INFORMATION TO USERS

This material was produced from a microfilm copy of the original document. While the most advanced technological means to photograph and reproduce this document have been used, the quality is heavily dependent upon the quality of the original submitted.

The following explanation of techniques is provided to help you understand markings or patterns which may appear on this reproduction.

1. The sign or “target” for pages apparently lacking from the document photographed is “Missing Page(s)”. If it was possible to obtain the missing page(s) or section, they are spliced into the film along with adjacent pages. This may have necessitated cutting thru an image and duplicating adjacent pages to insure you complete continuity.

2. When an image on the film is obliterated with a large round black mark, it is an indication that the photographer suspected that the copy may have moved during exposure and thus cause a blurred image. You will find a good image of the page in the adjacent frame.

3. When a map, drawing or chart, etc., was part of the material being photographed the photographer followed a definite method in “sectioning” the material. It is customary to begin photoing at the upper left hand corner of a large sheet and to continue photoing from left to right in equal sections with a small overlap. If necessary, sectioning is continued again – beginning below the first row and continuing on until complete.

4. The majority of users indicate that the textual content is of greatest value, however, a somewhat higher quality reproduction could be made from “photographs” if essential to the understanding of the dissertation. Silver prints of “photographs” may be ordered at additional charge by writing the Order Department, giving the catalog number, title, author and specific pages you wish reproduced.

5. PLEASE NOTE: Some pages may have indistinct print. Filmed as received.

Xerox University Microfilms
300 North Zeeb Road
Ann Arbor, Michigan 48106
REIF, Thomas Henry, 1950-
CROSSFLOW OVER A POROUS CIRCULAR CYLINDER
WITH UNIFORM BLOWING AT THE SURFACE.

The Ohio State University, Ph.D., 1977
Engineering, mechanical

Xerox University Microfilms, Ann Arbor, Michigan 48106

Copyright By
Thomas Henry Reif
1977
CROSSFLOW OVER A POROUS CIRCULAR CYLINDER
WITH UNIFORM BLOWING AT THE SURFACE

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

Thomas H. Reif, B.S.M.E., M.S.

* * * * *

The Ohio State University
1977

Reading Committee:
L. S. Han
M. J. Moran
F. A. Kulacki

Approved By
Francis A. Kulacki
Adviser
Department of Mechanical Engineering
ACKNOWLEDGMENT

The author wishes to acknowledge the many people who have contributed to this project. Special thanks are given to my adviser, Dr. F. A. Kulacki. I am also indebted to Eugene McCall, Polk Burleson, Dr. C.D. Jones, Dr. S. A. Korpela, and Thomas Wiley for their generous help throughout the study. Finally, I would like to thank my wife, Susan, and her sister, Francina, whose typing efforts expedited this work. This project was supported by the National Science Foundation under Grant GK-43727.
VITA

December 24, 1950 . . Born - Dayton, Ohio

June, 1973 . . . . . . B.S.M.E., The Ohio State University Columbus, Ohio

1973-1974 . . . . . . Graduate Research Associate, Departments of Mechanical Engineering and Aeronautical and Astronautical Engineering, The Ohio State University, Columbus, Ohio

June, 1974 . . . . . . M.S. in Mechanical Engineering, The Ohio State University, Columbus, Ohio

1974-1976 . . . . . . Graduate Research Associate, Department of Mechanical Engineering, The Ohio State University, Columbus, Ohio

1976-1977 . . . . . . Assistant Professor, Department of Mechanical Engineering, U.S. Naval Academy, Annapolis, Maryland

PUBLICATIONS


ABSTRACT

Crossflow over a porous circular cylinder, with uniform blowing at the surface, has been investigated experimentally and analytically. Two free stream conditions, Re = 4100 and 6200, and five dimensionless blowing rate parameters, \( u_w/U_\infty = 0.190, 0.154, 0.126, 0.102, \) and \( 0.0, \) were studied experimentally via hot wire measurements of the external velocity field. Time-averaged velocity profiles, at several radial positions, were obtained from the front to the rear stagnation point. The analytical study employed the Galerkin method to obtain an approximate solution for the flow field around the entire circumference of the cylinder. The theory was found to give good agreement with the experiments close to the surface of the cylinder and in the entire flow field upstream of separation. The predicted flow fields were found to be in poor agreement with the experiments downstream of separation and away from the wall. This was attributed to the presence of non-steady phenomena that the approximate method was not equipped to handle. Results for the theoretical drag coefficient were also obtained. Increased blowing was shown to increase the total drag.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>ii</td>
</tr>
<tr>
<td>VITA</td>
<td>iii</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>iv</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>vi</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>viii</td>
</tr>
<tr>
<td>COMPUTER LISTINGS</td>
<td>xvii</td>
</tr>
<tr>
<td>LIST OF SYMBOLS</td>
<td>xviii</td>
</tr>
<tr>
<td>Chapter</td>
<td></td>
</tr>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II. EXPERIMENTAL STUDY</td>
<td>5</td>
</tr>
<tr>
<td>III. THEORETICAL STUDY</td>
<td>17</td>
</tr>
<tr>
<td>IV. DISCUSSION</td>
<td>60</td>
</tr>
<tr>
<td>APPENDIX</td>
<td></td>
</tr>
<tr>
<td>A. EXPERIMENTAL VELOCITY DATA</td>
<td>75</td>
</tr>
<tr>
<td>B. COMPUTER PROGRAMS</td>
<td>88</td>
</tr>
<tr>
<td>C. CALIBRATION AND USE OF THE ROTAMETERS</td>
<td>122</td>
</tr>
<tr>
<td>D. EXTERNAL ORTHOGONALITY RELATIONS</td>
<td>127</td>
</tr>
<tr>
<td>E. OSEEN'S APPROXIMATION</td>
<td>130</td>
</tr>
<tr>
<td>F. FIGURES</td>
<td>136</td>
</tr>
<tr>
<td>G. TABLES</td>
<td>233</td>
</tr>
<tr>
<td>REFERENCE</td>
<td>251</td>
</tr>
<tr>
<td>TABLE</td>
<td>Description</td>
</tr>
<tr>
<td>-------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>1.</td>
<td>Velocity Profile at the Inlet of the Porous Pipe</td>
</tr>
<tr>
<td>2.</td>
<td>Pressure Distribution on the Surface, ( \frac{u_w}{u_\infty} = 0.102 ) and ( Re = 6200 )</td>
</tr>
<tr>
<td>3.</td>
<td>Pressure Coefficient on the Surface, ( \frac{u_w}{u_\infty} = 0.0 ) and ( Re = 5000 )</td>
</tr>
<tr>
<td>4.</td>
<td>Constants of the Trial Stream Function (Galerkin Solution 1), ( \frac{u_w}{u_\infty} = 0.190 )</td>
</tr>
<tr>
<td>5.</td>
<td>Constants of the Trial Stream Function (Galerkin Solution 1), ( \frac{u_w}{u_\infty} = 0.154 )</td>
</tr>
<tr>
<td>6.</td>
<td>Constants of the Trial Stream Function (Galerkin Solution 1), ( \frac{u_w}{u_\infty} = 0.126 )</td>
</tr>
<tr>
<td>7.</td>
<td>Constants of the Trial Stream Function (Galerkin Solution 1), ( \frac{u_w}{u_\infty} = 0.102 )</td>
</tr>
<tr>
<td>8.</td>
<td>Constants of the Trial Stream Function (Galerkin Solution 1), ( \frac{u_w}{u_\infty} = 0.0 )</td>
</tr>
<tr>
<td>9.</td>
<td>Constants of the Trial Stream Function (Galerkin Solution 3), ( \frac{u_w}{u_\infty} = 0.190 )</td>
</tr>
<tr>
<td>10.</td>
<td>Constants of the Trial Stream Function (Galerkin Solution 3), ( \frac{u_w}{u_\infty} = 0.154 )</td>
</tr>
<tr>
<td>Table</td>
<td>Description</td>
</tr>
<tr>
<td>-------</td>
<td>-------------</td>
</tr>
<tr>
<td>11.</td>
<td>Constants of the Trial Stream Function (Galerkin Solution 3), $u_w/U_\infty = 0.126$</td>
</tr>
<tr>
<td>12.</td>
<td>Constants of the Trial Stream Function (Galerkin Solution 3), $u_w/U_\infty = 0.102$</td>
</tr>
<tr>
<td>13.</td>
<td>Constants of the Trial Stream Function (Galerkin Solution 3), $u_w/U_\infty = 0.0$</td>
</tr>
<tr>
<td>A-1.</td>
<td>Experimental Velocity Data, $u_w/U_\infty = 0.190$ and $Re = 4100$</td>
</tr>
<tr>
<td>A-2.</td>
<td>Experimental Velocity Data, $u_w/U_\infty = 0.154$ and $Re = 4100$</td>
</tr>
<tr>
<td>A-3.</td>
<td>Experimental Velocity Data, $u_w/U_\infty = 0.126$ and $Re = 6200$</td>
</tr>
<tr>
<td>A-4.</td>
<td>Experimental Velocity Data, $u_w/U_\infty = 0.102$ and $Re = 6200$</td>
</tr>
<tr>
<td>A-5.</td>
<td>Experimental Velocity Data, $u_w/U_\infty = 0.0$ and $Re = 4100$</td>
</tr>
<tr>
<td>A-6.</td>
<td>Experimental Velocity Data, $u_w/U_\infty = 0.0$ and $Re = 6200$</td>
</tr>
<tr>
<td>B-1.</td>
<td>Constants of the Trial Stream Function (Galerkin Solution 1), $u_w/U_\infty = 0.0$</td>
</tr>
</tbody>
</table>
### LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURE</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-A. Front View of the Apparatus</td>
<td>137</td>
</tr>
<tr>
<td>1-B. Rear View of the Apparatus</td>
<td>137</td>
</tr>
<tr>
<td>2. Mounted Porous Pipe in the Test Section of the Wind Tunnel</td>
<td>138</td>
</tr>
<tr>
<td>3. Schematic of the Flow System</td>
<td>139</td>
</tr>
<tr>
<td>4. Coordinate System</td>
<td>140</td>
</tr>
<tr>
<td>5. Hot Wire Anemometer</td>
<td>141</td>
</tr>
<tr>
<td>6. Micromanometer</td>
<td>141</td>
</tr>
<tr>
<td>7. Traverse Mechanism</td>
<td>142</td>
</tr>
<tr>
<td>8. Reference Pitot Tube</td>
<td>143</td>
</tr>
<tr>
<td>9. Velocity Profile at the Inlet of the Porous Pipe, $\hat{T} = 78.0$ F, $\hat{P} = 10$ PSIG, and $Re' = 23000$</td>
<td>144</td>
</tr>
<tr>
<td>10. Hole Drilled in the Porous Pipe for the Insertion of the Plastic Bushing</td>
<td>145</td>
</tr>
<tr>
<td>11. Static Pressure Tap</td>
<td>146</td>
</tr>
<tr>
<td>12. Pressure Transducer</td>
<td>147</td>
</tr>
<tr>
<td>13. Analog and Digital Readout for the Pressure Transducer</td>
<td>147</td>
</tr>
<tr>
<td>14. Pressure Distribution on the Surface, $u_w/U_\infty = 0.102$ and $Re = 6200$</td>
<td>148</td>
</tr>
<tr>
<td>15. Pressure Coefficient on the Surface, $u_w/U_\infty = 0.0$ and $Re = 5000$</td>
<td>149</td>
</tr>
<tr>
<td>16. Streamlines (Galerkin Solution 1), $u_w/U_\infty = 0.190$ and $Re = 4100$</td>
<td>150</td>
</tr>
</tbody>
</table>
FIGURE

17. Streamlines (Galerkin Solution 1), 
   $u_w/U_\infty = 0.154$ and $Re = 4100$ ........ 151

18. Streamlines (Galerkin Solution 1), 
   $u_w/U_\infty = 0.126$ and $Re = 6200$ ........ 152

19. Streamlines (Galerkin Solution 1), 
   $u_w/U_\infty = 0.102$ and $Re = 6200$ ........ 153

20. Streamlines (Galerkin Solution 1), 
   $u_w/U_\infty = 0.0$ and $Re = 4100$ ........ 154

21. Streamlines (Galerkin Solution 1), 
   $u_w/U_\infty = 0.0$ and $Re = 6200$ ........ 155

22. Theoretical Velocity Profile 
   (Galerkin Solution 1), $u_w/U_\infty = 0.190$, 
   $Re = 4100$, and $\hat{f}/R = 1.17$ ........ 156

23. Theoretical Velocity Profile 
   (Galerkin Solution 1), $u_w/U_\infty = 0.190$, 
   $Re = 4100$, and $\hat{f}/R = 1.33$ ........ 157

24. Theoretical Velocity Profile 
   (Galerkin Solution 1), $u_w/U_\infty = 0.190$, 
   $Re = 4100$, and $\hat{f}/R = 1.50$ ........ 158

25. Theoretical Velocity Profile 
   (Galerkin Solution 1), $u_w/U_\infty = 0.190$, 
   $Re = 4100$, and $\hat{f}/R = 1.67$ ........ 159

26. Theoretical Velocity Profile 
   (Galerkin Solution 1), $u_w/U_\infty = 0.190$, 
   $Re = 4100$, and $\hat{f}/R = 1.83$ ........ 160

27. Theoretical Velocity Profile 
   (Galerkin Solution 1), $u_w/U_\infty = 0.190$, 
   $Re = 4100$, and $\hat{f}/R = 2.00$ ........ 161

28. Theoretical Velocity Profile 
   (Galerkin Solution 1), $u_w/U_\infty = 0.154$, 
   $Re = 4100$, and $\hat{f}/R = 1.17$ ........ 162

29. Theoretical Velocity Profile 
   (Galerkin Solution 1), $u_w/U_\infty = 0.154$, 
   $Re = 4100$, and $\hat{f}/R = 1.33$ ........ 163
| FIGURE | Theoretical Velocity Profile  
|        | (Galerkin Solution 1), $\frac{u_w}{U_\infty} = 0.154$,  
|        | $Re = 4100$, and $\hat{r}/R = 1.50$.  
|        | Page  
| 30.    | 164  
|        | Theoretical Velocity Profile  
|        | (Galerkin Solution 1), $\frac{u_w}{U_\infty} = 0.154$,  
|        | $Re = 4100$, and $\hat{r}/R = 1.67$.  
|        | Page  
| 31.    | 165  
|        | Theoretical Velocity Profile  
|        | (Galerkin Solution 1), $\frac{u_w}{U_\infty} = 0.154$,  
|        | $Re = 4100$, and $\hat{r}/R = 1.83$.  
|        | Page  
| 32.    | 166  
|        | Theoretical Velocity Profile  
|        | (Galerkin Solution 1), $\frac{u_w}{U_\infty} = 0.126$,  
|        | $Re = 6200$, and $\hat{r}/R = 2.00$.  
|        | Page  
| 33.    | 167  
|        | Theoretical Velocity Profile  
|        | (Galerkin Solution 1), $\frac{u_w}{U_\infty} = 0.126$,  
|        | $Re = 6200$, and $\hat{r}/R = 1.17$.  
|        | Page  
| 34.    | 168  
|        | Theoretical Velocity Profile  
|        | (Galerkin Solution 1), $\frac{u_w}{U_\infty} = 0.126$,  
|        | $Re = 6200$, and $\hat{r}/R = 1.33$.  
|        | Page  
| 35.    | 169  
|        | Theoretical Velocity Profile  
|        | (Galerkin Solution 1), $\frac{u_w}{U_\infty} = 0.126$,  
|        | $Re = 6200$, and $\hat{r}/R = 1.50$.  
|        | Page  
| 36.    | 170  
|        | Theoretical Velocity Profile  
|        | (Galerkin Solution 1), $\frac{u_w}{U_\infty} = 0.126$,  
|        | $Re = 6200$, and $\hat{r}/R = 1.67$.  
|        | Page  
| 37.    | 171  
|        | Theoretical Velocity Profile  
|        | (Galerkin Solution 1), $\frac{u_w}{U_\infty} = 0.126$,  
|        | $Re = 6200$, and $\hat{r}/R = 1.83$.  
|        | Page  
| 38.    | 172  
|        | Theoretical Velocity Profile  
|        | (Galerkin Solution 1), $\frac{u_w}{U_\infty} = 0.126$,  
|        | $Re = 6200$, and $\hat{r}/R = 2.00$.  
|        | Page  
| 39.    | 173  
|        | Theoretical Velocity Profile  
|        | (Galerkin Solution 1), $\frac{u_w}{U_\infty} = 0.102$,  
|        | $Re = 6200$, and $\hat{r}/R = 1.17$.  
|        | Page  
| 40.    | 174  

x
41. Theoretical Velocity Profile (Galerkin Solution 1), \( \frac{u_w}{U_\infty} = 0.102 \), 
Re = 6200, and \( \hat{r}/R = 1.33 \) .......................... 175

42. Theoretical Velocity Profile (Galerkin Solution 1), \( \frac{u_w}{U_\infty} = -0.102 \), 
Re = 6200, and \( \hat{r}/R = 1.50 \) .......................... 176

43. Theoretical Velocity Profile (Galerkin Solution 1), \( \frac{u_w}{U_\infty} = 0.102 \), 
Re = 6200, and \( \hat{r}/R = 1.67 \) .......................... 177

44. Theoretical Velocity Profile (Galerkin Solution 1), \( \frac{u_w}{U_\infty} = 0.102 \), 
Re = 6200, and \( \hat{r}/R = 1.83 \) .......................... 178

45. Theoretical Velocity Profile (Galerkin Solution 1), \( \frac{u_w}{U_\infty} = 0.102 \), 
Re = 6200, and \( \hat{r}/R = 2.00 \) .......................... 179

46. Streamlines (Galerkin Solution 3), 
\( \frac{u_w}{U_\infty} = 0.190 \) and Re = 4100 .......................... 180

47. Streamlines (Galerkin Solution 3), 
\( \frac{u_w}{U_\infty} = 0.154 \) and Re = 4100 .......................... 181

48. Streamlines (Galerkin Solution 3), 
\( \frac{u_w}{U_\infty} = 0.126 \) and Re = 6200 .......................... 182

49. Streamlines (Galerkin Solution 3), 
\( \frac{u_w}{U_\infty} = 0.102 \) and Re = 6200 .......................... 183

50. Streamlines (Galerkin Solution 3), 
\( \frac{u_w}{U_\infty} = 0.0 \) and Re = 4100 .......................... 184

51. Streamlines (Galerkin Solution 3), 
\( \frac{u_w}{U_\infty} = 0.0 \) and Re = 6200 .......................... 185

52. Theoretical Velocity Profile (Galerkin Solution 3), \( \frac{u_w}{U_\infty} = 0.190 \), 
Re = 4100, and \( \hat{r}/R = 1.17 \) .......................... 186

53. Theoretical Velocity Profile (Galerkin Solution 3), \( \frac{u_w}{U_\infty} = 0.190 \), 
Re = 4100 and \( \hat{r}/R = 1.33 \) .......................... 187
<table>
<thead>
<tr>
<th>FIGURE</th>
<th>Theoretical Velocity Profile</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>54.</td>
<td>(Galerkin Solution 3), $u_w/U_\infty = 0.190$</td>
<td>188</td>
</tr>
<tr>
<td></td>
<td>$Re = 4100$, and $\hat{r}/R = 1.50$</td>
<td></td>
</tr>
<tr>
<td>55.</td>
<td>(Galerkin Solution 3), $u_w/U_\infty = 0.190$,</td>
<td>189</td>
</tr>
<tr>
<td></td>
<td>$Re = 4100$, and $\hat{r}/R = 1.67$</td>
<td></td>
</tr>
<tr>
<td>56.</td>
<td>(Galerkin Solution 3), $u_w/U_\infty = 0.190$,</td>
<td>190</td>
</tr>
<tr>
<td></td>
<td>$Re = 4100$, and $\hat{r}/R = 1.83$</td>
<td></td>
</tr>
<tr>
<td>57.</td>
<td>(Galerkin Solution 3), $u_w/U_\infty = 0.190$,</td>
<td>191</td>
</tr>
<tr>
<td></td>
<td>$Re = 4100$, and $\hat{r}/R = 2.00$</td>
<td></td>
</tr>
<tr>
<td>58.</td>
<td>(Galerkin Solution 3), $u_w/U_\infty = 0.154$,</td>
<td>192</td>
</tr>
<tr>
<td></td>
<td>$Re = 4100$, and $\hat{r}/R = 1.17$</td>
<td></td>
</tr>
<tr>
<td>59.</td>
<td>(Galerkin Solution 3), $u_w/U_\infty = 0.154$,</td>
<td>193</td>
</tr>
<tr>
<td></td>
<td>$Re = 4100$, and $\hat{r}/R = 1.33$</td>
<td></td>
</tr>
<tr>
<td>60.</td>
<td>(Galerkin Solution 3), $u_w/U_\infty = 0.154$</td>
<td>194</td>
</tr>
<tr>
<td></td>
<td>$Re = 4100$, and $\hat{r}/R = 1.50$</td>
<td></td>
</tr>
<tr>
<td>61.</td>
<td>(Galerkin Solution 3), $u_w/U_\infty = 0.154$,</td>
<td>195</td>
</tr>
<tr>
<td></td>
<td>$Re = 4100$, and $\hat{r}/R = 1.67$</td>
<td></td>
</tr>
<tr>
<td>62.</td>
<td>(Galerkin Solution 3), $u_w/U_\infty = 0.154$,</td>
<td>196</td>
</tr>
<tr>
<td></td>
<td>$Re = 4100$, and $\hat{r}/R = 1.83$</td>
<td></td>
</tr>
<tr>
<td>63.</td>
<td>(Galerkin Solution 3), $u_w/U_\infty = 0.154$,</td>
<td>197</td>
</tr>
<tr>
<td></td>
<td>$Re = 4100$, and $\hat{r}/R = 2.00$</td>
<td></td>
</tr>
<tr>
<td>64.</td>
<td>(Galerkin Solution 3), $u_w/U_\infty = 0.126$,</td>
<td>198</td>
</tr>
<tr>
<td></td>
<td>$Re = 6200$, and $\hat{r}/R = 1.17$</td>
<td></td>
</tr>
<tr>
<td>65.</td>
<td>(Galerkin Solution 3), $u_w/U_\infty = 0.126$,</td>
<td>199</td>
</tr>
<tr>
<td></td>
<td>$Re = 6200$, and $\hat{r}/R = 1.33$</td>
<td></td>
</tr>
</tbody>
</table>

xii
<table>
<thead>
<tr>
<th>FIGURE</th>
<th>Theoretical Velocity Profile</th>
</tr>
</thead>
<tbody>
<tr>
<td>66.</td>
<td>(Galerkin Solution 3), ( u_w/U_\infty = 0.126 ), Re = 6200, and ( \hat{r}/R = 1.50 )</td>
</tr>
<tr>
<td>67.</td>
<td>(Galerkin Solution 3), ( u_w/U_\infty = 0.126 ), Re = 6200, and ( \hat{r}/R = 1.67 )</td>
</tr>
<tr>
<td>68.</td>
<td>(Galerkin Solution 3), ( u_w/U_\infty = 0.126 ), Re = 6200, and ( \hat{r}/R = 1.83 )</td>
</tr>
<tr>
<td>69.</td>
<td>(Galerkin Solution 3), ( u_w/U_\infty = 0.126 ), Re = 6200, and ( \hat{r}/R = 2.00 )</td>
</tr>
<tr>
<td>70.</td>
<td>(Galerkin Solution 3), ( u_w/U_\infty = 0.102 ), Re = 6200, and ( \hat{r}/R = 1.17 )</td>
</tr>
<tr>
<td>71.</td>
<td>(Galerkin Solution 3), ( u_w/U_\infty = 0.102 ), Re = 6200, and ( \hat{r}/R = 1.33 )</td>
</tr>
<tr>
<td>72.</td>
<td>(Galerkin Solution 3), ( u_w/U_\infty = 0.102 ), Re = 6200, and ( \hat{r}/R = 1.50 )</td>
</tr>
<tr>
<td>73.</td>
<td>(Galerkin Solution 3), ( u_w/U_\infty = 0.102 ), Re = 6200, and ( \hat{r}/R = 1.67 )</td>
</tr>
<tr>
<td>74.</td>
<td>(Galerkin Solution 3), ( u_w/U_\infty = 0.102 ), Re = 6200, and ( \hat{r}/R = 1.83 )</td>
</tr>
<tr>
<td>75.</td>
<td>(Galerkin Solution 3), ( u_w/U_\infty = 0.102 ), Re = 6200, and ( \hat{r}/R = 2.00 )</td>
</tr>
<tr>
<td>76.</td>
<td>(Galerkin Solution 3), ( u_w/U_\infty = 0.0 ), Re = 4100, and ( \hat{r}/R = 1.17 )</td>
</tr>
<tr>
<td>FIGURE</td>
<td>Theoretical Velocity Profile (Galerkin Solution 3), $\frac{u_w}{U_\infty}$ = 0.0, $Re = 4100$, and $\hat{r}/R$ =</td>
</tr>
<tr>
<td>--------</td>
<td>-------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>77.</td>
<td>$1.33$.</td>
</tr>
<tr>
<td>78.</td>
<td>$1.50$.</td>
</tr>
<tr>
<td>79.</td>
<td>$1.67$.</td>
</tr>
<tr>
<td>80.</td>
<td>$1.83$.</td>
</tr>
<tr>
<td>81.</td>
<td>$2.00$.</td>
</tr>
<tr>
<td>82.</td>
<td>$1.17$.</td>
</tr>
<tr>
<td>83.</td>
<td>$1.33$.</td>
</tr>
<tr>
<td>84.</td>
<td>$1.50$.</td>
</tr>
<tr>
<td>85.</td>
<td>$1.67$.</td>
</tr>
<tr>
<td>86.</td>
<td>$1.83$.</td>
</tr>
<tr>
<td>87.</td>
<td>$2.00$.</td>
</tr>
<tr>
<td>FIGURE</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
</tr>
<tr>
<td>88. Theoretical Drag Coefficient.</td>
<td>222</td>
</tr>
<tr>
<td>89. Flow Impinging on the Cylinder, from Reference (29), $u_w/U_\infty = 0.0$ and Re = 6200</td>
<td>223</td>
</tr>
<tr>
<td>90. Wake Region Behind the Cylinder, from Reference (29), $u_w/U_\infty = 0.0$ and Re = 6200</td>
<td>224</td>
</tr>
<tr>
<td>91. Flow Impinging on the Cylinder, from Reference (29), $u_w/U_\infty = 0.053$ and Re = 6200</td>
<td>225</td>
</tr>
<tr>
<td>92. Wake Region Behind the Cylinder, from Reference (29), $u_w/U_\infty = 0.053$ and Re = 6200</td>
<td>226</td>
</tr>
<tr>
<td>93. Flow Impinging on the Cylinder, from Reference (29), $u_w/U_\infty = 0.126$ and Re = 6200</td>
<td>227</td>
</tr>
<tr>
<td>94. Wake Region Behind the Cylinder, from Reference (29), $u_w/U_\infty = 0.126$ and Re = 6200</td>
<td>228</td>
</tr>
<tr>
<td>95. Control Volume for the Application of the Integral Mass and Momentum Equations</td>
<td>229</td>
</tr>
<tr>
<td>96. Approximate Drag Coefficient as Calculated from the Integral Mass and Momentum Equations</td>
<td>230</td>
</tr>
<tr>
<td>97. Velocity Components (Galerkin Solution 3), $u_w/U_\infty = 0.190$, Re = 6200, and $\ell/R = 1.17^W$</td>
<td>231</td>
</tr>
<tr>
<td>98. Theoretical Vorticity (Galerkin Solution 3), Re = 6200 and $\ell/R = 1.17$</td>
<td>232</td>
</tr>
<tr>
<td>C-1. Calibration Curve for the Upstream Rotameter.</td>
<td>124</td>
</tr>
<tr>
<td>FIGURE</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>C-2.</td>
<td>Calibration Curve for the Downstream Rotameter</td>
</tr>
<tr>
<td>C-3.</td>
<td>Downstream and Upstream Rotameters (Items A and B Respectively)</td>
</tr>
<tr>
<td>E-1.</td>
<td>Comparison of Galerkin Solution 3 (Re = 0.1) and the Oseen Approximation,</td>
</tr>
<tr>
<td></td>
<td>$u_w/U_\infty = 0.0$ and $\hat{r}/R = 1.17$</td>
</tr>
<tr>
<td>E-2.</td>
<td>Theoretical Velocity Profile (Galerkin Solution 3), $u_w/U_\infty = 0.0$</td>
</tr>
<tr>
<td></td>
<td>and $\hat{r}/R = 1.17$</td>
</tr>
</tbody>
</table>
## COMPUTER LISTINGS

<table>
<thead>
<tr>
<th>LISTING</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-1. Program for the Determination of the Constants in the Trial Stream Function (Galerkin Solution 1)</td>
<td>95</td>
</tr>
<tr>
<td>B-2. Program for the Determination of the Constants in the Trial Stream Function (Galerkin Solution 3)</td>
<td>105</td>
</tr>
<tr>
<td>B-3. Program for Checking the Results of Program B-1</td>
<td>115</td>
</tr>
<tr>
<td>B-4. Program for the Calculation of the Coordinates of the Streamlines (Galerkin Solution 1)</td>
<td>117</td>
</tr>
<tr>
<td>B-5. Program for the Calculation of the Coordinates of the Streamlines for the Special Case of $u_w/U_\infty = 0.0$ (Galerkin Solution 1)</td>
<td>118</td>
</tr>
<tr>
<td>B-6. Program for the Calculation of the Coordinates of the Streamlines (Galerkin Solution 3)</td>
<td>120</td>
</tr>
</tbody>
</table>
### LIST OF SYMBOLS

- **A** Region of Integration in equation (25)
- **A_k** Control surfaces for the application of the integral mass and momentum equations, defined in Figure 95
- **A(k)** Variables in Listings B-4, B-5, and B-6 corresponding to the physical quantities ($\alpha_k$)
- **B_k** Constants, defined in equations (47)
- **B_k[]** Arbitrary operator defining boundary conditions
- **C_D** Drag coefficient, defined by equation (101)
- **C_k** Constants, defined in equations (98)
- **C_P** Pressure coefficient, defined in Table 3
- **C_P** Constant-pressure specific heat; Btu/lbm·R (cal/g·K)
- **D_k** Dimensions, defined in Figure 10; in. (cm)
- **E_k** Dimensions, defined in Figure 95; in. (cm)
- **f(q)** Non-homogeneous part of differential equation (19)
- **F_DN** Drag force due to normal stresses; lbf (dyne)
- **F_DS** Drag force due to shear stresses; lbf (dyne)
- **g_i(q)** Non-homogeneous part of the boundary conditions described in equations (20a)
- **G[]** Arbitrary operator defining boundary conditions
\( h(\theta) \) Temperature distribution on the surface of the cylinder; \( F(C) \)

\( H[ ] \) Arbitrary differential operator

\( \hat{i} \) Current reading of the hot wire anemometer; \( ma \)

\( \text{IREC} \) Variable in Listings B-1 and B-2 corresponding to the physical quantity \((\text{Re}/100-1)\) or in Listing B-3 corresponding to \((\text{Re}/100)\)

\( k \) Thermal conductivity; \( \text{Btu/s-ft-F} \) (\( \text{cal/s-cm-C} \))

\( L \) Length of the porous pipe; \( \text{in. (cm)} \)

\( L[ ] \) Non-linear differential operator, defined by equation (17)

\( M \) Number of boundary conditions

\( N \) Number of constants in the trial function

\( n \) Reciprocal of the exponent of the power-law velocity profile, defined in equation (1)

\( \hat{P} \) Pressure; \( \text{psia (dynes/cm}^2 \))

\( P \) Non-dimensionalized pressure,

\( P = \frac{\hat{P}}{\rho U_\infty^2} \)

\( P_\infty \) Pressure of the crossflow; \( \text{psia (dynes/cm}^2 \))

\( P_c \) Pressure during the calibration of the flowmeters; \( \text{psia (dynes/cm}^2 \))

\( q \) Vector of independent variables in equation (19)

\( \hat{Q} \) Actual volume flow rate as measured in the flowmeters; \( \text{ft}^3/\text{s (cm}^3/\text{s)} \)

\( \hat{Q}_c \) Volume flow rate from the calibration curves for the actual scale reading; \( \text{ft}^3/\text{s (cm}^3/\text{s)} \)
\( \hat{r}, \hat{\theta}, \hat{z} \) Cylindrical coordinates; in. (cm), radians (degrees), and in. (cm)

\( r, \theta \) Non-dimensionalized polar coordinates, \( r = \hat{r}/R \) and \( \theta = \hat{\theta} \)

\( R \) Outside radius of the porous pipe; in. (cm)

\( R' \) Inside radius of the porous pipe; in. (cm)

\( Re \) Reynolds number, based on the outside diameter of the porous pipe, defined by equation (13)

\( Ré \) Reynolds number, based on the inside diameter of the porous pipe

\( s \) Boundary surface of region of interest

\( S \) Scale reading of the flowmeters

\( \hat{T} \) Temperature; F (C)

\( T_\infty \) Temperature of the crossflow; F (C)

\( \hat{T}_c \) Temperature during the calibration of the flowmeters; F (C)

\( \hat{u} \) Component of velocity in the \( \hat{r} \)-direction; ft/s (cm/s)

\( u \) Non-dimensionalized velocity in the \( \hat{r} \)-direction, \( u = \hat{u}/U_\infty \)

\( U_W \) Variable in Listings B-1, B-2, B-4, and B-6 corresponding to the physical quantity \( (u_w/U_\infty) \)

\( U_\infty \) Velocity of the crossflow; ft/s (cm/s)

\( u_w \) Velocity at the surface of the cylinder; ft/s (cm/s)

\( \hat{\theta} \) Component of velocity in the \( \hat{\theta} \)-direction; ft/s (cm/s)
$v$ Non-dimensionalized velocity in the $\hat{\theta}$-direction, $v = \hat{v}/U_\infty$

$\hat{v}_c$ Velocity during the calibration of the hot wire probe; ft/s (cm/s)

$\hat{v}_x$ Component of velocity in the $\hat{x}$-direction; ft/s (cm/s)

$\hat{v}_y$ Component of velocity in the $\hat{y}$-direction; ft/s (cm/s)

$\hat{w}$ Component of velocity in the $\hat{z}$-direction; ft/s (cm/s)

$w_j$ Linearly independent weighting functions used in the elimination of the unknown constants in the trial function

$w_{\text{max}}$ Maximum $\hat{z}$-component of velocity in the internal velocity field; ft/s (cm/s)

$\hat{x}, \hat{y}$ Rectangular coordinates; in. (cm) and in. (cm)

**Greek Symbols**

$\alpha_k$ Constants in the trial function

$\beta$ Dimensionless parameter dependent on the structure of the porous matrix

$\epsilon$ Non-dimensionalized velocity, defined in Figure 95, $\epsilon = -\hat{v}_x/U_\infty$

$\kappa$ Porosity of the porous wall; ft$^2$ (cm$^2$)

$\mu$ Absolute viscosity of the fluid; lbm/ft-s (g/cm-s)

$\nu$ Kinematic viscosity of the fluid; ft$^2$/s (cm$^2$/s)
\( \xi \) Dependent variable in equation (19)

\( \xi^* \) Trial function for the Galerkin solution of the equations (19) and (20a)

\( \rho \) Density of the fluid; lbm/ft\(^3\) (g/cm\(^3\))

\( \hat{\tau}_{rr} \) Component of the viscous stress tensor in the \( \hat{r}\hat{r} \)-direction; psi (dynes/cm\(^2\))

\( \hat{\tau}_{r\theta} \) Component of the viscous stress tensor in the \( \hat{r}\hat{\theta} \)-direction; psi (dynes/cm\(^2\))

\( \phi_0 \) Part of the trial function which satisfies the actual boundary conditions or the actual differential equation

\( \phi_k \) Part of the trial function which satisfies the homogeneous boundary conditions or the homogeneous differential equation

\( \psi \) Non-dimensionalized stream function

\( \psi^* \) Trial stream function, defined in equations (30) and (59)

\( \Omega_z \) Component of the non-dimensionalized vorticity in the \( \hat{z} \)-direction

**Superscripts**

\(^*\) Dimensional variable

\(^p\) Pertaining to dimensions at the inside radius or diameter of the porous pipe

\(^*\) Designating the function to be an approximate expression
Subscripts

c Pertaining to conditions at calibration

max Denoting a maximum condition

rr Component in the \( \hat{r}\hat{r} \)-direction

r\( \theta \) Component in the \( \hat{r}\hat{\theta} \)-direction

x Component in the \( \hat{x} \)-direction

y Component in the \( \hat{y} \)-direction

\( \infty \) Free stream conditions

\( - \) Denoting a vector quantity
I. INTRODUCTION

Crossflow over porous pipes, with momentum transport through the walls, occurs in a number of important industrial situations. For example, in the nuclear power industry, porous pipes are used in the gaseous diffusion process for the production of enriched uranium isotopes (21). Further examples can be found in the chemical process industry, where porous pipes are used in a number of filtration processes, such as the retention of catalyst fines, process fluid filtration, water purification, and steam filtration. A more thorough knowledge of the associated fluid mechanics is required in order to optimize the design and operation of such process equipment. The pursuit of this knowledge is the objective of this study.

