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COMPUTER GRAPHICS AS AN AID TO TEACHING MATHEMATICS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By
Bernard Louis Baltz, B.S., M.S.

***
The Ohio State University
1977

Reading Committee:
Professor Thomas G. Ralley
Professor Richard J. Shumway
Professor Harold C. Trimble

Approved By

Adviser
Department of Mathematics and Science
Education
For My Father
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VITA

July 12, 1948 .......... Born - Indianapolis, Indiana
1969 ..................... B.S., Mathematics, The Ohio State University, Columbus, Ohio
1969-70 .................. Woodrow Wilson Fellow, NSF Fellow
1970 ..................... M.S., University of Chicago, Chicago, Illinois
1971-1977 ................ Lecturer and Teaching Associate, Department of Mathematics, The Ohio State University, Columbus, Ohio

FIELDS OF STUDY

Major Field: Mathematics Education

Studies in Computer Graphics: Professor Charles Csuri and Professor Robert LaRue.

Studies in Film: Professor Richard B. Long.

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<td>Aptitude-Treatment Interaction.</td>
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<td>CBI</td>
<td>Computer-Based Instruction.</td>
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<td>CRT</td>
<td>Cathode Ray Tube.</td>
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<tr>
<td>Courseware</td>
<td>Software specifically for CAI uses, usually for a particular subject area.</td>
</tr>
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<td>Hardware</td>
<td>Computer hardware. The physical computer machinery.</td>
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<td>Iconicity</td>
<td>The degree of representativeness or reality of a figure or picture.</td>
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<td>Micro-Computer</td>
<td>Small computer based upon 8-bit words.</td>
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<tr>
<td>Mini-Computer</td>
<td>Small computer based upon 16-bit words.</td>
</tr>
<tr>
<td>Random vectors</td>
<td>Line segments drawn on a CRT between two specified points.</td>
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<tr>
<td>Raster Tube</td>
<td>CRT which displays an array of dots rather than line segments.</td>
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<td>Software</td>
<td>Written computer instructions. Programming.</td>
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CHAPTER I - INTRODUCTION

Computer Graphics as an Educational Medium:

Visual Media in Education:

Many educators have mentioned the importance of visual content in most types of learning (Bruner, 1964; Arnheim, 1970). This investigator became interested in the use of visual media for mathematics instruction. At the onset of the investigation two somewhat contrary opinions concerning media in education were evident. Some educators envisioned a versatile audio-visual technology, including computers, as having great impact. (Ashby et. al., 1972) (Bork, 1974). Others observed the failures of various media to live up to expectations (Gross, 1975), and warned of the pitfalls of relying on technology to provide instruction (Ottinger, & Marks, 1969; Hooper, 1974).

A report of the Carnegie Commission (Ashby et. al., 1972) lists as major educational media the printed word, film, multimedia classrooms, self-instruction units, radio, television (including videotape), and computer assisted instruction.
CAI in Education:

The relatively new medium, computer assisted instruction or CAI, has already become an important tool in education. (Suppes, Jerman & Brian, 1968), for instance. However the applications of CAI have not reached the levels predicted by Suppes among others (Octtinger & Marks, 1969).

Several reasons may account for the limited use of CAI to date. Among the principal reasons mentioned are:

(1) Computer time, even in a "time-sharing" environment, has been quite expensive. An estimate of costs for one project in 1970 was 94 dollars per student hour (Dunn and Wastler, 1972).

(2) Software has been lacking, and the time-consuming process of creating useable software has often been wasted since sharing has not been the rule and hardware changes tend to make software obsolete. One estimate in 1972 was that of hundreds of instructional programs for computer use, only about half a dozen were suitable for transport beyond the institution where they were developed. (Ashby et. al., 1972 , p. 47).

(3) Aside from its uses as a drill and practice medium, CAI has not been proved to be a generally effective teaching medium for mathematics (Riedeson & Suydam, 1967; Confer, 1971; Kanes, 1971).

Recent technological developments may alter these trends. First, the cost of hardware, particularly for small systems, is
definitely decreasing with the growth of the use of mini- and micro-computers. The quantity and quality of course software shared among users should tend to increase with the proliferation of systems in use. However, it seemed to this investigator that the apparent relative failure of useful applications of computers to materialize in teaching might be due in part to factors other than the above-mentioned problems of expense and lack of available software. Specifically the lack of capability to present information visually was seen as a crucial detriment to CAI.

**Graphics in CAI:**

Advances in processor and memory technology are making graphics via cathode ray tubes (CRT) or plasma panels an economical option in small computer systems. Thus, this investigator became interested in the educational potential of computer graphics, particularly featuring animation, as a component of CAI.

Use of computer graphics to teach mathematics within a CAI setting (Bork & Sherman, 1970) or otherwise (Graham & Read, 1971) Some large-scale systems in current use employ still computer graphic images in various subject areas. (McClintock & Kimberlin, 1973; King, 1975; Bork, 1976). Computer animated sequences have been made into films (Carlton, 1974) but have not been integrated into a standard CAI format of programmed instruction, again apparently due to high expense of the graphics hardware which could afford animation.
Animation:

The nature of most CRT's requires that they be "refreshed" many times each second, thus allowing for animation by incrementally changing the composition of the subsequent "frames".

Earlier media research (Silverman, 1958; Allen & Weintraub, 1968) indicated that animation can make a significant difference in teaching effectiveness if the concepts to be learned involve motion or time. Other research (Lefkowth, 1955; Dwyer, 1970) found the degree of iconicity to be of limited importance. Thus, weakly iconic drawings could be as effective as highly representational ones in teaching concepts.

Salomon (1972) found that when animated visual media were effective, a pattern of disordinal aptitude-treatment interaction occurred. Students of less aptitude were more able to internalize visual models, apparently because students of greater aptitude would tend to have self-provided models which the media presentations would not supplant.

Although much research had been done with CAI and with programmed learning with various visual media, this researcher found that the application of computer animated visuals in an individually-applied programmed learning situation was lacking. Reasons of expense alone could explain this lack. However, the
problem of expense is disappearing, and the possibility of low-cost micro-computers with graphics in the near future is very great. (Funk, 1976).

Questions concerning the value of this technology as an educational medium needed to be answered.

The Study:

It was decided to investigate the effectiveness of moderately iconic, animated graphics sequences drawn by computer in conveying selected topics in geometry. Here, by "moderately iconic graphics" is meant line drawings without shading, color, hidden lines, perspective, or other graphic features. The effects of such computer-drawn examples integrated into a programmed lesson were compared with standard classroom presentations.

The basic level of iconicity was chosen because such simple sequences would involve less memory and processing time by a computer, and thus would be representative of economical CAI systems which included graphics.

The animation features, too, were to be as basic as possible, while affording some power. Thus rigid pictures were able to be translated and rotated by computer manipulation, at controllable rates. Sophisticated scaling, truncating (or windowing), linkages, and other features, although available, were not employed.
At the outset of the study, the inexpensive graphics anticipated by this investigator were not available, and so having students learn directly from the computer output CRT was not economically feasible. In order to provide treatments which featured computer graphic output via CRT, as well as environmental factors such as the solitude of the student and student control of the mechanism, computer graphic sequences were edited onto a videotape cassette for individual playback.

In research involving applications of media which are not typical of standard practice, it is difficult to preserve both internal and external validity (Salomon & Clark, 1974). Design considerations in preserving internal validity such as random assignment, and elimination of effects due to history, novelty of treatments, Hawthorne effects, et. al. often preclude situations whose results readily generalize to actual practice.

Conversely the employment of treatments which reflect actual classroom practice would tend to involve too many uncontrollable influences on outcomes.

The desired comparisons between instruction based upon animated computer graphic examples and "standard" classroom presentations required an experimental treatment and a control. To provide results pertinent to instructional practice, the control treatment had to embody typical classroom procedures.
Statistical differences between two such treatments could be due to incidental differences, novelty, pedagogical differences, Hawthorne effect, etc. Therefore a control more exactly duplicating the experimental treatment was devised, employing the same medium of presentation, the same pedagogical development, virtually the same script, the same narrator's voice, and the same environment of presentation. The standard live classroom presentations were retained as a control treatment for purposes of external validity.

**Topic of the Lesson:**

The lesson was to be some topic or topics not covered in standard curricula. It had to involve time, motion, and/or some sequential processes. Furthermore, its representation had to be easily coordinatized. Fortunately, most topics in mathematics satisfy this last requirement. Finally, graphic approaches had to hold some promise for improved communication of the concepts to the students or improved modelling of objects, structures or relationships.

Transformational geometry does involve geometric results through motions or transformations of the plane, and is an important facet of mathematics which is not typically involved in the secondary curriculum. Thus, the general topic was transformational geometry and the lesson was restricted to "Congruence through rigid motions".
The Treatments:

Three treatments were employed; one whose visual content duplicated a CRT graphics output console as closely as possible, one that matched the computer-visual treatment in every way except for the exclusion of the computer graphic content, and one that provided a standard classroom situation and which matched the others in every way possible.

Treatment 1, the computer-visual treatment, was presented via videotape cassettes. Thus that was necessarily the format of one of the control treatments. Students receiving either of these treatments were given written notices to go to one of the two locations to view the taped lesson. With the video equipment at these locations were printed instructions for operating the mechanism. These instructions also included six short exercises which were assigned during the videotaped lessons. The video portion of the computer-visual treatment was white line-drawings on a dark background. The video content consisted of animated examples of the rigid motions described including figures and notations for the rigid motions described (See Chapter 4) as well as labels, terms, and the message "DO EXERCISES", at which the student was instructed to stop the tape player. The audio content was a narration coordinated with the visual content. The narrator
was the lecturer in treatment 2. Unstopped the treatment lasted 35 minutes. The six exercises involved the sketching of simple images and thus added only a few minutes at most.

Treatment 2 was also a videotaped lesson. The students receiving this treatment had the same instructions and exercises as the students receiving treatment 1. The video content was a lecturer teaching the same material, and the audio track was of course his voice. The scripts for treatments 1 and 2 were nearly identical (See Appendix D). Hence, the sequential development of concepts was identical to that of treatment 1. This tape lasted 43 minutes, 8 minutes longer than treatment 1, since certain illustrations and explanations took longer than on the computer-visual treatment.

Treatment 3 was a standard classroom presentation. Students receiving treatment 3 met with their regular geometry class at the regular time with their regular teacher. But on the day of the treatment, the teachers taught the topics covered in the other treatments. These topics were delimited by syllabi provided to the teachers, describing the topics to be covered, and including some examples (See Appendix C).

**Subjects:**

The subjects were 90 geometry students from a suburban secondary school. In the school three teachers taught geometry.
There were seven sections. One class of each teacher was selected to receive treatment 3, the intact classroom treatment. The remaining subjects were assigned to treatments 1 and 2, the taped treatments.

The Achievement Test:
Achievement in the subject area was to be measured by a test containing 39 items, comprised of true-false, short answer, and sketching questions. All but 17 of the questions were written and refined by the experimenter. The other 12 came from textbooks. For further scrutiny of effects several topical subtests were identified. These were (1) sketching, (2) direct/opposite, (3) visual recognition, (4) product/inverse, and (4) symmetry.

Blocking Students on Ability:
In order to discern aptitude-treatment interaction, it was necessary to block or stratify students within treatments according to mathematical aptitude. The measure of aptitude which was used was non-verbal IQ. The score was available for nearly all students, had a high reliability, and had probably as much validity for predicting a student's ability to learn mathematical concepts as any measure available. The IQ score used was found to correlate with the previous grade in geometry with a correlation coefficient of $\rho = .52$. 
In the analysis students were blocked into one of three ability levels based on IQ. So with the treatment classifications there were nine cells. Thus a scrutiny of ATI was possible by examining the interaction, or second level effects, in the two-way (treatment by aptitude) analyses of variance.

The selection of the classes for treatment 3 and the assignment of students to the taped treatments were governed by the consideration of the need to stratify by IQ. The distribution of IQ's had to be the same for the population of the three classrooms receiving treatment 3 as for the remaining students, who received the taped treatments. Students assigned to the taped treatments were paired in order by IQ and then randomly split into one of the two groups.

Hypotheses:
Principal hypotheses tested included:

(1) Effects of taped media treatments compared with classroom treatment, ( \( H_1 : \frac{T-1 + T-2}{2} - T-3 = 0 \) )
(2) The effect of the computer-visual treatment compared with the taped control, ( \( H_2 : T-1 - T-2 = 0 \) )
(3) The interaction of treatment with level of aptitude on performance on the achievement test.
The symbols $T-1$, $T-2$, and $T-3$ in the statements of null hypotheses refer to the mean performances of the computer-visual treatment group, the taped control, and the classroom lecture group respectively.

Furthermore, analyses were performed to discern possible effects of such variables as time of treatment, sex, teacher, place of treatment, and attitudes towards mathematics. Several topical subtests were analysed using multivariate techniques in an attempt to discern interactions of results with topical content.

**Attitudes:**

Research indicates that attitude towards mathematics can be a significant predictor of achievement in mathematics. (Aiken, 1971) Furthermore it is an important outcome variable in its own right. No prediction was made concerning the effect of the treatments on attitude, but it was deemed important to examine the effect of the treatments on particular attitudes.

Three attitude measures were taken using 13 items scored on a Lichart scale. These tests were:

1. Attitude towards mathematics in general
2. Attitude towards the presentation
3. Attitude towards the material
Attitudes (2) and (3) were measured only for the two taped treatments. All students took attitude test 1. A two-way ANOVA was performed for each test, and appropriate post-hoc comparisons were made.

The taped treatments were presented at two locations in the school. Each was isolated such that students could not converse with others and could avoid distraction. All taped treatments were presented during regular class hours, each subject having one whole class period to view the tape. The tapes were viewed during one week, (Monday through Friday) and treatment 3, the classroom lecture, was presented on Friday. The achievement test was given to all students on the following Monday.

In Chapter II the rationale is discussed, both from the point of view of literature on learning through media and of anticipated developments in computer graphics.

Chapter III outlines prior endeavors to determine the efficacy of computer graphics, and the steps involved in deciding upon the topics and instruments used in the final experiment.

Chapter IV deals with the technical aspects of preparing the experimental treatments, particularly the animated computer graphics.

Chapter V describes the design of the experiment, and the various instruments and procedures.
Results are presented in Chapter VI.

Chapter VII discusses the results and their implications and takes a look into the future of CAI systems with graphics.
CHAPTER II - RATIONALE AND LITERATURE REVIEW

Graphics in CAI Systems:

Computer assisted instruction (CAI) is just one of several facets of the application of computers in education. The computer can be used as a tool for educational design or management in various ways (Zinn, 1967; Gibb, 1973). Restricted to direct instructional uses, sometimes referred to as computer based instruction (CBI), CAI applications can be classified as tutorial or drill-and-practice modes, among others such as simulation, and problem solving. (Jurick, 1972; Kieren, 1973).

Effectiveness of CAI: Drill/Practice vs Tutorial:

The computer is an ideal individual administrator of drill problems (Palmer, 1973). Indeed it has been shown that CAI is quite effective for drill and practice tasks (Suppes, Jerman & Brian, 1968; Parkus, 1970). On the other hand, the use of the computer as a teacher of new facts is not as prevalent as for other purposes (Demb, 1974). In a review of CAI in mathematics education, Kieren (1973), observed that results of studies involving tutorial CAI were not uniformly positive, and indeed sometimes similar studies produced differing results. For example, studies by Isaac (1972)
and Melaragno (1966) produced respectively significant and non
significant differences between linear and branching strategies
in CAI. The failure of tutorial CAI to exhibit impressive
results can be blamed largely on the lack of quality software.
Although many functioning computer instructional packages have
been in use, (e.g., Dunn, 1974), and several institutions have
engaged in attempts at regional organization of CAI, (Zinn, 1967;
Hall & Mitzeł, 1974; Bork, 1976), there is a lack of useable quality
course materials (Ashby, et. al. 1972).

Factors in this lack include (1) the diversity of hardware
in use, (2) the necessary complexity of effective programmed
instructional materials (Igo, 1970; Bunderson, 1973), and
(3) the relatively high cost of computer hardware and time,
(Dunn & Wastler, 1972). Another drawback, related to each of the
three problems mentioned is that visual content, crucial in virtually
every aspect of learning (Arheim, 1970), has been absent in most
CAI applications. It would seem reasonable that in order to be
an effective instructional medium CAI ought to be able to provide
visual content.
Visual Content in CAI: Some Alternate Systems:

Some CAI systems have incorporated graphics by employing computer-controlled cueing of slides, (Kahne, 1969; Best, 1971; Hicks, 1971).

The Lincoln Terminal System utilized microfiche for both visual and audio content (Butman & Frick, 1972; Frick, 1973). Individual consoles had visual and audio playback facilities and could "stand alone" for periods of time, i.e. not require processing by the control computer. Further, the computer system could monitor the sequence and duration of frames viewed. However, the storage of images was limited, and special sensitive hardware was necessary. The system could not communicate completely over lines since the pictures and soundtracks making CAI were on hardcopy (microfiche).

Making CAI cost effective while providing versatile video content was the goal of the TICCIT program (Time Share Interactive Computer Controlled Information Television) (Rappaport, 1971; Stetten, 1972; Overview, 1974). Employing regular television tubes as output devices, TICCIT required extensive videotape hardware. Although fairly cost competitive, this system relied heavily on video hardware and a central media center. The number of simultaneous graphic activities was greatly limited, and the system was limited in interactive capabilities.
Cathode Ray Tubes

The capability of specifying the coordinates on the screen gives computer driven CRT's the power to provide theoretically unlimited graphics. The construction of complex visual images requires substantial computer processing.

Many CAI systems in use today employ CRT screens as alphanumeric output devices, as well as for some rudimentary plotting, e.g. (Wheaton, 1969; Beaujon, 1970; McClintock, 1971). Many mini- and micro-computer systems, also using raster type CRT's have this rudimentary graphics capability.

This capability is limited in that a small number of prepared figures can be put in one of a restricted number of lattice positions. (McClintock & Kimberlin, 1973) The capacity, for example, to draw a straight line segment does not generally exist.

Various types of CRT exist. Some must rapidly redraw (refresh) the screen cathode, some called storage tubes, can only project a constant image. Of the refresh type tubes, some draw line segments between specified coordinates, whereas some display dots of light at the coordinate points (Walker, 1970). Each of these refresh-type tubes, being capable of rapidly changing the screen image, is thus potentially capable of producing animated images (Halas, 1974). Another type of output device, whose image specification is identical in principle to raster-type CRT's, but which does not need to be refreshed is the Plasma Panel. Characteristics of these various direct computer-output devices are summarized in table 2.1.
Table 2.1

Types of Computer Graphics Output Devices and Their Characteristics

<table>
<thead>
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<th>Type of output device</th>
<th>Cathode Ray Tube</th>
<th>Refresh Tube</th>
<th>Plasma Panel</th>
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<tr>
<td></td>
<td>Characteristic</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Storage tube</td>
<td>Refresh</td>
<td>Plasma Panel</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Random</td>
<td>Vector</td>
</tr>
<tr>
<td>Relatively inexpensive</td>
<td>?</td>
<td>NO</td>
<td>YES</td>
</tr>
<tr>
<td>Draw Arbitrary lines</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
</tr>
<tr>
<td>Refresh:</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>(animation)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Early research, educational applications with large groups (Zinn, 1974), and other applications where cost is not a crucial factor have employed the graphic power of the random type tube.

For the more economical dot-oriented tubes, difficulties in constructing, and manipulating images are greater than for line-oriented tubes (Peloquin, 1968), (Bork, 1971b), (McGinnis, 1972), (McCintock & Kimberlin, 1973).
Some State-of-the-Art CAI Systems:

A variety of graphics systems exist, which could be employed as CAI terminals, although at a high cost. Reddy (1975) describes several.

One project, which attempted to employ standard graphic hardware, was the Graphics Terminal Display System (Hornbeck & Brock, 1972). This system employed storage CRT's. As indicated in table 2.1, there was no refresh capability, and so the entire screen had to be purged in order to erase one or more elements, and this does not happen rapidly. The project developed two software systems. 1. GRAIL: (Graphics Assisted Instructional Language) a course and graphics writing language, and 2. EXCALIBUR: (Extract and Collect Logically Indicated User Records) a student data maintenance program. The project included voice synthesizer output devices. It had the advantage of being portable to any IBM 360 system. Another major CAI software effort was PLANIT, (Romano, 1973) a coursewriting language.

Perhaps the largest integrated graphics-CAI project is the PLATO project (Sherwood & Stifle, 1975). This system uses prepared slides and recorded sound as well as computer drawn figures on a plasma panel (King, 1975). In PLATO, a variety of course software has been developed as well as a variety of visual materials.
Graphic, printed and audio sequences appear in an interactive feedback environment. Topics include elementary reading as well as mathematics and others. Graphically, animation is impossible but instructional programming is able to call up to 256 pictures.

The Physics Computer Development Project (PCDP) at the University of California is a sophisticated programmed computer tutorial system in undergraduate physics (Bork, 1971, 1971b, 1971c). Its efforts focus on four major areas of development. (1) The dialogue mechanism, (2) Graphics content, (3) The underlying software, and (4) an authoring system. (Arons & Bork, 1975) The graphics facility is a storage-type tube, and students are involved in the creation of graphic entities. The provision of graphics is a major commitment since on some of the more graphic-oriented sequences, 75% of the coding is for graphics. The development of effective graphic capabilities for CAI was seen as an important goal. Bork noted that other media; audio, slides, video and film sequences, leave much to be desired, and predicted that stand-alone computers, connectible to a large computer, will become inexpensive, and thus be used widely in education.
The Advent of Smaller CAI Systems:

Despite the existence of various graphics capabilities, CAI with graphics has not become a major educational medium. Brissendon and Davies (1975) attribute this to two factors, (1) the expense and (2) the lack of good generalized graphic programs. A development which may alleviate both problems will be the provision of versatile graphics in mini or micro-computer settings. (Rigney, et al., 1972; Breneman, 1973; Bork, 1975).

The growth of the computer industry and consequent development of production techniques has naturally caused prices of hardware to decrease. Dramatic examples of such development include the recent hand calculator boom, which was directly a result of the cheap mass production of micro-processor chips.

Funk (1976) anticipates CAI systems built around mini-computers with television as graphics output terminals, with interchangeable microprocessors. "Thus vendors will be able to fill the needs of different application oriented areas...because the microprocessor allows low development costs through common hardware and more flexibility through changeable programs." With cheap processors and peripheral devices, the only other major source of cost is memory.
In a recent meeting of CompCon, Oliphant (1976) cited four favorable trends in semiconductor memory: "(1) Higher bit densities per package, (2) lower cost/bit, (3) faster access time, and (4) lower power/bit." The trend of lower cost/bit is expected to continue at a decrease of more than 40% per year. Thus cheaper (Volz, 1974) and more sophisticated (Gunwaldsen, 1975) graphics capabilities can be expected in the not too distant future for small stand-alone CAI systems.

Attitudes:

Attitudes, which are an important outcome variable in their own right, have been shown to correlate with achievement in many cases (Aiken, 1970). Earle (1972) and Smith (1973) found no significant difference between attitudes of students receiving a CAI instructional program and those receiving a control. Ingle (1975) found that students regard for the expertise of computers involved a CAI higher than human sources of instruction.
The Power of Visual Media in Education

Since film became a viable educational medium in the 1940's, a great deal of research has sought either to evaluate the comparative effectiveness of visual media with other educational methods (Allen, 1971; Kalkofen, 1972) or to determine salient learner and media variables and their effect on the learning process (Travers, 1967; Lumsdaine, 1963; Levie & Dickie, 1969; Cleaves, 1976).

Early research efforts on learning from film established a certain broad basis of general results (Carpenter & Greenhill, 1956) concerning color, detail, audio-visual (2-channel) interaction (Coppen, 1972; Kalkofen, 1972).

With the advent of new media, especially television and videotape, much new research appeared (Razik & Ramroth, 1974; Kirschen, et al. 1975). Salomon and Clarke (1974) stated that much research involving media lacked either internal or external validity, and complained that no one had determined why media were effective. They noted that recent summaries of and comments on media research have, in fact, yielded very little (Seattler, 1968; Snow & Salomon, 1968; Gordon, 1970; Allen, 1971; Jamison, Suppes & Wells, 1974). It was further pointed out that there is a difference, often tacitly unrecognized, between research employing media and research with media. Wolf (1971) has noted
that controlled research into television and motion pictures has been limited by the dynamic and multivariant nature of the stimulus. This points out the fallacy some researchers have promoted by generalizing to for instance "films" from one or a few instances of films.

In studies which were both on media and with media, Rigney and Lutz (1974) (1975) examined the nature of learning through graphics and hypothesized that graphics improved learning by (1) facilitating dual coding (visual and verbal) of processes, and (2) causing the internalization of visual representations of invisible processes.

Since main effects differences in an analysis tell little about how differences in learning are produced, the importance of research designed to scrutinize ATI effects has been argued. Bracht (1970) observed the lack of results about ATI to date. Snow and Salomon (1971) provided some theoretical schema for producing a priori hypotheses. Hopefully interaction of treatment with aptitude would provide some insight into the mechanisms of learning.
Salomon (1972) hypothesized that visual media provided schemes and codes which could be internalized through imitation. In order for some demonstrated action to be internalized, (1) the demonstrated action needs to carry with it some promise for new adoptions, (2) the action must involve a certain degree of subjective novelty, and (3) the presentation of an action needs to be capable of replacing ("supplanting" is the term used) the mental operations already in the repetoir, or complementing them (p. 404). Such a supplanting process is likely to be internalized. In two studies involving tasks of selecting details and of unfolding solid objects, treatments which provided dynamic visual models, zooming-in and unfolding respectively, exceeded treatments which simply presented the original and final states. Interestingly, it was found that those initially scoring low on cue-attendance and verbal reasoning tests benefitted far more from filmic supplantation than did high aptitude subjects. Thus a pattern of disordinal Aptitude-Treatment interaction occurred.
The Application of Animated Graphics in Mathematics Education

The construction of a computer CRT image involves the specification of Cartesian coordinates. These coordinates must be integers in some finite range. (Walker, 1970) The nature of picture construction and manipulation is necessarily mathematical. Thus it would seem reasonable to apply computer graphic techniques to the representation of mathematical ideas for the following reasons:

1. Mathematical objects are suited to algorithmic or computational construction,
2. Many objects of mathematics are de facto co-ordinizized or coordinizable.
3. Examples could be constructed using the learners input of numerical parameters, proving an excellent feedback mechanism.

