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AN ORGANIZATIONAL CAPITAL BUDGETING
DECENTRALIZATION SYSTEM

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

Ed M. Sharon, M.B.A., M.S., B.S.

* * * * *

The Ohio State University
1976

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Adviser
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To my dear wife Hana.
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To my dear wife Hana my special thanks for her constant encouragement, support and patience.
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LIST OF SYMBOLS

APW - Alternative net Present Worth of funds applied for by a proposal, had they been invested in an alternative opportunity ($).

B - Budget (capital funds) available for investment ($).

DC - Divisional Cost incurred during the generation process of a single proposal ($).

ENOG - Expected Net Organizational Gain from capital-budgeting and investment ($).

HC - Headquarters Consideration Cost per proposal, incurred during a centralized reviewing process ($).

I - Initial Outlay required for the complete execution of a proposal for investment ($).

IC - Implementation Cost per proposal, incurred during a reorganization process of decision authority delegation ($).

i,j,k - Indices generally used to identify different sources of proposals (segments of opportunity space, cells).

LBE - Lower Break Even point along the "pa" axis, associated with two modes of decentralization (dimensionless).

n - Number of proposals submitted to a periodic batch of proposals by a source (units).

OENOV - Overall Expected Net Organizational Value of funds when associated with a proposal mix ($).
\( pA \) - Likelihood of a single proposal being accepted by headquarters in a centralized mode (dimensionless).

\( pa \) - Variance in Acceptability perceptions of division and headquarters (dimensionless).

\( pd \) - Likelihood of a single proposal being randomly drawn from a proposals batch (dimensionless).

\( ps \) - Probability of Survival of a single proposal during a centralized process of headquarters consideration (dimensionless).

\( PW \) - Present net Worth of a mutually (division and headquarters) acceptable proposal ($).

\( PWR \) - Present net Worth of an unacceptable proposal in headquarters perception (Rejected), which is acceptable in the divisional perception ($).

\( r \) - Symbol to indicate a Random selection process, outcome of which is uncertain.

\( R \) - Reward credited to a source of proposals upon acceptance of a single proposal of such origin by headquarters ($).

\( S \) - Index to identify the type of Sample drawn at random.

\( TDC \) - Total Divisional Cost incurred by divisions during the generation process of a combination of proposals ($).

\( U \) - Utility value associated with a dollar valued concept (dimensionless).

\( UBE \) - Upper Break Even point along "pa" axis, associated with two modes of decentralization (dimensionless).
$V_C$ - Gross Value of funds when associated with a Centralized single step decentralization mode, and a given proposals mix ($\$\$).

$V_A$ - Gross Value of funds when associated with a Complete Positive Decentralized single step decentralization mode, and a given proposals mix ($\$\$).

$V_R$ - Gross Value of funds when associated with a Complete Negative Decentralization single step decentralization mode, and a given proposals mix ($\$\$).

* - Symbol to indicate optimization or an optimal decision (optimal solution).
CHAPTER I
INTRODUCTION

1.0 The Problem

The issue of capital budgeting decentralization is in the intersection between the capital-budgeting issue, and the decentralization issue. By organizational decentralization we customarily refer to as the selection of some degree of decision authority delegation, in a multi-divisional organization (10, 11, 12, 13, 15, 37).* By capital budgeting we customarily refer to as the process of planning expenditures, whose uncertain results are expected to extend beyond one year (54).

Much work was done in an effort to determine the trade-offs in capital budgeting decentralization. The results from these efforts are in the form of sets of observations of a more qualitative nature, such as the work of Dearden (11, 12, 13, 14) and others. As a matter of fact, many of these discussions suggest that the problem of the proper degree of decentralization is non-trivial (37), and call for limits in capital budgeting decentralization (13). Unfortunately, not much help is offered, or suggestions made, as to the problem of measurement and explicit setting of these limits.

* Numbers in parentheses refer to Bibliography.
Two exceptions are the Size-Gate model suggested by Morris (37) and the Rate of Return model suggested by Litzenberger and Joy (32), both of which make very important steps in the right direction. But apart from the Morris and Litzenberger models no other explicit mathematical models are presented, as to deal more precisely with the problem.

A review of the capital-budgeting bibliography reveals that explicit mathematical models are suggested for dealing with sub-issues such as:

1) Discounting for time (2, 15, 25, 43, 50, 53, 54).
2) Interest rates (35).
3) Uncertainty (risk) (4, 5, 9, 21, 22, 26, 30, 31, 42, 49, 52, 54, 55).
4) Sensitivity (24, 42, 48).
5) Interaction with financing (39, 40, 41).
6) Value of information (38).
7) Allocation of resources in a fully decentralized firm (8).

The following characteristics are found to be very typical and common among these models:

1) Narrow definition of the boundaries of the sub-issue, which implies that a complex capital-budgeting problem can be resolved by separation into isolated sub-problems. Thus, each sub-problem will be solved independently from the others, and the combination of all solutions will represent the overall solution to
the complex problem. There still remains the question about the validity of the independence assumption.

2) Looking at the sources of uncertainty and variation outside the organization (in the environment) and not at such sources in the inside. For example: uncertainty and variation in divisional performance is not modeled.

3) De-emphasizing the unique and non-uniform role of the divisions in the capital budgeting process, in addition to the role of headquarters. As a matter of fact headquarters are never capable of dealing directly with the environment, but through the intermediating services of divisions (or agents).

4) Overlooking the decentralization (to a degree) economical option. For example, Hansssmann's book (18) titled "Operations Research Techniques for Capital Investment" says nothing about delegation of decision authority.

On the other hand, a review of the centralization-decentralization bibliography reveals that mathematical models in different formats do exist, however for specific decentralization problems such as:

1) Inventory (37, 45).

2) Transportation and distribution (28, 29).

Although these models take the simultaneous systems approach for dealing with the multi-dimensional problem, many of the
underlying assumptions make the models situation-specific in terms of external validity to capital budgeting decentralization.

As the problem of capital budgeting in large organizations becomes more and more complex, slight shifts in degrees of decentralization can cause significant impacts on the welfare of the firm. Evidence is available, which suggests that this problem is perhaps the most important (and conceptually complex) decision with which financial management is involved (54).

Haynes and Solomon (19) suggest that precise computation of the relative worth of investment alternatives is a misplaced emphasis in capital budgeting. They believe that emphasis should be placed in search for investment opportunities, search for information and estimation of incremental gains.

Kellough makes the observation that:

Traditionally, capital budgeting theory has centered attention on evaluation and selection, assuming that all alternatives were given, and that complete knowledge existed. However, when capital budgeting is to be both effective and efficient, control must be implemented over the total process by which proposals are derived, evaluated and committed (27).

Perhaps because of this reason Anderson (1) and other authors in managerial business positions have expressed the need for practical, explicit structured procedures, for efficient management and control of capital-budgeting processes, including the decentralization and search aspects of such.

This expressed need, when coupled with available qualitative observations, leads one to believe that quantitative systems approach to organizational capital-budgeting management is a goal
which might be worth looking for, as to provide a good answer to an important problem.

1.1 Typical Symptoms of the Problem under Study

The following phenomena can be observed in organizations, due to the lack of explicit guidelines for the proper degree of capital budgeting authority delegation:

1.1.1 The "hunting" phenomenon (37)

In contrast with a relatively efficient evolutionary growing process, organization oscillates from one level of decentralization to another. This conduct of a series of unsuccessful experiments, which fails to achieve a long run equilibrium, is regarded as the "hunting" phenomenon, and is accounted toward organizational inefficiency.

1.1.2 The "rubber stamp" effect (37)

This effect is very typical to highly centralized organizations, where headquarters have very little to contribute to the final capital-budgeting decisions. Thus, in many cases there is a very high likelihood for divisional proposals to be approved by headquarters. In this case, the costly (in terms of real headquarters reviewing cost, and in terms of delay and possible loss of investment opportunities) centralized systems are questionable.
1.1.3 The "I am the one division" phenomenon (3, 11, 13, 14, 37, 44)

In a highly decentralized organization, one may observe employment of financial resources in ways that:

1) Are not in line with overall organizational policy.
2) Do not maximize organizational objectives.
3) Do not take advantage of possible complementarities through coordination among divisions.
4) Assume different degrees of risk than acceptable by the organization.

Organizational policy (constraints), objectives, complementarities and risk attitude in this discussion are those perceived by headquarters (dominant coalition (51), chief executive, et cetera), as the responsible body in respect to the owners of the firm.

1.1.4 The "multiplets" phenomenon (twins, triplets, quadruplets, et cetera)

In many organizations, where divisional activity is, more or less, similar in nature (such as bank branches), there is a strong tendency to assume equal performance, and therefore to delegate equal authority. This, of course, may not be necessarily the case, and different divisional decision-makers are more likely to perform differently, thus an "equity" approach may not be the best one.
1.1.5 The "single dimension" phenomenon

Very frequently there is a tendency to view decision performance along one single dimension only (32, 37). For example—commercial banks delegate loan approval authority to their officers along one "loan size" dimension only. This approach, although relatively simple, fails to capture differences of performance along other important dimensions, which, in some cases, could be very relevant for improvement of total organizational performance.

1.1.6 The "performance continuity" phenomenon

Organizations tend to believe that decision performance, when measured along one dimension, is continuously acceptable up to an upper limit. From this point and on, performance is unacceptable. For example: Loan officers in commercial banks are delegated the approval authority up to an upper limit of loan size. As with previous assumptions, the assumption about continuity of performance may, or may not, be valid in specific situations. It is but reasonable to assume that in the general case, decision performance might differ at different ranges of any one dimension, and even more likely, in a multi-dimensional space.

At this point it would be important to point out that the "Gate Size" approach for capital budgeting decentralization suggested by Morris (37), is making the implicit assumptions of "single dimension" and "performance
continuity" as in 1.1.5 and 1.1.6, thus limiting validity to specific situations.

The model suggested by Carleton, Kendall and Tandon (8), assumes a complete decentralized corporate structure, thus fails to recognize the need for limits to decentralization. Furthermore, it fails to recognize the difficulties in making corporate constraints explicit (what do we need screening for, if explicit constraints are available?). Also, it explicitly assumes that corporate objectives are the algebraic sum of divisional objectives in a decentralized firm, an assumption which is challenged by many (3, 11, 13, 14, 37, 44), therefore its validity should be questionable.

Finally, the Rate of Return model suggested by Litz- enberger and Joy (32) suffers from the "single dimension" and "performance continuity" weaknesses, as well as from insufficient attention to the decision problem's constraints.

1.2 Issues to Be Addressed

Having the definitions of the problem as in 1.0 and its symptoms as in 1.1, one ought to address himself to the following issues:

1.2.1 What kind of problem is the problem under consideration in terms of objectives, constraints, boundaries and uncertainty?

1.2.2 Whose problem is it?
1.2.3 How can we model variations in perceptions of objectives between a division-manager and the chief-executive?

1.2.4 Similarly to 1.2.3, but in respect to variation in perceptions of constraints? (Note: These two issues represent the observation that divisional and corporate interests might differ (13, 14)).

1.2.5 How are we going to capture the effect of possible duplication in corporate efforts, in terms of headquarters proposal-consideration efforts on top of divisional efforts of the same nature?

1.2.6 How are we going to capture the negative effect of time-delay in decision making, in a highly centralized organization? (See 1.1.2.)

1.2.7 How are we going to differentiate among the different divisions' performances? (Note: This issue represents the observation that all divisional performances are not necessarily equal, as in 1.1.4.)

1.2.8 How are we going to model different dimensions in decision performance (see 1.1.5) and discontinuities within the dimensions? (See 1.1.6.)

1.2.9 What are the impacts of changes outside the boundaries of the problem (such as availability of funds, changes in economic environment, et cetera) on delegation decisions which are made within the boundaries?

1.2.10 Is the delegation of decision authority to the division associated with any other decision such as coordination of
divisional search efforts for information and/or investment opportunities?

1.2.11 How are we going to handle economies-of-scale and complementarities? (See 1.1.3.)

1.2.12 How are we going to model inefficiencies of re-organization? (See 1.1.1.)

1.2.13 What are the practical operational weaknesses of the system, once proposed, in terms of operational implementation, and what can be done to overcome these weaknesses?

1.2.14 How are we going to model uncertainties, different risk attitudes and value of information? (See 1.1.3.)

It is clear from the proposed set of issues to be addressed that a new conceptual framework has to be developed, in order to capture and describe phenomena in this context, and to provide answers to open questions.

1.3 Functions within the Problem Boundaries

Capital-budgeting process customarily includes the following functions (37):

1.3.1 Search for investment opportunities

This function is most frequently executed by the operating divisions. For example—in Butters (6), in the "Frontier Rubber Company" case, it is reported that: "Division managers sponsored nearly all project proposals and, in turn, were held accountable for their forecasts and projections . . . ." Nevertheless, in some other
cases, the R&D division is also a source for proposals. Because of its unique position in the organizational structure, it is assumed that if economy-of-scale and/or economy by complementarities are possible, they will be proposed by R&D (if, for some reason, not proposed by divisions).

1.3.2 Evaluation of opportunities--the divisional level

Done by the divisional-executive (D.E.), in terms of his perceptions about divisional constraints, objectives risk-attitude, et cetera, and also some explicit guidelines as per corporate constraints and preferences. Throughout this work it will be assumed that those proposals which are unacceptable to the division are screened out from further review by headquarters.

1.3.3 Allocation of further search efforts

In terms of the "technical horizon of choices," the D.E. is most frequently the person in charge of allocation of search efforts, for trivial reasons of competence. In terms of the "financial horizon of choices," the D.E. will make the choice in a "Profit Center" type of organizational structure, while central allocation of search resources will be coordinated in all other structures. Dunlop (16) reports that "Emphasis was placed also upon identifying higher return investment opportunities . . ." in the Sun Oil Company 1971 annual report. However, no indication is made on strategies which have been used to achieve this
goal. No further elaborations are made to the term "emphasis," as well.

1.3.4 Financing of capital investments

One way to look at the financing of capital investments, is to look at the cost of capital. Given that capital is always available for different costs, a corporation will stop borrowing for this purpose at a point where the marginal-internal-rate-of-return is equal to the marginal-cost-of-capital (54). In the course of the discussion, it will be assumed that these financial considerations are centralized, and so are the capital funds. Furthermore, it will be assumed, for the simplification of the discussion, that the total amount of available funds is determined prior to the reception of divisional proposals. This assumption is extremely close to past United States industry financial behavior, which displays a pattern not to reinvest all retained earnings. This simplification, though, is inconsistent with the marginal-cost-return argument, suggested previously. Or, as Myers puts it:

... there is a clear tendency to isolate financial decisions, in order to analyze them. But, financing decision is not independent of the investment decision. Financial management really requires simultaneous consideration of the investment, financing and dividend options facing the firm . . . . (41)

Perhaps an even more complete approach will be a simultaneous consideration of search, evaluation, decentralization, financing, dividend and investment, as it is apparent now that all these decisions are inter-dependent.
But, because of the complexity of such an approach, and time limitations, this most general approach will be suggested for further research.

1.3.5 Evaluation of available opportunities—corporate level

Unless capital budgeting decision authority is delegated to the D.E. (Divisional Executive) in a decentralized mode of operation, this function is done by the headquarters (dominant coalition, chief executive, et cetera). This evaluation is always on top of the divisional evaluation process as described in 1.3.2, and sometimes consists of duplication in efforts.

A) Screening

This process consists of comparisons against different kinds of explicit constraints for acceptability, as defined by headquarters (37, 54). For example:

1) Repay period less than T.
2) Internal Rate of Return greater than r*.
3) Net Present Value greater than zero.

March and Simon (33) suggest that "Most human decision-making, whether organizational or individual, is concerned with the discovery and selection of satisfactory alternatives . . . ." In this context, the screening stage is perhaps the most important stage in the decision process, and within this stage are the key issues of the decentralization problem. As mentioned
once before, not all headquarters constraints (policies) can be made explicit, and thus cannot be communicated to Division-Executives. These are the implicit constraints. Otherwise, how can one explain the following statement made by Butters in the "Frontier Rubber Company" case:

Despite these minimum standards, more proposals were submitted than could be met by the funds available. As a result, all sizeable proposals were given a critical appraisal throughout the review process. Typically, about one-third of all projects submitted by the division managers, were rejected during the screening process. (6)

The only possible interpretation of this statement is, that one-third of the submitted proposals were rejected in the headquarters screening process, due to differences in perceptions about the implicit constraints, and the term "acceptable proposal."

It has to be pointed out here, that in the screening process, no comparisons or preferences are made among the different proposals. The comparisons are only in respect to given and fixed constraints—the "Yard-stick for acceptability." Therefore, availability of funds is irrelevant here, as proposals are not in competition on scarce resources. Consequently, the screening process can also be looked upon as "The investment selection process in an ample funds situation." This situation is very frequently the case in commercial banks, where the amounts of funds
available for borrowing are larger than the demand for loans. Needless to say, that in an ample funds situation screening will be the only stage in headquarters evaluation process.

B) Preference

Proposals which survive the screening process, are considered "acceptable by headquarters in an ample-funds situation." But as most frequently is the case, only limited funds are available (for a reasonable cost). Thus, not all proposals which were considered "acceptable" in the ample-funds situation, will survive and be "acceptable" in the limited-funds situation. Here, all projects enter competition on the scarce funds, and only the preferred ones (in terms of expected yield) are approved. Here, the availability of funds is a prime factor in the decision process, and for any amount of funds, a set of approved proposals is associated.

1.3.6 Actual choice of investments

Having established a preference scale in 1.3.6, the best (highest expected yield) set of proposals is identified. But besides the highest yield figure, one has to recall that these figures are, in the best case, intelligent expectations (averages). Consideration is always given to the uncertainty coupled with these estimates. If
the associated risk is within corporate limits of acceptability, a final choice can be made. If not, more information has to be collected, as to economically decrease the associated risk to within the acceptable limits (37). Another way to rank alternative investments is by using utility figures, which will account both for the mean yields and their associated uncertainties.

1.3.7 Post auditing of past investments and estimates

A close surveillance is conducted on actual real cash-flows, in order to compare with the estimated cash-flows. This process enables improvements in predictions, and removal of personal biases.

1.4 Systems Analysis Approach

In the course of this discussion, it is suggested to view the problems of the interacting Capital-Budgeting elements as problems in a mathematical system. Following are the features, methodologies and assumptions underlying the proposed systems approach.

1.4.1 Types of organizational structure

Two major types of organizational financial structures will be discussed:

a) The simple structure.

b) The "Profit Center" structure.
Figures 1.01 and 1.02 describe the different interactions among the elements in the capital-budgeting system, for the two financial structures, respectively. In both cases it is assumed that capital funds are corporate controlled, and managed for the best interest of the firm, as perceived by the chief-executive.

1.4.2 Major problems

Three major problems are identified:

a) The optimal degree of capital-budgeting decision authority delegation. (Decentralization.)

b) The optimal allocation of search efforts for investment opportunities.

c) The optimal allocation of search efforts for decision information.

All three problems are interrelated, as any decision about one of them, might have an impact on one or more of the other. Therefore, these three problems should be looked upon and solved simultaneously.

1.4.3 Analysis by segmentation

The divisional investment opportunities sample space as a whole is divided into segments (sub spaces). The division into segments is situation specific, and is achieved by the use of a number of relevant performance dimensions. Each performance dimension is further economically divided (economically, in terms of M.I.S. costs)
Fig. 1.01. Elements within the capital budgeting system; the simple organizational structure.
Fig. 1.02. Elements within the capital budgeting system; the "Profit Center" Organizational structure.
into short intervals. Thus, the set of all possible combinations of dimensions and sections is the set of segments which describe a specific opportunities space. Each segment is subject to independent consideration in terms of all three major problems. (See 1.4.2 in previous paragraph.)

1.4.4 Decentralization—a Chief-Executive decision under risk

The decentralization problem is viewed as a decision under risk for the chief-executive (or dominant coalition, headquarters, et cetera). The major risk is of the "Type 2 error" in statistical tests of hypotheses. More specifically, the possible error of acceptance of a proposal by the division, when it should be rejected in terms of headquarters perception. This type of error might have a strong and long negative impact on the firm's well being, while "Type 1 error" has much less negative impact, and is more of a "loss of one opportunity" type.

1.4.5 Possible courses of action

It is proposed in the course of this discussion that headquarters may select one out of three possible courses of action, at each segment of the sample-space: (the concept "source" should be interpreted here as a specific segment of the capital-investment opportunities-space of a specific division).

a) Accept proposals from that source upon receipt, without further headquarters evaluation.
b) Consider and fully evaluate all proposals from that source, in batch with other proposals. (Complete centralization for the specific source.)

c) Reject proposals from that source upon receipt without headquarters evaluation. (Complete negative decentralization for the specific source.)

The preference of the best course of action is done by maximization of expected net-organizational gains due to the expected costs (of analysis and evaluation by headquarters) and revenues (of the investment yield).

### 1.4.6 Limits to decentralization of capital-budgeting

The limits to decentralization (13) will be specified by subsets of segments. As described in the previous paragraph, there will be a subset of segments, for which headquarters will decide that complete-positive decentralization is the most economical course of action. Another subset of segments will be completely centralized. And the third subset of segments will be completely-negatively-decentralized, thus capturing the whole investment opportunities sample space. Consequently, the subsets boundaries, as expressed by the different dimensions, are the limits for decentralization. These boundaries define "sub optimal limits," as for each segment in any of the
three subsets the expected net-organizational-gain is max­imized. On the limit, for an infinitely large number of segments, "sub optimal limits" to decentralization con­verge to the "optimal limits." A completely new conceptual framework is suggested in Chapter 2, in order to enable the quantitative formulation of the decision problem.

1.4.7 Ample and limited funds

Two separate financial situations are analyzed:

a) The ample funds situation

This situation is analyzed first, as it is much simpler from the conceptual point of view. This situation is very powerful for the purpose of demonstration of the concepts suggested in the discussion. Very important insights and conclusions are derived. In this situation all headquarters acceptable proposals are also preferable.

b) The limited funds situation

In this situation, not all headquarters acceptable proposals are also preferable, as there is competition among the acceptable proposals on the scarce capital resources. This situation is somewhat more complex, nevertheless very simple, given that the ample funds situation is fully understood.
The limited funds situation is typical to the capital-budgeting problem of many United States firms, which use only internally generated funds (depreciation and retained earnings, less dividend) for investment purposes. On the other hand, the ample funds situation is very typical to many commercial banks situations, where the total amount of funds they are willing to borrow exceeds their total demand.

1.4.8 Certainty equivalent and uncertainty

Two levels of analysis are suggested along the course of the discussion:

a) Certainty equivalent type of analysis

This approach is looking only at expected values (means) of variables. This kind of analysis is simple and straightforward, but it implicitly assumes a linear risk attitude of the decision maker. For all situations where linear utility (or profit maximization) is a close description of the risk behavior, the "certainty equivalent" type of analysis is recommended as the most practical one.

b) Uncertainty

This approach, which models uncertainty too, leads to a more complex analysis, nevertheless practical when applying computing aids.

Instead of previously maximized expected-net-
organizational-gain (in dollar scale) the preference among the three courses of decentralization action is done by maximization of organizational utility of net gain (on the utility scale). This approach is essential whenever a significant non-linear risk attitude is observed and modeled.

1.4.9 The search efforts for investment opportunities

The coordination and best allocation of search efforts for investment opportunities, is a very important decision, which should be better made prior to headquarters evaluation process. (The products or results from these efforts are the analyzed proposals.) As the decentralization decision is also made prior to headquarters evaluation, the same concepts and variables can be used to formulate the quantitative decision model for the best allocation of search efforts. (Again, "best" in terms of headquarters perception of organizational well being.) A model is proposed, and interesting insights and conclusions are derived.

1.4.10 The search efforts for investment information

The search for information is looked upon as a process and effort to reduce uncertainty (38). As the "standards for desirability" (51) are pretty "crystalized" in terms of uncertainty (the less uncertainty, the better), a non-linear decision maker will assign dollar-value to
any decrease in uncertainty. However, this will not be the case for a linear decision maker, who will not care less about uncertainty (refer to certainty equivalent approach at 1,4,8-a). Thus, it is suggested along the course of this discussion, to view uncertainty as a variable with explicit or implicit constraints (aspiration level). More specifically, March and Simon’s acceptability decision criteria is suggested to be used as a criteria for uncertainty reduction. This means that a proposal is going to be rejected if uncertainty about the different relevant variables was not reduced to an "acceptable" level. The cost of information is viewed as an integral cost, or as a non separable part from the divisional cost of generation of proposals. It should be pointed out that such an approach is necessary in order to avoid variations in proposals "acceptance" due to variations in the amount of attained information. Nevertheless, economical considerations should be made in the case of risk-averse behavior, as to the cost and value (thus, net gain) of purchased or investigated information. The Bayesian model is suggested for this consideration (31). It provides an answer to the question of the economical extent of allocation of investigative efforts.

1.4,11 Evaluation of time lags

Time delays and their negative effects are modeled in terms of the risks for proposals "death" during the
evaluation period. This model explicitly assumes that "life-time" of proposals or opportunities is limited, and if a decision is not reached within this "life-time," the opportunity disappears. This type of risk is also modeled into the proposed decentralization decision model, thus influencing the organizational well being, and thus the decentralization decision.

1.4.12 Operational and implementation implications

It is emphasized along the course of the discussion, that much of the needed information for these decisions is available within the boundaries of any organization. It is only the problem of the collection of the proper information, and the proper interpretation of this information through the use of proper concepts and models. A conceptual break-through was required, and this is probably the most significant contribution of this discussion. A dynamic learning evolutionary process is proposed, as to continuously update the variables estimates, the more relevant information becomes available. At the "start-up" point, when no information is available, subjective headquarters assessments are used. This approach implicitly assumes that even the worst intelligent guess is better than no guess at all (which, in turn, turns out to be an assessment of irrelevancy, which by itself is also a guess).

This observation could be written as an hypothesis, as follows: "There is no way one can escape from making
assessments, because by escaping he is making an assessment, too."
CHAPTER II
DELEGATION OF DECISION AUTHORITY—A DECISION UNDER RISK

2.0 Definition of Concepts

Following is a list of concept definitions, which will be used to come up with a set of hypotheses, in the next paragraph:

2.0.1 Degree of variation in the divisional "acceptability of a capital investment proposal" perception, when compared to headquarters (H.Q.)* perception of "acceptability."
(Degree of variation should be interpreted in terms of relative frequency.)

2.0.2 Expected value to the organization, of a capital-investment proposal, which is "acceptable" both by the divisional and H.Q. perceptions. (Dollars.)

2.0.3 Expected value to the organization, of a capital-investment proposal, which is only "acceptable" by the divisional perception, but "unacceptable" by H.Q. perception. (Dollars.)

2.0.4 Expected value to the organization, of funds applied for by the capital-investment proposal, had they been invested in an alternative opportunity. (Opportunity value, in dollars.)

2.0.5 Headquarters consideration costs per proposal (Dollars.)

*From here on, H.Q. stands for "headquarters."
2.0.6 Expected net organizational gain through capital investment—equals to the difference between expected value of the investment or alternative opportunity, and any cost incurred in the evaluation of such proposals by H.Q.

2.0.7 Level of Decentralization—point on a delegation scale, whose highest point is complete delegation, and lowest is no delegation at all.

2.1 Hypotheses Regarding Decentralization of Capital Budgeting Decisions

2.1.0 The smaller the degree of variation in divisional "acceptability" perception, the higher is the level of decentralization.

2.1.1 The smaller the expected value to the organization of a capital investment proposal which is divisional "acceptable" but H.Q. "unacceptable," compared to the expected value of a proposal which is "acceptable" by both perceptions, the lower is the level of decentralization.

2.1.2 The larger the cost of H.Q. consideration per proposal, the higher is the level of decentralization.

2.1.3 The smaller the difference between the value of a mutually "acceptable" proposal and the alternative value of the capital funds, the higher is the level of negative-decentralization (to delegate rejection authority, and not approval authority).

2.1.4 The higher the degree of H.Q. aversion to risks, the lower is the level of decentralization. (Risk aversion, in terms
of convexity of the utility function of the decision maker, in the dollar range of proposals expected values.)

2.1.5 The preferred level of decentralization is the level which maximizes Organizational Expected Net Gain (dollars, in the H.Q. linear utility case), or the one which maximizes Organizational Expected Utility of Net Gain (in the H.Q. non-linear risk attitude).

2.1.6 Proposals can be, and should be valued by an associated single valued Present Worth concept. A support to this hypothesis is provided by many, such as Hillier (21):

"Present Value criterion is usually the most appropriate one for evaluating investments . . . ."

It should be noted here that a comprehensive understanding of the Net Present Worth concept is required in order to fully interpret the system developed here. This, because throughout this report Net Present Worth concepts are used as elements, like atoms in a molecule.

2.2 Reformulation—Concepts Redefined

pa - Probability that a divisional generated and "acceptable" proposal, will also be considered "acceptable" in H.Q. perception ("Acceptability Variance").

PW - Expected Net Present Worth (time discounted expected net cash-flows) of a mutually (division and H.Q.) "acceptable" proposal. (Dollars.)

PWR - Expected Net Present Worth (time discounted expected net
cash-flows) of a divisional "acceptable" proposal, which is H.Q. "unacceptable." (Dollars.)

APW - Expected Net Present Worth (time discounted expected net cash-flows) of funds applied for by a proposal, had they been invested in an alternative opportunity. (Dollars.)

HC - H.Q. consideration cost per proposal. (Dollars.)

ENOG - Expected Net Organizational Gain from capital budgeting and investment. (Dollars.)

Levels of Decentralization:

(1) Complete Negative Decentralization:
Delegation of H.Q. authority to reject proposals prior to H.Q. consideration.

(2) Complete Centralization:
No delegation of decision authority. Central H.Q. consideration and final decision is required.

(3) Complete Positive Decentralization:
Delegation of H.Q. authority to accept and approve proposals at the divisional level, prior to H.Q. consideration.

