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INTERACTIVE CONTROL OF A SIX-LEGGED VEHICLE WITH OPTIMIZATION OF BOTH STABILITY AND ENERGY

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

by

David Edward Orin, B.E.E., M.S.

The Ohio State University

1976

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To My Beloved Parents
ACKNOWLEDGEMENTS

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CHAPTER I
INTRODUCTION

1.1 General Background

With the rapid technological advances in the past few years, areas of research are gradually emerging that were practically closed until recently to all except the imaginative science fiction writer. Among these fields is that of legged locomotion. It is envisioned that in the ensuing years, legged vehicles will perform tasks such as data collection in the forbidding regions of distant planets, mobility on the ocean floor, and bomb retrieval from the maze of rooms, hallways, and stairs in urban skyscrapers. These tasks will be open to legged vehicles because of their unique terrain adaptability.

Artificial legged locomotion systems typically possess many more independently controllable mechanical degrees of freedom than do conventional wheeled or tracked vehicles. Indeed, it is precisely these increased degrees of freedom which produce the generally acknowledged superior mobility characteristics of legged systems. However, this flexibility leads to a number of theoretical and practical difficulties in attempts to synthesize artificial legged systems. The basic difficulty may be stated as follows: How should the individual leg movements be coordinated so as to produce efficient locomotion with stability and a high degree of terrain adaptability?

A number of researchers have investigated this problem in the past decade. The first studies considered mechanical linkage systems
as an answer to the limb coordination control problem [1]. Early tests showed that although some success in off-road mobility was possible, a most serious shortcoming of this type of vehicle was its rather limited adaptability to terrain variations resulting from the relatively few mechanical degrees of freedom of the linkage system.

The next step, the possibility of realizing a versatile legged robot operating under computer control, has been greatly enhanced by recent developments in electronics. The successful introduction of large scale integration into the manufacture of computer and digital subsystems has resulted in sharply reduced costs and greatly increased usage of such devices. This is a trend which can be expected to continue for some time as the economics of large scale production come more and more into play. The result is that it is now possible to realistically consider the design and construction of electronic vehicle control systems of a complexity which would have been entirely out of the question only a few years ago. Such a synthetic "nervous system" seems to be an essential component of a highly adaptable legged vehicle.

The basis for this dissertation, then, is that it is now possible to design an interactive computer-control system for a robot in which a human supervises the higher level tasks of the robot such as route selection and speed determination, and the computer solves the problems of limb coordination and control [2]. The work of this dissertation is concerned with the modelling of a six-legged vehicle with its leg subsystems and with the development of algorithms that may be used in programming the control computer. The contributions of this
dissertation may be broken down into three general areas of the control problem:

1) Effective control of a six-legged vehicle implies that the control computer of the vehicle will need to be programmed with algorithms which permit the synthesis of a gait suitable for realization of the desired vehicle trajectory, taking into account the constraints imposed by terrain conditions. Such algorithms have been developed for automatic leg positioning and automatic body height, pitch, and roll regulation over undulating terrain. The human operator determines the speed, direction of travel, and heading of the vehicle and the control computer does the rest.

2) The problem of legged locomotion involves more than just solving the joint coordination control problem to successfully move the body in the desired trajectory; it also involves doing so efficiently without undue expenditure of energy. In legged locomotion systems, it is possible that legs arranged symmetrically about the forward or side body axes will tend to "fight" each other, while their net action is the desired body trajectory. For example, unless controlled, the legs on each side of the body can develop equal and opposing lateral body forces such that the body, in equilibrium, will remain in equilibrium with an unnecessary expenditure of energy. Also, when one of these legs is lifted from the ground, undesirable vibration of the vehicle may occur. Investigations in this dissertation into this force control problem include the use of mathematical programming techniques to obtain energy optimal solutions.
3) A final problem investigated in this dissertation involves the design of joint actuator systems. A 2-actuator mechanical simulation to a legged vehicle has been partially constructed and tested to study the nonlinear aspects of a worm gear driven by a universal series wound motor. Such actuator systems have potential for being used in robotic systems and their characteristics have been modelled and tested in this work.

In summary, the objective of this dissertation is to investigate the control of a six-legged vehicle with optimization of both stability and energy. Toward this end, a computer simulation has been programmed to develop these control algorithms. Also, a 2-actuator mechanical simulation of the six-legged robot has been constructed to investigate the force control problem and to study the development of joint actuator and leg sensory hardware. All of this work is in preparation for the construction and testing of an experimental legged vehicle presently under construction at this university. This vehicle will hopefully exhibit a greater degree of terrain adaptability than any existing vehicles.

1.2 Organization

Chapter I of this dissertation introduces the basic problem of legged locomotion with the various control problems encountered. Many of the previous investigations in legged locomotion studies are outlined in Chapter II with a survey of the extensive literature available in the field. This chapter includes the topics of 1) gait selection and implementation, 2) control techniques, and 3) a survey of
existing legged vehicles.

In Chapter III, the specific problem studied in this dissertation is formulated. The purpose of this dissertation is to design the control system for a six-legged vehicle that is being built at this university. The hexapod system is defined, and the body and leg subsystems are modelled. This includes a quasi-static analysis of the leg. Next the optimally stable gait to be used in the control design, the wave gait, is defined. Finally, a block diagram is given of the supervisory control system that is to be employed.

In Chapter IV, the first of the three areas of the control problem (as outlined in the first section of this chapter) is investigated. A kinematic command generator program has been developed to synthesize the body and leg trajectories of the vehicle while it moves over undulating terrain. The algorithms for position and rate command generation are detailed.

Chapter V investigates the second area of the control problem, that of the generation of joint bias torques. Included in this chapter is a section on static force calculations so that the non-zero static weight of the legs may be included. (Acceleration forces associated with the leg are ignored). The joint torques of the legs are generated by a linear programming technique with optimization of energy and load balancing among the supporting legs. The linear programming problem is formulated in this chapter with the constraints of the problem detailed. A final section of the chapter gives some theoretical results relating the existence of feasible solutions to
the linear programming problem to the static stability of the vehicle.

Chapter VI considers the third area of the control problem, the design and analysis of joint actuator systems. This chapter describes some of the initial investigations into the design of the actuators in the context of a mechanical simulation to the six-legged vehicle. The universal series wound motor, worm gear drive, and triac motor controller have been modelled. The two-actuator mechanical simulation model is then linearized about a specific operating point so that servo control gains can be set. Finally, a computer simulation of the mechanical simulation has been developed which includes all of the nonlinearities of the system (motor saturation, backlash in the worm gear, etc.). Some initial results from this program are given.

Chapter VII gives the results of the performance evaluation of the entire system as designed in the previous chapters. With the body and leg trajectories developed to maintain a high degree of stability, the force controls output bias torques to the joints that will optimize a weighted combination of the energy consumption of the vehicle over a gait cycle and load balancing among the legs. An energy model of the servo actuator is derived so that the optimization of energy is with respect to motor input voltage and current. The performance results consider different weights between energy consumption and load balancing.

Chapter VIII summarizes the results of this research and outlines topics for further work in this area. Finally, a list of references and an appendix containing a listing of the computer programs used in this research conclude the dissertation.
CHAPTER II
SURVEY OF PREVIOUS WORK

2.1 Introduction

This chapter attempts to give an overview of the available literature in the area of legged locomotion studies. Much of this work has been concerned with the establishment of a general mathematical theory of legged locomotion [3] so that these complex systems may be dealt with in more effective ways.

The first section of this survey considers the mathematical theory that has been developed for gait selection and implementation. The next section is concerned with the control techniques that have been used in designing legged locomotion systems. A prime consideration is that the vehicle be stable at all times. One part of this section deals with a quasi-static analysis of stability in which acceleration forces are assumed to be minimal and static stability is required in all phases of motion. Another part of this section includes a dynamic analysis of stability in which certain phases of the motion are statically unstable (that is, the vehicle could not stand still in this position—it would fall over), but the overall motion is stable.

A final section of this chapter is devoted to a survey of existing legged vehicles. In all cases, static stability has been assumed in the design of practical legged vehicles to date. High performance servo units on the legs and a more sophisticated control computer would be necessary before such vehicles will be able to
operate in other than statically stable modes.

2.2 Gait Selection and Implementation

Efforts in selecting gaits for artificial legged locomotion systems typically involve studies of human and animal gaits. Along with this approach, a general mathematical theory has been developed to describe the natural gait selection in animals and to define gaits to be used in artificial legged systems. Some of these investigations will be outlined in the following paragraphs.

The first major work on the kinematics of legged locomotion was contributed by Muybridge. His books on animal motion [4] and on human locomotion [5] are considered classics in the analysis of gaits of animals and human beings. Muybridge succeeded in producing the first revealing photographs of animals in natural and successive motion. Technically, they were not what are called motion pictures today. The exposures were made, not on a whirling reel of film in a camera that "followed" the subject, but with a battery of still cameras, as many as 48 of them, lined up and timed so that each would show a phase of the action in split-second succession.

Muybridge was able to achieve pictures with exposures as brief as 1/6000 of a second while an animal moved down a track in front of the row of cameras. Then, by arranging the pictures in proper order, it was possible for the first time to see what actually occurs when a horse or other animal walks, trots, or gallops.

While Muybridge's work was outstanding in the initial classification of gaits in animals, Hildebrand [6] further contributed to
the work in this area by formulating the definition of a gait and giving a more quantitative approach to the classification of symmetrical gaits. To make the description of a gait as complete and accurate as possible, he developed the concept of a gait formula. In order to do this, he first set down the definition of stride as the distance the body moves in one cycle of motion. Then, he used this definition to derive the essential information that was necessary to specify a symmetrical gait. The independent variables that are included in his "gait formula" are the percentage of each stride interval that each foot is on the ground and the percentage of each stride interval that the footfall of a fore foot lags behind the footfall of the hind foot on the same side of the body.

McGhee [7] extended the work of Hildebrand by generalizing the notion of a gait formula. He gives a stronger mathematical base for studying gaits by a further elaboration of the concepts presented by Hildebrand. McGhee converted the notions of stride length, duty factor, and phase to equivalent mathematical descriptions:

1) the stride length, \( \lambda \), is the distance by which the body is translated in one complete locomotion cycle of the gait,

2) the duty factor, \( \beta_i \), for leg \( i \), is the fraction of a locomotion cycle during which leg \( i \) is in contact with the ground,

3) and the phase, \( \phi_i \), is the fraction of a cycle by which the contact of leg \( i \) with the ground lags the contact of leg 1.
Tomovic and Karplus [8] applied the theory of finite state machines to legged locomotion systems. A leg is considered to be in one of two states; either on the ground in the supporting phase, or in the air in the transfer phase. McGhee [7] adopted this idealization by treating each leg as a sequential machine with two output states (1 and 0) with these corresponding to the transfer and support phases, respectively. McGhee then defined a particular representation of gaits called a gait matrix. A gait matrix, \( \mathbf{G} \), is formally defined as a \( k \)-column, \( (k\text{-legged system}) \) binary matrix in which no two successive rows are identical (including the first and last row) and in which each column corresponding to the state of a leg is composed of a string of zeros and a string of ones with a single change from zero to one and a single change from one to zero. Each row corresponds to a particular event (or set of events) in the locomotion cycle.

Another mathematical description of a gait, the "event sequence," has been defined by McGhee and Jain [9]. In the event sequence description, the legs of the machine are numbered, \( 1, 2, \ldots, k \). The event of placing leg \( i \) is then denoted event \( i \) while lifting of leg \( i \) is arbitrarily denoted event \( i+k \). Thus, for example, quadruped gaits can be represented by an ordering of the integers 1 through 8. When none of the \( 2k \) events associated with a given gait (lifting and placing of \( k \) legs) occurs simultaneously, the event sequence is said to be totally ordered. Gaits associated with totally ordered event sequences are called connected gaits while partially ordered sequences correspond to singular gaits [7]. It has been noted that only totally ordered
event sequences can actually be used in strictly periodic motion by an animal or machine and further that all other event sequences represent the limit of some totally ordered sequence as the phase relationships between various legs are altered. As a consequence of these facts, most analyses of gaits are limited to totally ordered event sequences.

McGhee [7] has shown that the number of distinct totally ordered event sequences for a k-legged system is given by

\[ N = (2k - 1)! \]

Thus, for quadrupeds, a total of \( 7! = 5040 \) connected gaits are possible. For hexapods, the number of such gaits is \( 11! = 39,916,800 \). The problem of selecting gaits from all of these possibilities is one of overwhelming combinatorial complexity and has been addressed by McGhee and Jain in 1972 [9]. They observed that if all legs operate with the same duty factor, then no leg can be placed and then lifted while another leg is on the ground. Gaits not containing such an event are said to be column compatible. They noted that with few exceptions, animals use only column compatible gaits.

McGhee and Jain [9] showed that out of the 5040 theoretically possible connected quadruped gaits, only 492 are column compatible. To simplify their work, they noted that column compatibility is a temporal property of a gait matrix which is not affected by column permutations. They also observed that complementation of a gait matrix
likewise does not alter compatibility. By using certain of these two operations, they reduced the 492 compatible gait matrices to a set of 45 equivalence classes. More recently, Sun has used a wider class of transformations to further reduce this set to only 14 equivalence classes [10].

McGhee and Jain [9] presented a condition called regular realizability which is advanced as an explanation of the gait preferences exhibited by animals. A gait matrix is regularly realizable if it is possible to assign a time duration to each row of $G$ so that $\beta$ is the same for all legs. Through the technique of linear programming, they determined the total number of regularly realizable connected gaits for bipeds and quadrupeds. The total number of regularly realizable connected gaits was found to be equal to 4 for bipeds and equal to 480 for quadrupeds. It was also found that these gaits accounted for all observed biped gaits and all but two quadruped gaits.

The above analysis and classification of quadruped gaits has been extended to hexapods by Sun [10]. His analysis shows that out of the 39,916,800 theoretically possible nonsingular hexapod gaits, there are only 148 equivalence classes of compatible gaits. Of these, exactly 135 classes are found to be regularly realizable, and the duty factor range for each of these classes is included in Sun's work. All 288 nonsingular regular symmetric hexapod gaits are contained in just 7 equivalence classes, a remarkably small number. Sun's results represent the furthest advance to date in analysis of the combinatorial aspects of gait. Similar results for machines or animals possessing
more than six legs are not yet available.

2.3 Control Techniques

In the design of artificial legged locomotion systems, the most important problem centers around the control of the vehicle. The major consideration in the solution to the control problem is that of system stability.

2.3.1 Gait Stability

Animals often solve the problem of stability by maintaining leg-body configurations which are statically stable at all times. This is true in almost all of the low speed gaits exhibited by animals. However, in higher speed gaits a more complex solution to the control problem is effected. For example, quadrupeds may use duty factors as low as \( \beta = .2 \) with the result that gait phases occur in which no legs at all are in contact with the ground. Evidently, the stability of such gaits depends upon very complex neural sensing and processing networks and cannot be accounted for by simple considerations of static stability. Since relatively little is known about the stability of higher speed gaits, only an analysis of statically stable gaits will be treated in the following paragraphs.

Static stability for legged systems has been defined by McGhee and Frank [11]. A legged vehicle is statically stable if the vertical projection of the vehicle center of gravity onto the supporting surface lies within the "support polygon" defined by the feet in contact with the ground. Gaits may be found in which the vehicle is statically stable at all times and can therefore be expected to be dynamically
stable provided that the vehicle speed is low enough to minimize the effect of the inertial forces associated with leg cycling.

The property of static stability can be made quantitative in the following way. Let the support polygon associated with a set of feet in contact with the ground be the smallest area convex point set enclosing all of the feet [11]. Then at any time, t, let \( s(t) \) be the shortest distance to the front or rear boundary of the support zone from the vertical projection of the center of gravity onto the supporting plane. Then the longitudinal stability margin, \( s \), associated with a given gait pattern is [11]

\[
\text{s} = \min_{0 \leq t < T} s(t) \tag{2-2}
\]

where \( T \) is the gait period.

For a given gait characterized by a specified sequence of foot liftings and placings, some kinematic degrees of freedom remain (the relative timing between the foot liftings and placings). These can be described by a "kinematic gait formula" [11], \( k \), and \( k \) can be varied to find the optimum kinematic relationships to yield the minimax longitudinal stability margin

\[
S^* = \max_{k \in K} \min_{0 \leq t < T} s(t) \tag{2-3}
\]

where \( K \) is the set of all gait formulas implying the given gait. This analysis has been carried out in detail for quadruped gaits and it has been shown that among 63,136 distinct possible quadruped gaits [12], there is a unique optimum. This optimum is the quadruped crawl [11].
This gait is a regular symmetric gait defined by the single parameter

\[ \phi_3 = \beta \quad \beta \geq 0.75 \quad (2-4) \]

where \( \phi_3 \) is defined as the fraction of a locomotion cycle by which the placing of the left rear leg follows the placing of the left front. More recently, Bessonov and Umnov [13] have shown that, for six-legged gait, \( S^* \) is maximized by a regular symmetric gait in which

\[ \phi_3 = \beta \quad \phi_5 = 2\beta - 1 \quad \beta \geq 0.5 \quad (2-5) \]

where \( \phi_3 \) is the time delay of the left middle leg and \( \phi_5 \) is the delay of the left rear leg, both measured as a fraction of a total leg cycle and relative to the placing of the left front leg. Both Eq. (2-4) and Eq. (2-5) describe "wave gaits" in which a wave of placing events runs from the rear to the front along either side of an animal or vehicle with a constant time interval between the action of adjacent legs on the same side.

In his comprehensive study of gait, Sun [10] considers the general regular symmetric wave gait defined for vehicles with \( k = 2K \) legs by

\[ \phi_{2n+1} = R(n\phi_0), \quad n = 1, 2, 3, \ldots, K-1 \quad 0 < \phi_0 < 1 \quad (2-6) \]

where \( R(x) \) represents the fractional part (residue) of a real number \( x \) and the subscripts \( n \) denote successive legs on the left side numbered from front to back. Under an assumption that the \( 2K \) legs are arranged in pairs along each side of a central body and that both the spacing between successive legs and the stroke of each leg is equal to a constant
distance, \( d \), Sun finds that \( S^* \) is maximized by using the phase increment, \( \phi \), given by
\[
\phi = \beta \quad \beta \geq 3/k
\] (2-7)

2.3.2 Kinematic Control

Kinematic control of a legged vehicle involves control of the motion of the vehicle-leg system without regard for the forces which produce this motion. Investigations in this area are concerned with an organizational structure for the algorithms of the control computer so that necessary kinematic control commands may be generated. Several computer simulations of "box" vehicles with "stick" legs have been produced to study the kinematic control of legged systems, and will be outlined in the following paragraphs.

Okhotsimsky, Platonov et al. [14] have investigated the movement of a walking machine over uneven terrain. The control computer of a six-legged vehicle was supplied with a set of "standpoints" (foot placement points) that would support the vehicle in locomotion. The task of the computer was to control the timing of the liftings and placements (tracking schedule) from these standpoints while maximizing the stability reserve (margin) of the moving vehicle.

To develop the algorithm for synthesizing the tracking schedule with a given movement of the body, two models were used. One included a two-degree-of-freedom leg and the other, a three-degree-of-freedom leg. The gaits employed were from the family of wave gaits.

Several algorithms were investigated for synthesizing the tracking schedule. One algorithm assumed that the vehicle had to move
by the tripod gait. A calculation of the optimum transfer time from one tripod of supporting legs to the other tripod of supporting legs was made. Another algorithm was based on the relative timing between the transfer waves for the right and left row of legs. The final algorithms developed may be used during movement of a walking machine over a support surface with complex relief. In further work, Okhotsimsky and Platonov [15] developed algorithms to control a walking machine climbing over obstacles. Algorithms for generation of standpoints are included along with consideration of the necessary terrain measurement system. Planning algorithms were designed which were able to generate standpoint sequences for a route comprising an arbitrary curve on a support surface with small-scale roughness. Also, algorithms were designed for generating special irregular standpoint sequences in the case of overcoming obstacles such as the "cleft" (domain forbidden for standpoints), "boulder", or "pit."

McGhee and Orin [16] have produced a digital computer simulation of an interactive computer-control system for a quadruped robot. Commands from a human operator were communicated to the simulated vehicle. Start and stop commands were furnished via a push-button while speed and direction commands were furnished by a joystick. The model used in this work consisted of a rectangular body with four three-degree-of-freedom legs. Development was aided by the use of a CRT display. An organizational structure for kinematic command generation was developed. Algorithms for turning, accelerating, and banking the vehicle were programmed. Automatic gait selection (crawl-walk-trot-gallop) was used with gait transition proceeding automatically as a
function of the longitudinal speed of the vehicle. The position of the legs while in the support phase and transfer phase was also controlled.

Petternella and Salinari [17], in a digital computer simulation, developed a multilevel control system for a legged robot. The upper levels of the control system determine the gait with its parameters and coordinate it. The gait is chosen from consideration of the stability and energy characteristics of the motion. The lower levels generate the laws of motion, establishing some interconnections between the servomechanisms. The motion of the vehicle is maintained at low speeds so that an elastic suspension system may be avoided. The task of reducing disturbances due to motion on uneven terrain is left entirely to the control system.

2.3.3 Dynamic Control

Much of the published work to date relating to the dynamics of complete legged locomotion systems has made use of the simplifying assumption that leg mass is negligible in comparison to body mass. This has been recognized as being clearly inappropriate to biped locomotion and questionable relative to quadruped locomotion, but the complexity of the dynamics of multi-jointed linkage structures has thus far prevented the derivation and use of more complex models. For example, McGhee [3] makes the following observation: "If a biped is modelled as a machine with two arms, two legs, and a central rigid body, with each limb possessing a two-degree-of-freedom hip (shoulder) joint and a single-degree-of-freedom knee (elbow) joint, then this
structure possesses eighteen degrees of freedom. It is, therefore, described by a 36-th order nonlinear differential equation. No computer simulation of machines of this degree of complexity is presently available."

Typically, the mass of a legged locomotion system is lumped into a single rigid body supported by massless legs. Frank and McGhee [13] have derived the equations of motion for a quadruped locomotion system of this type. Pai [19] successfully used the techniques of vibrational mode analysis to obtain stable postural control for this system.

The dynamic control of a quadruped vehicle has been investigated by Park [20]. Park modelled the vehicle as a single rigid body supported by massless legs. Using this model, he developed a computer simulation of a vehicle in which a human operator sets the desired velocity and either the desired direction in which the vehicle is to move or the desired rate of turn. The ideal linear and angular acceleration vectors which should be achieved through vehicle leg action are then determined. In general, it is impossible to achieve these ideal accelerations, since any real leg actuator can exert only a limited force and the force available from a leg depends upon the position of the leg and upon the velocity with which it is moving.

To overcome this problem, Park uses the technique of linear programming to generate the leg forces which will yield the linear and angular accelerations which are nearest to the optimum values. First, there are six equality constraints in the problem due to the
six net forces and torques appearing in Euler’s equations for a single rigid body [18]. Thus, for given values of the desired body translational and rotational accelerations, six required net forces and torques on the body are computed. These six equality constraints represent the way the required force and torque depend upon the positions of the legs and the forces they exert on the ground. The following equation involving one of the torques serves as an example [20]:

\[
\alpha_1 (F_{1XP} - F_{1XN}) + \beta_1 (F_{1YP} - F_{1YN}) + \gamma_1 F_{1ZP} + \alpha_2 (F_{2XP} - F_{2XN}) + \beta_2 (F_{2YP} - F_{2YN}) + \gamma_2 F_{2ZP}, \\
torque from leg \#1 + torque from leg \#2 + (TAERRP - TAERRN) = RTA + error = required torque.
\]

In this equation, \((F_{1XP} - F_{1XN}, F_{1YP} - F_{1YN}, F_{1ZP})\) and \((F_{2XP} - F_{2XN}, F_{2YP} - F_{2YN}, F_{2ZP})\) are the \(x, y, z\) components of the foot reaction forces for legs 1 and 2 respectively. The quantities \(\alpha_i, \beta_i,\) and \(\gamma_i\) are coefficients that relate the components of the foot reaction force to the net torque exerted on the body. The expression \((TAERRP - TAERRN)\) is the torque error, which is equal to the difference between the required torque and the torque exerted by the supporting legs. Finally, \(RTA\) is the required torque about the longitudinal axis of the body which is needed to achieve the desired body rotational acceleration. With these definitions, it can be seen that Eq. (2-8) is an equality constraint equation for the desired body torque about the longitudinal axis for the case of two legs, 1 and 2, in the supporting phase. The coefficients \(\alpha_i, \beta_i,\) and \(\gamma_i\)
depend on the present orientation between the body and leg i and are constant at any instant in time, thus giving a linear equation in the foot reaction forces. Since variables in linear programming must be greater than zero, a "p" subscript has been added to denote the positive part of the variable, and an "N" subscript has been added to denote the negative part.

There are four inequality constraints in the linear programming problem, as formulated in [20], and they represent the maximum force which each leg can exert. For leg 1, this constraint is:

\[ |F_{1X}| + |F_{1Y}| + |F_{1Z}| \leq F_{\text{MAX}} \]  

(2-9)

where \( F_{1X}, F_{1Y}, F_{1Z} \) are the components of the foot reaction force for leg 1 and \( F_{\text{MAX}} \) is the upper bound on the net foot reaction force for each leg. Using only positive variables, this equation may be rewritten for this leg as

\[ F_{1XP} + F_{1XN} + F_{1YP} + F_{1YN} + F_{1ZP} \leq F_{\text{MAX}} \]  

(2-10)

The performance function is optimized by minimizing the error between the desired and actual body forces and moments. Thus,

\[ P\text{FUNCTION} = F_{XERRP} + F_{XERRN} + F_{YERRP} + F_{YERRN} + F_{ZERRP} + F_{ZERRN} + T_{AERRP} + T_{AERRN} + T_{BERRP} + T_{BERRN} + T_{CERRP} + T_{CERRN}. \]  

(2-11)

where the terms on the right hand side of the equation are the positive and negative parts of the error variables for the Required body Forces and Torques—\( RFX, RFY, RFZ, RTA, RTB, \) and \( RTC. \)
Okhotsimsky, Golubev and Alekseeva [21] have studied the stabilization of a six-legged vehicle. The desired trajectory of the vehicle body is specified from one point to another and a control is generated to move the vehicle through this trajectory. This is in contrast to the work of McGhee and Orin [16] in which a human operator interactively selects the route by control over the vehicle velocity vector.

In [21], the control is generated under the effects of system perturbations such as deviations of geometrical parameters, of weight, of moments of inertia, errors in control moments produced by the actuators, and unknown external forces acting on the vehicle. (Leg mass is assumed to be zero). The desired linear and angular accelerations of the vehicle are determined by a linear interpolation from the present state to the next desired state (state after a discrete time interval). An estimate of the acceleration perturbation of the previous time interval is used in the computation of the desired acceleration components for the next time interval.

The net desired body accelerations, in general, may be produced by a number of sets of leg actuator torques (if at least three legs are in support phase). Freedom to choose the optimum set of torques remains. The authors have imposed the following two conditions on the surface reactions at the points of support:

1) To balance the load on each of the legs (desirable for soft soil locomotion, etc.), the maximum surface reaction among the feet was minimized.
2) The surface reactions were calculated to be as nearly perpendicular to the support surface as possible (for low coefficient of friction surfaces).

The vehicle has been simulated on a digital computer with perturbations due to errors of mass and moments of inertia included. Results show that the control algorithm provides locomotion over the prescribed trajectory in spite of perturbations.

2.3.4 Biped Control

As stated in the previous section, the dynamics of a biped locomotion system are extremely complex because of its inherent instability. Although achieving stable control is a difficult problem involving analysis of complex dynamic models, biped locomotion systems possess the possibility of a high degree of terrain adaptability. Overall gait stability can be effected with alternate phases of fall and recovery in the locomotion cycle.

A number of theoretical studies of biped locomotion control have appeared in the past few years. Several of these will be discussed in the following paragraphs as they relate to this dissertation, especially with regard to energy management and ground reaction constraints in legged locomotion systems.

Townsend and Seireg [22] have studied the influences of the model complexity, objective functions, and weighting factors on motion patterns, system forces causing locomotion (controls), and energy expenditure for bipedal locomotion. Three different models with 6, 7, and 9 mechanical degrees of freedom were investigated to determine what locomotion
characteristics can be expected of given models and what models are most applicable in the study of human and machine biped locomotion control, and 2) identify locomotion criterion functions which yield desirable trajectories for system design and recognizable gait characteristics relative to human motions.

The mathematical statement of the problem posed, to synthesize biped locomotion with a given model, yields more unknowns than independent relations. Therefore, an infinite number of solutions (controls) are possible. An optimal trajectory can be determined by finding a solution to the system dynamic equations (subject to any other constraints) which minimizes an objective function, $J$, the locomotion criterion. The most effective objective function was found to be the minimization of weighted combinations of 1) the size of the support base during each step, 2) an energy expenditure value, and 3) the magnitudes of the system external and internal angular motions.

System trajectories, control functions (at the moving joints), and parameters of foot placement were synthesized using a nonlinear programming technique for dynamic systems to solve the optimal control problem. The results give values of the three objectives as a function of the weighting factors for these objectives.

Yamashita and Yamada [23] have studied the dynamic stability of a bipedal locomotion system using a simplified model with first order approximations. The influence of ground reaction friction constraints was considered because of its significant effects on the dynamic stability of such systems (example: human walking on ice). In this work,
the linearized equations of motion for the model were derived. Admissible regions for the control moments were then obtained as a function of the angular state of the system so as to satisfy the ground friction requirements. A comparison of the walking pattern generated by the proposed control with that of a human walking was given. Calculated ground reactions of the system were compared with measured ones obtained in other research.

Vukobratovic et al. [24] presented the fundamentals of a synthesis approach to the gait of an anthropomorphic robot. In this approach, the trajectories of the leg joints are prescribed by preprogrammed algorithms. The trajectory of the zero moment point, the point at which the dynamic foot reaction force acts, is also prescribed throughout the gait cycle. The motion of the upper part of the body may thus be computed directly from the equations of dynamic connections (the deterministic equations that relate the motion of the upper body to the prescribed motion of the legs and zero moment point).

With the kinematics (synergy) of the biped and the position of the zero moment point already imposed, the joint torques may be computed by an appropriate free body analysis on each of the linkages of the system. That is, the system of equations relating the joint torques to the foot reaction forces, inertial forces, and linkage weights is deterministic for a given kinematic motion of all the joints and the zero moment point.

