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LEE, SHU HONG

GTD ANALYSIS OF REFLECTOR ANTENNAS WITH GENERAL RIM SHAPES: NEAR AND FAR FIELD SOLUTIONS

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CHAPTER I
INTRODUCTION

During the past three decades, the interest in research on reflector antennas has grown extensively. Most of the important work during this period has been collected in Love's book [1]. Traditionally, the surface current and aperture field methods [2] have been widely used for calculating reflector antenna patterns. A recent application of the aperture field method to offset circular reflectors has been reported by Kauffman [3]. In recent years, there has been increasing use of the Geometrical Theory of Diffraction (GTD) [4,5] to reflector antennas. Ratnasiri, Kouyoumjian and Pathak [6] and Rusch and Sorensen [7] applied two-point GTD to circularly symmetric reflector antennas. Mentzer [8] also used two-point GTD to calculate the far sidelobes and backlobes of offset circular reflectors. James and Kerdemelidis [9] used GTD and the equivalent edge current concept to analyze reflector antenna patterns. However, these papers and most of those in [1] analyzed only circular reflectors. The analysis of reflectors with other rim shapes which are often used in radar and communication systems has not been published.

In this paper, a combined technique of Aperture Integration (AI) and GTD is reported which can calculate both the near and far field patterns of a paraboloidal reflector with a general rim shape and which may be either offset or center-fed. AI is used to compute the main beam and near sidelobes; GTD is used to compute the wide-angle sidelobes and the backlobes.

The aperture integration uses an approach of overlapping subapertures which allows a piece-wise linear representation for the aperture distribution. Thus variations in the aperture fields can be represented with relatively few subapertures. Furthermore, the subapertures can be electrically large; thus minimizing the computer storage and also the amount of numerical integration required. For far field computations, a rotating grid method is employed in that the y-integrations are carried out for each column of the aperture and each one-dimensional integration result is stored. The stored values for the y-integration are then used for each pattern angle in the plane perpendicular to the y-axis; thus the efficiency approaches that of a one-dimensional integration.
By subdividing the reflector rim into straight segments, the GTD analysis of the reflector is similar to that of the diffraction by a flat plate \([10,11,12]\), except that the curvature of the reflector surface must be taken into account. The diffraction of energy from these segments is based on edge diffraction and slope diffraction from infinite straight edges. For reflectors with corners such as the rectangular reflectors the vertex diffraction from the corners \([12,13]\) must be included to compensate for the finiteness of the rim edges. Moreover, since the rim edges are further subdivided into small segments, the corner diffraction terms from the two end points of each straight segment (which are in fact modeled as the corners of a plate containing the edge segment) are calculated and added to the edge diffraction term and the slope diffraction term to obtain the total diffracted field from a segment.

Since the GTD analysis treats the rim as piece-wise linear segments by using corner diffraction, it can be used to treat all rim shapes including smooth curves such as elliptical or circular. Thus an integrated analysis is achieved whether the rim is subdivided for the purpose of approximating a smooth rim shape or to take into account the three dimensional curvature of projected rim sections which are straight.

Results for reflectors of different rim shapes as computed by this combined technique are compared with measured data. Computer run times are also given to show the efficiency of this method.
CHAPTER II

APERTURE INTEGRATION (APERTURE FIELD METHOD)

This method has been applied to compute the radiation patterns of reflector antennas [2] for many years. It states that the radiated field of an aperture antenna can be determined by the field distribution on the aperture. Thus

\[ E = \frac{jk}{2\pi} \iint \left[ F_x^a E_x^a + F_y^a E_y^a \right] \frac{e^{-jks}}{s} \, dx \, dy \]  \tag{1}

for a reflector with its axis coincident with the z-axis and with its aperture defined in the x-y plane. \( E_x^a \) and \( E_y^a \) are the x- and y-components of the aperture field. \( F_x^a \) and \( F_y^a \) are the modified vector element patterns associated with two Huygen's sources (crossed electric and magnetic dipoles) [14] each having its electric field vector parallel to the x- and y-axis, respectively. These vector element patterns are expressed by

\[ F_x = \hat{\theta} \cos \phi - \hat{\phi} \sin \phi \cos(\theta/2) \]  \tag{2}

and

\[ F_y = \hat{\theta} \sin \phi + \hat{\phi} \cos \phi \cos(\theta/2) \]  \tag{3}

Numerically, the aperture fields are calculated at the grid points and stored. Therefore they can be used for different pattern cuts without being recalculated.

Two efficient techniques have been employed to carry out the aperture integration which is performed over the portion of the aperture plane inside the reflector rim. One is the overlapping subaperture method; the other is the rotating grid method.
A. Overlapping Subaperture Method

For near field computations, a rectangular grid size (\(D_x\) and \(D_y\)) is chosen so that the aperture can be divided into a principal rectangular grid as shown in Figure 1. Using the approach of overlapping subapertures, the aperture is treated as a collection of overlapping subapertures. Each subaperture is rectangular in shape and consists of four adjacent grid rectangles. The aperture distribution for each subaperture is triangular. The use of overlapping, rectangular subapertures with triangular distributions permits a piecewise linear approximation to the overall aperture distribution of the reflector, as shown in Figure 2, and consequently gives excellent accuracy with relatively few aperture field samples. An example is shown in Figures 3 and 4 for a \(20\lambda\) aperture with a cosine distribution and sampled at \(2\lambda\) intervals. As can be seen the overlapping subapertures give much better agreement with the exact pattern than
Figure 2. Numerical integration using overlapping subapertures with triangular distributions.

