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STOCHASTIC ANALYSIS OF SEEPAGE IN HETEROGENEOUS SOILS

DISSertation

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

Elfatih M. Ali, B.Sc. (honors), M.S.

*******
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1979

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ACKNOWLEDGMENTS

The author wishes to express his sincere appreciation and gratitude to his adviser, Professor Tien Hsing Wu, for his patience, encouragement and guidance throughout this study.

Thanks are also due to Professors R.S. Sandhu, K.W. Bedford, and J. Rustagi for reviewing this dissertation and for constructive discussion on its contents.

This research was supported by a grant from the National Science Foundation. The author is indebted to the financial support of the N.S.F. in the form of a graduate research associateship.

Ms. Cheryl Helm's effort in typing this manuscript is appreciated.
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CHAPTER 1
INTRODUCTION

1.1 General

Properties of soil deposits are of major significance in the study of many geotechnical problems such as failure of slopes, settlement of foundations, and seepage. Soil deposits may be classified as homogeneous or heterogeneous. A homogeneous deposit is composed of one soil type whose properties are the same everywhere, while a heterogeneous deposit is composed of two or more soil types. In this study, we consider one of the soil types in a heterogeneous deposit as a medium in which the other types are scattered as inclusions or lenses. The size, type and location of all inclusions in the medium will be referred to as the stratification of the deposit. If this stratification is completely known, then we have a deterministic heterogeneous soil deposit. If, on the other hand, the stratification is not completely known, then we need to estimate it. Probabilistic methods may be used to represent the stratification by stochastic models and then we speak of a stochastic heterogeneous soil deposit.

The properties of a heterogeneous deposit depend on the properties of the individual soil types as well as the stratification. For many heterogeneous soils, we may make
a simplification and consider each individual soil type as homogeneous.

A representative sample is one that provides complete information about the stratification of a deposit. If a regular repeated pattern exists in the stratification, then a representative sample smaller than the deposit would give us the necessary details of such a pattern; otherwise, the representative sample would include all the deposit under consideration. For a deterministic heterogeneous soil deposit, a representative sample may be studied to characterize the soil properties of the whole deposit. This may be carried out experimentally by conducting a laboratory experiment on the sample and then the behavior of the deposit can be analyzed. For the case of seepage, a laboratory permeability test may be conducted on a representative sample. If the size of the representative sample is too large, then we may study the properties of the individual soil components. The seepage analysis may be made for the deposit using the electrical analog method or numerical techniques such as the finite element or finite differences methods.

A hypothetical homogeneous soil sample with the same dimensions, boundary conditions and discharge rate as the actual representative soil sample will then have an equivalent permeability, $k^*$. Thus, the deterministic heterogeneous soil deposit could be characterized by an equivalent homogeneous medium. There are two levels of investigation that are
apparent here. The first one is the **micro-level**, where the details of stratification are taken into consideration and a representative sample is analyzed for seepage. This is comparable to looking at a sample closely, such that all the details of inclusions are conspicuous. The second level is the **macro-level**, where the heterogeneous soil deposit is characterized by an equivalent homogeneous medium. This is comparable to observing the soil deposit from some distance away so that it appears to be composed of one material only.

Many of the heterogeneous soil deposits are considered stochastic and then the representation of the deposit by an equivalent medium is not a simple task. The main problem encountered is how to obtain a representative sample of the stochastic heterogeneous soil deposit. Any sample is only one realization from the population describing the deposit and does not have much significance by itself. It is thus necessary to investigate the stochastic nature of the soil deposit. The known procedure to date (1979) is to represent the soil stratification by some stochastic models. Many soil profiles may then be generated for the deposit by simulation from those stochastic models. Each soil profile would then be analyzed using a numerical technique such as the finite element method (see, for example, Lippman (1973) and Chang (1976)). The equivalent permeability of the deposit is then considered as a random variable with a certain probability distribution function. To achieve a good representation of the random
variable, a large number of soil profiles have to be analyzed. As those soil profiles are usually large in size, this approach is found to be very expensive and time-consuming. A new approach will be presented in this investigation to solve the problem of seepage in stochastic heterogeneous soils.

1.2 Objective and Scope

The goal of this investigation is two-fold. The first goal is to study and model the stratification of stochastic heterogeneous soil deposits. The theory of Markov chains is used to model such deposits. Case-studies are presented wherein the modeling procedure is applied to actual heterogeneous soil deposits.

The second goal is to develop a random flow model for the analysis of seepage in heterogeneous soils. This model takes into account the stochastic nature of the heterogeneous soil deposit and obeys the fundamental physical laws of seepage. The random flow model is applied to different soil deposits by using simulation methods. The model is then used in the characterization of the seepage properties of heterogeneous soil deposits. An equivalent medium whose permeability is a random variable is thus used to represent the stochastic heterogeneous soil deposit.

Chapter Two begins with a survey of the methods used to model stochastic heterogeneous soils. The proposed model in this investigation for the soil stratification is then presented. The procedures to estimate the model parameters
and to generate soil profiles are outlined. Those procedures are then applied to three soil deposits.

The random flow model developed to solve the problem of seepage in heterogeneous soils is presented in Chapter Three. The basis of the model is given in detail and it is then applied to the case of a deterministic heterogeneous soil deposit. The stochastic nature of the heterogeneous soil deposit is then considered next. The soil models developed in Chapter Two are incorporated in the random flow model and the one-dimensional confined flow problem is analyzed for the different soil deposits studied previously.

The random flow model is extended to analyze the case where the variations of stratification in the direction perpendicular to the vertical plane are significant. This is achieved by the three-dimensional random flow model presented in Chapter Four. A case-study is also given for that model. Chapter Five contains conclusions and recommendations for further research.
CHAPTER 2
STOCHASTIC MODELING OF HETEROGENEOUS SOILS

2.1 General

Researchers have been investigating geologic deposits composed of two or more rock types and existing in various stratigraphic patterns. If the pattern is everywhere the same and is precisely known, then the stratigraphy is deterministic. Otherwise, the pattern is random or stochastic. The stochastic nature of rock or soil deposits is due mainly to the randomness of the process causing the formation of the geologic deposits.

Stochastic or probabilistic methods have been utilized by geologists to model rock stratigraphic sequences (e.g., Vistelius (1940), Krumbein (1967) and Agterberg (1974)). Stochastic models that take into account the processes that produce the rock formation have also been investigated. Krumbein (1968) proposed a model for the lateral migration of beach sand. The model uses a continuous time Markov chain. Schwarzacher (1972) developed a semi-Markov process for sedimentation. He assumed a two-stage process. In the first stage, the environmental conditions determine the lithology. In the second stage, the mechanism of sedimentation determines the amount of sediment deposited.
Schwarzacher (1976) represented the variations in thickness of a sediment by a Poisson process.

Stochastic modeling of heterogeneous soils is also a recent development. A simple model of heterogeneous soils is composed of two components. One component is considered as the medium in which the other component is imbedded as inclusions. Vyas (1970) studied soil deposits where the medium is a silty clay, and the inclusions are sand lenses. Chang (1976) studied a varved clay deposit. He used Markov chains to model the vertical stratification and the variations in thickness of the silt and clay layers. Lippman (1973) modeled a two-dimensional heterogeneous soil system by Poisson lines which have a Markovian property.

The model considered in this investigation is an extension of Chang's model (1976). The proposed model will be used later in the analysis of seepage in heterogeneous soils. In this chapter, an introduction of Markov chains is given first. This is followed by the details of the proposed soil modeling technique and its application to different soil deposits.

2.2 Markov Chains

2.2.1 Basic Concepts

Stochastic processes have been used to represent many processes in nature. If a process can assume, randomly, any one of a number of states, then the process
is stochastic. The state space is the space containing all the states that the process can assume.

In some random processes, the previous events influence, but do not completely control, the subsequent events. Such processes are called Markov processes. A Markov chain is a Markov process which may be regarded as a sequence or chain of discrete states in time or space, in which the probability of transition from an existing state to the given state in the next step in the chain depends on the existing state.

In its simplest form, the first order discrete-state, discrete-time Markov chain can be visualized as representing a system with a finite number of discrete states A, B, C, ..., behaving in such a way that a transition occurs from state i to state j at each tick of a conceptual "Markovian clock." The Markovian clock is an imaginary concept whereby the transition from one state to itself or to another state takes place after a certain time or distance. The index space is the space of the "Markovian clock" time.

Consider, for example, a system with three states, A, B, C, that is described by a transition probability matrix, P, as shown in Equation (2.1) on the following page. The rows are represented by index i and the columns by index j, so that $p_{ij}$ is the transition probability from state i to state j.
If the transition from state $i$ to state $j$ depends only on the state of the system prior to the transition, then the Markov chain is of the first order. If the transition depends on more than one previous state, then the order of the Markov chain is greater than one. A stationary Markov chain is one in which the probabilities associated with transitions between states do not vary with time or space.

If the system is in state $i$, then the probability that it arrives at state $j$ after $n$ steps is $p_{ij}^{(n)}$ and the transition probability matrix is then $P^{(n)}$. It is known that $P^{(n)}$ is obtained by raising the original transition matrix $P$ to the power $n$. A limiting matrix is obtained by the successive powering of $P$ until all the rows of the transition matrix are identical. This is called the $T$-matrix, i.e.,

$$T = P^{(n)} = \begin{bmatrix} p_A & p_B & p_C \\ p_A & p_B & p_C \\ p_A & p_B & p_C \end{bmatrix}$$ (2.2)
This matrix says, in effect, that when the limit T has been reached, the probabilities of passing to either state A, B, or C are independent of the starting state. Therefore, the T-matrix gives us the overall probabilities of different states, A, B, C, ..., in the system.

2.2.2 Estimation of a Transition Probability Matrix

Transition probabilities are commonly estimated from frequency distributions. A frequency distribution of transitions is simply a tabulation of the number of transitions from each state to each other state. A step in the index space is defined as the interval inside which no change in state is assumed to occur. The number of observed transitions are tabulated in a "tally" matrix. Here, \( n_{ij} \) is the number of observed transitions from state i to state j.

The transition probability matrix is estimated by computing the frequency of each transition in a row of the tally matrix.

\[
p_{ij} = \frac{n_{ij}}{\sum_{j=1}^{m} n_{ij}} \quad (2.3)
\]

where:

\( m \) = total number of states

\( p_{ij} \) = transition probability from state i to state j.
The cumulative transition matrix is obtained by accumulating the transition probabilities along each row, i.e.,

\[
P_{ij} = \sum_{k=1}^{i} P_{kj}
\]

where \( P_{ij} \) = cumulative transition probability from state \( i \) to state \( j \).

2.2.3 Statistical Tests for Markov Chains

2.2.3.1 Testing for the Markov Property

An important property of first order Markov chains, called the "Markov property," is that the probabilities associated with each transition depend on the immediately preceding state.

A statistical test for the Markov property of a chain has been given by Anderson and Goodman (1957). The null hypothesis is that successive events are independent of each other while the alternative hypothesis is that they are not. If successive events are not independent, then they could form a first order Markov chain.

The test statistic, \( \lambda \), is:

\[
\lambda = \prod_{i,j=1}^{m} \left( \frac{p_{ij}}{P_{ij}} \right) ^{n_{ij}}
\]

(2.5)
where:

\[ p_{ij} = \text{probability of transition from state } i \text{ to state } j \]

\[ p_j = \text{marginal probability of the } j^{\text{th}} \text{ column} \]

\[ p_j = \left( \sum_{i=1}^{m} n_{ij} \right) / \left( \sum_{i,j=1}^{m} n_{ij} \right) \]

\[ n_{ij} = \text{transition frequency from state } i \text{ to state } j \text{ of the original tally matrix of observed transitions} \]

\[ m = \text{total number of states.} \]

Anderson and Goodman (1957) found that \(-2 \log_e \lambda\) is distributed asymptotically as a Chi-square \((\chi^2)\) distribution with \((m-1)^2\) degrees of freedom. An equivalent expression for \(-2 \log_e \lambda\), which is more convenient for computational work, is

\[ -2 \log_e \lambda = 2 \sum_{i,j=1}^{m} n_{ij} \log_e \left( \frac{p_{ij}}{p_j} \right) \quad (2.6) \]

### 2.2.3.2 Testing for Stationarity

As stated previously, stationarity of a Markov chain means that the transition probabilities do not change with the index space. To test for this property, the whole chain (or sequence) is divided into \(T\) subintervals. For each subinterval, a separate transition probability matrix is calculated. The null hypothesis is that \(p_{ij}(t) = p_{ij}\) for \(t = 1, 2, \ldots, T\); where \(p_{ij}\) is the transition probability from state \(i\) to state \(j\). The alternative is that the Markov chain is not stationary, i.e., \(p_{ij}(t) \neq p_{ij}\).
The test statistic, $\lambda$, suggested by Anderson and Goodman (1957) is

$$\lambda = \prod_{t=1}^{T} \prod_{i,j=1}^{m} \left[ \frac{p_{ij}(t)}{\hat{p}_{ij}(t)} \right]^{n_{ij}(t)} \tag{2.7}$$

where:

- $m$ = number of states;
- $T$ = number of subintervals;
- $n_{ij}(t)$ = frequency tally for transition from state $i$ to state $j$ in the $t$th subinterval;
- $p_{ij}(t)$ = transition probability from state $i$ to state $j$ in the $t$th subinterval; and
- $p_{ij}$ = transition probability from state $i$ to state $j$ in the whole sequence.

When the null hypothesis of stationarity is true, $-2 \log_{e} \lambda$ is distributed as a Chi-square ($\chi^2$) distribution with $(T-1)[m(m-1)]$ degrees of freedom. A convenient expression for the computation of $-2 \log_{e} \lambda$ is

$$-2 \log_{e} \lambda = 2 \sum_{t=1}^{T} \sum_{i,j=1}^{m} n_{ij}(t) \log_{e} \left( \frac{p_{ij}(t)}{\hat{p}_{ij}} \right) \tag{2.8}$$

2.2.3.3 Test of the Hypothesis that Two Samples are from the Same Markov Chain of a Given Order

Anderson and Goodman (1957) developed a procedure to test whether transition matrices obtained from several samples are similar, i.e., whether they
represent the same Markov chain of a given order.

The null hypothesis is that S samples (S \geq 2) are from the same r^{th} order Markov chain. That is, the S processes are identical. For the case of S = 2, the test statistic is:

\[ \chi^2_{ij...k} = \sum_{l} c_{ij...kl} \left[ \hat{p}^{(1)}_{ij...kl} - \hat{p}^{(2)}_{ij...kl} \right]^2 / \hat{p}^{(0)}_{ij...kl} \]

(2.9)

where:
- \( \hat{p}^{(1)}_{ij...kl} \) and \( \hat{p}^{(2)}_{ij...kl} \) are the estimates of the elements of transition matrices for the two samples;
- \( \hat{p}^{(0)}_{ij...kl} \) is the estimate of \( p_{ij...kl} \) obtained by pooling the data in the two samples;
- \( c_{ij...kl}^{-1} = \left[ \frac{1}{n^{*(1)}_{ij...k}} + \frac{1}{n^{*(2)}_{ij...k}} \right] \)
- \( n^{*(1)}_{ij...k} \) and \( n^{*(2)}_{ij...k} \) are sums of the rows of the tally matrices for the first and second samples respectively.

Anderson and Goodman (1957) found that \( \sum_{ij...k} \chi^2_{ij...k} \) has the limiting Chi-square (\( \chi^2 \)) distribution with \( m^r(m-1) \) degrees of freedom, where

\[ m = \text{number of states in the Markov chain}; \]
\[ r = \text{order of the Markov chain}. \]
2.3 Modeling of Two-Dimensional Deposits of Heterogeneous Soils

2.3.1 General

A two-dimensional stochastic model is proposed for heterogeneous soils. The stratification of the soil along a vertical line is considered to be random. The variations in the thicknesses of individual strata in the horizontal direction are also assumed to be random.

The vertical stratification is modeled by a stationary Markov chain. The soil deposit is considered to consist of two soil types. One soil type is the medium in which the other type, or inclusion, is located. The state space is therefore composed of all the soil types encountered, which include the medium and inclusions. The length of a step in the index space is the vertical distance in which no change in the soil type takes place. This is taken as the smallest thickness of an inclusion.

The variation in thickness of the inclusion in the horizontal direction is modeled by another stationary Markov chain. The state space is composed of all the possible values of the thickness of the inclusion. Discrete thickness values are chosen as discrete states. A zero thickness of an inclusion is considered as the first state. The length of a step in the index space is taken as the horizontal distance along which no change in the thickness state of an inclusion takes place.
2.3.2 **Modeling Procedure**

The stochastic modeling of heterogeneous soils is accomplished in three stages. In the first stage, the transition matrices, for both the vertical stratification and the variations in thickness of inclusions, are estimated from soil samples. In the second stage, statistical testing is carried out to test the stationarity and Markovian property, as explained in Sections 2.2.3.1 and 2.2.3.2. In the third stage, the transition matrices, obtained in the first stage, are used to simulate the soil stratification.

2.3.3 **Calculation of Transition Matrices**

To estimate the transition matrices for the vertical stratification and thickness of inclusions in the horizontal direction, we start with a two-dimensional profile of a soil sample. This may be a photograph, taken to a known scale, or a diagram drawn to scale, such that the details of thickness of inclusions in the medium are well defined.

The soil profile is first investigated to determine the length of steps in the index spaces of the Markov chains, representing both the vertical stratification and inclusion thicknesses. Figure 2.1 shows a fictitious two-component heterogeneous soil deposit. The dotted
Figure 2.1 A Hypothetical Heterogeneous Soil Deposit
areas are the inclusions (state 1) that lie in the medium (state 0). From inspection of the above-mentioned profile, distances \(a\) and \(b\) are chosen as the lengths of steps in the index spaces for the two Markov chains. Note that the values of \(a\) and \(b\) are arbitrary to some extent. Some judgment must, however, be exercised in the choice of \(a\) and \(b\). Very large values of \(a\) and \(b\) may lead to overlooking some transitions. Very small values, on the other hand, may give matrices that give too much weight to the transitions from a state to itself. In both these cases, the discrepancies will lead to inaccuracies when using the transition matrices in the simulation of soil profiles (see the next section).

A rectangular mesh whose nodal points are spaced at distances \(a\) and \(b\) vertically and horizontally is then drawn over the profile. This leads to the approximation of the shape of the inclusion by a sequence of straight lines. The approximate shape of inclusions is shown by the broken lines in Figure 2.1.

The vertical transition tally matrix is then calculated. This is done by counting the transition between states \(i\) and \(j\) \((i,j = 1,2)\) along the vertical lines of the mesh. The transition probability matrix is estimated from the tally matrix and the cumulative transition probability matrix is then calculated. At this stage, statistical
tests for both the Markovian property and stationarity are carried out (see Section 2.2.3).

It was noted that the smallest thickness of the inclusion is $a$. This is taken as state number 2 for the transition matrix representing the variations in inclusion thickness. States 3, 4, ... are taken as inclusion thicknesses of $2a$, $3a$, .... State number 1 is where the inclusion thickness is zero.

The variation in thickness of the inclusion in the horizontal direction is modeled by a Markov chain. The transitions from one thickness $t_n$ at nodal point $C_n$ to thickness $t_{n+1}$ at horizontally neighboring nodal point $C_{n+1}$ are counted to determine the tally matrix. The transition probability matrix and the cumulative transition probability matrix are then estimated. Stationarity and the Markovian property are then tested for as explained in Section 2.2.3.