Over the past twenty-five years, a number of investigators have studied flows internal and external to pipes with porous walls. The internal flow studies can be classified as dealing with either laminar or turbulent flows.

1The numbers in brackets designate references as cited in the List of References.
The problem of a fully developed laminar pipe flow as it enters a section with a porous wall, with uniform suction at the wall, has been studied theoretically by Berman (10), Sparrow, Beavers, and Hung (44), Raithby (36), and Quaile and Levy (35). Quaile and Levy (35) give a summary of previous work in this area as well as some experimental results which validate their theory. Most of the effort in the area of turbulent internal flows is empirical in nature. Investigators such as Weissberg and Berman (51), Kinney and Sparrow (25), Merkine, Solan, and Winograd (30), Aggarwal, Hollingsworth, and Mayhew (1), and Brosh and Winograd (12), have studied the turbulent entry problem with uniform wall suction experimentally. Only a few, such as Drake and Molz (15), have attempted even an approximate theoretical study in this area.

Most of the studies of the external flow field about porous circular cylinders pertain only to the region of boundary layer flow upstream of separation. Again, it is usually restricted to the case of uniform blowing at the wall. These boundary layer studies, as with internal flows, usually deal with either laminar or turbulent flows. Theoretical work for the laminar boundary layer region has been done by Sparrow, Torrance, and Hung (45). They assumed that the pressure gradient, imposed by the inviscid flow field external to the boundary layer, to be
that measured by Johnson and Harnett (20). The solution to
the boundary layer flow field was formulated by expanding
the free stream flow in a series of odd powers of the dis­
tance from the forward stagnation point and applying the
Blasius series to the stream function in the boundary
layer. Turbulent boundary layers have been studied theo­
retically by Pletcher (32) via a turbulence model based
on a new formulation of the exponential damping function
originally suggested by Van Driest (50). Jonsson and
Scott (22) investigated the turbulent boundary layer prob­
lem experimentally by conducting impact pressure probe
measurements of the time smoothed average velocity field.

In summary, the work to date can be classified as
dealing with either internal or external flows. The in­
ternal flow problem has been thoroughly studied both
theoretically and experimentally for the case of uniform
suction at the wall. On the other hand, the external flow
problem has been studied in depth only in the region of
the boundary layer upstream of separation. Thus, there is
a need for further information, both theoretical and ex­
periential, which includes the entire external flow field.

The objective of this investigation is to obtain a
more thorough knowledge of the external flow phenomena.
Experimentally obtained velocity profiles will be presented
for the entire flow field. An approximate analytical
method will be used to obtain a solution of the differential equations of conservation of momentum and of mass, valid everywhere in the external flow field where, at least, quasi-steady flow exists. Finally, a comparison of the experiments and the approximate theory will be made to gain insight into the utility and limitations of the theory. Both the experimental and the theoretical results are limited to the case of uniform crossflow impinging on a porous circular cylinder with uniform radial momentum transport at the wall.
II. EXPERIMENTAL STUDY

As stated in the Introduction, one objective of this study was to obtain experimental velocity data for the problem of uniform flow impinging on a porous circular cylinder with uniform blowing at the surface. To fulfill this objective, the ultra-low speed wind tunnel facility shown in Figures 1-A and 1-B was constructed.¹ This tunnel had an 18 x 18 in. (45.7 x 45.7 cm) test section and was designed to give time-smoothed average velocities in the range of 2-4 ft/s (61-122 cm/s), corresponding to the lower range of the practically important Reynolds numbers (4000-6000) encountered in the nuclear power industry. Soda straws and screens were used at the inlet section of the tunnel to reduce the turbulence intensity in the test section (16, 41). A cubic shaped bellmouth with a 10:1 contraction ratio was inserted between the inlet section and the test section to insure a separation free and low turbulence velocity field in the test section (14, 37). The tunnel was calibrated by McCall (29) prior to this study. Velocity and pressure profiles were found to be uniform, within the

¹All figures are contained in Appendix F.
error of the measurements,\textsuperscript{2} up to 1.5 in. (3.81 cm) from the walls of the test section. Turbulence intensity was measured at the center of the test section for the local time - smoothed velocity of 4.4 ft/s (134 cm/s). The result of this measurement was recorded as 3.0\% with the absolute error of ± 0.5\%.

Figure 2 shows the porous pipe mounted in the test section of the wind tunnel. The pipe was manufactured by the Mott Metallurgical Corporation of Farmington, Connecticut. It had a 2 in. (5.08 cm) inside diameter, a 3 in. (7.62 cm) outside diameter, and was 18 in. (45.7 cm) in length. The pipe was constructed of 316-L sintered stainless steel and its uniform porosity allowed it to retain all particles larger than 5 x 10^{-7} in. (1.27 x 10^{-6} cm) in diameter. Affixed to the ends of the porous pipe were fittings (items A and B of Figure 2) which allowed pressure and temperature measurements and the connections to the inlet and outlet pipes (items C and D of Figure 2 respectively). Care was taken in the machining of the fittings to prevent any step from the 2 in. (5.08 cm) inside diameter, solid wall inlet and outlet pipes to the porous pipe.

\textsuperscript{2}The maximum absolute error in the velocity measurements were ± 0.4 ft/s (± 12 cm/s) and ± 6 x 10^{-6} psi (± 0.4 dyne/cm²) for the pressure measurements.
A schematic of the flow system is shown in Figure 3. Shop air was forced through a filter and dryer and a regulator. It then passed through the upstream rotameter, a length of flexible air line, and entered the long inlet pipe. The length to diameter ratio for this pipe was 60, so that there was fully developed turbulent flow at the inlet of the porous pipe. As the fully developed internal flow entered the porous pipe, some of the air passed through the wall of the pipe where it interacted with the external flow field of the wind tunnel. The remainder of the internal flow passed through the solid wall outlet pipe and flexible air line to the downstream rotameter. Before the air flow exited the system, it passed through the downstream throttling valve. Adjustment of this valve permitted variation of the total efflux through the walls of the porous pipe, without changing the flow conditions at its inlet.

Figure 4 shows the coordinate system used in this study. The uniform free stream of the wind tunnel, with velocity $U_\infty$ and pressure $P_\infty$, is shown to be impinging on the porous pipe in the negative $\hat{x}$-direction. The

---

3 The calibration and use of both the rotameters are discussed in Appendix C.

4 This was experimentally verified and is discussed later in this chapter.
\( \hat{y} \)-coordinate is taken in the vertical direction and the axis of symmetry of the pipe is perpendicular to the impinging free stream. At the outside radius of the pipe, \( \hat{r} = R \), there is a uniform radial component of velocity and a zero circumferential component.\(^5\)

A Flow Corporation, Model HWB-3, constant current hot wire anemometer (Figure 5) was used to measure the local time-smoothed velocity in the external flow field near the porous cylinder. A Flow Corporation, Model B-1-C, single wire, general purpose probe was used. This probe was calibrated for these measurements with a United Sensor, Model PAC-8-KL, pitot-static probe in the isentropic nozzle discussed by Choi (13). A Flow Corporation, Model MM3, micromanometer was used to measure pressures during the calibrations. This instrument, with an accuracy of \( \pm 6 \times 10^{-6} \) psi (\( \pm 0.4 \) dynes/cm\(^2\)), is shown in Figure 6.

The calibration curve for the probe was found by applying a least squares, straight-line curve fit to the dependent variable \((\hat{P}\hat{\nu}_c)^{1/2}\) and the independent variable \((4\hat{I})^2\), where \( \hat{P} \) is the static pressure, \( \hat{\nu}_c \) the local time-smoothed velocity, and \( \hat{I} \) the current reading of the anemometer. The maximum absolute error in the velocity

\(^5\)These conditions were experimentally verified and are discussed at the end of this chapter.
measurements by this calibration procedure was ± 0.4 ft/s
(± 12 cm/s). This figure was calculated by assuming that
all of the error in calibration could be attributed to the
pressure measurement. Thus, even though the velocity
measurements are somewhat uncertain in absolute magnitude,
they were accurate on a relative basis. The hot wire
probe was positioned in the test section of the wind tunnel
with the traverse mechanism shown in Figure 7. After the
tip of the probe was located with the cross hairs at the
side wall of the tunnel, the probe could be positioned with
± 0.001 in. (± 0.0025 cm) accuracy.

Tables A-1 through A-6 of Appendix A show the \( \hat{x} \) and
\( \hat{y} \) velocity components and their resultant vector sum, as
measured with the hot wire anemometer.\(^6\) The \( \hat{y} \)-component
was obtained by positioning the axis of the hot wire
parallel with the free stream. The vector sum was mea-
sured by rotating the probe 90°, making the axis of the
wire parallel with the axis of the cylinder. The remain-
ing \( \hat{x} \)-component was calculated from these results. All
of the data were taken at the center axial position of the
porous cylinder and with the same inlet conditions for the
internal flow (Reynolds number based on inside diameter

\(^6\) This data was corrected for the effect of tunnel
blockage by the method of Allen and Vincenti (2).
of 33000). Experiments were conducted at two different free stream velocities and three different blowing rates, giving six distinct cases with which to compare to the theory. The presentation of these experimental results, along with the analytical results, will be made in Chapter III.

The free stream velocity was measured with the pitot-static probe and the micromanometer discussed before. The reference probe was mounted upstream of porous pipe as shown in Figure 8. The blowing rate at the surface of the porous pipe was calculated from the difference in the mass flow rates measured by the two rotameters, assuming the pressure on the surface of the cylinder was near atmospheric.⁷

To verify that the internal pipe flow upstream of the porous pipe was fully developed, velocity measurements were conducted at the upstream fitting. The pitot-static probe, discussed before, was used with a Dwyer, Model 115, inclined manometer. This manometer had an accuracy of ± 0.005 in. H₂O (± 0.013 cm H₂O). Thus, the maximum absolute error in the dimensionless velocity, \( \hat{w}/\hat{w}_{\text{max}} \), was ± 0.07.

⁷This assumption was verified experimentally and is discussed at the end of this chapter.
where \( \hat{w} \) is the \( \hat{z} \)-component of the velocity and \( \hat{z} \) is the axial coordinate with origin at the inlet of the porous pipe. The results of these measurements are shown in Table 1 and Figure 9.\(^8\) The experimental data were compared to the empirical relation given by Schlichting (40),

\[
\frac{\hat{w}}{\hat{w}_{\text{max}}} = (1 - \frac{\hat{z}}{R'})^{1/n},
\]

where \( R' \) is the inside radius of the inlet pipe. For \( Re = 23000, n = 6.6 \) (40). Figure 9 shows the internal flow to be symmetric and fully developed within the error of the measurements.

After all of the velocity measurements in the external flow field were complete, a hole was drilled in the porous pipe as shown in Figure 10. A plastic bushing, with an 0.052 in. (0.132 cm) inside diameter and an 0.126 in. (0.320 cm) outside diameter, was inserted into this hole. The ends of the plastic bushing were then filed flush with the surface of the cylinder. A number 18 gauge hypodermic needle was inserted into the bushing to form the static pressure tap shown in Figure 11. The entire assembly, composed of the porous pipe, the inlet and outlet pipes, and the fittings were free to rotate in the angular direction.

\(^8\)All tables are contained in Appendix G.
allowing all measurements to be made with a single tap.

Figures 12 and 13 show the Datametrics, Model 570D-10T-1Al-V1, pressure transducer and the Model 1173-ALA-10Al-A analog readout manometer used to measure the static pressure at the surface of the tube. This device had an accuracy of 1% of full scale, or $2 \times 10^{-6}$ psi ($0.13$ dynes/cm$^2$) for the scale used. Since the pressures were fluctuating, due to periodic vortex shedding, some method of time-averaging the signal was needed. A ThermoSystems Inc., Model 1076, digital voltmeter, with a variable time window served this purpose (also shown in Figure 13). Results are shown in Table 2 and Figure 14 for the case of $Re = 6200$ and $\frac{u}{U} = 0.102$. These data, uncorrected for the effects of tunnel blockage, show that the circumferential variation of pressure was of the order of $2 \times 10^{-4}$ psi ($13.8$ dynes/cm$^2$) above and below atmosphere.

As a check of the above measurements, the limiting case of no blowing was studied. These results are shown in Table 3 and Figure 15 with a maximum absolute error of $\pm 0.05$ for the pressure coefficient. Also, in Figure 15 are the experimental data of Linke (23). The data of this study was corrected for the effect of tunnel blockage by the method of Allen and Vincenti (2). Figure 15 shows good agreement, upstream of separation. The poorer agreement, downstream of separation, may be explained by
differences in surface roughness and turbulence intensity between the two studies (7).

For the blowing conditions of this study it was necessary to keep the pressures at the inlet of the porous pipe of the order of 29 psig (1.38 x 10^6 dynes/cm^2). For these conditions, the pressure rise, from inlet to outlet, across the porous pipe was of the order of 4 x 10^{-3} psi (2.76 x 10^2 dynes/cm^2). Further, as demonstrated above, the pressure variation around the outer circumference of the porous pipe was of the order of 2 x 10^{-4} psi (13.8 dynes/cm^2) above and below atmosphere. Thus, in order for the internal flow to have had a significant effect on the external flow, it could be concluded that the pressure gradient in the radial direction within the porous wall was very large relative to the pressure gradient in the circumferential and axial directions. This conclusion is supported by Darcy's law in the discussion which follows.

For flow through a uniform porous media, where the fluid has constant viscosity and the body forces are negligible, Darcy's law (11, 33) states:

\[
\dot{u} = -\frac{k}{\mu} \frac{\partial p}{\partial x},
\]
\[
\dot{v} = -\frac{k}{\mu} \frac{\partial p}{\partial \theta},
\]

(2)
and

\[ \dot{w} = - \frac{k}{\mu} \frac{\partial \hat{p}}{\partial z}, \]

where \( k \) is the permeability of the porous matrix. Since the experimental results just discussed imply that

\[ \frac{\partial \hat{p}}{\partial r} \gg \frac{1}{r} \frac{\partial \hat{p}}{\partial \theta}, \]

and

\[ \frac{\partial \hat{p}}{\partial \theta} \gg \frac{\partial \hat{p}}{\partial z}, \]

equations (2) predict that, within the porous wall, the radial velocity is a function only of the radius and that the circumferential and axial velocities are negligible.

---

\(^9\)The permeability of the porous pipe used in this study was \( 1.08 \times 10^{-10} \text{ in.}^2 \) \((6.97 \times 10^{-10} \text{ cm}^2)\). This figure was based on information obtained from Catalog No. 1000 of the Mott Metallurgical Corporation of Farmington, Connecticut.
Beavers and Joseph (8) have proposed that the circumferential slip velocity is actually

$$
\hat{v}(R, \theta) = \frac{\kappa^{\frac{3}{2}}}{\beta} \frac{\partial \varphi}{\partial r}(R, \theta)
$$

(3)

where $\beta$ is a dimensionless parameter dependent on the structure of the porous matrix. Defining the dimensionless variables

$$
u = \frac{\hat{u}}{U_\infty},$$

$$v = \frac{\hat{v}}{U_\infty},$$

and

$$r = \frac{\hat{r}}{R},$$

equation (3) can be written as

$$v(1, \theta) = \frac{\kappa^{\frac{3}{2}}}{\beta R} \frac{\partial v}{\partial r}(1, \theta).$$

(5)

Experiments (8, 9) with a porous material similar to that used in this study indicate that $\beta$ was of the order of $10^0$. Thus, the quantity $(\kappa^{\frac{3}{2}}/\beta R)$ was of the order of $10^{-5}$. 
Further, $\frac{\partial v}{\partial r}(1,\theta)$ was of the order of $10^0$. Therefore, $v(1,\theta)$ was of the order of $10^{-5}$. Finally, since $u(\infty,\theta)$ was of the order of $10^0$ and $u(1,\theta)$ was of the order of $10^{-1}$, then

$$v(1,\theta) \ll u(1,\theta)$$

and

$$v(1,\theta) \ll u(\infty,\theta).$$

These conclusions and those deduced from Darcy's law are the same since they both imply that for the conditions of this study, the external flow experienced the case of uniform radial velocity and zero slip at the surface of the porous pipe, i.e., the case of uniform blowing.
III. THEORETICAL STUDY

The objective of this investigation, as stated in Chapter I, was to obtain a more thorough knowledge of the flow field external to a porous pipe with uniform blowing at the surface placed in a uniform crossflow. Because of the complexity of this problem and since information was desired over the entire circumference of the cylinder, it was necessary to use an approximate technique to study the problem analytically. A discussion of the approximate analysis used and the viability of other analytical techniques will be given after a more formal definition of the problem.

The geometry of the problem is as shown in Figure 4. A uniform free stream, with velocity $U_\infty$, impinges on a circular cylinder, with radius $R$. The axis of symmetry of the cylinder is perpendicular to the free stream. As verified in Chapter II, at the surface of the cylinder there is a uniform emission of fluid in the radial direction and there is no circumferential component of velocity.

Assuming that the fluid in the external flow field is Newtonian and isotropic, with constant density and
viscosity, and that the flow is steady with negligible body forces, then the applicable equations (11) are:

\[ \hat{u} \frac{\partial \hat{u}}{\partial \hat{r}} + \frac{\hat{v}}{\hat{r}} \frac{\partial \hat{u}}{\partial \hat{\theta}} - \frac{\hat{v}^2}{\hat{r}} = - \frac{1}{\hat{\rho}} \frac{\partial \hat{p}}{\partial \hat{r}} \]

(6)

\[ + \nu \left[ \frac{\partial^2 \hat{u}}{\partial \hat{r}^2} + \frac{1}{\hat{r}} \frac{\partial \hat{u}}{\partial \hat{r}} - \frac{\hat{u}}{\hat{r}^2} + \frac{1}{\hat{r}^2} \frac{\partial^2 \hat{u}}{\partial \hat{\theta}^2} - \frac{2}{\hat{r}^2} \frac{\partial \hat{v}}{\partial \hat{\theta}} \right] , \]

\[ \hat{u} \frac{\partial \hat{v}}{\partial \hat{r}} + \frac{\hat{v}}{\hat{r}} \frac{\partial \hat{v}}{\partial \hat{\theta}} + \hat{u} \hat{v} = - \frac{1}{\hat{\rho} \hat{r}} \frac{\partial \hat{p}}{\partial \hat{\theta}} \]

(7)

\[ + \nu \left[ \frac{\partial^2 \hat{v}}{\partial \hat{r}^2} + \frac{1}{\hat{r}} \frac{\partial \hat{v}}{\partial \hat{r}} - \frac{\hat{v}}{\hat{r}^2} + \frac{1}{\hat{r}^2} \frac{\partial^2 \hat{v}}{\partial \hat{\theta}^2} + \frac{2}{\hat{r}^2} \frac{\partial \hat{u}}{\partial \hat{\theta}} \right] , \]

and

\[ \frac{\partial \hat{u}}{\partial \hat{r}} + \frac{\hat{u}}{\hat{r}} + \frac{1}{\hat{r}} \frac{\partial \hat{v}}{\partial \hat{\theta}} = 0 , \]

(8)

which are the equations of the conservation of momentum in the \( \hat{r} \)- and \( \hat{\theta} \)-directions and the conservation of mass respectively. Defining the dimensionless variable

\[ 1 \text{The validity of this assumption will be addressed in Chapter IV.} \]
and using those defined in equations (4), equations (6), (7), and (8) become

\[
\frac{u}{3r} + \frac{v}{r} \frac{3u}{3\theta} - \frac{v^2}{r} = - \frac{3P}{3r}
\]

\[
+ \frac{2}{Re} \left[ \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial v}{\partial r} \right]
\]

\[
\frac{u}{3r} + \frac{v}{r} \frac{3v}{3\theta} + \frac{uv}{r} = - \frac{1}{r} \frac{3P}{3\theta}
\]

\[
+ \frac{2}{Re} \left[ \frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2} + \frac{1}{r^2} \frac{\partial^2 v}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial u}{\partial r} \right]
\]

and

\[
\frac{3u}{3r} + \frac{u}{r} + \frac{1}{r} \frac{3v}{3\theta} = 0
\]

where Re is the Reynolds number, based on diameter, defined as

\[
Re = \frac{2RU_{\infty}}{v}
\]
Next, a new variable, \( \psi \), defined by

\[
u = -\frac{1}{r} \frac{\partial \psi}{\partial \theta}
\]

and

\[
v = \frac{\partial \psi}{\partial r}
\]

is introduced. This variable, physically the stream function, satisfies the continuity equation, equation (12). When equations (14) and (15) are substituted into equations (10) and (11), and the pressure terms are eliminated between them, the result is the single equation (18)\(^2\)

\[
L[\psi(r, \theta)] = 0
\]

where

\[
L[\psi(r, \theta)] = \frac{1}{r} \frac{\partial \psi}{\partial r} \frac{\partial}{\partial \theta} (\nabla^2) - \frac{1}{r} \frac{\partial \psi}{\partial \theta} \frac{\partial}{\partial r} (\nabla^2)
\]

\[
- \frac{2}{Re} \nabla^2 (\nabla^2)
\]

\(^2\)The pressure terms are eliminated by cross-differentiating the momentum equations and subsequently subtracting one from the other.
The boundary conditions which accompany equation (16) are

\begin{align*}
\text{u}(\infty, \theta) &= -\frac{1}{r} \frac{\partial \psi}{\partial \theta} \bigg|_{r \to \infty} = -\cos \theta, \\
v(\infty, \theta) &= \frac{\partial \psi}{\partial r} \bigg|_{r \to \infty} = \sin \theta, \\
u(1, \theta) &= -\frac{\partial \psi}{\partial \theta}(1, \theta) = \frac{u_w}{U_\infty}, \\
v(1, \theta) &= \frac{\partial \psi}{\partial r}(1, \theta) = 0, \\
\frac{\partial u}{\partial \theta}(r, 0) &= -\frac{1}{r} \frac{\partial^2 \psi}{\partial \theta^2}(r, 0) = 0, \\
v(r, 0) &= \frac{\partial \psi}{\partial r}(r, 0) = 0, \\
\frac{\partial u}{\partial \theta}(r, \pi) &= -\frac{1}{r} \frac{\partial^2 \psi}{\partial \theta^2}(r, \pi) = 0, \\
v(r, \pi) &= \frac{\partial \psi}{\partial r}(r, \pi) = 0.
\end{align*}

The first two boundary conditions indicate that far from the surface of the cylinder, the free stream is undisturbed by the presence of the cylinder. The third and fourth
boundary conditions state that there is a uniform radial velocity component and a zero circumferential component at the surface of the cylinder. The remaining four boundary conditions indicate symmetry at the leading and trailing edges, i.e., at $\theta = 0$ and $\theta = \pi$, respectively.

As stated in the beginning of this chapter, because of the complexity of the flow field, it was necessary to use an approximate technique to study the problem analytically. Approximate methods for the solution of nonlinear partial differential equations may be classified into four general categories: asymptotic, numerical, iterative, and weighted residual methods (3).

The asymptotic methods are based on the desire to find a solution when a physical parameter or variable becomes very small or very large. These methods may further be classified as perturbation procedures or asymptotic approximations (3). The perturbation procedures can be used to formulate a solution of the problem of interest by utilizing the results of a known or "standard" solution. Since the problem of interest in this study has no such "standard" solution, this approach is not a viable alternative. Asymptotic approximations utilize the idea that some equations, which are unsolvable analytically, could give some useful information if they were to be examined as a system parameter approached a certain limit (usually zero or infinity).
This method was rejected because no such system parameter exists which reduces the problem of this study to a form such that it can be solved more readily (this would be the "standard" solution or a starting place of the perturbation procedure).

The numerical methods are usually based on a finite difference approximation of the partial differential equations. A major disadvantage of these methods is that the solution is obtained at only a finite number of mesh points. A number of investigators have studied flows around solid circular cylinders (i.e., no blowing) by this method (19, 46, 48). In general, their results fail to show the finer features of the flow field, such as the development of the von Kármán vortex street. Further, numerical instabilities (related to computer storage and running time) usually prevent completely successful solutions in the range of Reynolds numbers encountered in the present study. Thoman and Szewczyk (48) were, however, able to extend the computations past the range of Reynolds numbers of this investigation by utilizing an ad hoc differencing scheme and mesh pattern. A typical solution by this method required approximately six hours using the Univac 1107 digital computer. Because of the non-homogeneous boundary conditions at the surface of the cylinder in the present problem, it is expected that even larger computation time and machine storage would be
required (47).

The iterative methods are analogous to the Picard method for ordinary differential equations (3). A repetitive calculation, via some operation (usually an integration), successively improves the approximation. Since this type of problem is most easily handled on a digital computer, it is subject to many of the same difficulties and limitations of the numerical methods just discussed.

Of the many weighted residual methods, such as the method of moments, collocation, subdomain, least squares, and Galerkin's method, Galerkin's method has been shown in practice to give superior convergence over those of the other methods (3, 43). That is, one can use fewer terms in the approximating or trial function. Hence, because of its superior convergence relative to the other weighted residual methods, and because of the limitations of the other approximate techniques which are not present in the weighted residual method (these advantages will become evident in the subsequent discussion), the Galerkin method was used to solve equation (16) with boundary conditions (18).

The Galerkin method is discussed at length in references (3, 4, 23, 26, 43). In essence, the method is useful for finding an approximate solution to a general differential equation
\[ H[\xi(q)] = f(q) \]  

with boundary conditions

\[ B_i[\xi(q_i)] = g_i(q_i), \quad i = 1, 2, 3, \ldots, M \]  

over some region \( A \) or

\[ G[\xi(q)] = 0 \]  

over some surface \( s \).

Generally, a trial function

\[ \xi^*(q) = \phi_0(q) + \sum_{k=1}^{N} \alpha_k \phi_k(q) \]  

is constructed such that \( \phi_0(q) \) and the \( \phi_k(q) \) satisfy

\[ B_i[\phi_0(q_i)] = g_i(q_i), \quad i = 1, 2, 3, \ldots, M \]  

and

\[ B_i[\phi_k(q_i)] = 0, \quad i = 1, 2, 3, \ldots, M \]  

Next, \( N \) linearly independent weighting functions (commonly taken as the \( \phi_j(q), \quad j = 1, 2, 3, \ldots, N \)),
are chosen and the integral conditions

\[
\int_A H[\xi^*(q)] w_j(q) \, dA = \int_A f(q) w_j(q) \, dA,
\]

\( j = 1, 2, 3, \ldots, N \) are used to eliminate the \( N \) undetermined constants. Equation (25) requires that the weighted residual of the differential equation be zero in the mean in region \( A \). This type of procedure is termed the "internal" method \( (^3, 43) \).

On the other hand, the trial function may be constructed such that \( \phi_0(q) \) and the \( \phi_k(q) \) satisfy

\[
H[\phi_0(q)] = f(q)
\]

(26)

and

\[
H[\phi_k(q)] = 0.
\]

(27)

Similar to the procedure for the internal method, \( N \) linearly
independent weighting functions are chosen and the integral equations

\[ \int_{s} G[\xi^{*}(q)] w_j(q) ds = 0, \]

(28)

\[ j = 1, 2, 3, \ldots, N \]

are used to solve for the N undetermined constants in the trial function. Here, s is the boundary surface of the region of interest. Solution of equations (19) and (20) by this scheme is referred to as the boundary or "external" method (3, 43).

In some cases it may be neither possible nor convenient to choose \( \phi_0(q) \) and the \( \phi_k(q) \) such that they satisfy either the boundary conditions or the differential equation. For such situations, both types of orthogonality relations, equations (25) and (28), are needed. This type of calculation is referred to as the "mixed" method (3, 43).

The major obstacle and challenge in obtaining a solution by the Galerkin method is in choosing an approximate trial function. Judgment, experience, and the criteria just set forth are the only guidelines available for this choice.
Thus, a reasonable starting place is to find a trial function used with some success in a similar problem.

Such a trial function for the present problem was found in the work of Kihara (24), who investigated the heat transfer characteristics of electric arcs. If this trial function is assumed to be of the form of equation (21) and is modified to take into account the presence of blowing, then

\[
\phi_{O}(r,\theta) = r \sin \theta - \frac{u_w \theta}{U_\infty},
\]

\[
\phi_{K}(r,\theta) = r^{-k} \sin \theta \quad (k = 1, 3, 5, 7),
\]

(29)

and

\[
\phi_{K}(r,\theta) = r^{-k} \sin 2\theta \quad (k = 2, 4, 6, 8).
\]

The trial stream function can then be written as

\[
\psi^*(r,\theta) = (r + \alpha_1 r^{-1} + \alpha_3 r^{-3} + \alpha_5 r^{-5})
\]

\[
+ \alpha_7 r^{-7}) \sin \theta + (\alpha_2 r^{-2} + \alpha_4 r^{-4})
\]

\[
+ \alpha_6 r^{-6} + \alpha_8 r^{-8}) \sin 2\theta - \frac{u_w \theta}{U_\infty}.
\]

(30)
Comparison of equations (29) with equations (16)-(18) shows that:

1) $\phi_o(r,\theta)$ satisfies all the actual boundary conditions, except at $r = 1$,

2) $\phi_o(r,\theta)$ satisfies the actual differential equation, i.e., $L[\phi_o] = 0$,

3) the $\phi_k(r,\theta)$ satisfy all of the homogeneous boundary conditions, except at $r = 1$, and

4) in general, except for $k = 1$ and 2, the $\phi_k(r,\theta)$ do not satisfy the homogeneous differential equation, e.g.,

$L[\phi_k] \neq 0$, $k \neq 1$ and 2.

Since $\phi_o(r,\theta)$ and the $\phi_k(r,\theta)$ do not satisfy the actual and homogeneous boundary conditions at $r = 1$, then the internal method does not apply. Further, because the $\phi_k(r,\theta)$ do not in general satisfy the homogeneous differential equation the external method cannot be used. Consequently, the mixed method must be utilized.

The internal orthogonality relations are

$$\int_0^1 \int_0^{2\pi} L[\psi^*]\sin\theta dr d\theta = 0 , \quad (31)$$
In equations (31) and (32) the weighting functions

\[ w_1(r, \theta) = r^{-1}\sin \theta \]  

(35)

and

\[ w_2(r, \theta) = r^{-2}\sin 2\theta \]  

(36)

have been used, respectively. In equations (33) and (34) the weighting functions

\[ w_3(r, \theta) = r^{-3}\sin \theta \]  

(37)
and

\[ w_4(r, \theta) = r^{-4} \sin 2\theta \]  \hspace{1cm} (38)

could have been used in principle. However, these equations are modified forms of equations (31) and (32) and are intended to bias the solution for precision near the surface of the cylinder. It may be noted that the flexibility to make such a modification is one of the chief advantages of the Galerkin method (43).

