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TEACHING RECONSTRUCTION MEMORY STRATEGIES TO SEVENTH GRADE STUDENTS IN A PROBLEM SOLVING SETTING

The Ohio State University

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TEACHING RECONSTRUCTION MEMORY STRATEGIES TO SEVENTH GRADE STUDENTS IN A PROBLEM SOLVING SETTING

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By
Phillip Edward Duren, B.S., M.A.

* * * * *

The Ohio State University
1980

Reading Committee:
Dr. F. Joe Crosswhite
Dr. Jon L. Higgins
Dr. Alan R. Osborne

Approved By

Advisor
Department of Science and Mathematics Education
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VITA

August 18, 1948. . . . . Born - Coshocton, Ohio
1970 . . . . . . . . . . B.S., Mathematics, University of Kentucky, Lexington, Kentucky
1975-1976. . . . . . Graduate Assistant, Department of Mathematics, University of Kentucky, Lexington, Kentucky
1976 . . . . . . . . . . M.A., Mathematics, University of Kentucky, Lexington, Kentucky
1976-1977. . . . . . Mathematics Teacher, Grove City High School, Grove City, Ohio
1977-1978. . . . . . Graduate Assistant, Department of Mathematics and Department of Science and Mathematics Education, The Ohio State University, Columbus, Ohio
1978-. . . . . . . . . . Mathematics Supervisor, Stark County Department of Education, Louisville, Ohio

FIELDS OF STUDY

Major Field: Mathematics Education

Studies in Mathematics Education. Professors Alan Osborne, F. Joe Crosswhite, Jon Higgins, and Harold Trimble

Studies in Educational Administration. Professor Roy Larmee

Studies in Supervision. Professor Elsie Alberty
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CHAPTER 1

INTRODUCTION

A serious problem in the learning of mathematics is that many students seem to forget the mathematical concepts, rules or principles after a period of a few weeks or months. Teachers often bemoan the fact that students cannot remember how to use a skill that was learned earlier in the school year. This is a major problem if the skill is a prerequisite to learning a new application, skill or concept. Teachers also complain concerning the inordinate amount of time spent on review each year because of this memory problem of students, especially following the summer recess. Although several factors may contribute to this memory failure, it is important to know if the teacher could employ specific instructional techniques that would improve long-term memory. The students would be the beneficiaries if these instructional moves could either significantly increase the retention of mathematical concepts and skills or reduce the amount of time the teacher needs to reteach forgotten concepts and skills.

Many researchers have been attempting to enhance long-term memory. No single variable has proven to work for every student in every learning situation. However, recent studies have shown that several factors can have a significant impact on retention of skills by students.
One of the most productive, recent constructs in the area of long-term memory has been the identification and verification of reconstruction as a type of memory different from recognition or recall. Reconstruction is the intentional reproduction of a particular schematized action and of its results. It is also a form of recall by action in which some sort of model is reconstructed that was no longer available for perception. For the purposes of this study, reconstruction of a mathematical rule will be operationally defined as the reproduction of that mathematical rule by using the process by which the rule was learned.

The existence of reconstruction as a mnemonic form wedged between the elementary level of recognition and the higher level of recall has been demonstrated by Piaget and Inhelder (1973), although some of the earlier psychologists such as Katona (1940) identified the importance of reconstruction in the retrieval process of long-term memory.

---

1 This reconstruction should not be confused with the reconstruction Dewey (1920) talks about. Dewey's reconstruction is the process by which a person or society uses logical thinking about past experiences from memory in a deliberate attempt to change or reform political and moral dogmas, prejudices, or intellectual ideas.

2 Piaget (1973) uses the term "action" throughout his discussions, but many educators should probably substitute the word "process" to understand Piaget's intended meaning.
memory. Piaget (1973) claims the fact that reconstruction fits between recall and recognition gives reason for its genetic importance because it demonstrates that the close links between the conservation of memories and the conservation of schemata apply to all forms of the memory.

But this mnemonic reconstruction has a much more important implication to the educator. Because a reconstruction is the intentional reproduction of a particular schematized action and of its results, it is a more powerful retrieval mechanism to the learner than recognition. This means that when recall fails, reconstruction is still able to retrieve information from long-term memory without the physical cue(s) that recognition needs. Piaget (1973) found in his studies that reconstruction was more useful over a broader range of information retrieval situations than recall, since much of the learning that is stored in long-term memory is not available for recall after a period of time has elapsed. Piaget asserts that recall is inferior to reconstruction because it reverses the order of events as they appeared to the learner: actions to schemata to memory-images. On the other hand, reconstruction follows the genetic order of the learning in which every partial coordination involves the next step in the process, as in the initial process of learning.

These arguments imply that it would be more useful to the learner if educators were to consider teaching many more concepts, principles, et cetera for reconstruction rather than for recall. Too often the teaching has been directed toward future recognition which will not be useful to the learner away from the school setting or toward future recall which has a tendency to lose its retrievability from
long-term memory. Most of what is taught involves the use of highly differentiated schemata that are quickly forgotten rather than promoting the learner's formation of more general schemata than can be used in a variety of situations, especially in the reconstruction process of memory.

The work of Piaget and Inhelder (1973) can be identified as an existence proof for reconstruction. But there remain the lingering questions of whether the theory actually holds up in the everyday situations of the classroom and especially in the mathematics classroom. Since the effects of memory are so important to learners and to teachers, it is surprising that so little attention has been given to the topic of reconstruction by researchers. In light of this fact, it is therefore appropriate to test this reconstruction theory on more general concept learning situations, such as in the learning of mathematics, in order to see if reconstruction really does make a difference in long-term memory.

Almost any person who has advanced through college mathematics or has taught mathematics at an abstract level has experienced reconstruction by having reproduced a proof, rule, formula or relationship through a series of actions when recall of the proof, rule, formula or relationship failed. Therefore, it does not seem worthwhile to question the existence of reconstruction in mathematics learning.

In addition, Piaget and Inhelder (1973) observed reconstruction in a very concrete, nonmathematical setting in children as young as 4 to 5 years old. This was at least verifiable with seventh graders as they demonstrated the usage of reconstruction when asked such
questions as: "How many windows are there altogether in the place where you live?" and "What did you do the last time you misplaced your book?" Therefore, it also does not seem worthwhile to question the existence of reconstruction in students at least as early as seventh grade.

Results of two pilot studies indicated that students who used reconstruction could be identified and distinguished from those who used recall. (Recognition was not a factor since the rule was not given to students during testing.) Three classes of seventh graders from each of two different school systems were presented a mathematical problem. A different problem was used at each school. A different teaching style (inductive, deductive, or rule/example) was used for each class within the school. The product of the problem solving in each case was a rule. The rules pertained to:

1. The number of handshakes between a given number of persons where each person shakes hands one and only one time with every other person, and

2. Multiplication of integers.

Following the initial instruction, a test was given within 48 hours to verify whether a student had learned the rule. The pool of students who had learned the rule was identified for each class. From this pool, five students were randomly selected for a follow-up interview 4 to 5 weeks later. The remainder of the class took a post-test similar to the initial test at the same time as the interviews. Evidence of reconstruction was exhibited when an interviewee did not instantaneously use the rule to find the answer to a problem, but instead reproduced a table, diagram, verbal reasoning or examples
from the instructional period in order to remember the rule to work the exercises. Seven out of the 30 interviewed used reconstruction even though reconstruction strategies were not discussed per se in the instruction. In addition, several of the students who recalled the rule instantaneously could reconstruct the rule when asked. It was possible that some of these subjects would have relied on reconstruction had a longer period of time elapsed between the initial instruction and the interview.

One interesting finding of the pilot studies was that both inductive and deductive reconstructions were observed. Most, if not all, reconstructions in mathematics would fall in the inductive or deductive categories. It seemed reasonable to expect that the style of teaching had a high correlation with the type of reconstruction that a student would employ. This raises the question as to whether the style of teaching (inductive, deductive or rule/example) had an effect on the frequency of reconstruction strategies employed by students. The results of the pilot experiments suggested that the use of inductive or deductive styles of teaching would increase the probability of reconstruction by students. The pilot results also suggested that the use of the rule/example style of teaching would push students toward dependence on recall.

The present study was an attempt to refine the pilot findings. In the next section of this chapter, six questions which were identified from the pilot experiments as worthwhile for further research are given. These questions have helped in the design of this study in which nine classes of seventh graders were involved.
The investigator prepared lessons on four problems and taught the problems to the classes in either an inductive, a deductive or a rule/example style. Four treatment/observation cycles were employed in the study to examine the effects of style of teaching and reconstruction instruction. In one cycle, students in four classes were taught two of the four problems, one inductively and one deductively, without any instruction on reconstruction. The same four classes progressed through a second cycle, but this time the students were instructed on reconstruction strategies before being instructed on two different problems, one inductively and one deductively. Four additional classes participated in a third cycle which was identical to the second cycle with the exception that the students in these four classes had no prior problem solving experience. Students in the ninth class took part in the fourth cycle twice by being instructed each time on two problems in a rule/example mode without receiving any instruction on reconstruction.

Following the instructional days, the remainder of the four treatment/observation cycles were identical. A criterion test was given to each class 2 days after the instruction of the two problems, and those who achieved criterion on this test by properly applying the rule for each problem were put in a pool for each class. Five of the students from the pool of each class were interviewed by the investigator 8 weeks later to see whether they employed reconstruction strategies to work the problems and to identify the cues that helped them to remember. The remainder of the pool in each class took a retention test similar to the criterion test under supervision of the regular classroom teacher.
If indeed reconstruction can be taught, the value to the mathematics learner could be substantial. If students' long-term memory of basic skills and concepts in mathematics can be improved, test scores should go up significantly and less time for review would be needed during the school year. This extra time would allow for more concentration of instruction in areas that do not receive sufficient attention such as problem solving. In addition, reconstruction should make it easier to reteach mathematical concepts or skills that have been forgotten, or at least are below the threshold of recall, especially those forgotten over the summer break.

Because the reconstruction process involves the same order or actions as the original instruction, it is possible that it will only take a cue, such as a figure, table, or object from the original instruction in order to trigger the reconstruction of the concept or skill by the student. If this be the case, then the student could reconstruct the concept or skill and proceed to work out the exercises pertaining to the concept or skill without the teacher having to repeat the entire instructional process.

It appears that reconstruction does take place in mathematics and that it can be identified. The important question to the mathematics educator is whether or not reconstruction processes can be taught. If they can, a related question is whether or not instructional styles of teaching or characteristics of students make any differences in the learning of the reconstruction processes.
Problem Statement

The major goal of this study is to learn more about how children acquire the ability to reconstruct in learning mathematics. Related to the broad goal are six questions that will be investigated.

1. What is the effect of teaching styles on long-term memory of mathematical rules?

2. If reconstruction strategies can be taught in mathematics, what is the effect of teaching styles on the frequency of reconstruction strategies employed by subjects?

3. What is the effect of teaching reconstruction strategies to access long-term memory of mathematical rules on the use of reconstruction by subjects?

4. What is the effect of teaching reconstruction strategies on the long-term retention of mathematical rules, regardless of whether the remembrance was by recall or reconstruction?

5. Do high achievers in mathematics tend to employ reconstruction strategies more frequently than low achievers?

6. What are the key characteristics of the reconstruction process used by mathematics learners?

Research Hypotheses

Based on the preceding questions, the following hypotheses will be tested in this study.

1. The remembrance of mathematical rules will be significantly higher among those receiving the inductive or deductive style of teaching than for those receiving the rule/example style of teaching.
2. The inductive style of teaching will result in a significantly higher frequency of reconstruction strategies employed by subjects than the deductive style, and the inductive and deductive styles will result in a significantly higher frequency of reconstruction strategies employed by subjects than the rule/example method.

3. The teaching of reconstruction strategies will significantly increase the subjects' use of reconstruction to access long-term memory of mathematical rules.

4. The teaching of reconstruction strategies will significantly improve the retention of mathematical rules where the rules are remembered by recall or reconstruction.

5. High achievers in mathematics will employ reconstruction strategies significantly more frequently than low achievers regardless of whether they have been taught reconstruction strategies or not.

Assumptions and Limitations of the Study

1. The public-school, seventh-grade students living in a middle-class, suburban setting of Stark County, Ohio were representative of students in the 13-14 age group.

2. An 8-week interval between final instruction and the interview-retention test was sufficient to allow the mathematical rules to have fallen below the threshold of recall so that reconstruction would be necessary if the subject was to remember the rules.

3. The four problems chosen gave a sufficient indication of the effect of teaching for reconstruction.
4. The classes used were intact even though the classes were randomly assigned to treatment groups and background information of the subjects and schools were known.

5. Because of the need to interview students, the sample size was relatively small.

Delimitations of the Study

1. One 40-minute session on reconstruction strategies prior to problem instruction and a 30-minute review session following problem instruction was used for teaching reconstruction strategies.

2. Subjects were identified as high or low achievers from their scores on the mathematics part of the California Test of Basic Skills.

3. Elicitation of reconstruction was limited to verbal and/or paper and pencil responses.

4. Groups of classes used in this study were selected from the same school district in as far as possible.

Definition of Terms

1. A cue is a signal to the mind to begin a mental action to attempt to recall a previously learned concept or skill. This cue may be physical, verbal or mental in nature.

2. Deductive style of teaching is the predominate use of known principles or known relationships in order to help the student make the appropriate logical conclusions. With older students, this may include postulates, theorems, and definitions as well. However, with seventh graders the deductions are more intuitive in nature. In relation to this study, the appropriate logical conclusion is a mathematical rule. In addition, the deductive style in this study
might also be referred to as primarily a geometrical style because of its heavy dependence on drawing figures or sketches.

3. Encoding is the process of transforming and organizing information to be stored in long-term memory.

4. Inductive style of teaching is the predominate use of specific examples, pattern completions or positive and negative instances in order to help the student make the appropriate generalization. In relation to this study, the appropriate generalization is a mathematical rule. In addition, the inductive style in this study might also be referred to as primarily a numerical style because of its reliance on numerical pattern completions in tables or specific examples.

5. Long-term memory is the part of the mind where information that has been learned is stored awaiting retrieval days, weeks, months or years later. (This definition is a modification of the definition used in the information processing format; the lower boundaries of time used by psychologists in defining long-term memory typically is in terms of a few seconds. The reason for this modification is that in the learning of mathematics it hardly seems worthwhile to talk of remembering a rule for a matter of seconds. Instead, it seems that the goal of teaching a rule would be for the student to be able to retrieve the rule from memory days, weeks or months following instruction.)

6. Recall is the instantaneous retrieval of a memory-image of previous learning without the presence of the model or object: of a mathematical rule, the instantaneous retrieval of the memory-image of the rule.
7. Recognition is the remembering of previous learning in the presence of a cue where the cue is a model, an object or a figure: the perception of the model or object as something known.

8. Reconstruction is the intentional reproduction of a particular schematized action from the past and of its results: a form of recall by action: of a mathematical rule, the reproduction of a mathematical rule by using the process in which the rule was learned.

9. Remembrance is the successful retrieval of information that has been stored in memory.

10. Retrieval is the process of accessing what has been stored in long-term memory in such a form as to be useful to the learner.

11. A rule is a mathematical relationship between two or more concepts or principles that can be expressed symbolically by the use of a formula.

12. Schema is a catalogued procedure: an instrument of generalization: a series of ordered mental or physical actions.

13. The threshold of recall is the point below which a person cannot retrieve from memory a previously learned concept, in this study a rule, independent of seeing or hearing its mathematical or verbal representation.

14. A treatment/observation cycle consisted of instruction on two problems followed by a 10-item criterion test and a review session, an 8-week interval of no practice, and then a retention test in which part of the criterion pool in each class was interviewed and part was given a 10-item paper and pencil test.
CHAPTER 2
REVIEW OF LITERATURE

The retrieval of information is an important part of the many cognitive processes that are employed in learning mathematics. One cannot be concerned about improving comprehension, intellectual skills, creative thinking or reasoning ability in mathematics without implicitly being concerned about retrievable information which forms the basis for these processes (E. Gagné, 1978). Thus, any useful construct which could enhance the retrieval process from long-term memory should have a significant positive effect on the learning and application of mathematics.

Historical Trends

Psychologists have been interested in the problem of retention for a long time. Historically, two main theoretical approaches have been taken in seeking to explain retention: an associationist view and a cognitive view. The origin of the associationist point of view came from Thorndike (1932) who used the term "associative shifting" to describe how learning takes place. He felt that a stimulus-response connection was strengthened by being accompanied with a satisfying state and by repetition, while the connection was weakened through lapse of time. Ebbinghaus's (1913) findings in his classical studies on the retention of nonsense syllables seemed to coincide with that
of Thorndike's work in that disuse was concluded to be the primary
cause of forgetting. More recently the work of Briggs (1957), Underwood
(1945) and others have continued in the association point of view but
have found many other factors involved with learning or forgetting.
Several interference theories such as pro-active and retro-active
interference have been put forward as a result.

The cognitive view acknowledges the stimulus and response operants
but also adds a third variable of mental processes which affect learn­
ing and retention. Those who hold this view see the learner as
receiving stimuli, reacting and fitting the stimuli into the existing
cognitive structure through various internal mental processes, and
then responding. Bartlett (1932) was one of the earlier researchers
who held this point of view. He viewed the past as operating like
an organized mass rather than as a group of special elements, each
operating independently of the other. He used the word schema to
refer to the active organization of past reactions or of past exper­
iences which have been serially organized and operate as a unitary
mass. These schemata would influence memory as they interact with
incoming stimuli.

