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THERMAL PERFORMANCE OF PARAFFIN PHASE CHANGE MATERIALS DISPERSED IN A MORTAR FILLER MATRIX.

THE OHIO STATE UNIVERSITY, PH.D., 1978

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1978
THERMAL PERFORMANCE OF PARAFFIN PHASE CHANGE MATERIALS
DISPERSED IN A MORTAR FILLER MATRIX

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By
Richard Davis Godfrey, B.M.E., M.Sc., P.E.

* * * * *

The Ohio State University
1978

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To my daughters Laura and Beth and my wife Janet.
ACKNOWLEDGMENTS

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"Single Family Residence Solar Heating Study", Ohio State University Engineering Experiment Station Report EES 507X to the Ohio Development Commission, 1975 (Coauthor)

"Energy Usage Comparison of Identical, Non-inhabited, Residential Homes with Various Types of Insulation and a Program Simulation of the Required Heating and Cooling Loads", Ohio State University Engineering Experiment Station Report EES 480X to Dow Chemical USA, Midland, Michigan, 1976, (Coauthor)

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NOMENCLATURE

Symbols

A  Area, ft² (m²)
B  Porosity, N.D.
C  Nodal Capacitance, Btu/F (kj/C)
E  Nodal Latent Heat Capacitance, Btu (kj)
G  Temperature Gradient at Phase Front, °F/ft (°C/m)
K  Thermal Conductance, Btu/hr (kj/hr)
L  Heat of Fusion or Latent Heat, Btu/lbm (kj/kg)
P  Porosity, N.D.
Q  Quantity of Heat, Btu (kj)
R  Thermal Resistance, hr-ft²-F/Btu (m²-C/W)
S  Void Pattern Spacing, in. (cm)
T  Temperature, °F (°C)
V  Volume, ft³ (m³)
c  Specific Heat, Btu/lbm²F (kj/kg°C)
d  Characteristic Diameter, ft (m)
f  Void or Paraffin Fraction, N.D.
g  Constant, 32.2 ft/sec² (10.0 m/sec²)
h  Convective Heat Transfer Coef., Btu/ft²-hr²°F (W/m²°C)
k  Thermal Conductivity, Btu/hr-ft²°F (W/m²°C)
l  Layer Thickness, ft (m)
Heat Flux Vector, Btu/ft²-hr (W/m²)
Radial Coordinate, ft (m)
Time, hr
Uncertainty Interval
Depth Coordinate, ft (m)
Coordinate, ft (m)
Thermal Diffusivity, ft²/hr (m²/hr)
Thermal Expansion Coef., 1/°F (1/°C)
Position of Phase Front, ft (m)
Tangential Coordinate, Radians
Permeability, ft² (m²)
Numerical Constant for Steffans Problem, N.D.
Kinematic Viscosity, ft²/hr (m²/hr)
Density, lbm/ft³ (kg/m³)

Subscripts
- c: Cold
- e: Effective Reduced Value
- f: Fusion Temperature
- h: Hot
- i: Initial Condition
- m: Matrix
- p: Phase Change Material
- o: Value at Surface
- l: Continuous Component or Solid State
- 2: Dispersed Component or Liquid State
- i, j, k: x, r, θ Indices in PCM Region
Indices in Matrix Region

Superscripts

- Average Composite Value
* Geometrically Modified Value
+ Normalized Parameter
\( \gamma \) Time Index

Other Definitions

Bi Biot Number, N.D.
C Counts
M Matrix
Nu Nusselt Number, N.D.
P(r, t) Time Step Parameter, hr/ft² (hr/m²)
P(x, t) Two Dimensional Phase Front Function
PCM Phase Change Material
R, X, Y Parameters in Error Analysis
Ra Rayleigh Number, N.D.
Ste Stefan Number, N.D.
T(I) Diamond Shaped Region Nodal Temperature, °F (°C)
TC Thermocouple
TC(I) Composite Nodal Temperature, °F (°C)
TM(I, L, K) Matrix Nodal Temperature, °F (°C)
TP(I, J, K) PCM Nodal Temperature, °F (°C)
X(x) Phase Change Analysis Function
r, x, y Deviation in Error Analysis
\( \beta^3 \) Fraction Associated With Component 2, N.D.
Word Definitions

Penetration Parameter \( \frac{x}{(at)^{1/2}} \), N.D.

Stefan Number \( \frac{c(T_f - T_o)}{L} \), N.D.

Temperature Ratio \( \frac{(T - T_o)}{(T_f - T_o)} \), N.D.

\( Q \) Matrix Heat Transfer Into a Semi-infinite Solid, Btu (kJ)

\( Q \) Neumann Heat Transfer Predicted by Neumann Model, Btu (kJ)

\( Q \) Numerical Heat Transfer Predicted by Numerical Model, Btu (kJ)
CHAPTER I

INTRODUCTION

Proposed Thermal Energy Storage Material

During preliminary investigations, the author developed a method of dispersing paraffin wax in a concrete mortar filler matrix as shown in Figure 1.1 (1)*. The motivation for this development was the enhancement of the thermal storage capacity of the "concrete" on the one hand and the improved effective thermal conductivity of the wax on the other hand. It was felt that by forming a composite of these materials, the resulting thermal energy storage material would have thermal performance advantages over either of its constituents.

Both concrete and paraffin waxes have been utilized separately as thermal energy storage materials for space heating and cooling applications (2,3). Concrete offers simplicity, low cost, and reliability. Waxes in a suitable

* References are listed in the bibliography. References in table and figure titles denote that all of the data shown came from that reference.
Figure 1.1 Paraffin Wax Dispersed In A Concrete Mortar Filler Matrix.
temperature range are attractive because the phase change is accompanied by a large latent heat effect, and they are reversible over a very large number of cycles (4,5,6). However, waxes have very low thermal conductivities.

Filler materials have been used to improve the effective thermal conductivity of waxes. Metallic fillers in various forms such as powders, foams, wools, and honeycombs have been investigated (5,7,8). A honeycomb has been found to be the most effective due to its continuous, direct path for heat transfer. The structure formed by mortar offers a continuous but tortuous path for heat transfer. Also, the thermal conductivity of concrete mortar is 10 times that of the wax (9,10).

Statement of the Problem

A paraffin phase change material (PCM) has a very low thermal conductivity, comparable to that of the best insulators. The temperature at the surface of such a PCM may raise far above the melting point with PCM solid still available but thermally isolated from the heated surface. That is, the low thermal conductivity of the wax limits the rate at which heat can be transferred. The use of fillers with waxes offers distinct advantages, the primary one being the improvement of thermal conductivity (5).

When a filler is used, the penalty resulting from the
displacement of some of the fusible material by the filler must be considered. This displacement substitutes the product of specific heat and temperature rise of the filler for the much larger heat of fusion of the wax.

Since the thermal conductivity of concrete mortar is only 10 times that of the wax, the enhancement of thermal conductivity may not compensate for the displacement of fusible material. It is therefore the objective of this research to investigate the rate at which heat can be transferred in these types of composites.

Study Approach

Initially, it was hoped that a well structured experimental research program could be developed to investigate the thermal performance of paraffin wax and concrete mortar composites. However, due to the lack of reliable thermophysical property data and the difficulty in controlling the composition, it was decided that this research should be conducted on a composite with thermophysical properties of a similar magnitude which were readily available and well documented.

A survey of potential matrix materials revealed that aluminum filled epoxies have thermophysical properties similar to those of concrete (11). Furthermore, casting methods could be used to form the desired test specimen
configuration.

A 99% pure normal paraffin, n-octadecane, was selected as the test PCM (12). Its selection was based on the abundance of property data and commercial availability. In addition, n-octadecane has a fusion temperature of 81° F (27.2°C) which is near room temperature, thus reducing thermal losses from the test specimen to the surroundings.

In order to have a mathematically specifiable geometry which also allowed visibility within the test specimen to observe the phase transition, a simulated semi-infinite region with a square array of circular voids was selected as shown in Figure 1.2.

Neumann (14) developed a solution for the freezing problem in a semi-infinite region whose initial temperature is uniform and whose surface temperature is stepped to a new value across the fusion temperature of the material at time zero as shown in Figure 1.2. If a heterogeneous composite, such as the test specimen, can be idealized as a homogeneous material with effective thermophysical properties, then the Neumann solution could be used to predict the rate at which heat is transferred into the test specimen. Therefore, throughout this study, the Neumann solution was used as a point of reference to compare the thermal behavior of the composite being investigated.

To make these comparisons, the test specimen was equilibrated at its fusion temperature, then the surface was
Figure 1.2 Idealized Composite and Thermal Exposure.
subjected to a step change in temperature. Thermal response was recorded via thermocouples cast into the epoxy matrix at the centers of the array. Visual observations were made of the progress of the phase front. During a limited number of tests, photographs were also taken.

A finite difference numerical model of the test specimen was developed using the Euler method to step forward in time (13). Comparisons were made between the measured and computed temperatures. These results were also compared to the Neumann solution (14).
CHAPTER II

LITERATURE SURVEY

Introduction

The literature survey was divided into two major parts: materials and thermal analysis. The materials part was limited to establishing the thermophysical characteristics of the proposed thermal energy storage material, a composite of paraffin wax in a concrete mortar filler matrix, and the materials used to experimentally model them, that is, paraffin and aluminum filled epoxy.

Since there are volumes of material on the general problems of heat transfer in composites and phase change, and it was not the intent of this research to cover this body of literature, this part was limited to considering classical solutions to these problems which would be used as a standard in subsequent experimentation and analysis. Specifically, conduction in composites by the equivalent network of resistances and the potential function methods were reviewed. Also, convection stability criteria for a plane layer heated from below with emphasis on porous media and phase change were included. Finally, the problem of
melting and solidification was discussed with emphasis on
the Neumann solution for the semi-infinite region.

**Paraffin Phase Change Materials**

Several surveys of phase change materials (PCM) have
identified paraffins and paraffin waxes as good PCM
candidates (4,5,6). To clarify, the term paraffin refers to
a particular hydrocarbon, whereas, the term paraffin wax is
used to describe a mixture of paraffins and other
hydrocarbons of various molecular weights. Some PCM
engineering design considerations are listed in Table 2.1.
Of these characteristics, paraffins are rated favorably
except for their low thermal conductivity and flammability.

Normal paraffins are hydrocarbons whose generalized
chemical formula is given by \( \text{C}_n \text{H}_{2n+2} \). Humphries (7)
presents an excellent summary of paraffin thermophysical
property data. Bailey and Liao (12) tabulated melting
temperature, transition temperature, latent heat of fusion,
heat of transition, specific heat, and thermal conductivity
of ultra-pure and commercial grades of selected paraffin
hydrocarbons. These data are the most recent data available
from the American Petroleum Institute for ultra-pure
hydrocarbons, and those measured by Bailey and Liao for
available materials of various grades provided by two
manufacturers, Phillips Petroleum and Humphrey Chemical
Table 2.1  PCM Engineering Design Considerations (5)

<table>
<thead>
<tr>
<th>Property or Characteristic</th>
<th>Desirable Value or Tendency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat of Fusion</td>
<td>High</td>
</tr>
<tr>
<td>Thermal Conductivity</td>
<td>High</td>
</tr>
<tr>
<td>Specific Heat</td>
<td>High</td>
</tr>
<tr>
<td>Density</td>
<td>High</td>
</tr>
<tr>
<td>Volume Change During Change of Phase</td>
<td>Low</td>
</tr>
<tr>
<td>Vapor Pressure</td>
<td>Low</td>
</tr>
<tr>
<td>Melting and Freezing Behavior</td>
<td>Dependable and Reversible</td>
</tr>
<tr>
<td>Availability</td>
<td>Readily Available</td>
</tr>
<tr>
<td>Cost</td>
<td>Low</td>
</tr>
<tr>
<td>Compatibility</td>
<td>Compatible with Container and Filler Material</td>
</tr>
<tr>
<td>Toxicity</td>
<td>Nontoxic</td>
</tr>
<tr>
<td>Hazardous Behavior</td>
<td>Not Exhibited</td>
</tr>
<tr>
<td>Property Data</td>
<td>Readily Available</td>
</tr>
<tr>
<td>Surface Tension</td>
<td>Low</td>
</tr>
</tbody>
</table>
Company. The property data was obtained using a Perkin-Elmer Differential Scanning Calorimeter using experimental procedures recommended by the manufacturer.

The paraffin selected for use in the experimental study was one of those investigated by Bailey and Liao, n-octadecane, supplied by Humphries Chemical Company. The reported data for n-octadecane are tabulated in Table 2.2. A discussion of paraffin thermophysical properties follows.

**Transition Temperature** - Paraffins with an even number of carbon atoms between 20 and 32, and those with any odd number of carbon atoms exhibit a lattice transition. The even numbered carbon atom paraffins exhibit this transition near their melting point, whereas odd numbered paraffins exhibit their transition in the solid state, as much as 30°F (16.7 °C) below their freezing/melting temperature (7). Bailey (12) reported that the commercial grade paraffin solid-liquid transition temperatures are close to those of the pure material although they are in general lower as shown in Figure 2.1. Also, there is little difference between the melting and freezing temperatures. Further, it was found that there was an absence of supercooling. These results were attributed to the relatively low level of impurities and the likelihood that the impurities were of a similar chain length. Freezing point temperatures were observed to increase in a nonlinear manner with increasing carbon chain length.
Table 2.2 Thermophysical Property Data For Humphrey Chemical Company, \( n \)-Octadecane (\( C_{18}H_{38} \)), as Reported by Bailey and Liao (12).

<table>
<thead>
<tr>
<th>Grade, Mole Percent Pure</th>
<th>99</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature of Fusion, (^\circ)F (^\circ)C</td>
<td></td>
</tr>
<tr>
<td>Heating Cycle (Solid to Liquid)</td>
<td>80.8 (27.1)</td>
</tr>
<tr>
<td>Cooling Cycle (Liquid to Solid)</td>
<td>81.0 (27.2)</td>
</tr>
<tr>
<td>Temperature of Transition, (^\circ)F (^\circ)C</td>
<td>No Data</td>
</tr>
<tr>
<td>Heat of Fusion, Btu/lbm (kj/kg)</td>
<td></td>
</tr>
<tr>
<td>Heating Cycle (Solid to Liquid)</td>
<td>107.0 (248.)</td>
</tr>
<tr>
<td>Cooling Cycle (Liquid to Solid)</td>
<td>105.6 (245.)</td>
</tr>
<tr>
<td>Heat of Transition, Btu/lbm (kj/kg)</td>
<td>No Data</td>
</tr>
<tr>
<td>Specific Heat, Btu/lbm-(^\circ)F (kj/kg-(^\circ)C)</td>
<td></td>
</tr>
<tr>
<td>Liquid Phase 134.5(^\circ)F (56.9(^\circ)C)</td>
<td>0.556 (2.33)</td>
</tr>
<tr>
<td>Solid Phase 44.6(^\circ)F (7.0(^\circ)C)</td>
<td>0.450 (1.88)</td>
</tr>
<tr>
<td>Thermal Conductivity, Btu/hr-ft(^2)F (W/m(^2)C)</td>
<td></td>
</tr>
<tr>
<td>Liquid Phase 81(^\circ)F (27.2(^\circ)C) (1)</td>
<td>0.0870 (0.151)</td>
</tr>
<tr>
<td>Solid Phase</td>
<td>No Data</td>
</tr>
<tr>
<td>Density, lbm/ft(^3) (kg/m(^3)) (2)</td>
<td></td>
</tr>
<tr>
<td>Liquid Phase 60(^\circ)F (15.6(^\circ)C)</td>
<td>49.0 (784.9)</td>
</tr>
<tr>
<td>Liquid Phase 68(^\circ)F (20.0(^\circ)C)</td>
<td>48.8 (781.7)</td>
</tr>
<tr>
<td>Liquid Phase 77(^\circ)F (25.0(^\circ)C)</td>
<td>48.6 (778.5)</td>
</tr>
<tr>
<td>Liquid Phase 100(^\circ)F (37.8 (^\circ)C)</td>
<td>48.1 (770.5)</td>
</tr>
<tr>
<td>Solid Phase (3)</td>
<td>53.3 (853.8)</td>
</tr>
<tr>
<td>Absolute Viscosity, lbf-sec/ft ( \times 10^5 ) (N-sec/m ( \times 10^3 )) (1)</td>
<td></td>
</tr>
<tr>
<td>86(^\circ)F (30.0(^\circ)C)</td>
<td>8.10 (3.88)</td>
</tr>
<tr>
<td>104(^\circ)F (40.0(^\circ)C)</td>
<td>6.44 (3.08)</td>
</tr>
<tr>
<td>122(^\circ)F (50.0(^\circ)C)</td>
<td>5.24 (2.15)</td>
</tr>
<tr>
<td>140(^\circ)F (60.0(^\circ)C)</td>
<td>4.35 (2.08)</td>
</tr>
</tbody>
</table>

(1) American Petroleum Institute
(2) Undercooled Liquid Below Normal Freezing Point, American Petroleum Institute
(3) Humphries, Reference 7
Figure 2.1 Effect of Carbon Chain Length on Phase Transition Temperature (12).
Latent Heat - Humphries (7) reported that the energy associated with solid-solid transitions are subtractive from the normal energy absorbed or liberated due to phase change. Paraffins exhibiting this phenomena have latent heats of fusion that are 10 to 20% below the latent heats of their neighboring paraffins. Neighboring paraffins refer to paraffins with one less or one more carbon atom in the chain. Bailey (12) observed that the latent heats of fusion for the commercial grade materials are close to those for the ultra high purity materials as shown in Figure 2.2. Also, there was little difference between heats of fusion for melting and freezing. Similar results were noted for solid-solid transitions. However, the heats of fusion for paraffins which exhibited solid-solid transitions were on the order of 30 percent lower than their neighboring paraffins. The heat of transition was only 20% lower.

Specific Heat - For data from the American Petroleum Institute, specific heat increased almost linearly with temperature and was independent of the number of carbon atoms in the chain (12). The specific heat data taken by Bailey showed considerable scatter about the ultra pure paraffin data as shown in Figure 2.3. Bailey attributed the scatter to sensitivity of specific heat determination to impurities. In general, however, specific heat increases with temperature.

Thermal Conductivity - Although there is general agreement
Figure 2.2 Effect of Carbon Chain Length on the Heat of Fusion and the Heat of Transition (12).
Figure 2.3 Effect of Temperature on Specific Heat (12).
on a thermal conductivity of paraffins in the liquid state of 0.087 Btu/hr-ft°F (0.151 W/m-°C) (7,12,15,16,17), there is little agreement on the value in the solid state. Powell (16) measured the thermal conductivity of n-octadecane using a guarded hot plate in the liquid and solid phases. These data are shown in Figure 2.4. Westwater (17) made microscopic studies of the phase front locations under carefully controlled conditions and observed freezing front velocities as much as 100% above those predicted by classical models (14) when Powell's solid state conductivity value of 0.111 Btu/hr-ft°F (0.193 W/m-°C) was used. Westwater noted that the crystal structure of n-octadecane, shown in Figure 2.5, is anisotropic and that in such materials the conductivity will vary from one direction to another. McAdams (18) listed a value of 0.14 Btu/hr-ft-F (0.24 W/m-°C) for paraffin wax at 32°F (0°C). No details were given. Assuming that the value given by the American Petroleum Institute for paraffin at the phase transition temperature applied for both the solid and liquid phases, Humphries (7) and Dyer (15) both measured freezing rates above that predicted by theory. Dyer's experimental apparatus was similar to that used in this study, i.e., a plane layer subjected to a step change in surface temperature across the fusion temperature of the paraffin. In order to obtain correlation with measured freezing front locations, thermal conductivities 2.3 and 3.5 times that of
Figure 2.4 Thermal Conductivity of Paraffins Vs. Temperature.
Figure 2.5 Freezing Front Dendrite Formation (7).
the liquid phase were required for nonadecane and hexadecane respectively. These paraffins bracket octadecane in carbon chain length. In addition, Dyer made direct thermal conductivity measurements of these paraffins in the solid state using a guarded hot pipe apparatus over a range of temperatures. These data are also shown in Figure 2.4. These measurements gave somewhat lower values. Factors of only 1.7 and 2.3 times the liquid state value for nonadecane and hexadecane respectively were measured.

Mixtures - Bailey and Liao (12) investigated nine binary paraffin systems. These systems were studied at eleven percents of solute in order to establish the temperature composition equilibrium phase diagram. Latent heats of fusion and transition were also measured. Three types of complex phase diagrams were observed depending on the difference in carbon chain length and whether the number of carbon atoms in the chain were odd-even or even-even pairs. For an odd-even adjacent binary system, a wide range of solid solutions appear to be formed with a single peritectic reaction near the 100% even paraffin concentration as shown in Figure 2.6. For an even-even binary system separated by a single odd paraffin, a wide range of solid solutions formed. In this system, two peritectic reactions at either extreme were observed. In addition, a complex eutectoid reaction was noted at low temperatures. For systems whose constituents have a difference in chain length of five or
Figure 2.6 Temperature-Composition Equilibrium Phase Diagrams for Adjacent Paraffin Binary System, Even-Odd and Even-Even (12).
more carbon atoms, whether even-odd or even-even, eutectoid systems form.

For all the systems studied, latent heat of fusion decreased with increased weight percent of solute until a minimum was reached as shown in Figure 2.7. Subsequently, heat of fusion increased. For binary systems which form extended solid solutions, that is neighboring paraffins, the minimum occurs at approximately 50 weight percent solute. On the average, this minimum was 34% of the average pure paraffin heats of fusion. For systems which form eutectics, the minimum appears to occur near the eutectoid composition. The average minimum in this case was 24% of the average pure component heat of fusion. Although similar trends were reported for heat of transition, the effect was less pronounced. Humphries (7) reported similar effects.

**Paraffin Waxes** - Due to the high cost of commercial and technical grade paraffins, up to $100.00 per lbm ($45.00 per kg) (7), paraffin waxes have been proposed as an economically feasible PCM for heating and cooling applications (4,5,6). Local petroleum distributors were surveyed as to type, availability, and cost of paraffin wax products. Bulk prices ranged from 15 to 20 cents per lbm (7 to 9 cents per kg). Although melting point temperature and oil content were listed in the technical bulletins (19,20), no data was available on composition and other thermophysical properties. The bulletins described these
\[ L^* = \frac{L_{\text{Composite}}}{\frac{L_{\text{Solute}}}{2} + \frac{L_{\text{Solvent}}}{2}} \]

Figure 2.7 Effect of Composition on Heats of Fusion and Transition for Neighboring Paraffins (12).
products as fully refined paraffins, usually used as coatings for food and food packaging. Clarification of the term fully refined was provided by Ourlian (21).

Waxes are distilled from crude oil in four temperature ranges. These products are called slack wax residuals or unrefined waxes and have oil contents ranging from 18 to 30%. The cost of the unrefined wax is currently around 7 cents per pound. Further refining reduces the oil content to approximately 0.5%. Finally, the wax is filtered to produce the fully refined paraffin wax.

A paraffin wax that has received much attention in the literature is Sun Oil Company P 116. University of Pennsylvania researchers (22) reported a heat of fusion of 90 Btu/lbm (209 kJ/kg) for this wax. However, another determination made by Mehalick and Tweedie (23) yielded a value of only 75 Btu/lbm (174 kJ/kg). Since this wax exhibits a range of melting temperatures from 112 °F to 118 °F (44.4 °C to 47.8 °C) with a peak at 116 °F (46.7 °C), it could be a composite of primarily docosane and tricosane. Both of these paraffins exhibit solid-solid transitions as well as solid-liquid phase changes. These effects coupled to the reported degradation of latent heat for composites and their sensitivity to proportions, make the reported disagreement in property value quite understandable.

Considering these reported discrepancies and the uncertainties inherent in a mixture of paraffins, waxes
appear to be unattractive from a thermophysical viewpoint especially for a research study.

Concrete

Concrete can be considered to be made of two components, aggregate and paste. Cement paste ordinarily constitutes 20 to 40% of the total volume of the concrete. It consists of cement (7 to 15%), and water (14 to 21%). The remainder, 60 to 80% is aggregate. Air content in air-entrained concrete ranges up to 8% of the volume (24).

In properly made concrete, each particle of aggregate is completely coated with paste and all the space between the aggregate is filled with paste. The quality (strength) of paste is dependent upon the ratio of water to cement and the extent of curing. The cementing properties of the paste are due to the chemical reaction between cement and water. These reactions, called hydration, require time and favorable conditions of temperature and moisture. More water is used in mixing concrete than is required in order to make the concrete plastic and workable.

A review of the literature, has shown that there are few comprehensive studies of the thermophysical properties of concrete with the possible exception of a U. S. Bureau of Reclamation study (25) and a Portland Cement Association study conducted by Lentz and Monfore (10). Zoldners (9) and
Bizri (26) present summaries of available concrete thermophysical data. In most studies, only one or two properties were measured or reported. This coupled with the variability of the components, mixes, and curing make specific, comprehensive determinations difficult at best.

**Thermal Conductivity** - Zoldners (9) reported that the thermal conductivity of concrete is influenced by five major variables: 1) conductivity of the cement paste, 2) conductivity of the aggregate, 3) mix proportions, 4) compactness, and 5) moisture. Of these, Zoldners, identified the conductivities of the cement paste and the aggregate as the most significant parameters. Furthermore, for high volume ratios of aggregate, the conductivity of the aggregate was noted to be most significant. Lentz and Monfore (10) concluded that the conductivity of concrete is largely determined by the conductivity of the aggregate at comparable moisture content. Bizri's survey (26) indicated a range of concrete thermal conductivities from 0.7 to 1.5 Btu/hr-ft-°F (1.2 to 2.6 W/m-°C) for temperatures up to 200 °F (93.3 °C). Bureau of Reclamation studies (25) show that thermal conductivity of moist mortar varies from 1.2 to 2.0 Btu/hr-ft-°F (2.1 to 3.5 W/m-°C).

Zoldners (9) reported that the thermal conductivity of hardened Portland cement paste varies between 0.5 and 0.7 Btu/hr-ft-°F (0.87 and 1.21 W/m-°C). Also, cement paste with higher water to cement ratios have lower thermal
conductivities. However, Lentz and Monfore noted no such dependence for moist cement paste specimens (10).

According to Zoldners (9) the thermal conductivity of aggregates range from 0.7 to 3.0 Btu/hr-ft°F (1.2 to 5.2 W/m-K). Quartzite, sandstone, and other quartzose rocks show the highest conductivities. Igneous rocks such as granities, gnesses, rhyolites as well as carbonate rocks such as limestone and dolomites have intermediate conductivities. Basalt, anorthosite, and barite are among the types having a low conductivity. Zoldners pointed out that rocks with crystalline structures show higher thermal conductivity than amorphous and vitreous rocks of the same composition.

Lentz and Monfore (10) studied three types of aggregates: marble, limestone, and sandstone (Table 2.3). Marble and limestone are both carbonate while sandstone is siliceous. Limestone and sandstone are relatively porous while marble is not. The thermal conductivity of the dense marble rock and the porous, moist limestone rock are nearly the same. However, in the oven dry condition the limestone conductivity was consistently lower. The lower conductivity was attributed to the large amount of empty pore space in the oven dry limestone as opposed to the small amount of voids in the oven dry marble. The effect of the nature of the mineral was investigated by comparing the sandstone and limestone rocks. Both rocks had a high porosity but the
Table 2.3 Thermophysical Properties of Concrete and Its Components Which Were Studied by Lentz and Monfore (10).

<table>
<thead>
<tr>
<th>Component</th>
<th>Moisture Condition</th>
<th>Water Percent at Test</th>
<th>Density at Test (lbm/ft(^3))</th>
<th>Thermal Conductivity at 75°F (23.9°C) (Btu/hr-ft-°F/W/m-°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>-</td>
<td>-</td>
<td>62.4 (1000)</td>
<td>0.39 (0.68)</td>
</tr>
<tr>
<td>Paste</td>
<td>Moist</td>
<td>-</td>
<td>-</td>
<td>0.67 (1.16)</td>
</tr>
<tr>
<td>Marble</td>
<td>Moist</td>
<td>0.06</td>
<td>-</td>
<td>1.83 (3.18)</td>
</tr>
<tr>
<td>Agg.</td>
<td>50 % RH</td>
<td>0.0</td>
<td>-</td>
<td>1.83 (3.18)</td>
</tr>
<tr>
<td></td>
<td>Oven Dry</td>
<td>0.0</td>
<td>171.0 (2739)</td>
<td>1.67 (2.90)</td>
</tr>
<tr>
<td>Sandstone</td>
<td>Moist</td>
<td>7.3</td>
<td>-</td>
<td>2.33 (4.04)</td>
</tr>
<tr>
<td>Agg.</td>
<td>50 % RH</td>
<td>0.5</td>
<td>-</td>
<td>1.42 (2.46)</td>
</tr>
<tr>
<td></td>
<td>Oven Dry</td>
<td>0.0</td>
<td>164.7 (2638)</td>
<td>1.33 (2.31)</td>
</tr>
<tr>
<td>Limestone</td>
<td>Moist</td>
<td>4.4</td>
<td>-</td>
<td>1.58 (2.66)</td>
</tr>
<tr>
<td>Agg.</td>
<td>50 % RH</td>
<td>0.1</td>
<td>-</td>
<td>1.33 (2.31)</td>
</tr>
<tr>
<td></td>
<td>Oven Dry</td>
<td>0.0</td>
<td>157.9 (2529)</td>
<td>1.25 (2.17)</td>
</tr>
<tr>
<td>Marble</td>
<td>Moist</td>
<td>5.2</td>
<td>152 (2435)</td>
<td>1.25 (2.17)</td>
</tr>
<tr>
<td>Concrete</td>
<td>50 % RH</td>
<td>2.3</td>
<td>148 (2371)</td>
<td>1.25 (2.17)</td>
</tr>
<tr>
<td></td>
<td>Oven Dry</td>
<td>0.0</td>
<td>143 (2291)</td>
<td>1.0 (1.74)</td>
</tr>
<tr>
<td>Sandstone</td>
<td>Moist</td>
<td>11.7</td>
<td>133 (2130)</td>
<td>1.67 (2.90)</td>
</tr>
<tr>
<td>Concrete</td>
<td>50 % RH</td>
<td>3.1</td>
<td>124 (1986)</td>
<td>1.25 (2.17)</td>
</tr>
<tr>
<td></td>
<td>Oven Dry</td>
<td>0.0</td>
<td>120 (1922)</td>
<td>0.83 (1.44)</td>
</tr>
<tr>
<td>Limestone</td>
<td>Moist</td>
<td>9.8</td>
<td>141 (2258)</td>
<td>1.25 (2.17)</td>
</tr>
<tr>
<td>Concrete</td>
<td>50 % RH</td>
<td>1.9</td>
<td>130 (2082)</td>
<td>0.92 (1.60)</td>
</tr>
<tr>
<td></td>
<td>Oven Dry</td>
<td>0.0</td>
<td>126 (2010)</td>
<td>0.83 (1.44)</td>
</tr>
</tbody>
</table>
conductivity of the moist sandstone was about 50% higher than that of the limestone. However, little difference was observed in the oven dry condition.

The thermal conductivities of the concretes made from these aggregates were found to depend mainly on the conductivity of the aggregate at comparable moisture contents. To illustrate, the conductivity of sandstone rock was observed to increase almost two fold as the moisture content was increased from oven dry to moist, while, for the same conditions, the conductivity of the crushed sandstone increased two fold. Also for each case studied by Lentz and Monfore (10), the conductivity of the moist concrete lies between the value for moist paste and that for moist rock. However, it was nearer the value for the rock. Tests performed by the Bureau of Reclamation (25) showed that rock contributed more per unit weight to the conductivity of concrete than either cement or water.

The conductivity of the types of concrete investigated by Lentz and Monfore were also dependent on moisture content. Large differences in the moist and oven dry conductivities were observed when the aggregate in the concrete was porous as opposed to much smaller differences when the aggregate was dense.

**Density** - For the concretes studied by Lentz and Monfore (10), density, moisture content, and the corresponding conductivity are tabulated in Table 2.3.
Specific Heat - None of the references indicated specific heats for the materials for which they were specifying the conductivity. Kreith (27) lists a specific heat of 0.2 Btu/lbm·°F (0.84 kj/kg·°C) for stone concrete at 70°F (21.1°C) with a corresponding conductivity of 0.54 Btu/hr·ft·°F (0.94 W/m·°C) and density of 144 lbm/ft³ (2307 kg/m³). The Handbook of Chemistry and Physics (28) lists thermal conductivity of rocks and their corresponding heat capacity at various temperatures and these values are listed in Table 2.4.

Concrete/Wax Composite - Jenkins and Butler (29) investigated the performance of waxes dispersed in concrete as a barrier to chloride penetration in bridge decks. This type of concrete is called internally sealed concrete and is made by mixing small discrete wax particles with the conventional components of Portland cement concrete. After the concrete has cured, heat is applied and the wax melts and flows into the capillaries and bleed channels of the concrete. Upon cooling, the wax solidifies in the pores and capillaries thereby blocking the capillary system.

A number of waxes and wax blends were investigated at concentrations up to 4% (percent weight of concrete or for normal weight concrete, approximately 12% of volume). It was found that a 25% Montan, 75% paraffin wax blend at a concentration of 3% had the most promising structural and sealing properties. Some of the characteristics observed
Table 2.4 Thermal Conductivity and Heat Capacity of Rocks
From the Handbook of Chemistry and Physics (28).

<table>
<thead>
<tr>
<th>Rock</th>
<th>Temperature, °F (°C)</th>
<th>Thermal Conductivity Btu/hr-ft-°F (W/m-°C)</th>
<th>Heat Capacity Btu/lbm-°F (kJ/kg-°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Granite</td>
<td>32 (0)</td>
<td>2.03 (3.52)</td>
<td>0.192 (0.81)</td>
</tr>
<tr>
<td></td>
<td>122 (50)</td>
<td>1.89 (3.28)</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>212 (100)</td>
<td>1.74 (3.02)</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>392 (200)</td>
<td>1.57 (2.72)</td>
<td>0.228 (0.96)</td>
</tr>
<tr>
<td>Marble</td>
<td>244 (118)</td>
<td>0.97 (1.68)</td>
<td>0.21 (0.88)</td>
</tr>
<tr>
<td></td>
<td>385 (196)</td>
<td>0.67 (1.16)</td>
<td>0.24 (1.01)</td>
</tr>
<tr>
<td>Dolomite</td>
<td>266 (130)</td>
<td>0.95 (1.65)</td>
<td>-</td>
</tr>
<tr>
<td>Limestone</td>
<td>358 (181)</td>
<td>0.92 (1.60)</td>
<td>-</td>
</tr>
<tr>
<td>Shale</td>
<td>32 (0)</td>
<td>1.11 (1.93)</td>
<td>0.17 (0.71)</td>
</tr>
<tr>
<td></td>
<td>212 (100)</td>
<td>1.01 (1.75)</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>370 (188)</td>
<td>0.95 (1.65)</td>
<td>0.24 (1.01)</td>
</tr>
<tr>
<td>Sandstone</td>
<td>32 (0)</td>
<td>3.29 (5.71)</td>
<td>-</td>
</tr>
<tr>
<td>(Quartzitic)</td>
<td>212 (100)</td>
<td>2.82 (4.89)</td>
<td>0.26 (1.09)</td>
</tr>
<tr>
<td></td>
<td>392 (200)</td>
<td>2.18 (3.78)</td>
<td>-</td>
</tr>
</tbody>
</table>
were: 1) no chloride penetration after 60 days of continuous ponding, 2) compressive strength and tensile splitting strength equal to that of the air entrained control, and 3) retardation of freeze/thaw scaling.

Heat treatment was studied by exposing prismatic slabs with aspect ratios of 3 to 1 and 4 to 1 to an ambient time temperature profile designed to keep the temperature from surface to center of the specimen at 50 °F (27.8 °C) or less and reach a maximum internal temperature 9 to 45 °F (5 to 25 °C) above the additive melting point. The thermal diffusivity of these composites was reported to be a quarter of the literature cited value of 0.02 ft²/hr (0.21 m²/hr). This reduction was attributed to free moisture content and the range of the thermal exposure, that is above 212 °F (100 °C). There was no discussion of the latent heat effect of the wax.

