strategy Use, Perceptual Ability, and Mathematical Ability.

<table>
<thead>
<tr>
<th></th>
<th>Strategy Use&lt;sup&gt;a&lt;/sup&gt;</th>
<th>GEFT College&lt;sup&gt;b&lt;/sup&gt;</th>
<th>GEFT 5 &amp; 7&lt;sup&gt;b&lt;/sup&gt;</th>
<th>Math Ability College&lt;sup&gt;b&lt;/sup&gt;</th>
<th>Math Ability 5 &amp; 7&lt;sup&gt;c&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimation Ability</td>
<td>.499***</td>
<td>.383*</td>
<td>.449*</td>
<td>.484**</td>
<td>.072</td>
</tr>
<tr>
<td>Strategy Use</td>
<td>.453*</td>
<td>.330*</td>
<td>.408*</td>
<td>.052</td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup><sub>n = 48</sub>

<sup>b</sup><sub>n = 24</sub>

<sup>c</sup><sub>n = 23</sub>

*<sub>p < .05</sub>

**<sub>p < .01</sub>

***<sub>p < .001</sub>
5.2 Grade Level and Sex Differences

Table A (Appendix G) contains the pretest estimation ability test means and standard deviations by grade level and sex. A two factor analysis of variance using the full regression method was conducted to test the viability of the following two hypotheses:

6. There is no difference between males and females on estimation ability.

8. There is no difference on estimation ability between grade levels (grades five, seven, and thirteen).

The results of the ANOVA are summarized in Table B (Appendix G). None of the results were significant: sex \( [F(1,42) = .034, p > .05] \); grade \( [F(2,42) = 1.685, p > .05] \); interaction of grade and sex \( [F(2,42) = 0.847, p > .05] \). Hence, null hypotheses 6 and 8 were not rejected.

The pretest means and standard deviations of STRUSE by grade level and sex are given in Table C of Appendix G. A two-way analysis of variance was conducted to test for grade level, sex, and interaction effects (Appendix G, Table D). None of the results were significant: sex \( [F(1,42) = 3.083, p > .05] \); grade \( [F(2,42) = 3.052, p > .05] \); nor interaction of grade and sex \( [F(2,42) = 2.544, p > .05] \). Hence, the null hypotheses 7 and 9 that there are no grade level or sex differences on strategy use were not rejected.
5.3 Estimation-olympic Competition

One of the purposes for holding the estimation-olympic competition on the last day of the treatment period was to give an additional measure of estimation ability. The results of the test were invalidated because of two problems. The seventh-grade guess-and-check treatment group did not complete the test because of time limitations. The source of the other problem was that the measurement for some items were given before subsequent estimations were made. For example, after the students estimated how far they threw the javelin (item 1), a measurement of the distance in feet and hundredths of a foot was made, and they wrote the estimate and the measurement down. During the shot put (item 2), some of the students made estimates in feet based upon the measured distance of item 1. In the grade five strategy treatment group, four of the 12 students apparently made estimates in feet on two or more items. In the grade seven strategy treatment group, five students apparently used this method. After the second class, the experimenter realized that some students were using feet instead of cm, dm, and m as the unit. Although the damage was already done, the use of metric units was emphasized during the two other classes. The motivational and practice objectives for the estimation-olympic competition were not invalidated by the circumstantial problems.
5.4 Treatment Effects on Strategy Use

The strategy use means and standard deviations by grade level (five and seven), testing group, and treatment group are given in Table 10. An examination of the means reveals a possibility that there was a difference in the pretest use of strategies by the two treatment groups. The pretest data were subjected to a two-way analysis of variance, summarized in Appendix G, Table E. The first-order interaction of grade level and treatment and the main effects of grade level and treatment were not found to be significant.

A three-way analysis of variance was conducted on post-test STRUSE by grade level, treatment, and pretest group (12 students in each grade had both the pretest and posttest and 12 students had the posttest only) to test hypotheses 11 and 13.

11. There is no difference on the use of estimation strategies between groups of fifth- and seventh-graders who have been taught to use estimation strategies and those who have been taught to use a guess-and-check method for estimating measurements.

13. There is no interaction effect on estimation strategy use by treatment and grade level (grades five and seven).

This analysis was expected to reveal if there were a pretest practice effect or a treatment effect. The results,
<table>
<thead>
<tr>
<th>Grade</th>
<th>Pretest</th>
<th>Posttest (Pretest Group)</th>
<th>Posttest Only</th>
<th>Overall Posttest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Five</td>
<td>17.68(6)</td>
<td>17.17(6)</td>
<td>16.00(6)</td>
<td>16.58(12)</td>
</tr>
<tr>
<td></td>
<td>4.85</td>
<td>2.93</td>
<td>3.41</td>
<td>3.08</td>
</tr>
<tr>
<td></td>
<td>15.50(6)</td>
<td>19.00(6)</td>
<td>18.83(6)</td>
<td>18.92(12)</td>
</tr>
<tr>
<td></td>
<td>2.17</td>
<td>3.16</td>
<td>2.98</td>
<td>2.23</td>
</tr>
<tr>
<td>Seven</td>
<td>16.83(6)</td>
<td>17.83(6)</td>
<td>19.17(6)</td>
<td>18.50(12)</td>
</tr>
<tr>
<td></td>
<td>2.32</td>
<td>1.94</td>
<td>2.56</td>
<td>2.28</td>
</tr>
<tr>
<td></td>
<td>17.67(6)</td>
<td>20.00(6)</td>
<td>20.50(6)</td>
<td>20.25(12)</td>
</tr>
<tr>
<td></td>
<td>4.89</td>
<td>2.53</td>
<td>8.94</td>
<td>3.17</td>
</tr>
<tr>
<td></td>
<td>Combined Posttest</td>
<td>17.54(24)</td>
<td>19.59(24)</td>
<td></td>
</tr>
</tbody>
</table>

\(^a\)Numbers in parentheses indicate the number of subjects in each subgroup.
Table 11, indicated that none of the interactions were significant and that there was no pretest practice effect \[ F(1,40) = 0.024 \]. Since the interaction of grade level and treatment (G X T) was not found to be significant \[ F(1,40) = 0.129 \], hypothesis 13 was not rejected. Significance was found for the treatment variable \[ F(1,40) = 6.315, p < 0.05 \]. Therefore, the null hypothesis that there is no difference on the use of estimation strategies between groups of fifth- and seventh-graders who have been taught to use estimation strategies and those who have been taught to use a guess-and-check method for estimating measurements was rejected. A check of the combined posttest means indicated that the strategy group mean was greater than that of the guess-and-check group. Hence, the children that were taught strategies used more strategies than the children that were taught to use the guess-and-check method.

