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THE OHIO STATE UNIVERSITY, PH.D., 1979

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1978
AN EVALUATION OF CONVERTIBLE DEBT FINANCING

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By
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* * * * *

The Ohio State University
1978

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To the memory of
my father
Basalingappa S. Yalawar
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LIST OF SYMBOLS

X - Firm's end-of-period realized cash flows in a single-period valuation framework

- Firm's end-of-current period realized operating earnings in a multi-period valuation framework

R_m - Unity plus rate of return on the market portfolio

r - Unity plus riskless rate of interest in a single-period valuation framework

- Riskless rate of interest in a multi-period valuation framework

λ - Market price of risk

θ - Corporate income-tax rate

α - Ratio of the number of common shares into which convertible bonds are convertible to the total number of outstanding common shares after conversion

I - Contractual one-period coupon interest on debt

D - Contractual principal amount of debt

B - Total bankruptcy cost

K - Fixed bankruptcy cost

k - Variable bankruptcy cost as a proportion of firm's value

Y_U - An unlevered firm's end-of-period after-tax cash flows

Y_E - End-of-period after-tax cash flows available to common stockholders in a single-period valuation framework

- End-of-current period wealth of common stockholders in a multi-period valuation framework

Y_DS - End-of-period cash flows available to straight bondholders
$V_{DC}$ - End-of-period cash flows available to convertible bondholders in a single-period valuation framework

- End-of-current period wealth of convertible bondholders in a multi-period valuation framework

$V$ - Firm's market value

$V_U$ - Market value of an unlevered firm

$V_E$ - Market value of Equity

$V_{DS}$ - Market value of straight debt

$V_{LS}$ - Market value of levered (straight debt) firm

$V_{DC}$ - Market value of convertible debt

$V_{LC}$ - Market value of levered (convertible debt) firm

$V_P$ - Market value of conversion premium on convertible debt

$CP$ - Call price

$CV$ - Stock-conversion value of a convertible bond

$ER$ - Expected return on underlying common stock

$VAR$ - Variance of return on underlying common stock

$COVAR$ - Covariance of return on underlying common stock with that of S&P Composite Index

$RETAN$ - Reciprocal of the ratio of retained earnings to total assets

$EBTAN$ - Reciprocal of the ratio of EBIT to total assets

$EVTDN$ - Reciprocal of the ratio of market value of equity to book value of long-term debt

$PQR$ - Reciprocal of numerical value of bond quality-rating

$CEQ_0(\bar{X})$ - $[E(\bar{X}) - \lambda \text{Cov}(\bar{X}, R_m)]$, the certainty-equivalent of the firm's end-of-period cash flows, $\bar{X}$ (truncated from zero upward)

$CEQ(\bar{B})$ - Certainty-equivalent of total bankruptcy cost
CEQ\(^b\)(\(\tilde{X}\)) - Certainty-equivalent of firm's end-of-period cash flows, \(\tilde{X}\), (truncated from b downward)

CEQ\(^{NC}\)_\(b\)(\(\tilde{X}\)) - Certainty-equivalent of firm's end-of-period cash flows being in between b and NC

CEQ\(_b\)(\(\tilde{X}\)) - Certainty-equivalent of firm's end-of-period cash flows, \(\tilde{X}\), (truncated from b upward)

CEQ\(^{NC}\)_\(C\)(\(\tilde{X}\)) - Certainty-equivalent of firm's end-of-period cash flows, \(\tilde{X}\), being between C and NC

F(b) - Probability of bankruptcy at the end-of-period

F(NC) - Probability of no conversion at the end-of-period

F(C)-F(b) - Probability of surrendering convertible bond upon call at the end-of-period

(\(\sim\)) - tilde denotes the randomness of the given variable
CHAPTER I
INTRODUCTION

In the legal sense, a convertible bond is a debt instrument issued by a corporation. The corporation promises to pay periodically a stated amount of interest until it matures or is surrendered for redemption and to pay the face value of the instrument on the date of maturity. A feature that distinguishes a convertible bond from a straight bond is that it can be converted into a stated number of common shares of the issuing corporation at the option of the bondholder at any time during the effective period of conversion privilege. A convertible bond may be callable as well. If it is callable, then the corporation has the option to call the bond back at any time during the effective call option period. Once the bond is called, the bondholder has the option either to surrender for redemption or to convert it into a stated number of shares of common stock as indicated in the indenture.

In recent years, the investment community has shown a growing and renewed interest in the trading of convertible bonds. The New York Stock Exchange Statistics (as given in the N.Y.S.E. 1976 Fact Book) show that of the 50 most actively traded bonds in 1975, 18 were convertible issues. During 1973-74, even though the new issues of convertible bonds were at their low ebb, of the 50 most actively traded bonds, 25 in 1973 and 31 in 1974 were convertible bonds. It has been
commented in the N.Y.S.E. 1976 Fact Book that the new convertible bond issues in 1975 enjoyed their best year since 1972 and the $1.3 billion of convertible bonds marketed (it forms only 3.03% of total new bond issues and 9.5% of new issues of common stock) was 177 percent above the 1974 total. Furthermore, the statistics show that during the past decade, a significant amount of new convertible bond issues has been made each year except in 1973-74. It suggests that there is an active market for the convertible bonds and firms are adopting this course of financing their investments more than ever before.

This growing and renewed interest of the investment community coupled with the growing interest shown by the academicians in recent years has made convertible-debt financing an important area of finance for both theoretical and empirical investigation. In particular, though the S-L-M's Capital Asset Pricing Model has been widely applied in the fields of finance and economics, the literature review (in Chapter II) indicates that surprisingly its application for the evaluation of convertible-debt financing and associated issues has so far been scanty.

A. Purposes of the Study

Noting the deficiencies in the existing literature, it is attempted in this study to accomplish primarily two purposes:

1. Evaluating the convertible-debt financing in the single-period as well as the multi-period capital asset pricing frameworks.
2. Based on the theoretical construct for the market value of convertible-debt, developing a multiple regression model for estimating the callable convertible bond prices and testing it empirically for its significance as well as for its stability over time and across firm sizes.

Besides, other interesting aspects of the research include the implications of the study for estimating premiums on callable convertible bonds, call policy of a firm and for a firm's optimal capital structure in the presence of convertible-debt instead of straight-debt in its capital structure.

The features distinguishing this study from the earlier studies are that it explicitly takes account of the possible costly bankruptcy and the firm's capital structure plan upon conversion. It treats call option in a proper perspective. And, with respect to capital structure theory, the study represents an extension of previous research that has dealt primarily with the capital structure comprising of common stock and straight-debt.

B. Importance of the Study

The general importance of the study stems from the growing use of convertible bond issues in the last decade but a lack of adequate answers to various questions confronting managers and investors alike in their decision making.

A careful review of the existing literature (summarized in Chapter II) suggests that previous studies on the valuation of convertible bonds have avoided the explicit consideration of the possible
costly bankruptcy.

Further, to our knowledge, Lewellen-Racette's [58] study seems to be the only study that has addressed the issue of the implications of convertible-debt financing for the wealth of existing stockholders and for the market value of the firm. Even Lewellen-Racette have failed to consider the callability feature of convertibles, costly bankruptcy and the firm's capital structure plan upon conversion, especially raising new debt, that are so crucial in determining the effect of this means of financing on the existing stockholders' wealth. Some studies do consider callability feature in their valuation procedures, but their treatment of this option has been inadequate.¹

This study incorporates, among others, these elements that have a great bearing on the valuation and importance of the convertible-debt financing but were either ignored or inadequately treated in previous studies. It provides a systematic and more complete evaluation of convertible-debt financing in both the single-period as well as the multi-period settings. Therefore, the study may be said to have at least the following novel features: a) the evaluation of convertible-debt financing utilizing a multi-period CAPM framework, b) the incorporation of the costly bankruptcy in evaluating convertible bonds within the context of an ideal capital market, c) the development of a theoretical pricing model for callable convertible bonds that is more comprehensive than is provided hitherto, and d) the empirical evidence in support of the significance or otherwise of call price and potential bankruptcy per se in determining the prices of callable convertibles.
C. **Scope and Method**

As said before, the study evaluates the convertible-debt financing. In the process, the corporate income tax and costly bankruptcy are explicitly considered. The evaluation is first carried out in a single-period setting. The Modigliani-Miller [66, 67] proposition of irrelevance of financial structure for a firm's market value, their tax model and Lewellen-Racette's [58] proposition of convertible-debt inferiority are re-examined in the context of the single-period valuation framework. The evaluation is then carried out in a multi-period setting. The effects of call option and a firm's capital structure plan upon conversion, which cannot be captured in the single-period setting, are captured in the multi-period analysis. Assuming that the firm's capital structure plan upon conversion is to maintain the same amount of leverage without retiring new issues and also that the newly raised debt is used to finance new investments, the valuation equations are derived for convertible bonds, equity and for the firm. The Lewellen-Racette proposition is again re-examined. Furthermore, in addition to discussing various implications, a multiple regression pricing model is developed for callable convertible bonds on the basis of the multi-period analysis. An empirical study is conducted to test the significance of the model and its stability over time and across firm size.
Our approach to the problem is simple. It begins by explaining the assumptions underlying the analysis and defining the events of bankruptcy and conversion in terms of the end-of-period cash flows or wealth. We then identify the end-of-period cash flows/wealth accruing to each group of security holders under different state-option combinations. Their present value is obtained by applying the S-L-M's single-period CAPM in the case of single-period analysis, and by applying Chen's simplified version of multi-period CAPM in the multi-period case. With respect to call option, the necessary and sufficient conditions to exercise the option are defined. The changes in the end-of-period wealth of security holders due to possible exercise of call option are recognized and simultaneously considered along with conversion and bankruptcy in deriving the valuation equations.

In the single-period analysis, especially for the consideration of bankruptcy, the study draws heavily on Kim [55] and Chen [20]. And, in the multi-period analysis, the study has greatly benefitted from Chen [22], Boness-Chen [13], and Gordon Pye [41].

The empirical part of the study deals with structuring and testing for significance of a multiple regression model for estimating the prices of callable convertible bonds. The structure of the model is based upon our theoretical evaluation of convertible-debt in the multi-period setting. Some modifications are made to the structure so as to obtain satisfactory results from the regression analysis. The empirical evidence is provided for the significance or otherwise of,
among others, the call price and potential bankruptcy in determining the prices of convertible bonds. The tests are also carried out for the stability of the model over time and across firm size.

D. Contributions of the Study

In the light of the above description of purposes and scope of the study, the main contribution of the study can be summarized as below:

1. Re-examining the MM proposition of financial structure irrelevance, their tax model and Lewellen-Racette's convertible-debt inferiority proposition in the capital asset pricing framework for convertible debt that incorporates corporate income taxes, costly bankruptcy and firm's capital structure plan upon conversion.

2. Deriving a more complete valuation model for convertible bonds than is provided hitherto.

3. Providing an alternative testable hypothesis for exercising the call option of convertible bonds.

4. Providing a theoretical valuation model for premiums on convertible bonds, and

5. Providing empirical evidence to support the consideration of the bankruptcy factor and call price in the evaluation of callable convertible bonds.
E. Framework of the Study

A careful review of the literature is presented in Chapter II. Some of the problems that have remained unresolved or have been treated inadequately in previous literature are pointed out and what we intend to accomplish in this study is thereby set forth.

Chapter III deals with a single-period evaluation of convertible-debt financing as opposed to financing by retained earnings or by issuing straight-debt. Besides re-examining the M-M and Lewellen-Racette propositions in a single-period setting, we have compared our single-period convertible-bond valuation model with the one proposed by Jennings [48]. Also included in this chapter are the implications of single-period convertible-debt valuation for a firm's cost of capital criterion.

The theoretical analysis is extended in Chapter IV to the consideration of convertible-debt valuation in a multi-period setting. The re-examination of the previously stated propositions in a multi-period context and the implications of the analysis for various firm decisions are also discussed in this chapter.

In Chapter V, we present the construction of multiple-regression model for estimating callable convertible-bond prices, formulate hypotheses to be tested and outline the testing procedure followed in our empirical study. The empirical results with respect to the determinants
of callable convertible-bond prices and the stability of the proposed (modified) regression model over time and across firm size are analyzed in Chapter VI. Lastly, a summary and conclusions of the study are given in Chapter VII.
Footnotes to Chapter I

1 See, for example, Jennings [1974], Duvel [1968], Ingersoll [1977], and Brennan-Schwartz [1977].
CHAPTER II

REVIEW OF EXISTING LITERATURE

To understand the current state of the literature on convertible-debt-financing, we undertake in this chapter a careful review of the existing literature. We confine ourselves in the process to those earlier studies that have direct relevance to the problem at hand. We also highlight, however, some of the other studies so as to bring home the point that the issues in this area of finance have been addressed by adopting various approaches.

It may be said at the outset that not until 1965 do we find an analytical study of bond valuation being made in the finance literature. The treatment of this subject before 1965 remained essentially in the realm of folklore, a typical description being that a convertible bond is a hybrid asset containing the best elements of both equity and straight bond and that the firms issue these bonds as they form a vehicle for raising equity capital at a higher than the current market price.

In our study, we address the issues of the valuation of and the premiums on convertible bonds, firms' call strategy and examine the value implications of the convertible-debt-financing as opposed to other means of financing. Therefore, what follows is a review of the literature relating to these issues.
A. On the Valuation of and the Premiums on Convertible Bonds

Poensgen [76] and Baumol-Malkiel-Quandt (BMQ) [7] are probably the first studies analyzing the convertible bond valuation. Using statistical properties of integration, Poensgen has shown that the expected value of a convertible bond is equal to either of the two following equal sums:

(i) The sum of the expected value of underlying common stock and the value of floor guarantee, or

(ii) The sum of the expected value of underlying straight bond and the value of conversion option.

This specification (which later came to be known as the conventional model) underscores the fact that the price of a convertible bond before maturity is higher than the greater of either the straight bond price or the price of underlying common stock. This is so because a convertible bond has the features that protect the holder against potential loss due to low stock price and allow him/her to capture any potential capital gains that could be obtained by conversion.

Without any further theoretical development, Poensgen then concentrates on the estimation of the value of conversion option. Having tried at various structural formulations of a regression model for the purpose, he finally showed that a simple regression model with share price, its future growth rate and volatility, bond rating, time of issue and the distance from the conversion price to stock price does a fairly good job of estimating the value of conversion option.
Introduction of all the four moments of the distribution of underlying common stock price truncated at the conversion price is shown to improve the predictive power of the model.

The theoretical analysis of BMQ [7] falls into a similar track to that of Poensgen [76]. There are two basic differences, however. First, BMQ assume that the straight bond value is given and constant, whereas Poensgen allows it to vary. Second, BMQ argue that the convertible bond value is the greater of either of the sums that are said to be equal according to Poensgen. Assuming that the investors would sell their convertible bond holdings after two or three years, and estimating the probability distribution of price relatives from their empirical distribution over a period of two or three years in the past, BMQ have tested the predictive ability of their model with a sample consisting of only seven securities. Their findings indicate that their normative valuation procedure estimates higher values for high-priced issues and systematically underestimates the market prices of low-priced bonds. These unsatisfactory findings could be attributed to, among others, the short horizon, the absence of risk consideration, the inadequate estimation of the distribution parameters of the price-relatives and the assumption of riskless straight bond.

Note that both the studies follow the expected value approach and do not treat the problem of convertible bond valuation within the context of capital market theory. They have ignored the callability feature though virtually all convertible bonds are callable. Further, the riskiness of bonds due to possible default of payment of interest and principal by the issuing corporations is apparently not taken into account by either study.
Brighma's [17] is the only other work of significance prior to 1967. His main contribution is the graphical presentation of the relative movement (over time) of dollar stock price, convertible bond price, call price and the floor value (straight bond). His graphical model recognizes the same underpinnings of the convertible bond valuation as are discovered by Poensgen [76] and BMQ [7]. It further implies that the stock in question is a steady growth stock. But the assumption of steady growth of stock price is restraining to the usefulness of his model, especially in view of the substantial evidence in the literature in support of the stock prices following the random walk hypothesis [30, 91, for example].

While the above studies focussed on the convertible bond valuation with empirical validation kept at a minimum, Weil-Segall-Green (WSG) [95] undertook the task of identifying and testing empirically the significance of the factors determining convertible-bond premiums. Based on the earlier theories and the consensus of practitioners, they initially identify seven factors as the determinants of conversion premium. They are: 1. transaction cost difference; 2. income difference (rate of return); 3. financing costs; 4. anti-dilution clauses; 5. floor variable (difference between convertible-bond price and its straight-bond value); 6. volatility of stock price, and 7. duration. Of these seven, WSG eliminate four factors, nos. 3, 4, 6 and 7 for various reasons and regress premiums against the remaining three factors. The results of their analysis are very poor. They have found the floor variable to be insignificant and the coefficient of transaction-cost difference being negative,
contrary to prior expectations. Though the coefficient of income-difference is found to be statistically significant with the expected sign, it is too small to be subjectively satisfactory. The reasons for the poor results as pointed out by West Largay [98] and others [25, 27, 74] include: poor definition of variables, misspecification of the model and inappropriate selection of the sample.

Jennings [48] and Duvel [26] differ from the earlier studies in that they have utilized the CAPM in deriving and/or estimating the value of a convertible bond. The purpose of Jennings' study is to estimate the premiums vis-a-vis convertible bond valuation. According to him, the expected value of a convertible bond at the end-of-period is equal to the weighted sum of its underlying (straight) bond value and the stock value. The bond value is weighted by the probability that the end-of-period stock value is less than the bond value; one minus this probability is the weight for stock value. The current market value of a convertible bond is the present value of the weighted sum. In the process, the bond value portion is discounted at the risk-free rate of interest and the stock value portion is discounted at the risk-free rate of interest, or the Sharpe's market model estimate of stock discount rate ($E(R)$), depending on whether the current bond price is greater than the stock price, or vice versa.

Conceptually, Jennings' valuation model is not radically different from that of Poensgen, or BMQ. There are two basic differences, however. First, Poensgen and BMQ have not adjusted for the risk of underlying common stock, whereas Jennings has. Second, BMQ have estimated the probabilities based on the empirical distribution of price relatives
in the past, whereas Jennings has estimated the probabilities in terms of the distribution of returns on the market portfolio which is assumed to be normal.

Since his purpose is to estimate premium, Jennings has adjusted the predicted value of a convertible bond for the transaction cost and income differences. The predicted premium is the difference of this adjusted value and the greater of either the current stock or the bond price. For those convertible bonds for which the underlying stock price has exceeded the effective call price, the forced conversion is assumed to occur in the near future and the premium is estimated to be equal to the transaction cost difference and one-half of the income difference. The empirical test of predictive ability, however, has not provided a good support for his model. Note that his model implies that the bondholders will never convert unless they are forced to do so (which is not always true). Further, he has treated the call provision outside the arena of bond valuation, probably because he has limited himself to a single-period analysis.

Unlike Jennings, Duvel [26] has evaluated the convertible bonds in a multi-period setting using dynamic programming and the single-period CAPM. As a dynamic programming problem, he evaluates a convertible bond starting from maturity and going backward one period at a time. At each decision point before maturity, the value of a convertible bond is the greater of either its current stock equivalent value or the present value of a convertible bond, if it is held for one more period of time. At maturity, it is the greater of either the face value of bond, plus one-period interest or the stock-equivalent value at maturity.
While estimating the present value of a convertible bond, Duvel makes three assumptions: (1) the stock prices follow a Markov process, (2) the bondholders decide whether or not to convert the bonds depending on the income differences between the two alternatives, and (3) the dividend is a constant proportion of the beginning stock price. Consequently, the convertible-bond value becomes a function of stock price only. Duvel has used the CAPM only in deriving an appropriate discount rate for the end-of-period value of a convertible bond. This discount rate is a linear combination of the weighted bond yield to maturity and stock discount rate ($E(R)$). With respect to the call provision, he has assumed that the probability of its exercise is 1.0 if the expected stock equivalent value is greater than $1,400, otherwise it is zero. Compared to Jennings', Duvel's empirical results are encouraging in so far as the estimation of bond premiums is concerned. But he has found certain anomalies such as the convertible-bond price predicted as decreasing when the underlying stock price (for some) was increasing. As is clear, Duvel's multiperiod analysis of convertible-bond valuation involves the limited application of the CAPM and the treatment of call provision is, in a naive way, with the a priori specification about its exercise.

Unlike the previous authors, Ingersoll [45] and Brennan-Schwartz [16] have recently discussed the convertible bond valuation, using the Black-Scholes Option Pricing Model. In this framework, the convertible bond value is a function of the firm's value and time. It is defined as a solution to a stochastic differential equation with certain boundary conditions that should simultaneously be satisfied. Since the
capital-asset pricing approach is used in our study, more discussion of these studies is unwarranted. An aspect of their studies which is of interest to us, however, is their proposed call strategy of a firm.

B. **On the Call Strategy of a Firm**

Excepting Ingersoll [45] and Brennan-Schwartz [16], the previous studies have, as noted before, either ignored or treated the call option with the a priori specification about its exercise. It is not stated whether forcing conversion at the stated stock-equivalent value of a convertible bond is in the best interest of the existing stockholders.

Ingersoll [45] and Brennan-Schwartz [16] define the optimal call strategy of a firm as: "(A convertible bond should be called) when the conversion value of the convertible is equal to the effective call price". ([45], p. 301). If the conversion value is less than the call price, it is advisable not to call the bond, otherwise the firm would end up paying unnecessarily more money. On the other hand, if the conversion value is greater than the call price, it is advantageous to issue equity shares instead of calling the bond. So, it is only optimal to call when the conversion value equals the call price.

This call strategy implies that the value of call option is a function of conversion value and call price only. By ignoring the potential role of the convertible-bond price at the time of call, Ingersoll and Brennan-Schwartz have in fact provided an incomplete strategy. If properly perceived, the stockholders are benefitted by the exercise of the call option whenever the convertible-bond value exceeds both the
conversion value and call price because the value foregone by the bondholders (i.e., the convertible-bond value) is greater than what they receive either by conversion or by surrender upon call. Therefore, for an optimal call strategy what is important is not the levels of conversion value and call price, but the distance of the convertible-bond value from the greater of either of these two. In other words, as Gordon Pye [41] has pointed out, for an on-going firm it is appropriate to weigh the benefits of calling now as opposed to the benefits of calling it later. In this context, Ingersoll's [46] conclusion that the firms in the past have sub-optimally exercised their call options is suspect.

In view of the above, Boness-Chen's study [13] is a step in the right direction. They address the issue of determining call option value to the firm at a point in time given a refunding policy. They consider two refunding policies: one, retiring new shares by raising straight debt; two, do not retire new shares.

When a convertible bond is callable, according to Boness-Chen [13], there are four possible combinations of actions by the issuer and the bondholder at each point in time: (a) the firm may call and the bondholder may convert upon call, (b) the firm may call but the bondholder may not convert upon call, (c) the bondholder may voluntarily convert even if it is not called, and (d) the bondholder may not convert but the firm may call at a future time. There is a value of call option associated with each of these combinations and under each refunding
policy, the value of call option to the firm is the maximum of these four values.

This study addresses only one aspect of the problem of convertible bond valuation, that is, the value of call option from the firm's viewpoint. The analysis is carried out in a dynamic programming framework. It is not extended to integrate it with other aspects of convertible-bond valuation. In an integrated valuation process, the loss incurred by the firm due to voluntary conversion, that is, the negative value of third combination of actions can not be attributed to the call option. This is so because the loss arises due to the conversion privilege granted to the bondholders but not because of the existence of call option. Moreover, if we make an assumption that the expected conditions prevailing at the end-of-period hence would fairly represent the conditions prevailing in all subsequent periods, then we can further simplify the derivation of the value of call option. Nevertheless, both the value and the strategy for call need to be defined within the context of an integrated convertible-bond valuation procedure.

C. On the Convertible-Debt Financing vs. Other Means of Financing

Another interesting issue for investigation is: What implications does the convertible-debt-financing have (as opposed to other means of financing) for the firm's value and for the existing stockholders' wealth? Surprisingly, with the exception of Lewellen-Racette [58], the literature is silent on addressing this problem directly.

Lewellen-Racette [58] have analyzed the implications of the convertible-debt-financing as opposed to the straight-debt-financing for
the firm's value and for the existing stockholders' wealth. In the process, they have assumed the existence of the MM world and that the future investments are financed only from retained earnings. The cash flows to each category of security holders and the combined cash flows at each point in time in the future are identified and valued by using the appropriate discount rate as MM have done.

In the no-corporate-tax case, Lewellen-Racette [58] have correctly shown that the levered firm value under the straight-debt-financing is just equal to the unlevered firm value and under the convertible-debt-financing, it is equal to the unlevered firm value plus the value of premium. Assuming that the premium is distributed among the existing stockholders, the stockholders' wealth is shown to be the same under both means of financing. According to them, the conditions are different in the presence of corporate income tax. Both the firm's value and the value of stockholders' equity are reduced under the convertible-debt-financing as opposed to its counterpart. The loss in the value is equal to the present value of lost tax subsidy due to possible conversion prior to maturity plus the present value of tax on the amortized premium amount. This loss is further aggravated if the interest rate on convertible bonds is lower than its counterpart. So their conclusion is that, "the corporate income tax renders convertible bonds a distinctly inefficient source of capital and for a fixed vector of real asset investment plans, the firm's present shareholders are better served by the issuance of (straight) debt obligations on the part of the bondholders". (p. 791)
This conclusion contrasts with the general belief as well as with the prevailing practice. Brigham's [17] survey showed that the majority of the firms could afford to issue either straight bonds or common stock, still they preferred to issue convertible bonds. Commenting on Lewellen-Racette [58] recently, Livingston [61] has correctly shown that if the firm issues either par or discount convertible-bonds, and if a sufficient amount of debt is raised upon conversion to negate the loss of tax subsidy (since nothing would prevent the firm from doing it), then the convertible-debt-financing does not reduce either the firm's value or the value of existing stockholders' equity. These values are also independent of any differences (relative to the premium bonds that are the subject of discussion by Lewellen-Racette [58]) in the interest rates, conversion premiums and in the proportions of stockholders' equity transferred upon conversion. Hence, their conclusion is not valid. Moreover, their analysis is confined to the case of non-callable convertible-debt-financing.

D. A Summary of Criticisms

From what we have understood from the above review, it is clear that the application of the capital-asset-pricing approach to the evaluation of convertible-debt-financing has so far been scanty. Most of the theoretical valuation models refer to the non-callable convertible bonds. Those studies that did consider the call option, have either made a priori specification about its exercise or have considered it inadequately. None of the existing estimation models for the value of convertible bonds is proved to be satisfactory. Furthermore, the effects
of potential costly bankruptcy and the firm's capital structure plan upon conversion on the convertible-bond value are largely unexplored.

We have addressed in this study these very unresolved issues.
Footnotes to Chapter II

1. This is true at least in recent years. The following table shows the number of convertible securities listed on the NYSE and AMEX by category as of June 30th of the respective years.

<table>
<thead>
<tr>
<th>YEAR</th>
<th>NEW YORK STOCK EXCHANGE</th>
<th>AMERICAN STOCK EXCHANGE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
<td>Callable</td>
</tr>
<tr>
<td>1976</td>
<td>353</td>
<td>353</td>
</tr>
<tr>
<td>1975</td>
<td>332</td>
<td>331</td>
</tr>
<tr>
<td>1974</td>
<td>335</td>
<td>334</td>
</tr>
<tr>
<td>1973</td>
<td>340</td>
<td>338</td>
</tr>
<tr>
<td>1972</td>
<td>333</td>
<td>333</td>
</tr>
</tbody>
</table>

(Source: Moody's Bond Records)

2. The graphical presentation of Brigham's model is given below:

3. The coefficient of determination for the test of association between actual and predicted dollar premiums is only 0.38 and for the association of relative premiums (dollar premiums divided by the larger of conversion or convertible-bond value) is 0.415. The coefficient of determination for the test of linear relationship is 0.38 and the coefficients of constant term and predicted premium are significantly different from the desired zero and one.

4. Duvel defines the appropriate rate for discounting the end-of-period value of a convertible bond as follows:
Appropriate discount rate = $\alpha E(\tilde{R})^{-1} + (1-\alpha)r^*_{-1}$

where,

$$
= \frac{P_j - P_{\min}}{1400 - P_{\min}}
$$

$E(\tilde{R})$ = Sharpe's market model estimate of stock discount rate assuming log-normal distribution of stock price relatives and with an adjustment for changes over time in the riskfree rate of interest.

