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VALUATION OF SAVINGS AND LOAN ASSOCIATIONS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By
Philip William Glasgo, B.S., M.B.A.

* * * * *

The Ohio State University
1980

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Adviser  
Faculty of Finance
To my parents

and to Dana
ACKNOWLEDGEMENTS

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CHAPTER I

INTRODUCTION

This study is concerned with an examination of the valuation of a firm in the absence of market price data for its stock. In the United States, there are more than two million active corporations; fewer than fifty thousand of these, however, have their shares traded on one of the organized stock exchanges or on the over-the-counter market.\(^1\) While an impressive quantity of literature in the leading finance and economic academic journals has addressed the question of valuation for firms whose shares are actively traded, little attention has been directed towards valuation of a firm which is closely held, has a mutual form of organization, or is a wholly-owned subsidiary of another firm.

However, it is often necessary to place a value upon a corporation for which there is not a market test of value. When the owner of a closely held business dies, a valuation must be placed upon the corporation for estate taxation purposes. If a nontraded corporation is merged into or acquired by another corporation, a value must be placed upon it,

or at least a relative value must be established between the two firms, so that the owners of the firm being acquired can be properly compensated for relinquishing their ownership. When a closely held company goes public, a proper value must be established so that an orderly sale of the shares can be accomplished. And when a corporation sells one of its divisions to another firm, a fair value must be established for the division; but the market price of the shares of the parent corporation provides little useful information concerning the value of the division.

In some industries, a small number of large corporations, with shares traded on the major stock exchanges, own virtually all of the assets of the industry. The availability of market information for a large portion of the industry makes available useful value guidelines for the untraded firms. But in other industries, the publicly traded firms represent only a small percentage of all firms and/or all assets in the industry, and the guidelines for valuation are not as strong.

In this study, the valuation question will be explored in the context of the savings and loan industry. There were more than 4700 savings associations in the United States at the end of 1978. However, fewer than twenty-five savings and loan associations or savings and loan holding companies are currently listed on the major exchanges, and active over-the-counter trading occurs in the shares of only

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2 United States League of Savings Associations, Savings and Loan Fact Book '79, p. 48 contains this estimate. Annual editions of this publication contain information about savings and loan associations, mortgage markets, and other mortgage lenders.
about twenty associations or holding companies. The associations with actively traded securities represent less than fifteen percent of the total assets of the savings and loan industry. This lack of market trading is due in part to the fact that more than eighty-three percent of all savings associations, holding over seventy-five percent of the total assets of all associations in the United States, have a mutual form of organization and hence have no stock which can be traded in the market place. It is also attributable to the fact that most stock-type savings associations are small, or at least have too small of an equity base to allow active, large-scale trading of their securities over a wide geographic area.

While the savings and loan industry is thus characterized by a large number of firms for which no test of market value has been established, the need for determinations of value has greatly increased in the past decade. Merger activity between associations and conversions of individual associations from the mutual form of organization to the stock form have become much more frequent. From 1960 to 1968, 317 mergers involving Federal Home Loan Bank member associations took place. During 1969 the Federal Home Loan Bank Board announced that it would look more favorably upon merger activity in the future than it had in the past. Since that announcement, merger activity has greatly increased.

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4 Ibid., pp. 52-55.

5 United States League of Savings Associations, Savings and Loan Fact Book '74, p. 61.
During the ten-year period from January 1, 1969 to December 31, 1978, there were 993 mergers in which the acquired association, the acquiring association, or both were members of the Federal Home Loan Bank system.

For mergers in which the acquired association is of the stock form of organization, the valuation question is directly raised, since a purchase price must be paid in either cash or in securities. Even when two mutual associations merge, the valuation question is implied. The practice currently authorized by the Federal Home Loan Bank Board is a simple joining of the associations through a balance sheet merger.

This may not be the appropriate action; it is possible that one association is relatively more valuable than the other, and thus the depositors, borrowers, employees, and managers of one organization may gain while those of the other association may suffer a loss in the merger.

Recent interest in the conversion of savings and loan associations from the mutual form to the stock form of organization has also frequently raised the question of valuation of the firm. In a few conversions the Federal Home Loan Bank Board allowed the free distribution of stock to depositors. However, the Board feared that this free distribution or "windfall gains" form of conversion would lead to serious disruptions in the flow of savings deposits due to depositors' anticipation of the potential windfall gains. Therefore, it has ruled that all conversions in the future will involve the sale of common stock, and will

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6 United States League of Savings Associations, Savings and Loan Fact Book '72, p. 54.

thus require a determination of value.

Similarities among firms in the savings and loan industry provide another justification for choosing that industry for examination in this study. Savings and loan associations are closely regulated either by the state (in the case of state chartered institutions) or by the federal government (in the case of federally chartered institutions). State chartered associations are, in fact, regulated by federal authorities as well if they are members of the Federal Home Loan Bank system; and eighty-eight percent of all savings associations, holding more than ninety-eight percent of the total assets of all associations, are members of the system. While it would be incorrect to contend that regulation makes the associations identical, and that they therefore belong to one homogeneous risk class, it does seem justifiable to contend that regulation does make them far more homogeneous in nature than would be a random sample of closely held firms from a nonregulated industry or a sample of closely held firms picked from various industries. The associations are required to hold certain minimum percentages of liquid assets based on their deposits and other borrowings. The types of loans they can make are specified, and the types of deposits they can accept and the maximum rates they can pay for these deposits are rigidly defined. Standardized reporting procedures are followed in the preparation of quarterly, semiannual, and annual reports to the regulatory agencies, so differences in financial statements due to differing accounting practices should be minor. Assets of all associations are similar, with a very high

concentration in mortgage loans (the average association has over eighty percent of its assets in mortgages), and opportunities for investment in the secondary markets for mortgages and government securities are also similar. Thus, for many reasons it appears that a valuation study which uses a sample of savings and loan associations may employ simplifying assumptions concerning the relative homogeneity of the firms that would not be warranted if an alternate source of data, such as new issues of securities through investment banking houses, were used.

For reasons discussed in detail in the previous paragraphs and summarized below, a study of valuation of firms for which there is not a market test of value, with specific concentration on the savings and loan industry, appears warranted, possible, and useful:

1. Most savings and loan associations are either closely held or mutual institutions for which a market test of value is not available.

2. Increased interest in mergers and conversions in the past decade has made the valuation question more important today than it was in the past.

3. Many savings and loan mergers have occurred recently; therefore, data are available for developing and testing models.

4. Regulation of the savings and loan industry results in similarities among institutions that should allow for useful simplifications in the models and should allow generalization of the results of the study to the entire industry.

**Objectives**

The objectives of this study are to:

1. Investigate the financial and economic literature relating to the valuation of the firm, especially the closely held firm;
2. Adapt recent developments in the theory of valuation to the case of the closely held firm;

3. Develop models of valuation for closely held firms in a specific industry—the savings and loan industry;

4. Test the ability of the models to predict market prices for a sample of savings and loan associations that are publicly traded;

5. Test the ability of the models to predict prices paid for savings associations that were acquired through merger during the period January 1, 1969, to December 31, 1975;

6. Compare the different models to determine if one is superior to the others.

Valuation of closely held organizations has not been a carefully examined topic in financial and economic literature. Most articles that do deal with the subject have focused on the valuation of securities for estate taxation purposes. Very simple approaches to the determination of value are usually taken by the tax courts when contested valuations are brought before them. Typically, the courts establish final prices by incorporating industry average price/earnings, price/dividends, and price/book value ratios to establish market-equivalent prices, and then discounting these figures by a factor of ten to twenty percent to compensate for a lack of marketability of the stock.

Very little attention has been given explicitly to risk considerations of the firm in the court decisions or in the literature commenting upon the court decisions. Yet a dominant theme of financial literature in the past fifteen years has been risk and the market price of risk. A theory of capital asset pricing which postulates a direct relationship between the nondiversifiable risk of an asset and the expected rate of return that can be earned on that asset has been
This study will incorporate risk considerations into valuation models for closely held firms.

Since the capital asset pricing model formulation of the price of risk employs a holding period return calculation which requires the market price of the firm's stock, it cannot be directly applied to the case of the closely held firm. Therefore, in this study, risk measures that do not require market prices of the firm's securities will be developed. These risk measures will then be used to develop estimates of value for the firms.

Scope

This study will explicitly consider only the valuation of the stock savings and loan association. The contention will be made that the models used could be used for mutuals as well; however, no actual determination of value is presently made for a mutual association, and so there are no data available for testing models of valuation for the mutual organization. By contrast, data are available to test valuation models for stock associations. Market price information for publicly traded savings and loan stocks is published in business newspapers and

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magazines, and merger price information for acquired associations is available from the Federal Home Loan Bank Board.

Not all researchers would agree that models of valuation developed for stock associations could be applied to the mutual sector of the industry. Hester 10 and Nicols 11 in studies comparing the mutual form with the stock form of organization, found significant differences in various performance measures and balance sheet characteristics. Brigham and Pettit 12 and Benston 13 did not reach the same conclusions. They could not conclude from their analyses that superior performance was demonstrated by one form of organization relative to the other. 14

Due to this lack of agreement, and also to the lack of market or acquisition price information for mutuals, only stock savings associations will be examined in this study. It is hoped, however, that the


14 For a summary and critique of studies relating to performance and new evidence relative to the question of stock performance compared to mutual performance, see Nelliyanik A. Padmarajan, "Cost Efficiency and Profit Performance of Savings and Loan Associations: The Mutuals Versus Stock Associations in Ohio" (Ph.D. Dissertation, The Ohio State University, 1976).
results of this study will lead to an examination of the valuation of mutual associations, and a determination of whether the present balance sheet merger practice is appropriate.

Limitations

The valuation models used in this study were developed from the theory of general capital market equilibrium referred to as the two-parameter capital asset pricing model. Empirical tests of this market model have not always strongly supported it.¹⁵ The usefulness and validity of this study will be reduced to the extent that the two-parameter capital asset pricing model does not provide a generally valid description of the asset pricing process in the capital markets.

A number of the holding companies originally used in the study had significant operations other than in the savings and loan industry. While many of the holding companies were eliminated from the study due to these irregularities, some of the remaining ones have some insurance, mortgage banking, and banking activities. To the extent that these activities distort the firms' balance sheets and income statements from what they would have been for "pure" savings and loan holding companies, the study is weakened.

The original data samples contained thirty-five holding companies for which market price information was available and 163 associations that were acquired during the period 1969 to 1975. The holding company

sample was reduced to fourteen due to screening procedures, and nonavailability of data reduced the sample size even further during the earlier years of the study. Screening and grouping mergers by year reduced the acquisitions samples to twelve or fewer for each year. As a result of such small sample sizes, the models that were developed and the conclusions that were drawn are not as strong as they would have been with larger sample sizes.

Information concerning prices paid for the acquired associations was obtained from merger documents provided to the Federal Home Loan Bank Board by the merging associations. If special considerations were given or side payments made as terms of a merger and these payments were not reported in the merger documents, then the price used in the study was incorrect, and the results of the study were biased accordingly.

Finally, savings and loan associations carry assets on their balance sheets at book values when, in fact, market values of the assets may differ significantly from book values. For example, a thirty-year mortgage made at a low rate of interest during a period when the cost of funds was low would be worth far less than its face value when higher interest rates prevail. This represents a reduction in the future earnings stream of the firm, since it must either sell the mortgage at a loss or suffer an opportunity loss by keeping funds tied up in the mortgage when the money could be invested at higher rates if it were not committed. If the action taken to compensate for this problem was not adequate, the conclusions of the study will be less meaningful than they could have been.
Plan of Study

Chapter II contains a review of relevant literature pertaining to the meaning of value, valuation and the cost of capital, valuation of closely held firms, and accounting substitutes for market risk measures. Model development and methodology are covered in chapter III. Procedures for estimating parameters of the models are established, the data bases used in the study are described, and research hypotheses are presented.

Chapter IV presents the analysis of the holding company sample, and chapter V contains the analysis of the acquisitions sample. Both chapters contain valuation estimates and tests of the research hypotheses. Chapter VI contains a summary of the study, a review of major results, and suggestions for future research.
CHAPTER II

REVIEW OF RELATED RESEARCH

In this chapter articles from finance and economics journals which study the topic of valuation and the closely related question of the cost of capital will be reviewed. The first section will examine the meaning of value. The term value is used in a number of contexts in our society. Different usages will be clarified, so that the usage of terminology will be standard throughout this chapter and the following chapters. The development of theories of valuation of the firm or the stock of the firm will be traced in the second section, and selected empirical tests of these theories will be presented. In the third section, literature which addresses the question of how to value a closely held firm will be examined. The final section will present literature pertaining to the development of risk measures for closely held firms. Synthesis of these sections will follow in Chapter III, where the ideas and concepts reviewed in this chapter will be used to develop valuation models for stock savings and loan associations.

I. Value

The value of a firm in a world of certainty is simply the net present value of all future cash flows, using the market (risk free) rate of interest as the discount factor. In such a situation, the price paid for the firm or a share of stock of the firm would be the value of the
firm or the share, since no one would be willing to sell for less or buy for more than that certain present value. In a world of uncertainty, however, the problem is not so simple. First of all, expectations of cash flows may differ significantly. Secondly, the discount rate applied to the cash flows may vary for different individuals. It therefore becomes appropriate to raise the question of whether, under uncertainty, there may be a difference between the price paid for a firm or a share of stock and the value of that firm or security.

It is possible to assume that price is equal to value and conduct all analyses on that basis. It is also possible, however, to contend that prices sometimes deviate from the intrinsic (or true) value of the firm or security. The idea that a security has an intrinsic value or worth was originally set forth in the classic book on investment value by John B. Williams: "Let us define the investment value of a stock as the present worth of all dividends to be paid upon it."¹ Hubbard and Hawkins² present a summary of the arguments of those who perceive a difference between price and value. In response to the above quotation by Williams, they state:

The truism that technical students of securities prices and others fail to realize is that if the theoretical foundation of the above statement is understood, to say the above statement is to say all. Value of a security is such, and nothing more. Technical analysts are dealing with prices, not values, . . . and bold and naive would be the economist, financial analyst, man-on-the-street who would in discussing objective value claim that "price is also value."

How can the market value of a security depart from its "intrinsic worth" or "investment value?" Investors don't know the true value of a security, and the price at a particular time reflects the consensus of investor opinion, which often is influenced by over pessimism or over optimism. Simply put, the answer is speculation. Traders speculate on profit from a rise in price of a security. Investors speculate on the uncertainty associated with the perpetual annuity that common stock represents. Both speculate on interest-rate changes, state of the national and world economy, health of the president, inflation, etc.3

An example of their contention, using data provided by Conklin,4 will emphatically illustrate the point. From their highest market price in 1961 trading to their lowest market price in 1966 trading, the ten listed savings and loan holding company stocks suffered declines of from 78 percent to 90 percent. Was the price of each of these securities equal to the value at all times, or were the securities overvalued in 1961, undervalued in 1966, or perhaps both? Certainly this is an extreme example, but security prices in the market may very well be more volatile, due to various psychological factors, than the underlying intrinsic value of the securities or assets.

Fama also acknowledges that a difference between price and intrinsic value can exist: "We stress, however, that actual market prices need not correspond to intrinsic value. In a world of uncertainty, intrinsic values are not known exactly."5

The above discussion has indicated that a difference may exist between the market price and intrinsic value of a firm's shares of

3Ibid., p. 139.
ownership. Another difference may exist between the computed market aggregate price for a block of shares, obtained by multiplying the market price per share times the number of shares in the block, and the actual price that would have to be paid for such a block. If a company's stock currently sells in the market for $25 per share, this does not mean that an investor can acquire all of the stock, or even controlling interest, for $25 per share. A considerable premium over the current market price may be necessary in order to acquire large blocks of a stock. Clear evidence of this phenomenon can be seen in the high premiums that are offered when a tender offer is made for securities.

The terminology for valuation can become confusing, as the preceding discussion has demonstrated. In this paper, the potentially different meanings will be separated in the following manner:

**Intrinsic Value** will refer to the theoretical worth of the firm, either on a per share or an aggregate basis.

**Market Price** or **Market Value** will refer to the price that exists in the market for shares of a firm's ownership. So frequently do authors use these two terms interchangeably that it is hopeless to try to change the usage, even though they probably should not be used synonymously.

**Computed Market Price** will refer to the product of the number of shares outstanding and the current market price per share.

Since most authors do not follow these conventions, quotations cited from other works may employ the terms differently. If the intent of the author is clear, the meaning will be explained. Most authors, however, take the position that while price may not equal value, it is the best estimate of value available. They thus ignore the potentially different meanings and use the terms synonymously.
II. Valuation and Cost of Capital Literature

Miller and Modigliani suggest that the literature of valuation contains four distinct approaches to the valuation of shares:

1. the discounted cash flow approach,
2. the current earnings plus investment opportunities approach,
3. the stream of dividends approach,
4. the stream of earnings approach.\(^6\)

Miller and Modigliani assumed the existence of perfect capital markets, rational investor behavior, and perfect certainty, and proceeded to demonstrate that once the investment policy of the firm is given, the four valuation approaches are, in fact, equivalent. Because of their general usage in financial literature, these four approaches will be summarized below.

Under perfect certainty and perfect market assumptions price would be equivalent to value, and the terminology used presents no opportunity for misunderstanding. In addition, the same assumptions guarantee that there is no need to distinguish between various types of securities; cash flows to all securities are assumed certain, and the value of the firm therefore is not influenced by the existence of different types of securities.

The following variables are standard throughout their analysis. Other variables will be introduced as they are needed.

\[ V_t = \text{the value of the firm at time } t \]
\[ D_t = \text{the total dividends paid during } t \text{ to shareholders of record at the start of period } t \]

\( p \) = the rate of return earned per dollar invested in each period. For simplicity it is assumed constant, but it could be allowed to vary

\( I_t \) = the firm's investment in period \( t \)

\( X_t \) = the firm's net profit in period \( t \)

Under the previously stated assumptions, the value of the firm is simply:

\[
V_0 = \sum_{t=0}^{\infty} \frac{1}{(1+p)^{t+1}} [X_t - I_t]
\]  
(Equation 2-1)

**Discounted Cash Flow Approach**

This approach to the valuation problem involves discounting at the market rate of interest the summation of all cash inflows and all cash outflows for each period in the life of the firm. Let:

\( R_t \) = all cash receipts of the firm during period \( t \)

\( O_t \) = all cash outflows of the firm during period \( t \)

Then the value of the firm can be expressed as:

\[
V_0 = \sum_{t=0}^{\infty} \frac{1}{(1+p)^{t+1}} [R_t - O_t]
\]  
(Equation 2-2)

but, \([R_t - O_t] = [X_t - I_t]\) for all \( t \), since cash receipts \((R_t)\), differ from profits \((X_t)\), by the same amount that cash outflows \((O_t)\), differ from investment \((I_t)\), namely, the summation of cost of goods sold and depreciation. Therefore, Equation 2-2 is equivalent to Equation 2-1. Because of this equivalence, the quantity \([X_t - I_t]\) is often referred to as [net] cash flow.

**Investment Opportunities Approach**

When an investor purchases a firm, he purchases the earning power of the assets currently held by the firm and also the opportunities that
the firm has or will have in the future to invest in assets that will yield more than the "normal" rate of return, p. Denote the excess rate of return as \( p_t^* \) for period t, and assume that this return is earned on all of \( I_t \). Then the "goodwill," or the present value of the excess earning opportunity is, for period t:

\[
V_t = \frac{I_t \left[ \frac{p_t^* - p}{p} \right]}{(1+p)^{t+1}}
\]

(Equation 2-3)

The value of the firm will be the summation of the present value of the normal earnings, \( X_0 \), and the present value of all goodwill, or:

\[
V_0 = \frac{X_0}{p} + \sum_{t=0}^{\infty} \frac{I_t \left[ \frac{p_t^* - p}{p} \right]}{(1+p)^{t+1}}
\]

(Equation 2-4)

and this quantity is, in fact, equivalent to Equation 2-1. 7

Stream of Dividends Approach

Since the investment policy of the firm is given (by assumption), the firm has to sell additional shares of ownership (or secure outside financing) to raise capital if it declares a large dividend in period t. Therefore, the notation \( D^V \) will be used to denote dividends paid to the shareholders of record as of time \( w \). Then, in general:

\[
V_w = \sum_{t=0}^{\infty} \frac{D^V_{w+t}}{(1+p)^{t+1}} \quad \text{and for } w=0, \text{ denoting the present:}
\]

\[
V_0 = \sum_{t=0}^{\infty} \frac{D^0_t}{(1+p)^{t+1}}
\]

(Equation 2-5)

7Ibid., p. 417 presents the derivation.
Miller and Modigliani demonstrate that this equation is equivalent to Equation 2-1 whether outside financing is undertaken or not.  

Stream of Earnings Approach

The stream of earnings approach expresses the value of the corporation as a function of the future earnings generated by the corporation. However, it is necessary to realize that additional capital may be required in the future in order to maintain the earnings stream. Under the simplifying assumption that the required rate is $p$, the cost of that additional capital raised in period $t$ (the capital is simply $I_t$), would be $pI_t$ in period $t$ and all future periods. The present value of the firm is:

$$
V_0 = \sum_{t=0}^{\infty} \frac{1}{(1+p)^{t+1}} (X_t - \sum_{t=1}^{\infty} pI_t) \tag{Equation 2-6}
$$

and this equation also reduces to Equation 2-1.

Notice that the cash flows approach and the investment opportunities approach are just variations of the stream of earnings approach and can be referred to, collectively, as the earnings approach. The alternative is the dividend approach, as presented in Equation 2-5.

Uncertainty

It should be no great surprise that the four approaches were shown to be equivalent by Miller and Modigliani. Under the idealized conditions of the model, the results were simply applied logic. However, considerable controversy exists in financial literature concerning whether the earnings and dividend approaches are equivalent under

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8 Ibid., p. 419.
uncertainty.

Two opposing positions have formed on the question. The "dividends" cause is championed by Professor Myron Gordon, who argues that under uncertainty (but still retaining the assumption of perfect markets) what is discounted does make a difference, and investors discount dividends, not earnings. Professors Miller and Modigliani champion the cause of those who argue for the earnings approach, and contend that dividend policy is, at best, irrelevant, and at worst, potentially misleading. Even when the perfect markets assumption is dropped, they contend, imperfections in the market do not lead to a systematic preference for current dividends. In fact, the major systematic imperfection in the market, in their opinion, is the tax differential in favor of capital gains, and this factor would lead the investor to prefer low, not high current dividends.9

Gordon's eloquent defense of the dividend approach attacks the Miller and Modigliani assumptions used in their statement of dividend irrelevance. His model is based on the valuation of shares of stock, rather than upon the valuation of the entire firm. Let:

\[ P_t = \text{the price of a corporation's share of stock at the end of period } t \]
\[ D_t = \text{dividends per share paid by the corporation during period } t \]
\[ Y_t = \text{income per share earned during period } t \]
\[ r = \text{return on investment the corporation is expected to earn in every future period} \]
\[ b = \text{fraction of income the corporation is expected to retain in every future period} \]

9Ibid., p. 432.
\( k = \text{return on investment that stockholders require on the corporation's stock} \)

Assuming certainty of \( r \) and \( b \), Gordon demonstrates that:\(^{10}\)

\[
P_0 = \frac{(1 - b)Y_0}{k - rb} \quad \text{(Equation 2-7)}
\]

This is the continuous compounding form of the model. A slightly different form, which uses discrete rather than continuous compounding, is frequently used:

\[
P_0 = \frac{D_1}{k - g} \quad \text{(Equation 2-8)}
\]

Recasting this equation in its cost of capital format (i.e., solving for \( k \)) yields:

\[
k = \frac{D_1}{P_0} + g \quad \text{(Equation 2-8a)}
\]

where the first term is the dividend yield and the second term is the growth rate of the dividend.

The fundamental difference between Gordon and Miller and Modigliani can be expressed in terms of Equation 2-8a. Gordon believes that under uncertainty, \( k \) is an increasing function of \( g \). That is, as the firm retains more of its earnings and pays less in dividends, \( k \), the cost of equity capital, increases. Because of the interaction between \( b \), \( k \), \( g \), and \( D \), the price of the share may rise or fall due to a change in \( b \).\(^{11}\)

Miller and Modigliani, by contrast, would contend that a change in the retention rate \( b \), and hence in \( g \) in Equation 2-8a, would be


\(^{11}\)Ibid., p. 53.
exactly offset by a commensurate change in the dividend yield, \( \frac{D_1}{P_0} \), leaving \( k \), and the price of the firm's stock, unchanged. Gordon, they argue, is guilty of confusing investment policy with dividend policy:

As should be abundantly clear by now, a change in dividend policy, given investment policy, implies a change only in the distribution of the total return in any period as between dividends and capital gains. If investors behave rationally, such a change cannot affect market valuations.\(^{12}\)

Gordon included in his book a summary of research efforts he has conducted to test the importance of dividends, earnings, and growth of dividends. While he was somewhat disappointed with his results, he concluded that the dividend and growth variables were highly significant, and that the addition of leverage and risk variables enriched the model.\(^{13}\)

Based on his thorough theoretical development of valuation, Gordon then developed and tested a more complex, nonlinear valuation model which included dividend and growth variables, plus five risk variables—an index of earnings instability, a debt/equity ratio, an asset liquidity index, a debt liquidity index, and a measure of corporate size. The results of the model were mixed. While for one industry sample the model was termed a "resounding success,"\(^{14}\) for other industries it was much less so. In general, the model was, despite its complexity, rather sample sensitive.

Miller and Modigliani empirically examined their hypothesis concerning irrelevance of dividend policy in their study of the cost of

\(^{12}\) Miller and Modigliani, p. 425.
\(^{13}\) Gordon, pp. 140-53.
\(^{14}\) Ibid., p. 174.
capital in the electric utility industry. While their study was concerned with the cost of capital, it was also an important contribution to valuation literature, due to the inverse relationship between cost of capital and value. Their model was a form of the investment opportunities approach (Equation 2-4 above) in which they assumed that the excess earning opportunities would be available for a finite, rather than an infinite, number of years.

They assumed that the electric utilities represented a homogeneous risk class or risk equivalent class. All firms in the industry were thus assumed to have the same cost of equity capital. Using this simplification, they developed a cross-sectional regression model to estimate empirically the cost of capital for the electric utility industry in 1954, 1956, and 1957.

Concern about the possibility that the crucial expected earnings variable was not adequately measured by published accounting statements led Miller and Modigliani to use an instrumental variables technique to improve the estimates (appendix A contains a review of instrumental variables methods). A two-stage least squares method was employed in which variables representing growth, size, common stock dividends, interest, and preferred dividends were included. These variables were selected because they were believed to be correlated with, or possess informational content about, the true (but unobservable) expected earnings. The


16Ibid., p. 344. The finite growth models, as Miller and Modigliani call them, are discussed in detail below.

17Ibid., pp. 351-56.
two-stage model provided results which were believed to be less biased than those provided by the direct least squares approach.

The instrumental variables approach allowed Miller and Modigliani to demonstrate strong support for their argument concerning dividend policy irrelevance. They believed that by employing the dividend as one of the instrumental variables in the estimation of true earnings, they had shown the informational content of dividends—the value of dividends to the market as an indication of future earnings. When the instrumental variables estimate of earnings was used in the valuation model instead of measured earnings, the dividend variable was not significant. This finding supported their contention that dividends are important because of their informational content, and not, as Gordon contended, because dividends are preferred to retained earnings.