The external orthogonality conditions are

\[ \int_0^{2\pi} \left[ \frac{\partial \psi^*}{\partial \theta} + \frac{\partial \psi^*}{\partial r} + \frac{u_\infty}{U_\infty} \right]_{r=1} \sin \theta d\theta = 0 \]  \hspace{1cm} (39)

and

\[ \int_0^{2\pi} \left[ \frac{\partial \psi^*}{\partial \theta} + \frac{\partial \psi^*}{\partial r} + \frac{u_\infty}{U_\infty} \right]_{r=1} \sin 2\theta d\theta = 0 \]  \hspace{1cm} (40)

3These conditions lead to computational difficulties which will be addressed later in this chapter.
In addition to the external equations (39) and (40), it is also necessary to add the following condition on physical grounds

\[
\frac{\partial \psi^*}{\partial \theta} (1, \theta) = - \frac{u_w}{U_\infty} \cdot 4
\]  

(41)

In summary, equations (30), (31)-(34), and (39)-(41) are the necessary relations for a solution of the external flow field. Using equation (30) directly with equation (39) and (40) results in the following equations:

\[
1 - \alpha_1 - 3\alpha_3 - 5\alpha_5 - 7\alpha_7 = 0
\]  

(42)

and

\[
\alpha_2 + 2\alpha_4 + 3\alpha_6 + 4\alpha_8 = 0
\]  

(43)

Similarly, from equations (30) and (41),

\[
1 + \alpha_1 + \alpha_3 + \alpha_5 + \alpha_7 = 0
\]  

(44)

and

\[\text{This point is discussed in detail in Appendix D.}\]
\[
\alpha_2 + \alpha_4 + \alpha_6 + \alpha_8 = 0 .
\]

When equation (30) is substituted into equations (16) and (17), the result is

\[
L[\psi^*(r,\theta)] = (B_1 B_3 - B_5 B_7) \sin\theta \cos\theta
\]

\[
+ (2B_1 B_4 - 2B_5 B_7) \sin\theta \cos 2\theta
\]

\[
+ (B_2 B_3 - B_5 B_8) \sin 2\theta \cos\theta
\]

\[
+ (2B_2 B_4 - 2B_6 B_8) \sin 2\theta \cos 2\theta
\]

\[
+ \left( -\frac{u_w B_7}{x U_\infty} - \frac{2B_9}{Re} \right) \sin\theta
\]

\[
+ \left( -\frac{u_w B_8}{x U_\infty} - \frac{2B_{10}}{Re} \right) \sin 2\theta
\]

where

\[
B_1 = r^{-1} - \alpha_1 r^{-3} - 3\alpha_3 r^{-5} - 5\alpha_5 r^{-7} - 7\alpha_7 r^{-9}
\]

\[
B_2 = -2\alpha_2 r^{-4} - 4\alpha_4 r^{-6} - 6\alpha_6 r^{-8} - 8\alpha_8 r^{-10}
\]

\[
B_3 = 8\alpha_3 r^{-5} + 24\alpha_5 r^{-7} + 48\alpha_7 r^{-9}
\]
\[ B_4 = 12\alpha_4 r^{-6} + 32\alpha_6 r^{-8} + 60\alpha_8 r^{-10}, \]
\[ B_5 = 1 + \alpha_1 r^{-2} + \alpha_3 r^{-4} + \alpha_5 r^{-6} + \alpha_7 r^{-8}, \]
\[ B_6 = \alpha_2 r^{-3} + \alpha_4 r^{-5} + \alpha_6 r^{-7} + \alpha_8 r^{-9}, \]
\[ B_7 = -40\alpha_3 r^{-6} - 168\alpha_5 r^{-8} - 432\alpha_7 r^{-10}, \]
\[ B_8 = -72\alpha_4 r^{-7} - 256\alpha_6 r^{-9} - 600\alpha_8 r^{-11}, \]
\[ B_9 = 192\alpha_3 r^{-7} + 1152\alpha_5 r^{-9} + 3840\alpha_7 r^{-11}, \]

and

\[ B_{10} = 384\alpha_4 r^{-8} + 1920\alpha_6 r^{-10} + 5760\alpha_8 r^{-12}. \]

Upon substitution of equation (46) into equation (31), the first internal condition, the result is

\[ \alpha_2 \left( -\frac{6}{8}\alpha_3 - \frac{24}{10}\alpha_5 - \frac{60}{12}\alpha_7 \right) \]

\[ + \alpha_4 \left( \frac{1}{2} + \frac{6}{8}\alpha_1 + \frac{2}{10}\alpha_3 - \frac{15}{12}\alpha_5 - \frac{51}{14}\alpha_7 \right) \]

\[ + \alpha_6 \left( \frac{3}{2} + \frac{20}{10}\alpha_1 + \frac{20}{12}\alpha_3 + \frac{6}{14}\alpha_5 - \frac{28}{16}\alpha_7 \right) \quad (48) \]
Similarly, equations (32)-(34), the second through fourth internal conditions become,

\[ a_1 \left( \frac{2}{8}a_3 + \frac{9}{10}a_5 + \frac{24}{12}a_7 \right) + a_3 \left( \frac{3}{6} + \frac{1}{10}a_3 + \frac{6}{12}a_5 + \frac{17}{14}a_7 \right) + a_5 \left( \frac{12}{8} + \frac{3}{14}a_5 + \frac{12}{16}a_7 \right) + a_7 \left( \frac{30}{10} + \frac{6}{18}a_7 \right) + u_w \left( - \frac{9}{8}a_4 - \frac{32}{10}a_6 - \frac{75}{12}a_8 \right) \]

\[- \frac{1}{Re} \left( \frac{96}{8}a_4 + \frac{480}{10}a_6 + \frac{1440}{12}a_8 \right) = 0 \]  

\[ a_2 \left( - 6a_3 - 24a_5 - 60a_7 \right) + a_4 \left( 3 + 6a_1 + 2a_3 - 15a_5 - 51a_7 \right) \]
\[ + \alpha_6 (12 + 20\alpha_1 + 20\alpha_3 + 6\alpha_5 - 28\alpha_7) \]

\[ + \alpha_8 (30 + 45\alpha_1 + 51\alpha_3 + 42\alpha_5 + 12\alpha_7) \]

\[ + \frac{u_w}{U_\infty} (-5\alpha_3 - 21\alpha_5 - 54\alpha_7) \]

\[ - \frac{1}{Re} (48\alpha_3 + 288\alpha_5 + 960\alpha_7) = 0 , \]

and

\[ \alpha_1 (2\alpha_3 + 9\alpha_5 + 24\alpha_7) \]

\[ + \alpha_3 (3 + \alpha_3 + 6\alpha_5 + 17\alpha_7) \]

\[ + \alpha_5 (12 + 3\alpha_5 + 12\alpha_7) \]

\[ + \alpha_7 (30 + 6\alpha_7) \]

\[ + \frac{u_w}{U_\infty} (-9\alpha_4 - 32\alpha_6 - 75\alpha_8) \]

\[ - \frac{1}{Re} (96\alpha_4 + 480\alpha_6 + 1440\alpha_8) = 0 . \]

Equations (42)-(45) and (48)-(51) form a set of eight simultaneous algebraic equations (four of which are non-linear), which will yield the eight constants of the trial
stream function. An algorithm, developed by Powell (33), which utilizes both the method of steepest descent and the Newton-Raphson method, was used to solve these equations. The results are presented in Tables 4-8 for the range of Reynolds numbers and blowing rates of interest in the experimental study and will be referred to as the "first Galerkin solution." The results for the limiting case of \( \frac{u_w}{U_\infty} = 0.0 \) agree with the results presented by Kihara (24), thus, serving as an independent check of the algebra and the numerical procedures.

Figures 16-21 depict the streamlines as calculated from the constants of Tables 4-8 and equation (30). The first four of these figures show streamlines that are physically reasonable. The stagnation point is shown to be upstream of the cylinder along the \( \theta = 0^\circ \) coordinate. Increasing the dimensionless blowing parameter, \( \frac{u_w}{U_\infty} \), is shown to move the stagnation point upstream. The

---

5 It should be noted that the use of the mixed method yields more equations than unknowns. Four more equations could have been written in the spirit of equations (31)-(34) to include the other weighting functions. Thus, to avoid an overdetermined system, some of the equations must be discarded. The higher order weighting functions have been discarded based on the suggestion of Ames (3) and Snyder, Spriggs, and Stewart (43).

6 This is discussed in more detail in Appendix B.

7 These calculations are also discussed in Appendix B.
streamlines at $r = 1$ are orthogonal to the surface, in keeping with the assumptions of uniform radial and zero circumferential emission. Far from the surface, the streamlines approach the conditions of the undisturbed free stream. The streamlines for the two zero blowing cases (Figures 20 and 21), however, are not physically reasonable. These figures show the streamlines to be closed upstream of the cylinder, depicting a region of recirculation. Further numerical experiments for the no blowing case showed that a forward recirculation region is predicted even at very low Reynolds numbers, e.g., $Re = 100$.

Velocity profiles can also be determined from equations (14), (15), and (30), e.g.,

$$u = -(1 + \alpha_1 r^{-2} + \alpha_3 r^{-4} + \alpha_5 r^{-6} + \alpha_7 r^{-8}) \cos \theta$$

$$- 2(\alpha_2 r^{-3} + \alpha_4 r^{-5} + \alpha_6 r^{-7} + \alpha_8 r^{-9}) \cos 2\theta$$

$$+ \frac{u_w}{rU_\infty}$$

and

$$v = -(1 - \alpha_1 r^{-2} - 3\alpha_3 r^{-4} - 5\alpha_5 r^{-6} - 7\alpha_7 r^{-8}) \sin \theta$$

$$+ (-2\alpha_2 r^{-3} - 4\alpha_4 r^{-5} - 6\alpha_6 r^{-7} - 8\alpha_8 r^{-9}) \sin 2\theta$$
With equations (52) and (53), the magnitude of the normalized vector velocity, \((u^2 + v^2)^{\frac{1}{2}}\), may be computed. These results are graphically presented in Figures 22-45. Also included in these figures are the experimental data and the theoretical prediction from potential flow theory. The approximate theory gives good agreement at \(r = 1.17\) (the radial position closest to the surface in the experimental study) for the four blowing rates \(u_w/U_\infty = 0.190, 0.154, 0.126,\) and \(0.102\). For the two highest blowing cases, \(u_w/U_\infty = 0.190\) and \(0.154\), the agreement is good. However, as the radial position variable is increased, the agreement between the approximate theory and the experiments becomes poor. Only in the region of \(\theta = 0^\circ\) to \(60^\circ\) is the accuracy reasonable. The approximate theory does approach the inviscid theory as \(r\) becomes large, thus, giving an asymptotic check of the solution in keeping with the boundary layer concept.

---

This prediction is based on the superposition of the solutions for a source and a solid circular cylinder in a free stream. The velocities are

\[
u = -1 + \frac{1}{r^2}\cos2\theta + \frac{u_w}{rU_\infty}\cos\theta
\]

and

\[
v = \frac{1}{r^2}\sin2\theta + \frac{u_w}{rU_\infty}\sin\theta
\]
Velocity profiles for the zero blowing case are not shown since the approximate theory predicts velocities from one to two orders of magnitude higher than the inviscid flow theory. The fact that the theory fails here is in keeping with the peculiar streamlines shown in Figures 20 and 21. Study of the results presented in Table 8 shows that there is clearly a dominance of the \( \sin 2\theta \) term in the trial stream function as \( \text{Re} \) increases. This is not the case, however, for the other results shown in Tables 4-7. In these cases there is a more equal weighting of the two sine terms.

To investigate this phenomena in more depth, the second two internal conditions, equations (33) and (34), were changed to the more general conditions

\[
\int_0^{2\pi} \int_0^\infty L[\psi^*] \frac{\sin \theta}{r^2} \,dr\,d\theta = 0.
\]

and

\[
\int_0^{2\pi} \int_0^\infty L[\psi^*] \frac{\sin 2\theta}{r^3} \,dr\,d\theta = 0.
\]

The solution for the velocity field resulting from these two
internal conditions will be referred to as the "second Galerkin solution."

Equations (54) and (55) reduce to

\[
\alpha_2 \left( - \frac{6}{10} \alpha_3 - \frac{24}{12} \alpha_5 - \frac{60}{14} \alpha_7 \right) \\
+ \alpha_4 \left( \frac{3}{8} + \frac{6}{10} \alpha_1 + \frac{2}{12} \alpha_3 - \frac{15}{14} \alpha_5 - \frac{51}{16} \alpha_7 \right) \\
+ \alpha_6 \left( \frac{12}{10} + \frac{20}{12} \alpha_1 + \frac{20}{14} \alpha_3 + \frac{6}{16} \alpha_5 - \frac{28}{18} \alpha_7 \right) \\
+ \alpha_8 \left( \frac{30}{12} + \frac{45}{14} \alpha_1 + \frac{51}{16} \alpha_3 + \frac{42}{18} \alpha_5 + \frac{12}{20} \alpha_7 \right) \\
+ \frac{u_w}{U_\infty} \left( - \frac{5}{8} \alpha_3 - \frac{21}{10} \alpha_5 - \frac{54}{12} \alpha_7 \right) \\
- \frac{1}{Re} \left( \frac{48}{8} \alpha_3 + \frac{288}{10} \alpha_5 + \frac{960}{12} \alpha_7 \right) = 0
\]

and

\[
\alpha_1 \left( \frac{2}{10} \alpha_3 + \frac{9}{12} \alpha_5 + \frac{24}{14} \alpha_7 \right) \\
+ \alpha_3 \left( \frac{3}{8} + \frac{1}{12} \alpha_3 + \frac{6}{14} \alpha_5 + \frac{17}{16} \alpha_7 \right) \\
+ \alpha_5 \left( \frac{12}{10} + \frac{3}{16} \alpha_5 + \frac{12}{18} \alpha_7 \right) \\
+ \alpha_7 \left( \frac{30}{12} + \frac{6}{20} \alpha_7 \right)
\]

(56)
respectively, when the procedures leading to equations (50) and (51) are followed. When equations (56), (57), (48), (49), and (42)-(45) were solved by the methods discussed in Appendix B, complete solutions were obtained only for the cases of $u_w/U_\infty = 0.190, 0.154, 0.126,$ and $0.102$. The results were two to three times the values of the inviscid solutions, even for the $r = 1.17$ radial position. In addition the solution of the no blowing case was found to be critically dependent on the range of Reynolds numbers studied. That is, solutions were found only below a certain Reynolds number. Above that value, either numerical difficulties or the non-existence of real roots prevented a successful solution. The actual reason behind the inability to find a solution was not pursued, since the results for the non-zero blowing cases failed to give any improvement over the results of the first Galerkin solution.

From the results of the first and second solutions, it was apparent that the general integral conditions, equations (54) and (55), lead to difficulties in obtaining solutions to the no blowing case and to decreased accuracy in the blowing cases.
It was, however, not clear what caused the difficulties with the streamlines in the first Galerkin solution at \( \frac{u_w}{U_\infty} = 0.0 \). That is, the problem could be in the form of the trial function or in an insufficient number of terms in this expression.

To gain insight into this problem, a new trial function was needed. This was so because the alteration of equations (31) and (32) to their degenerate cases at \( r = 1 \) lead to the same conditions set forth in equations (33) and (34). This gave only six independent equations to solve for the eight undetermined constants in equation (30).

The trial function chosen for further study was

\[
\phi_0(r, \theta) = r \sin \theta - \frac{u_w \theta}{U_\infty}
\]

\[
\phi_k(r, \theta) = r^{-k} \sin \theta \quad (k = 1, 2, 3)
\]

\[
\phi_k(r, \theta) = r^{-k+3} \sin 2\theta \quad (k = 4, 5, 6)
\]

(58)

and

\[
\phi_k(r, \theta) = r^{-k+6} \sin 3\theta \quad (k = 7, 8, 9)
\]

where, as before:

1) \( \phi_0(r, \theta) \) satisfies all the actual boundary conditions, except at \( r = 1 \),
2) $\phi_o(r, \theta)$ satisfies the actual differential equation, i.e., $L[\phi_o] = 0$.

3) $\phi_k(r, \theta)$ satisfies all of the homogeneous boundary conditions, except at $r = 1$, and

4) in general the $\phi_k(r, \theta)$ do not satisfy the homogeneous differential equation, e.g., $L[\phi_k] \neq 0$.

Thus,

$$\psi^*(r, \theta) = (r + \alpha_1 r^{-1} + \alpha_2 r^{-2} + \alpha_3 r^{-3}) \sin \theta$$

$$+ (\alpha_4 r^{-1} + \alpha_5 r^{-2} + \alpha_6 r^{-3}) \sin 2\theta$$

$$+ (\alpha_7 r^{-1} + \alpha_8 r^{-2} + \alpha_9 r^{-3}) \sin 3\theta - \frac{u_w \theta}{U_\infty}.$$

An extra sine term has been added to give a more equitable weighting to the $\theta$-dependence in the trial stream function. This extra term will also give information about the convergence of the approximate solution.

As a first attempt with this new trial function, and in keeping with the methods and the results of the first two Galerkin solutions, the following equations were used for the elimination of the nine undetermined constants:
\[ \int_{0}^{2\pi} \left[ \frac{\partial \psi^*}{\partial \theta} + \frac{\partial \psi^*}{\partial r} + \frac{u_w}{U_\infty} \right] \sin \theta d\theta = 0, \quad r=1 \] (60)

\[ \int_{0}^{2\pi} \left[ \frac{\partial \psi^*}{\partial \theta} + \frac{\partial \psi^*}{\partial r} + \frac{u_w}{U_\infty} \right] \sin 2\theta d\theta = 0, \quad r=1 \] (61)

\[ \int_{0}^{2\pi} \left[ \frac{\partial \psi^*}{\partial \theta} + \frac{\partial \psi^*}{\partial r} + \frac{u_w}{U_\infty} \right] \sin 3\theta d\theta = 0, \quad r=1 \] (62)

\[ \frac{\partial \psi^*}{\partial \theta} (1, \theta) = -\frac{u_w}{U_\infty} \] (63)

\[ \int_{0}^{2\pi} \left[ L[\psi^*] \right] \sin \theta d\theta = 0, \quad r=1 \] (64)

\[ \int_{0}^{2\pi} \left[ L[\psi^*] \right] \sin 2\theta d\theta = 0, \quad r=1 \] (65)

and

\[ \int_{0}^{2\pi} \left[ L[\psi^*] \right] \sin 3\theta d\theta = 0. \] (66)
The solution resulting from equations (59) and (60)-(66) will be referred to as the "third Galerkin solution."

Utilizing the methods outlined in the first Galerkin solution, these equations reduce to

\[ 1 - \alpha_1 - 2\alpha_2 - 3\alpha_3 = 0 \], \hspace{1cm} (67)

\[ - \alpha_4 - 2\alpha_5 - 3\alpha_6 = 0 \], \hspace{1cm} (68)

\[ - \alpha_7 - 2\alpha_8 - 3\alpha_9 = 0 \], \hspace{1cm} (69)

\[ 1 + \alpha_1 + \alpha_2 + \alpha_3 = 0 \], \hspace{1cm} (70)

\[ \alpha_4 + \alpha_5 + \alpha_6 = 0 \], \hspace{1cm} (71)

\[ \alpha_7 + \alpha_8 + \alpha_9 = 0 \], \hspace{1cm} (72)

\[ \alpha_4 (- 3 - 15\alpha_1 - 48\alpha_2 - 115\alpha_3 - 39\alpha_7 - 16\alpha_8 - 45\alpha_9) \]

\[ + \alpha_5 (- 30\alpha_2 - 96\alpha_3 - 96\alpha_7 - 70\alpha_8) \]

\[ + \alpha_6 (15 + 35\alpha_1 + 12\alpha_2 - 49\alpha_3 - 205\alpha_7 \]

\[ - 180\alpha_8 - 105\alpha_9) \]
\[ \frac{2u}{U_\infty} (-12\alpha_2 - 40\alpha_3) - \frac{4}{Re}(45\alpha_2 + 192\alpha_3) = 0, \]

\[ \alpha_2 (15 + 9\alpha_1 + 6\alpha_2 + 3\alpha_3 - 39\alpha_7 - 42\alpha_8 - 45\alpha_9) \]

\[ + \alpha_3 (48 + 32\alpha_1 + 24\alpha_2 + 16\alpha_3 - 128\alpha_7 - 136\alpha_8 - 144\alpha_9) \]

\[ + \alpha_7 (-48\alpha_1 - 72\alpha_2 - 96\alpha_3) \]

\[ + \alpha_8 (-5 - 35\alpha_1 - 50\alpha_2 - 65\alpha_3) \]

\[ + \frac{2u}{U_\infty}(9\alpha_4 - 25\alpha_6) - \frac{4}{Re}(-15\alpha_4 + 105\alpha_6) = 0, \]

and

\[ \alpha_4 (-15 - 3\alpha_1 + 24\alpha_2 + 81\alpha_3) \]

\[ + \alpha_5 (18\alpha_2 + 64\alpha_3) \]

\[ + \alpha_6 (35 + 15\alpha_1 + 20\alpha_2 + 51\alpha_3) \]

\[ + \frac{2u}{U_\infty}(24\alpha_7 + 20\alpha_9) - \frac{4}{Re}(-35\alpha_8) = 0, \]

where equations (67)-(69) resulted from equations (60)-(62),
equations (70)-(72) from equation (63), and equations (73)-(75) from equations (64)-(66), respectively.

Equations (67)-(75) were solved by using the same numerical methods leading to the first Galerkin solution. These results are presented in Tables 9-13 for the same conditions of Tables 4-8.

Figures 46-51 show the streamlines as calculated from the results of Tables 9-13 and equation (59) for the same conditions of Figures 16-21. Comparison of Figures 16-21 and 46-51 shows that there is both close quantitative and close qualitative agreement between the first and third Galerkin solutions for the blowing cases of $u_w/U_\infty = 0.190$, 0.154, 0.126, and 0.102. However, for the no blowing case, the results from the third Galerkin solution are much improved over the results of the first. Figures 50 and 51 show no closed streamlines upstream of the cylinder and, thus, are physically reasonable.

Velocity profiles, calculated from

$$u = - (1 + \alpha_1 r^{-2} + \alpha_2 r^{-3} + \alpha_3 r^{-4}) \cos \theta$$

$$- 2(\alpha_4 r^{-2} + \alpha_5 r^{-3} + \alpha_6 r^{-4}) \cos 2 \theta$$

$$- 3(\alpha_7 r^{-2} + \alpha_8 r^{-3} + \alpha_9 r^{-4}) \cos 3 \theta$$

(76)
and the constants of Tables 9-13, are shown in Figures 42-87. For the blowing cases of $u_w/U_\infty = 0.190, 0.154, 0.126, 0.102, \text{ and } 0.0$, the conclusions drawn from Figures 52-87 are similar to those drawn from Figures 22-45. The third solution is shown to be in fair agreement with the experiments close to the surface of the cylinder. For the blowing cases of $u_w/U_\infty = 0.190$ and 0.154 this agreement is quite good. As with the first Galerkin solution, this agreement becomes poor as the radial variable is increased, particularly in the region of $\theta = 60^\circ$ to $180^\circ$.

In general, the results of the first Galerkin solution are slightly better than those of the third Galerkin solution, for the blowing cases of $u_w/U_\infty = 0.190, 0.154, 0.126, \text{ and } 0.102$. This is because the peaks in the velocity curves of the first solution are between $\theta = 60^\circ$ and $80^\circ$, while those of the third are between $\theta = 80^\circ$ and $100^\circ$. This can be explained by an examination of Tables 9-12 and

\[
v = (1 - \alpha_1 r^{-2} - 2\alpha_2 r^{-3} - 3\alpha_3 r^{-4}) \sin \theta
\]

\[
+ (-\alpha_4 r^{-2} - 2\alpha_5 r^{-3} - 3\alpha_6 r^{-4}) \sin 2\theta
\]

\[
+ (-\alpha_7 r^{-2} - 2\alpha_8 r^{-3} - 3\alpha_9 r^{-4}) \sin 3\theta
\]
where it can be seen that the \( \sin \theta \) term dominates in the third solution, while no such dominance of the \( \sin \theta \) term exists for these blowing rates in the first solution. However, for the zero blowing case, the third Galerkin solution is far superior. It gives both realistic streamlines and physically reasonable velocity profiles, where the first approximate solution does not. In summary, the third Galerkin solution was found to be superior to the first Galerkin solution, because of its reasonable agreement with the experiments for all blowing rates \( u_w/U_\infty = 0.190, 0.154, 0.126, 0.102, \) and \( 0.0 \) close to the wall.

In an effort to improve the above results and to gain further insight as to why the third Galerkin solution is superior to the first for \( u_w/U_\infty = 0.0 \), the third Galerkin solution was modified by using

\[
\int_0^{2\pi} \int_0^\infty L[\psi^*] \sin \theta d\theta d\theta = 0 , \quad (78)
\]

and

\[
\int_0^{2\pi} \int_0^\infty L[\psi^*] \sin 2\theta d\theta d\theta = 0 , \quad (79)
\]
instead of equations (64), (65), and (66) respectively. The solution resulting from equations (59), (60), (61), (62), (63), and (78)-(80) will be referred to as the "fourth Galerkin solution."

Following the usual procedures, these equations give

\[ \alpha_4 \left( -\frac{3}{3} - \frac{15}{3} \alpha_1 - \frac{48}{6} \alpha_2 - \frac{115}{7} \alpha_3 - \frac{39}{5} \alpha_7 - \frac{16}{6} \alpha_8 + \frac{45}{7} \alpha_9 \right) \]

\[ + \alpha_5 \left( -\frac{30}{7} \alpha_2 - \frac{96}{8} \alpha_3 - \frac{96}{6} \alpha_7 - \frac{70}{7} \alpha_8 \right) \]

\[ + \alpha_6 \left( \frac{15}{5} + \frac{35}{7} \alpha_1 + \frac{12}{8} \alpha_2 - \frac{49}{9} \alpha_3 - \frac{205}{7} \alpha_7 \right) \]

\[ - \frac{180}{8} \alpha_8 - \frac{105}{9} \alpha_9 \]  

\[ + \frac{2u_w}{U_\infty} \left( -\frac{12}{5} \alpha_2 - \frac{40}{6} \alpha_3 \right) - \frac{4}{\text{Re}} \left( \frac{45}{5} \alpha_2 + \frac{192}{6} \alpha_3 \right) = 0 \]

\[ \alpha_2 \left( \frac{15}{4} + \frac{9}{6} \alpha_1 + \frac{6}{7} \alpha_2 + \frac{3}{8} \alpha_3 - \frac{39}{6} \alpha_7 - \frac{42}{7} \alpha_8 - \frac{45}{8} \alpha_9 \right) \]

\[ + \alpha_3 \left( \frac{48}{5} + \frac{32}{7} \alpha_1 + \frac{24}{8} \alpha_2 + \frac{16}{9} \alpha_3 \right) \]

\[ - \frac{128}{7} \alpha_7 - \frac{136}{8} \alpha_8 - \frac{144}{9} \alpha_9 \]
respectively.

When a solution of equations (67)-(72) and (81)-(83) was attempted by the usual methods, the difficulties encountered were similar to those in the second Galerkin solution. All combinations of the integral conditions (64)-(66) and (78)-(80) were then tried. The results in each instance were either not as good as those obtained in the third Galerkin solution or invalidated because of unreasonable or unobtainable solutions for the \( \frac{u_w}{U_\infty} = 0.0 \)
case. The introduction of the general internal condition,

$$
\int_0^1 \int_0^{2\pi} L[\psi^*] w_j(r, \theta) r \, dr \, d\theta = 0
$$

seemed to have a detrimental effect on the solution. In the fourth through tenth approximate solutions, whichever term (e.g., sin\(\theta\), sin2\(\theta\), or sin3\(\theta\)) had the double integral condition, that term was shown to have a strong influence on the solution.

The best combination of trial function and integral conditions was found in the third Galerkin solution where the sin\(\theta\) term dominated the sin2\(\theta\) term and the sin2\(\theta\) term dominated the sin3\(\theta\) term. As blowing was decreased, the sin2\(\theta\) and sin3\(\theta\) terms became more important relative to the sin\(\theta\) term. From an order of magnitude comparison between the sin\(\theta\), sin2\(\theta\), and sin3\(\theta\) terms, it is doubtful that a sin4\(\theta\) term would have had any significant effect or improvement over this solution. Thus, it was concluded that the third solution converged. Further, since convergence was not a problem in the third solution, one can argue that the difficulties encountered with the first solution, for \(u_w/U_\infty = 0.0\), were due to the presence of double integral conditions.
Since the third solution shows good agreement with the experimental data near the surface of the cylinder (where the quantities of engineering interest, such as, the total drag coefficient and ultimately the overall heat and/or mass transport rate are to be calculated) and since the solution converges and is valid for all the blowing rates of interest in the experimental study, then it can also be concluded that this combination of trial function and integral conditions is an appropriate stopping place for the analysis.

With these conclusions in mind and with a view towards meeting the objectives set forth in Chapter I, the total drag coefficient was calculated with the results from the third Galerkin solution. The drag force, due to the normal stresses exerted on the surface of the cylinder by the fluid, is

$$\hat{F}_{DN} = RL \int_{0}^{2\pi} (-\hat{P} + \hat{r}_{rr})_{r=R} \cos \theta \, d\theta , \quad (85)$$

where \(\hat{P}\) is the local thermodynamic pressure. The quantity \(\hat{r}_{rr}\) is the \(\hat{r}\)-component of the viscous stress tensor, defined by (27)
The expression for drag can be rewritten with the aid of equations (4), (9), (13), and (86) as

\[
\hat{F}_{DN} = - RL \rho U_\infty^2 \int_0^{2\pi} \left( p - \frac{4}{Re} \frac{\partial u}{\partial r} \right) r = 1 \cos \theta d\theta .
\]  

(87)

Similarly, the drag force due to shear stresses is

\[
\hat{F}_{DS} = - RL \int_0^{2\pi} \hat{\tau}_{r\theta}(R, \hat{\theta}) \sin \theta d\theta ,
\]  

(88)

where \( \hat{\tau}_{r\theta} \) is \( \hat{\theta} \)-component of the viscous stress tensor, defined by (27)

\[
\hat{\tau}_{r\theta} = \mu \left\{ \hat{\tau} \frac{\partial (\hat{v})}{\partial \hat{r}} + \frac{1}{\hat{r}} \frac{\partial \hat{u}}{\partial \hat{\theta}} \right\} .
\]  

(89)

Equations (88) and (89) can be alternatively expressed as

\[
\hat{F}_{DS} = - \frac{2RL \rho U_\infty^2}{Re} \int_0^{2\pi} (\frac{\partial v}{\partial r}) r = 1 \sin \theta d\theta .
\]  

(90)
The only unknown term in these equations is \( P(1, \theta) \), since the velocity terms can be evaluated from equations (76) and (77). To evaluate this term, consider the following argument:

\[
P = P(r, \theta) \quad , \tag{91}
\]

thus,

\[
dP = \frac{\partial P}{\partial r} dr + \frac{\partial P}{\partial \theta} d\theta \quad . \tag{92}
\]

and (5)

\[
\int_{(\infty, 0)}^{(1, \theta)} dP = \int_{\infty}^{1} \left( \frac{\partial P}{\partial r} \right)_{\theta=0} dr + \int_{0}^{\theta} \left( \frac{\partial P}{\partial \theta} \right)_{r=1} d\theta \quad , . \tag{93}
\]

or

\[
P(1, \theta) - P_{\infty} = \int_{0}^{\theta} \left( \frac{\partial P}{\partial \theta} \right)_{r=1} d\theta - \int_{1}^{\infty} \left( \frac{\partial P}{\partial r} \right)_{\theta=0} dr \quad . \tag{94}
\]

When the terms
\[ (\frac{\partial P}{\partial \theta})_{r=1} = \frac{2}{Re} \left( \frac{\partial^2 v}{\partial r^2} + \frac{\partial v}{\partial r} \right)_{r=1} - [u \frac{\partial v}{\partial r}]_{r=1} \] (95)

and

\[ (\frac{\partial P}{\partial \theta})_{\theta=0} = \frac{2}{Re} \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} \right) \]

\[ - \frac{2}{r} \frac{\partial v}{\partial \theta} \bigg|_{\theta=0} - [u \frac{\partial u}{\partial r}]_{\theta=0} \] (96)

obtained from equations (11) and (10) respectively, are substituted into equation (94), and equations (76) and (77) are utilized, the result is

\[ P(1, \theta) = P_\infty + \frac{2}{Re} [C_1 (\cos \theta - 1) + C_2 (\cos 2\theta - 1) \]

\[ + C_3 (\cos 3\theta - 1)] + \frac{u_w}{U_\infty} [C_4 (\cos \theta - 1) \]

\[ + C_5 (\cos 2\theta - 1) + C_6 (\cos 3\theta - 1)] - C_7 \] (97)

where

\[ C_1 = 4a_1 + 18a_2 + 48a_3 \] (98)

\[ C_2 = \frac{1}{2} (4a_4 + 18a_5 + 48a_3) \]
\[ C_3 = \frac{1}{3}(4\alpha_7 + 18\alpha_8 + 48\alpha_9) \]

\[ C_4 = 2\alpha_1 + 6\alpha_2 + 12\alpha_3 \]

\[ C_5 = \frac{1}{2}(2\alpha_4 + 6\alpha_5 + 12\alpha_6) \]

\[ C_6 = \frac{1}{3}(2\alpha_7 + 6\alpha_8 + 12\alpha_9) \]

and

\[ C_7 = \int_{1}^{\infty} \left( \frac{3\rho}{\theta} \right)_{\theta=0}^{\theta=\pi} \, d\theta = \text{constant} \]

Equations (97), (98), and (76) are substituted into equation (87) to give the final expression for the drag force due to normal stresses.

\[ F_{DN} = - RL\rho U_\infty^2 \pi \left[ \frac{2}{Re} (12\alpha_2 + 40\alpha_3) \right. \]

\[ + \frac{u_w}{U_\infty} (2\alpha_1 + 6\alpha_2 + 12\alpha_3) \]  

(99)

The final expression for the drag force due to shear stresses can be found by substituting equation (77) into equation (90). The result is,
Thus, the total drag coefficient, defined as

\[ C_D = \frac{(\hat{F}_{DN} + \hat{F}_{DS})}{2RL} \left( \frac{1}{2} \rho U_\infty^2 \right) \]  

(101)

can be expressed as

\[ C_D = \pi \left[ \frac{2}{Re} (2\alpha_1 + 18\alpha_2 + 52\alpha_3) \right. \\
\left. + \frac{u_w}{U_\infty} (2\alpha_1 + 6\alpha_2 + 12\alpha_3) \right] \]  

(102)

The drag coefficient is presented in Figure 88 as a function of Reynolds number for three blowing rates. Also in this figure are the experimental results in Tritton (49), for the limiting case of no blowing.
IV. DISCUSSION

As stated in Chapter I, the objective of this investigation was to obtain a more thorough knowledge of the external flow field for the problem of uniform cross-flow impinging on a porous circular cylinder with uniform radial momentum transport at the wall. This problem was studied experimentally via hot wire measurements and the results were first presented in Chapter II. In Chapter III, these results were graphically presented in Figures 52-87. The magnitude of the vector velocity was plotted as a function of the angular position from the leading edge for six radial positions and five blowing rates. In general, for the blowing rates of \( \frac{u_w}{U_\infty} = 0.190, 0.154, 0.126, \) and \( 0.102, \) many of the features of the flow were qualitatively the same. From \( \theta = 0^\circ \) to about \( 60^\circ - 80^\circ, \) the velocity was shown to increase as \( \theta \) was increased. A peak in the velocity field was observed between \( \theta = 60^\circ \) and \( 80^\circ. \) Downstream of the peak, there was a quick drop off and then a more gradual tapering off as the trailing edge of the cylinder was approached. Close to the wall of the cylinder, these changes in the flow field were less drastic. However, as the radial variable was increased, these changes became more pronounced.
As the blowing rate was increased, the velocity was shown to decrease over the entire circumference of the cylinder and for all radii. For the zero blowing case, the general shape of the velocity profiles were the same as those observed in the blowing cases. The velocity increased as $\theta$ increased to about $\theta = 60^\circ - 100^\circ$ and then sharply dropped off to a fairly constant value. Changes in the radius appeared to have little effect on the data.

For all of the blowing rates studied experimentally, a comparison was made with the inviscid theory. This theory was derived from the superposition of the solutions for a uniform flow about a solid cylinder and a source located at the center of the cylinder. The experiments were shown in general to give excellent agreement upstream of separation, within the error of the measurements, for the no blowing cases. This agreement suggests that measurements made for this limiting cases were outside of the boundary layer. This conclusion is consistent with calculations of the boundary layer thickness based on laminar flow over a circular cylinder (40). For the other blowing cases studied, $u_w/U_\infty = 0.190, 0.154, 0.126,$ and $0.102$, there was poor agreement, except for the regions far from the wall, between the inviscid theory and the experiments. This suggests that the region of viscous flow extends farther into the free stream as the blowing rate is
Further study of Figures 52-87 shows that in some cases, far from the wall and downstream of separation, the experimental data exceeds the inviscid theory. This is against physical reasoning, but can be explained by the fact that the hot wire anemometer is a non-linear measuring system. For the cases in question, the probe was located in a region of large scale fluctuations, causing an erroneous measurement.

In Chapter III, Galerkin's method was used to obtain approximate solutions for the external flow field. Two different trial functions and all possible variations of their internal conditions were investigated. Of all the approximate solutions studied, the third solution was found to give the best overall results. The theoretical velocity profiles for this solution were presented in Figures 52-87, as were the experimental data and the inviscid approximation. Comparison of these results showed that the approximate theory adequately modeled the flow field close to the surface of the cylinder for all of the angular positions and blowing rates studied experimentally. The theory also gave good agreement with the experiments for all of the radial positions and blowing rates upstream of separation. Further, the approximate theory was shown to approach the inviscid theory as the radial variable increased. This
is, of course, physically compatible with the boundary layer concept, thus giving credibility to the approximate solution. Downstream of separation and away from the wall, the theory was found to be in poor agreement with the experiments.