The STRUSE data were then analyzed to see if the pretested groups actually improved their use of strategies. A three-factor analysis of variance was conducted on the strategy use scores, using a two between-one within-subjects design. The three factors were grade level, treatment, and the repeated measures factor, test (Appendix G, Table F). The grade level, treatment and interaction effects were not found to be significant. The effects of the within subjects, test, variable was found to be significant \[ F(1,20) = 5.753, \]
p < .05). The overall grade level and treatment group means and standard deviations were: pretest, $M = 16.917$, $SD = 3.647$; posttest, $M = 18.500$, $SD = 2.735$. The results indicated that there was improvement in the use of estimation strategies between the pretest and posttest.

Table 11

ANOVA of Posttest Strategy Use by Grade, Treatment, and Pretest Group.

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade Level (G)</td>
<td>1</td>
<td>31.687</td>
<td>4.001</td>
</tr>
<tr>
<td>Treatment (T)</td>
<td>1</td>
<td>50.021</td>
<td>6.315*</td>
</tr>
<tr>
<td>Pretest Group (P)</td>
<td>1</td>
<td>0.187</td>
<td>0.024</td>
</tr>
<tr>
<td>G X T</td>
<td>1</td>
<td>1.021</td>
<td>0.129</td>
</tr>
<tr>
<td>G X P</td>
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<td>7.521</td>
<td>0.950</td>
</tr>
<tr>
<td>T X P</td>
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<td>0.021</td>
<td>0.003</td>
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<tr>
<td>G X T X P</td>
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<tr>
<td>Error</td>
<td>40</td>
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</table>

*p < .05

5.5 Treatment Effects on Estimation Ability

The estimation ability means and standard deviation by grade level, testing group, and treatment group are given in Table 12. An examination of the means reveals a possible
pretest difference in estimation ability. The pretest estimation ability data were subjected to a two-way analysis of variance, summarized in Appendix G, Table G. None of the results were found to be significant: grade \[F(1,20)=.202, p > .05]\; treatment group \[F(1,20) = .360, p > .05\]; nor interaction of grade and treatment group \[F(1,20) = 3.236, p > .05\].

<table>
<thead>
<tr>
<th>Table 12</th>
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**Estimation Ability Means and Standard Deviations by Grade, Test, and Treatment.**

<table>
<thead>
<tr>
<th>Grade</th>
<th>Guess &amp; Check</th>
<th>Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>SD</td>
</tr>
<tr>
<td>Five</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pretest</td>
<td>12.33(6)</td>
<td>2.25</td>
</tr>
<tr>
<td>Posttest (Pretest Group)</td>
<td>12.17(6)</td>
<td>3.06</td>
</tr>
<tr>
<td>Posttest Only</td>
<td>13.33(6)</td>
<td>3.88</td>
</tr>
<tr>
<td>Overall Posttest</td>
<td>12.75(12)</td>
<td>3.39</td>
</tr>
<tr>
<td>Seven</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pretest</td>
<td>10.83(6)</td>
<td>3.37</td>
</tr>
<tr>
<td>Posttest (Pretest Group)</td>
<td>13.83(6)</td>
<td>3.25</td>
</tr>
<tr>
<td>Posttest Only</td>
<td>12.67(6)</td>
<td>1.75</td>
</tr>
<tr>
<td>Overall Posttest</td>
<td>13.25(12)</td>
<td>2.56</td>
</tr>
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</table>

Table 13 contains a breakdown of the estimation ability means into separate means for the length and area items for
the various subpopulations. The area items seem to be about
twice as difficult as the length items for all subjects. No
further analysis of this data was conducted.

Table 13

<table>
<thead>
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<td></td>
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<tr>
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<tr>
<td>M</td>
<td>SD</td>
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<table>
<thead>
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<th>Grade Level</th>
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<tbody>
<tr>
<td>College</td>
<td>8.21</td>
<td>4.63</td>
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<td>SD</td>
<td>1.83</td>
<td>1.69</td>
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<tbody>
<tr>
<td>Fifth Grade</td>
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<tr>
<td>Pretest</td>
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<tr>
<td>Guess &amp; Check</td>
<td>8.83</td>
<td>3.50</td>
</tr>
<tr>
<td>Strategy</td>
<td>6.50</td>
<td>3.17</td>
</tr>
<tr>
<td>Posttest</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Guess &amp; Check</td>
<td>7.67</td>
<td>4.50</td>
</tr>
<tr>
<td>Strategy</td>
<td>8.83</td>
<td>6.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Grade Level</th>
<th>Length</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seventh Grade</td>
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<td></td>
</tr>
<tr>
<td>Pretest</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Guess &amp; Check</td>
<td>7.67</td>
<td>3.17</td>
</tr>
<tr>
<td>Strategy</td>
<td>8.00</td>
<td>4.17</td>
</tr>
<tr>
<td>Posttest</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Guess &amp; Check</td>
<td>9.00</td>
<td>4.83</td>
</tr>
<tr>
<td>Strategy</td>
<td>9.00</td>
<td>4.33</td>
</tr>
</tbody>
</table>

A three-way analysis of variance was conducted on post-
test estimation ability by grade level, treatment, and pre-
test group to test null hypotheses 10 and 12.
10. There is no difference on estimation ability between groups of fifth- and seventh-graders who have been taught to use estimation strategies and those who have been taught to use a guess-and-check method for estimating measurement.