Figure 3. Exact aperture distribution and approximation to it by subaperture techniques.
Figure 4. Exact far field pattern and calculations based on subaperture techniques.
do the other two approaches shown: array sampling and subaper-
tures with uniform distribution. Consequently, the grid spacings
\( D_X \) and \( D_Y \) can be electrically large, i.e., several wavelengths
in size provided that the aperture distribution is adequately
approximated in this piecewise linear fashion. This approach
minimizes the computation time. Thus the aperture integration
results in a sum of the pattern functions of the rectangular sub-
apertures weighted by the aperture field \( E^a \) and their respective
areas, and Equation (1) becomes

\[
\mathbf{E} = \frac{j}{\lambda} \sum_{M} \sum_{N} \left[ F_{XMN} E^{a}_{XMN} + F_{YMN} E^{a}_{YMN} \right] F_{RS} e^{-jks_{MN}} S_{MN}^{-1} \tag{4}
\]

where \( s_{MN} \) is the distance from the aperture point \( (X_M, Y_N, 0) \) to
the near field point and \( F_{XMN} \) and \( F_{YMN} \) are the vector element
patterns of the equivalent aperture currents. These currents
are assumed to radiate the same polarization as a Huygen's source
and thus the vector element patterns are expressed in rectangular
coordinates as

\[
F_{XMN} = \left\{ x \left[ 1 + (\cos \theta_{MN} - 1) \cos^2 \phi_{MN} \right]
+ \hat{y} (\cos \theta_{MN} - 1) \sin \phi_{MN} \cos \phi_{MN}
- \hat{z} \sin \theta_{MN} \sin \phi_{MN} \right\} \cos \left( \frac{\theta_{MN}}{2} \right) \tag{5}
\]

and

\[
F_{YMN} = \left\{ x (\cos \theta_{MN} - 1) \sin \phi_{MN} \cos \phi_{MN}
+ \hat{y} \left[ 1 + (\cos \theta_{MN} - 1) \sin^2 \phi_{MN} \right]
- \hat{z} \sin \theta_{MN} \sin \phi_{MN} \right\} \cos \left( \frac{\theta_{MN}}{2} \right). \tag{6}
\]

The angles \( \theta_{MN} \) and \( \phi_{MN} \) are the polar coordinate angles to the
near field point as referred to the aperture point \( (X_M, Y_N, 0) \).
The fields \( E_{XMN} \) and \( E_{YMN} \) are the X and Y components of the aperture
field sampled at the points \( (X_M, Y_N, 0) \) on the principal grid.
The basic pattern \( F_{RS} \) of each rectangular subaperture is given by
\[ F_{RS} = D_x D_y F_{XN} F_{YN} \]  \hfill (7)

where \( F_{XN} \) and \( F_{YN} \) are the horizontal and vertical patterns of each rectangular subaperture. The typical patterns for a basic subaperture with full triangular distribution as shown in Figure 5a, are given by

\[ f_F(x) \]  \hfill (a) full

\[ f_H(x) \]  \hfill (b) half

\[ \sin \frac{4\pi}{X} > x \]  \hfill (8)

\[ \sin \frac{2\pi}{Y} \]  \hfill (9)

where

\[ \phi_x = k D_x \sin \theta_{MN} \cos \phi_{MN} \]  \hfill (10)

and

\[ \phi_y = k D_y \sin \theta_{MN} \sin \phi_{MN} \]  \hfill (11)

Figure 5. Triangular subaperture distributions.
For subapertures near the reflector rim, the patterns with half triangular distribution, as shown in Figure 5b, should be used. These patterns are given by

\[
F_{\text{XN}} = \frac{1 - e^{-\frac{j \phi_x}{(\phi_y)^2}}}{} + \frac{j}{\phi_x} \tag{12}
\]

\[
F_{\text{YN}} = \frac{1 - e^{-\frac{j \phi_y}{(\phi_y)^2}}}{} + \frac{j}{\phi_y} \tag{13}
\]

B. Rotating Grid Method

The aperture integration method requires a two-dimensional integration for each pattern point as indicated in Equation (1). However, for far field computations, an appropriate rotation of the grids greatly improves the efficiency in carrying out the integration.

With this approach the y-integration is carried out for each column of the aperture and each one-dimensional integration result is stored. The stored values for the y-integration are then used for each pattern angle \( \Theta \) in the plane perpendicular to the y-axis; thus the efficiency approaches that of a one-dimensional integration.

Consider the far field expression of Equation (1) which is given by

\[
\mathcal{E} = \frac{jk}{2\pi R} \int \int \left( F_x E_x + F_y E_y \right) e^{jk(x \sin \Theta \cos \phi + y \sin \Theta \sin \phi)} dx dy \tag{14}
\]

where \((\Theta, \phi)\) represent the angular coordinates of a far field point. By rotating only the Y-axis in the rectangular grid system an angle \( \phi \), a non-orthogonal grid is set up as shown in Figure 6.

The coordinate transformation from the principal rectangular grid \((X, Y)\) to the rotated grid is shown in Figure 7 and is given by

\[
x = X + Y \tan \phi \tag{15}
\]

and

\[
y = Y / \cos \phi \tag{16}
\]
Figure 6. Rotated grid.

