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THE MEASUREMENT OF JOB SATISFACTION: A THREE-MODE FACTOR ANALYSIS

The Ohio State University

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THE MEASUREMENT OF JOB SATISFACTION:
A THREE-MODE FACTOR ANALYSIS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the
Degree of Doctor of Philosophy in the Graduate
School of The Ohio State University

By
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* * * * *

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PUBLICATIONS


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INTRODUCTION
Research Philosophy

Scientists have long been divided into the systematic and the intuitive. Szent-Gyorgi (1972) refers to these two types as Apollonians and Dionysians. These classifications reflect the extremes of two different attitudes that can be found equally in Art, Painting, Sculpture, Music, Dance, Science, etc. In Science the Apollonian tends to develop established theory and research to perfection while the Dionysian has a tendency to rely on intuition and is more likely to open new unexpected alleys for research. Organizational Science is presently characterized by an Apollonian mode of inquiry. The familiar paradigms of psychology and sociology are examples of the Apollonian model. The Dionysian model on the other hand is a problem solving approach which is open to new lines of research conforming to no existing paradigm. Other writers have also come to conclusions similar to those of Szent-Gyorgi (e.g., Hudson, 1966; Mitroff, 1974a; Mitroff and Kilmann, 1978).

Differences between the Apollonian and Dionysian models of inquiry have a very pragmatic implication for the development of organizational science. The Apollonian model of concentrating effort upon well established knowledge (as is
done, for example, in research on job satisfaction and the perception of job characteristics) permeates organizational science. This results in an ever increasing spiral of continued emphasis upon narrower and narrower views of the responses made by people in organizations, which generates more and more precise knowledge about increasingly trivial phenomena. It is therefore extremely difficult for the Apollonian researcher, who utilizes existing paradigms, to break new grounds of organizational knowledge. The Dionysian however, is better able to open new avenues of research because he is less constrained by the existing state of the art.

Roberts, Hulin and Rousseau (1978) argue strongly that a shift in emphasis from tightly constrained Apollonian Research models to less restricted Dionysian models is necessary for the advancement of organizational science, "A break with past tradition and paradigms seems necessary if organizational scientists are to continue to have any influence on either organizational practice or public policy" (Roberts et al., 1978, p. 147). If organizational scientists become more Dionysian they can still build upon what has been learned in the past but will also be able to deal with the large number of organizational phenomenon which have not been understood in the past.

Basic research implies venturing from the known into the unknown. To the extent that organizational scientists
allow themselves to become too enthralled with the known, safe and secure research areas investigations of the unpopular, the non-traditional, and the unknown will not be made (Roberts et al., 1978). The long term result will be disastrous for organizational science.

The research described in this thesis is more Dionysian than Apollonian. It utilizes a methodology which has never before been utilized in organizational science to investigate the general research problem of aggregation in job satisfaction measurement. What the results of this investigation might be and what implication they might have was, at the onset, somewhat unknown.

Overview of Research Problem

The basic problem of aggregation is one of the least concrete problems facing organizational science. It consists of a number of sub-problems for which there are presently very few solutions. These problems are especially important because they are potentially the most serious problems faced by organizational researchers. "At this time we can only say that potential impacts of aggregation on research and theory must be addressed before solutions can be developed" (Roberts et al., 1978, p. 139).

The thrust of this research is a Dionysian investigation into the uncharted area of the effects of aggregation
over several measures of the job satisfaction construct. There are two general aspects of this particular problem.

The first aspect is that job satisfaction is often conceptualized as a summary (aggregation) of a worker's perceptions of a number of facets of his job (this in the Turner and Lawrence, 1965; Hackman and Lawler, 1971; and Hackman and Oldham, 1975 tradition). Are the effects of aggregating various sets of job facets consistent and uniform across various aggregation techniques and across different types of individuals? And if the effects are patterned what are the implications of this patterning?

The second aspect is that, for a given set of job facets, various worker perceptions or combinations of perceptions of job facets are aggregated in order to derive a job satisfaction measure. Are the effects of these several aggregation techniques uniform and consistent across various sets of job facets and across different types of individuals? And if the effects are patterned what are the implications of these patterns?

Lawler (1971) has stated that the understanding of job satisfaction has not substantially increased over the past thirty years. A major reason for this is the existence of a plethora of conflicting empirical research results which utilize the job satisfaction construct. A major cause of these conflicting results is the lack of knowledge which we have about aggregation in job satisfaction measurement.
There are literally hundreds of studies within the job satisfaction literature which concern job satisfaction measurement and/or the relations between various measurement techniques, most of which are correlational and atheoretical. However, only a handful of these studies have been aimed at the job satisfaction measurement aggregation problem. The best examples of these efforts are those of Wanous and Lawler (1972, 1974). These authors made an unsuccessful (see Wall and Payne, 1973) attempt to investigate a portion of the job satisfaction measurement aggregation problem.

**Research Intent**

Given the importance of the job satisfaction construct within the organizational literature, the importance of the job satisfaction measurement aggregation problem and the failure of the Apollonian model to solve these particular problems, the intent of this research is to investigate this problem in a Dionysian fashion. This intent is broken down into three basic purposes.

The first purpose is to test in a certain fashion the appropriateness of the Hackman and Lawler (1971), Hackman and Oldham (1975) three-dimensional model of job characteristics. This portion of the research is Apollonian in that it is based upon a well established body of knowledge concerning a set of job facets. The second purpose is to demonstrate the usefulness of modelling organizational
phenomena in three dimensions. The third purpose is to present a computer program for a psychometric technique (three-mode factor analysis) which is capable of analysing data derived from a phenomenon which has been modeled in three dimensions. The second and third purposes are Dionysian in that they represent a complete breaking away from the extant paradigms of organizational science. The literature review, presented in Chapter I, is therefore broken down into three distinct parts which correspond to these three research purposes.
CHAPTER I
LITERATURE REVIEW

The Study of Satisfaction

Hoppock (1935) published the first intensive study of the construct to become formally known as job satisfaction. Hoppock emphasized the multiplicity of factors that could affect a worker's feelings toward his job. These factors encompassed items which had been previously investigated by the Hawthorne Studies (Roethlisberger and Dickson, 1939) (e.g., fatigue, monotony, working conditions, supervision), as well as items which were later emphasized by more advanced researchers (e.g., achievement).

Yet, despite the inclusion of such factors, it was the Hawthorne Studies rather than Hoppock's work which were to shape job satisfaction and management research for two decades. This work contributed to the development of a prescriptive body of literature known as the Human Relations Movement which emphasizes the work group as a major determinant of employee satisfaction and productivity. This movement also emphasized the effects of the supervisor upon group processes and sentiments and consequently led to the development of leadership studies, such as those conducted at The Ohio State University, (e.g., Stogdill and Coons,
1957), which developed the concept of "initiation of structure" and "consideration" and related these concepts to work group productivity and satisfaction. The Human Relations Movement reached its peak of influence in the late 1950's or perhaps early 1960's (Locke, 1976), then began to decline with the publication of Herzberg, Mausner, Peterson, and Capwell's (1957) review. The work of Herzberg et al. refocused attention on the work itself, rather than group processes. Task had been either ignored or de-emphasized since the decline of "Scientific Management" in the late 1920's. This task emphasis posited that job satisfaction could only be provided by giving workers enough responsibility and discretion to enable them to grow mentally.

These shifts in emphasis have contributed to the chaotic history of job satisfaction. General interest in this construct has waxed and waned since the Hawthorne Studies. There has, however, been a continual underlying flow of studies concerning this construct since 1936. At least 3,350 job satisfaction studies existed in 1976, and at least 111 new titles appear each year (Locke, 1976). Social scientists, managers and workers remain interested in job satisfaction because of job satisfaction's alleged relationship to variables which have distinct impacts upon organizations.

However, just as reviews of the literature have shown that job satisfaction is related to such important considerations as absences and turnover and various behavioral and
attitudinal items, such studies have also shown negligible, mostly neutral, relationships between satisfaction and the level of performance or productivity of workers (Brayfield and Crockett, 1955; Herzberg et al., 1957; Vroom, 1964).

Job satisfaction does not seem related to direct measures such as productivity but does seem related to indirect measures such as other attitudes. Even this marginal success has led to that plethora of academic studies noted earlier. These studies have utilized the job satisfaction construct as an independent variable, (e.g., Taylor and Weiss, 1972; Hrebiniai and Rodeman, 1973; Sheridan and Slocum, 1975; Gechman and Wiener, 1975), a dependent variable, (e.g., Sims and Szilagyi, 1975; Kim and Hammmer, 1975; Weiss and Sherman 1973; Baum and Youngblood, 1975) and on occasion as a moderating variable (e.g., Betz, 1971). In short, in close to 4,000 studies job satisfaction has been correlated with just about every variable that might cause it, be caused by it, or for some other reason be related to it (Lawler, 1971).

However despite intense empirical interest, our understanding of job satisfaction has not advanced substantially during the past decades. The failure to find a causal relationship between job satisfaction and productivity represents a clear example of this problem (e.g., Sheridan and Slocum, 1975; Lawler and Porter, 1967; Locke, 1970). This lack in causal knowledge results in severe problems in attempts to draw implications for practice. For example, if
satisfaction causes performance, (Schwab and Cummings, 1970) then it is clear that worker satisfaction is desirable. However, if performance causes satisfaction, then high worker satisfaction is not necessarily desirable (Porter and Lawler, 1968b) and there is much less reason to ensure that workers are satisfied. Lawler (1971) points out two reasons why our understanding of job satisfaction has not progressed: the research is typically both atheoretical, and correlational. Since research has not been guided by theory, most studies have simply reported unorganized and uninterpretable facts. Since the research is consistently correlational, much is known about what variables are related to job satisfaction but little is known about the causal basis for these relationships.

More recently, a "third" reason why the extant literature has not contributed to our overall understanding of job satisfaction has appeared. Since job satisfaction has been so atheoretical the concept has been defined in different ways. In practice it is common to find researchers using "operational definitions" of job satisfaction, that is "job satisfaction is whatever their arbitrarily chosen measure of it measures" (Locke, 1976, p. 1300). This has led to a large number of non-comparable job satisfaction measurement instruments. Typically, job satisfaction research assumes convergence among these measures and therefore the definitions which underlie them. Data collected with different
instruments, for example, have been pooled (aggregated) to reach conclusions about the relationships of satisfaction to other variables (e.g. Porter and Lawler, 1968a). Evidence is available however, to indicate that many of these measures are not in fact equivalent (e.g., Wanous and Lawler, 1972, 1974). This lack of "equivalence", which is the result of measurement aggregation, may be responsible for much of the confusion and lack of consistent findings in the literature and indicates that research is needed to map in detail the relationships among different job satisfaction measures. Such a mapping would resolve many of the conflicting results existent within the field and should begin by outlining what is currently known empirically and theoretically about job satisfaction.

Current State of the Art

Most authors begin their investigations of job satisfaction with definitions, either implicitly or explicitly. For example, Locke (1976, p. 1300) defines job satisfaction as, "a pleasurable or positive emotional state resulting from the appraisal of one's job or job experiences". While most organizational behaviorists might accept his definition, in practice it is common to find "operational definitions" of job satisfaction which evade the basic question of what it is that is being measured. All measurement
operations, either explicitly or implicitly, presuppose a conceptual definition of the phenomena being assessed. If this were not the case, the researcher would have no basis on which to choose among alternative measuring methods or questions. Consequently, an overview of job satisfaction measurement should begin with an examination of the conceptual definitions inherent in current measuring instruments and techniques in an effort to highlight points of noncomparability.

The major differences in job satisfaction measurement revolve around one major and three minor theoretical issues. Two measurement issues are also involved. The major theoretical issue is whether job satisfaction is a bipolar-unidimensional or a unipolar-bidimensional phenomena. The minor theoretical issues concern whether job satisfaction should be construed in terms of: (1) level of need fulfillment, (2) the difference between level of fulfillment and some desires or ideals, or (3) an equity comparison (assuming that job satisfaction is bipolar and unidimensional).

The measurement issues revolve around which dimensions or elements of a job should be included in an aggregate satisfaction measure, and whether the workers scores on these elements should be weighted by their importance to the worker before deriving the worker's overall job satisfaction score.
The Traditional Job Satisfaction Concept
(Bipolar - Unidimensional)

The bipolar-unidimensional conceptualization of job satisfaction is the oldest and most generally accepted view. A clearer understanding of this conceptualization can be obtained by investigating two basic job satisfaction theories: fulfillment theory and discrepancy theory.

Schaffer (1953), for instance, takes the fulfillment approach when he argues that job satisfaction varies directly with the extent to which needs of a worker which can be satisfied within the job context are actually satisfied or fulfilled. Morse (1953) also views satisfaction in terms of perceived need fulfillment. Vroom (1964), as well, sees job satisfaction in terms of the degree to which a job provides a worker with positively valued outcomes.

Other researchers argue however that a worker's satisfaction is a function of not only what is received but also of how much a worker feels he should receive or would like to receive (Locke, 1969; Lawler and Porter, 1963). For example, Lawler (1971) takes a discrepancy approach by arguing that a foreman can be satisfied with a salary of $12,000 while a corporate president can be dissatisfied with a salary of $100,000, even though the president perceives that he has higher need fulfillment.
Fulfillment theory then locates a worker along a simple satisfied/dissatisfied continuum on the basis of the percentage of the possible need fulfillment which the worker has achieved. The greater the percentage level of fulfillment, the greater the satisfaction level. Discrepancy theory, on the other hand, argues that job satisfaction is a function of the arithmetic distance between what a worker wants or feels he should receive (desired amount) from his job and what the job offers (actual amount). Discrepancy theory can be further subdivided into a "difference" and an "equity" approach. The distinguishing factor between these subdivisions of discrepancy theory revolves around how the "actual" and the "desired amount" levels are determined. The difference approach argues that desired and actual amounts are direct worker perceptions, while the equity approach argues that these amounts are the workers perceived outcome/input ratios for himself and a comparison individual or group.

Discrepancy theory then locates a worker along the satisfied-dissatisfied continuum according to the absolute size of the difference between the two factors described above. The smaller this difference the greater the job satisfaction. It has not, however, been determined whether this factor comparison is made by way of a simple difference comparison, a difference of ratios comparison, a ratio comparison, or some undiscovered manner. Yet theorists are constantly investigating various possibilities, in both the
equity and difference approaches. For example, Katzell (1964, p. 344) utilizing what is clearly a difference approach, models satisfaction as follows:

\[ \text{Satisfaction} = 1 - f \left( \frac{X-V}{V} \right) \]

Here \( X \) equals the actual amount of the stimulus and \( V \) equals the amount of the stimulus desired. While Katzell views satisfaction as the difference of what there actually is and some desired level. He assumes that this difference should be divided by the amount of the stimulus desired unlike most discrepancy theorists. This division process implies, assuming a linear function, that the more a worker desires a given stimulus the higher his satisfaction level will be for a given discrepancy. For example, for a difference level of one unit and a desired level of two units his satisfaction score would be equal to .5, while for a difference level of 1 unit and a desired level of 4 units the satisfaction level would be .75. Katzell however offers no evidence of this assumption and that there is no compelling argument in favor of it. In addition, Katzell refers to "actual" differences while most discrepancy theorists refer to "perceived" differences (Lawler, 1971).

Locke (1969) has also stated a discrepancy-difference theory of the meaning of job satisfaction. Locke's theory, however, differs from Katzell's in two important ways. First Locke does not argue that the difference should be
divided by what the person either wants or has; second, he emphasizes that it is the perceived difference that is important and not the actual difference. His approach, therefore, indicates that in order to determine one's job satisfaction, the researcher must consider what a person wants and not what he expects or feels he should receive.

However, a few researchers argue that satisfaction is determined by what one expects to receive rather than what one wants. Porter (1961) in measuring pay satisfaction, asked managers how much pay there should be for their job and how much there was, and considered the difference between the two levels of satisfaction. This form of the difference approach has been widely used. It differs from Locke's approach by focusing attention not on how much a person wants but by how much a person feels he should receive. This comparison of what one receives with what one expects to receive is an equity comparison, (Wanous and Lawler, 1972) thus this approach to the meaning of job satisfaction is the progeny of equity theory (e.g., Patchen, 1961; Homans, 1961; Sayles, 1958; Adams, 1963, 1965).

While many such theories exist, this section will concentrate on equity theory as it is conceived by Adams (1963, 1965). This is done for three reasons. First, as Vroom (1964) has pointed out, the various equity theories differ little. Second, Adams' presentation is the most explicit and extended of the theories, and third, his theory and the
research related to it has received most widespread attention in the job satisfaction literature (e.g., Pritchard, Dunnette, and Jorgensen, 1972; Lawler, 1971; Pritchard, 1969).

In its most recent formulation (Adams, 1965), the theory considers: 1) the nature of inputs and outcomes, 2) the nature of the social comparison process, 3) the conditions leading to equity or inequity and the possible effects of inequity, and 4) the possible responses one may make to reduce a condition of inequity.

Inputs include any and all factors perceived by a worker to be relevant for getting some return on his personal investment. For example, in a job situation they would include such things as how hard the person perceives he works, educational level, and his general qualifications for the job. The important point is that the worker actually perceives them as something of value that he brings or puts into his job. In contrast, outcomes include any and all factors perceived by the worker as returns to himself, this is to say factors that have utility or value to him. Outcomes in a job situation include such things as pay, fringe benefits, status, and the intrinsic interest of the job.

Outcomes and inputs form a ratio, and the individual outcomes and inputs are weighted according to their perceived importance in determining the final "value" of this outcome/input ratio.
A worker is said to consciously or unconsciously compare his outcome/input ratio to that of another person or persons. This comparison "person" does not necessarily have to be any one individual, it could be a broad class of individuals who are perceived by the worker as relevant for comparison.*

Equity is said to occur when PERSON perceives that the ratio of his outcomes to his inputs is equal to OTHERS outcome/input ratio. Inequity is conceptualized as the discrepancy between PERSON's outcome/input ratio and OTHER's outcome/input ratio. Satisfaction is seen to result when perceived equity exists and dissatisfaction to result when perceived inequity exists. There is emphasis in equity theory on the importance of OTHER's income/output balance in determining how PERSON will judge the equity of his own outcome/input balance. This emphasis does not enter into the difference approach to discrepancy theory. This points up a strength of the equity approach in that it rather clearly states how PERSON assesses his inputs and outcomes in order to develop his level of satisfaction with his outcome/input ratio. The difference approach, on the other hand, is very vague about how people decide what their outcomes should be or what they presently are (Lawler, 1971).

* Adams refers to this generalized comparison person as "OTHER", and to the comparer as "PERSON". This notation will be followed in the remainder of this section.
From equity theories such as Adams', a model of the meaning of job satisfaction evolves which would state that satisfaction is a function of a worker's perceived discrepancy between his outcome/input ratio and that of his comparison OTHER. This approach would locate a worker along the satisfied/dissatisfied continuum on the basis of the size of this perceived discrepancy.

The Two-Factor Job Satisfaction Concept
(UniPolar - Bidimensional)

The original basis for the unipolar - bidimensional conceptualization of job satisfaction was a study of some two hundred engineers and accountants who were asked to describe a time when they felt especially "good" and a time when they felt especially "bad" about their jobs (Herzberg, Mausner, and Snyderman, 1959). These "critical incidents" were classified by grouping together those that "seemed to go together" and recording the frequency with which each of these categories was mentioned. The work itself, achievement, promotion, recognition and responsibility, were frequently mentioned as sources of job satisfaction, but much less often mentioned as sources of job dissatisfaction. This group of incidents, labelled "motivators", were posited to involve job content characteristics.

Responses classified as supervision, interpersonal relations, working conditions, company policies, and salary
were frequently mentioned as causes of job dissatisfaction, but much less frequently mentioned as causes of job satisfaction. This group of critical incidents, labelled "hygienes", was asserted to involve primarily the context within which the work was performed.

Thus, job satisfaction and job dissatisfaction result from different causes; satisfaction depending on motivators and dissatisfaction depending on hygiene factors.

Even though two-factor theory has been the object of a great deal of research, it has been severely criticized (e.g., House and Wigdor, 1967; Locke, 1976), and is not the dominant conceptualization within the literature. In fact the unipolar - bidimensional conceptualization per se is no longer in favor, not merely Herzerg's theory.

Two Questions of Measurement

As noted above, the two measurement issues involved in the job satisfaction measurement instrument non-comparability problem revolve around 1) the selection of a set of job facets to measure a workers satisfaction with, and 2) whether or not to weight the worker's facet satisfaction scores. These two questions will be discussed in turn.

Job Facet Selection--Locke (1969) states that a job is not an entity but a complex interrelationship of tasks, roles, responsibilities, interactions, incentives and rewards. Thus an understanding of job attitudes such as job
satisfaction requires that the job be analyzed in terms of its constituent elements.

Factor analysis is one approach to identifying these elements. In this approach, a number of job facet items outlined by the researcher are responded to by a group of workers. These responses are then intercorrelated and grouped into "factors", each factor consisting of items that correlate more highly with each other than with other items. The basic dimensions of the job are then inferred from the content of the items in each factor.

Turner and Lawrence (1965) dealt with the problem of measuring job characteristics in depth. These authors developed operational measures of six "requisite task attributes" which, on the basis of a review of the then existing literature and an a-priori framework, were predicted to be positively related to worker satisfaction. The six attributes are: 1) variety, 2) autonomy, 3) required interaction, 4) optional interaction, 5) knowledge and skill required, and 6) responsibility. An examination of the relationships among these six requisite task attributes revealed that the attributes were very closely related to one another. Therefore, Turner and Lawrence developed a summary measure called the Requisite Task Attribute Index (RTA Index) by formulating a linear combination of the six separately measured attributes. This summary index is used to determine the relationships between job attributes and worker satisfaction.
Turner and Lawrence expected that employees working on jobs which were high on the RTA Index would have high job satisfaction. This hypothesis was not fully supported; instead, it appeared that the predicted relationship between the RTA Index and employee reactions held only for workers from factories located in small towns. Workers in urban settings reported less satisfaction with their jobs when the jobs were high on the RTA Index. These authors then argued that the obtained differences in reactions to good (i.e., high RTA Index) jobs were substantially moderated by differences in the cultural backgrounds of the workers.

Blood and Hulin (1967) and Hulin and Blood (1968) provide additional data on the importance of sub-cultural factors in determining a worker's response to job characteristics. These authors hypothesized that a worker's alienation from the traditional work norms which characterize the middle class is an important moderating factor between job satisfaction and task. When employees hold traditional values regarding the work and achievement in work settings (as would be expected of the employees in the small town factories of the Turner and Lawrence study) more complex jobs should be responded to positively. When employees are alienated from these norms (as might be expected of urban workers) more complex jobs should be responded to negatively. Blood and Hulin (1967) provide data supporting this general proposition and propose a three-dimensional explanation.
Hulin and Blood, 1968) which specifies the expected interrelationships among worker alienation, job level, and satisfaction with work.

Both Turner and Lawrence (1965) and Hulin and Blood (1968) choose to deal with individual differences in worker reactions to job characteristics on a sub-cultural or sociological level. That is, in terms of differences between town and city workers or in terms of the alienation of city workers from middle class work norms. However, Hackman and Lawler (1971) have presented an alternative strategy which attempts to conceptualize and measure the relevant individual differences in worker reactions to job characteristics at the individual level of analysis. These authors present a conceptual model which specifies a-priori what specific differences among workers are responsible for the results reported by Turner and Lawrence (1965) and Blood and Hulin (1967). They accomplish this by presenting a conceptualization of the interaction between job characteristics and individual differences which is primarily based upon the expectancy theory of motivation, as formulated by Lewin (1938) and Tolman (1959), and as applied to work settings by Vroom (1964), and Porter and Lawler (1968a). In particular, three conditions necessary for internal work motivation and hence job satisfaction are pointed out. A job must: 1) allow workers to feel personally responsible for an identifiable and meaningful portion of the work, 2) provide work outcomes
which are intrinsically meaningful or otherwise experienced as worthwhile, and 3) provide feedback about performance effectiveness.

**Weighting Facet Scores**—The second question in relation to the job satisfaction measurement instrument non-comparability problem is whether or not to weight the workers' job facet satisfaction scores. Weighting is a procedure used to take into account the relative importance of the various job facets in arriving at a total or composite job satisfaction score (e.g., Blood, 1971; Decker, 1955; Ewen, 1967; Mikes and Hulin, 1968; Schaffer, 1953). Mathematically, weighting involves multiplying a worker's score on each job facet by numbers whose relative values supposedly correspond with the importance of that job facet to the worker. Seashore and Taber (1975) contend that the magnitude of the weight assigned to a specific job facet may be determined by: 1) the worker's own report of the facet's importance, 2) empirical estimates of the importance for classes of workers, or 3) theoretical dictates.

Although acknowledging that the logic of weighting is conceptually appealing and the necessary mathematical operations simple, Seashore and Taber (1975) claim that an emerging consensus (e.g., Evans, 1969; Waters, 1969; Wanous and Lawler, 1972, 1974; Ronan and Marks, 1973) is that weighting seldom offers a significant gain in construct validity, measurement reliability or predictive power. Two possible
reasons that these authors offer for this are: 1) that the worker, whether on a conscious or unconscious level, inevitably weights the facets in responding, and 2) that the effects of even powerful weights may be cancelled out in the summated index because job facets are often numerous and positively correlated.

The Measurement of Job Satisfaction

Even though rather large differences exist between the bi-polar, uni-dimensional and the uni-polar, bi-dimensional conceptualizations of job satisfaction, as well as between the three approaches to the meaning of satisfaction within the traditional conceptualization; all agree that job satisfaction is an attitude. It is, therefore, appropriate to discuss the measurement of job satisfaction within the framework of attitude measurement and an appropriate beginning point is a general description of attitudes and their measurement.

Attitudes have generally been regarded as implicit predispositions that exert some general and consistent influence on a fairly large class of evaluative responses or behaviors (Zimbardo, Ebbesen and Maslach, 1969). Attitudes are internal, private events whose existence must be inferred from introspection or from some form of external manifestation. In addition, attitudes are seen as enduring predispositions, but ones that are learned rather than innate.
Thus, even though attitudes are not momentarily transient, they are susceptible to change.

Measuring an attitude — for example, job satisfaction — is not an easy task. How do you measure something that is conceptualized as existing within a person's mind? The solution to this assessment problem is to require that a person make his internal attitude external. To put it another way, it is necessary to get a person to translate an internal attitude into something external.

In studying attitudes then, it helps to conceptualize them as having three external manifestations: affect, cognition, and behavior (Rosenberg and Hovland, 1960). The affective manifestation consists of a person's evaluation of, liking of, or emotional response to some entity. The cognitive manifestation has been conceptualized as a person's beliefs about, or factual knowledge of the entity. The behavioral manifestation involves the person's overt behavior directed towards the entity (see Figure 1). This way of thinking about attitudes can serve as a guide about how to measure them. The affective manifestation can be measured by physiological responses or evaluative responses (e.g., "I like it", "I am dissatisfied", "I felt good", "I felt bad"). The cognitive manifestation can be measured by self ratings of beliefs (e.g., "I don't have very much of it", "There is a lot of variety in my job") or by the amount of knowledge a person has about some topic. The behavioral
manifestation can be measured by direct observation of how a person behaves in specific situations, or by statements about how he would act if he could.

Using this conceptual scheme, let us now turn our attention to the particular attitude of job satisfaction and how it has been operationally measured along each of these three different approaches.

**Affective Measures of Job Satisfaction**

Affective measures of job satisfaction (see Figure 1) include sympathetic nervous system responses and verbal statements of affect. Very few attempts have been made to measure the affective manifestation of job satisfaction. The only notable example is the "critical incident" technique of Herzberg, Mausner and Synderman (1959). This technique, which uses a format that asks workers to describe a time when they felt especially "good" and a time when they felt especially "bad" about their job, elicits evaluative statements. Hence it taps the affective component of job satisfaction. In addition, Herzberg et al. (1959) found that workers reported physical symptoms such as headaches, loss of appetite, indigestion and nausea following dissatisfying job incidents. These are also verbal statements of affect.
Behavioral Measures of Job Satisfaction

Behavioral measures of job satisfaction can be divided into two groups: overt actions, and statements concerning behavior.

Examples of overt behaviors which have been used as indicators of job satisfaction include: absences, termination, and complaints or grievances. The relation of these behaviors to cognitive measures of job satisfaction has been supported (Locke, 1976). It is interesting to note that all of these behaviors are indicative of job dissatisfaction; overt behaviors which are indicative of positive job satisfaction have not been investigated.

Locke (1976) cautions that one would be unwise to advocate the exclusive use of overt behavior as a measure of job satisfaction because it would be difficult to find a behavior which would satisfy the minimal criteria needed to satisfy it, namely: 1) the behavior must invariably follow the experience of satisfaction, that is, satisfaction is always expressed in this particular way; 2) the behavior occurs with a frequency or intensity that is directly proportional to the intensity of the attitude experienced; and 3) no causal factors other than satisfaction influence the behavior, or if so, their influence can be precisely calculated. In addition, Wood (1970) contends that the primary objective in measuring worker feelings (e.g., job satisfaction) lies
Attitude Manifestations

Measurable variables

Affect

Sympathetic nervous system responses
Verbal statements of affect

Cognition

Perceptual responses
Verbal statements of belief

Behavior

Overt actions
Verbal statements concerning behavior

FIGURE 1

A SCHEMATIC CONCEPTUALIZATION OF ATTITUDES
(After Rosenberg and Hovland, 1960)
in the identification of the dimensions of these feelings, rather than seeking behavioral correlates to feeling states.

The second method of measuring the behavioral manifestation of job satisfaction is a worker's statements concerning his behavior. Two such techniques are to be found in the literature: Other Work Indicators and Action Tendency Scales.

Other Work Indicators (e.g., Rosten, 1938; Morse and Weiss, 1955; Blauner, 1960; Wilensky, 1964) consist of single questions, which ask the worker if he would rather do some kind of work other than that in which he is currently engaged. The worker must choose between two response alternatives; "yes" (a dissatisfied response) or "no" (a satisfied response).

The essential ingredient of an Action Tendency Scale (e.g., Kornhauser, 1965) is that the worker is neither asked "how he feels as such, nor how he acts as such, but rather how he feels like acting" or how he would act if no other factors but his feelings were guiding his actions (Locke, 1976, p. 1335). Locke (1976) contends that only items which apply to attitudes about the job as a whole have to date been used in these types of scales.

Cognitive Measures of Job Satisfaction

Perceptual responses and statements of belief are the two methods of measuring the cognitive manifestation of job satisfaction.
There appears to be only one basic attempt towards measuring job satisfaction via perceptual responses; the "faces" scales. The General Motors Faces Scale (Kunin, 1955) is a graphic scale which consists of a number of faces ranging from a pronounced smile to a pronounced frown. The worker is required to check the face that best represents his feelings in general towards his job or some facet of his job. A female version of a "faces" scale has also been developed (Dunham and Herman, 1975). An example of a "faces" scale is shown in Figure 2.

The cognitive manifestation of job satisfaction is by far the most commonly measured. The technique most often utilized is to tap statements of "belief" via paper-and-pencil job satisfaction scales, which utilize Likert's method of summed ratings.

A Likert Scale (Likert, 1932) is made up of a series of opinion statements. The cognitive manifestation of a person's attitude is measured by asking him to indicate the extent of agreement or disagreement with each of a number of questionnaire items. This is done by having the person rate each item on a scale of responses (usually consisting of five points; strongly agree, agree, undecided, disagree, strongly disagree). A person's attitude score is the sum of his individual item ratings.

Likert assumes that each item or statement that is used is a linear function of the same attitude manifestation
Put a check under the face that expresses how you feel about your job in general, including the work, the pay, the supervision, the opportunities for promotion and the people you work with.

FIGURE 2
EXAMPLE OF A "FACES" SCALE
(Source Kunin, 1955)
(Zimbardo, Ebbesen and Maslach, 1969). This assumption is the basis for summing individual item scores to obtain the final score. It is important to note that at no point does Likert assume equal intervals between scale values. For example, it is quite possible that the difference between "agree" and "strongly agree" is much larger than the difference between "agree" and "undecided". This means that a Likert Scale can provide information on the ordering of people's attitudes on a continuum, but it is unable to indicate how close or how far apart different attitudes might be.

Such a large number of job satisfaction scales exist (see Robinson and Athanasiou and Head (1973) for a rather comprehensive listing) it is impossible to discuss them all. Therefore, since the focus of this study is first upon the cognitive manifestation (see Figure 1) of job satisfaction and second upon the traditional conceptualization (bipolar-unidimensional), the following discussion of job satisfaction measurement scales is limited to those Likert-type scales which are implicitly or explicitly based upon the fulfillment, discrepancy-difference and discrepancy-equity conceptualizations.

**Job Satisfaction Scales Based Upon Fulfillment Theory**

As noted above, the fulfillment approach to job satisfaction would place a worker along a satisfied/dissatisfied
continuum on the basis of percentage of possible need fulfillment which he believes he has achieved. Thus, all of the job satisfaction scales which, overtly or implicitly, ascribe to this approach attempt to elicit worker statements as to their degree of need fulfillment across a number of job facets. The means utilized to elicit a worker's statement of need fulfillment varies, however, most often the worker is asked to respond to a "How satisfied are you..." type of question utilizing Likert's (1932) technique.

Probably the earliest of the fulfillment job satisfaction scales was Hoppock's (1935) Job Satisfaction Blank (JSB). In filling out the JSB the worker places a check mark beside one of seven Likert-type scaled alternatives which describe how well he likes his job. It is assumed that the worker summates his likes and dislikes for his job and, in responding, weights them subjectively according to their importance to him (Crites, 1969). The worker's satisfaction score is calculated by totalling the values assigned to the various job facets.

The second example is the Brayfield-Rothe Index of Job Satisfaction (Brayfield and Rothe, 1951). This scale consists of eighteen statements which refer to the job in general. Each of these statements is provided with five response alternatives, ranging from "strongly agree" to "strongly disagree". The worker crosses out the alternative which best reflects his agreement with the statement. A
total satisfaction score is obtained by summing the values assigned to the endorsed alternatives.

Both of these scales, Hoppock's (JSB) and the Brayfield-Rothe Scale, are what can be termed **global** in nature. Global measures are those which are directed towards the worker's overall attitude toward his job. That is, a worker's overall satisfaction with his job is measured "by asking employees to assess their feelings towards their work situation as a whole" (Hoppock, 1935). These types of scales do not use questionnaire items which refer to specific aspects of a particular job or job environment in order to provide an instrument which is more suitable for use across a variety of applications (Brayfield and Rothe, 1951). The Brayfield-Rothe instrument, for instance, includes a total of eighteen such statements (e.g., Most days I am enthusiastic about my work).

On the other hand, it has been argued that a global measure of job satisfaction may be sufficient for some purposes but would be inadequate for an intensive study aimed at identifying the relations between different aspects of the job situation and individual and/or company characteristics (Locke, 1969, 1976). The same variables may be related quite differently to satisfaction with different aspects of the job, but these relationships would be diluted if only a global job satisfaction measure were used (Smith, Kendall, and Hulin, 1969). In other words, it is often argued that
global measures of job satisfaction not only fail to specify "sore spots" in the work-job/job-environment complex but also provide no suggestions for action to improve problem situations.

Thus, composite job satisfaction scales have been developed. Composite scales are those which attempt to overcome this problem by including a variety of questionnaire items which elicit worker responses to various elements of the job.