In response to a variety of extensions and criticisms of the Miller and Modigliani model, Higgins presented a paper which re-examined key questions raised in the original paper and in subsequent works by other authors concerning the cost of capital in the electric utility industry. 18 He substituted a growth variable based on the population of

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the area served by the utility for the often-criticized growth in total assets used by Miller and Modigliani. The resulting estimates were judged to be superior in several ways to the estimates when growth was defined as growth in total assets. Higgins used a technique involving nonlinear regression to serve as a check on the cost of capital estimates developed by a linear regression technique. The nonlinear estimates were "reassuringly similar" to the linear estimates, thus supporting the belief that the specification of the model was adequate. 19

To investigate the dividend relevance-irrelevance controversy, Higgins added a dividend variable to the model. The dividend variable entered the model with a negative value in all but one year of the 1960-1968 period for which the model was tested. He therefore concluded ". . . it would appear that dividends can safely be ignored without seriously distorting the model."20

Finite Growth Models

Higgins also examined works of other authors concerning the elusive growth term of the investment opportunities approach to valuation or cost of capital estimation. Many researchers have used this approach for empirical estimation. However, the "goodwill" or excess growth opportunity term of the model presented significant problems for them. The basic model assumes that opportunities to invest at a rate of return greater than the cost of capital will exist forever. Realistically, however, the potential investor in a firm considers such

19 Higgins, p. 1196.
20 Ibid., p. 1200.
opportunities in the distant future to be of negligible value. Excess earnings opportunities will attract new competitors to an industry, and their entry into the market will tend to reduce the rates of return available. Excess earnings in regulated industries will lead to great public concern, and eventually, to lower rates of return being allowed by regulatory authorities. Therefore, researchers have employed variations of the investment opportunities model in which the assumption is made that the opportunity to earn rates of return higher than the firm's cost of capital will exist for only a finite number of years.

Several attempts have been made to estimate the number of years (denoted here and in most of the studies as T) for which the especially profitable investment opportunities are expected to persist. In their study of the cost of capital in the electric utility industry, Miller and Modigliani used a model from which T could be calculated. The estimates, however, were not very stable; they ranged from one year to fifty-one years for different years and different specifications of the model.21 Robichek, McDonald, and Higgins suggested that this instability was due to misspecification of the growth variable in the model.22 Therefore, Litzenberger and Rao followed a procedure similar to Miller and Modigliani, but developed a more specific measure of the growth term. They found that T, the number of years of discounted earnings growth implicit in market prices of stocks, ranged from 1.5 to 5.7 years. This was consistent, they noted, with Malkiel's contention that profit growth

22 Robichek, McDonald, and Higgins, p. 1285.
should not be expected to be discounted for more than three to five years into the future.\(^{23}\)

McDonald used an estimation procedure which recognized the non-linearity of the coefficients of the growth model due to the appearance of \(p\), the cost of capital, in both the no-growth and growth terms of the equation. His cost of equity capital estimates were quite similar to those of Litzenberger and Rao. He did not directly estimate \(T\), but did report that the growth coefficient in his model was not statistically significant in eight of the twelve years of the study.

These cost of equity capital estimates of Litzenberger and Rao and of McDonald both implied that the growth term of the model was not very important. Litzenberger and Rao's estimates averaged only forty-eight basis points (0.48 of one percent) above the rates obtained from the no-growth formulation of the model. McDonald's estimates were even closer to the no-growth model, averaging only forty basis points above the simple earnings to price ratio.

Such small values attributable to the growth term caused Higgins to question the adequacy of prior formulations of the finite growth model to represent the true value of future growth to investors. So, in addition to the dividend policy question mentioned previously, Higgins also tested a new formulation of the finite growth model which he believed reflected more accurately the present value of future investment.

If opportunities to invest at the excess rate of return \(p^*\) exist for \(T\) years, after which such opportunities disappear, and if the

magnitude of such investments is $I_0$ for the present year with an exponential growth rate $g$, then the present value of this future growth to present equity shareholders can be expressed as:

$$\frac{(p^*-p)zI_0}{p} \int_0^t e^{(g-p)t} \, dt$$

(Equation 2-9)

where $z$ represents the percentage of assets financed by equity.\(^4\) Assuming that $(g-p)t$ is small, Equation 2-9 reduces to:

$$\frac{(p^*-p)zI_0T}{p}$$

(Equation 2-9a)

The finite growth version of Equation 2-4 then becomes:

$$S_0 = \frac{X_0}{p} + \frac{(p^*-p)zI_0T}{p}$$

(Equation 2-4a)

where $S_0$ represents the value of the equity of the firm.

Using this formulation of the growth model, and an estimation procedure for investment, $I_0$, that he believed superior to previously used methods, Higgins developed new cost of capital estimates for the electric utility industry. The estimates of the cost of equity capital were higher than those found by McDonald, and exceeded the earnings to price ratio by an average of 120 basis points. These estimates were judged to be reasonable, since year-to-year changes were similar to changes in the yields on comparable market instruments. The estimates of $T$ ranged from 2.2 years to 4.8 years, and averaged 3.9 years.\(^5\)

\(^4\) Higgins, p. 1190, and McDonald, p. 506 both developed this formula.

\(^5\) Higgins, pp. 1195-97.
Capital Asset Pricing Model

One of the greatest difficulties experienced by researchers investigating the questions of valuation and cost of capital has been the necessity for the homogeneous risk class assumption, which states that firms within the same industry, or firms grouped in some other manner have the same cost of capital for pure equity streams (i.e., before adjustments are made for leverage). Cross-sectional regression models have almost always been used to test models of valuation or cost of capital estimation. And in order to employ a cross-sectional model, the researcher must have a large data sample to avoid introducing severe bias into the results. Therefore, empirical tests have been conducted by contending that firms within the same industry should have identical costs of equity capital, and then using data from an industry containing many firms. Left unanswered has been the question of how to evaluate an industry with only a few companies, or a company which belongs to an unique risk class.

Developments in portfolio theory, based on the pioneering work of Markowitz, have eliminated the need for the homogeneous risk class

26See: Edwin J. Elton and Martin J. Gruber, "Improved Forecasting through the Design of Homogeneous Groups," Journal of Business 44 (October 1971): 432-50. The authors demonstrate that designing homogeneous groups based on similarity of key variables in an improvement over grouping by traditional industry classifications.

assumption. The capital asset pricing model, sometimes referred to as the Sharpe-Lintner-Mossin Model, is a statement of equilibrium pricing of all assets in the market. Each asset or firm is assumed to belong to a unique risk class, and the price of the asset is dependent upon this unique degree of risk.

The basic assumptions of the model are:

1. Capital markets are perfect in the sense that assets are infinitely divisible; there are no transactions costs or differential taxes; all investors can borrow or lend at the same rate of interest; information is available to all at no cost.

2. All investors are risk averse, expected utility of wealth maximizers who assess portfolios by their expected rate of return and the standard deviation of this rate of return.

3. All investors have the same expectations about expected rates of return and standard deviations of these rates.

4. All investors have the same planning horizon for portfolio decisions.

With these assumptions, it can be shown that:

\[
E(R_j) = R_F + \frac{[E(R^*_M) - R_F]}{\sigma^2(R^*_M)} \text{COV}(R^*_j, R^*_M) \quad \text{(Equation 2-10)}
\]

---


30 Fama, "Risk, Return and Equilibrium," p. 32.
where:

\[
E(R_j) = \text{the expected one period rate of return for asset } j
\]
\[
R_F = \text{the one period rate of return on the riskless asset}
\]
\[
E(R_M) = \text{the expected rate of return on the market portfolio}
\]
\[
\sigma^2(R_M) = \text{the variance of the market portfolio}
\]
\[
\text{COV}(R_j, R_M) = \text{the covariance of returns of asset } j \text{ and the market portfolio}
\]

Since \[\frac{\text{COV}(R_j, R_M)}{\sigma^2(R_M)}\] is a constant called "beta" for each firm \( j \),

the model can be expressed as:

\[
E(R_j) = R_F + \beta_j [E(R_M) - R_F] \quad (\text{Equation 2-11})
\]

or in risk premium form as:

\[
E(R_j) - R_F = \beta_j [E(R_M) - R_F] \quad (\text{Equation 2-11a})
\]

Direct estimation of the risk parameters is not possible, since the expected value or expectation of the returns is an \textit{ex ante} concept. However, if realized returns, or \textit{ex post} returns, are substituted for the expected returns, Equation 2-11 can be estimated by a time series regression of the form:

\[
\ln(R_{jt}) = \hat{\alpha}_j + \hat{\beta}_j [\ln(R_{Mt})] + e_{jt} \quad (\text{Equation 2-12})
\]

where \( e_{jt} \) represents a disturbance or error term for asset \( j \) in period \( t \).

The required rate of return for a firm is thus a function of two elements—the risk free rate of return and the firm's unique beta coefficient. No longer are firms in the same industry assumed to have the same required return. The return will vary as the riskiness of the firm, as measured by beta, varies.
The actual one period return for the firm's common stock is calculated as the sum of dividends received during the period and the capital gains realized during the period as a result of a change in the price of the stock. On a per share basis, this return can be expressed as:

\[
R_j = \frac{P_1 - P_0 + D_1}{P_0}
\]

(Equation 2-13)

Many empirical tests of the capital asset pricing model have been conducted. Results have, in general, shown that the model is a useful framework for investigation of the risk-return relationship.\(^3\)

Of particular relevance to this present study are works by Myers and by Gordon and Halpern which use forms of the capital asset pricing model similar to Equations 2-11 and 2-12 to express required yields on shares of common stock for cost of capital estimation in regulated industries.\(^3\)

### Beta and the Cost of Capital

The two studies by Myers recommended the beta approach to the cost of capital estimation for use in rate determination cases of public


utilities. The first paper presented a thorough analysis of the applicability of financial theory to such rate determination cases, and concluded that the approach was an appropriate method to apply to the problem.

Breen and Lerner strongly contested Myers' conclusions in a subsequent article. Beta, the systematic risk of the firm, is a theoretical construct that is not directly measurable. While the normal procedure is to regress past observations of holding period returns, as expressed in Equation 2-13, on an index of market returns, other corporate variables may be important as well, they contended. Therefore, they performed an analysis in which they demonstrated that the actual calculated beta will depend upon the index used, the general direction of the index during the time period used for estimation, and the length of time used for estimation of the coefficient. Even regulatory decisions themselves influence beta, they contend.33

Myers' rebuttal agreed with Breen and Lerner that using beta in regulatory hearings requires caution, but suggested that they were overly cautious in some instances, and incorrect in others. The index selection problem and the length of time (or stability) problem can be greatly reduced by a careful researcher; Breen and Lerner were raising questions that can be easily solved. Stability may be a problem, but not so severe a one as they suggested if reasonable care is taken, and the index chosen "... is not critical to the tasks of judging firms' relative risks or

computing their cost of capital." Myers suggested that researchers need to focus on the real problems in using beta in regulatory proceedings, which are as follows:

First, \( \beta \) cannot be measured precisely. The possible errors in \( \hat{\beta} \) limit the precision of the conclusions that can be drawn.

Second, \( \beta \) may not be stable. This may also limit the precision of any conclusions, unless ways can be found to explain and predict shifts in \( \beta \).

Third, the capital asset pricing model may not be the whole story about risk and return, on either a theoretical or an empirical basis. It would be foolish, given the present state of the art, to propose [Equation 2-11a] as a complete basis for regulation. On the other hand, that fact that the model may not be exactly true does not mean that it should be thrown away. A good case can be made for the use of \( \hat{\beta} \)'s as part of the evidence in regulatory proceedings.

In a recent article, Pettway examined the use of beta in regulatory hearings, and reported several instances when applications of the capital asset pricing model were used in rate determination cases. He focused in particular on the second point raised by Myers for future investigation, the question of structural stability of the parameters of the model. He discovered that although major shocks such as the Consolidated Edison dividend omission led to periods of instability in excess of one year for beta estimates of his sample of thirty-six electric utilities, the overall situation was one of reasonable stability:

But this study did find that there were some fairly long periods of stability when ex post estimates were not significantly different from observed values. Additionally, the period of instability

\[34\] Myers, "On the Use of \( \beta \) in Regulatory Proceedings: A Comment," p. 625.

\[35\] Ibid., pp. 626-27.

found, although somewhat long, was transitory. Thus, it may be possible, by observing the structural parameters carefully, to derive estimates of the future long-run observed structural values. Under these conditions, the use of β's might be of some value in the regulatory process as one of the many factors to consider in determining the required rate of return.37

Thus, several researchers have found the beta model to be useful for cost of capital estimation. And while there is by no means unanimous agreement among researchers that the capital asset pricing model is an adequate representation of the structure of expected returns on securities, extensions and alterations of the model have improved the estimates considerably, and the basic mean-variance approach to asset valuation "shows promise of a remarkable robustness."38

III. Valuation of Closely Held Firms

Many writers have addressed the question of how, in actual practice, the closely held firm should be valued. Most of these studies are concerned with valuing closely held securities for estate taxation purposes, with valuing the corporation in order to make an initial public offering of its common stock, or with valuing a firm which is a potential merger partner.

Articles focusing on the tax question are most frequently encountered because a price must be established for every closely held company that is a part of an estate. The U.S. government and the beneficiaries of the estate are pitted as adversaries, and if they cannot agree upon a taxable price for the securities, the tax courts will set a price. By contrast, in a merger situation, if the buyer and the seller cannot

37Ibid., p. 247.
reach accord, the marriage is simply called off and no price is estab-
lished. Similarly, if the underwriters and the sellers cannot agree upon
a price for the public offering, the firm's stock will not be offered to
the public, and no price is determined.

In this section, articles concerning valuation in the tax courts
are first reviewed, and then mergers and acquisitions studies are ex-
amined. Finally, pricing of the initial public offering of the capital
stock of a company is discussed.

Tax Court Valuations

Federal estate and gift tax laws require that property subject
to taxation be valued at its fair market value. Fair market value is
traditionally defined as:

That price at which a willing buyer will buy and a willing
seller will sell a property, neither being under any compulsion
to buy or sell and both having full knowledge of all relevant
facts.39

Thus, fair market value is an attempt to determine intrinsic
value, but the situation existing when the owner of a closely held busi-
iness dies does not conform very well to the standards set in this defi-
nition. The "sellers" are not willing sellers; they only require a
valuation for tax purposes, and they want the "sale price" to be as low
as possible. The "buyers" are not willing buyers, but rather federal
tax agents who do not want the company; they only want to collect taxes
on the value established. Both parties are under compulsion, and fre-
quently neither party has full knowledge of the facts—the facts were

39William A. McClellan, "Valuation of Closely-Held Securities: 
Accounting Know-how is the Key," Journal of Accountancy 12 (March
buried with the owner. Under such circumstances it should not be surprising to learn that many different techniques of valuation are used, widely different "values" are claimed by the adversaries, and, as a consequence, many of the cases are referred to the court system for final resolution.

Various guidelines for valuation have been issued by the Internal Revenue Service and by the courts. In 1920, the Treasury Department issued an Appeals and Review Memorandum (commonly known as ARM 34) which suggested that a formula approach should be used to define value. "Normal earnings" should be calculated as an arbitrary percentage (such as ten percent) return on tangible assets. Earnings in excess of this normal return should be capitalized at a higher rate (such as twenty percent). The value of the enterprise was the sum of the tangible assets and the excess capitalized earnings. In actual practice, this formula was not always adhered to; some courts considered it, and others chose to weigh other variables and arguments instead.

Numerous other formula approaches have been used. Cohen reports that the California Inheritance Tax Department once used a formula that examined publicly traded firms similar to the closely held firm and arrived at a valuation using several ratios. An average price/earnings ratio for similar firms was calculated and multiplied by the closely controlled firm's earnings. Next, average price/dividend and price/book ratios were calculated and used in a similar manner. Then, a final market equivalent price was developed by giving the price/earnings

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calculation a weight of five, the price/dividend calculation a weight of three, and the price/book calculation a weight of one. This average was then discounted by twenty percent to compensate for a lack of marketability.\footnote{\textsuperscript{1}Furman reported another case in which weights of five, three, and two were given to the same three industry average ratios as mentioned above, and then a marketability discount of 12.17 percent was subtracted.\footnote{\textsuperscript{2}Other similar approaches are reported frequently. The industry average comparison became a favorite technique of the courts, but a variety of different industry ratios were used.\textsuperscript{3}}}

In 1959 the Internal Revenue Service issued, in Revenue Ruling 59–60, suggested guidelines for approaching the valuation of closely held securities. The ruling listed valuation techniques that were considered sound, and suggested eight factors which, while not all-inclusive, are fundamental and should thus be considered in each case. The eight factors are:

1. the nature of the business and its history, with emphasis on risk, growth, and diversification;
2. the economic outlook, both general and for the specific industry;
3. the book value of the corporation;
4. the earnings capacity of the firm and the extent to which costs are fixed or variable;
5. the capacity to pay dividends;

\footnote{John R. Cohan, "Valuation of Interests in Closely Held Businesses," \textit{Taxes} \textit{44} (July 1966): 504–9.}
\footnote{Furman, p. 932.}
6. any actual recent sales of the firm's stock, and the size of the block being valued;

7. the existence of goodwill or any other intangible assets;

8. the market price of publicly traded stock of similar industries.\textsuperscript{43}

This list is quite comprehensive, and the ruling has been reaffirmed several times since 1959 by the Internal Revenue Service. However, in actual practice, the ruling gave little guidance because of its generality. The lack of suggestions concerning what constituted the most important factors has frequently been criticized. "It also seems to minimize the importance of the capitalizing and the discounting of future income, devices that many believe are the very essence of the valuation process."\textsuperscript{44}

The ruling, in specifically discouraging the use of a formula and mathematical weights, led to a wide variety of interpretations:

Because of a lack of definite valuation methodology, it is not surprising to find variations between the Internal Revenue Service guidelines and the factors actually considered in tax courts. In fact, a review of cases adjudicated in the tax courts reveals a noticeable absence of a common basis for decision.\textsuperscript{45}

Martin and Votta examined abstracts of 156 valuation cases during the period from 1949 through 1970. They produced a list of thirty-one factors which were considered in two or more of the cases. The most frequently used factors were: historical earnings; book value; dividend

\textsuperscript{43}Cohan, p. 507.


yield; the size of the block of stock; prior sales of the stock; tangible
assets; marketability; quality of management; the general economic condi-
tions; and the value suggested by expert witnesses. The last factor, ex-
pert testimony, was mentioned most frequently; it was a factor in forty-
five percent of the cases examined. 46

The major purpose of a dissertation by Huggins was to determine
if the courts were using any commonly known valuation model in arriving
at decisions. He tested simple capitalization of earnings models: the
familiar Graham, Dodd and Cottle model which includes both dividends and
earnings, and an excess growth model suggested by Malkiel. 47 Huggins
used a correlation analysis methodology to determine if there were a high
correlation between the prices suggested by these models and the price
established by the courts. He discovered that no significant correla-
tions existed, 48 which implied that no such valuation models were being
used by the courts.

A general consensus of opinion of most authors examining the valu-
ation of closely held firms for purposes of tax assessment can be
stated. 49 Their list of important factors to consider corresponds closely

46 Ibid., pp. 903-5.
48 Kenneth M. Huggins, "The Valuation of Closely Held Corporations
49 In addition to Cohan, Furman, Martin and Votta, and McClellan,
see: Ernest J. Lawinger, "Appraising Closely-held Stock--Valuation
Methods and Concepts," Trusts and Estates 110 (October 1971): 816-19;
Irving J. Olson, "Valuation of a Closely-held Corporation," Journal of
Accountancy 15 (August 1969): 35-47; Jeffrey E. Lamson, "Factors that
Will Substantiate the Valuation of a Closely-held Corporation," Journal
of Taxation 34 (April 1971): 226-29; Robert E. Moroney, "Most Courts
to the list presented by Martin and Votta. By far the most important factor to consider is the earnings potential of the firm. However, the appropriate measure or estimate of earnings is not generally agreed upon. Most writers, if they make a recommendation at all, suggest an average of the earnings of the past five years.

Dividends are frequently mentioned, but are less important unless the stock being valued is a minority interest and the owners of the stock do not participate in the management of the firm. Under such circumstances, the expected dividend stream assumes the primary role. Book value is criticized as a measure, but, nevertheless, shows up in many formula approaches. General acknowledgment is given to the opinion that a discount should be offered for a lack of control or, alternately, that a premium should be paid for a controlling interest in a firm.

Although capitalization or discounting of future earnings estimates is a favorite technique advocated, little attention is given to the capitalization or discount rate. Most authors resign themselves to the same practice followed by the tax courts—the use some form of an industry average earnings/price ratio computed for publicly traded companies as the capitalization rate.

**Mergers and Acquisitions**

Corporations interested in acquiring another firm or interested in being acquired are concerned with determination of fair market value as defined earlier. No transaction would take place if agreement between the parties involved did not occur. However, valuation of the enterprise is but one of many factors considered when merger negotiations do occur. There are many reasons why the acquiring firm acquires
and the selling company sells, and sensing a "good deal" or a tremendous opportunity is not usually one of them. Rather, after the two enterprises have decided, for other reasons, that they wish to combine, the price to be paid for the disappearing company is determined in negotiations between the two interested parties. Nevertheless, valuation is an important part of the acquisition process, and several articles provide important information concerning valuation in this context.

Beasley studied the mergers and acquisitions of closely-held corporations in his doctoral dissertation. While the study was more concerned with the tax and legal questions involved than with the actual valuation process, it did include a chapter on valuation in which the capitalization of net cash flows emerged as the recommended solution. No empirical test of this methodology was conducted.

Detailed studies of savings and loan association mergers do not exist. However, Conklin interviewed executives of savings and loan holding companies and learned that they used a "unique yardstick" when evaluating a savings and loan association as a potential merger candidate:

The most important single factor, in their opinion, was the book value per share. To this they were willing to add a premium of one percent of total loans outstanding plus one percent of total savings deposits.52


51 William Howard Beasley III, "Mergers and Acquisitions of Closely Held Corporations" (Ph.D. dissertation, University of Texas at Austin, 1971). This work is a very thorough and informative treatment of the merger situation. Sections on the legal and tax status of mergers are particularly informative.

52 Conklin, p. 40.
The rationale for this premium was quite simple. The executives estimated that the cost of establishing a new savings account or a loan was about one percent, so they were willing to pay this fee in order to acquire a going concern with an established loan portfolio and with existing savings deposits.

The most thorough study of mergers and acquisitions of financial institutions is Piper's study of bank holding company acquisitions.53 Commercial banks differ considerably from savings and loan associations in the type and the average maturity of the loans they offer, and to a lesser extent, in the average maturity of their liabilities. But significant similarities also exist. Both industries are characterized by their highly regulated environment and by the fact that the assets they hold are financial, rather than real assets. Because of these similarities, studies of bank merger activity should be especially relevant to the researcher investigating savings and loan valuation.

Piper's study had many objectives:

The researcher was interested in studying (1) the raison d'être of group banking, (2) the role of acquisitions in the expansion of holding companies, (3) the characteristics of the acquired banks, (4) the economic basis for bank acquisitions by holding companies, (5) the acquisition prices paid for banks, (6) the post-acquisition performance of the acquired banks, (7) the profitability of acquisitions by holding companies and any significant differences between the profitable acquisitions, as a group, and the unprofitable acquisitions.54

The fourth and fifth of these objectives are most relevant to the present study and will be discussed below.


54 Ibid., Abstract.
Piper developed a valuation model that expressed the value of the potential holding company acquisition as the summation of the present value of all future cash inflows from the bank to the holding company and the additional debt capacity which would be available to the holding company as a result of the acquisition. He then tested the variables hypothesized to be important to determine if high correlations existed between these variables and the prices paid for the bank acquisitions. The data sample consisted of 146 bank acquisitions by holding companies during the period 1946-1967. Fifty-one of these acquisitions involved payment in cash, and the other ninety-five acquisitions involved an exchange of stock.

Since tax considerations were different for the two types of acquisitions, and since Piper believed that the actual valuation process used by the seller was different, two slightly different specifications of the valuation model were used. For the model involving tax free exchanges (i.e., payment as made in stock of the acquiring holding company), the dependent variable was the exchange ratio of common stock. The independent variables were relative value ratios of the acquiring and acquired firms. Five relative value ratios were significant at the one percent level and had correlations of .9 or higher with the dependent variables: relative earnings per share; relative book value per share; relative market price per share (if the information was available); relative dividend rates; and book value of the acquisition divided by the market value of the holding company. Coefficients of relative size, profitability, deposit growth, and capital adequacy were each less than

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55 Ibid., p. 172.
.15 in absolute value and did not differ from zero at the five percent level of significance.  

Multiple regression analysis was then used to determine the combined influence of the variables. However, the high correlations among the explanatory variables made it impossible to reach any conclusions concerning the relative importance of the specific variables in contributing to the final valuation.

Piper hypothesized that book value and earnings per share would be the most important explanatory variables in the model involving purchase for cash. The cash price paid per share was the dependent variable in this model, and earnings per share capitalized at the earning multiple of the holding company, book value per share, and market price per share were independent variables. The remaining independent variables, involving size, profitability, deposit growth, and capital adequacy were defined as they were in the exchange of stock model— as relative values between the acquired bank and the holding company. Book value per share was the variable most highly correlated with the price paid, but capitalized earnings per share was also highly correlated. No other variable has a coefficient that differed from zero at a ten percent level of significance.  

Darnell examined the prices paid for twenty-two bank acquisitions in New Jersey and sixty bank acquisitions in Pennsylvania during the period 1968 to 1972. Most of the acquired banks were quite small; the average size of the acquired banks was about $17.5 million in total

56 Ibid., pp. 182-83.
57 Ibid., pp. 192-93.
deposits, while the average size of the acquiring bank was about $150 million in total deposits. 58

The price that was paid for the acquisition, Darnell discovered, was usually stated as a premium over book value. Book value, he suggested, was used for several reasons: it is readily accessible and frequently used; it is not as variable as market price; and it is not generally influenced by market trends. However, the price paid should be based on earnings potential: "The acquiring bank would pay up to an amount equal to the discounted future earnings that the acquired bank would bring it." 59

Going Public

The valuation of the initial public offering of the capital stock of a corporation represents a third classification of valuation studies for closely held firms. When the firm itself wishes to raise additional capital through a public offering of its stock, or when the existing shareholders of the firm wish to dispose of part or all of their stock, the services of an investment banker are obtained. Investment bankers are valuation experts who perform a thorough analysis of the company. Since most underwriting is done on a fixed commitment basis, meaning that the underwriter agrees to purchase all of the stock at a fixed price, a careful analysis is of paramount importance. If the underwriting firm pays too much for the stock and is unable to market it at the established price, it, not the seller, will lose money.


59 Ibid., p. 18.
McCarthy and Healy suggest that of primary importance to the underwriting firm when it decides to sponsor an initial public offering of common stock are:

1. The earnings record of the firm over a period of years. Considerable growth over the past five years is especially important.

2. If the stock is being offered by the company, rather than the selling stockholders, it is important that the proceeds be beneficially utilized.

3. The company's position in its industry and the apparent future of the industry. The market share and growth in market share are both critical.

4. The company's ability to pay dividends on the stock being offered, so that affirmative action in this regard may be indicated in the prospectus.

5. The adequacy of the number of shares being offered to create a reasonable after-market in the stock.

6. The amount of stock offered by the principal stockholders relative to their holdings. If it is substantial, this may indicate a lack of confidence in the future of the company.

