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A STUDY OF THE RADIATIVE CAPTURE REACTION MECHANISM IN $^{12}\mathrm{C}(p,\gamma)^{13}\mathrm{N}$

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in the Graduate School of The Ohio State University

By

Tsang Lin Tsin, B.S., M.S.

* * * *

The Ohio State University

1976

Reading Committee:

Professor S. L. Blatt
Professor T. R. Donoghue
Professor R. G. Seyler

Approved by

S. Leslie Blatt
Advisor
Department of Physics
DEDICATION

To my parents
A STUDY OF THE RADIATIVE CAPTURE REACTION MECHANISM IN $^{12}\text{C}(\vec{p},\gamma)^{13}\text{N}$

by

Tsang Lin Tsin, Ph.D.

The Ohio State University, 1976

Professor S. Leslie Blatt, Advisor

The radiative capture reaction $^{12}\text{C}(\vec{p},\gamma)^{13}\text{N}$ has been investigated to obtain information on this reaction mechanism as well as the structure of the nucleus $^{13}\text{N}$.

Gamma ray angular distributions for capture of unpolarized protons have been measured for six well-spaced angles at 1.8 MeV, incident proton energy, using a high resolution Ge(Li) detector. Through the use of a thick-target energy dispersive technique, excitation curves from 1.6 to 1.8 MeV could be extracted from the data. Vector analyzing powers of the same reaction have also been measured with a polarized beam, for three angles.

The theories of direct radiative capture, resonance radiative capture and their coherent interference, for both unpolarized and polarized cases, have been developed, and applied to the measured reaction. Complete sets of computer programs have been written for both the thick target data reduction and the theoretical radiative capture mechanism calculations.
ACKNOWLEDGEMENTS

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VITA

June 24 1951 ---------------------------------- Born, Kowloon, Hong Kong

1971 ---------------------------------- B.S. Magna Cum Laude
Southern Oregon College, Ashland, Oregon

1971-1973 ---------------------------------- Graduate Teaching Associate, The Ohio State University, Columbus, Ohio

1974 ---------------------------------- M.S. The Ohio State University, Columbus, Ohio

1974-1976 ---------------------------------- Graduate Research Associate, The Ohio State University, Columbus, Ohio.

Publications and Communications


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I. INTRODUCTION

Radiative capture reaction studies\(^1\) have long been an important source of information about the structure of light nuclei. At low energies, most reactions of this type have been analyzed satisfactorily by assuming a simple compound nucleus mechanism involving a few, possibly overlapping, resonances. In the reaction \(^{12}\text{C}(p,\gamma)^{13}\text{N}\) below 2 MeV bombarding energy, only two resonances seem to be involved, namely, those due to the first- \((1/2^+, T=0)\) and second- \((3/2^-, T=0)\) excited states, at \(E_p = 0.457\) and 1.699 MeV, respectively. The third excited state \((5/2^-, T=0)\), at around 1.75 MeV, does not appear to contribute to the radiative capture.\(^2,3\)

The properties of these \(^{13}\text{N}\) excited states have been investigated by many authors using numerous types of reactions, including proton elastic scattering \(^{12}\text{C}(p,p)^{12}\text{C}\)\(^{5,6,7}\) as well as the radiative capture\(^8,9,10\) while the resonance energies of these states are unambiguous, their widths are somewhat more in doubt. The capture reaction \(\gamma\) ray yield curve of this particular reaction at low energy was first described by using a single-level Breit Wigner formula with additional resonance background.\(^12\) Then, Young \textit{et al.}\(^3\) measured the \(\gamma\) ray angular distributions over the 1.699 MeV resonance \((E_p = 1.5-2.0\text{ MeV})\) and
Fig. I-1  The energy level diagram of $^{13}$N taken from Ajzenberg-Selove.$^{14}$)
The differential cross sections at 0° and 90° for the γ-ray transition to the ground state in $^{13}$N are shown as a function of beam energy ($E_p = 150-2500$ kev). The solid lines through the data points represent the optimal theoretical fit. The inset shows the γ-ray decay schemes involved in the yield curves. (Taken from Rolfs$^{13}$)
DIFFERENTIAL CROSS SECTION ($\mu$bar/ster)

PROTON ENERGY $E_p$ (MeV)

12C($p,\gamma$)N

157 MeV RESONANCE

1699 MeV RESONANCE

$\theta_p = 90^\circ$

$\theta_p = 0^\circ$

Diagram showing differential cross sections.
found the Legendre polynomial fit can be well defined by the two
caruncles \(1/2^+, 0; 3/2^-, 0\), taking into account this interference, without
the help of any background term. Later, however, Rolfs et al.\(^{13}\) sug-
gested that a direct capture contribution also had to be added to the
differential cross section to explain the reaction yield curve both above
and below the resonances. It is not obvious that by introducing the
direct capture component, one can preserve an acceptable Legendre poly-
nomial fit for the on-resonance angular distributions, however. Since
the significant term of interest is of interference nature, we felt that
by using the O.S.U. polarized ion source to perform a polarized
\(^{12}\text{C}(p, \gamma)^{13}\text{N} measurements, we should be able to resolve this particular
difficulty, as well as having additional information on the other nuclear
parameters involved.

The present work consists of a report on the experimental and theore-
tical work involved in this study. Data were taken on angular distribu-
tions of the \(^{12}\text{C}(p, \gamma)^{13}\text{N reaction over an energy range 1.6-1.8 MeV,}
utilizing a special thick-target technique to reduce the necessary ex-
perimental running time. Both unpolarized and polarized beams were em-
ployed. In order to properly analyze the data, the theoretical descrip-
tions of both compound-nucleus and direct capture mechanisms were em-
ployed. Considerable effort has been devoted to developing the formalism
for treating capture reactions when both mechanisms contribute coherently.
In particular, the analyzing powers produced under such circumstances
have been calculated, and applied to the \(^{12}\text{C}(p, \gamma)\) measurements.

In the following sections of this introduction, we present more
details on radiative capture reactions in general (section 1), criteria
of DC and CN reaction (section 2.), and polarization effects (section 3).
The second chapter describes the experimental set ups. After that, we discuss data reduction (Chapter III). The theoretical development begins in Chapter IV, with calculations of Direct Capture, resonance capture, and their coherent interference. Finally, we discuss the application of the theory to the $^{12}\text{C}(p,\gamma)$ data in Chapter V.

I-A. Radiative Capture Reactions and Their Importance

The radiative capture reaction is usually visualized as an incident particle $X$ (mass $M_1$, charge $Z_1$) captured by a target nucleus $A$ (mass $M_2$, charge $Z_2$) forming a final state $B$ of the combined system and emitting $\gamma$-radiation of energy $E_\gamma$. Symbolically one designates the radiative capture reaction by writing $A(X,\gamma)B$. Most nuclear reactions are mediated by nuclear forces for which the exact Hamiltonian is not known and perturbation theory is probably not applicable. However, there are some nuclear reactions which are of electromagnetic nature, i.e., photo-disintegration ($\gamma,p$), capture reactions ($p,\gamma$), ($n,\gamma$), Coulomb excitation, etc. Since we know the electromagnetic interaction Hamiltonian, and the force is small compared to nuclear forces, first or second order perturbation theory will give reasonably accurate results. We therefore believe that measuring the radiative capture reaction allows us to probe into some unknown properties of different nuclei.

I-B. Polarization and Its Importance

For simplicity let us consider the spin 1/2 system. It was observed that if an unpolarized proton beam (in which half of the particles have spin up and half have spin down) was incident on some par-
Fig. I-3A  Schematic diagram of resonance capture process. An intermediate state is formed then decays to its final state by emitting a $\gamma$-ray.

Fig. I-3B  Schematic diagram of the direct-capture process. The projectile (plane wave) is captured directly into a single particle orbit (standing wave) of the target nucleus by the emission of $\gamma$-radiation, without formation of an intermediate compound state. The reaction selects those projectiles from the total field of the elastic scattered projectiles which can jump into a final single-particle orbit by the emission of $\gamma$-radiation of multipolarity $L$. The usual selection rules for the $l_1 + l_f$ transition must, of course, be obeyed. Such a process can always be expected for such states in the final nucleus which can be described as target nucleus plus projectile.
Resonance Capture Process

**DIRECT CAPTURE PROCESS**
(GENTLE REACTION)

Projectile -> Target

Intermediate State

Final State

γ-Ray
ticular scatterer A (which is a nucleus with or without spin) which acted as a polarizer, the scattered particles turned out, according to their left-right position to have a preferential spin orientation. In other words, the left side may have more spin up particles and the right side may have more spin down particles or vice versa. This particular phenomenon is called polarization.

It may be easier for one to understand polarization in terms of electromagnetic interactions. Let us look at the scattering of electrons by nuclei. As electrons go toward a nucleus of charge $Ze$, they would be attracted by the Coulomb forces. In addition to the electrostatic force, there will be an interaction between the magnetic moment of the electron between the charged nucleus. In the reference frame where the electron is at rest, the moving nucleus would appear as a current which would produce a cylindrically symmetrical magnetic field. The force exerted by this field on the magnetic moment of the electron is either parallel or anti-parallel to the direction of the electrostatic force, depending on the orientation of the magnetic moment (see Fig. 1). If one assumes the magnetic field is up on the right hand side of the nucleus and down on the left hand side, the forces for electrons with spin up would add on the right and subtract on the left. The opposite is true for spin down electrons. Therefore electrons with spin up will be deflected to the left and spin down to the right. This is why in a given scattering direction, the number of electrons with magnetic moments pointing up and down is not equal. The strength of the electromagnetic spin-orbit interaction would be much less than that of the same type of interaction involving nuclear forces. To elucidate the nature
Fig. I-4

Electrons with magnetic moment $\mu$ are incident on a nucleus of charge $Ze$. The interaction energy between the magnetic moment and the nucleus adds to or subtracts from the electrostatic-interaction energy depending on the orientation of the magnetic moment and on the side of the nucleus on which the electron passes. This results in partial polarization of the scattered electrons. (Taken from Barchall's\textsuperscript{16} paper)
of the spin-orbit type interaction is one of the purposes of studying polarized nuclear reactions. Others\textsuperscript{15} include pinning down the spin and parity assignments of excited states, determining the type of reaction mechanism involved (either direct interaction or compound nucleus formation), testing parity conservation, time reversal invariance, or charge invariance associated with symmetries of the nuclear Hamiltonian, etc. In addition, because polarization is an interference phenomenon, the effect of small components of the nuclear interaction sometimes becomes more visible.

ii) Description of polarization: The Basel Convention

If $N_L$ is the number of particles emitted to the left of the nucleus and $N_R$ is the number of particles emitted to the right, the standard way of describing the reaction asymmetry is by defining the left-right asymmetry $A = \frac{N_L - N_R}{N_L + N_R}$. The asymmetry produced by the elastic scattering of a completely polarized beam is called the analyzing power of the scatterer. The sign convention adopted by the 1960 International Symposium on Polarization held in Basel\textsuperscript{16} was that, if the direction of the incident particles, the direction of the scattered particles and the preferred spin direction in the scattered beam form a right handed coordinate system, the polarization of the scattering beam is defined as positive. Also the sign of the analyzing power is positive if the direction of the incident particles, the direction of that portion of the scattering beam which had the greater number of particles, and the preferred spin direction in the incident beam form a right handed coordinate system. In the case of elastic scattering, the analyzing power is the same as the polarization produced by an unpolarized beam. This
means whether we have a polarized beam with polarization $P$ that goes
through an analyzer with analyzing power $A$, or instead have double
scattering, where an unpolarized beam interacts with an analyzer with
analyzing power $A' = P$ and then passes through a second analyzer with
analyzing power $A$, the results will be indistinguishable. This gives one
a way of checking the polarization of one's polarized ion source, if one
knows the analyzing power of a particular elastic scatterer.

$$P \cdot A = A' \cdot A = \frac{N_L - N_R}{N_L + N_R}$$

This is exactly the method we used to check our beam polarization within
every eight hours of our running time.

I-C. Direct Interaction vs. Compound Nuclear Formation

In the theory of nuclear reaction mechanisms, there are two extreme
cases. One of them is known as the direct interaction formalism (DI).
The other is known as the compound nucleus formalism (CN). In a given
physical situation, it may not be meaningful to draw a sharp line of
distinction between these two extreme conceptual mechanisms. Convention­
ally, one simply assumes the pure DI approach is valid at higher energies
(above ~ 12 MeV) where the incident particle goes right through the tar­
get nucleus without forming any intermediate states. The time it takes
would be the amount of time the incident particle requires to travel
through the nuclear diameter of the target nucleus, which is of the
order $10^{-22}$ seconds.

The CN approach, a comparatively more complicated method with
greater degrees of freedom, on the other hand, would be more valid at lower energies (below ~ 6 MeV). The incident particle would interact with the target nucleus to form a short-lived intermediate state which only carries with it the definite spin and parity information, but "forgets" the way it is formed (the "amnesia assumption" of Bohr). The CN lifetime can vary from ~ 10^{-20} to ~ 10^{-18} seconds. The newly formed state is neither bound nor stable and will decay by various available channels. There is not yet a general procedure for combining the two mechanisms in a coherent manner with a view to arriving at a physically meaningful admixture, but various attempts have been made, aiming at a mechanism which may be a partial resolution of the difficulty. One of the purposes of this experiment is to attempt to demonstrate the validity of such a mechanism in a very specific case, namely, the charged particle direct radiative capture vs. the charged particle radiative resonance capture reaction mechanisms.
II. EXPERIMENTATION

Proton beams with energies of 1.7 to 1.8 MeV were obtained from the Ohio State University 7 MV Van de Graaff accelerator. Angular distribution measurements were made of emitted γ-rays with Ge(Li) detectors, using a thick-target method previously used only for unpolarized work. Beam polarization were monitored with the $^4$He$(p, p)$ reaction between runs. Details of these procedures are presented in this chapter.

II-A. Thick Target Technique for Charged Particle Radiative Capture Studies

To start with, let us define what is meant by a "thick target." In a thick target, the incident particle beam of energy $E_p$ produces a continuous spectrum of particle energies from $E_p$ down to a lower limit $E_p^2 = 0$. The γ radiation energies are related to the incident proton energy by the following:

$$E_\gamma = Q + (M_{\text{target}}/[M_p + M_{\text{target}}])E_p$$

where $Q$ is the Q value of the reaction, and $M_p$ and $M_{\text{target}}$ are the masses of projectile and target nuclei, respectively. The thick target γ-radiation spectrum is directly related to the excitation curve vs. the proton capture cross section. This method has the following advantages:
1. The accelerator's energy resolution and stability as well as the target thickness and uniformity become relatively unimportant.

2. The equivalent particle energy resolution for the cross section measurements is essentially the \( \gamma \)-radiation detector energy resolution

\[
\frac{dE_p}{dE_\gamma} = \left( \frac{M_p + M_{\text{target}}}{M_{\text{target}}} \right) dE_\gamma
\]

3. One obtains a large amount of information in a small amount of time.

4. For isolated resonances, the number of counts at a given \( E_\gamma \), after adjusting for stopping power and detector efficiency, will be related to the cross section at the corresponding \( E_p \). The width of the peaks will be a direct measure of the resonance widths.

The thicknesses and other specifications of the targets used in the experiment are given in Appendix 9.

II-B. Detecting System

In the energy region of this experiment (0-1.8 MeV for \( E_p \)) the differential cross section for \( ^{12}\text{C}(p,\gamma_0) \) is around 10 \( \mu \text{b/sr} \) at energies around the resonances and considerably less elsewhere. It is therefore important to place the detecting system as close to the target chamber as possible. On the other hand, if reasonable angular measurements at back angles are to be made, the detecting system must be placed at some distance from the target chamber.
The total charge integration for the thick target runs was obtained by measuring directly on the target rod. In this case, for calibration, we also used a 10 μg/cm² ¹²C foil target. However, we measured our total charge collection on both the target rod and the rest of the target chamber. Since we were running with a low energy proton beam, placing a couple of pieces of 100 μ gold foil behind the target and in front of the target chamber wall prevents the proton beam from hitting the aluminum chamber, which avoids the strong ²⁷Al(p,γ) background reaction.

The principal detector used was a high-resolution Ge(Li) detector (Canberra model 7229). It is a 44.5 mm in diameter and 37 mm long right circular closed-end cylindrical detector with Li drifted coaxially. Its p-core diameter is 12 mm, the active volume is 49.5 cm³ and the front is 5 mm away from the entrance window. The detector is surrounded by an annular lead shield. The cart on which the detector is mounted is constrained to move from a position touching the target chamber to a position 1.2 meters away and angular motion of the entire assembly is possible from 0° to 140°. The track is constructed of 18.2 cm steel channels with hardened steel plates welded on. The plates were afterward ground flat and parallel. It was calculated that a maximum vertical load of 2000 Kg weight applied to the center of the track would produce a deflection of less than .25 mm. The tract is supported at the outer ends by steel wheels running on a circular steel plate with leveling screws every 5°.

A standard 3"x3" NaI(Tl) detector (Harshaw type 12512) with a RCA photomultiplier tube powered by a homemade high voltage divider was hung below the beamline, 2.154 cm away, and directly facing the target chamber. This particular high-efficiency, low-resolution detector was used mainly as a γ-radiation monitor which served as another check on the accuracy of the total charge integration.
Fig. II-1  Target chamber.
Mu Metal: anti-magnetic shield

O.D. 6.7935 cm
I.D. 5.8903 cm
Fig. II-2  Detector set-up.
Ge(Li) Detector
and shield

Target chamber position

Beam Direction

Track
II-C. **Electronics**

As we have noted, this is a fairly low count-rate experiment with little or no competing $\gamma$-ray emitting reactions. Therefore, the electronics are particularly simple. A block diagram of the electronics is shown in Fig. 2 and the corresponding components are tabulated.

We did try to experiment with two different types of pulses (i.e., the unipolar pulse and the bipolar pulse) on the Ge(Li) detector as well as the NaI detector. It was observed that with appropriate electronic arrangement, the unipolar signal gives a better energy resolution for Ge(Li) detector at low energies. As for the NaI detector, the bipolar pulse amplified by a double delay line amplifier served as the best compromise.

We did not use any pile-up rejection circuits. The PACE pulse-height analysis dead time remained less than 0.5% for all our runs.

II-D. **Polarimeter: Measurement of Polarization**

Since it is possible for the degree of polarization of a beam originating in the polarized ion source to vary during the experimental period, it is necessary to measure the polarization before and after each successful run. Usually, this is done with a second reaction or scattering process with a large, known analyzing power. In our case, $p^{4}\text{He}$ elastic scattering was chosen simply because of its well established experimental data as well as plane shift calculations\textsuperscript{19-21} and its large and slowly varying analyzing power (0.95) near 1.8 MeV.