An early example of this composite type of fulfillment based job satisfaction scale is the Tear Ballot for Industry (TBI) (Kerr, 1948), which is comprised of ten statements about various elements of the job and work situation. In responding to each of the ten items, the worker tears the edge of an answer sheet opposite one of five Likert type scale response alternatives. Scores are compiled by summing the values which correspond to the endorsed alternatives.

A second example of a composite type fulfillment measure results from the Minnesota Studies of Work Adjustment (Lofquist and Dawis, 1969). These authors define job satisfaction, in need fulfillment terms, as a correspondence between the reinforcer system of the work environment and the worker's needs. The Minnesota Satisfaction Questionnaire (MSQ) elicits worker responses to twenty different elements of the job environment: ability utilization, achievement, activity, advancement, authority, company policies and
practices, compensation, co-workers, creativity, independence, moral values, recognition, responsibility, security, social service, social status, supervision-human relations, supervision-technical, variety, and working conditions.

Satisfaction with each element of the job is measured by the sum of the worker's scores on five questionnaire items per job element with rating scales that range from "very satisfied" to "very dissatisfied" (Gillet and Schwab, 1975).

A third example is the Job Descriptive Index (JDI) developed by Smith and her associates (Smith, Kendall and Hulin, 1969). This scale is an adjective check-list which deals with five job elements. These elements are satisfaction with: 1) the work itself, 2) pay, 3) opportunities for promotion, 4) supervision, and 5) co-workers.

The JDI is a notable exception to the use of Likert's method of summated ratings to measure job satisfaction. In administering the JDI a list of adjectives and brief phrases concerning each of the five job elements is presented on a separate page. The worker is required to put a "Y" beside each item that describes a particular aspect of his job, or to put an "N" if the item does not describe that aspect of his job or, to place a "?" if he is unable to decide.

"Y" responses to "positive" items and "N" responses to "negative" items are given three points. A "?" response to any item is scored as one point, while "Y" responses to negative items and vice versa are scored as zero points.
Porter (1961) has also attempted to measure job satisfaction as the sum of goal attainment or need fulfillment in terms of Maslow's (1954, 1970) need hierarchy theory. The items of Porter's Need Satisfaction Questionnaire (PNSQ) are designed to tap the levels of Maslow's need theory. In Porter's (1961) terms, goal attainment or fulfillment can be thought of as a worker's response to a "how much is there now" questionnaire item for a particular job facet; hence, the use of the "Is Now" scale. This scale requires the worker to indicate the extent to which his current job provides for a number of goals or needs. This scale calculates an overall level of a worker's job satisfaction by summing the worker's responses across all of the job facets considered.

More recently, Hackman and Oldham (1975) have attempted to measure worker reactions to job characteristics, to include job satisfaction, in fulfillment terms. This measurement instrument is called the Job Diagnostic Survey (JDS) and is based upon the work of Turner and Lawrence (1965) and Hackman and Lawler (1971).

The basic theory underlying the JDS is shown in Figure 3. This theory proposes that personal and work outcomes, such as job satisfaction, result from the presence of three "critical psychological states" in a worker's job: 1) experienced meaningfulness of the work, 2) experienced responsibility for the outcomes of the work, and 3) knowledge of the results of the work activities.
FIGURE 3

THE THEORY UNDERLYING THE JOB DIAGNOSTIC SURVEY

(After Hackman and Oldham, 1975)
The theory also proposes that the three critical psychological states result from the presence of five "core" job dimensions. Experienced meaningfulness of the work results from the presence of three of these core dimensions: 1) skill variety, 2) task identity, and 3) task significance. Experienced responsibility for work outcomes results when a job has high autonomy. Knowledge of results is increased when a job is high on feedback. Following the theory shown in Figure 3, it is possible to create a summary "motivating potential score" (MPS) as follows:

\[
\text{MPS} = \left( \frac{\text{Skill} + \text{Task Variety} + \text{Task Identity}}{3} + \text{Task Significance} \right) \times \text{[Autonomy]} \times \text{[Feedback]} \]

As can be seen from the formula, an increase in any of the core dimensions will increase the MPS; but (because of the multiplicative relationship among its components) if any of the three major components of the MPS is low, the resulting MPS also must be low.

A job high in motivating potential will not affect all individuals in the same way. In particular, people who strongly value and desire personal feelings of accomplishment and growth should respond very positively (e.g., have high job satisfaction) to a job which is high on the core dimensions; individuals who do not value personal growth and accomplishment may find such a job anxiety arousing and may be uncomfortably "stretched" by it. Therefore individual
Growth Need Strength is shown in Figure 3 as a moderator of the other theory-specified relationships (Hackman and Oldham, 1975).

The JDS is the measurement instrument which has been developed to operationalize the MPS model. It utilizes seven-point Likert-type response scales and provides for the measurement of the concepts specified in the three-dimensional theoretical framework. It also utilizes many of the questionnaire items developed by Turner and Lawrence (1965) and Hackman and Lawler (1971).

Job Satisfaction Scales Based Upon Discrepancy Theory

Discrepancy Difference—Computed discrepancy scores represent the second category of job satisfaction measures. All of these measures, unlike the fulfillment based measures, are of the composite type.

Locke (1969) argues for the use of a discrepancy equation in the computation of job satisfaction. He believes that only unfulfilled desires can cause dissatisfaction and that satisfaction is the result of a comparison between a state of need fulfillment and a set of desires or ideals. Ronan and Marks (1973) also argue for a discrepancy rationale in job satisfaction measurement in that they contend that a discrepancy value more nearly reflects the positive or negative feeling states of the worker. Moreover, Seashore and Taber (1975) offer that discrepancy scores: 1) display
conceptual elegance, 2) have been known to "work" as representations of satisfaction in hypothesis testing (e.g., Porter and Lawler, 1968a; Locke, 1969), and 3) occasionally are found to work better than non-discrepancy scores (e.g., Wanous and Lawler, 1972).

The discrepancy approach, however, is not without opponents. A number of psychometricians have pointed out the problems that surround the use of discrepancy scores in psychological research (e.g., Lord, 1963; McNemar, 1958; Werts and Linn, 1970). Cronbach and Furby (1970, p. 68) observe that "although the unsuitability of such scores has long been discussed, they are still employed, even by some otherwise sophisticated investigators."

Wall and Payne (1973) have drawn particular attention to the way in which discrepancy score datum may be misrepresents in job satisfaction research by describing two inherent constraints within the derivation of such scores.

First, when a rating of an existing level of a job facet is subtracted from a desired level rating, a logical constraint comes into effect. For example, if a seven-point rating scale is being used, a worker scoring 5 on the existing level can attain a discrepancy score of -4 to 2. Whereas a worker scoring 2 on the existing level may obtain a discrepancy score of -1 to 5. Discrepancy scores for those workers with high perceived existing levels of a given job facet will consequently tend to be smaller than the
discrepancy scores of those workers with lower perceived existing levels of a given job facet. All other things being equal, there will be a negative relationship between the existing level scores and discrepancy scores, a discrepancy score being strongly, though not completely, determined by the existing level score. It follows that any independent variable which is positively related to the existing level score will tend to be negatively related to the discrepancy score, for no other reason than its original positive relation with the existing level score.

Second, in practice it is found that when workers are asked to rate how much of a "desirable" job facet is associated with their job, and then to rate how much of that facet should be associated with their job, they rarely state that there should be less than presently exists. Wall and Payne (1973) have labelled this a psychological constraint. Porter's (1962) research shows that discrepancy scores are predominantly positive, while in two studies involving 450 subjects, Wall and Payne (1973), found that only 5% of the workers obtained negative discrepancy scores. The appearance of the psychological constraint is relevant to the logical constraint in two respects. First, it means that discrepancy scores for those with high perceived existing levels of a job facet will fall within a more restricted range than they will for those with lower perceived existing levels. Second, when investigators have tried to overcome the
logical constraint by considering only the magnitude of the discrepancy score (ignoring whether it is positive or negative), the logical constraint will, in fact, still operate. There will still be a negative relationship between existing level scores and discrepancy scores (Wall and Payne, 1973).

**Discrepancy Equity**—As indicated above, the "equity" approach to the discrepancy theory of job satisfaction seeks to locate a worker along the satisfied/dissatisfied continuum in relation to the size of the arithmetic difference between the worker's perceived outcome/input ratio and the ratio of his comparison OTHER. This approach has been operationalized via the "Should Be-Is Now" scale (e.g., Porter, 1962, 1963; Porter and Lawler, 1968a; Wanous and Lawler, 1972; Hackman and Lawler, 1971). This scale requires the worker to respond to a number of questionnaire items concerning various facets of the worker's job by indicating the extent to which his job provides for these items. For example, one item might read: "My supervisor provides recognition for a job well done". The worker would respond by circling either: "almost always, often, usually, occasionally", or "not at all".

After completing the list of items in this fashion, the worker repeats the procedure, but this time must indicate how much of the particular job facet that his job should provide. The score is calculated as the difference between the two corresponding responses, summed across all of the
job facets. As such, the scale implies that satisfaction is the product of an equity comparison.

Further support for the "Should Be-Is Now" scale is presented by Rosen and Rosen (1955). These researchers view job satisfaction as a consequence of the discrepancy between precepts (Is Now) and value standards (Should Be). In particular, they hypothesize that satisfaction and dissatisfaction are related to the extent to which desires are perceived as being met (i.e., that satisfaction will tend to result when people see occurring what they feel should occur in a given situation, and that dissatisfaction will tend to result when people do not see occurring what they feel should occur).

On the other hand, the "difference" approach to the discrepancy theory of job satisfaction seeks to locate a worker along the satisfied/dissatisfied continuum in relation to the size of the arithmetic difference between the worker's level of need fulfillment and some psychological standard within the worker himself. This approach has been operationalized via the "Would Like-Is Now" scale (e.g., Locke, 1969). This scale envisions the worker as asking himself if his job comes close to his ideal or desired job (Jacques, 1961; Locke, 1969), and is utilized in the same manner as the "Should Be-Is Now" scale discussed above. A graphic summarization of these scales is shown in Figure 4.
FIGURE 4

GRAPHIC SUMMARIZATION OF JOB SATISFACTION MEASUREMENT TECHNIQUES AND THEIR ANTECEDENTS
Measurable Variables

Statements of Affect

Measurement Devices

Critical Incident Technique

Overt Actions

Absences, Termination Complaints, Grievances

Statements Concerning Behavior

Other Work Indicator Action Tendency Scales

Perceptual Responses

"Faces" Scale

Statements of Belief

Fulfillment Scales

Global
- Happock's (JSB)
- Brayfield-Roth Scale
- Bullock Scale

Kerr's (TBI)
Smith's (JDI)
Composite
Lofquist's (MSQ)
Porter's (PNSQ)
Hackman's (JDS)

Difference Approach
Would Like-Is Now Scale

Equity Approach
Should Be-Is Now Scale
The Study of Satisfaction Section above indicated that job satisfaction is conceptualized as an attitude consisting of three external manifestations, affect, behavior and cognition, and that this study is restricted to the measurement of the cognitive manifestation. Further, it can be argued that there are three sources of variation within the cognitive manifestation of an attitude, entities (people), characteristics (variables) and occasions. For example, within the area of personality theory, attitudes about other people (e.g., traits, attributes) are seen as being inferred from just such a three-dimensional process (Heider, 1958; Kelley, 1967). Kelley's model assumes that a person will cognitively respond in a certain way in the presence of a particular kind of stimulus, and points to three kinds of information which determine this response. The first, distinctiveness information, refers to whether the person makes the same kind of response to many different kinds of stimuli or whether the response is made only to a particular stimulus or class of stimuli. The second is consensus information which refers to whether other people will likely make the same response in the presence of the stimulus. The third is modalities information from which one infers consistency (i.e., whether or not a particular response occurs whenever a particular stimulus is present). Another important factor in consistency is
whether the response is made no matter how the stimulus is presented. Generally these three factors are considered in combination.

In a manner similar to three-dimensional personality theories such as Kelley's, a worker's cognitive satisfaction with his job can be inferred to result from three factors. The first is whether the worker makes the same kind of cognitive response (job satisfaction level) to many different aspects of his job (job facets) or whether the response is made only to a particular job facet or class of job facets (job dimension). This is analogous to Kelley's distinctiveness information. The second is whether other workers would likely indicate the same cognitive response (job satisfaction level) when faced with the same stimulus. This constitutes the area of individual worker differences and is analogous with Kelley's consensus information. The third factor is whether a worker would respond similarly to various job satisfaction measures, or the way in which one of these operationalizations is presented (i.e., various job satisfaction scales all of which are based upon the same theoretical definition). This is analogous to Kelley's consistency information.

The argument which is implied by the above analogy is that if people intuitively take three factors, such as Kelley's, into consideration in forming their attitude towards another person, it may be possible to take the same three,
or three analogous, factors into consideration when describing the cognitive manifestation of a worker's attitude towards his job. With this in mind, Rosenberg and Hovland's (1960) general conceptualization of attitudes, shown in Figure 1, can be expanded and particularized to the job satisfaction attitude as shown in Figure 5.

In summary, I argue that the concept of an attitude can be described as consisting of three external manifestations; affect, behavior and cognition. Further, the cognitive manifestation of a person's attitude towards another person has been described as being formulated via a three-dimensional process. The cognitive manifestation of a person's attitude towards his job (job satisfaction) is therefore also described via a similar three-fold method. In addition, it also becomes evident that the three elements of worker individual differences (entities), job facets (characteristics), and job satisfaction operational definitions (occasions) must be included in any investigation of the measurement of the cognitive manifestation of the job satisfaction attitude.

Thus it is evident that the job satisfaction phenomena must be described along three dimensions, which are referred to as "modes" (Tucker, 1966). This is not an unusual circumstance within the social sciences. Several examples are: 1) for the field of psychology; entities, characteristics and occasions (Rummel, 1970); persons, variables and occasions (Cattell, 1952); entities, attributes and occasions
FIGURE 5

AN EXPANSION AND PARTICULARIZATION OF ROSENBERG AND HOVLAND'S GENERAL ATTITUDE SCHEMA TO THE COGNITIVE MANIFESTATION OF THE JOB SATISFACTION ATTITUDE
(Horst, 1965); 2) for geography; characteristics, places and times (Berry, 1964); 3) for anthropology; characteristics, cultures and times (Berliner, 1962); 4) for history; topics, times and place (McCelland, 1958). In addition, other modes which have been proposed are operationalization and measurement instruments (Rummell, 1970). Thus many social science phenomena can be usefully described as consisting of three modes. However, commonly utilized social science data analysis techniques (e.g., correlation analysis, factor analysis) collapse these three-mode problems into a two-mode problem. Specifically, such techniques force an attitude which is generally accepted within the literature (e.g., Heider, 1958; Kelley, 1967) to be cognitively a three-mode phenomenon for the worker into an analytical space which has only two modes. This results in a loss of information which creates the scientific equivalent of a trivvet.

For the purpose of this study the three modes of interest are: entities (workers), characteristics (job facets) and operational definitions (job satisfaction measures). These modes of entity, characteristic and operationalization delimit the three-mode data cube illustrated in Figure 6. The phenomena of Figure 6 consist of datum cells defined by the intersection of a row (entity) and a column (characteristic) for an occasion. Figure 7 shows the data cube for the dimensions of this study, workers (entities), job facets
FIGURE 6
THE DATA CUBE
(After Rummel, 1970)
job satisfaction measures

workers

job facets

worker No. 304

"The opportunity for promotion" job facet

a datum cell (worker No. 304's perception of how much opportunity for promotion he should have)

"should be" operational measure

FIGURE 7
THE DATA CUBE FOR THIS STUDY
(characteristics) and job satisfaction measures (operational definitions).

To show how it is not possible to analyze the three-mode job satisfaction phenomena of Figure 7 via standard analysis techniques, the following discussion will first present several examples of the application of standard (two-mode) factor analysis to the problem. The discussion will then present a three-mode analysis technique in order to illustrate how the problems of two-mode analysis techniques can be overcome.

Analyzing the three-mode data set of Figure 7 across two modes consists of "slicing" or partitioning the data cube. This partitioning defines a data set to be analyzed, it is usually referred to as a matrix. Figures 8, 10, and 12 show the ways the data cube may be partitioned. The partition may be made across each of the three modes of the data cube to create three two-mode matrices of the phenomena. The columns of the ensuing matrices are usually seen as "variables" and the rows as "cases" (Rummell, 1970). Since the data in each of the three resulting matrices can be analyzed by utilizing the rows as cases and the columns as variables, or by transposing the data matrix, utilizing the rows as variables and the columns as cases; six possible ways - or perspectives - of orientating the phenomena for analysis result.
What are each of these six perspectives, and how are they applied to the three modes of workers, job facets, and job satisfaction operational measures? These perspectives are known as "R", "Q", "O", "P", "T", and "S" factor analysis. It is important to point out that these perspectives are not factor models or techniques of factor extraction but only the factor analysis of a specific "slice" of the data cube. Each of these two-mode factor analysis techniques results in "specialized" information and each is discussed below.

Two-Mode Factor Analytic Techniques

R and Q Factor Analysis— Figure 8 shows a "cross-sectional" partition of the data cube. The resulting matrix (slice of phenomena) is for a single operational measure and can be analyzed in two ways. The first way to analyze this matrix is to use the columns (characteristics) as variables and the rows (entities) as cases (see Figure 9). This is the most commonly analyzed matrix of applied factor analysis, and is known as R-factor analysis. R-factor analysis consists of factor analyzing the matrix of Figure 8 with variables (columns) referring to job facets, and rows (cases) referring to workers. The data in this partition are all for the same operational job satisfaction measure. This analysis determines the relationships between the job facets for a single chosen operational measure.
Thus if one wished to determine the degree of equivalence, with respect to the job facets, between each job satisfaction operational measure, one would perform a series of R-factor analyses (one for each of the possible partitions of the data cube shown in Figure 8) and compare the resulting factor structures. The greater the degree of the similarity of the resulting factor structures, the more the degree of "equivalence" of the respective operational measures.* At a more intuitive level, what one has accomplished via this multiple R-factor analysis technique is an identical factor analysis for each job satisfaction operational measure while holding workers (entities) and job facets (characteristics) constant and ascertaining the degree of equivalence of the results.

When the variables and cases from the data partition of Figure 8 are interchanged (transposed), the result is Q-factor analysis. Q-factor analysis consists of factor analyzing the matrix in which the variables (columns) refer to workers and cases (rows) refer to job facets, and the data are all for the same job satisfaction operational measure (see Figure 9). The equivalence of job satisfaction operational measures, with respect to the workers, can be ascertained via use of multiple Q-factor analyses. Each of the

* The degree of similarity of factor structures can, for example, be ascertained via vector comparisons or Procrustes rotation procedures (Rummel, 1970).
FIGURE 8
THE DATA CUBE, CROSS SECTIONAL SLICE
("R" or "Q" Factor Analysis)
(After Rummel, 1970)
R- Factor Analysis

workers (cases)

job facets (variables)

a single measure (e.g., "is now")

Q- Factor Analysis

job facets (cases)

workers (variables)

a single measure (e.g., "is now")

FIGURE 9
THE "R" AND "Q" FACTOR ANALYSIS MATRIX
(The slice of the data cube shown in figure 8)
single Q-factor analyses determines the relationships between the workers for a single job satisfaction operational measure.

**O and P Factor Analysis**— O-factor analysis consists of analyzing the matrix (slice) of the data cube of Figure 10 with variables (columns) referring to the operational measures and the cases (rows) referring to the job facets, for a single worker (see Figure 11). An O-factor analysis will determine the relationship between job satisfaction operational measures (occasions) for a worker (entity). O-factor analysis would be, in general, of little interest to organizational theorists since they are seldom interested in differences amongst theoretical or operational measurement techniques for a single individual but in relationships which are applicable to groups or classes of individuals.

**P-factor analysis** consists of analyzing the matrix of Figure 10 with the variables (columns) referring to job facets (characteristics) and cases (rows) referring to job satisfaction operational measures (occasions), for a single worker (entity) (see Figure 11). P-factor analysis delineates the relationships between job facets and job satisfaction measures for a single worker. Once again this type of analysis would be of little interest to the organizational theorist since it is concerned with a single individual.

**S and T Factor Analysis**— The third slice of the data cube (see Figure 12) is vertical along the operational
FIGURE 10
THE DATA CUBE, TIME SERIES SLICE
("O" or "P" Factor Analysis)
(After Rummel, 1970)
FIGURE 11
THE "O" AND "P" FACTOR ANALYSIS MATRIX
(The slice of the data cube of Figure 10)
measures (occasions) axis. This limits the analysis to one job facet (characteristic). The two ways of analyzing this partition result in T-factor analysis and S-factor analysis. T-factor analysis consists of analyzing this matrix with variables (columns) referring to operational measures (occasions), and cases (rows) referring to workers (entities), for a single job facet (characteristic) (see Figure 13). A T-factor analysis, then, determines the relationship between operational measures for a single job facet. Twenty-three T-factor analyses would therefore be required to investigate all of the possible interrelationships between the job satisfaction operational measures and the job facets of interest in this study. Once again, as was the case with R-factor analysis and Q-factor analysis, the results of the multiple T-factor analyses could be compared.

S-factor analysis consists of analyzing the matrix of Figure 12 with variables (columns) referring to workers and cases (rows) referring to job satisfaction measures, for a single job facet (see Figure 13). S-factor analysis determines the relationship between the workers for a single job facet, and can be used to determine if individual worker differences have any significant effects. Once again, comparison of results (multiple S-factor analyses) could be used to determine the equivalence of job facets.

At this point it is worthwhile to review the three slices of the data cube and their transposes. Figure 14
FIGURE 12
THE DATA CUBE, CASE STUDY SLICE
("T" or "S" Factor Analysis)
(After Rummel, 1970)
FIGURE 13
THE "T" AND "S" FACTOR ANALYSIS MATRIX
(The slice of the data cube of Figure 12)
shows the three partitions of the data cube which have been discussed, as well as the two types of factor analyses which are appropriate for each partition.

Two-Mode Factor Analysis and Job Satisfaction Measurement

Which of these six perspectives, then, can be applied to the problem of mapping the relationships between selected measures of job satisfaction in order to mitigate the aggregation problem that has led to "...our understanding of job satisfaction not being substantially increased in the last thirty years." (Lawler, 1971, p. 205). Usually, a researcher is interested in the relationships between job facets. This information is often obtained by collapsing the data cube along the operational measures (occasion) mode and factor analyzing (R-technique) the resultant two-mode matrix (e.g., Smith, Kendall and Hulin, 1969).

Another researcher may be interested in the effects of the differences between individual workers. These differences may revolve around a single job facet such as "pay" (e.g., Lawler, 1971), or a group of job facets such as "working conditions" (e.g., Korman, 1968). In this case, the data is collapsed along the operational measures mode and O-technique utilized. A third researcher may be interested in the relationship between several job satisfaction operational measures. The data cube is then collapsed along the
FIGURE 14
THE DATA CUBE, SUMMARY OF THE THREE SLICES
(After Rummel, 1970)
worker mode and factor analyzed via O-technique or collapsed along the job facet mode and analyzed via T-technique.*

**Operational Measures Mode**—Information concerning the pattern of relationships between job satisfaction operational measures has been obtained in two ways. The first method is to collapse the data cube along the worker mode, via the use of mean or total scores, and perform an O-factor analysis. The results of this analysis, however, are not able to account for any systematic variance among workers. Consequently, the effects of individual worker differences are masked, and information is lost. This is to say that non-comparability among operational job satisfaction measures can be the result of an "artificial" person—the mean score—being analyzed. The second method (collapsing the data cube along the job facet mode, via mean or total scores, and performing a T-factor analysis) also results in a loss of information for it cannot account for systematic variance among the job facets. It thus becomes evident that it is impossible to apply two-mode factor analytic techniques to the data cube to obtain results which are not susceptible to the criticism that these results are confounded by the systematic variance of the collapsed third mode. Consequently, a source of the lack of comparability among operational

* To the best of this writer's knowledge, neither of these two procedures have, as yet, been utilized within the job satisfaction literature.
measures of job satisfaction is this third mode problem. For example, if: 1) systematic individual worker psychological differences affect the worker's responses to a job satisfaction measurement scale, 2) there are significant differences in the levels of these worker individual differences across the samples of two studies, and 3) identical two-mode factor analytic techniques which ignore the worker mode of the data cube are utilized in the two studies; it is impossible to refute the alternate hypothesis that the obtained results were merely a function of the worker individual differences.

While it is possible to obtain information concerning the pattern of individual differences in the second mode of the data cube by collapsing the cube along the job facet mode and performing an S-factor analysis, or collapsing the cube along the operational measure mode and performing a Q-factor analysis, the problem of confounded systematic variance still applies. In addition, the use of multiple Q-factor analysis or multiple S-factor analysis is of little use unless workers are assumed to be equivalent.

Job Facet Mode—In order to obtain information concerning the pattern of interrelationships among the job facets, the third mode of the data cube, an R-factor analysis or a P-factor analysis can be performed. Once again, however, the problem of confounded systematic variance applies, but this time it applies to the collapsed
operational measures mode. Also the use of multiple R-factor analyses or multiple P-factor analyses would be of little use because both of these techniques assume that all of the job satisfaction operational measures within the collapsed mode of the data cube are equivalent, and it is known that this is not the case (Wanous and Lawler, 1972, 1974).

Therefore, due to the possibility of systematic confounded variance, even though information concerning the phenomena represented in a three-mode data cube can be obtained via the use of two-mode factor analysis, the two-mode factor analysis approach does not provide information concerning the interrelationships between the conceptual variables (factors) underlying the phenomena's three modes.

A very important question, therefore, emerges at this point. This question has to do with relationships among variations over workers, job facets, and operational measures. The job satisfaction literature has not yet addressed such questions as "do certain types of workers exhibit predictable and consistently different levels of job satisfaction over various combinations of job satisfaction operational measures and job facets; and if so, what are they?" It is the answers to such questions which can provide the basis for mapping the meaning of job satisfaction. Wanous and Lawler (1972, 1974) have indicated that this mapping is necessary in order to begin to untangle the plethora of conflicting results in the job satisfaction literature.
Three-Mode Factor Analysis

It is evident, from the above discussion, that two-mode factor analytic techniques are not adequate for the investigation of the job satisfaction measurement problem. However, Tucker (1966) has developed a method known as three-mode factor analysis, which provides the means of considering simultaneously all three modes of a phenomena, and develops an index which represents the relations between the conceptually underlying (structural) variables of each of a phenomena's three modes. This technique probes each of the three modes of a phenomena for their structural variables and determines the interrelations among the structural variables of each mode.

Principal Components Analysis—Conceptually three-mode factor analysis is a three-dimensional generalization of Principal Components Analysis. Principal components analysis begins with a two-mode observed score matrix consisting of observed variables (columns) and cases (rows). For example, the responses of a number of individuals to the items of a questionnaire (see Figure 15). Letting the number of variables, depicted in Figure 15, being equal to m and the number of cases being equal to n. Principal Components Analysis reduces the m variables to a smaller number p of conceptually underlying variables called "factors". The series of mathematical events which brings this about can be
A cell entry $z_{ij}$
(The score of the first subject upon the first observed variable)

"n" Cases
(Subjects)

"m" Observed
Variables

FIGURE 15
OBSERVED SCORE MATRIX "Z" FOR PRINCIPAL COMPONENTS ANALYSIS
outlined as follows: n questionnaire items are responded to by the n individuals, the responses are intercorrelated to yield a matrix R, the entries of this R matrix are then grouped into "factors", each factor consisting of questionnaire items that correlate more highly with each other than with any other items. The "conceptual meaning" of the factors is then inferred from the content of the questionnaire items which appear within each of the factors.

Thus, principal components analysis, in general, is a method of trying to explain the relationships among a set of observed variables, by identifying hypothetical (conceptual) variables, called factors, which underlie the observed variables. The model assumes that two observed variables are correlated because they both depend on some underlying factor.

The response of an individual for one of the observed variables (i.e., a cell entry of Figure 15) is seen to depend upon two things: First, the score of this observed variable upon each underlying factor. These scores are called factor loadings and are a characteristic of the observed variable. Second, how important each factor is in determining the individual workers response on an observed variable. These second items are called factor scores and are a characteristic of the individual worker. The larger the "factor score" that an individual has upon a factor, the more important that factor is in determining that individual's
response. The larger the "factor loading" of an observed variable upon a factor, the more important that observed variable is in determining the "conceptual meaning" of that factor.

We can represent this model mathematically in the following manner:

\[ z_{ij} = s_{i1}f_{j1} + s_{i2}f_{j2} + s_{i3}f_{j3} \ldots s_{ip}f_{jp} \]  

1) Let a standardized observed score (i.e., a cell entry of Figure 15) be represented by \( z_{ij} \) where \( i \) indicates the rows (cases) of the observed score matrix and ranges from one to \( n \), and \( j \) indicates the columns (observed variables) of the observed score matrix and ranges from one to \( m \).

2) Let \( s_{ib} \) represent the factor score of subject \( i \) on factor \( b \), with \( b \) ranging from one to \( P \) (\( P \) is the number of factors, which is less than or equal to \( m \), the number of observed variables).

3) Let \( f_{jb} \) represent the factor loading of observed variable \( j \) upon factor \( b \).

The model, equation [1], can also be represented in summational notation as:

\[ z_{ij} = \sum_{b=1}^{P} s_{ib}f_{jb} \]  

or in matrix notation as:
If we wish to obtain the factor scores matrix $S$ and the factor loadings matrix $F$, we first intercorrelate the entries of the observed score matrix $Z$ to obtain the correlation matrix $R$. It can be shown that $Z = SF'$ is equivalent to $R = FF'$. $R = FF'$ can be solved to obtain $F$ and $F'$, $F'$ is then substituted back into $Z = SF'$ to solve for $S$, which results in a two-mode solution, known as Principal Components Analysis.

**Three-Mode Analysis**—However, as indicated above, a minimum of three modes of data is required to completely describe many phenomena, and a three-mode data set can be more fruitfully analyzed via three-mode factor analysis. The three-mode model simultaneously analyzes all modes of the data cube to provide four output matrices. Three of these matrices are two-modal and relate the elements of each of the three modes of the data cube (entities, characteristics, and occasions) to a smaller number of underlying, corresponding structural variables. These three matrices therefore are analogous to the three factor matrices which are derived via collapsing the data cube, in turn, across each of its three modes and performing three separate principal components analyses. Three-mode factor analysis, however, does not collapse the data set and since no information is discarded, as in separate multiple principal components analyses, the alternate hypothesis of confounded variance is
eliminated. In addition, the fourth matrix which results is three-modal and provides information as to the relationships between the structural (conceptually underlying) variables of the phenomena's three modes. In order to describe this process more fully it is first necessary to extend the mathematical notation presented in the discussion of principal components analysis and to define some new terms.

**Definitions of Terms**— The first item is a further clarification of the use of the word "mode". This term denotes "a set of indices by which data might be classified", (Tucker, 1966, p. 112). For example, scores of a sample of individuals on a battery of tests can be classified by individuals and cross classified by the tests. The individuals are the elements of one set of indices by which the scores are classified; and, therefore constitute one mode of the data. A second mode of this data is the battery of tests. Test scores are then arranged in a rectangular table with rows for individuals and columns for tests. This arrangement is termed a **two-mode matrix**. If the battery of tests were administrated to the individuals on several occasions, the set of occasions are the third mode. The data could now be arranged in a rectangular prism with the horizontal strata of cells for individuals, vertical strata parallel to the end plains for tests, and vertical strata parallel to the front plains for occasions. Such an arrangement, described above as a data cube, is termed a **three-mode matrix**.
Each mode is identified by a lower case letter. For example, the letter $i$ may be used for the mode for individuals in a sample. This lower case letter is used in several related but distinct roles, two of which are: 1) as a general identification of the mode, 2) as a subscript identifying the mode to which an element belongs. An example of the first usage is the statement "mode $i$ is for the individuals in the sample". An example of the second usage is in the assignment of identification symbols $l_i, 2_i, 3_i, \ldots, N_i$ to individuals in the sample. A three-mode observed score matrix such as shown in Figure 16 is thus denoted $X_{ijk}$.

Since it is difficult to represent threemode matrices, it is convenient to reorganize them into two-mode matrices. This is accomplished by combining two elementary modes into one mode which is termed a combination mode. The three-mode observed score matrix $X_{ijk}$ of Figure 16 is shown in Figure 17 as a two-mode matrix $(ij)X_k$, consisting of an elementary mode and a combination mode. Thus, the three-mode matrix $X_{ijk}$ can be reorganized into a two-mode matrix $(ij)X_k$ where the: 1) capital letter $X$ indicates a matrix, 2) presubscripts $i$ and $j$ refer to the rows of the matrix, 3) order of the subscripts of the combination mode ($i$ and $j$ in this case) indicate which is the outer and which is the inner index (i.e., the first subscript indicates the outer index and the second subscript indicates the inner index), and 4) post subscripts refer to the columns of the matrix.
In addition, the three-mode matrix $X_{ijk}$ can be reorganized and represented as a two-mode matrix in several different ways. For example, $X_{ijk}$ can be represented as $(ij)^X_k$ as shown in Figure 17, or as $i^X(kj)$ as shown in Figure 18, or any other combination which is needed.

Assume that we have two two-mode matrices, $A$ which has $i$ rows and $m$ columns, and $B$ which has $j$ rows and $p$ columns. Remembering the subscripting notation presented above, these matrices can be represented as $iA^m$ and $jB^p$. The Kronecker product of these two matrices, denoted by the symbol $\otimes$, is indicated by:

$$iA^m \otimes jB^p = (ij)^H(mp)$$  \[4\]

Note that the number of rows of the resultant matrix is $(ij)$ and is the product of the number of rows of $A$ and the number of rows of $B$. This is true as well for the columns. In addition, both of the modes of the resultant matrix, denoted by $H$, are combination modes. A graphic example of the Kronecker product operation, illustrated in Figure 19, shows that an element or cell entry of the resultant matrix is equal to the simple product of an element of $A$ and an element of $B$ (e.g., $1_{h1}$ equals $1_{a1}$ times $1_{b1}$).
Mode k
(occasions)

Mode i
(entities)

(characteristics)

Mode j

FIGURE 16
A THREE-MODE OBSERVED SCORE MATRIX "X_{ijk}"
(After Rummel, 1970)
FIGURE 17

THE THREE-MODE MATRIX $X_{ijk}$ REPRESENTED

AS THE MATRIX $(i\; j)^{X_k}$

(An Elementary Mode for Columns and a Combination Mode for Rows)
FIGURE 18

THE THREE-MODE MATRIX $X_{ijk}$ REPRESENTED AS THE
TWO-MODE MATRIX $i^X(kj)$

(An Elementary Mode for Rows and
a Combination Mode for Columns)
Let \( i = 2 \), \( m = 2 \), \( j = 2 \), \( p = 3 \).

\[
A_i^\text{m} \times B_j^\text{p} = (i_j)^{H(\text{mp})}
\]

\[
2^A_2 \times 2^B_3 = 4^H_6
\]

**FIGURE 19**

A GRAPHIC EXAMPLE OF THE KRONECKER PRODUCT OF TWO MATRICES
Given that we have a three-mode observed score matrix \( X_{ijk} \) (as shown in Figure 16), an analysis of the data cube via principal components analysis would derive 12 output matrices. That is, a factor scores and a factor loadings matrix for each of the six possible types of two-mode factor analytic techniques.