2.3 Certainty Equivalent Model--Single Opportunity Space, Ample Funds, and Linear H.Q. Utility

Given the Certainty Equivalent assumption, where the Expected Present Worth of all relevant variables are available, the Expected Net Organizational Gain for each of the three levels of decentralization is computed and displayed in Table 2.1.
TABLE 2.1. Expected Net Organizational Gain; Single Segment, Ample Funds, Certainty Equivalent

<table>
<thead>
<tr>
<th>Possible Outcome</th>
<th>H.Q. &quot;Acceptable&quot;</th>
<th>H.Q. &quot;Unacceptable&quot;</th>
<th>Expected Net Organ. Gain ≡ ENOG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level of Decentralization</td>
<td>pa</td>
<td>(1-pa)</td>
<td>pa • PW + (1-pa) • PWR</td>
</tr>
<tr>
<td>Complete Positive</td>
<td>PW</td>
<td>PWR</td>
<td>pa • (PW-HC) + (1-pa) • (APW-HC)</td>
</tr>
<tr>
<td>Centralization</td>
<td>PW-HC</td>
<td>APW-HC</td>
<td></td>
</tr>
<tr>
<td>Complete Negative</td>
<td>APW</td>
<td>APW</td>
<td></td>
</tr>
</tbody>
</table>
This model is a rough, simple, first-cut presentation of the hypotheses listed in Section 2.1. As suggested in Hypothesis 2.1.5, the preference of the best level-of-decentralization is made by maximization of ENOG. For a given set of estimates of PW, APW, PWR and HC, ENOG remains to be a function of "pa." It is extremely interesting to review ENOG as a function of "pa" on one scale, and all other dollar variables on a second scale. A graphical and numerical analysis follows, which provides some interesting and important insights into the system.
3.0 The Decision Problem

The decentralization of Capital-Budgeting decision problem is the problem of the selection of the proper level of authority delegation to the different divisions to make capital investment decisions in different segments of their opportunity-space. This selection can be based on H.Q. past experience of divisional "performance." Here, "performance" is viewed as the divisional ability to predict H.Q. screening criteria when the explicit list of H.Q. constraints is incomplete. It should be noted here that the difficulty of developing a conclusive set of explicit constraints for capital-investments, is one of the major issues in the decentralization problem. Otherwise, if this was not the case, and a conclusive set of H.Q. constraints could be established, there would not be any need for limitation to complete decentralization in the ample-funds situation. (One may also look at different perceptions of some constraints as part of the "inconclusiveness" of the constraints.)
In any case, the decentralization decision has to be made prior to H.Q. screening stage of any new batch of investment proposals. (Here, it will be assumed that the centralized H.Q. reviewing process is a "batch" process.) There is no point in making the decentralization decision after the screening process, as the H.Q. screening process by itself is a centralized mode of operation. Having screened out all H.Q. "unacceptable" proposals, a complete centralized decision has been executed. It is for this reason that H.Q. decentralization decision has to be made prior to any insight into the specific batch of proposals under consideration. Therefore, this decision is interpreted as a decision with assumed uncertainty (or a decision under risk).

From Figures 1.1 and 1.2 it should be noted that all proposals received at the proposals "batch," are always considered as "Divisional Acceptable." Looking at the H.Q. decentralization decision under risk, one will observe the three possible courses of action available to H.Q., with their associated risks or costs:

3.0.1 Complete positive decentralization

All submitted proposals from a specific segment of the opportunity-space, are considered "acceptable" by H.Q., without further screening. The risk H.Q. takes by selecting this course of action is of the "Error type-2" or beta type of risk. The risk is of acceptance of "H.Q. unacceptable" proposals, which could result with a yield lower than originally thought, and lower than an
alternative opportunity on hand (outside the boundaries of the organization). Note that the yield here is compared to alternative opportunities, because the ample-fund situation is discussed here. Thus, funds which are not employed in investments, can only be employed outside the organization.

3.0.2 Complete centralization

All submitted proposals from a specific segment of the opportunity-space, are screened for "H.Q. acceptable" and "H.Q. unacceptable" proposals. The "H.Q. acceptable" are the only proposals which are finally executed. There is no risk of the previous nature involved with the selection of this course of action. Instead, there is a sure cost involved, which is the H.Q. screening cost, which, in turn, reduces the "Expected Net Organizational Gain."

In certain cases, this cost is considered as wasteful duplication of efforts, on top of extensive divisional screening efforts. (This happens when the variability in H.Q. and Divisional "acceptability" perceptions is minimal.) This course of action can be viewed as "trading the risk for a sure cost--or an insurance policy." The question is only whether the premium is high or low.

3.0.3 Complete negative decentralization

All submitted proposals from a specific segment of the opportunity-space are considered "unacceptable" by H.Q., without further screening. The risk H.Q. takes by
selection of this course of action is of the "Error type-1" or alpha type of risk. The risk is of rejecting an "H.Q. acceptable" type of proposal, which could result with a higher yield than the alternative value of the unemployed funds. (See discussion in 3.0.1.)

A graphical display of the H.Q. decentralization decision problem is presented in Figure 3.01. This diagram emphasizes the "prior" nature of the decision problem, the three possible courses of action, and the risks or costs associated with each selected course.

As suggested previously in 2.1.5, the preferred decentralization course of action will be the one which will maximize the Expected Utility of possible gains (in the general risk attitude), or the one which will maximize the Expected Net Organizational Gain (in the specific linear risk attitude). The gains and losses in the model take into consideration all types of possible gains, as well as all types of possible losses or costs. This brings us back to the need for a new conceptual structure, as suggested in Chapter 2, as to quantify all variables and risks for each possible course of action, in order to come up with an Expected Net Organizational Gain expression. An in-depth discussion follows in 3.4.
Fig. 3.01. Headquarters' decision problem, prior to its consideration process (single cell, ample funds).
3.1 Segmentation of Opportunity-Space

3.1.1 The Crudest Model

One crudest way to view the opportunity-space of a specific division is by viewing it as a single subspace. A union of all divisional opportunities subspaces will thus be the complete organizational opportunity-space. This approach, when coupled with the three possible decentralization courses of action, implies that H.Q. can either:

1) Delegate complete positive authority to the specific division.
2) Centralize all divisional investment decisions.
3) Discourage the specific division from generating any proposals.

This simplified crudest approach (perhaps oversimplified) is surely very inexpensive in terms of the collection and processing efforts of decision information (cost of M.I.S.).

Nevertheless, it is very inaccurate in terms of the convergence of its semi-optimal solution to the optimal one.

The inaccuracy is basically explained in terms of the "performance continuity" (refer to 1.1.6) error in assumption. More specifically, making a decentralization decision based on a crudest single divisional segment
assumption implies that divisional "performance" in terms of predictability of "H.Q. acceptability" (refer to 3.0) is uniformly distributed within the subspace. But as many people have already observed, most frequently this is not the case (13, 37). Divisional "performance" varies within one crude divisional opportunity-subspace. This means that different decisions would have been considered "optimal" at different sections of the subspace, had they been considered separately.

A support for this observation is found at Dearden's work (13), and Morris' work (37), who suggest that there should be limits to decentralization, and that the decentralization problem is not a trivial one, respectively.

Had the uniform distribution of divisional "performance" been the more frequent situation, there would not necessarily be limits to decentralization, and the decentralization problem could frequently become a trivial one (either you delegate full authority, or nothing at all).

3.1.2 The Single Dimensional, Less Crude Model

Having agreed to the observation that the crudest model is an oversimplification of divisional "performance," the second and less-crude model suggests to capture the non-uniformity of performance along a single dimension.
This might, or might not be a good approximation of divisional "performance" either, but no one will argue that this approach is less accurate. This, because it suggests to model the non-uniformity which the previous approach neglected, this time along a single dimension only. This approach, in turn, provides some answers to the "limits of decentralization" as suggested by Dearden (13), and to the non-trivial problem suggested by Morris (37).

Morris's "size gate" model, which suggests to measure divisional "performance" along the investment's initial outlay requirements dimension, is an excellent example to the single-dimension approach. This kind of approach implicitly implies that performance is uniformly distributed along all other possible dimensions. Again, the validity of this assumption should be tested for any specific situation.

The accuracy of the single dimensional model, in terms of convergence of its sub-optimal solution to the optimal one, should be far better than the accuracy of the crudest model. This, because this model captures non-uniformities in performance along that single dimension, thus recognizing and allowing the necessary degrees of freedom for different optimal decisions at different sections along that dimension.
Two additional weaknesses are observed at the "Size Gate" model: (37)

1) It allows for a single discontinuity in the recommended decentralization "best" course of action. In the general case, multiple discontinuities should be allowed for. (Refer to "Performance continuity" phenomenon, at 1.1.6). More specifically, what kind of evidence is available to support an assumption that organizational performance is always "poor" at the upper range of the initial-outlay dimension only, and cannot be "poor" at the low level of this same dimension?

2) It considers only two of the possible courses of action available to H.Q. It neglects the complete negative decentralization option, which might be useful in specific situations.

These two weaknesses are observed on top of single-dimension weakness. Therefore, if this model is to be implemented, ways have to be developed, in order to overcome these weaknesses. The multi-dimensional approach which is suggested along the course of this discussion, is capable of overcoming this kind of weakness. The single-dimensional model is to be viewed as a simple case of the multi-dimensional model.
3.1.3 The Multi-Dimensional Model

Pursuing the single dimension concept one step further, we arrive at the multi-dimensional model. This model suggests that the "Divisional Opportunity-Subspace" should be further divided along few dimensions, and should not be viewed as one uniform and homogenous unit. This suggested model is the result of an assumption that divisional "performance" (in terms of prediction of "H.Q. acceptability") might be different along one or more dimensions. Therefore, this model will be able to capture these non-uniformities in performance, thus will offer a sub-optimal decentralization decision, which will be closer to the optimal solution than previous solutions. Apart from the number of dimensions, the precision of the sub-optimal solution is also a function of the division of each dimension into bounded intervals. This issue is discussed in the following section. But as far as the number of modeled dimensions is concerned, it is simple to prove that the larger the number of dimensions, the closer is the sub-optimal solution to the optimal one. The proof uses the same argument which was used once before (see 3.2.2), that suggests that by adding more dimensions, one is adding more degrees of freedom for decisions in the sub-divided segments, thus being able to further sub-optimize decentralization decisions in these non-uniform sub-divisions. As long as the cost of M.I.S. is
not introduced, there is no reason whatsoever to limit the number of modeled dimensions. When this number is increased, the sub-optimal decentralization decision asymptotically converges to the optimal decision.

But as most frequently is the case, M.I.S. cost is not negligible—thus it should be introduced as a limiting factor to an endless growth of number of dimensions. By using the margin concepts in economics here, it could be suggested that the increase of modeled dimensions is "economical" as long as the marginal gain due to improved decisions exceeds the marginal M.I.S. cost due to an additional modeled dimension. This type of problem is very similar to a problem in accounting, where the choice has to be made for the most appropriate accounting structure, or the "best" list of accounts.

In accounting, this problem is considered as an "information economics" decision (expected value of information, vs. expected cost of information), and occurs at the implementation stage of the Management Information System. Horngren (23) suggests that trade-offs in "value" and "costs" should repeatedly be evaluated, as to make decisions about whether changes in an M.I.S. system are worth undertaking. A more analytical approach to this problem is suggested for further study.
3.1.4 **Sub division of dimensions into intervals**

As already mentioned in 3.1.3, each dimension whether in the single or multiple dimensional model should be divided into a finite number of bounded or unbounded intervals. As before, the problem of the "best" division strategy comes up. Again, using the same arguments as for the number of dimensions to be modeled, it can be proved that the more sections one has per dimension, the closer is his sub-optimal decentralization strategy to the optimal one. Once again, M.I.S. cost is the limiting factor, once it is recognized as non negligible. Hence, the "economically feasable" division into sections (in terms of implementation) is the best division strategy. Finally, after the selection of the "best feasible mixed strategy," in terms of number of dimensions as well as number of sections per dimension, one gets a network of segments, or "cells."* Cell's identity consists of the combination of the specific sections at the different dimensions, the intersection of which creates this cell. The higher the density of cells, the closer are the sub-optimal limits to decentralization, to the optimal limits.

*From here on, the term "cell" will replace the term "segment."
3.2 Segmentation in Banking and Industry

3.2.1 A proposition for segmentation in banking

Since the process of loans approval is equivalent to the capital-budgeting process (by definition of "planning expenditures whose uncertain yields are expected to extend beyond one year"), insights to the later process automatically provide insights to the first one.

Further support to the assumption that the problem of delegation of decision authority is of major concern to the banking industry, is provided by Schnitz:

The concern here is that of centralization and decentralization of authority. It is a question of the degree to which authority is concentrated or dispersed through the organization . . . . It is obvious that the extent to which authority is not delegated, it is centralized . . . . No bank is completely centralized or completely decentralized. It rather possesses a degree of each. Regardless of the extent of decentralization, central management still formulates policy. (46)

Based on the observation of similarity, an attempt is made here to provide the banking industry with some situation-specific recommendations.

Looking at possible dimensions, along which a "subordinate" or "division" might differ in its "performance" of prediction of H.Q. "acceptability" perception, one may propose the following dimensions to be tested:

1) Amount of loan applied for.
2) Repay period.
3) Purpose of loan.

*For definitions of ratios, refer to Weston (54).
4) Security provided.
5) Applicant Current-Ratio.
6) Applicant Quick-Ratio.
7) Applicant Debt-to-Total Assets Ratio.
8) Applicant Times Interest Earned.
9) Applicant Fixed Charge Coverage.
10) Applicant Inventory Turnover.
11) Applicant Average Collection Period.
12) Applicant Fixed Assets Turnover.
13) Applicant Total Assets Turnover.
14) Applicant Profit Margin on Sales.
15) Applicant Return on Total Assets.
16) Applicant Return on Net Worth.
17) Applicant or Firm Age.
18) Applicant location (area in town).
19) Applicant residency in same location (time), et cetera.

It should be noted, though, that no dimension has to be modeled, nor that this list of dimensions is conclusive.

The proper dimensions have to be carefully selected, using H.Q. experience. Whenever possible, past performance evidence could be very useful in making this selection.

It should be also noted that propositions 1 through 3 deal with the "loan characteristics." Proposition 4 deals with securities. Propositions 5 through 16 deal with the applicant financial position. Propositions 17 through
19 deal with other characteristics of the applicant. There might be other characteristic groups which were not listed here, the identifying of which might be important in specific situations. Breakdown of dimensions into sections should always be done to points which are economically-feasable. (See 3.2.4).

3.2.2 A proposition for segmentation in industry

Looking this time at possible dimensions in industry, along which divisional and H.Q. perceptions of "acceptability" might differ, one can propose the following dimensions for consideration and testing:

1) Initial Investment (Outlay) Required.
2) Horizon of Design (Economical Lifetime).
3) Expected Return on Equity. (Internal Rate of Return.)
4) Percent of Utilization by Division. (At a given point in time.)
5) Advantages to other divisions. (Complementarities.)
6) Specific End Product.
7) Specific Department within Division.
8) Specific Technical Function.
9) Magnitude of dependency on supplier.
10) Decrease in dependency on "environment," compared with current situation, et cetera.
Note that the breakdown into sections must not necessarily be numerical, nor must the breakdown be into segments of equal size. For example:

Dimension 5 could be divided into "Low, Medium, High."

Dimension 6 could be divided into "Small Cars, Middle Size, Full Size, Wagons, Trucks."

Dimension 7 could be divided into "Motors, Transmissions, Brakes, Steerings, Body, Accessories, et cetera."

Dimension 8 could be divided into "Materials Handling, Processing, Utilities," et cetera.

Most of the comments and notes which follow the Banking propositions are applicable for the Industry propositions, except those which are specific to Banking.

3.3 Precision Implications of Segmentation Strategies

One powerful way of demonstration of the impact of the different segmentation strategies on the precision of the limits to decentralization is through a graphical display of a two-dimensional space. Assuming a specific situation, where divisional "performance" is mostly variable along two dimensions as follows:

a) Amount of initial outlay - I.

b) Project economical horizon of design - H.

Also assuming the previously discussed three modes of
decentralization: 1) Accept - "A"
2) Reject - "R"
3) Centralize - "C"

Following are three graphical displays of phenomena one may anticipate under three different segmentation strategies (along given dimensions), which result in an explicit identification of optimal limits to decentralization, at different precision levels.

3.3.1 **No divisions into sections. (The "Crudest" Model)**

In Figure 3.02, the two-dimensional "Crudest" Model is presented. The two-dimensional opportunity subspace is not divided into segments at all. Consequently, it can either be fully "accepted," "rejected" or "centralized" as a whole. In this given example, the "centralized" mode was arbitrarily selected, for demonstrational purposes. It should be noted that in such a crude model, the limits to decentralization cannot be defined nor displayed. This, for the simple reason that the limits are on the boundaries of the union of all segments associated with the same optimal decentralization mode. Here, the formation of just one union is possible.

3.3.2 **Two sections per dimension (a "Less Crude" Model)**

In Figure 3.03, the two-dimensional "Less Crude" Model is displayed. This time, each dimension is divided into two intervals, thus creating four segments ("cells"). Each of the four cells can be independently associated with any of the three possible modes of decentralization.
Fig. 3.02. A two dimensional "Crudest" Model.
Fig. 3.03. A two dimensional "Less Crude" Model.
This association is done by solving a sub-optimal optimization problem for each of the four independent segments.

This model is the first model where both limits to decentralization can be identified, as well as displayed. In the arbitrarily selected example, Limit-1 (between "Accept" and "Centralize") runs from point "a" through point "c" to point "b." Limit-2 (between "Centralized" and "Reject") runs from point "c" to infinity, along both I and H dimensions. It should be noted that this model is more precise than the previous crudest model, in terms of maximization of the objective function and in terms of the solution to the problem of the limits to decentralization. However, when cost of M.I.S. is also taken into consideration, computation here is expected to cost four times more than costs in the previous model. Thus, the increase in precision is offset by an increase in M.I.S. cost, as previously discussed.

3.3.3 Eleven sections per dimension (a "Refined" Model)

In Figure 3.04 the "Refined" Model is displayed. Here, "I" dimension is divided into ten equal sections within the $0 to $10,000 range, plus one unbounded interval (open on one side) of $10,000 and above. Similarly, the "H" dimension is divided into ten equal sections within the 0 to 10 years range, and one "open" interval of 10 years and up. This segmentation strategy results with 121 "cells," where each cell can independently be associated
Fig. 3.04. A "Refined" two dimensional model.
with any of the three modes of decentralization. The more refined the segmentation, the more precisely can the optimal limits to decentralization be identified. Here, one can more precisely observe the true pattern of the two limits (Limit-1 and Limit-2), which were relatively crudely described in the second model, and not described at all in the "Crudest" Model. A quick comparison between the two "messages" Figure 3.03 and Figure 3.04 are trying to send reveals that the more precise limits (in Figure 3.04) call for more types of proposals to be "rejected," less to be "approved," and a clear shift in the types of proposals to be "Centralized." This new solution, though still "sub-optimal" in nature, is far closer to the optimal solution than the second example. When the size of segments tends to zero, the solution converges to optimum. (Refer to the mathematical rational of the differential and integral calculus, and their "limit" arguments).

3.3.4 Segmentation strategies for reduction in M.I.S. costs

Savings in M.I.S. costs without lowering limits precision could be achieved in certain circumstances. This cost reducing segmentation strategy can be used under the following two conditions:

1) There exists past information and current estimates about the limits trends and locations.

2) Evidence exists, to support an assumption that between the limits there is continuity in
association to decentralization modes (there is no reason to believe that there are isolated segments which are associated with different modes than their neighboring cells, except for those cells on the limits).

When these conditions are satisfied, an arbitrary and unequal segmentation strategy can be used, to identify the precise limits tracks.

As demonstrated in Figure 3.05, a fine segmentation will only be designed along the estimated tracks, and within a range of variation from the expected tracks. Crude segmentation will be designed in areas where the likelihood for crossing of any one of the two limits is very low. By doing so, the precision levels of identification of the limits tracks will be maintained, while unnecessary computational efforts in the "continuous areas" will be avoided. Needless to say that this approach can only be successfully implemented after a period of time, when experience has been accumulated through the use of a highly segmented network.

3.4 Practical Implications and Range Limitations

Looking at the six basic concepts defined in Section 2.2, and the realistic and practical problem of delegation of decision
Fig. 3.05. An M.I.S. cost minimizing segmentation model.
authority for capital investments, one may observe the following implications:

1) \( \text{p}_{a} \)  
This represents the variance in divisional versus H.Q. perception of "acceptability." Practically, this variable is defined as the probability that both perceptions are equal, for a particular segment "j" of the opportunity space. Consequently, as a probability term, all \( \text{p}_{a} \)'s can vary between zero and one.

\[
0 \leq \text{p}_{a} \leq 1, \quad \text{for all } j. \tag{3.01}
\]

As for the measurement of \( \text{p}_{a} \), at the initial stage of the system's operation, subjective estimates will be assigned by the decision maker. The more sampling information becomes available, \( \text{p}_{a} \) will be updated for all \( j \), by using prior/posterior updating techniques (such as the Bayesian model, as it will be explained later in this work).

2) \( \text{PW}_{j} \) and \( \text{PWR}_{j} \) 
These two variables stand for the present worth of a proposal in segment \( j \), when it is accepted by both H.Q. and division, or only by the division, respectively. These concepts are defined for H.Q. use (in this decision-aid), and as such, should represent H.Q. perception
about these two variables. The underlying assumption already made is that H.Q. believe they represent, and are responsible for the organizational "well being." Consequently, their perception about an "unacceptable" proposal, will always be equivalent to "less valuable to the organization" than the "acceptable." Therefore, it will be assumed here that $PWR_j$ can never be greater than $PW_j$ for any $j$, or

$$PW_j - PWR_j \geq 0.$$  
for all $j$ \hspace{1cm} (3.02)

When this assumption is not made, there might be situations where H.Q. screening process does not maximize the firm's "well being," but does the contrary. It is very unlikely that H.Q. perception will accept or admit such a situation. As for the measurement of $PW_j$ and $PWR_j$, these are two random variables, in each "cell." For the certainty equivalent model, we will be concerned about their expected values. Once again, the system can use subjective estimates for the means initially, and update them when more information becomes available.

3) $APW_j$

This represents the alternative present worth of funds which have been allocated for investment in a proposal
type-j, had they not been invested as planned but in the next best alternative. It is important to observe that "in the worst case" these funds could be re-invested in the bank. Consequently, when the difference between cost of capital to the firm and interest rate it can receive for deposits is non-significant, zero will always be the low limit for $\text{APW}_j$, for all j. (Investment in a bank account will generate the exact funds to pay for the cost of capital, thus a zero net cash-flow). In the ample funds situation, $\text{APW}_j$ will always be equal to zero, as funds are available for any generated proposal, and unused funds can only be deposited back in the bank. In the limited funds situation, $\text{APW}_j$ depends upon types and properties of existing proposals which are still available from the batch at the time of consideration. Therefore, it can always be suggested that

$$\text{APW}_j \geq 0$$
for all j

(3.03)

4) $\text{HC}_j$

Finally, looking at the H.Q. consideration costs per proposal type-j, one can immediately observe that these costs are non-negative. On the limit, these costs could be considered non-significant thus equal to zero, but negative they will never be.
Or

\[ HC_j \geq 0 \]

for all \( j \)  

(3.04)

As before, \( HC_j \) might be a random number within any given segment \( j \). In the certainty-equivalent model one should only become interested in the expected value of this random variable. The system will, once again, start with a subjective estimate for this expected value, while this estimate will be updated periodically, the more information becomes available.

3.5 Single Cell Decision Model, and the Five Mutually Exclusive Combinations of Decentralization Strategies

Once again, the reader is invited to refer to Table 2.1, where the three modes of decentralization and their associated ENOGs (Expected Net Organizational Gains) are presented. Following are two important observations as per these ENOG values:

1) All variables in the three ENOG expressions are either dimensionless, such as "pa," or in the dollar scale.

2) All three ENOG expressions are either linear with respect to "pa," or independent from "pa" (thus also linear, in essence).

These two observations have led to the idea that all the different relationships among the different variables can be
graphically displayed on a two-dimensional plane. One axis will thus represent the dimensionless variable "pa" throughout its 0 to 1 range. The other axis will be a dollar scale, for all the rest of the variables. The following discussion and analysis will take a significant advantage of this graphical aid.

3.5.1 Situations where (PW-HC) > APW

Having selected the range of situations where PW-HC > APW and all other constraints as per section 3.4 are satisfied, one might be interested to investigate and look for preferred course of decentralization, when PWR gets different values within its range of definition. In all future graphs and analysis, "line-1" will be referred to as the Positive Decentralization line, "line-2" as the Centralization line, and "line-3" as the Negative Decentralization line. Appendix A provides a proof to the fact that there are exactly five different sequences for strategy preference, sequences which will be identified here as "Case-1" through "Case-5." As it turns out, the PWR range is divided into three mutually exclusive sections, each associated with a different combination of decentralization strategies. The other two sequences are related to PW and HC, respectively.

Case-1: PWR > APW

From Figure 3.06 one can observe that in this case "line-1" has no intersection at all with any of the
Fig. 3.06. ENOG diagram for "Case-1."
other two lines. As "line-1" represents the largest ENOG values in this case, throughout the entire "pa" range, Positive Decentralization is preferred independently from the "pa" value.

Case-2: \( E_3 < \text{PWR} < \text{APR} \)

From Figure 3.07 one learns that "line-1" has exactly one intersect with "line-2." Consequently, for this PWR section there will always be two different ENOG maximizing strategies:

1) Positive decentralization—when "line-1" is higher than any other line.

2) Negative decentralization—when "line-3" is higher than any other line.

As long as "pa" is in the range

\[ 0 \leq \text{pa} \leq \text{BE} \]

negative decentralization will be the preferred strategy. But for

\[ \text{BE} \leq \text{pa} \leq 1 \]

positive-decentralization will maximize ENOG. The value of the break-even point for "pa" (BE), is determined by the intersection of "line-1" and "line-3," as follows:

\[ \text{ENO}_1 = \text{ENO}_3. \]  (3.05)
Fig. 3.07. ENOG diagram for "Case-2."
Therefore,

\[ p a^* \cdot PW + (1-pa^*) \cdot PWR = APW \]  

(3.06)

Rearranging for \( pa^* \),

\[ BE = pa^* = \frac{(APW-PWR)}{(PW-PWR)} . \]  

(3.07)

Here, both numerator and denominator are positive, by assumptions previously made. The lower limit for PWR in this case \((E_3)\) is determined from Figure 3.7 again. This limit is identified when "line-1" runs through point-B, which is the intersection of "line-2" and "line-3." (When "line-1" runs below that point, one gets the three-strategies case, which is defined as Case-3 in this discussion.)

Using the property of similarity between the two triangles \( E_1E_2B \) and \( D_1D_2B \), one can write the equation:

\[ \frac{pa_B}{HC} = \frac{1-pa_B}{(PW-HC-APW)}. \]  

(3.08)

Solving for \( pa_B \) one gets,

\[ pa_B = \frac{HC}{(PW-APW)} . \]  

(3.09)

Using, now, the similarity property of \( E_1E_3B \) and \( D_1D_3B \), one can write:

\[ \frac{pa_B}{(APW-E_3)} = \frac{1-pa_B}{(PW-APW)} . \]  

(3.10)
Solving for $E_3$, one gets:

$$E_3 = \frac{(APW-pa_B \cdot PW)/(1-pa_B)}{.} \quad (3.11)$$

Plugging back in the value for $pa_B$, one gets:

$$E_3 = \frac{(APW \cdot PW - APW^2 - PW \cdot HC)/(PW-APW-HC)}{.} \quad (3.12)$$

Case-3: $PWR < E_3$

From Figure 3.08 we find, as expected, that once $PWR$ gets a value below $E_3$, three different courses of action will represent the best mode of decentralization. This time there will be two break even points.

$LBE$ - or the Lower Break Even point.

$UBE$ - or the Upper Break Even point.

As long as "pa" remains within

$$0 < pa < LBE$$

"line-3" will be higher than any other line, thus negative-decentralization should be the preferred course of action.

When "pa" becomes larger, and remains within

$$LBE < pa < UBE$$

"line-2" becomes the highest, thus centralization should be preferred.
Fig. 3.08. ENOG diagram for "Case-3."
And finally, when "pa" goes up to

\[ UBE < pa < 1 \]

"line-1" becomes the highest, therefore positive
decentralization will maximize ENOG.

There still remains the problem of defining LBE and UBE,
which, once again, will be done by the use of ENOG ex­
pressions in Table 2.1, as follows:

For LBE (Low Break Even),

\[ ENOG_3 = ENOG_2 \]  \hspace{1cm} (3.13)

Alternatively,

\[ APW = pa^* \cdot (PW-HC) + (1-pa^*) \cdot (APW-HC) \] \hspace{1cm} (3.14)

Solving for \( pa^* \) one gets:

\[ LBE = pa^* = HC/(PW-APW) \] \hspace{1cm} (3.15)

Similarly, one can solve for UBE (Upper Break Even):

\[ ENOG_2 = ENOG_1 \] \hspace{1cm} (3.16)

or alternatively,

\[ pa^{**} \cdot (PW-HC) + (1-pa^{**}) \cdot (APW-HC) = pa^{**} \cdot PW + (1-pa^{**}) \cdot PWR \] \hspace{1cm} (3.17)
Solving for $p_a^{**}$ one gets:

$$UBE = p_a^{**} = 1 - \frac{HC}{(APW-PWR)}.$$  \hspace{1cm} (3.18)

3.5.2 Situations where $PW > APW > (PW-HC)$

**Case-1:** $PWR > APW$

Like in previous "Case-1," "Line-1" has no intersection at all with any other line. Therefore, Positive-Decentralization will be preferred independently from "pa." (Once again, please refer to Figure 3.06.)

**Case-2:** $PWR < APW$

In contrast with the situation in Section 3.5.1, "Line-2" and "line-3" have no possible intersection under the condition stated above. Consequently, "Point-B" in Figure 3.07 does not exist here, and the combination of the two decentralized levels of decentralization described as "Case-2" before, holds for the whole complementary range of PWR, namely for all $PWR < APW$. The previously developed equation for BE (Break Even) point, remains unchanged.

**Case-3**

As already explained in "Case-2," there is no possible intersection between "line-2" and "line-3." Consequently, the combination of all three levels of decentralization is also not possible. Therefore, "Case-3" as previously
defined does not exist under the prescribed conditions of Section 3.5.2.

3.5.3 Situations where APW > PW

Coupled with the second limitation in Section 3.4 (PWR < PW), the consequences of this situation are displayed in Figure 3.09. As one can see, the only intersection possible is between "line-1" and "line'2," but this intersection is not relevant for the decision making process. "Line-3" will always be superior to the other lines with respect to ENOG values, thus representing the preferred level of decentralization. The preference of Complete Negative Decentralization independently from "pa" is identified from here on as "Case-4," and displayed in Figure 3.9.

3.5.4 Situations where HC = 0

Although this type of situation is unrealistic, it can be viewed as realistic in the sense of relative orders of magnitude. For example: when HC is in the range of $0 - 1,000 and PW, APW and PWR are in the millions, then HC is considered negligible. For this reason it is worthwhile looking into this specific situation, especially when intuitive and also surprising conclusions are expected.

a) PW >_ APW and PWR >_ APW

This situation results with a "line-1" superiority, which means the preference of Complete Positive Decentralization, independently from "pa" (Case-1" and Figure 3.06).
b) \( PW \leq APW \) and \( PWR \leq APW \)

This situation results in a "line-2" superiority, which means the preference of Centralization, independently from "pa." Such a situation did not exist before, thus will be identified from here on as "Case-5" (see Figure 3.10).

c) \( PW \leq APW \)

This situation, coupled with limitation No. 2 in Section 3.4, results in a "line-3" superiority, which means the preference of Complete Negative Decentralization, independently from "pa" (Case-4," and Figure 3.09).