12. There is no interaction effect on estimation ability by treatment and grade level (grades five and seven). The results, summarized in Appendix G, Table H, indicated that none of the interactions or main effects were significant. Hence, no pretest practice effect was found and null hypotheses 10 and 12 were not rejected.

Since no difference was found in the estimation ability scores of fifth- and seventh-graders and no treatment difference was found, the treatment groups and grade levels were combined. Table 14 contains the estimation ability means and standard deviations by testing group. An examination of the means reveals a possible difference between the pretest and posttest scores of the fifth- and seventh-graders who took both tests. A three-way factor analysis of variance, using a two between-one within-subjects design, was conducted on the estimation ability scores of the fifth- and seventh-graders who took both tests. The three factors were grade level, treatment group, and the repeated measures factor, test. The only significant effect that was found (Appendix G, Table I) was the repeated measures factor
\[ F(1,20) = 11.626, p < .01 \]. Apparently, the two treatments were equally effective in producing a small but significant gain in estimation ability in the combined grades.

Table 14

<table>
<thead>
<tr>
<th>Group</th>
<th>Mean</th>
<th>Standard Deviation</th>
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<tbody>
<tr>
<td>College</td>
<td>12.833</td>
<td>2.899</td>
</tr>
<tr>
<td>Pretest 5th &amp; 7th</td>
<td>11.250</td>
<td>2.770</td>
</tr>
<tr>
<td>Posttest (Pretest Group)</td>
<td>13.5417</td>
<td>2.828</td>
</tr>
<tr>
<td>Posttest Only</td>
<td>13.458</td>
<td>3.0784</td>
</tr>
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</table>

\(^a_n = 24\) for each group
VI IMPLICATIONS OF THIS STUDY

6.1 Purpose

Recently there has been a call for more measurement estimation in the school mathematics curriculum. The N.C.S.M. Basic Skills list, the NCTM Agenda for Action and the metrification literature strongly support an increase in the teaching of estimation. Measurement and more particularly measurement estimation receive little emphasis in the curriculum. However, the educational research on measurement estimation is sparse and curriculum changes should be based on knowledge rather than hunches.

The purpose of this study was to investigate the use of estimation strategies and estimation ability. The particular questions of interest were:

1. What are the strategies that children and adults use when estimating length and area measurements?
2. What is the relationship between strategy use and estimation ability?
3. Is mathematical ability or perceptual ability related to strategy use or estimation ability?
4. Does strategy use or estimation ability vary with grade level or sex?
5. Is there a difference in strategy use or in estimation ability between children who have been taught to use strategies and those who have been taught to use a guess-and-check method?

6. Do either of the teaching methods (emphasis on strategy use or the guess-and-check method) have an effect on estimation ability or strategy use?

6.2 Procedures

The students within a fifth-grade class and within a seventh-grade class were randomly assigned to one of the two treatment groups. Half of each of the treatment groups were randomly assigned to the estimation ability pretest condition and the other half to the no pretest condition. To investigate whether there was a qualitative difference between the strategies used by school children and by young adults, a group of 24 college freshmen were given the same pretest. The pretest and the posttest consisted of an individually administered 24-item length and area estimation test. During the testing interview, the strategies that the subjects used and the estimations were recorded.

To investigate the relationship between perceptual ability, strategy use and estimation ability, the Group Embedded-figures Test was used as a measure of perceptual ability. The Group Embedded-figures Test was administered to all
subjects. To study the relationship between mathematical ability, strategy use and estimation ability, mathematical ability scores were gathered for all subjects except one seventh-grader and one fifth-grader for whom scores were not available. For the college students the Minimum Competency Test was used as a measure of mathematical ability. The Stanford Achievement scaled-total mathematics score was used for the school children.

The two treatments consisted of five, 41-minute lessons on length and area metric measurement. The strategy treatment emphasized explicit length and area estimation strategies. The guess-and-check treatment utilized the teaching technique suggested by Bright (1976); the children were given a list of objects to estimate and then check by measuring. All of the fifth- and seventh-graders were individually posttested using the estimation ability posttest form.

There are some limitations to the procedures that were used. Because of the amount of time required to interview individual subjects, the number of subjects in the study was small. The research design could have been improved by the inclusion of a control group of school children who did not receive the treatments nor a pretest, but were posttested. Perhaps a group estimation ability test could be developed; however, it is difficult to imagine gathering data on the estimation strategies without some type of individual
interview. There is a need for a standardized estimation
ability test and for an agreement upon whether relative error
scores or some other score should constitute estimation a-
bility.

The generalizability of the results may be limited by
the short duration of the instructional treatments and by
the concentration of the treatments in five consecutive
days. The normal practice is to spread instruction on a
particular topic over many days. The results of spreading
the teaching of estimation strategies and of using estima-
tion as a means for teaching about measurement throughout
the school year could be different from the results reported
in this study.

6.3 Results and Implications

Conclusion 1. Subjects in grades 5 and 7 ex-
hibit a variety of estimation strategies that
are not qualitatively different than those ex-
hibited by young adults.

It was reported in chapter 5 that the fifth- and
seventh-graders were about twice as likely to estimate
lengths correctly as they were areas. Corle (1960) found
that the mean relative errors for fifth- and sixth-graders
on length estimation was .52 and .42, respectively. In this
study it was found that the mean relative error for length and area estimation for fifth- and seventh-graders was .79 and .61. The difference between Corle's results and the current study results is another indicator of the difficulties that children have with the concept of area.

Except for wild guessing and unit size errors, the strategies utilized on length estimation items were varied but appropriate. A variety of inappropriate area estimation methods were observed. Since the subjects had more difficulty with area estimation, the area estimation strategies are given primary consideration in this discussion.

