INFORMATION TO USERS

This material was produced from a microfilm copy of the original document. While the most advanced technological means to photograph and reproduce this document have been used, the quality is heavily dependent upon the quality of the original submitted.

The following explanation of techniques is provided to help you understand markings or patterns which may appear on this reproduction.

1. The sign or “target” for pages apparently lacking from the document photographed is ”Missing Page(s)”. If it was possible to obtain the missing page(s) or section, they are spliced into the film along with adjacent pages. This may have necessitated cutting thru an image and duplicating adjacent pages to insure you complete continuity.

2. When an image on the film is obliterated with a large round black mark, it is an indication that the photographer suspected that the copy may have moved during exposure and thus cause a blurred image. You will find a good image of the page in the adjacent frame.

3. When a map, drawing or chart, etc., was part of the material being photographed the photographer followed a definite method in “sectioning” the material. It is customary to begin photoing at the upper left hand corner of a large sheet and to continue photoing from left to right in equal sections with a small overlap. If necessary, sectioning is continued again — beginning below the first row and continuing on until complete.

4. The majority of users indicate that the textual content is of greatest value, however, a somewhat higher quality reproduction could be made from “photographs” if essential to the understanding of the dissertation. Silver prints of “photographs” may be ordered at additional charge by writing the Order Department, giving the catalog number, title, author and specific pages you wish reproduced.

5. PLEASE NOTE: Some pages may have indistinct print. Filmed as received.

Xerox University Microfilms
300 North Zeib Road
Ann Arbor, Michigan 48106
MARCHON, Maurice Nicholas, 1947-
TAX AVOIDANCE: A THEORETICAL
ANALYSIS.

The Ohio State University, Ph.D., 1976
Economics, finance

Xerox University Microfilms, Ann Arbor, Michigan 48106

© Copyright by
Maurice Nicolas Marchon
1976
TAX AVOIDANCE: A THEORETICAL ANALYSIS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

BY

Maurice Nicolas Harchon, B.A., M.A.

The Ohio State University
1976

Reading Committee:
Edward J. Kane
Tetsunori Koizumi
William H. Oakland

Approved By

Edward J. Kane
Department of Economics
I wish to express gratitude to the members of my dissertation committee, Professors Edward J. Kane (Chairman), Tetsunori Koizumi and William H. Oakland.

I am especially grateful to Edward Kane for his intellectual stimulation and constant moral support. He read several drafts and enriched them with positive criticisms and helpful suggestions. I am grateful to William Oakland for his constructive criticisms and my knowledge in public finance. I owe my gratitude to Tetsunori Koizumi who taught me mathematical economics.

For financial support, I am grateful to The Graduate School of The Ohio State University that granted me a University Fellowship, without which I would not have been able to study in this country.
VITA

February 28, 1947
Born - Fribourg, Switzerland

1971
B.A., University of Fribourg, Switzerland

1971-1975
University Fellowship of the Graduate School of the Ohio State University

1972-1974
Teaching Associate, Department of Economics, The Ohio State University, Columbus, Ohio

1973
M.A., The Ohio State University, Columbus, Ohio

FIELDS OF STUDY

Major Fields:

Money and Banking. Professors Ernst Baltensperger, William Dewald and Edward J. Kane

Economic Theory and Mathematical Economics. Professors D. L. Frito, Tetsunori Koizumi, and David Tarr

Public Finance. Professor William H. Oakland

Finance. Professors Jerome Baesel, and Edward J. Kane
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acknowledgments</td>
<td>ii</td>
<td></td>
</tr>
<tr>
<td>Vita</td>
<td>iii</td>
<td></td>
</tr>
<tr>
<td>List of Tables</td>
<td>vii</td>
<td></td>
</tr>
<tr>
<td>List of Figures</td>
<td>viii</td>
<td></td>
</tr>
<tr>
<td>Introduction</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Chapter 1: A Microeconomic Model of Tax Avoidance</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>Section 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.1. How to Deal with Uncertainty</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td>1.2. Presentation of the Model</td>
<td>26</td>
<td></td>
</tr>
<tr>
<td>1.3. The Necessary and Sufficient Conditions for a maximum</td>
<td>29</td>
<td></td>
</tr>
<tr>
<td>1.4. The Optimal Level of Tax Avoidance</td>
<td>34</td>
<td></td>
</tr>
<tr>
<td>Section 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Comparative-static Analysis</td>
<td>38</td>
<td></td>
</tr>
<tr>
<td>2.1. The Impact of an Increase in Riskiness of Tax-avoidance Activity</td>
<td>39</td>
<td></td>
</tr>
<tr>
<td>2.2. The Qualitative Effects of an Increase in the Expected Value of the r.v.</td>
<td>46</td>
<td></td>
</tr>
<tr>
<td>2.3. The Qualitative Effects of Changes in the Effective Flat Tax Rate</td>
<td>48</td>
<td></td>
</tr>
<tr>
<td>2.4. The Qualitative Effects of Changes in the Price of Tax-avoidance inputs</td>
<td>49</td>
<td></td>
</tr>
<tr>
<td>2.5. The Qualitative Effects of Changes in the Taxpayer's Income with a Flat Effective Tax Rate</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>2.6. The Qualitative Effect of an Increase in the Exemption Level</td>
<td>51</td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER 2. TAX AVOIDANCE AND OPTIMAL INCOME-TAX STRUCTURE

Section 1

Optimal Tax Avoidance: A General Equilibrium Analysis

1.1. Setting of the Government's Problem


1.3. Existence of a Solution, the First-order Conditions for a Maximum and Economic Interpretation of Maximizing Conditions

Section 2

Optimal-tax Structure; Taxpayers having Different Earning Ability

2.1. The model

2.2. The Taxpayer's Reaction Function

2.3. Optimal Income-tax Structure and Social Welfare Maximization

2.4. Structure of Optimal Marginal Tradeoff across Taxable Incomes

CHAPTER 3 MACROECONOMIC EFFECTS OF TAX AVOIDANCE

Section 1

A Macroeconomic Model
Section 2

Comparative-static Analysis of Discretionary Policy Measures

2.1. Comparative-static Without the Budget Constraint

2.2. Comparative-static with the Government Budget Constraint

Section 3

Can an Increase in Tax Rate be Inflationary?

SUMMARY, CONCLUSIONS AND POSSIBLE EXTENSIONS

APPENDIX 1

APPENDIX 2

APPENDIX 3

BIBLIOGRAPHY
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 1</td>
<td>H &amp; R Block: Tax-return Preparation Revenues</td>
<td>4</td>
</tr>
<tr>
<td>Table 2</td>
<td>Statistics on Adjusted Gross Income for Personal Returns with Itemized Deductions (1958-72)</td>
<td>5</td>
</tr>
<tr>
<td>Table 3</td>
<td>Tax Expenditures Estimates, by Function</td>
<td>7</td>
</tr>
<tr>
<td>Table 4</td>
<td>Recapitulation of Comparative-static Results</td>
<td>68</td>
</tr>
<tr>
<td>Table 5</td>
<td>First-period Impact Multiplier for Aggregate Demand for the Model Ignoring the Government Budget Constraint</td>
<td>118</td>
</tr>
<tr>
<td>Table 6</td>
<td>First-period Impact Multiplier for Aggregate Demand for the Model Ignoring the Government Budget Constraint</td>
<td>119</td>
</tr>
<tr>
<td>Table 7</td>
<td>First-period Effects of Selected Monetary-fiscal Policies for Aggregate Demand, and the Endogenous Variable of the Budget Constraint</td>
<td>121</td>
</tr>
<tr>
<td>List of Figures</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>----------------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>Figure 1</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>Figure 2</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>Figure 3</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>Figure 4</td>
<td>22</td>
<td></td>
</tr>
<tr>
<td>Figure 5</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>Figure 6</td>
<td>41</td>
<td></td>
</tr>
<tr>
<td>Figure 7</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>Figure 8</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>Figure 9</td>
<td>71</td>
<td></td>
</tr>
<tr>
<td>Figure 10</td>
<td>76</td>
<td></td>
</tr>
<tr>
<td>Figure 11</td>
<td>84</td>
<td></td>
</tr>
<tr>
<td>Figure 12</td>
<td>92</td>
<td></td>
</tr>
<tr>
<td>Figure 13</td>
<td>92</td>
<td></td>
</tr>
<tr>
<td>Figure 14</td>
<td>94</td>
<td></td>
</tr>
<tr>
<td>Figure 15</td>
<td>105</td>
<td></td>
</tr>
<tr>
<td>Figure 16</td>
<td>107</td>
<td></td>
</tr>
</tbody>
</table>
INTRODUCTION

Progressive taxation is widely accepted as a secure policy to raise government funds and achieve certain equity targets among individuals. A progressive income tax is one whose average effective tax rate increases as the level of income increases. In such a system, taxpayers with five times the taxable income of another would pay something more than five times as much in taxes. On the other hand, a flat or proportional tax is one which taxes each dollar of income at the same marginal rate, so that a taxpayer with five times the income of another would pay five times as much in taxes. We can see immediately how progression complicates and obscures the operation of income-tax provisions. For example, under a flat tax, the exemption of interest paid on State and Municipal bonds would only permit State and Local governments to borrow at lower rates since after-tax interest rates equate in competitive financial markets. With rising marginal rates, however, the tax-exempt bonds are relatively more attractive to individuals with large incomes simply because the dollar-tax savings is higher than the difference in coupons on taxable issues to taxpayers whose marginal tax bracket would leave them indifferent between the two opportunities. High-income taxpayers doing this is one example of tax-avoidance activity.

Tax-avoidance activity is defined as outlays of resources that have no payoff in themselves except for achieving legal tax savings. These outlays can be expressed as some percentage of that tax avoided.
For example, investors in State and Local government bonds sacrifice, say, some 38 cents to gain tax savings of $38 \leq \Theta \leq 70$, where 70 cents is the maximum statutory marginal tax rate. The taxpayer can achieve tax savings equal to his marginal tax rate minus the average tax savings that is already discounted by the market. Some tax avoidance can be pursued at a very low cost, since some tax avoidance can be achieved simply by complying with the tax laws, i.e., a bulk of tax avoidance occurs through exclusions from adjusted gross income (AGI). The taxpayers face some tax-avoidance opportunities and they can achieve some amount of tax avoidance depending on the amount of resources they devote to such an economic activity. It is the focus of my dissertation to study the taxpayer's optimization problem and the economic consequences of tax avoidance.

Empirical Evidence

Tax-avoidance activity occurs mainly in the average and high-income tax brackets. Individuals and corporations buy tax-avoidance services, such as tax-accountants' and tax-lawyers' services, to achieve deliberate tax savings by planning activities benefiting of tax privileges, preferred tax treatment and so forth.

It is impossible to estimate the size of tax-avoidance industry. It not only comprises some part of tax preparation firms, tax lawyers and their supporting staff, but should include part of the operating cost of tax management departments of large corporations. That the frequency of this last type of job is growing can be seen by observing
trend in the classified adds of the Wall Street Journal.

At the same time, the tax-avoidance industry is flourishing. Although we know of no survey of the tax-avoidance industry, in Table 1, we present the dollar amount and the rate of growth of tax-return preparation fees collected by H & R Block. Block is the largest tax-preparation firm in the U.S. In 1974, the tax-return preparation revenue for H & R Block was about $75 million. The company prepared 8,669,000 U.S. and Canadian returns during 1974; representing 10 1/2 percent of all U. S. individual income-tax returns filed during the 1974 tax season. We have to be cautious about this statistic because not all of this represents increment in tax avoidance, since individuals would have carried some out on their own. It may also reflect changing comparative advantage or simply an Engel curve effect.

To provide some perspective on the extent of tax-avoidance opportunities in the modern U.S., we present, in Table 2 the dollar amount and percentage annual growth rate of adjusted gross income on all personal returns, and with itemized deductions. The last two columns, giving the amount of itemized deductions, show their growth between 1965 and 1972. This growth is 91 percent compared to a 63 percent growth in adjusted income for returns with itemized deductions. These figures show an increase in average productivity for this class of tax avoidance over the last decade. Average productivity being defined as the ratio of itemized deductions to AGI for returns with itemized deductions. This statistic is again somewhat deceptive because inflation introduces distortions. In addition, some tax avoidance occurs from exclusions from AGI and also part of tax-avoidance activities, such as
<table>
<thead>
<tr>
<th>Year</th>
<th>Revenue</th>
<th>Percentage annual growth rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1974</td>
<td>$75,692</td>
<td>22.28</td>
</tr>
<tr>
<td>73</td>
<td>61,901</td>
<td>16.40</td>
</tr>
<tr>
<td>72</td>
<td>53,181</td>
<td>4.28</td>
</tr>
<tr>
<td>71</td>
<td>50,996</td>
<td>9.73</td>
</tr>
<tr>
<td>70</td>
<td>46,472</td>
<td>43.07</td>
</tr>
<tr>
<td>69</td>
<td>32,482</td>
<td>47.60</td>
</tr>
<tr>
<td>68</td>
<td>22,008</td>
<td>53.17</td>
</tr>
<tr>
<td>67</td>
<td>14,368</td>
<td>63.88</td>
</tr>
<tr>
<td>66</td>
<td>8,767</td>
<td>52.36</td>
</tr>
<tr>
<td>65</td>
<td>5,754</td>
<td>74.84</td>
</tr>
<tr>
<td>64</td>
<td>3,291</td>
<td>---</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Year</th>
<th>Adjusted gross income for all personal returns ( $ millions)</th>
<th>Percentage annual growth rate</th>
<th>Adjusted gross income for personal returns with itemized deductions ( $ millions)</th>
<th>Percentage annual growth rate</th>
<th>Percentage Itemized annual deductions ( $ millions)</th>
<th>Percentage annual growth rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1972</td>
<td>748,924</td>
<td>10.73</td>
<td>436,254</td>
<td>- .45</td>
<td>96,665</td>
<td>5.22</td>
</tr>
<tr>
<td>71</td>
<td>676,334</td>
<td>6.63</td>
<td>438,109</td>
<td>-2.35</td>
<td>91,970</td>
<td>4.19</td>
</tr>
<tr>
<td>70</td>
<td>634,250</td>
<td>4.73</td>
<td>448,698</td>
<td>7.53</td>
<td>83,173</td>
<td>9.93</td>
</tr>
<tr>
<td>69</td>
<td>605,578</td>
<td>8.85</td>
<td>417,258</td>
<td>13.10</td>
<td>80,210</td>
<td>16.05</td>
</tr>
<tr>
<td>68</td>
<td>556,304</td>
<td>9.80</td>
<td>368,917</td>
<td>14.28</td>
<td>59,117</td>
<td>15.92</td>
</tr>
<tr>
<td>67</td>
<td>506,641</td>
<td>9.15</td>
<td>322,613</td>
<td>10.72</td>
<td>59,263</td>
<td>9.26</td>
</tr>
<tr>
<td>66</td>
<td>468,451</td>
<td>9.15</td>
<td>291,424</td>
<td>9.00</td>
<td>54,566</td>
<td>7.54</td>
</tr>
<tr>
<td>65</td>
<td>429,201</td>
<td>8.20</td>
<td>267,343</td>
<td>9.54</td>
<td>50,739</td>
<td>8.34</td>
</tr>
<tr>
<td>64</td>
<td>396,660</td>
<td>7.56</td>
<td>244,079</td>
<td>4.70</td>
<td>46,932</td>
<td>1.70</td>
</tr>
<tr>
<td>63</td>
<td>368,778</td>
<td>5.75</td>
<td>233,115</td>
<td>9.57</td>
<td>46,053</td>
<td>10.54</td>
</tr>
<tr>
<td>62</td>
<td>348,701</td>
<td>5.71</td>
<td>212,753</td>
<td>8.12</td>
<td>41,661</td>
<td>8.51</td>
</tr>
<tr>
<td>61</td>
<td>329,861</td>
<td>4.58</td>
<td>196,764</td>
<td>8.63</td>
<td>38,391</td>
<td>8.71</td>
</tr>
<tr>
<td>60</td>
<td>315,406</td>
<td>3.38</td>
<td>181,131</td>
<td>8.20</td>
<td>35,313</td>
<td>10.29</td>
</tr>
<tr>
<td>59</td>
<td>305,095</td>
<td>8.51</td>
<td>167,400</td>
<td>15.16</td>
<td>32,017</td>
<td>16.43</td>
</tr>
<tr>
<td>58</td>
<td>281,154</td>
<td>--</td>
<td>145,358</td>
<td>--</td>
<td>27,497</td>
<td>--</td>
</tr>
</tbody>
</table>


**TABLE 2**
response of tax avoiders through work-leisure choice, don't appear in these figures.

Another way to measure the amount of tax avoidance is to estimate forgone tax opportunities for the Treasury. Such estimates are presented for the first time in the Federal Budget documents. They are defined as tax expenditures. "Certain provision of the personal and corporate income tax result in tax expenditures which are defined as revenue losses attributable to a special exclusion, exemption or deduction from gross income or to a special credit, preferential rate of tax or deferral of tax liability" [30]. We present the government estimates of the major tax-avoidance opportunities for the years 1974, 1975, and 1976. They estimated the loss of budget receipts resulting from each particular feature of the tax system under two major assumptions:

1) Only the tax provision in question is deleted and all other features of the tax system, including the structure of rates remain unchanged.

2) Taxpayer's behavior and general economic conditions are assumed to remain unchanged in response to the hypothetical change in the tax bases.