Bruner (1973) views the major problem of human memory to be that
of retrieving information from long-term memory. He theorizes that
each individual has his or her own manner of grouping and relating
information which is constantly subject to change and organization.
He refers to this unique structure as a "coding system," which he
defines as "a set of contingently related, nonspecific categories"
(p. 222). According to Bruner, this coding system is close in meaning
to that of Bartlett's memory schemata. Bruner asserts that educators
should teach these schemata or formal coding systems so that the retrieval process will be improved.

Ausubel (1968) developed a subsumption theory about how a concept is learned and stored in memory. He states that a new concept as it is learned is attached to a more general anchoring idea which he refers to as a subsumer. The new concept and the anchoring idea form what he calls an "ideational unit." The degree to which the new concept can be retrieved from memory independent of the general anchoring idea depends on the relatedness of the two, the stability and clarity of the anchoring idea, and the degree to which the new concept can be discriminated from the anchoring idea. Ausubel asserts that the way an individual represents a particular subject matter discipline such as mathematics in his or her mind can best be explained by considering a hierarchically ordered pyramid in which the ideas occupying the apex of the pyramid are the most inclusive and general, and subsume progressively less general or more highly differentiated ideas. Each of the ideas in this pyramid is linked to the next higher step in the hierarchy in the same way that a newly learned idea is with its related anchoring idea(s). Thus, with everything else being equal, Ausubel concludes that the more general and inclusive an idea is, the more stable the idea is in memory.

**Current Trends**

During the past decade there has been a renewed interest in memory primarily due to the new point of view that a learner is an information-processor functioning in a manner similar to that of a highly complex computer. The most significant contribution of the
information-processing approach has been the moving away from the "black box" and speculate about the mental structures and processes underlying behavior. Thus, it is not a methodology for experimenting; rather, it is a methodology for theorizing (Anderson and Bower, 1974).

Consequently, both historical traditions are still discernible in the current theories of memory, although the original distinction in their empirical base is disappearing (E. Gagne', 1978). Anderson and Bower's (1974) "human associative memory" and Wicklegren's (1976) "network strength" theory are current associationist models in which attempts are made to draw action trees that would simulate how the learning process triggers recall of previously learned material by laws of association between "nodes" of the tree. Current cognitive theories have included such ideas as Moskovitch and Craik's (1976) assertion that the "depth of processing" at the original time of learning has a significant bearing on the retrieval process. The depth of processing hypothesis argues that presented material may be processed to a greater or lesser depth along a continuum ranging from superficial processing of perceptual features to processing involving semantic and cognitive analyses (Craik and Lockhart, 1972).

Regardless of which point of view, almost all researchers have adopted the information-processing format and model for theorizing. This study was also done on the basis that the mind is a complex information-processing system. A description of the model of learning and memory which forms a basis for information-processing theories is given in the next section.
The Information-Processing Model

When the initial conditions of learning are present, abrupt stimulus changes can arouse the learner's attention. Once the information is received in the sensory register, emphasis on features of the objects to be learned contributes to the process of selective perception. The perceived information is then transformed and placed in short-term memory, where it can persist for a limited period (generally thought to be up to 20 seconds without rehearsal). R. Gagné (1977, p. 54) states that "evidence exists for two forms of storage in short-term memory: (1) an acoustic form, in which the information is internally 'heard' by the learner, and (2) an articulatory form, in which the learner 'hears himself saying' the information." The capacity of the short-term memory is limited, and some studies indicate that the number of individual items that can be handled in short-term memory at any one time is seven plus or minus two (Miller, 1956).

According to the model (R. Gagné, 1977), the two most important factors influencing the ability of a learner's long-term memory are the encoding and retrieval processes. Encoding is the most critical transformation of the information to be learned, and it occurs when the information is taken from short-term memory and stored in long-term memory. The encoding process apparently causes the information to be stored as "concepts whose meaning is known and can be correctly referenced in the learner's environment" (p. 54). This means that not only is the information being changed to a conceptual mode, but it is also being organized in various ways as it is stored in long-term memory. Thus, encoding is the critical process by which information in
short-term memory is transformed into learned and memorable capabilities.

The real verification of whether an object has been learned is whether it can be retrieved from long-term memory (R. Gagné, 1977). This retrieval process requires that certain "cues" be provided, either externally or internally, and that these cues are used by the mind to match or "link" what is learned in a search process. After the information is retrieved from long-term memory, it is either returned to short-term memory to be worked with in a new learning endeavor, or it is transformed to activate the "response generator," which provides an organization for various human performances. Finally, feedback is provided via the learner's observations of this performance.

R. Gagné (1977) gives another dimension to the model by introducing the "executive processes" and "expectancies." He says both of these processes have been largely acquired in previous learning experiences, and they constitute another separate portion of long-term memory. The executive control processes are cognitive strategies that can come into play in any or all of the phases of information flow. They can influence what is attended and perceived by the learner, thus controlling what goes into short-term memory. They can determine what is rehearsed in short-term memory and what is retained to be put into long-term memory. They can affect how information is encoded, and they can affect the learner's search and retrieval processes, as well as his or her response mechanisms. Expectancies are also a type of executive control processes, but these have to do with motivations and attitudes of the learner.
According to Andre (1979), there are at least two distinct types of long-term memory stores, episodic and semantic. Tulving (1972) has theorized that episodic memory contains memories of events that the information-processor has personally encountered. These memories are tied to time, places and sequences. These events have passed through a perceptual screen and consequently have been interpreted and encoded in the system's cognitive structure.

The system's abstracted or generalized knowledge is contained in the semantic memory (Andre, 1979). This is where comprehension of input occurs and concepts, principles, rules and skills are represented as executive control processes, or as most psychologists refer to them, schemata. Thus, it is believed that the semantic memory is made up of a network of interrelated schemata where these networks contain nodes and connections between nodes (Anderson and Bower, 1973).

Current Findings About Memory

Melton says that "the coding concept is one of the most important causes, if not the most important cause, of the extraordinary advance in knowledge and theory of human memory in the last ten years" (1973, p. 508). The introduction and development of coding processes have been perhaps the most prominent topics of research in the psychology of learning in the 70's, and certainly the most important product. The coding process was introduced into the theory of memory in order to explain the interaction of the cognitive structure and processes of the learner with input information, that is, to explain how information in short-term memory is transformed for storage in long-term memory (Melton, 1973). Melton says that research evidence
has found that the nature of what was stored in memory varied as a function of modality of presentation, the availability of cross-modality or interclass transformations, the applicability of rule or relational representations, and the time allowed for these transformations to occur.

R. Gagné (1977) says that a way to aid the encoding process is to present or suggest an encoding scheme which affects the form in which the newly learned material will be placed in long-term memory. He suggests techniques of instruction that could do this. One way is for the teacher to provide the encoding scheme for the learner. This may be done by the utilization of images as well as words and propositions. The use of pictures and diagrams in instruction to provide concrete visual images can serve as an encoding function.

Melton (1970) states that studies have consistently shown that the relation between the frequency of presentation and recall under massed practice and delayed practice schedules is that a delayed practice schedule always produces a better recall than massed practice, and especially the greater the frequency of presentation. Tulving (1972) says that this is because delayed practice permits more different cues to be stored than does massed practice, and these additional cues aid retrieval. Bower (1970) concludes that improvement with practice in free recall is a concomitant of developing better integrated chunks in long-term memory. These findings indicate that practice is indeed an important factor in the retrievability of information from memory.

A key factor in the ability of a learner to retrieve information from storage is the presence of appropriate cues to trigger retrieval. Bower (1970) says that for a given stimulus to become an effective
retrieval cue for a given to-be-remembered object, the person must have thought of the relation between the cue and the to-be-remembered object at the time that learning took place. Eich (1980) observed that the remembering of an event does not occur "spontaneously." He stated that retrieval is always affected by a stimulus, a query, or a cue whose informational content from semantic memory matches or complements information that has been stored in episodic memory about the event.

R. Gagne' (1977) states that the retrieval processes of long-term memory are activated by external "cues," and the more of these cues that can be provided the learner, the better the success of retrieval will be. Since recall often takes place in situations different from that of the original learning situation, he advocates the use of a variety of contextual cues to increase the probability of successful retrieval. Wescourt and Atkinson (1975) also stress the influence of contextual cues on the fact retrieval process. They theorize that an object is not learned in isolation, but that a part of the context of the learning situation is also stored in memory with the object.

In an experiment where subjects were tested for recognition of famous surnames and then were tested for cued recall of the surnames, Muter (1978) found that subjects failed to recognize 53.4% of the names that they subsequently recalled. These studies point out the fact that the probability of retrieval from memory depends critically on the cues provided. For example, in a study involving a literary passage, Bransford and McCarrell (1975) found that comprehension and ability to remember a passage were greatly improved when a descriptive title or picture was given prior to the reading of the passage.
Reder (1980) concluded that comprehension was dependent on a schema or conceptual framework being activated to make the referents of the passage clear. It would seem that a picture used in the learning of a mathematical rule would enhance the comprehension of that rule and, hence, would help in remembering that rule for a longer period of time.

Winograd and Lynn (1979) found that imagined contexts could play an important role in determining the relative effectiveness of different memory strategies. The results of their experiments involving word pairs indicated that recall was four to six times greater when each pair was placed in a unique context rather than a shared one. If this were to be true in the learning of mathematics, mathematical rules would need to be learned in a unique context to insure longer recall. Problem solving contexts lend themselves very readily to this appearance of uniqueness, and so it may be the rules learned in a problem solving context may be remembered longer.

A major contributing factor to the efficiency of a retrieval process is its organization in memory. Bower (1969) states that the preferred strategy of adult learners when faced with a large body of material to learn is to "divide and conquer," that is, to divide the material somehow into smaller groups and learn these as integrated packets of information. He says that the level and type of this chunking are to a certain extent under the cognitive control of the learner. So what a person remembers are his or her prior cognitive acts which are the constructive elaborations one employs to relate items to be learned. Therefore, if a learner can discover or learn a simple rule or principle which associates the items to be learned and the present cognitive structure, then those same rules or
principles can be used as a retrieval plan in reconstructing the items from long-term memory with a consequent of improvement in performance of the retrieval process. Bower (1969) summarizes by stating that the evidence to date has shown that when a learner builds or is given a systematic retrieval plan, that such plans are sufficient to produce very high levels of recall. He uses this evidence to argue that material should be organized in a hierarchical manner when it is learned so that the retrieval process can take advantage of this organized hierarchy to perform more effectively and efficiently.

According to several studies, one factor that has been abused in the school setting is that of rehearsal. By rehearsal it is meant "any active processing that keeps information available in consciousness such that the information can be immediately and accurately recalled at any time during which it is rehearsed" (Dark and Loftus, 1976, p. 479). Craik and Lockhart (1972) suggest that rehearsal can be usefully broken down into its "maintaining" function and its "elaborating" function. If a learner is using rehearsal time to merely maintain the memory trace in some simple form, as the majority of instances in the schools would reflect, then further repetitions or a prolonged stay will not enhance long-term retention (Craik and Watkins, 1973). However, to the extent that the learner uses rehearsal time to enrich and elaborate the memory trace, Craik and Watkins have found that subsequent retention will be enhanced.

An observation needs to be made on behalf of mathematics educators concerning all the studies in psychology that have been done thus far. The studies used by the various researchers generally involved the learning of serial lists of words, word pairs, figure
relationships, sentences or paragraphs and usually did not involve any mathematics. Melton (1970) says that in the 40's and 50's the typical research study in psychology involved motor-skills, and in the 60's and 70's the typical research study involved simple verbal learning exercises mainly due to their ease of design and measurement as opposed to that associated with more broader concept learning. This does not mean that the studies were not useful. They led to the development of hypotheses and then to the theories that were mentioned in this study. But, these theories must be tested on more general concept learning situations, such as in mathematics learning, to see if the factors identified really do make a difference in long-term memory.

Even the research in mathematics education has not been interested in the process of memory. Though retention has been frequently included within the design of studies, it has been treated as a secondary variable with little attention paid to how memory works or how it can be improved. More concern has been focused on the use of advanced organizers, discovery learning, and such like, and memory has been something that has been taken for granted. Therefore, it seems important to focus on the operants involved with memory of mathematical learning.

For a great number of years, memory aids were looked upon as "tricks" or "gimmickry" by experimental psychologists and, hence, not a lot of research was devoted to studying them. But during the last 15 years, there has been a steady increase in the number of studies devoted to memory aids. In an experiment involving 30 college students (15 male and 15 female) and 30 women (mostly homemakers), Harris (1980)
found that the two most used internal aids in memory were a mental retracing of events or actions and alphabetical searching. Harris suggested that since both strategies were general retrieval strategies, they were not limited to a specific context. Therefore, subjects did not have to employ any special encoding schemes in order to use them. Harris concluded that "this preference among internal aids for retrieval strategies over schemes for learning or encoding may reflect a higher probability of usefulness for the retrieval strategies" (1980, p. 33). There is a hesitation on the part of a person to spend time learning and encoding schemes when the "normal" memory retrieval strategies may be sufficient.

This would also seem to apply in specific content areas such as mathematics. Of course, the alphabetical searching strategy would not be a very efficient one to use in mathematics. But the internal aid of a mental retracing of events or actions does lend itself to the learning of mathematics and this is what Piaget termed reconstruction.

The Theory of Reconstruction

There is one theory of memory that appears to explain many of the phenomena that researchers have found and that offers the possibility of exploring other operants of the memory process. This theory is based on Piaget and Inhelder's (1973) findings of reconstruction as a third type of remembrance distinct from recognition and recall. As an illustration of this reconstruction type, consider an example from my own experience. Someone asked me which of the Kenley Players' shows did I attend most recently. At first I could not recall the name of the show, although I would have recognized the name if I had
been shown a list of plays. But my mind started picking out bits of information from storage such as the stars' names, the music, and the plot. Finally, I had reconstructed the event and retrieved the name of the show. I have also used reconstruction on numerous occasions to retrieve a mathematical rule, proof or formula from memory.

In order to understand what Piaget theorizes about memory, one first must realize that he speaks of memory in two different senses (Piaget and Inhelder, 1973). First of all, he uses the term "memory in the wider sense" to refer to the "ability to reproduce whatever can be generalized in a system of actions or operations (habitual, sensori-motor, conceptual, operational and other schemata)" (p. 4). He says that this sense of memory involves "conservation of schemata." Secondly, he uses the term "memory in the strict sense" to refer to "reactions associated with recognition (in the presence of the object) and recall (in the absence of the object)" (pp. 4-5). The reason that he wants to make this distinction is that he feels that memory cannot be treated as an ability separate from the function of cognitive processes as a whole (Hilgard and Bower, 1975). Piaget views both these senses of memory as interdependent.

To illustrate the distinction between the two senses of memory, consider an example from calculus. If one employs the concept of implicit differentiation, one is simply conserving and applying a schema that he or she has acquired by external transmission. However, if one recalls finding that concept in one's college calculus text, then he or she is referring to a particular situation encountered in the past and which can be localized in time more or less accurately. In general, the differences between schema and memory in the strict
sense result mainly from the fact that the former reflects the internal organization and dynamics of behavior, that is, an organization the conservation of which is the expression of its own activity, and the latter is either a figurative interpretation or a reconstruction of the results of that activity (Piaget and Inhelder, 1973).

Thus, Piaget believes there is a fundamental functional difference between the two senses of memory that needs to be understood if conservation of memory (long-term memory) is to be enhanced. First of all, memory in the strict sense involves mental imagery or symbolism that can be employed in operational representations; in the reconstruction of past events; in recall of all sorts of ideas or information; or in creative activities such as fantasies, dreams, plays or artistic endeavors. On the other hand, memory in the wider sense makes use of imagery or symbols, but not in the same way. It is a mode of knowledge which is not bound up with the immediate data nor involved in the solution of new problems. Its special domain is the reconstruction of the past. However, Piaget states that this reconstruction cannot be done without some reflection, and so he says that memory cannot be divorced from intelligence (Piaget and Inhelder, 1973).

An important part of Piaget's theory is his ideas on the development of memory with age. He views this development as a history of gradual organizations closely dependent on the structuring activities of the intelligence and regulated by the unique schemata of the learner. He also asserts that the mnemonic schematizations develop by stages as the intelligence does, that is, they go through the pre-operations, concrete, and formal-abstract stages of development. The reason that he gives for this observation is that schemata used by the memory are
borrowed from the intelligence, and this has proven to be the case in studies that he has conducted.

In addition, Piaget (1973) claims that these schemata serve as instruments of mnemonic organization and are active during both the storage and retrieval processes of long-term memory. Thus, the conservation of memory involves two interdependent forms of generalizing function: conservation of schemata which is the constant restoration of general or specific processes learned from previous experiences, and conservation of memories (in the strict sense) which is the constant restoration of particular and past experiences. Each of the processes of conservation needs the support of the other with the former playing the principal role.

As mentioned previously, the fact that Piaget fits reconstruction between recall and recognition gives reason for its genetic importance because it demonstrates the links between conservation of schemata and conservation of memories. It shows the importance of schemata in the retrieval process. Since a given reconstruction is a special type of schema that calls for the reproduction of the process of learning to find the product of the learning, reconstruction strategies should have a significant effect on the retention of learning. Piaget's (1973) studies have given some proof of this.