$r^*_{-1}$ = equivalent straight-bond yield to maturity estimated by Moody's for a given convertible bond.

$P_j$ = a possible end-of-period stock price that lies between $P_{\min}$ and $\$1,400$, and

$P_{\min}$ = is the minimum stock price so low that once this level is reached, it would imply that the stock-equivalent value of the bond will never exceed its equivalent straight-debt value at expiration. This minimum level is defined as the level that is four standard deviations below the expiration value of the convertible bond.

5. This a priori specification is based on a study of called convertibles by Williams and Letwat [1963]. They have found out that for a sample of convertibles, the highs of underlying stock values in the week before the bonds were called ranged from 113 to 434 percent of par and were log-normally distributed. Since the mean of this distribution is approximately 140 percent of par, $\$1,400$. is taken to be the cut-off point for forced conversion.

6. The coefficient of determination for the test of linear relationship is 0.89. However, the longer the duration under consideration, the more are the cases of anomaly of predicted bond prices decreasing when the stock values of the bonds are increasing. For ten years to expiration, such cases are twice as that for five years to expiration. Since the convertible bonds are issued usually with 20 to 30 years maturity period, the utility of Duvel's model during the first, at least 10 - 15 years is therefore suspect.
CHAPTER III
A SINGLE-PERIOD EVALUATION OF
CONVERTIBLE-DEBIT-FINANCING

In this chapter, the convertible-debt-financing is evaluated in
the single-period CAPM framework that incorporates both the corporate
income tax and the costly bankruptcy. Modigliani-Miller's [66, 67] pro-
position of financial structure irrelevance, their tax model and
Lewellen-Racette's [58] convertible-debt inferiority proposition are
re-examined within this context. Our single-period convertible bond
valuation model is contrasted with the one suggested by Jennings [48].
Though it is our primary objective here, we have also examined the
implications of convertible-debt-financing for the firm's cost of
capital in a single-period setting.

A few points deserve their mentioning at the outset. In a single-
period valuation framework, the implied assumption is that a firm
starts its business anew at the beginning of each period and will
terminate its business at the end of each period. So, the end-of-period
cash flows of the firm include not only the operating earnings during
the period but also the liquidation value of real assets. It is
generally assumed that there is no cost associated with the termination
of business in the absence of bankruptcy. Secondly, in this framework,
the end-of-period value of call option is zero because the convertible
bonds are assumed to mature at the end of the period and at maturity the value of call option is zero. So, the call option is meaningless right in the single-period framework. Hence, the valuation models derived in this chapter do not capture the effects of call option on the value of outstanding securities. Thirdly, unlike Jennings, we have used the cash flow version of the single-period CAPM in deriving valuation models for the outstanding securities. It states that the equilibrium market value of a risky asset is equal to the present value of certainty-equivalent of the end-of-period total cash flows to the owners of that asset discounted at the one-period riskless rate of interest.

In Section A, the assumptions underlying our valuation procedure are described initially. The single-period definitions of bankruptcy and conversion are set forth. What follows in this section then is the analysis of the market value implications of the different means of financing investments and the re-examination of the previously stated propositions. In Section B, our single-period convertible bond valuation model is contrasted with the Jennings model to highlight the differences. And, we have examined in Section C, the implications of convertible-debt financing for the firm's cost of capital. A note on the limitations of our single-period analysis then follows.

A.1 Assumptions

Besides the assumptions underlying the S-L-M's CAPM (i.e. existence of riskless rate of interest, homogenous expectations,
risk-averse and single-period expected utility of terminal wealth maximizing investors in the capital market etc.), we make a few more assumptions in deriving the valuation models in this chapter. It is assumed that the firm's outstanding securities at the beginning of the period include only common stock and the firm has three alternatives to finance its investments during the period: i) by retained earnings, ii) by issuing straight bonds, and iii) by issuing convertible bonds. The firm's investment decisions are given and known to the investors. The corporate income tax rate is given and constant over the period. The bonds are risky due to possible bankruptcy. In case they are converted, all the convertible bonds are converted at one time. Further, no change is assumed in the conversion ratio during the period. The conversion ratio \( \alpha \) is the ratio of total number of common shares issued in conversion exchange to the total number of outstanding common shares after conversion. It is also assumed that the firm's security holders have limited liabilities.

A.2.a Single-Period Definition of Bankruptcy

Two of the key elements of our valuation procedure are the possible bankruptcy and the possible conversion of convertible bonds.

Bankruptcy is a situation where the firm becomes unable to meet its debt obligations. So, we accept for bankruptcy consideration the definition given by Stiglitz [88]. He defines bankruptcy as "(in a single-period case) a situation where the income of the firm is less
than the fixed obligations to bondholders" (p. 459). In terms of cash flows, this definition implies that bankruptcy occurs when the end-of-period cash flows of the firm are less than the contractual commitments to bondholders that include the principal amount of debt and accrued interest. For an all equity firm, it may be interpreted in two ways: first, an all equity firm has zero fixed obligations to bondholders; hence, it would go bankrupt if its end-of-period cash flows are less than zero; second, the question of an all equity firm going bankrupt does not arise because it does not have any fixed obligations to bondholders. The second interpretation is consistent with our assumption of limited liability of security holders. So, it is assumed that an all-equity firm does not declare bankrupt even if its end-of-period cash flows are negative.

The presence or absence of corporate tax makes the difference in the precise definition of bankruptcy obviously because the cash flows available to security holders are affected by the existence of corporate tax. Since we intend to recognize the existence of corporate income tax, the bankruptcy is, in precise terms, defined as a situation where the firm's before-tax cash flows are less than the accrued interest plus the contractual principal amount of debt expressed in terms of before-tax cash flows. Symbolically, the bankruptcy occurs when

\[(1 - \theta)[X - I] < D\]

i.e. \[X < I + D/(1 - \theta) \quad (\equiv b)\]  

(1)
where $X$ represents the end-of-period realized cash flows of the firm, $I$, the one-period interest on debt, $D$, the contractual principal amount of debt and $\theta$, the corporate income tax rate. Note that we assume for simplicity's sake that all the terminal cash flows of the firm net of interest on debt are taxable at the corporate income tax rate.\(^3\)

A firm may be reorganized or liquidated upon bankruptcy. But in a single-period framework, the question of considering a bankruptcy-reorganization case does not arise. So, our analysis is confined to the bankruptcy-liquidation case. Certain costs are associated with the bankruptcy-liquidation of a firm. As Kim [55] has correctly identified, these costs include: (i) loss due to liquidation of physical assets, (ii) the expenses arising from the bankruptcy proceedings such as the fees paid to lawyers, trustees, appraisers etc., and (iii) the loss of tax credits. Since tax credits are not included in the gross returns to bondholders of the bankrupt firm, as Kim [55] and Chen [20], we consider bankruptcy cost as comprising of the first two components. Letting $\tilde{B}$ denote the stochastic bankruptcy cost, it may be specified as follows:

\[
\tilde{B} = \begin{cases} 
0 & \text{if } X \geq I + D / (1 - \theta) \\
\tilde{B} & \text{if } X < I + D / (1 - \theta) 
\end{cases}
\]

(2)

The consideration of bankruptcy cost is relevant because of the evidence that it forms a significant portion of the terminal proceeds of a firm.\(^4\)
A.2.b. Single-Period Definition of Conversion

The conversion is an option available to the convertible bondholders to convert their bonds into a certain number of the shares of common stock according to mutually agreed and pre-determined ratio. The indenture agreement generally provides for a period of time during which this option is exercisable. It is assumed in our analysis that it is exercisable at anytime during the life of the bonds. However, the rational bondholders exercise this option at a time when they are better off (in terms of their terminal wealth) by converting than surrendering instead.

So, in precise terms, the convertible bondholders convert their holdings into common stock if the end-of-period proportionate cash flows they would receive due to their present conversion exceed the principal amount of debt plus one-period promised interest that would otherwise be available to them at the end-of-period. In the circumstance that the firm issues convertible bonds to finance its investments, its outstanding securities, after the financing decision, include the existing common stock and newly issued convertible bonds. In case the holders of convertible bonds decide to convert, the firm loses the benefit of tax deductability of interest payments. Consequently, the proportionate cash flows available to the bondholders (new stockholders) at the end-of-period are equal to the equivalent
unlevered firm's after-tax cash flows times the conversion ratio. It is this amount which should exceed the total promised debt payments for exercising the conversion option. Symbolically, the conversion takes place if

\[ \alpha (1 - \delta) X > I + D \]

i.e. \[ X > \frac{I + D}{\alpha (1 - \delta)} (\equiv NC) \] (3)

A.3 The Analysis

Given the above assumptions and definitions, in what follows is the analysis of the implications of the previously stated three alternative means of financing investments: a) financing by retained earnings, b) by issuing straight bonds, or c) by issuing convertible bonds. The analysis explores the possible effects on the firm's value and the existing stockholders' wealth.

a) Financing by Retained Earnings

If the firm chooses to finance its investments by retained earnings, then we have an all-equity firm even after the financing decision. All the end-of-period cash flows of the firm belong to the existing common stockholders. In the presence of corporate income tax, these cash flows equal \((1 - \delta)\tilde{X}\) when \(X > 0\). Note that the tilde \((\tilde{\cdot})\) denotes the stochastic nature of the variable \((X\) here). However, when \(X < 0\), the cash flows to common stockholders are only zero but never negative because they can exercise their limited liability right. Since the firm does not have any fixed obligations, the question of
considering bankruptcy risk and associated cost does not arise. Hence, there is no drain on the firm's cash flows other than the payment of income tax in case X is positive.

So, the end-of-period after-tax cash flows of the unlevered firm, \( \tilde{Y}_u \), may be specified as:

\[
\tilde{Y}_u = \begin{cases} 
(1-\theta)X & \text{if } X > 0 \\
0 & \text{if } X \leq 0 
\end{cases}
\]  

(4)

Using the cash flow version of the CAPM and assuming that \( \tilde{X} \) is jointly normally distributed with the rate of return on the market portfolio, \( R_m \), the equilibrium market value of the unlevered firm, \( V_U \), is derived as in (5):

\[
V_U = (r^{-1}) \left[ E(\tilde{Y}_u) - \lambda \text{Cov}(\tilde{Y}_u, R_m) \right] = (r^{-1}) \{(1-\theta)[E(\tilde{X}) - \lambda \text{Cov}(\tilde{X}, R_m)]\} \]  

(5)

where

\[
r = 1 + R_f, \text{ one plus the riskless rate of return}
\]

\[
E_0(\tilde{X}) = \int_0^{\infty} \tilde{X} f(\tilde{X}) d\tilde{X}, \text{the partial expectation of } \tilde{X} \text{ (truncated from zero upward)}
\]

\[
\lambda = \left[ E(R_m) - R_f \right]/\sigma_m^2, \text{the market price of risk}
\]

\[
\text{Cov}_0(\tilde{X}, R_m) = E[(\tilde{X}_0 - E_0(\tilde{X}))(R_m - E(R_m))]; \text{the partial covariance between } \tilde{X} \text{ (truncated from zero upward) and } R_m
\]
Letting $\text{CEO}_0(\tilde{X})$ denote the certainty equivalent of the firm's end-of-period cash flows, $\tilde{X}$ (truncated from zero upward), (5) is rewritten as in (6)

$$V_U = (r^{-1})(1-\theta)\text{CEO}_0(\tilde{X})$$

Thus, (6) suggests that the equilibrium market value of the unlevered firm is equal to the present value of the certainty-equivalent of the firm's end-of-period after-tax cash flows (truncated from zero upward). This in itself is the present value of the existing stockholders' wealth when the investments are financed by retained earnings.

b) **Straight-Debt-Financing**

In case the firm decides to issue straight bonds to finance its investments, then after the financing decision, the firm becomes a levered firm with positive fixed obligations to bondholders. So, the consideration of possible bankruptcy and associated cost becomes relevant for deriving the market value of outstanding securities and that of the firm as a whole. The market value of a levered firm is simply the sum total of the market values of its equity and debt. As such, in the following, the market values of equity and straight-debt are separately derived and the firm's value is obtained by summing up the market values of equity and straight-debt.

i) **Market Value of Equity**

With the possibility of bankruptcy, the market value of equity depends on whether or not the firm remains solvent at the end of the period. In other words, we have to consider two possible states
- no bankruptcy or bankruptcy - either of which the firm would be in, depending on the level of its end-of-period realized cash flows vis-a-vis its fixed obligations. If the firm's end-of-period realized cash flows are greater than the promised interest amount, plus the contractual principal amount of debt expressed in terms of the before tax cash flows, i.e., \( X > I + D/(1-\theta) \), then the firm remains solvent. In this situation, the end-of-period cash flows available to the common stockholders equal the net cash flows after taxes, promised interest and principal amount of straight debt. In an otherwise situation, the firm is declared bankrupt and the common stockholders neither receive anything nor do they bear the deficit of the firm's cash flows to meet fully the fixed obligations. This is so because of two reasons: 1) the bondholders have a preferential claim on the firm's cash flows after taxes and bankruptcy cost, and 2) the common stockholders have a limited liability.

So, the end-of-period after-tax cash flows available to the common stockholders, \( \tilde{Y}_E \), may be specified as follows:

\[
\tilde{Y}_E = \begin{cases} 
(1-\theta)(X-I) - D & \text{if } X > I + D/(1-\theta) \\
0 & \text{otherwise}
\end{cases} \quad (\equiv b)
\]

The equilibrium market value of equity, \( V_E \), therefore, is the present value of the certainty equivalent of \( \tilde{Y}_E \):

\[
V_E = (r^{-1}) \left[ E(\tilde{Y}_E) - \lambda \text{Cov}(\tilde{Y}_E, R_m) \right]
\]

\[
= (r^{-1}) \left\{ (1-\theta)\text{CEQ}_b(X) - [I(1-\theta)+D][1-F(b)] \right\} \quad (8)
\]
where \( F(b) = \int_{-\infty}^{b} f(x) \, dx \), the probability of bankruptcy

\[
CEQ_b(\tilde{X}) = E_b(\tilde{X}) - \lambda \text{Cov}_b(\tilde{X}, \tilde{R}_m),
\]

the certainty-equivalent of the firm's end-of-period cash flows (truncated from \( b \) upward).

Thus, the market value of equity is equal to the present value of the firm's after tax cash flows providing that it remains solvent at the end-of-period minus the present value of the (tax-benefit adjusted) debt payments made when the firm remains solvent.

ii) Market Value of Straight-Debt

As is true for equity, the market value of straight debt also depends on the state – no bankruptcy or bankruptcy – that the firm would be in. If the firm remains solvent (no bankruptcy) at the end of the period, the straight bondholders receive the promised interest plus the contractual principal amount of debt. On the other hand, if it is declared bankrupt, they receive the entire firm's cash flows after taxes and bankruptcy costs. Because the stockholders have limited liability, the bankruptcy costs (assumed to be tax deductible) shall have to be borne by the bondholders to whom the ownership of the firm is transferred upon bankruptcy. It is possible that the firm's end-of-period cash flows may be so low as to be inadequate even to fully meet the bankruptcy costs. Under these circumstances, the bondholders exercise their limited liability and will receive nothing. However, for simplicity of presentation, we ignore this possibility.

So, the end-of-period cash flows available to the straight bondholders, \( \tilde{Y}_{DS} \), may be expressed as follows:
\[ Y_{DS} = \begin{cases} I + D & \text{if } X > I+D/1-\theta \\ (1-\theta)(X-B) + I\theta & \text{otherwise} \end{cases} \]

The market value of straight debt, \( V_{DS} \), therefore is:

\[
V_{DS} = (r^{-1})[E(Y_{DS}) - \lambda \text{ Cov}(Y_{DS}, \tilde{R}_m)]
\]

\[
= (r^{-1}) \left\{ [I+D][1-F(b)] + (1-\theta)CEQ^b(X) + \theta IF(b) \right\} - (1-\theta)CEQ(B)
\]

where,

\( CEQ^b(X) \) = the certainty-equivalent of the firm's end-of-period cash flows (truncated from \( b \) downward).

\( CEQ(B) = E(B) - \lambda \text{ Cov}(B, \tilde{R}_m) \), the certainty-equivalent of the firm's bankruptcy costs. Note that \( B = 0 \) if \( X > b \) is implicit in this expression.

Thus (8) indicates that with the possible costly bankruptcy, the market value of straight debt equals a) the present value of total promised payments times the probability that the firm remains solvent plus b) the present value of the firm's certainty-equivalent after tax cash flows when the firm is in the bankruptcy state, plus c) the present value of the tax deductibility of interest payments times the probability of bankruptcy, minus d) the present value of the certainty-equivalent after tax bankruptcy costs. In the absence of bankruptcy risk and bankruptcy cost, the market value of straight debt is, however, simply the present value of total promised payments because \( F(b) = 0 \) and all the three terms within the braces on the RHS of (10) drop out. Given that the riskless rate of interest exists,
it indicates that the straight bonds are risky only if bankruptcy is possible.

**iii) Market Value of the Levered Firm**

As previously noted, the market value of the levered firm can be obtained by summing up the market values of equity and debt. So, by summing up $V_E$ in (8) and $V_{DS}$ in (10), the market value of the levered firm under straight-debt financing, $V_{LS}$, can be expressed as:

$$V_{LS} = (r^{-1}) \{ (1-\Theta)CEQ(\tilde{X}) + I^\Theta - (1-\Theta)CEQ(\tilde{B}) \}$$

$$= V_U + (r^{-1})I^\Theta - V(\tilde{B})$$  \hspace{1cm} (11)

where $V(\tilde{B})$ is the present value of the certainty-equivalent bankruptcy cost.

Thus, the market value of the levered firm under straight-debt financing is equal to its equivalent unlevered firm value plus the present value of tax deductibility of interest payments minus the present value of the risk-adjusted bankruptcy cost. In the absence of corporate income tax and bankruptcy cost, $V_{LS} = V_U$. So, it reinforces the M-M proposition that the financial structure is irrelevant for the firm's value. If the bankruptcy is costless, i.e., $V(\tilde{B}) = 0$, then (11) suggests that $V_{LS} = V_U + (r^{-1})I^\Theta$, i.e., market value of the levered firm is a positive function of interest payments implying that always more debt is preferable to less. This reinforces the MM tax model that a firm should use as much leverage as it can. It is also obvious from (11) that a costless bankruptcy does not affect the firm's value.
Thus, the existence of costly bankruptcy is one of the critical elements of the corporate capital structure theory. All these results are similar to what Kim [55] and Chen [20] have derived; hence, they are not new. The purpose of this analysis, however, is evident later when we examine the implications of convertible-debt-financing relative to the just discussed two means of financing.

c) **Convertible-Debt-Financing**

If the firm chooses to finance its investments by issuing convertible bonds, then we require to consider another dimension of valuation and that is the possible conversion of bonds into underlying common stock. Because of the dual features of convertible bonds, the derivation of the market value of outstanding securities gets somewhat complicated. The market value of the levered firm, however, can still be obtained by simply summing up the market values of equity and convertible debt.

i) **Market Value of Equity**

With the possible bankruptcy and the possible conversion of bonds, there are three state-option combinations either of which would prevail at the end of the period. They are: 1) The firm remains solvent and the bondholders convert their bonds into common stock. 2) The firm remains solvent but the bondholders do not convert, and 3) The Firm is declared bankrupt and the bondholders do not convert.

If the bonds are converted, the existing common stockholders shall have to share the firm's end-of-period cash flows with the bondholders (new common stockholders) in proportion to their respective holdings.
of the common shares. Since the conversion transforms the levered firm into an unlevered firm, under these circumstances, the firm's cash flows are simply equal to the after tax cash flows of an equivalent unlevered firm. The share of the existing common stockholders in the firm's cash flows then equals \( (1-\alpha) \), hence the end-of-period cash flows to them would be \( (1-\alpha)(1-\beta)\tilde{X} \) when \( X > [I+D]/\alpha(1-\beta) \). Recall the definition of conversion given in (3). If the bonds are not converted and the firm remains solvent, then the stockholders receive the firm's entire cash flows remaining after the payment of taxes and the total contractual obligation to the convertible bondholders. To simplify the comparison of convertible-debt-financing with the straight-debt-financing later in the chapter, it is assumed that the interest rate and the principal amount of debt are the same under both means of financing. We know from (3) that the bondholders do not convert if \( X \leq [I+D]/\alpha(1-\beta) \) \( (=NC) \) and from (1) that the firm remains solvent if \( X > I+D/1-\beta \) \( (=b) \). This means the second state-option combination occurs when \( b \leq X \leq NC \). In this case, the common stockholders receive \( (1-\beta)(X-I)-D \). In case the firm is declared bankrupt, the common stockholders receive nothing because the bondholders do not convert their holdings and exercise their prior claim as debt holders.

Thus, the end-of-period cash flows of the existing common stockholders may be
\[
\tilde{Y}_E = \begin{cases} 
(1-\alpha)(1-\beta)\tilde{X} & \text{if } X > NC \\
(1-\alpha)(1-\beta)\tilde{X} & \text{if } X > NC \\
(1-\beta)(X-I)-D & \text{if } b \leq X \leq NC \\
0 & \text{if } X < b 
\end{cases} \text{ conversion}
\]

\( \text{No conversion} \) \( \text{No bankruptcy} \) \( \text{bankruptcy} \) (12)
The market value of equity under convertible-debt-financing, therefore, is:

\[
V_E = (r^{-1}) \left[ E(\tilde{Y}_E) - \lambda \text{Cov}(\tilde{Y}_E, \tilde{R}_m) \right]
\]

\[
= (r^{-1}) \left\{ (1-\alpha)(1-\Theta)\text{CEQ}_{NC}(\tilde{x}) + (1-\Theta)\text{CEQ}_{b}^{NC}(\tilde{x}) 
- [1/(1-\Theta)+D] (F(NC)-F(b)) \right\}
\]

(13)

where

\[
F(NC) = \int_{-\infty}^{NC} f(x) \, dx \quad \text{the probability that the bondholders do not convert.}
\]

\[
\text{CEQ}_{NC}(\tilde{x}) = \text{the certainty-equivalent of the firm's end-of-period cash flows (truncated from NC upward)}
\]

\[
\text{CEQ}_{b}^{NC}(\tilde{x}) = \text{the certainty equivalent of the firm's end-of-period cash flows being in between b and NC.}
\]

The first term within the braces on the RHS of (13) is the proportionate certainty-equivalent of the firm's after-tax cash flows available to the stockholders when the bonds are converted. The second term is the certainty-equivalent after-tax cash flows of the firm when the firm remains solvent but the bonds are not converted. And, the third term is the tax-benefit adjusted total contractual obligations honored times the probability that neither the bonds are converted nor is the firm declared bankrupt.

11) **Market Value of Convertible-Debt**

From (3), we know that the condition to convert exists when \( X > NC \). Under these circumstances, the convertible bondholders convert
their holdings and receive $\alpha(1-\theta)X$ where $\alpha$ is their share in the firm's wealth after conversion. If $X \leq NC$, however, the firm remains solvent, i.e. $X > b$, then the convertible bondholders' end-of-period cash flow equals the promised interest plus the contractual principal amount of debt. In case the firm is declared bankrupt, their end-of-period cash flow is equal to the entire firm's cash flow remaining after payment of taxes and bankruptcy cost. Since the convertible bondholders are also assumed to have a limited liability, they receive nothing if the cash flow is insufficient to meet fully the bankruptcy cost. However, we continue to assume that these conditions do not arise, hence, it is ignored here as well.

To summarize, the end-of-period cash flow available to the convertible bondholders $\tilde{Y}_{DC}$ can be specified as:

$$\tilde{Y}_{DC} = \begin{cases} \alpha (1-\theta)X & \text{if } X > NC \\ I + D & \text{if } b \leq X \leq NC \\ (1-\theta)(\tilde{X}-\tilde{B}) + IO & \text{if } X < b \end{cases}$$

(14)

So the market value of the convertible-debt is:

$$V_{DC} = (r^{-1})[E(\tilde{Y}_{DC}) - \text{Cov}(\tilde{Y}_{DC}, \tilde{R}_m)]$$

$$= (r^{-1})\{\alpha(1-\theta)CEQ_{NC}(\tilde{X}) + [I+D][F(NC)-F(b)]$$

$$+ (1-\theta)CEQ_b(\tilde{X}) + IOF(b)-(1-\theta)CEQ(\tilde{B}) \}$$

(15)

The first term within the braces on the RHS of (15) is the bondholders' claim in the firm's certainty-equivalent after tax cash flow when the bonds are converted. The second term is their contractual claim times the probability that neither the bonds are converted nor
is the firm declared bankrupt. The third and fourth terms together represent the certainty-equivalent after tax cash flow of the firm in bankruptcy including the savings due to tax deductibility of interest payments. And, the last term is the risk-adjusted after-tax bankruptcy cost to be borne by the convertible bondholders.

iii) Market Value of the Levered Firm

By summing $V_E$ in (13) and $V_{DC}$ in (15), we obtain $V_{LC}$, the market value of the levered firm under convertible-debt-financing.

So,

$$V_{LC} = (r^{-1}) \{ (1-\theta)CEQ_0\tilde{X} + I\theta F(NC)-(1-\theta)CEQ(\tilde{B}) \}$$

$$= V_U + (r^{-1})I\theta F(NC) - (1-\theta)V(\tilde{B})$$

(16)

Thus, (16) indicates that the firm's market value under convertible debt-financing is equal to its equivalent unlevered firm value plus the present value of the tax deductibility of interest payments times the probability that the bonds are not converted minus the present value of the risk-adjusted after-tax bankruptcy cost.

iv) Re-Examination of the M-M and Lewellen-Racette Propositions

In view of the above analysis of the implications of convertible-debt-financing for the market value of a firm and its outstanding securities, we re-examine the M-M proposition of financial structure irrelevance, their tax model and Lewellen-Racette's convertible-debt-inferiority proposition.

In the absence of both the corporate income tax and bankruptcy costs, equation (16) reduces to $V_{LC} = V_U$. That means, the levered firm
value under convertible-debt-financing as well is simply equal to its equivalent unlevered firm value. So, the M-M proposition that the firm value is independent of its financial structure holds as long as there are no bankruptcy costs. There could exist a positive probability of bankruptcy, however. Since the costless bankruptcy does not entail any drain on the firm's cash flow, it does not affect the market value of the firm. But it does affect the market value of outstanding securities because the end-of-period cash flow available to different security holders is a function of the probability of bankruptcy. Thus, the M-M proposition of financial structure irrelevance holds in a taxless corporate income world with costless bankruptcy, no matter whether the debt in a firm's financial structure is straight or convertible.

From (16), we observe that in the presence of corporate income tax, but costless bankruptcy, the firm value under convertible-debt-financing is equal to its equivalent unlevered firm value plus the present value of the tax deductibility of interest payments times the probability that the bonds are not converted, i.e. $V_{LC} = V_U + (r^{-1}) I 0 F(NC)$. So, the M-M tax model stating that a firm should make maximum use of debt in the presence of corporate income tax is upheld provided, however, that the probability that the bonds are not converted, i.e. $F(NC)$, decreases at lower than the rate at which the promised interest payments increase at a subsequent higher level of debt. Given the mean and standard deviation of the firm's cash flow, $F(NC)$ is a direct positive function of the total contractual debt payments ($I + D$) and is an inverse
function of the conversion ratio, α (recall the definition of conversion given in (3)). So, F(NC) decreases when the rate of change in α with respect to D is greater than the rate of change in I+D. In other words, if more sweetener is added (by reducing conversion price) to sell the convertible bonds of higher total principal amount at the same promised interest rate such that the above condition holds, then F(NC) decreases. If it is further true that the decline in F(NC) more than negates the increased tax benefit, then the firm value is certainly lower at the given higher level of debt compared to its preceding level.

A hypothetical example makes clear the above arguments. Suppose a firm has 15,000 common shares outstanding and the current market price per share is $30. Consider two plans: (1) The firm raises $350,000 convertible debt at par with 10 percent promised interest rate and $56 conversion price per share to finance investments during the period. (2) The firm raises $500,000 convertible debt at par with 10 percent promised interest rate and $35.71 conversion price per share and uses the extra $150,000 to repurchase common shares at the current market price. Suppose further that the mean and standard deviation of the firm's end-of-period cash flow would be $1 million and $1.5 million respectively, given the investment decision.