2.3.4 Simulation of a Two-Dimensional Soil Profile

The simulation process is essentially the inverse of the process of obtaining the transition matrices. First we start with the random choice of an initial soil state (state $k$). This choice is based on the overall probability of different soil types in the soil deposit.

The original soil state is then used to simulate a stratigraphic column along a vertical line. This is called here the initial vertical column. A random number generator (see Dudwicz (1974)) is used to generate uniformly
distributed random number between 0.0 and 1.0. The generated random number \( \alpha \) is then compared with the entries of the \( k^{th} \) row of the cumulative transition matrix for the vertical stratification. Transition takes place from state \( k \) to state \( j \) if:

\[
P_{k,j} \leq \alpha \quad \text{and} \quad P_{k,j-1} \leq \alpha \quad (2.10)
\]

where \( \alpha \) is the generated random number (0.0 \( < \alpha \leq 1.0 \)).

The system is then in state \( j \), and we move to the next step of the stratigraphic column which is at distance \( a \) away. The generation of soil states is continued until the required thickness of the initial vertical column is reached.

The simulated initial vertical column is then used to obtain the initial states for simulating the variations of inclusion thickness in the horizontal direction. The thickness of an inclusion on the initial vertical column is first found. This is used as the starting state \( (p^{th} \) state). The cumulative transition matrix for the inclusion thickness, together with the randomly generated numbers, are used to generate the thickness \( (q^{th} \) state) at the next step. This is done for all the inclusions in the initial vertical column. The result gives the thickness of inclusions in the vertical column located at one step, or distance \( b \), from the initial vertical column. The inclusion
thicknesses in subsequent vertical columns are simulated in the same manner until the required width of the soil profile has been achieved.

It is desirable to test whether the simulated soil profile has the same statistical properties as the actual soil deposit. This will be accomplished by testing the hypothesis that the simulated soil profile and the actual soil profiles are both samples from the same first order Markov chain. To do this, the simulated two-dimensional soil profile is analyzed in order to obtain the transition matrix for its vertical stratification. This transition matrix is then used in the statistical tests explained in Section 2.2.2.3.

2.4 Application to Varved Clay

2.4.1 General

The first case study presented in this chapter is that of varved clay deposits. The stochastic model obtained for the varved clay deposit would be used in the next chapter to study seepage.

Over considerable areas of North America, Scandinavia, and the U.S.S.R., there are clay deposits with a definite stratified structure. Many of these strata were formed in lake basins where cyclic changes in temperature and sedimentation environment existed. These cyclic changes often produce a repetition of two distinct layers, one of which
is generally coarser in grain size and lighter in color than the other layer. These deposits are referred to as varved clays. The relatively coarse material, usually silt, was deposited during the melting season while the finer particles, usually clay, settled to the bottom during the colder season.

Some of the conditions for the formation of varved clays have been postulated by Antevs (1925). These include climatic, topographic and geologic factors. As most of these factors have random components in their nature, then the thicknesses and lengths of the silt and/or clay layers exhibit some random properties. Recently, Anderson (1963) and Agterberg and Banerjee (1964) used techniques of the Spectral Analysis to investigate the variations in varved clays and to relate them to the sedimentation processes. Chang (1976) used Markov chains to model the Toledo varved clay. He obtained transition matrices for the vertical stratification and for the variations in thickness of both the silt and clay layers.

The model proposed in this investigation considers the silt layers as inclusions scattered in a clay medium. Hence, only the transition matrices for the vertical stratification and for the variations in silt thickness are needed. This modification simplifies the soil model so that it could later be used in the analysis of seepage.
2.4.2 Soil Samples

The varved clay deposits studied in this investigation are located in the Lake Erie area in northern Ohio. Two groups of soil samples were studied; the first group was obtained from the Toledo area in northwestern Ohio and will be referred to as the Toledo varved clay. The second group of soil samples was obtained from the Cleveland area in northeastern Ohio and will be referred to as Cleveland varved clay.

Shelby tubes (3.0 inch) were used to obtain continuous Toledo varved clay samples from different boreholes. Chang (1976) presented detailed information about the location of boreholes and the samples obtained. The varved clay samples were studied in detail for stochastic modeling.

As the Toledo samples were 3.0 inches in diameter, it was felt that they did not provide a clear picture about the variations of the thickness of the silt layers in the horizontal direction. Samples with larger cross-sectional areas were then sought. Another varved clay deposit is the Cleveland deposit which was studied by Wu et al. (1975). A varved clay layer was exposed on the side of a highway cut near Cleveland, Ohio. Eight block samples, each of approximately 6.0 x 6.0 inch cross-section, were obtained and brought into the laboratory for analysis. The eight samples form approximately a square mesh with three samples in each row (see Figure 2.2).
N.B. Sample No. 1 was damaged and therefore discarded.

Figure 2.2 Arrangement of Soil Samples for the Cleveland Varved Clay
2.4.3 **Stochastic Modeling of Varved Clays**

The procedure outlined previously in Section 2.3.1 was used to obtain the transition matrices. First, the transition matrix for the vertical stratification was estimated. The transition matrix for the variations in thickness of the silt layers in the horizontal direction was then estimated. Statistical tests for stationarity and for the Markovian property were also carried out as explained in Section 2.2.3.

A two-dimensional soil profile was then simulated using the transition matrices and the Monte Carlo technique. Hypothesis testing was carried out to see if the simulated soil profile has the same statistical properties as the original soil deposit.

It should be noted that the Toledo and Cleveland varved clays are similar but not identical soil deposits. Therefore, the modeling procedures were applied to each deposit individually.

2.4.4 **Cleveland Varved Clay**

2.4.4.1 **Soil Description**

The varved clay layer is located between a silty sand layer at the top and a silty clay layer at the bottom. The thickness of the varved clay layer is about 20.0 ft. The soil is composed of gray silty clay with silt and fine sand and silt laminations.
Measurement of varve thicknesses was accomplished following the procedure established by Hughes (1965). The block samples obtained from the field were shaved to plane surfaces. They were then left to dry out slowly in the laboratory to obtain the maximum color contrast between silt and clay layers. Detailed sections were drawn to scale for all the samples. Figure 2.3 shows one of the sections while the others are presented in Appendix A.

2.4.4.2 Vertical Stratification

The sections presented in Figure 2.3 and Appendix A were used to obtain the tally matrix for the vertical transition. The state space is composed of two states: clay (state 0) and silt (state 1). It was found that the smallest thickness of a silt layer was about 0.1 inch. This was hence taken as the length of step in the index space. A rectangular mesh was then drawn on each section. The mesh is composed of a set of parallel horizontal lines 0.1 inch apart and an orthogonal set of vertical lines 1.0 inch apart. The tally matrix for the vertical stratification was determined by counting the soil transitions on all the vertical lines of each section. Table 2.1 gives the tally and probability transition matrices for the Cleveland varved clay deposit.

The overall percentages of clay and silt in the varved clay were obtained by raising the transition probability matrix to a large power 'n' as explained in Section
Figure 2.3 Section No. 6 for the Cleveland Varved Clay
TABLE 2.1 VERTICAL STRATIFICATION FOR THE CLEVELAND VARVED CLAY

<table>
<thead>
<tr>
<th></th>
<th>(0) clay</th>
<th>(1) silt</th>
</tr>
</thead>
<tbody>
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<td>(0) clay</td>
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<td>392</td>
</tr>
<tr>
<td>(1) silt</td>
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</table>

Vertical Tally Matrix

<table>
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<th></th>
<th>(0) clay</th>
<th>(1) silt</th>
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</thead>
<tbody>
<tr>
<td>(0) clay</td>
<td>0.887</td>
<td>0.113</td>
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<tr>
<td>(1) silt</td>
<td>0.362</td>
<td>0.638</td>
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Vertical Transition Probability Matrix

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<tr>
<td>(1) silt</td>
<td>0.362</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Vertical Cumulative Transition Probability Matrix
2.2.1. Calculations show that the clay is about 77% and the silt is about 23% of the Cleveland varved clay deposit.

The methods outlined in Section 2.2.3 were used to test whether the vertical stratification of the Cleveland varved clay exhibits both stationarity and the Markovian property. The stationarity was tested by dividing one section (section 2) into subintervals. Each subinterval was a vertical line in the section. The Chi-square statistic was then used as outlined earlier. Details of the computations are given in Appendix B.

It is concluded that at the significance level of $\alpha = 0.05$, the hypothesis that the chain is Markovian and stationary can be accepted.

2.4.4.3 Variations in Thickness of Silt Layers

The variation in the thicknesses of silt layers in the horizontal direction was modeled by a Markov chain. The state space is composed of all the different values of thicknesses of the silt layers. It was found that the thickness of silt layers varied from 0.0 in. (state 1) to more than 1.0 inch. However, we encountered few layers with thickness exceeding 0.5 inch, and it was thus decided to limit the state space to six discrete states. These represent silt layers of thicknesses 0.0, 0.1, 0.2, 0.3, 0.4 and 0.5 inch. Any layer whose thickness was more than 0.5 inch was included in state number 6.
The step length in the index space was taken as 1.0 inch, the distance between any two adjacent vertical lines of the rectangular mesh drawn on the varved clay sections.

To obtain the tally matrix, a silt layer with thickness, $t_n$, is considered to be at point $C_n$, which is the center of the layer (see Figure 2.4). Point $C_n$ lies on the $n^{th}$ vertical line. The thickness of the same silt layer on the $(n+1)^{th}$ vertical line is $t_{n+1}$, and its center lies at point $C_{n+1}$. Points $C_n$ and $C_{n+1}$ lie on adjacent vertical lines. One transition event from state $(t_n + 1)$ to state $(t_{n+1} + 1)$ is then recorded in the tally matrix. This procedure is followed for all the silt layers all over the soil section.

The tally matrix and the transition probability matrix for the variations in thickness of silt layers are given in Table 2.2.

The statistical testing for stationarity was carried out in two stages. In the first stage, each of the eight varved clay samples was considered as a subinterval and the stationarity tested. In the second stage, three horizontal varved clay layers lying one above the other were considered as subintervals. The first layer was composed of samples 2 and 3. The second layer included samples 4, 5 and 6, and the third layer was composed of samples 7, 8 and 9 (see Figure 2.2).
Figure 2.4 Rules for Computing the Tally Matrix for Variations in Inclusion Thickness
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**Horizontal Tally Matrix**

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**Horizontal Transition Probability Matrix**

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<td>0.0</td>
<td>0.05</td>
<td>0.11</td>
<td>1.0</td>
</tr>
</tbody>
</table>

**Horizontal Cumulative Transition Probability Matrix**
The details of the hypothesis testing for both the stationarity and the Markovian property are given in Appendix C. It is concluded that at the significance level of $\alpha = 0.05$, both the stationarity and the Markovian property can be accepted.

2.4.4.4 Simulation of Cleveland Varved Clay

A computer program was used to simulate a two-dimensional soil profile of the Cleveland varved clay. The inputs to the program are the cumulative transition matrices for both the vertical stratification and the variations in thickness of silt layers. An initial vertical stratigraphic column was simulated first. This was then used as an initial state to simulate the thicknesses of silt layers. Figure 2.5 shows a simulated profile for the Cleveland varved clay.

The transition matrix for the vertical stratification of the simulated soil profile was computed and is given in Table 2.3. This was used to test whether the simulated profile and the original soil samples were represented by the same Markov chain. The method presented in Section 2.2.3.3 was followed and the details of the computations are given in Appendix D. It is concluded that at the significance level of $\alpha = 0.05$, the hypothesis that the simulated profile has the same statistical properties as the original soil samples could be accepted.
Figure 2.5 A Simulated Soil Profile for the Cleveland Varved Clay
TABLE 2.3 VERTICAL STRATIFICATION FOR THE SIMULATED CLEVELAND VARVED CLAY

<table>
<thead>
<tr>
<th></th>
<th>(0)</th>
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</thead>
<tbody>
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<td>clay</td>
<td>silt</td>
</tr>
<tr>
<td>(0) clay</td>
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<td>(1) silt</td>
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<td>81</td>
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Vertical Tally Matrix

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</thead>
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<tr>
<td>(1) silt</td>
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<td>0.675</td>
</tr>
</tbody>
</table>

Vertical Transition Probability Matrix
2.4.5 **Toledo Varved Clay**

The Toledo varved clay deposit is located between a gray clayey silt layer at the top and a soft sensitive clay layer at the bottom. The total thickness of the varved clay layer is about 6.5 ft. The soil is composed of silty clay with thin layers of silt in it.

The varved clay samples were extruded from the Shelby tubes and a piece about three-quarters of an inch thick was sliced lengthwise from each Shelby tube sample (see Figure 2.6). These slices were dried slowly in the laboratory and full-scale photographs were taken. This was done for all the samples where the varved clay deposit was encountered.

It was found that no single silt layer was less than 0.1 inch in thickness. A grid of parallel lines 0.1 inch apart was drawn on a transparent sheet. This transparent sheet was then placed on the photograph. The changes in soil type at the vertical intervals of one-tenth of an inch were then observed and recorded. This was done along the centerline of the photograph and along two vertical lines at a horizontal distance of 1.0 inch on each side of the centerline (see Figure 2.6). Examples of such sections of the Toledo varved clay are given in Figure 2.7. Chang (1976) presented the details of all the Toledo varved clay samples.

Tables 2.4 and 2.5 give the matrices for the Markov chains for both the vertical stratification and the variations in thickness of silt layers as obtained by Chang (1976).
Figure 2.6  Sample Preparation for the Toledo Varved Clay
Figure 2.7 Soil Sections from the Toledo Varved Clay
TABLE 2.4 VERTICAL STRATIFICATION FOR THE TOLEDO VARVED CLAY
(From Chang (1976) )

<table>
<thead>
<tr>
<th></th>
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<th>(1) silt</th>
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<td>72</td>
</tr>
<tr>
<td>(1) silt</td>
<td>73</td>
<td>90</td>
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Vertical Tally Matrix

<table>
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<th>(1) silt</th>
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</thead>
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<td>0.34</td>
</tr>
<tr>
<td>(1) silt</td>
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<td>0.65</td>
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Vertical Transition Probability Matrix

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</thead>
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<tr>
<td>(1) silt</td>
<td>0.35</td>
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</table>

Vertical Cumulative Transition Probability Matrix
TABLE 2.5 HORIZONTAL VARIATIONS IN THICKNESS OF SILT LAYERS FOR THE TOLEDO VARVED CLAY (From Chang (1976))

<table>
<thead>
<tr>
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<td>12</td>
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<td>0</td>
</tr>
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<td>7</td>
<td>25</td>
<td>22</td>
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<td>0</td>
</tr>
<tr>
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<td>11</td>
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<td>7</td>
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<td>9</td>
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<tr>
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Horizontal Tally Matrix

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<td>0.21</td>
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<td>0.1</td>
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<tr>
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<td>0.0</td>
<td>0.05</td>
<td>0.32</td>
<td>0.41</td>
<td>0.23</td>
</tr>
<tr>
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<td>0.0</td>
<td>0.0</td>
<td>0.33</td>
<td>0.33</td>
<td>0.34</td>
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Horizontal Transition Probability Matrix

<table>
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<tbody>
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<td>1.0</td>
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<tr>
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<td>0.0</td>
<td>0.05</td>
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<td>0.77</td>
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<td>0.0</td>
<td>0.33</td>
<td>0.66</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Horizontal Cumulative Transition Probability Matrix
The overall percentages of silt and clay were found to be 50% for silt and 50% for clay in the whole deposit.

The cumulative transition matrices for the Toledo varved clay were used in the computer program to simulate a two-dimensional soil profile. Figure 2.8 shows a simulated Toledo varved clay profile.

The transition matrices for the vertical stratification of the simulated soil profile are given in Table 2.6. These were used to test the hypothesis that the simulated soil profile and original soil samples have the same statistical properties. Details of the hypothesis testing are given in Appendix D. It is concluded that, at the significance level of \( \alpha = 0.05 \), that hypothesis can be accepted.

2.5 Applications to Glacial Outwash Deposits

2.5.1 General

Melt-waters created by the melting of glaciers carry a load of rock and soil sediments. This load is transported, sorted and deposited by the melt-water to form fluvio-glacial deposits. The principal types of stratified drift deposits built within the glacial margin are eskers and kames; those deposited beyond the ice margin are outwash plains, deltas and marine and lake silts and clays. Outwash plains are formed by the deposition of stratified drift by melt-waters over a broad area. The grading of the outwash
Figure 2.8 A Simulated Soil Profile for the Toledo Varved Clay
TABLE 2.6 VERTICAL STRATIFICATION FOR THE SIMULATED TOLEDO VARVED CLAY

<table>
<thead>
<tr>
<th></th>
<th>(0) clay</th>
<th>(1) silt</th>
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</thead>
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<tr>
<td>(0) clay</td>
<td>196</td>
<td>86</td>
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<td>(1) silt</td>
<td>77</td>
<td>182</td>
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Vertical Tally Matrix

<table>
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</thead>
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<td>(1) silt</td>
<td>0.297</td>
<td>0.703</td>
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</table>

Vertical Transition Probability Matrix
layers is generally good. Across the outwash plain, the channels are unstable; they shift back and forth because of rapid choking by sediments.

The deposition process depends on many environmental factors, e.g., the amount of load and the change in velocity of the melt-water currents and are considered to have random components. Their nature and their effects on the soil deposits cannot be calculated with precision. Hence, the stochastic modeling techniques proposed earlier may be used to describe the changes in soil properties.

Wu (1957) studied the stratification in a glacial outwash deposit near Lansing, Michigan. The deposit consists of cohesionless materials ranging from fine silty sand to gravel. Test pits were excavated and subsoil profiles were drawn. One profile is shown in Figure 2.9 and the others are presented in Appendix E.

The grain size distribution, relative density and shear strength parameters for each layer were determined. The soil properties within each individual layer are very uniform, but change significantly from one layer to another. The variation in the stratification is modeled by the stochastic method proposed in Section 2.3.

2.5.2 Modeling of Outwash Deposit

The subsoil profiles for the outwash deposit were investigated to define the state and index spaces for
Figure 2.9 Subsoil Profile for the Glacial Outwash Deposit, Site B (From Wu (1957) )
the different Markov chains. First the vertical stratification of the deposit was represented by a Markov chain. The state space consists of four states representing the different soil types, shown in Figure 2.9. The medium sand layers are the largest component, and are therefore considered as the medium (state 0). The remaining three soil types (states 1, 2 and 3) are considered as inclusions. The length of the step in the index space is taken as the vertical distance of 1.0 ft. which is the smallest thickness of an inclusion.

The variations in thickness in the horizontal direction of each type of inclusion is represented by a Markov chain. The state space for each inclusion type is composed of all the various values of thickness of that inclusion. The thickness varies from 0.0 ft. to more than 10.0 ft. The state space (for each inclusion) was assumed to consist of six discrete states. These represent inclusion thicknesses of 0.0, 1.0, 2.0, 3.0, 4.0 and 5.0 ft. Any thickness greater than 5.0 ft. was taken to be in state number 6. The length of the step in the index space was taken as the horizontal distance of 5.0 ft.