Others have seen the potential of animated computer graphics for teaching mathematics. Several publishing companies have produced filmloops on topics in calculus. A more ambitious project was the Carlton College project Computer Graphics for Learning Mathematics (Carleton, 1974). Numerous animated filmloops were made on a range of topics; functions and graphs, methods of analysis, probability statistics and simulation, and mathematical models.
A wide range of mathematical topics are based on metric, spatial, or in some way physical, and hence visual, entities. These include: sets, counting numbers, different bases, integers, rational numbers, the real number line, the coordinate plane, graphs and curves, surfaces, derivatives, integrals, length, area, volume, construction, nearly all geometry, approximation of , congruence transformations of the plane, rigid motions, symmetry, similarity motions, linear operations, topological concepts, limit concepts, etc.

Computer graphics, typically via raster tube, have been employed for several years in various educational tasks (Dennis, 1969; Beaujon, 1970; Graham & Read, 1971; Harding, 1974; Lewis, 1974). In many of these instances the capability of interactive construction and analysis of examples are employed.

Conclusion:

There are indications that animation via CRT will be a viable feature for CAI systems of the not-too-distant future. Such a mode of presentation could be superior to standard modes if concepts to be taught involved motion related ideas. Also such a presentation might benefit students of less aptitude to a greater extent.
Mathematical topics appear to hold promise for applications of such graphic CAI. Appropriate research seems to include suitability of this mode of presentation for particular topics, its comparative effectiveness, and the effect on student attitudes.
CHAPTER III - PRELIMINARY INVESTIGATIONS

Introduction:

In this chapter are described preliminary investigations and pilot studies which preceded the experiment. These studies helped to determine the feasibility of the project as well as the media format to be used, the content of the lesson, and the design of the achievement instruments.

The feasibility of employing computer graphics in mathematics instruction could only be determined by actual production of materials, and employment of them as educational tools. Materials were produced on two computer systems. It was decided that videotape was the optimal medium for the experiment, and student reactions to a computer-graphic lesson via that medium were studied.

The topic of the lesson was determined both upon theoretical and practical criteria. Pilots were run to delimit the material appropriately, possibly eliminate bias from test items, develop a reliable test of achievement, and refine the practical aspects of the experiment.
The Practicality of Computer Graphics for Teaching Mathematics:

Among the computer facilities available for research at Ohio State University were two stand-alone graphic systems based upon 16-bit word mini-computers. Both computers were essentially devoted to graphics users. To answer some questions about the practicality of employing these computer graphics facilities to create mathematics teaching aides, a study was funded by the Ohio State University Task Force on Learning. Its purpose was to "develop a workable technique for producing computer graphics visual aids for the university mathematics teacher." Specifically:

(1) to develop a flexible mathematics graphics routine for the IBM 1130 graphics system at OSU,
(2) to produce videotapes useable in specific courses,
(3) to find the most economical ways of producing such tapes or films or whatever, and
(4) to produce a film demonstrating the usefulness and usability of computer graphic images " (Baltz, 1975).

The study uncovered several difficulties in the process of producing computer graphic sequences. These problems included the complexity of the programming required for sophisticated graphic effects, the relatively high cost of computer time, and the necessity of using other media. This necessity created new problems both in the acquisition of the screen image, and in
the presentation to the viewer-learners. As a practical matter, the required involvement with media and machinery prevented most teachers from availing themselves of this media.

Getting useful graphics was not impossible, and several versatile mathematical routines were written for both graphics systems. One, an IBM 1130 system, had Fortran type computation capabilities, and this was the one for which the mathematical programs were written. The other system, based on a PDP 11 computer, had many sophisticated graphics commands available including ones providing for a quality film image. Each of these two systems was used to produce a short color film sequence for use with televised calculus lessons at Ohio State. Figure 3.1 shows blown-up frames from the color film sequences.

Various differences existed between the two systems. The computational capabilities of the IBM system enabled easy graphing of mathematical functions and related derived graphical information. The PDP system was designed for non-technical use and afforded somewhat less in the way of Fortran-type computation. However the system was equipped with a synchronized camera, and many sophisticated graphics commands. Further technical details of these and other computer graphics systems are discussed in Chapter IV.
Graph of Curve with Tangents
Drawn on IBM 1130 System

Integration: Lower Approximation
Drawn on PDP System

Figure 3.1 Blown-Up Frames from Two Computer-Generated Films
Development of Methodology:

For economic reasons, the use of graphic CAI terminals was not practical. Several options were available for the presentation of instructional material drawn by computer on a CRT. Questions concerning feasibility had to be answered.

For the problem of instrumentation, the desire to create unbiased test items led to the investigation of a scheme for rating test items as to their degree of visuality.

The Medium for Presenting Computer Graphic Material:

The format of the computer-visual treatment was to duplicate the media features of a CAI system with animated graphics as closely as possible. Several possibilities for medium-of-presentation were evident. They were:

(1) Individual or small groups of students learn at computer graphic console.
(2) Animated graphics, filmed, projected in classroom with lecture.
(3) Still graphics, via slide show, or otherwise, with taped narration.
(4) Individual, or small group, screening of film with sound track.
(5) Closed circuit broadcast, repeated, of computer animated lesson with narration.
(6) Individually controlled lesson, with narration, on videocassettes.

Option (1) was not feasible for economic reasons. $50.00/hour is a low estimate of cost for the use of either graphics system. Options (4) and (5) eliminate student control of progress of the lesson, and are further from the nature of CAI interaction with students than (6). (2) also minimizes student control of progress and eliminate the independence of student treatments. (3) would lack the animation of images, and such a treatment would not suit the needs of the study.

Aside from the advantages of being individually applied, in an isolated setting, (6) featured a CRT (television) output, as did (5), but also involved student mechanical control of progress of the lesson. Certain additional logistical advantages existed with the use of videocassettes. Specifically:

(1) Various playback facilities were available including the University Learning Resources Center and portable equipment which could be borrowed from the University Teaching Aids Laboratory.
(2) Duplication of tapes was easy and inexpensive.
(3) Sound could be easily overdubbed and changed.
The visual content of the videotapes, i.e. the playback of the recorded CRT screen, was identical in pace and similar in intensity to computer CRT output. Resolution and tonal quality of the image (light on dark background), size, and proximity of screen were all closely similar to those aspects of the visual content of a CAI system with CRT graphics. Thus, many of the media features of the CRT were duplicated. The most important differences between a videotape playback and a computer CRT included the fact that the video system was not connected to a computer and the subsequent facts that branching and computational potentials of computers were absent. Although the tapes could not provide branching, no other feasible option could provide it, and linear control was possible. That is, the lesson could be stopped or backed up and replayed at the student's wish. One other major difference between the taped lesson and typical CAI systems was the provision of audial content along with the computer visuals on the tapes. This audio content eliminated the dependency upon reading skills and extensive auxiliary printed materials or large amounts of printing on the screen. It was not an unreasonable or unfair addition to the treatment, as such narration parallel to screen content was well within the range of current technology. Indeed various CAI systems have employed random access microfiche (Butman & Frick, 1972) or audio tape as in PLATO (King, 1972)
to provide recorded narration. Furthermore, computer voice simulation is by no means an impossibility (Hornbeck and Brock, 1972; Suppes, 1975; Rice, 1976).

Thus, a videotape of computer-generated CRT output with accompanying narration was adjusted to be an optimally representative medium to duplicate the effects of computer-generated animated graphics within a CAI setting.

A First Computer-Generated Visual Lesson:

Once videotape cassettes were determined to be the most accessible and appropriate means available for presenting a computer graphic mathematics lesson, a small pilot activity utilizing this medium was planned as a feasibility study.

A 15-minute long videotape with audio narration, concerning aspects of graphing (translations and distortions of functions, especially trigonometric functions) was produced on the IBM system described above. The lesson consisted of various graphs with explanations, and, although not employing animation as such, did feature rapid juxtaposition of figures.

Eighteen students enrolled in a pre-calculus trigonometry course at Ohio State in Spring, 1975 voluntarily viewed the tape, worked five problems, and answered items concerning the treatment. Distributions and means of their responses are presented in Table 3.1.
Table 3.1

**Attitudes Towards Trigonometry Videotape Lesson**

(Strongly Disagree = -2 , Disagree = -1 , Undecided = 0 ,
Agree = 1 , Strongly Agree = 2)

The statements are listed in order of greatest mean response:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Responses</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>The videoplayer is easy to operate.</td>
<td>9 9 0 0 0</td>
<td>1.50</td>
</tr>
<tr>
<td>I like to see graphs and diagrams in a lesson</td>
<td>6 10 0 1 0</td>
<td>1.24</td>
</tr>
<tr>
<td>I have found the tapes helpful.</td>
<td>6 9 4 0 0</td>
<td>1.11</td>
</tr>
<tr>
<td>The worksheet helped.</td>
<td>4 7 3 0 0</td>
<td>1.07</td>
</tr>
<tr>
<td>Other tapes of this nature might be helpful.</td>
<td>2 9 2 0 0</td>
<td>1.00</td>
</tr>
<tr>
<td>I learned some mathematics previously unknown.</td>
<td>0 7 3 0 0</td>
<td>0.70</td>
</tr>
<tr>
<td>Audio quality was good.</td>
<td>5 4 0 4 1</td>
<td>0.57</td>
</tr>
<tr>
<td>This method (videotape) could be even more helpful.</td>
<td>2 7 5 3 0</td>
<td>0.47</td>
</tr>
<tr>
<td>The material was poorly presented.</td>
<td>0 6 2 6 0</td>
<td>0.00*</td>
</tr>
<tr>
<td>Video quality was good</td>
<td>2 4 1 5 2</td>
<td>-0.07</td>
</tr>
<tr>
<td>I enjoy mathematics.</td>
<td>1 1 2 2 3</td>
<td>-0.11</td>
</tr>
<tr>
<td>I couldn't understand the material.</td>
<td>0 4 1 7 2</td>
<td>-0.50*</td>
</tr>
<tr>
<td>I don't like lessons from a machine.</td>
<td>1 0 2 1 3</td>
<td>-0.76*</td>
</tr>
</tbody>
</table>

* Items to be reflected.

The results were encouraging. The only negative responses were for (1) enjoyment of mathematics in general (\( \bar{x} = -.11 \)) and (2) the video quality (\( \bar{x} = -.07 \)). The dissatisfaction with video was very likely due to the fact that editing on videotape causes several seconds of jumbled picture for each "cut". It was because
of such editing problems that it was decided not to tape directly from the CRT. The lack of enjoyment of mathematics could be attributed to the population in general, rather than to effects of the treatment. The other positive responses seem to support that view. Thus, videotape cassettes were felt to be an acceptable and viable medium for presenting the experimental mathematics lesson.

**Classifying Test Items as to Degree of Visuality:**

In a test of concept attainment, some items might be biased toward students whose instruction had a visual or physical emphasis as opposed to a computational one, or conversely towards those whose did not. A scheme for rating items according to their visual bias was tried, and tested for effectiveness.

The rating scale was a Likert scale with five levels going from strongly visual or physically manipulative to strongly computational. The items to be rated were those from the post-test employed in the pilot activity described in the previous section. The rating sheet is shown in figure 3.2.

Two different sets of raters were employed. First, the students who did the problems were subsequently asked to rate them. Secondly, a group of graduate students in mathematics
education was asked to rate the same items along the same scale. These were 10 students in a graduate seminar in mathematics education at Ohio State, Spring, 1975. The results are summarized in Table 3.2.

Table 3.2

<table>
<thead>
<tr>
<th>Question</th>
<th>Student Rating n = 18</th>
<th>Graduate Rating n = 10</th>
<th>z-score of difference (significance)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Mean: 3.7, Stand. Dev.: 1.1</td>
<td>Mean: 2.5, Stand. Dev.: 0.50</td>
<td>4.04 (p &lt; .01)</td>
</tr>
<tr>
<td>2.</td>
<td>Mean: 3.1, Stand. Dev.: 0.83</td>
<td>Mean: 2.3, Stand. Dev.: 0.46</td>
<td>2.53 (p &lt; .01)</td>
</tr>
<tr>
<td>3.</td>
<td>Mean: 4.2, Stand. Dev.: 0.76</td>
<td>Mean: 3.1, Stand. Dev.: 1.09</td>
<td>2.56 (p &lt; .01)</td>
</tr>
<tr>
<td>4.</td>
<td>Mean: 3.2, Stand. Dev.: 1.6</td>
<td>Mean: 2.1, Stand. Dev.: 0.54</td>
<td>3.81 (p &lt; .01)</td>
</tr>
<tr>
<td>5.</td>
<td>Mean: 2.7, Stand. Dev.: 1.29</td>
<td>Mean: 2.6, Stand. Dev.: 0.49</td>
<td>0.23 (N.S.)</td>
</tr>
</tbody>
</table>
Below are questions to be used on the post-test for the pilot experiment. Please solve each of the problems, and then classify each as one of the five categories. (The distinction to be made is between rote computational techniques and physical or visual manipulation.)

1. relies too heavily on visual or physical manipulation.
2. more visual but an appropriate item.
3. appropriate.
4. more computational but appropriate.
5. overly computational.

A. Sketch the graph of \( y = -3 \sin(2x) \)

B. Below is the graph of \( y = f(x) \). Sketch the graph of \( y = -3f(2x) \)

C. If the graphs of \( y = f(x) \) and \( y = g(x) \) intersect at \((3,1)\) and \((-2,3)\), where do the graphs \( y = f(2x) \) and \( y = g(2x) \) intersect?

D. Circle the operations which do not change the shape of a graph: XD, XT, YD, YT, XR, IR

E. Sketch the graph of \( y = 3(x-2)^2+1 \) using translations and distortions.

Figure 3.2 Rating Sheet Used by Graduate Raters
Due to the relatively large standard deviation ($\hat{\sigma} = 1$, with range of 4), as well as the failure of the scores to conform (four of the five ratings were significantly different $p < .01$), it was decided to not use this technique for classifying the test items.

Although the scores differed between the two groups doing the rating, the coefficient of correlation was .63, indicating a degree of consistence or reliability in the raters.

The strong positive correlation indicated that this technique for rating items has some value for determining the relative bias of an item. But in order to determine the absolute bias of an item, some objective standard had to be established. However it is questionable if it is possible to operationally define what would constitute a bias in favor of a student who had a visually oriented instruction.

Failure to provide a check on the bias of items on the achievement test in favor of one treatment population might invalidate results. However the lack of control of the degree of visuality of items was not harmful.

(1) Questions would be constructed which were general or categorical, having an objective mathematical nature. (See Appendix C). Thus a "bias" in the questions, would indicate "bias" in the material. And so whatever incidental advantages which say, a
visual approach might have concerning that material would be substantial advantages and should be recognized as such.

(2) A major concern of the analysis is the question of the presence of aptitude-treatment interaction. Results on this phenomenon would not differ under the biasing of a test towards a treatment unless the degree of the bias interacted with ability. This (although possibly a real phenomenon) would be virtually impossible to detect. So again, although first-order effects might be affected by a bias in the instrumentation, interaction effects would not. In other words, a test possessing content validity will be considered to be unbiased.

**Decision of Content:**

Several criteria based either upon research or practical conditions were applied in determining a subject area for the experiment involving computer graphics:

(1) In order for the potential value of animation to be maximized, the principal concepts needed to have elements which were strongly spatial, pictorial, or time or motion oriented.

(2) The graphics needs of presenting the particular material had to be met by an available graphics system. Two systems were available for producing animation sequences to be put onto
videotape. One had sophisticated graphics commands but limited computational power. The other had Fortran-type calculation power but limited graphic commands. The chosen system had to provide suitable iconicity and graphic versatility.

(3) A population of students had to be available to receive the treatments in the particular subject area at the particular level. Furthermore, the topics of the lesson had to be uniformly unfamiliar to the students.

Geometric concepts come to mind as being representable graphically. In addition, many students at both secondary and college level take geometry. Concepts in transformational geometry are quite useful in a variety of applications, and involve the processes of rigid motions of the plane. A transformational approach is still non-standard and hence was expected to be unfamiliar to students.

Another reason for deciding upon topics in transformational geometry is criterion 2 above. The system which afforded easy film animation and image manipulation had graphic facility for rigid motions in 3-space. Pictures were stored and manipulated as 3-dimensional objects, although they were displayed projected onto a 2-dimensional screen. This consequently provided convenient translations, rotations, and particularly reflections (flips) of
the plane. Therefore generation of appropriate examples would be quite straightforward.

In order to put a time-related or action emphasis on the material, each of the three basic rigid motions was represented as a continuous transformation. Products were presented as sequential. Congruence was shown through continuous motion to juxtaposition. It should be noted again that a graphical approach could be employed without using animation, but rather only still figures. In the case of rigid motions the original figure, the notation and the image could be presented in some appropriate sequential format, the action of the particular rigid "motion" being inferred by the learner. It is felt that such an approach although perhaps effective would be different from the one employed. This point is discussed further in Chapter VII.

The content decided upon included the basic notions of congruence developed through rigid motions of the plane. Included also was the nature of these motions, generating motions, direct and opposite motions, compositions or products, inverses, and basic notions of symmetry and symmetry groups. (See Appendix B). The development was consistent with that of the UICSM Motion Geometry series (Phillips and Zwoyer, 1969).
With the assistance of university faculty members who had taught methods courses in geometry for teachers, a first outline of topics was prepared. The content was outlined as follows:

I. Definitions and examples
   A. Slide, flip, turn
   B. Direct, opposite

II. Composition
   A. Definition
   B. All rigid motions are a product...

III. Congruence in terms of rigid motions

IV. Economy and representation
   A. direct is slide or turn
   B. slide is 2 flips
   C. turn is 2 flips
   D. opposite is flip and slide
   E. opposite is flip and turn
   F. every rigid motion is 2 or 3 flips

V. Group properties
   A. product
   B. closure
   C. identity
   D. inverse
   E. not commutative
   F. associative?

VI. Symmetry

More carefully refined organization of content can be seen in Appendix B, the syllabus, and in Appendix D, the scripts of the taped treatments.
Pilot Activities

A First Trial

Originally, it was decided to use the topics "Congruence through rigid motions" with undergraduate education majors in a geometry course at Ohio State University for Autumn Quarter 1975. This group of students provided the most convenient population for several reasons. Among these was the availability of a similar population taking the geometry course during Summer 1975, with whom pilot study could be conducted.

In order to decide how much material could be covered in a one-hour lecture, and to measure the difficulty and reliability of some of the test items, the experimenter taught these topics to a class of 13 students in Math 106 (Geometry for Teachers) in Summer 1975 for three periods. Four multiple-part questions concerning this material were included on the final examination, comprising 13 separate items (Figure 3.3). Summary statistics are presented in Table 3.3.

It was found that some of the items were too easy, (had difficulty 0.) and some were apparently ambiguous and did not correlate well with performance (Table 3.4). When these were eliminated, the resulting test of seven items had reliability $KB_{20} = .76$. It was felt that this was a satisfactory beginning. A substantially longer test would be used in the experiment.
13. Sketch the image of the following figures under the motion indicated.

14. Match the image of A with its corresponding transformation.

i. _______ translation

ii. _______ reflection

iii. _______ rotation

A. 5

15. Name five capital letters which are reflections of themselves.

16. True - False

_________ The product of two translations is a translation.

_________ The product of two flips is a flip.

_________ No quadrilateral is congruent to a triangle.

_________ Every congruence is a product of translations.

_________ A circle of radius 2 is congruent to any other circle of radius 2 by a translation.

Figure 3.3 Test Items on Transformational Geometry for Summer Class
Table 3.3
Performance of Summer Geometry Students on 13-Item Test

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of subjects</td>
<td>n = 10</td>
</tr>
<tr>
<td>mean</td>
<td>$\bar{x} = 9.62$ (out of 13)</td>
</tr>
<tr>
<td>range</td>
<td>6.4 (6.6 to 13)</td>
</tr>
<tr>
<td>variance</td>
<td>$\sigma^2 = 3.51$</td>
</tr>
<tr>
<td>reliability</td>
<td>KR20 = .52</td>
</tr>
</tbody>
</table>
Table 3.4

<table>
<thead>
<tr>
<th>Problem</th>
<th>Item</th>
<th>Discrimination</th>
<th>Difficulty</th>
<th>Item-test correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>13i</td>
<td>1</td>
<td>0.00</td>
<td>0.2</td>
<td>-0.02</td>
</tr>
<tr>
<td>13ii</td>
<td>2</td>
<td>1.00</td>
<td>0.7</td>
<td>0.65</td>
</tr>
<tr>
<td>13iii</td>
<td>3</td>
<td>0.33</td>
<td>0.1</td>
<td>0.54</td>
</tr>
<tr>
<td>14i</td>
<td>4</td>
<td>0.00</td>
<td>0.0</td>
<td>0.00</td>
</tr>
<tr>
<td>14ii</td>
<td>5</td>
<td>1.00</td>
<td>0.6</td>
<td>0.86</td>
</tr>
<tr>
<td>14iii</td>
<td>6</td>
<td>0.50</td>
<td>0.6</td>
<td>0.54</td>
</tr>
<tr>
<td>15i</td>
<td>7</td>
<td>0.2</td>
<td>0.06</td>
<td>0.34</td>
</tr>
<tr>
<td>15ii</td>
<td>8</td>
<td>0.3</td>
<td>0.38</td>
<td>0.47</td>
</tr>
<tr>
<td>16i</td>
<td>9</td>
<td>0.0</td>
<td>0.0</td>
<td>0.00</td>
</tr>
<tr>
<td>16ii</td>
<td>10</td>
<td>0.66</td>
<td>0.3</td>
<td>0.22</td>
</tr>
<tr>
<td>16iii</td>
<td>11</td>
<td>0.0</td>
<td>0.0</td>
<td>0.00</td>
</tr>
<tr>
<td>16iv</td>
<td>12</td>
<td>0.66</td>
<td>0.5</td>
<td>0.57</td>
</tr>
<tr>
<td>16v</td>
<td>13</td>
<td>0.0</td>
<td>0.0</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Items with positive correlation with the total test, and difficulty greater than or equal to .1 were included on subsequent versions of the achievement test. These were the seven items indicated with arrows. (†) This selection corresponds to selecting items with discrimination greater than .25 (Discrimination = fraction of upper quartile passing the item minus the fraction of the lower quartile passing the item.) Selecting items by these criteria provided for a test with reliability KR 20 = .76.

To perform the experiment as planned in Autumn 1975, the topics had to be delineated, content had to be specified, and the
preparation of the treatments had to be accomplished. This production procedure (See Chapter IV) took more time than originally anticipated. Since it was impossible to perform the experiment with the Autumn geometry class at Ohio State, and since there was no section of this course during Winter Quarter 1976, it was decided to do the experiment with secondary students enrolled in geometry during the winter.

Such a changing of subject population did not effect the content or scope of the subject matter in the treatments. The material required some background in geometry, but was essentially an independent topic. In particular, the differences in history and of maturity between the education majors and the high school students were not expected to present vast differences in ability to learn the topics in transformational geometry.

However, the appropriateness of the test items with respect to difficulty certainly might have changed. Therefore new reliability measures were necessary.
Expanding the Achievement Test:

In order (1) to provide a highly reliable test, and (2) to provide a measurement of achievement on the many facets of the topic, a longer test had to be employed. The experimenter sought items concerning rigid motions and congruence at a difficulty level appropriate for secondary students. Sources such as Buros (1974) and Suydam (1974) were examined. However suitable items were not located. Thus the achievement test was to be comprised of items written by (1) the experimenter, (2) the classroom teachers involved, and (3) the authors of appropriate texts.

Certain of the easier questions were retained or modified. (13i,14i,15i,16i). Questions 14 i, ii, and iii were combined into one item. In addition to the three true-false items, the three short answer items, and the three sketching items, the experimenter wrote twelve true-false, five short answer, and two sketching items. From the texts Anderson, Garen & Gremillion (1966) and Krause (1972) six true-false and 5 short answer questions were taken.

The resulting test was scored as a 39 point test. Questions 33 and 34 involved both a sketching and a positional aspect. Question 36 was scored as two items. Thus there were 36 numbered questions. The 39 items scored involved 21 true-false, 5 image sketching, and 13 short answer items.
The Pilot:

The new student population was from the geometry classes in a four-year suburban secondary school. In order to test the feasibility of the experimental treatment, and to eliminate confusing content, instructions, and/or test items, a small pilot was performed with a group of students in the honors section of geometry at the school. The same instructions were given, the same procedures followed, virtually the same test items used, and virtually the same experimental treatment given was as were to be given in the main experiment. The achievement test and computer-drawn treatment were not in final form, as they were to be improved based upon results of this pilot study.

The 13 students were different from the actual subject population in that they were generally younger (mostly freshmen rather than mostly sophomores). The amount of mathematical experience was similar to that of the general subject population.

The treatment which corresponded to Treatment I or the computer-visual treatment in the main experiment was presented over a period of three days. Students had been asked for a list of free periods, and were notified by note of the time and place to appear. The video apparatus was located in a booth in the library. It was possible to schedule each of the 13
students for one of the 17 periods of a 2\(\frac{3}{2}\) day span. Eleven of the students were able to come at the appointed time. (The two others had unforeseen conflicts.) Completion of the post-test was voluntary. Seven of the students returned completed tests. In addition to the post-test, students were asked to comment on the aspects of the treatment which might be improved. Results on the achievement test and the results of the pilot study were summarized as follows: Performance statistics are presented in Table 3.5.

(1) The notification procedure was adequate. No student failed to come except because of conflicts, i.e. no student "forgot".

(2) The presentation format was certainly feasible. The students had little trouble operating the videotape player, and there seemed to be an appreciable amount of learning, judging from performance on the test.

(3) The time restrictions in the main experiment were adjudged not to be a problem. That is, no student needed more than the one class period to view the tape to his/her satisfaction, and several used much less. Also the achievement test was completed in a half hour or less.

(4) The test could be improved. (See Table 3.6). Certain deficiencies in the wording of the test were identified using both students' comments and their performance.
(5) An additional check of the test reliability was called for.

Table 3.5
Statistics of the Pilot Group on the Achievement Test

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of subjects</td>
<td>n = 7</td>
</tr>
<tr>
<td>Mean</td>
<td>( \bar{x} = 19.0 ) (out of 26)</td>
</tr>
<tr>
<td>Range</td>
<td>6.5  (15.5 - 22)</td>
</tr>
<tr>
<td>Variance</td>
<td>( \sigma^2 = 5.125 )</td>
</tr>
<tr>
<td>Reliability</td>
<td>KR 20 = .32</td>
</tr>
</tbody>
</table>

Due to the results of the study, five items were dropped from the achievement test. Three of these items had exceedingly high relative difficulty (1.1, 1.9). One was too easy (relative difficulty = 0). Another item was ambiguous and correlated badly with performance.