The estimates, in Table 3 give an idea of the importance of tax-avoidance activity. For example, one major item is the net exclusion of pension contributions and earnings for employer plans of $5.7 billion in 1976. Capital gains are estimated at $4.1 billion in 1976. This underestimates capital gains since it ignores loss from not taxing unrealized capital gains. In the section of housing investment, deductibility of property taxes account for $5.2 billion and mortgage interest
# TABLE 3

Tax Expenditure Estimates, by Function

<table>
<thead>
<tr>
<th>Description</th>
<th>Corporations</th>
<th></th>
<th>Individuals</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural resources:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Excess of percentage over cost depletion</td>
<td>1,315</td>
<td>2,200</td>
<td>2,610</td>
<td>305</td>
</tr>
<tr>
<td>Commerce and transportation</td>
<td>3,270</td>
<td>3,590</td>
<td>3,770</td>
<td>---</td>
</tr>
<tr>
<td>$25,000 corporate surtax ex.</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Bad debt reserve of fin. institutions in excess of actual</td>
<td>1,000</td>
<td>1,020</td>
<td>920</td>
<td>---</td>
</tr>
<tr>
<td>Health:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exclusion of employer cont. to medical ins. premiums and medical care</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>2,340</td>
</tr>
<tr>
<td>Deductibility of med. expenses</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>2,135</td>
</tr>
<tr>
<td>Deduction of pension contributions and earnings:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Employer plans</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>4,750</td>
</tr>
<tr>
<td>Plans for self-empl. and others</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>230</td>
</tr>
<tr>
<td>Revenues sharing and pen. purpose fiscal assistance:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Excl. of interest on State and local debt</td>
<td>2,305</td>
<td>3,153</td>
<td>3,505</td>
<td>1,060</td>
</tr>
<tr>
<td>Ded. of nonbusiness State and local taxes</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>5,250</td>
</tr>
<tr>
<td>Business investment:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investment credit</td>
<td>3,690</td>
<td>4,160</td>
<td>4,420</td>
<td>860</td>
</tr>
<tr>
<td>Personal investment:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital gain: indiv.</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>6,150</td>
</tr>
<tr>
<td>Exclusion of int. on life insurance savings</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>1,420</td>
</tr>
<tr>
<td>Deferral of capital gain on home sales</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>255</td>
</tr>
<tr>
<td>Deduct. of mortgage int. on owner-occupied homes</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>4,870</td>
</tr>
<tr>
<td>Deductibility of prop. taxes on owner-occup. homes</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>4,000</td>
</tr>
<tr>
<td>Ded. of chrit. contrib.</td>
<td>290</td>
<td>295</td>
<td>297</td>
<td>3,520</td>
</tr>
<tr>
<td>Ded. of int. on consumer credit</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>2,035</td>
</tr>
</tbody>
</table>

* Source: Special Analysis of the Budget for Fiscal year 1976.
on owner-occupied homes for $6.5 billion in 1976. Finally, the estimated tax loss from charitable contributions is $5.1 billion for the 1976 tax year.

If the taxpayers respond to price incentives in tax-avoidance activity, it is clear that progression in statutory tax rates induces certain type of tax-avoidance, especially in high-income tax brackets. This leads to the conjecture that tax-avoidance activity is able to weaken the progressivity in effective tax rates. However, some degree of progression might be reintroduced to the extent that tax-avoidance outlays increase with the level of income.

Empirical evidence supports the proposition that effective tax rates diverge from statutory rates for both personal and corporate income tax. A study by Pechman and Okner [18] presents evidence that effective tax rates are proportional or slightly progressive for families classified by percentiles in the income distribution. As shown in figure 1, there is little difference in overall effective tax rates between the tenth and ninety-seventh percentiles of family units. The difference in effective tax rates under the most and the least progressive set of assumptions is very small over practically the entire income scale. Over a broad range of the income distribution, between $2,000 and $30,000 and including 82 percent of all family units, the tax system is either proportional or only slightly progressive. But this overlooks "outlays" for tax avoidance, and to the extent that tax-avoidance costs increase as a function of income it may reintroduce some degree of progressivity.

Studies by Siegfried [28] and Kane [12] show that the null hypoth-
Effective tax rates (percent)

100 80 60

Least progressive

20

Most progressive

40 50 60

Population percentile

S

ou k

So u k

* ;

Computed from the 1966 MERGE data file.

FIGURE 1. EFFECTIVE RATES OF FEDERAL, STATE, AND LOCAL TAXES UNDER THE MOST AND LEAST PROGRESSIVE INCIDENCE VARIANTS, BY POPULATION PERCENTILES, 1966

esis of a flat effective tax rate cannot be rejected for corporate income tax. Siegfried has computed the effective average corporation income tax rate for the Internal Revenue Service "minor" industries in mining and manufacturing for the 1963 tax year. Effective tax rates take into consideration tax provisions, tax shelters and preferences which cause the actual tax structure to deviate from a simple basic corporation income tax schedule. The average effective tax rate for
110 industries for 1963 is 39 percent. The deviation between the average and 52 percent (the statutory tax rate on income in excess of \$25,000) provides evidence for the existence of tax-avoidance activity. According to his evaluation a single proportional tax rate of 28.3 percent without deductions would be sufficient to maintain the same tax revenues.

Kane's study on tax avoidance by large commercial banks also reaches the conclusion that the null hypothesis of a proportional or flat effective tax rate cannot be rejected at a 5 percent level.

The focus of this dissertation is to develop a mathematical model to analyze the behavior of the honest taxpayer attempting to pay his taxes wisely. This analysis is in contrast to another aspect of income-tax collection: tax evasion. Tax evasion occurs when the individual declares an income \( X \) different from \( Y \), the true income.\(^1\) Our focus is the far less cynical and publicly less shameful activity of tax avoidance. We present some estimates of the size of aggregate tax-avoidance activities. A studious taxpayer can reap rewards for being clever without being criminal.

---

1) This aspect of income tax has been studied by Allingham and Sandmo [24] and Srinivasan [29]. Allingham and Sandmo present the taxpayer with a binary gamble of which he can buy a small lottery ticket as he wants, but one only as large as his income.
A crude theory of tax avoidance

It is appropriate at this point to present a heuristic model of tax avoidance. A crude theory of tax avoidance has been developed by Kane in the introduction of his study [12] on tax avoidance by large commercial banks. His analysis is carried out for the corporate income tax case. The statutory tax rates depicted by the step function, MB, in figure 2 represents the current federal corporate-tax structure: 22 percent in the first $25,000 of taxable income and 48 percent on all higher amounts. Tax-avoidance activity, X, measures the reduction in taxable income. The distance \( X_{\text{max}} \) equals the taxpayer's maximum potential income. Kane focuses on maximum potential income to recognize that some forms of tax avoidance reduces the taxpayer's reported flow of net income below what it would be if all receipts were taxable as ordinary income. This recognizes what we said earlier on tax-exempt bonds. A difference in effective interest rate equals the amount of taxes payable on taxable issues by investors whose marginal tax bracket would leave them indifferent between the two opportunities. The cost of such reductions are subsumed under the category of avoidance costs. On the vertical axis, we plot the marginal cost and marginal benefits of each dollar of X achieved. MC is increasing and drawn with the X axis to the extent, we think that some deductions are readily available without information costs. With the existing corporate income tax structure, the taxpayer would pay taxes equal to the sum of the areas labelled A, B, and C. The tax avoidance costs are equal to the sum of D, E, and F. From the point of view of the individual taxpayer, tax-avoidance inputs represent informations costs and payments made to
someone other than the Treasury to carry out tax-avoidance strategy.
These information costs accrue to tax-avoidance industry, which consists primarily of tax layers, tax accountants and their supporting staffs.

Comparative statics for a "reform" tax structure, MB' gives a new tax-avoidance level, $X_1$. Tax avoidance decreases to $F$, freeing resources for other uses but also reducing incentive payments such as charitable giving. The new tax bill would be $A$, $B$, and $E$. The amount paid to the Treasury under the new tax structure will be less then, equal to or greater than before according to whether $E \leq C$. This
simple graphical analysis clarifies the problem in hand and illustrates Kane's conclusion in [12]: "Because a less steeply progressive corporate tax structure or even a flat proportional tax on net corporate income of about 25 to 30 percent that admitted no privileged or exempted categories of income could be designed to raise approximately the same aggregate tax and implicit-subsidy revenue at lower social cost; the burden of proof should fall on opponents of tax simplification to prove that the distortions and direct resource costs attributable to corporate tax avoidance are counter-balanced by identifiable social benefits transmitted to small corporations".

The previous analysis implicitly assumed a utility function linear in income. McKenzie [15] presents his heuristic approach using a disutility function in two variables: tax payments and tax-avoidance costs. He views tax-avoidance activity as a linear trade-off between tax payments on the one hand and deliberate acts to avoid tax payments on the other hand. The major weakness is his unrealistic assumption of a constant opportunity cost, $M'$, between the two activities. According to his model, the taxpayer chooses the point on the frontier which minimizes his disutility, $P$, in figure 3.

In his attempt to carry out comparative-static analysis, McKenzie assumes a tax reform designated to increase the statutory tax payments from $A'$ to $B'$. On the other hand, he assumes that taxpayers can still avoid completely the tax reform at previous avoidance costs, $A$. The new equilibrium point, $P'$, actually implies a decrease, $B_0P_1$, in taxes collected by the Treasury. This is defined by McKenzie as a perverse effect of tax avoidance. Our task is to put this analysis in a rigorous mathematical model and study the implications of tax avoidance.
in a systematic way.

Figure 3
The plan of this dissertation

This dissertation is composed of three chapters. The first chapter is devoted to a microeconomic model of tax-avoidance activity. Formally, we limit our analysis to two classes of tax-avoidance expenditures. The first class is composed of resources spent by the taxpayer on information about tax laws and calculation of optimal-tax strategies. The second class of expenditures measures the dollar amount spent to carry out an optimal strategy. These two inputs combined together produce, through a transformation function, a level of tax-avoidance activity aimed at removing "bites" from his taxable income. However, the first class of activity is risky (even though not illegal), because the taxpayer is never sure of finding profitable tax-avoidance strategies. Even the second class of expenditures is risky, because there is always the risk of a government audit and that as a result of the audit a seemingly legitimate deduction will be rejected by the examiners, leading the taxpayer to pursue the risky costs and benefits of litigation. Formally, we focus on the risk growing out of government control and enters the tax-avoidance function as a type-B of output uncertainty. This interpretation is useful for the applications in the theory of optimal taxation.

In an environment of uncertainty, the taxpayer maximizes the expected value of his utility function with respect to the risky inputs. The taxpayer's expected utility comes from income available for consumption or investment after deduction of tax-avoidance expenditures and also from ancillary rewards under the form of direct gratification.
and/or social consideration as benefactor. These rewards are generated by some of his second class of tax-avoidance expenses, such as charitable contributions.

We derive the effects of risk on the optimal level of tax-avoidance inputs. Comparative-static analysis allows us to find the qualitative effects of small changes in the exogenous variables. These comparative-static results are used to derive the impact on tax avoidance of an expected compensated-revenue increase in tax progression. In section 3, we introduce the work-leisure tradeoff as a special case of tax-avoidance activity, to obtain the marginal effect on working and tax-avoiding time of a revenue-compensated change in tax progression in a certainty world.

Chapter Two scrutinizes the optimal response of the government given the existence of tax-avoidance activity. The first section shows the effects of tax avoidance on the level of public goods that should be provided when the government wants to determine an income-tax policy that maximizes social welfare. The existence of a meaningful solution is proved. The next step studies the implications of tax avoidance for a government that wants to establish vertical equity. Some simplifying assumptions have to be made, but the essence of the trade-off is fully respected.

Chapter Three looks at macroeconomic implications. The traditional macroeconomic model overlooks the implications of tax avoidance on fiscal and monetary policy. We introduce tax-avoidance activity into an IS/LM framework and derive the qualitative impact of tax avoidance on governmental policy measures. For example, a measure of fiscal policy increasing the statutory rates will not be fully reflected in the amount
of taxes effectively collected. The Treasury should take account of
tax avoidance in its forecast of the impact of fiscal policy. Com­
parative-static results are derived for the case including the govern­
ment budget constraint explicitly as well as for the case without it.

In the last section, another aspect of fiscal policy in the presence
of tax avoidance is taken up. We combine an inflationary or deflationary
environment with an endogenous labor supply. We derive the conditions
under which an increase in statutory tax rates to fight inflation has
a deflationary or a perverse inflationary effect on the economy.
CHAPTER 1

A MICROECONOMIC MODEL OF TAX AVOIDANCE

Section 1

Kane [12] sketches some of the elements that a theory of individual or firm's tax avoidance should contain; as seen in the introduction of this work. We draw on this work to build a rigorous economic theory of tax avoidance. We deal with a one-period model. As any economic activity, tax avoidance uses resources to generate a sum of benefits. This activity is not free of risk. There are two major reasons why tax avoidance is a risky activity.

First, when an individual spends time studying tax laws, he does not automatically receive a tax-avoidance benefit. The productivity of his time depends on his intelligence, his level of assimilation and his ability to figure out good tax-avoidance strategies from the knowledge he does accumulate, and his personal circumstances, e.g., source of income. Similarly, a taxpayer can spend some money on a lawyer specializing in tax avoidance or on a specialized tax accountant without being sure that the expenditures will return a generous payoff.

Second, there is always the risk of a government audit and that as a result of the audit even seemingly legitimate deductions will be rejected by the examiner, leading the taxpayer to consider the risky costs and benefits of litigation. Courts control the interpretation of
the tax laws and the government controls the frequency rate of formal audits. Therefore, if the taxpayer follows a previously planned strategy, he faces the risk that as time goes on, the law will be changed or be subjected to restricted interpretation by the tax collectors or the tax courts.

1.1 How to deal with uncertainty

We limit ourselves to two broad classes of tax-avoidance expenditures: tax-avoidance information costs and expenses to carry out the best tax-avoidance strategy.

Two classes of tax-avoidance information can be identified. First, if the taxpayer spends his own time, then his opportunity cost is the marginal value of his leisure time. Second, this class of information costs includes opportunities to purchase outside services of tax lawyers, tax-accountant specialists and/or tax return preparation firms. If the taxpayer hires outside services the opportunity cost is given by the price of such type of services. At the taxpayer's optimum the two opportunity costs are equal. To simplify our model (to keep a two tax-avoidance inputs model), we assume that all information costs are purchased and considered as pecuniary expenses to be subtracted from his income. This class of expenditures is denoted by \( q_1 \), where \( q \) is the unit opportunity cost of resources employed in that way. His first class of tax-avoidance expenditures represents those outlays of resources that have no payoff in themselves except for achieving legal tax-savings.

The second class of tax-avoidance expenditures is measured by the
cost of carrying out optimal tax-avoidance strategy. There are always
costs associated with a particular strategy. For example, the Pension
Reform Act permits employees not covered by private pension plans to
save or invest up to 15 percent of their income tax-free, for a maximum
of $1,500 each year to finance retirement. Consequently, if a taxpayer
wants to take advantage of such an opportunity, he still has to support
the cost of his tax-free investment, which is simply the time preference
rate in excess of interest.

There are some tax-avoidance activities which give the taxpayer
utility but which are tax deductible. Examples of this type of tax-
avoidance activities are the dollar amounts spent on tax-deductible
gifts, contribution to cultural or philanthropic enterprises. Tax-
deductible professional expenses (such as attending conventions in
exotic locales) providing directly enjoyable benefits are examples of
this class of tax-avoidance expenditures. The opportunity cost of these
activities is the price net of tax. We take account of these ancillary
rewards by introducing part of the second class of tax-avoidance ex-
penditures into the taxpayer's utility function. We denote the dollar
amount of the second class of tax-avoidance expenditures by P. This
second class of tax-avoidance outlays represents the traditional con-
ception of tax avoidance.

As pointed out previously, tax-avoidance is a risky activity. The
risk in tax-avoidance activity arises mainly from government behavior.
In a regime of self assessment of tax returns, the government acts ran-
domly within classes of taxpayers to detect any excess in filling out
tax returns. Depending on which aspects we want to emphasize, we can
distinguish two types of risk.

First, we have that internally generated by the uncertain process of collecting tax-avoidance information. A taxpayer's lengthy study of tax laws need not yield directly fruitful results. We can also consider homemade income as a form of tax-avoidance activity which does not reduce the taxpayer's potential income but reduces the dependence of tax-avoidance benefits upon the state of nature. For this type of uncertainty, the more one invests the better-informed the taxpayer is about new developments on the tax scene. Consequently, from this point of view homemade income and tax-avoidance information reduce the variance of the payoff of tax-avoidance activity.

The second type of risk deals with the uncertainty created by the government's behavior. The productivity of the taxpayer's investment depends on which state of nature obtains, i.e., whether or not he is audited and overturned. Which state of the world obtains depends on the partial or total rejection of the taxpayer's claims by the tax-collecting agency.

To formalize this phenomenon, we shall draw on a concept developed by Diamond [10] to deal with technological uncertainty. We shall restrict our attention to government generated uncertainty by introducing risk in a special form. This form is defined as type-B uncertainty. We present the concept in a general form which is best defined as input uncertainty. We refer to it as type-A uncertainty. In this case tax-avoidance activity depends on the two classes of tax-avoidance expenditures and the state of nature \( \gamma \). We write:

\[
X = X(I, P, \gamma)
\] (1)
where $X$ is the level of tax-avoidance activity, and we assume:

$$X_1 > 0, \quad X_P > 0, \quad X_Y > 0$$

and

$$X_{11} < 0, \quad X_{PP} < 0$$

i.e., a positive decreasing marginal productivity for both tax-avoidance inputs. The positive partial derivative, $X_Y$, implies uniformity in the direction of changes of an increase in $Y$ for all levels of tax-avoidance activity. A crucial assumption is that the cross-partial derivative, $X_{1Y}$, is negative or positive reflecting the correlation between marginal productivity of tax-avoidance information and the state of nature. The assumption of a negative cross-partial derivative, $X_{1Y}$, reflects the decrease in uncertainty of tax avoidance when the taxpayer invests more in information. The economic meaning of this assumption is more easily grasped with the aid of figure 4.
Suppose there are only two states $\mathcal{Y}_1$ and $\mathcal{Y}_2$. $\mathcal{Y}_1$ represents a situation of no tax reform. $\mathcal{Y}_2$ is the state of a tax reform aiming at removing tax loopholes. The average productivity of tax-avoidance information is lower at state $\mathcal{Y}_2$ for all levels of $I$. (The convention $X_{\mathcal{Y}}$ positive implies that the assigned indexes satisfy $\mathcal{Y}_1 > \mathcal{Y}_2$ for all level of $I$). We see, however on the graph that the difference in output is smaller for higher levels of information. The well-informed taxpayer can adjust faster to the new situation. The marginal productivity of information expenditures is increased in state $\mathcal{Y}_2$. Thus, a negative value of $X_{I\mathcal{Y}}$ corresponds to a negative correlation between average and marginal rate of returns of tax-avoidance information. Furthermore, this assumption implies that the variance decreases with the level of $I$:

$$
\delta^2 \delta I \text{ Var } X = \delta^2 \delta I \left\{ E \left[ X(I, P, \mathcal{Y}) - E(X(I,P, \mathcal{Y})) \right]^2 \right\} \\
= 2 E \left\{ \left[ X - E(X) \right] \left[ X_I - E(X_I) \right] \right\} \\
= 2 \text{ Cov}(X, X_I) < 0 \quad (3)
$$

The result obtains given the assumptions on $X$, listed in (2), and $X_{I\mathcal{Y}}$ negative implying a negative covariance between $X$ and $X_I$.