Under plan no. 1, the conversion ratio

\[ \alpha = \frac{6250}{21250} = 0.294, \quad I + D = 440,000. \]

Let the corporate income tax rate be 0.5. So, assuming that the firm's cash flow is normally distributed, then
\[ F(\text{NC}) = F\left( \frac{440}{0.294 \times 0.5 - 1000} \right) = 0.908 \]

Hence the tax benefits would amount to:

\[ 10F(\text{NC}) = 40,000 \times 0.5 \times 0.908 = $18,160 \]

Under plan no. 2, the conversion ratio,

\[ \alpha = \frac{14000}{24000} = 0.583 \text{ and } I + D = $550,000. \]

So,

\[ F(\text{NC}) = F\left( \frac{550}{0.583 \times 0.5 - 1000} \right) = 0.722 \text{ and} \]

the tax benefits would amount to

\[ 10F(\text{NC}) = 50,000 \times 0.5 \times 0.722 = $18,050. \]

Therefore, the market value of the firm under the second plan, which involves more use of convertible debt, is lower than that under the first plan. This is because of the decline in the F(\text{NC}), caused by the sharp decline in the conversion price. However, there is no reason why the firm should not promise a higher rate of interest on the $500,000 debt instead of reducing the conversion price. This is especially so when it can afford to raise the same amount of straight debt at a higher than 10 percent promised interest rate. In fact, the rational management promises a higher rate of interest rather than reducing the conversion price so low that it involves a threat of loss of their control. When a higher rate of interest is promised, the market value of the firm increases because of the increase in both the total promised interest amount as well as the F(\text{NC}).
In view of the above arguments, it may be concluded that the M-M tax model holds for convertible-debt-financing when bankruptcy is costless and management either promises a higher rate of interest at higher levels of convertible-debt or does not reduce the conversion price such as to more than negate the incremental tax benefits vis-a-vis a decline in F(NC). Once the costliness of bankruptcy is recognized, it is fairly obvious that the M-M tax model does not hold. This is so because with the increase in convertible-debt, given the rational management policy, two offsetting forces set in - an increase in the tax shelter due to the tax deductibility of interest payments, and an increase in the bankruptcy cost. Kim [55] has shown in the case of straight-debt that at some level below 100 percent debt in the financial structure beyond which the increase in bankruptcy cost is greater than the increase in tax shelter. Given the same total promised debt payments under both means of debt financing, it certainly holds for convertible-debt-financing as well because generally F(NC) < 1 except in bankruptcy (when F(NC) = 1) causing lower tax shelter under convertible-debt-financing as opposed to straight-debt financing.

To re-examine Lewellen-Racette's proposition that convertible-debt-financing is less efficient than straight-debt financing, we subtract \( V_{LS} \) given in (11) from \( V_{LC} \) given in (14) to determine the differential impact of convertible-debt-financing on the firm value.
Thus, we observe that the market value of the levered firm under convertible-debt-financing is less than under its counterpart straight-debt-financing by the amount shown in (17). The loss of firm value is equal to the present value of the tax shelter lost upon conversion. So, if there is a positive probability of conversion (i.e., \(1 - F(NC) > 0\)), the amount of tax shelter is smaller, hence the firm value is lower under convertible-debt-financing than under its counterpart. The higher the probability of conversion, the larger is the decline in the firm's value. Thus, Lewellen-Racette's proposition is upheld even in the presence of costly bankruptcy and even if we do not consider the tax paid on the amortized premium. Due to our assumption of the same promised interest rate and principal amount of debt, the costly bankruptcy does not have any differential impact on the firm's value.

Let us relax this assumption for the moment and consider the case of lower promised interest rate on convertible bonds. Let \(\delta\) represent the difference in the one-period interest on straight debt and convertible debt. The contractual principal amount is, however, assumed to be the same. If the bankruptcy is costless, then the differential impact of convertible debt financing on the firm value is:

\[
-(r^{-1} \Theta I [1-F(NC)]) - (r^{-1} \Theta \delta F(NC)), \text{ where } F(NC) < F(NC), \quad (18)
\]
Thus, the less in the firm's value is much larger than in the previous case and the additional loss comes from the difference in the tax deductibility of interest payments, and small F(NC). In other words, the lower interest rate on the convertible-debt-financing makes it a much more inefficient means of financing relative to straight-debt-financing. So, Lewellen-Racette's proposition is upheld again. This is also true when the promised interest rate on convertible-debt is smaller but the total contractual debt payments (i.e., I + D) are the same under both means of debt financing. Under these circumstances, the differential impact equals (18) with F(NC) replaced by F(NC), which reinforces our conclusion.

But with the costly bankruptcy the validity of Lewellen-Racette's proposition in the case of lower promised interest rate on convertible-debt is no more clear. If the firm raises convertible-debt at lower interest rate such that I + D is the same as under straight-debt alternative, then the costly bankruptcy does not have any differential impact. The reason is that the bankruptcy risk and the associated cost are the same under both means of debt financing. Hence, the proposition holds. If instead only (I) is the same, then I + D is smaller, hence the probability of bankruptcy is lower under convertible-debt than under straight-debt-financing. Since this causes a reduction in bankruptcy cost, V(B), there is some advantage in issuing convertible bonds at lower interest rate. Whether this advantage far outweights the loss of the tax shelter depends on the firm specific conditions with respect to tax rate, bankruptcy cost and the probability that the bonds will be converted.
By subtracting $V_E$ given in (8) from $V_E'$, given in (13), we obtain the differential impact of convertible-debt-financing on the stockholders' wealth. This differential impact, $\Delta V_E$, is:

$$\Delta V_E = (r^{-1})(1+D)[1-F(NC)] - (r^{-1}) \alpha (1-\Theta)CEQ_{NC}(\tilde{X})$$

$$\quad - (r^{-1})I\Theta[1-F(NC)]$$  

(19)

The first two terms on the RHS of (19) are the present value of interest and the principal amount of debt, which the stockholders are saved from, and the present value of cash flow relinquished by the stockholders in the conversion exchange respectively. Since the bondholders do not convert unless it amounts to an increase in their wealth, the second term is greater than the first term, when $1 - F(NC) > 0$. The last term on the RHS of (17) indicates that in the presence of corporate income tax, the stockholders' wealth is further reduced by the amount of tax shelter lost upon conversion. The net loss of stockholders' wealth is not given by (19), however. This is so because the convertible bondholders pay some premium for the conversion privilege granted to them, and like Lewellen-Racette [58], the premium may be assumed to have been distributed among the existing stockholders. Consequently, the net loss of stockholders' wealth equals (19), minus the value of the conversion premium. The market value of the conversion premium is expressed by (20), by subtracting $V_{DS}$ given in (10) from $V_{DC}$ given in (15):

$$(r^{-1}) \alpha (1-\Theta)CEQ_{NC}(\tilde{X}) - (I+D)[1-F(NC)](r^{-1})$$  

(20)
Notice that this is the same amount as the sum of the first two terms in (19) with signs altered. So, the net loss of stockholders' wealth is simply the present value of tax shelter lost upon conversion. In a world of taxless corporate income, this loss also vanishes. These results uphold Lewellen-Racette's conclusion that in the no-corporate tax case, the stockholders' wealth is the same and in the presence of corporate tax, the stockholders' wealth is lower under convertible-debt than under straight-debt-financing. Their conclusion is shown to hold even in the presence of costly bankruptcy if the total promised debt payment is the same. Once the difference in promised interest rates is recognized along with the costliness of bankruptcy, we obtain results parallel to those of firm value. Hence, the validity of Lewellen-Racette's proposition is ambiguous under these circumstances.

In the above, we confined ourselves to the examination of the relative efficiency of two means of financing. A comment is in order with respect to their efficiency relative to financing by retained earnings - another alternative we have analyzed. From equations (6), (11) and (16), it is clear that all three means of financing are equally efficient in a taxless corporate income world with costless bankruptcy. If the bankruptcy is costless, but corporate income is taxable, both means of debt-financing are more efficient than retained earnings, solely because of the tax-deductibility of interest payments. If, in addition, the bankruptcy is costly, then this is not always true because at some level (below 100 percent) of debt the bankruptcy cost starts increasing more than the tax shelter for each of the means of debt-financing. This causes them to be less efficient than retained earnings beyond their respective
optimum levels. In other words, either of the debt-financing is preferable to retained earnings only if it leads to the use of debt at or below its level in the optimal financial structure. Otherwise, financing by retained earnings is preferable.

B. Our Valuation Model vs. Jenning's Model for Convertible Bonds

\( V_{DC} \) given in (15) is the convertible-debt valuation model which is derived by using the cash flow version of the single-period CAPM that incorporates corporate income tax and costly bankruptcy. Because the analysis in Section A assumes \( I + D \) being the same for both straight-debt and convertible-debt financing, \( V_{DS} \) given in (10) represents the equivalent straight-debt-value of convertible-debt. So, by substitution, (15) may be rewritten as:

\[
V_{DC} = V_{DS} + (r^{-1}) \left\{ \alpha (1-\theta) CEQ_{NC}(\bar{X}) - (I+D)[1-F(NC)] \right\}
\]

Thus, the market value of convertible-debt is equal to its equivalent straight-debt value plus the present value of the conversion option (or conversion premium) represented by the second term on the RHS of (21). This is the same as Poensgen [76] and BMQ [7] have shown, but unlike Poensgen, and BMQ, we have presented in terms of the present value of the risk-adjusted cash flows accruing to convertible bondholders by using the single-period CAPM. In an extreme situation of a single convertible bond outstanding, (21) is the theoretical expression for the market value of a convertible bond. But in the case of plurality of convertible bonds outstanding, the RHS of (21) should be
devided by the number of outstanding convertible bonds to obtain the equilibrium market value of a convertible bond.

Using the rate of return version of the single-period CAPM, Jennings [48] has derived a convertible bond valuation model which is reproduced below:

\[
V_{DC}(0) = B(0)P^* + \frac{(1-P^*)S_0[(1-\beta)R_f] + S_0\beta \int_{R^*}^{\infty} f(\tilde{R}_m) d\tilde{R}_m}{r \text{ or } E(\tilde{R})}
\]

where

- \(B(0)\) = the current market value of the underlying straight bond (riskless).
- \(S_0\) = the current stock-conversion value of convertible bond.
- \(\beta\) = systematic risk of underlying common stock, i.e. \(\frac{\text{Cov}(R_i, \tilde{R}_m)}{\text{Var}(\tilde{R}_m)}\)
- \(E(\tilde{R})\) = required return on the underlying common stock
- \(R^*\) = minimum market return relative required for the end-of-period stock-conversion value to exceed the straight-bond value
- \(P^*\) = \(\int_{R^*}^{\infty} f(\tilde{R}_m) d\tilde{R}_m\), the probability that stock conversion value is less than the bond value,

and the denominator in the second term is:

\[
E(\tilde{R}) \text{ if } B(0) < S_0 \quad r \text{ if } B(0) > S_0.
\]

Thus, Jennings' model suggests that the market value of a convertible bond is the weighted sum of the underlying straight bond value and stock-conversion value, the bond value being weighted by the probability \((P^*)\) that the stock conversion value is less than the bond value at the end-of-period and \((1-P^*)\) is the weight for stock-conversion value.
As is clear from the discount rate used for the conversion portion of the convertible bond, Jennings' model makes an implicit assumption that the conversion portion is risk-free if stock-conversion value does not dominate straight bond value and if it does, then the conversion portion is risky. Since the conversion portion of the convertible bond value is stochastically generated, by the probability that market return is relatively greater than $R^*$, as Frankle [39] has rightly pointed out, it has to be converted into a certainty-equivalent value in order that the risk-free rate of interest is the appropriate discount rate when $B(0) < S_0$. In our model, the conversion portion of the convertible bond value refers to the value of the conversion option and is generated by the probability that the bondholders' cash flow in conversion exchange is greater than their cash flow if they do not convert. But unlike Jennings, we have converted the cash flows to their certainty-equivalent before discounting at the risk-free rate of interest. We notice that the equivalent straight bond value under Jennings' model is riskless, whereas it is risky under our model due to the recognition of possible bankruptcy. Since his model is derived on the basis of underlying stock and bond prices, the implications of corporate income tax for convertible bond value can not be shown explicitly as we have done in (15). Furthermore, Jennings' model is not well suited for deriving a criterion for capital budgeting and other firm decisions, whereas our model is a convenient form to derive such a criterion. However, from the security investors' point of view, his model is simple and convenient to work with because it requires market data only. But note that the evaluation will not be as rigorous as it otherwise would be.
C. Firm's Cost of Capital Under Convertible-Debt-Financing

We have addressed in the above the question of arriving at an efficient financing decision, given the investment decision. Now let us address the question: What cost of capital criteria should the firm adopt for an investment decision, given that it would be financed by raising convertible-debt?

As Chen [20] has pointed out, in a single-period setting, the firm's cost of capital is a required rate of return such that the firm's expected end-of-period cash flow, when discounted at this rate, equals the current market value of the firm. Since the firm's expected cash flow is simply the sum of the expected cash flow available to its security holders, the firm's cost of capital, \( E(R_0) \), in the case of convertible-debt-financing can be specified as:

\[
E(R_0) = \left( V_{LC}^{-1} \right) \left[ \bar{Y}_E + \bar{Y}_{DC} \right]^{-1} \tag{22}
\]

since

\[
V_{LC} = \left[ 1 + E(\bar{R}_0) \right]^{-1} (\bar{Y}_E + \bar{Y}_{DC})
\]

where

\[
\bar{Y}_E + \bar{Y}_{DC} = (1-\Theta)E_0(\bar{X}) + (\bar{r}^{-1})\Theta IF(NC) - (1-\Theta)E(\bar{B}) \text{ from} \tag{12}, (13), (14) \text{ and } (15).
\]

Similarly from (5) and (6), the cost of capital of an unlevered firm, \( E(R_0^U) \) can be specified as:
\[ E(R_0^u) = (V_U^{-1})(1-\Theta)E_0(\tilde{X}) - 1 \]  

(23)

since \[ V_U = [1+E(\tilde{R}_0^u)]^{-1}(1-\Theta)E_0(\tilde{X}). \]

This is the same as (18) in Chen [20]. Using the relationships in (22) and (23), we can rewrite \( V_{LC} \), given in (16), as follows:

\[ V_{LC} = V_U + (r^{-1})\Theta F(NC) - (1-\Theta)V(B) \]

\[ = [1+E(\tilde{R}_0^u)]^{-1}(\tilde{Y}_E + \tilde{Y}_{DC} + (r^{-1})[E(\tilde{R}_0^u) - R_f]. \]

\[ \cdot [\Theta F(NC) - (1-\Theta)V(B)] \]

\[ + (r^{-1})(1-\Theta)[1+E(\tilde{R}_0^u)] \lambda \text{Cov}(\tilde{B}, \tilde{R}_m) \} \]  

(24)

So,

\[ E(\tilde{R}_0) = E(\tilde{R}_0^u) - \Theta [E(\tilde{R}_0^u) - R_f] F(NC) \phi_1 q_1 - \phi_2 q_2 \]  

(25)

where

\[ \phi_1 = (r^{-1})[1-(1-\Theta)I^{-1}E(\tilde{B})] \]

\[ \phi_2 = (r^{-1})(1-\Theta)[1+E(\tilde{R}_0^u)] \lambda \text{Cov}(\tilde{B}, \tilde{R}_m) \]

\[ q_1 = 1/V_{LC} \]

\[ q_2 = 1/V_{LC}. \]

(25) indicates that the firm's cost of capital is a decreasing function of the probability that bonds are not converted, the leverage factor and the positive covariance between bankruptcy cost and \( \tilde{R}_m \). On the other hand, it is an increasing function of the expected bankruptcy cost. When bankruptcy is costless and \( F(NC) = 1 \), (25) reduces to (26).
\[ E(\tilde{R}_0) = E(\tilde{R}_0) - \left( r^{-1} \right) \Theta [E(\tilde{R}_0^u) - R_f] q_1 \]  

(26)

This is the same as Eq. (22) in Chen [20] and Eq. (11.3) in MM [63], except that (26) is derived in a single-period setting as opposed to debt-perpetuity assumption underlying Eq. (11.3) in MM [63] and treating only interest as tax deductible as opposed to the tax deductibility of the total promised debt payments' assumption underlying Eq. (22) in Chen [20].

D. A Note on the Limitations of the Single-Period Analysis

We have evaluated the convertible-debt-financing in this chapter rather extensively in a single-period CAPM framework. Utilizing the cash flow version of the S-L-M CAPM that incorporates both corporate income tax and costly bankruptcy, the implications of financing by retained earnings, straight-debt and convertible-debt for firm value and stockholders' wealth are analyzed and compared. The analysis generally supports Lewellen-Racette's proposition of convertible-debt-inferiority. But, the validity of their proposition is shown to be ambiguous once the lower promised interest rate on convertible bonds is recognized in conjunction with the bankruptcy cost. The results of the analysis, however, can not be said to be conclusive on this subject.

Our single-period analysis has certain limitations. It assumes simultaneous payment of interest and dividend which is not generally observed in practice. It considers neither the transaction cost differences nor the taxes paid on the amortized premium. However, all
these are ignored as a matter of expositional simplicity. Since we have worked within the static valuation framework, the maturity-period of convertible bonds is apparently not considered in evaluating convertible-debt-financing. There is some evidence, though, that this is not a significant factor in determining convertible bond value.\footnote{8}

Furthermore, our single-period analysis does not recognize the effects of call option and possible issue of new debt when the convertible-debt is converted into equity. Because almost all the convertible bonds are callable and since nothing can prevent the firm from issuing new debt upon conversion, this certainly is a serious limitation of our analysis in this chapter. An attempt has been made in the next chapter to capture, among other things, the effects of call option and the capital structure plan that involves the issue of new debt upon conversion.
Footnotes to Chapter III

1. See for example Boneas-Chen [1970].

2. For derivation, see Hamada [1969], footnotes to Rubinstein [1973], and Chen [1977].

3. This treatment lies in between two approaches to the consideration of corporate income tax that we find in the finance literature. In one approach, the cost of physical assets acquired during the period are deducted from the firm's end-of-period cash flow to obtain the operating earnings. From this amount, the interest on debt is deducted to obtain the taxable income. (For details see Kim [1978]). In the second, the total contractual obligations including principal amount of debt are assumed to be tax deductible. (For example, see Rubinstein [1973], and Chen [1977]). However, our treatment of the subject does not alter materially the results of the analysis.

4. For example, the bankruptcy statistics of the Administrative Office of the U.S. Courts [1969] show that in 1969 the total administrative expenses formed 23.4 percent of the total realization from bankruptcies. Stanley-Girth [1971] estimate it to be 20 percent of the estate. See also Baxter [1967], Van Horne [1976], and Kim [1978] for extensive discussion.

5. According to Section 171 of the Federal Revenue Code, the premium on convertible bonds should be amortized over the period of their maturity. The amount amortized each year is taxable as income. And this amortized sum is taxed until the bonds are actually converted. After conversion, the unamortized premium is regarded as equity. Therefore, it is not taxed. Lewellen-Racette [1973] have considered the tax paid on the premium amortized up to conversion in deriving the loss of firm's value under convertible debt-financing.

6. For extensive discussion of the optimal financial structure in the case of risky straight-debt, see Scott [1976], Chen [1978], and Kim [1978]. Given the same total promised payments (i.e. I+D) to bondholders under both means of debt-financing, it is needless to show that the optimal financial structure (below 100 percent debt) exists in the case of risky convertible-debt. This is so because the amount of tax shelter is lower
under convertible-debt than under straight-debt financing at all levels of debt due to possible conversion (i.e. \( F(\text{NC}) < 1 \)) but the bankruptcy costs are the same for both means of debt-financing.

7. Like Chen [1977], it may be easily shown that the firm's cost of capital under convertible-debt-financing is also the weighted average cost of capital of equity and convertible debt.

From equation (12) and (13), the equity cost of capital, \( E(\tilde{R}_E) \), is:

\[
E(\tilde{R}_E) = (V_E^{-1}) \left\{ (1-\Theta)E_b(\tilde{x}) - \alpha(1-\Theta)E_{\text{NC}}(\tilde{x}) - [I(1-\Theta) + D][F(\text{NC}) - F(b)] \right\} - 1
\]

From (14) and (15), the cost of convertible debt is:

\[
E(\tilde{R}_{DC}) = (V_{DC}^{-1}) \left\{ \alpha(1-\Theta)E_{\text{NC}}(\tilde{x}) + (I+D)[F(\text{NC}) - F(b)] + I\Theta F(b) + (1-\Theta)E^b(\tilde{x}) - (1-\Theta)E(b) \right\} - 1
\]

Therefore,

\[
E(\tilde{R}_0) = E(\tilde{R}_E)h_1 + E(\tilde{R}_{DC})h_2
\]

where \( h_1 = \frac{V_E}{V_E + V_{DC}} \) and \( h_2 = \frac{V_{DC}}{V_E + V_{DC}} \).

8. See for example Felheim [1974].
CHAPTER IV
A MULTIPERIOD EVALUATION OF
CONVERTIBLE-DEBT-FINANCING

It is noted in the preceding chapter that the single-period analysis carried out there has not recognized the significance of call option nor has it captured the implications of firm's capital structure plan upon conversion (specifically, the possible issue of new debt in place of the old convertible-debt). The reason is that the analysis does not extend beyond one-period. It generally supports Lewellen-Racette's proposition that the convertible bonds are an inefficient source of capital as opposed to straight bonds. But in the circumstance where the lower promised interest rate on convertible bonds is recognized in conjunction with the bankruptcy cost, the validity of their proposition is shown to be ambiguous. It means that if the bankruptcy is costless, convertible-debt-financing is always an inefficient means relative to straight-debt-financing. But this conclusion of the single-period analysis can only be accepted with suspicion because the above stated limitations are severe. Most of the convertible bonds listed on both the New York Stock Exchange and the American Stock Exchange are, at least in recent years, callables.\(^1\) Brigham's [17] survey showed that the firms preferred to issue convertible bonds even though they could afford to issue common stock or straight bonds, contrary to the LR's proposition and to the results of our single-period analysis.
The convertible-debt valuation theory discussed in the preceding chapter and by Lewellen-Racette [58] is missing a point in that it ignores the firm's ability to reward its stockholders for the loss in their wealth upon conversion. This may be done either by following a financial plan that helps stockholders recover their loss upon conversion or by diluting the anti-dilution clauses at the time of indenture agreement or by both. While the course of action may differ, intent is the same. The recognition of this intent in the convertible-debt valuation theory is important, particularly when we are dealing with a stockholder's wealth maximizing firm. Any findings on the efficiency of a means of financing in the absence of its recognition become inconclusive and hence, they are suspect.

To recognize this intent and the callability feature of convertible bonds, the analysis is extended in this chapter, to a multi-period setting. In so doing, we assume that the firm pursues a policy of maintaining either the same capital structure or the same leverage upon conversion. In the former case, the firm retires new issues and raises simultaneously the same amount of debt as the old convertible-debt, whereas, it does not retire new issues but raises the same amount of debt in the latter case. The combination of these two policies would also yield the same results. However, we don't need to consider those combinations to make our point.

One might argue that the firm's capital structure plan upon conversion can only be known ex-post. But, it is not unreasonable to assume that the firm pursues either of the above stated plans because nothing prevents a stockholder's wealth maximizing firm from so doing.
Note that these plans are consistent with the capital structure theory which states that leverage is advantageous in the presence of corporate income-tax and a firm should use it optimally so that the firm's market value is maximized. If the firm pursues a plan of maintaining the same capital structure, it means that the current capital structure is not-over-optimal and the firm intends to benefit from maintaining it. On the other hand, if it pursues a plan of maintaining the same leverage upon conversion, it implies that the firm's current capital structure is not optimal and the pursuance of this plan is an attempt at optimizing its capital structure.

By incorporating explicitly the firm's capital-structure plan upon conversion into the analysis, we show in this chapter that Lewellen-Racette's (LR) proposition does not hold under the circumstances where it is shown to hold in the preceding chapter. It may be possible to prove the inappropriateness of LR's proposition by considering firm's legal violation of the me-first rules as applied to convertible bonds. We do not, however, intend to analyze this aspect in order to not detract from the main purpose of the study, i.e., the evaluation of convertible-debt-financing when the firm operates within the confines of the me-first rules.5

The Chen's [22] simplified version of multi-period CAPM is utilized here to carry out multi-period evaluation of convertible-debt-financing. In a way, this portion of the study is an extension of his recent work [22] on the theory of bankruptcy and capital structure into the area of convertible-bond valuation.

In Section A, we describe in brief the Chen's simplified version of
the multi-period CAPM and the assumptions underlying our multi-period analysis. Multi-period definitions of bankruptcy and conversion are presented. The convertible-debt-financing is then analyzed considering each of the previously stated firm's capital-structure plan upon conversion. The analysis is carried out with or without call option. In Section B, we present the implications of our multi-period analysis for the premiums on convertible bonds, firm's optimal call-strategy and for the existence of optimal capital-structure when convertible bonds are outstanding.

A.1 Chen's multi-period CAPM and the Assumptions

Many academicians have diverted their attention in recent years towards the development of a multi-period pricing model for a risky capital asset. While many of these models are complex in their own right, the Chen's simplified version of the multi-period-valuation model is convenient to work with. The simple multi-period-valuation model is needed especially when we are evaluating a complex asset like a callable convertible bond.

Chen's model is basically a combination of Fama [33], Bogue-Roll [11] and Steven's [84] approaches. The general form of his model is:

\[ V(0) = \sum_{t=0}^{\infty} \{ E[\tilde{Y}(t)] - \lambda(t) [\text{Cov}(\tilde{Y}(t), \tilde{R}_m(t))] \} \frac{1}{s=0} \left[ 1 + r(s) \right]^{-1} \]  

where the expression in the braces is the certainty-equivalent of firm's net operating earnings at time 't', \( r(s) \) is the riskless rate of interest at time 's' and it assumes infinite horizon. It states that the market value of a firm at time '0' is equal to the sum of all its discounted risk-adjusted future income streams. Chen simplifies (27)
to the following under a set of assumptions:

\[ V(0) = (r^{-1}) \left[ E(\tilde{Y}) - \lambda \text{Cov}(\tilde{Y}, R_m) \right] \]

The first term of the product on the RHS of (28) is the reciprocal of one-period riskless rate of interest. The second term is the per period certainty-equivalent net operating earnings of the firm.

To derive (28), Chen assumes that the capital market is efficient and perfect. All investors are assumed to be risk averse and to share the same subjective probability distributions of a firm's future operating earnings and the states a firm might possibly be in. Firm's investment decisions are assumed to have been given and its operating earnings are assumed to have a temporally independent identical distribution in all future periods. This implies that the risk associated with the reassessments through time of the expected operating earnings is constant.\(^9\) Further, the model assumes riskless interest rate and corporate income-tax rate as given and constant in the future. No change is assumed in the market price of risk (\(\lambda\)) over time.

In addition, our multi-period analysis assumes that after the financing decision, the firm's outstanding securities consist of only common stock and convertible bonds. All the security holders have limited liability. The convertible bonds are assumed to be perpetual\(^10\) and are all converted at one time. Both call and conversion options are also assumed to be exercisable in perpetuity with no change in the conversion ratio. Further, it is assumed that the corporate management acts according to its pre-determined capital-structure plan upon conversion which is assumed to be known to the investors.
A.2 Multi-Period Definitions of Bankruptcy and Conversion Conditions

Like in the single-period analysis, two of the key elements of our multi-period analysis are the possible bankruptcy and the possible conversion of bonds into common stock. The conditions for their occurrence in a multi-period setting are hence defined in the following paragraphs. The discussion of another key element - call option - is deferred to a later part of this section where the analysis is extended to include this element.

Multi-period definition of bankruptcy is sine qua non with the definition of bankruptcy for an on-going firm. Legally, the term bankruptcy (of an on-going firm) is used generally to describe the proceedings undertaken in a federal court when a debtor is unable to pay or to reach agreements with his creditors outside of court.\(^\text{11}\)

Consistent with this interpretation are the definitions provided by Stiglitz [88], Scott [81] and Chen [22]. They set forth that "an on-going firm can only be declared bankrupt if it is unable to meet its current obligation (i.e., interest payments) to the bondholders".

Chen [22] and Scott [81] have stated that this situation occurs when any possibility of selling equity, assets and borrowing money is inadequate to meet the current obligation of interest payments. So, they define in precise terms the bankruptcy condition as:

\[
(1-\theta) (X-I) + V_E \leq 0
\]

(29)

i.e. \(X \leq I - V_E/1-\theta\) \(\equiv b\) \hspace{1cm} (29')
where $X$ denotes the firm's realized operating earnings at the end-of-current period and other notations are the same as defined previously. (29) states that the firm is declared bankrupt when the total end-of-period stockholders' wealth, defined by the sum of after-tax operating earnings available to stockholders and the value of equity, is zero or negative. We utilize this definition to incorporate possible bankruptcy into our multiperiod analysis. Note also that unlike in the single-period analysis, $X$ denotes the end-of-period realized operating earnings in all our subsequent analysis.