7. The all-important aspects of the strengths, abilities, and progressiveness of the management, management successors, and technical executives and personnel.60

Potential earnings, they agree, are the primary factor in the actual pricing of the issue. However:

Inasmuch as these [potential earnings] are largely conjectural, some known factors are used in lieu thereof—that is, the latest year's earnings of the company, the trend of earnings in recent years, general stock market conditions, market prices of stocks of other companies in the industry, and the company's performance compared with its competitors.61

These factors are then incorporated into an offering price based on capitalization of the latest year's earnings at an appropriate price-earnings

60 McCarthy and Healy, pp. 77-78.
61 Ibid., p. 79.
ratio developed from the other variables.

While the valuation process for going public as described above appears similar to the methodology used by the tax courts and by merger-minded firms, the actual valuation problem is different in at least two important aspects: there is a bias towards conservatism in the setting of the price; and rarely is a controlling interest offered.

As noted above, the underwriter of the issue will lose money if the price established is higher than the price the market will pay. The underwriter makes, in effect, an unilateral determination of value, since he is both the valuation expert and the initial purchaser of the issue (but then sells it in the market). The seller, of course, has the right to choose not to go public if the price offered is unsatisfactory, but will probably do so only if the price offered is far below his estimate of intrinsic value. In fact, it may even be in the best interests of the company and its principal stockholders if the price is fairly conservative. An active aftermarket is desirable, and is more likely to occur if the issue is slightly underpriced. Future public offerings will be easier to undertake if the initial offering is quickly absorbed by the market and if prices increase in the aftermarket.

At what price should the initial offering be made?

The answer is startlingly simple. It is purely a matter of judgment, exercised by the underwriter and his associates in the deal. Theirs is a merchandising problem. At what price will the public buy the stock, so that after the period of market-stabilizing operations the underwriters will not be the unwilling owners of substantial amounts of the offered stock? Like merchandisers in other lines they do not wish to have their capital unproductively tied up.62

62 Ibid.
The valuation of a controlling interest of a firm involves different considerations than the valuation of a minority interest. The discussion of valuation as determined by the tax courts placed considerable emphasis on the control factor. Since the initial public offering usually represents only a small percentage of the total stock outstanding, a significant part of the valuation problem, the value of the controlling interest, or the value of the entire corporation, is never directly addressed.

IV. Accounting Substitutes for Market Risk Measures

The widespread recognition in the past decade of beta, the systematic risk of a security or asset, as an appropriate measure of the riskiness of the security or asset has led to a new area of research—what relationships, if any, exist between the market based risk measure, beta, and the traditional measures of risk developed from accounting data. Many studies have focused on this question, and several of them provide insight into solutions to the question of valuation of the closely held firm.

Beaver, Kettler and Scholes examined the correlation between the market based risk measure, beta, and seven accounting based risk

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measures: Dividend payout; growth; leverage; size; liquidity; variability of earnings; and covariability of earnings. They considered the accounting risk measures to be surrogates for the total variability of return of a firm's equity securities. The total variability includes both systematic and individualistic components. However, as long as the two risk components are positively correlated (and most empirical evidence suggests that they are), the accounting risk measures can be viewed as surrogates for systematic risk as well. 65

The data sample for the Beaver, Kettler and Scholes study consisted of the 307 firms for which both financial statement data from the Compustat Industrial Tape and dividend and security price data from the Center for Research in Security Prices (CRSP) tape were available for the years 1947 through 1965. Correlations between accounting and market risk measures were calculated at both the individual security and portfolio (five securities were included in each portfolio) levels for two periods, 1947 to 1956 and 1957 to 1965. All seven accounting variables had statistically significant correlations with the market variable in at least one of the two periods, and four of the seven were significant in both periods. These four variables, listed in order from strongest association to weakest, were: earnings variability; dividend payout; earnings covariability; and liquidity. 66

After establishing that high correlations existed between market and accounting-based betas, Beaver, Kettler and Scholes examined the

65 Ibid., p. 659.
66 Ibid., p. 669.
forecasting ability of the accounting-based betas. They tested the
ability of the accounting variables to forecast the future market risk
measures. Accounting variables from the first period were used to fore-
cast the market betas for the second period. The results of this fore-
casting model were compared with a naive forecasting model which stated,
simply, that the market beta in the second period would be the same as
in the first period. The naive model was actually quite strong, since
a preliminary study had shown a high degree of stationarity in the
betas. However, the forecasting model developed using accounting
variables was consistently superior to the naive model, both on an
individual security basis and on a portfolio basis.

Even if the technique used had not been able to outperform the
naive model, the authors suggested, it was an appropriate, and perhaps
superior, forecasting approach. The model used was, in fact, an instru-
mental variables technique. As indicated above, an instrumental vari-
ables estimation procedure is an econometric technique used to reduce
or remove measurement errors when there is reason to believe the observed
estimates of a parameter were subject to error. A frequent criticism of
the beta model is that it may be misspecified due to the use of ex post
rather than ex ante returns, and therefore, measurement error exists.
Thus, the instrumental variables technique used by Beaver, Kettler and
Scholes may not have just been acceptable, but may actually have been
superior to the direct market estimation procedure, since it used

67 Ibid., p. 665.

68 See footnote 17 above and Appendix A.
accounting risk measures as instruments. 69

Studies since the Beaver, Kettler and Scholes paper have focused mainly on issues of methodology and variable specification. Differences of opinion have surfaced concerning the strength of the association and the correct way to define the accounting variables, but the central hypothesis of the Beaver, Kettler and Scholes work, that accounting variables contain important information about the market determined measures of risk, has not been seriously challenged.

Rosenberg and McKibbon reached conclusions similar to Beaver, Kettler and Scholes—accounting variables can be used as instrumental variables to remove estimation error and provide superior forecasts of the market risk measure. 70 This study reached the same conclusion as Beaver, Kettler and Scholes concerning the accounting covariability of earnings (the accounting beta) measure also—it is not the most important variable for predicting the market beta. Other variables were found to be more important, probably due both to multicollinearity with the other accounting risk measures and also to the small number of annual observations of the earnings variable used to calculate it.

Gonedes questioned the wisdom of using the market value of the common stock as the deflator of the accounting variables, and suggested that spurious correlation was induced in the Beaver, Kettler and Scholes model when a market measure was used in the specification of the accounting returns. He also suggested that the betas should be defined in

69 Beaver, Kettler and Scholes, p. 671.
terms of first differences of returns, rather than in terms of the actual return series. When accounting betas were computed with earnings deflated by book value of assets, the association between market betas and accounting betas was reduced to a nonsignificant level. For betas defined in terms of first differences of returns, the association between market and accounting betas was small but statistically significant. 71

Bildersee extended the Beaver, Kettler and Scholes methodology to include certain dummy variables that represented the firm's industry and management decisions concerning dividend policy and diversification. His data sample included a number of preferred stocks as well as common stocks. Bildersee concluded from his analysis that better associations exist between market and nonmarket measures of risk when the nonmarket variables are formed with accounting and decision variables than when they are formed with accounting variables only. 72

In response both to criticisms of the earlier Beaver, Kettler and Scholes study and to new developments in forecasting methodology, Beaver and Mangold conducted a further examination of the association between market determined and accounting determined measures of risk. Several issues or questions were investigated, including: 73

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1. Would a Bayesian adjustment procedure, incorporating cross-sectional information into the beta estimates, improve the estimates?  

2. How should the accounting returns be defined? What definition of earnings should be used? What denominator should be used to deflate the earnings figure? The previous Beaver, Kettler and Scholes study used market value of common stock; Gonedes used total assets; book value of common equity would also be a theoretically defensible deflator.

3. How should autocorrelation be treated? Should first differences be used, rather than simple returns? Should a two-stage least squares method be used?

4. Should accounting betas be computed from all information available, or from some subperiod of time, so that recent changes in operating characteristics will be weighted more strongly? Would the reduction in sampling error achieved through the use of additional years of earnings data more than compensate for the loss of accuracy due to nonstationarity of the accounting betas over time?

Beaver and Mangold's study used two data samples—one sample consisted of 254 of the original 307 firms examined in the Beaver, Kettler and Scholes paper; the second sample consisted of ninety-four of the ninety-nine firms studied by Gonedes. The reduction in size of each sample was due to problems with missing data. For each firm ten different market betas were calculated, reflecting combinations of different time periods and Bayesian adjustments. Fifty-four different accounting betas were defined, reflecting all combinations of the specification procedures (three time periods, three ways to define the return series, three transformations, and two adjustment procedures).

74 Previous research cited by Beaver and Mangold, O. Vasichek, "A Note on Using Cross-Sectional Information in Bayesian Estimation of Security's Beta," Journal of Finance 28 (December 1973): 1233-39 and M. Bogue, "The Behavior and Estimation of Systematic Risk," (Graduate School of Business, Stanford, 1972), concluded that such an adjustment procedure was an improvement over ordinary least squares or instrumental variables techniques.
Correlations between the accounting betas and market betas were computed, and the following conclusions were made:

1. Regardless of the exact specification of the betas, significant correlations existed between accounting and market betas.

2. The Bayesian adjustment procedure strengthened the degree of association. However, the improvement was not substantial.

3. The returns specified as earnings deflated by market value demonstrated the highest correlations, but deflation by net worth was only slightly inferior to the market value technique.

4. There was no clear superiority among the regular, first difference, and two-stage least squares methods of defining the returns.

5. Total period accounting betas had stronger associations with the market betas than did subperiod accounting betas.\(^75\)

Based upon this exhaustive analysis and a partial reconstruction of the Gonedes study (in which they found significant errors\(^76\)), the authors concluded that while there is significant correlation between market and accounting betas regardless of the specification of the variables, accounting betas appear to be only one of the explanatory variables for the market betas.\(^77\) Future studies, they suggest, should realize that measurement error in accounting betas cannot be reduced as much as in the market betas, and thus, the inclusion of other variables would be appropriate.

Beaver, Kettler and Scholes concluded their paper with the suggestion that the methodology they used might be useful for decisionmaking.

\(^75\) Beaver and Mangold, pp. 262-63.

\(^76\) Ibid., p. 264.

\(^77\) Ibid., p. 265.
when market determined risk measures are not available. Specific applications recommended included the case of the privately held firm going public for the first time, and the case of the multi-division firm with divisions operating at different levels of risk. Gordon and Halpern subsequently examined the second of these suggestions. They used a variation of the instrumental variables technique developed by Beaver, Kettler and Scholes to estimate the cost of capital for nontraded firms or for a division of a publicly traded firm.

In Equation 2-11 above, the yield investors require on shares of a firm's stock was expressed as a function of the risk free rate of return, the return on the market, and the firm's unique beta coefficient. The risk free return and the market return are available information if ex post data can be substituted for an ex ante concept. But since the normal procedure for estimation of beta involves regressing the ex post market rate of return on the ex post rate of return of the firm, the beta calculation presents a problem when dealing with closely held firms. The return calculation involves the price of the firm's stock, information that is not normally available for the closely held firm. As a solution, Gordon and Halpern suggested:

Assume that there is a statistic $\hat{\beta}_j$ based on non-market data with which $\beta_j$ is highly correlated. Specifically, let

$$\hat{\beta}_j = \lambda_0 + \lambda_1 \tilde{\varepsilon}_j$$

subject to a margin of error that we can ignore.

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78 Beaver, Kettler and Scholes, p. 680.
79 Gordon and Halpern, pp. 1153-63.
80 Ibid., pp. 1154-55.
Their analysis then suggested that this estimate of $\hat{\delta}_j$ developed from $\hat{\delta}_j$ could be used as the estimate of systematic risk for the non-traded firm. Although they did not identify this procedure by name, it was in fact, simply an instrumental variables technique using only one instrument, $\hat{\delta}_j$. The actual accounting variable selected as the instrument was the rate of growth of earnings, a variable which they demonstrate should be highly correlated with the rate of return.

Gordon and Halpern tested their model for the period 1957 to 1968 using a sample of 49 industrial companies and public utilities. The correlation between $\hat{\beta}$ and $\hat{\delta}$ was .72 and was significant at the .1 percent (.001) level. The authors realized that a higher correlation would have been desirable, and that a problem with omitted variables was evident, but the results were adequate for their intended objective, the estimation of the cost of capital for the mail division of the airline industry.

In a comment on the Gordon and Halpern article, Weston and Lee suggested a more direct method for estimating the required rate of return or the cost of capital for the firm or for a division of the firm. Equation 2-10 above can be restated as:

$$E(R_j) = R_f + \lambda \text{COV}(R_j, R_M)$$

(Equation 2-10a)

where $\lambda = \frac{(E(R_M) - R_f)}{\sigma^2(R_M)}$, commonly called the market price of risk.

They then contend that the market value of the firm or division can be

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81 Ibid., p. 1158.

expressed as:

\[ V_j = \frac{\mu_j - \lambda \text{COV}(\mu_j, \mu')}{\sigma_f} \]  
(Equation 2-14)

This certainty equivalent formulation of the valuation equation can then be used to obtain the cost of capital:

\[ E(R_j) = \frac{\mu_j}{\sigma_j} \]  
(Equation 2-15)

Gordon and Halpern's reply agreed that on a theoretical basis Weston and Lee's comment had merit, but they questioned the empirical usefulness of the comment on several grounds. Even if Equation 2-14 properly calculates \( V_j \), the value of the firm, Equation 2-15 is a proper measure of the cost of capital only if the firm "is not expected to undertake any investments with a return above or below its cost of capital."\(^{83}\) The authors were even more concerned with the estimation of \( V_j \) provided. \( \lambda \) is difficult to measure, due to uncertainty concerning estimation of \( \mu_M \), the return of the market portfolio. Even more troublesome, though, is the assumption on which Equation 2-15 is based, that;

\[ \text{COV}(\mu_j, \mu') = \sigma_j \text{COV}(\mu_j, \mu') \]  
(Equation 2-16)

This equation, they point out, was derived by assuming that \( \mu_j \), which here represents operating income for the period, is equal to holding period income (dividends plus change in value, as expressed in Equation 2-13). Observations of actual firms' returns do not support this assumption. Therefore, Gordon and Halpern believe that the left

hand side of Equation 2-16 is "such a poor approximation of the right hand side as to make . . . [Equation 2-14] worthless for the estimation of the market value of the non-traded firm or division."

Gordon and Halpern noted that both Equation 2-14 and Equation 2-16 were taken from Hamada's work. An examination of this article will provide additional insight into the topic, and will clarify several omissions or oversights of the papers by Weston and Lee and by Gordon and Halpern.

Both of these papers expressed the market value of the firm as \( V_j \), the net operating income as \( X_j \), and the required return or cost of capital as \( R_j \). These symbols and the resulting equations are correct interpretations of Hamada's work only if the assumptions are made that there are no taxes and that the firm is a debt-free firm. Otherwise, the equations must be recast to reflect taxation, leverage, and the difference between the cost of equity capital and the overall cost of capital.

Since this study is concerned with valuation of the equity portion of the firm, the more complex formulations will be needed, and are developed below. Equation 2-14 above, the expression of the value of the equity of the leveraged firm in a world of taxation would become:

\[
V < 3
\]

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84 Ibid., p. 1782.


86 The use of \( V \) by Weston and Lee illustrates a common problem in finance literature—the failure to distinguish between \( S_d \), the market
\[ S_j = \frac{E(X^\gamma_B) - \lambda COV(X^\gamma_B, \tilde{R}^\gamma_M)}{\tilde{R}_F} \]  

(Equation 2-14a)

and Equation 2-15, the required rate of return or cost of equity capital would become:

\[ \tilde{E}(R^\gamma_j) = \frac{E(X^\gamma_B)}{S_j^\gamma} \]  

(Equation 2-15a)

where \( X_B \) is used to represent \(((X_j - IB)(1-\tau))\), income after interest and taxes.  

V. Summary

This chapter contained a discussion of the concept of value and a review of key articles addressing cost of capital and valuation. Specific attention was focused on valuation of closely held firms, and the development of risk measures for firms whose securities are not publicly traded. These ideas and concepts will form the basis for the models of valuation developed in the next chapter.

87 These equations were derived from those of Hamada. Equation 2-15a is Hamada's Equation 4b, and 2-14a can be developed from his Equations 6 and 7. It should be noted that Equation 2-14a would not be universally accepted as a valuation formula. Technically, the one period form of the capital asset pricing model would require that the denominator represent \((1 + R_P)\) rather than \(R_B\), and the numerator would include an estimate of terminal value, \(S_j\). Also, changing expectations over time can influence the covariance term, and this revision will usually increase the covariance term, thus decreasing the valuation estimate. A thorough discussion of this phenomenon can be found in: Charles W. Haley and Lawrence D. Schall, The Theory of Financial Decisions (New York: McGraw-Hill Book Company, 1979), pp. 195-200.
CHAPTER III

MODEL DEVELOPMENT AND METHODOLOGY

Valuation models for the closely held firm, and specifically, for the savings and loan association, will be developed in this chapter. Relevant information from the studies reviewed in chapter II will be synthesized into a framework which will be both theoretically defensible and empirically testable.

The first section will deal with the selection of a valuation approach from among those described in section II of the previous chapter. The second section will focus on the development of models for the determination of the cost of capital. The third section will contain an integration of the material in the first two sections into valuation models for the savings and loan association. The fourth section will present a description of the data to be used for developing and testing the models, and the final section will develop the hypotheses to be tested and the methods for testing them.

I. Valuation Approach

In chapter II, four distinct approaches to valuation of the firm were set forth. Three of them employed variations of the earnings stream of the firm, and were referred to as the earnings approach. A fourth approach focused on the stream of dividends the firm pays to its owners, and was called the dividend approach. Under conditions of certainty, the
approaches were shown to be equivalent. Under uncertainty, however, controversy remains. While the greater amount of research results appears to support those who argue for dividend irrelevance, the issue is by no means closed, and the argument remains a spirited one.

Fortunately, the controversy can be avoided in this study. For while dividend policy may be important in pricing shares of a company, it is clearly not so when valuing the firm as a whole, or valuing controlling interest in a firm. The owner of a firm has the right to set whatever dividend rate he wishes, subject to capital constraints. Thus, if a new owner acquires control of a firm, he can be expected to set whatever dividend policy he considers to be optimal, given the firm's earnings.

An earnings approach to valuation is therefore more appropriate for the valuation of an entire firm. And of the three earnings approaches, the investment opportunities approach:

\[ V_0 = \frac{x_0}{p} + \sum_{t=0}^{\infty} \frac{I_t [P^*_t - P]}{(1 + p)^{t+1}} \]  

(Equation 2-4)

will be chosen as the most appropriate. This formulation expresses the value of the firm as the sum of two terms: the capitalized value of current earnings; and the present value of all future opportunities of the firm to invest at a rate of return greater than its cost of capital. If the firm possesses no opportunities to invest at such excess rates of return, the second term of the equation reduces to zero, and the value of the firm is simply the capitalized value of current earnings.

This approach offers several benefits to the researcher. Estimates of current earnings are readily available. While true earnings of the firm may differ from those published in accounting statements,
present reported earnings certainly represent an appropriate point of departure. The second term of the expression, the goodwill or excess earnings opportunities term, clearly focuses on the critical elements involved—the normal rate of return \((p)\), the above average rate \((p^*)\), and the amount of the investment opportunity that will earn the excess rate of return in each period \((I_t)\). Such estimates, while certainly difficult, seem less difficult than the estimates required in other approaches—items such as dividends, earnings, cash receipts, or cash outflows for each year in the future.\(^1\)

The separation of the normal earnings term and the excess earnings term is the most important benefit. For the possibility exists that certain firms, or certain industries, do not possess significant opportunities to invest at rates of return above the cost of capital. And savings and loan associations would appear to be likely candidates. Savings and loan associations have been faced with rising interest rates throughout most of the past twenty years. With most of their assets consisting of long term commitments (twenty-five and thirty year mortgages with effective lives of six to seven years), and their liabilities consisting of certificates of deposit with maturities of from ninety days to eight years and passbook accounts subject to withdrawal in ninety days or less, savings and loan associations have faced a serious refinancing problem. That is, the associations have not been able to reinvest maturing mortgage funds at higher rates to the extent

\(^1\)See appendix B for a discussion of the equivalence of cash flows and earnings approaches.
that they have been forced to refund liabilities at higher rates.²

Savings and loan associations have been further handicapped by their inability to diversify their holdings into loans other than long term mortgage loans. Regulations have restricted their lending and forced them primarily into these long term assets. Only a small percentage of other types of loans have been allowed.³ While recently enacted legislation will allow greater diversification, mortgage loans will remain the primary assets of savings and loan associations for the foreseeable future.

Intense competition also exists, both for the depositor's dollar and for the right to provide the home mortgage. Commercial banks, mutual savings banks (in some eastern states), and credit unions are in direct competition for savings deposits, and many other investments such as life insurance, U.S. Treasury securities, and high quality municipal and corporate bonds also represent viable savings alternatives. Less competition is faced in the mortgage market, since many of the other financial institutions prefer not to invest in such assets. Nevertheless, sufficient competition does exist, either from competing savings and loan associations in the area, or from other institutions, to assure that the association cannot charge a mortgage interest rate far in excess of prevailing rates throughout the country.

²United States League of Savings Associations, Savings and Loan Fact Book '79, pp. 15, 74 provides information concerning rates paid on savings deposits and rates earned on new mortgage instruments.

³As of December 31, 1978, mortgage loans represented 82.7 percent of the average association's assets, insured mortgages and mortgage backed securities represented another 3.2 percent, and no other loan classification contained more than .8 percent of total assets. See: Savings and Loan Fact Book '79, p. 80.
Considering all the unfavorable circumstances faced by savings and loan associations, and the considerable difficulties experienced by previous researchers in estimating the growth term, the researcher might simply contend that true growth opportunities are not significant, drop the growth term, and focus on the capitalization of current earnings. The results of such an approach could then be examined to see if, in retrospect, such a simplification was warranted.

II. Estimating the Cost of Capital

Two different approaches to empirical estimation of the cost of capital of the firm seem justifiable. Both will be discussed below.

The assumption could be made that the savings and loan industry represents a homogeneous risk class, and therefore, all firms in the industry have the same cost of capital. A finite growth model could be specified which includes both an allowance for normal earnings capitalization and a term which represents the value of future growth opportunities. This was the approach used by Miller and Modigliani, Litzenberger and Rao, McDonald, and Higgins. The model offers the obvious advantage of considering the growth potential of the industry in the estimate, and does provide an estimate of $T$, the period for which growth is discounted. However, it does not allow for different growth potentials or for different costs of capital for firms within the industry.

The second possible approach is to assume that each firm within an industry is in a unique risk class, and thus has a unique cost of

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These articles were summarized in section II of chapter II. It should be noted that Litzenberger and Rao did allow for some variability among firms. The formulation used allowed unique costs of capital for each firm to be calculated from an industry-wide marginal rate of time preference.
capital. The required rate of return could be computed in the following manner. First, derive the firm's unique beta coefficient, \( \hat{\beta}_j \), based on the correlation between returns of the firm and the returns of the market; then, determine the risk-free rate \( \hat{R}_F \) and the expected return on the market portfolio, \( \hat{R}_M \). Finally, apply Equation 2-11 of chapter II, compute the required return, \( \hat{R}_j \), as:

\[
\hat{R}_j = \hat{R}_F + \hat{\beta}_j (\hat{R}_M - \hat{R}_F).
\]

If the capital asset pricing model adequately describes the return-generating process in the market place, and if \textit{ex post} returns adequately approximate the \textit{ex ante} concept from which beta is developed, \( \hat{R}_j \) will be an unbiased estimate of the cost of equity capital for the firm.

The ability of this approach to treat each firm as an unique entity, with a distinct cost of capital, is an appealing one. Not so appealing, however, is the failure of this approach to deal specifically with the growth term. For with the cost of equity capital calculated in this manner, no estimate of the growth period \( T \) is provided. Thus, after the [unique] cost of capital is calculated for each firm, additional steps would be necessary to determine what excess growth opportunities are available to the firm and for what length of time these opportunities would exist.

### III. The Models

The investment opportunities approach to valuation of the firm was shown in section I to be an attractive form, due to its separation of the firm's value into two distinct parts—a no growth component and a growth component, both of which are a function of the cost of equity
capital. The simplified finite growth model presented in chapter II:

\[ S_0 = \frac{X_0}{p} + \frac{(p^* - p)zI_0T}{p} \]  \hspace{1cm} (Equation 2-4a)

is a form which is conducive to empirical testing.

In section II, two different approaches to the estimation of the cost of equity capital were reviewed. While the homogeneous risk class form did provide an estimate of the growth period T, the estimation of \( I_0 \), the amount of investment expected to earn an above-average return, and \( p^* \), the excess return, have not been adequately addressed by researchers using either approach. The estimation of the growth term is probably worthy of a major research study by itself. But it may be possible to avoid that difficulty here. As noted in section I, the savings and loan industry does not, at the present time, seem to possess significant opportunities to invest at excessive rates of return. Even if they do, the example shown below in table 1 demonstrates that the contribution of the "growth" term to the overall value of the firm is small.

Given this fact, and the formidable problems encountered in accurately estimating the components of the growth term, it seems prudent to proceed without the growth term and return to it later only if its omission seems to seriously distort the models.

With concern for the growth term greatly reduced, the capital asset pricing model formulation emerges as the preferred method of estimation of the cost of capital. It allows for the possibility that each firm has a distinct cost of capital. If this is not true, then the individual estimates will be identical, or nearly so, and the results will support the homogeneous risk class assumption.
TABLE 1

THE GROWTH TERM

Assume that a savings and loan association has total assets of $100 million. Assume that it has a return on assets of .6 percent, and an equity to total assets ratio of six percent (these figures represent approximations that are similar to industry averages). Assume that through loans maturing, loans being paid off, or new deposits flowing in, new investment of $20 million, or twenty percent of assets, will be possible during the year. Make the assumption that this entire new investment will earn a return of eleven percent, one percent above the firm's cost of equity capital of ten percent. Finally, assume that this excess earning opportunity will persist for four years (using the estimates of Higgins and of Litzenberger and Rao from their studies of the electric utility industry). Using the Higgins model developed in chapter II:

$$S_0 = \frac{X_0}{p} + \frac{(p^* - p) z I_0 T}{p}$$

and with:

- $X_0 = $600,000
- $p = 10$ percent
- $p^* = 11$ percent
- $z = 6$ percent
- $T = 4$ years
- $I_0 = 20$ million

$$S_0 = \frac{600,000}{10} + (0.11 - 0.10) \cdot 0.06(20,000,000) \cdot 4 \cdot 0.10$$

$$= $6,000,000 + $480,000$$

$$= $6,480,000$$

Thus, under the conditions established, the growth term contributes only 7.4 percent of the value of the firm. Over ninety-two percent of the value is due simply to capitalization of normal earnings.

The Cost of Capital

As noted in the final section of chapter II, the capital asset pricing model cannot be applied directly to the closely held firm because
no market price information is available, and therefore, no holding period return calculations can be made. However, the problem is not insolvable. Beaver, Kettler and Scholes demonstrated that an instrumental variables technique could be employed to provide estimates of beta which might actually be superior to betas calculated solely from market holding period returns. Gordon used such a technique to estimate the cost of capital for the mail division of the airline industry. Similar techniques will be used in this study of the savings and loan industry.