Jenkins and Butler made microscopic studies, shown in Figure 2.8, before and after heat treating. They noted that before heat treatment, the porosity of the cement paste and the appearance of the wax were clearly delineated. After heat treatment, the central portion was now a void but the surface retains a coating of wax and the additive flowed into and coated the spherical air bubbles. Also, observed was the affinity of this additive for the cement matrix seen by the wetting and film retention by the chamber and capillary walls.
Figure 2.8 Photomicrographs of Mantan Sealed Specimens Before and After Fusion (29).
A commercially available aluminum filled epoxy system was selected because: 1) its gross thermophysical properties were similar to those of concrete, 2) property data with the exception of specific heat were available from the manufacturer, 3) property data for similar materials were reported in the literature, 4) casting methods could be used to form the desired shape, 5) curing times were short, and 6) well specified boundaries could be established between the matrix and the phase change material.

**System Description** - Epocast 31-D with hardener 9216 (Furane Plastics Inc.) was proportioned according to manufacturer's recommendations, that is, 100 parts of resin to 6 parts of hardener (30). This system was reported to contain approximately 56% aluminum powder by weight. Technical data available from the manufacturer are summarized in Table 2.5. It should be pointed out that the technical bulletin noted the reported values as typical.

**Density** - Based on the reported system specific gravity of 1.74, the approximate aluminum powder content by volume was estimated to be 0.35. Lee and Neville (11) reported unfilled epoxy specific gravities in the range of 1.2 to 1.3. Based upon these data and an aluminum of density 169 lbm/ft$^3$ (2707 kg/m$^3$), anticipated composite specific gravity would be between 1.73 and 1.79.

**Specific Heat** - Since specific heat data were not available
Table 2.5 Manufacturer Reported Property Data for Epocast 31-D/9216 Aluminum Filled Epoxy System (30,56).

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Test Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal Conductivity, Btu/hr·ft²·F (W/m²·C)</td>
<td>0.435 (0.753)</td>
<td>ASTM D-1674</td>
</tr>
<tr>
<td>Specific Gravity</td>
<td>1.74</td>
<td>ASTM D-792</td>
</tr>
<tr>
<td>Coefficient of Thermal Expansion, ft/ft²·F (m/m·C) (2.8 to 3.0x10⁻⁵)</td>
<td>1.6 to 1.7x10⁻⁵</td>
<td>ASTM D-696</td>
</tr>
</tbody>
</table>
from the manufacturer, samples were taken from the material used to cast the specimens. These samples were sent to Dow Chemical, Midland, Mich. for analysis in their thermophysical laboratory. The sample chips were pulverized and sealed in a pressure weld aluminum pan. The specific heat was determined by comparison with a reference standard of alpha aluminum oxide in a differential scanning calorimeter over a range of temperatures from 32 to 212°F (0 to 100°C). These data are shown in Figure 2.9. An accuracy of 2% was reported (31).

The specific heat increased almost linearly with temperature up to 158°F (70°C) where a step change in value was recorded. Subsequently, it increased nearly linearly again but at a reduced slope. Lee and Neille (11) reported specific heat vs. temperature for several unfilled epoxy systems. Although none of these epoxies show an abrupt change, one type does exhibit a change in slope. It is possible that the presence of the aluminum filler caused the observed behavior. Using these data and an aluminum volume fraction of 0.35, a range of possible specific heats were calculated. These results are shown in Figure 2.10 along with the measured data. It appears that the measured data lies above the calculated values over the entire range of temperatures investigated.

**Thermal Conductivity** - A thermal conductivity of 0.435 Btu/hr-ft-°F (0.753 W/m-°C) was reported by the manufacturer
Figure 2.9 Specific Heat Vs. Temperature for Epocast 31-D/9216 System.
Figure 2.10  Comparison of Measured and Computed Specific Heat Vs. Temperature for Aluminum Filled Epoxy System.
(56). This value was believed to have been determined using the procedures specified in ASTM D-1674 which is a transient method based on the response of low Biot number systems. Since this method measures apparent conductivity over a range of temperature the temperature dependence of conductivity remains to be established.

Kline (32) studied the thermal conductivity of epoxy/aluminum systems over a range of temperatures 32 to 212 °F (0 to 100 °C). A steady state, guarded hot pipe method was used. The specimens were prepared from Epon (Shell Chemical Co., specific gravity 1.19) with m-phenylenediamine as catalyst in the respective ratio of 85:15 by weight. In filled epoxy, fine aluminum powder was added as filler. Kline used the notation specifying the relative parts of resin, catalyst, and filler. The results, shown in Figure 2.11, for various epoxy/aluminum systems indicated that the thermal conductivity increased with temperature. This trend was noted to be consistent with the specimens amorphous structure. The conductivity varied strongly with the amount of filler. The unfilled epoxy conductivity was 0.097 to 0.109 Btu/hr-ft-°F (0.167 to 0.188 W/m-°C). Lee and Neville reported unfilled epoxy conductivity near 0.121 Btu/hr-ft-°F (0.209 W/m-°C) (11). The conductivity of the specimen with the most filler (56% aluminum by weight) was approximately four times that of the unfilled epoxy.

Reported thermal conductivity data are summarized in
Figure 2.11 Thermal Conductivity Vs. Temperature (32).
Figure 2.12  Epoxy/Aluminum Filler System Thermal Conductivity Versus Aluminum Content.
Figure 2.12 along with that for Epocast 31-D/9216. Also shown are computed effective conductivities using the parallel-series (33) and Maxwell (34) approaches. It appears that the reported value for the Epocast system is reasonable. However, there is considerable scatter in these data and there appears to be a strong dependence on aluminum content.

Conduction in Composites

Many investigators have studied the problem of evaluating the effective conductivity of two-component systems. In these systems, an otherwise continuous and homogeneous material has discontinuous inclusions of a second homogeneous material that are dispersed in a regular array. In general, two types of analysis have been employed. An equivalent network of resistances which are connected in parallel and/or series, and the superposition of potential functions which satisfy the Laplace equation and associated boundary conditions.

Duga (33) compiled developments using the network approach for two and three dimensional inclusions. Rohsenow and Hartnet (35) summarized some of these results and they are shown in Table 2.6. Maxwell (34), Rayleigh (36) and Runge (37), as well as Meredith and Tobias (38) developed potential solutions for a cubic array of spheres, see Table
2.7. Powers (39) surveyed various methods and geometries including one of particular interest in this study, cylindrical inclusions with heat flow normal to their axes. These equations are listed in Table 2.8.

Network Approach - As a simple first approximation, one may rearrange the composite to one of two different forms as shown in Figure 2.13. With the heat flow so designated, the equivalent conductivity of the two lumped conductors are listed in Table 2.6. These two expressions for the effective conductivity are the minimum and maximum values possible for a treatment of various combinations of series and parallel contributions (33).

One of the classical problems considered using the simple network approach related to the conductivity of porous media. Russell (40) assumed that the two component system is composed of a simple cubic array of regular cubes. The network can be connected in two ways: parallel/series or series/parallel. In the former, adjacent paths are treated as parallel elements and then this series of effective elements are summed. For the series/parallel network, the elements in line with the direction of heat flow are summed in series first and then these effective elements are treated as parallel elements to obtain the overall effective conductivity. These two approaches do not yield the same result as is shown in Figure 2.14 for a simple cubic array of regular cubes: very high conductivity inclusions, \( k_1/k_2 \).
Figure 2.13  Typical Dispersion and First Approximation Lumped Networks.
Table 2.6 Effective Reduced Conductivity for Various Types of Inclusions Calculated by Means of the Network Approach (35).

<table>
<thead>
<tr>
<th>Type</th>
<th>Effective Reduced Conductivity</th>
<th>Comment/Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lumped</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$k_e = (1 - \Gamma)k_1 + \Gamma k_2$ (parallel)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$k_e = \frac{k_1 k_2}{\Gamma k_1 + (1 - \Gamma) k_2}$ (series)</td>
<td></td>
</tr>
<tr>
<td>Square Inclusions in Square Array</td>
<td>$\frac{k_e}{k_1} = \frac{\kappa + \beta(1 - \kappa)}{\kappa + \beta(1 - \beta)(1 - \kappa)}$ (parallel/series)</td>
<td>$\kappa = k_1/k_2$, Square Side D Spacing $d$, $\beta = D/d$ Porosity $P = \beta^2$</td>
</tr>
<tr>
<td>Cubic Inclusions in Cubic Array</td>
<td>$\frac{k_e}{k_1} = \frac{1 - \beta(1 - \beta)(1 - \kappa)}{1 - \beta(1 - \kappa)}$ (series/parallel)</td>
<td>Cube Side $D$, Spacing $d$ Porosity $P = \beta^3$</td>
</tr>
<tr>
<td>Spherical Inclusions in Cubic Array</td>
<td>$\frac{k_e}{k_1} = \left{1 - \beta \left[\frac{1 - \gamma'}{2\gamma'+1} - \ln \left(\frac{\sqrt{\gamma'+1}+1}{\sqrt{\gamma'+1}-1}\right)\right]\right}^{-1}$ $(\kappa &lt; 1)$</td>
<td>Spheres of Diameter $D$ Cubic Spacing $d$ $\gamma' = (4/\pi \beta^2)k/(1-\kappa)$ $P = (\pi/6) \beta^3$ for Spherical Pores</td>
</tr>
</tbody>
</table>
Figure 2.14 Effective Reduced Conductivity Vs. Porosity for $k_1/k_2 = 0.001$ and 10.
= 0.001, representing the aluminum filled epoxy; and a moderately low conductivity inclusion, wax in concrete, \( \frac{k_1}{k_2} = 10.0 \).

**Potential Function Method** - In 1881 Maxwell (34) proposed a solution valid for very dilute dispersions of spheres. When the obstacles are no longer very small in comparison to the distance between them, the effects of fields around each particle on each other must be considered. Lord Rayleigh (36) considered the case of a cubic array of uniform size spheres. Rayleigh solved the Laplace equation for the potential inside and outside a sphere by using the principle of superposition to take into account the effects of its neighbors. Runge (37) corrected Rayleigh's derivation in 1925 for an omitted factor of \( \pi \).

Meredith and Tobias (38) pointed out that as the volume fraction of the dispersed component approaches zero, Rayleigh's expression reduced to Maxwell's. However for maximum packing (spheres touching) and infinite inclusion conductivity, Rayleigh's expression for the effective reduced conductivity did not become infinite. Meredith and Tobias reviewed Rayleigh's development and attributed this defect to the number of terms used in the potential series representation. By use of a different function for the potential and by consideration of higher terms, Rayleigh's results were modified yielding an expression which better represented their experimental data than does Rayleigh's
<table>
<thead>
<tr>
<th>Investigator</th>
<th>Effective Reduced Conductivity</th>
<th>Comment/Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maxwell</td>
<td>$K_m = \frac{K_d + 2 - 2f(1-K_d)}{[K_d + 2 + f(1-K_d)]}$</td>
<td>Very Dilute Dispersion</td>
</tr>
<tr>
<td>Rayleigh</td>
<td>$K_m = \frac{[2+K_d/(1-K_d)] - 2f - 0.525 \left[ \frac{3-K_d}{4+3K_d} \right] f^{10/3}}{[2+K_d/(1-K_d)] + f - 0.525 \left[ \frac{3-K_d}{4+3K_d} \right] f^{10/3}}$</td>
<td>Two Term Expansion</td>
</tr>
<tr>
<td>Meredith and Tobias</td>
<td>$K_m = \left( \frac{2+K_d}{1-K_d} - 2f + 0.409 \frac{6+3K_d}{4+3K_d} f^{7/3} - 2.133 \frac{3-3K_d}{4+3K_d} f^{10/3} \right) \div \left( \frac{2+K_d}{1-K_d} + f + 0.409 \frac{6+3K_d}{4+3K_d} f^{7/3} - 0.906 \frac{3-3K_d}{4+3K_d} f^{10/3} \right)$</td>
<td>Three Term Expansion</td>
</tr>
</tbody>
</table>

**Comment/Notation**

- $f =$ Volume Fraction
- $k_c =$ Continuous
- $k_d =$ Dispersed
- $K_m =$ Effective
- $K_m = k_m/k_c$
- $K_d = k_d/k_c$
Figure 2.15  Effective Reduced Conductivity for a Cubic Array of Spheres.
model, especially near maximum packing as shown in Figure 2.15.

**Circular Inclusions in a Square Array** - In the experimental portion of this research, the geometry of the dispersed component was two dimensional circular cylinders arranged in a square array. This configuration along with other composites is discussed in the survey paper by Powers (39). Formulations developed by Rayleigh (36), Bruggeman (41), and Duga (33) are summarized in Table 2.8.

As in the case of spheres, Rayleigh (36) solved the Laplace equation for the potential inside and outside the cylinders. The expression shown in Table 2.8 is for the simplified case of a dilute dispersion in which higher order interaction terms have been neglected. Bruggeman (41) developed a mixture of potential solutions so that for dilute dispersions the effective reduced conductivity favored the lower bound series approach while for maximum packing, $k$ approached the upper bound parallel model as shown in Figure 2.16. Also shown in Figure 2.16 are the Rayleigh (36) potential solution and the Duga (33) parallel series results for high conductivity inclusions ($k_1/k_2 = 0.01$) and low conductivity inclusions $k_1/k_2 = 10$. For these ratios, the series model yields the lowest $k_e$ over the entire range of porosities. Similarly, the parallel model gives the highest $k_e$. The Rayleigh, Bruggeman, and Duga expressions exchanged ranks for various $k_1/k_2$ and
Table 2.8 Effective Reduced Conductivity for a Square Array of Circular Inclusions Calculated by Various Means.

<table>
<thead>
<tr>
<th>Investigator</th>
<th>Effective Reduced Conductivity</th>
<th>Comment/Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rayleigh (36)</td>
<td>( a = p \frac{k_2 - k_1}{k_2 + k_1} )</td>
<td>Potential Method</td>
</tr>
<tr>
<td></td>
<td>( k_2 = \frac{\alpha + 1}{\alpha - 1} )</td>
<td>Dilute Dispersion</td>
</tr>
<tr>
<td>Bruggeman (41)</td>
<td>((1 - p)\left(\frac{k_1 - k_e}{k_1 + k_e} + p \frac{k_2 - k_e}{k_2 + k_e}\right) = 0)</td>
<td>( k_1 ) and ( k_2 ) Continuous (1)</td>
</tr>
<tr>
<td></td>
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<td>and Dispersed (2) Conductivities</td>
</tr>
<tr>
<td>Duga (33)</td>
<td>( \frac{k_e}{k_1} \left{1 - \beta \left[1 + \frac{y}{\sqrt{1-\gamma^2}} \ln \left(\frac{1 - \sqrt{1 - \gamma^2}}{\gamma}\right)\right]\right}^{-1} )</td>
<td>Porosity ( P = \frac{\pi D^2}{4 d^2} )</td>
</tr>
<tr>
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<td>Diameter, D Spacing, d</td>
</tr>
<tr>
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<td>( \frac{k_e}{k_1} \left{1 - \beta \left[1 + \frac{y}{\sqrt{1-\gamma^2}} \left(\tan^{-1} \frac{1}{\sqrt{2-\gamma^2}} + \frac{\pi}{2}\right)\right]\right}^{-1} )</td>
<td>Mixture of Potentials</td>
</tr>
<tr>
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<td>with ( k_1 ) and ( k_2 ) inclusions</td>
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<td>Parallel/Series</td>
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<tr>
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<td>( \beta^2 = 4P/\pi )</td>
<td>( \gamma = \frac{k_1/k_2}{1-k_1/k_2} )</td>
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<tr>
<td></td>
<td></td>
<td>( \frac{1}{\beta} )</td>
</tr>
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</table>
Figure 2.16 Effective Reduced Conductivity for a Square Array of Circles.
porosities.

Examination of the Duga (33) parallel/series expression shows that there is a pole for \( k_e/k_2 = 1 \), and depending on the porosity, the form of the expression changed as shown in Table 2.8. In the vicinity of the pole, these expressions yield unreasonably low effective conductivities. In fact, they yield \( k_e 's \) that are lower than the series model, as is shown in Figure 2.17. Since the experimental material conductivities fell in this range \( k_1/k_2 = 3.2 \) and \( 6.3 \), this approach was not considered further.

Both the Rayleigh (36) and Bruggeman (41) expressions yield reasonable results over the entire range of interest. However, since the Bruggeman mixture method appears to introduce an artificial transition, the Rayleigh expression was adopted to compute effective reduced conductivities throughout the remainder of this study.

Convection in Composites

For the case of a fluid between infinite parallel plates, numerous experimental and analytical studies have been conducted and criteria have been established for the onset of convection. In 1916, Lord Rayleigh (42) analyzed the stability of a horizontal fluid layer heated from below. Silveston's (43) experimental results shown in Figure 2.18 confirmed Rayleigh's prediction. For Rayleigh numbers less
Figure 2.17 Comparison of Duga Method with Series Method Near Pole in Duga's Method at $k_1/k_2 = 1.0$. 

Porosity, $P = \pi/2$

$\pi/4$

$\pi/6$

$\pi/8$
Figure 2.18 Silveston's Experimental Results in the Neighborhood of Instability in Various Liquids (43).
than the critical value of 1708, heat transfer occurs by
conduction only. Above 1708, the Nusselt number increases
which is indicative of the onset of buoyancy driven
convection.

Porous Media — Although the dynamics of fluids in porous
media are less well known, Katto and Masuoka (44) have
reported results of their studies which will be used to
estimate the hydrodynamic stability in the concrete
mortar/paraffin composite. Katto and Masuoka found that the
onset of convection is governed by

\[
\frac{\text{Ra} \, \kappa}{l^2} = 2 \, \pi^2
\]  

(2.1)

where \( \kappa \) is the permeability and \( l \) is the vertical thickness
of the porous layer. The thermal diffusivity included in Ra
is defined as the thermal conductivity of the porous media
divided by the specific heat capacity of the fluid. The
permeability \( \kappa \), was related to the porosity \( B \) and the
diameter of the fill particles \( d \) by the semi-emperical
Blake-Kozeny Equation (44).

\[
\kappa = \frac{B^2 \, d^2}{(150(1 - B))^2}
\]  

(2.2)
Katto and Masuoka conducted experiments on layers composed of glass, steel, or aluminum spheres and nitrogen gas as the fluid. By varying the gas pressure, (up to 200 atm.), convection currents were generated at low temperature difference. Their data and theory are shown in Figure 2.19.

Using Katto and Masuoka's criteria, Equation 2.1, the stability limits of concrete mortar/paraffin layers have been estimated and are shown in Figure 2.20. The conductivity of the porous media was estimated using the network approach for a cubic array of cubes, Figure 2.14: \( \frac{k_1}{k_2} = 10 \), and \( k_1 = 1.0 \text{ Btu/hr-ft}^2\text{F} \approx 1.74 \text{ W/m}^2\text{C} \). Although the aggregate/cement paste labyrinth is more restrictive than the smooth spheres, Equation 2.2 was used to estimate the permeability. Paraffin properties were evaluated at 105° F (40.5° C). The porosity of \( \pi/6 \) represents a cubic array of spheres (an upper bound). A more practical value is \( \pi/12 \). The two sieve sizes span the normal grading of fine aggregates. It appears that for realizable porosities, i.e. around \( \pi/12 \), it is unlikely that convective motion will be initiated by buoyancy in concrete mortar/paraffin composites.

**Phase Change** - Since the manner in which density varies with height plays a key role in the instability phenomenon, and since the temperature and therefore density variation in a melting layer heated from below is not linear as is the rest-state for a single-phase fluid, Sparrow, Lee, and
Figure 2.19 Comparison of Katto and Masuoka's Experimental and Theoretical Critical Rayleigh Number (44).
Figure 2.20 Stability Criteria of Katto and Masuoka's (44) for a Horizontal Layer of Concrete/Paraffin Composite.
Shamsundar (45) analyzed a melting layer initially at the fusion temperature to determine the conditions marking the onset of convection. They also considered the effect of a convective boundary condition at the lower surface in addition to the step change characterizing Neumann's solution (14). The analysis was carried out for liquid melts such as paraffins whose density decreased with increasing temperature. Linear stability theory was employed.

The results of the analysis are expressed in terms of the internal Rayleigh number which is based on the instantaneous thickness, $l$, and instantaneous temperature difference across the layer.

$$\text{Ra} = \frac{g \beta (T_0 - T_f) l^3}{\alpha \nu}$$  \hspace{1cm} (2.3)

The Rayleigh numbers making the onset of instability are presented in Figure 2.21 as a function of Biot number, $Bi$. The curves are parameterized by the Stefan number, $Ste$. The critical internal Rayleigh number is significantly affected by Biot number but the Stefan number effect is small. The right margin of the figure contains line segments which represent the results for a step change in wall temperature.
Figure 2.21 Results for the Internal Rayleigh Number Marking the Onset of Instability (45).
For the idealized composite, the minimum Biot number was above 10 and the Stefan number was near 0.25. Therefore, the effect of these factors should not exceed 16/17.

Phase Change

The first published discussion of the one dimensional phase change problem was by Stefan (46) in a study of the thickness of polar ice, and for this reason the freezing problem is frequently referred to as Stefan's Problem. Of the few exact solutions, the most important is that of Neumann (47). Numerous other solutions are available in the technical literature. Sunderland and Muehlaur (48) give an excellent bibliography.

Neumann Solution - Following the development of the Neumann solution presented in Carslaw and Jaeger (14), the system shown in Figure 2.22 is defined by the following postulates: 1) the material has a definite fusion temperature, $T_f$, 2) initially the liquid is at a uniform temperature, $T_i$, above $T_f$, 3) all the thermal properties of the material are uniform and constant but can have different values in the solid and liquid phases with the exception of density. The symbols $\rho$, $c$, $k$, and $\alpha$ are the density, specific heat, thermal conductivity, and thermal diffusivity. Subscript 1 denotes the solid phase and 2 denotes the liquid phase. $T_1$
Figure 2.22 Special Case of Stefan's Problem Solved by Neumann.
and $T_2$ are the temperatures in the solid and liquid regions respectively. Finally, the material has a latent heat of fusion, $L$.

At the fusion front $x = X(t)$, one boundary condition is

$$T_1 = T_2 = T_f.$$  \hspace{1cm} (2.4)

A second boundary condition concerning the absorption or liberation of latent heat at the fusion front is

$$k_1 \frac{\partial T_1}{\partial x} - k_2 \frac{\partial T_2}{\partial x} = L \rho \frac{dx}{dt}.$$  \hspace{1cm} (2.5)

For conductive heat flow, the temperatures in the solid and liquid regions must satisfy

$$\frac{\partial^2 T_1}{\partial x^2} - \frac{1}{\alpha_1} \frac{\partial T_1}{\partial t} = 0.$$  \hspace{1cm} (2.6)

and $$\frac{\partial^2 T_2}{\partial x^2} - \frac{1}{\alpha_2} \frac{\partial T_2}{\partial t} = 0.$$  \hspace{1cm} (2.7)
Initially the region \( x > 0 \) is in the liquid phase at constant temperature, \( T^1 \), with the surface \( x = 0 \) maintained at \( T_0 \) for \( t > 0 \). \( T_0 \) will be taken to be zero for convenience. The Equations to be satisfied are 2.4, 2.5, 2.6, and 2.7 with the additional boundary conditions

\[ T_2 + T^1, \quad x \to \infty \]  \hspace{1cm} (2.8)

and

\[ T^1 = 0, \quad x = 0. \]  \hspace{1cm} (2.9)

It can be shown that

\[ T_1 = A \text{erf} \frac{x}{2(\alpha_1 t)^{1/2}} \]  \hspace{1cm} (2.10)

satisfies Equation 2.6 and 2.9 where \( A \) is a constant. Also, if \( B \) is a constant,

\[ T_2 = T^1 - B \text{erfc} \frac{x}{2(\alpha_2 t)^{1/2}} \]  \hspace{1cm} (2.11)
satisfies Equations 2.7 and 2.8. Equation 2.4 requires that

\[ A \frac{x}{2(\alpha_1 t)^{1/2}} = T_i - B \frac{x}{2(\alpha_2 t)^{1/2}} = T_f. \]  

Since Equation 2.12 must be satisfied for all time, \( X \) must be proportional to \( t^{1/2} \)

\[ X = 2 \lambda (\alpha_1 t)^{1/2} \]  

(2.13)

where \( \lambda \) is a numerical constant to be determined from the remaining condition Equation 2.5. Using Equations 2.12 and 2.13, \( \lambda \) can be determined from the expression

\[ \frac{e^{-\lambda^2}}{\text{erf} \lambda} - \frac{k_2 \alpha_1^{1/2} (T_i - T_f) e^{-\alpha_1^2/\alpha_2}}{k_1 \alpha_2^{1/2} T_f \text{erfc} \lambda (\alpha_1/\alpha_2)^{1/2}} = \frac{\lambda L}{c_1 T_f^{1/2}}. \]  

(2.14)

When \( \lambda \) has been found from Equation 2.14, \( T_1 \) and \( T_2 \) can be written down from Equations 2.10, 2.11, 2.12, and 2.13. They are
\[
T_1 = \frac{T_f \text{ erf} \frac{x}{2(\alpha_t t)^{1/2}}} \text{ erf} \lambda \quad (2.15)
\]

\[
T_2 = T_i - \frac{(T_i - T_f)}{\text{ erfc} \frac{x}{2(\alpha_t t)^{1/2}}} \text{ erfc} \lambda \quad (2.16)
\]

For the special case when the liquid is initially at the fusion temperature, i.e., \( T_i = T_f \), Equation 2.14 reduces to

\[
\lambda \text{ erf} \lambda = \frac{c_1 T_f}{L \pi^{1/2}} \quad (2.17)
\]

Solutions for this special case are shown in Figure 2.23.

**Idealized Material** - If a heterogeneous composite, such as paraffin and concrete can be idealized as a homogeneous material with properties equal to the composite average, then the Neumann solution could be used to predict the rate at which heat is transferred into the idealized material. This approach is an extension of a method outlined by Ingersoll (49) for semi-infinite solids developed by the author of this dissertation. The heat flux can be evaluated by

\[
\frac{Q}{A} = \int_0^t -k \frac{dT}{dx} \bigg|_{x=0} \; dt \quad (2.18)
\]
Figure 2.23 Neumann Solution for Various Values of Stefan Number.

\[ \text{Ste} = \frac{c(T_f - T_o)}{L} \]
For the special case of the Neumann solution, the gradient at the surface of the region is

\[
\frac{dT}{dx}igg|_{x=0} = \frac{T_f - T_0}{\text{erf} \lambda (\pi \alpha t)^{1/2}}
\]  

(2.19)

Substituting Equation 2.19 into Equation 2.18 and integrating yields

\[
\frac{Q/A}{\pi \alpha} = 2 \frac{k (T_f - T_0)}{\text{erf} \lambda (\pi \alpha t)^{1/2}}
\]

(2.20)

By idealizing the composite as a cubic array of cubic inclusions and using the parallel-series approach, the effective reduced conductivity \( k \) can be computed by the expression

\[
k = \frac{k_1 \left( k_1/k_2 + (1 - k_1/k_2) \beta^2 \right)}{k_1/k_2 + (1 - k_1/k_2) \beta^2 (1 - \beta)}
\]

(2.21)

where the subscripts 1 and 2 denote the continuous and dispersed components respectively, and \( \beta^3 \) is the volume fraction of the dispersed component.
Average density, $\rho$, specific heat, $c$, and latent heat, $L$, can be calculated with the expressions

$$
\rho = (1 - \beta^3) \rho_1 + \beta^3 \rho_2 \quad (2.22)
$$

$$
c = \frac{(1 - \beta^3) \rho_1 c_1 + \beta^3 \rho_2 c_2}{\rho} \quad (2.23)
$$

$$
L = \frac{\beta^3 \rho_2 L_2}{\rho} \quad (2.24)
$$

Since time appears in Equation 2.20 as a multiplying factor, it may be divided out by forming ratios of heat absorbed. Using concrete as a reference, the effect of increased paraffin fraction for various step changes in surface temperature are shown in Figure 2.24.

Apparently for low wax volumes, the latent heat effect increases the gradient at the wall and this effect dominates the decreasing conductivity. Beyond the maximum, increased paraffin fraction causes the lower thermal conductivity effect to dominate. Also, it is interesting to note that the optimum paraffin fractions correspond to the range of achievable mixes shown in Figure 1.1.
Figure 2.24 Heat Transfer Ratio, Composite to Mortar, Versus Paraffin Fraction
CHAPTER III

EXPERIMENTAL STUDY

Overview

The specimen was a horizontal composite wall made up of an aluminum filled epoxy matrix and a square array of circular cylindrical voids running parallel to the heat exchange surface as shown in Figure 3.1. The void cylinders were filled with paraffin through a Plexiglas header. The transparent header also allowed visual observation and photography of the interior of the specimen.

At the heat exchange surface, the epoxy was bonded to an aluminum plate which served as an isothermal surface. Step changes in surface temperature were achieved by pouring water at the desired temperature into an open reservoir and circulating the water through a heat exchanger adjacent to the aluminum plate as shown in Figure 3.2. The water was maintained at the desired temperature by circulating it over ice in the open reservoir or heating with a thermostatically controlled emersion heater.
Figure 3.1 Test Apparatus.
Figure 3.2 Apparatus and Instrumentation.
The thermal response of the specimen was measured with thermocouples cast into the matrix at the centers of the square array. The thermocouple wires were run out from the center of the specimen parallel to the voids in order to minimize conduction effects. Details of positioning these thermocouples will be discussed in detail in the following section. Accuracy will be discussed under experimental error. Two references and the reservoir outlet were also instrumented with thermocouples. The thermocouple leads were interfaced with the leads to the data logger in an isothermal junction box. The data logger automatically polled the active channels at one minute intervals and converted the millivolt input to an IBM 8 level binary punched paper tape record.

These data were converted to punched card form using an IBM 1130 computer system. Subsequent processing on the IBM 370 computer reduced the data to engineering units and finally to dimensionless groups for plotting. Notes were also taken on the position of the fusion front, and in a limited number of runs photographs were taken.

Each specimen was subjected to a basic series of tests including: heating and cooling without paraffin in the voids, heating and cooling with paraffin in the voids, and melting and freezing with paraffin in the voids. Selected tests with various specimens at constant ratios of sensible to latent heat were also run.
Specimen

The specimens were designed to simulate a semi-infinite, horizontal region with various degrees of porosity and yet allow visual observation of the fusion front inside the paraffin region. Also, a configuration was selected that could be mathematically described. In order to achieve these objectives, a square array of circular voids whose axes are parallel to the heat exchange surface was selected. A single column of the arrays that were cast along with the center to center spacing and void fraction are shown in Figure 3.3. The overall configuration is shown in Figure 3.4 including the aluminum plate heat exchanger surface, Plexiglas header, and a representative array of voids and thermocouples. The thickness of 1.4 in. (3.56 cm) was based on ten times the nominal diameter of the plastic straws used to form the cylindrical voids. The lateral dimension of 4.5 in. (11.4 cm) was selected on the bases of a compromise between possible lateral heat flow, deflection of the straws during casting, and photographic depth of field.

Forming - The specimens were cast in an aluminum and Plexiglas form as shown in Figure 3.5. Plastic straws were used to form the voids. The straw array and thermocouples were located relative to an aluminum base with a pair of Plexiglas locator plates. The form members were assembled on a rigid board to insure dimensional stability.

The 4.5 in. x 4.5 in. x 0.05 in. (11.4 cm x 3.6 cm x 0.13
Figure 3.3 Single Column of Void Pattern with Spacing and Voidage.
Figure 3.4 Configuration of a Representative Specimen.
Figure 3.5 Two Row Specimen Form.
cm) aluminum plate was ground on the casting side to insure a good bond with the epoxy. The removable form sides were 0.25 in. (0.64 cm) Plexiglas which was treated with a fluorocarbon release agent.

Originally it was hoped that the epoxy would bond to the Plexiglas locator plates as shown in Figure 3.5. However, tests with the two row specimen revealed that the bond was inadequate. In subsequent specimens, epoxy flanges were cast using 0.5 in. x 0.5 in. (1.3 cm x 1.3 cm) aluminum angles treated with a release agent as shown in Figure 3.6. The locator plates and angles were spaced with double nutted bolts. Lateral surfaces of the flanges were taped.

The 2.5 in. x 5.5 in. x 0.25 in. (6.4 cm x 14.0 cm x 0.64 cm) predrilled Plexiglas locator plates were machined with a tape controlled drill press which had a locational accuracy of 0.005 in. (0.013 cm). A grading of the straws, shown in Figure 3.7, revealed that the average diameter was 0.148 in. (0.376 cm) instead of 0.140 in. (0.356 cm) for straws used in a prototype specimen. Instead of recoding the tapes, the drill size was increased to a number 23 to accomodate the larger straws. This resulted in the odd voidages and spacings shown in Figure 3.3. The thermocouple lead locator holes were manually drilled using a 0.022 in. (0.056 cm) diameter drill.

In order to fix the location of the thermocouples, the leads were strung through the locator holes, around a
Figure 3.6 Epoxy Bolt Flange Form.
Figure 3.7 Plastic Straw Size Distribution.
tensioning spring, or ring, at the dead end and back through the plate into the flange region where they were grouped and led to the side of the specimen. The free end of the spring was tied to a nail in the rigid board. By bending the nail, shown in Figure 3.5, each wire could be individually tensioned.

In order to insure a uniform distribution of aluminum powder in the resin, it was thoroughly mixed. Portions of resin weighing 0.11 to 0.44 lb (0.05 to 0.20 kg) were premeasured into paper cups. The resin was then heated to 120°F (48.9°C) in a water bath to lower its viscosity. As required, epoxy hardening agent was metered into each batch and the components were thoroughly mixed. A complete description of the epoxy system is included in Chapter II.

Starting at a lower edge, one straw was positioned and epoxy was poured between the straw and the wall of the form. The straw was then rotated to draw the epoxy under the straw while reducing the deflection of the straw. Another straw was then positioned in that row and the pouring and rotating was repeated. This sequence was repeated back and forth across the form until the main body of the casting was completed. The top flange angles were then positioned and the upper flange was poured. The casting was allowed to cure in the mold for twelve hours at 70°F (21°C). The casting was then stripped from the form.

The specimen was completed by bolting clear 0.25 in. 
Plexiglas cover plates on the open ends. A red rubber gasket provided the seal. The space between the cover and locator plates served as a manifold between voids. Each cover plate was fitted with a port to fill the specimen with paraffin. The completed six row specimen is shown in Figure 3.8. The end view of all of the specimens is shown in Figure 3.9.

Thermal Control Loop

At time zero, water at the desired temperature was poured into an open cylindrical reservoir 6 in. (15.3 cm) in diameter and 7 in. (17.8 cm) high. A 0.25 in. (0.64 cm) diameter copper tube outlet was soldered to the reservoir near its base. The water was drawn from the reservoir through a heat exchanger by a rotary pump and discharged back into the reservoir as shown in Figure 3.2. The water was maintained at a nearly constant temperature by an emersion heater or an ice cube bed. The emersion heater, a Scientific Apparatus Corp. Bath-A-Trol, is shown in Figure 3.10.

Water from the reservoir was circulated through an "S" shaped 0.75 in. x 1.25 in. (1.9 cm x 3.2 cm) passage formed by clamping an open baffled box against the specimen as shown in Figure 3.11. The cast epoxy box was fitted with 0.75 in. x 0.75 in. x 2.5 in. (1.9 cm x 1.9 cm x 6.4 cm)
Figure 3.8 Six Row Specimen.
Figure 3.9  End View of Specimens.
Figure 3.10 Bath-A-Trol.
Figure 3.11  Heat Exchanger.
aluminum baffles. The inlets were 1.5 in. (3.8 cm) sections of 0.5 in. (1.27 cm) diameter copper tube. A neoprene rubber gasket sealed the rim of the box.

The water was circulated with an Eastern Rotary Pump, model VT-9 modified to operate on 120 volt AC. A flow rate of 11.2 ft³/hr (0.318 m³/hr) was measured. The housing was electrically grounded. In order to isolate the vibration of the pump; the pump, reservoir, and specimen were mounted on 2 in. (5.1 cm) thick foam pads. In addition, all couplings in the thermal loop were fabricated with flexible tubing.

Temperature Measurements

The transient thermal response of the specimen was measured with thermocouples cast into the epoxy matrix. In addition, the reservoir and two reference temperatures were automatically recorded. The cold temperature reference was a thermos of ice-water. The hot reference was a thermos of water near room temperature i.e. 80°F (27.7°C) to minimize drift during the test. Periodically, the temperature of the hot reference and reservoir were measured with a mercury thermometer and recorded manually.