In the above work, results of this theoretical approach to the synthesis of an artificial anthropomorphic gait have been verified on
a walking machine of the exoskeleton type. The results give the power demand and torque requirements for two different artificial gaits—the "bending foot" gait and the "flat-foot" gait. Also, a discussion of the effects of different programmed synergies on the torque and power requirements of the biped is given.

2.4 Survey of Existing Legged Vehicles

This portion of this dissertation provides a brief state-of-the-art review of legged vehicle technology. Since this discussion is included for the purpose of establishing a reliable foundation for the present study, only research programs which have produced successful experimental vehicles are included. While the following presentation encompasses all such vehicles known to the writer, it is certainly possible that some have been inadvertently omitted. Nevertheless, it is believed that all of the major approaches to legged vehicle design are represented by their most advanced implementation.

2.4.1 Linkage-Controlled Machines

Prior to recent developments in electronics and actuators, the only feasible way to obtain joint coordination in a legged vehicle was by means of mechanical linkages. The first successes along these lines were undoubtedly achieved by toymakers who, beginning many years ago, produced a seemingly endless number and variety of battery or spring powered walking machines. The writer has made no attempt to survey these toys exhaustively, but casual observation leads to the following general conclusions regarding walking toys of either the
quadruped or biped type:

1. Stability is always achieved by using large and sometimes overlapping feet and, if necessary, rocking the toy from side to side to keep its center of gravity over the foot or feet in contact with the ground.

2. Typically, the legs of such machines are rigidly interconnected so that only one mechanical degree of freedom remains. This degree of freedom is a rotation angle which determines forward motion.

3. Forward motion is very slow and adaptability to terrain irregularities is generally poor.

Beginning approximately fifteen years ago, Prof. J. E. Shigley of the University of Michigan undertook an extensive study of linkage mechanisms for legged locomotion [25,26]. Shigley was interested in realizing improvements over toy walking machines in terms of speed, efficiency, and smoothness of gait. He carried his work through to the design and construction of a quadruped vehicle which made use of a set of four rectangular frames as legs. These frames were nearly as long as the body of the vehicle and were exchanged in pairs with a stroke short enough to ensure static stability. While this machine did function, its concept required the use of non-circular gears and this was found to be impractical. No further evolution of walking machines along such lines has occurred since Shigley's investigation.

A few years after Shigley's work, a group at Space General Corporation at Azusa, California, became interested in linkage-controlled walking machines as a concept for lunar locomotion. This group first
produced a six-legged externally powered vehicle [27] and then an eight-legged self-contained machine capable of carrying a small child [1,28]. Both Space General machines utilized legs arranged in pairs operating in exact phase opposition so that a minimum of either three or four legs were in contact with the ground at all times, thus assuring stability. Unlike Shigley's machine, the Space General vehicles were cam-driven and did prove to be quite effective within their design goals. The eight-legged vehicle is especially interesting because it possessed a type of differential rotation controlled by a joystick to enable turning of the vehicle within its own length. Alternatively, turning with a six foot radius during forward locomotion was possible.

In many ways, the Space General eight-legged vehicle is the most successful walking machine constructed to date. Early tests showed that it was able to climb ordinary stairs rather well. In a later series of tests, reported in 1968 [1], this machine demonstrated exceptional off-road mobility. In particular, it exhibited an unusually high ratio of drawbar pull to vehicle weight. This characteristic of walking machines had previously been theoretically predicted, but not experimentally demonstrated. The most serious shortcoming of this vehicle, and indeed of all linkage machines, was its limited adaptability to terrain variations resulting from its possessing so few mechanical degrees of freedom.
2.4.2 Manually-Controlled Machines

The largest walking machine ever built is also the world's largest off-road vehicle. "Big Muskie", a coal-mining dragline constructed by Bucyrus-Erie Company, weighs a total of twenty-seven million pounds and is propelled by four hydraulically powered legs located at each of the four corners of the machine [29]. During normal mining operations, Big Muskie rests on a cylindrical base 105 feet in diameter. During walking, this machine utilizes twenty-four electric motors of 600 horsepower each to provide hydraulic power for raising its base off the ground while transferring the weight of the machine to four feet, each having dimensions of sixty-five by twenty feet. Once raised, a second set of actuators moves the machine backward for a stride of up to fourteen feet at which time it again settles on its base. This walking action is accomplished with the aid of an electronic sequencer which continues to cycle the legs until the operation commands a halt via a control panel push button. A legged locomotion system was chosen for Big Muskie in preference to wheeled or tracked systems because it provided the greatest degree of flexibility at the least cost under the expected vehicle operating conditions.

Perhaps the second largest walking machine demonstrated to date is the General Electric Quadruped Transporter. This vehicle is approximately the size of an elephant and weighs about 3000 pounds. It possesses four legs, each with three degrees of freedom and is also entirely
manually controlled [30,31]. In operation, the operator is strapped into a seat from which he controls a system of levers with his hands and feet to direct the motions of the vehicle limbs. He is aided in this action by force reflecting servomechanisms which provide him with an indication of the interaction of the vehicle with the supporting terrain. This machine first walked in 1968 and later exhibited a significant ability to climb obstacles and to traverse difficult terrain. Unfortunately, the task of coordinating twelve independent joints proved to be an extremely difficult one which few operators could master. Moreover, even for those who could operate the vehicle, the joint coordination task was so demanding that it required nearly all of the operator's attention and was so exhausting that vehicle operation was limited to a few minutes at a time. Thus, even though the mechanical capabilities of this machine were in many ways impressive, its most important contribution to the field of walking machines may have been in the nature of a demonstration that computer control is essential for machines with this many degrees of freedom.

2.4.3 Computer-Controlled Vehicles

The first legged vehicle to walk autonomously under full computer control was the "Phoney Pony" constructed by Frank and McGhee at the University of Southern California in 1966 [32,33]. This experimental machine was furnished with four electrically powered legs, each with a single-degree-of-freedom hip joint and an independent single-degree-of-freedom knee joint. A passive suspension system was also included to permit vertical excursion of each leg relative to the vehicle body.
This machine was approximately one hundred pounds in weight and was roughly the size of a small pony. It was powered by two twelve-volt automobile batteries connected to the vehicle by a trailing cable.

The eight independent joints of the Phoney Pony's legs were coordinated by a small special purpose digital computer. The machine had no purpose except to demonstrate that the joint coordination control problem could indeed be solved by an "electronic linkage" rather than a mechanical linkage. It did this with two different gaits, the quadruped trot and the quadruped walk, and was then retired [33].

In parallel with the work at the University of Southern California, an affiliated group at Institute Mihailo Pupin in Belgrade, Yugoslavia, developed a powered biped exoskeleton intended for application to the locomotion of paraplegics. This device was pneumatically powered and was coordinated by an analog computer using angle vs. time joint commands derived from measurements of normal gait. Successful operation of this brace both with and without the inclusion of a patient was reported in 1972 [34].

Both of the above machines were true robots in the sense that they were fully autonomous; human interaction was neither necessary nor permitted. More recently, two experimental systems employing interactive computer control have been reported. One of these, developed by Prof. M. Petternella and his associates at the University of Rome [35] is a six-legged electrically powered system similar to the Phoney Pony except that a degree of operator interaction is eventually intended. A somewhat more advanced research program, directed
by Prof. I. Kato at Waseda University in Tokyo, Japan, has produced a series of computer controlled biped robots with stair climbing ability and with operator control of direction. One of these machines is hydraulically powered, weighs 130 kg, and is able to carry a load of 30 kg [36]. Unfortunately, at their present stage of development, Kato's machines are very slow, requiring up to 90 seconds per step. With respect to computer control, however, they do at present represent the furthest advance in terms of operating systems.

2.4.4 Summary

Table I provides a listing of the salient characteristics of all successful walking machines known to the writer. Photographs of most of these machines and further engineering details can be found in a research monograph by Vukobratovic [37]. In addition to these six machines, the author knows of only one other successful legged robot with independently powered joints. This is the hexapod system constructed in the Moscow Physio-Technical Institute and first reported in 1974 by Schneider et al. [38]. This machine makes use of electronic joint control in some ways similar to the USC Phoney Pony, but further details are not available to the author at the time of this writing.

The main conclusions to be drawn from these machines and the above discussion are as follows:

1. Actuator components powerful enough to permit the construction of useful legged vehicles are commercially available.
2. Electronic coordination of joint motions to achieve stable legged locomotion is feasible and has been demonstrated by at least four independent research groups.

3. Manual coordination of all joint motions can be accomplished successfully only for systems with very limited degrees of freedom or for very slow locomotion with more adaptable vehicles.

2.5 Summary

This chapter has given an overview of the available literature in the area of legged locomotion studies, especially as it relates to this dissertation. The results of the mathematical theory that has been developed for gait selection and implementation are used in this design work. The optimally stable wave gait has been selected for use in the six-legged vehicle under construction at this university. The section on control techniques in this chapter gives the necessary background to provide a versatile control design for the vehicle. Finally, the vehicle is being constructed with due consideration of the capabilities of existing legged vehicles.

The remainder of this dissertation builds upon the theory presented in this chapter to design the interactive control system for the six-legged vehicle, with optimization of both stability and energy. Considerable work in computer simulation and testing of real hardware characteristics has completed most of the initial design phase for the six-legged vehicle, and has permitted construction of the mechanical components to begin.
<table>
<thead>
<tr>
<th>Vehicle</th>
<th>Reference</th>
<th>Date of First Test</th>
<th>No. of Legs</th>
<th>Approx. Weight</th>
<th>Approx. Top Speed</th>
<th>Payload</th>
<th>Actuator Type</th>
<th>Coordination Method</th>
<th>Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shigley Quadruped</td>
<td>[26]</td>
<td>1961</td>
<td>4</td>
<td>100 lbs</td>
<td>.5 mph</td>
<td>none</td>
<td>electric</td>
<td>gear train</td>
<td>none</td>
</tr>
<tr>
<td>Space General Octaped</td>
<td>[28]</td>
<td>1965</td>
<td>8</td>
<td>200 lbs</td>
<td>2 mph</td>
<td>100 lbs</td>
<td>electric</td>
<td>cam drive</td>
<td>joystick</td>
</tr>
<tr>
<td>Phoney Pony</td>
<td>[32]</td>
<td>1966</td>
<td>4</td>
<td>100 lbs</td>
<td>.5 mph</td>
<td>none</td>
<td>electric</td>
<td>computer</td>
<td>none</td>
</tr>
<tr>
<td>Big Muskie</td>
<td>[29]</td>
<td>1969</td>
<td>4</td>
<td>27,000,000 lbs</td>
<td>.1 mph</td>
<td>500,000 lbs</td>
<td>hydraulic</td>
<td>manual</td>
<td>manual</td>
</tr>
<tr>
<td>Mihailo Pupin Exoskeleton</td>
<td>[34]</td>
<td>1972</td>
<td>2</td>
<td>50 lbs</td>
<td>.5 mph</td>
<td>200 lbs</td>
<td>pneumatic</td>
<td>computer</td>
<td>none</td>
</tr>
<tr>
<td>Waseda Biped</td>
<td>[36]</td>
<td>1972</td>
<td>2</td>
<td>290 lbs</td>
<td>.02 mph</td>
<td>65 lbs</td>
<td>hydraulic</td>
<td>computer</td>
<td>interactive</td>
</tr>
<tr>
<td>Univ. of Rome Hexapod</td>
<td>[35]</td>
<td>1972</td>
<td>6</td>
<td>100 lbs</td>
<td>.5 mph</td>
<td>none</td>
<td>electric</td>
<td>computer</td>
<td>interactive (planned)</td>
</tr>
</tbody>
</table>
CHAPTER III

PROBLEM FORMULATION

3.1 Introduction

The hexapod locomotion system that is described in this chapter is presently under construction at this university. It is the work of this dissertation to lay the foundation for the design of the control system for the hexapod vehicle. Toward this end, the vehicle has been modelled, and the mathematical model of the body and leg subsystem has been described in the first section of this chapter.

The second section describes the gait that is to be used in the locomotion of the vehicle. It is the wave gait—a statically stable, low speed gait used by many animals. The final section of this chapter outlines the overall control scheme that is to be implemented. The present state-of-the-art in legged locomotion research has suggested a supervisory control system for the vehicle in which a human operator develops a strategy for route selection and provides steering and speed commands to the vehicle. A control computer then automatically solves the limb coordination control problem to effectively move the vehicle over the desired trajectory.

3.2 Hexapod System Definition

The hexapod model to be investigated consists of a single rigid body supported by six three-degree-of-freedom legs. A description of the body with the leg subsystems is outlined in the following paragraphs.
3.2.1 Body Model

Figure 1 on the following page shows the simulated model of the hexapod vehicle. The body consists of a rectangular box with the legs adjoined to the body at the outside edges of the lower surface. Since much of the work to follow makes use of this model for the vehicle, it is detailed in the following paragraphs taken from [19].

The following convention has been assumed for the coordinate axes. The $x_E$ coordinate axis is directed toward the desired initial direction of travel, the $z_E$ coordinate axis is in the direction of gravitational acceleration (positive downward), and the $y_E$ axis is in the direction of the vector cross product

$$j_E = k_E \times i_E$$

The equations of motion are defined with respect to a flat, nonrotating earth, so that $i_E$, $j_E$, and $k_E$ are regarded as the unit vectors defining an earth-fixed frame.

The total state of the locomotion system is described by the twelve-element body state vector

$$\dot{\mathbf{x}} = (x_E, y_E, z_E, u, v, w, \theta, \phi, \psi, \rho, \varsigma, \tau)$$

where

$x_E, y_E, z_E$ = position of the center of gravity of the system relative to an inertial frame $i_E, j_E, k_E$.

$u, v, w$ = components of the translational velocity of the center of gravity expressed in body coordinates.
Figure 1. The Hexapod Locomotion System Consisting of a Rigid Body Supported by Six Three-Degree-of-Freedom Legs.
\[ \theta, \phi, \psi \text{ = the body Euler angles.} \]

\[ p, q, r \text{ = body rotation rates expressed in body coordinates.} \]

The body Euler angles are unambiguously defined in the following manner: a right-handed body fixed coordinate system with unit vectors \( \hat{i}, \hat{j}, \hat{k} \), is established with its origin fixed at the center of gravity of the model. This body fixed coordinate system is defined such that when the body angles \( \theta, \phi, \psi \), are all simultaneously reduced to zero, the \( i, j, \text{ and } k \) axes are parallel to the \( \hat{i}_E, \hat{j}_E, \hat{k}_E \) axes of the earth fixed frame, respectively. The \( x, y, z \) coordinates are measured relative to the body fixed coordinate system \( \hat{i}, \hat{j}, \hat{k} \).

Let the rotation from the earth fixed \( (x_E, y_E, z_E) \) system to the body fixed system \( (x, y, z) \) be accomplished by first rotating about the \( \hat{k}_E \) axis (azimuth), then about the rotated \( \hat{j} \) axis (elevation), and finally about the \( \hat{i} \) axis (roll). Then, for any arbitrary point \( (x_a, y_a, z_a) \) in the earth fixed system, the corresponding coordinates in the body fixed coordinate system are

\[
\begin{bmatrix}
x_b \\
y_b \\
z_b
\end{bmatrix} = T_1 \begin{bmatrix}
x_a - x_E \\
y_a - y_E \\
z_a - z_E
\end{bmatrix}
\]

where

\[
T_1 = \begin{bmatrix}
\cos \psi \cos \theta & \sin \psi \cos \theta & -\sin \theta \\
\cos \phi \sin \theta - \sin \phi \sin \psi & \cos \phi \cos \psi + \sin \phi \sin \psi \sin \theta & \cos \phi \sin \psi \\
\sin \phi \sin \theta + \cos \phi \sin \psi \cos \theta & \sin \phi \sin \psi \cos \phi - \cos \phi \sin \psi & \cos \phi \cos \psi
\end{bmatrix}
\]
The equations of motion for this system may be found in full in Pai's dissertation [19].

3.2.2 Leg Model

Each leg consists of two rigid links joined with a one-degree-of-freedom knee. Two other degrees of freedom at the hip make a total of three kinematic degrees of freedom for each leg. Figure 2 shows the model for the leg. In this section, a quasi-static analysis for the leg is given. That is, the acceleration forces associated with the leg are assumed to be negligible. Only the static weight is included in the analysis.

In the quasi-static analysis of the leg system, three additional coordinate systems for each of the legs are defined. The hip actuator coordinate system for leg $i$, $(x_A^i, y_A^i, z_A^i)$, has its origin at the hip socket. The $z_A^i$ axis is parallel to the body $z$ axis and directed downward. The $y_A^i$ axis is perpendicular to the plane of the leg segments, and the $x_A^i$ axis is uniquely determined by maintaining a right hand coordinate system. The $(x,y,z)^i$ coordinate system has its origin at the hip socket of leg $i$ and rotates with the upper limb segment of the leg. The $x_i^i$ coordinate is directed along the upper limb segment. The $y_i^i$ coordinate is perpendicular to the plane of the upper and lower limb segments. Finally, the $z_i^i$ axis is uniquely determined by maintaining a right hand coordinate system. The $(x,y,z)^{i'}$ coordinate system has its origin at the knee of leg $i$ and rotates with the lower limb segment of the leg. The $z_i^{i'}$ coordinate is directed along the lower limb segment into the supporting surface. The $y_i^{i''}$ coordinate axis is parallel to the
Figure 2. Model for a Leg Shown from Three Views
and $y^i_1$ axes, and the $x^i$ axis is uniquely determined by maintaining a right hand coordinate system. One may note that when all leg angles are reduced to zero, the $(x^A_1, y^A_1, z^A_1)$, $(x, y, z)_1^1$, and $(x, y, z)_1^1$ coordinate systems are all parallel to the body coordinate system $(x,y,z)$.

The kinematic state of the leg may be described by the Joint Angle Vector

$$
\mathbf{JAV} = (\psi_i, \theta_{li}, \theta_{2i}, \dot{\psi}_i, \dot{\theta}_{li}, \dot{\theta}_{2i})
$$

for $i = 1, 2, \ldots, 6$

where

$\psi_i = \text{leg azimuth angle, the angle between the leg plane and the body longitudinal axis, } x,$

$\theta_{li} = \text{hip elevation angle, the angle between the body plane, xy, and the upper limb segment,}$

$\theta_{2i} = \text{knee elevation angle, the angle at the knee between the lower limb and a perpendicular to the upper limb.}$

$\dot{\psi}_i, \dot{\theta}_{li}, \dot{\theta}_{2i} = \text{rates of the joint angles.}$

These angles and their rates may be calculated from a knowledge of the state of the body and the position of the foot.

Let the position of the foot of leg $i$ be given by the vectors $(x, y, z)_1^T$, $(x, y, z)_1^T$, and $(x^E_1, y^E_1, z^E_1)$ as expressed in the three different coordinate systems. Thus,

$$
\begin{bmatrix}
    x_1 \\
    y_1 \\
    z_1
\end{bmatrix}
= T_1
\begin{bmatrix}
    x^E_1 - x^E \\
    y^E_1 - y^E \\
    z^E_1 - z^E
\end{bmatrix}
$$

(3-6)
and
\[
\begin{bmatrix}
  x'_i \\
  y'_i \\
  z'_i
\end{bmatrix} = T_3 \begin{bmatrix}
  x_i - a_i \\
  y_i - b_i \\
  z_i - c_i
\end{bmatrix}
\] (3-7)

where
\[
T_3 = \begin{bmatrix}
  \cos \theta_{1i} \cos \psi_i & \sin \psi_i \cos \theta_{1i} & -\sin \theta_{1i} \\
  -\sin \psi_i & \cos \psi_i & 0 \\
  \cos \psi_i \sin \theta_{1i} & \sin \psi_i \sin \theta_{1i} & \cos \theta_{1i}
\end{bmatrix}
\] (3-8)

and
\[
h_i = (a_i, b_i, c_i)^T
\] (3-9)

are the coordinates of the hip socket for leg $i$ as expressed in the body fixed coordinate system.

From Figure 2, it can be noted that
\[
\begin{bmatrix}
  x'_i \\
  y'_i \\
  z'_i
\end{bmatrix} = \begin{bmatrix}
  l_1 + l_2 \sin \theta_{2i} \\
  0 \\
  l_2 \cos \theta_{2i}
\end{bmatrix}
\] (3-10)

where
\[
l_1 = \text{length of the upper limb segment},
\]
\[
l_2 = \text{length of the lower limb segment}.
\]

Combining Eqs. (3-7) and (3-10),
\[
\begin{bmatrix}
  x_i - a_i \\
  y_i - b_i \\
  z_i - c_i
\end{bmatrix} = \begin{bmatrix}
  l_1 + l_2 \sin \theta_{2i} \\
  0 \\
  l_2 \cos \theta_{2i}
\end{bmatrix}
\] (3-11)
Finally,

\[
\begin{bmatrix}
  x_i - a_i \\
  y_i - b_i \\
  z_i - c_i 
\end{bmatrix} =
\begin{bmatrix}
  l_1 \cos \theta_{1i} + l_2 \sin(\theta_{1i} + \theta_{2i}) \cos \psi_i \\
  l_1 \cos \theta_{1i} + l_2 \sin(\theta_{1i} + \theta_{2i}) \sin \psi_i \\
  -l_1 \sin \theta_{1i} + l_2 \cos(\theta_{1i} + \theta_{2i}) 
\end{bmatrix}
\] (3-12)

By further manipulation of the components of Eq. (3-12), expressions are obtained for the leg angles as functions of the foot position expressed in body coordinates as follows:

\[
\psi_i = \tan^{-1} \left( \frac{y_i - b_i}{x_i - a_i} \right),
\] (3-13)

\[
\sin \psi_i \neq 0 \quad \theta_{2i} = \sin^{-1} \left[ \frac{(y_i - b_i)^2 + (z_i - c_i)^2 - l_1^2 - l_2^2}{2 \ l_1 \ l_2} \right]
\] (3-14)

or

\[
\cos \psi_i \neq 0 \quad \theta_{2i} = \sin^{-1} \left[ \frac{(x_i - a_i)^2 + (z_i - c_i)^2 - l_1^2 - l_2^2}{2 \ l_1 \ l_2} \right]
\] (3-15)

\[
\theta_{1i} = \sin^{-1} \left[ \frac{-AC + B \sqrt{A^2 - C^2 - B^2}}{A^2 + B^2} \right]
\] (3-16)

where

\[
A = l_1 + l_2 \sin \theta_{2i}
\]

\[
B = l_2 \cos \theta_{2i}
\]

\[
C = z_i - c_i
\] (3-17)
Let \( (x_k, y_k, z_k)_1 \) and \( (x_k, y_k, z_k)_1^T \) be the vector position of the knee of leg 1 as expressed in body coordinates and upper limb segment coordinates. Then,

\[
\begin{bmatrix}
  x_{k1} - a_1 \\
y_{k1} - b_1 \\
z_{k1} - c_1
\end{bmatrix} = T_3 \begin{bmatrix}
  x_{k1}' \\
y_{k1}' \\
z_{k1}'
\end{bmatrix} = T_3^T \begin{bmatrix}
  l_1 \\
0 \\
0
\end{bmatrix}.
\]

Finally, the position of the knee for leg 1 in body coordinates is:

\[
\begin{bmatrix}
  x_{k1} \\
y_{k1} \\
z_{k1}
\end{bmatrix} = \begin{bmatrix}
a_1 + l_1 \cos \theta_{i1} \cos \psi_i \\
b_1 + l_1 \cos \theta_{i1} \sin \psi_i \\
c_1 - l_1 \sin \theta_{i1}
\end{bmatrix}.
\]

The joint angle rates may be obtained by differentiating Eqs. (3-13) through (3-16). The results are:

\[
\dot{\psi}_i = \sin^2 \psi_i \left[ \frac{(x_i-a_i)\dot{x}_i - (y_i-b_i)\dot{y}_i}{(y_i - b_i)^2} \right]
\]

\[
\cos \psi_i \neq 0 \quad \dot{\theta}_{21} = \frac{1}{2l_1l_2 \cos \theta_{21}} \left[ \frac{2(z_1-c_1)\dot{z}_1 + 2l_1l_2 \cos \theta_{21} \dot{z}_1 + 2 \cos^2 \psi_i (x_1-a_1)\dot{x}_1 + (x_1-a_1)^2 \sin 2\psi_i \dot{\psi}_i}{\cos^4 \psi_i} \right]
\]
or

$$\sin \psi_i \neq 0 \quad \dot{\theta}_{2i} = \frac{1}{2z_i \sin \theta_{2i}} \left[ 2(z_i - c_i^2)z_i \right]$$

$$\frac{2\sin^2 \psi_i (y_i - b_i) \dot{y}_i - (y_i - b_i)^2 \sin 2\psi_i \dot{\psi}_i}{\sin^4 \psi_i} \right]$$

$$\theta_{11} = \frac{1}{\cos \theta_{11} (A_i^2 + B_i^2)} \left[ -\dot{A} - \dot{C} + \dot{b} \sqrt{A_i^2 - C_i^2 + B_i^2} \right.$$

$$+ \frac{B}{2} (A_i^2 + C_i^2 + B_i^2) \left( 2\dot{A} - 2C - 2\dot{B} \right) - \sin \theta_{11} (2\dot{A} + 2\dot{B}) \right]$$

(3-23)

where

$$\dot{A} = \dot{z}_2 \cos \theta_{2i} \dot{\theta}_{2i} \dot{C} = \dot{z}_1$$

(3-24)

Eqs. (3-20) through (3-24) give the joint angle rates as a function of the foot velocity (with respect to the hip) for any instantaneous position of the leg. This relationship may be written

$$\begin{bmatrix}
\dot{\psi}_i \\
\dot{\theta}_{11} \\
\dot{\theta}_{2i}
\end{bmatrix} = J_i^{-1} \begin{bmatrix}
\dot{x}_i \\
\dot{y}_i \\
\dot{z}_i
\end{bmatrix}$$

(3-25)

where $J_i^{-1}$ is the inverse of the Jacobian matrix for leg i. The elements of this matrix may be obtained by manipulating Eqs. (3-12) and (3-20) through (3-24); these elements are a direct function of the present leg position:
3.2.3 Quasi-Static Analysis of the Leg

The relationship between the resultant force, $f$, and moment, $m$, acting on the vehicle body and the weight and desired rotational and translational accelerations of the body is given by Euler's equations:
\[ f = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \begin{bmatrix} m_i + mwq - mvr + mg \sin \theta \\ mw + mur - mwq - mg \cos \theta \sin \phi \\ mw + mvp - muq - mg \cos \theta \cos \phi \end{bmatrix} \] (3-27)

\[ m = \begin{bmatrix} L \\ M \\ N \end{bmatrix} = \begin{bmatrix} I_{xx} \dot{p} + (I_{zz} - I_{yy}) qr \\ I_{yy} \dot{q} + (I_{xx} - I_{zz}) rp \\ I_{zz} \dot{r} + (I_{yy} - I_{xx}) pq \end{bmatrix} \] (3-28)

where

- \( m \) = body mass,
- \( I_{xx}, I_{yy}, I_{zz} \) = the principle moments of inertia about the body axes.

These forces and moments result from the net action of the legs on the body at each of the hip sockets. That is,

\[ f = \sum_{i=1}^{6} f_i \] (3-29)

and

\[ m = \sum_{i=1}^{6} \mathbf{T}_i + \sum_{i=1}^{6} (\mathbf{h}_i \times f_i) \] (3-30)

where

- \( f_i \) = the force applied to the body by leg \( i \) at the hip socket
- \( \mathbf{T}_i \) = the torque applied to the body by leg \( i \) at the hip socket
- \( \mathbf{h}_i = [a_i, b_i, c_i]^T \), the position of hip socket \( i \) in body coordinates.

In order to determine the force, \( f_i \), and torque, \( \mathbf{T}_i \), applied to the body by leg \( i \), a free-body analysis of the leg is necessary.
In this analysis, the static weight of the leg is included while leg acceleration forces are ignored. The final results give the total force and torque acting on the body as a function of the leg joint torques and leg static weights.

The actuators at each joint produce three torques in each leg which may be described by the leg torque vector,

\[ \tau_i = [\tau_{\psi_i}, \tau_{\theta_i}, \tau_{ki}]^T \]

where

- $\tau_{\psi_i} =$ azimuth torque applied to the upper limb segment by the hip about the $z_{Ai}$ axis.
- $\tau_{\theta_i} =$ elevation torque applied to the upper limb segment by the hip about the $y_{Ai}$ axis.
- $\tau_{ki} =$ elevation torque that the upper limb segment applies to the lower limb segment about the $y_{Ai}$ axis.

Figure 3 shows the forces and moments acting on each of the limb segments.