An intuitive conclusion is that \( HC = 0 \) is a necessary and sufficient condition for a Complete Centralization mode preference, independently from "pa." However, surprisingly as it may seem, a proof is provided to the fact that this condition is an insufficient one. Other conditions have to be satisfied too, in order to make this particular course of action the preferred one. The other surprising conclusions are, that under certain conditions the two Decentralization courses of action are preferred (each at a time), thus \( HC = 0 \) cannot be automatically associated with Complete Centralization.
Fig. 3.09. ENOG diagram for "Case-4."
Fig. 3.10. ENOG diagram for "Case-5."
Finally, Figure 3.11 displays a flow-chart for the selection of proper "Case," once the values for PW, APW, PWR and HC are given. When these dollar concepts are considered relatively stable (time invariant), then the whole decision problem becomes a problem of measurement (or estimation) of "pa," and the proper assignment of such into the correct "Case" (out of the five sequences).

3.6 Interaction Among Cells, and Organizational Objectives

In this analysis, the "ample-funds" situation is considered. By "ample-funds," one customarily refers to a situation where all generated proposals could, ultimately, be financed, had they been accepted. Consequently, the utilization of funds in one project, will not create financial shortage for any other potential project. Using the same reasoning, when a project proposal is rejected, funds which otherwise would have been employed (or committed) to its financing, will have to seek "employment" outside the organization (for an "alternative value"—APW), assuming all other proposals already got their financing from the "unlimited source."

Using both observations simultaneously, one can argue that in the "ample-funds" situation, proposals and/or "cells" are not in competition (for scarce resources). Therefore, APW should always be interpreted as an Alternative Present Worth of Capital Funds outside the organization (and not inside), as their alternative use will be "outside" as long as funds are "ample."
Fig. 3.11. Flow Chart for case selection.

\[
E_3 = \frac{APW \cdot PW - APW^2 - PW \cdot HC}{PW - APW - HC}
\]
Consequently, the ENOG function for any one "cell" is independent from the ENOG, PW, APW, PWR, HC, "pa" or any other term in any other proposal or "cell."

Finally, the selection of the "best" level of decentralization for any one "cell" (by the ENOG criteria) is independent from the selection of such in any other "cell." Naturally, this will not be the case in the "limited funds" situation, when competition among "cells" for funds does exist, and financing of one proposal could be the end of hope for financing of a different one. The latter situation will be discussed in detail in the coming chapters.

As for the overall organizational objectives in the "ample funds" situation, the more proposals on hand--the better (provided that the proper level of decentralization is selected). Furthermore, the increase in gross-value (to the organization) of a marginal proposal is not diminishing with the number of proposals already on hand. It remains the same as the gross-value of the first proposal from the same source ("cell"). Consequently, as long as the "cost" to produce (generate) a proposal is lower than its "expected gross-value" (and the associated best level of decentralization is selected) the overall-expected-net-organizational gain will increase linearly with the number of proposals from the different "cells." Once again, this will not be the case when funds are limited, where the marginal gross value of proposals is diminishing.
Incidentally, the "ample-funds" situation is not unrealistic. As a matter of fact, it is very likely that the Banking business can be viewed as in the "ample funds" situation. (As suggested before, long range loans can be viewed as capital-investments.) Support to this hypothesis is provided by a recent Ohio National Bank of Columbus commercial, which makes a statement as follows: "We, at Ohio National Bank, want to make loans . . . ." However, availability of funds is a phenomenon which varies from time to time, as well as the associated interest rate (cost of capital).

3.7 Analysis of Proposals with Limited Life-Time

Up to this point in the analysis, there was an implicit assumption that proposal life-time is unlimited, thus should not be taken into consideration. The moment a limit is set to proposal life-time, one has to question the validity of previous models, as well as their associated insights and conclusions. A quick investigation into this question reveals that limited lifetime of proposals has different impacts on the different levels of decentralization, and their associated economical considerations. For example: a proposal is submitted for H.Q. consideration (under a preferred Complete Centralization mode). By the time it is approved for execution by the division (with an associated PW value), it turns out that this opportunity is not valid (does not exist) any more, because of its limited lifetime. Thus, the previously expected value of funds cannot be materialized, and the net effect of such an event on the organization is a loss.
The question now becomes: "How can we model this limited life-time? How can we capture this phenomenon in order to take it into consideration in our economical analysis, as to be reflected in our selection of decentralization levels?" By the introduction of an additional concept, this problem is resolved.

"ps" - Probability of Survival, is the additional concept.

It should be interpreted as the probability of a proposal from a specific source ("cell") to survive during a possible H.Q. consideration process, in terms of its validity (not acceptability). On the limits, when it is certain that it will survive (such as a situation where H.Q. consideration period equals to zero) or die (when the lifetime of a proposal equals zero), "ps" will get the values 1 and 0, respectively. Practically, the numerical value for the Probability of Survival can be estimated from the distributions of both the "Reviewing Period" ($t_R$) and the "Life Span" ($t_1$) of proposals generated from a specific "cell."

When the distribution of the difference ($t_1 - t_R$) is established from the two separate distributions, one can define ($ps$) as the probability that the difference ($t_1 - t_R$) is positive.

$$ps = Pr[(t_1 - t_R) > 0]. \quad (3.19)$$
For example, if both time variables could be approximated by Normal distributions such that:

\[ t_1 \sim N(u_1, v_1) \]
\[ t_r \sim N(u_r, v_r) . \]  
\[ (3.20) \]

Consequently, their difference would also be Normally distributed, such that:

\[ (t_1 - t_r) \sim N((u_1 - u_r), (v_1 + v_r)) . \] 
\[ (3.21) \]

Therefore,

\[ ps = \Phi[(-u_1 + u_r)/(v_1 + v_r)^{1/2}] \]
\[ (3.22) \]

As we are looking now at two different and independent processes ("acceptability by H.Q.," "survival in environment"), each at two levels (yes, no), one can expect to have 2 x 2 combinations of possible outcomes, as follows:

1) "Acceptable, survive."
2) "Acceptable, die."
3) "Unacceptable, survive."
4) "Unacceptable, die."

The differentiation between possible outcomes 3 and 4 will not be important here, in terms of their economical consequences. Both will be associated with an (APW-HC), value, in the Centralized
mode of operation. Also, it is very important to note that the survival process has no effect on both Complete Positive and Negative Decentralization modes. This happens because both modes are assumed to be considered as "zero time consuming" modes, thus assuring proposals survival (here time is measured from proposal departure to corporate consideration, till its final return to its generating division).

Taking all these definitions and observations into consideration, one comes up with a new set of three equations for ENOG, as to replace those in Table 2.1. This new set is displayed in Table 3.1. The significant difference between the two is presented by the separation of the "H.C. acceptable" column into two sub-columns (using the two possible survival process outcomes). Another difference is the reformulation of the different terms in the Centralization mode, and the resulting expression for ENOG.

A quick comparison between the ENOG expressions for the three decentralization modes in Tables 2.1 and 3.1 reveals that:

a) There is no change whatsoever in the ENOG terms for both Decentralization modes (which are independent from the survival process, as expected).

b) In the Centralization mode, the product \( (p_a \cdot p_s) \) term replaces the \( (p_a) \) term in the unlimited lifetime model. This means that when the Centralization mode is of concern, the proposals' "equivalent probability of H.Q. acceptance" equals to such probability in the unlimited
TABLE 3.1. Expected Net Organizational Gain, Single Segment, Limited Proposals’ Lifetime

<table>
<thead>
<tr>
<th>Possible Outcome</th>
<th>H.Q. &quot;Acceptable&quot;</th>
<th>H.Q. &quot;Unacceptable&quot;</th>
<th>Probability</th>
<th>Expected Net Organ. Gain = ENOG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Survival</td>
<td>pa'ps</td>
<td>pa*(1-ps)</td>
<td>(1-pa)</td>
<td>pa*PW + (1-pa) * PWR</td>
</tr>
<tr>
<td>Death</td>
<td>(1-pa)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability Level of Decentralization</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Complete Positive</td>
<td>PW</td>
<td>PW</td>
<td>PWR</td>
<td></td>
</tr>
<tr>
<td>Centralization</td>
<td>PW-HC</td>
<td>APW-HC</td>
<td>APW-HC</td>
<td>pa * ps * (PW-HC) + (1-pa*ps) (APW-HC)</td>
</tr>
<tr>
<td>Complete Negative</td>
<td>APW</td>
<td>APW</td>
<td>APW</td>
<td></td>
</tr>
</tbody>
</table>
lifetime situation, corrected (discounted) for the limits in lifetime.

The second observation suggests that limited life-time has a negative effect on the preference for Centralization, in contrast with the two Decentralization modes (which is the kind of phenomenon that was also expected).

An effort to graphically analyze this new set of equations (similarly to the previous analysis) becomes more complex, because this time one faces a three dimensional problem (due to the additional interest in "ps"). However, when sensibly selected, a two dimensional graphical analysis provides, once again, the insights looked for.

As already noticed, the introduction of "ps" and the limited opportunity lifetime has an adverse effect on the priority of Centralization, while the other two modes are not affected. Consequently, there is not much sense in the investigation of situations where Decentralization was preferred before (in the unlimited lifetime situation), as it will be further preferred, for sure. The only doubt is about previously preferred Centralized situations, which might change to one of the Decentralized modes, once "ps" is introduced. Consequently, only two of the cases previously defined, which include Centralization as a possible preferred mode, are worth looking and investigating into. A detailed discussion as well as a proof to this fact are provided in Appendix B.
Case-3: $HC > 0$

\[ PW - HC > APW \]

\[ PWR < E_3 \]

\[ PWR < PW \] (Conditions in Section 3.4 and 3.5.1)

For ($ps=1$), satisfaction of these conditions was sufficient to allow for the preference of Centralization for "pa" values between LBE and UBE points (refer to Section 3.5.1).

As one recalls, LBE was defined as the Low Break Even point, for which ENOG's for both Complete Negative Decentralization and Centralization were equal. Similarly, UBE was the Upper Break Even point between Centralization and Complete Positive Decentralization.

Once "ps" is introduced, these same calculations for LBE and UBE can be repeated for different values of "ps" (down from 1). Following this procedure, one observes that the "pa" range between LBE and UBE, for which Centralization is preferred, diminishes.

Finally, at a certain point for "ps," LBE and UBE become equal, thus completely eliminating the Centralization mode. At this point (combination of "ps" and "pa") all three decentralization modes have the same ENOG value, thus equally preferred.

Further reduction of "ps" below this triple break even point, completely excludes Centralization.
from competition on preference, as it becomes inferior to the other modes for any "pa" value. As only the two modes of Decentralization become candidates for preference at that low range of "ps," Case-3 is said to be converted into Case-2. Figure 3.12 displays this set of observations on a two dimensional orthogonal axes system. Point LL (low limit) on the "ps" axis is the point for which all three ENOGs are equal. This is the "ps" solution to the simultaneous set of linear equations as follows:

1) \( \text{ENOG}_1 = \text{ENOG}_2 \),
2) \( \text{ENOG}_1 = \text{ENOG}_3 \). \hspace{1cm} (3.23)

For (1) one gets:

\[
\text{pa} \cdot \text{PW} + (1-\text{pa}) \cdot \text{PWR} = \text{pa} \cdot \text{ps} \cdot (\text{PW}-\text{HC}) + (1-\text{pa} \cdot \text{ps}) \cdot (\text{APW}-\text{HC}) .
\] \hspace{1cm} (3.24)

Solving for \( \text{ps} \), one gets

\[
\text{ps} = \frac{[(\text{PW} - \text{PWR}) + (\text{PWR} - \text{APW} + \text{HC})/\text{pa}]/(\text{PW} - \text{APW})}.
\] \hspace{1cm} (3.25)

From (2) one gets:

\[
1/\text{pa} = (\text{PW} - \text{PWR})/(\text{APW} - \text{PWR}) .
\] \hspace{1cm} (3.26)
Fig. 3.12. Modifications in "Case-3 due to Limited Proposals Lifetime."
Finally, the solution for $ps^{**}$ is:

$$LL = ps^{**} = (PW-PWR) \cdot HC/(PW-APW)(APW-PWR)$$  \hfill (3.27)

where all factors in this equation are positive by assumptions.

**Case-5:** $HC = 0$

$$PW \geq APW$$

$$PWR < APW \quad \text{(Conditions in Section 3.5.4 - (b))}.$$  

When checking out this case, one comes up with the conclusion that as long as these conditions are satisfied, independently from "pa" and also independently from "ps," Centralization is always preferred to Complete Negative Decentralization. This is for the simple reason that:

$$ENOG_2 > ENOG_3,$$  \hfill (3.28)

since always

$$pa \cdot pa \cdot PW + (1-pa\cdot ps) \cdot APW \geq APW,$$  \hfill (3.29)

for all $PW \geq APW$.

As for Centralization in contrast with Complete Positive Decentralization, a break even function can be identified.

Here,

$$ENOG_1 \equiv ENOG_2.$$  \hfill (3.30)
By substitution,

\[ pa \cdot PW + (1-pa) \cdot PWR = pa \cdot ps \cdot PW + (1-pa \cdot ps) \cdot PW. \] (3.31)

Solving for "ps" one gets,

\[ ps = \frac{[ (PW-PWR) + (PWR-APW)/pa ]/(PW-APW)}{ (PW-APW)}. \] (3.32)

which means that (pa, ps) combinations form an hyperbola. A graphical presentation of these findings is displayed in Figure 3.13. The end result is that for a given "ps" value (ps<1), there exist a point in "pa" above which Complete Positive Decentralization becomes the preferable level (in contrast to the original Case-5, where Centralization was preferred independently with "pa," for a given set of conditions. The surprising and non trivial phenomenon here is the fact that only Complete Positive Decentralization takes its "bite" from the previously Centralized share, while Complete Negative Decentralization does not expand its boundaries at all.

3.8 Reorganization of Authority Delegation, and the Implementation Costs

Once a change in one or more of the four dollar concepts (PW, APW, PWR, HC) is observed, or a change in "pa" is observed, the previous discussion suggests that a different mode of decentralization might seem more appropriate (preferred). Thus, an
Fig. 3.13. Modifications in "Case-5" due to limited proposals lifetime.
immediate "reorganization" in the level of authority delegation is expected.

This should not cause any discomfort, unless there is a high likelihood that a reverse process might quickly follow. This is in particular the case in a close neighborhood to the break-even points. Therefore, it might be worth while looking into the sensitivity problem of implementation, which will be done in this section. When these back and forth changes in the state of the system (level of decentralization in any cell) occur rather frequently, one faces the "Hunting" Phenomenon (refer to Section 1.1.1). As suggested once before, lack of "memory" for the previous states of the system, allows for the organizational failure from achieving a long range steady state equilibrium. This phenomenon would not have been considered a problem at all, unless a real discomfort (in terms of organizational efforts) was involved each time a new mode is implemented. One way to reduce this discomfort is by the creation of a "memory" to work with the system, and by modeling the "discomfort." This will eventually reduce the kind of oscillations allowed for previously, to a practical and economical level.

Two simple and crude proposals are made:

3.8.1 Memory: A simple "memory" should be added to the model, as to always remember the current mode of decentralization. A more sophisticated memory, as to recall previous modes for all cells, could be considered,
with additional complexity to the system.

3.8.2 Implementation Costs: The discomfort involved in any re-organizational effort, will be modeled by the dollar value of the Expected Implementation Cost per proposal to the organization. This cost will not be recovered, and will occur each time the system moves from one level of decentralization to another. Costs at different cells and at different directions of change within cells, might be different, and modeled separately.

This new addition to the model, actually models the "inefficiencies" involved in reorganization. A "round trip" from any initial state to another state, and back to origin, is not going to be "free" any more. From here on, there will be a net loss to the organization in the execution of such a "cycle." (For analogy, please refer to the "Hysteresis" Phenomenon in the theory of electro-magnetics.) There will be, however, the instrumental problem of measuring these costs, and the allocation problem of allocating these costs to the different proposals in the cell, once a re-organization takes place. However, these problems are of the implementation kind, and will be discussed in a later chapter, together with other problems of a similar kind.

Assuming the new concept of "IC" will stand for Implementation Cost per Proposal, the ENOG of being in any
"state" will from now on include two components: the real component and an "alternative" component, which is the IC associated with the possible change in state. For example, being in "state-2" (Centralization) initially. An "alternative" value of IC$_{2,1}$ is added to the ENOG expression (refer to Table 2.1) when a change to "state-1" (Complete Positive Decentralization) is considered. Similarly, IC$_{2,3}$ is added when a change to "state-3" (Complete Negative Decentralization) is considered. (These terms could be, alternatively, deducted from the state being considered, since what finally matters is the difference between the relevant ENOGs).

Consequently, compared to the other modes of decentralization, the "current" mode always becomes somewhat more attractive, thus discouraging changes in the "marginal" situations. A display of the impact of Implementation Costs on the original decision table (Table 2.1) is presented in Table 3.2, for the situation described in the previous example (being in "state-2" initially).

An interesting problem in particular is the sensitivity of reorganization to variations in "pa," assuming PW, PWR, APW, HC remain constant. Figure 3.14 presents a graphical display of the three ENOGs, when "pa" is in a downward trend, and all IC are assumed to be identical in a specific cell (a simplifying assumption). When being
<table>
<thead>
<tr>
<th>Possible Outcome</th>
<th>H.Q. &quot;Acceptable&quot;</th>
<th>H.Q. &quot;Unacceptable&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>Survive</td>
<td>Die</td>
</tr>
<tr>
<td>Decentralization</td>
<td>pa·ps</td>
<td>pa·(1-ps)</td>
</tr>
<tr>
<td>Complete Positive</td>
<td>PW</td>
<td>PW</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Centralization</td>
<td>PW-HC</td>
<td>APW-HC</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Complete Negative</td>
<td>APW</td>
<td>APW</td>
</tr>
</tbody>
</table>

**TABLE 3.2.** Expected Net Organizational Gain; Implementation Costs Included (Being in Centralized "Mode-2")
Fig. 3.14. The impact of implementation costs on decentralization stability ("pa" in a downward trend).
initially in "state-1," ENOG_1 has to be at least IC dollars lower than ENOG_2, in order to make the transition to "state-2" economical. Thus, the revised UBE point is to the left of the original (IC=0) UBE point for "pa." Being now in "state-2," for further reduction in "pa" values, a similar consideration will be given to a possible "reorganization" into "state-1." ENOG_3 has to be at least IC dollars higher than ENOG_2, in order to make a transition to "state-3" attractive. Again, there is a shift to the left of the IBE point for "pa."

Figure 3.15 presents a graphical display of the ENOG curves in "Case-3," but for the opposite trend in "pa" (upward). As expected, both break even points in "pa" are shifted to the right, by the introduction of IC. Consequently, independently from the nature of the decentralization level, the range of "pa" values for which any "current" state is preferred, is larger than previously. The boundaries for Centralization are expanded to both sides, and the boundaries for the two Decentralization modes are expanded to the one possible side (the other is fixed). As expected, this phenomenon represents an increase in stability, and a decrease in preference sensitivity to variations in "pa." At this point one can say that the "Hunting" Phenomenon, if not completely eliminated, is at least reduced to an economical level (can be economically justified).
Fig. 3.15. The impact of implementation costs on decentralization stability ("pa" in an upward trend).
Analytical derivations of boundary expansions of "pa" can be developed, once again, by the use of graphical aids. Using similarity properties of triangles in Figure 3.15 one can come up with an expression Δpa, which will describe the shift to the right of the UBE, when considering a transition from Centralization to Complete Positive Decentralization.

\[
\frac{Δpa}{(1-UBE)} = \frac{IC}{HC} \quad (3.33)
\]

for \( HC > IC \).

From here, the new break even point will be at \( UBE' \), as follows:

\[
UBE' = UBE + Δpa = UBE + \frac{IC \cdot (1-UBE)}{HC} \quad (3.34)
\]

for \( HC > IC \).

Using this derivation as an illustration, one can develop the proper expressions for the revised break even points, for all the different cases, and for all initial conditions.

3.9 **Conclusions**

Looking backward to the analysis of the basic model and its peripheral sophistications which were introduced in this chapter, one comes up with the following summary of conclusions:

1) There are **three** possible courses of action in terms of decentralization modes, and not just two, as suggested by several authors (13, 37) (Section 3.0).
2) There are always two limits to decentralization, when the divisional proposals generation "performance" can be assumed to be continuous. This, in contrast with the single limit which is suggested by different authors (13, 37) (Section 3.1).

3) The limits to decentralization can be quantitatively or qualitatively defined, with an optimization criterion.

4) The more dimensions are modeled for "performance" measure, and used for the purpose of authority delegation, the closer is the overall sub-optimal ENOG to the optimal value. (ENOG converges to optimum directly with the number of dimensions) (Section 3.3).

5) The more refined are the dimensional divisions, the closer is the overall sub-optimal ENOG to the optimal value. (ENOG converges to optimum directly with the number of divisions per dimension, for any dimension) (Section 3.3).

6) Discontinuities in divisional proposal generation "performance," when in existence, can easily be spotted. Consequently, when practical, authority can be delegated as to optimize local performance, thus increasing overall sub-optimal ENOG (Section 3.3).

7) Segmentation of opportunity space need not necessarily be into cells of equal size. Unequal segmentation strategies might result with no adverse effect on the
overall sub-optimal ENOG, but with significant savings in Management Information Systems' costs (computational efforts) (Section 3.3).

8) There is a high limit for PWR (as a function of PW, APW, HC), above which Complete Positive Decentralization is always preferred, regardless of "pa." For this situation, Complete Negative Decentralization as well as Centralization will never be preferred (Case-1) (Section 3.5).

9) There is a bounded range of value for PWR (as a function of PW, APW, HC), within which one of the two Complete Decentralization modes is preferred each at a time. The exact preference among the two is determined by the value of "pa." Complete Centralization cannot be preferred (Case-2) (Section 3.5).

10) When APW ≥ PW-HC, Complete Centralization can never be preferred, regardless of "pa" and PWR values (subject to the practical implications and range limitations in Section 3.4) (Section 3.5).

11) For APW ≥ PW, Complete Negative Decentralization is preferred regardless of "pa," PWR and HC values (Case-4) (Section 3.5).

12) A necessary (but insufficient) condition for Centralization to be the preferred mode regardless of "pa," is
the condition $HC=0$. The other required conditions are:

a) $PW > APW$ and b) $PWR < APW$ (Case-5).

13) When headquarters consideration costs are negligible ($HC=0$), but either of the additional conditions required for Centralization preference is not satisfied, one of the Complete Decentralization modes will be preferred, regardless of "pa" (Case-1 or Case-4) (Section 3.5).

14) In all cases where the selection of the preferred level of decentralization does depend upon "pa" value, Complete Negative Decentralization is always associated with the low range of "pa" values (up to, and including the low limit), while Complete Positive Decentralization is always associated with the high range of "pa" values (up to, and including the high limit) (Cases 2 and 3) (Section 3.5).

15) In the basic model, when the preference among the three modes of decentralization does depend upon the "pa" value, one level of Decentralization and the Centralization mode alone can never collectively exhaust the "pa" range. The second level of Decentralization will always be preferred at some range of "pa" (Cases 2 and 3) (Section 3.5).
16) For a given and fixed set of variables \(PW, PWR, APW, HC\), a procedure can be designed, as to determine whether the preferred mode is a function of "pa." If it is not, the proper preferred mode can immediately be identified. If the preferred mode is also a function of the "pa" value, break even points for "pa" can be determined, as to divide the "pa" range into sections with a predetermined mode of preference (Figure 3.11) (Section 3.5).

17) In Case-3 there is always a low limit (LL) for the variable "ps"—(probability of survival through a headquarters consideration process), below which Centralization can never be preferred. When this happens, Case-3 changes into a Case-2 situation (Section 3.7).

18) Case-5, where Centralization is preferred regardless of "pa," holds as long as the survival of a proposal through headquarters consideration is certain. Once uncertainty is introduced to survival (\(ps \neq 1\)), preference is shared between Complete Positive Decentralization and Centralization, as a function of both "ps" and "pa." In this special event, Complete Negative Decentralization is never preferred. This is in contrast with conclusion 15 about the basic model (Section 3.7).

19) The larger the implementation costs \(IC\) (subject to \(IC < HC\)), the larger all boundaries expansions for
"pa." Consequently, the larger IC the lower the sensitivity of the selection of the preferred mode to variations in "pa" is (and the smaller the frequency of "hunting") (Section 3.7).
CHAPTER IV

DECENTRALIZATION OF CAPITAL-BUDGETING DECISIONS:
A CERTAINTY EQUIVALENT APPROACH IN
A LIMITED FUNDS SITUATION

4.0 The Decision Problem

Once again, as in Section 3.0, the decentralization of the Capital-Budgeting decision problem is the problem of the selection of the proper level of authority delegation to the different divisions, to make capital investment decisions in different segments of their opportunity space.

Once again, this is a decision under risk from the chief executive perspective, which has to be made prior to H.Q. screening process, and which has the same three possible courses of action. Things which were previously written about segmentation (Section 3.1), about application in Banking and Industry (Section 3.2), about precision implications of segmentation strategies (Section 3.3), about practical implications and range limitations (Section 3.4), about the single cell model (Section 3.5), about proposals' limits of life-time (Section 3.7) and about implementation costs of reorganization (Section 3.8) are all as valid as in the limited funds situation.
However, what was previously written in Section 3.6 about interaction among "cells" and organizational objectives, does hold only for the ample funds situation. This makes the whole difference between the ample and limited funds situations, difference which turns out to be very significant and far reaching in terms of its consequences. Here are some of the important issues which should be raised in connection with the limited funds situation, which make this situation different from the previous one:

4.0.1 Continuous versus batch generation and decision process

In the ample funds situation, the question about the nature or type of the proposal evaluation process in respect to time, was never raised. It was assumed that independently from the nature of the process, whether it is continuous or in batches, there will always be sufficient funds for all acceptable proposals. Thus, the decentralization problem became to be only a function of the identity of the source ("supplier" or "generator") of proposals, independently from the rate or kind of process through which these were submitted.

Once a limitation is enforced upon availability of capital funds in absolute dollars (not as a portion of demand), the problem of time period is raised automatically. The question which is raised is: "For how long should these funds last, and when are the new funds due?"
Independently of the reply, one can recognize that capital funds are allocated periodically, and investment decisions are expected to be made during these periods, thus periodically, with an identical period. For simplification of the interpretation of the problem, one shall assume that a periodic dead-line is made public by H.Q. to the different divisional "proposal writers," by which interested divisions must submit all their proposals for possible H.Q. consideration. The number of proposals on hand by this deadline, will be considered and identified from here on as the "Batch of Proposals."

4.0.2 The interaction and dependency among "cells"

In contrast with the ample funds situation, the different sources of proposals are in competition for the limited funds available. Consequently, the approval of a proposal from one source, might be the end of hopes for approval of other competing proposals, from other "cells."

Similarly, delegation of Complete Positive Decision Authority to one source (for a limited number of proposals), is a sure cut in the total available funds remaining for allocation to other projects.

Therefore, one can say that in the limited funds situation "cells" do interact, and decentralization decisions about one "cell" have always an impact on other "cells." As we shall see in future sections, these decisions might have an impact on divisional motivations to
generate proposals of certain kinds in the future, as well.

4.0.3 The alternative value of funds

Again, in contrast with the ample funds situation, the alternative value of funds, once a proposal is rejected by H.Q., is not evaluated outside the organization, but on the inside. When a proposal is rejected, funds which were originally intended to be allocated for investment in such, can be reallocated for investment internally. The possible reallocation is to other project proposals which are on hand, but for which funds have not been previously allocated. Therefore, the alternative value of such funds is not necessarily equal to zero, but dependent upon the kind of project proposals on hand, for which funds were not allocated prior to the specific rejection act.

4.0.4 The diminishing marginal effect of proposals on batches' gross value

In the limited funds situation, one additional (a marginal) proposal will increase the batch gross value much less than previous proposals of the same source. This occurs when the demand for capital funds by the set of proposals already on hand, exceeds the amount of funds available for investment.

The rationale for this observation can be explained in the following way: one additional proposal will probably not increase the total number of financed
proposals, nor will it improve their expected return when found "acceptable." Its only contribution to the value of funds and their associated proposal-mix is in respect to "safety," by having the property of a "spare proposal," in case all previously submitted proposals are rejected. (in the model suggested in this work, the probability for such an event to happen is non-zero, although asymptotically converges to zero with the increase in the number of proposals in the batch).

4.0.5 Preference of proposals, versus acceptance

Finally, perhaps the most important difference between the ample and limited funds situations is in terms of the selection process.

While previously all proposals had to run through a "screening" process only, to identify the "acceptable" versus the "unacceptable" proposals, this time there is an additional process. The "preference" process is added on top of the "screening" process, as to select the preferred proposals among the previously "acceptable" ones. Thus, not all "acceptable" proposals in the ample funds situation, can be also considered "acceptable" when funds are limited. The latter are only a subset of the first.

Having considered all these differences, we shall proceed to the derivation of the expression for the "Gross Value to H.Q. of capital funds, when associated with a combination of proposals."
4.1 The Expected Gross Value to Headquarters, of a Batch of Proposals

It is intuitive to observe that the "value" (in terms of present worth) of a limited fund with an empty batch of proposals equals zero. When this situation occurs, the only potential employment of the funds is outside the organization, and as explained previously in Section 3.4, the present worth of such funds equals zero.

Also intuitive is an observation which suggests that the "value" of any possible combination of proposals in a batch, when associated with a nil amount of funds, once again equals zero. This time the reason is even simpler, as no proposal can be financed and executed.

By making these two observations, the boundary conditions for the "value" function were defined. As a consequence of these, one realizes that in order for a batch of proposals to be associated with a non-zero "value," a necessary condition is to have a non-empty set of proposals, and a non-empty amount of funds.

This "value" should be related directly with the number of proposals from any source in the set, however with a diminishing marginal effect, as suggested previously in Section 4.0.

As it is clear that the more funds become available, the more "acceptable" proposals can be funded, one may suggest that the "value" should be related directly with the amount of funds available, as well.
The two other factors which should be considered for the determination of the "value" expression of a batch of proposals are:

a) The quality (in terms of present worth) of proposals from the various sources ("cells"), when considered mutually acceptable or only acceptable to the division.

b) The variance in acceptability perception ("pa") of proposals from various sources, and the strategy selected for the treatment of those variations.

A valid question might demonstrate the importance of these two factors: "What is the difference, in terms of the proposal contribution to the batch's "value," between a proposal with high quality (present worth) but with a large variance in acceptability perception, and a different proposal with lower values in both dimensions?" It is clear that the first proposal could be highly valuable to H.Q., if there was a way to assure its "acceptability." However, high screening costs might reduce its "value" contribution considerably.

For the situation where Headquarters Consideration Costs are non-zero, one cannot suggest that a systematic consideration of all proposals in the batch is a priori the best strategy to go about identifying the preferred ones. When the systematic review turns out to be questionable in terms of cost, the next equitable strategy in line, as to provide all proposals with an "equal opportunity" to be reviewed for acceptance, is through a random selection process. When followed by a careful H.Q.
decision, whether to immediately "accept," immediately "reject" or fully investigate the proposal by H.Q. staff before making a final decision, it can be suggested that this process is responsive to management information and decision systems costs, as well as to the need to come up with equitable reviewing opportunities (at least initially) for all proposals sources.

Here it will be assumed that the random selection of proposals from the batch is cost-free, and the source identity remains on each proposal.

When funds required for execution of "acceptable" proposals selected through this process exceed available amounts, the selection process is terminated.