Figure 7. Coordinate transformation from principal rectangular grid to rotated grid.
or
\[ X = x - y \sin \phi \] (17)

and
\[ Y = y \cos \phi \] (18)

The rotated grid sizes as shown in Figure 6 are expressed by
\[ d_x = D_x \] (19)
\[ d_y = D_y / \cos \phi. \] (20)

Note that the rotated grid size \( d_y \) becomes quite large if the rotated angle is close to 90°. This may affect the accuracy of the result. Consequently, the rotated angle is restricted to be not greater than 45°. Thus, for PHI cuts in the interval \((45°, 135°)\), the x-axis is effectively rotated instead of the y-axis. This is done indirectly in the code by transforming rim points such that the x- and y-coordinates of the rim points are interchanged and the indices are adjusted to stay in a counterclockwise order. Then the new y-axis is rotated by an angle \( 90° - \phi \) which is less than 45°. For PHI cuts in the other quadrants, a similar procedure is followed. Using Equations (17)-(20), Equation (14) becomes
\[
\mathbf{E} = \frac{jk}{2\pi} \frac{e^{-jkR}}{R} \sum_{k=x,y} \overline{F}_k(\theta, \phi) \int I_k(x) e^{jкс\sin\theta\cos\phi} dx \] (21)

where
\[ I_k(x) = \int E_k^0(x, y) dy \] (22)

and \( \overline{F}_{x,y} \) are defined in Equations (2) and (3). It can be seen in Equation (22) that the y integrations denoted as \( I_x \) and \( I_y \) are independent of \( \theta \) and can be stored. Consequently, the far field pattern in the plane perpendicular to the rotated y-axis is reduced to a one-dimensional integration; this provides greatly improved efficiency over the many two-dimensional integrations that would otherwise be required.

In fact, the number of sum operations for the far field integration is \( M(M+N) \) where \( M \) represents the maximum total number of rotated grid lines and \( N \) the number of far field observation angles. The number of sum operations for the usual two-dimensional integration would be \( M^2 x N \) for a square grid. The ratio of the number of operations for the two methods is \( MxN \) to \( M+N \). As an example, a 40x40 grid was used with 80 AI pattern angles for the
35\lambda square reflector antenna as will be shown later. For that case, the rotating grid method is 27 times more efficient than the non-rotated grid model.

The choice of a nonorthogonal grid system over an orthogonal one is based on the following considerations. Since the aperture fields are calculated and stored for the principal grid points any rotation of the principal grid system would make the aperture fields on the rotated grid different from the stored values. The nonorthogonal grid system shown in Figure 6 needs only a one dimensional interpolation of the stored aperture fields to find the aperture field at a rotated grid point rather than a two dimensional one for an orthogonal rotation grid system. The other advantage is that the horizontal grid lines are unchanged such that it simplifies the work of setting up the new coordinate system for implementing the Y-integration. The overlapping subaperture method as discussed in part (A) is also used to obtain maximum efficiency in the numerical summation of the x-integration for far field computations.
Since AI is valid only in the main beam region, GTD is used to calculate the diffracted fields for the wide angle sidelobes and backlobes. For near field calculations, GTD is sometimes used for the whole region including the near axis region if the near field points are close to the aperture.

By subdividing the reflector rim into straight segments, as shown in Figure 8 for the example of a rectangular shape for the projected rim, the GTD analysis of the reflector is similar to that of diffraction by a flat plate in that each segment is treated as an edge of a flat plate which is tangent to the reflector surface. The total diffracted field is obtained by superimposing the edge diffracted field, the slope diffracted field and the corner diffracted field from each rim segment. A suitable criterion for each segment of the reflector rim is that it be small enough that the focus of the reflector lies in the far field of the rim segment. Thus the maximum length of each rim segment is approximately

\[ \frac{\Delta L}{\lambda} = \sqrt{\frac{F}{2\lambda}} \]

where \( F \) is the focal length of the reflector. For example, the 35\( \lambda \) square reflector which is used for comparison in the results section has a focal length \( F = 27.5\lambda \) and thus the maximum segment length is \( \Delta L = 3.7\lambda \). This results in 10 segments on each side for a total of 40 segments for the GTD far field computations. For near field calculations, the segment length is reduced to half of that for far field so that diffraction for straight edge segments can also be used in the close near field. To take the curvature of the reflector surface into account, the normal of a plate is approximated by the normal of the paraboloidal reflector surface at the mid point of the rim segment. Thus the unit vector of each plate is given by

\[ \hat{V_N} = -\hat{\rho} \sin \frac{\psi}{2} + \hat{z} \cos \frac{\psi}{2} \]  \hspace{1cm} (23)

where \( \hat{\rho} = \hat{x} \cos \phi_0 + \hat{y} \sin \phi_0 \) and \( \hat{x}, \hat{y}, \hat{z} \) are the unit vectors of the reflector coordinate system as shown in Figure 9 and \( \psi \) and
Figure 8. Illustration of subdivision of a reflector rim into straight segments.

Figure 9. Unit vectors associated with the reflector rim.
\( \phi_0 \) denote the angular coordinates of the midpoint of the rim segment. The three orthogonal unit vectors associated with the edge fixed coordinate system on each segment of the reflector rim are the edge unit vector \( \mathbf{V} \) along the rim segment, the unit normal vector \( \mathbf{VN} \) and the unit binormal vector \( \mathbf{VP} = \mathbf{VN} \times \mathbf{V} \) as shown in Figure 9. A diffraction point is a point on the rim segment for which a ray on the cone of the diffracted rays passes through the observation point such that the diffraction angle \( \beta_0 \) is equal to the incident angle \( \beta_0' \) as shown in Figure 10a. If there is a diffraction point within the limits of a particular rim segment, this segment gives a contribution to the edge diffraction. Once the diffraction point is located, the incident ray unit vector \( \mathbf{VI} \) and diffracted ray unit vector \( \mathbf{d} \), as shown in Figure 10, are calculated. Then the incident angles \( \beta_0 \) and \( \phi \) and the diffraction angles \( \beta_0' \) and \( \phi' \) and the associated unit vectors \( \beta_0', \phi' \), \( \beta_0 \) and \( \phi \) which define the ray fixed coordinate system are determined by