A second type of uncertainty can be defined. This type-B uncertainty is more interesting for the application of the model to the theory of public finance. Uncertainty enters in a multiplicative form:

$$
X(\mathcal{Y}) = X(I, P, \mathcal{Y}) = \mathcal{Y} X(I, P) \quad (4)
$$
In this specification, $X = X$ is positive and implies that if for example a state of the world is more favorable to tax-avoidance activity, it is uniformly better for all level of tax-avoidance activity. Furthermore, we have: $X = X^T$ and consequently $X^T = X$ is positive. This sign is opposite for type-A uncertainty. We have by similar reasoning that there is a positive correlation between the average and the marginal productivity of inputs. This captures government's ability to influence or "control" tax-avoidance activity. The government intervenes at the end of the process of tax avoidance. In the U.S., the IRS uses a random selection within classes of taxpayers to sort out tax returns to be audited. An automatic screening classifies the tax returns estimated to contain errors with a high probability. Some tax returns are subject to a manual screening. In both cases, tax returns with large amount of tax avoidance have a higher probability of being selected for audit. Such tax returns have a greater chance to deviate significantly from the computerized norms. With this process, a high level of tax avoidance increases the probability of audit and consequently imposes further costs on the audited taxpayer. Knowledge of this tends to reduce the final level of tax avoidance. Our specification implies an increasing variance of tax-avoidance activity for an increase in the level of one of the input. This is shown as follows:

$$
\begin{align*}
\frac{\partial}{\partial I} \text{Var } X &= \frac{\partial}{\partial I} \mathbb{E} \left\{ \left[ X(I, P) - \mathbb{E}( X(I, P) ) \right]^2 \right\} \\
&= 2 \mathbb{E} \left\{ [X(\gamma - \mathbb{E}(\gamma))] [X_I (\gamma - \mathbb{E}(\gamma))] \right\} \\
&= 2 X X_I \sigma_\gamma^2 > 0. \quad (5)
\end{align*}
$$

Uncertainty is endogenous in our model because the taxpayer can always remove uncertainty by forgoing tax avoidance and choosing zero level of
tax-avoidance inputs.

The random variable \( Y \) is assumed to have a known distribution on the interval \([0, 1]\). The nature of our problem imposes restrictions on the possible range of values taken by \( Y \). The upper limit is one because the tax-collacting agency would not grant the taxpayer more than he claims. A lower bound, at zero, exists because in most cases there are some strategies with a positive payoff. This is true because we don't include tax evasion where there is a chance of being caught and of incurring pecuniary and/or jail penalties. The only case in which tax avoidance could lead to a negative outcome is the case of litigation and the tax court rules against the taxpayer. The taxpayer not only loses his tax-avoidance benefits, but has to support extra costs of litigation. However, the taxpayer may have a comparative advantage in an out-of-court settlement, allowing him to avoid the cost of litigation. This justifies the zero lower bound.

The random variable has a known distribution and a positive expected value less than one:

\[
E(Y) = \int_0^1 Y f(Y) dY = \bar{Y} \tag{6}
\]

where \( Y \) denotes a particular realization of the random variable associated with the known probability density function. A zero expected value would be infinitely costly to monitor and in contradiction with tax-avoidance opportunities created by the government. The government controls the expected value of \( Y \) because it can increase the frequency rate of investigation, make audits more unpleasant and even interpret the tax laws in unexpectedly harsh ways. The government is also able to decrease the mean value of \( Y \) by tax reforms designed to eliminate
one or more loopholes. The individual faces uncertainty and cannot be sure of competitive rates of return on his investment in tax-avoidance. In principle, the mean value is determined at the national level of optimal tax-collection practice within a general-equilibrium framework.

1.2. Presentation of the model

Before developing the model, we recapitulate and define the major variables:

\(Y\) : the total potential taxpayer's income, which is taken temporarily as exogenously given (can be viewed as maximum income).

\(X = X(I, P)\): the tax-avoidance activity transformation function

\(I\) : the first class of resources measured in man/hours

\(q\) : the opportunity cost per man/hour

\(P\) : the amount spent on the second class of resources

\(\gamma\) : a random variable which captures the tax-avoidance output uncertainty. \(\gamma\) has a known probability density function defined on the closed interval \([0, 1]\). The expected value, \(E(\gamma) = \bar{\gamma}\) lies between 0 and 1.

\(A\) : the exemption level

\(\Theta\) : the average effective tax rate

\(S = U(C, mP)\) : the utility function with arguments; \(C\) : the net income, after deduction of tax-avoidance expenditures, available for consumption or investment. We assume a maximum potential income and don't introduce explicitly work-leisure choice in this model (see section 3)
The taxpayer's objective is to maximize his expected utility. His expected utility comes from income available for consumption after deduction of expenditures on tax avoidance and also from non-monetary rewards under the form of direct gratification and/or social consideration as benefactor generated by part of the second class of tax-avoidance expenses. It may include psychological benefits of outwitting the government. We assume the utility function to be thrice-differentiable with the following properties:

\[ U_C > 0, \quad U_P > 0 \]

\[ U_{CC} < 0, \quad U_{PP} < 0 \quad \text{and} \quad U_{PC} = U_{CP} = 0 \quad (7) \]

\[ U_{CCC} > U_{CC}^2/U_C, \quad U_{PPP} > U_{PP}^2/U_P \]

The assumption \( U_{CP} = 0 \) implies an additive utility function in consumption and ancillary rewards. This assumption is necessary to deal with uncertainty. There is a one-to-one correspondence between the sign of \( U_{CC} \) and the sign of the measure of absolute risk aversion, \( R_A(C) = -(U_{CC}/U_C) \), developed by Arrow [2] and Pratt [21]. In many proofs, we have to know the sign of \( R_A(C) \); that is if the taxpayer has an increasing or a decreasing absolute risk aversion. This tells us about the attitude of the individual towards risky bets when the taxpayer becomes richer. There is considerable precedent in economic analysis (see Arrow [2], Levhari and Weiss [14]) for the assumption of a decreasing absolute risk aversion, i.e., \( R_A'(C) \) negative. What are the implication of this assumption on the utility function? Take the derivative of \( R_A(C) \) with respect to \( C \). We obtain:
\[ \frac{d(R_A(C))}{dC} = \frac{d(-U_{CC}/U_C)}{dC} = \frac{U_C U_{CCC} - U_{CC}^2}{U_C^2} < 0 \] 

which implies \( U_C U_{CCC} - U_{CC}^2 > 0 \) or precisely the assumption on the third derivative of the utility function in (7), i.e., \( U_{CCC} > U_{CC}^2/U_C \).

Having developed the main building blocks, we finish the formalization of the model and derive the necessary and sufficient conditions for a maximum.

First, the effective tax payment is:

\[ T = \Theta \left[ Y - A - \gamma X(I,P) \right] \] 

where \( \Theta \) is the flat effective tax rate and \( A \) is the exempt-tax income. The introduction of an exemption is a way to introduce progression in the tax structure and will allow us to define a revenue-compensated change in the degree of progression.

Second, the level of income available for consumption or investment is defined for a particular state of the world:

\[ C(Y) = Y - \Theta \left[ Y - A - \gamma X(I,P) \right] - qI - P \] 

The level of income, \( Y \), can be considered as a maximum income, i.e., the taxpayer is in the labor market for forty hours and may earn some non-labor income. This assumption is strong, but not unrealistic because in many jobs both work and remuneration are fixed, and the taxpayer cannot earn more by working harder or longer to compensate for his tax burden. The relaxation of this assumption is studied, for the certainty case, in section 3. Finally, the taxpayer buys outside tax-
avoidance services, i.e., these costs are subtracted from disposable income. The taxpayer's disposable income is a random variable since it depends on which state of the world obtains through tax-avoidance activity. We have a case of endogenous uncertainty because the uncertainty is not removed by the time the taxpayer decides. However, he can remove the uncertainty by forgoing tax avoidance. He can choose \( I = P = 0 \).

1.3. The necessary and sufficient conditions for a maximum

The taxpayer maximizes his expected value of the utility function with respect to \( I \) and \( P \):

\[
S = \max_{I, P} \mathbb{E}\{U(C, mP)\} \quad \text{subject to (10)}
\]

or

\[
S = \max_{I, P} \mathbb{E}\{U(Y - \Theta[Y - A - X(I, P)] - qI - P, mP)\} \quad (11)
\]

We derive the first-order conditions for an interior solution in the two level of tax-avoidance inputs, \( I \) and \( P \):

\[
\frac{\delta S}{\delta I} = \mathbb{E}\{U'(\Theta Y X_I - q)\} = 0 \quad (12)
\]

\[
\frac{\delta S}{\delta P} = \mathbb{E}\{U'(\Theta Y X_P - 1)\} + \mathbb{E}U_P = 0. \quad (13)
\]

We can rework conditions (12) and (13) by taking the expected value of both equations, to get:

\[
\Theta X_I \mathbb{E}(U_C Y) - q \mathbb{E}U_C = 0 \quad (14)
\]

\[
\Theta X_P \mathbb{E}(U_C Y) - \mathbb{E}U_C + m\mathbb{E}U_P = 0 \quad (15)
\]
Taking the ratio of equation (14) and (15), we get the following ratio of marginal conditions for productivity of the two classes of tax-avoidance expenditures:

\[
\frac{X_I}{X_P} = \frac{q \cdot EU_C}{EU_C - mEU_P} \quad \text{(16)}
\]

or

\[
\frac{X_I}{X_P} = \frac{q}{1 - \frac{mEU_P}{EU_C}} \quad \text{(16')}
\]

The optimal values I*, and P* are found by solving (16') and one of equations (14) and (15). At the optimum the taxpayer should equalize the ratio of marginal rate of transformation and the ratio of marginal cost of transformation and the ratio of marginal cost of these inputs. The marginal cost for P expenditures is lower than the nominal value of dollars spent by the value of non-monetary rewards of such expenditures measured in terms of expected utility of income. The value of ancillary rewards is weighted by the fraction of P giving rise to such benefits. After deriving the sufficient conditions for a maximum, we shall show explicitly how optimal values I* and P* are altered by risk.

The second-order conditions to ensure a maximum are the following:

\[
H_{11} = \frac{\partial^2 S}{\partial I^2} = EU_{CC}(\Theta Y_{I} - q)^2 + EU_C \Theta Y_{II} < 0 \quad \text{(17)}
\]

\[
H_{22} = \frac{\partial^2 S}{\partial P^2} = EU_{CC}(\Theta Y_{P} - 1)^2 + m^2EU_{PP} + EU_C \Theta Y_{PP} < 0 \quad \text{(18)}
\]

\[
H_{12} = \frac{\partial^2 S}{\partial I \partial P} = EU_{CC}(\Theta Y_{I} - q)(\Theta Y_{P} - 1) + EU_C \Theta Y_{PI} \quad \text{(19)}
\]

\[
H_{21} = \frac{\partial^2 S}{\partial P \partial I} = EU_{CC}(\Theta Y_{I} - q)(\Theta Y_{P} - 1) + EU_C \Theta Y_{IP} \quad \text{(20)}
\]
Since $H_{12} = H_{21}$, we observe that the second-order conditions for a maximum are:

$$H_{11} < 0, \quad H_{22} < 0 \quad \text{and}$$

$$H = \begin{vmatrix} H_{11} & H_{12} \\ H_{12} & H_{22} \end{vmatrix} = H_{11}H_{22} - (H_{12})^2 > 0$$

$H$ is a dominant diagonal matrix with negative diagonal elements and we show that:

$$|H_{22}| > |H_{11}| > |H_{12}| = |H_{21}| \quad (22)$$

To show the above result, the marginal conditions on both tax-avoidance yield the following relationship:

$$a = \Theta Y X - \eta = \Theta Y X P(X_{1}/X_{P}) - (1 - mE_{U}P/E_{U}C)(X_{I}/X_{P})$$

$$= (\Theta Y X - 1)(X_{I}/X_{P}) + g(X_{I}/X_{P}) \quad \text{where} \quad g = mE_{U}P/E_{U}C$$

$$= b(X_{I}/X_{P}) + g(X_{I}/X_{P}) \quad \text{where} \quad b = \Theta Y X - 1$$

or \quad $$b = a(X_{I}/X_{P}) - g \quad (23)$$

This equality permits us to rewrite the second-order conditions in a useful way for future calculations:

$$H_{11} = EU_{CC}a^2 + EU_{C}\Theta Y X_{II} < 0 \quad (24)$$
\[ H_{22} = \text{EU}_{CC} \alpha^2(x_I/x_I)^2 - 2\text{EU}_{CC} \alpha \gamma (x_I/x_I) + \text{EU}_{CC} \gamma^2 + \]
\[ m^2\text{EU}_{PP} + \text{EU}_{C} \Theta \gamma x_{PP} < 0 \quad (25) \]
\[ H_{12} = H_{21} = \text{EU}_{CC} \alpha^2(x_I/x_I) - \text{EU}_{CC} \gamma \gamma + \text{EU}_{C} \Theta \gamma x_{IP} \quad (26) \]

We immediately see that by (23) the first three right-hand-side terms of (25) are equivalent to \( \text{EU}_{CC} \alpha^2 \) and since \( m^2\text{EU}_{PP} \) is negative: \( |H_{22}| > |H_{11}| \). Further, to sign and to provide an economic interpretation for the second-order conditions, we need to determine the sign of
\[ \text{EU}_{CC} (\Theta \gamma x_I - q) (m\text{EU}_{P}/\text{EU}_{C}) = \text{EU}_{CC} \alpha \gamma. \]

The expression, \( \text{EU}_{CC} (\Theta \gamma x_I - q) \), is related to the taxpayer's attitude towards risk. It is proved in A1 that under the assumption of decreasing absolute risk aversion, the expression is positive:
\[ \text{EU}_{CC} (\Theta \gamma x_I - q) > 0. \quad \text{A1}(1) \]

It is shown in A2 and in section 1.4 that under uncertainty the expected marginal value of productivity of tax-avoidance input must exceed its price. There is an economic reward for risk-taking. When the taxpayer enjoys higher income levels, his decreasing absolute risk aversion induces him to reduce his level of economic rewards for risk-bearing. To reduce the level of economic reward the taxpayer demands a greater quantity of input.

2) We have to derive several technical results, but to ease the exposition, we shall establish them in separated appendices: A1, A2, and A3 at the end of this chapter. An equation in a particular appendix is referred, for example, as A2(4).
The two components of $H$ have the following economic content:

1) $EU_{CC}(\Theta y_{I} - q)^2$: a negative income effect, i.e., when the taxpayer becomes wealthier, the marginal utility of an extra dollar decreases.

2) $EU_{CC} \Theta y_{II}$: a negative technological effect, i.e., the taxpayer experiences diminishing returns for his tax-avoidance expenditures.

The value of $H_{22}$ is negative and the economic content of each term is the following:

1) $EU_{CC}(\Theta y_{I} - q)^2(x_p/x_I)^2 + EU_{CC}a^2$: a negative income effect

2) $EU_{CC} \Theta y_{pp}$: a negative technological effect for his second class of input expenses

3) $-EU_{CC}(\Theta y_{I} - q)a(x_p/x_I)$: a negative income-risk effect since by $\Pi(1)$, $EU_{CC}(\Theta y_{I} - q)$ is positive, i.e., when the taxpayer is wealthier, his decreasing attitude towards risk induces him to reduce the economic reward for risk-taking. Multiplied by a clearly negative term, $-2a(x_p/x_I)$, we have a negative income-risk effect.

4) $m^2EU_{pp}$ is negative. The taxpayer experiences diminishing marginal utility for social consideration or activities (the marginal value of an extra dollar spent on charitable contributions or to be a member of an extra tax-exempt country club decreases.

The cross terms $H_{12}$ and $H_{21}$ of the second-order conditions have the following significance:

1) $EU_{CC}(\Theta y_{I} - q)^2$: a negative income effect

2) $-EU_{CC}ag$: a negative income-risk effect

3) $EU_{CC} \Theta y_{IP}$: a substitution or complementary technological effect.
If the two classes of inputs are substitutes, the term is negative. The whole cross term $H_{12}$ is negative. If the two classes of input are complements, the term is positive. Since resources spent on the first class of input allows the taxpayer to figure out better tax-avoidance strategies and the second class represents the cost of carrying them out, the case for complementarity is strong.

1.4. **The optimal level of tax avoidance**

The results of the previous section allow comparisons between the optimal level of tax avoidance under uncertainty and the case of certainty. We derive the effect of uncertainty on the optimal level of tax-avoidance inputs in an alternative way. At the optimum, conditions (12) and (13) hold and may be rewritten as:

\[
E(U_C)E(\Theta Y X_{I^*}) - E(U_C)q = - \text{Cov}(U_C, \Theta Y X_{I^*})
\]

\[
= - \Theta X_{I^*} \text{Cov}(U_C, Y) \tag{27}
\]

\[
E(U_C)E(\Theta Y X_{P^*}) - E(U_C) + E(U_P) = - \text{Cov}(U_C, \Theta Y X_{P^*})
\]

\[
= - \Theta X_{P^*} \text{Cov}(U_C, Y) \tag{28}
\]

for optimal values of $I^*$ and $P^*$.

We can compute the covariance between $C$ and $Y$:

\[
\text{Cov}(C, Y) = E \left\{ (B + \Theta X)Y \right\} - E(B + \Theta Y X)E(Y)
\]

\[
= \Theta X \left\{ E(Y^2) - [E(Y)]^2 \right\}
\]

\[
= \Theta X \sigma_Y^2 > 0 \tag{29}
\]
where $B = Y(1 - \Theta) + \Theta A - q I^* - P^*$. Given the concavity assumption on the utility function, we have $\text{Cov} (U_C, Y) = -\text{Cov}(C, Y) < 0$. The right-hand side of equation (27) is positive, and we obtain:

$$E(\Theta Y X_P) > q \text{ or } X_{I^*} \tilde{Y} > q/\Theta.$$  \hspace{1cm} (30)

Similarly for the second class of tax-avoidance expenditures, the right-hand side of equation (28) is positive:

$$E(\Theta X_P) > 1 - EU_p.$$  \hspace{1cm} (31)

The existence of ancillary rewards precludes any definite answer on the value of direct marginal benefits, $E(\Theta Y X_P)$.

To compare with the certainty case, we define $E(Y) = \tilde{Y}$. Then, under certainty the taxpayer will invest up to the point where:

$$X_{I^*} \tilde{Y} = q/\Theta$$  \hspace{1cm} (32)

We compare equations (30) and (32). We conclude from the assumption of decreasing marginal productivity in tax avoidance that:

$$I^* < I^0$$  \hspace{1cm} (33)

if and only if $X_{I^*}$ is positive, implying a positive $\text{Cov}(X, X_P)$.

