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D I S S E R T A T I O N

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By
Prasert Tantayanondkul, B.E.,M.S.

The Ohio State University
1976

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CHAPTER 1
INTRODUCTION

The deterioration of concrete bridge decks due to the use of deicing salts has been a major problem. Elimination of new bridge deck's deterioration and rehabilitation of deteriorated bridge decks has become one of the highest priorities. In recent years a number of protective systems have been used to protect concrete bridge decks. One of them is the application of Latex Modified Concrete (LMC) overlays to seal the deck and to provide a more durable wearing surface. Using LMC in place of conventional concrete is very costly, even on new bridges. In order to reduce the deterioration rate of reinforced concrete bridge decks without increasing the cost beyond an acceptable level, only part of the conventional concrete clear cover to the top rebars (3/4" to 1-1/2") is replaced by LMC.

Structural members of conventional reinforced concrete overlayed by a thin layer of LMC had been tested by the writer and Dr. A. Bishara\(^{(1)}\) to study their corrosion resistance and flexural characteristics. The experimental study of the flexural characteristics of the tested slabs under static short term loading will be presented in Chapter 6.
The inherent complexity in analysis of this composite structural system is mainly due to the nonlinear behavior of concrete, and the continuous changing of boundary conditions due to cracking of the concrete under increasing load. Finite element methods for the analysis of structural and continuum mechanics problems offer a new tool to be used in understanding the behavior of such complex structural systems. An analytical study of this composite flexural system under static short term loading by the finite element method, and a comparison of experimental and analytical results will be presented in Chapter 7.

The accuracy of finite element solutions depends on how closely the analytical models represent the actual structures. A review of related work done in this area is presented in Chapter 2.
CHAPTER 2
REVIEW OF LITERATURE

2.1 BEHAVIOR OF CONCRETE

Considerable research\(^{(2,4,5,6,7,8,9)}\) has been conducted to study the instantaneous deformations of concrete under load. A typical stress-strain diagram of concrete under axial load is presented in Fig. 2.1. Under compression the stress-strain relationships is linear up to about 30 percent of the ultimate load. Beyond this stage the curve deviates gradually from the straight line to the horizontal up to about 80 percent of the ultimate load, then the curve starts to bend more sharply to the horizontal up to the ultimate load and it ends in a descending tail. Nonlinear behavior of concrete is in general due to internal microcracks, which initiate at the interface between aggregates and the surrounding mortar. At a load about 30 percent of ultimate load, there is an increase in the number and length of the bond cracks between aggregates and surrounding mortar. The bond cracks continue to grow in length and number until a load of 80 percent of the ultimate load. At this point, the bond cracks begin to join together.
FIG. 2.1 Typical Stress-Strain Diagram of Concrete Under Uniaxial Load.
forming mortar cracks. The mortar cracks grow until continuous crack patterns develop, and the ultimate load is reached. An illustration of crack initiation and propagation in concrete is presented in Fig. 2.2. A rather complete review of stress-strain relationships for concrete under axial compression with several empirical formulas obtained by curve fitting is reported by Popovics. Under uniaxial tension the stress-strain relationships of concrete is almost linear up to ultimate load, and concrete fails by splitting normal to the direction of the applied load. At a load about 70 percent of ultimate tensile load there is a discontinuity point which corresponds to the formation of the first crack.

Liu studied the behavior of concrete under biaxial compression. He compared the experimental failure mode of concrete with the existing failure theories and found that none of those theories seem to fit. Carino extended the work done by Liu to study the behavior of concrete under biaxial tension and biaxial compression-tension, and concluded that despite many years of research, there still is no universal failure theory which can predict the failure of concrete under the action of different stress states. Empirical stress-strain relationships of concrete under biaxial load at different stress states are also presented by Liu and Carino.
FIG. 2.2 Illustration of Crack Initiation and Propagation in Concrete Under Uniaxial Compressive Stress (5).
According to the present informations on the behavior of concrete under load it seems unnecessary to use any complicated failure theory or stress-strain relationships. A simple failure mode and a simplified stress-strain relationships are used in this study, it will be presented in Chapter 3.

2.2 **FINITE ELEMENT MODELS**

Recently, finite element methods have been applied to reinforced concrete members by several authors. Ngo and Scordelis\(^{(10)}\) developed an analytical model for two dimensional analysis of flexural reinforced concrete members. Both concrete and steel were assumed to be homogeneous, isotropic with linearly elastic stress-strain relationships. The steel and concrete were represented by two dimensional triangular finite elements. The bond between concrete and steel was represented by a series of finite spring elements (Linkage-Element) spaced along the bar length. Idealized crack patterns were imposed on the beam to study the effect of cracking on the resulting stress distribution. A crack was represented by separating the concrete elements on either side of the crack.

Nilson\(^{(11)}\) investigated reinforced concrete flexural members loaded from zero to their ultimate capacity. Nonlinear stress-strain (or displacement) relationships of concrete,
steel, and bond-slip between concrete and steel were represented by piecewise linear relationships for each incremental loading. Nilson's analytical model for the two dimensional finite element analysis was similar to the one that was presented by Ngo and Scordelis. In this analysis Nilson assumed that concrete cracks when the principal tensile stress exceeds its modulus of rupture. After a crack was established, the member was completely unloaded and the newly defined body reloaded incrementally. As noted by him, this may not be necessarily true in the actual loading case.

Franklin\textsuperscript{(12)} performed a nonlinear analysis in which cracking within the finite elements and redistribution of stresses was automatically accounted for, so that the response from initial loading to failure was obtained in one continuous computer analysis. Franklin assumed isotropic behavior during each increment of loading prior to cracking. When tensile stress exceeds the tensile strength of concrete, the element is cracked normal to the tensile stress direction, and this stress is redistributed to the remaining structure. Subsequently, the element is assumed to be anisotropic and to have zero modulus of elasticity normal to the crack. Unlike the preceding studies, the crack was not predefined, and progressive cracking was assumed to occur over an entire element rather than along a single line.
Valliapan and Doolan\(^{(13)}\) studied the behavior of reinforced concrete members under continuous incremental load by considering a crack to be only the consequence of the inability of concrete to sustain tensile stress. The limiting tension before cracking is the prescribed tensile strength of concrete and after cracking it is zero. As the concrete is assumed incapable of sustaining more than the limiting tension, the excess tension is removed. These excess tensile stresses are converted into nodal forces. During the next cycle, these restraining nodal forces are applied to the structure. The stress and strain relationships of steel and concrete (in compression) were assumed to be bilinear, elastic-perfectly plastic. The "initial stress" finite element approach\(^{(14,15)}\) was employed to obtain the solutions of the problems.

Nam\(^{(16)}\) compared the results between "constant stiffness method" and "variable stiffness method" in nonlinear finite element analysis. He concluded that the variable stiffness approach in the evaluation of nonlinear behavior of reinforced concrete due to cracking (Franklin's\(^{(12)}\) approach), was superior to the constant stiffness approach (Valliapan and Doolan's\(^{(13)}\) approach).

Adham, Bhaumik and Isenberg\(^{(18)}\) developed a composite element of reinforced concrete in the form of a variable modulus model for use in a finite element analysis. Before
cracking occurs, the properties of the element depend on the entire stiffness of steel and concrete. After cracking, a composite modulus is used which reflects the combined stiffness of steel and concrete and takes into account the extent to which bond between steel and concrete is broken. Unlike previous studies no linkage-element is needed.
CHAPTER 3
IDEALIZATION OF STRUCTURAL MATERIALS

In this chapter the behavior of the structural materials under load is idealized so that the finite element analysis can be carried out within a reasonable time. The idealization is based on experimental results or where experimental results are not available assumptions which seem reasonable are made.

3.1 CONCRETE

In the finite element analysis the nonlinear stress-strain relationships of concrete are commonly approximated by the multilinear relationships such as bilinear\(^{(13,16)}\) or trilinear or piecewise linear,\(^{(11,12)}\) relationships. In this study the stress-strain relationships of both Latex Modified Concrete (LMC) and air-entrained concrete (AEC) under uniaxial tensile load are assumed linear up to a failure stress equal to the modulus of rupture of concrete (\(f_{r}^{\prime}\)). In uniaxial compression a bilinear stress-strain diagram as shown in Fig. 3.1 is assumed. Both AEC and LMC under uniaxial compression are assumed linearly-elastic until the compressive stress equal to 85 percent of the compressive strength of
FIG. 3.1 Idealized Stress-Strain Curve of Concrete
concrete ($f'_c$), and perfectly-plastic until crushing when the compressive strain equal to ultimate crushing strain ($\varepsilon_{cu}$).

The cracking stress of concrete for a reinforced concrete member in general will differ from the modulus of rupture obtained from the flexural tests and the tensile strength obtained from the pulling tests. Since, the major cracks of the slabs under investigation are flexural cracks it is reasonable to assume that the cracking stress of concrete is equal to the modulus of rupture of concrete.

The modulus of elasticity of concrete ($E_c$) under tension and compression are assumed to be the same. The modulus of elasticity of AEC is assumed as Eq. 3.1, according to ACI - 318-71(19):

$$E_c = 57000 \sqrt{f'_c} \text{ psi}$$

where

$$f'_c = \text{ compressive strength of concrete, psi}$$

According to Eash and Shafer(20) the modulus of elasticity of LMC is less than the value given by Eq. 3.1. In this investigation the modulus of elasticity of LMC is assumed as 75 percent of the value given by Eq. 3.1.
Concrete is assumed as a linear isotropic homogeneous material with a constant Poisson's ratio (\(v\)) when the maximum principal tensile stress is less than the modulus of rupture of concrete. Hence the stress-strain relations of uncracked concrete under plane stresses are equal to:

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_{xy}
\end{bmatrix} = \frac{E_c}{1-v^2} \begin{bmatrix}
1 & v & 0 \\
v & 1 & 0 \\
0 & 0 & 1-v^2/2
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix}
\]  

(3.2)

where \(\sigma_x, \sigma_y, \sigma_{xy}\), and \(\varepsilon_x, \varepsilon_y, \gamma_{xy}\) are stresses and strains, respectively.

Concrete cracks when the maximum principal stress exceeds the modulus of rupture. The crack direction is assumed to be fixed and perpendicular to the maximum principal axis just prior to crack formation. The cracked concrete is considered continuous, anisotropic and capable of resisting normal stress parallel to the crack direction only. Shear along the crack, known as aggregate interlock, is neglected. The stress-strain relations in the direction of the principal axes are assumed as: \((12,16)\)
\[
\begin{pmatrix}
\sigma'_x \\
\sigma'_y \\
\sigma'_{xy}
\end{pmatrix} =
\begin{bmatrix}
0 & 0 & 0 \\
0 & E_c & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{pmatrix}
\varepsilon'_x \\
\varepsilon'_y \\
\gamma'_{xy}
\end{pmatrix}
\tag{3.3}
\]

where \( \sigma'_x, \sigma'_y, \sigma'_{xy} \) and \( \varepsilon'_x, \varepsilon'_y, \gamma'_{xy} \) are principal stresses and strains, respectively. It is shown in Eq. 3.3 that Poisson's ratio, the shear modulus, and the modulus of elasticity in the maximum principal direction are assumed to be zero. The maximum principal stress which existed just prior to cracking is removed and transferred to the surrounding structure by converting it to unbalanced nodal forces. This will be discussed in Chapter 4, Section 4.3.

After concrete cracks in one direction, and if the tensile stress parallel to the crack direction exceeds the modulus of rupture of concrete, concrete cracks in a second direction. The stress-strain relations of concrete cracked in two direction are assumed as Eq. 3.4.

\[
\begin{pmatrix}
\sigma'_x \\
\sigma'_y \\
\sigma'_{xy}
\end{pmatrix} =
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{pmatrix}
\varepsilon'_x \\
\varepsilon'_y \\
\gamma'_{xy}
\end{pmatrix}
\tag{3.4}
\]

This mean, the concrete element cracked in two directions is incapable of resisting any stress.
3.2 **STEEL**

The stress-strain relations of reinforcing steel are assumed elastic and perfectly-plastic as shown in Fig. 3.2. The yield strength of reinforcing steel \((f_y)\) is considered as the elastic limit in both tension and compression.

3.3 **BOND BETWEEN CONCRETE AND STEEL**

The bond-slip relations between steel and concrete in the direction parallel to the steel axis used by Wilson \(^{(11)}\) and the idealized bond-slip relations for tension reinforcement used in this research are shown by a dotted line and a solid line, respectively, in Fig. 3.3. There are no experimental data on the bond between concrete and steel in the direction perpendicular to the steel axis available. The bond between tension steel reinforcement and concrete in the direction perpendicular to the steel axis is assumed as almost perfect with the ratio of local bond stress to local bond-slip equal to 10 times that of Fig. 3.3 in the elastic range. This assumption is based on the fact that the vertical movement of steel is resisted by the surrounding concrete. Compression steel reinforcement is assumed to have perfect bond with the surrounding concrete in both directions.

3.4 **BOND BETWEEN OVERLAY AND SUBSTRATE**

Latex Modified Concrete is assumed to have perfect bond with the air-entrained concrete if the tensile and shear stresses at the interface are less than the corresponding
FIG. 3.2 Idealized Stress-Strain Curve of Reinforcing Steel
FIG. 3.3 Idealized Bond-Slip Curve
strengths. Cracking along the interface may occur either when the tensile stress or shear stress at the interface exceeds its corresponding strength.
CHAPTER 4

FINITE ELEMENT FORMULATION

In this study the slabs are considered under plane stresses. The finite element representation is based on a slice of the slab of unit width. Three types of elements have been used:

1. Linear isoparametric quadrilateral element which represents concrete
2. One-dimensional bar element which represents reinforcing steel
3. Linkage-element which represents bond-slip relations.

The bar elements which represent the tension steel reinforcement are connected to the quadrilateral elements at the nodes by linkage-elements, while the bar elements which represent the compression steel reinforcement are connected directly to the quadrilateral elements at the nodes. It is assumed that both the bar elements and linkage-elements do not occupy any cross-sectional area of the quadrilateral elements. The reinforced concrete slab and its finite element representation are shown in Fig. 4.1 and Fig. 4.2, respectively.
FIG. 4.1 Reinforced Concrete Slab
FIG. 4.2 Finite Element Representation of Slab
4.1 FORMULATION OF THE PROBLEM\(^{(21,22)}\)

The standard procedure in the displacement method is to assume displacement components internally in the element, \(\{u\}\) as interpolation polynomials between nodal displacements, \(\{q\}\):

\[
\{u\} = [N] \{q\} \tag{4.1}
\]

where \([N]\) is a matrix of interpolation functions, also known as shape functions. The strains in the element, \(\{\varepsilon\}\) are expressed in terms of nodal displacements by using the strain-displacement relations and Eq. 4.1:

\[
\{\varepsilon\} = [B] \{q\} \tag{4.2}
\]

where \([B]\) is obtained by proper differentiation of the shape functions. The stresses in the element, \(\{\sigma\}\) are related to the strains in the element by the constitutive relations:

\[
\{\sigma\} = [C] \{\varepsilon\} \tag{4.3}
\]

The stresses in the element can be expressed in terms of nodal displacements by substituting Eq. 4.2 into Eq. 4.3:

\[
\{\sigma\} = [C] [B] \{q\} \tag{4.4}
\]
The strain energy $U_e$ of the element and the potential energy $W_e$ of the applied loads are obtained by:

$$U_e = \frac{1}{2} \int_{V_e} \{ \varepsilon \}^T \{ \sigma \} \, dV_e \quad \text{(4.5)}$$

$$W_e = -\int_{S_e} \{ u \}^T \{ T \} \, dS_e - \int_{V_e} \{ u \}^T \{ X \} \, dV_e \quad \text{(4.6)}$$

where $V_e$ is the volume of the element, $S_e$ is the surface area of the element, $\{ T \}$ the external surface tractions applied and $\{ X \}$ the body forces. The total potential energy of the element, $\Pi_e$ is the sum of its strain energy and the potential energy of the applied loads:

$$\Pi_e = \frac{1}{2} \int_{V_e} \{ \varepsilon \}^T \{ \sigma \} \, dV_e - \int_{S_e} \{ u \}^T \{ T \} \, dS_e$$

$$- \int_{V_e} \{ u \}^T \{ X \} \, dV_e \quad \text{(4.7)}$$

The total potential energy of the element can be expressed in terms of nodal displacements and element stiffness by substituting Eq. 4.1, Eq. 4.2 and Eq. 4.4 into Eq. 4.7:

$$\Pi_e = \frac{1}{2} \left| q \right|^T \left[ k_e \right] \left| q \right| - \left| q \right|^T \left[ f_e \right] \quad \text{(4.8)}$$
where \( [k_e] = \int_{V_e} [B]^T [C] [B] \, dV_e \) the element stiffness and 
\[
\{f_e\} = \left[ \int_{S_e} [N]^T \{T\} \, dS_e + \int_{V_e} [N]^T \{X\} \, dV_e \right]
\] the load vectors.