Accepting these "rules of game," the only remaining problem will be to determine the "value" of a batch of proposals, and to identify the best course of decentralization, once a proposal is selected at random from an existing batch. (The content of the batch is known in terms of numbers of proposals and their associated origins, and the available amount of funds is known as well).

As suggested previously in the ample funds situation, there should be ways to express "values" as a function of the selected mode of decentralization. In the next section an effort is made to use many of the previously defined concepts, as to come up with these three different value expressions. The single step value-maximizing decentralization strategy will be identified, and the associated gross batch "value" as well.
4.2 The Expected Gross Value Maximization Criteria, and the Associated Best Delegation Strategy

Assuming a single proposal from source-k is drawn at random from a batch of proposals-S, known as containing a combination of proposals from m different sources, as follows:

\[ S(n_1, n_2, n_3, \ldots, n_k, \ldots, n_m) \]

Using a similar structure and concepts as in Table 2.1 at the "ample funds" situation, one can easily come up with analytical expressions, to describe the "value" of a single sample (proposal), which was drawn at random from a proposals' batch, in the "limited funds" situation.

However, this time the "value" expressions will not only include the immediate expected reward from the drawn proposal, but also "future-rewards" from samples which are expected to be drawn in the future. This major difference in the definition of the "value" concepts is required, because approval of any proposals in the initial stages of the selection process, lowers the balance of available capital-funds for the remaining unconsidered proposals in the batch, in the "limited funds" situation (thus reducing their chances of being approved). Therefore, instead of viewing the "value" concept as the value of a specific drawn sample, we will view it as the value of the whole batch, given that the specific sample was drawn.

Recalling that the models we wish to develop are of the "certainty equivalent" kind (using average values), and the fact
that the content of the proposals batch is known with respect to
the proposal's origin, following is a symbolic presentation of
the concept we wish to express analytically:

\[ V\left[ \frac{B}{s_k}, n_1, n_2, n_3, \ldots, n_m \right] \]

or, the Expected Value of Funds (B), given Sample "type-k" was
drawn, and the content of the batch prior to the recent drawing
is \( S(n_1, n_2, n_3, \ldots, n_m) \).

As expected, three different expressions for \( V \) (Expected
Value of Funds and their Associated Proposal Mix) will be devel­
oped, as per the three different decentralization modes which
were previously identified. These will be presented as \( V_c \), \( V_A \),
and \( V_R \), for the "Centralize," "Approve" and "Reject" modes,
respectively. Each term will consist of two parts; the expected
immediate reward and the expected future reward, with much simi­
larity to expressions in Dynamic-Programming expressions.

Following is a list of definition of concepts to be used
in the three expressions:

\( V_A \) - Expected Gross Value of funds when associated with Com­
plete Positive Decentralization mode for the next de­
cision. (Dollars.)

\( V_R \) - Expected Gross Value of funds when associated with a Com­
plete Negative Decentralization mode for the next de­
cision. (Dollars.)

\( V_C \) - Expected Gross Value of funds when associated with a Centrali­
zation mode for the next decision. (Dollars.)
v* - Optimal Expected Gross Value of funds, when associated with
the value maximizing decentralization strategy. (Dollars.)

p_{a_k} - Variance in "acceptability" perception for proposals
type - k. (Dimensionless.)

_{PW_k} - Expected Present Worth of Future net cash flows from a
"mutually acceptable" proposal type - k. (Dollars.)

_{PWR_k} - Expected Present Worth of future net cash flows from a
"divisional accepted only" proposal type - K. (Dollars.)

_{HC_k} - Expected Headquarters Consideration Costs per proposal
type - k.

B - Available Amount of Funds. (Dollars.)

_{I_k} - Expected Initial Outlay required for the execution of an
investment, proposal type - k. (Dollars.)

(S_k) - Index to identify the type of sample randomly selected from
a proposals batch. (Here, type - k is indicated.)

(r) - Symbol to indicate a random selection process, outcome
of which is uncertain.

(n_j) - Number of proposals on hand in a proposals batch, from
source - j, (for j = 1, 2, 3, ..., k, ..., m).

Having defined the necessary concepts, following is the ex-
pression for the Expected Value of Funds, when Complete Posi-
tive Decentralization is selected for the immediate proposal
analysis:

\[
v_A \left[ \frac{B}{(S_k)}, n_1, ..., n_m \right] = p_{a_k} \cdot _{PW_k} + (1-p_{a_k}) \cdot _{PWR_k} + \\
+ v^* \left[ \frac{(B-I_k)}{(r)}, n_1, ..., (n_k-1), ..., n_m \right], \quad (4.01)
\]
where the first two terms are the expected immediate reward (when the proposal is mutually or only divisional acceptable, respectively), and the third term stands for the optimal expected value of the balance of funds, given the remaining proposals in the batch.

Similarly, one can develop the second V expression for the Centralization mode, as follows:

\[
V_C[B/(s_k), n_1, n_2, \ldots, n_m] = \alpha_k \cdot (PW_k + V^*[(B-I_k)/(r), n_1, \ldots, (n_k-1), \ldots, n_m]) + (1-\alpha_k) \cdot V^*[B/(r), n_1, n_2, \ldots, (n_k-1), \ldots, n_m] - HC_k, \tag{4.02}
\]

where the first term represents the expected immediate and future reward when the proposal is approved, the second term when rejected, and the third term stands for headquarters consideration costs (for sure).

Finally, for Complete Negative Decentralization one develops the third V expression, as follows:

\[
V_R[B/(s_k), n_1, n_2, \ldots, n_m] = V^*[B/(r), n_1, n_2, \ldots, (n_k-1), \ldots, n_m], \tag{4.03}
\]

where there is no immediate reward (authority was delegated to the division to reject proposals), and the only possible value is associated with the expected value of future drawings from the remaining of proposals' batch, having available the original amount of funds.
Having defined all three "values" of funds, it becomes rather intuitive to suggest that the preferred course of action should be the one which maximizes the funds' expected value (precisely as done in the "ample funds" situation). Or, the optimal value $V^*$ is the one for which

$$V^* = \max[V_A, V_C, V_R] = \max_{A, C, R}[V]. \quad (4.04)$$

Consequently, we have defined the optimal expected value of funds, given a specific (origin wise) sample was drawn from a specific proposal-batch composition. The decentralization mode which is associated with this maximum value, will from here on, be considered as the best decentralization course of action for the immediate proposal analysis. (A "Single Step" decision.)

However, in order to completely identify this "best delegation strategy," one has to come up with quantitative values for "future rewards" in the different expressions. Therefore, these have to be evaluated first. Carrying this argument one step further, it becomes apparent that the only way to solve the problem is by a recursion formula, starting at the boundary conditions.

Having arrived at this point, one can see the similarity between the required solution procedure and the commonly used dynamic-programming procedure. A discussion about this follows in the next section.
4.3 The Dynamic Programming Technique for Solution

In the "limited funds" situation, the following hypotheses about boundary and other conditions will be used:

a) The optimal expected value (in terms of present worth of future net cash flows) of any amount of capital funds \( B \), while having no single proposal of any source on hand, equals zero. Symbolically, this hypothesis is presented by

\[
V^*[B/(r), 0, 0, \ldots, 0, \ldots, 0] = 0. \tag{4.05}
\]

b) The optimal expected value (in terms of present worth of future net cash flows) of any composition of a proposals batch \( S(n_1, n_2, n_3, \ldots, n_k, \ldots, n_m) \), while having completely no funds available for investment, equals zero. Symbolically, this hypothesis is presented by

\[
V^*[0/(r), n_1, n_2, n_3, \ldots, n_k, \ldots, n_m] = 0. \tag{4.06}
\]

Once again, this hypothesis makes sense, since no project can be funded with no funds, nor can any proposal generate revenues in future periods. This is once again a situation where all future net cash flows are zero, thus the associated discounted net present worth equals zero as well.
c) The amount of proposals from any source (k) (which is identified by \( n_k \)) has no impact whatsoever on the decision problem, as long as the Initial Expected Outlay \( (I_k) \) for the execution of one single project from this source is larger than the available funds (B).

Symbolically,

\[
\text{when } I_k > B, \quad \text{then}
\]

\[
V^*[B/(r), n_1, ..., n_k, ..., n_m] = V^*[B/(r), n_1, ..., 0, ..., n_m]
\]

for all \( n_k \).  \hspace{1cm} (4.07)

This means that because no project can be funded when it comes from source-k, even when it was successfully drawn at random from the proposals batch, and was considered "acceptable" to H.Q., optimal expected values of compositions of proposals containing k's, equal to optimal expected values of similar compositions which contain no k's at all.

In this case, the associated best single step course of action in terms of the selection of the decentralization best mode, is trivial. Here Complete Negative Decentralization will always turn out to be the most economical, when a proposal type-k is randomly drawn.
Having developed and agreed upon these three boundary conditions, one can use a recursion formula to calculate the expected value of "future rewards" at any "state" of the batch system, when there are still some steps to go.

The solution, thus, gets the following form:

a) For Complete Positive Decentralization

\[
V_A[B/(1),1,0,0,0] = p_a_1 \cdot PW_1 + (1-p_a_1) \cdot PWR_1 + V*[B-I_1/(r),1,0,0,0]
\]

\[
V_A[B/(1),2,0,0,0,0] = p_a_1 \cdot PW_1 + (1-p_a_1) \cdot PWR_1
\]

\[
V_A[B/(1),3,0,0,0,0,0,0] = p_a_1 \cdot PW_1 + (1-p_a_1) \cdot PWR_1 + V*[B-I_1/(r),2,0,0,0,0,0,0]
\]

\[\text{et cetera.}\]

b) For Complete Centralization

\[
V_C[B/(1),1,0,0,0] = p_a_1 \cdot (PW_1 + V*[B-I_1/(r),0,0,0,0,0]) +
\]

\[
+ (1-p_a_1) \cdot V*[B/(r),0,0,0,0,0,0,0] - HC_1 = p_a_1 \cdot (PW_1 - 0) + (1-p_a_1) \cdot 0 - HC_1 =
\]

\[
= p_a_1 \cdot PW_1 - HC_1,
\]

\[
V_C[B/(1),2,0,0,0,0] = p_a_1 \cdot (PW_1 + V*[B-I_1/(r),1,0,0,0,0,0,0])
\]

\[
+ (1-p_a_1) \cdot V*[B/(r),1,0,0,0,0,0] - HC_1,
\]

\[
V_C[B/(1),3,0,0,0,0,0,0] = p_a_1 \cdot (PW_1 + V*[B-I_1/(r),2,0,0,0,0,0,0,0]) +
\]

\[
+ (1-p_a_1) \cdot V*[B/(r),2,0,0,0,0,0,0] - HC_1, \text{et cetera.}
\]

c) For Complete Negative Decentralization

\[
V_R[B/(1),1,0,0,0,0,0,0] = V*[B/(r),0,0,0,0,0,0,0,0] = 0
\]

\[
V_R[B/(1),2,0,0,0,0,0,0] = V*[B/(r),1,0,0,0,0,0,0,0]
\]
In a similar way one can develop appropriate expressions for any other drawn sample, and for any combination of proposals in the batch.

The selection of the best course of action, when a specific sample is drawn, is done by maximization of the expected gross value of funds and their associated proposal-mix, as viewed from that specific "state" of the system.

\[
V^*[B/(1), 1, 0, \ldots, 0] = \max_{A, C, R} V[B/(1), 1, 0, \ldots, 0]
\]
\[
V^*[B/(1), 2, 0, \ldots, 0] = \max_{A, C, R} V[B/(1), 2, 0, \ldots, 0]
\]
\[
V^*[B/(1), 3, 0, \ldots, 0] = \max_{A, C, R} V[B/(1), 3, 0, \ldots, 0]
\]

In general,

\[
V^*[B/(k), n_1, n_2, \ldots, n_k, \ldots, n_m] = \max_{A, C, R} V[B/(k), n_1, n_2, \ldots, n_k, \ldots, n_m]
\]

In words: The optimal expected gross value of funds \((V)^*\), which is associated with a proposal-mix \(S(n_1, n_2, n_3, \ldots, n_k, \ldots, n_m)\) and an amount of funds \((B)\) given that a proposal type - \(k\) was drawn, is that value of the three possible values (which are associated with the three decentralization modes) which is the highest of all three.

Going one step further, one would like to define the optimal expected value of funds with their associated proposal-mix prior to the drawing process.
Here, we shall look into the uncertainty that is involved in the drawing process. As samples are drawn at random, one can assume that the likelihood which is associated with any drawing outcome, is equal to the relative frequency of the specific type of proposal in the proposal-mix. Therefore, the unconditional optimal expected gross value of available funds with their associated mix of proposals will be the weighted average of individual optimal values for the different possible drawing outcomes.

\[ V^*[B/(r), n_1, n_2, \ldots, n_m] = \sum_{k=1}^{m} \frac{n_k}{\sum_{i=1}^{n_1} n_i} V^*[B/(k), n_1, n_2, \ldots, n_m] \]  \hspace{1cm} (4.08)

This expected value of funds and their associated proposal-mix is the optimal gross value prior to the drawing process. Therefore, it is unconditioned with respect to the drawing outcome.

This last \( V^*[B/(r), n_1, n_2, \ldots, n_m] \) expression is the expression we were looking for, to be used in the recursion formula as "optimal future reward" from remaining funds and remaining proposals in the batch. (Refer to the three \( V \) expressions in Section 4.2).

Consequently, a methodology has been laid out, to identify the best immediate (next step) decentralization course of action at any given proposal-mix, for any type of sample drawn and for any given amount of limited funds. This mathematical model is, once again, based on the same fundamental concepts previously defined for the ample-funds situation, but the treatment is obviously different.
Having estimated the relevant variables values, it is evident that the selection of "best" courses of action can be predetermined for all possible drawing outcomes, prior to actual drawing. Therefore, one can say that headquarters evaluation strategy can be programmed "off-line." Starting from a known (or alternatively-estimated or even controlled) proposal-mix, strategies for a "best" one step decentralization mode can be developed and recorded for use during the various steps of the drawing process.

In contrast with the ample-funds situation, one will observe in the limited-funds situation two types of single-step decentralization strategies behavior, in specific segments of the opportunity space:

A) The unchanged strategy

Here, the single step decentralization strategy remains unchanged from the initial state of a proposal-mix, to the final emptying state of a batch of proposals, for a specific type of proposal. These strategies are considered independent (or unconditioned) from the proposal-mix, therefore could be defined as "absolute strategies," and could be administered by headquarters prior to the evaluation process, similarly to the administration in the "ample-funds" situation.

For example: when certain types of proposals are always accepted without consideration, independently from the "state" of the remaining proposal-mix, one should
administer Complete Positive Decentralization of authority to those specific sources, respectively.

B) The dependent strategy

Here, the best single step decentralization strategy does change as a function of the content of the remaining proposal-mix. As the decentralization strategies are conditioned upon the content (or the "state" of the system), they cannot be a priori fixed, thus the delegation of authority to the division is somewhat more difficult. It is important to note that this kind of behavior is only possible in the "limited-funds" situation.

For example: when proposals from source type-k are rejected as long as there are proposals of a different origin-j in the remaining mix. Once type-j proposals disappear, type-k begins to be associated with centralized evaluation, as its best decentralization strategy. In this case, one may suggest that ranking types of proposals by a preference criteria, instead of sampling proposals at random, might be a better idea. In the latter example, both processes bring about precisely the same outcomes.

As we believe that a numerical example will perfectly illustrate the phenomena previously described, and will make it easier for the reader to interpret the concepts as well as the powerful model and the associated conclusions, we have decided to submit such an example. This example is presented in the next section. By no
means should this example limit the general validity of the suggested model, nor should it be considered as part of the direct analytical effort.

4.4 A Numerical Example

The problem

Given are two sources of proposals, which generate proposals for headquarters consideration. The sources are identified as "Cell-1" and "Cell-2" respectively, and their respectively generated proposals have the characteristics as displayed in Table 4.1.

Table 4.1 Input Data for a Numerical Example

<table>
<thead>
<tr>
<th></th>
<th>Cell-1</th>
<th>Cell-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>PW ($)</td>
<td>200</td>
<td>1000</td>
</tr>
<tr>
<td>PWR ($)</td>
<td>-150</td>
<td>-1000</td>
</tr>
<tr>
<td>HC ($)</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>I ($)</td>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>pa (-)</td>
<td>0.9</td>
<td>0.7</td>
</tr>
<tr>
<td>B $</td>
<td></td>
<td>50</td>
</tr>
</tbody>
</table>

The problem is to identify the best mode of decentralization at any proposal-mix, and for any type of proposal drawn, which will maximize the expected gross value of the funds, or
alternatively, will maximize organizational gross expected gain.

The solution

4.4.0 Boundary and other conditions

Based on the three boundary conditions previously discussed in Section 4.3, the following conditions hold:

\[ V^*\left[\frac{B}{r}, 0, 0\right] = 0 \quad \text{for all } B , \]
\[ V^*\left[\frac{0}{(r)}, n_1, n_2\right] = 0 \quad \text{for all } n_1 \text{ and } n_2 , \]
\[ V^*\left[\frac{B<50}{(r)}, n_1, n_2\right] = V^*\left[\frac{B<50}{(r)}, n_1, 0\right] \quad \text{for all } n_1 . \]

4.4.1 Solution for B=10, n_2=0 all n_1

In this specific situation, no proposals of "Cell-2" are available, consequently none of these can be randomly drawn.

Using the three \( V \) equations 4.01 through 4.04, one gets for \( n_1=1 \):

\[ V_C\left[\frac{10}{(1)}, 1, 0\right] = 0.9(200+0) + 0.1(0)-10 = 170 , \]
\[ V_A\left[\frac{10}{(1)}, 1, 0\right] = 0.9(200) + 0.1(-150) + 0 = 165 , \]
\[ V_R\left[\frac{10}{(1)}, 1, 0\right] = 0 , \]

\[ \text{Max}_{A, C, R} V\left[\frac{10}{(1)}, 1, 0\right] = 170 @ "C" \text{ as best associated decentralization strategy.} \]
Similarly, for $n_1=2$ one gets:

\[
V_C[10/(1),2,0] = 0.9(200 + V*[0/(r),1,0]) + 0.1(V*[10/(r),1,0])
\]

\[-10 = 0.9(200+0) + 0.1(170) = 187 ,
\]

\[
V_A[10/(1),2,0] = 0.9(200) + 0.1(-150) + V*[0/(r),1,0] = 165 ,
\]

\[
V_R[10/(1),2,0] = V*[10/(r),1,0] = 170
\]

\[
\begin{align*}
\text{Max}_{A,C,R} & \ V[10/(1),2,0] = 187 \ @ \ "C" \ \\
& \text{as best associated}
\end{align*}
\]

\[
\text{decentralization strategy.}
\]

Similarly, for $n_1=3,4,5,6,7$ one gets:

\[
\begin{align*}
\text{Max}_{A,C,R} & \ V[10/(1),3,0] = 188.7 @ "C" \\
\text{Max}_{A,C,R} & \ V[10/(1),4,0] = 188.87 @ "C" \\
\text{Max}_{A,C,R} & \ V[10/(1),5,0] = 188.88 @ "C" \\
\text{Max}_{A,C,R} & \ V[10/(1),6,0] = 188.888 @ "C" \\
\text{Max}_{A,C,R} & \ V[10/(1),7,0] = 188.8888 @ "C"
\end{align*}
\]

On the limit, for $N_1 \to \infty$, one gets:

\[
0.9(200) + 0.1(V) - 10 = V_C ,
\]

\[
V_C = 170/0.9 .
\]

Therefore,

\[
\text{Max}_{A,C,R} V[10/(1),n_1,0] = 170/0.9 = 188.89 @ "C" .
\]

$n_1 \to \infty$
4.4.2 Solution for \( B = \$10 \), all \((n_1, n_2)\)

No proposal from "Cell-2" can be financed even when drawn and found acceptable, because \( I_2 = \$50 > \$10 = B \), which means that the expected outlay for a proposal type-2 is larger than available funds.

Therefore, type-2 proposals even when existing in the batch, do not have any contribution to the batch's value, and have to be rejected when drawn. As such they have no impact on the decentralization strategies previously identified for type-1 proposals.

Consequently, the best operational strategy for any \((n_1, n_2)\) combination will be identical to the respective \((n_1, 0)\) optimal strategy, results of which are displayed in Table 4.2.
<table>
<thead>
<tr>
<th>Proposal-Mix</th>
<th>Best Decentralization Strategy, when Drawing Sample Type</th>
<th>$V^*[10/(x), n_1, n_2]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_1$</td>
<td>$n_2$</td>
<td>(1)</td>
</tr>
<tr>
<td>0</td>
<td>any</td>
<td>C</td>
</tr>
<tr>
<td>1</td>
<td>any</td>
<td>C</td>
</tr>
<tr>
<td>2</td>
<td>any</td>
<td>C</td>
</tr>
<tr>
<td>3</td>
<td>any</td>
<td>C</td>
</tr>
<tr>
<td>4</td>
<td>any</td>
<td>C</td>
</tr>
<tr>
<td>5</td>
<td>any</td>
<td>C</td>
</tr>
<tr>
<td>6</td>
<td>any</td>
<td>C</td>
</tr>
<tr>
<td>7</td>
<td>any</td>
<td>C</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$\infty$</td>
<td>any</td>
<td>C</td>
</tr>
</tbody>
</table>

Table 4.2. Optimal Expected Value and Single Step Decentralization Strategy, $10$ Budget
4.4.3 Solution for $B = $20, any $(n_1,n_2)$

Table 4.3 displays the results for a $20$ budget, results which are based on a similar analysis to that in 4.4.2.

Table 4.3. Optimal Expected Value and Single Step Decentralization Strategy, $20$ Budget

<table>
<thead>
<tr>
<th>$n_1$</th>
<th>$n_2$</th>
<th>(1)</th>
<th>(2)</th>
<th>$V^*\left[\frac{30}{r}, n_1, n_2\right]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>any</td>
<td>C</td>
<td>R</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>any</td>
<td>C</td>
<td>R</td>
<td>170</td>
</tr>
<tr>
<td>2</td>
<td>any</td>
<td>C</td>
<td>R</td>
<td>340</td>
</tr>
<tr>
<td>3</td>
<td>any</td>
<td>C</td>
<td>R</td>
<td>372.3</td>
</tr>
<tr>
<td>4</td>
<td>any</td>
<td>C</td>
<td>R</td>
<td>377.03</td>
</tr>
<tr>
<td>5</td>
<td>any</td>
<td>C</td>
<td>R</td>
<td>377.69</td>
</tr>
<tr>
<td>6</td>
<td>any</td>
<td>C</td>
<td>R</td>
<td>377.76</td>
</tr>
<tr>
<td>7</td>
<td>any</td>
<td>C</td>
<td>R</td>
<td>377.777</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$\infty$</td>
<td>any</td>
<td>C</td>
<td>R</td>
<td>377.7/9</td>
</tr>
</tbody>
</table>
4.4.4 Solution for $B = 30$, any $(n_1, n_2)$

Table 4.4 displays the analysis results for a $30$ budget.

Table 4.4. Optimal Expected Value and Single Step Decentralization Strategy, $30$ Budget

<table>
<thead>
<tr>
<th>$n_1$</th>
<th>$n_2$</th>
<th>(1)</th>
<th>(2)</th>
<th>$v^*_{[30/(x), n_1, n_2]}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>any</td>
<td>C</td>
<td>R</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>any</td>
<td>C</td>
<td>R</td>
<td>170</td>
</tr>
<tr>
<td>2</td>
<td>any</td>
<td>C</td>
<td>R</td>
<td>340</td>
</tr>
<tr>
<td>3</td>
<td>any</td>
<td>C</td>
<td>R</td>
<td>510</td>
</tr>
<tr>
<td>4</td>
<td>any</td>
<td>C</td>
<td>R</td>
<td>557.07</td>
</tr>
<tr>
<td>5</td>
<td>any</td>
<td>C</td>
<td>R</td>
<td>564.93</td>
</tr>
<tr>
<td>6</td>
<td>any</td>
<td>C</td>
<td>R</td>
<td>566.41</td>
</tr>
<tr>
<td>7</td>
<td>any</td>
<td>C</td>
<td>R</td>
<td>566.63</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>+∞</td>
<td>any</td>
<td>C</td>
<td>R</td>
<td>566 2/3</td>
</tr>
</tbody>
</table>
4.4.5 Solution for B = $40, any \( (n_1,n_2) \)

Table 4.5 displays the optimal strategies and values associated with $40 funds.

Table 4.5. Optimal Expected Value and Single Step Decentralization Strategy, $40 Budget

<table>
<thead>
<tr>
<th>( n_1 )</th>
<th>( n_2 )</th>
<th>(1)</th>
<th>(2)</th>
<th>( V^*[40/(r),n_1,n_2] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>any</td>
<td>C</td>
<td>R</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>any</td>
<td>C</td>
<td>R</td>
<td>170</td>
</tr>
<tr>
<td>2</td>
<td>any</td>
<td>C</td>
<td>R</td>
<td>340</td>
</tr>
<tr>
<td>3</td>
<td>any</td>
<td>C</td>
<td>R</td>
<td>510</td>
</tr>
<tr>
<td>4</td>
<td>any</td>
<td>C</td>
<td>R</td>
<td>680</td>
</tr>
<tr>
<td>5</td>
<td>any</td>
<td>C</td>
<td>R</td>
<td>738.46</td>
</tr>
<tr>
<td>6</td>
<td>any</td>
<td>C</td>
<td>R</td>
<td>752.28</td>
</tr>
<tr>
<td>7</td>
<td>any</td>
<td>C</td>
<td>R</td>
<td>755.00</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td></td>
</tr>
<tr>
<td>+( \infty )</td>
<td>any</td>
<td>C</td>
<td>R</td>
<td>755 5/9</td>
</tr>
</tbody>
</table>
4.4.6 Solution for $B = 50$, all $(n_1, 0)$

Thus far, proposals "type-2" could not be financed, making the problem solution rather simple. A capital budget of $50$ is the lowest $B$ value for which proposals "type-2" could be financed when so desired (recall that $I_2$ equals $50$), making the solution somewhat more complex.

However, by using the recursion procedure previously described, one can routinely solve $V^*$ for all $(n_1 n_2)$, and identify best decentralization modes as well.

The only difference from previous situations is the need to solve for more combinations. Here, we will fix $n_2$, then solve for different $n_1$'s, increment $n_2$, and solve for different $n_1$'s, and so on, sequentially.

We shall start from $(n_1, 0)$ combinations, and come up with expressions similar to those developed in the previous subsections. Table 4.6 displays the results of such analysis, when $B = 50$ and $n_2 = 0$. 
Table 4.6. Optimal Expected Value and Single Step Decentralization Strategy, No Proposals "Type-2," $50 Budget

<table>
<thead>
<tr>
<th>Proposal-Mix</th>
<th>Best Decentralization Strategy, when Drawing Sample Type</th>
<th>(V^*[50/(r),n_1,0])</th>
</tr>
</thead>
<tbody>
<tr>
<td>(n_1)</td>
<td>(n_2)</td>
<td>(1)</td>
</tr>
<tr>
<td>0</td>
<td>any</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>any</td>
<td>C</td>
</tr>
<tr>
<td>2</td>
<td>any</td>
<td>C</td>
</tr>
<tr>
<td>3</td>
<td>any</td>
<td>C</td>
</tr>
<tr>
<td>4</td>
<td>any</td>
<td>C</td>
</tr>
<tr>
<td>5</td>
<td>any</td>
<td>C</td>
</tr>
<tr>
<td>6</td>
<td>any</td>
<td>C</td>
</tr>
<tr>
<td>7</td>
<td>any</td>
<td>C</td>
</tr>
<tr>
<td>(\infty)</td>
<td>any</td>
<td>C</td>
</tr>
</tbody>
</table>
4.4.7 Solution for \( B = \$50 \), all \((n_1, 1)\)

For a fixed value of \( n_2 = 1 \), we shall evaluate \( V \) for the different values of \( n_1 \), starting from \( n_1 = 0 \) and sequentially incrementing it by one.

For \( n_1 = 0 \), when "type-2" is drawn:

\[
V_c[50/(2),0,1] = 0.7(1000+V*[0/(r),0,0]) + 0.3(V*[50/(r),0,0])
- 20 = 0.7(1000) + 0.3(0) - 20 = 680,
\]

\[
V_a[50/(2),0,1] = 0.7(1000) + 0.3(-1000) + V*[0/(r),0,0] = 400,
\]

\[
V_r[50/(2),0,1] = V*[50/(r),0,0] = 0,
\]

\[
Max_{A,C,R}V[50/(2),0,1] = 680 @ C_2.
\]

Similarly, for \( n_1 = 1 \) and "type-1" drawn:

\[
V_c[50/(1),1,1] = 0.9(200+V*[40/(r),0,1]+0.1(V*[50/(r),0,1]-10)
= 0.09(200)+0.1(680)-10 = 238,
\]

\[
V_a[50/(1),1,1] = 0.9(200)+0.1(-150)+V*[40/(r),0,1] =
= 180-15+0 = 165,
\]

\[
V_r[50/(1),1,1] = V*[50/(r),0,1] = 680,
\]

\[
Max_{A,C,R}V[50/(1),1,1] = 680 @ R_1.
\]

For \( n_1 = 1 \) and "type-2" drawn:

\[
V_c[50/(2),1,1] = 0.7(1000+V*[0/(r),1,0]) + 0.3(V*[50/(r),1,0])
-20 = 0.7(1000) + 0.3(170)-20 = 731,
\]

\[
V_a[50/(2),1,1] = 0.7(1000) + 0.3(-1000) + V*[0/(r),1.0] = 400,
\]

\[
V_r[50/(2),1,1] = V*[50/(r),1,0] = 170,
\]

\[
Max_{A,C,R}V[50/(2),1,1] = 731 @ C_2.
\]
The expected unconditional value of $B = \$50$ when associated with a $(1,1)$ combination of proposals, prior to the drawing process, is the weighted average of the conditional values, when weighted by the likelihood that each outcome will occur. (See Equation 4.08.)

Therefore, 

$$V^*[50/(r),1,1] = \frac{n_1}{n_1+n_2} V^*[50/(1),1,1] + \frac{n_2}{n_1+n_2} V^*[50/(2),1,1]$$

$$= 0.5 \cdot 680 + 0.5 \cdot 731 = 705.5 .$$

Similarly, we will increment $n_1$ for the fixed $n_2 = 1$, and solve. Here we get:

$$V^*[50/(1),2,1] = 705.5 @ R_1$$

$$V^*[50/(2),2,1] = 782 @ C_2$$

and for the expected prior value,

$$V^*[50/(r),2,1] = 0.667 \cdot 705.5 + 0.333 \cdot 782 = 731 .$$

A further increment in $n_1$ will result with the following values:

$$V^*[50/(1),3,1] = 731 @ R_1 ,$$

$$V^*[50/(2),3,1] = 833 @ C_2 ,$$

$$V^*[50/(r),3,1] = 756.5 .$$

Similarly, we shall proceed for $n_1 = 4, 5, 6, 7, \ldots\ldots,$ $\infty$ then set $n_1$ to zero, increment $n_2$, and repeat this whole process again and again. Doing so, we will cover
the whole \((n_1,n_2)\) plane (in the general case—a multi-dimensional space) to points of interest (one can stop the procedure at any point).