Thermocouples - All thermocouples were made of 30 gauge copper constantan wire. The beads were made by stripping the fabric and lacquer insulation, mechanically twisting the wires, and soldering them with a resin core solder as shown
in Figure 3.12. For the thermocouples cast into the specimen, the fabric was stripped back to the flange region. The thermocouples were located in the specimen as described above. The typical staggered thermocouple pattern is shown in Figures 3.1 and 3.4. Two thermocouples were positioned at the surface of the aluminum plate. One on the centerline and the other near the edge of the specimen. The next thermocouple was located between the first and second row, two voids over toward the centerline. This pattern was continued across the specimen with the thermocouple at mid-thickness also at the centerline. Two thermocouples were cast into the top surface. One at the outer edge and the second at the centerline. In addition, an external thermocouple was placed between the top of the specimen and the foam insulation 1 in. back of the front flange.

The data logger processed thermocouple emf was investigated over the range of temperatures 32°F to 140°F. Thermocouples were emersed in water which was kept uniformly at the desired temperature ±0.5°F (± 0.3°C) with a thermostatically controlled heater/stirrer shown in Figure 3.10. The millivolt output was recorded through the data logger system. As shown in Figure 3.13, a linear increase in indicated emf with temperature was observed. Temperatures in the specimen were then calculated by linearly interpolating between reference emf's. This procedure minimized the effects of data logger drift.
Figure 3.12 Copper/Constantan Thermocouples.
Figure 3.13 Thermocouple Calibration.
A 5 in. x 6 in. x 15 in. (13 cm x 15 cm x 38 cm) aluminum box, shown in Figure 3.14, with 0.25 in. (0.64 cm) walls and insulated with 0.75 in. (1.9 cm) of foam served as an isothermal environment to interface the specimen thermocouples with the data logger input leads. The terminal blocks were fastened to a central 1 in. x 2.25 in. (2.5 cm x 5.7 cm) aluminum bar. Silicone grease was applied to the bottom of the terminal blocks to reduce thermal resistance.

Data Logger – An S. Sterling Company compact data logger, shown in Figure 3.2, was used to record the thermocouple millivolt output. The logger performed the functions of acquisition, preamplification, visual digital display, and storage of the millivolt data as punched paper tape. The logger had a capacity of 30 channels with any multiple of 2 up to 30 being selectable for reading and punching. Scan modes of manual, continuous, and internally timed intervals from 1 to 69 minutes were attainable. For most data runs, the scan mode was set on single and an interval of one minute was selected. The amplifier range was set at 10 volts and the display time was set at infinity.

Phase Front Location

During freezing and melting runs, the location of the phase front was tracked visually and noted in the run log.
Figure 3.14 Junction Box.
In a limited number of runs, photographs were taken of the interior of the specimen. Typical freezing and melting fronts are shown in Figure 3.15.

**Front Characteristics** - During freezing runs, three stages of front development, shown in Figure 3.16, could be identified: development of opaque solid in the bottom of the void, completion of a ring of solid around the circumference of the void, and finally complete solidification in a given void. Although the freezing front was nearly uniform, there was some variation in the progress of the front from void to void. Therefore, the time at which a certain stage was first observed in a given row as well as the time when all voids in that row had reached the same state were noted. Times were read from a large clock with a second hand, shown in Figure 3.2, which was started concurrently with the data logger.

During melting runs, the progress of the phase front was less well defined and the variations from void to void in a given row were more pronounced. However, three stages were again noted: first appearance of transparent liquid; settling of the solid phase core to the bottom of the void; and complete liquification. Once again, extremes within a given row were noted.

**Photographs** - In order to photograph the development of the phase front inside the 0.148 in. (0.376 cm) diameter, 5 in. (12.7 cm) long voids, a 135 mm bellows extension lens system
Figure 3.15 Typical Freezing and Melting Fronts.
First Liquid  Complete Liquid

Solid Settles  Heating from Below

Complete Liquid

Cooling from Below

Complete Ring of Solid

Complete Solid

Figure 3.16 Freezing and Melting Phase Front Stages.
was used. The film format was 35 mm. Photographs were taken at one minute intervals concurrent with the start of the data logger poll.

Consideration of depth of field and image size established the system optics (50). Experimentation showed that a setting which included the front face of the specimen through the centerline in its depth of field gave the best results. Also by selecting a magnification of 0.5, almost all of the voids could be viewed. However, only the voids directly in line with the lens axis were recorded without distortion.

Using these guidelines, the following procedure was used. The bellows was adjusted to give a magnification of 1/2. With the lens set at f4, the entire camera lens system was moved until the system was focused on an object at 1/3 the depth of the specimen. The lens was then stopped down to f22 yielding a depth of field of about 3.5 in. (8.89 cm) which included both the face of the specimen and its centerline.

For a common 60 W light bulb mounted in a hemispherical reflector and heat shielded with a translucent yellow plastic plate, Kodak Tri-X ASA 400 black and white film exposed for 1 second was found to give excellent exposures.

As the phase front moved across the specimen, the camera and lens system was aimed at the thermocouple lead nearest the front and aligned so that the near and far end
of the nearby voids were concentric.

Testing

The specimen was insulated and mated with the heat exchanger box. This assembly was clamped in a threaded rod and plywood press. Glass funnel paraffin reservoirs were mounted and connected to the header ports. In order to keep the funnel neck passage, shown in Figure 3.1, free of solid paraffin, a nichrome heater wire was passed through the funnel, across the top outside void, and out the other funnel.

The water circulating loop was connected and tested for leaks. The numbered thermocouples were connected to the junction box and tested. When the specimen was charged with paraffin, the liquid paraffin was poured into the preheated specimen through the glass funnel. Although most of the air was displaced by the paraffin, bubbles were trapped in some voids. In order to dislodge these bubbles, the specimen was rocked.

Test Schedule - Each specimen was subjected to a basic series of tests shown in Figure 3.17. In all tests, the thermal exposure was a step change in surface temperature with the specimen initially at a uniform temperature near the fusion temperature of the paraffin. Both heating and cooling pulses were investigated. First the thermal
Figure 3.17 Basic Run Matrix.
response of the specimen without paraffin in the voids was determined. These runs are referred to as void heating and cooling. The specimen was then charged with paraffin and the response of the composite was determined without phase change. These runs are referred to as liquid and solid. Finally, the specimen was exposed to step changes in surface temperature which crossed the fusion temperature of the paraffin. Both freezing (cooling) and melting (heating) runs were made.

The basic series temperature step was nearly 49°F (27.2°C). Specific specimens were exposed to larger or smaller steps so that comparisons between specimens at constant Stefan number could be made. Each thermal exposure was run twice.

The run number indicates the specimen investigated and the chronological test number on that specimen. For example, 0812 designated the twelfth test on the eight row specimen. The 4, 6, and 8 row specimens were designated 04XX, 06XX, and 08XX respectively. The solid matrix specimen was designated by 05XX.

Test Procedure - The test procedure is itemized in Table 3.1. Water at the desired initial specimen temperature was circulated through the heat exchanger. The internal specimen temperature was monitored by manually polling the data logger and reading the output on the digital voltmeter. When conditions had stabilized, the water was pumped out of
Table 3.1 Test Procedure

<table>
<thead>
<tr>
<th>Item</th>
<th>Time Period</th>
<th>Procedure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Pretest</td>
<td>Stabilize Room Temperature Near 81°F (27.2°C)</td>
</tr>
<tr>
<td>2</td>
<td>Pretest</td>
<td>Stabilize Specimen Temperature at Initial Condition</td>
</tr>
<tr>
<td>3</td>
<td>Pretest</td>
<td>Melt Paraffin In Reservoir and Manifold</td>
</tr>
<tr>
<td>4</td>
<td>Pretest</td>
<td>Check Reference Temperature Baths</td>
</tr>
<tr>
<td>5</td>
<td>Pretest</td>
<td>Remove Air Bubbles from Voids and Manifold</td>
</tr>
<tr>
<td>6</td>
<td>Pretest</td>
<td>Check Data Logger and Put Leader on Tape</td>
</tr>
<tr>
<td>7</td>
<td>Pretest</td>
<td>Preheat or Precool Water</td>
</tr>
<tr>
<td>8</td>
<td>Pretest</td>
<td>Start Data Logger and Clock (Establish Initial Condition, 2 Scans)</td>
</tr>
<tr>
<td>9</td>
<td>t = 0</td>
<td>Pour Pretemperted Water into Reservoir, Start of Scan 3</td>
</tr>
<tr>
<td>10</td>
<td>0 to 10 Sec.</td>
<td>Position Bath-A-Trol</td>
</tr>
<tr>
<td>11</td>
<td>Test</td>
<td>Read Reference and Reservoir Thermometers</td>
</tr>
<tr>
<td>12</td>
<td>Test</td>
<td>Make Visual Observations or Photographs</td>
</tr>
<tr>
<td>13</td>
<td>Test</td>
<td>Monitor Apparatus</td>
</tr>
<tr>
<td>14</td>
<td>Post Test</td>
<td>Read Thermometers</td>
</tr>
<tr>
<td>15</td>
<td>Post Test</td>
<td>Remove Tape and Identify</td>
</tr>
</tbody>
</table>
the system. While this process was under way, the paraffin in the reservoirs and manifolds was melted to allow thermal expansion and visual observation within the voids.

The circulating water was preheated in an auxiliary reservoir using the thermostatically controlled heater. Precooling was accomplished by filling the auxiliary reservoir with ice cubes and then adding water.

Prior to runs in which the paraffin was initially in a liquid state, air bubbles were removed from the voids by rocking the specimen.

When conditions were stabilized, the data logger and clock were started. Two scans were recorded to establish the initial condition. Between the second and third scan, the pump was started. At the beginning of the third scan, the preconditioned water was poured into the reservoir. During heating runs, the thermostatically controlled heater was then positioned and started. The reference and reservoir thermometers were read and recorded. During phase change tests, visual observations or photographs were made. The test was terminated when the temperature pulse reached the backside of the specimen. Thermometers were reread and the tape was identified.
Data Reduction

The temperature data was reduced in three steps. First the millivolt data recorded on punched paper tape was transferred to cards using an IBM 1130 computer system. Next, a Fortran Program named CONVERT was used to convert the raw millivolt data to temperatures by linearly interpolating the millivolt data at the desired channel with those of the known reference baths. Time in hours was calculated from the scan number and the known interval between scans. Details are included in Appendix A.

Using average composite properties and reference temperatures, dimensionless temperatures, \((T-T_o)/(T_i-T_o)\), and penetration parameter, \(x/(\alpha t)^{1/2}\) were calculated where \(t\) is time. If phase change occurred during the run, the temperature, \(T_i\), was taken to be the fusion temperature, \(T_f\). However, if no change of phase occurred, the initial temperature was taken to be the average measured specimen temperature from the scan prior to time zero. The surface temperature, \(T_o\), was taken to be the average measured temperature at the interface between the aluminum plate and the epoxy for that scan.

The position, \(x\), in the penetration parameter was computed based on the specimen thickness, i.e. 1.4 in. (3.56 cm) and the number of rows.

Normalized data plots of dimensionless temperature versus penetration parameter are included in Chapter V.
Also shown are the idealized response based on computed average composite properties. The Fortran program used to compute these values as well as the idealized response is included in Appendix A.

The visual observations were normalized using the penetration parameter \( x/(at)^{1/2} \). Since the fusion front was irregular, the depth was assumed at the locations shown in Figure 3.16 for the various stages of freezing and melting. A complete series of melting and freezing photographs are included in Chapter V. For freezing runs, solidification never began in subsequent rows until it was complete in the current row. Therefore, the assumed phase front depths corresponding to first solid and complete solid events were accurate descriptions of the phase front location. The depth associated with complete ring of solid results in an assumed depth greater than the average phase front depth. However, due to the melting settling process, melting began in adjacent rows while settled solid was still melting. Therefore, individual assumed phase front depths are less accurate in describing the average phase front depth. For example, the first liquid and solid settles events occurred in rapid succession. However, the assumed phase front locations were one row apart. The depth associated with complete liquid is less than the average depth since melting has begun in the next row. In general, these assumed phase front locations would tend to bias the observed phase front
locations toward greater depth for freezing and toward lesser depth for melting. For this reason, the visual observations should be considered qualitative.

Experimental Error

The thermal response of the specimen was measured with thermocouples cast into the epoxy at the centers of the square array of circular voids. The thermocouple emf was processed through the data logger system which converted the emf signal to a digital count. These data were subsequently reduced to temperatures in °F by linearly interpolating these counts with those of two known references. The precision with which each of these operations was made affect the accuracy of the measured profile.

Precision Index - For phenomenon which can be described by functions that are continuous and have derivatives everywhere, precision indices can be derived by expanding the function in a Taylor series (51). Say,

\[ R_c + r_1 = f(x_c + x_1, y_c + y_1) \]  

(3.1)

where upper case letters denote parameters or results and lower case letters denote deviation from the correct value.
Then the uncertainty interval $w$ is

$$w = \left( \left( \frac{\partial R}{\partial x_c} w_x \right)^2 + \left( \frac{\partial R}{\partial y_c} w_y \right)^2 \right)^{1/2}. \tag{3.2}$$

Uncertainty interval analysis will be used to evaluate experimental and normalization errors. Measurement, geometry, and thermophysical property uncertainties used in this analysis are summarized in Table 3.2.

**Thermocouple Position** - During the forming process, the thermocouples were located relative to the heat exchange surface by stringing the leads through predrilled holes in the Plexiglas locator plates as shown in Figures 3.5 and 3.6. To minimize deflection due to epoxy placement, the epoxy was poured one row at a time. The tensioned thermocouple leads were plucked to insure that they were not displaced. In addition, the straws were rotated about their axes to eliminate deflection. In the case of the 8 row specimen, the adjacent straws nearly surround the thermocouple leads thus insuring their position within the diamond shaped region between straws.

In order to determine the accuracy of thermocouple placement, a trial six row specimen was cut transverse to the axis of the leads at midspan. Thermocouple locations were measured with a machinist rule divided into 1/64 in.
Table 3.2 Measurement, Geometry, and Property Uncertainties, Deviation from Published Mean Value.

**Measurement**

<table>
<thead>
<tr>
<th>Data Logger Counts</th>
<th>± 1 Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference Temperatures</td>
<td>± 0.2°F</td>
</tr>
<tr>
<td>Time</td>
<td>± 5 Sec</td>
</tr>
</tbody>
</table>

**Geometry**

<table>
<thead>
<tr>
<th>Thermocouple Position</th>
<th>± 0.03 in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Void Diameter</td>
<td>± 0.002 in.</td>
</tr>
<tr>
<td>Void Spacing</td>
<td>± 0.005 in.</td>
</tr>
</tbody>
</table>

**Thermophysical Properties**

<table>
<thead>
<tr>
<th>Matrix:</th>
<th>Conductivity</th>
<th>± 0.06 BTU/hr-ft°F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>± 2.0 lbm/ft³</td>
<td></td>
</tr>
<tr>
<td>Specific Heat</td>
<td>± 0.005 Btu/lbm°F</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PCM:</th>
<th>Conductivity</th>
<th>± 0.06 BTU/hr-ft°F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>± 1.0 lbm/ft³</td>
<td></td>
</tr>
<tr>
<td>Specific Heat</td>
<td>± 0.05 BTU/lbm°F</td>
<td></td>
</tr>
<tr>
<td>Latent Heat</td>
<td>± 1.0 BTU/lbm</td>
<td></td>
</tr>
<tr>
<td>Fusion Temp.</td>
<td>± 0.1°F</td>
<td></td>
</tr>
</tbody>
</table>

Conductivity: W/m²C multiply by 1.74
Density: kg/m³ multiply by 16.0
Sp. Heat: kj/kg°C multiply by 4.19
Latent Heat: kj/kg multiply by 2.33
(0.04 cm). Listed in Table 3.3 are these measurements along with the assumed locations based on specimen thickness and number of rows. Although all measured locations were less than the calculated value, fractional locational accuracy improved away from the specimen surface. Based on this sample, average thermocouple position accuracy was +0.0 in., -0.03 in. (-0.076 cm).

Temperature Interpolation - The interpolation formula for temperature is

\[ T = \left( \frac{C - C_c}{C_h - C_c} \right) (T_h - T_c) + T_c \]  (3.3)

where \( C \) denotes counts and \( T \) temperature. The subscripts \( c \) and \( h \) indicate hot and cold references. The data logger resolution was 1 count. The reference bath thermometer minimum division was 0.2°F. Uncertainty intervals are listed in Table 3.4 at various temperatures. It was assumed that all count and temperature uncertainties were identical. The temperature range from 81 to 32°F corresponds to cooling runs. The minimum in uncertainty is a result of interpolation between reference temperatures. On the other hand increasing uncertainty above 81°F is a result of extrapolation beyond the hot reference temperature. This range corresponds to heating runs.
Table 3.3 Measured and Calculated Thermocouple location, Trial Six Row Specimen.

<table>
<thead>
<tr>
<th>Thermocouple Between</th>
<th>Measured, in.</th>
<th>Calculated, in.*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface and Row 1</td>
<td>0.02, 0.02</td>
<td>0.0</td>
</tr>
<tr>
<td>Row 1 and Row 2</td>
<td>0.20, 0.19</td>
<td>0.23</td>
</tr>
<tr>
<td>Row 2 and Row 3</td>
<td>0.44</td>
<td>0.47</td>
</tr>
<tr>
<td>Row 3 and Row 4</td>
<td>0.66</td>
<td>0.70</td>
</tr>
<tr>
<td>Row 4 and Row 5</td>
<td>0.91</td>
<td>0.93</td>
</tr>
<tr>
<td>Row 5 and Row 6</td>
<td>1.14</td>
<td>1.17</td>
</tr>
<tr>
<td>Row 6 and Top</td>
<td>1.32, 1.33</td>
<td>1.40</td>
</tr>
</tbody>
</table>

* Location = (1.4 in./6)*Row

Table 3.4 Temperature Interpolation Uncertainty Interval

<table>
<thead>
<tr>
<th>Temperature, °F</th>
<th>0.2°F/1 Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>0.37°F</td>
</tr>
<tr>
<td>57</td>
<td>0.30</td>
</tr>
<tr>
<td>81</td>
<td>0.37</td>
</tr>
<tr>
<td>106</td>
<td>0.52</td>
</tr>
<tr>
<td>130</td>
<td>0.70</td>
</tr>
</tbody>
</table>
**Time** - The 1 minute scan interval of the data logger was compared with an electric clock. No scan deviated by more than 1 second. After an hour, the logger was also accurate to within a second.

The experiment was initiated by pouring water into the reservoir at the beginning of the third scan. The maximum time delay between the beginning of the scan and water reaching the specimen surface was estimated to be no more than 5 seconds.

**Normalization Error**

In Chapter V, the experimental data are presented in dimensionless form. The three dimensionless variables plotted are those which characterize the Neumann solution. They are the temperature ratio

\[
\frac{T - T_0}{T_f - T_0} \quad (3.4)
\]

the penetration parameter

\[
\frac{x}{(a t)^{1/2}} \quad (3.5)
\]

and the Stefan number

\[
\frac{c (T_f - T_0)}{L} \quad (3.6)
\]
The normalization of the experimental data introduces additional uncertainties in material properties determination, calculation of average composite properties, and simulation of a step change in surface temperature. First, uncertainties in determining the temperature ratio will be investigated.

**Temperature Ratio** - The total derivative of the temperature ratio is

\[
D\left[\frac{T - T_o}{T_f - T_o}\right] = \frac{T - T_o}{T_f - T_o} \left( \frac{1}{T - T_o} \frac{dT}{dt} - \frac{1}{T_f - T_o} \frac{dT_f}{dt} + \frac{T - T_f}{(T_f - T_o)(T - T_o)} \frac{dT_o}{dt} \right) \tag{3.7}
\]

Measured temperatures, T and T_o, had an uncertainty of 0.7°F (0.4°C). T was known to within 0.1°F (0.06°C). For semi-infinite solid runs, T_i replaces T_f in the temperature ratio. Since T was measured, it also had an uncertainty of 0.7°F (0.4°C). Assuming an ideal step change in surface temperature, uncertainty intervals are listed in Table 3.5 for both the semi-infinite solid and phase change runs.

**Penetration Parameter** - The total derivative of the penetration parameter is

\[
D\left[\frac{x}{(\alpha t)^{1/2}}\right] = \frac{x}{(\alpha t)^{1/2}} \left( \frac{1}{x} \frac{dx}{dt} - \frac{1}{2 \alpha} \frac{d\alpha}{dt} - \frac{1}{2} \frac{dt}{t} \right). \tag{3.8}
\]
Table 3.5 Temperature Ratio Uncertainty Interval

<table>
<thead>
<tr>
<th>Temperature, °F</th>
<th>Temperature Ratio</th>
<th>Semi-infinite Solid</th>
<th>Phase Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>0.0</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>57</td>
<td>0.51</td>
<td>0.017</td>
<td>0.016</td>
</tr>
<tr>
<td>81</td>
<td>1.0</td>
<td>0.020</td>
<td>0.014</td>
</tr>
<tr>
<td>106</td>
<td>0.49</td>
<td>0.017</td>
<td>0.016</td>
</tr>
<tr>
<td>130</td>
<td>0.0</td>
<td>0.020</td>
<td>0.020</td>
</tr>
</tbody>
</table>

Temperature Ratio = \( \frac{(T - T_o)}{(T_i - T_o)} \)
for Semi-infinite Solid

= \( \frac{(T - T_o)}{(T_f - T_o)} \)
for Phase Change
Thermocouple position uncertainty was estimated to be 0.03 in. (0.08 cm). Time was at most in error by 5 sec. Examination of Equation 3.8, shows that for small $x$ and $t$ error in penetration parameter can be large. Except for the solid matrix specimen, thermal diffusivity must be estimated by calculating average thermal properties with equations like Equations 2.21, 2.22, 2.23, and 2.24. Uncertainties in matrix properties will be examined first.

In Chapter II, epoxy thermophysical properties were discussed. Density was calculated from reported specific gravity listed in Table 2.5. As shown in Figure 2.10, specific heat was measured over the temperature range of interest with a reported accuracy of 2%. The thermal conductivity reported by the manufacturer was determined by a transient test method over a range of temperatures. However, steady state studies on a similar material indicated that the conductivity of aluminum filled epoxies are weakly dependent on temperature but are strongly dependent on aluminum content, especially near maximum packing as shown in Figure 2.12. A variability in epoxy thermal diffusivity of 11 percent was calculated using the thermophysical property deviations listed in Table 3.2. Penetration parameter uncertainties intervals are listed in Table 3.6 for various depth thickness ratios, $x/l$, of 0.25, 0.5, and 0.75 and at times from 1 to 5 minutes.

The 8 row specimen was used to estimate errors in
Table 3.6 Penetration Parameter Uncertainty Interval for the Matrix Material at Various Depths, $x$, and Times, $t$.

<table>
<thead>
<tr>
<th>Depth, in.</th>
<th>Time, Min.</th>
<th>$x/(at)^{1/2}$</th>
<th>Uncertainty Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35</td>
<td>1</td>
<td>1.68</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.19</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.97</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.84</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.75</td>
<td>0.08</td>
</tr>
<tr>
<td>0.70</td>
<td>1</td>
<td>3.37</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.38</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.94</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1.68</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>1.51</td>
<td>0.11</td>
</tr>
<tr>
<td>1.05</td>
<td>1</td>
<td>5.05</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3.57</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2.92</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>2.53</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>2.26</td>
<td>0.14</td>
</tr>
</tbody>
</table>
penetration parameter properties. For a square array of circular voids, average composite thermal conductivity can be estimated by the Rayleigh expression (36)

\[
\alpha = \left( \frac{k_2 - k_1}{k_2 + k_1} \right) \rho \\
\frac{k}{k_1} = \frac{\alpha + 1}{\alpha - 1} 
\]

(3.9)

which is listed in Table 2.6. The size distribution of straws used to form the specimen is shown in Figure 3.7. The maximum deviation in diameter, d, was 0.002 in. Center to center spacing, S, was accurate to 0.005 in. Based on these uncertainties, the uncertainty in porosity was 0.036.

Although the thermal conductivity of paraffin in the liquid phase is well known, the solid phase conductivity has not been firmly established. Therefore, in order to make a conservative estimate of error, the solid phase was considered, \( k_2 = 0.15 \pm 0.04 \text{ Btu/hr-ft}^\circ\text{F} \). Uncertainties in epoxy conductivity, \( k_1 \), were estimated to be \( 0.5 \pm 0.06 \text{ Btu/hr-ft}^\circ\text{F} \). The uncertainty interval for \( k_1 / k_2 \) was therefore 0.94 about a mean value of 3.2.

For these uncertainties, uncertainty in average composite thermal conductivity, \( k \), was 0.05 Btu/hr-ft\(^\circ\)F about a mean value of 0.26 Btu/hr-ft\(^\circ\)F.

Equation 2.22 was used to calculate average composite density, \( \rho \). Uncertainties in epoxy density, \( \rho_1 \), and
paraffin density, $\rho_2$, were estimated to be $2.0 \text{ lbm/ft}^3$ and $1.0 \text{ lbm/ft}^3$ respectively. The resultant average density uncertainty interval was $2.47 \text{ lbm/ft}^3$ about a mean value of $78 \text{ lbm/ft}^3$.

Specific heat, $c$, was calculated using Equation 2.23. Estimates of error in epoxy specific heat, $c_1$, and paraffin specific heat, $c_2$, were $0.005 \text{ Btu/lbm}^2\text{F}$ and $0.05 \text{ Btu/lbm}^2\text{F}$ respectively. The uncertainty interval for $c$ was $0.02 \text{ Btu/lbm}^2\text{F}$ about a mean value of $0.33 \text{ Btu/lbm}^2\text{F}$.

These estimates of uncertainties in $k$, $\rho$, and $c$ yield an uncertainty interval for $\alpha$ of $0.002 \text{ ft}^2/\text{hr}$ about a mean value of $0.01 \text{ ft}^2/\text{hr}$. This value and the previously estimated errors in $t$ and $x$ were used to determine the uncertainty intervals for the penetration parameter. For the 8 row specimen with solid phase paraffin, uncertainty intervals are listed in Table 3.7. As in Table 3.6, the uncertainties are tabulated for various depths, but time ranges up to an hour.

**Stefan Number** - The total derivative of the Stefan number is

$$D \left( \frac{c(T_f-T_o)}{L} \right) = \frac{c(T_f-T_o)}{L} \left( \frac{1}{c} \frac{dc}{dT} - \frac{1}{L} \frac{dT_f}{dT_o} \frac{dT_o}{dT} + \frac{1}{T_f-T_o} \frac{dT_f}{dT} \right)$$

(3.10)

Uncertainties in $T_f$, $T_o$, and $c$ were estimated to be $0.1^\circ\text{F}$, $0.7^\circ\text{F}$, and $0.02 \text{ Btu/lbm}^2\text{F}$ respectively. Average composite
Table 3.7 Penetration Parameter Uncertainty Interval for the 8 Row Specimen with Solid Phase Paraffin at Various Depths and Times.

<table>
<thead>
<tr>
<th>Depth, in.</th>
<th>Time Min.</th>
<th>$x/(at)^{1/2}$</th>
<th>Uncertainty Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35</td>
<td>1</td>
<td>2.26</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>1.01</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.71</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>0.41</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>0.29</td>
<td>0.04</td>
</tr>
<tr>
<td>0.70</td>
<td>1</td>
<td>4.52</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>2.02</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>1.42</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>0.83</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>0.58</td>
<td>0.06</td>
</tr>
<tr>
<td>1.05</td>
<td>1</td>
<td>6.78</td>
<td>0.76</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>3.30</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>2.14</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>1.24</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>0.88</td>
<td>0.09</td>
</tr>
</tbody>
</table>
latent heat, \( L \), was calculated using Equation 2.24. The paraffin latent heat, \( L_2 \), was estimated to be \( 106 \pm 1 \) Btu/lbm. The resulting uncertainty in \( L \) was \( 3.6 \) Btu/lbm about a mean value of \( 40.8 \) Btu/lbm. For a \( T_f - T_o \) of \( 49^\circ F \), the uncertainty interval for Stefan number was \( 0.05 \) Btu/lbm.

Variable Surface Temperature - Due to a finite heat transfer coefficient at the surface of the specimen and the thermal mass of the apparatus, an ideal step change in surface temperature was not achieved. Normalized surface temperatures near \( t=0 \) are shown in Figure 3.18.

For phase change runs, the duration of the test greatly exceeded this transient phase. However for semi-infinite solid runs, the duration of the test was the same order of magnitude as the transient. In order to evaluate the effect of variable surface temperature, a specimen with known thermal properties was tested. The specimen was polystyrene laminates instrumented between layers with thermocouples. The specimen was clamped in the apparatus as were the epoxy/paraffin specimens. The thermal diffusivity of this polystyrene, Dow Styrofoam, was \( 0.03 \text{ ft}^2/\text{hr} (0.003 \text{ m}^2/\text{hr}) \) (52) which was nearly identical to that of the epoxy matrix material. The experimental response was compared in Figure 3.19 with the analytical solution for a semi-infinite solid. The correlation between measured response and the analytical solution is very good.
Figure 3.18 Normalized Surface Temperatures Response.
Figure 3.19 Comparison of Measured Response of Polystyrene Specimen with the Analytical Solution for the Semi-infinite Solid.
CHAPTER IV

NUMERICAL ANALYSIS

Overview

In Chapter II, a procedure was developed to evaluate the thermal response of composites based on the Neumann solution to the Stefan problem using average composite properties. It was shown that at achievable void fractions, a composite of paraffin and concrete stores more energy than either of its constituents alone as shown in Figure 2.24. An experimental investigation of a composite with similar thermophysical properties, Chapter III, indicated that the phase front was two dimensional. Therefore, in order to evaluate the multidimensional effects for the experimental composite, a finite difference numerical model of the composite, listed in Appendix B, was developed.
Physical Model

For solids neglecting thermal energy generation within the control volume, the law of conservation of thermal energy states (53)

\[
\begin{bmatrix}
\text{rate of accumulation} \\
\text{of internal energy}
\end{bmatrix} = \begin{bmatrix}
\text{net rate of heat addition by} \\
\text{conduction across the boundary}
\end{bmatrix}.
\]

This statement can be expressed formally as

\[
\rho c \frac{\partial T}{\partial t} = - \left( \nabla \cdot \overline{q} \right) \tag{4.1}
\]

where \( \rho \) is density, \( c \) is specific heat, \( T \) is temperature, \( t \) is time, and \( \overline{q} \) is the heat flux vector. After insertion of Fourier's law of heat conduction, Equation 4.1 becomes

\[
\rho c \frac{\partial T}{\partial t} = \nabla \cdot \left( k \nabla T\right). \tag{4.2}
\]

If the thermal conductivity is independent of temperature and position, Equation 4.2 becomes
\[
\frac{\partial T}{\partial t} = \alpha \nabla^2 T
\]

(4.3)

in which \(\alpha = k/\rho c\) is the thermal diffusivity of the solid.

In Chapter II, it was assumed that the two component system could be considered to be a homogeneous material with average bulk properties and that the thermal response of this homogeneous material could be described by the Neumann solution (14). In this chapter, each component will be considered to be distinct with well defined boundaries. When each component is considered to be distinct, the development is considerably more complicated. The problem is two dimensional in a series of two material regions. The unit cell is shown in Figure 4.1 for the freezing case. At each matrix/PCM interface, temperature varies with position and time. Within the PCM region, a two dimensional moving interface exists between the solid and liquid phases.

Within the PCM region, the following assumptions were made to simplify the analysis: 1) the PCM has a definite fusion temperature, 2) all the thermal properties of the PCM are uniform and constant but can have different values in the solid and liquid phases with the exception of density. The symbols \(\rho_1, c_1, k_1,\) and \(\alpha_1\) are the density, specific heat, thermal conductivity, and thermal diffusivity for the material in the solid phase. The symbols: \(\rho_2, c_2, k_2,\) and \(\alpha_2\) represent the corresponding quantities for the liquid phase.
Figure 4.1 Two Dimensional Freezing.
$T_1$ and $T_2$ are the temperatures in the respective regions. The PCM has a well-defined fusion temperature, $T_F$, and a latent heat of fusion, $L$. The thermal properties of the matrix are uniform and constant. The symbols: $\rho_m$, $c_m$, $k_m$, and $\alpha_m$ are the density, specific heat, thermal conductivity, and thermal diffusivity for the matrix.

At the fusion front $y = P(x,t)$, one boundary condition is

$$T_1 = T_2 = T_f. \quad (4.4)$$

A second boundary condition concerning the absorption or liberation of latent heat at the fusion front for a two-dimensional boundary was developed by Rathjen and Jiji (54) and is

$$\left( k_1 \frac{\partial T_1}{\partial y} - k_2 \frac{\partial T_2}{\partial y} \right) \left( 1 + \left( \frac{\partial P}{\partial x} \right)^2 \right) = \rho L \frac{\partial P}{\partial t}. \quad (4.5)$$

Equation 4.5, which is derived in References 7 and 54, asserts conservation of energy at the interface and relates the velocity of the free boundary to the rate at which energy diffuses toward the boundary. For the simplified Neumann solution discussed in Chapter II, $P(x,t)$ is a function of $t$ only therefore Equation 4.5 reduces to
Equation 2.5 when \( x \) and \( y \) are interchanged. For two dimensional heat conduction in the solid and liquid PCM, Equation 4.3 becomes

\[
\frac{\partial^2 T_1}{\partial x^2} + \frac{\partial^2 T_1}{\partial y^2} = \frac{1}{\alpha_1} \frac{\partial T_1}{\partial t} \quad (4.6)
\]

and

\[
\frac{\partial^2 T_2}{\partial x^2} + \frac{\partial^2 T_2}{\partial y^2} = \frac{1}{\alpha_2} \frac{\partial T_2}{\partial t} \quad (4.7)
\]

Similarly in the matrix region

\[
\frac{\partial^2 T_m}{\partial x^2} + \frac{\partial^2 T_m}{\partial y^2} = \frac{1}{\alpha_m} \frac{\partial T_m}{\partial t} \quad (4.8)
\]

At the PCM/matrix interface

\[
T_m(x,y,t) = T_1(x,y,t) \quad (4.9)
\]

and

\[
T_m(x,y,t) = T_2(x,y,t) \quad (4.10)
\]
At \( y = 0 \),

\[
T_m(x, 0, t) = T_0 \quad (4.11)
\]

for \( t > 0 \). At \( x = 0 \),

\[
\frac{\partial T_m}{\partial x} = \frac{\partial T_1}{\partial x} = \frac{\partial T_2}{\partial x} = 0 \quad (4.12)
\]

and at \( x = S/2 \),

\[
\frac{\partial T_m}{\partial x} = 0. \quad (4.13)
\]

Along the boundary \( y = S \), the matrix temperatures match those in the next unit above shown schematically in Figure 3.3.

\[
T_m(x, S, t) \bigg|_{\text{Row 1}} = T_m(x, 0, t) \bigg|_{\text{Row 2}} \quad (4.14)
\]

\[
\frac{\partial T_m(x, S, t)}{\partial y} \bigg|_{\text{Row 1}} = \frac{\partial T_m(x, 0, t)}{\partial y} \bigg|_{\text{Row 2}} \quad (4.15)
\]
Row 1 refers to the region between the heat exchange surface, \( x = 0 \), and a unit depth, \( S \). Row 2 is the region between depths \( S \) and \( 2S \), and so on. The initial condition, for the region \( y > 0 \), is that the liquid phase is at a uniform temperature, \( T_i \). At the surface \( y = 0 \), the temperature is reduced to and maintained at some temperature, \( T_o \), below the fusion temperature for all \( t > 0 \). Due to the complexity of this problem, it has not been solved in closed form. In non ideal problems, such as this, a numerical technique is typically used.