The strategy that was most frequently used on both length estimation and area estimation items was unit iteration. In area estimation problems centering on one dimension or on a limited aspect of the problem (Piaget et al., 1960) was not observed. The subjects that used centration in the Piagetian investigations were younger than the subjects in this study. The length + width rule (Anderson & Cuneo, 1978) was observed being used by 33% of the fifth-graders and 17% of the seventh-graders on the pretest. The Anderson & Cuneo information integration theory does not explain why 33% of the fifth-graders, 42% of the seventh-graders, and 25% of the college students (on the pretest) turned the unit when estimating the area of a rectangle using a rectangular unit. The finding that no fifth- or seventh-
grader and only 3 college students used the Length X Width strategy while estimating the area of a triangle concurs with the N.A.E.P. results indicating that 4% of the 13-year olds and 18% of the 19-year olds could correctly find the area of a triangle. The finding that the Count Around strategy was used 36 times (Table 5) on the pretest by the fifth- and seventh-graders concurs with Wagman's findings that 1/3 of a group of subjects of about the same age confused area and perimeter.

Because of the observed decrease in the more primitive area estimation strategies and increase in the more appropriate strategies between the pretest and posttest (Table 5), the hypothesis that children progress through stages of misperceptions about area is advanced. The proposed stages are: (1) centering, (2) length + width, (3) count around, (4) count around plus some for the middle, (5) unit iteration, (6) length times width-unit turned, (7) length times width. A need for further study is indicated.

Conclusion 2. Estimation ability and strategy use are correlated with perceptual ability.

The correlations that were found between estimation ability, strategy use, mathematical ability and perceptual ability support a few conclusions and raise many questions.
The correlations between the Group Embedded-figures Test scores and estimation ability ($r = .38$ for college students and $r = .45$ for the fifth- and seventh-graders) supports Eisner's (1972) conclusion that line length and angle size judgements correlate with the Embedded-figures Test ($r = .38$). The suggestion by Witkin et al. (1977) that field-dependent subjects may ignore cues in problem solving is indicated by the finding that strategy use and GEFT are positively correlated ($r = .45$ for college students and $r = .33$ for fifth- and seventh-graders). Further study is needed to ascertain the strategies that are used by subjects who score high on the GEFT and by those who score low. It is possible that field-dependent students would gain more from the treatment that explicitly emphasized strategies and field-independent persons would gain more from the guess-and-check treatment. This aptitude-treatment-interaction study is left for future research.

Conclusion 3. Estimation ability and strategy use are related to mathematical ability for young adults and unrelated to mathematical ability for fifth- and seventh-graders.

The correlation that was found between mathematical ability and estimation ability ($r = .48$) for the college
students corresponds with Pauli's (1971) finding that length and area estimation ability is correlated with the PSAT Mathematical score. There are two plausible reasons for the failure to find significant correlations between mathematical ability and estimation ability and between mathematical ability and strategy use at the fifth- and seventh-grade levels while significant correlations were found for the college population. First, it is possible that the two tests that were used for measuring mathematical ability did not measure the same abilities. This conclusion is supported by the fact that the 46-item Minimum Competency Test included nine measurement items and the 112-item Stanford Achievement Test used for the fifth-graders included two measurement items. The second possible explanation of conclusion 3 is that for the less mature subjects there is no relationship between mathematical ability and the abilities to choose appropriate estimation strategies and to estimate lengths and areas, while for the more intellectually mature young adults a relationship does exist between these abilities. The younger subjects may not have learned to apply the mathematical skills and knowledge that they have to novel situations; thus the correlation between mathematical ability and strategy use is near zero.
Conclusion 4. Estimation ability and strategy use are related.

A correlation ($r = .50$) was found between estimation ability and strategy use. The use of strategies accounts for approximately 25% of the variance in estimation ability. Estimating is not a skill that depends solely upon luck and prior experience. Estimating is similar to other problem solving skills; it is related to the strategies that one uses to attack the problem. A multiple regression analysis could be conducted on estimation ability to discover the relative contributions of strategy use, mathematical ability, perceptual ability and age. In the current study, grade level and sex were not found to have a significant effect on estimation ability or strategy use; however, further study on the grade level variable seems to be indicated.

Conclusion 5. Length and area estimation strategies can be taught.

Fifth- and seventh-grade children that were taught to use strategies exhibited more strategy use than the children that were taught the guess-and-check method. Significant differences were found between the strategy use pretest scores and the posttest scores for the 24 children that were
given both tests. A pretest practice effect was ruled out when no difference was found between the posttest means of pretested subjects and the posttest only subjects. It is possible that the difference between the pretest means and posttest means is due to maturation since the study took place over a four month period. Since no grade level differences were found, it is unlikely that a significant growth in strategy use could be attributed to four months of maturation. It is concluded that length and area estimation strategies can be taught and there is a difference between the two teaching methods favoring the treatment that emphasized explicit length and area estimation strategies. A follow-up study with control groups and with the instructional treatments spread over a longer period of time could strengthen these conclusions.

Conclusion 6. The ability to estimate length and area can be improved by providing instruction that includes estimation practice.

Although no posttest differences were found between the two treatment groups on estimation ability, a small but significant difference was found between pretest estimation ability and posttest estimation ability. A pretest practice effect was ruled out when no difference was found between
the posttest estimation ability means of the pretested sub-
jects and the posttest only subjects. Since the grade level
differences on estimation ability were not significant, it
is unlikely that a significant growth in estimation ability
could be attributed to maturation during the four months of
the study. The conclusion that instruction has a positive
effect on estimation ability could be strengthened by fur-
ther studies which include control groups and instructional
treatments that are spread over a longer period of time.

Two of the purposes for including measurement estima-
tion in the curriculum are: to increase measurement concept
learning and to help students develop a mental frame of re-
ference for the sizes of units of measure relative to each
other and to the real world (Bright, 1976). The increase
in the use of strategies that were noted in the study, par-
ticularly with area items, indicated that there was an in-
crease in measurement concept learning. The inclusion of
measurement concept learning as a dependent variable might
be fruitful in future investigations. The increase in esti-
mation ability between the pretest and posttest and the de-
crease in unit size errors is an indication that the sub-
jects in the study developed a mental frame of reference
for the size of the units.