Perhaps, the inability of the approximate theory to precisely represent the entire flow field can best be explained by studying the streamlines about the porous cylinder. Figures 89-94 are pictures from a preliminary flow visualization study by McCall (29), using the apparatus of this investigation. Figure 89 shows the external flow impinging on the cylinder for the case of no blowing. The wake region for this case is presented in Figure 90. Figures 91-94 show the porous cylinder from similar angles and the same free stream Reynolds number, but for the cases of $u^c/U^c = 0.053$ and 0.126. For all blowing cases, the flow upstream of separation appears to be steady and the flow downstream of separation appears to be dominated by a periodic vortex shedding. Further, the presence of blowing is shown to cause a more pronounced displacement of the external flow from the surface upstream of the separation point. Downstream of separation, blowing causes the unsteady vortex structure to be shifted slightly away from the surface. In general, blowing causes the region of quasi-steady flow, near the surface of the cylinder, to be extended
farther into the free stream.

It may be recalled that in Chapter III, the steady form of the momentum equation, equations (6) and (7), were used to formulate the approximate theory. It follows, then that the theory should agree best with the experiments for the regions where steady, or at least quasi-steady flow exists. As shown in Figures 89-94, these conditions are upstream of separation, for all radial positions, and close to the surface, for all angular positions. It is no coincidence, then, that the theory is shown to give good agreement with the experiments in these regions. The fact that the approximate theory gives good results slightly farther from the surface, for the higher blowing cases studied, is also consistent with these arguments, since the region of quasi-steady flow extends farther into the free stream.

The drag coefficient calculated from the results of the third Galerkin solution is presented in Figure 88. Three blowing rates were shown: \( u_w/U_\infty = 0.190, 0.086, \) and \( 0.0 \). Comparison of these curves shows that as the dimensionless blowing rate parameter, \( u_w/U_\infty \), is increased, the drag coefficient increases. To show that this result is physically reasonable, consider the following. In Figure 95 is shown a control volume for the application of the integral mass and momentum equations. Also in this figure are approximations of the \( \hat{x} \)-components of the velocity field at the
control surface boundaries. Upstream of the cylinder, at control surface $A_1$ the flow field is uniform. At surface $A_2$, the velocity distribution is assumed to be as shown and at surfaces $A_3$ and $A_4$ the $x$-component of velocity is taken as $-U_\infty$. With these assumed velocity profiles and the further assumption that the viscous stresses are negligible at the flow boundaries, the drag coefficient based on a combination of the integral mass and momentum equations (42) is

$$C_D = \frac{2E_1}{3R} (1 - \varepsilon) \left( \frac{1}{2} + \varepsilon \right)$$

From equation (103), the drag coefficient is a function of the two parameters of the velocity field, $\varepsilon$ and $E_1/R$, and the pressure field, which is also related to these parameters. As a further simplification, it is assumed that $\hat{P} = P_\infty$ for $-E_2 \leq \hat{y} \leq E_2$. Thus, equation (103) can be reduced to

$$C_D = \frac{2E_1}{3R} (1 - \varepsilon) \left( \frac{1}{2} + \varepsilon \right)$$

Equation (104) is plotted in Figure 96 as a function of $\varepsilon$
for three different values of $E_1/R$. Equation (104) indicates that as $\varepsilon$ is increased, $C_D$ varies quadratically. The drag is shown to increase for $0 \leq \varepsilon \leq 0.25$ and then decrease for $0.25 \leq \varepsilon \leq 1.0$. Further, for $0 \leq \varepsilon \leq 0.5$, $C_D$ is a relatively weak function of $\varepsilon$.

Comparison of Figures 90, 92, and 94 shows that the presence of blowing causes the scale parameter $E_1/R$ to increase. The preliminary flow visualization study by McCall (29) showed that there was as much as a 50% increase in $E_1/R$ between the cases of $u_w/U_\infty = 0.0$ and $0.102$. It is suggested by Figures 52-87 that $\varepsilon$ increases only slightly as the blowing is increased and never exceeds 0.5. Equation (104) and Figure 96 imply that $C_D$ is a relatively weak function of $\varepsilon$ for $0 \leq \varepsilon \leq 0.5$. Thus, since $E_1/R$ increases with increased blowing, it follows that $C_D$ also increases. This conclusion is consistent with the results of Figure 88.

The experimental data of Tritton (49) was also shown in Figure 88 for the no blowing case. In general, except at Reynolds numbers less than about 100, the agreement between the drag coefficient predicted by the third Galerkin method and these data was poor. From equation (102),

$$C_D = \pi [\frac{2}{Re} (2\alpha_1 + 18\alpha_2 + 52\alpha_3)$$
$$+ \frac{u_w}{U_\infty} (2\alpha_1 + 6\alpha_2 + 12\alpha_3)] ,$$

(102)
it is seen that as Re becomes large, the drag coefficient approaches the form

\[ C_D = \frac{u}{U_\infty} \left(2\alpha_1 + 6\alpha_2 + 12\alpha_3\right) \text{ (105)} \]

Equation (105) suggests that the drag coefficient is a linear function of \( u_w/U_\infty \). This means that as \( Re \to \infty \), then \( C_D \to 0 \) for \( u_w/U_\infty = 0.0 \), which leads to the poor agreement between theory and experiment for \( Re > 100 \).

Equations (99) and (100) further show that at very large Reynolds numbers, the total drag is due completely to the normal stresses acting on the cylinder with no contribution from shear stresses. This is in keeping with physical reasoning. However, it leads to practical difficulties. Equation (99), the drag force due to normal stresses, was derived from the more general expression, equation (87). Equation (87) shows the drag due to normal stresses to be dependent on the pressure field, which was derived from the motion equations as discussed in equations (92)-(98). Even though the Galerkin method is capable of predicting reasonably accurate velocity profiles, it appears to fail to predict accurate derivatives. This leads to inaccuracies in the pressure field and as a consequence, to errors in the drag force due to normal stresses. This is roughly analogous to the difficulties encountered when numerically
differentiating on an analog computer, where noise problems usually prevent an accurate solution.

In Chapter III, the approximate theory was compared with the experiments by plotting the magnitude of the vector velocity (Figures 52-87) as a function of the angular and radial position, blowing rate, and Reynolds number. In Figure 97 a similar comparison (analogous to Figure 52) is made between theory and experiment. However, the magnitudes of the \( \hat{x} \) and \( \hat{y} \) velocity components are presented instead of the magnitude of the vector velocity. The agreement between theory and experiment, when the data is presented in this manner, is poor. This is attributed to the possibility of probe interference during the measurement of \( |\hat{v}_y/U_\infty| \).\(^1\) No such interference occurred for the vector velocity measurement, since it was in its normal measuring position. Thus, these measurements are accurate (within the error of the calibration) as previously shown. Because \( |\hat{v}_x/U_\infty| \) was calculated from \( |\hat{v}_y/U_\infty| \) and \( (\hat{v}_x^2 + \hat{v}_y^2)\frac{1}{2}/U_\infty \), it follows that \( |\hat{v}_x/U_\infty| \) should be in error also, as is observed in Figure 97. Thus, because of the possibility of probe interference in the measurement of \( |\hat{v}_y/U_\infty| \), the magnitude of the vector velocity is the only criterion by which to fairly evaluate

\(^1\)It may be recalled from Chapter II, that for this measurement, the hot wire probe was positioned such that the wire was perpendicular to the axis of the porous pipe.
the theory.

It can be argued that another method by which the approximate theory can be compared to the experiments is by computing the vorticity (twice the angular velocity of the fluid particle). For the two-dimensional problem at hand, the \( \hat{z} \)-component of the dimensionless vorticity (the \( \hat{x} \) and \( \hat{y} \) or \( \hat{r} \) and \( \hat{\theta} \)-components are zero) is

\[
\Omega_z = r \frac{\partial v}{\partial r} + v - \frac{\partial u}{\partial \theta} .
\] (106)

Using equations (76) and (77), then equation (106) becomes

\[
\Omega_z = (3a_2 r^{-3} + 8a_3 r^{-4}) \sin \theta
+ (-3a_4 r^{-2} + 5a_6 r^{-4}) \sin 2\theta
+ (-8a_7 r^{-2} - 5a_8 r^{-3}) \sin 3\theta ,
\] (107)

from which the theoretical vorticity can be computed. These results are shown in Figure 98 for the two blowing rates, \( u_w/U_\infty = 0.190 \) and \( 0.0 \), where \( \hat{r}/R = 1.17 \) and \( \text{Re} = 6200 \).

To compute the experimental vorticity, it is necessary to use some sort of finite difference scheme to calculate
the values of the derivatives in equation (106). ² An error analysis applied to equation (106) shows that the absolute error in this calculation (due to truncation errors in the finite difference expressions) is of the order of the value computed from equation (107). Further, it was previously shown that \(|v'/U_\infty|\) and \(|v''/U_\infty|\) were not reliable measurements because of the possibility of probe interference (these quantities are needed for the computation of the derivatives in equation (106)). Thus, it is concluded that any calculations based on experimental vorticity are unreliable and cannot be used to further evaluate the theory.

Thus far, the accuracy of the Galerkin solution has been studied by comparing the theoretical results to the experimental results for the magnitude of the vector velocity. Another, but perhaps less critical, evaluation of the approximate theory is to compare the theoretical results obtained in Chapter III to the exact solution obtained by Lamb (27)³ for the special case of creeping flow (Re \(<< 1\)) around a solid circular cylinder (i.e., \(u_\infty/U_\infty = 0.0\)).

Such a comparison was made (see Appendix E), and it was shown that the agreement between the Galerkin solution

²For these computations, a centered difference scheme was used.

³This solution is usually referred to as "Oseen's solution" for a circular cylinder (6).
and Oseen's approximation was poor. The poor agreement between the two solutions tends to discount the value of the Galerkin solution only at very low Reynolds numbers for the case of no blowing. Since the problem of practical significance pertains to the case of blowing at much higher Reynolds numbers, then the experimental results presented in Chapter II must be considered as a more rigorous test of the theory presented in Chapter III. Therefore, the disagreement between the Galerkin solution and the Oseen solution does not influence, from a practical standpoint, the conclusions reached previously, with regard to the adequacy of the theory.

In summary, an approximate theoretical solution for crossflow over a porous circular cylinder, with uniform blowing at the surface, has been presented. The accuracy of the theory was evaluated by comparison to the experimental results. This comparison showed the theory to be adequate close to the surface of the cylinder for all of the angular positions and blowing rates studied experimentally. Further, the theory gave good agreement with the experiments for all of the radial positions and blowing rates upstream of separation. The failure of the theory downstream of separation and away from the wall was attributed to the presence of non-steady state phenomena that the approximate method was not equipped to handle. Finally, the
drag coefficient was predicted to increase as the blowing rate increased, and this was attributed to an increase of the wake region due to blowing.

It can be argued that, near the surface of the cylinder, periodic vortex shedding is minimized. Thus, the conditions existing in such regions approach those of the steady state, allowing the approximate theory to be applicable there. Since the quantities of potential future interest, such as the overall heat and/or mass transport, are extremely dependent on the flow field near the surface, the theoretical results and methods of this study can be used in their computation.

Hence, a suggestion for further study is to take the results of the velocity field obtained from the third Galerkin solution and apply them in an approximate Galerkin solution of the heat and/or mass transport problem in the external flow field. For example, the energy equation for constant thermal conductivity becomes (11).

\[ \hat{u} \frac{\partial \hat{T}}{\partial \hat{x}} + \hat{v} \frac{\partial \hat{T}}{\partial \hat{y}} = \frac{k}{\rho \hat{C}_p} \hat{v}^2 \hat{T} \]

(108)

where the viscous dissipation has been neglected. The boundary conditions could be as follows:
\[ \hat{T}(\infty, \hat{\theta}) = T_\infty, \]
\[ \hat{T}(R, \hat{\theta}) = h(\hat{\theta}), \]
\[ \frac{\partial \hat{T}}{\partial \hat{\theta}}(\hat{r}, 0) = 0, \quad (109) \]

and
\[ \frac{\partial \hat{T}}{\partial \hat{\theta}}(\hat{r}, \pi) = 0, \]

where \( h(\hat{\theta}) \) is some assumed or measured temperature distribution at the surface. This problem could be approached by assuming a trial temperature function of the form of equation (21) and solving for the unknown constants by using the methods discussed in Chapter III and the results of the velocity field obtained from the third Galerkin solution. This problem can also be studied experimentally with very minor changes to the apparatus discussed in Chapter II.

As stated in Chapter I, the purpose of this study was to gain insight into the complicated external flow phenomena. With the theoretical methods discussed in this
study (verified by the experiments), \(^4\) it is possible to extend this investigation to solve additional problems of practical significance, as discussed above. It was shown in Chapter II that in order for the internal pipe flow to have a significant effect on the external crossflow, it was necessary that the flow through the porous wall was uniform, varying only in the radial direction. Thus, for the conditions of this study, the external flow experienced the case of a uniform radial velocity component and a zero circumferential component at the surface. This information can be a powerful tool. It permits the designer to utilize the wealth of knowledge of the internal flows with the results of the limited knowledge in the external region and/or the results of the present study to further investigate the industrial processes discussed in Chapter I.

\(^4\)This experimental verification is necessary since no convergence theorems or error bounds are available for the solution of non-linear partial differential equations (3, 23, 43).
APPENDIX A

EXPERIMENTAL VELOCITY DATA

Tables A-1 through A-6 contain the experimental velocity data of this study as discussed in Chapter II.
TABLE A-1

Experimental Velocity Data, 
$u_w/U_\infty = 0.190$ and $Re = 4100$

| $r/R$ | $\theta$ (DEG) | $|\hat{v}_x/U_\infty|$ | $|\hat{v}_y/U_\infty|$ | $(\hat{v}_x^2 + \hat{v}_y^2)^{1/2}/U_\infty$ |
|-------|----------------|----------------|----------------|----------------------------------|
| 1.17  | 0              | 0.063          | 0.071          | 0.095                            |
| 1.17  | 20             | 0.00           | 0.268          | 0.268                            |
| 1.17  | 40             | 0.00           | 0.479          | 0.479                            |
| 1.17  | 60             | 0.185          | 0.420          | 0.459                            |
| 1.17  | 80             | 0.204          | 0.204          | 0.315                            |
| 1.17  | 100            | 0.126          | 0.187          | 0.226                            |
| 1.17  | 120            | 0.086          | 0.153          | 0.175                            |
| 1.17  | 140            | 0.059          | 0.153          | 0.164                            |
| 1.17  | 160            | 0.059          | 0.153          | 0.164                            |
| 1.17  | 180            | 0.00           | 0.122          | 0.122                            |
| 1.33  | 0              | 0.00           | 0.071          | 0.071                            |
| 1.33  | 20             | 0.200          | 0.348          | 0.401                            |
| 1.33  | 40             | 0.566          | 0.732          | 0.926                            |
| 1.33  | 60             | 0.901          | 0.812          | 1.213                            |
| 1.33  | 80             | 0.710          | 0.499          | 0.868                            |
| 1.33  | 100            | 0.311          | 0.254          | 0.401                            |
| 1.33  | 120            | 0.142          | 0.175          | 0.226                            |
| 1.33  | 140            | 0.059          | 0.153          | 0.164                            |
| 1.33  | 160            | 0.00           | 0.153          | 0.153                            |
| 1.33  | 180            | 0.00           | 0.112          | 0.112                            |
| 1.50  | 0              | 0.110          | 0.122          | 0.164                            |
| 1.50  | 20             | 0.268          | 0.348          | 0.439                            |
| 1.50  | 40             | 0.652          | 0.657          | 0.926                            |
| 1.50  | 60             | 1.177          | 0.812          | 1.430                            |
| 1.50  | 80             | 1.393          | 0.840          | 1.627                            |
| 1.50  | 100            | 0.695          | 0.365          | 0.785                            |
| 1.50  | 120            | 0.222          | 0.175          | 0.283                            |
| 1.50  | 140            | 0.086          | 0.153          | 0.175                            |
| 1.50  | 160            | 0.092          | 0.122          | 0.153                            |
| 1.50  | 180            | 0.00           | 0.112          | 0.112                            |
TABLE A-1 (Continued)

| $r/R$ | $\theta$ (DEG) | $|\hat{v}_x/U_\infty|$ | $|\hat{v}_y/U_\infty|$ | $(\hat{v}_x^2 + \hat{v}_y^2)^{\frac{1}{2}}/U_\infty$ |
|-------|-----------------|-----------------|-----------------|----------------------------------|
| 1.67  | 0               | 0.269           | 0.164           | 0.315                            |
| 1.67  | 20              | 0.374           | 0.331           | 0.499                            |
| 1.67  | 40              | 0.716           | 0.586           | 0.926                            |
| 1.67  | 60              | 1.123           | 0.758           | 1.355                            |
| 1.67  | 80              | 1.393           | 0.840           | 1.627                            |
| 1.67  | 100             | 1.304           | 0.586           | 1.430                            |
| 1.67  | 120             | 0.345           | 0.240           | 0.420                            |
| 1.67  | 140             | 0.109           | 0.153           | 0.188                            |
| 1.67  | 160             | 0.092           | 0.122           | 0.153                            |
| 1.67  | 180             | 0.0             | 0.112           | 0.112                            |
| 1.83  | 0               | 0.369           | 0.200           | 0.420                            |
| 1.83  | 20              | 0.484           | 0.331           | 0.586                            |
| 1.83  | 40              | 0.765           | 0.520           | 0.926                            |
| 1.83  | 60              | 1.114           | 0.707           | 1.319                            |
| 1.83  | 80              | 1.439           | 0.758           | 1.627                            |
| 1.83  | 100             | 1.650           | 0.707           | 1.795                            |
| 1.83  | 120             | 0.566           | 0.283           | 0.633                            |
| 1.83  | 140             | 0.148           | 0.153           | 0.213                            |
| 1.83  | 160             | 0.092           | 0.122           | 0.153                            |
| 1.83  | 180             | 0.0             | 0.095           | 0.095                            |
| 2.00  | 0               | 0.475           | 0.213           | 0.520                            |
| 2.00  | 20              | 0.568           | 0.331           | 0.657                            |
| 2.00  | 40              | 0.779           | 0.499           | 0.926                            |
| 2.00  | 60              | 1.157           | 0.633           | 1.319                            |
| 2.00  | 80              | 1.407           | 0.732           | 1.586                            |
| 2.00  | 100             | 1.660           | 0.682           | 1.795                            |
| 2.00  | 120             | 0.934           | 0.401           | 1.016                            |
| 2.00  | 140             | 0.185           | 0.153           | 0.240                            |
| 2.00  | 160             | 0.092           | 0.122           | 0.153                            |
| 2.00  | 180             | 0.0             | 0.095           | 0.095                            |
TABLE A-2

Experimental Velocity Data,
u_w/U_∞ = 0.154 and Re = 4100

<table>
<thead>
<tr>
<th>( \hat{r}/R )</th>
<th>( \hat{\theta} ) (DEG)</th>
<th>( \hat{v}<em>x/U</em>∞ )</th>
<th>( \hat{v}<em>y/U</em>∞ )</th>
<th>( (\hat{v}_x^2 + \hat{v}<em>y^2)^{\frac{1}{2}}/U</em>∞ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.17</td>
<td>0</td>
<td>0.0</td>
<td>0.064</td>
<td>0.064</td>
</tr>
<tr>
<td>1.17</td>
<td>20</td>
<td>0.0</td>
<td>0.282</td>
<td>0.282</td>
</tr>
<tr>
<td>1.17</td>
<td>40</td>
<td>0.0</td>
<td>0.584</td>
<td>0.584</td>
</tr>
<tr>
<td>1.17</td>
<td>60</td>
<td>0.412</td>
<td>0.477</td>
<td>0.630</td>
</tr>
<tr>
<td>1.17</td>
<td>80</td>
<td>0.229</td>
<td>0.282</td>
<td>0.364</td>
</tr>
<tr>
<td>1.17</td>
<td>100</td>
<td>0.095</td>
<td>0.175</td>
<td>0.199</td>
</tr>
<tr>
<td>1.17</td>
<td>120</td>
<td>0.0</td>
<td>0.142</td>
<td>0.142</td>
</tr>
<tr>
<td>1.17</td>
<td>140</td>
<td>0.044</td>
<td>0.103</td>
<td>0.112</td>
</tr>
<tr>
<td>1.17</td>
<td>160</td>
<td>0.0</td>
<td>0.103</td>
<td>0.103</td>
</tr>
<tr>
<td>1.17</td>
<td>180</td>
<td>0.0</td>
<td>0.078</td>
<td>0.078</td>
</tr>
<tr>
<td>1.33</td>
<td>0</td>
<td>0.0</td>
<td>0.086</td>
<td>0.086</td>
</tr>
<tr>
<td>1.33</td>
<td>20</td>
<td>0.111</td>
<td>0.346</td>
<td>0.364</td>
</tr>
<tr>
<td>1.33</td>
<td>40</td>
<td>0.624</td>
<td>0.679</td>
<td>0.922</td>
</tr>
<tr>
<td>1.33</td>
<td>60</td>
<td>0.989</td>
<td>0.864</td>
<td>1.313</td>
</tr>
<tr>
<td>1.33</td>
<td>80</td>
<td>0.894</td>
<td>0.654</td>
<td>1.108</td>
</tr>
<tr>
<td>1.33</td>
<td>100</td>
<td>0.322</td>
<td>0.267</td>
<td>0.418</td>
</tr>
<tr>
<td>1.33</td>
<td>120</td>
<td>0.103</td>
<td>0.142</td>
<td>0.175</td>
</tr>
<tr>
<td>1.33</td>
<td>140</td>
<td>0.044</td>
<td>0.103</td>
<td>0.112</td>
</tr>
<tr>
<td>1.33</td>
<td>160</td>
<td>0.0</td>
<td>0.103</td>
<td>0.103</td>
</tr>
<tr>
<td>1.33</td>
<td>180</td>
<td>0.0</td>
<td>0.078</td>
<td>0.078</td>
</tr>
<tr>
<td>1.50</td>
<td>0</td>
<td>0.147</td>
<td>0.152</td>
<td>0.212</td>
</tr>
<tr>
<td>1.50</td>
<td>20</td>
<td>0.257</td>
<td>0.330</td>
<td>0.418</td>
</tr>
<tr>
<td>1.50</td>
<td>40</td>
<td>0.607</td>
<td>0.654</td>
<td>0.893</td>
</tr>
<tr>
<td>1.50</td>
<td>60</td>
<td>1.055</td>
<td>0.782</td>
<td>1.313</td>
</tr>
<tr>
<td>1.50</td>
<td>80</td>
<td>1.327</td>
<td>0.782</td>
<td>1.540</td>
</tr>
<tr>
<td>1.50</td>
<td>100</td>
<td>0.869</td>
<td>0.457</td>
<td>0.981</td>
</tr>
<tr>
<td>1.50</td>
<td>120</td>
<td>0.183</td>
<td>0.175</td>
<td>0.253</td>
</tr>
<tr>
<td>1.50</td>
<td>140</td>
<td>0.044</td>
<td>0.103</td>
<td>0.112</td>
</tr>
<tr>
<td>1.50</td>
<td>160</td>
<td>0.0</td>
<td>0.086</td>
<td>0.086</td>
</tr>
<tr>
<td>1.50</td>
<td>180</td>
<td>0.0</td>
<td>0.078</td>
<td>0.078</td>
</tr>
<tr>
<td>$\hat{r}/R$</td>
<td>$\hat{\theta}$ (DEG)</td>
<td>$\hat{v}<em>x/U</em>\infty$</td>
<td>$\hat{v}<em>y/U</em>\infty$</td>
<td>$(\hat{v}_x^2 + \hat{v}<em>y^2)^{1/2}/U</em>\infty$</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>1.67</td>
<td>0</td>
<td>0.272</td>
<td>0.187</td>
<td>0.330</td>
</tr>
<tr>
<td>1.67</td>
<td>20</td>
<td>0.400</td>
<td>0.330</td>
<td>0.518</td>
</tr>
<tr>
<td>1.67</td>
<td>40</td>
<td>0.654</td>
<td>0.607</td>
<td>0.893</td>
</tr>
<tr>
<td>1.67</td>
<td>60</td>
<td>1.024</td>
<td>0.704</td>
<td>1.242</td>
</tr>
<tr>
<td>1.67</td>
<td>80</td>
<td>1.356</td>
<td>0.729</td>
<td>1.540</td>
</tr>
<tr>
<td>1.67</td>
<td>100</td>
<td>1.516</td>
<td>0.799</td>
<td>1.661</td>
</tr>
<tr>
<td>1.67</td>
<td>120</td>
<td>0.360</td>
<td>0.212</td>
<td>0.418</td>
</tr>
<tr>
<td>1.67</td>
<td>140</td>
<td>0.073</td>
<td>0.121</td>
<td>0.142</td>
</tr>
<tr>
<td>1.67</td>
<td>160</td>
<td>0.0</td>
<td>0.086</td>
<td>0.086</td>
</tr>
<tr>
<td>1.67</td>
<td>180</td>
<td>0.0</td>
<td>0.078</td>
<td>0.078</td>
</tr>
<tr>
<td>1.83</td>
<td>0</td>
<td>0.389</td>
<td>0.199</td>
<td>0.437</td>
</tr>
<tr>
<td>1.83</td>
<td>20</td>
<td>0.455</td>
<td>0.330</td>
<td>0.562</td>
</tr>
<tr>
<td>1.83</td>
<td>40</td>
<td>0.711</td>
<td>0.540</td>
<td>0.893</td>
</tr>
<tr>
<td>1.83</td>
<td>60</td>
<td>1.056</td>
<td>0.654</td>
<td>1.242</td>
</tr>
<tr>
<td>1.83</td>
<td>80</td>
<td>1.370</td>
<td>0.704</td>
<td>1.540</td>
</tr>
<tr>
<td>1.83</td>
<td>100</td>
<td>1.516</td>
<td>0.679</td>
<td>1.661</td>
</tr>
<tr>
<td>1.83</td>
<td>120</td>
<td>0.687</td>
<td>0.313</td>
<td>0.755</td>
</tr>
<tr>
<td>1.83</td>
<td>140</td>
<td>0.073</td>
<td>0.121</td>
<td>0.142</td>
</tr>
<tr>
<td>1.83</td>
<td>160</td>
<td>0.0</td>
<td>0.086</td>
<td>0.086</td>
</tr>
<tr>
<td>1.83</td>
<td>180</td>
<td>0.0</td>
<td>0.078</td>
<td>0.078</td>
</tr>
<tr>
<td>2.00</td>
<td>0</td>
<td>0.508</td>
<td>0.239</td>
<td>0.562</td>
</tr>
<tr>
<td>2.00</td>
<td>20</td>
<td>0.537</td>
<td>0.330</td>
<td>0.630</td>
</tr>
<tr>
<td>2.00</td>
<td>40</td>
<td>0.776</td>
<td>0.497</td>
<td>0.922</td>
</tr>
<tr>
<td>2.00</td>
<td>60</td>
<td>1.071</td>
<td>0.630</td>
<td>1.242</td>
</tr>
<tr>
<td>2.00</td>
<td>80</td>
<td>1.350</td>
<td>0.654</td>
<td>1.501</td>
</tr>
<tr>
<td>2.00</td>
<td>100</td>
<td>1.471</td>
<td>0.679</td>
<td>1.620</td>
</tr>
<tr>
<td>2.00</td>
<td>120</td>
<td>1.375</td>
<td>0.497</td>
<td>1.462</td>
</tr>
<tr>
<td>2.00</td>
<td>140</td>
<td>0.122</td>
<td>0.142</td>
<td>0.187</td>
</tr>
<tr>
<td>2.00</td>
<td>160</td>
<td>0.0</td>
<td>0.086</td>
<td>0.086</td>
</tr>
<tr>
<td>2.00</td>
<td>180</td>
<td>0.0</td>
<td>0.078</td>
<td>0.078</td>
</tr>
</tbody>
</table>
### TABLE A-3

Experimental Velocity Data, \( u_w/U_\infty = 0.126 \) and \( Re = 6200 \)

<table>
<thead>
<tr>
<th>( \hat{R}/R )</th>
<th>( \hat{\theta} ) (DEG)</th>
<th>( \hat{v}<em>x/U</em>\infty )</th>
<th>( \hat{v}<em>y/U</em>\infty )</th>
<th>((\hat{v}_x^2 + \hat{v}<em>y^2)^{1/2}/U</em>\infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.17</td>
<td>0</td>
<td>0.0</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>1.17</td>
<td>20</td>
<td>0.0</td>
<td>0.180</td>
<td>0.180</td>
</tr>
<tr>
<td>1.17</td>
<td>40</td>
<td>0.176</td>
<td>0.427</td>
<td>0.462</td>
</tr>
<tr>
<td>1.17</td>
<td>60</td>
<td>0.358</td>
<td>0.347</td>
<td>0.498</td>
</tr>
<tr>
<td>1.17</td>
<td>80</td>
<td>0.153</td>
<td>0.131</td>
<td>0.202</td>
</tr>
<tr>
<td>1.17</td>
<td>100</td>
<td>0.060</td>
<td>0.047</td>
<td>0.076</td>
</tr>
<tr>
<td>1.17</td>
<td>120</td>
<td>0.034</td>
<td>0.024</td>
<td>0.042</td>
</tr>
<tr>
<td>1.17</td>
<td>140</td>
<td>0.023</td>
<td>0.010</td>
<td>0.024</td>
</tr>
<tr>
<td>1.17</td>
<td>160</td>
<td>0.030</td>
<td>0.012</td>
<td>0.032</td>
</tr>
<tr>
<td>1.17</td>
<td>180</td>
<td>0.021</td>
<td>0.012</td>
<td>0.024</td>
</tr>
<tr>
<td>1.33</td>
<td>0</td>
<td>0.027</td>
<td>0.018</td>
<td>0.033</td>
</tr>
<tr>
<td>1.33</td>
<td>20</td>
<td>0.283</td>
<td>0.203</td>
<td>0.348</td>
</tr>
<tr>
<td>1.33</td>
<td>40</td>
<td>0.711</td>
<td>0.464</td>
<td>0.849</td>
</tr>
<tr>
<td>1.33</td>
<td>60</td>
<td>1.151</td>
<td>0.619</td>
<td>1.307</td>
</tr>
<tr>
<td>1.33</td>
<td>80</td>
<td>1.173</td>
<td>0.501</td>
<td>1.275</td>
</tr>
<tr>
<td>1.33</td>
<td>100</td>
<td>0.353</td>
<td>0.141</td>
<td>0.380</td>
</tr>
<tr>
<td>1.33</td>
<td>120</td>
<td>0.070</td>
<td>0.058</td>
<td>0.091</td>
</tr>
<tr>
<td>1.33</td>
<td>140</td>
<td>0.028</td>
<td>0.025</td>
<td>0.037</td>
</tr>
<tr>
<td>1.33</td>
<td>160</td>
<td>0.0</td>
<td>0.025</td>
<td>0.025</td>
</tr>
<tr>
<td>1.33</td>
<td>180</td>
<td>0.013</td>
<td>0.021</td>
<td>0.025</td>
</tr>
<tr>
<td>1.50</td>
<td>0</td>
<td>0.106</td>
<td>0.036</td>
<td>0.113</td>
</tr>
<tr>
<td>1.50</td>
<td>20</td>
<td>0.293</td>
<td>0.148</td>
<td>0.328</td>
</tr>
<tr>
<td>1.50</td>
<td>40</td>
<td>0.579</td>
<td>0.343</td>
<td>0.673</td>
</tr>
<tr>
<td>1.50</td>
<td>60</td>
<td>0.919</td>
<td>0.439</td>
<td>1.019</td>
</tr>
<tr>
<td>1.50</td>
<td>80</td>
<td>1.128</td>
<td>0.474</td>
<td>1.223</td>
</tr>
<tr>
<td>1.50</td>
<td>100</td>
<td>0.921</td>
<td>0.286</td>
<td>0.964</td>
</tr>
<tr>
<td>1.50</td>
<td>120</td>
<td>0.114</td>
<td>0.041</td>
<td>0.121</td>
</tr>
<tr>
<td>1.50</td>
<td>140</td>
<td>0.013</td>
<td>0.021</td>
<td>0.024</td>
</tr>
<tr>
<td>1.50</td>
<td>160</td>
<td>0.013</td>
<td>0.012</td>
<td>0.017</td>
</tr>
<tr>
<td>1.50</td>
<td>180</td>
<td>0.0</td>
<td>0.007</td>
<td>0.007</td>
</tr>
<tr>
<td>( \hat{r}/R )</td>
<td>( \hat{\theta} ) (DEG)</td>
<td>(</td>
<td>\hat{v}<em>x/U</em>\infty</td>
<td>)</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>1.67</td>
<td>0</td>
<td>0.242</td>
<td>0.046</td>
<td>0.246</td>
</tr>
<tr>
<td>1.67</td>
<td>20</td>
<td>0.416</td>
<td>0.138</td>
<td>0.439</td>
</tr>
<tr>
<td>1.67</td>
<td>40</td>
<td>0.663</td>
<td>0.272</td>
<td>0.716</td>
</tr>
<tr>
<td>1.67</td>
<td>60</td>
<td>0.976</td>
<td>0.373</td>
<td>1.045</td>
</tr>
<tr>
<td>1.67</td>
<td>80</td>
<td>1.262</td>
<td>0.373</td>
<td>1.315</td>
</tr>
<tr>
<td>1.67</td>
<td>100</td>
<td>1.399</td>
<td>0.373</td>
<td>1.447</td>
</tr>
<tr>
<td>1.67</td>
<td>120</td>
<td>0.289</td>
<td>0.075</td>
<td>0.299</td>
</tr>
<tr>
<td>1.67</td>
<td>140</td>
<td>0.049</td>
<td>0.014</td>
<td>0.051</td>
</tr>
<tr>
<td>1.67</td>
<td>160</td>
<td>0.017</td>
<td>0.005</td>
<td>0.017</td>
</tr>
<tr>
<td>1.67</td>
<td>180</td>
<td>0.017</td>
<td>0.004</td>
<td>0.017</td>
</tr>
<tr>
<td>1.83</td>
<td>0</td>
<td>0.349</td>
<td>0.075</td>
<td>0.357</td>
</tr>
<tr>
<td>1.83</td>
<td>20</td>
<td>0.431</td>
<td>0.148</td>
<td>0.456</td>
</tr>
<tr>
<td>1.83</td>
<td>40</td>
<td>0.560</td>
<td>0.285</td>
<td>0.629</td>
</tr>
<tr>
<td>1.83</td>
<td>60</td>
<td>0.981</td>
<td>0.357</td>
<td>1.044</td>
</tr>
<tr>
<td>1.83</td>
<td>80</td>
<td>1.212</td>
<td>0.421</td>
<td>1.283</td>
</tr>
<tr>
<td>1.83</td>
<td>100</td>
<td>1.328</td>
<td>0.373</td>
<td>1.380</td>
</tr>
<tr>
<td>1.83</td>
<td>120</td>
<td>0.939</td>
<td>0.210</td>
<td>0.962</td>
</tr>
<tr>
<td>1.83</td>
<td>140</td>
<td>0.063</td>
<td>0.028</td>
<td>0.069</td>
</tr>
<tr>
<td>1.83</td>
<td>160</td>
<td>0.013</td>
<td>0.012</td>
<td>0.017</td>
</tr>
<tr>
<td>1.83</td>
<td>180</td>
<td>0.010</td>
<td>0.009</td>
<td>0.009</td>
</tr>
<tr>
<td>2.00</td>
<td>0</td>
<td>0.449</td>
<td>0.082</td>
<td>0.456</td>
</tr>
<tr>
<td>2.00</td>
<td>20</td>
<td>0.511</td>
<td>0.138</td>
<td>0.529</td>
</tr>
<tr>
<td>2.00</td>
<td>40</td>
<td>0.747</td>
<td>0.247</td>
<td>0.786</td>
</tr>
<tr>
<td>2.00</td>
<td>60</td>
<td>0.998</td>
<td>0.313</td>
<td>1.046</td>
</tr>
<tr>
<td>2.00</td>
<td>80</td>
<td>1.275</td>
<td>0.328</td>
<td>1.317</td>
</tr>
<tr>
<td>2.00</td>
<td>100</td>
<td>1.376</td>
<td>0.328</td>
<td>1.415</td>
</tr>
<tr>
<td>2.00</td>
<td>120</td>
<td>1.600</td>
<td>0.286</td>
<td>1.624</td>
</tr>
<tr>
<td>2.00</td>
<td>140</td>
<td>0.102</td>
<td>0.024</td>
<td>0.104</td>
</tr>
<tr>
<td>2.00</td>
<td>160</td>
<td>0.022</td>
<td>0.009</td>
<td>0.024</td>
</tr>
<tr>
<td>2.00</td>
<td>180</td>
<td>0.014</td>
<td>0.004</td>
<td>0.014</td>
</tr>
</tbody>
</table>
TABLE A-4