\[
\beta_0' = \beta_0 = \sin^{-1}|\mathbf{d} \times \mathbf{V}|, \tag{24}
\]

\[
\phi' = \tan^{-1} \left( \frac{-\mathbf{V} \cdot \mathbf{VN}}{-\mathbf{V} \cdot \mathbf{VP}} \right), \tag{25}
\]

\[
\phi = \tan^{-1} \left( \frac{\mathbf{d} \cdot \mathbf{VN}}{\mathbf{d} \cdot \mathbf{VP}} \right), \tag{26}
\]

\[
\phi' = -\mathbf{VP} \sin \phi' + \mathbf{VN} \cos \phi', \tag{27}
\]

\[
\phi = -\mathbf{VP} \sin \phi + \mathbf{VN} \cos \phi, \tag{28}
\]

\[
\beta_0' = \phi' \times \mathbf{VI}, \tag{29}
\]

and

\[
\hat{\beta}_0 = \hat{\phi} \times \hat{d}, \tag{30}
\]

as illustrated in Figure 10.

Thus the edge diffracted field from each segment, expressed in parallel and perpendicular components referred to the ray fixed system, is given by [5]

\[
E_{\|}^d(s) = -E_{\|}(X_D) D_s(L) A(s) e^{-jks} \tag{31}
\]

and
Figure 10. Geometry for three dimensional diffraction of a half plane.
\[ E_\perp^d(s) = -E_\perp^i(x_0) \, D_h(L) \, A(s) \, e^{-jks} \]  

(32)

where

\[ D_{s,h} = \frac{e^{-j\frac{\pi}{4}}}{2\sqrt{2\pi k} \sin\beta_0} \begin{bmatrix} F[k\tan(\beta^-)] & F[k\tan(\beta^+)] \end{bmatrix} \left[ \begin{array}{c} \cos \frac{\beta^-}{2} \\ \cos \frac{\beta^+}{2} \end{array} \right] \]  

(33)

\[ \beta^\tau = \phi - \phi' \]  

(34)

\[ a = 2 \cos^2(\frac{\beta}{2}) \]  

(35)

\[ F(X) = 2j\sqrt{|x|}e^{jX} \int_{-\infty}^{\infty} e^{-j\tau^2} d\tau \]  

(36)

is the transition function,

\[
\begin{cases}
A(s) = \frac{s'}{\sqrt{s(s+s')}} \\
L = \frac{ss'}{s+s'} \sin^2\beta_0
\end{cases}
\]  

for near field, \hspace{1cm} \hspace{1cm} \hspace{1cm} (37)

or

\[
\begin{cases}
A(s) = \frac{\sqrt{s'}}{s} \\
L = s' \sin^2\beta_0
\end{cases}
\]  

for far field \hspace{1cm} \hspace{1cm} \hspace{1cm} (38)

and

\[ s \text{ and } s' \text{ are the slant distances from the diffraction point to the observation and source points, respectively.} \]

Slope diffraction \cite{15,16,17} can be used to model variations in the incident field at the diffraction point. For reflectors using a tapered feed pattern with a steep slope at the reflector rim the slope diffraction term must be included in the edge diffraction part. The slope diffraction fields are calculated in a similar way to that of edge diffraction except that the slope diffraction coefficients \( \partial D_s/\partial \phi' \) and \( \partial D_h/\partial \phi' \) and the slope of the incident field \( \partial E/\partial n \) at the
edge are used. Thus the respective parallel and perpendicular components of the slope diffracted field are given by

\[ E_{\parallel}^{SD}(s) = \frac{1}{jks\sin \beta_0} \frac{\partial E_{\parallel}^i(X_D)}{\partial \alpha n} \frac{\partial D_s(L)}{\partial \phi} A(s)e^{-jks} \tag{39} \]

and

\[ E_{\perp}^{SD}(s) = \frac{1}{jks\sin \beta_0} \frac{\partial E_{\perp}^i(X_D)}{\partial \alpha n} \frac{\partial D_h(L)}{\partial \phi} A(s)e^{-jks} \tag{40} \]

where

\[ \frac{\partial D_{sh}}{\partial \phi} = j \sqrt{\frac{k}{2\pi}} \frac{e^{-j\frac{\pi}{4}L}}{\sin \beta_0} \left\{ \sin\left(\frac{\beta^-}{2}\right) [1-F[kL\alpha(\beta^-)]] \right\} \pm \sin\left(\frac{\beta^+}{2}\right) [1-F[kL\alpha(\beta^+)]] \tag{41} \]

and \( n \) is the distance in the direction of \( \phi' \) normal to the plane of incidence of the edge.

Since each rim segment is finite, the corner effect at the end points of the segment must be taken into account. A corner diffraction solution has been proposed by Burnside [12,13] which is based on the asymptotic evaluation of the radiation integral which employs the equivalent edge currents that would exist in the absence of the corners. The corner diffraction term is then found by appropriately (but at present empirically) modifying the asymptotic result for the radiation integral which is characterized by a saddle point near an end point. The corner diffraction compensates for the discontinuity which occurs when the diffraction point moves off of the rim segment. The corner diffraction field is given by [13]

\[ \begin{bmatrix} E_{\parallel}^c \\ E_{\perp}^c \end{bmatrix} = \begin{bmatrix} IZ_0 \\ MY_0 \end{bmatrix} \sin \beta_0 e^{-j\frac{\pi}{4}} \frac{e^{-jks_c}}{2\pi (\cos \beta_{oc} + \cos \beta_c)} F[kL_c a(\beta_{oc} + \beta_c)] e^{-jks_s} \frac{e^{-jks_s}}{s_s} \]