However, when three-mode factor analysis is applied to the same data cube, only four output matrices are derived. Three of the output matrices are two-modal and refer to the three observational modes of the observed score matrix of Figure 16 (i.e., entities, characteristics and occasions). Each of these three matrices is composed of the hypothetical variables which underlie the observational mode to which they correspond. Each set of these underlying conceptual variables is called a derivational mode. These output matrices are denoted by \( iA_m \), \( jB_p \) and \( kC_q \), with the \( iA_m \) matrix corresponding to the observational mode \( i \) and its derivational mode \( m \), the \( jB_p \) matrix corresponding to the observational mode \( j \) and its derivational mode \( p \), and the \( kC_q \) matrix corresponding to the observational mode \( k \) and its derivational mode \( q \).

Each of these three output matrices are analogous to a combination of four of the output matrices that would be obtained from six two-mode analyses of the same three-mode observed score matrix. Two of these four matrices would be factor score matrices and two of them factor loadings.
matrices. Specifically, the $iA_m$ matrix (which corresponds to the observational mode $i$) is analogous to a combination of the factor loadings matrices which would result from Q-analysis (see Figure 9) and from S-analysis (see Figure 13); and the factor scores matrices which would result from T-analysis (see Figure 13) and from R-analysis (see Figure 9). Therefore the $iA_m$ matrix contains the same information as is contained in the four matrices which result from multiple two-mode analyses. The information contained within this matrix, however, is not fragmented and is not subject to the problems of confounded variance.

This argument is also true for the $jB_p$ matrix which corresponds to the observational mode $j$, and the $kC_q$ matrix which corresponds to the observational mode $k$. Here the $jB_p$ matrix is analogous to the factor loadings matrices of P-analysis and R-analysis, and the factor scores matrices of O-analysis and Q-analysis, while the $kC_q$ matrix is analogous to the factor loadings matrices of O-analysis and T-analysis, and the factor scores matrices of P-analysis and S-analysis. Thus the $iA_m$ matrix relates the observational mode $i$ (workers) to its derivational mode $m$ (individual differences), the $jB_p$ matrix relates the observational mode $j$ (job facets) to its derivational mode $p$ (job elements), and the $kC_q$ matrix relates the observational mode $k$ (job satisfaction operational measures) to its derivational mode $q$ (theoretical bases). These relations are shown in Figure 20.
It can be seen that in the three-mode factor analysis model the three derivational modes, m, p, and q, are conceptually more basic than the modes employed in making the observations. Each derivational mode corresponds to one of the observational modes, and each derivational mode represents a set of factors in the domain of the corresponding observational mode. An alternate interpretation is to think of each derivational mode as containing conceptual categories or variables corresponding to the observational mode. Thus, if the observational mode I is used to designate individuals in a sample, the derivational mode m can be thought of as consisting of factors among individuals or of conceptual, or idealized groups of individuals. As in principal components analysis the three-mode model is prefaced upon the notion that the number of elements in each derivational mode is markedly less than the number of elements in the corresponding observational mode.

The fourth output matrix is three modal, called the core matrix and is denoted by G. The three modes of the core matrix are the three derivational modes m, p, and q. The core matrix, while three modal, is always represented as a two-mode matrix via the use of an elementary mode and a combination mode. This is denoted as:

$$m^G (pq)$$  \[5\]

The cell entries $g_{mpq}$ of the core matrix each represent a unique combination of categories from the derivational
FIGURE 20

A GRAPHIC REPRESENTATION OF THE
RELATIONS OF THE DERIVATIONAL MODE
MATRICES $iA_m$, $jB_p$ AND $kC_q$ TO THE
OBSERVATIONAL MODE MATRIX $X_{ijk}$
**Observational Mode j** (job facets)

**Derivational Mode p** (job elements)

**Observational Mode i** (workers)

**Derivational Mode m** (classes of workers)

**Observed Score Matrix** $X_{ijk}$

**A Matrix**

**B Matrix**

**C Matrix**

**FIGURE 20**
modes. The core matrix therefore describes the *basic relations between the observational modes*. The core matrix can be used to ascertain the relationship between classes of workers, job elements, and the theoretical basis of the job satisfaction operational measures. A graphical representation of the core matrix and its relations to the observed score matrix is shown in Figure 21.

In summary the three mode model can be presented as:

\[ x_{ijk} = \sum_m \sum_p \sum_q a_{im} b_{jp} c_{kq} e_{mpq} \]  

where: \( i, j \) and \( k \) are the observational modes, \( m, p, \) and \( q \) are the derivational modes or the sets of factors derived from each of the observational modes respectively.

Alternatively, in matrix form, the model can be represented as:

\[ i^X(jk) = i^A_m G(pq)(p^B_j \otimes q^C_k) \]  

Since the derivation of this model is quite complex it is discussed more completely in Appendix A.

**Idealized Individual Analysis**

Given that three-mode factor analysis can ascertain the relationships between derivational modes, the question remains concerning how best to make use of the information provided. This question can be answered, through example, by concentrating on the \( i^A_m \) matrix, (i.e., the worker derivational mode). This mode indicates a number of individual differences in the sample of workers, and gives each worker
FIGURE 21

A GRAPHIC REPRESENTATION OF THE
RELATION OF THE CORE MATRIX $G_{mpq}$ TO
THE OBSERVATIONAL MODE MATRIX $X_{ijk}$
Observational Mode $k$ (operational measures)

Derivational Mode $q$ (bases of operational measures)

Observational Mode $i$ (workers)

Derivational Mode $m$ (classes of workers)

Derivational Mode $p$ (job elements)

Observational Mode $j$ (job facets)

Core Matrix $G_{mpq}$ (cell entries contain the relationships between the derivational modes)

Observed Score Matrix $X_{1jk}$

FIGURE 21
a score on these conceptual variables. However, without a-priori knowledge as to what the differences in these scores mean for job satisfaction research, the researcher is left to speculate about the impact of a specific individual difference or set of differences. With three-mode analysis, however, this speculation process is not a blind venture because the core matrix of the three-mode model provides information about the relations between the worker derivational mode and the other two derivational modes of the phenomena. Given that the researcher has some indication as to the "conceptual meaning" of the other two derivational modes, changes in relations among the factors of these modes may be mapped by "changing" a worker's scores upon the worker mode factors. This provides specific information concerning what happens to a worker's scores on the factors of the remaining two derivational modes (job facets and measurement instruments) when his scores are systematically variated across individual difference factors. This process, then, provides information concerning the effect of individual differences upon the factors of the other two modes. Obviously, knowing the effects of the individual differences upon known or at least partially understood variables, can greatly aid the process of understanding and ultimately identify those individual differences.

Within the context of three-mode factor analytic model this type of information can be obtained through the use of
Idealized Individual Analysis. In this procedure, an individualized two-mode core matrix is computed for each of a number of hypothetical individuals based upon their scores on the worker mode factors. These two-mode core matrices contain the scores of the hypothetical individuals upon the factors of the other two modes which result from their assigned scores upon the worker mode factors. These two-mode core matrices can then be compared to observe the effect of various levels of hypothetical individual differences.

This analysis is accomplished as follows: given that the three mode model may be stated as:

\[ \hat{x}_{ijk} = \sum_{m} \sum_{p} \sum_{q} a_{im} b_{jp} c_{kq} g_{mpq} \]  

where:

1) \( \hat{x}_{ijk} \) is the fitted score of individual \( i \) on operational measurement method \( j \) at job facet item \( k \).
2) \( a_{im} \) is the loading of individual \( i \) on worker factor \( m \).
3) \( b_{jp} \) is the loading of operational measurement method \( j \) on operational measurement method factor \( p \).
4) \( c_{kq} \) is the loading of job facet item \( k \) on job facet factor \( q \).
5) \( g_{mpq} \) is the entry in the three-mode core matrix.

Given this, it can be demonstrated that a two-mode core matrix may be defined for each hypothetical individual by:

\[ (h_{i})_{pq} = \sum_{m} a_{im} g_{mpq} \]  

so that the score for this hypothetical individual may be
expressed as:

\[ \hat{x}_{ijk} = \sum_{p} \sum_{q} b_{jp} (h_{p})_{pq} c_{kq} \]  

Thus, in matrix form, the matrices B of \( b_{jp} \)'s and C of \( c_{kq} \)'s form a constant framework for workers. The matrices \( H_{i} \) of \( (h_{i})_{pq} \)'s contain the individual information for the different hypothetical workers.

A more intuitive explanation of the technical process of the Idealized Individual Analysis is as follows:

1) Make a scatter plot, or a series of bi-variet scatter plots if more than two factors are involved, of the scores of the individual workers upon the worker mode factors (i.e., plot the \( iA_{m} \) matrix).

2) Locate a number of idealized or hypothetical individuals at extreme points within the scatter plot (see Figure 22 for a simple example).

3) Calculate an individual two-mode core matrix for each of the hypothetical individuals.

4) Compare the two-mode core matrices of the hypothetical individuals to determine the effects of the individual differences.

The effect of this analysis when applied to the results of a three-mode factor analysis of job satisfaction (represented by a three-mode data set consisting of workers, job facets and operational measures) is to denote: in what manner, and to what degree, individual differences systematically affect a worker's indicated level of satisfaction with
various elements (clusters of job facets) of his job; and in what manner and to what degree individual differences systematically affect a worker's indicated satisfaction level across various types of job satisfaction measurement instruments.
Idealized individuals are indicated by a circle, their coordinates are (1, 1.5), (1,0) and (1, -1.5).

FIGURE 22
AN EXAMPLE OF THE LOCATION OF HYPOTHETICAL INDIVIDUALS FOR A WORKER MODE WITH TWO FACTORS
(A plot of an $A_m$ matrix)
CHAPTER II
RESEARCH PROBLEM AND HYPOTHESES

Research Problem

The general problem of aggregation is one of the least concrete problems facing organizational science. It consists of a number of sub-problems for which there are presently very few solutions. These problems are especially important because they are potentially the most serious problems presently confronting organizational science (Roberts et al., 1978).

The general problem of aggregation can be broken down into several components: 1) whether to aggregate at all, 2) conceptual aggregation, 3) aggregation over samples, 4) aggregation over time, 5) aggregation over measurements, 6) aggregation in data analysis, and 7) aggregation in interpretation (Roberts et al., 1978). The literature review of Chapter I has implications for three of these sub-areas of the general aggregation problem: 1) aggregation over measurement, 2) conceptual aggregation and, 3) aggregation in data analysis. These will be discussed in turn.
Aggregation in Measurement

The study of satisfaction section of Chapter I indicated that the most often utilized measures of job satisfaction attempt to tap the cognitive manifestation of this attitude in a composite manner. It is the composite nature of these measures which is relevant to the job satisfaction measurement aggregation problem. This problem contains two components which are relevant to this research. The first component evolves from the selection of a set (an aggregation) of job facets over which workers perceptions are assessed. The second component evolves from the selection of which worker perceptions (or combination of perceptions) of a given set of job facets to assess. These two components result in an equivalence of measures problem and will be discussed in turn.

Aggregating Job Facets

It is clearly impossible to investigate all of the possible combinations of the large number of job facets that have been utilized in measuring job satisfaction; a narrowing of focus is necessary. It was therefore decided to begin this research in an Apollonian manner by selecting a set of job facets which are well grounded in theory, often utilized and considered to be psychometrically valid. A set of 23 job facets derived from the work of Turner and Lawrence (1965), Hackman and Lawler (1971), and the Job Diagnostic
Survey of Hackman and Oldham (1975) was chosen on the basis of these criteria.

These job facets are well grounded in theory in that, as noted in Chapter I, both the Hackman and Lawler and Hackman and Oldham efforts are based upon the expectancy theory of motivation which is one of the most well developed theories of organizational science. The psychometric properties of these job facets have also been heavily researched (e.g., Wanous and Lawler, 1972, 1974; Wanous and Zwany, 1977; Hackman, Oldham, Janson, and Purdy, 1975) and been found to consist of the three theoretically defined factors of "meaningfulness", "autonomy/responsibility" and "feedback".

These 23 job facets therefore represent a well defined and well understood base from which to begin a Dionysian foray into the unknown.

The Equivalence of Operational Measures Problem

In making comparisons across job satisfaction studies one glaring fact can be noted - the lack of common worker perceptions utilized for measuring this attitude. To illustrate the chaos that can result from the failure to adopt common standards, we need only to outline the variety of measures that can be utilized to define a change in job satisfaction level. Job satisfaction change may be described in terms of any one of the following measures: 1) percentage of workers showing any positive change at all,
2) percentage of workers showing "large", "moderate", "small", or "no change" (categories arbitrarily defined), 3) net percentage change (positive minus negative changes), 4) any of the above for an arbitrarily determined combination of opinion items, 5) the absolute need scale distance changed, 6) distance changed relative to amount of change possible, and 7) scale distance change weighted (corrected) for the subjective distance between scale points (e.g., two units moved across neutral is "worth more" than two units within one side of the scale) (Zimbardo, Ebbesen and Maslach, 1969).

In the weakest sense equivalence of measures means comparability of score distributions. Two "equivalent" measures have the same means and dispersions, as in the case of two equated intelligence tests. This type of comparability has been defined in terms of identical true-score distributions of measures. But there is another requirement for meaningful equivalence, that the content of the measures be similar. Otherwise by arithmetic manipulations one can create comparable score distributions for an intelligence test and eye color; it is obvious that these two examples are not equivalent.

There is also a much stronger sense of equivalence in which equivalent measures have identical true scores but no overlap of error variance (Thorndike, 1951). This stricter requirement states that two equivalent measures of job
satisfaction share a common factor analytic composition (i.e., factorial invariance) (Rummel, 1970). The specific requirements for obtaining such a factorial equivalence across two measures can include: similar item types, the same difficulty levels for items, the same procedures for item selection and the same content distribution. In the extreme, one has essentially two identical measures. In general, then, equivalence requires that measures behave in the same way for individual and situational changes about which generalizations are going to be attempted. Lack of equivalence implies that given the circumstances where two investigators were attempting to test precisely the same hypothesis with all other variables being operationalized in the same manner, yet operationalizing job satisfaction by way of two different measurement techniques, there is a distinct possibility that such research would simply add to the confusion of conflicting results in the literature rather than advance knowledge (e.g., Evans, 1969; Smith, Kendall and Hulin, 1969; Wanous and Lawler, 1972, 1974; Wall and Payne, 1973).

In other words, empirical studies have used different transformation techniques (i.e., aggregation techniques) to change common, constant worker responses to a common set of job facets into quantitatively derived variables. Thus, because researchers often declare that the measure of job satisfaction that they are using is measuring what it is
measuring and proceed upon their way, the literature is replete with measures which are often not equivalent.

Thus it could be argued that a step toward the possibility of "sorting out" some of the conflicting results in the extant job satisfaction literature would be to begin to formulate the pattern of interrelationships between various and often utilized operational measures of job satisfaction which have a common respondent and job facet base. Such a set of relations should provide the information required to, at least in part, reconcile some of the conflicting results which presently exist within the literature. This portion of the research is a Dionysian attempt to formulate such a pattern of interrelationships between a selected number of job satisfaction operational measures. The pattern of interrelationships to be investigated is three-mode factor analytic in nature and thus goes beyond the correlational types of interrelationships which have been studied thus far (e.g., Wanous and Lawler, 1972, 1974).

Job facets--Which operational measures of job satisfaction to include in this investigation now becomes a problem. As was indicated above, it is unequivocably clear from their preponderance within the literature that the composite as opposed to global job satisfaction measures deserve first consideration, and that a common characteristic of these composite measures is that they revolve around a set of job facets. However, in order to make comparisons between
measures it is necessary to have a common set or pool of job facets to be utilized across all of the operationalizations. As indicated above, a set of 23 job facets derived from the work of Turner and Lawrence (1965), Hackman and Lawler (1971) and Hackman and Oldham (1975) was chosen. These 23 job facets are as follow:

1. The feeling of self-esteem or self-respect a person gets from being in my job.
2. The opportunity for personal growth and development in my job.
3. The prestige of my job inside the company (that is, the regard received from others in the company).
4. The amount of close supervision I receive.
5. The opportunity for independent thought and action in my job.
6. The feeling of security in my job.
7. The opportunity to find out how well I am doing my job.
8. The prestige of my job outside the company (that is, the regard received from others not in the company).
9. The opportunity to complete work I start.
10. The opportunity to do challenging work.
11. The feeling that I know whether I am performing my job well or poorly.
12. The opportunity to do a number of different things.
13. The opportunity on my job to get to know other people.
14. The opportunity to do a job from the beginning to the end (that is, the chance to do a whole job).
15. The freedom to do pretty much what I want on my job.
16. The amount of variety in my job.
17. The pay for my job.
18. The feeling of worthwhile accomplishment in my job.
19. The opportunity, in my job, to give help to other people.
20. The opportunity, in my job, for participation in the determination of methods, procedures, and goals.
21. The opportunity to develop close friendships in my job.
22. The opportunity for promotion.
23. The amount of respect and fair treatment I receive from my boss.

**Operational Measures**—As was the case with job facets, it is impossible to investigate all of the possible job satisfaction operational measures, this necessitates a further narrowing of focus. Two criteria are utilized for the selection of job satisfaction operationalizations to be investigated. First is the frequency of use of a measure within the literature and second is the degree of controversy over or interest in the measure as personified by articles within the literature. These criteria led to the selection of the following two types of measures.

1. Fulfillment based measures,
   - Job facet satisfaction (JFS)
   - Porter's need satisfaction (Is Now)
2. Discrepancy based measures,
   - Discrepancy-difference (Would Like-Is Now)
   - Discrepancy-equity (Should Be-Is Now)
Conceptual Aggregation

Conceptual aggregation refers to the effect of discipline oriented paradigms upon theory and research development. The following is an example of conceptual aggregation. To varying degrees people in organizations follow rules. Psychologists observe this response and develop theories of conformity to explain it. Theories of conformity are closely tied to observations of individual rule following. Observing the same response sociologists develop the construct "formalization" and develop a theory about how formalization develops. The theory is distant from the observations. Conceptual aggregation is thus linked to the levels of analysis associated with the various discipline oriented paradigms.

To clarify the role that conceptual aggregation plays in organizational studies it is necessary that organizational scientists explicitly state the conceptual aggregations that they make. Organizational scientists' failure to do this has resulted in a literature replete with contradictory results that are literally impossible to integrate (Roberts et al. 1978).

As was indicated in Chapter I, the literature on job satisfaction is a prime example of this problem for two basic reasons. First, even though nearly all researchers of job satisfaction consider it to be an attitude, few have explicitly linked their theories and research to the
attitude literature—there has been an insistence upon re-inventing the wheel. Second, practically all job satisfaction researchers are psychologists and most of them are also Apollonians. This has resulted in, as was indicated in the Multidimensional Approaches to Organizational Phenomena section of Chapter I, persistent attempts to force the three-dimensional job satisfaction attitude into a two-dimensional conceptual space. This research attempts to avoid these errors.

Aggregation in Data Analysis

The majority of writings on aggregation focus upon its effects on statistical data analysis (e.g., Blalock 1964; Hammond 1973). Aggregation must be considered in interpreting results of data analysis because the way data are grouped more often than not affects the values that statistics take. The confounded variance of the third-mode problem, described in the Multidimensional Approaches to Organizational Phenomenon section of Chapter I, clearly indicated the importance of this aggregation problem in job satisfaction research. Three-mode factor analysis is a valuable statistical tool in overcoming this problem.

Hypotheses

The three observational modes of the data cube for this study consist of 304 workers (entities), 23 job facets
(characteristics), and 8 job satisfaction operational measures (occasions). This section examines each of these three modes in light of current literature in order to arrive at testable propositions and derived hypotheses.

The Job Satisfaction Operational Measures Mode

The job satisfaction operational measures mode contains 8 possible operational definitions of job satisfaction. Overall job satisfaction (JS) has been operationalized as the sum of a worker's indicated level of satisfaction across a number of job facets (JFS) as shown in equation [11].

$$JS = \sum (JFS)$$  \[11\]

For example, Ewen (1967) summed satisfaction scores from the five components of the Job Descriptive Index (JDI) (Smith, Kendall and Hulin, 1969) and correlated the sums with two measures of overall job satisfaction, the Brayfield-Roth Scale (Brayfield and Roth, 1951) and the General Motors Faces Scale (Kunin, 1955). The correlations between the JDI sums and the two measures of overall job satisfaction range from .50 to .74 for three separate samples. In addition, Schaffer (1953) reported a correlation of .44 between mean satisfaction for 12 needs and an overall satisfaction measure. Thus, overall job satisfaction has been viewed as a function of the sum of job facet satisfaction and is a fulfillment measure.
Job satisfaction has also been operationalized as the sum of goal attainment or need fulfillment across job facets. In Porter's (1961) terms, goal attainment or fulfillment can be thought of as a response to a "How much is there now" item for a particular job facet. Alderfer (1969) has also measured job facet satisfaction by having workers agree or disagree on a 6 point scale with descriptive statements of their jobs. Both are fulfillment measures. Hence, the use of "Is Now" in equation [12].

\[ JS = \sum_{\text{facets}} (\text{Is now}) \]  

[12]

Job satisfaction has been operationalized as an equity-discrepancy as shown in equation [13]. Porter (1961) operationally defined satisfaction as the difference between responses to a "How much is there now" item, and responses to a "How much should there be" item, when these two items are asked over a number of job facets. The difference between these two types of items is computed by the researcher, and the differences are summed across the job facets to yield a measure of overall job satisfaction.

\[ JS = \sum_{\text{facets}} (\text{Should Be} - \text{Is Now}) \]  

[13]

Locke (1969) also argues for the use of a discrepancy equation. He believes that only unfulfilled desires can cause dissatisfaction and that satisfaction is the result of a comparison between fulfillment ("Is Now" in this study)
and desires or ideals ("Would Like" in this study). Thus we have the difference-discrepancy measure of equation [14].

\[
JS = \sum \frac{\text{facets researcher computed}}{\text{(Would Like - Is Now)}}
\]  

[14]

As noted earlier, a number of psychometricians have pointed out the problems that surround the use of discrepancy scores in psychological research, such as those defined in equations [13] and [14] above, (e.g. Lord, 1963; McNemar, 1958; Werts and Linn, 1970).

Wall and Payne (1973) have drawn particular attention to the way in which discrepancy score datum may be misinterpreted in job satisfaction research by describing a logical and a psychological constraint, and conclude that job satisfaction discrepancy scores as traditionally operationalized (e.g., equations [13] and [14] above) are not more than the sum of their parts.

Conceptually however, they have been treated as such. Nevertheless, the conceptual logic behind the discrepancy score operationalization has considerable support within the literature (e.g., Porter, 1961, 1962; Locke, 1969; Wall and Payne, 1973). Wall and Payne (1973, p. 326) conclude that:

The operationalization of the concept (of job facet satisfaction discrepancy scores) might be better achieved (while continuing to use the present questionnaire format), by allowing our subjects to do their own arithmetic. That is: How much is there now? How much would you like? And (having considered the above two questions), How satisfied are you? It is just possible that in the mathematics
of affect, 7-5 does not necessarily equal 2. The usefulness of this alternative needs to be explored.

Thus two more operational definitions of job satisfaction, which represent Wall and Payne's (1973) suggested operationalization, are included in this study and are shown in equations [15] and [16] respectively.

\[
JS = \sum_{\text{facets}} (\text{Should Be} - \text{Is Now}) \quad [15]
\]

\[
JS = \sum_{\text{facets}} (\text{Would Like} - \text{Is Now}) \quad [16]
\]

Finally, two more operational measures of job satisfaction were included as shown in equations [17] and [18].

\[
JS = \sum_{\text{facets}} (\text{Should Be}) \quad [17]
\]

\[
JS = \sum_{\text{facets}} (\text{Would Like}) \quad [18]
\]

These formulations, which represent component parts of equations [13], [14], [15] and [16], presented above, were included in the analysis in order to determine if equation [17] relates differently to equations [13] and [15], and to determine if equation [18] relates differently to equations [14] and [16] -- thus providing some information in relation to the Wall and Payne (1973) proposition.

**Proposition One, Operational Measures Mode**— If the eight job satisfaction operational measures included in this mode of the data cube are in fact measuring what their advocates contend they are measuring, two conceptual underlying
variables (factors) should be derived. Specifically, in terms of the three-mode factor analysis model, the output $jBp$ matrix should contain two factors. These two factors should indicate the existence of a fulfillment measure (Porter, 1961) and a discrepancy measure (Porter, 1961; Locke, 1969).

**Hypothesis 1-A**— If two factors are in fact derived from the operational measures observational mode, these factors should be empirically independent (i.e., orthogonal). The fulfillment and discrepancy operational job satisfaction measures are supposedly independent and conceptually distinct formulations of the unidimensional-bipolar conceptualization (Locke, 1969; Wanous and Lawler, 1972; Wall and Payne, 1973).

**Hypothesis 1-B**— The "Is Now" operationalization, equation [12] above, should load heavily on the fulfillment factor and near zero on the discrepancy factor. Porter (1961) has argued that the "Is Now" operationalization is in fact a fulfillment measure.

**Hypothesis 1-C**— The "Job Facet Satisfaction" operationalization, equation [11] above, should load heavily on the fulfillment factor and near zero on the second factor. Wanous and Lawler (1972, 1974) report average correlations between "Is Now" scores and "Job Facet Satisfaction" scores of .60, .71, and .60. Thus there is some indication that this operationalization like Porter's (1961) "Is Now" operationalization is conceptually a fulfillment measure.
Hypothesis 1-D— The "Should Be" and the "Would Like" operationalizations, equations [17] and [18] above, should load heavily on the discrepancy factor and near zero on the fulfillment factor. These concepts supposedly measure "desires" or "ideal standards" and as such are not measures of fulfillment (Porter, 1961; Locke, 1969).

Hypothesis 1-E— The "Should Be - Is Now" and the "Would Like - Is Now" operationalizations where the computation is carried out by the researcher, (equations [13] and [14] above), represent a discrepancy approach to job satisfaction. If in fact these operational definitions are measuring what their advocates (e.g., Wood, 1970; Locke, 1969; Porter, 1961) say they are measuring, they should load heavily on the discrepancy factor and near zero on the fulfillment factor.

Hypothesis 1-F— The "Should Be - Is Now" and the "Would Like - Is Now" operationalizations which are generated subjectively by the worker should load heavily on the discrepancy factor and near zero on the fulfillment factor. This is a direct test of Wall and Payne's (1973) proposition quoted above.

The Job Facet Mode

The 23 job facets which comprise the second observational mode were derived from the work of Hackman and Lawler (1971) and Hackman and Oldham (1975). These authors, as
previously described, provide a model of job characteristics consisting of three components; meaningfulness, autonomy/responsibility, and feedback.

**Proposition Two, Job Facet Mode**—The job facet observational mode should resolve into three conceptually underlying variables. Specifically, in terms of the three-mode factor analysis model, the output $kC_q$ matrix should contain three factors.

**Hypotheses 2-A**—If three factors are in fact derived from the job facet observational mode, they should be readily identifiable as the three "critical psychological states" of "meaningfulness," "autonomy/responsibility," and "feedback" of Hackman and Oldham (1975).

**Hypothesis 2-B**—Hackman and Oldham (1975) have conceptualized the three "critical psychological states" as being independent (i.e., this is an implicit assumption of the linear combination format of the Motivating Potential Score model). Hence, if three factors are derived from the job facet mode, they should be empirically independent (i.e., orthogonal).

**The Worker Mode**

The output $iA_m$ matrix of the three-mode analysis will provide a number of conceptual individual differences among the sample of workers. Even though the three-mode analysis does not give direct indication as to the nature of the
individual difference variables which it identifies within the data, one can seek the identity of these differences by relating the scores which the workers obtain upon these factors to other information about the workers.

There are two basic classifications of information about individual differences which have historically been hypothesized to affect a worker's reaction to his job, psychological variables (e.g., growth needs, intrinsic/extrinsic motivation, internal/external orientation) and demographic variables (e.g., urban versus rural background, age, marital status, number of dependants).

Information concerning several worker demographic variables was available from the company's personnel records. These include: 1) years with the company, 2) educational level, 3) number of dependants, 4) age, 5) marital status, 6) number of previous factory work experiences, 7) job classification, and 8) absenteeism.

Information on several psychological variables was also obtained, these include: 1) internal/external locus of control (Rotter, 1966), 2) social desirability (Crown and Marlow, 1964), 3) job involvement (Lodahl and Kejner, 1965), 4) growth need strength (Hackman and Lawler, 1971), and 5) performance evaluation (immediate supervisor rating).

Proposition Three - Worker Mode—Only one individual difference variable, Growth Need Strength (Hackman and Lawler, 1971; Hackman and Oldham, 1975), has obtained
reasonable support for its effect upon a worker's reaction to characteristics of his job. It therefore appears that the only viable literature based proposition concerning the worker observational mode is that one of the individual difference variables derived by the three-mode factor analysis should be identifiable as the level of a worker's growth needs.

**Hypothesis 3-A**-- There should be a significant positive correlation between the workers' scores upon one of the individuals difference factors and their scores upon Hackman and Lawler's (1971) Growth Need Strength Scale.
CHAPTER III
METHODOLOGY
Research Setting and Subjects

Data was collected from three divisions of a midwestern manufacturing company from 354 blue collar and clerical employees. The sample is 96 percent male. Jobs surveyed include: secretary, press operator, welder, driver, mill operator, shipping clerk, quality control, painter, maintenance. The jobs ranged widely in complexity and in the level of employee skill required.

Data Collection Procedure

Data were collected via three methods: 1) on site administration of a questionnaire, 2) a mail questionnaire and, 3) an examination of the company's personnel records.

On site questionnaire

On site questionnaire data were collected at each of the company's three divisions over a two-day period (48 continuous hours). During the first 24 hour period each of three work shifts was administered a questionnaire which required about 30 minutes to complete. The administration was accomplished via a continuous series of administrations to
"logical work groups" of four to fifteen employees. During the second 24-hour period an attempt was made to obtain data from all those employees who were not contacted the previous day. During all administrations a standard set of instructions was read to the subjects. Data for 92 percent of the workers were obtained.

The questionnaire consisted of a single legal size sheet which asked the workers to rate each of 23 job facet items six different times. Seven-point scales were used. This was accomplished by listing the 23 facet items down the left side of the page and providing six columns to the right of the items. Each column contained different instructions, which were as follow:

Column 1: "How much of each quality or characteristic is present on your job?"

Column 2: "How much of each quality or characteristic do you think should be associated with your job?"

Column 3: "How much of each quality or characteristic would you like to be associated with your job?"

Column 4: "Considering your answers to columns one and two how satisfied are you with these aspects of your job?"

Column 5: "Considering your answers to columns one and three how satisfied are you with these aspects of your job?"

Column 6: "How satisfied are you in general with these aspects of your job?"

Two further measures were derived by computing the difference between columns one and two above (i.e., Should Be -
Is Now), and columns one and three above (i.e., Would Like - Is Now). In addition, a direct rating measure of overall job satisfaction was obtained via a response to a single item, "How satisfied are you in general with your job as a whole?" which was located on the reverse side of the questionnaire. The questionnaire and the instructions utilized for its administration are shown in Appendix B.

In addition, scores for the Hackman and Lawler (1971) twelve item Growth Need Strength Scale were obtained from this questionnaire.

**Mail Questionnaire**

Information on several psychological variables was obtained via a mail questionnaire which was sent out after the onsite data collection. Each worker was sent a six page questionnaire including a cover letter. Two additional mailings (follow-ups) were employed. The first follow-up started 30 days after the original mailing. The second follow-up started 30 days after the first follow-up. Each follow-up contained a questionnaire. All mailings also included a stamped self-addressed reply envelope. Outgoing and return postage was by first class stamped mail. The total response rate was 223 of the possible 304 or 73%. The questionnaire and cover letter are shown in Appendix B.

In this manner, information was obtained for: 1) internal/external locus of control (Rotter, 1966), 2) social
desirability (Crown and Marlow, 1974), and 3) job involvement (Lodahl and Kejner, 1965).

Data from Company Personnel Records

Information concerning several worker demographic variables was available from the company's personnel records. These include: 1) years with the company, 2) educational level, 3) number of dependants, 4) age, 5) marital status, 6) number of previous factory work experiences, 7) job classification, and 8) absenteeism.

A worker performance evaluation score (immediate supervisory rating) was also obtained. The performance evaluation scale utilized is shown in Appendix B.

Data Configuration

Since the three-mode factor analysis program utilized is incapable of accepting missing data, all subjects with missing data were eliminated from the analysis. Three hundred and four of the 354 subjects' data were usable (86 percent). This data results in a three-mode rectangular data set with one mode representing the 304 workers (entities), a second mode representing the 23 job facet items (characteristics), and a third mode representing the eight operational job satisfaction measures (occasions).
Computational Procedure, the Three-Mode Analysis

Computations were accomplished via Tucker's (1966) method-III. Since no extant computer program was of sufficient size to accommodate the data set, a FORTRAN-G program was created to carry out the analysis. A computer listing of the compiled program appears in Appendix C. Since this FORTRAN-G program is very complex and relatively untested, the data was also analyzed via a series of linear algebra manipulations as an accuracy/error check -- the results of the two methods of analysis are accurate to the third decimal place.

Choosing the Number of Factors

The problem of deciding how many factors are significant in some sense, and therefore set the number of factors to be retained for further analysis, has not been completely solved within the context of the three-mode factor analysis model (Tucker, 1966). The best procedure presently available is to make the plot between factor number and factor root size for the two-mode output matrices. Then inspect the resulting series of points for a break from a steep slope to a more gentle slope and retain all factors whose roots precede this break.
Factor Rotation

Having chosen the number of factors, the rotation of the factors to simple structure is at hand. The simple structure goal of factor rotation is to rotate the factors (represented by vectors in n-space around their origin until each factor is maximally colinear with a distinct cluster of variables, which are also represented by vectors. This shifts the factors, which as a result of factor analytic extraction techniques, maximize the total variance explained to factors delineating distinct groups of highly intercorrelated variables.

For example, consider the two orthogonal unrotated factors $S_1$ and $S_2$ of Figure 23. The simple structure goal of rotation is to rotate the factors about their origin to delineate the two groups of variables (denoted I and II in Figure 23). This rotation is shown graphically in Figure 24, in which the new factor positions $S_1^*$, and $S_2^*$ clearly define the two clusters of the eight variables (delineated $X_1$ to $X_8$).

Rotating factors so that they define distinct clusters of intercorrelation has resulted in several explicit simple structure criteria. These were developed by Thurstone. There are five requirements that simple structure should satisfy (Thurstone, 1947, pp. 335):

1. Each variable should have at least one zero loading in the factor matrix.
1) The factors are denoted $S_1$ and $S_2$
2) The vectors of the variables are denoted $X_1$, to $X_8$
3) There are two clusters of variables denoted I and II

FIGURE 23
A GRAPHIC REPRESENTATION OF TWO UNROTATED FACTORS OF EIGHT VARIABLES
(Source Rummel, 1970)
1) The rotated factors are denoted $S_1^*$ and $S_2^*$.

**FIGURE 24**
A GRAPHICAL ORTHOGONAL ROTATION OF THE FACTORS OF FIGURE 23
(Source Rummel, 1970)
2. For a factor matrix of \( p \) factors, each column of factor loadings should have at least \( p \) variables with zero loadings.

3. For each pair of columns of loadings (factors), several variables should have zero loadings in one column but not in the other.

4. For each pair of columns of loadings (factors), a large proportion of the variables should have zero loadings in both columns.

5. For each pair of columns of loadings (factors), only a small proportion of variables should have nonzero loadings in both columns.