**Finite Difference Model**

**Node Pattern** — Cylindrical coordinates as shown in Figure 4.2 were selected in order to accurately model the interface between the PCM core and the matrix shell. Each concentric, composite cylinder was sectioned into pie shaped segments of \( \Delta \theta = \pi/4 \). These sections were alternately tangent to other cylinders and to diamond shaped matrix regions. The centers of the diamond shaped region correspond to thermocouple locations in the experimental study. Since planes of symmetry exist along PCM centers and cylinder tangencies, only half of one column of cylinders, shown in Figure 3.3, was modeled. Both the PCM and matrix were further divided in the radial direction. As shown in Figure 4.3, symmetry
Figure 4.2 Grid Pattern.
boundary nodes were used to generate the adiabatic plane along the PCM center lines. Imaginary boundary nodes, shown in Figure 4.3, were used to interface with other cylinders and adjacent diamond shaped regions.

**Heat Balance** - Heat balances were written for each node type. Details of the development are outlined below for an interior PCM node shown in Figure 4.4. The heat balance is

\[
\begin{align*}
\frac{k_p}{r_p} \frac{dr_p}{r_p} \frac{1}{\pi/4} \left( T_{p_i,l,k+1} - T_{p_i,l,k} \right) + k_p \frac{dr_p}{r_p} \frac{1}{\pi/4} \left( T_{p_i,l,k-1} - T_{p_i,l,k} \right) \\
\frac{\left(r_{p_1} + r_{p_1-1}\right)}{2} \pi \frac{1}{4 dr_p} \left( T_{p_i,l-1,k} - T_{p_i,l,k} \right) + \frac{\left(r_{p_1} + r_{p_1+1}\right)}{2} \pi \frac{1}{4 dr_p} \left( T_{p_i,l+1,k} - T_{p_i,l,k} \right) \\
\frac{d}{dt} T_{p_i,l,k} = \rho_p c_p \frac{\pi}{4 r_{p_1}} \frac{dr_p}{dt} T_{p_i,l,k}.
\end{align*}
\]

(4.16)

The temperature at the next time, \( t + \Delta t \), was found by the Euler method (13).

\[
T_p^{(\gamma+1)} = T_p^{(\gamma)} + \frac{\Delta t}{dt} T_p^{(\gamma)}
\]

(4.17)
No. Radial Div. Plus 1 in Matrix Region (NM) = 2
No. Radial Div. in PCM Region (NP) = 3

* Imaginary Annulus of Matrix Nodes Used to Calculate Insulated Boundary and Interface With Adjacent Row

Figure 4.3 Node Pattern for a Typical Row Including PCM and Matrix Regions.
Figure 4.4 Typical Node Arrangement
By defining

\[ P = \frac{\Delta t}{dr_m^2} \]  

(4.18)

and net heat transfer as

\[ Q_{i,l,k} = P \ \frac{dr_m}{r_{P1}} \left\{ \frac{8dr_P}{r_{P1}} + \frac{\pi (r_{P1-1} + 2r_{P1} + r_{P1+1})}{8dr_P} \right\} T_{i,l,k} \]

(4.19)

and node capacitance as

\[ C_{i,l,k} = \frac{\pi \rho P r_{P1} dr_P}{4} \]  

(4.20)

Equation 4.16 can be rewritten as

\[ T_P^{(\gamma + 1)} = T_P^{(\gamma)} + \frac{Q_P^{(\gamma)}}{C_P} \]  

(4.21)

At the phase front, the temperature was held constant until sufficient heat had been stored or rejected by the
node to change phase as shown in Figure 4.5. This was accomplished by monitoring the total energy stored in each node so that when the nodal temperature reached the fusion temperature, it was maintained at this value until sufficient energy was absorbed or rejected to balance the latent heat of fusion

\[
E_{i,1,k} = \frac{\pi}{4} \rho \int_0^L r_p L \rho \, dr_p.
\]  

(4.22)

Except for latent heat effects, matrix nodes were treated in a similar manner. The geometric factor used to account for the space between the cylindrical and diamond shaped regions is shown in Figure 4.6.

**Numerical Stability** - The space and time increments were not selected arbitrarily but were established by a method presented by Rohsenow and Hartnett (35). Stability criteria were developed by the following argument. Let us consider the general Euler difference equation

\[
T_i^{(\gamma+1)} = \Delta t \left[ \sum K_{ij} T_j^{(\gamma)} + \left( \frac{\rho c V_i}{\Delta t} - \sum K_{ij} \right) T_i^{(\gamma)} \right]
\]  

(4.23)

where \( K_{ij} \) are the thermal conductances and \( V_i \) is the node volume. The coefficients of \( T_j^{(\gamma)} \) are positive. If the coefficient of \( T_i^{(\gamma)} \) was negative, then the higher the
Figure 4.5 Heat Stored Vs. Temperature.
Void Area = $S^2 - \pi \left(\frac{S}{2}\right)^2 - (\sqrt{2}-1)^2 S^2$

= $\pi \left( S^* - S^2 \right) / 4$

$S^* = 1.027027835 \, S$

Figure 4.6 Void Space Geometric Factor.
temperature $T_i^{(\gamma)}$ at the present time, the lower the temperature $T_i^{(\gamma+1)}$ at the future time. This would be in violation of thermodynamic principles. Therefore, $\Delta t$ was chosen so that the coefficient of $T_i^{(\gamma)}$ was positive or zero. For the interior PCM node this argument yields a time increment of

$$\Delta t = \frac{d_r^2}{\alpha \left[ \left( \frac{dr_m}{dr_p} \right)^2 \left( \frac{32}{\pi^2} \left( \frac{dr_p}{r_p} \right) + \frac{r_p}{P_{1-1}} + \frac{2r_p}{P_1} + \frac{r_p}{P_{1+1}} \right) \right]^{1/2}}.$$  (4.24)

For each node type and size, a stable time increment was calculated. Of these, the minimum time increment was selected.

Run Time - Due to the small capacitance of some nodes, numerical stability criteria yield very small time steps. This coupled with the large number of nodes gave very long computing times. In order to reduce the run time several simplifications were investigated. During preliminary runs, it was noted that the temperature field behind the row in which the phase front was located was essentially one dimensional. Furthermore, the matrix temperature field was unaltered more than one row ahead of the phase front. Therefore while analyzing freezing runs with an initial temperature equal to the fusion temperature, two approximations were made: 1) only row 1 through the row just
beyond the phase front was included in the calculation; and
2) rows behind the phase front were lumped into large nodes, as shown in Figure 4.7, with a stored energy equal to the
sum of its component nodes and with properties equal to the
average composite. These nodes coincide with centers of the
diamond shaped region (thermocouple locations) and extend
half a spacing (S) on either side except at the specimen
surface and next to the phase front.

Limiting Cases - For the geometry investigated in this
research, there is no analytical solution to either the
composite semi-infinite solid problem or the composite semi-
infinite phase change problem. Therefore, in order to
verify the numerical solution some simplified limiting cases
were studied. The composite semi-infinite solid problem can
be reduced to a single material by defining identical
material properties in the PCM and matrix regions. For this
case, a series of numerical solutions, shown in Figure 4.8,
were calculated and the results are compared with the
analytical solution for the semi-infinite solid. The nodal
pattern investigated was three PCM radial nodes, NP=3, and
two matrix radial nodes, NM=2 (one real and one imaginary as
shown in Figure 4.3). The imaginary annulus of matrix nodes
was used to calculate insulated boundary conditions and
interface with adjacent rows as shown in Figure 4.3. All
the numerical solutions were stable and compared favorably
with the analytical solution. A slight increase in response
Figure 4.7 Composite Nodes.
Figure 4.8 Comparison of Numerical Solution for Solid Matrix Limiting Care With Analytical Solution For Semi-Infinite Solid.
was observed for composites with fewer rows. This was attributed to the size of the diamond shaped boundary node. At $t=0$, the diamond shaped node $T(1)$ and the adjacent matrix node $TM(1,NM,2)$ were assigned a constant temperature of 32 °F. Therefore the simulated boundary was irregular. For the 4 row configuration, this initial temperature penetration is greater than that of either the 6 or 8 row configurations. This defect of the hybrid nodal pattern was considered to be minor weighted against the direct analog to the experimental configuration and its accurate description of the matrix PCM interface.

The only analytical solution for the freezing problem in cylindrical coordinates is that by Kreith and Romie (55). This problem is similar to the special case of Stefan's problem discussed in Chapter II as shown in Figure 4.9. Within the solid phase

$$
\nabla^2 T = \frac{1}{\alpha} \frac{\partial T}{\partial t}.
$$

(4.25)

At the solid-liquid interface, $r=r_0-\varepsilon$, the temperature is equal to the fusion temperature

$$
T = T_f.
$$

(4.26)

A second condition at the solid-liquid interface is obtained from equating the latent heat effect to the rate of heat
Figure 4.9 Semi-infinite and Cylindrical Freezing.
transfer into the phase front

\[ k \frac{\partial T}{\partial r} = -\rho L \frac{dc}{dt}. \] (4.27)

The initial conditions are that no solidification has occurred and that the temperature of the liquid is uniform at the fusion temperature, i.e. at \( t=0 \)

\[ \varepsilon = 0 \quad \text{and} \quad T = T_f. \] (4.28)

An additional boundary condition is required at the outer radius, \( r_o \).

In order to obtain a solution, Kreith and Romie (55) eliminated the non-linearity at the phase front by forcing the temperature gradient in the solid at the phase front to be a constant, i.e. at \( r = r_o - \varepsilon \)

\[ \frac{\partial T}{\partial r} = G. \] (4.29)

It then follows from Equation 4.27 that the rate of travel of the interface is constant. It can easily be shown from Equations 4.27 and 4.29 that the position of the interface at any time, \( t \), is
In order to determine the temperature distribution in the solid at any time and, of more interest, the temperature history that must be imposed at the surface, \( r = r_0 \) to cause a constant rate of solidification, Equations 4.25, 4.26, 4.27, 4.28, and 4.29 must be solved. Kreith and Romie (55) obtained the solution in the form of a dimensionless series

\[
T^+ = T_1^+ + T_2^+ + T_3^+ + \cdots + T_n^+ + \cdots
\]  

Each term of the series is related to the preceding term by

\[
\frac{1}{r^+} \frac{\partial}{\partial r^+} \left( r^+ \frac{\partial T_n^+}{\partial r^+} \right) = G^+ \frac{\partial T_{n-1}^+}{\partial \phi}
\]  

where the normalized parameters are defined as

\[
\begin{align*}
T^+ &= \frac{T - T_f}{r_o G}, \\
\theta^+ &= \frac{t k G}{\rho L r_o}, \\
r^+ &= \frac{r}{r_o}, \\
G^+ &= \frac{G c r_o}{L}, \\
\phi &= 1 + \theta^+.
\end{align*}
\]
For values of the parameter $G^+$ between 0 and -1.5 the series was reported to converge rapidly. An accuracy of 2% was reported using the first five terms. An error was detected in the published expression for $T_2$ and $T_4$ (Equation 17, Ref. 55), by substitution into Equation 4.32. The negative signs should be plus. The correct expressions are,

\[
T_1 = \phi \ln \frac{r}{\phi}
\]

\[
T_2 = + G \left[ \left( \frac{\phi^2}{2} + \frac{3}{4} \phi^3 \right) \ln \frac{r}{\phi} + \frac{\phi^2}{2} - \frac{r^2}{2} \right]
\]

\[
T_3 = G^2 \left[ \left( \frac{5}{16} \phi^3 + \frac{3}{8} \phi^4 \right) \ln \frac{r}{\phi} + \frac{21}{64} \phi^3 - \frac{5}{16} \phi^5 - \frac{r^4}{64 \phi} \right]
\]

\[
T_4 = + G^3 \left[ \left( \frac{35}{384} \phi^4 + \frac{15}{64} \phi^5 \phi^2 + \frac{3}{128} \phi^4 \right) \ln \frac{r}{\phi}
+ \frac{278}{2304} \phi^4 - \frac{17}{256} \phi^2 r^2 - \frac{7}{128} r^4 + \frac{r^2}{2304} \phi^2 \right]
\]

\[
T_5 = G^4 \left[ \left( \frac{198}{9216} \phi^5 + \frac{35}{384} \phi^7 \phi^2 + \frac{15}{512} \phi^5 r^2 \right) \ln \frac{r}{\phi}
+ \frac{2325}{73728} \phi^5 + \frac{31}{4608} \phi^7 \phi^2 - \frac{77}{2048} \phi^5 r^2
- \frac{1}{1536} \phi^7 - \frac{1}{73728} \phi^9 \right].
\]

Following the notation used by Kreith and Romie (55), the superscript + is omitted from the moduli $G^+$ and $r^+$ in Equation 4.34.

In order to validate the numerical solution for the phase change problem developed in this study, all of the
matrix nodes surrounding the PCM region where assigned temperatures calculated from Equation 4.34 in subroutine "Kreith" listed in Appendix B. In Figure 4.10, the numerically determined freezing front velocity is compared with the rate calculated by combining Equations 4.27 and 4.29.

\[
\frac{d\xi}{dt} = -\frac{k}{\rho L} G \quad (4.35)
\]

At the outer and inner radii (\(r/r_o = 1.0\) and \(0.0\) respectively), the numerically determined freezing rate is nearly identical to that obtained from Equation 4.35. In the central region, the numerically determined rate slightly exceeds the predicted value. On the average, the numerical solution yields a rate of 0.58 ft/hr compared to a calculated value of 0.51 ft/hr.
Figure 4.10 Comparison of Numerically Determined Freezing Front Velocity With That Predicted by Kreith's Solutions.
CHAPTER V

DISCUSSION OF RESULTS

Overview

In Chapter II, it was proposed that the thermal performance of composites such as paraffin and concrete could be described by an effective media with average composite properties. Furthermore, it was shown that these composites store more energy in a given time interval than either of the constituents alone when this composite media, initially at the fusion temperature, was subjected to a step change in surface temperature. This problem was solved analytically by Neumann for a pure substance. The Neumann solution was therefore used as a basis to design an experimental study of a composite media with thermophysical properties similar to paraffin and concrete, but structured in such a way that the void fraction was well established, as were the thermophysical properties. In addition, the configuration, a square array of circular cylindrical voids, allowed visual observation of the fusion front.

In this chapter, the experimental results will be compared with the Neumann model starting with the limiting
This case corresponds to the semi-infinite solid problem. Next, freezing results will be discussed. For selected cases, the experimental results will be compared with the multi-dimensional numerical model developed in Chapter IV. The numerical model will also be compared with the composite media Neumann model. Finally, the melting experimental results will be discussed and compared with the Neumann and numerical models.

**Semi-infinite Solid**

As described in Chapter III, each specimen was subjected to a basic series of tests as shown in Figure 3.17. Initially the thermal response of the specimens without paraffin in the voids was determined. These runs are referred to as void heating and cooling. The specimen was then charged with paraffin and the composite was tested without phase change. These runs are referred to as liquid and solid. In all tests, the thermal exposure was a step change in surface temperature with the specimen initially at a uniform temperature near the fusion temperature of the paraffin. However, prior to investigating composite specimens, the thermal response of the solid matrix specimen was investigated.

**Matrix** - In Chapter II, thermophysical property data for the matrix material, an aluminum filled epoxy, was summarized.
Based on these data, the normalized measured response of the solid matrix specimen was to the right of the analytical solution as shown in Figure 5.1. The uncertainty interval for the matrix specimen was developed in Chapter III, and it is shown graphically in Figure 5.2. The horizontal and vertical bars indicate the magnitude of the root squared uncertainties for the penetration parameter and temperature ratio respectively. The measured response showed a slight bias near the right most limit of the uncertainty interval.

In a letter from the manufacturer of the epoxy concerning thermophysical properties (56), it stated that "I have examined our files and find that we have run thermal conductivity on the 31-D obtaining a value of 18x10^{-4} \text{cal/cm/cm}^2/\text{oC/sec}. Unfortunately, the test method was not specified so I must assume that ASTM D-1674 was used as that is our standard method of testing for thermal conductivity. The note also does not specify which hardener was used and, again, I must assume either the 927 or the 9216 since they are our standard hardeners."

If ASTM D-1674 was used, the thermal conductivity reported would be an average value over a range of temperature. It was shown in Chapter II that these types of materials are temperature dependent. Furthermore, it was shown that conductivity varied strongly with aluminum content. It is possible that aluminum content varied from
Figure 5.1 Comparison of Basic Run Matrix Solid Matrix Specimen Data Normalized Using Epocast Thermal Conductivity With Neumann Solution, Ste = ∞.
Figure 5.2 Solid Matrix Specimen Uncertainty Interval About Neumann Solution, $Ste = \infty$, for Various Depths and Times.
batch to batch. Therefore, since the reliability of the single thermal conductivity value available from the manufacturer was not clearly documented and the thermal conductivity of these types of materials are sensitive to temperature dependence and aluminum content, the thermal conductivity of the matrix was adjusted to achieve a better fit to the analytical semi-infinite solution. The justification for this procedure was partially based on the successful correlation achieved with the polystyrene specimen, shown in Figure 3.19, whose properties were well known. The resulting data fits for the cooling and heating solid matrix runs as shown in Figures 5.3-1 and 5.3-2, respectively. The adjusted conductivity values are shown in Table 5.1 along with calculated effective properties for all of the specimens. These values were used throughout the remaining data normalization and analysis.

Composites - The semi-infinite "solid" response of the 4 row, 6 row, and 8 row specimens was measured for the void cooling, void heating, solid, and liquid composites. These data were normalized using effective composite properties listed in Table 5.1. Since a void system and a second material introduce additional uncertainties, these effects were evaluated by analyzing the specimen with maximum voidage, and the least well known paraffin properties, the 8 row specimen with solid paraffin. The analysis of this case
Figure 5.3-1 Comparison of Basic Run Matrix Solid Matrix Specimen Cooling Data Normalized Using Adjusted Thermal Conductivity With Neumann Solution, Ste = ∞.
Figure 5.3-2 Comparison of Basic Run Matrix Solid Matrix Specimen Heating Data Normalized Using Adjusted Thermal Conductivity With Neumann Solution, $S_t e = \infty$. 

Runs: 0503 (○), 0504 (△)
Table 5.1 Specimen Average Composite Properties.

<table>
<thead>
<tr>
<th>Property</th>
<th>Matrix</th>
<th>4 Row</th>
<th>6 Row</th>
<th>8 Row</th>
<th>Air/PCM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voidage</td>
<td>0.0</td>
<td>0.140</td>
<td>0.317</td>
<td>0.562</td>
<td>1.000</td>
</tr>
<tr>
<td>Conductivity, Btu/hr-ft-(^\circ)F</td>
<td>0.480*</td>
<td>0.368</td>
<td>0.259</td>
<td>0.147</td>
<td>0.014 (1)</td>
</tr>
<tr>
<td></td>
<td>0.550*</td>
<td>0.421</td>
<td>0.296</td>
<td>0.168</td>
<td>0.015 (2)</td>
</tr>
<tr>
<td></td>
<td>0.480*</td>
<td>0.414</td>
<td>0.343</td>
<td>0.262</td>
<td>0.150 (3)</td>
</tr>
<tr>
<td></td>
<td>0.550*</td>
<td>0.448</td>
<td>0.334</td>
<td>0.231</td>
<td>0.087 (4)</td>
</tr>
<tr>
<td>Density, lbm/ft(^3)</td>
<td>108.7</td>
<td>93.45</td>
<td>74.28</td>
<td>47.68</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>108.5</td>
<td>93.27</td>
<td>74.14</td>
<td>47.59</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>108.7</td>
<td>100.9</td>
<td>91.14</td>
<td>77.58</td>
<td>53.3</td>
</tr>
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<td></td>
<td>108.5</td>
<td>100.0</td>
<td>89.36</td>
<td>74.57</td>
<td>48.1</td>
</tr>
<tr>
<td>Sp. Heat, Btu/lbm-(^\circ)F</td>
<td>0.250</td>
<td>0.250</td>
<td>0.250</td>
<td>0.250</td>
<td>0.250</td>
</tr>
<tr>
<td></td>
<td>0.280</td>
<td>0.280</td>
<td>0.280</td>
<td>0.280</td>
<td>0.280</td>
</tr>
<tr>
<td></td>
<td>0.250</td>
<td>0.265</td>
<td>0.287</td>
<td>0.327</td>
<td>0.450</td>
</tr>
<tr>
<td></td>
<td>0.280</td>
<td>0.299</td>
<td>0.327</td>
<td>0.380</td>
<td>0.556</td>
</tr>
<tr>
<td>Diffusivity, ft(^2)/hr</td>
<td>0.018</td>
<td>0.016</td>
<td>0.014</td>
<td>0.012</td>
<td>0.700</td>
</tr>
<tr>
<td></td>
<td>0.018</td>
<td>0.016</td>
<td>0.014</td>
<td>0.013</td>
<td>0.765</td>
</tr>
<tr>
<td></td>
<td>0.018</td>
<td>0.015</td>
<td>0.012</td>
<td>0.008</td>
<td>0.003</td>
</tr>
<tr>
<td>Latent Heat, Btu/lbm</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>7.83</td>
<td>19.57</td>
<td>40.76</td>
<td>105.6</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>7.23</td>
<td>18.25</td>
<td>38.77</td>
<td>107.0</td>
</tr>
<tr>
<td>Stefan Number, N.D.</td>
<td>1.66</td>
<td>0.72</td>
<td>0.39</td>
<td>0.21</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.02</td>
<td>0.88</td>
<td>0.48</td>
<td>0.25</td>
<td></td>
</tr>
</tbody>
</table>

*Adjusted values, manufacturer quoted value was 0.435 BTU/hr-ft-\(^\circ\)F.

(1) Cooling Void, (2) Heating Void, (3) Solid and Cooling, (4) Liquid and Heating

Conductivity: W/m\(^\circ\)C multiply by 1.74
Density: kg/m\(^3\) multiply by 16.0
Sp. Heat: kJ/kg\(^\circ\)C multiply by 4.19
Diffusivity: m\(^2\)/hr multiply by 0.093
is included in Chapter III. The measured data and uncertainty analysis results are shown in Figures 5.4 and 5.5, respectively. The measured data clusters around the analytical solution within the bounds of the uncertainty interval. It appears that the effective media concept modeled this particular composite and thermal exposure quite well.

The remaining semi-infinite solid data are shown in Figure 5.6 through 5.9 for the void cooling, void heating, solid, and liquid composites. Specimen types are noted in the figure titles. In general the correlation between the normalized measured data and the semi-infinite solution was remarkably good over the wide range of voidage investigated. However, there was a slight bias to the right of the semi-infinite solution.

One hypothesis that might explain the observed shift is that the temperature at the thermocouple location responds more rapidly than the general cross section (The matrix = 0.02 as compared to = 0.006 for the PCM). In order to investigate this hypothesis, the numerical model of the 8 row specimen with solid paraffin was utilized. The results of this study are summarized in Table 5.2. Differences in normalized temperatures between the thermocouple centerline and the PCM centerline are listed at depths corresponding to PCM centers and thermocouple
Figure 5.4 Comparison of Basic Run Matrix 8-Row Specimen Solid Data With Neumann Solution, \( \text{Ste} = \infty \).
Figure 5.5 8-Row Specimen, Solid, Uncertainty Interval About Neumann Solution, $\text{Ste} = \infty$, For Various Depths and Times.
Figure 5.6-1 Comparison of Basic Run Matrix 4-Row Specimen, Void Cooling Data With Neumann Solution, Ste = ∞.
Runs: 0601 (○), 0602 (△)

Figure 5.6-2 Comparison of Basic Run Matrix 6-Row Specimen, Void Cooling Data With Neumann Solution, Ste = ∞.
 Runs: 0803 (O), 0804 (△)

Figure 5.6-3 Comparison of Basic Run Matrix 8-Row Specimen, Void Cooling Data With Neumann Solution, Ste = ∞.
Figure 5.7-1  Comparison of Basic Run Matrix 4-Row Specimen, Void Heating Data With Neumann Solution, Ste = ∞.
Figure 5.7-2 Comparison of Basic Run Matrix 6-Row Specimen, Void Heating Data With Neumann Solution, Ste = $\infty$. 

Runs: 0603 (O), 0604 (△)
Figure 5.7-3 Comparison of Basic Run Matrix 8-Row Specimen, Void Heating Data With Neumann Solution, Ste = ∞.
Figure 5.8-1  Comparison of Basic Run Matrix 4-Row Specimen, Solid Data With Neumann Solution, Ste = ∞.
Figure 5.8-2 Comparison of Basic Run Matrix 6-Row Specimen, Solid Data With Neumann Solution, Ste = ∞.
Figure 5.9-1 Comparison of Basic Run Matrix 4-Row Specimen, Liquid Data With Neumann Solution, Ste = \infty.
Figure 5.9-2 Comparison of Basic Run Matrix 6-Row Specimen, Liquid Data With Neumann Solution, Ste = ∞.
Runs: 0806 (○), 0814 (△)

Figure 5.9-3 Comparison of Basic Run Matrix 8-Row Specimen, Liquid Data With Neumann Solution, Ste = ∞.
Table 5.2  Difference Between PCM and Thermocouple Centerline Normalized Temperature for an Idealized 8 Row Solid Run.

<table>
<thead>
<tr>
<th>Row</th>
<th>(x/(at)^{1/2})</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>at TC 1 to 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.282</td>
<td>0.0024</td>
<td></td>
</tr>
<tr>
<td>1.480</td>
<td>0.0031</td>
<td></td>
</tr>
<tr>
<td>1.813</td>
<td>0.0046</td>
<td></td>
</tr>
<tr>
<td>2.564</td>
<td>0.0082</td>
<td></td>
</tr>
<tr>
<td>2 to 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.546</td>
<td>0.0010</td>
<td></td>
</tr>
<tr>
<td>1.709</td>
<td>0.0012</td>
<td></td>
</tr>
<tr>
<td>1.938</td>
<td>0.0014</td>
<td></td>
</tr>
<tr>
<td>2.292</td>
<td>0.0017</td>
<td></td>
</tr>
<tr>
<td>2.960</td>
<td>0.0019</td>
<td></td>
</tr>
<tr>
<td>3.625</td>
<td>0.0017</td>
<td></td>
</tr>
<tr>
<td>At PCM 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.641</td>
<td>0.0185</td>
<td></td>
</tr>
<tr>
<td>0.907</td>
<td>0.0490</td>
<td></td>
</tr>
<tr>
<td>1.282</td>
<td>0.1186</td>
<td></td>
</tr>
</tbody>
</table>

2 | | |
| 1.160 | 0.0090 |
| 1.282 | 0.0115 |
| 1.454 | 0.0151 |
| 1.720 | 0.0207 |
| 2.200 | 0.0284 |
| 2.719 | 0.0296 |
| 3.846 | 0.0179 |
locations as shown in Figure 5.10. No appreciable difference was observed at depths corresponding to thermocouple locations. However at PCM centers, a significant difference exists in row 1. In row 2, the differences were greatly reduced. It appears that the peak variation occurred just behind the wave front and diminished rapidly as the wave front moved through the composite.

Void heating and liquid runs are shifted somewhat more than are the void cooling and solid runs. Since these tests were conducted with the heat exchanger below the specimen, it is possible that convective motion was induced by the thermal gradient along the wall. However, the duration of the test was so short (2 to 4 minutes) that it is unlikely that significant convective motion would have developed. Further discussion of this phenomenon will follow in the discussion of melting.

Phase Change

During phase change runs, the specimen was allowed to stabilize at a temperature near the fusion temperature of the paraffin. Once equilibrium was established, the heat exchange surface was exposed to sudden convective heating or cooling which resulted in nearly a step change in surface temperature crossing the fusion temperature of the paraffin.
Figure 5.10 Thermocouple and PCM Centerlines in Row 1 and 2.
Details of the procedure are discussed in Chapter III. For freezing runs also referred to as cooling runs, the specimen was stabilized at a temperature just above 81°F. The surface was then exposed to convective cooling by water maintained at 32°F. Thus, freezing began at the heat exchange surface and proceeded inward. For melting runs also referred to as heating runs, the specimen was equilibrated at a temperature just below 81°F and then exposed to convective heating by water maintained at 130°F. In these runs, the heat was transferred to the specimen from below for both heating and cooling.

Freezing - During cooling runs, freezing began in the row of voids next to the heat exchange surface and proceeded inward. A minute by minute sequence of phase front propagation for the 8 row specimen is shown in Figure 5.11. At time, \( t=0 \) (top of figure), all of the paraffin was transparent to light (white circular regions) while the matrix was opaque. In order to picture sufficient detail, only the row in which the phase front was located and the row just beyond are shown at each time step. Therefore, the bottom row of voids in the top frame is row 1 and the one above is row 2. Heat is being transferred out of the specimen along the lower edge of row 1. At \( t=1 \) min., nearly all of the paraffin in row 1 had frozen and had become opaque to light transmission. In the next frame, \( t=2 \) min.,
<table>
<thead>
<tr>
<th>Time, Phase Front</th>
<th>Min.</th>
<th>Row</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>

White regions liquid paraffin, Black regions solid paraffin and epoxy.

Figure 5.11-1  Phase Front Photographs for 8 Row Specimen, Freezing Run 0812.
White regions liquid paraffin, Black regions solid paraffin and epoxy.

Figure 5.11-2  Phase Front Photographs for 8 Row Specimen, Freezing Run 0812.
White regions liquid paraffin, Black regions solid paraffin and epoxy.

Figure 5.11-3  Phase Front Photographs for 8 Row Specimen, Freezing Run 0812.
<table>
<thead>
<tr>
<th>Time, Phase Front Min.</th>
<th>Row</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>21 4</td>
</tr>
<tr>
<td></td>
<td>22 4</td>
</tr>
<tr>
<td></td>
<td>23 5</td>
</tr>
<tr>
<td></td>
<td>24 5</td>
</tr>
<tr>
<td></td>
<td>25 5</td>
</tr>
<tr>
<td></td>
<td>26 5</td>
</tr>
<tr>
<td></td>
<td>27 5</td>
</tr>
</tbody>
</table>

White regions liquid paraffin, Black regions solid paraffin and epoxy.

Figure 5.11-4 Phase Front Photographs for 8 Row Specimen, Freezing Run 0812.


<table>
<thead>
<tr>
<th>Time, Min.</th>
<th>Phase Front Row</th>
</tr>
</thead>
<tbody>
<tr>
<td>28</td>
<td>5</td>
</tr>
<tr>
<td>29</td>
<td>5</td>
</tr>
<tr>
<td>30</td>
<td>5</td>
</tr>
<tr>
<td>31</td>
<td>5</td>
</tr>
<tr>
<td>32</td>
<td>5</td>
</tr>
<tr>
<td>33</td>
<td>5</td>
</tr>
<tr>
<td>34</td>
<td>5</td>
</tr>
</tbody>
</table>

White regions liquid paraffin, Black regions solid paraffin and epoxy.

Figure 5.11-5 Phase Front Photographs for 8 Row Specimen, Freezing Run 0812.
The phase front was located in row 2. The solid appears as black crescent shaped regions in the bottom of the voids. By $t=5$ min., freezing was nearly complete in row 2 and at $t=6$ min. the phase front had moved to row 3 and so on through the specimen.

Three stages of phase front development were identified: development of opaque solid in the bottom of the void, completion of a ring of solid around the circumference of the void, and finally complete solidification in a given void. Visual observations of these events are tabulated in Table 5.3. Since the phase front is two dimensional, it was assumed to be at the depth indicated by the arrow shown in the schematic for each stage. Based on these depths, penetration parameter values were calculated for each observation. For the 8 row runs, both the first (F) occurrence of the front and completion of that stage in that row (A) were noted. Simple statistics were calculated for each stage. Also listed are the predicted penetration parameters based on the average property Neumann model.

These observations indicate that the phase front was propagating through the composite media more rapidly than was predicted by the Neumann model. By squaring the average mean observed penetration parameter divided by the predicted Neumann model value, the ratio of observed penetration time to that predicted can be calculated. For the 4, 6, and 8
Table 5.3 Normalized Observations of Freezing Phase Front Location.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Stage</th>
<th>Mean $x/(at)^{1/2}$</th>
<th>Standard Deviation</th>
<th>Number Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 Row</td>
<td>1A**</td>
<td>1.80</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2A</td>
<td>1.92</td>
<td>0.11</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>3A</td>
<td>1.46</td>
<td>0.41</td>
<td>4</td>
</tr>
<tr>
<td>6 Row</td>
<td>1F***</td>
<td>1.14</td>
<td>0.27</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>2F</td>
<td>1.33</td>
<td>0.18</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>3F</td>
<td>1.20</td>
<td>0.05</td>
<td>9</td>
</tr>
<tr>
<td>8 Row</td>
<td>1F</td>
<td>1.07</td>
<td>0.10</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>1A</td>
<td>1.08</td>
<td>0.08</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>2F</td>
<td>1.07</td>
<td>0.04</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>2A</td>
<td>1.01</td>
<td>0.02</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>3F</td>
<td>0.99</td>
<td>0.03</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>3A</td>
<td>0.97</td>
<td>0.11</td>
<td>8</td>
</tr>
</tbody>
</table>

* Penetration Parameter for Neumann Model.

** All Voids in Row.

*** First Void in Row

![Diagram](image-url)
row specimens, these ratios were 1.3, 1.2, and 1.5, respectively.

Although the freezing front was nearly uniform, there was some variation from void to void. However, the 8 row specimen pictured here exhibited the largest deviation which was small. In fact it should be pointed out that due to lens optics, only the void directly in line with the centerline of the lens was photographed without distortion. This is most evident in the initial stage of solidification. Voids to the left of center appear to have only the lefthand portion of the crescent while those to the right appear to have only the righthand portion. In the final stage of freezing, light appears to be coming through only the center voids while in fact it was visible in most voids. Noting these limitations, these photographs demonstrate the uniformity of the phase front propagation.

Normalized thermocouple temperature data for the 4, 6, and 8 row specimen are plotted in Figure 5.12. Also shown is the Neumann solution for the corresponding Stefan number based on a $\Delta T$ of 49°F. Both Stefan number and penetration parameter are based on the calculated average composite properties listed in Table 5.1. Once again, as in the case of the semi-infinite runs, these normalized data cluster just to the right of the Neumann model. It should be pointed out that these data represent all depths and times
Figure 5.12-1 Comparison of Basic Run Matrix 4 Row Specimen, Cooling Data With Neumann Solution, Ste = 1.66.
Figure 5.12-2 Comparison of Basic Run Matrix 6 Row Specimen, Cooling Data With Neumann Solution, Ste = 0.72.
Runs: 0810 (○), 0812 (△)

\[ T_i - T_f = 4.3^\circ F \]  
\[ (2.4^\circ C) \]

Figure 5.12-3 Comparison of Basic Run Matrix 8 Row Specimen, Cooling Data With Neumann Solution, Ste = 0.39.
until the backside of the specimen fell below a temperature ratio of 0.95. The deviation of the 8 row data above a temperature ratio of 1.0 was due to an initial condition 4.3 °F above the fusion temperature of 81°F. Typical initial conditions were on the order of 1 to 2 degrees °F above the fusion temperature. Considering the complexity of the geometry and the resulting phase front, the simplified Neumann model predicted the temperature at these locations in the matrix exceptionally well.

Uncertainties in the normalization process were developed in Chapter III and are pictured graphically for the 8 row specimen during freezing in Figure 8.13. In addition to the uncertainties in the temperature ratio and the penetration parameter discussed earlier for the 8 row solid specimen, freezing introduces uncertainties in the Stefan number. The band of uncertainty in Stefan number lies between the dotted lines. Since the uncertainty in penetration parameter and temperature ratio form a rectangular region around the measured data and the uncertainty in Stefan number shift the model laterally, it appears that the sum of the lateral uncertainties could explain the observed shift to the right of the Neumann model. However before drawing any conclusions, the data will be compared with the numerical model.

**Numerical Freezing Results** - Since the actual response of
Figure 5.13 8 Row Specimen, Freezing, Uncertainty Intervals for the Penetration Parameter and Temperature Ratio, Crosses, at Various Depths and Times, and Stefan Number Dashed Line About Neumann Solution, Ste = 0.39.
the heat exchange surface, $x = 0$, was not an ideal step, measured surface temperatures were used as boundary conditions at the surface nodes and the response at interior nodes was calculated. For run 0812, both measured thermocouple response and photographs of the phase front location were available. In Figure 5.14, computed and measured response are compared for the first 4 minutes of this freezing run. At each time, the numerical grid is drawn next to the photograph of the PCM region adjacent to the thermocouples. The measured temperatures are shown in parenthesis next to computed values. The phase front location was transposed to the outline and is shown dotted. The computed phase front is outlined with a dash/dot line. The computed and measured temperatures were within 1.4°F (0.8°C) with an average difference of 0.9°F (0.5°C). The computed and observed phase front locations were in good agreement.