In addition, several items were modified to increase their effectiveness, as indicated in Table 3.6.
With the 5 items dropped, the scores, variance, and reliability were computed. (See Table 3.5.) The reliability was unacceptably low. However the estimation of reliability could have been spuriously low due to the small variance of the scores of the pilot group. So it was decided to obtain another measure of the test's reliability. The results of the subsequent administration of the test are discussed in a following section.

Results of the Attitude Tests:

Two types of attitudes were measured. The first was attitude towards mathematics (Attitude I). It consisted of only three items. The second (Attitude II) contained eight items concerning
the students' evaluation of the taped presentations. The test was only for response by subjects receiving videotaped treatments (Treatments I and II), and was split into Attitude 2 and Attitude 3 for the main experiment.

The Likert scale responses were converted to dichotomous data in order to use the Kuder-Richardson formula 20 for the estimation of test reliability.

Attitude II was found to have reliability = .68. Attitude I had only three items and the fact of having only ten subjects might account for its computed reliability being only .23. To improve this, three items from Aiken's Revised Math Attitude Scale (1963, reliability .94) were added to Attitude I.

Questions in the second part of the attitude test dealt with the tape presentation itself. In general the attitude of these students to the treatment was positive. Average responses are indicated by the vertical bars in Table 3.7.
Table 3.7
Attitude Test and Responses, Pilot Group

Please indicate your degree of agreement with each of the following statements. Circle one of the five options: SD...Strongly Disagree D...Disagree U...Unable to say A...Agree SA...Strongly agree

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
<td>SA</td>
<td>1. Mathematics is one of my better subjects.</td>
<td></td>
</tr>
<tr>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
<td>SA</td>
<td>2. I feel at easy with mathematics.</td>
<td></td>
</tr>
<tr>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
<td>SA</td>
<td>3. I enjoy the challenge of mathematics problems.</td>
<td></td>
</tr>
<tr>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
<td>SA</td>
<td>4. I would enjoy learning more about rigid motions and symmetries.</td>
<td></td>
</tr>
<tr>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
<td>SA</td>
<td>5. I would enjoy learning more mathematics from videotapes.</td>
<td></td>
</tr>
</tbody>
</table>

Concerning the presentation of the material concerning rigid motions and congruences:

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
<td>SA</td>
<td>6. The material was too hard.</td>
<td></td>
</tr>
<tr>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
<td>SA</td>
<td>7. The presentation was too fast.</td>
<td></td>
</tr>
<tr>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
<td>SA</td>
<td>8. The presentation was too slow.</td>
<td></td>
</tr>
<tr>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
<td>SA</td>
<td>9. The presentation was interesting.</td>
<td></td>
</tr>
<tr>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
<td>SA</td>
<td>10. I feel I learned a lot.</td>
<td></td>
</tr>
<tr>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
<td>SA</td>
<td>11. The subject matter seemed interesting.</td>
<td></td>
</tr>
</tbody>
</table>
Additional Reliability Check:

Another administration of the achievement test (with the improvements, and on a new group of students) was felt necessary to obtain a more representative reliability score.

Twenty-seven college students in Mathematics Education were given a one half hour lecture on rigid motions and congruence, and then were given the revised achievement test. The results are summarized in Table 3.8.

Table 3.8
Statistics on Achievement Test (OSU Juniors)

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>$\bar{x} = 29.6$ (37 possible)</td>
</tr>
<tr>
<td>Range</td>
<td>20 - 37</td>
</tr>
<tr>
<td>Variance</td>
<td>$\sigma^2 = 12.08$</td>
</tr>
<tr>
<td>Reliability</td>
<td>$KR_{20} = .75$</td>
</tr>
</tbody>
</table>

This group differed from the experimental population in age, (by approximately five years) and experience in mathematics, (most had taken a geometry course in college). It would thus be expected that the mean performance would exceed that of secondary students. However, the topics and approach were still unfamiliar to the students, and the instruction they received
was only cursory. So their performance was expected to be reasonably similar to that of secondary students and the reliability of the test could be inferred from a high reliability with the college students.

After examining each item (for correlation with the total test and for difficulty) it was decided that the test was satisfactory. Both pilot groups achieved approximately 80% on the test, and it was felt that the increased difficulty that the typical secondary students might encounter would make the test quite appropriate. The reliability of .75 provided some reassurance that an earlier low estimate of the reliability was spuriously low. Validity of the test is discussed later.

Conclusion:

In this chapter the steps involved in deciding upon the format, selecting the content of the treatments, determining the subject population, and developing instrumentation were described. The process of creating the treatments, particularly Treatment 1, a videotape simulating graphic output from a computer CRT, is described in the following chapter.
CHAPTER IV - THE PRODUCTION OF COMPUTER GRAPHIC MATERIALS

Introduction:

In Chapter II, it was argued that the most appropriate experiment would involve three treatments:
(1) The computer-drawn visual treatment, employing animated examples;
(2) A videotaped lecture following the identical pedagogical development of the computer-drawn treatment but without the computer-generated visual content;
(3) A standard lecture in classroom conveying the same content as the videotaped lectures.

In Chapter III, videotape in the format of 3/4 inch cassettes was determined to be a feasible and seemingly optimal choice among available media for simulating a CAI system with animated graphics. Preparation for this experiment, then, involved the utilization of computer graphic facilities to produce visual sequences which were integrated into a lesson on videotape.

In this chapter, some pertinent aspects of computer-cathode ray tube technology are discussed, not only in terms of the immediate problem of producing animated computer graphic sequences, but also in terms of the direction of development of computer graphics hardware. The process of production of the computer-graphic treatment, as well as the other treatments, comprises the rest of the chapter.
Aspects of Computer Graphics Technology:

Any computer technology involves hardware (processors, memory, peripheral devices) and software (programming). Hardware determines and limits what can be done, but appropriate software is required in order to utilize the computer's capabilities. So first hardware involved in computer graphics will be considered, with implications for animation, and then software.

Hardware: Two Types of Cathode Ray Tubes:

In a cathode ray tube photic information is created from electronic information as a stream of electrons strikes a phosphor screen and emits light. Screens can be classified as "storage" tubes, on which a point or line will remain until the entire screen is purged, or as "refresh" tubes, on which the graphic content must be redrawn constantly (usually 30 times a second). It is this second type, the refresh tube, which has the capability for versatile animation. The trick is to construct the succession of "frames" in the graphic sequence and have them change at an appropriate rate (Halas, 1974).

Two different kinds of refresh tubes are in standard use. The random type can connect arbitrary points of a $2^n \times 2^n$ grid creating line segments (random vectors). Arrays of such vectors make-up the picture. Graphic systems of this type require a vector generator, a piece of hardware, for operation. The
other type of refresh tube is the raster scanning type tube.
The television picture tube is of this type. A typical computer output raster tube covers the screen row-by-row, with \( 2^n \) rows of \( 2^n \) points, each point being "on" or "off". Some systems have variable intensities. In a raster tube, the picture is a union of dots, so line segments must be constructed by program software. The differences of the tubes for the task of representing line segments can be illustrated by the 2 examples on \( 4 \times 4 \) grids in figures 4.1 and 4.2. In figure 4.1 the points \((1,4)\) and \((4,1)\) are connected on each of the two tubes.

![Diagram of raster tube and random tube]

Figure 4.1
Schematic Line Segment from \((1,4)\) to \((4,1)\) on Two Types of CRT
In figure 4.2 the segment from (2,4) to (4,1) is drawn. We see that the raster type tube cannot construct actual straight lines. Notice on the raster tube in figure 4.2 dots are colored which do not actually lie on the line segment. However, since the grids are generally at least 256 x 256, virtual straight lines can be made as indicated, and will be perceived as straight. So, no substantial graphic deficiency exists. But of course these "straight" lines require software in order to be constructed. Indeed a variety of types of straight lines is possible, since a variety of conceivable algorithms, with different criteria for "incidence" do exist. The importance of line-segment orientation rather than point orientation is in the capability manipulation of pictures. To rotate a picture, it would possibly be easier to rotate the endpoints and "fill in" the rest, rather than rotate each point.
Software: Examples of Animation Systems:

In many refresh systems, the array of vertices or "vectors" can be changed dynamically, e.g. by rapid substitution of coordinate values, thus affording animation. The nature of the animation possible is a function of the structure of the data in the graphics array, and of the software commands available.

A very basic system would allow the user to construct an array of \((x,y)\) coordinates to be connected, in order, with line segments. Of course the option to "jump" or draw with the electron gun off would be necessary. In such a system changes in a picture could be effected by, for example, providing a different \((x,y)\) value somewhere in the array, which would have the effect of moving some vertex and, consequently, the two line segments attached to it. A more natural type of manipulation would be to add a certain value, say \(N\), to all \(x\) coordinates. This would have the effect of translating the entire screen contents or at least all elements so transformed, to the right by \(N\) raster units.

A more sophisticated system might allow for incremental changes in \((x,y)\) coordinates rather than only absolute endpoints. An entire picture drawn with relative coordinates could thus be shifted about the screen rigidly with only having to determine its first set of coordinates.
With sophisticated hardware and appropriate software, separate pictures could be stored and recalled upon command, duplicated, scaled, distorted, shaded, merged, rotated, translated, or flipped in real time (i.e. dynamically) according to digital or analog inputs. Data could be maintained as three dimensional and projected onto a two-dimensional screen. The PDP graphics system described earlier possessed these features. (Csuri, 1973)

More sophisticated software systems would involve hidden line or hidden surface algorithms. Any of the graphic functions mentioned above can potentially be performed entirely by software.

For raster graphics, the optimal data organization and manipulation schemes have not been developed. There is much to be said for defining pictures as unions of flat surfaces defined by their vertices for such graphic features as algorithms for shading, hidden edges (lines), and hidden surfaces. (Myers, 1975).

Of course, there is virtually no limit to the versatility of graphic systems, save cost of hardware and development of software. For large-scale educational applications, cost would certainly be a crucial factor.
Directions of Development of Computer Graphics:

It is difficult to predict accurately the future development of computer and video technology. But in the near future several conclusions are clear:

(1) Of the refresh type, only raster-scanning-type tubes are likely to become inexpensive computer graphic output CRTs. Plasma panels will probably also be moderate in cost. The television type raster enjoys the benefit of large scale research and development including the potential home-entertainment computer games market. Vector generators are expensive now, and are not being developed for a mass market.

(2) Although raster tubes are in a sense limited in image construction capability, as mentioned, with improved software and increased processing potential, virtually any figure possible on a random-type tube will be possible on a raster tube. It seems quite reasonable to expect that processors to construct line segments on raster tubes will be mass produced.

(3) Animation will be possible. Given sufficient processing power, images could be dynamically calculated and constructed. If highly complex images were desired, computations could be made in advance and the successive arrays for the raster could be stored in a mass memory device for recall and display.
(4) Hardware costs should be moderate since (1) raster CRT's or televisions are a mass consumer item, (2) processors, if mass produced, become quite inexpensive (Funk, 1976), and (3) memory is also becoming less expensive with time (Oliphant, 1976).

Two Software Systems:

It is clear from the earlier discussion that for line-drawings, the random-type refresh type is easier to implement than raster tubes. Both graphic systems available for general computer-graphic research at Ohio State were of the random vector type. One was an IBM 1130 with a Fortran package of instructions, operated by the university's Instructional and Research Computer Center. The other, operated by the Computer Graphics Research Group, used a PDP 11 system. It had facilities for constructing, storing, recalling, and manipulating specific pictures.

The IBM 1130 system was of the more basic type described. The array of points had to be reconstructed if the picture were to be changed. Thus extensive or complex programming was required if animation was to occur. Incremental instructions were available, so translation of pictures in the operations on plane was easy. However features such as rotations were not at all simple.
The software system for the PDP system (called "ART") was suited to reproducing rigid motions and linear distortions of the plane or 3-space. Each picture was stored as an array of 3-D coordinates and could be recalled from memory as needed, displayed, moved, or rotated about virtually any axis. These movements could be controlled by program, by computation, or by hand using analog control devices.

Since the topics selected emphasized elements of motion rather than interactive computation, the system with the sophisticated picture manipulation language, albeit with inferior computation, was to be used. This decision was made essentially simultaneously with the decision to deal with the subject of transformational geometry.

It is relevant to question whether the graphics obtained from a sophisticated random-type tube are of the same nature in terms of conveying information, particularly mathematical concepts, as would be images obtained from scanning tubes based on mini-systems of the future that might be envisioned. In terms of sophistication of graphic operations and iconicity of pictures, the answer would have to be "yes". The capabilities employed were all graphically simple, but not trivial. These basic motion and manipulation potentialities would be available in any basic "good" graphics system. It is assumed that line segment
construction would be available as well as the capability of rigid motions. The fundamental importance of rigid motions is discussed in Chapter VII.

**Getting the Screen Image onto Videotape:**

For reasons discussed already, the experimental treatments were to be put on videotape cassettes. Two options were available to get the graphics onto the tape. The first was taping from the screen image. The second was to film the screen, and to make a tape copy of the film.

It was difficult to get a strong video image directly from the CRT due to the relatively weak intensity of the beam. (On the IBM system the beam was stronger, took longer to fade, and consequently could be taped directly as was done in the pilot activity on trigonometry. See Chapter III. Furthermore there were potential sync and strobe problems.\(^1\) The PDP system was designed for synchronized filming from the screen. Hence it seemed to make sense to film from that system, edit the film, and then transfer that image onto videotape.

\(^1\) This is typical of advances in the field. The graphics system now employed by the Computer Graphics Research Group incorporates the capability of producing a video signal that can be put directly onto tape from the computer without any photo stages.
The Production of the Treatments:

The production of computer animated materials involves several levels of development. Halas (1974) identifies conception, development, production and editing as the four major stages. For this experiment the tasks were as follows:

CONCEPTION: Determine the extent of the content. Produce a syllabus. Determine graphic approaches.

DEVELOPMENT: Create a narrative script with accompanying story board. Specify needs of graphic programs. Write the graphic programs (MACRO's). Create picture data.

PRODUCTION: Make timed shooting script from narrative script and story board. Film the computer animation according to script. Record narration.

EDITING: Edit computer graphics sequences. Match with narration. Transfer to videotape, and dub in sound.

The entire production process of the three treatments is summarized on the flow chart in figure 4.6. The important steps and appropriate practical, theoretical, and pedagogical considerations are listed below.
The Syllabus:

The first task in scripting was selecting the subject matter. This was determined (Chapter III) by subject population, experimental rationale, and computer capabilities. The rigid motion approach to congruence was both modern and different enough from standard treatments to be rather independent of previously learned material, and quite worthwhile.

It was decided by the experimenter and consulting faculty members to include facts about rigid motions, especially their products, as the bulk of the treatment. The notion of symmetry was included. An outline of topics was made. Several proofs of results were included originally, but scrutiny of this material revealed that its inclusion would be too difficult for the students to handle. This observation was made by a mathematics professor in geometry who had taught the university geometry course which contained the originally planned subject population. A second consideration in this decision was time. The experimental treatment had to be completed in a period of time less than one hour. The syllabus originally was simply an outline of topics. From this original syllabus, the narration script and video-storyboard were made. After these were finished, a summary of the topics, presented in the logical order of the taped treatments narrative exposition, was written. (Appendix B). This summary
included basic notation and terminology. It was provided to the classroom teachers as a syllabus for their 1 hour lecture. The syllabus did not provide examples, suggestions for models, heuristics, or exercises.

The Narrative Script and Storyboard:

The narrative script was simply an explanation and elucidation of the topics specified in the syllabus. This scripting for the experimental lesson involved software considerations, (of the computer graphics system), pedagogical considerations, (experience of students, heuristics, interdependency of concepts), and media considerations, (pace, visibility, iconicity, redundancy, integration of video and audio). The capabilities of the graphics system are described earlier in this section. The nature of strong pedagogy is difficult to define operationally. It is not a crucial question here since both treatments 1 and 2 are based upon the same pedagogical development. However the media considerations, upon which the storyboard examples were based, can be specified. The following principles based upon the body of research on media were followed generally in the creation of the graphic sequences:

(1) The content was principally examples for definitions, or heuristic illustrations of certain results,

(2) Examples were as general as possible, rather than special cases,
(3) The figures used were simple, although not regular or symmetric, except, of course, when symmetry was a desired attribute,

(4) Audio information did not compete with important visual information,

(5) Repetition was minimized. Students could review or re-listen to any explanatory sequence. And so,

(6) Terminology was printed on the screen at appropriate times.

(7) Clarity through visual juxtaposition was sought, i.e. comparisons and contrasts with earlier results and figures were made by superposition, juxtaposition, or sequential presentation so that salient differences and similarities might easily be observed.

**Shooting Script:**

From the narration script and the accompanying visual material in the storyboard, a succession of visual sequences was determined. These made up the shooting script. This script was used to make the filmed images. An example of a page from the actual shooting script used is shown in Figure 4.3. Each entry consists of

(1) the shot number, identifying the place of the shot in the entire sequence,
(2) background pictures,
(3) the main picture, usually the one(s) manipulated, along with attribute information or modifications,
(4) the macro used, i.e. the name of a program,
(5) comments on the action in the sequence,
(6)-(9) timing information, durations in frames of the first still sequence, then the number of frames for the movement, then the number of final still frames and followed by the total duration in seconds = frames/24,
(10) status of the shot... R = ready... D = done (omitted from figure)
(11) coordinates, beginning and terminal for the moved figure, and
(12) notes or comments on the actual filming.

The decision of timing was a function of the scripting pace. However, since it was possible to view sequences at their actual speed before filming, some of the timing variables were determined at the shooting.

A variety of graphics programs, or macros, were employed. Some of these were written for only one usage, others were quite general and were used many times. These macros allowed for input of different pictures as well as other parameters. The macro system for the "ART" language is described in the following section.
<table>
<thead>
<tr>
<th>Shot</th>
<th>Backgr.</th>
<th>Pict-arrow</th>
<th>Macro</th>
<th>Comment</th>
<th>Hold</th>
<th>Move</th>
<th>Hold</th>
<th>T</th>
<th>Begin</th>
<th>End</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Cone 1?</td>
<td>Plat4 rotng</td>
<td>SHADE</td>
<td>superim.</td>
<td>250</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Plat1</td>
<td>HIDE</td>
<td>if neces.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Pine 1</td>
<td></td>
<td>ROT/D pl</td>
<td>by hand</td>
<td>50</td>
<td>2</td>
<td>0</td>
<td>1/4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>STACK</td>
<td></td>
<td>group N</td>
<td></td>
<td>50</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>N-from 3</td>
<td>ROT/D N</td>
<td></td>
<td></td>
<td>100</td>
<td>25</td>
<td>4/1</td>
<td>edit</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Pine 1</td>
<td>TRNS 1</td>
<td>camera</td>
<td>same as 4</td>
<td>50</td>
<td>75</td>
<td>50</td>
<td>12</td>
<td>(0,0)</td>
<td>off screen</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Pine 1</td>
<td>TRNS 2</td>
<td></td>
<td></td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>6</td>
<td>600</td>
<td>slow</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Pine 1</td>
<td>ROT/D,P,Z,</td>
<td></td>
<td></td>
<td>25</td>
<td>50</td>
<td>75</td>
<td>6</td>
<td>600°</td>
<td>ends good</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Slidrl</td>
<td>TRNS 1</td>
<td>off</td>
<td></td>
<td>36</td>
<td>72</td>
<td>12</td>
<td>(-3000,4000)</td>
<td>tilt</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Tri 1</td>
<td>TRNS 1</td>
<td>Detroit</td>
<td>(0,0)</td>
<td>15</td>
<td>54</td>
<td>30</td>
<td>27</td>
<td>3 (-400,200)(900,-500)</td>
<td>ends good</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Tri 1+</td>
<td>TRNS1</td>
<td>arrow w.</td>
<td></td>
<td>20</td>
<td>40</td>
<td>40</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>point</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Copy ar+mov</td>
<td>dotty</td>
<td>line/pmt</td>
<td>24*</td>
<td>move arrow to vertex</td>
<td>#deleteTri</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Vector w. point</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tri 1+</td>
<td>leave aw</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Pnt 1</td>
<td>JSDRAW</td>
<td>visible</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.3 Sample Page from Shooting Script
Macro Programs:

The graphics language called "ART" provided a variety of picture construction commands, as well as picture manipulation commands. Further, it incorporated commands which enabled an animation camera to make short timed exposures for each frame, thus enabling the user to obtain very sharp non-flickering images without huge time expenditures (Csuri, 1973).

Single pictures were constructed using the "putpoint" command. (PUTPOI x,y,z,k) where k determines whether the point is the end of a picture, a draw, or a jump. Pictures could be stored in the user's library on disc and recalled as necessary.

The system allowed the user to input numerical values by digital keyboard, analog devices, or program computation.

A macro was simply a sequence of these graphic instructions which could be stored, with a name, and called by the user. It could provide for inputs at particular points, such as the name of a picture to be rotated, names of stationary background pictures, dial setting of the pictures on the screen, number of degrees to be rotated, number of frames the rotation should take, etc.

Since the same types of basic motions recurred throughout the script, a few general macros could suffice for most of the sequence. A list of macros used is given in table 4.1 along with the use of each macro. A sample macro is shown in figure 4.4.
<table>
<thead>
<tr>
<th>Name of Macro</th>
<th>Number of commands</th>
<th>Inputs</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARMOV</td>
<td>12</td>
<td></td>
<td>Sub-macro Moves arrow</td>
</tr>
<tr>
<td>ARW1</td>
<td>23</td>
<td>ø</td>
<td>Constructs arrow to scale</td>
</tr>
<tr>
<td>ARW2</td>
<td>23</td>
<td>ø</td>
<td>Constructs arrow to scale</td>
</tr>
<tr>
<td>CRVARI1</td>
<td>75</td>
<td>degrees, direction, center of rotation, picture, radius</td>
<td>Rotates picture to specification with curved arrow drawn.</td>
</tr>
<tr>
<td>ASIN</td>
<td>19</td>
<td>number of degrees</td>
<td>Produces sin(A)</td>
</tr>
<tr>
<td>HILIT1</td>
<td>12</td>
<td>Any picture</td>
<td>Increases picture's intensity 10-fold</td>
</tr>
<tr>
<td>HILIT2</td>
<td>12</td>
<td>Any picture</td>
<td></td>
</tr>
<tr>
<td>STACK</td>
<td>20</td>
<td>Separation, number of copies</td>
<td>Stacks copies of plane figure in third dimension</td>
</tr>
<tr>
<td>TRNS1</td>
<td>50</td>
<td>Background, timing, picture, coordinates</td>
<td>Translates figure against background</td>
</tr>
<tr>
<td>TRNS2</td>
<td>50</td>
<td>Background, timing, picture, angles</td>
<td>Rotates figure against background</td>
</tr>
<tr>
<td>TRBL1</td>
<td>65</td>
<td>Figure coordinates, timing, position of arrow</td>
<td>(Like TRNS1) options on visibility; figure, image, arrows</td>
</tr>
<tr>
<td>TRNS3</td>
<td>11</td>
<td>Angle, figure</td>
<td>Constructs flip line and flips image (hand device)</td>
</tr>
<tr>
<td>TRFL1</td>
<td>45</td>
<td>Angle, figure</td>
<td>(Like TRNS3) Visibility options included</td>
</tr>
</tbody>
</table>
Name of Macro....TRNS 3

<table>
<thead>
<tr>
<th>Command</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>GETDISK D0TD1</td>
<td>Picture of dotted line named D0TD1, retrieved from Disk</td>
</tr>
<tr>
<td>PROMPT &quot;INPUT ANGLE&quot;</td>
<td>Words &quot;input angle&quot; are shown on teletype screen</td>
</tr>
<tr>
<td>INPUT A</td>
<td>User types in numerical value, system waits for this input. Value is recorded as variable A</td>
</tr>
<tr>
<td>A = -90 - A</td>
<td>Value is transformed</td>
</tr>
<tr>
<td>B = A x 64 x 1024/360</td>
<td>Arithmetic to scale units for analog devices.</td>
</tr>
<tr>
<td>ROT/D D0TD1,Z,B</td>
<td>Puts line at indicated angle i.e. rotates D0TD1 about the Z axis A degrees (B units).</td>
</tr>
<tr>
<td>PROM &quot;INPUT PICTURE&quot;</td>
<td>Words on screen,</td>
</tr>
<tr>
<td>INPUT C</td>
<td>Name of picture is typed and recorded as $C (picture variable)</td>
</tr>
<tr>
<td>GETDISC $C</td>
<td>Picture is retrieved.</td>
</tr>
<tr>
<td>TICK 2</td>
<td>Pause to allow for picture retrieval.</td>
</tr>
<tr>
<td>ROT/D $C,Z,D1,B</td>
<td>Rotates the picture about the axis previously specified by hand control D1 (not Z axis, but line drawn in step 6)</td>
</tr>
</tbody>
</table>

This macro is employed to perform arbitrary flips of any picture in the plane.

Figure 4.4 Sample Macro Program
Pedagogical Use of the Medium:

When a learner is confronted with a novel situation, and perceives some phenomenon, this phenomenon must be assimilated into the learning set of the learner, or his learning set must alter to accommodate the perception. Thus, an important factor in conveying a concept is providing a situation in which the crucial aspects of the concept can be made to be perceived.

In a lesson about various rigid motions, their products, congruence, and symmetry, the viewer-learners can be compelled to perceive the action of a rigid motion, the process and end result of a product, the establishment of congruence by juxtaposition, or symmetry as self-congruence under a rigid motion. Each of these examples involves an action which defines or describes general properties or principles. It is the animation which enables the action to be portrayed.

In Figure 4.5 are 12 blow-up frames from the film used to make Treatment 1, with descriptions of the animation sequence along with the pedagogical purposes of the sequence.
4.5a Rigid Motions of the Plane
Figure transformed to its image continuously. Rigidity observable
Examples of non-rigid motions provided.

4.5b The Slide
Any parallel arrow of the
same length represents the
same slide. Slide repeated
tracing out different arrows.

4.5c Nature of the Inverse Motions
The product of two opposite arrows
does return the figure to its original
position.