The polarimeter\textsuperscript{22} was made with a helium gas cell and four pairs of surface barrier detectors which were symmetrically located inside a
Fig. II-3    Electronics
<table>
<thead>
<tr>
<th>Figure Label</th>
<th>Descriptive Name</th>
<th>Manufacturer's Model Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ge(Li) PREAMP</td>
<td>Preamplifier</td>
<td>Canberra 970</td>
</tr>
<tr>
<td>NaI PREAMP</td>
<td>Preamplifier</td>
<td>Ortec 113</td>
</tr>
<tr>
<td>SPEC. AMP</td>
<td>Spectroscopy Amplifier</td>
<td>Canberra 7010</td>
</tr>
<tr>
<td>DELAY AMP.</td>
<td>Double Delay Line Amplifier</td>
<td>Ortec 460</td>
</tr>
<tr>
<td>Bipolar AMP</td>
<td>Bipolar Adder</td>
<td>Ortec</td>
</tr>
<tr>
<td>S.C.A.</td>
<td>Timing single Channel Analyzer</td>
<td>Ortec 455</td>
</tr>
<tr>
<td>STR1,STR2*</td>
<td>Stretcher Multiplexer</td>
<td>Tennelec TC 620</td>
</tr>
<tr>
<td>MP*</td>
<td>Multiplexer Control</td>
<td>Tennelec TC 520</td>
</tr>
<tr>
<td>ADC2*</td>
<td>Analog to Digital Converter</td>
<td>Tennelec TC 501</td>
</tr>
<tr>
<td>ADC1</td>
<td>Multichannel Analyzer</td>
<td>Northern Scientific NS 625</td>
</tr>
<tr>
<td>SCALER</td>
<td>Scaler</td>
<td>Tennelec 559P</td>
</tr>
</tbody>
</table>

* These units comprise the Tennelec PACE system.
Fig. II-4  Scattering chamber (taken from J.L. Regner's thesis)
ANTI-SCATTERING SNOUT
FRONT SLITS
REAR SLITS TARGET ASSEMBLY
~40 cm
FARADAY CUP
4He Cell
SCATTERING CHAMBER
DETECTOR BLOCKS
~2 m
SLIT BOX
Fig. II-5 Electronics for the polarimeter.
POLARIMETER ELECTRONICS

Left 65°—— preamp—— DDL amp—— STR 1 ————

Right 65°—— preamp—— DDL amp—— STR 2 ———

Left 75°—— preamp—— DDL amp—— STR 3 ————

Right 75°—— preamp—— DDL amp—— STR 4 ———

Left 85°—— preamp—— DDL amp—— STR 5 ————

Right 85°—— preamp—— DDL amp—— STR 6 ———

Left 95°—— preamp—— DDL amp—— STR 7 ————

Right 95°—— preamp—— DDL amp—— STR 8 ———

Scaler 1

Scaler 2

Scaler 3

Scaler 4

Scaler 5

Scaler 6

Scaler 7

Scaler 8

TTL

IBM

1800
rotating scattering chamber which achieves a "proper flip" for the left-right detectors. The helium gas cell is a brass cell (0.9525 cm wide, 2.54 cm long and 2.54 cm tall). 0.635 cm by 1.905 cm entrance and exit holes were milled through the 2.54" by 2.54" cm surfaces. These were covered with a nickel foil of 1/20 μ thickness that contained helium gas at 0.5 atmospheres. The scattering chamber is 54 cm in diameter and is constructed of 1.3 cm thick stainless steel. It is supported at both ends by ball bearings which allow it to rotate about the beam axis. The detector blocks, each of which holds 4 detectors spaced 10° apart were mounted on a 5.08 cm thick aluminium plate which was precision-aligned to the bottom of the chamber. Both of the detector blocks can be moved independently from outside the scattering chamber from 18° to 162° allowing one to measure the analyzing power at four sets of angles simultaneously.

The detector geometry was also fixed in the following way. Each of the detector holders had a 0.1778 cm wide, 0.254 cm high rectangular front slit 3.81 cm away from the gas cell. A 0.3175 cm diameter collimator was put 8.89 cm behind the front slit to make the half angle subtended by each detector to be 1.5°.

The electronics are shown in Fig. II-5. The low counting rate again simplifies everything. The output of the 8 - 512 channel spectrum was fed to the Tennelec PACE ADC system, which in turn routed the signal for storage in an IBM 1800 computer.

II-E. Experimental Procedure

For the unpolarized $^{12}$C(p,γ) reactions, the thick target was first
bombarded by 1.8 MeV incident protons at the 30° N beam line target chamber. Then, for line-shape calibration, the thin target spectrum was measured with 1.7 MeV incident protons. Both thick and thin targets were positioned at a +45° angle with respect to the incident proton momentum axis. The NaI detector was placed 2.154 cm away from the target chamber at the 180° backward angle. The Ge(Li) was placed at 1.514 cm from the target chamber and a well-spaced set of 6 angles (-90°, -60°, -30°, 0°, +110°, +120°) were chosen to perform the angular distribution measurements.

For the polarized $^{12}$C(p,γ) reaction, the same detecting system was used. The polarization spin flip process was performed by changing the ionizer from one polarity to the other. We subdivided each run into 8 sections varying the ionizer polarity in the following sequence: + - - + + - - +. The intention was to minimize small fluctuations and to observe large ones. The beam polarization was monitored before and after each successful run by the detecting system in the polarimeter in the 0° beam line. The four pairs of 500-micron partially-depleted silicon surface barrier detectors were set at 65°, 75°, 85°, 95°, on one side and -65°, -75°, -85°, 95° on the other side. Each polarization check involved a "proper spin flip" measurement on both polarities of the ionizer.
Fig. II-6  
Beam line set-up.
III. DATA REDUCTION AND ANALYSIS

III-A. Thick Target Data Reduction Method

The idea of converting a thick target spectrum to an equivalent of a number of thin target spectra\textsuperscript{18}) at corresponding bombarding energies is a relatively simple one. A complication, however, comes from the fact that the detector resolution is not an ideal S-function, and the thin-target γ-radiation spectrum is not simple.

Basically, a thin target spectrum (the "line shape") in the yield curve of the γ-radiation over a particular small energy region ΔE, where ΔE approaches zero as the thickness is diminished. Its shape consists of a full energy peak, the compton distribution, and the annihilation-radiation-escape peaks. Assuming one possesses a perfect experimental apparatus, a thick target spectrum can be obtained using a point by point addition over every single thin target spectrum of the corresponding energies. Because the detector resolution is finite and because of the existence of coherent statistical errors, there is a limit on the fineness of the energy subdivision ΔE one can extract from a thick target spectrum. In our particular case, ΔE was chosen as 18 keV.
Fig. III-1 Ge(Li) (detector resolution) $^{228}$Th spectrum.
SPEC AMP UNIPOLAR + OUTPUT BLR
GE(Li) + 4000 VOLT
Fig. III-2  Ge(Li) thin target spectrum.
$^{12}\text{C} (p, \gamma)^{13}\text{N}$

$E_\gamma = 1.7 \text{ MeV}$

full energy peak

1st. escape peak

2nd. escape peak

Compton edge
Fig. III-3  Ge(Li) thick target spectrum.
THICK TARGET SPECTRUM
AT - 60 DEGREE

COUNTS (X 10^6)

2.614 MeV → 0
Full Energy Peak

3.51 MeV → 0
D.E.

3.51 MeV → 0
S.E.

3.51 MeV → 0
Full Energy Peak
Fig. III-4 Three peaks from NaI for normalization purpose.
THE THREE $\gamma$ RAY PEAK CHOSEN FROM N$_{2}$ FOR MONITORING PURPOSE.
Fig. III-5  Thick target spectrum centered on the region of interest.
$^{12}\text{C}(p, \gamma)^{13}\text{N}$

EP = 1.8 MEV

COMPTON "EDGE"  
2nd EXCITED STATE  
$^{13}\text{N}$  
$E_\gamma$ MAXIMUM  

COUNTS

10^0
10^1
10^2

1536 1586 1636 1686 1736 1786 1836 1886 1936 1986

CHANNEL NUMBER  
RUN NO. 138 - 21 - 4
Fig. III-6 Step-by-step data reduction technique.
STEP 1
BACKGROUND SUBTRACTION

CHANNEL 0 1603

STEP 2
DOPPLER SHIFT CORRECTION

CHANNEL 0 1200

STEP 3
LINE SHAPE

CHANNEL 0 1200

STEP 4
THICK TARGET SPECTRUM AFTER STRIPPING
III-B. Unpolarized $(p,\gamma)$ Angular Distribution Analysis

A typical experimental run consists of the following:

A run using a $^{228}$Th source for calibration purposes as well as for checking the Ge(Li) detector resolution.

A set of thick target spectra using the Ge(Li) including 6 different angular position with the incident proton energy set at 1.8 MeV, to cover the whole resonance region of interest (1.6-1.8 MeV).

A thin target spectrum with the Ge(Li) at 90° with the incident proton energy set at the peak of the resonance, namely 1.7 MeV.

The thick target spectra contain all the vital information while the thin target spectrum carries the "line shape" information. In order to check on the total charge integration, the NaI detector is used at the 180° backward angle at every thick target run as a monitor. During each run of our experiment, the sums of the last three peaks in the NaI spectrum (Fig. III-4) remains within 1% of each other.

A set of programs were written to perform the data reduction. Step 1 is to perform a background subtraction. Step 2 is to correct for the Doppler Shift Effect. Step 3 is to construct from the thin target spectrum a line shape respond function. Step 4 is to strip the thick target spectrum. Step 5 is to extract angular distribution information from the data. Step 6 we fit the angular distribution information to a linear superposition of Legendre polynomials by the method of least squares. This gives the coefficients in the expression

$$W(E,\theta) = \sum_{k=0}^{N} A_k(E)P_k(\theta)$$
Fig. III-7

Angular distributions for the ground-state $\gamma$-rays from the $^{12}\text{C}(p,\gamma)^{13}\text{N}$ reaction. (Taken from Young et al. $^3$)
Fig. III-8 Comparison of resonance capture prediction and the values extracted from unpolarized measured angular distribution. (Taken from Young et al.3)
THE $^{12}\text{C}(p,\gamma)^{13}\text{N}$ REACTION

Fig. 4a.

Fig. 4b.
Fig. III-9

Resonance capture calculations of the ratio of the yield of ground-state $\gamma$-rays from the $^{12}\text{C}(p,\gamma)^{13}\text{N}$ reaction at 45° to that at 135°. The solid curves are based on interference between the $^{13}\text{N}$ 2.37, 3.51 and 3.56 MeV levels. The parameter $\alpha_2$ is a measure of the strength of the 3.56 MeV level contribution.

(Taken from Young et al.\textsuperscript{3})
where the $A_k(E)$'s are the coefficients of the Legendre polynomials. In our case, fits up to maximum order $N = 2$ and 4 were performed.

1. Geometrical Errors

The solid angle $\Omega$ subtended by the detector in this thick target method turns out to be very important. In our case, a 9% correction to the data was required due to a slight error in target centering.

One knows $\Omega$ is defined to be $A/R^2$. If

$$R + R \pm \Delta R(\theta, R),$$

then

$$\Delta \Omega = \frac{\delta \Omega}{\delta R} \Delta R = -\frac{2}{R} (\pm \Delta R).$$

The total value then equals $\frac{4\Delta R}{R}$. Of course, this can be reduced by increasing $R$.

Let us assume we know the exact coordinate $(x, y)$ where the beam has exactly the energy $E_p$. With a little trigonometry, one sees the following:

$$x_0 = x - y \cos \theta,$$

$$R_0 = R - \sqrt{x^2 + y^2},$$

$$\sigma = \sqrt{x_0^2 + R_0^2 - 2x_0R_0 \cos \theta} \quad \text{where} \quad R = R - \sigma$$

2. Detector Efficiency

Due to the uncertainty of the position and extent of the active volume of the Ge(Li) detector, the determination of the attenuation coefficient is particularly difficult. Furthermore, the detector efficiency is energy dependent. Since the energy region we are examining is relatively small ($< 200$ keV), we simply use those values translated in Nuclear Reaction and Analysis (Fig. III-11).
Fig. III-10  Geometrical correction factor.
Fig: Top View of Target Chamber  True beam axis
\[ X_0 = X - Y \cos \theta \]
\[ R_0 = R - \sqrt{X^2 + Y^2} \]

By cosine law
\[ \cos \theta = \frac{X_0^2 + R_0^2 - r^2}{2 X_0 R_0} \]
3. Statistical Error

At each energy, the error involved in the thick target technique is estimated by the square root of the number of counts peeled for that particular energy.

III-C. Polarized \((p, \gamma)\) Angular Distribution Analysis

A typical set of our polarized experimental data looks like that of an unpolarized one except for one difference. We have two sets of thick target Ge(Li) spectra at each of three different angular positions, one of which was measured with the ionizer set of the "+" position,\(^{23}\) the other of which was measured with it set at the "-" position.

The data reduction is similar to that of the unpolarized runs. The analysis focused on the analyzing power instead. Again it is conventional for one to fit the polarized angular distribution information to a Legendre's polynomial plus an associated Legendre's polynomial like that of the following:

\[
W_{\text{pol}}(E, \theta) = \sum_{k=0}^{N} A_k(E) P_k(\theta) + \sum_{j=1}^{M} b_j(E) P_j'(\theta) \left[ \hat{p} \cdot \hat{A} \right]
\]

\[
= \sum_{k=0}^{N} A_k(E) P_k(\theta) \left[ 1 + \frac{\sum_{j=1}^{M} b_j(E) P_j'(\theta) \left( \hat{p} \cdot \hat{n} \right)}{\sum_{k=0}^{N} A_k(E) P_k(\theta)} \right]
\]

\[
= W_{\text{unpol}}(E, \theta) \left[ 1 + \hat{p} \cdot \hat{A} \right]
\]

In our case \(W_{\text{pol}}(E, \theta) = W_{\text{unpol}}(E, \theta) [1 + P_y A_y]\)

where

\[
A_y = \sum_{j=1}^{M} \frac{b_j(E)}{\sum_{k=0}^{N} A_k(E) P_k(\theta)} P_j'(\theta)
\]
Fig. III-11  Ge(Li) efficiency
ABSORPTION COEFFICIENTS FOR GERMANIUM

Gamma Ray Energy (keV) vs. \( \tau \) (cm\(^{-1}\))

- Photoelectric
- Compton
- Pair Production
- Total

Multiply \( \tau \) by 1000
Fig. III-12  Polarized $^{12}$C(p,$\gamma$)$^{13}$N thick target spectrum.
IONIZER - DETECTOR AT -90 DEGREE

IONIZER - DETECTOR AT -90 DEGREE
Fig. III-13  Analyzing power measurements at three different angles.
Fig. III-14 A,B  Associate Legendre's polynomial fit for analyzing power and the solid curve is the resonance capture prediction.
The sets of analyzing power measurements at three different angles are plotted in Fig. 3-13A,B. As before a least-squares program was used, but this time the series includes the associate Legendre polynomials. Assuming the $a_n(E)$'s from the corresponding unpolarized part is correct, one gets a set of $b_n(E)$'s, the coefficients of the associate Legendre polynomials. In our case maximum orders $N = 1$ and 2 were tried.

1. **Polarized Case: Geometrical Errors**

By using our particular one-detector method\(^{24}\) to measure analyzing power, one gets rid of the misalignment problem we had in the unpolarized case, since at each angle, the distance from target remains constant for both spin up and spin down runs. But there exists another important requirement, namely, $nN = n'N'$, which must be fulfilled ($n$ is the number of particles incident on the target and $N$ is the number of target nuclei per square centimeter). The following will make this obvious.

If the detector is at the left side of the incident beam, and the proton polarization is up,

$$L_1 = N_1(\theta + \Delta \theta_1, 0) = NnE_1\Omega(\Delta \theta_1, \Delta r_1)\sigma_0(\theta + \Delta \theta_1)[1 + PA_y(\theta + \Delta \theta_1)].$$

If a proper flip is executed by reversing the ionizer,

$$R = N_1(\theta + \Delta \theta_1, \pi) = N'n'E_1\Omega(\Delta \theta_1, \Delta r_1)\sigma_0(\theta + \Delta \theta_1)[1 - PA_y(\theta + \Delta \theta_1)].$$

Then if $Nn = N'n'$,

$$PA_y = \frac{L - R}{L + R}.$$
In our particular case the target density is uniform and therefore is the same for both runs, so

\[ n = n'. \]

The integrated beam of the two runs is monitored by the charge integration as well as the NaI \( \gamma \)-radiation spectrum; it was checked that, in fact,

\[ N' = N. \]

2. The efficiency is the same in the unpolarized case.

3. Statistical error: The error in measured analyzing power is given by the expression

\[ \Delta A_y = A_y \left[ \frac{\Delta P A_y}{P A_y} \right]^2 + \left( \frac{\Delta P}{P} \right)^2 \right]^{1/2}, \]

where \( P \) and \( \Delta P \), the polarization and uncertainty in polarization, will be discussed in the next section.

where

\[ \Delta(P A_y) = \left[ \frac{1 - (P A_y)^2}{L + R} \right]^{1/2} \]

III-D. Analysis of Beam Polarization Measurements

Before we start, we would like to point out that our measurements and analyses are that of spin 1/2 particles. We assume the usual Cartesian coordinates with \( \hat{k}_{\text{in}} \) along the momentum 2 axis, \( \hat{k}_{\text{in}} \times \hat{k}_{\text{out}} \) to be the \( \hat{z} \)-axis and the right hand rule defines the x-axis. Furthermore, we use angles \( \beta \) and \( \phi \) to define the spin quantization axis \( \hat{S} \).
Fig. III-15  Polarization convention adopted from Kenton et al.'s paper.27)
Fig. III-16  $^4\text{He}(p,p)$ elastic proton scattering data.
P-\(^{4}\)He ELASTIC SCATTERING

\(E_p = 1.8\) MeV

COUNTS ( x 10 )

CHANNEL NUMBER

RUN NO: 139 - 63 - 59

Ni

p-\(\alpha\)

p-\(\alpha\)

Ni

512 562 612 662 712 762 912 962 1012 1062
The angular distribution function at any particular energy, then can be written as follows:

\[ W(\theta) = W_{\text{unpol}}(\theta) [1 + P \cdot \hat{n}] \]  

By choosing appropriate source parameters, one can set \( \beta \approx 90^\circ \) and \( \phi \approx 0^\circ \), hence \( P \) differs from \( P_y \) by at most a second order term.

\[ W(\theta) = W_{\text{unpol}}(\theta) [1 + P_y A_y] \]

In order to find \( P \), let us first look at the actual number of counts recorded by each detector.

\[ N(\theta, \phi) = nN\Delta \Omega \sigma(\theta, \phi) \]

where \( n \) is the number of particles incident on the target, \( N \) is the number of target nuclei per square centimeter, \( \Delta \Omega \) is the solid angle subtended by the detector, \( E \) is the detector efficiency and \( \sigma(\theta, \phi) \) is the cross section for scattering in the direction \( (\theta, \phi) \).

Suppose dissimilar detectors were used. Then

\[ N_1(\theta, 0) \equiv L_1 = nn'N_1E_1\sigma_0(\theta) [1+pA_y(\theta)] \quad \text{and} \quad R_2(\theta, \pi) \equiv R_2 = nn'N_2E_2\sigma_0(\theta) [1-pA_y(\theta)]. \]

Now flip the polarization, i.e., let \( p \rightarrow -p \). Then

\[ N_1(\theta, \pi) \equiv R_1 = n'n'N_1E_1\sigma_0(\theta) [1-pA_y(\theta)] \quad \text{and} \quad N_2(\theta_2, 0) \equiv L_2 = n'n'N_2E_2\sigma_0(\theta) [1+pA_y(\theta)]. \]

If one forms the geometrical means,

\[ L = \sqrt{(L_1L_2)} = [nn'n'N_1N_2E_1E_2]^{1/2} \sigma_0[1+pA_y] \quad \text{and} \quad R = \sqrt{(R_1R_2)} = [nn'n'N_1N_2E_1E_2]^{1/2} \sigma_0[1-pA_y]. \]
then \[ p_{\lambda y} = \frac{L-R}{L+R}. \]

By knowing \( A_y \) for the particular reaction used, namely \( \alpha(p,p)\alpha \), we have a direct measurement of the beam polarization \( P \). A plot of the left-right asymmetry of p-p scattering is shown in Fig. 3-16. Our experimental polarization depends on the ionizer polarity; it was found to be about 0.71 when the ionizer was "+", and around 0.69 when the ionizer was "-"; the exact numbers, as monitored before and after each \((\vec{\phi}, \gamma)\) run, varied slightly, and the average of before/after was used in each case.