One additional problem remains, though, to calculate the values of \(B\) when \(n_1\) and/or \(n_2\) increase to very large numbers. Here we will prove that \(B\)'s value asymptotically converges to a limit, with a declining marginal increase in value.

As the best decentralization mode in this problem is \(R_1\) (reject "type-1" when it is drawn), one will reject "type-1" proposals till he draws a "type-2" one, assuming it is known that "type-2" proposals are included in the batch. These rejections do not cause any cost to the evaluation process. Once a "type-2" proposal is randomly drawn, centralization will be the preferred course of action (refer to previous numerical examples). Assuming that this event happens prior to running out of "type-1" proposals, we can write the following:

\[
\text{Limit } V^*[50/(r), n_1, 1] = \text{Limit } V^*_C[50/(2), n_1-x, 1] = \\lim_{n_1 \to \infty} \quad n_1 \to \infty
\]

\[
= 0.7(1000) + 0.3 \cdot \text{Limit } V^*[50/(r), n_1-x, 0] - 20, \quad n_1 \to \infty
\]

where \(x\) is the number of "type-1" proposals rejected prior to the discovery of "type-2."
Assuming \((n_1 - x)\) is still a large number of "type-1" proposals,

\[
\text{Limit } V^*[50/(r), n_1-x, 0] = 944 \frac{4}{9} \quad \text{ (from Table 4.6)}.
\]

\(n_1 \to \infty\)

Therefore, we get

\[
\text{Limit } V^*[50/(r), n_1, 1] = 700 + 0.3 \cdot (944 \frac{4}{9}) - 20 = 963 \frac{1}{3}.
\]

\(n_1 \to \infty\)

Similarly, we can come up with the following limits:

\[
\text{Limit } V^*[50/(r), n_1, 2] = 969 ,
\]

\(n_1 \to \infty\)

\[
\text{Limit } V^*[50/(r), n_1, 3] = 970.7 ,
\]

\(n_1 \to \infty\)

\[
\text{Limit } V^*[50/(r), n_1, 4] = 971.21 ,
\]

\(n_1 \to \infty\)

and so on.

4.4.8 Solution for \(B = $50\), all \((n_1 n_2)\)

Similarly, we can solve for different fixed \(n_1\), when \(n_2\) is sequentially increased to infinity. For large \(n_2\) one can assume that because of the \(R_1\) and \(C_2\) strategies, the remaining "type-1" proposals after an extensive analysis of the "type-2" proposals have very little contribution to \(B\)'s total value.
Therefore

\[
\text{Limit } v^*_C[50/(r),0,n_2] = 0.7(1000) + 0.3 \cdot v^*_C[50/(r),0,n_2-1] - 20.
\]

\[
n_2 \rightarrow \infty
\]

On the limit,

\[
\begin{align*}
v^* &= 700 + 0.3 \cdot v^* - 20, \\
v^* &= 680/0.7 = 971 \frac{3}{7}.
\end{align*}
\]

A similar figure will apply to all other limits, as long as \( n_2 \rightarrow \infty \).

Finally, a summary of all best strategies, as well as the unconditional expected values of \( B = 50 \) for the different proposals mixes is presented in Table 4.7.

Three major insights do emerge from the table of unconditional expected gross values (Table 4.7):

1) Independently from the proposals' mix, whenever a "type-2" proposal is randomly drawn, the best course of action calls for the submittal of such for headquarters consideration. Therefore, as long as "source-2" (or "cell-2") is concerned, the best mode of decentralization is fixed to be "Complete-Centralization" (\( C_2 \)).

2) The best mode of decentralization associated with "type-1" proposals is variable. When "type-2" proposals do exist in the mix, "Complete
Table 4.7. Expected Unconditional Optimal Value of $50 Funds Associated with a Dual Source, and the Conditional Optimal Modes of Decentralization

<table>
<thead>
<tr>
<th>No. of &quot;Type-2&quot; proposals $n_2$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>$n_2 + \infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of &quot;Type-1&quot; proposals $n_1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>680 $C_2$</td>
<td>884 $C_2$</td>
<td>945.2 $C_2$</td>
<td>963.6 $C_2$</td>
<td>971.3 $C_2$</td>
</tr>
<tr>
<td></td>
<td>$C_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
</tr>
<tr>
<td>1</td>
<td>170 $C_1$</td>
<td>705.5 $C_2$</td>
<td>888.8 $C_2$</td>
<td>953.8 $C_2$</td>
<td>965.5 $C_2$</td>
<td>971.3 $C_2$</td>
</tr>
<tr>
<td></td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
</tr>
<tr>
<td>2</td>
<td>340 $C_1$</td>
<td>731 $C_2$</td>
<td>894 $C_2$</td>
<td>950.4 $C_2$</td>
<td>965.2 $C_2$</td>
<td>971.3 $C_2$</td>
</tr>
<tr>
<td></td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
</tr>
<tr>
<td>3</td>
<td>510 $C_1$</td>
<td>756.5 $C_2$</td>
<td>899.2 $C_2$</td>
<td>950.0 $C_2$</td>
<td>965.1 $C_2$</td>
<td>971 $C_2$</td>
</tr>
<tr>
<td></td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
</tr>
<tr>
<td>4</td>
<td>680 $C_1$</td>
<td>782 $C_2$</td>
<td>904.3 $C_2$</td>
<td>950.5 $C_2$</td>
<td>965.2 $C_2$</td>
<td>971.3 $C_2$</td>
</tr>
<tr>
<td></td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
</tr>
<tr>
<td>5</td>
<td>850 $C_1$</td>
<td>807.5 $C_2$</td>
<td>909.5 $C_2$</td>
<td>951.4 $C_2$</td>
<td>965.2 $C_2$</td>
<td>971.3 $C_2$</td>
</tr>
<tr>
<td></td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
</tr>
<tr>
<td>6</td>
<td>919.6 $C_1$</td>
<td>828.6 $C_2$</td>
<td>914.2 $C_2$</td>
<td>952.4 $C_2$</td>
<td>965.4 $C_2$</td>
<td>971.3 $C_2$</td>
</tr>
<tr>
<td></td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
</tr>
<tr>
<td>7</td>
<td>939.0 $C_1$</td>
<td>845.2 $C_2$</td>
<td>918.5 $C_2$</td>
<td>953.3 $C_2$</td>
<td>965.6 $C_2$</td>
<td>971.3 $C_2$</td>
</tr>
<tr>
<td></td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
</tr>
<tr>
<td>$n_1 + \infty$</td>
<td>944.4 $C_1$</td>
<td>963.1 $C_2$</td>
<td>969 $C_2$</td>
<td>970.7 $C_2$</td>
<td>971.2 $C_2$</td>
<td>971.3 $C_2$</td>
</tr>
<tr>
<td></td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
<td>$R_1$</td>
</tr>
</tbody>
</table>
Negative Decentralization" for "type-1" proposals should be preferred \((R_1)\). Whenever "type-2" proposals run out, "Complete-Centralization" for "type-1" should be the preferred mode \((C_1)\). Thus, "type-1" proposals are associated with two different optimal modes of decentralization, depending upon the content of the proposals' mix.

3) The phenomenon of decline in the marginal expected gross value of funds in respect to increases in the number of associated proposals, is beautifully demonstrated. On top of that, an upper limit to such value is identified and proved to exist. Increases in numbers of proposals, asymptotically increase the expected gross value of available capital-funds, in a way that converges with the upper limit.

Besides the tabular way of presentation, there might be an advantage to display the results in some graphical ways.

One good way to display the results of a dual source problem is on a Cartesian system of axes, where each axis will represent the number of proposals from a single source, and "Iso-Gross-Value" curves will be drawn through all points (combinations of proposals) of equal
expected gross value. This kind of display is presented in Figure 4.01.

The other way to display those same results is by using a three-dimensional perpendicular system of axes. Here, the first two axes will represent the different proposal-mixes while the third axis will stand for the expected gross value of such combinations. A display of the "Expected Gross Value" surface is provided in Figure 4.02.

4.5 Conclusions

1) An Expected Conditional Value Function was developed for a single step decision at any "state" of the proposals' mix, and for any possible drawing outcome and possible mode of decentralization.

2) By maximization of the Expected Conditional Gross Value Function, a "best" course of decentralization is identified, for each possible drawing outcome, at any "state" of the proposals' mix.

3) A weighted average of the Optimal Expected Conditional Values associated with the different drawing outcomes and their likelihood is defined, in order to remove the outcomes conditions. The Unconditional Expected Optimal Value of funds, associated with a certain proposals' mix, can be interpreted as the value of funds and
Fig. 4.01. Iso expected gross value of funds curves, associated with different proposal combinations.
Fig. 4.02. Expected unconditional gross value of funds with their associated proposal combinations.
4) The Unconditional Expected Optimal Value of funds and proposals has a diminishing marginal value, and it asymptotically converges to an upper limit, which is determined by the type of proposals with the best potential. This means that the marginal value of additional proposals in a batch, when capital funds are limited, is declining, although the Expected Optimal Value itself does increase. This phenomenon should raise the question whether the effort to generate the marginal proposals is worth while undertaking, a question which will be raised again in the next chapter.

It should be important to note that in contrast with the limited-funds situation, in the ample-funds situation there is no marginal decline in value in respect to increases in proposals content. Theoretically, there are no limits to the Expected Unconditional Value of funds, as there are no limits to the funds themselves.

5) Optimal decentralization modes are identified and associated with the optimal conditional expected values. These decentralization strategies may, or may not, remain fixed and independent from the content of the proposals' mix. In the numerical example (refer to Section 4.4) Complete Negative Decentralization is
associated with all events resulting with a drawing of "type-1" proposals, as long as "type-2" proposals are included in the batch. Once "type-2" proposals run out, the best mode for "type-1" changes to Centralization. However, Centralization is the best (optimal) decentralization strategy associated with "type-2" proposals independently from the "state" of the proposals' mix.

6) The general model is structured to be multi-dimensional in respect to different sources of proposals. By no means is it designed to restrict the number of sources which compete for financing of their proposals from the limited capital funds. The dual source example in Section 4.4 is designed for illustrative purposes only, and does not reflect a limitation to the model.

7) A random sequential sampling process was selected to increase management decision productivity, without giving up much in equity. This process could be considered as a semi-equitable reviewing process, second in equity to complete consideration to all proposals. However, additional savings in cost could result from a priority and discriminative sequential sampling procedure, resulting with further loss in equity.

For example, in Section 4.4 priority could be given to the review and analysis of all "type-2" proposals in any batch, with a secondary preference to
the analysis of "type-1" proposals. This, in turn, would discourage "source-1" from its efforts to come up with its proposals, because of "unequal opportunity for consideration,"

8) Proposals which are a priori rejected, should be saved for possible consideration at a later stage. This observation is beautifully demonstrated in a numerical example in Section 4.4, where centralized consideration of "type-1" proposals is not the best mode of decentralization as long as proposals"type-2" are available. This preference does change later, when "type-2" proposals are no longer available. Therefore, if the previously rejected proposals (because of the "economy" or "cost" of consideration, and not for any other reason) could be saved for a later stage, these same proposals would have received a very serious Centralized consideration, due to the change in the "state" of the economical system.

9) Three different techniques have been used to display the results of a dual-source situation; the first is by a table and the other two are graphical. Examples are presented in Section 4.4. However, multi-source results could only be displayed in tables while the graphical display is only good for the specific dual-source situation.

10) Up to this point we were discussing expected gross values of combinations of proposals to the organization,
being aware of potential costs incurred during centralized consideration processes, which could be reduced by delegation of authority (for a "trade-off" in higher risk for the chief executive). We have implicitly assumed that all proposals are generated and submitted for headquarters approval at no "generation" or "submittal" cost to the organization. This might be the case for organizations which receive proposals from the "environment" (another word to describe all economies outside the specific organization) like book publishers, professional journal publishers, research or philanthropic foundations, et cetera.

However, this might not be the case in many business firms, banks and other financial institutions, et cetera. In these cases, some cost is incurred in the process of generating a proposal, and this expense is charged to the business firm itself. Looking at the capital budgeting decentralization problem from this extended point of view, one should ask himself two questions:

1) Why should not we look at the net value to the organization of any combination of proposals, where the cost of generating those proposals will be discounted from the gross value?
2) Secondly, once we have come up with those net terms, why should not we identify this specific combination of proposals which maximizes our net organizational gain, and control our generation process accordingly? (Customarily, the process of generating proposals is not thought of as a "controllable process").

The second question turns out to be very provocative in terms of the suggestion that there might be a structured and consistent way to generate control recommendations on processes which were previously considered as "uncontrollable." Even further than that, the claim that these "controls" will provide some kind of optimal allocation of search and generation resources in a highly uncertain environment seems provocative by itself.

However, these doubts are irrelevant to the claim. The contribution of this model is not in its success to reduce uncertainty. The contribution is reflected in its power to provide consistent decision-aids, thus improving the decision making process. (This, of course, implicitly assumes that a rational decision has to be consistent, and that contributions to consistency are of appreciated value.)

A more detailed discussion on the referred topic is presented in Chapter 5.
CHAPTER V

COORDINATION OF SEARCH EFFORTS FOR INVESTMENT OPPORTUNITIES: THE CENTRALIZED FIRM

5.0 Divisional Search Cost for Investment Opportunities or the Generation Cost of Proposals

As indicated previously in Section 4.5, by definition of Expected Unconditional Gross Value of Funds, associated with a certain proposals' mix, we have assumed that all proposals are available to headquarters at no "generation cost."

When this is not the case, as in organizations where opportunities have to be searched for, researched, investigated and carefully planned, a certain cost to the organization (usually to the division, which is on the organizational frontiers with the environment and the opportunities-space) is associated with the "generation" process.

One can try and trace the components of such costs through the accounting information system. These could be identified as (for example):

1) Employees direct salaries and benefits (economists, typists, programmers, et cetera).

2) Materials (paper, laboratory materials, et cetera).
3) Other direct costs (traveling, telephone, mail, et cetera).

4) Overhead (administration, buildings, utilities, et cetera).

5) Services from other departments (computer services).

6) External information and counseling, et cetera.

Whenever these costs turn out to be significant in contrast with other costs and benefits, the validity of the implicit assumption that proposals are provided cost free becomes questionable. At the same time, maximization of the previously defined Expected Unconditional Gross Value of Funds will lead to the selection of strategies which will not, necessarily, maximize the net values of funds. More specifically, an infinitely large proposals' mix will maximize Expected Unconditional Gross Value of Funds, but the generation of such mix might not be the best strategy, when a component in the organization will have to "pick up the tab" for generating those proposals.

As these costs become prohibitive, one should implement economic tests in order to make a decision when to stop the generation process. Generally speaking, one should continue to generate proposals as long as the marginal expected return from that specific proposal is larger than the marginal expected cost to generate it.

The important observation which is made here suggests that the number of proposals to be generated from each source is
a non-trivial problem, which has to be decided and controlled in some way or another. Once this optimal mix is identified, different control measures will have to be applied, depending on the organizational structure. Wherever search strategies are centrally controlled, the generation of the optimal mix could be imposed on the divisions. Reward methods should be implemented in "profit-center" structured organizations.

By expansion of the model to account for possible proposal generation costs, one moves from a position which cares for "headquarters welfare" to a position which cares for "overall organizational welfare."

As a first degree of approximation, we shall assume that generation costs per proposal from any source-\(i\) (\(DC_i\)) are constant, and no "quantity discounts" apply to these costs. Different types of proposals (source wise) will be associated with different generation costs. Proposals from a similar source will be associated with a single cost distribution, commonly identified by a mean and a variance. Customarily, these costs are incurred by the divisions, therefore Divisional Cost (or in short, "DC") was selected to represent this concept.

5.1 Environmental Limitations on Opportunities' Availability

It might be reasonable to assume that the marginal cost of generating proposals from a certain source increases continuously, with the number of proposals already generated. This, for the simple reason that it becomes more and more difficult to
generate proposals of a similar "nature," the more proposals of the same "nature" have already been generated.

Usually, it becomes difficult to identify and monitor these marginal differences, however there is no better substitute for precise and accurate information.

Nevertheless, in order to try and avoid complex and expensive accounting systems, we shall make a simplified assumption that proposals could be available for a fixed and constant cost (which will vary from one source to another), up to a point where this cost suddenly rises to an absolutely prohibitive amount of money. This means that the marginal cost will be constant within a certain quantity range of proposals, and will suddenly change to an infinitely large number, at a certain quantity point. The interpretation of the sudden increase in marginal cost should be as a limitation on the availability of proposals of a similar source (and consequently, a similar "nature"). Figure 5.01 illustrates the cost approximation which will be used in following sections, in contrast to the real cost behavior, as expected from precise accounting procedures.

The availability limitation, thus, will always be the value for the relevant "n" associated with the point of discontinuity on the accumulated generation-cost curve. It should be interpreted as the highest n value, above which no proposals of the specific type could be generated during a specific "batch period," for a feasible cost.
Fig. 5.01. Real versus approximate cumulative divisional cost of proposals generation (single source).
It should be noted, however, that limits to availability are not a necessary condition, therefore there should not always be a point of discontinuity, or it could be for very large "n" values.

5.2 Overall Expected Net Organizational Value of Funds

Having defined the unconditional expected gross value of funds when associated with a certain proposals' mix, and also the expected divisional cost of generating different types of proposals, one may come up with an expression for the overall expected net organizational value of such funds and associated proposals, by taking the difference between the gross value and the generation cost.

Defining OENOV as the Overall Expected Net Organizational Value, one can write:

\[
OENOV[B/(r), n_1, n_2, \ldots, n_m] = V^*[B/(r), n_1n_2, \ldots, n_m] - TDC[n_1, n_2, \ldots, n_m].
\]

(5.01)

Recalling the assumption that DC_i are constant for all (i) within respective availability ranges, one can write an expression for the total divisional cost:

\[
TDC[n_1n_2, \ldots, n_m] = \sum_{k=1}^{m} n_i \cdot DC_i,
\]

(5.02)

such that

\[ n_i \leq n_i' \]

for all \((i=1,2,\ldots,m)\),
where \( n_i' \) are defined as all upper limits for availability, and 
\( \text{TDC} \) is the total divisional generation cost. Therefore, by 
plugging in the explicit expression for \( \text{TDC}[n_1, n_2, n_3, \ldots, n_m] \), 
the OENOV expression can be rewritten as:

\[
\text{OENOV}[B/\langle r \rangle, n_1, n_2, \ldots, n_m] = V^*\left[B/\langle r \rangle, n_1, n_2, \ldots, n_m\right] - \sum_{i=1}^{m} n_i \cdot \text{DC}_i,
\]

(5.03)
such that 
\[ n_i \leq n_i' \]
for all \((i = 1, 2, \ldots, m)\).

5.3 The OENOV Maximizing Search Strategy 
for Investment Opportunities

Previously in Section 5.0 we had suggested that by expand- 
ing our model from the "expected gross value" concept to the 
"expected net value" concept, we are moving to describe the 
"welfare" of the organization as a whole, instead of the "wel­
fare" of the corporate headquarters, as a separate economical 
entity.

Now, when an "expected net value" expression has been de- 
veloped, it could be used to identify those non-trivial combina- 
tions of proposals which are expected to maximize the overall 
organizational welfare. By maximizing OENOV in respect to 
possible proposals-mixes, we get:

\[
\text{OENOV}^* = \max_{n_1, n_2, \ldots, n_m} \text{OENOV}[B/\langle r \rangle, n_1, n_2, \ldots, n_m],
\]

(5.04)
such that

\[ n_i \leq n_i' \]

for all \( i=1,2,\ldots,m \).

The set \( S(n_1^*, n_2^*, \ldots, n_m^*) \) of proposal-mix that maximizes this OENOV expression is the optimal combination of proposals, toward which search efforts should be allocated.

The optimal expected net value of the search (generation), evaluation and investment process associated with a given opportunity-space and a given amount of capital-funds will therefore be \( OENOV^* \). The associated total amount of funds which should be allocated to the generation process of proposals will be

\[ TDC^* = \sum_{i=1}^{m} n_i^* \cdot DC_i, \quad (5.05) \]

such that

\[ n_i \leq n_i' \]

for all \( i=1,2,\ldots,m \).

An extension to the previous (Section 4.4) numerical example is believed to provide an excellent illustration to these ideas. This extension is presented in the next section (Section 5.4).

5.4 A Numerical Example

Assuming that expected divisional costs associated with the generation of a single proposal are as following:

"type-1" proposal: \( DC_1 = \$10 \) such that \( n_1 \leq 6 \),
"type-2" proposal: \( \text{DC}_2 = $20 \) such that \( n_2 \leq 2 \).

It is also assumed that all numerical data and conclusions given in previous examples to this point are valid and hold for this numerical example, as well.

Using the expression developed in Section 5.2 for the expected total Divisional Cost for generating proposals, one can come up with Table 5.1, which displays the total generation costs for different combinations of proposals.

As the OENOV matrix will consist of the matrix of differences between the \( V^*[B/(r),n_1,n_2] \) matrix entries and the \( \text{TDC}(n_1,n_2) \) matrix entries (see Section 5.2), Table 5.2 displays OENOV entries with which one will come up in this numerical example.

By looking at Table 5.2, one can identify that specific proposals' mix which maximizes OENOV, simply by scanning over the matrix entries. Here, the combination \( (6,0) \) provides the largest OENOV to the given $50 capital funds. In other words, the solution to the problem is:

\[
(n_1^*, n_2^*) = (6,0) .
\]

Once the problem becomes more complex in terms of the number of different sources involved, an iterative solution technique might be more efficient in terms of computational efforts (like the Simplex method in linear programming). However, computational efficiency is not the main point of concern in this
Table 5.1. Expected Total Divisional Generation Cost of Investment Proposals' Mixes

<table>
<thead>
<tr>
<th>n2</th>
<th>TDC ($n_1, n_2$) in $</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>... et cetera</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>20</td>
<td>40</td>
<td>60</td>
<td>80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>30</td>
<td>50</td>
<td>70</td>
<td>90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>40</td>
<td>60</td>
<td>80</td>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>50</td>
<td>70</td>
<td>90</td>
<td>110</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>60</td>
<td>80</td>
<td>100</td>
<td>120</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>50</td>
<td>70</td>
<td>90</td>
<td>110</td>
<td>130</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>60</td>
<td>80</td>
<td>100</td>
<td>120</td>
<td>140</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>70</td>
<td>90</td>
<td>110</td>
<td>130</td>
<td>150</td>
<td></td>
<td></td>
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<tr>
<td>...</td>
<td>...</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>...</td>
<td>...</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>et cetera</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 5.2. Overall Expected Net Organizational Value of Funds Associated with a Given Constrained Opportunity Space (6.2)

<table>
<thead>
<tr>
<th>n₁</th>
<th>n₂</th>
<th>OENOV ((n₁,n₂)) in $</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>660</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>844</td>
</tr>
<tr>
<td>1</td>
<td>160</td>
<td>675.5</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>838.8</td>
</tr>
<tr>
<td>2</td>
<td>320</td>
<td>691</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>834</td>
</tr>
<tr>
<td>3</td>
<td>480</td>
<td>706.5</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>829.2</td>
</tr>
<tr>
<td>4</td>
<td>640</td>
<td>722</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>824.3</td>
</tr>
<tr>
<td>5</td>
<td>800</td>
<td>737.5</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>819.5</td>
</tr>
<tr>
<td>6</td>
<td>859.6</td>
<td>748.6</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td></td>
</tr>
</tbody>
</table>
analytical study. The main point is in the observation that there might be an optimal mix, which should be identified in one way or another.

It should be worth-while to point out here, that the associated headquarters decentralization strategy with this solution is $C_1$, which means that when proposals "type-1" are sequentially and randomly drawn, Complete Centralization will be the best mode for consideration.

When one accepts the recommended search, consideration and investment strategy in this given opportunities space, he may expect to receive a net gain of $859.6.

A quick check confirms this figure:

1) **Generation Cost (Certain):**

$$\sum_{i=1}^{2} n_i \cdot DC_i = 6 \cdot 10 + 0 \cdot 20 = $60.$$ 

2) **Headquarters Consideration Cost (Uncertain):**

Likelihood that all first 5 drawn are "acceptable"

$$= (0.9)^5 .$$

Likelihood that a 6'\text{th} draw will be required

$$= [1-(0.9)^5] .$$

**Total Expected Consideration Cost:**

$$5 \cdot 10 \cdot (0.9)^5 + 6 \cdot 10 \cdot [1-(0.9)^5] = $54.1$$

3) **Proposals' Gross Value (Uncertain):**

(Please refer to Table 5.3.)
### Table 5.3. Total Expected Gross and Net Value of $50 Funds, When Associated with 6 Proposals "Type-1"

<table>
<thead>
<tr>
<th>i</th>
<th>Likelihood that exactly i-proposals out of 6 are &quot;acceptable&quot;</th>
<th>Conditional present worth ($)</th>
<th>Likelihood times present worth ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$^6_0$ (0.9) (0.1)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>$^6_1$ (0.9) (0.1)</td>
<td>200</td>
<td>0.0108</td>
</tr>
<tr>
<td>2</td>
<td>$^6_2$ (0.9) (0.1)</td>
<td>400</td>
<td>0.486</td>
</tr>
<tr>
<td>3</td>
<td>$^6_3$ (0.9) (0.1)</td>
<td>600</td>
<td>9.748</td>
</tr>
<tr>
<td>4</td>
<td>$^6_4$ (0.9) (0.1)</td>
<td>800</td>
<td>78.72</td>
</tr>
<tr>
<td>5</td>
<td>$^6_5$ (0.9) (0.1)</td>
<td>1000</td>
<td>885.00</td>
</tr>
<tr>
<td>6</td>
<td>$^6_6$ (0.9) (0.1)</td>
<td>1000*</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Total Expected Gross Value:</th>
<th>$972.96</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)+(2) Less Expected Costs:</td>
<td>114.1</td>
<td></td>
</tr>
<tr>
<td><strong>Expected Net Value</strong></td>
<td><strong>$858.86</strong></td>
<td></td>
</tr>
</tbody>
</table>

*No more than five proposals could be financed.*
As one can observe, this figure is similar to the figure indicated in the OENOV matrix (Table 5.2), within a 0.1% accuracy range.

It is important to note that limits on proposals' availability are never necessary, however possible. Therefore, this numerical example does include a situation where such limits do exist.

For the unconstrained problem, where unlimited numbers of proposals could be generated for the same fixed divisional costs DC_i, there should be a different optimal solution, however once again a non-trivial one.

In this given numerical example, when all availability constraints are removed, the OENOV matrix will include entries as in Table 5.4.

By scanning, once again, through an OENOV(n_1,n_2) matrix in Table 5.4 or alternatively by identifying the highest point in a OENOV(n_1,n_2) graph as presented in Figure 5.02, it becomes evident that the combination (0,3) provides the maximum OENOV value for the unconstrained space, at the fixed amount of $50 capital funds.

Therefore, one can say that the solution to the unconstrained problem is

\[(n_1^*,n_2^*) = (0,3)\].
Table 5.4. Total Expected Net Value of $50 Funds, When Associated with Different Proposals' Mixes

<table>
<thead>
<tr>
<th>$n_2$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>... et cetera</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>660</td>
<td>844</td>
<td>885.2</td>
<td>Max</td>
<td>883.6</td>
</tr>
<tr>
<td>1</td>
<td>160</td>
<td>675.5</td>
<td>838.8</td>
<td>883.8</td>
<td>875.5</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>320</td>
<td>691</td>
<td>834</td>
<td>870.4</td>
<td>865.2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>480</td>
<td>706.5</td>
<td>829.2</td>
<td>860.0</td>
<td>855.1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>640</td>
<td>722</td>
<td>824.3</td>
<td>850.5</td>
<td>845.1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>800</td>
<td>737.5</td>
<td>819.5</td>
<td>841.4</td>
<td>835.2</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>859.6</td>
<td>748.6</td>
<td>814.2</td>
<td>832.4</td>
<td>825.4</td>
<td></td>
</tr>
</tbody>
</table>

... et cetera
Fig. 5.02. Unconstrained OENOV \((n_1*, n_2*)\) and the OENOV maximizing proposal mix \((n_1*, n_2*)\).
The associated optimal headquarters decentralization strategy will be $C_2$, which means that whenever proposals "type-2" are randomly drawn, Complete Centralization will be the best course of action for the next single step evaluation process. (One has to remember that the referred optimal solution for the best search strategy, recommends the generation of three "type-2" proposals, only.)

The expected net gain associated with the optimal strategy in the unconstrained opportunity space is $885.2. This figure could be confirmed in a similar way to the one used to test the precision of the solution in the constrained space problem.

5.5 Conclusions

1) An Overall Expected Net Organizational Value of funds with an associated opportunity space (OENOV) expression for any proposals' mix has been developed. The expression consists of the difference between the previously developed Unconditional Expected Optimal Gross Value of funds and proposals' mix ($V^*[n_1,n_2]$) and the Divisional Cost ($TDC[n_1,n_2]$) associated with the generation of those proposals.

2) The proposal mix which maximizes $OENOV(n_1,n_2)$ is defined as the optimal mix of proposals, which when generated, maximizes the welfare of the organization as a whole.
3) In a "simple" organizational structure (see Figure 1.01) corporate headquarters supposedly have control and can coordinate the allocation of divisional search efforts for investment opportunities. In many cases this option is not exercised, efforts are not coordinated, and the end result is a batch of capital investment proposals, the content of which nobody can predict (nor can one predict the cost involved in the generation effort).

In this chapter it is suggested that headquarters can use their control option by imposing a "quota" of proposals on each "source" (or segment) so as to conform with the optimal proposals' mix.

This control can be practically implemented by central budgetary appropriations of carefully predetermined amounts of money to the different financial entities, in a way that source "type-i" will receive the amount AP\textsubscript{j} such that

\[ AP_i = n_i^* \cdot DC_i, \]

and that the total allocation of search funds will equal TDC\textsuperscript{*}, such that

\[ TDC^*(n_1,n_2,n_3...,n_m) = \sum_{i=1}^{m} n_i^* \cdot DC_i. \]

This implies that there is an optimum to the total allocation for search efforts, as well as an optimal
distribution of such funds. (This is on top of previous conclusions about optimal authority delegation strategies.)

4) In a "profit center" organizational structure (see Figure 1.02) corporate headquarters have no direct control over divisional search strategies and the allocation of their efforts for such purposes, as these decisions are within centers' array of authority and responsibility. They are judged by their profit "performance," and all risks associated with centers' decisions, are fully assumed by the "profit centers."

   However, indirect or implicit control measures could be developed, as to create an "artificial economy," where centers' most profitable search strategies will conform with those desired by firms' corporate headquarters.

   In order to achieve this objective, one has to understand how "profit centers" do make their economical decisions regarding the generation of investment proposals, and the competition with other "centers" on scarce resources. Having understood "profit centers" decision making process, a discriminative "rewards system" could be implemented by headquarters, so as to attract "profit centers" to adjust their
search and generation strategies to the already identified overall optimal strategy.

A detailed discussion about this topic follows in Chapter VI.