Experimental Velocity Data,  
\( \frac{u_w}{U_\infty} = 0.102 \) and \( \text{Re} = 6200 \)

<table>
<thead>
<tr>
<th>( \hat{r}/R )</th>
<th>( \hat{\theta} ) (DEG)</th>
<th>( \hat{v}<em>X/U</em>\infty )</th>
<th>( \hat{v}<em>Y/U</em>\infty )</th>
<th>( (\hat{v}_X^2 + \hat{v}<em>Y^2)^{\frac{1}{2}}/U</em>\infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.17</td>
<td>0</td>
<td>0.003</td>
<td>0.0</td>
<td>0.003</td>
</tr>
<tr>
<td>1.17</td>
<td>20</td>
<td>0.070</td>
<td>0.206</td>
<td>0.218</td>
</tr>
<tr>
<td>1.17</td>
<td>40</td>
<td>0.461</td>
<td>0.501</td>
<td>0.681</td>
</tr>
<tr>
<td>1.17</td>
<td>60</td>
<td>0.706</td>
<td>0.414</td>
<td>0.819</td>
</tr>
<tr>
<td>1.17</td>
<td>80</td>
<td>0.283</td>
<td>0.118</td>
<td>0.307</td>
</tr>
<tr>
<td>1.17</td>
<td>100</td>
<td>0.072</td>
<td>0.036</td>
<td>0.081</td>
</tr>
<tr>
<td>1.17</td>
<td>120</td>
<td>0.014</td>
<td>0.009</td>
<td>0.017</td>
</tr>
<tr>
<td>1.17</td>
<td>140</td>
<td>0.0</td>
<td>0.009</td>
<td>0.009</td>
</tr>
<tr>
<td>1.17</td>
<td>160</td>
<td>0.007</td>
<td>0.002</td>
<td>0.007</td>
</tr>
<tr>
<td>1.17</td>
<td>180</td>
<td>0.002</td>
<td>0.002</td>
<td>0.003</td>
</tr>
<tr>
<td>1.33</td>
<td>0</td>
<td>0.067</td>
<td>0.045</td>
<td>0.081</td>
</tr>
<tr>
<td>1.33</td>
<td>20</td>
<td>0.226</td>
<td>0.231</td>
<td>0.323</td>
</tr>
<tr>
<td>1.33</td>
<td>40</td>
<td>0.604</td>
<td>0.485</td>
<td>0.775</td>
</tr>
<tr>
<td>1.33</td>
<td>60</td>
<td>1.100</td>
<td>0.560</td>
<td>1.234</td>
</tr>
<tr>
<td>1.33</td>
<td>80</td>
<td>1.335</td>
<td>0.503</td>
<td>1.427</td>
</tr>
<tr>
<td>1.33</td>
<td>100</td>
<td>0.334</td>
<td>0.111</td>
<td>0.352</td>
</tr>
<tr>
<td>1.33</td>
<td>120</td>
<td>0.059</td>
<td>0.017</td>
<td>0.062</td>
</tr>
<tr>
<td>1.33</td>
<td>140</td>
<td>0.019</td>
<td>0.007</td>
<td>0.020</td>
</tr>
<tr>
<td>1.33</td>
<td>160</td>
<td>0.017</td>
<td>0.004</td>
<td>0.017</td>
</tr>
<tr>
<td>1.33</td>
<td>180</td>
<td>0.024</td>
<td>0.002</td>
<td>0.024</td>
</tr>
<tr>
<td>1.50</td>
<td>0</td>
<td>0.239</td>
<td>0.045</td>
<td>0.243</td>
</tr>
<tr>
<td>1.50</td>
<td>20</td>
<td>0.410</td>
<td>0.186</td>
<td>0.450</td>
</tr>
<tr>
<td>1.50</td>
<td>40</td>
<td>0.756</td>
<td>0.384</td>
<td>0.848</td>
</tr>
<tr>
<td>1.50</td>
<td>60</td>
<td>1.079</td>
<td>0.467</td>
<td>1.175</td>
</tr>
<tr>
<td>1.50</td>
<td>80</td>
<td>1.386</td>
<td>0.467</td>
<td>1.462</td>
</tr>
<tr>
<td>1.50</td>
<td>100</td>
<td>1.664</td>
<td>0.400</td>
<td>1.712</td>
</tr>
<tr>
<td>1.50</td>
<td>120</td>
<td>0.170</td>
<td>0.041</td>
<td>0.175</td>
</tr>
<tr>
<td>1.50</td>
<td>140</td>
<td>0.0</td>
<td>0.009</td>
<td>0.009</td>
</tr>
<tr>
<td>1.50</td>
<td>160</td>
<td>0.0</td>
<td>0.009</td>
<td>0.009</td>
</tr>
<tr>
<td>1.50</td>
<td>180</td>
<td>0.008</td>
<td>0.005</td>
<td>0.009</td>
</tr>
<tr>
<td>r/R</td>
<td>( \hat{\theta} ) (DEG)</td>
<td>( \hat{\nu}<em>x/U</em>\infty )</td>
<td>( \hat{\nu}<em>y/U</em>\infty )</td>
<td>( (\hat{\nu}_x^2 + \hat{\nu}<em>y^2)^{1/2}/U</em>\infty )</td>
</tr>
<tr>
<td>-----</td>
<td>-----------------</td>
<td>-----------------</td>
<td>-----------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>1.67</td>
<td>0</td>
<td>0.332</td>
<td>0.068</td>
<td>0.339</td>
</tr>
<tr>
<td>1.67</td>
<td>20</td>
<td>0.435</td>
<td>0.176</td>
<td>0.469</td>
</tr>
<tr>
<td>1.67</td>
<td>40</td>
<td>0.674</td>
<td>0.339</td>
<td>0.755</td>
</tr>
<tr>
<td>1.67</td>
<td>60</td>
<td>0.977</td>
<td>0.417</td>
<td>1.063</td>
</tr>
<tr>
<td>1.67</td>
<td>80</td>
<td>1.262</td>
<td>0.434</td>
<td>1.335</td>
</tr>
<tr>
<td>1.67</td>
<td>100</td>
<td>1.385</td>
<td>0.369</td>
<td>1.433</td>
</tr>
<tr>
<td>1.67</td>
<td>120</td>
<td>0.411</td>
<td>0.075</td>
<td>0.417</td>
</tr>
<tr>
<td>1.67</td>
<td>140</td>
<td>0.042</td>
<td>0.017</td>
<td>0.046</td>
</tr>
<tr>
<td>1.67</td>
<td>160</td>
<td>0.014</td>
<td>0.004</td>
<td>0.014</td>
</tr>
<tr>
<td>1.67</td>
<td>180</td>
<td>0.008</td>
<td>0.004</td>
<td>0.009</td>
</tr>
<tr>
<td>1.83</td>
<td>0</td>
<td>0.368</td>
<td>0.111</td>
<td>0.384</td>
</tr>
<tr>
<td>1.83</td>
<td>20</td>
<td>0.521</td>
<td>0.208</td>
<td>0.561</td>
</tr>
<tr>
<td>1.83</td>
<td>40</td>
<td>0.732</td>
<td>0.324</td>
<td>0.800</td>
</tr>
<tr>
<td>1.83</td>
<td>60</td>
<td>1.019</td>
<td>0.384</td>
<td>1.089</td>
</tr>
<tr>
<td>1.83</td>
<td>80</td>
<td>1.192</td>
<td>0.433</td>
<td>1.268</td>
</tr>
<tr>
<td>1.83</td>
<td>100</td>
<td>1.377</td>
<td>0.384</td>
<td>1.430</td>
</tr>
<tr>
<td>1.83</td>
<td>120</td>
<td>1.277</td>
<td>0.243</td>
<td>1.299</td>
</tr>
<tr>
<td>1.83</td>
<td>140</td>
<td>0.054</td>
<td>0.017</td>
<td>0.056</td>
</tr>
<tr>
<td>1.83</td>
<td>160</td>
<td>0.011</td>
<td>0.004</td>
<td>0.012</td>
</tr>
<tr>
<td>1.83</td>
<td>180</td>
<td>0.006</td>
<td>0.004</td>
<td>0.007</td>
</tr>
<tr>
<td>2.00</td>
<td>0</td>
<td>0.509</td>
<td>0.119</td>
<td>0.523</td>
</tr>
<tr>
<td>2.00</td>
<td>20</td>
<td>0.593</td>
<td>0.186</td>
<td>0.622</td>
</tr>
<tr>
<td>2.00</td>
<td>40</td>
<td>0.854</td>
<td>0.282</td>
<td>0.900</td>
</tr>
<tr>
<td>2.00</td>
<td>60</td>
<td>1.092</td>
<td>0.353</td>
<td>1.147</td>
</tr>
<tr>
<td>2.00</td>
<td>80</td>
<td>1.280</td>
<td>0.369</td>
<td>1.332</td>
</tr>
<tr>
<td>2.00</td>
<td>100</td>
<td>1.413</td>
<td>0.384</td>
<td>1.465</td>
</tr>
<tr>
<td>2.00</td>
<td>120</td>
<td>1.525</td>
<td>0.369</td>
<td>1.569</td>
</tr>
<tr>
<td>2.00</td>
<td>140</td>
<td>0.105</td>
<td>0.036</td>
<td>0.111</td>
</tr>
<tr>
<td>2.00</td>
<td>160</td>
<td>0.013</td>
<td>0.012</td>
<td>0.017</td>
</tr>
<tr>
<td>2.00</td>
<td>180</td>
<td>0.013</td>
<td>0.005</td>
<td>0.014</td>
</tr>
</tbody>
</table>
TABLE A-5
Experimental Velocity Data,
$w/U_\infty = 0.0$ and $Re = 4100$

| $\hat{r}/R$ | $\hat{\theta}$ (DEG) | $|\hat{v}_x/U_\infty|$ | $|\hat{v}_y/U_\infty|$ | $(\hat{v}_x^2 + \hat{v}_y^2)^{\frac{1}{2}}/U_\infty$ |
|-----------|-----------------|-----------------|-----------------|---------------------------------|
| 1.17      | 0               | 0.077           | 0.132           | 0.153                           |
| 1.17      | 20              | 0.465           | 0.500           | 0.683                           |
| 1.17      | 40              | 1.091           | 0.869           | 1.395                           |
| 1.17      | 60              | 1.624           | 0.869           | 1.841                           |
| 1.17      | 80              | 1.806           | 0.683           | 1.931                           |
| 1.17      | 100             | 1.846           | 0.565           | 1.930                           |
| 1.17      | 120             | 0.239           | 0.153           | 0.284                           |
| 1.17      | 140             | 0.092           | 0.122           | 0.153                           |
| 1.17      | 160             | 0.0             | 0.176           | 0.176                           |
| 1.17      | 180             | 0.0             | 0.142           | 0.142                           |
| 1.33      | 0               | 0.355           | 0.188           | 0.402                           |
| 1.33      | 20              | 0.684           | 0.440           | 0.813                           |
| 1.33      | 40              | 1.099           | 0.733           | 1.321                           |
| 1.33      | 60              | 1.555           | 0.813           | 1.755                           |
| 1.33      | 80              | 1.806           | 0.683           | 1.931                           |
| 1.33      | 100             | 1.800           | 0.565           | 1.890                           |
| 1.33      | 120             | 1.714           | 0.543           | 1.798                           |
| 1.33      | 140             | 0.0             | 0.153           | 0.153                           |
| 1.33      | 160             | 0.0             | 0.176           | 0.176                           |
| 1.33      | 180             | 0.0             | 0.142           | 0.142                           |
| 1.50      | 0               | 0.561           | 0.240           | 0.611                           |
| 1.50      | 20              | 0.750           | 0.440           | 0.869                           |
| 1.50      | 40              | 1.171           | 0.611           | 1.321                           |
| 1.50      | 60              | 1.559           | 0.708           | 1.712                           |
| 1.50      | 80              | 1.663           | 0.683           | 1.798                           |
| 1.50      | 100             | 1.800           | 0.565           | 1.886                           |
| 1.50      | 120             | 1.784           | 0.611           | 1.886                           |
| 1.50      | 140             | 0.193           | 0.142           | 0.240                           |
| 1.50      | 160             | 0.0             | 0.176           | 0.176                           |
| 1.50      | 180             | 0.0             | 0.176           | 0.176                           |
| \( \hat{r}/R \) | \( \hat{\theta} \) (DEG) | \( |\hat{v}_x/U_\infty| \) | \( |\hat{v}_y/U_\infty| \) | \( (\hat{v}_x^2+\hat{v}_y^2)^{\frac{1}{2}}/U_\infty \) |
|----------|-------------|----------------|----------------|----------------|
| 1.67     | 0           | 0.682          | 0.269          | 0.733          |
| 1.67     | 20          | 0.816          | 0.440          | 0.927          |
| 1.67     | 40          | 1.131          | 0.611          | 1.285          |
| 1.67     | 60          | 1.501          | 0.634          | 1.629          |
| 1.67     | 80          | 1.636          | 0.634          | 1.755          |
| 1.67     | 100         | 1.707          | 0.565          | 1.798          |
| 1.67     | 120         | 1.792          | 0.587          | 1.886          |
| 1.67     | 140         | 0.726          | 0.366          | 0.813          |
| 1.67     | 160         | 0.0            | 0.176          | 0.176          |
| 1.67     | 180         | 0.229          | 0.240          | 0.332          |
| 1.83     | 0           | 0.816          | 0.299          | 0.869          |
| 1.83     | 20          | 0.884          | 0.440          | 0.897          |
| 1.83     | 40          | 1.165          | 0.543          | 1.285          |
| 1.83     | 60          | 1.457          | 0.634          | 1.589          |
| 1.83     | 80          | 1.572          | 0.565          | 1.671          |
| 1.83     | 100         | 1.661          | 0.565          | 1.755          |
| 1.83     | 120         | 1.700          | 0.587          | 1.798          |
| 1.83     | 140         | 1.600          | 0.480          | 1.671          |
| 1.83     | 160         | 0.0            | 0.176          | 0.176          |
| 1.83     | 180         | 0.248          | 0.269          | 0.366          |
| 2.00     | 0           | 0.877          | 0.299          | 0.927          |
| 2.00     | 20          | 0.918          | 0.440          | 1.018          |
| 2.00     | 40          | 1.184          | 0.500          | 1.285          |
| 2.00     | 60          | 1.433          | 0.587          | 1.549          |
| 2.00     | 80          | 1.572          | 0.565          | 1.671          |
| 2.00     | 100         | 1.661          | 0.565          | 1.755          |
| 2.00     | 120         | 1.714          | 0.543          | 1.798          |
| 2.00     | 140         | 1.823          | 0.480          | 1.886          |
| 2.00     | 160         | 0.248          | 0.269          | 0.366          |
| 2.00     | 180         | 0.248          | 0.269          | 0.366          |
| $\hat{r}/R$ | $\hat{\theta}$ (DEG) | $|\hat{v}_x/U_\infty|$ | $|\hat{v}_y/U_\infty|$ | $(\hat{v}_x^2 + \hat{v}_y^2)^{\frac{1}{2}}/U_\infty$ |
|-----------|-----------------|------------------|------------------|-----------------------------------------------|
| 1.17      | 0               | 0.042            | 0.017            | 0.045                                         |
| 1.17      | 20              | 0.273            | 0.268            | 0.383                                         |
| 1.17      | 40              | 0.667            | 0.559            | 0.870                                         |
| 1.17      | 60              | 1.474            | 0.230            | 1.492                                         |
| 1.17      | 80              | 1.407            | 0.383            | 1.458                                         |
| 1.17      | 100             | 1.574            | 0.268            | 1.597                                         |
| 1.17      | 120             | 0.234            | 0.062            | 0.242                                         |
| 1.17      | 140             | 0.054            | 0.051            | 0.074                                         |
| 1.17      | 160             | 0.0              | 0.136            | 0.136                                         |
| 1.17      | 180             | 0.193            | 0.185            | 0.268                                         |
| 1.33      | 0               | 0.178            | 0.051            | 0.185                                         |
| 1.33      | 20              | 0.552            | 0.230            | 0.599                                         |
| 1.33      | 40              | 0.756            | 0.432            | 0.870                                         |
| 1.33      | 60              | 1.154            | 0.432            | 1.232                                         |
| 1.33      | 80              | 1.407            | 0.383            | 1.458                                         |
| 1.33      | 100             | 1.425            | 0.308            | 1.458                                         |
| 1.33      | 120             | 0.869            | 0.218            | 0.896                                         |
| 1.33      | 140             | 0.087            | 0.068            | 0.111                                         |
| 1.33      | 160             | 0.156            | 0.136            | 0.207                                         |
| 1.33      | 180             | 0.176            | 0.218            | 0.281                                         |
| 1.50      | 0               | 0.358            | 0.081            | 0.367                                         |
| 1.50      | 20              | 0.569            | 0.185            | 0.599                                         |
| 1.50      | 40              | 0.824            | 0.352            | 0.896                                         |
| 1.50      | 60              | 1.139            | 0.383            | 1.202                                         |
| 1.50      | 80              | 1.250            | 0.337            | 1.295                                         |
| 1.50      | 100             | 1.225            | 0.308            | 1.263                                         |
| 1.50      | 120             | 1.582            | 0.218            | 1.597                                         |
| 1.50      | 140             | 0.308            | 0.136            | 0.337                                         |
| 1.50      | 160             | 0.276            | 0.136            | 0.308                                         |
| 1.50      | 180             | 0.142            | 0.242            | 0.281                                         |
| \( \hat{r}/R \) | \( \hat{\theta} \) (DEG) | \( |\hat{v}_{x}/u_{\infty}| \) | \( |\hat{v}_{y}/u_{\infty}| \) | \( (|\hat{v}_{x}|^2 + |\hat{v}_{y}|^2)^{\frac{3}{2}}/u_{\infty} \) |
|---|---|---|---|---|
| 1.67 | 0 | 0.511 | 0.103 | 0.521 |
| 1.67 | 20 | 0.569 | 0.185 | 0.599 |
| 1.67 | 40 | 0.841 | 0.308 | 0.896 |
| 1.67 | 60 | 1.154 | 0.337 | 1.202 |
| 1.67 | 80 | 1.250 | 0.337 | 1.295 |
| 1.67 | 100 | 1.290 | 0.308 | 1.326 |
| 1.67 | 120 | 1.476 | 0.218 | 1.492 |
| 1.67 | 140 | 1.067 | 0.196 | 1.084 |
| 1.67 | 160 | 0.276 | 0.136 | 0.308 |
| 1.67 | 180 | 0.234 | 0.242 | 0.337 |
| 1.83 | 0 | 0.507 | 0.119 | 0.521 |
| 1.83 | 20 | 0.569 | 0.185 | 0.599 |
| 1.83 | 40 | 0.776 | 0.268 | 0.821 |
| 1.83 | 60 | 1.035 | 0.322 | 1.084 |
| 1.83 | 80 | 1.138 | 0.281 | 1.172 |
| 1.83 | 100 | 1.290 | 0.308 | 1.326 |
| 1.83 | 120 | 1.341 | 0.218 | 1.359 |
| 1.83 | 140 | 1.308 | 0.218 | 1.326 |
| 1.83 | 160 | 0.392 | 0.136 | 0.415 |
| 1.83 | 180 | 0.204 | 0.267 | 0.337 |
| 2.00 | 0 | 0.588 | 0.111 | 0.599 |
| 2.00 | 20 | 0.703 | 0.185 | 0.727 |
| 2.00 | 40 | 0.776 | 0.268 | 0.821 |
| 2.00 | 60 | 0.981 | 0.308 | 1.028 |
| 2.00 | 80 | 1.232 | 0.281 | 1.263 |
| 2.00 | 100 | 1.121 | 0.218 | 1.142 |
| 2.00 | 120 | 1.408 | 0.218 | 1.424 |
| 2.00 | 140 | 1.408 | 0.218 | 1.424 |
| 2.00 | 160 | 0.432 | 0.255 | 0.502 |
| 2.00 | 180 | 0.292 | 0.294 | 0.415 |
To determine the constants in the trial stream function, for a given blowing rate and free stream Reynolds number, it was necessary to solve a set of simultaneous non-linear algebraic equations, such as, equations (42)-(45) and (48)-(51) or (67)-(75). The Fortran programs used to solve these sets of equations are shown in Listings B-1 and B-2. The other programs required for the second and fourth through tenth Galerkin solutions are not presented since they require only slight modification of Listings B-1 and B-2.

The only inputs to these programs are the two cards

\[ UW = \left( \frac{u_w}{U_\infty} \right) \]  

and

\[ IREC = \left( \frac{Re}{100} - 1 \right) \],

\[ ^1 \text{The quantities in the parentheses should be replaced by their numerical values before insertion into the computer programs.} \]
which designate that for the prescribed blowing rate, the following quantities will be calculated:

1) the constants of the trial stream function for \(100 \leq \text{Re} \leq 100 (\text{IREC} + 1)\),

2) the drag coefficient for \(100 \leq \text{Re} \leq 100 (\text{IREC} + 1)\),

and

3) the velocity profiles at \(\text{Re} = 100(\text{IREC} + 1)\) for \(1.17 \leq r \leq 2\).

To change either the blowing rate or the Reynolds number, at which the velocity profiles are calculated, only these two cards need to be changed. The constants of the trial stream function and the drag coefficient are automatically computed over the range of Reynolds numbers \(100 \leq \text{Re} \leq 100(\text{IREC} + 1)\).

These programs use the subroutine "NONLIN," modified to use double precision, from the FORTLIB section of the Ohio State University Instructional and Research Computer Center Library. Double precision was required, since round off errors prevented satisfactory solution of the equations for some combinations of free stream and blowing conditions. Use of the subroutine "NONLIN" is fully documented in reference (33).

Listing B-1 (the first Galerkin solution) can be checked by solving equations (42)-(45) and (48)-(51) by a
different method, for the limiting case of no blowing. In this special case, $u_w/U_\infty = 0.0$. Thus, the third and fourth internal conditions (equations (50) and (51) respectively) reduce to

\[ \alpha_3 + 6\alpha_5 + 20\alpha_7 = 0 \quad \text{(B-1)} \]

and

\[ \alpha_4 + 5\alpha_6 + 15\alpha_8 = 0 \quad \text{(B-2)} \]

These equations can be solved with equations (42)-(45) to give

\[ \alpha_3 = -4 - \frac{22}{9}\alpha_1 \quad \text{(B-3)} \]

\[ \alpha_4 = -\frac{5}{2}\alpha_2 \]

\[ \alpha_5 = 4 + \frac{17}{9}\alpha_1 \]

\[ \alpha_6 = 2\alpha_2 \]

\[ \alpha_7 = -1 - \frac{4}{9}\alpha_1 \]

and
\( \alpha_8 = -\frac{1}{2}\alpha_2 \).

Equations (B-3) can be substituted into the first and second internal conditions (equations (48) and (49) respectively) to give

\[
\alpha_2 = \frac{(16 + \frac{52}{9}\alpha_1)}{\text{Re}\left(\frac{1}{140} - \frac{13}{5040}\alpha_1\right)} \quad (B-4)
\]

and

\[
\frac{151}{1890}\alpha_1^2 + \frac{59}{84}\alpha_1 + \frac{138}{70} - \frac{54}{\text{Re}\alpha_2} = 0 \quad (B-5)
\]

Upon substituting equation (B-4) into (B-5), the result is

\[
-\frac{1963}{95256}\alpha_1^3 - \frac{2627}{21168}\alpha_1^2 - \left(\frac{1}{147} + \frac{31200}{\text{Re}^2}\right)\alpha_1
\]

\[
+ \left(\frac{69}{49} - \frac{86400}{\text{Re}^2}\right) = 0
\]

which is a cubic equation in \( \alpha_1 \). This equation was solved, as a function of \( \text{Re} \), by using the root finding subroutine "POLRT," in the SSP section of the Ohio State University Instructional and Research Computer Center Library. This Fortran program is shown in Listing B-3. Its only input is
from which the same output quantities as before are computed for the range of Reynolds numbers $100 \leq Re \leq 100(IREC)$. Table B-1 shows the values of the constants in the trial stream function, as calculated by the program of Listing B-3. The fact that these results compare favorably with the results of Table 8, verifies that program of Listing B-1 is functioning properly.

After the constants of the trial stream function were calculated, it was possible to determine the coordinates of the streamlines. Listing B-4 shows the Fortran program used to compute the coordinates for the cases of $\frac{u_w}{U_\infty} = 0.190, 0.154, 0.126, \text{ and } 0.102$ in the first Galerkin solution. The only inputs for this program are the cards

- $UW = \left( \frac{u_w}{U_\infty} \right)$,
- $A(1) = (\alpha_1)$,
- $A(2) = (\alpha_2)$,
- $A(3) = (\alpha_3)$,
\[ A(4) = (\alpha_4), \]
\[ A(5) = (\alpha_5), \]
\[ A(6) = (\alpha_6), \]
\[ A(7) = (\alpha_7), \]

and

\[ A(8) = (\alpha_8). \]

For the limiting case of no blowing in the first Galerkin solution, Listing B-5 was used. The only inputs are the cards

\[ A(1) = (\alpha_1), \]
\[ A(2) = (\alpha_2), \]
\[ A(3) = (\alpha_3), \]
\[ A(4) = (\alpha_4), \]
\[ A(5) = (\alpha_5), \]
\[ A(6) = (\alpha_6), \]
\[ A(7) = (\alpha_7), \]

and
Listing B-6 was used to compute the coordinates of the streamlines in all cases for the third solution. The inputs are:

$$A(8) = (\alpha_8)$$

$$UW = (u_w/U_\infty)$$

$$A(1) = (\alpha_1)$$

$$A(2) = (\alpha_2)$$

$$A(3) = (\alpha_3)$$

$$A(4) = (\alpha_4)$$

$$A(5) = (\alpha_5)$$

$$A(6) = (\alpha_6)$$

$$A(7) = (\alpha_7)$$

$$A(8) = (\alpha_8)$$

and

$$A(9) = (\alpha_9)$$

Programs B-4, B-5, and B-6 utilize the subroutine "RTMI" of the SSP section of the Ohio State University Instructional and Research Computer Center Library.
LISTING B-1

Program for the Determination of the Constants in the Trial Stream Function (Galerkin Solution 1)

```
1. THIS PROGRAM CALCULATES THE FOLLOWING QUANTITIES:
2. THE CONSTANTS OF THE TRIAL STREAM FUNCTION
3. THE LINEAR COEFFICIENT
4. THE WEIGHT AT WHICH GENERATION OCCURS
5. 61-UPPRECISION A(I), C(I), P(I), AINV, DMAX, ACC, MAXFUN, IPRINT

6. IF THE FOLLOWING TWO CARDS ARE THE ONLY INPUTS FOR THIS PROGRAM
7. UK=1653
8. REC=9
9. THE FOLLOWING TWO CARDS ARE THE ONLY INPUTS FOR THIS PROGRAM

10. UK=1653
11. REC=9

12. A(I) = 1.1
13. STEP = .01
14. MAXE = 6
15. MAXFUN = 200
16. IPRINT = 0
17. RF=105
18. CALL DCHAIN(A+F, AINV, DMAX, ACC, MAXFUN, IPRINT, I)
19. PRINT2(RE)
20. PRINT2((A(I), J = 1, 68)
21. 2 = 1.0 = 10
22. A(I) = 0.9
23. A(I) = 1.9
24. A(I) = 1.9
25. CALL (C(K) A(K))
26. PRINT2(RE)
27. CALL (C(K) A(K))
28. PRINT2(RE)
29. CC = 1.4 + 15.93 + (2.02 + 0.11) + 52 + A(I) + 198 + A(I) + 488 + A(I) + 171 / REC + LK0(2)
30. SEPC = C(A(I)) + 14 + 15 + A(I) + 28 + A(I) + 171 / 6 + A(I) + 20 + A(1) + 42
31. SEP1 = C(A(I) + 14) + 16
32. PRINT2(RE)
33. CC = 1.4 + 15.93 + (2.02 + 0.11) + 52 + A(I) + 198 + A(I) + 488 + A(I) + 171 / REC + LK0(2)
34. CC = 1.4 + 15.93 + (2.02 + 0.11) + 52 + A(I) + 198 + A(I) + 488 + A(I) + 171 / REC + LK0(2)
```

This program calculates the following quantities:

1. The constants of the trial stream function
2. The linear coefficient
3. The weight at which generation occurs

The following two cards are the only inputs for this program:

- UK = 1653
- REC = 9

If the following two cards are the only inputs for this program:

- UK = 1653
- REC = 9

Example input:

- A(I) = 1.1
- STEP = .01
- MAXE = 6
- MAXFUN = 200
- IPRINT = 0
- RF = 105
```fortran
96

```
S-(1./RCC>=*(96.FC(A)*AIA) + ARC.3C16!*A(6)*IAC.9C(0)<'A(0) )

RETURN
C START ITERATION BY PREDICTING THE DESCENT AND NEWTON MINIMA

GO TO 17

C TEST WHETHER A NEARBY STATIONARY POINT IS PREDICTED

IF (F1X-PFLA1<p0P511) 1.477 00001370

GO TO 17

C TEST WHETHER TO APPLY THE FULL NEWTON CORRECTION

IF (1X-DS-DE ) 47.67 48 00001500

GO TO 80

C CALCULATE THE LENGTH OF THE STEEPEST DESCENT STEP

K=0

GO TO 52

C TEST WHETHER TO USE THE STEEPEST DESCENT DIRECTION

IF (DS-DE 00001780

GO TO 52
C

SET THE MULTIPLIER OF THE STEEPEST DESCENT DIRECTION

C

INTERPOLATE BETWEEN THE STEEPEST DESCENT AND THE NEAREST DIRECTION

C

CALCULATE THE CHANGE IN X AND ITS ANGLE WITH THE FIRST DIRECTION

C

TEST WHETHER AN EXTRA STEP IS NEEDED FOR INDEPENDENCE

C

EXPRESS THE NEW DIRECTION IN TERMS OF THOSE OF THE DIRECTION MATRIX AND UPDATE THE COORDS IN W(DEC+1) ETC.
GO TO 38.
END

SUBROUTINE INVI A, N, C, L, K

DOUBLE PRECISION A, C, SINGA, SINC

C = 1.
N = 1.
K = 1.
L = 1.

IF (SINGA) .LE. 0.98 .AND. SINC .GE. 0.98
RETURN

DO 90 J = 1, N

IF (SINGA) .LE. 0.98 .AND. SINC .LT. 0.98
RETURN

I = J.

DO 80 I = 1, N

IF (SINGA) .LE. 0.98 .AND. SINC .LT. 0.98
RETURN

CONTINUE

J = N.

I = 1.