1. Geometrical Error

The misalignment that is associated with the geometrical imperfection of the two left-right detectors can be reduced to a second order effect by use of a "proper flip" procedure. In the proper flip procedure, either the polarization of the beam is reversed, or the analyzer is rotated 180° in such a way that the beam direction and the position are held invariant with respect to each of the detectors. Experimentally, such a flip can be carried out, either by reversing the ionizer position or by actually rotating the scattering chamber, with the same amount of beam current intercepted by each slit before and after the rotation. Mathematically

\[
L_1 \equiv N_1(\theta + \Delta \theta_1, 0) = nN_E \Omega_1(\Delta r_1, \Delta \theta_1) \sigma_\theta (\theta + \Delta \theta_1) [1 + p_{\lambda y} (\theta + \Delta \theta_1)]
\]

\[
R_2 = N_2(\theta + \Delta \theta_2, \pi) = nN_E \Omega_2(\Delta r_2, \Delta \theta_2) \sigma_\theta (\theta + \Delta \theta_2) [1 - p_{\lambda y} (\theta + \Delta \theta_2)]
\]

where \( \Delta \theta_1, \Delta r_1, \Delta \theta_2, \Delta r_2 \) denote the total error quantities of detectors 1 and 2 respectively. If a proper flip is executed,
In our case, \( \lambda \) varies very slowly between the scattering angles \( \theta + \Delta \theta_1 \) and \( \theta + \Delta \theta_2 \). We have

\[
P_A \lambda = \frac{L-R}{L+R}
\]

Further details can be obtained from Ohlsen et al.'s paper.

2. **Statistical Error**

The statistical error calculations are made in the usual way using the following formula

\[
[\Delta f(x_1, x_2, \ldots, x_n)]^2 = \sum_{i=1}^{n} \left( \frac{\partial f}{\partial x_i} \right)^2 (\Delta x_i)^2
\]
we obtain

$$\Delta P = P \left[ \left( \frac{\Delta PA_y}{PA_y} \right)^2 + \left( \frac{\Delta A_y}{A_y} \right)^2 \right]^{1/2}$$

where

$$\Delta (PA_y) = \frac{1}{L+R} \sqrt{(1-PA_y)^2 (\Delta L)^2 + (1+PA_y)^2 (\Delta R)^2}$$

$$\Delta L = \frac{1}{2L} \left[ \frac{1}{L_1} + \frac{1}{L_2} \right]^{1/2}, \quad L = \sqrt{L_1 L_2}$$

$$\Delta R = \frac{1}{2R} \left[ \frac{1}{R_1} + \frac{1}{R_2} \right]^{1/2}, \quad R = \sqrt{R_1 R_2}$$

In our experiment $\Delta P \approx 0.004$. 
A. Resonance Radiative Capture Mechanism

Even though the exact strong nuclear interaction is not yet known, one can use the quantum mechanical properties of angular momentum without explicit treatment of the dynamical process. The angular distribution, which is the probability that the reaction occurs with the outgoing radiation in a given direction, can be obtained by the following method: first, choosing a general wave function whose asymptotic form is governed by the scattering matrix (the wave functions are taken as incident plane waves and outgoing spherical waves with definitive spin, parity and angular dependence, then averaging over all possible magnetic quantum numbers of the initial states and summing over the corresponding final states. This is physically obvious, but the process of averaging and summing for reactions involving arbitrary spin angular momentum often proves to be very tedious.

Instead of averaging and summing over the non-observables, a more logical procedure is to avoid introducing pure states to describe the initial and final conditions. Instead one could incorporate the limitation of the measurements within the theory. The introduction of the density matrix $\rho$ provides a compact description of the assembly of the
system (the new state is not the same QM state as before but a mixed one) and the introduction of the efficiency matrix as characteristic of the modes of detection of the outgoing radiation and nuclei. The efficiency matrix is further assumed to be diagonal in the Q-representation such that the probability of detection is

\[ W = \sum_{q} |E_q(q|p|q) = T_{p}(Ep) | \]

with the aid of vector coupling coefficients, such as CG, Racah, 6J., Wigner and 9J symbols, and irreducible tensors, the complexity of summing of magnetic quantum numbers is transformed into a neatly arranged mathematical problem.\(^{28,29}\)

In a \((p,\gamma)\) reaction it is conventional for one to use a "mixed" representation like the following (Fig. 4-1):

\[
\omega = \sum_{k_{s},k_{s}} \sum_{k_{s},k_{s}} f_{k_{s}k_{s}}(c_{c}) f_{k_{s}k_{s}}(c_{c}) \gamma_{k_{s}k_{s}}(c_{c}) \gamma_{k_{s}k_{s}}(c_{c}) \]

Thus far, we have made no assumption as to whether the target radiation or incident beam is prepared in some definite polarization state and
$^{12}\text{C}(p,\gamma)^{13}\text{N}$ resonance capture process can best be described by the Mix representation which consists of the following:

Channel spin representation

\[ b = S_1 + \lambda_1 \]
\[ S_1 = s_1 + a \]

L-representation

\[ b = L_2 + C \]
\[ L_2 = \lambda_2 + s_2 \]
$^{12}\text{C ( p, }\gamma\text{ )}^{13}\text{N}$

$J (1)$

$\frac{5}{2}^+ + \frac{3}{2}^- \rightarrow 3.5491 \text{ MeV}$

$\frac{1}{2}^+ \rightarrow 3.5078$

$\frac{1}{2}^+ \rightarrow 2.9882$

$1 = 1, s = \frac{1}{2}$

$1 = 0, s = \frac{1}{2}$

$\gamma^+$

$^{12}\text{C + p}$

$1.941 \text{ MeV}$

$\frac{1}{2}^- \rightarrow 1.941 \text{ MeV}$

$^{13}\text{N}$
whether the detectors of the outgoing radiation are sensitive to polarization.

It is convenient to pick a coordinate system (since we have the freedom to do so) whose axes of quantization coincide with the direction of propagation. Mathematically speaking,

\[
\rho_{k_k'k'_k}(LL') = \sum_{k_2} C_{k_k'k'_k} C_{k_k'k'_k}(LL') D_{k_2k_2}^{k_k'}(R_1) = \frac{e^{2\pi i}}{4\pi} \langle L0L'0|k_20\rangle D_{k_20}^{k_k'}(R_1)
\]

where \( C_{k_k'k'_k} \) is the Racah radiation parameter and \( D(R_1) \) denotes the rotation that carries the \( z \)-axis into the direction of radiation.

If the detector of the outgoing radiation is not polarization sensitive one can again have the freedom to pick a convenient coordinate such that the outgoing \( \gamma \)-radiation has the angle \( (\theta, \phi, 0) \):

\[
E_{k_k'k'_k}^{\theta \phi} (LL') = \sum_{k_L} C_{k_k'k'_k} C_{k_k'k'_k}^{k_Lk_L'} D_{k_Lk_L'}^{k_k'}(R_2) \theta_k
\]

where \( C_{k_k'k'_k}^{k_Lk_L'}(LL') \) is again the Racah radiation parameter, \( \theta_k \) is the detector efficiency and \( D(R_2) \) denotes the rotation that carries the outgoing radiation quantization axis to the \( z \)-axis.

Now let us examine the case where one has an unpolarized proton beam incident on the unpolarized target nuclei. Mathematically speaking, the random spin state of the beam and that of the target can be written as

\[
\rho_{k_k k_k}(ss') = \frac{1}{8} \delta_{k_k} \delta_{k_k'} \delta_{ss'}
\]

\[
\rho_{k_k k_k}(aa') = \frac{1}{8} \delta_{k_k} \delta_{k_k'} \delta_{aa'}
\]

where \( \rho_{k_k k_k}(ss') = \rho_{k_k k_k}(aa') \times \rho_{k_k k_k}(ss') = \frac{1}{8} \delta_{k_k} \delta_{k_k'} \delta_{ss'} \).
and the unobserved final nucleus can also be written as

$$\xi_{E}^c(\omega') = \delta_{K} \delta_{\omega} \delta_{\omega'}$$

$$\mathcal{S}_{E}^c(\omega) = \frac{4 \pi}{\mathcal{A}} \left< \omega, \omega' \mid \delta_{E} \right>$$

$$\mathcal{E}^c_{E} = \frac{4 \pi}{\mathcal{A}} \left< \omega', \omega \mid \delta_{E} \right> \mathcal{D}^c_{E}$$

Keeping the following rotational matrix property in mind

$$\sum_{K} D_{E}^K = D_{E}$$

one derives

$$\omega_{E} = \frac{1}{4 \pi} \sum_{K=0}^{\infty} \left< \omega, \omega' \mid \delta_{E} \right> = \mathcal{A} \left< \omega, \omega' \mid \delta_{E} \right> \mathcal{D}^c_{E}$$

It is worthwhile to note the following:

1. The final result confirms that the angular distribution only involves the angles between the direction of each radiation, not the arbitrary coordinate system one's using when one carries out the process.

2. The complexity of the condition actually is limited by the properties of the CG coefficients, Racah 6J coefficients and the Wigner
9J symbols. To be more precise,

\[ k \leq \min(2l, 2L, 2b), \text{ or,} \]

if \( b \) is a half integer,

\[ k \leq \min(2l, 2L, 2b + 1). \]

3. The angular distribution obeys the law of reciprocity, i.e., it is time reversal invariant.

In our particular case \( \) there are three nearby excited states of interest with spin parity assignments \( 1/2^+, 3/2^-, \) and \( 5/2^+ \) respectively. Using channel-spin representation for the incident channel, the conservation of parity and total angular momentum allows \( l = 0 \) for the \( 1/2^+ \) state, \( l = 1 \) for the \( 3/2^- \) state and \( l = 2 \) for the \( 5/2^+ \) state. As for the exit channel, using the \( L \)-representation, the conservation of parity and total angular momentum again allows only \( E1 \) decays for the \( 1/2^- \) state, \( M1, E2 \) for the \( 3/2^- \) state and \( M2, E2 \) for the \( 5/2^+ \) state.

The reaction matrix element is

\[ \langle c_{12}, \ell_{14} \rangle = e^{i[-\ell(1) + \ell(2)]} \frac{i \sqrt{\Gamma(1) \Gamma(2)}}{(E_{c} - E)^{2} + \Gamma^{2}/4} e^{ip_{14}} \]
where $\xi(\ell) = \phi_\ell + \psi_\ell + \sigma_\ell$ is the phase shift for nuclear and potential scattering and

$$e^{\ii \xi_\ell} = \frac{G_{\ell}(\ii \varphi) - \ii P_{\ell}(\ii \varphi)}{G_{\ell}(\ii \varphi) + \ii P_{\ell}(\ii \varphi)}$$

$$e^{\ii \chi_\ell} = \frac{(1+\ii \gamma)(1-\ii \gamma) \ldots (1+\ii \gamma)}{(1-\ii \gamma)(1-\ii \gamma) \ldots (1-\ii \gamma)}$$

$$e^{\ii \alpha_{\chi}} = \frac{\ii \gamma!}{(-\ii \gamma)!}, \quad \gamma = \frac{2\ii}{\hbar\omega}$$

$\xi(\alpha)$ is determined by the screening radius of the Coulomb field which is independent of $\ell$. The $\gamma$-radiation reaction matrix element is real and therefore does not contribute to the phase. We have

$$\beta(b) = \tan^{-1} \left( \frac{2}{\Gamma(b)} (E - E_0(b)) \right), \quad \text{where}$$

$E_0(b) =$ is the effective resonance energy; $E$ is the c.m. energy of the system,

$\Gamma(b) =$ Total width of the state,

$\Gamma_p(\ell_1) =$ proton width, and

$\Gamma_\gamma(b, L) =$ partial decay width of the $\gamma$-radiation.

The energy-dependent angular distribution for the two levels takes the form:
\[ W(\theta) = \frac{2^k \Gamma_k(\ell, \ell')}{{4}^{1/2} a_1^2 a_2^2} \left\{ C_1 \left[ W_{10} \left( \frac{\theta}{2} \right) + \beta W_{20} \left( \frac{\theta}{2} \right) \right] + \alpha \beta W_{10} \frac{\theta}{2} \right\} \]

\[ + \frac{2 \alpha \gamma \delta}{\ell_1 \ell_2} \left[ \left( \frac{\theta}{2} \right) W_{10} \left( \frac{\theta}{2} \right) + \beta W_{20} \left( \frac{\theta}{2} \right) \right] \]

\[ + \frac{2 \gamma \delta}{\ell_1 \ell_2} \left[ \left( \frac{\theta}{2} \right) W_{10} \left( \frac{\theta}{2} \right) + \beta W_{20} \left( \frac{\theta}{2} \right) \right] \]

(F)* (G)

\[ m_{1/2} = C_1 \left[ (E, E_{1}) + \frac{2}{5} \Gamma_{1/2} \right] \]

\[ \alpha_{1/2} = \frac{\Gamma_{1/2}(\ell, \ell_1)}{\Gamma_{1/2}(\ell, \ell_1)} \quad \beta_{1/2} = \frac{\Gamma_{1/2}(\ell, \ell_1)}{\Gamma_{1/2}(\ell, \ell_1)} \]

\[ \frac{\tilde{f}(\ell) - \tilde{f}(\ell')} = \tilde{R} \left( \frac{\theta}{2} \right) - \frac{\tan^{-1} \left( \frac{\tilde{R}(\theta)}{\tilde{R}(\theta)} \right) - \tan^{-1} \left( \frac{\tilde{R}(\theta)}{\tilde{R}(\theta)} \right)}{\tilde{R}(\theta)} \]

\[ P_1(R) \text{ and } G_1(R) \text{ are the regular and irregular Coulomb functions.} \]

For our case, the individual \( W \) 's are as follows:

\[ W_{1/2}(\theta) = 4P_0 - 2P_2, \]

\[ W_{2/2}(\theta) = 4P_0 + 2P_2, \]

\[ W_{1/2\text{le}}(\theta) = 2\sqrt{3}P_2, \]

\[ W\text{le}(\theta) = 2P_0 \]

\[ W_{1/2\text{le}}(\theta) = 2P_1, \text{ and} \]

\[ W_{2/2\text{le}}(\theta) = 2\sqrt{3}P_1. \]

The energy dependent angular distribution for the three levels takes the form

\[ W_{\text{ang}}(\theta) = \frac{2^k \Gamma_k(\ell, \ell')}{{4}^{1/2} a_1^2 a_2^2} \left\{ C_1 \left[ W_{10} \left( \frac{\theta}{2} \right) + \beta W_{20} \left( \frac{\theta}{2} \right) \right] + \alpha \beta W_{10} \frac{\theta}{2} \right\} \]
The polarized general formula for capture reactions is given in the following:

\[
W(\theta, E) = \sum_{\alpha_0} \left[ A_k (1b 1'b', S_{11}) A_k (LL'cbb') Q_k p_k (\cos \theta) \right] \nonumber
\]

where \( S_{11} = \frac{\Delta_s^2}{\Delta_a^2 + \Delta_s^2} \)

\[
\begin{align*}
W_{\alpha_0}(\theta) &= 6P_3 + \frac{3}{7} P_2 - \frac{5}{7} P_0, \\
W_{\alpha_0}(\theta) &= \frac{3\sqrt{5}}{2} (9P_1 - 4P_0), \\
W_{\alpha_0}(\theta) &= \frac{12}{3} (P_1 + \frac{1}{2} P_2), \\
W_{\alpha_0}(\theta) &= \frac{3}{2} P_3.
\end{align*}
\]

\[
\alpha_0^2 = \frac{P_0^2 (\alpha_0 m_1)}{P_0^2 (\alpha_1 m_1)} = 0.306 \frac{P_0^2 (\alpha_0 m_1)}{P_0^2 (\alpha_1 m_1)} \left( \frac{E_{\text{lab}}}{E_{\text{lab}}} \right)^2 = \alpha_0^2 \left( \frac{E_{\text{lab}}}{E_{\text{lab}}} \right)^2
\]

\[
\begin{align*}
W(\theta, E) &= \sum_{\alpha_0} \left[ A_k (1b 1'b', S_{11}) A_k (LL'cbb') Q_k p_k (\cos \theta) \right] \nonumber
\end{align*}
\]

\[
W(\theta, E) = \sum_{\alpha_0} \left[ A_k (1b 1'b', S_{11}) A_k (LL'cbb') Q_k p_k (\cos \theta) \right] \nonumber
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W_{\alpha_0}(\theta) &= \frac{3}{2} P_3.
\end{align*}
\]

\[
\alpha_0^2 = \frac{P_0^2 (\alpha_0 m_1)}{P_0^2 (\alpha_1 m_1)} = 0.306 \frac{P_0^2 (\alpha_0 m_1)}{P_0^2 (\alpha_1 m_1)} \left( \frac{E_{\text{lab}}}{E_{\text{lab}}} \right)^2 = \alpha_0^2 \left( \frac{E_{\text{lab}}}{E_{\text{lab}}} \right)^2
\]

\[
\begin{align*}
W(\theta, E) &= \sum_{\alpha_0} \left[ A_k (1b 1'b', S_{11}) A_k (LL'cbb') Q_k p_k (\cos \theta) \right] \nonumber
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\end{align*}
\]

\[
\alpha_0^2 = \frac{P_0^2 (\alpha_0 m_1)}{P_0^2 (\alpha_1 m_1)} = 0.306 \frac{P_0^2 (\alpha_0 m_1)}{P_0^2 (\alpha_1 m_1)} \left( \frac{E_{\text{lab}}}{E_{\text{lab}}} \right)^2 = \alpha_0^2 \left( \frac{E_{\text{lab}}}{E_{\text{lab}}} \right)^2
\]

\[
\begin{align*}
W(\theta, E) &= \sum_{\alpha_0} \left[ A_k (1b 1'b', S_{11}) A_k (LL'cbb') Q_k p_k (\cos \theta) \right] \nonumber
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W_{\alpha_0}(\theta) &= \frac{3}{2} P_3.
\end{align*}
\]

\[
\alpha_0^2 = \frac{P_0^2 (\alpha_0 m_1)}{P_0^2 (\alpha_1 m_1)} = 0.306 \frac{P_0^2 (\alpha_0 m_1)}{P_0^2 (\alpha_1 m_1)} \left( \frac{E_{\text{lab}}}{E_{\text{lab}}} \right)^2 = \alpha_0^2 \left( \frac{E_{\text{lab}}}{E_{\text{lab}}} \right)^2
\]

\[
\begin{align*}
W(\theta, E) &= \sum_{\alpha_0} \left[ A_k (1b 1'b', S_{11}) A_k (LL'cbb') Q_k p_k (\cos \theta) \right] \nonumber
\end{align*}
\]

\[
\begin{align*}
W_{\alpha_0}(\theta) &= 6P_3 + \frac{3}{7} P_2 - \frac{5}{7} P_0, \\
W_{\alpha_0}(\theta) &= \frac{3\sqrt{5}}{2} (9P_1 - 4P_0), \\
W_{\alpha_0}(\theta) &= \frac{12}{3} (P_1 + \frac{1}{2} P_2), \\
W_{\alpha_0}(\theta) &= \frac{3}{2} P_3.
\end{align*}
\]
In the case of an incident proton beam (spin 1/2), the only chance one needs to make is

\[ \sum_{E_1} \Delta E \frac{1}{E_1} \delta_{E_1, E_1'} \delta_{E_1 E_2} \delta(E_1 \pm E_2) \]

where

\[ \phi = \frac{1}{2} \delta_{E_1, E_1'} \delta_{E_1 E_2} \]

After some simplification, the polarized angular distribution for the two levels takes the form:

\[ W = \text{unpol.} + \frac{A^2}{4} \Gamma_y(b') \{(2\alpha_1 \mu(b') \mu(b') \sin(\delta(3/2) - \delta(h)))(w_{E_1 M_1}^{X} + w_{E_1 E_2}^{X}) P_x \]

\[ + \frac{A^2}{4} \Gamma_y(b') \{(2\alpha_1 \mu(b') \mu(b') \sin(\delta(3/2) - \delta(h)))(w_{E_1 M_1}^{Y} + w_{E_1 E_2}^{Y}) P_y \]

<table>
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<tr>
<th>W_{M_1}</th>
<th>W_{M_1}</th>
<th>W_{M_1}</th>
</tr>
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<tbody>
<tr>
<td>M1 _0</td>
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</tr>
<tr>
<td>W_{E_2}</td>
<td>W_{E_2}</td>
<td>W_{E_2}</td>
</tr>
<tr>
<td>E2 _0</td>
<td>E2 _0</td>
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<tr>
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<tr>
<td>W_{E1E2}</td>
<td>W_{E1E2}</td>
<td>W_{E1E2}</td>
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<tr>
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<tr>
<td>W_{E1M1}</td>
<td>W_{E1M1}</td>
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<tr>
<td>E1M1 _0</td>
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<td>W_{E1}</td>
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</tr>
<tr>
<td>E1 _0</td>
<td>E1 _0</td>
<td>E1 _0</td>
</tr>
</tbody>
</table>
One noted that our result was further checked against the following:

1. Transverse Polarization Theorem.

\[ (-1)^{l+l'}(-1)^{L+L'+P+P'} <10,1'0|k_10> \neq 0 \quad \text{implies } k_1+k=\text{even} \]

\[ (1 \pm (-1)^{L+L'+P+P'+k}) \neq 0 \]

But \( <10,k_10|k0> = 0 \) Therefore no \( P_z \) term.