4.5d The Flip
Can be characterized as a
reflection with corresponding
object and image distances
to flip line equal.

Figure 4.5 Blown-up Frames from the Computer-Animated Treatment
4.5e Concept of Direct Motion
Clockwise sense of labelled vertices can be seen to be invariant under a slide.

4.5f Concept of Opposite Motion
Clockwise sense of labelled vertices seen to change under a flip.

4.5g Rigid Image of Irregular Figure
Internal distances are preserved. Student can observe, following a series of products that distances are preserved.

4.5h Using Tracing Paper...
...To physically perform a turn. The student can see the mechanics as well as underlying geometry.

Figure 4.5 (Continued)
4.5i Opposite Motion - Not a Flip
This glide reflection is shown to be a product of a slide and a flip. And thus an opposite motion.

4.5j Another Glide Reflection Is shown to be a product of a flip and a turn.

4.5k and l Rigid Motions of Equilateral Triangle
Two of the symmetry motions are shown. These motions are shown to be closed under products.

Figure 4.5 (Continued)
Filming:

The graphics language included the command "FILM...", which enabled the user to halt the program until the camera had exposed each frame for 1/4 second. It was thus a straightforward matter to film most of the sequences once they were put onto the shooting script. In some cases double exposures were used, and in some cases the manual controls were employed. Generally two hours or less was required to expose a 100 foot roll of film. (2 min. 45 sec). This included determining parameters, practice runs, constructing new figures, and adjusting the camera. So approximately one hour of filming was employed to obtain one minute of finished film.

Editing:

Since sequences were precisely controlled by the computer, there was little need for editing. Sequences could be shot in order and made in the exact length. Of course some sequences were faulty for one reason or another and were thus eliminated, reshot, shortened, or otherwise edited into the entire sequence. After any provisional edition was completed, the narrator read the script along with the film in order to ascertain proper length and pace.
Transfer to Tape:

The technical process whereby a film image is transferred onto videotape requires a "film chain", a mechanic-optic device. A film image would be difficult to tape directly because of the relative weakness of a reflected image and because of strobe problems. The video content for Treatment 1, which was the computer-drawn film was thus transferred onto 3/4 inch videotape cassettes by the Teachers Education Laboratory at Ohio State University.

Dubbing the Sound:

Since the film had been matched to the narration, it was a fairly easy matter to dub the sound track onto the film. The narrator merely taped the entire script, and then the sound track was matched to the video portion at the studio of the Teachers Education Laboratory. This process was much easier than that of editing the live videotaped lecture, since with the live lecture the sound track was synchronous with the video and could not be handled separately.
The Videotaped Lecture (Treatment 2):

For this treatment, the lecturer read from the script and/or paraphrased as appropriate to the examples which were used. The lecturer used chalk diagrams at a blackboard, and solid objects on the desk or blackboard.

Two cameras were used. They were not moved, although zooms and dissolves were employed. The technician was a PhD candidate in English and Cinema with experience in both film and videotape. Since the lesson was made on videotape, each segment could be immediately reviewed before accepting it and going on to the next.

The entire 43 minute sequence was shot in two sessions. Of course more than 43 minutes of content was shot. The tapes were edited with the help of the Teaching Aids Laboratory at Ohio State University. The original recording was made on 1/2-inch tape, a standard gauge for videotape reels in educational use. The tapes were edited onto a master 1/2-inch tape, and then transferred onto 3/4-inch cassettes at the Teachers Education Laboratory.
Figure 4.6 Production Flow-Chart for the Three Treatments
CHAPTER V - THE EXPERIMENT

The purpose was to compare the effectiveness of instruction based upon computer-graphic animation with standard lecture presentations in conveying mathematical concepts. Reasons for the study included determining the feasibility of presenting certain mathematical material via computer-animated cathode ray tubes, evaluating the efficacy of teaching certain subject areas with this medium, identifying the needs of such education graphics systems, and determining the effects of such teaching on performance and attitudes. Although CAI systems with animated graphics were not available on a large-scale basis a satisfactory imitation was a videotape cassette of CRT output, which allowed the user to stop, rewind, and review material at his/her own discretion.

Thus, computer-drawn visual material presented via videotape cassettes was compared with standard lectures presented both via videotape and in regular classrooms. Content of the lessons was to be "congruence through rigid motions".

In this chapter are descriptions of (1) the treatments, (2) the subject population, which had to be assigned to treatments and aptitude levels equally by IQ, (3) the instruments to measure
performance variables, (4) the analysis, and (5) the procedures followed in the experiment.

Treatments:

In an attempt to maintain both internal and external validity, two control treatments were employed: one which differed from the computer-drawn treatment only in visual content, the other being a normal classroom situation. The same subject matter was taught in one lesson to students randomly assigned to one of the three treatment groups.

Treatment 1: The Computer-Visual Treatment:

The treatment was a 35 minute videotape, with a narrative sound track. The visual content was white animated line-drawings on black background, demonstrating the various concepts and relationships of the topic, utilizing simple geometric figures, and providing some verbal information. Every image was represented as it appeared on the computer driven cathode ray tube. Figures were simple, appropriate notation was employed, and no animation effects were employed which could be considered "flashy". Narration was in a deliberate matter-of-fact tone. The content of the narration is transcribed in Appendix D. The organization and development of the material for Treatment 1 was the same as outlined on the teacher's instruction sheet (See Appendix B).
At a prescribed time, students receiving treatment 1 went to a table on which was a videotape player, a television, and instructions. The student was instructed to view the tape, stop it and review it as necessary, and to stop the tape and do several examples. (See Appendix B).

Treatment 2: The Videotaped Lecture:

This treatment was designed to duplicate Treatment 1 in every feature, especially medium of presentation, environmental situation, and content of instruction. The exterior circumstances, locations, instructions, examples, and equipment were identical to those in the computer-visual treatment. The content of the lessons were virtually identical, being based on the same script. Slight differences existed, however, since the lecturer spoke ad lib, rather than reading directly from the script. Parallel transcriptions of the two narration tracts are printed in Appendix D. The narration in Treatment 2 is the voice of the lecturer, who narrated Treatment 1.

The mode of instruction was different in the two taped treatments. Treatment 2, the videotaped lecture, was an edited tape of a teacher at a blackboard and a table. Two camera angles were used for the blackboard diagrams and for tabletop demonstrations. The lecturer employed cut-out figures, and simple construction apparati, ruler, compass, and mirror.
Differences of the Treatments:

The classroom treatment, treatment 3, differed from the taped treatments in that no real novel situation occurred. Regular live teachers provided the instruction. The specific content of those lectures, although generally specified by syllabus, was not controlled. Another difference was that Treatment 1 and 2 utilized work sheets, which were referred to on the tapes. No provision for worksheets or class exercises was made for treatment 3. The development of the lessons in treatment 1 and 2 were identical, and followed the outline of the teachers' instructions for treatment 3. So the material content was made as equal among the treatments as possible. The essential difference of treatments 1 and 2, the taped treatments, was the nature of the visual content. Chalk-drawn and physical examples were employed in one, the other consisted entirely of computer-animated images. Besides that, there were differences in the number and type of examples, due basically to constraints imposed by the particular medium.

Subjects:

The subjects were students in geometry at a suburban high school. Most were sophomores, but freshmen, juniors, and seniors were included in the subject population. The school is in an upper-middle-class community, and the academic level of the students compares quite favorably with norms.
The tape lasted 43 minutes, 8 minutes longer than the computer-visual tape. The explanation of the topic took a little longer than on the computer tape, since examples and models were manipulated, labelled, and/or drawn by hand.

**Treatment 3: Regular Classroom Lectures:**

In order to provide a comparison of the taped treatments with real-world educational situations, a treatment employing regular classroom instruction under normal conditions, was included. In the school, three teachers taught geometry. Each was asked to prepare a one-period (56 minutes) lecture concerning the material outlined in Appendix B. The tapes for Treatment 1 and Treatment 2 were available for viewing by the teachers, and two of them viewed at least part of the tapes. The experimenter was available for consultations on matters of content, although none of the teachers did consult on substantive questions of content. One regular geometry period was designated for the lecture. No homework was to be given on the material. No particular work sheet was prepared for the class lectures, since the content of Treatment 3 depended upon the particular teacher, and any materials of that sort were left to the discretion of the particular teacher. One lecturer xeroxed the teachers' notes, and worked through those materials with the students. Another employed the problems from the notes as a basis for his lecture. The third gave a blackboard lecture to his class.
Table 5.1

Similarities and Differences of Treatments

<table>
<thead>
<tr>
<th></th>
<th>Treatment 1</th>
<th>Treatment 2</th>
<th>Treatment 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Medium of</strong></td>
<td>Videotape</td>
<td>Cassette</td>
<td>Classroom</td>
</tr>
<tr>
<td><strong>Presentation</strong></td>
<td></td>
<td>Playback</td>
<td></td>
</tr>
<tr>
<td><strong>Duration of</strong></td>
<td>1 period</td>
<td>1 period</td>
<td>1 period</td>
</tr>
<tr>
<td><strong>Treatment</strong></td>
<td>35 min. tape</td>
<td>43 min. tape</td>
<td>56 minutes</td>
</tr>
<tr>
<td><strong>Content</strong></td>
<td>Defined by Syllabus</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Pedagogical</strong></td>
<td>Experimenter's Script</td>
<td>Teacher's Choice</td>
<td></td>
</tr>
<tr>
<td><strong>Development</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Visual Content</strong></td>
<td>Computer</td>
<td>Taped</td>
<td>Classroom</td>
</tr>
<tr>
<td></td>
<td>Drawn</td>
<td>Lecture</td>
<td>Lecture</td>
</tr>
<tr>
<td><strong>Exercises</strong></td>
<td>6 Exercises</td>
<td></td>
<td>Not in General</td>
</tr>
</tbody>
</table>

Subjects:

The subjects were students in geometry at a high school located in an upper-middle-class community. Most of the students were sophomores, but freshmen, juniors, and seniors were included in the subject population.
The geometry course used the text *School Mathematics Geometry*, by Anderson, Garen, and Gremillion. The course followed a modern SMSG approach. The experiment took place in February, 1976, so the courses were approximately 3/5 through the year. The course did not involve any aspects of transformational geometry.

The students came from seven sections of geometry. (3 teachers, two or three classes each.) 127 students from these classes took the achievement test. 109 received one of the treatments. 90 of these were included in the analyses. In addition there was an honors section, and this group received the pilot treatment. This pilot was discussed in Chapter III.

**Blocking on IQ:**

In order to be able to discern aptitude-treatment interaction (ATI), a variable was employed which measured aptitude, and which was used as a blocking variable in the analysis. Two important decisions had to be made: (1) the choice of the blocking variable, and (2) number of levels in the analysis.

The variable had to be (1) reliable, (2) available, and (3) shown to correlate highly with performance in mathematics. For the subjects, scores were available for most students for verbal, non-verbal, and composite IQ on the SFTAA-5 (Short Form Test of Academic Aptitude), a revision of the California Test of Mental Maturity. For the purposes of the experiment, and for the
topic of transformational geometry the non-verbal IQ seemed to have the most validity, since the concepts involved were quantitative, relational, and/or spatial rather than verbal.

In order to test the ability of the variable to predict performance in mathematics, or rather geometry, the Pearson correlation of the non-verbal IQ score with first semester grades in geometry (A=4, B=3, C=2, D=1, E=0) was computed. The coefficient was calculated to be .52. It was therefore felt that this variable was appropriate for use as a blocking variable in the analysis.

**Number of Blocking Levels:**

In determining the number of levels of the blocking variable to use, two criteria had to be applied. The first was obtaining maximum power for the experiment, and the second was conformance of the design to the rationale. Given the sample size, the number of treatment groups, and appropriate correlation of dependent variable with blocking variable, criteria exist for selecting the optimal number of blocking levels in terms of power of the experiment (Feldt, 1958; Kennedy, 1974). However, an optimization of the likelihood of discovering main effects differences may not be the most desirable goal of an experiment. In this study, the interaction of aptitude level with treatment was an important
effect to be scrutinized. But the criteria presented by Kennedy and Feldt were based upon discovering main, or first order, effects, and so the resulting original design which involved five levels of aptitude effectively "factored" out aptitude effects optimally in terms of degrees of freedom. Therefore, the original design, based upon the criteria optimizing first order effects, was changed shortly after it became clear that interaction effects had been obscured. And so, the final design reflected the more standard procedure of division into only 3 ability levels, and was much more suited to discerning interaction effects. It should be noted that changing the number of ability levels did not alter procedures of assignment to treatment.\(^1\)

The original assignment of subjects to treatments was done with a design involving five levels of aptitude, and then descriptions of procedures in the following section reflect this fact. Cell means for IQ for both designs are presented in table 5.2.

\(^1\)The reassignment to levels involved no difficulties, and was a straightforward process.

1. The subjects (after dropping) were linearly ordered by IQ, and where applicable, subject number. (Alphabetical information was not present on the data cards) and the new level numbers were appended directly onto the cards.

2. Level 1's went into the low group, level 3's went into the medium group, and level 5's went into the high group. The 6 level 2's in each treatment were split 4-2 into Low-Medium respectively. The level 4's were split 2-4 into medium high. Hence each of the 3 levels of the 3 treatments had 10 subjects.
Assignment of Subjects to Treatments:

The assignments to treatments was done as follows:

1. As many students as possible were to be included in the sample, so slightly less then one-third of the total would be allocated to each treatment.

2. Distributions of the IQ scores (the level classification variable) had to be homogeneous, i.e. the means and variance of the blocking variable had to be nearly equal across treatments at all levels of the blocking variable (IQ). For this to be possible, the pooled classes designated for Treatment 3 had to exhibit closely the same distribution of IQ scores as the remaining 2/3 of the students. (Those to be assigned to treatments 1 or 2.)

3. The "intact" classrooms for Treatment 3 were selected first, to allow for random splitting of the remaining subjects. For Treatment 3, each of the three geometry teachers was to teach one of his classes, or a subset of one, since it was necessary to split a class to satisfy the IQ distribution requirement.

4. The remaining subjects to be randomly split into Treatments 1 and 2 were linearly ordered by IQ, and alphabetically in case of ties. In that order, then, the students were paired off. From each pair, one was assigned to Treatment 1, one to Treatment 2, according to whether an entry in a table of random numbers was even or odd.
The IQ score used was the non-language score on the SFTAA-5 (Short form test of academic aptitude). The score was available for nearly all students in the school. The experimenter recorded these scores from school records, and deleted from consideration students who did not have this score on their record. The distribution of IQ scores of the six sections whose rosters were made available are listed on Table 5.1. The total number of students involved in this assignment procedure was 110. Hence approximately 37 students were to be allocated to each treatment.

Three classes involving the three different teachers, or subsets of each classes, had to be designated for Treatment 3. The first division that would appear to be right would have classes a, b, and c put into Treatment 3. (See Table 5.2.) However, due to a consequent lack of students for Treatment 3 in level IV, this selection was unacceptable.

The split adjudged to be most desirable put class b into the taped treatments pool and split class b into an intact class group for Treatment 3 (13 students) with the remaining 15 students going into the pool for 1 or 2. This division could be made such that proper distributions of IQ scores existed, and indeed, by matching distributions as far as possible the group means matched quite well, as was required above. (See Table 5.3.) Table 5.3 also includes cell means of IQ's after dropping subjects from the analysis to obtain equal cell sizes of six, as well as after realignment into three treatment levels.
### Table 5.2

Assignment of Classes and Subjects to Treatments by IQs

<table>
<thead>
<tr>
<th>Students IQ</th>
<th>Students Classroom</th>
</tr>
</thead>
<tbody>
<tr>
<td>87</td>
<td>CC</td>
</tr>
<tr>
<td>91</td>
<td>B b b</td>
</tr>
<tr>
<td>93</td>
<td>C</td>
</tr>
<tr>
<td>94</td>
<td>B</td>
</tr>
<tr>
<td>96</td>
<td>A</td>
</tr>
<tr>
<td>97</td>
<td>b</td>
</tr>
<tr>
<td>98</td>
<td>b</td>
</tr>
<tr>
<td>99</td>
<td>A C</td>
</tr>
<tr>
<td>100</td>
<td>A B B b</td>
</tr>
<tr>
<td>101</td>
<td>a</td>
</tr>
<tr>
<td>102</td>
<td>A a B C</td>
</tr>
<tr>
<td>103</td>
<td>B' b&quot;C' C&quot; c'</td>
</tr>
<tr>
<td>104</td>
<td>B b c</td>
</tr>
<tr>
<td>105</td>
<td>A</td>
</tr>
<tr>
<td>106</td>
<td>B B c</td>
</tr>
<tr>
<td>107</td>
<td>a a B</td>
</tr>
<tr>
<td>108</td>
<td>A a B C</td>
</tr>
<tr>
<td>109</td>
<td>b</td>
</tr>
<tr>
<td>110</td>
<td>b</td>
</tr>
<tr>
<td>111</td>
<td>B b C</td>
</tr>
<tr>
<td>112</td>
<td>A B B</td>
</tr>
<tr>
<td>113</td>
<td>C C C C C c</td>
</tr>
<tr>
<td>114</td>
<td>b C c c</td>
</tr>
<tr>
<td>115</td>
<td>B C</td>
</tr>
<tr>
<td>116</td>
<td>AA a B B e</td>
</tr>
<tr>
<td>117</td>
<td>A'B&quot; C' C&quot;</td>
</tr>
<tr>
<td>118</td>
<td>A A a B B B C C</td>
</tr>
<tr>
<td>119</td>
<td>A B B B B B c</td>
</tr>
<tr>
<td>120</td>
<td>B C C</td>
</tr>
<tr>
<td>121</td>
<td>A a a B</td>
</tr>
<tr>
<td>122</td>
<td>a a</td>
</tr>
<tr>
<td>123</td>
<td>B b</td>
</tr>
<tr>
<td>124</td>
<td>B c</td>
</tr>
<tr>
<td>125</td>
<td>a</td>
</tr>
<tr>
<td>126</td>
<td>B b</td>
</tr>
<tr>
<td>127</td>
<td>a</td>
</tr>
<tr>
<td>128</td>
<td>A</td>
</tr>
<tr>
<td>129</td>
<td>C c</td>
</tr>
<tr>
<td>130</td>
<td>A A a a B</td>
</tr>
<tr>
<td>131</td>
<td>b</td>
</tr>
<tr>
<td>132</td>
<td>c</td>
</tr>
<tr>
<td>133</td>
<td>b</td>
</tr>
<tr>
<td>134</td>
<td>c</td>
</tr>
<tr>
<td>135</td>
<td>C</td>
</tr>
</tbody>
</table>

**Class sizes:**
- A-18
- a-12
- B-28
- b-15
- C-25
- c-12

1. Upper and lower cases of the same letter indicate the same teacher.
2. Underlined entries received Treatment 3.
3. (*) Indicates a cut off value for a level. The symbols ('') and ("') were used for values. (') indicates higher, (") lower level classification.
Table 5.3
Cell Means of IQ by Treatment Group and Aptitude Level

Before Random Assignment to Tape Treatments

<table>
<thead>
<tr>
<th>Aptitude Level</th>
<th>Rel. File</th>
<th>Range</th>
<th>Classroom (3)</th>
<th>Taped Treatment (1 and 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0-19</td>
<td>87-103</td>
<td>n 7</td>
<td>Mean IQ 98.42</td>
</tr>
<tr>
<td>II</td>
<td>20-39</td>
<td>103-111</td>
<td>n 8</td>
<td>Mean IQ 106.75</td>
</tr>
<tr>
<td>III</td>
<td>40-59</td>
<td>112-117</td>
<td>n 7</td>
<td>Mean IQ 114.42</td>
</tr>
<tr>
<td>IV</td>
<td>60-79</td>
<td>117-123</td>
<td>n 7</td>
<td>Mean IQ 119.57</td>
</tr>
<tr>
<td>V</td>
<td>80-99</td>
<td>123-150</td>
<td>n 8</td>
<td>Mean IQ 128.75</td>
</tr>
</tbody>
</table>

After Random Assignment and Dropping of Subjects\textsuperscript{a}

<table>
<thead>
<tr>
<th>Aptitude Level</th>
<th>Comp-Vis(1)</th>
<th>Tape Lect(2)</th>
<th>Classroom(3)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>96.5</td>
<td>95.0</td>
<td>98.1</td>
<td>96.9</td>
</tr>
<tr>
<td>2</td>
<td>106.5</td>
<td>108.0</td>
<td>105.3</td>
<td>106.6</td>
</tr>
<tr>
<td>3</td>
<td>113.3</td>
<td>116.0</td>
<td>114.5</td>
<td>114.3</td>
</tr>
<tr>
<td>4</td>
<td>118.3</td>
<td>121.3</td>
<td>119.8</td>
<td>120.0</td>
</tr>
<tr>
<td>5</td>
<td>128.8</td>
<td>130.0</td>
<td>131.3</td>
<td>130.1</td>
</tr>
<tr>
<td>Totals</td>
<td>112.8</td>
<td>114.3</td>
<td>113.8</td>
<td>113.6</td>
</tr>
</tbody>
</table>

After Adjusting to Three Aptitude Levels\textsuperscript{b}

<table>
<thead>
<tr>
<th>Aptitude Level</th>
<th>Comp-Vis(1)</th>
<th>Tape Lect(2)</th>
<th>Classroom(3)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>125.2</td>
<td>126.8</td>
<td>127.1</td>
<td>126.4</td>
</tr>
<tr>
<td>Medium</td>
<td>113.3</td>
<td>115.8</td>
<td>113.7</td>
<td>114.3</td>
</tr>
<tr>
<td>Low</td>
<td>99.9</td>
<td>100.2</td>
<td>100.7</td>
<td>100.3</td>
</tr>
<tr>
<td>Totals</td>
<td>112.8</td>
<td>114.3</td>
<td>113.8</td>
<td>113.6</td>
</tr>
</tbody>
</table>

\textsuperscript{a} n = 6 for each cell.

\textsuperscript{b} n = 10 for each cell.
Instrumentation:

The Achievement Test:

In order to have a measure of achievement in the subject, "Congruence through rigid motions", it was necessary to construct a test, since the experimenter could locate neither a source test of this topic, nor a general test which contained appropriate examples. The questions were to be short-answer, easily determined as right or wrong and had to deal with general factual information relating to congruence via rigid motions. It was planned to take questions from three sources. (1) the experimenter, (2) appropriate textbooks, and (3) the classroom teachers. The employment of classroom teacher questions proved not to be feasible.

The textbook questions came from two sources, the book used in the classes at the school (Anderson, Garen, Gremlion, 1965) and a text for secondary geometry teachers (Krause, 1972). Portions of this text were included in the teachers' notes.

The experimenter's questions comprised 28 of the 39 questions in the final achievement test form. The entire test included 21 True-False, 13 short answer, and 5 sketching questions.

The achievement test was assembled by the experimenter, and 72% of the items were written by the experimenter. Thus, careful scrutiny of the validity of the achievement test was in order.
Test Content Analysis:

In order to provide an analysis of the content of the achievement test, the following were employed:

(1) The lecture notes prepared for the classroom teachers are presented in Appendix B. Superimposed on these notes are outline labels. These are provided as an organizational and labelling device. Each paragraph corresponds to a main heading. Each sentence corresponds to a subheading.

(2) Table A in Appendix C is an evaluation by the experimenter of which questions on the achievement test pertain to which topics on the outline. The outline format is presented in the first or "Topic" column. Next to selected indices are question numbers in parentheses. According to the experimenter's judgment, the particular question(s) listed pertain principally to the particular statement of the lecturer's notes outlined in Appendix B, as indicated. Further, part 2 of table A performs the inverse listing. i.e. For each test question, the portion of the syllabus outline which provides the information to answer the question is indicated. So if the questions are distributed about the items evenly, then the notes were an accurate indication of the content of the achievement test.
(3) Another content breakdown of the test items by concepts involved was made, but from specific verbal content. Table B provides evidence of the subject or topic of questions. Concepts overly involved (i.e. verbally or otherwise obviously) in the statement of the problem are indicated. This table B does not deal with the nature of the solution or the analysis required for solution. Table B provides additional evidence of the pertinence of the outlined teachers notes, as well as the distribution of the topics of the questions on the test.

Since the distribution of items exhibits a fairly reasonable correspondence to the syllabus, the content validity of the tests is verified, at least to the degree that the experimenter's evaluation is accurate.

Subtests:

The achievement test was constructed as an instrument to measure overall achievement in "Congruence through rigid motions". A given treatment might produce better results in a certain sub-area while producing worse results in another.

To glean an insight into the nature of the questions that might have made a difference, several topical subtests were identified. The method of designation was similar to that of the content analysis. Certain skills (or behavioral objectives) were perceived by the experimenter to be requisite in answering certain subsets of these questions.
For example some items required sketching of an image of some sort. Other questions involved the notion of direct vs. opposite motions. Some questions involved visual (or physical) manipulations of figures, some notion of product or inverse, and finally some questions involved symmetry. These five topics determined five subsets, not necessarily disjoint. For example the question:

"12. T-F The product of two flips is a flip" is listed on both the Direct/Opposite test and the Product/Inverse.

The subtests are made up as indicated in Table 5.4 and were found to have the indicated reliabilities.

<table>
<thead>
<tr>
<th>Subtest Name</th>
<th>N of items</th>
<th>Item numbers from main test</th>
<th>Rel. KR 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sketching</td>
<td>5</td>
<td>22,23,24,33,35</td>
<td>.54</td>
</tr>
<tr>
<td>Direct/Opposite</td>
<td>8</td>
<td>4,12,13,14,16,24,31,37</td>
<td>.51</td>
</tr>
<tr>
<td>Visual Recognition</td>
<td>10</td>
<td>25,26,28,29,30,31,34,37,38,39</td>
<td>.59</td>
</tr>
<tr>
<td>Product/Inverse</td>
<td>9</td>
<td>5,8,10,12,13,14,15,16,30</td>
<td>.39</td>
</tr>
<tr>
<td>Symmetry</td>
<td>8</td>
<td>4,20,28,29,33,34,35,36</td>
<td>.53</td>
</tr>
</tbody>
</table>
The test items were selected as satisfying (in the experimenter's judgment) the following criteria.

**Sketching:** The item involves the sketching of geometric figures; the image of a specified motion; or a figure with specified symmetries.

**Direct/Opposite:** Answer to the question involves facts about direct and/or opposite motions and their particular attributes.

**Visual Recognition:** Problem involves sight manipulation or recognition of figure(s) or spatial relationships in a non-trivial way.