In comparing Piaget's ideas about memory with other theorists, reconstruction offers the possibility of being a key factor to improving memory. Reconstruction strategies could be thought of as formal coding systems that Bruner advocates. The process of reconstruction in some sense resembles Ausubel's subsumers and because of its more general nature would thus be easier to remember than a particular
concept or rule. Reconstruction strategies can be equated with the executive control processes of the information-processor and would be both a valuable and efficient retrieval mechanism. With all the indications from psychologists that general schemata can and should be taught, it would appear that reconstruction strategies could become an important part of instruction in mathematics.

A closer examination of recent studies that have been successful in improving long-term memory of mathematical learning indicated that reconstruction could have been an important reason for their success. In a study which compared various types of schematic "rote" learning, Skemp (1971) found that schematically learned material was not only learned better but retained better over a period of 4 weeks. In this study, the only difference between the groups was in the mental structures which they had available for the learning task. While Skemp was interested in demonstrating that the use of schemata could make learning more efficient, reconstruction strategies are in fact schemata and could be what subjects really employed to help them score higher on the retention test.

In a study involving learning disabled (LD) children, Myers and Thornton (1977) hypothesized that what seemed to be apparent memory failure of basic facts in LD students might actually be a failure on their part to use relationships to aid recall of facts. Myers and Thornton went on to say that the teacher must provide more than simply telling a child to memorize. They found that when relationships were clearly defined and included as an integral part of instruction that retrieval from memory was more successful. It may well be that reconstruction was involved in the LD students' successes in
recalling facts. The relationships that were carefully taught to the LD students could have acted as reconstruction strategies in helping retrieval of facts. Thornton (1978) found in another study that the use of thinking strategies facilitated the learning and retention of basic facts. She also concludes that children in this study seemed to adopt strategies that had been explicitly taught, encouraged or suggested during instruction. Again, these thinking strategies may well have acted as reconstruction strategies in the recalling of basic facts.

All of this evidence lends support to the idea that reconstruction strategies can and should be taught as an integral part of mathematics instruction. Recent emphasis in mathematics education on strategies in the teaching of estimation and problem solving gives evidence of the importance of general processes to the long-term retention and use of mathematics. An examination of the memory processes involved in the encoding and retrieval of mathematical learning is needed, and a study of reconstruction seems a good avenue to gain insight into these processes. In addition, the study of reconstruction will also give some evidence as to what kinds of cues are important in the retrieval process. Since many researchers have indicated how critical cues are in remembering, the types of cues that are used to trigger reconstruction should give an indication of what factors are important to include in the instructional process.
CHAPTER 3

METHODS AND PROCEDURES

Three major tasks were undertaken in this study. First, there was a search for evidence of reconstruction and its characteristics in the learning of mathematical rules. Secondly, the effects of reconstruction instruction on the remembrance of mathematical rules was examined. Thirdly, since the mathematical rules were learned in a problem solving setting and the students had had but limited experience with this type of class activity, there was a need to ascertain the effects of problem solving practice on the remembrance of mathematical rules. Thus, the design of this study was developed to gather data relative to these tasks.

The following sections of this chapter explain the way in which the study was done. There is an overview of the design of the study, an identification of the subjects that took part in the experiment, an explanation of the major treatment/observation cycles, a discussion of the problems selected for instruction, a detailed description of how each problem was taught in an inductive style and a deductive style, an instrumentation section, an explanation of how the interview was conducted along with three specific examples, and a summary of the procedures.
Design

The design of this study involved the use of three groups of classes to test the hypotheses (see Figure 1). However, to better communicate the design and for ease of analyses, the design is pictured by treatment/observation cycles in Figure 2 which involved four distinct cycles. Each treatment/observation cycle had the common components of instruction on two problems, a 10-item quiz used as a criterion test, a review session following the quiz, an 8-week interval, and then a retention test for which one part of those attaining criterion in a class was interviewed and the remainder attaining criterion took a pencil and paper test.

The four distinct cycles which were formed to accomplish the three previously mentioned tasks of this study were:

1. Cycle W in which students in Group A₁ participated twice by being instructed each time on two problems in a rule/example style of teaching,

2. Cycle X in which students in Group A₂ were instructed in a deductive style on one problem and an inductive style on a second problem without being instructed on reconstruction strategies,

3. Cycle Y in which the same students who participated from Group A₂ in Cycle X were instructed this time on reconstruction strategies before being instructed on two different problems, one inductively and one deductively, and

4. Cycle Z in which students from Group A₃ were instructed on reconstruction strategies before being instructed on two problems, one inductively and one deductively.
Groups of classes | Classes | Treatment/Observation cycles
---|---|---
A_1 | 1 | W (No reconstruction instruction)
 | 2 |  
 | 3 |  
 | 4 | X  
 | 5 | (No reconstruction instruction)  

A_2

 | 6 |  
 | 7 |  
 | 8 | (Usual class instruction)  
 | 9 | (Reconstruction instruction)  

A_3

Figure 1. Major design of study.

Figure 2. Contrast groups defined by treatment/observation cycles.
As shown in Figure 3, Group A₁'s initial involvement with Cycle W occurred at approximately the same time period in the school year as A₂'s involvement with Cycle X, and A₁'s second involvement with Cycle W was approximately the same time period in the school year as A₂'s involvement with Cycle Y and A₃'s involvement with Cycle Z. Since Groups A₁ and A₂ both were administered two treatment/observation cycles, a 6-week interval was allowed between the completion of the first cycle and the beginning of the second cycle to insure that there would be no contamination between the different sets of two problems during the retention tests.

<table>
<thead>
<tr>
<th>Groups of classes</th>
<th>Treatment/Observation cycles</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>W</td>
<td>October-December</td>
</tr>
<tr>
<td>A₂</td>
<td>X</td>
<td>October-December</td>
</tr>
<tr>
<td>A₁</td>
<td>W</td>
<td>January-March</td>
</tr>
<tr>
<td>A₂</td>
<td>Y</td>
<td>January-March</td>
</tr>
<tr>
<td>A₃</td>
<td>Z</td>
<td>February-April</td>
</tr>
</tbody>
</table>

Figure 3. Time line when groups of classes were administered treatment/observation cycles.

Except for Cycle W, each of the four problems was treated inductively and deductively within a cycle but with different students. The assignment of the four problems for classes participating in Cycles X, Y and Z are given in Figure 4. Students in Group A₁ were assigned problems 1 and 2 for the first time through Cycle W and problems 3 and 4 the second time through Cycle W. A description of
Classes

<table>
<thead>
<tr>
<th>Treatment/observation cycles</th>
<th>Instructional Style</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cycle X</td>
<td>I 1 2 3 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D 2 1 4 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cycle Y</td>
<td>I 3 4 1 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D 4 3 2 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cycle Z</td>
<td>I 1 2 3 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D 2 1 4 3</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Figure 4. Problem assignment within treatment/observation cycles for Cycles X, Y and Z.

The problems and their characteristics are given in the section following the detailed description of the treatment/observation cycles. Thus, each treatment/observation cycle received a different type of instruction and/or problem solving experience.

Group A\textsubscript{1} was a quasi-control group for the experiment. The term quasi was used because the basic intent of this study was to examine the effects of reconstruction instruction. The term control was used because the rule/example style of teaching is the predominant style of teaching used by teachers at all grade levels. The
results from the four treatment/observation cycles were compared to test the hypotheses. Cycle W was compared with X to determine the effects of styles of teaching on remembrance. W was compared with Z to determine the effects of reconstruction instruction and teaching styles together on remembrance. X was compared with Y to look at the effects of reconstruction instruction within the same classes. Y was compared to Z to examine the effects of problem solving experience on the use of reconstruction. X was compared to Z to determine the effects of reconstruction instruction on remembrance and on use of reconstruction. In addition, the comparisons of X, Y and Z helped to determine whether the reconstruction strategies employed by students could be attributed to the reconstruction session or to the instructional sessions.

Subjects

The nine, seventh-grade, mathematics classes that participated in this study were from the Stark County Public School System in suburban Canton, Ohio. Groups of two or three of the classes were selected from the same school district in order to reduce the time for the investigator since he was doing all the teaching and interviewing in the study in addition to working full time. An additional requirement for the selection of mathematics classes was that the students must have taken the California Test of Basic Skills (CTBS) in order to assure that the background information concerning mathematical achievement of the students was comparable.

The CTBS scores were used to check whether the students in the nine classes were similar in mathematical achievement. The CTBS
means for the nine classes are listed in Table 24 in Appendix A.
It was found that eight of the classes were essentially equivalent
in mathematical achievement, but Class 6 was higher than the others.
This was found by performing a one-way analysis of variance for the
CTBS scores of students in each of the nine classes and a one-way
analysis of variance for the CTBS scores of all the students except
for those in Class 6 (see Tables 25 and 26 in Appendix A). Although
the students in Class 6 did well on the criterion and retention tests,
they did not perform better than students in several of the other
classes with lower mathematical achievement levels. Since the
students were being equated on the basis of passing a criterion
test and not on the CTBS scores, the classes were treated as essen­
tially similar.

Several considerations led to the selection of seventh-grade
classes for the experiment. With the recent controversy over compe­
tency testing and basic skills, it seemed to be more important to
look at what might help the retention of mathematical concepts and
skills of the "average" student rather than the mathematically
oriented student in the college-prep classes of grades 8 through 12.
In addition, the seventh grade was chosen instead of one of the
elementary grades because Piaget (1973) found in his research that
mnemonic schematizations develop by stages as the intelligence does;
they appear in pre-operations, concrete, and formal/abstract stages
of development. It was concluded that the best place to start
investigation was at or around the age when children could be expected
to be shifting to the stage of formal operations. In several of
Piaget's studies, the reconstruction abilities were demonstrated by subjects at all age levels 5 and up. The schemata employed by these subjects in reconstructing coincided with their developmental age. Thus, in selecting subjects from the 13-14 age group, reconstruction abilities should be possessed by all subjects and the teaching for reconstruction of mathematical rules at somewhat of a formal level appeared appropriate. It also seemed important to study the subjects at or near the formal operations stage before they have had enough mathematical experiences to develop mnemonic reconstruction strategies on their own. In this way, the effects of teaching for reconstruction could better be determined.

Treatment/Observation Cycles

Two distinct types of instructional weeks were used in this experiment for the treatment/observation cycles. One type had a day for instruction on reconstruction strategies and the other type did not (see Figure 5).

1. The instructional week for Cycles X and W did not include a day of instruction on reconstruction strategies. In Cycle X, instruction on two of the four problems were given on consecutive days to each class. The first problem was taught in an inductive or a deductive style and the second problem was taught in the other mode. Approximately 30-35 minutes of each period were devoted to the instruction of a problem with a mathematical rule as the objective of each problem session. The remaining 10-15 minutes of each period were used to provide practice for the rule of the day. Two days following the instruction of the two problems, the criterion
<table>
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<th>Treatment/observation cycles W and X</th>
<th>Day 1</th>
<th>Day 2</th>
<th>Day 3</th>
<th>Day 5</th>
<th>8 weeks later</th>
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<td>Practice</td>
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<th>Day 2</th>
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<th>Day 5</th>
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Figure 5. The two types of instructional weeks for the treatment/observation cycles.
test was given to each class. Following the test, the remainder of
the period was used to go over the test items and review the process
by which the rules were derived with special attention given to how
the process was related to the exercises.

In Cycle W, the quasi-control class, the problems were taught
by the rule/example method. The instruction for each problem entailed
the giving of the rule after the background of the problem had been
introduced. This was followed by several examples worked as a class
and a 10-15 minute period of practicing the rule. The total number
of examples that were worked was roughly equivalent to those in the
other treatment groups. Two days after the instruction of two of the
problems, the criterion test was given to the class. Following the
quiz, time was given to going over the items on the test with the
major attention being given to the rule.

2. The instructional week was identical for Cycles Y and Z
and included a 40-minute instructional session on reconstruction
strategies. The session began with an explanation of reconstruction
and this was followed by several examples. A discussion was then
conducted to have students give their strategies for remembering.
The final part of the session was devoted to pointing out the advan-
tages of reconstruction and the important points in using reconstruc-
tion. Instruction on two of the four problems was given on the two
consecutive days following the session on reconstruction with one
problem being taught on each day. Again, the first problem was
taught using either an inductive or deductive style of teaching and
the second problem was taught in the other mode. Approximately
30-35 minutes of each period were devoted to the instruction of a problem with a mathematical rule as the objective of each problem session. The last 10-15 minutes of each period were used to provide practice for the rule of the day. Two days following the instruction of the two problems, the criterion test was given to each class. Following the test, the remainder of the period was used to go over the test items and review the process by which the rules were derived with special attention given to how the process related to the exercises. The objective of the review session was to relate the review of the processes with the reconstruction session. Students were asked what they would want to remember in order to reconstruct the rule.

As can be seen from Figure 5, the instructional days for the problems, the criterion tests that were given 2 days later, and the emphasis on the processes in the review following the quiz were identical in all four treatment/observation cycles. The only difference in the instructional week was the addition in Cycles Y and Z of the 40-minute session on reconstruction and the referral to that session in the review of processes used with the rules.

In Cycles W, X, Y and Z, a pool of criterion-attaining students was formed in each class of those students who demonstrated remembrance of both rules on the criterion test. The pool itself acted as a means of equating the classes because only the subjects in the pools were eligible for later interviews and retention test comparisons.
Problems

Two variables in problem solving instruction were identified in the literature which may have an effect on retention. They were types of cues available to the student during a learning situation, and the instructional style employed during a learning situation. In order not to introduce other factors into the study, problems were chosen so that they could be taught in an inductive or deductive style and so that a range of cues from concrete/semi-concrete to symbolic were available to the students.

The four problems used in this study were:

1. For a given number of people, how many handshakes are there altogether if each person shakes hands one and only one time with every other person?

2. For a right triangle, what is the relationship of the three sides?

3. What does \((a+b)^2\) equal?

4. What is the sum of the interior angles of a polygon?

The problem solving situations that were selected had not previously been studied by the students. Interviews with the regular mathematics teachers indicated the content of the problems would not be practiced or used during the interval of the study, except during treatment sessions.

Each of the four problems were taught in an inductive and a deductive style in Cycles X, Y and Z so that the effects of style of teaching would be spread across each group. A detailed description of how each instructional session was conducted is provided in the next section.
Problems 1 and 2 were paired together and problems 3 and 4 were paired together in each treatment/observation cycle. Problems 1 and 4 were similar in difficulty, prerequisites, and type of cues available for the students; problems 2 and 3 were similar in difficulty, prerequisites, and type of cues available for the students. By this pairing, an attempt was made to distribute the error that might result from types of cues available and the attributes of types of problems across treatment/observation cycles.

A frequency table on the successful use of reconstruction was constructed from the interview data to check the effects of the problem types (see Table 27 in Appendix B). A similar table was made from the class data except the frequency count was on the successful recall or reconstruction (see Table 28 in Appendix B). On each set of data a separate log-linear analysis was performed for the inductive and deductive styles of teaching on the use of reconstruction instruction by problem type (see Tables 2, 3, and 4 in chapter 4). In each case, the effects of problem type were not found to be significant at the .05 level. Where significance was found, the model which provided the best fit of the data involved the main effects of reconstruction instruction with the dependent variable. Thus, the attempt to distribute error due to problem type across the treatment/observation Cycles W, X, Y and Z apparently was successful.

Instructional Strategies

All classes in this study were taught by the researcher. The lesson format was the same for all four problems regardless of which
instructional mode was employed. The first 5 to 10 minutes of the period were devoted to the introduction of the problem. The introduction began by posing the problem to the class in the form of a question and asking the students to make an educated guess at the solution. If guesses were made, they were written down. Regardless of whether there were any guesses, the students were then asked to make suggestions for how one might solve the problem. After pros and cons were discussed for each suggestion (if there were any), a brief review of any necessary prerequisite skills was undertaken. For example, in the Pythagorean theorem problem and the squaring of the binominal problem, the squaring of numbers was reviewed.

Following the introduction of the problem, 15 to 20 minutes were used to develop the rule for solving the problem. Of course, the instructional mode for the development of the rules varied for each problem and for the inductive and deductive mode of each problem.

As the classes were being taught problems in an inductive style, a deliberate effort was made to have each student make predictions and hypothesize a rule without receiving help from other students. This was done to keep the teaching inductive rather than have someone provide the rule for others. Class discussion was encouraged in the inductive style of teaching but only after everyone had committed themselves to a prediction. This discussion centered on the verification of correct responses and on encouraging students to look for a general rule. Students were constantly reminded not to convey their hunches to other students, but they were permitted to show their ideas nonverbally to the instructor. Since the students were seventh
graders, most of the deductions made in the deductive style of teaching were intuitive in nature rather than being formal deductions from theorems or postulates as in a tenth-grade geometry course.

In the inductive mode for the handshake problem, instruction began by suggesting that students sketch a diagram where points represented people and line segments represented handshakes. It was also suggested to the students that they start with the smallest class where handshakes were possible (a class size of two) and record their findings in a table. After a diagram was drawn for a class size of two, three and four, each student was asked to write a prediction, without consultation, about the total number of handshakes in a class of five students. They were then instructed to draw a sketch and check their prediction. Responses were then elicited from the class for that specific exercise and class discussion was centered on finding which response was correct. This process was repeated for a class size of six. Each student was then asked to look at their table to see if they could find a pattern that might help them predict how many handshakes there would be in a class size of 10. The skipping of numbers for class size was intentional because most of the students would observe addition patterns at first to make their predictions. These strategies were limited by the fact that the table would need to be filled in for all previous integral values and these strategies would not lead to a general rule. The direction was given to continue to make predictions on their paper for a class size of 15 with the correct total of handshakes given to them after all predictions were written down. This process continued for class hypothesized sizes of 20 and 30 until the allotted time was up for
this part of the period or until everyone had a strategy which appar-ently gave them the correct number. Then students were asked to verbalize their strategy for finding the total number of handshakes and a rule was formed from the verbalizations \[N(N - 1) \div 2].