2. If polarized nucleon incident on \( 0^+ \) target, and if one looks at the single isolated resonance, one should not be able to observe asymmetry.

\[ <10,1'0|k_10> + 1+1'+k=\text{even} \]

\[ (1 \pm (-1)^{L+L'+P+P'+k}) \rightarrow k=\text{even} \]

according to the formula \( k_1+k+1=\text{odd} \) means one picks \( \text{iIm}(RR^{*}) \).

But for single isolated resonance \( \text{iIm}(RR^{*}) = 0 \).

IV-B. **Direct Radiative Capture Mechanism**

The direct capture reaction is a direct interaction; during the course of the reaction, no intermediate nuclear states are formed. A particle \( X \) of mass \( M_1 \), charge \( Z_1 \) is captured by a target nuclear \( A \) of mass \( M_2 \), charge \( Z_2 \) to form a final state \( B \) of the mass \( M_3 \) and charge \( Z_3 = Z_2 + Z_1 \) with the emission of \( \gamma \)-radiation of energy \( E_\gamma \), mediated by the electromagnetic interaction Hamiltonian.\(^{30-32}\) The direct capture cross section is calculated in first order perturbation theory by considering the matrix elements of the electromagnetic multipole.
\[ ^{12}C(\bar{p},\gamma)^{13}N \]

\[ J(\pi) \]

\[ \frac{1}{2}^+ \quad 3.5491 \text{ MeV} \]

\[ \frac{3}{2}^+ \quad 3.5078 \]

\[ \gamma_1 \]

\[ \frac{1}{2}^+ \quad 2.3632 \]

\[ \gamma_2 \]

\[ 0^+ \quad 1.941 \text{ MeV} \]

\[ ^{12}C + p \]

\[ \frac{1}{2}^- \quad \text{g.s.} \]

\[ ^{13}N \]

Fig. IV-2 \[ ^{12}C(p,\gamma)^{13}N \] direct capture process
operators between initial scattering states and the final bound states of the incident and target nuclei. The differential cross section for the unpolarized direct capture from continuum to bound states was given by Tombrello and Parker\textsuperscript{32} (We have generalized the formula slightly by the inclusion of the density matrix explicitly.):

\begin{equation}
\frac{d\sigma}{d\Omega} = \frac{K_\gamma}{2\pi \hbar v_1} \sum_{m_i m_f} (f m_f | H_{\text{int}}^P | i m_i) (m_i | P | m_f) \nonumber \end{equation}

\begin{equation}
(f m_f | H_{\text{int}}^P | i m_i)^* \nonumber \end{equation}

where \(j_1\) and \(j_2\) are the spins of the projectile and target respectively, \(K_\gamma\) is the wave number for the outgoing systems, \(i\) and \(f\) refer to the initial continuum states and final bound states of the combined system, and the \(P\) sum is for the circular polarization of the \(\gamma\)-radiation.

The electromagnetic interaction Hamiltonian, in the case of El radiation, is given by

\begin{equation}
H_{\text{int}}^P = -i \left(\frac{4\pi}{3}\right)^{1/2} K_\gamma e \mu \left(\frac{Z_1}{M_1} - \frac{Z_2}{M_2}\right) \sum_M D_M^1 (\phi, \theta_\gamma \theta_{\text{El}} (\gamma) \gamma_M^\ast (\theta, \phi) \nonumber \end{equation}

where \(\theta_{\text{El}}\) is the radial part of the electric-dipole operator. It is equal to \(\frac{3}{2\rho^3} (\rho \cos \rho - (1 - \rho^2) \sin \rho) r\) and in the long wavelength approximation (i.e., for \(K_\gamma r \ll 1\), \(\theta_{\text{El}}\) approaches \(r\). \((\theta_\gamma, \phi_\gamma)\) specifies the direction of the emitted \(\gamma\)-radiation with respect to the beam and \((r, \theta, \phi)\) are the coordinates of the incident particle in the center of mass coordinate system. The effect of the nuclear
recoil is reflected by the reduced mass times $\left(\frac{Z_1}{M} - \frac{Z_2}{M_2}\right)$ factor.

The initial wave function for the incident particle is chosen to be along the z-axis and is represented by a distorted Coulomb wave.

$$\phi_{\text{i} mi} = \sum_{\ell_i \text{Si}} i \ell_1 \sqrt{4\pi} (2\ell_1 + 1) e^{i(\sigma_{\ell_1} - \sigma_0 + \delta_{\ell_1})} \frac{U_{\ell_i}(kr)}{k_i^r} Y_{\ell_i}^0(\theta_\phi) \chi_{\text{Si} mi}$$

where $\ell_i$ is the orbital angular momentum of a partial wave, $\chi_{\text{Si} mi}$ is the spin function with channel spin $s_i$, $k_i$ is the wave number for the incident particle, $\sigma_{\ell_1} - \sigma_0$ is the usual difference of Coulomb phase and $\delta_{\ell_1}$ represents the nuclear phase shift.

In the case of the perturbed square well potential (Appendix 5), the initial (scattering) state radial wave function $\frac{U(k_i(r))}{k_i^r}$ is given analytically by the confluent hypergeometric function inside the nuclear radius ($r \leq R_0$) and the Coulomb wave function $33-37$ ($F_{\ell_1} \cos \delta_{\ell_1} + G_{\ell_1} \sin \delta_{\ell_1}$) outside the nuclear radius ($r \geq R_0$). The initial wave functions are normalized by matching the inside wave functions to the already normalized outside wave functions.

The final wave function of the combined system is of the form

$$\phi_{\text{f} m_f} = \sum_{\ell_f S_f} \theta_f \frac{U_{\ell_f}(k_f r)}{k_f r} \sum_{\beta} (\ell_f m_f - \beta S_f \beta | J_f M_f ) Y_{\ell_f}^{\text{m}_f - \beta} \chi_{S_f}^{\beta}$$

where $\theta_f$ is the fractional parentage coefficient (spectroscopy factor) of $^{13}\text{N} + \gamma$. Also

$$\sigma_{\text{expt}} = \sum_{\ell_f} C^2 \theta_f^2 \sigma_{\text{theoretical}}$$
where $C^2$ is the isospin Clebath-Gordan Coefficient given by $$\langle t_p^M_p t_t^M_t | T_f^M_f \rangle^2$$ where $t_p$, $t_t$ and $t_f$ represent the isospin of the projectile, target nucleus and final state of the system respectively.

$S_f$ is the final channel spin and $l_f$ and $J_f$ are the final orbital and total angular momentum. The wave function $\frac{\psi_f(k_f r)}{k_f}$ for the bound state is again a confluent hypergeometric function inside the nuclear radius ($r < R_0$) and a Whittaker function outside the nuclear radius ($r > R_0$). The bound state normalization is accomplished by matching the inside and outside wave function at the boundary and making use of the relation

$$\int_0^{R_0} \frac{\psi_f(\text{inside})}{r} \frac{\psi_f(\text{inside})}{r} r^2 dr + \int_{R_0}^{\infty} \frac{\psi_f(\text{outside})}{r} \frac{\psi_f(\text{outside})}{r} r^2 dr = 1$$

In the present case, the nucleus radius $R_0$, along with the particular constant $\psi_f$, is determined by the following two reactions:

$$^{12}C + E_{Bn} \rightarrow ^{12}C + n$$

$$^{13}N + E_{Bp} \rightarrow ^{12}C + p$$

The first one is a neutron capture reaction. Knowing the binding energy $E_B$ of the $^{N+1}_{2A}$ nucleus, the outside bound state wave function, which is a spherical Hankel function, can be calculated. Matching the logarithmic derivative of the wave function outside, which is a Bessel function, one has one equation and two unknowns. Next, one applies the second reaction which is a proton capture reaction. Again, knowing the binding energy $E_B$ of $^{N+1}_{2A+1}$ nucleus, the outside bound state wave function, which is a Whittaker function, can be calculated. Matching the logarithmic derivative of the inside wave function which now is a confluent hypergeometric function, one has the second independent equation. Hence,
one can now solve for the two unknowns. One thing one needs to be particularly careful of is that the value of the logarithmic derivative is $l_1$ dependent,\(^{39}\) i.e., the first zero belongs to $(\ell = 0)$ S wave, the second zero belongs to $(\ell = 1)$ P wave and so on.

Up to this point everything has been quite general. From here on, we will discuss our specific problem: the process of $^{12}$C capturing the protons from S and D waves and decaying by means of El radiation to the P-wave ground state of $^{13}$N. There are two different approaches we would like to examine in the following:

1) **Extranuclear direct capture:**

It was first pointed out by Christy and Duck\(^{10}\) that if the $\gamma$-radiation energy was sufficiently low ($< 2$ MeV) and the reaction was of non-resonant type, then the capture matrix element could be determined in the region external to the nuclear radius. The fact that the interior contribution was small led to the name "extra-nuclear" direct capture process. For the scattering wave, one usually equates $\delta_\lambda$ with the Coulomb hard sphere phase shift ($\tan^{-1}(-F_\lambda/G_\lambda)$) which drew on the assumption that there was no interior wave function in the hard sphere case. The continuity condition of the wave function which forced the scattering wave to be zero at the hard sphere surface normalized it to unit flux at the same time. The bound state function is exactly like what has been described before. The radial integral looks like the following:

$$R = \int_{R_0}^{\infty} \frac{U_b(kr)}{r} \frac{d}{dE_\lambda} \frac{U_f(kr)}{r} r^2 dr$$

and the corresponding unpolarized differential cross section is
Fig. IV-3 Reproduction of Phase-shift data curve by effective square well potential.
Fig. IV-4 Simple particle resonance wave functions.
2. Single-particle-resonance type direct capture

The nuclear phase shift $\delta_{ij}$, on the other hand, can be obtained from a phase shift analysis of elastic scattering data (12C(p,p)12C). The analysis of the available data shows that the single particle S\textsubscript{1/2} phase shift which resonates around 0.461 MeV gives a large $\delta_{ij}$ ($l_2 = 0$) contribution at the energy region of interest (1.6-1.8 MeV). But the d\textsubscript{3/2} single particle resonance phase shift which resonates around 6.7 MeV (Fig. IV-3) is very small (< 3°) at our energy region. It is therefore not apparent that the latter resonance phase shift can be used as a test for the hypothesis against the hard sphere phase shift.

Resulting from the use of single-particle-resonance phase shifts, two modifications are required in the previous formalism:

a) The phase shift $\delta_{ij}$ is not only $l$ dependent but also $j$ dependent. This means the scattering wave must then take into account its $j$-dependency as follows:
\[ \phi_{i m_1} = \sum_{(l_1 s j_1) m_{2} m_{s 1}} (i)^{l_1} \frac{\sqrt{4\pi}}{k_{1 r}} e^{i \phi_{l_{j 1}}} \frac{U_{l_{j 1}}(k_{1 r})}{k_{1 r}} \hat{\ell}_{1}(l_{1} s_{1} m_{1} | j_{1} m_{1}) \]

\[ (l_{1} m_{2} s_{1} m_{s 1} | j_{1} m_{1}) Y_{l_{1}}^{m_{2}} \chi_{s_{1}}^{m_{s 1}} \]

b) Assuming that \( r_0 \) does not vary much and matches the logarithmic derivative to the outside Coulomb wave function, one can solve for the effective nuclear potential \( V \). In other words, the inside wave function of the scattering state is no longer zero. It is a confluent hypergeometric function. Fig. IV-4A, IV-4C gives the inside wave function for the \( S, D \) scattering waves, Fig. IV-4B gives the inside wave function for the bound state, Fig. IV-4D, IV-4F show the outside Coulomb wave function for the \( S, D \) scattering waves and Fig. IV-4E represents the outside (Whittaker) bound state wave function.

The radial integral takes on the form

\[ R_{s00} = \int_{0}^{\infty} \frac{d}{d r} \left( \frac{u_{s}^{2}(r)}{r} \right) a_{e_{i}} \frac{u_{f}^{2}(r)}{r} r^{2} dr + \int_{r_{0}}^{\infty} \frac{d}{d r} \left( \frac{u_{s}^{2}(r)}{r} \right) a_{e_{i}} \frac{u_{f}^{2}(r)}{r} r^{2} dr \]

\[ R_{d00} = \int_{0}^{\infty} \frac{d}{d r} \left( \frac{u_{d}^{2}(r)}{r} \right) a_{e_{i}} \frac{u_{f}^{2}(r)}{r} r^{2} dr + \int_{r_{0}}^{\infty} \frac{d}{d r} \left( \frac{u_{d}^{2}(r)}{r} \right) a_{e_{i}} \frac{u_{f}^{2}(r)}{r} r^{2} dr \]

and the unpolarized differential cross section then becomes

\[ \frac{d \sigma}{d \Omega} (\theta, \phi) = \frac{k_{r}}{\pi k_{l} V_{c}^{2}} \sum \left[ \frac{e^{i \theta_{j_{1}}}}{\sin^{2} \theta_{j_{1}}} \left( \frac{2 \pi}{\lambda_{0}} \right)^{2} \frac{1}{\lambda_{0}^{2}} \frac{1}{\sin^{2} \theta_{j_{1}}} \right] e^{i \lambda_{1} \theta_{j_{1}} - i \lambda_{2} \phi_{j_{1}}} \int_{t_{0}}^{\infty} \left( \frac{d}{d r} \left( \frac{u_{s}^{2}(r)}{r} \right) \right)^{2} \frac{d r}{r} \]

\[ \sum_{k} \left[ \frac{e^{i \lambda_{1} \theta_{j_{1}} - i \lambda_{2} \phi_{j_{1}}} \frac{d}{d r} \left( \frac{u_{s}^{2}(r)}{r} \right)^{2}}{\sin^{2} \theta_{j_{1}}} \right] \]
The polarized differential cross section is

\[
\frac{d\sigma}{d\Omega} (\theta, \phi) = \frac{\delta_{2}}{\sin \theta \sqrt{L_{\text{PR}}}} \sum_{M_{3}M_{3}^{*}} \frac{1}{R_{M_{R}^{*}M_{D}}} \frac{1}{R_{M_{R}^{*}M_{D}}} \sum_{J_{1}M_{1}} \left\langle J_{3}M_{3} | T_{M_{R}M_{D}} | J_{2}M_{2} \right\rangle \left\langle J_{1}M_{1} | D_{M_{R}^{1}}^{(LR)} \right\rangle (\theta, \phi, 0)
\]

\[
+ \left\langle J_{3}M_{3} | T_{M_{D}M_{D}} | J_{1}M_{1} \right\rangle \left\langle D_{M_{D}^{1}}^{(LD)} \right\rangle (\theta, \phi, 0)
\]

IV-C Interference between Direct Radiative Capture and Resonance Radiative Capture

It was mentioned in Chapter I section 3 that the two processes may not be separated in many cases. A coherent way of calculating the differential cross section is presented in the following (for details see Appendix 1):

\[
\sigma(E, \theta) \propto \sum_{M_{1}M_{3}P_{R}P_{R}} \sum_{M_{2}M_{R}^{*}M_{D}} \left\langle J_{3}M_{3} | T_{M_{R}M_{R}} | J_{2}M_{2} \right\rangle \left\langle J_{1}M_{1} | D_{M_{R}^{1}M_{R}^{1}}^{(LR)} \right\rangle (\theta, \phi, 0)
\]

\[
+ \left\langle J_{3}M_{3} | T_{M_{D}M_{D}} | J_{1}M_{1} \right\rangle \left\langle D_{M_{D}^{1}M_{D}^{1}}^{(LD)} \right\rangle (\theta, \phi, 0)
\]
In the above formula:

Term 1 is the differential cross section of resonance capture

Term 2 is the differential cross section of direct capture

Term 3 is the differential cross section of interference between resonance capture and direct capture, and

Term 4 is C.C. of Term 3.

Loosely speaking, one may write the following:

\[
\sigma(E, \theta) = \sigma_{R}(E)W_{R}(\theta) + \sigma_{D}(E)W_{D}(\theta) + 2\sqrt{\sigma_{R}(E)\sigma_{D}(E)}\cos(\phi_{R} - \phi_{D})W_{int}(\theta),
\]

\[
\sigma(E, \theta) = \sigma_{R1}(E)W_{R1}(\theta) + \sigma_{R2}(E)W_{R2}(\theta) + \sigma_{DC}(E)W_{DC}(\theta)
+ 2\sqrt{\sigma_{R1}(E)\sigma_{DC}(E, s \rightarrow p)}\cos\delta_{1}(E)W_{1}(\theta)
+ 2\sqrt{\sigma_{R1}(E)\sigma_{DC}(E, d \rightarrow p)}\cos\delta_{2}(E)W_{2}(\theta)
+ 2\sqrt{\sigma_{R1}(E)\sigma_{R2}(E)}\cos\delta_{3}(E)[W_{3}(\theta) + \delta_{R}W_{4}(\theta)]
+ 2\sqrt{\sigma_{R2}(E)\sigma_{DC}(E, s \rightarrow p)}\cos\delta_{4}(E)[W_{5}(\theta) + \delta_{R}W_{6}(\theta)]
+ 2\sqrt{\sigma_{R2}(E)\sigma_{DC}(E, d \rightarrow p)}\cos\delta_{5}(E)[W_{7}(\theta) + \delta_{R}W_{8}(\theta)].
\]

where \(\delta_{1}(E) = \tan^{-1}\frac{2}{\Gamma_{1}(E)}(E-E_{R})\)

\(\delta_{2}(E) = \delta_{1}(E) + \Delta\phi(E) + \Delta\eta(E)\) where \(\Delta\phi(E)\) represents the usual differences in Coulomb phases

\(\Delta\eta(E)\) represents the usual differences in hard sphere phases.