Let:

\( \beta \) denote the theoretical beta, which is unobservable because it is formed from ex ante expectations

\( \hat{\beta} \) denote the empirical estimate of \( \beta \), computed by a linear regression of the form expressed in Equation 2-12

\( \hat{\beta} \) denote an estimate of \( \beta \) derived using an instrumental variables approach with non market variables

\( z_1 \ldots z_n \) denote a series of \( n \) accounting risk variables

Using holding period return information for publicly traded savings and loan associations and holding companies and a market index, \( \hat{\beta}_j \) can be calculated for each of the holding companies or associations. Then, a cross-sectional regression equation can be developed which expresses \( \hat{\beta}_j \) as a function of the accounting risk variables:

\[
\hat{\beta}_j = \hat{\beta}_0 + \hat{\beta}_1 z_{1j} + \ldots + \hat{\beta}_n z_{nj} + e_j \quad (Equation \ 3-1)
\]

The instrumental variables beta, or nonmarket beta, can then be computed for firm \( j \) by using the coefficients of Equation 3-1 in the following manner:

\[
\hat{\beta}_j = \hat{\beta}_0 + \hat{\beta}_1 z_{1j} + \ldots + \hat{\beta}_n z_{nj} \quad (Equation \ 3-2)
\]
\( \hat{\beta}_j \) and \( \hat{\gamma}_j \) are both estimates of the unobservable \( \beta_j \), the systematic risk of firm \( j \). Using these estimates, two (undoubtedly) different costs of equity capital, or required rates of return, can be calculated:

\[
\hat{R}_{j1} = \hat{R}_F + \hat{\beta}_j (\hat{R}_M - \hat{R}_F) \quad \text{(Equation 3-3)}
\]

\[
\hat{R}_{j2} = \hat{R}_F + \hat{\gamma}_j (\hat{R}_M - \hat{R}_F) \quad \text{(Equation 3-4)}
\]

The two different estimates, the first developed from market price information, and the second developed from nonmarket information, can be used to calculate estimates of value for each firm. If estimates of value developed from the nonmarket cost of capital estimates \( \hat{R}_{j2} \) are as accurate, or more so than the market-based estimates \( \hat{R}_{j1} \), in predicting the computed market price of the holding companies, then the nonmarket model is indeed a strong one. It should be stressed, however, that to employ the models in such a manner is to compare estimates of intrinsic value with computed market prices. And, as explained in chapter II, these may not be the same. Nevertheless, such a test should be useful in its own right, and should serve as a validation procedure if, indeed, price is an unbiased estimate of intrinsic value.

Whether or not \( \hat{R}_{j2} \) provides a better estimate than \( \hat{R}_{j1} \) of the cost of equity capital for publicly traded firms, it is (if the instrumental variables \( z_1 \ldots z_n \) were selected properly) an appropriate estimate of the cost of equity capital for nontraded firms, since no market-based estimate exists. By using the parameters \( \hat{\gamma}_0, \hat{\gamma}_1, \ldots \hat{\gamma}_n \) developed in Equation 3-1, and hypothesizing that the same risk measures apply to nontraded firms as to traded firms, cost of equity capital estimates can be developed for the closely held savings and loan associations. The nonmarket beta, \( \hat{\gamma}_j \), can be estimated using Equation 3-2, and
this risk measure can be used to calculate $\hat{R}_j$, the cost of equity capital for firm $j$.

**The Earnings Estimate**

The no-growth valuation model suggested in section I of this chapter requires only estimates of the cost of equity capital and the current earnings of the firm in order to estimate the current value of the firm. In such an earnings capitalization model, the selection of the correct earnings figure and the correct cost of capital figure are obviously critical; any error will be greatly increased in the capitalization process. Current published earnings may not be the best estimate of $X_0$, the earnings of the firm, for several reasons. For example, certain industries (of which the savings and loan industry is one) have highly volatile earnings performance, due to business cycles. Some averaging of several years of reported earnings is thus warranted. A rather common convention seems to be to use a simple average of the past five years earnings. However, this procedure probably results in an estimate of earnings which is too conservative for most firms. Even if a firm is not a growth company in the sense of being able to invest at rates of return higher than the cost of capital, it is usually an expanding firm, one with earnings increasing over time.\(^5\) If the firm invests in new projects each year, the earnings stream will grow, even if the project earns a return equal to the cost of capital.

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A variation of a technique used by Higgins to estimate the earnings of the firm was employed in this study. A "trend value" was computed which realized that earnings need to be smoothed, yet also realized that growth over time makes simple averages inappropriate.

Let:

\[ X = \text{earnings} \]
\[ t = \text{time} \]
\[ a & b = \text{constraints} \]

Then, develop the linear regression model:

\[ \ln(X) = \ln(a) + b[\ln(t)] + e \]  
(Equation 3-5)

from all available earnings data for the firm. The trend value for current earnings, \( \hat{X}_0 \), can then be estimated as:

\[ \ln(\hat{X}_0) = \ln(\hat{a}) + \hat{b}[\ln(t)] \]  
(Equation 3-6)

where \( t_0 \) is the current year and \( \hat{a} \) and \( \hat{b} \) are estimates of the parameters.

Using the estimate of trendline earnings and the appropriate cost of equity, an estimate of the intrinsic value of the firm can be computed. Five specific valuation models are developed below, incorporating the techniques discussed in the preceding sections.

The Market Beta Model

Estimates of market betas were calculated for a sample of holding companies whose securities were actively traded. These betas were developed using monthly holding period returns for the five years prior to

the dates when the value estimates were to be made\(^7\) and the Fisher value-weighted return index (which will be described in section IV). A linear regression of the form specified by Equation 2-12 was used to estimate the market beta. Then, the cost of equity capital was calculated using Equation 3-3 (the estimates of the risk free rate, \(R_p\), and the risk premium, \((R_m-R_p)\), will also be discussed in section IV). Finally, this cost of equity capital for firm \(j\), \(\hat{R}_{j1}\), was used to capitalize the trend value of present earnings to obtain the market beta estimate of the intrinsic value of the equity of the firm:

\[
\hat{S}_{j1} = \frac{\hat{X}_0}{\hat{R}_{j1}} \quad \text{for all firms, } j=1,m \quad \text{(Equation 3-7)}
\]

The subscript denoting time was dropped from this and all subsequent estimates of \(S\), the value of the equity of the firm, since the time period will always be clear. The second subscript now denotes the model number, so that easy reference can be made to the different value estimates.

The Beaver, Kettler and Scholes Model

In this model, variables believed to be related to the riskiness of the firm were selected to serve as instruments to develop an instrumental variables estimate of the true but unobservable beta, \(\beta_j\). These variables were used as independent variables in Equation 3-1, and the

\(^7\)Originally, the betas were calculated using all available price information, which usually involved the period from January 31, 1966 to the month prior to the period of valuation. However, considerable non-stationarity was observed in the betas calculated in this manner. Investigation revealed that this non-stationarity was attributable to the fact that holding period returns for savings and loan stocks were very volatile in the 1960's. Thus, any beta calculation that included these earlier years tended to overstate the degree of systematic risk that had been observed more recently. It is believed that using the shorter period of time to estimate the betas better reflects the firm's systematic risk at the time of the valuations.
market beta estimate for each holding company, \( \hat{\beta}_j \), was used as the dependent variable. The parameters \( \hat{c}_0, \hat{c}_1, \ldots, \hat{c}_n \) developed in this manner were used in Equation 3-2 to calculate the nonmarket beta, \( \hat{b}_j \), for each firm. Then, this \( \hat{b}_j \) was used in Equation 3-4 to compute the cost of equity capital, \( \hat{R}_{j2} \). Finally, this cost of capital estimate was used to capitalize the trend value of present earnings to obtain the instrumental variables estimate of the intrinsic value of the equity of the firm:

\[
\hat{S}_{j2} = \frac{\hat{X}_0}{\hat{R}_{j2}} \quad \text{for all firms, } j=1,m \quad \text{(Equation 3-8)}
\]

The instruments selected for this model were similar to those used by Beaver, Kettler and Scholes, but were not identical. The variables used in the model are listed below. Although Beaver, Kettler and Scholes found a dividend payout variable to be highly significant, a payout variable was not included in this study. Since very few of the holding companies or acquired savings and loan associations paid a cash dividend, the inclusion of a dividend payout variable would have created severe econometric problems. Instead, an interest rate variable was included.

1. **Interest rate.** The average interest rate paid to depositors and other creditors should give a firm-specific, as opposed to market-wide, indication of the cost of funds to the savings and loan. This variable should be positively related to the risk level of the firm. It was calculated as the two-year average of total interest and dividends paid to depositors and other creditors divided by total liabilities.

2. **Leverage.** The volatility of the firm's earnings is directly related to the level of debt used in the capital structure. Thus, the
debt/equity ratio was included as a measure of the leverage the firm employed. The variable was calculated as the two-year average debt/equity ratio.

3. **Liquidity.** The level of liquidity of the firm is often used as a measure of riskiness. The hypothesis is that (within reasonable limits) higher levels of liquid assets prevent the firm from experiencing distress situations where it cannot pay its bills. High levels of liquidity also allow the firm to seize upon favorable investment opportunities. However, a large portion of a savings and loan's cash and liquid securities represent forced liquidity required by regulatory authorities; thus, liquidity could be either directly or inversely related to firm riskiness. The variable was calculated as the two-year average of cash and securities divided by total assets.

4. **Size.** Total assets of the firm are often believed to be a measure of the risk level of the firm. To the extent that the returns of the assets of the firm are not perfectly correlated, larger firms will have lower variances of rates of returns than smaller firms. The variable was calculated as the natural logarithm of the average size of the firm over a two-year period.

5. **Profitability.** The profitability of the firm should be related to the market measure of risk. It has been demonstrated frequently that there is a direct relationship between risk and return. Thus, the firm's return on equity, a measure of profitability, was included as an instrument in the model. This variable was used by Bildersee, but not by Beaver, Kettler and Scholes.

Two-year averages were used for these five variables to smooth out any extreme observations that might be associated with just one year,
and yet keep the variables relatively current. This practice was suggested by Bilderssee, who believed that using nine or ten-year averages, as Beaver, Kettler and Scholes did, would deemphasize important recent changes. The remaining variables were calculated using all available information, since many observations were needed to reduce measurement error in them.

6. Growth. The growth rate of the firm's earnings per share was shown by Gordon and Halpern to be strongly correlated with the firm's beta coefficient. Beaver, Kettler and Scholes used growth in total assets, and found a much lower degree of association. Therefore, growth in earnings per share adjusted for all stock splits and stock dividends was used in this study. The trend line growth rate (6 in Equation 3-6) was the value used for this variable.

7. Variability of earnings. Since systematic variability of returns is what a firm's beta coefficient attempts to measure, a variable reflecting the total variance of returns, which includes both systematic and unsystematic components, should be highly correlated with beta. Bilderssee and Beaver, Kettler and Scholes used the variance of the firm's price/earnings ratio as their measure of variance, and found that it was the variable most highly correlated with the market beta. Since this study wished to avoid the use of market price information, the variance of the firm's return on equity was used instead. Beaver and Mangold demonstrated that this formulation was nearly as good as the measure.

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involving market price information.9

8. Covariability of earnings. The covariance of earnings represents the accounting parallel to the market covariance measure, beta. The covariance of the firm's earnings with the earnings of a large number of firms should approximate the market risk measure. However, this expected relationship may not materialize due to measurement error, caused by the use of only a small number of observations. Thus, other variables may prove to be better instruments, even though this covariance measure, or "accounting beta," has the strongest theoretical relationship to the market beta.10 The accounting beta was calculated as the covariance of the firm's return on equity with the average return on equity of a large number of firms listed on the COMPSTAT tapes (the construction of the market return on equity index will be discussed in section IV).

The Gordon and Halpern Model

Gordon and Halpern found that using one instrumental variable, the growth of earnings, was sufficient to estimate the market beta coefficient for their calculation of the cost of capital for the mail division of the airline industry. Thus, their model represents a special case of the Beaver, Kettler and Scholes approach. It is possible that the Beaver, Kettler and Scholes model is overspecified, and that Gordon and Halpern's simpler model will work as well, or even better. Therefore, a

9See footnote 75 in chapter II.

cost of equity capital, \( R_{j3} \), was calculated using Equations 3-2 and 3-4 with only one instrumental variable, the trend line growth of earnings, to calculate the estimate of the nonmarket beta.\(^{11}\) Then, the estimate of the intrinsic value of the equity of the firm was calculated as:
\[
\hat{S}_{j3} = \frac{\hat{X}_0}{R_{j3}} \quad \text{for all firms, } j=1,m \quad \text{(Equation 3-9)}
\]

The Certainty Equivalent Model

In addition to the three models using the beta method for estimation of the cost of capital, a fourth model was developed using a certainty equivalent approach. Weston and Lee recommended this approach as a more direct technique for the estimation of the cost of capital than the method used by Gordon and Halpern.

Equation 2-14a can be restated using the symbology developed in this chapter as:
\[
\hat{S}_{j4} = \frac{\hat{X}_0 - \lambda \text{COV}(\hat{X}, \hat{R}_M)}{R_F} \quad \text{for all firms, } j=1,m \quad \text{(Equation 3-10)}
\]
since the expected value of the firm's earnings is \( \hat{X}_0 \). The other parameters are observable, or can be estimated from available data (the

\(^{11}\)It should be emphasized that the model used here is not the same one used by Gordon and Halpern. They used a true "earnings beta" concept in their work, regressing the natural logarithm of the growth of the firm's earnings against the natural logarithm of the growth in earnings of all companies in the market, as measured by the National Income and Product Accounts. Since Beaver, Kettler and Scholes demonstrated that such earnings betas were subject to large sampling error, a preliminary test was conducted in which the original Gordon and Halpern method was compared to a method in which a smoothed estimate of the growth rate in earnings was used. The model using the trend line growth of earnings as the single instrument was superior in every year, and thus was used in the final model in place of the variable used by Gordon and Halpern. However, the concept being tested is clearly theirs—that earnings growth and beta are related. Only the method of measuring that growth has been altered.
estimation procedures will be discussed in section IV). Thus, $S_{ij4}$ is a direct estimate of the intrinsic value of the firm obtained by using a risk adjustment procedure to reduce expected earnings to a certainty equivalent value, and then capitalizing this quantity by the risk free rate.

The Price/Earnings Model

The four models described above were developed from accepted theories concerning valuation and the cost of capital. They will be compared against one another to ascertain which one develops estimates of intrinsic value most consistent with the computed market price of savings and loan holding companies whose shares are actively traded. The models that provide an acceptable estimate of value for the holding company sample will be used to estimate the value of the closely held firm, and comparisons will be made to determine which model gives the best estimates. Before the models could be considered useful to the financial manager, however, they should be able to outperform a simple model which capitalizes the firm's current earnings at the industry's average price/earnings ratio (this index will be described in section IV). Otherwise, the practitioner could be expected to prefer the more familiar (and much simpler) industry average price/earnings method of valuation.

Therefore, a fifth model was developed which estimated the intrinsic value of the firm as:

$$S_{ij5} = \text{Current Earnings} \times \text{Industry Average P/E Ratio} \quad (\text{Equation 3-11})$$
IV. Data Description

Many different data bases were used in this study. A description of each of these sets of data is provided in this section, along with an explanation of how the data were used.

Market Price Information for Holding Companies

Market price information was required to calculate market betas and computed market prices for the holding companies. Monthly stock price information was compiled for savings and loan associations or holding companies whose securities were actively traded during all or part of the period January 1961 through December 1975. Information was obtained for thirty-five associations that had their securities traded on major exchanges or actively traded in the over-the-counter markets during this period.\(^{12}\) Table 2 lists these companies, the periods for which data were available, and the exchange (if any) upon which the securities were listed. Price information was available for thirteen associations and holding companies continuously from 1965 through 1975. These associations were strong candidates for inclusion in the study, since information was available during the entire period for which balance sheet and income statement data were available from the Federal Home Loan Bank Board.

Most savings and loan associations and holding companies did not pay cash dividends during the period, but did declare stock dividends

\(^{12}\)Information concerning listing and trading activity was obtained from annual issues of Moody's Bank and Finance Manual. Then, price information was obtained from monthly issues of Standard and Poor's Stock Guide, from weekly issues of Barrons, and from monthly issues of the Bank and Quotation Record. Any prices that seemed unreasonable were double-checked with other sources. Stock dividend and stock split information was also obtained from these sources.
TABLE 2

HOLDING COMPANY DATA AVAILABILITY AND LISTING

<table>
<thead>
<tr>
<th>COMPANY</th>
<th>DATA FIRST AVAILABLE</th>
<th>LISTING</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Charter</td>
<td>1/61</td>
<td>NYSE prior to 1964</td>
</tr>
<tr>
<td>Great Western Financial</td>
<td>1/61</td>
<td>NYSE prior to 1964</td>
</tr>
<tr>
<td>Imperial</td>
<td>1/61</td>
<td>NYSE prior to 1964</td>
</tr>
<tr>
<td>Financial Federation</td>
<td>10/61</td>
<td>NYSE prior to 1964</td>
</tr>
<tr>
<td>Gibraltar Financial</td>
<td>2/62</td>
<td>NYSE prior to 1964</td>
</tr>
<tr>
<td>Transworld Financial</td>
<td>1/62</td>
<td>NYSE prior to 1964</td>
</tr>
<tr>
<td>California Financial</td>
<td>11/62</td>
<td>NYSE prior to 1964</td>
</tr>
<tr>
<td>Wesco Financial</td>
<td>2/63</td>
<td>NYSE prior to 1964</td>
</tr>
<tr>
<td>Empire Financial</td>
<td>12/63</td>
<td>ASE 6/68, NYSE 1/72</td>
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<td>Equitable Savings and Loan</td>
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<td>Not listed 12/75</td>
</tr>
<tr>
<td>Financial Corporation of Santa Barbara</td>
<td>12/63</td>
<td>ASE 4/68, NYSE 12/75</td>
</tr>
<tr>
<td>Hawthorne Financial</td>
<td>12/63</td>
<td>Not listed 12/75</td>
</tr>
<tr>
<td>United Financial</td>
<td>12/63</td>
<td>NYSE prior to 1964</td>
</tr>
<tr>
<td>Far West Financial</td>
<td>7/64</td>
<td>NYSE 8/64</td>
</tr>
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<td>First Western</td>
<td>9/67</td>
<td>Not listed 12/75</td>
</tr>
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<td>Western Financial</td>
<td>8/68</td>
<td>Not listed 12/75</td>
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<td>6/69</td>
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</tr>
<tr>
<td>Homestead Financial</td>
<td>12/69</td>
<td>Not listed 12/75</td>
</tr>
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<td>Affiliated Capital</td>
<td>4/70</td>
<td>ASE 4/71</td>
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<tr>
<td>Bass Financial</td>
<td>3/71</td>
<td>Not listed 12/75</td>
</tr>
<tr>
<td>State Savings and Loan</td>
<td>5/71</td>
<td>ASE 8/72</td>
</tr>
<tr>
<td>TransOhio Financial</td>
<td>7/71</td>
<td>NYSE 6/74</td>
</tr>
<tr>
<td>Downey Savings and Loan</td>
<td>7/71</td>
<td>ASE 3/72</td>
</tr>
<tr>
<td>Fidelity Financial</td>
<td>12/71</td>
<td>NYSE 7/73</td>
</tr>
<tr>
<td>Mercury Savings and Loan</td>
<td>12/71</td>
<td>Not listed 12/75</td>
</tr>
<tr>
<td>General of Ohio</td>
<td>5/72</td>
<td>Not listed 12/75</td>
</tr>
<tr>
<td>Gulf Republic Financial</td>
<td>5/72</td>
<td>ASE 11/72</td>
</tr>
<tr>
<td>University Savings and Loan</td>
<td>5/72</td>
<td>ASE 5/72</td>
</tr>
<tr>
<td>Nevada Savings and Loan</td>
<td>6/72</td>
<td>Not listed 12/75</td>
</tr>
<tr>
<td>Northern California S&amp;L</td>
<td>7/72</td>
<td>Not listed 12/75</td>
</tr>
<tr>
<td>H. F. Ahmanson &amp; Company</td>
<td>10/72</td>
<td>NYSE 1/73</td>
</tr>
<tr>
<td>Broadview Financial</td>
<td>2/73</td>
<td>Not listed 12/75</td>
</tr>
</tbody>
</table>

with great regularity. The price data were adjusted for all stock splits and stock dividends. The adjustment factor used for this calculation was also used to adjust earnings per share data.
Balance Sheet and Income Statement Data

Balance sheet and income statement data for the savings and loan associations that were publicly traded, for associations that were affiliates of the holding companies that were traded, and for associations that were acquired by merger during the period 1969 through 1975 were obtained from the Federal Home Loan Bank Board. The reporting format for these financial statements changed several times during the period for which data were available, 1965 through 1975. Accordingly, extensive realignment and adjustment of the data were required. The resulting adjusted data base should accurately reflect the values of the variables used in this study.

All eleven years of data were obtained for the holding company affiliates. For each of the acquired associations, data were obtained from December 1965 until the year in which the association was merged out of existence.

Summary balance sheet and income statement data for the holding companies were obtained from annual issues of Moody's Bank and Finance Manual. As will be explained more fully in chapter IV, a screening procedure was used to compare the financial statements found in Moody's with those provided by the Federal Home Loan Bank Board. Holding companies were eliminated from the study if substantial discrepancies were evident.

Purchase Prices for Acquired Associations

The actual price paid for each of the acquired associations was determined by examining the merger documents filed by the associations involved in the merger. Of the 820 mergers involving members of the
Federal Home Loan Bank system during the period January 1, 1969, through December 31, 1975, 163 involved an acquired association that was of the stock form of organization.\textsuperscript{13} Access to the confidential merger documents for these stock associations was gained through a special authorization granted by the Federal Home Loan Bank Board.

After examination of documents relating to the 163 mergers, fifty-four were eliminated either because the purchase price was not reported or because the merger was a "balance sheet merger" in which a holding company formally merged an affiliate whose stock it already owned. Another twelve were eliminated because the purchase prices were paid in closely held stock for which no market prices were available. Of the remaining ninety-seven mergers, thirty-seven involved acquisition for stock in companies whose securities were traded actively enough that a market price of the stock on the date of acquisition was reported or could be found in published sources. The other sixty mergers were acquisitions for cash. Additional screening procedures used to arrive at the final sample will be described in chapter V.

\textbf{Market Return Index}

A number of stock market return indices were available for consideration as the $R_M$ used for the beta calculations and for the estimates of the risk premium ($R_M - R_F$). While the Dow Jones averages and the Standard and Poor's averages are better known to the man on the

\textsuperscript{13}United States League of Savings Associations, \textit{Savings and Loan Fact Book '79}, p. 54, contains information on mergers. The mergers involving acquisition of stock associations were determined from a Federal Home Loan Bank Board internal publication titled: "Mergers and Deletions [of] FSLIC Insured Associations Jan-60 thru Dec-75." (Washington, D. C. 1976).
street, the Fisher value-weighted return index with dividends, available with the Center for Research in Security Prices (CRSP) stock tapes was chosen as the index to be used in this study. This index of returns includes information from each company listed on the New York Stock Exchange, and weights the results according to the total market value of the company. It is believed that this index represents the true "market return" better than any other source, since it includes more companies and uses value weights, rather than simple averages.

**Market Return on Equity**

An index of return on equity for all firms was required for calculation of covariability of earnings in the Beaver, Kettler and Scholes model. Since no acceptable index was found in published sources, one was developed using the COMPUSTAT data tapes. The return on equity was calculated for each company on the tape that had a fiscal year ending in December (a serious problem of data alignment arises if companies with different fiscal years are mixed). Then, returns in excess of two hundred percent in absolute value were screened out to avoid problems created by firms with very low book value of equity. The remaining values were averaged to obtain the annual return on equity index. The index was developed from at least 1,300 companies in each year, and thus should be representative of all firms.

**Market Earnings Index**

An index of aggregate corporate earnings was needed in order to calculate the growth in earnings for the original formulation of the Gordon and Halpern model. The Corporate Income Tax Returns publication of
the Internal Revenue Service contains very precise estimates of corporate income. However, this information is published with a three or four year delay; earnings information for more current years would not be available for the decisionmaker. An estimate of such earnings could be computed from the COMPUSTAT tapes, but this would include only about 2,400 companies, and would be subject to errors due to additions and deletions, particularly in the earlier years of the tapes. The source chosen was that used by Gordon and Halpern, the National Income and Product Accounts published in the Business Statistics supplement to the Survey of Current Business.\(^{14}\) The raw profits after tax measure reported in this source was used, rather than a variety of other measures which contain inventory valuation adjustments and capital consumption allowances, since the correlation of the chosen index with the firm's unadjusted earnings was desired. This index was also used in the final formulation of the certainty equivalent model, for reasons that will be explained in chapter IV.

The Risk Free Rate

Since annual return data and annual earnings data were used, an annual risk free rate should be used for \(R_p\) in the models. The rate of return on a one-year U.S. Treasury bill was obtained from the Federal Reserve Bulletin for each month for which \(R_p\) was needed. Since this rate is reported on a bank discount basis, it must be converted to a coupon equivalent value before it is used. Appendix C contains an explanation

\(^{14}\) The data used in the study was taken from the biennial statistical supplement: United States Department of Commerce, Business Statistics 1977 (March 1978), p. 7.
of the procedure involved in this conversion.

An examination of other empirical studies involving the risk free rate would seem to indicate that this conversion has not been used. However, the risk free rate is substantially understated if it is not converted to a coupon equivalent basis. This understatement of \( R_p \) would lead to an overstatement of the risk premium \( (R_M - R_f) \) if \textit{ex post} data were used to estimate that quantity.

The Risk Premium and Market Price of Risk

One of the most difficult tasks encountered in this study was the estimation of the risk premium, \( (R_M - R_f) \), and the market price of risk, \( \lambda \). Even if it is deemed acceptable to use past holding period return data as estimates of future expectations, considerable problems still exist.

A number of different sources contain information concerning historical rate of return indices for stocks and/or bonds. These sources differ in the time period they cover, the number of securities used to construct the indices, the inclusion or exclusion of dividends, the reinvestment assumptions, the weighting scheme used (value weights or equal weights), and the compounding method used.

Ibbotson and Sinquefield presented a series of return indices and a thorough discussion of other available indices in a 1976 \textit{Journal of Business} article.\(^{15}\) Their data were presented in both holding period

return and risk premium form. The results they reported closely parallel those calculated from other data series, and thus were used in this study.

Their results differ from what was desired in this study, however, in two ways. First, they used a one-month holding period return for a U.S. Treasury bill of slightly more than one month to maturity. This figure would tend to be lower than the one-year rate, and hence the risk premium would tend to be overstated. However, their use of holding period return data did eliminate the frequent understatement of yields due to the use of bank discount rates for bills.

Second, their risk premium was calculated as a net premium for investing in common stocks. The common formation of the risk premium, \((R_M - R_F)\), was divided by the quantity \((1 + R_F)\) to obtain the monthly premium. Thus, this procedure led to an understatement of the risk premium as it is normally expressed. It was assumed in this study that these two differences would approximately cancel each other, and that the Ibbotson and Sinquefield estimates were adequate.

For almost an entire decade, from early 1940 until March 1951, U.S. Treasury bill rates were pegged at artificially low rates by the government. Therefore, data prior to 1951 were largely ignored in this study. For estimates of the risk premium prior to the end of 1971, all data since 1951 were used. Beginning with 1971, an average of all available data since 1951 and the twenty most recent years was calculated, in order to reflect greater importance of more recent returns (which had been very different from the period from 1951 to 1970). The resulting estimated risk premiums are presented in table 3.
TABLE 3
ESTIMATE OF \( (R_m - R_f) \) AT YEAREND

<table>
<thead>
<tr>
<th>Year</th>
<th>Risk Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>1969</td>
<td>10.76</td>
</tr>
<tr>
<td>1970</td>
<td>10.10</td>
</tr>
<tr>
<td>1971</td>
<td>9.78</td>
</tr>
<tr>
<td>1972</td>
<td>9.83</td>
</tr>
<tr>
<td>1973</td>
<td>8.73</td>
</tr>
<tr>
<td>1974</td>
<td>5.80</td>
</tr>
<tr>
<td>1975</td>
<td>6.37</td>
</tr>
</tbody>
</table>

These averages were then changed based on subjective estimates of how investors would have felt on the date when they had to make their investment decisions. This procedure, while admittedly ad hoc, was believed to provide estimates that a knowledgeable investor might make.