It is recommended that measurement estimation activi-
ties be included in the school mathematics curriculum
beginning in the first grade. The research literature and the current study seem to support a wide variety of activities. For example, students who turn the unit when estimating the area of rectangles need activities requiring the use of rectangular units; such as, estimating the number of rectangular sheets of construction paper required to cover a bulletin board and then attempting to cover the bulletin board with the estimated number of sheets of paper. Students who use repeated addition on a rectangular area problem need activities requiring them to count the number of units in the length and width as well as counting the number of units required to cover the entire target rectangle. Transparent plastic grids and leading questions would be helpful.

The estimation-interview procedure of the study has proven to be valuable in gathering information about how children perceive measurement. It is recommended that teachers observe their students doing estimation tasks as a means of learning about the conceptual errors of the children. It is recommended that in-service and preservice teachers be informed of the necessity of teaching measurement and of the potentials of using estimation as a teaching method as well as a goal in and of itself.
BIBLIOGRAPHY


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Burton, G. M. Metrification of elementary mathematics textbooks in the seventies—the 1870s, that is. Arithmetic Teacher, 1980, 27, 28-31.


REFERENCE NOTES

### Schedule of Testing and Treatments

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<tr>
<th>Dates</th>
<th>Activity</th>
<th>Subjects</th>
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<td>Pretest</td>
<td>4 fifth-graders and 4 seventh-graders--2 from each of the two treatment groups</td>
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<tr>
<td>Feb. 4-8</td>
<td>Pretest</td>
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<tr>
<td>Feb. 11-14</td>
<td>Pretest</td>
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<td>Feb. 18-Feb. 29</td>
<td>Pretest</td>
<td>11 college students</td>
</tr>
<tr>
<td>March 3-March 7</td>
<td>Treatments</td>
<td>fifth- and seventh-grade classes</td>
</tr>
<tr>
<td>March 10-14</td>
<td>Posttest</td>
<td>4 fifth-graders and 4 seventh-graders--2 from each of the two treatment groups</td>
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<td>March 18-24</td>
<td>Posttest</td>
<td>same</td>
</tr>
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<td>March 25-31</td>
<td>Posttest</td>
<td>same</td>
</tr>
<tr>
<td>April 14-25</td>
<td>Pretest</td>
<td>13 college students</td>
</tr>
<tr>
<td>April 28-May 2</td>
<td>Posttest</td>
<td>4 fifth-graders and 4 seventh-graders--2 from each of the two treatment groups</td>
</tr>
<tr>
<td>May 5-May 9</td>
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</tr>
<tr>
<td>May 12-16</td>
<td>Posttest</td>
<td>same</td>
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</tbody>
</table>
APPENDIX B

Estimation Ability Pretest

(NOTE: The actual measurements of the objects are given for the benefit of the reader. The measurements were not displayed to the subjects. For each question a description of the object that was displayed, the question that was read to the subjects, and the answer for the item are given.)

1. A rectangle made of white posterboard with a black line drawn on it was displayed. The line had 3 black dots on it as indicated. The rectangle was placed on the easel tray 36 in. above the floor and approximately 48 in. from the subjects' eyes.

Q.: How long is this black line to the nearest inch?

A.: 29 in.

2. A dowel 31 in. long and 3/8 in. in diameter was held in the middle at eye level to the subject and approximately
36 in. away.

Q.: How long is this stick to the nearest 1/2 foot? This means that you may use a half foot in your answer. For example, if it were a little longer than 50 feet, you might say 50 1/2 feet.
A.: 2 1/2 ft.

3. A Muppets poster was placed on the easel. The poster had fold marks as shown.

Q.: Estimate the distance around the edge of the poster in feet.
A.: 9 ft.
4. A rectangle made of white posterboard with a black line drawn on it was placed on the easel. The line had 3 black dots on it as indicated.

Q.: How long is this black line in centimeters?
A.: 29 cm

5. An unsharpened pencil 19 cm in length was held in the middle with the printing toward the subject approximately 36 in. from the subject at eye level.

Q.: How long is this pencil to the nearest centimeter?
A.: 19 cm

6. The subject was asked to stand up and move to a marked spot in the room. A cash register paper tape 355 cm long and 5.7 cm wide was unfolded and stretched out in front of the subject. The tape had seven fold marks on it at 44.4 cm intervals.

Q.: How long is this strip of paper to the nearest 1/2 meter? This means that you may use 1/2 in your answer.
A.: 3.5 m
7. A wooden dowel (6 cm long) was held up by the experimenter and then placed on the desk. A cardboard box (42 cm long, 33.6 cm wide and 29.5 cm high) was placed on the desk with the length of the box parallel to the subject and the dowel in line with the bottom edge of the box and 15 cm between the box and the dowel. The box and the dowel were approximately 18 in. from the subject.

Q.: How many of these dowels (pointing at the dowel) could be laid end to end to go from one end of the box to the other (length of the box indicated with a sweeping motion)?
A.: 7 dowels

8. Q.: I want you to pretend that this (the experimenter placed the longer cardboard strip on the easel) black line is a curb. We are going to park cars along this curb bumper to bumper. How many cars just like this one (card with car on it is placed 10 cm to the left of
the "curb") can be parked bumper to bumper along the curb?
A.: 11 cars

9. A roll of cash register tape like that used in question 6 and a pair of scissors were placed next to the subject.
Q.: I want you to cut off a strip of this paper that is as long as I am tall.
A.: 72 in.

10. A strip of cash register tape (12 cm long) was placed on the easel and a 132 cm strip was held by the experimenter. The longer strip was held approximately 36 in. from the subject at eye level and parallel to the subject.
Q.: How many of the smaller strips of paper could be cut from this longer one?
A.: 11 strips

11. A piece of 1/4 inch graph paper 9 3/4 in. by 6 in. was mounted on a sheet of poster board that was the same size. The mounted graph paper was placed on the easel after the subject was told that "you will have no more than five seconds to see this problem".
Q.: How many squares are there along the top edge (the experimenter pointed to the edge) of this graph paper?
A.: 39 squares
12. A 42 cm by 20 cm rectangle was placed on the right-hand corner of the desk nearest the experimenter. A book (3.5 cm thick, 16 cm wide and 24 cm tall) was first placed on the rectangle and then placed 15 cm to the left of the rectangle.
Q.: I want you to pretend that his rectangle is a book shelf. How many books like this one could be placed on the book shelf?
A.: 12 books

13. A 14 in. by 10 in. rectangle made of white posterboard was placed on the easel.
Q.: How many one inch squares could be cut from this rectangle?
A.: 140 sq. in.