It is possible to consider the bankruptcy-reorganization case in a multi-period setting. However, we confine in our analysis to the bankruptcy-liquidation case as we did in the single-period analysis. To facilitate direct comparison with the Chen's [22] analysis of straight-debt-financing and thereby to examine Lewellen-Racette's proposition in a multi-period context, the costs of bankruptcy-liquidation are assumed to consist of a fixed portion (administrative expenses), $K$ and a variable portion (loss due to distress-selling of assets) that forms $k$ proportion of firm's value. Recall that they were merged together and were represented by $\tilde{B}$ in the preceding chapter.

Moreover, whether the bankruptcy proceedings are initiated by the debtor (voluntary bankruptcy) or by the creditors (involuntary bankruptcy) does not alter the definition in (29) because it is assumed that if the debtor does not initiate, the creditors will do so when (29) holds.\footnote{13}

Regarding possible conversion, we have said in the single-period analysis that the condition for conversion exists when the proportion-
ate cash flows of the firm received by the bondholders in conversion exchange exceed the sum of contractual principal amount of debt and one-period interest. Though the basic idea is sound, the precise definition of conversion condition as given in (3) is not an appropriate definition in a multi-period setting. This is so for two reasons: First, since we are dealing with an on-going firm in a multi-period setting, the proportionate operating earnings and value of equity that the bondholders receive in conversion exchange and the market value of convertible-debt that they forego are relevant but not the amounts we considered previously. Second, the wealth transferred to bondholders in conversion exchange depends on the firm's capital structure plan upon conversion. If the firm plans to raise new debt either for retirement of new shares or for maintaining the same leverage, then the bondholders' (new stockholders) wealth in conversion is different from that under the circumstance where the firm neither retires new issues nor does it raise new debt.

As is already stated, we assume that the firm pursues either of the two capital structure plans upon conversion: i) maintaining the same capital structure or (ii) maintaining the same amount of leverage. Under both plans, the firm continues to obtain the same tax shelter after conversion as before because it maintains its leverage by raising new debt equivalent to the old.

In view of the above, the condition for conversion exists in a multi-period setting if the proportionate stockholders' wealth received by the bondholders in conversion exchange exceeds the market value of convertible-debt and due interest foregone by them.
Symbolically,

$$\alpha[(1-\theta)(X-I) + V_E] > I + V_{DC}$$  \hspace{1cm} (30)

i.e.,

$$X > I + \frac{(I+V_{DC})}{\alpha} - V_E(1-\theta)^{-1} \hspace{1cm} (\equiv NC)$$  \hspace{1cm} (30')

where \(V_{DC}\) denotes the market value of convertible-debt.

(30) means that for the bondholders to convert their holdings, the proportionate EAIT and equity value available to them in conversion should be greater than the value of convertible-debt plus interest due. (30), in fact, is the condition for voluntary conversion. If the convertible bonds are callable, then there could occur an involuntary (or forced) conversion. The condition for involuntary conversion is defined later while discussing the condition for exercising call option by the issuer-corporation.

A.3 Valuation of Equity, Convertible-debt and Firm with Call-option

Two points deserve mentioning before undertaking the multi-period analysis in the following paragraphs. First, it is shown in the preceding chapter that if the bankruptcy is costless, the presence of corporate income-tax renders retained earnings the most inefficient means of financing. Assuming that the use of either straight-debt or convertible-debt to finance current investments is not over-optimal, it is fair to conclude that both the means of debt-financing are more efficient than retained earnings even while bankruptcy is costly within the multi-period framework as well. Hence, it is not warranted to undertake the multi-period analysis of the alternative of financing by retained earnings. However, we have discussed in footnote No. 18 the implications of financing the investments subsequent to conversion by
retained earnings. Secondly, although it is necessary to undertake the multi-period analysis of straight-debt-financing to re-examine LR's proposition, we do not need to do so because the same is carried out by Chen 22. We can simply borrow his results for the purpose. So, the analysis is carried out below only with respect to convertible-debt-financing.

Note that, given the assumptions underlying Chen's simplified version of the multi-period CAPM, we are only required to define all the possible state-option combinations in terms of the end-of-current period operating earnings of the firm, identify the wealth of security holders under each state-option combination and discount the identified wealth (adjusted for risk) at the riskless rate of interest to derive the valuation equations. 15

With the call option excluded and the possible bankruptcy and conversion included, we know that there are three possible state-option combinations:

1) conversion, no bankruptcy
2) no conversion, no bankruptcy, and
3) no conversion but bankruptcy.

So in the following, the multi-period valuation equations are derived by identifying the wealth of security holders under each of the above state-option combinations. The analysis is carried out separately for: 1) maintaining the same capital structure upon conversion, and 2) maintaining the same leverage amount upon conversion.
A.3.a Maintaining the Same Capital Structure

1) Market Value of Equity

Note that this plan calls for retiring new issues of common stock and simultaneously raising new debt equivalent to the old. So if the firm remains solvent but the bonds are converted, the post-conversion wealth of stockholders is the same as pre-conversion. It consists of net earnings and value of equity. The stockholders' wealth can be expressed by the same quantity if the end-of-period realized operating earnings of the firm, \( X \), are such that \( b < X \leq NC \) where \( b \) and \( NC \) are respectively defined by (29*) and (30*). This is the circumstance where the firm remains solvent but the bondholders do not convert. When the firm is declared bankrupt, however, the stockholders receive nothing. As indicated in (29), the stockholders' wealth is either zero or negative when the firm is in bankruptcy condition. If their wealth is negative, they can exercise their limited liability and let the ownership of the firm be transferred to the creditors.\(^{16}\)

So the stochastic end-of-period stockholders' wealth, \( \tilde{Y}_E \), can be specified as follows:

\[
\tilde{Y}_E = \begin{cases} 
(1-\theta)(\bar{X}-I) + V_E & \text{if } X > NC \\
(1-\theta)(\bar{X}-I) + V_E & \text{if } b < X \leq NC \\
0 & \text{if } X \leq b 
\end{cases}
\]

Utilizing Chen's simplified version of the multi-period CAPM, the equilibrium market value of equity can be expressed as:

\[
V_E = \{(1-\theta)CEQ_b(\bar{X}) - I(1-\theta)[1-F(b)]\}[r+F(b)]^{-1} \tag{32}
\]

where \( CEQ_b(\bar{X}) = E_b(\bar{X}) - \lambda Cov_b(\bar{X}, \bar{R}_m) \), the per period certainty-equivalent operating earnings of the firm truncated from \( b \) upward, \( F(b) \) is the firm's probability of bankruptcy in a multi-period setting, \( I \), one-
period promised interest on the convertible debt and unlike in a single-period framework, $r$ represents here riskless interest rate (without +1). Incidentally, it may be noted that we have continued to use the same notations for the certainty-equivalent quantities and cumulative distributions as in the preceding chapter to avoid confusion arising from numerous notations. However, their definitions in a multi-period context have been made explicit wherever appropriate. The expression in the braces on the RHS of (32) is the per-period operating earnings after taxes and interest payments times the probability that the firm remains solvent. The second term of the product on the RHS is the reciprocal of the appropriate discount rate. Because of the explicit consideration of possible bankruptcy, the discount rate equals the riskless rate of interest plus a default (bankruptcy) premium equal to $F(b)$. Thus, (32) suggests that the market value of equity in a multi-period context is equal to the present value of the per-period net operating earnings available to the stockholders when the firm remains solvent.

The market value of equity under straight-debt-financing as derived by Chen [22] is reproduced below:

$$V_E = (1-)[E_b(\bar{x}-I) - \lambda Cov_b(\bar{x}, \bar{\bar{R}}_m)][r+F(b)]^{-1} \tag{8'}$$

Notice that our equation (32) coincides with his equation (8'). So, the convertibility feature of outstanding debt does not have any bearing on the value of equity. This is so because the share (equal to conversion ratio) of wealth that otherwise would have been lost to new stockholders (old convertible bondholders) upon conversion is retained by the existing stockholders since the firm retires new issues of common stock right away. Similarly, the tax shelter which otherwise would.
have been lost upon conversion is saved by issuing new debt simultaneously. In an efficient capital market, the firm should be able to raise new debt of the same magnitude with the same terms of interest payments, maturity, etc. as the debt before conversion. In fact, that is what we have assumed so far as the new debt is concerned. Therefore, given the same promised interest for both types of debt and that the firm pursues a plan of maintaining the same capital structure upon conversion, the existing stockholders should be indifferent to the type of debt-financing.

ii) Market Value of Convertible-debt and Firm

Given the firm's capital structure plan upon conversion under consideration, the convertible bondholders cannot become the stockholders because the common shares issued to them in conversion-exchange are retired right away. Since the new-debt is issued simultaneously, they can be presumed to have bought the new debt instruments and have continued to possess the same fixed claim. The only difference is that they will be holding new certificates of debt after conversion. So, under the first two state-option combinations, the end-of-period total wealth of bondholders equals one-period interest plus the value of debt. But if the firm is declared bankrupt, since the stockholders exercise their limited liability, the bondholders become owners of the firm and receive the liquidation proceeds net of bankruptcy cost. It is assumed that there are no priority claims other than bankruptcy costs for simplification. It is further assumed for the same reason, as we have done in the preceding chapter, that the liquidation proceeds of the firm in bankruptcy exceed the bankruptcy costs. So, we ignore the
possible circumstance of bankruptcy costs exceeding the liquidation pro-
ceedings in which case the bondholders exercise their limited liability
and receive nothing.

So, the end-of-period wealth of the convertible bondholders, $\tilde{Y}_{DC}$,
can be specified as:

$$
\tilde{Y}_{DC} = \begin{cases} 
I + V_{DC} & \text{if } X \geq NC \\
\tilde{I} + V_{DC} & \text{if } b < X \leq NC \\
\tilde{X} + V(l-k) - K & \text{if } X \leq b
\end{cases}
$$

(33)

where $V$ denotes the firm's market value. The market value of conver-
tible-debt can therefore be expressed as:

$$
V_{DC} = \left\{ (I[1-F(b)] + [V(l-k) - K]F(b) + CEQ^b(\tilde{X})) \right\} \left[ r + F(b) \right]^{-1}
$$

(34)

where $CEQ^b(\tilde{X}) = E^b(\tilde{X}) - \lambda \text{Cov}^b(\tilde{X}, \tilde{R}_m)$, the certainty-equivalent of the
firm's operating earnings (truncated from $b$ downward), $K$ and $k$ are
respectively fixed and variable costs of bankruptcy as defined earlier.

The market value of the firm is simply the sum of the market values
of its equity and debt. So,

$$
V = V_E + V_{DC}
$$

$$
= \left\{ (1-\theta)CEQ(\tilde{X}) + [V(l-k) - K]F(b) + \theta I[1-F(b)] + \theta CEQ^b(\tilde{X}) \right\} \left[ r + F(b) \right]^{-1}
$$

or

$$
V = V_U + \theta V_D^* - \left\{ [KV + K + I\theta]F(b) - \theta CEQ^b(\tilde{X}) \right\} \left[ r^{-1} \right]
$$

(35)

where

$$
V_U = (1-\theta) CEQ(\tilde{X})(r^{-1}), \text{ the equivalent unlevered firm value in the presence of corporate income-tax.}
$$

$$
V_D^* = (r^{-1})I, \text{ the market value of perpetual interest payments.}
$$

The equations (34) and (35) coincide exactly with those derived by
Chen [22] for the corresponding values in the case of straight-debt-
financing. Although our equation (34) is exactly the same as Eq. (22) in Chen [22], this does not imply that there is zero premium on convertible bonds. The results are obtained here by the simultaneous treatment of the firm's capital-structure plan upon conversion and the conversion privilege. Hence, in the process, it gets netted out and does not appear in our equation (34). The conversion premium is equal theoretically to the present value of the excess of wealth received by the bondholders in conversion exchange over the wealth foregone by them. Letting \( V_p \) represent the value of conversion premium on the convertible-debt, it can be expressed as below under the given firm's plan:

\[
V_p = \{a(1-\theta)\text{CEQ}_{NC}(\tilde{X}-I) + [\alpha V_{E-I-V_{DC}}][1-F(\text{NC})]\}(r^{-1})
\]  

(36)

where \( \text{CEQ}_{NC}(\tilde{X}-I) = E_{NC}(\tilde{X}-I) - \lambda \text{Cov}_{NC}(\tilde{X}, \tilde{R}_m) \), the certainty-equivalent of the firm's taxable operating earnings when \( X > NC \).

The convertible bondholders pay initially this much premium. This is also the value of excess of the proceeds from retired new shares issued to them over the value of new debt subscribed by them. Similarly, from the standpoint of stockholders, (36) represents their initial gain when the convertible bonds are issued instead of straight bonds. It also represents the present value of deficit borne by them when the firm retires new shares issued in conversion-exchange. So, there is no net gain to either party compared to their respective wealth under straight-debt-financing. Thus, the analysis implies that the stockholders should be indifferent to the type of debt-financing so long as the firm pursues the stated plan upon conversion and the promised interest is the same for both types of debt.
Notice that when the convertible bonds are issued, the firm uses more leverage than it otherwise would have because the convertibles fetch a higher price than straight bonds due to the existence of premium shown in (36). And, in an infinite horizon, compared to straight-debt-financing, the firm has nothing to lose by issuing now convertible bonds and pursuing later the stated plan upon conversion. In that regard, the convertible-debt-financing may be regarded by the firm as a better means of financing. The Lewellen-Racette's proposition does not hold.

A.3b Maintaining the Same Leverage

1) Market Value of Equity

We know that this plan calls for not retiring new shares, but raising new debt in place of the old so that the same leverage is maintained. To keep capital stock constant, the firm is assumed to have distributed additional cash dividend at the same time when new debt is raised and equal to the amount raised. So, the existing stockholders' wealth upon conversion includes proportionate total wealth of stockholders plus the amount of dividend distributed to them (which equals end-of-period $I + V_{DC}$). But for other state option combinations, their wealth is the same as under the first plan.

So, the end-of-period wealth of existing stockholders, $\tilde{Y}_E$, can be specified as:

$$
\tilde{Y}_E = \begin{cases} 
(1-\alpha)[(1-\theta)(\tilde{X}-I) + V_E] + I + V_{DC} & \text{if } X > NC \\
(1-\theta)(\tilde{X}-I) + V_E & \text{if } b < X \leq NC \\
0 & \text{otherwise} 
\end{cases} \quad (37)
$$
The market value of equity, therefore, can be expressed as:

\[ V_E = \left( (1-\theta) CEO_b (\tilde{X}) - \theta (1-\theta)[1-F(b)] - F(b)V_E \right) (r^{-1}) \]

\[ - \{ \alpha (1-\theta) CEO_{NC} (\tilde{X}-I) + [\alpha V_E - I - V_{DC}] [1-F(\text{NC})] \} (r^{-1}) \] (38)

The first term on the RHS of (38) is the equivalent-equity value, that is, the value of equity if the debt were a straight-debt. This is the same as Chen's [22] derivation for the market value of equity under straight-debt-financing. The second term is the net value relinquished by the existing stockholders when the bonds are converted.

The first expression within the second set of braces is the certainty-equivalent operating earnings after-tax and interest that are transferred to the new shareholders. The second expression is value of equity (adjusted for the extra-dividend distributed upon conversion) that is transferred to new shareholders times the probability that bonds are converted. It is shown in the next paragraph that the total net value relinquished by the existing stockholders is exactly equal to the conversion premium that the bondholders pay. If we assume, as Lewellen-Racette have done, that the premium is distributed among the existing stockholders, then the second term on the RHS of (38) nets out and the existing stockholders remain as well off as they would have been if the straight-debt were issued instead. As a result, the existing stockholders remain again indifferent to type of debt-financing used by the firm.

ii) Market Value of Convertible-Debt and Firm

In the second plan, the existing bondholders will own the shares of common stock issued upon conversion. In other words, they surrender their debt claims to become the share owners of the firm, their share
being equal to the conversion ratio. Since the firm simultaneously issues new debt instruments equivalent in claims to the old, the convertible bondholders may be presumed to have bought these instruments and have continued to hold the debt claims. So, when \( X > NC \), the end-of-period wealth of the convertible bondholders is

\[
\alpha[(1-\theta)(\tilde{X}-I) + V_E] - (I + V_{DC}) + (I + V_{DC}).
\]

Whereas, in other state-option combinations, their wealth is the same as under the first plan. In summary,

\[
\tilde{Y}_{DC} = \begin{cases} 
\alpha[(1-\theta)(\tilde{X}-I)+V_E - (I+V_{DC}) + (I+V_{DC}) & \text{if } X > NC \\
I + V_{DC} & \text{if } b < X \leq NC \\
\tilde{X} + V(1-k) - K & \text{otherwise} 
\end{cases}
\]  

(39)

So, the market value of convertible-debt can be expressed as:

\[
V_{DC} = \{I[1-F(b)] + [V(1-k)-K]F(b) + CEQ^b(\tilde{X}) - V_{DC}F(b)\} \left(\frac{1}{r}\right)
+ \{\alpha(1-\theta)CEQ_{NC}(\tilde{X}-I) + [\alpha V_{E}-I-V_{DC}][1-F(NC)]\} \left(\frac{1}{r}\right)
\]  

(40)

The first term on the RHS of (40) is the equivalent straight-debt value of the convertible-debt. This is the same expression as derived by Chen [22] for the market value of straight debt. The second term represents the market value of (conversion) premium on the convertible-debt.

Notice that this term is exactly equal to the value relinquished by the stockholders (given by the second term on the RHS of (38)). Notice further that this term is exactly equal to the value of conversion premium as given in (36). So, both the capital structure plans upon conversion are no different from the standpoint of both the groups of security holders.

Adding equations (38) and (40), we obtain the market value of the firm, \( V \):
\[ V = V_U + \theta V^*_D - \{[kV+K+I\theta]F(b) - \theta CEQ^b(X) \} (r^{-1}) \]  

(41) is the same as (35). Therefore, we end up with the same conclusions as we have for the first plan. The convertible-debt-financing is not different from the straight-debt-financing from the standpoint of the market values of equity and firm when the firm pursues a plan of maintaining the same leverage amount after conversion.

Thus, our results show that if the firm pursues either of the stated plans and if the promised interest on both the types of debt is the same, then the convertibility feature does not adversely affect the existing stockholders' wealth nor does it adversely affect the firm's value. Besides, two interesting attributes of the convertible-debt-financing as opposed to its counterpart are: first, it enables the firm to use more leverage, second, it provides flexibility to adjust the firm's capital-structure upon conversion towards the optimum use of leverage. In these respects, it has an edge over the straight-debt-financing. Certainly, we find that Lewellen-Racettes's proposition does not hold.

iii) **Use of new-debt to finance new investment: A More Realistic Case**

A more realistic case would be to assume that the firm raises new debt upon conversion neither for retirement of new shares nor for paying extra cash dividend, but to undertake new investment projects. Let us suppose that the new investment is made in the existing lines of business; hence the cash flows from the new investment are perfectly correlated with the cash flows from the existing investments. They are temporally independent and identically distributed over time. Further, the risk associated with the reassessment of the expected cash flows
from new investment is assumed to be constant through time. Of course, we do assume that the market has the knowledge of this policy of the firm.

This case is analyzed below by letting $Q$ represent one plus the constant growth, $q$, in the firm's operating earnings due to the new investment upon conversion.

Under the circumstance, the conversion takes place if:

$$\alpha[(1-\theta)(Q\tilde{X}-1)+V_E] > I + V_{DC} \quad \text{(42)}$$

i.e.,

$$X > \left[\frac{I+V_D}{\alpha} - V_E + I(1-\theta)[Q(1-\theta)]^{-1}\right] (\equiv NC)' \quad \text{(42')}$$

The condition for bankruptcy does not change, however. This is so because we are evaluating the firm that has convertible bonds still outstanding, hence the operating earnings from the existing investments are relevant for defining bankruptcy condition. It will change only in the ex-post conversion situation, that is, if the bonds have already been converted and new investment has already been undertaken.

So, given the condition for conversion defined by (42') and for bankruptcy defined by (29'), the end-of-period wealth of the existing stockholders can be specified as:

$$\tilde{Y}_E = \begin{cases} 
(1-\alpha)[(1-\theta)(Q\tilde{X}-1) + V_E] & \text{if } X > NC' \\
(1-\theta)(\tilde{X}-1) + V_E & \text{if } b < X < NC' \\
0 & \text{otherwise}
\end{cases} \quad \text{(43)}$$

The market value of equity, therefore, is:

$$V_E = V_E^{(\text{equivalent})} + \{q(1-\theta)CEQ_{NC},(\tilde{X}) - (I+V_{DC})[1-F(NC')]\}(r^{-1})$$

$$- \{\alpha(1-\theta)CEQ_{NC},(Q\tilde{X}-1) + [\alpha V_E-I-V_{DC}] [1-F(NC')]\}(r^{-1}) \quad \text{(44)}$$

where $V_E^{(\text{equivalent})}$ is the same as the first term on the RHS of (38).

The equation (44) suggests that the market value of equity in this
case is equal to its equivalent-equity value under straight-debt-financing plus the present value of all the excess earnings from new investment over the cost of investment minus the value relinquished to bondholders upon conversion.

Similarly, the end-of-period wealth of bondholders, $\tilde{Y}_{DC}$, in this case may be expressed as:

\[
\tilde{Y}_{DC} = \begin{cases} 
\alpha(1-\theta)(QX-I) + V_E & \text{if } X > NC'

I + V_{DC} & \text{if } b < X \leq NC'

X + V(1-k) - K & \text{otherwise}
\end{cases}
\]

So, the market value of convertible-debt is:

\[
V_{DC} = V_D(\text{equivalent}) + \{\alpha(1-\theta)CEQ_{NC'}(QX-I)

+ [\alpha V_E-I-V_{DC}] [1-F(NC')]\}[r^{-1}]
\]

where $V_D(\text{equivalent})$ is the same as the first term on the RHS of (40).

Thus, (46) indicates that the market value of convertible-debt under the circumstance where the new debt is used to finance new investment, is equal to its equivalent straight-debt value plus a premium equal to the second term on the RHS. Notice that this conversion premium is exactly equal to the value relinquished by stockholders in the event of conversion. As is clear, the conversion premium in this case is larger than the conversion premium under either of the previously discussed capital-structure plans upon conversion. This is due to the possible share in the extra earnings from new investment that the bondholders would enjoy by conversion.

Combining equations (44) and (46), we obtain the market value of the firm:
\[ V = V_u + \theta V_D^* - \{[kV+K+I\theta]F(b) - \theta CEQ^b(\bar{x})\}(r^{-1}) \]
\[ + \{q(1-\theta)CEQ_{NC'}(\bar{x}) - (I+\theta V_{DC})[1-F'(NC')]\}(r^{-1}) \]  

(47) is derived under the assumption that the current investments are financed by issuing convertible-debt and the new debt that is raised subsequent to conversion is used to finance new investment. This policy is equivalent to a possible alternative of issuing straight-debt now and issuing later the same number of common shares as are otherwise issued to convertible bondholders at the time of new investment. One might argue that if instead new investment were also financed by new straight-debt, the firm value would increase because of the additional tax shelter. But a counter argument is that if it is optimal to do so, then there is no reason why the firm cannot issue debt upon conversion equivalent to the old and new debt required for new investment, retire new common shares and achieve the same purpose.

Thus the above analysis and the arguments put forward lead us to conclude that the existing stockholders are as well off under the convertible-debt-financing as they would be under its counterpart, straight-debt-financing. The market value of the firm is the same. Moreover, with a potential growth in the firm's earnings, the value of convertible bonds will be much higher allowing the firm to use more leverage besides enjoying the flexibility afforded by these instruments. So, the closer examination of the convertible-debt financing reveals that Lewellen-Racette [58] are wrong in saying that the typical folklore story about convertible bonds is arrant nonsense. Those folks who have said that the convertible bonds contain the best elements of both equity and straight-debt have a great sense of imagination and
imagination and intuition, though they may not have put it in an analytical form.

Another alternative available to the firm is: to neither retire new shares nor raise new debt, but to finance new investment subsequent to conversion by retained earnings. Both the existing stockholders' wealth and the firm value decline in this alternative if the current investments were financed by convertible-debt instead of straight-debt. See for the analysis footnote No. 18. This is in fact what is implied in Lewellen-Racette's analysis of convertible-debt-financing. But one wonders why a stockholders' wealth maximizing firm should opt for this course of action when it is possible to avoid the ensuing loss. So, like Frankle [39], it may be argued that LR's assumption of financing subsequent investments by retained earnings is inconsistent with the firm's rational behavior, that is, maximizing existing stockholders' wealth.

A.4 Valuation of Equity, Convertible-Debt and Firm With Call-Option Included

Since most of the convertible bonds are callables, it is only appropriate to analyze the implications of the call option for the market value of equity, convertible-debt and of the firm.

Call option is a right of the issuer-firm to call back its outstanding convertible bonds. By exercising this right, the firm forces the bondholders either to convert or to surrender for redemption at the call price. Unlike other security options (convertibility, warrant, etc.), the call option cannot be resold; it only has to be exercised to realize its value. A stockholders' wealth maximizing firm exercises
this option only when such an action yields some benefits to its stockholders; if not, at least it has zero effect on the stockholders' wealth. In an efficient and perfect capital market, the value gained by the stockholders upon call is exactly equal to the value relinquished by the convertible bondholders. So, the call option by itself does not have any impact on the market value of the firm. Though this is a matter of indifference from the standpoint of the firm value, it needs to be shown whether this is also a matter of indifference from the standpoint of the security holders, the reason being that the call policy of the firm subsequent to the issuance matters for the market value of both the convertible bonds and the equity.

From the standpoint of the stockholders' wealth maximizing firm, the necessary and sufficient conditions to exercise the call option are:

(i) the convertible debt value should be greater than its call price and

(ii) the sum of one-period interest and convertible-debt value should exceed the share of stockholders' wealth to be transferred to the bondholders in conversion exchange.

In these circumstances, the bondholders would not otherwise convert. But once the bonds are called, the bondholders will convert if the share of stockholders' wealth that they would receive in conversion exchange exceeds the call price plus interest; otherwise they will surrender for redemption. In any case, the bondholders are the losers because the value relinquished by them is greater than the value received. The stockholders are the beneficiaries. If the firm calls back the bonds when both the inequalities defined in (i) and (ii) above
are equality conditions, then neither party loses nor does anyone gain. Because of the reverse effects of exercising call under other conditions, the firm would not call, hence the value of call option is zero under the conditions other than those described in (i) and (ii) above.

With the call option included, there are five possible state-option combinations.

(1) Call, conversion, no bankruptcy
(2) Call, no conversion, no bankruptcy
(3) No call, conversion, no bankruptcy
(4) No call, no conversion, no bankruptcy
(5) No call, no conversion, but bankruptcy

The above suggests that call, conversion or both may be exercised only if the firm remains solvent at the end of the period. If instead the firm is declared bankrupt, then neither the firm call nor do the bondholders convert because the former will be unable to meet obligations arising from call and the latter can exercise their prior claim as bondholders on the firm's liquidation proceeds net of bankruptcy cost.

We were concerned in the preceding analysis regarding the implications of the last three state-option combinations. To treat both conversion and call options simultaneously, we simply superimpose the implications of the first two-state-option combinations on the preceding analysis.

In the following, the analysis with the inclusion of call option is confined to the case of firm maintaining the same leverage upon conversion but using new debt to finance new investment. There is some evidence that the new shares are not retired upon conversion in the
majority of the cases. So, it is reasonable to consider the second plan.