Computed and measured temperatures are compared for the first 40 minutes of freezing run 0810 in Figure 5.15. As in the analysis of run 0812, measured surface temperatures were used as boundary conditions. The temperatures at all interior nodes were calculated. As was discussed earlier, the initial condition was 4.3°F (2.4°C) above the fusion temperature of the paraffin. Therefore, as the run progressed the measured temperatures beyond the phase front
Figure 5.14-1  Computed and Measured Response, Run 0812.
Figure 5.14-2 Computed and Measured Response, Run 0812.
Figure 5.15 Comparison of Measured Data For Freezing Run 0810 With Numerical Solution.
approached the fusion temperature of 81°F (27.2°C). Since the numerical model considered only nodes one row beyond the phase front, there was some difference at times greater than 20 minutes.

The measured data show an increase in temperature starting at 75 °F (24 °C) followed by an abrupt drop in temperature to 70°F (21°C) in all rows. This was the result of a deficiency in the data logger system when it was measuring temperatures near the scanner temperature. Therefore, these anomalies should be ignored. This effect can also be seen in the normalized void cooling, solid, and cooling runs at temperature ratios near 0.9.

As was discussed in Chapter IV in order to decrease the computation time, nodes in rows behind the phase front were consolidated. The justification for this procedure was based on the observation that isotherms in rows behind the phase front were essentially parallel to the heat exchange surface. Therefore, this portion of the composite could be modeled with a more efficient one dimensional approach. The transition from the two dimensional gridding to a one dimensional model manifests itself as an abrupt offset in the numerical solution due to the averaging technique used. Details are discussed in Chapter IV. For the node between rows 1 and 2, this transition occurred at approximately 8 minutes and 60°F (16°C). This transition occurred at 15
minutes and 68°F (20°C) between rows 2 and 3, and at 25 minutes and 70°F (21°C) between rows 3 and 4.

The numerical model was in good agreement with the measured temperatures. Prior to the transition to the one dimensional model, the numerical result was nearly exact. However, after the transition the numerical model slightly lagged the measured data between rows 1 and 2, as well as between rows 2 and 3. This shift is not evident between rows 3 and 4 but was reestablished between rows 4 and 5.

Normalized results for run 0810 are compared with the Neumann model in Figure 5.16. The normalized numerical results plot just to the right of the Neumann model as did the normalized measured data shown in Figure 5.12-3. In fact, the normalized numerical results directly overlay the normalized measured data except for penetration parameters above 1.1 where the numerical effect of considering only one row beyond the phase front occurs.

Since the experiment did not precisely achieve the Neumann solution assumptions of: 1) an initial condition just above the fusion temperature, and 2) a step change in surface temperature; idealized freezing was investigated with the numerical model. In Figure 5.17, numerical results for an ideal step change in surface temperature of 49°F (9.4°C) from an initial condition of 81.0°F (27.2°C) are compared with the Neumann solution for this case, Ste =
Figure 5.16 Comparison of Numerical Solution With Stefan Solution, Ste = 0.39, Run 0810.
Figure 5.17 Comparison of Numerical Solution at Thermocouple Locations and at PCM Centers With the Neumann Solution, Ste = 0.39. Also Plotted is Neumann Solution, Ste = 0.21.
0.39. In addition to the response at the thermocouple locations, temperature ratios at PCM centers are also plotted. For these ideal conditions, the response at the thermocouples is clustered to the right of the Neumann model for all rows. However, the PCM center response varies from row to row. In row 1, the PCM center response nearly follows the Neumann model for pure paraffin, Ste = 0.21 shown dotted. In row 2, the PCM center froze completely at the penetration parameter corresponding to that predicted by the average composite model. It appears that the PCM center response delay diminishes at greater depth. This pattern would be expected to continue with subsequent nodes. Beyond the first row, both PCM center and thermocouple temperature ratios are below the Neumann model for Ste = 0.39. These results clearly show that the two dimensional composite responds more rapidly than predicted by the average composite model. More rapid thermal response indicates that the heat flux from the composite exceeds that predicted for the average composite. Two dimensional effects will be discussed in detail later in this chapter.

**Volume Change** - Paraffins exhibit a change in volume when they freeze and melt (7). During these experiments, an influx of liquid paraffin from the reservoir was observed during freezing runs and an efflux was observed during
melting runs. Tien and Koump (57) studied volume change effects in one dimensional freezing problems and concluded that this effect will cause both the freezing and melting processes to be retarded. This is due to the ingress of warm fluid during freezing and egress of warm fluid during melting. For a fictitious system where the solid density is 25% greater than the liquid density, Tien and Koump calculated only a 10% reduction in the freezing rate due to volume change effects. Since paraffins experience only a 5 to 10% volume increase on melting (7), it can be inferred that an even smaller effect can be expected in paraffin for similar conditions. Even if a 10% rate reduction is assumed, the effect on the normalized results such as Figure 5.17 would only be a shift to the left of 5%.

**Melting** - As shown in Figure 5.18, the melting phase front is less well defined and more complex than the freezing front discussed earlier. Since paraffin in the solid state is opaque, the first frame shown is at time, \( t = 3 \) minutes when melting was nearly complete in the first row (white circular region). The bottom edge of row 1 was adjacent to the heat exchanger. Therefore, as heat was transferred to the specimen, melting propagated upward. After 4 minutes, melting was complete in row 1 and in the next frame, \( t = 5 \) min., melting began in row 2. Due to the optical distortion
White regions liquid paraffin, Black regions solid paraffin and epoxy.

Figure 5.18-1 Phase Front Photographs for 8 Row Specimen, Melting Run 0811.
White regions liquid paraffin, Black regions solid paraffin and epoxy.

Figure 5.18-2  Phase Front Photographs for 8 Row Specimen, Melting Run 0811.
<table>
<thead>
<tr>
<th>Time, Phase Front</th>
<th>Min.</th>
<th>Row</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>17</td>
<td>4</td>
</tr>
<tr>
<td></td>
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<td>4</td>
</tr>
<tr>
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<td>22</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>23</td>
<td>5</td>
</tr>
</tbody>
</table>

White regions liquid paraffin, Black regions solid paraffin and epoxy.

Figure 5.18-3 Phase Front Photographs for 8 Row Specimen, Melting Run 0811.
White regions liquid paraffin, Black regions solid paraffin and epoxy.

Figure 5.18-4  Phase Front Photographs for 8 Row Specimen, Melting Run 0811.
<table>
<thead>
<tr>
<th>Time, Phase Front</th>
<th>Min.</th>
<th>Row</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>31</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>32</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>33</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>34</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>35</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>36</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>37</td>
<td>7</td>
</tr>
</tbody>
</table>

White regions liquid paraffin, Black regions solid paraffin and epoxy.

Figure 5.18-5 Phase Front Photographs for 8 Row Specimen, Melting Run 0811.
discussed above, it is difficult to distinguish the various stages of melting visible to the naked eye. However during melting of the 4th row starting at $t = 15$ min., phase front propagation was slow enough to identify the various stages shown schematically in Figure 3.16. The first melting was visible in the bottom of the void (white crescent shaped region) at times, $t = 15$ and 16 min. Melting proceeded around the edge of the void until the solid paraffin core was detached from the top of the void. Due to the higher density of the solid, the core settled to the bottom of the void where it continued to melt as shown at times, $t = 18$ and 19 min.

While in the solid state, air infiltrated through the crystalline structure of the paraffin shown in Figure 2.5. After melting, this air collected in the top of the void which appears as opaque regions in the photographs.

During freezing runs, solidification never began in subsequent rows until it was complete in the current row. However, due to the melting settling process, melting began in adjacent rows while the settled solid was completing melting. This effect is evident at $t = 20$ min. in the 5th row and at $t = 28$ min. in row 6.

Normalized observations of the melting front location are tabulated in Table 5.4. The ratio of observed to predicted front penetrations are 0.7, 0.9, and 1.0 for the
4, 6, and 8 row specimens respectively. The slower apparent response was anticipated as discussed in Chapter III. The qualitative nature of the visual observations should also be kept in mind.

Normalized thermocouple temperature data for the 4, 6, and 8 row specimens are plotted in Figure 5.19. The measured response in the 4 row specimen led the predicted average response. Replicate runs were not in good agreement. For the 6 row specimen, replicate runs responded similarly, and the normalized data correlated with the Neumann model in a manner similar to the freezing counterpart shown in Figure 5.12-2. However, the 8 row specimen exhibited a marked change in character.

The initial response of the 8 row specimen correlated well with the Neumann model. After 30 minutes, normalized response between rows 1 and 2 ceased tracking down the predicted response and hooked to the left of the predicted curve. The response of successive thermocouple locations plotted along individual paths well to the right of the Neumann model. Shifts in this direction are indicative of more rapid response. It is evident from these data that the liquid paraffin within the voids was in motion causing convective transport across the liquid composite layer being heated from below. The transition from a conduction temperature profile to that of a convective layer is more
Table 5.4 Normalized Observations of Melting Phase Front Location.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Stage</th>
<th>Mean $\frac{x}{(\alpha t)^{1/2}}$</th>
<th>Standard Deviation</th>
<th>Number Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 Row</td>
<td>1A**</td>
<td>-</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>2A</td>
<td>1.32</td>
<td>0.26</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>3A</td>
<td>1.36</td>
<td>0.19</td>
<td>4</td>
</tr>
<tr>
<td>6 Row</td>
<td>1A</td>
<td>1.16</td>
<td>0.28</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>2A</td>
<td>0.77</td>
<td>0.52</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>3A</td>
<td>1.13</td>
<td>0.16</td>
<td>5</td>
</tr>
<tr>
<td>8 Row</td>
<td>1F***</td>
<td>0.98</td>
<td>0.20</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>1A</td>
<td>0.92</td>
<td>0.25</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>2F</td>
<td>0.98</td>
<td>0.12</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>2A</td>
<td>0.80</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>3F</td>
<td>0.98</td>
<td>0.14</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>3A</td>
<td>0.94</td>
<td>0.17</td>
<td>8</td>
</tr>
</tbody>
</table>

* Penetration Parameter for Neumann Model.

** All Voids in Row.

*** First Void in Row.

![Diagram of stages](image)
Figure 5.19-1 Comparison of Basic Run Matrix 4 Row Specimen, Heating Data With Neumann Solution, Ste = $2.02$. 

Runs: 0414 ($\bigcirc$), 0416 ($\triangle$)
Figure 5.19-2 Comparison of Basic Run Matrix 6 Row Specimen, Heating Data With Neumann Solution, Ste = 0.88.
Figure 5.19-3 Comparison of Basic Run Matrix 8 Row Specimen, Heating Data With Neumann Solution, Ste = 0.48.
clearly shown in the dimensional plot of these data shown in Figure 5.20.

In Chapter II, stability criteria developed by Katto and Masuoka (44) were summarized. An analysis of the concrete/paraffin composite using this theory showed that the onset of convection was unlikely. This approach was used to investigate the stability limits for the experimental composite. The results are shown in Figure 5.21. Although the experimental composite had a different geometry and in fact the pores were closed, Equation 2.2 was used to estimate the "permeability". Paraffin properties were evaluated at 105°F (40.5°C). For the nominal thickness of 1.4 in. (3.56 cm) and a temperature difference of 49°F (27.2 °C), convective motion is predicted for the 8 row specimen, is marginal for the 6 row specimen, and is unlikely for the 4 row specimen. In order to confirm that the measured response of the 6 row specimen did not include convection, the apparatus was inverted so that the specimen was heated from above. As shown in Figure 5.22, no difference was observed between heating from above and heating from below. Next, while heating from below, the temperature difference was increased to 96°F (53.3°C). The results are shown in Figure 5.23. As predicted by Katto and Masuoka, convection was evident after 12 minutes as shown in Figure 5.24.
Figure 5.20 Temperature Profiles for the 8 Row Specimen, Heated from Below.
To Determine Critical Temperature Difference
Locate Layer Thickness and Move Up To Composite Porosity, \( P \), and Read Critical Temperature Difference on Left.

4 Row, \( P = 0.14 \)
\[ 1\Delta T = 1405 \text{ in} \^\circ \text{F} \]

6 Row, \( P = 0.32 \)
\[ 1\Delta T = 55 \text{ in} \^\circ \text{F} \]

8 Row, \( P = 0.56 \)
\[ 1\Delta T = 2.9 \text{ in} \^\circ \text{F} \]

Thickness, \( \text{in} \) (cm)

Figure 5.21  Stability Criteria of Katto and Masuoka (44) for the Experimental Composite of Epocast and Paraffin.
Figure 5.22 Comparison of 6 Row Specimen Heated From Above and From Below, Ste = 0.88.
\[ \Delta T = 49\,^\circ F (27.2\,^\circ C), \Delta T = 96\,^\circ F (53.3\,^\circ C) \]

Runs: 0608 (○), STE = 0.88, 0615 (△), STE = 1.72

Figure 5.23 Comparison of 6 Row Specimen Heating From Below With Variable Stefan Number.
Figure 5.24 Temperature Profiles for the 6 Row Specimen, Heated from Below with a 96°F (53.3°C) Temperature Difference.
Numerical Model of Convective Layer – During melting runs, PCM convective heat transfer was modeled by lumping PCM nodes behind the phase front into a single node as shown in Figure 5.25. The stability criteria for the composite layer was based on the results of Katto and Masuoka (44). The layer thickness, \( l \), was assumed to be the overall spacing between the heat exchange surface and the diamond shaped region just below the phase front as shown in Figure 5.26. The "porous" layer characteristic dimension, \( d \), was taken to be the PCM inclusion diameter. The temperature difference used was that over the thickness, \( l \). If the stability criteria was met, convection within each PCM inclusion was assumed. The Nusselt number, \( Nu \), in each row was calculated using the infinite flat plate correlations of Silveston and O'Toole (58). The PCM inclusion diameter, \( d \), and the temperature difference from bottom to top of each inclusion were used to calculate the Rayleigh number, \( Ra \).

\[
Ra < 1700 \quad Nu = 1 \\
1700 < Ra < 3500 \quad Nu = 0.00238 Ra \\
\quad 5 \quad 0.252 \quad (5.1) \\
3500 < Ra < 10 \quad Nu = 0.229 Ra \\
\quad 5 \quad 0.305 \quad 0.084 \\
10 < Ra \quad Nu = 0.104 Ra \quad Pr
\]

In Figure 5.27, computed and measured response for
\[ Ra = \frac{g \beta (T_h - T_c)}{\alpha \nu} d^3 \]

Figure 5.25 PCM Lump Node for Convective Analysis.
Figure 5.26 Convective Layer Geometry for Analysis.
Figure 5.27-1 Comparison of Measured Data For Heating Run 0809 With Numerical Solution For Conduction Only.
Figure 5.27-2  Comparison of Measured Data For Heating Run 0809 With Numerical Solution Including Convective Algorithm.
melting run 0809 are plotted. Computed response with and without convection are shown in Figures 5.27-2 and 5.27-1, respectively. Between rows 1 and 2 both models overestimate the response while between rows 2 and 3, the measured response lies between the computed values initially but favors the convective model for times beyond 30 minutes. Starting around 30 minutes, between rows 3 and 4, and rows 4 and 5, a transition appears to be taking place. The convective model assumes that convection builds up instantaneously to the levels dictated by the Silveston and O'Toole correlation. This abrupt transition appears to be overestimating the response. A more gradual transition might yield better correlation.

Two Dimensional Effects - It has been demonstrated experimentally and numerically that the Neumann model under predicts the thermal response of the experimental composite. Since the two dimensional numerical prediction correlated well with the measured data for the freezing case and tracked the response for the melting case until convective effects became dominant, the numerical model was investigated over a range of parameters to evaluate the magnitude of the heat transfer enhancement relative to the simplified Neumann model. The initial matrix of cases investigated corresponded to experimental freezing runs. In addition to specimen voidage effects, grid size and scale
effects were studied. Comparisons were made between the 
heat transfer predicted by the numerical model and the 
Neumann approximation by making ratios of these values. 
These ratios are summarized in Table 5.5. The notation 
6:3,4 indicates the six row specimen with three PCM radial 
nodes and four matrix nodes, as shown in Figure 4.3. For 
all of the cases investigated, the rate of heat transfer 
predicted by the two dimensional model exceeded that 
predicted by the Neumann approximation.

Initially, the numerically determined value does not 
appear to be proportional to the square root of time as is 
predicted by the simplified Neumann approach. However, it 
appears to approach this type of dependence asymptotically. 
It also appears to depend somewhat on specimen type and grid 
pattern. The grid dependence is probably due to the way in 
which the amount of heat transfer was computed. In lieu of 
summing the heat transferred at the surface node interface 
the amount of heat transferred was computed at discrete 
times by determining the difference in energy at each node 
from that at time zero. For nodes that were undergoing 
phase change, latent heat effects were not included until 
transition was completed. Therefore, these values lag the 
actual value and are somewhat irregular with time.

In order to verify that this was indeed the cause of 
the irregularity, temperature response was analyzed.
Table 5.5 Ratio of Heat Transfer Predicted by the Numerical Model to that Predicted by the Neumann Approximation.

<table>
<thead>
<tr>
<th>Time, Min.</th>
<th>4:3,4*</th>
<th>4:3,6</th>
<th>4:4,6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.14</td>
<td>1.15</td>
<td>1.25</td>
</tr>
<tr>
<td>2</td>
<td>1.23</td>
<td>1.24</td>
<td>1.28</td>
</tr>
<tr>
<td>3</td>
<td>1.23</td>
<td>1.24</td>
<td>1.32</td>
</tr>
<tr>
<td>4</td>
<td>1.28</td>
<td>1.29</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>1.32</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>1.30</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>1.33</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>1.32</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>9</td>
<td>1.33</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>6:3,4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.16</td>
<td>1.35</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.34</td>
<td>1.38</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1.38</td>
<td>1.41</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>1.48</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>1.48</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-</td>
<td>1.51</td>
<td></td>
</tr>
</tbody>
</table>

* 4:3,4 Notation for Four Row Specimen with Three PCM Radial Nodes and Four Matrix Radial Nodes.
Normalized temperatures for various mesh sizes are summarized in Table 5.6. Values at thermocouple locations between rows one and two and at the centers of the PCM node in row one are listed. Although somewhat more responsive, the x:3,2 pattern yield results within 4 percent of the finest mesh with a substantial savings in computing time.

The lack of PCM size effect or scale effect can be seen by examining the temperature ratios at constant penetration parameter for different PCM inclusion sizes, i.e., half (1/2X), equivalent (1X), and twice (2X) that of the experimental configuration listed in Table 5.7. These comparisons were made at two composite locations: the thermocouple location between rows 1 and 2 and at the center of the PCM inclusion in row 1. There was no detectable size effect at either location. Since neither the mesh size nor the scale effect caused significant variation in the thermal response, the noted variation in heat transfer ratio was ascribed to the manner in which latent heat was summed.

It is not clear from the runs listed in Table 5.5 which composite properties have the greatest effect on the heat transfer enhancement. Some insight may be gained by reinvestigating the ratio of heat transfer predicted by the Neumann approximation for a composite ($Q_{\text{Neumann}}$) to that of the matrix component alone ($Q_{\text{Matrix}}$).
Table 5.6 Node Size Sensitivity Summary.

<table>
<thead>
<tr>
<th>Time, Min.</th>
<th>4:3,2*</th>
<th>4:3,4</th>
<th>4:3,6</th>
<th>4:4,6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.8460 (1)</td>
<td>0.8766</td>
<td>0.8730</td>
<td>0.8757</td>
</tr>
<tr>
<td></td>
<td>1.0002 (2)</td>
<td>1.0002</td>
<td>1.0002</td>
<td>1.0002</td>
</tr>
<tr>
<td>2</td>
<td>0.7215</td>
<td>0.7605</td>
<td>0.7551</td>
<td>0.7509</td>
</tr>
<tr>
<td></td>
<td>0.5351</td>
<td>0.5666</td>
<td>0.5581</td>
<td>0.5329</td>
</tr>
<tr>
<td>3</td>
<td>0.6201</td>
<td>0.6588</td>
<td>0.6516</td>
<td>0.6485</td>
</tr>
<tr>
<td></td>
<td>0.3778</td>
<td>0.3894</td>
<td>0.3817</td>
<td>0.3768</td>
</tr>
<tr>
<td>6:3,2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.8317</td>
<td>0.8408</td>
<td>0.8434</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.0002</td>
<td>1.0002</td>
<td>1.0002</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.6890</td>
<td>0.7018</td>
<td>0.6922</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.4719</td>
<td>0.4723</td>
<td>0.4548</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.5825</td>
<td>0.5887</td>
<td>0.5831</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.3342</td>
<td>0.3297</td>
<td>0.3252</td>
<td></td>
</tr>
<tr>
<td>8:3,2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.8999</td>
<td>0.8757</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.0002</td>
<td>1.0002</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.7497</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.5106</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.6263</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.3505</td>
<td>-</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* The notation 4:3,2 indicates the four row specimen with three PCM radial nodes and two matrix nodes.

1) Thermocouple Location

2) PCM Center
### Table 5.7 PCM Inclusion Size Effect.

<table>
<thead>
<tr>
<th>(x/(at)^{1/2})</th>
<th>1/2 X</th>
<th>X</th>
<th>2 X</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.641</td>
<td>0.5767</td>
<td>0.5769</td>
<td>0.5767 at TC</td>
</tr>
<tr>
<td>0.453</td>
<td>0.3956</td>
<td>0.3956</td>
<td>0.3956 Rows 1 to 2</td>
</tr>
<tr>
<td>0.427</td>
<td>-</td>
<td>0.3784</td>
<td>0.3789</td>
</tr>
<tr>
<td>0.3205</td>
<td>0.8972</td>
<td>0.8978</td>
<td>0.8972 at PCM</td>
</tr>
<tr>
<td>0.6795</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000 Center Row 1</td>
</tr>
</tbody>
</table>

1/2 X = One Half Experimental Dia.

1X = Experimental Dia.

2X = Twice Experimental Dia.

Experimental Dia. = 1.48 in. (0.376 cm.)
\[
\frac{Q_{\text{Neumann}}}{Q_{\text{Matrix}}} = \frac{1}{\text{erf} \ \frac{\bar{x}}{k}} \left( \frac{\alpha}{\bar{\alpha}} \right)^{1/2}
\]  \hspace{1cm} (5.2)

where

\[
\bar{x} e^{\bar{x}^2} \text{erf} \ \bar{x} = \frac{\text{Ste}}{\pi^{1/2}}
\]  \hspace{1cm} (5.3)

and \( k = \) Matrix conductivity

\( \bar{k} = \) Composite effective conductivity

\( \alpha = \) Matrix diffusivity

\( \bar{\alpha} = \) Composite effective diffusivity

\( \text{Ste} = \) Composite effective Stefan number

From Equations 5.2 and 5.3, three dimensionless ratios are evident \( \bar{k}/k, \ \bar{\alpha}/\alpha, \) and \( \text{Ste}. \) By manipulation of Equations 2.21, 2.22 and 2.23, it can be shown that the diffusivity ratio is related to the conductivity ratio by

\[
\frac{\bar{\alpha}/\alpha}{\bar{k}/k} = \frac{1}{1 - \beta^3 + \beta^3 \frac{c_2}{c_1}}.
\]  \hspace{1cm} (5.4)

For the experimental composite, the product of density and specific heat ratios is 0.89. For concrete and paraffin,
this product is 0.87. Therefore, over the entire range of possible paraffin fractions, i.e. $\beta^3 = 0$ to 1, the ratio of diffusivities to conductivities varies only from 1.0 to 1.15. Or over the range of interest, it can be assumed to be a constant value. Conductivity ratio and Stefan numbers were selected to parameterize the composite.

First the ratios of heat transfer predicted by the Neumann model to that transferred to a solid matrix specimen were calculated for a series of conductivity ratios, $\kappa/k$, corresponding to those of the experimental specimens at $\text{St} = 0.2$, 1.0, and 2.0. Next, a series of numerical solutions was obtained for the same conditions. From these numerical solutions, ratios of the heat transfer predicted from the numerical model to that predicted by the Neumann model were calculated. As previously noted, this ratio varies with time but it was found that the ratio was fairly stable by the time that the phase front had reached the mid plane of the specimen. Therefore, the maximum ratio calculated to that point was used.

In order to calculate the heat transfer ratio including two-dimensional effects, these two ratios were multiplied, i.e.

$$\frac{Q_{\text{Numerical}}}{Q_{\text{Matrix}}} = \frac{Q_{\text{Numerical}}}{Q_{\text{Neumann}}} \times \frac{Q_{\text{Neumann}}}{Q_{\text{Matrix}}} \quad (5.5)$$
These ratios are listed in Table 5.8 and indicate that two dimensional effects are significant. These effects become more important as the Stefan number decreases, i.e. as latent heat effects become dominate. Conductivity ratio effects are not as strong.

The ratio of the heat transfer predicted by the numerical model to that predicted by the Neumann model can be interpreted as multi-dimensional effects. Multiple regression techniques were used to fit a plane to the values listed in Table 5.8 and the following equation was developed

\[
\frac{Q_{\text{Numerical}}}{Q_{\text{Neumann}}} = 1.324 + 0.1831 \frac{k}{k} - 0.1816 \text{Ste}^0. \quad (5.6)
\]

Multiplying this expression times the Neumann model will yield an approximate equation including two dimensional effects for the experimental configuration. Equation 5.6 was included in the second program listed in Appendix B. These ratios along with those predicted by the Neumann model are shown in Figure 5.28. Since it is geometrically impossible for the paraffin fraction to exceed \(\pi/4\) for this configuration, a normalized paraffin fraction, \(4f/\pi\), was used. This figure corresponds to Figure 5.24 which was developed for a cubic array of cubic inclusions of paraffin
Table 5.8 Ratio of Heat Transfer Predicted by Various Models.

<table>
<thead>
<tr>
<th>Stefan No.</th>
<th>Conductivity Ratio, $\bar{\kappa}/k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Ste} = \bar{c} \Delta T/L$</td>
<td>0.55</td>
</tr>
<tr>
<td>0.2</td>
<td>2.19 (1)</td>
</tr>
<tr>
<td></td>
<td>1.41 (2)</td>
</tr>
<tr>
<td></td>
<td>3.09 (3)</td>
</tr>
<tr>
<td>1.0</td>
<td>1.19</td>
</tr>
<tr>
<td></td>
<td>1.20</td>
</tr>
<tr>
<td></td>
<td>1.43</td>
</tr>
<tr>
<td>2.0</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td>1.07</td>
</tr>
</tbody>
</table>

1) Neumann/Matrix
2) Numerical/Neumann
3) Numerical/Matrix
Figure 5.28  Heat Transfer Ratio, Composite to Matrix, Versus Paraffin Fraction With and Without Two Dimensional Effects for the Experimental Configuration.
in mortar. Of course, for that geometry, paraffin fractions up to 1.0 are possible. As opposed to the cubic array of cubes, the experimental configuration never reached the point where lower conductivity effects dominated the latent heat induced increase in the temperature gradient at the wall, i.e., the optimum paraffin fraction was the geometric maximum. However, it is worth noting that the heat transfer ratio approaches this maximum slowly. Since the paraffin is the costly component in this composite, a fraction lower than that based purely on heat transfer ratios should be considered.

On the average, the two dimensional model predicted 41% more heat transfer than the Neumann model for a 10°F (5.6°C) T and 29% more for the 50°F (27.8°C) case. For the optimum paraffin fraction, i.e. \( \pi/4 \), heat transfer ratios are plotted as a function of \( \Delta T \) in Figure 5.29. On the average, two-dimensional effects accounted for a 35% increase in heat transfer. Of course for small \( \Delta T \)'s, the heat transfer advantage of the composite over the solid matrix specimen becomes very large.
Figure 5.29  Heat Transfer Ratio, Composite to Matrix Versus $\Delta T$ With and Without Two Dimensional Effects for the Experimental Configuration, at a Void Fraction of $\pi/4$. 
CHAPTER VI

SUMMARY

Overview

In Chapter I, the author proposed a thermal storage media which consisted of a dispersion of paraffin wax in a concrete mortar filler matrix. Both of these materials have been used separately as a thermal storage media and are readily available in industrialized regions. Separately, these materials have limitations. Over a limited temperature range, the energy storage density of concrete is poorer than phase change materials such as waxes. However, waxes are noted for their extremely low thermal conductivity. It was hoped that a mixture of these materials would have dynamic thermal storage advantages over either of the materials alone.

To investigate the potential of the proposed composite, a literature survey was conducted which included a review of the thermophysical properties, heat transfer in composite media, and the Stefan phase-change problem. Based upon these findings, an experimental investigation, parameterized using effective thermophysical properties, was conducted.
with an idealized composite. These well defined composites were analyzed analytically and numerically. In this chapter, the results will be related back to the proposed thermal storage media, a dispersion of paraffin wax in a concrete mortar filler matrix.

Discussion

Idealized Material - By assuming that the heterogeneous composite could be idealized as a homogeneous material with effective thermophysical properties, the Neumann solution could be used to calculate the rate of heat transfer at the surface. By forming the ratio of heat transferred into the composite, to that into the solid mortar,

\[
\frac{Q_{\text{Composite}}}{Q_{\text{Matrix}}} = \frac{1}{\text{erf} \left( \frac{\alpha}{\sqrt{\lambda(\text{Ste})}} \right)} \frac{\alpha}{\alpha}^{1/2}
\]  

(6.1)

and evaluating this expression for a series of paraffin fraction, it was shown in Figure 2.24 that some composites would absorb more heat than either of the constituents alone. A paraffin fraction of 0.0 indicates solid mortar while a value of 1.0 indicates solid paraffin. The paraffin fraction corresponding to the maximum heat transfer rate varied as a function of temperature step amplitude. The maximum shifted toward solid mortar for large temperature
steps. For a temperature step of 25°F (13.9°C), the heat transferred into the composite was 1.7 times that which would have been transferred into the solid mortar. With smaller temperature steps, the heat transfer ratio increased in magnitude and shifted toward larger paraffin fractions. In the limiting case with an infinitesimal temperature step, the optimum paraffin fraction would be 1.0. For temperature steps greater than 25°F (13.9°C), the range of optimum paraffin fractions was found to correspond to achievable paraffin fractions for the proposed composite material.

Based upon these encouraging results, it appeared that evaluation of a proposed composite would depend upon the ability to obtain reliable thermophysical property data and combine these data by some appropriate model into effective properties i.e. k, c, and L.

**Thermophysical Properties** - A review of the literature has shown that there are few comprehensive studies of the thermophysical properties of concrete. In most studies, only one or two properties were measured. This, coupled with the variability of the components, mixes, and curing makes specific, comprehensive determinations difficult at best. Thermal conductivity is the most difficult property to establish.

Although there is considerable thermophysical property data for pure normal paraffins, there is only limited information on commercial grade products, and even less
information is available on paraffin waxes which are mixtures of paraffin hydrocarbons. There is general agreement on the thermal conductivity of paraffins in the liquid state. However, in the solid state, much less is known. There appears to be some dependence on hydrocarbon chain length as shown in Figure 2.4. Due to its crystalline structure, some investigators have observed anisotropy (17). Bailey and Liao (12) measured specific heats for commercial grade paraffin that were considerably higher than that reported by the American Petroleum Institute for the ultra-pure materials as shown in Figure 2.3.

For economic reasons, it was hoped that slack waxes could be used in the proposed composite. This material is a residue of petroleum refining. However, it was found that mixtures of paraffins with comparable chain length can have substantially lower latent heats than the pure hydrocarbon (7, 12). Since the additional costs associated with further refining would be prohibitive, this fact alone makes the development of this particular composite unlikely.

Effective Properties - Many investigators have studied the classical problem of evaluating the effective conductivity of two component systems. In general, two types of analysis have been employed: the equivalent network of resistances and potential function method. As is shown in Figure 2.15 for a cubic array of spheres and in Figure 2.16 for a square array of circles, there is much disparity among the various
approaches. It was observed that Duga's (33) parallel/series expression yields unreasonable results for a considerable range of conductivity ratios about the pole in its functional form.

Calculation of effective density and specific heat appear to be more routine. However, combining these effective properties with an effective thermal conductivity to compute an effective thermal diffusivity is open to question. It is recognized that thermal response of composite materials is frequency dependent. For example, suppose we consider a series layer of two pure materials of known conductivity. There is no question about how an effective conductivity should be calculated nor its effective density or specific heat. However, its response to varying surface temperature will be frequency dependent. Of course, for steady heat conduction, the series model is exact.

This fact may be one reason why an effective thermal diffusivity reasonably approximate the thermal response for the phase change Stefan problem. For the materials investigated in this study, the relaxation times for the conduction problem were much smaller than those of the phase front propagation. Therefore, the temperature profiles in the composite were nearly that of steady conduction for which effective properties yield reasonable approximations. Of course, the accuracy of the approximation was
demonstrated in the experimental phase of this study.

**Convection** - When heated from below, a composite layer such as liquid wax in a concrete mortar matrix may become unstable and convection will begin. For the case of a pure fluid between infinite parallel plates, numerous experimental and analytical studies have been conducted and the criteria have been established for the onset of convection (42,43). The parameter that characterizes this transition is the Rayleigh number. For infinite parallel plates the critical value is 1708.

Although the dynamics of fluids in porous media are less well known, Katto and Masuoka (44) have reported a correlation that predicts the onset of convection in porous media which was discussed in Chapter II. The thermal diffusivity included in the Rayleigh number is defined as the effective thermal conductivity of the porous media divided by specific heat and density of the fluid component. Using this criteria and the Blake-Kozeny Equation to calculate the permeability, it was shown that for normal concrete aggregate sieve sizes and realistic porosities i.e. up to $\pi/12$, convection is unlikely in the proposed composite. Since the extremely low thermal conductivity of the liquid paraffin was one of the motivating factors for adding the filler mortar matrix, it is unfortunate that the matrix which was to enhance the heat transfer may in fact be suppressing it for some conditions.
Experimentation - Based on the findings of the literature survey, an idealized concrete/paraffin experimental layer was designed. The specimen was an aluminum filled epoxy casting which had thermophysical properties similar to those of concrete. A square array of circular cylindrical voids ran parallel to the heat exchange surface as shown in Figures 3.1 and 3.4. The void cylinders were filled with n-octadecane through a Plexiglas header.

This configuration allowed visual observation of the progress of the phase front. Temperature measurements were made between each row of voids with thermocouples cast into the centers of the diamond shaped matrix regions, one per row except at the heat exchange surface and at the top of the specimen. At these locations, two thermocouples were used, one at the side and one at the centerline.

At the heat exchange surface, the epoxy was bonded to an aluminum plate which served as an isothermal surface. The step change in surface temperature was accomplished by pouring water at the desired temperature into an open reservoir and circulating the water through a heat exchanger adjacent to the aluminum plate. The water was maintained at the desired temperature by circulating it over ice in the open reservoir or heating it with a thermostatically controlled emersion heater. Although the surface temperature profile, shown in Figure 3.18, was not an ideal step, it was a reasonable approximation. It was very good
for phase change runs which were up to an hour in duration.

The thermal response of the specimen was recorded using an automatic data logger which converted the millivolt signal of the thermocouples into punched paper tape. This tape was later processed to punched cards for use in data analysis.

Each specimen was subjected to a basic series of tests including: heating and cooling without paraffin in the voids, heating and cooling with paraffin in the void but without crossing the phase change temperature, and melting and freezing. This series is shown schematically in Figure 3.17. The first four tests in this series do not involve phase change and until the thermal wave reaches the back of the specimen, their response is that of a semi-infinite region. Of course, this case represents a limiting case of the Stefan problem where the Stefan number is infinite.

Prior to each run, the specimen was brought to an isothermal condition near the fusion temperature of n-octadecane i.e., \( T_F = 81^\circ F \) (27.2°C).

The data was normalized assuming that effective composite properties could be calculated and organized into non-dimensional groups which characterize the Stefan problem, i.e., temperature ratio, penetration parameter, and Stefan number. Effective properties that were calculated are listed in Table 5.1. In addition to the non-dimensional temperature profiles, visual observations of the progress of
the phase front were normalized. However, these observations should be considered as qualitative data as was discussed in Chapter III.