5) One of the weaknesses of the "quota" (or rationing) system suggested for the "simple" structured firm (see conclusion number 3) is in its rigidity. Once search resources have been allocated for a given period, there are not many degrees of freedom for the division to take the chance, and come up with proposals different than those contracted for.

This would mean that it will be extremely difficult for divisions to change headquarters' attitudes toward specific sources, especially when these attitudes are negative, for some reasons.

Therefore, it might be important for headquarters to design procedures divisions could use in order to bring up revolutionary, extraordinary or unprecedented investment ideas for which moneys have not been originally allocated, without giving up in productivity and profitability.

One way which could be used to overcome this weakness is by giving additional weight to subjective beliefs about divisional future performance, compared to hard evidence on past divisional performance.
(One should interpret divisional performance as the "pa" concept.)

6) An optimal non-trivial search strategy was identified both for the unconstrained and constrained opportunity space. If the solution to the unconstrained problem violates any of the constraints, the two optimal strategies are different. Like in other optimization problems, if such violation takes place, one or more of the constraints will be included in the solution to the constrained problem.

7) The search, generation and evaluation process of investment opportunities has an impact on organizational welfare. Therefore it should be carefully controlled and coordinated, like other important processes the organization is involved in.

Sometimes, past information and future estimates are available somewhere in the organization, but when not properly treated, they are considered useless. It has been demonstrated here how certain kinds of data could be put together in a meaningful structured model, in order to identify consistent economic decisions. However, it has to be pointed out that as with all other decision aids, the quality of the recommendation on the "output" side is always as good as the quality of the "input" data, for a given and fixed model.
As to the quality of the model itself, this has to be judged by and compared to common analytical practices, as this model is purely analytical and is developed through analytical observations and not through experimental design.

To this point, a "linear" risk attitude was assumed for the chief executive (decision maker). "Non linear" attitudes will be discussed in Chapter VII. In that chapter, the "certainty equivalent" model will also be relaxed, allowing for uncertainty and imperfect information.
CHAPTER VI

THE DISCRIMINATING REWARDS SYSTEM FOR COORDINATION OF
SEARCH EFFORTS FOR INVESTMENT OPPORTUNITIES:
THE "PROFIT CENTERS" STRUCTURED FIRM

6.0 Proposal Sources in Competition for Funds

The main effort in this chapter is devoted to the analytical structuring of the decision making process "profit centers" follow when making their choices about numbers and types of proposals they ought to generate in a given period and from a certain given opportunity space, as to maximize their "welfare."

It is assumed that corporate headquarters have the final word about project approval either by direct control and decision, or by its indirect decision to delegate certain kinds of authorities to "profit centers."

Therefore, from the point of view of the "center," any decisions to allocate funds for the purpose of search and generation of investment proposals are decisions under risk, for which the "center" assumes full responsibility. A proposal could end up being approved thus generating future positive net cash flows, or it could end up being rejected, thus causing only
negative net cash flows at present (generation and consideration costs).

As in previous chapters, it will be assumed that head­quarters reviewing process is a periodic one, where proposals are sequentially drawn at random from a batch, which is periodically accumulated. This process is selected in order to assure, if not absolute equity by across-the-board consideration, at least a just and productive way of consideration, where all proposal "sources" have equal opportunities to submit proposals and expect them to be reviewed.

As before, our effort will concentrate around "linear" risk attitude on the part of the "centers'" decision behavior. Therefore, it will be sufficient to look at trade-offs between expected values, without looking at the whole range of possible outcomes and associated probabilities.

The following hypotheses and assumptions will be used in the proposed model:

6.0.1 The likelihood of a single proposal "type-j" to be drawn from a batch \((p_d_j)\) equals the relative frequency of proposals of same type in the batch, prior to the drawing act.

\[
p_d_j = \frac{n_j}{\sum_{i=1}^{m} n_i} = \frac{n_j}{N}, \quad (6.01)
\]
where \( N \) is the number of proposals in the batch such that
\[
N = \sum_{i=1}^{m} n_i, \quad N \neq 0.
\]

6.0.2 The drawing process is statistically independent from the consideration process. In order for a specific proposal to be qualified for approval, it has to meet two conditions.

a) It has to be drawn.

b) Once drawn, it has to be "acceptable" by headquarters perception.

Therefore, the likelihood for a single proposal "type-\( i \)" to be approved \( (pA_i) \) equals the likelihood it will be drawn, times the conditional likelihood that once drawn it will be "acceptable."

\[
pA_i = \text{Likelihood ("Acceptable"/drawn)}_i \cdot \text{Likelihood (drawn)}_i
\]

Symbolically,

\[
pA_i = pA_i \cdot pd_i.
\]  \hspace{1cm} (6.02)

6.0.3 Defining \( S \) as the total number of proposals drawn from the batch for consideration, as counted at the end of the sequential drawing process. The expected relative composition of \( S \) (type wise) is equal to the relative composition of the original batch. Therefore, the expected number of
"type-i" proposals in $S,(l_i')$, will be equal to:

$$l_i' = \left(\frac{n_i}{N}\right) \cdot S = p_{d_i} \cdot S,$$

such that

$$S \leq N$$

and

$$n_i \leq N. \quad (6.03)$$

6.0.4 Only a fraction ($a_i$) of "type-i" proposals in $S$ ($l_i$) is expected to be approved.

When headquarters' decision option not to consider proposals (for any reason) does not exist, the "mapping" from the drawn set of proposals to the approved set of proposals is through the use of the "variance in acceptance" concept ($p_a$). Consequently, the expected number of proposals "type-i" which are finally approved ($a_i'$), is equal to:

$$a_i' = p_{a_i} \cdot l_i' = p_{a_i} \cdot p_{d_i} \cdot S = p_{A_i} \cdot S,$$

such that

$$S \leq N. \quad (6.04)$$

6.0.5 Looking at the expected relative frequency of approved "type-i" in contrast to the total number of proposals approved, one gets for ($r_i'$):

$$r_i' = a_i'/\sum_{j=1}^{m} a_j' = p_{A_i} \cdot S/\sum_{j=1}^{m} p_{A_j} \cdot S = p_{A_i} \cdot S,$$
= \frac{p_a_i}{\sum_{j=1}^{m} p_a_j} = \frac{p_a_i \cdot p_d_i}{\sum_{j=1}^{m} (p_a_j \cdot p_d_j)} . \quad (6.05)

By definition of relative frequency, one can observe that the following condition should hold as well:

\[ \sum_{i=1}^{m} r_i = 1. \quad (6.06) \]

6.0.6 A prerequisite in the evaluation process is assumed to be the full utilization of available capital funds. Consequently, the sequential random drawing and consideration of proposals continues as long as portions of funds (B) are still available.

Therefore, in order to exhaust B, the following equation should be satisfied:

\[ \sum_{i=1}^{m} a_i \cdot I_i = B , \]

or, alternatively,

\[ \sum_{i=1}^{m} a_i \cdot \left( \frac{I_i}{B} \right) = 1 . \quad (6.07) \]

Having made these assumptions, as well as set of hypotheses, one is ready to define "source-k's" decision problem. Initially, we shall assume that perfect information is given to "profit
center-k" on "input" variables as following:

\[ n_i - (i=1,2,3,..,m/ i \neq k) \] All competitors updated
decisions for the number of proposals they intend
to generate.

\[ I_i - (i=1,2,3,..,m) \] Initial outlays required by all
types of proposals.

\[ p_{a_i} - (i=1,2,3,..,m) \] Variance in "acceptance" perception
for all types of proposals.

B - Available funds.

\[ R_k \] Expected reward credited to "source-k" for any
approved "type-k" proposal.

\[ DC_k \] Expected Divisional Cost for generating a single
"type-k" proposal.

The perfect information requirement will be relaxed later
(Chapter VII) and will be substituted by beliefs and assessments.

Given that "source-k" is by itself or within a "profit
center," it is most likely that its objectives are to maximize
profits. However, nothing could be learned from that statement
about the time distribution of profits (preferences between
profits in the short or long run). Therefore, we shall assume
that "profit centers" wish to maximize the Time-Discounted Net
Present Worth of future cash-flows. The previously defined
R_k and DC_k concepts should be interpreted as negative and positive cash flows, respectively.

The decision variable will thus be n_k, and the decision problem will be to maximize "centers" discounted net gain, with respect to possible n_k values.

Evidently, there will not be a trivial solution to the "center's" decision problem, as neither n_k = 0 can maximize net gain (no possible rewards), nor can an extremely large value of n_k do (because of the prohibitive generation costs and the limited amount of rewards).

Consequently, the "center's" problem can be formulated as following:

\[
\max n_k \{Z_k\} ,
\]

where

\[
Z_k = a_k' \cdot R_k - n_k \cdot DC_k ,
\]

such that

\[
\sum_{j=1}^{m} a_j (I_j/B) = 1 . \tag{6.08}
\]

From this point, x will replace n_k as the single decision variable, the value of which has to be determined.

By solving for a_i' in 6.05, one gets:

\[
a_i' = (\sum_{j=1}^{m} a_j') \cdot p_i / \sum_{j=1}^{m} p_j . \tag{6.05'}
\]
By substitution of 6.02 into 6.05' one gets:

\[ a_i' = \left( \frac{\sum a_j'}{\sum n_j \cdot p_{a_j}} \right) / \left( \sum n_i \cdot p_{a_i} \right) \leq p_{a_i} n_i . \]

(6.09)

\((p_{a_i} n_i\) is the upper limit for \(a_i'\), when all proposals in the batch have been sampled.)

This set of equations consists of \((m-1)\) independent equations (the \(m\)'s equation is trivial, or dependent), and \((m+1)\) unknowns \((a_{i}'\) to \(a_{m}'\), as well as \(x\)). The same set of equations can be rewritten as following:

\[ a_i' / a_k' = \frac{n_i \cdot p_{a_i}}{(x \cdot p_{a_k})} , \]

or alternatively as:

\[ a_i' = \left( n_i \cdot p_{a_i} / x \cdot p_{a_k} \right) \cdot a_k' , \]

for all \(i\) (\(i = 1,2,\ldots,m\)) .

(6.09')

(Note that the trivial equation in this set is for \(i = k\)).

Using equation 6.07 one can substitute \(a_{i}\) by its expected value \(a_{i}'\), and use the expression for \(a_{i}'\) which was previously developed in 6.09' . Thus, one gets:

\[ \left( a_{k}' / B \cdot x \cdot p_{a_k} \right) \sum_{j=1}^{m} n_j \cdot p_{a_j} \cdot I_j = 1 . \]

(6.07')
By separation of the sum, and rearranging the expression one gets:

\[
a_k' \left( \sum_{j=1}^{m} n_j \cdot p_{a_j} \cdot I_j + x \cdot p_{a_k} \cdot I_k \right) = B \cdot x \cdot p_{a_k},
\]

or finally:

\[
a_k' = \frac{B \cdot x \cdot p_{a_k}}{\sum_{j=1}^{m} n_j \cdot p_{a_j} \cdot I_j + x \cdot p_{a_k} \cdot I_k} \quad (6.07')
\]

By substitution of the \( a_k' \) expression in 6.08 one can rewrite the objective function \( Z_k \) as a function of the single decision variable---\( x \), as follows:

\[
Z_k = \frac{(B \cdot x \cdot p_{a_k} \cdot R_k)}{\sum_{j=1}^{m} n_j \cdot p_{a_j} \cdot I_j + x \cdot p_{a_k} \cdot I_k} - x \cdot DC_k \quad (6.08')
\]

The final step will be to identify the optimal solution. As \( Z_k \) is a function of one discrete variable \( x \), a good approximation to the optimal solution will be the solution to a similar problem, where \( x \) is assumed to be a continuous variable. (One of the bounding integers will be the optimal solution to the discrete problem.)

Therefore, the solution to the "continuous" problem, coupled with a non-negativity constraint on the solution, results in the following expression:

\[
X^* = \frac{(B \cdot R_k / DC_k \cdot I_k)^{1/2}}{(L_k)^{1/2} - I_k},
\]

such that

\[I_k \leq B,\]
and where
\[ I_k = \sum_{j=1, j \neq k}^{m} \frac{(n_j \cdot p_{aj} \cdot I_j)}{(pa_k \cdot I_k)}. \]  \hspace{1cm} (6.10)

Incidentally, \( I_k \) could be interpreted as the total "amount of competitive effort" on the part of \( k \)-th competitors, which is invested in the competition.

Recalling the upper limit on \( a_i' \) \( (a_i' \leq pa_i \cdot n_i) \), in 6.09, equation 6.07" will be revised as follows (for \( i = k \)):
\[ a_k' = \frac{(B \cdot x \cdot pa_k)}{\left( \sum_{j=1, j \neq k}^{m} n_j \cdot pa_j \cdot I_j + x \cdot pa_k \cdot I_k \right)} \leq x \cdot pa_k. \]  \hspace{1cm} (6.07''')

By further simplification of this expression, one comes up with a lower limit for \( x \), as follows:
\[ x \geq \frac{(B/pa_k \cdot I_k) - I_k}{I_k}, \]  \hspace{1cm} (6.11)
where \( I_k \) is the total amount of "competitive effort" as previously defined.

This low limit for \( x \) is important, because without it one may conclude that when there is no competition at all, the best strategy on \( k \)'th part is to generate no proposals at all (see \( X^* \) for \( I_k = 0 \)). However, this violates the prerequisite that all available funds (\( B \)) will be exhausted. By imposing this condition, the low limit for \( x \) comes up, which means that whenever the competitive effort diminishes, \( k \)'th best strategy converges to:
\[ x_{\text{low limit}} = \frac{B}{pa_k \cdot I_k} \] \hspace{1cm} (6.11')
\[ I_k \rightarrow 0 \]
Consequently, the optimal solution to "center-k's" decision problem is approximated by the combination of 6.10 and 6.11:

\[ X^* = \left( \frac{B \cdot R_k}{D C_k \cdot I_k} \right)^{1/2} \cdot (L_k)^{1/2} - L_k , \]

such that

\[ I_k \leq B , \]

\[ X^* \geq \frac{B}{p a_k \cdot I_k} - L_k , \]

and a non-negativity constraint for \( x \):

\[ L_k \leq \frac{B \cdot R_k}{D C_k \cdot I_k} . \] (6.12)

The "low-limit" test could be further simplified, as to suggest that an optimal solution exists only when:

\[ X^* \geq X_{\text{low-limit}} . \]

In other words,

\[ \left( \frac{B \cdot R_k}{D C_k \cdot I_k} \right)^{1/2} \cdot (L_k)^{1/2} - L_k \geq \frac{B}{p a_k \cdot I_k} - L_k , \]

which can be reduced to

\[ L_k \geq \frac{B \cdot D C_k}{R_k \cdot p a_k^2 I_k} . \] (6.13)

In practical solution procedures, the "low limit test," the "non-negativity test" and the "outlay versus funds test" could
be run first, to make sure that all conditions for an optimal solution do exist. The flow chart in Figure 6.01 might be an efficient procedure to follow in terms of minimizing computational efforts.

It should be noted, however, that this optimization model is based on the assumption that all proposals which are sequentially and randomly selected from the batch, do receive "Centralized" consideration, and neither "Complete Positive Decentralization" nor "Complete Negative Decentralization" are possible. When these modes of decentralization are introduced, and headquarters preference is established, a different model should be developed. Guidelines for such a model are provided in the next section. However, one property remains in common—which is the fact that there is one \( x \) value which maximizes "center's" profits \( (Z_k) \).

6.1 A Numerical Example

Given a two dimensional source problem, as follows:

<table>
<thead>
<tr>
<th>Source-1</th>
<th>Source-2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>( p_{a_1} = 0.9 )</td>
<td>( p_{a_2} = 0.7 )</td>
</tr>
<tr>
<td>( I_1 = 50 ) ($)</td>
<td>( I_2 = 10 ) ($)</td>
</tr>
<tr>
<td>( R_1 = 2 ) ($)</td>
<td></td>
</tr>
<tr>
<td>( DC_1 = 1 ) ($)</td>
<td></td>
</tr>
<tr>
<td>( n_1 ) = Decision Variables = ( (x) ); ( n_2 = 100 ) (units)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( B = 700 ) ($)</td>
</tr>
</tbody>
</table>
\[ L_k = \sum_{j=1}^{m} \frac{(n_j \cdot p_{a_j} \cdot I_j)}{(p_{a_k} \cdot I_k)} - L_k \]

**Fig. 6.01.** "Profit Center's" optimal strategy solution procedure.
The problem is to identify the optimal number of proposals "Source-1" should produce under the rules previously stated, if such an optimum does exist.

Using the procedure suggested in Figure 6.01, we get:

a) \( I_1 = 50 \), \( B = 700 \),
   therefore \( I_1 < B \).

b) \( L_1 = \frac{(100 \cdot 0.7 \cdot 10)}{(0.9 \cdot 50)} = 15.55 \),
   \( \frac{(B \cdot DC_1)}{(R_1 \cdot pa_1 \cdot I_1)} = \frac{(700 \cdot 1)}{(2 \cdot 0.81 \cdot 50)} = 8.64 \),
   therefore \( L_1 > \frac{(B \cdot DC_1)}{(R_1 \cdot pa_1 \cdot I_1)} \).

c) \( \frac{(B \cdot R_1)}{(DC_1 \cdot I_1)} = \frac{(700 \cdot 2)}{(1 \cdot 50)} = 28 \),
   therefore \( L_1 < \frac{(B \cdot R_1)}{(DC_1 \cdot I_1)} \).

d) Consequently, a profit maximizing solution does exist:

\[
X^* = \frac{(700 \cdot 2 \cdot 1 \cdot 50)^{1/2}}{(15.55)^{1/2}} - 15.55 \\
X^* \approx 5.3 .
\]

Two integers should, therefore, be considered for the solution of the discrete problem:

a) \( X^*_1 = 5 \);

For this solution:

\[
a_1' = \frac{(B \cdot X \cdot pa_1)}{\sum_{j=1}^{m} (r_{n_j} \cdot pa_j \cdot I_j)} = \frac{700 \cdot 5 \cdot 0.9}{(5 \cdot 0.9 \cdot 50 + 100 \cdot 07 \cdot 10)} ,
\]

\[
a_1' = 3.405 ,
\]
and the expected profit,

\[ Z_1' = a_1' \cdot R_1 - X_1 \cdot DC_1 = 3.405 \cdot 2 - 5 \cdot 1 = $1.81 \]

b) \( X_1^* = 6 \);

Similarly, we get

\[ a_1' = 3.897 \]

\[ Z_1' = $1.794 \]

Therefore, we can conclude that because the expected profit associated with \( X^* = 5 \) is greater than the expected profit associated with \( X_1^* = 6 \), the first is the final solution to the discrete problem.

One may wonder how should "source-1"s best strategy be adjusted, when "source-2" decides to enter competition with "source-1" at different degrees of intensiveness. The following Table 6.1 was developed, using the same given data, but for different \( n_2 \) values, using the same techniques and procedures described before.

From "source-1" standpoint, there are three different ranges of \( n_2 \), bounded by two break-even points, within which it has to take different kinds of responsive measures.
Table 6.1. "Source-1" Optimal Conditional Proposals Generation Strategy, in a Competitive Market

<table>
<thead>
<tr>
<th>$n_2$ range</th>
<th>$x_1^*$</th>
<th>$a_1'$</th>
<th>$z_1'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>180 &amp; up</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>150</td>
<td>2</td>
<td>1.100</td>
<td>0.2</td>
</tr>
<tr>
<td>100</td>
<td>5</td>
<td>3.405</td>
<td>1.81</td>
</tr>
<tr>
<td>75</td>
<td>6</td>
<td>4.755</td>
<td>3.51</td>
</tr>
<tr>
<td>56</td>
<td>7</td>
<td>6.238</td>
<td>5.475</td>
</tr>
<tr>
<td>50</td>
<td>8</td>
<td>7.099</td>
<td>6.197</td>
</tr>
<tr>
<td>25</td>
<td>12</td>
<td>10.573</td>
<td>9.146</td>
</tr>
<tr>
<td>10</td>
<td>14</td>
<td>12.600</td>
<td>11.200</td>
</tr>
<tr>
<td>0</td>
<td>16</td>
<td>14.400</td>
<td>12.800</td>
</tr>
</tbody>
</table>

The set of all optimal "source-1" responses to different competition intensiveness on the part of "source-2" is "Source-1
Conditional Best Strategy." This set (function) could be noted as \( F\left(\frac{n_1^*}{n_2}\right) \).

Similarly, one can expect to be able to develop a similar function \( G\left(\frac{n_2^*}{n_1}\right) \), which will represent the same conditional best strategy, but this time from "source-2"'s standpoint.

Points of intersection between these conditional functions are the only points in the \( (n_1, n_2) \) space, which have the special property of being in both "Conditional Best Strategy" sets. One would wonder whether these points could be stable steady state solutions to the bargaining process between the sources, where each source can add or subtract proposals to and from the batch. A discussion on this topic follows in the following sections.

As previously indicated, the previous model is valid as long as headquarter's option to decentralize the selection process is not used. For demonstration purposes, let us assume from here on that this restriction is relaxed, and that "type-2" proposals are rejected as long as "type-1" proposals are available. (A similar phenomenon was identified in the previous chapter.) Under the new situation, "source-1" and "source-2" will, once again, have the dilemma about the best course of action they should select in terms of the best (optimal) number of proposals to be generated. However, the previously developed "mapping" equations cannot be applied here, as the "rules of game" violate the assumptions the previous model is based upon.
As "source-1" is assumed to have priority over "source-2," all "type-1" proposals will be considered first. Therefore, the number of "type-2" proposals in the batch has no impact on the chances of "type-1" proposals to get headquarters approval. Therefore, $n_1^*$ will be completely independent from $n_2$ (in contrast with the previous model, where such a phenomenon was not possible).

The best (optimal) $n_1$ will be the number which will satisfy the "utilization of funds" constraint, assuming no proposals "type-2" are in the batch. In this example, full utilization will occur (on the average) when:

$$X_1^* \approx B/(I_1pa_1) = \frac{700/(50\cdot0.9)}{15.55},$$

$$X_1^* = 16 \text{ (independent of } n_2).$$

In contrast with "source-1" which has the priority, "source-2" strategy will be designed to take advantage of "leftovers" of the capital funds, which have not been allocated to "source-1" because of insufficient acceptable (by headquarters) "type-1" proposals.

In the two-dimensional problem:

$$[n_2^*/(i)] = (i-n_1-B/I_1)I_1/(pa_2 \cdot I_2).$$

In other words, the optimal conditional expected number of proposals "type-2" to be generated, equals the remaining funds $(i-n_1-B/I_1) \cdot I_1$, (where $i$ is the number of "type-1" proposals being
rejected), divided by \((p_{a2} \cdot I_2)\) to assure, once again, average utilization of funds.

At the same time, the likelihood that exactly \(i\) proposals "type-1" will be rejected is equal to:

\[
LK[(n_1 - i) "acceptable"; (i) "unacceptable"/ n_1 proposals "type-1")] = \frac{(n_1) \cdot (1 - p_{a1})^i \cdot (p_{a1})^{n_1 - i}}{i}
\]

Consequently, if one agrees that the unconditional optimal expected number of proposals "type-2" to be generated is equal to the average value of the conditional optimal numbers when weighted by their associated likelihoods, then one can write:

\[
n_2^* = \Sigma LK[(i) "type-1" rejected/ i submitted] \cdot [n_2^*/(i)] = \sum_{i=n_1-B/I_1+1}^{n_1} (n_1) \cdot (1 - p_{a1})^i \cdot (p_{a1})^{n_1 - i} \cdot (i - n_1 - B/I_1) \cdot I_1 / (p_{a2} \cdot I_2).
\]

The following Table (6.2) demonstrates the procedure one has to follow in order to determine "source-2"'s optimal strategy, using the numerical example given in this chapter.

The sum of the products of entries in the second and fourth columns (for \(i = 0, 1, 2, \ldots, 16\)), is the unconditional optimal expected number of "type-2" proposals, given that 16 "type-1" proposals have been generated, and the "variance of acceptance" values, as well as initial outlays and total available funds, are those given in the numerical example.
Table 6.2. "Source-2" Optimal Conditional Proposals Generation Strategy, when Proposals "Type-1" Have Priority for Review

<table>
<thead>
<tr>
<th>(i) &quot;type-1&quot; rejected</th>
<th>( \text{Likelihood of event} )</th>
<th>Funds Remaining at event</th>
<th>Conditional Optimal Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( \binom{16}{0} (1-0.9)^0 (0.9)^{16} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>( \binom{16}{1} (1-0.9)^1 (0.9)^{15} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>( \binom{16}{2} (1-0.9)^2 (0.9)^{14} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>( \binom{16}{3} (1-0.9)^3 (0.9)^{13} )</td>
<td>50</td>
<td>50/10\cdot 0.7</td>
</tr>
<tr>
<td>4</td>
<td>( \binom{16}{4} (1-0.9)^4 (0.9)^{12} )</td>
<td>100</td>
<td>100/10\cdot 0.7</td>
</tr>
<tr>
<td>5</td>
<td>( \binom{16}{5} (1-0.9)^5 (0.9)^{11} )</td>
<td>150</td>
<td>150/10\cdot 0.7</td>
</tr>
<tr>
<td>j</td>
<td>( \binom{16}{j} (1-0.9)^j (0.9)^{16-j} )</td>
<td>(j-2) 50</td>
<td>(j-2)50/10\cdot 0.7</td>
</tr>
<tr>
<td>16</td>
<td>( \binom{16}{16} (1-0.9)^{16} (0.9)^0 )</td>
<td>700</td>
<td>700/10\cdot 0.7</td>
</tr>
</tbody>
</table>
6.2 The Homogeneous Steady State Solution in a Multi-
Source Competitive Situation

As suggested in Section 6.1, the intersection of the conditional optimal functions $F(n_1^*/n_2)$ and $G(n_2^*/n_1)$ in the two-source situation will provide a solution to the steady state problem, for a competitive market situation as well as a completely centralized mode of operation.

In analytical expressions, the pair $(n_1^{**}, n_2^{**})$ is a steady state solution to the dual-source problem, if it satisfied the simultaneous equations:

1) $n_1 = -\left(\frac{pa_2 I_2}{pa_1 I_1}\right) \cdot n_2 + \left(\frac{B \cdot R_1}{DC_1 \cdot I_1}\right)^{1/2} \cdot \left(\frac{pa_2 I_2}{pa_1 I_1}\right)^{1/2} \cdot n_2^{1/2}$ \hspace{1cm} (6.14)

2) $n_2 = -\left(\frac{pa_1 I_1}{pa_2 I_2}\right) \cdot n_1 + \left(\frac{B \cdot R_2}{DC_2 \cdot I_2}\right)^{1/2} \cdot \left(\frac{pa_1 I_1}{pa_2 I_2}\right)^{1/2} \cdot n_1^{1/2}$ \hspace{1cm} (6.15)

This set of equations can be rewritten as:

1) $n_1 = -A_{12} \cdot n_2 + C_{12} \cdot n_2^{1/2}$, \hspace{1cm} (6.14')

2) $n_2 = -A_{21} \cdot n_1 + C_{21} \cdot n_1^{1/2}$, \hspace{1cm} (6.15')

where

$A_{12} = \frac{pa_2 I_2}{pa_1 I_1}$, \hspace{1cm} $C_{12} = \left(\frac{B \cdot R_1 \cdot pa_2 I_2}{DC_1 \cdot I_1^2 \cdot pa_1}\right)^{1/2}$, \hspace{1cm} (6.16)

$A_{21} = \frac{pa_1 I_1}{pa_2 I_2} = (A_{12})^{-1}$, \hspace{1cm} $C_{21} = \left(\frac{B \cdot R_2 \cdot pa_1 I_1}{DC_2 \cdot I_2^2 \cdot pa_2}\right)^{1/2}$, \hspace{1cm} (6.17)
A simultaneous solution results with:

\[
n_1^{**} = \left( \frac{C_{12}^2 \cdot C_{21} \cdot A_{21}}{A_{21}^2 \cdot C_{12}^2 + A_{12} \cdot C_{21}^2} \right)^2, \tag{6.18}
\]

\[
n_2^{**} = \left( \frac{C_{12} \cdot C_{21}^2}{A_{21}^2 \cdot C_{12}^2 + A_{12} \cdot C_{21}^2} \right)^2, \tag{6.19}
\]

in addition to the trivial (0,0) solution.

Consequently, it was proven that in the original rules of random selection with centralized review, there is always a non-trivial steady-state solution to the dual-source proposal generation problem, a solution which maximizes both profit-centers' objectives.

Similarly, one can identify a steady-state solution, which maximizes both profit-centers' objectives in the consideration process, where decentralization is permitted.

The trivial solution is always unacceptable, as it violates the "utilization of funds" constraint, although it may, indeed, maximize centers' objectives.

A graphical presentation of the described concept of steady-state may help in the interpretation of the idea. The following data* will be used for illustration.

*This data has been used for illustrative purposes in Sections 4.4 and 5.4.
pa_1 = 0.9 \quad I_1 = $10 \quad R_1 = $200 \quad DC_1 = $10
pa_2 = 0.7 \quad I_2 = $50 \quad R_2 = $1000 \quad DC_2 = $20

In Figure 6.02 the different values for the conditional optimal expected number of "type-2" proposals G(n_2*/n_1) were displayed. On the same "map," iso-profit lines for "source-2" were displayed as well. At the lower left corner, the "utilization of funds" constraint was also displayed.

In Figure 6.03, both conditional strategies were displayed one on top of the other. One non-trivial point of intersection was identified, a "point" which satisfies both optimal requirements. This point satisfies exactly the same requirements as the solution to the simultaneous set of equations, thus represents the "non-trivial steady state" solution. In Section 6.3 it will be proven that this solution is stable, and will be practically achieved by a simple bargaining process between the two parties, as long as agreements between the parties to refrain from competition will be outlawed. (Note that by allowing the parties to refrain from competition, the underlying assumptions in the model will, once again, be violated, making the conclusions invalid).