![Figure 3. Quasi-Static Analysis of the Leg Showing the Forces and Moments Acting on Each of the Limb Segments](image-url)
The following 12 equations may be written.

\[ \Sigma_{x}^{'} = M_{k_{x}}^{'} - F_{y_{1}}^{'} \frac{x_{2}}{y_{1}} \cos \theta_{2_{1}} - (m_{g})'' \frac{x_{1}}{y_{1}} \cos \theta_{2_{1}} = 0 \]  
\[ \Sigma_{y}^{'} = \tau_{k_{y}}^{'} + F_{x_{1}}^{'} \frac{x_{2}}{y_{1}} + (m_{g})'' \frac{x_{1}}{y_{1}} = 0 \]  
\[ \Sigma_{z}^{'} = M_{k_{z}}^{'} + y_{2}^{'} \frac{x_{2}}{y_{1}} \sin \theta_{2_{1}} + (m_{g})'' \frac{x_{1}}{y_{1}} \sin \theta_{2_{1}} = 0 \]

\[ \Sigma_{x}^{''} = F_{x_{1}}^{''} - F_{z_{2}}^{'} \cos \theta_{2_{1}} + F_{z_{2}}^{'} \sin \theta_{2_{1}} + (m_{g})'' \frac{x_{1}}{y_{1}} = 0 \]  
\[ \Sigma_{y}^{''} = F_{y_{2}}^{'} - (m_{g})'' = 0 \]  
\[ \Sigma_{z}^{''} = F_{z_{2}}^{''} - F_{z_{2}}^{'} \cos \theta_{2_{1}} - F_{z_{2}}^{'} \sin \theta_{2_{1}} + (m_{g})'' \frac{x_{1}}{y_{1}} = 0 \]

\[ \Sigma_{x_{A}}^{'} = M_{k_{x_{A}}}^{'} - M_{x_{2}^{''}} \sin \theta_{1_{1}} - M_{x_{2}^{''}} \cos \theta_{1_{1}} + F_{y_{2}^{'} \frac{x_{1}}{y_{1}} \sin \theta_{1_{1}}} + (m_{g})'' \frac{x_{1}}{y_{1}} \sin \theta_{1_{1}} = 0 \]  
\[ \Sigma_{y_{A}}^{'} = \tau_{\theta_{1}}^{'} - \tau_{k_{y_{1}}}^{'} - F_{z_{2}^{'} \frac{x_{1}}{y_{1}} \cos \theta_{1_{1}}} - (m_{g})'' \frac{x_{1}}{y_{1}} = 0 \]  
\[ \Sigma_{z_{A}}^{'} = \tau_{\psi_{1}}^{'} - M_{k_{z_{A}}}^{'} \cos \theta_{1_{1}} + M_{k_{z_{A}}}^{'} \sin \theta_{1_{1}} + F_{y_{2}^{'} \frac{x_{1}}{y_{1}} \cos \theta_{1_{1}}} + (m_{g})'' \frac{x_{1}}{y_{1}} \cos \theta_{1_{1}} = 0 \]

\[ \Sigma_{x}^{'} = F_{x_{1}}^{'} - F_{x_{2}^{'} \frac{x_{1}}{y_{1}} \cos \theta_{1_{1}}} = 0 \]  
\[ \Sigma_{y}^{'} = F_{y_{1}}^{'} - F_{y_{2}^{'} \frac{x_{1}}{y_{1}} \cos \theta_{1_{1}}} = 0 \]  
\[ \Sigma_{z}^{'} = F_{z_{1}}^{'} - F_{z_{2}^{'} \frac{x_{1}}{y_{1}} \cos \theta_{1_{1}}} = 0 \]
where

\((M_{kx}^0, M_{kz})^T_i\) = the reaction moment applied by the upper limb segment to the lower limb segment.

\((F_x^T, F_y^T, F_z)^T_i\) = reaction force applied by the supporting surface to the lower limb segment.

\((F_{x2}^T, F_{y2}^T, F_{z2})^T_i\) = reaction force applied by the lower limb segment to the upper limb segment.

\(M_{Hi}^T\) = reaction moment applied by the body to the upper limb segment about the \(x_{Al}\) axis.

\(f'_i = (F_x^T, F_y^T, F_z)^T_i\) = reaction force applied by the upper limb segment to the body at the hip.

\([(m_1g)_x, (m_1g)_y, (m_1g)_z]^T\) = weight vector of the upper limb segment expressed in upper limb segment coordinates.

\([(m_2g)_x, (m_2g)_y, (m_2g)_z]^T\) = weight vector of the lower limb segment expressed in lower limb segment coordinates.

From these equations, it is possible to calculate the force applied by the leg to the body at the hip, \(f'_i\), as a function of the actuator torques and leg weights.

\[ f'_i = T^T_3 f'_i = T^T_3 T_4 T_3 + D_4 \quad (3-33) \]
If leg $i$ is in the transfer phase, Eq. (3-33) reduces to

$$f_i = (m_1 g)_i + (m_2 g)_i$$  \hspace{1cm} (3-36)

where $(m_1 g)_i$ and $(m_2 g)_i$ are the weight vectors of the upper and lower limb segments as expressed in body coordinates. For a leg in the support phase, Eq. (3-33) gives a linear relationship between the joint torques and applied force to the body for any set leg position. The foot reaction applied by the supporting surface to the foot, $f_{Ri}$, is
\[
\frac{f_{Ri}}{m} = - (m_1 g)_i - (m_2 g)_i + \frac{f_1}{m} \tag{3-37}
\]

\[
\frac{f_{Ri}}{m} = T_3^T T_4 \mathbf{I}_i + \mathbf{D}_i - (m_1 g)_i - (m_2 g)_i \cdot
\]

In Eq. (3-37), if the leg weights were zero, then the value of the foot reaction force would be \(T_3^T T_4 \mathbf{I}_i\). For the leg subsystems, we may apply the principle of conservation of energy and find a relationship between \(T_3^T T_4\) and the Jacobian matrix, \(\mathbf{J}_i\), relating the joint rates to the foot velocity:

\[
T \begin{bmatrix}
\dot{x}_i \\
\dot{y}_i \\
\dot{z}_i
\end{bmatrix} = T \begin{bmatrix}
\dot{\psi}_i \\
\dot{\phi}_{1i} \\
\dot{\phi}_{2i}
\end{bmatrix} \tag{3-38}
\]

Combining Eqs. (3-25), (3-37), and (3-38), the desired result is

\[
[J_i^{-1} T] = -T_3^T T_4 \cdot
\]

(3-39)

Eqs. (3-33) and (3-37) may be written using this result

\[
\begin{align*}
\mathbf{f}_i &= [J_i^{-1} T] \mathbf{I}_i + \mathbf{D}_i \\
\frac{f_{Ri}}{m} &= [J_i^{-1} T] \mathbf{I}_i + \mathbf{D}_i - (m_1 g)_i - (m_2 g)_i \\
\end{align*}
\]

(3-40)

From Eqs. (3-32) the total torque applied to the body by leg \(i\), \(T_4\), may be calculated.
This expression may be rewritten.

\[ T_4 = E_4 \hat{I}_4 + G_4 \]  

(3-43)
There are many gaits that are used by animals that have potential application for legged vehicles. Generally, they may be placed in two categories: those that are statically stable and those that are not. While dynamically stable gaits such as the trot and the gallop in horses may be implemented in future vehicles, the complexity of the dynamics of multi-jointed linkage structures has thus far prevented the derivation and use of more complex vehicle models necessary for the effective implementation of these higher speed gaits. Thus, only statically stable gaits are considered in the design of practical vehicles at this time.

In setting up kinematic models for gaits, certain definitions have continued to be useful. These are:

1. **Stride Length**, \( \lambda \): the distance by which the body is translated in one complete locomotion cycle of the gait.

2. **Period**, \( \tau \): the time required for one complete locomotion cycle of the gait.
3. **Duty Factor**, $\beta_1$: the relative amount of time spent on the ground by each leg during one locomotion cycle.

4. **Relative Phase**, $\phi_i$: the amount of time by which leg $i$, $i = 2, 3, 4, 5, 6$, lags behind that of leg 1 expressed as a fraction of the time required to complete one locomotion cycle.

For the legged vehicle under consideration, the family of gaits called **optimal wave gaits** have been implemented in the control computer. These have been described mathematically by Bessonov and Umnov [13] as the regular symmetric gaits in which

$$
\phi_3 = \beta, \quad \phi_5 = 2\beta - 1, \quad \beta \geq 0.5 \quad (3-46)
$$

where $\phi_3$ is the time delay of the left middle leg and $\phi_5$ is the delay of the left rear leg relative to the placing of the left front leg. These have become known as "wave" gaits because a wave of placing events runs from the rear to the front along either side of an animal or vehicle with a constant time interval between the action of adjacent legs on the same side.

With $\beta = 0.5$, the vehicle is supported by alternating tripods. This gait is described by a gait matrix and event sequence in Figure 4. As $\beta$ is increased, the vehicle is successively supported by more legs during all phases of the locomotion cycle, as shown in Figure 5. The family of wave gaits has been chosen because it has been shown to optimize the minimax longitudinal stability criterion [10,13], as described in Chapter II.
Figure 4. Event Sequence and Gait Matrix for the Tripod Gait

Figure 5. Event Sequence and Gait Matrix for the Parallelogram Gait
3.4 Control System

The control system that is presently being designed for the hexapod vehicle is a supervisory control system in which the higher level commands of steering, speed, etc., are supplied to the control computer by a human operator. The control computer effectively implements a gait that will move the vehicle in the desired trajectory. Figure 6 gives a block diagram of the control system to be implemented. It is expected that a legged vehicle under operator supervision with effective programming of the control computer will be able to

1) climb stairs and slopes
2) ride smoothly over rough terrain
3) avoid obstacles directly ahead
4) maneuver over large obstacles
5) cross ditches of length comparable to the vehicle length
6) maneuver in small spaces such as halls or stair landings.

Again noting Figure 6, the human operator in this system receives orders from some higher authority in the form of mission objectives. Once the operator has developed a strategy for carrying out a sequence of operations to accomplish the given objectives, he begins to provide commands to the control computer. Such commands can be either continuous in nature or they may be logical commands (mode selection). For the control scheme proposed, the commands from the operator to the computer can be broken into four basic categories:

1. Velocity commands: a 2-axis joystick can be used to furnish the control computer with a desired trajectory for the center of
Figure 6. Control System for the Legged Vehicle
2. Heading commands: the same joystick under proper mode selection can be used to specify the desired heading of the vehicle.

3. Individual leg control: in certain modes of operation, such as obstacle climbing, it will be essential that the operator be able to control individual legs. This could be done by changing the function of the joystick.

4. Mode selection: this is a discrete input to the computer which could be accomplished through a keyboard. Available computer modes could include some of the functions given in Table II.

Table II: Typical Control Computer Modes

<table>
<thead>
<tr>
<th>Mode</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vector Velocity Control</td>
<td>The heading of the vehicle remains constant while the center of gravity of the vehicle moves as commanded by two axes of a joystick.</td>
</tr>
<tr>
<td>Turning Control</td>
<td>The heading turns with a rate commanded by one axis of a joystick and the forward (backward) velocity in the direction of the heading is determined by the position of the other axis of the joystick.</td>
</tr>
<tr>
<td>Follow-the-Leader Control</td>
<td>The operator manually places the front foot on each side and the others follow in these footprints as the vehicle moves forward.</td>
</tr>
<tr>
<td>Manual Control</td>
<td>The operator exercises individual control over each leg in a sequence determined by the operator.</td>
</tr>
<tr>
<td>Stairclimb Control</td>
<td>The operator controls speed and direction and also provides step length and height inputs.</td>
</tr>
</tbody>
</table>
The control computer program contains two major functional blocks:

1) The **Kinematic Command Generator (KCG)** accepts commands from the operator, and with a knowledge of the present state of the vehicle relative to the supporting surface, it synthesizes joint commands that direct the vehicle over the desired trajectory (desired angles, angle rates, bias torques). That is, the KCG is filled with algorithms that automatically implement the gait pattern that will properly move the vehicle.

2) The **Force Accommodation Controller (FAC)** compares the actual state of the vehicle with the desired state from the KCG and appropriately adjusts the desired joint commands to account for conditions such as terrain variation, leg servo errors, and the complex dynamics of leg cycling.

The actuator for each of the joints is a universal series would electric motor driving a worm gear. Command information from the FAC in the form of an analog signal contains the commanded rate and bias joint torque at the output of the worm gear. The actual rate is compared to the desired rate and automatically adjusted.

For high performance servo loops on the legs, it is expected that only minor adjustments will be effected by the FAC while the vehicle is walking over smooth terrain. That is, the leg servo would closely follow those values furnished by the KCG. However, when traveling over rough terrain, the full power of force feedback into the FAC is expected to be necessary to move the vehicle efficiently.
and rapidly over the desired trajectory.

3.5 Summary

In this chapter, some of the necessary background information in the formulation of the legged vehicle control design problem is given. Complete mathematical models for the body-leg system are detailed. The optimum statically stable gait, the wave gait, has been given. Finally, the supervisory control system to be employed has been outlined.

In Chapter IV, a complete explanation of the algorithms for position and rate command generation that have been programmed for the KCG are given. Chapter V details a method for generating joint bias torques with considerations of energy, load balancing, and the coefficient of friction of the supporting surface. Chapter VI outlines the approach taken in this work to the design of joint actuator systems. These chapters give the basis for a control scheme that should successfully use the power of the operator, control computer, and vehicle to effectively provide for locomotion of the vehicle system.
4.1 Introduction

In the control of a legged vehicle, it is necessary to specify both the trajectory over which the vehicle is to travel and the trajectory of the individual legs. In the supervisory control system presently being designed for the six-legged vehicle, the overall trajectory of the vehicle is specified by the human operator and the leg coordination is controlled by a digital computer. The operator furnishes the desired mode of travel along with speed and direction commands. The control computer accepts the operator commands and effectively implements the wave gait. It has been shown that the wave gait is an optimum gait for the static stability of the vehicle over level ground [10,13]. Since no research has addressed the question of stability over other types of terrain, the wave gait will be used throughout.

The basic structure of the control computer is shown in Figure 6. It is functionally broken into two segments. First of all, there is the Kinematic Command Generator (KCG). It is filled with algorithms that translate the operator commands into the machine level commands of the desired leg joint angles and rates (JAV), and bias torques (TV), and the body state variables describing orientation and position (X). Secondly, there is the Force Accommodation Controller (FAC) which
accepts these desired body and leg trajectories, and with feedback from
the leg subsystems, adjusts these to account for system perturbations.
The purpose of the next two chapters is to describe the algorithms
that have been programmed for the KCG. This chapter describes algo-
rithms for mode control, automatic body height control (above the sup-
porting surface), pitch and roll regulation, and automatic leg posi-
tioning to implement a gait with adjustments to account for terrain
variation. The generation of bias torques is treated in Chapter V.
The design of the FAC awaits studies into the use of force feedback
in legged locomotion systems and will not be discussed in this
dissertation.

A Kinematic Command Generator program has been written in
Fortran on a PDP-10 computer. Its basic organization is given in
Figure 7, showing a flow chart of the main program, ROBOT. The main
program consists of an initialization segment and calls to subroutines
which contain the algorithms for generation of the trajectory for the
system. Those subroutines concerned with joint position and rate com-
mand generation will be outlined in the remainder of this chapter.

With those subroutines that have been programmed, the simulated
vehicle is able to travel in any direction and heading over rough ter-
rain while automatically adjusting its body height, pitch, roll for
terrain variation. With the basic program organization established,
algorithms for obstacle negotiation, stair climbing, etc., may be
added in future research investigations.
Figure 7. Elementary Flow Chart for the Main Program of the Kinematic Command Generator
4.2 Kinematic Command Generator Organization

The software for the KCG has been organized into subroutines as noted in Figure 7. Each of these subroutines operates on some of the vectors that describe the state of the body, leg subsystems, etc. For their input, the subroutines may use any of the global parameter vectors that define the system model. There are three parameter vectors and they are 1) Body Physical Parameter Vector (BPPV); a vector with 28 parameters defining the physical parameters of the body-leg system. 2) Terrain Parameters Vector (TPV); a vector with 9 parameters defining the terrain over which the vehicle travels, and 3) Motor Parameters Vector (XMPV); a vector with 11 parameters of the joint actuator motor-gear system. BPPV is completely defined by its elements:

\[
BPPV = (H_1, H_2, H_3, L, L_1, L_2, DT, XMAS_1, XMAS_2) \tag{4-1}
\]

where

\[H_1(i) = a_i, \text{ for } i=1,2,\ldots,6: \text{ the } x \text{ coordinate of the hip socket for leg } i \text{ as expressed in body coordinates.}\]

\[H_2(i) = b_i, \text{ for } i=1,2,\ldots,6: \text{ the } y \text{ coordinate of the hip socket for leg } i \text{ as expressed in body coordinates.}\]

\[H_3(i) = c_i, \text{ for } i=1,2,\ldots,6: \text{ the } z \text{ coordinate of the hip socket for leg } i \text{ as expressed in body coordinates.}\]

\[L = \text{ the absolute value of the } y \text{ component of the equilibrium foot position about the hip socket as expressed in body coordinates.}\]

\[L_1 = \text{ the length of the upper limb segment for each of the legs.}\]
L2 = the length of the lower limb segment for each of the legs.
DT = \Delta t, time increment for integration in the control computer.
XMAS = the mass of the body.
XI(3) = the three components of the moment of inertia about the principle axes of the body.
XMAS1 = the mass of the upper limb segment of each of the legs.
XMAS2 = the mass of the lower limb segment of each of the legs.

The TPV has nine elements:

\[ TPV = (FRCTN,G1,G2,G3,G4,G5,G6,G7,BALNCE) \] (4-2)

where, if \((x_E, y_E, z_E)\) are the coordinates for any point of the terrain as expressed in earth fixed coordinates, then

\[ z_E(x_E, y_E) = G1 + G2[G3(x_E - G7) + G4 y_E] + G5[\cos(G6(x_E - G7)) - 1] \] (4-3)

defines an undulating terrain superimposed upon a slope. Also,
FRCTN = the static coefficient of friction of the terrain, and
BALNCE = the weight given to load balancing in the linear programming problem (See Chapter V).

The quantity XMPV will be explained in succeeding chapters.

In the next sections of this chapter, the control algorithms will be outlined as the subroutines of the KCG are explained. In each case an elementary flow chart for each subroutine will be given along with the vectors that it uses and operates upon.
4.3 Mode Control

The first two subroutine calls from the main program are the subroutines INPUT and INTERpret. These subroutines provide the software interface between the human operator's commands and the other subroutines of the KCG. Elementary flow charts are given in Figures 8 and 9 along with the input and output vectors. The input vectors are those that are used by the subroutine while the output vectors are those that are altered by that subroutine.

The function of the INPUT subroutine is to read the commands from the operator. The Input Vector (XIV) is updated by this subroutine; it is defined as

\[ \text{XIV} = (\text{AXIS1}, \text{AXIS2}, \text{MODE}) \]  

(4-4)

where

- **AXIS1** = the forward/backward rotation angle of the control joystick about the vertical.
- **AXIS2** = the side to side rotation angle of the control joystick about the vertical.
- **MODE** = the present operating mode as commanded through the keyboard by the operator.

Table III defines the two operating modes employed: Turning Control and Vector Velocity Control. In Turning Control, the rotational rate of the heading is commanded by AXIS2 of the joystick while the forward/backward translational velocity is furnished through AXIS1.

It may be noted that with AXIS1 = 0, the vehicle will turn in place at a rate determined by AXIS2. For other values of AXIS1, the vehicle...
Figure 8 (above): Elementary Flow Chart for the INPUT Subroutine

Input Data: $x_{IV}$
Output Data: $x_C$

Figure 9 (below): Elementary Flow Chart for the INTERpret Subroutine

Input Data: $x_{IV}, x_C$
Output Data: $x_C$

\[ u_0 = \frac{u + u \Delta t + u}{2} \]

\[ v_0 = \frac{v + v \Delta t + v}{2} \]
Table III: Vehicle Operating Modes

<table>
<thead>
<tr>
<th>Turn Control</th>
<th>Mode</th>
<th>Axis1</th>
<th>Axis2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>Forward</td>
<td>Azimuth</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Velocity</td>
<td>Rate</td>
</tr>
<tr>
<td>Vector Velocity Control</td>
<td>2</td>
<td>Forward</td>
<td>Sidestep</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Velocity</td>
<td>Velocity</td>
</tr>
</tbody>
</table>

will turn with a characteristic radius of curvature. For Vector Velocity Control, the heading of the vehicle remains constant while the center of gravity of the vehicle moves as commanded by the two axes of the joystick. AXIS1 is the forward/backward velocity in the direction of the heading. AXIS2 is the sidestep velocity of the body. Such a mode of control allows the vehicle to easily sidestep obstacles that would cause conventional vehicles more maneuvering time (trees in a forest, etc.).

Subroutine INTERpret uses the information received from the operator to update certain of the body state variables. The desired state of the body generated by the KCG is defined as the vector $X_C$. It has the same variables as the vector $X$ (equation 3-2) with a "C" added to denote Commanded or desired body state. Noting Figure 10, AXIS1 is the steady-state value that the desired forward velocity of the vehicle ($u$) approaches. For mode 2, AXIS2 is the steady-state value for the sidestep velocity ($v$). If there is a difference in the desired velocity and the value furnished through the joystick,
then the vehicle accelerates or decelerates to remove the difference. This scheme is much like that employed by automobiles. The exact mathematical relationships are

\[
\begin{align*}
\dot{u} &= K_1(u - \text{Axis 1}) \\
\dot{v} &= K_2(v - \text{Axis 2}) \quad \text{(mode 2)}
\end{align*}
\]

For mode 1, the sidestep velocity is reduced to zero over some short time interval. For mode 2, the body azimuth rate is commanded to be zero. The final steps of the subroutine updates the forward/backward and side velocities through integration.

Figure 10. Control of Vehicle Velocity
4.4 Automatic Body Height, Pitch, and Roll Regulation

Certain of the body state variables were updated through the INPUT and INTERpret subroutines outlined in the previous section of this chapter. The variables associated with body translational velocity in the forward/backward and side directions and the body azimuth rotational rate were updated by commands furnished by the operator. The body height above the supporting surface and the body pitch and roll angles are adjusted automatically by subroutine BODY, (see Figure 11). The desired values for these variables \((z_E, \theta, \phi)\) are determined by fitting an approximate plane to the points of support by a least squares method. The body, then, is commanded to be parallel to this estimated feet plane and at a constant perpendicular height above it. The BODY subroutine operates on the desired body state vector, \(X_C\). To calculate the approximate plane of the supporting feet, the coordinates of the feet are taken from the Leg Position Vector, \(X_{LPV}\). It is defined as

\[
(X_{LPV}) = (x_{iE}, y_{iE}, z_{iE}, M_i)
\]

for \(i = 1, 2, \ldots, 6\)

where \((x_{iE}, y_{iE}, z_{iE})^T\) = the position of foot \(i\) expressed in earth fixed coordinates.

\(M_i\) = indicator of foot position

\(M_i = 1\) (on ground)
\(M_i = 0\) (off ground).

The estimated plane of the feet that are in contact with the ground may be written in the earth fixed coordinate system as
\[ \hat{z}_E = \hat{a}_0 + (\hat{a}_1 - \hat{a}_4 \hat{a}_3) \bar{x}_E + \hat{a}_4 \bar{y}_E - \hat{a}_2 \hat{a}_4 \]  
\[ (4-7) \]

where
\[ \hat{a}_1 = \frac{6}{\sum_{i=1}^{6} \frac{M_i(x_{iE} - \bar{x})}{M_i(x_{iE} - \bar{x})^2}} \]
\[ \hat{a}_2 = \frac{6}{\sum_{i=1}^{6} \frac{M_i(x_{iE} - \bar{x})}{M_i(x_{iE} - \bar{x})^2}} \]
\[ \hat{a}_3 = \bar{y} - \hat{a}_2 \bar{x} \]
\[ \hat{a}_4 = \frac{6}{\sum_{i=1}^{6} \frac{M_i[(y_{iE} - \hat{y}_{1E}) - (y_{1E} - \hat{y}_{1E})] [z_{iE} - \hat{z}_{1E}] - (z_{1E} - \hat{z}_{1E})]}{\sum_{i=1}^{6} \frac{M_i[(y_{iE} - \hat{y}_{1E}) - (y_{1E} - \hat{y}_{1E})]^2}{M_i}} \]

and
\[ n = \sum_{i=1}^{6} M_i \]
\[ \bar{x} = \frac{6}{\sum_{i=1}^{6} M_i} \frac{x_{iE}}{n} \]
\[ \bar{y} = \frac{6}{\sum_{i=1}^{6} M_i} \frac{y_{iE}}{n} \]
\[ \bar{z} = \frac{6}{\sum_{i=1}^{6} M_i} \frac{z_{iE}}{n} \]
\[ \hat{z}_{1E} = \hat{a}_0 + \hat{a}_1 \bar{x}_{1E} \]
\[ \hat{y}_{1E} = \hat{a}_2 + \hat{a}_3 \bar{x}_{1E} \]
Input Data: $\tilde{X}_{LPV}$, $\tilde{X}_C$

Output Data: $\tilde{X}_C$

Figure 11. Elementary Flow Chart for the Body Subroutine
These equations follow from doing a linear regression on the points-of support; the technique may be found in any linear regression textbook [39].

The next step is to calculate the steady-state values for the height, pitch, and roll of the body from Eq. (4-7). This may be resolved by expressing the unit vector normal to this plane, \( \hat{N} \), in two ways and equating the components. A unit vector normal to the desired body orientation will also be normal to the body in steady-state and thus have a body z component only. That is, \( \hat{N} \) may be expressed in the earth fixed coordinate system as

\[
\hat{N} = \begin{bmatrix}
0 \\
\sin\phi \sin\theta + \cos\phi \sin\theta \cos\phi \\
\sin\phi \sin\theta \cos\phi - \cos\phi \sin\theta \\
\cos\phi \cos\theta
\end{bmatrix}
\]

(4-8)

The equation for the unit vector normal to the feet plane given by Eq. (4-7) may be drawn from an advanced geometry textbook [40]:

\[
y_{1E} = \frac{\sum_{i=1}^{6} M_i (y_{1E} - y_{1E})}{n}
\]

\[
z_{1E} = \frac{\sum_{i=1}^{6} M_i (z_{1E} - z_{1E})}{n}
\]
Equating Eqs. (4-8) and (4-9), for a given value of the body azimuth angle, \( \psi \), the roll and pitch of the feet plane, \( \phi_{FP} \) and \( \theta_{FP} \), which determine the desired orientation of the body, are given by:

\[
\phi_{FP} = \sin^{-1} \left( A \sin \psi - B \cos \psi \right) \quad (4-10)
\]

\[
\theta_{FP} = \begin{cases} 
\sin^{-1} \left( \frac{B + \cos \psi \sin \psi}{\sin \psi \cos \psi} \right) & \text{for } \sin \psi \neq 0 \\
\sin^{-1} \left( \frac{A - \sin \psi \sin \psi}{\cos \psi \cos \psi} \right) & \text{for } \cos \psi \neq 0 
\end{cases}
\]

or

For automatic height regulation, the perpendicular distance between the center of gravity of the body \((x_E, y_E, z_E)^T\), and the plane of the feet is

\[
D = -A \, x_E - B \, y_E - C \, z_E + \frac{(\hat{a}_0 \hat{a}_4)}{\sqrt{1 + (\hat{a}_1 \hat{a}_4 \hat{a}_3)^2 + \hat{a}_4^2}} \quad (4-11)
\]
The orientation angles of the feet plane, θ_{FP} and φ_{FP}, along with the distance, D, from above are used to determine the commanded body state variables, x, y, z, w, θ, φ, p, and q. The specific control laws implemented are given in Figures 12 and 13. Noting these figures θ_{FP}(φ_{FP}) is the steady state value for the body state variable θ(φ).

The body height above the feet plane, D, is compared to the desired height, z_0, and the body vertical velocity is appropriately calculated.

4.5 Automatic Leg Positioning

4.5.1 Gait Implementation

With the desired state of the body generated by the previous subroutines, it is now necessary to implement a gait and define leg trajectories that move the body in the desired path. The purpose of the LEG POSITION subroutine then is to effectively coordinate the motion of the legs (see Figure 14); this is achieved by implementing the optimum wave gait for forward/backward movement. The same phase relationships among the legs described by the wave gait are used in sidestep and turning in place movement. Specifically, for β > 1/2,

\[ \phi_2 = \frac{1}{2}, \phi_3 = β, \phi_4 = β - \frac{1}{2}, \phi_5 = 2β - 1, \phi_6 = R(2β - \frac{1}{2}) \]  \hspace{1cm} (4-12)

where R(x) is the fractional part of a real number x. For any specific value of the duty factor, β, the gait may be described by the particular event sequence. One such example, for β = 0.75, is shown in Figure 15. By investigating such event sequences, it may be found that as the duty factor increases from 0.5 to 1.0, a greater average number of legs are in contact with the ground over a gait cycle. These relationships are
Figure 12. Control Law for the Body State Variables Associated With Translation

Figure 13. Control Law for the Body State Variables Associated with Rotation (\(T_2\) is defined in [19])
Input Data: $X_C, X_{LPV}, X_{IV}$

Output Data: $X_{LPV}$

Figure 14. Elementary Flow Chart for the LEGPOS Subroutine

Calculate $\tau, \beta$

Calculate $M_1$

1. $M_1$?
   0. Mode?
      1. Calculate $(X_1, Y_1)$
      2. Calculate $(X_1, Y_1)$

Calculate $Z_{1E}, Z_{i1E}, \tilde{z}_{1E}$

Calculate $(X_{1E}, Y_{1E}, \tilde{z}_{1E})$

0. $M_1$?
   1. No

Foot i in contact?
   Yes
   $M_1 = 1$

No
   Set $Z_{1E} = Z_E(X_{E}, Y_{E})$

Foot i in contact?
   Yes
   $M_1 = 0$

No

Return
Figure 15. Event Sequence for the Optimum Wave Gait with $\beta = 0.75$

\[\begin{align*}
\beta &= 1/2 & 3 \text{ legs in contact (tripod gait)} & \text{(4-13)} \\
1/2 < \beta < 2/3 & 3 \text{ or 4 legs in contact} \\
\beta &= 2/3 & 4 \text{ legs in contact (parallelogram gait)} \\
2/3 < \beta < 5/6 & 4 \text{ or 5 legs in contact} \\
\beta &= 5/6 & 5 \text{ legs in contact} \\
5/6 < \beta < 1 & 5 \text{ or 6 legs in contact}
\end{align*}\]

It may also be shown that the percentage of time that only 3 legs are in contact with the ground is linearly related to the duty factor, $\beta$, for the interval $1/2 \leq \beta \leq 2/3$. Thus, three and only three legs are in contact with the ground 100% of the cycle time for $\beta = 1/2$ (tripod gait) while this occurs 0% of the cycle time for $\beta = 2/3$. The same linear relationship of the number of legs in contact with the ground as a function of the duty factor holds for each of the prescribed duty factor intervals (4-5 legs or 5-6 legs).
In Chapter II, the stride length, \( \lambda \), was defined for straight line locomotion as the distance by which the body is translated in one complete locomotion cycle of the gait. In this research, the definition of stride length has been generalized to include locomotion in three dimensions:

- \( \lambda_x \) = The longitudinal distance by which the body is translated in one complete locomotion cycle.
- \( \lambda_y \) = The lateral distance by which the body is translated in one complete locomotion cycle.
- \( \lambda_\psi \) = The angular distance by which the body is rotated in one complete locomotion cycle.