\(\delta_{3}, \delta_{4}, \delta_{5}, \text{ etc.}\)

\(\delta_{R}\) involves the multipole mixing ratio.
In order to be precise, we give the exact formula for unpolarized and polarized interference tensor as the following:

\[
\frac{d}{dQ} (\mathbf{F}) = \frac{1}{4 \pi} \frac{1}{\mathbf{x}} \oint \mathbf{G} \cdot \mathbf{E} \cdot \mathbf{E}' \cdot \mathbf{G}' \cdot \frac{1}{\omega^2} \mathbf{H} \cdot \mathbf{H}' \cdot \frac{1}{\mathbf{y}} \mathbf{H} \cdot \mathbf{H}' \cdot \frac{1}{\omega^2} \mathbf{G} \cdot \mathbf{E} \cdot \mathbf{E}' \cdot \mathbf{G}'
\]

+ Complex Conjugate

\[
\frac{d}{dQ} (\mathbf{F}) = \frac{1}{4 \pi} \frac{1}{\mathbf{x}} \oint \mathbf{G} \cdot \mathbf{E} \cdot \mathbf{E}' \cdot \mathbf{G}' \cdot \frac{1}{\omega^2} \mathbf{H} \cdot \mathbf{H}' \cdot \frac{1}{\mathbf{y}} \mathbf{H} \cdot \mathbf{H}' \cdot \frac{1}{\omega^2} \mathbf{G} \cdot \mathbf{E} \cdot \mathbf{E}' \cdot \mathbf{G}'
\]

+ Complex Conjugate

\[
\left[ \begin{array}{c}
\text{Re} (\mathbf{F}) \\
\text{Im} (\mathbf{F})
\end{array} \right] = \left[ \begin{array}{c}
\text{Re} (\mathbf{F}) \\
\text{Im} (\mathbf{F})
\end{array} \right]
\]

\[
\text{Re} (\mathbf{F}) = \left[ \begin{array}{c}
\text{Re} (\mathbf{F}) \\
\text{Re} (\mathbf{F})
\end{array} \right]
\]

\[
\text{Im} (\mathbf{F}) = \left[ \begin{array}{c}
\text{Im} (\mathbf{F}) \\
\text{Im} (\mathbf{F})
\end{array} \right]
\]

where \(e^i\theta\) a constant phase between RC and DC that was not been introduced.
V. RESULTS AND CONCLUSIONS

In the following discussion, we will attempt to compare the resonance capture calculation with experimental results and show why this mechanism by itself cannot satisfactorily interpret the data. Then we will introduce the direct capture calculation and try to give a general idea of the shape and magnitude of such a term. Finally, we will present the resonance capture and direct capture interference calculation and its adequacy to interpret the data. The parameters used, as taken from the literature, are shown in Table V-1.

The last section of this chapter is devoted to the remaining unsolved problems.

V-A. Resonance Capture Angular Distribution Analysis

The unpolarized resonance capture analysis (Fig. V-1) confirms Young et al.'s findings. It seems to indicate that the incident proton interacts with $^{12}$C to form $^{13}$N$^*$ which in turn undergoes $\gamma$ decay from its second ($3/2^-, T=0$, $E_\gamma = 3.5078$ MeV) and first ($1/2^+, T=0$, $E_\gamma = 2.368$ MeV) excited states through $M1, E2$ and $E1$ transitions, respectively. Furthermore, a 3 level resonance capture calculation placed an upper limit (of $W_{\gamma} < 0.006$ eV) on the partial $\gamma$ decay width on the third excited
<table>
<thead>
<tr>
<th>$E_p$ (MeV)</th>
<th>$E_\gamma$ (MeV)</th>
<th>Spin, Parity, Isospin</th>
<th>$\Gamma_{\text{tot}}$ (keV)</th>
<th>$W_{\gamma}$ (eV)</th>
<th>Proton Spectroscopic Factor $\theta_p^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.459</td>
<td>2.3682</td>
<td>$1/2^+, 0$</td>
<td>3.609 ± 2.8</td>
<td>0.67$^\dagger$</td>
<td>0.54$^{**}$</td>
</tr>
<tr>
<td>1.699</td>
<td>3.5078</td>
<td>$3/2^-, 0$</td>
<td>54.8 ± 11.5</td>
<td>1.06$^\dagger$</td>
<td>0.031$^{**}$</td>
</tr>
<tr>
<td>1.734</td>
<td>3.5491</td>
<td>$5/2^+, 0$</td>
<td>46.5 ± 7.1</td>
<td>&lt;0.006$^*$</td>
<td>0.21$^{**}$</td>
</tr>
</tbody>
</table>

The parameter is mainly taken from Ref. 9.

* Ref. 1

** Ref. 14.

$^\dagger$ Ref. 41.
Fig. V-1  Unpolarized $^{12}\text{C}(p,\gamma)^{13}\text{N}$ resonance capture
Legendre's polynomial prediction.
Fig. V-2

Polarized $^{12}\text{C}(p,\gamma)^{13}\text{N}$ resonance capture

Associate Legendre's polynomial prediction.
B1 IN MICROBARNS

0.00E+00
0.20E-01
0.40E-01

1.6
1.7
1.8 MEV
state (5/2+, T=0, E_γ = 3.549 MeV), also in agreement with Young et al. The total cross section calculations generated from this code were further checked with Seagraves' results and we obtained exact agreement.

The polarized resonance capture calculation, however, turns out to be significantly different from the experimentally observed vector analyzing power values (Fig. III-14). This large discrepancy leads us to believe another important reaction process is presented in the reaction.

V-B. Direct Capture Angular Distribution Analysis

Direct capture of S, D waves by means of an El transition to a ground state of 13N has been studied. This process is calculated from a first order perturbation theory. At this point, only the extranuclear direct capture has been considered. The hard sphere phases were around -8° for S waves and -2° for D waves. The nuclear radius was chosen to be 3.41 fm. The bound state effective potential was -26.77 MeV, while the corresponding values for S, D waves were -47.84 MeV and -32.86 MeV, respectively.

The S_{1/2} (r>a) and D_{3/2} (r>a) wave functions are shown in Fig. V-3 while P_{1/2} (r<a) and P_{3/2} (r>a) are shown in Fig. V-4. We use the Romberg Quadrature formula for the numerical integration with increment steps finer than half a fermi. We integrated from the nuclear radius (3.41 fm) up to around 44 fermi, where the integrand sizes were less than 0.5 x 10^{-5} fm^{1/2}. The total cross section for the reaction contributed by El from the S_{1/2} (L=0) potential wave as well as the D_{3/2} (L=2) potential wave is plotted in the figures. The unpolarized and polarized angular distribution is very small. Fig. V-7 and Fig. V-8 show the polynomial
Fig. V-3  Radial wave function for partial S, D waves ($\ell = 0, 2$, respectively) of ($^{12}$C + p) system at $E_p = 1.8$ MeV. The nucleon phase shift $\delta$ has been assumed to be given by hard sphere phase shifts at $R_0 (3.41$ fm).
Fig. V-4 Normalized radial wave function of 1p single particle state in $^{13}N$ for a square well potential of radius $R_0(3.41$ fm) and well depth $V_1(-26.77$ MeV) both the well depth $V$, and the nucleus radius $R_0$ was adjusted to reproduce the observed bound energies $E_{b^{13}C}$ and $E_{b^{13}N}$. 
RADIAL WAVEFUNCTION (fm)
Fig. V-5 Radial intergrand for direct proton capture from both S and D to p single particle state in $^{13}$N at $E_p = 1.8$ MeV. Assume extranuclear capture process.
$^{12}$C ($p, \gamma$)$^{13}$N

Radial integrand (fm$^2$)

$^{12}$C ($p, \gamma$)$^{13}$N

Radial integrand (fm$^2$)

$^{12}$C ($p, \gamma$)$^{13}$N

Radial integrand (fm$^2$)
Fig. V-6 Theoretical total cross sections is a function of beam energy \(^{12}\text{C}(p,\gamma)^{15}\text{N}\) reaction deduced from the direct capture mechanism.
Fig. V-7 Unpolarized $^{12}$C(p,$\gamma$)$^{13}$N direct capture Legendre's polynomial prediction.
B2 in microbarns

$0.00E+00$ $1.6$ $1.7$ $1.8$ MEV

$-0.20E+03$
Fig. V-8 Polarized $^{12}\text{C}(p,\gamma)^{13}\text{N}$ direct capture Associated Legendre's polynomial prediction.
angular distribution.

We did check our program by applying it to calculate the total cross section ratio of the $S+p$ and $D+p$ processes of the $^3\text{He}(\alpha,\gamma)^7\text{Be}$ reaction, and got exact agreement with the results of Tombrello and Parker's paper.\textsuperscript{32)}

V-C. **Resonance Capture and Direct Capture Interference Angular Distribution Analysis**

A fairly large contribution was predicted by this interference in the unpolarized, as well as polarized, calculations.

Its contribution in the $a_1$ term of the unpolarized angular distribution (Fig. V-9) alone is larger than the experimental result. This suggests that there is a phase difference between the direct capture and the resonance capture mechanism. Furthermore, this also allows one to have an estimation on other nuclear parameters like the spectroscopic factor, partial decay widths, etc.

The interference in the $b_1$ term predicted by the theory is again larger than the measured polarized result (Fig. V-10). This allows one to have an extra check on what one has picked to fit the unpolarized case.

V-D. **Conclusion**

Even though we only assume the El direct capture process without taking into account the higher order transitions, its combination with the two level resonance capture process satisfies most of the experimental data. We should also obtain the relative phase between the direct capture and the resonance capture mechanism, as well as the spectroscopic
Fig. V-9

Unpolarized $^{12}\text{C}(p,\gamma)^{13}\text{N}$ direct and resonance interference Legendre's polynomial prediction.
AO IN MICROBARNs

0.10E 00

0.00E 00

1.6  1.7  1.8 MEV

A1 IN MICROBARNs

0.40E 00

0.30E 00

0.20E 00

0.10E 00

0.00E 00

1.6  1.7  1.8 MEV

A2 IN MICROBARNs

0.00E 00

1.6  1.7  1.8 MEV

A3 IN MICROBARNs
Fig. V-10  Polarized $^{12}\text{C}(p,\gamma)^{13}\text{N}$ direct and resonance Associated Legendre's polynomial prediction.
Fig. V-11  Unpolarized $^{12}\text{C}(p,\gamma)$ Legendre's polynomial prediction.
Fig. V-12  Polarized $^{12}\text{C}(p,\gamma)^{13}\text{N}$ Associated Legendre's polynomial prediction.
B1 IN MICROBARNs

B2 IN MICROBARNs

B3 IN MICROBARNs
factor for the ground state of $^{13}\text{N}(0^+_2 \approx 0.49)$.

We have not tested the single particle resonance phase shift approach even though we obtained the resonance phase shift by $^{12}\text{C}(p,p)^{12}\text{C}$ proton elastic scattering data.

The data suggest that both direct capture and resonance capture mechanisms are required. Further work continues to find a consistent set of parameters to describe this reaction fully.
APPENDIX A

PART 1 The unpolarized $^{12}\text{C}(p,\gamma)^{13}\text{N}$ angular distribution

(Excluding the $j$-dependent nuclear phases)
Let us first consider $\frac{\partial^2 \Sigma}{\partial \phi^2}$.

$$\frac{\partial^2 \Sigma}{\partial \phi^2} = \frac{K_{\phi}}{\delta_{\phi} V_{\phi}} \left[ \sum \frac{1}{\pi \phi J_{m}} \left\langle f_{M_{i}} H_{m,1}^{P} \right\rangle \right]_{\pi \phi \phi}$$

While $H_{m,1}^{P}(x_{i}) = -\frac{z}{\sum_{j}^{*} (\phi \phi)} \sum_{j}^{*} D_{m,1}^{P}(x_{i}, y_{j}, 0, \theta_{i}) \left( \phi_{i}, Y_{i}, \theta_{i}, \phi_{j} \right)$

$$\phi_{j} = \left\{ \frac{3}{2\rho} \left[ \rho \phi_{j} - (\phi - \rho) \phi_{j} \right] \right\}_{\rho} \text{ exact}$$

$$\left\langle f_{M_{i}} H_{m,1}^{P} \right\rangle_{\pi \phi \phi} = \sum_{j}^{*} \phi_{j} \frac{U_{M_{i}}(k_{j}, r)}{r} \sum_{j}^{*} \left\langle f_{M_{i}} Y_{i}, s_{i}, \theta_{i}, \phi_{j} \right\rangle_{\rho} \chi_{M_{i}}^{\phi}$$

$$\left\langle f_{M_{i}} H_{m,1}^{P} \right\rangle_{\pi \phi \phi} = \left[ -i \left( \frac{z}{\sum_{j}^{*}} \right)^{k_{j}} \phi_{j} \sum_{j}^{*} \sum_{j}^{*} \left\langle f_{M_{i}} Y_{i}, s_{i}, \theta_{i}, \phi_{j} \right\rangle_{\rho} \chi_{M_{i}}^{\phi} \right] \phi_{j}$$

$$\left\langle f_{M_{i}} H_{m,1}^{P} \right\rangle_{\pi \phi \phi} = \left[ \int \frac{U_{M_{i}}^{*}(k_{j}, r)}{r} \frac{U_{M_{i}}(k_{j}, r)}{r} \right]_{\rho} \phi_{j}$$

Knowing $Y_{i}^{M_{i}} Y_{i}^{M_{i}} = (-)^{M_{i}} Y_{i}^{M_{i}} Y_{i}^{M_{i}} = (-)^{M_{i}} \frac{2^{1/2}}{\pi} \left\langle f_{01} \theta_{1} \right\rangle \left\langle f_{01} \theta_{1} \right\rangle \chi_{M_{i}}^{\phi}$
\[ = \sum_{j} \frac{2 \epsilon_{j}^{2}}{3\gamma_{ij}} \left( \delta_{\alpha} - m_{\alpha} \right) \left( \delta_{\epsilon} - m_{\epsilon} \right) \left( y_{j}^{-M} \right) \]

and

\[ = \sum_{j} \frac{2 \epsilon_{j}^{2}}{3\gamma_{ij}} \left( \delta_{\alpha} - m_{\alpha} \right) \left( \delta_{\epsilon} - m_{\epsilon} \right) \left( y_{j}^{-M} \right) \]

\[ \langle \Delta \phi_{1}^{\alpha} \Delta \phi_{1}^{\alpha} \Delta \phi_{1}^{\beta} \rangle = \int \left[ -i \phi_{1}^{\alpha} \phi_{1}^{\beta} \right] \left[ i \phi_{1}^{\gamma} \phi_{1}^{\delta} \right] \delta_{\gamma_{ij}} \delta_{\epsilon_{ij}} \delta_{\alpha_{ij}} \delta_{\beta_{ij}} \langle \Delta \phi_{1}^{\epsilon} \Delta \phi_{1}^{\gamma} \Delta \phi_{1}^{\delta} \rangle \]

\[ \sum_{j} \left( \epsilon_{j}^{2} \right) \phi_{1}^{\alpha} \phi_{1}^{\beta} \left( \delta_{\epsilon} - m_{\epsilon} \right) \left( \delta_{\alpha} - m_{\alpha} \right) \left( y_{j}^{-M} \right) \]

\[ \langle \Delta \phi_{1}^{\alpha} \Delta \phi_{1}^{\alpha} \Delta \phi_{1}^{\beta} \rangle = \int \left[ -i \phi_{1}^{\alpha} \phi_{1}^{\beta} \right] \left[ i \phi_{1}^{\gamma} \phi_{1}^{\delta} \right] \delta_{\gamma_{ij}} \delta_{\epsilon_{ij}} \delta_{\alpha_{ij}} \delta_{\beta_{ij}} \langle \Delta \phi_{1}^{\epsilon} \Delta \phi_{1}^{\gamma} \Delta \phi_{1}^{\delta} \rangle \]

\[ \sum_{j} \left( \epsilon_{j}^{2} \right) \phi_{1}^{\alpha} \phi_{1}^{\beta} \left( \delta_{\epsilon} - m_{\epsilon} \right) \left( \delta_{\alpha} - m_{\alpha} \right) \left( y_{j}^{-M} \right) \]

The formula would like the following

\[ \phi_{1} \text{ is not detecting the photon, } \phi_{2} \text{ is not detecting } \phi_{1} \text{, } (\phi \cdot \phi') \]
\[
\sum_{\alpha=1}^{N} \sum_{\beta=1}^{M} \frac{1}{N+M} \left[ \left( \frac{\alpha^* e^{i\theta} + \beta e^{-i\theta}}{N+M} \right) \right]^2 \left[ \left( \frac{\alpha^* e^{-i\theta} + \beta^* e^{i\theta}}{N+M} \right) \right]^2
\]

\[
\sum_{\alpha=1}^{N} \sum_{\beta=1}^{M} \left( \frac{\alpha^* e^{i\theta} + \beta e^{-i\theta}}{N+M} \right) \left( \frac{\alpha e^{-i\theta} + \beta^* e^{i\theta}}{N+M} \right)
\]

\[
\sum_{\alpha=1}^{N} \sum_{\beta=1}^{M} \frac{1}{N+M} \left[ \left( \frac{\alpha e^{i\theta} + \beta^* e^{-i\theta}}{N+M} \right) \right]^2 \left[ \left( \frac{\alpha e^{-i\theta} + \beta^* e^{i\theta}}{N+M} \right) \right]^2
\]

\[
\sum_{\alpha=1}^{N} \sum_{\beta=1}^{M} \left( \frac{\alpha e^{i\theta} + \beta^* e^{-i\theta}}{N+M} \right) \left( \frac{\alpha e^{-i\theta} + \beta^* e^{i\theta}}{N+M} \right)
\]

\[
\sum_{\alpha=1}^{N} \sum_{\beta=1}^{M} \frac{1}{N+M} \left[ \left( \frac{\alpha e^{i\theta} + \beta^* e^{-i\theta}}{N+M} \right) \right]^2 \left[ \left( \frac{\alpha e^{-i\theta} + \beta^* e^{i\theta}}{N+M} \right) \right]^2
\]

\[
\sum_{\alpha=1}^{N} \sum_{\beta=1}^{M} \left( \frac{\alpha e^{i\theta} + \beta^* e^{-i\theta}}{N+M} \right) \left( \frac{\alpha e^{-i\theta} + \beta^* e^{i\theta}}{N+M} \right)
\]
\[ \sum_{k} \langle \psi_{i} \psi_{j} | \mathbf{P}^{(k)} | \psi_{k} \psi_{l} \rangle \mathbf{P}_{k}^{(k)} \mathbf{P}_{l}^{(k)} \mathbf{P}_{m}^{(k)} \mathbf{P}_{n}^{(k)} \mathbf{P}^{(k)} \rangle \langle \psi_{i} \psi_{j} \rangle \]
\[ W = \frac{1}{\hbar} \frac{1}{\sqrt{\hbar}} \sum_{\text{d.o.}} \sum_{\text{d.o.'s}} (-)^{S+1} e^{i \phi} \phi^2 \sum_{\text{d.o.}} \sum_{\text{d.o.'s}} \left( \sum_{\text{d.o.}} \sum_{\text{d.o.'s}} \phi^2 \right) \]

\[ \left( \phi \right) = \left( \phi \right) \sum_{\text{d.o.}} \sum_{\text{d.o.'s}} \left( \sum_{\text{d.o.}} \sum_{\text{d.o.'s}} \phi^2 \right) \]

\[ \frac{d^2}{d \chi^2} = (2\pi \hbar)^2 \Delta \]

Where \( \Delta = \frac{\hbar^2}{\sqrt{\hbar} E_{\text{c.m.}}} \)

\[ \chi = \frac{4.287 \times 10^{-10} \text{cm}}{[E_{\text{c.m.}}]} \]

"Note"

\[ H = \sum_{m_{d.o.}} \sum_{m_{d.o.'s}} \left( \sum_{m_{d.o.}} \sum_{m_{d.o.'s}} \phi^2 \right) \]

\[ \sum_{\text{d.o.}} \sum_{\text{d.o.'s}} \left( \sum_{\text{d.o.}} \sum_{\text{d.o.'s}} \phi^2 \right) \]

Where \( \sum_{m_{d.o.}} \sum_{m_{d.o.'s}} \phi^2 \) = \( \sum_{m_{d.o.}} \sum_{m_{d.o.'s}} \phi^2 \)

\[ = \sum_{m_{d.o.}} \sum_{m_{d.o.'s}} \left( \sum_{m_{d.o.}} \sum_{m_{d.o.'s}} \phi^2 \right) \]

\[ \left( \chi, \phi \right) = \left( \chi, \phi \right) \sum_{m_{d.o.}} \sum_{m_{d.o.'s}} \left( \sum_{m_{d.o.}} \sum_{m_{d.o.'s}} \phi^2 \right) \]

\[ \left( \chi, \phi \right) = \left( \chi, \phi \right) \sum_{m_{d.o.}} \sum_{m_{d.o.'s}} \left( \sum_{m_{d.o.}} \sum_{m_{d.o.'s}} \phi^2 \right) \]
1-C. Difference between direct capture and remote capture

\[
\frac{d}{dt} \left( \theta \right) = \sum_{m, m^{'}} \sum_{n, n^{'}} \left[ \begin{array}{c} \left( \mathbf{M}_1 \mathbf{M}_2 \right) \mathbf{M}_3 \mathbf{M}_4 \\
\mathbf{M}_5 \mathbf{M}_6 \mathbf{M}_7 \mathbf{M}_8 \end{array} \right] + \frac{K_s}{x} \left( \mathbf{M}_9 \mathbf{M}_10 \mathbf{M}_11 \right)
\]

\[
\frac{d\theta}{dt} = \text{Interaction} + \text{C.C.}
\]

\[
\text{Interaction} = \sum_{m, m^{'}} \sum_{n, n^{'}} \left[ \frac{\Delta \theta_{m n}}{x} \left[ \frac{K_s}{x} \right] \right] \left( \mathbf{M}_1 \mathbf{M}_2 \mathbf{M}_3 \mathbf{M}_4 \right)
\]

\[
\left( \mathbf{M}_5 \mathbf{M}_6 \mathbf{M}_7 \mathbf{M}_8 \right) \mathbf{M}_9 \mathbf{M}_{10} \mathbf{M}_{11}
\]

\[
\left( \mathbf{M}_1 \mathbf{M}_2 \mathbf{M}_3 \mathbf{M}_4 \right) \mathbf{M}_5 \mathbf{M}_6 \mathbf{M}_7 \mathbf{M}_8
\]

\[
\left( \mathbf{M}_1 \mathbf{M}_2 \mathbf{M}_3 \mathbf{M}_4 \right) \mathbf{M}_5 \mathbf{M}_6 \mathbf{M}_7 \mathbf{M}_8 \]
We would like to point out that \( \sum_{\text{fin} \leq \text{colo}} \) in the proxy omitted.