It is interesting to point out that in contrast to portfolio theory, the assumption of risk aversion is not sufficient to ensure

---

3) By definition $\text{Cov} (X, Y) = E(XY) - E(X)E(Y)$ and at the optimum $E(XY) = 0$, by equations (12) and (13). Thus the relationship $E(X)E(Y) = -\text{Cov}(X, Y)$ holds at the optimum.
an interior solution: \( E(\Theta X_{I*}) > q \). It is worth elaborating this point by showing that if we had only input uncertainty in tax-avoidance information with the assumption \( X_{IY} < 0 \), the result in equation (30) is reversed. This case is pictured in Figure 4, where the marginal and average rates of return are negatively correlated. The first-order condition for \( I \) becomes:

\[
E \left\{ U_C(\Theta X_{I}(I, P, \gamma) - q) \right\} = 0 \tag{34}
\]

and we rewrite equation (27) as follows:

\[
E(U_C)E(\Theta X_{I*}) - E(U_C)q = -\Theta Cov(U_C, X_{I*}) < 0. \tag{35}
\]

Therefore, we get \( E(\Theta X_{I*}) < q \) and the assumption of decreasing marginal productivity in tax-avoidance inputs, we have that under type A of uncertainty \( I^* > 1 \).

The economic meaning of these results deserves emphasis. The special role of \( X_{IY} \) becomes clear. In the model where \( X_{IY} \)

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]

\[\]
is positive, the assumption of expected utility maximization states
that the contribution of an additional unit of tax-avoidance information
is weighted by the marginal utility in each state of nature. A positive
correlation between $X_\text{i}$ and $X$ implies that states of the world with high
marginal rates of return on tax avoidance are given relatively lower
weights. In other words, the increase in $I$ implies an increase in $\text{Var} X$,
as shown in equation (5). In the case of type-3 uncertainty, the
taxpayer wants a higher risk premium to protect himself against the
increased variance; thus the result: $E(\Theta X_{Y_\text{i}X}) > q$. 
Section 2

Comparative-static analysis

In this section, we determine the qualitative effects on the optimal solution of changes in parameters. We use the qualitative-economic standard forms derived by Lancaster, Quirk and Morishima in [17] to establish our results. Before going into the derivation and the evaluation on the qualitative effects of changes in parameter for the model, we give a list of the cases studied.

2.1 The qualitative effects of an increase in the riskiness of tax-avoidance activity, i.e., the government intervenes in a more erratic way, or unpredictable fashion towards such an activity.

2.2 The qualitative effects of an increase in the expected value of the random variable $\gamma$, i.e., in the average chances of getting a higher proportion of its tax-avoidance activity accepted by the tax-collecting agency are increased.

2.3 The effects of a change in the flat tax rate, $\Theta$.

2.4 The effects of a change in the exemption lever, $\Lambda$.

2.5 The qualitative effects of changes in the price of tax-avoidance input.

2.6 The qualitative effects of changes in the taxpayer's income.

In the following subsections, where particular qualitative comparative-static effects for marginal adjustment in parameters are developed, we refer to the basic framework exposited below. We have a 2x2 matrix system, which is solvable if the second order conditions are satisfied,
\( H \) is positive. The general pattern for a marginal change in an arbitrary parameter \( \alpha \), is as follows:

\[
\begin{bmatrix}
H_{11} & H_{12} \\
H_{21} & H_{22}
\end{bmatrix}
\begin{bmatrix}
\delta I/\delta \alpha \\
\delta P/\delta \alpha
\end{bmatrix}
=
\begin{bmatrix}
\delta G_1/\delta \alpha \\
\delta G_2/\delta \alpha
\end{bmatrix}
\] (34)

where \( G_i, i = 1, 2 \) represents the implicit functions defined by the system of first-order conditions (12) and (13). Consequently, the standard problem of qualitative comparative statics is to determine the sign of:

\[
\delta I/\delta \alpha = \frac{1}{H} \left\{ (- \delta G_1/\delta \alpha)H_{22} - (- \delta G_2/\delta \alpha)H_{12} \right\} \] (35)

\[
\delta P/\delta \alpha = \frac{1}{H} \left\{ (- \delta G_2/\delta \alpha)H_{11} - (- \delta G_1/\delta \alpha)H_{21} \right\} \] (36)

These equations ultimately determine the marginal reaction of tax-avoidance activity since:

\[
X_{\alpha} = \frac{\delta X/\delta \alpha}{X_1 \delta I/\delta \alpha + X_P \delta P/\delta \alpha} \] (37)

2.1. The impact of an increase in riskiness of tax-avoidance activity.

In analyzing the impact of an increase in riskiness of tax-avoidance activity, we must begin by explaining the meaning of riskiness. In general, an increase in riskiness means an alteration or a spread of the probability density function. Spreading involves rearranging the weights assigned to two or more possible outcomes in the probability density function. To finesse this problem, Stiglitz and Rothschild in [22] and [23] focused on spreads that affect the variability without changing the expected value of the distribution: so-called mean-
preserving spreads. Let us define, $f$ and $s$, two probability density function on the interval $[0, 1]$. Let $s(x)$ be a step function defined by:

$$s(x) = \begin{cases} \infty & \text{for } a < x < a + t \\ \alpha & \text{for } a + d < x < a + d + t \\ 0 & \text{otherwise} \end{cases}$$

(38)

where $0 \leq a \leq a + t \leq a + d \leq a + d + t \leq 1$.

Such a function is pictured in figure 5. It is easy to verify that

$$\int_0^1 s(x)dx = \int_0^1 s\cdot s(x)dx = 0.$$ Thus, if $f$ is a density function and if $g = f + s$, then

$$\int_0^1 g(x)dx = \int_0^1 f(x)dx + \int_0^1 s(x)dx = 1,$$

and

$$\int_0^1 xg(x)dx = \int_0^1 xf(x)dx + \int_0^1 xf(x)dx.$$ It follows that if $g(x) \geq 0$

for all $x$ ($f(x) \geq \infty$ for $a + d \leq x \leq a + d + t$), $g$ is a density function with the same mean as $f$. Adding a function like $s$ to $f$ shifts the probability weights from the center to the tails. See figure 6.

![Figure 5](image-url)
A function satisfying that condition is called a mean-preserving spread. Stiglitz and Rotschild showed in [22] that this "weak" definition implies only a partial orderings of distribution function. However, in our work, we assume a more restrictive definition of mean-preserving spread. It implies multiplicative shift in the density function leaving the mean unchanged. This definition implies a complete ordering. To formalize this type of mean-preserving spread, we posit:

\[ \gamma^*(h) = (1 + h)\gamma - h\bar{\gamma} \]  \hspace{1cm} (39)

The new random variable \( \gamma^* \) is viewed as a small alteration of the spread of the density function. For \( h > 0 \), we take some "bites" out of the original probability density function and locate these chunks of density further out from the mean. However, a priori there is no certainty that the spread of chunks of density would not alter the mean.

5) It is easy to show that for positive values of \( h \) the random variable \( \gamma^* \) is riskier. The variance of \( \gamma^* \) is: \( \text{Var}(\gamma^*) = (1 + h)^2\text{Var}\gamma \). A small increase in \( h \): \( \Delta\text{Var}(\gamma^*)/\Delta h = 2(1 + h)\). \( \text{Var}\gamma \) makes \( \gamma^* \) riskier.
The change in the mean of the density is captured by the parameter $\bar{\gamma}$. In our specification, the mean-preserving spread defines a shift in the density function $\gamma^*$ such that the mean, $E(\gamma^*)$, of the random variable $\gamma^*$ is the same as the original random variable $\gamma$. That is:

$$E(\gamma^*) = E \left\{ (1 + h)\gamma - h \bar{\gamma} \right\} = \bar{\gamma} \quad (40)$$

Using this restrictive specification of a mean-preserving spread, which is necessary for our comparative-static analysis, leads to the possibility of violating the upper and lower bound of the economic meaningful values of $\gamma$. To utilize our specification we assume small distortion of the probability density function ($h \to 0$) and that in the neighborhood of the optimal solution a small multiplicative shift of the density function do not violate the constraint on the range of economic meaningful values of the random variable; i.e., $[0, 1]$. A mean-preserving spread of a two states of the world case is portrayed in figure 7.

![Figure 7](image-url)
The increased riskiness comes from the shift to \( y_0' \) and \( y_1' \) of the two states of the world, shift preserving the expected value of the random variable \( Y \). An increase in riskiness in tax avoidance may be generated by a revised audit procedure producing a more unpredictable impact on the final level of \( X \), as examiners are directed to be more picky. One can also envisage one or more changes in tax legislation.

To evaluate the marginal effects of a change in riskiness, we substitute \( Y' \) to \( Y \) in (10); the function to maximize with respect to \( I \) and \( P \). The modified problem is:

\[
S = \max_{I, P} \left\{ U(Y - \Theta \left[ Y - A - ((1 + h)Y - h \bar{Y})X(I, P) \right] - qI - P, mP) \right\}
\]

and the first-order conditions are:

\[
E \left\{ U_C \Theta [(1 + h)Y - h \bar{Y}] \right\} X_T - q = 0 \quad (42)
\]

\[
E \left\{ U_C \Theta [(1 + h)Y - h \bar{Y}] \right\} X_P - 1 + EU_P = 0 \quad (43)
\]

However, since we are interested in marginal adjustments, it is intuitively clear that when the parameter \( h \) goes to zero, one has the original optimal solution given by equations (12) and (13). Furthermore, taking the differential of equations (42) and (43) with respect to \( h \), we obtain a system of two equations, which, written in the standard form given by equation (35) and (36) are:

\[
\frac{\delta I}{\delta h} = \frac{1}{H} \left\{ H_{22} G_{1h} - H_{12} G_{2h} \right\} \quad (44)
\]

\[
\frac{\delta P}{\delta h} = \frac{1}{H} \left\{ H_{11} G_{2h} - H_{21} G_{1h} \right\} \quad (45)
\]

where
In these equations, \( H \) is the determinant of the Hessian in equation (21).

This approximation holds by letting \( h \) go to zero. \( g_{1h} \) represents the terms directly dependent on the parameter \( h \). Notice that we have an increase in riskiness without changes in the mean of the distribution.

Using the relationship (23), we can derive the following relation between \( g_{1h} \) and \( g_{2h} \):

\[
g_{2h} = - \left\{ E_{cc}(\Theta X_P - 1) \Theta X(\gamma - \bar{\gamma}) + E_{cc} \Theta X_P(\gamma - \bar{\gamma}) \right\}
\]

\[
= - \left\{ E_{cc}(\Theta X_P - 1) \Theta X(\gamma - \bar{\gamma}) (X_P/X_I) - E_{cc} \Theta X(\gamma - \bar{\gamma})g \right. \\
+ \left. E_{cc} \Theta X_P(\gamma - \bar{\gamma}) (X_P/X_I) \right\}
\]

\[
= g_{1h} (X_P/X_I) + E_{cc} \Theta X(\gamma - \bar{\gamma})g \text{ by (46)}
\]

The sign of the last right-hand side term is proved to be positive in A3 (3). This term states that an increase in riskiness has a favorable effect on the second class of tax-avoidance expenditures producing ancillary rewards.

The two components of \( g_{1h} \), equation (46), have the following economic interpretation. The first right-hand side term, \( E_{cc}(\Theta X_P - q) \Theta X(\gamma - \bar{\gamma}) \), is shown to be negative in A3(2), under the assumption of decreasing absolute risk aversion. An increase in riskiness defined as taking some "bites" out of the original probability density function and locating these chunks of density further out has the following implication. The taxpayer experiences a higher probability of doing extremely well or poorly assigns, under decreasing absolute risk aversion, a
weight to the poor outcomes than he appreciates extremely good outcome. The taxpayer getting no compensation in terms of mean value is worse-off.

The second term, $EU \cdot \delta X_1 (y - \bar{y})$ is proved to be negative in A2(3). Given the level of disposable income, an increase in riskiness of tax avoidance increases the risk premium demanded on inputs. The two terms being negative, the expression $G_{1h}$ and by (48) $G_{2h}$ are unambiguously positive, taking account of the sign in front of them.

Now, we can evaluate the qualitative effects of an increase in riskiness on the level of demand for tax-avoidance inputs, $I^h$ and $P^*$. We use the fact that $H$ is a dominant diagonal matrix with negative elements and the ranking established in (22). We rewrite equations (44) and (45) to obtain the following qualitative effects:

$$\delta I/\delta h = \frac{1}{H} \left\{ G_{1h} \left( H_{21} - H_{12} \left( \frac{X_f}{X} \right) \right) - H_{21} EU \cdot \delta (y - \bar{y}) \right\} < 0$$

$$\delta P/\delta h = \frac{1}{H} \left\{ G_{1h} \left( H_{11} \left( \frac{X_f}{X} \right) - H_{12} \right) + H_{11} EU \cdot \delta (y - \bar{y}) \right\} < 0$$

An increase in riskiness has negative effects on the quantity invested in the first class of expenditures. The second class of tax-avoidance expenditures also decreases. There is a bias in favor of the second class of tax-avoidance input producing ancillary rewards which are independent of the level of risk. To get the unlikely positive effect on $P$ expenditures, $- G_{1h} \left( H_{11} \left( \frac{X_f}{X} \right) - H_{12} \right) + H_{11} EU \cdot \delta (y - \bar{y})$ have to be satisfied. We conclude that an increase in riskiness has qualitative negative effects on the demand for the two inputs used in tax-avoidance activity:

$$\delta I/\delta h < 0, \quad \delta P/\delta h < 0.$$
2.2. The qualitative effects of an increase in the expected value of the random variable \( Y \).

A change in the mean of \( Y \) is defined by:

\[
Y^* = Y + dY
\]  
(52)

The new random variable \( Y^* \) has a mean value equal to: \( E(Y^*) = \bar{Y} + d\bar{Y} \), and the same variance as the original random variable \( Y \).

This allows us to derive the reaction of demand for the two tax-avoidance inputs to changes in the mean value of the probability density function determined by governmental behavior and tax-collection practices. In other words, if (in the United States) the IRS becomes more severe and/or the government increases the rate of audits and/or the legislature closes one or more loopholes, we want to predict the direction of changes in tax-avoidance activity to variations \( d\bar{Y} \).

At this point, we should make the same qualification about the possible violation of upper and lower bounds. We assume that in the neighborhood of the optimal solution, a small increase \( (d\bar{Y} \rightarrow 0) \) in the mean value is not violating the lower or upper bound.

We substitute the new random variable \( Y^* \) in, and differentiate the first-order conditions (12) and (13) with respect to \( d\bar{Y} \). We obtain a system of two equations written in the standard form given by (35) and (36), where the value of the Hessian is evaluated for \( d\bar{Y} = 0 \).

\[
\delta I/\delta (d\bar{Y}) = \frac{1}{H} \left\{ H_{22}G_{1d\bar{Y}} - H_{12}G_{2d\bar{Y}} \right\}
\]  
(53)

\[
\delta P/\delta (d\bar{Y}) = \frac{1}{H} \left\{ H_{11}G_{2d\bar{Y}} - H_{21}G_{1d\bar{Y}} \right\}
\]  
(54)

where
The first term of $G_{1d\bar{y}} = EU_C(\Theta X_I - q) \Theta X$ is positive, as proved in AI (1), and measures the decrease in risk premium on input caused by the income effect on risk-bearing. Notice that this effect is weighted by the product of tax rate times the level of tax-avoidance activity. The second term measures the increase in the marginal value productivity due to a rise in the mean value of rewards from tax-avoidance activity. Taking account of the sign in front of the whole expression, we have $G_{1d\bar{y}}$ and $G_{2d\bar{y}}$ negative.

Similarly to equation (48), we derive the following relationship between $G_{1d\bar{y}}$ and $G_{2d\bar{y}}$:

$$G_{2d\bar{y}} = G_{1d\bar{y}}(X_P/X_I) + EU_C \Theta X$$

and by the concavity of the utility function, we have $EU_C \Theta X < 0$, and:

$$G_{2d\bar{y}} < G_{1d\bar{y}}(X_P/X_I)$$

To evaluate equations (53) and (54), we rework them using the relationship (57) to get:

$$bI/\delta (d\bar{y}) = \frac{1}{H} \left\{ G_{1d\bar{y}} (H_{22} - H_{12}(X_P/X_I)) - H_{12} EU_C \Theta X \right\} > 0$$

(59)

$$bP/\delta (d\bar{y}) = \frac{1}{H} \left\{ G_{1d\bar{y}} (H_{11}(X_P/X_I) - H_{12}) + H_{11} EU_C \Theta X \right\} > 0$$

(60)

An increase in the mean $d\bar{y}$, brings about a positive effect on the demand for both risky inputs of tax-avoidance activity.

$$bI/\delta (d\bar{y}) > 0, \quad bP/\delta (d\bar{y}) > 0.$$  

(61)
2.3. The qualitative effects of changes in the effective flat tax rate, \( \Theta \).

The qualitative effects are obtained by differentiating the system of first-order conditions, (12) and (13), with respect to \( \Theta \). The standard form states the marginal effect on the demand for the two inputs and eventually the direction of change in tax-avoidance activity:

\[
\begin{align*}
\frac{\delta I}{\delta \Theta} &= \frac{1}{H} \left\{ H_{22} G_{1\Theta} - H_{12} G_{2\Theta} \right\} \quad (62) \\
\frac{\delta P}{\delta \Theta} &= \frac{1}{H} \left\{ H_{11} G_{2\Theta} - H_{21} G_{1\Theta} \right\} \quad (63)
\end{align*}
\]

where

\[
\begin{align*}
\delta G_{1\Theta} &= G_{1\Theta} = -\left\{ -EU_{CC}(\Theta Y X_I - q) (Y - X) + EU_C Y X_I \right\} \quad (64) \\
\delta G_{2\Theta} &= G_{2\Theta} = -\left\{ -EU_{CC}(\Theta Y X_P - 1) (Y - X) + EU_C Y X_P \right\} \quad (65)
\end{align*}
\]

\(-EU_{CC}(\Theta Y X_I - q) (Y - X)\) expresses the increase in economic rent on tax-avoidance input caused by the marginal tax increase. The second term, \( EU_C Y X_I \), values the marginal benefit in tax-avoidance information due to a small increase in the effective flat marginal tax rate. The net sum of marginal benefits is positive and \( G_{1\Theta} \) and \( G_{2\Theta} \) are negative after taking account of the negative sign in front of them. Then using the same technique as in subsection 2.1., we have qualitatively positive effects on the two tax-avoidance inputs caused by an increase in the marginal effective tax rate:

\[
\frac{\delta I}{\delta \Theta} > 0, \quad \frac{\delta P}{\delta \Theta} > 0. \quad (66)
\]
2.4. The qualitative effects of changes in the price of tax-avoidance inputs.