14. A 20 9/16 in. by 28 in. white posterboard rectangle was placed on the easel.
Q.: How many square feet of material would it take to cover this rectangle?
A.: 4 sq. ft.

15. An equilateral triangle (altitude = 11 in. and base = 10 in.) made of white posterboard was placed on the easel.
Q.: How many square inches would it take to cover this triangle?
A.: 55 sq. in.
16. A white posterboard rectangle (14 cm by 10 cm) was placed on the easel.
Q.: How many one centimeter squares could be cut from this rectangle?
A.: 140 cm²

17. A rectangular piece of posterboard with a triangle drawn on it, as indicated, was placed on the easel.
Q.: How many square centimeters would it take to cover this triangle?
A.: 60 cm²

18. The subject's attention was directed to a sheet of corrugated paper that was leaning against the wall opposite the subject. The subject was asked to stand up and move to a spot that was approximately 2 m from the middle of the paper. The paper was 125 cm high by 320 cm long and had dotted lines on it that formed squares that were 5 cm on each side.
Q.: What is the area of this sheet of paper in square meters?
A.: 4 m²

19. The pentagon and the rectangle were placed on the easel five inches apart.
Q.: How many of the orange cards would it take to cover the white five-sided figure?
A.: 24 orange cards

20. A paperback book (21 cm wide and 28 cm tall) with a torn cover was placed on the easel tray and a square piece of cloth material (5 cm by 5 cm) was taped to the easel 8 cm above the book and 2 cm in from the edge of the book.
Q.: The jacket on this book is torn and I want to cover
it. How many pieces of material this size (pointing to the material) will I need in order to cover the front of this book.

A.: 24 pieces of material

21. A rectangle and a triangle made of white posterboard were placed on the easel as indicated.

Q.: How many triangles would it take to completely cover the rectangle, assuming that the triangle could be cut up?

A.: 35 triangles
22. Two rectangles made of white posterboard were placed on the easel about five inches apart.

Q.: How many of the smaller rectangles would it take to cover the larger one?
A.: 7 smaller rectangles

23. Two Lego blocks were placed on the desk 2 inches apart and approximately 18 inches from the subject. The red block was 2 x 10 and the green block was 10 x 20.

Q.: This is another question in which you will have only five seconds to see the objects. (The Legos were then placed on the desk.) How many of the red Legos will fit on the green one?
A.: 10 red Legos
24. The larger rectangle was placed on the easel tray. The smaller rectangle was taped to the easel in the position indicated in the display.

Q.: In this problem I want you to pretend that we are going to put new wallpaper on this wall. Of course, we would not put wallpaper on the door (door indicated) or window (window indicated). How many pieces of wallpaper like this (pointing toward small rectangle) would it take to cover the wall?

A.: 12 pieces of wallpaper
APPENDIX C

Estimation Ability Posttest

(NOTE: The actual measurements of the objects are given for the benefit of the reader. The measurements were not displayed to the subjects. For each question a description of the object that was displayed, the question that was read to the subjects, and the answer for the item are given.)

1. A rectangle made of white posterboard with a black line drawn on it was displayed. The line had 3 black dots on it as indicated. The rectangle was placed on the easel tray 36 in. above the floor and approximately 48 in. from the subjects' eyes.

Q.: How long is the black line to the nearest inch?

A.: 29 in.

2. A dowel 32 in. long and 3/8 in. in diameter was held in the middle at eye level to the subject and approximately
36 in. from the subject.

Q.: How long is this stick to the nearest half-foot? This means that you may use a half-foot in your answer. For example, if it were a little longer than 50 feet you might say 50 1/2 feet.
A.: 2 1/2 ft.

3. A rectangular sheet of posterboard (28 in. by 22 in.) was placed on the easel.
Q.: Estimate the distance around the edge of the sheet of posterboard.
A.: 8 ft.

4. A rectangle made of white posterboard with a black line drawn on it was placed on the easel. The line had 3 black dots on it as indicated.
Q.: How long is this black line in centimeters?
A.: 29 cm

5. A felt tip marker 13.6 cm in length was held in the middle with the clip toward the subject approximately 36 in. from the subject at eye level.
Q.: How long is this marking pen to the nearest centimeter?
A.: 14 cm

6. The subject was asked to stand up and move to a marked spot in the room. A cash register paper tape 335 cm long and 5.7 cm wide was unfolded and stretched out in front of the subject. The tape had 9 fold marks on it at 33.5 cm intervals.

Q.: How long is this strip of paper to the nearest 1/2 meter? This means that you may use 1/2 in your answer.
A.: 3.5 m

7. A wooden dowel (4.8 cm long) was held up by the experimenter and then placed on the desk. A cardboard box (42 cm long, 33.6 cm wide, and 29.5 cm high) was placed on the desk with the width of the box parallel to the subject and the dowel in line with the bottom edge of the box and 15 cm between the box and the dowel. The box and the dowel were approximately 18 in. from the subject.

Q.: How many of the dowels (pointing at the dowel) could be laid end to end to go from one side of the box to the other (width of the box indicated with a sweeping motion)?
A.: 7 dowels
8. Q.: (The longer cardboard strip was placed on the easel) I want you to pretend that this black line is a curb. We are going to park truck along this curb bumper to bumper. How many trucks just like this one (the card with the truck on it was placed 10 cm to the left of the "curb") can be parked bumper to bumper along the curb?  
A.: 11 trucks

9. A roll of cash register tape like that used in question 6 and a pair of scissors were placed next to the subject.

Q.: I want you to cut off a strip of this paper that is as long as your teacher is tall.

A.: 64 in.