The deductive mode for the handshake problem was started in the same manner as the inductive mode with the suggestion for sketching a diagram using points to represent people and line segments to represent handshakes. These classes were also instructed to begin by looking at class sizes of two, three and four. But instead of suggesting the building of a table, students were asked to look for reasons why the totals came out as they did. Unlike the inductive mode in which responses were expected on paper from each student, this mode proceeded as a group discussion following each student’s sketching of the diagram and the finding of the total number of handshakes. The first observation that was made by the students was that you could not shake hands with yourself, so that each person shook hands with \(N - 1\) other people in the class. This was an intuitive deduction made by the students from what was known about the custom of handshaking. The class was asked to draw the sketches for class sizes of five, six and ten with discussion following each one. The first hypothesis that was given by the students was that \(N(N - 1)\) would give them the total number of handshakes. But they quickly found that this gave them twice as many as they counted on the diagrams so the students were asked to go back to their diagrams and find out why. The students then suggested that the \(N(N - 1)\) was counting handshakes between two people twice. This was an intuitive deduction made by the students from the diagrams and the given
of the problem. They reasoned that if they counted Paul shaking hands with Pam, then they could not count Pam shaking hands with Paul. Thus, they reasoned that they must divide $N(N - 1)$ by 2 in order to get rid of the duplications. In this manner the rule $N(N - 1) : 2$ was derived.

The inductive mode for the Pythagorean theorem problem was begun by asking the students to construct a table with the sides of a right triangle on one side of the table and their squares on the other side of the table. Three right triangles along with the lengths of each side were given in a handout to the students. The students were instructed to take the lengths from the drawings and put the values in the table for $a$, $b$ and $c$. The students were then asked to find the squares of $a$, $b$ and $c$ and enter them in the table. Following this, the students were shown a right triangle in which only the lengths of the legs were known. They were then asked to record $a$ and $b$ in their tables and also to find $a^2$ and $b^2$. After they recorded $a^2$ and $b^2$, the students were instructed to look at the previous entries in their tables and see if they could find a pattern that would help them make a prediction for $c^2$. Students were asked not to consult with other students in making their predictions. After each student had made a written prediction, responses were elicited and the correct response was indicated by the instructor. This procedure was repeated in four more triangles with $c$, $a$, $b$ and a missing respectively. By the end of the fourth such missing side, most, if not all, of the students had discovered on their own the relationship $a^2 + b^2 = c^2$. The rule was then formalized from students
verbally telling the relationship, and instruction was given on how to find the missing side length once the square of that side was known.

The deductive mode for the Pythagorean theorem problem was begun by giving each student a handout in which three right triangles were drawn and the lengths of the sides indicated. The students were instructed to sketch a square on the sides of the right triangle using each side as the base of that square. The students were then told to find and label the area of each of the three squares for each of the three triangles. The students were asked which square on each triangle had the largest area. Students identified the square on the hypotenuse as the largest in area and from questioning they pointed out that the sum of the area of the two smaller squares equaled the area of the largest square. This was then put into the rule \( a^2 + b^2 = c^2 \). Following the identification of the relationship between the squares of the sides of a right triangle, the students were given five triangles (one at a time) in which one of the side lengths was unknown. The classes were instructed to sketch the squares on each side of the triangles and find the areas of the squares whose side lengths were known. The students were then asked to find the area of the square on the side of unknown length by using the relationship \( a^2 + b^2 = c^2 \). Instruction was then given on how to find the missing side length once the square of that side was known.

The inductive mode for the squaring of a binomial began by the researcher asking the students for a guess as to what \((a + b)^2\) might be. Invariably, students predicted that \((a + b)^2 = a^2 + b^2\).
Specific numbers were then used to test this guess. After three trials with specific numbers, the students would recognize that \(a^2 + b^2\) was not large enough. In each case, the students were asked how much more would have to be added to \(a^2 + b^2\) to equal \((a + b)^2\). Students were then told that there was a way to find this "extra" amount from \(a\) and \(b\) without having to go through all the computation. Each student was instructed to see if he or she could see a way to come up with the amount that was needed to add to \(a^2 + b^2\) in each of the first three pairs of specific values for \(a\) and \(b\). A fourth pair of specific values was then given to the students and each student was asked to make a prediction on paper without consultation as to what "extra" amount would be needed to add to \(a^2 + b^2\) to equal \((a + b)^2\). The students were then directed to do the computation to check their predictions. This procedure was repeated for three more pairs of specific values for \(a\) and \(b\) with the exception that instead of the students being asked to check their predictions with computation, students read their predictions orally. The correct answer was confirmed by the researcher following each specific case. Following these specific examples, most, if not all, of the students had discovered a way of finding the term 2ab. The students were asked to verbalize their method of finding the "extra" amount needed to add to \(a^2 + b^2\) and up to eight versions of 2ab were given. (For example, "double \(b\) and then multiply by \(a\).") It was then concluded by the class that \((a + b)^2 = a^2 + b^2 + 2ab\).

Instruction began on the deductive mode for squaring of a binomial by drawing two line segments of different but unknown lengths on the board. The two line segments were labeled \(a\) and \(b\).
respectively. The students were told that the key to finding \((a + b)^2\) was to make a square whose sides are \(a + b\) long. This square was then sketched on the board and copied by the students on their papers. Perpendicular lines were then sketched from the points that joined line segments \(a\) and \(b\) on each side of the square. This divided the square into four regions, two squares and two rectangles. In the introduction the students had already learned that \((a + b)^2\) was equal to \((a + b)\) times \((a + b)\), so they were instructed that the area of the square was equal to \((a + b)^2\). They were then told to find the area of the four regions. The students were then asked how they could find the area of the large square whose side length was \(a + b\). Students responded by indicating that the areas of the four regions could be added together to find the area of the large square. This was an intuitive deduction by the students based on their knowledge of area being additive. Thus, as a class the areas of the four regions were calculated to be \(a\) times \(a\) or \(a^2\), \(a\) times \(b\), \(a\) times \(b\), and \(b\) times \(b\) or \(b^2\) (using the formula length times width for the area of a rectangle). These regions were then added together to find that \((a + b)^2 = (a + b)(a + b) = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2\). Three specific pairs of values for \(a\) and \(b\) were then tried to verify that this rule does work. Finally, three binomials were squared in class by having the students make the squares as it was done for \(a\) and \(b\). They were \((c + d)^2\), \((a + 3)^2\) and \((5 + b)^2\).

In both the inductive and deductive mode for the problem of finding the sum of the interior angles of a polygon, the students already were aware that the sum of the angles of a triangle was \(180^\circ\). Instruction in the inductive mode began by giving each
student a handout sheet on which there was one acute triangle, one obtuse triangle, one right triangle, one square, one rectangle, a pentagon, and an hexagon. The researcher gave the values for the measures of the angles of each triangle so that the students could verify the fact that the sum of the angles of any triangle is 180°. The students already knew that the angles of a square and a rectangle were right angles with a measure of 90°. So they quickly found that the sum of the angles of a four-sided polygon was 360°. At this point the students were asked to construct a table in which the number of sides was kept on the left side of the table, and the sum of the angles was on the right side. Each student was then requested to write on paper a prediction, without consultation, on the sum of the angles of a five-sided polygon. Everyone predicted either 540° or 720°. The researcher gave the measures of the angles of the pentagon on their handout sheets so that each could check their prediction with the actual sum. The sample procedure was repeated for the hexagon. All of the students recognized that the sum of the angles of the polygon was going up by multiples of 180 in the table. Each student was then told to look carefully at the number of sides in the polygon and the multiple of 180 that was the sum of the angles of the polygon. Each student was asked to make a prediction on paper, without consultation, for the sum of the angles of a 10-sided polygon. The students were told not to count on by 180 but to look for a quicker means of making their prediction. The researcher confirmed the correct total after the students gave their predictions orally. This procedure was then continued with a 15-sided polygon, a 20-sided polygon, and a 30-sided polygon. Most, if not all, of the students were now
correctly predicting the total so they were asked to verbalize their methods of making their predictions. The responses were formalized into the rule \((N - 2) \times 180\).

Instruction on the sum of the interior angles of a polygon in the deductive mode was started with the same handout sheet that the inductive mode received. The researcher gave the values for the measures of the angles of each triangle so that the students could verify the fact that the sum of the angles of any triangle is \(180^\circ\). As in the other classes, the students already knew that the angles of squares and rectangles were right angles, so they quickly figured the sum of a four-sided polygon was \(360^\circ\). The students were asked to select one of the vertices of the square and rectangle and draw as many diagonals as they could from that vertex for each figure. Of course they could only draw one, so they observed that the four-sided figures had each been divided into two triangles. It was pointed out that since the sum of the angles in each triangle was \(180^\circ\), then the sum of the angles of the four-sided polygons could be calculated by \(2 \times 180^\circ\). This gave the same result of \(360^\circ\) and coincided with what the students had already figured. The students were then asked to pick a vertex in the pentagon and draw as many diagonals from that point as they could. The students observed that two diagonals could be drawn and that the pentagon had been divided into three triangles. It was concluded that since the sum of the angles of each triangle was \(180^\circ\), then the sum of the angles of the pentagon must be \(3 \times 180^\circ\) or \(540^\circ\). The same procedure was repeated for the hexagon. The rule was then formalized by observing that the number of triangles formed by drawing all possible diagonals of a
polygon from one vertex was always two less than the number of sides. Thus, the rule was \((N - 2)\) times 180. The students were then asked to apply that rule to polygons of 10, 15, 20 and 30 sides.

Instruction in the rule/example mode was similar for each of the four problems. Following the same introduction to the problems as the inductive and deductive styles, the class was told that a fictitious student had come up with a possible solution to the problem. The solution was the correct rule, and the examples used to come up with the rule in the inductive or deductive mode were used in this class by the students to "verify" that the rule was the right one.

After the development of the rule for each of the four problems, 5-10 minutes were given to practice the rule by working examples in each class regardless of mode of instruction. Within each problem, the number of examples after the development of the rule was approximately the same. If anything, the rule/example class worked more examples due to less time spent in developing a rule. For each problem and each mode of instruction, five homework problems were given. In this way, sufficient practice was provided for the students to learn the rule and to work with it comfortably without introducing a practice variable into the experiment. The students in each class were instructed not to practice or worry about the problems after they had taken the criterion test.

**Instrumentation**

The California Test of Basic Skills (CTBS) was used to determine the mathematical achievement of the students involved in this study. This background data was used to identify high and low mathematical achievers in order to examine the correlation between mathematical
achievement and the use of reconstruction strategies. The reliability of the CTBS as determined by the KR-20 test was .855 overall for the mathematics section with .81 for computation and .82 for concepts and problems in the subtests (Buros, 1978).

A 10-item test was used to measure whether a student had learned a rule following instruction and whether he or she could recall the rule that had been learned 8 weeks later. This test which was given 2 days after instruction was referred to as the criterion test. The test (see Appendix C) consisted of five exercises that called for the direct application of one of the four problem solutions used in this study and five exercises that called for the direct application of a second of the four problems. The two problem types that appeared on the test were determined by which treatment/observation cycle the class was in (see Figure 5). The five exercises that called for a direct application on each problem were identical regardless of treatment/observation cycle. A reliability test was done on problems 1 and 2 together and on problems 3 and 4 together since that was the way they were grouped for testing. By calculating the scores for the odd and even numbered exercises, a split-half reliability correlation was found for each pair of problems. For problems 1 and 2 it was .99, and for problems 3 and 4 it was 1.00.

The results from the criterion test were used to establish a pool in each class. This restriction was taken since it cannot be expected that weeks later a subject will reconstruct a rule that he or she has not learned. The criterion pool was formed from those students who demonstrated remembrance of both rules on the criterion test. A subject was placed in the criterion pool if he or
she attempted to apply the correct rule to any one of the five exercises pertaining to each of the two problems on the test. Computational mistakes were ignored because the primary interest was whether or not the subject learned the rule sufficiently well to remember it for application. In the pilot studies, 85-100% of the subjects in each class demonstrated remembrance of each problem, so it was anticipated that a sizeable majority of the subjects from each class would be able to remember the rules for two problems.

The test given 8 weeks after instruction was referred to as the retention test (see Appendix C). The retention test had the same format as the criterion test. The numbers in the exercises were changed to insure that correct responses were not due to prior experience with specific problems. A split-half correlation was calculated by totaling student scores on the odd and even numbered problems. For problems 1 and 2 it was .95, and for problems 3 and 4 it was .98.

The retention test was administered by the regular classroom teacher. The investigator was not present in the classroom when the test was given but remained in another room in the school. This allowed the investigator to conduct interviews with students who had been selected randomly from the criterion pool of the class. It also was done so that the investigator would not become a cue for some students to remember the rule and possibly tell others before the test was given.

The Interview

An interview was conducted with five of the students selected randomly from the criterion pool formed in each class. It is possible
that the investigator, who was also the instructor, was a cue factor in helping students who were being interviewed to remember the rule. However, this cue factor was the same for all students who were interviewed from each of the treatment/observation cycles. Therefore, it should not have favored any particular cycle.

The interview had a protocol outline that was followed for each student regardless of treatment. The student was asked to solve exercises that called for a direct application of the solution strategy of one of the two problems. The subject was asked to verbalize his or her process of remembrance as well as to use paper and pencil to show all work. The major objectives of the interview were:

1. To determine whether a subject initially used recall or reconstruction in remembering the rule. If the subject remembered the rule spontaneously, remembrance was classified as recall. If a part or all of the instructional process was reproduced either verbally or on paper in order to come up with the rule, the memory process was classified as reconstruction. The part of the instructional process that was reproduced could have been a table, figure, specific examples, or line of reasoning.

2. To determine whether a subject could subsequently reconstruct the rule even though the subject initially recalled the rule.

3. To identify the kinds of cues subjects employed to help them remember the rule.

Only five students were selected to be interviewed from each class since each interview averaged about 8 minutes and five interviews were all that could be completed in a 40-minute class period typical of each cooperating school.
When a student came into the interview room, he or she was given paper and half of the retention test containing five exercises associated with one of the four problems. The student was told to begin working any of the five exercises after reading the directions. The student was asked to verbalize what he or she was doing as much as possible and to do all work on the paper provided.

As the student began to work the selected exercise, the first determination that was made by the researcher was whether the student recalled the rule or whether a reconstruction process was being attempted. If the determination was recall, the student was asked what helped him or her to remember the rule. If the recall of the rule was correct, the student was also asked if he or she could reconstruct the rule as had been done during the instructional period. If the determination was made that a reconstruction process was being attempted, the researcher tried to determine what part of the original instructional presentation was remembered. If reconstruction was successful or partially successful, the student was asked what helped him or her to remember the process of reconstructing or finding the rule. If this was indicated by the student first, then the question was not necessary. If the student could not recall the rule or initiate an attempt at reconstruction, an attempt was made by the researcher to provide a cue by asking the student if he or she remembered a line of reasoning, chart, diagram, sketch or table used in the instructional period. If the student could not proceed with the problem, then he or she was asked if any part of the problem solving process was remembered. The student was then given the other half
of the retention test containing five exercises of a second of the four problems. The interview procedure and protocol was repeated for the second problem.

The investigator made the final determination in labeling a successful remembrance of the rule as recall or reconstruction. During the two pilot studies, the investigator labeled students as having recalled or reconstructed the rule and then consulted with others who listened to the tapes and read the comments of the students. In every case, the labeling by others agreed with that of the investigator, and thus, no outside help was deemed necessary in labeling students' remembrances during the study. The determination of whether a remembrance was a recall or reconstruction was straightforward in all but three cases. But two of the three more difficult decisions involved the labeling of remembrance as reconstructions from Cycle X in which the students had not received any reconstruction instruction. If the remembrances had been labeled recalls instead, it would have made the differences even larger in favor of Cycles Y and Z in which students had received instruction on reconstruction.

In order to give a flavor of the interviews, excerpts from three of the 70 interviews follow. The first case selected was a successful reconstruction, the second was a successful case of recall, and the third case was one which was difficult to determine whether it was recall or reconstruction. In each case, the students had been given the problem and had worked on it a few moments before the conversation began. The letter S is used to label the subject's responses and the letter I is used for the interviewer's questions and statements.
In the first case, the subject was working on problem 3, squaring the binomial. She had chosen to do \((g + h)^2\) and began by drawing a square whose side lengths were \(g + h\). She had also sketched in the perpendicular line segments joining the common endpoints of segments \(g\) and \(h\) on each side but had not labeled anything. It was apparent that she was attempting a reconstruction and that the square was the memory cue. Therefore, questions designed to reveal whether it was recall or reconstruction did not need to be asked. The researcher concentrated the questions to see if she could successfully reconstruct. The conversation follows:

I - "Do you remember the lengths of this piece here and this piece here (pointing to segments \(g\) and \(h\))?"

S - "Okay, it's a..."

I - "Okay, pick this one."

S - "Okay, this is a...g, g times...g times...yeah, g times h. I have this backwards."

I - "Uh-huh."

S - (Subject erases first attempt.) "There, okay..." (Subject then fills in \(g\) and \(h\) and the areas of the four regions \(g \times g\), \(g \times h\), \(g \times h\) and \(h \times h\).)

I - "Okay, very good. If you were to find the area of this square...(points to large square)."