The total potential energy, \( \Pi \) of the structure is the sum of the total potential energy of the elements:
\[
\Pi = \sum_{e=1}^{n} \Pi_e = \sum_{e=1}^{n} \left[ \frac{1}{2} \{q\}^T [k_e] \{q\} - \{q\}^T \{f_e\} \right]
\] ------(4.9)

where \( n \) is the total number of elements in the structure.

The direct assemblage of element stiffnesses, nodal displacements and load vectors in Eq. 4.9 yields:
\[
\Pi = \frac{1}{2} \{r\}^T [K] \{r\} - \{r\}^T \{F\}
\] ------(4.10)

where \( [K] \) is the stiffness of the structure, \( \{r\} \) and \( \{F\} \) are the generalized nodal displacements and nodal forces, respectively. Applying the theorem of minimum total potential energy to Eq. 4.10 (variational principle):
\[
\delta \Pi = \left[ \delta \{r\} \right]^T \left[ [K] \{r\} - \{F\} \right] = 0
\] ------(4.11)

where \( \delta \Pi \) is the first variation of the total potential energy and \( \{\delta r\} \) is the first variation of the generalized nodal displacements. The total potential energy is stationary for arbitrary variations of the generalized nodal displacements, the terms in the bracket of Eq. 4.11 must vanish.
\[
\begin{bmatrix}
K
\end{bmatrix}
\begin{bmatrix}
r
\end{bmatrix}
= \begin{bmatrix}
F
\end{bmatrix}
\quad \text{(4.12)}
\]

Eq. 4.12 represents a set of linear algebraic simultaneous equations which can be solved for the generalized nodal displacements. Once the generalized nodal displacements are obtained, displacements, strains and stresses may be computed from Eq. 4.1, Eq. 4.2 and Eq. 4.4, respectively.

4.2 FORMULATION OF ELEMENT STIFFNESSES

The stiffness matrices of the three types of elements used in this study (linear isoparametric quadrilateral element, one-dimensional bar element, linkage-element) are formulated in a form that is convenient for programming.

4.2.1 Linear Isoparametric Quadrilateral Element \((23,24)\)

The natural or local coordinate system of the element, Fig. 4.3a is a set of dimensionless numbers whose magnitude varies from -1 to 1. The global cartesian coordinate system, Fig. 4.3b and local coordinate system are related by:

\[
x(s,t) = \sum_{i=1}^{4} N_i x_i
\]

\[
y(s,t) = \sum_{i=1}^{4} N_i y_i
\]

where \(x_i\) and \(y_i\) are the global coordinates of the nodal points, and the shape functions, \(N_i\) are:

\[N_i = \frac{1}{4} (1-s) (1-t)\]
FIG. 4.3 Linear Isoparametric Quadrilateral Element

a. Local Coordinates

b. Cartesian Coordinates (global)
\[ N_2 = \frac{1}{4} (1+s) (1-t) \]
\[ N_3 = \frac{1}{4} (1+s) (1+t) \]
\[ N_4 = \frac{1}{4} (1-s) (1+t) \]

It can be seen from the above expression that the shape functions, \( N_i \) is equal to one at nodal point \( i \) and zero at all other nodal points. The same shape functions are used to relate the assumed displacements, \( \{ u \} \) and nodal displacements, \( \{ q \} \):

\[
\begin{align*}
  u(s,t) &= \sum_{i=1}^{4} N_i \, q_{ix} \\
  v(s,t) &= \sum_{i=1}^{4} N_i \, q_{iy}
\end{align*}
\]

where \( u \) and \( v \) are the displacements in the \( X \) and \( Y \) directions, respectively, \( q_{ix} \) and \( q_{iy} \) are the nodal displacements in the \( X \) and \( Y \) directions, respectively. The strain displacement relations can be expressed in matrix form as:

\[
\begin{pmatrix}
  \varepsilon_x \\
  \varepsilon_y \\
  \gamma_{xy}
\end{pmatrix}
= \begin{pmatrix}
  u_x \\
  v_y \\
  u_y + v_x
\end{pmatrix}
\]

where \( u_x, u_y, v_x \) and \( v_y \) are the partial derivative of \( u \) and \( v \) with respect to \( x \) or \( y \), which can be obtained by using the chain rule of partial differentiation.
\[
\begin{bmatrix}
  u, s \\
  u, t
\end{bmatrix}
= \begin{bmatrix} J \end{bmatrix}
\begin{bmatrix}
  u, x \\
  u, y
\end{bmatrix}
\tag{4.16}
\]

where \([J]\) is the Jacobian matrix:

\[
[J] = \begin{bmatrix}
  x, s & y, s \\
  x, t & y, t
\end{bmatrix}
\tag{4.17}
\]

Premultiply Eq. 4.16 by the inverse of the Jacobian matrix:

\[
\begin{bmatrix}
  u, x \\
  u, y
\end{bmatrix}
= \frac{1}{|J|} \begin{bmatrix}
  y, t & -y, s \\
  -x, t & x, s
\end{bmatrix}
\begin{bmatrix}
  u, s \\
  u, t
\end{bmatrix}
\tag{4.18}
\]

where \(|J|\) is the determinant of the Jacobian matrix.

\[
|J| = x, s y, t - x, t y, s
\tag{4.19}
\]

Substituting Eq. 4.13 into Eq. 4.19 and rewriting in matrix form:

\[
|J| = \{X\}^T \{N, s\} \{N, t\}^T \{Y\} - \{X\}^T \{N, t\} \{N, s\}^T \{Y\}
\tag{4.20}
\]

where \([X]\) and \([Y]\) are the global coordinates of the nodal points, \([X]^T = [x_1\ x_2\ x_3\ x_4]\), \([Y]^T = [y_1\ y_2\ y_3\ y_4]\) and \(\{N, s\}\) and \(\{N, t\}\) are the partial derivative of the shape functions with respect to \(s\) and \(t\).
Let $K f M = \frac{1}{4} \left[ -(1-t) (1-t) (1+t) -(1+t) \right]$ and $M = \frac{1}{4} \left[ -(1-s) -(1+s) (1+s) (1-s) \right]$

Let $[P] = \left\{ N_s \right\} \left[ N_t \right]^T - \left\{ N_t \right\} \left[ N_s \right]^T$ -----(4.21)

Substituting values of $\left\{ N_s \right\}$ and $\left\{ N_t \right\}$ into Eq. 4.21:

$$[P] = \frac{1}{8} \begin{bmatrix} 0 & 1-t & -s+t & -1+s \\ -1+t & 0 & 1+s & -s-t \\ s-t & -1-s & 0 & 1+t \\ 1-s & s+t & -1-t & 0 \end{bmatrix}$$ -----(4.22)

Eq. 4.20 can be written as:

$$|J| = \left\{ X \right\}^T [P] \left\{ Y \right\}$$ -----(4.23)

Substituting Eq. 4.22 into Eq. 4.23 and carrying out the matrix multiplication:

$$|J| = \frac{1}{8} \left( x_{13} y_{24} - x_{42} y_{31} \right) + s(x_{34} y_{12} - x_{21} y_{43})$$
$$+ t(x_{23} y_{14} - x_{41} y_{32}) \right)$$ -----(4.24)

where $x_{ij} = x_i - x_j$ and $y_{ij} = y_i - y_j$.

Substituting Eq. 4.13 and Eq. 4.14 into Eq. 4.18 and rewriting in matrix form as:
\[
\begin{align*}
\begin{bmatrix}
u_x' \\ u_y'
\end{bmatrix} &= \frac{1}{|J|} \begin{bmatrix}
\end{bmatrix} \begin{bmatrix}
[N_s]^T [q_x] \\ [N_t]^T [q_x]
\end{bmatrix} \\
\begin{bmatrix}
u_x'' \\ u_y''
\end{bmatrix} &= \frac{1}{|J|} \begin{bmatrix}
[y]^T [P] [q_x] \\ [x]^T [P] [q_x]
\end{bmatrix} \\
\begin{bmatrix}
v_x' \\ v_y'
\end{bmatrix} &= \frac{1}{|J|} \begin{bmatrix}
[y]^T [P] [q_y] \\ [x]^T [P] [q_y]
\end{bmatrix}
\end{align*}
\]

where \( \begin{bmatrix} q_x \end{bmatrix}^T = [q_{1x} \ q_{2x} \ q_{3x} \ q_{4x}] \), are the nodal displacements in X-direction. \( \text{Eq. 4.25} \) can be rewritten as:

\[
\begin{align*}
\begin{bmatrix}
u_x' \\ u_y'
\end{bmatrix} &= \frac{1}{|J|} \begin{bmatrix}
\end{bmatrix} \begin{bmatrix}
[q_x]
\end{bmatrix} \\
\begin{bmatrix}
u_x'' \\ u_y''
\end{bmatrix} &= \frac{1}{|J|} \begin{bmatrix}
[y]^T [P] [q_x] \\ [x]^T [P] [q_x]
\end{bmatrix}
\end{align*}
\]

Substituting \( \text{Eq. 4.21} \) into \( \text{Eq. 4.26} \):

\[
\begin{align*}
\begin{bmatrix}
u_x' \\ u_y'
\end{bmatrix} &= \frac{1}{|J|} \begin{bmatrix}
[y]^T [P] [q_x] \\ [x]^T [P] [q_x]
\end{bmatrix} \\
\begin{bmatrix}
v_x' \\ v_y'
\end{bmatrix} &= \frac{1}{|J|} \begin{bmatrix}
[y]^T [P] [q_y] \\ [x]^T [P] [q_y]
\end{bmatrix}
\end{align*}
\]

The partial derivative of \( v \) with respect to \( x \) and \( y \) can be obtained in a similar manner:

\[
\begin{align*}
\begin{bmatrix}
v_x' \\ v_y'
\end{bmatrix} &= \frac{1}{|J|} \begin{bmatrix}
[y]^T [P] [q_y] \\ [x]^T [P] [q_y]
\end{bmatrix}
\end{align*}
\]

where \( \begin{bmatrix} q_y \end{bmatrix}^T = [q_{1y} \ q_{2y} \ q_{3y} \ q_{4y}] \), are the nodal displacements in Y-direction. Substituting \( u, x', u, y' \) and \( v, x', v, y' \) from \( \text{Eq. 4.27} \) and \( \text{Eq. 4.28} \) into \( \text{Eq. 4.15} \) and rearranging the matrix form as:
\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} = \frac{1}{|J|} \begin{bmatrix}
-x \{y\}^T [P] & 0 \\
0 & x \{x\}^T [P] \\
x \{x\}^T [P] & -y \{y\}^T [P]
\end{bmatrix} \begin{bmatrix}
q_x \\
q_y
\end{bmatrix}
\]

where:

\[
x \{x\}^T [P] = \frac{1}{8} \left[(x_{42} + sx_{34} + tx_{23}) (x_{13} + sx_{43} + tx_{41})
(x_{24} + sx_{21} + tx_{14}) (x_{31} + sx_{12} + tx_{32})\right]
\]

\[
y \{y\}^T [P] = \frac{1}{8} \left[(y_{42} + sy_{34} + ty_{23}) (y_{13} + sy_{43} + ty_{41})
(y_{24} + sy_{21} + ty_{14}) (y_{31} + sy_{12} + ty_{32})\right]
\]

Rearranging the right hand side of Eq. 4.29 by changing the columns and rows of the first and second matrix, respectively as follow: 2 to 3, 3 to 5, 4 to 7, 5 to 2, 6 to 4 and 7 to 6, gives:

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} = [B] \begin{bmatrix}q\end{bmatrix}
\]
where:

\[
[B]^T = \frac{1}{8|J|} \begin{bmatrix}
(y_{24} + sy_{43} + ty_{32}) & 0 & (x_{42} + sx_{34} + tx_{23}) \\
0 & (x_{42} + sx_{34} + tx_{23}) & (y_{24} + sy_{43} + ty_{32}) \\
(y_{31} + sy_{34} + ty_{14}) & 0 & (x_{13} + sx_{43} + tx_{41}) \\
0 & (x_{13} + sx_{43} + tx_{41}) & (y_{31} + sy_{34} + ty_{14}) \\
(y_{42} + sy_{12} + ty_{41}) & 0 & (x_{24} + sx_{21} + tx_{14}) \\
0 & (x_{24} + sx_{21} + tx_{14}) & (y_{42} + sy_{12} + ty_{41}) \\
(y_{13} + sy_{21} + ty_{23}) & 0 & (x_{31} + sx_{12} + tx_{32}) \\
0 & (x_{31} + sx_{12} + tx_{32}) & (y_{13} + sy_{21} + ty_{23})
\end{bmatrix}
\]

and \[q^T = [q_{1x} \ q_{1y} \ q_{2x} \ q_{2y} \ q_{3x} \ q_{3y} \ q_{4x} \ q_{4y}]\].

According to the previous presentation, section 4.1, Eq. 4.8, the element stiffness matrix, \([k_e]\) for a unit thickness is obtained by:

\[
[k_e] = \int_{A_e} [B]^T [C] [B] \text{d}A_e
\]

or

\[
[k_e] = \int_{-1}^{1} \int_{-1}^{1} [B]^T [C] [B]|J| \text{d}s \text{d}t
\]  \quad (4.31)

where \(\text{d}A = \text{d}x \text{d}y = |J| \text{d}s \text{d}t\). The numerical integration of Eq. 4.31 is obtained by Gauss quadrature formula:

\[
\int_{-1}^{1} \int_{-1}^{1} f(s,t) \text{d}s \text{d}t = \sum_{i=1}^{n} \sum_{j=1}^{n} a_i b_j f(s_i, t_j) \quad (4.32)
\]
for a 2 x 2 integration points, n = 2, the weighting coefficients, a_i = b_j = 1, and the integration points, s_i = ± \frac{1}{\sqrt{3}}, t_j = ± \frac{1}{\sqrt{3}}. This gives the results of numerical integration of Eq. 4.31 as:

\[ [k_e] = 4 \sum_{i=1}^{4} [B(s_i, t_i)]^T [C] [B(s_i, t_i)] |J(s_i, t_i)| \quad (4.33) \]

where \((s_i, t_i)\) are the four integration points.

For the uncracked element, concrete is considered as a linear isotropic homogeneous material, according to Eq. 3.2:

\[ [C] = \frac{E_c}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \quad (4.34) \]

The stiffness matrix, \([k_e]\) is obtained by Eq. 4.33 with \([C]\) equal to Eq. 4.34.

For the element cracked in one direction, concrete is considered as a linear anisotropic homogeneous material with the constitutive relations in the direction of the principal axes as Eq. 3.3, accordingly:

\[ [C'] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & E_c & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (4.35) \]
The constitutive matrix in the principal direction, \([C']\) can be transformed to the global direction by equating the strain energy stored in the element in the two coordinate systems:

\[
\{\varepsilon\}^T \{C\} \{\varepsilon\} = \{\varepsilon'\}^T \{C'\} \{\varepsilon'\}\\ 
\text{(4.36)}
\]

But the strain in the two coordinate systems are related by:

\[
\{\varepsilon'\} = [T] \{\varepsilon\}\\ 
\text{(4.37)}
\]

where \([T]\) is the strain transformation matrix,

\[
[T] = \begin{bmatrix}
\cos^2\theta & \sin^2\theta & \cos\theta\sin\theta \\
\sin^2\theta & \cos^2\theta & -\cos\theta\sin\theta \\
-2\cos\theta\sin\theta & 2\cos\theta\sin\theta & (\cos^2\theta - \sin^2\theta)
\end{bmatrix}
\text{(4.38)}
\]

and the angle \(\theta\) is measured from the global coordinate \((x, y)\) to the principal coordinate \((x', y')\), in the counter-clockwise direction, see Fig. 4.4. Substituting Eq. 4.37 into the right hand side of Eq. 4.36 yields the constitutive matrix in the global coordinate, \([C]\) as:

\[
[C] = [T]^T \{C'\} \{T\}\\ 
\text{(4.39)}
\]

Substituting Eq. 4.35 and Eq. 4.38 into Eq. 4.39, the constitutive matrix of the element cracked in one direction, in the global coordinate is given by:
FIG. 4.4 Cracked Concrete Element
\[
[C] = E_c \begin{bmatrix}
\sin^4 \theta & \cos^2 \theta \sin^2 \theta & -\cos \theta \sin^3 \theta \\
\cos^2 \theta \sin^2 \theta & \cos^4 \theta & -\cos^3 \theta \sin \theta \\
-\cos \theta \sin^3 \theta & -\cos^3 \theta \sin \theta & \cos^2 \theta \sin^2 \theta
\end{bmatrix}
\]

The stiffness matrix of the element cracked in one direction is obtained by Eq. 4.33 with \([C]\) equal to Eq. 4.40.

For the element cracked in two directions, the constitutive matrix is assumed as a null matrix, Eq. 3.4, this implies that the element stiffness matrix is equal to the null matrix.