The first step in the LEG POSITION subroutine is to calculate the period, \( \tau \), of the locomotion cycle. For straight line locomotion, the period is equal to the stride length divided by translational rate. For the turn control and vector velocity control modes, there are two rates associated with each. For these cases, the period has been calculated for constant stride length as follows:

\[
\tau = \min(\tau_1, \tau_2) \quad \text{for mode 1 (turn) control} \tag{4-14}
\]

\[
\tau = \min(\tau_1, \tau_3) \quad \text{for mode 2 (vector velocity) control}
\]

where

\[
\tau_1 = \frac{\lambda}{u}
\]

\[
\tau_2 = \frac{\lambda_\psi}{\tau}
\]

\[
\tau_3 = \frac{\lambda_y}{\nu}
\]

If \( \tau_1 < \tau_2 \), then the effective rotational stride is less than \( \lambda_\psi \) while the effective longitudinal stride is equal to \( \lambda_x \). The reverse case is true and similar arguments hold for vector velocity control.
Similar to that which is evidenced in nature, the duty factor is allowed to vary to yield a constant leg transfer or return time, $\tau_R$. With the gait period equal to $\tau$, then

$$\beta = (\tau - \tau_R) / \tau .$$

(4-15)

To explain the algorithm for gait implementation the event sequence diagram is broken into 6 diagrams, one for each leg:

Consider a piece of hardware in which 6 discs are placed concentrically on a shaft with 6 fingers connected to the shaft, one riding on each disc. Let there be metal plating on the perimeter of each of the discs (shown with darker ink above). Let $M_i = 1$ if the finger is riding on the plated metal of disc $i$. Otherwise, let $M_i = 0$.

The rotation of the shaft (and fingers) could be used to implement a gait if proper electrical connections were made. A wave gait in the forward direction is implemented when the shaft rotates clockwise. A wave gait in the reverse direction is implemented when the shaft rotates counterclockwise. A varying value of the duty factor...
would require a different amount of plating for each disc. A changing phase relationship will result in relative rotations between the discs. This is a description of a kind of "cam" control. The software gait implementation algorithms are used in place of such a hardware control.

4.5.2 Foot Positioning

The next step in defining the leg trajectories is to calculate the desired coordinates of each of the feet for the leg in the transfer phase ($M_1 = 0$). The following definition is helpful. Let $CT_i$ be the current time in the gait cycle for leg $i$ as a fraction of the period. Time zero for leg $i$ is defined by the placement of leg $i$. Thus, $CT_i$ has a value of 0 as the finger for disc $i$ begins to touch the metal plating; it has value $\beta$ as the finger rotates and just loses contact with the plating again. Thus, $0.0 \leq CT_i \leq 1.0$.

Mode 1 (turn control)

The desired steady-state $x$-$y$ coordinates of foot $i$ as expressed in the body coordinate system $(X,Y)$ are given as follows:

$$X_i = \pm \sqrt{a_i^2 + (b_i + d_i)^2} \sin \gamma + \tau \beta \left( \frac{CT_i - \beta}{1.0 - \beta} - 0.5 \right)$$ (4-17)

$$Y_i = \pm \sqrt{a_i^2 + (b_i + d_i)^2} \cos \gamma$$

where

$$\gamma = \tan^{-1} \left( \frac{a_i}{b_i + d_i} \right) + \tau \beta \left( \frac{CT_i - \beta}{1.0 - \beta} - 0.5 \right)$$

and

$$d_i = \pm L = \text{equilibrium lateral displacement of foot } i \text{ from hip socket } i \text{ as expressed in body coordinates.}$$
In these equations, the "+" sign denotes right side legs and the "−" sign denotes left side legs in the "±" coefficient.

The equations above describe superimposed forward/backward motion and rotational motion about an equilibrium x-y foot position \((a_1, b_1 + d_1)\), as illustrated in Figure 17. The product \((rr)\) is just the effective rotational stride; thus \((rr\beta)\) is the effective rotational stroke—the angular distance the body rotates when the foot is on the ground.

Figure 17. Top View of the Body-Leg System Showing Leg 1 in the Transfer Phase

When \(CT_1 = \beta\), the angular displacement is 1/2 of a rotational stroke away from the equilibrium angle, \(\gamma_0 = \tan^{-1}(a_1/(b_1 + d_1))\). When \(CT_1 = 1.0\), the angular displacement is 1/2 of a rotational stroke to the other side of the equilibrium angle, \(\gamma_0\). Similar arguments hold for the forward/backward motion about the equilibrium position. The foot is transferred to a placement point such that symmetrical motion about the equilibrium position occurs in the support phase for that leg.
Mode 2 (vector velocity control)

The desired steady-state x-y coordinates of foot i expressed in the body coordinate system may be calculated as follows:

\[
X_i = \tau u \beta \left( \frac{CT_i - \beta}{1.0 - \beta} - 0.5 \right) + a_i
\]

\[
Y_i = \tau v \beta \left( \frac{CT_i - \beta}{1.0 - \beta} - 0.5 \right) + (b_i + d_i).
\]

These equations describe symmetric motion about an equilibrium position, \((a_i, b_i + d_i)\).

For both modes 1 and 2, the steady-state z coordinate for foot i is given as

\[
z_i = -\frac{z_0}{4.0} \left( 1.0 + \sin[2\pi \left( \frac{CT_i - \beta}{1.0 - \beta} - \frac{\pi}{2} \right)] \right) + z_0
\]

(4-19)

The z coordinate of a foot in the transfer phase describes a sine trajectory. With \(CT_i = \beta\), the foot is a z distance of \(z_0\) away from the body. In the middle of the transfer phase, it is lifted to a distance \(z_0/2\) away from the body and finally placed at \(z_0\) distance. (Refer to Figure 18).

The coordinates for foot i calculated above are expressed in body coordinates \((X_i, Y_i, Z_i)^T\). These are the desired steady-state values for the foot. The control law for foot positioning is shown in Figure 19. The desired steady-state values are transformed to earth coordinates, \((X_{1E}, Y_{1E}, Z_{1E})^T\). These values are compared with the present commanded foot position coordinates \((x_{1E}, y_{1E}, z_{1E})^T\) and the
Figure 18. Side View of the Body-Leg System Showing Leg 1 in the Transfer Phase

Figure 19. Control Law for Foot Positioning
control removes the difference. The controls for $x_{1E}$ and $y_{1E}$ are simple first order systems. The control for $z_{1E}$ is a second order system with the damping coefficient $\xi$ and the natural frequency $\omega_n$ set for an underdamped response. This allows $z_{1E}$ to reach the steady-state value $Z_{1E}$ in finite time.

The value for $M_1$ has been calculated as a function of $CT_1$ without any regard for the variation in terrain. That is, it may be anticipated that the foot is in contact with the supporting surface but a "hole" in the terrain would prevent such from occurring. On the other hand, the leg may contact a small "hill" before $M_1 = 1$. To account for terrain variation the last few steps in the subroutine alter $M_1$. If a "hill" is encountered the transfer phase is ended and $M_1$ is set equal to 1. If the foot is expected to be in contact and is not, the steady-state $z$ coordinate for the foot, $Z_{1E}$, is set equal to $Z_E(X_E, Y_E)$, the coordinate of the terrain, while $M_1$ remains at 0 until foot contact.

4.6 Commanded Joint Angles and Rates

The subroutine OUTPUT uses the information contained in the body state vector, $X_C$, and the leg position vector, $XLPV$, and calculates the desired steady-state values for the joint angles and rates. The equations appear in Chapter III, Eqs. (3-13) through (3-17) and (3-25) and (3-26). The position of the knee for leg 1, $(x_k, y_k, z_k)^T$, is also calculated from Eq. (3-19) for display purposes.
4.7 Summary

This chapter has given a description of the algorithms that have been developed for the Kinematic Command Generator (KCG). The KCG is a functional block of the control computer that synthesizes the body and leg trajectories of the six-legged vehicle under operator control. Algorithms for mode control, automatic body height, pitch, roll regulation, and leg positioning have been programmed for locomotion in any direction over an undulating terrain. With the basic structure of the KCG defined, algorithms for stair climbing, obstacle negotiation, etc., can be added with considerably less effort in future work.

In order to successfully develop the algorithms described above, the control has been simulated on a PDP-10 computer with the simulation programs written in Fortran. Output from a CRT display and other results of this work are given in Chapter VII.

In synthesizing the body and leg trajectories, static stability has always been insured by implementing the optimum statically stable gait, the wave gait. With stability having first been considered, other design questions may be addressed. One of the most important is as follows: with a given trajectory, how should the torques needed for support and propulsion of the vehicle be distributed among its joints? This question is considered in the next chapter.
5.1 Introduction

In the design of the control system for the six-legged vehicle, two considerations are of prime importance. First and of most importance is the consideration of system stability. It is desirable that the vehicle maintain stability at all times throughout locomotion.

In Chapter IV, the trajectory of the vehicle (including body and leg subsystems) was generated by the operator with the assistance of the control computer. The operator with a knowledge of the terrain selected a route and guided the vehicle along this path. The control computer was assigned the task of implementing a gait and generating individual leg trajectories. The trajectories of the body and leg subsystems were developed by the computer to maintain a statically stable leg-body configuration. Also, in a general way, the operator was expected to select an appropriate route in which the system could always be stable.

Having selected the body trajectory and a specific gait to optimize stability, the second consideration in the control design is addressed. Namely, given the position of the limbs and the desired vehicle trajectory, how should the torques needed for support and propulsion of the vehicle be distributed among its joints? A number of researchers have investigated this problem and results from
some of these investigations will be given in the next few paragraphs.

In deterministic dynamic linkage systems, where the number of unknowns (joint torques) is equal to the number of constraint equations of motion of the system, the joint torques have been generated by use of the inverse plant [41,42]. That is, with the joint positions, rates, and accelerations all given at any instant in time, the exact joint torques to produce this motion may be obtained by solving the dynamic equations of motion for the joint torques. In the absence of errors, if torques are generated continuously throughout the motion of the system, then the system can exactly follow the desired reference trajectory. Since there are errors in all practical systems (non-ideal joint actuators, modelling errors in the control system, etc.), closed loop feedback is necessary to adjust these bias torques in order to stabilize about the given trajectory.

In Park's computer simulation for a four-legged robot [20], the dynamic equations of motion for the system gave an overspecified problem—fewer unknown torques than constraint equations. Certain phases of the locomotion cycle were statically unstable and the combined action of the joint torques of the support legs could not produce the exact values of body forces and moments to achieve the desired system trajectory. In this work, joint torques were generated by a linear programming technique in order to optimize dynamic stability over the locomotion cycle.

In computer simulation work by Okhotsimsky, Golubev and Alekseeva for a six-legged robot [21], the dynamic equations of motion for the
system gave an underspecified problem—more unknown joint torques than constraint equations. This resulted because the system was statically stable. Because of the underspecified nature of the problem, there is freedom to obtain an optimal solution from the set of infinite number of solutions possible. In this work, the joint torques were chosen to minimize system energy, to balance the system load among the legs, and to minimize components of the foot reactions along the supporting surface (necessary in locomotion over ice).

In the present research, a linear programming (LP) technique for generating joint bias torques has been developed and is described in this chapter. Because the vehicle is assumed to always be statically stable, the problem is underspecified. In this regard, a weighted combination of power consumption and load balancing is optimized. Linear programming is appropriate to this problem for the following reasons:

1) Specification of a vehicle trajectory imposes, at any instant in time, a set of linear equality constraints defining a subspace in the space of individual joint torques.

2) Motor and terrain limitations (maximum joint torques and coefficient of friction of the support system) impose linear inequality constraints in this same space.

3) The electrical power consumed by a joint consisting of a worm gear driven by a universal series wound motor is linearly related to the output torque for any specific value of joint rate (See Chapter VII).
The purpose of this chapter, then, is to consider the optimization of energy of the six-legged vehicle over the locomotion cycle. This is accomplished by realizing that the energy is just the integral of power, and energy may be minimized over an enforced trajectory by minimizing the power consumption at any instant in time. If trajectory planning for energy optimization were included, dynamic programming techniques employing the calculus of variations could be used in the optimization procedure. However, in this research, the trajectory is generated considering stability only and energy is then minimized over this prescribed trajectory by minimizing the power at each point in time.

The remainder of this chapter details the formulation of the LP problem. First of all, the static weights of the legs are considered, and the next section of this chapter includes static force calculations giving the effects of the nonzero leg weights on the body. The section following sets up the LP problem specifically with descriptions of the constraints and criterion function. Finally, some theoretical results are given relating feasible solutions to the LP problem with the static stability of the vehicle.

In order to perform the control optimization relative to real hardware characteristics, joint actuator systems are introduced in Chapter VI and the performance evaluation of the entire system is given in Chapter VII.
5.2 Static Force Calculations

The next subroutine in the Kinematic Command Generator (see Figure 7) is the Static Force subroutine (STCFOR). The flow chart for the STCFOR subroutine is shown in Figure 20. The subroutine uses the joint angles furnished by the Commanded Joint Angle Vector (CJAV), the body parameters given in the Body Physical Parameters Vector (BPPV), the body state variables given in the body state vector (XC), and the leg support information from the Leg Position Vector (XLPV). The subroutine calculates the joint torques for those legs in the transfer phase and outputs these through the Commanded Torque Vector (CTV). CTV is defined to be

\[
CTV = (\tau_{\psi_1}, \tau_{\theta_1}, \tau_k, \tau_{\psi_2}, \ldots, \tau_{\psi_6})^T.
\]

This subroutine generates the 6 components of the desired force and moment acting on the body that are to be produced by the joint torques of the legs. These forces and moments bear the static weight of the body and legs and provide any desired translational and rotational acceleration to the vehicle body. The subroutine also generates the components of the foot reactions due to the static weight of the leg. This information is used in the inequality constraints of the LP problem. The 3 components of desired body force, the 3 components of desired body moment, and the 18 components of the foot reactions are contained in the Force Vector:

\[
\mathbf{FV} = (F(3), T(3), FR(3,6))
\]
**Input Data:** \( \tilde{\text{CJAV}}, \tilde{\text{XLPV}}, \tilde{\text{XC}} \\
**Output Data:** \( \tilde{\text{CTV}}, \tilde{\text{FV}} \)

Figure 20. Elementary Flow Chart for the STCFOR Subroutine
where

\[ F(3) = \text{the three components of the desired body force (to be produced by the joint torques of the supporting legs) as expressed in body coordinates.} \]

\[ T(3) = \text{the three components of the desired body torque (to be produced by the joint torques of the supporting legs) as expressed in body coordinates.} \]

\[ FR(3,6) = \text{the three components of the foot reaction force for each of the six legs as expressed in body coordinates.} \]

The above may be computed from Eqs. (3-27), (3-28), (3-40), and (3-43).

\[ FR_i = D_i - (m_1 g)_i - (m_2 g)_i \]  

(5-3)

\[ F = \mathbf{\bar{f}} - \sum_{i=1}^{6} \left[ D_i + (1-M_i)(-J_i^{-1}) T_i \right] \]  

(5-4)

\[ T = \mathbf{\bar{m}} - \sum_{i=1}^{6} \left[ G_i + (1-M_i)E_i T_i \right] \]  

(5-5)

where, as defined before,

\[ M_i = \text{indicator of foot position} \]

\[ M_i = 1 \text{ (on ground)} \]

\[ M_i = 0 \text{ (off ground)}. \]

In Eqs. (5-4) and (5-5), the first term in each of the summations is just the static force or torque bearing on the body due to the weight of each leg. The second term is the force or torque on the body due to the joint torques of the legs in the transfer phase. Subroutine STCFOR calculates the static terms \( D_i \) and \( G_i \) and the terms \( \mathbf{\bar{f}} \) and \( \mathbf{\bar{m}} \) in these equations. These equations and the equations from which
they are derived involve the static weights of the leg segments expressed in the body coordinate system, \((m^g_1)\) and \((m^g_2)\), the static weight of the upper limb segment expressed in upper limb segment coordinates, \((m'^g_1)\)', and the static weight of the lower limb segment expressed in lower limb segment coordinates, \((m''^g_1)\). These may be computed from the following equations:

\[
(m^g_1) = T_1 \begin{bmatrix} 0 \\ 0 \\ (m^g_1) \end{bmatrix} \\
(m'^g_1) = T_1 \begin{bmatrix} 0 \\ 0 \\ (m^g_1) \end{bmatrix} \\
(m''^g_1) = T_3 T_1 \begin{bmatrix} 0 \\ 0 \\ (m^g_1) \end{bmatrix} \\
(m''^g_1) = T_4 T_3 T_1 \begin{bmatrix} 0 \\ 0 \\ (m^g_1) \end{bmatrix}
\]

where

\[
T_4 = \begin{bmatrix} \cos \theta_{21} & 0 & -\sin \theta_{21} \\ 0 & 1 & 0 \\ \sin \theta_{21} & 0 & \cos \theta_{21} \end{bmatrix}
\]

= transformation matrix from the upper limb segment coordinate system to the lower limb segment coordinate system.

The STCFOR subroutine calculates the joint torques for the legs in the transfer phase. These values are used in Eq. (5-5). These
torques bear the static weight of the leg and may be obtained by setting the ground reaction at the foot, $f_{R1}$, equal to zero in Eq. (3-40). The torques computed are a function of the leg position and static weights:

$$\tau_{ki} = -\frac{(m_2 g)^{''} x_i}{2} \xi_i \xi_2 / 2$$

$$\tau_{\theta_i} = \xi_1 \cos \theta_{2i} \left[ \frac{(m_1 g)_{zi}}{2 \cos \theta_{2i}} + (m_2 g)^{''} x_i \right] - \frac{(m_2 g)^{''} x_i}{2} \xi_2 / 2$$

$$\tau_{\psi_i} = -\frac{(m_1 g)^{'} y_i \xi_1 \cos \theta_{1i}}{2} - \frac{(m_2 g)^{''} y_i}{2} \xi_2 \cos \theta_{1i}$$

- \frac{(m_2 g)^{''} y_i \xi_2 \sin (\theta_{1i} + \theta_{2i})}{2}

5.3 Mathematical Programming Formulation

The next subroutine in the KCG is the FORCE subroutine. Its purpose is to set up the linear programming (LP) problem and call subroutine LINear PROgramming (LNNPRO) to solve the problem and give the resulting joint torques for the legs in the support phase. An elementary flow chart for the FORCE subroutine is given in Figure 21.

The subroutine uses the joint angles and rates information contained in the Commanded Joint Angle Vector (CJAV), the desired body forces and moments to be produced by the joint torques of the legs and the foot reaction forces due to static leg weight both contained in the Force Vector (FV) the body parameters contained in the Body Physical Parameters Vector (BPPV), the static coefficient of friction of the terrain contained in the Terrain Parameters Vector.
Input Data: CJAV, FV
Output Data: CTV, MPS

Start

Compute
\[ [-J_4^{-1}]^T, E_4 \]

Set
\[ F = F - \Sigma (I - M_1)([-J_4^{-1}]T_i) \]
\[ T = T - \Sigma (I - M_1)(E_4 T_4) \]

Set up 6 equality constraints (6n variables & costs)

Set up 3n maximum torque inequality constraints

Set up n load balancing inequality constraints (add 1 variable & cost)

Set up 4n terrain friction inequality constraints

Add 6 artificial variables

Call LINPRO

Compute CTV

Return

Figure 21. Elementary Flow Chart for the FORCE Subroutine
(TPV), leg support information from the Leg Position Vector (XLPV), and motor-gear characteristics obtained in the Motor Parameters Vector (XMPV). The Mathematical Programming System vector (MPS) contains the linear programming tableau and is used by LIMPRO to solve the LP problem. The resultant joint torques for the legs in the support phase are output through the Commanded Torque Vector (CTV). The MPS vector is defined as follows:

\[
\text{MPS} = (\text{MDIM}, \text{NDIM}, c, A, \text{CB}, \text{NBS})
\]

where

\[
\text{MDIM} = \text{the number of constraint equations in the LP problem equal to } (6 + 8n) \text{ where } n \text{ is the number of legs in the support phase.}
\]

\[
\text{NDIM} = \text{the number of variables in the LP problem equal to } (6n + 1) \text{ (excludes all slack, surplus, or artificial variables).}
\]

\[
c(37) = \text{the vector of costs for the variables of the LP problem; it has a maximum dimension of 37 for the case where 6 legs are in the support phase.}
\]

\[
A(55,92) = \text{the matrix containing the simplex tableau (See Figure 22); it contains columns for the } 6n+1 \text{ variables for the LP problem plus columns for the (8n) slack variables and 6 artificial variables.}
\]

\[
\text{CB}(54) = \text{the vector of costs for the variables presently in the basis for a feasible solution to the LP problem.}
\]
NBS(54) = the vector of variables presently in the basis;  
each element of this vector denotes a column number  
in the A matrix corresponding to one of the variables  
of the LP problem.

The Motor Parameters Vector (XMPV) defines the characteristics of the  
motor-gear system:

$$XMPV = (R, GAFV, GAFT, XN, ETAF, ETAR, \tau_{\psi M}, \tau_G M, \tau_K M) \quad (5-13)$$

where  
$$R = \text{combined resistance of the field and armature windings}$$  
of the universal series wound motor.  

$$GAFV = G_{afv}, \text{ the back emf motor constant.}$$  

$$GAFT = G_{afT}, \text{ the motor torque constant.}$$  

$$XN = \text{the speed reduction ratio of the worm gear system.}$$  

$$ETAF = \text{the forward efficiency of the worm gear system.}$$  

$$ETAR = \text{the reverse efficiency of the worm gear system.}$$  

$$\tau_{\psi M}, \tau_G M, \tau_K M = \text{maximum absolute values for the joint torques.}$$  

These parameters will be discussed in greater detail in Chapter VI.

5.3.1 Equality Constraints

There are 6 equality constraints in the LP problem. They are  
taken from Eqs. (3-40), (3-43), (5-4), and (5-5):

$$\mathbf{F}_{3x1} = \sum_{i=1}^{6} M_i [-J_i^{-1}]^T \mathbf{1}_i \quad (5-14)$$

$$\mathbf{T}_{3x1} = \sum_{i=1}^{6} M_i E_i \mathbf{1}_i$$
For any leg-body configuration prescribed by a given trajectory, the matrices \([J_1^{-1}]^T\) and \(E_i\) are constant and Eqs. (5-14) define 6 linear equations. These 6 equations form the first 6 rows of the simplex tableau as shown in Figure 22. This tableau is for the case where 3 legs, legs 1, 4, and 5, are in the support phase.

In the simplex method for the solution of the LP problem, the variables must all be greater than or equal to zero. Since the joint torques appearing in Eqs. (5-14) are free variables and may take on both positive and negative values, these variables have been split into positive and negative parts

\[
\tau_i = (\tau_i^+) - (\tau_i^-) . \tag{5-15}
\]

Eqs. (5-14) may thus be rewritten:

\[
\mathbf{F} = \sum_{i=1}^{6} \mathbf{M}_i \{ [-J_1^{-1}]^T \mathbf{\tau}_i^+ + [J_1^{-1}]^T \mathbf{\tau}_i^- \} \tag{5-16}
\]

\[
\mathbf{T} = \sum_{i=1}^{6} \mathbf{M}_i [E_i \mathbf{\tau}_i^+ - E_i \mathbf{\tau}_i^-]
\]

Again noting Figure 22, the first column of the first 6 rows of the \(A\) matrix gives the left hand side of Eqs. (5-16). The next 18 columns give the elements of the \([-J_1^{-1}]^T\) or \(E_i\) matrices. Finally, columns 45 through 50 add the necessary artificial variables for the equality constraints. These artificial variables are placed in the initial basic feasible solution at a large negative cost.
Figure 22. The Initial Simplex Tableau (A) for the Linear Programming Problem (CB and NBS are also shown)
5.3.2 Maximum Torque Constraints

The next 3n constraints in the LP problem have to do with the maximum allowable torque at each joint. The absolute value of each joint torque is constrained to be less than some maximum value \((\tau_{\psi M}, \tau_{\theta M}, \text{ or } \tau_{KH})\). This results from the heating characteristics of the universal series wound motor. Three inequality constraints are necessary for each of the legs in the support phase,

\[
\begin{align*}
\tau_{\psi i}^+ + \tau_{\psi i}^- & \leq \tau_{\psi M} \\
\tau_{\theta i}^+ + \tau_{\theta i}^- & \leq \tau_{\theta M} \\
\tau_{Ki}^+ + \tau_{Ki}^- & \leq \tau_{KM}.
\end{align*}
\]

These equations provide a constraint on the absolute value of the joint torque if one or the other of the positive and negative parts is always equal to zero.

Slack variables \((S_{\psi i}, S_{\theta i}, S_{Ki})\) must be added to these equations to provide an initial basic feasible solution. For the case of three legs on the ground, rows 7-15 of the A matrix shown in Figure 22, are associated with the maximum torque constraints given in Eqs. (5-17). The slack variables are added in columns 21 through 29 and are placed in the initial basis at zero cost.

5.3.3 Load Balancing

In locomotion over soft soil, it is desirable to balance the load over the legs as much as possible so that none of the legs penetrate too far into the surface. One way of accomplishing this is to minimize the maximum component of the foot reaction forces. Minimax
criteria are nonlinear but may be manipulated into linear program inequality constraints by adding a variable to the LP problem, \( f_{RZM} \), equal to the maximum z component of all the foot reaction forces. This produces one inequality constraint equation for each leg or a total of \( n \) equations for load balancing. The constraint equations may be written for the three leg case:

\[
\begin{align*}
    f_{Rz1} & \leq f_{RZM} \\
    f_{Rz4} & \leq f_{RZM} \\
    f_{Rz5} & \leq f_{RZM}
\end{align*}
\]  

These three equations are associated with rows 16-18 in the simplex tableau. The variable \( f_{RZM} \) is added in column 20. Slack variables are added in columns 30-32 and are placed in the initial basic feasible solution at zero cost.

5.3.4 Terrain Friction Constraints

In locomotion over ice or other types of slippery terrain, it is necessary to constrain the reaction forces at the foot-terrain interface to be as closely normal to the terrain surface as possible to reduce slippage of the feet. In order to do this, each foot reaction may be constrained within a friction cone as shown in Figure 23. As long as the resultant force at the foot lies within the friction cone, slippage will not occur. That is,

\[
\frac{-\frac{f_{R_{1i}}}{z}}{\left|\frac{f_{R_{1i}}}{z}\right|} > \cos \alpha = (1 + \mu^2)^{-1/2}
\]  

\[(5-19)\]
where

\[ \alpha = \text{vertex angle of the friction cone} \]
\[ \mu = \text{coefficient of friction of the terrain} \]
\[ \hat{z} = \text{unit vector downward along the body z axis} \]

(assumed to be normal to the terrain)

Figure 23. Friction Cone Constraint for the Foot Reaction Force

Eq. (5-19) assumes that the body is parallel to the terrain, and this constraint is with respect to the body z-axis. If a ball and socket joint at the ankle was used along with an angular measurement system, the unit vector normal to the terrain surface could be deduced and used in this constraint.

Eq. (5-19) is nonlinear in the components of the vector, 
\[ \mathbf{f}_{Ri} = [f_{Rx}, f_{Ry}, f_{Rz}]^T. \] To provide linear constraints for the LP problem, a rectangular pyramid may be inscribed in the friction
cone. The resultant foot reaction must lie within this pyramid.

This fact is described mathematically by the equations:

\[ f_{Rx1} + \mu \frac{h}{l} f_{Rzi} \leq 0 \]  \hspace{1cm} (5-20)

\[ f_{Rx1} + \frac{h}{l} f_{Rzi} \leq 0 \]

\[ f_{Ry1} + \mu \frac{h}{l} f_{Rzi} \leq 0 \]

\[ f_{Ry1} + \frac{h}{l} f_{Rzi} \leq 0 \]

where

\[ \mu = \frac{h}{\sqrt{2}}. \]

Eqs. (5-20) amount to a stricter constraint than Eq. (5-19) since the pyramid is inscribed in the friction cone.

In the simplex tableau for three supporting legs, 4 rows are associated with each of the legs to give a total of \( 4n = 12 \) friction constraint equations for the 3 legs. Using Eqs. (3-40), (5-3), and (5-20), the exact constraint equations may be found and are given in the simplex tableau in rows 19 through 30. The left side of the equations are due to the static weight of the leg and the right sides are produced by the joint torques of the supporting legs. Slack variables are added to these equations to provide an initial basic feasible solution. These are added in columns 33 through 44 and are placed in the initial basis at zero cost.

5.3.5 Criterion Function

As was explained earlier in the chapter, the solution to the LP problem gives the bias joint torques for the legs in the support
phase. A number of feasible solutions exist. It is desirable to find the solution that both minimizes the power consumed and the maximum foot reaction force among the legs (load balancing) while staying within the friction pyramid constraints and maximum torque constraints. The criterion function alluded to is a weighted function of the joint torques ($t_i$) for power minimization and the maximum foot reaction force ($f_{RZM}$) for load balancing. The costs, $C$, given to these variables will be explained in detail in Chapter VII. The objective function, then, is defined as

$$ \phi = \begin{bmatrix} T_1 & T_4 & T_5 & f_{RZM} \end{bmatrix} C \quad (5-21) $$

for the case where there are 3 legs on the ground.

5.3.6 Method of Solution

The linear programming problem formulated in this chapter has the general form:

$$ \text{Max } \phi = x^T C \quad (5-22) $$

$$ Ax = b $$

$$ x \geq 0 $$

This problem has been set up in the Mathematical Programming System (MPS) vector. The problem has been solved by the "simplex method" in the subroutine LINPRO and the resulting joint torques are output through the Commanded Torque Vector (CTV). The results of this problem are given in detail in Chapter VII for various weights between power consumption and load balancing.
5.4 Theoretical Results

In the computer simulation of the six-legged robot as described in Chapter IV, the trajectories of the body and leg subsystems were developed by the computer to maintain a statically stable leg-body configuration at all times. Within the bounds of this enforced trajectory, this chapter describes a linear programming formulation of the problem of torque distribution among the joints. A weighted combination of power consumption and load balancing is optimized. In this section some theoretical results are given relating the stability of the instantaneous vehicle leg-body configuration to the feasibility of solutions to the basic linear programming problem.

Static stability has been previously defined in Chapter II for locomotion over even terrain in which the points of support of the system lie in a plane. This gives rise to the following definition.

**Definition 1.** The support plane for a locomotion system traveling over even terrain (level or sloping) is defined by the points of support of the feet.

The problem of joint torque distribution has been solved through linear programming. The most basic constraints are contained in the following definition:

**Definition 2.** The Positive Force Linear Programming Problem is the basic optimization problem consisting of any linear criterion function defined on the joint torques together with the following linear constraints:
1) six equality constraints due to desired vehicle acceleration and the relative positioning of the body and legs and the orientation of the vehicle system with respect to the gravitational vector and

2) inequality constraints imposed upon the reaction forces of the support surface onto the foot to be out of the surface.