S - "Multiply, the whole area?"

I - "Since you've got the pieces..."

S - "Okay, equals, you multiply these \(g \times h\) and, uh, equals..."

I - "Okay, so you've got all the pieces there. Now if...I think you still haven't understood me now. What you have done is find the
area of these pieces here. Now, if you want to get the area of the big square, what would you do?"

S - "Add them together...okay."

I - "Okay, that's all you need to do."

In the second case, the subject was working on problem 1, the handshake problem. She had chosen the problem with 16 in a class and had quickly calculated 16 times 15 divided by 2 which was correct. The major task became the determination of whether she had worked the problem from recall or had mentally reconstructed the rule. The conversation follows:

I - "Why did you use the 15 there?"

S - "Well, I was thinking of how to do it, and I remembered that we would multiply it by one less than the number that was given and divide by 2."

I - "Do you remember why we subtracted by 1?"

S - "Because, you don't shake hands with yourself."

I - "That's right. Now, did you think about not shaking hands with yourself before you got the 15, or did you just automatically put it there?"

S - "I just put it down."

I - "You didn't think about why?"

S - "No."

I - "How about dividing by 2?"

S - "Well, it seemed like...I remembered that we had to divide by 2, but when I think about it, it's just like, you know, like when you're shaking hands with everyone...It would narrow down so much that it would split it in half."
I - "So you divided by 2 because of that reasoning?"

S - "No."

I - "You didn't think of it?" (Subject shakes head no.)

S - "Well, I knew you had to subtract 1 and multiply, then divided by 2. I just remembered it."

In the third case, the subject was also working problem 1, the handshake problem. She had also selected the class size of 16 and after a few moments had worked it correctly by multiplying 16 times 15 and dividing the product by 2. Once again the task was to determine whether she had worked the problem from recall or had mentally reconstructed the rule. The conversation follows:

I - "What I am going to do is ask you questions to try to find your reasoning or way you remembered the problem."

S - "I don't...I didn't exactly remember this right off. So I just remember I had to subtract one person because you can't shake hands with yourself. And I multiplied those and then I divided by 2. (laughs) I wasn't too sure."

I - "The reason you divided by 2 was?"

S - "You got two hands, I guess. I remember something along that line, but I don't know."

I - "Let me ask you this question. Did you think about the reason you gave me that you can't shake hands with yourself as you wrote the 15 down or before you wrote the 15 down? Or, did you already do that and think about the reason later?"

S - "Well, I had remembered that (the reason) right off so... It took longer I guess because I was thinking about it as I was working it."
I - "Okay, so you thought of the reason while you were doing it?"
S - "Uh-huh, yeah."
I - "Okay. The dividing by 2, you are not real sure on the reason for that?"
S - "No."
I - "But you just remembered doing it?"
S - "Yeah."

In this third case, it was decided to label her as one who used reconstruction even though she obviously only recalled dividing by 2. The reason for this was the fact that she had taken some time before writing $16 \times 15$ and she had stated that she thought about the reason for the $N - 1$ factor before multiplying. Thus, she apparently was using reconstruction where she needed it with the line of reasoning as the cue.

**Summary of Procedures**

The four classes who began with the treatment/observation Cycle X were randomly assigned to receive instruction on two of the four problems according to the schedule in Figure 4. The problems were taught on consecutive days. In each class the first problem listed was taught in an inductive style and the second problem was taught in a deductive style. Two days following the instruction on these two problems, a 10-item criterion test was administered. Following the test, the remainder of the period was used to go over the test items and review the process by which the rules were derived with special attention given to how the process relates to the exercises.
For each class, those subjects who demonstrated remembrance of both rules were placed in a criterion pool. Eight weeks following the initial instruction, five subjects were randomly selected for an interview from the criterion pool formed in each class. The remainder of each class took a 10-item retention test parallel to the initial criterion test.

The same four classes who had finished the treatment/observation Cycle X began a second treatment/observation Cycle Y following a 6-week interlude. The day prior to instruction on problems, a 40-minute session was held with each class to give instruction on reconstruction strategies. Then instruction on the remaining two problems was given on the following 2 days (see Figure 5). Again, the first problem listed was taught in an inductive style and the second in a deductive style. Similar to the other treatment/observation Cycle X, a 10-item criterion test was administered 2 days later. Following the test, the remainder of the period was used to go over the test items and review the process by which the rules were derived with special attention given to how one might remember these rules for reconstruction.

As in treatment/observation Cycle X, a criterion pool was formed in each class of those students who demonstrated remembrance of both rules on the criterion test. Eight weeks following the initial instruction, five students were randomly selected from this criterion pool in each class for an interview. The remainder of each class took a 10-item retention test parallel to the initial criterion test. The five questions that called for a direct application on each problem
were identical regardless in which treatment/observation cycle the problem was taught.

Following the instruction of the classes who were participating in treatment/observation Cycle Y, instruction began for the four classes who received treatment/observation Cycle Z only (see Figure 3). The procedure followed for these classes was the same as for those classes which participated in treatment/observation Cycle Y.

The schedule for the one class participating in Cycle W twice was the same time frame as the initial four classes involved with Cycles X and Y. The first two problems were taught in a rule/example mode to this class when the four classes of Cycle X were receiving instruction and the interview/retention tests were conducted the same day as those of two of the classes of Cycle X. The last two problems were taught in a rule/example mode to this class in Cycle W when the four classes of Cycle Y were receiving instruction and the interview/retention tests were conducted the same day as those two of the classes of Cycle Y. The procedures for administering the criterion test, the forming of the criterion pool, the selecting of students for interviews, the conducting of the interviews, and the administering of the retention test were the same as the other three cycles.
CHAPTER 4
ANALYSIS AND RESULTS

Planning the Analysis

Two crucial questions had to be addressed to guide the analyses of data before detailed interpretations could be made. First, did a sufficient number of students in each class unit achieve criterion immediately after treatment to allow meaningful data analysis and interpretation? The generalizability and practical implications of the study would be severely limited if not. Second, the previous research on memory suggested a number of variables of potential importance. The design of the study controlled or collected evidence concerning those variables that appeared to have greatest relevance. However, the data collected were typically in a form most suitable for nonparametric or chi-square analyses. It is difficult to recognize the effects of specific variables and their interpretations if the data are in nonparametric form. Consequently, a log-linear analysis was conducted in order to test the strength of the relationship of the variables and to find the model most parsimonious in identifying effects of variables. The a priori significance level for this study was set at $p < .05$, and hence the log-linear analysis would help eliminate any nonsignificant relationships. Thus, the log-linear analysis provided guidance in the selection of subsequent
analysis in order to acquire a more detailed interpretation of the variables in question.

The remainder of the chapter is organized in four sections. The first reports the results of the critical two questions that served to guide the remainder of the analyses, the portion of students achieving criterion in each class, and the selection of the most parsimonious model identifying the salient variables and their interaction. The second section examines the evidence for remembrance (interview and class test data) and reconstruction (interview data) in terms of the style of teaching without the intervention of a treatment that promotes reconstruction. The third section examines the data for remembrance and reconstruction under the impact of an intervention treatment designed to promote reconstruction. A fourth section provides two types of supplementary information about memory processes. Correlates of mathematical achievement with achievement of criterion, successful remembrance, and successful reconstruction are reported, and an informal analysis of the use of cues by subjects who were interviewed is given.

Preliminary Analysis

The percent of the number of students in each class who achieved criterion immediately after treatment was computed in order to determine the answer to the first critical question. The results from the criterion test indicated that a majority of each of the classes made criterion (see Table 1). The number of students in the criterion pools formed in each class ranged from a low of 52% to a high of 84%. Thus, a more than adequate number of students was available for the
Table 1
Percent of Class
Making Criterion

<table>
<thead>
<tr>
<th>Class</th>
<th>Number in class</th>
<th>Number achieving criterion</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cycle W</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>20</td>
<td>11</td>
<td>55%</td>
</tr>
<tr>
<td>2</td>
<td>24</td>
<td>15</td>
<td>63</td>
</tr>
<tr>
<td>3</td>
<td>22</td>
<td>12</td>
<td>55</td>
</tr>
<tr>
<td>4</td>
<td>29</td>
<td>15</td>
<td>52</td>
</tr>
<tr>
<td>5</td>
<td>28</td>
<td>17</td>
<td>61</td>
</tr>
<tr>
<td>Cycle X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>24</td>
<td>15</td>
<td>63</td>
</tr>
<tr>
<td>3</td>
<td>22</td>
<td>14</td>
<td>64</td>
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<tr>
<td>4</td>
<td>29</td>
<td>18</td>
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<tr>
<td>5</td>
<td>28</td>
<td>18</td>
<td>64</td>
</tr>
<tr>
<td>Cycle Y</td>
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<td>6</td>
<td>27</td>
<td>20</td>
<td>74</td>
</tr>
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<td>7</td>
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</tr>
<tr>
<td>9</td>
<td>31</td>
<td>26</td>
<td>84</td>
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</tbody>
</table>
administration of the interview or paper and pencil tests. It was concluded that the interview and class test data formed a sufficient base for analysis and interpretation of results.

Log-linear analyses were computed in order to answer the second critical question concerning which specific main effects of the independent variables warranted further analysis. The log-linear analyses were conducted on the frequency of successful reconstruction usage by reconstruction instruction, style of teaching and problem type from the interview data, and on the frequency of successful remembrance by reconstruction instruction, style of teaching and problem type from the class test data (see Appendix B). A separate log-linear analysis was performed on the frequencies under the inductive and deductive columns for each of the interview and class test data. Separate analyses were conducted due to the lack of independence of the scores.

The log-linear analysis was performed using the BIOMED (1977) statistical program BMDP3F. Significant treatment effects were found for the inductive style of teaching from both the interview and class data and for the deductive style of teaching from the class data (see Tables 2, 3 and 4). For the deductive interview data, significance was not achieved for any log-linear model. The same table and the same model provided the best "fit" of the data in each case where significance was found. There were two choices of tables possible depending upon which term to add to line three with the reconstruction instruction-problem type effect (AC): either the problem type-successful reconstruction (CB) or the reconstruction instruction-successful reconstruction (AB). In each
Table 2
Log-Linear Analysis on Frequency of Reconstruction
for Inductive Style of Teaching: Interview Data

<table>
<thead>
<tr>
<th>Model</th>
<th>df</th>
<th>LR  $\chi^2$</th>
<th>p</th>
<th>Component $\chi^2$</th>
<th>df</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>A, C, $\overline{B}$</td>
<td>17</td>
<td>22.53</td>
<td>.165</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AC, $\overline{B}$</td>
<td>11</td>
<td>17.94</td>
<td>.083</td>
<td>4.59</td>
<td>6</td>
<td>.70</td>
</tr>
<tr>
<td>AC, A$\overline{B}$</td>
<td>9</td>
<td>8.95</td>
<td>.442</td>
<td>8.99</td>
<td>2</td>
<td>.02*</td>
</tr>
<tr>
<td>AC, C$\overline{B}$, A$\overline{B}$</td>
<td>6</td>
<td>5.16</td>
<td>.524</td>
<td>3.79</td>
<td>3</td>
<td>.30</td>
</tr>
<tr>
<td>A$\overline{B}C$</td>
<td>0</td>
<td>0.00</td>
<td>1.000</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Note. A = reconstruction instruction, $\overline{B}$ = successful reconstruction (dependent variable), C = problems.

*P < .02.
Table 3
Log-Linear Analysis on Frequency of Reconstruction for Inductive Style of Teaching: Class Test Data

<table>
<thead>
<tr>
<th>Model</th>
<th>df</th>
<th>LR $X^2$</th>
<th>$p &lt;$</th>
<th>Component $X^2$</th>
<th>df</th>
<th>$p &lt;$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A, C, \bar{B}$</td>
<td>17</td>
<td>28.99</td>
<td>.037</td>
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</tr>
<tr>
<td>$AC, \bar{B}$</td>
<td>11</td>
<td>24.39</td>
<td>.011</td>
<td>4.60</td>
<td>6</td>
<td>.70</td>
</tr>
<tr>
<td>$AC, AB$</td>
<td>9</td>
<td>9.92</td>
<td>.357</td>
<td>14.47</td>
<td>2</td>
<td>.001*</td>
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<tr>
<td>$AC, CB, AB$</td>
<td>6</td>
<td>8.11</td>
<td>.230</td>
<td>1.81</td>
<td>3</td>
<td>.70</td>
</tr>
<tr>
<td>$ABC$</td>
<td>0</td>
<td>0.00</td>
<td>1.000</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Note. $A =$ reconstruction instruction, $\bar{B} =$ successful reconstruction (dependent variable), $C =$ problems.

* $p < .001$. 
Table 4
Log-Linear Analysis on Frequency of Reconstruction
for Deductive Style of Teaching: Class Test Data

<table>
<thead>
<tr>
<th>Model</th>
<th>df</th>
<th>LR $\chi^2$</th>
<th>$p &lt;$</th>
<th>Component $\chi^2$</th>
<th>df</th>
<th>$p &lt;$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A, C, B</td>
<td>17</td>
<td>56.03</td>
<td>.0001</td>
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<tr>
<td>AC, B</td>
<td>11</td>
<td>51.44</td>
<td>.0001</td>
<td>4.59</td>
<td>6</td>
<td>.70</td>
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<tr>
<td>AC, AB</td>
<td>9</td>
<td>39.10</td>
<td>.0001</td>
<td>12.34</td>
<td>2</td>
<td>.01*</td>
</tr>
<tr>
<td>AC, CB, A B</td>
<td>6</td>
<td>33.60</td>
<td>.0001</td>
<td>5.50</td>
<td>3</td>
<td>.20</td>
</tr>
<tr>
<td>A B C</td>
<td>0</td>
<td>0.00</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note. A = reconstruction instruction, B = successful reconstruction (dependent variable), C = problems.

*p < .01.
case, the partial and marginal associations for $C \overline{B}$ were not significant at the .05 level, but the partial and marginal associations for $A \overline{B}$ were. In addition, the selection of $A \overline{B}$ for line three resulted in a more parsimonious model. Thus, it was determined to add the $A \overline{B}$ effects to $AC$ in line three (Kennedy, 1980).

In working upward from the saturated model in each table, the first significant goodness-of-fit $x^2$ was found in line two. Thus, this model ($AC, \overline{B}$) and all models above line two were eliminated from consideration in choosing the best "fit" (Kennedy, 1980). Again working upwards from the component $x^2$'s, the first significant component $x^2$ was that of line three ($AC, A \overline{B}$). This indicated that all models below line three were eliminated from consideration in choosing the best "fit" (Kennedy, 1980). This left the model $AC, A \overline{B}$ in line three as the model which provided the best "fit" of the data in each case. Thus, the addition of the $A \overline{B}$ effects to $AC$ significantly reduced the amount of $x^2$ which indicated that the $A \overline{B}$ effects were significant in each table. Since $A$ was the independent variable, reconstruction instruction, and $\overline{B}$ was the dependent variable, successful reconstruction usage, the results indicated the existence of significant main effects of reconstruction instruction. The effects of which problems ($C$) the students encountered were not significant in any of the analyses. Therefore, the results indicated that reconstruction instruction effects and style of teaching effects were the important variables for further statistical comparisons.
Effects of Teaching Style Variables

The effects of inductive, deductive and rule/example styles of teaching on the remembrance of mathematical rules and the frequency of reconstruction were examined in two contexts: with and without reconstruction. Comparisons of the data from Cycles W and X gave an indication of the effects of the teaching style variables without any interference from reconstruction instruction. Since Cycle Z received reconstruction instruction, comparisons of the data from Cycles W and Z gave corroborating evidence of the effects of teaching style variables to the results of the comparisons of Cycles W and X. Data from both the interviews and class tests were examined for each context. Two hypotheses were tested concerning the effects of teaching style variables: Hypothesis 1 predicted the effects on remembrance, and Hypothesis 2 predicted the effects on reconstruction.

Remembrance. Hypothesis 1 was tested to determine the effects of the teaching style variables on remembrance of mathematical rules. The first hypothesis, stated in the null form, was:

$H_0^1$: There is no significant difference in the remembrance of mathematical rules between students receiving an inductive or deductive style of teaching and those receiving a rule/example style of teaching.

To test Hypothesis $H_0^1$, the mean score of Cycle W was compared to the mean scores of Cycles X and Z by using a t-test. A score of 1 was given to those students who successfully remembered, either by recall or reconstruction, the rule for application, and a score of 0 was given to those who did not successfully remember the rule for application. From the interview data, the means of Cycles W, X
and Z were .15, .55 and .80 respectively. The mean of Cycle X was significantly higher than that of Cycle W, $t_{(58)} = 3.15, p < .005$. Also, the mean of Cycle Z was significantly higher than that of Cycle W, $t_{(58)} = 6.05, p < .0001$. For the classroom groups, those students taking the written test, the means of Cycles W, X and Z were .00, .34 and .54 respectively. Again, the mean of Cycle X was significantly higher than that of Cycle W, $t_{(84)} = 3.32, p < .001$, and the mean of Cycle Z was significantly higher than that of Cycle W, $t_{(140)} = 5.05, p < .0001$. With this strong evidence, the null Hypothesis $H_0$ was rejected. It was concluded that students remembered mathematical rules for application significantly better after 8 weeks if the rules were taught in an inductive or deductive style rather than in a rule/example mode.