Savings and Loan Price/Earnings Index

An industry average price/earnings index was needed for the valuation estimates of the price/earnings model. Available published indices contain only a very small number of associations. Therefore, an index was created from data for the companies used in this study. Quarterly earnings per share information was obtained from issues of Standard and Poor's Stock Guide, and price/earnings ratios were computed by dividing the stock price at the end of the month by the sum of the four most recent quarterly earnings figures, adjusted for any splits or
dividends. Ratios were eliminated if earnings were negative or if the ratio was out of a reasonable range due to extraordinary circumstances. Table 4 presents the index constructed and the number of observations from which it was composed.

It should be obvious that the price/earnings model will be a strong one, since the ratio used by it was constructed from the actual price/earnings ratios of the companies in this study. While this model will thus tend to overstate the ability of a price/earnings model to value a savings and loan, it will serve as a strong model against which to compare the models developed in this study.

V. Hypotheses

The hypotheses to be tested will be presented in this section, and the procedures used to test the hypotheses will be explained.

Holding Companies

The first set of tests was designed to determine if the intrinsic value estimates developed from the first four models presented in section III were unbiased estimates of the computed market prices of the holding companies. Specific hypotheses were:

A. Market Beta model

\[ H_0: \frac{\hat{S}_{j1}}{\text{Computed Market Price}} = 1.0 \]

B. Beaver, Kettler and Scholes model

\[ H_0: \frac{\hat{S}_{j2}}{\text{Computed Market Price}} = 1.0 \]
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>10.34</td>
<td>15.44</td>
<td>11.90</td>
<td>9.26</td>
<td>6.08</td>
<td>5.71</td>
</tr>
<tr>
<td></td>
<td>(9)*</td>
<td>(8)***</td>
<td>(11)##</td>
<td>(13)###</td>
<td></td>
<td></td>
</tr>
<tr>
<td>February</td>
<td>13.74</td>
<td>15.94</td>
<td>11.38</td>
<td>8.47</td>
<td>6.35</td>
<td>5.38</td>
</tr>
<tr>
<td>March</td>
<td>12.87</td>
<td>16.70</td>
<td>11.64</td>
<td>8.32</td>
<td>5.41</td>
<td>5.67</td>
</tr>
<tr>
<td>April</td>
<td>10.29</td>
<td>16.43</td>
<td>11.72</td>
<td>6.67</td>
<td>5.43</td>
<td>5.49</td>
</tr>
<tr>
<td>May</td>
<td>9.32</td>
<td>14.68</td>
<td>11.00</td>
<td>6.04</td>
<td>4.98</td>
<td>6.78</td>
</tr>
<tr>
<td>June</td>
<td>9.02</td>
<td>14.19</td>
<td>11.09</td>
<td>5.88</td>
<td>4.11</td>
<td>6.70</td>
</tr>
<tr>
<td>July</td>
<td>10.33</td>
<td>13.04</td>
<td>9.73</td>
<td>5.78</td>
<td>4.10</td>
<td>6.36</td>
</tr>
<tr>
<td>August</td>
<td>12.07</td>
<td>14.31</td>
<td>11.36</td>
<td>5.88</td>
<td>3.35</td>
<td>5.87</td>
</tr>
<tr>
<td>September</td>
<td>12.64</td>
<td>15.32</td>
<td>12.22</td>
<td>6.39</td>
<td>3.73</td>
<td>5.45</td>
</tr>
<tr>
<td>October</td>
<td>12.28</td>
<td>13.96</td>
<td>11.69</td>
<td>6.03</td>
<td>4.94</td>
<td>5.41</td>
</tr>
<tr>
<td>November</td>
<td>13.35</td>
<td>13.17</td>
<td>12.19</td>
<td>4.75</td>
<td>4.80</td>
<td>5.57</td>
</tr>
<tr>
<td>December</td>
<td>14.84</td>
<td>12.98</td>
<td>11.82</td>
<td>4.64</td>
<td>4.54</td>
<td>5.74</td>
</tr>
</tbody>
</table>

*Nine observations were used: California Financial, Empire, Far West, Fin. Corp. of Santa Barbara, First Charter, Gibraltar, Great Western, Hawthorne, Western.

**Golden West added.

***California Financial deleted due to negative earnings and Great Western deleted due to abnormality.

#Great Western returned to index.

##California Financial returned to index and State added.

###Equitable and Fidelity added.

@California Financial and Far West removed due to negative earnings and Empire removed due to announcement of merger plans.
C. Gordon and Halpern model

\[ H_0: \frac{\hat{S}_{13}}{\text{Computed Market Price}} = 1.0 \]

D. Certainty Equivalent model

\[ H_0: \frac{\hat{S}_{14}}{\text{Computed Market Price}} = 1.0 \]

Since there was no reason to believe in advance that any deviation would be in a particular direction, the alternative hypothesis in each case was that the ratio would be unequal to one. The hypotheses were tested by forming the ratio of estimated intrinsic value to computed market price for each company and for each model, summing these values for each model, and calculating the mean value of this ratio for each model. The means for each of the models were then tested to determine if they were significantly different from one using the Student's t test. If that premise could not be rejected, then the conclusion was that the model provided an unbiased estimate of the computed market price of the company.

The second set of tests was designed to make paired comparisons of the different models to determine which one provided estimates that were most consistent with computed market prices. First, each of the other four models was compared to the price/earnings model. Then, the Beaver, Kettler and Scholes model, the Gordon and Halpern model, and the certainty equivalent model were compared to the market beta model to determine if the models that did not make direct use of market price information performed as well as the one that did. Finally, comparisons were made among the three nonmarket models to determine if one of them provided estimates that were superior to the estimates of the other two.
While a parametric test was appropriate for the mean values of the ratios expressed in hypotheses A, B, C, and D, it was not judged appropriate for the paired comparison tests. The individual estimates of value developed by the models, or even the ratios of these estimates to the computed market prices, may not conform to distributions for which parametric tests are applicable. Therefore, a non-parametric test, the Wilcoxon matched-pairs signed-ranks test was chosen as the appropriate one for these analyses. This test was applied to the individual intrinsic value to computed market price ratios calculated in order to estimate mean values for tests of hypotheses A, B, C, and D.

The results of the t tests and the Wilcoxon matched-pairs signed-ranks tests will be presented in chapter IV.

Acquisitions

A third set of tests was designed to determine if the intrinsic value estimates developed by the nonmarket models and the price/earnings model were unbiased estimates of the prices paid for savings and loan associations that were acquired through a purchase for cash. Of course, the market beta model could not be used for this analysis, and, for reasons that will be explained in chapter V, the certainty equivalent model was not used either. Specific hypotheses were:


17. Ibid., pp. 75-83 describes this test.
E. Beaver, Kettler and Scholes model

$$H_0: \frac{\hat{S}_{j2}}{\text{Acquisition Price}} = 1.0$$

F. Gordon and Halpern model

$$H_0: \frac{\hat{S}_{j3}}{\text{Acquisition Price}} = 1.0$$

G. price/earnings model

$$H_0: \frac{\hat{S}_{j4}}{\text{Acquisition Price}} = 1.0$$

These hypotheses were tested by forming ratios of each individual company's estimated intrinsic value to acquisition price, summing these values, and calculating the mean value of the ratio for each model. The means for each of the models were then tested for a significant difference from one, again using the Student's t test.

The fourth and final set of tests was equivalent to the second set, but compared the three models used to estimate intrinsic value for the acquisitions, rather than for the holding companies. Similar procedures were used--the Wilcoxon test was used to compare the Beaver, Kettler and Scholes model and the Gordon and Halpern model to the price/earnings model, and then the two models were compared to each other.

The results of the acquisition tests will be presented in chapter V.

VI. Summary

In this chapter, the models of valuation used in this study were developed. Alternate procedures for cost of capital estimation and valuation were discussed, and five valuation models were constructed. The
data bases created and/or used in this study were described, and the hypotheses to be tested and the procedures for testing them were presented. The next chapter will present the application of the models to the holding company sample. Chapter V will present the application of the models to the sample of merger acquisitions.
CHAPTER IV

THE HOLDING COMPANY RESULTS

Application of the five models developed in the previous chapter to the holding company sample will be described in this chapter. Estimates of firm value provided by the models will be presented and compared, and results of the tests of the research hypotheses will be reported.

The first section will describe the screening procedures used to select the holding company sample and present data relating to the associations selected. The second section will present parameters developed in the models or used in developing the models. The third section will contain the estimates of value provided by the five models, and the final section will contain tests of the research hypotheses.

I. Selection of Holding Companies

Selection of the holding company sample required a compromise between a desire for pure data and a desire for a large enough sample to develop meaningful models. An initial condition was established that price information should be available for at least five years at the end of 1975, so that thirty-six, forty-eight, and sixty monthly observations would be available for estimation of market betas at the end of 1973, 1974, and 1975 respectively. Estimation of betas with fewer observations has been shown to provide very dissimilar results, depending upon
the time period selected.\textsuperscript{1} This requirement reduced the potential sample from thirty-five to twenty-eight companies.

Then, data were requested for all of the savings and loan affiliates of these holding companies from the Federal Home Loan Bank Board. Some data were not sent, causing the elimination of seven of the holding companies. For the remaining twenty-one firms, the data for each association in the holding company were added together to give a composite balance sheet and income statement for each year. These composite figures were then compared to summary balance sheet and income statement data provided in Moody's \textit{Bank and Finance Manual}. While differing accounting practices at the association and holding company levels could result in differences between the two sets of figures, any significant irregularities would indicate potential problems in the data. Seven holding companies were removed from the sample on this basis. The differences in the two figures could usually be traced to involvement of the holding company in operations other than the savings and loan business, including real estate ventures, insurance companies or agencies, and banking.

The remaining fourteen holding companies were used in all or part of the analyses presented in this chapter; some holding companies were removed from the sample in specific years because insufficient data were available for accurate estimation of some of the required variables. Information pertaining to the associations' size and data availability is presented in table 5. Equitable is located in Oregon, and Western

Financial is located in Arizona. The other holding companies are based in California.

**TABLE 5**

**HOLDING COMPANY SAMPLE**

<table>
<thead>
<tr>
<th>Company</th>
<th>Total Assets Dec. 31, 1973 (millions)</th>
<th>Useful Price Information Available</th>
</tr>
</thead>
<tbody>
<tr>
<td>California Financial</td>
<td>820</td>
<td>11/62 - 1/76</td>
</tr>
<tr>
<td>Empire Financial</td>
<td>295</td>
<td>12/63 - 2/74</td>
</tr>
<tr>
<td>Equitable Savings and Loan</td>
<td>607</td>
<td>12/63 - 5/69</td>
</tr>
<tr>
<td>Far West Financial</td>
<td>506</td>
<td>7/64 - 1/76</td>
</tr>
<tr>
<td>Fidelity Financial</td>
<td>717</td>
<td>12/71 - 1/76</td>
</tr>
<tr>
<td>Financial Corporation of Santa Barbara</td>
<td>641</td>
<td>12/63 - 1/76</td>
</tr>
<tr>
<td>First Charter</td>
<td>4,211</td>
<td>1/61 - 1/76</td>
</tr>
<tr>
<td>Gibraltar Financial</td>
<td>1,443</td>
<td>2/62 - 1/76</td>
</tr>
<tr>
<td>Golden West Financial</td>
<td>734</td>
<td>12/68 - 9/75</td>
</tr>
<tr>
<td>Great Western Financial</td>
<td>4,763</td>
<td>1/61 - 1/76</td>
</tr>
<tr>
<td>Hawthorne Financial</td>
<td>125</td>
<td>12/63 - 1/76</td>
</tr>
<tr>
<td>Homestead Financial</td>
<td>91</td>
<td>12/69 - 1/76</td>
</tr>
<tr>
<td>State Savings and Loan</td>
<td>310</td>
<td>5/71 - 12/74</td>
</tr>
<tr>
<td>Western Financial</td>
<td>732</td>
<td>7/68 - 1/76</td>
</tr>
</tbody>
</table>

The holding company sample was analyzed at the end of 1969, 1970, 1973, 1974, and 1975. These periods were selected for analysis because a large number of mergers of stock associations took place during 1970, 1971, 1974, and 1975. Therefore, models of valuation were needed at the beginning of each of these years to establish the parameters for models to be used throughout the year. The year 1975 was also included in order
to test the models for a period which was more favorable to the savings and loan industry than were the years 1973 and 1974.

The actual valuation estimates were made for January 31 following the end of each of the years chosen for study. It was believed that on this date all information pertaining to operations of the previous year would be reflected in the market price, and yet operations during the new year would not yet have had a significant impact on the price.

II. The Models

Estimates of the risk premium \((R_M - R_F)\) were required for each of the models except the price/earnings model. These estimates were made by incorporating the calculations presented in table 3 with an estimate of how investors might have viewed the risk premium on the date for which estimates were required. An estimate of eight percent for the risk premium was used for the 1969 models, which used year-end 1969 information to estimate the intrinsic value of the holding company on January 31, 1970. This value was chosen after considering the historical risk premium from 1926 to 1969, 10.26 percent, the twenty year average as developed in table 3, and the fact that the observed risk premium for 1969, -14.1 percent, was the worst since 1937. Eight percent was again chosen for the 1970 model, incorporating the information used for 1969 and an observed risk premium of -2.4 percent in 1970.

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3 Ibid., p. 37. The remaining references to specific risk premia in this section were also taken from this source, unless otherwise noted.
The estimate of the 1973 models was seven percent, reflecting the value shown in table 3, 8.73 percent, an observed risk premium of -20.2 percent in 1973, and premia of 9.6 percent and 14.6 percent in 1971 and 1972 respectively. The estimate for 1974 was six percent, chosen to reflect reduced expectations as shown in table 3 and attributable to an observed risk premium of -31.9 percent in 1974. A value of six percent was also chosen in 1975. The observed risk premium was highly positive, about thirty-two percent, but the large negative premia of the preceding years were not likely to be forgotten immediately by investors.

Estimates of the market price of risk, \( \lambda \), were obtained by dividing the risk premium as specified above by 3.24 percent. Regardless of the time period analyzed or the return series used to calculate the market return \( R_M \), the standard deviation of the return series was found in this analysis to be very close to eighteen percent. Therefore, 3.24 percent was used as the estimate of the variance for each year. The resulting values of \( \lambda \) were: 2.5 in 1969 and 1970; 2.15 in 1973; and 1.85 in 1974 and 1975. The risk premia and market prices of risk listed here are low compared to those used in many previous studies. However, they are consistent with recent conclusions by other researchers.

Since the Ibbotson and Sinquefield estimates were derived only through 1974, this value was derived from the Fisher index and Federal Reserve Bulletin data.

Richard W. MKenally and David E. Upton, "A Reexamination of the Ex Post Risk-Return Tradeoff on Common Stocks," Journal of Financial and Quantitative Analysis 14 (June 1979): 395-417. This article contains a review of other works, as well as a new analysis. The authors conclude that the ex ante risk-return tradeoff has been overstated. An explanation of theoretical reasons for this overstatement can be found in: Andrew H. Chen and Edward J. Kane, "Corporate Tax Integration and Uncertain Inflation," paper presented at the meetings of the Western Economics Association, June 1978. The authors show that under a variety of
The Market Beta Model

Estimates of the market beta for the fourteen holding companies are presented in table 6. These values were used with Equation 3-3 to develop the cost of capital estimates for use in Equation 3-7 to provide the market beta estimate of the intrinsic value of firm j, $\hat{s}_{j1}$.

The Beaver, Kettler, Scholes Model

The market beta estimates presented in table 6 were used to develop instrumental variables estimates of beta, $\hat{b}_j$, the corresponding estimate of the cost of capital, $\hat{\beta}_{j2}$, and the intrinsic value of the equity of the firm, $\hat{s}_{j2}$, as expressed by Equations 3-2, 3-4, and 3-8, respectively. Due to the small number of observations, only a small number of independent variables were allowed to enter the stepwise regression model from which estimates of the betas were derived. The resulting models therefore appear to exhibit more nonstationarity than was really the case. Table 7 shows the simple correlations of each of the instrumental variables with the observed market beta, and also the order of entry of the variables into the annual models.

Interpretation of the coefficients would be difficult, and is not really necessary, according to Miller and Modigliani. Rather, the predictive power of the models is the relevant concern. Nevertheless, it is alarming that leverage was inversely correlated with the measured beta in all but one year, and total variance of returns was inversely correlated with beta in all years, contrary to accepted theory. In addition, conditions the normal capital asset pricing model formulation will overstate the market price of risk.

6See appendix A.
**TABLE 6**

**CALCULATED MARKET BETAS**

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
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<tbody>
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<td>1. California</td>
<td>1.47</td>
<td>1.58</td>
<td>1.63</td>
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<td>1.62</td>
</tr>
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<td>-</td>
<td>1.76</td>
</tr>
<tr>
<td>3. Equitable</td>
<td>-</td>
<td>-</td>
<td>1.48</td>
<td>1.19</td>
<td>1.26</td>
<td>(115)</td>
</tr>
<tr>
<td>4. Far West</td>
<td>2.26</td>
<td>2.13</td>
<td>2.00</td>
<td>1.16</td>
<td>1.09</td>
<td>1.53</td>
</tr>
<tr>
<td>5. Fidelity</td>
<td>-</td>
<td>-</td>
<td>1.12</td>
<td>1.03</td>
<td>1.18</td>
<td>-</td>
</tr>
<tr>
<td>6. Santa Barbara</td>
<td>1.38</td>
<td>1.58</td>
<td>1.55</td>
<td>1.55</td>
<td>1.78</td>
<td>1.70</td>
</tr>
<tr>
<td>7. First Charter</td>
<td>2.41</td>
<td>2.18</td>
<td>1.86</td>
<td>1.45</td>
<td>1.44</td>
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<tr>
<td>8. Gibraltar</td>
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<td>2.36</td>
<td>1.65</td>
<td>1.33</td>
<td>1.83</td>
</tr>
<tr>
<td>9. Golden West</td>
<td>-</td>
<td>-</td>
<td>1.84</td>
<td>1.66</td>
<td>1.40</td>
<td>1.64</td>
</tr>
<tr>
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<td>2.09</td>
<td>1.99</td>
<td>1.71</td>
<td>1.51</td>
<td>1.30</td>
<td>1.60</td>
</tr>
<tr>
<td>11. Hawthorne</td>
<td>1.60</td>
<td>1.43</td>
<td>1.73</td>
<td>.98</td>
<td>1.22</td>
<td>1.28</td>
</tr>
<tr>
<td>12. Homestead</td>
<td>-</td>
<td>-</td>
<td>1.37</td>
<td>.71</td>
<td>.51</td>
<td>.74</td>
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<td>13. State</td>
<td>-</td>
<td>-</td>
<td>.72</td>
<td>1.09</td>
<td>1.31</td>
<td>-</td>
</tr>
<tr>
<td>14. Western</td>
<td>-</td>
<td>-</td>
<td>1.39</td>
<td>1.09</td>
<td>.97</td>
<td>1.22</td>
</tr>
</tbody>
</table>

**NOTES:**

1969 betas were calculated from 48 months of data.

The 1970, 1973, 1974 and 1975 betas were calculated from 60 months of data unless fewer months of data were available. These special cases have the number of months of data available listed in parentheses below the data.

The total period (1966-1975) betas were calculated from 120 months of data unless fewer price relatives were available, in which case the number of months used is listed in parentheses below the data.
### TABLE 7
CORRELATIONS OF VARIABLES WITH MARKET BETA

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest</td>
<td>0.574**</td>
<td>0.058</td>
<td>0.412**</td>
<td>0.287***</td>
<td>-0.087</td>
</tr>
<tr>
<td>Leverage</td>
<td>-0.050#</td>
<td>0.214</td>
<td>-0.373</td>
<td>-0.330</td>
<td>-0.498</td>
</tr>
<tr>
<td>Liquidity</td>
<td>-0.590</td>
<td>-0.554#</td>
<td>-0.171***</td>
<td>-0.418</td>
<td>-0.587*</td>
</tr>
<tr>
<td>Size</td>
<td>0.344</td>
<td>0.254</td>
<td>0.237#</td>
<td>0.718*</td>
<td>0.472</td>
</tr>
<tr>
<td>Profitability</td>
<td>0.378</td>
<td>0.280</td>
<td>-0.289</td>
<td>0.137</td>
<td>-0.277#</td>
</tr>
<tr>
<td>Variability</td>
<td>-0.385</td>
<td>-0.806*</td>
<td>-0.414*</td>
<td>-0.537**</td>
<td>-0.459</td>
</tr>
<tr>
<td>Covariability</td>
<td>-0.006</td>
<td>-0.009</td>
<td>0.403</td>
<td>0.062#</td>
<td>0.054</td>
</tr>
<tr>
<td>Growth</td>
<td>0.727*</td>
<td>0.322</td>
<td>-0.225</td>
<td>0.583</td>
<td>0.291**</td>
</tr>
<tr>
<td>Number of Observations</td>
<td>7</td>
<td>7</td>
<td>14</td>
<td>13</td>
<td>11</td>
</tr>
</tbody>
</table>

* denotes the first variable to enter the model.

** denotes the second variable to enter the model.

*** denotes the third variable to enter the model.

# denotes the variable that would have entered next.
the positive correlation of size and beta was not the expected relationship. The positive correlation between beta and growth and the negative correlation between beta and liquidity were consistent with results found in other works. Also consistent with results of other authors, the earnings beta, or accounting beta, demonstrated very low correlation with the market beta in most years.

The parameters of the models developed for each year are presented in table 8, and the resulting instrumental variables estimates of betas are presented in table 9.

**TABLE 8**

PARAMETERS OF THE BEAVER, KETTLER SCHOLES MODEL

<table>
<thead>
<tr>
<th>Year</th>
<th>Variables and Coefficients</th>
<th>Adjusted $R^2$</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1969*</td>
<td>$\hat{\beta} = -12.895 + 297.98 \text{ Interest} + .0564 \text{ Growth}$</td>
<td>.892</td>
<td>.157</td>
</tr>
<tr>
<td>1970</td>
<td>$\hat{\beta} = 3.017 - .3690 \text{ Variability}$</td>
<td>.579</td>
<td>.326</td>
</tr>
<tr>
<td>1973</td>
<td>$\hat{\beta} = -7.438 + 177.65 \text{ Interest} + 13.86 \text{ Liquidity} - .2124 \text{ Variability}$</td>
<td>.305</td>
<td>.368</td>
</tr>
<tr>
<td>1974</td>
<td>$\hat{\beta} = -3.630 + 39.00 \text{ Interest} + .1573 \text{ Size} - .1303 \text{ Variability}$</td>
<td>.709</td>
<td>.158</td>
</tr>
<tr>
<td>1975</td>
<td>$\hat{\beta} = 2.219 - 16.13 \text{ Liquidity} + .0249 \text{ Growth}$</td>
<td>.550</td>
<td>.226</td>
</tr>
</tbody>
</table>

*Two variables were allowed to enter the 1969 model so that it would not be identical to the Gordon and Halpern model.

Despite the fact that the beta estimates were derived from different models in each year, they appeared reasonably stable, or at least no less stable than the market beta estimates shown in table 6.
TABLE 9
BEAVER, KETTLER SCHOLES MODEL BETA ESTIMATES

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. California</td>
<td>1.38</td>
<td>1.64</td>
<td>1.39</td>
<td>1.23*</td>
<td>1.42</td>
</tr>
<tr>
<td>2. Empire</td>
<td>2.57</td>
<td>2.52</td>
<td>2.26</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3. Equitable</td>
<td>-</td>
<td>-</td>
<td>1.60</td>
<td>1.33</td>
<td>1.37</td>
</tr>
<tr>
<td>4. Far West</td>
<td>-</td>
<td>-</td>
<td>1.78</td>
<td>1.19</td>
<td>1.05</td>
</tr>
<tr>
<td>5. Fidelity</td>
<td>-</td>
<td>-</td>
<td>1.36</td>
<td>1.11</td>
<td>1.09</td>
</tr>
<tr>
<td>6. Santa Barbara</td>
<td>1.53*</td>
<td>2.18*</td>
<td>1.85</td>
<td>1.40</td>
<td>1.34*</td>
</tr>
<tr>
<td>7. First Charter</td>
<td>2.25*</td>
<td>1.85*</td>
<td>1.86</td>
<td>1.62*</td>
<td>1.70*</td>
</tr>
<tr>
<td>8. Gibraltar</td>
<td>2.07</td>
<td>2.41</td>
<td>1.95*</td>
<td>1.68</td>
<td>1.39</td>
</tr>
<tr>
<td>9. Golden West</td>
<td>-</td>
<td>-</td>
<td>1.52</td>
<td>1.40</td>
<td>-</td>
</tr>
<tr>
<td>10. Great Western</td>
<td>2.26*</td>
<td>2.02</td>
<td>1.33*</td>
<td>1.46</td>
<td>1.23</td>
</tr>
<tr>
<td>11. Hawthorne</td>
<td>1.52</td>
<td>1.40</td>
<td>2.03</td>
<td>1.08</td>
<td>1.39</td>
</tr>
<tr>
<td>12. Homestead</td>
<td>-</td>
<td>-</td>
<td>1.46</td>
<td>.68</td>
<td>.67</td>
</tr>
<tr>
<td>13. State</td>
<td>-</td>
<td>-</td>
<td>1.47**</td>
<td>1.21</td>
<td>-</td>
</tr>
<tr>
<td>14. Western</td>
<td>-</td>
<td>-</td>
<td>1.28</td>
<td>1.13</td>
<td>1.08</td>
</tr>
</tbody>
</table>

* denotes an estimate more than one standard deviation from the observed value.

** denotes an estimate more than two standard deviations from the observed value.
The Gordon and Halpern Model

The market beta estimates presented in table 6 were also used to estimate the intrinsic value of the firm, $S_{j3}$, as expressed by Equation 3-9. The same procedure was followed as in the Beaver, Kettler, Scholes model, except that only one independent variable, the growth of earnings, was used as an instrument. Table 10 shows the parameters of the model for each year, and table 11 presents the resulting estimates of betas.