We turn now to determining the direction of changes in demand for inputs when the price of tax-avoidance expenditures changes. We hypothesize a change in the price of the first class of expenses, with the opportunity cost of the second class of input remaining unaltered.

Actually, the price of the second class of expenditures as a dollar amount can only change in absolute value. Dollars are chosen as numéraire of the system. Our analysis is carried out in real terms. In our case, we want a small increase in the relative price of tax-avoidance information in terms of dollars (the numéraire).

Differentiating the system of first-order conditions (12) and (13) with respect to \( q \) gives the following qualitative effects:

\[
\begin{align*}
\delta I / \delta q &= \frac{1}{H_{22}} \left\{ H_{12} \ G_{1q} - H_{12} \ G_{2q} \right\} < 0 \\
\delta P / \delta q &= \frac{1}{H_{22}} \left\{ H_{11} \ G_{2q} - H_{21} \ G_{1q} \right\} 
\end{align*}
\]

(67)

(68)

where

\[
\begin{align*}
\delta G_{1} / \delta q &= G_{1q} = - \left\{ - EU_{CC}(\Theta Y_{I} - q) \ I - EU_{C} \right\} \\
\delta G_{2} / \delta q &= G_{2q} = - \left\{ - EU_{CC}(\Theta Y_{P} - 1) \ I \right\}
\end{align*}
\]

(69)

(70)

The first term of \( G_{1q} \), \(-EU_{CC}(\Theta Y_{I} - q)I\) expresses the increase in economic rent on input \( I_{q} \), caused by the increase in its relative price. The price increase diminishes his net income and makes the taxpayer more risk averse. The second term, \(-EU_{C}\), evaluates the
marginal increase in the opportunity cost of I in terms of consumption expected utility. \( G_{1q} \) is clearly positive when we take account of the negative sign in front of it. \( G_{2q} \) captures the indirect effect on risk-bearing on the second input caused by a small increase in the relative price of input I. A price increase in tax-avoidance input I has a qualitative negative impact on its demand since \( G_{1q} \) and \( G_{2q} \) are positive and the dominant diagonal matrix of the Hessian has negative diagonal elements.

The marginal impact on the second class of tax-avoidance input depends on the substitutability or the complementarity nature of the second class of expenditures involves the cost of carrying out tax-avoidance strategy based on tax-avoidance information collected. In case of complementarity, equation (68) implies: \( \delta P/\delta q < 0 \). If we had a substitution relationship between the two inputs, \( x_{1p} < 0 \), then we would have: \( \delta P/\delta q > 0 \).

2.5. The qualitative effects of change in the taxpayer's income with a flat effective tax rate.

To analyze the change in optimal values of \( I^* \) and \( P^* \) for an increase in taxpayer's income, we take the total differential with respect to \( Y \) of the first-order conditions (12) and (13). We obtain a system of two equations, and solve for the two partial derivatives of I and P with respect to \( Y \), which written in standard form given by (35) and (36) are as follows:
\[ \frac{\delta I}{\delta Y} = \frac{1}{H} \left\{ H_{22} G_{1Y} - H_{12} G_{2Y} \right\} \]  
(71)  
\[ \frac{\delta P}{\delta Y} = \frac{1}{H} \left\{ H_{11} G_{2Y} - H_{21} G_{1Y} \right\} \]  
(72)  
where  
\[ \frac{\delta G_1}{\delta Y} = G_{1Y} = -EU_{CC}(\tilde{B}YX_I - \tilde{q}) (1 - \Theta) \]  
(73)  
\[ \frac{\delta G_2}{\delta Y} = G_{2Y} = -EU_{CC}(\tilde{B}YX_P - 1) (1 - \Theta). \]  
(74)  

The taxpayer's disposable income increases by \((1 - \Theta)\). From equation \( A(1) \), a taxpayer with decreasing absolute risk aversion is induced to lower his risk premium required on tax-avoidance inputs, \( G_{1Y} \) and \( G_{2Y} \), are negative. Taking account of the dominant diagonal Hessian matrix with negative diagonal elements, we have a positive qualitative effect on both risky tax-avoidance inputs:  
\[ \frac{\delta I}{\delta Y} > 0, \quad \frac{\delta P}{\delta Y} > 0. \]  
(75)  

2.6. The qualitative effect of an increase in the exemption level.  

An increase in the exemption level with a flat tax rate is one way to increase the degree of tax progression. We want to know the unconstrained impact of changes in the exemption level. Differentiating totally the first-order conditions \((12)\) and \((13)\) with respect to the exemption level and solving, we obtain:  
\[ \frac{\delta I}{\delta A} = \frac{1}{H} \left\{ H_{22} G_{1A} - H_{12} G_{2A} \right\} \]  
(76)  
\[ \frac{\delta P}{\delta A} = \frac{1}{H} \left\{ H_{11} G_{2A} - H_{21} G_{1A} \right\} \]  
(77)  
where
The expression, \(-EU_{CC}(\Theta \gamma x_I - q)\), represents the negative income-risk effect on the demand for tax-avoidance expenditures. A one dollar increase in the exemption level raises the after tax disposable income by the amount of the effective flat tax rate. This increase in disposable income induces the taxpayer to lower the risk premium on I and P. The result holds under the condition that the exemption level, \(A\) is smaller than the maximum taxpayer's potential income, \(Y\), because if \(A = Y\) no taxes one collected and the level of tax-avoidance activity is optimally zero.

In the case of an interior solution for tax-avoidance inputs, and \(A < Y\) we have \(G_{1A}\) and \(G_{2A}\) to be negative. Taking account of the dominant diagonal Hessian matrix with negative diagonal components, we have a positive qualitative effect on both risky tax-avoidance expenditures:

\[
\frac{\delta I}{\delta A} > 0, \quad \frac{\delta P}{\delta A} > 0.
\]
An expected revenue-compensated increase in the degree of
effective tax progression

We have derived the unconstrained qualitative impact of changes in
the effective flat tax rate, the level of income, and the exemption
level, among others, in the last section. We are interested, however,
to predict the effect on the amount of tax avoidance of a tax reform,
where the government collects the same amount of expected-tax revenue
but with a different effective tax rate mixture. The degree of tax pro­
gression is increased by a larger exemption level, A. To raise the
same amount of expected-tax revenue, the effective flat tax rate has to
be increased. We want to predict the impact on tax-avoidance inputs of
such a change under tax-revenue constraint.

The expected tax payment is defined by:
\[
E(T) = E \left[ Y - \Lambda - X(I, P) \right]
\] (81)
The requirement of an expected compensated-revenue or of a compensated-
expected-tax liability can be expressed as:
\[
E(dT) = E(Z - A) d\theta - dA - \Theta E(\gamma X_I) dI - \Theta E(\gamma X_P) dP = 0 \] (82)
where \(E(Z - A) = E \left[ Y - \Lambda - X(I, P) \right]\). We can also express this require­
ment taking account of the partial derivative of I and P:
\[
E(Z - A) d\theta - \Theta E \left\{ \gamma X_I \frac{\delta I}{\delta \theta} + \gamma X_P \frac{\delta P}{\delta \theta} \right\} d\theta
\]
\[= \Theta dA - \Theta E \left\{ \gamma X_I \frac{\delta I}{\delta A} + \gamma X_P \frac{\delta P}{\delta A} \right\} dA = 0\]
or \[ d\theta = \frac{1}{E(Z-A) - \theta E(Z-A)} \times \left[ \frac{E(Z-A) \delta I/\delta \theta + \delta P/\delta \theta}{\left( \frac{E(Z-A) \delta I/\delta \theta + \delta P/\delta \theta \right)} \right] \] \hspace{1cm} (83)

We substitute the above value of \( d\theta \) into equation (82). We set, \( dP = 0 \), and this yields:

\[
\frac{dI}{dA} \left|_{E(dT) = 0} \right. = \frac{\frac{1}{X_I H} \left\{ \frac{E(U_X Y)}{X_I H} \left( X_I^2 H_{22} - 2X_I X_P H_{12} + X_P^2 H_{11} \right) \right\}}{E(Z-A) - \tilde{\theta} \tilde{V}(X_I \delta I/\delta \theta + X_P \delta P/\delta \theta) \right) \] \hspace{1cm} (84)

To determine the sign of the numerator, we substitute the values of the comparative static results derived in section 2. We substitute in the values of \( \delta I/\delta A \), \( \delta P/\delta A \) and \( \delta I/\delta \theta \), \( \delta P/\delta \theta \) given by equations (76), (77), (62), and (63) respectively. We obtain:

\[
\frac{dI}{dA} \left|_{E(dT) = 0} \right. = \frac{\frac{1}{X_I H} \left\{ \frac{E(U_X Y)}{X_I H} \left( X_I^2 H_{22} - 2X_I X_P H_{12} + X_P^2 H_{11} \right) \right\}}{E(Z-A) - \tilde{\theta} \tilde{V}(X_I \delta I/\delta \theta + X_P \delta P/\delta \theta) \right) \] \hspace{1cm} (85)

The numerator is always positive by the second-order conditions and the negative dominant diagonal Hessian matrix. Thus, an expected revenue-compensated change in the degree of tax progression induces an increase in tax-avoidance risky input \( I \), if the denominator is positive:

\[
E(Z-A) - \tilde{\theta} \tilde{V}(X_I \delta I/\delta \theta + X_P \delta P/\delta \theta) > 0
\]

or in elasticity form:

\[
\frac{\tilde{V} X}{Y - A} + \left( \frac{\tilde{V} X I}{Y - A} \right) \tilde{E}_I + \left( \frac{\tilde{V} X P}{Y - A} \right) \tilde{E}_P < 1
\] \hspace{1cm} (86)

This condition states that the sum of the percentage (in terms of gross taxable income) expected tax avoidance plus the percentage of expected tax-avoidance inputs weighted by their elasticity with respect to the flat tax rate should be less than one. This conditions appears to be
satisfied for plausible values, e.g., a 15 percent of tax avoidance associated with an homogeneous tax-avoidance function of degree one (say 2/3 of tax-avoidance information and 1/3 of tax-avoidance expenditures of the second class). Finally, we suppose an elasticity of 2.

We obtain: \(0.15 + 0.10(2) + 0.05(2) = 0.45 < 1\).

Similarly, for the second class of tax-avoidance expenditures. We substitute the value of \(d\theta\), equation (83) into equation (82). We set \(dI = 0\), and we obtain:

\[
\frac{dP}{dA} E(dT) = 0 = \frac{\frac{1}{X_p} \left\{ X_I \left[ E(Z-A) \delta I/\delta A + \theta \delta I/\delta \theta \right] + X_p \left[ E(Z-A) \delta P/\delta A + \delta P/\delta \theta \right] \right\}}{E(Z-A) - \theta \delta \left( X_I \delta I/\delta \theta + X_p \delta P/\delta \theta \right)}
\]  

(87)

To determine the sign of the numerator, we substitute the values of the comparative-static results derived in section 2. We obtain an equation identical to (85) except for \(1/X_p\):

\[
\frac{dp}{dA} \bigg|_{dI = 0} = \frac{-\theta \xi \left( U_{C, \gamma} \right)}{X_p H} \left\{ X_I^2 H_{22} = 2X_I X_p H_{12} + X_p^2 H_{11} \right\}
\]  

(88)

Consequently, the same condition (86) applies for an increase in second class of tax-avoidance expenditures.

The demand for both tax-avoidance inputs increases for an expected compensated-revenue increase in tax progression. The result has an important policy implication. The increase in tax progression increases the level of tax avoidance and the taxpayer's reaction to the degree of tax progression introduces greater distortionary effects. This result bring forth the efficiency cost of a tax reform aiming towards a de-
crease in after-tax income distribution inequality. One reason for such a strong result is that the maximum taxpayer's potential income is given in this model. In the next section, we examine the problem from another point of view where the taxpayer's income is endogeneous and tax-avoidance information input is one element of the taxpayer's time constraint; then the sign of tax-avoidance information input for a revenue-compensated (no uncertainty) increase in the degree of tax progression is subject to more qualifications.
Section 4

**Tax Avoidance, Progression and Leisure**

Our first model of tax-avoidance activity used two risky inputs. The two outlays undertaken to achieve tax-avoidance benefits were tax-avoidance information costs, q, and expenditures, P, to carry out the chosen tax-avoidance strategy. These two risky-input costs are subtracted from the maximum potential income, Y.

To analyze the impact of tax-avoidance activity on the work-leisure decision, we treat this particular tradeoff as a special case of tax avoidance. We simplify the model by neglecting the second class of expenditures, P, that produce ancillary monetary rewards and focus on a certainty environment. But we complicate our analysis by endogenizing taxpayer income. The taxpayer sells his labor services in the market at the exogenous wage rate, \( w \). His gross income is \( Y = wN \), where \( N \) is his working time.

Furthermore, the taxpayer also spends time gathering and analyzing information (or studying tax laws) to figure out ways to minimize his tax bill. He can also generate some nontaxable homemade income, e.g., by painting his house, repairing his car and so forth. The amount of tax avoidance is a function of his time input, \( X(I) \). We posit that the taxpayer operates under a time constraint of this kind:

\[
k = L + N + I
\]

where \( L = \text{leisure time} \)
\[ N = \text{working time} \]
\[ I = \text{tax-avoiding time} \]
\[ k = \text{total time available.} \]

We assume an effective tax rate structure that is approximated by a linear tax function:
\[
t = \Theta + \pi Z
\]
where \( \Theta \) is a flat tax rate and \( \pi Z \) is the progressive part. \( Z = Y - X(I) \) is the effective taxable income. A tax is said to be progressive if its effective rate, \( t \), increases with taxable income and regressive if it decreases with taxable income. We define the degree of progression for a taxable income \( Z \) as \( t'(Z) \). In our case, \( t'(Z) = \pi \). Thus, we consider \( \pi \) as the degree of tax progression. The total effective tax payment is defined by:
\[
T = t(Z) \cdot Z
= (\Theta + \pi Z) \cdot Z
= (\Theta \pi (Y - X)) (Y - X)
= [\Theta \pi (wN - X(I))] (wN - X(I))
\]
where \( Z = Y - X = wN - X(I) \) is the effective taxable income.

The taxpayer's problem is to maximize a real-valued utility function \( U(C, L) \), where \( C \) is the disposable income including the homemade income and \( L \) is his leisure time. The utility function has continuous second derivatives, is strictly increasing in \( C \) and \( L \), strictly quasi-concave, and unbounded from below as \( C \) and \( L \) tend to zero. The last characteristic rules out no consumption and no leisure at the optimum. The taxpayer maximizes \( U(C, L) \) under the time constraint \( L = k - N - I \) and his disposable income:
C = wN + bI - \left[ \Theta + \pi(wN - X(I)) \right] (wN - X(I)). \tag{4}

where bI represents homemade income. We substitute the time constraint into the utility function and set up the following Lagrangian function:

\[ L = U(C, k - N - I) + \lambda \left[ wN + bI - \left[ \Theta + \pi(wN - X(I)) \right] (wN - X(I)) \right] \tag{5} \]

The first-order conditions for an interior solution are:

\[ U_1 - \lambda = 0 \tag{6} \]
\[ -U_2 + \lambda w(1 - \Theta - 2\pi \gamma) = 0 \tag{7} \]
\[ -U_2 + \lambda (b + (\Theta + 2\pi \gamma)X) = 0 \tag{8} \]
\[ wN + bI - (\Theta + \pi \gamma) Z - C = 0 \tag{9} \]

Let us define \( \mu = \Theta + 2\pi \gamma \) which is the effective marginal tax rate \( \delta T/\delta z \), then the sufficient condition for a maximum is that the bordered Hessian:

\[
\begin{bmatrix}
0 & -1 & w(1 - \mu) & b + X_1 \mu \\
-1 & U_{11} & -U_{12} & -U_{12} \\
w(1 - \mu) & -U_{21} & U_{22} - 2\lambda w^2 \pi & U_{22} + 2\lambda wX_1 \pi \\
b + X_1 \mu & -U_{21} & U_{22} + 2\lambda X_1 \pi & U_{22} + 2\lambda X_1^2 \pi \\
\end{bmatrix}
\begin{bmatrix}
d\lambda \\
dC \\
dN \\
dI \\
\end{bmatrix}
\]
be negative definite under constraint. We denote by $D$ the determinant of this bordered Hessian. $D$ is negative and we denote by $D_{ij}$ the $ij$th cofactor of $D$.

**Uncompensated changes in $\Theta$**

By differentiating the equilibrium conditions with respect to $\Theta$, and solving, we obtain the effect of an increase in $\Theta$, the flat rate on work effort $N$, and on tax-avoiding time:

$$
\delta N/\delta \Theta = \frac{ZD_{13}}{D} + \frac{wD_{33}}{D} - \lambda X_{14}/D \quad (11)
$$

$$
\delta I/\delta \Theta = \frac{ZD_{14}}{D} + \frac{wD_{34}}{D} - \lambda X_{14}/D. \quad (12)
$$

The first RHS term of equation (11) measures the income effect. This is positive, if $D_{13}$ is negative since $D$ is negative. The condition for $D_{13}$ to be negative is:

$$
L \eta_{yy} < c \eta_{yL} \quad (13)
$$
where $\eta_{yy}$ and $\eta_{yL}$ are written in place of $-CU_{11}/U_1$ and $-LU_{12}/U_2$. There are the elasticities (multiplied by $-1$) of the marginal utility of income and the cross-elasticity of income with respect to leisure respectively. By the increase of the flat tax rate, his budget constraint is tighter and the immediate reaction is to increase working time, if the elasticity condition (13) is satisfied.

The two other terms measure substitution effects. The second RHS term is negative since $D_{33}$ is positive. The increased flat effective tax rate induces the taxpayer to take more leisure. The third RHS term is unsigned. It measures the cross-substitution effect between tax-avoiding time and working time. Thus the sign of $\delta U/\delta \theta$ is unsigned.

The economic interpretation of equation (12) is similar. The first RHS term is a negative income effect, $D_{14}$ being positive, if the same elasticity condition (13) holds. The taxpayer's budget constraint is relaxed by the increased return on tax-avoidance activity caused by the marginal increment in the flat effective tax rate. The two other terms measure substitution effects. The third RHS term is positive because tax-avoiding time brings higher rewards. $D_{14}$ is positive. The second RHS term measures the unsigned cross-substitution effect. The sign of $\delta U/\delta \theta$ is undetermined.