DO 70 I = 1, N

IF (SINGA) .LE. 0.98 .AND. SINC .LT. 0.98
RETURN

CONTINUE
CONTINUE
KJ=KJ-A
CD 75 J=1,N
KJ=KJ=N
IF(J-K) 70,75,70
75 CONTINUE
K=1
K1=N
70 CONTINUE
K=N-1
IF(K) 150,150,105
105 CD 11
106 JC=J-K=11
CD 11 J=J-1
JK=J-K
MOLD=AIJK
JI=JR+J
110 AIJKI=AIJK
120 AIJKI=HOLD
125 CD 130 I=I+1
AIJKI=AIJK
130 CD TO 100
150 RETUR
END
/*
** DUMMY EXEC PRCC=RUNCRT,CCM=(4,LT),TIME,LED=(0,5),TIME,GE=(1,20)
** GO SYSIN CD *
** */
LISTING B-2

Program for the Determination of the Constants in the Trial Stream Function (Galerkin Solution 3)

```
// 4000. CLASS=A
// EXEC PRGC=FCRTRUN,
// TIP=CM=(0.5),
// PRAM=EMAP*
// CMPS=SYS=0.
// THIS PROGRAM CALCULATES THE FOLLOWING QUANTITIES:
// 1) THE DMCX COEFFICIENT
// 2) THE ANGLE AT WHICH SEPARATION OCCURS
// 3) VELOCITY PROFILES
// 4) VELOCITY PROFILES
// DOUBLE PRECISION ALG1,ALG2,ALG3,ALG4,ALG5,ALG6,ALG7,ALG8,ALG9,ALG10,
// REC,UX,CE,SEP
// EXTERNAL FCT
// THE FOLLOWING TWO CARDS ARE THE ONLY INPUTS FOR THIS PROGRAM
// UC=1.903
// REC=46
// THE FOLLOWING TWO CARDS ARE THE ONLY INPUTS FOR THIS PROGRAM
// C1(1)=4.8394
// C2(1)=4.7907
// C3(1)=1.8594
// C4(1)=1.7431
// C5(1)=0.9556
// C6(1)=0.9265
// C7(1)=0.7947
// DO 1 =1+10
// 1 AI(1)=DSTEP=0.001
// ACC=1.E-4
// PAXFUN=200
// DMAX=1.
// EPS=1.E-6
// IENC=100
// XL=0.
// REC=100.
// CALL CNM1(M=NX,AY,AJ,N,DSTEP,DMAX,ACC,PAXFUN,IPRINT,K)
// PRINT2(RE)
// PRINT3((AI(1))=1+10)
// 2 FORMAT(9(3,5))
// 3 FORMAT(I(K=11))
// 4 FORMAT(44(
// 5 FORMAT(3(11))
// 6 FORMAT(6(11))
// 7 FORMAT(6(11))
// 8 FORMAT(6(11))
// 9 FORMAT(6(11))
// 10 FORMAT(6(11))
// 11 FORMAT(6(11))
// 12 FORMAT(6(11))
// 13 FORMAT(6(11))
// 14 FORMAT(6(11))
// 15 FORMAT(6(11))
// 16 FORMAT(6(11))
// 17 FORMAT(6(11))
// 18 FORMAT(6(11))
// 19 FORMAT(6(11))
// 20 FORMAT(6(11))
// 21 FORMAT(6(11))
// 22 FORMAT(6(11))
// 23 FORMAT(6(11))
// 24 FORMAT(6(11))
// 25 FORMAT(6(11))
// 26 FORMAT(6(11))
// 27 FORMAT(6(11))
// 28 FORMAT(6(11))
// 29 FORMAT(6(11))
// 30 FORMAT(6(11))
// 31 FORMAT(6(11))
// 32 FORMAT(6(11))
// 33 FORMAT(6(11))
// 34 FORMAT(6(11))
// 35 FORMAT(6(11))
// 36 FORMAT(6(11))
// 37 FORMAT(6(11))
// 38 FORMAT(6(11))
// 39 FORMAT(6(11))
// 40 FORMAT(6(11))
// 41 FORMAT(6(11))
// 42 FORMAT(6(11))
// 43 FORMAT(6(11))
// 44 FORMAT(6(11))
// 45 FORMAT(6(11))
// 46 FORMAT(6(11))
// 47 FORMAT(6(11))
// 48 FORMAT(6(11))
// 49 FORMAT(6(11))
// 50 FORMAT(6(11))
// 51 FORMAT(6(11))
// 52 FORMAT(6(11))
// 53 FORMAT(6(11))
// 54 FORMAT(6(11))
// 55 FORMAT(6(11))
// 56 FORMAT(6(11))
// 57 FORMAT(6(11))
// 58 FORMAT(6(11))
// 59 FORMAT(6(11))
// 60 FORMAT(6(11))
// 61 FORMAT(6(11))
// 62 FORMAT(6(11))
// 63 FORMAT(6(11))
// 64 FORMAT(6(11))
// 65 FORMAT(6(11))
// 66 FORMAT(6(11))
// 67 FORMAT(6(11))
// 68 FORMAT(6(11))
// 69 FORMAT(6(11))
// 70 FORMAT(6(11))
// 71 FORMAT(6(11))
// 72 FORMAT(6(11))
// 73 FORMAT(6(11))
// 74 FORMAT(6(11))
// 75 FORMAT(6(11))
// 76 FORMAT(6(11))
// 77 FORMAT(6(11))
// 78 FORMAT(6(11))
// 79 FORMAT(6(11))
// 80 FORMAT(6(11))
// 81 FORMAT(6(11))
// 82 FORMAT(6(11))
// 83 FORMAT(6(11))
// 84 FORMAT(6(11))
// 85 FORMAT(6(11))
// 86 FORMAT(6(11))
// 87 FORMAT(6(11))
// 88 FORMAT(6(11))
// 89 FORMAT(6(11))
// 90 FORMAT(6(11))
// 91 FORMAT(6(11))
// 92 FORMAT(6(11))
// 93 FORMAT(6(11))
// 94 FORMAT(6(11))
// 95 FORMAT(6(11))
// 96 FORMAT(6(11))
// 97 FORMAT(6(11))
// 98 FORMAT(6(11))
// 99 FORMAT(6(11))
// 100 FORMAT(6(11))
```
```
106

5

A ( I >= 1 .
. PR I NT 2 . R EC
P R I N T 2 - , I A ( I ) . I = 1 , M --------------------------------------------------------------------------------------CC 6 K - 1 . N
ACK>-C(K)-»A(KI
6 PIKJ=A(K)
P R I N T 2 , ( F( 1 | , I = i , M

J«UwP(2.PA(l l . f c . P A I 2 ) * l 2 . P A ( ? ) ) l
- C - A L t -0 4 T M H S T A F F S .-F-€4-rXL-,XR-'rFPST -IFNDT-l-6fl-»------------S E P = 1 ‘! C . * S l p / 3 « 1 A 1 6
PRINT2,CC,SEP,FCR
7 CONTI NUE

R-*Wb

CG 8 J = 1 1 7
R = P.+ 1 . / 6 .
CTHET A = “ 1 0 •

-CO—6—I-M7-

CTF,ETA = D T H £ T A * 1 0 .
ThCTA = 3 . 1A 1SPCTHETA. / 1 BO.
U = - ( l . » A I 1! / ( R P P 2 ) ♦ A ( Z > / < 9 P P 3 ) . A t 3 I / ( PPP4 ) 1PCCS ( THET A 1 - 2 . * I A « A ) / C
V=< l . - A I 1 ) / I R P P 2 I - 2 . P A ( 2 ) / < P 9 9 3 ) - 3 . 9 A ( 3 ) / ( R 9 P 4 ) I P S I M THFTA ) ♦ I - A ( A
J)/(R P P 21-2.P A t9l/P 'P P 3l-2.P A (S l/(9P P il)9S IN (? .9T H eT A l*< -A < 71/(R 99
— J2 -I—2-.PAF-S l-/-F F ~ -m -l - -3-.-PA-I 9 )-/-(-RP.*444-9 S44M-3-. P-TUE-7-A4-------------------------------------------veCT=SCRT(LPP2.V*P2)
PRINT2.R.D1HETA,VECT
3 CONTI NUE
-S7U4P---------------------------------------------------END
0 0 U 8 L E P R E C I S I O N F U N C T I ON F C T I X )
COUPLE PR EC IS Il'r . 2 ( 9 ) , C ( 3 ) , X , O F C . U W , C I 9 I
_CC.AiCN-R-fcX-i UN’ .C + 0 -----------------------------------------------------D ( 1 1 = 2 . PE I 1 I * f c . P 1( 2 ) + 9 . P P n I
D I 2 I = 2 • P 2 I 4 I ♦ fc • P l! I S I ♦ 9 • P 3 I > I
01 31 = 2 . 9 ? I 7 ) * 6 . PP. I 2 1 * 9 . 9 3 1 " I
F-CJ = 1 2 . 9 2 12 .M U , . 9 3|.4.)4LDCC^4-X-)4-gCCaS I X ) - C l 31 * 04 4 4 RETURN
END
S UERCUTI N E C ALTUN ( “1, A . F )
-OCUC-Lc—PRE-CIS n ,.'--A t-9i.».E-L94-,X-(-9 )-,-UV.»RFXCC*'I'GN R F C . U R . C , "
F I 11 = 1 . * C I 1 I * A I I ) * C | 2 ) PAI 2 ) *C ( 3 ) PA I 3 )
F ( 2 | = C ( A l * A ( A h C I 2 I S A ! r> ) *C ( 2 ) P A I N )
-F I -3-1 - C- < 7-1-■ A I 7 )-* C (-0-1 .a A ( -14 -. C ( 94-a A ( "1 F I A »= l . - C t I I PA I 1 I - ? . P C ( 2 ) 9 . ' I 2 1 - 3 . PC ( 3 I PA ( 3 )
F I e-» = - C I 7 I 9 '• I 7 I - 2 . P C I ■>I PA I - 1 - 3 .PC I 9 ) 9 A I 9 I
-F-I -7 I =-C-I A7-P-A I 4 ) P ( - 3 . - 1 3 .9 .9 ( I ) PA ( 1 ) - 4 P . P C I 2 I P A I 7 - l - l J W C I 3 )-* A4-34=3.<iS . 9 C I 7 | P A ( 7 I - I F . PC I 3 I PA I 2 I
5 . P C I ’ IP-A I 9 I I
$« C ( 5 I P / . I S I P I - 3 0 . P C ( 2 )PA 12 1 - 9 ' . . P C I 2 I PA I 3 ) - 9 6 . P C ( 7. 1PAI 7 I - 7 0 . P C I B )PA

110)1

H 2 . f i / U l - I - 1 2 . PC ( 2 I PA I 2 I - 4 0 . PC I 3 IP A < 3 I I
* - I A . / R f C I P I A 9 . PC I 2 I PA I 21 » I 9 -2 . P C I 3 IS A ( 3 I I


34 CONTINUE
36 IF (IC(N) .LT. 35) 35,35,35
37 CALCULATE THE INVERSE OF THE JACOBIAN AND SET THE DIRECTION MATRIX
38 CONTINUE
39 X(NC+1)=1.0
40 CONTINUE
41 CALL DINV4(INVNC,WINL+1,WINM+1)
42 START ITERATION BY PREDICTING THE DESCENT AND NECKED MINIMA
43 DPS=0
44 DSN=0
45 DFP=0
46 DPD=0
47 D=35.1=1.0
48 CONTINUE
49 CALL DINV4(INVNC,WINL+1,WINM+1)
50 START ITERATION BY PREDICTING THE DESCENT AND NECKED MINIMA
51 CONTINUE
52 IF (IC(N) .LT. 35) 35,35,35
53 CALCULATE THE INVERSE OF THE JACOBIAN AND SET THE DIRECTION MATRIX
54 CONTINUE
55 X(NC+1)=1.0
56 CONTINUE
57 CALL DINV4(INVNC,WINL+1,WINM+1)
58 START ITERATION BY PREDICTING THE DESCENT AND NECKED MINIMA
59 CONTINUE
60 IF (IC(N) .LT. 35) 35,35,35
61 CALCULATE THE INVERSE OF THE JACOBIAN AND SET THE DIRECTION MATRIX
62 CONTINUE
63 X(NC+1)=1.0
64 CONTINUE
65 CALL DINV4(INVNC,WINL+1,WINM+1)
66 START ITERATION BY PREDICTING THE DESCENT AND NECKED MINIMA
67 CONTINUE
68 IF (IC(N) .LT. 35) 35,35,35
69 CALCULATE THE INVERSE OF THE JACOBIAN AND SET THE DIRECTION MATRIX
70 CONTINUE
71 X(NC+1)=1.0
72 CONTINUE
73 CALL DINV4(INVNC,WINL+1,WINM+1)
74 START ITERATION BY PREDICTING THE DESCENT AND NECKED MINIMA
75 CONTINUE
76 IF (IC(N) .LT. 35) 35,35,35
77 CALCULATE THE INVERSE OF THE JACOBIAN AND SET THE DIRECTION MATRIX
78 CONTINUE
79 X(NC+1)=1.0
80 CONTINUE
81 CALL DINV4(INVNC,WINL+1,WINM+1)
82 START ITERATION BY PREDICTING THE DESCENT AND NECKED MINIMA
83 CONTINUE
84 IF (IC(N) .LT. 35) 35,35,35
85 CALCULATE THE INVERSE OF THE JACOBIAN AND SET THE DIRECTION MATRIX
86 CONTINUE
87 X(NC+1)=1.0
88 CONTINUE
89 CALL DINV4(INVNC,WINL+1,WINM+1)
90 START ITERATION BY PREDICTING THE DESCENT AND NECKED MINIMA
91 CONTINUE
92 IF (IC(N) .LT. 35) 35,35,35
93 CALCULATE THE INVERSE OF THE JACOBIAN AND SET THE DIRECTION MATRIX
94 CONTINUE
95 X(NC+1)=1.0
96 CONTINUE
97 CALL DINV4(INVNC,WINL+1,WINM+1)
98 START ITERATION BY PREDICTING THE DESCENT AND NECKED MINIMA
99 CONTINUE
100 IF (IC(N) .LT. 35) 35,35,35
101 CALCULATE THE INVERSE OF THE JACOBIAN AND SET THE DIRECTION MATRIX
102 CONTINUE
103 X(NC+1)=1.0
104 CONTINUE
105 CALL DINV4(INVNC,WINL+1,WINM+1)
106 START ITERATION BY PREDICTING THE DESCENT AND NECKED MINIMA
107 CONTINUE
108 IF (IC(N) .LT. 35) 35,35,35
109 CALCULATE THE INVERSE OF THE JACOBIAN AND SET THE DIRECTION MATRIX
DO 10 T=1,N
GO TO 59
       IF (T.EQ.1) GO TO 60
       IF (T.EQ.N) GO TO 61

       X(1)=0
       GO TO 59

60  X(1)=C
61  X(N)=1.
62  CONTINUE
       WIND=1

       REDUCE THE DIRECTIONS SO THAT KK IS FIRST
63  IF (KK.EQ.M+1) GO TO 71
       K=KK
       DO 73 J=2,KK
          W(J)=W(K-J)
          K=K-J
       73  CONTINUE
       C
       CONTINUE

       GENERATE THE NEW ORTHOGONAL DIRECTION MATRIX
64  DO 74 I=1,N
          H(I,T)=0.
       74  CONTINUE
       SP=X(1)*X(1)
       DO 75 I=2,N
          G=0.5*(X(I)*SP+X(I)*X(I))
          DT=SP/G
          SP=SP-DT*G
          DS=DS+G
          DO 76 J=1,N
             K=K-J
             H(I,J+1)=X(I)*H(J+1,K)
             H(J+1,I)=H(I,J+1)
       76  CONTINUE
       C
       CONTINUE

       CALCULATE THE NEXT VECTOR X, AND PREDICT THE RIGHT HAND SIDES
65  IF (T.EQ.1) GO TO 77
       W(1,T)=0.
       DO 78 J=2,N
          W(J,T)=W(J-1,T)+F(J)
       78  CONTINUE

       CONTINUE
CALL CALFUN USING THE NEW VECTOR CF VARIABLES

UPDATE THE STEP SIZE

- 27.DMULT = 0.9*FMIN + 0.1*FNAP = FSC
- 82.GS = MAX(WSS + 0.25*CD)

IF (IOPULT) = 22, 23, 24

TRY THE TEST TO DECIDE WHETHER TO INCREASE THE STEP LENGTH

- 81 SPQ = 0.
- 84.1 = 1

IF IFIX) IMPROVES STORE THE NEW VALUE OF X

- 87 IF IFIX = FMIN + 0.2*SPQ
- 83 FMIN = FSC

- DC 84.1 = 1

CALCULATE THE CHANGES IN F AND IN X

- 89 CONTINUE

UPDATE THE APPROXIMATION TO J AND TO AJINV

- 92 CONTINUE
```
50  IK=IK+1
   A(IK)=A(IK)/BIGA
55  CONTINUE
   DO 65  I=1,N
   IK=IK+1
   HOLD=A(IK)
   GO TO 55
   IF(I-K) 60,65,60
60  IF(I-J)<.5,155,62
   IF(I-J)<.5,65,65
   A(IJ)=HOLD+A(KJ)+A(IJ)
   CONTINUE
   KJ=K-N
   DO 75  J=1,N
   KJ=KJ+N
   IF(I-J)<.75,170,70
   A(IJ)=A(IJ)/BIGA
   CONTINUE
   A(KK)=1/BIGA
80  CONTINUE
   K=N
100  K=K-1
105  I=IK
   IF(I-K)<.150,150,105
108  J0=N0(I-K-1)
   JR=J0(I-1)
   GO TO 105
   J0=J0+1
   HOLD=A(JK)
   JI=J0+J
   A(IJKI)=A(IJ)
110  A(J0)=HOLD
120  J=IK
   IF(J-K)<.100,100,125
125  KJ=K-N
   IF(K-J)<.100,125
   KI=KI+1
   HOLD=A(KJ)
   JI=KI+J
   A(KJ)=A(IJ)
130  A(IJ)=HOLD
   GO TO 120
150  RETURN
END
```

LISTING B-3

Program for Checking the Results of Program B-1

```
// EXEC
PROG=PORTIXXX
// TIME=CMPP=MAP
TIME=GC=(1,0)

** THIS PROGRAM CALCULATES THE FOLLOWING QUANTITIES **
1) THE CONSTANTS OF THE TRIAL STREAMFUNCTION
2) THE NPHS COEFFICIENT
3) THE ANGLE AT WHICH SEPARATION OCCURS

** THE FOLLOWING CARG IS THE ONLY INPUT FOR THIS PROGRAM **
REC=4

** THE ABOVE CARG IS THE ONLY INPUT FOR THIS PROGRAM **
REC=0.
DO 4 K=1,REC
XGOF(1)=674/452-9430/10C702)
XGOF(2)=1/177.32C701/10C702)
XGOF(3)=2.93/21.69.
CALL PCLRT(XGOF,CCF,RCST,RECT,IER)
4 CONTINUE
R = 5 ./6.
BEGIN=1
PRINT3,REC
R = P.« 1 / L.
CTHETA=-1.
C5 = 1./125.
SEP=1CCS((141)6,A151)15.,A6(4)+2R,A6(2))1/6.*A121+2C,«A141)+42.0
SEP=RE/3.03C00)+141593
PRINT3,REC,SEP
CONTINUE
R=5./6.
BEGIN=1.
PRINT3,REC
R = P.« 1 / L.
CTHETA=-1.
C5 = 1./125.
SEP=1CCS((141)6,A151)15.,A6(4)+2R,A6(2))1/6.*A121+2C,«A141)+42.0
SEP=RE/3.03C00)+141593
PRINT3,REC,SEP
CONTINUE
R=5./6.
BEGIN=1.
PRINT3,REC
R = P.« 1 / L.
CTHETA=-1.
C5 = 1./125.
SEP=1CCS((141)6,A151)15.,A6(4)+2R,A6(2))1/6.*A121+2C,«A141)+42.0
SEP=RE/3.03C00)+141593
```

PRINT3, R, DT=STA, VECT
S CONTINUE
END
//GG $SYSLIB DD DSM=$SYS2.FCGTLIB, DISP=SHR
//GG $SYSSIN DD DSM=$SYS2.FCGTLIB, DISP=SHR
//
LISTING B-4

Program for the Calculation of the Coordinates of the Streamlines
(Galerkin Solution 1)

```
/ /CLASS=A
/ /TIME=CM=11
/ /TIME=GO=11.1
/ /COMP=SYIN
DO
THIS PROGRAM CALCULATES THE COORDINATES OF THE STREAMLINES
COMPAR A(2),SIGMA,THETA
C
EXTERNAL FCNT
THE FOLLOWING 9 CARDS ARE THE ONLY INPUT FOR THIS PROGRAM
WOM=1983
A(1)=3.346446
A(2)=1.063165
A(3)=.646406
A(4)=2.666280
A(5)=3.255039
A(6)=2.547225
A(7)=.631742
A(8)=.799639
A(9)=.865743
THE ABOVE 9 CARDS ARE THE ONLY INPUT FOR THIS PROGRAM
EPS=.0.5
END=100
X=1.
A=10.
STL=4.
CC 3 =E=1.16
DTHETA=-5.
D THETA=0THETA+5.
CALL FCNT(X,FCT,SIGMA,THETA,EPS,END)
PRINT1.IER
PRINT2.F.DTHETA.SIGMA.FER
STOP
END

FUNCT CN
COMPAR A(2),SIGMA,THETA
FCT = SIGMA*SIGMA/(X**2)*A(2)/(X**4) . . + A (6)/(X**6)*A(9)/(X**9)
RETURN
END

/GO=SYSTEC
DC DSK=SYSTEC,FCT=1L1,DISP=SHR
/GO=SYSTEC
DC DSK=SYSTEC,FCT=1L1,DISP=SHR
/GO=SYSTEC
DC DSK=SYSTEC,FCT=1L1,DISP=SHR
/GO=SYSTEC
DC DSK=SYSTEC,FCT=1L1,DISP=SHR
/GO=SYSTEC
DC DSK=SYSTEC,FCT=1L1,DISP=SHR
```
LISTING B-5

Program for the Calculation of the Coordinates of the Streamlines for the Special Case of \( u_w/U_\infty = 0.0 \)
(Galerkin Solution 1)

```
// A0C0,CLASS=A
//FTCM
6
_/ /
---_ ----------
P - A  p
. c y y a
p
. . — —--------------------------------------
/> / TIME.CCM l.SA)
//CMP.SYSIN D C = f
THIS PROGRAM CALCULATES THE COORDINATES OF THE STREAMLINES FOR THE SPECIAL CASE OF UWC.
EXTERNAL FCT
THE FOLLOWING 2 CARDS ARE THE ONLY INPUT FOR THIS PROGRAM
A(1)=2-741663
A(2)=34.2260
A(3)=-16.9273
A(4)=9.83465
A(6)=656.5500
A(7)=2.37402
A(8)=-17.6122
THE ABOVE 2 CARDS ARE THE ONLY INPUT FOR THIS PROGRAM
UWC=C
EPS=l.E-2
ENC=100
X=1
XL=2.
XR=3.
S.I=-55.
S2=1.121
S3=5.
CO 3=1.137
THETA=CTHETA*5.
CALL R1(FE.E,FC-.ET,XL,XR,CPS,IEP,IFR)
PRINT 1,FP
1 FP=010(E20.7)
CONTINUE
2 FP=010(E20.7)
3 CONTINUE
XL=2.
XR=3.
S1=-55.
S2=1.121
S3=5.
CO 3=1.137
THETA=CTHETA*5.
THETA=01752232*THETA
PRINT 1,EP
4 EP=010(E20.7)
CONTINUE
STOP
END
FUNCTION FCT(X)
COMMON A(0),S1,THETA,UN
S1=X**2
RETURN
END
```
LISTING B-6

Program for the Calculation of the Coordinates of the Streamlines (Galerkin Solution 3)

```
// 4000 CLASS=A
// EXEC  PREC=FCRTRUN
// TIME=CPU=109:00:01
// XP=CPU=MPAD1
// CMP,SYSIN,DC=0
C THIS PROGRAM CALCULATES THE COORDINATES OF THE STREAMLINES
COMMENT A(91),GI,THETA,AM
EXTERNAL FCT
C THE FOLLOWING 10 CARDS ARE THE ONLY INPUT FOR THIS PROGRAM
UN=1003
A(1)=4.4997
A(2)=4.9774
A(3)=1.63E-5
A(4)=1.631E-5
A(5)=1.631E-5
A(6)=1.631E-5
A(7)=1.631E-5
A(8)=1.631E-5
A(9)=1.631E-5