Given that the firm pursues the above stated plan, the condition for exercising call option may be specified as:

\[ (i) \quad V_{DC} > CP \quad \text{and} \quad \]
\[ (ii) \quad I + V_{DC} > \alpha[(1-\theta)(Q\tilde{x}-I) + V_E] \quad (48) \]

where CP denotes the call price of convertible-debt. The bondholders' decision in the event of call will be:

Convert, if \[ \frac{[I+CP}{\alpha} - V_E + I(1-\theta)](Q(1-\theta))^{-1} \quad \Xi(c) \leq X \leq NC' \]
Surrender, if \[ b < X < C \]

\[ (49) \]

1) Market Value of Equity:

Summarizing the above discussion, the end-of-period wealth of the existing stockholders, \( \tilde{Y}_E \), may be now written as:

\[ \tilde{Y}_E = \begin{cases} 
(1-\alpha)[(1-\theta)(Q\tilde{x}-I) + V_E] & \text{if } X > NC' \\
(1-\theta)(\tilde{x}-I) + V_E & \text{if } b < X \leq NC' \\
I + V_{DC} - \alpha[(1-\theta)(Q\tilde{x}-I) + V_E] & \text{if } C < X < NC' \\
V_{DC} - CP & \text{if } b < X < C \\
0 & \text{otherwise} 
\end{cases} \quad (50) \]

So, the market value of equity can be expressed as:

\[
V_E = V_E(\text{equivalent}) + \{q(1-\theta)CEQ_{NC'}(\tilde{x}) - (I+V_{DC})[1-F(\text{NC'})]\}(r^{-1}) \\
- \{\alpha(1-\theta)CEQ_{NC'}(Q\tilde{x}-I) + [\alpha V_E - I-V_{DC}[1-F(\text{NC'})]\}(r^{-1}) \\
+ \{[I+V_{DC} - \alpha V_E][F(\text{NC'}) - F(C)] - \alpha(1-\theta)CEQ_{NC'}(Q\tilde{x}-I) \\
+ (V_{DC} - CP)[F(C) - F(b)]\}(r^{-1}) \quad (51)
\]
We notice that the first three terms on the RHS of (51) are the same as in (44) which represents the market value of equity in the absence of call option. The last term is the present value of call option. The first two expressions together within the braces in the last term represent the net gain to the stockholders when the bonds are converted upon call. The last expression is their net gain when the bonds are surrendered for redemption at call price. Thus, the market value of equity is higher in the presence of call option by the amount indicated by the last term on the RHS of (51).

ii) Market Value of Convertible-debt and Firm

As previously noted, when the call is exercised, the conditions are such that irrespective of whether the bonds are converted or surrendered upon call, the bondholders incur a loss in their wealth. This loss is equal to the stockholders' gain simply because what is gained by the stockholders is an opportunity lost by the bondholders. So, the end-of-period wealth of the convertible bondholders can now be written as:

\[
\tilde{Y}_{DC} = \begin{cases} 
\alpha[(1-\theta)(Q\tilde{X}-I) + V_E] & \text{if } X > NC' \\
I + V_{DC} & \text{if } b < X \leq NC' \\
-(I+V_{DC}) + \alpha[(1-\theta)(Q\tilde{X}-I) + V_E] & \text{if } C \leq X \leq NC' \\
-V_{DC} + CP & \text{if } b < X < C \\
\tilde{X} + V(1-k) - K & \text{otherwise }
\end{cases}
\]

The market value of convertible-debt, therefore, can now be expressed as:
The first two terms on the RHS of (53) denote the market value of convertible-debt without call option as given in (46). And, the last term represents the loss of bondholders' wealth due to the possible exercise of call option. Notice that this loss is equal to what the stockholders gain. Further, in a perfect capital market where the information is costlessly available, the bondholders anticipate this loss efficiently; hence they offer less premium on the callable convertible-debt than they do if there were no call option. The market value of premium on the callable convertible-debt is represented by the difference between the second term and the third term on the RHS of (53). It is the excess of the present value of possible gain due to conversion privilege over the present value of the possible opportunity loss due to the exercise of call option. Because of the assumption that the premium is distributed among the existing stockholders, we observe that the net effect of call option is zero, for both the parties. That is to say, overall the exercise of call option is a zero-sum game between the issuer and the holders of callable convertible bonds.

Combining (51) and (53), we obtain the market value of the firm.

\[
V = V_u + \theta V^*_D - \left\{ [kV+K+I\theta]F(b) - \theta CEQ^b(x) \right\}(r^{-1}) \\
+ (V_{DC}-CP)[F(C)-F(b)](r^{-1})
\]

(54)

Thus, (54) coincides with (47). It means that the market value of the firm is indifferent to the existence of call option. It also means that
our previous conclusions regarding the efficiency of convertible-debt-financing hold for the callable convertible-debt as well. If anything, the call option reduces the premium on convertible bonds. Of course, it provides more flexibility to the firm in so far as the use of leverage is concerned.

B. Other Implications

We have placed in the preceding section a major thrust on analyzing the implications of convertible-debt-financing for the existing stockholders' wealth and for the market value of the firm. There, our intention is to address one question: Is convertible-debt less efficient than straight-debt as a source of capital? In this section, our intention is to discuss some of the other implications of our multi-period analysis that interest researchers, managers and the investors alike.

B.1 For Optimal Capital Structure

In the presence of corporate income-tax, bankruptcy risk and bankruptcy cost, the optimal capital structure is shown to exist by some academicians. While these studies provide proofs of the existence of optimal capital structure with respect to the use of straight debt, we provide an intuitive argument for its existence with respect to the use of convertible-debt. As is clear later, no mathematical proof is necessary.

The proofs of the existence of optimal capital structure involve the trade-offs between the tax shelter and the bankruptcy costs
associated with the straight debt. Although possible conversion causes the total amount of tax shelter to be less when the convertible-debt is used instead of the straight-debt, we have shown in the preceding section that the firm can avoid this loss of tax shelter by taking a countermeasure of raising new debt upon conversion sufficient enough to negate this loss. Hence, the market value of the firm is not affected by the convertibility feature of convertible-debt. The amount of tax shelter and bankruptcy costs are the same as are associated with its equivalent straight-debt. It clearly suggests that the optimal level of convertible-debt can be obtained by treating this debt as if it were a straight-debt and by considering the trade-offs between the associated tax shelter and bankruptcy costs disregarding the possible conversion. Chen's [22] proof of the existence of optimal capital structure (which is provided by treating the problem in a multi-period CAPM framework that assumes risk-averse investors in the market) holds for the convertible-debt as well.

B.2 For Optimal Call Policy of the Firm

Ingersoll [45] and Brennan-Schwartz [16] have proposed that it is optimal for the firm to call as soon as the underlying stock value rises to equal the call price. Since normally the convertible bond price is higher than the greater of either the underlying stock value or the straight-debt value, we observe that this is a special case of what we have defined [in (48)] as the condition for exercising the call option. We further observe that their point strategy ignores a situation when it is more profitable to call the bonds than it is to do so
when the underlying stock value equals the call price. Such a situation exists when the underlying stock value may be higher than the call price but the distance of convertible-bond price from its underlying stock value is more than what it was when the underlying stock value equalled the call price. As if to support the latter viewpoint, Ingersoll's [46] later study has shown that the firms never call the bonds at the point proposed by him and Brennan-Schwartz [16]. It is found in the majority of cases that the underlying stock value exceeded the call price when the call option was exercised. In this context, it is not unreasonable to say that their point strategy is incomplete.

Gordon Pye [41] has in fact set an appropriate optimal call policy in a statement that runs as: "(under certainty) the bond should be called as soon as the profit from doing so is greater than the value of doing so later" (p. 202). In our analysis, the third term on the RHS of (51) represents the present value of exercising call option in the subsequent period. The value of exercising call option right now can be figured out from the observed values. This value is equal to the excess of the value of convertible-debt at current price over the greater of either call price or the underlying current stock value. If the observed value of call option is greater than its theoretical value as defined by the third term on the RHS of (51), then it is profitable to exercise it right now; otherwise it is profitable to do so later. Thus, our multi-period analysis provides a hypothesis to test a firm's efficiency in exercising call option.
B.3 For Premiums on Convertible-Bonds

Besides, it must be already evident that our analysis provides a model to estimate conversion-premiums on both the callable and non-callable convertible bonds. We have expressed the market value of convertible-debt through our analysis as the sum of its equivalent-straight-debt value and the present value of conversion (and call) option. Therefore, it is not difficult to derive a conversion premium model. Since the convertible-debt-financing is evaluated under the varied presumptions with respect to firm's plan upon conversion, in fact, there is more than one model that might be derived.

The market value of convertible-debt, given in Eq. (53) is derived assuming that the convertibles are callable and that the firm pursues a plan of maintaining the same leverage upon conversion and using the new debt to finance new investment. Since this is a comprehensive valuation model, we derive below a conversion-premium model from Eq. (53):

Denoting $V_{D}$ (equivalent) by $V_{DS}$ and rearranging the terms in (53), $V_{DC}$ can be expressed as:

$$V_{DC} = V_{DS} + \{\alpha(1-\theta)CEQ_{NC'}(QX-I) + \alpha V_{E}[1-F(NC')]\}
+ \alpha(1-\theta)CEQ_{NC}(QX-I) + \alpha V_{E}[F(NC')-F(C)]
+ [I+CP][F(C)-F(b)] - (I+V_{DS}[1-F(b)])[r+1-F(b)]^{-1}$$

(55)

So the premium model can be specified as:

$$V_{p} = \{\alpha(1-\theta)CEQ_{NC'}(QX-I) + \alpha V_{E}[1-F(NC')]\}
+ \alpha(1-\theta)CEQ_{NC}(QX-I) + \alpha V_{E}[F(NC')-F(C)]
+ (I+CP)[F(C)-F(b)] - (I+V_{DS}[1-F(b)])[r+1-F(b)]^{-1}$$

(56)
Though the expression is long, the logic is clear. The value of premium on callable convertible-debt is basically the difference between what is received in conversion-exchange (voluntary or involuntary) or surrender for redemption at call price and what is foregone by the bondholders in that exchange. The first two terms within the braces on the RHS of (56) represent the share of stockholders' wealth to be received in a voluntary conversion-exchange. The next two terms represent the share of stockholders' wealth to be received in an involuntary conversion-exchange. The fifth term is the sum of one-period interest and call price to be received upon surrender subsequent to call and the last term is the amount of debt claim to be foregone when the bonds are converted and/or called back. Since there exists a possibility of converting and/or calling the bonds when the firm remains solvent, the debt claim to be foregone is multiplied by the probability of no bankruptcy. Similarly, the appropriate discount rate for the sum given within the braces on the RHS of (56) includes riskless rate of interest plus a premium equivalent to \(1-F(b)\) for the risk of call and conversion. Note that the premium for the risk of possible bankruptcy is included in the discount rate used while deriving the equivalent straight-debt value of convertible-debt.

Thus, (56) indicates explicitly that the premiums on callable convertible bonds are not only a function of the equivalent straight-debt value, the risk-return characteristics of firm's cash flows, and the growth rate of cash flows, but also a function of possible bankruptcy, conversion and call. Most of the variables (such as \(V_{DS}\), income differential, stock return and volatility, stock price growth
rate, etc.) identified by the previous studies [36, 76, 94, 98] to be significant in explaining convertible bond premiums are all embodied in the above model. Plus it states explicitly the role of possible bankruptcy and exercise of call option in determining conversion premiums on callable convertible bonds. So, it may be concluded that our premium model is more comprehensive than is provided hitherto.

We have carried out in this chapter a multi-period evaluation of the convertible-debt-financing by utilizing the Chen's simplified version of multi-period CAPM. In the process, we have recognized: a) the existence of corporate income-tax; b) the possible costly bankruptcy; c) the existence of call option; and d) firm's capital-structure plan upon conversion.

We have been able to establish that Lewellen-Racette's proposition of convertible-debt-inferiority is invalid. This is so because their proposition rests on a critical assumption of the firm's future investments being financed by the retained earnings, but the assumption is inconsistent with the firm's rational behavior. By recognizing the firm's rational behavior vis-a-vis, its capital-structure plan upon conversion, it is shown that the convertible-debt is at least as an efficient means of financing as straight-debt. Certainly, the argument of tax shelter difference does not have any substance. Once its other attributes - flexibility, potential for greater use of leverage, less severe restrictions, and subordination - are considered, the convertible-debt is indeed a more attractive means of financing. Consideration of costly bankruptcy does not make it less attractive.
In our analysis, no a priori specification is made regarding the exercise of call option. Instead, the necessary and sufficient conditions for exercising call option are defined. It is pointed out that Ingersoll's [45] and Brennan-Schwartz's [16] point call-strategy as a special case of the broader condition defined by us. The existence of call option reduces the conversion premium and affects the wealth of security holders. However, the market value of the firm is shown to be unaffected by its existence as it involves a zero-sum game between the stockholders and the convertible bondholders. Another interesting result of our multi-period analysis concerns the optimal capital structure. In the presence of costly bankruptcy, the optimal capital structure exists and it can be obtained by treating the convertible-debt as if it were a straight-debt. No consideration need be given to the possible conversion. This conclusion is, however, based on our assumption of firm issuing immediately a new debt equivalent to the converted debt. Notwithstanding this, we have been able to present a conversion-premium model that is more comprehensive than is provided hitherto.

Although these are all interesting results, the valuation of convertible bonds is still our main interest. Hence, what follows in the next two chapters is the presentation of a multiple regression pricing model for callable convertible bonds that is based on the convertible-debt-valuation equation given in Eq. (53) and the results of an empirical study testing the significance of the said model.
Footnotes To Chapter IV

1 See the footnote (1) of Chapter II.

2 Kaplan [1965] provides an interesting analysis of anti-dilution clauses in convertible bond Indenture Agreements. He reports a surprising variance in these provisions.

3 Much of the modern finance theory underscores the point that a firm's objective is to maximize the wealth of its stockholders. However, some studies have questioned this objective based on the theory of agency which focuses on the separation of management from ownership in the case of a corporation. The underpinning of the theory of agency is that if the manager's goal is to maximize their utility, then there will be times when manager's and stockholders' interests are in conflict with each other. For a detailed exposition, see Jensen and Meckling [1976]. We do, however, continue to believe in the point underscoring the main stream of finance theory.

4 It does not matter whether the firm issues straight bonds or convertible bonds upon conversion as long as it follows either of the stated policies if the convertible bonds are outstanding at a point in time. This is analogous to Kraus [1973]'s argument on the refunding policy of a firm that has issued callable straight bonds.

5 Kim, McConnell and Greenwood [1977] have done an empirical study by considering one-way (i.e. forming captive finance subsidiaries) of violation of me-first-rules as applied to straight bonds. A study with similar empirical approach to the problem of violation of me-first-rules as applied to convertible bonds would be interesting.


7 Fame [1970] has shown that under the myopic condition and quadratic utility function, the single-period CAPM is adequate to evaluate multi-period consumption-investment decisions. Assuming perfect secondary markets, Bogue and Roll [1974] have observed that multi-period problem collapses to a single-period problem and for an n-period problem, an n-period infinite state dynamic programming can be used; solution at each step is obtained by the application of single-period valuation model, with the parameters depending on the state of the world at the beginning of
that period. And Stevens [1974]' approach is analogous to Bogue and Roll and states that the value of the firm under uncertainty is the infinite sum of risk-adjusted expected dividends, discounted by the riskless rate of interest. If we assume that the stockholders know future riskless interest rates and market prices of risk with certainty, then the problem becomes much simpler.

Actually, Chen [1978b] has used $\tilde{M}$ to represent the market-wide factor. $M$ denotes the sum of end-of-period stochastic operating earnings of all the firms in the market. It does not matter in theory whether we use $\tilde{M}$ or $\tilde{R}$ to represent the market-wide factor. Since $R$ is already introduced in the preceding chapter, it is used in place of $\tilde{M}$ while presenting his model. We have also done the same thing in the subsequent analysis whenever an equation from his paper [1978b] is reproduced or referred.

For extensive discussion of the significance of reassessments for capital budgeting decisions, see Fama [1977] for example. Fama states that the risk adjustments in discount rates through time arise due to uncertainty about the reassessment through time of expected cash flows and their relationships with the reassessments of the expected cash flows of all the firms. He argues that it is probably reasonable to assume that the risks in the reassessments of the expected value of a cash flow through time and across cash flows are constant at least in the case of a firm whose activities are not anticipated to change much in nature through time.

The convertible bonds have a finite period of maturity no doubt, but it is usually 20-30 years — long enough to treat the bonds as perpetual debt instruments. Besides, the issuers as well as the holders of bonds have a chance in perpetuity of substituting similar assets for bonds as and when they mature, are converted or surrendered. So, it is not an unreasonable assumption.

See the definition in Stanley and Girth [1971].

For arguments leading to this condition, see Scott [1976] and Chen [1978b].

The optimal initiation of bankruptcy proceedings either by debtor or creditors is an interesting issue. Van Horne [1976] has addressed this issue from the creditors' viewpoint. But the optimal initiation presupposes that the firm is, at the time of evaluation, unable to pay its current obligations. In our case, there is no bankruptcy situation at the time of evaluation, rather we would like to incorporate probability of bankruptcy in the subsequent period. Further, we are confining to bankruptcy-liquidation case only. So, we don't need to worry about the optimal initiation of bankruptcy proceedings.
14 Any combination of these two plans upon conversion might yield the desired results. Besides, another possibility is that the firm neither retires new issues nor does it raise new debt upon conversion. That means, it opts for being an all-equity firm upon conversion, in which case the tax subsidy will be lost upon conversion and the entire amount of this loss is borne by the existing stockholders. This would result in the reduction in stockholders' wealth and that of the firm. For analysis, see footnote no. 18. But note that a stockholders' wealth maximizing firm would never opt for this plan upon conversion.

15 Scott [1976] has used the same procedure, under the assumption of risk neutral investors in the market, in deriving the market value of equity and debt when both corporate income-tax and costly bankruptcy exist.

16 In the ex-post sense, the equity owners receive the remaining proceeds after the claims of bondholders and other creditors are satisfied. But in the ex-ante sense, as Scott [1976] has correctly pointed out, the bankruptcy occurs precisely because there will not be any positive net proceeds available to shareholders upon bankruptcy.

17 This assumption is not a limitation of our analysis. The assumption is only for the purpose of clarity. Even if the new bonds are subscribed by investors other than the existing bondholders, that does not affect our analysis, because we are concerned with the effect of convertible-debt-financing in conjunction with this plan on the wealth of existing stockholders. It is essential to know that the capital market evaluates convertible bond issue followed by this firm's plan upon conversion as a single package consisting of convertible bonds now and new bonds upon conversion with the same claim. Prices based on any variation of this package would only lead to arbitrage process and in equilibrium, the offer of convertible bonds is valued as a single package having a potential of continuing to hold debt instruments of the same firm even after conversion.

18 Under the plan of neither retiring new issues nor raising new debt, but financing new investment subsequent to conversion by retained earnings, the end-of-period wealth of stockholders, $\bar{Y}_E$, is:

$$\bar{Y}_E = \begin{cases} 
(1-\alpha)[(1-\theta)Q\bar{X}+V] - A & \text{if } X > \left[\frac{I+V_{DC}}{\alpha} - V\right]((1-\theta)Q)^{-1} \\
(1-\theta)\bar{X} - I + V_T & \text{if } I - V_T (1-\theta)^{-1} < X \leq \left[\frac{I+V_{DC}}{\alpha} - V\right]((1-\theta)Q)^{-1} \\
0 & \text{otherwise}
\end{cases}$$

where $A$ denotes the amount of new investment.
Notice that the conversion strategy now is to convert if \[ X > \left[ \frac{I+V_{DC}}{\alpha} - V \right] [(1-\theta)\alpha]^{-1}. \] So the market value of equity is:

\[
V_E = \{(1-\theta)[E_b(\bar{X}-I) - \lambda \text{Cov}_b(\bar{X}, \bar{R}_m)] - F(b)V_E \}(r)^{-1}
- \{(1-F(NC''))(\alpha V - V_e - I) + \alpha Q(1-\theta)[E_{NC''}(\bar{X}) - \text{Cov}_{NC''}(\bar{X}, \bar{R}_m)]\}(r)^{-1}
- \{1-F(NC'')\}I\theta(r)^{-1} + \{q(1-\theta)\text{Ceq}_{NC''}(\bar{X}) - A[1-F(NC'')]\}(r)^{-1}
\]

Similarly, the end of period wealth of bondholders is:

\[
\tilde{Y}_D = \begin{cases} 
\alpha[(1-\theta)Q\bar{X}+V] & \text{if } X > \left[ \frac{I+V_{DC}}{\alpha} - V \right] [(1-\theta)\alpha]^{-1} \\
I + V_{DC} & \text{if } I - V_e(1-\theta)^{-1} < X \leq \left[ \frac{I+V_{DC}}{\alpha} - V \right] [(1-\theta)\alpha]^{-1} \\
V(1-k) - K + \bar{X} & \text{otherwise}
\end{cases}
\]

so the market value of convertible debt is:

\[
V_{DC} = \{(1-F(b))I+F(b)[V(1-k)-K] + [E^b(\bar{X})-\text{Cov}^b(\bar{X}, \bar{R}_m)] - F(b)V_{DC} \}(r)^{-1}
+ \{(1-F(NC''))(\alpha V - V_{DC} - I) + \alpha Q(1-\theta)[E_{NC''}(\bar{X}) - \text{Cov}_{NC''}(\bar{X}, \bar{R}_m)]\}(r)^{-1}
\]

The market value of the firm is the sum of these two values.

\[
V = V_E + V_{DC}
= V_u + \theta V^* - \{F(b)[kV + K + I\theta] - \theta[E^b(\bar{X}) - \text{Cov}^b(\bar{X}, \bar{R}_m)]\}(r)^{-1}
- \{1-F(NC'')\}I\theta(r)^{-1} + \{q(1-\theta)\text{Ceq}_{NC''}(\bar{X}) - A[1-F(NC'')]\}(r)^{-1}
\]

Therefore, the market value of both equity and firm decline under this plan upon conversion. LR's proposition holds. But one wonders whether the stockholders wealth maximizing firm would ever opt for this course of action upon conversion.


Brigham [1966]'s survey showed that the dominant reason for issuing convertible was to obtain equity in future. Seventy-three per cent of the respondent-firms in his survey had the delayed equity motive for issuing convertibles. See also Browman [1963]. This apart, recently Mehta [1976] has theoretically substantiated by using Hamiltonian Dynamics that the motive of "issuing convertibles for raising tomorrow's equity today" leads to normative dividend policies that are generally acceptable to management. (steady increase or at least maintaining dividend over time may be regarded as a normative dividend policy that is adequately evidenced in the literature, e.g. Lintner [1956]).
21  See, Scott [1976], Elton, Gruber and Lightstone [1977] and Chen
[1978b] for example.

22  See also Boness-Chen [1970] and Alan Kraus [1973].
CHAPTER V

EMPIRICAL HYPOTHESES AND TESTING PROCEDURE

A multi-period convertible-debt-valuation model [Eq. No. (53) on p. 88 or (55) on p. 92] is presented in the preceding chapter. In an extreme situation of a single convertible bond outstanding, the equation (55) is a theoretical model for a callable convertible-bond price. In a realistic case of the plurality of convertible bonds outstanding, the RHS of (55) shall have to be divided by the number of convertible bonds outstanding. In any case, the variables determining the prices of callable convertible bonds are explicit in (55).

We have also presented in the preceding chapter some of the interesting implications of the multi-period evaluation of convertible-debt financing. One of the main implications is for the estimation of convertible bond prices based on the model given in Eq. (55). Although some of the other implications (e.g. for the firm's call policy) can be examined by an empirical study, we have confined our examination to the callable convertible-bond price estimating equation (55).

Ideally, we should estimate the convertible-bond prices directly, using the model given in (55). But this cannot be done for two reasons: first, there is a lack of empirical evidence to support one of our basic assumptions that the cash flows (EBIT) of a firm are normally distributed, and second, a firm normally uses different forms of debt (such as
bank loans) besides convertible debt. This makes it difficult for us to estimate efficiently the probabilities of state-option combinations. However, Eq. (55) is a useful model in the sense that unlike the conventional convertible-bond valuation models, it indicates more specifically a comprehensive set of factors determining convertible-bond prices. So, in the following a multiple-regression model to estimate convertible-bond prices is developed on the basis of our knowledge from Eq. (55) of the determining factors. Empirical hypotheses concerning this model and test procedures are described in this chapter, with the results of our empirical study presented in the next chapter.

Equation (55) is reproduced below for further consideration:

\[
V_D = V_{DS} + (\alpha(1-\theta)CE_{NC}(QX-I) + \alpha V_{E}[1-F(NC)]) \\
+ \alpha(1-\theta)CE^{\mathcal{N}_C}(QX-I) + \alpha V_{E}[F(NC)-F(C)] \\
+ (I+CP) [F(C)-F(b)] - (I+V_{DS}[1-F(b)]) [r+1-F(b)]^{-1} \]

As is clear, (55) involves partial expectations and partial covariances, that are embodied in the certainty-equivalent expressions on the RHS. Due to Winkler et. al. [100], the partial expectations of a normally distributed variable can be derived in terms of the first and second moments of the distribution of that variable, the cumulative probability distribution and the probability density at a given truncation point and the truncation point itself.\(^1\) Similarly, due to Mood-Graybill [69] and Kim [55], the partial covariance between a variable with another jointly normally distributed variable can be derived in
terms of the covariance between those variables, the cumulative probability and the probability density at a given truncation point and the given truncation point itself.\(^2\) Using these results, equation (55) can be written in functional form as follows:

\[
V_n + f(V_n) + \frac{1}{2} \sum_{X} \text{Cov}(X, \tilde{X}) + \alpha, F(b), F(C), F(NC), NC, C, f(NC), f(C), r, q, \theta, I)
\]

That is, the convertible-bond price is a function of equivalent straight-debt value, call price, return-risk characteristics of firm's cash flows, conversion ratio, probability of bankruptcy, the truncation points of call and conversion, cumulative probability and probability density functions at those truncation points, market rate of interest, tax rate, growth rate of cash flows and the interest income from holding the convertible bond.

Of these variables, market rate of interest and tax rate are exogenously given and interest income, conversion ratio and call price are known with certainty on the date of issue of a convertible bond. If the market-related value variables are already present in the model, the exogenously given variables (i.e., \( r, \theta \)) may be presumed to have been reflected in those variables. Though the interest income is shown to be a separate variable, it may be presumed to have been reflected (as are \( r \) and \( \theta \)) in the equivalent straight-debt value \( V_{DS} \). This is because, the equivalent straight-debt value is the value of a straight bond that matches with its counterpart convertible bond with respect to interest income, maturity and quality rating. Further, \( \alpha, F(NC), f(NC) \)
and NC are all related to the conversion option. And, CP, C, F(C) and f(C) are all related to the call option. Especially, the presence of truncation points, cumulative probabilities and density functions at those truncation points is due to our assuming a normal distribution of cash flows and the recognition of truncated distribution of cash flows accruing to the convertible bondholders. Now, if we were to test the model by multiple regression introducing all the variables, we run into a problem of linear dependence between some of the regressors. Moreover, as is already noted, to our knowledge, there is no known empirical evidence to suggest that a firm's EBIT are in fact normally distributed. So, it does not seem appropriate for empirical testing to estimate the truncation points and moments of distribution of cash flows directly by using firm's annual EBIT.

To circumvent these problems, we propose to include in the proposed multiple-regression model a single variable to represent conversion, a single variable to represent call option and proxies for return-risk measures, growth rate and for the possible bankruptcy.

A possible proxy for the exercise of conversion option is the ratio of stock-equivalent value (i.e., conversion value) to straight-debt value of a convertible bond. The higher this ratio, the greater is the probability of conversion. But when this ratio is included along with the straight debt value as another regressor, the assumption of independence between the regressors is violated because they are linearly dependent. Other possible proxies are CV/V_{DC} and (CV-V_{DC})/V_{DC}. In both cases, the higher the ratio, the greater is the probability of
conversion. But again, if we use either of these proxies, we run into a problem of a misleading specification because of the simultaneous consideration of the same variable on both sides of the regression equation. In view of these difficulties, the conversion value, CV, itself is taken to represent the conversion option. Similarly, CP/V_{DC} or [CP-V_{DC}]/V_{DC} could be a proxy for call-option exercise. The higher either of these ratios, the lower is the probability of calling a convertible bond. Since call price (CP) is indicated to be one of the determining variables in Eq. (57) and because the introduction of either of the above ratios as an additional variable would lead to the problems of linear dependence and simultaneous-equations bias, only the call price (CP) is taken to represent the call option in the regression model.