**Semi-infinite Solid** - When manufacturer thermophysical property data was used the thermal response of the solid matrix specimen plotted to the right of that predicted as shown in Figure 5.1. Although the data was within the uncertainty interval, as shown in Figure 5.2, the thermal conductivity of the Epocast matrix was adjusted to achieve a better fit to the theoretical curve. The adjustment was made because of lack of clear documentation from the manufacturer, and the good correlation achieved with the experimental apparatus on a polystyrene specimen as shown in Figure 3.19. All subsequent effective conductivities were calculated using these conductivities and are listed in Table 5.1.

Since a void system and a second material introduce additional uncertainties over the solid matrix specimen, these effects were analyzed using root squared error procedures. These uncertainties are shown graphically in Figure 5.5 for the 8 row specimen. The horizontal bars represent the uncertainty in evaluation of the penetration parameter while the vertical bars show the uncertainties in the temperature ratio. Comparison of this region with the measured response, shown in Figure 5.4, indicates that the measured results were well within the uncertainty which
surrounds the predicted response. The remaining semi-infinite solid data are shown in Figures 5.6 through 5.9. In general, the correlation between the normalized measured data and the semi-infinite solution was remarkably good over the wide range of voidage investigated. However, there appeared to be a slight bias to the right of the semi-infinite solution.

It was hypothesized that since the temperature was measured in the higher conductivity component, the measured response leads that of the general composite which is predicted by the effective properties. However, a finite difference model showed that temperature differences between thermocouple and PCM centerline paths at common depths were small as shown in Table 5.2 and Figure 5.10.

Freezing - During phase change runs, visual observations of the progress of the phase front were made. For a limited number of runs, photographs were made. In Figure 5.11, photographs show the two dimensional nature of the phase front. Three stages of front development were identified: formation of opaque solid in the bottom of the void, completion of a ring of solid around the circumference of the void, and finally complete solidification. Normalized visual observations classified by these stages are summarized in Table 5.3. On the normalized Stefan curve, the point of departure from a temperature ratio of 1.0 indicates the phase front location. These calculated values
are shown for each specimen type. These observations indicate that on the average the phase front was propagating through the composite media more rapidly than was predicted by the Neumann model. A time multiplying factor can be calculated by forming ratios of the square of the average mean penetration parameter to the square of the calculated value. The factor is 1.3, 1.2 and 1.5 for the 4 row, 6 row and 8 row specimens respectively.

Normalized thermocouple temperature data are plotted in Figure 5.12 along with Neumann solutions for the corresponding Stefan numbers. Once again, these normalized data cluster just to the right of the Neumann model. Although it is not possible to identify a specific penetration parameter to characterize the shift as was possible for phase front location, the shift in penetration parameter at a temperature ratio of 0.8 was used. Multiplying factors of 1.1, 1.2 and 1.2 were calculated. These values are in substantial agreement with the phase front observations. Although uncertainties could explain the observed shift, clearly two dimensional effects are present. In order to evaluate these effects, numerical analysis was used.

**Numerical Model** - Although a physical model of the experimental configuration can be developed, as outlined in Chapter IV, solution of this boundary value problem was not possible. In non-ideal problems such as this, numerical
techniques are usually employed. This approach was used in this study.

The selection of a coordinate system presented a dilemma. A finely meshed rectangular coordinate system could have been used but computing time would have been unacceptable. As shown in Figure 4.2, cylindrical coordinates efficiently describe the circular void region and adjacent matrix region out to the point of tangency between cylinders. However, treatment of the diamond shaped region between cylinders is not possible except by noting that if the tangential grid interval is selected to be $\pi/4$, as shown in Figure 4.2, cylindrical coordinate nodes adjacent to the diamond shaped region are approximately orthogonal. Since this unique grid pattern, offered significant computation time advantages while retaining accuracy in the phase change region, it was adopted in this study.

The heat balance approach was used to develop the finite-difference model. The Euler method was used to step forward in time. Time increments were established based on thermodynamic principles. For each node type and size, a stable time increment was calculated. Of these, the minimum was selected and used throughout the calculation. At the phase front, the temperature was held constant until sufficient heat had been stored or rejected to account for the latent capacities of that node.
In order to verify the numerical model, two limiting cases were investigated for which analytical solutions were available. One was the case of a semi-infinite solid initially at a uniform temperature whose surface temperature was changed to a new temperature and held constant, i.e. the limiting case of the Stefan problem, \( \text{Ste} = \infty \). As shown in Figure 4.8, numerical results were found to track the analytical solution very well. However, there appeared to be some specimen void fraction dependence which was traced to the size of the diamond shaped region and its treatment at time equal zero.

The other limiting case was freezing in cylindrical coordinates. By specifying a specific temperature history at the surface, Kreith and Romie (55) eliminated the non-linearity at the phase front. However, the published expression appeared to include a typographical error. When this error was corrected the analytical solution yielded reasonably good correlation with the numerical model developed in this study as shown in Figure 4.10.

Since the actual response of the surface was not an ideal step, measured surface temperatures were used as boundary conditions and thermal response was calculated at interior nodes. For run 0812, both measured thermocouple responses and photographs of the phase front were available. These data were compared with numerically computed responses for the first 4 minutes and are shown in Figure 5.14.
Excellent agreement was achieved. In Figure 5.15 up to 40 minutes of calculated and measured thermal response are shown. The deviation in measured data around 75°F was traced to a deficiency of the data logger. The offsets in the numerical results were due to transition from a full nodal matrix to consolidated nodes as was discussed in Chapter IV. The justification for this procedure was based on the observation that isotherms in rows behind the phase front were essentially parallel to the heat exchange surface. Overall, the agreement between measured and numerically calculated response was very good. These data were also plotted in normalized form in Figure 5.16. Normalized results plot just to the right of the Neumann model as did the normalized measured data as shown in Figure 5.12-3.

In Figure 5.17, numerical results for an ideal step change are compared with the Neumann solution for a Ste = 0.39. In addition to thermocouple location response, response at the PCM center is also plotted. Once again, thermocouple location response plots just to the right of the Neumann model. However, PCM center response varies from row to row. In row 1, the PCM center response nearly follows the Neumann model for pure paraffin, Ste = 0.21. In subsequent rows, the PCM center response gradually approaches that at the thermocouple location. The enhanced rate of heat transfer is clearly due to two dimensional
effects.

The numerical model was investigated over a range of parameters to evaluate the magnitude of heat transfer enhancement relative to the simplified Neumann model. It was found that the numerically determined heat transfer was not proportional to the square root of time as is predicted by the simplified Neumann approach. However, it appears to approach this type of dependence asymptotically.

By examining the Neumann model, it was shown that over the material property range of interest, the ratio of conductivity and diffusivity ratios was nearly constant. Due to their more basic nature, conductivity ratio and Stefan number were selected to parameterize a study using the numerical model.

Table 5.5 lists asymptotic ratios of heat transfer calculated by the numerical model to that predicted by the Neumann approach. It appears that multi-dimensional models are necessary to accurately predict the thermal performance of phase change composites over a wide range of conditions.

For the experimental configuration, a multiplicative term was developed, Equation 5.6, to modify the Neumann model. Over a rather wide range of conditions, this empirically developed factor predicts heat transfer enhancement ranging for 30 to 40%. It was also shown that although the optimum paraffin fraction for this geometry was the geometric maximum, heat transfer ratios approached these
values asymptotically. However, economics may dictate paraffin fraction considerably lower.

For the random dispersion, characterizing the composite shown in Figure 1.1, the average conductivity based on a cubic array of cubes will yield a conservative estimate of the heat transfer enhancement as shown in Figure 2.24. Due to the nature of the dispersion, it is anticipated that the geometric effects will be at least as large as those established for the experimental configuration.

Due to the complexity of the required numerical models and the large amount of computing time necessary to obtain a numerical solution, it is recommended that proposed composites be screened using the conservative, simplified Neumann approach prior to conducting detailed analysis.

**Melting** - As shown in Figure 5.18, the propagation of the melting phase front was less well defined than the freezing front. In addition to two-dimensional phase fronts, buoyancy effects of the solid paraffin and convection were present. As shown in Figure 3.16, three stages of melting phase front development were identified: 1) first liquid, 2) solid settlement, and 3) complete liquid. Normalized visual observations classified by these stages are summarized in Table 5.4. Also shown are phase front location predicted by the Neumann model. These observations indicate that the phase front was propagating more slowly than predicted by the Neumann model except for the 8 row
specimen. Time multiplying factor for the 4, 6 and 8 row specimens were 0.7, 0.7 and 1.4 respectively. However, as discussed in Chapter III, these observations are qualitative in nature.

For the 4 and 6 row specimens, the thermocouple location thermal response was to the right of the Neumann model. Time-multiplying factors determined at temperature ratios of 0.8 were 1.3 and 1.2 for the 4 and 6 row specimens. However for the 8 row specimen, the thermal response showed a dramatic change in form. Initially the response followed the form of the Neumann model. After 30 minutes, the response began to deviate in a manner indicative of more rapid phase front propagation. This transition is shown in the form of temperature profiles in Figure 3.20. It is evident from these data that the liquid paraffin within the voids was in motion which resulted in convective transport across the liquid composite layer while being heated from below. This transition had been predicted by stability criteria developed by Katto and Masuoka (44). Since these methods had indicated that the stability of the 6 row specimen was marginal, the apparatus was inverted so that the specimen could be heated from above. No difference in response was observed. Next, while heating from below, the 6 row specimen was subjected to a larger temperature difference (96°F or 53.3°C). As predicted by the stability criteria, convection was evident as shown in Figure 5.24.
The numerical computer program incorporated algorithms to account for stability and convective transport. The stability criteria was that of Katto and Masuoka. Convective layer characteristic dimensions are shown schematically in Figure 5.26. If stability criteria were met, convection within each PCM inclusion was assumed. The Nusselt's number in each row was calculated using the infinite flat plate correlations of Silveston and O'Toole (58). Computed and measured response for melting run 0809 are plotted in Figure 5.27. Two numerical models are included: conduction only and convection. As shown in Figure 5.27-1, the conduction only model traces the response reasonably well, up to 30 minutes where convection became apparent. The convective model assumes that convection builds up immediately to the levels dictated by the Silveston and O'Toole's correlations. This abrupt transition appears to overestimate the response as shown in Figure 5.27-2.

Conclusions

Composites of concrete and paraffin wax can transfer more heat than either of the constituents alone when exposed to a step change in surface temperature away from the fusion temperature of the composite. A conservative estimate of the relative rate at which heat is transferred can be made by use of the Neumann solution using average composite
properties. However, multi-dimensional effects can be significant which must be assessed by either experimental or numerical approaches.

Reliable thermophysical property data were difficult to obtain if they existed at all. Paraffin properties are available for ultra pure materials but these data are scarce for commercial products. Little is known about the thermal conductivity of solid paraffin. It was found that mixtures of paraffins with comparable chain length can have substantially lower latent heat than the pure hydrocarbons. This precludes use of slack waxes which were originally considered for economic reasons as a logical industrial by-product to mix with the concrete.

Concrete thermophysical properties are difficult to generalize due to the variability of components, mixes and curing. Although trends were established, specific determinations are recommended to obtain reliable data.

Computation of average composite thermophysical properties should be done with care, especially thermal conductivity. Many investigators have studied this problem and these methods result in widely varying values.

Effective thermal diffusivity reasonably approximates the thermal response for the Stefan phase change problem for concrete/wax types of composites because of the relative relaxation times of the conduction and phase change problems. However, care should be exercised when applying
this concept to other types of transient boundary conditions.

Although convection was predicted and observed for the experimental composite, it is unlikely in the proposed concrete/wax composite. Since the extremely low thermal conductivity of the liquid paraffin was one of the primary factors for adding the mortar filler matrix, it is unfortunate that the matrix which was to enhance the heat transfer may in fact be suppressing it for some conditions.

Visual observations of the phase front were invaluable in studying the nature of the phase front. Without automated data acquisition, accurate measurement of the thermal response of the specimen would have been impossible. In general, the experimental apparatus was found to be quite adequate with the possible exception of only one freezing AT. Also use of heat meter instrumentation at the heat exchange surface would have aided in making direct comparisons between measured results and the Neumann model.

The systematic test sequence used was vital in confirming the thermophysical properties and averaging approach used, i.e., testing the specimen void, then filling the voids with paraffin and retesting but not crossing the fusion temperature, and finally testing by freezing and melting. In general, the correlation between the normalized measured data and the semi-infinite solid solution was remarkably good over the wide range of void fractions.
Both visual observations of the position of the phase front and measured temperature response of freezing runs indicated that the two dimensional composite responded more rapidly than predicted by the Neumann approach by a factors of 1.2 to 1.5. This enhanced heat transfer rate could be predicted by using numerical methods.

The unique two-dimensional numerical model developed in this study was found to correlate well with measured freezing data while reducing computing time over conventional grid patterns. A matrix of cases was investigated with the numerical model. For some combinations of conductivity ratio and Stefan number, significantly enhanced rates of heat transfer were calculated, see Table 5.8. Based on the numerical solutions, a Neumann model multiplicative factor was developed to account for two dimensional freezing effects. This expression yields predictions of 30 to 40% enhancement in heat transfer over the Neumann model for a wide range of conditions. Although the optimum paraffin fraction was found to be the geometric maximum, heat transfer ratios approached these values asymptotically and economics may dictate optimum paraffin fractions considerably lower.

Melting phase fronts propagation was more complex than the freezing front. In addition to two dimensional phase fronts, buoyancy effects of the solid paraffin and
convection were present. Until convection became dominate, it was found that the conduction only numerical model predicted the thermal response of the experimental specimens quite well during melting. Steady state convective algorithms used to model the convection were found to over predict the thermal response. A transitional model appears to be required.

Finally, although there were some discouraging findings, it was conclusively shown that these types of composites have heat transfer advantages for the ideal conditions investigated in this study. Further research should be conducted on these types of composites including other geometries and PCM's, convection, and other types of boundary conditions.
Appendix A

Data Reduction Programs
A Fortran Program named Tally/IRCC was used to convert the coded paper tape, Figure A.1, into punched card output. The format of the punched cards was run number (16), scan number (16), ten channels of millivolt data times 100 (10I6), and card number sequence x of y, Figure A.2. Table A.1 lists the processing steps required to operate the program.

After entering run data through the console keyboard, one scan of data was read into the memory. The binary coded data was checked to see that it contained the correct number of channels and that the first channel in the set was channel one. If either of these criteria failed, the system would type a message 'YOU HAVE LOADED THE TAPE WRONG' and would index the scan counter. The tape was then reloaded at the beginning of the next scan and the processing was continued. Note that the scan in which the error was detected would be skipped. If the set was free of errors, the program would convert the binary data and punch a set of up to 3 cards for that scan. Another scan was then read through the tape reader and the process was repeated.
Figure A.1  Tape Code.
Figure A.2  Millivolt Punched Card Output for Run 0813, Scan 1 and 2, 16 Channels of Data.
Table A1.1 Paper Tape to Cards Processing Steps.

<table>
<thead>
<tr>
<th>Step</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Place 0011 Disk in Outboard Disk Drive</td>
</tr>
<tr>
<td>2</td>
<td>Turn on 1130 System</td>
</tr>
<tr>
<td>3</td>
<td>Enable Console and Outboard Disks</td>
</tr>
<tr>
<td>4</td>
<td>Load Paper Tape (Last Row of Header or Last End Symbol of Previous Scan on the Sprocket in the 12:00 Position)</td>
</tr>
<tr>
<td>5</td>
<td>Load Program in Hopper</td>
</tr>
<tr>
<td>6</td>
<td>Press Stop, Reset, Load (Program Read to XEQ Card)</td>
</tr>
<tr>
<td>7</td>
<td>Press Start on Reader</td>
</tr>
<tr>
<td>8</td>
<td>Enter Run Number and Number of Channels from Keyboard (I4,1X,I2)</td>
</tr>
<tr>
<td>9</td>
<td>Press NPRO and Remove Program Deck</td>
</tr>
<tr>
<td>10</td>
<td>Load Blank Cards and Press Start</td>
</tr>
<tr>
<td>11</td>
<td>Enter 1 from Keyboard</td>
</tr>
</tbody>
</table>

**Note:** The tape will now be read scan by scan and the data will be punched on cards (One card per 10 channels, up to 3 cards per scan).

**Note:** If the processor detects an error in the punch sequence, the message 'YOU HAVE LOADED THE TAPE WRONG' will be typed. Repeat steps 4 and 11.
THE PURPOSE OF THIS PROGRAM IS TO READ THE TALLY
CODED PAPER TAPE. THE FORTRAN PROGRAM CALLS AN
ASSEMBLER SUBROUTINE CALLED PAPTX VIA ANOTHER
SUBROUTINE CALLED INTPR.

DIMENSION KARRA(240), IDATI(30)

ENTERING THE RUN NO. AND NO. OF CHANNELS FROM
THE KEYBOARD

READ (6,110) IRUN, NCH

FORMAT (I4, 1X, I12)

COMPUTING THE NO. OF FRAMES AND THE NO. OF
cards required

NF = 8 * NCH
NS = 0
IF (NCH - 10) 61, 61, 2
   NCA = 1
   GO TO 4
2 IF (NCH - 20) 62, 62, 3
   NCA = 2
   GO TO 4
3 NCA = 3
4 CONTINUE

STARTING THE TAPE

READ (6,120) IGO

FORMAT (I11)
IF (IGO - 9) 5, 999, 5
5 NS = NS + 1
DO 6 I = 1, 30
6 IDATI(I) = 0

READ IN ONE DATA SET

DO 10 I = 1, NF
   KARRA(I) = INPTF(8)
10 CONTINUE

CHECK TO SEE IF THE TAPE HAS BEEN LOADED CORRECTLY

IF (KARRA(1) - 32) 21, 18, 21
18 IF (KARRA(2)-1) 21,19,21
19 CONTINUE
   KO=-NCH*128
20 CONTINUE
   DO 20 I=8,NF,8
   KO=KO+KARRA(I)
      CONTINUE
21 IF (KO) 21,28,21
22 WRITE (1,100)
23 FORMAT (1X,'YOU HAVE LOADED THE TAPE WRONG')
   GO TO 4
28 CONTINUE
   DO 53 I=1,NF
      IF (KARRA(I)-32) 51,50,51
50 KARRA(I)=0
      GO TO 53
51 IF (KARRA(I)-16) 53,53,52
52 KARRA(I)=KARRA(I)-16
53 CONTINUE
J=-4
   DO 32 I=1,NCH
      J=J+8
      IDATI(I)=KARRA(J)*1000+KARRA(J+1)*100+KARRA(J+2)*100+KARRA(J+3)
31 IF (KARRA(J-1)-48) 32,31,32
32 CONTINUE
5 NC=1
   WRITE (2,113) IRUN,NS,(IDATI(I),I=1,10),NC,NCA
113 FORMAT (I4,I6,10I6,4X,' OF ',12)
5 NC=2
   WRITE (2,113) IRUN,NS,(IDATI(I),I=11,20),NC,NCA
63 NC=3
   WRITE (2,113) IRUN,NS,(IDATI(I),I=21,30),NC,NCA
40 GO TO 5
999 STOP
END
// XEQ
Millivolt to Engineering Units
And Dimensionless Ratios

A Fortran Program named CONVERT was used to convert the raw millivolt data cards to engineering units. Run characteristics were read from cards images, Figure A.3. Millivolt data was converted to temperatures in °F by linearly interpolating the data at channel X with those of the known reference baths. Time in hours was calculated from the scan number and the known interval between scans.

Using average composite properties and reference temperatures, dimensionless temperatures \( \frac{T-T_o}{T_i-T_o} \), and penetration parameter, \( x/(\alpha t)^{1/2} \) were also calculated by CONVERT. If phase change occurred during the run, the temperature, \( T_i \), was taken to be the input fusion temperature, \( T_f \). However, if no change of phase occurred, the initial temperature was taken to be the average measured specimen temperature from the scan prior to time zero. The surface temperature, \( T_o \), was taken to be the average measured temperature at the interface between the aluminum plate and the epoxy (channels 5 and 6) for that scan.

The position, \( x \), in the penetration parameter was calculated based on the thickness of the specimen and the number of rows. Time was computed from the scan number and the known interval between scans.
<table>
<thead>
<tr>
<th>Card 1</th>
<th>Card 2</th>
<th>Card 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Ch.'s</td>
<td>Scan at t = 0</td>
<td>Run No.</td>
</tr>
<tr>
<td>15</td>
<td>60.0</td>
<td>0813-1</td>
</tr>
</tbody>
</table>

Time Interval, Sec. No. of Scans

<table>
<thead>
<tr>
<th>Low Temp. Ref. Ch. No.</th>
<th>Low Ref Temp., F</th>
<th>Stabilized Inlet Temp., F</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4</td>
<td>32.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>High Temp. Ref. Ch. No.</th>
<th>High Ref Temp., F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diffusivity, Sq. Ft./Hr.</td>
<td>Time Interval, Ch's, Sec.</td>
</tr>
<tr>
<td>0.0063</td>
<td>8.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>No. of Rows</th>
<th>Depth, Ft.</th>
<th>First</th>
<th>Last</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ch. No.,'s, Initial Cond.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure A.3 Input Run Characteristic Data, Run 0813.
The output was written onto magnetic tape for later analysis and plotting.
THE PURPOSE OF THIS PROGRAM IS TO CONVERT THE RAW MILLIVOLT DATA TO ENGINEERING UNITS AND CALCULATE NONDIMENSIONAL TIME AND TEMPERATURE.

DIMENSION ID(90,15), T(90,15), TI(90,15), TN(90,15), CINDEX(90,15),
CTS(90,9), TIS(90,9), TNDS(90,9), TINDS(90,9), X(15),
CTT(7), TTI(7)

KRUN=1

READ (5,300) NOCH, STEP, ISTART, NSCAN
300 FORMAT (I10, F10.1, I110)
IF (NOCH.LT.0) GO TO 999
READ (5,310) M, N, TL, TH, TINF, TF
310 FORMAT (2I10, 4F10.1)
READ (5,320) ALPHA, NROW, DTCH, XL, ITYPE, ICF, ICL
320 FORMAT (F10.4, I10, F10.1, F10.4, I110)
DO 10 I=1, NSCAN
READ (5,*) IR1, IS1, (ID(I,J), J = 1, 10)
IF (NOCH.LE.IO) GO TO 10
READ (5,*) IR2, IS2, (ID(I,J), J = 11, 20)
IF (IR2.NE.IR1.OR.IS2.NE.IS1) GO TO 999
10 CONTINUE
IR=IR1
DO 20 I=1, NSCAN
C CONVERTING MILLIVOLTS TO TEMPERATURES
DO 20 J=1, NOCH
T(I,J) = (TH-TL) * (ID(I,J)-ID(I,M))/(ID(I,N)-ID(I,M)) + TL
C CONVERTING SCANS TO TIME
20 TI(I,J) = (I-ISTART)*STEP/3600. +(J-1)*DTCH/3600.
IF (IR1.GT.499) GO TO 29
C REASSIGNING CHANNEL NO'S FOR THE 4 ROW SPECIMEN TO CONFORM WITH THE STANDARD.
DO 25 I=1, NSCAN
T(I,8) = T(I,10)
TI(I,8) = TI(I,10)
T(I,9) = T(I,12)
TI(I,9) = TI(I,12)
T(I,10) = T(I,14)
TI(I,10) = TI(I,14)
T(I,11) = T(I,13)
25 TI(I,11) = TI(I,13)
29 CONTINUE
C COMPUTING THE REFERENCE TEMPERATURE, TREF. TREF = TF FOR PHASE CHANGE RUNS, AND THE AVERAGE TEMPERATURE ON SCAN BEFORE ISTART FOR SEMI-INFINITE SOLID RUNS.
SUM=0.0
IAVE=ISTART-1
DO 30 J=ICF, ICL
30 SUM=SUM+T(IAVE,J)
TREF=SUM/(ICL-ICF+1)

IF (ITYPE.EQ.1) TREF=TF

C COMPUTING DEPTHS, X
DO 40 J=6,N0CH
X(J)=(J-6)*XL/(NROW)
40 IF (X(J).GT.XL) X(J)=XL
X(5)=X(6)

C COMPUTING DIMENSIONLESS PARAMETERS
DO 50 I=1,NSCAN

C COMPUTING DIMENSIONLESS TEMPERATURE (T-T0)/(TREF-T0)
T0=(T(I,5)+T(I,6))/2.
DO 50 J=5,N0CH
TND(I,J)=(T(I,J)-T0)/(TREF-T0)

C COMPUTING THE PENETRATION PARAMETER, X/SQRT(ALPHA*TIME)

50 TIND(I,J)=X(J)/SQRT(ALPHA*ABS(TI(I,J)))

C COMPUTING CUTOFF FOR SEMI-INFINITE RESPONSE
JJ=6+NROW-1
J1=JJ+1
J2=JJ+2
NTEST=NSCAN
DO 55 I=1,NSCAN
TEST=(TND(I,J1)+TND(I,J2))/2.
55 IF (TEST.GT.0.95) NTEST=I

C COMPILING SUMMARY
DO 70 I=1,NTEST
TS(I,1)=(T(I,5)+T(I,6))/2.
TIS(I,1)=60*(T(I,5)+TI(I,6))/2.
TNDS(I,1)=(TND(I,5)+TND(I,6))/2.
TINDS(I,1)=(TIND(I,5)+TIND(I,6))/2.
DO 60 J=7,JJ
KK=J-5
TS(I,KK)=T(I,J)
TIS(I,KK)=60*TI(I,J)
TNDS(I,KK)=TND(I,J)
60 TINDS(I,KK)=TIND(I,J)
NRP1=NROW+1
TS(I,NRP1)=(T(I,J1)+T(I,J2))/2.
TIS(I,NRP1)=60*(TI(I,J1)+TI(I,J2))/2.
TNDS(I,NRP1)=(TND(I,J1)+TND(I,J2))/2.
70 TINDS(I,NRP1)=(TIND(I,J1)+TIND(I,J2))/2.
WRITE (6,90) IR
90 FORMAT ('!','RUN ',I5,/) 
DO 80 I=1,NTEST
WRITE (6,100) IR
80 FORMAT (/,I10,' 0-1 1-2 2-3 3-4 ','
C'4-5 5-6 6-7 7-8 8-9')
WRITE (6,200) (TS(I,J),J=1,NRP1)
200 FORMAT (' TS(I,J) ','9F8.3')
WRITE (6,210) (TIS(I,J),J=1,NRP1)
210 FORMAT (' TIS(I,J) ',9F8.3)
WRITE (6,220) (TNDS(I,J),J=1,NRP1)
220 FORMAT (' TNDS(I,J) ',9F8.3)
80 WRITE (6,230) (TINDS(I,J),J=1,NRP1)
230 FORMAT (' TINDS(I,J) ',9F8.3)
NROW=(NTEST-2)*(NRPL-1)
NCOL=4
GO TO (101,102,103,104,105,106,107,108,109,110),KRUN
101 WRITE (11,400) NROW,NCOL,IR
GO TO 111
102 WRITE (12,400) NROW,NCOL,IR
GO TO 111
103 WRITE (13,400) NROW,NCOL,IR
GO TO 111
104 WRITE (14,400) NROW,NCOL,IR
GO TO 111
105 WRITE (15,400) NROW,NCOL,IR
GO TO 111
106 WRITE (16,400) NROW,NCOL,IR
GO TO 111
107 WRITE (17,400) NROW,NCOL,IR
GO TO 111
108 WRITE (18,400) NROW,NCOL,IR
GO TO 111
109 WRITE (19,400) NROW,NCOL,IR
GO TO 111
110 WRITE (20,400) NROW,NCOL,IR
CONTINUE
400 FORMAT (315)
DO 140 I=3,NTEST
DO 140 J=2,NRP1
GO TO (121,122,123,124,125,126,127,128,129,130),KRUN
121 WRITE (11,410) TIS(I,J),TS(I,J),TINDS(I,J),TNDS(I,J)
GO TO 140
122 WRITE (12,410) TIS(I,J),TS(I,J),TINDS(I,J),TNDS(I,J)
GO TO 140
123 WRITE (13,410) TIS(I,J),TS(I,J),TINDS(I,J),TNDS(I,J)
GO TO 140
124 WRITE (14,410) TIS(I,J),TS(I,J),TINDS(I,J),TNDS(I,J)
GO TO 140
125 WRITE (15,410) TIS(I,J),TS(I,J),TINDS(I,J),TNDS(I,J)
GO TO 140
126 WRITE (16,410) TIS(I,J),TS(I,J),TINDS(I,J),TNDS(I,J)
GO TO 140
127 WRITE (17,410) TIS(I,J),TS(I,J),TINDS(I,J),TNDS(I,J)
GO TO 140
128 WRITE (18,410) TIS(I,J),TS(I,J),TINDS(I,J),TNDS(I,J)
GO TO 140
129 WRITE (19,410) TIS(I,J),TS(I,J),TINDS(I,J),TNDS(I,J)
GO TO 140
130 WRITE (20,410) TIS(I,J),TS(I,J),TINDS(I,J),TNDS(I,J)
410 FORMAT (4F10.3)
140 CONTINUE
KRUN=KRUN+1
IF (KRUN.GT.10) GO TO 999
GO TO 1
999 WRITE (6,*), IR
WRITE (6,*), NTEST
STOP
END
Average Properties

Average composite properties were calculated with a fortran program named APRAY. This program calculated average conductivity, \( \kappa \), using the Rayleigh method for a square array of circular inclusions, see Table 2.8, average density, \( \bar{\rho} \), average specific heat, \( \bar{c} \), and average latent heat, \( \bar{L} \), see Equations 2.22, 2.23, and 2.24 respectively. Average diffusivity, \( \bar{\alpha} \), was calculated for \( \kappa, \bar{\rho}, \) and \( \bar{c} \), i.e. \( \bar{\alpha} = \kappa / (\bar{\rho} \bar{c}) \). Average Stefan number, \( \bar{\text{Ste}} \), was calculated from \( \bar{c}, \bar{L}, \) and a given temperature difference, \( \Delta T, \bar{\text{Ste}} = \bar{c} \Delta T/\bar{L} \).

These values were calculated for each specimen type and test condition and were printed out in the format shown in Table 5.1. In addition, using these average properties, Neumann solution thermal response was calculated and written onto magnetic tape for later analysis and plotting.
THE PURPOSE OF THIS PROGRAM IS TO CALCULATE AVERAGE COMPOSITE PROPERTIES FOR A SQUARE ARRAY OF CIRCULAR INCLUSIONS. RAYLEIGH'S METHOD WAS USED TO CALCULATE THE THERMAL CONDUCTIVITY. IN ADDITION, USING THESE AVERAGE PROPERTIES, THE THERMAL RESPONSE BASED ON THE NEUMANN SOLUTION IS CALCULATED.

REAL KM,KP,LP,KB,LB,K,L
REAL LAM,LAMDA
DIMENSION KM(4),RM(4),CM(4),KP(4),RP(4),CP(4),LP(4),CTF(4),SA(6)
DIMENSION V(6),K(6,4),R(6,4),C(6,4),A(6,4),L(6,4),CS(6,4),B(6,4)
DIMENSION Q(6,4)
DIMENSION Z(250),T(6,4,250),LAM(6,4)
DATA KM /0.48,0.55,0.48,0.55/
DATA RM /108.7,108.5,108.7,108.5/
DATA CM /0.25,0.28,0.25,0.28/
DATA KP /1.0,1.0,1.0,1.0/
DATA RP /1.0,1.0,1.0,1.0/
DATA CP /1.0,1.0,1.0,1.0/
DATA LP /100.,100.,100.,100./
DATA LP /0.0,0.0,105.6,107.0/
DATA X,RR,DT /1.4,0.074,100. /
DATA X,RR,DT /1.4,0.074,49.0/
DATA SA /1.0,0.7,0.35,0.233,0.175,1.0/
PI=3.1415927
DO 10 NC=1,6
SP=SA(NC)
BS=PI*(RR/SP)**2.0
IF (NC.EQ.1) BS=0.0
IF (NC.EQ.6) BS=1.0
DO 10 J=1,4
1 CONTINUE
ALPHA=BS*(KP(J)-KM(J))/(KP(J)+KM(J))
IF (ALPHA.EQ.1.0) GO TO 2
KB=KM(J)*(ALPHA+1.)/(1.-ALPHA)
GO TO 3
2 KB=KP(J)
3 CONTINUE
IF (NC.EQ.6) KB=KP(J)
RB=(1.-BS)*RM(J)+BS*RP(J)
CB=((1.-BS)*RM(J)*CM(J)+BS*RP(J)*CP(J))/RB
AB=KB/RB/CB
LB=BS*RP(J)*LP(J)/RB
SB=LB/(DT*CB)
CALL NUMAN (AB,SB,KB,DT,QN,LAMDA)
LAM(NC,J)=LAMDA
V(NC)=BS
K(NC,J)=KB
R(NC,J)=RB
C(NC,J)=CB
A(NC,J)=AB
L(NC,J)=LB
S(NC,J)=SB
Q(NC,J)=QN
10 CONTINUE
WRITE (6,102)
102 FORMAT ('1','AVERAGE COMPOSITE PROPERTIES /',
C' SQ. ARRAY OF CIRCLES')
WRITE (6,110)
110 FORMAT ('0','PROPERTY',16X,'MATRIX',4X,
C'2 ROWS',4X,'4 ROWS',
C4X,'6 ROWS',4X,'8 ROWS',3X,'AIR/PCM')
WRITE (6,120)
120 FORMAT ('0','VOIDAGE')
WRITE (6,130) (V(I),I=1,6)
130 FORMAT ('0',20X,6F10.3)
WRITE (6,140)
140 FORMAT ('0','CONDUCTIVITY')
WRITE (6,150) ((K(I,J),I=1,6),J=1,4)
150 FORMAT ('*',20X,6F10.3)
WRITE (6,160)
160 FORMAT ('0','DENSITY')
WRITE (6,170) ((R(I,J),I=1,6),J=1,4)
170 FORMAT ('*',20X,6F10.2)
WRITE (6,180)
180 FORMAT ('0','SP. HEAT')
WRITE (6,190) ((C(I,J),I=1,6),J=1,4)
190 FORMAT ('*',20X,6F10.3)
WRITE (6,200)
200 FORMAT ('0','DIFFUSIVITY')
WRITE (6,210) ((A(I,J),I=1,6),J=1,4)
210 FORMAT ('*',20X,6F10.4)
WRITE (6,220)
220 FORMAT ('0','LATENT HEAT')
WRITE (6,230) ((L(I,J),I=1,6),J=1,4)
230 FORMAT ('*',20X,6F10.3)
WRITE (6,240) DT
240 FORMAT ('0','STEYAN/F',F4.0,'F')
WRITE (6,250) ((S(I,J),I=1,6),J=1,4)
250 FORMAT ('*',20X,6F10.3)
WRITE (6,260)
260 FORMAT ('0','Q NEUMANN')
WRITE (6,270) ((Q(I,J),I=1,6),J=1,4)
270 FORMAT ('*',20X,6F10.2)
DO 20 NC=1,6
DO 20 J=1,4
DO 20 IX=1,250
Z(IX)=0.02*IX
T(NC,J,IX)=ERF(Z(IX)/2.)/ERF(LAM(NC,J))
20 IF (T(NC,J,IX).GT.1.0) T(NC,J,IX)=1.0
DO 30 J=1,4
WRITE (6,410)
DO 30 KK=1,250
GO TO (21,22,23,24),J
GO TO (30,30,23,24),J
21 WRITE (11,400) Z(KK),(T(I,J,KK),I=1,6)
GO TO 30
22 WRITE (12,400) Z(KK),(T(I,J,KK),I=1,6)
GO TO 30
23 WRITE (13,400) Z(KK),(T(I,J,KK),I=1,6)
GO TO 30
24 WRITE (14,400) Z(KK),(T(I,J,KK),I=1,6)
WRITE (6,400) Z(KK),(T(I,J,KK),I=1,6)
30 CONTINUE
400 FORMAT (10X,7F10.3)
410 FORMAT ('0')
STOP
END

SUBROUTINE NUMAN (AB,SB,KB,DT,Q,LAM)
REAL KB,LB,LAM
FUNCY(F,X)=F-X*EXP(X**2.)*ERF(X)
DATA N1,N2,N3 /10,10,10/
PI=3.141592654
IF (SB.LE.0.0) GO TO 10
FL=1./(SB*PI**0.5)
DXEP=0.001
TEST=0.0001
XMIN=0.0
YMIN=FL
X2=XMIN
IF (YMIN.LT.0.0) GO TO 6
XMAX=1.0-DXEP
YMAX=FUNCY(FL,XMAX)
X2=XMAX
IF (YMAX.GT.0.0) GO TO 6
DX=(1.-2.*DXEP)/N1
X1=XMIN
Y1=YMIN
1 X2=X1+DX
Y2=FUNCY(FL,X2)
IF (ABS(Y2).LT.TEST) GO TO 6
IF ((Y1/Y2).LE.0.0) GO TO 2
X1=X2
Y1=Y2.
GO TO 1
2 DX=(X2-X1)/N2
3 X2=X1+DX
Y2=FUNCY(FL,X2)
IF (ABS(Y2).LT.TEST) GO TO 6
IF ((Y1/Y2).LE.0.0) GO TO 4
X1=X2
Y1=Y2
GO TO 3
4 DX=(X2-X1)/N3
5 X2=X1+DX
Y2=FUNCY(FL,X2)
IF (ABS(Y2).LT.TEST) GO TO 6
IF ((Y1/Y2).LE.0.0) GO TO 6
X1=X2
Y1=Y2
GO TO 5
6 CONTINUE
LAM=X2
Q=(2.*KB*DT)/(ERF(LAM)*SQRT(PI*AB))
RETURN
10 Q=(2.*KB*DT)/SQRT(PI*AB)
LAM=10.0
RETURN
END
Appendix B

Numerical Analysis Programs
Program Description

This program computes the transient temperature field in a thick wall with specified temperature on one face and insulated on the other. The wall is a composite of two materials. In cross section, a square array of circular regions of a phase change material (PCM) are surrounded by a continuum of an impervious solid material (MATRIX).