**Product/Inverse:** Problem involves product of rigid motions, or the inverse of a particular rigid motion.

**Symmetry:** Problem specifically or implicitly involves the notion of symmetry.

Again, these tests were not for the purpose of major analyses. They were constructed for the purpose of shedding some information on effects peculiar particular aspects of the material. The reliabilities are high enough to be considered as containing worthwhile information. The tests have some face validity inasmuch as the evaluations of the experimenter were appropriate.
Attitude Tests:

The attitude items were administered along with the achievement test. There were 13 of these items. Six of these pertained to attitude towards mathematics in general, the final seven pertained either to the subject matter of the experimental lesson or to its presentation.

The items concerning attitude towards mathematics came from Aiken (1963) and Schultz (1972). This six item test, called Att 1, was administered to all students who took the achievement test and its reliability was calculated to be KR$_6$ = .95.

The other items were administered only to those who received the taped treatments. These items were classified as pertaining to the presentation, or pertaining to the material learned. Responses by treatment group 3 would have been in effect critiques of their teachers, and this type of information was not desired.

The test items appear in Appendix A. From the thirteen items, the three tests are made up as follows:

- Att 1 (Toward Mathematics) 1,2,3*,4,5*,6
- Att 2 (Toward the presentation) 8*,9*,10
- Att 3 (Toward the material) 7*,11,12,13

Here (*) indicates the item is scored negatively.

The items can be seen to have a face validity. The items in Att 1 are a varied survey of attitudes towards mathematics.
Att 2 contains simple evaluations of the appropriateness of the presentation, specifically its rate, and interestingness. Att 3 involves attitudes toward the subject matter.

Reliability of the tests cannot be computed by the standard KR 20 or 21 formula since the data is not dichotomous but Likert scale. A reliability measure for this data is available. This reliability estimate is applicable to data which are not dichotomous, and is based upon equation 8 of Kuder and Richardson, (1937). Rather than the expression \( p(1-p) \) for the item variance, the actual item variance can be computed. Based upon this formula, the following estimates were computed:

- Att 1 (6 items) had reliability \( KR_3 = .95 \)
- Att 2 (3 items) had reliability \( KR_3 = .61 \)
- Att 3 (4 items) had reliability \( KR_3 = .87 \)
Analysis:

The original design provided for five levels of IQ. Subjects were assigned to one of three treatment groups. Hence a two-way ANOVA was to be performed, using students as the unit of analysis. The variable to be studied was achievement in the subject area as measured on the achievement test. (See Table 5.5)

This design involved three treatments with five levels for 15 cells with 7 students each or 105 subjects. Several subjects were available as substitutes, and if for any reason a cell size was smaller, cell sizes were to be equalized by random dropping of subjects. Cell sizes were dropped to 6 students per cell or 90 subjects. The new design involved 3 levels, so there were 9 cells and 10 students per cell.

The variables measured and recorded for the analysis are listed in Table 5.6.

<table>
<thead>
<tr>
<th>Aptitude Level</th>
<th>Treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Computer-Visual</td>
</tr>
<tr>
<td>High</td>
<td>10</td>
</tr>
<tr>
<td>Medium</td>
<td>10</td>
</tr>
<tr>
<td>Low</td>
<td>10</td>
</tr>
</tbody>
</table>
Table 5.6
Variables in the Analysis

I. Criterion variables:

<table>
<thead>
<tr>
<th>Name of Test</th>
<th>Number of Item</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Achievement Test</td>
<td>39</td>
</tr>
<tr>
<td><strong>Subtests of the Achievement Test</strong></td>
<td></td>
</tr>
<tr>
<td>2. Non-text Test</td>
<td>28</td>
</tr>
<tr>
<td>3. Sketching</td>
<td>5</td>
</tr>
<tr>
<td>4. Direct/Opposite</td>
<td>8</td>
</tr>
<tr>
<td>5. Visual Recognition</td>
<td>10</td>
</tr>
<tr>
<td>6. Product/Inverse</td>
<td>9</td>
</tr>
<tr>
<td>7. Symmetry</td>
<td>8</td>
</tr>
<tr>
<td>8. Exercise Performance&lt;sup&gt;a&lt;/sup&gt;</td>
<td>6</td>
</tr>
</tbody>
</table>

Attitude Tests
9. Att<sub>1</sub>(toward mathematics) 6
10. Att<sub>2</sub><sup>a</sup>(toward the presentation) 3
11. Att<sub>3</sub><sup>a</sup>(toward the material) 4

II. Classification Variables:

<table>
<thead>
<tr>
<th>Name of Variable</th>
<th>Number of Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Aptitude level (based on IQ)</td>
<td>3</td>
</tr>
<tr>
<td>2. Treatment</td>
<td>3</td>
</tr>
<tr>
<td>3. Sex</td>
<td>2</td>
</tr>
<tr>
<td>4. Location of Treatment&lt;sup&gt;a&lt;/sup&gt;</td>
<td>2</td>
</tr>
<tr>
<td>5. Day of Treatment&lt;sup&gt;a&lt;/sup&gt;</td>
<td>5</td>
</tr>
</tbody>
</table>

III. Covariates:

1. IQ

a. Variable measured only for treatments 1 and 2, the taped treatments.
Hypotheses:

Primary questions concerned the effect of treatment on achievement. Following are statements of the Null hypotheses:

H-1: Achievement of students receiving a taped treatment would not differ from that of students receiving a classroom presentation. \[ (T-1 + T-2 - 2T-3) = 0 \]

H-2: There would be no difference in performance of students whose treatment differed only in the visual content. \[ (T-1 - T-2) = 0 \]

H-3: No significant interaction of treatment with level of aptitude on performance will occur.

In addition, information concerning the effect of other variables was desired. To answer these questions appropriate ANOVA's (and ANCOVAs and MANOVAs) were performed, and post-hoc analyses were made when appropriate.

Q-1: How do the respective treatments affect attitude toward mathematics?

Q-2: How does the computer-visual treatment affect particular attitudes (toward the content and its presentation)?

Q-3: Do performance outcomes vary depending on the nature of the material. (Do results vary with subtests.)

Q-4: What is the effect of the treatments on retention?

Q-5: Are other variables (sex, location of treatment) involved in a significant way?
Students as Unit of Analysis:

The design of the study was rather unusual in that it employed two different "control" treatments. The necessity of two control treatments reflects the difficulty mentioned by Salomon (1972) in obtaining both internal and external validity. Experiments on media which control important variables tend to lack generalizability to actual practice (external validity). And conversely experiments comparing media treatments with actual practice tend to violate internal validity. Treatment 2 provided a well controlled "control" for important comparisons with Treatment 1, the computer-visual treatment. But to afford some degree of generalizability to actual practice, the live classroom treatment was included. Analysis must be made with these design considerations in mind.

One evident problem presented by the inclusion of Treatment 3 was that it violated the assumption that the statistical units are treated independently. In the taped treatments students randomly assigned received their treatments individually and hence independently. Consequently, the student had to be the unit of analysis. Students receiving Treatment 3 did so in regular classroom groups, and so these subjects did not receive independent treatments. It would not be possible to consider classrooms the unit. Thus, the violation was abided. Insofar as the classroom presentations were normal and uncontaminated by negative influences, the resulting analysis would be robust.
In the analysis many of the principal tests did not involve Treatment 3. i.e. Interaction was tested between treatment groups 1 and 2, as well as the principal test concerning the efficacy of the computer graphic treatment (H-2). The only important test involving Treatment 3 was the pooled tape performances compared against the classroom lectures. i.e. H-1: T1/2 + T2/2 - T3 = 0.

Scheduling of Students to Treatment Times:

Students were scheduled for treatments during free periods, i.e. study halls. Each student in treatment 1 or 2 was given a form (See Appendix B) on which he/she indicated his/her free periods. The experimenter then assigned students to free periods. Since there were some systematic patterns in schedules, it was impossible to schedule some students. Fortunately, an additional section of geometry existed which was not on the experimenter's original list. Therefore substitutes were available to be assigned to receive the taped treatments. Also, in one case, two students from the same
level were "traded" between treatments 1 and 2 to accommodate schedules. Hence each period in the one-week schedule, that is 5 (days) x 7 (periods) x 2 (stations) = 70 assignments, was filled, 35 students being scheduled to receive each of the taped treatments.

The treatments were all given during one week to minimize differences in performance on the achievement test due to different lag times. The test was given on the following Monday. The possible effect on performance of lag time was examined for each of the two taped treatments.

Two locations were used for presenting the taped treatments. One was within a glass listening room in the library; the other was in an unused foyer area in the school. Both locations were effectively isolated from noise and traffic. To cancel any possible effect due to the nature of the environment of the tape replay set-up, the tapes were used at alternate locations on successive days. The effect of the different locations was statistically examined.
**Procedures:**

**Experimenter Involvement:**

The videotape players and monitors were transported, assembled, and maintained by the experimenter. Worksheets and videotapes were placed at the appropriate locations daily, and collected by the experimenter or the aide. The forms explaining the experiment, or rather "academic activity", requesting hours of students' free periods, and indicating to individual students their time of treatment, were all distributed by the experimenter through the classroom teachers. The experimenter did not interact with subjects, except to be available in case of difficulty with either the instructions or the operation of the videotape player, during the actual treatment (1 or 2). An aide was employed to help fulfil this function of experimenter overseer. Either he or the experimenter was always present to ensure that the appropriate subject indeed received the proper treatment at the assigned time, and that technical problems be taken care of. Data about students were collected by the experimenter from school files in the main office.
Student (Subject) Involvement:

The interaction of the student with the experiment consisted of:

(1) Notification of involvement with the experiment: The experiment was described as an "academic activity" (See Appendix B). Along with this description a request for a list of free periods was made. This notice was distributed and collected during a regular geometry class by the teacher.

(2) Notification of time of treatment: (See Appendix B). The students were given a form, which described the activity in more detail and gave the time and place of the treatment. This form was given to each subject one or two days prior to the date of treatment by their geometry teacher.

(3) The Treatment: At the particular location, on the table were instructions with the student's name on the first page. Inside were problems or exercises referred to in the videotape. An aide was nearby in case of any problems. Those subjects receiving Treatment 3 did not have the interaction indicated in (1) or (2). On Friday of the week of the experiment, each of the three geometry teachers taught their geometry class which was selected for Treatment 3: the material "Congruence through rigid motions".
(4) All students in geometry (even those who received no experimental Treatments) were given the combined attitude and achievement tests on the Monday following the treatments.

**Teacher Involvement:**

Each of the three teachers of geometry was informed of the experiment several weeks in advance. Before the week of the experiment, each was given a syllabus of the topics to be covered in the lecture. Each was asked to prepare a lecture on the material with the emphases as indicated on the notes (Appendix B).

The teachers also distributed and collected the forms as mentioned above, and administered and collected the achievement test.

**Handling of Data:**

Data processed for each geometry student involved was name, non-verbal IQ, previous geometry grade, teacher, hour of math class, day of treatment, sex, treatment, scores on exercises, 13 attitude items (1 through 5) and 39 achievement items (right or wrong) as appropriate. Student data, name, ID, IQ, previous geometry grade, teacher, treatment, hour, sex were recorded on a mimeographed form by the experimenter. Tests were scored by hand by the experimenter and recorded onto computer cards by a keypuncher. Student data were recorded onto cards by the experimenter. The data taken are indicated in Table 5.7, along with the number of digits required and the purpose of the information.
Table 5.7

<table>
<thead>
<tr>
<th>Data Item</th>
<th>Digits</th>
<th>Use, Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name, ID</td>
<td>3</td>
<td>For identification and sorting, transferred to 3-digit ID number where data was required.</td>
</tr>
<tr>
<td>Non-verbal IQ</td>
<td>3</td>
<td>The non-verbal portion of California (SPTAA-5) test. Used as blocking variable and covariate in analysis.</td>
</tr>
<tr>
<td>Previous Math Grade</td>
<td>1</td>
<td>Used to evaluate correlation with non-verbal IQ.</td>
</tr>
<tr>
<td>Teacher</td>
<td>1</td>
<td>Used for analysis and classification.</td>
</tr>
<tr>
<td>Treatment</td>
<td>1</td>
<td>For analysis.</td>
</tr>
<tr>
<td>Attitude</td>
<td>13</td>
<td>Scored on a range of 1-5, broken into subsets of 6, 3, and 4 items.</td>
</tr>
<tr>
<td>Test Items</td>
<td>39</td>
<td>Marked as right or wrong broken into six subsets, including text items, and the 5 topical subtests.</td>
</tr>
<tr>
<td>Day of Treatment</td>
<td>1</td>
<td>For taped treatments only, for analysis of regression on time.</td>
</tr>
<tr>
<td>Performance on exercises</td>
<td>6</td>
<td>Right or wrong, for taped treatments only.</td>
</tr>
</tbody>
</table>
CHAPTER VI - RESULTS

Treatments:

The treatments were presented during the one academic week from February 23 through February 27, 1976. The achievement test was administered to all geometry students on the following Monday, March 1.

Technical performance of the videotape equipment and attendance of the students scheduled to receive the taped treatments were monitored by one of two experiment-assistants. The only problem encountered was that the tape for Treatment 2 jammed in the machine several times. The assistants were able to extricate the tape on each of these occasions. However for two students, too much time was lost, and so they were subsequently dropped from the analysis. So actually, 34 students viewed at least parts of Treatment 2, but only 32 were considered to have received the treatment.

Subjects:

Thirty-five students were scheduled to receive each of the taped treatments, Treatments 1 and 2. Of these, 33 students received Treatment 1, and 32 received Treatment 2. Forty-eight received
treatment 3, the classroom treatment. Of these 32, 32, and 42 respectively completed the attitude and achievement tests. In order to have equal cell sizes, subjects were dropped from the analysis according to the following procedures:

1. Cell size was to be 6. (That was for the original design, i.e., 5 levels of 10. Since 5 x 6 = 3 x 10, the changing of the number of levels would not involve further manipulation of the sample, only of classification to levels.)

2. No cells had fewer than six. If a particular cell was too large, then the subjects were arranged in a linear order (by IQ and alphabet again) labelled 1-n (where n was less than or = 10), and the subjects represented by the labels were dropped as those digits occurred on a table of random numbers.

Comparison of Achievement Involving All Treatments:

The first test was to compare the two taped treatments against classroom lectures. Specifically, the null hypothesis tested was that performance on the achievement test by students receiving classroom instruction, Treatment group III, will not differ from the mean performance of students receiving taped instructions.

However, in order to perform this test, and other tests involving all three treatments, without bias a slight revision of the achievement test had to be made.
Abridgement of the Achievement Test:

As part of the teachers' notes were included pages from a text (Anderson et. al) On these pages were some sample items which were also used on the achievement test. Two of the classroom teachers used these questions as part of their presentation of this material. Hence it was felt that the part of the test excluding these items might be more informative when comparing the performance of treatment group 3 against group 1 and 2.

Considered as a separate test, the (28 item) portion of the test which did not involve problems from the texts had reliability: KR 20 = .78.

An analysis of variance was performed on the non-text portion of the achievement test. Results are summarized in Table 6.1. Mean scores and within cell variances on the achievement test without text questions are indicated on Table 6.2.

As expected aptitude level had a significant effect on performance. According to Tukeys HSD criterion the pooled high level group scored significantly higher (p < .05) than the middle level. The middle level exceeded the low group but only with significance p < .15.

Interaction of treatment and level was only significant at the .2 level.
Table 6.1

ANOVA of Achievement (Non-text)
by Method of Instruction and Aptitude Level

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>DF</th>
<th>Mean Square</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment (A)</td>
<td>95.555</td>
<td>2</td>
<td>47.778</td>
<td>4.26</td>
</tr>
<tr>
<td>Aptitude (B)</td>
<td>222.423</td>
<td>2</td>
<td>111.212</td>
<td>9.91</td>
</tr>
<tr>
<td>Treat x Aptitude (AB)</td>
<td>68.443</td>
<td>4</td>
<td>17.111</td>
<td>1.52</td>
</tr>
<tr>
<td>Residual (S/AB)</td>
<td>909.392</td>
<td>81</td>
<td>11.227</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>1295.813</td>
<td>89</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*p < .02  **p < .01

Table 6.2

Cell Performance Statistics:
Achievement Test (Non-text Portion)

<table>
<thead>
<tr>
<th>Aptitude Level</th>
<th>Treatment</th>
<th>Computer</th>
<th>Taped</th>
<th>Class</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Vis.</td>
<td>Lecture</td>
<td></td>
<td>Lecture</td>
</tr>
<tr>
<td>High</td>
<td>$x = 19.0$ (76%)</td>
<td>21.7 (87)</td>
<td>19.5 (78)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$s^2 = 11.8$</td>
<td>12.5</td>
<td>8.1</td>
<td></td>
</tr>
<tr>
<td>Medium</td>
<td>$x = 18.7$ (75)</td>
<td>18.4 (74)</td>
<td>16.4 (66)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 14.7$</td>
<td>10.9</td>
<td>11.8</td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td>$x = 18.1$ (72)</td>
<td>16.7 (67)</td>
<td>13.9 (56)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 10.1$</td>
<td>2.6</td>
<td>11.7</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>18.6</td>
<td>18.9</td>
<td>16.6</td>
<td></td>
</tr>
</tbody>
</table>
The ANOVA indicated the existence of significant treatment differences. A Sheffe test shows that null hypothesis H-1 can be rejected with less than .01 probability of alpha error. Thus the test indicated that the pooled mean of Treatment 1 and 2 was significantly higher than that of Treatment 3.

Since both treatment and aptitude variables were found to indicate significant differences, all cell means were compared using Tukeys (HSD) honest significant difference criterion. Differences significant at the .05 level are indicated.

Table 6.3

<table>
<thead>
<tr>
<th>Differences Significant at .05 Level</th>
<th>Notation for Cell Means</th>
</tr>
</thead>
<tbody>
<tr>
<td>T2H &gt; T2L</td>
<td>Aptitude</td>
</tr>
<tr>
<td>T2H &gt; T3M</td>
<td>High</td>
</tr>
<tr>
<td>T2H &gt; T3L</td>
<td>Med</td>
</tr>
<tr>
<td>T3H &gt; T3L</td>
<td>Low</td>
</tr>
<tr>
<td>T1H &gt; T3L</td>
<td></td>
</tr>
<tr>
<td>T1M &gt; T3L</td>
<td></td>
</tr>
<tr>
<td>Comp-Vis</td>
<td>Treatment</td>
</tr>
<tr>
<td>T1H</td>
<td>T2H</td>
</tr>
<tr>
<td>T1M</td>
<td>T2M</td>
</tr>
<tr>
<td>T1L</td>
<td>T2L</td>
</tr>
</tbody>
</table>
Although the high-aptitude cell in the computer-visual treatment group scored higher than the lower cells, it did not significantly exceed, at the .05 level, the low aptitude cell in that group. In the other treatments the high aptitude cell did significantly exceed the low.

Comparisons of Achievement Involving Only Taped Treatments:

For comparisons involving only the two taped treatments (1 and 2) the full achievement test was used. Neither of these groups saw any of the test items in their treatments.

The ANOVA table for performance on this test is presented in Table 6.4.

Table 6.4

Analysis of Variance of the Complete Achievement Test By the Two Taped Treatments and by Aptitude Level

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>DF</th>
<th>Mean Squares</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment (A)</td>
<td>0.067</td>
<td>1</td>
<td>0.067</td>
<td>0.005</td>
</tr>
<tr>
<td>Aptitude (B)</td>
<td>117.434</td>
<td>2</td>
<td>58.717</td>
<td>4.29*</td>
</tr>
<tr>
<td>TreatmentxAptitude(AB)</td>
<td>58.635</td>
<td>2</td>
<td>29.317</td>
<td>2.14**</td>
</tr>
<tr>
<td>Residual</td>
<td>39.192</td>
<td>54</td>
<td>13.689</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>915.328</td>
<td>59</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*p < .01  **p < .125
An important question concerned aptitude-treatment interaction or, in this experiment, level-treatment interaction. Although the significance of the interaction is not at the standard .05 level of confidence, it is noteworthy that the chance of type I error (alpha error) is only .125. The pattern of interaction is the same as predicted by earlier studies, (e.g. Salomon).

Table 6.1 shows the disordinal pattern of cell means. Although the classroom lectures group was not included in this analysis, the cell means for that group were included for the readers information. An identical pattern of interaction existed for means on the (Non-Text) test.

The overall correlation of level IQ with performance on the achievement test was .45. For Treatments 1, 2 and 3 the coefficients were .16, .57, and .57, respectively.
This difference provides some evidence that interaction effects are due to the cancellation of performance differences due to aptitude within the computer-visual treatment.

Results on Attitude:

From the 13 attitude items, three separate tests were constructed as indicated in table 6.5.

Table 6.5
The Three Attitude Tests

<table>
<thead>
<tr>
<th>Test Name</th>
<th>Test description &quot;Attitude towards...&quot;</th>
<th>Number of Items</th>
<th>Reliability KR8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Att 1</td>
<td>Mathematics in General</td>
<td>6</td>
<td>.95</td>
</tr>
<tr>
<td>Att 2</td>
<td>Presentation of the Material</td>
<td>3</td>
<td>.61</td>
</tr>
<tr>
<td>Att 3</td>
<td>Subject matter of the lesson</td>
<td>4</td>
<td>.87</td>
</tr>
</tbody>
</table>

The first test (Att 1) was given to all students. The last two were only given to treatment groups 1 and 2. Thus students in treatment group 3 had only six attitude items on their tests. Items were scored on a 1-5 scale, where 1 = strongly negative, 2 = negative, 3 = neutral, 4 = positive, 5 = strongly positive.
Each test was "normalized" to a 1-5 scale by appropriately reflecting items and computing the mean response. For each test a two-way ANOVA was performed.

\textbf{Attitude Towards Mathematics (Att1):}

Results of the ANOVA on Att1, attitude towards mathematics, are summarized in table 6.6. Cell means are plotted in figure 6.2. There was significant ($F(4,81) = 2.9\ p < .05$) aptitude-treatment interaction on this attitude. Treatment had no effect on this attitude, although ability level did. Generally, higher aptitude levels had more positive attitudes. However, the high-aptitude-level cell receiving the computer-visual treatment failed to exceed either of the other cells in that treatment group and was significantly lower than the high aptitude cell of the taped control (Treatment 2).

Other significant differences of cells (at the .05 level) as determined by Tukey's HSD are $T2H > T1H$, $T1L$, $T2L$, and $T3L$. 
Table 6.6

Analysis of Variance of Att(l)

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment (A)</td>
<td>1.113</td>
<td>2</td>
<td>0.556</td>
<td>0.69</td>
</tr>
<tr>
<td>Aptitude (B)</td>
<td>11.706</td>
<td>2</td>
<td>5.853</td>
<td>7.34*</td>
</tr>
<tr>
<td>Treatment x Aptitude (AB)</td>
<td>9.287</td>
<td>4</td>
<td>2.322</td>
<td>2.91**</td>
</tr>
<tr>
<td>Residual (S/AB)</td>
<td>64.605</td>
<td>81</td>
<td>0.798</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>86.711</td>
<td>89</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*p < .01  **p < .05

Figure 6.2  Plots of Cell Means of Att(l)
Attitude Towards the Presentation (Att2):

Only the two taped treatments were given this instrument. There was little difference between either groups or levels of ability. All six cell means fell between 3.4 and 3.94. However the 2-way interaction was significant at the .1 level (Table 6.7); the means are plotted on Figure 6.3. The recurrent pattern of lower scores by the high-aptitude cell of treatment group 1 is present.

Attitude Towards the Material (Att3):

Analysis of this test is summarized in Table 6.8. Means are plotted in Figure 6.4. The Computer-visual treatment group, T1, had a better attitude towards the material than T2. (p < .05). And again the same pattern of interaction with aptitude is present (p < .14). (See Figure 6.4).

The consistent pattern of aptitude-treatment interaction deserves to be noticed. Both performance and attitude were higher for T1 than T2 for the lower and middle ability groups. But the high ability group again scored lower than the comparable group receiving the control treatment.

For each attitude test, the high aptitude cell in treatment group 1 failed to exceed the mean of the group as a whole.
Table 6.7
ANOVA of Att 2

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>DF</th>
<th>Mean Squares</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment (A)</td>
<td>0.224</td>
<td>1</td>
<td>0.224</td>
<td>0.776</td>
</tr>
<tr>
<td>Aptitude (B)</td>
<td>0.281</td>
<td>2</td>
<td>0.141</td>
<td>0.483</td>
</tr>
<tr>
<td>Treatment x Aptitude (AB)</td>
<td>1.348</td>
<td>2</td>
<td>0.674</td>
<td>2.335</td>
</tr>
<tr>
<td>Residual</td>
<td>15.589</td>
<td>54</td>
<td>0.289</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>17.443</td>
<td>59</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 6.3  Plots of Cell Means of Att2
Table 6.8

ANOVA of Att3

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment (A)</td>
<td>2.204</td>
<td>1</td>
<td>2.204</td>
<td>4.95**</td>
</tr>
<tr>
<td>Aptitude (B)</td>
<td>0.331</td>
<td></td>
<td>0.416</td>
<td>.94</td>
</tr>
<tr>
<td>Treatment x Aptitude (AB)</td>
<td>1.852</td>
<td>2</td>
<td>0.926</td>
<td>2.08 *</td>
</tr>
<tr>
<td>Residual (S/AB)</td>
<td>24.025</td>
<td>54</td>
<td>0.445</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>28.912</td>
<td>59</td>
<td></td>
<td>** p &lt; .05    * p &lt; .13</td>
</tr>
</tbody>
</table>

Computer-Visual

Taped Control

Figure 6.4 Plots of Cell Means of Att3
The consistency of the interaction pattern can be seen if first level effects are "factored out". In the two-way ANOVA model a performance score is determined as
\[ y_{i} = \alpha_{i} + \beta_{j} + \delta_{i:j} + \epsilon_{i:j} \]
where \( \delta_{i:j} \) is resolved into an interaction component \( \alpha_{i} \beta_{j} \) and an error term \( \epsilon_{i:j} \) (Kennedy, 1974). The means of the \( \alpha_{i} \beta_{j} \) (or equivalently the \( \delta_{i:j} \)) for each attitude test as well as for the achievement test and the abridged achievement test are plotted in Figure 6.5. In each case the high-aptitude cell from the computer-visual treatment had the lowest value.