Uncompensated changes in $\pi$

We may similarly obtain the effect of changes in $\pi$ by differentiating the equilibrium conditions with respect to $\pi$ and solving for
the impact on work, N and tax-avoiding time I:

\[ \delta N/\delta \Pi = 2^2D_{13}/D + 2\lambda wZD_{33}/D - 2\lambda X_{I}ZD_{43}/D \]  \hspace{1cm} (14)

\[ \delta L/\delta \Pi = 2^2D_{14}/D + 2wZD_{34}/D - 2\lambda X_{I}ZD_{44}/D \]  \hspace{1cm} (15)

The comparative-static results may be interpreted similarly to equations (11) and (12).

The first RHS term of equation (14) measures the positive income effect of an increase in the degree of tax progression, if condition (13) is satisfied. The two other terms measure substitution effects. The second RHS term is negative since \( D_{33} \) is positive. The increase in the degree of tax progression induces the taxpayer to take more leisure. The third RHS is unsigned. It evaluates the cross-substitution effect between tax-avoiding time and working time. The sign of \( \delta N/\delta \Pi \) is unsigned.

The economic content of equation (15). It has a negative income effect, if elasticity condition (13) is satisfied, and substitution effects. The third RHS term is positive because tax-avoiding time brings higher returns. The second RHS term measures the unsigned cross-substitution effect. The sign of \( \delta L/\delta \Pi \) is undetermined.

**Revenue-compensated changes in \( \Pi \)**

Now, as the degree of progression is measured by \( \Pi \), we may investigate the effect of a change in this by considering a revenue-compensated change in \( \Pi \). The revenue-compensated change requires the effective tax payment to stay the same. This means that the total
differential of tax payment, \( dT \) is constrained to zero. From equation (3), we get:

\[
\begin{align*}
\Delta T &= Z \frac{\partial \Theta}{\partial \pi} + Z^2 \Delta \pi + \mu w \delta N / \delta \pi - \mu X_I \Delta I = 0 \\
&= Z^2 + \mu w \delta N / \delta \pi - \mu X_I \Delta I / \Delta \pi
\end{align*}
\]

Taking into account that \( N \) and \( I \) are affected by changes in \( \Theta \) and \( \Pi \).

We substitute the value of \( d \Theta \) obtained in equation (17) into equation (16). Then, solving for the total derivative of \( N \) with respect to \( \Pi \), given \( dT = 0 \), and \( dI = 0 \), we obtain:

\[
\frac{dN}{d\Pi} \bigg|_{dT=0 \text{ and } dI=0} = -\frac{\left\{ - w \delta N / \delta \Pi + w Z \delta I / \delta \Pi + X_I \delta I / \delta \Pi - X_I \delta I / \delta \Theta \right\}}{Z + \mu w \delta N / \delta \Theta - \mu X_I \delta I / \delta \Theta}
\]

Moreover, we can substitute the values of \( \delta N / \delta \Pi \), \( \delta N / \delta \Theta \), \( \delta I / \delta \Pi \) and \( \delta I / \delta \Theta \) derived in our previous comparative-static analysis to obtain the following result:

\[
\frac{dN}{d\Pi} \bigg|_{dT=0 \text{ and } dI=0} = -\frac{Z^2 \left\{ - \frac{w^2 D_{33} + w X_I D_{43} + w X_I D_{34} - X_I^2 D_{44}}{w D} \right\}}{Z + \mu w \delta N / \delta \Theta - \mu X_I \delta I / \delta \Theta}
\]

where the expression in brackets is always negative. \( D \) is the negative determinant of the bordered Hessian. Therefore, we have that a decrease in the amount of labor supplied occurs if the denominator is positive:
\[ z + \mu w \delta N / \delta \theta - \mu x \delta I / \delta \theta > 0 \quad (20) \]

or in elasticity terms:

\[ \varepsilon^N_\theta > - \eta \mu + \frac{X}{Y} (\eta \mu + \varepsilon^I_\theta). \quad (21) \]

We may conclude that a small revenue-compensated change in the degree of tax progression has a disincentive effect on work if, and only if, the elasticity of work with respect to the flat tax rate exceeds minus the ratio of the flat to the marginal effective tax rate, plus the ratio of tax avoidance to gross income which multiplies the ratio of the flat to the marginal effective tax rate plus the elasticity of tax-avoiding time with respect to the flat effective tax rate.

When no tax avoidance is allowed, the condition (21) for a decrease in work effort reduces to:

\[ \varepsilon^N_\theta > - \eta \mu. \quad (22) \]

This condition was first derived by Allingham in [1]. The result implies that, when no tax avoidance is allowed, a small revenue-compensated change in the degree of tax progression has a disincentive effect on work if, and only if, the elasticity of work with respect to the flat tax rate exceeds minus the ratio of the flat to the marginal tax rate.

We derive the impact of a revenue-compensated change in the degree of tax progression on tax-avoiding time. We substitute the value of \( d\theta \), equation (17), into equation (16). Then, solving for the total
derivative of I with respect to \( \pi \), given \( dT = 0 \), and \( dN = 0 \), we obtain:

\[
\frac{dI}{d\pi} \bigg|_{dT=0} = \frac{-\frac{Z}{X} \left\{ -w \frac{\delta I}{\delta \pi} + \frac{X}{X} \frac{\delta I}{\delta \pi} + wE \frac{\delta N}{\delta \theta} - \frac{X}{X} \frac{\delta I}{\delta \theta} \right\}}{Z + \mu \frac{\delta I}{\delta \theta} - \mu \frac{X}{X} \frac{\delta I}{\delta \theta}}
\]

Moreover, we can substitute the value of \( \frac{\delta I}{\delta \pi} \), \( \frac{\delta I}{\delta \pi} \), \( \frac{\delta N}{\delta \theta} \) and \( \frac{\delta I}{\delta \theta} \) derived in equations (14), (15), and (11), and (12), respectively, to obtain:

\[
\frac{dI}{d\pi} \bigg|_{dT=0} = \frac{\frac{Z^2}{X} \left\{ -\frac{w^2 D_3}{X} + \frac{wX D_3}{X} + \frac{wX D_4}{X} - \frac{X^2 D_4}{X} \right\}}{Z + \mu \frac{\delta N}{\delta \theta} - \mu \frac{X}{X} \frac{\delta I}{\delta \theta}}
\]

where the expression in brackets and D are negative. Therefore, we have an increase in the amount of tax-avoiding time if, and only if, the denominator is positive:

\[
Z + \mu \frac{\delta I}{\delta \theta} - \mu \frac{X}{X} \frac{\delta I}{\delta \theta} > 0
\]

or in elasticity terms:

\[
\xi_\theta^N > -\frac{\theta \mu}{Y} + \frac{X}{Y} \left( \frac{\theta \mu}{Y} + \xi_\theta^I \right)
\]

This condition is the same as the condition for a decrease in working time caused by a revenue-compensated increase in tax progression. The taxpayer has an incentive to increase his tax-avoiding time if, and only
if, the elasticity of work with respect to the flat tax rate exceeds minus the ratio of the flat to the marginal effective tax rate, plus the ratio of tax avoidance to gross income which multiplies the ratio of the flat to the marginal effective tax rate plus the elasticity of tax-avoiding time with respect to the flat tax rate.

From the above analysis, we can state: a revenue-compensated change in the degree of tax progression induces an opposite relation between working time and tax-avoiding time. Furthermore, the taxpayer decreases working time and tax-avoidance time increases if, and only if, condition (21) holds.

Conditions (21) and (22) are stated in terms of observable quantities. They do not require utility measurement. However, the elasticity of tax-avoiding time with respect to the flat tax rate is difficult to measure.

Finally, if substitution effects are dominant, then, $\delta W/\theta$ is negative and $\delta I/\theta$ is positive. We see that condition (21) becomes more stringent, or impossible to meet if $X/Y(\theta_{I,\mu} + \xi_{I,\theta}) > \theta_{I,\mu}$.

On the other hand, if the income effect is dominant, $\delta W/\theta$ is positive and $\delta I/\theta$ negative. The necessary and sufficient condition (21) for a decrease in work effort and an increase in tax-avoiding time is always satisfied since $\theta_{I,\mu} (X/Y - 1)$ is negative and $X/Y \xi_{I,\theta}$ is always negative.
Recapitulation

In this chapter, we developed a microeconomic model of tax-avoidance activity. We assumed two classes of tax-avoidance expenditures, and the taxpayer decided how much to invest in tax savings expenditures under uncertainty since he is subject to audit and/or rejection of his tax savings activities by the tax collecting agency.

In section 2, we derived unconstrained comparative-static results. Table 4 presents the impact on the two tax-avoidance risky inputs of changes in the exogenous variables. The results are derived assuming an additive utility function in consumption and the second class of tax-avoidance expenditures generating some ancillary rewards. Taxpayers have a decreasing absolute risk aversion. Furthermore, risk enters the tax avoidance function in a multiplicative form implying that a larger level of tax avoidance increases the overall variance of tax-avoidance activity.

In section 3, we presented the effect of an expected revenue-compensated change in the degree of tax progression. A tax reform aiming at reducing inequality in after-tax-income distribution should measure the increase in inefficiency introduced by the taxpayer's reaction through an increase in tax-avoidance activity.

Finally, in section 4, we analyzed the impact of tax-avoidance activity on the work-leisure choice. The taxpayer can devote his time to working time generating gross income or to tax-avoiding time allowing to cut in his tax bill. The taxpayer operates under his time constraint.
The major result states an opposite relationship between tax-avoiding time and working time for a revenue-compensated increase in the degree of tax progression.
In this chapter, we analyze the effects of tax avoidance on optimal provision of public goods and its implications for optimal taxation. We use a general-equilibrium model to determine the optimal tax rate and "surveillance" parameter. In a second stage, we study the redistributional aspects of income taxes, when we consider the distribution of earning ability.

Before presenting the model, we briefly locate this study in modern theory of public goods and optimal taxation. The Conventional Rule for provision of public goods, stated in its modern form by Samuelson [25], [26] among other economists, is that the sum of individual marginal rate of substitution between public and private goods is equal to the marginal rate of transformation, or \( \sum MRS^1 = MRT \). Figure 8 illustrates the Conventional Rule for provision of public good, where individual demand curves add vertically.

It is well known in public finance that taxes introduce distortionary effects. Atkinson and Stern [3] restated the theory of excess burden associated with commodity tax as opposed to lump-sum taxation. The use of a commodity tax introduces a deviation from the
Conventional Rule of this nature:

\[
MRT = \alpha \frac{\sum MRS}{\lambda}
\]  

(1)

where \( \alpha \) measures the marginal utility of an additional dollar used for private consumption and \( \lambda \) measures the social marginal cost of producing an additional unit of public good. \( \lambda \) is always larger than \( \alpha \). Thus, we graphically observe the impact on the optimal amount of public good of the excess burden of raising commodity taxes, in figure 9.
In section 1 and 2, the distortionary effects of tax avoidance and its consequences for optimal taxation are studied.
1.1. Setting of the government's problem

To derive an optimal tax system, we must endogenize the tax spending and financing decisions of government. In Chapter I, we studied, in a one-period model, the optimal taxpayer's tax-avoidance level given the flat effective tax rate, the "exemption level", and type-B uncertainty generated by tax-collecting agency. $\Theta \tilde{X}$ indicated the expected benefits from tax-avoidance activity.

In normative economics, the government must abandon the myopic assumption that the taxpayer ignores the provision of public goods. The Treasury should integrate all costs and benefits involved in maximizing social welfare. In our model, the government controls the tax rate and the expected value of the uncertainty component of tax-avoidance activity.

A somewhat similar approach has been taken by Kolm in his artice on optimal tax evasion [13]. In our one-period model the taxpayer's problem is:

$$\max_{I,P} \mathbb{E} \left\{ U \left[ Y - \Theta(Y - A - X(K,P) - qI - P, mP) \right] \right\}$$

(2)

The maximization problem assumes a concave additive utility function
and a concave tax-avoidance transformation function. The utility function further satisfies the conditions of decreasing absolute risk aversion, i.e., $U_{CC} < 0$ and $U_{CCC} > U_{CC}'U_{C}$. The random variable $\gamma$ introduces technological uncertainty and is supposed to have a given expected value set by matters outside the individual's control. The solution to this problem represents the optimal taxpayer's reaction to the existing tax structure.

For the government, the problem of optimal taxation is to maximize a social welfare function given individual's preferences, maximum income, and tax-collection costs. We assume a society composed of $N$ similar individuals. We focus on a conceptualized "representative" taxpayer to derive the expected revenue from income-tax collection. We have discussed extensively why the taxpayer faces uncertainty. At the national level, the aggregate-tax revenue tends to be equal to the expected value of the random component, which is known and government-determined. Thus, the total income-tax revenue for the Treasury is equal to the sum of the individual's expected tax yield:

$$E(T) = E\left\{\sum_{i=1}^{n} \Theta(Y - \gamma_{i}X)\right\}$$

$$= N \Theta(Y - E\Theta(Y)) = N \Theta(Y - \bar{Y}X). \quad (3)$$

The government can alter the expected value of tax-avoidance activity either by increasing the frequency of audits or by rewriting tax legislation to close tax loopholes. We assume that all governmental actions are costly. In the United States, the IRS would reap additional benefits in tax payment from additional audits, but it would also generate some additional costs; more employees, more office space, more
paper work, etc. According to the Wall Street Journal [31] the audit percentage rose, in 1973, for the first time in a decade to 2.4 percent from 2 percent in 1972. In 1974, the IRS was shooting for 2.6 percent, but the realized percentage was 2.32. To maintain uncertainty about the actual percentage of audits is part of the government strategy. Audit coverage will level off in fiscal year 1976, according to Ford's budget.

Penalties and interest charges are forecast at $4.75 billion in fiscal 1976. Based on the fiscal year 1974, we can calculate the additional cost of collecting an extra dollar of tax-revenue based on the total cost of the department "audit of tax returns" and the total additional tax and penalties recommended after audit examination. The total operating cost of the "audit of tax returns" department was $495,152,000, and the total additional tax and penalties recommended was $5,909,198,000 [30]. This statistic gives an average estimate of the marginal cost to collect an additional $100 of tax revenue equal to $8.2.

We define the function of "surveillance" cost to achieve a given level of expected value $\bar{\gamma}$ by:

$$J(\bar{\gamma})$$  \hspace{1cm} (4)

where $\bar{\gamma}$ represents the expected value for the "representative" citizen of the risk component of tax-avoidance activity. The audited taxpayer will very likely have only a fraction of his original claims accepted and will presumably suffer additional costs as a result of the audit. The taxpayer might also find his investment in tax-avoidance
inputs unexpectedly beneficial and not be audited. The random effects are captured at the individual level by the random variable \( \gamma \). At the national level the expected value of \( \gamma \) is determined by governmental actions. We already mentioned the reliance placed on computers to select returns for audit. The system assures uniformity by using the same criteria to screen all tax returns within classes of taxpayers. This might satisfy the equality-of-treatment criterion, but is probably not efficient because high-income taxpayers enter into tax avoidance on a grander scale. The costs would be a function of the level of income. However, our model uses the concept of "representative" individual and we don't consider the income-distribution aspect in this section.

Any action to reduce the expected value of \( \gamma \) is costly. We saw that in 1974, it cost in average $8.2 to collect an additional $100 of tax revenue. The marginal cost associated with a small increase in the expected value, \( \tilde{\gamma} \) is assumed to be negative:

\[
J'(\tilde{\gamma}) < 0 \quad \text{and} \quad J'(1) = 0, \quad |J'(0)| < \infty \tag{5}
\]

Figure 10 represents the assumed shape of this "surveillance" cost curve defined in equation (4).

The goal of the government is to make \( \tilde{\gamma} \) as small as possible, after allowing for the cost associated with a specific value of \( \gamma \). This Treasury's attitude is correct because by our comparative-static results, an increase in \( \tilde{\gamma} \) implies an increase in taxpayer's tax-avoidance activity.

The aggregate-tax revenue available to the government (we assume)
to produce public goods is:

\[ E(G) = E(T) - J(\bar{\gamma}). \]  

(6)

1.2. A specification of the social welfare function

In a general-equilibrium framework, the social welfare function should not only recognize the expected utility of its citizens, but acknowledge the social value of public-goods provision as well as the social cost of tax-avoidance activity. We can write the utility of the "representative" citizen as \( EU + EV \), where \( EU \) is, as before, the expected utility of private-goods consumption and \( EV \) is the expected utility obtained from public goods.
There is no perfect symmetry in uncertainty between individuals and the government. For each taxpayer, tax avoidance is a risky activity and the final result depends on the state of nature. The risk-averse taxpayer requires a risk premium in case of uncertainty generated by governmental attitude. This decreases the amount of tax avoidance. It is rational for the Treasury to maintain uncertainty, given the existence of tax avoidance, because our comparative-static results show that an increase in uncertainty decreases the amount of tax-avoidance expenditures and consequently of tax avoidance. The second reason why the Treasury should not minimize in the variance space is as follows.

At the national level, we have a large number of taxpayers and the variance of the average amount of taxes collected tends toward its expected value with a zero variance. From equation (3), the aggregate expected tax yield is: \( E(T) = N \theta(Y - \bar{x}) \). The expected average-tax payment collected by the Treasury is:

\[
E(T/N) = E(T) = E\left\{ \frac{1}{N} \sum_{i=1}^{n} (Y - \bar{y}_i) \right\} = \theta(Y - \bar{x}). \tag{7}
\]

The variance of the expected average-tax payment is:

\[
\text{Var}(T) = E\left\{ \frac{1}{N} \sum_{i=1}^{n} \left[ \frac{\theta(Y - \bar{y}_i) - \theta(Y - \bar{x})}{N} \right]^2 \right\} = \frac{1}{N^2} \sum_{i=1}^{n} E\left[ - X(\bar{y}_i - \bar{x}) \right]^2 = \frac{X^2}{N} \sigma_x^2. \tag{8}
\]
Clearly, for a large number of taxpayers, \( \lim_{N \to \infty} \frac{1}{N} \sigma^2 = 0 \), the variance of the expected average tax collected tends towards zero.