Ignoring exogenously given variables and the interest income for the reasons cited earlier and postponing the discussion of proxies for other variables to the section on the estimation of variables, (57) can be reduced to the following:

\[ V_D = f(V_{DS}, CV, CP, E(\tilde{X}), \sigma^2_X, COV(\tilde{X}, \tilde{R}_m), q, F(b)) \]  

(58) is the basis of the regression model proposed later for estimating the prices of callable convertible bonds. Unlike the previous studies, we wish to investigate empirically, among other things, the significance of call price and the probability of bankruptcy.
A. Hypotheses

Our purpose is to test in general the significance of the regression model proposed later. It is further intended to test the significance of each of the included variables in determining convertible-bond prices. We also wish to test the stability of the proposed regression model over time and across firm size, so that at least some initial conclusions may be drawn regarding the applicability of the model as an estimating device. The following hypotheses are offered for examination:

Null hypothesis 1: Variation in the prices of callable convertible bonds is not significantly explained by the proposed regression model.
Alternative hypothesis 1: The proposed regression model explains significantly the variation in the prices of callable convertible bonds.

Null hypothesis 2: None of the regression coefficients in the proposed model is significant.
Alternative hypothesis 2: The coefficients of at least some of the regressors in the proposed model are significant.

Null hypothesis 3: The proposed regression model is not stable over time.
Alternative hypothesis 3: The proposed regression model is stable over time.

Null hypothesis 4: The proposed regression model is not stable across firm size.
Alternative hypothesis 4: - The proposed regression model is stable across firm size.

B. The Sample

The following criteria governed our choice of convertible-bond issues to be included in the sample:

(1) The financial year of the firms should end on December 31st because the dates of evaluation were decided to be December 31st of 1973, 1974, 1975 and 1976. This facilitates obtaining the relevant balance-sheet items required for calculating proxies for the possible bankruptcy.

(2) The firm's common stock and convertible bond must have been listed on the NYSE so that monthly return data would be available on CRSP tapes and relevant data on convertible bond would be available either in Moody's Bond Records or S&P Bond Guides. The common stock must have been listed on the NYSE at least five years prior to December 31st, 1973.

(3) The convertible-bond issue must have been included in the list of those evaluated by Moody's Investment Service to determine their equivalent straight-debt value during December, each year under consideration.

(4) Both conversion and call options should be in effect on all dates of evaluation.

Only 26 issues satisfied these criteria. All are included in our sample. See Appendix A for the list. It should be noted that of these
26 issues, only five issues are found not to appear on Moody's lists of evaluated convertibles for all four years. So, the equivalent straight-debt values of these issues for one or two years could not be obtained from the Moody's Convertible Bond Survey. The missing data are, however, interpolated by selecting matching straight bonds by a procedure explained later.

Though the sample is small, there is broad representation of different industry groups. Appendix A lists: three issues from utilities, three from railroads, one from transportation, and the rest from the general "industry" category. Within the category of "industry", a variety of industries are represented. However, note that the sample is an all-inclusive and non-stratified one. Hence, the representation of issues from different industry groups in the proportion of their listing on the NYSE cannot be assured.

So far as the maturity composition is concerned, the earliest year in which a bond matures is 1980. The latest year of maturity is 1999. Excepting three issues, all the issues mature after January 1, 1990. It means the sample is dominated by issues with long terms-to-maturity. Incidentally, conversion privileges apply to all included issues until the end of their respective life. Since the conversion expiry date is not close at hand for most of the issues included in our sample, it may be assumed that the term-to-maturity is not a significant variable in determining prices of the included convertible bonds. However, a multiple regression including, among others, term-to-maturity as one of the regressors was carried out initially.
And the coefficient of term-to-maturity was not found to be significant at $\alpha=0.05$. This provides some justification for not including term-to-maturity as one of the regressors in the proposed regression model.

The sample is also dominated by issues of large-size firms. There are 17 issues of the firms whose total assets on December 31, 1973 exceed $1.2$ billion and the remaining issues are of the firms with total assets below $1.2$ billion, the smallest asset size being $271.1$ million. The cut-off point of $1.2$ billion of total assets in the 1973 figures is obtained by using Quandt's [77] test for maximum likelihood estimate of cut-off point. This test procedure is explained in the next chapter.

Appendix B gives the beta values of underlying common shares for the years under consideration. These beta values are estimated by fitting Sharpe's market model, $\tilde{R}_{ji} = a_j + b_j \tilde{R}_{Mi} + e_{ji}$, to the monthly-return data of the underlying common share and S&P Composite Index for 60 months prior to each date of evaluation. A mere glance at the appendix suggests that only four stock issues are defensive (beta less than 1). The rest may be classified as either moderate or aggressive common shares.

C. Data Collection

The primary sources of data are Moody's Bond Records, S&P Bond Guides, CRSP tapes and COMPUSTAT. The equivalent straight-debt value, call price, quality rating, and the number of shares per $1000$ bond data are collected from Moody's Bond Records and Bond Surveys. The convertible bond price data are gathered from S&P Bond Guides, CRSP
tapes provided monthly-return data (adjusted for dividend) on common
stocks and S&P Composite Index. The return data are used in estimating
the expected return, beta, variance and covariance of a stock's return
with the return on S&P Composite Index. COMPUSTAT provided stock prices
on December 31st of each year and the balance sheet and income state­
ment items that were required for calculating the proxies for probabil­
ity of bankruptcy. Moody's Manuals (industrial etc.) were consulted for
the verification and missing data given in COMPUSTAT.

D. Estimation of parameters and variables

\[ V_{DS} \] - The equivalent straight-debt values estimated by Moody's
Investment Service are considered as the first estimates of \( V_{DS} \). Since
these estimates are usually made in the first week of December, an
adjustment is needed for changes in bond prices during the remaining
three weeks of December to establish comparability with other data.
For this purpose, weekly averages of S&P corporate-bond yields (given
according to industry category in S&P Bond Guide: "industry", trans­
poration, utility, and railroads) are used as the basis of adjustment.
Since the bond yields are inversely related to bond prices, the
adjusted \( V_{DS} \) estimates are obtained as follows:

\[
\text{Adjusted } V_{DS} = \frac{V_{DS} \text{ given in Moody's Bond Survey}}{S&P \text{ corporate bond yield for the respective industry category in the week of Moody's evaluation}} \times \frac{S&P \text{ corporate bond yield for the respective industry category in the last week of December.}}{V_{DS} \text{ given in Moody's Bond Survey}}
\]
Normally the above adjustment is appropriate for consol bonds. Since most of the bonds included in our sample have long terms-to-maturity, the suggested adjustment is thought to provide close approximates of the equivalent straight-debt values on respective dates of evaluation.

As is previously noted, some $V_{DS}$ data are missing from Moody's Bond Survey for five convertible issues.

They are:

2. Great No. Nekoosa for 1974
3. Pittston for 1976
5. Trane Co. for 1976

To interpolate these missing data, matching straight bonds were selected according to the procedure suggested by Kim-McConnell-Greenwood [56]. As indicated by them, the following four characteristics were considered for matching:

(a) bond quality rating determined by the Moody bond quality rating service
(b) term-to-maturity
(c) coupon interest rate and
d) coupon interest payment dates.

The justification for matching on these characteristics is very well explained in their paper. Though perfect matching is not possible, we attempted to match as many characteristics as possible and to find as close a solution as possible for other characteristics. The result is the following:
<table>
<thead>
<tr>
<th>Convertible bond</th>
<th>Matched straight bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Georgia Pacific 5 1/4s, 96</td>
<td>Idaho Power 5 1/4s, 96, A&amp;O 1, Aa</td>
</tr>
<tr>
<td>A&amp;O 1, Baa</td>
<td></td>
</tr>
<tr>
<td>2. Great No. Nekoosa 4 1/4s, 91</td>
<td>Balt &amp; O.R.R. 4 1/4s, 95, A&amp;O 1, Baa</td>
</tr>
<tr>
<td>A&amp;O 1, Baa</td>
<td></td>
</tr>
<tr>
<td>3. Pittston 4s, 97, J&amp;J 1, Baa</td>
<td>Hocking Valley Ry. 4 1/4s, 99 A&amp;O 1, Baa</td>
</tr>
<tr>
<td>4. St. Regis Paper 4 7/8s, 97</td>
<td>Georgia Power 47/8s, 95 M&amp;S 1, Baa</td>
</tr>
<tr>
<td>A&amp;O 1, Baa</td>
<td></td>
</tr>
<tr>
<td>5. Trane Co. 4s, 92 M&amp;S 15, Baa</td>
<td>Alabama Power 4 1/2s, 91 M&amp;S 1, Baa</td>
</tr>
</tbody>
</table>

The prices of these matched straight bonds on December 31st of the respective year(s) are considered as the estimates of equivalent straight-debt values of the respective convertible bonds.

Estimation of equivalent straight-debt values by finding matching straight bonds is definitely a better procedure than using the estimates of Moody's Investment Service. This is so because the former procedure gives market determined figures whereas the latter gives the figures estimated by an outside agency. But it is found to be impossible to find matching straight bonds on the above set of criteria for a majority of the convertible bonds in our sample. Therefore, we are left with the option of considering the estimates of Moody's Investment Service.

$E(X)$ and $\sigma^2_x$: The expected return (ER) and the variance of returns (VAR) on the respective common stocks are proposed to be used as the proxies for these parameters. These are thought to be reasonable
proxies because the returns are supposed to reflect the underlying earnings and also because of the existence of literature supporting the assumption of normal distribution being a fair approximation of actual distribution of stock returns (Fama [31] for example). First, expected monthly returns and variance of monthly returns are calculated using return data for sixty months prior to each date of evaluation. The values thus obtained are then transformed into the annual figures to establish comparability with other data.

\[ \text{Cov}(\tilde{X}, \tilde{R}_m): \] The covariance of the respective common-stock returns with the returns on S&P Composite Index (COVAR) is proposed to be used as the proxy for the covariance of a firm's cash flow with the return on a market index. The reasons for proposing this are noted above. Sharpe's market model is first fitted to monthly-return data for sixty months prior to each date of evaluation to estimate beta coefficients of common stocks. Covariance is the product of beta coefficient and the variance of returns on the S&P Composite Index. The covariances thus obtained are then transformed into annual figures to bring about comparison.

\[ \text{CV}: \] This is the stock-equivalent value of a convertible bond. So this is equal to the number of shares into which a convertible bond of a face value of $1,000 is exchangeable on each date of evaluation times the common stock closing price prevailing on that date.

**Proxy for probability of bankruptcy:** Altman's study [1] of the usefulness of financial ratios for predicting corporate bankruptcy has
received a good deal of attention in recent years. His purpose was to use multiple-discriminant analysis to identify which financial ratios distinguish between bankrupt and non-bankrupt firms. In his study, three ratios proved to be major contributing factors in explaining bankruptcy: retained earnings to total assets, EBIT to total assets and market value of equity to total book value of debt. These ratios consistently deteriorated for bankrupt firms over the five years preceding the actual bankruptcy. These ratios represent the relative equity base provided by internally generated funds, profitability and leverage or solvency position of a firm. Bankruptcy, of course, is a situation resulting from the deterioration in all of these positions.

Altman [1] provided scale factors to suggest the relative contribution of the financial ratios included in his discriminant function. Based on these scale factors, one could construct a composite index to be used as a proxy for a firm's probability of bankruptcy. We do not, however, use the composite index for the following reasons:

(1) Altman's sample consisted of only small firms with the largest asset size being $25.9 million, whereas our sample consists predominantly of large firms with the smallest asset size being $271.1 million and the largest asset size being $3.989 billion in 1973 figures. Moyer's [73] recent study re-examining Altman's model suggests that the model is not generally suitable when applied to a sample of larger firms outside the original sample period.

(2) The procedure used by Altman for determining the scale factors is questionable.
(3) As shown by Altman-Eisenbeis [3], inferences about the relative importance of individual financial ratios are extremely sensitive to the criterion of importance being employed.

We do, however, recognize the previously stated three financial ratios as proxies for probability of bankruptcy. Besides, we would like to consider Moody's bond-quality rating as an additional proxy because of evidence to the effect that defaults are significantly and consistently related to the ratings assigned by Moody and S&P.\textsuperscript{10} Since defaults are indicative of a firm's financial distress, the significance or otherwise of quality ratings is believed to suggest the importance given to a firm's financial distress. Note that it is difficult to distinguish between probable bankruptcy and financial distress of on-going firms. Consideration of bond quality rating, therefore, in no way implies that we intend to recognize default risk as separate from the risk of bankruptcy.

To consider bond quality ratings, we need to assign numbers to quality ratings of the convertible issues included in our sample that range from Aa to Ba. For this purpose, the average monthly corporate bond yields classified according to quality ratings are gathered for 60 months prior to December 31st, 1973 from Moody's Bond Record. This source gives the bond yields summary data only for investment-grade bonds (i.e., Aaa, Aa, A and Baa rated bonds). Since none of the issues in our sample have Aaa quality rating, we have gathered data for Aa, A and Baa classifications and fitted a multiple regression (without intercept) using dummy variables. Because the F statistics (univariate) are the standardized measures of importance, the criterion
for assigning numerical values is chosen to be the F statistics of the respective dummy variables. The F statistics of the dummy variable of Aa quality rating was initially equated to 1.0. Since the bond yields are inversely related to quality ratings, the numerical value for A quality rating is estimated by the ratio of F for Aa to the F for A. For Baa quality rating, it is the ratio of F for Aa to the F for Baa. The numerical values thus obtained are as follows: 1.0 for Aa, 0.95 for A and 0.84 for Baa. To interpolate the numerical value for Ba quality rating, we have considered the decline in the numerical value from one quality rating to another in the above analysis. We notice that the value has declined by 0.05 from Aa to A and by 0.09 from A to Baa -- a 0.04 difference in the decline. Assuming this difference in the decline rate, the numerical value for Ba quality rating is estimated to be 0.71.

The three financial ratios: retained earnings to total assets, EBIT to total assets and market value of equity to total book value of debt as defined by Altman [1] and the bond quality rating are all proposed to represent the probability of bankruptcy. Their reciprocals are separately introduced into the proposed regression model to establish a direct correspondence between probability of bankruptcy and these variables. The purpose of introducing them separately is to investigate which of these variables are perceived to be important in determining convertible-bond prices.
This represents the expected growth rate of cash flows upon conversion due to presumed additional investment supported by incurring new debt equivalent to that converted. One of the possible proxies for this variable could be the ratio of current EBIT to total assets. Since the reciprocal of this ratio is one of the explanatory variables, the introduction of a separate variable representing the growth rate is unwarranted.

In view of this discussion, the first functional form of the convertible-bond price model to be tested may be written as follows:

\[ V_{DC} = f (V_{DS}, CV, CP, ER, VAR, COVAR, RETAN, EBTAN, EVTDN, PQR) \] (59)

where

- \( V_{DC} \) = convertible bond price
- \( V_{DS} \) = equivalent straight-debt value
- \( CV \) = conversion value
- \( CP \) = call price
- \( ER, VAR, COVAR \) = respectively expected annual return, variance of returns and covariance of returns on underlying common stock with the returns on S&P Composite Index.
- \( RETAN \) = reciprocal of the ratio of retained earnings to total assets
- \( EBTAN \) = reciprocal of the ratio of EBIT to total assets
- \( EVTDN \) = reciprocal of the ratio of market value of equity to total book value of debt
- \( PQR \) = reciprocal of the quality rating.
E. **Testing Procedure**

The linear regression model of the function of convertible-bond price given in (59) is first carried out using the SPSS program. A step-wise (forward) regression procedure is used. This procedure allows a single regressor-variable to enter the equation at each step on the basis of its partial correlation with the dependent variable (when controlled for the effects of those that have already entered the equation) subject to minimum F-level to enter. Several of the variables that have finally entered the regression equation are found to be highly correlated. This condition, known as "multicollinearity", makes the results of statistical tests on individual coefficients unreliable. So, we need to correct for multicollinearity. To do so, a formal test, suggested by Farrar-Glauber [40], is first conducted to locate the harmful multicollinearity. This test involves determining by multiple regressions how much variation in each of the regressors is explained by the other regressors in the set. Harmful multicollinearity is said to exist among those regressor-variables that are significantly explained by the other regressor-variables. Other formal tests include: Klien's test criterion and Farrar-Glauber's simple criterion based on multiple correlations. Since the multiple correlation of $V_{DC}$ with all the regressors is very high ($R_y = 0.8988$), these formal tests did not suggest the presence of harmful multicollinearity. But, in view of the high simple correlation coefficients, we would not otherwise have been comfortable with the results of the
statistical tests. The test we have applied indicated the presence of harmful multicollinearity, however. Having located it, we have then applied an elimination of variables procedure to correct for multicollinearity. Between a pair of variables that are highly correlated and that are members of the set of variables where harmful multicollinearity exists, the one that is less partially correlated with the dependent variable \( V_{DC} \) when controlled for the effects of some of the other variables is eliminated. Though this procedure retains apparent interpretability in terms of included variables, it surrenders interpretability in terms of the original structure of the regression model.

The regression model corrected for multicollinearity is then put to two more tests: a test for heteroscedasticity and a test of simultaneous-equations bias. Goldfeld-Quandt's F-statistic is used to test for heteroscedasticity. And some regressions are separately carried out with regressors that are believed to be by definition dependent in one direction. The tests indicated the presence of both problems. Especially, the simultaneous-equations bias is found to exist due to COVAR being significant in explaining the variation in conversion values. So, the simultaneous-equations bias was first corrected by fitting a bivariate regression of CV against COVAR and introducing in place of CV and COVAR, the estimated conversion value \( ECV = a + b \cdot COVAR \) and the residuals of the fitted bivariate regression (UCV) into the model.

Heteroscedasticity was then fixed up by a procedure suggested by Taylor [90]. This procedure involves fitting separate regressions for each subgroup of sample to obtain the mean square error estimates and
then deflating dependent as well as regressor-variables by their respective mean square error estimates. These corrections turned the originally proposed model into a deflated model with some changes in the set of regressor-variables. The modified model satisfies the test of heteroscedasticity and therefore, the results are considered reliable and efficient.

The modified regression model is further tested for stability over time and across firm size by calculating Chow's F-statistic. Furthermore, significant shifts in the regression coefficients are identified by fitting differential regression equation with dummy variables. Analysis of all our empirical results are presented in the next chapter.
Footnotes to Chapter V

1. Winkler et al. [1972] have derived the following expression for the partial moment of $\tilde{X}$ which is assumed to be normally distributed:

$$E^Z(X^n) = -\sigma^2 Z^{n-1} \int_{-\infty}^{\infty} f_n(z) + (n-1)\mu \sigma^2 Z^{n-2} + \mu Z^{n-1}$$

where $Z$ is the point of truncation, $\mu$ and $\sigma^2$ respectively are the expected value and variance of $X$. This is Eq. (3.4) in their paper.

2. In Appendix B to his paper, Kim [1978] has derived the partial covariance of $\tilde{X}$ that is assumed to be jointly normally distributed with $\tilde{R}_m$ to show that:

$$\text{COV}(\tilde{b}X, \tilde{R}_m) = \text{COV}(\tilde{X}, \tilde{R}_m) [F(F) - \bar{F} f(F)] - (B-4)$$

where $b$ is the bankruptcy operator defined as:

$$\tilde{b} = \begin{cases} 0 & \text{if } X > \bar{F} \\ 1 & \text{if } X < \bar{F} \end{cases}$$

and $\bar{F}$ represents the total promised debt payment - the point of truncation for the firm to be declared as bankrupt.

3. The recognition of truncated distribution of cash flows accruing to the convertible-bondholders is consistent with the thesis of Poensgen [1965], BMQ [1966], Jennings [1973] and others that the distribution of convertible-bond returns is truncated.

4. This was one of the explanatory variables considered by Walter-Que [1973].

5. Walter-Que [1973] found the term-to-maturity of a convertible bond to be a significant variable in explaining the variation in convertible-bond premiums. Both Poensgen [1966] and Felheim [1974] did not find it to be significant, however. One of the reasons advanced, especially by Felheim, for this discrepancy is that the conversion expiry date for many of the issues included in the Walter-Que sample was close at hand.

6. This procedure does not involve any trend adjustments in view of the way people form expectations. The economic literature is rich with mechanisms for the formation of expectations by economic units. Generally, some form of extrapolative expectation mechanism...
is suggested in which higher weight is given to the recent past values of the variable than that for more distant ones whenever past values are used in forming expectations. More generally the "rational expectations" hypothesis is assumed to hold. This hypothesis states that forecasts use information optimally. Nonetheless, the procedure we have used is not uncommon in the finance literature.

7. The attempt at predicting possible bankruptcy, more so the default risk, goes back to 1904. Norton suggested in "The Theory of Loan Credit in Relation to Corporation Economics" publications of the American Economic Association, 3rd ver., vol. (1904), that the probability that a firm will fail to pay interest on its bonds in a particular year could be found by computing the coefficient of variation of the firm's income in past years, over and above the amount required for fixed charges and by looking up the probability in a normal distribution table. Irving Fisher suggested in "The Nature of Capital and Income", New York, The MacMillan Company, 1906, p. 409, that the chance that earnings should fall below the amount required to pay interest on bonds, is the probability corresponding to that relative deviation obtained by dividing the difference between the mean expected earnings and the interest by two-thirds of the standard deviation. The reason why I. Fisher suggested to divide the difference by two-thirds of standard deviation is, however, not clear. Drawing from Norton, Lawrence Fisher [1959] used the coefficient of variation of earnings as a proxy for the variability of earnings (which implied the risk of default in his case). He found it significant in determining the risk premiums on corporate straight bonds. All these measures assume the normal distribution of firm's earnings. But to our knowledge, this does not seem to have been empirically evidenced yet.

8. Working capital to total assets and sales to total assets also appeared in the Altman discriminant function. But these two ratios did not deteriorate consistently over five years prior to actual bankruptcy as the others did.


10. See for details Atkinson [1967], for example.

11. Altman-Eisenbeis [1978] have noted that this criterion is one of several that have been proposed in the literature for assessing the importance of the variables.
Klein's criterion of harmful multicollinearity is:
harmful if \( r_{ij} > R_y \), i.e., the simple correlation coefficient
between two regressors is greater than the multiple correlation
of the fitted regression. A simple criterion suggested by
Farrar-Glauber [1967] is:
harmful if \( R_x > R_y \), i.e. if the multiple correlation of a
regressor-variable with other variables in the set is greater
than the dependent variable's multiple correlation with the
entire set of regressor-variables.
CHAPTER VI

ANALYSIS OF EMPIRICAL RESULTS

A general form of the function of convertible-bond prices is presented in the preceding chapter (Eq. No. 59, on p. 117). We have constructed a multiple regression model of this function and have empirically investigated it. So, what follows in this chapter is the analysis of the results of our empirical study. The empirical study is concerned with the determinants of callable convertible-bond prices and the stability of the proposed model over time and across firm size.

A. Determinants of Callable Convertible-Bond Prices

A linear model of the function of convertible-bond prices given in Eq. (59) can be written in terms of a regression equation as follows:

\[
V_{DC_i} = a + b_1 V_{DS_i} + b_2 CV_i + b_3 CP_i + b_4 ER_i + b_5 VAR_i + b_6 COVAR_i + b_7 RETAN_i + b_8 EBTAN_i + b_9 EVTDN_i + b_{10} PQR_i + e_i
\]  

(60)

where
\[ a = \text{constant term} \]
\[ i = i^{th} \text{ observation} \]
\[ b_1, b_2, \ldots, b_{10} = \text{regression coefficients of the respective variables} \]
\[ e = \text{error term.} \]

Other notations are the same as defined before.

As stated in the testing procedure, the multiple regression presented in equation (60) is first carried out using the SPSS step-wise regression program. Pooled data with 104 observations for each variable are used for fitting the regression. The results are summarized in Table 1. They indicate that this all-inclusive regression model explains more than three-fourths of variation in the callable convertible-bond prices \((R^2 = 0.7895)\). This is a significant explanation. In the step-wise regression process, EVTDN did not enter the equation because of insufficient F-value to enter.\(^1\) Also perhaps because RETAN with which EVTDN is highly correlated did already enter the equation. The fact that it is eliminated, however, could be interpreted to mean that on the whole, during the sample period, the market was not concerned about the threat of long-term insolvency of the firms included in our sample.

A problem, though, is that some of the included variables are highly correlated. See Table 2 for single correlation coefficient matrix. For example, the coefficient of correlation between VAR and COVAR is 0.8009. It is 0.6778 between ER and CV. The presence of such multicollinearity makes the results of statistical tests on individual coefficients unreliable.
Table 1

Summary of the results of pooled multiple regression explaining the variation in prices of callable convertible bonds.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Regression Coefficient</th>
<th>t-Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-912.5547</td>
<td>-0.8069</td>
</tr>
<tr>
<td>VDS</td>
<td>0.3568</td>
<td>2.7619*</td>
</tr>
<tr>
<td>CV</td>
<td>0.6114</td>
<td>10.4837*</td>
</tr>
<tr>
<td>CP</td>
<td>1.3839</td>
<td>1.2509</td>
</tr>
<tr>
<td>ER</td>
<td>-119.5679</td>
<td>-0.7035</td>
</tr>
<tr>
<td>VAR</td>
<td>-460.9735</td>
<td>-0.845</td>
</tr>
<tr>
<td>COVAR</td>
<td>226.6331</td>
<td>1.3568</td>
</tr>
<tr>
<td>RETAN</td>
<td>-14.957</td>
<td>-4.067*</td>
</tr>
<tr>
<td>EBTAN</td>
<td>-1.0337</td>
<td>-0.4016</td>
</tr>
<tr>
<td>FVTDN</td>
<td>Insufficient F-level to enter</td>
<td></td>
</tr>
<tr>
<td>PQR</td>
<td>-233.173</td>
<td>-1.8209</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.7895</td>
<td></td>
</tr>
<tr>
<td>SEE</td>
<td>113.209</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>43.9152*</td>
<td></td>
</tr>
</tbody>
</table>

*significant at $\alpha = 0.05$
Table 2

Simple Correlation Coefficient Matrix

<table>
<thead>
<tr>
<th></th>
<th>VDS</th>
<th>CV</th>
<th>CP</th>
<th>COVAR</th>
<th>ER</th>
<th>VAR</th>
<th>RETAN</th>
<th>EBTAN</th>
<th>EVTDN</th>
<th>PQR</th>
</tr>
</thead>
<tbody>
<tr>
<td>VDS</td>
<td>0.3649</td>
<td></td>
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<td></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>CV</td>
<td>0.8611</td>
<td>0.2835</td>
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<tr>
<td>CP</td>
<td>0.2747</td>
<td>0.0953</td>
<td>0.236</td>
<td></td>
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</tr>
<tr>
<td>COVAR</td>
<td>-0.1613</td>
<td>-0.2617</td>
<td>-0.2467</td>
<td>-0.0571</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>ER</td>
<td>0.5573</td>
<td>0.0908</td>
<td>0.6778</td>
<td>0.0754</td>
<td>-0.0161</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>VAR</td>
<td>-0.2422</td>
<td>-0.2851</td>
<td>-0.3261</td>
<td>0.0536</td>
<td>0.8009</td>
<td>0.0087</td>
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</tr>
<tr>
<td>RETAN</td>
<td>-0.297</td>
<td>0.0216</td>
<td>-0.1457</td>
<td>-0.1325</td>
<td>-0.0853</td>
<td>-0.2273</td>
<td>-0.1984</td>
<td></td>
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</tr>
<tr>
<td>EBTAN</td>
<td>-0.0401</td>
<td>0.1148</td>
<td>0.0314</td>
<td>-0.1046</td>
<td>-0.3774</td>
<td>-0.024</td>
<td>-0.2245</td>
<td>0.193</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EVTDN</td>
<td>-0.3847</td>
<td>0.051</td>
<td>-0.3546</td>
<td>-0.1835</td>
<td>-0.2449</td>
<td>-0.4518</td>
<td>-0.1566</td>
<td>0.4288</td>
<td>0.6461</td>
<td></td>
</tr>
<tr>
<td>PQR</td>
<td>-0.1941</td>
<td>-0.1827</td>
<td>-0.1579</td>
<td>0.0519</td>
<td>0.062</td>
<td>-0.0058</td>
<td>0.0845</td>
<td>-0.1862</td>
<td>-0.1239</td>
<td>-0.0151</td>
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<tr>
<td>VDC</td>
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<tr>
<td>VDS</td>
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<tr>
<td>COVAR</td>
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<td>RETAN</td>
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<td>EBTAN</td>
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<tr>
<td>EVTDN</td>
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</tbody>
</table>
To reduce multicollinearity, we first applied a formal test of the location of harmful multicollinearity suggested by Farrar-Glauber [40]. The test involves determining by multiple regressions how much variation in each of the regressors is explained by the other regressors in the set. Harmful multicollinearity is said to exist among those regressor-variables that are significantly explained by the other regressor-variables. So, separate multiple regressions were fitted for each regressor-variable and tested for significance. The relevant results are summarized in Table 3. CV, ER, VAR, and COVAR were found to be significantly explained by the other variables in the set at 0.05 level of significance. This makes correlations between CV and ER and between VAR and COVAR certifiable as harmful. In the next step, between a pair of these highly correlated variables, the one that is less partially correlated with the dependent variable ($V_{DC}$) is eliminated. Since we used the step-wise regression procedure in carrying out equation (60), we simply considered the order in which the variables entered the equation. The first variable to enter was CV and when controlled for it, the partial correlation between $V_{DC}$ and ER was -0.0705. In the subsequent steps, COVAR entered the equation before VAR. When controlled for the effects of variables entered previously, the partial correlation between $V_{DC}$ and VAR was -0.1218. So, both ER and VAR were eliminated from the regression to reduce multicollinearity.