**Exercises on the taped treatments:**

Six exercises which involved sketching images under various rigid motions were part of the work sheets for subjects receiving Treatments 1 or 2 (See Appendix B). Work sheets were corrected and the sketched answers were scored 1 or 0 by the experimenter. 60% of the questions were answered correctly. An ANOVA of scores by treatment and level is given in Table 6.9.

Treatment 1 was significantly better \( (p < .01) \) than Treatment 2 for performance on the exercises. Means are plotted in Figure 6.5.
### Table 6.9

**ANOVA of Performance on Exercises**

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>DF</th>
<th>Mean Square</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment (A)</td>
<td>16.017</td>
<td>1</td>
<td>16.017</td>
<td>7.54*</td>
</tr>
<tr>
<td>Aptitude (B)</td>
<td>12.033</td>
<td>2</td>
<td>6.017</td>
<td>2.833</td>
</tr>
<tr>
<td>Treatment x Aptitude (AB)</td>
<td>5.433</td>
<td>2</td>
<td>2.717</td>
<td>1.279</td>
</tr>
<tr>
<td>Residual (S/AB)</td>
<td>114.700</td>
<td>54</td>
<td>2.124</td>
<td></td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td>148.183</td>
<td>59</td>
<td>*p &lt; .01</td>
<td></td>
</tr>
</tbody>
</table>

*Computer Visual → Taped Control

Exercise Total

- Low
- Med
- High

Figure 6.5 Plots of Cell Means on Exercises
Figure 6.6 Plots of Cell Means Corrected for First-Level Effects
PLEASE NOTE:

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UNIVERSITY MICROFILMS
The total score on the six exercises correlated with the score on the achievement test with a coefficient of \( \rho = .46 \).

Individually the items were strong predictors of related test items. The single exercises to perform a motion using tracing paper correlated with corresponding "sketch the image" problems on the achievement test with coefficients of .33, .39, and .42 for slides, turns and flips respectively.

The cell means corrected for first-level effects are included with the others on Figure 6.6. Again the same pattern of interaction was present.

Analysis of Subtests:

A two-way multivariate analysis of variance was performed on the five subtests (sketching, direct/opposite, visual product/inverse, and symmetry). Interaction was not significant. Aptitude level was significant \( (F(10,154) = 2.43 \ p < .05) \) as well as for each test separately. \( (F(2,81) = 3.9^* , 5.9, 7.2, 3.8^* , 6.2 \ \text{resp.}) \) \( (*p < .05 \ \text{others} \ p < .01) \). Overall effect for treatment was significant \( (F(10,154) = 2.5 \ p < .01) \). A summary of these results is presented in table 6.10. Plots of means on these subtests are presented in Figure 6.7.
Table 6.10
Multivariate and Univariate Analysis of Variance for Topical Subtests of the Achievement Test

<table>
<thead>
<tr>
<th>Variable(s)</th>
<th>Test</th>
<th>Source</th>
<th>df</th>
<th>F</th>
<th>p &lt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Subtests</td>
<td>M</td>
<td>Trt x Apt</td>
<td>20,254.3</td>
<td>.92</td>
<td>.55</td>
</tr>
<tr>
<td>Sketching</td>
<td>U</td>
<td>h, 81</td>
<td>.75</td>
<td>.56</td>
<td></td>
</tr>
<tr>
<td>Dir/ Opp</td>
<td>U</td>
<td>h, 81</td>
<td>1.32</td>
<td>.27</td>
<td></td>
</tr>
<tr>
<td>Vis Rec</td>
<td>U</td>
<td>h, 81</td>
<td>2.16</td>
<td>.08</td>
<td></td>
</tr>
<tr>
<td>Prod/ Inv</td>
<td>U</td>
<td>h, 81</td>
<td>.71</td>
<td>.59</td>
<td></td>
</tr>
<tr>
<td>Symmetry</td>
<td>U</td>
<td>h, 81</td>
<td>.95</td>
<td>.44</td>
<td></td>
</tr>
<tr>
<td>All Subtests</td>
<td>M</td>
<td>Aptitude</td>
<td>10,154</td>
<td>2.34</td>
<td>.01**</td>
</tr>
<tr>
<td>Sketching</td>
<td>U</td>
<td>2, 81</td>
<td>3.95</td>
<td>.03*</td>
<td></td>
</tr>
<tr>
<td>Dir/ Opp</td>
<td>U</td>
<td>2, 81</td>
<td>5.93</td>
<td>.004**</td>
<td></td>
</tr>
<tr>
<td>Vis Rec</td>
<td>U</td>
<td>2, 81</td>
<td>7.23</td>
<td>.001**</td>
<td></td>
</tr>
<tr>
<td>Prod/ Inv</td>
<td>U</td>
<td>2, 81</td>
<td>3.77</td>
<td>.03*</td>
<td></td>
</tr>
<tr>
<td>Symmetry</td>
<td>U</td>
<td>2, 81</td>
<td>6.22</td>
<td>.003**</td>
<td></td>
</tr>
<tr>
<td>All Subtests</td>
<td>M</td>
<td>Treatment</td>
<td>10,154</td>
<td>2.46</td>
<td>.009**</td>
</tr>
<tr>
<td>Sketching</td>
<td>U</td>
<td>2, 81</td>
<td>5.46</td>
<td>.006**</td>
<td></td>
</tr>
<tr>
<td>Dir/ Opp</td>
<td>U</td>
<td>2, 81</td>
<td>2.82</td>
<td>.06</td>
<td></td>
</tr>
<tr>
<td>Vis Rec</td>
<td>U</td>
<td>2, 81</td>
<td>.55</td>
<td>.58</td>
<td></td>
</tr>
<tr>
<td>Prod/ Inv</td>
<td>U</td>
<td>2, 81</td>
<td>.94</td>
<td>.39</td>
<td></td>
</tr>
<tr>
<td>Symmetry</td>
<td>U</td>
<td>2, 81</td>
<td>.30</td>
<td>.74</td>
<td></td>
</tr>
</tbody>
</table>

*p < .05

**p < .01
Figure 6.7 Plots of Cell Means of Subtests
Sex

Sex and other variables relating to personality often have an important effect on performance. Although the scope of this study precluded considering such variables as motivation, aggressiveness, locus of control, a cursory examination as to the influence or interaction of sex on performance was possible.

An analysis of covariance (by IQ) of achievement indicates sex had no effect or interaction or performance. The ANOVA summary is presented in Table 6.11.

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>DF</th>
<th>Mean Squares</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>IQ (^a)</td>
<td>138.412</td>
<td>1</td>
<td>138.412</td>
<td>9.82 *</td>
</tr>
<tr>
<td>Treatment (A)</td>
<td>0.971</td>
<td>1</td>
<td>0.971</td>
<td>0.07</td>
</tr>
<tr>
<td>Sex (B)</td>
<td>0.035</td>
<td>1</td>
<td>0.035</td>
<td>0.002</td>
</tr>
<tr>
<td>Treatment x Sex (AB)</td>
<td>0.898</td>
<td>1</td>
<td>0.898</td>
<td>0.06</td>
</tr>
<tr>
<td>Residual (S/AB)</td>
<td>774.997</td>
<td>55</td>
<td>14.091</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>915.328</td>
<td>59</td>
<td></td>
<td>*p &lt; .01</td>
</tr>
</tbody>
</table>

a. Beta = 0.13 multiple r = .39
**Day of Treatment**

In the analysis it was assumed that differences of time lag due to differing days of treatment would not be a crucial factor since all of the delays were of the same order of magnitude (from three to seven days).

The treatment days were coded as 1, 2, 3, 4, 5 for Monday through Friday respectively, and thus some positive correlation of "day" with achievement might be expected, since higher week days mean shorter lag time.

An analysis of covariance reveals no significant effect of day. See table 6.12.

For each treatment an analysis of regression of Achievement-corrected-for-aptitude on the variable "day" was computed.

For Treatment 1,
\[
\text{Corrected Achievement} = 1.07 - 0.104 \times \text{Day}.
\]

For Treatment 2,
\[
\text{Corrected Achievement} = -1.04 + 0.486 \times \text{Day}.
\]

Neither regression accounted for a significant amount of the variance, nor did the pooled regression. The algebraic signs of the correlation coefficients of achievement with day indicate that Treatment 1 exceeded Treatment 2 in the effect of retention, but neither the difference nor either regression was significant.
Table 6.12
Analysis of Covariance Achievement by Treatment and Day, Covariate IQ

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>DF</th>
<th>Mean Square</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariate (IQ)^a</td>
<td>125.137</td>
<td>1</td>
<td>125.137</td>
<td>8.84*</td>
</tr>
<tr>
<td>Treatment (A)</td>
<td>1.140</td>
<td>1</td>
<td>1.140</td>
<td>0.08</td>
</tr>
<tr>
<td>Day (B)</td>
<td>82.475</td>
<td>1</td>
<td>22.619</td>
<td>1.45</td>
</tr>
<tr>
<td>Treatment Day (AB)</td>
<td>4.928</td>
<td>1</td>
<td>4.928</td>
<td>0.08</td>
</tr>
<tr>
<td>Residual</td>
<td>678.770</td>
<td>48</td>
<td>14.141</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>893.181</td>
<td>58</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. Multiple r = .48

* p < .01
Place of Treatment

There were two separate facilities for the videotape treatments. In a sense, they were physically different. One was in a glass booth in the library. The other was located in an unused foyer of the school. In order to test if the different locations had any effect on learning, an analysis of variance of corrected achievement by location and treatment was made. The summary table, presented in Table 6.13, indicates that the effect of location was not significant. However, the interaction of location with treatment was indicated to be significant ($F = 5.4$, df = (1,55)). Means are plotted in Figure 6.8. Cell n's were unequal. The pattern indicates that the library location enhanced performance on the computer-visual treatment, and the foyer location enhanced the control.
Table 6.13
ANOVA of Achievement (Corrected) by Location of Treatment

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>DF</th>
<th>Mean Squares</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location (A)</td>
<td>2.082</td>
<td>1</td>
<td>2.082</td>
<td>0.16</td>
</tr>
<tr>
<td>Treatment (B)</td>
<td>3.374</td>
<td>1</td>
<td>3.374</td>
<td>0.26</td>
</tr>
<tr>
<td>Loc x Treatment (AB)</td>
<td>69.004</td>
<td>1</td>
<td>69.004</td>
<td>5.37*</td>
</tr>
<tr>
<td>Residual</td>
<td>706.949</td>
<td>55</td>
<td>12.854</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>780.346</td>
<td>58</td>
<td></td>
<td>#p &lt; .01</td>
</tr>
</tbody>
</table>

Figure 6.8 Cell Means of Corrected Achievement by Location of Treatment
CHAPTER VII - CONCLUSIONS AND DISCUSSION

Introduction:

Several conclusions concerning learning from cathode ray tubes with animated graphics are drawn immediately from the data. A presentation of these conclusions is followed by a discussion of other trends which were indicated by the data, but did not produce differences significant at the .05 level. Limitations to generalizations are then discussed. Following these discussions are suggestions for replications, refinements and for other related studies.

In addition to conclusions and discussion based upon the statistics gathered in the experimental study, conclusions and observations concerning the technical and pedagogical problem involved in creating computer animation are presented; technical aspects of current computer technology are discussed. In connection with these ideas the basic importance of the capability of future CAI systems with graphics to picture rigid motions is discussed.
Finally, the author examines the potential of CAI systems with graphics in educational research, and the possibilities and problems involved in integrating such teaching machines into school curriculums.

**Data-Based Conclusions and Discussion:**

**Conclusions:**

The first and most important conclusion is that the basic graphics employed provided a suitable and valuable medium for conveying certain mathematical concepts. Specifically, the line drawings as produced by CRTs that are capable of dynamic translation and rotation:

1. provided sufficient iconicity for the illustration of mathematical concepts,
2. provided sufficient graphic versatility to denote at least a large class of spatio-temporal relationships, and
3. were at least as effective as certain principal alternate presentation modes in teaching the particular topics involved.

At least for the topics presented, teaching via computer graphics tended to be more effective for students of moderate ability than for students of high aptitude; furthermore, for students of moderate ability receiving a computer graphic treatment, there is a positive effect upon attitude. Thus there
were strong indications that computer controlled instruction featuring graphics holds substantial promise as an instructional tool, particularly for students of lesser ability. The extent of this differential effect, as well as the domain of the topics for which it occurs remains to be determined.

**Discussion: Aptitude-Treatment Interaction:**

The most interesting pattern in the data was the occurrence of aptitude-treatment interaction, often designated as ATI. The pattern was disordinal and appeared consistently in the data. Specifically, when cell means on the achievement test, as well as on each of the three attitude tests were corrected by treatment and by level so that means across treatments and across levels were equalized, (i.e. if main effects were eliminated) then the high-aptitude cell for the computer-visual treatment scored lowest of the nine cells on each test. Although this interaction was not significant at the .05 level on all tests, the risk of \(\alpha\), or type I, error on each was less than .15. The form of the interaction was to make the low and middle ability cells exceed the values predicted by first-order effects and the high ability cell fall below expectation.

This pattern of interaction conforms to the results of Salomon (1972). His theoretical model for learning involved a mechanism whereby animated visual material would be incorporated.
into the learner's internal codes by way of imitation. Thus the visual code presented via some medium becomes an internal perception code assimilated into the student's dynamic conception of reality. Salomon found that a necessary condition for this to happen is that the visual activity supplant the previous internal scheme, i.e. it must either provide an original function consistent with the learner's perceptual schemes, or it must displace existing schemes of thought by being apparently superior.

The last point, that the graphic image must exceed existing images in apparent utility to the learner is a telling one. Bright students would tend to provide their own efficient internal models, and so the imagery provided on the animated medium would be less likely to supplant their existing internal codes. Indeed, it has been found that such mediation had a tendency to interfere with learning for brighter students.

The computer-visual treatment provided models of the action of rigid motions in the plane. The notations for the various basic operations (slides, flips, and turns) were schematic and denoted the physical action of various figures. The learners could see the correspondence of the notation with the action of the motion, as well as the nature of the action itself. Also visual presentations were given for the concepts of product and inverse and direct/opposite motions. It seems that Salomon's
analysis of the learning situation might be quite appropriate. These visual models, although being correct and pedagogically sound, would very likely not be as helpful in defining the above-mentioned subtle concepts to learners who already possessed some analytic understanding of the concepts.

Thus it appeared that the animated media treatment employed indeed had the predicted effect. But before concluding that Salomon's model explains the cause of the effect we should be careful to identify any other possible sources of the ATI.

The fact that the pattern of regression of performance on IQ was disrupted in the computer-visual treatment indicates that the source of the ATI lies in effects within the visual treatment. That treatment was the only one which altered the slope of regression, or the relative achievements of the aptitude-level cells. Hence we should search for factors in the computer-visual treatment which might:

1. bias the outcome in favor of students of moderate aptitude, or
2. bias the outcome against students of high aptitude, or
3. disrupt internal validity through some other violation of statistical design.

These three effects are considered in order.
(1) The scores of the low and middle cells of the computer visual group on the attitude tests indicated high satisfaction with the material and its presentation, and relatively positive attitudes towards mathematics in general. These high scores suggest that the animated computer-visual instruction was more appropriate or at least more appreciated by these students.

The fact that students from these lower cells of the computer-visual group scored higher than their counterparts in other groups, is a further indication that the interaction effects were due largely to improved performances of students of moderate ability. So explanations of this improved performance are called for.

Besides providing appropriate models for internal coding, the treatment may have facilitated the interpretation of certain definitions or helped in the task of classifying and discriminating among the new concepts. Put in a Piagetian framework, the computer-visual treatment may have provided appropriate symbolic/concrete operational experience. All of these functions can be considered to have helped learning by providing visual codes which could be internalized, and thus the results can be explained by Salomon's model.

An alternative explanation for improved performances of average students, which is not subsumed by Salomon's model, is that the pictoral content might have served to keep students entertained. Perhaps the animated examples provided a different
atmosphere, say recreational rather than instructional, causing students who traditionally exhibit less attention to instruction to continue to attend to the instruction. Or, perhaps, the removal of the effects of tedium improved the performances.

A fine line of distinction can be drawn between functions of diversion and maintenance of attention. The performances indicated that any "entertainment" value permitted and possibly enhanced maintenance of attention. Whether there was indeed "entertainment" value and whether it was due merely to the novelty of the situation was not determined.

(2) The extremely low scores of the high-ability students in treatment group 1 on attitude both towards the presentation and towards the material are worthy of note, and seem to indicate some dissatisfaction of these students with the treatment.

Several explanations of this dissatisfaction come to mind, and most of these go hand-in-hand with the relatively poor performance of these typically higher-achieving students. Specifically, then, it is felt that relative failure of these students to excel, due to whatever reason, might tend to lower their attitudes towards the presentations. Perhaps the high-aptitude students were in some way upset by the unusual presentation format or by the uncertainty of successful performance on the achievement test. Features of the treatment which might have been inappropriate, tedious, or otherwise disturbing to these
students could have been the pace, tone, or level of difficulty either of examples or of other material in the treatment. Or, possibly, the failure to interest or challenge them with difficult exercises or examples might account for the relatively lower scores of the high-aptitude group in the computer-visual treatment.

The fact that the high aptitude students in the taped control treatment performed very well indicated that, the pace, tone, or level of difficulty of examples could not explain the pattern of cell means, since each of their functions were at least roughly equivalent in the two taped treatments.

Another related plausible explanation is that the instruction in the computer-visual treatment was limited in some way in which the control was not. However, the demonstrated equality of verbal content of the two taped treatments makes that seem unlikely.

Thus it seems that the explanation of the ATI effect, specifically the relatively lower performances of higher-aptitude students, is best attributed to the effect described by Salomon.

(3) Several conditions of the study may have resulted in loss of internal validity. One was the possibility of mutual contamination. Subjects were not controlled outside the classroom and certainly could have interacted with one another; so, possibly, either negative or positive reinforcement could have occurred. A second possibility was the so-called Hawthorne effect. Expectations
or preconceptions on the part of students could have affected responses on the attitude tests especially. Finally it is conceivable that the consistently low values of the high aptitude cell of the computer visual group were due to a statistical aberration; for example, possibly an abnormally large number of students of higher aptitude who happened to dislike mathematics were put into treatment group 1. Were this true, it might explain some of the interaction effects on the achievement test, since achievement is somewhat related to attitude. A preattitude test on the subject population would have helped to resolve this question. Even without such a test, since the assignment of students to treatments 1 and 2 was done randomly, it is unlikely that the strength of the interaction was due to a statistical difference before the fact.

It seems to this author that there is not sufficient cause from any or all of the potential sources of biasing to explain the interaction by any means other than Salomon's model of imagery imitation via supplantation. In addition to any implications for learning theorists, this result has important implications to instructional designers. These implications are discussed in a later paragraph of this section.
Discussion: Differences Between Taped Treatments and Classrooms:

The principal aim of the study was to help delimit the effectiveness of CAI with graphics. Careful comparisons were made between a treatment employing computer-generated animated graphics and a prepared lecture using an identical development of ideas. No significant difference in first-order effect was found.

In order to compare the effectiveness of these somewhat contrived treatments with real-world educational settings, that is, in order to obtain a higher degree of external validity, a treatment employing regular geometry instructors in the school was included. On the achievement test the prepared tape groups exceeded the regular classroom group. However it would be wrong for several reasons to conclude that either or both modes of taped treatment were superior to regular classroom instruction.

First, the students receiving classroom instruction could not, and thus did not, receive independent treatments. This violated one of the ANOVA assumptions, since students were the units of analysis.

Secondly, the instruction in the taped treatments was presented by the person who had written most of the items on the test, and so the instruction would certainly be a possible source of bias.
within the experiment. Although the teachers viewed the lessons and had accurate outlines of the material, certain emphases would not automatically be incorporated into their teaching, nor did the teachers have prior knowledge of the test items. And so there was certainly a potential bias against the classroom treatment.

The purpose of the comparison between the media treatments and the classroom treatment was not to determine which mode was superior. Such a question would be inappropriate since it is in no way suggested that a machine could perform the functions of motivation, concern, discipline, and insight into personality which good teachers must perform. The purpose of the comparison was to determine whether the experimental treatments were indeed effective. It seems rather obvious that results on the effectiveness of one teaching mode versus another would be fairly meaningless if neither was particularly effective. What has been shown is that the taped presentations were effective, and that results involving those treatments, specifically the ATI, were indeed meaningful.

Aside from the possible biases mentioned, other possible sources for the significant differences of the taped treatments and the classroom treatment bear mentioning. First, the subject matter included concepts which involved spatial relationships as well as concepts dependent on sequential processes. Earlier research has shown the advantages of physical or visual examples in the teaching of such concepts. Both experimental lessons were produced with
this thought in mind. The classroom lecturers, on the other hand, did not employ physical examples to the extent that the taped treatments did. Secondly, prepared media presentations involve certain built-in advantages such as the (1) careful pre-organization of ideas of the presentation, (2) editing of the script and the finished product, and (3) close student involvement with the source of information including the ability or responsibility of controlling the progress of the lecture. Finally, there may have been a novelty effect benefitting the two taped, individually presented treatments.

Discussion: Performance on Sketching Exercises:

The worksheets for the taped treatments included six sketching exercises. The computer-visual treatment group exceeded the control group on performance of these examples, which correlated strongly with performance on the sketching items of the achievement test. The differences provided some indications that visual presentation of concepts facilitates better performance on graphics-related problems. However alternative explanations exist such as the different durations of the two taped lessons, the wording of the introduction of the problems on the tape, or the subsequent provision of some solutions on the computer-visual tape. Certainly there was insufficient control of the situation to make any inferences from the data, but the differences do at least warrant further investigation.
In conclusion, the media features of CAI with animated graphics seem to provide a possible improvement over other more standard modes of presentation, particularly for students of lesser aptitude. This advantage may be dependent upon the nature of the material to be learned. Nevertheless if, or rather when, computer-generated animated graphics become economically feasible within CAI settings, mathematics educators should be prepared to take advantage of its potentials.

Implications of the Results for Educational Development:

We have concluded that the ATI can be most plausibly explained by Salomon's model of internalizing visual codes. Among the premises of this model are that visual codes serve as internal codes. This premise would seem to demand that creators of visual media strive to make visual sequences which are not only interesting and effective in immediate learning, but, more importantly, can be assimilated into more basic and general schemes and accommodate extensions to other related topics including more advanced levels of the given topic. That is, the graphic models must be consistent with both higher and lower level concepts, including their visual models.

Thus it seems important that the idea of standard visual or graphic notations be considered seriously. Properly developed, these graphic symbols could help greatly to improve the technology
of mathematics education. However, dangerous pitfalls exist. The most serious one which comes to mind is the limitation of generalizability of models, which could possibly result in stagnation of thought processes.

One frustration which seems to face the creator of visual educational materials is that any media presentation which facilitates learning is doomed, at some level of competency or aptitude, to be ineffectual, or even detrimental. Thus great care should be taken in matching audience with visual media content in curriculum development.

Limitations to Generalizability of Conclusions:

At this point, several limitations of the design should be mentioned:

(1) Results were dependent upon subject matter. Thus the same pattern of interaction would not necessarily be expected if the nature of the lesson or performance criteria were different. Research in other potentially appropriate areas is therefore strongly recommended.

(2) The subjects were not from a typical high school, but from an environment which was probably somewhat richer and more academically-oriented than average. Thus, for example, the students might be more receptive to unusual educational treatments.
So, generalizations to the entire population of high school students (or learners of mathematics) should perhaps be tempered somewhat.

(3) The computer visual treatments were designed to duplicate computer consoles of the foreseeable future. They differed from such consoles in several ways. [a] Actual physical equipment was different. Specifically, computer input/output facilities, such as keyboards, were lacking. [b] Student awareness of computer control was absent. [c] Visual content was presented via raster tube, although constructed via random tube. [d] Branching in the pedagogical sequence did not occur. [e] Due to threading mechanism on the video cassette players, the linear control was not exact as a computer system would be, i.e., if the student would "STOP" the tape and then just "PLAY" the tape, he/she would hear 10-15 seconds of material over again. To the degree to which these differences would create performance outcomes different from those of the hypothetical CAI systems, we must weaken the conclusions specific to that medium. It is felt that this degree is small and that indeed the effects of CAI instruction with animated graphics would produce the same pattern of results.

(4) The video equipment employed was somewhat old, and the video pictures included a small amount of "white" video noise, i.e., soft visual static or fuzzing of the background. This could
have been due to the hardware, the signal connections, or a source of electronic noise in or near the school. Two different signal circuit systems were tried. (co-axial and 8-pin) Neither rendered a perfect playback of the tapes. This might have had a deleterious effect on either treatment 1 or 2, but very possibly not.

These last two points ((3) and (4)) indicate that the power of computer animation when actively employed in real CAI situations may be greater than indicated by the data.

In weighing all of these various threats to external validity, it is tentatively assumed that the totality of their effects would not be great enough to invalidate the observed results. Thus, the conclusions and implications are taken to be valid.

Suggestions for Further Research

Transfer of Learning:

The transfer of learned concepts to other topics is recognized as an important higher-level educational goal, and where it occurs to a greater extent it is supposed that the learning is "better". Some research indicated that transfer effects are greater when a visual treatment is utilized.
Topics in transformational geometry seem to be well suited to examination for transfer effects. The instruction given in this study involved the somewhat subtle notion of groups of rigid motions, and such associated concepts as "group", and "invariance"; other concepts of transformational geometry may have been conveyed to some degree in the learning process. Specifically, the lesson dealt directly with the notions of product, identity, and inverse, but the term "group" was not made explicit. It would be interesting to determine whether such instruction concerning the special group of rigid motions of the plane would enhance the facility with which students could obtain a working concept of abstract group. Other notions whose learning might be facilitated are invariance, permutations and, certainly, rigid motions of 3-space.

Retention:

Some research suggested that one aspect of the value of visual models is in long-term retention of concepts, that is, that learning enhanced by visual correlates or models is more fortuitously internalized and consequently retained.

A simple retention test on the original subject population could provide information concerning the effect of the visual presentation on long-term retention. There might be some problems with intervening history introducing some systematic error into
the data. Regression analysis indicated that retention did not
deteriorate in any significant way over the short span of the
experiment, although this was certainly far from being compelling
evidence of retention.

A follow-up test on attitude could help serve the function of
an attitude pre-test. Since short-term effects of the treatments
would be obliterated by the time interval, attitudes should
closely reflect pre-treatment attitudes. This could only help
to determine if indeed the strong level effects in treatment 1
were due to the treatment. Much maturation may have occurred, and
some of the attitude changes could have been long-term ones.
These would have to be negligible if such a test were used meaning-
fully as an effective pre-test.