The procedures outlined in Section 2.3.2 to estimate the transition matrices of the Markov chains were followed here. The transition matrices for the vertical stratification and for the variations in thickness of each type of inclusion are given in Tables 2.7 through 2.10.
### TABLE 2.7 VERTICAL STRATIFICATION OF THE GLACIAL OUTWASH DEPOSIT

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<td>30</td>
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<td>4</td>
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<td>70</td>
<td>20</td>
</tr>
<tr>
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<td>20</td>
<td>45</td>
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Tally Matrix

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<th>3</th>
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<td>0.05</td>
<td>0.05</td>
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Transition Probability Matrix

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<td>0.95</td>
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</tr>
<tr>
<td>1</td>
<td>0.17</td>
<td>0.92</td>
<td>0.97</td>
<td>1.0</td>
</tr>
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<td>0.81</td>
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</tr>
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Cumulative Transition Probability Matrix

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<td>0.97</td>
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</tr>
<tr>
<td>2</td>
<td>0.48</td>
<td>0.92</td>
<td>0.97</td>
<td>1.0</td>
</tr>
<tr>
<td>3</td>
<td>0.60</td>
<td>0.97</td>
<td>0.97</td>
<td>1.0</td>
</tr>
</tbody>
</table>

N.B. Soil No. 0: Medium-fine sand
N.B. Soil No. 1: Fine sand-silt
N.B. Soil No. 2: Medium-coarse sand
N.B. Soil No. 3: Coarse sand-gravel
TABLE 2.8 HORIZONTAL VARIATIONS IN THICKNESS OF LAYERS FOR SOIL # 1 (FINE SAND-SILT)

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**Horizontal Tally Matrix**

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**Horizontal Transition Probability Matrix**

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**Horizontal Cumulative Transition Probability Matrix**
TABLE 2.9  HORIZONTAL VARIATIONS IN THICKNESS OF LAYERS FOR
SOIL #2 (MEDIUM-COARSE SAND)

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Horizontal Tally Matrix

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Horizontal Transition Probability Matrix

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Horizontal Cumulative Transition Probability Matrix
TABLE 2.10  HORIZONTAL VARIATION IN THICKNESS OF LAYERS FOR
SOIL #3 (COARSE SAND-GRAVEL)

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Horizontal Tally Matrix

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Horizontal Transition Probability Matrix

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Horizontal Cumulative Transition Probability Matrix
Hypothesis testing for both the Markovian property and stationarity of the Markov chain representing the vertical stratification were carried out. It is concluded that at the significance level of $\alpha = 0.05$, those hypotheses can be accepted. Details of the computations are given in Appendix F. Due to the limited amount of data available for the Markov chains representing the variations in thickness of inclusions, the hypothesis testing procedures were not carried out.

A two-dimensional soil profile for the outwash deposit was then simulated. This was done using the computer program described in Section 2.4.4.4. The program, however, was modified to include more than one inclusion type. The simulated soil profile is shown in Figure 2.10. It was then analyzed to obtain the transition matrix for its vertical stratification. Hypothesis testing was carried out as explained in Section 2.2.3.3 to test whether the simulated soil profile has the same statistical properties as the original soil samples. It was found that the above-mentioned hypothesis can not be accepted at the significance level of $\alpha = 0.05$.

It was therefore concluded that more information and field data are needed to obtain better estimates of the transition matrices for the variations in thickness of inclusions.
Figure 2.10 A Simulated Profile for the Glacial Outwash Deposit
3.1 Introduction

The passage of matter — solid, gas or fluid — through porous media is a general process encountered in many fields of science and engineering. Examples are given in Table 3.1. One of these processes is the flow of fluids through porous media or seepage.

There are two main approaches to the analysis of seepage. The first is the deterministic approach. Here, the medium is considered to have completely known properties and the motion of fluid is governed by fundamental physical laws. These laws are Darcy's law and the law of conservation of mass.

Darcy's law relates the velocity of flow to the hydraulic gradient as:

\[ v_x = -k_x \frac{dh}{dx} \]  

(3.1)

where:

- \( v_x \) = velocity of flow in the x-direction;
- \( k_x \) = coefficient of permeability in the x-direction;
- \( h \) = total head
- \( \frac{dx}{dh} \) = hydraulic gradient
<table>
<thead>
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<th>Form of Energy</th>
<th>Name of Law</th>
<th>Quantity</th>
<th>Storage</th>
<th>Resistance</th>
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<td>Current (voltage)</td>
<td>Capacitor</td>
<td>Resistor</td>
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<td>Newton's Law</td>
<td>Force (velocity)</td>
<td>Mass</td>
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<tr>
<td>Thermal</td>
<td>Fourier's Law</td>
<td>Heat flow (temperature)</td>
<td>Heat Capacity</td>
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<tr>
<td>Fluid</td>
<td>Darcy's Law</td>
<td>Flow rate (pressure)</td>
<td>Liquid Storage</td>
<td>Permeability</td>
</tr>
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</table>
Similar equations for the \( y \) and \( z \) directions are also available.

The law of conservation of mass is used to analyze the flow in and out of an infinitesimal element as shown in Figure 3.1. For incompressible fluids, we obtain the continuity equation:

\[
\frac{\partial}{\partial x} (\varrho v_x) + \frac{\partial}{\partial y} (\varrho v_y) + \frac{\partial}{\partial z} (\varrho v_z) = 0 \quad (3.2)
\]

where \( \varrho \) is the mass density of the fluid.

Substituting Equation (3.1) into Equation (3.2), we get:

\[
\frac{\partial}{\partial x} (\varrho k_x \frac{\partial h}{\partial x}) + \frac{\partial}{\partial y} (\varrho k_y \frac{\partial h}{\partial y}) + \frac{\partial}{\partial z} (\varrho k_z \frac{\partial h}{\partial z}) = 0 \quad (3.3)
\]

For incompressible fluids, \( \varrho \) is constant and then we get:

\[
k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0 \quad (3.4)
\]

and for an isotropic medium where

\[
k_x = k_y = k_z
\]

we have:

\[
\nabla^2 h = \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0 \quad (3.5)
\]

which is the Laplace equation.
Figure 3.1 Flow in an Infinitesimal Element
Thus we have obtained a partial differential equation which, together with the known values of head h on the boundaries, give us what is known as the boundary value problem. Analytical and numerical solutions of boundary value problems are the concern of many investigators. Usually the problems become more complicated for heterogeneous soils.

The second approach to seepage analysis is the stochastic approach. Underlying this approach is the concept that soil properties are not completely known. Two main areas of research have emerged in this field. The research in each area depends on the scale at which the porous medium is investigated.

If the porous medium is investigated as an assembly of small particles with pores between them, then we are looking at the micro-level. Scheidegger (1974) and Bear (1972) summarized the available research literature in this field. The flow of fluids in the pores is approximated by the flow in pipes. The size, length and direction of pores are considered as random variables. The medium as a whole is assumed to be homogeneous. These analyses provided analytical verification for Darcy's law which was originally empirical in nature.

In the second area of stochastic research, the variations in the permeability of the medium are investigated at the macro-level. Freeze (1975) studied statistically homogeneous soils where the permeability of the soil
deposit is considered as a random variable. He analyzed a soil deposit composed of many soil blocks. The coefficient of permeability of each block was one realization of the random variable describing the permeability of the medium. The blocks were considered to be connected in series, and the seepage in that system of blocks was solved as a boundary value problem. By generating many systems of blocks, Freeze obtained the mean and variance for the permeability of the medium.

A more recent approach has been presented by Gelhar (1976). He used spectral analysis methods to study seepage in statistically homogeneous soils. The permeability \( k \) of a soil deposit and the pressure head \( h \) were both considered as random variables. By using the methods of spectral analysis (Jenkins and Watts, 1968), the variations in head were obtained from the known variations in permeability. The variations in flow discharge, \( Q \), were then computed.

Heterogeneous soils may be considered as stochastic media. The soil deposit is composed of two or more components arranged according to certain stochastic laws. Individually, each component is considered to be isotropic. Vyas (1970) studied seepage in heterogeneous soil deposits. He modeled the soil deposit statistically and then used a 'network model' to solve for seepage. By simulating many soil profiles and analyzing them by the network model, Vyas
presented a stochastic analysis of seepage in a heterogeneous soil deposit. Lippman (1973) and Chang (1976) analyzed seepage in heterogeneous soils by using the finite elements method to solve for flow in heterogeneous soil profiles that were simulated from stochastic models.

A random flow model for the analysis of seepage in heterogeneous soils is presented in this chapter. This flow model takes into account the stochastic nature of the heterogeneous soil deposit. The proposed model is then used to analyze seepage in the heterogeneous soils that were modeled in Chapter 2.

3.2 Random Walk Problem

3.2.1 General

The random movement of pollens in water was first discovered by Brown in 1827. This random movement was hence called the Brownian motion and its discovery is considered as the starting point for the study of the random movement of particles.

Consider a particle (or walker) in a s-dimensional euclidean space such that its initial position at time $t_0$ is given by the vector $R_0$. At subsequent times $t_1$, $t_2$, ..., $t_n$, the particle undergoes the displacements $r_1$, $r_2$, ..., $r_n$; so that at time $t_n$, its position is given by:

$$ R_n = R_0 + \sum_{k=1}^{t_n} r_k $$

(3.6)
This equation defines a random walk if the sequence of jumps \( \{r_k\} \) are mutually independent random variables with a distribution function given as:

\[
p_r(k) = P_r [r \leq r_k \leq r + dr]
\]  

(3.7)

The position of the particle after n-steps is then a random variable. The basic problem in the theory of random walks is to calculate the probability, \( P_n(R)dR \) such that after n-steps, the particle is in the interval \((R, R + dR)\). This definition describes a random walk as a stochastic process in discrete time. Its state space may be either continuous if the steps \( \{r_k\} \) are continuous random variables or discrete if they are discrete. A walk with a discrete state space will be denoted by a lattice walk.

The random walk on a mesh was first studied by Polya (1921). It has since found greater popularity, especially with the advent of digital computers. In a two-dimensional case (see Figure 3.2), the walk starts at point 0 and the particle has probabilities \( p_1, p_2, p_3 \) and \( p_4 \) of moving to points 1, 2, 3 and 4 respectively. The walk continues until the particle reaches an absorbing boundary where the particle is absorbed. If the particle impinges on a reflecting boundary, then it is reflected back. Some of the problems of interest to researchers in
Figure 3.2 Random Walk on a Mesh
the random walk problem is the probability of returning to the starting point and the probability of absorption at an absorbing boundary.

The use of fast digital computers and random number generators have enabled researchers to execute a large number of walks easily and quickly. Thus, the Monte Carlo technique (Hammersley and Handscomb (1964)) has been widely used to obtain solutions to random walk problems.

From Figure 3.2, assume that the probabilities that a particle is at points 1, 2, 3 or 4 are \( p_1 \), \( p_2 \), \( p_3 \) and \( p_4 \) respectively. Assume furthermore that the particle moves with equal probability (\( \frac{1}{4} \)) in all four directions. Therefore, the probability that the particle in the next step will be at point 0 is \( p_0 \) where

\[
p_0 = \frac{1}{4}(p_1 + p_2 + p_3 + p_4) \quad (3.8)
\]

This equation is similar to the standard equation obtained from the finite differences solution of Laplace equations (see, for example, Scott (1963), p. 135), i.e.,

\[
h_0 = \frac{1}{4}(h_1 + h_2 + h_3 + h_4) \quad (3.9)
\]

where \( h_0 \), \( h_1 \), . . . , \( h_4 \) are the values of the head at points 0, 1, 2, 3 and 4. The similarity of these two equations led many researchers (e.g., Brown (1956), A.H. Sheikh (1967) and Shih (1973)) to use the random walk process and the Monte Carlo technique to find a solution of the Laplace
equation at a certain point (or points).

3.3 Random Flow Model

3.3.1 Introduction

A random flow model is proposed in this investigation. The randomness is attributed to both the heterogeneous soil and to the flow of fluid in that soil. The proposed model is based generally on two main concepts. The first concept is the Random Walk process that was explained in Section 3.2. The second concept is that of statistical mechanics.

One of the classical problems in statistical mechanics is the kinetic theory of gases. The number of molecules in a certain volume of gas is very large. The laws of collision and the equations of motion of individual molecules are known but it is practically impossible to find a solution for the whole volume of gas with such a very large number of molecules. Thus, statistical mechanics was invented and the deterministic calculations of the path of each molecule were replaced by consideration of the probable behavior of a large number of molecules. The probability refers to our incomplete knowledge about the paths of the molecules.

The application of the statistical mechanics approach to seepage through porous media, as outlined by Scheidegger (1974), consists of three steps:
(a) First choose an ensemble of states. This consists of all those states that cannot be distinguished macroscopically because of one's ignorance of the microscopic details.

(b) Define the dynamic equations of motion on the microscopic scale for transition between states in the ensemble.

(c) Calculate the resulting observable quantities and analyze them statistically.

At a certain time, the whole system is in a certain state, defined by pertinent values of its characteristic variables. Instead of the system, one now considers the whole ensemble of states and then the expected value of an observable quantity in the system is the average of that quantity over all states of the ensemble. Often in time-stationary processes, one can make the hypothesis that a particular system will, in time, pass through all the states that are possible at any one given time. Then the ensemble averages and the averages of observables can be interchanged: this is the famous "Ergodic" hypothesis.

Scheidegger (1974) applied the statistical mechanics approach to flow through homogeneous porous media. He considered the medium to consist of particles with pores between them. The pores are approximated by pipes and thus, the ensemble of states consists of all those pipes that are
randomly distributed. The flow of fluid through the pores was analyzed and the flow through the whole medium was then predicted.

The model proposed in the present investigation uses the concept of the Random Walk process to define the flow path of a hypothetical flow unit. This flow unit is assumed not to subdivide along a flow path. The flow path is assumed to take place in steps. The flow begins randomly at boundary A and terminates at boundary B (for example, see Figure 3.3). In each step, the flow is governed by Darcy's law. The direction that the flow path follows in every step depends on the soil type encountered which is considered probabilistic. The rules controlling the flow path will be given in detail in the next section.

The ensemble of states consists of all the flow paths that might take place in the soil profile. The equations of motion include Darcy's law and the continuity condition that are both followed in each step of the path. The observable quantities to be computed are the details of every path and the average velocity of the flow unit. Therefore, by executing a large number of flow paths, we are approximating the flow condition in the flow region. The details of the computational procedures will be given in the following sections.

Although this application of the statistical mechanics approach to flow in heterogeneous soils is basically similar to Scheidegger's application, differences do exist between
Figure 3.3 Details of A Flow Path
the two. These differences arise because the assumptions concerning the states at the micro-level are not the same. Scheidegger considered flow through pores at the micro-level and used that to predict flow through a homogeneous porous medium. In this investigation, it is assumed that the flow path takes place in steps. In each step, the flow is Darcian and passes through one homogeneous soil block. The types of soils in the blocks are governed by a certain stochastic model. Thus, the flows in many flow paths are used to predict the total flow in the heterogeneous soil deposit.

3.3.2 Rules for the Flow Path

The model proposed applies to heterogeneous soils. The soil deposit is composed of two soil types. One type is in the form of inclusions that are scattered in the second type, the medium. The permeability of the inclusions is much greater than that of the medium. For example, for the varved clays consisting of silt inclusions in a clay medium, the permeability of silt is about 200 times that of clay.

The flow path is considered to take place in steps. It is assumed that the flow path starts at a point chosen randomly on boundary A and terminates when it reaches a point on exit-boundary B (see Figure 3.3). Boundaries A and B are equipotential lines and are both vertical in this problem. To make computations manageable, the flow path is
assumed to take place in steps from one nodal point to
another on a rectangular mesh. The distances between nodal
points on the mesh are chosen to be equal to the length of
steps in the index spaces of the soil transition matrices.

The rules governing the procession of a flow path
will be given here and they are explained in Figures 3.3 and
3.4. For every individual step in the flow path, the direc-
tion that the path takes depends on the soil types at the
adjacent nodal points. This is based on the understanding
that the flow path will follow the line of 'least resistance.'
Thus, the flow path would rather go horizontally and into an
inclusion if possible. Note that, due to the continuity
condition, a flow path that enters an inclusion must exit
from it and proceed until it reaches an exit boundary. In
the case of the soil types being the same along two or more
potential paths, then the flow path follows any one of them
with equal probability.

A rectangular mesh is drawn on the soil profile with
 spacings between nodal points Dx and Dy in the horizontal
and vertical directions respectively. The lengths Dx and
Dy are chosen so that no soil transition would pass undetec-
ted between any two adjacent nodal points. The starting
point for a flow path is chosen at random from the nodal
points that lie on boundary A. The random flow model assumes
that if the flow path is at point M, then in the next step,
the flow path direction is governed by the soil types at
Figure 3.4 Rules for the Steps of a Flow Path

(A)

(B)

(C)

(D)

- Inclusion
- Medium
neighboring points I, J or K (see Figure 3.5) and by the rules already mentioned. The next step is then executed and the information is stored. The flow path is continued until the exit boundary B is reached. A computer program was written to simulate the flow paths and carry out the analysis. The flow chart outlining this computer program is given in Figure 3.6.

3.3.3 Computation of Head and Flow

The head loss between boundaries A and B, \( H_A - H_B \), is assumed to dissipate uniformly along an equivalent length of the flow path, \( l_e \), such that

\[
 l_e = l_m + \frac{k_m}{k_i} l_i
\]  

(3.10)

where

- \( l_m \) = length of the flow path in the medium
- \( l_i \) = length of the flow path in the inclusion
- \( k_m \) = permeability of the medium
- \( k_i \) = permeability of the inclusion.

The average hydraulic gradient \( X_r \) for the \( r^{th} \) flow path is

\[
 X_r = \frac{H_A - H_B}{l_e}
\]  

(3.11)

and the average velocity for the \( r^{th} \) flow path, \( v_r \), is

\[
v_r = X_r k_m
\]  

(3.12)
Figure 3.5 One- and Two-Step Foresight Cases
Figure 3.6 Flow Chart for Computer Program to Apply the Random Flow Model to Deterministic Heterogeneous Soils
Note that if the whole flow path is in the medium, then the equivalent length, $l_e$, is

$$l_e = l_m$$

and the average hydraulic gradient, $X_{rm}$, for the flow path is

$$X_{rm} = \frac{H_A - H_B}{l_m}$$

The average velocity, $v_m$, is then

$$v_m = X_{rm} k_m = \frac{H_A - H_B}{l_m} \cdot k_m \quad (3.13)$$

On the other hand, if all the flow path is in the inclusion, then the equivalent length is

$$l_e = \frac{l_i k_m}{k_i}$$

and the average hydraulic gradient, $X_{ri}$, is

$$X_{ri} = \frac{H_A - H_B}{l_i} \cdot \frac{k_i}{k_m}$$

and the average velocity, $v_i$, is

$$v_i = X_{ri} k_m = \frac{H_A - H_B}{l_i} \cdot k_i \quad (3.14)$$

It is clear that each of Equations (3.13) and (3.14) is identical to Darcy's law for flow in a homogeneous material.
The head at any point $j$ in the $r^{th}$ flow path may then be computed if the details of the path itself are all known, i.e.,

$$H_j = H_A - X_r \left[ l_{jm}^j + \frac{k_m}{k_i} l_{jm}^j \right]$$ \hspace{1cm} (3.15)

where the index $j$ refers to conditions at point $j$.