The positive force linear programming problem includes the most basic constraints for a vehicle without grasping feet in which torque limiting due to motor heating effects is ignored and the terrain coefficient of friction is assumed to be infinite.

The following theorem may be proven:

Theorem 1. A legged locomotion system with the points of support defining a support plane is statically stable in a given configuration if and only if the positive force linear programming problem has a feasible solution.

Proof. The variables of the LP problem are the joint torques. The joint torques are related to the foot reaction forces by Eq. (3-40). Since a feasible solution for the joint torques in the LP problem implies that there be a set of reaction forces that satisfy Eq. (3-40) under the constraint that the reactions be out of the surface, an analysis of the foot reaction forces is sufficient. A solution of foot reaction forces uniquely determines a set of joint torques.

Let the feet coordinate system \((x_f, y_f, z_f)\) be defined such that the \(x_f - y_f\) plane is parallel to the support plane and the origin is located at the center of gravity of the total vehicle system (body and legs). The \(z_f\) axis is directed into the support plane; \(x_f\) and \(y_f\) are chosen to
maintain a right hand coordinate system. For three legs in contact with the ground, Figure 24 is appropriate.

Let the reaction of the supporting surface onto the foot expressed in feet coordinates be $(R_{x1}, R_{y1}, R_{z1})^T$ for leg $i$. Also, let the components of the weight vector be $(W_x, W_y, W_z)^T$ and the position of the point of support of leg $i$ be $(X_i, Y_i, Z_i)^T$, both as expressed in feet coordinates. Then the force and moment equations about the center of gravity for equilibrium for a 3 legged support phase are:
Eqs. (5-23) through (5-28) may be decoupled in the following way. First, from the definition of the feet coordinate system, 

\[ Z_{2f} = Z_{3f} = Z_{6f} = Z_{1f} \]. 

This position value may be factored out of Eqs. (5-23) and (5-24) giving the results that

\[ R_{z2}Y_{2f} + R_{z3}Y_{3f} + R_{z6}Y_{6f} - (R_{y2} + R_{y3} + R_{y6})Z_{1f} = 0 \] (5-29)

\[ -R_{z2}X_{2f} - R_{z3}X_{3f} - R_{z6}X_{6f} + (R_{x2} + R_{x3} + R_{x6})Z_{1f} = 0 \] (5-30)

Eqs. (5-26) and (5-27) may be substituted into Eqs. (5-29) and (5-30) to give

\[ R_{z2}Y_{2f} + R_{z3}Y_{3f} + R_{z6}Y_{6f} = -W_y Z_{1f} \] (5-31)

\[ -R_{z2}X_{2f} - R_{z3}X_{3f} - R_{z6}X_{6f} = W_x Z_{1f} \] (5-32)

Eqs. (5-25) through (5-27) are three independent equations with six unknowns \( R_{x2}, R_{x3}, R_{x6}, R_{y2}, R_{y3}, \) and \( R_{y6} \). Thus a number of solutions are possible since these variables are not constrained.
Eqs. (5-28), (5-31), and (5-32) provide three equations with three unknowns $R_{z2}$, $R_{z3}$, and $R_{z6}$. These may be satisfied only under certain conditions since

$$R_{z2}, R_{z3}, R_{z6} \leq 0$$ (5-33)

(normal reactions to the supporting surface are constrained to be out of the surface). To determine under what conditions Eqs. (5-31) and (5-32), the moment equations, are satisfied, consider each normal reaction component taken individually. From Figure 25, taking moments about line $(X_f, Y_f)_{2-6}$, the projection of the origin of the coordinate system onto the support plane must lie in the half plane to the left of line $(X_f, Y_f)_{2-6}$ since

$$\sum_{2-6} = 0 \quad R_{z3} = -W_z \frac{d_1}{l_1} \quad (5-34)$$

and $W_z \geq 0$.

Pictorially, this means that the projection of the center of gravity of the vehicle system onto the support plane must lie somewhere in the shaded area in Figure 25. Applying the same arguments to moments about line $(X_f, Y_f)_{3-6}$ and $(X_f, Y_f)_{2-3}$, it is found that the projection of the center of gravity must lie within the support zone of the three feet, which is the condition for static stability. Thus, it is proven that Eqs. (5-25) through (5-28) and Eqs. (5-31) through (5-33), which are the constraints of the positive force linear programming problem, have a solution if and only if the system is in a
statically stable configuration. The proof may be generalized to include more than three legs.

The above analysis relating the static stability of the system to feasible solutions to the positive force linear programming problem may be extended to include the case of nonzero system acceleration. The following definitions are useful:

Definition 3. The zero moment point (center of pressure) associated with the ground reactions of a legged locomotion system is the point in the support plane about which the moment due to the ground reaction forces is equal to zero [24].

The effective action of the foot reactions is found by summing these forces and applying this net force at the zero moment point.
Definition 4. For locomotion on a plane, a legged locomotion system is **intrinsically stable** if its zero moment point is contained within the support polygon defined by feet in contact with the ground.

With the above definitions, the following theorem may be proven.

**Theorem 2.**

A legged locomotion system is intrinsically stable if and only if there exists a feasible solution to the positive force linear programming problem.

**Proof.**

The proof of Theorem 2 is much like that of Theorem 1. Let the desired linear acceleration on the center of gravity of the vehicle as expressed in feet coordinates be \((\ddot{x}, \ddot{y}, \ddot{z})_f\). These components may be multiplied by the total mass, \(m\), of the system and added to the weight terms in Eqs. (5-26) through (5-28). Considering system acceleration, Eqs. (5-26) through (5-28) may be rewritten for this case to give

\[
\Sigma F_{xf} = R_{x2} + R_{x3} + R_{x6} + W_x = m \ddot{x}_f
\]

(5-35)

\[
\Sigma F_{yf} = R_{y2} + R_{y3} + R_{y6} + W_y = m \ddot{y}_f
\]

(5-36)

\[
\Sigma F_{zf} = R_{z2} + R_{z3} + R_{z6} + W_z = m \ddot{z}_f
\]

(5-37)

The analysis in this proof follows that of Theorem 1 at this point with \((W_x - m \ddot{x}_f, W_y - m \ddot{y}_f, W_z - m \ddot{z}_f)^T\), the force components due to gravitational and vehicle acceleration, substituted for \((W_x, W_y, W_z)^T\) in all cases. From previous analysis, the projection of the force
vector (due to acceleration) from the center of gravity onto the support plane must lie within the support polygon of the feet in order to find a solution to the positive force linear programming problem. This projection by definition is the zero moment point, the effective point of application of the net force due to the foot reactions. It follows that the vehicle must be intrinsically stable in order for a solution to exist to the positive force linear programming problem. The reverse is also true, and the theorem is proven.

5.5 Summary

This chapter describes the linear programming technique that was used to generate the bias torques for each of the joints of the legged vehicle. A weighted combination of power consumption and load balancing is optimized. The constraints of the problem are due to the imposed vehicle trajectory and motor and terrain limitations (maximum torque and coefficient of friction of the surface).

The optimization described in this chapter is with respect to real hardware characteristics. In order to proceed with this, the characteristics of joint actuator systems are introduced in Chapter VI. Finally, the performance evaluation of the system is given in Chapter VII. Chapter VII also includes a section on typical results along with validation of the software for bias torque calculations.
6.1 Introduction

In the design of control systems for legged vehicles, a fundamental consideration is that of the design of the legs with their associated joint actuators, sensory hardware, and physical characteristics. In at least two respects, the legs of a vehicle are its most critical component. First of all, this is trivially true, since the legs are the components which must produce both the lift and thrust needed for vehicle support and propulsion. It is therefore their speed, range of motion, and efficiency which determine more than anything else the utility and adaptability of the vehicle. Secondly, an automatically controllable leg of the type needed for the present vehicle was not available prior to this research. A development program has been required to obtain a leg with adequate performance.

The work of developing a suitable vehicle leg has been divided into three parts: the mechanical design, the design of the electronics, and the control design. The particular task that is investigated in this chapter is the control design, which has consisted in most part of modelling each of the elements of the leg subsystem in order to study its characteristics within a control
system. The legs of the vehicle have been constructed and a photograph showing the general arrangement of components is given in Figure 26. The mechanical and electronics design along with an initial experimental evaluation of this leg may be found in [43].

![Photograph of a Leg for the Hexapod Vehicle](image)

Figure 26. Photograph of a Leg for the Hexapod Vehicle

When the legs of a vehicle are acting together to move it along a desired trajectory, certain coordination problems exist. In particular, it is desirable that legs arranged symmetrically about the body share the load of support and propulsion without opposing each other and thus expending unnecessary energy. That is, it is possible that one leg will drag another across the supporting surface ("binding" effect), or that legs on each side of the body will push at the body
unnecessarily although the body remains in equilibrium. These phenomena need to be studied and appropriate controls need to be implemented to eliminate such potential problems.

To investigate this "opposing actuator" problem of pairs of legs, and also to gain an increased understanding of joint control systems for the individual legs, a one-axis, two-actuator mechanical simulation has been partially constructed and tested. A block diagram of this system is given in Figure 27. Such a mechanical simulation has enough complexity to reveal the essentials of the problems of leg development and coordination control, but is relatively simple (sixth order system) so that the results may be effectively analyzed. In this chapter, a series of investigations dealing with the design and analysis of joint control systems in the context of the mechanical simulation are outlined and results are given.

6.2 Mechanical Simulation--System Configuration

Again referring to Figure 27, each actuator is implemented with a worm gear driven by a universal series wound motor. The motor rate is sensed by a tachometer, and the output angle is measured by a potentiometer. These outputs are fed back to the input nodes to give two closed loop servo units. Each motor-gear system is coupled through a rubber hose to a flywheel inertia load mounted along the axis. The commanded input to each of the servo units is furnished by a computer with the value proportional to the desired joint rate and angle. The servo units, rubber coupling, and flywheel inertia simulate the legs,
Figure 27. Mechanical Simulation Block Diagram
a hip suspension system, and the body of a legged vehicle, respectively.

6.3 System Model

The model for the system is diagrammed in Figure 28. The two-servo unit system provides a symmetric arrangement about the flywheel load. The flywheel is assumed to be a pure inertia load, \( J \). That is,

\[
T = J\ddot{\theta} = J\dot{\omega} \quad .
\]

where

\( T \) = torque applied to the flywheel
\( \omega \) = angular rate of flywheel
\( \theta \) = angular position of the flywheel.

Each rubber coupling is assumed to have linear characteristics. The appropriate equations for the left and right couplings are

\[
T_{\theta L} = B_{3L} (\omega_L - \omega) + K_{3L} (\theta_L - \theta) \quad (6-2)
\]

\[
T_{\theta R} = B_{3R} (\omega_R - \omega) + K_{3R} (\theta_R - \theta)
\]

where

\( T_{\theta L}, T_{\theta R} \) = torque across the coupling
\( B_{3L}, B_{3R} \) = damping coefficient of the coupling
\( K_{3L}, K_{3R} \) = stiffness coefficient of the coupling
\( \theta_L, \theta_R \) = angular position at the output of the servo unit
\( \omega_L, \omega_R \) = angular rate at the output of the servo unit.
Figure 28. Model of the Mechanical Simulation
6.3.1 Universal Series Wound Motor

The basic characteristics of a universal series wound motor are modelled in Figure 28. The back emf equation is

\[ e_L = G_{afv} \omega_m L_i_L \]

\[ e_R = G_{afv} \omega_m R_i_R \]

where

- \( e_L, e_R \) = back emf
- \( G_{afv} \) = back emf constant (sign dependent upon the relative wiring polarity between the armature and field windings)
- \( i_L, i_R \) = current through the windings
- \( \omega_m L, \omega_m R \) = motor output rate.

The input voltage is related to the back emf as follows:

\[ v_L = R_{1L} i_L + L_L \frac{di_L}{dt} + e_L \]

\[ v_R = R_{1R} i_R + L_R \frac{di_R}{dt} + e_R \]

where

- \( v_L, v_R \) = motor input voltage
- \( R_{1L}, R_{1R} \) = sum of the armature and field winding resistance
- \( L_L, L_R \) = sum of the armature and field winding inductance.

Since the armature and field windings are wired in series, the output torque of the motor is proportional to the current squared. That is,
\[ T_{mL} = C_{afT} i_L^2 - T_s \]  
\[ T_{mR} = C_{afT} i_R^2 - T_s \]

where

- \( T_{mL}, T_{mR} \) = output torque of the motor
- \( C_{afT} \) = torque constant (sign dependent upon the relative wiring polarity between the armature and field windings)
- \( T_s \) = static torque loss due to friction where \( \text{sgn}(T_s) = \text{sgn}(T_m) \).

The equation relating the output torque of the motor to the torque into the gear train is

\[ T_{mL} = J_{mL} \ddot{\omega}_{mL} + B_{mL} \omega_{mL} + T_{\text{in}(L)} \]  
\[ T_{mR} = J_{mR} \ddot{\omega}_{mR} + B_{mR} \omega_{mR} + T_{\text{in}(R)} \]

where

- \( J_{mL}, J_{mR} \) = internal motor inertia
- \( B_{mL}, B_{mR} \) = internal motor damping
- \( T_{\text{in}(L)}, T_{\text{in}(R)} \) = driving torque into the gear train.

Equations (6-3) to (6-6) are approximate equations, with the saturation effects of the motor ignored. To obtain values for the motor parameters from these equations and to correctly account for saturation effects of the motor windings, a series of tests was made and the results obtained will be given in this section.
A Black and Decker drill motor, type #1174, was chosen because of its exceptional power capabilities for its size. The quoted factory specifications for the motor are given in Table IV. Four basic tests were made on the motor to determine its operating characteristics. They were

1) Impedance test to measure the armature and field winding resistance and inductance,

2) No load test to find the relationship between the back emf and motor current and speed,

3) Near-stalled motor test to find the torque-current relationship, and

4) Step voltage response to determine the mechanical time constant.

Table IV: Factory Specifications for a Black and Decker Model 1174 Drill Motor

Input: 3a, 110v = 330 watts
Output: 1.7 ft-lb, 650 rpm = 150 watts
Stalled torque (110 v AC input): 13.6 ft-lb

The results of these tests are given in Table V. One may note that saturation effects are apparent. As the current-speed product increases, the back emf is related to that product to a power less than one. Also, as the current increases, the torque is proportional to the current raised to a value less than two.
Table V: Tests of Motor Characteristics

Test I: Armature and Field Winding Impedance

Results: $L_f = 20 \, \text{mh}$ \hspace{1cm} $R_f = 3.82 \, \Omega$

$La = 8.6 \, \text{mh}$ \hspace{1cm} $Ra = 3.71 \, \Omega$

Test II: No Load Test

Results:

\[ \epsilon_L = \left[ \frac{0.0821 \, \text{V}}{\text{amp-rpm}} \frac{I_L}{\Omega_{\text{mL}}} \right] \] \hspace{1cm} $\epsilon_L < 52\, \text{v}$

\[ \epsilon_L = \left[ \frac{0.0821 \, \text{V}}{\text{amp-rpm}} \frac{I_L}{\Omega_{\text{mL}}} \right]^{0.935} + 11.727\, \text{v} \] \hspace{1cm} $52 < \epsilon_L < 65\, \text{v}$

\[ \epsilon_L = \left[ \frac{0.0821 \, \text{V}}{\text{amp-rpm}} \frac{I_L}{\Omega_{\text{mL}}} \right]^{0.87} + 24.50\, \text{v} \] \hspace{1cm} $65 < \epsilon_L$

($I_L$, $\Omega_{\text{mL}}$ are steady-state values for the motor current and speed).

Test III: Near-Stalled Motor Test

Results:

\[ T_{\text{mL}} = 0.244 \, \frac{\text{ft-lb}}{\text{amp}^2} \frac{I_L^2}{L} - 0.141 \, \text{ft-lb} \] \hspace{1cm} $I_L < 1.30 \, \text{amps}$

\[ T_{\text{mL}} = 0.244 \, \frac{\text{ft-lb}}{\text{amp}^{1.8}} \frac{1.8}{I_L} - 0.113 \, \text{ft-lb} \] \hspace{1cm} $1.30 < I_L < 2.00 \, \text{amps}$

\[ T_{\text{mL}} = 0.244 \, \frac{\text{ft-lb}}{\text{amp}^{1.6}} \frac{1.6}{I_L} \] \hspace{1cm} $2.00 < I_L < 4.25 \, \text{amps}$

Results of Tests II and III:

\[ B_{\text{mL}} = 0.425 \times 10^{-3} \, \frac{\text{ft-lb}}{\text{rpm}} \]

Test IV: Mechanical Time Constant

Results:

\[ \tau_L = 0.25 \, \text{sec} \]

\[ J_{\text{mL}} = B_{\text{mL}} \tau_L \]

\[ J_{\text{mL}} = 1.375 \times 10^{-3} \, \text{kg-m}^2 \]
The equations in Table V relate the motor torque and speed to the current and input voltage. From these equations, torque-speed characteristics for each value of input voltage may be drawn. A computer program was written to solve the equations obtained from the No Load and Near-Stalled Motor Tests. The output torque-speed characteristics were plotted and are shown in Figure 29.

6.3.2 Worm Gear Modelling

A worm gear speed reduction unit was chosen to increase the maximum torque capabilities of the actuator unit. The worm gear is especially appropriate for legged vehicle systems because of its self locking characteristic. That is, with no power into the gear unit, the unit will sustain large external reaction moments at the output while remaining at zero joint rate. The unit is irreversible under certain conditions in that the reaction moment at the output cannot drive the input. When the power is turned off, a vehicle with worm gear units will not "droop" to the supporting surface.

Efficiency

An analysis of the dynamics of worm gears is a rather difficult one which is best understood by studying the reaction forces at the worm and gear interface. Figure 30 shows these forces. The force $P$ is the force tangential to the worm which causes rotation. The force $L$ is the reaction force of the worm thrust bearing. The quantity $R$ is the reaction force of the gear onto the worm. Thus, assuming acceleration forces are small,
Figure 29. Torque-Speed Characteristics of Universal Series-Wound Motor Derived from Experimental No Load and Locked Rotor Tests (DC voltage source).
Figure 30. Worm-Gear Force Reactions—Forward Drive

Figure 31. Worm-Gear Force Reactions—Reverse Drive
\[ \dot{F}_R = \dot{F} + \ddot{F} \quad (6-7) \]

The force \( F_R \) acts at an angle of \( \alpha \), where \( f = \tan(\alpha) \) is the friction coefficient, so from elementary geometrical considerations,

\[ \frac{P}{L} = \tan(\alpha + \alpha) \quad (6-8) \]

where \( \alpha \) = lead angle of the worm. The forward efficiency of the worm gear is defined as

\[ \eta_f = \frac{L_x}{P_y} = \frac{L}{P} \tan(\alpha) \quad (6-9) \]

where \( \eta_f \) = forward worm gear efficiency

\( L_x \) = power out of the gear

\( P_y \) = power into the worm.

Combining Eqs. (6-8) and (6-9), the forward efficiency as a function of the lead angle and friction angle is thus

\[ \eta_f = \frac{\tan(\alpha)}{\tan(\alpha + \alpha)} \quad (6-10) \]

When power is supplied to the input from the output, the worm gear operates with a characteristic reverse efficiency. From Figure 31, for reverse conditions

\[ \frac{P}{L} = -\tan(\alpha - \lambda) \quad (6-11) \]

The reverse efficiency may thus be deduced as

\[ \eta_R = \frac{P_y}{L_x} = \frac{P}{L} \frac{1}{\tan \lambda} \quad (6-12) \]
Combining Eqs. (6-11) and (6-12) the reverse efficiency as a function of the lead angle and friction angle is

\[ \eta_R = -\tan(a-\lambda)/\tan(\lambda) \]  

(6-13)

The input and output torques may be related from geometrical considerations. From Eqs. (6-8) and (6-11),

\[ \frac{P}{L} = \frac{r}{R} \tan(\lambda+a) \] (forward)  

(6-14)

and

\[ \frac{P}{L} = -\frac{r}{R} \tan(\lambda) \] (reverse)

where  

\[ r = \text{mean radius of the worm}, \]

\[ R = \text{mean radius of the gear}. \]

Noting that Pr is just the input torque and LR is the output torque, it follows that

\[ \frac{T_{in}}{T_{out}} = \frac{r}{R} \tan(\lambda+a) \] (forward)  

(6-15)

and

\[ \frac{T_{in}}{T_{out}} = -\frac{r}{R} \tan(a-\lambda) \] (reverse)

The torques act in opposite directions in the reverse drive mode.

From Mark's Mechanical Engineer's Handbook [44], for worm gears

\[ \tan \lambda = \frac{R}{nr} \]  

(6-16)

where \( n \) = speed reduction ratio of the gear.

Combining Eqs. (6-15) and (6-16),

\[ \frac{T_{in}}{T_{out}} = \frac{\tan(\lambda+a)}{n \tan \lambda} \] (forward)  

(6-17)
Further combination of Eqs. (6-10), (6-13), and (6-17) gives

\[
\frac{\tau_{in}}{\tau_{out}} = \frac{\tan(a-\lambda)}{n \tan \lambda} \quad \text{ (reverse)}
\]

When the acceleration forces associated with the gear train are small in comparison with the applied forces and the worm is not in the reverse locking condition, then Eqs. (6-18) give the relationship between the input and output torques. If the efficiency is a constant, then the gear has linear characteristics in that operation region.

For a test of worm gear efficiency, a five foot lever arm with weights was attached to the output of the worm gear. The input torque was calculated from motor current readings and from torque-current relationships in Table V. The output torque was calculated from the mechanical reaction moment at the worm gear output (proportional to the weight). Applying Eqs. (6-18), the forward efficiency was obtained by lifting the weights, and the reverse efficiency was found by lowering the weights. The results are shown in Figure 32. From this graph, \( \eta_f = 0.53 \) and \( \eta_R = -0.34 \).

**Reverse-locking conditions**

A worm gear is said to be self locking if it is essentially "locked" when the power is disconnected from the input. That is, the output cannot drive the input. Not all worm gears are self locking.
Figure 32. Experimental Worm Gear Efficiency Results
To derive the condition for self locking, refer to Figure 33. For a small value of the friction coefficient, the reaction force, \( K_R \), acts at an angle of \( \alpha \) to the normal of the surface. The component \( P \) is greater than zero and drives the input to the worm. For a large value of the friction coefficient, the reaction force, \( K_R \), cannot develop a driving force component; that is, \( P = 0 \). It follows that the condition for self locking is

\[
f = \tan(\alpha) > \tan \lambda \tag{6-19}
\]

In practice, \( \tan \lambda \) can be made greater than \( f \), and the gear will be self locking because of the friction in other parts of the drive. A lead angle less than about 4° usually insures self locking.

A self locking worm gear is not irreversible under all operating conditions. The condition for irreversibility is given by

\[
\frac{n_R}{n} \leq \frac{\tau_{mL}}{\tau_{GL}} \leq \frac{1}{n_{\eta_f}} \tag{6-20}
\]

If the magnitude of the torque ratio, \( \frac{\tau_{mL}}{\tau_{GL}} \), lies outside the range given in Eq. (6-20), then the steady-state relationship between input and output torque is given by Eqs. (6-18). The transient time constants are small and can be ignored. If the torque ratio lies within the range of Eq. (6-20), then the worm gear is irreversible, and reverse locking occurs when the joint rate goes to zero. The condition of reverse locking will remain until the threshold for the torque ratio is again exceeded.
Figure 33. Worm Gear Self-Locking Conditions
Backlash

Large amounts of backlash in worm gears may cause them to "chatter" when made a part of a closed loop servo system. It is important to model the backlash in order to study its effects in closed loop.

The teeth of the worm gear with characteristic backlash, b, (backlash referred to in the input), may be engaged or disengaged with the teeth in the backlash region. The following equations apply (for the left side worm gear):

\[
\theta_L = \theta_{mL} / n \quad (6-21)
\]

or

\[
\theta_L = \frac{\theta_{mL} + bL}{n} \quad \text{(teeth engaged)}
\]

\[
\frac{\theta_{mL}}{n} < \theta_L < \frac{\theta_{mL} + bL}{n} \quad \text{(teeth disengaged)}
\]

The worm gear enters the backlash region when the output torque changes sign; that is, the switching conditions are

\[
\theta_L = \theta_{mL} / n \quad \text{and} \quad T_{\theta L} < 0 \quad (6-22)
\]

or

\[
\theta_L = \frac{(\theta_{mL} + bL)}{n} \quad \text{and} \quad T_{\theta L} > 0 .
\]

To effectively analyze the dynamics of the disengaged system, the inertia and damping coefficient of the output gear must be modelled and included:

\[
T_{\theta L} = -J_L \omega_L - B_L \omega_L \quad (6-23)
\]
where  
\[ J_g = \text{gear inertia} \]
\[ B_g = \text{gear damping coefficient}. \]

The gear teeth are again engaged when

\[ \Theta_L = \Theta_{\text{mL}} / n \quad \text{and} \quad T_{\Theta L} \geq 0 \tag{6-24} \]

or

\[ \Theta_L = (\Theta_{\text{mL}} + b_L) / n \quad \text{and} \quad T_{\Theta L} \leq 0 \]

When the gear teeth are engaged, the acceleration forces associated with the worm gear may be ignored.

6.3.3 Triac Motor Controller

The block diagram for the triac motor controller is given in Figure 34. A discussion of its operation may be found in [45]. The overall characteristics of this nonlinear power amplifier have been modelled and are detailed in the following paragraphs.

The time delay associated with the controller is on the order of a few milliseconds which is much smaller than the mechanical time constant of the motor and has thus been ignored. The controller has been modelled as a pure nonlinear gain.

Although the output of the controller is a phase controlled half rectified wave, it has been modelled as an effective DC voltage by making tests of the output motor speed as a function of the controller input voltage. By comparing the values of the controller input voltage with the motor input voltages (from Test II in Table V) that produced the same output speed, the relationship between controller input and output was deduced (see Figure 35). A polynomial regression
was done on the resultant data points resulting in the following nonlinear equation relating the effective motor voltage, $v_L$, to the motor controller input signal, $\xi$:

$$v_L = 2.84 + 69.63\xi - 45.79\xi^2 + 15.95\xi^3$$

$$- 2.854\xi^4 + 0.2490\xi^5 - 0.00836\xi^6$$  \hspace{1cm} (6-25)

### 6.4 Control Gain Analysis

With each of the components of the system modelled, the next step is to set the control gains $K_1$, $K_2$ and $K_3$. It is desirable to obtain a fast well damped response at the output under all operating
Figure 35. Triac Motor Controller Characteristics (The output of the controller, a phase controlled half rectified wave, has been modelled as an effective DC voltage by tests of motor speed as a function of the input error voltage).
conditions. However, since the system is nonlinear, the transient response for constant control gain values will vary with the operating point. It would be necessary to set different gains for each operating point to obtain the same transient response. In order to investigate this phenomenon, the system transient response is derived for an arbitrary operating point.

Let \( V_L, I_L, T_{ml}, \) and \( \Omega_{mL} \) be the variables associated with any arbitrary operating point for the universal series wound motor. Let \( v_L, i_L, \tau_{ml}, \) and \( \omega_{mL} \) be small signal deviations away from the operating point. Eqs. (6-4) and (6-5) give

\[
V_L + v_L = G_{afv} (\Omega_{mL} + \omega_{mL})(I_L + i_L)
\]

\[
T_{ml} + \tau_{ml} = G_{afT} (I_L + i_L)^2 - T_s
\]

Separating out the small signal and operating point variables,

\[
V_L = G_{afv} \Omega_{mL} I_L + R_1 I_L
\]

\[
T_{ml} = G_{afT} I_L^2 - T_s
\]

and

\[
V_L = G_{afv} \Omega_{mL} I_L + G_{afv} I_L \omega_{mL} + R_1 i_L
\]

\[
\tau_{ml} = 2 G_{afT} I_L i_L
\]

where all second order terms involving small signal variables have been dropped. If

\[
R = G_{afv} \Omega_{mL} + R_1
\]
\[ K_T = 2 \alpha fT I_L \]
and
\[ K_b = \alpha f \nu I_L, \]
then Eqs. (6-28) become
\[ v_L = R i_L + K_b \omega_{mL} \]
\[ \tau_{mL} = K_T i_L \]

These linearized equations have the same form as the equations for a DC permanent magnet motor with \( K_T \) being the torque constant and \( K_b \) being the back emf constant.

Although the overall characteristics of the worm gear are non-linear, they are linear in each of the forward and reverse driving modes and the disengaged mode. In the following paragraphs, the state equations are derived for the linearized system for the three conditions of the worm gear. In the small signal analysis of the system with the worm gear in the forward drive mode and the universal series wound motor set at some arbitrary operating point, it is helpful to realize that the linearized equations describe a DC permanent magnet motor driving a spur gear with characteristic efficiency. The parameters of these linearized equations vary with the operating point.