<table>
<thead>
<tr>
<th>Year</th>
<th>Coefficients</th>
<th>Adjusted $R^2$</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1969</td>
<td>$\beta = 1.693 - .0529$ Growth</td>
<td>.434</td>
<td>.358</td>
</tr>
<tr>
<td>1970</td>
<td>$\beta = 1.995 + .0133$ Growth</td>
<td>-.076</td>
<td>.521</td>
</tr>
<tr>
<td>1973</td>
<td>$\beta = 1.802 - .0114$ Growth</td>
<td>-.029</td>
<td>.448</td>
</tr>
<tr>
<td>1974</td>
<td>$\beta = 1.112 + .0180$ Growth</td>
<td>.280</td>
<td>.249</td>
</tr>
<tr>
<td>1975</td>
<td>$\beta = 1.157 + .0124$ Growth</td>
<td>-.017</td>
<td>.339</td>
</tr>
</tbody>
</table>

The low adjusted $R^2$ values and high standard errors were alarming, but the predictive power of the model, rather than these statistics, is the relevant measure of its usefulness. The changes in the estimates from year to year were more radical than the changes in the estimates from the Beaver, Kettler and Scholes model, but were usually consistent, at least in direction, with observed changes in the market betas.
TABLE II
GORDON AND HALPERN MODEL BETA ESTIMATES

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. California</td>
<td>1.67</td>
<td>1.76</td>
<td>1.65</td>
<td>1.32</td>
<td>1.27*</td>
</tr>
<tr>
<td>2. Empire</td>
<td>2.47</td>
<td>2.14*</td>
<td>1.64*</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3. Equitable</td>
<td>-</td>
<td>-</td>
<td>1.69</td>
<td>1.26</td>
<td>1.26</td>
</tr>
<tr>
<td>4. Far West</td>
<td>-</td>
<td>-</td>
<td>1.89</td>
<td>.89*</td>
<td>.98</td>
</tr>
<tr>
<td>5. Fidelity</td>
<td>-</td>
<td>-</td>
<td>1.61*</td>
<td>1.24</td>
<td>1.25</td>
</tr>
<tr>
<td>6. Santa Barbara</td>
<td>2.01*</td>
<td>2.19*</td>
<td>1.59</td>
<td>1.38</td>
<td>1.32*</td>
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<td>7. First Charter</td>
<td>2.23</td>
<td>2.16</td>
<td>1.62</td>
<td>1.37</td>
<td>1.33</td>
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<td>1.65*</td>
<td>1.94*</td>
<td>1.72*</td>
<td>1.23*</td>
<td>1.24</td>
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<tr>
<td>9. Golden West</td>
<td>-</td>
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<td>1.46</td>
<td>1.58</td>
<td>-</td>
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<td>10. Great Western</td>
<td>2.04</td>
<td>1.88</td>
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<td>1.29</td>
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<tr>
<td>11. Hawthorne</td>
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<td>1.96</td>
<td>1.72</td>
<td>1.22</td>
<td>1.23</td>
</tr>
<tr>
<td>12. Homestead</td>
<td>-</td>
<td>-</td>
<td>1.70</td>
<td>1.05*</td>
<td>1.23**</td>
</tr>
<tr>
<td>13. State</td>
<td>-</td>
<td>-</td>
<td>1.65**</td>
<td>1.20</td>
<td>-</td>
</tr>
<tr>
<td>14. Western</td>
<td>-</td>
<td>-</td>
<td>1.52</td>
<td>1.42*</td>
<td>1.33*</td>
</tr>
</tbody>
</table>

* denotes an estimate more than one standard deviation from the observed value.

** denotes an estimate more than two standard deviations from the observed value.
The Certainty Equivalent Model

The certainty equivalent model intrinsic value estimates for the holding companies were calculated using equation 3-10. The annual earnings estimate $\hat{x}_0$ was reduced (or increased if the covariance was negative) by the product of the market price of risk, $\lambda$, and the covariance of the firm's earnings with a measure of market earnings. Hamada's development of this approach to valuation suggested an index of returns on the market portfolio should be used for $R^*$\textsuperscript{M}.\textsuperscript{7} Initially, therefore, the Fisher value-weighted index described in chapter III was used in this study. However, for twenty-two of fifty calculations made in a pilot study, the covariance term was negative, and most of the remaining ones were only slightly positive. The resulting estimates of value, $\hat{S}_{j4}$, were far in excess of the other intrinsic value estimates developed in this study, and were several times as large as the computed market prices of the holding companies.

This problem is at least partly attributable to the use of annual earnings figures and annual return data in this study. The covariance calculations were thus made with five to eleven observations, which left them subject to substantial measurement error.

An alternate measure of market returns, the market earnings index described in chapter III was therefore used in order to develop more rational estimates of value. The premise was that firm earnings would be more highly correlated with aggregate earnings than with aggregate

market holding period returns. While theoretical justification for this approach may be weak, the results were more consistent with the expectation that firms with positive market betas (and hence positive correlations between holding period returns and market holding period returns) should also exhibit positive covariances between their earnings stream and the market return measure.

The Price/Earnings Model

The price/earnings model estimate of the intrinsic value of the firm, \( \hat{S}_j \), was developed using equation 3-11. The reported earnings of the firm for the twelve months ending on December 31 were multiplied by the industry average price/earnings ratio on January 31 of the new year to estimate the value of the firm at the end of January. If the earnings of the firm were negative for the year, no estimate of value was made.

III. Intrinsic Value Estimates

The estimates of value developed by the five models are presented in this section. The models are presented in reverse order, because the more recent ones were developed with a greater amount of information, and thus should have stronger and more reliable results. Mean values and standard deviations of the ratio of estimated intrinsic value to computed market price for each model are also presented in the tables. This information provides a concise comparison of the models, and will be used in section IV for testing the research hypotheses for each of the models in each year.

---

8 An interesting discovery made in this study was that, at least when using annual data, the choice of an index does make a considerable difference. Appendix D contains a discussion of this topic.
1975 Models

Table 12 presents the estimates of intrinsic value for January 31, 1976. Because each of the models was so inaccurate in estimating the value of Western Financial, the mean and standard deviations of the value to price ratios were calculated with and without that holding company. Subsequent investigation revealed that Western Financial took a substantial writeoff for discontinued operations in 1976, reducing earnings for the year to about five percent of what they were in 1975. It is likely that the market was aware of this pending writeoff, and thus the computed price of the firm was much less than the value predicted by the models used in this study. Based on these facts, it seemed appropriate to treat Western Financial as an outlier, and exclude it from tests of the model to prevent it from unduly distorting the results. With that one exception, the estimated intrinsic values appear similar to the computed market prices.

1974 Models

Table 13 contains the estimates of intrinsic value given for each of the models for January 31, 1975, and mean and standard deviation statistics for the value to price ratios. Because the estimates for State Savings and Loan and for Western Financial were so excessive for all models, these two were treated as outliers, and summary statistics were calculated with and without these two. A possible explanation for the Western Financial results was offered above. By the end of 1974, more than eighty-five percent of State Savings and Loan's shares were controlled by Budget Industries. Perhaps market demand for the remaining minority interest was very slight, and the low computed price was a
TABLE 12
1975 VALUATION ESTIMATES
(values in millions)

<table>
<thead>
<tr>
<th></th>
<th>Price/ Earnings</th>
<th>Gordon Halpern</th>
<th>Certainty Equivalent</th>
<th>Beaver Ketter Scholes</th>
<th>Market Beta</th>
<th>Computed Market Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equitable</td>
<td>26.946</td>
<td>27.344</td>
<td>60.424</td>
<td>26.128</td>
<td>27.426</td>
<td>20.605</td>
</tr>
<tr>
<td>Fidelity</td>
<td>37.507</td>
<td>38.295</td>
<td>85.205</td>
<td>41.203</td>
<td>39.499</td>
<td>29.100</td>
</tr>
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<td>28.424</td>
<td>36.455</td>
<td>84.853</td>
<td>36.111</td>
<td>30.246</td>
<td>22.497</td>
</tr>
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<td>First Charter</td>
<td>363.578</td>
<td>391.310</td>
<td>918.948</td>
<td>337.481</td>
<td>373.104</td>
<td>471.738</td>
</tr>
<tr>
<td>Gibraltar</td>
<td>73.416</td>
<td>75.676</td>
<td>167.313</td>
<td>70.939</td>
<td>72.750</td>
<td>59.771</td>
</tr>
<tr>
<td>Great Western</td>
<td>165.673</td>
<td>250.228</td>
<td>549.109</td>
<td>256.517</td>
<td>248.933</td>
<td>249.588</td>
</tr>
<tr>
<td>Homestead</td>
<td>7.343</td>
<td>4.852</td>
<td>10.842</td>
<td>6.548</td>
<td>7.232</td>
<td>5.005</td>
</tr>
<tr>
<td>Western</td>
<td>27.910</td>
<td>37.248</td>
<td>86.055</td>
<td>41.660</td>
<td>44.111</td>
<td>16.676</td>
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</tbody>
</table>

Value/Price

<table>
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<tr>
<th></th>
<th>( \mu_{10} )</th>
<th>( \sigma_{10} )</th>
<th>( \mu_{11} )</th>
<th>( \sigma_{11} )</th>
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<td>.523</td>
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<tr>
<td></td>
<td>1.096</td>
<td>.307</td>
<td>1.237</td>
<td>.550</td>
</tr>
</tbody>
</table>

NOTES: Empire was not included because it was acquired by another holding company in 1974.

Golden West was not included because it acquired another savings and loan holding company in 1975, making the resulting association unlike the Golden West used in this study.

State was not included because it was acquired by another holding company in 1975.
TABLE 13
1974 VALUATION ESTIMATES
(values in millions)

<table>
<thead>
<tr>
<th></th>
<th>Price/Earnings</th>
<th>Gordon Halpern</th>
<th>Certainty Equivalent</th>
<th>Beaver Kettler</th>
<th>Market Scholes</th>
<th>Beta</th>
<th>Computed Market Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>California</td>
<td>-</td>
<td>22.374</td>
<td>41.750</td>
<td>23.252</td>
<td>21.196</td>
<td>23.014</td>
<td></td>
</tr>
<tr>
<td>Equitable</td>
<td>17.118</td>
<td>23.280</td>
<td>46.470</td>
<td>22.660</td>
<td>24.074</td>
<td>17.140</td>
<td></td>
</tr>
<tr>
<td>Far West</td>
<td>-</td>
<td>12.481</td>
<td>22.513</td>
<td>10.820</td>
<td>10.962</td>
<td>12.772</td>
<td></td>
</tr>
<tr>
<td>Fidelity</td>
<td>19.737</td>
<td>33.024</td>
<td>66.392</td>
<td>34.933</td>
<td>36.295</td>
<td>22.089</td>
<td></td>
</tr>
<tr>
<td>Santa Barbara</td>
<td>18.633</td>
<td>34.322</td>
<td>70.183</td>
<td>32.112</td>
<td>30.263</td>
<td>18.667</td>
<td></td>
</tr>
<tr>
<td>First Charter</td>
<td>250.487</td>
<td>329.434</td>
<td>695.027</td>
<td>295.605</td>
<td>314.440</td>
<td>328.574</td>
<td></td>
</tr>
<tr>
<td>Gibraltar</td>
<td>46.897</td>
<td>66.068</td>
<td>132.161</td>
<td>55.386</td>
<td>56.122</td>
<td>43.078</td>
<td></td>
</tr>
<tr>
<td>Golden West</td>
<td>44.741</td>
<td>56.992</td>
<td>132.991</td>
<td>60.998</td>
<td>55.446</td>
<td>.53.583</td>
<td></td>
</tr>
<tr>
<td>Great Western</td>
<td>201.415</td>
<td>240.780</td>
<td>497.876</td>
<td>231.243</td>
<td>226.680</td>
<td>235.035</td>
<td></td>
</tr>
<tr>
<td>Homestead</td>
<td>1.669</td>
<td>3.392</td>
<td>6.201</td>
<td>4.111</td>
<td>4.035</td>
<td>2.816</td>
<td></td>
</tr>
<tr>
<td>State</td>
<td>12.289</td>
<td>17.983</td>
<td>36.114</td>
<td>17.905</td>
<td>18.895</td>
<td>8.435</td>
<td></td>
</tr>
<tr>
<td>Western</td>
<td>18.998</td>
<td>34.032</td>
<td>73.793</td>
<td>38.413</td>
<td>39.082</td>
<td>14.824</td>
<td></td>
</tr>
</tbody>
</table>

Value/Price

\[ \begin{align*}
\mu_{11} & = 0.908 & 1.280 & 2.571 & 1.270 & 1.267 \\
\sigma_{11} & = 0.171 & 0.304 & 0.640 & 0.320 & 0.336 \\
\mu_{13} & = 0.992 & 1.424 & 2.887 & 1.437 & 1.448 \\
\sigma_{13} & = 0.244 & 0.448 & 0.979 & 0.511 & 0.542
\end{align*} \]

NOTES: Empire was not included because it was acquired during 1974.

A dash indicates that no value could be estimated because the firm had negative earnings.
reflection of this. Most of the remaining estimates appear reasonably similar to the market prices, but, on average, overstate the price.

1973 Models

Table 1 presents the estimates of intrinsic value made by each model for January 31, 1974, and the means and variances of the value to price ratios computed from these estimates. No significant outliers appear in the estimates, and all but one of the mean value statistics appear reasonable.

1970 Models

Estimates of value for January 31, 1971, and summary statistics are contained in table 15. While three of the five models systematically understated the computed market prices of the seven holding companies included in the analysis, results for California Financial and Great Western were particularly poor for all five models. Both of these holding companies suffered precipitous declines in earnings in 1970. Far West reported negative earnings, and Great Western suffered a sixty-five percent decrease from the previous year. The models used in this study may have incorporated this information and given estimates of earnings far different from those anticipated by market forces. Accordingly, the mean and variance statistics are reported both with and without these two holding companies.

1969 Models

Table 16 presents estimates of intrinsic value and summary statistics for the holding companies on January 31, 1970. Three of the five models again appear to consistently understate the price of the holding
### Table 14

1973 Valuation Estimates
(values in millions)

<table>
<thead>
<tr>
<th></th>
<th>Price/Earnings</th>
<th>Gordon Halpern</th>
<th>Certainty Equivalent</th>
<th>Beaver Kettler Scholes</th>
<th>Market Beta</th>
<th>Market Computed Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equitable</td>
<td>27.141</td>
<td>16.510</td>
<td>38.941</td>
<td>17.111</td>
<td>17.877</td>
<td>22.598</td>
</tr>
<tr>
<td>Fidelity</td>
<td>36.904</td>
<td>29.960</td>
<td>71.074</td>
<td>33.083</td>
<td>36.668</td>
<td>28.927</td>
</tr>
<tr>
<td>Santa Barbara</td>
<td>28.037</td>
<td>26.909</td>
<td>64.482</td>
<td>24.537</td>
<td>27.291</td>
<td>18.882</td>
</tr>
<tr>
<td>First Charter</td>
<td>218.373</td>
<td>233.557</td>
<td>526.326</td>
<td>214.977</td>
<td>215.187</td>
<td>383.177</td>
</tr>
<tr>
<td>Gibraltar</td>
<td>68.101</td>
<td>44.429</td>
<td>106.729</td>
<td>40.809</td>
<td>35.895</td>
<td>55.839</td>
</tr>
<tr>
<td>Golden West</td>
<td>46.472</td>
<td>45.880</td>
<td>102.979</td>
<td>44.645</td>
<td>39.850</td>
<td>65.681</td>
</tr>
<tr>
<td>Great Western</td>
<td>257.713</td>
<td>165.744</td>
<td>379.427</td>
<td>187.535</td>
<td>162.247</td>
<td>302.188</td>
</tr>
<tr>
<td>Homestead</td>
<td>3.300</td>
<td>3.263</td>
<td>7.952</td>
<td>3.570</td>
<td>3.653</td>
<td>3.858</td>
</tr>
<tr>
<td>Western</td>
<td>37.315</td>
<td>30.147</td>
<td>68.856</td>
<td>33.265</td>
<td>31.777</td>
<td>22.472</td>
</tr>
</tbody>
</table>

Value/Price

\[
\mu_{14} = 1.121, \quad \sigma_{14} = 0.333
\]

\[
\mu_{14} = 0.914, \quad \sigma_{14} = 0.314
\]

\[
\mu_{14} = 2.172, \quad \sigma_{14} = 0.762
\]

\[
\mu_{14} = 0.916, \quad \sigma_{14} = 0.314
\]

\[
\mu_{14} = 0.957, \quad \sigma_{14} = 0.438
\]
### TABLE 15

1970 VALUATION ESTIMATES  
(values in millions)

<table>
<thead>
<tr>
<th></th>
<th>Price/Earnings</th>
<th>Gordon Halpern</th>
<th>Certainty Equivalent</th>
<th>Beaver Kettler Scholes</th>
<th>Market Beta</th>
<th>Computed Market Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>California</td>
<td>-</td>
<td>3.706</td>
<td>3.466</td>
<td>3.909</td>
<td>4.013</td>
<td>32.173</td>
</tr>
<tr>
<td>Empire</td>
<td>32.442</td>
<td>10.656</td>
<td>52.179</td>
<td>9.354</td>
<td>8.704</td>
<td>30.631</td>
</tr>
<tr>
<td>First Charter</td>
<td>423.044</td>
<td>116.880</td>
<td>575.274</td>
<td>132.094</td>
<td>116.032</td>
<td>501.380</td>
</tr>
<tr>
<td>Gibraltar</td>
<td>55.247</td>
<td>19.608</td>
<td>79.875</td>
<td>16.540</td>
<td>18.987</td>
<td>61.915</td>
</tr>
<tr>
<td>Great Western</td>
<td>98.198</td>
<td>54.889</td>
<td>213.735</td>
<td>51.774</td>
<td>52.506</td>
<td>379.814</td>
</tr>
<tr>
<td>Hawthorne</td>
<td>13.986</td>
<td>4.404</td>
<td>18.842</td>
<td>5.630</td>
<td>5.550</td>
<td>10.072</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>μ₅</th>
<th>σ₅</th>
<th>μ₇</th>
<th>σ₇</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value/Price</td>
<td>1.069</td>
<td>.320</td>
<td>1.487</td>
<td>.332</td>
</tr>
<tr>
<td></td>
<td>.220</td>
<td>.080</td>
<td>.300</td>
<td>.128</td>
</tr>
<tr>
<td></td>
<td>.934</td>
<td>.265</td>
<td>1.158</td>
<td>.274</td>
</tr>
<tr>
<td></td>
<td>.385</td>
<td>.113</td>
<td>.626</td>
<td>.144</td>
</tr>
</tbody>
</table>

**NOTES:** A dash indicates that no value could be estimated because the firm had negative earnings.

The remaining associations were not included because insufficient data were available.
TABLE 16
1969 VALUATION ESTIMATES
(values in millions)

<table>
<thead>
<tr>
<th></th>
<th>Price/Earnings</th>
<th>Gordon Halpern</th>
<th>Certainty Equivalent</th>
<th>Beaver Kettler</th>
<th>Market Scholes</th>
<th>Market Beta</th>
<th>Computed Market Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>California</td>
<td>17.066</td>
<td>4.987</td>
<td>10.648</td>
<td>5.585</td>
<td>5.390</td>
<td>28.890</td>
<td></td>
</tr>
<tr>
<td>Santa Barbara</td>
<td>17.620</td>
<td>4.223</td>
<td>13.417</td>
<td>5.007</td>
<td>5.338</td>
<td>17.026</td>
<td></td>
</tr>
<tr>
<td>First Charter</td>
<td>287.441</td>
<td>87.784</td>
<td>267.764</td>
<td>82.275</td>
<td>78.362</td>
<td>292.057</td>
<td></td>
</tr>
<tr>
<td>Gibraltar</td>
<td>51.264</td>
<td>20.930</td>
<td>52.666</td>
<td>18.026</td>
<td>18.308</td>
<td>43.344</td>
<td></td>
</tr>
<tr>
<td>Great Western</td>
<td>190.853</td>
<td>63.354</td>
<td>190.115</td>
<td>59.158</td>
<td>62.408</td>
<td>228.443</td>
<td></td>
</tr>
</tbody>
</table>

Value/Price

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mu_7)</td>
<td>.997</td>
<td>.353</td>
<td>.991</td>
<td>.346</td>
<td>.346</td>
<td></td>
</tr>
<tr>
<td>(\sigma_7)</td>
<td>.221</td>
<td>.151</td>
<td>.382</td>
<td>.138</td>
<td>.133</td>
<td></td>
</tr>
</tbody>
</table>

NOTE: The remaining associations were not included because insufficient data were available.
companies. A possible explanation of this phenomenon will be offered in section IV, along with tests of the research hypotheses.

IV. Tests and Conclusions

As explained in section V of chapter 3, two sets of tests were conducted. The first set tested the null hypothesis that, on average, the model under examination gave an intrinsic value estimate that was equal to the computed market price. Student's t statistics for tests of the hypothesis that the mean value of the ratios of estimated intrinsic value to computed market price is 1.00 are presented in table 17.

Analysis of 1973 and 1975 Models

The null hypothesis could not be rejected for any of the models except the certainty equivalent model for 1973 and 1975. Thus, the conclusion can be drawn that the other four models gave estimates consistent with computed market prices in 1973 and 1975.

Analysis of 1974 Models

The null hypothesis was rejected at the .05 or greater level of significance for all models except the price/earnings model in 1974. The conclusion could be drawn that the risk premium and the market price of risk were understated for the 1974 model. Raising this rate to a value higher than six percent would yield higher cost of equity capital for the Gordon and Halpern, Beaver, Kettler and Scholes, and market beta models, and thus would provide lower valuation estimates. The same results would be noticed in the certainty equivalent model, since an increase in the risk premium would increase the value of \( \lambda \), which would decrease the valuation estimate of all firms with positive covariances between the firm's
TABLE 17

T STATISTICS FOR THE HYPOTHESES THAT \( \mu = 1.00 \)

<table>
<thead>
<tr>
<th>Year</th>
<th>Market Beta</th>
<th>Beaver Kettler Scholes</th>
<th>Gordon Halpern</th>
<th>Certainty Equivalent</th>
<th>Price/Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1969</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n = 7</td>
<td>13.03***</td>
<td>12.51***</td>
<td>11.30***</td>
<td>.06</td>
<td>.03</td>
</tr>
<tr>
<td>1970</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n = 6</td>
<td>13.10***</td>
<td>12.76***</td>
<td>20.94***</td>
<td>4.02*</td>
<td>.77</td>
</tr>
<tr>
<td>n = 7</td>
<td>13.22***</td>
<td>13.30***</td>
<td>17.13***</td>
<td>.67</td>
<td>.45</td>
</tr>
<tr>
<td>1973</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n = 14</td>
<td>.37</td>
<td>1.01</td>
<td>1.03</td>
<td>5.76***</td>
<td>1.35</td>
</tr>
<tr>
<td>1974</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n = 11</td>
<td>2.64*</td>
<td>2.08*</td>
<td>3.06*</td>
<td>8.14***</td>
<td>.79</td>
</tr>
<tr>
<td>n = 13</td>
<td>2.98*</td>
<td>3.08*</td>
<td>3.41**</td>
<td>6.95***</td>
<td>.12</td>
</tr>
<tr>
<td>1975</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n = 10</td>
<td>1.36</td>
<td>.95</td>
<td>1.00</td>
<td>6.24***</td>
<td>1.07</td>
</tr>
<tr>
<td>n = 11</td>
<td>1.43</td>
<td>1.43</td>
<td>1.37</td>
<td>5.19***</td>
<td>1.54</td>
</tr>
</tbody>
</table>

* denotes significant at .05 level.

** denotes significant at .01 level.

*** denotes significant at .001 level.
earnings and the market return measure.

Such a conclusion must be rejected, however. It is not logical to contend that the risk premium was understated for the end of 1974, when observed market risk premia had been highly negative for two consecutive years, and then contend that the same value of the risk premium was correct at the end of 1975, when the observed risk premium for the year was highly positive. Yet the six percent estimate for 1975 seems correct, since the 1975 models performed satisfactorily.

It is more logical to conclude that no models based on earnings estimates estimated from past trends, betas estimated from past holding period returns, or instrumental variables estimates of beta calculated from past accounting data could adequately reflect the pessimism the market held for savings and loan stocks at that time. In retrospect, January 1975 may have been one of those times when intrinsic values were substantially different from market prices.

Analysis of 1969 and 1970 Models

None of the beta-based models gave estimates of intrinsic value consistent with computed market prices in either 1969 or 1970, while the price/earnings model, as was expected, did. It is tempting to conclude that the certainty equivalent model provided satisfactory estimates in both years; the statistics presented in table 17 support this conclusion. Another conclusion, however, may be more correct. The tendency of this

9 See section II of this chapter for discussion of the observed risk premiums in the market.

10 See chapter II, footnote 5, and the accompanying text, for a discussion of this point.
The failure of the beta approach must be attributed to misspecification of the models. The results shown in tables 15, 16, and 17 suggest that the no-growth formulation was not an appropriate one. Investors were anticipating future growth opportunities when they purchased shares in savings and loan holding companies in 1969 and 1970. This can be clearly seen by investigating the implications of the price/earnings ratios for savings and loan securities during these years. The average price/earnings ratios for January 31, 1970, and January 31, 1971 were 10.34 and 15.44 respectively, implying no-growth capitalization rates, or costs of equity capital, of 9.67 and 6.48 percent. Assuming that the risk premium estimates of eight percent were correct (if not, they were probably higher, which would lend even greater support to this analysis), and observing that the risk free rates were 8.05 percent and 4.60 percent, the average beta coefficients implied by the data can be calculated. The average beta for January 1970 would be .2025, and the average coefficient for January 1971 would be .235. These values are much too low to be believed.

Thus, the conclusion must be reached that investors were capitalizing anticipated future earnings in excess of present earnings \( \hat{x}_0 \).
and that the no-growth model was not an adequate representation of the valuation process at that time.

**Model Comparisons**

The second set of tests involved making paired comparisons of the five models to determine which one or ones, if any, outperformed the others. For purposes of this analysis, one model was considered to have outperformed another model if it provided an estimate of value that was closer to the computed market price than that predicted by the other model.

Predicting values that were too high was considered no worse than predicting a value that was too low, so the absolute values of the deviations from 1.00 of the predicted value to computed market value ratios were used as the statistics for comparison. The difference between the two deviations was calculated for each holding company, and then ranked according to magnitude. These ranks were then given a negative sign if the first model was outperformed by the second, and a positive sign if the first model outperformed the second. If the models were equal in estimation ability, the sum of the negative ranks should equal the sum of the positive ranks, except for random error.

If one model outperformed the other for all n observations, then the sum of the ranks of the inferior model would be zero, and the sum of the ranks of the superior model (ignoring the sign) would be the highest possible value, n(n+1)/2. Probabilities that the smaller of these sums of ranks will be less than or equal to particular values have been calculated, so that if a computed sum is less than or equal to that value, the hypothesis that the two samples are equal can be rejected at the
associated probability level. This procedure, the Wilcoxon matched pairs signed-rank test, is a more powerful nonparametric test than the chi-square test, since it considers the magnitude of deviations, not just the number of times that one model outperforms the other. It has a power-efficiency of nearly 95 percent relative to the t test, without making the restrictive assumptions of the t test.\(^\text{11}\)

The Wilcoxon test was applied to all possible paired comparisons for each year, and the results are reported in tables 18, 19, 20, 21, and 22. The sum for the row model is reported first, followed by the sum for the model listed in the column above the value. The following abbreviations are used:

- MKT refers to the market beta model
- BKS refers to the Beaver, Kettler, Scholes model
- GH refers to the Gordon and Halpern model
- CE refers to the certainty equivalent model
- P/E refers to the price/earnings model

For some years the price/earnings model could not be used due to negative earnings. In these instances, an indication is given of how the model performed both including the negative value (which involved assigning the "worst" possible rank to the observation) and without that value. The data are reported so that the larger sum represents the stronger model. Thus, the entry "40/15" in table 18 denotes that in a comparison between the Gordon and Halpern model and the market beta model, the sum of ranks for the GH model was 40 and for the MKT model was 15.

### TABLE 18

**WILCOXON TEST FOR 1975**

<table>
<thead>
<tr>
<th></th>
<th>MKT</th>
<th>BKS</th>
<th>GH</th>
<th>CE</th>
</tr>
</thead>
<tbody>
<tr>
<td>BKS</td>
<td>31/24</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GH</td>
<td>40/15</td>
<td>36/19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CE</td>
<td>1/54***</td>
<td>1/54***</td>
<td>1/54***</td>
<td></td>
</tr>
<tr>
<td>P/E</td>
<td>39/16</td>
<td>33/22</td>
<td>32/23</td>
<td>55/0***</td>
</tr>
</tbody>
</table>

NOTE: n = 10 and the maximum sum = 55

*** denotes significant at .01 level.

### TABLE 19

**WILCOXON TEST FOR 1974**

<table>
<thead>
<tr>
<th></th>
<th>MKT</th>
<th>BKS</th>
<th>GH</th>
<th>CE</th>
</tr>
</thead>
<tbody>
<tr>
<td>BKS</td>
<td>35/31</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GH</td>
<td>45/21</td>
<td>42/24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CE</td>
<td>0/66***</td>
<td>0/66***</td>
<td>0/66***</td>
<td></td>
</tr>
<tr>
<td>P/E</td>
<td>36/9</td>
<td>35/10</td>
<td>35/10</td>
<td>45/0***</td>
</tr>
</tbody>
</table>

NOTES: n = 11 and the maximum sum = 66 for all comparisons not involving the price/earnings model.

n = 9 and maximum sum = 45 for the price/earnings comparisons, because two price/earnings ratios were negative. Excluding these two items resulted in the values shown above. Including these two items resulted in there being no significant differences in the models.