In contrast to using artificial orthogonalization to reduce multicollinearity, this procedure retains at least as a working hypothesis the direct interpretability of the coefficients of the included economic variables in our regression equation. It should, however, be
### Table 3

Summary results of the multiple regressions where each regressor is regressed against all remaining regressors in equation (1) of Table 1.

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Multiple R</th>
<th>R²</th>
<th>Adjusted R²</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. VDS</td>
<td>0.3877</td>
<td>0.1503</td>
<td>0.0884</td>
<td>7.123</td>
</tr>
<tr>
<td>2. CV</td>
<td>0.7892</td>
<td>0.6229</td>
<td>0.5954</td>
<td>19.82*</td>
</tr>
<tr>
<td>3. CP</td>
<td>0.4172</td>
<td>0.1741</td>
<td>0.1045</td>
<td>2.529</td>
</tr>
<tr>
<td>4. ER</td>
<td>0.7356</td>
<td>0.5411</td>
<td>0.5076</td>
<td>14.148*</td>
</tr>
<tr>
<td>5. VAR</td>
<td>0.8471</td>
<td>0.7176</td>
<td>0.6971</td>
<td>30.498*</td>
</tr>
<tr>
<td>6. COVAR</td>
<td>0.8329</td>
<td>0.6938</td>
<td>0.6715</td>
<td>27.188*</td>
</tr>
<tr>
<td>7. RETAN</td>
<td>0.4218</td>
<td>0.1779</td>
<td>0.1179</td>
<td>2.597</td>
</tr>
<tr>
<td>8. EBTAN</td>
<td>0.4395</td>
<td>0.1932</td>
<td>0.1344</td>
<td>2.874</td>
</tr>
<tr>
<td>9. PQR</td>
<td>0.3364</td>
<td>0.1131</td>
<td>0.0485</td>
<td>1.531</td>
</tr>
</tbody>
</table>

* significant at $\alpha = 0.05$
noted that the fact of multicollinearity reduces the interpretability of the original structure of our regression model. Since ER is found to be highly correlated with both \( V_{DC} \) and CV, when CV is significant in a regression model without ER, we cannot rule out the possibility that not only is the immediate exercise of conversion option valuable but also the expected return on underlying common stock. So, in a model without ER, CV should be regarded as representing both factors.

Eliminating ER and VAR from the set of regressors, a multiple regression was carried out using the pooled data. The results are summarized in Table 4. We observe some improvement in the goodness-of-fit statistics: \( R^2 \), standard error of estimate (SEE) and F. Eliminating these variables obviously did not cause any decline in the explanatory power of the model. Another notable feature is that unlike Duvel [26], we do not find any systematic anomalies in the estimated convertible-bond prices relative to the stock-conversion values. As shown in Table 5, except in a few cases, the estimated convertible-bond price increases when the stock-conversion value increases and it decreases when the stock-conversion value decreases.

Since we wish to have the efficient model in terms of the major assumptions underlying the linear regression model, the proposed model was further put to the following tests:

a) a test of heteroscedasticity to examine whether the variance of error terms is constant throughout the sample;

b) a test of simultaneous-equations bias due to the possible existence of additional relationships between some of the regressor variables.
Summary results of pooled multiple regression (after correcting for multicollinearity) explaining the variation in callable convertible-bond prices.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Regression Coefficient</th>
<th>t-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-785.0157</td>
<td>-0.7236</td>
</tr>
<tr>
<td>VDs</td>
<td>0.3757</td>
<td>2.9318*</td>
</tr>
<tr>
<td>CV</td>
<td>0.5944</td>
<td>15.4784*</td>
</tr>
<tr>
<td>CUVAR</td>
<td>97.4652</td>
<td>0.9588</td>
</tr>
<tr>
<td>RETAN</td>
<td>-13.6542</td>
<td>-3.8879*</td>
</tr>
<tr>
<td>EBTAN</td>
<td>-1.5946</td>
<td>-0.635</td>
</tr>
<tr>
<td>PQR</td>
<td>-238.4201</td>
<td>-1.8828</td>
</tr>
</tbody>
</table>

Adjusted $R^2$: 0.7902

*significant at $\alpha = 0.05$
Table 5
Movement of the estimated convertible-bond prices over time relative to their respective underlying stock-conversion values.

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Air Reduction</td>
<td>649.6</td>
<td>428.8</td>
<td>575.4</td>
<td>329.6</td>
<td>752.7</td>
<td>560.0</td>
<td>1028.9</td>
<td>982.4</td>
</tr>
<tr>
<td>Alcoa</td>
<td>989.2</td>
<td>853.8</td>
<td>745.6</td>
<td>524.2</td>
<td>828.1</td>
<td>679.5</td>
<td>1079.9</td>
<td>1021.6</td>
</tr>
<tr>
<td>Burlington North</td>
<td>997.1</td>
<td>894.5</td>
<td>875.4</td>
<td>727.2</td>
<td>792.2</td>
<td>581.7</td>
<td>952.1</td>
<td>799.9</td>
</tr>
<tr>
<td>Dart Inds.</td>
<td>550.2</td>
<td>188.1</td>
<td>514.9</td>
<td>134.4</td>
<td>630.6</td>
<td>301.0</td>
<td>701.2</td>
<td>369.8</td>
</tr>
<tr>
<td>FMC</td>
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<td>568.4</td>
<td>252.9</td>
<td>723.5</td>
<td>484.4</td>
<td>811.9</td>
<td>588.0</td>
</tr>
<tr>
<td>Georgia Pacific</td>
<td>633.8</td>
<td>447.6</td>
<td>661.3</td>
<td>489.1</td>
<td>726.2</td>
<td>563.2</td>
<td>821.7</td>
<td>668.4</td>
</tr>
<tr>
<td>Gran C. Steel</td>
<td>926.5</td>
<td>875.2</td>
<td>746.5</td>
<td>604.1</td>
<td>1028.0</td>
<td>1041.9</td>
<td>1169.2</td>
<td>1192.3</td>
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<td>Greyhound</td>
<td>885.8</td>
<td>761.9</td>
<td>789.5</td>
<td>555.1</td>
<td>906.7</td>
<td>712.9</td>
<td>996.7</td>
<td>816.3</td>
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<tr>
<td>Grt. N. Nekoosa</td>
<td>888.2</td>
<td>764.6</td>
<td>818.7</td>
<td>777.0</td>
<td>899.2</td>
<td>936.8</td>
<td>817.5</td>
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<tr>
<td>North Ind. P.S.</td>
<td>767.8</td>
<td>496.8</td>
<td>581.1</td>
<td>230.7</td>
<td>687.3</td>
<td>362.9</td>
<td>851.1</td>
<td>575.2</td>
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<tr>
<td>North Amer. Philips</td>
<td>742.5</td>
<td>724.0</td>
<td>561.5</td>
<td>564.0</td>
<td>672.3</td>
<td>708.0</td>
<td>771.2</td>
<td>788.0</td>
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<tr>
<td>Owens-Illinois</td>
<td>563.8</td>
<td>327.6</td>
<td>491.2</td>
<td>217.2</td>
<td>618.9</td>
<td>403.3</td>
<td>875.6</td>
<td>748.4</td>
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<td>Pepsi Co.</td>
<td>757.9</td>
<td>518.8</td>
<td>760.8</td>
<td>567.8</td>
<td>946.7</td>
<td>873.7</td>
<td>1022.5</td>
<td>952.6</td>
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<td>Pfizer</td>
<td>1104.0</td>
<td>1091.5</td>
<td>826.2</td>
<td>637.9</td>
<td>1121.4</td>
<td>1102.5</td>
<td>1240.9</td>
<td>1252.1</td>
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<td>Pittston</td>
<td>1013.9</td>
<td>905.2</td>
<td>862.0</td>
<td>688.3</td>
<td>802.8</td>
<td>578.9</td>
<td>846.9</td>
<td>616.8</td>
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<tr>
<td>RCA</td>
<td>605.6</td>
<td>311.9</td>
<td>482.0</td>
<td>179.1</td>
<td>583.8</td>
<td>323.7</td>
<td>708.1</td>
<td>452.6</td>
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<td>Rochester Tel.</td>
<td>938.4</td>
<td>795.6</td>
<td>675.5</td>
<td>456.5</td>
<td>741.9</td>
<td>521.7</td>
<td>873.6</td>
<td>669.6</td>
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<tr>
<td>Santa Fe Inds.</td>
<td>1134.2</td>
<td>1062.5</td>
<td>973.6</td>
<td>856.3</td>
<td>1016.2</td>
<td>946.9</td>
<td>1206.9</td>
<td>1231.3</td>
</tr>
<tr>
<td>So. Cal. Edison</td>
<td>691.5</td>
<td>423.0</td>
<td>672.1</td>
<td>400.0</td>
<td>714.8</td>
<td>448.3</td>
<td>818.6</td>
<td>553.6</td>
</tr>
<tr>
<td>St. Regis Paper</td>
<td>1047.5</td>
<td>1133.3</td>
<td>737.8</td>
<td>634.4</td>
<td>1117.8</td>
<td>1126.7</td>
<td>1232.4</td>
<td>1272.0</td>
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<tr>
<td>Trane</td>
<td>673.8</td>
<td>405.3</td>
<td>506.3</td>
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<td>596.9</td>
<td>267.9</td>
<td>788.9</td>
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<td>Union Pacific</td>
<td>1500.9</td>
<td>1615.3</td>
<td>1219.2</td>
<td>1206.5</td>
<td>1268.6</td>
<td>1356.3</td>
<td>1564.7</td>
<td>1792.0</td>
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<tr>
<td>Va. Elect. &amp; Pr.</td>
<td>558.7</td>
<td>436.3</td>
<td>375.1</td>
<td>280.4</td>
<td>582.8</td>
<td>483.6</td>
<td>694.1</td>
<td>591.4</td>
</tr>
<tr>
<td>Will Ross</td>
<td>670.3</td>
<td>474.5</td>
<td>655.1</td>
<td>296.7</td>
<td>698.5</td>
<td>329.5</td>
<td>679.7</td>
<td>285.8</td>
</tr>
<tr>
<td>Xerox</td>
<td>1361.8</td>
<td>1332.6</td>
<td>861.2</td>
<td>558.7</td>
<td>880.8</td>
<td>551.1</td>
<td>961.9</td>
<td>634.8</td>
</tr>
</tbody>
</table>
Goldfeld-Quandt's F-statistic was used to test for heteroscedasticity. To do this, the data were rearranged in the descending order of the magnitude of CV. By eliminating the middle 20 observations, separate regressions (with the same variables as in the equation of Table 4) were fitted to the first 42 observations with high conversion values and to the last 42 observations with low conversion values. Goldfeld-Quandt's F-statistic is the ratio of the error sum of squares of the former regression to that of the latter. The test results are given in Appendix C. We found F to be significant at $\alpha = 0.05$ signaling that the variance of error terms was not constant throughout the sample. In turn, this implied that our estimates of individual coefficients were not efficient.

Simultaneous-equations bias was suspected because, as shown by our theory, the equivalent straight-debt value should be a function of, among other things, the probability of bankruptcy. Similarly, conversion value is nothing but the stock-equivalent value of convertible bond, hence is a function of, among other things, the covariance (systematic-risk measure) and the variables representing probability of bankruptcy. Hence, we would expect additional relationships to exist between some regressor variables: $V_{DS}$ with RETAN, EBTAN, and PQR and CV with COVAR, RETAN, EBTAN and PQR. The presence of these additional relationships would cause simultaneous-equations bias in the estimates of regression coefficients given in Table 4. To test the extent of this bias, $V_{DS}$ was regressed against RFTAN, FBTAN and PQR and so was CV against COVAR, RETAN, EBTAN and PQR. The results show that the
former regression is not significant at $\alpha = 0.05$. Among the regressors in the latter equation, the coefficient of COVAR is significant signalling the presence of simultaneous-equations bias in the estimates of coefficients given in Table 4 due to significant relationship between CV and COVAR.

We were required therefore, to correct for simultaneous-equations bias as well as for heteroscedasticity to obtain efficient estimates of regression coefficients. First, simultaneous-equations bias was corrected for by carrying out a bivariate regression of CV against COVAR and introducing thus estimated conversion values (ECV) and the residuals of that regression (UCV) in place of CV and COVAR in the previous model.

Having done so, we proceeded to correct for heteroscedasticity. To do this, we used Taylor's [90] procedure of correction for heteroscedasticity. This procedure involves determining mean square error estimates by multiple regressions fitted to sub-groups of sample observations that are supposed to have different variances of errors. The observations of both dependent and regressor-variables in each sub-group are then deflated by their respective mean square error estimates.

Because the sample consists of the issues of different-size firms, we hypothesized that the variance of errors of regression fitted to large size firms' data differed from that of the regression fitted to small-to-medium size firms' data. To classify the observations into two sub-groups based on firm size, we used Quandt's [77] test for finding the maximum likelihood estimate of the cut-off point. This procedure
involved in our case rearranging the data according to firm size measured by total assets. It further involves dividing the data arbitrarily into left hand group and right hand group, estimating separate regression lines for the two groups, moving then the point of division between the two groups by one unit to the left and one unit to the right, calculating for each of these new divisions, separate regression lines for left hand group and the right hand group, moving the dividing point again and proceeding in analogous fashion, evaluating at each iteration the maximum likelihood estimate function:

\[ L(t) = -T \log(H) - t \log(\hat{\sigma}_1) - (T-t) \log(\hat{\sigma}_2) - T/2 \]

where \( T \) denotes total number of observations, \( t \), the point of division, and \( \hat{\sigma}_1 \) and \( \hat{\sigma}_2 \) respectively are standard error estimates of regressions fitted to \( t \) and \( T-t \) observations and finally to locate the cut-off point, \( t \), that maximizes \( L(t) \). We repeated this procedure separately for each year of the sample period as well as for pooled data. The results indicated that the most satisfactory cut-off point would be $1.2 billion of total assets in the 1973 figures.

With this cut-off point, observations were classified into two sub-groups and the multiple regression with \( V_{DS} \), ECV, UCV, CP, RETAN, EBTAN, and PQR as regressor-variables was separately fitted and mean square error estimates were obtained. The observations on the dependent variable and the above regressor-variables were then deflated by their respective mean square error estimates and the following modified regression model was fitted to the pooled data:
\[ V_{DCNi} = a + b_1 V_{DSNi} + b_2 ECV_{Ni} + b_3 UCV_{Ni} + b_4 CP_{Ni} \]
\[ + b_5 NRETA_{i} + b_6 NEBTA_{i} + b_7 PQRN_{i} + e_i \]  
(61)

where

\[ a = \text{mean square error estimate} \]
\[ b_1, b_2, \ldots, b_7 = \text{regression coefficients of the respective variables} \]
\[ V_{DC}, V_{DS}, ECV, UCV, CP, NRETA, NEBTA \text{ and PQRN are the deflated } V_{DC}, V_{DS}, ECV, UCV, CP, \text{ RETA, EBTAN and PQR respectively.} \]
\[ i = i^{th} \text{ observation} \]
\[ e = \text{error term.} \]

The results of (61) are summarized in Table 6 under Equation (1). In each column, the first row gives the regression coefficient of the respective variable and the second row gives its t-statistics. This model passed Goldfeld-Quandt's F-test for heteroscedasticity. See Appendix D for test results. Hence, the coefficient estimates given in Equation (1) of Table 6 are considered efficient. The results indicate that the modified model is a good fit as \( R^2 = 0.9847 \) and \( F = 950.6082 \). Excepting constant term and NEBTA, all are significant at \( \alpha = 0.05 \).

This suggests that equivalent straight-debt value, conversion value, call price, firm's retained earnings relative to total assets and bond quality rating all prove to be significant variables determining callable convertible-bond prices during 1973-1976. Since ECV is by definition equal to \( a + b \text{COVAR} \) (i.e., the systematic part of conversion value explained by COVAR), the covariance of underlying common stock is also a significant determining factor. As a result, our null hypotheses 1 and 2 are certainly rejected.
Table 6
Summary of the results of different regressions using modified model.

<table>
<thead>
<tr>
<th>Equation No.</th>
<th>1/MSR</th>
<th>V_{D0}</th>
<th>ZCVN</th>
<th>UCVN</th>
<th>CPM</th>
<th>NKETA</th>
<th>NETA</th>
<th>PCRN</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (Pooled)</td>
<td>0.0002</td>
<td>0.4707</td>
<td>0.3312</td>
<td>0.6607</td>
<td>0.8424</td>
<td>-13.7615</td>
<td>-1.0224</td>
<td>-417.8951</td>
</tr>
<tr>
<td></td>
<td>(0.078)</td>
<td>(6.037)*</td>
<td>(3.293)*</td>
<td>(27.522)*</td>
<td>(7.36)*</td>
<td>(-8.032)*</td>
<td>(-0.789)</td>
<td>(-5.397)*</td>
</tr>
<tr>
<td>2. (Cross-sectional 1973)</td>
<td>0.0012</td>
<td>0.5377</td>
<td>0.1461</td>
<td>0.5777</td>
<td>0.9999</td>
<td>-18.134</td>
<td>-1.8104</td>
<td>-443.5076</td>
</tr>
<tr>
<td></td>
<td>(0.223)</td>
<td>(2.425)*</td>
<td>(0.409)</td>
<td>(8.908)*</td>
<td>(3.382)*</td>
<td>(-3.559)*</td>
<td>(-0.672)</td>
<td>(-2.715)*</td>
</tr>
<tr>
<td>3. (Cross-sectional 1974)</td>
<td>-0.0014</td>
<td>0.4965</td>
<td>0.2263</td>
<td>0.4043</td>
<td>1.0266</td>
<td>-11.3425</td>
<td>0.7667</td>
<td>-569.3949</td>
</tr>
<tr>
<td></td>
<td>(-0.494)</td>
<td>(3.364)*</td>
<td>(1.429)</td>
<td>(10.59)*</td>
<td>(6.853)*</td>
<td>(-4.555)*</td>
<td>(0.402)</td>
<td>(-5.763)*</td>
</tr>
<tr>
<td>4. (Cross-sectional 1975)</td>
<td>0.0007</td>
<td>0.7134</td>
<td>-0.053</td>
<td>0.6213</td>
<td>0.9604</td>
<td>-13.2438</td>
<td>3.1113</td>
<td>-505.0911</td>
</tr>
<tr>
<td></td>
<td>(0.178)</td>
<td>(4.237)*</td>
<td>(-0.262)</td>
<td>(15.036)*</td>
<td>(4.261)*</td>
<td>(-4.91)*</td>
<td>(1.325)</td>
<td>(-3.197)*</td>
</tr>
<tr>
<td>5. (Cross-sectional 1976)</td>
<td>0.0016</td>
<td>0.6882</td>
<td>0.2106</td>
<td>0.7089</td>
<td>0.9774</td>
<td>-17.4423</td>
<td>-0.5155</td>
<td>-630.299</td>
</tr>
<tr>
<td></td>
<td>(0.231)</td>
<td>(4.016)*</td>
<td>(0.936)</td>
<td>(18.338)*</td>
<td>(3.621)*</td>
<td>(-5.172)*</td>
<td>(-0.131)</td>
<td>(-3.403)*</td>
</tr>
</tbody>
</table>
### Table 6 (continued)

<table>
<thead>
<tr>
<th>Adjusted $R^2$</th>
<th>SEE</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9847</td>
<td>0.0129</td>
<td>950.6082*</td>
</tr>
<tr>
<td>0.9842</td>
<td>0.0143</td>
<td>223.3541*</td>
</tr>
<tr>
<td>0.9927</td>
<td>0.0074</td>
<td>489.8892*</td>
</tr>
<tr>
<td>0.9884</td>
<td>0.0105</td>
<td>306.3767*</td>
</tr>
<tr>
<td>0.9891</td>
<td>0.0123</td>
<td>325.5489*</td>
</tr>
</tbody>
</table>

*Significant at 0.05 level of significance

MSE = mean square error estimate
B. Stability of the proposed (modified) model over time

Equation (1) of Table 6 was tested for its stability over time by employing dummy variables. 1973 was considered as the base year and dummy variables for 1974, 1975 and 1976 were represented by assigning subscripts 1, 2 and 3 respectively for each of the variable names. For example, $V_{DS}N_1$ is a dummy variable whose coefficient represents the change in importance of deflated straight-debt value of convertible bonds in 1974 from its importance in 1973. $V_{DS}N_2$'s coefficient represents the change in $V_{DS}N_1$'s importance in 1975 as opposed to its importance in 1973 and so on. But the changes in the intervening terms of 1973 to 1974, 1975 and 1976 were represented respectively by $D_1$, $D_2$ and $D_3$. A multiple regression with dummy variables is called a differential regression.

Initially, analysis of covariance was used to test the significance of shifts in the intercept term when adjusted for the regressors (which means holding slope coefficients constant over time) and to test the significance of interactions representing the changes in slope coefficients. The results are summarized in Table 7. As indicated in the table, the sum of squares (SS due to intercept dummies when adjusted for regressors is not significant but the SS due to interactions is significant at $\alpha = 0.05$. These results are indicative of non-stability of the model due to significant shifts in the slope coefficients over time. Nonetheless, we then applied Chow's F-test which is considered an efficient test of stability of a regression model. Separate
Table 7


<table>
<thead>
<tr>
<th>Sources of variation</th>
<th>sum of squares</th>
<th>d.f.</th>
<th>F.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. SS due to dummies, regressors with the interactions included, i.e. saturated model $(SS_y[R^2_{Z,X,ZX}]$)</td>
<td>1.11555</td>
<td>32</td>
<td>263.59*</td>
</tr>
<tr>
<td>2. SS due to dummies and regressors with the interactions excluded, i.e. additive model $(SS_y[R^2_{Z,X}]$)</td>
<td>1.10973</td>
<td>11</td>
<td>762.81*</td>
</tr>
<tr>
<td>(a) SS due to dummies (Z) adjusted for regressors (X) $(SS_y[R^2_{Z,X}-R^2_X]$)</td>
<td>0.00078</td>
<td>3</td>
<td>1.966</td>
</tr>
<tr>
<td>(b) SS due to regressors (X) adjusted for dummies (Z) $(SS_y[R^2_{Z,X}-R^2_Z]$)</td>
<td>1.08464</td>
<td>8</td>
<td>1025.15*</td>
</tr>
<tr>
<td>3. SS due to interactions $(SS_y[R^2_{Z,X,ZX}-R^2_{Z,X}]$)</td>
<td>0.00582</td>
<td>21</td>
<td>2.096*</td>
</tr>
<tr>
<td>4. SS residuals $(SS_y[1-R^2_{Z,X,ZX}]$)</td>
<td>0.00939</td>
<td>71</td>
<td></td>
</tr>
</tbody>
</table>

*significant at $\alpha = 0.05$
regressions with the same variables as in Equation (1) of Table 6 were fitted to the cross-sectional data of each year, i.e., 1973 through 1976. The test was carried out using the error sums of squares of these cross-sectional regressions and that of the pooled regression (i.e., Equation (1) of Table 6). Appendix E contains the test result. Calculated Chow's F is significant at $\alpha = 0.05$ indicating that the model was not stable over time.

To identify the significant shifts in estimated coefficients, we refer to the results of a differential regression with dummies for both intercept and slope coefficients. These are given in Table 8. As indicated there, significant shifts occur only in the coefficients of the unsystematic portion of conversion value (UCV) in 1974 and 1976. This is possibly because of the market's perceptions of systematic-risk's (COVAR) importance being different in 1974 and 1976 as opposed to its importance in 1973 and 1975. These significant shifts should be added to the basic model to estimate convertible-bond prices in 1974 and 1976. In view of Chow's F-test and the results of differential regression, we cannot reject the null hypothesis 3 completely. Still the proposed (modified) model is fairly stable over time.

C. Stability of the proposed (modified) model across firm size

Another important dimension of market evaluation may be firm size. Does the market perceive that different variables influence the evaluation of financial instruments of different sizes of firms? This question is relevant in pricing callable convertible bonds because these instruments are issued by firms of all sizes. Implications of
Table 8
Summary results of the differential regression (modified) carried out to locate significant shifts in the regression coefficients: 1973 to 1974, 1975 and 1976.

<table>
<thead>
<tr>
<th>Variables</th>
<th>1973 base year</th>
<th>regression coefficients</th>
<th>t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.0012</td>
<td>0.28</td>
<td></td>
</tr>
<tr>
<td>VDSM</td>
<td>0.5377</td>
<td>3.041*</td>
<td></td>
</tr>
<tr>
<td>ECVM</td>
<td>0.1461</td>
<td>0.313</td>
<td></td>
</tr>
<tr>
<td>UCVM</td>
<td>0.5777</td>
<td>11.171*</td>
<td></td>
</tr>
<tr>
<td>CPF</td>
<td>0.9999</td>
<td>4.242*</td>
<td></td>
</tr>
<tr>
<td>NRTA</td>
<td>-18.134</td>
<td>-4.463*</td>
<td></td>
</tr>
<tr>
<td>NRTA</td>
<td>-1.8104</td>
<td>-0.843</td>
<td></td>
</tr>
<tr>
<td>PQRM</td>
<td>-443.5078</td>
<td>-3.405*</td>
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</table>

<table>
<thead>
<tr>
<th>Dummies for constant:</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
</tr>
<tr>
<td>D2</td>
</tr>
<tr>
<td>D3</td>
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</table>

<table>
<thead>
<tr>
<th>Dummies for VDSM:</th>
</tr>
</thead>
<tbody>
<tr>
<td>VDSM1</td>
</tr>
<tr>
<td>VDSM2</td>
</tr>
<tr>
<td>VDSM3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dummies for ECVM:</th>
</tr>
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<tbody>
<tr>
<td>ECVM1</td>
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<tr>
<td>ECVM2</td>
</tr>
<tr>
<td>ECVM3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dummies for CPF:</th>
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</thead>
<tbody>
<tr>
<td>CPF1</td>
</tr>
<tr>
<td>CPF2</td>
</tr>
<tr>
<td>CPF3</td>
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<table>
<thead>
<tr>
<th>Dummies for UCVM:</th>
</tr>
</thead>
<tbody>
<tr>
<td>UCVM1</td>
</tr>
<tr>
<td>UCVM2</td>
</tr>
<tr>
<td>UCVM3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dummies for NRTA:</th>
</tr>
</thead>
<tbody>
<tr>
<td>NRTA1</td>
</tr>
<tr>
<td>NRTA2</td>
</tr>
<tr>
<td>NRTA3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dummies for NBEQA:</th>
</tr>
</thead>
<tbody>
<tr>
<td>NBEQA1</td>
</tr>
<tr>
<td>NBEQA2</td>
</tr>
<tr>
<td>NBEQA3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dummies for PQRM:</th>
</tr>
</thead>
<tbody>
<tr>
<td>PQRM1</td>
</tr>
<tr>
<td>PQRM2</td>
</tr>
<tr>
<td>PQRM3</td>
</tr>
</tbody>
</table>

| Adjusted R²        | 0.9881             |                      |
| SEE                | 0.0114             |                      |
| F-statistic        | 275.8866*          |                      |

*Significant at 0.05 level of significance
size difference for callable convertible bond pricing are investigated by testing the stability of the proposed (modified) model across firm sizes.