where

\[ \begin{bmatrix} I \\ M \end{bmatrix} = \begin{bmatrix} E_{\parallel}^i(X_D) \\ E_{\perp}^i(X_D) \end{bmatrix} \begin{bmatrix} C_s(X_D)Y_0 \\ C_h(X_D)Z_0 \end{bmatrix} \sqrt{s} e^{jks'} \tag{43} \]
and

\[
C_{s,h}(x_D) = \frac{-e^{-j \pi \frac{1}{4}}}{2 \sqrt{\pi} \sin \beta_0} \left\{ F \left| \frac{kL a(\beta^-)}{\cos \frac{\beta^-}{2}} \right| \left[ \frac{kL a(\beta^-)/\lambda}{kL a(\beta_0 + \beta_c)} \right] \right\}
\]

\[
\bar{F} \left| \frac{kL a(\beta^+)}{\cos \frac{\beta^+}{2}} \right| \left[ \frac{kL a(\beta^+)/\lambda}{kL a(\beta_0 + \beta_c)} \right] \right\} \tag{44}
\]

in which

\[
L_C = S_c \quad \text{for far field} \quad \tag{45}
\]

and

\[
L_C = \frac{S_c S_s}{S_c + S_s} \quad \text{for near field} \quad \tag{46}
\]

The other variables associated with geometry are shown in Figure 11.

Figure 11. Geometry for corner diffraction problem for near field.
For near field calculations, the geometrical optics reflected field must also be included in the total field if the observation point is inside the projected aperture. Since the reflected fields from a parabolic reflector with a focussed feed are those of a plane wave with its wavefront parallel to the aperture plane, the magnitude of the reflected fields can be calculated from the aperture field by adding the appropriate phase term.
CHAPTER IV
COMPUTER CODE DESCRIPTION

A computer code has been developed based on the theories discussed in the previous sections. This code is documented in two manuals: the user's manual [18] describes the operation of the code and the code manual [19] describes the theory used in the code and gives a detailed explanation of the code. After the geometry of the reflector, the feed pattern data and other optional inputs have been specified, the code outputs the calculated near or far field pattern using the combined AI and GTD techniques.

Sampled data from each measured feed pattern cut is input and stored in the code. Linear interpolation is then used to obtain a piecewise linear representation of the input pattern cut. The feed patterns in planes other than those corresponding to the input pattern cuts are calculated by linear interpolation. This method provides a computationally efficient way of calculating the aperture field without requiring huge amounts of computer storage for the measured feed pattern.

Two criteria has been developed to determine whether AI or GTD should be used for specified observation points. These criteria are based on numerous comparisons in which it was found that the AI and GTD solutions overlapped in certain regions. Thus the code automatically switches from AI to GTD according to the following criteria: The angle criterion which is used for the near field as well as the far field, is defined as

\[ \theta_x = \sin^{-1} \frac{1}{\sqrt{A_w}} \]  

(47)

where \( A_w \) is the aperture width in the specific pattern cut as shown in Figure 12. Thus AI is used when \( 0 < \theta < \theta_x \) and GTD is used when \( \theta > \theta_x \).

The range criterion is used solely for the near field and is defined by

\[ Z_x = \frac{A_w}{2 \tan \theta_x} \]  

(48)

AI is used only when the near field point is located in the region for which \( z > Z_x \) and \( \theta < \theta_x \) as illustrated in Figure 12, otherwise, GTD is used.
Examples which verify the validity of these switching criteria are shown in the following chapters.
CHAPTER V
RESULTS AND COMPARISONS

Far as well as near field results and their comparisons with measurements or calculations by other approaches are presented in this chapter. Figure 13 shows the geometry of an offset reflector of which the measured far field patterns of two principal planes were published in Reference [2] and are reproduced in Figure 14. The calculated patterns from the general reflector code using AI is shown in Figure 15. The measured and computed patterns are in good agreement considering that feed strut scattering effects are not included in the reflector antenna calculations. The lack of symmetry of the measured pattern in the H-plane (the plane containing the strut) is caused by the feed blockage and the feed strut scattering.

The next example shows the depolarization properties of offset reflector antennas which were studied by Chu and Turrin [20]. They calculated the maximum cross polarization produced by a linearly polarized feed and checked their result with a measured cross polarized pattern of the offset reflector with an offset angle $\theta=45^\circ$ shown in Figure 16. Kauffman, et al. [3] also checked this case with calculated patterns from their computer program for circular reflector antennas. Their computer program is based on an aperture integration analysis involving the quantizing and sorting of numerous aperture field data. In the calculations of [20] and [3] for this offset reflector example, a circularly symmetric feed pattern was assumed. The patterns computed by the general reflector code, as shown in Figures 17 through 19, are in excellent agreement with those computed in [3] and with the measured patterns of [20]. Note that excellent agreement is achieved for the cross polarized beam as shown in Figure 18 for the $\phi=0^\circ$ plane. There is no cross polarized beam in the $\phi=90^\circ$ plane for a symmetrical feed pattern.