4.2.2 Bar element:

The stiffness matrix of the bar element may be obtained in a similar way to the linear isoparametric quadrilateral element. But it is easier to obtain the strain-nodal displacement matrix, \([B]\) directly by considering the bar element in Fig. 4.5 where \(xy\) and \(xy\) are the global and local coordinates, respectively. The strain, \(\varepsilon\) in the bar element is obtained by:

\[
\varepsilon = \frac{1}{L} (\bar{q}_2 - \bar{q}_1)
\]

where \(L\) is the length of the bar element, \(\bar{q}_1\) and \(\bar{q}_2\) are the nodal displacements in the local coordinate parallel to the axis of the bar element. The nodal displacements, \(\bar{q}_1\) and \(\bar{q}_2\) are related to the nodal displacements in the global coordinate by:

\[
\bar{q}_1 = q_{1x} \cos \theta + q_{1y} \sin \theta
\]

\[
\bar{q}_2 = q_{2x} \cos \theta + q_{2y} \sin \theta
\]
FIG. 4.5 Bar Element
where $q_{1x}$, $q_{2x}$ and $q_{1y}$, $q_{2y}$ are the nodal displacements of nodes 1, 2 in the $x$ and $y$ directions, respectively, and $\theta$ is the angle between the two coordinate systems, see Fig. 4.5. Substituting Eq. 4.42 into Eq. 4.41 and writing in matrix form:

$$
\varepsilon = \frac{1}{L} \begin{bmatrix}
-Cos\theta & -Sin\theta & Cos\theta & Sin\theta
\end{bmatrix} \begin{bmatrix}
q_{1x} \\
q_{1y} \\
q_{2x} \\
q_{2y}
\end{bmatrix} \quad (4.43)
$$

From the above equation the strain-nodal displacement matrix, $[B]$ is equal to:

$$
[B] = \frac{1}{L} \begin{bmatrix}
-Cos\theta & -Sin\theta & Cos\theta & Sin\theta
\end{bmatrix} \quad (4.44)
$$

The element stiffness is obtained by:

$$
[k_e] = \int_{V_e} [B]^T[C][B]dV_e = [B]^T[C][B]AL \quad (4.45)
$$

where $A$ is the cross-sectional area of the bar element.

For the element whose stress from the previous loading is less than the yield strength of steel:

$$
[C] = E_s \quad (4.46)
$$

where $E_s$ is the modulus of elasticity of steel. The element stiffness is obtained by substituting Eq. 4.44 and Eq. 4.46 into Eq. 4.45 to give:
\[
\begin{bmatrix}
\mathbf{k_e}
\end{bmatrix} = \frac{AE_s}{L} \begin{bmatrix}
\cos^2 \theta & \cos \theta \sin \theta & -\cos^2 \theta & -\cos \theta \sin \theta \\
\cos \theta \sin \theta & \sin^2 \theta & -\cos \theta \sin \theta & -\sin^2 \theta \\
-\cos^2 \theta & -\cos \theta \sin \theta & \cos^2 \theta & \cos \theta \sin \theta \\
-\cos \theta \sin \theta & -\sin^2 \theta & \cos \theta \sin \theta & \sin^2 \theta
\end{bmatrix}
\]
\[\text{(4.47)}\]

For the element whose stress from the previous loading exceeds the yield strength of steel, \([C] = 0\), this implies that the element stiffness matrix is equal to the null matrix.

4.2.3 Linkage-Element:

The Linkage element has two nodal points, 1 and 2, and it contains two infinitesimal bar elements perpendicular to each other, Fig. 4.6. Each infinitesimal bar element represents the bond in that direction. The local stiffness of the infinitesimal bar element \((k_x^-, k_y^-)\) is obtained by multiplying the ratio of local bond stress and local bond slip by the bond area which the linkage element represented.

The local stiffness of the infinitesimal bar element in the \(\bar{x}\) - direction can be transformed to the global coordinate by Eq. 4.47:

\[
\begin{bmatrix}
\mathbf{k_{ex}}
\end{bmatrix} = k_x^- \begin{bmatrix}
\cos^2 \theta & \cos \theta \sin \theta & -\cos^2 \theta & -\cos \theta \sin \theta \\
\cos \theta \sin \theta & \sin^2 \theta & -\cos \theta \sin \theta & -\sin^2 \theta \\
-\cos^2 \theta & -\cos \theta \sin \theta & \cos^2 \theta & \cos \theta \sin \theta \\
-\cos \theta \sin \theta & -\sin^2 \theta & \cos \theta \sin \theta & \sin^2 \theta
\end{bmatrix}
\]
\[\text{(4.48)}\]
FIG. 4.6 Linkage Element
where \( [k_{ex}] \) is the global stiffness of the infinitesimal bar element in the \( \bar{X} \) - direction. The local stiffness of the infinitesimal bar element in the \( \bar{Y} \) - direction can be transformed to the global coordinate by replacing \( \theta \) by \( \pi/2 + \theta \) in the above transformation matrix:

\[
[k_{ey}] = k_{y} 
\begin{bmatrix}
\sin^2 \theta & -\sin \theta \cos \theta & -\sin^2 \theta & \sin \theta \cos \theta \\
-\sin \theta \cos \theta & \cos^2 \theta & \sin \theta \cos \theta & -\cos^2 \theta \\
-\sin^2 \theta & \sin \theta \cos \theta & \sin^2 \theta & -\sin \theta \cos \theta \\
\sin \theta \cos \theta & -\cos^2 \theta & -\sin \theta \cos \theta & \cos^2 \theta
\end{bmatrix}
\]

-----(4.49)

where \( [k_{ey}] \) is the global stiffness of the infinitesimal bar element in the \( Y \) - direction. The global stiffness of the linkage element, \( [k_{e}] \) is equal to the sum of \( [k_{ex}] \) and \( [k_{ey}] \).

\[
[k_{e}] = [k_{ex}] + [k_{ey}]
\]

-----(4.50)

For the linkage element which represents bond between concrete and steel, the infinitesimal bar elements in the \( \bar{X} \) - direction and \( \bar{Y} \) - direction represent the bond in the directions parallel and perpendicular to the steel reinforcement, respectively. The local stiffness of the infinitesimal bar element in the \( \bar{X} \) - direction, \( k_{x} \) is obtained from Fig. 3.3, the local stiffness of the infinitesimal bar element in the \( \bar{Y} \) - direction, \( k_{y} \) is assumed equal to \( 10 k_{x} \). If the stress in the infinitesimal bar element in the \( \bar{X} \) - direction from
previous loading exceeds the elastic limit (500 psi., see Fig. 3.3), \( k_{X} \) is assumed as zero. Generally the bond stress in the \( Y \) - direction is very much lower than the elastic limit, it is reasonable to assume that \( k_{Y} \) remains constant.

For the linkage element which represents the bond between Latex Modified Concrete and air entrained concrete, the local stiffnesses in both directions (\( k_{X} \), \( k_{Y} \)) are set to be very large values in order to represent perfect bond between the two concrete. If the stress in the infinitesimal bar element in the \( X \) - direction exceeds the shear-bond strength or the stress in the infinitesimal bar element in the \( Y \) - direction exceeds the normal-bond strength both \( k_{X} \) and \( k_{Y} \) are set to zero to represent the crack along the interface.

**4.3 REDISTRIBUTION OF EXCESSIVE STRESSES**

The excessive stresses in the element are the stresses or portion of stresses which the element is unable to carry. For a cracked element the excessive stress is the principal tensile stress perpendicular to the crack direction in the element before crack. For an uncracked element the excessive stresses are the portions of the stresses which exceed the elastic limit of the material (compressive strength of concrete, yield strength of steel etc.). The excessive stresses in the element are redistributed to the remaining elements of the structure by converting it to unbalanced nodal forces applied to the structure. The excessive
stresses in the element may be converted to unbalanced nodal forces by considering the equilibrium equation of the structure obtained in Section 4.1, Eq. 4.12, \[ \{K\} \{r\} = \{F\}, \]
which may be written as:

\[
\sum_{e=1}^{n} \int_{V_e} [B]^T [C][B] \{q\} \, dV_e = \sum_{e=1}^{n} \{f_e\} \tag{4.51}
\]

where \( n \) is the number of elements in the structure and \( \{f_e\} \) the nodal forces vector of each element. Substituting Eq. 4.4 into Eq. 4.51 and considering each element separately, gives:

\[
\int_{V_e} [B]^T \{\sigma\} \, dV_e = \{f_e\} \tag{4.52}
\]

Eq. 4.52 relates the stresses in the element to nodal forces and may be used to convert the excessive stresses to unbalanced nodal forces.

In this study the stresses at the centroid of the element are considered as the average stresses of the element and used as the base in computing excessive stresses. The excessive stress are assumed to be uniformly distributed in the element. From Eq. 4.52, the excessive stresses are converted to unbalanced nodal forces by:

\[
[B]^T \begin{bmatrix} \sigma_{ex} \end{bmatrix} v_e = \{f_{ex}\} \tag{4.53}
\]
where \( \mathbf{B} \) is evaluated at centroid of the element, \( \{ \sigma_{ex} \} \) is the excessive stresses in the global coordinates, \( V_e \) is the volume of the element and \( \{ f_{ex} \} \) the unbalanced nodal forces in the global coordinates. The excessive stresses in the principal coordinates are transformed to the global coordinates by the stress transformation matrix:

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_{xy}
\end{bmatrix}
= \begin{bmatrix}
\cos^2\theta & \sin^2\theta & -2\sin\theta\cos\theta \\
\sin^2\theta & \cos^2\theta & 2\sin\theta\cos\theta \\
\cos\theta\sin\theta & -\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta
\end{bmatrix}
\begin{bmatrix}
\sigma_x' \\
\sigma_y' \\
0
\end{bmatrix}
\]

\[ \text{(4.54)} \]
CHAPTER 5
COMPUTER PROGRAM

5.1 PROGRAM OUTLINE

The computer program is developed for nonlinear analysis of planar reinforced concrete structures where the nonlinear problem is approximated by the incremental step-by-step linear analysis.

According to the idealization of the structural materials presented in Chapter 3, the load-midspan deflection curve may be approximated roughly as shown by a solid line in Fig. 5.1. The horizontal increments of the curve (at loads $P_2$, $P_4$ and $P_8$) represent the increase in deflections with constant loads due to cracking of the slab and resulting reduction of its stiffness. The changes of slope of the load-midspan deflection curve at loads $P_6$, $P_{10}$ etc. may be caused by the yielding of a bar element or a linkage element. The nonlinear behavior in the last portion of the curve starts at load $P_m$, where the maximum compressive stress in concrete attains 85 percent of its cylinder compressive strength and failure occurs when the maximum compressive strain in concrete reaches its ultimate flexural compressive strain (see Fig. 3.1).
FIG. 5.1 Approximation of Load - Deflection Curve
The loading procedure is designed so that the path of the load-midspan deflection relation in the numerical analysis closely follows the approximation of load-midspan deflection curve as shown by the dotted line in Fig. 5.1. The load increments from loads $P_0$ to $P_2$, $P_2$ to $P_4$, $P_4$ to $P_6$ etc. are divided into two parts. A small load increment ($P_1$) is applied first so that the nearest cracking or yielding load ($P_2$) can be approximated, then the remaining load ($P_2 - P_1$) is applied. The path that has been chosen from $P_0$ to $P_m$ is called the variable stiffness approach, because new stiffnesses are used every time there is a new crack or yield. Load increments higher than $P_m$ (where the maximum compressive stress of concrete attains 85 percent its cylinder compressive strength) are equal. The path from $P_m$ to $P_u$ is called the constant stiffness approach because the same structural stiffness is used all the time.

5.2 INCREMENTAL PROCEDURE

The basic steps of the numerical solution adopted may be summarized as follow:

1. Form the structural stiffness and a vector of load increment.

2. The load vector or the multiplication of load vector by a constant is applied and the corresponding displacements, strains and stresses are computed.
3. Update the current displacements, strains and stresses by adding the displacements, strains and stresses that just have been obtained to the existing displacements, strains and stresses.

4. Compute the principal stresses and strains from the updated current stresses and strains. If the magnitude of principal compressive strain in the concrete element is larger than its ultimate compressive strain the execution is terminated.

5. Compute the constant of multiplication for the load vector in step 2.

6. Check whether there is more than one concrete element where the principal tensile stress exceeds the tensile strength of concrete, or if there is any concrete element where the principal compressive stress exceeds the compressive strength of concrete. If not, go to step 10.

7. If the number of iterations are larger than ten, go to step 10.

8. Convert the portions of the stresses in the concrete elements which exceed the tensile and compressive strength of concrete to unbalanced nodal forces.

9. The unbalanced nodal forces from step 8 are applied to the structure and the corresponding displacements, strains and stresses are computed. Go back to step 3.
10. Check for cracking in concrete elements and yielding in bar and linkage elements. If there is no new cracking or yielding, go back to step 2. If there is new cracking or yielding, go to the next step.

11. Form the new structural stiffness and the unbalanced nodal forces due to the new cracking and yielding.

12. Compute displacements, strains and stresses, and go back to step 3.

The flow chart of the basic steps is presented in Fig. 5.2.

5.3 SUBROUTINES

The flow diagram of the subroutines of the computer program is presented in Fig. 5.3. The primary functions of the subroutines are as follows:

- **MAIN** and **INPUT**: read input data.
- **SOLVE**: controls the incremental loading step, forms the structural stiffness matrix from the element stiffness matrices.
- **ELEM**: generates the quadrilateral element stiffness (cracked or uncracked) and the unbalanced nodal forces for the newly cracked element.
- **ONED** and **LINK**: generate the element stiffnesses (elastic or yield) and the unbalanced nodal forces for the bar and linkage elements, respectively.
FIG. 5.2 Flow Chart of The Basic Steps
FIG. 5.3  Flow Diagram of Subroutines of Computer Program
SYMBAN: triangulizes the structural stiffness matrix and back substitution to obtain nodal displacements.

Stress: computes strains, stresses, principal stresses, direction of principal axes and the smallest load that will cause an element to crack or yield.

VECTORS: converts the portions of the stresses in the quadrilateral elements which exceed the tensile and compressive strength of concrete to unbalanced nodal forces and solves the equilibrium equations by using the previous structural stiffness, the iterations are terminated when the number of iterations are larger than ten, and reassigns codes to all elements (crack or yield or elastic).

SKET: shows the locations of the cracked elements.

The listing of the computer program is presented in Appendix I. The development of the computer program is based on Dr. Sandhu and his students' program for linear two-dimensional stress analysis.
CHAPTER 6

EXPERIMENTAL RESULTS

6.1 INTRODUCTION

The composite structural members of conventional reinforced concrete bridge slabs overlayed by a thin layer of Latex Modified Concrete had been tested by the writer and Dr. A. Bishara(1) to study their corrosion resistance and flexural characteristics. In this chapter the flexural characteristics of the test slabs are reviewed. In Chapter 7 the test results will be compared to the analytical results obtained from the finite element analysis developed in Chapters 3-5 .

6.2 TEST PROGRAM

6.2.1 Description of Test Slabs and Test Procedure

Thirty two slabs 72" long, 18" wide and 7\(\frac{1}{4}\)" total height, composed of two layers of concrete were tested. The first layer which had all reinforcing was of normal weight air-entrained concrete Type C, conforming to the Ohio Department of Transportation Specification of concrete for rigid pavement (AEC) . The second layer was of Latex Modified Concrete or Mortar (LMC) with the thickness either 3/4", 1", 1\(\frac{1}{4}\)" or 1\(\frac{1}{2}\)" . The interface between the two layers was rough
with a full amplitude of approximately \( \frac{1}{4} \)". All slabs were reinforced longitudinally by two layers of 3 - #5 at 6" deformed bars of Grade 60 steel and 20 - #4 distribution bars, 13 in the top and 7 in the bottom. A 2" clear cover over the top distribution bars and 1" clear cover under the bottom longitudinal bar was provided. The slabs were tested as simply supported beams of span 69" with a concentrated load at midspan. Most of the slabs were tested with the LMC overlay in the tension zone. However a small number was tested with the overlay in the compression zone. Fig. 6.1 gives details of the test slabs and test set up.

6.2.2 Description of Test Prisms and Test Procedure:

Prisms 3" x 4" x 16" were used to determine the modulus of rupture of concrete and direct shear-bond strength of the LMC overlay cast against the AEC substrate. The modulus of rupture tests were carried out according to ASTM C78. The shear-bond tests were carried out as direct shear tests.

6.3 TEST RESULTS

Results from flexural tests of slabs with LMC overlay either in the tension or the compression zone are tabulated in Tables 1 and 2, respectively, with the following notations:

\[ P_{cr} \]

- measured load at midspan which produced the first visible flexural crack
a. Slabs Tested with Overlay in Compression.
(slabs#11, 15, 18, 24, 25, 30)

b. Slabs Tested with Overlay in Tension.
(slabs#1-10, 12, 13, 14, 16, 17, 19, 20, 21, 22, 23, 26, 29, 31, 32, 33, 34)

where
T is the thickness of latex modified concrete (or mortar),
equal to 0.75, 1.0, 1.25 or 1.5 in.

when
T is less than or equal to 1", it is a latex modified mortar
T is greater than or equal to 1½", it is a latex modified concrete.