A unique difference nodal pattern is used to describe this geometry. A series of two dimensional cylindrical coordinate regions describe the PCM regions, TPCM(X,R,THETA), and adjacent matrix regions, TM(X,R,THETA) outward to the point of tranfugeney of the cylinders. The remaining diamond shaped regions, T(X), are approximately orthogonal to adjacent cylindrical nodes of arch length (P1/4)*S. The radius of the outer ring of matrix nodes is adjusted to account for the remaining matrix material. The explicit Euler Method is used to step forward in time. The temperature of the PCM nodes is held at the fusion temperature, TF, until sufficient heat is transferred to balance the nodal latent heat.

Convective heat transfer in the liquid PCM is computed based on Silveston Nusselt number correlations. Convective stability is based on the Katto and Musoka stability criteria for a porous layer heated from below.

For the freezing problem, lumped nodes of thickness, S,
are used between the temperature boundary and the row in which the phase front is located. Average composite thermophysical properties are used for these lumped nodes.

Since initial conditions being studied are near the fusion temperature, $TF$, of the PCM, only rows between the specified temperature surface and one row beyond where the phase front is loaded are included in the computation for freezing and melting runs. All rows are included in non phase change runs i.e. runs in which temperatures do not cross the fusion temperature of the PCM.

The data for the run is listed as follows:
Void radius ($R$) = 0.074 in. Void spacing ($S$) = 0.350 in.
No. of rows (NROWS) = 4
Ave. Composite Conductivity (KBAR) = 0.342 BTU/HR-FT-F
Ave. Composite Density (RBAR) = 100.92 LBM/CU FT
Ave. Composite Specific Heat (CBAR) = 0.265 BTU/LBM-F
Ave. Composite Latent Heat (LBAR) = 7.832 BTU/LBM
No. R Divisionsd in PCM Region (NP) = 3
No. R Divisions Plus 1 in Matrix Region (NM) = 2
Fusion Temperature ($TF$) = 81.0 F
Initial Condition Delta Offset (DTF) = 0.00 F
PCM Conductivity (KP) = 0.150 BTU/HR-FT-F
PCM Density (RHOP) = 53.3 LBM/CU FT
PCM Specific Heat (CAP) = 0.45 BTU/LBM-F
PCM Latent Heat (LH) = 105.6 BTU/LBM
PCM Beta Group (BETAP) = 0.0005 1/F
PCM Kinematic Viscosity (VISP) = 0.000039 SQ FT/SEC
Matrix Conductivity (KM) = 0.48 BTU/HR-FT-F
Matrix Density (RHOM) = 108.7 LBM/CU FT
Matrix Specific Heat (CAM) = 0.25 BTU/LBM-F
Convection Analysis Control Word (NCONV) = 0 if NCONV=0 No
Convection
No. of Data Points in Surface Temperature and Time Arrays
TAF and TIF (NDP) = 2
Surface time Array (TIH), HR
   0.0     0.017
Surface Temperature Array (TAF), F
   21.890  21.890
Delta Time Between Printing (DTIPR) = 0.50 Min.

The limiting case of one dimensional radial PCM
freezing with a constant phase front velocity is also
included. This solution by Kreith and Romie can be
specified by selecting the Kreith indicator, KRE, and the
temperature gradient at the phase front, GK. For this run
KRE = -1 and GK = 10927.

For each node type and size, a stable time step is
calculated by a method based on second law reasoning
outlined by Rohsenow and Hartnett. Of these, the minimum
time step is selected and used throughout the computation.
For this run, the time step is 0.27947 seconds.
At selected time intervals, DTIPR, the complete temperature field is printed. Temperatures are in degrees F. Also normalized data are presented. Array notations correspond to that used in the Fortran program. For phase change runs, only detailed cylindrical coordinate nodal temperatures are presented for the row in which the phase front is located and the next further row. For non phase change runs, all rows are printed.

The notation used is defined below:

Time = Time, T
QS = Net heat transfer at the specified temperature boundary, X=0, from T = 0
QN = Net heat transfer at X = 0 Calculated from the Neumann solution using average composite properties.
DEPTH = Depth, X, from the specified temperature boundary.
TN(I) = Temperature at the center of the diamond shaped regions, these points correspond to thermocouple location in the experimental portion of this research. These temperatures are aligned under the corresponding X locations.
TPCMN(I) = Temperatures at the center of the PCM regions i.e. TP(I,1,1)
TIND(I) = Normalized penetration parameters, DEPTH/SQRT(ALPHA*TIME). ALPHA is the average composite thermal diffusivity.
TNND(I) = Normalized TN(I) temperatures, \((TN(I) - TREF)/(TF - TREF)\) where TF is the PCM fusion temperature and TREF is the surface temperature, TN(1).

TPCND(I) = Normalized TPCMNI(I) Temperatures.

TPN(I,J,K) = Temperatures in the cylindrical PCM region, I = Row, I = 1 is the row next to the specified temperature boundary, J = radial position. J = 1 is at the center of the PCM region, and K = THETA position. K = 2 and 6 are in line with the depth coordinate K = 2 is nearest X = 0.

TPNND(I,J,K) = Normalized TPN(I,J,K) temperatures.

TMN(I,J,K) = Temperatures in the cylindrical coordinate matrix region, J = 1 is at the matrix PCM interface.

TMNND(I,J,K) = Normalized TMN(I,J,K) temperatures

TPK(J) = KREITH solution PCM temperatures
PROGRAM DESCRIPTION:

THIS PROGRAM COMPUTES THE TRANSIENT TEMPERATURE FIELD IN A THICK WALL WITH SPECIFIED TEMPERATURE ON ONE FACE AND INSULATED ON THE OTHER. THE WALL IS A COMPOSITE OF TWO MATERIALS. IN CROSS SECTION, A SQUARE ARRAY OF CIRCULAR REGIONS OF A PHASE CHANGE MATERIAL (PCM) ARE SURROUNDED BY A CONTINUUM OF AN UNPREVIOUS SOLID MATERIAL (MATRIX).

A UNIQUE FINITE DIFFERENCE NODAL PATTERN IS USED TO DESCRIBE THIS GEOMETRY. A SERIES OF TWO DIMENSIONAL CYLINDRICAL COORDINATE REGIONS DESCRIBE THE PCM REGIONS, TPCM(R,R,T), AND ADJACENT MATRIX REGIONS, TM(R,R,T). THE REPAIRING DIAMOND SHAPED REGIONS, TDL, ARE APPEARENTLY ORTHOGONAL TO ADJACENT CYLINDRICAL NODES OF ARCH LENGTH (P/A). THE RADII OF THE OUTTER RING OF MATRIX NODES ARE ADJUSTED TO ACCOUNT FOR THE REMAINING MATRIX MATERIAL. THE EXPLICIT EULER METHOD IS USED TO STEP FORWARD IN TIME. THE TEMPERATURE OF THE PCM NODES IS HELD AT THE FUSION TEMPERATURE, TF, UNTIL SUFFICIENT HEAT IS TRANSFERRED TO BALANCE THE NODAL LATENT HEAT.

CONVECTIVE HEAT TRANSFER IN THE LIQUID PCM IS COMPUTED BASED ON SILVESTON NUSSELTS NUMBER CORRELATIONS. CONVECTIVE STABILITY IS BASED ON THE KATTO AND MUSOKA STABILITY CRITERIA FOR A POROUS LAYER HEATED FROM BELOW.

FOR THE FREEZING PROBLEM, LUMPED NODES OF THICKNESS, B, ARE USED BETWEEN THE TEMPERATURE BOUNDARY AND THE ROW IN WHICH THE PHASE FRONT IS LOCATED. AVERAGE COMPOSITE THERMOPHYSICAL PROPERTIES ARE USED FOR THESE LUMPED NODES.

SINCE INITIAL CONDITIONS BEING STUDIED ARE NEAR THE FUSION TEMPERATURE, TF, OF THE PCM, ONLY ROWS BETWEEN THE SPECIFIED TEMPERATURE SURFACE AND ONE ROW BEYOND WHERE THE PHASE FRONT IS LOCATED ARE INCLUDED IN THE COMPUTATION FOR FREEZING AND MELTING RUNS. ALL ROWS ARE INCLUDED IN MON PHASE CHANGE RUNS I.E., RUNS IN WHICH TEMPERATURES DO NOT CROSS THE FUSION TEMPERATURE OF THE PCM.

THE DATA FOR THIS RUN WAS:

VOID RADIUS (R) = 0.074 IN.
VOID SPACING (S) = 0.175 IN.
NO. OF ROWS (MROWS) = 8
AVG. COMPOSITE CONDUCTIVITY (KBAR) = 0.262 BTU/HR-FT-F
AVG. COMPOSITE DENSITY (RBAR) = 77.58 LB/M-CU FT
AVG. COMPOSITE SPECIFIC HEAT (CBAR) = 0.327 BTU/LBM-F
AVG. COMPOSITE LATENT HEAT (LBAR) = 40.755 BTU/LBM
NO. R DIVISIONS IN PCM REGION (KP) = 3
NO. R DIVISIONS PLUS 1 IN MATRIX REGION(NM) = 2
FUSION TEMPERATURE (TF) = 81.0 F
INITIAL CONDITION DELTA OFFSET (DTP) = 0.00 F
PCMC CONDUCTIVITY (KP) = 0.150 BTU/HR-FT-F
PCMC DENSITY (BKP) = 53.3 LBM/CU FT
PCMC SPECIFIC HEAT (CFP) = 0.45 BTU/LBM-F
PCMC LATENT HEAT (LFP) = 105.6 BTU/LBM
PCMC KINEMATIC VISCOSITY (VF) = 0.00039 SF/SEC
MATRIX CONDUCTIVITY (RM) = 0.48 BTU/HR-FT-F
MATRIX DENSITY (RM) = 108.7 LB/M-CU FT
MATRIX SPECIFIC HEAT (CM) = 0.25 BTU/LBM-F
CONVECTION ANALYSIS CONTROL WORD (NCONV) = 0 IF NCONV=0 NO CONVECTION
NO. OF DATA POINTS IN SURFACE TEMPERATURE AND TIME ARRAYS TAF AND TIF (NDF) = 2
SURFACE TIME ARRAY (TAF), HR
0.0 0.030

Figure B.1-1 Example of Numerical Program Output
SURFACE TEMPERATURE ARRAY (TAF), F
56.080 56.080

DELTA TIME BETWEEN PRINTING (DTIPR) = 1.00 MIN.

THE LIMITING CASE OF ONE DIMENSIONAL RADIAL PCM FREEZING WITH A CONSTANT PHASE FRONT VELOCITY IS ALSO INCLUDED. THIS SOLUTION BY KREITH AND ROMIE CAN BE SPECIFIED BY SELECTING THE KREITH INDICATOR, KRE, AND THE TEMPERATURE GRADIENT AT THE PHASE FRONT, G. FOR THIS RUN KRE = -1.00 AND G = -19027.

FOR EACH NODE TYPE AND SIZE, A STABLE TIME STEP IS CALCULATED BY A METHOD BASED ON second Law reasoning outlined by Borewski and Hartnett. OF THESE, THE MINIMUM TIME STEP IS SELECTED AND USED THROUGHOUT THE COMPUTATION.

FOR THIS RUN, THE TIME STEP IS 0.00405 SECONDS.

THE MINIMUM TIME STEP IS AT LOCATION 1

AT SELECTED TIME INTERVALS, DTIPR, THE COMPLETE TEMPERATURE FIELD IS PRINTED. TEMPERATURES ARE IN DEGREES F. ALSO NORMALIZED DATA ARE PRESENTED. ARRAY NOTATIONS CORRESPOND TO THAT USED IN THE FORTRAN PROGRAM. FOR PHASE CHANGE RUNS, ONLY DETAILED CYLINDRICAL COORDINATE NODE TEMPERATURES ARE PRESENTED FOR THE ROW IN WHICH THE PHASE FRONT IS LOCATED AND THE NEXT FURTHER ROW. FOR NON PHASE CHANGE RUNS, ALL ROWS ARE PRINTED.

THE NOTATION USED IS DEFINED BELOW:
TIME = TIME, T
QS = NET HEAT TRANSFER AT THE SPECIFIED TEMPERATURE BOUNDARY, X = 0, FROM T = 0
QN = NET HEAT TRANSFER AT X = 0 CALCULATED FROM THE NEUMANN SOLUTION USING AVERAGE COMPOSITE PROPERTIES.
DEPT = DEPT, X, FROM THE SPECIFIED TEMPERATURE BOUNDARY.
TH(I) = TEMPERATURE AT THE CENTER OF THE DIAMOND SHAPED REGIONS. THESE POINTS CORRESPOND TO THERMOCOUPLE LOCATION IN THE EXPERIMENTAL PORTION OF THIS RESEARCH. THESE TEMPERATURES ARE ALIGNED UNDER THE CORRESPONDING X LOCATIONS.
TCMN(I) = TEMPERATURES AT THE CENTER OF THE PCM REGIONS I.E. TP(I,1,1)
TIND(I) = NORMALIZED PENEtrATION PARAMETERS, DEPT/SQRT(ALPHA'TIME). ALPHA IS THE AVERAGE COMPOSITE THERMAL DIFFUSIVITY.
THOD(I) = NORMALIZED T(I) TEMPERATURES, (TH(I)-TREF)/(TF-TREF) WHERE TF IS THE PCM FUSION TEMPERATURE AND TREF IS THE SURFACE TEMPERATURE, TH(I).
TCNO(I) = NORMALIZED TCMN(I) TEMPERATURES.
TPH(1,J,) = TEMPERATURES IN THE CYLINDRICAL PCM REGION, J = ROW, I = 1 IS THE ROW NEXT TO THE SPECIFIED TEMPERATURE BOUNDARY, J = RADIAL POSITION, J = 1 IS AT THE CENTER OF THE PCM REGION, AND R = THETA POSITION, R = 2 AND 6 ARE IN LINE WITH THE DEPTH COORDINATE R = 2 IS NEAREST X = 0.
THOD(1,J,K) = NORMALIZED TPH(1,J,K) TEMPERATURES.
TMCN(1,J,K) = TEMPERATURES IN THE CYLINDRICAL COORDINATE MATRIX REGION, J = 1 IS AT THE MATRIX PCM INTERFACE.
THOD(1,J,K) = NORMALIZED TMCN(1,J,K) TEMPERATURES.
TPH(J) = KREITH SOLUTION PCM TEMPERATURES

Figure B.1–2 Example of Numerical Program Output.
### Figure B.1-3 Example of Numerical Program Output

<table>
<thead>
<tr>
<th>TIME, MIN.</th>
<th>QH. BTU/SP</th>
<th>CH. BTU/SP</th>
<th>QH/CH</th>
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<tr>
<td>0.7694</td>
<td>-24.73</td>
<td>-24.49</td>
<td>1.09</td>
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<table>
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<tr>
<th>DEPTH, IN</th>
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<tr>
<td>0.0</td>
<td>0.09</td>
<td>0.18</td>
<td>0.26</td>
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<table>
<thead>
<tr>
<th>TN(J), F</th>
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<th></th>
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<tbody>
<tr>
<td>0.00</td>
<td>79.44</td>
<td>81.00</td>
<td>81.00</td>
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<table>
<thead>
<tr>
<th>TPCNH(1), F</th>
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<th></th>
<th></th>
</tr>
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<tbody>
<tr>
<td>0.00</td>
<td>81.00</td>
<td>81.00</td>
<td>81.00</td>
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<td>1.247</td>
<td>1.901</td>
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<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
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</table>

<table>
<thead>
<tr>
<th>TPH(1), J,K,F</th>
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</thead>
<tbody>
<tr>
<td>J/K</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>81.00</td>
<td>81.00</td>
<td>81.00</td>
</tr>
<tr>
<td>2</td>
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</tr>
<tr>
<td>3</td>
<td>67.07</td>
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<td>76.94</td>
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<table>
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<th>TPHH(1), J,K,F</th>
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<tbody>
<tr>
<td>J/K</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
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<td>66.37</td>
<td>72.93</td>
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<table>
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<tbody>
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<td>J/K</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>81.00</td>
<td>81.00</td>
<td>81.00</td>
</tr>
<tr>
<td>2</td>
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</tr>
<tr>
<td>3</td>
<td>81.00</td>
<td>81.00</td>
<td>81.00</td>
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<table>
<thead>
<tr>
<th>TPHH(2), J,K,F</th>
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<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>J/K</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>80.05</td>
<td>79.88</td>
<td>80.62</td>
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Figure B.1-3 Example of Numerical Program Output.
THE PURPOSE OF THIS PROGRAM IS TO COMPUTE THE
TRANSIENT TEMPERATURE FIELD IN A THICK WALL WITH
SPECIFIED TEMPERATURE ON ONE FACE AND INSULATED
ON THE OTHER. THE WALL IS A COMPOSITE OF TWO
MATERIALS. IN CROSS SECTION, A SQUARE ARRAY OF
CIRCULAR REGIONS OF A PHASE CHANGE MATERIAL (PCM)
ARE SURROUNDED BY A CONTINUUM OF AN IMPERVIOUS
SOLID MATERIAL (MATRIX). A DESCRIPTION OF THE SOLUTION
AND THE PRINTED FORMAT ARE INCLUDED IN THE OUTPUT.

DIMENSION TP0(8,6,7),TPN(8,6,7),TM0(8,6,7),TMN(8,6,7),
CTO(9),TN(9)
DIMENSION RP(6),RM(6),CP(6),EL(6),Y(6),W(6),X(6),Y(6),
CZ(6),A(6),B(6),C(6),D(6),E(6)
DIMENSION QP(8,6,7),QPS(8,6,7)
DIMENSION PT(IO)
DIMENSION TIH(16),TAF(16)
DIMENSION TNN(9),TNNN(8,6,7),TPNN(8,6,7),TIND(17)
DIMENSION TPCM(8),TPCN(8),TPCN(8)
DIMENSION RAS(8),XNU(8),APCM(8),BPCM(8),CPCM(8),DPCM(8)
DIMENSION NKA(5),DEPTH(17)
DIMENSION TPK(7)
REAL LH,KP,KM,LAM,LBAR,KBAR
DATA NKA /2,3,4,5,6/

READING INPUT DATA

DATA R,S,NROW /0.074,0.233,6/
DATA R,S,NROW /0.074,0.175,8/
DATA R,S,NROW /0.074,0.350,4/ 4 ROW
R = VOID RADIUS, IN.
S = VOID SPACING, IN.
NROW = NO. OF ROWS
DATA KBAR, RBAR, CBAR, LBAR /0.343,91.14,0.287,19.57/
DATA KBAR, RBAR, CBAR, LBAR /0.262,77.58,0.327,40.755/
DATA KBAR, RBAR, CBAR, LBAR /0.48,108.7,0.25,40.755/
DATA KBAR, RBAR, CBAR, LBAR /0.231,74.57,0.380,38.77/
DATA KBAR, RBAR, CBAR, LBAR /0.342,100.92,0.265,7.832/
DATA KBAR, RBAR, CBAR, LBAR /0.414,100.9,0.265,7.83/
KBAR = AVE. COMPOSITE CONDUCTIVITY, BTU/HR-FT-F
RBAR = AVE. COMPOSITE DENSITY, LB/M/CU FT
CBAR = AVE. COMPOSITE SPECIFIC HEAT, BTU/LB-F
LBAR = AVE. COMPOSITE LATENT HEAT, BTU/LB
DATA NP,NM /5,2/
NP = NO. R DIVISIONS IN PCM REGION
NM = NO. R DIVISIONS PLUS 1 IN MATRIX REGION
DATA TF,DTF /81.0,0.0001/
TF = FUSION TEMPERATURE, F
DTF = INITIAL CONDITION DELTA OFFSET, F
DATA KP, RHOP, CAP, LH /0.15, 53.3, 0.45, 105.6/FREEZING
DATA KP, RHOP, CAP, LH /0.087, 48.1, 0.556, 107.0/
DATA KP, RHOP, CAP, LH /0.15, 53.3, 0.45, 105.6/
KP = PCM CONDUCTIVITY, BTU/HR-FT-F
RHOP = PCM DENSITY, LBM/CU FT
CAP = PCM SPECIFIC HEAT, BTU/LBM-F
LH = PCM LATENT HEAT, BTU/LBM
DATA BETAP, VISP /0.0005, 0.000039/
BETAP = PCM BETA GROUP, 1/F
VISP = PCM KINEMATIC VISCOSITY, SQ FT/SEC
DATA KM, RHOM, CAM /0.48, 108.7, 0.25/FREEZING
DATA KM, RHOM, CAM /0.55, 108.5, 0.28/
DATA KM, RHOM, CAM /0.48, 108.7, 0.25/
KM = MATRIX CONDUCTIVITY, BTU/HR-FT-F
RHOM = MATRIX DENSITY, LBM/CU FT
CAM = MATRIX SPECIFIC HEAT, BTU/LBM-F
DATA NCONV /0/
NCONV = CONVECTION ANALYSIS CONTROL WORD,
NCONV=0 NO CONVECTION
DATA NDP /2/
NDP = NO. OF DATA POINTS IN SURFACE TEMPERATURE AND TIME ARRAYS TAF AND TIF
DATA TIH /0.0, 0.0167, 0.0333, 0.05, 0.0667, 0.0833,
C 0.1667, 0.25, 0.333, 0.4167, 0.5, 0.5833, 0.6667/RUN 609
DATA TIH /0.0, 0.0167, 0.0333, 0.05, 0.0667, 0.0833, 0.10,
C 0.1167, 0.1333, 0.15, 0.1667, 0.1833, 0.25, 0.25, 0.5, 0.75, 1.0/
DATA TIH/0.0, 0.091/
TIH = SURFACE TIME ARRAY, HR
DATA TAF /32.0, 44.2, 40.4, 38.9, 38.4, 37.8, 36.5, 36.0,
C 36.0, 36.0, 35.8, 35.1, 36.2/RUN 609
DATA TAF /32.0, 44.8, 41.1, 40.5, 39.5, 38.6, 38.2, 37.8,
C 37.1, 36.9, 36.6, 36.2, 35.8, 35.5, 35.3, 35.5/
DATA TAF /32.0, 44.8, 41.1, 40.5, 39.5, 38.6, 38.2, 37.8,
C 37.1, 36.9, 36.6, 36.2, 35.8, 35.5, 35.3, 35.5/
DATA TAF /130.0, 120.3, 123.1, 125.4, 126.8, 127.2, 127.4,
C 127.6, 128.3, 127.9, 128.3, 129.2, 129.6, 129.4, 129.6, 129.6/
DATA TAF/56.08, 56.08/
TAF = SURFACE TEMPERATURE ARRAY
DATA DTIPR /1.0/
DTIPR = DELTA TIME BETWEEN PRINTING, MIN.
DATA KRE, GK /-1,-19027.1/
KRE = KREITH SOLUTION INDICATOR KRE.GT.0
FOR SPECIAL CASE, KRE.LT.0 FOR CONVENTIONAL RUN.
GK = TEMPERATURE GRADIENT AT THE PHASE FRONT FOR THE KREITH SOLUTION, F/FT.
KS=1
DATA PT /10*0.5/
PT = INITIALIZATION OF PT ARRAY
*********************************************************
WRITING INPUT DATA
C  ***********************************************************************
WRITE (6,2000)
2000 FORMAT ('I', 'PROGRAM DESCRIPTION:', '/
  C' THIS PROGRAM COMPUTES THE TRANSIENT TEMPERATURE FIELD ',
  C' IN A THICK WALL WITH SPECIFIED TEMPERATURE ON ONE ', /
  C' FACE AND INSULATED ON THE OTHER. THE WALL IS ',
  C' A COMPOSITE OF TWO MATERIALS. IN CROSS SECTION ', /
  C' A SQUARE ARRAY OF CIRCULAR REGIONS OF A PHASE ',
  C' CHANGE MATERIAL (PCM) ARE SURROUNDED BY A ', /
  C' CONTINUUM OF AN IMPERVIOUS SOLID MATERIAL (MATRIX). ', /
WRITE (6,2001)
2001 FORMAT ('O',
  C' A UNIQUE FINITE DIFFERENCE NODAL PATTERN IS USED ',
  C' TO DESCRIBE THIS GEOMETRY. A SERIES OF TWO ', /
  C' DIMENSIONAL CYLINDRICAL COORDINATE REGIONS DESCRIBE ',
  C' THE PCM REGIONS, TPCM(X,R,THETA), AND ADJACENT ', /
  C' MATRIX REGIONS, TM(X,R,THETA) OUTWARD TO THE POINT OF ',
  C' TRANGENCY OF THE CYLINDERS. THE REMAINING DIAMOND ', /
  C' SHAPED REGIONS, T(X), ARE APPROXIMATELY ORTHOGONAL ',
  C' TO ADJACENT CYLINDRICAL NODES OF ARCH LENGTH ', /
  C' (PI/4)*S. THE RADIUS OF THE OUTER RING OF MATRIX ',
  C' NODES ARE ADJUSTED TO ACCOUNT FOR THE REMAINING ', /
  C' MATRIX MATERIAL. THE EXPLICIT EULER METHOD IS ',
  C' USED TO STEP FORWARD IN TIME. THE TEMPERATURE ', /
  C' OF THE PCM NODES IS HELD AT THE FUSION ',
  C' TEMPERATURE, TF, UNTIL SUFFICIENT HEAT IS TRANSFERRED ', /
  C' TO BALANCE THE NODAL LATENT HEAT.' /
WRITE (6,2002)
2002 FORMAT ('O',
  C' CONVECTIVE HEAT TRANSFER IN THE LIQUID PCM IS COMPUTED ',
  C' BASED ON SILVESTON NUSSELT'S NUMBER CORRELATIONS. ', /
  C' CONVECTIVE STABILITY IS BASED ON THE KATTO AND ',
  C' MUSOKA STABILITY CRITERIA FOR A POROUS LAYER ', /
  C' HEATED FROM BELOW. ', /
  C' FOR THE FREEZING PROBLEM, LUMPED NODES OF THICKNESS ',
  C' S, ARE USED BETWEEN THE TEMPERATURE BOUNDARY AND ', /
  C' THE ROW IN WHICH THE PHASE FRONT IS LOCATED ',
  C' AVERAGE COMPOSITE THERMOPHYSICAL PROPERTIES ARE ', /
  C' USED FOR THESE LUMPED NODES.' /
WRITE (6,2003)
2003 FORMAT ('O',
  C' SINCE INITIAL CONDITIONS BEING STUDIED ARE NEAR THE ',
  C' FUSION TEMPERATURE, TF, OF THE PCM, ONLY ROWS BETWEEN ', /
  C' THE SPECIFIED TEMPERATURE SURFACE AND ONE ROW BEYOND ',
  C' WHERE THE PHASE FRONT IS LOCATED ARE INCLUDED IN ', /
  C' THE COMPUTATION FOR FREEZING AND MELTING RUNS. ALL ',
  C' ROWS ARE INCLUDED IN NON PHASE CHANGE RUNS I.E. RUNS ', /
  C' IN WHICH TEMPERATURES DO NOT CROSS THE FUSION ',
  C' TEMPERATURE OF THE PCM.' /
WRITE (6,2010)
2010 FORMAT ('01','THE DATA FOR THIS RUN WAS:
WRITE (6,2020) R,S,NROW
2020 FORMAT ('01','VOID RADIUS (R) = ',F5.3,' IN.',/
C' VOID SPACING (S) = ',F5.3,' IN.',/
C' NO. OF ROWS (NROWS) = ',12)
WRITE (6,2030) KBAR,RBAR,CBAR,LBAR
2030 FORMAT ('01','AVE. COMPOSITE CONDUCTIVITY (KBAR) = ',
CF5.3,' BTU/HR-FT-F',/
C' AVE. COMPOSITE DENSITY (RBAR) = ',F6.2,' LBM/CU FT',/
C' AVE. COMPOSITE SPECIFIC HEAT (CBAR) = ',F5.3,' BTU/LBM-F',/
C' AVE. COMPOSITE LATENT HEAT (LBAR) = ',F6.3,' BTU/LBM')
WRITE (6,2040) NP,NM
2040 FORMAT ('01','NO. R DIVISIONS IN PCM REGION (NP) = ',12,/
C' NO. R DIVISIONS PLUS 1 IN MATRIX REGION(NM) = ',12)
WRITE (6,2050) TF,DTF
2050 FORMAT ('01','FUSION TEMPERATURE (TF) = ',F5.1,' F',/
C' INITIAL CONDITION DELTA OFFSET (DTF) = ',F6.2,' F')
WRITE (6,2060) KP,RHOP,CAP,LH
2060 FORMAT ('01','PCM CONDUCTIVITY (KP) = ',F6.3,' BTU/HR-FT-F',/
C' PCM DENSITY (RHOP) = ',F5.1,' LBM/CU FT',/
C' PCM SPECIFIC HEAT (CAP) = ',F5.2,' BTU/LBM-F',/
C' PCM LATENT HEAT (LH) = ',F6.1,' BTU/LBM')
WRITE (6,2070) BETAP,VISP
2070 FORMAT ('01','PCM BETA GROUP (BETAP) = ',F7.4,' 1/F',/
C' PCM KINEMATIC VISCOSITY (VISP) = ',F9.6,' SQ FT/SEC')
WRITE (6,2080) KM,RHOM,CAM
2080 FORMAT ('01','MATRIX CONDUCTIVITY (KM) = ',F5.2,' BTU/HR-FT-F',/
C' MATRIX DENSITY (RHOM) = ',F6.1,' LBM/CU FT',/
C' MATRIX SPECIFIC HEAT (CAM) = ',F5.2,' BTU/LBM-F')
WRITE (6,2090) NCONV,NDP,(TIH(I),I=1,NDP)
2090 FORMAT ('01','CONVECTION ANALYSIS CONTROL WORD (NCONV)',
C' = ',12,' IF NCONV=0 NO CONVECTION',/
C' NO. OF DATA POINTS IN SURFACE',/
C' TEMPERATURE AND TIME ARRAYS TAF AND TIF (NDP) = ',12,/
C' SURFACE TIME ARRAY (TIH), HR',/
WRITE ( 6,2100) (TAF(I),I=1,NDP)
2100 FORMAT ('01','SURFACE TEMPERATURE ARRAY (TAF), F',/
C' TIME BETWEEN PRINTING (DTIPR) = ',F4.2,' MIN')
WRITE (6,2102) KRE,GK
2102 FORMAT ('01','THE LIMITING CASE OF ONE DIMENSIONAL',
C' RADIAL PCM FREEZING WITH A CONSTANT PHASE FRONT',/
C' VELOCITY IS ALSO INCLUDED. THIS SOLUTION BY KREITH AND',
C' ROMIE CAN BE SPECIFIED BY SELECTING THE KREITH',/
C' INDICATOR, KRE, AND THE TEMPERATURE GRADIENT AT THE',
C' PHASE FRONT, GK. FOR THIS RUN KRE = ',13,' AND GK = ',F7.0)
DSTEP=TAF(1)-TF
IF (DSTEP.LT.0.0.AND.DTF.LT.0.0) NTYPE = 1
IF (DSTEP.LT.0.0.AND.DTF.GT.0.0) NTYPE = 2
IF (DSTEP.GT.0.0.AND.DTF.GT.0.0) NTYPE = 3
IF (DSTEP.GT.0.0.AND.DTF.LT.0.0) NTYPE = 4
2110 FORMAT ('11',50X,11,' ROW SPECIMEN, SOLID')
2120 FORMAT ('11',50X,11,' ROW SPECIMEN, COOLING')
2130 FORMAT ('11',50X,11,' ROW SPECIMEN, LIQUID')
2140 FORMAT ('11',50X,11,' ROW SPECIMEN, HEATING')
C
COMPUTING NEUMANN SOLUTION LAMMDA PARAMETER
C
ABAR=KBAR/RBAR/CBAR
SBAR=LBAR/(CBAR*(TF-TAF(1)))
SBAR=ABS(SBAR)
CALL NUMAN(SBAR,LAM)
C
INITIALIZING PRINTING
C
DTIPR=DTIPR/60.
NPR=0
NPRD=0
PRTI=NPR*DTIPR
C
CONVERTING TO FT
C
R=R/12.
S=S/12.
C
COMPUTING MATERIAL PARAMETERS
C
PRP=3600.*VISP*RHOP*CAP/KP
AP=KP/RHOP/CAP
AM=KM/RHOM/CAM
PI=3.141592654
C
SETTING INDICES AND COMPUTING DEPTH
C
KPR=NP
NPM1=NP-1
NMM1=NM-1
NRP1=NROW+1
NPX=2*NROW+1
DO 5 1=1,NPX
      DEPTH(1)=(1-1)*(S*12.)/2.
5   DEPTH(1)=(1-1)*(S*12.)/2.
NPF=1
NPFP1=NPF+1
IF (NTYPE.EQ.1.OR.NTYPE.EQ.3) NPFP1=NROW
NPFC=0
C
INITIALIZING NODE TEMPERATURES
C
DO 20 1=1,8
DO 10 J=1,6
DO 10 K=1,7
TP0(I,J,K)=TF+DTF
TPN(I,J,K)=TF+DTF
TMO(I,J,K)=TF+DTF
TMN(I,J,K)=TF+DTF
QP(I,J,K)=0.0
QPS(I,J,K)=0.0
TPCMO(I)=TF+DTF
TPCMN(I)=TF+DTF
TO(I)=TF+DTF
10 TN(I)=TF+DTF
TO(NRP)=TF+DTF
TN(NRP)=TF+DTF

C

DRP=(2.*R)/(2.*NP-1)
C

C CALCULATE RADII
C

DO 30 L=1,NP
30 RP(L)=(L-1)*DRP
SS=1.027027835*5
DRM=2.*(SS/2.-R)/(2.*(NM-2)+1)
DO 40 J=1,NM
40 RM(J)=R+(J-1)*DRM
C

C COMPUTE TIME STEP, DTIME
C

HP=KP/DRP/2.
S2M1=SQR(T(2.))-1
PT(I)=1./(4.*AP*(DRM/DRP)**2.)
DO 41 L=2,NPM1
41 PT(L)=1./((AP*(DRM/DRP)**2.)*((32./PI**2.)*(DRP/RP(L))
C**2.+(RP(L-1)+2.*RP(L)+RP(L+1))/(2.*RP(L))))
L=NP
PT(L)=1./((AP*(DRM/DRP)**2.)*((32./PI**2.)*(DRP/RP(L))
C**2.+(RP(L-1)+3.*RP(L)+2.*RM(J))/(2.*RP(L))))
J=1
1=NP+J
PT(I)=1./((AM*((32./PI**2.)*(DRM/RM(J))**2.+(RM(J)+
CRM(J+1))/RM(J)+2.*HP*DRM/KM))
IF (NM.EQ.2) GO TO 43
DO 42 J=2,NM1
1=NP+J
42 PT(I)=1./((AM*((32./PI**2.)*(DRM/RM(J))**2.+(RM(J-1)+
C2.*RM(J)+RM(J+1))/(2.*RM(J))))
43 CONTINUE
1=NP+NMM1+1
PT(I)=1./((AM*(8.*DRM**2.)/((S2M1*S)**2.+S2M1*S*DRM))
P=0.5
1IP=1
MP=NP+NMM1+1
DO 50 I=1,MP
  IF (PT(I).LT.P) IIP=I
    P=P/2.
    DTIME=P*DRM**2.
    TIME=DTIME
  DTIMES=DTIME*3600.
  WRITE (6,51) DTIMES, IIP
51 FORMAT ('0', 'FOR EACH NODE TYPE AND SIZE, A STABLE ',
  'TIME STEP IS CALCULATED BY A METHOD BASED ON ',
  'SECOND LAW REASONING OUTLINED BY ROHENNOW AND ',
  'HARTNETT. OF THESE, THE MINIMUM TIME STEP IS ',
  'SELECTED AND USED THROUGHOUT THE COMPUTATION.',
  'FOR THIS RUN, THE TIME STEP IS ',F10.5,'SECONDS.',
  'THE MINIMUM TIME STEP IS AT LOCATION',I2,')
  WRITE (6,890)
  WRITE (6,891)
  WRITE (6,892)
  WRITE (6,893)
C *****************************************************
C CALCULATE NODAL CAPACITANCES CP(L) AND
C LATENT HEATS EL(L)
C *****************************************************
CP(1)=(PI/4.)*RHO*CAP*DRP**2.
EL(1)=(PI/4.)*RHO*LH*DRP**2.
DO 60 L=2, NP
  CP(L)=(PI/4.)*RHO*CAP*RP(L)*DRP
  EL(L)=(PI/4.)*RHO*LH*RP(L)*DRP
60 VXY=P*DRM**2.*KP*PI/4.
V(L)=-P*DRM**2.*KP*(8.*DRP/(PI*RP(L))+(PI/8.)*(RP(L-1)
  C+2.*RP(L)+RP(L+1))/DRP)
W(L)=P*DRM**2.*KP*(4.*DRP)/(PI*RP(L))
Y(L)=P*DRM**2.*KP*(4.*DRP)/(PI*RP(L))
X(L)=P*DRM**2.*KP*(PI/8.)*(RP(L)+RP(L+1))/DRP
Z(L)=P*DRM**2.*KP*(PI/8.)*(RP(L)+RP(L-1))/DRP
L=NP
V(L)=-P*DRM**2.*KP*(8.*DRP/(PI*RP(L))+(PI/8.)*(RP(L-1)
  C+2.*RP(L)+RM(1))/DRP)
W(L)=P*DRM**2.*KP*(4.*DRP)/(PI*RP(L))
X(L)=P*DRM**2.*KP*(PI/8.)*(RP(L)+RM(1))/DRP
Y(L)=P*DRM**2.*KP*(4.*DRP)/(PI*RP(L))
Z(L)=P*DRM**2.*KP*(PI/8.)*(RP(L)+RP(L-1))/DRP
J=1
A(J)=(1.-P*AM*((32./PI**2.)*(DRM/RM(J))**2.+(RM(J)+
  CRM(J+1)))/RM(J)+2.*HP*DRM/KM))
B(J)=P*AM*(16./PI**2.)*(DRM/RM(J))**2.
C(J)=P*AM*(RM(J)+RM(J+1))/RM(J)
D(J)=P*AM*(16./PI**2.)*(DRM/RM(J))**2.
E(J)=P*AM*2.*HP*DRM/KM
IF (NM.EQ.2) GO TO 80

DO 79 J=2,NM1

A(J)=(1.-P*AM*(32./PI**2.)*(DRM/RM(J))*2.+(RM(J+1)+C2.*RM(J)+RM(J-1))/(2.*RM(J))**2.)