We write the expected utility function for public goods as a function of \( G \), equation (6), where \( G \) is the net expected aggregate value of taxes collected from taxpayers and used to produce public goods. We assume an additive utility function for simplification. The "representative" citizen's social welfare function is written:

\[
S^* = EU + EV
\]

\[
= E\left\{ U \left[ Y - \Theta(Y - \gamma X(I, P)) - qI - P, mP \right] \right\} + E\left\{ V \left[ N \Theta(Y - \gamma X(I, P)) - J(\bar{Y}) \right] \right\}
\]

(9)

where \( N \) is the number of "representative" taxpayers.

The government has incomplete knowledge of the taxpayer's utility function. The shape of taxpayer's utility function together with the tax-avoidance transformation function determines the optimal amount of tax avoidance for any given \( \bar{Y} \) and \( \Theta \). In addition, the government faces the familiar problem of correct revelation of demand for public goods.

In a general-equilibrium model the aggregate constraint should be satisfied. In the basic microeconomic model, we assumed individuals to be endowed with a maximum potential income \( Y \). The aggregate national income is the sum of individual's income, \( \sum Y_i = N Y \). Taxpayers choose \( I \) and \( P \) so as to maximize the level of consumption after tax payments and tax-avoidance expenditures. The aggregate constraint from the individuals' point of view is:
NY = E \left[ N(C + qI + P) \right] + E(T) \quad (10)

where \( E(T) = N \Theta (Y - \bar{y} X) \) is the expected aggregate taxes paid by individuals. The government provides a public good \( G \). In a pure exchange model, we assume a unitary marginal rate of transformation between private and public good. Thus, the amount of public goods provided is equal to: \( E(G) = E(T) - J(\bar{y}) \). If we substitute the last expression into (10), we see that the national accounting equation is satisfied:

\[
NY = E \left[ N(C + qI + P) \right] + E(T) = E \left[ N(C + qI + P) \right] + E(G) + J(\bar{y}). \quad (11)
\]

1.3. Existence of a solution, the first-order conditions for a maximum and economic interpretation of maximizing conditions

Before deriving the first-order conditions for maximizing social welfare, we must explore the possibility of corner solutions. First, we can rule out \( E(\bar{y}) = \bar{y} = 0 \), since the marginal cost of such a governmental action, \( J'(0) \) is infinite. At the other extreme, let \( E(\bar{y}) = \bar{y} = 1 \), then, in view of equations (30) and (31) of Chapter I, which can be rewritten as:

\[
q < \Theta X_I
\]

AND

\[
l < \frac{mEU_P}{EU_C} + \Theta X_P, \quad (13)
\]
these conditions give positive optimal values of \( I \) and \( P \) such that the level of tax-avoidance activity is determined. In principle, tax laws and tax administration might be so defective as to allow a corner solution where \( X = \text{Max } X = Y \). But surely this would be unstable. Zero tax payments would force an immediate government reaction. Zero tax collection would be infuriating and would devastate the production of public goods. We are, therefore, assured of a unique interior solution to the taxpayer's problem for \( \gamma \in (0, 1) \).

The government's problem is to choose \( \Theta \) and \( \bar{\gamma} \) so as to maximize \( S^* \) in equation (9). The optimal choice of \( \Theta \) and \( \bar{\gamma} \) will determine the amount of taxes collected and consequently determine the optimal rule for public-goods provision. The first-order condition for maximum with respect to \( \Theta \) is:

\[
\frac{\partial S^*}{\partial \Theta} = - \frac{\partial E_C'(Y - \gamma X)}{\partial \Theta} + \frac{\partial E_C'(\gamma Y X - q)}{\partial \Theta} \frac{\partial I}{\partial \Theta} \\
+ \left[ \frac{\partial E_C'(\gamma Y X - P - 1)}{\partial P} \frac{\partial P}{\partial \Theta} + \frac{\partial E_C'(\gamma Y X - P - 1)}{\partial P} \frac{\partial P}{\partial \Theta} \right] = 0 \quad (14)
\]

The second and third RHS terms are zero by the first-order conditions (12) and (13) in Chapter I. By equation (37) in Chapter I, \( X_I(\delta I/\delta \Theta) + X_P(\delta P/\delta \Theta) = X_\Theta \). Thus, we can rewrite the first-order condition as follows:

\[
\frac{\partial S^*}{\partial \Theta} = - \frac{\partial E_C'(Y - \gamma X)}{\partial \Theta} + \frac{\partial E_C'(\gamma Y X - P)}{\partial \Theta} - \frac{\partial E_C'(\gamma Y X - P)}{\partial \Theta} \frac{\partial P}{\partial \Theta} = 0 \quad (15)
\]

Furthermore, from the Treasury's point of view, we defined the "representative" taxpayer's expected average-tax payment. We extend
the concept to define the expected average income for consumption:

\[ E(C) = E\left\{ \sum_{i=1}^{n} \frac{c_i}{N} \right\} = E\left\{ \frac{1}{N} \sum_{i=1}^{n} (B + \theta \gamma_i x_i) \right\} \]

\[ = B + \theta \bar{\gamma} x \]  \hspace{1cm} (16)

where \( B = Y (1 - \theta) + \theta A - qI - P \). We can compute the following covariance:

\[ \text{Cov}(\bar{C}, \gamma_1) = E\left\{ \left[ \sum_{i=1}^{n} \frac{(B + \gamma_i x_i)}{N} \right] \gamma_1 \right\} - E\left\{ \left[ \sum_{i=1}^{n} \frac{(B + \gamma_i x_i)}{N} \right] E(\gamma_1) \right\} \]

\[ = \frac{\theta X}{N} E\left\{ \gamma_1 \gamma_1 + \gamma_2 \gamma_1 + \ldots + \gamma_2 \gamma_1 + \ldots + \gamma_n \gamma_1 \right\} \]

\[ - \theta X \left\{ E(\gamma_1) \right\}^2 \]

\[ = \frac{\theta X}{N} \left\{ (N - 1)\bar{\gamma}^2 - N\bar{\gamma}^2 \right\} + \frac{\theta X}{N} E(\gamma_1^2) \]

\[ = \frac{\theta X}{N} \left\{ E(\gamma_1^2) - E(\gamma_1)^2 \right\} = \frac{\theta X}{N} \sigma_y^2 \]  \hspace{1cm} (17)

The government can ignore the covariance between average after tax consumption and a particular taxpayer's random component \( \gamma_1 \), since

\[ \lim_{N \to \infty} \text{Cov}(\bar{C}, \gamma_1) = \frac{\theta X}{N} \sigma_y^2 = 0. \]

Similar result holds for \( \text{Cov}(\bar{C}, \gamma_1) = N\text{Cov}(\gamma_1, \gamma_1) \). In addition, we know that if \( \text{Cov}(X, Y) = 0 \), then \( E(XY) = E(X)E(Y) \). These derivations permit us to rewrite the first-order condition(15) as follows:
The economic interpretation of this result is that the Treasury can diversify individuals' uncertainty and must not require a risk premium in setting the optimal tax rate. The government acts only on the basis of expected value, since the variance on individuals' tax collections is diversified away.

The first-order condition for a maximum with respect to is:

\[
\frac{\delta S}{\delta \Theta} = -EU_C(y - E(Y)X) + EV_GN(Y - E(Y)X - EV_G N E(Y)\Theta X_{\Theta} \\
= [NEV_G - EU_C](y - \bar{\bar{y}}X) - EV_G N \bar{\bar{y}}X_{\Theta} = 0 \quad (15')
\]

The second and third RHS terms are zero by the first-order conditions (12) and (13) in Chapter I. Furthermore, by similar arguments as for the first-order condition with respect to, we can take the product of expected values and rewrite equation (13) as follows:

\[
\frac{\delta S}{\delta \gamma} = -\gamma \left[ NEV_G - EU_C \right] - NEV_G \bar{\bar{y}}X_{\gamma} - EV_G J' = 0, \quad (18')
\]

Equations (15') and (18') give the first-order conditions for a social welfare optimum. We already know why the government must not require a risk premium for optimal income taxation. However, the government that wishes to maximize the social welfare of the nation should take account of the distortionary effects of income taxation.
We know from our one-period comparative-static results that an increase in the effective tax rate, $\Theta$, rises the level of both tax-avoidance inputs: $\delta I/\delta \Theta$, and $\delta F/\delta \Theta$ are positive. Consequently, by equation (37) of Chapter I, we have $\delta X/\delta \Theta = X_\Theta$ positive. Similarly, an increase in the mean value of $Y$, denoted by $\delta Y$ in our comparative-static results, induces $\delta I/\delta Y$, and $\delta F/\delta Y$ to be positive. These terms measure the impact of changes in the mean value of the probability density function and thus allow us to predict the impact on tax-avoidance activity: $\delta X/\delta Y = X_\delta$ is positive. In addition, knowing that $\delta X$ is bounded by $Y$, we derive from equation (15') that $\text{NEV}_G > \text{EU}_C$, since $\text{EV}_G \delta Y X_\Theta$ is positive.

To obtain a clear expression for the distortionary effects, we rework equation (15') to obtain the sum of marginal rate of substitution between public good and private consumption:

$$\text{NEV}_G (Y - \bar{X} - \Theta \bar{X} X_\Theta) = \text{EU}_C (Y - \bar{Y}X)$$

or

$$\frac{\text{NEV}_G}{\text{EU}_C} = \frac{Y - \bar{X}}{Y - \bar{YX} - \Theta \bar{X} X_\Theta}$$

$$= \frac{1}{1 - (\bar{X}/(Y - \bar{X})) X_\Theta}$$

(19)

We interpret the economic significance of the first-order conditions of our government's maximization problem. Dealing with a pure exchange model, the marginal rate of transformation between private and public goods is unity. Without tax avoidance the Conventional Rule for provision of public goods is: $\sum MFS^1 = MRT$. We picture this result in figure 11.
The existence of tax-avoidance activity modifies the result by \(- \frac{\bar{y}_x}{y - \bar{y}_x} \xi_x^X\), which is the tax-avoidance elasticity with respect to the effective tax rate weighted by its relative size. This term being negative implies that the right-hand side expression of (19) is greater than one. Consequently, the distortionary effect of changes in \(\Theta\), for a given \(Y\), on the amount of tax-avoidance activity requires at the optimum a sum of benefits, \(\sum \text{MRS}^i\) greater than the unitary marginal rate of transformation, MRT. We can read this result in figure 11. A public dollar must have a higher social value than the marginal utility of income. In other words, if the government recognizes the cost of income-tax collection caused by tax avoidance, an
underprovision of public goods, $G^*$, compared with the conventional Rule, $G_o$ is required. The cost of raising public funds puts a halt to the provision of public goods before the equality $\sum \text{MPS}^1 = \text{MRT}$ is reached.

The optimal income-tax structure is determined by solving equation (15') and (18') simultaneously. An interior solution for $\Theta$ and $\tilde{\gamma}$ exists. $\partial S^* / \partial \Theta$ is positive for small values of $\Theta$ because the expected value of public goods $\text{NEV}_G$ is very large and becomes dominant. On the other hand, for values of $\Theta$ close to one, the expected value of consumption goods $\text{EU}_C$ is dominant and $\partial S^* / \partial \Theta$ is negative. Similarly, we can rule out values of $\tilde{\gamma}$ close to zero because the marginal cost of "surveillance" is very large and $\partial S^* / \partial \tilde{\gamma}$ is positive. At zero an increase in the expected value of forces the social welfare to improve. Finally, for values of $\tilde{\gamma}$ in the neighborhood of one, $J'(1) = 0$, equation (5), and the first-order condition becomes:

$$\frac{\partial S^*}{\partial \tilde{\gamma}} = -\Theta (\text{NEV}_G - \text{EU}_C) - \text{EV} \Theta \tilde{\gamma} X \frac{\partial}{\partial \tilde{\gamma}} < 0$$

taking into account of $(\text{NEV}_G - \text{EU}_C) > 0$ and $X \frac{\partial}{\partial \tilde{\gamma}} > 0$. Therefore, a simultaneous solution of equations (15') and (18') exists. The solution determines $\Theta$, and $\tilde{\gamma}$ maximizing social welfare. The values of $\Theta$, and $\tilde{\gamma}$ allow the government to raise funds to provide public goods up to the optimal rule, which is lower when tax avoidance is allowed.
Section 2

Optimal-tax structure; taxpayers having different earning ability

We have derived optimal effective tax rate $\theta$ and government controlled parameter $\bar{\gamma}$. Taxes being collected to provide public goods, we derived optimal rule to maximize a utilitarian social welfare function. We assumed away the distributional effects of taxation by using the concept of "representative" taxpayer.

In this section, we investigate what can be said about the optimal variation of the marginal tax rates over different ranges of individual incomes. In this analysis taxpayers are utility maximizers trying to pay their taxes wisely. However, the government wants to achieve some redistributional goals through income taxation. The social distributional benefits are captured by the social welfare criterion, as pointed out later.

The assumption of $N$ individuals with the same level of income is relaxed. We assume a distribution of earning ability across individuals. However, to address the difficult problem of finding an optimal tax structure in present of tax-avoidance activity, we simplify our model developed in Chapter I.

Phelps [19] studied the problem of "just" taxation of wage income. The major normative result of his paper shows that the marginal tax rate should be decreasing and be precisely zero for taxpayers in the highest skill class. Our goal is to reconsider a simple case of Phelps
model in presence of tax-avoidance activity.

2.1. The model

Consider individuals having a whole range of earning ability. The difference in maximum income level among individuals is taken as determined outside of the model. We ignore inter-temporal aspects, as before. Taxpayers have identical preferences. They differ only in ability to earn income according to differences in \( n \) that varies from 0 to \( N < \infty \). We are not studying the education-work choice since we are interested in the impact of tax avoidance on the optimal-tax structure.

Let \( F(n) \) a distribution function which denotes the proportion of individuals whose ability to earn income is less than or equal to \( n \) (a quantile). We assume \( dF(n) = f(n) \) to be compact density function, so that \( n \) is restricted to the range of \( 0 \leq n \leq N \), where \( N \) is the maximum plausible index of innate ability. A taxpayer of type \( n \) exploits his opportunities by selecting a variable \( x \) within his control, \( x \) denotes the level of his tax-avoidance activity. In our microeconomic model, we viewed tax avoidance as a risky two-input activity. However, for the purpose at hand, we consider \( x \) as the level of tax-avoidance activity achieved by the taxpayer at a known cost. Let the cost function be \( b(x) \) with \( b'(x) \) and \( b''(x) \) positive.

We also want to capture the government reaction to tax-avoidance activity. The government is controlling auditing rates and changing legislation to cut back tax avoidance. We let \( \bar{y} \) capture the mean value of the distribution function over \( \bar{y} \). At the national level, the mean
value of the "surveillance" parameter is known to the government. To build a deterministic model, we have to assume \( \tilde{y} \) to be perfectly known to the government. The effective taxable income becomes

\[
y = n - \tilde{y}x
\]  

(20)

Every person's earning power and earnings are given, i.e., no "dropped out" is allowed. The taxpayer only alters his tax bill by entering into tax-avoidance activity at the expense of leisure and income spent to carry out tax-avoidance activity. These costs are captured by the tax-avoidance cost function \( b(x) \).

In our model taxpayers act in their own best interest, however, the society attributes social value to income redistribution. Measurement of inequality have been explored recently by Atkinson, [4] and [5]. Drawing on concept developed in the literature on decision-making under uncertainty, Atkinson explored the use of particular transformation on individuals' utility to measure the social inequality-aversion. He used the particular iso-elastic transformation:

\[
G(U) = \frac{U^{1-\rho}}{1-\rho}
\]  

(21)

where \( G \) is a social welfare function defined on individuals' utility. This transformation defines a constant inequality-aversion.

Pioneering work in optimal income taxation by Pigou [20] led to a 100 percent tax rate so as to achieve complete equalization of after-tax income under the maximization of the sum of individuals' utility. A social welfare function defined as the sum of individuals' utility
implies a zero inequality-aversion, $p = 0$. However, even when we maximize the sum of individuals' utility, optimal rules don't ask for complete equalization when we consider the influence of taxation on the work-leisure choice (see Mirrlees [16]. When $p = 1$, we have a unitary inequality-aversion and implies a logarithmic transformation on individuals' utility.

As $p$ rises, we attach more sensitivity to transfers at the lower end of the distribution and less weight to transfers at the top. We can explore this fact. The rate of change of social marginal utility of income, $G'' = - u'(1 + p)$, for an increase in $p$ is defined by:

$$\frac{\delta G''}{\delta p} = \frac{p \ln U - 1}{U^{1+p}}$$

(22)

The value of $\frac{\delta G''}{\delta p}$ when $p \to \infty$ is:

$$\lim_{p \to \infty} \frac{p \ln U - 1}{U^{1+p}} = \lim_{p \to \infty} \frac{\ln U}{(1+p)U^p} = 0$$

(23)

using l'Hôpital's rule.

We can interpret this result as stating that the social value of any increase in utility (by transfer of income) above the very lowest group is completely discounted. The social welfare function normalized in term of the worse-off individual, $V_0$, becomes:

$$\text{Max} \quad V_0 \quad N \int_0 V_n/V_0 f(n) \, dn$$

$$= \text{Max} \quad V_0 \quad N \int_0 f(n) \, dn = \text{Max} \quad V_0$$

$$= \text{Maximin} \quad V_n$$

(24)
since the social value of \( V_n > V_o \) is completely discounted. Consequently, the maximin is the limiting case of the utilitarian framework, where the social welfare function is based on individual's utilities. This is the maximum concern for social inequality within the framework allowing to achieve Pareto optimality. Any further concern for inequality per se would require to introduce an explicit trade-off between the utilitarian measure (a function of the individual utilities) and equality per se. Atkinson's investigation shows that the maximin principle may well be the best compromise between pure egalitarian and pure economic efficiency criteria. This justifies the use of the maximin principle in the following analysis. The government achieves the maximin principle by using income-tax revenue to make transfer payments.

The government is constrained by its budget constraint that aggregate-tax revenue net of transfers, cost of monitoring tax avoidance and other social costs created by tax avoidance, covers any fixed government expenditures, \( C \). We remove the possibility of money or bond financing. The budget constraint is written:

\[
\int_0^N \left[ k(y(n, x)) - C(y(n)) - I(y) \right] dP(n) = \text{constant} \quad (25)
\]

We may view \( k \) as equal to a gross tax \( t(y) \) with \( t(0) = 0 \), less a minimum disposable-income transfer or lump-sum grant paid to all individuals. The disposable income is then:

\[
z(y) = n + g - t(y); \quad t(0) = 0. \quad (26)
\]
The model does not consider the implementation cost of an income redistribution. This model is such that every taxpayer receives a minimum income through the government redistribution mechanism. It is the major difference with respect to a negative income tax proposal because the transfer payment is only for individuals below the minimum guaranteed income. The negative income tax is preferred if there are any distribution costs depending on the volume of funds transferred. An illustration of the major difference between the two systems is presented in figure 12 and 13 for the case of a linear tax structure. In our model, figure 13, each taxpayer receives the optimal transfer payment; independently of his location on the gross income-tax function. On the other hand, in the negative income tax system the taxpayer receives or pays the net tax payment.