Knowing: \( (-)^{\mu_0} D_{\mu_0} \rho_{\lambda_0} = (-)^{\mu_0} \rho_{\mu_0} \sum_{\mu_0} \frac{\lambda_0}{\rho_{\mu_0}} \lambda_0 (\lambda_0 - 16 \mu_0)(\lambda_0 - 16 \mu_0)(\lambda_0 - 16 \mu_0)(\lambda_0 - 16 \mu_0) \partial_{\lambda_0} \rho_{\lambda_0} \)

\[ \sum_{\mu_0} \left( \frac{\lambda_0}{\rho_{\mu_0}} \lambda_0 (\lambda_0 - 16 \mu_0)(\lambda_0 - 16 \mu_0)(\lambda_0 - 16 \mu_0)(\lambda_0 - 16 \mu_0) \partial_{\lambda_0} \rho_{\lambda_0} \right) = \frac{\lambda_0}{\rho_{\mu_0}} \sum_{\mu_0} \lambda_0 (\lambda_0 - 16 \mu_0)(\lambda_0 - 16 \mu_0)(\lambda_0 - 16 \mu_0)(\lambda_0 - 16 \mu_0) \partial_{\lambda_0} \rho_{\lambda_0} \]

Knowing: \( \sum_{\mu_0} \sum_{\lambda_0} \frac{\lambda_0}{\rho_{\mu_0}} \lambda_0 (\lambda_0 - 16 \mu_0)(\lambda_0 - 16 \mu_0)(\lambda_0 - 16 \mu_0)(\lambda_0 - 16 \mu_0) \partial_{\lambda_0} \rho_{\lambda_0} \)
\[
\sum_{n=0}^{\infty} \left( \text{L}^n \text{V}_{n}(x) \text{H}_{n}(y) \text{H}_{n}(z) \right) \int \frac{e^{-K_z y}}{y} \frac{e^{-K_y z}}{z} \text{d}y \text{d}z dx
\]

\[
= \frac{2}{\pi} \frac{e^{-K_z y}}{y} \frac{e^{-K_y z}}{z} \int \text{d}y \text{d}z dx
\]

\[
\sum_{n=0}^{\infty} \left( \text{L}^n \text{V}_{n}(x) \text{H}_{n}(y) \text{H}_{n}(z) \right) \int \frac{e^{-K_z y}}{y} \frac{e^{-K_y z}}{z} \text{d}y \text{d}z dx
\]

\[
= \frac{2}{\pi} \frac{e^{-K_z y}}{y} \frac{e^{-K_y z}}{z} \int \text{d}y \text{d}z dx
\]
Part 2  The unpolarized $^{12}$C(p,γ)$^{13}$N angular distribution/

(Including j-dependent nuclear phases)
2. \[ A \]

\[
|f_{\alpha'\beta'}(r)\rangle = \sum_{J} e^{iJ\xi(r)} \sum_{s,s'^{\prime}} Y_{\alpha'}^{*}(r) X_{s'} \sum_{v} \int_{0}^{2\pi} \int_{0}^{\pi} e^{i\alpha'_{v} r} Y_{\beta'}^{*}(r) Y_{s} \sin \frac{\theta}{2} d\theta d\phi.
\]

\[
|a_{\alpha'\beta'}\rangle = \sum_{J} e^{iJ\xi(r)} \sum_{s,s'^{\prime}} Y_{\alpha'}^{*}(r) X_{s'} \sum_{v} \int_{0}^{2\pi} \int_{0}^{\pi} e^{i\alpha'_{v} r} Y_{\beta'}^{*}(r) Y_{s} \sin \frac{\theta}{2} d\theta d\phi
\]

\[
\langle f_{\alpha'\beta'}(r) | a_{\alpha'\beta'} \rangle = \left[ e^{i\xi(r)} \sum_{s,s'^{\prime}} e^{iJ\xi(r)} \sum_{s^{\prime}} Y_{\alpha'}^{*}(r) X_{s'} \sum_{v} \int_{0}^{2\pi} \int_{0}^{\pi} e^{i\alpha'_{v} r} Y_{\beta'}^{*}(r) Y_{s} \sin \frac{\theta}{2} d\theta d\phi \right] e^{i\xi(r)} \sum_{s,s'^{\prime}} Y_{\alpha'}^{*}(r) X_{s'} \sum_{v} \int_{0}^{2\pi} \int_{0}^{\pi} e^{i\alpha'_{v} r} Y_{\beta'}^{*}(r) Y_{s} \sin \frac{\theta}{2} d\theta d\phi.
\]

\[
\sum_{J} e^{iJ\xi(r)} \sum_{s,s'^{\prime}} Y_{\alpha'}^{*}(r) X_{s'} \sum_{v} \int_{0}^{2\pi} \int_{0}^{\pi} e^{i\alpha'_{v} r} Y_{\beta'}^{*}(r) Y_{s} \sin \frac{\theta}{2} d\theta d\phi.
\]

\[
\sum_{J} e^{iJ\xi(r)} \sum_{s,s'^{\prime}} Y_{\alpha'}^{*}(r) X_{s'} \sum_{v} \int_{0}^{2\pi} \int_{0}^{\pi} e^{i\alpha'_{v} r} Y_{\beta'}^{*}(r) Y_{s} \sin \frac{\theta}{2} d\theta d\phi.
\]

\[
\sum_{J} e^{iJ\xi(r)} \sum_{s,s'^{\prime}} Y_{\alpha'}^{*}(r) X_{s'} \sum_{v} \int_{0}^{2\pi} \int_{0}^{\pi} e^{i\alpha'_{v} r} Y_{\beta'}^{*}(r) Y_{s} \sin \frac{\theta}{2} d\theta d\phi.
\]
\[ \sum_{\mathbf{q}} \langle J_{\mathbf{q}, \alpha} (\mathbf{p}, \mathbf{p}) | \mathcal{D}_{\mu} \langle \mathbf{p} | \mathbf{p} \rangle \rangle \]

\[ = \sum_{\mathbf{q}} \frac{e^{i \mathbf{q} \cdot \mathbf{L}_d}}{\mathbf{q} !} \left( \frac{2\pi}{\mathbf{L}_d} \right)^{\mathbf{q} !} \left[ i e^{i \mathbf{q} \cdot \mathbf{L}_d} \cdot \frac{\mathbf{L}_d}{\mathbf{q} !} \right] e^{i \mathbf{q} \cdot \mathbf{L}_d} \cdot \frac{\mathbf{L}_d}{\mathbf{q} !} \]
\[ \sum_{\mathbf{m}, \mathbf{m}'=\pm 1} \left< \mathbf{m} | \mathbf{J} \mathbf{m}' \right> = \sum_{\mathbf{m}, \mathbf{m}'=\pm 1} \frac{1}{2} \left< \mathbf{m} | \mathbf{J} \mathbf{m}' \right> \left< \mathbf{m}' | \mathbf{J} \mathbf{m} \right> \]

For polarized case 
\[ \left< \mathbf{m} | \mathbf{J} \mathbf{m}' \right> = \frac{1}{\sqrt{2}} \delta_{\mathbf{m} \mathbf{m}'} \]

\[ \sum_{\mathbf{m}, \mathbf{m}'=\pm 1} \left< \mathbf{m} | \mathbf{J} \mathbf{m}' \right> \left< \mathbf{m}' | \mathbf{J} \mathbf{m} \right> \]

\[ \left< \mathbf{m} | \mathbf{J} \mathbf{m}' \right> \left< \mathbf{m}' | \mathbf{J} \mathbf{m} \right> \]
\[\sum_{\mu} \left( \cos \theta_{\nu \mu} X_{\nu \mu} \right) = \frac{1}{4 \pi} \left[ \left( \frac{2}{3} \right) \text{Re} \left( \text{e}^{i \phi \theta_{\nu \mu}} \right) \right] \text{e}^{i \phi \theta_{\nu \mu}} \]

\[\sum_{\mu} \left( \cos \theta_{\nu \mu} X_{\nu \mu} \right) = \frac{1}{4 \pi} \left[ \left( \frac{2}{3} \right) \text{Re} \left( \text{e}^{i \phi \theta_{\nu \mu}} \right) \right] \text{e}^{i \phi \theta_{\nu \mu}} \]

\[\sum_{\mu} \left( \cos \theta_{\nu \mu} X_{\nu \mu} \right) = \frac{1}{4 \pi} \left[ \left( \frac{2}{3} \right) \text{Re} \left( \text{e}^{i \phi \theta_{\nu \mu}} \right) \right] \text{e}^{i \phi \theta_{\nu \mu}} \]
\[ O_{\text{exp}} \ (p. \text{dependent}) \]

\[ M_{\text{con}} = \sum_{m_1 \in M_f} \sum_{m_2 \in M_o} \frac{\Xi_{m_1, m_2} X \left( \frac{1}{2} \Xi_{m_1, m_2} X \right)}{\Xi_{m_1, m_2} X} \langle \sum_{m_1} T_{m_1} \mid L_{\text{exp}} \rangle \langle \sum_{m_2} T_{m_2} \mid L_{\text{exp}} \rangle \left( \frac{\Xi_{m_1, m_2}}{\Xi_{m_1, m_2} X} \right)^k \]

\[ \times \left( \frac{\Xi_{m_1, m_2} X \left( \frac{1}{2} \Xi_{m_1, m_2} X \right)}{\Xi_{m_1, m_2} X} \right) \]

\[ = \sum_{m_1 \in M_f} \left[ \frac{1}{\Xi_{m_1, m_2} X} \sum_{m_2 \in M_o} \frac{\Xi_{m_1, m_2} X \left( \frac{1}{2} \Xi_{m_1, m_2} X \right)}{\Xi_{m_1, m_2} X} \sum_{f} \right. \]

\[ \left. \langle \sum_{m_1} T_{m_1} \mid L_{\text{exp}} \rangle \langle \sum_{m_2} T_{m_2} \mid L_{\text{exp}} \rangle \left( \frac{\Xi_{m_1, m_2}}{\Xi_{m_1, m_2} X} \right)^k \right] \]

\[ \times \left( \frac{\Xi_{m_1, m_2} X \left( \frac{1}{2} \Xi_{m_1, m_2} X \right)}{\Xi_{m_1, m_2} X} \right) \]

\[ + \left[ \frac{\Xi_{m_1, m_2} X \left( \frac{1}{2} \Xi_{m_1, m_2} X \right)}{\Xi_{m_1, m_2} X} \right] \sum_{f} \left( \frac{1}{2} \Xi_{m_1, m_2} X \right)^k \]

\[ \times \left( \frac{\Xi_{m_1, m_2} X \left( \frac{1}{2} \Xi_{m_1, m_2} X \right)}{\Xi_{m_1, m_2} X} \right) \]

\[ \times \left( \frac{\Xi_{m_1, m_2} X \left( \frac{1}{2} \Xi_{m_1, m_2} X \right)}{\Xi_{m_1, m_2} X} \right) \]

\[ + \left( \frac{\Xi_{m_1, m_2} X \left( \frac{1}{2} \Xi_{m_1, m_2} X \right)}{\Xi_{m_1, m_2} X} \right) \sum_{f} \left( \frac{1}{2} \Xi_{m_1, m_2} X \right)^k \]

\[ \times \left( \frac{\Xi_{m_1, m_2} X \left( \frac{1}{2} \Xi_{m_1, m_2} X \right)}{\Xi_{m_1, m_2} X} \right) \]

\[ + \left( \frac{\Xi_{m_1, m_2} X \left( \frac{1}{2} \Xi_{m_1, m_2} X \right)}{\Xi_{m_1, m_2} X} \right) \sum_{f} \left( \frac{1}{2} \Xi_{m_1, m_2} X \right)^k \]

\[ \times \left( \frac{\Xi_{m_1, m_2} X \left( \frac{1}{2} \Xi_{m_1, m_2} X \right)}{\Xi_{m_1, m_2} X} \right) \]

\[ + \left( \frac{\Xi_{m_1, m_2} X \left( \frac{1}{2} \Xi_{m_1, m_2} X \right)}{\Xi_{m_1, m_2} X} \right) \sum_{f} \left( \frac{1}{2} \Xi_{m_1, m_2} X \right)^k \]
1. \[ \sum_{m} S \chi_{m}^{2} \gamma_{m}^{2} \left( \chi_{m} \gamma_{m} \right) \left( \chi_{m} \gamma_{m} \right) \left( \chi_{m} \gamma_{m} \right) \]

\[ = \left( \sum_{m} \chi_{m}^{2} \right) \left( \sum_{m} \gamma_{m}^{2} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \]

\[ \neq \left( \sum_{m} \chi_{m} \right) \left( \sum_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \]

\[ \neq \left( \sum_{m} \chi_{m} \right) \left( \sum_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \]

\[ \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \]

2. \[ \sum_{m} \left( \sum_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \]

\[ \neq \left( \sum_{m} \chi_{m} \right) \left( \sum_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \]

\[ \left( \sum_{m} \chi_{m} \right) \left( \sum_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \]

3. \[ \sum_{m} \left( \sum_{m} \chi_{m} \right) \left( \sum_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \]

\[ \neq \left( \sum_{m} \chi_{m} \right) \left( \sum_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \]

\[ \left( \sum_{m} \chi_{m} \right) \left( \sum_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \]

4. \[ \sum_{m} \left( \sum_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \]

\[ \neq \left( \sum_{m} \chi_{m} \right) \left( \sum_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \]

\[ \left( \sum_{m} \chi_{m} \right) \left( \sum_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \left( \sum_{m} \chi_{m} \gamma_{m} \right) \]
\[ 1 + 2 + 3 + 4 = \left( \frac{3^2 + 2^2 + 3^2 + 6 \times 3 + 1}{2} \right) - \frac{3^2}{2} - \frac{2^2}{2} - \frac{3^2}{2} - \frac{6 \times 3}{2} = 10 \]

\[ \Psi(\omega) = \frac{1}{2\pi^2} \left( \mathcal{A}^2 \mathcal{B}^2 \mathcal{C}^2 \mathcal{D} \right) \int \int \int \frac{k_0}{2\pi^2} \left[ \frac{k_0}{\omega^2} \right] \frac{1}{\left( \frac{\omega_0}{\omega} \right)^2} \left( \frac{1}{\omega_0^2} \right) e^{i k_0 \cdot \mathbf{r}} \frac{\omega}{\omega_0} \]

\[ R(\mathbf{r}) = \frac{1}{2\pi} \int \int \int \frac{k_0}{2\pi^2} \left[ \frac{k_0}{\omega^2} \right] \frac{1}{\left( \frac{\omega_0}{\omega} \right)^2} \left( \frac{1}{\omega_0^2} \right) e^{i k_0 \cdot \mathbf{r}} \frac{\omega}{\omega_0} \]

\[ \mathcal{W}(x_0, y_0, z_0, f_0) \mathcal{W}(x_0, y_0, z_0, f_0) \mathcal{W}(x_0, y_0, z_0, f_0) \]

\[ \sum_{\mathbf{r}} \left( \frac{\omega_0}{\omega} \right) \cdot \mathcal{W}(x_0, y_0, z_0, f_0) \left( \mathcal{W}(x_0, y_0, z_0, f_0) \right)^{-1} \left( \frac{1}{\omega_0^2} \right)^{-1} \]
Part 3 Polarized $^{12}\text{C}(p,\gamma)^{13}\text{N}$ Angular distribution

(Consider j-dependent nuclear phases)
polarized direct capture angular distribution (p dependent nucleon phase \( \theta_{\text{p}} \))

\[
\frac{d\sigma}{d\Omega} = \frac{K_p}{2\pi} \sum_{\text{final}} \left[ \langle \psi_{\text{f}} \mid \hat{H}_{\text{int}} \mid \psi_{\text{i}} \rangle \langle \psi_{\text{f}} \mid \hat{H}_{\text{int}} \mid \psi_{\text{i}} \rangle^* \right] \langle \text{m} \rangle \langle \text{m} \rangle^*
\]

where \( H_{\text{int}}^{\text{p}} \) \((\hat{\mathbf{e}}_\text{p}) = -i(\frac{\hbar}{2}) K_p \gamma_p \left( \frac{\hbar}{2m_c} \right) \sum_m \Delta \mu^m \left( \frac{\hbar}{2m_c} \right) Y^m_l \chi^m_l \)

\[
| \psi_{\text{f}} \rangle = \sum_j | \text{f}, j \rangle e^{i \phi_{\text{f}, j}} \frac{1}{r_j} \sum_m \langle \text{f}, m_j \rangle Y^m_l \eta^m_l
\]

\[
| \psi_{\text{i}} \rangle = \sum_j | \text{i}, j \rangle e^{i \phi_{\text{i}, j}} \frac{1}{r_j} \sum_m \langle \text{i}, m_j \rangle Y^m_l \eta^m_l
\]

\[
\langle \text{f}, j \mid \hat{H}_{\text{int}} \mid \text{i}, j \rangle = \sum_{m_{\text{f}}, m_{\text{i}}} e^{i \phi_{\text{f}, j}} \frac{1}{r_{\text{f}}} \sum_{m_{\text{i}}} \langle \text{f}, m_{\text{f}} \rangle \langle \text{i}, m_{\text{i}} \rangle Y^m_{\text{f}, l} \chi^m_{\text{i}, l}
\]

\[
\sum_{m_{\text{f}}, m_{\text{i}}} e^{i \phi_{\text{f}, j}} \frac{1}{r_{\text{f}}} \sum_{m_{\text{i}}} \langle \text{f}, m_{\text{f}} \rangle \langle \text{i}, m_{\text{i}} \rangle Y^m_{\text{f}, l} \chi^m_{\text{i}, l} \times \Delta \mu^m \left( \frac{\hbar}{2m_c} \right) \Delta \mu^m \left( \frac{\hbar}{2m_c} \right)
\]