Refined Replications:

It is strongly recommended that as graphic capabilities
provide the opportunity, research similar to this study but
involving other topics and methods should be pursued.

Certain changes in the design of the experiment are suggested
for replications or extensions:

(1) The pre-testing of attitude would have made the patterns
of post-treatment attitudes easier to analyze.

(2) The use of another treatment, a taped lecture presenting
only formal and no physical or visual development of the material
might further have isolated visual-graphic-physical relationships—
to/or dependencies-on presentation of the material or subtopics.

(3) The use of two computer-drawn treatments, one employing only still graphics, the other animation would correspond to the differences between the storage tube capabilities (still graphics) and the potential for animated raster-tube graphics. The additional treatment, i.e. non-animated graphics, would correspond to the "short-circuit" treatment in Salomon's experiments. Since the ATI occurred between the visual and "short-circuit" treatments we could expect that there would be differences in the learning of concepts involving motion-related processes likely in the nature of aptitude-treatment interaction disordinal in the animation treatment. Such an experiment could help determine the importance of being able to observe the process of a sequential event such as a rigid motion of the plane.

Implications for Graphic CAI Technology:

Besides conclusions from the performance data, observations based upon the experience of the experimenter can be made concerning both the technology and pedagogy. The results of this study would be meaningless if the technology proved to be inadequate or impracticable for educational use.

For the problem of graphical-data manipulation, the graphic activities in this experiment concerning transformational geometry
have some intrinsic importance. Specifically, since the movement of solid (rigid) objects through space (or on the plane) requires only the three basic motions (translations, rotations, and reflections) and products of these motions, any system which provides these motions as basic operations on pictures has the capacity to perform virtually any rigid motion.

Actually, reflections are not even requisite for a graphics system depicting real phenomena since (1) reflections in the plane are just 180 rotations in space, "flips", in the terminology of this experiment, and (2) reflection, or any opposite motion in 3-space, do not occur as transformations of solid objects except as mirror images.

**Animated Rigid Motions as Graphics Options: Hardware Problems**

Thus it is critical whether a graphics system would be able to provide translations and rotations in 3-space. It should be fairly obvious that the problems of translations is a trivial one, since virtually all computer graphic pictures are coordinatized, and can be easily translated by adding the same constant to all coordinates. Rotations could be harder. In the system employed in this experiment, rotations were hard-wired in the vector generator, and were actually analog phenomena rather than a digital data transformation. The digital problem seems to have an unavoidable inclusion of trigonometry, i.e. the necessity of
storing or generating trigonometric values, and the modifying of
the data by multiplications, which are not as fast as additions.

If animation were to be employed, it seems that incremental
rotations for each frame would be necessary. The number of
calculations would be quite large.

Of course this problem is alleviated somewhat if the picture
data are arrays of endpoints rather than the full sets of points
comprising the pictures, since many fewer computations would be
involved for each rotation if the picture were determined by a
relatively small number of endpoints.

From the inception of the concept of computer graphics until
a very short while ago, the vast majority of research in computer
graphics was with random type tubes. However, in order for CRT
graphics to be utilized on a large scale it seems that raster
tubes will have to be the standard output device.

Advances such as the recent microprocessor boom could
radically eliminate some economic/technological roadblocks. An
important technical development would be mass production of cheap
processor chips which function as vector generators. As mentioned
before, line segments cannot be drawn on raster tubes, but could
be constructed by algorithm. This "software" activity is clearly
so important as to be an integral part of the graphics processor,
and seemingly could be performed by microprocessor chips. As
mentioned above, this would mean, for example, that rotations
as well as other affine transformations (e.g. scaling, perspective) need be performed only on the endpoints. Hopefully the development of such chips will come soon, if it has not already. As implied earlier in this chapter, it remains to be seen if the graphic effects of raster screens are the same as random tubes. To be specific:

Although (1) the computer graphic images used in this study, were constructed by simple state-of-the-art random vector algorithms, and (2) the images were presented on a raster tube, preserving the iconic properties of the original CRT image and (3) algorithms to produce the same effects (e.g. animated translations, rotations, and flips of the plane) for computer generated video images are clearly feasible for raster tubes; it is not clear that the same graphic qualities such as smoothness and apparent rigidity would be present with a computer driven video (scanning) tube. Likely the preservation of such qualities would depend on the design and speed of the particular algorithms.

**Animated Computer Graphics: The Software Problem**

Should the requisite image generation and manipulation capacities develop, through processor technology, several levels of software problems come into existence:

(1) Picture construction: If indeed pictures are constructed by endpoints or otherwise, the problem of determining the endpoints or just points for pictures is a non-trivial one. In mathematics
this problem is not great since most ideas can be imbedded in a
cartesian representation, and thus can be constructed by programmed
algorithms.

(2) Picture qualities: Can figures be shaded, be colored, be
made opaque, be distorted by perspective, be moved smoothly ???
The answer is a qualified yes in each case. Each of those
functions could be performed if suitable calculations could be
made within the real time constant of the system.

(3) Picture manipulation: Can pictures be stored, altered,
duplicated, merged, or scaled ? How can they be moved ?

(4) Picture manipulation by user: Can facilities be created
for student ability to construct graphic images ? The student
commands would be different from the coursewriter commands which
would be different from the machine's graphic commands.

(5) Courseware: Sophisticated branching, information processing,
input-output schemes, as well as graphic capabilities will be
available for CAI use. However coursewriting languages will be
necessary in order for teachers to employ these potentials.

It seems the questions of complex picture manipulation might
be solved by some buffering of graphic data. (Myers, 1976).

Of course cheap mass memory is a key to the solution of the problem.

Finally it should be said that in order for these capabilities
to be utilized, educators with sensibility to the information-handling
capabilities of graphic CAI systems will need to involve themselves
in designing suitable graphic presentations of topics.
Computer Graphic Terminals in Educational Research

For several reasons it seems that CAI with graphics holds great promise both as a research tool and as a practical instruction device. Use of computers in instructional research would allow for accurate measurement of time intervals, sequences of responses, and other student input variables as well as the option of possible immediate analysis of performance and the determination of subsequent feedback operations, without experimenter influence. Every aspect of the instruction would be completely controllable, recordable and replicable.

With computer graphic terminals, there will be a real potential for controlled instructional experiments with consistent quality visual material whose effect would not be due to novelty but rather to a true integration into the instruction. Further, the experimental conditions would be identical with the actual "classroom" conditions, since the "classroom" being studied would be the terminal as instructional medium.

Such a system would allow for research at every step of development. Beneficial byproducts features would indicate standardized presentation of material, evolution of a easy replication of conditions in diverse locations, and the resulting refinement of presentation, branching, and graphic models. Possibly standardized presentations and graphic models will evolve.
Hopefully, and probably, many teachers would participate in writing course material to be used across the country. This is no easy task, and much work and research would be necessary to make routines sophisticated enough for the diverse abilities of students. The danger of insufficiently sophisticated material has been indicated in this experiment.

The difference between a restricted study on media effects, and the potential for research using such a computer graphic system (i.e. internally-externally valid research using media and on media), can be indicated by the situations described in the following sections.

Situation (1). The Design of this Experiment:

Figure 7.1 is a schematic drawing of the information flow in an experiment similar in design to the one contained in this dissertation. The experimenter-teacher is interested in the effect of certain variables in the learning of mathematics, so that better instructional techniques might be developed. So he creates certain fixed instructional materials, and graphics as well.

The student views this material with control of progress and/or repetition of the material. The student learning is measured by scores on post-tests. Conclusions are made upon these performances, and instructional techniques are modified accordingly or perhaps the experimenter will make another study with modified materials.
Diagram of Information Flow in Simple Experiment on the Medium of Computer Graphics
Situation (2). An On-Going Experiment on a System of the Future:

In a projected system of the future, for roughly the same
amount of effort, time, and commitment of facilities, (i.e. equipment
cost) the following scheme of interactions indicated in figure 7.2
will occur.

Again the experimenter-educator is interested in effect of
pedagogical sequences and graphic content on the learning of
certain concepts. But now, since micro-computer systems with
graphic capability are fairly common, he is also interested in
developing or refining graphic-instructional sequences utilizing
the instructional software available.

Certain aspects of the instructional package can be controlled
or varied; number of examples, extent of definitions, type of
instances (positive or negative), frequency of student exercises,
amount and nature of student feedback, types of graphic explanations,
etc. Through a scheme input by the research, one or various
strategies or combinations might be tried, and indeed could be
generated by some stochastic operator. And with the possibility
of on-the-spot testing and contingent branching, unproductive
activities (or rather the algorithms used by the machine which
produced them) could be terminated. Thus the teaching algorithms
could be improved internally in certain statistically obvious cases,
and future algorithms could thus be improved dynamically.
Figure 7.2
Diagram of Information Flow in Sophisticated Learning Research Involving Computer Graphic Presentation of Branching-Student-Interactive Instructional Material
Performance data could be maintained for each student user, so that upon log-on the student's history and progress would be recalled and he would resume where he left off. This performance data, which could include response times and indeed any electronically measureable quantity, could be kept on disk or tape and ultimately analyzed by a large machine.

Again, the teaching algorithms could be reflected, especially to suit student characteristics, aptitudes, interests, personality types, etc. The researcher could review the performance statistics and the resultant changes in teaching algorithms, and could make changes in the algorithms himself; or, at one higher level, change the method (i.e. algorithm) whereby the teaching algorithms are modified. This may all seem far-fetched, but the information processing capabilities indicated are real. Only the actual hardware, software, and technical methodology are undeveloped.

Storage Requirements for Recording Histories

For discussion purposes, the particular instructional sequence could be considered a succession of "frames" where a frame would be a particular picture (or animated scene) with explanations and/or inputs from the students, or a question (probably with multiple choice answers), or perhaps a request for the student to select the type of activity. In other words, a frame would be a unit of processing which involves approximately 4-8 options.
Then, the entire history of a student's sequence and performance on a module of approximately 100 frames might require 100x3 bits, and certainly could be stored in 100 bytes. This is a small fraction of the memory of most micro-systems. Student histories, including current status or progress would probably have to be preserved on a disk or cassette tape or other mass storage device.

The information handling capabilities described are available today. As is indicated in figure 7.2, a large computer may be needed for processing of the huge amounts of data that the experimentation described could generate. However, very likely, for less complex systems, the data files could be maintained and analyzed by the micro-system itself.

In any case, the research could monitor accurately many completely controlled variables of instruction in a non-contrived setting where the design of the treatment is pedagogically oriented. High quality data recording, equality of conditions, mobility to any location, and non-deterioration of graphic materials (such as occurs with film and videotape) would be benefits of such an instructional/research environment.

**Necessary Future Developments:**

For such teaching systems to develop and be used widely, it seems that certain trends would need to continue or develop:
(1) Prices of systems (including peripherals such as tape and disk drives and CRT's) would have to remain relatively moderate.

(2) Appropriate graphics driving systems (hardware and software) need to be developed, as mentioned above. Clearly some are in use today (e.g. the arcade games). With processors functioning as vector generators, and with processors functioning as line-segment-constructors, much of the random vector technology could be applied to the raster tube graphics.

(3) Educators would have to begin using these systems for serious instructional purposes. This has already been done with time-sharing CAI, including several instances with graphics. A conversion to micro-systems, although extremely economical in the long run, will be difficult to initiate.

(4) In order for teachers to employ these systems, suitable software would have to be developed such that "courseware" is easy for non-programmers to write. Again, research and development is proceeding, although based upon the larger machines.

(5) Research on the methodology of CAI must be refined to the point that materials utilize the medium effectively, and that the coursewriting algorithms are attuned to this application of learning theory.
It is clear that some of these points depend on others in a "vicious cycle" situation. For example, machines must be in general use for serious coursewriting efforts to be worthwhile. On the other hand, the availability of quality instructional materials would be necessary before schools could commit their resources to buying hardware.

A summary of some of the important necessary developments and their interrelationships is presented in figure 7.3.

From the point of view of the mathematics educational researcher-curriculum developer, the next steps are waiting for systems suitable for micro-CAI to be developed. It is debatable whether the inexpensive 8 bit/word machines of today are suitable to the task, particularly handling graphics. Mini-computer systems employing 16 bit words will be on the market within two years at competitive micro prices. It seems that these could possibly become the standard, since much software will have been developed already for machines of that size.

In any case, an important function of mathematics educators should be the coordination of research and development activities to produce and promulgate, and eventually standardize, quality software for writing course material, as well as the course material itself.

Hopefully, software symposia will occur. Such a symposium would have to be based upon compatibility of systems, and so the
Figure 7.3 The Development of Micro-computer CAI Materials
issue of the word size used by machines could be a very important one. Whether such standard languages as BASIC or APL would be suitable for the needs would also have to be investigated.

Probably several "gauges" of educational software will arise, and eventually sort themselves out in terms of usability and effectiveness. However, if no impetus is given to the development of standards (word size, assembler language, other (?)) the value of the technology may be lost.

Again without the capability of exchanging and co-refining educational software, the potential value of CAI, particularly with the more economically feasible micro-computers, will be lost.
APPENDIX A

Attitude and Achievement Tests

(The Attitude Items:)

Please indicate your degree of agreement with each of the following statements. Circle one of the five options for each statement. SD - Strongly Disagree   D - Disagree
U - Unable to say  A - Agree   SA - Strongly Agree

SD D U A SA 1. Mathematics is one of my better subjects.
SD D U A SA 2. I feel at ease with mathematics.
SD D U A SA *3. I approach math with a feeling of hesitation, from a fear of not being able to do math.
SD D U A SA 4. I enjoy the challenge of mathematics problems.
SD D U A SA *5. I feel a sense of insecurity when attempting mathematics.
SD D U A SA 6. Mathematics is very interesting to me, and I enjoy math courses.

Concerning the presentation of the material concerning rigid motions and congruences:
SD D U A SA *7. The material was too hard.
SD D U A SA *8. The presentation was too fast.
SD D U A SA *9. The presentation was too slow.
SD D U A SA 10. The presentation was interesting.
SD D U A SA 11. I would enjoy learning more about rigid motions and symmetries.
SD D U A SA 12. I feel I learned a lot.
SD D U A SA 13. The subject matter seemed interesting.

* These items are scored negatively.
TRUE-FALSE ....... In front of each statement write T if it is always true, and write F if sometimes it is false.

____ 1. Two figures are congruent if they have the same perimeter and area.

____ 2. If two angles have equal measure, then the angles are congruent.

____ 3. Every plane is congruent to its mirror image.

____ 4. A flip is the same as a 180° clockwise turn.

____ 5. If A is congruent to B, and B is congruent to C, then A must be congruent to C.

____ 6. A turn will sometimes change the measure of the angles of a triangle.

____ 7. If all the corresponding sides of two figures have equal length, then the figures must be congruent.

____ 8. The product of two slides is a slide.

____ 9. If \( \triangle ABC \sim \triangle BCA \), then \( \triangle ABC \) is equilateral.

____ 10. Every congruence is a product of slides.

____ 11. A clockwise turn of 360° has the same effect as the zero slide.

____ 12. The product of two flips is a flip.

____ 13. The product of two direct motions is a direct motion.
14. The product of two opposite motions is an opposite motion.
15. Any rigid motion is a product of flips.
16. If a rigid motion is direct, then its inverse is direct.
17. Every half-plane is congruent to every other half plane.
18. If $\triangle XYZ \cong \triangle CAB$, then $\angle A \cong \angle X$.
19. There are only three different correspondences between the sets of vertices of two triangles.
20. There are at least four congruence correspondences of the set of vertices of a square with itself.
21. If two segments have the same length, then they are congruent.
22. Sketch the image of the figure under the indicated motion.

23. Sketch the image of the figure under the indicated motion.

24. Sketch the image of the figure under the indicated motion.

25. Indicate the image of point $A$, under the rigid motion which takes Figure 1 to Figure 2.
26. Which of the following figures is congruent to Figure A by a flip?

![Diagrams of figures A to E]

27. The correspondence $ABCD \leftrightarrow QRSP$ describes a congruence between two rectangles. Which of the correspondences below do not describe a congruence?

a. $DACB \leftrightarrow PQRS$

b. $CDAB \leftrightarrow SRQP$

c. $DBAC \leftrightarrow PQRS$

d. $CDAB \leftrightarrow SPQR$

28. Name (draw) three capital letters of the alphabet which are flips of themselves. (have bilateral symmetry)

29. Name (draw) three capital letters which are turns of themselves. (have radial symmetry)

30. Describe precisely the inverse of a $72^\circ$ clockwise rotation about the point $P$. 
31. Using tracing paper, decide which of the other figures (b), (c), (d) are congruent to figure (a).

(a) (b) (c) (d)

32. Match the label of the figure with the motion which takes figure A to that figure.

___ TURN ___ FLIP ___ SLIDE

5 5

For each of the following problems, sketch an object whose image is itself, under the indicated motion.

33. (FLIP)

34. (90 clockwise turn)
35. Will at least one flip be required to make the following pairs of figures coincide?

(a) yes no
(b) yes no
(c) yes no
(d) yes no
(e) yes no

36. Illustrated below are pairs of congruent figures. Fill in the missing letters so that the indicated correspondence describes a congruence.

a. Q ___ ___ Y

b. EABCD ___ ___ ___ ___
APPENDIX B

Materials Used in the Experiment

<table>
<thead>
<tr>
<th>Section</th>
<th>Page number</th>
</tr>
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<tbody>
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<td>VII. Answers to exercises</td>
<td>194</td>
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</tbody>
</table>
APPENDIX B

Materials Used in the Experiment

(I. Letter to Teachers and Lecture Notes)

In order to evaluate the importance of visual perception in learning certain concepts, I have designed an experiment comparing several modes of presentation of certain mathematical topics. These are outlined on the following page. The approach should be different from the one your students have seen. The development should complement what they know, rather than compete with, or detract from it. All of the concepts covered, although fairly elementary, lead to important concepts in higher level mathematics.

I realize that the material is probably too much to cover in one class period. However we are limited by time and other constraints one of them being the design of the experiment. Comparable time and content are essential for any meaningful conclusions to be gotten from this effect. I therefore request that you make a serious effort to cover all the topics indicated on the following sheet, as effectively as possible.

Although we cannot spend a great deal of time on this material, which I feel is quite worthwhile, whatever success we attain in conveying these concepts will be worth our efforts.

I truly appreciate your time and effort in performing this experiment.

Sincerely,

184
(Teachers' Notes Continued:)

RIGID MOTIONS AND CONGRUENCE

The one-period lecture is to contain descriptions of the three fundamental rigid motions of the plane, slides, flips and turns.

I  aStudents should understand the effect of each type of motion on arbitrary objects in the plane, bnoting such facts as the preservation of lengths and angles, cand the notion that the entire plane is moved (taking all figures or points with it).

II  aThe notion of product is introduced. (Examples: product of slides is a slide; products of turns about the same axis is another turn about that axis.) bThe Identity is seen to be the "zero slide" or "zero motion". cThe inverse os these motions are determined. (Noting that the inverse of a flip is itself, the inverse of a slide is a slide of parallel but opposite direction and of the same length, and the inverse of a turn is a turn about the same axis of the same angle in the opposite direction.)

III  aEach motion is categorized as direct or opposite, depending on respectively whether or not clockwise sense of vertices is preserved.

The Roman numerals and small case Latin letters are used to outline the material in the notes, for the purpose of discerning the emphasis of the achievement test.  (See Appendix C)
bSlides and turns are direct. cFlips are opposite. dThe product of direct motions is direct. eThe product of two opposite motions is direct. fThe product of a direct and an opposite motion is opposite. (In a sense they are like positive and negative.)

IV aProducts of these rigid motions are also rigid motions. (Distances are preserved by both operations and hence by the product)

bIt can be proved that every rigid motion is a product of slides, flips and/or turns. (Every direct motion is a slide or a turn. Some opposite motions are not just a flip. These are called glide reflections. Each can be represented either as a flip and a slide, or as a turn and a flip.)

V aTwo figures are congruent if one is the image of the other by a rigid motion. b(That is by a product of slides, flips, and turns... often just one.) cA good exercise is to determine for particular congruences, the particular rigid motion which determines the congruence. That is given figure and image, determine the motion.

VI In fact, since every slide is a product of two flips, and since every turn is also the product of two flips, then every rigid motion is the product of flips.

VII aA figure has symmetry if it is its own image under a "non-zero" rigid motion. bProducts of such symmetry motions of a figure are again symmetry motions of that figure. cExercise: Determine all the
(Teachers' Notes Continued:)
symmetry motions of an equilateral triangle; a square; \( ^e \) a rectangle; 
\( ^f \) an isosoles triangle. Obtain products of these...they should correspond 
to one of the other symmetry motions.

\[ g \text{ radial symmetry} \quad \text{----turn} \]
\[ h \text{ bilateral symmetry} \quad \text{----flip} \]

????? ????--\text{-----turn}

Working Vocabulary

I. Concepts and terms with which students are presumed conversant:

- 2-dimensional (intuitive)
- 3-dimensional
- distance
- direction (planar...intuitive)
- clockwise
- counterclockwise
- correspondence
- measure of an angle
- parallel

II. New terminology introduced:

- motion of the plane
- rigid motion
- slide
- slip
- turn
- direct
- opposite
- image
- product (of rigid motions)
- zero slide (identity)
- inverse (of rigid motion)
- symmetry \ldots \text{radial}
- bilateral
(Teachers' Notes Continued:)

The teachers' noted also included photocopies of pp. 110 - 115 of *A Second Course in Mathematics for Teachers*, Eugene F. Krause, Harper and Row.
Geometry students:

A graduate student from Ohio State University, Mr. Bernard Baltz, is interested in how students learn mathematics. As part of his studies, he has prepared a mathematics lesson on videotape. You will be asked to view this lesson, and later in math class you will be given a test on the material covered in that lesson. The test score is only for Mr. Baltz, and in no way will affect your grade in geometry.

Please indicate periods which you will have free during the week of Feb. 23 - Feb. 27. Circle your free periods for each particular day. If you wish to keep a study period for some reason, then leave it uncircled. You will be asked to view the tape during one of the circled periods.

Monday 1 2 3 4 5 6 7
Tuesday 1 2 3 4 5 6 7
Wednesday 1 2 3 4 5 6 7
Thursday 1 2 3 4 5 6 7
Friday 1 2 3 4 5 6 7
NAME ____________________ __________________

Last         First

Geo. hr. __________
Room __________________

ID # __________
Treatment __________
Day __________
Period __________
IQ __________
Prev. Course __________
Course grade __________
Test 1. __________
Test 2. __________
Test 3. __________
Att. 1. __________
Att. 2. __________
To: __________________________

You are being asked to view a mathematics lesson on videotape, prepared by a graduate student from Ohio State University.

During _______th period, on _________ Feb. ______, please go to ___________________________________ where you will find a television and a videotape player. A set of instructions with your name at the top will be next to the machines on the table.

Read those instructions and immediately start viewing the tape. (Make sure that you arrive promptly at the beginning of the period.) A man will be available to help you if there are any difficulties with the videotape machine.

If, for any reason, you will not be able to view the tape, please notify your geometry teacher or Mr. Ridenour immediately.

On the following Monday (______, March 1) we will have a test about the material from the tapes in geometry class. Since we are interested in how much you learn from the tapes, please do not discuss these topics with fellow students after viewing the tape. Your performance on this test will have no effect on any of your regular course grades. But please try to do as well as you can. I sincerely thank you for your cooperation.

Bernard Baltz
Department of Mathematics
The Ohio State University
During this period you will be viewing a videotape lesson concerning certain topics in geometry. It is important that you make a serious effort during this period. No outside work is asked.

Begin the tape immediately after reading this page. You may have to rewind the tape before beginning, so push the REWIND button to get to the beginning of the tape. And when you are finished viewing the tape, be sure to rewind the tape.

During the tape, certain exercises which are on the attached sheets will be assigned. Stop the machine, (push the STOP button) do the exercise, and then restart the lesson (push the PLAY button). Answers will either appear on the screen or be on an attached sheet.

The tape takes less than a full period, and if you do not stop the machine for too long a time, you should finish before the end of the period. If any segment of the lesson is confusing, feel free to stop the machine, backup the tape and review the material.

The material on the tapes should be new and fairly difficult. So do not expect to master every idea mentioned. A test concerning this material will be given at a later time. Your performance can in no way affect your regular mathematics grade.

PLEASE DO NOT DISCUS THIS MATERIAL WITH OTHER STUDENTS; AND DO NOT STUDY ANY OF THESE TOPICS FURTHER UNTIL AFTER THE TEST.

If you have any questions or problems, someone will be available to help you. He will be either in the library, or in the foyer of the auditorium.

When you are finished, please leave your worksheets. I thank you for your help and cooperation.

Bernard Baltz
The Ohio State University
Exercise 4.
Use tracing paper to perform the indicated slide.

Exercise 5.
Use tracing paper to perform the indicated turn.

Exercise 6.
Use tracing paper to perform the indicated flip.
ANSWERS TO THE EXERCISES

Ex. 1.

Ex. 2.

Ex. 3.

Ex. 4.

Ex. 5.

Ex. 6.
APPENDIX C

Classification of Test Items According to Topics in the Teachers' Notes

The teachers' notes were outlined by paragraph and sentence for this approach. (The notes themselves were actually just an outline, so this organization is a valid one.) Then each question on the achievement was examined and was labelled with the topic (sentence) from the notes which provides the concept involved in answering that question (when possible).

Of course, this process is quite arbitrary and somewhat imperfect in that many questions involve more than one fact, or many facts implicitly involve preceding concepts. In such cases, the main fact or most complex concept involved was to be selected. Question marks indicate items the experimenter-classifier found difficult to classify.

The topic headings are found on the teachers' notes (page ), the question numbers are those of the achievement test (page ).

The reader can verify that many of these classifications are clearcut, whereas some are quite arguable. Nonetheless, the investigation reveals that the scope of the achievement test was restricted to the notes, and that all areas, and most concepts mentioned were tested.