Chu and Turrin [20] found that no cross polarization occurs when the offset reflector is fed by a circularly polarized feed; but the depolarization property manifests itself by a beam shift for circular polarization. The left and right circularly polarized beams shift in opposite directions. The corresponding circularly polarized cases as computed by the general reflector code are shown in Figures 20 and 21. The calculated beam shift in each case was $0.35^\circ$, or a total shift of $0.70^\circ$ between left and right circular polarizations. This compares well with the value of $0.75^\circ$ as determined from measured patterns [20]. The latter are reproduced in Figure 22.
The previous examples used only AI for far field calculations. The following example illustrates the validity of GTD for both near and far field calculations. The reflector [21] is offset at approximately 40° and has a 50.8 cm projected rim shape as shown in Figure 23. The feed used for the reflector is a conical corrugated horn. The measured patterns of the feed horn are given in [21] for both operating frequencies at 20.6 GHz and 31.65 GHz. The feed is x-polarized for 20.6 GHz and y-polarized for 31.65 GHz as referred to the coordinate system shown in Figure 23. The near field measurements were taken in the plane at a distance 1.07 m from the center of the reflector, corresponding to a plane at z=0.91 m referred to the aperture center defined in the code manual [19]. The calculated near field data using GTD are shown in Figures 24 and 25 and the data measured by Hogg et al. [21] are reproduced in Figure 26. A comparison between these figures shows that the agreement is excellent for 20.6 GHz and very good for 31.65 GHz. Note that only GTD is used in computing these near field data since the distance satisfies the criterion $z < z_X$ as explained in the previous chapter. Figures 27 and 28 show the calculated far field patterns which use both AI and GTD. The angles $\theta_X$ for switching from AI to GTD are 9.75° and 7.85° for 20 GHz and 31.65 GHz, respectively. To obtain the computed results, the feed was slightly defocused along the feed axis as was done in their experiment [21]. The far field results obtained by using the near field data [21] are reproduced in Figure 29. The agreement between these far field patterns is also very good.
Figure 13. Offset reflector rim shape and cross-section.
Figure 14. Measured E- and H-plane patterns of the antenna shown in Figure 13 (from Ref 2).
Figure 15. Far field patterns of offset reflector example computed by general reflector code. Main beam and near sidelobes.
Figure 16. Offset reflector geometry for example of Chu and Turrin [20]. $D=20$ inches, $\psi_T=\theta=45^\circ$, $f/D=0.25$ and frequency = 18.5 GHz.
Figure 17. Far field pattern for example of Reference 18, computed by general reflector code. Principal polarization in $\phi=0^\circ$ plane.
Figure 18. Far field pattern for example of Reference 18, computed by general reflector code. Cross polarization in $\phi=0^\circ$ plane.
Figure 19. Far field pattern for example of Reference 18, computed by general reflector code. Principal polarization in $\phi=90^\circ$ plane.
Figure 20. Far field pattern for example of Reference 18, computed by general reflector code. Left circular polarization in \( \phi=0^\circ \) plane.
Figure 21. Far field pattern for example of Reference 18, computed by general reflector code. Right circular polarization in $\phi=0^\circ$ plane.
Figure 22. Measured far field patterns from Chu and Turrin [20]. Circular polarization in $\phi=0^\circ$ plane.

Figure 23. Geometry of a square reflector from Reference 21.
Figure 24. Calculated near field data in plane \( z=1.07 \) m of the antenna shown in Figure 23 at frequency = 20.6 GHz.
Figure 25. Calculated near field data in plane $z=1.07$ m of the antenna shown in Figure 23 at frequency $= 31.65$ GHz.
Figure 26. Measured near field data from Reference 21, (a), (b) at 20.6 GHz, (c), (d) at 31.65 GHz.
Figure 27. Calculated far field patterns of the antenna shown in Figure 23 at frequency = 20.6 GHz.
Figure 28. Calculated far field patterns of the antenna as shown in figure 23 at frequency = 31.65 GHz.
Figure 29. Calculated far field patterns from Reference 21.
As mentioned in Chapter II, the use of overlapping rectangular subapertures with triangular distributions for the AI calculations permits a piecewise linear approximation to the overall aperture distribution of the reflector and gives excellent accuracy with relatively few aperture field sample grids. A criterion for the choice of the grid size is that the sampled field values at the grid points give a good representation of the true aperture distribution. However, the required grid size also depends on the shape of the aperture. As one recalls, referring to the grid system shown in Figure 1, the aperture integration method first integrates the sampled aperture fields at the grid points along the rotated y-axis, then the Y-integration sums are integrated at the grid points along the x-axis. Since overlapping rectangular subapertures are used for these integrations, the areas that the subapertures cover do not always represent the actual shape of the aperture as illustrated in Figure 30. Fortunately, the extra area in each subaperture is compensated by the part missed by the adjacent subaperture as shown by the two shaded areas in Figure 30a for the straight rim sections. The adjacent areas tend to compensate each other in the pattern as well, especially since triangular distributions are used for the sub-apertures. That is, the triangular distribution goes to zero at the edge of each subaperture where the phase or the path length to the far field will have the most errors for the compensation. Therefore, the grid size may not have a significant effect on the resulting pattern for sampling along a straight rim section. If the rim section contains a corner as shown in Figure 30b, the area covered by the two adjacent subapertures do not compensate well if there is not a grid line near the corner. Thus errors are introduced not only in the phase information contained in the subapertures but also in the area. The accuracy of the resulting pattern tends to be affected more seriously when the area is not well approximated. Thus a sufficiently small grid size should be used to correctly sample the rim shape. As an example to show the effect of the grid size, consider a 50λx25λ rectangular aperture with a uniform field distribution. For principal plane cuts, relatively large grid sizes can be used and excellent agreement is achieved compared with the analytical solution for which the pattern is

\[
F = AB \left( \frac{\sin \phi_A}{\phi_A} \right) \left( \frac{\sin \phi_B}{\phi_B} \right) \cos \left( \frac{\theta}{2} \right)
\]
(a) Grids sampling along straight rim sections.

(b) Grids sampling across a corner.