The expansion of these conclusions to the multi-source situation is from here on rather simple. From equation 6.14 we have for the set of m conditional optimal strategies:

\[ n_1 = -A_{12} \cdot n_2 - A_{13} \cdot n_3 + \cdots - A_{1m} \cdot n_m + B_{12} \cdot n_2^{1/2} + \cdots + B_{1m} \cdot n_m^{1/2}, \]

\[ n_2 = -A_{21} \cdot n_1 - A_{23} \cdot n_3 + \cdots - A_{2m} \cdot n_m + B_{21} \cdot n_1^{1/2} + \cdots + B_{2m} \cdot n_m^{1/2}, \]
Fig. 6.02. "Source-2" conditional best strategy, random selection, complete centralization.
Fig. 6.03. "Profit Centers" in competition for funds.
Here $m$ is the number of independent sources, and the coefficients $A_{ij}$ are as follows:

$$A_{ij} = \frac{p_{ai} I_i}{\sum_{i=1}^{m} p_{ai} I_i} \quad \text{for} \quad i = 1, 2, \ldots, m \; / \; j \neq i,$$

$$A_{ij} = 0 \quad \text{for} \quad i = j.$$  \hspace{1cm} (6.21)

At the same time $B_{ij}$ are:

$$B_{ij} = \frac{(B R_i)}{\sum_{i=1}^{m} (B R_i)} \cdot \frac{p_{aj} I_j}{\sum_{i=1}^{m} p_{ai} I_i} \quad \text{for} \quad i = 1, 2, \ldots, m \; / \; i \neq j,$$

$$B_{ij} = 0 \quad \text{for} \quad i = j.$$  \hspace{1cm} (6.22)
Arranging the equations in a matrix form results with:

\[
\begin{bmatrix}
  n_1 \\
  n_2 \\
  n_3 \\
  \vdots \\
  n_m
\end{bmatrix} =
\begin{bmatrix}
  0 & A_{12} & A_{13} & \cdots & A_{1m} \\
  A_{21} & 0 & A_{23} & \cdots & A_{2m} \\
  A_{31} & A_{32} & 0 & \cdots & A_{3m} \\
  \vdots & \vdots & \vdots & \ddots & \vdots \\
  A_{m1} & A_{m2} & A_{m3} & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
  n_1 \\
  n_2 \\
  n_3 \\
  \vdots \\
  n_m
\end{bmatrix}
+ \begin{bmatrix}
  0 & B_{12} & B_{13} & \cdots & B_{1m} \\
  B_{21} & 0 & B_{23} & \cdots & B_{2m} \\
  B_{31} & B_{32} & 0 & \cdots & B_{3m} \\
  \vdots & \vdots & \vdots & \ddots & \vdots \\
  B_{m1} & B_{m2} & B_{m3} & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
  n_1^{1/2} \\
  n_2^{1/2} \\
  n_3^{1/2} \\
  \vdots \\
  n_m^{1/2}
\end{bmatrix}
\]

(6.20')

In vector form, the previous matrix form is presented by:

\[
\overline{n} = -A \cdot \overline{n} + B \cdot (\overline{n}^{1/2})
\]

(6.20'')

where by $\overline{n}^{1/2}$ one has to interpret a vector which consists of entries which are the square root of the entries in $\overline{n}$, respectively.

The vector equation can further be rearranged, as:

\[(I+A) \cdot \overline{n} = B \cdot (\overline{n}^{1/2}) .\]

Alternatively

\[C \cdot \overline{n} = B \cdot (\overline{n}^{1/2}) ,\]

where

\[C = I + A .\]

(6.21)
By definition, all entries in A and B are non-negative. Therefore, all entries in C will be non-negative as well.

As this vector equation is quadratic in form, it implies that:

1) One cannot use linear algebra in order to solve the problem.

2) There might be either no solution, exactly two solutions or an infinite number of solutions.

For the reason that we have m independent equations, an infinite number of solutions is ruled out.

For the reason that all entries are non-negative in the final simple form of the vector equation, no solution is also ruled out. As a matter of fact the trivial solution (\( \bar{n} = 0 \)) is one solution, which proffs that exactly two solutions should result from the set of equations.

As the trivial solution violates the utilization constraint, one will only be interested in the second, non-trivial solution. There remains the algebraic problem of identifying the process which will generate the desired non-trivial solution. An iterative and sequential approximation procedure will be suggested here, to identify the second solution within any arbitrary selected degree of precision.
Solution Procedure

a) Pick up an arbitrary set of \( (m-1) \) values for \( n_2, n_3, \ldots, n_m \) which satisfy the non-negativity constraint (see Section 6.0) such that

\[
\sum_{j=2, j\neq 1}^{m} \frac{P_{aj}I_j}{P_{a1}I_k} \leq \frac{B \cdot R_1}{DC_1 \cdot I_1} .
\]

b) Calculate the optimal "source-1" strategy \( n_1^\ast \) associated with the previously selected set of competitors strategies.

\[
n_1^\ast = \frac{(B \cdot R_1/ DC_1 \cdot I_1)^{1/2} \cdot (L_1)^{1/2} - L_1}{(L_1)^{1/2}}.
\]

A non-negative optimal value for \( n_1 \) will be identified.

c) Select the closest integer as the first approximation to the \( n_1 \) component of the steady state solution vector.

d) Plug in the identified estimate of \( n_1 \) into the initially selected set (as the first-in-line entry) while dropping the second-in-line entry of \( n_2 \).

e) Calculate the optimal "source-2" strategy \( n_2^\ast \) associated with the previously selected or computed set of competitors strategies.

\[
n_2^\ast = \frac{(B \cdot R_2/ DC_2 \cdot I_2)^{1/2} \cdot (L_2)^{1/2} - L_2}{(L_2)^{1/2}} .
\]
f) Repeat (c) and (d) for \( n_2 \), increment counter, go to (b), and stop when counter equals \( m \).

g) Repeat the procedure once again, and check for convergence to a limit. Repeat again and again until the degree of desired convergence is achieved.

A graphical display of the iterative and sequential approximation procedure is presented for the dual-source problem in Figure 6.03.

Step (a) is represented by point A on the \( n_2 \) axis.
Step (b) is represented by point B.
Step (e) is represented by point C.
Step (b) in the second cycle is represented by point D.
Step (e) in the second cycle is represented by point E.

Points F and G represent the approximations for \( n_1 \) and \( n_2 \) in the third cycle, respectively. Consequently, one can observe the convergence process to the "steady-state" solution \( (n_1^{**}, n_2^{**}) \).

6.3 Conditions for Solution Stability

One commonly used way for testing system's stability is by observation of its behavior once pulled out from its steady state by an arbitrary external factor. A stable solution is commonly interpreted as a steady state condition toward which the system will always converge in response to a perturbation, independently from the magnitude of the perturbation, its direction or its timing.
However, an identified stable solution set of "state variables" in a certain array of forces like the concept of magnetic "field", may not be necessarily stable nor at steady state under a different array.

Consequently, if not proved otherwise, steady state solutions and their stability are dependent upon the "field" of forces. Therefore, any stability test to a solution set is valid to the extent that the "field" remains unchanged, and its results can only be associated with the specific "field."

In economic systems one may observe a broad spectrum of relations among separate economic entities (in this context, "profit centers" are separate economic entities). This wide spectrum of relations could be identified by its two limits, which are:

a) "Perfect Market" - An ideal market, where no entity is dominant, and pure competition is practiced.

b) "Monopoly or Oligopoly" - A market dominated by few entities, which coordinate their strategies and agree (explicitly or implicitly) to refrain from competition for the benefit of the participating members.

If a concept such as "The Degree of Competition" in a certain market could be defined, and if it could be described on an 0 to 1 continuous scale, the "Perfect Market" would be associated with the value of "1," and the "Oligopoly" with "0."
"In between" situations are those situations where entities agree to refrain from competition to a certain extent, but not to the full extent, thus not enjoying the complete advantages of monopoly.

Typically, economic entities behave in ways which are sometimes similar to the first market type, and sometimes more similar to the second type. One can suggest that a typical market consists of a combination of both types, and the precise behavior of entities in any specific market could be described as a point on the "0 to 1" "degree of competition" scale.

This long introductory discussion was necessary to explain the limitations of previously identified steady-state solutions and their associated stability.

Using these terms for the types of market behavior it should be noted that the models previously developed have assumed perfect-market behavior (pure competition), and their validity is limited to the extent that this behavior is actually observed in the specific market.

As for imperfect information, the system may find itself in steady states which are different than those suggested when having perfect information. However, this should not be interpreted as a limitation to model's validity.

Two different models have been suggested for the identification of steady-state solutions; one for the completely centralized firm (the basic model) and another, where the two modes
of decentralization could also be used by headquarters. These two identified solution sets are mutually exclusive and are associated with two different "fields."

As for the basic model (Section 6.0), the limits to the stability of the solution set were established and identified in the approximation procedure, which is an iterative sequential process (refer to Section 6.2). One recalls that in order for an optimal steady-state set to exist, the non-negativity constraint has to be satisfied. However, when initial conditions are selected, or a perturbation is imposed on the system such that the non-negativity constraints are violated, no optimal set can exist and the problem of stability becomes irrelevant.

As long as all constraints are satisfied, one uses the approximation procedure to identify the solution set. However, this procedure by its nature operates as a stability tester. In order for a solution set to exist, the sequential iterative procedure has to converge to the solution within any arbitrary selected level of precision. The same set has to be identified independently from the selected initial conditions. These conditions are identical to the conditions for stability.

The fact that the solution set has to satisfy a quadratic equation which has a trivial solution is a proof that one cannot expect more than one non-trivial solution set, which means, in turn, that if this solution is stable, there will not be more limitations to stability than those identified as limitations for
existence of the set. (A move from one steady state solution to another is impossible.)

Consequently, in the basic model—stability of the optimal solution set, and the existence of such a set are related one to one.

An excellent demonstration to this effect can be given through the dual-source example in Section 6.2. As long as the initial conditions do not violate the non-negativity constraint, a solution set will be identified. Here, the initial "state" \((n_1^0, n_2^0)\) should not violate the conditions as following:

a) \(n_1^0 < 195\)

b) \(n_2^0 < 27\)

Independently from the magnitude or direction of the initial perturbation, when this constraint is satisfied, the solution approximation procedure will converge to the solution set. A graphical support to this conclusion is provided by Figure 6.03.

Incidentally, the two limits have been identified by:

a) \(F(n_1^*/n_2) = 0\);

b) \(G(n_2^*/n_1) = 0\);

, respectively.

However, when the "field" of forces does change (for example, by changing the rules of competition, or by changing
the rules of headquarters consideration by introducing the de-
centralization modes), this change could be associated with
changes in the steady-state solution set. This shift in solu-
tion might be interpreted by mistake as instability on the part
of the original solution. However, this interpretation is wrong,
since the stability judgment is made through the use of two
completely different "fields" of forces.

Once again, one can use the numerical example in 6.2
to demonstrate this point. Having made the "Perfect Market"
assumption, one comes up with the optimal solution set of:

\[(n_1^{**}, n_2^{**}) = (23, 11) \]

This solution set is expected to provide the two profit-
centers with a net expected gain of:

\[z_1' = \$120\]
\[z_2' = \$430\]
respectively.

A change in market behavior may encourage the centers
to negotiate a deal as to refrain from competition. This could
result, for example, by a decision to generate a smaller amount
of proposals such as:

\[(n_1, n_2) = (3, 1)\]
which will, in turn, generate a larger expected net gain for the centers:

\[ z_1' = 406, \]
\[ z_2' = 544. \]

If the new rules are legitimate, the previously identified solution set will be considered as unstable, as an increase in both centers profits will result from a shift to the new suggested set.

A new steady-state optimal solution set might exist, which will be associated with the "Oligopoly" market, the identification of which is left for further study.

However, the point which was made here is that any optimal solution set is associated with one "field" of economic forces. One should not test the stability of a solution set by relaxing or changing the "field" of forces, an action which is very likely to lead to misleading conclusions.

In order for headquarters to be in a position to indirectly coordinate divisional search and generation strategies, it is important for headquarters to be able to correctly identify the kinds of economic relationships among the entities ("the rules of the game"), in order to be able to predict the associated steady-state solution. This, in turn, will be used to develop selective inducement strategies, as to encourage certain sources and discourage other sources from generating proposals.
These selective inducement strategies will be developed having in mind the overall organizational welfare, as viewed from headquarters' standpoint (in contrast with the different "profit centers'" standpoints).

A discussion on this subject follows in Section 6.4.

Given that such inducement strategies can be developed for all kinds of economic market behaviors, the control of such market behaviors becomes of no interest. However, when such strategies can be developed for a specific kind of economic market behavior, there might be an interest for headquarters to be in a position to impose this certain kind of behavior.

For example, in order for an "Oligopoly" to exist, all dominant members (dominant divisions) in the market have to agree to a certain contract. It is a required and sufficient condition to break such an agreement, to convince one dominant member to violate the agreement (or alternatively—not to participate in it). A strong and competitive research and development department, when organized as a separate economic entity, could be a good choice for this purpose. Headquarters may take advantage of this phenomenon when planning to impose perfect competition, or when planning to move closer to this kind of economic market behavior.
6.4 The Discriminating Rewards System, and the Enforced Steady State Solution

As previously described (Section 6.3), one inducement strategy cannot be developed to suit all kinds of market behaviors. However, it is very likely that different inducement strategies can be developed to meet the needs of specific market environments.

A strategy will be developed here, as to provide headquarters with an indirect decision and for coordination, in a situation where the only mode used by headquarters for proposals consideration is the "Centralized" mode, the organizational structure consists of "profit-centers," and the "centers" economical relations are purely competitive (a "Perfect Economy"). This model, when adjusted to the various kinds of consideration processes and economic relationships, could become a prototype for other coordination procedures, the development of which is left for further study.

In contrast with the simple centralized organizational structure (refer to Figure 1.01), in the "profit-center" organizational structure (refer to Figure 1.02) headquarters is not in a position to directly coordinate "centers'" search efforts by direct assignment. One way to beat this problem is through the use of a discriminating rewards method, which introduces a new set of forces into the economic "field." These economic forces, when carefully designed, will shift the steady-state
solution set to the combination of proposals most preferred by headquarters.

It is important to observe that Reward Ratios \( (RR_j) \),

\[
RR_j = \frac{R_j}{DC_j} \quad \text{for all } j \ (j=1,2,\ldots,m), \tag{6.14}
\]

have a significant role in the final determination of the steady-state solution set.

By using the functional relationships already developed in Section 6.0, one recalls that "source-k" optimal strategy can be expressed by 6.10 as follows:

\[
n_k^* = \left(\frac{B \cdot R_k}{DC_k \cdot I_k}\right)^{1/2} \cdot \left(\frac{I_k}{L_k}\right)^{1/2} - L_k = \frac{L_v}{3} \left(\frac{L_v}{3} - \sum_{j=1}^m \frac{n_j \cdot p_a_j \cdot I_j}{p_a_k \cdot I_k} \right).
\]

For a given set of values for \( L_k, I_k \) and \( B \), the optimal strategy \( n_k^* \) can be rewritten as a function of \( RR_k \) as follows:

\[
n_k^* = \left(\frac{B \cdot L_k}{I_k}\right)^{1/2} \cdot \left(\frac{RR_k}{L_k}\right)^{1/2} - L_k \tag{6.10'}
\]

This new functional presentation provides two conclusions:

a) \( n_k^* \) is positively correlated with \( RR_k \).

b) The marginal increase of \( n_k^* \) in response to an
increase in \( RR_k \) is diminishing. (Refer to the first derivative of \( n_k^* \) with respect to \( RR_k^* \).)

This in turn would mean that an increase in the centers' perceived Reward Ratio will result in an increase in its proposals' output (and vice versa), but the effectiveness of such a motivator is marginally diminishing.

The problem, now, is how to determine the adjustments required to all sources "homogeneous expected rewards" (those rewards each source expects to receive from each "accepted" proposal, such as PW) in order to impose a desirable solution set.

From this point and on one should interpret \( RR_k \) as an "effect" (dependent variable) rather than a "cause" (independent variable), interpretation which perceives \( RR_k \) as a decision set. The independent variable will thus be \( n_k \) \( (k = 1, 2, \ldots, m) \).

Assuming a desired optimal solution set \( (n_1, n_2, \ldots, n_m) \) is defined by headquarters, from its standpoint, as to maximize the welfare of the organization as a whole. We shall look for the "non-homogeneous expected reward ratio" set \( RR_k^* \), which is associated with the solution set.

By rearranging the functional presentation of \( RR_k \) and \( n_k^* \) such that \( RR_k^* \) is the subject of the formula, one gets:

\[
RR_k^* = \frac{(n_k + I_k)^2 \cdot I_k^*}{(B \cdot I_k)^*},
\]

where \( I_k = (p_{k_k}^2 \cdot I_k)^{-1} \cdot \sum_{j=1}^{m} n_j \cdot p_{aj} \cdot I_j \). \text{(6.15)}
It turns out that the solution for $\text{RR}_k^*$ is straightforward, as all entries in the $\text{RR}_k^*$ vector are independent of one another.

Three different strategies could be used by headquarters for adjusting the homogeneous Reward Ratios $\text{RR}_k^0$ into $\text{RR}_k^*$:

a) Pay an artificial reward on top of the perceived homogeneous reward, each time a proposal is "accepted." This strategy is recommended in situations where incentives are needed to encourage a source to come up with more proposals than in its homogeneous steady state strategy. Here, the amount to be paid for each accepted proposal will be:

$$\Delta R_k = R_k^* - R_k^0 = (\text{RR}_k^* - \text{RR}_k^0) \cdot DC_k.$$  \hfill (6.15)

b) Require "consideration fees" each time a proposal is submitted by the source for headquarters consideration. This strategy is recommended in situations where headquarters wants to discourage sources from submitting many proposals as seems from their homogeneous steady-state strategy. Here, the expected amount sources will be required to pay will be:

$$\Delta DC_k = DC_k^* - DC_k^0 = [(\text{RR}_k^* )^{-1} - (\text{RR}_k^0 )^{-1}] \cdot R_k$$  \hfill (6.16)

c) A combination of (a) and (b). Could be used in order to balance expected total pay with expected total income (left for further study).
A valid question could be asked regarding possible impacts of such rewards or fees on the overall welfare of the organization. More specifically, if one decides to deliberately change the economical structure of the "profit-centers'" environment, how can one be sure that this change will improve organizational welfare (on the average, at least), and from what kind of moneys is one going to pay for it.

The answers to those questions would be rather simple:

a) Headquarters is in the best position to decide about the definition of "organizational welfare," they have the authority to make such definitions, and are in the best position to maximize this "welfare" (whatever will its definition be). Who else can maximize organizational welfare when in conflict with divisional ("profit-center") welfare?

b) Like intra-company transactions which do not enter into any "consolidated income statement," rewards or fees which will be paid internally, from one economic entity to another, will not have any effect on organizational welfare. (Expense to one entity will become a revenue to another entity.) However, if one still worries about such a problem, he could come up with a rewards/fees system which will be expected to be balanced (refer to the third strategy suggested in this section), thus all moneys required for rewards will be generated through fees.
c) When bonuses and actual incentives are paid to managers or employees, argument (b) becomes invalid, since these are not intra-company transactions, but real organizational expense. Further study is recommended to optimally tie these in.

6.5 Conclusions

1) "Profit Centers'" decisions about their optimal search and generation strategies for investment proposals are non-trivial.

2) Optimal Centers' search strategies depend upon the kind of economic relations with other entities who compete on the limited capital funds (competition versus coordination and agreement), as well as upon optional decentralization modes headquarters is using for proposals evaluation.

3) For different competitive relations (among Centers) and different optional modes for headquarters evaluation, there exist different non-trivial steady-state optimal (from Centers' perspective) proposal mixes. The analytical structure of models to generate such solutions is essential and prerequisite for the design of indirect control procedures, as to enable headquarters to optimally (from their perspective) coordinate Centers' efforts.
4) A model for the optimal Centers' decision has been developed in Section 6.0, for the competitive market behavior of Centers, and for Centralized consideration of proposals. Guidelines for equivalent decisions, but when all three modes of consideration are optional for headquarters, have been laid out and demonstrated in a numerical example in Section 6.1. The development of a general model is suggested for further study.

5) A model for the identification of the homogeneous steady-state solution in a multi-source competitive market has been developed in Section 6.2, for the Centralized mode of headquarters proposals evaluation process. Guidelines for the development of a similar model when all three modes of evaluation are optional to headquarters, were laid out.

6) Conditions for existence and stability of previously developed steady-state solutions were studied and analyzed in Section 6.3. The main steady-state model was found to be stable throughout its existence range.

7) A methodology has been developed for headquarters to indirectly encourage a desired steady-state solution set for the proposal-mix, by payment of rewards or collection of consideration-fees from the profit-centers. This methodology substitutes the coordination
by assignments which was used in the simple organizational structure, but is impossible in the "profit-center" structure.

8) The allocation of search and generation efforts by "profit-centers" can be controlled. They should be controlled only if such efforts, or their outcomes, are a significant factor in the definition of organizational welfare, and if such welfare has to be maximized.

The likelihood that the steady-state solution set of Centers in competition will be identical with the optimal proposal-mix which maximizes organizational net-gain, is very small. Support to this observation is provided by Haynes and Solomon (19) and Kellough (27) in sections which were already quoted in Section 1.0.

9) Information about updated strategies \( n_j \), initial required outlays \( I_j \) and variances of acceptability \( p_{aj} \) should be made public for all \( j = 1, 2, \ldots, m \), in order to improve Centers' decisions, thus improving headquarters predictions, and in turn--improving control measures and precision of final results. Obviously, the total amount of funds available \( B \) has to be publicly spelled out.
The best strategy headquarters can pursue in respect to information, is to make it available cost free and as soon as it becomes available to headquarters. The more uncertainty is involved in Centers' decisions, and the more remote is the information at Centers' disposal from being perfect, the less is the steady-state solution set predictable, and the less is the system controllable.

Support to this hypothesis is found at the Litzenberger and Joy (32) observation, that: "... the firm is a group of individuals, and each member has objectives that are not necessarily coincidental to the organization's goal." Marschak (34) observes that "It is found that a general preference for one of the subclasses cannot be defended without further restricting the model substantially."

In other words, strategies which optimize separate divisional objectives do not necessarily optimize organizational objectives when such objectives and constraints can be explicitly spelled out. When both objectives and constraints are partially explicit and partially implicit, the likelihood for inconsistencies is even higher.
7.0 Non Linear Risk Attitude, and the Certainty Equivalent Model

For simplification purposes, two underlying assumptions were explicitly made in previously developed models:

a) A linear risk attitude describes decision makers' decision behavior.

b) Consequently, point estimates of the expected values of the state-variables are sufficient for modeling purposes, as well as for the derivation of conclusions and guidelines.

In this section, the first attempt will be made to relax these assumptions, as to expand the limits for models' validity, thus making them much more general than previously.

In this section we will still assume that statistical distributions for the estimation of state-variables are not available. State-variables will be identified by their expected value point estimates, assuming that the complete distribution
was suppressed, for one reason or another (cost of information, availability of information, et cetera).

However, this will be the first time we shall introduce the non-linear risk attitude as a factor in the decision making process.

For illustration purposes only, the decentralization problem for the single cell in the ample-funds situation will be discussed. However, similar considerations could be implemented into other models used as "decision-aids" for decisions with assumed risk.

The basic difference between the models we will use from here on and previously developed models, is in the definition of the objective function. Instead of maximization of expressions such as "Expected Net Organizational Gain," we shall maximize expressions such as "Expected Organizational Utility." (One should note that the risk of delegation of decision authority is not suppressed, since it is modeled by the "pa" concept.)

The decentralization decision model for a single cell in the ample-funds situation will have to be adjusted in the following way:

Starting from a given non-linear utility function (such as the one displayed in Figure 7.01), one will determine (analytically) the utility values associated with all expected dollar concepts, as follows.
Fig. 7.01. Figuring utility values associated with non-linear risk attitudes.
Figure 7.01 presents a graphical association procedure, while the analytical procedure calls for the substitution of all dollar values (x) by their associated utility values $U = u(x)$, where $u$ is the given utility function of the specific chief executive.

Having developed all utility values associated with the relevant dollar values, one will replace all dollar values by their associated utility values. The transformation of the concepts in Table 2.1 results with a similar utility map, which is presented in Table 7.1.

This time, the selection of the preferred course of action will be done by identifying the course of action which is associated with the maximum EOU (Expected Organizational Utility) in the extreme right column.

The impact of a non-linear risk attitude could be analyzed once again both analytically or graphically. It was observed that the graphical analysis is very powerful in displaying the contrast between the linear and non-linear behavior. Figure 7.02 displays the impact of a "risk-averse" attitude (the one presented
Table 7.1. Expected Organizational Utility—Single Segment, Ample Funds

<table>
<thead>
<tr>
<th>Possible Outcome</th>
<th>H.Q. &quot;Acceptable&quot;</th>
<th>H.Q. &quot;Unacceptable&quot;</th>
<th>Expected Organ. Utility = EOU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability Level of Decentralization</td>
<td>pa</td>
<td>(1−pa)</td>
<td>pa * U[PW] + (1−pa) * U[PWR]</td>
</tr>
<tr>
<td>Complete Negative</td>
<td>U[APW]</td>
<td>U[APW]</td>
<td>U[APW]</td>
</tr>
</tbody>
</table>
in Figure 7.01) on the upper and lower break-even points (UBE and LBE, respectively).

As it turns out from Figure 7.02, a "risk averse" attitude causes both UBE and LBE a shift to the right on the "pa" axis. This, in turn, means that certain kinds of proposals which have previously been in the lower part of the Centralization zone, will now be in the upper part of the Negative Decentralization zone. Similarly, proposals which have previously been in the lower part of the Positive Decentralization zone, will now be in the upper part of the Centralization zone.

Consequently, one may conclude that the more averse is the decision maker toward risks, the more kinds of proposals should be a priori rejected (more negative authority will be delegated), and the less kinds of proposals should a priori be accepted (less positive authority will be delegated). A reverse shift will be caused by a "risk seeking" attitude.

Comparing these conclusions with one's intuition suggests that these kinds of insights do make sense, however non-trivial.

The shift to the right of the UBE point means that some types of proposals will now be centrally reviewed, for a certain cost (sure cost) of HC, instead of taking the risk of positive decentralization. This phenomenon is similar to buying additional insurance, and paying a sure premium independently from the events, behavior which is associated with risk averse attitude.
Fig. 7.02. Shifts in break-even points due to risk averse attitudes.
The shift to the right of the LBE point, although making sense, is even less trivial than the shift of the UBE. One can interpret the negative decentralization mode, as headquarters' lowest risk. This because when proposals are not generated, there may be neither loss nor cost to the organization. Once a proposal is generated, headquarters' risk could be in terms of potential loss (when in the positive decentralization mode) or in terms of potential consideration cost (when in the centralization mode, and projects are rejected). Under such unfavorable situations, shifts to negative decentralization seems to reduce risks.

It should be noted that due to these shifts in preference of modes of decentralization, new limits to decentralization should be plotted. Therefore, one can argue that optimal limits to decentralization are associated with decision makers' risk attitudes.

The statement that limits to decentralization are a matter of personal "style" of the chief executive, can now be defended by a similar statement—that the limits are a matter of personal "risk attitude." As risk attitude is purely subjective, and because of the fact that no quality judgment can be made on it, limits to decentralization could remain a matter of subjective personal style.
7.1 All Concepts--Random Variables

The second step in the efforts to account for non-linearity in decision makers' risk attitude is to associate utilities with complete distributions of "costs" or "values."

This association is expected to be far more accurate in terms of risk modeling than the initial approach, in which important portions of uncertainty (except from the risk of delegation) were suppressed into "expected values."

Here, it will be suggested that all monetary valued concepts \( (x) \) will be modeled by their associated probability distributions.

\[ x \sim f(x) \]  \hspace{1cm} (7.01)

The decision makers' risk attitude will be modeled, as before, by his Utility Function:

\[ U = u(x) \]  \hspace{1cm} (7.02)

The definition of the expected utility associated with a specific dollar value (point estimate for utility) will be:

\[ U' = \int u(x) \cdot f(x) \cdot dx \quad \text{for continuous variables,} \]
\[ U' = \sum_{x} u(x) \cdot f(x) \quad \text{for discrete variables.} \]  \hspace{1cm} (7.03)

This definition is in contrast to the previously used definition of \( U' \):

\[ U' = u(E[x]) \]  \hspace{1cm} (7.04)
which in turn, was equal to:

\[ U' = u (\int x \cdot f(x) \cdot dx) , \]

or

\[ U' = u (\Sigma x \cdot f[x]) , \]

however, for a "suppressed" \( f(x) \).

Having done these changes in the model, the rest of the analysis procedure remains almost unchanged, and all steps previously suggested in Section 7.0, apply.

Consequently, one may expect to arrive at somewhat different recommendations, especially in borderline cases. Slight shifts in the two "Limits to Decentralization" are anticipated, in addition to the shifts already anticipated when non-linearity in risk attitude was initially introduced.

It should be noted that in places where a difference between two variables is of concern (such as \([APW-HC]\), for example), the distribution of the difference has to be developed prior to any attempt to associate utility. When both random variables are approximated by Normal distributions, the distribution of the difference is also normal.

For example, when both APW and HC are normally distributed, \((APW-\text{HC})\) is distributed as follows:

\[ (APW-\text{HC}) \sim N[ (u_{APW} - u_{HC}) , (v_{APW} - v_{HC}) ] . \]
7.2 The Bayesian Approach for Updating

The purpose of this section is to discuss practical implications of the models' operation, in particular the process of assigning quantitative values to the different concepts used. More specifically, how should one go about "measuring" values?

We shall resolve this question by looking at the worst case, first. Other cases will be resolved similarly.

One extreme situation, which is the most difficult one, is the situation where, for one reason or another, there is absolutely no information which could be used to identify probability distribution functions for the different concepts used in the model. (Some authors define this decision situation as "uncertainty"). This situation can happen when no historical data is available (have not been recorded or the operation is young), or where such data is invalid (reorganization took place, executives have been replaced, et cetera).

The problem, thus, becomes to be a "starting" problem. In other words, how should one start the operation of the model with no information on hand?

The main, and most significant contribution of modeling and decision-aids is in their ability to increase consistency between conclusions on one hand, and information and beliefs on the other hand. When information is not available, consistency between conclusions and beliefs is still important. It is this strength of decision-aids which will be used initially.
Here, it will be suggested to start with subjective estimates provided by the organizations' decision maker (Chief Executive), thus assuring the consistency of derived conclusions with his beliefs.

These estimates ought to be purely subjective, and will include allowances for uncertainty in judgment. Commonly, these estimates will be expressed in a form of a probability distribution function, with a certain mean and a certain variance. However, uncertainty about the mean can, once again, be expressed by a probability distribution function, for which the mean and the variance will be identified as the "Prior Beliefs. (38)

These initial subjective estimates will be required as well as sufficient to generate conclusions, which will provide operational guidelines for decentralization as well as for the allocation of search efforts, consistent with the Chief Executive's subjective prior beliefs.

As time passes, relevant information becomes available, information which ought to be taken into consideration for the adjustment process of conclusions.

The next problem will be to identify a procedure which will allow for the adjustment of conclusions, as a function of updated information, as well as prior beliefs. One way to resolve this problem is through the use of the Bayesian model, a detailed discussion on which is given in Morris's book. (38)
Conclusions will be adjusted through the use of "Posterior-Beliefs," which will be generated by the Bayesian model, based on "Prior Beliefs," available information and the reliability of the source of information.

The collection process of information can be described as a sampling process. Decisions as for the proper frequency of sampling, for the purpose of updating the used estimates, could be based on different criteria. One useful criterion is the economical criterion, by which the cost of information is evaluated in contrast with its expected benefit, when reflected by a change in decisions. Sampling frequency could be selected as to maximize expected net gain. (38)

Another criterion could be the degree of remaining uncertainty. When information is "purchased," it is considered valuable as long as it can contribute to the reduction in uncertainty of a specific estimate. Some decision makers cannot make their choices, unless uncertainty is brought to within certain given limits. It is this aspiration-level of certainty which could be the determining factor in some information purchasing decisions.