If many flow paths pass through point $j$, then the probability distribution function for the head at point $j$ may be obtained. The average head at that point will then be

$$\bar{H}_j = \frac{1}{n} \sum_{p=1}^{n} \left[ H_A - X_p(l_{jm}^{jp} + \frac{k_m}{k_i} l_{jm}^{jp}) \right]$$ \hspace{1cm} (3.16)

where

- $n$ = number of flow paths passing through point $j$;
- $l_{jm}^{jp}$ = length in the medium for the $p^{th}$ path until point $j$;
- $l_{jm}^{jm}$ = length in the inclusion for the $p^{th}$ path until point $j$;
- $X_p$ = average hydraulic gradient for the $p^{th}$ path

To analyze the discharge rate along the $r^{th}$ flow path, we use Equation (3.13) to get

$$q_r = X_r \cdot k_m \cdot D_y$$ \hspace{1cm} (3.17)

where $D_y$ is the vertical distance between nodal points.

The discharge rate for the whole soil profile is then

$$Q_r = X_r \cdot k_m \cdot D_y \cdot N_y$$ \hspace{1cm} (3.18)
where $N_y$ = number of nodal points in boundary A.

Consider a hypothetical soil profile with the same size and boundary conditions as the one already analyzed and composed of a uniform homogeneous deposit. Assume, furthermore, that this hypothetical soil profile has the same discharge rate as the actual profile of the heterogeneous soil deposit. Then we may consider that the hypothetical uniform homogeneous soil has an equivalent permeability, $k^*$, such that

$$Q_T = k^* \frac{H_A - H_B}{L}$$  \hspace{1cm} (3.19)

Therefore, for every flow path we can compute an equivalent permeability, $k_T^*$. The probability distribution function of $k^*$ could thus be obtained after simulating a large number of flow paths. The average equivalent permeability, $\overline{k^*}$, is then

$$\overline{k^*} = \frac{1}{m} \sum_{i=1}^{m} k^*_i$$  \hspace{1cm} (3.20)

where $m$ = total number of simulated flow paths.

3.3.4 Flow in Deterministic Heterogeneous Soils

One way to test the validity of the random flow model is to compare its solution for the problem in a deterministic heterogeneous soil with other known solutions. A deterministic soil profile is one where the locations and sizes of all inclusions in the medium are completely known.
An illustration is given by a hypothetical deterministic varved clay profile shown in Figure 3.7. The soil deposit is composed of silt inclusions in a clay medium. Both the silt and clay components are isotropic and uniform, and their permeabilities are

\[
\begin{align*}
    k_{\text{silt}} &= 300 \times 10^{-8} \text{ in/sec.} \\
    k_{\text{clay}} &= 1.5 \times 10^{-8} \text{ in/sec.}
\end{align*}
\]

The random flow model was used to calculate the flow through such a deterministic soil profile. The solution arrived at was then compared with the solution obtained using the electrical analog method.

A flow path starts at random on the entrance boundary and terminates at the exit boundary. The information about each flow path is analyzed to obtain the equivalent permeability, \( k^* \), and the head at all nodal points of the flow path. A large number of flow paths are simulated and the average values of head at nodal points together with the probability distribution function of \( k^* \) are computed.

The same profile was analyzed by the electrical analog method. The values of head obtained from this method are plotted as a flow net in Figure 3.7. In the same figure, the results obtained from the random flow model are also shown. Figure 3.8 shows the results of the probability distribution function for \( k^* \) obtained from the random flow model. The values of the average equivalent permeability \( \overline{k^*} \)
Figure 3.7 Flow Net for Deterministic Heterogeneous Deposit
Equivalent Permeability for a Deterministic Heterogeneous Soil Deposit

\[ k^* \times 10^{-8} \text{ in/sec.} \]

- \( \bar{k}_{\text{rfm}}^* \): average equivalent permeability obtained from random flow model
- \( \bar{k}_{\text{analog}}^* \): average equivalent permeability obtained from the electrical analog method

**Figure 3.8** Equivalent Permeability for a Deterministic Heterogeneous Soil Deposit
obtained from the model was found to be about 65% of that obtained from the electrical analog method. Three more heterogeneous profiles were analyzed and the values of the average equivalent permeability, $k^*$, obtained from the model were 69%, 93% and 96% of those obtained from the electrical analog method.

In the preceding analysis, the direction a flow path takes was assumed to be governed totally by the soil types at the immediately adjacent points $I, J, K$ (see Figure 3.5). This we will call a one-step foresight case. If the direction of the flow path depends on soil types farther than one step away, then we have an $n$-step foresight case. In the two-step foresight case, the flow direction depends on soil types at nodal points $I, J, K, II, IJ, KK$ (see Figure 3.5). This condition was incorporated in the computer program using the random flow model. It was found that the two-step foresight case gave a value for $k^*$ that was about 90% of the value obtained from the electrical analog method.

We note that the $n$-step foresight case — when $n$ is very large — is actually a deterministic way to obtain the flow lines. However, as the one-step foresight case gave reasonable results and as it is more economical than the two-step foresight case, it will be used exclusively in the remainder of this investigation.

An investigation was also carried out to test the effect of decreasing the spacing $Dx$ of the mesh on $k^*$. The initial value used for $Dx$ was equal to the length of step in the index
space of the transition matrix for the inclusion thickness. When smaller values of \(Dx\) were used, it was found that the change occurring in \(K^*\) was insignificant. Hence, the original choice of \(Dx\) was confirmed.

It is thus concluded that the random flow model gives results that are not very different from the results obtained from other known solutions.

3.3.5 Flow in Stochastic Heterogeneous Soils

After analyzing the seepage in a deterministic heterogeneous soil deposit, the next step is to include the random nature of the soil in the analysis. Two methods are used to achieve this goal. The first method used is to simulate a two-dimensional soil profile using the stochastic model for the soil deposit as explained in Section 2.3.3. This simulated soil profile is one realization of the stochastic process describing the soil deposit. The seepage through the profile is solved as proposed in Section 3.3.4. By simulating many soil profiles and computing the seepage, we are taking the stochastic nature of the soil deposit into consideration. This method will be referred to as the simulated profile method. The flow chart outlining the computer program using this method is shown in Figure 3.9 and the program listing is given in Appendix G.

Thus, the simulated profile method is equivalent to obtaining many soil profiles and analyzing each one of them. The validity of the solution for a given soil profile had already
Figure 3.9 Flow Chart for Computer Program for the Simulated Profile Method
been demonstrated in Section 3.3.4. Statistical analysis of the results of the seepage in all the profiles may be added to obtain the probability distribution function, mean and variance for the equivalent permeability of the soil profile and for the head at any specific point on that profile.

If we, however, incorporate the random nature of the soil deposit in the individual steps of the flow path, then we are dealing with a random walk process. The flow path is started, randomly, from a nodal point at the entrance boundary and the soil types in the adjacent nodal points are simulated using the known soil transition matrices. The direction that the flow path takes depends on the soil types in the vicinity of the tip of the flow path. Then the next step in the flow path is executed according to the rules given in Section 3.3.2. This procedure is repeated until the flow path terminates at an exit boundary. Many flow paths are simulated and the results analyzed. This method will be referred to as the simulated flow path method. The flow chart outlining the computer program used for this method is shown in Figure 3.10 and the program listing is given in Appendix H.

The simulated flow path method is a random walk process that uses the statistical mechanics approach. By simulating a large number of flow paths and by invoking the ergodicity principle, we may interchange the average equivalent permeability of the flow paths with that of the soil profile.
Figure 3.10 Flow Chart for Computer Program for the Simulated Flow Path Method
The following sections describe the analysis of seepage in the heterogeneous soil deposit whose stratification was already modeled in Chapter 2. The analysis is carried out using both the simulated profile and simulated flow path methods, and the results of both methods are compared. The analysis is carried out to obtain the probability distribution function of the equivalent permeability, \( k^* \), and its average value, \( \overline{k^*} \). One example is provided for the determination of the probability distribution function for the head at a point.

3.4 Seepage in Varved Clays

3.4.1 General

The two-dimensional random flow model presented in Section 3.3 was used to analyze one-dimensional confined flow in the varved clay deposits that were modeled in Sections 2.4.4 and 2.4.5. The simulated profile and the simulated flow path methods were both used in the analysis.

Seepage was considered to occur through a rectangular soil profile with dimensions \( Ax \) and \( Ay \) (see Figure 3.11). The horizontal boundaries were considered impervious and the vertical boundaries are equipotentials with a head difference \( DH \) between them. A mesh with rectangular elements \( Dx \) by \( Dy \) was drawn on each profile. The lengths \( Dx \) and \( Dy \) were taken to be equal to the length of steps in the index
Figure 3.11 Mesh for the Random Flow Model
spaces of the Markov chains representing the vertical stratification and the variations in thickness of silt layers. Many different profiles with different dimensions were analyzed.

In the simulated profile method, 50 different soil profiles were simulated for each profile size. For each simulated profile, $5 N_y$ flow paths were simulated where $N_y$ is the number of nodal points on the entrance boundary. It was found that this gave a large enough statistical sample. This means that by simulating more profiles and/or flow paths, the results of the average equivalent permeability, $k^*$, did not change appreciably. In the simulated flow path method, 2000 flow paths were simulated for each profile size. This was also found to give a large enough statistical sample.

Both the silt and clay components of the varved clay were considered to be individually isotropic and uniform. The permeabilities of both components of the Toledo varved clay were determined by Chang (1976). He found that for confining pressures below the preconsolidation pressure, the permeabilities for the silt and clay were:

$$k_{\text{silt}} = 300.0 \times 10^{-8} \text{ in/sec.}$$
$$k_{\text{clay}} = 1.5 \times 10^{-8} \text{ in/sec.}$$

These values were used in the simulation analysis for seepage in both the Toledo and Cleveland varved clays.
Figure 3.12 Results of the Two-dimensional Random Flow Model for the Toledo Varved Clay
Figure 3.13 Results of the Two-dimensional Random Flow Model for the Cleveland Varved Clay
3.4.2 Analysis of Simulation Results

We first consider the values of $k^*$ obtained by the simulated profile method. These are given in Figures 3.12 and 3.13 for the Toledo and Cleveland varved clays respectively. It is clear that the variation in the dimension $A_y$ does not produce large variations in $k^*$. Hence, for values of $A_x$ greater than 40.0 inches, calculations were made only for $A_y = 5.0$ in. On the other hand, it is found that as the dimension $A_x$ increases, $k^*$ decreases to a limiting value.

The values of $k^*$ obtained by the simulated flow path method are then considered. These are also shown in Figures 3.12 and 3.13. Calculations by this method show that $k^*$ does not generally depend on the dimension $A_y$. Thus, results for soil profiles with the dimension $A_y = 5.0$ inches only are shown.

It is clear that the two methods gave results for $k^*$ that are nearly the same. The simulated profile method has already been shown to give solutions that are comparable with solutions obtained by the electrical analog method. Hence, the similarity between the results of the two simulation methods lends more credibility to the simulated flow path method. While the simulated profile method is convenient to use in the seepage analysis of small-size profiles, the simulated flow path method is more economical with respect to computer time where profiles of large dimensions
must be analyzed. Thus, the latter method was used to develop a three-dimensional random flow model as described in Chapter 4.

The calculated trend of decreasing $k^*$ with increasing $Ax$ is the same for the Toledo and Cleveland deposits. The values of $k^*$ obtained for the Toledo varved clay were, however, larger than those obtained for the Cleveland varved clay. This may be attributed to the higher overall percentage of silt in the former deposit.

The general trend of decreasing $k^*$ with increasing $Ax$ may be due to the fact that as $Ax$ increases, more discontinuities in the silt layers are expected to exist. This would cause a reduction in the equivalent permeability $k^*$ for a flow path. The ratio of the average equivalent permeability, $\bar{k}^*$, for a small profile ($Ax = 3.0$ inches) to that for a large profile ($Ax = 40.0$ inches) is between 2 and 4. This may be compared with the permeability ratio, $M$, defined as:

$$M = \frac{k_h}{k_v} \quad (3.21)$$

where

- $k_h =$ permeability of varved clay in the horizontal direction; and
- $k_v =$ permeability of varved clay in the vertical direction.

If we assume that $k_v$ for varved clay is very much less sensitive to changes in the dimension $Ax$ than is $k_h$, then the change in $\bar{k}^*$ with $Ax$ may be compared with the change in $M$ with $Ax$. 
Wu et al. (1978) found that, for the Toledo varved clay, the values of M obtained from laboratory samples (Ax = 3.0 inches) were larger than those obtained from in-situ tests (Ax = 40.0 inches). Both laboratory and in-situ tests represent radial flow which is assumed to be close to the one-dimensional flow analyzed. Thus, the findings of Wu et al. (1978) are in general agreement with the results obtained in this investigation.

The probability distribution function of the equivalent permeability k* for soil profiles with Ay = 5.0 in. and different values of Ax are presented in Figures 3.14 and 3.15. These were obtained using the simulated profile and simulated flow path methods respectively for the Toledo varved clay. It should be noted at the outset that the values of k* obtained from the simulated profile method are for individual profiles with many flow paths generated for each profile. The simulated flow path method, on the other hand, produces values of k* for individual flow paths. Hence, if we lump a group of fifty flow paths together and find their average equivalent permeability and then find the average permeability for all the groups of flow paths, then we obtain the probability distribution functions shown in Figure 3.16. Although this averaging procedure does not take the vertical correlation between flow paths into consideration, it produces distribution functions that are not very different from those
Figure 3.14 Probability Distribution for $k^*$ of the Toledo Varved Clay (The Simulated Soil Profile Method)
Figure 3.15  Probability Distribution Function for $k^*$ of the Toledo Varved Clay (The Simulated Flow Path Method)
Figure 3.16  Probability Distribution Function for $k^*$ of the Toledo Varved Clay (The Simulated Flow Path Method with the Averaging Procedure)
shown in Figure 3.14. The general trend of decreasing variance of $k^*$ with increasing $Ax$ is apparent in Figures 3.14 and 3.16.

The results mentioned above constitute a major product of this investigation, namely, the statistical characterization of the permeability of the whole deposit. The permeability of the varved clay deposit may be represented by a random variable with a probability distribution function. The mean and variance of $k^*$ could then be used in the analysis of flow in the soil deposit under consideration.

Another way to statistically characterize the permeability of the deposit is to obtain the auto-correlation function (ACF) of the permeability. This was achieved by simulating a soil profile, $nb \times y$. This profile was then divided into $n$ sections, each with dimensions $b \times y$. The simulated profile method was then used to compute $k^*$ for each section. The values of $k^*$ were then plotted as realizations of a random process in the upper parts of Figures 3.17 and 3.18 for both the Toledo and Cleveland varved clays. The ACF's were then computed and plotted in the lower parts of Figures 3.17 and 3.18. These auto-correlation functions may then be used to study seepage by the spectral analysis method (Gelhar (1976)).

The results of the simulated profile method may be used to obtain the probability distribution function for the head at any point in the profile by means of Equation
Figure 3.17 Auto-Correlation Function of $k^*$ for the Toledo Varved Clay Deposit
Figure 3.18 Auto-Correlation Function of $k^*$ for the Cleveland Varved Clay Deposit
(3.15). As an example, we consider a central point P in a Toledo varved clay. The probability distribution function and the cumulative distribution function for the head at point P are shown in Figure 3.19. It should be noted that the probability distribution function may vary with the location of point P. This approach may help in estimating the reliability of piezometric readings.

3.5 Seepage in a Glacial Outwash Deposit

The random flow model was used to study seepage through the outwash deposit modeled in Section 2.5. For the analysis of flow, the four soil types considered in the modeling process were combined to form a medium and an inclusion. The fine sand-silt and the medium sand layers were considered as the medium. The inclusion consisted of the medium-coarse sand and the coarse sand-gravel layers. This modification was done to simplify the seepage analysis for the glacial outwash deposit.

The data on the permeability of the different soils were not available. Hence, the permeability was estimated by the relationship:

\[ k = c D_{10}^2 \]  

(3.22)

where

- \( k \) = coefficient of permeability in cm/sec.
- \( c \) = a coefficient whose value lies between 100 and 150 (Terzaghi and Peck (1967))
- \( D_{10} \) = effective soil diameter in centimeters.
Figure 3.19 Probability Distribution Function for Head at Point P in a Toledo Varved Clay Profile
The coefficient of the permeability of the medium, \( k_{\text{med}} \), was taken as:

\[
k_{\text{med}} = k
\]

(3.23)

and that of the inclusion, \( k_{\text{incl}} \), is then:

\[
k_{\text{incl}} = k \left( \frac{D_{10 \text{ incl}}}{D_{10 \text{ med}}} \right)^2
\]

(3.24)

Wu (1957) obtained grain size distribution curves for the different soil types in the glacial outwash deposit. The inclusion was considered to have the same gradation as the coarse sand-gravel and hence,

\[
D_{10 \text{ incl}} = 0.044 \text{ cm}
\]

The medium, on the other hand, was assumed to have the same gradation as the fine sand-silt and hence:

\[
D_{10 \text{ med}} = 0.015 \text{ cm}
\]

Hence, from Equation (3.24),

\[
k_{\text{incl}} = k \left( \frac{0.044}{0.015} \right)^2 = 10k
\]

(3.25)

Soil profiles with different dimensions were studied. Both the simulated profile and the simulated flow path
methods were used. The results of the average equivalent permeability $\bar{k}^*$ are shown in Figure 3.20. The probability distribution functions of $k^*$ were computed for soil profiles with the dimension $Ay = 20.0$ ft. and different $Ax$ values. Figure 3.21 shows the probability distribution functions obtained from the simulated profile method. The averaging procedure presented in Section 3.4.2 was used to obtain the probability distribution function for $k^*$ from the simulated flow path method and 20 flow paths were lumped together in each group. The results obtained are given in Figure 3.22. Similar trends of decrease in variance as $Ax$ increases are shown in both figures.

The auto-correlation function of the permeability of the glacial outwash deposit was then computed. This was done following the same procedure used in Section 3.4.2 for the varved clays. The values of $k^*$ were plotted as realizations of a random process in the upper part of Figure 3.23. The ACF was then computed and plotted in the lower part of the same figure.

Figure 3.24 shows the probability distribution function and the cumulative distribution function for the head at a central point $P$ in a soil profile.

It is clear that, although modeling of the glacial outwash deposit was found to produce unsatisfactory simulated profiles, both the simulated profile and the simulated flow path methods gave similar results for $\bar{k}^*$. However, to achieve
Figure 3.20 Results of the Two-dimensional Flow Model for the Glacial Outwash Deposit
Figure 3.21 Probability Distribution Function for k* of the Glacial Outwash Deposit (The Simulated Profile Method)
Figure 3.22 Probability Distribution Function for $k^*$ of the Glacial Outwash Deposit (Simulated Flow Path Method with Averaging Procedure)
Figure 3.23 Auto-Correlation Function of $k^*$ for the Glacial Outwash Deposit
Figure 3.24 Probability Distribution Function for Head at Point P in a Glacial Outwash Soil Profile
a more reliable statistical characterization of the permeability of the glacial outwash, more information on the soil stratification and the permeability of the different soil layers is needed.
4.1 General

The random flow model presented in Chapter 3 is extended to study the effect of variations in inclusion thickness in the direction perpendicular to the vertical plane. This is achieved by a three-dimensional random flow model. An illustrative example for the analysis of seepage in the Toledo varved clay is given in this chapter.