Figure 30. Illustration of rotating grids sampling a rectangular aperture.
where

\[ \phi_A = \frac{kA}{2} \sin \theta \cos \phi \]  \hspace{1cm} (50)

\[ \phi_B = \frac{kB}{2} \sin \theta \sin \phi \]  \hspace{1cm} (51)

with \( A \) and \( B \) being the width and height of the aperture, respectively. For off-principal plane cuts, the result calculated by using a large grid size does not agree well with the analytical solution. An example is shown in Figure 31 where a \( \phi=30^\circ \) plane pattern is computed by using a grid size \( D_x=10\lambda \). The lack of sufficient information about the rectangular rim shows up in the \( Y \)-integration sum distribution along the \( x \)-axis as shown in Figure 32a. If the grid size is reduced to \( 2\lambda \), the \( Y \)-integration sum distribution approaches the exact distribution as shown in Figure 32b. The computed far field pattern using \( D_x=2\lambda \) agrees very well with the analytical solution as shown in Figure 33. Normally, 20 to 40 grid lines will give good results for rectangular or square apertures with reasonably tapered aperture distributions (up to 15 dB tapers).

For circular apertures, the same grid size can be used for both principal and off-principal plane pattern calculations. The grid size required to obtain accurate results should be small enough to ensure that the curvature of the rim is not distorted significantly after being sampled by the grids, especially at the left and right edges of the aperture (upper and lower edges for horizontal grids) where the slope of the rim is almost parallel to the grid lines. A typical circular aperture will usually get very accurate results with about 40 grid lines across the diameter for the far field or about 60 grid lines for the near field.

So far only apertures with uniform field distributions have been discussed. In practice, both the aperture field distribution and the aperture shape affect the choice of appropriate grid size. For low sidelobe designs, the field distribution concentrates near the center and tapers toward the edge. Thus it is advantageous to use denser grids at the center than near the rim for these cases. On the other hand, the above discussions for the rectangular and circular apertures indicates that more grids may be required near the rim for apertures with certain shapes. Therefore, as this program is designed to solve general reflector problems, a uniformly spaced grid system was chosen to avoid the complexity of using an adaptive system subject to some complicated conditions.
Figure 31. Far field patterns of a 50λx25λ aperture with a uniform field distribution in φ=30° plane.
Figure 32. Y-integration sum distributions.
Circular reflectors are also an excellent test case for the GTD analysis in which the reflector rim is simulated by straight segments and the total diffracted field is obtained by superimposing the edge diffracted fields and corner diffracted fields from each rim segment. Comparisons between the calculated and measured data shown in the previous chapter verify that this approach works very well for most practical problems. But since those comparisons are only in the principal planes, it is interesting to see the effect when a pattern plane cuts through a corner of two adjacent straight segments. As an example, a 22.35\lambda diameter dish having an F/D=1/3 is illuminated by an axially symmetrical feed pattern which has a -15 dB taper at the reflector rim. According to the segment length criterion discussed in Chapter III, the circular rim is subdivided...
into 36 straight segments. Figure 34 shows the circular aperture cut by a \( \phi = 0^\circ \) plane which cuts through a vertical edge segment and a \( \phi = 5^\circ \) plane which cuts across a corner formed by two adjacent edge segments. The calculated patterns in these two planes are shown in Figures 35a and 35b, respectively. These two patterns are almost identical except in the region from approximately 130° to 160° where the levels are below -60 dB as referred to the main beam. The difference in this region arises from the fact that a slight shift of the plane cut changes the diffraction structure due to the straight segment approximation. The change in the diffraction structure between these two cuts shows the extent to which circular symmetry is lacking in the model; but the total fields come out the same as more rim segments are used. Indeed, an increase from 36 to 64 rim segments does give identical results in these two planes as shown in Figure 35c. This pattern also compares perfectly with that computed by the two-point GTD approach [6], considering the fact that slope diffraction is not included in the two-point analysis. The pattern from the two-point analysis has a "kink" at the incident shadow boundary as does the pattern from the general reflector code if the slope diffraction option is not used.

Figure 34. Rim segment geometry of a circular aperture cut by two pattern planes.
Figure 35a. Far field pattern of a circular reflector antenna with $D=22.35\lambda$ and $F/D=1/3$, $\phi=0^\circ$ (with 36 rim segments).
Figure 35b. Far field pattern of a circular reflector antenna with \( D=22.35 \lambda \) and \( F/D=1/3 \). 
\( \phi=5^\circ \) (with 36 rim segments).
Figure 35c. Far field patterns of a circular reflector antenna with \( D = 22.35 \lambda \) and \( F/D = 1/3 \)
\( \phi = 0^\circ, 5^\circ \) (with 64 rim segments).
From this example, one can see that the GTD analysis using the straight segment approximation for a general rim shape reflector works very well. Only a moderate amount of segments, as automatically set up by the code, are required to get results with good accuracy. The error caused by using 36 straight segments to simulate a 22\(\lambda\) circular reflector rim is very minor and only appears in a narrow region in the pattern with extremely low dB level as indicated by Figures 35. More rim segments may be used if very high accuracy is desired as shown by the pattern in Figure 35c. Note that a circular reflector is used in this example which serves as a worse case test of this code. Reflectors with other rim shapes usually require less rim segments than a circular one with comparable aperture size and very good results are obtained as illustrated in the previous chapter.