10. A strip of cash register tape (13 cm long) was placed on the easel and a 143 cm strip was held by the experimenter. The longer strip was held parallel to the
subject and approximately 36 in. from the subject at eye level.

Q.: How many of the smaller strips of paper could be cut from the longer one?
A.: 11 smaller strips of paper

11. A sheet of 5 mm graph paper (26.6 cm by 20.2 cm) was placed on the easel after the subject was told that "you will have no more than five seconds to see this problem".

Q.: How many squares are there along the top edge (pointing to the 20.2 cm edge) of this graph paper? (After five seconds, the graph paper was removed from sight.)
A.: 40 squares

12. A 49.5 cm by 20 cm rectangle was placed on the right-hand corner of the desk nearest the experimenter. A book (4.5 cm thick, 21.5 cm wide, and 24 cm tall) was first placed on the rectangle and then placed 15 cm to the left of the rectangle.

Q.: I want you to pretend that this rectangle (pointing at the rectangle) is a book shelf. How many books like this one (pointing at the book) could be placed on the book shelf?
A.: 11 books
13. A 13 in. by 10 in. rectangle made of white posterboard was placed on the easel.
   Q.: How many one inch squares could be cut from this rectangle?
   A.: 130 one inch squares

14. A poster (31 in. by 20.5 in.) with a large dog on it was placed on the easel. The poster had fold marks on it like item 3 of the pretest.
   Q.: How many square feet of material would it take to cover this rectangle?
   A.: 4 sq. ft.

15. An equilateral triangle (altitude = 11 in. and base = 10 in.) made of white posterboard was placed on the easel.
   Q.: How many square inches would it take to cover this triangle?
   A.: 55 sq. in.

16. A white posterboard rectangle (13 cm by 10 cm) was placed on the easel.
   Q.: How many square centimeters could be cut from this rectangle?
   A.: 130 cm²
17. The subjects' attention was directed to a section of the wall in front of them. The wall was divided into sections as shown.

Q.: I would like you to estimate the area of the wall in square meters from this line (pointing to the line) to the corner (pointing to the corner) from the floor to the ceiling (pointing at each).
A.: 5 m²

18. Same as pretest, item 17.

Q.: How many square centimeters would it take to cover this rectangle?
A.: 60 cm²
19. The pentagon and the rectangle were placed on the easel five inches apart.

Q.: How many of the orange cards would it take to cover the white five-sided figure?

A.: 20 orange cards

20. A paperback book (20.5 cm wide and 27.2 cm high) with a torn cover was placed on the easel tray and a rectangular piece of material (2.8 cm x 5 cm) was taped to the easel 8 cm above the book and 2 cm from the edge of the book.

Q.: The jacket on this book is torn and I want to cover it. How many pieces of material this size (pointing to the material) will I need in order to cover the front of the book?
21. A rectangle and a triangle made of white posterboard were placed on the easel as indicated.

Q.: How many triangles would it take to completely cover the rectangle, assuming that the triangles could be cut up?

A.: 32 triangles
22. Two rectangles made of white posterboard were placed on the easel 10 cm apart.
Q.: How many of the smaller rectangles would it take to cover the larger one?
A.: 7 smaller rectangles

23. Two Lego blocks were placed on the desk 2 inches apart and approximately 18 inches from the subject. The yellow block was 2 x 3 and the red block was 6 x 10.
Q.: This is another question in which you will have only five seconds to see the objects. (The Legos were placed on the desk.) How many of the yellow Legos will fit on the red one?
A.: 10 yellow Legos
24. A white rectangle with a floor plan and a tile drawn on it was placed on the easel.

Q.: This is a floor plan for a kitchen (pointing at the sketch). We are going to cover the floor with tiles like this one (pointing at the tile). We are not going to cover the fireplace, any of the cabinets or the closet floor (pointing to each). How many tiles would it take to cover the open floor (indicated with a sweeping motion)?

A.: 12 tiles
APPENDIX D

Estimation-olympics Team Competition Test

1. JAVELIN THROW  Materials: plastic straw, tape on the floor for starting line and a piece of tape for each contestant to write their name on. Instructions: While standing behind the starting line, throw your javelin as far as you can. Write your name on the tape that the judge will place at the point where your javelin lands. Estimate the distance that you threw your javelin in cm, that is how far it is in cm from the starting line to your tape. Write this estimate down in the proper place on the sheet that I gave you. After everyone has tossed their javelin and recorded their estimates, the judge will measure each of the distances. When the judge calls your name, write the number he gives you on the line marked measurement.

2. SHOT PUT  Materials: balloon, tape on the floor for starting line and a piece of tape for each contestant to write their name on. Instructions: Similar to item one; balloon is tossed; and distance is estimated in cm.

3. MARATHON  Instructions: Estimate the distance from one end of the main hall in the school building to the other end in meters. Write this estimate down on the sheet next to number 3.

4. ICE HOCKEY  Materials: large rectangle resembling a hockey goal drawn on the blackboard. Instructions: Estimate the area of the ice hockey goal drawn on the blackboard in square decimeters. Write your estimate down.

5. ICE HOCKEY  Materials: A sheet of paper attached to the answer form with a rectangular ice rink drawn on it. Instructions: Estimate the area of the ice rink drawn on the next page in sq. cm. Write your estimate on the line.

7. **DOWNHILL SKIING** Materials: Sheet of paper with a downhill skiing course drawn on it. The course consisted of three rectangles drawn in a zig-zag pattern. Instructions: Estimate the area of the downhill skiing course on page 3 in sq. cm. Write your estimate on the line.

8. **DISCUS THROW** Materials: Paper plate, tape on the floor for starting line, and a piece of tape for each contestant to write their name on. Instructions: Similar to item one; paper plate is tossed like a discus; and the distance is estimated in dm.

9. **400 CM DASH** Materials: tape on the floor for starting line and a piece of tape for each contestant to write their name on. Instructions: Beginning at the starting line, walk as far as you think 400 cm would be. Place the piece of tape with your name on it 400 cm from the starting line. When the judge calls your name, write down the number that he gives you on the line marked "estimation" next to number nine.