**Worm Gear Teeth Engaged**

With the teeth of the worm gear engaged, the worm gear operates at a characteristic output efficiency, \( \eta \), where
\[ n = \eta_F \quad \text{(forward drive mode)} \quad (6-31) \]
\[ n = 1/\eta_R \quad \text{(reverse drive mode)} . \]

Rearranging Eq. (6-6),
\[ \omega_{\text{mL}} = \frac{\tau_{\text{mL}}}{J_{\text{mL}}} - \frac{B_{\text{mL}}}{J_{\text{mL}}} \omega_{\text{mL}} - \frac{\tau_{\text{in(L)}}}{J_{\text{mL}}} . \quad (6-32) \]

Combining Eq. (6-32) with Eqs. (6-18),
\[ \omega_{\text{mL}} = \frac{\tau_{\text{mL}}}{J_{\text{mL}}} - \frac{B_{\text{mL}}}{J_{\text{mL}}} \omega_{\text{mL}} - \frac{\tau_{\text{out(L)}}}{n \eta J_{\text{mL}}} . \quad (6-33) \]

Noting that \( \tau_{\text{out(L)}} = \tau_{\Theta_L} \) and making the substitutions from Eqs. (6-2) and (6-30), it follows that
\[ \omega_{\text{mL}} = \frac{K_T i_L}{J_{\text{mL}}} - \frac{B_{\text{mL}}}{J_{\text{mL}}} \omega_{\text{mL}} - \frac{B_{3L}(\omega_L \cdot \omega) + K_{3L}(\theta_L \cdot \theta)}{n \eta J_{\text{mL}}} . \quad (6-34) \]

From Figure 28, with no inputs into the system,
\[ v_L = -K_1 K_2 \omega_{\text{mL}} - K_1 K_4 \theta_L . \quad (6-35) \]

From Eqs. (6-30),
\[ i_L = \frac{v_L - K_4 \omega_{\text{mL}}}{R} . \quad (6-36) \]

Substituting Eq. (6-35) into (6-36),
\[ i_L = -\left(\frac{K_1 K_2}{R} + \frac{K_4}{R}\right) \omega_{\text{mL}} - \frac{K_1 K_4 \theta_L}{R} . \quad (6-37) \]

Finally substituting Eq. (6-37) into (6-34) and noting that \( \omega_{\text{mL}} = n \omega_L \) and \( \omega_{\text{mL}} = n \omega_L \), the resulting state equation for \( \omega_L \) is:
For one-half of the mechanical simulation,

\[ \tau = \tau_{\Theta_L} = B_{3L}(\omega_L - \omega) + K_{3L}(\Theta_L - \Theta) \]  

(6-39)

Combining Eq. (6-1) with Eq. (6-39), the state equation for \( \omega \) is:

\[ \dot{\omega} = \frac{B_{3L}\omega_L}{J} + \frac{K_{3L}\Theta_L}{J} - \frac{B_{3L}\omega}{J} - \frac{K_{3L}\Theta}{J} \]  

(6-40)

For the mechanical simulation with one actuator, the state equations may be written in matrix form:

\[
\begin{bmatrix}
\dot{\Theta}_L \\
\dot{\theta} \\
\dot{\omega}_L \\
\dot{\omega}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
A & B & C & D \\
E & F & G & H
\end{bmatrix}
\begin{bmatrix}
\Theta_L \\
\theta \\
\omega_L \\
\omega
\end{bmatrix}  
\]

(6-41)

with

\[
A = - \left( \frac{nK_{1}K_{3}K_{T}}{Rn^2J_{mL}} + \frac{K_{3L}}{n^2J_{mL}} \right) \quad B = \frac{K_{3L}}{n^2J_{mL}} \\
C = - \left( \frac{K_{1}K_{3}K_{T}}{RJ_{mL}} + \frac{K_{3L}}{J_{mL}} + \frac{B_{3L}}{n^2J_{mL}} \right) \\
D = \frac{B_{3L}}{n^2J_{mL}} \quad E = \frac{K_{3L}}{J} \quad F = - \frac{K_{3L}}{J} \\
G = \frac{B_{3L}}{J} \quad H = - \frac{B_{3L}}{J}
\]
Worm Gear Teeth Disengaged

For the worm gear teeth of the system in the disengaged mode of operation, a similar analysis to the one in the preceding paragraphs will produce the state equations for the linearized system about an arbitrary motor operating point. The system order for one-half of the mechanical simulation is increased by two and the inertia and damping coefficient of the output gear, \( J_g \) and \( B_g \), must be modelled. With \( \theta_{mL} \) and \( \omega_{mL} \) included as state variables, the resulting state equations for the disengaged mode are

\[
\begin{bmatrix}
\dot{\theta}_{mL} \\
\dot{\theta}_L \\
\dot{\theta}_g \\
\dot{\omega}_{mL} \\
\dot{\omega}_L \\
\omega
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & A & 0 & B & 0 & 0 \\
0 & C & D & 0 & E & F \\
0 & G & H & 0 & J & K
\end{bmatrix}
\begin{bmatrix}
\theta_{mL} \\
\theta_L \\
\theta_g \\
\omega_{mL} \\
\omega_L \\
\omega
\end{bmatrix}
\]

(6-42)

with

\[
A = -\frac{K_1 K_4 K_T}{R J_{mL}} \\
B = -\left( \frac{K_1 K_2 K_T}{R J_{mL}} + \frac{K_2 K_T}{R J_{mL}} + \frac{B_{mL}}{J_{mL}} \right) \\
C = -\frac{K_3}{J_g} \\
D = \frac{K_3}{J_g} \\
E = -\frac{B_3 + B}{J} \\
F = \frac{B_3}{J} \\
G = \frac{K_3}{J} \\
H = -\frac{K_3}{J} \\
I = \frac{B_3}{J} \\
K = -\frac{B_3}{J}
\]
With the state equations for the linearized system derived, the next step is to set the control gains for the system, $K_1$, $K_2$, and $K_4$, so that the transient response for the system will be as desired. The following operating point was chosen with Table VI giving the linearized parameters:

\[
T_{\text{out}(L)} = 0 
\]
\[
\xi_L = 1.15v 
\]
\[
v_L = 25v  
\]
\[
I_L = .96a 
\]
\[
\Omega_{mL} = 22 \text{ rad/sec} 
\]
\[
\Omega_L = .152 \text{ rad/sec} 
\]

With the control gains set at $K_{1L} = 358$, $K_{2L} = 1.57 \times 10^{-2} v/\text{(rad/sec)}$, and $K_{4L} = 4v/\text{rad}$, the following eigenvalues were obtained for one-half of the mechanical simulation:

**eigenvalues:**

- $-115.99$ forward worm gear mode (6-44)
- $-1.507$
- $-6.273 \pm 6.266$

**eigenvalues:**

- $-115.98$ reverse worm gear mode (6-45)
- $-1.513$
- $-6.278 \pm 6.271$

For this operating point, the gain, $K_{1L}$, includes a gain of 35.8 for the triac controller and a gain of 10 for a series adjustable gain.
Table VI: Mechanical Simulation Model Parameters for No Load
\( (n_{ML} = 200 \text{ rpm}) \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R )</td>
<td>26</td>
</tr>
<tr>
<td>( K_T )</td>
<td>0.635 nt-m/amp</td>
</tr>
<tr>
<td>( K_b )</td>
<td>0.752 v/(rad/sec)</td>
</tr>
<tr>
<td>( B_m )</td>
<td>5.5 \times 10^{-3} \text{ nt-m/(rad/sec)}</td>
</tr>
<tr>
<td>( J_m )</td>
<td>1.375 \times 10^{-3} \text{ kg-m}^2</td>
</tr>
<tr>
<td>( n )</td>
<td>145</td>
</tr>
<tr>
<td>( J )</td>
<td>0.108 kg-m^2</td>
</tr>
<tr>
<td>( K_3 )</td>
<td>8.502 nt-m/rad</td>
</tr>
<tr>
<td>( B_3 )</td>
<td>1.356 nt-m/(rad/sec)</td>
</tr>
<tr>
<td>( J_g )</td>
<td>2.93 \times 10^{-4} \text{ kg-m}^2</td>
</tr>
<tr>
<td>( B_g )</td>
<td>0.0</td>
</tr>
<tr>
<td>( n_f )</td>
<td>0.35</td>
</tr>
<tr>
<td>( n_R )</td>
<td>0.28</td>
</tr>
</tbody>
</table>

6.5 Computer Simulation

In order to test the control gain analysis of the previous section and to understand the nonlinear characteristics of the closed loop MECHANICAL Simulation, a computer simulation program (MECHS) was developed for this system. All of the nonlinear elements described in this chapter were included (motor saturation, triac nonlinearity, worm gear operating modes, etc.).

The total simulation program is outlined in Appendix A and consists of the main program, MECHS, a subroutine for Commanded input, CMDM, and a data file containing all of the system parameters, MECHS.DAT. The program has been sufficiently generalized to simulate either the full mechanical system of one-half of it. Typical results obtained from this simulation are shown in Figure 36.
Figure 36. Flywheel Response to a Commanded Step Input of Angular Rate (Results are from the computer simulation)
The inertia is initially rotating at $\omega = 0.138$ rad/sec. For the case of one-half of the mechanical simulation, the commanded rate input is given a step input so that $\omega$ will have a final steady-state value of 0.1517 rad/sec. Figure 36 gives plots of $\omega$ and $\theta$, the rate and position of the flywheel, as a function of time. Note that because of the nonlinear nature of the worm gear backlash, the rate of the flywheel sustains a constant error.

6.6 Summary

In this chapter, a model for an electric joint actuator was derived in the context of a mechanical simulation for a legged vehicle. The joint actuator consists of a triac controller, universal series wound motor, and worm gear drive. This model along with a computer simulation of the mechanical system provides the initial control design steps necessary in the leg development program for the legged vehicle system under construction at this university.

With the analysis presented in this chapter, further tests may be made to determine the characteristics of the electric joint actuators under varying load conditions. Finally, the control system may be set such that the leg subsystems optimize a set of operating objectives.

In Chapter VII, the model of the electric joint actuator is used to determine the energy consumption for any torque-speed operating point. In the analysis of the locomotion of the six-legged vehicle, these real hardware characteristics are used to calculate the total energy consumption of the vehicle over a gait cycle.
CHAPTER VII
EVALUATION OF THE PROPOSED CONTROL SCHEME

7.1 Introduction

In the previous three chapters, the elements of the design of the interactive computer control system for the six-legged vehicle have been developed. These include: 1) the development of a set of computer algorithms for the synthesis of body and leg trajectories under operator control, 2) the generation of bias torques for the joints of the vehicle with consideration of energy minimization and load balancing, and 3) the development of the leg servo units. In this chapter, some results will be given concerning the overall evaluation of the proposed control scheme developed in the previous chapters.

First of all, from the model of the joint actuator units given in Chapter VI, an energy model for the worm gear driven by a universal series wound motor is obtained. The results given show that at any instant in time for any operating point, the input power of the joint actuator unit is linearly related to the output power of the worm gear if saturation effects of the motor are ignored. For an enforced vehicle trajectory (positions and rates given at any time), the input power to the joint actuator in terms of voltage and current is linearly related to the output joint torque. Thus this relationship
may be used in the energy criterion function for the linear programming problem.

The third section of this chapter outlines some of the results obtained from the computer simulation developed in this work. Two additional subroutines have been used to obtain these results. First, the trajectory algorithms have been debugged with the aid of a CRT display through the subroutine \texttt{KINematic DIsplay} (KINDIS). Second, the power, energy, and foot reaction forces associated with the solution to the linear programming problem are computed in the \texttt{Foot Reaction} subroutine (FRTCTN). Output from the two subroutines is given.

The fourth section of this chapter presents a brief verification and evaluation of the computer simulation. A check is made on the set of foot reaction forces to insure that the static force calculations of Section 5.2 give correct results in this case. Also, an evaluation of the linear programming optimization problem is given for different weights between energy minimization and load balancing. Finally, the last section of this chapter gives a summary of the evaluation results.

7.2 Energy Model for the Joint Actuators

The relationship between the input power and the output torque and speed of the joint actuator unit may be obtained from the analysis in Chapter VI. The input power is just the product of the input motor voltage and current. From Eqs. (6-3) and (6-4) (with the inductance assumed to be small),

\[
\text{input power} = V i = (R_l + G_{afv} \omega_m i)i .
\]  

(7-1)
Since from Eq. (6-5),

\[ i^2 = \frac{T_m + T_s}{G_{afT}} \]  

(7-2)

then

\[ vi = \left( R_1 + G_{afv} \omega_m \right) \frac{T_m + T_s}{G_{afT}} \]  

(7-3)

Further, for steady-state conditions, Eq. (6-33) gives

\[ T_m = \frac{T_{out}}{n_\eta} + \frac{B_m \omega_m}{n_\eta} \]  

(7-4)

Substituting Eq. (7-4) into (7-3) gives

\[ vi = \left( \frac{R_1 + G_{afv} \omega_m}{G_{afT}} \right) \frac{T_{out}}{n_\eta} + \left( \frac{T_s + B_m \omega_m}{G_{afT}} \right) \]  

(7-5)

For a given value of the motor rate and worm gear efficiency, Eq. (7-5) gives a linear relationship between the output torque and input power.

Eq. (7-5) may be used to derive an appropriate criterion function for the power minimization in the linear programming problem. Since the second term in the equation involves a constant power loss for each joint that does not depend upon the joint torque, it may be ignored.

The efficiency, \( \eta \), for each joint basically has two values depending upon the relative signs between the joint rate and torque. That is,

\[ \eta = \begin{cases} \eta_f & \text{for } T_m \omega_m \text{ positive (forward drive)} \\ \frac{1}{\eta_R} & \text{for } T_m \omega_m \text{ negative (reverse drive)} \end{cases} \]  

(7-6)
With the above discussion, the power minimization criterion function, $\phi_p$, may be written

$$\phi_p = - \sum_{i=1}^{6} M_i \left[ \frac{R_1 + nG_{afT} \psi_i}{G_{afT}} \frac{\tau_{\psi i}}{n_{\psi i}} + \frac{(R_1 + nG_{afT} \theta_{1i})}{G_{afT}} \frac{\tau_{\theta i}}{n_{\theta i}} + \frac{(R_1 + nG_{afT} \theta_{2i})}{G_{afT}} \frac{\tau_{k i}}{n_{k i}} \right] \tag{7-7}$$

where $M_i = \text{indicator of foot position}$

$M_i = 1 \text{ (on the ground)}$

$M_i = 0 \text{ (in the air)}$

and $n_{\psi i}, n_{\theta i}, n_{k i} = \text{the efficiencies of the 3 joints for leg } i$.

In the following sections of this chapter, the power minimization criterion function given in Eq. (7-7) is used in the linear programming problem and the results are studied.

7.3 Computer Simulation

The control for the vehicle has been simulated on a PDP-10 computer. The flow chart for the main program of the Kinematic Command Generator is given in Figure 7 of Chapter IV. The specific functions of all of the subroutines called from the main program except for the Foot Reaction (FTRCTN) and Kinematic Display (KINDIS) subroutines have been discussed in the previous chapters. These two subroutines will be treated in this section.
The Fortran listing for the simulation programs may be found in the Appendix. The computation time for each loop through the program varies with the present state of the vehicle. Much of the computation time is involved in solving the linear programming problem with the total loop time typically varying between 5 and 15 seconds. Usually 15 to 30 iterations are necessary within the LINear PROgramming subroutine (LINPRO) itself to find the optimal solution. Most of these iterations involve finding a feasible solution (eliminating the artificial variables) for the joint torques. The total memory used for execution of the programs along with the necessary system library routines is about 25K of core.

7.3.1 Vehicle Trajectories

A photograph of the operator's control station for the simulated system is given in Figure 37. The figure shows a CRT display interfaced to the PDP-10 computer, along with a teletype unit. Since a joystick has not been interfaced to the computer, a row of keys on the teletype keyboard was used in place of the forward/backward joystick axis to furnish speed commands to the computer while another row of keys was used in place of the side to side joystick axis. Mode commands were designated by still other keys.

The algorithms for adaptation to terrain variation were successfully programmed so that the vehicle could travel under operator control over an undulating terrain with an overall average slope. Figure 38 shows a picture of the simulated vehicle travelling over the terrain with perspective and side views shown. Figure 39 shows the vehicle successfully turning in place in a "valley" of the terrain.
Figure 37. Photograph of the Operator's Control Station for the Simulated System
Figure 38. Perspective and Side Views of the Simulated Vehicle Traveling over an Undulating Terrain.
Figure 39. The Simulated Vehicle Turning in Place in a "valley" of the Terrain.
The kinematic outputs of joint position and rate that are used by the display subroutine could be furnished to the leg servos of the real vehicle for leg control purposes. Only a few changes at most would have to be completed in order for this segment of the simulation to be used directly in the synthesis of the trajectories for the actual legged vehicle.

7.3.2 Generation of Joint Torques

The generation of joint torques is accomplished by solving a linear programming (LP) problem as outlined in Chapter V. The solution to the LP problem minimizes a combination of power consumption and load balancing. The criterion function for the LP problem is:

\[ \phi = \phi_p + (\text{BALNCE}) f_{RZM} \]  

(7-8)

where

BALNCE = the cost weight of the load balancing criterion with respect to that of the power minimization criterion.

and

\[ f_{RZM} \] = the maximum normal component of any of the six foot reaction forces.

Large negative values of BALNCE (BALNCE) give solutions in which the load is more nearly balanced among the legs while small negative values give solutions to the joint torques in which power consumption is more nearly minimized.

Figure 40 gives graphs of the 18 joint torques as a function of time. The results are given for an 18-second interval for locomotion.
Figure 40. Bias Torques for the Over a Locomotion Cycle Three Joints of the Six Legs of the OSU Hexapod Given (Torque in ft/lbs, time in seconds)
over level terrain. The stride length is set at 3 feet while the vehicle is moving 1/6 ft/sec in a straight line forward direction, so that the graphs cover one complete period of locomotion.

BALNCE has been set equal to 0 for power minimization. The Foot Reaction subroutine (FTRCTN) has been programmed to compute the instantaneous power consumed, the total energy consumed through the locomotion cycle, and the foot reaction forces for any set of joint torques from Eq. (3-40). Graphs for the instantaneous power and net energy consumed are given in Figures 41 and 42. Typical tabular results for the output of the FTRCTN subroutine are given in Table VI. The joint positions and joint rates are included for completeness.

7.4 Performance Evaluation

The equations derived in Chapter III and used in the static force calculations of Section 5.2 involve complex functions of the joint positions and static leg weights. The accuracy of these equations may be checked by considering the tabular results given in Table VI. The sum of the foot reaction forces for the legs should be equal to the total weight of the vehicle system since acceleration forces are assumed to be zero. Since this locomotion is over level terrain,

\[ \sum_{i=1}^{6} f_{Rx_i} = 0 \]  
\[ \sum_{i=1}^{6} f_{Ry_i} = 0 \]  

(7-9)
Figure 41. Instantaneous Power Consumed Through One Locomotion Cycle (Straight line forward direction over level terrain).
Figure 42. Energy Consumed Through One Locomotion Cycle
(Straight line forward direction over level terrain)
Table VI: Typical Tabular Results for the Output of the FTRCTN Subroutine Showing the Instantaneous Values for the Joint Torques, Foot Reaction Forces, and Joint Angles and Rates.

<table>
<thead>
<tr>
<th>TIME = 4.0</th>
<th>POWER = 175.8519</th>
<th>ENERGY = 0.0</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>τ̇_θ_1</strong></td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td><strong>τ̇_θ_1</strong></td>
<td>-40.3537</td>
<td>-39.9131</td>
</tr>
<tr>
<td><strong>τ̇_θ_1</strong></td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td><strong>ḟ_Rx_1</strong></td>
<td>-1.1592</td>
<td>0.7661</td>
</tr>
<tr>
<td><strong>ḟ_Ry_1</strong></td>
<td>2.7819</td>
<td>-2.3284</td>
</tr>
<tr>
<td><strong>ḟ_Rz_1</strong></td>
<td>-30.1622</td>
<td>-30.1622</td>
</tr>
<tr>
<td><strong>θ̇_1</strong></td>
<td>-1.1760</td>
<td>1.8925</td>
</tr>
<tr>
<td><strong>θ̇_1</strong></td>
<td>0.3760</td>
<td>0.3788</td>
</tr>
<tr>
<td><strong>θ̇_2</strong></td>
<td>-0.2737</td>
<td>-0.2954</td>
</tr>
<tr>
<td><strong>θ̇_2</strong></td>
<td>-0.0710</td>
<td>0.0750</td>
</tr>
<tr>
<td><strong>θ̇_2</strong></td>
<td>-0.0034</td>
<td>0.0023</td>
</tr>
<tr>
<td><strong>θ̇_2</strong></td>
<td>0.0240</td>
<td>-0.0194</td>
</tr>
</tbody>
</table>

Torque in ft/lbs, force in lbs, angles in radians, rates in rad/sec
and \[ \sum_{i=1}^{6} f_{RI} = -(m + 6m_1 + 6m_2)g = -185 \text{ lbs.} \]

where

\[ m = \text{vehicle mass (2.5 slugs)} \]

\[ m_1 = \text{mass of the leg upper limb segment (0.5 slugs)} \]

\[ m_2 = \text{mass of the leg lower limb segment (0.05 slugs)} \]

Noting Table VI, Eqs. (7-9) are satisfied.

Figure 41 showing the instantaneous power over a locomotion cycle shows that a minimum of about 225 watts peak power is required to power the actuators of the vehicle as it travels over level terrain at \( \frac{1}{6} \text{ ft/sec} \). Figure 43 shows that as the relative weighting between power minimization and load balancing is increased, the nominal power required is increased. In actual vehicle control, the weighting between power minimization and load balancing could be easily varied to adapt to varying soil conditions.

7.5 Summary and Conclusions

This chapter gives an evaluation of the results of the rather extensive computer simulation studies which have been carried out by the author for the design of the control system for the six-legged robot. Algorithms for generation of the joint positions, rates, and torques have been successfully programmed to control the vehicle.

The vehicle could not presently be controlled in real time by the simulation programs since the total loop computation time for the control program (approximately 10 seconds) is two or three orders of magnitude slower than real time. However, in the evaluation, it
Figure 43. Nominal Power Required as a Function of the Weight Between Power Minimization and Load Balancing.
is found that most of this time involves setting up and solving the linear programming problem. It is anticipated that in real vehicle control, the linear programming problem could be solved off-line for locomotion over a cycle with results placed in tables for use in real time control. It is also possible that the linear programming solution may be implemented more efficiently if each new solution used the feasible solution from the previous program loop. More will be said about this in the next chapter.

The amount of memory required is manageable (approximately 25K words). The estimated total electrical power required at a speed of 1/6 ft/sec is about 1/3 horsepower. Although this estimate is based on a number of simulation models with their inherent approximations, this result should be a reasonably accurate figure since model parameters were derived from actual hardware tests.

Altogether, the author feels that the feasibility of the proposed approach to interactive computer-control of a legged vehicle has been demonstrated by the results presented in this chapter. While considerable work remains to be done to complete the mechanical and electrical assemblies of the machine now under construction, it is felt that the control programs developed in the research of this dissertation will be found to be satisfactory when applied to this vehicle.
CHAPTER VIII
SUMMARY AND CONCLUSIONS

The central problem treated in this dissertation is the design of the interactive computer control system for a six-legged vehicle under construction at this university. Toward this end, the generation of joint positions, rates, and bias torques has been investigated with consideration of real hardware characteristics. Computer algorithms have been developed that should allow the vehicle to move over uneven terrain with a high degree of stability and with a minimum amount of energy consumed.

Several of the specific contributions that have been made to legged locomotion studies will be discussed in the next section of this chapter. The last section indicates other research problems to be investigated that grow out of this work.

8.1 Research Contributions

On the macroscopic level, a vital link has been completed in a "supervisory control system" for legged locomotion systems. This is the necessary link that brings together man and machine at their own levels and allows them to successfully communicate to solve a given task, namely that of legged locomotion. The kinematic command generator has been sufficiently developed to allow the vehicle to travel in two different modes over uneven terrain. In the turn control mode, the
forward/backward speed of the vehicle along with the azimuth turning rate are furnished by the operator. In the vector velocity control mode, the heading of the vehicle does not change while the operator controls the forward/backward speed and side step speed of the vehicle.

In the software of the kinematic command generator, several algorithms have been developed. Among these are:

1) Automatic body height, pitch, and roll regulation. One subroutine of the kinematic command generator automatically controls the state variables of the body associated with height, pitch, and roll to maintain the body parallel to the supporting surface and at a constant height above it. For uneven terrain, a linear regression is done on the points of support of the vehicle in order to compute an estimated plane for the terrain.

2) Gait implementation. The optimum statically stable wave gait is automatically implemented to insure locomotion with a high degree of stability.

3) Automatic foot positioning. Within the bounds of the wave gait, an algorithm has been developed to allow the foot placement and positioning to adapt to irregular terrain conditions. Also, the transfer phase of leg motion is controlled so that the foot will not "bang" into the ground upon placement.

Another contribution to the area of automatic trajectory synthesis is the design of the basic organization of the kinematic command generator. This organization includes 1) the definition and modelling...
of the system consisting of parameter vectors defining the vehicle, terrain, and leg systems, 2) the description of the system consisting of state vectors totally defining the present state of the body and legs, and 3) the partitioning of the problem of locomotion into functional subroutines that operate upon certain of the state vectors through prescribed algorithms. These include the man-machine interface, body trajectory synthesis, and leg positioning subroutines. With the basic structure of the kinematic command generator developed, additional algorithms for obstacle negotiation, ditch clearance, stair climbing, etc., may be effectively implemented.

Another basic theoretical contribution has been to use linear programming to generate the bias torques for the joints. The solution of joint torques optimizes a combination of power consumption and load balancing. Since energy is just the time integral of power and the trajectory of the vehicle has already been prescribed, the minimization of instantaneous power over the enforced trajectory also minimizes energy. The weight between power minimization and load balancing may be interactively adjusted for variations in soil conditions. Under soft soil conditions, the weight would optimize load balancing while it would minimize power consumption, otherwise.

The constraints in the linear programming problem also characterize real world conditions. For locomotion over low coefficient of friction surfaces (ice, for example), linear constraints have been placed upon the foot reaction vectors to lie within a friction pyramid.
The vertex angles of the pyramid may be adjusted to allow for various surface conditions. Bounds have been placed upon the joint torques to allow for saturation effects of the motors. Thus, a joint will not have to deliver more torque than it is capable of providing.

A final area of contribution of this dissertation has been to model the nonlinear effects of a worm gear driven by a universal series wound motor. Such electric actuators are suitable for robotic applications in particular because of the self-locking characteristic of the worm gear. That is, with no power applied to the unit, it may sustain large output reaction torques while in an essentially locked condition. Thus an unpowered legged vehicle equipped with worm gears would not droop to the ground.

The characteristics of the electric actuators have been modelled, and their performance inside a servo loop have been simulated. To set the control gains for the servo loop, the system has been linearized about an operating point. The analysis shows that the actuator system performs like a spur gear driven by a DC permanent magnet motor for small signal variations from the operating point. The power characteristics from output to input have been derived. These are found to be linear and may be used as a basis for the criterion function in the linear programming problem. Practical tests have been made on a Black and Decker drill motor, type #1174, and all models are with respect to real hardware characteristics.
8.2 Research Extensions

A number of research problems have become increasingly clear as the initial phases of this study have been completed. Some of these research problems manifest themselves with the development of the control system for the present vehicle while others are more advanced. Some of these problems will be outlined in the following paragraphs.

One of the major problems that must be solved is to include leg acceleration forces in the control analysis since these forces are considered to be significant. Even at modest vehicle speeds, with leg accelerations ignored, the body of the vehicle may be unduly affected by these unaccounted forces. The basic theoretical work in computing such accelerations has been studied by Jaswa [41]. Extensions of this work as it applied to the six-legged vehicle need to be investigated for locomotion of the practical vehicle.

A second major problem to be investigated is the design of the Force Accommodation Controller (FAC) as outlined in Chapter IV. In locomotion over uneven terrain, it is highly probable that force feedback will be necessary in addition to simple foot contact sensing. Force feedback will be necessary to account for conditions such as system model inaccuracies, terrain variation, leg servo errors, and the complex dynamics of leg cycling.

A third major problem involves the generation of joint bias torques in real time. Presently, the control is 100 to 1000 times
slower than real time because of the long computation time for the solution of the linear programming problem. However, this problem may be eliminated in two possible ways.

First of all, the computation time may be decreased by making the linear programming algorithm more efficient. Most of the solution time is spent in just obtaining a feasible solution (eliminating the artificial variables). This time may be virtually eliminated by using the previous feasible solution through the program loop and incrementally changing it. Only at points in the locomotion cycle when a leg is placed or lifted would a markedly different solution need to be realized.

A second approach to the possibility of real time control is to solve the linear programming problem off-line and store the leg force solutions in a table. For each distinct phase of the gait cycle, each of the supporting legs would be assigned a set of forces that are to be generated. These required forces could be translated into joint torques for various leg positions within the specific gait phase.

Another very interesting approach to the problem of generating joint torques in real time lies in the application of learning control. One specific approach is provided by the work of Albus [46,47,48], the cerebral model articulation controller (CMAC). If such a controller were used it could be trained off-line by comparing its joint torque output to the desired values of torque generated by the linear programming problem. Any difference would be used to adjust CMAC weights.
using the iterative algorithm of Albus [47]. After such initial training, the CMAC controller (implemented in terms of a computer program) could then be placed on-line to control the vehicular system. Training could then be continued based upon actual control performance.

Other research problems that may be investigated include the following:

1) In the friction pyramid constraints in the linear programming problem, it was assumed that the normal to the terrain was also normal to the body. Thus, friction components were computed with respect to the body coordinate system. In order to correctly constrain the components normal and tangential to the surface at the foot, an ankle measurement system could be constructed and the surface normal could be derived.

2) The basic organization for the kinematic command generator has been defined. Within this structure, algorithms for obstacle negotiation, ditch clearance, stair climbing, etc., may be developed to take advantage of the legged vehicle's unique off-the-road mobility.

3) The present model for the vehicle body and leg subsystems should be adequate for its effective control. However, the parameters defining the system may be adjusted with dynamic tests on the model. Good parameter estimation may eliminate some of the need for force feedback.
4) The response of the leg servos varies with the torque and speed operating point. In particular, the transfer and support phases for the leg define distinctively different operating points for the leg servos. In the transfer phase, joint rates are relatively high while bias torques are small. In the support phase, the reverse conditions hold. Investigations into good transient response of the leg may conclude that switchable control gains are necessary especially between the two phases for a leg.

5) In the theoretical results presented in Chapter V, relationships between feasible solutions to the linear programming problem and stability over even terrain were derived. Extensions to uneven terrain should be investigated in the future. The concept of "potential energy surfaces" introduced for analysis of gait stability in [49] should be useful in such work.

6) In the constraints for the linear programming problem outlined in Chapter V, joint torque was limited due to the saturation effects of the motor. Specifically, the saturation effects of the motor and the heat dissipation characteristics of the windings limit the total power capability of the motor. Thus, the maximum torque allowed for a joint should vary with the rate of that joint.

7) In this work, stability was optimized by selecting the best leg and vehicle trajectory. Within this limitation, power was minimized through the solution of a linear programming problem. If trajectory planning for both stability and energy optimization were employed, dynamic programming techniques employing the calculus of
variations could be used in the optimization procedure.

8) The model reference control approach taken in this work involves the computation of joint angles and rates from the specification of a desired system trajectory. While the generality and flexibility of this approach is very great, it is not efficient from the point of view of computer storage or execution time requirements. An alternative approach that could be investigated is the use of finite-state control [50,51] in which no computations at all are made in the usual sense. That is, in this control approach, the desired motions are composed in robot coordinates and a program consists simply of a list of successive joint positions together with conditions which must be satisfied to proceed from one point to the next. This approach has been used successfully to control an artificial quadruped [32,33], so an extension to hexapod locomotion ought to be possible.