*** denotes significant at .01 level.
### TABLE 20

**WILCOXON TEST FOR 1973**

<table>
<thead>
<tr>
<th></th>
<th>MKT</th>
<th>BKS</th>
<th>GH</th>
<th>CE</th>
</tr>
</thead>
<tbody>
<tr>
<td>BKS</td>
<td>90.5/14.5**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GH</td>
<td>76/29</td>
<td>46/59</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CE</td>
<td>7/98***</td>
<td>7/98***</td>
<td>5/100***</td>
<td></td>
</tr>
<tr>
<td>P/E</td>
<td>62/43</td>
<td>40.5/64.5</td>
<td>42/63</td>
<td>105/0***</td>
</tr>
</tbody>
</table>

**Note:** n = 14 and the maximum sum = 105

**** denotes significant at .02 level.

*** denotes significant at .01 level.

### TABLE 21

**WILCOXON TEST FOR 1970**

<table>
<thead>
<tr>
<th></th>
<th>MKT</th>
<th>BKS</th>
<th>GH</th>
<th>CE</th>
</tr>
</thead>
<tbody>
<tr>
<td>BKS</td>
<td>12/16</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GH</td>
<td>11.5/16.5</td>
<td>14/14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CE</td>
<td>21/7</td>
<td>20/8</td>
<td>22/6</td>
<td></td>
</tr>
<tr>
<td>P/E</td>
<td>21/0*</td>
<td>21/0*</td>
<td>21/0*</td>
<td>16/5</td>
</tr>
</tbody>
</table>

**Notes:** n = 7 and the maximum sum = 28 for all comparisons not involving the price/earnings model.

n = 6 and maximum sum = 21 for the price/earnings comparisons, because of one negative price/earnings ratio. Excluding this company resulted in the values shown above. Including this company resulted in there being no significant differences in the models.

* denotes significant at .05 level.
### TABLE 22

**WILCOXON TEST FOR 1969**

<table>
<thead>
<tr>
<th></th>
<th>MKT</th>
<th>BKS</th>
<th>CH</th>
<th>CE</th>
</tr>
</thead>
<tbody>
<tr>
<td>BKS</td>
<td>13.5</td>
<td>14.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GH</td>
<td>25/3</td>
<td>17/11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CE</td>
<td>27/1*</td>
<td>27/1*</td>
<td>27/1*</td>
<td></td>
</tr>
<tr>
<td>P/E</td>
<td>28/0**</td>
<td>28/0**</td>
<td>28/0**</td>
<td>28/0**</td>
</tr>
</tbody>
</table>

**NOTE:** n = 7 and the maximum sum = 28.

* denotes significant at .05 level.

** denotes significant at .02 level.
The Gordon and Halpern model was thus stronger, but not at a statistically significant level.

The price/earnings model was clearly the most effective one in 1969 and 1970, even after taking into account its inability to make a prediction when the firm's earnings were negative. The certainty equivalent model clearly outperformed the beta-based models in both years, although the differences were not statistically significant in 1970. No clearly strongest model emerged from among the beta-based models; and since each of them significantly understated the value of the firm, and each was outperformed by the price/earnings model, they should not be relied upon for accurate prediction of value.

The price/earnings model was at least as strong as any other model for the 1973, 1974, and 1975 tests, and was clearly superior to the certainty equivalent model in all three years. This result is not too surprising, since the index used was constructed from the same associations used in the actual study. Nevertheless, it does raise the question of whether the other models are useful to the financial manager. Whether the beta-based models are useful to the manager or not (and that decision is for him or her to make after analyzing these results), they clearly did dominate the certainty equivalent model. Each of the three models outperformed the certainty equivalent model at a .01 level of significance for each of the three years.

Among the three beta-based models, no dominant one emerged. However, it is encouraging to note that the Gordon and Halpern model and the Beaver, Kettler and Scholes model performed at least as well as the market beta model. These results are important, because they imply that the
absence of market price information from which to compute market betas is not an insurmountable problem for the analyst who wishes to employ beta-based estimates of the cost of capital in the valuation process. If a sufficiently large sample of publicly traded firms is available from which to calculate parameters, an instrumental variables estimate of beta may be substituted for a market-based estimate without a deterioration in the accuracy of the prediction.

No real difference can be perceived between the Gordon and Halpern and Beaver, Kettler and Scholes models. The greater statistical significance of the Beaver, Kettler and Scholes model\textsuperscript{12} did not translate into better estimates. If anything, the Gordon and Halpern model was slightly better, since its values on the Wilcoxon test were stronger in two of the three years.

**Discussion of the Certainty Equivalent Model Results**

The failure of the certainty equivalent model to provide satisfactory results for either the tests of the null hypotheses or the paired model comparisons was surprising, and warrants further attention. In retrospect, it seems apparent that this model was too sensitive to measurement error in the earnings figures. Because annual data were used, the covariance estimates were formed from a small number of observations, and the resulting figures could thus be greatly influenced by one or two years of abnormal or incorrectly measured earnings. This possibility is consistent with the discovery in this study that a trendline growth rate of earnings was a better instrument in the Gordon and Halpern model

\textsuperscript{12}See tables 8 and 10.
than was the earnings beta variable they originally proposed.\textsuperscript{13} It is also consistent with the results obtained by Beaver, Kettler and Scholes, who found several variables which demonstrated higher correlations with their market beta than did the variable which was most strongly linked to beta on a theoretical level.\textsuperscript{14} Whether the failure of this model is in fact due to the form of its specification in this study or to a more fundamental problem cannot be reliably determined here, and will be an interesting topic for future research.

V. Summary

This chapter contained the analysis of the holding company sample. Procedures for selection of the final sample were described, and the parameters to be used in the models were estimated. The estimates of intrinsic value made by each model were presented, and tests were performed to determine if each model's estimates of value were consistent with computed market prices. Paired comparisons of the five models were also made, using the Wilcoxon matched pairs signed-rank test, to determine if one or more of the models gave results that were superior to those given by the other models.

The price/earnings model emerged as the strongest model in both sets of tests, but the Gordon and Halpern model and the Beaver, Kettler and Scholes model also gave satisfactory results in the more recent time periods. One important discovery emerged—the instrumental variables models performed at least as well as the market beta model.

\textsuperscript{13} See chapter II, footnote 11.

CHAPTER V

THE ACQUISITIONS

This chapter contains an analysis of associations acquired through merger. The valuation approaches that proved satisfactory for the holding company analysis presented in chapter IV will be used to estimate intrinsic values of acquired savings and loan associations on the date of their mergers. These value estimates will be compared to the prices paid for the associations, and the results of tests of the research hypotheses will be reported.

The first section will contain a description of the screening procedures used to select the final sample of acquisitions, and also will contain data relating to the associations included. The second section will present the models used in the analysis and the parameters of these models. The third section will contain the estimates of value developed by the models, and the final section will contain tests of the research hypotheses.

I. Selection of the Sample

As explained in section IV of chapter III, sixty mergers involving acquisition of a stock association in exchange for cash and thirty-seven mergers involving acquisition of a stock association in exchange for actively traded stock occurred during the seven year period analyzed in this study. These mergers are categorized according to the year in which they took place in table 23.
TABLE 23
ACQUIRED SAVINGS AND LOANS

<table>
<thead>
<tr>
<th>Year</th>
<th>Mergers for Stock</th>
<th>Mergers for Cash</th>
</tr>
</thead>
<tbody>
<tr>
<td>1969</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>1970</td>
<td>4</td>
<td>15</td>
</tr>
<tr>
<td>1971</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>1972</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>1973</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>1974</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>1975</td>
<td>1</td>
<td>14</td>
</tr>
<tr>
<td>TOTAL:</td>
<td>37</td>
<td>60</td>
</tr>
</tbody>
</table>

NOTE: The date of acquisition was considered to be the month in which the merger was announced as tentative by the Federal Home Loan Bank Board.

Originally, this study was designed to analyze both forms of acquisition and to compare the two. However, this proved to be impossible to carry out. Several factors led to the exclusion of the stock acquisitions from the analysis. First, only in 1971 were there enough mergers of both types to allow a meaningful comparison. Second, the price data for the securities in which payment was made for the stock associations were of variable quality; some of the data represented prices of listed securities, while other data were reported by the
acquiring associations themselves, along with the claim that their securities were actively traded. Third, as the Piper study\textsuperscript{1} emphasized, there are fundamental differences between acquisitions for stock and acquisitions for cash that make comparison of the two forms difficult, even under ideal conditions. And finally, the results of the holding company analysis in 1969 and 1970 were so poor that useful comparisons could not be developed for the stock and cash models for those years anyway. Thus, only the acquisitions for cash are analyzed in this chapter.

Insufficient data were available to develop models for the end of 1968, which would have been required in order to establish values for acquisitions made during 1969. Covariances, earnings betas, and growth rates would have been estimated from only four observations (1965 to 1968). Thus, the 1969 acquisitions had to be excluded from the study.

There were not enough acquisitions in 1972 or 1973 to justify development of models for those years. Therefore, 1970, 1971, 1974, and 1975 were the only years for which analyses could be performed. Accordingly, the holding company analyses were conducted for the end of 1969, 1970, 1973, and 1974 in order to develop models for use with the acquisitions throughout the following year. After the tests of the research hypotheses were conducted for the holding company sample, the 1970 and 1971 acquisitions were deleted from the sample as well. Since the null hypotheses that the beta-based models provided unbiased estimates of computed market

\textsuperscript{1}Section III of chapter II contains a discussion of this study.
prices were rejected at the .001 level of significance in both 1969 and
1970 (see table 17), these models could not be counted upon to provide
accurate estimates of intrinsic value. Acceptable estimates could only
be obtained by restructuring each of the models to include a true growth
term, and that requirement exceeds the scope of this study. Therefore,
only the eight cash acquisitions that occurred in 1974 and twelve of
the fourteen acquisitions made in 1975 are analyzed in this chapter. 2

Data pertaining to these acquisitions are presented in table
24. The Federal Home Loan Bank Board required that the identity of
the associations not be revealed. Therefore, they are referred to in
this table and all subsequent discussions by a unique letter assigned
to them.

II. Models and Parameters

The market beta model could not be used in these analyses
because price relatives could not be calculated. The certainty
equivalent model was also not used, since the null hypothesis that
this model provided unbiased estimates of computed market prices
was rejected at the .001 level of significance in both 1973 and 1974,
and the model was judged to be inferior to the other models at the
.01 level of significance by the Wilcoxon test in both years. 3
Thus, only the price/earnings model, the Beaver, Kettler and Scholes

2 The other two associations were eliminated in 1975 due to
significant differences between data contained in the merger documents
and data provided by the Federal Home Loan Bank Board.

3 See tables 17, 19, and 20.
<table>
<thead>
<tr>
<th>Code</th>
<th>Acquisition Date</th>
<th>Book Value in Millions</th>
<th>Total Assets in Millions on December 31 prior to Acquisition</th>
<th>Purchase Price in Millions</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4/74</td>
<td>9.2</td>
<td>122.4</td>
<td>12.6</td>
</tr>
<tr>
<td>B</td>
<td>4/74</td>
<td>.4</td>
<td>7.3</td>
<td>.9</td>
</tr>
<tr>
<td>C</td>
<td>7/74</td>
<td>.2</td>
<td>3.9</td>
<td>.1</td>
</tr>
<tr>
<td>D</td>
<td>6/74</td>
<td>1.3</td>
<td>35.2</td>
<td>2.3</td>
</tr>
<tr>
<td>E</td>
<td>1/74</td>
<td>1.1</td>
<td>16.1</td>
<td>3.2</td>
</tr>
<tr>
<td>F</td>
<td>1/74</td>
<td>.8</td>
<td>13.0</td>
<td>1.6</td>
</tr>
<tr>
<td>G</td>
<td>1/74</td>
<td>2.8</td>
<td>43.0</td>
<td>9.0</td>
</tr>
<tr>
<td>H</td>
<td>1/74</td>
<td>.3</td>
<td>4.2</td>
<td>.4</td>
</tr>
<tr>
<td>I</td>
<td>4/75</td>
<td>3.8</td>
<td>81.1</td>
<td>3.8</td>
</tr>
<tr>
<td>J</td>
<td>10/75</td>
<td>.7</td>
<td>10.8</td>
<td>1.0</td>
</tr>
<tr>
<td>K</td>
<td>10/75</td>
<td>1.8</td>
<td>25.0</td>
<td>2.9</td>
</tr>
<tr>
<td>L</td>
<td>3/75</td>
<td>.5</td>
<td>17.2</td>
<td>.9</td>
</tr>
<tr>
<td>M</td>
<td>12/75</td>
<td>21.0</td>
<td>294.0</td>
<td>21.1</td>
</tr>
<tr>
<td>N</td>
<td>12/75</td>
<td>1.8</td>
<td>19.0</td>
<td>2.4</td>
</tr>
<tr>
<td>O</td>
<td>12/75</td>
<td>.7</td>
<td>20.5</td>
<td>1.6</td>
</tr>
<tr>
<td>P</td>
<td>12/75</td>
<td>2.8</td>
<td>43.3</td>
<td>4.4</td>
</tr>
<tr>
<td>Q</td>
<td>11/75</td>
<td>.8</td>
<td>23.5</td>
<td>1.7</td>
</tr>
<tr>
<td>R</td>
<td>7/75</td>
<td>1.9</td>
<td>33.9</td>
<td>4.8</td>
</tr>
<tr>
<td>S</td>
<td>11/75</td>
<td>1.8</td>
<td>19.1</td>
<td>2.0</td>
</tr>
<tr>
<td>T</td>
<td>1/75</td>
<td>1.2</td>
<td>49.8</td>
<td>3.6</td>
</tr>
</tbody>
</table>
model and the Gordon and Halpern model were used to develop intrinsic value estimates. The 1973 and 1974 Beaver, Kettler and Scholes and Gordon and Halpern models, as presented in tables 8 and 10, were used to estimate the beta coefficients for the associations acquired in 1974 and 1975. While the models were specifically developed for January 31 of those years, the parameters of the models were relatively stable from year to year. Thus, there is little reason to believe that the models would be inaccurate in predicting an instrumental variables beta estimate in November or December. The instrumental variables estimates of beta given by the two models are presented in table 25.

The same values were used for the risk premium as were used in the holding company models, seven percent in 1974 and six percent during 1975. The value used for the risk free rate was the coupon equivalent yield on a one year U. S. Treasury bill at the close of the month prior to the date of acquisition, and the trendline value of earnings was used as the estimate of \( \hat{X}_0 \). For the price/earnings model, the price/earnings ratio for the month prior to the date of acquisition was used to capitalize the firm's reported earnings for the previous year.

III. Intrinsic Value Estimates

The estimates of value developed by the three models for the 1974 acquisitions are presented in table 26, and the values for the 1975 acquisitions are presented in table 27. Also included are the actual acquisition prices paid, and mean values and standard deviations of the ratios of estimated value to acquisition price.
### TABLE 25

**INSTRUMENTAL VARIABLES BETA ESTIMATES**

<table>
<thead>
<tr>
<th>Association</th>
<th>Beaver, Kettler Scholes Model</th>
<th>Gordon and Halpern Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2.20</td>
<td>1.77</td>
</tr>
<tr>
<td>B</td>
<td>2.58</td>
<td>1.54</td>
</tr>
<tr>
<td>C</td>
<td>2.03</td>
<td>1.69</td>
</tr>
<tr>
<td>D</td>
<td>1.70</td>
<td>1.56</td>
</tr>
<tr>
<td>E</td>
<td>2.34</td>
<td>1.56</td>
</tr>
<tr>
<td>F</td>
<td>.49</td>
<td>1.26</td>
</tr>
<tr>
<td>G</td>
<td>1.19</td>
<td>1.20</td>
</tr>
<tr>
<td>H</td>
<td>-.09</td>
<td>1.32</td>
</tr>
<tr>
<td>I</td>
<td>.87</td>
<td>1.60</td>
</tr>
<tr>
<td>J</td>
<td>.46</td>
<td>1.50</td>
</tr>
<tr>
<td>K</td>
<td>.34</td>
<td>1.70</td>
</tr>
<tr>
<td>L</td>
<td>.59</td>
<td>1.01</td>
</tr>
<tr>
<td>M</td>
<td>1.27</td>
<td>1.15</td>
</tr>
<tr>
<td>N</td>
<td>1.03</td>
<td>1.25</td>
</tr>
<tr>
<td>O</td>
<td>.34</td>
<td>.99</td>
</tr>
<tr>
<td>P</td>
<td>.89</td>
<td>1.41</td>
</tr>
<tr>
<td>Q</td>
<td>.17</td>
<td>1.15</td>
</tr>
<tr>
<td>R</td>
<td>.60</td>
<td>1.33</td>
</tr>
<tr>
<td>S</td>
<td>.61</td>
<td>1.45</td>
</tr>
<tr>
<td>T</td>
<td>1.19</td>
<td>1.02</td>
</tr>
</tbody>
</table>
TABLE 26

VALUATION ESTIMATES FOR 1974 ACQUISITIONS

<table>
<thead>
<tr>
<th>Association</th>
<th>Price/Earnings</th>
<th>Gordon Halpern</th>
<th>Beaver Kettler</th>
<th>Scholes</th>
<th>Merger Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2,931,201</td>
<td>2,510,607</td>
<td>2,199,324</td>
<td>12,600,000</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>387,436</td>
<td>286,563</td>
<td>208,824</td>
<td>924,226</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>159,139</td>
<td>116,364</td>
<td>104,302</td>
<td>142,000</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>914,258</td>
<td>1,133,122</td>
<td>1,078,964</td>
<td>2,307,873</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>494,893</td>
<td>942,778</td>
<td>726,757</td>
<td>3,250,000</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>656,996</td>
<td>1,220,268</td>
<td>1,822,038</td>
<td>1,597,000</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>1,335,234</td>
<td>3,075,473</td>
<td>3,085,112</td>
<td>9,000,000</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>231,916</td>
<td>185,138</td>
<td>451,451</td>
<td>444,500</td>
<td></td>
</tr>
</tbody>
</table>

Value/Price

\[
\begin{align*}
\mu_8 &= 0.425 \\
\sigma_8 &= 0.312 \\
\end{align*}
\]
### TABLE 27

VALUATION ESTIMATES FOR 1975 ACQUISITIONS

<table>
<thead>
<tr>
<th>Association</th>
<th>Price/Earnings</th>
<th>Gordon Halpern</th>
<th>Beaver Kettler</th>
<th>Merger Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>453,282</td>
<td>1,890,455</td>
<td>2,625,454</td>
<td>3,774,000</td>
</tr>
<tr>
<td>J</td>
<td>562,050</td>
<td>635,608</td>
<td>1,042,056</td>
<td>970,000</td>
</tr>
<tr>
<td>K</td>
<td>1,739,769</td>
<td>1,769,618</td>
<td>3,372,545</td>
<td>2,946,944</td>
</tr>
<tr>
<td>L</td>
<td>22,606</td>
<td>154,430</td>
<td>195,460</td>
<td>865,948</td>
</tr>
<tr>
<td>M</td>
<td>3,163,871</td>
<td>9,346,262</td>
<td>8,839,458</td>
<td>21,084,000</td>
</tr>
<tr>
<td>N</td>
<td>728,562</td>
<td>987,348</td>
<td>1,092,308</td>
<td>2,400,000</td>
</tr>
<tr>
<td>O</td>
<td>109,150</td>
<td>224,617</td>
<td>329,103</td>
<td>1,600,000</td>
</tr>
<tr>
<td>P</td>
<td>1,557,121</td>
<td>2,057,478</td>
<td>2,607,248</td>
<td>4,399,995</td>
</tr>
<tr>
<td>Q</td>
<td>41,781</td>
<td>379,508</td>
<td>660,076</td>
<td>1,690,000</td>
</tr>
<tr>
<td>R</td>
<td>1,518,240</td>
<td>1,271,864</td>
<td>1,930,144</td>
<td>4,777,000</td>
</tr>
<tr>
<td>S</td>
<td>1,057,552</td>
<td>1,500,883</td>
<td>2,226,538</td>
<td>1,950,000</td>
</tr>
<tr>
<td>T</td>
<td>-</td>
<td>591,719</td>
<td>550,570</td>
<td>3,600,000</td>
</tr>
</tbody>
</table>

Value/Price

\[ \mu_{11 \text{ or 12}} = 0.280, \quad \mu_{12} = 0.402, \quad \sigma_{11 \text{ or 12}} = 0.219, \quad \sigma_{12} = 0.209, \quad \sigma_{12} = 0.575 \]

**NOTE:** The price/earnings model could not place a value on association T because earnings were negative. Thus, the mean and standard deviation were estimated from eleven observations for that model.
While the Beaver, Kettler and Scholes model appears to give better estimates than the Gordon and Halpern model in both years, the 1975 results must be viewed with caution. Because the models tend to understate the acquisition prices, whichever model predicted the lower beta predicted the higher value, and this was invariably the one most consistent with the acquisition price. The Beaver, Kettler and Scholes model usually predicted the lower beta due to the inclusion of the size variable as an instrument. In retrospect, it probably would have been prudent to not include a size variable in the analysis, due to the great disparity in size between the holding companies that were used to build the models and the acquisitions that were then appraised using the models. The size variable tended to overpower the models and give results that perhaps understate the betas of the smaller companies.

IV. Tests and Conclusions

The Student's t statistics for the hypotheses that, on average, each of the models gave intrinsic value estimates consistent with the acquisition prices of the associations are presented in table 28. All of the null hypotheses could be rejected at the .05 level of significance, and all could be rejected at the .01 level except the Beaver, Kettler, Scholes model in 1974.

These results are not surprising. The fact that a large control premium is justified when the entire firm, or at least a majority interest in the firm, is acquired was discussed in chapter II. In chapter IV, computed market prices were used to test the models, and these did not reflect control premiums, whereas the acquisition prices used in this chapter do.
TABLE 28

<table>
<thead>
<tr>
<th>Year</th>
<th>Beaver Kettler Scholes</th>
<th>Gordon Halpern</th>
<th>Price/Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1974</td>
<td>3.43*</td>
<td>6.83***</td>
<td>5.21***</td>
</tr>
<tr>
<td>1975</td>
<td>4.06**</td>
<td>9.92***</td>
<td>11.40***</td>
</tr>
</tbody>
</table>

* denotes significant at .05 level.

** denotes significant at .01 level.

*** denotes significant at .001 level.

The results are also consistent with those in a recent study of premiums paid for controlling shares of closely held commercial banks. Meeker and Joy found that "On average, control premiums ranged from about 50% to 70%" in the 1970-1975 period which they studied.

The mean values reported in tables 25 and 26 would imply control premiums of 73.8 percent in 1975 and 84.9 percent in 1974 for the Beaver, Kettler and Scholes model, and 144.8 percent in 1975 and 120.2 percent in 1974 for the Gordon and Halpern model. While the Gordon and Halpern model thus seems to overstate the premiums, the Beaver, Kettler and Scholes model implications appear possible.

Tables 29 and 30 present the results of the paired comparisons of the models. These comparisons show that the price/earnings model was inferior to the beta-based models in 1975, but was not different from them in 1974. They also show the Beaver, Kettler and

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### TABLE 29
WILCOXON TEST FOR 1975

<table>
<thead>
<tr>
<th></th>
<th>BKS</th>
<th>GH</th>
</tr>
</thead>
<tbody>
<tr>
<td>GH</td>
<td></td>
<td>3/75***</td>
</tr>
<tr>
<td>P/E</td>
<td>0/78***</td>
<td>2/76***</td>
</tr>
</tbody>
</table>

*NOTE: n = 12 and the maximum sum = 78.
*** denotes significant at .01 level.*

### TABLE 30
WILCOXON TEST FOR 1974

<table>
<thead>
<tr>
<th></th>
<th>BKS</th>
<th>GH</th>
</tr>
</thead>
<tbody>
<tr>
<td>GH</td>
<td>20/16</td>
<td></td>
</tr>
<tr>
<td>P/E</td>
<td>10/26</td>
<td>12/24</td>
</tr>
</tbody>
</table>

*NOTES: n = 8 and the maximum sum = 36.
None of the comparisons were significant.*
Scholes model was superior to the Gordon and Halpern model in 1975, but not in 1974.

The fact that the beta-based models appear to have performed at least as well as the price/earnings model in 1974 and significantly better in 1975 is an important discovery. It suggests that the procedures developed in this study do indeed outperform a simple price/earnings method of valuation if the associations for which value is to be established are not used to develop the price/earnings ratio. Thus, although the small sample sizes used in the study make generalization of the results difficult, it appears that the beta-based models do possess some predictive power in excess of that provided by the price/earnings technique.

V. Summary

Analysis of the acquisitions sample was presented in this chapter. Procedures for selecting the final sample were established, and estimates of intrinsic value made by the Beaver, Kettler and Scholes model, the Gordon and Halpern model, and the price/earnings model were presented. Tests of the research hypotheses revealed that none of the models gave estimates consistent with the acquisitions prices paid for the merger candidates. This result was expected, since the models were designed to estimate current intrinsic value of the firm, but were not designed to cope with control premiums. The beta-based models provided estimates that were more consistent with acquisition prices and that also were more consistent with a recent study concerning control premiums than were the estimates made by the price/earnings model.
This chapter contains a review of the first five chapters. The first section presents a summary of the problems addressed, the models developed, and the tests of these models. The second section reviews the conclusions that were reached, and the final section suggests ideas for future research that arise out of this work.

I. Review

This study was designed to develop procedures for valuation of the firm in the absence of market price information for its stock. Market price information is often not available for smaller businesses, and yet determination of value must often be made for purposes of estate taxation, issuance of an initial block of stock to the market, or merger. Even when market prices are available, they may not adequately reflect intrinsic value, due to abnormal market conditions or to infrequent trading. Accordingly, valuation techniques which do not directly rely upon market price information are frequently needed, and were investigated in this study.

The savings and loan industry was chosen as the focus of this work for several reasons. Most of the savings and loan associations in the United States are closely held or mutual organizations. Yet because of mergers and conversions, proper values must often be determined. Secondly, the industry is highly regulated, and thus there are...
certain uniformities among associations that facilitate comparisons.
And finally, data were available for testing because there were a
number of mergers during the past decade in which a purchase price
was determined.

Several possible approaches to valuation were investigated,
and the current earnings plus investment opportunities approach was
selected as the best one for this study, due to its separation of
the value of the firm into two distinct parts—the value of the
present earnings stream, and the value of all future opportunities
to invest at rates of return in excess of the firm's cost of capital.
It was then demonstrated that under conditions facing the savings and
loan industry in recent years, the magnitude of this second component
was likely to be very small relative to the first component, and thus
the excess earnings opportunities were assumed to be nonexistent.

Under this simplifying assumption, valuation requires only two
tasks: estimation of the true present earnings (for which observed
present earnings may be a poor proxy), and estimation of an appropriate
capitalization rate for those earnings. The first of these steps was
accomplished by using a trendline value of earnings which recognizes
the fact that earnings may show different values from year to year
due to a variety of possible reasons, but an underlying trend exists.

The second step, estimating the capitalization rate, was
approached in a number of ways, in order to test various methods that
have been proposed by researchers and practitioners. Three of the
proposed models were derived from the beta concept common to financial
literature. One model directly employed the market beta concept, and
the other two relied upon a reference group of market betas for publicly
traded firms from which to develop instrumental variables estimates of the true, but unobservable, beta.