# Estimation—olympic Competition

<table>
<thead>
<tr>
<th>Team</th>
<th>Name</th>
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<tbody>
<tr>
<td>1. Javelin Throw</td>
<td>Estimate _____________ cm</td>
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<tr>
<td>3. Marathon</td>
<td>Estimate _____________ m</td>
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<td>4. Ice Hockey</td>
<td>Estimate the area of the Goal drawn on the blackboard in sq. dm.</td>
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<td>Estimate _____________ sq. dm</td>
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<td>5. Ice Hockey</td>
<td>Estimate the area of the ice rink drawn on the next page in sq. cm.</td>
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<td>Estimate _____________ sq. cm</td>
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<tr>
<td>6. 15 cm Speed Skating</td>
<td>Draw a line that is 15 cm long in this space.</td>
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<tr>
<td>7. Downhill Skiing</td>
<td>Estimate the area of the downhill course on page 3 in sq. cm.</td>
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<td>Estimate _____________ sq. cm</td>
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<td>8. Discus Throw</td>
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<td>9. 400 CM Dash</td>
<td>Estimate _____________</td>
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<td>10. Figure Skating</td>
<td>a) Draw a triangle that has an area of 50 sq. cm.</td>
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<td>b) draw a rectangle that has an area of 75 sq. cm.</td>
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<td>c) draw a 5-sided figure that has a perimeter of 50 cm (50 cm around the edge).</td>
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APPENDIX E

Estimation Strategy Form

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APPENDIX F

Teaching Materials for the First Day

G1

(Guess-and-Check) Name__________________________

For each of the following you should estimate the length, width, or height of the object, then measure the object using your meter stick. I will help you with measuring.

1. Width of your hand (nearest cm): Estimate_________
   Measurement_________

2. Length of your pencil or pen (nearest cm):
   Estimate_________
   Measurement_________

3. Length of this sheet of paper from top to bottom (nearest cm):
   Estimate_________
   Measurement_________

4. Width across your desk (nearest cm): Estimate_________
   Measurement_________

5. Width across this sheet of paper (nearest cm):
   Estimate_________
   Measurement_________

6. Height of your desk (nearest cm):
   Estimate_________
   Measurement_________

7. Your height (nearest cm):
   Estimate_________
   Measurement_________
8. Length of the eraser (nearest cm): Estimate__________
    Measurement__________

9. Mr. Hildreth's height (nearest cm): Estimate__________
    Measurement__________

10. Height of the door (nearest half of a meter):
    Estimate__________
    Measurement__________

11. Length of the blackboard (nearest m):
    Estimate__________
    Measurement__________

12. Length of the room from front to back (nearest m):
    Estimate__________
    Measurement__________

13. Length of your shoe (nearest cm):
    Estimate__________
    Measurement__________

14. Length of paper tape on the blackboard (nearest dm):
    Estimate__________
    Measurement__________
For each of the following you should estimate the length, width or height of the object using several different methods. After you have written down your estimates, you should measure the object with the meter stick and write this measurement down. I will help you.

1. Width of your hand (nearest cm):
   Estimates: Counting (using small finger width)
   Clues (four fingers)
   Measurement:

2. Length of your pencil (nearest cm):
   Estimates: Counting
   Compared with hand width
   Chunking (find middle)
   Clues
   Squeezing
   Measurement:

3. Length of this sheet of paper from top to bottom (nearest cm):
   Estimates: Counting
   Clues
   Squeezing
   Chunking
   Compared with hand
   Compared with pencil
   Measurement:
4. Width of this sheet of paper (nearest cm):
   Estimates:  Counting
   Clues
   Squeezing
   Chunking
   Compared with hand
   Compared with pencil
   Measurement: ________________

5. Width across your desk (nearest cm):
   Estimates:  Counting
   Chunking
   Clues
   Squeezing
   Compared with hand, pencil or paper
   Measurement: ________________

6. Height of your desk (nearest cm):
   Estimates:  Counting
   Chunking
   Clues
   Squeezing
   Compared with pencil, paper, or width of desk
   Measurement: ________________
7. Your height (nearest cm):
   Estimates: Clues
   Squeezing
   Compared with desk height
   Knowledge (1 inch = 2 1/2 cm)

   Measurement: ______________

8. Mr. Hildreth's height (nearest cm):
   Estimates: Counting
   Clues
   Squeezing
   Compared with your height
   Knowledge

   Measurement: ______________

9. Length of eraser (nearest cm):
   Estimates: Counting
   Clues
   Chunking
   Squeezing
   Compared with__________? __________

   Measurement: ______________
10. Length of blackboard (nearest m):

   Estimates:  Counting
              Chunking
              Clues
              Squeezing
              Compared with?____________________

Measurement: ________________________

11. Door height (nearest dm):

   Estimates:  Counting
              Chunking
              Clues
              Squeezing
              Compared with?____________________

Knowledge____________________

Measurement: ________________________

12. Length of room from front to back (nearest m):

   Estimates:  Counting
              Chunking
              Clues
              Squeezing
              Compared with?____________________

Knowledge____________________

Measurement: ________________________
### Table A

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<tr>
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Pretest Estimation Ability Means and Standard Deviations by Grade Level and Sex.
Table B

ANOVA of Pretest Estimation Ability Scores by Grade and by Sex.

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<thead>
<tr>
<th>Source of Variation</th>
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Table C

Pretest Strategy Use Means and Standard Deviations by Grade Level and Sex.

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<th>Female M</th>
<th>Female SD</th>
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ANOVA of Pretest Strategy Use Scores by Grade and by Sex.

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### Table E
ANOVA of Pretest Strategy Use Score by Grade and Treatment Group.

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Table F
ANOVA of Strategy Use Scores by Grade, Treatment, and Test.

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*P < .05
Table G

ANOVA of Pretest Estimation Ability by Grade and Treatment Group.

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Table H

ANOVA of Posttest Estimation Ability Scores by Grade, Treatment, and Pretest Group.

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Table I

ANOVA of Estimation Ability Scores by Grade Level, Treatment Group and Test.

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*P < .05