FIG. 6.1 Details of Test Slabs and Test Set up.
### TABLE 1 Results from Flexural Tests (Overlay in Tension)

<table>
<thead>
<tr>
<th>Slab No.</th>
<th>Depth of Slab (in.)</th>
<th>( f'_c ) (psi.)</th>
<th>( f_r ) (psi.)</th>
<th>Pu in kips</th>
<th>Deflection at 8 kips (inches)</th>
<th>Max. Crack Width at 8 kips at level of steel (in.)</th>
<th>No. of cracks in overlay at 8 kips</th>
<th>Pu in kips</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,2</td>
<td>7½</td>
<td>6800</td>
<td>467</td>
<td>19.4</td>
<td>0.09</td>
<td>0.0134</td>
<td>5</td>
<td>3.8</td>
</tr>
<tr>
<td>23</td>
<td>7½</td>
<td>5520</td>
<td>556</td>
<td>18.0</td>
<td>0.06</td>
<td>0.0043</td>
<td>1</td>
<td>7.2</td>
</tr>
<tr>
<td>3,4</td>
<td>6½ 3/4&quot;A&quot;</td>
<td>6800 4820</td>
<td>467 638</td>
<td>19.4</td>
<td>0.09</td>
<td>0.0137</td>
<td>1</td>
<td>5.7</td>
</tr>
<tr>
<td>20,21,22</td>
<td>6½ 3/4&quot;A&quot;</td>
<td>4070 6558</td>
<td>512 697</td>
<td>18.0</td>
<td>0.09</td>
<td>0.0129</td>
<td>2</td>
<td>5.3</td>
</tr>
<tr>
<td>17,19</td>
<td>6½ 1 &quot;A&quot;</td>
<td>4070 6558</td>
<td>512 697</td>
<td>17.6</td>
<td>0.09</td>
<td>0.0122</td>
<td>2</td>
<td>4.5</td>
</tr>
<tr>
<td>26</td>
<td>6½ 1 &quot;A&quot;</td>
<td>5520 6180</td>
<td>556 737</td>
<td>19.1</td>
<td>0.07</td>
<td>0.0043</td>
<td>1</td>
<td>7.6</td>
</tr>
<tr>
<td>7,8</td>
<td>6 1½ &quot;A&quot;</td>
<td>6800 4890</td>
<td>467 756</td>
<td>19.8</td>
<td>0.09</td>
<td>0.0124</td>
<td>2</td>
<td>5.5</td>
</tr>
<tr>
<td>14,16</td>
<td>6 1½ &quot;A&quot;</td>
<td>4070 7480</td>
<td>512 734</td>
<td>17.2</td>
<td>0.09</td>
<td>0.0114</td>
<td>2</td>
<td>5.7</td>
</tr>
<tr>
<td>12,13</td>
<td>5 3/4 1½ &quot;A&quot;</td>
<td>4070 7480</td>
<td>512 734</td>
<td>17.9</td>
<td>0.07</td>
<td>0.0040</td>
<td>1</td>
<td>7.1</td>
</tr>
<tr>
<td>5,6</td>
<td>6½ 3/4&quot;B&quot;</td>
<td>6800 6680</td>
<td>467 567</td>
<td>20.3</td>
<td>0.10</td>
<td>0.0130</td>
<td>3</td>
<td>4.6</td>
</tr>
<tr>
<td>29,31</td>
<td>6½ 3/4&quot;B&quot;</td>
<td>5520 7230</td>
<td>556 943</td>
<td>19.3</td>
<td>0.07</td>
<td>0.0122</td>
<td>2</td>
<td>5.0</td>
</tr>
<tr>
<td>9,10</td>
<td>6 1½ &quot;B&quot;</td>
<td>6800 7300</td>
<td>467 650</td>
<td>20.3</td>
<td>0.10</td>
<td>0.0153</td>
<td>4</td>
<td>4.3</td>
</tr>
<tr>
<td>32,33,34</td>
<td>6 1½ &quot;B&quot;</td>
<td>5520 7980</td>
<td>556 642</td>
<td>19.2</td>
<td>0.07</td>
<td>0.0071</td>
<td>2</td>
<td>6.2</td>
</tr>
</tbody>
</table>
### TABLE 2  Results from Flexural Tests (overlay in Compression)

<table>
<thead>
<tr>
<th>Slab No.</th>
<th>Depth of Slab (in.)</th>
<th>$f'_c$ (psi.)</th>
<th>$f_r$ (psi.)</th>
<th>$P_u$ in kips</th>
<th>Deflection at 8 kips (inches)</th>
<th>Max. Crack Width at 8 kips (in.)</th>
<th>No. of cracks at 8 kips</th>
<th>$P_{cr}$ in kips</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>5 3/4</td>
<td>4070</td>
<td>7480</td>
<td>512</td>
<td>35.4</td>
<td>0.06</td>
<td>0.006</td>
<td>3</td>
</tr>
<tr>
<td>15</td>
<td>6</td>
<td>4070</td>
<td>7480</td>
<td>512</td>
<td>33.1</td>
<td>0.05</td>
<td>0.002</td>
<td>3</td>
</tr>
<tr>
<td>18</td>
<td>6 1/2</td>
<td>4070</td>
<td>6558</td>
<td>512</td>
<td>31.0</td>
<td>0.07</td>
<td>0.003</td>
<td>3</td>
</tr>
<tr>
<td>24</td>
<td>7 1/2</td>
<td>----</td>
<td>5520</td>
<td>556</td>
<td>31.6</td>
<td>0.05</td>
<td>0.002</td>
<td>3</td>
</tr>
<tr>
<td>25</td>
<td>6 1/4</td>
<td>5520</td>
<td>6180</td>
<td>556</td>
<td>31.2</td>
<td>0.06</td>
<td>0.003</td>
<td>3</td>
</tr>
<tr>
<td>30</td>
<td>6 3/4</td>
<td>5520</td>
<td>7980</td>
<td>556</td>
<td>31.1</td>
<td>0.06</td>
<td>0.003</td>
<td>3</td>
</tr>
</tbody>
</table>

* Modifier "A" is Styrene Butadiene.

** Modifier "B" is a blend of 75% Saran and 25% Styrene Butadiene.
\( P_u \) = measured load at midspan at the onset of failure

\( f'_{c} \) = concrete compressive strength measured on 6" x 12" cylinders

\( f_r \) = concrete modulus of rupture measured on 3" x 4" x 16" prisms.

The results from shear-bond tests are tabulated in Table 3, the loading diagram is shown below the table. The stress-strain relationships of reinforcing steel (#5 rebar) is shown in Fig. 6.2. The load versus midspan deflection of some slabs with LMC overlay in tension zone is shown in Fig. 6.3.

The following conclusions may be drawn from the analysis of the results obtained from the experimental study:

1. Results in Table 1 show that the average first crack load of slabs with LMC overlay is higher than that of slabs without overlay. The thicknesses of the overlay do not seem to affect the cracking load, which might be affected by shrinkage of LMC overlay.

2. The number of cracks in slabs tested with the LMC overlay in tension are less than the number of cracks in slabs without overlay for any specified load level. Comparison of the number of cracks at a load of 8 kips are shown in Table 1.

3. There is no significant difference in the maximum crack width whether the slab is overlayed or not.
### TABLE 3. Results from Shear Bond Tests

<table>
<thead>
<tr>
<th>Types of Specimens</th>
<th>Shear Bond Strength (psi.) at (weeks)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>AEC. 4&quot; (monolithic)</td>
<td>352</td>
</tr>
<tr>
<td>AEC. 3 1/4&quot; + LMM. 3/4&quot; &quot;A&quot; *</td>
<td>181</td>
</tr>
<tr>
<td>AEC. 2 3/4&quot; + LMC. 1 1/4&quot; &quot;A&quot;</td>
<td>260</td>
</tr>
<tr>
<td>AEC. 3 1/4&quot; + LMM. 3/4&quot; &quot;B&quot; **</td>
<td>387</td>
</tr>
<tr>
<td>AEC. 2 3/4&quot; + LMC. 1 1/4&quot; &quot;B&quot;</td>
<td>389</td>
</tr>
</tbody>
</table>

Each value is an average of 3 specimens.
FIG. 6.2 Stress-Strain Curve of Reinforcing Steel
(# 5 Rebars)
FIG. 6.3 Load-Deflection Diagram of Slabs with Overlay in Tension

- Aver. of Slabs #1 and 2
- Aver. of Slabs #3 and 4
- Aver. of Slabs #7 and 8

First Crack
The average maximum crack width at the level of the center line of tension reinforcing steel at a load of 8 kips was 0.013 in. for the slabs with the overlay in tension (Table 1) and 0.003 in. for the slabs with the overlay in compression (Table 2). It should be noted that the slabs with the overlay in compression have a greater effective depth and smaller clear cover over tension reinforcement.

4. The average deflections at midspan at a load level of 8 kips were 0.09 in. for the slabs with the LMC overlay in tension and 0.06 in. for the slabs with the LMC overlay in compression.

5. The average ultimate midspan load for the slabs in Table 1 is about 19 kips. The average ultimate midspan load for the slabs in Table 2 is about 32 kips. As mentioned before the effective depth of the slabs in Table 2 is larger than that of the slabs in Table 1.

6. Load-deflection curves of slabs with the overlay and slabs without overlay as shown in Fig. 6.3 were almost identical. This is an indication that the LMC overlay and AEC substrate work together as a composite section till failure. However 6 out of 23 slabs which have been tested with the LMC overlay
in tension, have developed some delamination cracks, i.e. the interface between the substrate and the overlay separates, at about 80 percent of ultimate load. All the slabs which had been tested with the LMC overlay in compression showed no sign of delamination.
CHAPTER 7

ANALYSIS OF TEST SLABS

The test slabs presented in Chapter 6 are analyzed using the proposed finite element models developed in Chapters 3-5. The major objective in this analysis is to obtain information on shear stresses and normal stresses at the interface between the overlay and the substrate concretes. The analytical and experimental results are compared to verify the applicability and accuracy of the finite element analysis.

7.1 SLABS WITHOUT OVERLAY

The 72 in. long slabs were tested as simply supported beams of span 69 in. with a concentrated load at midspan. The dimensions of the slab and its reinforcement are shown in Fig. 7.1. Due to symmetry, the analysis is carried out on only half of the slab from midspan to the support. The finite element representation of the slab with node numbers is shown in Fig. 7.2. The element numbers of the finite element representation are shown in Fig. 7.3. The effect of lateral reinforcement (#4 Rebars) is neglected.

The compressive strength of air entrained concrete measured on 6" x 12" cylinders varied from 4000 psi. to 6800 psi. (Table 3, Chapter 6). Therefore two separate finite
FIG. 7.1 Details of Test Slab without Overlay
FIG. 7.2 Finite Element Representation of Slab without Overlay
### (a) Quadrilateral Elements

<table>
<thead>
<tr>
<th></th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
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<tbody>
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<td>9</td>
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</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td></td>
<td></td>
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<td>6</td>
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<td></td>
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<td>71</td>
<td>81</td>
<td>91</td>
<td>101</td>
<td></td>
</tr>
</tbody>
</table>

### (b) Bar Elements and Linkage Elements

<table>
<thead>
<tr>
<th></th>
<th>8</th>
<th>18</th>
<th>28</th>
<th>38</th>
<th>48</th>
<th>58</th>
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<th>78</th>
<th>88</th>
<th>98</th>
<th>108</th>
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<tbody>
<tr>
<td>4</td>
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<td>34</td>
<td>44</td>
<td>54</td>
<td>64</td>
<td>74</td>
<td>84</td>
<td>94</td>
<td>104</td>
<td></td>
</tr>
</tbody>
</table>

**FIG. 7.3 Element Numbers of Slab without Overlay.**
element analyses are made with the compressive strengths of concrete equal to 4000 psi and 6800 psi to study the effect of modulus of elasticity and compressive strength of concrete on the nonlinear behavior of the tested reinforced concrete slabs. The idealized stress-strain relations of structural materials presented in Chapter 3 are used in these analyses with the following material properties:

**Air-Entrained Concrete (Substrate Concrete):**

- Compressive Strength \( (f'_c) \) = 6800 psi and 4000 psi
- Modulus of Elasticity = \( 4.70 \times 10^6 \) psi and \( 3.64 \times 10^6 \) psi
- Elastic Limit \( (0.85 f'_c) \) = 5780 psi and 3400 psi
- Modulus of Rupture = 467 psi
- Ultimate Compressive Strain = 0.008 in./in.
- Poisson's Ratio\(^{(25)}\) = 0.20

**Reinforcing Steel (#5 Rebar):**

- Modulus of Elasticity = \( 29 \times 10^6 \) psi
- Elastic Limit = 65000 psi

**Bond Between Concrete and Tension Steel:**

- Elastic Limit = 500 psi
- Bond Stress/Bond Slip = \( 2 \times 10^6 \) lb/in.

For slabs without overlay, the load versus midspan deflection diagrams of slabs obtained from the finite element analyses and those obtained experimentally are plotted in Fig. 7.4. The load-deflection curves of slabs obtained from the finite element analyses with two different
FIG. 7.4 Comparison of Load-Deflection of Slab without Overlay

- Test Results (Aver. of Slabs #1 and 2)
- F.E.M. with $f_c' = 6800$ psi
- F.E.M. with $f_c' = 4000$ psi
compressive strengths of concrete are quite close and are also close to those obtained experimentally for loads below 15 kips. At loads above 15 kips the maximum compressive stress in concrete obtained from the finite element analysis with compressive strength of concrete equal to 4000 psi reached the concrete elastic limit which caused the load-deflection curve to bend more to the horizontal. In the finite element analysis with compressive strength of concrete equal to 6800 psi, the maximum compressive stress in the concrete reached its elastic limit at a load of 20 kips. This indicates that the effect of modulus of elasticity of concrete on the load-deflection curves obtained from the finite element analyses was small and the effect of the cracks in concrete was predominant.

At loads below 7 kips and above 16 kips the midspan deflections predicted by the finite element analysis with compressive strength of concrete equal to 4000 psi are closer to the experimental results than those with compressive strength of concrete equal to 6800 psi. At loads between 8 kips and 16 kips the contrary is observed. In order to compare the results numerically, midspan deflections of the slab at various loading stages are tabulated in Table 4, where \( \Delta_1 \) and \( \Delta_2 \) are midspan deflections predicted by the finite element analyses with concrete compressive strength equal to 6800 psi and 4000 psi, respectively, and \( \Delta_m \) measured midspan deflections. The
Table 4 Comparison of Load-Deflection Curves

<table>
<thead>
<tr>
<th>Load  (kips)</th>
<th>$\Delta_1$ (in.)</th>
<th>$\Delta_2$ (in.)</th>
<th>$\Delta_m$ (in.)</th>
<th>$\frac{\Delta_1}{\Delta_m}$</th>
<th>$\frac{\Delta_2}{\Delta_m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.010</td>
<td>0.013</td>
<td>0.02</td>
<td>0.50</td>
<td>0.65</td>
</tr>
<tr>
<td>6</td>
<td>0.04</td>
<td>0.04</td>
<td>0.05</td>
<td>0.80</td>
<td>0.80</td>
</tr>
<tr>
<td>8</td>
<td>0.09</td>
<td>0.09</td>
<td>0.09</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>10</td>
<td>0.12</td>
<td>0.13</td>
<td>0.12</td>
<td>1.00</td>
<td>1.08</td>
</tr>
<tr>
<td>12</td>
<td>0.15</td>
<td>0.18</td>
<td>0.15</td>
<td>1.00</td>
<td>1.20</td>
</tr>
<tr>
<td>14</td>
<td>0.20</td>
<td>0.22</td>
<td>0.19</td>
<td>1.05</td>
<td>1.15</td>
</tr>
<tr>
<td>16</td>
<td>0.25</td>
<td>0.27</td>
<td>0.26</td>
<td>0.96</td>
<td>1.04</td>
</tr>
<tr>
<td>18</td>
<td>0.29</td>
<td>0.33</td>
<td>0.39</td>
<td>0.74</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Average 0.88 0.97
averages of $\Delta_1/\Delta_m$ and $\Delta_2/\Delta_m$ are equal to 0.88 and 0.97 respectively.

Loads producing first cracks predicted by the finite element analyses are 5.17 kips and 5.25 kips for the compressive strength of concrete equal to 6800 psi and 4000 psi, respectively, the average measured first crack load of slabs without overlay was about 4.0 kips. Ultimate loads predicted by the finite element analyses are 20.9 kips and 20.1 kips for the compressive strength of concrete equal to 6800 psi and 4000 psi, respectively. At ultimate load, a larger number of concrete elements are cracked and the slab is unstable. The average ultimate load of slabs without overlay obtained from the experiment was about 19.3 kips.