\[
\Delta \mu^m \left( \frac{\hbar}{2m_c} \right) \Delta \mu^m \left( \frac{\hbar}{2m_c} \right) \Delta \mu^m \left( \frac{\hbar}{2m_c} \right) \Delta \mu^m \left( \frac{\hbar}{2m_c} \right)
\]
\[
\sum_{\mu, \nu, \omega} \left( K_{\mu \lambda} K_{\lambda \nu} K_{\nu \omega} + \sum_{\mu', \nu', \omega'} \left[ \left( \frac{\lambda}{2} \right) \rho_{\mu \mu'} \left( \frac{\lambda}{2} \right) \rho_{\nu \nu'} \left( \frac{\lambda}{2} \right) \rho_{\omega \omega'} \right] \right) \}
\]
\[
\sum_{k=1}^{\infty} \sum_{n=1}^{\infty} \int_{\mathbb{R}^d} f(x) \phi_n(x) \, dx = \sum_{k=1}^{\infty} \sum_{n=1}^{\infty} \left[ \frac{1}{k^2} \int_{\mathbb{R}^d} g(x) \phi_n(x) \, dx \right] \cdot \int_{\mathbb{R}^d} h(x) \phi_n(x) \, dx
\]
\[
\sum_{\text{min}_0} \langle J_m \mid H \mid J_m \rangle_{\text{min}_0} = \sum_{p=0}^{\infty} \left( \frac{1}{k_i^2} \left[ \frac{1}{e^\hbar - 1} \right] \right)^2 \frac{1}{\eta_0} \rho_{\Delta a} \rho_{\Delta b} \rho_{\Delta c} \rho_{\Delta d} \rho_{\Delta e} \rho_{\Delta f} \rho_{\Delta g} \rho_{\Delta h}
\]
\[ X(abc,def,ghi) = \sum_k k^2 W(\text{aikf} \text{h} \text{k}) W(\text{aih} \text{d} \text{kg}) W(\text{d} \text{fh} \text{ke}) \]

\[ \sum_f (-5)^f \cdot W(h, s, f, p_i) W(h, s, f, p_j) W(h, s, f, p_k) \]

\[ \Rightarrow \begin{pmatrix} (h', s, p_i') \\ (j, k, f) \end{pmatrix} \begin{pmatrix} (h', s, f) \\ (j, k, f) \end{pmatrix} \begin{pmatrix} (s, p_i f) \\ (j, k, f) \end{pmatrix} \]

\[ \Rightarrow \begin{pmatrix} h', k, f \\ j, s, p_i' \end{pmatrix} \begin{pmatrix} h', k, f \\ j, s, f \end{pmatrix} \begin{pmatrix} s, f \\ j, k, s \end{pmatrix} \]

\[ = (-5)^f \sum_f W(h, s, f, p_i) W(h, s, f, p_j) W(s, j, k, f, s) \]

\[ = (-5)^f X(h, s, f', i, s, j, f, k, k) \]

\[ = (-5)^f \begin{pmatrix} h', s, p_i' \\ j, s, f \end{pmatrix} \begin{pmatrix} h, s, f \\ j, k, k \end{pmatrix} \]
\( \Omega_{pd}(cm) \)

\[
W = \sum_{m'_1, m'_2, m'_e} \sum_{p=1} \langle cm | T_{L_1} | m'_1, m'_2, m'_e \rangle \langle b m'_1 | T_{R_0} | s m'_e \rangle D_{p}^{L_1} (\Omega_{pd})
\]

\[
+ \sum_{p=1}^{P_{\Omega}} \langle cm | T_{L_1} | m'_1, m'_2, m'_e \rangle \langle b m'_1 | T_{R_0} | s m'_e \rangle D_{p}^{L_1} (\Omega_{pd})
\]

\[ \times \frac{x_2 z_2}{y^2} \frac{\hat{b}'_2 z'_2}{y'^2} \]

\[
\langle cm | T_{L_1} | m'_1, m'_2 \rangle = \langle cm | \omega_{cm} | b m'_1 \rangle \langle c l, u b \rangle
\]

\[
\langle b m'_1 | T_{R_0} | s m'_e \rangle = \langle b m'_1 | \omega_{b m'_1} | s m'_e \rangle \langle s l, u b \rangle
\]
\[ L = \sum \lim_{\mu^s \mu^c \mu^l \mu^c} \sum_{k_1 k_2} \frac{1}{2 \pi} \frac{i}{2 \pi} \frac{\gamma_{\mu^c}}{2 \pi} \sum_{k_1 k_2} (-)^{s-m^c} \left< s \mu^c s-m^c i k_1 k_2 \right> f_{\mu^c} k (s) \left< s \mu^c s-m^c l k_1 k_2 \right> \]

\[ \left< c_{\mu^c} l_{\mu^c} \left| c_{\mu^c} l_{\mu^c} \right> \left< c_{\mu^c} l_{\mu^c} \right| \right> \Delta_{\mu^c}^{s} (s) \]

\[ \left< c_{\mu^c} l_{\mu^c} \right> \left< c_{\mu^c} l_{\mu^c} \right> \left< s \mu^l s \mu^l \right> \left< s \mu^l s \mu^l \right> \]

\[ \sum_{\mu^c \mu^l} \frac{1}{2 \pi} \frac{i}{2 \pi} \frac{\gamma_{\mu^c}}{2 \pi} \sum_{k_1 k_2} (-)^{s-m^c} \left< s \mu^c s-m^c i k_1 k_2 \right> f_{\mu^c} k (s) \left< s \mu^c s-m^c l k_1 k_2 \right> \]

\[ \left< c_{\mu^c} l_{\mu^c} \right> \left< c_{\mu^c} l_{\mu^c} \right> \left< s \mu^l s \mu^l \right> \left< s \mu^l s \mu^l \right> \]

\[ (-)^{s-m^c} \left< \mu^c \mu^c \mu^l \mu^c i k_1 k_2 \right> \left< \mu^c \mu^c \mu^l \mu^c \right> \left< \mu^c \mu^c \mu^c \mu^l \right> \left< \mu^c \mu^c \mu^c \mu^l \right> \]

\[ \frac{1}{2 \pi} \frac{i}{2 \pi} \frac{\gamma_{\mu^c}}{2 \pi} \sum_{k_1 k_2} (-)^{s-m^c} \left< s \mu^c s-m^c i k_1 k_2 \right> f_{\mu^c} k (s) \left< s \mu^c s-m^c l k_1 k_2 \right> \]

\[ \left< c_{\mu^c} l_{\mu^c} \right> \left< c_{\mu^c} l_{\mu^c} \right> \Delta_{\mu^c}^{s} (s) \]

\[ \left< c_{\mu^c} l_{\mu^c} \right> \left< c_{\mu^c} l_{\mu^c} \right> \left< s \mu^l s \mu^l \right> \left< s \mu^l s \mu^l \right> \]

\[ \left< c_{\mu^c} l_{\mu^c} \right> \left< c_{\mu^c} l_{\mu^c} \right> \Delta_{\mu^c}^{s} (s) \]

\[ \left< c_{\mu^c} l_{\mu^c} \right> \left< c_{\mu^c} l_{\mu^c} \right> \Delta_{\mu^c}^{s} (s) \]
\[ \omega = \sum_{\theta = 0}^{1} \frac{1}{\lambda \eta^2} \langle c_{1} \lambda n h \rangle < c_{1} \lambda n h' \rangle \langle c_{1} \lambda n h \rangle < c_{1} \lambda n h' \rangle \langle s_{1} \lambda n h \rangle < c_{1} \lambda n h' \rangle \sum_{k_{1} k_{3} k_{5}} s_{k_{1} k_{3} k_{5}} \langle s_{k_{1} k_{3} k_{5}} \rangle \]

\[ * (s_{k_{1} k_{3} k_{5}}) \lambda n h \langle c_{1} \lambda n h' \rangle \langle c_{1} \lambda n h \rangle < c_{1} \lambda n h' \rangle \langle s_{1} \lambda n h \rangle < c_{1} \lambda n h' \rangle \langle s_{k_{1} k_{3} k_{5}} \rangle s_{k_{1} k_{3} k_{5}} \]

\[ * D_{k_{1} k_{3} k_{5}} \langle s_{k_{1} k_{3} k_{5}} \rangle \]

\[ \langle 1 + 1^{+} + \ell \rangle \]
\[
\sum_f \hat{f}^2 \mathcal{W}(k_s, l, k, f) \mathcal{W}(l, k, k^\prime, f) \mathcal{W}(k^\prime, l, f, g)
\]

\[
\rightarrow \begin{pmatrix} k_s & s_i & s_i \\ l & s_j^f & \end{pmatrix} \hspace{1cm} \begin{pmatrix} l^\prime & s_i & l^\prime \end{pmatrix} \hspace{1cm} \begin{pmatrix} l^\prime & k & f \end{pmatrix} \hspace{1cm} \begin{pmatrix} k_s & l & g \end{pmatrix}
\]

\[
\rightarrow \begin{pmatrix} k_s & l & f \\ l & s_i & s_i \end{pmatrix} \hspace{1cm} \begin{pmatrix} l^\prime & k & f \end{pmatrix} \hspace{1cm} \begin{pmatrix} k_s & l & g \end{pmatrix}
\]

\[
\rightarrow \begin{pmatrix} l & s_i & f \\ k_s & l & s_i \end{pmatrix} \hspace{1cm} \begin{pmatrix} l & s_i & f \end{pmatrix} \hspace{1cm} \begin{pmatrix} k_s & l & g \end{pmatrix}
\]

\[
\rightarrow \sum_f \hat{f}^2 \mathcal{W}(k s, l, k, f) \mathcal{W}(l, k, k^\prime, f) \mathcal{W}(k^\prime, l, f, g)
\]

\[
\left( \begin{array}{c} l^\prime & s_i & l^\prime \\ k_s & l & g \end{array} \right) \begin{array}{c} k_s & l & g \end{array}
\]
\[ \mathcal{W} = \sum_{\text{all}} \frac{1}{167} \left( \mathcal{W}_{\text{analytical}} \right) \left( \mathcal{W}_{\text{numerical}} \right) \sum_{k_1, k_2, k_3} f_{k_1, k_2, k_3} \]

\[ \Phi \left( k_0, l_0, \mathbf{R}_0 \right) \left( \Phi_{k_1, k_2, k_3} \left( \mathbf{R}_{k_1, k_2, k_3} \right) \right) \left[ 1 + \text{other terms} \right] \]

\[ \left\{ \begin{array}{c}
\phi_{s, k, \ell, \mathbf{R}} \\
\phi_{s', k', \ell', \mathbf{R}'}
\end{array} \right\} \left( \mathcal{D}_{k_0, l_0, \mathbf{R}_0} \right) \]
\[ O_{\text{design pre.}} \]

\[
\mathcal{F} = \prod_{m_0} \left( \sum_{m} \left( \sum_{m' \neq m} \left( \sum_{m'' \neq m, m'} \left( \sum_{m''' \neq m, m', m''} \right) \right) \right) \right) \left( \sum_{m_0} \left( \sum_{m} \left( \sum_{m'} \left( \sum_{m''} \right) \right) \right) \right) \left( \sum_{m_0} \left( \sum_{m} \right) \right) \]

\[
\left( \mathcal{F}_m \right) \left( \mathcal{F}_{m'} \right) \left( \mathcal{F}_{m''} \right) \left( \mathcal{F}_{m_0} \right) \]

\[
\mathbf{D}_{m_0} \mathbf{D}_{m'} \mathbf{D}_{m''} \mathbf{D}_{m_0}^T \]

\[
\mathbf{D}_{m_0}^T \mathbf{D}_{m'} \mathbf{D}_{m''} \mathbf{D}_{m_0} \]

\[
\mathbf{T} \mathbf{R} \mathbf{T}^T \]
\[
1. \sum_{m_3} (-)^{\frac{1}{2} n_0 + n_3 + m_3} \langle \ell_n \mu_0^* j_3 M_3 \mu_3^* m_3^* | (j_{i_1} \ell_{i_1} f_{i_1} s_{i_1} | j_{i_2} \ell_{i_2} f_{i_2} s_{i_2} ) \rangle \\
2. \sum_{l_0 k} (-)^{l_0} \langle \ell_{m_0} \mu_0^* l_0 s_{l_0} f_{l_0} t_{l_0} k_0 \rangle \langle \ell_{n_0} \mu_0^* l_0 s_{l_0} f_{l_0} t_{l_0} k_0 \rangle = (-)^{l_0 - k} \frac{f_{l_0} t_{l_0}}{h_{l_0}} \sum_{l_0} \int \hat{S}_l \psi(l_0, \beta, \alpha) \hat{S}_l \psi(l_0, \beta, \alpha) \\
3. \sum_{m_1} (-)^{s_{m_1}} \langle s, m_1, s_{-m_1}, f_{s_0} k_s \rangle \langle q_i, m_i^* s_{i_1} \rangle \langle j_{i_1} \ell_{i_1} f_{i_1} s_{i_1} \rangle = (-)^{s_{m_1}} \sum_{l_0} \int \hat{S}_l \psi(l_0, \beta, \alpha) \hat{S}_l \psi(l_0, \beta, \alpha) \\
4. \sum_{n_1} (-)^{j_{i_1}} \langle s, m_1^* j_{i_1} - m_{1'}, f_{i_1} s_{i_1} \rangle \langle l_0, s_{m_1} f_{i_1} s_{i_1} \rangle = (-)^{l_0} \sum_{l_0} \int \hat{S}_l \psi(l_0, \beta, \alpha)
\[
\frac{dA}{d\Omega_2}(\theta_0) = \frac{2e}{3} \left( \delta_{1L_{1}} \delta_{1L_{2}} \right) \left( \pi L_{1} L_{2} R \right) \left[ \frac{k_{x}}{2\pi\hbar c} \right] [e^{i\frac{2\pi}{3}}k_{0}g_{\mu} \left( \frac{m_{e}}{m_{e}^{2}} \right)]
\]

\[
\times \left[ e^{-i\frac{2\pi}{3}} \left( \frac{1}{\hbar} \right) \theta_{1} R_{1, L_{0}} + \sum_{f_{1}, k_{y}} \frac{f_{1}}{f_{1}k_{z}} \delta_{2}k_{z} \delta_{2} \right] \left( \frac{e^{-i\frac{2\pi}{3}}}{\hbar} \right) \left( \delta_{1L_{1}} \delta_{1L_{2}} \right) \left( \delta_{1L_{1}} \delta_{1L_{2}} \right) \left( \delta_{1L_{1}} \delta_{1L_{2}} \right)
\]

\[
\times \left[ \sum_{f_{1}, k_{y}} \frac{f_{1}}{f_{1}k_{z}} \delta_{2}k_{z} \delta_{2} \right] \left( \frac{e^{-i\frac{2\pi}{3}}}{\hbar} \right) \left( \delta_{1L_{1}} \delta_{1L_{2}} \right) \left( \delta_{1L_{1}} \delta_{1L_{2}} \right) \left( \delta_{1L_{1}} \delta_{1L_{2}} \right)
\]

\[
+ \left( \delta_{2}k_{z} \delta_{2} \right) \left( \frac{e^{-i\frac{2\pi}{3}}}{\hbar} \right) \left( \delta_{1L_{1}} \delta_{1L_{2}} \right) \left( \delta_{1L_{1}} \delta_{1L_{2}} \right) \left( \delta_{1L_{1}} \delta_{1L_{2}} \right)
\]
\[
\frac{d\sigma}{d\Omega}(\tau,\phi) \sim \frac{2}{a} \sum_{i=1}^{N} \frac{k_i}{\sin \theta_c} \frac{1}{(M^2 - m_i^2)^{1/2}} \cdot \left( \frac{\epsilon_m}{\sigma_{\text{in}}} \right)^2 \cdot \frac{1}{\mathbf{k}_0 \cdot \mathbf{k}_f} \cdot \sum_{k, f} \left( \begin{array}{c} \nu_f \ \nu_i \ \nu_j \ \nu_k \ \nu_\ell \ \nu_m \ \nu_n \ \nu_o \ \nu_p \ \nu_q \ \nu_r \ \nu_s \ \nu_t \ \nu_u \ \nu_v \ \nu_w \ \nu_x \ \nu_y \ \nu_z \ \nu^{\dagger} \end{array} \right) \]

[...mathematical expressions...]

\[
\frac{1}{\mathbf{k}_0 \cdot \mathbf{k}_f} \cdot \sum_{k, f} \left( \begin{array}{c} \nu_f \ \nu_i \ \nu_j \ \nu_k \ \nu_\ell \ \nu_m \ \nu_n \ \nu_o \ \nu_p \ \nu_q \ \nu_r \ \nu_s \ \nu_t \ \nu_u \ \nu_v \ \nu_w \ \nu_x \ \nu_y \ \nu_z \ \nu^{\dagger} \end{array} \right) \]

...
APPENDIX B

The total width of a level $\Gamma_{\text{tot}}$ is equal to $\Sigma \Gamma_i$ over all channels $i$ and that is model independent.

1) Proton width (in R-matrix formalism)

$$\Gamma_{\alpha l} = \frac{2\rho_\alpha \gamma^2_{\alpha l}}{A_{\alpha l}(1 + \frac{1}{\rho_\alpha} \gamma^2 S_{\alpha l})}$$

where $S_{\alpha l} = \frac{\rho_\alpha}{2A_{\alpha l}^2} \frac{d}{d\rho_\alpha} (A_{\alpha l}^2)$ the shift function

Proton width (in S-matrix formalism)

$$\Gamma_{\alpha s l} = \frac{\partial \rho_\alpha \gamma^2_{\alpha s l}}{A_{\alpha l}^2}$$

Both define $\rho = ky$

$A_{\alpha l} = $ penetrability $= F_{\alpha l}^2 + G_{\alpha l}^2$

$\gamma^2_{\alpha s l} = $ reduce width

$\theta^2 = \gamma^2_{\text{expt}}$

$\gamma^2_{\text{Wigner limit}} = \frac{3\hbar^2}{2\mu R^2}$

If $\theta^2$ is less than .2 then equations 1 and 2 will have negligible difference.
2) $\gamma$ partial decay width. $\Gamma_\gamma(b,L)$

$$\Gamma_\gamma^R(b,L) = \Gamma_\gamma^R(b,L) \left( \frac{E_y}{E_R} \right)^{2L+1}$$

where $\Gamma_\gamma^R(b,L)$ means the measure of $\gamma$ partial decay width on resonance. The factor $\frac{E_y}{E_R}$ shows the strength of the decay width and it behaves just like the Weisskopf unit off resonance. General motivation of using such units is given in many texts and is skipped here. We would like to indicate that either their ratios or the quantity $\omega \Gamma_\gamma$ is of common use.

For example:

$$\alpha_i^z = \frac{\Gamma_\gamma(\frac{3}{2},\frac{M_1}{2})}{\Gamma_\gamma(\frac{1}{2},E_1)} = \frac{\Gamma_\gamma^R(\frac{3}{2},M_1)}{\Gamma_\gamma^R(\frac{1}{2},E_1)} \left( \frac{E_y}{E_R} \right)^{\frac{3}{2}}$$

$$\beta_i^z = \frac{\Gamma_\gamma(\frac{3}{2},E_1)}{\Gamma_\gamma(\frac{1}{2},M_1)} = \frac{\Gamma_\gamma^R(\frac{3}{2},E_1)}{\Gamma_\gamma^R(\frac{1}{2},M_1)} \left( \frac{E_y}{E_R} \right)^{\frac{3}{2}}$$

and $\omega \Gamma_\gamma(1/2,E_1) = 0.67$ eV where $\omega$ is the statistical factor.
APPENDIX C

For an incident proton with 1.8 MeV going through a gas cell, of 5/8" x 5/8" x 5/8" cube with 1/3 atm. "He covered by 1/20μ Ni foils, the energy at the center of this cell can be obtained in the following ways:

1. 1/20μ thick Ni foils alternation.
   Atomic stopping power A for Ni at 1.8 MeV is 8.8 x 10^{-15} eV-cm²
   Conversion factor B = 9.13 MeV/cm.
   1/20μ = 8 x 10^{-5} in. x 2.54 cm/in. = 1.27 x 10^{-4} cm.
   Energy loss 8.8 x 9.3 x 1.27 x 10^{-4} = 1.0203 x 10^{-1} MeV
   1.8 MeV - 0.1020 MeV = 1697.97 keV.