These conditions insure that factors will be rotated to positions identifying distinct clusters of variables. A simple structure rotation exhibits the pattern of loadings depicted in Figure 25.

It is important to note that the two rotated factors of Figure 24 have remained graphically perpendicular (i.e., the angle between them is \( 90^\circ \)). This indicates that the factors are statistically independent or orthogonal. If two factors are rotated in such a manner that they are not perpendicular, they will not be independent and are termed oblique. Thus, there are two basic types of factor rotation procedures: orthogonal procedures which require that the rotated factors remain independent, and oblique procedures which do not make this requirement. Orthogonal rotation is therefore
FIGURE 25

THE PATTERN OF LOADINGS OF A SIMPLE STRUCTURE ROTATION
a special case of oblique rotation. The independence of a set of factors can therefore be empirically tested by rotating them by both orthogonal and oblique methods. If the orthogonally and obliquely rotated factor structures are the "same" the factors are empirically independent.

The "sameness" of the results of such a comparison can be determined by way of Procrustes matrix rotation procedures or vector comparison procedures (Rummel, 1970). An example of a vector comparison procedure is the Root Mean Square Coefficient (Harman, 1967, p. 269). This coefficient compares a factor from one matrix to a factor from another matrix. The factors are treated as vectors in a Euclidian space and the coefficient measures the distance between the two factors within a proportionality constant. If the distance is zero, the two factors are similar in magnitude and direction. Thus, as this coefficient parts from zero the two factors are less alike, a coefficient of less than 0.1 can be considered small (Rummel, 1970). The coefficient is denoted by the capital letter U with two post subscripts. The first subscript refers to the factor of the first matrix and the second subscript refers to the factor of the second matrix.

**Factor Interpretation**

Although the interpretation phase of any factor analysis research design has no data preparation or computational
function, it is one of the most important steps of any factoring process. Results have to be interpreted, meaning has to be given to the factor structures, labels have to be given to the factors, and the findings have to be given a visual representation as an aid to their understanding and communication to other researchers.

The perspective that a researcher has upon his factor results colors their interpretation. Basically, three perspectives have been identified (Cattell, 1957). The first perspective considers the factors as descriptive of the interrelationships within the data. The factor structure is then a typology, and the factors are classifications to which descriptive names are assigned. The second perspective is a causal approach, in which the factors are looked at as underlying causes of interrelationships delineated and are causally labeled. The third perspective is symbolic, that is, the factors represent new concepts or variables that are designated by algebraic symbols only.

Several criteria are relevant to the naming of factors regardless of the approach which is taken. One of the most important reasons for naming a factor is to communicate to others. The names should encapsulate the substantive nature of the factor and enable others to grasp its meaning. Factor names that communicate the essence of the results are important in enabling the rapid identification of similar factors across studies. The factor names can also serve a
heuristic function. The name can be theoretically suggestive or invoke hypothesis for further testing. Moreover, it can relate to the major theoretical issues of the field, stimulate wide interest, and promote additional research. A third criteria of factor naming is future use. The purpose of the research and the subsequent use of the factors should govern the labeling. If the factor analytic study is classificatory, factor names should be selected that are descriptive of the loadings. And finally, the choice of factor names should be related to the basic purpose of the factor analytic study. If the goal of the study is to describe or simplify the complex interrelationships in the data, which is the purpose of this study, a descriptive factor label should be applied. The descriptive approach to factor naming involves selecting a label that best reflects the substance of the variables loaded highly and near zero on a particular factor. The factors are classificatory and names to define each category are sought.

There are a number of considerations involved in descriptively naming factors, three of which are directly relevant to this study (Rummell, 1970). First, those variables with a zero or near zero loadings are unrelated to the factor. In interpreting a factor these unrelated variables should also be taken into consideration. The name should reflect what is as well as what is not involved in a factor. Second, some factors that are difficult to name can be
better interpreted by reversing the sign of some of the loadings. Reversing the sign for a variable has the effect of reversing the scaling. And third, for oblique factors, the primary pattern matrices are best for interpretation. They display the saturation of the variables with the factors, whereas the loadings of primary structure matrices give the correlations of the variables with the factors and are influenced by the interactions between the factors (Cattell, 1962).

Idealized Individual Analysis

The idealized individual analysis was undertaken in order to discern the effect of the individual difference factors (the worker observational mode), if any, upon the relationships between the factors of the other two modes.

The analysis was carried out by first systematically locating eight hypothetical workers within a scatter plot of the 304 workers' scores upon the two individual difference variables (see Figure 26). Second, two-mode core matrices were calculated for each of the hypothetical workers via the technique described in Chapter I. Third, plots of the changes between the hypothetical workers' two-mode core matrix scores, in relation to changes in the level of the individual difference variables, were made. This was done in order to discern the effect of changes in level of the
individual difference variables upon the relationships between the factors of the operational measure and job facet modes.
FIGURE 26

THE LOCATIONS OF THE EIGHT HYPOTHETICAL WORKERS
CHAPTER IV

RESULTS AND DISCUSSION

The Three-Mode Analysis

As in any factor analytic method, there are several decisions which must be made in the course of the three-mode analysis. These decisions and their justifications will now be presented.

Within Computational Method-III all "non-zero or almost non-zero" eigen values are to be retained when determining the $p_1 p_1$, $q_1 q_1$, and $m_1 m_1$ matrices. In order to be conservative only roots that were unquestionably zero were dropped. Only two such roots developed. These were the seventh and eighth roots of the $jP_j$ matrix; all other roots were retained until the final step of the Method-III procedure.

Number of Factors

After the $i_1 A_m$, $j_1 B_p$, $k_1 C_q$, and $p_1 q_1 G_m$ matrices have been computed, the number of factors to be extracted must be determined in order to obtain the $iA_m$, $jB_p$, and $kC_q$ matrices and thus fixing the $pq G_m$ matrix. The characteristic root number versus root size plot for the $jP_j$ matrix, shown in Figure 27, gives a very clear indication that two factors...
are appropriate. The interpretability of the rotated two factor solution (discussed below) also supports this contention.

The plot of the characteristic roots of the $kQ_k$ matrix, shown in Figure 28, would indicate that three factors are appropriate. However, in order to be doubly sure of this choice, both a three-factor and a four-factor solution were subjected to both orthogonal and oblique rotation procedures. The four-factor solution was discarded when it was discovered that the fourth factor exhibited a significant loading upon only one of the twenty-three job facets.

The plot of the first ten characteristic roots of the $(jk)^R(jk)$ matrix, shown in Figure 29, would indicate a choice of either two or three factors for the $iA_m$ matrix. Two factors were chosen when it was discovered that the third factor was correlated 0.84 with the first factor and only accounted for 0.81 percent of the cumulative variance. Thus, the third factor was considered to be a residual and therefore dropped.

To summarize, the $iA_m$ (304x2) matrix relating the 304 subjects to conceptualized individual differences contains two factors. The $jB_p$ (8x2) matrix relating the eight operational measurement techniques to theoretical differences in these techniques contains two factors. The $kC_q$ (23x3) matrix relating the 23 job facet items to theoretical differences in these items contains three factors.
FIGURE 27

PLOT OF EIGEN VALUE VERSUS EIGEN VALUE NUMBER FOR THE \( \hat{J}^P_J \) MATRIX
<table>
<thead>
<tr>
<th>Points</th>
<th>Eigen Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>42.043</td>
</tr>
<tr>
<td>2</td>
<td>37.1</td>
</tr>
<tr>
<td>3</td>
<td>27.2</td>
</tr>
<tr>
<td>4</td>
<td>19.2</td>
</tr>
<tr>
<td>5</td>
<td>17.7</td>
</tr>
<tr>
<td>6</td>
<td>15.7</td>
</tr>
<tr>
<td>7</td>
<td>15.0</td>
</tr>
<tr>
<td>8</td>
<td>13.7</td>
</tr>
<tr>
<td>9</td>
<td>12.5</td>
</tr>
<tr>
<td>10</td>
<td>11.7</td>
</tr>
<tr>
<td>11</td>
<td>10.9</td>
</tr>
<tr>
<td>12</td>
<td>10.5</td>
</tr>
</tbody>
</table>

**FIGURE 28**

PLOT OF EIGEN VALUE VERSUS EIGEN VALUE NUMBER FOR THE $kQ_k$ MATRIX
<table>
<thead>
<tr>
<th>Points</th>
<th>1</th>
<th>414</th>
<th>4.3</th>
<th>6</th>
<th>10</th>
<th>9.9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
<td>67.6</td>
<td></td>
<td>7</td>
<td></td>
<td>9.9</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>19.8</td>
<td></td>
<td>8</td>
<td></td>
<td>8.8</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>12.4</td>
<td></td>
<td>9</td>
<td></td>
<td>7.7</td>
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<tr>
<td></td>
<td>5</td>
<td>11.7</td>
<td></td>
<td>10</td>
<td></td>
<td>7.6</td>
</tr>
</tbody>
</table>

**FIGURE 29**

PLOT OF EIGEN VALUE VERSUS EIGEN VALUE NUMBER FOR THE $^{(jk)}R^{(jk)}$ MATRIX
Factor Rotation

Operational Measures Mode— The \( jB_p \) matrix denoted by the eigen value plot of Figure 27 clearly contains two factors. Consequently, these two factors were rotated using both orthogonal (Normal Varimax) and oblique (Binormamin) methods. Both methods produced the "same" factor structure/pattern. A comparison of the structure and pattern matrices, the transformation matrices, and as well, an inspection of the intercorrelations of the obliquely rotated factors, indicates that the two factors contained within the \( jB_p \) matrix are empirically independent (see Table 1).

As a means of comparing these factors for orthogonality root mean square coefficients were calculated for the results of the two rotation procedures, these are:

\[
U_{11} = 0.005 \\
U_{22} = 0.003
\]

These root mean squares are so small as to leave no doubt that the two factors are independent.

Job Facet Mode— The \( kC_q \) matrix was also subjected to both Varimax and Binormamin rotations for both a three and four factor solution. An examination of the output of each method (shown in Tables 2 and 3) provides further support for the contention that three factors are appropriate.

A comparison was undertaken to determine the degree of "sameness" of the results of the three factor Varimax and Binormamin rotations of the \( kC_q \) matrix.
TABLE 1

ROTATION COMPARISON OF THE $jB_p$ MATRIX

(OPERATIONAL MEASURES MODE)

**Input Matrix of Factor Loadings**

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.380</td>
<td>0.285</td>
</tr>
<tr>
<td>2</td>
<td>-0.427</td>
<td>-0.241</td>
</tr>
<tr>
<td>3</td>
<td>-0.439</td>
<td>-0.334</td>
</tr>
<tr>
<td>4</td>
<td>-0.395</td>
<td>0.165</td>
</tr>
<tr>
<td>5</td>
<td>-0.397</td>
<td>0.165</td>
</tr>
<tr>
<td>6</td>
<td>-0.399</td>
<td>0.180</td>
</tr>
<tr>
<td>7</td>
<td>-0.046</td>
<td>-0.526</td>
</tr>
<tr>
<td>8</td>
<td>-0.059</td>
<td>-0.619</td>
</tr>
</tbody>
</table>

**VARIMAX**

<table>
<thead>
<tr>
<th>Factor Structure</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.095</td>
<td>0.465</td>
</tr>
<tr>
<td>2</td>
<td>0.400</td>
<td>0.283</td>
</tr>
<tr>
<td>3</td>
<td>0.489</td>
<td>0.255</td>
</tr>
<tr>
<td>4</td>
<td>0.019</td>
<td>0.428</td>
</tr>
<tr>
<td>5</td>
<td>0.019</td>
<td>0.429</td>
</tr>
<tr>
<td>6</td>
<td>0.007</td>
<td>0.437</td>
</tr>
<tr>
<td>7</td>
<td>0.496</td>
<td>-0.182</td>
</tr>
<tr>
<td>8</td>
<td>0.585</td>
<td>-0.210</td>
</tr>
</tbody>
</table>

**BINORMAMIN**

<table>
<thead>
<tr>
<th>Primary Factor Pattern</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.102</td>
<td>0.466</td>
</tr>
<tr>
<td>2</td>
<td>0.396</td>
<td>0.279</td>
</tr>
<tr>
<td>3</td>
<td>0.486</td>
<td>0.250</td>
</tr>
<tr>
<td>4</td>
<td>0.013</td>
<td>0.428</td>
</tr>
<tr>
<td>5</td>
<td>0.013</td>
<td>0.429</td>
</tr>
<tr>
<td>6</td>
<td>0.001</td>
<td>0.437</td>
</tr>
<tr>
<td>7</td>
<td>0.499</td>
<td>-0.187</td>
</tr>
<tr>
<td>8</td>
<td>0.588</td>
<td>-0.216</td>
</tr>
</tbody>
</table>

**Transformation Matrix**

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.426</td>
<td>-0.904</td>
</tr>
<tr>
<td>2</td>
<td>-0.904</td>
<td>0.426</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.412</td>
<td>-0.900</td>
</tr>
<tr>
<td>2</td>
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<td>0.435</td>
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</table>

**Intercorrelations of Primary Factors**

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.024</td>
<td>1.000</td>
</tr>
</tbody>
</table>
### Table 2

**Three Factor Rotation Comparison of the $kC_q$ Matrix (Job Facet Mode)**

<table>
<thead>
<tr>
<th>Input Matrix of Factor Loadings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>1. -0.202</td>
</tr>
<tr>
<td>2. -0.202</td>
</tr>
<tr>
<td>3. -0.202</td>
</tr>
<tr>
<td>4. -0.191</td>
</tr>
<tr>
<td>5. -0.208</td>
</tr>
<tr>
<td>6. -0.222</td>
</tr>
<tr>
<td>7. -0.208</td>
</tr>
<tr>
<td>8. -0.197</td>
</tr>
<tr>
<td>9. -0.217</td>
</tr>
<tr>
<td>10. -0.205</td>
</tr>
<tr>
<td>11. -0.218</td>
</tr>
<tr>
<td>12. -0.211</td>
</tr>
<tr>
<td>13. -0.220</td>
</tr>
<tr>
<td>14. -0.216</td>
</tr>
<tr>
<td>15. -0.203</td>
</tr>
<tr>
<td>16. -0.200</td>
</tr>
<tr>
<td>17. -0.196</td>
</tr>
<tr>
<td>18. -0.212</td>
</tr>
<tr>
<td>19. -0.211</td>
</tr>
<tr>
<td>20. -0.203</td>
</tr>
<tr>
<td>21. -0.217</td>
</tr>
<tr>
<td>22. -0.197</td>
</tr>
<tr>
<td>23. -0.222</td>
</tr>
</tbody>
</table>
### TABLE 2 (Continued)

<table>
<thead>
<tr>
<th>VARIMAX</th>
<th>BINORMANIN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor Structure</td>
<td>Primary Factor Pattern</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>0.287</td>
</tr>
<tr>
<td>2</td>
<td>0.396</td>
</tr>
<tr>
<td>3</td>
<td>0.231</td>
</tr>
<tr>
<td>4</td>
<td>-0.192</td>
</tr>
<tr>
<td>5</td>
<td>0.044</td>
</tr>
<tr>
<td>6</td>
<td>0.092</td>
</tr>
<tr>
<td>7</td>
<td>0.205</td>
</tr>
<tr>
<td>8</td>
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</tr>
<tr>
<td>9</td>
<td>-0.047</td>
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<td>10</td>
<td>0.275</td>
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<td>13</td>
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<tr>
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<tr>
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<tr>
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<tr>
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</tr>
<tr>
<td>20</td>
<td>0.224</td>
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<tr>
<td>21</td>
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<tr>
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<tr>
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</table>

<table>
<thead>
<tr>
<th>Transformation Matrix</th>
<th>Transformation Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>-0.528</td>
</tr>
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<td>0.795</td>
</tr>
<tr>
<td>3</td>
<td>0.296</td>
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</table>

<table>
<thead>
<tr>
<th>Intercorrelations of the Primary Factors</th>
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</thead>
<tbody>
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<td>1</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>
TABLE 3
FOUR FACTOR ROTATION COMPARISON OF THE
$kC_q$ MATRIX
(JOB FACET MODE)

<table>
<thead>
<tr>
<th>Input Matrix of Factor Loadings</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.202</td>
<td>0.170</td>
<td>0.151</td>
<td>0.072</td>
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<td>2</td>
<td>-0.202</td>
<td>0.317</td>
<td>0.122</td>
<td>0.121</td>
</tr>
<tr>
<td>3</td>
<td>-0.202</td>
<td>0.131</td>
<td>0.067</td>
<td>0.217</td>
</tr>
<tr>
<td>4</td>
<td>-0.191</td>
<td>-0.395</td>
<td>0.071</td>
<td>0.739</td>
</tr>
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<td>5</td>
<td>-0.208</td>
<td>-0.067</td>
<td>-0.039</td>
<td>0.015</td>
</tr>
<tr>
<td>6</td>
<td>-0.222</td>
<td>-0.103</td>
<td>0.193</td>
<td>-0.057</td>
</tr>
<tr>
<td>7</td>
<td>-0.208</td>
<td>-0.026</td>
<td>0.393</td>
<td>0.000</td>
</tr>
<tr>
<td>8</td>
<td>-0.197</td>
<td>0.013</td>
<td>-0.070</td>
<td>0.212</td>
</tr>
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TABLE 3 (Continued)

VARIMAX

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Transformation Matrix

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### TABLE 3 (Continued)

**BINORMAMIN**

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**Transformation Matrix**

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**Intercorrelations of the Primary Factors**

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The root mean square coefficients for the Varimax versus the Binormamin rotations were again used as a means of comparison, the coefficients were calculated for the results of the two rotation procedures; these are:

\[
\begin{align*}
U_{11} &= 0.005 \\
U_{22} &= 0.008 \\
U_{33} &= 0.008
\end{align*}
\]

These root mean square coefficients are so small that there is little doubt that the three factors are independent.

**Worker Mode**—Thus the two factors of the $jB_p$ matrix of job satisfaction operational measures, as well as the three factors of the $kC_q$ matrix of job facets were determined to be independent (orthogonal). Therefore the Varimax orthogonal rotations of these matrices were accepted. The inverses of the orthogonal transformations applied to the $jB_p$ and $kC_q$ matrices were applied to the core matrix. This transformed core was then rotated both obliquely and orthogonally for first a three factor solution and then a two factor solution, because the eigen vector plot of Figure 29 was not unequivocal in deciding between a two or a three factor solution for the $iA_m$ matrix. The results of these rotations appear in Tables 4 and 5 respectively. The three factor solution is unsatisfactory. The third factor only accounts for 0.81 percent of the variance attributable to the three factors. Thus the third factor was deemed to be a residual
TABLE 4
THREE FACTOR ROTATION COMPARISON OF THE CORE MATRIX

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**VARIMAX**

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**BINORMAMIN**

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**Intercorrelations of the Primary Factors**

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</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>0.756</td>
<td>0.653</td>
</tr>
<tr>
<td>2.</td>
<td>0.653</td>
<td>0.756</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Transformation Matrix</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>0.284</td>
<td>0.143</td>
</tr>
<tr>
<td>2.</td>
<td>-0.958</td>
<td>0.989</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Intercorrelations of the Primary Factors</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. 0.908</td>
<td>1.000</td>
<td></td>
</tr>
</tbody>
</table>
and discarded. In addition, since there is no evidence of orthogonality, the oblique rotation of the core matrix was accepted.

When the inverses of the transformation matrices of the oblique rotation of the core matrix are applied to the A matrix (the worker mode), the rotated A matrix contains two factors which are correlated 0.15. This correlation is of a small enough absolute level that it is reasonable to contend that the two individual difference factors of the worker mode are independent.

Factor Interpretation

As was noted above the number of factors chosen for the three two-mode output matrices of the three-mode analysis (i.e., the \( iA_m \), \( jB_p \), and \( kC_q \) matrices) were two, two, and three respectively.

Operational Measures Mode-- The loadings of the two orthogonally rotated factors of the operational measures mode, the \( jB_p \) matrix, are shown in Table 6. These loadings present clear simple structure with the fulfillment based job satisfaction measures (i.e., Is Now and Job Facet Satisfaction) loading heavily on the second factor and near zero on the first factor, and the discrepancy based job satisfaction measures (i.e., the researcher generated, "Is Now - Should Be" and "Is Now - Would Like") loading in an opposite
TABLE 6

THE LOADINGS OF THE ORTHOGONALLY ROTATED OPERATIONAL MEASURES MODE, THE \( jB_p \) MATRIX*

<table>
<thead>
<tr>
<th>Variables (Operational Measures)</th>
<th>Factor Number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>#1</td>
</tr>
<tr>
<td>Is Now</td>
<td>47</td>
</tr>
<tr>
<td>J.F.S.</td>
<td>44</td>
</tr>
<tr>
<td>Is Now - Should Be (Subject)</td>
<td>42</td>
</tr>
<tr>
<td>Is Now - Would Like (Subject)</td>
<td>42</td>
</tr>
<tr>
<td>Should Be</td>
<td>40</td>
</tr>
<tr>
<td>Would Like</td>
<td>48</td>
</tr>
<tr>
<td>Is Now - Should Be (Researcher)</td>
<td>50</td>
</tr>
<tr>
<td>Is Now - Would Like (Researcher)</td>
<td>59</td>
</tr>
</tbody>
</table>

*Decimals have been dropped.
manner. Thus, the first factor can be descriptively named "discrepancy" and the second "fulfillment".

Job Facet Mode—The three orthogonally rotated factors of the job facet mode, the $kC_q$ matrix, are shown in Table 7. The clear simple structure pattern shown in Table 7 readily lead to the naming of the three factors as "meaningfulness", "autonomy/responsibility," and "feedback" respectively.

Worker Mode—The naming of the two worker mode individual difference factors proves to be somewhat more difficult than the naming of factors of the other two modes. None of the demographic variables of age, marital status, number of previous factory work experiences, number of dependents, education level, or number of years with the company were found to be related (correlated) to either of the two worker individual difference factors. As well, neither job involvement, performance rating, internal/external locus of control or social desirability were found to be related to the individual difference factors. However, the workers' scores upon Hackman and Lawler's (1971) Growth Need Strength Scale were correlated 0.70 with their scores upon the second individual difference factor. Thus, the second factor has been tentatively named "Growth Need Strength".

This leaves one to ponder as to the nature of the first individual difference factor. However, Locke (1969, 1976) and Mobley and Locke (1970) provide a possible basis for its interpretation. These authors, among others (e.g., Katzell,
### Table 7

**The Loadings of the Orthogonally Rotated Job Facet Mode, the $kCq$ Matrix**

<table>
<thead>
<tr>
<th>Variables (Job Facets)</th>
<th>Factor Number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\theta_1$</td>
</tr>
<tr>
<td>Opportunity for promotion</td>
<td>49</td>
</tr>
<tr>
<td>Pay</td>
<td>40</td>
</tr>
<tr>
<td>Opportunity for personal growth and development</td>
<td>40</td>
</tr>
<tr>
<td>Feeling of self esteem or self respect</td>
<td>29</td>
</tr>
<tr>
<td>The prestige of my job inside the company</td>
<td>23</td>
</tr>
<tr>
<td>Opportunity for participation in the determination of methods, procedures, and goals</td>
<td>23</td>
</tr>
<tr>
<td>Opportunity to do challenging work</td>
<td>28, 22</td>
</tr>
<tr>
<td>Opportunity to do a number of different things</td>
<td>43</td>
</tr>
<tr>
<td>Amount of variety</td>
<td>39</td>
</tr>
<tr>
<td>Opportunity to give help to other people</td>
<td>34</td>
</tr>
<tr>
<td>Opportunity to get to know other people</td>
<td>30</td>
</tr>
<tr>
<td>Freedom to do pretty much what I want on my job</td>
<td>29</td>
</tr>
<tr>
<td>Opportunity to develop close friendships</td>
<td>28</td>
</tr>
<tr>
<td>Feeling of worthwhile accomplishment</td>
<td>23</td>
</tr>
<tr>
<td>Prestige of my job outside the company</td>
<td>18</td>
</tr>
<tr>
<td>Opportunity for independent thought and action</td>
<td>18</td>
</tr>
<tr>
<td>Opportunity to do a job from the beginning to the end</td>
<td>18, 23</td>
</tr>
<tr>
<td>Opportunity to find out how well I am doing</td>
<td>40</td>
</tr>
<tr>
<td>Amount of close supervision</td>
<td>37</td>
</tr>
<tr>
<td>Amount of respect and fair treatment that I receive</td>
<td>37</td>
</tr>
<tr>
<td>Feeling that I know whether I am performing my job well or poorly</td>
<td>35</td>
</tr>
<tr>
<td>Opportunity to complete work that I start</td>
<td>35</td>
</tr>
<tr>
<td>Feeling of security in my job</td>
<td>30</td>
</tr>
</tbody>
</table>

* Decimals have been dropped.
1964; Likert, 1961; Pelz and Andrews, 1966) state that it is the job situation in relation to the worker's values that is the most direct determinant of job satisfaction, and further, that the view that job satisfaction results entirely from value attainment is not the whole story. Among other issues, these writers contend that the relation of value importance to satisfaction is crucial.

Locke (1969) has argued that every emotional response reflects a dual value judgement; the relation between what the worker wants and what he perceives himself as getting, and the importance of what is wanted. He further states that estimates of job satisfaction reflect both the relation between what the individual wants and what he perceives himself as getting and value importance. It is to be noted here that value importance affects the relative intensity of job satisfaction or dissatisfaction. Importance affects the range of affect (i.e., the variance of job satisfaction) which a given value can produce. Given a random distribution of job satisfaction levels more important values will lead to greater overall variability in job satisfaction than less important values. This relationship is represented graphically in Figure 30. The influence of importance on the variability of job satisfaction may explain the findings that: the correlation between satisfaction with more important values or needs and overall satisfaction is higher than the corresponding correlations for less important values.
(Ewen, 1967; Schaffer, 1953), and the correlation between various job facets and satisfaction is higher for individuals who want them more than those who want them less (Hackman & Lawler, 1971). The tremendous similarity between the hypothetical function of Figure 30 and the plot of the workers scores upon the two individual difference factors, the $i_A$ matrix, shown in Figure 31 indicates that the first factor is affecting the range of affect of the second factor. Thus, the first factor is behaving in a manner which is characteristic of "importance" variables. This could lead to the contention that the first individual difference factor can be named "Value Potency".

Results of Hypotheses Tests

With the factors being tentatively named, attention now turns to the results which the three-mode analysis has provided in relation to the propositions and hypotheses which were presented in Chapter II.

Operational Measures Mode

Proposition one stated that the operational measures observational mode would resolve into two factors. This is clearly the case. Further, as indicated above, these two factors can be named "fulfillment" and "discrepancy".

Hypothesis 1-A contended that the two factors would be independent. The extremely small mean square coefficients
FIGURE 30
HYPOTHEtical FUNCTION RELATING IMPORTANCE
TO THE VARIABILITY OF JOB SATISFACTION
(After Locke, 1976)
FIGURE 31

PLOT OF THE WORKERS SCORES UPON THE
TWO INDIVIDUAL DIFFERENCE VARIABLES
INDICATING THE "IMPORTANCE" EFFECT
between the orthogonal and oblique rotations of these factors offer strong support for their orthogonality.

Hypotheses 1-B and 1-C stated that the job satisfaction operational measures which are based upon fulfillment theory (i.e., Is Now and Job Facet Satisfaction) should obtain high loadings on the "fulfillment" factor and near zero loadings on the "discrepancy" factor. The factor loadings shown in Table 8 clearly support these two hypotheses.

Hypotheses 1-D indicated that the "Should Be" and "Would Like" measures would load heavily on the "discrepancy" factor and near zero on the "fulfillment" factor, because these concepts measure "desires" or "ideal standards" which as such are not measures of fulfillment (Porter, 1961; Locke, 1969). The loadings shown in Table 8 do not strongly support this hypothesis. There is limited support in that the "Should Be" and "Would Like" loadings of .40 and .48 respectively upon the "discrepancy" factor are, in an absolute sense, quite large. However, their respective loadings of .28 and .25 upon the "fulfillment" factor are too large to be discounted.

The large loadings upon the "discrepancy" factor can be accounted for in that these "ideal standards" are theoretically included within discrepancy types of job satisfaction measures. The fulfillment types of job satisfaction measures, however, do not theoretically include these "ideal standards", they seek only an absolute level. The results
### TABLE 8

**THE ORTHOGONALLY ROTATED FACTORS OF THE OPERATIONAL MEASURES MODE, THE $jB_p$ MATRIX**

<table>
<thead>
<tr>
<th>Operational Measure</th>
<th>Discrepancy</th>
<th>Fulfillment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Is Now</td>
<td>-.09</td>
<td>.47</td>
</tr>
<tr>
<td>Job Facet Satisfaction</td>
<td>.00</td>
<td>.44</td>
</tr>
<tr>
<td>Is Now - Should Be (Subject)</td>
<td>.01</td>
<td>.42</td>
</tr>
<tr>
<td>Is Now - Would Like (Subject)</td>
<td>.01</td>
<td>.42</td>
</tr>
<tr>
<td>Should Be</td>
<td>.40</td>
<td>.28</td>
</tr>
<tr>
<td>Would Like</td>
<td>.48</td>
<td>.25</td>
</tr>
<tr>
<td>Is Now - Should Be (Researcher)</td>
<td>.50</td>
<td>-.18</td>
</tr>
<tr>
<td>Is Now - Would Like (Researcher)</td>
<td>.59</td>
<td>-.21</td>
</tr>
</tbody>
</table>
would indicate, post hoc, that the age old philosophical argument that there are no absolutes and that everything is relative also applies to fulfillment based job satisfaction measures — fulfillment measures are at least in part affected by "desires" or "ideal standards".

Hypothesis 1-E stated that the researcher generated arithmetic difference discrepancy measures (i.e., Should Be - Is Now and Would Like - Is Now) would load heavily on the "discrepancy" factor and near zero on the "fulfillment" factor. The loadings of Table 8 strongly support this hypothesis, for even though the respective loadings of .50 and .59 of these two measures are the highest of the entire table and upon the "discrepancy" factor, these measures also obtain small negative loadings on the "fulfillment" factor.

Hypothesis 1-F was a direct test of Wall and Payne's (1973) hypothesis that "letting the subjects do their own arithmetic" would result in a "better" operational method of measuring the discrepancy conceptualization of job satisfaction. If Wall and Payne's hypothesis is valid, two things should be evident within Table 8. First, the two worker generated operational measures should load heavily on the "discrepancy" factor and near zero upon the "fulfillment" factor. And second, if this suggested operational measurement technique is superior to the arithmetical model, the loadings of the two worker generated measures upon the "discrepancy" factor should be larger than their respective
researcher generated measures upon the "discrepancy" factor. The results strongly reject this hypothesis, in that the pattern of loadings are exactly the opposite of that hypothesized.

A possible, post hoc, explanation for these inverted findings is that the manner of presentation of the questions "fooled" the respondents in this study. The three items comprising this measurement method were as follows:

(1) How much of each quality or characteristic is present in your job?
(2) How much of each quality or characteristic do you think should be (would like) associated with your job?
(3) Considering your answers to the two above items, how satisfied are you?

This series of questions can be conceptualized in two different ways. First one could think of them as measuring a discrepancy as did Wall and Payne (1973), as shown by the schematic of the top half of Figure 32. This same series of three questions could, however, be conceptualized as measuring fulfillment as shown in the bottom half of Figure 32.

In the first model the worker is presumed to determine the magnitude of the discrepancy and state how satisfied he is (the smaller the discrepancy, the greater presumably the satisfaction). The second model assumes that the worker responds to the "how satisfied" question by making a
"percentage comparison" of the "Is Now" component to the "Should Be" or "Would Like" component. Thus when the subject responds to "how satisfied" he is telling you "how much of the amount he desires (Should Be or Would Like) is represented by the amount he has (Is Now)".

The results of Table 8 support the percentage comparison fulfillment conceptualization in two ways. First, the two Wall and Payne measures load heavily on the "fulfillment" factor and about zero on the "discrepancy" factor. Second, the percentage comparison fulfillment conceptualization incorporates an "ideal standard". Therefore one would expect that the "ideal standards" of "Would Like" and "Should Be" would also load to some positive degree upon the "fulfillment" factor, which is the case.

**Job Facet Mode**

The job facet mode proposition and its two derived hypotheses stated that three independent factors, which should be identifiable as "meaningfulness", "autonomy/responsibility", and "feedback" should be derived. As was indicated above, clear support was obtained.

**Worker Mode**

As was indicated above, none of the ten demographic or psychological variables were found to be related to either of the two individual difference factors except the Hackman
FIGURE 32
A SCHEMATIC OF TWO ALTERNATE CONCEPTUALIZATIONS OF THE WALL AND PAYNE HYPOTHESIS
and Lawler (1971) Growth Need Strength Scale. This finding supports the one meager hypothesis of the worker mode.

**Results of Idealized Individual Analysis**

The idealized individual analysis was undertaken in order to discern the effect, if any, of the individual difference factors upon the relationships between the factors of the other two data modes. More specifically stated, this analysis determines if the individual difference factors of "growth need strength" and "value potency" have any effect upon: 1) the relation between the operational measures factors of "discrepancy" and "fulfillment", 2) the relation of the "discrepancy" measures to the three job facet factors of "meaningfulness", "autonomy/responsibility", and "feedback", and 3) the relation of "fulfillment" measures to the three job facet factors of meaningfulness, autonomy/responsibility, and feedback.

The analysis was carried out by first systematically locating eight "hypothetical workers" within a scatter plot of the 304 workers' scores upon the two individual difference variables (see Figure 26). Second, two-mode core matrices were calculated for each of the hypothetical workers via the technique described in Chapter I. Third, plots of the changes between the hypothetical workers' two-mode core matrix scores, in relation to changes in the level of the individual difference variables, were made. This was done
in order to discern the effect of changes in level of the individual difference variables upon the relationships between the factors of the operational measures and job facet modes.

The rotated three-mode core matrix, the locations of the eight hypothetical workers, and the hypothetical workers' two-mode core matrices are shown in Table 9.