A fourth model provided direct estimates of the value of the firm through a method in which earnings were adjusted to certainty equivalent values and then capitalized at the risk free rate. The final model developed was a simple price/earnings model in which the firm's observed earnings for the most recent year were capitalized by the industry average earnings to price ratio.

These models were applied to two different samples of firms. The first sample was a group of savings and loan holding companies for which market price information was available. The intrinsic value estimates provided by the five models were compared to computed market prices (the product of the number of shares outstanding and the market price per share) to determine if the model estimates were consistent with observed prices. While intrinsic values may deviate from observed prices, efficient market hypotheses suggest that prices are unbiased estimates of value. The second sample used was a number of stock-form savings and loan associations which were acquired by merger and for which acquisition prices were available. The holding company sample was used as a reference group from which parameters were estimated, and the resulting instrumental variables estimates of unobservable betas were used to estimate the cost of capital for the closely held associations. The price/earnings model was also applied to these associations.

Screening procedures reduced the two samples to rather small numbers of firms, but were necessary in order to assure that problems
with the data did not distort the models. However, the small sample sizes did undoubtedly weaken the generalizations that can be made from the models developed.

II. Results and Conclusions

The results of the holding company analysis must be separated into two parts. For 1973, 1974, and 1975, all of the models except the certainty equivalent model gave results that were reasonably consistent with observed market prices. The hypotheses that each model, on average, provided an estimate of intrinsic value consistent with the market price could not be rejected for the 1973 and 1975 models, and was rejected, but not at a high level of significance, for the 1974 models. This rejection, however, may have been attributable to market conditions or error in estimation of the risk premium, rather than fundamental breakdown of the models.

When paired comparisons of the models were made, the certainty equivalent model was judged significantly weaker than the other four models for each of the three years. Among the other models, the price/earnings model was at least as strong as any other model, even after considering its inability to make an estimate when earnings were negative. This performance, however, was probably artificially strong due to the fact that the price/earnings ratio used was constructed with data from the associations which were valued by it.

No clearly superior model emerged from the three beta-based models, but the two that employed instrumental variables were found to be at least as good as the market beta model, and were reasonably stable over time. This discovery led to the conclusion that the
absence of market price information is not a significant handicap for beta-based estimation of value. As long as enough publicly traded firms of a similar nature are available for estimation of the parameters, an instrumental variables estimate of beta can be substituted for the unobservable market estimate without deterioration in the accuracy of the prediction of value.

The results of the 1969 and 1970 models, however, were very different. The price/earnings model was clearly superior to all other models in both years. It and the certainty equivalent model provided estimates consistent with computed market prices, while the three beta-based approaches significantly understated the prices. The source of the deficiency was easy to pinpoint—the assumption made in this study that investors in savings and loan shares did not anticipate "true growth" investment opportunities was not correct. An analysis of prevailing price/earnings ratios, risk free rates, and risk premiums at that time reveals that significant growth expectations must have prevailed.

Thus, the models developed were adequate for the 1973-1975 period, but were not adequate for 1969 and 1970. Accordingly, 1969 and 1970 had to be ruled out of the analysis of the merger acquisitions, and only 1974 and 1975 acquisitions were studied.

The certainty equivalent model was not used in the acquisitions analysis due to its weak performance in the holding company study, and the market beta model could not be used. The three remaining models all, on average, provided estimates of value that were lower than acquisition prices. This result is logical, since the models were designed to estimate the value of the earnings
stream. They were not designed to estimate the premium that buyers would be willing to pay for the right to control the operations of the firm and (hopefully) increase the earnings stream in the future.

However, the value estimates made by the two beta-based models, especially the Beaver, Kettler and Scholes model, implied control premiums that were similar to those found in a recent study of bank acquisitions. The beta-based models did provide estimates that appeared superior to the estimates made by the price/earnings model. It appears that when the price/earnings model was not made artificially strong (as it was in the holding company analysis), the more complex models did indeed provide better estimates.

III. Suggestions for Future Research

A number of topics worthy of future investigation were alluded to in this study or now appear logical as a result of this study. The focus in this work was on the value of present earnings, and a very restrictive assumption, that future true growth opportunities were negligible, was made. While this assumption seemed acceptable for the savings and loan industry in 1973, 1974, and 1975, it would not be appropriate for other industries and other time periods. Therefore, development of richer models which include growth opportunities is in order.

This analysis limited itself to stock form associations that were acquired for cash. Other important areas of study exist for the savings and loan industry. The large number of conversions of mutual associations to the stock form of organization in recent years provides another rich data base for testing. Acquisitions
in which the purchase price was paid in stock, rather than cash, deserve study, although they had to be excluded from this work. And the issue of whether the valuations implied by a balance sheet merger of two mutual associations are the correct ones should be investigated.

The techniques developed here need not be restricted to the savings and loan industry, and could and should be extended to other industries, or other relatively homogeneous groups. The only real restriction on the selection of the group would be the requirement that the variables selected as instruments should be correlated with the market betas in a relatively homogeneous way—that is, since liquidity for a bank is very different from liquidity for a manufacturer of steel, steel producers and banks should not be used together as the market reference group if liquidity is to be one of the instruments.

Estimation of the risk free rate, $R_p$, and the risk premium, $(R_M - R_p)$, have been addressed by other researchers, but unanimous conclusions have not been reached. Certainly additional study of these topics is warranted. The issue of what data series to use as a proxy for the market return, $R_M$, is also a fertile field for new research. As was demonstrated in appendix D and by the results of the certainty equivalent model, the choice of an index does make a considerable difference, at least when using annual data. Would results using different series tend to converge as more observations were available? Would quarterly earnings figures be much more highly correlated with the market return series than the annual figures were?
The whole issue of empirical estimation using the certainty equivalent approach warrants more intense study. A number of reasons for the failure of this model were cited in chapter IV, and it is still much too soon to agree with Gordon and Halpern that the entire concept is inappropriate for empirical use.

This study has merely exposed the tip of the iceberg on many issues. Follow-up study is in order not only for the questions raised above, but other ideas as well that the reader will hopefully conceive as he reads this work.

IV. Conclusion

The task was formidable, and the assumptions were heroic. But, the results presented in this study seem to show that there are techniques with which estimates of value that are consistent with observed market prices can be made, and that these same basic techniques, suitably modified, can be used to estimate the value of closely held firms.
SELECTED BIBLIOGRAPHY


"Security Prices, Risk, and Maximal Gains from Diversification."


APPENDIX A

INSTRUMENTAL VARIABLES

An instrumental variables technique is an econometric method used to cope with problems in the variables used in linear regression models. These problems generally take one of two forms. One or more of the independent variables may not be independent of the (supposedly random) disturbance or error term. This problem is likely to occur, for example, if a lagged value of the dependent variable is used as one of the independent variables. The result of this correlation between the independent variable and the disturbance term is that the estimate of the regression coefficient will be inconsistent. This means that even for large data samples, the sample regression coefficient will not converge towards the true (but unknown and unobservable) coefficient.¹

The second type of problem occurs when the variables in the regression model are subject to measurement error. The result of this measurement error is that the estimates of regression coefficients are both inconsistent and biased, which means that the expected value of the coefficient will not be the true value.

The instrumental variables technique used to attempt to solve either of these problems involves finding other independent variables which are uncorrelated with either the disturbance term or the measurement

error, but are correlated with the explanatory variables. The proper use of these variables can lead to coefficients which are unbiased, consistent estimators of the true regression coefficients.

Finding such instrumental variables may not be an easy task. If the correlations between the instruments and the original explanatory variables are low, the sampling variances will be very large. As Johnston states, "with only a small correlation between ... [the instruments and the explanatory variables] ... we may be paying a very high price for consistency."²

Miller and Modigliani's use of an instrumental variables technique was designed to cope with measurement error in reported earnings in the electric utility industry.³ Their cost of capital estimates depended upon a variable representing the long run, future, tax-adjusted earning power of the firm's assets. This expectational variable was neither observable nor measurable, and had to be approximated using earnings from accounting statements. Thus, they used several instrumental variables to develop "better" estimates of earning power than those provided by the accounting information. Then, they used these first-stage estimates in the second stage of their two-stage least-squares model to better represent the crucial long-run earning power of the firm. The results of this procedure were judged to be superior to the results obtained by using the accounting earnings figures directly.

²Ibid.

Beaver, Kettler and Scholes also used an instrumental variables technique to cope with measurement error in the estimation of beta coefficients for the securities of a large sample of industrial firms. They used accounting risk measures of the firm from an earlier period of time (1947-1956) to develop instrumental variables forecasts of the market determined risk measure in a later period (1957-1965). They discussed the problem of measurement error in beta estimation, and identified one particular source of error—the use of ex post data to approximate an ex ante concept, a problem which was discussed in chapter II of this study.

The instrumental variables model was tested against a naive forecasting model which hypothesized that the beta coefficient in period two was the same as in period one. Prior knowledge that the period one and period two measured betas were highly correlated (.626 for individual securities, using the Spearman rank correlation measure) caused the authors to speculate that the naive model would not be easy to beat. Nevertheless, the overall performance of the instrumental variables model was "consistently superior to that of the naive model." The model demonstrated particularly strong superiority in predicting the outliers, or values in the tails of the distribution of values, where the authors believed that accurate forecasts were most needed.

As both the Miller and Modigliani paper and the Beaver, Kettler and Scholes paper imply, using an instrumental variables technique is not an exact science, but rather a combination of science and art. There is

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2 Ibid., p. 678.
no precise decision rule for selecting instruments. Low correlations with the explanatory variables leave the model subject to large sampling variances. But correlations that are too high are also undesirable, since this would probably defeat the purpose of attempting to remove measurement errors. In addition, interpretation of the individual regression coefficients is difficult, and probably relatively meaningless. "The relevant concern, rather must be whether they seem to behave 'sensibly' taken as a whole."\(^6\)

\(^6\)Miller and Modigliani, pp. 361-62.
APPENDIX B

EARNINGS AND CASHFLOWS

A number of authors have stressed the fact that valuation of the firm should involve the discounting of cashflows, not accounting earnings. However, estimation of cashflows far into the future is a most difficult task. Some formulation which focuses on the firm's present earnings may appear less formidable to the financial manager. It will be shown in this appendix that discounting of the firm's earnings stream is an appropriate method for valuation of the equity portion of the firm.

This analysis closely parallels those of Miller and Modigliani¹ and Haley and Schall.² The symbology and equations used are those of Haley and Schall. The analysis differs from those two, however, by focusing specifically on the valuation of the equity portion of the firm, S, rather than the valuation of the entire firm, V. It also differs by developing an equivalence between the cashflow variables and the accounting variables of the firm.

Equations in this appendix are numbered in the following manner: if the equation is one given by Haley and Schall, it will carry the same number as it does in that text; if the equation is an extension of their

¹Merton H. Miller and Franco Modigliani, "Dividend Policy, Growth, and the Valuation of Shares," Journal of Business 34 (October 1961): 415. See section II of chapter II for a summary of this paper.

work, it will be given an additional letter (for example, Equation 9-50a). Unless otherwise noted, variable definitions are consistent with those already used in this study.

Haley and Schall provide a most thorough analysis of the valuation of the firm under certainty and uncertainty, and demonstrate that not all future earnings of the firm belong to the present owners. They show that earnings must be adjusted to a quantity equivalent to net cash flows to reflect the fact that "... any investment $N_t$ that must be made to sustain future earnings has to be deducted at time $t$ as a real cost to the firm, just like any other cost."\textsuperscript{3} Hence, their formulation of the value of the firm is an extension of that shown in Equation 1 of chapter II, one that allows the rate of return to vary over time. Their equation uses $i_t$, rather than $p$, to denote the rate of return:

$$V = X_0 - I_0 + \sum_{t=1}^{T} \frac{X_t - I_t}{\prod_{j=1}^{t} (1+i_j)}$$

(Equation 2A-7)

Assume that all cash flows occur at the end of the period, and that the firm has no preferred stock, and let:

- $Y_t = \text{net cash flows of the firm for period } t$,
- $Y_t^B = \text{net cash flows to bondholders for period } t$,
- $Y_t^S = \text{net cash flows to stockholders for period } t$.

Then, ignoring taxes for the moment, the following relationship holds:

$$X_t = I_t + Y_t^B + Y_t^S$$

(Equation 9-44)

since cash inflows must equal cash outflows. Rearranging,

\textsuperscript{3}Ibid., p. 37.
\[ X_t - I_t = y_t^B + y_t^S \]  
(Equation 9-44a)

But, since:
\[ y_t = y_t^B + y_t^S \]

then,
\[ X_t - I_t = y_t \]  
(Equation 9-45)

Now, the analysis can be extended to introduce taxation. Let:

- \( \text{Tax}_t \) = the firm's tax payment,
- \( \tau \) = the tax rate of the firm,
- \( \text{DP}_t \) = the depreciation deduction permitted for tax purposes,
- \( R_t \) = the interest payments in period \( t \) to bonds outstanding in period \( t \) (notice that \( R \) is not cash receipts as it was in chapter 2),
- \( K_t \) = all other bond-related cash flows in period \( t \). It represents the net effect of new bonds issued and old bonds retired. The value will be negative if the firm sells more new bonds than it retires old bonds.

Then:
\[ y_t^B = R_t + K_t \]

and the firm's taxes (assuming that there are no noncash expenses other than depreciation) will be:
\[ \text{Tax}_t = \tau(X_t - \text{DP}_t - R_t) \]  
(Equation 9-46)

The cash inflow and outflow equations above must be modified for taxation. Equation 9-44 becomes:
\[ X_t = I_t + y_t^B + y_t^S + \text{Tax}_t \]  
(Equation 9-47)

Therefore,
\[ y_t = X_t - I_t - \tau(X_t - \text{DP}_t - R_t) \]
\[ = (1 - \tau)X_t - I_t + \tau\text{DP}_t + \tau R_t \]  
(Equation 9-49)
The net cash payments to the stockholders alone will be:
\[ Y^S_t = Y_t - Y^B_t \]
\[ = \left[ (1 - \tau)X_t - I_t + \tau DP_t + \tau R_t \right] - (R_t + K_t) \]
\[ = (1 - \tau)X_t - I_t + \tau DP_t + (1 - \tau)R_t - K_t \quad \text{(Equation 9-50)} \]

Notice that \( \tau DP_t \) could be expressed as \( \left[ DP_t - (1 - \tau)DP_t \right] \). Then:
\[ Y^S_t = (1 - \tau)X_t - I_t + DP_t - \left( (1 - \tau)DP_t - (1 - \tau)R_t - K_t \right) \]
\[ = (1 - \tau)(X_t - DP_t - R_t) + DP_t - I_t - K_t \quad \text{(Equation 9-50a)} \]

But the first term of this equation is merely the firm's profit after taxes (PAT). Therefore:
\[ Y^S_t = PAT_t + DP_t - I_t - K_t \quad \text{(Equation 9-50b)} \]

These net cash payments to the shareholders will take the form of dividends and stock repurchases by the firm less any cash inflows to the firm from the sale of new shares of stock at the end of the period. But, the sale of new shares of stock at the end of period \( t \) will obligate the firm to distribute part of all future cash flows to these new shareholders in all future periods. Let:
\[ Y^S(n)_t = \text{cash flows paid at time } t \text{ to shareholders who purchased their shares at the end of period } n. \text{ This value will be an outflow to the shareholder when } n = t. \text{ That is, when } n = t, \text{ the individual pays cash to the firm in order to "buy into" the future cash disbursements to shareholders.} \]
\[ Y^{S_0}_t = \text{cash flows paid at time } t \text{ to "old" shareholders, those who already owned their shares at the beginning of period } 0. \]

Then in general, the following holds:
\[ Y^S_t = Y^{S_0}_t + \sum_{n=0}^{t} Y^S(n)_t \quad \text{(Equation B-1)} \]

If no new shares are issued, Equation B-1 simply states that all cash
flows in period $t$ go to stockholders who already owned the stock at the beginning of period 0.

The following analysis will demonstrate that while the actual cash flows received by shareholders at a particular time may take on a number of values depending upon the level of investment and the firm's financing decisions, the value of these cash flows to the present shareholders will be a function of the firm's present level of after tax profits. Assume that in all cases the level of depreciation taken is just sufficient to maintain the present earnings stream, and that the firm's investment budget $I_t$ has been determined.

Case 1

Consider first the case when all firm profits are paid out as dividends. Inserting the appropriate value into Equation 9-50b yields:

$$Y_0^S = PAT_0 + DP_0 - I_0 - K_0$$

$$PAT_0 = PAT_0 + DP_0 - I_0 - K_0$$

$$0 = DP_0 - I_0 - K_0$$

or

$$-K_0 = I_0 - DP_0$$

(Equation B-2)

Case 2

Next, consider the situation when the firm makes a decision to pay no dividends, but rather to retain all earnings, and again does not wish to issue additional stock. Then the actual cash flow paid to stockholders during the period is 0, so that:

$$Y_0^S = PAT_0 + DP_0 - I_0 - K_0$$

$$0 = PAT_0 + DP_0 - I_0 - K_0$$
Case 3

Finally, consider the situation when the firm makes the decision to pay part of profits as a dividend, and to finance any additional need for funds with both debt and the sale of new stock. Let:

\[ \text{RE}_{t} = \text{the portion of profits in period } t \text{ that are retained by the firm} \]
\[ \text{D}_{t} = \text{the portion of profits paid out as dividends, so that} \]
\[ \text{PAT}_{t} = \text{D}_{t} + \text{RE}_{t}. \]

Then:

\[ y^{S} = y^{S_{0}} + y^{S(0)} \]  
and since old shareholders will actually be paid \( D_{0} \) this period:

\[ y^{S_{0}} = y^{S} - y^{S(0)} \]
\[ = \text{PAT}_{0} + D_{0} - I_{0} - K_{0} - y^{S(0)} \]
\[ D_{0} = (D_{0} + \text{RE}_{0}) + D_{0} - I_{0} - K_{0} - y^{S(0)} \]

or:

\[ -K_{0} = D_{0} - (D_{0} + \text{RE}_{0}) - D_{0} + I_{0} + y^{S(0)} \]
\[ = I_{0} - D_{0} - \text{RE}_{0} + y^{S(0)} \]  

(Equation B-4)

Since \( K_{t} \) as defined represents a cash payment to bondholders, a negative value, as denoted by Equations B-2, B-3, and B-4, represents a cash inflow to the firm from the sale of new bonds. Thus, these equations show the amount of bonds that must be sold to meet the (predetermined) investment budget under different dividend policies of the firm. The retention of part of the firm's profits in Case 3, and all of the profits in Case 2, reduced the actual amount of financing that the firm...
needed to raise by selling new bonds. But it did not negate the fact that the cash flows owned in that period by the old shareholders was precisely $\text{PAT}_0$. It merely reflected the fact that, rather than taking their cash now, they loaned part or all of it to the firm to reduce the firm's need for additional external financing at this time.

Will profit after taxes (and hence cash flows to old shareholders) increase in the future? The answer depends upon how well the firm selects investment opportunities. If the rate earned on new investments is merely the firm's cost of capital, the profits in the future will be the same as they are at present for Case 1. In Case 2, profits will be larger in future years by an amount sufficient to compensate the shareholders for loaning all of their time 0 profits to the firm as retained earnings. In Case 3, profits will be larger by an amount sufficient to compensate old shareholders for loaning funds to the firm as retained earnings, plus an amount sufficient to compensate the new shareholders who purchased the right to receive cash flows in the future by buying shares in period 0. The portion of the new profits belonging to the old shareholders will still be equivalent to $\text{PAT}_0$.

Only if the new investments earn a rate of return greater than the firm's cost of capital will future profit after taxes (and hence cash flows) truly grow. Therefore, the investment opportunities approach to valuation, as developed in chapter 2, represents an appropriate method for empirical estimation of the value of the equity of the firm. To the extent that determining the firm's current level of profits and its future opportunities to invest at rates of return in excess of its cost of capital is easier than estimating all future cash flows, the results shown in this appendix simplify the difficult task of valuation of the firm.
TREASURY BILL COUPON EQUIVALENT YIELDS

Treasury bill yields reported in most publications understate the true yield of the instrument, because they are reported on a bank discount, rather than a coupon equivalent (or bond equivalent) basis. This understatement increases with higher interest rates and with term to maturity. Since this study used annual risk free rates, the bank discount rate badly understated the true investment yield of the risk free asset. Accordingly, rates were converted to coupon equivalent values before they were used in this analysis.

The following example will illustrate the problem and the procedure for correcting it. At the Treasury Department's auction of 13-week bills on March 4, 1980, the average price of accepted bids was 96.174. The bank discount formula expresses this as a yield of 15.136%, calculated as follows:

\[ \frac{100 - 96.174}{100} \times \frac{360}{91} = 15.1358\% \]  

(Equation C-1)

The actual formula used to calculate yields is generally rearranged to isolate the price, rather than the yield, but the above presentation divides the calculation into two components, the per-period yield and the number of such periods in a year. The bank discount yield understates the true yield in two ways. First, it uses the full par value, rather than the actual price paid, as the denominator of the first term; and second, it uses a 360-day year rather than a 365-day year in the
second term.

To calculate the coupon equivalent yield for Treasury securities with six months or less to maturity, the price is substituted for par value in the denominator of the first term, and 365 (or 366 if the security will be outstanding on a February 29) is used in place of 360 in the second term. The coupon equivalent yield of the bills described above was therefore:

\[
\frac{100 - \frac{96.174}{96.174} \times \frac{365}{91}}{96.174} = 15.9565\% 
\]

(Equation C-2)

For bills with maturities in excess of six months, the two corrections shown above would lead to an overstatement of the yield, because interest on the bill is received only when it matures, while interest on a coupon issue is received every six months. The Treasury and Federal Reserve therefore use a complex interpolation method to arrive at a closer approximation of the true yield. The calculation is made as follows:¹

Let:

\[P = \text{price of the bill}\]

\[C = \text{number of periods (i.e., 2)}\]

\[N = \text{number of days in excess of one-half year}\]

\[S = \text{number of days in semi-annual period (normally 182.5, but 183 if term of bill includes February 29)}\]

\[I_0 = \text{the initial estimate of the coupon equivalent yield calculated in the manner shown by equation C-2}\]

\[i_1, i_2 = \text{two estimates of the true investment equivalent or coupon equivalent yield, and } i_2 \text{ is larger}\]

¹U.S. Treasury Department, "Method of Calculating Equivalent Coupon Issue Yield for Treasury Bills with a Term to Maturity of More than a Half Year and Less than a Whole Year," Washington, D.C., 1961. (mimeographed)
First, \( I_0 \) is calculated as shown above. Then, select values of \( i_1 \) and \( i_2 \) which will "bracket" the true rate,\(^2\) and perform the following calculation for each one:

\[
X_m = \left(1 + \frac{i_m}{2}\right) \left(1 + \frac{N \times i_m}{C} \left(\frac{P}{100}\right)\right), m = 1, 2
\]

Then, perform the following steps, carrying calculations to seven decimal places:

a. Compute \((X_2 - X_1)\)
b. Compute \((1.0000000 - X_1)\)
c. Divide difference in b. by difference in a.
d. Multiply quotient in c. by \((i_2 - i_1)\)

Finally, add the product of d. to \( i_1 \). This gives the approximation of the investment yield or coupon equivalent yield.

As an example, consider the Treasury auction of 52-week bills on October 8, 1980. The average yield was 11.136 percent, quoted on a bank discount basis. The price was thus 88.74. \( I_0 \) then, was:

\[
\frac{100 - 88.74}{88.74} \times \frac{365}{360} = 12.86\%
\]

Selecting 12.30 for \( i_1 \) and 12.40 for \( i_2 \) and performing the calculations as shown above:

\[
X_1 = 1 + \frac{.1230}{2} = 1 + \frac{181.5}{182.5} \times \frac{.1230}{2} \times \frac{88.74}{100}
\]

\[
= (1.0615)(1.061163)(.8874)
\]

\[
= .9995891
\]

\(^2\)The attempt to "bracket" the true rate requires some practice, just as in an internal rate of return calculation. Such estimates will become easy with experience, but initial estimates can be found in tables such as those found in: *Handbook of Securities of the United States Government and Federal Agencies*, 29th ed. (New York: The First Boston Corporation, 1980), pp. 210-11.
\[ x_2 = 1 + \frac{.1240}{2} \left( 1 + \frac{181.5}{182.5} \right) \frac{.1240}{2} \frac{88.74}{100} \]

\[ = (1.062)(1.0616603)(.8874) \]

\[ = 1.0005286 \]

a. \[ 1.0005286 - .9995891 = .0009395 \]

b. \[ 1.0000000 - .9995891 = .0000109 \]

c. \[ \frac{.000109}{.0009395} = .4373603 \]

d. \[ .4373603 (.1240 - .1230) = .04373603 \]

Adding this value to \( i_1 \) gives a coupon equivalent yield of 12.343736, or 12.34 percent, which is more than one 120 basis points higher than the bank discount rate.
APPENDIX D

CORRELATION OF INDICES

As explained in chapter IV, an index of market holding period returns is usually used as the $R^*_M$ figure in beta estimation or certainty equivalent calculations. However, illogical covariance estimates were obtained when this measure was used; although beta estimates were highly positive for all of the holding companies, the covariance term, which measured the covariance of the firm's earnings and the Fisher market index, was often negative.

Thus, an analysis was performed in order to determine the degree of correlation of various indices of market returns. In general, it was discovered that when annual return series are used, the degree of correlation between any two indices is not great, and, in fact, is sometimes negative. Also, the relationship varies greatly from year to year. Correlations between five different potential measures of $R^*_M$ are presented in tables 31, 32, 33, and 34 for four different time periods: 1961-1973, 1961-1974, 1961-1975, and 1965-1975, respectively.

The indices used are abbreviated as follows in the tables:

- **FISHEQ** stands for the Fisher equally-weighted return index with dividends
- **FISHVW** stands for the Fisher value-weighted return index with dividends
- **EARN** stands for the National Income and Product Accounts Profits after Valuation Allowance and Capital Consumption Allowance measure

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S&P stands for the Standard and Poor's index
RAWE stands for the National Income and Product Accounts raw earnings figure, before any adjustments

**TABLE 31**
CORRELATIONS OF INDICES, 1961-1973

<table>
<thead>
<tr>
<th></th>
<th>FISHEQ</th>
<th>FISHVW</th>
<th>EARN</th>
<th>S&amp;P</th>
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</thead>
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<td></td>
<td></td>
</tr>
<tr>
<td>EARN</td>
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</tr>
<tr>
<td>S&amp;P</td>
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<tr>
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<td>-.0530</td>
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**TABLE 32**
CORRELATIONS OF INDICES, 1961-1974

<table>
<thead>
<tr>
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<td>.3379</td>
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<td></td>
</tr>
<tr>
<td>S&amp;P</td>
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<td>.2733</td>
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</table>

**TABLE 33**
CORRELATIONS OF INDICES, 1961-1975

<table>
<thead>
<tr>
<th></th>
<th>FISHEQ</th>
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</tr>
</thead>
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<tr>
<td>S&amp;P</td>
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<td>.7577</td>
<td>.5924</td>
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<tr>
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<td>-.1644</td>
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<td>.2656</td>
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</tbody>
</table>
TABLE 34

CORRELATIONS OF INDICES, 1965-1975

<table>
<thead>
<tr>
<th></th>
<th>FISHEQ</th>
<th>FISHVW</th>
<th>EARN</th>
<th>S&amp;P</th>
</tr>
</thead>
<tbody>
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<td>FISHVW</td>
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<tr>
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