C THE ABOVE 10 CARDS ARE THE ONLY INPUT FOR THIS PROGRAM
EPS=1.E-5
IEND=100
NL=15
XR=10.
SL=6
DO 1=1,16
   SIGMA=3.1415/180.
   CTHERA=CTHERA+SIGMA
   CALL RTPI(R,FER,FCT,XL,XR,EP,R,IEND,IERI)
   PRINT(1)
   1 FORMAT(5E20.7)
   CONTINUE
STOP
END
FUNCTION FCT(X)
COMPO N A(91),SIGMA,THETA,AM
FCT=SIGMA(X+A(1)/X*A(2)/X*X+2+X(2)/(X*X)+A(3)/(X*X)+A(4)/(X**2)+A(5)/(X*X)+A(6)/(X*X)+A(7)/(X*X)+A(8)/(X*X)+A(9)/(X*X))
RETURN
END
```

// GO.SYS1F DC DSN=SYS1,FCT,ISSP,TISP=SHR
// GO.SYS1 DC DSN=SYS1,FCT,TILIE,TISP=SHR
//
TABLE B-1

Constants of the Trial Stream Function
(Galerkin Solution 1), $\frac{u}{u_\infty} = 0.0$

<table>
<thead>
<tr>
<th>Re</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
<th>$\alpha_4$</th>
<th>$\alpha_5$</th>
<th>$\alpha_6$</th>
<th>$\alpha_7$</th>
<th>$\alpha_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>-2.4545</td>
<td>1.3496</td>
<td>1.9999</td>
<td>-3.3739</td>
<td>-0.6363</td>
<td>2.6991</td>
<td>0.0909</td>
<td>-0.6748</td>
</tr>
<tr>
<td>200</td>
<td>-1.1155</td>
<td>4.7680</td>
<td>-1.2733</td>
<td>-11.920</td>
<td>1.8930</td>
<td>9.5360</td>
<td>-0.5042</td>
<td>-2.3840</td>
</tr>
<tr>
<td>300</td>
<td>0.9230</td>
<td>14.932</td>
<td>-6.2561</td>
<td>-37.330</td>
<td>5.7434</td>
<td>29.864</td>
<td>-1.4102</td>
<td>-7.4660</td>
</tr>
<tr>
<td>400</td>
<td>1.7794</td>
<td>25.735</td>
<td>-8.3497</td>
<td>-64.338</td>
<td>7.3612</td>
<td>51.470</td>
<td>-1.7909</td>
<td>-12.868</td>
</tr>
<tr>
<td>500</td>
<td>2.1512</td>
<td>35.668</td>
<td>-9.2585</td>
<td>-89.169</td>
<td>8.0634</td>
<td>71.335</td>
<td>-1.9561</td>
<td>-17.834</td>
</tr>
<tr>
<td>600</td>
<td>2.3458</td>
<td>45.096</td>
<td>-9.7341</td>
<td>-112.74</td>
<td>8.4309</td>
<td>90.193</td>
<td>-2.0426</td>
<td>-22.548</td>
</tr>
<tr>
<td>700</td>
<td>2.4606</td>
<td>54.230</td>
<td>-10.015</td>
<td>-135.57</td>
<td>8.6479</td>
<td>108.46</td>
<td>-2.0936</td>
<td>-27.115</td>
</tr>
<tr>
<td>800</td>
<td>2.5342</td>
<td>63.177</td>
<td>-10.195</td>
<td>-157.94</td>
<td>8.7868</td>
<td>126.35</td>
<td>-2.1263</td>
<td>-31.588</td>
</tr>
<tr>
<td>900</td>
<td>2.5842</td>
<td>72.000</td>
<td>-10.317</td>
<td>-180.00</td>
<td>8.8812</td>
<td>144.00</td>
<td>-2.1485</td>
<td>-36.000</td>
</tr>
<tr>
<td>1000</td>
<td>2.6197</td>
<td>80.735</td>
<td>-10.404</td>
<td>-201.84</td>
<td>8.9484</td>
<td>161.47</td>
<td>-2.1643</td>
<td>-40.368</td>
</tr>
<tr>
<td>1100</td>
<td>2.7322</td>
<td>166.17</td>
<td>-10.679</td>
<td>-415.42</td>
<td>9.1607</td>
<td>332.34</td>
<td>-2.2143</td>
<td>-83.084</td>
</tr>
<tr>
<td>1300</td>
<td>2.7600</td>
<td>334.62</td>
<td>-10.747</td>
<td>-836.54</td>
<td>9.2133</td>
<td>669.23</td>
<td>-2.2267</td>
<td>-167.31</td>
</tr>
<tr>
<td>1400</td>
<td>2.7604</td>
<td>343.03</td>
<td>-10.748</td>
<td>-857.57</td>
<td>9.2141</td>
<td>686.05</td>
<td>-2.2269</td>
<td>-171.51</td>
</tr>
<tr>
<td>1500</td>
<td>2.7633</td>
<td>418.57</td>
<td>-10.755</td>
<td>-1046.4</td>
<td>9.2196</td>
<td>837.15</td>
<td>-2.2281</td>
<td>-209.29</td>
</tr>
<tr>
<td>1600</td>
<td>2.7651</td>
<td>502.49</td>
<td>-10.759</td>
<td>-1256.2</td>
<td>9.2230</td>
<td>1005.0</td>
<td>-2.2289</td>
<td>-251.25</td>
</tr>
<tr>
<td>1700</td>
<td>2.7654</td>
<td>519.32</td>
<td>-10.760</td>
<td>-1298.3</td>
<td>9.2235</td>
<td>1038.6</td>
<td>-2.2291</td>
<td>-259.66</td>
</tr>
</tbody>
</table>
Rotameters are flow measuring devices designed to give a linear relationship between scale reading and actual flow. The calibration curves for the upstream and downstream rotameters are shown in Figures C-1 and C-2 respectively. These curves show the volume flow rate, at standard temperature and pressure (70.0 °F and 14.70 psia), as a function of the flowmeter scale reading. The following correction must be applied for flows at conditions different than those of calibration

\[ \hat{Q} = \hat{Q}_c \left( \frac{\hat{T}}{T_c} \frac{\hat{P}}{P_c} \right)^{\frac{1}{2}} \]  

(C-1)

where \( \hat{T}_c \) and \( \hat{P}_c \) are the temperature and pressure at calibration (for this case S.T.P.), \( \hat{T} \) and \( \hat{P} \) the temperature and pressure of the actual flow, \( \hat{Q}_c \) the volume flow rate from the calibration curve for the actual scale reading,

\[ \text{For further information, consult the manufacturer's literature.} \]
and $\hat{Q}$ is the actual volume flow rate. Figure C-3 shows the rotameters used in the study. Item A is the down-stream meter and item B is the upstream meter.
FIGURE C-1

Calibration Curve for the Upstream Rotameter
Calibration Curve for the Downstream Rotameter

FISCHER-PORTER ROTAMETER
FLOAT: SVT-64
MATER.: 316 S.S.
TUBE: FP-1-35-G-10/80
MAX.: S = 100, \( \hat{Q}_c = 27.0 \) SCFM
FIGURE C-3

Downstream and Upstream Rotameters
(Items A and B Respectively)
The external orthogonality relations for the first Galerkin solution (equations (39) and (40)) were given as

\[
\int_{0}^{2\pi} \left[ \frac{\partial \psi^*}{\partial \theta} + \frac{\partial \psi^*}{\partial r} + \frac{u_w}{U_\infty} \right] \sin \theta \, d\theta = 0 \quad \text{(D-1)}
\]

and

\[
\int_{0}^{2\pi} \left[ \frac{\partial \psi^*}{\partial \theta} + \frac{\partial \psi^*}{\partial r} + \frac{u_w}{U_\infty} \right] \sin 2\theta \, d\theta = 0 \quad \text{(D-2)}
\]

These equations have used the weighting functions (see equations (35) and (36))

\[
w_1(1, \theta) = \sin \theta \quad \text{(D-3)}
\]

and
\[ w_2(l, \theta) = \sin 2\theta \quad \text{(D-4)} \]

Use of the higher order weighting functions, such as (see equations (37) and (38))

\[ w_3(l, \theta) = \sin \theta \quad \text{(D-5)} \]

and

\[ w_4(l, \theta) = \sin 2\theta \quad \text{(D-6)} \]

fail to give any independent equations.

If equation (30) is substituted into equation (D-1) and subsequently integrated, the result is

\[ 1 - a_1 - 3a_3 - 5a_5 - 7a_7 = 0 \quad \text{(D-7)} \]

Similarly, substituting equation (30) into equation (D-2) gives

\[ a_2 + 2a_4 + 3a_6 + 4a_8 = 0 \quad \text{(D-8)} \]

Equations (D-7) and (D-8) can also be derived by forcing equation (30) to satisfy the boundary condition
Because only two independent external orthogonality relations can be found and since these relations, together with equations (31) - (34), are not sufficient to solve for the eight undetermined constants in the trial stream function, equation (41) must be added on "physical grounds."
APPENDIX E
OSEEN'S APPROXIMATION

In Chapter III, the equations governing the external flow field were given as (see equations (10) and (12))

\[ u \frac{\partial u}{\partial r} + \frac{v}{r} \frac{\partial r}{\partial \theta} - \frac{v^2}{r} = - \frac{\partial P}{\partial r} \]  
\[ + \frac{2}{\text{Re}} \left[ \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{1}{2} \frac{\partial^2 u}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial v}{\partial \theta} \right] \]  
\[ u \frac{\partial v}{\partial r} + \frac{v}{r} \frac{\partial v}{\partial \theta} + uv = \frac{1}{r} \frac{\partial P}{\partial \theta} \]  
\[ + \frac{2}{\text{Re}} \left[ \frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2} + \frac{1}{2} \frac{\partial^2 v}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial u}{\partial \theta} \right] \]

and

\[ \frac{\partial u}{\partial r} + \frac{u}{r} + \frac{1}{r} \frac{\partial v}{\partial \theta} = 0 \]  

Oseen's approximation (31) linearizes equations (E-1) and (E-2) such that they simplify to

\[ - \cos \theta \frac{\partial u}{\partial r} + \frac{\sin \theta}{r} \frac{\partial u}{\partial \theta} - \frac{v \sin \theta}{r} = - \frac{\partial P}{\partial r} \]
This linearization technique takes the inertia effects into account in the region where they are comparable to the viscous forces, but neglects them in the region near the wall where the viscous forces dominate (34).

As in Chapter III, equations (E-4) and (E-5) can be contracted to a single differential equation by implementing the stream function defined in equations (14) and (15). The resulting equation is linear and can be solved by the method of separation of variables (27, 34). A simplified solution is given by Batchelor (6) for small Reynolds numbers as

\[ \psi = \frac{1}{2 \ln(7.4/Re)} [2 \ln r - r + r^{-1}] \sin \theta \quad , \quad (E-6) \]

where the absolute error in the magnitude of the vector velocity is of the order of \( Re \).
Equation (E-6) and equations (14) and (15) were used to generate the curves depicting the Oseen approximation in Figure E-1 for Re = 0.1, 2.0, 3.0, 4.0, and 4.4 at \( \hat{r}/R = 1.17 \). Also shown in Figure E-1 is the theoretical velocity profile as predicted by the third Galerkin solution for \( u_w/U_\infty = 0.0 \), Re = 0.1, and \( \hat{r}/R = 1.17 \). Only one curve is shown in Figure E-1 for the third Galerkin solution, owing to the fact that it exhibits a weak dependence of the velocity profile on the Reynolds number for \( u_w/U_\infty = 0.0 \). Figure E-2 further illustrates this fact. In Figure E-2 the magnitude of the vector velocity, as predicted by the third Galerkin solution, is plotted as a function of \( \theta \) and Re for \( u_w/U_\infty = 0.0 \) and \( \hat{r}/R = 1.17 \).

In general, the quantitative agreement between the third Galerkin solution and the Oseen approximation is poor. Only at Re = 4.4 do the solutions match, and there the solution given by equation (E-6) is in error by an amount of the order of Re. The general form of the two solutions are, however, qualitatively the same, in that both solutions appear to be sinusoidal in nature.

The generally poor agreement between the Oseen approximation and the third Galerkin solution tends to discount the value of the Galerkin solution at low Reynolds numbers, for the case of no blowing. This disagreement, however, does not influence the conclusions drawn in Chapters III
and IV, for the practically important cases at higher Reynolds numbers and with blowing. For such cases, the approximate Galerkin theory was compared with experiments and found to be adequate.
Comparison of Galerkin Solution 3 (Re = 0.1) and the Oseen Approximation, $u_w/U_\infty = 0.0$ and $\hat{\xi}/R = 1.17$.
FIGURE E-2

Theoretical Velocity Profile
(Galerkin Solution 3),
\( \frac{u_w}{U_\infty} = 0.0 \) and \( \hat{r}/R = 1.17 \)
FIGURE 1-A
Front View of the Apparatus

FIGURE 1-B
Rear View of the Apparatus
FIGURE 2

Mounted Porous Pipe in the Test Section of the Wind Tunnel
FIGURE 3
Schematic of the Flow System
FIGURE 4

Coordinate System
FIGURE 5
Hot Wire Anemometer

FIGURE 6
Micromanometer
FIGURE 7

Traverse Mechanism
FIGURE 8

Reference Pitot Tube
Velocity Profile at the Inlet of the Porous Pipe, $T = 78.0 \, ^\circ F$, $P = 10 \, PSIG$, and $Re' = 23000$
FIGURE 10

Hole Drilled in the Porous Pipe for the Insertion of the Plastic Bushing

\[ D_1 = 1.218 \text{ IN. (3.093 CM)} \]
\[ D_2 = 1.750 \text{ IN. (4.445 CM)} \]
\[ D_3 = 0.125 \text{ IN. (0.318 CM)} \]
FIGURE 11

Static Pressure Tap
FIGURE 12
Pressure Transducer

FIGURE 13
Analog and Digital Readout for the Pressure Transducer
FIGURE 14
Pressure Distribution on the Surface,
$u_{w}/U_{\infty} = 0.102$ and $Re = 6200$
Pressure Coefficient on the Surface, 
\( u_w/U_\infty = 0.0 \) and \( \text{Re} = 5000 \)
Streamlines (Galerkin Solution 1),
$u_w/U_\infty = 0.190$ and $Re = 4100$

FIGURE 16
FIGURE 17

Streamlines (Galerkin Solution 1),

\[ \frac{u_w}{U_\infty} = 0.154 \text{ and } Re = 4100 \]
Streamlines (Galerkin Solution 1),
\( u_w/U_\infty = 0.126 \) and \( \text{Re} = 6200 \)
FIGURE 19

Streamlines (Galerkin Solution 1),
\[ \frac{u_w}{U_\infty} = 0.102 \text{ and } Re = 6200 \]
Streamlines (Galerkin Solution 1),
\( u_w/U_\infty = 0.0 \) and \( Re = 4100 \)
FIGURE 21

Streamlines (Galerkin Solution 1),

$u_w/U_\infty = 0.0$ and $Re = 6200$
Theoretical Velocity Profile (Galerkin Solution 1), $u_w/U_\infty = 0.190$, Re = 4100, and $r/R = 1.17$
Theoretical Velocity Profile (Galerkin Solution 4), \( \frac{u_w}{U_\infty} = 0.190 \), Re = 4100, and \( \hat{r}/R = 1.33 \)
Theoretical Velocity Profile (Galerkin Solution 1), $\frac{u'_w}{U_\infty} = 0.190$, $Re = 4100$, and $\hat{r}/R = 1.50$
Theoretical Velocity Profile (Galerkin Solution 1), \( u_w/U_\infty = 0.190 \), \( Re = 4100 \), and \( \hat{r}/R = 1.67 \)
Theoretical Velocity Profile (Galerkin Solution 1), $u_w/U_\infty = 0.190$, $Re = 4100$, and $x/R = 1.83$
Theoretical Velocity Profile (Galerkin Solution 1), $u_w/U_\infty = 0.190$, $Re = 4100$, and $\hat{r}/R = 2.00$
Theoretical Velocity Profile (Galerkin Solution 1), $u_{\theta}/U_\infty = 0.154$, $Re = 4100$, and $\hat{r}/R = 1.17$
Theoretical Velocity Profile (Galerkin Solution 1), $u_W/U_\infty = 0.154$, $Re = 4100$, and $\hat{f}/R = 1.33$
Theoretical Velocity Profile (Galerkin Solution 1), $u_w/U_\infty = 0.154$, Re = 4100, and $\hat{\gamma}/R = 1.50$
FIGURE 31

Theoretical Velocity Profile (Galerkin Solution 1), $u_w/U_\infty = 0.154$, $Re = 4100$, and $r/R = 1.67$
Theoretical Velocity Profile (Galerkin Solution 1), $u_w/U_\infty = 0.154$, $Re = 4100$, and $\hat{r}/R = 1.83$.
Theoretical Velocity Profile (Galerkin Solution 1), $u_w/U_\infty = 0.154$, $Re = 4100$, and $\hat{r}/R = 2.00$
Theoretical Velocity Profile (Galerkin Solution 1), $u_\infty/U_\infty = 0.126$, $Re = 6200$, and $r/R = 1.17$
Theoretical Velocity Profile (Galerkin Solution 1), $u_{w}/U_{\infty} = 0.126$, $Re = 6200$, and $d/R = 1.33$
Theoretical Velocity Profile (Galerkin Solution 1),
\( \frac{u_w}{U_\infty} = 0.126, \quad \text{Re} = 6200, \quad \text{and} \quad r/R = 1.50 \)
Theoretical Velocity Profile (Galerkin Solution 1), $u_w/U_\infty = 0.126$, Re = 6200, and $\hat{r}/R = 1.67$
Theoretical Velocity Profile (Galerkin Solution 1), $u_W/U_\infty = 0.126$, $Re = 6200$, and $\hat{r}/R = 1.83$
Theoretical Velocity Profile (Galerkin Solution 1), \( \frac{u_w}{U_\infty} = 0.126 \), \( Re = 6200 \), and \( \hat{x}/R = 2.00 \)
Theoretical Velocity Profile (Galerkin Solution 1), $u_w/U_\infty = 0.102$, $Re = 6200$, and $\hat{r}/R = 1.17$
Theoretical Velocity Profile, (Galerkin Solution 1), $u_w/U_\infty = 0.102$, $Re = 6200$, and $\hat{z}/R = 1.33$
Theoretical Velocity Profile (Galerkin Solution 1), $u_w/U_\infty = 0.102$, $Re = 6200$, and $\hat{r}/R = 1.50$
Figure 43

Theoretical Velocity Profile (Galerkin Solution 1), $u_w/U_\infty = 0.102$, $Re = 6200$, and $\hat{r}/R = 1.67$
Theoretical Velocity Profile (Galerkin Solution 1), $u_W/U_\infty = 0.102$, $Re = 6200$, and $\hat{r}/R = 1.83$
Theoretical Velocity Profile (Galerkin Solution 1), $u_0^2/U_\infty = 0.102$, $Re = 6200$, and $\hat{r}/R = 2.00$
Streamlines (Galerkin Solution 3),
$u_w/U_\infty = 0.190$ and $Re = 4100$
FIGURE 47

Streamlines (Galerkin Solution 3),
\( u_w/U_\infty = 0.154 \) and \( \text{Re} = 4100 \)
FIGURE 48
Streamlines (Galerkin Solution 3), $u_w/U_\infty = 0.126$ and $Re = 6200$
FIGURE 49

Streamlines (Galerkin Solution 3),
\( u_w/U_\infty = 0.102 \) and \( Re = 6200 \).
FIGURE 50

Streamlines (Galerkin Solution 3), $u_w/U_\infty = 0.0$ and Re = 4100
FIGURE 51

Streamlines (Galerkin Solution 3),

\[ u_w/U_\infty = 0.0 \text{ and } Re = 6200 \]
Theoretical Velocity Profile (Galerkin Solution 3), $u_w/U_\infty = 0.190$, Re = 4100, and $\hat{\rho}/R = 1.17$
Theoretical Velocity Profile (Galerkin Solution 3), 
$u_{w}/U_\infty = 0.190$, $Re = 4100$, and $F/R = 1.33$
The theoretical velocity profile (Galerkin Solution 3), $u_w/U_\infty = 0.190$, $Re = 4100$, and $F/R = 1.50$.
Theoretical Velocity Profile (Galerkin Solution 3), $u_w/U_\infty = 0.190$, $Re = 4100$, and $\ell/R = 1.67$
Figure 56

Theoretical Velocity Profile (Galerkin Solution 3), $u_w/U_w = 0.190$, $Re = 4100$, and $r/R = 1.83$
Theoretical Velocity Profile (Galerkin Solution 3), $u_w/U_\infty = 0.190$, $Re = 4100$, and $\vec{r}/R = 2.00$
Theoretical Velocity Profile (Galerkin Solution 3), $u_w/U_\infty = 0.154$, $Re = 4100$, and $\hat{r}/R = 1.17$.

**Figure 58**

- INVISCID THEORY
- APPROXIMATE THEORY
- EXPERIMENT
Figure 59

Theoretical Velocity Profile (Galerkin Solution 3), 
\( u_w/U_\infty = 0.154 \), \( Re = 4100 \), and \( r/R = 1.33 \)
Theoretical Velocity Profile (Galerkin Solution 3), $u_w/U_\infty = 0.154$, Re = 4100, and $\hat{x}/R = 1.50$. 

**Figure 60**

**Theoretical Velocity Profile (Galerkin Solution 3), $u_w/U_\infty = 0.154$, Re = 4100, and $\hat{x}/R = 1.50$.**
Figure 61

Theoretical Velocity Profile (Galerkin Solution 3),
\( u_w/U_\infty = 0.154, \text{ Re } = 4100, \text{ and } \frac{\tilde{z}}{R} = 1.67 \)
Theoretical Velocity Profile (Galerkin Solution 3), \( \frac{u_w}{U_\infty} = 0.154 \), \( Re = 4100 \), and \( \frac{\theta}{R} = 1.83 \).
Theoretical Velocity Profile (Galerkin Solution 3), \( u_w/U_\infty = 0.154 \), \( \text{Re} = 4100 \), and \( \hat{z}/R = 2.00 \)
FIGURE 64

Theoretical Velocity Profile (Galerkin Solution 3), $u_w/U_\infty = 0.126$, $Re = 6200$, and $E/R = 1.17$
Theoretical Velocity Profile (Galerkin Solution 3), $u_w/U_\infty = 0.126$, $Re = 6200$, and $\hat{r}/R = 1.33$
Theoretical Velocity Profile (Galerkin Solution 3), $u_w/U_\infty = 0.126$, $Re = 6200$, and $\hat{r}/R = 1.50$
Theoretical Velocity Profile (Galerkin Solution 3), $u_w/U_\infty = 0.126$, Re = 6200, and $R/R = 1.67$.
Theoretical Velocity Profile (Galerkin Solution 3), \( u_w/U_\infty = 0.126 \), \( \text{Re} = 6200 \), and \( \hat{r}/R = 1.83 \).
Theoretical Velocity Profile (Galerkin Solution 3), \( \frac{u_w}{U_\infty} = 0.126, \) \( Re = .6200, \) and \( \hat{r}/R = 2.00 \)
Theoretical Velocity Profile (Galerkin Solution 3), $u_w/U_\infty = 0.102$, $Re = 6200$, and $\tilde{r}/R = 1.17$
Theoretical Velocity Profile (Galerkin Solution 3), \( \frac{u_w}{U_\infty} = 0.102 \), Re = 6200, and \( \hat{r}/R = 1.33 \)
Theoretical Velocity Profile (Galerkin Solution 3), $u_w/U_\infty = 0.102$, $Re = 6200$, and $\hat{\varphi}/R = 1.50$
Theoretical Velocity Profile (Galerkin Solution 3),
$u_w/U_\infty = 0.102$, Re = 6200, and $\hat{f}/R = 1.67$
Theoretical Velocity Profile (Galerkin Solution 3), $u_w/U_\infty = 0.102$, $Re = 6200$, and $f/R = 1.83$
Theoretical Velocity Profile (Galerkin Solution 3), $u_w/U_\infty = 0.102$, $Re = 6200$, and $r/R = 2.00$
Figure 76

Theoretical Velocity Profile (Galerkin Solution 3), $u_w/U_\infty = 0.0$, $Re = 4100$, and $\hat{x}/R = 1.17$
Theoretical Velocity Profile (Galerkin Solution 3), $u_w/U_\infty = 0.0$, $Re = 4100$, and $\hat{r}/R = 1.33$
Theoretical Velocity Profile (Galerkin Solution 3), \( \frac{u_w}{U_\infty} = 0.0 \), \( \text{Re} = 4100 \), and \( \hat{r}/R = 1.50 \)
Theoretical Velocity Profile (Galerkin Solution 3),
$u_w/U_\infty = 0.0$, $Re = 4100$, and $\hat{r}/R = 1.67$
Theoretical Velocity Profile (Galerkin Solution 3), $u_w/U_\infty = 0.0$, Re = 4100, and $\hat{z}/R = 1.83$
Theoretical Velocity Profile (Galerkin Solution 3), $u_w/U_\infty = 0.0$, $Re = 4100$, and $r/R = 2.00$
Theoretical Velocity Profile (Galerkin Solution 3), $u_w/U_{\infty} = 0.0$, $Re = 6200$, and $\hat{r}/R = 1.17$
Theoretical Velocity Profile (Galerkin Solution 3), \( u_w/U_\infty = 0.0, \) Re = 6200, and \( \hat{r}/R = 1.33 \)
Theoretical Velocity Profile (Galerkin Solution 3), $u_w/U_\infty = 0.0$, $Re = 6200$, and $\tilde{r}/R = 1.50$
Theoretical Velocity Profile (Galerkin Solution 3), $u_w/U_\infty = 0.0$, Re = 6200, and $\hat{r}/R = 1.67$
Theoretical Velocity Profile (Galerkin Solution 3),
$u_w/U_\infty = 0.0$, $Re = 6200$, and $\hat{r}/R = 1.83$
Theoretical Velocity Profile (Galerkin Solution 3), $u_w/U_\infty = 0.0$, $Re = 6200$, and $r/R = 2.00$
FIGURE 88

Theoretical Drag Coefficient
Flow Impinging on the Cylinder,
from Reference (29),
$u_w/U_\infty = 0.0$ and $Re = 6200$
FIGURE 90

Wake Region Behind the Cylinder, from Reference (29),
$u_w/U_\infty = 0.0$ and $Re = 6200$
Flow Impinging on the Cylinder,
from Reference (29),
$u_w/U_\infty = 0.053$ and $Re = 6200$
FIGURE 92

Wake Region Behind the Cylinder, from Reference (29),

$u_w/u_\infty = 0.053$ and $Re = 6200$
Flow Impinging on the Cylinder,
from Reference (29),
$u_w/U_\infty = 0.126$ and $Re = 6200$
FIGURE 94

Wake Region Behind the Cylinder,
from Reference (29),
$u_w/U_\infty = 0.126$ and $Re = 6200$
FIGURE 95

Control Volume for the Application of the Integral Mass and Momentum Equations
FIGURE 96

Approximate Drag Coefficient as Calculated from the Integral Mass and Momentum Equations
Velocity components (Galerkin Solution 3), $u_g/U_\infty = 0.190$, $Re = 6200$, and $\hat{f}/R = 1.17$
FIGURE 98

Theoretical Vorticity
(Galerkin Solution 3),
Re = 6200 and \( \hat{f}/R = 1.17 \)
TABLE 1
Velocity Profile at the Inlet of the Porous Pipe

\( \hat{T} = 78.0 \, ^\circ F \)
\( \hat{P} = 10 \, \text{PSIG} \)
\( \text{Re}^* = 23000 \)

<table>
<thead>
<tr>
<th>( \hat{r}/R^* )</th>
<th>( \hat{w}/\hat{w}_{\text{max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Top) 0.75</td>
<td>0.84</td>
</tr>
<tr>
<td>0.50</td>
<td>0.89</td>
</tr>
<tr>
<td>0.25</td>
<td>1.00</td>
</tr>
<tr>
<td>0.0</td>
<td>1.00</td>
</tr>
<tr>
<td>0.25</td>
<td>0.97</td>
</tr>
<tr>
<td>0.50</td>
<td>0.92</td>
</tr>
<tr>
<td>(Bottom) 0.75</td>
<td>0.84</td>
</tr>
</tbody>
</table>
TABLE 2
Pressure Distribution on the Surface,
\( \frac{u_w}{U_\infty} = 0.102 \) and \( Re = 6200 \)

<table>
<thead>
<tr>
<th>( \hat{\theta} ) (DEG)</th>
<th>( \hat{P}_p ) (PSI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1.41 ( \times 10^{-5} )</td>
</tr>
<tr>
<td>20</td>
<td>4.53 ( \times 10^{-5} )</td>
</tr>
<tr>
<td>40</td>
<td>2.22 ( \times 10^{-5} )</td>
</tr>
<tr>
<td>60</td>
<td>-1.13 ( \times 10^{-4} )</td>
</tr>
<tr>
<td>80</td>
<td>-1.66 ( \times 10^{-4} )</td>
</tr>
<tr>
<td>100</td>
<td>-1.40 ( \times 10^{-4} )</td>
</tr>
<tr>
<td>120</td>
<td>-2.19 ( \times 10^{-4} )</td>
</tr>
<tr>
<td>140</td>
<td>-2.03 ( \times 10^{-4} )</td>
</tr>
<tr>
<td>160</td>
<td>-1.99 ( \times 10^{-4} )</td>
</tr>
<tr>
<td>180</td>
<td>-1.93 ( \times 10^{-4} )</td>
</tr>
</tbody>
</table>
### TABLE 3

Pressure Coefficient on the Surface, $u_w/U_\infty = 0.0$ and $Re = 5000$

<table>
<thead>
<tr>
<th>$\hat{\theta}$ (DEG)</th>
<th>$C_p = \frac{p-p_\infty}{\frac{1}{2} \rho U_\infty^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.98</td>
</tr>
<tr>
<td>20</td>
<td>0.60</td>
</tr>
<tr>
<td>40</td>
<td>-0.22</td>
</tr>
<tr>
<td>60</td>
<td>-1.05</td>
</tr>
<tr>
<td>80</td>
<td>-1.17</td>
</tr>
<tr>
<td>100</td>
<td>-0.98</td>
</tr>
<tr>
<td>120</td>
<td>-1.01</td>
</tr>
<tr>
<td>140</td>
<td>-0.98</td>
</tr>
<tr>
<td>160</td>
<td>-1.03</td>
</tr>
<tr>
<td>180</td>
<td>-0.97</td>
</tr>
</tbody>
</table>
TABLE 4

Constants of the Trial Stream Function
(Galerkin Solution 1), \( \frac{u_w}{U_\infty} = 0.190 \)

<table>
<thead>
<tr>
<th>Re</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( a_3 )</th>
<th>( a_4 )</th>
<th>( a_5 )</th>
<th>( a_6 )</th>
<th>( a_7 )</th>
<th>( a_8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>-3.1873</td>
<td>0.5785</td>
<td>3.8158</td>
<td>-1.4922</td>
<td>-2.0698</td>
<td>1.2491</td>
<td>0.4412</td>
<td>-0.3353</td>
</tr>
<tr>
<td>200</td>
<td>-3.2925</td>
<td>0.7639</td>
<td>4.0954</td>
<td>-1.9966</td>
<td>-2.3134</td>
<td>1.7016</td>
<td>0.5105</td>
<td>-0.4689</td>
</tr>
<tr>
<td>300</td>
<td>-3.3370</td>
<td>0.8530</td>
<td>4.2179</td>
<td>-2.2458</td>
<td>-2.4248</td>
<td>1.9327</td>
<td>0.5439</td>
<td>-0.5399</td>
</tr>
<tr>
<td>400</td>
<td>-3.3612</td>
<td>0.9045</td>
<td>4.2859</td>
<td>-2.3926</td>
<td>-2.4881</td>
<td>2.0715</td>
<td>0.5635</td>
<td>-0.5835</td>
</tr>
<tr>
<td>500</td>
<td>-3.3770</td>
<td>0.9379</td>
<td>4.3305</td>
<td>-2.4887</td>
<td>-2.5302</td>
<td>2.1638</td>
<td>0.5766</td>
<td>-0.6129</td>
</tr>
<tr>
<td>600</td>
<td>-3.3877</td>
<td>0.9612</td>
<td>4.3612</td>
<td>-2.5566</td>
<td>-2.5595</td>
<td>2.2295</td>
<td>0.5859</td>
<td>-0.6341</td>
</tr>
<tr>
<td>700</td>
<td>-3.3955</td>
<td>0.9784</td>
<td>4.3838</td>
<td>-2.6068</td>
<td>-2.5812</td>
<td>2.2785</td>
<td>0.5929</td>
<td>-0.6501</td>
</tr>
<tr>
<td>800</td>
<td>-3.4011</td>
<td>0.9904</td>
<td>4.3997</td>
<td>-2.6422</td>
<td>-2.5966</td>
<td>2.3133</td>
<td>0.5979</td>
<td>-0.6615</td>
</tr>
<tr>
<td>900</td>
<td>-3.4062</td>
<td>1.0102</td>
<td>4.4151</td>
<td>-2.6763</td>
<td>-2.6115</td>
<td>2.3469</td>
<td>0.6026</td>
<td>-0.6725</td>
</tr>
<tr>
<td>1000</td>
<td>-3.4100</td>
<td>1.0110</td>
<td>4.4262</td>
<td>-2.7012</td>
<td>-2.6223</td>
<td>2.3716</td>
<td>0.6062</td>
<td>-0.6806</td>
</tr>
<tr>
<td>2000</td>
<td>-3.4273</td>
<td>1.0842</td>
<td>4.4771</td>
<td>-2.8151</td>
<td>-2.6725</td>
<td>2.4855</td>
<td>0.6227</td>
<td>-0.7187</td>
</tr>
<tr>
<td>3000</td>
<td>-3.4331</td>
<td>1.0607</td>
<td>4.4941</td>
<td>-2.8530</td>
<td>-2.6894</td>
<td>2.5240</td>
<td>0.6283</td>
<td>-0.7318</td>
</tr>
<tr>
<td>4000</td>
<td>-3.4361</td>
<td>1.0674</td>
<td>4.5033</td>
<td>-2.8734</td>
<td>-2.6987</td>
<td>2.5447</td>
<td>0.6314</td>
<td>-0.7388</td>
</tr>
<tr>
<td>4100</td>
<td>-3.4365</td>
<td>1.0682</td>
<td>4.5044</td>
<td>-2.8758</td>
<td>-2.6997</td>
<td>2.5472</td>
<td>0.6317</td>
<td>-0.7397</td>
</tr>
</tbody>
</table>
### TABLE 5

Constants of the Trial Stream Function (Galerkin Solution 1), $u_w/U_\infty = 0.154$

<table>
<thead>
<tr>
<th>Re</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
<th>$\alpha_4$</th>
<th>$\alpha_5$</th>
<th>$\alpha_6$</th>
<th>$\alpha_7$</th>
<th>$\alpha_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>-3.1186</td>
<td>0.6476</td>
<td>3.6397</td>
<td>-1.6643</td>
<td>-1.9237</td>
<td>1.3857</td>
<td>0.4025</td>
<td>-0.3690</td>
</tr>
<tr>
<td>200</td>
<td>-3.2037</td>
<td>0.8955</td>
<td>3.8647</td>
<td>-2.3314</td>
<td>-2.1182</td>
<td>1.9763</td>
<td>0.4573</td>
<td>-0.5404</td>
</tr>
<tr>
<td>300</td>
<td>-3.2389</td>
<td>1.0243</td>
<td>3.9611</td>
<td>-2.6871</td>
<td>-2.2056</td>
<td>2.3013</td>
<td>0.4833</td>
<td>-0.6385</td>
</tr>
<tr>
<td>400</td>
<td>-3.2556</td>
<td>1.1027</td>
<td>4.0086</td>
<td>-2.9072</td>
<td>-2.2505</td>
<td>2.5063</td>
<td>0.4975</td>
<td>-0.7018</td>
</tr>
<tr>
<td>500</td>
<td>-3.2660</td>
<td>1.1549</td>
<td>4.0389</td>
<td>-3.0555</td>
<td>-2.2797</td>
<td>2.6463</td>
<td>0.5069</td>
<td>-0.7457</td>
</tr>
<tr>
<td>600</td>
<td>-3.2729</td>
<td>1.1898</td>
<td>4.0587</td>
<td>-3.1553</td>
<td>-2.2991</td>
<td>2.7414</td>
<td>0.5132</td>
<td>-0.7760</td>
</tr>
<tr>
<td>700</td>
<td>-3.2783</td>
<td>1.2196</td>
<td>4.0752</td>
<td>-3.2417</td>
<td>-2.3154</td>
<td>2.8245</td>
<td>0.5185</td>
<td>-0.8024</td>
</tr>
<tr>
<td>800</td>
<td>-3.2821</td>
<td>1.2410</td>
<td>4.0866</td>
<td>-3.3039</td>
<td>-2.3268</td>
<td>2.8846</td>
<td>0.5224</td>
<td>-0.8218</td>
</tr>
<tr>
<td>900</td>
<td>-3.2850</td>
<td>1.2571</td>
<td>4.0951</td>
<td>-3.3506</td>
<td>-2.3355</td>
<td>2.9301</td>
<td>0.5253</td>
<td>-0.8366</td>
</tr>
<tr>
<td>1000</td>
<td>-3.2875</td>
<td>1.2719</td>
<td>4.1030</td>
<td>-3.3941</td>
<td>-2.3435</td>
<td>2.9726</td>
<td>0.5280</td>
<td>-0.8503</td>
</tr>
<tr>
<td>2000</td>
<td>-3.2983</td>
<td>1.3349</td>
<td>4.1359</td>
<td>-3.5804</td>
<td>-2.3774</td>
<td>3.1567</td>
<td>0.5397</td>
<td>-0.9111</td>
</tr>
<tr>
<td>3000</td>
<td>-3.3019</td>
<td>1.3561</td>
<td>4.1469</td>
<td>-3.6439</td>
<td>-2.3891</td>
<td>3.2205</td>
<td>0.5438</td>
<td>-0.9326</td>
</tr>
<tr>
<td>4000</td>
<td>-3.3036</td>
<td>1.3668</td>
<td>4.1525</td>
<td>-3.6760</td>
<td>-2.3950</td>
<td>3.2529</td>
<td>0.5459</td>
<td>-0.9435</td>
</tr>
<tr>
<td>4100</td>
<td>-3.3038</td>
<td>1.3679</td>
<td>4.1532</td>
<td>-3.6793</td>
<td>-2.3956</td>
<td>3.2562</td>
<td>0.5461</td>
<td>-0.9446</td>
</tr>
<tr>
<td>Re</td>
<td>(a_1)</td>
<td>(a_2)</td>
<td>(a_3)</td>
<td>(a_4)</td>
<td>(a_5)</td>
<td>(a_6)</td>
<td>(a_7)</td>
<td>(a_8)</td>
</tr>
<tr>
<td>-----</td>
<td>----------</td>
<td>----------</td>
<td>----------</td>
<td>----------</td>
<td>----------</td>
<td>----------</td>
<td>----------</td>
<td>----------</td>
</tr>
<tr>
<td>100</td>
<td>-3.0517</td>
<td>0.7152</td>
<td>3.4696</td>
<td>-1.8315</td>
<td>-1.7843</td>
<td>1.5175</td>
<td>0.3663</td>
<td>-0.4011</td>
</tr>
<tr>
<td>200</td>
<td>-3.1059</td>
<td>0.1038</td>
<td>3.6127</td>
<td>-2.6931</td>
<td>-1.9076</td>
<td>2.2709</td>
<td>0.4008</td>
<td>-0.6162</td>
</tr>
<tr>
<td>300</td>
<td>-3.1183</td>
<td>0.1223</td>
<td>3.6485</td>
<td>-3.1972</td>
<td>-1.9422</td>
<td>2.7245</td>
<td>0.4120</td>
<td>-0.7506</td>
</tr>
<tr>
<td>400</td>
<td>-3.1205</td>
<td>0.1342</td>
<td>3.6572</td>
<td>-3.5261</td>
<td>-1.9529</td>
<td>3.0259</td>
<td>0.4162</td>
<td>-0.8419</td>
</tr>
<tr>
<td>500</td>
<td>-3.1194</td>
<td>1.4245</td>
<td>3.6564</td>
<td>-3.7569</td>
<td>-1.9546</td>