In Fig. 7.5 the load and midspan deflection relations obtained from the finite element analysis (with compressive strength of concrete equal to 4000 psi) and the experiment are compared with the empirical method(19) based on the effective moment of inertia at the midspan section. The effective moment of inertia ($I_e$) is equal to(19):

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \leq I_g \quad \text{---(7.1)}$$

where

$M_{cr} =$ cracking moment  
$M_a =$ maximum moment in member at stage for which deflection is being computed
FIG. 7.5 Comparison of Load-Deflection of Slab without Overlay ($f'_c = 4000$ psi)
\[ I_g = \text{moment of inertia of gross concrete section about the centroidal axis, neglecting the reinforcement} \]
\[ I_{or} = \text{moment of inertia of cracked section transformed to concrete.} \]

The midspan deflections \( (\Delta) \) are computed by:

\[ \Delta = \frac{PL^3}{48E_cI_e} \quad \text{(7.2)} \]

where

\[ P = \text{concentrate load at midspan} \]
\[ L = \text{span length} \]
\[ E_c = \text{modulus of elasticity of concrete} \]
\[ I_e = \text{effective moment of inertia, form Eq. 7.1} \]

At loads below 18 kips midspan deflections of the cracked slab predicted by the empirical method are conservative but reasonable, as shown in Fig. 7.5.

The relationships between loads and stresses in tension steel reinforcement at midspan section obtained from the finite element analyses and the experiment (by using strain gauges) are shown in Fig. 7.6. The sudden variations in the stresses without increasing load correspond to the first crack in the slab. The stresses in the steel predicted by
FIG. 7.6 Comparison of Maximum Stress in Tension Steel

- F.E.M. with $f_c' = 4000$ psi
- F.E.M. with $f_c' = 6800$ psi
- Test Results (Slab #1)
the finite element analyses with two different compressive strengths of concrete are close to each other and both are lower than the experimental results. In the analyses the steel started to yield at loads 19.8 kips and 18.5 kips for compressive strengths of concrete equal to 6800 psi and 4000 psi, respectively. Experimentally tension steel started to yield at a load of 13.8 kips.

The underestimation of steel stresses in the analysis might be attributed to inaccurate evaluation of the first cracking load and the fact that the analysis assumes steel stress to be constant within an element. Also, smoothing the surface of the reinforcing steel before placing the strain gage might cause a local reduction in its cross sectional area.

The crack patterns of slab obtained from the finite element analyses with compressive strengths of the substrate concrete equal to 4000 psi and 6800 psi are almost identical. Only the crack pattern obtained from the finite element analysis for slabs with substrate concrete compressive strength equal to 4000 psi will be compared with the experimental crack pattern. Since results obtained from the finite element analysis with compressive strength of concrete equal to 4000 psi are closer to the experimental results.

The crack pattern of the slab without overlay obtained experimentally is shown in Fig. 7.7. The numbers in the figure near the end of the cracks indicates the loads at which the crack is detected. The loads correspond to the number of loading steps are as follows:
FIG. 7.7 Experimental Crack Pattern of Slab without Overlay
(Slab #1)
<table>
<thead>
<tr>
<th>No. of Step</th>
<th>Load in kips</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>4.3</td>
</tr>
<tr>
<td>5</td>
<td>5.4</td>
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<tr>
<td>6</td>
<td>6.6</td>
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<tr>
<td>7</td>
<td>7.8</td>
</tr>
<tr>
<td>8</td>
<td>8.8</td>
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<tr>
<td>9</td>
<td>10.0</td>
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<tr>
<td>10</td>
<td>11.3</td>
</tr>
<tr>
<td>11</td>
<td>12.3</td>
</tr>
<tr>
<td>12</td>
<td>14.0</td>
</tr>
<tr>
<td>13</td>
<td>15.0</td>
</tr>
<tr>
<td>14</td>
<td>19.5</td>
</tr>
<tr>
<td>F</td>
<td>Failure</td>
</tr>
</tbody>
</table>

The crack pattern is almost symmetric with respect to the load at midspan. There are three major cracks on each side of the load and a secondary crack at midspan at a load equal to 15 kips (step 13). A diagonal crack occurs after the ultimate load have been reached. It extended from the southern end of the slab to about one third of the length of the slab.

At a load of 7.8 kips there are two major creacks on each side of the load (cracks numbering 7 or less in Fig. 7.7). A similar crack pattern is obtained by the finite element analysis at a load of 7.5 kips, as shown in Fig. 7.8a. As the load increases from 7.8 kips to 14.0 kips the existing cracks extend upward to the top of the slab, the third major
(a) At Load 7.5 kips

(b) At Load 12.9 kips

FIG. 7.8 Analytical Crack Pattern of Slab without Overlay
(a) At Load 18.7 kips

(b) At Load 20.1 kips (Ultimate Load)

FIG. 7.9 Analytical Crack Pattern of Slab without Overlay
crack near the southern support starts at a load of 10.0 kips and the third major crack near the northern support occurs at a load of 14.0 kips. At that load there are three major cracks on each side of the load (cracks numbering 12 or less in Fig. 7.7). A similar crack pattern and crack development are obtained from the finite element analysis at a load of 12.9 kips are shown in Fig. 7.8b.

For loads exceeding 14 kips, the existing cracks extend upward, a secondary crack occurs at midspan, and failure is caused by the major diagonal crack. In the finite element analysis for loads between 12.9 kips and 19 kips, there are no new cracks except extension of existing ones. The crack pattern at a load of 18.7 kips is shown in Fig. 7.9a. At the ultimate load cracks extend all over which cause the slab to become an unstable structure, as indicated by the random cracks shown in Fig. 7.9b. The failure mode predicted by the finite element analysis does not seem to produce the seemingly post failure shear bond crack observed experimentally.

7.2 SLABS WITH 1\(\frac{1}{2}\) in. LMC OVERLAY

The slabs with 1\(\frac{1}{2}\) in. Latex Modified Concrete (LMC) overlay were tested as simply supported beams with a concentrated load at midspan with the overlay in tension. The details of the test slabs are shown in Fig. 7.10. The finite element representation of the slab with node numbers is shown in Fig. 7.11. The element numbers of the quadrilateral elements which represent the substrates slab and the LMC overlay are shown in Fig. 7.12a. The element numbers of bar elements
FIG. 7.10 Details of Test Slab with $\frac{1}{2}$ in. LMC Overlay
FIG. 7.11 Finite Element Representation of Slab with 1\% in. LMC Overlay
### FIG. 7.12 Element Numbers of Slab with 1 1/2 in. LMC Overlay

**Part (a) Quadrilateral Elements**

<table>
<thead>
<tr>
<th>11</th>
<th>12</th>
<th>23</th>
<th>34</th>
<th>45</th>
<th>56</th>
<th>67</th>
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<table>
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<td>7</td>
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<td>114</td>
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<tr>
<td>3</td>
<td>113</td>
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<tr>
<td>1</td>
<td>12</td>
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</table>

**Part (b) Bar Elements and Linkage Elements**

<table>
<thead>
<tr>
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<th>20</th>
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<th>75</th>
<th>86</th>
<th>97</th>
<th>106</th>
<th>119</th>
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<td>39</td>
<td>50</td>
<td>61</td>
<td>72</td>
<td>83</td>
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<td>50</td>
<td>71</td>
<td>82</td>
<td>93</td>
<td>104</td>
<td>115</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
<td>24</td>
<td>35</td>
<td>46</td>
<td>57</td>
<td>68</td>
<td>79</td>
<td>90</td>
<td>101</td>
<td>112</td>
</tr>
</tbody>
</table>


and linkage elements are shown in Fig. 12b where the bar elements represent reinforcing steel and the linkage elements represent bond.

The idealized stress-strain relations of the structural materials presented in Chapter 3 are used in this analysis with the following material properties:

<table>
<thead>
<tr>
<th>Material Description</th>
<th>Compressive Strength ($f'_c$)</th>
<th>Modulus of Elasticity</th>
<th>Elastic Limit ($0.85 f'_c$)</th>
<th>Modulus of Rupture</th>
<th>Ultimate Compressive Strain</th>
<th>Poisson's Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air-Entrained Concrete (substrate concrete)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compressive Strength ($f'_c$)</td>
<td>4000</td>
<td>3.64 x 10^6 psi</td>
<td>3400 psi</td>
<td>467 psi</td>
<td>0.008 in./in.</td>
<td>0.20</td>
</tr>
<tr>
<td>Latex Modified Concrete</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compressive Strength ($f'_c$)</td>
<td>4800</td>
<td>3 x 10^6 psi</td>
<td></td>
<td>756 psi</td>
<td></td>
<td>0.20</td>
</tr>
<tr>
<td>Reinforcing Steel</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Modulus of Elasticity</td>
<td>29 x 10^6 psi</td>
<td>65000 psi</td>
<td></td>
<td></td>
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<tr>
<td>Bond Between: Concrete and Positive Steel</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elastic Limit</td>
<td>500</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bond Stress/Bond Slip</td>
<td>$2(10)^6$ lb/in.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Bond Between AEC and LMC:

Shear Bond Strength = 442 psi
Normal Bond Strength = 265 psi

The shear bond strength between the LMC overlay and the substrate is taken from Table 3 of Chapter 3 and the normal bond strength is assumed as 60 percent of the shear bond strength.

The loads versus midspan deflections obtained from the finite element analysis are compared with the experimental results in Fig. 7.13. Deflections seem to be underestimated by the finite element analysis. The change of slope of the load-deflection curve obtained from the finite element analysis at a load of 7.9 kips is due to a crack in a substrate element above the LMC overlay at midspan. At a load of 8.5 kips this crack extended above and below which increased the deflection as shown. The average measured first crack load of slabs with 1\frac{1}{4} in. LMC overlay was 6 kips. At loads exceeding 17.3 kips the load-deflection curve obtained from the finite element analysis is almost horizontal, maximum tensile stress in steel and maximum compressive stress in AEC reach the elastic limit. At a load of 18.4 kips the compressive stress in five elements on the top of the slab reach the elastic limit. Therefore it is considered as the ultimate load. The average ultimate load of the slab with 1\frac{1}{4} in. LMC overlay obtained experimentally was about 19.4 kips.
FIG. 7.13 Comparison of Load-Deflection of Slab with 1\(\frac{1}{4}\) in. LMC Overlay
The relationships between loads and stresses in the tension reinforcement at midspan section obtained from the finite element analysis and the experiment (by using strain gages) are shown in Fig. 7.14. The finite element analysis underestimated the stresses in the steel. According to the finite element analysis the steel starts to yield at a load of 17.3 kips. Experimentally the steel started to yield at a load of 12 kips.

As mentioned before, the reasons for the underestimation of the steel stresses might be the inaccurate evaluation of the first cracking load and the fact that in the analysis the stress in a steel element is assumed constant. Also, smoothing the surface of the reinforcing steel before placing the strain gage might cause stress concentration at that location.

The crack pattern of the slab with 1\(\frac{1}{2}\) in. LMC overlay obtained experimentally is shown in Fig. 7.15. The numbers in the figure near the end of the cracks indicate the loading steps at which the crack is detected. The loads correspond to the number of loading steps are as follows:

<table>
<thead>
<tr>
<th>No. of Step</th>
<th>Load in kips</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>6.3</td>
</tr>
<tr>
<td>6</td>
<td>7.2</td>
</tr>
<tr>
<td>7</td>
<td>9.0</td>
</tr>
<tr>
<td>8</td>
<td>10.1</td>
</tr>
<tr>
<td>9</td>
<td>10.9</td>
</tr>
</tbody>
</table>
FIG. 7.14 Comparison of Maximum Stress in Tension Steel
FIG. 7.15 Experimental Crack Pattern of Slab with 1\(\frac{1}{2}\) in. LMC Overlay (Slab #7)
At a load of 6.3 kips there is one major crack at midspan. As the load increases from 6.3 kips to 12.5 kips the crack at midspan extends upward, leftward and rightward, the second major crack in the southern half starts at a load of 10.9 kips and the third major crack in the northern half starts at a load of 12.5 kips. At loads exceeding 13.7 kips diagonal cracks start in both halves of the slab and failure is caused by the diagonal crack in the northern half at load of 19.4 kips.

In the finite element analysis a crack in a substrate element above LMC at midspan starts at a load of 7.9 kips as shown in Fig. 7.16a. At a load of 8.5 kips this crack extends above and below as shown in Fig. 7.16b. The second crack starts in the substrate at a load of 13.0 kips similar to the first crack and it extends above and below at a load of 13.5 kips. The crack pattern at a load of 13.5 kips is shown in Fig. 7.17a. The crack pattern at a load of 18.4 kips (ultimate load) is shown in Fig. 7.17b. The crack at midspan does not extend horizontally similar to that observed

<table>
<thead>
<tr>
<th>No. of Step</th>
<th>Load in kips</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>12.5</td>
</tr>
<tr>
<td>11</td>
<td>13.7</td>
</tr>
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<td>12</td>
<td>16.4</td>
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<tr>
<td>13</td>
<td>18.6</td>
</tr>
<tr>
<td>F</td>
<td>Failure</td>
</tr>
</tbody>
</table>
FIG. 7.16 Analytical Crack Pattern of Slab with 1\(\frac{1}{2}\) in. LMC Overlay
FIG. 7.17 Analytical Crack Pattern of Slab with 1\(\frac{1}{2}\) in. LMC Overlay
experimentally. A crack similar to the experimental diagonal crack has been obtained by the finite element analysis but it is not the one that caused failure.

The shear stresses along the interface between the LMC overlay and the AEC substrate predicted by the finite element analysis at a load of 7.9 kips are shown in Fig. 7.18. Before cracking the shear stress distribution is almost uniform and its magnitude is about 22 psi, as shown by a solid line in the figure. After a crack is developed in the substrate (see the crack pattern in Fig. 7.16a), the redistribution of the stresses carried by the concrete before cracking, increases the longitudinal stresses in the LMC overlay which in turn results in an increase in the shear stresses along the interface, as shown by a dotted line in Fig. 7.18. The longitudinal stress distributions in the LMC overlay before and after cracking are also shown by a solid line and a dotted line, respectively, in the same figure.

As the load increases from 7.9 kips to 8.5 kips the shear stresses along the interface which do not seem to be affected by the crack development, increase uniformly from about 22 psi to 24 psi. On the other hand, the shear stresses which are affected by the crack development, seem to increase non-uniformly, more so near the crack itself than away from it. The shear stresses along the interface at a load of 8.5 kips before the extension of the crack are shown by a solid line
FIG. 7.18 Distributions of Shear Stress at The Interface and Longitudinal Stress in The LMC Overlay at a Load of 7.9 kips
in Fig. 7.19. As the crack extends into the LMC overlay (see the crack pattern in Fig. 7.16b) the shear stresses near the crack location reverse their sign as shown by a dotted line in Fig. 7.19. This phenomena is due to the fact that the longitudinal stress distribution in the LMC overlay before cracking is maximum at the crack location and decreases at a distance away from it, as shown by a solid line in the figure. After cracking the longitudinal stresses in the cracked element are zero, they reach their maximum value where the longitudinal stresses are not affected by the crack formation, as shown by a dotted line in the figure.

The shear stresses along the interface at a load of 13 kips before and after cracking of the substrate are shown by a solid line and a dotted line, respectively, in Fig. 7.20. The effect of the substrate crack under this load on the shear stress distribution is similar to that at a load of 7.9 kips. The longitudinal stress distributions in the LMC overlay before and after cracking are also shown by a solid line and a dotted line, respectively, in the same figure.

The shear stress distribution at a load of 13.5 kips before and after the extension of the substrate crack into the LMC overlay are shown by a solid line and a dotted line, respectively, in Fig. 7.21. The effect of such a crack on the shear stress distribution is similar to that at a load of 8.5 kips. The longitudinal stress distributions in the LMC
FIG. 7.19 Distributions of Shear Stress at The Interface and Longitudinal Stress in The LMC Overlay at a Load of 8.5 kips
FIG. 7.20 Distributions of Shear Stress at the Interface and Longitudinal Stress in the LMC Overlay at a Load of 13 kips
FIG. 7.21 Distributions of Shear Stress at the Interface and Longitudinal Stress in the LMC Overlay at a Load of 13.5 kips.
overlay before and after cracking are also shown by a solid line and a dotted line, respectively, in the same figure. The shear stress distribution at a load of 17.3 kips is shown in Fig. 7.22.

According to the finite element analysis the maximum shear stress at the interface between the LMC overlay and the AEC substrate is about 235 psi and it occurs at a distance of 20 in. from the support (see Fig. 7.22). The shear bond strength between LMC and AEC obtained from the tests (Table 3) is almost twice this maximum shear stress. The tensile stresses at the interface between the LMC overlay and the AEC substrate are very small before cracking. The maximum tensile stress between the LMC overlay and the AEC substrate is about 36 psi and it occurs near the first crack at midspan at a load of 13.5 kips before the second crack extends into the LMC overlay. Accordingly, cracking does not take place at the interface, according to the finite element analysis.

It is important to note that the shear stress of a cracked slab at the interface between the LMC overlay and the AEC substrate obtained from the beam theory (VQ/Ib) based on the uncracked concrete section (neglecting the reinforcing steel and assumed that the modulus of elasticity of the LMC overlay equal to modulus of elasticity of the AEC substrate) is about 25 percent of the maximum shear stress at the
FIG. 7.22 Shear Stress Distribution at a Load of 17.3 kips
interface obtained from the finite element analysis. For example, at a load of 8.5 kips the shear stress at the interface obtained from the beam theory is equal to 28 psi, and the maximum shear stress at the interface obtained from the finite element analysis is equal to 105 psi (see Fig. 7.19). The shear stress at the interface obtained from the beam theory is close to the shear stress at the interface obtained from the finite element analysis before cracking (which was 24 psi at a load of 8.5 kips).