2. 1/3 Atm. of "He gas.
   Stopping power A = 1.3 x 10^{-15} eV-cm²
   Conversion factor B = 0.02547 MeV/cm²
   Energy loss 5/16" x 2.54 x 1.3 x 0.02547 MeV = 8 keV
   Final energy 1.697.97 keV - 8 keV = 1.68997 keV ~ 1.69 MeV

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APPENDIX D

For γ-ray (to bound state)

\[ \Gamma = \frac{2\pi}{\hbar} \left\langle f \nu_f \mid H_{im} \mid i \right\rangle \left( \varepsilon_i - \varepsilon_f \right) \]

\[ d\Gamma \] (the rate of transition into dΩ) = \( \frac{1}{\mathcal{V}} \frac{2\pi}{\hbar} K \left\langle f \nu_f \mid H_{im} \mid i \right\rangle \left( \varepsilon_i - \varepsilon_f \right) \)

= \( \frac{\sqrt{\delta \omega}}{r} \int k'' dk \left\langle f \nu_f \mid H_{im} \mid i \right\rangle \left( \varepsilon_i - \varepsilon_f \right) \)

knowing \( E_\gamma = h\nu = \frac{hc}{\lambda} = hc\kappa \gamma \)

\[ \Gamma = \frac{\sqrt{\delta \omega}}{r} \int k'' dk \left\langle f \nu_f \mid H_{im} \mid i \right\rangle \left( \varepsilon_i - \varepsilon_f \right) \]

= \( \frac{\sqrt{\delta \omega}}{r} \int \frac{k''}{(hc)^2} \left\langle f \nu_f \mid H_{im} \mid i \right\rangle \left( \varepsilon_i - \varepsilon_f \right) \)

incident flux: \( S = \frac{r}{2\mathcal{M}} (\bar{\psi}^* \bar{\psi} - \psi \bar{\psi}^*) \) Assuming \( \psi = \alpha \epsilon \kappa \gamma \)

= \( \frac{\hbar \kappa}{m} \mathcal{A} \alpha^* \)

= \( \mathcal{V} \mathcal{A} \alpha^* \)

\[ \frac{d\sigma}{d\Omega} = \frac{d\Gamma}{\mathcal{A} \nu \mathcal{V}^2} = \frac{\mathcal{V} \kappa}{\delta \omega \hbar \nu} \left( \frac{\kappa''}{(hc)^2} \right) \left\langle f \nu_f \mid H_{im} \mid i \right\rangle \]

knowing: we average over initial states and sum over final states.

g factor = \( \frac{1}{2j_L + 1} (2j_p + 1) \)

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\[
\frac{d\sigma}{d\Omega} = V \sum_{\delta \Delta k} \left( \frac{k_F}{2\Delta \Delta} \right) \left( \frac{1}{2\Delta \Delta} \right) \sum_{m_i} |f_{m_i}|^2 |m_i>^2
\]

Futhermore, \( |f_{m_i}| \) are bound states and \( H \) properly normalized, if we choose \( |m_i> \sim (1)e^{ikz} \) then \( AA^* = 1 \). That is why we pick

\[
|m_i> = \frac{1}{\sqrt{Z_m s_i}} \sum (\frac{\hbar}{k_F}) Y_{0}(\theta) X_{s_i} e^{ikz}
\]

The Hamiltonian\(^5\)

\[
H = \left( \frac{\hbar^2}{2m} \right)^2 + \int \Phi^2 + \Phi t^2
\]

picking the transverse gauge \( \nabla A = 0 \), \( \Phi(t) = 0 \) (free space)

dropping second order term \( \theta(A^2) \)

\[
H = H_o + \frac{\hbar^2}{2m} (\frac{\partial}{\partial i})^2
\]

Look only at the electric part of the multipole\(^9\)

\[
\hat{A}_e = \sum_{LM} |A^*_{LM} e^{-ikr} \rangle
\]

\[
= \sum_{K} \left( A_{k} \frac{1}{\sqrt{n_{k}}} e^{ikr} + A_{k}^* \frac{1}{\sqrt{n_{k}}} e^{-ikr} \right)
\]

\[n_{k} = 0 \Rightarrow \hat{B} = A_{k} e^{-ikr} \text{ and } A_{k} A_{k}^* = \frac{2\pi \hbar c}{k_F} N \]

\[
H_{el} = \sum_{LM} A_{k}^* A_{k} e^{ikr} \left( \frac{ -ik}{(\omega - l + 1)!} \right) \sqrt{k_F/2\Delta} \sqrt{k_F/2\Delta + 1} \ Y_{LM} \ D_{LM}^L
\]
Knowing in the field \( |1;00000...\rangle \rightarrow |0,00...010...\rangle \) means \( N_{\kappa \lambda} = 1 \)

If we normalize the initial wave function using

\[
H_{\text{init}} = \epsilon \sum_{\ell m} \frac{(-ik)^l}{(2\ell)!} \sqrt{\frac{\ell+1}{2\ell}} \Omega_{\ell \kappa} \sqrt{\frac{\ell+1}{2\ell+1}} Y_{\ell m} D_{\ell m}^L \quad \star
\]

\[
\frac{d\sigma}{d\Omega} = \frac{k^2}{2\pi \hbar^2} \frac{1}{(2\ell)!} \Omega_{2\ell} \sum_{m_{\ell \kappa}} \left| \langle \ell m_{\ell \kappa} | H_{\text{init}} | \ell 0 \rangle \right|^2
\]
In order to derive the differential cross section formula, one first assumes \( N(E_\gamma) \, dE_\gamma \) to be the number of counts in a small energy region \( dE_\gamma \) about the \( \gamma \)-ray energy \( E_\gamma \). Then

\[
N(E_\gamma) \, dE_\gamma = \frac{d\sigma}{d\Omega}(E_p, \theta) \, N_p \, dx(E_p) \cdot \varepsilon(E_\gamma) \, d\Omega
\]

where \( N_p \) is the total number of incident particles, \( n \) is the number of nuclei/cm\(^3\) of target material \( dx(E_p) \) is the target thickness and \( \varepsilon(E_\gamma) \, d\Omega \) is the efficiency of the detector for \( \gamma \) rays of energy \( E_\gamma \). If one substitutes \( dx(E_p) = (dx/dE_p) \, dE_p \), the above eq. becomes

\[
N(E_\gamma) = \frac{d\sigma}{d\Omega}(E_p, \theta) \, N_p \, (dE_p/dE_\gamma) \{ \varepsilon(E_\gamma)/\xi(E_p) \} \, d\Omega
\]

where \( \xi = 1/n(dE_p/dx) \) the target stopping power.

Substitute \( dE_p = \left( \frac{M_p + M_{\text{target}}}{M_{\text{target}}} \right) \, dE_\gamma \) and rearrange the above equation one gets.

\[
\frac{d\sigma}{d\Omega}(E_p, \theta) = \frac{N(E_\gamma) \cdot \varepsilon(E_\gamma) \cdot M_{\text{target}}}{N_p \cdot \varepsilon(E_\gamma) \cdot d\Omega \cdot (M_p + M_{\text{target}})}
\]
APPENDIX F

$^{12}\text{C} + \text{n}$ problem:

\[ H\psi = E\psi \quad \text{where } E = E_{\text{En}} \]

\[ [-\frac{\hbar^2}{2m} \nabla^2 + V]\psi = E\psi \quad \text{where } V = -V_0 \quad r < R_0 \]

\[ 0 \quad r > R_0 \]

$\psi(r<a)$ is the spherical Bessel function and $\psi(r>a)$ is the spherical Hankel function.

$^{12}\text{C} + \text{p}$ problem:

\[ [-\frac{\hbar^2}{2m} \nabla^2 + V']\psi = E\psi \quad \text{where } E = E_{\text{Ep}} \]

where $V' = V + \frac{3}{2} \frac{Ze^2}{a} - \frac{1}{2} \frac{Ze^2}{a^3} r^2 \quad r < R_0$

\[ \frac{Ze^2}{r} \quad r > R_0 \]

$\psi(r<a)$ is a confluent hypergeometric function and $\psi(r>a)$ is a Whittaker function.

Upon receiving $r_1$, a set of programs was written to calculate $v_0$ and $v_2$.

Subprogram HANKL calculates $\frac{d\psi}{dr}/\psi$ for outside $^{12}\text{C} + \text{n}$

Subprogram SPBES calculates $\frac{d\psi}{dr}/\psi$ for inside $^{12}\text{C} + \text{n}$
Subprogram WHITT calculates $\frac{d\psi}{dr}/\psi$ for outside $^{12}\text{C} + p$

Subprogram CONTY calculates $\frac{d\psi}{dr}/\psi$ for inside $^{12}\text{C} + p$

The main program steps up from initial value of $r_0$ and $v_0$ to $r_1$ and $v_1$ and finds $R$ and $V$ such that it fits both $^{12}\text{C} + n$ and $^{12}\text{C} + p$ simultaneously.
APPENDIX G ::

The code that performs the Radiative Capture calculation is as follows:

**DCANG:** Direct capture angular distribution coefficients.
Input: One needs to supply all the spin parity assignment in all the data cards.
Output: The Racah Alg would be profound and the result would be stored on file #369.

**DCANP:** Polarized version of DCANG; same type of input was required. The result is then stored on file #469.

**PHASE:** The kinematic part of the different cross section was performed in this program for the Direct-Direct type contribution.
Input: Masses, charges, energies level information.
Output: The result is stored in common.

**DCCST:** Direct capture cross section. This program first will read file #368 first, then calculates cross section.
Output: The result is then stored on file #368.
Option TOGSW (8.6) Up - hard sphere phase Down - single particle phase; TOGSW (7.5) Up - polarized, Down - unpolarized version.
CNANG:  Resonance capture angular distribution coefficient
         (compound nucleus type).  Like the DCANG, one needs to
         define all spin parity assingments in the data cards.
         The result is stored on file #371.

CNANP:  Polarized version of CNANG.  Same type of input was
         required.  The result was stored on file #471.

PARA1:  This program is an analog to phase except it performs
         CN-CN type calculations.
         Input:  mass, charges energies level information
         Output:  The result was stored on file #372.

CNCST:  Resonance capture cross section.  The program read file
         #371 first and depended on TOGSW (7.5) up or down to
         read polarized or unpolarized files.  The result was
         stored on file #373.

DCCNA:  Direct capture and resonance capture interference angu-
         lar distribution coefficients.
         Input:  Spin parity assignemnt in data statement
         Output:  File #374.

DCCNPA: Polarized counterpart of DCCNA
         Input:  Same as DCCNA
         Output:  File #474.
Another analog version of PARAl. This one performs DC-CN type interference.

Input: like PHASE or PARAl

Output: File #367

**DCCNI:** Direct capture and resonance capture interference cross sections. Option: TOGSW (7.5) up or down for polarized or unpolarized case. Up - hard sphere; Down single particle resonance.

Output: File #375.

**DCCNC:** Direct capture and resonance capture total cross section. This program will sum up all the DC-DC, DC-CN, CN-CN contributions and present a total angular distribution. Again, it depends on TOGSW (7.5) for polarized or unpolarized case.

Option 1: TOGSW (7.5) Up - polarized, Down - unpolarized

2: TOGSW (8.6) Up - Differential cross section, Down - skip that

Input: A = spectroscopy factor, B = γ-ray partial decay width, C := relative phase between CN and DC, D = the angles for differential cross sections.

Output: Will be stored on file #376.

For more details, one should see the program printout from the Ohio State University Van de Graaff Lab.
TOGSW(8.6) UP: HARD SPHERE PHASE

DOWN: SINGLE PARTICLE RESONANCE PHASE

TOGSW(8.6) SAME AS ABOVE
The code that performs the radial integrals is as follows:

MASTR: Master starting program.
Input: Starting energy in MeV.
Output: Link to SLAVE1.

SLAVE1: Function: Solving confluent hypergeometric function of all orders from \( r = 0 \) to \( r = r_0 \) for our case \( \lambda = 1,2,3 \) is required.
Option: TOGSW (8.6) Up, by-pass (S, D waves)
Input: \( r_0 \), nucleus radius and \( V^* \), effective nuclear potentials.
Output: Stored results in common and Link the above.

SLAVE2: Function: Calculated Coulomb wave functions (both Bernard's code and Goss' Code were used depending on the region of interest) from \( r = r_0 \) to \( r = r_{\text{max}} \)
Option: TOGSW (8.6) Up - hard sphere phased was used,
Down - single particle resonance phase was used.
Input: \( r_{\text{max}} \): predetermined radius such that the radial integrand would be \(< 0.5 \times 10^{-5} \text{fm}^4 \)
Output: in common and Link to SLAVE3.

SLAVE3: Function: Calculated Whittaker function for \( \lambda = 1 \), from \( r = r_0 \) to \( r = r_{\text{max}} \)
Input: same as SLAVE2
Output: in common with Link and SLAVE4.
SLAVE4: Matching the inside and outside wave functions.
Input: None
Output: Link to SLAVE5

SLAVE5: The wave function still carries an extra \( \frac{1}{k} \), and this program is going to get rid of it. Then link to SLAVE6.

SLAVE6: Function: Normalization of the bound state using Romberg Quadrature formula.\(^*\).
Input: None
Output: Link to SLAVE7.

SLAVE7: Function: Use Romberg Quadrature formalism with step less than half of a fermi.
Option: TOGSW (8.6) Up - extranuclear, Down - otherwise
Input: Final energy
Output: Store results in file #370 or #390 depending on TOGSW 8.6 then either exit or Link back to SLAVE1.

The code that performs single particle resonance phase shift

CHECK: Function: Find effective square well potential that can reproduce the resonance phase shift data.
Input: \( r_0 \) the nuclear radius
Output: the potential in MeV.
SET OF PROGRAMS THAT GENERATE RADIAL INTEGRALS

MASTER

SLAVE0

SLAVE1 $S, P, D$

SLAVE2 $S, D$

SLAVE3 $P$

SLAVE4 (MATCH)

SLAVE5 (GRIDK)

SLAVE6 (NOMAL)

SLAVE7 (RLILE)

TOGSW(8.6) up: S, D = 0

TOGSW(8.6) up: HARD SPHERE PHASE

TOGSW(8.6) up: EXTRANUCLEAR RADIAL INTEGRAL
SINGLE PARTICLE RESONANCE PHASE SHIFT PROGRAM

FILE 400(202,4,U,ICB1)
FILE 401(202,4,U,ICB2)
and so on...
stores phase shift informat
APPENDIX H

The thick target data reduction code:

HICAL: Calibrate the detector to obtain information on energy vs. channel number.

The rest of these programs display on the storage scope the spectrum on disk and allow the user to have visual feedbacks after each successful operation.

PG000: Background subtraction for the thick and thin target spectrum.

PG001: Doppler shift correction for the spectra.

PG002: From the thin target spectrum, one tries to create a perfect line shape as an experimental ideal detector response function.

PG003: After obtaining the information from the response functions, (total counts under the peak, starting + end channel number) one attempts to use it to unpeel the thick target spectrum. This can be done either automatically or manually by
picking starting and ending points in the thick target spectrum by the users) depending on TOGSW (7.5)'s position.

PG004: This program allows one to normalize the data to any other previous adjusted data (for comparison purposes only).

PG005: It is also known as PCPGN which stands for polarized $^{12}\text{C}$arbon (proton,$\gamma$) $^{13}\text{N}$itrogen reaction analysis. It consists of the following programs.

ADANA: See N. Tsoupas' thesis with a theoretical prediction of resonance capture mechanism added to it.

CONVS: Conversion

PLOCO: A plotting routine for output of data analysis through Legendre's or Associate Legendre's polynomial and its comparison from theoretical predictions.
APPENDIX I

Carbon Thick Targets Specifications

ERROR:

1. For a 1.8 MeV incident proton beam, the thickness that is required to stop the proton beam can be calculated provided one knows the molecular stopping power of carbon ($12.3 \times 10^{-15}$ eV·cm$^2$)

\[
113 \times 12.3 \times 10^{-15} \text{ (eV·cm$^2$)} \times \text{MeV/cm} = 1389.9 \text{ MeV/cm.}
\]

Assuming its linear, it takes 0.012950572 mm to stop the 1.8 MeV beam. Since the thick target is considerably thicker than that, using the center of the thick target as the true center of the target chamber, we introduced a source of error. Furthermore, if the beam is not carefully centered at the true z-axis, the false center gets farther away from the true center.

Suppose $R = 3''$ is the radius from true center to the surface of the detector. $\Delta R = 1/16''$ is the false center away from the true center. The $\Delta \Omega$ (solid angle) = $\frac{3 \Delta \Omega}{3R} = \pm \frac{1}{24}$ which is approximately 9.5%.

2. $\gamma$-ray Attenuation (~ 2%)

Photon attenuation coefficient.\textsuperscript{52)}
For $1.5$ MeV photon on $^{12}\text{Carbon}$ (graphite)

$$\sigma_{\text{tot}} = 1.03 \text{ barns/atom}$$

For $^{12}\text{C}$ $1 \text{ barns/atom} = 0.050140 \text{ cm}^2/\text{gm}$

$$\sigma_{\text{tot}} = 0.050140 \times 1.03 = 0.0516442 \text{ cm}^2/\text{gm}$$

For $^{12}\text{C} \rho dx = 0.2 \text{ cm} \times 2.25 \text{ gm/cm}^3 = 0.45 \text{ gm/cm}^2$

$$I = I_0 e^{-\sigma \rho dx} \Rightarrow \frac{I}{I_0} = 0.9770280764$$

So for $1.8$ MeV photons the amount of attenuation would be less than $0.977$.

3. Doppler Shift Correction ($\sim 1\%$)

The total error of about $10\%$ can result if one fails to perform the thick target technique carefully.

**Carbon Thin Target Specifications**

Thin target: $10$ S.S. ($10 \mu\text{g/cm}^2$)

$$\left(\frac{dE}{dx}\right) = 150 \text{ keV/mg/cm}^2 \text{ (Values for incident proton energy equals 1.8 MeV on carbon)}$$

$$dE = 150 \times \frac{\text{keV}}{\text{mg/cm}^2} \times 10 \times 10^{-3} \text{ mg/cm}^2$$

$$dE = 1.5 \text{ keV}$$

Thick target impurities will be given in Table G-1.

The thin target comes from the carbon backing plate of the Yissum Research Development Company.
## Ash and Impurity Contents of Artificial Graphite

<table>
<thead>
<tr>
<th>Grade</th>
<th>Ash</th>
<th>B</th>
<th>Al</th>
<th>Ca</th>
<th>Ti</th>
<th>Fe</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>AGX*</td>
<td>1500</td>
<td>1.3</td>
<td>20</td>
<td>2000</td>
<td>50</td>
<td>40</td>
<td>70</td>
</tr>
<tr>
<td>C-18*</td>
<td>1200-2000</td>
<td>1.0</td>
<td>10</td>
<td>1200</td>
<td>35</td>
<td>100</td>
<td>200</td>
</tr>
<tr>
<td>AGHT†</td>
<td>700-1700</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>&lt;100</td>
</tr>
<tr>
<td>AGOT†</td>
<td>350-700</td>
<td>...</td>
<td>...</td>
<td>100-200</td>
<td>&lt;100</td>
<td>&lt;100</td>
<td>&lt;100</td>
</tr>
<tr>
<td>CB†</td>
<td>350-700</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>&lt;100</td>
</tr>
</tbody>
</table>

*Typical commercial graphite
†Normal reactor graphite
REFERENCES


3. F.C. Young, J.S. Armstrong, and J.B. Marion, Nucl. Phys. 44 (1903) 486.


10. J.D. Seagrave, Phys. Rev. 84 (1951) 1219.


28. S. Devons and L.J.B. Goldfarb, Handbook of Physics, XLII.


41. J.D. Seagrave, Phys. Rev. 85 (1952) 197.


44. A. Ferguson, Angular Correlation Method and Gamma-ray Spectroscopy, Wiley.


47. M. Rotenberg, R. Bivins, N. Metropolis, and J.K. Wooten, Jr., The 3j and 6j Symbols, M.I.T.


51. J.B. Marion and F.C. Young, Nuclear Reaction Analysis, Wiley.