The basic GTD and AI analyses do not have a limitation on the maximum size of the reflector. As long as there is enough storage space for the required grids and rim segments, this code can handle rather large size reflectors. However, there is a basic limitation on the minimum size of the aperture for the GTD analysis. When the aperture size is small (several wavelengths), the effect of double diffraction from one side of the rim due to the single diffracted ray from the opposite side becomes significant. Since double diffraction is not included in this analysis, it is not surprising that for very small apertures the GTD results are not as accurate as those for larger apertures. To find out the lower limit on aperture size, the patterns of different size apertures are examined. Figure 36 shows the computed E-plane patterns for a 10\(\lambda\) circular aperture with F/D=1/3 and an assumed feed pattern having the form of \(\cos^4(\psi/2)\) which gives about a 12 dB aperture field taper at the rim. As can be seen from Figure 36, the AI and GTD results overlap very well in the region between 10° and 30°. The switching angle \(\theta_X=18°\) calculated by Equation (47) falls right in the overlapping region as shown in the figure. Figure 37 shows the E-plane patterns for a 5\(\lambda\) circular aperture with the same F/D ratio and feed pattern. The AI pattern still overlaps with the GTD result although in a narrower region as compared to the 10\(\lambda\) case. The calculated switching angle is at 26° which is also inside the overlapping region as shown in Figure 37a. The AI and GTD results do not agree well beyond this region since the incident term (the term associated with the \(\beta^+\) in the diffraction coefficient, see Equation (33)) in the GTD solution starts to dominate. Note that only the reflected term (\(\beta^+\)) in GTD corresponds to AI as pointed out by Rudduck and Chen [22]. This is confirmed by the excellent agreement between AI and the reflected term in GTD as shown in Figure 37b.

For an aperture with only 3\(\lambda\) in diameter, a \(\cos(\psi/2)\) feed pattern which gives a 6 dB aperture field taper is used. The AI and GTD patterns roughly overlap in a region where the switching angle \(\theta_X=35°\) is located as shown in Figure 38.
Figure 36. Far field patterns of a $10\lambda$ circular reflector.
Figure 37a. Far field patterns of a 5\lambda circular reflector.
Figure 37b. Far field patterns of a $5\lambda$ circular reflector.
Hodge [23] recently developed a rigorous eigenfunction solution for the problem of plane wave scattering by a circular disk. In order to compare his exact solution results, a flat plate is modeled as a circular reflector by letting the focal distance $F=20\lambda$ such that the feed is in the far field zone of the reflector. Also a uniform feed pattern is used to serve as a uniform plane wave in the code. The calculated AI and GTD patterns as well as Hodge’s results for a disk with diameter $D=3.18\lambda$ ($ka=10$) are shown in Figure 39. It can be seen that the AI pattern agrees well with the rigorous solution in the $H$-plane. GTD also gives good results for $\theta>\theta_X=34^\circ$ in this plane. Note that the same patterns are predicted in both the $E$- and $H$-planes when either AI or GTD are used because double diffraction is not included. The difference between the $E$- and $H$-plane patterns for the rigorous solution indicates that double diffraction has a significant effect in the $E$-plane but almost no effect in the $H$-plane due to the incident wave polarization.

Although reflector antennas with aperture size less than $10\lambda$ are not common in practice, the above comparisons and discussions show that this analysis is accurate for reflector as small as $3\lambda$ to $5\lambda$. The switching angle criterion given by Equation (47) also works well for very small apertures.
Figure 39a. Far field pattern of plane wave scattering by a circular disk with $D=3.18\lambda$ — AI.
Figure 39b. Far field pattern of plane wave scattering by a circular disk with $D=3.18\lambda$ — GTD.
Figure 39c. Far field patterns of plane wave scattering by a circular disk with D=3.18λ — Hodge's calculations.
In addition to accuracy, this code also provides good efficiency because of the features used in AI and GTD as discussed in Chapters II and III. Typical CPU times for far field computations are 0.05 seconds per pattern angle for AI and 0.6 seconds for GTD on a VAX 11/780 computer. The highly efficient AI results from the fact that rotating grid is used for far field computations. As an example, the patterns of a square reflector shown in Figure 27 required about 60 seconds each of CPU time for 161 pattern points. In other words, it requires only 0.37 seconds per pattern angle when both AI and GTD are used. For near field computation, the typical CPU times are about 2 seconds per pattern point for AI and 1.2 seconds for GTD. For example, the near field results of the same antenna as shown in Figure 24 require about 106 seconds each of CPU times for 81 pattern points.
A technique which combines AI and GTD for reflector analysis has been presented. One important feature of this technique is its capability for computing the patterns of a reflector with general rim shape. By approximating the reflector rim with piecewise linear rim segments and by applying vertex diffraction [12,13] to take into account the corner effects at the end points of each segment, GTD provides an integrated approach for analyzing reflector antennas with an arbitrary rim shape including smooth curves such as elliptical or circular. Two simple but effective criteria are used to switch between AI and GTD. AI uses a rectangular subaperture method and provides an efficient approach to calculate the near field and far field patterns. For far field AI computations, the rotating grid method is employed such that the efficiency approaches that of a one-dimensional integration rather than a two-dimensional one. Good agreement in the comparisons between the calculated patterns and the measured data demonstrates the validity of the theory as well as the computer code.

While most reflector antenna designs are based on the far field pattern performance, the measured far field patterns are inefficient to obtain either directly or by using scale modeling, and it is very tedious and expensive to transform near field data to get far field patterns. The near field and far field capability of this code offers an efficient way for reflector antenna designs in that the computed near field data can be verified by near field measurements which are much easier to obtain, then the far field patterns are calculated from the code and used in an iterative manner to seek a practical design which meets the far field requirement. In addition, the near field capability of the code can be used for EMC and radiation hazard applications.

The present computer code for this analysis calculates results only for paraboloidal reflector surfaces with focused feeds. However, the GTD approach can be extended with rather straightforward modifications to analyze most reflector surfaces and feed positions. Furthermore, the AI approach is independent of the reflector surface and feed position, once the aperture fields are calculated. Thus the computer code is designed so that it can be extended to analyze other reflector geometries.
REFERENCES