7.3 EFFECT OF DIFFERENTIAL SHRINKAGE BETWEEN THE OVERLAY CONCRETE AND SUBSTRATE CONCRETE ON THE ANALYSIS OF SLAB WITH 1 1/2 IN. LMC OVERLAY

The 1 1/2 in. LMC overlay was placed about 4 weeks after the AEC substrate had been placed. The differential shrinkage between the LMC overlay and the AEC substrate causes tensile stresses to develop in the LMC overlay before loading. This might be a major source of the discrepancy between the calculated and measured first cracking load of slabs with LMC overlay, presented in Section 7.2. Although shrinkage is a time dependent phenomenon, in this section the same slab with 1 1/2 in. LMC overlay is analyzed by assuming the differential shrinkage between the LMC overlay and the AEC substrate to be uniformly distributed and equal to 0.0002 in./in. before loading.
The initial shear stresses at the interface between the LMC overlay and the AEC substrate and the longitudinal stresses in the LMC overlay due to the assumed differential shrinkage as obtained from the finite element analysis are shown in Fig. 7.23.

In Fig. 7.24, the load-deflection curve obtained from the finite element analysis for the slab including the differential shrinkage between the LMC overlay and the substrate concrete, is compared with the load-deflection curves obtained from the previous analysis (Section 7.2) and that obtained experimentally. The first cracking load of the slab including differential shrinkage effects is 5.6 kips, as compared to 7.9 kips obtained from the previous analysis. The ultimate load is 21 kips which is higher than 18.4 kips obtained from the previous analysis and higher than the 19.4 kips measured ultimate load.

The relationships between loads and stresses in the tension reinforcement at midspan section from the two finite element analyses and the measured values are shown in Fig. 7.25. The stresses in the tension reinforcement of the slab including differential shrinkage are higher than those excluding differential shrinkage at loads below 13 kips.

The shear stress distribution along the interface between the LMC overlay and the AEC substrate at a load of 17.3 kips are shown in Fig. 7.26. The shear stress distribution along the interface of the slab including differential shrinkage is
FIG. 7.23 Initial Shear Stress at The Interface and Longitudinal Stress in The LMC Overlay Due to Differential Shrinkage Between The LMC Overlay and The Substrate Concrete
Test Results (Slabs #7&8)
F.E.M. Without Differential Shrinkage Effects
F.E.M. Including Differential Shrinkage Effects

FIG. 7.24 Load-Deflection Curves of Slabs with 1½ in. LMC Overlay (Including Differential Shrinkage)
FIG. 7.25 Load-Maximum Steel Stress (Including and Excluding Differential Shrinkage Between The LMC Overlay and Substrate Concrete)
FIG. 7.26 Comparison of Shear Stress Distribution at a Load of 17.3 kips
(Including and Excluding Effects of Differential Shrinkage)
shown by a dotted line. The shear stress distribution along the interface without considering differential shrinkage as obtained from the previous analysis is shown by a solid line on the same figure. The maximum shear stress at the interface of the slab including differential shrinkage is less than that computed without considering differential shrinkage.
CHAPTER 8

CONCLUSIONS AND AREAS FOR FUTURE RESEARCH

8.1 CONCLUSIONS

The application of the finite element method to the analysis of reinforced concrete slabs with and without Latex Modified Concrete overlays have been studied. Comparison between analytical and experimental results show that:

1. The load-deflection curves of slabs with and without LMC overlay obtained from the finite element analyses are quite close to those obtained experimentally. Under short term loads, the nonlinear relationships of loads and deflections are essentially caused by cracking of concrete.

2. Load producing first cracks predicted by the finite element analysis are slightly higher than those obtained experimentally. Analysis confirms that first crack loads of slabs with LMC overlays are higher than those for slabs without overlay.

3. The ultimate load of a slab without overlay obtained from the finite element analysis is slightly higher than that obtained experimentally but the ultimate load of slab with LMC overlay obtained from the
finite element analysis is slightly lower than that obtained experimentally.

4. The stresses in the tension reinforcement predicted by the finite element analyses are lower than those measured experimentally. According to the finite element analysis the steel starts to yield at a load of 17.3 kips and 18.5 kips for slabs with and without LMC overlay, respectively. Test results show that the steel start to yield at loads of 12 kips and 13.8 kips for slabs with and without LMC overlay, respectively.

5. The crack patterns of slabs with and without LMC overlay obtained from the finite element analyses are similar to the experimental crack patterns. However, the diagonal cracks which caused failure in the test slabs, have not been predicted by the finite element analysis. The analysis confirms however that number of cracks in slabs with LMC overlay is less than the number of cracks in slabs without overlay.

6. The finite element analysis of slabs with LMC overlay shows that there are no delamination between the LMC overlay and the AEC substrate. The normal stresses at the interface between the overlay and the substrate are small. Nevertheless the shear stresses of the cracked slab at the interface near the crack locations are much higher than what would be expected, using classic composite beam elastic analysis.
7. Differential shrinkage between the LMC overlay and the substrate concrete reduces the cracking load of the slab, also reduces the maximum shear stress at the interface between the LMC overlay and the substrate concrete.

8.2 AREAS FOR FUTURE RESEARCH

The following major areas of research should be considered:

(a) Dynamic Loading:
The actual loading on bridge decks is dynamic in nature. Material properties such as bond between the LMC overlay and the AEC substrate under dynamic loading may differ from those under static loading.

(b) Effects of Shrinkage and Temperature:
A more accurate analysis of shrinkage problem should be considered since shrinkage is a time dependent phenomenon. The combined effects of shrinkage and temperature may result in premature cracking of the overlay.
APPENDIX I

LISTING OF PROGRAM
**ANALYSIS OF COMPOSITE FLEXURAL MEMBER**

**QUADRILATERAL ELEMENT, GAUSS-LICHTER ELEMENT**

**ELASTIC - PERFECTLY PLASTIC STRESS-STRAIN RELATIONSHIPS**

**LINEAR PRESSURE BOUNDARY**

**LINKAGE ELEMENT IX(N,1)=IX(1,3) AND IX(N,2)=IX(2,4)**

**GND IX(N,2)=IX(1,3) AND IX(1,1)=IX(N,4)**

**-----------------------------------------------------------------------**

**IMPLICIT REAL 64 (A-H, O-Z)**

**COMMON PAT, TOT, IA(600)**

**COMMON/NEZ, XI(N), X(N), Y(N), Z(N), D(N), T(N), S(N), R(N,1), R(N,2)**

**COMMON/THI, THI(6), TOP, FACT, PL, C**

**C**

**LOAD = ML, AL, M**

**CALL E (N, 1, 1, 1, 1)**

**DEFINE FILE Z (1, 4, 10)**

**DEFINE FILE Z (1, 4, 10)**

**READ (.1, 1000) HFD, NUMF, NUMEL, NUMMAT, NUMPC, ACER, ACEL, 0**

**WRITE (.1, 1000) HFD, NUMF, NUMEL, NUMMAT, NUMPC, ACER, ACEL, 0**

**NUMT = 100**

**XTOT = 100**

**NPC = NUMPC**

**IF (.1, 1000, 1, 0)**

**PC = 1**

**XI = 1**

**N2 = N1 + NUMF**

**N3 = N2 + NUMF**

**N4 = N3 + NUMF**

**R5 = N4 + NUMP**

**N6 = N5 + NPC**

**N7 = N6 + NPC**

**N8 = N7 + NPC**

**N9 = N8 + NPC**

**N10 = N9 + NPC**

**JJ = N10**

**-----------------------------------------------------------------------**
IF (JJ .LE. 0) GO TO 100
WRITE (6,3000) JJ
CALL EXIT
100 CONTINUE
CALL INPUT (AA(M1), AA(M2), AA(M3), IA(M4), AA(M5),
* IA(M1), IA(M2), IA(M3))
1000 FORMAT (1H4/4, 3F10.2)
2000 FORMAT (1H1/16A4/)
1 3OH0 NUMBER OF NODEAL POINTS------ I3 /
2 3OH0 NUMBER OF ELEMENTS--------- I3 /
3 3OH0 NUMBER OF DIFF. MATERIALS-- I3 /
4 3OH0 NUMBER OF PRESSURE CAPES-- I3 /
5 3OH0 X-ACCELERATION------------- E12.4/
6 3OH0 Y-ACCELERATION------------- E12.4/
7 3OH0 REFERENCE TEMPERATURE------ E12.4/
3000 FORMAT (7OH PROGRAM EXECUTION TERMINATED, REQUIRED CORE EXCEEDS "TRAIN")
*PT BY
END
SUBROUTINE 1

SUBROUTINE INPUT (R, Z, U, KODE, T, P, X, IRC, JRC)

IMPLICIT REAL*8(A-H,O-Z)
COMMON /A000/ IA(800)
COMMON /UMP/ UMP, UMEL, NUKAT, NUMPC, RANG, RUMBLK, MTYPE, N, VOL, ACFL
*ACFLZ, H, IEP (12), NL
*C
COMMON /TWO/ C (2, 3), S(0, 3), SIG(6), P(8), ST(3, 9), RR(4), ZZ(4), LM(4),
*EN(4), XC, YC, E(6, 6), KG(3), ITC(3), S(5), EFS(5),
COMMON/THP/ THP, JTP, IT0, J0, STOP, FACT, ILOAD,
*LOAD, NLOAD, NEX, KOUNT, KNLY
DIMENSION R(0, 2), Z(N, 0), UJ(1, 0), KODE(UMP),
*T(G, M, P), P(0, M, P), I0(X, 0, M, P), LTE(0, P), J0L(0, P),
10, 6 10, N=1, M=1
READ (5, 1C16) NTYPE, ITC(NTYPE), KG(NTYPE),
WRITE (6, 2010) NTYPE, ITC(NTYPE), KG(NTYPE),
50 CONTINUE
WRITE (6, 2020)!
60 READ (5, 1020) N, KODE(N), X(N), Z(N), UJ(N, 1), UJ(N, 2), T(N)
NL=L+1
IF (N.EQ.1) GO TO 70
ZX=N-L
DZ=(Z(N)-Z(L))/ZX
DT=(T(N)-T(L))/ZX
70  L=L+1
IF (N-L) 100, 90, 80
80 KODE(L)=0
KL=K(L-1)+1
Z(L)=Z(L-1)+DZ
T(L)=T(L-1)+DT
UJ(L, 1)=0.
UJ(L, 2)=0.
GO TO 70
90 WRITE (6, 2040) K, KODE(K), RIK, Z(K), UJ(K, 1), UJ(K, 2), T(K), J=NL, 13
IF (J.EQ.MAP) 100, 110, 60
100 WRITE (6, 2050) N
CALL EXIT
SUBROUTINE 1

110 CONTINUE
WRITE (6,2040)
N=0
120 READ (5,1040) M,(IX(M,I),I=1,N)
IX(M,5) = IABS(IX(M,5))
140 N=N+1
IF (N.LE.N) GO TO 170
IX(N,1)=IX(N-1,1)+1
IX(N,2)=IX(N-1,2)+1
IX(N,3)=IX(N-1,3)+1
IX(N,4)=IX(N-1,4)+1
IX(N,5)=IX(N-1,5)
170 WRITE (6,2070) N,(IX(N,I),I=1,N)
IF (K.GT.N) GO TO 140
IF (N.LT.NUMEL) GO TO 130
IF (NUMPC.GT.200) GO TO 210
WRITE (6,2060)
DO 300 L=1,NUMEL
READ (5,1050) IC(L),JC(L),PR(L,1),PR(L,2)
300 CONTINUE
310 CONTINUE
J=0
DO 340 : = 1,NUMEL
DO 340 J=1,N
DO 340 L=1,N
KK=IABS(IX(N,I)-IX(N,L))
IF (KK.GT.J) J=KK
340 CONTINUE
MBAND=2*J+2
WRITE (6,3010) MBAND
LEQ=Z*NUMEL
LL=(NMIN-4980-N6+1)/MBAND
IF (LL.GT.(NEG+2)) NL=NEQ+2
NL=NL/4
NL=4*NL
IF (NL.LT.0) GO TO 250
NL2=NL
IF (NL.GT.250) GO TO 250
NL2=NL
WRITE (6,4010) NL2,MBAND
SUBROUTINE I

CALL EXIT

250 CONTINUE
N7=N6+NEQ
N8=N7+KEQ
N9=N8+NEQ
N10=N9+KEQ
N11=N10+W Keep Mand
JJ=11+INT
IF(JJ.LE.4) GO TO 360
WRITE (6,3050) JJ
CALL EXIT

360 CONTINUE
WRITE(6,4000) N11, H5
CALL SOLVL (RZ, UU, KODE, TPF, AA(N6), AA(N7), AA(N8), AA(N9), AA(N10),
  *IX, 1GC, JHC)
RETURN

1010 FORMAT (215, 1F10.2)
1020 FORMAT (6F10.5)
1030 FORMAT (215, 5F10.5)
1040 FORMAT (6F15)
1050 FORMAT (215, 2F10.3)
2010 FORMAT (17Hmaterial number= I2, 20H, number of temperature cards= I2)
1 15, 16D, MSL Density= E14.4}
2020 FORMAT (13Htemperature 10X 5HE 9X 6HEU 10X 5HEA 12X I6)
*20P1 1X 2HFC/Z/F15.2, 5E15.5))
2030 FORMAT (10Hinitial point type X ordinate y ordinate X load or displacement y load or displacement temperature )
1051 FORMAT (2112, 92F12.6, 2F24.7, 5F17.3)
2050 FORMAT (6F10.5)
2060 FORMAT (45Hgeneral point card error for n= 15)
2061 FORMAT (I6Helement number x j material)
2070 FORMAT (1112, 16, 1312)
2080 FORMAT (25H0P0 PRESSURE BOUNDARY CONDITIONS. I4H i j pressure)
15 F10.4)
2090 FORMAT (4F10.0)
2100 FORMAT (4F10.0)
3000 FORMAT (20H fail with-------------
3010 FORMAT (70H program execution terminated. required core exceeds INT))
"01 BY
4000 FORMAT (47H for this program the location used in aa is = I5, INT)
*17H and in ia is = I
4010 FORMAT (25HEq hl is less than 2*MAND
  */10123
SUBROUTINE 1

*SHO 'IL=
*PHG MBAND=
END
SUBLROUTINE _2


IMPLICIT REAL*A-H,N-Z)

COMMON AA(6000), IA(6000)


*ACFLZ,E,HEC(16),NL*

*ACFL*.

COMMON/IND(C(3,3),S*)

COMMON/VFL,VC(6),P(E),ST(3,1),PR(4),2Z(4),LX(4)

*EF(5),XL,YC,E(6,12),K1(12),MTC(9),S(E),FFS(E)

COMMON/THKF,MTN*,VTCT,NS*,MF*,STOP*,FAX*,DLOAD*

*ILOAD*,"LN0","XCT",KOUNT*,KNT*

DIMENSION (RQMP),Z* (NUMKP),UU* (NUMERG),IX* (NUMLL*,E) ,IBC* (NPC),JBC* (NPC)

DIMENSION : UL,STRAND*,R(REQ),PO(REQ),SR(REQ),PR(REQ)

R(0)=0.0

REWIND F

NFZ=NL

NL=NL/2

N2=C/2

STOP=0.0

GO.1 N=1,REC

BR(0)=0.0

51 ED(0)=C.

IF (NUMPC) 70,120,70

70 GO.10 I=1..NUMPC

I=ILC(L)

J=JBC(L)

K1=KSKF(I)

KDJ=KSKC(J)

CE=Z(I)-Z(J)

P2=K(J)-K(I)

PPZ=PR(L,2)+PR(L,1)/6.

PP1=PP2+PR(L,1)/6.

PPZ=PP2+PR(L,2)/6.