4.2 Three-Dimensional Soil Simulation

The simulation of the stratification in a three-dimensional soil block is based on the assumption that the variations in the thickness of the inclusions in the horizontal plane are independent of direction. This means that the variations in the thickness of inclusions along any two horizontal and orthogonal directions can be represented by the same Markov chain. Hence, to simulate the stratification of a three-dimensional soil block, the transition matrices for the vertical stratification and for the variations in inclusion thickness in the horizontal direction given in Chapter 2 may be used.
The procedure used to simulate the stratification in a three-dimensional soil block is similar to the two-dimensional soil profile simulation. The soil type at point M (see Figure 4.1) is chosen at random, if M lies on the entrance boundary. If M is at an interior point, it is already known from the preceding step. The thickness, $T_M$, of an inclusion at point M is simulated using the transition matrix for vertical stratification. The thickness $T_1$ of the inclusion at Section 1 is then simulated using the transition matrix for the variation in thickness of inclusions. This simulated inclusion thickness $T_1$ is then used to simulate the inclusion thicknesses, $T_2$ and $T_3$, at Sections 2 and 3 respectively. Hence, the soil types at the nine neighboring nodal points II, JJ, KK, II2, JJ2, KK2, II3, JJ3 and KK3 are all known and are used to simulate the next step in the flow path.

4.3 Three-Dimensional Flow Path Simulation

The simulated flow path method is used in the three-dimensional random flow model and the case of a single-step foresight is also followed. The direction that a flow path takes in a certain step depends on the soil types at the neighboring points. As in the case of the two-dimensional random flow model, the flow path is assumed to follow the line of "least resistance." If two or more potential flow paths have the same soil conditions, then the direction that the flow path takes is chosen at random with equal probability.
Figure 4.1 Details of One Step in the Three-dimensional Random Flow Model
The results of the simulation analysis for the Toledo varved clay are given in Figures 4.2 and 4.3. Soil blocks with different sizes were analyzed. The dimension Ay was kept constant while the other dimensions, Ax and Az, were varied. The number of flow paths simulated was taken as 2000 which gave a "large enough" statistical sample.

Figure 4.2 shows the values of the average equivalent permeability, $k^*$, for many soil blocks with different sizes. It is clear that except for very small values of the dimension Az, the two- and three-dimensional random flow models do not produce similar trends. Figure 4.3 gives the probability distribution functions of $k^*$ for four soil blocks with different dimensions. This was obtained by the averaging procedure used in Section 3.4.2.

4.4 Analysis of Results

The differences in the characteristics of the equivalent permeabilities for the cases represented by the two- and three-dimensional random flow models have been mentioned before by Gelhar (1976). He, however, studied the differences in the auto-covariance functions for the two cases.

The average equivalent permeability, $k^*$, obtained from the three-dimensional random flow model is found to be always larger than that obtained from the two-dimensional case. It is clear from Figure 4.2 that as the dimension Az decreases, the values of $k^*$ approach the two-dimensional case. For
Figure 4.2 Results of Three-dimensional Random Flow Model on the Toledo Varved Clay
Figure 4.3  Probability Distribution Function for $k^*$ of the Toledo Varved Clay
large Az dimensions, the values of $\overline{k^*}$ increase as Ax increases until they reach a constant value. This trend may be due to the fact that a flow path would avoid the clay and pass to silt inclusions that may exist on the horizontal plane and that may not have been available in the case of the two-dimensional soil simulation. As Az decreases, however, the effect of stratification in the z-direction becomes less important and the flow approaches the case represented by the two-dimensional random flow model.

The probability distribution functions shown in Figure 4.3 show that as the dimension Ax increases, the variance of $k^*$ decreases. The variance for the three-dimensional case was also found to be generally smaller than that for the two-dimensional case. Hence, for the cases where the flow may be represented by a three-dimensional random flow model, namely when the effects of stratification in the z-direction are significant, the probability distribution function shown in Figure 4.3(d) may be used in the analysis of seepage in the whole deposit.
CHAPTER 5
SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

A stochastic model was proposed for the stratification of heterogeneous soil deposits. Markov chains were used in the modeling procedure and case-studies were presented for different soil deposits.

A two-dimensional random flow model was then developed for the analysis of seepage in heterogeneous soils. The stratification in the vertical plane was taken into consideration and it was assumed not to change in the direction perpendicular to that plane. The flow model was thus used to study the confined flow in two-dimensional soil profiles. Seepage analyses were then carried out for the soil deposits whose stratifications were already modeled. The permeability of the heterogeneous soil deposit was represented by a random variable $k^*$ whose probability distribution function and autocorrelation function were determined. It was found that the values of the average permeability $\overline{k^*}$ obtained from the two-dimensional random flow model were similar to experimental values obtained for the radial flow case.

A three-dimensional random flow model was then used to analyze seepage in soil deposits where the variations in inclusion-thicknesses in the direction perpendicular to the
vertical plane were taken into consideration. This model was hence used to study the confined flow in three-dimensional soil blocks. The values of $k^*$ obtained by the use of this model were found to be larger than the values obtained from the two-dimensional flow model. The auto-correlation function of $k^*$ for three-dimensional soil blocks needs to be estimated. This can be achieved by extending the simulated profile method to the three-dimensional random flow model.

A detailed investigation of the probability distribution function of the head at points in a soil profile (or a soil block) is needed. Research in this direction can help in understanding the reliability of field piezometric readings.

The statistical characterization of the permeability of a heterogeneous soil deposit can be used in the reliability analysis of seepage in that deposit. In such a study, the probability distribution function and/or the auto-correlation function of $k^*$ may be used.

The general methodology presented in this investigation may be used to study different soil behavior problems for heterogeneous soil deposits. The stochastic model for the stratification of a deposit is used to generate realizations of that deposit. These realizations are then analyzed to obtain the statistical characterization of the soil property under investigation.
REFERENCES


APPENDIX A

SOIL SECTIONS FOR THE CLEVELAND
VARVED CLAY DEPOSIT
Figure A.1  Section No. 2, Cleveland Varved Clay
Figure A.2  Section No. 3, Cleveland Varved Clay
Figure A.3  Section No. 4, Cleveland Varved Clay
Figure A.4  Section No. 5, Cleveland Varved Clay
Figure A.5  Section No. 7, Cleveland Varved Clay
Figure A.6  Section No. 8, Cleveland Varved Clay
Figure A.7  Section No. 9, Cleveland Varved Clay
APPENDIX B

HYPOTHESIS TESTING FOR THE VERTICAL STRATIFICATION
OF THE CLEVELAND VARVED CLAY

B.1 Testing for the Markov Property

Null Hypothesis $H_0$: States are independent

Alternative Hypothesis $H_1$: States are not independent

According to the procedure explained in Section 2.2.3.1,

$$-2 \log_e \lambda = 2 \sum_{i,j=1}^{m} n_{ij} \log_e \left( \frac{p_{ij}}{p_j} \right) \quad i,j = 1,2$$

$$= 1144.62$$

d.f. $= (2-1)^2 = 1$

$$\chi^2_{a=0.05} (1) = 3.84 \ll 1144.62$$

:. Reject $H_0$ and accept $H_1$
<table>
<thead>
<tr>
<th>Section #2</th>
<th>Section #3</th>
<th>Section #4</th>
</tr>
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<td><strong>N(I,J)</strong></td>
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<th>Transition Matrices for all Sections</th>
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**N(I,J): Tally matrix**

**P(I,J): Transition probability matrix**
B.2 Testing for Stationarity

Analysis was carried out for Section #2 only. The subintervals are the eight vertical columns of the mesh drawn on the section.

Null hypothesis $H_0$: $p_{ij}(t) = p_{ij}$ \quad $t=1,2,\ldots,8$

Alternate hypothesis $H_1$: $p_{ij}(t) \neq p_{ij}$

According to the procedure explained in Section 2.2.3.2, the statistic is:

$$-2 \log_e \lambda = 2 \sum_{t=1}^{8} \sum_{i,j=1}^{2} n_{ij}(t) \log_e \left( \frac{p_{ij}(t)}{p_{ij}} \right)$$

$$= 14.6$$

d.f. \quad = (8-1)(2(2-1)) \quad = 14

$$\chi^2_{14, \alpha=0.05} = 23.7 > 14.6$$

\therefore Accept $H_0$
TABLE B.2  TRANSITION MATRICES FOR THE VERTICAL STRATIFICATION IN SECTION #2

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<td>( N(I,J) )</td>
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<td>0 50 11</td>
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<tr>
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<td>1 0.33 0.67</td>
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<tr>
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<td>( N(I,J) )</td>
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\( N(I,J) \): Tally matrix

\( P(I,J) \): Transition probability matrix
C.1 **Testing for the Markov Property**

\[ H_0: \text{States are independent} \]

\[ H_1: \text{States are not independent} \]

\[
-2 \log_e \lambda = 2 \sum_{i,j=1}^{m} n_{ij} \log_e \left( \frac{p_{ij}}{p_j} \right), \quad i,j=0,1,2,\ldots,5
\]

\[
= 556.9
\]

\[ \text{d.f.} = (6 - 1)^2 = 25 \]

\[ \chi^2_{0.5} (25) = 37.7 \ll 556.9 \]

\[ \therefore \text{Reject } H_0 \text{ and accept } H_1 \]
C.2 Testing for Stationarity

Case 1  Consider the eight sections as 8 subintervals

\[ H_0: \ p_{ij}(t) = p_{ij} \quad t = 2, 3, \ldots, 8 \]

\[ H_1: \ p_{ij}(t) \neq p_{ij} \]

\[-2 \log_e \lambda = 2 \sum_{t=2}^{9} \sum_{i,j=0}^{5} n_{ij} \log \left( \frac{p_{ij}(t)}{p_{ij}} \right)\]

\[ = 117.11 \]

\[ \text{d.f.} = (8 - 1)(6)(6 - 1) = 210 \]

\[ \chi^2 \]

\[ = 0.05(210) = 251.54 > 117.11 \]

\[ \therefore \ \text{Accept} \ H_0 \]

Case 2  Consider three subintervals only, i.e.,

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<tr>
<td>3</td>
<td>7, 8, 9</td>
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\[ H_0: \ p_{ij}(t) = p_{ij} \quad t = 1, 2, 3 \]

\[ H_1: \ p_{ij}(t) \neq p_{ij} \]

\[-2 \log_e \lambda = 2 \sum_{t=1}^{3} \sum_{i,j=0}^{5} n_{ij}(t) \log \left( \frac{p_{ij}(t)}{p_{ij}} \right)\]

\[ = 55.27 \]

\[ \text{d.f.} = (3 - 1)(6)(6 - 1) = 60 \]

\[ \chi^2 \]

\[ = 0.5(60) = 82.82 > 55.27 \]

\[ \therefore \ \text{Accept} \ H_0 \]
# Table C.1: Transition Matrices for Variation in the Thickness of Silt Layers of the Cleveland Varved Clay

### Section Number 2

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<td>0 0 5 1 0 0</td>
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<td>0 0 0 0 0 0 0</td>
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TABLE C.1  Continued  TRANSITION MATRICES FOR VARIATION IN
THICKNESS OF SILT LAYERS OF THE CLEVELAND VARVED
CLAY

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<td>3 0 0 1 0 0</td>
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SECTION NUMBER 4

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SECTION NUMBER 5
TABLE C.1  Continued TRANSITION MATRICES FOR VARIATION IN THE THICKNESS OF SILT LAYERS OF THE CLEVELAND VARVED CLAY

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<th>P(I,J)</th>
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SECTION NUMBER 6

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<td>0.17</td>
<td>0.0</td>
<td>0.83</td>
</tr>
</tbody>
</table>

**SECTION NUMBER 9**

N(I,J) = Tally matrix  
P(I,J) = Transition probability matrix
### TABLE C.2 TRANSITION MATRICES FOR ALL THE SECTIONS

$$N(I,J) \quad P(I,J)$$

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>14</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>124</td>
<td>4</td>
<td>4</td>
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<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>9</td>
<td>26</td>
<td>7</td>
<td>0</td>
<td>0</td>
</tr>
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<td>3</td>
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<td>4</td>
<td>5</td>
<td>30</td>
<td>4</td>
<td>2</td>
</tr>
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<td>4</td>
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<td>0</td>
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<td>3</td>
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<td>0</td>
<td>0</td>
<td>3</td>
<td>4</td>
<td>59</td>
</tr>
</tbody>
</table>

<table>
<thead>
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<th></th>
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<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.74</td>
<td>0.21</td>
<td>0.05</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>1</td>
<td>0.05</td>
<td>0.89</td>
<td>0.03</td>
<td>0.03</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>0.05</td>
<td>0.2</td>
<td>0.59</td>
<td>0.16</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>3</td>
<td>0.0</td>
<td>0.09</td>
<td>0.11</td>
<td>0.67</td>
<td>0.09</td>
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</tr>
<tr>
<td>4</td>
<td>0.0</td>
<td>0.0</td>
<td>0.07</td>
<td>0.20</td>
<td>0.33</td>
<td>0.4</td>
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<tr>
<td>5</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.05</td>
<td>0.06</td>
<td>0.89</td>
</tr>
</tbody>
</table>
APPENDIX D

HYPOTHESIS TESTING FOR THE SIMULATED VARVED CLAY PROFILES

D.1 Cleveland Varved Clay

Consider that the actual soil profiles form one sample (S1) and the simulated profile another sample (S2).

\( H_0: \) S1 and S2 are from the same first order Markov chain

\( H_1: \) S1 and S2 are not from the same first order Markov chain

As explained in Section 2.2.3.3:

\[
\chi^2_i = \sum_{i=1}^{2} c_i \left[ \hat{p}_{ij}^{(1)} - \hat{p}_{ij}^{(2)} \right]^2 / \hat{p}_{ij}^{(0)} \quad i,j=1,2
\]

and

\[
\begin{pmatrix}
\chi_1^2 \\
\chi_2^2
\end{pmatrix} =
\begin{pmatrix}
0.00375 \\
0.6853
\end{pmatrix}
\]

\[
\begin{pmatrix}
\chi_1^2 \\
\chi_2^2
\end{pmatrix} =
\begin{pmatrix}
0.00375 \\
0.6853
\end{pmatrix}
\]

i.e., \( \chi^2 = \chi_1^2 + \chi_2^2 = 0.689 \)

d.f. = \( 2^1 (2-1) = 2 \)

\[ \chi^2 (2) = 5.99 > 0.689 \]

\( \alpha=0.05 \)

\( \therefore \) Accept \( H_0 \)
D.2 Toledo Varved Clay

$H_0$: S1 and S2 are from the same first order Markov chain

$H_1$: S1 and S2 are not from the same first order Markov chain

\[
\begin{pmatrix}
\chi_1^2 \\
\chi_2^2
\end{pmatrix} =
\begin{pmatrix}
0.681 \\
1.370
\end{pmatrix}
\]

\[
\chi^2 = \chi_1^2 + \chi_2^2 = 2.051
\]

d.f. = $2(2-1) = 2$

$\chi^2 (2) = 5.99 > 2.051$

$\alpha=0.05$

$\therefore$ Accept $H_0$
APPENDIX E

SUBSOIL PROFILES FOR THE GLACIAL OUTWASH DEPOSIT
Figure E.1 Subsoil Profile for Glacial Outwash Deposit, Site A (from Wu (1957))
Figure E.2 Subsoil Profile for Glacial Outwash Deposit, Site C (from Wu (1957))
Figure E.3 Subsoil Profile for Glacial Outwash Deposit, Site D (from Wu (1957))
F.1 Testing for the Markov Property

\( H_0: \) States are independent

\( H_1: \) States are not independent

\[
-2 \log_e \lambda = 2 \sum_{i,j=0}^{m} n_{ij} \log_e \left( \frac{p_{ij}}{p_j} \right)
\]

\[
= 572.16
\]

d.f. \( = (4 - 1)(4 - 1) = 9 \)

\[
\chi^2(9) = 16.9 << 572.16
\]

\( \alpha = 0.05 \)

\( \therefore \) Reject \( H_0 \) and accept \( H_1 \)
TABLE F.1  TRANSITION MATRICES FOR VERTICAL STRATIFICATION
OF GLACIAL OUTWASH DEPOSIT

<table>
<thead>
<tr>
<th>N(I,J)</th>
<th>P(I,J)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>1  56 2 3 4</td>
<td>1  0.86 0.03 0.05 0.06</td>
</tr>
<tr>
<td>2  5 21 0 27</td>
<td>2  0.19 0.78 0.0 0.03</td>
</tr>
<tr>
<td>3  3 0 1 0</td>
<td>3  0.75 0.0 0.25 0.0</td>
</tr>
<tr>
<td>4  0 0 0 7</td>
<td>4  0.125 0.0 0.0 0.875</td>
</tr>
</tbody>
</table>

SITE A

<table>
<thead>
<tr>
<th>N(I,J)</th>
<th>P(I,J)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>1  23 4 2 2</td>
<td>1  0.74 0.13 0.06 0.07</td>
</tr>
<tr>
<td>2  2 30 4 2</td>
<td>2  0.05 0.79 0.11 0.05</td>
</tr>
<tr>
<td>3  4 0 18 4</td>
<td>3  0.15 0.0 0.70 0.15</td>
</tr>
<tr>
<td>4  2 0 6 18</td>
<td>4  0.08 0.0 0.23 0.69</td>
</tr>
</tbody>
</table>

SITE B
TABLE F.1 Continued TRANSITION MATRICES FOR VERTICAL STRATIFICATION OF THE GLACIAL OUTWASH DEPOSIT

<table>
<thead>
<tr>
<th>N(I,J)</th>
<th>P(I,J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 37 15 0</td>
<td></td>
</tr>
<tr>
<td>2 18 5 0</td>
<td></td>
</tr>
<tr>
<td>3 7 49 16</td>
<td></td>
</tr>
<tr>
<td>4 3 13 15</td>
<td></td>
</tr>
<tr>
<td>1 0.86 0.02 0.12 0.0</td>
<td></td>
</tr>
<tr>
<td>2 0.04 0.75 0.21 0.0</td>
<td></td>
</tr>
<tr>
<td>3 0.10 0.0 0.68 0.22</td>
<td></td>
</tr>
<tr>
<td>4 0.10 0.0 0.42 0.48</td>
<td></td>
</tr>
</tbody>
</table>

SITE C

<table>
<thead>
<tr>
<th>N(I,J)</th>
<th>P(I,J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 38 14 0 5</td>
<td></td>
</tr>
<tr>
<td>2 22 62 0 1</td>
<td></td>
</tr>
<tr>
<td>3 1 0 2 0</td>
<td></td>
</tr>
<tr>
<td>4 3 2 1 5</td>
<td></td>
</tr>
<tr>
<td>1 0.67 0.24 0.0 0.09</td>
<td></td>
</tr>
<tr>
<td>2 0.26 0.73 0.0 0.01</td>
<td></td>
</tr>
<tr>
<td>3 0.33 0.0 0.67 0.0</td>
<td></td>
</tr>
<tr>
<td>4 0.27 0.18 0.09 0.16</td>
<td></td>
</tr>
</tbody>
</table>

SITE D

N(I,J) = Tally matrix
P(I,J) = Transition probability matrix
TABLE F.2 TRANSITION MATRICES FOR ALL THE PROFILES