Several plots of changes between hypothetical worker two-mode core matrix scores were derived to represent changes in the levels in the two individual difference variables. The plots for the effect of "value potency" upon the relations of discrepancy and fulfillment operational measures to meaningfulness, autonomy/responsibility, and feedback job facets are derived for the shift from hypothetical worker 4 to 3 to 2 to 1 (see Figure 26). This represents a shift from highest to lowest value potency along the line of greatest dispersion, which is also the mean of the growth need strength individual difference variable. Figure 33 shows the effect of changes in the level of value potency upon discrepancy operational measures of meaningfulness, autonomy/responsibility, and feedback job facets. The "level" nature of these plots would clearly indicate that value potency, as an individual difference variable, has no effect upon discrepancy measures of any of the three types of job facets.
TABLE 9

IDEALIZED INDIVIDUAL ANALYSIS,
TWO-MODE CORE MATRICES

Rotated Three-Mode Core Matrix

<table>
<thead>
<tr>
<th>Discrepancy</th>
<th>Meaningfulness</th>
<th>Potency</th>
<th>Growth Need Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Autonomy/Resp.</td>
<td>-2.19</td>
<td>20.05</td>
<td></td>
</tr>
<tr>
<td>Feedback</td>
<td>1.23</td>
<td>16.78</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.95</td>
<td>10.12</td>
<td></td>
</tr>
<tr>
<td>Fulfillment</td>
<td>Meaningfulness</td>
<td>32.75</td>
<td>-3.64</td>
</tr>
<tr>
<td>Autonomy/Resp.</td>
<td>36.37</td>
<td>3.08</td>
<td></td>
</tr>
<tr>
<td>Feedback</td>
<td>22.65</td>
<td>8.79</td>
<td></td>
</tr>
</tbody>
</table>

Hypothetical Worker Locations

<table>
<thead>
<tr>
<th>Hypothetical Worker No.</th>
<th>Value Potency</th>
<th>Growth Need Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.078</td>
<td>.055</td>
</tr>
<tr>
<td>2</td>
<td>.056</td>
<td>.055</td>
</tr>
<tr>
<td>3</td>
<td>.034</td>
<td>.055</td>
</tr>
<tr>
<td>4</td>
<td>.023</td>
<td>.055</td>
</tr>
<tr>
<td>5</td>
<td>.056</td>
<td>.076</td>
</tr>
<tr>
<td>6</td>
<td>.034</td>
<td>.099</td>
</tr>
<tr>
<td>7</td>
<td>.056</td>
<td>.036</td>
</tr>
<tr>
<td>8</td>
<td>.034</td>
<td>.013</td>
</tr>
</tbody>
</table>
TABLE 9 (Continued)

Two-Mode Core Matrices

<table>
<thead>
<tr>
<th>Hypothetical Worker No.</th>
<th>Discrepancy</th>
<th>Fulfillment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(.078, .055)</td>
<td></td>
</tr>
<tr>
<td>Hypothetical Worker No.</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Meaningfulness</td>
<td>.95</td>
<td>2.35</td>
</tr>
<tr>
<td>Autonomy/Resp.</td>
<td>1.04</td>
<td>3.01</td>
</tr>
<tr>
<td>Feedback</td>
<td>.87</td>
<td>2.26</td>
</tr>
<tr>
<td>Hypothetical Worker No.</td>
<td>2</td>
<td>(.056, .055)</td>
</tr>
<tr>
<td>Meaningfulness</td>
<td>.99</td>
<td>1.63</td>
</tr>
<tr>
<td>Autonomy/Resp.</td>
<td>1.01</td>
<td>2.21</td>
</tr>
<tr>
<td>Feedback</td>
<td>.79</td>
<td>1.76</td>
</tr>
<tr>
<td>Hypothetical Worker No.</td>
<td>3</td>
<td>(.034, .055)</td>
</tr>
<tr>
<td>Meaningfulness</td>
<td>1.07</td>
<td>.91</td>
</tr>
<tr>
<td>Autonomy/Resp.</td>
<td>.98</td>
<td>1.41</td>
</tr>
<tr>
<td>Feedback</td>
<td>.70</td>
<td>1.26</td>
</tr>
<tr>
<td>Hypothetical Worker No.</td>
<td>4</td>
<td>(.023, .055)</td>
</tr>
<tr>
<td>Meaningfulness</td>
<td>1.07</td>
<td>.55</td>
</tr>
<tr>
<td>Autonomy/Resp.</td>
<td>.97</td>
<td>1.01</td>
</tr>
<tr>
<td>Feedback</td>
<td>.66</td>
<td>1.01</td>
</tr>
<tr>
<td>Hypothetical Worker No.</td>
<td>5</td>
<td>(.056, .076)</td>
</tr>
<tr>
<td>Meaningfulness</td>
<td>1.41</td>
<td>1.55</td>
</tr>
<tr>
<td>Autonomy/Resp.</td>
<td>1.35</td>
<td>2.27</td>
</tr>
<tr>
<td>Feedback</td>
<td>.99</td>
<td>1.94</td>
</tr>
<tr>
<td>Hypothetical Worker No.</td>
<td>6</td>
<td>(.034, .099)</td>
</tr>
<tr>
<td>Meaningfulness</td>
<td>1.91</td>
<td>.75</td>
</tr>
<tr>
<td>Autonomy/Resp.</td>
<td>1.71</td>
<td>1.54</td>
</tr>
<tr>
<td>Feedback</td>
<td>1.14</td>
<td>1.64</td>
</tr>
<tr>
<td>Hypothetical Worker No.</td>
<td>7</td>
<td>(.056, .036)</td>
</tr>
<tr>
<td>Meaningfulness</td>
<td>.60</td>
<td>1.70</td>
</tr>
<tr>
<td>Autonomy/Resp.</td>
<td>.68</td>
<td>2.15</td>
</tr>
<tr>
<td>Feedback</td>
<td>.59</td>
<td>1.59</td>
</tr>
<tr>
<td>Hypothetical Worker No.</td>
<td>8</td>
<td>(.034, .013)</td>
</tr>
<tr>
<td>Meaningfulness</td>
<td>.20</td>
<td>1.07</td>
</tr>
<tr>
<td>Autonomy/Resp.</td>
<td>.27</td>
<td>1.28</td>
</tr>
<tr>
<td>Feedback</td>
<td>.27</td>
<td>.89</td>
</tr>
</tbody>
</table>
Figure 34 shows the effect of changes in value potency upon fulfillment measures of the three types of job facets. These plots indicate that value potency has a clear effect upon fulfillment measures, and this effect is consistent across all three types of job facets.

The plots for the effect of "growth need strength" upon the relations of discrepancy and fulfillment measures to meaningfulness, autonomy/responsibility, and feedback are derived from the shift from hypothetical worker 6 to 3 to 8 and/or 5 to 2 to 7 (see Figure 26). This represents a shift from highest to lowest growth need strength. Figure 35 indicates the effect of the growth need strength individual difference variable upon discrepancy measures of the three types of job facets for the shift from hypothetical worker 6 to 3 to 8. These plots indicate a consistent effect of growth need strength upon discrepancy measures of the three types of job facets.

Figure 36 indicates the effect of the growth need strength upon fulfillment measures of the three types of job facets. The evidence here, though not as clear as that of Figure 33, could support the contention of no effect.

Similar results are obtained for the effect of growth need strength upon both fulfillment and discrepancy measures for the shift from hypothetical worker 5 to 2 to 7.

To summarize, these plots indicate that value potency consistently affects fulfillment measures but not
FIGURE 33
INDEALIZED INDIVIDUAL ANALYSIS PLOTS,
EFFECT OF VALUE POTENCY ON DISCREPANCY MEASURES
FIGURE 34
IDEALIZED INDIVIDUAL ANALYSIS PLOTS, EFFECT OF VALUE POTENCY ON FULFILLMENT MEASURES
FIGURE 35
IDEALIZED INDIVIDUAL ANALYSIS PLOTS, EFFECT OF GROWTH NEED STRENGTH ON DISCREPANCY MEASURES
FIGURE 36
IDEALIZED INDIVIDUAL ANALYSIS PLOTS, EFFECT OF GROWTH NEED STRENGTH ON FULFILLMENT MEASURES
discrepancy measures and that growth need strength consistently affects discrepancy measures but most likely not fulfillment measures.

Secondary Analysis

One possible interpretation of the effects of the individual difference variables is that they moderate the relationship between job satisfaction operational measures and other variables of interest. Since absenteeism is often found to be related to job satisfaction, the proposition that value potency and growth need strength act as moderating variables was tested.

Two arithmetical discrepancy measures (Should Be - Is Now and Would Like - Is Now) and two fulfillment measures (Is Now and Job Facet Satisfaction) were correlated with absenteeism (mean hours per week absent for the twelve month period immediately prior to the on site data collection), under five different groupings: 1) the total sample, 2) high value potency, 3) low value potency, 4) high growth need strength, and 5) low growth need strength. This in order to test the four hypotheses of:

1) There should be no statistically significant difference in correlation between discrepancy measures and absenteeism for the high value potency workers and the low value potency workers.
2) There should be a statistically significant difference in correlation between the fulfillment measures and absenteeism for the high value potency workers and the low value potency workers.

3) There should be a statistically significant difference in correlation between the discrepancy measures and absenteeism for the high growth need strength workers and the low growth need strength workers.

4) There should be no statistically significant difference in correlation between the fulfillment measures and absenteeism for the high growth need strength workers and the low growth need strength workers.

The pattern and levels of statistical significance of the correlations shown in Table 10 strongly support all four of these hypotheses. This in turn supports the proposition that value potency moderates the relationship between fulfillment based job satisfaction measures and absenteeism but not discrepancy based measures and absenteeism. And conversely, growth need strength moderates the relationship between discrepancy based job satisfaction measures and absenteeism but not fulfillment based measures and absenteeism.
TABLE 10

JOB SATISFACTION - ABSENTEEISM CORRELATIONS

AS MODERATED BY VALUE POTENCY

<table>
<thead>
<tr>
<th>JOB SATISFACTION MEASURE</th>
<th>DISCREPANCY</th>
<th>FULFILLMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Should Be (Would Like - Is Now) - Is Now)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTAL SAMPLE</td>
<td>-.03</td>
<td>-.05</td>
</tr>
<tr>
<td>HIGH VALUE POTENCY</td>
<td>-.06</td>
<td>-.09</td>
</tr>
<tr>
<td>LOW VALUE POTENCY</td>
<td>-.04</td>
<td>-.05</td>
</tr>
</tbody>
</table>

AS MODERATED BY GROWTH NEED STRENGTH

<table>
<thead>
<tr>
<th>JOB SATISFACTION MEASURE</th>
<th>DISCREPANCY</th>
<th>FULFILLMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Should Be (Would Like - Is Now) - Is Now)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTAL SAMPLE</td>
<td>-.03</td>
<td>-.05</td>
</tr>
<tr>
<td>HIGH G.N.S.</td>
<td>.10</td>
<td>.09</td>
</tr>
<tr>
<td>LOW G.N.S.</td>
<td>-.15</td>
<td>-.10</td>
</tr>
</tbody>
</table>

Statistical Significance Legend (two-tail test)

Non-significant = NS

\[ a < .05 = * \]
\[ a < .01 = ** \]
\[ a < .001 = *** \]
CHAPTER V

SUMMARY AND IMPLICATIONS FOR FUTURE RESEARCH

Summary

Two major conclusions can be drawn from this research. First, job satisfaction instruments affect data relationships in predictable directions (i.e., attitude measurement tools leave their mark on the work just as a hammer leaves a mark on soft metal). Second, individual worker differences moderate the relationship between the cognitive measures of job satisfaction and behavioral measures of that attitude such as absenteeism.

The two independent job satisfaction operational measures factors (fulfillment and discrepancy) appeared indicative of the theoretical conceptualizations upon which the eight operational measures were based. In addition, three-mode factor analysis of the 23 job facets utilized in this study resulted in three independent factors, also indicating the impact which measuring instruments have upon collected data. It should be remembered that these twenty-three job facets were derived primarily from the work of Hackman and Lawler (1971) and Hackman and Oldham (1975), who hypothesized that the JDS instrument tapped three such independent factors: meaningfulness, autonomy/responsibility, and
feedback. While consistent with existing literature these findings support the contention that data can not be separated from its mode of collection. It might also be added that attempts at "getting around" such problems through more sophisticated questioning techniques and complex instruments is also less than successful. Wall and Payne's suggested operational measurement technique of "letting the subjects do their own arithmetic" did not result in a discrepancy measure at all, let alone a "better" discrepancy measure, than the usual computational techniques. Their suggested measurement technique clearly results in a fulfillment measure which also leaves its own "impact" on the data.

Such conclusions are not surprising. However, they have been ignored by otherwise sophisticated organizational scientists who have not used common and well understood job satisfaction measurement instruments. The use of tools whose impacts are ignored or not understood has led to the impossibility of integrating the plethora of conflicting results within the extant job satisfaction literature.

These results are most interesting when viewed in relation to the worker mode individual difference factors found in this study. One of the two derived individual difference variables was identifiable as the level of the workers' growth needs. The second individual difference variable while not related to a number of traditionally described
demographic and psychological variables exhibits the characteristics of "importance" or "value potency".

Idealized individual analysis clearly indicates that value potency affects fulfillment job satisfaction measures in a consistent manner across the three types of job facets, but does not affect discrepancy measures. Conversely, growth need strength affects discrepancy job satisfaction measures in a consistent manner across the three types of job facets, but does not affect fulfillment measures. The strong support for the four hypotheses of the secondary analysis indicate that the two individual difference variables moderate the relationship between their respective types of job satisfaction measures and absenteeism. Such results are important because they indicate that instrument effects interact with individual differences, and they serve as weather vanes of the conceptual and methodological problems which currently exist in job satisfaction and other attitudinal research (e.g., Leadership).

These methodological problems seem to follow a pattern. First, researchers assume common links between the three attitude manifestations of affect, behavior and cognition. This often results in the "cramming" of the attitude into a theoretical space of one or two dimensions (i.e., a problem of conceptual aggregation). Second, the problem is compounded by the use of multiple measurement instruments which suffer from two types of measurement difficulties:
1) aggregating perceptions across different sets of facets, and 2) given a common set of job facets, aggregating different perceptions or combinations of perceptions. Third, the problem is further compounded by an adherence to use of paradigm dictated data analysis techniques which are often incapable of illuminating the relationships which we critically need to understand.

In short, Lawler (1971) is correct in stating that our understanding of attitudes such as job satisfaction has not advanced in over three decades, and he is correct because of the Apollonian pre-occupation of organizational scientists. Further, our understanding of such attitudes as job satisfaction will not be advanced until we have solved measurement aggregation problems such as those addressed in this study. The solution of these problems is an effort which Roberts et al. (1978) contend can only be accomplished via more Dionysian methods.

Implications For Future Research

Because of the large amount of information provided by the three-mode factor analysis methodology and the Dionysian nature of this research, many implications for future research can be derived.

The most obvious implications for future research concern the moderating effect of the growth need strength and
value potency individual differences, where several ques-
tions deserve further investigation:

First, are the results concerning the individual dif-
ferences replicable? That is, will the growth need strength
and value potency individual differences be found in other
samples of workers, and will their moderating effects be the
same?

Second, why do growth need strength and value potency
moderate the relationship of the fulfillment and discrepancy
job satisfaction measures to absenteeism?

Third, since the moderate loadings of the "Should Be"
and "Would Like" measures upon the fulfillment factor, imply
that the traditional conceptualization of a fulfillment mea-
sure (e.g., Porter, 1961; Alderfer, 1969) (representing an
absolute level of fulfillment) is not entirely accurate, is
a reformulation of the fulfillment conceptualization as
representing a percentage comparison in relation to an
"ideal standard" a useful alternative?

Lastly, it becomes obvious that Dionysian approaches
and models must be utilized if the job satisfaction litera-
ture in particular and attitude research in general is to be
lifted out of the quagmire in which it has wallowed for the
past four decades. As Mitroff (1974a, 1974b, 1977) has
pointed out, scientists themselves consistently consider the
Dionysians among them to be the most prestigious in terms of
having advanced their field.
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APPENDIX A

MATHEMATICAL DEVELOPMENT OF THE
THREE-MODE FACTOR ANALYSIS MODEL*

Remarks on Mathematical Notation

In the development of the three-mode factor analysis model, Tucker (1966) has introduced several unique features of mathematical notation. Some of these notational items are at variance with common mathematical usage but have been found to be helpful in consideration of some of the relatively complex relations which are a part of the three-mode model. However, much of standard summational and matrix notation has been retained. The following is a summary of the mathematical notational items which are relevant to the presentation of the model which is to follow.

The first item is the use of the word "mode", Tucker (1964, p.112) introduced this term to denote "a set of indices by which data might be classified". For example, the individual scores on a battery of tests could be classified by the individuals in the sample and cross-classified by tests in the battery. The individual samples are elements of one set of indices by which the scores are classified, and therefore constitute one mode of the data. The battery of tests is the second mode of this data. Test scores are arranged in a rectangular table with rows for individuals and columns for tests. Such an arrangement of tests and subjects is termed a **two-mode matrix**. If the test battery is administered on several occasions, the set of occasions
is considered to be the third mode. The data is now represented via a cube with horizontal strata of cells for individuals, vertical strata parallel to the end planes for tests, and vertical strata parallel to the front plane for occasions. This is a three-mode matrix (see Figure 16).

Each mode in this matrix is identified by a lower case letter. The letter \( i \), for example, may be used to denote the mode for individuals. A lowercase letter, then, is used in several related but distinct roles. First it serves as a general identification of the mode; second, as a subscript identifying the mode to which an element belongs; and third, as a variable identification symbol for the mode elements. An example of the first usage is the statement "mode \( i \) is for the individuals in the sample". An example of the second usage is in the assignment of identification symbols, \( 1_i, 2_i, 3_i \ldots, N_i \) to the individuals in the sample. An element's identification symbol is composed of two parts, one being a number termed the \textit{index value} of the element, designated by \( i \), and the other part being the identification subscript for the mode. The series of index values for an element in a mode consists of the integers from one to the number of elements in the mode. The number of elements in a mode is designated by the capital letter \( N \) with a subscript identifying the mode. For example in the notation \( N_i \) \( i \) is a generalized mode identification, and \( N_i \) is the number of elements in mode \( i \). The index values for the elements in a
mode are designated by $V_i = 1, 2, 3, \ldots, N_i$. The elements in a mode are designated by $i = 1_i, 2_i, 3_i, \ldots, N_i$. In the third role, the lowercase letter is used as a general, unspecified identification symbol which may be particularized to the identification symbol of each of the elements in turn. For example, $x_{ijk}$ is used as the generalized entry in the three-mode matrix $X$ with the letters $i, j$ and $k$ being used as generalized identification symbols for the elements in the three modes.

This use of subscripts is at variance with common mathematical practice. Commonly, a matrix is defined to have rows for one mode and columns for a second mode. The subscripts for the elements are used in common mathematical notation solely as indices; thus, $i$ could be used as a row index to designate an individual if the row mode were individuals and be used as a column index to indicate a factor, if the column mode were factors. In the first case $i$ would be the first subscript and in the second case $i$ would be the second subscript. In contrast, in the notation herein presented any given letter will designate a particular mode and the arrangement of the matrix will change with a change in location of the subscripts. This change in notation permits the use of matrix letters to designate classes of matrices with the particular matrix in the class being designated by the modes involved.
This form of notation can be used to represent three-mode matrices. In contrast to the elementary modes a combination mode is defined as the Cartesian product of two elementary modes. A combination mode is denoted by the letters of the two elementary modes enclosed in parentheses. A combination mode formed by the Cartesian product of the elementary modes \( i \) and \( j \) is denoted as \((ij)\). Each element in this combination mode corresponds to a unique pair of elements from the elementary modes, one element of the pair from each of the two modes.

An illustration of this computation is given below for \(N_i=2\) and \(N_j=3\).

Identification Symbols for:

<table>
<thead>
<tr>
<th>Elementary Modes</th>
<th>Combination Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) (j)</td>
<td>((ij))</td>
</tr>
<tr>
<td>(1_i) (1_j)</td>
<td>((1ij))</td>
</tr>
<tr>
<td>(1_i) (2_j)</td>
<td>((2ij))</td>
</tr>
<tr>
<td>(1_i) (3_j)</td>
<td>((3ij))</td>
</tr>
<tr>
<td>(2_i) (1_j)</td>
<td>((4ij))</td>
</tr>
<tr>
<td>(2_i) (2_j)</td>
<td>((5ij))</td>
</tr>
<tr>
<td>(2_i) (3_j)</td>
<td>((6ij))</td>
</tr>
</tbody>
</table>

The index-value part of the identification symbol for each element in a combination mode is computed over the index values of the identification symbols of the corresponding paired elements of the elementary modes. The indexing equation is:

\[ v(ij) = [v(i) - 1] N_i + v(j). \]  

[19]
The order \((ij)\) is read as "i-outer loop, j-inner loop." This definition is compatible with computer calculation of subscripts, for example, \((ij)\) and \((ji)\) contain the same elements but with a change in order and corresponding index values. The number of elements in a combination mode is the product of the numbers of elements in the elementary modes:

\[
N(ij) = N_i N_j \quad [20]
\]

By utilizing the combination mode \((ij)\) a three-mode matrix can be written as a two-mode matrix with rows as a combination mode and columns as an elementary mode. This matrix can be denoted as \((ij)^{X_k}\). The same matrix can also be written as the two-mode matrix \((ik)^{X_j}\) by use of the combination mode \((ik)\) for rows and mode \(j\) for columns (see Figures 17 and 18 for a graphic example).

Another notational device that is especially useful for two-mode matrices is to pre-subscript the matrix letter with the letter for the row mode and to post-subscript the matrix letter with the letter for the column mode. Thus, \(iA_m\) is the matrix having entries \(a_{im}\) with rows for mode \(i\) and columns for mode \(m\). Note that the matrix \(mA_i\) with entries \(a_{mi}\) is the transpose of matrix \(iA_m\). Thus, for fixed values of \(i\) and \(m\), \(a_{im}\) and \(a_{mi}\) are two ways of denoting the same
quantity, the first in matrix $iA_m$ and the second in the transposed matrix $mA_i$. This notational device is utilized to designate the transpose of a matrix.

In a matrix product of two-mode matrices, the post-subscript of the first matrix must conform with the pre-subscript of the second matrix. This matrix multiplication notation follows the usual convention of an entry of the product matrix equalling the sum of products between entries in a row of the first matrix with entries in a column of the second matrix. Thus, the common subscript for the two matrices is written only once. For example, the matrix product of matrices $iA_j$ and $jB_k$ is written as $iA_jB_k$. This product could yield a matrix $iC_k$ with row mode $i$ and column mode $k$.

Another matrix operation which is very necessary in the development of the three-mode factor analysis model is the Kronecker product of two matrices, which is denoted by the symbol $\otimes$. For example, consider the two matrices $iA_m$ and $jB_p$ with elements $a_{im}$ and $b_{jp}$. Let the entries of a matrix $(ij)^H(mp)$ be $h(ij)(mp)$ which are defined in terms of the entries of $iA_m$ and $jB_p$ by:

$$h(ij)(mp) = a_{im}b_{jp} \quad [21]$$

The matrix notation for this operation is:

$$(ij)^H(mp) = iA_m \otimes jB_p \quad [22]$$

The Kronecker product matrix is written as a supermatrix containing sub-matrices proportional to the matrix $jB_p$, the
second matrix of the product. The constants of proportionality are the elements of the first matrix \( iA_m \). Thus, rectangular representation of the Kronecker product matrix is given by:

\[
(ij)^H(mp) = \begin{bmatrix}
(a_{11} 1m \, jB_p) & (a_{11} 2m \, jB_p) & (a_{11} 3m \, jB_p) & \cdots \\
(a_{21} 1m \, jB_p) & (a_{21} 2m \, jB_p) & (a_{21} 3m \, jB_p) & \cdots \\
(a_{31} 1m \, jB_p) & (a_{31} 2m \, jB_p) & (a_{31} 3m \, jB_p) & \cdots \\
\vdots & \vdots & \vdots & \ddots
\end{bmatrix}
\]

Note that the modes of the Kronecker product matrix are combination modes obtained from the modes of the matrices involved in the product. The row mode of the Kronecker product is obtained from the row modes of the matrices involved, the outer loop mode of the combination being the row mode for the first matrix and the inner loop being the row mode for the second matrix. The column mode of the Kronecker product is similarly a combination mode of the column modes of the matrices involved in the product.

Several propositions concerning the Kronecker product are important to the use of this operation in three-mode factor analysis:

(a) The transpose of a Kronecker product matrix equals the Kronecker product of the transposes of the original matrices in the same order as in the original product. Compare equation [22] with equation [23].
\[
(mp)^H(ij) = m^A_i \otimes p^B_j \quad \text{[23]}
\]

(b) If the matrices \( iA_m \) and \( jB_p \) are square and symmetric, then their Kronecker product \((ij)^H(mp)\) will be a square, symmetric matrix.

(c) If the matrices \( iA_m \) and \( jB_p \) are diagonal matrices, their Kronecker product will be a diagonal matrix containing products of pairs of diagonal elements, one member of the pair being a diagonal entry in \( iA_m \) and the other member of the pair being a diagonal entry in \( jB_p \), for all possible such pairs.

(d) The proposition of equation [24] is true for Kronecker products.

\[
(iA_m S_n) \otimes (jB_p T_q) = (iA_m \otimes jB_p) (mS_n \otimes pT_q) \quad \text{[24]}
\]

(e) If matrices \( iA_m \) and \( jB_p \) possess left inverses, the left inverse of their Kronecker product is the Kronecker product of their left inverses in the same order; that is, if:

\[
m^A_i^{-1} A_m = m^{-1} I_m \quad \text{and} \quad p^B_j^{-1} B_p = p^{-1} I_p \quad \text{[25]}
\]

where \( m^A_i^{-1} \) and \( p^B_j^{-1} \) are the left inverses of \( iA_m \) and \( jB_p \) and where \( m^{-1} I_m \) and \( p^{-1} I_p \) are identity matrices, then:

\[
(m^A_i^{-1} \otimes p^B_j^{-1})(iA_m \otimes jB_p) = (mp)^I(mp) \quad \text{[26]}
\]

where \((mp)^I(mp)\) is an identity matrix. Note that the left inverse matrix is denoted by an asterisk and that the subscripts indicate row and column modes of this matrix.

(f) If the matrices \( iA_m \) and \( jB_p \) are column-wise sections of orthonormal matrices, their Kronecker product will
be a column-wise section of an orthonormal matrix. This proposition is a special case of the preceding proposition and occurs when \( mA^*_i \) is the transpose of \( iA_m \) and \( pB^*_j \) is the transpose of \( jB_p \).

\( g \) The relation of the characteristic roots and vectors of two square, symmetric matrices to the roots and vectors of their Kronecker product is of considerable importance. Let the two matrices be \( jPj \) and \( kQk \) and let their Kronecker product be \( (jk)^S(jk) \) as given in equation [27].

\[
jPj \times kQk = (jk)^S(jk). \tag{27}
\]

Let the matrices \( jPj \) and \( kQk \) be resolved into their characteristic roots and vectors as in equation [28].

\[
jPj = jB_pPpBj \text{ and } kQk = kC_qQqC_k \tag{28}
\]

where the matrices \( pPp \) and \( qQq \) are diagonal matrices containing the characteristic roots of \( jPj \) and \( kQk \) respectively, and the matrices \( jB_p \) and \( kC_q \) contain, as column vectors, the characteristic vectors of \( jPj \) and \( kQk \), respectively. The characteristic vectors are unit vectors so that:

\[
pBjB_p = pIp \text{ and } qC_kC_q = qIq. \tag{29}
\]

Let the matrix \( (jk)^S(jk) \) be resolved into its characteristic roots and vectors as in equation [30].

\[
(jk)^S(jk) = (jk)^V(pq)^S(pq)^V(jk) \tag{30}
\]

where \( (pq)^S(pq) \) is a diagonal matrix containing the characteristic roots and \( (jk)^V(pq) \) contains, as column vectors, the characteristic vectors of \( (jk)^S(jk) \). The characteristic
Vectors are unit vectors so that:

\[(pq)^{v}(jk)^{v}(pq) = (pq)^{I}(pq)\]  \[31\]

The important relations are given by equation [32] and equation [33].

\[(pq)^{S}(pq) = p^{P}p \times q^{Q}q\]  \[32\]

\[(jk)^{V}(pq) = j^{B}p \times k^{C}q\]  \[33\]

These relations are developed from substitution from equation [28] into equation [27] and the use of propositions (c) and (f) above. Thus, the matrix of characteristic roots of the Kronecker product matrix is the Kronecker product of the matrices of characteristic roots of the two matrices involved in the product; and the matrix of characteristic vectors of the Kronecker product matrix is the Kronecker product of the matrices of characteristic vectors of the matrices that enter into the product.

The Basic Mathematics of the Three-Mode Factor Analysis Model

The type of data set to which three-mode factor analysis can be applied can be recorded in a three-mode matrix \(X\), which has cell entries \(x_{ijk}\). An example of this type of data is the scores of a sample of individuals on a battery of tests on several occasions. A second example would be ratings by a sample of individuals on a selection of traits by several raters. A third example would be ratings by a
sample of individuals on a selection of bipolar adjective scales over a selection of concepts, as is done in studies using the semantic differential developed by Osgood, Suci and Tannenbaum (1957). In each of these cases, the data consists of numerical values which are identified by three modes of classification. These modes are directly related to the observation of the data and may be termed observational modes. These observational modes are designated as mode $i$, mode $j$, and mode $k$.

A convenient form in which to list the data is as a two-mode matrix with elementary mode $i$ for rows and combination mode $(jk)$ for columns. This is the matrix $i^X(jk)$.

Since an allowance must be made for discrepancies in fitting a model to observed data, such an allowance is symbolized in the model by letting $\hat{x}_{ijk}$ be the value obtained from the model and $e_{ijk}$ be the value of the discrepancy. Then:

$$ x_{ijk} = \hat{x}_{ijk} + e_{ijk} \quad [34] $$

In the simplest conceptualization of the model, which is a three dimensional extension of principle components analysis, $\hat{x}_{ijk}$ is approximated by $\tilde{x}_{ijk}$.

The model for $\tilde{x}_{ijk}$, written in summational notation, is

$$ \tilde{x}_{ijk} = \sum_m \sum_p \sum_q a_{im} b_{jp} c_{kq} m_{pq}. \quad [35] $$

In this model, three derivational modes, $m$, $p$, and $q$, are defined as conceptually more basic than the modes employed in making the observations. Each of these derivational modes
corresponds to one of the observational modes: \( m \) corresponding to \( i \), \( p \) corresponding to \( j \), and \( q \) corresponding to \( k \).

Each of these derivational modes can be thought of as a set of factors in the domain of the corresponding observational mode. An alternate interpretation is to think of each derivational mode as consisting of conceptual, or idealized categories corresponding to the observational mode. Thus, if the observational mode \( i \) is used to designate individuals in a sample, the derivational mode \( m \) can be thought of consisting of factors among individuals or of conceptual, or idealized individuals. It is hoped that the number of elements in each derivational mode will be markedly less than the number of elements in the corresponding observational mode. This hope is prefaced by the condition that a sufficiently large number of elements are included in each observational mode from the domain of elements that might be included.

The coefficients \( a_{im} \), \( b_{jp} \), and \( c_{kq} \) are entries in the two-mode matrices \( iA_m \), \( jB_p \), and \( kC_q \). These coefficients describe the elements in the observational modes in terms of the elements in the derivational modes. The coefficients \( g_{mpq} \) are entries in the three-mode matrix \( G \) which is termed the "core matrix". Just as in the original three-mode matrix, \( X \), in which each cell represents a particular combination of categories from the observational modes and the entry is a measure of a phenomenon whose value depends
on the combination of categories, in the same way each cell in the core matrix, $G$, represents a unique combination of categories from the derivational modes and the entry is a measure of the phenomenon for this combination of categories. The core matrix can be thought of as describing the basic relations existent in the measures of the phenomenon being observed. The two-mode matrices $iA_m$, $jB_p$, and $kC_q$ transform the statements of these relations from applying to the more basic derivational modes to applying to the observational modes. The interrelations among elements of one of the observational modes depend, in part, on the similarity of their relations to the derivational modes and, in part, to the relations in the core matrix.

The fundamental three-mode factor analysis model given in equation [35] can be written in terms of two-mode matrices by use of combination modes and Kronecker products. Three interpretations are:

\[
\begin{align*}
X_{jk} &= iA_m G_{pq} (pB_j \otimes qC_k) \quad [36a] \\
X_{ik} &= jB_p G_{mq} (mA_i \otimes qC_k) \quad [36b] \\
X_{lj} &= kC_q G_{mp} (mA_i \otimes pB_j) \quad [36c]
\end{align*}
\]

The core matrix $G$ is arranged to the matrix $\tilde{X}$ in each of these equations, taking into account the correspondence of observational modes to derivational modes. Also, the elementary derivational mode used as the row mode for the matrix $G$ is transformed by the appropriate two-mode matrix to the observational mode used as the row mode of matrix $\tilde{X}$. 
For example, in equation (36a), the two-mode matrix $iA_m$ transforms the row mode $m$ of matrix $G$ to the row mode $i$ of the matrix $X$.

The practical objective is, then, to solve for the $iA_m$, $jB_p$, $kC_q$, and $mG_{pq}$ matrices. This can be accomplished by defining three cross product matrices of the original data:

$$i^M_i = iX(jk)X_i$$  \[37a\]
$$j^P_j = jX(ik)X_j$$  \[37b\]
$$k^Q_k = kX(ij)X_k$$  \[37c\]

Consider the characteristic roots and vectors of these observed product matrices and let the modes for these dimensions be $m_2$, $p_2$, and $q_2$ with $N_{m_2}$, $N_{p_2}$, and $N_{q_2}$ as the number of nonzero roots. Note that all roots of these product matrices will be diagonal entries in the diagonal matrices $m_2M_{m_2}$, $p_2P_{p_2}$ and $q_2Q_{q_2}$. Also, let the roots for each matrix be arranged in descending order. Further, let the vectors corresponding to the roots be entered as columns in the matrices $iA_{m_2}$, $jB_{p_2}$, and $kC_{q_2}$. Then:

$$i^M_i = iA_{m_2}M_{m_2}A_i$$  \[38a\]
$$j^P_j = jB_{p_2}P_{p_2}B_j$$  \[38b\]
$$k^Q_k = kC_{q_2}Q_{q_2}C_k$$  \[38c\]

The core matrix is obtained by:

$$m_2^G(p_2q_2) = m_2A_iX(jk)(jB_{p_2}X_kC_{q_2}).$$  \[39\]

Since all nonzero roots are retained, a precise fit to the
observed data matrix $iX(jk)$ is obtained by an extension of equation [36a].

$$iX(jk) = iA_{m_2 G(p_2 q_2)}(p_2 Bj \boxtimes q_2 Ck)$$ \[40\]

The foregoing analysis is the complete model for the observed data.

The approximation is obtained by reducing the number of vectors in each of the three modes. One method of accomplishing this reduction is to make a plot between root number and root size for each of the product matrices and to inspect the series of points which result for a break from a steep slope to a more gentle slope, and to retain all roots preceding this break.

Note that if any of the observational modes ($i, j, \text{or } k$) are large, for example, 300 it would be necessary to factor a 300 by 300 cross products matrix containing 90,000 entries! This is impossible to accomplish by means of extant factoring methods within the capabilities of current computing equipment. However, computations can be carried out by use of another method.

This alternate method solves the problem of computations when the mode $i$ for individuals in the sample is very large.

First, a matrix $(jk)^R(jk)$ is defined as:

$$(jk)^R(jk) = (jk)X_i X(jk).$$ \[41\]

It is a product matrix of the observed data matrix $X$, and has rows and columns of combination modes. If submatrices
of the form \( k R_k \) are defined they would represent
correlations among elements of mode \( k \) for specific values of
mode \( j \). Remember that mode \( j \) formed the outer loop for the
combination mode \((jk)\) and that mode \( k \) formed the inner loop
for the combination mode \((jk)\). The sectioned matrix may be
represented as follows:

\[
\begin{bmatrix}
1_j^k R_k 1_j & 1_j^k R_k 2_j & 1_j^k R_k 3_j & \cdots \\
2_j^k R_k 1_j & 2_j^k R_k 2_j & 2_j^k R_k 3_j & \cdots \\
3_j^k R_k 1_j & 3_j^k R_k 2_j & 3_j^k R_k 3_j & \cdots \\
\end{bmatrix}
\]

The general form of the sections is \( j, k R_k, j' \) where \( j \) is used
as a variable index for the elements in mode \( j \) and \( j' \) is
used as an alternate variable index for the elements in mode
\( j \). The entries in the matrix \((jk) R(jk)\) are given by

\[
r_{jk j' k'} = \sum_i x_{ijk} x_{ij'k'}
\]

The entries in section \( j, k R_k, j' \) are for the specified values
of \( j \) and \( j' \) for the specified section.