II=1

JJ=7*X

KK=10

IF (KDI=GE*KK) G0 TO 80

KDI=KDI+10

F0+11-0)=F0+11-0)+PR1*DR

80 KKK=KK/10

KDI=KDI-10

SOLV 1

SOLV 2

SOLV 3

SOLV 4

SOLV 5

SOLV 6

SOLV 7

SOLV 8

SOLV 9

SOLV 10

SOLV 11

SOLV 12

SOLV 13

SOLV 14

SOLV 15

SOLV 16

SOLV 17

SOLV 18

SOLV 19

SOLV 20

SOLV 21

SOLV 22

SOLV 23

SOLV 24

SOLV 25

SOLV 26

SOLV 27

SOLV 28

SOLV 29

SOLV 30

SOLV 31

SOLV 32

SOLV 33

SOLV 34

SOLV 35

SOLV 36

SOLV 37

SOLV 38

SOLV 39

SOLV 40

SOLV 41
SUBROUTINE 2

IF (KDI*GE.*KKK) GO TO 90
PO(I1)=FO(I1)+PP1*DZ
90  KKK=10
IF (KDI*GE.*KKK) GO TO 100
KDJ=KDJ+10
FO(JJ-1)=FJN(JJ-1)+FF2*DR
100  KKK=KKK/10
KJ=KJ-10
IF (KDJ*GE.*KKK) GO TO 110
PO(JJ)=FO(JJ)+PF2*DZ
110  CONTINUE
120  CONTINUE
DO 140 I=1,140,L
K=Z*N-1
KKK=0
KP=KDF(N)
DO 150 J=1,2
IF (KP*GE.*KKK) GO TO 130
PO(J)=FC(J)+UU(K,H)
KP=KP+10
CO TO 140
130  PO(K)=0.
140  KKK=KKK/10
KD=KD-10;
K=K+1
150  CONTINUE
WRITE (1) (FO(I), I=1, MC)
GO TO 157 N= 1, MUXEL
I1 = N,
FIN(1,10)
KLOAD = 0
MLOAD = 0
MEX=0
GO 151 I = 1,6
151  SIG(I) = 0.0
GO 154 I = 1,7
154  FJS(I) = 0.0
WRITE(1,10) (SIG(I),I=1,6),(FJS(I),I=1,5),KLOAD,MLOAD,MEX
152  CONTINUE
KPST=0
KPST=0

SOLV 42
SOLV 43
SOLV 44
SOLV 45
SOLV 46
SOLV 47
SOLV 48
SOLV 49
SOLV 50
SOLV 51
SOLV 52
SOLV 53
SOLV 54
SOLV 55
SOLV 56
SOLV 57
SOLV 58
SOLV 59
SOLV 60
SOLV 61
SOLV 62
SOLV 63
SOLV 64
SOLV 65
SOLV 66
SOLV 67
SOLV 68
SOLV 69
SOLV 70
SOLV 71
SOLV 72
SOLV 73
SOLV 74
SOLV 75
SOLV 76
SOLV 77
SOLV 78
SOLV 79
SOLV 80
SOLV 81
SOLV 82

121
SUBCUTINE 2

DO 500 III = 1,200
LOAD = III
IF (LOAD .LE. .FO.1) GO TO 155
IF(MLEO.EQ.0) GO TO 158
KOUNT=KOUNT+1
KNUM=G
GO TO 155

155 REIND 6
KREAD (5)(50(I), I=1, NEQ)
IF(KNUM.GT.0) GO TO 159
PLAD=PLAD+PLAD
KNUM=1
IF(PLAD.GT.26000.0) KNUM=0
GO TO 450

159 DO 10 I=1,NLE
10 60(I)=50(I)*FACT*1.01
PLAD=PLAD+PLAD
KNUM=0
GO TO 450

156 DO 157 I=1,NEQ
157 66(I)=0.6
RKC=RL
NL=SL/2
N魫=NL/2
KMAXLK=0
DO 50 N=1,NLE
50 N=1,N NUMLK
50 M=1,NEQ
AMX.M)=0.6
REIND 1
N NUMLK=NUMLK+1
N魫=N魫(NUMLK+1)
WRL=NW+8
N魫=N魫+1
KSHIFT=ZK+Z-2
DO 240 N=1,NUMEL
IN=IX(N,1)
DO 160 I=2,4
IF (IX(N,1).LT.NN) NN=IX(N,1)
160 CONTINUE
SUBROUTINE 2

IF((NN.LT.12).OR.(NN.GT.NM)) GO TO 240
IF (IX(N,2).NE.1X(N,1)) GO TO 170
CALL LINK (K,Z,UU, *KOE,T,P3,A,E,IX,IBC,JE)
NM=2
GO TO 260
170 IF (IX(1,3).NE.1X(N,2)) GO TO 190
CALL DQED (K,Z,UU, *KOE,T,PR,A,E,IX,IBC,JE)
NM=2
GO TO 260
150 CALL ELMEN (K,Z,UU, *KOE,T,PR,A,E,IX,IBC,JE)
IF (VOL.GT.0.) GO TO 190
WRITE (*,2000) N
STOP=1.0.
190 NM=4
200 DO 210 I=1,NM
210 MJ=M(I)+2*(I-1)-2
DO 230 J=1,NM
DO 230 K=1,MJ
II=LJ(I)+K-1
JJ=LJ(J)+L+1-1-K
LL=Z*J-Z*L
IF (JJ.LT.C) GO TO 230
IF (JJ.GT.C) GO TO 220
WRITE (*,2000) N
STOP=1.0.
GO TO 240
220 A(I,JJ)=A(I,JJ)+S(KK,LL)
220 CONTINUE
240 CONTINUE
WRITE (2) ((A(K,M)+M=1,MMAND),N=1,ND)
DO 420 K=1,ND
K=K+NU
GO TO 420
M=M+1
A(K,M)=A(K,M)
420 A(K,M)=A(K,M)
IF (NM.LT.NUMP) GO TO 60
SUBROUTINE 

SUBROUTINE ELEMEN (R, Z, JJU, KODE, T, PR, A, B, IX, IBC, JSC)  

IMPLICIT REAL*8 (A-H, O-Z)  

COMMON A (ILOCC), IA(ILOO)  
COMMON/JUMP/, KUMP, TNUME, NUMMAT, JUMPC, ME AND, NUMBLK, MTYPE, N, VORL, ACER, L, NL  
*ACER, L, NL  
*JUMP, I, N  
COMMON/THC(1), THC(1), THC(1), THC(1), THC(1), THC(1), THC(1), THC(1)  
*THC(1), THC(1), THC(1), THC(1), THC(1), THC(1), THC(1), THC(1)  
*THC(1), THC(1), THC(1), THC(1), THC(1), THC(1), THC(1), THC(1)  
DIMENSION MAT (4, 4), V (N), X (N), Y (N), Z (N), M (N), K (N), J (N), T (N)  
DIMENSION THC (4), THC (4), THC (4), THC (4), THC (4), THC (4), THC (4), THC (4)  
DATA HI/1.00000, 1.00000, 1.00000, 1.00000, 1.00000, 1.00000, 1.00000, 1.00000/  
1  S5 = -0.4077550256916967  
2  S5 = -0.4077550256916967  
4  S5 = -0.4077550256916967  
   IP =  
   F1 = 0.0  
   IF(IP) = 0.0  
   READ (*, IVI) (SIG (I), I = 1, N), (ESC (I), I = 1, N), KLOAD, MLOAD, MX  
10 I = IX(N+1)  
J = IX(N+2)  
K = IX(N+3)  
L = IX(N+4)  
M TYPE = IX(N+5)  
10 40 I = 1, N  
NPP = IX(N+7)  
KRI = R (NPP)  
40 Z2II = R (NPP)  
   VOL =  
   TEMP = (T (I) + T (J) + T (K) + T (L)) / 4.0  
   KATI = 0.0  
   NUTC = NUTC (MTYPE)  
IF (NUTC .EQ. 1) GO TO 90  
GO TO 90  
   SEC = 2.4, NUTC  
SUBROUTINE 3
SUBROUTINE 3

IF ( (E(M,1,MYTYPE),GE,TEMP) GO TO 60
60 CONTINUE
60 DIF=K(E(M,1,MYTYPE),E(M-1,1,MYTYPE)
IF (DIF,EC(0),) GO TO 70
KATIOI=(TEMP-E(M-1,1,MYTYPE))/DIF
70 DO 80 KK=1,3
80 (E(KK)=E(N-1,1,MYTYPE)+KATIOI*(E(M,KK+1,MYTYPE)-E(M-1,KK+1,MYTYPE)
80 GO TO 110
90 DO 100 KK=1,5
100 EF(KK)=E(1,1,MYTYPE)
110 COMM=EE(1)/(1.-EF(2)*2)
IF (KLDIF,CT.6) GO TO 111
C(1,1)=COMM
C(1,2)=C(1,1)/EF(2)
C(1,3)=C(1,1)/EF(2)
C(2,1)=C(1,2)
C(2,2)=C(1,1)
C(2,3)=C(1,2)
C(3,2)=C(1,3)
C(3,3)=C(1,2)+COM**2*(1.-EE(2))
110 GO TO 119
111 COMC=COM**2*(SIG(6))/57.296
COS=COMC*(SIG(6))/57.296
IF (KLDIF,CT.1) GO TO 117
C
CONSTITUTIVE RELATION FOR CRACK ELEMENT
C(1,1)=E(1,1)*SIN**4
C(1,2)=E(1,1)*COS**2*SIN**2
C(1,3)=E(1,1)*COS**3
C(2,1)=C(1,2)
C(2,2)=E(1,1)*COS**4
C(2,3)=E(1,1)*SIN*COS**3
C(3,1)=C(1,3)
C(3,2)=C(1,2)
C(3,3)=C(1,2)
119 GO TO 119
117 DO 119 J=1,3
119 DO 120 J=1,3
120 GO TO 119
<table>
<thead>
<tr>
<th>Line</th>
<th>Code</th>
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<tbody>
<tr>
<td>120</td>
<td>[ S(I,J) = 0. ]</td>
</tr>
<tr>
<td></td>
<td>[ X42 = X42 + X34 * S + X23 * ETA. ]</td>
</tr>
<tr>
<td></td>
<td>[ X41 = X41 + X22 * S + X34 * ETA. ]</td>
</tr>
<tr>
<td></td>
<td>[ X13 = X13 + X42 * S + X41 * ETA. ]</td>
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<tr>
<td></td>
<td>[ X34 = X3 + X41 * S + X23 * ETA. ]</td>
</tr>
<tr>
<td>200</td>
<td>[ X12 = X12 + X13 * S + X34 * ETA. ]</td>
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</tbody>
</table>
SUBROUTINE 2

DO 240 L=1,6
LD 240 I=1,3
DO 240 J=1,2

240 S(K,L)=(ST(I,K)*D(I,J)*ST(J,L))/XJ
CONTINUE

DO 160 J=1,3
DO 160 I=1,2

160 S(I,J)=0.
XJ=2.*VCL
PX(1)=X42
PX(2)=X13
PX(3)=-PX(1)
FY(1)=Y24
FY(2)=Y21
FY(3)=FY(1)
FY(4)=-FY(2)

LD 160 L=1,4
LL=2*L
ST(I,LL-1)=FY(L)/XJ
ST(I,LL)=PX(L)/XJ
ST(I,LL+1)=X(L)/XJ

310 EX=SIG(4)
GO TO 370
320 EX=SIG(5)
GO TO 380

370 FX(1)=COS**2*EX
FX(2)=SIG**2*EX
FX(3)=COS*SIG*FX
ST(1)=ST(1)+FX(1)
SIG(2)=SIG(2)+FX(2)
SIG(3)=SIG(3)+FX(3)
GO TO 400

380 FX(1)=SIN**2*EX
SUBROUTINE 3

\[
\begin{align*}
FEX(2) &= \cos^2 \times 2 \times EX \\
FEX(3) &= -\cos \times \sin \times EX \\
SIG(1) &= SIG(1) - FEX(1) \\
SIG(2) &= SIG(2) - FEX(2) \\
SIG(3) &= SIG(3) - FEX(3)
\end{align*}
\]

400 IF 420 I = 1,6 THEN 410 J = 1,2

410 PI(I) = P(I) + SIG(J,I) \times FEX(J)

420 PI(I) = P(I) \times VDL

GO TO 540

WRITE (4,10) (SIG(I), I=1,6), (FFS(I), I=1,5), KLOAD, MLOAD, MEX

GO TO 540

500 IX = TE*P - Q

DX = E(2) * LT

DY = E(3) * U1

SIG(1) = -C(1,1) \times DX - C(1,2) \times DY

SIG(2) = -C(2,1) \times DX - C(2,2) \times DY

SIG(3) = 0.

GO TO 520

P(I) = 0.0

GO TO 410

510 PI(I) = PI(I) - SIG(J,I) \times SIG(J)

520 PI(I) = P(I) \times VDL

KMM = 4

MXX = 4

DX = VOL \times AC \times L2 \times KD(MTYPE) / XMM

DX = VOL \times AC \times LN(MTYPE) / XMM

GO TO 530

I = 1, 2

530 PI(2*I-1) = PI(2*I) \times DY

540 GO TO 520

NPP = IX(1,1)

KD = KONE(NPP)

KKK = 10

II = 2*I-1

GO TO 560 J = 1, 2

IF (KD.LT.KKK) GO TO 570

GO TO 560

IF (JJ.JT.KKK) GO TO 570

GO TO 560

JJ.I = II(JJ,JT) \times U(NPP,J) \times S(JJ,IT)

S(JJ,JT) = 0.

560 \text{STOP}
SUBROUTINE 3

P(II)=UU(NPP,J)
S(II,II)=1.
KD=KD-10
570 KKK=KKK/10
II=II+1
600 CONTINUE
610 RETURN
1000 FORMAT(I10/(10X,3F20.8))
END
SUBROUTINE 4

SUBROUTINE LINK (KZ,IU,KODE,IPR,PR,A,B,I5C) 
IMPLICIT REAL*8(A-H,O-Z) 
COMMON AA(6600), IA(6600) 
COMMON/ONE/NUMP,HNUM,NUME,NUMAT,NUMPC,MNDM,NUMBLK,MTYPE,N,VOL,AC,LNL 
*ACE=N1,H1(NL),NL 
*NUMC,NPC 
COMMON/TWO/C(2,3),S1,R1,E1,F1,ST(3,8),SR(8),SS(4),ZZ(4),LM(4),L1 
*E(1),XL,YC,L(6,12),RC(12),MIC(8),SK(5),EPS(5) 
COMMON/THREE/MTOT,MT5,MT,MTS,STOP,FACT,PL squirrel 
*LOAD,PL,LOD,NEXT,KOUNT,KNUM 
DIMENSION K(UMAP),Z(UMAP),IU(UMAP),4.0,KODE(UMAP), 
*TN(UMAP),FR(MPC,2),IX(UMAP,5),IEC(MPC),JEC(MPC) 
*DIMENSION A(NL,MNDM),B(NL) 
ID = N 
FIND(4'1D) 
DO 100 I=1,8 
F(I)=0.6 
DO 100 J=1,4 
100 S(I,J)=0.6 
MTYPE=IX(N+5) 
I=IX(N+1) 
J=IX(N+2) 
GO TO 120 
120 IF(K(IK)=1,1+1,MTYPE) 
IF(AL[4*IL] SIG(T),I=1,6,(EPS(I),I=1,5),KLOAD,LOAD,ME 
IF(KLOAD.LT.6) GO TO 140 
C/K=FC(1)*E(I) 
C/K=FR(2)*E(3) 
GO TO 160 
140 C/K=0.0 
C/K=FR(2)*E(3) 
IF(I(N+5)*.T.5) CKV=0.0 
160 S(1,1)=C/K 
S(1,2)=0.0 
S(1,3)=C/K 
S(1,4)=0.0 
S(2,1)=S(1,1) 
S(2,2)=C/K 
S(2,3)=0.0 
S(2,4)=C/K 
S(3,1)=S(1,2)
SUBROUTINE 4

S(3,2) = S(2,3)
S(3,3) = S(1,3)
S(3,4) = S(1,4)
S(4,1) = S(1,4)
S(4,2) = S(2,4)
S(4,3) = S(3,4)
S(4,4) = S(2,2)

IF (KLOAD = 10, 0) GO TO 200
IF (XIN(5,5) = 0.5) GO TO 196
IF (SIG(4)) IF (C, 200, 180)

160 EX = SIG(4) - EE(4)
SIG(4) = EE(4)
GO TO 160

190 EX = SIG(4) + EE(4)
SIG(4) = -EE(4)

192 P(1) = -EX*EE(3)
P(2) = 0.0
P(3) = -P(1)
P(4) = 0.0

196 MLOAD = 0

RETURN

END
SUBROUTINE ONEP (R, Z, UU, KONE, T, PER, A, B, IX, IPC, ISC)

IMPLICIT REAL*8(A-H, O-Z)
COMMON AA(600), IA(600)
COMMON/DONEZ, UUX, NUMEL, NUPM, MBRAND, NUBLK, MTYPE, N, VOL, ACFRLQUNI,
*ACFZ, G, HFL(19), RL N
Common/TWO/C(12, 3), $S(*, 6), SIG(6), P(*), ST(3, 8), RR(4), ZZ(4), LN(4), GMC,
*EF(5), XE, YC, E(*, 12), LON(12), VTC(6), SS(5), EFFC(5)
COMMON/THREE/FACT, NTOT, X6, X7, STOP, FACT, LOAD
*LOAD, CLOT, LEX(KNOT), KNUM
DIMENSION X(NUMP, 1, Z(NUMP), UU(NUMP, 2), KONE(NUMP),
*T(NUMP), PR(NPC, 2), IX(NUMP, 5), IPC(NPC), JEC(NPC)
DIMENSION AIL, MBAND, B(IPEG)