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>N(I,J)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>P(I,J)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>154</td>
<td>21</td>
<td>10</td>
<td>11</td>
<td>1</td>
<td>0.79</td>
<td>0.11</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>132</td>
<td>9</td>
<td>4</td>
<td>2</td>
<td>0.17</td>
<td>0.75</td>
<td>0.05</td>
<td>0.03</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>0</td>
<td>70</td>
<td>20</td>
<td>3</td>
<td>0.14</td>
<td>0</td>
<td>0.67</td>
<td>0.19</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>2</td>
<td>20</td>
<td>45</td>
<td>4</td>
<td>0.12</td>
<td>0.03</td>
<td>0.26</td>
<td>0.59</td>
</tr>
</tbody>
</table>
F.2 Testing for Stationarity

Stationarity was tested for Site A. Four vertical sections of the profile were considered as the subintervals.

\[ H_0: \quad p_{ij}(t) = p_{ij} \quad t = 1, 4 \]
\[ H_1: \quad p_{ij}(t) \neq p_{ij} \quad i, j = 0, 1, 2, 3 \]

\[-2 \log_e \lambda = 2 \sum_{t=1}^{4} \sum_{i,j=0}^{3} n_{ij} \log_e \left( \frac{p_{ij}(t)}{p_{ij}} \right) \]
\[ = 17.372 \]

\[ \text{d.f.} = 3 \times 4(4 - 1) = 36 \]

\[ \chi^2(36) = 53.94 > 17.372 \quad \alpha = 0.05 \]

\[ . . . \text{Accept } H_0 \]
TABLE F.3  TRANSITION MATRICES FOR VERTICAL STRATIFICATION IN FOUR VERTICAL SECTIONS AT SITE A

<table>
<thead>
<tr>
<th>Vertical Section 1</th>
<th>N(I,J)</th>
<th>P(I,J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.25</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0.0</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Vertical Section 2</th>
<th>N(I,J)</th>
<th>P(I,J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16</td>
<td>0.89</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1.0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0.0</td>
</tr>
</tbody>
</table>
TABLE F.3 Concluded TRANSITION MATRICES FOR VERTICAL STRATIFICATION IN FOUR VERTICAL SECTIONS AT SITE A

<table>
<thead>
<tr>
<th>N(I, J)</th>
<th>P(I, J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2 3 4</td>
<td>1 2 3 4</td>
</tr>
<tr>
<td>1 15 0 1 1</td>
<td>1 0.88 0.0 0.06 0.06</td>
</tr>
<tr>
<td>2 1 3 0 0</td>
<td>2 0.25 0.75 0.0 0.0</td>
</tr>
<tr>
<td>3 1 0 0 0</td>
<td>3 1.0 0.0 0.0 0.0</td>
</tr>
<tr>
<td>4 0 0 0 4</td>
<td>4 0.0 0.0 0.0 1.0</td>
</tr>
</tbody>
</table>

VERTICAL SECTION 3

<table>
<thead>
<tr>
<th>N(I, J)</th>
<th>P(I, J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2 3 4</td>
<td>1 2 3 4</td>
</tr>
<tr>
<td>1 1 1 1 0</td>
<td>1 0.85 0.075 0.075 0.0</td>
</tr>
<tr>
<td>2 1 8 0 1</td>
<td>2 0.1 0.8 0.0 0.1</td>
</tr>
<tr>
<td>3 1 0 1 0</td>
<td>3 0.5 0.0 0.5 0.0</td>
</tr>
<tr>
<td>4 0 0 0 1</td>
<td>4 0.0 0.0 0.0 1.0</td>
</tr>
</tbody>
</table>

VERTICAL SECTION 4

N(I, J) = Tally matrix  
P(I, J) = Transition probability matrix
APPENDIX G

LISTING OF COMPUTER PROGRAM FOR
THE SIMULATED PROFILE METHOD
### G.1 Description of Important Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AKA</td>
<td>Equivalent permeability for a flow path</td>
</tr>
<tr>
<td>AK</td>
<td>Permeability of medium</td>
</tr>
<tr>
<td>Ax</td>
<td>Horizontal dimension of soil profile (see Figure 3.3)</td>
</tr>
<tr>
<td>Ay</td>
<td>Vertical dimension of soil profile (see Figure 3.3)</td>
</tr>
<tr>
<td>DCLAY</td>
<td>Length of flow path in medium</td>
</tr>
<tr>
<td>DH</td>
<td>Head difference between entrance and exit boundaries</td>
</tr>
<tr>
<td>DSILT</td>
<td>Length of flow path in inclusion</td>
</tr>
<tr>
<td>Dx</td>
<td>Horizontal distance between adjacent nodal points</td>
</tr>
<tr>
<td>Dy</td>
<td>Vertical distance between adjacent nodal points</td>
</tr>
<tr>
<td>IALI</td>
<td>Code for computing head at a point</td>
</tr>
<tr>
<td>ICASE</td>
<td>Number of case under consideration</td>
</tr>
<tr>
<td>IX</td>
<td>Array for nodal points adjacent to point M (see Figure 3.3)</td>
</tr>
<tr>
<td>IX(M,1)</td>
<td>II</td>
</tr>
<tr>
<td>IX(M,2)</td>
<td>JJ</td>
</tr>
<tr>
<td>IX(M,3)</td>
<td>KK</td>
</tr>
<tr>
<td>MBAR</td>
<td>Nodal point where head is to be analyzed</td>
</tr>
<tr>
<td>N</td>
<td>Total number of nodal points in the profile</td>
</tr>
<tr>
<td>NCASE</td>
<td>Total number of cases to be analyzed</td>
</tr>
<tr>
<td>NCODE</td>
<td>Code for nodal points</td>
</tr>
<tr>
<td>Variable</td>
<td>Description</td>
</tr>
<tr>
<td>------------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>NMESH</td>
<td>Number of simulated profile under investigation</td>
</tr>
<tr>
<td>NMMESH</td>
<td>Total number of profiles to be simulated</td>
</tr>
<tr>
<td>NMWALK</td>
<td>Total number of flow paths to be simulated in each profile</td>
</tr>
<tr>
<td>NSILT</td>
<td>Simulated inclusion thickness in the next step</td>
</tr>
<tr>
<td>NSTATE</td>
<td>Soil type at a nodal point</td>
</tr>
<tr>
<td>NTSILT</td>
<td>Thickness of inclusion thickness in current step</td>
</tr>
<tr>
<td>NWALK</td>
<td>Number of simulated flow path under investigation</td>
</tr>
<tr>
<td>Nx</td>
<td>Number of steps in each flow path</td>
</tr>
<tr>
<td>Ny</td>
<td>Number of nodal points in entrance boundary</td>
</tr>
<tr>
<td>TITLE</td>
<td>Title of Problem</td>
</tr>
</tbody>
</table>

### Input Format

<table>
<thead>
<tr>
<th>Card No.</th>
<th>Variable</th>
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<td>1-5</td>
<td>I5</td>
</tr>
<tr>
<td>3</td>
<td>Ny</td>
<td>1-5</td>
<td>I5</td>
</tr>
<tr>
<td>3</td>
<td>N</td>
<td>6-10</td>
<td>I5</td>
</tr>
<tr>
<td>3</td>
<td>NMWALK</td>
<td>11-15</td>
<td>I5</td>
</tr>
<tr>
<td>3</td>
<td>NMMESH</td>
<td>16-20</td>
<td>I5</td>
</tr>
<tr>
<td>3</td>
<td>Ax</td>
<td>21-25</td>
<td>F5.1</td>
</tr>
<tr>
<td>3</td>
<td>Ay</td>
<td>26-30</td>
<td>F5.1</td>
</tr>
<tr>
<td>3</td>
<td>Dx</td>
<td>31-35</td>
<td>F5.1</td>
</tr>
<tr>
<td>3</td>
<td>Dy</td>
<td>36-40</td>
<td>F5.1</td>
</tr>
<tr>
<td>3</td>
<td>DH</td>
<td>41-45</td>
<td>F5.1</td>
</tr>
<tr>
<td>Card No.</td>
<td>Variable</td>
<td>Columns</td>
<td>Format</td>
</tr>
<tr>
<td>---------</td>
<td>----------</td>
<td>---------</td>
<td>--------</td>
</tr>
<tr>
<td>3</td>
<td>IALI</td>
<td>46-50</td>
<td>I5</td>
</tr>
<tr>
<td>3</td>
<td>MBAR</td>
<td>51-55</td>
<td>I5</td>
</tr>
<tr>
<td>3</td>
<td>Nx</td>
<td>56-60</td>
<td>I5</td>
</tr>
</tbody>
</table>
RANDOM FLOW MODEL

ANALYSIS OF SEEPAGE IN STOCHASTIC HETEROGENEOUS SOIL DEPOSITS

DEPOSIT IS COMPOSED OF A MEDIUM AND AN INCLUSION

THE SINGLE-STEP FORESIGHT CASE IS CONSIDERED

PROGRAM WRITTEN BY ELFATIH M. ALI *

THE OHIO STATE UNIVERSITY 1979

COMMON X(9000), Y(9000), NCODE(9000), NX(9000), NSTATE(9000),

TRANSV(2, 2), TRS(6, 6), NSTEP(200, 2), AKA(5000), TRC(7, 7), NODE(5000),

3 NY, N, NMMWK, NY, DX, DX, DH, DSIIL, DCLAY, ISTEP, NTSLT, IS,

4 NSLIT, JJ, KK, X1, Y1, X2, Y2, DL, DL1, N, PM, NWAL, VEL, R, AK,

6 NX, NWALK(200, 2), NWALK(200), IALI, IMAR, HMB(5000), MBAR, MB1, NMESI,

5 NEW, NCASE, ICASE, NCLAY, NTCLAY, NMMESH, NMBH(5000)

DIMENSION TITLE(18)

DIMENSION UBO(3), FREQ(50), PCT(50), STATS(3), STATS1(1000, 3)

DIMENSION FREQ2(50), PCT2(50), STATS2(3), AKMSH(5000)

DIMENSION FREQ3(50), PCT3(50), STATS3(3)

DIMENSION UBO(3)

DIMENSION AVMSH(500), STATS5(3), PCT3(50), FREQ5(50)

READ AND WRITE TITLE

READ(5, 1001) TITLE

WRITE(6, 2001) TITLE

READ(5, 901) NCASE

ICASE=1

*I*1=50*219481

IX=206641355

DO 999 ICASE=1, NCASE

WRITE(6, 901) ICASE

C

READ AND WRITE INPUT DATA

IF IALI=1 WE ANALYZE PRESSURE AT MBAR

READ(5, 1002) NY, N, NMMWK, NMESH, AX, DX, DX, DH, IALI, MBAR, NX

WRITE(6, 2002) NY, N, NMMWK, NMESH, AX, DX, DX, DH, IALI, MBAR, NX

UBO(1)=0.

UBO(2)=32.

UBO(3)=300.

UBO(1)=0.

UBO(2)=22.

UBO(3)=100.

GENERATE AND WRITE NODAL POINT DATA

X(1)=0.0

Y(1)=0.0

K=1

AY1=AY+.01

AY2=AY+.01

DO 10 I=K, K+NY-1

Y(I+1)=Y(I)+DY

X(I+1)=X(I)

IF(IXK).GE.AXD GO TO 11
**Release 2.0**

**Main**

\[ K = K + N \]

\[ X(K) = X(K - NY) + DX \]

\[ Y(K) = 0, 0 \]

**Date = 79005**

**09/02/48**

---

**C***

FIND CODE FOR NODAL POINTS

**C***

DO 20 I = 1, N

IF \((Y(I).EQ.0.)\) GO TO 22

IF \((Y(I).GE.AY1)\) GO TO 22

IF \((Y(I).GE.AY2)\) GO TO 22

IF \((X(I).EQ.AX)\) GO TO 23

GO TO 21

**C***

IF \((X(I).EQ.AX)\) GO TO 23

NCODE(I) = 1

GO TO 20

**C***

IF \((X(I).EQ.AX)\) GO TO 23

NCODE(I) = 2

GO TO 20

**C***

NCODE(I) = 0

**C***

CONTINUE

---

**C***

TO GENERATE AND WRITE STEP DATA

**C***

DO 30 I = 1, N

IF \((Y(I).EQ.0.)\) GO TO 31

IF \((Y(I).GE.AY1)\) GO TO 32

IF \((Y(I).GE.AY2)\) GO TO 32

IF \((X(I).EQ.AX)\) GO TO 33

**C***

M = I

IX(M, 1) = M + NY + 1

IX(M, 2) = M + NY

IX(M, 3) = M - NY - 1

GO TO 30

**C***

IF \((X(I).EQ.AX)\) GO TO 33

M = I

IX(M, 1) = M + NY

IX(M, 2) = M + NY

IX(M, 3) = M + NY + 1

GO TO 30

**C***

M = I

IX(M, 1) = I

IX(M, 2) = I

IX(M, 3) = I

**C***

CONTINUE

---

**C***

CLEAR ALL SOIL STATES

**DO 797 I = 1, N**

**797**

**NSTATE(I) = 0**

**C***

GENERATE SOIL TYPES AT NODAL POINTS

**C***

NNESH = 0
C GENERATE RANDOM FLOW PATH

NWALK = 0
M1 = 0
M1 = M1 + 1
C WALK STARTS AT M1
M = M1
NWALK = NWALK + 1
ISTEP = 0

CALL FLOW

C STORE DATA ABOUT FLOW PATH AND CHECK FOR PRESSURE AT MBAR

IF(IAL1.EQ.0) GO TO 87
CALL PRESS1

C IS FLOW PATH FINISHED?

87 IF(NCODE(N).EQ.2) GO TO 89
GO TO 88

C ANALYZE DATA ABOUT FLOW PATH

CALL DATA1(DSILT, DCLAY, NWALK, AKA, DB, AX)

C IF FLOW PATH PASSES OVER MBAR ANALYZE FOR PRESSURE

IF(IAL1.EQ.0) GO TO 66
IF(10MAR.NE.1) GO TO 66
CALL PRESS2

C REPEAT FLOW PATH?

66 IF(NWALK.GE.NNWALK) GO TO 91
IF(M1.GE.NY) GO TO 94
GO TO 99

C ANALYZE DATA FOR MESH

CALL DATAN2(AKA, UBO, FREQ, PCT, STATS, NWALK)

C ANALYZE PRESSURE DATA FOR MESH

IF(IAL1.EQ.0) GO TO 97
CALL PRESS2

97 DO 95 I = 1, 3
95 STATS(NMESH, I) = STATS(I)
AKSHTH(NMESH) = STATS(2)
AVSHT(NMESH) = STATS(3)

C GENERATE MORE MESHES?

IF(NMESH.GE.NMESH) GO TO 98
GO TO 92
C WRITE ALL RESULTS
93 WRITE(6,9001)
   DO 96 J=1,NMESH
   WRITE(6,9002)J,(STATS1(J,I),I=1,3)
96    CONTINUE
C ANALYZE DATA FOR ALL MESHES
C ANALYZE AND WRITE RESULTS FOR PRESSURE AT MBAR
C
C ANALYZE AND WRITE RESULTS FOR PRESSURE AT MBAR
901 FORMAT(15)
902 FORMAT(30HI CASE NO I5///)
1001 FORMAT(18A4)
1002 FORMAT(415,5F5.1,3I5)
2001 FORMAT(1HI 18A4)
2002 FORMAT(30H0 NY----------------- I5/)
2 30H0 N ------------------ I5/
3 30H0 NNMVLK------------------ I5/
9 30H0 NMESH ----------------- I5/
4 30H0 AX---------------------- E12.5/
5 30H0 AY---------------------- E12.5/
6 30H0 DX---------------------- E12.5/
7 30H0 DY---------------------- E12.5/
8 30H0 DH-------------------- E12.5///)
4001 FORMAT(1HI/'INTER. FREQ. ')
4002 FORMAT(15,F10.4)
4003 FORMAT(1HI/'INTER. PCT. FREQ. ')
4004 FORMAT(15,F10.4)
4005 FORMAT(30H0 TOTAL---------------- E12.5/
2 30H0 MEAN----------------- E12.5/
5 30H0 ST. DEV.-------------- E12.5///)
5000 FORMAT(1HI/'ANALYSIS FOR PRESSURE AT MBAR ')
5001 FORMAT(30H0 MBAR---------------- I5///)
9001 FORMAT(1HI/'NMESH TOTAL MEAN ST.DEV. ')
9002 FORMAT(15,3E12.5)
999 CONTINUE
STOP
END
SUBROUTINE MESH
C************************************************************* ********
C TO GENERATE SOIL TYPES AT NODAL POINTS OF MESH
C**************************«******************************************
COPDEON XC90O0),  Y(9000), NCODEC9000),  IXC9O0O.3), NSTATEC9000),
2TRANSV(2,2), TRS(6,6), NSTEP(200,2), AKA(5000), TRC(7,7), NODE(5000,5),
3 NY, NNRWALK, AX, AY, DX, DY, DS1LT, TCLAY, ISTATE, NTSILT, IS,
4 NSILT, II, JJ, KK, LL, IX1, IY1, IXX, IYY, DL, DL1, M, NH, NRWALK, VEL, R1, AK,
6 N1, AHWL(200,2), NNLW(200), IAL1, IAMAR, HMA(5000), MBAR, MB1, NMESH,
5 NNEW, NCASE, ICASE, NCLAY, NTCLAY, NMESH, HMB(5000)
C************************************************************* ********
C FIRST GENERATE SOIL AT FIRST COLUMN
DO 1999 III=1, NY
C CHOOSE INITIAL STATE AT RANDOM
C THIS DEPENDS ON THE OVERALL PROBABILITY OF SOIL TYPES IN THE
C DEPOSIT
IF(1. NE. 1) GO TO 1900
IHIGH=4
ILOW1=1
NY3=N-NY
CALL IRIANU(IX1, IHIGH1, ILOW1, IY1)
IF(IY1.EQ. 1) GO TO 19
NSTATE(I) = 0
GO TO 1999
19 NSTATE(I) = 1
GO TO 1999
1900 MM=1-1
LL=1
CALL TRSV
1999 CONTINUE
C************************************************************* ********
C NOW THAT FIRST SOIL COLUMN IS GENERATED WE LOOK FOR HORIZONTAL
C TRANSITION OF INCLUSIONS ONLY
111=1
11=111
NY1=NY
1799 IL=11
IF(11.GT.NY) GO TO 1901
IF(NSTATE(11).NE.0) GO TO 1800
11=11+1
GO TO 1799
1800 ITS=1
1802 IL=11+1
IF(11.LT. NY) GO TO 1821
GO TO 1803
1821 IF(NSTATE(11).EQ.0) GO TO 1803
ITS=ITS+1
GO TO 1802
1803 NTSILT=ITS
NTSLT=NTSILT
IF(NTSILT.LT.2) GO TO 111
KK1=(NTSILT-1)/2
MA=IL+KK1
GO TO 1500
111 MA=IL
C************************************************************* ********
C NTSILT IS THE THICKNESS OF INCLUSION IN THE CURRENT STEP
NSILT IS THE SIMULATED INCLUSION THICKNESS AT THE NEXT STEP

CALL TRANS

IF(NSILT.EQ.0) GO TO 1870

NN1=NY-NY1+MA
NN2=NY1-MA
KKK1=NSILT/2
KKK2=(NSILT-1)/2
IF(NN1.GE.KKK2) GO TO 114
KKK2=NN1

IF(NN2.GE.KKK1) GO TO 113
KKK1=NN2
GO TO 113

DO 110 I1=1,KKK2
L1=MA+NY-I1

NSTATE(L1)=1
K4=KKK1+1
DO 120 I1=1,K4
J2=I1-1
L2=MA+NY+J2

NSTATE(L2)=1

NTSILT=NSILT
NY1=NY1+NY
MA=MA+NY
IF(NCODE(MA).EQ.2) GO TO 1960
IF(MA.GT.NY3) GO TO 1960
GO TO 1500

110

120

1870

1960

1901

RETURN

END
DO 108 KS1 = 1, 6
  IF (YFL LE. TRS(1M1, KS1)) GO TO 106
108  CONTINUE
106  IM2 = KS1
    NSILT = IM2 - 1
    RETURN
END
SUBROUTINE TRANS