Discussing the process of updating, a unique problem might be the process of estimating (pa)—the "Degree of Variation in Acceptability" concept. In order to "purchase" information relevant to this variable, proposals have to be brought for
Centralized consideration, a course of action which might be in conflict with the recommended optimal strategy. However, one has to separate Centralization of samples as a mean for uncertainty reduction, from Centralization as a screening device for all proposals of a certain kind, with no exception. The difference between the two is similar to the difference between quality control through statistical inference, and quality control through 100% screening.

In order to demonstrate the Bayesian process as applied to the reduction of uncertainty of estimates about the "Degree of Variation in Acceptability," a numerical example has been designed (refer to next section, 7.3). A prior distribution is given, indicating prior subjective beliefs, about "pa." Then, information is provided, which suggests that a randomly selected proposal has been submitted for headquarters' consideration, and was found to be "acceptable." Through the use of "Likelihood" (or reliability) figures, a posterior distribution is developed, a distribution with the property of a reduced variance (therefore—reduced uncertainty), as well as an updated mean. This updated mean shall be used as a posterior point estimate for "pa," in all the analysis past the latest sample, and prior to the next sample.

7.3 Updating "pa"—a Numerical Example

Assuming prior beliefs, which are subjectively assigned by the decision maker, about the value of "pa" for a certain
"cell," are expressed in the form of a discrete probability distribution function as follows:

Table 7.2. Data on Prior Beliefs About "pa"

<table>
<thead>
<tr>
<th>Index for possible value</th>
<th>&quot;pa&quot; possible value</th>
<th>Prior Belief (probability)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9</td>
<td>0.20</td>
</tr>
<tr>
<td>2</td>
<td>0.8</td>
<td>0.30</td>
</tr>
<tr>
<td>3</td>
<td>0.7</td>
<td>0.20</td>
</tr>
<tr>
<td>4</td>
<td>0.6</td>
<td>0.12</td>
</tr>
<tr>
<td>5</td>
<td>0.5</td>
<td>0.07</td>
</tr>
<tr>
<td>6</td>
<td>0.4</td>
<td>0.05</td>
</tr>
<tr>
<td>7</td>
<td>0.3</td>
<td>0.03</td>
</tr>
<tr>
<td>8</td>
<td>0.2</td>
<td>0.02</td>
</tr>
<tr>
<td>9</td>
<td>0.1</td>
<td>0.01</td>
</tr>
</tbody>
</table>

The likelihood that a randomly submitted proposal from the same source will be "acceptable" by headquarters (out of a sample size-1) is equal to "pa." Therefore, the list of Likelihoods associated with the different components of the possible "pa" set, will be as follows:

\[
\text{LK} (1 \text{ "accepted" out of sample size-1}/\text{pa}=0.9) = 0.9
\]
\[
\text{LK} (\quad \quad \quad \quad \quad \quad \quad \quad \quad /\text{pa}=0.8) = 0.8
\]
\[
\text{LK} (\quad \quad \quad \quad \quad \quad \quad \quad \quad /\text{pa}=0.7) = 0.7
\]
Given the information that one proposal from the same source has been submitted at random for headquarters' consideration, and was found to be "acceptable," one can compute the posterior distribution of beliefs using the Bayesian model, as follows:

$$P_0(\text{pa belief/1 out of 1 "acceptable"}) =$$

$$\frac{\sum \text{LK}(1 \text{ out of 1 "acceptable"}/\text{pa belief}) \cdot \text{PR}(\text{pa belief})}{\text{LK}(1 \text{ out of 1 "acceptable"}/\text{pa belief}) \cdot \text{PR}(\text{pa belief})},$$

all pa beliefs

where $P_0$ stands for POSTERIOR, LK for LIKELIHOOD an PR for PRIOR.

Using the data available, one gets the following posterior beliefs, in Table 7.3.

Consequently, a shift in the expected value is recorded (from 0.711 to 0.7476) and a small decrease in the variance as well. From this point on, 0.7476 represents "pa" in this numerical example.

As already suggested in 7.2, whether to continue taking samples or not is a matter of criteris (economics or aspiration level for certainty).
Table 7.3. Posterior Beliefs About "pa"

<table>
<thead>
<tr>
<th>Index for possible value</th>
<th>&quot;pa&quot; possible value</th>
<th>Posterior Belief (probability)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9</td>
<td>0.256</td>
</tr>
<tr>
<td>2</td>
<td>0.8</td>
<td>0.342</td>
</tr>
<tr>
<td>3</td>
<td>0.7</td>
<td>0.200</td>
</tr>
<tr>
<td>4</td>
<td>0.6</td>
<td>0.103</td>
</tr>
<tr>
<td>5</td>
<td>0.5</td>
<td>0.050</td>
</tr>
<tr>
<td>6</td>
<td>0.4</td>
<td>0.029</td>
</tr>
<tr>
<td>7</td>
<td>0.3</td>
<td>0.013</td>
</tr>
<tr>
<td>8</td>
<td>0.2</td>
<td>0.006</td>
</tr>
</tbody>
</table>
| 9                        | 0.1                 | 0.001

7.4 Conclusions

1) Non linear risk attitude can be modeled at two levels of precision:

a) When uncertainty about monetary variables is suppressed into expected values.

b) When monetary variables are expressed in their complete form, with the uncertainty associated in the estimate.
2) Updating procedures (such as the Bayesian model) can be established, to allow for initial operation of the model without information, but with subjective beliefs instead. The more valid information becomes available, the more estimates can be updated. When the source for information is reliable, uncertainty can be reduced.

3) Different preferences of optimal limits to decentralization by different individuals can be explained by different risk attitudes or by differences in subjective prior beliefs.

4) Limits to decentralization could be adjusted to risk attitudes of different decision makers, to availability of information and to changes in organizational structure. However, implementation costs (see Section 3.8) do act as absorbers for some of the fluctuations, and provide a degree of long term stability.
8.0 Conclusions

A systematic analytical structure has been developed here, to resolve the kind of problems raised in Section 1.2.

Following is an outline of the structure's main properties and conclusions:

1) The delegation of authority for capital budgeting decisions is a decision problem of the chief executive.
2) This decision is a decision under risk. Once authority is delegated, the chief executive has no screening control over the acceptance process of proposals.
   (Refer to Section 3.0.)
3) Three choices are available to the chief executive:
   A) Complete Positive Decentralization.
   B) Centralization.
   C) Complete Negative Decentralization.
   (The third choice has been identified in this study.)
   (Refer to Section 3.0.)
4) Headquarters objectives are to maximize the chief executive's expected utility. It is the chief executive's authority to define corporate objectives,
therefore his utility has to be maximized. In the case of linear utility function, expected net organizational gain is to be maximized. (See Chapter 7.)

5) The decision problem can be structured as a certainty equivalent model (see Chapters 3 through 6) or as two different models with assumed uncertainty in monetary parameters. (See Chapter 7.)

6) Differences in constraints perceptions between headquarters and divisions are modeled by "pa"—"The probability that a proposal from a certain source is mutually acceptable." This concept is one of the most important sources' performance measures. (See Section 2.2). The higher the "pa"—the more authority should be delegated, in the "ample-funds" situation.

7) Effects of time delays in headquarters consideration can be modeled by "ps"—"The probability of proposals' survival." (See Section 3.7.) The lower the "ps" the more authority should be positively or negatively delegated, in the ample funds situation. Below a certain "ps" value, centralization can never be economically superior.

8) Duplication in corporate evaluation efforts are modeled by "HC"—"Headquarters consideration costs per proposal" which come on top of similar efforts by the divisions. (See Section 2.2.) High "HC" values should
be associated with both types of complete decentralization.

9) Implementation costs of authority reorganization could be modeled by "IC"--"Implementation Costs per Proposal." The higher "IC" values, the higher the likelihood that a certain degree of stability will be achieved in the authority system.

10) The investment opportunities space could be identified along several relevant dimensions, not just one. One of those dimensions could be the classical "Initial Required Outlay," previously thought to be the only relevant dimension along which authority should be delegated. (See Sections 1.1.5 and 3.1.)

11) Different combinations of sections, resulting from division of dimensions into intervals, define different segments ("cells") in the opportunity subspace. The smaller are the segments, the closer is the sub-optimal solution to optimum. (See Section 3.3.)

12) Optimal levels of authority delegation are a function of funds availability. Two different models are developed, in order to identify optimal delegation strategies:

   a) For "ample-funds" situations. (See Chapter 3.)
b) For "limited-funds" situations. (See Chapter 4.)

13) Positive decision authorities are delegated to divisions in all the cells which are associated with Complete Positive Decentralization optimal decisions. Similarly, divisional Complete Negative Decentralized decision authority consists of the union of all "cells" associated with respective optimal preference.

14) Types of proposals which ought to be submitted for Centralized headquarters consideration, are those within the union of divisional "cells" which are associated with "Centralization" as their optimal strategy.

15) Two optimal limits to decentralization are identified, instead of the "classical" single limit; one along the boundaries between divisional Complete Positive Decentralization subspace and the Centralization subspace (the "classical" limit), and the other along the boundaries between Centralization subspace and Complete Negative Decentralization subspace. (See Chapter 3.)

16) Limits to decentralization can explicitly be defined. In the two dimensional opportunities space, these limits can be graphically displayed. (See Section 3.3.)

17) Neither the subspaces nor their boundaries need to be continuous. Discontinuities in divisional proposal generation "performances," when existing, can easily
be spotted. (See Sections 1.1.6 and 3.3.)

18) Segmentation of opportunities subspaces need not necessarily be in cells of equal size. Unequal segmentation strategies might result with savings in computational efforts, with no adverse effect on the degree of precision of the decentralization process, nor of its objectives. (See Section 3.3.)

19) Authorities delegation to different divisions need not be identical. Proven divisional "performance" is one major factor which makes one division different from the other. (See Chapter 3.) This option is a response to the "multiplets" phenomenon ("twins, triplets, quadruplets, et cetera," observed in Section 1.1.4.

20) When "standard" authority is to be uniformly delegated to all divisions, this suggested model could be used as a testing device. Arrays of authority delegated can be compared with arrays of authority to be delegated, once optimal decentralization is to take place. A variance criteria could be established for "fitness tests," to be taken by candidates for divisional manager positions. When contrasts between the standard array and the optimal array exceed a certain limit, individuals should be considered "unfit" for the specific position.
21) There is a high limit for PWR--"Expected Present Worth of proposals which are unacceptable by headquarters" for any given set of PW, APW and HC in the "ample-funds" situation, above which Centralization or Complete Negative Decentralization can never be preferred, regardless of "pa." (See Case-1, Section 3.5.)

22) When APW ≥ PW-HC, Complete Centralization is uneconomical, regardless of "pa" and PWR values. (See Section 3.5.) (Once again--in the "ample funds" situation.)

23) When APW ≥ PW, Complete Negative Decentralization ought to be preferred, regardless of "pa," PWR and HC, ("ample-funds.") (See Case-4, Section 3.5.)

24) A necessary but surprisingly insufficient condition for Centralization to be the preferred mode in the "ample-funds" situation regardless of "pa," is cost free consideration by headquarters (HC = 0). The other required conditions are:
   a) PW ≥ APW
   b) PWR < APW (See Case-5, Section 3.5.)

25) An unconditional optimal expected gross value of funds expression, associated with a given set of proposals, has been developed, for the random drawing process. This expression consists of similar concepts used for APW which disappears, and B--"The amount of funds available" which shows up. This "value" has
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a diminishing margin, and asymptotically converges
to an upper limit. (Section 4.1.)

26) Optimal decentralization modes could be identified
and associated with optimal conditional expected
values, for the "limited-funds" situation. (See
Section 4.2.)

27) An expression for the overall expected net organiza­
tional value of funds, with their associated oppor­
tunities space, has been developed. A proposal mix
which maximizes that value, subject to possible
limits to proposals availability, can be identified.
Consequently, a decision aid is developed for head­
quarters to optimally coordinate divisional search and
generation efforts for capital investment proposals,
a process which customarily is believed to be uncon­
trollable. (See Chapter 5.)

28) In contrast with the simple organizational structure,
search and generation efforts by "profit centers"
cannot be directly controlled, as these decisions are
within centers' authority. However, these efforts
should be indirectly controlled by headquarters, for
the sake of overall organizational welfare. (See
Chapter 6.)

29) Models have been developed to identify centers'
homogeneous generation strategies in steady state,
under different competitive market relationships, and
different modes of headquarters consideration. (See
Chapter 6.)

30) Indirect incentive reward systems have been developed,
to artificially shift homogeneous steady state genera-
tion strategies into desired combinations. (See
Chapter 6.)

31) The release to the public of all information and
estimates known to headquarters in respect to the
values of different variables associated with the
different sources in competition, has a positive im-
pact on precision of centers' decisions (due to de-
crease in centers' uncertainty). This, in turn, would
result in improvement in steady states predictability,
thus enabling better adjustments in the reward sys-
tems, which should result in proposals mixes which
are closer to the desired ones. (See Chapter 6.)

32) Updating procedures are suggested, to allow for the
"start-up" of the model with no initial information,
but with prior subjective assessments. (See Chapter
7.2.)

33) Different preferences of optimal limits to decentrali-
zation by different decision makers could be explained
by different risk attitudes, or by different subjective
prior beliefs (biases). (See Chapter 7.2.)
34) Lack of sufficient inside information, and the limited access to corporate data, made it impossible to provide experimental evidence to support the validity of even the crudest portions of the suggested models.

35) The main contribution of this research is in its conceptual, structural and analytical developments, and in its proposition that the process of generation of capital investment proposals, as well as the process of evaluation of such, could be quantified and consequently optimized. The specific models suggested here by no means are the only possible models, and maybe not the best models to describe specific portions of those processes. The only proposition made here is that this model is consistent with the theory of decision analysis, and provides many insights into the problem of delegation of financial authority, insights which were not provided by existing models.

8.1 Topics for Further Research

The various topics for further research could be classified into three main groups:

8.1.1 Expansion of the model beyond the delegation of authority for capital investments decisions

In this group of topics, one could investigate the possibilities for the implementation of similar concepts and structures to the delegation problems in other kinds of activities like the following.
a) **Engineering Design** - where different people at different levels are authorized to approve (or disapprove) various designs and specifications. Questions like: "Who has to approve what? and why?" commonly seek explicit, consistent and optimal answers. It is believed that the "pa" concept, which was developed in this paper, could be a key concept in other delegation problems, as well.

b) **Engineering Construction** - where different people at different levels are authorized to approve (or disapprove) various parts of construction. Once again, questions like: "Who should have the authority to approve a specific section? and why?" could be raised. A similar decision model with assumed risk, could be developed.

c) Similarly, the delegation problem of any type of authority in organizations such as public administrations, medical centers, et cetera:

8.1.2 **Expansion of the model into an hierarchial model, within the capital budgeting decentralization problem**

All previous discussions have assumed two levels in the organizational hierarchy: headquarters and divisions. A valid question should be asked about the
delegation process of authorities further down the line, to the lower levels in the organization.

Serious questions should be raised about authorities to make such decentralization decisions, and the assumption of risks resulting from such delegations.

It seems (on the surface) that standard authorities which are commonly delegated to the different levels in the organization by the chief executive, are in some respects inconsistent. A decision by a decision maker at level (i) to delegate part of his own authority to his subordinate at level (i+1) seems more appropriate.

However, standard delegation could be compared with optimal delegation, thus creating a testing device, which could be used to test one’s performance in a job, and his degree of fitness to the standard authority.

8.1.3 Enrichment of the model

a) Collection of experimental evidence, to support the validity of the models. (To what extent is the real world situation close to optimum?)

Experimental assessments on corporate performance, providing answers to questions such as: "We spend about 1% of our capital budget (B) for search, generation and consideration. Is it too much, reasonable, too little?" "We receive requests for capital funds which exceed three
times the funds available. Should we ask for more proposals, same amount, less proposals?"
b) A simultaneous consideration of search efforts, evaluation efforts, decentralization, investment, financing and dividend.
c) The impact of personal rewards to divisional managers for maximizing "profit centers" net gains, when such maximization cannot be assumed without such compensations. The possibilities of writing a managerial compensation contract, which will induce a manager to act in the best interest of an "owner," was researched by Heckerman (20), assuming that the owner is unable to form his own assessment of the information value.

A similar study should be conducted for situations where "owners" (in this situation—corporate headquarters) are able to form their own assessment, thus exercising their control over the business process, and maximizing organizational welfare as perceived from their perspective.
d) The development of an analytical structure for the homogeneous, steady state solution, when headquarters is using its three modes options for decentralization.
APPENDIX A
PROOF OF FIVE CASES

The following notation will be used in the appendices for concepts already defined in the text:

\[ \alpha - \text{will represent PWR} \]
\[ \beta - \text{will represent HC} \]
\[ \gamma - \text{will represent APW} \]
\[ \delta - \text{will represent PW} \]
\[ x - \text{will represent } \text{pa} \]

\[ y_i(x) - \text{will represent } \text{ENOG}_i(\text{pa}) \]
where \( i = 1, 2, 3 \) for the three possible modes of decentralization.

The problem, as introduced in Chapter 2 and discussed in detail in Chapter 3, will thus be:

\[
\max_i[y_i(x)] \quad (i = 1, 2, 3),
\]
where

\[
y_1(x) = \alpha + (\delta - \alpha) \, x,
\]
\[
y_2(x) = (\gamma - \beta) + (\delta - \gamma) \, x,
\]
\[
y_3(x) = \gamma.
\]

For comparison, please refer to Table 2.1.
The relationship between all three dependent variables $y_i(x)$ and their single independent variable $x$ is linear, with slopes and $y$ intercepts (for $x = 0$) as follows:

<table>
<thead>
<tr>
<th>$i$</th>
<th>slope</th>
<th>$y$ intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$(\delta - a)$</td>
<td>$a$</td>
</tr>
<tr>
<td>2</td>
<td>$(\delta - \gamma)$</td>
<td>$(\gamma - \beta)$</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>$\gamma$</td>
</tr>
</tbody>
</table>

From Section 3.4 the following practical implications apply:

a) $(\delta - a) \geq 0$ or $\delta \geq a$,

b) $\gamma \geq 0$

c) $\beta \geq 0$

Also, by definition, the variable $x$ does exist as following:

$$0 \leq x \leq 1.$$

Let us define $P$ as the sequence of preferred strategies, when $x$ varies continuously from its low limit ($x = 0$) to its upper limit ($x = 1$), and when all other factors remain unchanged.

$$P = [P_1, P_2, P_3, \ldots, P_j, \ldots, P_n].$$

Here, $n$ describes the total number of subsections in $x$, which are associated with different decentralization strategies, and cause discontinuities in decentralization in respect to $x$. 
The three possible modes of decentralization will be identified as follows:

- \( S_1 \) - Complete Positive Decentralization strategy.
- \( S_2 \) - Complete Centralization strategy.
- \( S_3 \) - Complete Negative Decentralization strategy.

Following are several facts which are derived from the presented problem and its implications:

**Fact 1**

Since all three equations are continuous (between \( 0 < x < 1 \)) and linear in \( x \), the sequence of preferred strategies \( P \) cannot consist of more than three subsections \( n \leq 3 \) and two discontinuities. Therefore,

\[
P = [p_1, p_2, p_3], \text{ at most.}
\]

**Fact 2**

Any preferred strategy \( S_i \) \( (i = 1, 2, 3) \) can only appear in \( P \) once, if at all.

**Fact 3**

\( S_2 \) can never follow \( S_1 \) when \( x \) is in an upward trend.

**Proof:** Assuming \( S_2 \) can follow \( S_1 \). This implies that:

1) \( y_2(x_A) > y_1(x_A) \),
2) \( y_2(x_B) < y_1(x_B) \),
3) \( x_A > x_B \).
By substitution of the y's with their explicit expressions one gets:

1') \((\alpha - \gamma) \cdot x_A > (\alpha + \beta - \gamma)\),

2') \((\alpha - \gamma) \cdot x_B < (\alpha + \beta - \gamma)\),

3') \(x_A > x_B\).

As it turns out, for positive \((\alpha - \gamma)\) inequality (1') is violated. (Note that \(\beta\) is non-negative.) For negative \((\alpha - \gamma)\) inequality (2') is violated, for precisely the same reasons.

Consequently, (1') and (2') cannot co-exist, which means that the initial assumption is invalid. Therefore \(S_2\) can never follow \(S_1\).

**Fact 4**

\(S_3\) can never follow \(S_2\).

**Proof:** Assuming \(S_3\) can follow \(S_2\). This implies that:

1) \(y_3(x_A) > y_2(x_A)\),

2) \(y_3(x_B) < y_2(x_B)\),

3) \(x_A > x_B\).

By substitution of all y's with their explicit expressions one gets:

1') \((\delta - \gamma) \cdot x_A > \beta\),

2') \((\delta - \gamma) \cdot x_B < \beta\),

3') \(x_A > x_B\).

Assuming, for a minute, that \((\delta - \gamma) \geq 0\), implies:

1") \(x_A < \beta/(\delta - \gamma)\),

2") \(x_B > \beta/(\delta - \gamma)\),

1" + 2") \(x_A > x_B\) \quad \text{which is in conflict with (3')}.

(Note that \(\beta \geq 0\).)
If the reverse is correct, and \((\delta - \gamma) < 0\), this implies:

1''') \(x_A > \beta/(\delta - \gamma)\),  
2''') \(x_B < \beta/(\delta - \gamma)\).

As the expressions on the right side of both inequalities become negative, (2''') suggests that \(x_B\) should get negative values, a requirement which violates the range of definition for \(x\).

Consequently, \(S_3\) can never follow \(S_2\), independently from the values of all different factors.

Fact 5

\(S_3\) can never follow \(S_1\).

Proof: Assuming \(S_3\) follows \(S_1\). This implies that:

1) \(y_3(x_A) > y_1(x_A)\),  
2) \(y_3(x_B) < y_1(x_B)\),  
3) \(x_A > x_B\).

By substitution one gets:

1') \((\delta - \alpha) \cdot x_A < (\gamma - \alpha)\),  
2') \((\delta - \alpha) \cdot x_B > (\gamma - \alpha)\),  
3') \(x_A > x_B\).

Since \((\delta - \alpha) \geq 0\) (see initial implications), this set of inequalities can be rewritten as follows:

1"") \(x_A < (\gamma - \alpha)/(\delta - \alpha)\),  
2") \(x_B > (\gamma - \alpha)/(\delta - \alpha)\),

1" + 2") \(x_B > x_A\), which is in conflict with (3'). Therefore \(S_3\) can never follow \(S_1\).
Fact 6

\( S_1 \) can follow \( S_2 \) only when \( S_3 \) preceeds \( S_2 \), thus creating a three stage strategy \([S_3, S_2, S_1]\).

**Proof:** Assuming \( S_1 \) follows \( S_2 \) but \( S_3 \) does not preceed \( S_2 \),

This implies that:

1) \( y_1(x_A) > y_2(x_A) \),
2) \( y_1(x_B) < y_2(x_B) \),
3) \( y_2(0) > y_3(0) \),
4) \( x_A > x_B > 0 \).

By substitution one gets:

1') \( a + (\delta - \alpha) \cdot x_A > (\gamma - \beta) + (\delta - \gamma) \cdot x_A \),
2') \( a + (\delta - \alpha) \cdot x_B < (\gamma - \beta) + (\delta - \gamma) \cdot x_B \),
3') \( (\gamma - \beta) > \gamma \),
4') \( x_A > x_B > 0 \).

Since inequality (3') is violated by a positive \( \beta (\beta > 0) \), which is required to satisfy (1') and (2'),

\( S_1 \) can only follow \( S_2 \) if \( S_3 \) preceeds \( S_2 \).

Fact 7

\( S_2 \) can follow \( S_3 \) only when \( S_1 \) follows \( S_2 \), thus creating, once again, a three stage strategy \([S_3, S_2, S_1]\).

**Proof:** Assuming \( S_2 \) follows \( S_3 \), but \( S_1 \) does not follow \( S_2 \),

This implies that:

1) \( y_2(x_A) > y_3(x_A) \),
2) \( y_2(x_B) < y_3(x_B) \),
3) \( y_2(1) > y_1(1) \),
4) \( 1 > x_A > x_B \).
By substitution one gets:

1') \((\gamma - \beta) + (\delta - \gamma) \cdot x_A > \gamma\),

2') \((\gamma - \beta) + (\delta - \gamma) \cdot x_A < \gamma\),

3') \((\gamma - \beta) + (\delta - \gamma) \cdot 1 < \alpha + (\delta - \alpha) \cdot 1\),

4') \(1 > x_A > x_B\).

Since inequality (3') can be rearranged to read:

3") \((\delta - \beta) > \delta\),

it is violated by a positive \(\beta (\beta > 0)\). Therefore, \(S_2\) can only follow \(S_3\) when it is also followed by \(S_1\).

Conclusions

Having considered facts 1 through 7, as well as the various possible combinations of strategies (a total of fifteen combinations), one can come up with those combinations which satisfy all requirements.

Following is a list of all possible sequences, with their associated existence references.

Apparently, only five sequences turn out to exist (as a result of this analysis), the same five sequences which are identified in the text as the "five cases."

Three Stage Combinations

<table>
<thead>
<tr>
<th>Combination</th>
<th>Exists</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>(S_1, S_2, S_3)</td>
<td>No</td>
<td>Facts 3 and 4</td>
</tr>
<tr>
<td>(S_1, S_3, S_2)</td>
<td>No</td>
<td>Fact 5</td>
</tr>
<tr>
<td>(S_2, S_3, S_1)</td>
<td>No</td>
<td>Fact 4</td>
</tr>
<tr>
<td>(S_2, S_1, S_3)</td>
<td>No</td>
<td>Fact 5</td>
</tr>
</tbody>
</table>
(Continued)

<table>
<thead>
<tr>
<th>Combination</th>
<th>Exists</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_3, S_1, S_2$</td>
<td>No</td>
<td>Fact 3</td>
</tr>
<tr>
<td>$S_3, S_2, S_1$</td>
<td>Yes</td>
<td></td>
</tr>
</tbody>
</table>

**Two Stage Combinations**

<table>
<thead>
<tr>
<th>Combination</th>
<th>Exists</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1, S_2$</td>
<td>No</td>
<td>Fact 3</td>
</tr>
<tr>
<td>$S_1, S_3$</td>
<td>No</td>
<td>Fact 5</td>
</tr>
<tr>
<td>$S_2, S_1$</td>
<td>No</td>
<td>Fact 6</td>
</tr>
<tr>
<td>$S_2, S_3$</td>
<td>No</td>
<td>Fact 4</td>
</tr>
<tr>
<td>$S_3, S_1$</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>$S_3, S_2$</td>
<td>No</td>
<td>Fact 7</td>
</tr>
</tbody>
</table>

**One Stage Combinations**

<table>
<thead>
<tr>
<th>Combination</th>
<th>Exists</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>$S_2$</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>$S_3$</td>
<td>Yes</td>
<td></td>
</tr>
</tbody>
</table>
As already suggested in Section 3.7, what "ps" does is to make $S_2$ less attractive compared to $S_1$ and $S_3$. Therefore, there will be no impact on the analysis of those strategy sequences which do not include $S_2$. This leaves one with just two sequences:

$$[S_3, S_2, S_1] \quad \text{and} \quad [S_2],$$

two combinations which are identified in the text as Case-3 and Case-5, respectively.

Using the same notation as in Appendix A, and identifying "ps" as $z$ ($0 \leq z \leq 1$), one can make the following break-even analysis:

$$[S_3, S_2, S_1]$$

The explicit expressions for $y_1(x)$ will be as following:

$$y_1(x) = \alpha + (\delta - \alpha) \cdot x,$$

as before.

$$y_3(x) = \gamma,$$

as before,

$$y_2(x, z) = xz (\delta - \beta) - (1 - xz) \cdot (\gamma - \beta),$$

$$= (\gamma - \beta) + xz \cdot (\delta - \gamma).$$
The effect of $0 < z < 1$ is to:

a) Decrease slope of line 2 when $(\delta-\gamma) > 0$.

b) Increase slope of line 2 when $(\delta-\gamma) < 0$.

For UBE one can write:

$$y_1(x) = y_2(x, z),$$
or,

$$\alpha + (\delta-\alpha) \cdot \text{UBE} = (\gamma-\beta) + z \cdot (\delta-\gamma) \cdot \text{UBE}.$$

Consequently,

$$\text{UBE} = (\alpha+\beta-\gamma)/ [z(\delta-\gamma) - (\delta-\alpha)].$$

(compared to the value of $(\alpha+\beta-\gamma)/(\alpha-\gamma)$ for $z = 1$).

For LBE one gets:

$$y_2(x, y) = y_3(x)$$
or

$$(\gamma-\beta) + z \cdot \text{LBE} \cdot (\delta-\gamma) = \gamma$$

Consequently,

$$\text{LBE} = \beta/ z \cdot (\delta-\gamma)$$

(compared to $\beta/(\delta-\gamma)$ for $z = 1$).

The phenomena that are observed for $(\delta-\gamma) > 0$, $(\delta-\alpha) > 0$ and $(\gamma-\alpha) > 0$ are a decline in UBE and an increase in LBE due to declines in $z$. At one point in $z$ (which we shall identify as $z^*$, having in mind $\text{ps}^{**}$) these two points turn into one. For that point,
\[
\frac{(a + b - \gamma)}{[z^* (\delta - \gamma) - (\delta - \alpha)]} = \frac{\beta}{z^* (\delta - \gamma)} ,
\]

\[z^* = \frac{\beta (\delta - \alpha)}{(\delta - \gamma)(\gamma - \alpha)}
\]

which is identical to the \(ps**\) equation in Section 3.7.

For all \(z\) values below \(z^*\), \(S_2\) can never be preferred, therefore the sequence \([S_3, S_2, S_1]\) (Case-3) turns into the sequence \([S_3, S_1]\) (Case-2), for which only one break-even point does exist, point which is independent from \(z\) and \(S_2\).

\(S_2\)

Originally in Case-5, for \(S_2\) to be superior to \(S_3\) and \(S_1\), the following conditions had to be satisfied:

a) \(\beta = 0\),

b) \((\delta - \gamma) > 0\),

c) \((\gamma - \alpha) > 0\), (refer to Section 3.5.4)

This, on top of the general practical implication,

d) \((\delta - \alpha) > 0\).

In the uncertain lifespan situation, for \(S_2\) to be superior to \(S_3\) is sufficient and necessary that:

\[(\gamma - \beta) + xz \cdot (\delta - \gamma) > \gamma ,
\]

or alternatively,

\[xz \cdot (\delta - \gamma) > \beta .
\]

As it turns out, the original conditions (a) through (d) are also sufficient to maintain \(S_2\)'s superiority over \(S_1\) in the limited lifespan situation.
However, for $S_2$ to maintain superiority over $S_1$ the following conditions are necessary:

$$(\gamma - \beta) + xz (\alpha - \gamma) > \alpha + (\delta - \alpha) \cdot x.$$ 

Assuming conditions (a) through (d) still hold, one gets for $x$:

$$x < \frac{(\alpha - \gamma)/(z\cdot[\delta - \gamma] - [\gamma - \alpha])}{},$$

which means that for any given value for $z$, there exists an upper limit in $z$ above which $S_2$ is not preferred, but $S_1$ is. As suggested in the text, this $[S_2, S_1]$ combination is very unusual, however very specific for conditions (a) through (d), and only in situations when there is uncertainty in proposals' surviving during the reviewing process.