As a sidelight to Hypothesis 1, comparisons were made between the inductive and deductive styles of teaching with regards to remembrance of mathematical rules. From the interview data, those students who had received the inductive style of teaching successfully remembered the rules for application 58% of the time compared to 65% for those receiving the deductive style of teaching. From the class data, a similar difference was observed with those receiving the inductive style of teaching successfully applying the rule 33% of the time compared to 43% for those receiving the deductive style of teaching. From this data, there was an indication that the deductive mode was somewhat more effective than the inductive mode. However, significant conclusions could not be drawn from these percentages due to the design of this experiment. An attempt was made in the design to spread out the effects of the inductive and
deductive style of teaching and not to look at which of the two styles would help students better remember mathematical rules. However, further study regarding the effects of inductive versus deductive styles of teaching is warranted.

Reconstruction. The second hypothesis predicted the relationship between instances of successful reconstruction with teaching styles. The data used for the analyses were collected from interviews of samples of criterion-achieving students. The null for Hypothesis 2 is stated in two parts:

$\bar{\theta}_2(a):$ There is no significant difference in the frequency of reconstruction strategies employed by students between those who were instructed in an inductive style and those instructed in a deductive style.

$\bar{\theta}_2(b):$ There is no significant difference in the frequency of reconstruction strategies employed by students between those who were instructed in an inductive or deductive style and those instructed in a rule/example style.

To test the frequency of reconstruction usage between the inductive and deductive styles of teaching, a correlated t-test was used. Each student had received instruction on two problems, one inductively and one deductively. Thus, the data were not independent. The product-moment, correlation coefficient was found to be .10 between the scores from the students on the problem in which they had received the instruction inductively as compared to their scores on the problem in which they had received the instruction deductively.

Of those students who had not recalled the rule for application, 47% of the students who had received the inductive style of teaching
successfully reconstructed the rule for application as compared to 52% of those who had been taught in a deductive style. This difference was not found to be significant at the .05 level, \( t (21) = .50 \). Therefore, the first part of the null of Hypothesis 2, \( \phi_2(a) \), was not rejected. Whether a mathematical rule was taught in an inductive or deductive style apparently made no significant difference in the frequency of reconstruction strategies employed by students. Further study is needed to see if there are other factors which may explain more carefully the effects of inductive and deductive teaching on students' usage of reconstruction strategies.

In order to test the second part of the null of Hypothesis 2, students from Cycles W, X and Z who had been interviewed and had successfully recalled the rule for application were ignored. For the remainder of the interview data from these cycles, students who had successfully reconstructed the rule for application were given a score of 1, and a score of 0 was given to those who did not successfully remember the rule for application. Cycle means were calculated for W, X and Z which were .00, .28 and .71 respectively. The mean of Cycle X was significantly higher than the mean of Cycle W, \( t (40) = 2.51, p < .01 \), and the mean of Cycle Z was significantly higher than the mean of Cycle W, \( t (43) = 6.33, p < .0001 \). Therefore, the second part of the null of Hypothesis 2, \( \phi_2(b) \), was rejected. Students who had received instruction in an inductive or deductive style did employ reconstruction strategies significantly more often than those who had been instructed in a rule/example mode.

Summary. Evidence of successful remembrance was found in Cycles W and X which had received no instruction on reconstruction
strategies. Instances of successful reconstruction were found in Cycle X which had received reconstruction instruction but had received instruction on one problem in an inductive style and another in a deductive style. No instances of reconstruction attempts were found in Cycle W. Two hypotheses were tested to determine the effects of style of teaching variables.

Hypothesis 1 was concerned with the effects of style of teaching on remembrance of mathematical rules. From both the interview and class test data, those classes receiving an inductive or deductive style of teaching remembered mathematical rules for application significantly better than those classes which had been taught in a rule/example style. Thus, the null of Hypothesis 1 was rejected.

Hypothesis 2 was concerned with the effects of style of teaching on the frequency of reconstruction strategies employed. These comparisons were made because problem effects were found to be nonsignificant. It was found from the interviews that those students who had received instruction in an inductive or deductive style employed reconstruction strategies significantly more often than those who had been instructed in a rule/example mode. There were no significant differences in the frequency of reconstruction use between students who had received instruction inductively and those who had received instruction deductively. Thus, the null of Hypothesis 2(b) was rejected but the null of Hypothesis 2(a) was not rejected.

**Effects of Reconstruction Instruction**

The effects of reconstruction instruction on frequency of reconstruction and on remembrance of mathematical rules were examined.
For Cycles Y and Z, reconstruction as a retrieval strategy was overtly taught to the students while in Cycle X no reconstruction strategies were taught. Comparisons between Cycles X and Z examined whether reconstruction instruction made a difference without interference from other factors. The comparisons between Cycles X and Y measured growth since the same classes were in each group, and the comparisons of Cycles Y and Z looked at differences due to problem solving experience. The differences between Cycles X and Y and Cycles Y and Z were used as supportive evidence for the differences found between Cycles X and Z. Two hypotheses were tested to determine the effects of reconstruction instruction: Hypothesis 3 predicted the effects on reconstruction, and Hypothesis 4 predicted the effects on remembrance.

Reconstruction hypothesis. Hypothesis 3 examined the impact that reconstruction instruction had on the use of reconstruction by students to remember mathematical rules for application. The null form of Hypothesis 3 was:

\[ \theta_3 : \text{The teaching of reconstruction strategies will not significantly increase the students' use of reconstruction to access long-term memory of mathematical rules.} \]

The interview data were used to test this hypothesis. Those students who had successfully recalled the rule for application were ignored since only reconstruction was of interest in this case. Students who successfully reconstructed the rule for application were given a score of 1 for that problem, and those who missed the application of the rule were given a score of 0 for that problem. The class means from Cycles X and Y and the class means from X and Z
were compared to determine the effects of reconstruction instruction (see Tables 5 and 6). Since each student received instruction on one problem in an inductive manner and the second problem in a deductive manner or vice versa in each treatment/observation cycle, it was possible that the order of style of teaching received may have effected reconstruction usage. A one-factor, repeated-measures analysis of variance was performed on X and Y and a two-factor, fixed-effects analysis of variance was conducted on X and Z with the class as the unit of analysis in each case (see Tables 7 and 8).

Treatment/observation Cycles X and Y involved the same classes, so the comparison between X and Y was a measure of growth as well as the effects of reconstruction instruction. No significant differences between Cycles X and Y were found at the .05 level (see Table 7). However, there was a significant main effect of reconstruction instruction between Cycles X and Z, $F(1) = 9.56, p < .0365$ (see Table 8).

One question that arose since the same classes were involved in treatment/observation Cycles X and Y was whether the additional two problems the classes had done had increased the usage of reconstruction strategies. In order to test problem experience effects, class means of Cycles Y and Z were compared (see Table 9). Order of style of teaching was again of concern, so a two-factor, fixed-effects analysis of variance was performed with the class as the unit of analysis (see Table 10). Significant, first-order interaction was found. The greater cell mean involving the deductive-inductive order of teaching style with Cycle Z which had no prior problem solving experience was the reason for this interaction. Since the
Table 5
Class Means for Successful Reconstruction by Order of Teaching Style: X versus Y

<table>
<thead>
<tr>
<th>Order of teaching style</th>
<th>Cycle X</th>
<th>Cycle Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class</td>
<td></td>
<td></td>
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<tr>
<td>c₂</td>
<td>.57</td>
<td>.50</td>
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<td>Inductive-deductive</td>
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<td></td>
</tr>
<tr>
<td>c₄</td>
<td>.00</td>
<td>.56</td>
</tr>
<tr>
<td>c₃</td>
<td>.14</td>
<td>.50</td>
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<tr>
<td>Deductive-inductive</td>
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<td></td>
</tr>
<tr>
<td>c₅</td>
<td>.25</td>
<td>.40</td>
</tr>
</tbody>
</table>

Table 6
Class Means for Successful Reconstruction by Reconstruction Instruction and Order of Teaching Style: X versus Z

<table>
<thead>
<tr>
<th>Order of Teaching Style</th>
<th>Inductive-deductive</th>
<th>Deductive-inductive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cycle X</td>
<td>.57</td>
<td>.14</td>
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<tr>
<td></td>
<td>.00</td>
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<td>Cycle Z</td>
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<td></td>
<td>.50</td>
<td>.83</td>
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Table 7
Analysis of Variance for Successful Reconstruction by Order of Teaching Style and Reconstruction Instruction: X versus Y

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F-ratio</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between groups</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Order of teaching style (A)</td>
<td>1</td>
<td>.014</td>
<td>.014</td>
<td>0.44</td>
<td>.5737</td>
</tr>
<tr>
<td>Class (C)/A</td>
<td>2</td>
<td>.065</td>
<td>.033</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Within groups</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reconstruction instruction (B)</td>
<td>1</td>
<td>.125</td>
<td>.125</td>
<td>2.27</td>
<td>.2711</td>
</tr>
<tr>
<td>A X B</td>
<td>1</td>
<td>.0005</td>
<td>.0005</td>
<td>0.00</td>
<td>.9787</td>
</tr>
<tr>
<td>B X C/A</td>
<td>2</td>
<td>.110</td>
<td>.055</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 8
Analysis of Variance for Successful Reconstruction by Reconstruction Instruction and Order of Teaching Style: X versus Z

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F-ratio</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reconstruction instruction (A)</td>
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<td>.437</td>
<td>.437</td>
<td>9.56</td>
<td>.0365*</td>
</tr>
<tr>
<td>Order of teaching style (B)</td>
<td>1</td>
<td>.053</td>
<td>.053</td>
<td>1.15</td>
<td>.3431</td>
</tr>
<tr>
<td>A X B</td>
<td>1</td>
<td>.128</td>
<td>.128</td>
<td>2.79</td>
<td>.1703</td>
</tr>
<tr>
<td>Class/A X B</td>
<td>4</td>
<td>.183</td>
<td>.046</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*p < .0365.
Table 9
Class Means for Successful Reconstruction by Problem Solving Experience: Y versus Z

<table>
<thead>
<tr>
<th>Order of Teaching Style</th>
<th>Inductive-deductive</th>
<th>Deductive-inductive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cycle Y</td>
<td>.50</td>
<td>.50</td>
</tr>
<tr>
<td></td>
<td>.56</td>
<td>.40</td>
</tr>
<tr>
<td>Cycle Z</td>
<td>.50</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>.50</td>
<td>.83</td>
</tr>
</tbody>
</table>

Table 10
Analysis of Variance for Successful Reconstruction by Problem Solving Experience: Y versus Z

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F-ratio</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem solving experience (A)</td>
<td>1</td>
<td>.095</td>
<td>.095</td>
<td>17.81</td>
<td>.0135**</td>
</tr>
<tr>
<td>Order of teaching style (B)</td>
<td>1</td>
<td>.056</td>
<td>.056</td>
<td>10.56</td>
<td>.0313*</td>
</tr>
<tr>
<td>A X B</td>
<td>1</td>
<td>.123</td>
<td>.123</td>
<td>23.06</td>
<td>.0086***</td>
</tr>
<tr>
<td>Class/A X B</td>
<td>4</td>
<td>.021</td>
<td>.005</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*p < .0313.

**p < .0135.

***p < .0086.
significance was in favor of Cycle Z which had no prior problem solving experience instead of Cycle Y which had had two previous problems, it was concluded that prior problem solving experience did not significantly affect the use of reconstruction strategies by students.

Therefore, based on the significance between Cycles X and Z, the null Hypothesis $\theta_3$ was rejected. The teaching of reconstruction strategies apparently did significantly increase the students' use of reconstruction to access long-term memory of mathematical rules.

**Remembrance.** The second hypothesis that involved reconstruction instruction effects was Hypothesis 4. This hypothesis predicted the relationship between reconstruction instruction and remembrance of mathematical rules. The null of Hypothesis 4 was:

$\theta_4$: The teaching of reconstruction strategies will not significantly improve the retention of mathematical rules.

Both the interview and class data were used to test this hypothesis as all students who had successfully remembered the rule for application were of interest. For each student, a score of 1 was given for the problem if the rule was successfully recalled or reconstructed for application while a score of 0 was given for an unsuccessful remembrance of the rule for application. The class means from Cycles X and Y, and the class means from Cycles X and Z were compared for both the interview and class data to determine the effects of reconstruction instruction on remembrance (see Tables 11, 12, 13 and 14). Again, since order of teaching style was a concern, a one-factor, repeated-measures analysis of variance was performed on Cycles X and Y, and a two-factor, fixed-effects analysis of variance was conducted
Table 11
Class Means from Interviews for Successful Remembrance
by Order of Teaching Style: X versus Y

<table>
<thead>
<tr>
<th>Order of teaching style</th>
<th>Cycle X</th>
<th>Cycle Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inductive-deductive</td>
<td>.70</td>
<td>.60</td>
</tr>
<tr>
<td></td>
<td>.60</td>
<td>.60</td>
</tr>
<tr>
<td>Deductive-inductive</td>
<td>.40</td>
<td>.50</td>
</tr>
<tr>
<td></td>
<td>.40</td>
<td>.40</td>
</tr>
</tbody>
</table>

Table 12
Class Means from Class Test for Successful Remembrance
by Order of Teaching Style: X versus Y

<table>
<thead>
<tr>
<th>Order of teaching style</th>
<th>Cycle X</th>
<th>Cycle Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inductive-deductive</td>
<td>.33</td>
<td>.30</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>.55</td>
</tr>
<tr>
<td>Deductive-inductive</td>
<td>.75</td>
<td>.11</td>
</tr>
<tr>
<td></td>
<td>.72</td>
<td>.15</td>
</tr>
</tbody>
</table>
Table 13
Class Means from Interviews for Successful Remembrance by Reconstruction Instruction and Order of Teaching Style: X versus Z

<table>
<thead>
<tr>
<th>Order of Teaching Style</th>
<th>Inductive-deductive</th>
<th>Deductive-inductive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cycle X</td>
<td>.70</td>
<td>.40</td>
</tr>
<tr>
<td></td>
<td>.60</td>
<td>.40</td>
</tr>
<tr>
<td>Cycle Z</td>
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<td>1.00</td>
</tr>
<tr>
<td></td>
<td>.70</td>
<td>.90</td>
</tr>
</tbody>
</table>

Table 14
Class Means from Class Tests for Successful Remembrance by Reconstruction Instruction and Order of Teaching Style: X versus Z

<table>
<thead>
<tr>
<th>Order of Teaching Style</th>
<th>Inductive-deductive</th>
<th>Deductive-inductive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cycle X</td>
<td>.33</td>
<td>.75</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>.72</td>
</tr>
<tr>
<td>Cycle Z</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>.87</td>
<td>1.35</td>
</tr>
</tbody>
</table>
on Cycles X and Z with the class as the unit of analysis in each case (see Tables 15, 16, 17 and 18).

Significant main effects of order of teaching style were found between Cycles X and Y from the interview data, $F(1) = 32.00$, $p < .0297$ (see Table 15). The inductive-deductive order of teaching apparently helped students remember better than the deductive-inductive order of teaching. No significant differences were found between Cycles X and Y from the class data (see Table 16). The first-order interaction of reconstruction instruction with order of teaching style was significant between Cycles X and Z from the interview data. The deductive-inductive order apparently was less effective in helping students in Cycle X to remember the rule for application than the inductive-deductive order but was more effective with Cycle Z than the inductive-deductive order. This first-order interaction also explained why there were significant main effects of reconstruction instruction since the deductive-inductive order of teaching had helped the group which had received reconstruction instruction to remember better. There was no significant difference found between Cycles X and Z from the class data.

The effects of problem solving experience on the remembrance of mathematical rules was also tested by comparing the class means of Cycles Y and Z (see Tables 19 and 20). From the interview data, significant, first-order interaction was found $F(1) = 27.00$, $p < .0065$ (see Table 21). The deductive-inductive order of teaching seemed to have been more effective with Cycle Z which had no prior problem solving experience than with Cycle Y which had prior problem solving experience. The first-order interaction also explained why
Table 15
Analysis of Variance on Interviews for Remembrance
by Order of Teaching Style: X versus Y

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F-ratio</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between groups</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Order of teaching style (A)</td>
<td>1</td>
<td>.080</td>
<td>.0800</td>
<td>32.00</td>
<td>.0299*</td>
</tr>
<tr>
<td>Class (C)/A</td>
<td>2</td>
<td>.005</td>
<td>.0025</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Within groups</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reconstruction instruction (B)</td>
<td>1</td>
<td>0.000</td>
<td>0.0000</td>
<td>0.00</td>
<td>1.0000</td>
</tr>
<tr>
<td>A X B</td>
<td>1</td>
<td>.005</td>
<td>.0050</td>
<td>2.00</td>
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</tr>
<tr>
<td>B X C/A</td>
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<td>.005</td>
<td>.0025</td>
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<td></td>
</tr>
</tbody>
</table>

*p < .0299.

Table 16
Analysis of Variance on Class Tests for Remembrance
by Order of Teaching Style: X versus Y

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F-ratio</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between groups</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Order of teaching style (A)</td>
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<td>.025</td>
<td>.025</td>
<td>.239</td>
<td>.6732</td>
</tr>
<tr>
<td>Class (C)/A</td>
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<td>.212</td>
<td>.106</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Within groups</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reconstruction instruction (B)</td>
<td>1</td>
<td>.357</td>
<td>.357</td>
<td>15.753</td>
<td>.0580</td>
</tr>
<tr>
<td>A X B</td>
<td>1</td>
<td>.067</td>
<td>.067</td>
<td>2.939</td>
<td>.2286</td>
</tr>
<tr>
<td>B X C/A</td>
<td>2</td>
<td>.045</td>
<td>.023</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 17
Analysis of Variance from Interviews for Remembrance by Reconstruction Instruction and Order of Teaching Style: X versus Z

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F-ratio</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reconstruction instruction (A)</td>
<td>1</td>
<td>.151</td>
<td>.151</td>
<td>40.33</td>
<td>.0032*</td>
</tr>
<tr>
<td>Order of teaching style (B)</td>
<td>1</td>
<td>.001</td>
<td>.001</td>
<td>.33</td>
<td>.5946</td>
</tr>
<tr>
<td>A X B</td>
<td>1</td>
<td>.151</td>
<td>.151</td>
<td>40.33</td>
<td>.0032*</td>
</tr>
<tr>
<td>Class/A X B</td>
<td>4</td>
<td>.015</td>
<td>.004</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*p < .0032.