The entries in the product matrices \( j P_j \) and \( k Q_k \) have
simple relations to the entries in the matrix \((jk) R(jk)\).

The diagonal entries in the matrix \( j P_j \) are the traces
of the corresponding diagonal sections in the matrix
\((jk) R(jk)\) and the off-diagonal entries in \( j P_j \) are traces of
corresponding off-diagonal sections of \((jk) R(jk)\).

\[
P_{jj'} = \text{tr}(j, k R_k, j')
\]
The entries in the matrix $kQ_k$ are:

$$q_{kk'} = \sum_j^r r_{jk} jk'$$  \[44\]

The entries $r_{jk} jk'$ occur only in the diagonal section of $(jk)^R_{(jk)}$ and the summation for specific values of $k$ and $k'$ is over entries in corresponding locations in the diagonal sections of $(jk)^R_{(jk)}$. Thus, the matrix $kQ_k$ can be expressed as the element wise sum of the diagonal sections of the matrix $(jk)^R_{(jk)}$.

$$kQ_k = \sum_j^r j, kR_{kj} jk'$$  \[45\]

Computational Procedures

Using the above relations, computations may be accomplished via the following steps:

(1) For many applications of three-mode factor analysis, one mode of the data will consist of individuals in a sample. In these cases mode $i$ may be used to represent the sample with $N_i$ being the number of individuals in the sample. It appears to be desirable to consider all scores divided by $\sqrt{N_i}$ before entering it into the three-mode factor analysis.

(2) Compute the matrix $(jk)^R_{(jk)}$ by equation [41].

(3) Compute the product matrices $jP_j$ and $kQ_k$ by equation [43] and equation [44].

(4) Determine the characteristic roots and vectors of these product matrices and form the diagonal matrices of
roots $p_1 q_1$ and matrices of vectors $jB p_1$ and $kC q_1$.

All nonzero or non-almost-zero roots are retained. A truncation of the characteristic roots and vectors is envisaged with the number of roots being between the numbers of all nonzero roots, $N_{p_2}$ and $N_{q_2}$, and the numbers of roots retained in the final approximation $N_p$ and $N_q$. It is important to discard only the very small roots at this time in addition to the zero roots. Elimination of very small roots, however, does aid in reduction of computations in subsequent steps and should not materially affect the results of these steps which assume that all nonzero roots are retained at this point.

(5) Compute the matrix $(p_1 q_1)^S(p_1 q_1)$ by:

$$ (p_1 q_1)^S(p_1 q_1) = (p_1 B_j \otimes q_1 C_k) (j k)^R(j k) (jB p_1 \otimes kC q_1) $$

(6) Determine the characteristic roots and vectors of $(p_1 q_1)^S(p_1 q_1)$ and for the diagonal matrix $m_1 S m_1$ containing the nonzero, or non-almost zero roots and the matrix $(p_1 q_1)^V m_1$ containing the corresponding unit-length vectors.

(7) Compute the core matrix $G$ by:

$$ (p_1 q_1)^G m = (p_1 q_1)^V m_1 S^l_2 $$

(8) Compute the matrix $i^A m_1$ by:

$$ i^A m_1 = i^X(j k) (jB p_1 \otimes kC q_1) (p_1 q_1)^V m_1 S^{-l_2} m_1 $$

(9) Reduce the number of elements in each derivational mode to those elements that will be used in the approximation. Retain only the roots considered to be significant in
some sense. The problem of how to arrive at the decisions as to which roots to retain has not been solved. The best procedure available may be to make the plot between root number and root size for each of the product matrices and to inspect the resulting series of points for a break from a steep slope to a more gentle slope and to retain all roots preceding this break.

(10) Rotate the matrices if desired.

It is now appropriate to discuss the transformation or rotation of the derivalional modes.

Let the matrices \( m^* \), \( p^* \), and \( q^* \) be square, non-singular matrices and let:

\[
\begin{align*}
 i^A &= i^A_m = i^A_{m^*} \quad [49a] \\
 j^B &= j^B_p = j^B_{p^*} \quad [49b] \\
 k^C &= k^C_q = k^C_{q^*} \quad [49c]
\end{align*}
\]

Where \( m^* \), \( p^* \) and \( q^* \) are transformed derivalional modes and where the matrices \( i^A_m \), \( j^B_p \), and \( k^C_q \) contain coefficients describing the observational mode elements in terms of the transformed derivalional modes. The inverse transformations are:

\[
\begin{align*}
 i^A_m &= (i^A_m)^{-1} = i^A_m \quad [50a] \\
 j^B_p &= (j^B_p)^{-1} = j^B_p \quad [50b] \\
 k^C_q &= (k^C_q)^{-1} = k^C_q \quad [50c]
\end{align*}
\]

Note that the row modes and column modes must be interchanged when inverting a matrix. Substitution from equation [50] into equation [36a] yields:
\[ i\tilde{X}(jk) = iA_{m\star}(m_{m\star})^{-1} m_{G}(pq) [(p T_{p})^{-1} \circ (q T_{q})^{-1}] \]

Use of the proposition of equation [24] concerning Kronecker products yields:

\[ i\tilde{X}(jk) = iA_{m\star}(m_{m\star})^{-1} m_{G}(pq) [(p T_{p})^{-1} \circ (q T_{q})^{-1}] \]

\[(p \circ B_{j} \circ q \circ C_{k})\]

Let:

\[ m \star G(p \star q \star) = (m_{m \star})^{-1} m_{G}(pq) [(p T_{p})^{-1} \circ (q T_{q})^{-1}] \]  

\[ p \star G(m \star q \star) = (p T_{p})^{-1} p_{G}(mq) [(m_{m \star})^{-1} \circ (q T_{q})^{-1}] \]  

\[ q \star G(m \star p \star) = (q T_{q})^{-1} q_{G}(mp) [(m_{m \star})^{-1} \circ (p T_{p})^{-1}] \]

Substitution of equation [53a] into equation [51] yields:

\[ i\tilde{X}(jk) = iA_{m \star} G(p \star q \star) (p \circ B_{j} \circ q \circ C_{k}) \cdot \]

Equation [53b] gives the transformed G matrix that is developed in steps involving equations [51] and [52]. Similar statements may be made for the other two modes. Equation [54] indicates that the use of the transformed two-mode coefficient matrices and transformed core matrix G reproduces the three-mode model in the form of equation [36a]. The other two equations of equation group [54] would demonstrate the same point.

Equations [53] yield the same transformed G matrix written in the three ways as two-dimensional matrices. Once
the transformation matrices are determined by some rotation of axes procedure, the transformed G matrix can be determined by some one of the equation set [53].

Determination of the transformation matrices \( p^T_p^* \) and \( q^T_q^* \) by rotation of matrices \( j^B_p \) and \( k^C_q \) to simple structure has been reasonably successful. But, it is generally difficult to rotate the \( i^A_m \) matrix to obtain the \( m^T_m^* \) matrix (assuming that the \( i \) mode is the subject mode) due to the usually large size of \( N_i \). Hence one can rotate the \( j^B_p \) and \( k^C_q \) matrices, apply the inverses of the obtained transformations to the core matrix, rotate the core matrix and then apply the inverse of the core transformations to the \( i^A_m \) matrix. This approach to rotation has met with the greatest success.

Since no extant computer program was of sufficient size to accommodate the data set, a FORTRAN-G program was created to carry out the analysis. A listing of the compiled program is shown in Appendix C.
APPENDIX B

DATA COLLECTION INSTRUMENTS

211
THE INSTRUCTIONS UTILIZED IN ADMINISTERING
THE ON SITE QUESTIONNAIRE

1. Introduce yourselves.

2. You are from The Ohio State University.

3. We are trying to find a best method to learn how satisfied a person is with his job.

4. Worthington feels that this is an important issue and has allowed us company time for this study. THEY FEEL IT IS IMPORTANT.

5. It is also important to you because the Company will get feedback about the satisfaction of its people and will act on that information as it has done in the past. (There was a satisfaction questionnaire about two years ago...the Co. posted the results and took remedial action).

6. The Co. will get satisfaction information by plant, by trick, and by job classification, but not by individuals...your names are in strict confidence with us at O.S.U.

7. If there are so few people in your job classification that you feel that it would identify you, put OTHER on the line asking that question.

8. If you don't believe that we will keep your name confidential...don't put it on the questionnaire.

9. But if you don't put your name on it we won't be able to get back to you if the questionnaire gets messed up somehow...no matter how hard we try some of them always get messed up...and it doesn't take too many messed up questionnaires to ruin our study.

10. Also, we will be mailing you another SHORT questionnaire later on...and without your names we won't know who to send them to.

11. Please look at the questionnaire...
   - It has two sides...look at them...
   - Can you read it...if a bad copy get another one...
   - Has anyone forgotten his glasses...

12. As I said before we are trying to find the best way we can to write a questionnaire to find out how satisfied a person is with his job.
13. On the questionnaire you will find a number of questions which we are asking you to answer in six different ways...We are not trying to trick you or to see how consistent you are...We are trying to find out which of the questions are the best ones to use and which of the six ways or combinations of ways is the best way to ask these questions.

14. You may find the different ways we ask the questions in the six columns are a bit confusing...but this is one of the things we are trying to find out and we will be able to tell from the way you answer...so just do the best you can and don't worry about it.

15. We have about half an hour to finish this...I'll keep you posted on the amount of time we have left. You shouldn't have any trouble finishing.

16. Oh yes...you can keep the pencils...but please give us back the questionnaires...

17. ANY QUESTIONS???. . .field them...

18. Possible questions...
- on age...grade down
- on education...grade up

19. If you have any questions after we start I'll answer them individually.
THE ON SITE QUESTIONNAIRE
Below you will see a list of characteristics or qualities that might be connected with your job. To the right you will find six columns. Each of these columns asks a question about each of the qualities listed below.

Each of the questions in each column asks you to give your answer in the form of a number; use the numbers indicated at the top of each column to answer the question for that column.

<table>
<thead>
<tr>
<th>COLUMN ONE</th>
<th>COLUMN TWO</th>
<th>COLUMN THREE</th>
</tr>
</thead>
<tbody>
<tr>
<td>How much of each quality is present on your job?</td>
<td>How much of each quality do you think should be present on your job?</td>
<td>How much of each quality would you like to be present on your job?</td>
</tr>
<tr>
<td><strong>COLUMN ONE</strong></td>
<td><strong>COLUMN TWO</strong></td>
<td><strong>COLUMN THREE</strong></td>
</tr>
<tr>
<td><strong>USE THESE NUMBERS</strong></td>
<td><strong>USE THESE NUMBERS</strong></td>
<td><strong>USE THESE NUMBERS</strong></td>
</tr>
<tr>
<td>very little</td>
<td>very little</td>
<td>very little</td>
</tr>
<tr>
<td>1/2</td>
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<td>1/2</td>
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<tr>
<td>a lot</td>
<td>a lot</td>
<td>a lot</td>
</tr>
<tr>
<td>1 2 3 4 5 6 7</td>
<td>1 2 3 4 5 6 7</td>
<td>1 2 3 4 5 6 7</td>
</tr>
</tbody>
</table>

The feeling of self esteem or self respect a person gets from being in my job.

The opportunity for personal growth and development in my job.

The prestige of my job inside the company (that is, the regard received from others in the company).

The amount of close supervision I receive.

The opportunity for independent thought and action in my job.

The feeling of security in my job.

The opportunity to find out how well I am doing in my job.

The prestige of my job outside the company (that is, the regard received from others not in the company).

The opportunity to complete work that I start.

The opportunity to do challenging work.

The feeling that I know whether I am performing my job well or poorly.

The opportunity to do a number of different things.

The opportunity on my job to get to know other people.

The opportunity to do a job from the beginning to the end (that is, the chance to do a whole job).

The freedom to do pretty much what I want on my job.

The amount of variety in my job.

The pay for my job.

The feeling of worthwhile accomplishment in my job.

The opportunity in my job to give help to other people.

The opportunity in my job for participation in the determination of methods, procedures, and goals.

The opportunity to develop close friendships in my job.

The opportunity for promotion.

The amount of respect and fair treatment that I receive from my boss.
COLUMN ONE

How much of each quality do you think should be present on your job?

USE THESE NUMBERS

1/2 very little 1/2 a lot

1 2 3 4 5 6 7

COLUMN TWO

How much of each quality would you like to be present on your job?

USE THESE NUMBERS

1/2 very little 1/2 a lot

1 2 3 4 5 6 7

COLUMN THREE

Look at your answers in column one and two. Considering those answers, how satisfied are you with each quality?

USE THESE NUMBERS

not 1/2 very satisfied 1/2 satisfied

1 2 3 4 5 6 7

COLUMN FOUR

Look at your answers in column one and three. Considering those answers, how satisfied are you with each quality?

USE THESE NUMBERS

not 1/2 very satisfied 1/2 satisfied

1 2 3 4 5 6 7

COLUMN FIVE

How satisfied are you in general with each quality?

USE THESE NUMBERS

not 1/2 very satisfied 1/2 satisfied

1 2 3 4 5 6 7

COLUMN SIX

How satisfied are you in general with each quality?

USE THESE NUMBERS

not 1/2 very satisfied 1/2 satisfied

1 2 3 4 5 6 7
Dear Sir:

First of all, we would like to thank you for your time and trouble in completing the recent job satisfaction survey. The enclosed questionnaire is being sent to everyone because the information we got from the satisfaction survey will be much more useful when combined with your responses to this mail questionnaire. As was true with the satisfaction form, whether or not you put your name on it is up to you; however, it would help us if you did. Please get this back as soon as you can, because we're anxious to process this information, and we feel the results can be beneficial to both you and the company. Thanks again.

Sincerely,

Tom Zenisek
The Ohio State University

TZ/ckh

Name __________________________
THE MAIL QUESTIONNAIRE
The following items are comments people have made or might make about their work. Think about yourself and your job. If you strongly agree with the statement, put a check in the "strongly agree" column; if you strongly disagree, put a check in the "strongly disagree" column. Use the "agree" and "disagree" columns for intermediate values.

<table>
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<tr>
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<td>Y</td>
<td>E</td>
<td>E</td>
<td>Y</td>
</tr>
</tbody>
</table>

a. The major satisfaction in my life comes from my job ( ) ( ) ( ) ( )
b. The most important things that happen to me involve my work .......................................................... ( ) ( ) ( ) ( )
c. I'm really a perfectionist about my work.............. ( ) ( ) ( ) ( )
d. I live, eat and breathe my work......................... ( ) ( ) ( ) ( )
e. Most things in life are more important than work... ( ) ( ) ( ) ( )

1) How would you feel if you heard (or read about) someone criticizing your company or company products or comparing your company unfavorably to other companies?

_____ a. It would not really bother me. I do not care much what other people think of the company.

_____ b. It would bother me a bit.

_____ c. It would bother me quite a bit; I am anxious to have people think well of the company.

2) If someone asked you to describe yourself and you could tell only one thing about yourself, which of the following answers would you be most likely to give?

_____ a. I come from (my home state)

_____ b. I work for (this company)

_____ c. I am a (occupation as work type)

_____ d. I am a (church membership or preference)

_____ e. I am a graduate of (my school)
Listed below are a number of statements concerning personal attitudes and feelings. Read each item and decide whether the statement is true or false as it pertains to you personally. Check each item in the appropriate column marked true or false (BUT NOT BOTH).

<table>
<thead>
<tr>
<th>True</th>
<th>False</th>
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</thead>
<tbody>
<tr>
<td>1. Before voting I thoroughly investigate the qualifications of all the candidates.</td>
<td></td>
</tr>
<tr>
<td>2. I never hesitate to go out of my way to help someone in trouble.</td>
<td></td>
</tr>
<tr>
<td>3. It is sometimes hard for me to go on with my work if I am not encouraged.</td>
<td></td>
</tr>
<tr>
<td>4. I have never intensely disliked someone.</td>
<td></td>
</tr>
<tr>
<td>5. On occasion I have had doubts about my ability to succeed in life.</td>
<td></td>
</tr>
<tr>
<td>6. I sometimes feel resentful when I don't get my way.</td>
<td></td>
</tr>
<tr>
<td>7. I am always careful about my manner of dress.</td>
<td></td>
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<tr>
<td>8. My table manners at home are as good as when I eat out in a restaurant.</td>
<td></td>
</tr>
<tr>
<td>9. If I could get into a movie without paying and be sure I was not seen I would probably do it.</td>
<td></td>
</tr>
<tr>
<td>10. On a few occasions, I have given up doing something because I thought too little of my ability.</td>
<td></td>
</tr>
<tr>
<td>11. I like to gossip at times.</td>
<td></td>
</tr>
<tr>
<td>12. There have been times when I felt like rebelling against people in authority even though I knew they were right.</td>
<td></td>
</tr>
<tr>
<td>13. No matter who I'm talking to, I'm always a good listener.</td>
<td></td>
</tr>
<tr>
<td>14. I can remember playing sick to get out of something.</td>
<td></td>
</tr>
<tr>
<td>15. There have been occasions when I took advantage of someone.</td>
<td></td>
</tr>
<tr>
<td>16. I'm always willing to admit it when I make a mistake.</td>
<td></td>
</tr>
<tr>
<td>17. I always try to practice what I preach.</td>
<td></td>
</tr>
<tr>
<td>18. I don't find it particularly difficult to get along with loud-mouthed, obnoxious people.</td>
<td></td>
</tr>
</tbody>
</table>
19. I sometimes try to get even rather than forgive and forget................................................................. True False
20. When I don't know something I don't at all mind admitting it................................................................. True False
21. I am always courteous, even to people who are disagreeable.................................................................. True False
22. At times I have really insisted on having things my own way................................................................. True False
23. There have been occasions when I felt like smashing things................................................................. True False
24. I would never think of letting someone else be punished for my wrongdoings........................................ True False
25. I never resent being asked to return a favor................................................................................. True False
26. I have never been irked when people expressed ideas very different from my own.............................. True False
27. I never make a long trip without checking the safety of my car............................................................. True False
28. There have been times when I was quite jealous of the good fortune of others..................................... True False
29. I have almost never felt the urge to tell someone off............................................................................ True False
30. I am sometimes irritated by people who ask favors of me..................................................................... True False
31. I have felt that I was punished without a cause.................................................................................... True False
32. I sometimes think when people have a misfortune they only got what they deserved........................ True False
33. I have never deliberately said something that hurt someone's feelings................................................ True False

Each question below has two responses. Select one response within each question which agrees most closely with your views and check that response.

1. _____ (A) Children get into trouble because their parents punish them too much
         _____ (B) The trouble with most children nowadays is that their parents are too easy with them
2. (A) Many of the unhappy things in people's lives are partly due to bad luck
   (B) People's misfortunes result from the mistakes they make

3. (A) One of the major reasons why we have wars is because people don't take enough interest in politics
   (B) There will always be wars, no matter how hard people try to prevent them

4. (A) In the long run people get the respect they deserve in this world
   (B) Unfortunately, an individual's worth often passes unrecognized no matter how hard he tries

5. (A) The idea that teachers are unfair to students is nonsense
   (B) Most students don't realize the extent to which their grades are influenced by accidental happenings

6. (A) Without the right breaks one cannot be an effective leader
   (B) Capable people who fail to become leaders have not taken advantage of their opportunities

7. (A) No matter how hard you try some people just don't like you
   (B) People who can't get others to like them don't understand how to get along with others

8. (A) Heredity plays the major role in determining one's personality
   (B) It is one's experiences in life which determine what they're like

9. (A) I have often found that what is going to happen will happen.
   (B) Trusting to fate has never turned out as well for me as making a decision to take a definite course of action

10. (A) In the case of the well prepared student there is rarely, if ever, such a thing as an unfair test
    (B) Many times exam questions tend to be so unrelated to course work that studying is really useless

11. (A) Becoming a success is a matter of hard work; luck has little or nothing to do with it
    (B) Getting a good job depends mainly on being in the right place at the right time
12. (A) The average citizen can have an influence in government decisions
(B) This world is run by the few people in power, and there is not much the little guy can do about it

13. (A) When I make plans, I am almost certain that I can make them work
(B) It is not always wise to plan too far ahead because many things turn out to be a matter of good or bad fortune anyhow

14. (A) There are certain people who are just no good
(B) There is some good in everybody

15. (A) In my case getting what I want has little or nothing to do with luck
(B) Many times we might just as well decide what to do by flipping a coin

16. (A) Who gets to be boss often depends upon who was lucky enough to be in the right place first
(B) Getting people to do the right thing depends upon ability; luck has little or nothing to do with it

17. (A) As far as world affairs are concerned, most of us are victims of forces we can neither understand nor control
(B) By taking an active part in political and social affairs the people can control world events

18. (A) Most people don't realize the extent to which their lives are controlled by accidental happenings
(B) There is really no such thing as luck

19. (A) One should always be willing to admit mistakes
(B) It is usually best to cover up one's mistakes

20. (A) It is hard to know whether or not a person really likes you
(B) How many friends you have depends on how nice a person you are

21. (A) In the long run the bad things that happen to us are balanced by the good ones
(B) Most misfortunes are the result of lack of ability, ignorance, laziness, or all three

22. (A) With enough effort we can wipe out political corruption
(B) It is difficult for people to have much control over the things politicians do in office
23. ______ (A) Sometimes I can't understand how teachers arrive at the grades they give
 ______ (B) There is a direct connection between how hard I study and the grades I get

24. ______ (A) A good leader expects people to decide for themselves what they should do
 ______ (B) A good leader makes it clear to everybody what their jobs are

25. ______ (A) Many times I feel that I have little influence over the things that happen to me
 ______ (B) It is impossible for me to believe that chance or luck plays an important part in my life

26. ______ (A) People are lonely because they don't try to be friendly
 ______ (B) There's not much use in trying too hard to please people; if they like you, they like you

27. ______ (A) There is too much emphasis on athletics in high school
 ______ (B) Team sports are an excellent way to build character

28. ______ (A) What happens to me is my own doing
 ______ (B) Sometimes I feel that I don't have enough control over the direction my life is taking

29. ______ (A) Most of the time I can't understand why politicians behave the way they do
 ______ (B) In the long run the people are responsible for bad government on a national level as well as a local level
THE PERFORMANCE EVALUATION SCALE

(Immediate Supervisor Rating)
PERFORMANCE EVALUATION PROGRAM

Evaluation Date __________________________ Evaluated by ________________

Name _______________________ Social Security # ________________

Position Title & Department ____________________________

Cost Center (if any) ____________________________ Length of Service Date ______

Instructions: For each item below, circle the number on the line at that point which best expresses your evaluation. Circle only one number on each line. Answer each item unless you have absolutely no basis for arriving at any evaluation, in which case, check the NA (not applicable) box to the right of the item you cannot evaluate.

<table>
<thead>
<tr>
<th>Item</th>
<th>Description</th>
<th>Circle Numbers</th>
<th>Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Quality</td>
<td>(Consider both routine and priority-emergency jobs.)</td>
<td>1 2 3 4 5 6 7</td>
<td>very poor average outstanding NA</td>
</tr>
<tr>
<td>2. Quantity</td>
<td>(Consider both routine and priority-emergency jobs.)</td>
<td>1 2 3 4 5 6 7</td>
<td>very poor average outstanding NA</td>
</tr>
<tr>
<td>3. Speed of Response</td>
<td>(Time to react, on both routine and priority-emergency tasks.)</td>
<td>1 2 3 4 5 6 7</td>
<td>very poor average outstanding NA</td>
</tr>
<tr>
<td>4. Effort Expended</td>
<td>(On both routine and priority-emergency jobs.)</td>
<td>1 2 3 4 5 6 7</td>
<td>very poor average outstanding NA</td>
</tr>
<tr>
<td>5. Interest</td>
<td>(Asks questions, makes suggestions, provides alternatives.)</td>
<td>1 2 3 4 5 6 7</td>
<td>very poor average outstanding NA</td>
</tr>
<tr>
<td>6. Flexibility</td>
<td>(Is open to new ideas, can assimilate new information rapidly, is able to adjust to rapid change.)</td>
<td>1 2 3 4 5 6 7</td>
<td>very poor average outstanding NA</td>
</tr>
<tr>
<td>7. Commitment</td>
<td>(Is dedicated to the Company, to his work group, and to his job.)</td>
<td>1 2 3 4 5 6 7</td>
<td>very poor average outstanding NA</td>
</tr>
<tr>
<td>8. Job Knowledge</td>
<td>(Possession of knowledge and ability and willingness to share it with others.)</td>
<td>1 2 3 4 5 6 7</td>
<td>very poor average outstanding NA</td>
</tr>
<tr>
<td>9. Cooperation</td>
<td>(Cooperativeness, acting as a good team member, working well with co-workers and with other levels.)</td>
<td>1 2 3 4 5 6 7</td>
<td>very poor average outstanding NA</td>
</tr>
<tr>
<td>10. Good Housekeeping</td>
<td>(Concern for and attention to neatness; cleanliness; orderliness; care of equipment, materials, and supplies.)</td>
<td>1 2 3 4 5 6 7</td>
<td>very poor average outstanding NA</td>
</tr>
<tr>
<td>11. Safety</td>
<td>(Work habits, actions, and methods which demonstrate consideration for the safety of self and fellow workers; making safety suggestions.)</td>
<td>1 2 3 4 5 6 7</td>
<td>very poor average outstanding NA</td>
</tr>
<tr>
<td>12. Punctuality and Attendance</td>
<td></td>
<td>1 2 3 4 5 6 7</td>
<td>very poor average outstanding NA</td>
</tr>
<tr>
<td>13. Summary</td>
<td>(Overall effectiveness in current position.)</td>
<td>1 2 3 4 5 6 7</td>
<td>very poor average outstanding NA</td>
</tr>
</tbody>
</table>
APPENDIX C

A FORTRAN-G PROGRAM FOR
THREE-MODE FACTOR ANALYSIS
This computer program provides a factor analytic solution for a three-dimensional i by j by k data matrix. The computational procedures employed are based upon those presented in Method III of Tucker (1966). This method provides most efficient analysis when the i mode, usually individuals, is quite large, though this size requirement is certainly not a necessary condition.

The mathematical notation used in this section is that of Appendix A.

The Theoretical Model

In this section, i, j, and k represent the modes of classification that are directly related to the observation of the data; i, j, and k are thus termed observational modes. An example would be the observation of scores for i individuals on j tests given under k different conditions.

Through factoring, one wishes to reduce the observational modes i, j, and k to corresponding derivational modes m, p, and q, respectively. Each of the derivational modes can be thought of as a set of factors in the domain of the corresponding observational mode. The core matrix G then serves to describe the relationships among the derivational modes.

The fundamental three-mode factor analysis model is represented by the equation:
\[ \tilde{x}_{ijk} = \sum_{m} \sum_{p} \sum_{q} a_{im} b_{jp} c_{kq} g_{mpq}, \]  

in which:

(a) \( \tilde{x}_{ijk} \) is an approximation to the observed score \( x_{ijk} \).

(b) \( a_{im}, b_{jp}, \) and \( c_{kq} \) are entries in the two-mode matrices \( iA_m, jB_p, \) and \( kC_q \) describing the elements in the observational modes \( i, j, \) and \( k \) in terms of the dimensions in the derivational modes \( m, p, \) and \( q \) respectively.

(c) the coefficients \( g_{mpq} \) as entries in a three-mode core matrix \( G \) represent the measures of the phenomenon being observed for each combination of the dimensions of the derivational modes.

In matrix form, the model can be represented as

\[ iX(jk) = iA_m G(pq) (pB_j \times qC_k), \]

in which:

(a) \( \times \) indicates a Kronecker product, and

(b) the matrices \( A, B, \) and \( C \) are factor solutions for modes \( i, j, \) and \( k \) respectively, which serve to transform the core matrix \( G \) of the 3 derivational modes into the matrix \( X \) representing the 3 observational modes.

Program Limitations

The program is designed for use on an IBM system 360/370 that is equipped with a FORTRAN-G or FORTRAN-H compiler that provides at least 1536-K of core storage. This amount
of core storage is usually provided via a VS-1 or VS-2 virtual memory system. Time requirements are central processor unit and data set size dependent; for large data sets, a 165 or 168 central processor unit is highly recommended.

The user must specify the number of factors to be extracted from each of the three observational modes. This procedure is employed, as the use of other factor-stopping criteria (e.g., percentage of variance accounted for, or eigenvalues below unity) could easily lead to the computation of a great many useless factors and to a very large and unmanageable core matrix. The user is also cautioned against specifying large numbers of factors, for this specification would cause substantial increases in time required to factor the various modes and to compute the core matrix.

It is strongly recommended that the user be thoroughly familiar with Tucker's (1966) article and with factor analysis in general before attempting to utilize this program.

Physical size limitations of the program are as follow:

1. The maximum allowable number of variables in the i mode is 305.
2. The maximum allowable number of variables in the j mode is 41.
3. The maximum allowable number of variables in the k mode is 41.
4. The product of the number of variables in the j and k modes cannot exceed 185.
5. The maximum allowable number of factors to be removed from the R matrix (the i mode) is 32.

6. The maximum allowable number of factors to be removed from the P matrix (the j mode) is 21.

7. The maximum allowable number of factors to be removed from the Q matrix (the k mode) is 21.

8. The product of the number of factors to be removed from the P and Q matrices cannot exceed 100.

Program Input

The input data may consist of a three-dimensional raw data matrix in the two-dimensional form $i^{\text{X}(jk)}$, where $i$ is assumed to be the largest mode (usually subjects) and $jk$ represents the combination mode, with mode $k$ nested within mode $j$, or a Gramian matrix usually consisting of correlation coefficients, covariances, or cross-products in the form $(jk)^{R(jk)}$.

Control Parameters

There are six input parameter cards which must precede the input data deck.

Card No. 1 - this card contains the fifteen input parameters to be described; they must be in fortran I-4 format, right justified.

The 15 input parameters of input parameter Card No. 1 are as follow:
1. \# = the number of variables in the i mode = the number of subjects: Max = 305

2. \# = the number of variables in the j mode: Max = 41

3. \# = the number of variables in the k mode: Max = 41 (the product of parameters 2 and 3 above, cannot exceed 185)

4. \# = the number of factors to be removed from the \((jk)^R(jk)\) matrix = the number of factors in the \(A\) matrix: The i-mode: Max = 32

5. \# = the number of factors to be removed from the \(P\) matrix: The j mode: Max = 21

6. \# = the number of factors to be removed from the \(Q\) matrix: The k mode: Max = 21 (the product of parameters 5 and 6 above cannot exceed 100)

7. \#1 = compute and print the \(iA_m\) matrix
   2 = do not do so

8. \#1 = the input data is an \(iX(jk)\) matrix
   2 = the input data is a \((jk)^R(jk)\) matrix

9. \#1 = print the input \(iX(jk)\) matrix
   2 = do not do so

10. Concerning the \((jk)^R(jk)\) matrix
    \#1 = standardize by dividing by the sample size
        2 = do not do so

11. \#1 = print the \((jk)^R(jk)\) matrix
    2 = do not do so

12. \#1 = punch the \(iA_m\) matrix
    2 = do not do so

13. \#1 = punch the \(jB_p\) matrix
    2 = do not do so

14. \#1 = punch the \(kC_q\) matrix
    2 = do not do so
15. #1 = punch the \((pq)G_m\) matrix 
   2 = do not do so

   The default option on parameters 7 to 15 above is one.

   The user specified parameters appear on program output where
   a \# appears above.

Card No. 2 - this card must be as follows:

   Cols. 1-8 I-format
   Cols. 9-80 The format of the input data.*

Card No. 3 - this card must be as follows:

   Cols. 1-8 A-format
   Cols. 9-80 The format of the punched output \(A_m\)
   matrix, leave blank if not desired.*

Card No. 4 - this card must be as follows:

   Cols. 1-8 B-format
   Cols. 9-80 The format of the punched output \(B_p\)
   matrix; leave blank if not desired.*

Card No. 5 - this card must be as follows:

   Cols. 1-8 C-format
   Cols. 9-80 The format of the punched output \(C_q\)
   matrix; leave blank if not desired.*

Card No. 6 - this card must be as follows:

   Cols. 1-8 G-format
   Cols. 9-80 The format of the punched output
   \((pq)G_m\) matrix; leave blank if not desired.*

* Use fortran F-type format on input parameter cards 2 to 6.
Program Output

The printed output consists of the following:

1. The input $i^X(jk)$ or $(jk)^R(jk)$ matrix, if desired.
2. The $(jk)^R(jk)$ matrix, if desired.
3. All of the eigenvalues of the $(jk)^R(jk)$ matrix.
4. The requested number of principal axis factors of the $(jk)^R(jk)$ matrix.
5. The $jP_j$ matrix.
6. All of the eigenvalues of the $jP_j$ matrix.
7. The requested number of eigenvectors of the $jP_j$ matrix. This set of eigenvectors is the $jB_p$ matrix where $p$ represents the derivational mode corresponding to the observational mode $j$. The $jB_p$ matrix will also be punched onto cards, according to a user specified format, if desired.
8. The $kQ_k$ matrix.
9. All of the eigenvalues of the $kQ_k$ matrix.
10. The requested number of eigenvectors of the $kQ_k$ matrix. This set of eigenvectors is the $kC_q$ matrix where $q$ represents the derivational mode corresponding to the observational mode $k$. The $kC_q$ matrix will also be punched onto cards, according to a user specified format, if desired.
11. The core matrix $(pq)G_m$, where $m$, $p$, and $q$ represent the derivational modes corresponding to the observational
modes i, j, and k, respectively. The \((pq)^G_m\) matrix will also be punched onto cards, according to a user specified format, if desired.

12. The \(iA^m_m\) matrix where \(m\) is the derivational mode corresponding to the observational mode \(i\). The \(iA^m_m\) matrix will also be punched onto cards, according to a user specified format, if desired.

Availability

The program in card deck form with complete documentation including a sample card setup and sample output is available from the author at the Faculty of Management, The University of Calgary, Calgary, Alberta T2N 1N4, Canada.