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Table A

Correspondence of Test items to the Syllabus

<table>
<thead>
<tr>
<th>Topic</th>
<th>Question(s) on Achievement Test</th>
<th>Question Number on Achievement Test</th>
<th>Topics from Teachers' Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>a(22)(23)(24)(33)(35)</td>
<td>1 V a</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b(6)(21)(27)(38)(39)</td>
<td>2 V a</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c(25)</td>
<td>3 V b</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4 III b,c</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>a(8)(10)</td>
<td>5 IV a</td>
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<td></td>
<td>b(11)</td>
<td>6 I b</td>
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<td>c(16)(30)</td>
<td>7</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>8 II a, V b</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>9 VII f</td>
<td></td>
</tr>
<tr>
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<td></td>
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<td></td>
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<td>39 I b</td>
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</tr>
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In geometry we study such things as the sizes, shapes and positions in space of objects. Generally we restrict ourselves to a two-dimensional situation for the sake of simplicity, and because many important results will generalize to three dimensions.

We will be interested in motions of the plane which move objects but do not change their shape or size. We call these rigid motions. Let's examine some of these rigid motions.

Now not every motion is a rigid motion. This motion changes the shape of figures. This one does not change the shape but changes the size of the figure.

The first type of rigid motion we are going to look at is called the slide. This motion is one which moves a figure by a particular distance in a particular direction, without changing its size or shape or its angle with respect to the horizontal.

Notice that the distance and direction from each point to its image can be indicated by an arrow. This arrow determines the slide. If we put the tail of the arrow on a point, the tip of the arrow will lie on the image of that point.

Notice that the slide has the same effect on every figure in the plane.

We are now ready to try exercise one. So, if you would, stop the tape player, and restart it when you have finished the exercise.... Your answer to exercise one should look like this. If it does not, back up the tape player and review material as you feel needed.

Now notice that many arrows can indicate the same slide. You don't have to be touching the figure. However, notice they all have the same length and the same direction.

For example, what would be the image of this disc under this slide? Well, we could take a guess. It goes about this far. (It would be about here.) The way to do it accurately would be to make an accurate parallel copy of this arrow in the same direction. We have to measure it, and have the corresponding point of the disc move to the tip of the arrow.
APPENDIX D

Scripts (Transcriptions) of Narration of Treatments I and II

In order to provide evidence of the equivalence of the treatment of the material in Treatment I with that of Treatment II, the scripts are presented here in parallel. It is to be noted that in several instances verbal explanations were longer in the control lecture.

**Treatment I (Computer-Visual)**

In geometry we study such things as the sizes, shapes and positions in space of objects. Generally we restrict ourselves to a two-dimensional situation for the sake of simplicity, and because many important results will generalize to three dimensions.

We will be interested in notions of the plane which move objects but do not change their shape or size. We call these rigid motions. Let’s examine some of these rigid motions.

Not every motion is a rigid motion. This motion changes the shape of figures. This one does not change the shape but changes the size of the figure.

The first type of rigid motion we are going to look at is called a slide. A slide moves an object a particular distance in a particular direction, without changing its size or shape or its angle with respect to the horizontal.

Notice that the distance and direction from each point to its image can be indicated by an arrow. This arrow determines the slide. If we put the tail of the arrow at a point, the tip of the arrow will lie on the image of that point.

Notice that the slide has the same effect on every figure in the plane.

We are now ready to try exercise one. So, if you would, stop the tape player, and restart it when you have finished the exercise... Your answer to exercise one should look like this. If it does not, back up the tape player and review material as you feel needed.

You notice that many arrows can represent the same slide. It leaves the figure to its image. We have here from three different arrows, and notice that each of the arrows has the same length, they are all parallel, and they all go in the same direction.

If we have a group of arrows, which have the same length, are parallel, and in the same direction, then they all represent the same slide. And in fact all of these can be considered to be the same arrow. They all represent the same slide.

**Treatment II (Control Lecture)**

In geometry we study such things as the sizes, shapes and positions in space of objects, and their interrelationships. Generally we consider a two-dimensional situation, for the sake of simplicity, and because many important results will generalize to similar results in three dimensions.

We will be interested in notions of the plane, which move objects, but do not change their shape or size. We call these rigid motions.

Not every motion is a rigid motion. We will look at several kinds of rigid motions.

The first type that we will consider is called a slide. A slide moves a figure by a particular distance in a particular direction, without turning it and without distorting it in any way.

A given slide has the same effect on every object in the plane.

Well, let’s look at this. We have a figure and its image under a slide. We notice that each vertex...

...A, B, C... is moved to a corresponding vertex A', B', C'. We notice the distances are all the same. (In fact, for any point), and the direction is the same. How if we are given a figure, and an indelible slide, we see it takes an arrow, a direction and a distance to indicate a slide. We can determine the image of this figure under the given slide. We have a point A, it moves to its point, the image of A.

Now this is a basic type of rigid motion, and I would like you to try to perform a slide on an object, so, I would like you to perform exercise one. Stop the tape machine and restart it when you finish the exercise.

Now any one of many arrows can indicate the same slide. You don’t have to be touching the figure.

However, notice they all have the same length and the same direction.

For example, what would be the image of this disc under this slide? Well, we could take a glance. It goes about this far. (It would be about here.) The way to do it accurately would be to make an accurate parallel copy of this arrow in the same direction. We have to measure it, and have the corresponding point of the disc move to the tip of the arrow.
If we are given a point $A$ and a point $B$, is there a slide which will map $A$ to $B$? Well, yes. We just take the arrow which originates at $A$ and ends at $B$, and this will give us a slide which will map $A$ onto $B$.

Now we are ready to do exercise two. So stop the machine and restart it when you are finished. Your answer to number two should look like this. If it does not, back up and review some of the material.

Now notice that we can have a slide in any direction. Also we can have a slide of any length. Here are slides in the same direction of various lengths. They are getting shorter and shorter. Would it be possible to have a slide of length zero?

And if so what would the effect of such a slide be? Well, if we had a figure which a point $A$ it would be moved a distance zero so the image of $A$ would be point $A$. $A$ and the image of $A$ will coincide. We call this motion the zero slide. This is of interest even though its effect on a figure is nothing.

Now if we perform two consecutive slides on a figure, i.e. if we slide a figure and then slide its image, what is the total result? Well, here we look at a few. Notice carefully: if we slide a figure, take that new image, slide it, the result is a slide of the original figure. Taking this composition of slides we call the product of the two slides. We have just seen that the product of two slides is another slide.

Now we'll look at some more examples of products of slides. Since the product of two slides is a slide, the product of any number of slides is always going to be a slide of the original figure. The product of all these slides will get us back to the original position. This product is the zero slide.

Now if we have a point $A$ and $B$, is there a slide which will move $A$ to $B$? Well, of course there is. It's not hard to figure out that if we just take the slides whose arrow's tail is at $A$ and tip is at $B$, it's going to give us a slide which takes $A$ to $B$.

Well, now we're ready to try exercise two, so again stop the machine. Do exercise two. Restart the machine when you have finished.

A motion of this type is also called a translation. We will use the term "slide".

Well, here is a slide. Notice we can have a slide in any direction. We can have a slide down this way... here is the image. Now we can have a slide of any length. We can have one in the same direction of maybe half the distance, and its image would be the figure here. We could go half that distance, or even less...maybe we could have a slide of this distance, in fact, and here is its image. It doesn't hurt for the image to overlap with the figure. We could have a slide of length zero.

Now what would that be like? Well, the tip of the arrow would coincide with the tail of the arrow, and the image of this point would be at the same point. The figure actually doesn't move. The image of the figure is the figure itself. This is a particularly important type of motion, type of slide, and we call it the zero slide. We'll be coming back to this.

Now we want to consider the situation in which we perform one slide and then another slide on its image. What is the total result? Well, let's draw a couple slides here. Here's a slide. Here's another slide. To perform this slide on this figure... takes us up to about here... to perform this slide on this figure... to preserve the distance, and the direction...that point will be about here. (So let's draw copies.) Now this figure which I'll call $C$ is the image of $B$ under this second slide, and $B$ is the image of $A$.

Now we're not really concerned about $B$. We're concerned about the total effect on figure $A$ of performing the two slides. The image of $A$ under those two slides is $C$, and the important thing to notice is that this is, also, a slide... and what we call this slide is the product of the other two slides. If I call this slide 1 and slide 2, this slide will be the "product" of slide 1 and slide 2.
Now, if a slide takes A to B, is there a slide which will take B back to A? Of course, there is. It's just the arrow that goes from B to A. How do these two slides compare? We see the arrows are parallel, of the same length, but, in the opposite direction. So, if we are given a slide, and if we take this opposite arrow, and take the product of the first slide with the second slide, what is the effect?

Doing the first slide and the second slide takes the figure back to its original position. The product of these two motions is the zero slide.

We give a special name to this motion. It is called the inverse of another one. Whenever the product of two slides is the zero slide, they are called inverses.

If we take a figure and label the vertices clockwise, its image under a slide is also clockwise. We call this a direct motion.

Again, we take another figure, we slide it, and since both the original figure and the image are counterclockwise, it is a direct motion. The clockwise or counterclockwise sense is the same.

A motion which is not direct is the flip. We have a line, and we are flipping the plane over this line. Notice that if we label the vertices clockwise, and we take a flip, we look at the image of A, the image of B, the image of C, and so forth, and we see that these vertices go counterclockwise around the figure. Since it changes this is called an opposite motion.

Now a slide takes A to B. Now there is a slide that takes B back to A? Well, sure there is, we know what slide it is; the slide that originates at B and comes back to point A. The question is: How do these two slides compare? Well, maybe we should eliminate the figures and just look at the arrows. We see these two lines are parallel. If I drew it carefully they should be the same length...although in opposite directions.

Now what would we get if we took the product of these slides? Once again maybe do it at this point in the middle. One slide will take you from here to here. On this particular figure I'll be performing this slide. The image of this figure will come right back to the original figure.

Maybe I should draw that in once again. Here's my figure...Now I redrew this arrow. If I drew it carefully the product of this arrow with its opposite is the zero slide. Now this arrow, if I call this slide 1, this arrow is called the inverse of slide 1.

And the product of a slide with its inverse is going to be the zero slide.

We remember the inverse is a slide in the opposite direction, of the same length.

If we label the vertices of a polygon clockwise, let's label this... A, B, going clockwise around the figure... C, and D, and then take a slide image of it, and we label the corresponding vertices. Here's the image of A, call it A', B, here, C' is here and D' is here. We notice that these are also clockwise.

This is important. We call such a motion that maintains a clockwise sense a direct motion. And what we're going to do now is to look at a different kind of motion. One which reverses the clockwise sense, and is called an opposite motion.

This motion we're going to look at is the flip. And it works basically like this. You take the figure, and along some line flip it. We're taking the figure and flipping it over the line. It is important that we maintain its rigidity.
This opposite motion, the flip, takes a figure to the opposite side of the flip line. Notice it always changes an R to a backwards R. Notice any line can be used as the flip line.

If we examine a flip we notice each point goes to a point on the opposite side of the line, but an equal distance to the flip line.

The flip line can cross the figure, but always the image of each point is on the opposite side of the flip line.

Now we are ready to try the next exercises, so stop the machine, and restart it when you finish.

Your answer to exercise three should look like this. If not, go back and review some of the material about flips.

If we perform a flip and perform it again, do we notice anything particular about this? It seems to return us to our original position.

Now, if you remember, if we have a particular motion, whatever it might be, we are interested in a motion which will take it back to the original position, that is, so that the product is a zero slide.

We have already seen the so-called inverse of a slide. Now we see that the inverse of a flip is that same flip itself.

If we are given two points, A and B, we can find a flip which will map A onto B, by connecting A and B with a segment, and taking the perpendicular bisector to that segment as the flip line. This will flip point A onto point B.

Again, remember a flip will always change the clockwise sense of a figure from clockwise to counterclockwise, or from counterclockwise to clockwise. It will change a left hand to a right hand. We call such a motion a reflection, sometimes, but, again, we will use the term flip.

If we label these vertices...this is where A goes, this is where B goes. C goes here, and D goes here. If we look at these, these go counterclockwise whereas A, B, C and D go clockwise. We call this type of motion an opposite motion.

This motion is also referred to as a reflection. As we can see we can get the same type of image by putting a mirror along the line, which we call the flip line. Now there's the flip, if I hold the mirror perpendicular. But we will use the term flip rather than reflection.

Notice each point is transformed to a point directly opposite from it, on the other side of the flip line.

Now I'd like you to do exercise three on your worksheet. So stop the machine and restart it when you finish the exercise.

A flip is determined entirely by the flip line. A straight line in the plane. So we'll draw in a line. What will the flip image of the figure be? It's flipped across the line like so. Again notice all of the vertices...Let's label them counterclockwise this time. The image A', B', and C'... If this is counterclockwise, then this is clockwise. So they are always opposite.

Now it is important to notice that we can use any line as the flip line.

Here I draw a line through the figure. And the image of the figure, flipped over the line, would be here. There is some overlap.

Let's do one more. And ask the question, what is the product of a flip with itself? Well let's draw a flip line, copy the figure. I'm going to take this figure and flip it along this line. Its image is about here...A', B', C', the images of the points. Now what is the image of this figure under a flip along this line? Every point will be flipped to the opposite side the same distance, and we see the image will be the original position. So the product of a flip with itself is the zero-slide.

So we have the result that the inverse of a flip is that same flip. And this is true for any flip.

Well, now we're going to leave flips for a moment and consider another type of rigid motion.
Another type of motion is called a turn. What this does is to rotate, or turn, or spin, the entire plane about a fixed center.

For example, we have a triangle here, ABC. It is turned through a fixed angle around a fixed center. Notice that if ABC is clockwise, its image is labelled clockwise, so, a turn is a direct motion; the image of a left hand is a left hand.

How do we specify this motion? We need a center. We, also, need an angular distance, a measurement of the angle that we turn, and a direction. So what we will use is a curved arrow, indicating the original side and final side of the angle through which the figure is to be rotated. This indicates the action on any figure in the plane. Any figure is rotated through that angle, about that center.

If we are given a particular motion, how can we determine where the figure goes? The curved arrow determines an angle, and each point is rotated through that angle, and remains the same distance from the center. So, here we can see how the images of A, B, and C are determined, and that is where the triangle goes under that motion. We'll see later how we can perform this motion easier using tracing paper.

Now, what is the product of two turns about the same center? We can see the product of two turns, if they are around the same center, is going to be another turn about that same center whose angular distance is the sum of the other two angles.

If we are given a particular turn, what would its inverse be? It is not too hard to reason that it would be a turn about the same center, the same angle, in the opposite direction.

For this motion we're going to take the entire plane and spin it or rotate it about a particular point. Here's the point. I'm going to fix it with the point of my compass, and turn it about the point. We will refer to this motion as a turn or a rotation, but we will use the term turn.

Now what can we notice about this? We notice that under this motion everything is rotated a certain angle, and notice that if we take a point, its distance to the center of the turn, or the so-called vertex, remains constant.

Well let's go back to the board and look at one of these turns. Let's take a figure, and to determine a turn we need a center, the center of the turn or the vertex. And we need an angle through which everything is to be turned. Let me draw in an angle here. And we're going to go from one ray to the angle to another so we indicate a direction within the angle.

Now each point is going to be rotated through the same angle. So if I copy down this distance d, and I construct a copy of the angle here.

If I call this A, this is the image of A. Point B here. Its image is going to be what we get when we rotate...this angle, 30 or 35 degrees...The image of B should be up here. And for point C, its image will be rotated to about here. C', B', and A'.

If I did this well, I should be able to fit C', B' and A' onto the figure about like this. Again if I took a copy of this figure and rotate it about this point, it will coincide with its image.

Is a turn direct like a slide or opposite like a flip? Well let's perform a turn. Notice that we have A, B, and C labelled clockwise on this figure. Now if we perform a turn, the image is located here. Let's check out the vertices. A, B, and C are still clockwise, so a turn is a direct motion.

Now if we perform a turn, what is the inverse of this motion? What do we have to do to get back where we started? Well, it's not too hard to image that what we have to do is turn it about the same center the same angle, but in the opposite direction. So the inverse of a turn is another turn about the same center.
Each of these motions, (slides, flips, and turns), has an important connection with the idea of congruence. We'll say two figures are congruent if we can pick one up and without changing its shape and size, move it rigidly onto another. To do this we need the idea of a rigid motion.

A rigid motion is one in which distances between points of the figure are not changed. So through these motions, for example, those slides, or this flip, the distances AB and BC, for example, do not change. Here is a slide. Here is a turn. Observe the lengths from A to B, B to C. These remain the same under each of these basic rigid motions.

We can look at a product of motions, here we took a slide, and now a turn, and now we are going to flip it.

The product of these rigid motions is going to be another rigid motion, because for each of the mappings the distances between points is preserved.

Since paper doesn't stretch, copies of figures on tracing paper can be used to perform these various slides, flips, and turns, easily.

For example, suppose we had a slide, indicated by an arrow, given a figure, we can find the image of this figure under this slide. We cover both with tracing paper. Trace the figures. Then draw a line in the

Take a center. We perform one turn about that point. Perform another turn about the point. What is the total product? Again it is not too hard to reason that it is a turn about that same point.

Well, these motions we have seen, slides, flips and turns, are all rigid motions, and by rigid we mean that distances are preserved. If I pick a point A and a point B on this figure, if I slide it or turn it, or flip it, the distance from A to B remains the same.

If I take a product of these motions... if I take a flip, and then a turn, and then a slide, and then another flip, the distance from A to B remains the same throughout.

So we see that any slide, flip, or turn, or any product of slides, flips, and turns, will be a rigid motion. So, if I can take one figure and move to another figure using slides, or by using a slide and a flip, or any combination, then I know these figures are congruent.

And it turns out that any rigid motion can be obtained as a product of slides, flips, and turns. (We won't prove this.)

These basic rigid motions can be constructed with straightedges and compass, but also are easy enough for elementary students using tracing paper. A paper tracing is a good way to manipulate a rigid image of any figure.

So suppose the slide indicated on this figure. To perform this slide we first copy the figure, and then we draw a line along the translation arrow or the slide arrow, and we put a mark on the tail of the
direction of the arrow. Put a mark at the tail of the arrow. Then if we slide the tracing paper along the line until the tail mark gets to the tip, then the figure will lie at the image.

For a turn, we put the tracing paper down, copy the figure, and the center. We put a mark at the tail of the arrow. If we fix the center with a pen or compass point, and then rotate the tracing paper until the mark on the paper goes to the tip of the arrow, then we can copy the image of the figure using carbon paper or some other means.

And finally for a flip, we put down the paper, copy the flip-line and the figure, and after we flip it over, we are going to match up the lines. So that we can match them right we put a reference mark on the line and on the paper. We will match these up after we flip the paper over.

We flipped it over...We match up the lines.

Now you are ready to do the next exercises, so stop the machinery and restart it when you are done.

Now it turns out, but we won't prove it, that every rigid motion is a product of slides, flips and turns.

Now let's look at this slide. We can represent it as the product of two flips. Here's one flip. The second flip gives us the same effect as that slide.

Now we slide the line in the direction of the arrow until the point at the tail is on the tip. We have slid the paper the length of the arrow in the direction of the arrow, and the location of the tracing is the location of the image of this figure under this slide.

Now do the next exercise on the worksheet, on which you will use this technique to perform a slide.

Now for a turn, indicated by an angle with a direction and a center...(in fact let's eliminate these rays so we can indicate the angle with this curved arrow)...given the direction and the center...To perform this turn, we copy the figure, mark the center, and then we mark the tail of the curved arrow, or mark the initial ray of the angle.

We fix the center and rotate until the tail of the arrow is at the tip of the arrow. Again...we rotate until the tail marking goes to the tip of the arrow...and our image is the traced figure here. This is the image of this figure under the indicated turn.

Now I would like you to perform a similar exercise on the worksheet. So please stop the machine......

And now for a flip, we do some of the same things. We copy the figure, of course. And again the point is...a paper tracing of the figure is rigid. The paper doesn't stretch. We trace the figure. We're going to trace the flip line. Now we have to do one thing, because if we flip it and just put the flip-line where it is, it could go anywhere. So we have to put a reference mark on the flip line, anywhere along the line... a dot. And we mark that reference point on our tracing. And when we perform this flip, we match the dot on the line with the dot on our tracing and the image we obtain is the flip image of our figure.

Now do the next exercise and I'll see you in a moment.

Well now, we know that every congruence is a product of slides, flips, and turns.

Something we mentioned before was the notion of direct and opposite congruences. This congruence between these capital letters is a direct congruence, and we can tell that..... They are sort of in the
Let's do this again and notice some facts. Here's our slide. We flip once and flip again, along two lines we've got the same effect as a slide. Notice that the two flips lines are perpendicular to the direction of the slide.

How about a turn? Can we represent it as the product of two flips? Well, let's blow it up a little bit, and look at one flip. Notice that the flip line goes through the original center. And another flip, yes we've achieved our turn.

So once again, a slide can be achieved as two flips, and a turn can be achieved as two flips. And so it follows that every rigid motion, which is the product of slides, flips, and turns, can be represented as a product of flips.

Here's a slide and a flip, it could be represented as one, two, three flips.

So every rigid motion is a product of flips. If we look at these flips... Here are several opposite motions, that are flips, taking a right hand to a left hand. And here's an opposite motion. Can it be represented as a flip?

We will try a few. And it should be clear that we won't be able to find a single flip that works. For certain opposite motions it's going to require a product of two motions. Here we see a slide and a flip. Here's two opposite figures. With a turn and a flip, we establish their congruence.

Now if we have an opposite motion as this figure and it image are, there is no way we can arrive at it as a product of slides and turns, as we are trying too do here. Slides and turns are direct motions, and any product of direct motions will be direct.

To achieve such an opposite motion, then we may have to slide and then flip. In fact a slide and a flip will always do. We can see if we have two triangles, in this particular case, we can slide point B to B' and then flipping along the line which is the bisector of the angle ABA', we will arrive at the congruence.

We can also get at such opposite motions, which are called glide-reflections, as a product of a flip and a turn, in this way. There is a flip which will take B to B', (We can find the segment and its perpendicular bisector and flip along that line.)

Every direct motion is a product of slides and turns. (Because if we included a flip, it would change the clockwise sense or change the orientation.) And it turns out that every direct motion is a slide or a turn.

I think that we can find the center of this turn. (It's obviously not a slide.) Can you tell where the center of the turn is?

Connect corresponding points,... draw the perpendicular bisectors. The center of this turn would be where these bisectors intersect because these two points would have been the same distance from the center, as would this point... and its image.

And so every direct rigid motion is a slide or a turn. Now how about opposite motions. Opposite motions are those whose images look backwards.

Here's an opposite motion. Is it a flip? Well, let's see. Try a flip about this line... no, the image would look like this.

In fact there is no flip that gets from this figure to this figure. So we are going to have to rely on a product. What will work? Well, if we look at the slide that takes this point to this point, ...Perform that slide,... And now there's a flip which will map this onto this. And see the flip is about this line, so that the congruence I had, an opposite congruence, was the product of a slide and a flip.
And after we do that there will be a simple turn which will take A to A' and C to C' around the center B (or B').

Now we are going to consider the idea of symmetry.

Some congruent figures have more than one congruence mapping which will take one onto the other. There we saw a slide. Here's a turn taking that square to its image square. And of course there is a flip which will map that square onto its image.

Often congruences can be represented in more ways than one. This one is both a slide and a flip. This congruence is both a slide and a turn. (Not a flip if you notice.) And this third congruence is both a flip and a turn. Figures that have this property are said to have symmetry.

Every figure is congruent to itself by the zero slide. Any figure congruent to itself by any other mapping we say has symmetry.

This equilateral triangle is congruent to itself by various rigid motions.

Now every figure is congruent to itself by the zero slide. Any figure which is congruent to itself by another mapping we say has "symmetry".

This equilateral triangle is congruent to itself by several mappings... several different flips... a turn here of 120° is a congruence mapping... and so on...

Now a figure that is congruent to itself by a flip we say has bilateral symmetry.

A figure that is congruent to itself by a turn, we say has radical symmetry.

Now some congruences can be represented in more ways than one as a rigid motion. For example, this particular congruence between these two figures is determined as a slide, or it could be determined as a flip along this line.

An object that has this property has what we call "symmetry". Generally, the way we look at symmetry is if a figure is its own image under a rigid motion that is not the zero motion. (Every figure is its own image under the zero motion.) But this figure, for example, is its own image under a flip along this line... Flip it over the line... There's its image... the same figure.

A figure that under a flip is its own image has bilateral symmetry. (Bilateral meaning two-sided.)

There is another kind of symmetry, associated with a turn. Let's look at that... another figure which is its own image under a turn.

Let's take the figure, trace its outline. Look at the center. We rotate the object. It is its own image under that turn. (A clockwise turn of 90°.) .... or a clockwise turn of 180°, which is the product of two clockwise turns of 90°.

Now we say something like this has radical symmetry.
Now some figures, for example an equilateral triangle, have both flips and turns...

If we think about it... if we take a product of these symmetry motions, the product will also be a symmetry motion, because each of these motions takes the figure to itself.

We can keep track of these motions by looking at the action of the various motions on the vertices. For example, the 120° turn takes A, B, and C to C, A, and B.

What is the action of a 240° turn? We can see it there. It turns out to be the same as a counterclockwise turn of 120°.

Now suppose we perform a 360° turn. It brings us back to where we started, just the same as a zero slide.

Now suppose we flip about one axis and then flip about another flip line. This is a symmetry motion. Is it the same as another one we have seen? Yes, it has the same effect as a clockwise turn of 120°, you may want to run back and look at that again.

Now if we follow that with a flip, we get another motion. Do we know where this one came from?

Some figures have both bilateral and radical symmetry.

Let's take an equilateral triangle,... trace its outline. Can you think of motions that will take this figure to itself?

Well, a 120 degree clockwise turn....
or a 240 degree clockwise turn....
or a flip along this line, will all take this figure to itself.

An interesting aspect of this is, we can look at products of these particular symmetry motions.

So for example, if I do a rotation, and then a flip, the product is a motion of the triangle to itself and another symmetry motion.

Let's see if we can determine what it is. If I label these vertices A, B, and C, and label my triangle A, B, C.

I'll call a flip along this line flip #1. I also have the 120° clockwise turn. If I do this motion first, and then this motion second, what do I have?... 120° clockwise turn,.. and then a flip. Let's see what we do have. This is B, this is A. This is C. Is this the same as some single rigid motion?

Well, let's see. I had C here, B here and A here. Let's go back to the original position, a back at A, B at B, C at C. How can I obtain this motion as a single motion? I leave B fixed, A and C switch. That's a flip along this axis.

So we can see that the products of symmetries give us other symmetries. And these ideas lead to some very important ideas in higher mathematics.

This concludes our lecture. I thank you for your attention and time.
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