Table 18
Analysis of Variance from Class Tests for Remembrance by Reconstruction Instruction and Order of Teaching Style: X versus Z

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F-ratio</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reconstruction instruction (A)</td>
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<td>.252</td>
<td>.252</td>
<td>3.422</td>
<td>.1380</td>
</tr>
<tr>
<td>Order of teaching style (B)</td>
<td>1</td>
<td>.048</td>
<td>.048</td>
<td>.652</td>
<td>.4646</td>
</tr>
<tr>
<td>A X B</td>
<td>1</td>
<td>.014</td>
<td>.014</td>
<td>.196</td>
<td>.6807</td>
</tr>
<tr>
<td>Class/A X B</td>
<td>4</td>
<td>.295</td>
<td>.074</td>
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</tr>
</tbody>
</table>
Table 19
Class Means from Interviews for Successful Remembrance by Problem Solving Experience: Y versus Z

<table>
<thead>
<tr>
<th>Order of Teaching Style</th>
<th>Inductive-deductive</th>
<th>Deductive-inductive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cycle Y</td>
<td>.60</td>
<td>.50</td>
</tr>
<tr>
<td></td>
<td>.60</td>
<td>.40</td>
</tr>
<tr>
<td>Cycle Z</td>
<td>.60</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>.70</td>
<td>.90</td>
</tr>
</tbody>
</table>

Table 20
Class Means from Class Tests for Successful Remembrance by Problem Solving Experience: Y versus Z

<table>
<thead>
<tr>
<th>Order of Teaching Style</th>
<th>Inductive-deductive</th>
<th>Deductive-inductive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cycle Y</td>
<td>.30</td>
<td>.11</td>
</tr>
<tr>
<td></td>
<td>.55</td>
<td>.15</td>
</tr>
<tr>
<td>Cycle Z</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>.87</td>
<td>1.35</td>
</tr>
</tbody>
</table>
Table 21
Analysis of Variance on Interviews for Remembrance by Problem Solving Experience: Y versus Z

<table>
<thead>
<tr>
<th>Source</th>
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<th>SS</th>
<th>MS</th>
<th>F-ratio</th>
<th>p &lt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem solving experience (A)</td>
<td>1</td>
<td>.151</td>
<td>.151</td>
<td>40.33</td>
<td>.0032**</td>
</tr>
<tr>
<td>Order of teaching style (B)</td>
<td>1</td>
<td>.011</td>
<td>.011</td>
<td>3.00</td>
<td>.1583</td>
</tr>
<tr>
<td>A X B</td>
<td>1</td>
<td>.101</td>
<td>.101</td>
<td>27.00</td>
<td>.0065*</td>
</tr>
<tr>
<td>Class/A X B</td>
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<td>.015</td>
<td>.004</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*p < .0065.

**p < .0032.

Table 22
Analysis of Variance on Class Tests for Remembrance by Problem Solving Experience: Y versus Z

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F-ratio</th>
<th>p &lt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem solving experience (A)</td>
<td>1</td>
<td>1.209</td>
<td>1.209</td>
<td>47.529</td>
<td>.0023*</td>
</tr>
<tr>
<td>Order of teaching style (B)</td>
<td>1</td>
<td>.002</td>
<td>.002</td>
<td>.060</td>
<td>.8194</td>
</tr>
<tr>
<td>A X B</td>
<td>1</td>
<td>.143</td>
<td>.143</td>
<td>5.626</td>
<td>.0767</td>
</tr>
<tr>
<td>Class/A X B</td>
<td>4</td>
<td>.102</td>
<td>.025</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*p < .0023.
there were significant main effects of problem solving experience
where the cell mean of Cycle Z was found to be significantly higher
than the cell mean of Cycle X. There was no significant difference
found between Cycles Y and Z from the class data (see Table 22).
The data seemed to indicate that remembrance of mathematical rules
was at least as good, if not better, in Cycle Z that had no prior
experience as in Cycle X which had. Therefore, problem solving
experience did not help students remember any better the mathematical
rules for application, so any differences between Cycles X and Y and
between X and Z were apparently due to the session on reconstruction.

There was evidence that at least one of the teachers who had
given the paper and pencil retention test was not as insistent about
the students taking their time and trying to remember the rule for
application as would be preferred. This lack of trying was reflected
in the unusually low class means in the deductive-inductive cell for
Cycle Y and may explain the main effects of order of teaching style
between Cycles X and Y in the interviews. Consequently, since paper
and pencil retention tests formed the basis for the class data, more
weight was put on the interview data. Because of the significant
differences in the interview data between Cycles X and Z, there was
a good indication that reconstruction instruction does improve remem-
brance of mathematical rules for application. But since this only
occurred in the deductive-inductive order of teaching, the evidence
was not strong enough to reject the null of Hypothesis 4, $\theta_4$.
Further study is needed to determine the effects of reconstruction
instruction on remembrance of mathematical rules.
Summary. The instruction of reconstruction strategies did have an impact on the use of reconstruction by students, and there was an indication that it may have had a positive effect on remembrance. Two hypotheses were tested to determine the effects of reconstruction instruction.

Hypothesis 3 was concerned with the effects of reconstruction instruction on the frequency of reconstruction strategies employed by students. It was found from the interviews that students who had received instruction on reconstruction strategies employed reconstruction significantly more often than students who had received no instruction on reconstruction strategies. In addition, problem solving experience was not found to significantly increase the frequency of usage of reconstruction strategies. Therefore, the null of Hypothesis 3 was rejected.

Hypothesis 4 was concerned with the effects of reconstruction instruction on the remembrance of mathematical rules for instruction. It was found from the interviews that there were several significant differences between the groups of classes pertaining to remembrance of mathematical rules. Students from the group of classes that had had problem solving experience followed by a session on reconstruction and instruction on two more problems remembered the mathematical rule for applications significantly better if they had received the inductive-deductive order of teaching rather than the deductive-inductive order. With students who had received the deductive-inductive order of teaching, the group of classes which had received only the session on reconstruction and instruction on two problems remembered the rule for application significantly better than the group of classes which
had received only instruction on two problems. No significant differences were found between any of the groups from the class test data. Also, problem solving experience was not found to significantly increase remembrance of mathematical rules for application. The null of Hypothesis 4 was not rejected.

Other Supportive Evidence About Memory Processes

Students' background information on mathematical achievement was examined to see if higher achievement might explain some of the differences in remembrance and reconstruction usage. Correlates of achievement with criterion, remembrance, and reconstruction were computed to test the differences due to mathematical achievement. In addition, an informal analysis on the type of cues students employed to reconstruct mathematical rules was conducted on the data from the interviews. This was done in an attempt to gain insight on the nature of the reconstruction process as it pertains to mathematical rules learned in a problem solving setting. The percent of students who could reconstruct the rule even though they had successfully recalled it for application were reported. This was to give an indication of what results might have been found had the interval between the initial instruction on the problems and the retention test been extended.

Correlates with achievement. Hypothesis 5 predicted the relationship between mathematical achievement and the use of reconstruction strategies. The null form of Hypothesis 5 was:

$\mathcal{H}_5^0$: There is no significant difference in the frequency of usage of reconstruction strategies between high and low achievers
in mathematics regardless of whether they have been taught reconstruction strategies.

Scores from the California Test of Basic Skills (CTBS) were obtained for each of the students participating in this study. These scores were a measure of mathematical achievement. Each student who had been interviewed was assigned a score of 1 if he or she had successfully reconstructed the rule for application on a given problem and a 0 if he or she had missed the problem. Students who had recalled the rule for application were ignored. In order to test Hypothesis 5, a Pearson correlation coefficient was computed between the CTBS scores and successful reconstruction using the Statistical Package for the Social Sciences (Nie, 1975) subprogram PEARSON CORR. The correlation between mathematical achievement and successful reconstruction was positive and significant, \( r = .15, p < .001 \). Therefore, the null of Hypothesis 5, \( H_5 \), was rejected. Those students who were high achievers as measured by CTBS did tend to use reconstruction strategies more frequently than did low achievers.

Two other correlations were run involving mathematical achievement. One correlation was performed to find the strength of the relationship between mathematical achievement and achieving criterion. In order to test this relationship, each student who participated in the study was assigned a score of 1 if he or she achieved criterion and a score of 0 if he or she did not achieve criterion. A Pearson correlation coefficient was computed between the CTBS scores and achievement of criterion. The correlation between mathematical achievement and achievement of criterion was positive and significant, \( r = .14, \)
High achievers, as measured by CTBS, did tend to achieve criterion more often than low achievers.

The other correlation was done to find the strength of the relationship between mathematical achievement and successful remembrance of a mathematical rule for application. Each student who had achieved criterion was assigned a score on the basis of whether he or she was able to remember a given rule pertaining to one of the four problems for application. A score of 1 was given for a successful remembrance of the rule for application whether by recall or reconstruction. A score of 0 was given for a miss. Again, a Pearson correlation coefficient was computed between the CTBS scores and successful remembrance. The correlation between mathematical achievement and remembrance of a mathematical rule was positive and significant, \( r = .22, p < .0005 \). High achievers, as measured by CTBS, did tend to remember the mathematical rule for application more often than low achievers.

Informal Observations. One of the tasks of this study was to identify factors that help students reconstruct a mathematical rule. One of the important factors observed during the interviews was the type of cue employed by the student to initiate an attempt at reconstruction. Four basic types of cues were observed in this study. They were drawing a figure, making a table, trying a specific example, and using a line of reasoning.

The students found varying degrees of success with each type of cue (see Table 23). The most often used cue that students employed to attempt a reconstruction was drawing a figure. This was followed in order by making a table, using a line of reasoning, and then
Table 23
Raw Data from Interviews on
Types of Cues Employed

<table>
<thead>
<tr>
<th>Cue</th>
<th>Number attempting reconstruction</th>
<th>Successful reconstruction</th>
<th>Percent success</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line of reasoning</td>
<td>8</td>
<td>6</td>
<td>75%</td>
</tr>
<tr>
<td>Drawing a figure</td>
<td>32</td>
<td>20</td>
<td>63%</td>
</tr>
<tr>
<td>Making a table</td>
<td>22</td>
<td>17</td>
<td>77%</td>
</tr>
<tr>
<td>Using specific examples</td>
<td>5</td>
<td>3</td>
<td>60%</td>
</tr>
</tbody>
</table>

Trying a specific example. But in terms of successful reconstruction, the cues in order of most successful to least successful were making a table, line of reasoning, drawing a figure, and using specific examples. Also, it should be noted that of the students who missed the application of a problem, 46% of them employed one of the cues to attempt a reconstruction. It was difficult to draw any solid conclusions from this information on cues, but it appeared that the type of cue used by a student may have a significant effect on the success of a reconstruction attempt. A note of caution should be issued regarding Table 23 and the temptation to draw unwarranted conclusions from it. No conclusions can be made as to whether making a table heuristic is better than drawing a figure heuristic, or vice versa, due to the different teaching styles used in each problem. Making a table and trying specific examples was a part of an inductive
style of teaching while drawing a figure and line of reasoning was a part of a deductive style of teaching.

In only one case did a student use a reconstruction strategy that was different than the instructional development in class. This strongly suggests that students will employ cues that have been provided for them in the course of their instruction.

It was interesting to note that 100% of the students who had received the deductive style of teaching on problems 2 and 3 used the drawing of a figure to help them reconstruct the rule. Problems 2 and 3 involved the relationship between the sides of a right triangle and the squaring of a binomial respectively. These two problems were more difficult for seventh graders than the other two problems. Thus, it may be that the more difficult the problem, the more important the type of cue that a student employs.

Students who demonstrated recall in the interview setting were pushed to see whether they could also reconstruct. The percentage of students who recalled the rule for application but could also reconstruct the rule is noted below. Of those students who had received an inductive style of instruction and had recalled the rule for application, 15% could reconstruct the rule. For the deductive style, the percentage of students who could reconstruct the rule following successful recall was 38%. This information may indicate that had the interval between observations been extended beyond 8 weeks, a significant number of the students receiving instruction in a deductive style would have been able to successfully apply the rule. This is in contrast to the small number that probably would have been able to reconstruct if they had received instruction inductively.
This suggests that those students receiving instruction in a deductive mode may have remembered the mathematical rule significantly better than those receiving instruction in an inductive mode. Further study of these factors is needed.

A third observation that can be made from the interviews was that students were not self-reliant in asking themselves the right questions which would have helped them reconstruct the rule for application. As indicated in the interview protocol, the interviewer would sometimes have to probe by asking the question, "Do you remember a line of reasoning, figure, table or trying specific examples that we used to come up with the rule?" This was done after a period of time had elapsed in which the students were unable to proceed with the problem. Seventeen percent of the successful reconstructions followed the asking of this question. It would seem that one possible factor in increasing the frequencies of students using reconstruction, which would in turn increase remembrance, would be to teach them how to ask the proper questions to themselves. This process was begun in the session on reconstruction but it was very difficult to change in one session the effect of years of training of reliance on the teacher.

**Summary.** Hypothesis $H_5$ was tested to determine the strength of the relationship between mathematical achievement and the use of reconstruction strategies. A significant, positive correlation was found between mathematical achievement as measured by CTBS scores and successful reconstruction. The null of Hypothesis 5 was rejected. In addition, significantly positive correlations were found between mathematical achievement and achievement of criterion, and mathematical achievement and successful remembrance of mathematical rules. These
results indicated that mathematical achievement was found to explain some of the differences in remembrance and reconstruction usage. However, since the positive correlations were small and higher achievers were spread throughout the criterion pools, these results did not take away from the differences found between Cycles W, X, Y and Z.

Four types of cues were identified during the interviews as having been used by students to attempt a reconstruction. The type of cue used by a student may have a significant effect on the success of a reconstruction attempt. An observation was made from the interviews regarding how many students who had successfully recalled the rule for application could also reconstruct the rule. A higher percentage of students who had received the deductive style of teaching were able to reconstruct the rule compared to those students who had been instructed inductively. A third observation was made concerning the need for students to ask themselves the necessary questions to initiate the reconstruction process.
Summary

Improving the long-term retention of mathematical learning should be an important goal of the school mathematics program. As identified in the review of literature, the accomplishment of this goal depends primarily on two major processes: coding and retrieval. For students to be able to remember mathematical learning better, the learning must have taken place in such a way as to be encoded into the cognitive structure in a retrievable form, and there must be an adequate retrieval strategy available to the learner to access that learned information. This study focused on selected variables from each of the two processes in order to identify instructional strategies which the teacher of mathematics could employ to help students remember and retrieve mathematical learning better.

Three styles of teaching were examined to find their effects on the encoding process of students. Four problems were taught to groups of students in an inductive, a deductive and a rule/example style of teaching to determine their effects on the remembrance of mathematical rules. One retrieval strategy, reconstruction, was tested to see if it could be learned and used by students and to see if the teaching of reconstruction would improve students' remembrance of mathematical rules. Reconstruction was identified by
Piaget and Inhelder (1973) and for this study their definition was modified to mean the intentional reproduction of a mathematical rule by using the process in which the rule was learned. Half of the groups of students received a session on reconstruction before the instruction on two problems, and half of the groups did not have a session on reconstruction before instruction on two problems. The interaction between style of teaching and reconstruction instruction was also investigated.

Nine seventh-grade classes were randomly assigned to different treatment/observation cycles. Each treatment/observation cycle had the following in common: instruction on two problems, a 10-item quiz used as a criterion test, a review session following the quiz, an 8-week interval, and then a retention test for those attaining criterion in each class. To test the retention of those achieving criterion in each class, five students were randomly selected for an interview and the remainder took a paper and pencil test. The treatment/observation cycles differed in the type of instruction received and/or problem solving experience.

Four treatment/observation cycles were designed to test the hypotheses. Three groups of classes were formed and each group participated in one or two of the cycles. The group assignments to the cycles were: (1) One group participated twice in a cycle by being instructed each time on two problems in a rule/example style of teaching without instruction on reconstruction, (2) a second group was instructed in a deductive style on one problem and an inductive style on a second problem without being instructed on reconstruction, (3) the second group also progressed through another cycle but this
time received instruction on reconstruction before being instructed on two different problems, one inductively and one deductively, and (4) a third group was instructed on reconstruction before being instructed on two problems, one inductively and one deductively.

Data were collected relevant to the effects of the treatment/observation cycles on (1) remembrance of mathematical rules for application whether by recall or reconstruction and (2) use of reconstruction strategies to remember the mathematical rules for application. Data were collected only from students in these four groups who "passed" the criterion test by demonstrating remembrance of both rules for application of the two problems on which they had received instruction. A pool of criterion-achieving students was formed in each class for purposes of making comparisons. This manner of equating the classes eliminated the statistical problems of matching groups on background information. Five students were randomly selected from the criterion pool in each class to be interviewed. The remainder of students in each criterion pool were administered a paper and pencil retention test. The data were analyzed using analysis of variance and t-test techniques. Additional observations concerning remembrance-generating cues and students who were able to reconstruct the rule after successful recall were noted from the interviews. Scores from the California Test of Basic Skills were collected on each student participating in the study in order to make correlational tests with achievement of criterion, remembrance of mathematical rules, and the use of reconstruction strategies. The details and results of the data analysis are presented in chapter 4.
Conclusions

The results of the data analysis support several conclusions. The generalizability of these conclusions is affected by the characteristics of the sample, the experimental setting, and instruments used for measurement.