Program Listing

A compiled listing of the program is as follows:
DIMENSION A (130,185), B (185,335)
REAL*8 Z(165,185)
DIMENSION IPAR(17), IFORM(18), IAOR(18), IBOR(18), ICOR(18), IGOR(18)
COMMON /ONE/ IAOR, IBOR, ICOR, IGOR, IPAR
COMMON /TWO/ JMX, KMX, MMX, LMX, NMX, TMX
COMMON /THREE/ IFORM

READ(5,35) (IPAR(I), I=3,17)
READ(5,75) (IFORM(I), I=1,18)
READ(5,75) (IAOR(I), I=1,18)
READ(5,75) (IBOR(I), I=1,18)
READ(5,75) (ICOR(I), I=1,18)
READ(5,75) (IGOR(I), I=1,18)

IF (IPAR(9).NE.2) IPAR(9) = 1
IF (IPAR(10).NE.2) IPAR(10) = 1
IF (IPAR(11).NE.2) IPAR(11) = 1
IF (IPAR(12).NE.2) IPAR(12) = 1
IF (IPAR(13).NE.2) IPAR(13) = 1
IF (IPAR(14).NE.2) IPAR(14) = 1
IF (IPAR(15).NE.2) IPAR(15) = 1
IF (IPAR(16).NE.2) IPAR(16) = 1
IF (IPAR(17).NE.2) IPAR(17) = 1

IMX = IPAR(3)
JMX = IPAR(4)
KMX = IPAR(5)
MMX = IPAR(6)
LMX = IPAR(7)
NMX = IPAR(8)
M = IPAR(3)
L = IPAR(4) * IPAR(5)
IL = IPAR(7) * [IPAR(8)]

WRITE(6,51)
WRITE(6,52)
WRITE(5,53)
WRITE(6,54)
WRITE(6,55)
WRITE(5,56)
WRITE(6,57)
WRITE(6,58)
WRITE(6,59)
WRITE(6,60)
WRITE(6,61)
WRITE(6,62)
WRITE(6,63)
WRITE(6,64)
WRITE(6,65)
WRITE(6,57)
WRITE(6,200)
WRITE(6,201)
WRITE(6,71)
WRITE (6,101)
WRITE(6,202)
WRITE (6,203)
WRITE(6,204)
WRITE (5,205)
WRITE (6,206)
WRITE (6,207)
WRITE (5,410)
WRITE (6,208)
WRITE (6,209)
WRITE (6,210)
WRITE (5,411)
WRITE (6,211)
WRITE (6,212)
WRITE (6,213)
WRITE (5,412)
WRITE (6,214)
WRITE (6,215)
WRITE (6,216)
WRITE (5,413)
WRITE (5,1)
WRITE (5,3) I PAR(3)
WRITE (5,4) I PAR(4)
WRITE (6,5) I PAR(5)
WRITE (5,14)
WRITE (6,19) L
WRITE (6,6) I PAR(6)
WRITE (5,402)
WRITE (6,21)
WRITE (6,7) I PAR(7)
WRITE (5,414)
WRITE (6,8) I PAR(8)
WRITE (6,415)
WRITE (5,18)
WRITE (6,20) L
WRITE (5,66)
WRITE (5,9) I PAR(9)
WRITE (6,416)
WRITE (6,19)
WRITE (6,10) I PAR(10)
WRITE (5,401)
WRITE (6,11)
WRITE (6,16)
WRITE (5,402)
WRITE (5,11) I PAR(11)
| WRITE (6,403) | WRITE (6,15) | WRITE (5,12) IPAR(12) | WRITE (5,404) | WRITE (6,17) | WRITE (6,15) | WRITE (5,13) IPAR(13) | WRITE (5,405) | WRITE (6,15) | WRITE (5,84) IPAR(14) | WRITE (5,406) | WRITE (6,15) | WRITE (5,85) IPAR(15) | WRITE (5,407) | WRITE (6,15) | WRITE (5,96) IPAR(16) | WRITE (5,408) | WRITE (5,15) | WRITE (5,97) IPAR(17) | WRITE (6,409) | WRITE (5,15) | WRITE (5,103) |
| WRITE (5,22) [IFORM(I),I=1,13] | WRITE (6,94) [IAOR(I),I=1,18] | WRITE (5,417) | WRITE (6,95) [IBOR(I),I=1,18] | WRITE (5,418) | WRITE (6,96) [ICOR(I),I=1,18] | WRITE (5,419) | WRITE (6,97) [IGOR(I),I=1,18] | WRITE (5,420) | WRITE (5,303) | WRITE (5,304) | WRITE (5,305) | WRITE (5,306) | WRITE (5,307) | WRITE (5,308) | WRITE (15,309) | WRITE (15,310) | WRITE (15,311) | WRITE (15,312) |

C CALL DR1V1 (*AR,RR,M,L) C

301 FORMAT(15X,'THIS PROGRAM IS DESIGNED FOR THE USE OF AN IBM SERIES 36 OR 370 SYSTEM (OR COMPATIBLE SYSTEM).')
302 FORMAT (3X,'EQUIPPED WITH A FORTRAN-G COMPILER AND PROVIDING AT LEAST 1935-K OF CORE STORAGE -- USUALLY VIA A VIRTUAL MEMORY SYSTEM.')
303 FORMAT (3X,'TIME REQUIREMENTS ARE CPJ AND DATA SET SIZE DEPENDENT',M)
 1X, FOR LARGE DATA SETS')
364 FORMAT(3X,'A 165 OR 168 CPU IS RECOMMENDED.')
365 FORMAT(1X,'A 304 BY 184 INPUT RAW DATA SET, UTILIZING ALL P
 1PROGRAM OPTIONS, REQUIRES ABOUT 35 MINUTES OF CPU TIME ON')
366 FORMAT (3X,'A 143 CPU-- UTILIZING THE ENTIRE MACHINE.')
367 FORMAT (1X,'MULTIPLE COPIES OF THE PRINTED OUTPUT ARE OBTAINED BY')
1CHANGING THE PARAMETER ON THE THREE (3) SYSPRINT JCL CARDS.')
370 FORMAT (3X,'18A4)
371 FORMAT (1514)
372 FORMAT (1X,'*********** NOTE ***********/',)
373 FORMAT (1X,'THE NUMBER OF VARIABLES IN THE I MODE = THE N',-1)
13, OF SUBJECTS : MAX=305,')
374 FORMAT (1X,'THE NUMBER OF VARIABLES IN THE J MODE (THE OUT',-1)
1R LOOP, SEE THE TUCKER ARTICLE PP. 281 & 300) MAX=41,')
375 FORMAT (1X,'THE NUMBER OF VARIABLES IN THE K MODE (THE INNE',-1)
1R LOOP, SEE THE TUCKER ARTICLE PP. 281 & 300) MAX=41,')
376 FORMAT (5X,'THE PRODUCT OF J & K IS',-1)
377 FORMAT (5X,'THE PRODUCT OF J & K CANNOT EXCEED 185...')
378 FORMAT (1X,'THE NO. OF FACTORS TO BE REMOVED FROM THE .O. MAT',-1)
379 FORMAT (1X,'THE J MODE : MAX=21')
380 FORMAT(1X,'THE K MODE : MAX=21')
381 FORMAT (1X,14,2X,1 = COMPUTE & PRINT THE .A. MATRIX')
382 FORMAT (7X,2 = THE INPUT DATA IS A (...) MATRIX')
383 FORMAT (1X,'THE INPUT DATA IS AN (...) MATRIX')
384 FORMAT (1X,'PRINT THE INPT. (...) MATRIX')
385 FORMAT (1X,'CONCERNING THE (...) MATRIX')
386 FORMAT (7X,1 = STANDARDIZE BY DIVIDING BY THE SAMPLE SIZE')
387 FORMAT (1X,'PRINT THE (...) MATRIX')
388 FORMAT (5X,'...THE PRODUCT OF THE ABOVE TWO (2) PARAMETERS IS',-1)
389 FORMAT (1X,'...CANNOT EXCEED 100...')
390 FORMAT (1X,'THE NO. OF FACTORS TO BE REMOVED FROM THE (...) MAT',-1)
391 FORMAT (1X,'THE NO. OF FACTORS IN THE .A. MATRIX :')
392 FORMAT (1X,'THE I MODE : MAX=32')
393 FORMAT (7X,2 = DO NOT DO SO')
394 FORMAT (1X,'THE INPUT DATA FORMAT IS',18X,18A4,/)
395 FORMAT(1X,'THE PUNCHED OUTPUT .C. MATRIX FORMAT IS',2X,18A4)
396 FORMAT(1X,'THE PUNCHED OUTPUT .B. MATRIX FORMAT IS',2X,18A4)
397 FORMAT(1X,'THE PUNCHED OUTPUT .G. MATRIX FORMAT IS',2X,18A4)
398 FORMAT (21X,'K 0')
399 FORMAT (21X,'J P')
400 FORMAT (21X,'I M')
401 FORMAT (20X,'PO M')

303 FORAMT (3X,'TIME REQUIREMENTS ARE CPJ AND DATA SET SIZE DEPENDENT-M')
1IT, FOR LARGE DATA SETS')
364 FORMAT(3X,'A 165 OR 168 CPU IS RECOMMENDED.')
365 FORMAT(1X,'A 304 BY 184 INPUT RAW DATA SET, UTILIZING ALL P
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370 FORMAT (3X,'18A4)
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373 FORMAT (1X,'THE NUMBER OF VARIABLES IN THE I MODE = THE N',-1)
13, OF SUBJECTS : MAX=305,')
374 FORMAT (1X,'THE NUMBER OF VARIABLES IN THE J MODE (THE OUT',-1)
1R LOOP, SEE THE TUCKER ARTICLE PP. 281 & 300) MAX=41,')
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1R LOOP, SEE THE TUCKER ARTICLE PP. 281 & 300) MAX=41,')
376 FORMAT (5X,'THE PRODUCT OF J & K IS',-1)
377 FORMAT (5X,'THE PRODUCT OF J & K CANNOT EXCEED 185...')
378 FORMAT (1X,'THE NO. OF FACTORS TO BE REMOVED FROM THE .O. MAT',-1)
379 FORMAT (1X,'THE J MODE : MAX=21')
380 FORMAT(1X,'THE K MODE : MAX=21')
381 FORMAT (1X,14,2X,1 = COMPUTE & PRINT THE .A. MATRIX')
382 FORMAT (7X,2 = THE INPUT DATA IS A (...) MATRIX')
383 FORMAT (1X,'THE INPUT DATA IS AN (...) MATRIX')
384 FORMAT (1X,'PRINT THE INPT. (...) MATRIX')
385 FORMAT (1X,'CONCERNING THE (...) MATRIX')
386 FORMAT (7X,1 = STANDARDIZE BY DIVIDING BY THE SAMPLE SIZE')
387 FORMAT (1X,'PRINT THE (...) MATRIX')
388 FORMAT (5X,'...THE PRODUCT OF THE ABOVE TWO (2) PARAMETERS IS',-1)
389 FORMAT (1X,'...CANNOT EXCEED 100...')
390 FORMAT (1X,'THE NO. OF FACTORS TO BE REMOVED FROM THE (...) MAT',-1)
391 FORMAT (1X,'THE NO. OF FACTORS IN THE .A. MATRIX :')
392 FORMAT (1X,'THE I MODE : MAX=32')
393 FORMAT (7X,2 = DO NOT DO SO')
394 FORMAT (1X,'THE INPUT DATA FORMAT IS',18X,18A4,/)
395 FORMAT(1X,'THE PUNCHED OUTPUT .C. MATRIX FORMAT IS',2X,18A4)
396 FORMAT(1X,'THE PUNCHED OUTPUT .B. MATRIX FORMAT IS',2X,18A4)
397 FORMAT(1X,'THE PUNCHED OUTPUT .G. MATRIX FORMAT IS',2X,18A4)
398 FORMAT (21X,'K 0')
399 FORMAT (21X,'J P')
400 FORMAT (21X,'I M')
401 FORMAT (20X,'PO M')
1. FORMAT(IX, 'THE FIFTEEN (15) INPUT PARAMETERS OF INPUT PARAMETER CARD NO. 1 ARE...
1ST NO. 1 ARE:',/)
2. FOR THIS THREE MODE FACTOR (PRINCIPAL COMPONENTS) ANALYSIS PROGRAM:
3. 'THREE MODE FACTOR (PRINCIPAL COMPONENTS) ANALYSIS'
4. SUMMARY PROGRAM: FEBRUARY 1978 VERSION
5. FORMAT(IX, '/I ', 'PLEASE REPORT ALL MALFUNCTIONS TO:')
6. FORMAT(IX, '/I ', 'PROFESSOR THOMAS J. ZEMISEK')
7. FORMAT(IX, '/I ', 'THE FACULTY OF MANAGEMENT')
8. FORMAT(IX, '/I ', 'THE UNIVERSITY OF CALGARY')
9. FORMAT(IX, '/I ', 'CALGARY ALBERTA, T2N 1N4')
10. FORMAT(IX, '/I ', 'CANADA')
11. FORMAT(IX, '/I ', 'A THOROUGH FAMILIARITY WITH THE ARTICLE:')
12. FORMAT(IX, '/I ', 'BUT THE ABOVE THREE (3) PARAMETERS SHOULD BE
13. 'LARGE.')
14. FORMAT(IX, '/I ', 'THE DEFAULT ON THE ABOVE NINE (9) PARAMETERS IS ONE (1)')
15. FORMAT(IX, '/I ', 'THE THERE ARE SIX (6) INPUT PARAMETER CARDS WHICH...
16. MUST PRECEDE THE DATA DECK')
17. FORMAT(IX, '/I ', 'THE INPUT PARAMETERS DESCRIBED BELOW')
18. FORMAT(IX, '/I ', 'THEY MUST BE IN 14 FORMAT, RIGHT JUSTIFIED')
19. FORMAT(IX, '/I ', 'THE FORMAT OF THE INPUT DATA')
20. FORMAT(IX, '/I ', 'THIS CARD MUST BE AS FOLLOWS:')
21. FORMAT(IX, '/I ', 'THE FORMAT OF THE PUNCHED OUTPUT A MATRIX')
22. FORMAT(IX, '/I ', 'LEAVE BLANK IF NOT DESIRED')
23. FORMAT(IX, '/I ', 'THE FORMAT OF THE PUNCHED OUTPUT R MATRIX')
24. FORMAT(IX, '/I ', 'LEAVE BLANK IF NOT DESIRED')
25. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:')
26. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:')
27. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:')
28. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:')
29. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:')
30. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:')
31. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:')
32. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:')
33. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:')
34. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:')
35. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:')
36. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:')
37. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:')
38. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:')
39. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:')
40. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:')
41. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:')
42. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:')
43. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:')
44. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:')
45. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:')
46. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:')
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81. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:')
82. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:')
83. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:')
84. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:')
85. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:')
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99. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:')
100. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:')
101. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:')
102. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:')
103. FORMAT(IX, '/I ', 'THE CARD MUST BE AS FOLLOWS:)
212 FORMAT (21X,'COLS. 1-8 G-FORMAT')
213 FORMAT(21X,'COLS. 9-9G THE FORMAT OF THE PUNCHED OUTPUT .C. MATR)
214 FORMAT (IX,/,10X,'CARD NO. 5 - THIS CARD MUST BE AS FOLLOWS:')
215 FORMAT (21X,'COLS. 1-8 G-FORMAT')
216 FORMAT (21X,'COLS. 9-8U THE FORMAT OF THE PUNCHED OUTPUT .G. MATR)
161 FORMAT(IX,/,10X,') USE FORTRAN F-TYPE FORMAT ON INPUT PARAM
400 FORMAT (49X,'JK JK',36X,'I M')
401 FORMAT (34X,'I JK')
402 FORMAT (33X,'JK JK',/)
403 FORMA (29X,'I JK')
404 FORMAT (24X,'JK JK')
405 FORMAT (22X,'JK JK')
406 FORMAT (66X,'K O')
407 FORMAT (56X,'J P')
408 FORMAT (66X,'I M')
409 FORMAT (65X,'PO M',/)
410 FORMAT (49X,'J J',/)
411 FORMAT (49X,'K K',/)
412 FORMAT (32X,'I M')
STOP
END
SUBROUTINE DPM1 (A,B,R,M,L)  

DIMENSION A(M,L), B(L,M)  
REAL*8 R(L,L), P(41,41), Q(41,41)  
DIMENSION IPA(17), IPARM(18)  
DIMENSION IADP(19), IBOR(18), ICOR(19), IGOR(18)  
COMMON /ONE/ IANR, IBOR, ICOR, IGOR, IPARM  
COMMON /TWO/ JMX, KMX, WWX, LMX, NMX, IMX  
COMMON /THREE/ IFORM  

IF(IPAR(10).EQ.2) GO TO 333  

DO 7 I=1,M  
7 READ(5,IFORM) (A(I,J),J=1,L)  
WRITE(6,100)  
WRITE(6,502)  
IF (IPAR(11).EQ.2) GO TO 4  
WRITE(6,111)  
WRITE (5,503)  
CALL XPRTIA (A,M,L)  
4 CONTINUE  

DO 20 JFK=1,M  
DO 20 LB=1,L  
20 BI(LB)JF(K)=A(JFK,LBJ)  
CALL MATA (B,A,R,L,M,L)  
IF (IPAR(12).EQ.2) GO TO 500  

DO 15 LB=1,L  
DO 15 JFK=1,L  
15 RJF(K,LBJ)=R(JFK,LBJ)/P  
WRITE (5,15)  
WRITE(6,504)  
GO TO 44  

500 WRITE (5,501)  
WRITE(6,504)  
GO TO 44  

333 DO 9 I=1,L  
9 READ(5,IFORM) (R(I,J),J=1,L)  
WRITE(6,11)  
WRITE (5,505)  
IF (IPAR(12).EQ.2) GO TO 17  
WRITE (5,19)
WRITE(6,505)
DO 16 LBJ=1,L
DO 16 JFK=1,L
16 R(JFK,L3J) = R(JFK,LB.J)/M
GO TO 44
17 CONTINUE
WRITE(6,18)
WRITE(6,505)
44 CONTINUE
IF(IPAR(13).EQ.2) GO TO 5
20C WRITE (6,541)
WRITE(6,504)
C CALL XPRIT (R,L,L)
5 CONTINUE
WRITE (5,112)
WRITE (5,933)
I X1 = IPAR(4)
IX2 = IPAR(5)
CALL DdV2 (R,L,M,A,IX1,IX2,P,Q)
10C FORMAT(IHI,' THE INPUT DATA IS IN THE FORM OF AN .X.. MATRIX')
111 FORMAT (1X,/' THE INPUT .X.. MATRIX')
15 FORMAT (IHI,' THE .R.. MATRIX ELEMENTS HAVE BEEN DIVIDED BY THE SAMPLE SIZE')
11 FORMAT (IHI,' THE INPUT DATA IS IN THE FORM OF AN .R.. MATRIX')
1TRIX)
19 FORMAT (1X,/' THE USER INPUT .R.. MATRIX ELEMENTS HAVE BEEN DIVIDED BY THE SAMPLE SIZE')
18 FORMAT (1X,/' THE USER INPUT .R.. MATRIX ELEMENTS HAVE NOT BEEN DIVIDED BY THE SAMPLE SIZE')
541 FORMAT (1X,/' THE .R.. MATRIX')
112 FORMAT (1X,/' THE EIGENVALUES OF THE .P.. MATRIX')
9C FORMAT (33X,'JK JK',/)
5C1 FORMAT(IHI,' THE .R.. MATRIX ELEMENTS HAVE NOT BEEN DIVIDED BY THE SAMPLE SIZE')
1 THE SAMPLE SIZE')
5C6 FORMAT (1X,33X,'JK JK',//)
5C4 FORMAT (1X,7X,'JK JK',//)
5C5 FORMAT (1X,17X,'JK JK',//)
5C3 FORMAT (1X,13X,'JK JK',//)
5C2 FORMAT (1X,39X,'JK JK',//)
RETURN
END
SUBROUTINE DRIV2 (R,L,IPP,A,I1,I2,P,O)

IMPLICIT REAL*8 (8-H-D-Y)

DIMENSION BETA (185),VEC (185, 32), Q1(185), Q2(185), Q3(185),
Q4(185), RV(185, 32), EV(185, 32), BET(185), R(L,L), P(I1,I1),
Q(I2,I2), C(I1,I1)

CALL FACTOR (L,L, P4, O, R, BETA, VEC, Q1, Q2, Q3, Q4, RV, EV, P, Q)

CALL PCOM (P,P2,P,L,P2,P4,P5,P6,P7)

CALL FACTOR (P2,P5, O,TWO,P, BETA, VEC, Q1, Q2, Q3, Q4, RV, EV, P, Q)

CALL JCOM (O,P3,R,L,P2,P3,P4,P5,P6,P7)

CALL FACTOR (P3,P6, O, SCAT, O, BETA, VEC, Q1, Q2, Q3, Q4, RV, EV, BET)

N99 = N4X * LMX

CALL DRIIV3 (P2,P3,P4,P5,P6,P7, N99,F,B,C,G,L,H,R1,C1,A,Z
17BB,ZCC,1GG,ZS,ZH2,ZH1,ZH3,ZV)

RETURN
END
SUBROUTINE DRV3 (J2,J3,J4,J5,J6,J7,J8,J9,J10,J11,J12,J13,J14,J15,J16,J17,J18,J19,J20)
N99,F,B,C,G,L,H,B1,C1,3-M 409
1J7-ZBB,ZCC,ZGG,ZS,ZH2,ZH1,ZH3,ZV1

C IMPLICIT REAL*8(B-H,O-Y)
C DIMENSION F(L,J4), B(J2,J5), C(J3,J61), G(N99,J4), H(N99,L)
C DIMENSION B1(J5,J2), C1(J6,J3)
C DIMENSION A(J7,L), ZBB(J2,J5), ZCC(J3,J6), ZGG(N99,J4)
C DIMENSION ZV(N99,J4), ZS(J4,J4), ZH2(J7,N99), ZH1(L,N99)
C DIMENSION T(A(J8,J18), JBD(J8,J18), JCSR(J8,J18), JG(J8,J18), IPAR(J8), IFORM(J8)
C INTEGER UND/1, TWD/2, SCRAT/3/
C COMMON /ONE/ IAOR, IBOR, ICOR, IGOR, IPAR
C COMMON /TWO/ JMX, KMX, MX, LMX, NMX, LMX, M
C COMMON /THREE/ IFORM
C
C JKMX=J4*KMX
C
C REWIND JNO
C READ (VNO) ((F(JK,M), J=1,JKMX), M=1,MNX)
C REWIND JNO
C
C REWIND TWO
C READ (TWJ) ((BTJ,L1), J=1, JMX), L1=1,L4X)
C REWIND TWO
C
C DO 2 I = 1, JMX
C DO 2 J = 1, LMX
C BT(1,J1) = BT(1,J)
C 2 CONTINUE
C
C REWIND SCRAT
C READ (SCRAT) ((C(K,N), K=1, KMX), N=1, NX)
C REWIND SCRAT
C
C DO 3 I = 1, KMX
C DO 3 J = 1, NMX
C CI(J1,J1) = CI(I,J1)
C 3 CONTINUE
C
C COMPUTE COPE MATRIX G
C
C CALL <RD (B1,C1,H,J5,J2,J6,J3,N99,JKMX)
C CALL MAT (H,F,G,N99,JKMX,J4)
C
C WRITE (5,1)
C WRITE (5,7C3)
C
C PRINT ..G. MATRIX
ILLEGAL
DO 117 J=1,NMX
DO 117 I=1,NMX
ZCC(I,J) = C(I,J)
117 CONTINUE
C
DO 118 J=1,MX
DO 118 I=1,N99
ZGG(I,J) = G(I,J)
118 CONTINUE
C
GET THE A MATRIX
CALL SETVS (ZGG,ZV,ZS,N99,J4)
CALL KROS (ZBR,ZCC,ZH1,J2,J5,J3,J6,J<MX,N99)
CALL MATS (A,ZH1,ZH2,J7,JKMX,N99)
CALL MATS (ZH2,ZV,ZH3,J7,N99,J4)
CALL MATS (ZH3,ZS,Z,J7,J4,J4)
C
WRITE (5,119)
WRITE (6,754)
CALL XPRITA (Z,J7,J4)
C
GO TO 112
112 WRITE (5,111)
WRITE (6,755)
GO TO 112
113 WRITE (5,905)
WRITE (6,906)
WRITE (5,114)
WRITE (6,706)
WRITE (6,129)
WRITE (5,707)
112 CONTINUE
C
IF (IPAR(14).EQ.21) GO TO 900
IF (IPAR(14).EQ.2) GO TO 905
WRITE (5,883)
WRITE (6,715)
WRITE(7,886)
C
JC 883 < 7 =1,MX
WRITE(7,140) (Z(KP,K9),K9=1,MMX)
886 CONTINUE
C
901 IF (IPAR(16), EQ. 2) GO TO 907
WRITE (6, 882)
WRITE (5, 712)
WRITE (7, 762)
C
DO 887 I = 1, NMX
WRITE (7, 1078) (G(I, J), J = 1, NMX)
887 CONTINUE
C
902 IF (IPAR(17), EQ. 2) GO TO 903
WRITE (6, 884)
WRITE (7, 767)
C
DO 888 I = 1, NMX
WRITE (7, 1018) (G(I, J), J = 1, NMX)
888 CONTINUE
903 CONTINUE
C
1 FORMAT (1HI, //'THE COPE MATRIX .G.'
1.4 FORMAT (1X, //'6X,7115)
139 FORMAT (1H, '2I3,3X,7F15.5)
119 FORMAT (1HI, '1X,4HTHE .A. MATRIX')
111 FORMAT (1HI, //'THE COMPUTATION OF THE .A. MATRIX WAS NOT FENDED')
1STED')
765 FORMAT (24X, 'I M', //)
767 FORMAT (24X, 'I M', //)
126 FORMAT (1X, 'THE COMPUTATION OF THE .A. MATRIX IS THEREFORE IMPOSSIBLE')
1PLE')
766 FORMAT (29X, 'JK JK')
114 FORMAT (1X, 'THE INPUT DATA WAS IN THE FORM OF AN .P.. MATRIX')
1X')
765 FORMAT (1HI, //'THE USER HAS SPECIFIED THAT THE .A. MATRIX BE COMputed AND PRINTED')
9.6 FORMAT (33X, 'I M')
886 FORMAT ('THE .A. MATRIX FOLLOWS THIS CARD')
7.1 FORMAT ('THE .C. MATRIX follows this card')
700 FORMAT ('THE .B. MATRIX follows this card')
702 FORMAT ('THE .G. MATRIX follows this card')
886 FORMAT (1X,'THE .G. MATRIX has been punched')
881 FORMAT (1X,'THE .C. MATRIX has been punched')
882 FORMAT (1X,'THE .B. MATRIX has been punched')
883 FORMAT (1X,'///',1X,'THE .A. MATRIX has been punched')
703 FORMAT (27X,'PO M',///)
704 FORMAT (15X,'I M',///)
715 FORMAT (5X,'I M')
713 FORMAT (6X,'J P')
712 FORMAT (6X,'K O')
714 FORMAT (5X,'PO M')
    RETURN
END
SUBROUTINE FACTOR (N,M,LENGTH,K,R,BETA,VEC,Q1,Q2,Q3,Q4,RVL,IEV,BET)

C

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION BETA(N),VEC(N,M),Q1(N),Q2(N),Q3(N),Q4(N),
R(N),RVL(N,5),EV(N,M),BET(N)
DIMENSION IAOR(18), IBOR(18),ICOR(18),IGOR(18),IPAR(17),IFORM(18)
INTEGER*4 Q
INTEGER UNO/1/,TWO/2/,SCRAT/3/
COMMON /ONE/ IAOR,IBOR,ICOR,IGOR,IPAR
COMMON /TWO/ JMX,KMX,MPX,LMX,NMX,IMX
COMMON /THREE/ IFORM

GET M ROOTS AND M VECTORS

600 CALL LRSVSM(N,M,R,EV,BET,Q1,Q2,Q3,Q4,RVL,IER)
GO TO 700
600 WRITE (6,1)

IF (LENGTH) .LT. 650,660,661
601 CALL LRSVSM(N,M,R,EV,BET,Q1,Q2,Q3,Q4,RVL,IER)
IF(IERR.NE.0)GO TO 3
GO TO 700
600 CALL LRSVSM(N,M,R,EV,BET,Q1,Q2,Q3,Q4,RVL,IER)
GO TO 3
700 WRITE (6,1)

C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C

IF (LENGTH) .LT. 116,116,118
116 DO 117 I=1,N
117 WRITE (5,2) I,BET(I)

C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C

22 REWIND <
WRITE (K) ((EV(I,J),I=1,N),J=1,M)
END FILE K
REWIND <

C
GO TO 100

C
118 DO 17 I=1,N
17 WRITE (5,2) I,BET(I)

C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C

23 VEC(I,J)=VEC(I,J)*DSORT(BETA(J))
22 CONTINUE

C
REWIND <
WRITE (K) ((VEC(I,J),I=1,N1,J=1,M))
END FILE K
REWIND <
C
C 100 IF (K.EQ.UND) GO TO 200
IF (K.EQ.TWO) GO TO 300
IF (K.EQ.SCRAT) GO TO 400
C
C 200 WRITE (5,5)
WRITE (5,802)
GO TO 500
C
C 300 WRITE (6,6)
WRITE (5,801)
GO TO 500
C
C 400 WRITE (5,7)
WRITE (5,800)
C
C 500 LL=M-(M/8)*A
C
C IF(LL)=502,502,503
C
C 503 LL=1
C
C 502 LL=(M/8)+LL
C
LK=0
C
DO 513 II=1,LL
NC=LK+1
NI=LK+A
N1=MIN(N1,4)
WRITE (5,8) (J,J=4,N1)
C
C IF(LENGTH)=305,305,304
C
C 3,5 DO 303 I=1,N
C
C 303 WRITE (5,9) I, (EV(I,J),J=1,N1)
GO TO 513
C
C 3.4 DO 508 I=1,N
5C6 WRITE (5,9) I, (VEC(I,J),J=NO,NI)
5C6 LK=L<+8
C
C
RETURN
3 WRITE(6,4) IERR
WRITE(6,334)
334 FORMAT(1X,///,1X,74HSEE COMMENTS ON THE VARIABLE IERR IN SUBRUTABLE13-M 712
1YES L=SVSM INTOL1 TINVIT)
4 FORMAT(1X,75H** FATAL ERROR, COMPUTATION OF THE EIGENVALUES OR THE3-M 720
1EIGENVECTORS FAILED ,/21X,49H THIS SUGGESTS SEVERE ANOMALIES IN 3-M 722
2THE INPJT DATA.///,1X,21H CONSULT IN=3: TERR=,I6)
9 FORMAT (1H ,/13,3X,8F15.5)
8 FORMAT (1H ,/3X,8I15)
5 FORMAT (1X,///,15X,*THE PRINCIPAL AXIS FACTORS OF THE ..R.. MATRI3-M 726
1X*)
8G2 FORMAT (49X,*JK JK',///)
6 FORMAT (1X,///,15X,*THE EIGENVECTORS OF THE .P. MATRIX ..THE .B3-M 729
1. MATRIX ..'*)
8G1 FORMAT(39X,*J J',16X,*J P',///)
7 FORMAT (1X,///,15X,*THE EIGENVECTORS OF THE .O. MATRIX ..THE .C3-M 732
1. MATRIX ..*)
8G2 FORMAT (39X,*K K',16X,*K O',///)
1 FORMAT (1H*,21H FACTOR ROOT/) 3-M 734
2 FORMAT (1H*,16,F18.5) 3-M 736
RETURN
END
**C**

**SUBROUTINE LRSVSM**

**C**

**TITLE:** LATENT ROOTS AND SOME VECTORS OF A SYMMETRIC MATRIX.

**C**

**PURPOSE:** FIND ALL LATENT ROOTS (EIGENVALUES) AND A SMALLER NUMBER OF LATENT VECTORS (EIGENVECTORS) OF A SYMMETRIC MATRIX.

**C**

**SYMBOLS:**

**C**

**N** = ORDER OF INPUT SYMMETRIC MATRIX [MUST BE LE. NM]

**C**

**NRV** = NUMBER OF LATENT VECTORS TO BE COMPUTED (CORRESPONDING TO THE NRV LARGEST LATENT ROOTS)

**C**

**A** = CONTAINS THE INPUT SYMMETRIC MATRIX IN ITS FIRST N ROWS

**C**

**Z** = CONTAINS THE OUTPUT LATENT VECTORS OF A IN ITS FIRST NRV COLUMNS

**C**

**W** = ALL N LATENT ROOTS OF A

**D** = TEMPORARY STORAGE ARRAYS OF LENGTH AT LEAST N, CONTAIN THE DIAG, SUBDIAGONAL, AND SQUARED SUBDIAGONAL ELEMENTS

**C**

**IND** = AN INTEGER*4 ARRAY OF AT LEAST N+5 ELEMENTS IN IT.

**C**

**ERR** = ERROR FLAG. IF ERR = 0 ON OUTPUT, THE COMPUTATIONS SUCCEEDED.

**C**

**A** = DIAGONAL, SUBDIAGONAL, AND SQUARED SUBDIAGONAL FORM

**C**

**TRI** = THE TRIAGONAL FORM OF A.

**C**

IF 

**TRI** = THE TRIAGONAL FORM OF A.

**N** = 0 FOR IDENTICAL OR NEAR IDENTICAL LATENT ROOTS. THIS SUBROUTINE IS FASTER THAN LRSVM FOR FINDING NRV LARGEST LATENT VECTORS OF A.