As was outlined in the preceding paragraphs, a number of theoretical and practical extensions to this work may be studied. Many of these will be associated with the completion and testing of the vehicle presently under construction. The theoretical design results of this dissertation should eventually be tested in relation to this machine. Tests of power consumption, draw bar pull, speed, and terrain adaptation capability should uncover still further research problems. With the development of control systems for legged locomotion systems advancing at a moderate rate, the future will hopefully see practical use of legged vehicular systems in off-the-road applications.
APPENDIX

COMPUTER PROGRAMS

This Appendix lists the computer programs used in the course of this research. Both an explanation of the various symbols used and a description of the programs may be found in the body of this dissertation. Many of the symbols and program organization were taken from [2]. The computer programs include the following:

I The Fortran Main Programs
   A. ROBOT.F4
   B. MECHANical Simulation (MECHS.F4)

II The Fortran Subroutines
   A. INPUT.F4
   B. INTERpret (INTER.F4)
   C. BODY.F4
   D. LEG POSITION (LEGP0S.F4)
   E. OUTPUT.F4
   F. Static FORCE (STCFOR.F4)
   G. FORCE.F4
   H. LINEar PROgramming (LINPRO.F4)
   I. Foot Reaction (FTRCTN.F4)
   J. KINematic DISplay (KINDIS.F4)
   K. Command Mechanical (CMHM.F4)

III The Assembly Subroutine—KEYBRD.MAC

IV The Data Arrays—A. ROBOT.DAT; B. MECHS.DAT
THIS IS THE MAIN PROGRAM FOR THE KINEMATIC COMMAND GENERATOR. THE MAIN PROGRAM CONSISTS OF AN INITIALIZATION SEGMENT, CALLS TO SUBROUTINES WHICH CONTAIN THE ALGORITHMS FOR GENERATION OF JOINT ANGLES, RATES, AND TORQUES, AND AN OUTPUT SEGMENT.

REAL L, L1, L2
LOGICAL K
DIMENSION XC(12), CJAV(36), XLPIV(42), TPV(8),
IXIV(4), BPPV(32), T1(3,3), CTV(21), F(3), T(3), FR(3,6),
ENERGY(4), XMPV(3)
COMMON/BPPV/BPPV/BSTATE/XC/CJAV/CJAV/TPV/TPV/XMPV/XMPV
COMMON/XLPV/XLPV/XIV/XIV/T1/T1/CTV/CTV/F/T, FR

***INITIALIZATION SEGMENT***

ACCEPT 80, NPRINT
FORMAT(I4)
EQUIVALENCE (DT, BPPV(22))
PARAT=5HR0BOT
CALL IFILE(I, PARAT)
READ(1,70) BPPV, TPV, XMPV
70 FORMAT(6F11.4)
K=.FALSE.
TIME=0.0
NPRINT=NPRINT
DO 100 I=1,3
100 ENERGY(I)=0.0

***CALLS TO SUBROUTINES***

900 CALL INPUT(K, IDEN)
CALL INTER(K)
CALL BODY(K, IDEN)
CALL LEGPOS(K, IDEN)
CALL OUTPUT

***OUTPUT SEGMENT***

IF(TIME.LT.3.995) GO TO 60
IF(NPRINT-NPRINT.LT.0) GO TO 60
CALL STCFOR(K, IDEN)
CALL FORCE
CALL FTRCTN
POWER=CTV(19)+CTV(23)+CTV(21)
WRITE(3,90) TIME, POWER, ENERGY(4)
WRITE(3,90) (CTV(J), J=1, 6), CTV(19), ENERGY(1)
WRITE(3,90) (CTV(J), J=7, 12), CTV(24), ENERGY(2)
WRITE(3,90) (CTV(J), J=13, 18), CTV(21), ENERGY(3)
90 FORMAT(1X, 3Fl-3.4)
WRITE(3,91) (FR(J, I), I=1, 6), J=1, 3)
FORMAT(1X,6F10.4)

ENERGY(1)=ENERGY(1)+CTV(19)*DT*FLOAT(NPRINT)
ENERGY(2)=ENERGY(2)+CTV(20)*DT*FLOAT(NPRINT)
ENERGY(3)=ENERGY(3)+CTV(21)*DT*FLOAT(NPRINT)
ENERGY(4)=ENERGY(1)+ENERGY(2)+ENERGY(3)

WRITE(3,91) CJAVA

TIME=TIME+DT
IF(TIME.GT.22.3) STOP
NPRINT=NPRINT+1
K=.TRUE.
GO TO 900
CALL KINDISK(IDEN)
STOP
END

SUBROUTINE INPUT READS THE COMMANDS FURNISHED BY
THE OPERATOR.
SUBROUTINE INPUT(K,IDEN)
LOGICAL K
COMMON/XIV/MODE,AXIS1,AXIS2,AXIS3

***INITIALIZATION***
IF(K) GO TO 10
DATA MODE,AXIS1,AXIS2,AXIS3,SGN/1,0.0,0.0,0.0,1.0/
AXIS1=1.5
RETURN
10 CONTINUE
900 IDEW=0

***READ THE OPERATOR'S COMMANDS***
CALL KEYBD(IDEN)
CONTINUE
IF(IDEN.EQ.84) MODE=1
IF(IDEN.EQ.72) CALL EXIT
IF(IDEN.EQ.83) MODE=2
IF(IDEN.EQ.43) SGN=1.0
IF(IDEN.EQ.45) SGN=-1.0
IF(IDEN.EQ.49) AXIS1=3.0
IF(IDEN.EQ.50) AXIS1=SIGN(2.50,SGN)
IF(IDEN.EQ.51) AXIS1=SIGN(1.50,SGN)
IF(IDEN.EQ.52) AXIS1=SIGN(1.50,SGN)
IF(IDEN.EQ.53) AXIS1=SIGN(2.50,SGN)
IF(IDEN.EQ.54) AXIS1=SIGN(2.50,SGN)
IF(IDEN.EQ.55) AXIS1=SIGN(3.00,SGN)
IF(IDEN.EQ.56) AXIS1=SIGN(3.50,SGN)
SUBROUTINE INTERPRET USES THE INFORMATION RECEIVED
FROM THE OPERATOR TO UPDATE CERTAIN OF THE BODY
STATE VARIABLES.

SUBROUTINE INTER(K)

REAL L,L1,L2
LOGICAL K

DIMENSION H1(6),H2(6),H3(6)
COMMON/BPPV/Hi,H2,H3,L,L1,L2,DT/XIV/MODE,
1AXIS1,AXIS2,AXIS3/STATE/XR,YR,ZR,VEL,VELY,VELZ,THR,
2PHIR,PSIR,DPHIR,DPSIR,DPDPSIR

***INITIALIZATION***

IF(K) GO TO 10
VEL=1.0/6.0
VELY=0.2
DPSIR=0.0
RETURN

10 CONTINUE

***UPDATE THE FORWARD/BACKWARD VELOCITY,***

*** SIDESTEP VELOCITY, AND TURNING RATE ***
C SUBROUTINE BODY AUTOMATICALLY ADJUSTS THE BODY HEIGHT
above the supporting surface and the body pitch and
roll by keeping the body parallel to an estimated plane
through the supporting feet.

SUBROUTINE BODY(K, IDEN)
REAL L, L1, L2
LOGICAL K, M(6)
DIMENSION XC(12), H1(6), H2(6), H3(6)
I, XF(6), YF(6), ZF(6), XDATA(6), YDATA(6), ZDATA(6)
COMMON/STATE/XR, YR, ZR, VEL, VELY, VELZ, THR, PHIR, PSIIR,
1 DTHR, DPHIR, DPSIR
COMMON/PPV/H1, H2, H3, L, L1, L2, DT
COMMON/XLPV/XF, YF, ZF, A
EQUIVALENCE (XR, XC(1))

C ***INITIALIZATION***

IF(K) GO TO 10
DO 20 I = 1, 12
XF(I) = H1(I) - 35.0
YF(I) = H2(I) + SIGN(L, H2(I))
ZF(I) = 0.0
M(I) = .TRUE.
20 XC(I) = 0.0
ZR = -2.50
XR = -35.0
VEL = 1.0/6.0
RETURN
CONTINUE

C ***COMPUTE THE PARAMETERS OF THE***
C *** ESTIMATED FEET PLANE ***
CPS = COS(PSIR)
SPS = SIN(PSIR)
XBAR = 0.0
YBAR = 0.0
ZBAR = 0.0
N = 0
DO 30 I = 1, 6
IF (.NOT. M(I)) GO TO 30
N = N + 1
XDATA(I) = XF(I)
YDATA(I) = YF(I)
ZDATA(I) = ZF(I)
XBAR = XBAR + XF(I)
YBAR = YBAR + YF(I)
ZBAR = ZBAR + ZF(I)
CONTINUE
XBAR = XBAR / FLOAT(N)
YBAR = YBAR / FLOAT(N)
ZBAR = ZBAR / FLOAT(N)
XNUM1 = 0.0
XNUM2 = 0.0
XDEN = 0.0
IF (N .LT. 3) PAUSE "TOO FEW LEGS ON GROUND"
DO 40 I = 1, N
XDATA(I) = XDATA(I) - XBAR
YDATA(I) = YDATA(I) - YBAR
ZDATA(I) = ZDATA(I) - ZBAR
XNUM1 = XNUM1 + XDATA(I) * ZDATA(I)
XDEN = XDEN + XDATA(I)**2
ALPHA1 = XNUM1 / XDEN
ALPHA2 = XNUM2 / XDEN
ALPHA3 = ZBAR - ALPHA1 * XBAR
ALPHA0 = YBAR - ALPHA2 * XBAR
YRBAR = 0.0
ZRBAR = 0.0
DO 50 J = 1, N
XDATA(I) = XDATA(I) + X3AR
YDATA(I) = YDATA(I) + YBAR - ALPHA2 - ALPHA3 * XDATA(I)
ZDATA(I) = ZDATA(I) + ZBAR - ALPHA0 - ALPHA1 * XDATA(I)
YRBAR = YRBAR + YDATA(I)
ZRBAR = ZRBAR + ZDATA(I)
CONTINUE
YRBAR = YRBAR / FLOAT(N)
ZRBAR = ZRBAR / FLOAT(N)
XNUM1 = 0.0
XDEN = 0.0
DO 60 I = 1, N
YDATA(I) = YDATA(I) - YRBAR
ZDATA(I) = ZDATA(I) - ZRBAR
XNUM1 = XNUM1 + ZDATA(I) * ZDATA(I)
XDEN = XDEN + YDATA(I)**2
CONTINUE
YDATA(I) = YDATA(I) + YBAR - ALPHA2 - ALPHA3 * XDATA(I)
ZDATA(I) = ZDATA(I) + ZBAR - ALPHA0 - ALPHA1 * XDATA(I)
CONTINUE
YDEN = XDEN + YDATA(I)**2
\[
\text{ALPHA4} = \frac{XNUM1}{XDEN}
\]
\[
XDEN = \sqrt{1.3 + (\text{ALPHA1} - \text{ALPHA4} \times \text{ALPHA3})^2 + \text{ALPHA4}^2}
\]
\[
XBAR = \frac{\text{ALPHA1} - \text{ALPHA4} \times \text{ALPHA3}}{XDEN}
\]
\[
YBAR = \frac{\text{ALPHA4}}{XDEN}
\]
\[
ZBAR = \frac{1.3}{XDEN}
\]

***UPDATE THE BODY HEIGHT, PITCH, AND ROLL VARIABLES***

\[
SPH = XBAR \times SP - YBAR \times CPS
\]
\[
DPHIR = \arcsin (SPH) - \phiIR
\]
\[
\text{IF} (\text{ABS}(DPIRH) < 1.0E-12) DPHIR = 0.0
\]
\[
DTHR = \arcsin ((YBAR + CPS \times SPH) / (SPS \times \sqrt{1.3 - SPS^2})) - THR
\]
\[
\text{IF} (\text{ABS}(DTHR) < 1.3E-12) OTHR = 0.0
\]
\[
\text{GO TO 80}
\]
\[
70 DTHR = \arcsin ((XBAR + CPS \times SPH) / (SPS \times \sqrt{1.3 - SPS^2})) - THR
\]
\[
\text{IF} (\text{ABS}(DTHR) < 1.0E-12) OTHR = 0.0
\]
\[
\text{GO TO 80}
\]
\[
80 \phiIR = DPHIR \times DT + \phiIR
\]
\[
\text{THR} = 30.0 \times DTHR \times DT + \text{THR}
\]
\[
\text{VELZ} = -XBAR \times XR - YBAR \times YR - ZBAR \times ZR + (\text{ALPHA0} - \text{ALPHA2} \times \text{ALPHA4}) / XDEN - 2.53
\]
\[
\text{110 FORMAT}(1X, 5F11.4)
\]

***UPDATE THE OTHER BODY STATE VARIABLES***

\[
ZR = \text{VELZ} \times DT + ZR
\]
\[
XR = XR + \text{VEL} \times DT \times CPS - \text{VELY} \times DT \times SPS
\]
\[
YR = YR + \text{VEL} \times DT \times SPS + \text{VELY} \times DT \times CPS
\]
\[
PSIR = \text{DPSIR} \times DT + PSIR
\]
\[
\text{RETURN}
\]
\[
\text{END}
\]

C SUBROUTINE LEG POSITION IMPLEMENTS THE OPTIMUM WAVE
C GAIT AND SYNTHESIZES THE TRAJECTORIES FOR EACH OF THE
C LEGS. SEE CHAPTER IV.
C SUBROUTINE LEGPOS(K,DEN)
REAL L,L1,L2
LOGICAL M(6),K
DIMENSION PHI(6),H1(6),H2(6),H3(6),
1CT(6),XF(6),YF(6),ZF(6),ZFD(6)
COMMON/SPV/H1,H2,H3,L,L1,L2,DT
COMMON/FPV/FRCTN,G1,G2,G3,G4,G5,G6,G7
COMMON/STATE/XR,YR,ZR,VEL,VELY,VELZ,THR,PHIR,PSIR,
1DTHR,DPHIR,DPSIR
COMMON/XLPV/XF,YF,ZF,H/XIV/MODE,AXIS1,AXIS2,AXIS3
COMMON/TV/TV1,TV11,TV12,TV13,TV14,TV15,TV16,TV17,TV18,TV19,TV20,TV21
COMMON/ZERR/XR,YR=G1*G2*(G3*(XR-G7)+G4*YR)+
105*(COS(G6*(XR-G7))-1.0))
***INITIALIZATION***

IF (K) GO TO 10

DATA (PHI(I), I=1,6)/3.0,0.5,0.333,3.667,3.157/
STD=3.0
STDY=1.50
TSTD=.72
DO 20 I=1,6
PHI(I)=1.0-PHI(I)+.9
XF(I)=H1(I)+XH
YF(I)=H2(I)+SIGN(L,H2(I))
ZF(I)=0.0
ZFD(I)=0.0

20 M(I)=.TRUE.
BETA=0.833
RETURN

10 CONTINUE

***COMPUTE THE PERIOD, DUTY FACTOR, AND RELATIVE***

*** LEG PHASES FOR THE OPTIMUM WAVE GAIT ***

GO TO (33,40), MODE

30 IF(ABS(VEL/STO).LT.ADS(DPSIR/TSTD) ) GO TO 35

31 PER=SIGN(I000.3,VEL)
IF(ABS(VEL).LT.0.0001) GO TO 63
PER=STD/VEL
GO TO 60

35 PER=SIGN(I0003.0,DPSIR)
IF(ABS(DPSIR).LT.0.0001 ) GOTO 60
PER=TSTD/DPSIR
GO TO 60

40 IF(ABS(VEL/STO).GT.ABS(VELY/STODY)) GO TO 31
PER=SIGN(I0003.0,VELY)
IF(ABS(VELY).LT.0.0001 ) GOTO 60
PER=STODY/VELY
GO TO 60

60 DEN1=BETA
BETA=1.0-3.0/P3

310 DO 330 I=1,6
PHI(I)=PHI(I)+DT/PER
PHI(I)=PHI(I)+(DEN1-BETA)*FLOAT((I-1)/2)
NDEN=INT(PHI(I))
DEN=FLOAT(NDEN)
CT(I)=PHI(I)-DEN
IF(CT(I).LT.0.0) CT(I)=.0+CT(I)

330 CONTINUE

***COMPUTE THE ANTICIPATED FOOT POSITION***

*** (ON THE GROUND OR OFF) ***
DO 358 I=1,6
M(I)=.FALSE.
IF(CT(I).LE.BETA) M(I)=.TRUE.
IF(CT(I).LT.0.0OR.CT(I).GT.1.0) PAUSE 'CT IS NEGATIVE'
110 FORMAT(1X,GF11.4)
358 CONTINUE

CTH=COS(THR)
STH=SIN(THR)
CPH=COS(PHIR)
SPH=SIN(PHIR)
CPS=COS(PSIR)

CTH=CTH*SPH-SPS*CPH
T112=CPS*CPH+SPS*SPH
T123=CTH*SPH
T131=SPS*SPH+STH*CPH
T132=SPS*STH-CPH*SPH
T133=CPH*CTH

C ***COMPUTE THE NEW FOOT POSITION FOR THE LEGS***
C *** IN THE TRANSFER PHASE ***
C
DO 320 I=1,6
IF(M(I).GT.0) GO TO 320

XBNF=PER*VSL*3ETA*((CT(I)-BETA)/(1.0-BETA)-0.5)
GO TO (130,140), MODE
140 XBNF=XBNF+H1(I)

YBNF=PER*V5LY*BETA*((CT(I)-BETA)/(1.0-BETA)-0.5)
1+H2(I)+SIGN(L,H2(I))
GO TO 859

DEN=SQRT(H1(I)**2+H2(I)**2+SIGN(L,H2(I)))

859 DEN=(CT(I)-BETA)/(1.0-BETA)
ZBNF=-2.5*25*SIN(2.3*3.14159*DEH-3.14159/2.0)
1-2.50*0.25+2.55

CONTINUE
***AUTOMATIC FOOT PLACEMENT TO ADAPT***
*** TO TERRAIN VARIATION ***

DO 952 I=1,6
IF (.NOT. M(I)) GO TO 951
IF (XF(I).GT.G7.AND.ZF(I).GE.ZTERR(XF(I),YF(I)))
GO TO 951
IF (XF(I).LE.G7.AND.ZF(I).GE.0.0) GO TO 951
M=30.0
U=0.5
DEN=ZF(I)*DT+ZF(I)
IF (XF(I).LE.G7)
ZF(I)= ((A**2)*(ZF(I))**2-2.0*U*ZF(I)*DT+ZF(I)
2-2.0*U*ZF(I)**2)*DT+ZF(I)
IF (ZF(I).GE.0.0.AND.XF(I).LE.G7) ZF(I)=0.0
CONTINUE
951 M(I)=.FALSE.
952 CONTINUE
IF (IDEN.EQ.71) TYPE 533,1
503 FORMAT (1X,6F11.4)
RETURN
END

C SUBROUTINE OUTPUT COMPUTES THE DESIRED STEADY-STATE
C VALUES FOR THE JOINT ANGLES AND RATES WITH A KNOWLEDGE
C OF THE POSITION OF THE FOOT FOR EACH LEG WITH RESPECT
C TO THE BODY CENTER OF GRAVITY.
SUBROUTINE OUTPUT
REAL L,L1,L2
LOGICAL M(6)
DIMENSION H1(6),H2(6),H3(6),XF(6),YF(6),ZF(6)
DIMENSION XK(6),YK(6),ZK(6),
IPSIC(6),TH1C(6),TH2C(6),DPSIC(6),DTH1C(6),DTH2C(6)
COMMON/BPPV/H1,H2,H3,L,L1,L2,DT
COMMON/STATE/XR,YR,ZR,VEL,VELY,VELZ,
THR,PHIR,PSIR,DTHR,DPHIR,DPSIR
COMMON/XLPV/XF,YF,ZF,XX,YY,ZZ
COMMON/CJAV/PSIC,TH1C,TH2C,DPSIC,DTH1C,DTH2C
COMMON/T1/T111,T121,T131,T112,T122,T132,T113,T123,T133
***COMPUTE THE TRANSFORMATION MATRIX***

CTH = COS(THR)
STH = SIN(THR)
CPH = COS(PHIR)
SPH = SIN(PHIR)
CPS = COS(PSIR)
SPS = SIN(PSIR)
T111 = CTH * CPS
T112 = CTH * SPS
T113 = -STH
T121 = CPS * STH * SPH - SPS * CPH
T122 = CPS * CPH + SPS * SPH * STH
T123 = CTH * SPH
T131 = SPS * SPH + STH * CPH
T132 = SPS * STH * CPH - CPS * SPH
T133 = CPH * CTH

DO 1100 I = 1, 6
    XT1 = XF(I) - XR
    XT2 = YF(I) - YR
    XT3 = ZF(I) - ZR
    XB1 = XT1 * T111 + XT2 * T112 + Xt3 * T113
    XB2 = XT1 * T121 + XT2 * T122 + XT3 * T123
    XB3 = XT1 * T131 + XT2 * T132 + XT3 * T133
    01 = XEl - H1(I)
    02 = XB2 - H2(I)
    03 = XB3 - H3(I)

***COMPUTE THE HIP AZIMUTH ANGLE***

IF(ABS(01).GT.0.001) GO TO 993
PSIC(I)=3.14159/2.0*SIGN(I,0,H2(I))
GO TO 991
990 PSIC(I)=ATAN(D2/D1)
991 IF(D1.LE.-3.0) PSIC(I)=PSIC(I)+3.14159*SIGN(I,0,D2)

***COMPUTE THE KNEE ANGLE***

IF(ABS(SIN(PSIC(I))).LT.0.5) GO TO 992
DEN=D2/SIN(PSIC(D))
GO TO 993
992 DEN=D1/COS(PSIC(I))
993 IF((DEN**2+D3**2-L1**2-L2**2)/(2.0*L1*L2).LT.0.99)
    GO TO 994
    TYPE 503,I
503 FORMAT(1X,13)
    TYPE 50, XF, YF, ZF
    TYPE 50, XR, YR, ZR, VEL, VELY, VELZ, THR, PHIR, PSIR,
    DTHR, DPHIR, DPSIR
50 FORMAT(1X,6F11.4)
PAUSE 'LEG IS BEING STRETCHED'

994 TH2C(I)=ASIN((DEN**2+DS**2+L3**2)/(2*L1*L2))

***COMPUTE THE HIP ELEVATION ANGLE***

A=L1+L2*SIN(TH2C(I))
B=L2*COS(TH2C(I))
DEN=SQR(TH1C(I)**2+B**2-2*L1*L2*COS(TH1C(I)))
TH1C(I)=ASIN(DEN)

***COMPUTE THE KNEE POSITION***

XK(I)=L1*COS(TH1C(I))*COS(PSIC(I))
YK(I)=L1*COS(TH1C(I))*SIN(PSIC(I))
ZK(I)=-L1*SIN(TH1C(I))
DXF=(-DTH2*CPS-DPSIC*T112)*XT1+DPSIR*T111*DTHR*X(T1)*XT2
DYF=(DTH2*SPH-DPSIR*T122)*XT1+(DTH2*SPH+DPSIR*T132)*XT2-DTHR*CTH*X(T1)*XT3
DZF=(DTH2*CPH-DPSIR*T123)*XT1+(DTH2*CPH+DPSIR*T131)*XT2-DTHR*SPH*X(T1)*XT3

CPSI=COS(PSIC(I))
SPSI=SIN(PSIC(I))
CTH2=COS(TH2C(I))
STH2=SIN(TH2C(I))

***COMPUTE THE HIP AZIMUTH ANGLE RATE***

IF(ABS(D2).LT.0.0031) GO TO 872
DEN=(CPSI/D2)**2
GO TO 873
872 DEN=(CPSI/D2)**2
873 DPSIC(I)=DEN/(D1*DYF-D2*DXF)

***COMPUTE THE KNEE ANGLE RATE***

IF(Abs(SPSI).LT.0.5) GO TO 874
DEN=(2.0*SPSI**2*D2*DYF+D2**2*SIN(2.0*PSIC(I)))/SPSI**4
GO TO 875
874 DEN=(2.0*PSIC(I)**2*D1*DXF+D1**2*SIN(2.0*PSIC(I)))/SPSI**4
875 DTH2C(I)=(DEN+2.0*D3*DZF)/(2.0*L1*L2*CTH2)

***COMPUTE THE HIP ELEVATION ANGLE RATE***

TEMP1=L2*CTH2*DTH2C(I)
TEMP2 = −L2 × STH2 × DTH2C(I)
DEN = A × DZF × TEMP1 × D3 + TEMP2 × SORT(A × TEMP1 − D3 × ZF + P × TEMP2)
1 + 3 / SORT(A × TEMP1 × D3 × ZF + P × TEMP2)
TEMP3 = SIN(TH1C(I)) × (2.3 × A × TEMP1 + 3 × B × TEMP2)
DTH1C(I) = (DEN − TEMP3) / (COS(TH1C(I)) × (A × D3 + B × 2))

1100 CONTINUE
RETURN
END

SUBROUTINE STATIC FORCE GENERATES THE 6 COMPONENTS
OF THE DESIRED FORCE AND MOMENT ACTING ON THE BODY
THAT ARE TO BE PRODUCED BY THE JOINT TORQUES OF THE
LEGS. IT ALSO CALCULATES THE JOINT TORQUES FOR
THOSE LEGS IN THE TRANSFER PHASE AND THE COMPONENTS
OF THE FOOT REACTIONS DUE TO THE STATIC WEIGHT OF
THE LEG.

SUBROUTINE STCFOR(K, IDEN)
REAL L, L1, L2, M, H
LOGICAL K(6), K
DIMENSION H1(6), H2(6), H3(6), TPSI(6), TTH(6), TK(6)
DIMENSION PSI(6), TH1C(6), TH2C(6), T1(3, 3),
T3(3, 3), T9(3, 3), XMM(3), XM1(3), XM2(3),
2XF(6), YF(6), ZF(6), XI(3), FR(3, 6)
DIMENSION F(3), T(3)
COMMON/CJAV/PSI, TH1C, TH2C
COMMON/TV/TPSI, TTH, TK, X/F, T, FR
COMMON/BPPV/H1, H2, H3, L, L1, L2, DT, XMAS, XI, XMAS1, XMAS2
COMMON/T1/T1
COMMON/XLPV/XF, YF, ZF, M
DO 800 I = 1, 3
F(I) = 3, 3
T(I) = 0, 3

800 ***COMPUTE THE COMPONENTS OF THE STATIC WEIGHTS***
***OF THE LEG IN LEG COORDINATES***
DO 900 I = 1, 6
TH1 = COS(TH1C(I))
STH1 = SIN(TH1C(I))
TH2 = COS(TH2C(I))
STH2 = SIN(TH2C(I))
CPSI = COS(PSIC(I))
SPSI = SIN(PSIC(I))
CTH = COS(TH1C(I) + TH2C(I))
STH = SIN(TH1C(I) + TH2C(I))
T3(1, 1) = CTH × CPSI
T3(2, 1) = −SPSI
T3(3, 1) = CPSI × STH1
T3(1, 2) = SPSI × CTH1
T3(2, 2) = CPSI
T3(3, 2) = SPSI × STH1
\[ T3(1,3) = -STH1 \]
\[ T3(2,3) = 0.0 \]
\[ T3(3,3) = CTH1 \]
\[ T9(1,1) = CTH2 \]
\[ T9(1,2) = 0.0 \]
\[ T9(1,3) = -STH2 \]
\[ T9(2,1) = 0.0 \]
\[ T9(2,2) = 1.0 \]
\[ T9(2,3) = 0.0 \]
\[ T9(3,1) = T9(1,3) \]
\[ T9(3,2) = 0.0 \]
\[ T9(3,3) = T9(1,1) \]
\[ XM(1) = 2.0 \]
\[ XM(2) = 0.0 \]
\[ XM(3) = 32.0 \times XMAS2 \]
\[ DO 10 J = 1,3 \]
\[ FR(J,1) = 0.0 \]
\[ DO 10 K = 1,3 \]
\[ FR(J,1) = FR(J,1) + T1(J,K) \times XMM(K) \]
\[ DO 20 J = 1,3 \]
\[ XM(J) = 0.0 \]
\[ DO 20 K = 1,3 \]
\[ XM(J) = XM(J) + T3(J,K) \times FR(K,1) \]
\[ DO 30 J = 1,3 \]
\[ XM2(J) = 0.0 \]
\[ DO 30 K = 1,3 \]
\[ XM2(J) = XM2(J) + T9(J,K) \times XM(K) \]
\[ FPX = XM2(J) / 2.0 / CTH2 - XM1(J,3) / 2.0 - STH2 / CTH2 + XM1(J,1) \]
\[ FPY = FPY / (L1 * CTH1 + L2 * STH) + XM1(J,2) \]
\[ FPZ = XM1(J,3) / 2.0 \]
\[ FX = T3(1,1) \times FPX + T3(2,1) \times FPY + T3(3,1) \times FPZ \]
\[ FY = T3(1,2) \times FPX + T3(2,2) \times FPY + T3(3,2) \times FPZ \]
\[ FZ = T3(1,3) \times FPX + T3(2,3) \times FPY + T3(3,3) \times FPZ \]

*** COMPUTE THE COMPONENTS OF THE FOOT REACTION ***

*** DUE TO THE STATIC HEIGHT OF THE LEG ***

\[ DO 40 J = 1,3 \]
\[ FR(J,1) = -FR(J,1) \times (1.0 + XMAS1 / XMAS2) \]
\[ FR(1,1) = FX + FR(1,1) \]
\[ FR(2,1) = FY + FR(2,1) \]
\[ FR(3,1) = FZ + FR(3,1) \]
\[ XM = H2(1) \times FZ - H3(1) \times FY \]
\[ YM = H3(1) \times FX - H1(1) \times FZ \]
\[ ZM = H1(1) \times FX + H2(1) \times FX \]
\[ MH = FPY \times (-L1 \times STH1 + L2 \times CTH) - XM1(2) \times L1 / 2.0 \times STH1 \]
\[ TX = -MH \times CPSI \]
\[ TY = -MH \times SPSI \]
\[ TZ = 0.0 \]
***COMPUTE THE JOINT TORQUES FOR THOSE LEGS***
*** IN THE TRANSFER PHASE ***

IF(M(I)) GO TO 893

TK(I)=-XM2(1)*L2/2.
TTH(I)=(L1/L2*STH2*1.3)*TK(I)-XM2(1)/2.0*STH2*L1
1+XM2(3)*L1/2.
TPSI(I)=-XM2(2)*L2/2.
890 CONTINUE

***COMPUTE THE 6 COMPONENTS OF DESIRED BODY***
*** FORCES AND MOMENTS ***

T(1)=T(I)+TX+XM
T(2)=T(2)+TY+YM
T(3)=T(3)+TZ+ZN
F(1)=F(1)+FX
F(2)=F(2)+FY
F(3)=F(3)+FZ
900 CONTINUE

RETURN

END

SUBROUTINE FORCE SETS UP THE LINEAR PROGRAMMING
PROBLEM AND CALLS SUBROUTINE LINEAR PROGRAMMING TO
SOLVE THE PROBLEM AND GIVE THE RESULTING JOINT
TORQUES FOR THE LEGS IN THE SUPPORTING PHASE.