1. Students remembered mathematical rules for application significantly better if the rules were taught in an inductive or deductive style rather than in a rule/example style.

2. The teaching of reconstruction strategies significantly increased the students' use of reconstruction to access long-term memory of mathematical rules.

3. Students who had received instruction in an inductive or deductive style did employ reconstruction strategies significantly more often than those who had been instructed in a rule/example style.

4. The inductive and deductive style of teaching did not differentially affect the frequency of reconstruction strategies employed by students.

5. Higher mathematical achievers tended to use reconstruction strategies more often than low achievers.

In addition to these conclusions that were warranted from the data analysis, there were other conclusions that were made in which the evidence was not as strong. However, there was at least a strong indication from the results of the study that these may be true.

1. The teaching of reconstruction strategies significantly improves the retention of mathematical rules.
2. Low mathematical achievers can learn reconstruction strategies and will use them to access long-term memory of mathematical rules.

3. Students will employ cues that have been provided for them in the course of their instruction.

4. The type of cue used by a student has an effect on the success of a reconstruction attempt.

A note of caution should be made concerning the generalizability of these conclusions and hunches. All of the instruction done was in a problem solving setting. The problem selection may have favored either the inductive or deductive style of teaching. That is, for a given problem, one of the two styles of teaching, either inductive or deductive, may have helped students remember the mathematical rule better than the other. In addition, there was evidence that problem solving strategies may have a stronger effect on long-term memory than other types of mathematical learning because of the nature of the condition-action sequences that are implicit in problem solving activities (Larkin, McDermott, Simon, D. P. and Simon, H. A., 1980). Thus, it is not known whether these same results would occur if the instruction had been on teaching mathematical skills or concepts.

A second note of caution should be made because several tests were conducted on the same set of data and these tests may have increased the alpha error. Two justifications are offered for running the tests despite the alpha risk. First, in planning the design, some compromises had to be made in order to carry out the experiment involving style of teaching, reconstruction instruction and their interaction. Because of the relative lack of experimentation in this area of study, it was not known which factors would prove to be salient.
Secondly, in looking at the data analysis, most of the tests that were conducted showed significant differences and both the significant and nonsignificant differences were in the same direction.

It also should be pointed out that the analyses were conducted on a very conservative basis due to the use of the class as the unit of analysis. If the individual student had been used as the unit of analysis, the degrees of freedom for a given ANOVA would have been far greater and the chances of finding significant differences would have been much higher.

**Implications**

The results of this study have indicated ways in which the classroom teacher can help students improve retention of mathematical learning over the long term. In order for this to be put into practice, a major change in the present mode of instruction would be needed as the two major factors of long-term memory, the encoding and retrieval processes, are not being considered by the majority of teachers in their instruction. An indication that memory is currently receiving inadequate attention in the schools comes from two observations:

1. An inordinate amount of time is being spent in reteaching the same skills, concepts, principles and rules from grade level to grade level.

2. There is an apparent inability by most students to apply the mathematics they have learned to situations outside the classroom.

One way in which the classroom teacher can apply the results of this study is by incorporating an inductive or deductive style of teaching into each lesson. The majority of teachers presently teach
in a rule/example mode. This experiment has shown that the rule/example style of teaching is inferior to either an inductive or deductive style in helping students' long-term retention of mathematical learning. One reason for this is that the rule/example mode pushes students toward dependence on memorization or recall. When students who were taught in this style could not recall the rule for application, they had nothing on which to fall back to help them remember. The rule/example style does not seem to provide the type of cues or the type of learning process to give an alternative retrieval strategy to the student if recall fails, and with this style, recall fails a high percentage of the time.

Another way in which the classroom teacher can apply the results of this study is by teaching retrieval strategies, especially reconstruction. This experiment has demonstrated that reconstruction strategies are very well suited for the mathematics classroom and that they can be taught to students. Students of both high and low mathematical achievement were able to reconstruct rules for application successfully. If these kind of results were found from one 40-minute session on reconstruction, then a continuing emphasis by the teacher on reconstruction strategies through the course of the year should improve the frequency of use of reconstruction and the long-term retention of mathematical learning dramatically. In addition, reconstruction is a general retrieval strategy, and thus the students should be able to use it in many other learning situations in the mathematics classroom.

The implementation of the use of an inductive or deductive style of teaching and reconstruction instruction will not only improve the
long-term retention of mathematics students but should lesson the amount of reteaching time that a teacher needs throughout the year. In many instances the teacher may find that the asking of a question which provides a cue, such as a line of reasoning, making a table, drawing a figure, or trying a specific example, is all that is needed to start students' reconstruction of the mathematical relationship that is desired. In such cases, the teacher will not have to repeat the entire instructional process. This should help students become more self-reliant on their own ability to remember rather than depending on the teacher to repeatedly tell them what to do.

The results of this study have also indicated important topics for the mathematics educator to include in the preservice and in-service training of teachers. An emphasis needs to be placed on the need to find ways to teach mathematical skills, concepts, principles, and rules in a deductive or inductive way. Teachers need to both observe and practice teaching in these styles. In addition, an emphasis needs to be placed on the long-term retention of mathematical learning rather than on what one can do to help students pass tomorrow's quiz. Teachers need to be aware that retrieval strategies such as reconstruction exist, that they are important for the students to learn to use, and that these strategies will ultimately help the teacher accomplish more in the long run. While obtaining an understanding of mathematics and how best to teach it, teachers also need to learn how to teach retrieval strategies such as reconstruction. They need to learn to ask their students at the end of an important lesson, "How would you try to reconstruct this mathematical relationship, if you could not recall it?"
The results of this study substantiated some and contradicted none of the educational psychologists and researchers' findings on memory. The results suggested that those principles which were substantiated warrant emphasis in providing psychological instruction for educators in general and for teachers of mathematics in particular. Some of these principles are:

1. R. Gagné (1977) suggested that to enhance long-term memory, teachers should provide an encoding scheme for the learner. He also recommended the use of pictures and diagrams to provide concrete visual images to serve as an encoding function. The inductive and deductive styles of teaching in this study were the successful, teacher-provided, concrete cues and an organizational structure which aided students in the encoding process. The evidence for this conclusion was the significantly better remembrance of the mathematical rules by those students who had been instructed in an inductive or deductive style compared to those who had been instructed in a rule/example style. The rule/example style of teaching provided few cues and little organizational structure to act as an encoding function.

2. Eich (1980) stated that remembrance does not occur spontaneously, but retrieval of information is always affected by a stimulus, query or a cue whose informational content from semantic memory matches or compliments information that has been stored in episodic memory about the event. This was borne out in the study as most instances of successful remembrance resulted from a line of reasoning, a table, a figure, specific examples, or the investigator acting as a cue. In other instances, the query provided to the student by the investigator successfully affected the remembrance of the mathematical rule.
The successful recall or reconstruction of the rule was the apparent result of the students matching these cues or queries with what had taken place in the classroom during the instruction.

3. Winograd and Lynn (1979) theorized that if the learning took place in a unique context, then the remembrance of that learning would be enhanced. The problem solving contexts used in this study were unique contexts for the students as the problems had not been seen by the students before. Furthermore, the number of successful remembrances in this study were much higher than might be expected since no pages of practice at the conclusion of the lesson were given to the students nor was any practice or review allowed during the 8-week interval. This fact lends support to the idea that unique contexts aid memory.

4. Piaget and Inhelder's (1973) identification of reconstruction as a type of remembrance distinct from recognition or recall was verified in this study. In addition, the importance of reconstruction as a means of accessing long-term memory when recall fails was seen in the many successful instances of reconstruction by students participating in this study.

5. Bower (1970) recommended that the learner should be encouraged to build or should be given systematic retrieval plans. One of the major findings of this study was that reconstruction was a viable retrieval plan that could be taught to students and would be employed by students to enhance long-term memory.

6. Craik and Lockhart (1972) concluded from their studies that elaborating during rehearsal time was better than simply trying to maintain the memory trace. Evidence from this study to support their conclusions can be found by examining the review period in each cycle
that followed the administration of the criterion test. In the rule/example cycle, the review period was used to practice or maintain the rules which had been learned that week. In the cycle which had the inductive and deductive style of teaching without reconstruction instruction, the review period was used to go over the process by which the rules had been derived. This was an elaboration of rehearsal time in that students were not simply practicing the rule but were deepening their understanding of how and why the rule worked. Finally, in the two cycles in which students received reconstruction instruction before instruction on two problems, one inductively and one deductively, the rehearsal time was used to ask students how they would try to reconstruct the rule if they could not recall it. This called for a higher degree of elaboration on the part of the students than in either of the other two cycles. Students not only increased their understanding of how and why the rule worked, but they had to identify for themselves the key factors to be remembered for each problem if they wanted to be able to reconstruct the rule in the future.

Recommendations for Further Study

It is apparent from the conclusions of this study and the substantiations of other research findings that more research needs to be done in the area of memory and, in particular, on reconstruction as it pertains to mathematical learning. The present study has identified some of the factors that can improve the remembrance of mathematical rules. It has also identified some variables which may potentially improve long-term retention of mathematical learning. Further studies into the effects of teaching style and reconstruction
instruction on the long-term retention of mathematical learning would be beneficial. The following are suggested areas in which the study may be extended:

1. The design of the study was intended to spread out the effects of the inductive and deductive style of teaching. This was done by teaching each problem both ways within each treatment/observation cycle. Studies which would have inductive and deductive styles of teaching as an independent variable would help determine which, if any, of the two styles helps students remember mathematical learning better and which, if any, of the two styles helps students reconstruct better.

2. The instruction in this study was done in a problem solving setting. The data were collected on whether students could remember the rules from the solution to the problems in order to apply them. Studies in different settings in which other types of mathematical learning are taught such as skills, concepts or principles are needed to see if the same results are found.

3. There was some evidence in the study that reconstruction instruction helped students remember the mathematical rules for application better regardless of whether the remembrance was by recall or reconstruction. Due to the small number of classes, the evidence was not substantial enough to warrant any strong conclusions. It may be that the attention to retrieval strategies made students more sensitive to the need of remembering over a long period of time. Studies which would control for this sensitivity would help determine the effects of reconstruction on long-term retention of mathematical rules.
4. The design of the study was intended to spread out the effects of types of cues. This was done by choosing and pairing problems so that each type of cue was available to students in each treatment/observation cycle. Studies which would examine the effects of type of cue on the success of reconstruction attempts would increase our understanding of the retrieval process as it pertains to mathematical learning.

5. Reconstruction instruction was given only to classes receiving instruction on problems in an inductive or deductive mode. It is not known what effects, if any, would be found if reconstruction instruction was given to classes receiving instruction on problems in a rule/example mode. It may be that the emphasis on long-term memory may improve the retention of mathematical rules taught in a rule/example style. Studies in which the interaction between reconstruction instruction and the rule/example style of teaching would help to further define the memory processes in a nonproblem solving situation.

6. The size of the sample for the interviews was limited in both the number of classes involved in the study and the number of students interviewed from the criterion pool in each class. In addition, the students in the classes were primarily from a rural-suburban, middle-class, socio-economic background. Therefore, a study which would increase the sample size in number of students from a broader range of socio-economic backgrounds, in the number of classes involved in the study, and in the number of students interviewed from the criterion pools in each class would help improve the generalizability of the findings.
7. The focus on reconstruction instruction was for only one 40-minute session in this study. Examination of the results of more than one reconstruction session on the frequency of reconstruction usage and the remembrance of mathematical learning would help to further clarify the effects of reconstruction instruction.
## Table 24

CTBS Means of the Nine Classes

<table>
<thead>
<tr>
<th>Class</th>
<th>CTBS means</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.3</td>
</tr>
<tr>
<td>2</td>
<td>7.2</td>
</tr>
<tr>
<td>3</td>
<td>6.7</td>
</tr>
<tr>
<td>4</td>
<td>7.8</td>
</tr>
<tr>
<td>5</td>
<td>7.1</td>
</tr>
<tr>
<td>6</td>
<td>9.4</td>
</tr>
<tr>
<td>7</td>
<td>7.7</td>
</tr>
<tr>
<td>8</td>
<td>6.8</td>
</tr>
<tr>
<td>9</td>
<td>7.7</td>
</tr>
</tbody>
</table>

Mean = 7.5
Table 25
Analysis of Variance for the CTBS
Scores of the Nine Classes

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F-ratio</th>
<th>p &lt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class</td>
<td>8</td>
<td>98.49</td>
<td>12.31</td>
<td>5.38</td>
<td>.0001*</td>
</tr>
<tr>
<td>Subjects</td>
<td>137</td>
<td>313.19</td>
<td>2.29</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

p < .0001

Table 26
Analysis of Variance for the CTBS Score Minus Class 6

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F-ratio</th>
<th>p &lt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class</td>
<td>8</td>
<td>22.78</td>
<td>2.85</td>
<td>1.18</td>
<td>.3184</td>
</tr>
<tr>
<td>Subjects</td>
<td>120</td>
<td>290.22</td>
<td>2.42</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX B

Table 27
Frequency Results on Successful Use of Reconstruction from Interview Data

<table>
<thead>
<tr>
<th></th>
<th>Inductive Problems</th>
<th>Deductive Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P1</td>
<td>P2</td>
</tr>
<tr>
<td>Cycle X</td>
<td>R</td>
<td>2</td>
</tr>
<tr>
<td>M</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>P1</td>
<td>P2</td>
</tr>
<tr>
<td>Cycle Y</td>
<td>R</td>
<td>3</td>
</tr>
<tr>
<td>M</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>P1</td>
<td>P2</td>
</tr>
<tr>
<td>Cycle Z</td>
<td>R</td>
<td>3</td>
</tr>
<tr>
<td>M</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Subscripts:

\( a_R \) = successful reconstruction

\( b_M \) = missed
Table 28
Frequency Results on Successful Recall or Reconstruction from Class Data

<table>
<thead>
<tr>
<th></th>
<th>Inductive Problems</th>
<th>Deductive Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P1</td>
<td>P2</td>
</tr>
<tr>
<td>Cycle X</td>
<td>Ra</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Mb</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9</td>
</tr>
<tr>
<td>Cycle Y</td>
<td>R</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11</td>
</tr>
<tr>
<td>Cycle Z</td>
<td>R</td>
<td>6</td>
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<td></td>
<td>M</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11</td>
</tr>
</tbody>
</table>

\[a_{R} = \text{recall}\]

\[b_{M} = \text{missed}\]
APPENDIX C

Criterion Test

Problem Number 1

Name _____________________________
(Please print)

Directions: Find the number of handshakes that there would be altogether if each student shook hands exactly once with every other student in the class. The number of students in the class is given below. Please show all work.

1. 18
2. 35
3. 70
4. 54
5. 500
Retention Test  

Problem Number 1

Name ____________________________  
(Please print)

Directions: Find the number of handshakes that there would be altogether if each student shook hands exactly once with every other student in the class. The number of students in the class is given below. Please show all work.

1. 16  
2. 45  
3. 60  
4. 52  
5. 300
Directions: Below you are given two of the three sides of a right triangle. Find the missing side. Please show all work.

1. \[ \begin{align*} \text{hypotenuse} &= 13 \\ \text{opposite} &= 5 \end{align*} \]

2. \[ \begin{align*} \text{adjacent} &= 10 \\ \text{hypotenuse} &= 15 \end{align*} \]

3. \[ \begin{align*} \text{opposite} &= 7 \\ \text{adjacent} &= 24 \end{align*} \]

4. \[ \begin{align*} \text{adjacent} &= 14 \\ \text{hypotenuse} &= 48 \end{align*} \]

5. \[ \begin{align*} \text{hypotenuse} &= 25 \\ \text{opposite} &= 15 \end{align*} \]
Directions: Below you are given two of the three sides of a right triangle. Find the missing side. Please show all work.

1. \( \sqrt{12^2 + 13^2} \)

2. \( \sqrt{8^2 + 6^2} \)

3. \( \sqrt{24^2 + 25^2} \)

4. \( \sqrt{11^2 + 61^2} \)

5. \( \sqrt{30^2 + 50^2} \)
Criterion Test

Problem Number 3

Name ___________________________ (Please print)

Directions: Find the squares of the following quantities. Please show all work.

1. \((c + d)^2\)

2. \((s + t)^2\)

3. \((y + 3)^2\)

4. \((a + 5)^2\)

5. \((w + y)^2\)
Name ___________________________ (Please print)

Directions: Find the squares of the following quantities. Please show all work.

1. \((g + h)^2\)
2. \((b + 4)^2\)

3. \((d + 6)^2\)
4. \((m + p)^2\)

5. \((c + 7)^2\)
<table>
<thead>
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<th>Problem Number</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>2</td>
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<td>4</td>
<td>48</td>
</tr>
<tr>
<td>5</td>
<td>22</td>
</tr>
</tbody>
</table>
Retention Test

Problem Number 4

Name _____________________________
(Please print)

Directions: Find the sum of the interior angles of the polygon with the given number of sides. Please show all work.

1. 42
2. 19

3. 26
4. 23

5. 37


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