**C**

**HOWEVER,** IF **NRV** .GT. **N** - 8, THEN LRSVM USES LESS CODE THAN THIS.

**C**

**SUBROUTINE AND SO IS TO BE PREFERRED IN THESE CIRCUMSTANCES.**
SUBROUTINE LRSVSML(NM,N,NRV,A,IZW,DO,FE,FZ,IND,
                  RV,IER) 3-M 736
C IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(NM,N),D(N),E(N),Z(N),W(N),RV(N,5),IND(N),Z(NM,NRV)
REAL*8 MACHEP
C CALL EPSAV(268,ERRNO)
CALL ERRSET(208,256,-1,0)
C MACHEP IS A MACHINE DEPENDENT PARAMETER SPECIFYING THE RELATIVE
C PRECISION OF FLOATING POINT ARITHMETIC.
C MACHEP = .5*15
C IERR = -(N+1)
C*RESET NUMBER OF VECTORS REQUESTED IF INCORRECTLY
C SPECIFIED AND IF IT IS POSSIBLE TO DEDUCE A CORRECTION
C IF(NLTVRV)NRV=N
IF(NRV.LT.3)NRV=0
C DIMENSIONALITY CHECK
C IF(N.GT.NM).OR.N.LE.0)GO TO 31
C TRIDIAGONALIZE A
C CALL TRED(NM,N,A,D,E,E2)
C TEST FOR SMALL SUBDIAGONAL ELEMENTS AND FIND LATENT ROOTS FOR
C SUBMATRICES AND BUILD IND TO HAVE SUBMATRIX NUMBERS.
C IND1=1
IND(1)=1
E2(1)=2.0G
W(1)=D(1)
L=1
DO 20 IJ=2,N
IF(IDABS(E(IJ)).LT.*MACHEP*(IDABS(D(IJ))+IDABS(D(IJ-1))))GO TO 30
V(IJ)=D(IJ)
RV(IJ)=E(IJ)
IND(IJ)=INDI
20 CONTINUE
IND=IND+1
IND=IND
20 CONTINUE
IF(IJ.NE.N+1)GO TO 30
C LK=IJ-L
C 3-M 736
*FIND LATENT ROOTS OF SUBMATRIX
CALL IMTOL1(LK,W(L),RV(L+2),IERR)
IF(IERR.NE.0)GO TO 10
IF(IJ.EQ.N+1)GO TO 40

*SET NEW IND VALJE AND SET E2 ELEMENT TO TRUE 0.
INDI=INDI+1
IND(IJ)=INDI
E2(IJ)=.DG
W(IJ)=D(IJ)
L=IJ
GO TO 20

*SORT LATENT ROOTS AND REORDER IND CORRESPONDINGLY (RUBBLE SORT)
40 NM1=N-1
1 K=N-1
2 J1=J+1
IF(W(J).GE.W(J1))GO TO 3
TEMP=W(J)
W(J)=W(J1)
W(J1)=TEMP
L=IND(J)
IND(J)=IND(J1)
IND(J1)=L
3 J=J1
IF(J.LE.K)GO TO 2
I=I+1
IF(I.LE.NM1)GO TO 1

*OBTAIN LATENT VECTORS (IF REQUESTED) OF TRIDIAGONAL MATRIX
*BY INVERSE ITERATION.
70 IF(NRV.EQ.0)GO TO 10
CALL TINVT(NM,N,D,E,E2,NRV,W,IND,Z,IERR,RV,RV(1,2),RV(1,3),
1PV(1,4),RV(1,5))
IF(IERR.NE.0)GO TO 33

*RECK TRANSFORM THE LATENT VECTORS SO THEY ARE LATENT VECTORS OF A.
CALL TR3AK1(NM,N,A,E,NRV,Z)
GO TO 10

*RESTORE A
33 WRITE(6,34)
  WRITE(6,35)
C
10 DO 150 I=1,N
  J=I-1
C
  DO 150 J=I-2,J
150 A(I,K)=A(K,I)
C
  GO TO 50
31 WRITE(6,36)
C
  CALL ERSTR(208,ERRN)
C
  FORMAT(1X,//1X,39HTHE PROGRAM BOMBED IN SUBROUTINE TINVIT)
36 FORMAT(1X,//1X,68HTHE PROGRAM BOMBED ON THE DIMENSIONALITY CHECK IN)
  1 SUBROUTINE LRSVSM
35 FORMAT(1X,//1X,61HTHERE WAS NO CONVERGENCE TO AN EIGENVECTOR AFTER)
  1 ER 5 ITERATIONS)
RETURN
END
C  SUBROUTINE TRED1
C
C*TITLE: REDUCE A SYMMETRIC MATRIX TO TRIDIAGONAL FORM
C*SOURCES: Aiken, R.: <AIRCRAFT ENGINEERING> 25, No. 5 (1963), 3-M 969
C*SYMBOLS:
C*  M = ROW DIMENSION OF A IN THE CALLING PROGRAM.
C*  N = THE ORDER OF A, MUST BE .GE. M.
C*  A = THE REAL SYMMETRIC MATRIX TO BE TRIDIAGONALIZED. ONLY THE FULL
C*  LOWER TRIANGLE OF THE MATRIX NEED BE SUPPLIED. ON OUTPUT, THE
C*  STRICT LOWER TRIANGLE CONTAINS TRANSFORMATION INFORMATION.
C*  P = THE DIAGONAL ELEMENTS OF THE TRIDIAGONAL FORM OF A.
C*  Q = SAME AS E IN TRBAK1.
C*  E2 = SAME AS E2 IN TINVIT. E2 AND E NEED NOT BE DISTINCT IN THE
C*  * * * CALLING PROGRAM.
C*  * * * PROCEDURE: THE TRIDIAGONAL REDUCTION IS PERFORMED IN THE FOLLOWING
C*  * * * WAY. STARTING WITH J=N, THE ELEMENTS IN THE JTH ROW TO THE LEFT OF
C*  * * * THE DIAGONAL ARE FIRST SCALING, TO AVOID POSSIBLE UNDERFLOW IN THE
C*  * * * TRANSFORMATION THAT MIGHT RESULT IN SEVERE DEPARTURE FROM
C*  * * * ORTHOGONALITY. THE SUM OF SQUARES SIGMA OF THESE SCALING ELEMENTS IS
C*  * * * NEXT FORMED. THEN, A VECTOR U AND A SCALAR H (=U'U/2) DEFINE AN
C*  * * * OPERATOR P (=I-UU'H) WHICH IS ORTHOGONAL AND SYMMETRIC AND FOR
C*  * * * WHICH THE SIMILARITY TRANSFORMATION PA0 ELIMINATES THE ELEMENTS IN
C*  * * * THE JTH ROW OF A TO THE LEFT OF THE SUBDIAGONAL AND THE SYMMETRICAL
C*  * * * ELEMENTS IN THE JTH COLUMN. THE NON-ZERO COMPONENTS OF U ARE THE
C*  * * * ELEMENTS OF THE JTH ROW TO THE LEFT OF THE DIAGONAL WITH THE LAST
C*  * * * OF THEM AUGMENTED BY THE SQUARE ROOT OF SIGMA PREFIXED BY THE SIGN
C*  * * * OF THE SUBDIAGONAL ELEMENT. BY STORING THE TRANSFORMED SUBDIAGONAL
C*  * * * ELEMENT IN E(J) AND NOT OVERWRITING THE ROW ELEMENTS ELIMINATED IN
C*  * * * THE TRANSFORMATION, FULL INFORMATION ABOUT P IS SAVED FOR LATER USE
C*  * * * IN TRBAK1, THE TRANSFORMATION SETS E2(J) EQUAL TO SIGMA AND E(J)
C*  * * * THE TRANSFORMATION, FULL INFORMATION ABOUT P IS SAVED FOR LATER USE
C*  * * * IN TRBAK1, THE TRANSFORMATION SETS E2(J) EQUAL TO SIGMA AND E(J)
C*  * * * IS ELIMINATED TO THE SQUARE ROOT OF SIGMA PREFIXED BY SIGN OPPOSITE TO THAT
C*  * * * OF THE REPLACED SUBDIAGONAL ELEMENT. THE ABOVE STEPS ARE REPEATED
C*  * * * IN FURTHER ROWS OF THE TRANSFORMED A IN REVERSE ORDER UNTIL A IS
C*  * * * REDUCED TO TRIDIAGONAL FORM; THAT IS, REPEATED J=N-1,N-2,...,3.
C*  * * * THE ELEMENTS IN THE LOWER TRIANGLE OF A ARE ACCESSED, AND
C*  * * * ALTHO THE DIAGONAL ELEMENTS ARE MODIFIED IN THE ALGORITHM, THEY ARE
C*  * * * RESTORED TO THEIR ORIGINAL CONTENTS BY THE END OF THE SUBROUTINE,
C*  * * * THUS PRESERVING THE FULL UPPER TRIANGLE OF A.
C*  * * * REFERENCES: SAME AS TRBAK1
C
SUBROUTINE TRED1 (M,N,A,D,E2)

C*    IMPLICIT REAL*(A-H,N-Z)
DIMENSION A(M,N),D(M,N),E(N,E2(N))

CALL ERRSAV(208,ERPO1)
CALL ERRSET(2C8,256,-1,C)

3-M 907
3-M 908
3-M 909
3-M 910
3-M 911
3-M 912
3-M 913
3-M 914
3-M 915
3-M 916
3-M 917
3-M 918
3-M 919
3-M 920
3-M 921
3-M 922
3-M 923
3-M 924
3-M 925
3-M 926
3-M 927
3-M 928
3-M 929
3-M 930
3-M 931
3-M 932
3-M 933
3-M 934
3-M 935
3-M 936
3-M 937
3-M 938
3-M 939
3-M 940
3-M 941
3-M 942
3-M 943
3-M 944
3-M 945
3-M 946
3-M 947
3-M 948
3-M 949
3-M 950
3-M 951
3-M 952
3-M 953
C   DO 100 I=1,N
100  D(I)=A(I,I)
C*FOR I=N STEP -1 UNTIL 1 DO
C
C   DO 350 I=1,N
   I=N+1-I
   L=I-1
   H=0.00

   SCALE=5.00
   IF(L.LT.I)GO TO 130
C
C*SCALE ROW
C
C   DO 120 K=1,L
120  SCALE=SCALE+DABS(A(I,K))
C
   IF(SCALE.GE.J)GO TO 140
130  E(I)=0.00
   E2(I)=5.00
   GO TO 290
C
C   DO 150 K=1,L
   A(I,K)=A(I,K)/SCALE
   H=H+A(I,K)*A(I,K)
   CONTINUE
C
   E2(I)=SCALE*SCALE*H
   F=A(I,L)
   G=-DSIGN(DSQR(T(H),F)
   E(I)=SCALE*G
   H=H-F*G
   A(I,L)=F-G
   IF(L.LT.I)GO TO 270
   F=G.00
   DO 240 J=1,L
   G=G.00
C
C*FORM ELEMENT OF A*U
C
C   DO 180 K=1,J
180  G=G+A(J,K)*A(I,K)
C
   JP1=J+1
   IF(L.LT.JP1)GO TO 220
SUBROUTINE IMTOL1

* TITLE: EIGENVALUES OF TRIDIAGONAL MATRIX
* PURPOSE: DETERMINES EIGENVALUES OF A SYMMETRIC TRIDIAGONAL MATRIX
* USING THE IMPLICIT QL METHOD
* SYMBOLS:
  * N = ORDER OF THE TRIDIAGONAL MATRIX
  * D = A ONE DIMENSIONAL ARRAY CONTAINING THE DIAGONAL ELEMENTS OF THE
    TRIDIAGONAL MATRIX. ON OUTPUT, IT CONTAINS THE EIGENVALUES IN ASCENDING
    ORDER.
  * E = A ONE DIMENSIONAL ARRAY CONTAINING THE SUBDIAGONAL ELEMENTS OF
    THE TRIDIAGONAL MATRIX IN ITS LAST N-1 ELEMENTS. E(1) IS
    ARBITRARY. IMTOL1 DESTROYS THE CONTENTS OF E.
  * IERR = CONTAINS ERROR CODE ON OUTPUT. IF MORE THAN 30 ITERATIONS
    ARE REQUIRED, IMTOL1 TERMINATES IMMEDIATELY WITH IERR SET TO
    3.0.
  * THE INDEX OF THE EIGENVALUES WHICH THE FAILURE OCCURS.
  * IF EVERYTHING IS OK, IERR IS RETURNED AS 0.
* PROCEDURE: THE EIGENVALUES ARE DETERMINED BY THE IMPLICIT QL METHOD.
* THE ESSENCE OF THIS METHOD IS A PROCESS WHEREBY A SEQUENCE OF
  SYMMETRIC TRIDIAGONAL MATRICES, UNITARILY SIMILAR TO THE ORIGINAL
  TRIDIAGONAL MATRIX, IS FORMED WHICH CONVERGES TO A DIAGONAL MATRIX.
* THE RATE OF CONVERGENCE IS IMPROVED BY IMPLICITLY SHIFTING THE
  ORIGIN AT EACH ITERATION. BEFORE EACH ITERATION, THE TRIDIAGONAL
  MATRIX IS CHECKED FOR A POSSIBLE SPLITTING INTO SUBMATRICES. IF A
  SPLIT OCCURS, ONLY THE UPPERMOST SUBMATRIX IS USED IN THE NEXT
  ITERATION. THE EIGENVALUES ARE ORDERED IN ASCENDING ORDER AS THEY
  ARE FOUND. THE ORIGIN SHIFT AT EACH ITERATION IS THE EIGENVALUE OF
  THE CURRENT UPPERMOST 2 X 2 PRINCIPAL MINOR CLOSER TO THE FIRST
  DIAGNOL ELEMENT OF THIS MINOR. WHEN THE UPPERMOST 1 X 1 SUBMATRX
  FINALLY SPLITS OFF, IT IS TAKEN TO BE AN EIGENVALUE AND EXECUTION
  PROCEEDS WITH THE REMAINING SUBMATRIX. THIS PROCESS IS CONTINUED
  UNTIL THE MATRIX HAS SPLIT COMpletely INTO SUBMATRICES OF ORDER 1.
* THE TOLERANCES IN THE SPLITTING TEST ARE A TINY PROPORTION OF THE
  1.
  DIAGONAL ELEMENTS.
* REFERENCES: WILKINSON, J.H., & REINSCH, C. HANDBOOK FOR AUTOMATIC
  COMPUTATION, VOL. II, LINEAR ALGEBRA, PART 2, SPRINGER-VERLAG,

IMPLICIT REAL*8(A-H,O-Z)
REAL*8 MACHEP,0(N),E(N)
CALL ERRSAV(238,E,RND)

IMPLICIT REAL*8(A-H,O-Z)
REAL*8 MACHEP,0(N),E(N)
CALL ERRSAV(238,E,RND)
CALL ERRSET(208,256,-1,3)

C *MACH2P IS A MACHINE DEPENDENT PARAMETER SPECIFYING THE RELATIVE
C * PRECISION OF FLOATING POINT ARITHMETIC.
C
MACH2P=.5D-15
ERR2P=14.0D16
GO TO 1001

100   DO 120 I=2,N
       E(I-1)=E(I)
       E(N)=.0
   120   DO 290 L=1,N
          J=L

C *LOOK FOR SMALL SUB-DIAGONAL ELEMENT
C
145   DO 110 M=L,N
          IF(M.EQ.L)GO TO 120
          IF(DABS(E(M)).LE.MACH2P*(DABS(D(M))+DABS(D(M+1))))GO TO 120
       CONTINUE
C
120   P=E(L)
       IF(M.EQ.L)GO TO 215
       IF(J.EQ.30)GO TO 1000
       J=J+1

C *FORM SHIFT
C
G=(D(L+1)-P)/(2.0D0*E(L))
R=DSORT(G*G+1.0D0)
G=(M)-P+E(L)/(G+DSIGN(P,G))
S=1.0D0
C=1.0D0
P=C*G
MML=M-L

C *FOR I=M-1 STEP -1 UNTIL L DO
C
135   DO 233 II=1,MML
          I=M-II
          MML=M-II

233   IF(II.EQ.1)GO TO 233
          IF(II.EQ.0)GO TO 233
F=S*E(I)
B=C*E(I)
IF(DABS(F).LT.DABS(G))GO TO 150
C=G/F
R=DSORT(C*C+1.0D0)
E(I+1)=F*R
S=1.0D0/R
C=C*S
GO TO 150
150 S=F/G
R=DSORT(S*S+1.0D0)
E(I+1)=G*R
C=1.0D0/R
S=S*C
160 G=D(I+1)-P
R=(D(I)-G)*S+2.0D0*C*R
P=S*R
D(I+1)=G+P
G=C*R-P
200 CONTINUE
C
D(L)=D(L)-P
E(L)=G
E(M)=3.0D0
GO TO 1105
C*ORDER EIGENVALUES
C
215 IF(L.EQ.1) GO TO 250
C
C*FOR I=L STEP -1 UNTIL 2 DO
C
DO 230 II=2,L
I=I+2-II
IF(P.GE.D(I-1))GO TO 270
D(I)=D(I-1)
230 CONTINUE
C
250 I=1
270 D(I)=P
290 CONTINUE
GO TO 1991
C
C*SET ERROR - NO CONVERGENCE TO AN EIGENVALUE AFTER 30 ITERATIONS
C
C
C IERR = L
  WRITE (5,2000)
  WRITE (5,3000)
2000 FORMAT (1X, 7H THE PROGRAM BOMBE IN SUBROUTINE TMTOL 1)
3000 FORMAT (1X, 7H THERE WAS NO CONVERGENCE TO AN EIGENVALUE AFT E 
            1R 30 ITERATIONS)
C
1001 CALL ERRSTP (2D8, ERRND)
RETURN
END
SUBROUTINE TINVIT
C TITLE: EIGENVECTORS OF A TRI DIAGONAL MATRIX BY INVERSE ITERATION
C PURPOSE: DETERMINE THOSE EIGENVECTORS OF A TRI DIAGONAL MATRIX
C * CORRESPONDING TO A SET OF ORDERED EIGENVALUES, USING INVERSE
C * ITERATION.
C SYMBOLS:
C * RM = ROW DIMENSION OF Z IN THE CALLING PROGRAM
C * N = ORDER OF THE TRI DIAGONAL MATRIX
C * D = DIAGONAL ELEMENTS OF THE TRI DIAGONAL MATRIX
C * E = SUB DIAGONAL ELEMENTS OF THE TRI DIAGONAL MATRIX IN THE LAST N-1
C ** POSITIONS OF E. E(1) IS ARBITRARY.
C * M = SQUARES OF E WITH 2 CORRESPONDING TO NEG LIGIBLE ELEMENTS OF E.
C ** IF 2 CONTAINS 2 IF EIGENVALUES ARE IN ASCENDING ORDER AND 0 IF
C ** THEY ARE IN DESCENDING ORDER.
C * K = # OF THE FIRST EIGENVALUES FOR WHICH CORRESPONDING EIGENVECTORS
C ** ARE OBTAINED.
C * W = CONTAINS THE M EIGENVALUES IN ASCENDING OR DESCENDING ORDER
C ** DEPENDING ON THE VALUE OF E2(I).
C * MD = SUBMATRICES INDICES ASSOCIATED WITH THE M EIGENVALUES IN W.
C ** SUBMATRICES ARE NUMBERED FROM 1... AND ARE DETERMINED BY THE
C ** FOLLOWING VALUES IN E2.
C * Z = CONTAINS THE OUTPUT ORTHONORMAL EIGENVECTORS IN AN ORDER
C ** CORRESPONDING TO THE ORDER OF THE EIGENVALUES. Z IS DIMENSIONED
C ** BY AT LEAST M.
C * IERR = 3 IF EXECUTION TERMINATES NORMALLY.
C ** = -2 IF MORE THAN 5 ITERATIONS ARE REQUIRED FOR AN EIGENVECTOR,
C ** WHERE R IS THE INDEX OF THE LAST EIGENVECTOR FOR WHICH THIS
C ** OCCURRED.
C * RV1, RV2, RV3, RV4, RV6 = TEMPORARY STORAGE ARRAYS.
C PROCEDURE: FIRST THE E2 ARRAY IS INSPECTED FOR THE PRESENCE OF C
C ** ELEMENTS DEFINING SUBMATRICES, EIGENVALUES BELONGING TO A GIVEN
C ** SUBMATRIX ARE IDENTIFIED BY THEIR COMMON SUBMATRIY INDICES IN MD.
C ** EIGENVECTORS OF A SUBMATRIX ARE THEN COMPUTED BY INVERSE ITERATION.
C ** FIRST THE LU DECOMPOSITION OF THE SUBMATRIX WITH AN EIGENVALUE
C ** IS SUBTRACTED FROM ITS DIAGONAL ELEMENTS IS ACHIEVED BY GAUSSIAN
C ** ELIMINATION WITH PARTIAL PIVOTING. THE MULTIPLIERS DEFINING THE
C ** LOWER TRIANGULAR MATRIX L ARE STORED IN RV4 AND THE UPPER
C ** TRIANGULAR MATRIX U IS STORED IN RV1, RV2, AND RV3. THUS IF FURTHER
C ** ITERATIONS ARE REQUIRED, THE LU DECOMPOSITION NEED NOT BE REPEATED.
C ** AN APPROXIMATE VECTOR, STORED IN RV6, IS COMPUTED STARTING FROM AN
C ** INITIAL VECTOR, AND THE NORM OF THE APPROXIMATE VECTOR IS COMPARED
C ** WITH A LOW OF THE SUBMATRIX TO DETERMINE WHETHER THE GROWTH IS
C ** SUFFICIENT TO ACCEPT IT AS AN EIGENVECTOR. IF ACCEPTED, ITS
C ** EUCLIDEAN NORM IS MADE 1, IF NOT, THIS VECTOR IS USED AS AN INITIAL
C ** VECTOR IN COMPUTING THE NEXT APPROXIMATE VECTOR. THIS ITERATION
C ** IS REPEATED AT MOST 5 TIMES. EIGENVECTORS COMPUTED IN THIS
C * WAY ARE ORTHOGONAL IF THE CORRESPONDING EIGENVALUES ARE WELL
C * SEPARATED. IF THE EIGENVALUES ARE CLOSE, BUT NOT IDENTICAL,
C * ORTHOGONALITY IS INSURED BY ORTHOGONALIZING EACH APPROXIMATE VECTOR
C * WITH RESPECT TO THE PREVIOUSLY COMPUTED "CLOSE" VECTORS. IF THIS
C * ORTHOGONALIZATION RESULTS IN A ZERO VECTOR, A COLUMN OF THE IDENTITY
C * MATRIX IS USED AS AN INITIAL VECTOR FOR THE NEXT ITERATION. IF THE
C * EIGENVALUES ARE IDENTICAL, THEY ARE PERTURBED SLIGHTLY AND THESE
C * PERTURBATIONS ARE NOT RECORDED IN W. THE ABOVE STEPS ARE REPEATED
C * ON EACH SUBMATRIX UNTIL ALL REQUESTED EIGENVECTORS ARE COMPUTED.
C *REFERENCE: SAME AS TRBAK1.
SUBROUTINE TINVIT(INM,N,ID,E2,M,W,IND,Z,IERR,)
   1R1V1,1R2V2,1R3V3,1R4V4,1R5V5
C
   IMPLICIT REAL*8(A-H,O-Z)
   INTEGER P,O,R,S,TAG,GM,IND(M)
   REAL*8 MACHEP,NORM,D(IN),E(N),E2(N),W(M),Z(NM,P),R1(N),R2(N),
       1R3(N),R4(N),R5(N),R6(N)
C
C *THE NEXT STEP WAS ADDED TO ALLOW TINVIT TO HANDLE THIS TRIVIAL SITUATION
C
C IF(IN.GT.11)GO TO 1
C 7(1,1)=1.D0
C RETURN
C
C 1 CALL ERSAVE(ZG8,ERRNO)
C CALL EPRSET(ZG8,256,-1,0)
C
C *MACHEP IS A MACHINE DEPENDENT PARAMETER SPECIFYING THE RELATIVE
C *PRECISION OF FLOATING POINT ARITHMETIC
C
MACHEP=5D-15
IERE=0
TAG=0
ORDER=1.0D-E2(1)
DEC=0
C
C *ESTABLISH AND PROCESS NEXT SUBMATRIX
C
C 10E P=0+1
C
C 00 12G C=P,N
C 12G CONTINUE
C
C *FIND VECTORS BY INVERSE ITERATION
C
C 14G TAG=TAG+1
DO 320 R=1,4
IF(IN3(R).NE.TAG)GO TO 320
ITR=1
X1=W(R)
IF(S.VE.0)GO TO 510
C*CHECK FOR ISOLATED ROOT
XU=1.0E-30
IF(P.VE.0)GO TO 490
RV6(P)=1.0E-30
GO TO 870
49C NORM=DABS(D(P))
IP=P+1
DO 500 I=IP,9
50C NORM=NORM+DABS(D(I))+DABS(E(I))
C*EPS2 IS THE CRITERION FOR GROUPING, EPS3 REPLACES 3 PIVOTS AND EQUAL
C*ROOTS ARE MODIFIED BY EPS3, EPS4 IS TAKEN VERY SMALL TO AVOID
C*OVERFLOW.
EPS2=1.0E-3*NORM
EPS3=MACHEP*NORM
UK=0-P+1
EPS4=JK*EPS3
UK=EPS4/FSCRT(UK)
S=P
5C5 GROUP=0
GO TO 520
C*LOOK FOR CLOSE OR COINCIDENT ROOTS
510 IF(DABS(X1-X0).GE.EPS2)GO TO 525
GROUP=GROUP+1
IF(ORDER*(X1-X0).LE.0)GO X1=X0+ORDER*EPS3
C*ELIMINATION WITH INTERCHANGES AND INITIALIZATION OF VECTOR
52C V=0.05
C
DO 53C I=1,P,2
C

RV6(I) = JK
IF(I.EQ.1) GO TO 56C
IF(DABS(E(I)).LT.DABS(U)) GO TO 540
C
C* WARNING - A DIVIDE CHECK MAY OCCUR HERE IF E2 ARRAY HAS NOT BEEN
C* SPECIFIED CORRECTLY.
C
XU = U/E(I)
RV4(I) = XU
RV1(I-1) = E(I-1)
RV2(I-1) = D(I-1) - X1
RV3(I-1) = 3.0
IF(I.NE.0) RV3(I-1) = E(I+1)
U = V - XU * RV2(I-1)
V = -XU * RV3(I-1)
GO TO 56C
545 XU = E(I)/U
RV4(I) = XJ
RV1(I-1) = U
RV2(I-1) = V
RV3(I-1) = 0.33
565 U = D(I-1) - X1 * XU * V
IF(I.NE.0) V = E(I+1)
560 CONTINUE
C
C IF(U.EQ..D) U = EPS3
RV1(0) = J
RV2(0) = -.D0
RV3(0) = J.D0
C
C BACK SUBSTITUTION FOR I=0 STEP -1 UNTIL P DN
C
600 DO 620 I = P, 0
I = P + Q - II
RV6(I) = (RV6(I) - U * RV2(I) - V * RV3(I))/RV1(I)
V = U
U = RV6(I)
620 CONTINUE
C
CORTHOMOGONALIZE WITH RESPECT TO PREVIOUS MEMBERS OF GROUP
C
IF(GROUP.EQ.0) GO TO 76C
J = P
C
C DO 683 JJ = 1, GROUP
683 J = J - 1
IF(IND(J).NE.TAG)GO TO 530
XU=0.D0

640 DO 640 I=P,Q
640 XU=XU+RV6(I)*Z(I,J)

660 DO 660 I=P,Q
660 RV6(I)=RV6(I)-XJ*Z(I,J)

680 CONTINUE

700 NORM=0.D0

720 DO 720 I=P,Q
720 NORM=NORM+DABS(RV6(I))

740 CONTINUE

760 DO 760 I=P,Q
760 RV6(I)=RV6(I)*XU

780 DO 800 J=IP,0
800 RV6(I)=EPS4/NORM

820 CONTINUE

840 CONTINUE

860 IF(RV1(I-1).EQ.E(I))A ROW INTERCHANGE WAS PERFORMED EARLIER IN THE
860 TRIANGULARIZATION PROCESS.

880 CONTINUE
IT S = ITS+1
GO TO 650
C                           C*SET ERROR - NONCONVERGED EIGENVECTOR
C 830 IF(ERR=-R
XU=0,0)
GO TO 670
C                           C*NORMALIZE SQ THAT SUM OF SQUARES IS 1 AND EXPAND TO FULL ORDER.
C 840 U=0.0)
850 DO 990 I=P,Q
860 U=U+RV6(I)**2
C                        XU=1.0C/DSQRT(U)
C 870 DO 990 I=1,N
880 Z(I,R)=U**0
C                           C            
910 DO 990 I=P,Q
920 Z(I,R)=RV6(I)*XU
C                         XQ=X1
930 CONTINUE
C                         C
940 IF(Q.LT.M)G5 TO 1G5
C CALL ERRST9(2G8,ERRNO)
RETURN
END
SUBROUTINE TRBAK1

C TITLE: BACK TRANSFORMATION OF EIGENVECTORS
C PURPOSE: FORMS THE EIGENVECTORS OF A REAL SYMMETRIC MATRIX FROM THE
C EIGENVECTORS OF THAT SYMMETRIC TRIDIAGONAL MATRIX DETERMINED BY TRED3.
C SYMBOLS:
C **NM = ROW DIMENSION OF BOTH A AND Z IN THE CALLING PROGRAM.
C **NM = ORDER OF A, MUST BE LE. NM.
C **A = SOME INFORMATION ABOUT THE ORTHOGONAL TRANSFORMATIONS USED IN
C **B = THE REDUCTION TO TRIDIAGONAL FORM CONTAINED ONLY IN THE STPICT
C **C = LOWER TRIANGLE OF A. THE REMAINDER OF THE MATRIX IS ARBITRARY.
C **M = THE NUMBER OF COLUMNS OF Z TO BE BACK TRANSFORMED.
C **7 = AT LEAST M EIGENVECTORS TO BE BACK TRANSFORMED. ON OUTPUT, 7
C **7 = CONTAINS THE TRANSFORMED EIGENVECTORS.
C **PROCEDURE: LET C BE THE ORIGINAL SYMMETRIC MATRIX WHICH HAS BEEN
C **REDUCED TO TRIDIAGONAL FORM - F - BY THE SIMILARITY TRANSFORMATION:
C **GIVEN AN ARRAY Z OF COLUMN VECTORS WHICH ARE EIGENVECTORS OF F,
C **TRBAK1 COMPUTES OZ WHICH IS THE EIGENVECTORS OF C. THIS SUBROUTINE
C **SHOULD BE USED IN CONJUNCTION WITH THE SUBROUTINE TRED1.
C **REFERENCE: WILKINSON, J.H., "REINSCH, C. HANDBOOK FOR AUTOMATIC
C **COMPUTATION, VOL. II, LINEAR ALGEBRA" PART, SPRINGER-VERLAG,
C **NEW YORK, HEIDELBERG, BERLIN, 1971.
C **SMITH, B.T., ROYLE, J.M., GARBOU, B.S., IKBEL, Y., KLEMA, V.C., MOLER, C.
C **MATHEMATICAL SYSTEMS ROUTINES - EISPACK GUIDE IN LECTURE NOTES IN
C **COMPUTER SCIENCE, EDITED BY G. GODS AND J. HARTMANS, SPRINGER-
C **VERLAG, NEW YORK, BERLIN, HEIDELBERG, 1974.
C
C IMPLICIT REAL*8(A-H,O-Z)
C DIMENSION A(NM,N),E(N),Z(NM,4)
C IF(N.EQ.1) RETURN
C CALL EPSAV(208,ERRNO)
C CALL EPSSET(208,256,-1,C)
C DO 140 I=2,N
C L=I-1
C IF(N.EQ.3.D0)GO TO 140
C DO 140 K=1,L
C IF(1.LT.I)GO TO 140

C END
      C      DO 130  J=1,M
            S=0.0
      C      DO 110  K=1,L
            110  S=S+A(I,K)*Z(K,J)
            S=S/H
      C      DO 120  K=1,L
            120  Z(K,J)=Z(K,J)+S*A(I,K)
      C      CONTINUE
      C      CONTINUE
      C      CALL ERRSTR(208,ERRNO)
      RETURN
      END
SUBROUTINE PCOM  (P,L1,R,L,LMX,MMX,LMX,MMX,NMX,IMX)

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION R(L,L),P(L1,L1)

DO 11 JP=1,JMX
JKPT=(JP-1)*KMX

11 CONTINUE

DO 12 J=1,JMX
JKT=(J-1)*KMX

12 CONTINUE

DO 13 K=1,KMX
JKP=JKPT+K
JK=JKT+K

WRITE (5,1)
WRITE (5,4)

NRW=JMX
CALL XPRIT (P,NROW,NROW)

WRITE (5,2)
WRITE (5,3)

1 FORMAT (1HI1, //, 1XLX, ' THE P. MATRIX')
2 FORMAT (1X, //, LUX, ' THE EIGENVALUES OF THE P. MATRIX')
3 FORMAT (13X, 'J J',PP)
4 FORMAT (16X, 'J J',PP)
RETURN
END
SUBROUTINE QCOM (0,L1,R,L,LMX,KMX,MMX,LMX,NMX,IMX)

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION R(L,L),O(L1,L1)

DO 15 KP=1,KMX

DO 16 K=1,KMX
Q(K,KP)=0.0

DO 17 J=1,LMX
JKP=(J-1)*KMX+KP
JK=(J-1)*KMX+K
17 Q(K,KP)=Q(K,KP)+R(JK,JKP)

16 CONTINUE

15 CONTINUE

WRITE (5,1)
WRITE (5,4)

NROW=KMX
CALL XPRIT (Q,NROW,NROW)

WRITE (5,2)
WRITE (5,3)
1 FORMAT (1H1,10X,'THE .Q. MATRIX')
2 FORMAT (1X,10X,'THE EIGENVALUES OF THE .Q. MATRIX')
3 FORMAT (13X,'K K',/)
4 FORMAT (16X,*K K',/)
RETURN
END
SUBROUTINE XPRIT (X,NROW,NCOL)
C REAL*8 X(NROW,NCOL)
C
LL=NCOL-(NCOL/8)*8
IF(LL)103,102,103
103 LL=1
102 LL=(NCOL/8)*LL
K=9
C
DO 110 II=1,LL
NC=K+1
N1=K+3
N1=INT(N1,NCOL)
WRITE (5,104) (J,J=NO,N1)
C
DO 103 I=1,NROW
103 WRITE (6,109) I, (X(I,J),J=NO,N1)
C
110 K=K+8
C
104 FORMAT (1X,/,3X,8I15)
109 FORMAT (1H,16,8F15.5)
RETURN
END
SUBROUTINE XPRITA (X, NROW, NCOL)
C
DIMENSION X(NROW, NCOL)
C
LL=NCOL-(NCOL/8)*8
IF(LL lt=2,1,2,103)
103 LL=1
102 LL=(NCOL/8)+LL
K=C
C
DO 113 I=1,LL
N0=K+1
N1=K+8
N1=MIN(N1, NCOL)
WRITE (5,1C4) (J, J=NO, N1)
C
DO 108 I=1, NROW
108 WRITE (5,1C9) I, (X(I,J), J=NO, N1)
C
116 K=K+8
C
1C4 FORMAT (1X, /, 3X, 8I15)
1C9 FORMAT (1H, 16, 8F15, 5)
RETURN
END

C
SUBROUTINE MATA (A,B,R,L,M,N)

DIMENSION A(L,M), B(M,N)
REAL *P R(L,N)
REAL *B TEMP

DO 15 J = 1,N
   DO 15 I = 1,L
      TEMP=0.
      DO 13 K = 1,M
         TEMP = (A(I,K)) * (B(K,J)) + TEMP
      13 CONTINUE
      R(I,J) = TEMP
   15 CONTINUE

RETURN
END
SUBROUTINE CRN (A, B, H, I, J, K, L, M, N)
C
INTEGER AR, AC, BR, BC, HR, HC
REAL*8 A(I, J), B(K, L), H(M, N)
C
DO 1 AC = 1, J, 1
DO 1 BC = 1, L, 1
DO 1 AR = 1, I, 1
DO 1 BR = 1, K, 1
HR = (AR - 1) * K + BR
HC = (AC - 1) * L + BC
1 H(HR, HC) = A(AR, AC) * B(BR, BC)
C
RETURN
END

3-M 1686
3-M 1687
3-M 1688
3-M 1689
3-M 1690
3-M 1691
3-M 1692
3-M 1693
3-M 1694
3-M 1695
3-M 1696
3-M 1697
3-M 1698
3-M 1699
3-M 1700
SUBROUTINE MAT (A,B,R,L,M,N)
C
REAL *A(L,M), B(M,N), R(L,N)
REAL *R TEMP
C
DO 15 J=1,N
DO 15 I=1,L
TEMP=3.
C
DO 15 K=1,M
TEMP=(A(I,K)) * (B(K,J)) + TEMP
15 CONTINUE
C
R(I,J) = TEMP
15 CONTINUE
C
RETURN
END
SUBROUTINE GETVS (G,V,S,NROW,NCOL)

INTEGER X,Y,Z
DIMENSION G(NROW,NCOL), V(NROW,NCOL), S(NCOL,NCOL)

DO 1 L = 1, NCOL
DO 2 K = 1, NCOL
S(K,L) = 0.
1 CONTINUE

DO 10 J = 1, NCOL
SUM = C.

DO 50 I = 1, NROW
SUM = SUM + (G(I,J) * G(I,J))
50 CONTINUE

S(J,J) = (1.0 / SORT (SUM))

DO 60 X = 1, NROW
V(X,J) = SORT (G(X,J) * G(X,J)) / SUM
60 CONTINUE

100 CONTINUE

DO 3 Z = 1, NCOL
DO 3 Y = 1, NROW
IF (G(Y,Z) .LT. G(J)) V(Y,Z) = -V(Y,Z)
3 CONTINUE

RETURN
END
SUBROUTINE CROS(A, B, H, I, J, K, L, M, N)

INTEGER AR, AC, BR, BC, HR, HC

DIMENSION A(I, J), B(K, L), H(M, N)

DO 1 AC = 1, J, 1
DO 1 BC = 1, L, 1
DO 1 AR = 1, I, 1
DO 1 BR = 1, K, 1

HR = (AR - 1) * K + BR
HC = (AC - 1) * L + BC

1 H(HR, HC) = A(AR, AC) * B(BR, BC)

RETURN

END
SUBROUTINE MATS(A,B,R,L,M,N)

REAL A(L,M), B(M,N), R(L,N)

REAL TEMP

DO 15 J=1,N
  DO 15 I=1,L
    TEMP=0.0
    DO 15 K=1,M
      TEMP=TEMP+A(I,K)*B(K,J)
    15 CONTINUE
    R(I,J)=TEMP
  15 CONTINUE

RETURN
END