C **********************************************************************
C THIS SUBROUTINE GENERATES THE VARIATIONS IN THICKNESS OF
C INCLUSIONS IN THE HORIZONTAL DIRECTION
C **********************************************************************

CONJO X(9000), Y(9000), NCODE(9000), IX(9000,3), NSTATE(9000),
2TRANSV(2,2), TRS(6,6), NSTEP(200,2), AKA(5000), TRC(7,7), NODE(5000,5),
3NY,N,NWALK, AX,AY,DX,DY,DL,DSILT,DCLAY, LSTEP, NTSILT, IS,
4 NSILT, IJ, J, K, LL, IX1, IY1, IXX, IYY, DL, DL1, N, N1, N2, NWALK, VEL, RI, AK,
6NX, NWILK(200,2), NWLK(200), IAL1, IOMAR, HMBA(5000), MBAR, MB1, NMESH,
5MNW.NCASE, ICASE , NCLAY, NTCLAY, NMESH , HMB(5000)

C **********************************************************************
C TRS(I,J) IS THE CUMULATIVE TRANSITION MATRIX FOR INCLUSION
C THICKNESS
C USE THE PROPER MATRIX FOR THE SOIL DEPOSIT UNDER INVESTIGATION
C **********************************************************************

TRS(1,1) = .41
TRS(1,2) = .82
TRS(1,3) = 1.
TRS(1,4) = 1.
TRS(1,5) = 1.
TRS(1,6) = 1.
TRS(2,1) = .13
TRS(2,2) = .69
TRS(2,3) = .9
TRS(2,4) = 1.
TRS(2,5) = 1.
TRS(2,6) = 1.
TRS(3,1) = .11
TRS(3,2) = .48
TRS(3,3) = .82
TRS(3,4) = .98
TRS(3,5) = 1.
TRS(3,6) = 1.
TRS(4,1) = .6.
TRS(4,2) = .24
TRS(4,3) = .45
TRS(4,4) = .76
TRS(4,5) = .9
TRS(4,6) = 1.
TRS(5,1) = 6.0
TRS(5,2) = 9.0
TRS(5,3) = 0.0
TRS(5,4) = .36
TRS(5,5) = .77
TRS(5,6) = 1.
TRS(6,1) = 6.0
TRS(6,2) = 6.0
TRS(6,3) = 0.0
TRS(6,4) = .33
TRS(6,5) = .66
TRS(6,6) = 1.
IF(NTSILT.NE.5) GO TO 92

92  IN1 = NTSILT + 1
199 CALL RN2CIXX, IYY, YFL)
1XX=1YY
SUBROUTINE TRSV

C **SUBROUTINE GENERATES THE VERTICAL SOIL TRANSITION**
C **COMMON X(9000), Y(9000), NCODE(9000), IX(9000), ISTATE(9000),**
C **TRANSV(2,2), TRS(6,6), NSTEP(200,2), AK(5000), TRC(7,7), NODE(5000,5),**
C **AK, TRC(7,7), NODE(5000,5),**
C **NX, NY, NWH, AX, AY, DX, DY, DSI, DCLAY, ISTEP, NSTATE, ISTATE, IS,**
C **NSIL, JJ, KK, LL, IY, IY, BL, DL, MM, NWH, VEL, R1, AK,**
C **TRANSV(1,1) = .66**
C **TRANSV(1,2) = 1.**
C **TRANSV(2,1) = .35**
C **TRANSV(2,2) = 1.**
C **STATE(ND+1)**
C **CALL RN2(IXX, IYY, YFL)**
C **STATE(ND+1)**
C **DO 103 KTJ=1,2**
C **IF(YFL.LE.TRANSV(IM,KTJ)) GO TO 102**
C **CONTINUE**
C **STATE(LL) = KTJ-1**
C **RETURN**
C **END**
SUBROUTINE IRANU( IX, IHIGH, ILOW, IY)
C ***************************************************************
C THIS SUBROUTINE GENERATES UNIFORMLY DISTRIBUTED RANDOM INTEGERS
C BETWEEN IHIGH AND ILOW
C ***************************************************************
C
CALL RN2( IX, IZ, YFL)
IX=IZ
IY=( IHIGH-ILOW+1)*YFL
RETURN
I
IY=( ILOW-IHIGH+1)*YFL
IY=IY+ILOW
RETURN
END
SUBROUTINE RN2( IX, IY, YFL)

C THIS SUBROUTINE GENERATES UNIFORMLY DISTRIBUTED RANDOM NUMBERS

C BETWEEN 0.0 AND 1.0

C

IY= IX*65539
IF( IY)< 6, 6
   5 IY= IY+2147483647+1
   6 YFL= IY
   YFL=YFL*.465613E-9
RETURN
END
SUBROUTINE FLOW

THIS SUBROUTINE GENERATES THE STEPS OF THE FLOW PATH
IT ALSO STORES DATA ABOUT THE FLOW

COMMON X(9000),Y(9000),NCODE(9000),IX(9000,3),NSTATE(9000),
2TRANS(2,2),THS(6,6),NSTEP(200,2),AKA(5000),TRC(7,7),NODE(5000,5),
3NY,N,NUVALK,AX,AY,DX,DY,DH,DSILT,DCLAY,ISTEP,NTSILT,IS,
4NSILT,II,JI,KK,LL,IXI,IXJ,IXK,IALI,OMAR,HMB(5000),MMA,MB1,NMESH,
5MNEW,NCASE,ICASE,NCLAY,NTCLAY,NMESH,HMB(5000)

DL1=(DX**2+DY**2)**0.5
ISTEP=ISTEP+1
IF(ISTEP.NE.1) GO TO 96
DSILT=0.0
DCLAY=0.0
96 IF(NSTATE(JJ).EQ.I) GO TO 130
IF(NSTATE(KK).EQ.1) GO TO 131
GO TO 132

C FLOW GOES TO JJ
M=JJ
IF(NSTATE(MM).EQ.0) GO TO 1301
DSILT=DSILT+DX
RETURN
1301 DSILT=DSILT+DX/2
DCLAY=DCLAY+DX/2
RETURN
131 IF(NSTATE(KK).EQ.1) GO TO 133
GO TO 134

C FLOW GOES TO II
M=II
IF(NSTATE(MM).EQ.0) GO TO 1341
DSILT=DSILT+DL1
RETURN
1341 DSILT=DSILT+DL1/2
DCLAY=DCLAY+DL1/2
RETURN

C FLOW GOES TO KK
M=KK
IF(NSTATE(MM).EQ.0) GO TO 1321
DSILT=DSILT+DL1
RETURN
1321 DSILT=DSILT+DL1/2
DCLAY=DCLAY+DL1/2
RETURN

C FLOW IS RANDOM TO II OR KK
FL=0.5
PK=0.5
CPI=PI
CPE* 1.
CALL RN2(IXX, IYY, YFL)
IXX= IYY
IF(YFL.LE.CPI) GO TO 134
GO TO 132
139 IF(NSTATE(N).EQ.1) GO TO 140
GO TO 141
C FLOW GOES TO JJ
141 M=JJ
DCLAY=DCLAY+DX
RETURN
C FLOW IS RANDOM TO I, J, K
140 PI=0.333
PJ=0.334
PK=0.333
CPI=PI
CPJ=PI+PJ
CPK=1.
CALL RN2(IXX, IYY, YFL)
IXX= IYY
IF(YFL.LE.CPI) GO TO 710
IF(YFL.LE.CPJ) GO TO 720
GO TO 730
C FLOW GOES TO II
710 M=II
IF(NSTATE(N).EQ.0) GO TO 711
DSILT=DSILT+DL1
RETURN
711 DCLAY=DCLAY+DL1/2
DSILT=DSILT+DL1/2
RETURN
C FLOW GOES TO JJ
720 M=JJ
IF(NSTATE(N).EQ.0) GO TO 721
DSILT=DSILT+DX
RETURN
721 DCLAY=DCLAY+DX/2
DSILT=DSILT+DX/2
RETURN
C FLOW GOES TO KK
730 M=KK
IF(NSTATE(N).EQ.0) GO TO 731
DSILT=DSILT+DL1
RETURN
731 DCLAY=DCLAY+DL1/2
DSILT=DSILT+DL1/2
RETURN
END
SUBROUTINE DATAB1 (DSILT, DCLAY, NVALK, AKA, DH, AX)
C
C THIS SUBROUTINE ANALYZES THE DATA ABOUT THE FLOW PATH
C AND COMPUTES THE EQUIVALENT PERMEABILITY OF THE PATH
C
DIMENSION AKA(5000)
R1 = 0.005
DL = DCLAY + DSILT * R1
AK = 1.5
DLL1 = DSILT * DCLAY
VEL = AK * DH / DL
AKA(NVALK) = VEL * DLL1 / DH
RETURN
END
SUBROUTINE DATAN2(AKA, UBO, FREQ, PCT, STATS, NMWALK)

C THIS SUBROUTINE COMPUTES THE MEAN, VARIANCE AND DISTRIBUTION
C FUNCTION OF ANY SET OF DATA
C
C DIMENSION AKA(5000), UBO(3), FREQ(50), PCT(50), STATS(3)
C DIMENSION AVMSH(500), STATS5(3), PCT5(50), FREQ5(50)

C CLEAR OUTPUT AREAS
INN=UB0(2)
DO 45 I=1, INN
FREQ(I)=0.0
PCT(I)=0.0
DO 50 I=1, 3
STATS(I)=0.0

C CALCULATE INTERVAL SIZE
SINT=ABS((UBO(3)-UBO(1))/(UBO(2)-2.))

C DEVELOP TOTAL AND FREQUENCIES
STATS(1)=STATS(1)+AKA(J)
STATS(3)=STATS(3)+AKA(J)*AKA(J)
TEMP=UB0(1)-SINT
INTX=INN-I
DO 60 I=1, INTX
TEMP=TEMP+SINT
IF(AKA(IJ)-TEMP)70,60,60
60 CONTINUE
IF(AKA(IJ)-TEMP)75,65,55
65 FREQ(INN)=FREQ(INN)+1.
GO TO 75
70 FREQ(I)=FREQ(I)+1.
75 CONTINUE
IF(SCNT)79,105,79

C CALCULATE PER CENT FREQ.
79 DO 80 I=1, INN
80 PCT(I)=FREQ(I)*100.0/SCNT

C CALCULATE MEAN AND ST. DEVIATION
IF(SCNT-1)85,85,90
85 STATS(2)=STATS(1)
STATS(3)=0.
80 CONTINUE
IF(SCNT)105,100,105
90 STATS(2)=STATS(1)/SCNT
STATS(3)=SQRT(ABS((STATS(3)-STATS(1)*STATS(1)/SCNT)/(SCNT-1.)))
105 RETURN
END
SUBROUTINE PRESS1

TO STORE DATA ABOUT FLOW PATH AND CHECK FOR MBAR

COMMON X(9000),Y(9000),NCODE(9000),IX(9000,3),NSTATE(9000),
TRANSV(2,2),TR(6,6),NSTEP(200,2),AKA(5000),TRC(7,7),NODE(5000,5),
NY,N,NWALK,AX,AY,DX,DY,DS1LT,DCLAY,ISTEP,NTS1LT,IS,
NSILT,II,JK,LL,IX1,IXY,DL,DL1,MM,NWALK,VEL,R1,AK,
NX,AWLK(200,2),NLKL(200),JL1,1OMAR,HMB(5000),MBAR,MB1,NMESH,
NEW,NCASE,ICASE,NCLAY,NTCLAY,NMESH,IMD(5000)

IF(ISTEP.NE.1) GO TO 12

CLEAR ARRAYS
10 MAR=0
10 DO 10 I=1,NX
10 NWLKI(I)=0
10 AWLK(I,1)=0.
10 AWLK(I,2)=0.

12 IW=ISTEP
12 NWLKI(IW)=M
12 AWLK(IW,1)=DS1LT
12 ABLK(IW,2)=DCLAY

CHECK FOR MBAR

MD=MBAR-M

IF(ND.EQ.0) GO TO 17

GO TO 16

10 MAR=1

MB=-1,STEP

RETURN

END
SUBROUTINE PRESS2

ANALYZE PRESSURE AT MBAR AND STORE IT

COMMON X(9000), Y(9000), RCODE(9000), RX(9000, 3), RSTATE(9000),
2TRANSV(2, 2), TRS(6, 6), NSTEP(200, 2), AKA(5000), TRC(7, 7), NODE(5000, 5),
2NY, N, NINWALK, AX, AY, DX, DY, DH, DSDIL, DSLAY, ISTEP, NTSILT, IS,
4NSL, LT, JJ, KK, LL, 1X1, 1Y1, 1XX, 1YY, DL, DL1, N, MM, NWALK, VEL, R1, AK,
6NX, AMLK(200, 2), NWLK(200), IALJ, 1OMAR, HMBA(5000), MBAR, MB1, NMESH,
5NMEN, NCASE, ICASE, NCLAY, NTCLAY, NMESH, HMBA(5000)

IF (1OMAR.EQ.0) GO TO 9
IS=IS+1
DLA=DCLAY+DSILT*0.005
DH=MWLT+MBAR,2)+AWLK(MB1,1)*0.005
HMBA(IS)=DH/DL*ALASH
9 IF (NWALK.CE.NMWALK) GO TO 10
GO TO 11

FIND AVERAGE PRESSURE AT MBAR FOR MESH

H MBA=0.0
IF (IS.EQ.0) GO TO 13
DO 12 I=1, IS
12 HMBA=HMBA+HMBA(I)
13 HMBA(JM)=HMBA/IS
GO TO 14

INITIALIZE IS

ISM(JM)=0.
IS=0
RETURN
END
APPENDIX H

LISTING OF COMPUTER PROGRAM FOR
THE SIMULATED FLOW PATH METHOD
H.1 **Description of Variables**

The same variables described in Appendix G, Section G.1, were used in this computer program.