IF = N
END(4, 19)
DO 100 J = 1, 8
P(I) = 0.6
DO 100 J = 1, 8
100 S(I, J) = 0.6
MTYPE = IX(N, 5)
I = IX(N, 1)
I1 = IX(N, 2)
I2 = IX(N, 7)
I3 = IX(N, 3)
I4 = IX(N, 4)
I5 = IX(N, 5)
X = DSKRT((IX**2 + IY**2)
COSA = DX/XL
SINA = IY/XL
RADI (E**L) (SIG(I), I = 1, 6), (EFS(I), I = 1, 5), KLOAD, VLOAD, VSK
IF (KLOAD = ST, 0) GO TO 120
COMME = (1, 2, MTYPE)*E(1, 1, MTYPE)/XL
GO TO 120
120 COMM = 0.6
130 S11, J) = COSA * SINA * COMM
S11, 1) = COSA * SINAA * COMM
S11, 3) = -S(1, 2)
S11, 5) = -S(1, 2)
S12, 1) = S(1, 2)
S12, 2) = S11A * SINAA * COMM
S12, 3) = -S(1, 2)
S12, 4) = -S(1, 2)
S13, 1) = S(1, 2)
SUBROUTINE 5

S(3,2)=S(2,3)
S(3,3)=S(1,1)
S(3,4)=S(1,2)
S(4,1)=S(1,4)
S(4,2)=S(2,4)
S(4,3)=S(2,2)
S(4,4)=S(2,2)

C CONVERT THE EXCESS STRESS INTO NODAL FORCES
IF (MLOAD .LE. 0) GO TO 200
EE(5)=EE(1,6,MTYPE)
IF(SIG(4)) 150, 200, 140
140 EX = SIG(4) - EE(5)
GO TO 160
150 EX = SIG(4) + EE(5)
GO TO 160
160 EE(3) = F(1,4,MTYPE)
P(1) = -EX * F(3) * COSA
P(2) = -EX * F(3) * SINA
P(3) = -P(1)
P(4) = -P(2)
WRITE(6,1) (SIG(I),I=1,6),(EFS(I),I=1,5),KLOAD,MLOAD,MEX
200 DO 250 I=1,7
PP=PP(I)
KK=KONE(KP)
KKK=10
II=II+1
IF (II .LE. J) GO TO 250
IF(KP .LE. KKK) GO TO 250
PP=PP(I)
II=II+1
240 S(IJ,II)=0
P(II)=PP*F(JJ)*F(JJ)
S(IJ,II)=1*
KK=KKK/10
II=II+1
300 CONTINUE
SUBROUTINE 5

RETURN
END
SUBROUTINE SYMBAN (B, A, MBAND, NUMELK, KKK, NE, NK)
IMPLICIT REAL*8 (A-H, O-Z)
DIMENSION A(NK+1,MBAND), N(NEQ)
N=1/NK
N=1/NL+1
A=1/NN
GO TO (1000,2000), KKK
1000 INERED 2
GO TO 150
150 N=1/N+1
DO 125 N=1, NN
M=M+1
DO 125 N=1, MBAND
A(N,M)=A(N,M)
125 A(N,M)=0.
C
IF (NUMELK-KK. 150, 200, 150
150 IF (M. 1, MBAND) 150, 200, 150
IF (N. 100, 200
200 DO 300 N=1, NN
IF (A(N,1)) 225, 200, 225
225 DO 275 L=2, MBAND
IF (A(N,L)) 225, 275, 230
230 C=A(N,L)/A(N,1)
I=N-L-1
J=0
DO 250 K=L, MBAND
J=J+1
250 A(I,J)=A(I,J)-C*A(N,K)
A(N,L)=C
275 CONTINUE
300 CONTINUE
WRITE(1) ((A(N,M), N=1, MBAND), M=1, NN)
IF (NUMELK-NK) 350, 900, 350
350 GO TO 100
2000 NN=0
IF =0
GO TO 450
SUBROUTINE 6

400 NB=NB+1
405 GO TO 425
410 N=N+1,NN
415 NB=NB+N
420 GO TO 425
425 M=1,NBAND
430 A/N,M/=A/N,N,M
425 A(N,M,M)=0.
430 IF(NUMBLK=NL) 450,500,450
450 READ (1) ((A(N,M),M=1,NBAND),N=NL,NN)
460 IF(N') 500,400,500
500 GO TO 540
505 N=1,NN
510 J=NU+N
515 L=2,MBand
520 IF(NEC-I) 545,540,540
540 B(I)=S(I)-A(N,L)*I(I)
545 IF(A(N,1),ECI.N.) A(N,1)=1.
550 E(I)=S(I)^/-N,N,1)
555 IF('NUMBLK=NL) 660,0,600
600 N=N+1
605 GO TO 400
650 BACKSPACE 1
700 IF(N=750) 705,750,700
705 N=MIN1-M
710 J=NU+N
715 L=2,MBand
720 IF(A(N,L)) 740,750,740
740 I=J+1
745 IF(NEC-I) 750,745,745
750 CONTINUE
755 K=K-1
760 IF(NK) 775,500,775
775 BACKSPACE 1
800 IF(NK=1,NN)
805 NK=NK+1
810 GO TO 800
815 M=1,MBand
820 A(N,M)=A(N,N)
825 A(N,M)=N.
830 IFAD(I)((A(N,M),M=1,MBand),N=I,NN)
835 BACKSPACE 1
840 Q=Q=Q-N!
SUBROUTINE 6

GO TO 700
900 RETURN
END
SUBROUTINE 7

SUBROUTINE VECT(R*,Z*,UU,KODE,T,PR,A,B,B*,DB,IX,IBC,JEC,NQ2) VECT 1
IMPLICIT REAL*(A-H,O-Z) VECT 2
COMMON (A(1000),IA(1000)) EGF VECT 3
COMMON/HNUMP,NUMEL,NUMMAT,BAND,NUMPLK,NUTYPE,N,VOL,ACTL) VECT 4
* ACEL2,F1,F11(16),NL VECT 5
* NPC,TUC VECT 6
COMMON/HNUMP(3,3),(I,1,F),(P,F),SIG(6),P(6),ST(3*,P),DK(4),Z(4),LM(4)) VECT 7
* EE,(E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,E,
SUBROUTINE 7

DO 10 I=1,NEQ

10  BO(I)=0.0

DO 300 N=1,NUMEL

IF (IX(N,2).EQ.IX(N,3)) GO TO 300

ID = N

IF (ID(4).EQ.15)

READ (4*IC) (SIG(I), I=1,6), (EE(I), I=1,5), XLOAD, MLOAD, MEX

IF (MEX.EQ.0) GO TO 300

DO 70 I=1,6

20 P(I)=0.0

M_TYPE=IX(N,3)

DO 100 KK=1,5

100 IF (EX(K)) = F(K)+1,M_TYPE)

DO 200 I=1,5

200  EX=SIG(4)-F(4)

IF (MEX.EQ.0) EX=SIG(4)+EE(5)

FLX(1) = COS*VE*EX

FLX(2) = SIN*VE*EX

FLX(3) = COS*ST*FX

SIG(1) = SIG(1)-FLX(1)

SIG(2) = SIG(2)-FLX(2)

SIG(3) = SIG(3)-FLX(3)

IF (MEX.EQ.0) GO TO 220

MEX=0

GO TO 220

200  EX=SIG(5)-EE(4)

IF (MEX.EQ.0) EX=SIG(5)+EE(5)

FX(1) = SIN*VE*EX

FX(2) = COS*VE*EX

FFX(3) = -COS*ST*EX

SIG(1) = SIG(1)-FFX(1)

SIG(2) = SIG(2)-FFX(2)

SIG(3) = SIG(3)-FFX(3)

MEX=0

GO TO 220

220  DO 225 I=1,6
SUEROUTINE 7

DO 225 J=1,3
225 P(I) = P(I) + ST(J,I) * FEX(J)
IF (MEX.EQ.9) GO TO 200
GO TO 250 I=1,8
230 P(I) = P(I) * VOL
DO 240 I=1,4
KPP=IX(I,1)
KD=KDEF(KPP)
KKK=10
II=2*I-1
DO 240 J=1,2
IF (KD.LT.KKK) GO TO 235
P(I) = 0.0
K=KD-10
235 KK=KKK/10
II=II+1
240 CONTINUE
MR=9
GO TO 250
245 IF (PA<.5 * (SIG(4)) .LT. EE(4)) GO TO 300
IF (SIG(4) .LT. C.G) EX=SIG(4)+EE(4)
IF (SIG(4) .GT. 0.0) EX=SIG(4)-EE(4)
P(I) = -EX*EE(3)
P(4) = 0.0
F(3) = -P(1)
P(4) = 0.0
SIG(4) = SIG(4)-EX
MEX=0
MK=2
250 GO TO 260 I=1,MR
260 LR(II)=2*IA(I)+1-2
GO 280 I=1,MR
FC 280 K=1,2
KK=2*I-2+K
IK=LM(I)+K
ER(IK)=CO(IK)+P(KK)
280 CONTINUE
:RITL(4,II) (SIG(I),I=1,6) (EFS(I),I=1,5),KLOAD,MLOAD,MEX
300 CONTINUE
GO TO 1
500 GO TO 600 J=1,NUMEL
SUBROUTINE 7

ID = N
FIND(4*ID)
MSTYLE=IX(N*5)
GO TO 510 KK=1,5

510 IF(KK)=F(1,5K+1,MTYPE)
READ (4*ID) (SIG(I),I=1,6),(EEF(I),I=1,5),KLOAD,MLOAD,MAX
*EX*0
IF(IX(N,1).LE. IX(N,3) AND IX(N,2).LE. IX(N,3)) GO TO 540
IF(KLOAD.NE.0) GO TO 530
IF(IX(N,5).LT.5) GO TO 515
IF(SIG(5).LT.F(5)) GO TO 515
KLOAD=1
MLOAD=1
GO TO 550

515 IF(SIG(4).LT.0.0) GO TO 520
IF(SIG(4).LT. EE(4)) GO TO 520
KLOAD=KLOAD+1
MLOAD=1
GO TO 560

520 IF((EEF(4)+SIG(4)).GT.0.0) GO TO 530
KLOAD=KLOAD+1
MLOAD=1
GO TO 560

530 MLOAD=0
GO TO 550

540 IF(KLOAD.NE.0) GO TO 560
IF(EXT.6L.1) IF(4)=EF(4)X1.004
IF(SIG(4).LT. EE(4)) GO TO 580
KLOAD = KLOAD+1
MLOAD = 1
GO TO 560

560 IF(KLOAD.GT.1) GO TO 580
IF(SIG(5).LT. EE(4)) GO TO 580
KLOAD = KLOAD+1
MLOAD = 1
GO TO 560

580 MLOAD = 0
KLOAD = KLOAD
MLOT = KLOT + MLOAD

CONTINUE

VECT124
VECT125
VECT126
VECT127
VECT128
VECT129
VECT130
VECT131
VECT132
VECT133
VECT134
VECT135
VECT136
VECT137
VECT138
VECT139
VECT140
VECT141
VECT142
VECT143
VECT144
VECT145
VECT146
VECT147
VECT148
VECT149
VECT150
VECT151
VECT152
VECT153
VECT154
VECT155
VECT156
VECT157
VECT158
VECT159
VECT160
VECT161
SUBROUTINE 7

IF (NLOT GT 0) CALL SKETCH
RETURN
2000 FORMAT(1H1,10X,'LOADING NO.*12.4X,'LOAD = ',F10.4,
*4X,'MODIF NO.*12.4X,'ITERA NO.*12 //
*1X,'N,P. NUMBER*16X,'HX*,16X,'VY* // (1110,2F20.7))
3000 FORMAT(1H1,12X,'LOADING NO.*12// 1X,'N,P. NUMBER*16X,'RX*,18X,
*9Y* // (1110,2F20.7))
END
SUBROUTINE P

SUBROUTINE STRESS (R, Z, UU, KODE, I, PR, A, P, IX, TRC, JRC)
IMPLICIT REAL*8 (A-H, O-Z)
COMMON AA(800), IA(800)
COMMON/ONE/UIK, NUMEL, NUMMAT, NUMPC, MBAND, NUMPLK, MTYPE, N, VOL, ACELP
* ACELZ, Q, HED(10), NL
* RPC, NRO
COMMON/TWO/C(3, 3), S(3, 3), SIG(6), P(6), ST(3, 3), PR(4), ZZ(4), L(14), STPM
* EL(5), XG, YC(5, 12), R(12), NT(3, 1), CS(5), EPS(5)
COMMON/THR/NTOT, NT(6, 5), STOP, FACT, PLAD
* ID, LOAD, KLDT, MIXT, KOUNT, KNUM
DIMENSION K(100, 100), Z(NUMNP), UU(NUMNP, 2), KODE(NUMNP),
* IX(NUMEL), PR(NPC, 2), IX(NUMEL), JRC(NPC), IRC(NPC)
DIMENSION A(NL, MBAND), SINENQ
FACT=1.0000, 0
STOP=0.0
MPRINT=0
P(0, 1)=0
DO 300 I=1, NUMEL
IC=*
FIND (4, I)
FINC (3, I)
MTYPE=IX(N, 5)
DO 40 KK=1, 5
40 FF(KK) = F(I, KK+1, MTYPE)
DO 50 I = 1, 5
50 SG(I) = 0.0
READ (4, I) (SIG(I), I=1, 6), (EPS(I), I=1, 5), KLOAD, MLOAD, NEX
IF (IX(N, 3) NE IX(N, 1)) GO TO 60
I=IX(N, 1)
J=IX(N, 7)
XC=(R(I)+K(J))/2.0
YC=(Z(I)+Z(J))/2.0
DO=B(2#J-1) E(2#I-1)
EV=(2#J-1) (2#I)
EPS(4)=EFS(4)+EV
F=KLOAD, M, IX(N, 5) .GT. 5 GO TO 200
IF (KLOAD, LE. 0) GO TO 50
SG(4)=SIG(4)+FF(1)
52 SG(4)=SIG(4)+SG(4)
SUBROUTINE 8

IF(KNUM.EQ.0) GO TO 260
IF(IN1.EQ.IX(IN,3)) GO TO 260
IF(KLOAD.GT.0) GO TO 260
IF(SIG(4).LE.0.0) GO TO 260
FACT=(EE(4)-SIG(4))/SIG(4)
IF(FACT.LE.1.0) FACT=FACT

260 WRITE(6,2000) IN, X, Y, SIG(4), KLOAD, MEX
WRITE(4,10) SIG(I), I=1,6), (FFS(I), I=1,5), KLOAD, MLOAD, MEX
300 CONTINUE
IF(INLOAD.GT.3) FACT=FACT
IF(SIG .EQ. 0.0) CALL EXIT
RETURN

7000 FORMAT (7HILL.NO. 7X 1HX 7X 1HY 4X 8HXX-STRESS 4X 8HXY-STRESS 3X
1 8HXY-STRESS 2X 10HMAX-STRESS 2X 10HMIN-STRESS 7H ANGLE 4X
2 8HLOAD 3X 8MXX)
2010 FORMAT (11,2E14.4,1PE12.4,6P1E7.2,2I7)
2020 FORMAT (5I7,5F10.6)
2030 FORMAT (5I10)
END
SUBROUTINE 9

SUBROUTINE SKETCH
IMPLICIT REAL*8(A-H,O-Z)
COMMON/CNL/NUMPL,RUMAT,NUMPC,KBAND,NUMBLK,MTYPE,N,VOL,ACEL,SKET
*,ACELZ,C,HED(16),NL
*,TPC,MFG
COMMON/TWO/2(2,3),S(8,F),SIG(6),P(F),ST(3,P),RR(4),ZZ(4),LM(4),SKET
*,RL(5),XL,YL,E(R,6,12),RO(12),NTC(8),SG(5),FFS(5)
COMMON/THREE/NTOT,NTOT,NT,MM,STOP,FACT,PLDAM
*,ILoad,KLOT,KEXT,KOUNT,KNUM
DIMENSION KET(160)
KLOT=0
DO 10 N=1,NUMPL
ID = N
FIND(4,IP)
READ (4,IP) (SIG(I),I=1,6),(FFS(I),I=1,5),KLOAD,MLOAD,MEX
KLOT=N
KLOT=KLOT+KLOAD
10 CONTINUE
WRITE(6,3000) ILOAD,PLDAM
KLOT=0
DO 20 I=1,11
IF(I.EQ.3,.OR.,I.EQ.6,.OR.,OR.,I.EQ.10) GO TO 15
WRITE(6,3010) (KET(J*I-M),J=1,11)
15 M=M+1
20 CONTINUE
IF(KLOT.GT.50) CALL EXIT
RETURN
3000 FORMAT(1H1,10X,'CRACK PATTERN AFTER LOADING NO.',I7,
*X,'LOAD = ',F10.4,'/')
3010 FORMAT(1X,213,317,615)
END
LIST OF REFERENCES


8. Hannant, D. J., Failure Criteria for Concrete in Compression, Mag. of Concrete Research, V. 20, No. 64, September 1968, pp. 137-144.


25. Neville, M. A., Hardened Concrete: Physical and Mechanical Aspects, ACI Monograph No. 6, American Concrete Institute, 1971.