SUBROUTINE FORCE

REAL L,L1,L2
LOGICAL M(6),K
DIMENSION H1(6),H2(6),H3(6),TPSI(6),TH1(6),TK(6)
DIMENSION PSIC(6),TH1C(6),TH2C(6),DPSIC(6),
1DTH1C(6),DTH2C(6),T5(3,3),T3(3,3),2X(6),YF(6),ZF(6),XI(3)
2X(6),YF(6),ZF(6),XI(3)
DIMENSION F(3),T(3),C(37),A(55,92),
1CB(54),NBS(54),FR(3,6)
COMMON/CJAV/PSIC,TH1C,TH2C,DPSC,DT1C,DT2C
COMMON/CTV/TPSI,TH1,TK/F/F,T,F
COMMON/BPV/H1,H2,H3,L1,L2,DT,XMAS,XI,XMAS1,XMAS2
1,TPSI1,TH1C,TH2C,BALANCE/TPV/FRCTM
COMMON/XLPV/XP,F,ZF
COMMON/MPS/ADIM,NDIM,C,A,CS,NBS
COMMON/STATE/XR,YR,XF,VEL,VLY,VELZ
1THR,PHIR,PSIR,DTHR,DPHIR,DPISIR
COMMON/XMPV/R,GAFV,GAFT,XN,ETAF,ETAR
DO 5 I=1,55
DO 5 J=1,92
5 A(I,J)=0.0
***COMPUTE THE INVERSE JACOBIAN MATRIX AND WEIGHTS***
*** VECTORS FOR THE EQUALITY CONSTRAINTS ***

NDIM=0
MDIM=0

CPH=COS(PHI*R)
SPH=SIN(PHI*R)

CTH=COS(THI*R)
STH=SIN(THI*R)

T2(1,1)=0.0
T2(1,2)=CPH
T2(1,3)=SPH
T2(2,1)=1.0
T2(2,2)=STH/CTH*SPH
T2(2,3)=STH/CTH*CPH
T2(3,1)=0.4
T2(3,2)=SPH/CTH
T2(3,3)=CPH/CTH

PR=T2(2,1)*THI*R+T2(2,2)*DPHI*R+T2(3,1)*DPSI*R
QR=T2(1,2)*THI*R+T2(1,3)*DTHI*R+T2(2,3)*DPSI*R

F(1)=F(1) + XMAS*32.2 + STH - XMAS*
1(VELY*PR-VELZ*QR)
F(2)=F(2) - XMAS*32.2 + CTH*SPH - XMAS*
1(VELZ*PR-VEL*QR)
F(3)=F(3) - XMAS*32.2 + CTH*SPH - XMAS*
1(VEL*QR-VELY*PR)

T(1)=T(1) - (XI(2) - XI(3)) * QR * RR
T(2)=T(2) - (XI(1) - XI(3)) * RR * PR
T(3)=T(3) - (XI(1) - XI(2)) * PR * QR
D0 20 I=1,3

CTHI=COS(THIC(I))
STHI=SIN(THIC(I))

CTH2=COS(TH2C(I))
STH2=SIN(TH2C(I))

CPSI=COS(PSIC(I))
SPSI=SIN(PSIC(I))

CTH=COS(THIC(I)) + TH2C(I))
STH=SIN(THIC(I)) + TH2C(I))

T2(1,1)=CTH1*CPSI
T2(1,2)=SPSI
T2(1,3)=CTH1
T2(2,1)=CTH1*STH1
T2(2,2)=CPSI
T2(2,3)=SPSI*STH1
T2(3,1)=STH1
T2(3,2)=SPSI*STH1
T2(3,3)=CTH1

TB(1,1)=0.0
TB(2,1)=-1.3/(L1*CTH1 + L2*STH)
TB(3,1)=0.3
TB(1,2)=STH2/CTH2/L1
T8(2,2)=3.3
T8(3,2)=1.3/L1
T8(1,3)=-1.3/CTH2/L2=STH2/CTH2/L1
T8(2,3)=3.3
T8(3,3)=-1.3/L1
DO 30 II=1,3
DO 30 JJ=1,3
T5(II, JJ)=0.3
DO 30 J=1,3
30 T5(II, JJ)=T5(II, JJ)+T2(J, II)*T8(J, JJ)
T2(1,1)=0.3
T2(2,1)=H3(1)
T2(3,1)=-H2(1)
T2(1,2)=H3(1)
T2(2,2)=0.3
T2(3,2)=H1(1)
T2(1,3)=H2(1)
T2(2,3)=-H1(1)
T2(3,3)=0.3
DO 40 II=1,3
DO 40 JJ=1,3
T8(II, JJ)=0.3
DO 40 J=1,3
40 T8(II, JJ)=T8(II, JJ)+T2(II, J)*T5(J, JJ)
T8(1,1)=CPSI*(L1*STH1-L2*CTH)/L1*CTH1+L2*STH)+T3(1,1)
T8(2,1)=SPSI*(L1*STH1-L2*CTH)/L1*CTH1+L2*STH)+T3(2,1)
T8(3,1)=-1.0+T8(3,1)
T8(1,2)=SPSI+T8(1,2)
T8(2,2)=CPSI+T3(2,2)
IF(A(I)) GO TO 10
DO 50 J=1,3
F(J)=F(J)-T5(J, 1)*TPSI(I)-T5(J, 2)*TTH(I)-T5(J, 3)*TK(I)
50 T(J)=T(J)-T8(J, 1)*TPSI(I)-T3(J, 2)*TTH(I)-T8(J, 3)*TK(I)
GO TO 20
10 CONTINUE

***SIX EQUALITY CONSTRAINTS***

DO 70 II=1,3
MDIM=MDIM+1
DO 70 JJ=1,2
NDIM=NDIM+1
A(MDIM+NDIM+1)=1.3
DO 70 J=1,3
A(J, NDIM+1)=T5(J, II)
A(J+3, NDIM+1)=T8(J, II)
IF(JJ.EQ.2) A(J, NDIM+1)=-A(J, NDIM+1)
IF(JJ.EQ.2) A(J+3, NDIM+1)=-A(J+3, NDIM+1)
70 CONTINUE
ETAP=ETAP
ETAN=1.3/ETAN
IF(DPSIC(I).GT.3) GO TO 71
ETAP=1.0/ETA1
ETAN=ETA1
71 C(NDIM-5)=-(R+GAFV*ABS(DPSIC(I))*XN)/ETAP
C(NDIM-4)=C(NDIM-5)*ETAP/ETAN
ETAP=ETA1
ETAN=ETA1
IF(DTHIC(I).GT.3) GO TO 72
ETAP=1.0/ETA2
ETAN=ETA2
72 C(NDIM-3)=-(R+GAFV*ABS(DTHIC(I))*XN)/ETAP
C(NDIM-2)=C(NDIM-3)*ETAP/ETAN
ETAP=ETA2
ETAN=ETA2
IF(DTH2C(I).GT.3) GO TO 73
ETAP=1.0/ETA3
ETAN=ETA3
73 C(NDIM-1)=-(R+GAFV*ABS(DTH2C(I))*XN)/ETAP
C(NDIM)=C(NDIM-1)*ETAP/ETAN
CONTINUE
DO 95 J=1,3
A(J,1)=F(J)
95 A(J+3,1)=F(J)
C
C ***MAXIMUM TORQUE INEQUALITY CONSTRAINTS***
C
II=1
DO 110 I=1,(MDIM-6)/3
II=II+1
A(II+5,NDIM+II+1)=1.3
CB(II+5)=I,0
NBS(II+5)=NDIM+II+1
GO TO (120,130,140), J
120 A(II+5,1)=IPSI:
GO TO 150
130 A(II+5,1)=ITHM
GO TO 150
140 A(II+5,1)=IK:
150 CONTINUE
110 CONTINUE
C
C ***LOAD BALANCING INEQUALITY CONSTRAINTS***
C
II=1
NDIM=NDIM+1
C(NDIM)=BALNC
DO 165 I=1,6
IF(.NOT.M(I)) GO TO 166
MDIM=MDIM+1
MM=MDIM-6+NDIM
165 CONTINUE
**VARIOUS VARIABLES**

```
** CONTINUE 001
DO 90 J=1,INFO+1
IF (A(I,J).GT.2) GO TO 100
DO 10 I=1,INFO
** CONTINUE 124
II=II+6
** CONTINUE 153
A(MDNM-2*(I))=A(I)**2+2*(A(I)+A(I+1))/2+(A(I)+A(I+1))**2
A(MDNM-3*(I))=A(I)**2+2*(A(I)+A(I+1))/3+(A(I)+A(I+1))**2
0*0=0
A(MDNM-4*(I))=A(I)**2+2*(A(I)+A(I+1))/4+(A(I)+A(I+1))**2
A(MDNM-5*(I))=A(I)**2+2*(A(I)+A(I+1))/5+(A(I)+A(I+1))**2
A(MDNM-6*(I))=A(I)**2+2*(A(I)+A(I+1))/6+(A(I)+A(I+1))**2
** CONTINUE 166
IF (NOT(I)) GO TO 152
DO 191 J=1,6
II=II+6
** CONTINUE 167
A(MDNM-2*(I))=A(I)**2+2*(A(I)+A(I+1))/2+(A(I)+A(I+1))**2
A(MDNM-3*(I))=A(I)**2+2*(A(I)+A(I+1))/3+(A(I)+A(I+1))**2
A(MDNM-4*(I))=A(I)**2+2*(A(I)+A(I+1))/4+(A(I)+A(I+1))**2
A(MDNM-5*(I))=A(I)**2+2*(A(I)+A(I+1))/5+(A(I)+A(I+1))**2
A(MDNM-6*(I))=A(I)**2+2*(A(I)+A(I+1))/6+(A(I)+A(I+1))**2
```

```
DO 160 I=1,6
CB(I)=-1000.3
NBS(I)=MDIM-5+NDIM+I
160 A(I,NBS(I))=1.0

***CALL THE LINEAR PROGRAMMING SUBROUTINE***

CALL LINPRO

***COMPUTE THE JOINT TORQUES FOR THE LEGS***
*** IN THE TRANSFER PHASE ***

DO 170 I=1,NDIM
170 A(MDIM+1,I)=3.0
DO 180 I=1,NOIN
180 A(MDIM+1,NBS(I))=A(I,1)

NDIM=0
DO 190 I=1,6
IF(.NOT.M(I)) GO TO 139
TPSI(I)=A(MDIM+1,NDIM+1)-A(MDIM+1,NDIM+2)
TTH(I)=A(MDIM+1,NDIM+3)-A(MDIM+1,NDIM+4)
TK(I)=A(MDIM+1,NDIM+5)-A(MDIM+1,NDIM+6)
NDIM=NDIM+6
189 CONTINUE
190 CONTINUE
RETURN
END

SUBROUTINE LINEAR PROGRAMMING SOLVES THE LP
PROBLEM BY THE SIMPLEX METHOD

SUBROUTINE LINPRO
DIMENSION C(37),CN(92),CB(54),NBS(54),A(55,92),TEMP(55)
REAL MN
COMMON/MPS/M,N,C,A,CB,NBS
NX=0
N1=M-6+N
N3=6
N2=N3+N1
NN1=N1+1
DO 120 J=1,NN1
120 CW(J)=0.0
NN1=NN1+2
NN2=NN1+1
DO 130 J=NN1,NN2
130 CW(J)=-1000.0

*** CALCULATE ZJ - CJ ***
NFLAG=1
NW=N2+1
500 DO 140 J=1,N
A(M+1,J)=-C(J)
DO 140 I=1,N
140 A(M+1,J)=A(M+1,J)+CB(I)*A(I,J)

*** FIND MIN(ZJ - CJ) ***

1000 MIN=A(M+1,2)
NX=NX+1
GO TO 116

TYPE I11,NX
111 FORMAT(15)
DO 112 III=1,10
DO 114 JJJ=1,55
WRITE (3,113) (ACJJJ, (III-1)*10+JJ),JJ=M0)
113 FORMAT(1X,10F10.4)
114 CONTINUE
WRITE (3,115) III
115 FORMAT(1H1,I4)
112 CONTINUE
116 CONTINUE

K=2
NNW=NW-1
DO 151 J=2,NNW
IF(A(M+1,J+1).GE.MIN) GO TO 150
K=J+1
MIN=A(M+1,J+1)
150 CONTINUE
151 CONTINUE

*** OPTIMUM SOLUTION POSSIBLE? ***

IF(NFLAG.EQ.1) GO TO 160
IF(MIN.GE.0.0) GO TO 2000
GO TO 170

160 IF((MIN.GE.0.0).AND.(A(M+1,1).LT.0.0)) GO TO 4000
IF((MIN.LT.0.0).AND.(A(M+1,1).LT.0.0)) GO TO 170

180 CONTINUE
IF(NTEMP.GT.(N1+1)) GO TO 5000
NW=N1+1
NFLAG=2
DO 190 J=1,N
190 CW(J+1)=C(J)
DO 200 I=1,M
NTEMP=NBS(I)
200 CB(I)=CW(NTEMP)
GO TO 530
*** FIND VECTOR GOING OUT OF BASIS ***

MIN=1000.0
MM=M+1
NR=0
DO 250 I=1,M
IF(A(I,K).LE.0.0) GO TO 210
IF((A(I,1)/A(I,K)).GT.MIN) GO TO 210
NR=I
MIN=A(I,1)/A(I,K)
210 CONTINUE
250 CONTINUE
IF(NR.EQ.0) GO TO 3000

*** CALCULATE A(R,J) ***
Pivot=A(NR,K)
DO 220 J=1,NW
220 A(NR,J)=A(NR,J)/Pivot
DO 240 I=1,MM
240 TEMP(I)=A(I,K)

*** CALCULATE A(I,J) ***
DO 231 I=1,MM
IF(I.EQ.NR) GO TO 231
DO 230 J=1,NW
A(I,J)=A(I,J)-TEMP(I)*A(NR,J)/A(NR,K)
230 CONTINUE
231 CONTINUE

*** CHANGE NBS,CB ***
NBS(NR)=K
CB(NR)=CWOC)
GO TO 1000

3000 WRITE(5,3001)
3001 FORMAT(20X,'NO OPTIMAL SOLUTION EXISTS')
RETURN
4000 WRITE(5,4001)
4001 FORMAT(20X,'NO FEASIBLE SOLUTION EXISTS')
RETURN
5000 WRITE(5,5001)
5001 FORMAT(20X,CASE III OF PHASE I RESULTED')
RETURN
2000 TYPE 111,NX
DO 2002 I=1,M
NBS(I)=NBS(I)-1
2002 CONTINUE
RETURN
END
SUBROUTINE FOOT REACTION Calculates the foot reaction
vectors for those legs in support of the vehicle.
Also, the instantaneous motor input power is calculated
from a knowledge of motor and gear train
characteristics.

SUBROUTINE FTRCTN
REAL L, L1, L2
LOGICAL H(6), K
DIMENSION H1(6), H2(6), H3(6), TPSI(6), TTH(6), TK(6)
DIMENSION PSIC(6), THIC(6), TH2C(6), DPSIC(6), DTHIC(6),
DTH2C(6), T5(3,3), T8(3,3), T2(3,3), POWER(3)
DIMENSION F(3), T(3), FR(3,6)
COMMON/CJAV/PSIC, THIC, TH2C, DPSIC, DTHIC, DTH2C
COMMON/CTV/TPSI, TTH, TK, POWER/F, F, FR
COMMON/XMPV/Hi, H2, H3, L, L1, L2
COMMON/XMPV/R, GAFV, GAFT, XN, ETA F, ETA R, BM, TS
POWER(1)=2.0
POWER(2)=1.0
POWER(3)=3.0

***Compute the inverse Jacobian matrix***

DO 20 I=1,6
CTHI=COS(THIC(I))
STHI=SIN(THIC(I))
CTH2=COS(TH2C(I))
STH2=SIN(TH2C(I))
CPSI=COS(PSIC(I))
SPSI=SIN(PSIC(I))

CTH=COS(THIC(I)+TH2C(I))
STH=SIN(THIC(I)*TH2C(I))

T2(1,1)=CTHI*CPSI
T2(2,1)=-SPSI
T2(3,1)=CPSI*STHI
T2(1,2)=SPSI*CPSI
T2(2,2)=CPSI
T2(3,2)=SPSI*STHI
T2(1,3)=-STHI
T2(2,3)=0.0
T2(3,3)=CTHI
T8(1,1)=0.0
T8(2,1)=-1.0/(L1*CTHI+L2*STHI)
T8(3,1)=0.0
T8(1,2)=STH2/CTH2/L1
T8(2,2)=1.0/L1
T8(3,2)=1.0/CTH2/L2-STH2/CTH2/L1
T8(2,3)=0.0
T8(3,3)=-1.0/L1
DO 30 II=1,3
DO 30 JJ=1,3
T5(II, JJ)=0.0
DO 30 J=1,3
T5(I,J,J)=T5(I,J,J)+T2(J,I)*T8(J,J,J)

***COMPUTE THE FOOT REACTION FORCES***

DO 50 J=1,3
FR(J,1)=FR(J,1)+T5(J,1)*TPSI(I)+T5(J,2)*TTH(I)+T5(J,3)*T'<(I)

*** COMPUTE THE INSTANTANEOUS POWER ***
***REQUIREMENT FOR EACH OF THE JOINTS***

ETAP=ETAP
IF(DPSIC(I)>TPSI(I),LT.0.0) ETAP=1./ETAR
POWER(I)=POWER(I)+(R+GAFV*XN)*ABS(DPSIC(I))/GAFT*1
ABS(TPSI(I))/XH/ETAP+B*M*ABS(DPSIC(I))*XH+TS)

ETAP=ETAP
IF(DTH1C(I)*TTH(I),LT.0.0) ETAP=1./ETAR
POWER(2)=POWER(2)+(R+GAFV*XN)*ABS(DTH1C(I))/GAFT*1
ABS(TTH(I))/XH/ETAP+B*M*ABS(DTH1C(I))*XH+TS)

ETAP=ETAP
IF(DTH2C(I)*TTH(I),LT.0.0) ETAP=1./ETAR
POWER(3)=POWER(3)+(R+GAFV*XN)*ABS(DTH2C(I))/GAFT*1
ABS(TTH(I))/XH/ETAP+B*M*ABS(DTH2C(I))*XH+TS)

CONTINUE
RETURN
END

THE KINEMATIC DISPLAY SUBROUTINE IS THE SOFTWARE INTERFACE BETWEEN THE CRT DISPLAY AND SIMULATION PROGRAMS.

SUBROUTINE KINDIS(K,IDEN)
REAL L,L1,L2,G1,G2,G3,G4,G5,G6,G7
LOGICAL K(5),K
DIMENSION X(12),YF(6),ZF(6),XK(6),YK(6),ZK(6)
DIMENSION H(2700),PIC1(3),PIC2(3),PIC3(3),
IT(4,4),SST(4,4),ELEV(4,4),AZIM(4,4),
RROLL(4,4),AZIMP(4,4),ROLLP(4,4),TII(4,4)
COMMON/STATE/XR,YR,ZR,VEL,VELY,VELZ,
ITHR,PHIR,PSIR,THHR,DPHR,DPSIR
COMMON/XLRV/XF,YF,ZF,XX,YK,ZK
COMMON/BPPV/H1,H2,H3,L1,L1,L2
COMMON/TPV/FRCTH,G1,G2,G3,G4,G5,G6,G7
COMMON/TI/T111,T121,T131,T112,T122,T132,T113,T123,T133
ZTER=(X,DENY)=G1+G2*(G3*(DEN XX-G7)+
G4*DEN YY)+G5*(COS(G6*(DEN XX-G7))-1.0)
IF(IDEN.EQ.80) K=.FALSE.
IF(K) GO TO 1963
PITFAC=-30.0
ROLFAC=60.3
IF (IDEN.EQ.88) ACCEPT 50, PITFAC, ROLFAC

CALL DSPINI(0,2703)
CALL SETM3(SST)
CALL DCLPIC(PIC1,3)
CALL DCLPIC(PIC2,3)
CALL DCLPIC(PIC3,3)
CALL SETWINO(-60.0, 60.0, -60.0, 60.0)
CALL SETPORT(8,1625,3,1623)
CALL PITCH(PITFAC, AZIMP)
CALL ROLL(ROLFAC, ROLLP)
CALL MULM3(2, AZIMP, ROLLP, SST)
CALL GENINI(PIC3)
CALL PARMS(1,1)

DENXX=-40.0
DENYY=-40.0
DO 1077 J=1,31
DENZZ=ZTERR(DENXX, DENYY)
IF (DENXX.LE.7) DENZZ=0.0
CALL PGEN3 (DENXX, DENYY, DENZZ,1)
DENZZ=ZTERR(DENXX, DENYY+80.0)-DENZZ
IF (DENXX.LT.7) DENZZ=0.0
CALL VGEN3(0.0, 30.0, DENZZ,1)
1077 DENXX=DENXX+1.0
CALL PGEN3 (-40.0, -40.0, 3.0, 1)
CALL VGEN3(80.0, 0.0, 3.0, 1)
CALL VGEN3(0.0, 0.0, ZTERR(43.0, 40.3), 1)
CALL GENEND (PIC3)
CALL SETM3(T)
LLL=1

1060 THRR=THR*130.0/3.14159
PSIRR=PS12*130.0/3.14159
PHIRR=PH12*183.0/3.14159
CALL YAN(-THRR, ELEV)
CALL PITCH (PSIRR, AZIMP)
CALL ROLL(-PHIRR, ROLLP)
CALL TRANSL(XR, -YR, ZR, TTH)
CALL MULM3(6, AZIMP, ROLLP, ELEV, TTH, AZIMP, ROLLP, T)
GO TO (1061, 1062), LLL

1061 CALL GENINI(PIC1)
GO TO 1063
1062 CALL GENINI(PIC2)
1063 CALL PARMS(1,5)
DEN=-H3(2)
DENXX=-2.3*H1(2)
DENYY=2.0*H2(2)
DENZZ=2.0*H3(2)
CALL PGEN3 (H1(1), H2(1), H3(1), 0)
DO 1064 I=1,2
CALL VGEN3(J, 0, DENYY, 0.0, 1)
CALL VGEN3(DENXX, 0.0, 0.0, 1)
DENYY=-DENYY
CALL VGEN3(0.0,0.0,0.0,1)
DENXX=-DENXX
CALL VGEN3(DENXX,0.0,0.0,1)
DENXX=-DENXX
DENYY=-DENYY
1064 CALL PG3N3(H1(1),H2(1),DEN,0)
   DO 1065 I=1,6
   CALL PGEN3(H1(I),-H2(I),DEN,0)
   CALL VGEN3(0.0,0.0,DENZZ,1)
   XTi=XF(I)-XR
   XT2=YT(I)-YR
   XT3=ZP(I)-ZR
   XBI=XT1*T111+XT2*T112+XT3*T113
   XBI2=XT1*T121+XT2*T122+XT3*T123
   XB1=XT1*T131+XT2*T132+XT3*T133
   D1=XBI-H1(I)
   D2=XB2-H2(I)
   D3=XB3-H3(I)
   TEMP1=D1-XK(I)
   TEMP2=D2-YX(I)
   TEMP3=D3-ZX(I)
   CALL VGEN3(X(1),-YK(I),ZK(I),1)
   CALL VGEN3(TEMP1,-TEMP2,TEMP3,1)
   CONTINUE
   GO TO (1066,1067),LLL
   1066 CALL GENEND(PIC1)
   IF(K) CALL DESTROY (PIC2)
   GO TO 1068
   1067 CALL GENEND(PIC2)
   CALL DESTROY (PIC1)
   1068 LLL=3-LLL
   IF(XR.GT.40.0.OR.XR.LT.-40.0) CALL EXIT
   IF(YR.GT.40.0.OR.YR.LT.-40.0) CALL EXIT
   RETURN
   END

C THIS IS THE MAIN PROGRAM, MECHS.F4, FOR THE COMPUTER
C SIMULATION OF THE MECHANICAL SIMULATION. ALL NON-
C LINEARITIES (BACKLASH, MOTOR SATURATION, NORM GEAR
C EFFICIENCY, ETC.) ARE INCLUDED.
C REAL K1,K2,K3,K4,JM,N,J,JO
C DIMENSION X(2,6),DX(2,6),DX%(2,6),Y(24),XC(2,7)
C K1(2),K2(2),K3(2),K4(2),MODE(2),TM(2),T(2)
C COMMON/Y/K1,K2,K3,K4,JO,B1,JM,N,J
C COMMON/XC/XC
C EQUIVALENCE(K1,Y(1))
I. SYSTEM INITIALIZATION

A. SET THE PRINT PERIOD

ACCEPT 80, NPRINT
FORMAT(2I5)

B. ARRAY INITIALIZATION

PARAT=5HMECHS
CALL IFILE(1, PARAT)
READ(1, 70) Y, X

PARAT=5HMECHS
CALL IFILE(1, PARAT)
READ(1, 70) Y, X

C. SET THE TIME INCREMENT

ACCEPT 90, DT
FORMAT(F)

D. SET THE NUMBER OF ACTUATORS

ACCEPT 80, NN
MODE(1)=1
MODE(2)=1
TIME=0.0
NPRINT=NPRINT

IF(NNPRINT-NPRINT.LT.0) GO TO 60
NPRINT=0
TYPE 110, ((X(I,K), I=1, NN), K=1, 6), TIME
GO TO 60
TYPE 110, ((XC(I,K), I=1, NN), K=1, 6)
TYPE 80, MODE
FORMAT(1X, 7F10.4)

II. INPUT SYSTEM COMMANDS

CALL CMDM(TIME)

DO 800 K=1, NN

III. MOTOR VOLTAGE(V), CURRENT(XI), TORQUE(TM)

E=(XC(K,4)-X(K,4))*K2(K)+(XC(K,2)-X(K,2))*K4(K)+XC(K,7)
V=0.0
IF(ABS(E).LT.0.80) GO TO 810
E=E-SIGN(0.74, E)
EE=ABS(E)
V=2.84+69.63*EE-45.79*EE**2+15.95*EE**3-2.854*EE**4
1+0.2490*EE**5-2.836*EE**6
V=SIGN(V, E)
CONTINUE

810 CONTINUE
CONS=GV*ABS(X(K,4))
GO TO (260,270,280), MODE(K)

B. EFFICIENCY

ETA=E0TAF
IF(I(M(K),LT.0,0)) ETA=-1.0/ETA
GO TO 285

ETA=E0TAF
IF(I(M(K),GT.0,0)) ETA=-1.0/ETA

C. UNLOCKED

DX(K,2)=X(K,5)
   DX(K,3)=X(K,6)
   DX(K,5)=THETA(K)/N/JV-T(K)/N/2.0/JM/ETA-X(K,5)*BM*JN
   DX(K,6)=B3*(X(K,5)-X(K,6))/J+K3(K)*(X(K,2)-X(K,3))/J
GO TO 290

D. NOT ENGAGED

DX(K,1)=X(K,4)
   DX(K,2)=X(K,5)
   DX(K,3)=X(K,6)
   DX(K,4)=THETA(K)/JH-3H/JH*X(K,4)
   DX(K,5)=-(33+8G)/JG*X(K,5)+33/JG*X(K,6)+K3(K)/JG*1*(X(K,2)-X(K,3))
   DX(K,6)=B3/J*(X(K,5)-X(K,6))+K3(K)/J*(X(K,2)-X(K,3))

VII. NEW STATE VECTOR

XOLD=X(1,6)
DO 440 I=1,6
   X(K,1)=X(K,1)+DX(K,1)+DXM(K,1)*DT/2.0
440   DXM(K,1)=DX(K,1)
GO TO (310,320,330), MODE(K)

A. GEAR ENGAGED FORWARD

DXM(K,1)=N**DXM(K,2)
DXM(K,4)=N**DXM(K,5)
X(K,1)=X(K,2)*N
X(K,4)=X(K,5)*N
GO TO 800

B. GEAR NOT ENGAGED?

IF((N*X(K,2),GT.X(K,1)) .AND. 
1(N*X(K,2)-BACKL,LT.X(K,1))))
1 GO TO 800
IF(X(K,1),LE.N*X(K,2)-BACKL) GO TO 303
   MODE(K)=1
   GO TO 310
END
RETURN
XC(2,0)=DIH/145°
XC(1,0)=DIH/145°
XC(2,5)=DIH/145°
XC(1,5)=DIH/145°
XC(2,1)=DIH/145°
XC(1,4)=DIH/145°
XC(2,4)=DIH/145°
XC(1,1)=DIH/145°
XC(2,2)=DIH/145°
XC(1,2)=DIH/145°"'

THL=(1.0-EXP(-11E6/5)+22.0*TIME)/145°
THL=22.0*TIME/145°

DTHL=22.0
XC(2,7)=1.342779
XC(1,7)=1.342779
XC(1,4)=1.342779

DIMENSION X(2,7)
SUBROUTINE COMMON TIME
COMMON/X/XC
MAIN PROGRAM
C
C
REFERENCE (COMAND TO THE MECHANICAL SIMULATION)
C
C
SUBROUTINE COMMAND (MECHANICAL PROVIDES KINEATICS)
C
C
COPYRIGHT 1985

END
STOP
GO TO 060
X(2,0)=X(1,0)
X(1,0)=X(2,0)-XOLD
CONTINUE
GO TO 000

X(K+4)=X(K,4)
X(K,4)=X(K,3)-BACKL
DAXA(X,4)=DAXA(X,3)+330
MODE(X)=K+3
C
C
C
C
C
C
C

203
### ROBOT.DAT

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### MECHS.DAT

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LIST OF REFERENCES


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