In so doing, using our result of Quandt's test for the best likelihood estimate of cut-off point, firms with total assets of $1.2$ billion or more in 1973 were classified as large-size firms and others were classified as small-to-medium (S-M) size firms. We then followed the same procedure used in the previous section. The S-M firms' data are used as the base. Coefficients of dummy variables in the differential regressions are the changes in the values of coefficients of respective variables for large size firms from those for S-M firms. Interaction dummy variables are denoted by assigning a subscript 1 to the respective variable names. The intercept dummy is denoted by DUMMY. Analysis of covariance was first used to examine whether there were any significant shifts in intercept and/or slope coefficients across firm sizes. Table 9 contains the results of the analysis. As shown in the table, the sum of squares due to dummy variable for intercept (DUMMY) when adjusted for the regressors and the sum of squares due to interaction terms are both not significant at $\alpha = 0.05$, signalling that the model is stable across firm size. Nonetheless, Chow's F-statistic was calculated by fitting regression lines separately for the data of large and S-M firms. As shown in Appendix F, the calculated F-statistic of Chow is not significant at $\alpha = 0.05$, indicating that the proposed (modified) regression model is stable across firm size. As a result, we can reject null hypothesis 4.
Table 9
Analysis of Covariance: Large and S-M firms being the categorical variables called "factors".

<table>
<thead>
<tr>
<th>Sources of variation</th>
<th>sum of squares</th>
<th>d.f.</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. SS due to dummies, regressors with the interactions included, i.e. saturated model (SSy[R^2 Z,X,ZX])</td>
<td>1.10911</td>
<td>16</td>
<td>380.97*</td>
</tr>
<tr>
<td>2. SS due to dummies, and regressors with interactions excluded, i.e. additive model (SSy[R^2 Z,X])</td>
<td>1.10896</td>
<td>9</td>
<td>677.19*</td>
</tr>
<tr>
<td>(a) SS due to dummies (Z) adjusted for regressors (X) (SSy[R^2 Z,X-R^2 X])</td>
<td>0.00001</td>
<td>1</td>
<td>0.055</td>
</tr>
<tr>
<td>(b) SS due to regressors (X) adjusted for dummies (Z) (SSy[R^2 Z,X-R^2 Z])</td>
<td>0.24241</td>
<td>8</td>
<td>166.53*</td>
</tr>
<tr>
<td>3. SS due to interactions (SSy[R^2 Z,X,ZX-R^2 Z,X])</td>
<td>0.00015</td>
<td>7</td>
<td>0.118</td>
</tr>
<tr>
<td>4. SS residuals (SSy[1-R^2 Z,X,ZX])</td>
<td>0.01583</td>
<td>87</td>
<td></td>
</tr>
</tbody>
</table>

*significant at α = 0.05
that the proposed (modified) model is not stable across firm size.

As an overall test of stability of the modified model, a
differential regression with intercept dummies for both size and year
effects, besides the regressors given in equation (61), was fitted to
the pooled data. Table 10 contains the results of this differential
regression. Since none of the coefficients of dummy variables is
significant at \( \alpha = 0.05 \), it may be concluded that overall the modified
model is stable.

D. A Summary of findings

This study presents some interesting findings and implications
for further research. It shows that a multiple regression model with
\( V_{DS}, CV, CP, COVAR, RETAN, EBTAN, \) and PQR adequately explains variation
in the prices of callable convertible-bonds. However, estimated co­
efficients are not efficient because of a significant relationship
between \( CV \) and \( COVAR \) that causes simultaneous-equation bias and also
because of a heteroscedasticity problem that seems to exist when the
coefficients are estimated by using the time-series data across firms.
If the conversion value estimates of bivariate regression of \( CV \) against
COVAR and the residuals of that regression are used in place of \( CV \) and
COVAR, and if heteroscedasticity is corrected for by the procedure
used in this study, then the multiple regression thus modified pro­
vides efficient estimates of coefficients. The modified model is also
shown to be a better fit as indicated by the goodness-of-fit statistics.
Furthermore, the stability tests suggest that it is stable across firm
size and fairly stable over time. An overall test of stability
Table 10

Summary results of the differential regression with dummy variables for size and year effects introduced simultaneously.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Regression coefficient</th>
<th>t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.0115</td>
<td>0.553</td>
</tr>
<tr>
<td>D1</td>
<td>-0.0026</td>
<td>-0.662</td>
</tr>
<tr>
<td>D2</td>
<td>-0.0074</td>
<td>-1.951</td>
</tr>
<tr>
<td>D3</td>
<td>-0.007</td>
<td>-1.71</td>
</tr>
<tr>
<td>Dummy</td>
<td>0.0476</td>
<td>0.344</td>
</tr>
<tr>
<td>VDSN</td>
<td>0.5085</td>
<td>5.646*</td>
</tr>
<tr>
<td>ECVN</td>
<td>0.2483</td>
<td>2.297*</td>
</tr>
<tr>
<td>UCVN</td>
<td>0.6071</td>
<td>25.554*</td>
</tr>
<tr>
<td>CPN</td>
<td>0.6588</td>
<td>1.064</td>
</tr>
<tr>
<td>NRETA</td>
<td>-13.7848</td>
<td>-7.94*</td>
</tr>
<tr>
<td>NEBTA</td>
<td>-0.4375</td>
<td>-0.328</td>
</tr>
<tr>
<td>PQRN</td>
<td>-423.3973</td>
<td>-5.434*</td>
</tr>
</tbody>
</table>

Adjusted $R^2 = 0.9849$
See = 0.0128
$F = 610.8976*$

*Significant at $\alpha = 0.05$
conducted by simultaneously introducing dummies for size and year effects in the regression equation is also satisfied.

The study has shown that on the whole, investors value significantly conversion option, equivalent straight-debt value representing the protection against downside risk, call price, retained earnings to total assets and bond quality rating representing firm's financial distress and the systematic risk of underlying common stocks (COVAR) as well. Results of year-wise analysis given in equations (2) through (5) of Table 6, suggest that during the sample period, market gave significant consideration to straight-debt worth, call price and unsystematic portion of conversion value of a convertible bond. Firm's retained earnings relative to total assets and possible default of meeting debt obligations were also considered important in determining callable convertible bond prices. Since some of the variables representing the probability of bankruptcy prove significant, the study provides some justification for the role of a bankruptcy factor (which is not generally distinguished from financial distress in theoretical discussions) in the evaluation of the firm's convertible bond issues. With respect to call price per se, findings of the study emphasize the need for its explicit consideration in evaluating callable convertible bonds.

E. Limitations of the Study

We recognize below some of the limitations of our empirical study.

A main limitation is that, although it covers a fairly wide range of industries, the sample consists of only 26 callable convertible
issues. This is a small number on which to base generalizations. We did not deliberately restrict the sample size. Our sampling was all-inclusive, with these 26 proving the only issues that met a priori criteria for inclusion.

The sample consists predominantly of issues of large-size firms whose common stocks can be categorized as moderate or aggressive. It includes 9 issues of S-M firms, with the smallest firm's asset-size being $271.11 million and 17 of the large firms. This predominance of large firms and high-beta stocks affects our pooled-regression estimates. In spite of the test results suggesting the stability of the modified model across firm size, we cannot comfortably generalize those results because of the few issues of S-M firms being included in our sample. It would be useful to re-examine our model with a larger sample, including a larger number of S-M firms of still smaller asset size.

The period 1973-1976 was certainly not a normal period. It could be characterized as a cyclical turning point from economic recession to recovery. We cannot be sure that the same results would hold during other cyclical phases.

An element of bias may also have been introduced in the study by the manner in which we measure the bonds' equivalent straight-debt value. Market prices of matching straight-bonds definitely seem a better alternative to the estimates of Moody's Investment Service which we used in our study. But a perfect matching of bonds on the basis of the comprehensive set of criteria suggested by Kim-McConnell-Greenwood [56] proved impossible. Imperfect matching introduces
elements of measurement bias. Though the exact criteria applied by Moody's Investment Service to estimate \( V_{DS} \) is not known, it is more likely that their estimates of \( V_{DS} \) differ from the values of perfectly matched straight bonds.

In spite of these limitations, the study indicates that the proposed (modified) regression model has a high explanatory power and it provides efficient estimates of coefficients. At least for the firms included in our sample, the threat of actual bankruptcy proved of less concern than financial distress, based on the statistical tests of the significance of the proxies we used to represent the probability of bankruptcy. This provides some justification for considering a financial distress or a bankruptcy factor in evaluating financial instruments of on-going firms. Further, an explicit consideration of call price variable in estimating callable convertible bond prices is shown to be important as it is one of the significant variables determining the prices of such financial instruments.
Footnotes to Chapter VI

1. The minimum F-Value to enter a variable in the equation was 0.01.

2. The results of the regressions are summarized below:

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Regressor-variables</th>
<th>Adjusted $R^2$</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{DS}$</td>
<td>constant COVAR RETAN EBTAN PQR</td>
<td>0.0142</td>
<td>1.494</td>
</tr>
<tr>
<td></td>
<td>815.23 -0.849 1.886 -178.218</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(6.573)* (0.30) (0.987) (-1.762)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$CV$</td>
<td>-708.731 -18.551 -3.783 -641.998</td>
<td>0.0872</td>
<td>3.459*</td>
</tr>
<tr>
<td></td>
<td>1797.993</td>
<td>(4.186)* (-2.688)* (-1.974)(-0.555) (-1.91)</td>
<td></td>
</tr>
</tbody>
</table>

*significant at 0.05 level of significance
CHAPTER VII
SUMMARY AND CONCLUSION

A. A Summary of the Study

This study has evaluated convertible-debt-financing in both the single-period and the multi-period valuation frameworks. In the process, the existence of call option, corporate income-tax, costly bankruptcy and firm's capital structure plan upon conversion are all incorporated. Modigliani-Miller's proposition of irrelevance of financial structure, their tax model and Lewellen-Racette's convertible-debt-inferiority proposition have been re-examined in the context of our evaluation of convertible-debt-financing. Also, the study has empirically investigated the determinants of callable convertible-bond prices and tested the significance or otherwise of call price and the variables signalling possible bankruptcy per se.

The problem was approached by identifying cash flows accruing to security holders in different state-option combinations and evaluating thus identified cash flows by employing the cash flows version of the CAPM.

The theoretical analysis was carried out in two stages. In the first stage, the implications of different means of financing - retained earnings, straight-debt and convertible-debt - were analyzed utilizing
the cash flow version of the S-L-M's CAPM that incorporates corporate income-tax and costly bankruptcy. As in Kim [55] and Chen [20], it is shown that M-M's proposition of irrelevance of financial structure and their tax model do not hold in the presence of costly bankruptcy even for the convertible-debt-financing case. But, Lewellen-Racette's convertible-debt-inferiority proposition is shown to hold generally in a single-period setting, trade-offs between tax shelter and bankruptcy cost always favor straight-debt-financing if the interest rate were the same for both convertible bonds and straight bonds. However, the validity of their proposition is shown to be ambiguous under the circumstance where the interest rate on convertible bonds is lower than straight bonds but the contractual principal amount of debt is the same.

The single-period analysis could capture neither the value of call option - a common feature of convertible bonds listed on the NYSE - nor the firm's capital-structure plan upon conversion that has significant implications for market valuation of convertible bonds. So, the analysis was extended, in the second stage, to include the implications of these factors for valuation of convertible bonds. This required the analysis to be carried out in a multi-period setting. The ideal capital market conditions were again assumed and the analysis was carried out by employing the Chen's simplified version of multi-period CAPM. It is shown that if a firm follows a policy of either maintaining the same capital structure or maintaining the same amount of leverage upon conversion, then convertible-debt-financing is as good as straight-debt-financing. These presumed policies upon conversion are consistent with a firm's rational behavior of maximizing its shareholders' wealth. They
are also consistent with the prevailing practice of issuing convertible bonds as a delayed equity-financing. Note that the investors are assumed to have known firm's policy upon conversion before they start bidding for convertible bonds. So then, they bid for convertible bonds as a package of convertible bonds now and equity shares plus convertible bonds later upon conversion. Under either of the presumed policies, the firm continues to avail the same tax shelter even after conversion. In an ideal capital market, in the process of bid and offer, shareholders of the firm would gain in terms of premium that is equivalent to the value that they would relinquish upon conversion. So, the shareholders are at least as well-off under convertible-debt-financing as they otherwise would be under straight-debt-financing. Moreover, the firm's market value remains the same under convertible-debt-financing as it would be under its counterpart. Hence, Lewellen-Racette's convertible-debt-inferiority proposition is flatly rejected. In fact, if other features of convertible-debt, such as flexibility, subordination and less severe restrictions are recognized, then it may be considered as a better means of financing than straight-debt. This explains why the firms issue convertible bonds even though they can afford to issue straight bonds.

Besides the refutation of Lewellen-Racette's proposition, the study provides some interesting implications for some of the firm's decisions. The study indicates that a firm can determine its optimal capital structure in the case of convertible-debt simply by treating it as if it were a straight-debt and looking at the trade-offs between tax shelter and bankruptcy cost associated with its equivalent straight-debt.
This conclusion is, however, based on our assumption of firm issuing immediately the new debt equivalent to the converted debt.

We have defined the condition for exercising call option that is broader than the one proposed by Ingersoll [46] and Brennan-Schwartz [16]. It is shown that their call-strategy is a special case of the one proposed in this study. Furthermore, it is shown that the gains from exercising call option are exactly offset by reduction in premium precisely because bondholders expect a firm to call the bonds once the condition for exercising call option prevailed. So, there will not be any net gain from the exercise of call option if we visualize convertible-debt-financing in its entirety. This is consistent with the theses of Elton-Gruber [28] and Kraus [57] that the call option has a zero effect on the market value of firm. The study also provides a conversion-premium model that is more comprehensive than is provided hitherto.

Based on the theoretical convertible-debt valuation model derived in a multi-period setting, a multiple-regression model was constructed to estimate callable convertible-bond prices. A sample of 26 callable convertible-bonds was selected to test empirically the significance of the proposed regression model. The sample period runs from 1973 to 1976, with dates of evaluation on December 31st each year. The results show that a regression model with equivalent straight-debt value, conversion value, call price, covariance of returns on the underlying common stock with the returns on S&P Composite Index, the reciprocals of retained earnings to total assets, EBIT to total assets and of bond quality rating explains more than three-fourths of variation in the prices of callable convertible bonds. Because of the problems of
multicollinearity, heteroscedasticity and simultaneous-equations bias, the initially-proposed model, however, would not provide efficient estimates of coefficients. When these problems are corrected and the model modified, we obtain encouraging results. The goodness-of-fit statistics are better after modification than before. Modified regression model explains more than 90 per cent of variation in the prices of callable convertible bonds.

The results indicate that equivalent straight-debt value, call price, conversion value, systematic risk of underlying common stock, bond quality rating and the firm's retained earnings to total assets ratio are all significant variables determining callable convertible bond prices. The fact that some of the variables representing probability of bankruptcy prove significant provides some justification for the role of a bankruptcy factor (which is not generally distinguished from the firm's distress in the case of on-going firms) in evaluating financial instruments traded in the capital markets. Call price is found significant supporting our stand for its explicit consideration in evaluating callable convertible bonds.

The results further show that the proposed (modified) model is stable across firm size and is fairly stable over time. Our results are not, however, adequate to generalize for estimating the prices of small size firms' issues. This is so because our sample consists of only 9 issues of S-M firms and it does not contain any issue of a firm having total asset less than $270 million. In this context, our modified estimation equation may be considered suitable generally for estimating the prices of large firms' issues.
B. Further Research and Possible Extensions

Further research is needed in at least three respects. First, our sample consists of very few issues of small-to-medium size firms. Therefore, as noted above, the results of our stability are not adequate to generalize that the modified estimation equation is equally a good fit for the issues of small-size firms as it is so for large-size firms' issues. An investigation should be made into the goodness-of-fit of the suggested estimation equation by selecting a large sample of the issues of S-M firms with still smaller asset sizes. Second, we have assumed throughout our theoretical exposition that once the bankruptcy condition prevailed, the firm would go bankrupt and liquidate its business. But, in reality, a majority of the firms would attempt to reorganize their capital structure to stave off actual liquidation. So, the valuation procedure needs to be extended to include bankruptcy-reorganization case. Third, the sample period we were concerned with in this study – 1973 through 1976 – was certainly not a normal period. It could be characterized as a period involving cyclical turning-point from economic recession to recovery. So, a re-examination should be made in the context of other cyclical phases of the economy with respect to the explanatory power of the modified regression model that is proposed here.

Furthermore, the study has provided a theoretical conversion-premium model and a strategy for firm to call back its convertible bonds. An empirical investigation could be made into the determinants of conversion-premiums utilizing the suggested theoretical model. Also
interesting would be the examination of firms' call policies on convertible issues in the light of the proposed call strategy. Both are still unsettled issues.

Our valuation procedure is useful for examining implications of issuing various financial instruments carrying options. It can easily be extended to evaluating preferred stocks with conversion option and straight bonds with call option. The differences between convertible bonds, preferred stocks and straight bonds need no explanation. A study on the implications, for stockholders' wealth, of financing a business activity such as acquisitions by these alternative means of financing or a combination thereof would be an interesting contribution. Our procedure can also be applied to evaluating other kinds of bonds such as extendible bonds and retractable bonds. An extendible bond is a bond which contains a provision to the effect that the bondholder has an option to exchange the bond at its maturity date for another bond with a maturity date in the distant future. The new bond issued in the exchange generally carries a different rate of interest (usually a higher rate of interest). Such bonds have been popular with the Government of Canada for the last decade. Under the premise that they could be issued by industrial firms in any country, it would be interesting to examine the implications of issuing these bonds for firm's market value and for stockholders' wealth. Similarly, a retractable bond contains a provision to the effect that the bondholder has an option to ask for redemption or recall of the bond at a specified date before maturity. This is a straight bond with a call option provided to the bondholder instead of being enjoyed by the issuing-corporation.
These bonds are popular with Canadian companies. In both the case of extendible bonds and retractable bonds, however, an advance notice of intention of exercising the option shall have to be given. These extensions will, no doubt, provide valuable insights into the effects of different means of debt-financing.

C. Conclusion

Convertible debt is at least as an efficient means of financing as the straight debt provided the firm follows a capital-structure policy that ensures regaining the tax subsidy that would otherwise be lost upon conversion. If our empirical results are any indication, the dominant variables determining callable convertible-bond prices include equivalent straight-debt value, conversion value of convertible bond, call price, and firm's financial distress represented by retained earnings to total assets ratio and bond quality rating. The fact that some of these financial distress variables prove significant indicates the market's concern for the financial distress of on-going firms. However, this evidence needs to be re-examined in the context of a larger sample and selecting sample period different from the one we investigated. Similarly, further research on the lines outlined in this chapter would provide added insights into the pricing of complex financial instruments such as callable convertible bonds.
Appendix A

The Sample

<table>
<thead>
<tr>
<th>Particulars of the issue</th>
<th>Type of Industry</th>
<th>Asset-size of the firm (Total Assets in 1973) in million $</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air Reduction 3 7/8s, 8/15/87</td>
<td>Industrial inorganic chemicals</td>
<td>590.231</td>
</tr>
<tr>
<td>Alcoa 5 1/4s, 12/1/91</td>
<td>Prim. smelt-refin. Non for MTL</td>
<td>2806.54</td>
</tr>
<tr>
<td>Burlington North 5 1/4s, 1/5/92</td>
<td>Railroads</td>
<td>2990.05</td>
</tr>
<tr>
<td>Dart Inds.</td>
<td>Retail-Misc.</td>
<td>1035.34</td>
</tr>
<tr>
<td>FMC Corp. 4 1/4s, 7/15/92</td>
<td>Machinery-Const. &amp; Mat. Handling</td>
<td>1380.4</td>
</tr>
<tr>
<td>Georgia Pacific 5 1/4s, 4/1/96</td>
<td>Forest Products</td>
<td>2002.02</td>
</tr>
<tr>
<td>Gran C. Steel 4 5/8s, 12/1/94</td>
<td>Blast furnace-steel works</td>
<td>2024.38</td>
</tr>
<tr>
<td>Greyhound Corp. 6 1/2s, 1/15/90</td>
<td>Transportation and food-meat packing</td>
<td>1309.16</td>
</tr>
<tr>
<td>Great No. Nekoosa, 4 1/4s, 4/1/91</td>
<td>Paper</td>
<td>551.36</td>
</tr>
<tr>
<td>NCR 6s, 5/1/95</td>
<td>Office and Business Equipment</td>
<td>1833.92</td>
</tr>
<tr>
<td>North Ind. P.S. 4 1/4s, 2/8/92</td>
<td>Elect. Utilities</td>
<td>1348.34</td>
</tr>
<tr>
<td>North American Philips 4s, 6/1/92</td>
<td>Elect. &amp; Electr. Mach. Equip. &amp; Supplies</td>
<td>524.64</td>
</tr>
<tr>
<td>Owens-Illinois 4 1/2s, 11/1/92</td>
<td>Containers - metal and glass</td>
<td>1642.47</td>
</tr>
<tr>
<td>Pepsi Co. 4 3/4s, 8/1/96</td>
<td>Beverages - soft drinks</td>
<td>1207.28</td>
</tr>
<tr>
<td>Pfizer Inc. 4s, 2/15/97</td>
<td>Drugs - Ethical</td>
<td>1407.81</td>
</tr>
<tr>
<td>Pittston 4s, 7/1/97</td>
<td>Coal - Bituminous</td>
<td>495.99</td>
</tr>
<tr>
<td>RCA 4 1/2s, 8/1/92</td>
<td>Radio - TV manufacturing</td>
<td>3300.8</td>
</tr>
<tr>
<td>Rochester Tel. 4 3/4s, 3/1/94</td>
<td>Telephone</td>
<td>297.69</td>
</tr>
<tr>
<td>Santa Fe Inds. 6 1/4s, 8/1/98</td>
<td>Railroads</td>
<td>2483.25</td>
</tr>
<tr>
<td>So. Cal. Edison 3 1/8s, 8/15/80</td>
<td>Elect. Utilities</td>
<td>3989.35</td>
</tr>
<tr>
<td>St. Regis Paper 4 7/8s, 4/1/97</td>
<td>Paper</td>
<td>1172.7</td>
</tr>
<tr>
<td>Trane Co. 4s, 9/15/92</td>
<td>Machines - Service</td>
<td>271.11</td>
</tr>
<tr>
<td>Union Pacific 4 3/4s, 4/1/99</td>
<td>Railroads</td>
<td>2828.38</td>
</tr>
<tr>
<td>Va. Elect. &amp; Power 3 5/8s, 5/1/86</td>
<td>Elect. Utilities</td>
<td>2990.21</td>
</tr>
<tr>
<td>Will Ross 4 1/2s, 9/1/92</td>
<td>Drugs - Ethical</td>
<td>577.33</td>
</tr>
<tr>
<td>Xerox 6s, 11/1/95</td>
<td>Office and Business Equipment</td>
<td>3089.0</td>
</tr>
</tbody>
</table>
### Appendix B

**Beta Coefficients of the Underlying Common Stocks**

<table>
<thead>
<tr>
<th>Name of the firm</th>
<th>1973</th>
<th>1974</th>
<th>1975</th>
<th>1976</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Air Reduction</td>
<td>1.307</td>
<td>1.099</td>
<td>1.22</td>
<td>1.186</td>
</tr>
<tr>
<td>2. Alcoa</td>
<td>1.17</td>
<td>0.661</td>
<td>0.661</td>
<td>0.691</td>
</tr>
<tr>
<td>3. Burlington North</td>
<td>1.082</td>
<td>1.707</td>
<td>0.901</td>
<td>0.805</td>
</tr>
<tr>
<td>4. Dart Inds.</td>
<td>1.564</td>
<td>1.034</td>
<td>1.893</td>
<td>1.993</td>
</tr>
<tr>
<td>5. FMC</td>
<td>1.626</td>
<td>1.41</td>
<td>1.399</td>
<td>1.448</td>
</tr>
<tr>
<td>6. Gran C. Steel</td>
<td>1.175</td>
<td>0.846</td>
<td>0.609</td>
<td>0.59</td>
</tr>
<tr>
<td>7. Georgia Pacific</td>
<td>1.161</td>
<td>1.271</td>
<td>1.539</td>
<td>1.658</td>
</tr>
<tr>
<td>8. Greyhound Corp.</td>
<td>0.716</td>
<td>0.506</td>
<td>0.609</td>
<td>0.622</td>
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<tr>
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<td>1.195</td>
<td>0.793</td>
<td>0.584</td>
<td>0.668</td>
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<tr>
<td>10. NCR</td>
<td>1.255</td>
<td>1.088</td>
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<tr>
<td>11. North Ind. P.S.</td>
<td>0.713</td>
<td>0.832</td>
<td>0.868</td>
<td>0.958</td>
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<tr>
<td>14. Pepsi Co.</td>
<td>0.926</td>
<td>1.391</td>
<td>1.602</td>
<td>1.525</td>
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<tr>
<td>15. Pfizer</td>
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<td>1.138</td>
<td>1.076</td>
<td>1.066</td>
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<tr>
<td>16. Pittston</td>
<td>1.262</td>
<td>1.385</td>
<td>1.414</td>
<td>1.386</td>
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<tr>
<td>17. RCA</td>
<td>1.57</td>
<td>1.329</td>
<td>1.387</td>
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<td>18. Rochester Tel.</td>
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<td>0.581</td>
<td>0.666</td>
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<td>19. Santa Fe Inds.</td>
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<td>20. So. Cal. Edison</td>
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<tr>
<td>21. St. Regis Edison</td>
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<td>1.282</td>
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<tr>
<td>24. Va. Elect. &amp; Pr.</td>
<td>1.007</td>
<td>0.93</td>
<td>1.091</td>
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<td>25. Will Ross</td>
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<td>1.084</td>
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<td>1.182</td>
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<tr>
<td>26. Xerox</td>
<td>0.577</td>
<td>0.918</td>
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</table>
Appendix C

Test of heteroscedasticity in the original model

Test statistic: Goldfeld-Quandt's F.

\[ F = \frac{\text{SSE}_1}{\text{SSE}_2} \text{ with } \frac{(N-P-2K)}{2}, \frac{N-P-2K}{2} \text{ d.f.} \]

where

\( \text{SSE} = \text{error sum of squares} \)
\( N = \text{total no. of observations} \)
\( P = \text{no. of middle observations eliminated} \)
\( K = \text{No. of coefficients estimated} \)

Subscripts 1, and 2 respectively stand for the high value group and the low value group data rearranged according to the magnitude of the conversion value.

Calculated \( F = \frac{458,497.84}{206,807.72} \)

\[ = 2.217^* \text{ with 34,34 d.f.} \]

*significant at \( \alpha = 0.05 \)
Appendix D

Test of heteroscedasticity in the modified model

Test statistic: Goldfeld-Quandt's F.

\[
F = \frac{SSE_1}{SSE_2} \text{ with } \frac{N-P-2K}{2}, \frac{N-P-2K}{2} \text{ d.f. where}
\]

- SSE = error sum of squares
- N = total no. of observations
- P = no. of middle observations eliminated
- K = no. of coefficients estimated

Subscripts 1 and 2 respectively stand for the high UCVN value group and the low UCVN value group.

Calculated \( F = \frac{0.00344}{0.00258} \)

= 1.333 with 34, 34 d.f.

This is not significant at 0.05 level of significance.
Appendix E

Test of stability of the modified model over time.

Test statistic: Chow's F.

\[
F = \frac{[SSE_N - (SSE_1 + SSE_2 + SSE_3 + SSE_4)]/K}{(SSE_1 + SSE_2 + SSE_3 + SSE_4)/N-4K}
\]

where

- \( SSE \) = error sum of squares
- \( N \) = total no. of observations
- \( K \) = no. of coefficients estimated.

Subscripts \( N 1, 2, 3 \) and \( 4 \) respectively stand for the pooled, the cross-sectional 1973, 1974, 1975 and 1976 data.

Calculated \( F \) = \[
\frac{[0.016 - (0.00369 + 0.00099 + 0.00199 + 0.00272)]/8}{0.00939/72}
\]

= 6.335* with 8,72 d.f.

*significant at \( \alpha = 0.05 \)
Appendix F

Test of stability of the modified model across firm size.

Test statistic: Chow's F

\[
F = \frac{[\text{SSE}_N - (\text{SSE}_L + \text{SSE}_\text{SM})]/K}{(\text{SSE}_L + \text{SSE}_\text{SM}) / N - 2K}
\]

where

SSE = error sum of squares
N = total no. of observations
K = no. of coefficients estimated.

Subscripts N, L, SM, respectively, stand for the pooled, large and small-to-medium size firms' data.

Calculated F = \[\frac{0.016 - (0.00089 + 0.01494)}{0.01583/88}\]

= 0.1181 with 8.88 d.f.

This is not significant at \(\alpha = 0.05\)
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