<td>3.2402</td>
<td>0.4176</td>
<td>-0.9078</td>
</tr>
<tr>
<td>600</td>
<td>-3.1176</td>
<td>1.4848</td>
<td>3.6534</td>
<td>-3.9270</td>
<td>-1.9539</td>
<td>3.3997</td>
<td>0.4181</td>
<td>-0.9575</td>
</tr>
<tr>
<td>700</td>
<td>-3.1162</td>
<td>1.5254</td>
<td>3.6504</td>
<td>-4.0420</td>
<td>-1.9526</td>
<td>3.5085</td>
<td>0.4183</td>
<td>-0.9919</td>
</tr>
<tr>
<td>800</td>
<td>-3.1142</td>
<td>1.5665</td>
<td>3.6465</td>
<td>-4.1600</td>
<td>-1.9505</td>
<td>3.6204</td>
<td>0.4182</td>
<td>-1.0270</td>
</tr>
<tr>
<td>900</td>
<td>-3.1125</td>
<td>1.5955</td>
<td>3.6430</td>
<td>-4.2433</td>
<td>-1.9485</td>
<td>3.7000</td>
<td>0.4180</td>
<td>-1.0523</td>
</tr>
<tr>
<td>1000</td>
<td>-3.1113</td>
<td>1.6172</td>
<td>3.6403</td>
<td>-4.3054</td>
<td>-1.9469</td>
<td>3.7598</td>
<td>0.4178</td>
<td>-1.0715</td>
</tr>
<tr>
<td>2000</td>
<td>-3.1039</td>
<td>1.7296</td>
<td>3.6238</td>
<td>-4.6328</td>
<td>-1.9363</td>
<td>4.0778</td>
<td>0.4163</td>
<td>-1.1744</td>
</tr>
<tr>
<td>3000</td>
<td>-3.1008</td>
<td>1.7704</td>
<td>3.6170</td>
<td>-4.7535</td>
<td>-1.9318</td>
<td>4.1966</td>
<td>0.4155</td>
<td>-1.2134</td>
</tr>
<tr>
<td>4000</td>
<td>-3.0994</td>
<td>1.7896</td>
<td>3.6137</td>
<td>-4.8101</td>
<td>-1.9296</td>
<td>4.2531</td>
<td>0.4152</td>
<td>-1.2322</td>
</tr>
<tr>
<td>4100</td>
<td>-3.0993</td>
<td>1.7911</td>
<td>3.6135</td>
<td>-4.8144</td>
<td>-1.9295</td>
<td>4.2574</td>
<td>0.4152</td>
<td>-1.2337</td>
</tr>
<tr>
<td>5000</td>
<td>-3.0984</td>
<td>1.8029</td>
<td>3.6115</td>
<td>-4.8503</td>
<td>-1.9281</td>
<td>4.2927</td>
<td>0.4149</td>
<td>-1.2453</td>
</tr>
<tr>
<td>6000</td>
<td>-3.0976</td>
<td>1.8125</td>
<td>3.6098</td>
<td>-4.8796</td>
<td>-1.9270</td>
<td>4.3217</td>
<td>0.4147</td>
<td>-1.2548</td>
</tr>
<tr>
<td>6200</td>
<td>-3.0975</td>
<td>1.1814</td>
<td>3.6096</td>
<td>-4.8840</td>
<td>-1.9269</td>
<td>4.3262</td>
<td>0.4147</td>
<td>-1.2563</td>
</tr>
</tbody>
</table>
### TABLE 7

Constants of the Trial Stream Function  
(Galerkin Solution 1), $\frac{u_{w}}{U_{\infty}} = 0.102$

<table>
<thead>
<tr>
<th>$Re$</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
<th>$\alpha_4$</th>
<th>$\alpha_5$</th>
<th>$\alpha_6$</th>
<th>$\alpha_7$</th>
<th>$\alpha_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>-2.9833</td>
<td>0.7839</td>
<td>3.2972</td>
<td>-2.0009</td>
<td>-1.6443</td>
<td>1.6500</td>
<td>0.3305</td>
<td>-0.4330</td>
</tr>
<tr>
<td>200</td>
<td>-2.9917</td>
<td>1.2029</td>
<td>3.3211</td>
<td>-3.1073</td>
<td>-1.6670</td>
<td>2.6058</td>
<td>0.3376</td>
<td>-0.7014</td>
</tr>
<tr>
<td>300</td>
<td>-2.9658</td>
<td>1.4716</td>
<td>3.2567</td>
<td>-3.8309</td>
<td>-1.6159</td>
<td>3.2469</td>
<td>0.3250</td>
<td>-0.8877</td>
</tr>
<tr>
<td>400</td>
<td>-2.9366</td>
<td>1.6592</td>
<td>3.1825</td>
<td>-4.3428</td>
<td>-1.5552</td>
<td>3.7080</td>
<td>0.3093</td>
<td>-0.1024</td>
</tr>
<tr>
<td>500</td>
<td>-2.9109</td>
<td>1.7969</td>
<td>3.1165</td>
<td>-4.7222</td>
<td>-1.5004</td>
<td>4.0536</td>
<td>0.2947</td>
<td>-1.1284</td>
</tr>
<tr>
<td>600</td>
<td>-2.8888</td>
<td>1.9025</td>
<td>3.0594</td>
<td>-5.0153</td>
<td>-1.4524</td>
<td>4.3230</td>
<td>0.2818</td>
<td>-1.2102</td>
</tr>
<tr>
<td>700</td>
<td>-2.8703</td>
<td>1.9857</td>
<td>3.0113</td>
<td>-5.2475</td>
<td>-1.4118</td>
<td>4.5379</td>
<td>0.2707</td>
<td>-1.2761</td>
</tr>
<tr>
<td>800</td>
<td>-2.8547</td>
<td>2.0529</td>
<td>2.9707</td>
<td>-5.4358</td>
<td>-1.3773</td>
<td>4.7131</td>
<td>0.2613</td>
<td>-1.3301</td>
</tr>
<tr>
<td>900</td>
<td>-2.8433</td>
<td>2.1015</td>
<td>2.9408</td>
<td>-5.5721</td>
<td>-1.3517</td>
<td>4.8406</td>
<td>0.2542</td>
<td>-1.3700</td>
</tr>
<tr>
<td>1000</td>
<td>-2.8316</td>
<td>2.1501</td>
<td>2.9100</td>
<td>-5.7094</td>
<td>-1.3253</td>
<td>4.9694</td>
<td>0.2468</td>
<td>-1.4100</td>
</tr>
<tr>
<td>2000</td>
<td>-2.7718</td>
<td>2.3864</td>
<td>2.7517</td>
<td>-6.3843</td>
<td>-1.1882</td>
<td>5.6095</td>
<td>0.2082</td>
<td>-1.6116</td>
</tr>
<tr>
<td>3000</td>
<td>-2.7511</td>
<td>2.4656</td>
<td>2.6966</td>
<td>-6.6118</td>
<td>-1.1399</td>
<td>5.8288</td>
<td>0.1944</td>
<td>-1.6822</td>
</tr>
<tr>
<td>4000</td>
<td>-2.7389</td>
<td>2.5122</td>
<td>2.6637</td>
<td>-6.7475</td>
<td>-1.1109</td>
<td>5.9598</td>
<td>0.1860</td>
<td>-1.7243</td>
</tr>
<tr>
<td>5000</td>
<td>-2.7375</td>
<td>2.5176</td>
<td>2.6599</td>
<td>-6.7634</td>
<td>-1.1075</td>
<td>5.9752</td>
<td>0.1850</td>
<td>-1.7291</td>
</tr>
<tr>
<td>5600</td>
<td>-2.7311</td>
<td>2.5417</td>
<td>2.6428</td>
<td>-6.8338</td>
<td>-1.0924</td>
<td>6.0434</td>
<td>0.1807</td>
<td>-1.7512</td>
</tr>
<tr>
<td>6000</td>
<td>-2.7270</td>
<td>2.5576</td>
<td>2.6315</td>
<td>-6.8796</td>
<td>-1.0824</td>
<td>6.0883</td>
<td>0.1778</td>
<td>-1.7659</td>
</tr>
<tr>
<td>6200</td>
<td>-2.7262</td>
<td>2.5602</td>
<td>2.6297</td>
<td>-6.8872</td>
<td>-1.0808</td>
<td>6.0958</td>
<td>0.1773</td>
<td>-1.7684</td>
</tr>
</tbody>
</table>
TABLE 8

Constants of the Trial Stream Function
(Galerkin Solution 1), $u_w/u_\infty = 0.0$

<table>
<thead>
<tr>
<th>Re</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
<th>$\alpha_4$</th>
<th>$\alpha_5$</th>
<th>$\alpha_6$</th>
<th>$\alpha_7$</th>
<th>$\alpha_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>-2.4545</td>
<td>1.3503</td>
<td>1.9998</td>
<td>-3.3756</td>
<td>-0.6362</td>
<td>2.7005</td>
<td>0.0909</td>
<td>-0.6751</td>
</tr>
<tr>
<td>200</td>
<td>-1.1137</td>
<td>4.7612</td>
<td>-1.2771</td>
<td>-11.903</td>
<td>1.8952</td>
<td>9.5223</td>
<td>-0.5045</td>
<td>-2.3805</td>
</tr>
<tr>
<td>300</td>
<td>0.9266</td>
<td>14.950</td>
<td>-6.2652</td>
<td>-37.374</td>
<td>5.7506</td>
<td>29.899</td>
<td>-1.4120</td>
<td>-7.4748</td>
</tr>
<tr>
<td>400</td>
<td>1.7744</td>
<td>25.704</td>
<td>-8.3380</td>
<td>-64.260</td>
<td>7.3527</td>
<td>51.408</td>
<td>-1.7891</td>
<td>-12.852</td>
</tr>
<tr>
<td>500</td>
<td>2.1440</td>
<td>35.647</td>
<td>-9.2296</td>
<td>-89.118</td>
<td>8.0272</td>
<td>71.294</td>
<td>-1.9416</td>
<td>-17.824</td>
</tr>
<tr>
<td>600</td>
<td>2.3392</td>
<td>45.072</td>
<td>-9.7084</td>
<td>-112.68</td>
<td>8.3993</td>
<td>90.145</td>
<td>-2.0301</td>
<td>-22.536</td>
</tr>
<tr>
<td>700</td>
<td>2.4756</td>
<td>54.373</td>
<td>-10.062</td>
<td>-135.93</td>
<td>8.6967</td>
<td>108.75</td>
<td>-2.1105</td>
<td>-27.186</td>
</tr>
<tr>
<td>800</td>
<td>2.5409</td>
<td>63.260</td>
<td>-10.215</td>
<td>-158.15</td>
<td>8.8068</td>
<td>126.52</td>
<td>-2.1329</td>
<td>-31.630</td>
</tr>
<tr>
<td>900</td>
<td>2.5882</td>
<td>72.059</td>
<td>-10.329</td>
<td>-180.15</td>
<td>8.8928</td>
<td>144.12</td>
<td>-2.1523</td>
<td>-36.029</td>
</tr>
<tr>
<td>1000</td>
<td>2.6218</td>
<td>80.783</td>
<td>-10.409</td>
<td>-201.96</td>
<td>8.9519</td>
<td>161.57</td>
<td>-2.1650</td>
<td>-40.392</td>
</tr>
<tr>
<td>2000</td>
<td>2.7334</td>
<td>166.23</td>
<td>-10.682</td>
<td>-415.58</td>
<td>9.1631</td>
<td>332.47</td>
<td>-2.2148</td>
<td>-83.117</td>
</tr>
<tr>
<td>3000</td>
<td>2.7540</td>
<td>250.65</td>
<td>-10.732</td>
<td>-626.64</td>
<td>9.2020</td>
<td>501.31</td>
<td>-2.2240</td>
<td>-125.33</td>
</tr>
<tr>
<td>4000</td>
<td>2.7612</td>
<td>334.82</td>
<td>-10.750</td>
<td>-837.04</td>
<td>9.2156</td>
<td>669.63</td>
<td>-2.2272</td>
<td>-167.41</td>
</tr>
<tr>
<td>4100</td>
<td>2.7617</td>
<td>343.23</td>
<td>-10.751</td>
<td>-858.06</td>
<td>9.2165</td>
<td>686.45</td>
<td>-2.2274</td>
<td>-171.61</td>
</tr>
<tr>
<td>5000</td>
<td>2.7646</td>
<td>418.87</td>
<td>-10.758</td>
<td>-1047.2</td>
<td>9.2219</td>
<td>837.75</td>
<td>-2.2287</td>
<td>-209.44</td>
</tr>
<tr>
<td>6000</td>
<td>2.7664</td>
<td>502.88</td>
<td>-10.762</td>
<td>-1257.2</td>
<td>9.2253</td>
<td>1005.8</td>
<td>-2.2295</td>
<td>-251.44</td>
</tr>
<tr>
<td>6200</td>
<td>2.7667</td>
<td>519.10</td>
<td>-10.763</td>
<td>-1297.7</td>
<td>9.2258</td>
<td>1038.2</td>
<td>-2.2296</td>
<td>-259.55</td>
</tr>
</tbody>
</table>
### TABLE 9

Constants of the Trial Stream Function  
(Galerkin Solution 3), $u_w/U_\infty = 0.190$

<table>
<thead>
<tr>
<th>Re</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
<th>$\alpha_4$</th>
<th>$\alpha_5$</th>
<th>$\alpha_6$</th>
<th>$\alpha_7$</th>
<th>$\alpha_8$</th>
<th>$\alpha_9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>-4.2522</td>
<td>4.5044</td>
<td>-1.2522</td>
<td>-1.7590</td>
<td>3.5180</td>
<td>-1.7590</td>
<td>5.9709</td>
<td>-1.1942</td>
<td>5.9709</td>
</tr>
<tr>
<td></td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-6}$</td>
</tr>
<tr>
<td></td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-8}$</td>
<td>$\times 10^{-7}$</td>
<td>$\times 10^{-8}$</td>
</tr>
<tr>
<td>300</td>
<td>-4.3872</td>
<td>4.7745</td>
<td>-1.3872</td>
<td>-2.7496</td>
<td>5.4992</td>
<td>-2.7496</td>
<td>-5.8900</td>
<td>1.1780</td>
<td>-5.8900</td>
</tr>
<tr>
<td></td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-8}$</td>
<td>$\times 10^{-7}$</td>
<td>$\times 10^{-8}$</td>
</tr>
<tr>
<td>400</td>
<td>-4.4102</td>
<td>4.8217</td>
<td>-1.4110</td>
<td>-2.5855</td>
<td>5.1710</td>
<td>-2.5855</td>
<td>-5.7325</td>
<td>1.1465</td>
<td>-5.7325</td>
</tr>
<tr>
<td></td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-8}$</td>
<td>$\times 10^{-7}$</td>
<td>$\times 10^{-8}$</td>
</tr>
<tr>
<td>500</td>
<td>-4.4252</td>
<td>4.8525</td>
<td>-1.4265</td>
<td>-1.9084</td>
<td>3.8168</td>
<td>-1.9084</td>
<td>1.5255</td>
<td>-3.0511</td>
<td>1.5255</td>
</tr>
<tr>
<td></td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-8}$</td>
<td>$\times 10^{-7}$</td>
<td>$\times 10^{-8}$</td>
</tr>
<tr>
<td>600</td>
<td>-4.4380</td>
<td>4.8759</td>
<td>-1.4380</td>
<td>-1.6450</td>
<td>3.2901</td>
<td>-1.6450</td>
<td>1.5843</td>
<td>-3.1686</td>
<td>1.5843</td>
</tr>
<tr>
<td></td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-8}$</td>
<td>$\times 10^{-7}$</td>
<td>$\times 10^{-8}$</td>
</tr>
<tr>
<td>700</td>
<td>-4.4451</td>
<td>4.8912</td>
<td>-1.4458</td>
<td>-1.5302</td>
<td>3.0604</td>
<td>-1.5302</td>
<td>6.5616</td>
<td>-3.1233</td>
<td>6.5616</td>
</tr>
<tr>
<td></td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-8}$</td>
<td>$\times 10^{-7}$</td>
<td>$\times 10^{-8}$</td>
</tr>
<tr>
<td>800</td>
<td>-4.4523</td>
<td>4.9045</td>
<td>-1.4523</td>
<td>-1.3396</td>
<td>2.6791</td>
<td>-1.3396</td>
<td>6.1664</td>
<td>-1.2333</td>
<td>6.1664</td>
</tr>
<tr>
<td></td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-8}$</td>
<td>$\times 10^{-7}$</td>
<td>$\times 10^{-8}$</td>
</tr>
<tr>
<td>900</td>
<td>-4.4559</td>
<td>4.9133</td>
<td>-1.4569</td>
<td>-1.2112</td>
<td>2.4225</td>
<td>-1.2112</td>
<td>5.4781</td>
<td>-1.0956</td>
<td>5.4781</td>
</tr>
<tr>
<td></td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-8}$</td>
<td>$\times 10^{-7}$</td>
<td>$\times 10^{-8}$</td>
</tr>
<tr>
<td>1000</td>
<td>-4.4598</td>
<td>4.9211</td>
<td>-1.4608</td>
<td>-1.0765</td>
<td>2.1531</td>
<td>-1.0765</td>
<td>6.0067</td>
<td>-1.2013</td>
<td>6.0067</td>
</tr>
<tr>
<td></td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-8}$</td>
<td>$\times 10^{-7}$</td>
<td>$\times 10^{-8}$</td>
</tr>
<tr>
<td>2000</td>
<td>-4.4781</td>
<td>4.9582</td>
<td>-1.4794</td>
<td>-1.3067</td>
<td>2.6134</td>
<td>-1.3067</td>
<td>2.6002</td>
<td>-5.2004</td>
<td>2.6002</td>
</tr>
<tr>
<td></td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
</tr>
<tr>
<td>Re</td>
<td>$\alpha_1$</td>
<td>$\alpha_2$</td>
<td>$\alpha_3$</td>
<td>$\alpha_4$</td>
<td>$\alpha_5$</td>
<td>$\alpha_6$</td>
<td>$\alpha_7$</td>
<td>$\alpha_8$</td>
<td>$\alpha_9$</td>
</tr>
<tr>
<td>-----</td>
<td>--------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
</tr>
<tr>
<td>3000</td>
<td>-4.4841</td>
<td>4.9707</td>
<td>-1.4858</td>
<td>7.6971</td>
<td>-1.5394</td>
<td>7.6971</td>
<td>7.0631</td>
<td>-1.4126</td>
<td>7.0631</td>
</tr>
<tr>
<td></td>
<td>x 10^{-6}</td>
<td>x 10^{-5}</td>
<td>x 10^{-6}</td>
<td>x 10^{-9}</td>
<td>x 10^{-8}</td>
<td>x 10^{-9}</td>
<td>x 10^{-9}</td>
<td>x 10^{-9}</td>
<td></td>
</tr>
<tr>
<td></td>
<td>x 10^{-5}</td>
<td>x 10^{-5}</td>
<td>x 10^{-5}</td>
<td>x 10^{-9}</td>
<td>x 10^{-8}</td>
<td>x 10^{-9}</td>
<td>x 10^{-9}</td>
<td>x 10^{-9}</td>
<td></td>
</tr>
<tr>
<td>4100</td>
<td>-4.4877</td>
<td>4.9779</td>
<td>-1.4894</td>
<td>1.6031</td>
<td>-3.2062</td>
<td>1.6031</td>
<td>8.8855</td>
<td>-1.7771</td>
<td>8.8855</td>
</tr>
<tr>
<td></td>
<td>x 10^{-5}</td>
<td>x 10^{-5}</td>
<td>x 10^{-5}</td>
<td>x 10^{-9}</td>
<td>x 10^{-8}</td>
<td>x 10^{-9}</td>
<td>x 10^{-9}</td>
<td>x 10^{-9}</td>
<td></td>
</tr>
<tr>
<td>( \text{Re} )</td>
<td>( \alpha_1 )</td>
<td>( \alpha_2 )</td>
<td>( \alpha_3 )</td>
<td>( \alpha_4 )</td>
<td>( \alpha_5 )</td>
<td>( \alpha_6 )</td>
<td>( \alpha_7 )</td>
<td>( \alpha_8 )</td>
<td>( \alpha_9 )</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>100</td>
<td>-4.2206</td>
<td>4.4412</td>
<td>-1.2206</td>
<td>1.3966</td>
<td>-2.7932</td>
<td>1.3966</td>
<td>-3.0715</td>
<td>6.1430</td>
<td>-3.0715</td>
</tr>
<tr>
<td>200</td>
<td>-4.3195</td>
<td>4.6390</td>
<td>-1.3195</td>
<td>1.3276</td>
<td>-2.6552</td>
<td>1.3276</td>
<td>8.1938</td>
<td>-1.6388</td>
<td>8.1938</td>
</tr>
<tr>
<td>300</td>
<td>4.3667</td>
<td>4.7334</td>
<td>-1.3667</td>
<td>1.3570</td>
<td>-2.7139</td>
<td>1.3570</td>
<td>3.7275</td>
<td>-7.4550</td>
<td>3.7275</td>
</tr>
<tr>
<td>400</td>
<td>-4.3943</td>
<td>4.7886</td>
<td>-1.3943</td>
<td>1.3739</td>
<td>-2.7479</td>
<td>1.3739</td>
<td>2.3373</td>
<td>-4.6746</td>
<td>2.3373</td>
</tr>
<tr>
<td>500</td>
<td>-4.4125</td>
<td>4.8249</td>
<td>-1.4125</td>
<td>1.4192</td>
<td>-2.8384</td>
<td>1.4192</td>
<td>2.2065</td>
<td>-4.4131</td>
<td>2.2065</td>
</tr>
<tr>
<td>600</td>
<td>-4.4253</td>
<td>4.8056</td>
<td>-1.4253</td>
<td>1.4615</td>
<td>-2.9231</td>
<td>1.4615</td>
<td>2.3321</td>
<td>-4.6642</td>
<td>2.3321</td>
</tr>
<tr>
<td>700</td>
<td>-4.4346</td>
<td>4.8694</td>
<td>-1.4347</td>
<td>1.5772</td>
<td>-3.1545</td>
<td>1.5772</td>
<td>1.8529</td>
<td>-3.7058</td>
<td>1.8529</td>
</tr>
<tr>
<td>800</td>
<td>-4.4422</td>
<td>4.8844</td>
<td>-1.4422</td>
<td>1.6220</td>
<td>-3.2440</td>
<td>1.6220</td>
<td>1.8405</td>
<td>-3.6809</td>
<td>1.8405</td>
</tr>
<tr>
<td>1000</td>
<td>-4.4529</td>
<td>4.9058</td>
<td>-1.4529</td>
<td>1.6961</td>
<td>-3.3921</td>
<td>1.6961</td>
<td>1.7858</td>
<td>-3.5716</td>
<td>1.7858</td>
</tr>
<tr>
<td>Re</td>
<td>$\alpha_1$</td>
<td>$\alpha_2$</td>
<td>$\alpha_3$</td>
<td>$\alpha_4$</td>
<td>$\alpha_5$</td>
<td>$\alpha_6$</td>
<td>$\alpha_7$</td>
<td>$\alpha_8$</td>
<td>$\alpha_9$</td>
</tr>
<tr>
<td>------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
</tr>
<tr>
<td>3000</td>
<td>-4.4815</td>
<td>4.9650</td>
<td>-1.4829</td>
<td>2.7072</td>
<td>-5.4144</td>
<td>2.7072</td>
<td>2.2165</td>
<td>-4.4330</td>
<td>2.2165</td>
</tr>
<tr>
<td></td>
<td>$x 10^{-7}$</td>
<td>$x 10^{-7}$</td>
<td>$x 10^{-7}$</td>
<td>$x 10^{-11}$</td>
<td>$x 10^{-11}$</td>
<td>$x 10^{-11}$</td>
<td>$x 10^{-11}$</td>
<td>$x 10^{-11}$</td>
<td>$x 10^{-11}$</td>
</tr>
<tr>
<td>4000</td>
<td>-4.4856</td>
<td>4.9731</td>
<td>-1.4869</td>
<td>-3.0262</td>
<td>-6.0524</td>
<td>3.0262</td>
<td>2.3777</td>
<td>-4.7553</td>
<td>2.3777</td>
</tr>
<tr>
<td></td>
<td>$x 10^{-7}$</td>
<td>$x 10^{-7}$</td>
<td>$x 10^{-7}$</td>
<td>$x 10^{-11}$</td>
<td>$x 10^{-11}$</td>
<td>$x 10^{-11}$</td>
<td>$x 10^{-11}$</td>
<td>$x 10^{-11}$</td>
<td>$x 10^{-11}$</td>
</tr>
<tr>
<td>4100</td>
<td>-4.4862</td>
<td>4.9740</td>
<td>-1.4873</td>
<td>3.0752</td>
<td>-6.1505</td>
<td>3.0752</td>
<td>2.3950</td>
<td>-4.7900</td>
<td>2.3950</td>
</tr>
<tr>
<td></td>
<td>$x 10^{-7}$</td>
<td>$x 10^{-7}$</td>
<td>$x 10^{-7}$</td>
<td>$x 10^{-11}$</td>
<td>$x 10^{-11}$</td>
<td>$x 10^{-11}$</td>
<td>$x 10^{-11}$</td>
<td>$x 10^{-11}$</td>
<td>$x 10^{-11}$</td>
</tr>
</tbody>
</table>
### TABLE 11

Constants of the Trial Stream Function  
(Galerkin Solution 3), $u_w/U_\infty = 0.126$

<table>
<thead>
<tr>
<th>Re</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
<th>$a_5$</th>
<th>$a_6$</th>
<th>$a_7$</th>
<th>$a_8$</th>
<th>$a_9$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
<td>$\times 10^{-5}$</td>
</tr>
<tr>
<td>200</td>
<td>-4.2926</td>
<td>4.5852</td>
<td>-1.2926</td>
<td>-4.2383</td>
<td>8.4767</td>
<td>-4.2383</td>
<td>-1.4274</td>
<td>2.8549</td>
<td>-1.4272</td>
</tr>
<tr>
<td></td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-7}$</td>
<td>$\times 10^{-7}$</td>
<td>$\times 10^{-7}$</td>
<td>$\times 10^{-7}$</td>
<td>$\times 10^{-7}$</td>
</tr>
<tr>
<td>300</td>
<td>-4.3443</td>
<td>4.6886</td>
<td>-1.3443</td>
<td>-5.5976</td>
<td>1.1195</td>
<td>-5.5976</td>
<td>-4.5511</td>
<td>9.1021</td>
<td>-4.5511</td>
</tr>
<tr>
<td></td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-7}$</td>
<td>$\times 10^{-8}$</td>
<td>$\times 10^{-8}$</td>
<td>$\times 10^{-8}$</td>
<td>$\times 10^{-8}$</td>
<td>$\times 10^{-8}$</td>
</tr>
<tr>
<td>400</td>
<td>-4.3754</td>
<td>4.7508</td>
<td>-1.3754</td>
<td>-3.5308</td>
<td>7.0616</td>
<td>-3.5308</td>
<td>-1.1107</td>
<td>2.2213</td>
<td>-1.1107</td>
</tr>
<tr>
<td></td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-8}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
</tr>
<tr>
<td>500</td>
<td>-4.3947</td>
<td>4.7911</td>
<td>-1.3957</td>
<td>-1.7091</td>
<td>3.4182</td>
<td>-1.7091</td>
<td>7.6776</td>
<td>-1.5355</td>
<td>7.6776</td>
</tr>
<tr>
<td></td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-10}$</td>
<td>$\times 10^{-10}$</td>
<td>$\times 10^{-10}$</td>
<td>$\times 10^{-10}$</td>
<td>$\times 10^{-10}$</td>
</tr>
<tr>
<td>600</td>
<td>-4.4107</td>
<td>4.8216</td>
<td>-1.4108</td>
<td>-1.5648</td>
<td>3.1295</td>
<td>-1.5648</td>
<td>8.9607</td>
<td>-1.7921</td>
<td>8.9607</td>
</tr>
<tr>
<td></td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-10}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
</tr>
<tr>
<td>700</td>
<td>-4.4220</td>
<td>4.8441</td>
<td>-1.4220</td>
<td>-1.4244</td>
<td>2.8488</td>
<td>-1.4244</td>
<td>8.1524</td>
<td>-1.6305</td>
<td>8.1524</td>
</tr>
<tr>
<td></td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-10}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
</tr>
<tr>
<td></td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-10}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
</tr>
<tr>
<td></td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-10}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
</tr>
<tr>
<td>1000</td>
<td>-4.4415</td>
<td>4.8849</td>
<td>-1.4427</td>
<td>-1.2978</td>
<td>2.5955</td>
<td>-1.2978</td>
<td>4.7040</td>
<td>-9.4080</td>
<td>4.7040</td>
</tr>
<tr>
<td></td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-10}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
</tr>
<tr>
<td>2000</td>
<td>-4.4685</td>
<td>4.9389</td>
<td>-1.4697</td>
<td>-1.6816</td>
<td>3.3632</td>
<td>-1.6816</td>
<td>5.2770</td>
<td>-1.0554</td>
<td>5.2770</td>
</tr>
<tr>
<td></td>
<td>$\times 10^{-7}$</td>
<td>$\times 10^{-7}$</td>
<td>$\times 10^{-7}$</td>
<td>$\times 10^{-10}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
</tr>
<tr>
<td>3000</td>
<td>-4.4778</td>
<td>4.9578</td>
<td>-1.4792</td>
<td>1.1393</td>
<td>-2.2787</td>
<td>1.1393</td>
<td>1.9761</td>
<td>-3.9522</td>
<td>1.9761</td>
</tr>
<tr>
<td></td>
<td>$\times 10^{-7}$</td>
<td>$\times 10^{-7}$</td>
<td>$\times 10^{-7}$</td>
<td>$\times 10^{-10}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
</tr>
<tr>
<td>4000</td>
<td>-4.4825</td>
<td>4.9673</td>
<td>-1.4840</td>
<td>5.0164</td>
<td>-1.0033</td>
<td>5.0164</td>
<td>5.8457</td>
<td>-1.1691</td>
<td>5.8457</td>
</tr>
<tr>
<td></td>
<td>$\times 10^{-7}$</td>
<td>$\times 10^{-7}$</td>
<td>$\times 10^{-7}$</td>
<td>$\times 10^{-10}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-9}$</td>
</tr>
</tbody>
</table>
TABLE 11 (Continued)

<table>
<thead>
<tr>
<th>Re</th>
<th>(a_1)</th>
<th>(a_2)</th>
<th>(a_3)</th>
<th>(a_4)</th>
<th>(a_5)</th>
<th>(a_6)</th>
<th>(a_7)</th>
<th>(a_8)</th>
<th>(a_9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4100</td>
<td>-4.4829</td>
<td>4.9680</td>
<td>-1.4844</td>
<td>5.5850</td>
<td>-1.1170</td>
<td>5.5850</td>
<td>6.3960</td>
<td>-1.2792</td>
<td>6.3960</td>
</tr>
<tr>
<td></td>
<td>x 10^{-7}</td>
<td>x 10^{-6}</td>
<td>x 10^{-7}</td>
<td>x 10^{-10}</td>
<td>x 10^{-9}</td>
<td>x 10^{-9}</td>
<td>x 10^{-9}</td>
<td>x 10^{-10}</td>
<td>x 10^{-9}</td>
</tr>
<tr>
<td>5000</td>
<td>-4.4853</td>
<td>4.9731</td>
<td>-1.4870</td>
<td>1.1917</td>
<td>-2.3834</td>
<td>1.1917</td>
<td>1.2028</td>
<td>-2.4055</td>
<td>1.2028</td>
</tr>
<tr>
<td></td>
<td>x 10^{-6}</td>
<td>x 10^{-6}</td>
<td>x 10^{-9}</td>
<td>x 10^{-9}</td>
<td>x 10^{-9}</td>
<td>x 10^{-9}</td>
<td>x 10^{-9}</td>
<td>x 10^{-9}</td>
<td>x 10^{-9}</td>
</tr>
<tr>
<td>6000</td>
<td>-4.4874</td>
<td>4.9772</td>
<td>-1.4890</td>
<td>1.9422</td>
<td>-3.8844</td>
<td>1.9422</td>
<td>1.8310</td>
<td>-3.6621</td>
<td>1.8310</td>
</tr>
<tr>
<td></td>
<td>x 10^{-6}</td>
<td>x 10^{-6}</td>
<td>x 10^{-9}</td>
<td>x 10^{-9}</td>
<td>x 10^{-9}</td>
<td>x 10^{-9}</td>
<td>x 10^{-9}</td>
<td>x 10^{-9}</td>
<td>x 10^{-9}</td>
</tr>
<tr>
<td>6200</td>
<td>-4.4878</td>
<td>4.9779</td>
<td>-1.4894</td>
<td>2.1099</td>
<td>-4.2199</td>
<td>2.1099</td>
<td>1.9608</td>
<td>-3.9216</td>
<td>1.9608</td>
</tr>
<tr>
<td></td>
<td>x 10^{-6}</td>
<td>x 10^{-6}</td>
<td>x 10^{-6}</td>
<td>x 10^{-9}</td>
<td>x 10^{-9}</td>
<td>x 10^{-9}</td>
<td>x 10^{-9}</td>
<td>x 10^{-9}</td>
<td>x 10^{-9}</td>
</tr>
</tbody>
</table>
## TABLE 12

Constants of the Trial Stream Function (Galerkin Solution 3), \( u_w/U_\infty = 0.102 \)

<table>
<thead>
<tr>
<th>( Re )</th>
<th>( \alpha_1 )</th>
<th>( \alpha_2 )</th>
<th>( \alpha_3 )</th>
<th>( \alpha_4 )</th>
<th>( \alpha_5 )</th>
<th>( \alpha_6 )</th>
<th>( \alpha_7 )</th>
<th>( \alpha_8 )</th>
<th>( \alpha_9 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>-4.1573</td>
<td>4.3146</td>
<td>-1.1573</td>
<td>7.2869</td>
<td>-1.4574</td>
<td>7.2869</td>
<td>-5.6658</td>
<td>1.1332</td>
<td>-5.6658</td>
</tr>
<tr>
<td></td>
<td>x 10^{-5}</td>
<td>x 10^{-4}</td>
<td>x 10^{-5}</td>
<td>x 10^{-5}</td>
<td>x 10^{-5}</td>
<td>x 10^{-4}</td>
<td>x 10^{-6}</td>
<td>x 10^{-6}</td>
<td>x 10^{-5}</td>
</tr>
<tr>
<td>200</td>
<td>-4.2629</td>
<td>4.5257</td>
<td>-1.2629</td>
<td>-1.2307</td>
<td>2.4613</td>
<td>-1.2307</td>
<td>1.6642</td>
<td>-3.3285</td>
<td>1.6642</td>
</tr>
<tr>
<td></td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-6}</td>
<td>x 10^{-6}</td>
<td>x 10^{-6}</td>
</tr>
<tr>
<td>300</td>
<td>-4.3187</td>
<td>4.6374</td>
<td>-1.3187</td>
<td>-6.7677</td>
<td>1.3535</td>
<td>-6.7677</td>
<td>3.9789</td>
<td>-7.9577</td>
<td>3.9789</td>
</tr>
<tr>
<td></td>
<td>x 10^{-5}</td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-6}</td>
<td>x 10^{-6}</td>
<td>x 10^{-6}</td>
</tr>
<tr>
<td>400</td>
<td>-4.3523</td>
<td>4.7056</td>
<td>-1.3529</td>
<td>1.5795</td>
<td>-3.1591</td>
<td>1.5795</td>
<td>1.7944</td>
<td>-3.5888</td>
<td>1.7944</td>
</tr>
<tr>
<td></td>
<td>x 10^{-5}</td>
<td>x 10^{-5}</td>
<td>x 10^{-5}</td>
<td>x 10^{-5}</td>
<td>x 10^{-5}</td>
<td>x 10^{-5}</td>
<td>x 10^{-6}</td>
<td>x 10^{-6}</td>
<td>x 10^{-6}</td>
</tr>
<tr>
<td></td>
<td>x 10^{-5}</td>
<td>x 10^{-5}</td>
<td>x 10^{-5}</td>
<td>x 10^{-5}</td>
<td>x 10^{-5}</td>
<td>x 10^{-5}</td>
<td>x 10^{-7}</td>
<td>x 10^{-7}</td>
<td>x 10^{-7}</td>
</tr>
<tr>
<td>600</td>
<td>-4.3930</td>
<td>4.7868</td>
<td>-1.3935</td>
<td>9.5899</td>
<td>-1.9180</td>
<td>9.5899</td>
<td>2.0424</td>
<td>-4.0848</td>
<td>2.0424</td>
</tr>
<tr>
<td></td>
<td>x 10^{-5}</td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-7}</td>
<td>x 10^{-7}</td>
<td>x 10^{-7}</td>
</tr>
<tr>
<td>700</td>
<td>-4.4066</td>
<td>4.8133</td>
<td>-1.4066</td>
<td>1.1214</td>
<td>-2.2429</td>
<td>1.1214</td>
<td>1.3856</td>
<td>-2.7712</td>
<td>1.3856</td>
</tr>
<tr>
<td></td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-7}</td>
<td>x 10^{-7}</td>
<td>x 10^{-7}</td>
</tr>
<tr>
<td>800</td>
<td>-4.4167</td>
<td>4.8335</td>
<td>-1.4167</td>
<td>1.2398</td>
<td>-2.4797</td>
<td>1.2398</td>
<td>1.4390</td>
<td>-2.8780</td>
<td>1.4390</td>
</tr>
<tr>
<td></td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-7}</td>
<td>x 10^{-7}</td>
<td>x 10^{-7}</td>
</tr>
<tr>
<td></td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-7}</td>
<td>x 10^{-7}</td>
<td>x 10^{-7}</td>
</tr>
<tr>
<td>1000</td>
<td>-4.4300</td>
<td>4.8615</td>
<td>-1.4310</td>
<td>1.4905</td>
<td>-2.9810</td>
<td>1.4905</td>
<td>1.7613</td>
<td>-3.5226</td>
<td>1.7613</td>
</tr>
<tr>
<td></td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-4}</td>
<td>x 10^{-7}</td>
<td>x 10^{-7}</td>
<td>x 10^{-7}</td>
</tr>
<tr>
<td></td>
<td>x 10^{-5}</td>
<td>x 10^{-5}</td>
<td>x 10^{-5}</td>
<td>x 10^{-5}</td>
<td>x 10^{-5}</td>
<td>x 10^{-5}</td>
<td>x 10^{-8}</td>
<td>x 10^{-8}</td>
<td>x 10^{-8}</td>
</tr>
<tr>
<td></td>
<td>x 10^{-5}</td>
<td>x 10^{-5}</td>
<td>x 10^{-5}</td>
<td>x 10^{-5}</td>
<td>x 10^{-5}</td>
<td>x 10^{-5}</td>
<td>x 10^{-8}</td>
<td>x 10^{-8}</td>
<td>x 10^{-8}</td>
</tr>
<tr>
<td>4000</td>
<td>-4.4797</td>
<td>4.9609</td>
<td>-1.4807</td>
<td>2.7161</td>
<td>-5.4323</td>
<td>2.7161</td>
<td>-5.3204</td>
<td>1.0641</td>
<td>-5.3204</td>
</tr>
<tr>
<td></td>
<td>x 10^{-5}</td>
<td>x 10^{-5}</td>
<td>x 10^{-5}</td>
<td>x 10^{-5}</td>
<td>x 10^{-5}</td>
<td>x 10^{-5}</td>
<td>x 10^{-8}</td>
<td>x 10^{-8}</td>
<td>x 10^{-8}</td>
</tr>
<tr>
<td>Re</td>
<td>( \alpha_1 )</td>
<td>( \alpha_2 )</td>
<td>( \alpha_3 )</td>
<td>( \alpha_4 )</td>
<td>( \alpha_5 )</td>
<td>( \alpha_6 )</td>
<td>( \alpha_7 )</td>
<td>( \alpha_8 )</td>
<td>( \alpha_9 )</td>
</tr>
<tr>
<td>-----</td>
<td>----------------</td>
<td>----------------</td>
<td>----------------</td>
<td>----------------</td>
<td>----------------</td>
<td>----------------</td>
<td>----------------</td>
<td>----------------</td>
<td>----------------</td>
</tr>
<tr>
<td>4100</td>
<td>-4.4802</td>
<td>4.9618</td>
<td>-1.4812</td>
<td>2.6809</td>
<td>-5.3618</td>
<td>2.6809</td>
<td>-4.5422</td>
<td>9.0844</td>
<td>-4.5422</td>
</tr>
<tr>
<td></td>
<td>( \times 10^{-5} )</td>
<td>( \times 10^{-5} )</td>
<td>( \times 10^{-5} )</td>
<td>( \times 10^{-9} )</td>
<td>( \times 10^{-9} )</td>
<td>( \times 10^{-9} )</td>
<td>( \times 10^{-9} )</td>
<td>( \times 10^{-9} )</td>
<td>( \times 10^{-9} )</td>
</tr>
<tr>
<td>5000</td>
<td>-4.4830</td>
<td>4.9679</td>
<td>-1.4843</td>
<td>2.5843</td>
<td>-5.1686</td>
<td>2.5843</td>
<td>-1.4454</td>
<td>2.8908</td>
<td>-1.4454</td>
</tr>
<tr>
<td></td>
<td>( \times 10^{-5} )</td>
<td>( \times 10^{-5} )</td>
<td>( \times 10^{-5} )</td>
<td>( \times 10^{-9} )</td>
<td>( \times 10^{-9} )</td>
<td>( \times 10^{-9} )</td>
<td>( \times 10^{-9} )</td>
<td>( \times 10^{-9} )</td>
<td>( \times 10^{-9} )</td>
</tr>
<tr>
<td>6000</td>
<td>-4.4855</td>
<td>4.9728</td>
<td>-1.4867</td>
<td>2.5163</td>
<td>-5.0326</td>
<td>2.5163</td>
<td>-1.1368</td>
<td>2.7326</td>
<td>-1.1368</td>
</tr>
<tr>
<td></td>
<td>( \times 10^{-5} )</td>
<td>( \times 10^{-5} )</td>
<td>( \times 10^{-5} )</td>
<td>( \times 10^{-9} )</td>
<td>( \times 10^{-9} )</td>
<td>( \times 10^{-9} )</td>
<td>( \times 10^{-9} )</td>
<td>( \times 10^{-9} )</td>
<td>( \times 10^{-9} )</td>
</tr>
<tr>
<td>6200</td>
<td>-4.4859</td>
<td>4.9736</td>
<td>-1.4871</td>
<td>2.5052</td>
<td>-5.0105</td>
<td>2.5052</td>
<td>-1.1078</td>
<td>2.2156</td>
<td>-1.1078</td>
</tr>
<tr>
<td></td>
<td>( \times 10^{-5} )</td>
<td>( \times 10^{-5} )</td>
<td>( \times 10^{-5} )</td>
<td>( \times 10^{-9} )</td>
<td>( \times 10^{-9} )</td>
<td>( \times 10^{-9} )</td>
<td>( \times 10^{-9} )</td>
<td>( \times 10^{-9} )</td>
<td>( \times 10^{-9} )</td>
</tr>
</tbody>
</table>
TABLE 13

Constants of the Trial Stream Function
(Galerkin Solution 3), \( \frac{u_w}{U_\infty} = 0.0 \)

<table>
<thead>
<tr>
<th>( k )</th>
<th>( a_k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-3.8823</td>
</tr>
<tr>
<td>2</td>
<td>3.7647</td>
</tr>
<tr>
<td>3</td>
<td>-8.8233 \times 10^{-1}</td>
</tr>
<tr>
<td>4</td>
<td>-2.6309 \times 10^{-7}</td>
</tr>
<tr>
<td>5</td>
<td>5.2617 \times 10^{-7}</td>
</tr>
<tr>
<td>6</td>
<td>-2.6309 \times 10^{7}</td>
</tr>
<tr>
<td>7</td>
<td>-1.0376 \times 10^{-5}</td>
</tr>
<tr>
<td>8</td>
<td>2.0752 \times 10^{-5}</td>
</tr>
<tr>
<td>9</td>
<td>-1.0376 \times 10^{-5}</td>
</tr>
</tbody>
</table>
REFERENCES


47. Thoman, D.C., Private Communication, Bendix Corp., South Bend, Ind., 1976.