B(J)=P*AM*(16./PI**2.)*(DRM/RM(J))**2.

C(J)=P*AM*(RM(J)+RM(J+1))/(2.*RM(J))

D(J)=P*AM*(16./PI**2.)*(DRM/RM(J))**2.

79 CONTINUE

AA=1.-P*AM*(8.*DRM**2.)/((S2M1*S)**2.+S2M1*S*DRM)

BB=P*AM*4.*DRM**2./((S2M1*S)**2.+S2M1*S*DRM)

DO 84 I=1,NROW

XNU(I)=1.0

APCM(I)=1.-XNU(I)*AP*P*(DRM/R)**2.

BPCM(I)=XNU(I)*AP*P*(DRM/R)**2./8.

CPCM(I)=(1.-P*AM*(32./PI**2.)*(DRM/RM(1))**2.+(RM(1)+CRM(2))/RM(1)+(XNU(I)*KP/KM)*(DRM/R))

DPCM(I)=P*AM*(XNU(I)*KP/KM)*(DRM/R)

QS=0.0

84 DPCM(I)=P*AM*(XNU(I)*KP/KM)*(DRM/R)

C ****************************************************

C CALCULATE BOUNDARY TEMPERATURES, TA

C ****************************************************

81 IF (KRE.GT.0) GO TO 86

T11=TH(KT)

T12=TH(KT+1)

TA1=TAF(KT)

TA2=TAF(KT+1)

82 IF (TIME.LT.T12) GO TO 83

K=KT+1

GO TO 81

83 TA=(TA2-TA1)*(TIME-T11)/(T12-T11)+TA1

GO TO 88

86 CALL KREITH (TF,RM(1),RM(1),TIME,KP,RHOP,LH,CAP,GK,TA,KS)

IF (KS.EQ.0) GO TO 999

DO 87 I=1,NPF1

DO 87 K=1,7

TMN(I,1,K)=TA

87 TMO(I,1,K)=TA

88 CONTINUE

C ****************************************************

C CALCULATE PHASE CHANGE MATERIAL (PCM) NODE

C TEMPERATURES, TPN(I,L,K)

C ****************************************************

DO 150 I=1,NPF1

150 IF (I.EQ.NPF) GO TO 99

TPCMN(I)=APCM(I)*TPCMO(I)+BPCM(I)*(TMO(I,1,2)+2.*CTMO(I,1,3)+2.*TMO(I,1,4)+2.*TMO(I,1,5)+TMO(I,1,6))

GO TO 120

99 L=1
DO 100 K=2,6
    QP(I,L,K)=VXY*(-4.*TP0(1,1,1)+TP0(1,2,2)/2.+TP0(1,2,3)
        +TP0(1,2,4)+TP0(1,2,5)+TP0(1,2,6)/2.)
    QPS(I,L,K)=QPS(I,L,K)+QP(I,L,K)
100 CALL TEMP (TP0(I,L,K),QP(I,L,K),QPS(I,L,K),EL(L),
        CCP(L),TF,TPN(I,L,K))
    IF (NP.EQ.2) GO TO 111
DO 110 L=2,NPM1
DO 110 K=2,6
    QP(I,L,K)=V(L)*TP0(I,L,K)+W(L)*TP0(I,L,K+1)+X(L)*
        CTPO(I,L+1,K)+Y(L)*TP0(I,L,K-1)+Z(L)*TP0(I,L-1,K)
    QPS(I,L,K)=QPS(I,L,K)+QP(I,L,K)
110 CALL TEMP (TP0(I,L,K),QP(I,L,K),QPS(I,L,K),EL(L),
        CCP(L),TF,TPN(I,L,K))
111 L=NP
DO 120 K=2,6
    QP(I,L,K)=V(L)*TP0(I,L,K)+W(L)*TP0(I,L,K+1)+X(L)*
        CTMO(I,J+1,K)+Y(L)*TP0(I,L,K-1)+Z(L)*TP0(I,L-1,K)
    QPS(I,L,K)=QPS(I,L,K)+QP(I,L,K)
120 CONTINUE
    IF (KRE.GT.0) GO TO 150
C CALCULATE MATRIX NODE TEMPERATURES, TMN(I,J,K)
C *****************************************************
        J=1
    IF (J.GE.NPF) GO TO 129
DO 121 K=2,6
121 TMN(I,J,K)=CPCM(1)*TM0(I,J,K)+B(J)*TM0(I,J,K+1)+C(J)*
        CTMO(I,J+1,K)+D(J)*TM0(I,J,K-1)+DPCM(I)*TPCMO(I)
    GO TO 131
129 DO 130 K=2,6
130 TMN(I,J,K)=A(J)*TM0(I,J,K)+B(J)*TM0(I,J,K+1)+C(J)*
        CTMO(I,J+1,K)+D(J)*TM0(I,J,K-1)+E(J)*TP0(I,NP,K)
131 IF (NM.EQ.2) GO TO 141
DO 140 J=2,NNM1
DO 140 K=2,6
140 TMN(I,J,K)=A(J)*TM0(I,J,K)+B(J)*TM0(I,J,K+1)+C(J)*
        CTMO(I,J+1,K)+D(J)*TM0(I,J,K-1)+E(J)*TM0(I,J-1,K)
C *****************************************************
C CALCULATE DIAMOND SHAPED REGION MATRIX NODE
C TEMPERATURE, TN(I)
C *****************************************************
141 IF (J.EQ.1) GO TO 149
    TN(I)=AA*TO(I)+BB*TM0(I-1,NNM1,5)+BB*TM0(I,NNM1,3)
    GO TO 150
149 TN(I)=TA
150 CONTINUE
    TN(NRP1)=AA*TO(NRP1)+2.*BB*TM0(NROW,NNM1,5)
**EQUATE SYMMETRY NODE TEMPERATURES**

DO 160 I=NPF, NPF1
DO 160 L=N, NP
TPN(I,L,1)=TPN(I,L,3)
160 TPN(I,L,7)=TPN(I,L,5)
DO 180 I=NPF, NPF1
DO 170 J=1, NM1
TMN(I,J,1)=TMN(I,J,3)
170 TMN(I,J,7)=TMN(I,J,5)
IF (I.EQ.1) GO TO 171
TMN(I,NM,2)=TMN(I-1,NM-1,6)
GO TO 172
171 TMN(I,NM,2)=TA
172 CONTINUE
TMN(I,NM,3)=TMN(I,NM1,3)+DRM*(TN(I)-TMN(I,NM1,3))
C((S2M1*S+DRM)/2.)
TMN(I,NM,4)=TMN(I,NM1,4)
TMN(I,NM,5)=TMN(I,NM1,5)+DRM*(TN(I+1)-TMN(I,NM1,5))
C((S2M1*S+DRM)/2.)
IF (I.EQ.NROW) GO TO 173
TMN(I,NM,6)=TMN(I+1,NM1,2)
GO TO 180
173 TMN(I,NM,6)=TMN(I,NM1,6)
180 CONTINUE

**CHECK FOR PRINT TIME**

IF (TPN(I,KPR,3).LT.TF) GO TO 185
IF (TIME.LT.PRTI) GO TO 190
NPR=NPR+1
PRTI=NPR*DTPR
GO TO 186
185 CONTINUE
KPR=KPR-1
IF (KPR.LT.0) GO TO 999
186 CONTINUE
TREF=TA

**CALCULATE Q SUM AND Q NEUMANN**

CALL QSUM (RHOP,CAP,LH,RHQM,CAM,DRP,DRM,NPF1,NP,NM1
C,S,TF,DTF,TPN,TMN,TN,RP,RM,QS)
QN=2.*KBAR*(TREF-TF)*TIME**0.5/(ERF(LAM)*(P1*ABAR)
C**0.5)
Q RAT=QS/QN

**NORMALIZE TEMPERATURES, (TEMP-TREF)/(TF-TREF)**
IF (KRE.GT.0) TREF=2*TA
DO 181 I=1,NRP1
181 TNNND(I)=(TN(I)-TREF)/(TF-TREF)
DO 182 I=1,NROW
TPCND(I)=(TPCMN(I)-TREF)/(TF-TREF)
DO 182 J=1,NMM1
DO 182 K=2,6
182 TMNNND(I,J,K)=(TMN(I,J,K)-TREF)/(TF-TREF)
DO 183 I=1,NROW
DO 183 J=1,NP
DO 183 K=2,6
183 TPNND(I,J,K)=(TPN(I,J,K)-TREF)/(TF-TREF)
DO 184 I=1,NPX
184 TIND(I)=(I-1)*(S/2.)/(SQRT(ABAR*TIME))
C
**********************************************************************
C WRITE OUTPUT
C
**********************************************************************
890 FORMAT ('0',AT SELECTED TIME INTERVALS, DTIPR, THE ',
C COMPLETE TEMPERATURE FIELD IS PRINTED. TEMPERATURES ',/
C ARE IN DEGREES F. ALSO NORMALIZED DATA ARE PRESENTED. ',
C ARRAY NOTATIONS CORRESPOND TO THAT USED IN THE ',
C FORTRAN PROGRAM. ',/
C FOR PHASE CHANGE RUNS, ONLY DETAILED CYLINDRICAL COORDINATE ',
C NODAL TEMPERATURES ARE PRESENTED FOR THE ROW IN WHICH ',/
C THE PHASE FRONT IS LOCATED AND THE NEXT FURTHER ROW. ',
C FOR NON PHASE CHANGE RUNS, ALL ROWS ARE PRINTED. ',/)
891 FORMAT ('0',
C THE NOTATION USED IS DEFINED BELOW: ',/
C TIME = TIME, T',/
C QS = NET HEAT TRANSFER AT THE SPECIFIED TEMPERATURE ',
C BOUNDARY, X=0, FROM T = 0',/
C QN = NET HEAT TRANSFER AT X = 0 CALCULATED FROM THE ',
C NEUMANN SOLUTION USING AVERAGE COMPOSITE PROPERTIES.',/
C DEPTH = DEPTH, X, FROM THE SPECIFIED TEMPERATURE ',
C BOUNDARY.',/
C TN(I) = TEMPERATURE AT THE CENTER OF THE DIAMOND SHAPED ',
C REGIONS. THESE POINTS CORRESPOND TO THERMOCOUPLE ',/
C LOCATION IN THE EXPERIMENTAL PORTION OF THIS RESEARCH. ',
C THESE TEMPERATURES ARE ALIGNED UNDER THE CORRESPONDING',/
C X LOCATIONS.',/)
892 FORMAT ('0',
C TPCMN(I) = TEMPERATURES AT THE CENTER OF THE PCM ',
C REGIONS I.E. TP(I,1,1)',/
C TIND(I) = NORMALIZED Penetration PARAMETERS, ',
C DEPTH/SQRT(ALPHA*TIME). ALPHA IS THE AVERAGE COMPOSITE ',/
C THERMAL DIFFUSIVITY.',/
C TNND(I) = NORMALIZED TN(I) TEMPERATURES, (TN(I)-TREF)/',
C (TF-TREF) WHERE TF IS THE PCM FUSION TEMPERATURE AND',/
C TREF IS THE SURFACE TEMPERATURE, TN(I).',/
C TPCND(I) = NORMALIZED TPCMN(I) TEMPERATURES.')/
FORMAT ('O',
C' TPN(I,J,K) = TEMPERATURES IN THE CYLINDRICAL PCM ',
C' REGION, I = ROW, I = 1 IS THE ROW NEXT TO THE ',/
C' SPECIFIED TEMPERATURE BOUNDARY, J = RADIAL POSITION, ',
C' J = 1 IS AT THE CENTER OF THE PCM REGION, AND K = ',/
C' THETA POSITION, K = 2 AND 6 ARE IN LINE WITH THE ',
C' DEPTH COORDINATE K = 2 IS NEAREST X = 0.',/
C' TPNND(I,J,K) = NORMALIZED TPN(I,J,K) TEMPERATURES.',/
C' TMN(I,J,K) = TEMPERATURES IN THE CYLINDRICAL ',
C' COORDINATE MATRIX REGION, J = 1 IS AT THE MATRIX ',/
C' PCM INTERFACE.',/
C' TMNND(I,J,K) = NORMALIZED TMN(I,J,K) TEMPERATURES',/
C' TPK(J) = KREITH SOLUTION PCM TEMPERATURES')
GO TO (910,920,930,940),NTYPE
910 WRITE (6,2110) NROW
GO TO 950
920 WRITE (6,2120) NROW
GO TO 950
930 WRITE (6,2130) NROW
GO TO 950
940 WRITE (6,2140) NROW
950 CONTINUE
TIME M = TIME M *60.
WRITE (6,1000)
1000 FORMAT ('O',5X,'TIME, MIN.',4X,'QS, BTU/SF',4X,
C'QN, BTU/SF',9X,'QS/QN')
WRITE (6,1010) TIME M, Q S, Q N, Q R A T
1010 FORMAT (' ',F14.4,3F14.2)
WRITE (6,1020)
1020 FORMAT ('O',9X,'DE P T H, I N')
WRITE (6,1030) (DEPT H(I),I=1,NPX)
1030 FORMAT (' ',17F7.2)
WRITE (6,1040)
1040 FORMAT ('O',9X,'TN(I), F')
WRITE (6,1050) (TN(I),I=1,NRP1)
1050 FORMAT (' ',F7.2,7F14.2)
WRITE (6,1060)
1060 FORMAT ('O',9X,'TPCMN(I), F')
WRITE (6,1070) (TPCMN(I),I=1,NROW)
1070 FORMAT (' ',8F14.2)
WRITE (6,1080)
1080 FORMAT ('O',9X,'TIND(I), ND')
WRITE (6,1090) (TIND(I),I=1,NPX)
1090 FORMAT (' ',17F7.3)
WRITE (6,1100)
1100 FORMAT ('O',9X,'TNND(I), ND')
WRITE (6,1110) (TNND(I),I=1,NRP1)
1110 FORMAT (' ',F7.3,8F14.3)
WRITE (6,1120)
1120 FORMAT ('O',9X,'TPCND(I), ND')
WRITE (6,1130) (TPCNND(I),I=1,NROW)
1130 FORMAT (' ',8F14.3)
C **************************************************************
C WRITING TO DISK FOR PLOTS
C **************************************************************
NPM1=NPX=1
DO 3010 I=3,NPX,2
IF (TIND(I).GT.2.0) GO TO 3010
D11X=TIND(I)
N11Y=(I+1)/2
D11Y=TPCNDN(I1Y)
WRITE (11,3000) D11X,D11Y
3000 FORMAT (2F10.3)
3010 CONTINUE
DO 3020 I=2,NPM1,2
IF (TIND(I).GT.2.0) GO TO 3020
D12X=TIND(I)
N12Y=1/2
D12Y=TPCNDN(I12Y)
WRITE (12,3000) D12X,D12Y
3020 CONTINUE
IF (NPRD.GT.0) GO TO 3050
WRITE (13,3030) TREF,TF,ABAR,NPX
3030 FORMAT (3F10.3,110)
WRITE (13,3040) (DEPTH(I),I=1,NPX,2)
3040 FORMAT (8F10.3)
WRITE (13,3040) (DEPTH(I),I=2,NPM1,2)
NPRD=NPRD+1
3050 CONTINUE
WRITE (13,3040) TIMEM,(TN(I),I=1,NRP1),QRAT
WRITE (13,3040) (TPCMN(I),I=1,NROW)
C **************************************************************
C CONTINUE TO WRITE TO PRINTER
C **************************************************************
DO 1210 I=NPF,NFPF1
WRITE (6,1140) I,1
1140 FORMAT ('0',TPN(' ',11,'J,K'),F',36X,TPNND(' ',11,'J,K'),ND')
WRITE (6,1150) (NKA(I1),I1=1,5),(NKA(I1),I1=1,5)
1150 FORMAT ('0',4X,J/K,517,11X,'J/K',517)
DO 1170 J=1,NP
WRITE (6,1160) J,(TPN(I,J,K),K=2,6),J,(TPNND(I,J,K),K=2,6)
1160 FORMAT ('0',17,5F7.2,7X,17,5F7.3)
1170 CONTINUE
WRITE (6,1180) I,1
1180 FORMAT ('0',TMN(' ',11,'J,K'),F',36X,TPMND(' ',11,'J,K'),ND')
WRITE (6,1190) (NKA(I1),I1=1,5),(NKA(I1),I1=1,5)
1190 FORMAT ('0',4X,'J/K',517,11X,'J/K',517)
DO 1210 J=1,NMM1
WRITE (6,1200) J,(TMN(I,J,K),K=2,6),J,(TPMND(I,J,K),K=2,6)
1200 FORMAT ('0',17,5F7.2,7X,17,5F7.3)
1210 CONTINUE
   IF (KRE.LT.0) GO TO 1250
   DO 1220 L=2,NP
      RPK=(RP(L-1)+RP(L))/2.
1220 CALL KREI TH (TF, RM(1), RPK, TIME, KP, RHOP, LH, CAP, GK, CTPK(L), KS)
   IF (KS.EQ.0) GO TO 999
   KL=NP+1
   TPK(KL)=TA
   WRITE (6,1230)
1230 FORMAT ('O', TPK(J), F')
   WRITE (6,1240) (TPK(J), J=2,KL)
1240 CONTINUE
1250 CONTINUE
190 NPFM1=NP-1
   IF (NPFM1.LT.1) GO TO 191
   DO 189 I=1,NPFM1
      DO 189 L=1,NP
         DO 189 K=2,6
189  TPN(I,L,K)=TPCMN(I)
   C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
   C SET OLD TEMPERATURES EQUAL TO NEW TEMPERATURES
   C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
191 CONTINUE
   DO 220 I=1,NPFP1
      DO 220 L=1,NP
         DO 220 K=1,7
220  TPO(I,L,K)=TPN(I,L,K)
   DO 210 J=1,NM
      DO 210 K=1,7
210  TM0(I,J,K)=TMN(I,J,K)
   TPCMO(I)=TPCMN(I)
   TO(I)=TN(I)
   TO(NRP1)=TN(NRP1)
   TIME=TIME+DTIME
   C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
   C CHECK FOR MAXIMUM REAL TIME
   C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
   IF (TIME.GT.TIH(ND P)) GO TO 999
   IF (DTF.GT.0.0) GO TO 229
   DO 228 L=1,NP
228  IF (TPN(NPF,L,6).GT.TF) NPFC=NPFC+1
      GO TO 231
229  DO 230 L=1,NP
230  IF (TPN(NPF,L,6).LT.TF) NPFC=NPFC+1
231  IF (NPFC.EQ.NP) GO TO 240
      NPFC=0
      GO TO 82
240 CONTINUE
QZ=0.0
QZ=CP(1) *(TPO(NPF,1,1)-TF)
DO 250 L=2,NP
250 QZ=QZ+CP(L) *(TPO(NPF,L,2)+2.*TPO(NPF,L,3)+2.*TPO(NPF,L,4)+2.*TPO(NPF,L,5)+TPO(NPF,L,6)-8.*TF)
TPCMN(NPF)=QZ/(RHOP*CAP*PI*R*R)+TF
C
C CHECK FOR CONVECTION
C
IF (DTF.GT.0.0) GO TO 350
IF (NCONV.EQ.0) GO TO 350
XP=PI*(R/S)**2.0
XL=NPF*S
RACK=(4.*PI**2.0)/(XP**2.0.*((150.*(1.-XP)**2.0))
C(2.*R/XL)**2.)
RACK=3600.*((32.2*BETAP*X)**3.0.*(TN1-TN(NPF)))/
C(VISP*KBAR/(RHOP*CAP))
IF (RACK.LT.RACK) GO TO 350
DO 350 I=1,NP
RAS(I)=3600.*32.2*BETAP*(2.*R)**3.*(TMN(I,1,2)-CTMN(I,1,6))(/VISP*AP)
IF (RAS(I).GT.1700.) GO TO 310
XNU(I)=1.0
GO TO 340
310 IF (RAS(I).GT.3500.) GO TO 320
XNU(I)=0.00238*RAS(I)**0.816
GO TO 340
320 IF (RAS(I).GT.500000.) GO TO 330
XNU(I)=0.229*RAS(I)**0.252
GO TO 340
330 XNU(I)=0.104*RAS(I)**0.305*PRP**0.084
340 CONTINUE
APCM(I)=1.-XNU(I)*AP*(DRM/R)**2.
BPCM(I)=XNU(I)*AP*(DRM/R)**2./8.
CPCM(I)=(1.-P*AM*((32./PI**2.0)*(DRM/RM(1))**2.+C(RM(1)+RM(2))/RM(1)+(XNU(I)*KP/KM)*(DRM/R)))/
DPCM(I)=P*AM*(XNU(I)*KP/KM)*(DRM/R)
350 CONTINUE
NPF=NPF+1
NPFP1=NPF+1
NPFC=0
IF (NTYPE.EQ.2.OR. NTYPE.EQ.4) GO TO 82
NPF=1
NPFP1=NRW
GO TO 82
999 STOP
END
C
C SUBROUTINE
C
C
**THIS SUBROUTINE CHECKS FOR PHASE TRANSITION AND CALCULATES THE NEW NODAL TEMPERATURE**

SUBROUTINE TEMP (TPO, QP, QPS, EL, CP, TF, TPN)

10 IF (TPO .NE. TF) GO TO 50
   QPS = QPS + QP
   IF ((ABS(QPS)) .GT. EL) GO TO 20
   TPN = TF
   GO TO 60
20 IF ((QPS/EL) .LT. 0.0) GO TO 30
   QP = QPS - EL
   GO TO 40
30 QP = QPS + EL
40 TPN = TPO + QP / CP
   QPS = 0.0
   GO TO 60
50 TPN = TPO + QP / CP
   TEST = (TPN - TF) / (TPO - TF)
   IF (TEST .GT. 0.0) GO TO 60
   QPD = (TF - TPO) * CP
   QP = QP - QPD
   TPO = TF
   QPS = 0.0
   GO TO 10
60 RETURN

END

**SUBROUTINE FOR THE NEUMANN SOLUTION**

SUBROUTINE NUMAN (SB, LAM)

REAL LAM
FUNCY(F, X) = F - X * EXP(X**2.) * ERF(X)
DATA N1, N2, N3 / 10, 10, 10/
PI = 3.141592654
FL = 1. / (SB * PI**0.5)
DXEP = 0.001
TEST = 0.0001
XMIN = FL
X2 = XMIN
IF (YMIN .LT. 0.0) GO TO 6
XMAX = 1.0 - DXEP
YMAX = FUNCY(FL, XMAX)
X2 = XMAX
IF (YMAX.GT.0.0) GO TO 6
DX=(1.-2.*DXEP)/N1
X1=XMIN
Y1=YMIN:
1  X2=X1+DX
   Y2=FUNCY(FL,X2)
   IF (ABS(Y2).LT.TEST) GO TO 6
   IF ((Y1/Y2).LE.0.0) GO TO 2
   X1=X2
   Y1=Y2
   GO TO 1
2  DX=(X2-X1)/N2
3  X2=X1+DX
   Y2=FUNCY(FL,X2)
   IF (ABS(Y2).LT.TEST) GO TO 6
   IF ((Y1/Y2).LE.0.0) GO TO 4
   X1=X2
   Y1=Y2
   GO TO 3
4  DX=(X2-X1)/N3
5  X2=X1+DX
   Y2=FUNCY(FL,X2)
   IF (ABS(Y2).LT.TEST) GO TO 6
   IF ((Y1/Y2).LE.0.0) GO TO 6
   X1=X2
   Y1=Y2
   GO TO 5
6  CONTINUE
   LAM=X2
   RETURN
END
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C  SUBROUTINE
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C ****************************************************
0 THIS SUBROUTINE SUMS THE HEAT TRANSFER FROM TIME = 0
C ****************************************************
SUBROUTINE QSUM (RHO,P,CA,LM,HR,AM,DRP,DRM,ROW,PN,NM1,IM,TF,DTF,TNP,TMN,TNP,TNP,TNP,TNP)
REAL LH
DIMENSION TPN(8,6,7),TMN(8,6,7),TN(9),RP(6),RM,QS
CL(8,6,7)
QS=0.0
T1=TF+DTF
P1=3.141592654
S2M1=SQRT(2.)-1.
C SUM SENSIBLE HEAT IN PCM
C
QSP = 0.0
DO 10 I = 1, NROW
QSP = QSP + DRP * DRP * TPN(I, 1, 2) / 0.25
DO 10 L = 2, NP
10 QSP = QSP + 0.25 * RP(L) * DRP * TPN(I, L, 2) / 0.25
CTPN(I, L, 4) = 2. * TPN(I, L, 6) - TPN(I, L, 5) - 8. * T1
QSP = P1 * RHOP * CAP * QSP
C SUM SENSIBLE HEAT IN MATRIX
C
C
QSM = 0.0
DO 20 J = 1, NROW
QSM = QSM + (S2M(J) * S) / 2. * (T1 - T1) / PI
QSM = QSM + 0.125 * RM(J) * DRM(TMN(I, J, 2) + 2. * TMN(I, J, 3) + 2. * TPN(I, J, 6) - 8. * T1)
IF (NMM1.EQ.1) GO TO 20
DO 20 J = 2, NMM1
QSM = QSM + 0.25 * RM(J) * DRM(TMN(I, J, 2) + 2. * TMN(I, J, 3) + 2. * TPN(I, J, 6) - 8. * T1)
CTMN(I, J, 4) = 2. * TMN(I, J, 6) - TMN(I, J, 5)
IF (TMN(I, J, 1) .EQ. 1) GO TO 20
DO 20 J = 2, NMM1
QSM = QSM + 0.25 * RM(J) * DRM(TMN(I, J, 2) + 2. * TMN(I, J, 3) + 2. * TPN(I, J, 6) - 8. * T1)
CTMN(I, J, 4) = 2. * TMN(I, J, 6) - TMN(I, J, 5)
IF (TMN(I, J, 1) .EQ. 1) GO TO 20
20 CONTINUE
QSM = QSM - (S2M(J) * S) / 2. * (T1 - T1) / (2. * PI)
C SUM LATENT HEAT
C
C
OLP = 0.0
DO 80 J = 1, NROW
OLP = OLP + DRP * DRP * LI(I, 1, 2) / 0.25
DO 80 L = 2, NP
80 OLP = OLP + 0.25 * RP(L) * DRP * LI(I, L, 2) / 0.25
OLP = P1 * RHOM * LH * OLP
C SUM TOTAL
C
C
QS = QSP + QSM + OLP
SUBROUTINE
C
C THIS SUBROUTINE CALCULATES THE TEMPERATURE FIELD FOR
C THE CASE OF CONSTANT PHASE FRONT VELOCITY IN CYLINDRICAL
C COORDINATES, REF. KREITH AND ROMIE CORRECTED.
C
SUBROUTINE KREITH (TF,R0,SR,STH,K,RHO,L,C,SG,ST,KS)
DOUBLE PRECISION ARG
REAL K,L
R=SR/R0
G=SG*C*R0/L
TH=STH*K*SG/(RHO*L*R0)
P=1+TH
IF (P.LT.0.0) GO TO 99
ARG=R/P
G2=G*G
G3=G2*G
G4=G2*G2
R2=R*R
R3=R2*R
R4=R2*R2
R5=R4*R
R6=R3*R3
R7=R3*R4
R8=R4*R4
P2=P*P
P3=P2*P
P4=P2*P2
P5=P3*P2
P6=P3*P3
P7=P3*P4
P8=P4*P4
T1=(*P*DLOG(ARG)
T2=G*((R2/4.+3.*P2/4.)*DLOG(ARG)+P2/2.-R2/2.)
T3=G2*((5.*P3/16.+3.*P*R2/8.)*DLOG(ARG)+21.*P3/64.-
C5.*P*R2/16.-R4/(64.*P))
T4=G3*((35.*P4/384.+15.*P2*R2/64.+3.*R4/128.)*
T5=G4*((198.*P5/9216+35.*P*R2/384.+15.*R4*P/512.)*
C*DLOG(ARG)+2325.*P5/73728.+31.*P3*R2/4608.-
C77.*P*R4/2048+R6/(1536.*P)-R8/(73728.*P))
TS=T1+T2+T3+T4+T5
ST=TS*(R0*SG)+TF
C  ST=32.
RETURN
99  KS=0
RETURN
END
THE PURPOSE OF THIS PROGRAM IS TO COMPUTE THE RATIO
OF HEAT TRANSFER INTO A TWO DIMENSIONAL ARRAY OF
CIRCULAR INCLUSIONS OF PCM IN A SOLID MATRIX DIVIDED
BY THE HEAT TRANSFER INTO THE 100% SOLID MATRIX
FOR THE BOUNDARY CONDITIONS OF THE SIMPLIFIED NEUMANN
SOLUTION. TWO DIMENSIONAL EFFECTS WERE ACCOUNTED
FOR BY AN EMPERICAL EQUATION DERIVED FOR NUMERICAL
ANALYSIS. THE NEUMANN APPROXIMATION IS ALSO INCLUDED.

DIMENSION QR(IOO),PFA(IOO)
REAL K1,K2,L2,KB,LAM,LB,KBK
DATA K1,R1,C1 /0.48,108.7,0.25/
DATA K2,R2,C2,L2 /0.150,53.3,0.45,105.6/
PI=3.141593
WRITE (5,1)
1 FORMAT (' NEUMANN ? IF YES ENTER POSITIVE INTEGER.')
READ (5,*) NM
10 WRITE (5,100)
100 FORMAT (' DELTA T = ?')
READ (5,*) DT
IF (DT.LT.0.0) GO TO 999
DO 20 I=1,100
F=I*0.01*PI/4.
RB=(1-F)*R1+F*R2
CB=((1-F)*R1*C1+F*R2*C2)/RB
LB=F*R2*L2/RB
STE=CB*DT/LB
SB=1./STE
CALL NUMAN (SB,LAM)
ALP=F*(K2-K1)/(K2+K1)
IF (ALP.EQ.1.0) GO TO 11
KB=K1*(ALP+1)/(1.-ALP)
GO TO 12
11 KB=K2
12 CONTINUE
KBK=KB/K1
AB=KB/(RB*CB)
ABA=AB/(K1/(R1*C1))
QR(I)=(1.324+0.1831*KBK-0.1816*STE)*KBK
C*SQR(1./ABA)/ERF(LAM)
IF (NM.GT.0) QR(I)=KBK*SQR(1./ABA)/ERF(LAM)
20 CONTINUE
QRT=QR(I)
DO 40 I=2,100
IF (QR(I).GT.QRT) GO TO 30
GO TO 40
30 QRT=QR(I)
40 CONTINUE
PF=0.01*I
DO 50 I=1,100
50 PFA(I)=I*0.01
WRITE (6,200)
200 FORMAT (30X,'4F/PI',/)
WRITE (6,210) (PFA(I),I=1,100,10)
210 FORMAT (4X,'DT',10F6.2,'4F(OPT)/PI·QR',/)
WRITE (6,220) DT,(QR(I),I=1,100,10),PF,QRT
220 FORMAT (F6.1,12F6.2,/) 
GO TO 10
999 STOP
END

SUBROUTINE NUMAN (SB,LAM)
REAL LAM
FUNCY(F,X)=F-X*EXP(X**2.)*ERF(X)
DATA N1,N2,N3 /10,10,10/
PI=3.141592654
FL=1./(SB*PI**0.5)
DXEP=0.001
TEST=0.0001
XMIN=0.0
YMIN=FL
X2=XMIN
IF (YMIN.LT.0.0) GO TO 6
XMAX=1.0-DXEP
YMAX=FUNCY(FL,XMAX)
X2=XMAX
IF (YMAX.GT.0.0) GO TO 6
DX=(1.-2.*DXEP)/N1
X1=XMIN
Y1=YMIN
1 X2=X1+DX
Y2=FUNCY(FL,X2)
IF (ABS(Y2).LT.TEST) GO TO 6
IF ((Y1/Y2).LE.0.0) GO TO 2
X1=X2
Y1=Y2
GO TO 1
2 DX=(X2-X1)/N2
3 X2=X1+DX
Y2=FUNCY(FL,X2)
IF (ABS(Y2).LT.TEST) GO TO 6
IF ((Y1/Y2).LE.0.0) GO TO 4
X1=X2
Y1=Y2
GO TO 3
4 DX=(X2-X1)/N3
5 X2=X1+DX
Y2=FUNCY(FL,X2)
IF (ABS(Y2).LT.TEST) GO TO 6
IF (((Y1/Y2).LE.0.0) GO TO 6
X1=X2
Y1=Y2
GO TO 5
CONTINUE
LAM=X2
RETURN
END
REFERENCES


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52. Wang, F. S., Telephone Conversation to Dow Chemical USA, Midland, Michigan, 1975.