H.2 **Input Format**

<table>
<thead>
<tr>
<th>Card No.</th>
<th>Variable</th>
<th>Columns</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TITLE</td>
<td>1-72</td>
<td>18A4</td>
</tr>
<tr>
<td>2</td>
<td>NCASE</td>
<td>1-5</td>
<td>I5</td>
</tr>
<tr>
<td>3</td>
<td>Ny</td>
<td>1-5</td>
<td>I5</td>
</tr>
<tr>
<td>3</td>
<td>N</td>
<td>6-10</td>
<td>I5</td>
</tr>
<tr>
<td>3</td>
<td>NMWALK</td>
<td>11-15</td>
<td>I5</td>
</tr>
<tr>
<td>3</td>
<td>NMMESH</td>
<td>16-20</td>
<td>I5</td>
</tr>
<tr>
<td>3</td>
<td>Ax</td>
<td>21-30</td>
<td>F10.5</td>
</tr>
<tr>
<td>3</td>
<td>Ay</td>
<td>31-40</td>
<td>F10.5</td>
</tr>
<tr>
<td>3</td>
<td>Dx</td>
<td>41-50</td>
<td>F10.5</td>
</tr>
<tr>
<td>3</td>
<td>Dy</td>
<td>51-60</td>
<td>F10.5</td>
</tr>
<tr>
<td>3</td>
<td>DH</td>
<td>61-70</td>
<td>F10.5</td>
</tr>
</tbody>
</table>
COMMON X(50000), Y(50000), NCODE(50000), IX(50000, 3), NSTATE(50000), TRANSV(2, 2), TRS(6, 6), NSIP(200, 2), AKA(5000), NODE(200, 5), NY, N, NMWALK, AX, AY, DX, DY, DH, DSILT, DCLAY, ISTEP.  
V, W, R, M, MN, NCASE, ICASE, NCASE, NCLAY, NTCLAY, NMMESH, NN1, LL1, MA 
DIMENSION TITLE(18) 
DIMENSION UBOC(3), FREQ(50), PCT(50), STAT(3), STAT2(100, 3) 
DIMENSION FREQ2(50), PCT2(50), STAT2(3), AKMSH(5000) 
DIMENSION AKA1(5000), AKA2(5000) 
C READ AND WRITE TITLE 
READ(5, 1001) TITLE 
WRITE(6, 2001) TITLE 
READ(5, 901) NCASE 
ICASE=1 
IXI=060219481 
IXX=288641359 
DO 999 ICASE=1, NCASE 
NWALK=0 
WRITE(6, 992) ICASE 
C**********************************************************************: 
C READ AND WRITE INPUT DATA 
C**********************************************************************: 
READ(5, 1002) NY, N, NMWALK, NMMESH, AX, AY, DX, DY, DH 
WRITE(6, 2002) NY, N, NMWALK, NMMESH, AX, AY, DX, DY, DH 
UBO(1)=.0. 
UBO(2)=.32. 
UBO(3)=.300. 
C UBO(1)=SMALLEST VALUE FOR EQUIVALENT K 
C UBO(2)= OF INTERVALS FOR THE STATISTICAL ANALYSIS OF K 
C UBO(3)=LARGEST VALUE OF EQUIVALENT K 
Y(1)=.0.0 
X(1)=.0.0 
AY1=AY-.01 
AY2=AY+.01 
K=1 
C**********************************************************************: 
C GENERATE NODAL POINT DATA 
C**********************************************************************: 
9 K11=K+NY-1 
DO 10 I=K, K11 
Y(I+1)=Y(I)+DY 
10 IF(X(I).LE.AX.XD) GO TO 11 
K=K+NY 
X(K)=X(K-NY)+DX 
Y(K)=.0.
C**********************************************************************
C FIND CODE FOR NODAL POINTS
C**********************************************************************
11 DO 20 M=If  N
12 IF(YC M) .EQ.0.)  GO TO 22
12 IF(YC M) .CE. AY1) GO TO 22
12 IFCY(M) .CE.AY2) GO TO 22
12 IFXC M> .  EQ. AX) GO TO 23
12 GO TO 21
22 IFC XC M) .  EQ. AX) GO TO 23
12 NCODECM)3 1
12 GO TO 20
23 NCODEC M) =2
12 GO TO 20
21 NCODEC M)*0
20 CONTINUE
C**********************************************************************
C GENERATE STEP DATA
C**********************************************************************
DO 30 M= 1 ,  N
12 IFCYCM) .EQ.O.)  GO TO 31
12 IFCYCM>,GE.AY1 >  GO TO 32
12 IFCYCM) .GE.AY2) GO TO 32
12 IFCXCM) .Eft.AX) GO TO 33
12 IXC M, I  >  = M+NY+1
12 IXC n,2> «M+NY
12 IXC M,3) = M+ NY-1
12 GO TO 30
83 IFCXCM) .EQ.AX) GO TO 33
12 IXCM, l) = M+IfY+l
12 IXC M,2)»M+NY
12 IXCM,3)=M+NY
12 GO TO 30
83 IFCXCM) .EQ.AX) GO TO 33
12 IXCM, 1) 3 M+NY
12 IXC M.2) BM+NY
12 IXCM,3)=M+NY-1
12 GO TO 30
83 IXCM, 1)«M
12 IXC M.2)=M
12 IXCM,3)*M
30 CONTINUE
C**********************************************************************
C START RANDOM FLOW PATH AT A NODAL POINT IN THE ENTRANCE AT RANDOM
C**********************************************************************
99 IHIGH2=NY
99 ILOW2=1
99 N$=0
99 CALL IRANUC1X1,IHICH2,ILOW2,1Y1)
C WALK STARTS AT M=1Y1
99 ISTEP=0
99 NWALK=NWALK+1
99 M=1Y1
C**********************************************************************
C EMPTY ARRAY FOR SOIL TYPE
C**********************************************************************
DO 1011 l=1,N
1011 NSTATE(I) = 0
GO TO 1012
88 IF(ISTEP.EQ.0) GO TO 99
GO TO 1012
C***********************************************************
C GENERATE SOIL TYPES AT ADJACENT NODAL POINTS
C***********************************************************
1012 CALL MESH
C***********************************************************
C GENERATE STEP OF RANDOM FLOW PATH
C***********************************************************
CALL FLOW
C***********************************************************
C IS FLOW PATH AT EXIT BOUNDARY?
C***********************************************************
IF(NCODE(M),EQ.2) GO TO 89
GO TO 88
89 CALL DATAN1(DSILT,DCLAY,NVALK,AKA,DX,AKA)
C REPEAT FLOW PATH?
C***********************************************************
IF(NVALK.GE.NMWALK) GO TO 91
GO TO 99
C***********************************************************
C ANALYZE DATA FOR ALL FLOW PATHS
C***********************************************************
91 CALL DATAN2(AKA,UBO,FREQ,PCT,STATS,NMWALK)
C***********************************************************
C WRITE ALL RESULTS
C***********************************************************
WRITE(6,4001) CASE NO 15///)
WRITE(6,4002) (I,FREQ(I) ,I=1,32)
WRITE(6,4003) (I,PCT(I) ,I=1,32)
WRITE(6,4004) (STATS(I) ,I=1,3)
901 FORMAT(15)
902 FORMAT(30H1) CASE NO 15///)
1001 FORMAT(18H4)
1002 FORMAT(415,5F10.5)
2001 FORMAT(1NH1) NY------------------ 15/)
2 30H0 N ------------------ 15/
3 30H0 NWALK------------------ 15/
9 30H0 NMMESH ------------------ 15/
4 30H0 AX---------------------E12.5/
5 30H0 AY---------------------E12.5/
6 30H0 DX---------------------E12.5/
7 30H0 DY---------------------E12.5/
8 30H0 DH---------------------E12.5///)
4001 FORMAT(1NH1) INTER. FREQ. ')
4002 FORMAT(15,F10.4)
4003 FORMAT(1NH1) PCT. FREQ. ')
4004 FORMAT(15,F10.4)
4005 FORMAT(30H0 TOTAL--------------E12.5/
2 30H0 MEAN----------------------E12.5/
3 30H0 ST. DEV. ------------------E12.5///)
9001 FORMAT(1NH1) 'MMESH TOTAL MEAN ST. DEV. ')
9002 FORMAT(15,3E12.5)
<table>
<thead>
<tr>
<th>RELEASE 2.0</th>
<th>MAIN</th>
<th>DATE = 79004</th>
<th>11/46/24</th>
</tr>
</thead>
<tbody>
<tr>
<td>999</td>
<td>CONTINUE</td>
<td>STOP</td>
<td>END</td>
</tr>
</tbody>
</table>
SUBROUTINE MESH
C**********************************************************************
C TO GENERATE SOIL TYPE AT NODAL POINTS FOR CURRENT STEP
C**********************************************************************
COMMON (XC(50000), YC(50000), NCODE(50000), IX(50000,3), NTBS(6,6),
2 NSSTATE(50000), TRANSV(2,2), TRS(6,6), NSTEP(200,2), AKA(50000),
3 NODE(2000,5), NY, N, NMWALK, AX, AY, DX, DY, DL, DSILT, DCLAY, ISTEP,
4 NTSLT, NSILT, IJ, JJ, KK, LL, IX1, IY1, IXX, IYY, DL, DL1, MM, MM, NWALK,
5 VEL, R1, AK, MNEW, NCASE, ICASE, NCLAY, NTCLAY, NMMEH, MM1, LL1, MA
1 ISTEP=ISTEP+1
2 NY2=NY1+1
3 NY1=NY1+NY
4 MM=N
5 JJ=IX(M,1)
6 KK=IX(M,3)
7 IHIGH1=2
8 ILOW1=1
9 CALL IRANUC(IK1, IHIGH1, ILOW1, IY1)
10 IF(IY1.EQ.1) GO TO 9
11 NTSTATE(M)=0
12 GO TO 1900
13 NSTATE(M)=1
14 Go TO 1900
1900 IF(NSILT.EQ.0) GO TO 110
15 IF(NSILT.EQ.0) GO TO 110
16 NTSLT=NSILT
17 GO TO 1500
10 C**********************************************************************
C GENERATE SOIL VERTICALLY FIRST
110 IF(NSTATE(MM).EQ.0) GO TO 200
12 NTU=1
13 MM1=MM
14 LL1=MM1+1
15 CALL TRSV
16 IF(NSTATE(LL1).EQ.1) GO TO 19
17 NTU=NTU+1
18 MM1=MM1+1
19 LL1=MM1+1
20 IF(LL1.LT.NY1) GO TO 21
21 NTL=1
22 MM1=MM
23 LL1=MM1-1
24 CALL TRSV
25 IF(NSTATE(LL1).EQ.1) GO TO 22
26 NTL=NTL+1
27 MM1=MM1-1
28 LL1=MM1-1
29 IF(LL1.GT.NY2) GO TO 23
30 NTSLT=NTU+NTL-1
31 GO TO 25
200 NTSLT=0
32 NTL=1
33 CONTINUE
34 IL=MM-NTL+1
R E L E A S E  2.0  M E S H  D A T E = 7 9 0 0 4  11/46/24

IF(NTSILT.LT.2) GO TO 111
KK1=(NTSILT-1)/2
MA=IL+KK1
IF(MA.LT.NY1) GO TO 112
MA=(NY1+IL)/2
GO TO 112
111 MA*IL
112 CONTINUE

C *******************************************************
C NTSILT IS THE THICKNESS OF INCLUSION IN THE CURRENT STEP
C NSILT IS THE SIMULATED INCLUSION THICKNESS AT THE NEXT SECTION
C *******************************************************

1560 CALL TRANS
IF(NTSILT.EQ.0) GO TO 1870
NN1=NY-NY1+MA
NN2=NY1-MA
KKK1=NTSILT/2
KKK2=(NTSILT-1)/2
IF(NN1.GE.KKK2) GO TO 114
KKK2*NN1
114 IF(NN2.GE.KKK1) GO TO 113
KKK1=NN2
GO TO 113
113 DO 119 11=1,KKK2
119 L1=MA+NY-11
119 NSTATE(L1)=1
K4=KKK1+1
DO 120 11=1,K4
J2=11-1
L2=MA+NY+J2
120 NSTATE(L2)=1
GO TO 1901
1870 IF(NTSILT.GE.3) GO TO 1871
IF(NTSILT.GE.2) GO TO 1872
GO TO 1882
1871 IF(NTU.LE.1) GO TO 1885
IF(NTL.LE.1) GO TO 1886
GO TO 1880
1885 NSTATE(JJ)=0
NSTATE(KK)=0
C GENERATE SOIL AT II
1895 MM1=JJ
LL1=II
CALL TRSV
GO TO 1901
1886 NSTATE(IJ)=0
NSTATE(JJ)=0
C GENERATE SOIL AT KK
1896 MM1=JJ
LL1=KK
CALL TRSV
GO TO 1901
1872 IF(NTU.LE.1) GO TO 1888
IF(NTL.LE.1) GO TO 1889
GO TO 1901
1888 NSTATE(JJ)=0
NSTATE(KK)=0
C GENERATE SOIL AT II
RELEASE 2.0

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GO TO 1895
1889 NSTATE(II)=0
NSTATE(JJ)=0
GO TO 1896
1890 NSTATE(II)=0
NSTATE(JJ)=0
NSTATE(KK)=0
GO TO 1901
1881 NSTATE(II)=0
NSTATE(JJ)=0
GO TO 1891
1882 NSTATE(JJ)=0
GO TO 1892
C GENERATE SOIL AT II
1892 MM1=JJ
LL1=II
CALL TRSV
C GENERATE SOIL AT KK
1891 MM1=JJ
LL1=KK
CALL TRSV
GO TO 1901
1800 FORMAT(215)
1801 FORMAT(15)
1901 RETURN
END
SUBROUTINE TRANS

***********************************************************************************
THIS SUBROUTINE GENERATES THE VARIATIONS IN THICKNESS OF INCLUSIONS
IN THE HORIZONTAL DIRECTION
***********************************************************************************
COMMON X(50000), Y(50000), NCODE(50000), IX(50000), 3), NTRS(6, 6),
2 NSTATE(50000), TRANSV(2, 2), TRS(6, 6), NSTEP(200, 2), AKA(5000),
3 NODE(2000, 5), NY. N, NWALK, AX, AY, DX, DY, DB, DSILT, DCLAY, 1STEP,
4 NTSILT, NSILT, JJ, KK, LL, IX, IY, IXX, IYY, DL, DL1, IT, MM, NWALK,
5 VEL, R1, AK, MNEW, NCASE, ICASE, NCLAY, NMMMESH, MM1, LL1, MA

TRC(1, J) IS THE CUMULATIVE TRANSITION MATRIX FOR INCLUSION THICKNESS
USE THE PROPER MATRIX FOR THE SOIL DEPOSIT UNDER INVESTIGATION
***********************************************************************************

TRC(1, 1) = .41
TRC(1, 2) = .82
TRC(1, 3) = 1.
TRC(1, 4) = 1.
TRC(1, 5) = 1.
TRC(1, 6) = 1.
TRC(2, 1) = .13
TRC(2, 2) = .69
TRC(2, 3) = .9
TRC(2, 4) = 1.
TRC(2, 5) = 1.
TRC(2, 6) = 1.
TRC(3, 1) = 1.1
TRC(3, 2) = .48
TRC(3, 3) = .82
TRC(3, 4) = .98
TRC(3, 5) = 1.
TRC(3, 6) = 1.
TRC(4, 1) = 1.
TRC(4, 2) = .24
TRC(4, 3) = .45
TRC(4, 4) = .76
TRC(4, 5) = .9
TRC(4, 6) = 1.
TRC(5, 1) = 1.
TRC(5, 2) = 1.
TRC(5, 3) = .65
TRC(5, 4) = .36
TRC(5, 5) = .77
TRC(5, 6) = 1.
TRC(6, 1) = 1.
TRC(6, 2) = 1.
TRC(6, 3) = 1.
TRC(6, 4) = 1.
TRC(6, 5) = 1.
TRC(6, 6) = 1.
IF(NTSILT. LT. 5) GO TO 92
IM1 = 6
GO TO 199
92 IM1 = NTSILT + 1
199 CALL RN2CIXX, IXX, IYY, YFL
IXX = 1YY
DO 108 KSI = 1, 6
IF(YFL. LE. TRC(1, KSI)) GO TO 106
108 CONTINUE
106 IM2 = KS1
NSILT = IM2 - 1
RETURN
END
SUBROUTINE TRSV

THIS SUBROUTINE GENERATES THE VERTICAL SOIL TRANSITION

TRANSV(I,J) IS THE CUMULATIVE TRANSITION MATRIX FOR VERTICAL
STRATIFICATION OF THE SOIL DEPOSIT

TRANSV(1,1) = .66
TRANSV(1,2) = 1.
TRANSV(2,1) = .35
TRANSV(2,2) = 1.

CALL RN2(IXX, IYY, YFL)

DO 103 KTJ = 1, 2
IF(YFL .LE. TRANSV(1M, KTJ)) GO TO 102
103 CONTINUE
102 NSTATE(LL1) = KTJ - 1
RETURN
END
SUBROUTINE IRANU( IX, IHIGH, ILOW, IY)

**SUBROUTINE IRANU (IX, IHIGH, ILOW, IY)**

C
C THIS SUBROUTINE GENERATES UNIFORMLY DISTRIBUTED RANDOM INTEGERS
C BETWEEN IHIGH AND ILOW
C
CALL RN2( IX, IZ, YFL)
IX* IZ
IF( IHIGH.LT. ILOW) GO TO 1
IY=( IHIGH- ILOW+1 )*YFL
IY* IY+ ILOW
RETURN
1 IY=( ILOW-IHIGH+1 )*YFL
IY= IY* IHIGH
RETURN
END
SUBROUTINE RN2 (IX, IY, YFL)

C THIS SUBROUTINE GENERATES UNIFORMLY DISTRIBUTED RANDOM NUMBERS
C BETWEEN 0.0 AND 1.0

C

IY = IX * 65539
IF (IY) 5, 6, 6
5 IY = IY + 2147483647 + 1
6 YFL = IY
    YFL = YFL * .465613E-9
RETURN
END
SUBROUTINE FLOW
*********
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
THIS SUBROUTINE GENERATES THE NEXT STEP IN THE RANDOM FLOW PATH
AND STORES THE DATA ABOUT THAT STEP
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
COMMON X(50000), Y(50000), NCODE(50000), IX(50000, 3), NTRS(6, 6),
NSTATE(50000), TRANSV(2, 2), TRS(6, 6), NSTEP(200, 2), AKA(5000).
NODE(2000, 5), NY, N, NMWALK, AX, AY, DX, DY, DH, DSILT, DCLAY, ISTEP,
NTSILT, NSILT, JJ, KK, LL, IX1, IY1, IX2, IY2, DL, DL1, MM, MMW, NMWALK,
VEL, R1, AK, MNEW, NCASE, ICASE, NCLAY, NTCLAY, NMEMS, MM1, LL1, MA
M=(DX**2+DY**2)**0.5
IF(ISTEP.NE.1) GO TO 96
DSILT=0.0
DCLAY=0.0
96 CONTINUE
120 IF(NSTATE(JJ).EQ.1) GO TO 130
 IF(NSTATE(II).EQ.1) GO TO 131
 IF(NSTATE(KK).EQ.1) GO TO 132
GO TO 139
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
FLOW GOES TO JJ
130 M=JJ
MA=MA+NY
IF(NSTATE(MM).EQ.0) GO TO 1301
DSILT=DSILT+DX
RETURN
1301 DSILT=DSILT+DX/2
DCLAY=DCLAY+DX/2
RETURN
131 IF(NSTATE(KK).EQ.1) GO TO 133
GO TO 134
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
FLOW GOES TO II
134 M=II
MA=MA+NY
IF(NSTATE(MM).EQ.0) GO TO 1341
DSILT=DSILT+DL1
RETURN
1341 DSILT=DSILT+DL1/2
DCLAY=DCLAY+DL1/2
RETURN
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
FLOW GOES TO KK
132 M=KK
MA=MA+NY
IF(NSTATE(MM).EQ.0) GO TO 1321
DSILT=DSILT+DL1
RETURN
1321 DSILT=DSILT+DL1/2
DCLAY=DCLAY+DL1/2
RETURN
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
FLOW IS RANDOM TO II OR KK
133 PI=0.5
PK=0.5
CP1=PI
CPK=1
CALL RN2C(IX2, IY2, YFL)
**RELEASE 2.0**

**FLOW**

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```
1XX= IYY
IF(YFL.LE.CPI) GO TO 134
GO TO 132
139 IF(NSTATE(MM).EQ.1) GO TO 140
GO TO 141
C FLOW GOES TO JJ
141 M=JJ
MA=MA+NY
DCLAY=DCLAY+DX
RETURN
C **********************************************************************
C FLOW IS RANDOM TO I, J, K
140 PI=0.333
PJ=0.334
PK=0.333
CPI=PI
CPJ=PI+PJ
CPK=1.
CALL RN2CIXX,IYY,YFL)
IXX= IYY
IF(YFL.LE.CPI) GO TO 710
IF(YFL.LE.CPJ) GO TO 720
GO TO 730
C **********************************************************************
C FLOW GOES TO II
710 M=II
MA=MA+NY
IF(NSTATE(II).EQ.0) GO TO 711
DSILT=DSILT+DL1
RETURN
711 DCLAY=DCLAY+DL1/2
DSILT=DSILT+DL1/2
RETURN
C **********************************************************************
C FLOW GOES TO JJ
720 M=JJ
MA=MA+NY
IF(NSTATE(JJ).EQ.0) GO TO 721
DSILT=DSILT+DX
RETURN
721 DCLAY=DCLAY+DX/2
DSILT=DSILT+DX/2
RETURN
C **********************************************************************
C FLOW GOES TO KK
730 M=KK
MA=MA+NY
IF(NSTATE(KK).EQ.0) GO TO 731
DSILT=DSILT+DL1
RETURN
731 DCLAY=DCLAY+DL1/2
DSILT=DSILT+DL1/2
RETURN
END
```
SUBROUTINE DATAN1 (DSILT, DCLAY, NWALK, AKA, DH, AX)
C
C THIS SUBROUTINE ANALYZES THE DATA ABOUT THE FLOW PATH
C IT COMPUTES THE EQUIVALENT PERMEABILITY OF THE PATH
C
DIMENSION AKA(NWALK)

R1 = 0.003
DL = DCLAY + DSILT * R1
AK = 1.5
DLL = DSILT + DCLAY
VEL = AK * DH / DL

AKA(NWALK) = VEL * DLL / DH

RETURN
END
SUBROUTINE DATAN2(AKA, UBO, FREQ, PCT, STATS, NMWALK)

C THIS SUBROUTINE COMPUTES THE MEAN, VARIANCE AND DISTRIBUTION
C FUNCTION OF ANY SET OF DATA

DIMENSION AKA(5000), UBO(3), FREQ(50), PCT(50), STATS(3)

C CLEAR OUTPUT AREAS
INH=UB0(2)
DO 43 I=1,INH
FREQ(I)=0.0
43 CONTINUE
DO 50 I=1,3
50 STATS(I)=0.0

C CALCULATE INTERVAL SIZE
SINT=ABS((UB0(3)-UB0(1))/(UB0(2)-2.))

C TEST SUBSET VECTOR
I,J=0
DO 73 J=1,NMWALK
IJ=IJ+1
IF(AKA(IJ)) 33,75,55
E 5 SEN I = 3CN7--
C DEVELOP TOTAL AND FREQUENCIES
STATS(1)=STATS(1)+AKA(IJ)
STATS(3)=STATS(3)+AKA(IJ)*AKA(IJ)
TEMP=UB0(1)-SINT
INTX=INH-1
DO 60 I=1,INTX
TEMP=TEMP+SINT
IF(AKA(IJ)-TEMP) 70,60,60
60 CONTINUE
65 FREQ(IJ)=FREQ(IJ)+1.
70 CONTINUE
75 CONTINUE
GO TO 79
79 CONTINUE
GO TO 75

C CALCULATE PER CENT FREQ.
DO 80 I=1,INH
80 PCT(I)=FREQ(I)*100.0/SCNT

C CALCULATE MEAN AND ST. DEVIATION
IF(SCNT-1) 85,65,90
65 STATS(2)=STATS(1)
STATS(3)=0.
GO TO 105
90 STATS(2)=STATS(1)/SCNT
STATS(3)=SQRT(ABS((STATS(3)-STATS(1)*STATS(1)/SCNT)/(SCNT-1.)))
105 RETURN
END