The problem is to find the function \( t(y) \) that maximizes the minimum utility subject to the relation:

\[
g = \max_{n} \left\{ \int_{0}^{\infty} \left[ t(y) - C(\bar{y}(y)) - I(y) \right] dF(n) - G \right\}.
\]

An individual of ability \( n_2 > 0 \) can and will earn more utility than persons of type \( n_1, 0 \leq n_1 \leq n_2 \) for every \( g \) and \( t \) functions. This is because we can say that taxpayers with a higher income ability can do at least as well, in terms of tax-avoidance activity, as the taxpayer right below them. It follows from such reasoning that the minimum utility for every meaningful \( t \) function is the one received by taxpayers with \( n = 0 \).
Figure 12

Figure 13
2.2. The taxpayer's reaction function.

Each taxpayer acts to maximize his utility which, given government expenditures, $G$, depends only upon his disposable income, $c$. We start with a utility function linear in disposable income: $u = u(c) = c$. We assume no ancillary rewards from tax-avoidance activity. There is a cost function for tax avoidance capturing the information costs and the costs of carrying out optimal strategies:

$$b(x) > 0 \text{ and } b'(x) > 0, \quad b''(x) > 0 \text{ for all } x. \quad (28)$$

The taxable income is defined by (20). Assuming linearity in disposable income for simplicity, we can write the taxpayer's maximization problem in the following way:

$$c + b(x) = n - t(y) + g$$

or

$$c = n - t(y) - b(x) + g. \quad (29)$$

The first-order conditions for an interior maximum are as follows:

$$\frac{dc}{dx} = t'(y) - b'(x) = 0 \quad 0 \leq n \leq N. \quad (30)$$

The taxpayer must invest in tax-avoidance activity up to the point where the marginal revenue from it, $\gamma t'(y)$, is equal to his marginal cost, $b'(x)$. Let us assume that governmental tax-revenue maximization to achieve maximal social welfare through income redistribution implies $t(y)$ to be twice continuously differentiable with marginal tax rate:

$$(y) = t'(y) < 1 \quad \text{for all } y. \quad (31)$$
Figure 14
We also make the provisional assumption that the "maximum" tax function causes the second-order conditions for a relative maximum to be satisfied:

\[
\frac{\partial^2 c}{\partial x^2} = -\frac{\partial^2 t''(y)}{\partial x^2} - b''(x) < 0. \tag{32}
\]

Subject to the condition that \( t'(y) < 1 \) and (31) holds for all \( x \), we have the taxpayer's utility maximum. This holds because we are studying the taxpayer's optimal reaction to the tax structure. In the next section, \( \Theta \) and \( \bar{\gamma} \) will be optimally chosen so as to maximize a social welfare criterion, but are given to the taxpayer.

The first-order conditions (30) can be rewritten as:

\[
\Theta \bar{\gamma} - b'(x) = 0 \tag{33}
\]

where \( \Theta = t'(n - \bar{\gamma}x) \) and makes \( x \) an implicit function of \( \Theta \), \( \bar{\gamma} \) and \( n \), say \( x = \chi(\Theta, \bar{\gamma}, n) \). In addition, the relationship (20) between earning ability and taxable income makes \( y \) another function of \( \Theta, \bar{\gamma} \) and \( n \):

\[
y = n - \bar{\gamma} \chi(\Theta, \bar{\gamma}, n) = \psi(\Theta, \bar{\gamma}, n). \tag{34}
\]

Differentiating the first-order conditions (33), we get the marginal change in tax-avoidance activity or the taxpayer's reaction to changes in parameters \( \Theta, \bar{\gamma} \) and \( n \):

a) \( \bar{\gamma} - b'' \frac{\partial x}{\partial \Theta} = 0 \Rightarrow \frac{\partial x}{\partial \Theta} = \chi_{\Theta} = \bar{\gamma}/b'' > 0. \tag{35} \)

An increase in the marginal tax rate increases tax-avoidance activity just as in our more-sophisticated model.
b) \[ \theta - b'' \delta x/\delta \theta = 0 \implies \delta x/\delta \theta = \chi_{\theta} = \theta/b'' > 0. \] (36)

An increase in the value of \( \theta \), which implies that the government becomes looser in its surveillance of tax-avoidance activity, raises the incentive to engage in this activity, again as in our earlier model.

c) The partial derivative with respect to \( n \) of the first-order conditions gives:

\[ -b'' \delta x/\delta n = 0 \implies \delta x/\delta n = 0 \quad \text{since } b'' > 0 \] (37)

d) The total differentiation of \( x = \chi(\theta, \theta, n) \) gives:

\[ \frac{dx}{dn} = \chi_\theta \theta (1 - \delta x/\delta \theta) + \chi_n \]

\[ \frac{dx}{dn} (1 + \delta \theta \chi_\theta) = \chi_n + \theta' \chi_\theta \]

\[ \frac{dx}{dn} = \frac{\chi_n}{1 + \frac{\theta' \chi_\theta}{\delta \theta \chi_\theta}} \]

\[ = \frac{\theta' \chi_\theta}{b'' + \delta^2 \theta'}. \] (38)

There are two cases to consider:

1) An increase in the earning ability the level of tax-avoidance activity for sure if the marginal tax rate is an increasing function of taxable income.

2) An increment in earning ability decrease the level of tax-avoidance activity if \( \theta' \) is negative and the second-order conditions for a maximum (32) are satisfied.
So far, we have derived the optimal reaction of tax-avoidance activity to changes in $\theta$, $\bar{y}$, and $n$. However, for further derivations, we are interested in the relationships between $y$ and $\theta$, $\bar{y}$, and $n$ as well. It is easy to establish such a functional since, by definition, the taxable income is related to $x$ as follows:

$$y = n - \bar{y}x = n - \bar{y}X(\theta, \bar{y}, n) = \psi(\theta, \bar{y}, n) \quad (39)$$

a) The marginal change of $y$ for an increase in the effective tax rate, by substituting (35) for $\delta x/\delta \theta$, is:

$$\delta y/\delta \theta = \psi(\theta, \bar{y}, n) = - \bar{y} \delta x/\delta \theta = - \bar{y}^2/b'' < 0 \quad (40)$$

For an individual with given earning ability, an increase in the tax rate will reduce his taxable income.

b) Differentiating with respect to $\bar{y}$, and substituting (36) for the value of $\delta x/\delta \bar{y}$, we obtain:

$$\delta y/\delta \bar{y} = \psi(\theta, \bar{y}, n) = - x - \bar{y} \delta x/\delta \bar{y}
= - x - \bar{y} \theta/b'' = - \frac{bx + \bar{y} \theta}{b''} \leq 0 \quad (41)$$

An increase in the effective rewards of tax-avoidance activity decreases the taxable income.

c) The partial differentiation with respect to $n$ is:

$$\delta y/\delta n = \psi_{n}(\theta, \bar{y}, n) = 1 - \bar{y} \delta x/\delta n = 1 > 0. \quad (42)$$

d) Finally, totally differentiating $y$ with respect to $n$ and substituting (40), (41) and (42) for $\psi_{\theta}$, $\psi_{\bar{y}}$ and $\psi_{n}$ respectively, we get:
\[
\frac{dy}{dn} = \psi_0 \theta'(dy/dn) + \psi_n + \psi_\theta y(dy/dn) \quad (43)
\]

\[
= \frac{\psi_n}{1 - \theta'\psi_\theta - \psi_\theta} = \frac{b'}{b''(1 + \bar{\theta}x) + \bar{\theta}(\bar{\theta}'\bar{\theta} + \bar{\theta}'\theta)} > 0
\]

An increment in the earning ability always increases taxable income because there is no drop out and we assume continuity at the tax-avoidance cost function.

Our problem is to find that distribution of tax burden that maximizes aggregate tax revenue and thus the lump-sum grant, so as to make minimum consumption as large as possible. Taxes paid by individuals are to be a direct function of taxable income \(t(y)\). However, income is a function of the earning ability, \(F(n)\). There is a relationship between \(y\) and \(n\), deducible from (39) to (43). These equations give the relationships between \(y\) and \(\theta, \bar{\theta}\), and \(n\). Therefore, we can express the aggregate tax revenue as the integral over taxable income of taxpayers rather than over ability. At this point, we are able to recognize that tax-avoidance activity is an information problem for the government. Actually, the government could close all loopholes and enforce statutory rates, but the political costs are prohibitive even if on efficiency grounds we could argue otherwise. If the Finance Minister is fully aware of tax-avoidance activity, he can and should design an optimal tax structure taking account of such a phenomenon. Actually, this may explain the phenomenon of high statutory rates. Statutory rates are much higher than they would be in absence of tax-avoidance opportunities.
Furthermore, costs of administering the tax should also be recognized in the social welfare maximization process. The first type of costs is directly associated with the management of tax avoidance (auditing, tax courts) that are a function of the level of $\bar{\gamma}$. $\bar{\gamma}$ represents the mean value of the fraction of effective tax-avoidance activity for the taxpayer. We denote by $C(\bar{\gamma}(y))$ these "surveillance" costs and are dependent on the level of taxable income because tax-avoidance activity arises mainly in high income-tax brackets.

The second type of costs are those measured by the misallocation of resources created by tax-avoidance activity. A crude measure of these social costs is the money spent on tax-avoidance information. We denote these costs by $I(y)$. We know the sign of marginal increase in income to be positive by analogy to our basic microeconomic model, equation (75) in Chapter I.

2.3 Optimal income-tax structure and social welfare maximization.

We can set up the problem facing the government as one of designing a structure of marginal tax rates on taxable income, taking account of a welfare criterion and of costs of collecting taxes. In terms of taxable income (after tax avoidance), our maximization problem is:

$$\max_{t(y), \bar{\gamma}} \int_0^N \left[ t(y) - C(\bar{\gamma}(y)) - I(y) \right] dD(y)$$

subject to $t(0) = 0$; where $D(y)$ is the proportion of taxpayers with taxable income below or equal to $y$. To have suitable relation between
n and y, we invert \( y = \psi(\Theta, \bar{y}, n) \) to obtain \( n = \phi(\Theta, \bar{y}, n) \)

which gives the relation between ability, \( n \), and taxable income, \( y \), the marginal tax rate, \( \Theta \), and the mean value of the effective portion of tax-avoidance activity, \( \bar{y} \), assuming optimizing behavior from the taxpayer.

Using the original relation \( y = \psi(\Theta, \bar{y}, n) \), we get:

\[
dy = \psi_\Theta d\Theta + \psi_n dn + \psi_{\bar{y}} d\bar{y}.
\]

Substituting values of equations (40) and (42), one obtains:

\[
\frac{\partial n/\partial \Theta}{\partial y} = -\frac{\psi_\Theta}{\psi_n} = \phi_\Theta = \frac{\bar{y}^2}{b''} > 0 \tag{45}
\]

and also \( \phi_{\Theta y} = 0 \).

The partial derivative with respect to \( y \) is:

\[
\frac{\partial n}{\partial y} = \frac{1}{\psi_n} = \phi_y = 1 > 0 \tag{46}
\]

and also \( \phi_{y\Theta} = \phi_{yy} = 0 \).

The partial derivative with respect to \( \bar{y} \), substituting values (41) and (42), is:

\[
\frac{\partial n}{\partial \bar{y}} = -\frac{\psi_{\bar{y}}}{\psi_n} = \phi_{\bar{y}} = \frac{b''x + b''\bar{y}\Theta}{b''} > 0. \tag{47}
\]

Now, we have the following relationships between \( D(y) \) and \( F(n) \):
\[ D(y) = F(\phi(\theta, \tilde{y}, y)) \]
\[ D(0) = F(0) \geq 0; \quad D(\infty) = F(\infty) = 1 \]
\[ d(y) = D'(y) = F'(\phi) \left[ \phi^* \right] \]
\[ = F'(\phi) \left[ \phi_y' + \phi_{\theta}' + \phi_{\tilde{y}}' \right] \]
\[ = f(\phi) \left[ b''(1 + \gamma'x) + \tilde{\gamma}'(\gamma\theta' + \gamma'\theta) \right] /b'' > 0 \]
\[ = f(\phi) \left[ \phi^* \right], \]

where \( d(y) \) is the density of taxpayers with taxable income \( y > 0 \) and \( \gamma \) is the largest taxable income after tax-avoidance activity.

We can rewrite (44) as a problem in optimal control:

\[
\begin{align*}
\max_{\theta'(y), \tilde{\gamma}'(y)} R &= \int_0^Y \left[ t(y) - C(\tilde{\gamma}(y)) - I \right] f(\phi(\theta, \tilde{\gamma}, y)) \\
&\quad \left[ 1 + \phi_\theta' + \phi_\tilde{\gamma}' \right] dy
\end{align*}
\]

given \( t'(y) = \theta(n - \tilde{\gamma}x) = \theta(y) \) and \( t(0) = 0 \).

The rate of change of the marginal tax rate and the rate of change of the government-controlled variable \( \tilde{\gamma} \) ("surveillance costs") are the control variables.

The state variables are \( \theta(y), \gamma(y) \) and \( \tilde{\gamma}(y) \). Let \( \theta'(y) = u_1 \) and \( \tilde{\gamma}'(y) = u_2 \). Then, we write the Hamiltonian as follows:

\[
\begin{align*}
H(\theta, \tilde{\gamma}, y, u_1, u_2) &= \left[ t(y) - C(\tilde{\gamma}(y)) - I(y) \right] f(\phi(\theta, \tilde{\gamma}, y)) \\
&\quad \left[ 1 + \phi_\theta u_1 + \phi_\tilde{\gamma} u_2 \right] + p \theta(y) + q_1 u_1 + q_2 u_2
\end{align*}
\]

where \( p, q_1 \) and \( q_2 \) are the adjoint variables and \( t, \theta, \tilde{\gamma} \) are the state variables. We denote by \( \phi^* \), the total differentiation of \( \phi \) with
respect to $y$ when convenient:

$$d \phi(\phi, \bar{\phi}, y)/dy = \phi^* = 1 + \phi_\theta^{\theta'} + \phi_{\bar{\phi}}^{\bar{\phi}'}$$

$$= 1 + \phi_{u_1}^{u_1} + \phi_{u_2}^{u_2}.$$

The adjoint variables are defined as follows:

$$dp/dy = p^* = - \frac{\delta H}{\delta \phi'} = - f\phi^*.$$  (51)

$$dq_1/dy = q_1^* = - \frac{\delta H}{\delta \phi} = - \left[ t - C - I \right] (f'\phi_\theta^* + f'\phi_{\bar{\phi}}^*) + p$$  (52)

$$dq_2/dy = q_2^* = - \frac{\delta H}{\delta \bar{\phi}'} = - \left[ t - C - I \right] (f'\phi_\theta^* + f'\phi_{\bar{\phi}}^*) - C_{\bar{\phi}} f\phi^*.$$  (53)

The maximization with respect to the control variables is:

$$\frac{\delta H}{\delta u_1} = \left[ t - C - I \right] f\phi_\theta + q_1 = 0$$  (54)

$$\frac{\delta H}{\delta u_2} = \left[ t - C - I \right] f\phi_{\bar{\phi}} + q_2 = 0$$  (55)

and the boundary conditions:

$$p(Y) = 0$$

$$q_1(Y)\Theta(Y) = 0$$  (56)

$$q_2(Y)\bar{\phi}(Y) = 0$$

$$p(0(t(0) + q_1(0)\Theta(0) = 0$$

Now, we have to derive the optimal policies. Differentiating (54) with respect to $y$, we get:
\[ q_1^* = - \left( \Theta - C_8 u_2 - I_y \right) \phi_\theta - \left( t - C - I \right) \phi_\theta \phi_\phi + \phi_\phi^* \]  

Equating (57) with (52), noting that:

\[ \delta \phi^u/\delta \Theta = \phi_\Theta u_1 + \phi_\Theta = \phi_\Theta^* \]

we obtain:

\[ \left[ \Theta - C_8 u_2 - I_y \right] \phi_\theta = p \]  

(58)

Similarly, differentiating (55) with respect to \( y \), we get:

\[ q_2^* = - \left( \Theta - C_8 u_2 - I_y \right) \phi_\theta - \left( t - C - I \right) \phi_\theta \phi_\phi + \phi_\phi^* \]  

Equating (59) with (53), noting that:

\[ \delta \phi^s/\delta \Phi = \phi_\Phi u_2 = \phi_\Phi^* \]

we obtain:

\[ \left[ \Theta - C_8 u_2 - I_y \right] \phi_\theta = - C_8 \phi_\phi^* \]  

(60)

We rewrite (51) as \( p^* = - dF/dy \) and integrate, to get:\[ p(y) = c - F(c) \]

which recognizing the boundary condition \( p(Y)t(Y) = 0 \) and \( t(Y) > 0 \) implies \( p(Y) = 0 \) so that \( p(Y) = d - F(d) = 0 \) and since for the highest taxable income, the proportion of taxpayers with equal or lower taxable income is one, we get \( c = 1 \).

We have determined the first-order conditions that must hold for the whole range of earning ability and not only for a particular level.
of income:

\[ [\theta - C^\theta u_2 - I_y]f = 1 - F(\phi) = 1 - D(y). \]

The right-hand side is the increment to aggregate tax revenue from a small increase in the marginal tax rate \( \theta \) at a given \( y \) owing to the presence of \( 1 - D(y) \) individuals who have their tax bill increased by that amount given the marginal tax rate \( \theta \) for higher level of \( y \).

The left-hand side is the loss of tax revenue from the same small increase of the marginal tax rate \( \theta \). The marginal loss is equal to the net marginal tax collection \( [\theta - C^\theta u_2 - I_y] \) times the density of taxpayers with taxable income \( y \), weighted by the marginal reaction of tax-avoidance activity due to a marginal change in \( \theta \). This equality should hold for all \( y \).

Observe that when \( y \to Y \), we have \( 1 - D(Y) = 0 \), and consequently the optimal marginal tax rate becomes:

\[ [\theta - C^\theta u_2 - I_y] = 0 \text{ or } \theta = C^\theta u_2 + I_y \]  

(62)

The marginal tax rate must never be zero because the welfare-maximizing government should consider all social costs of income-tax collection. This result is different from Phelps' result where the marginal tax rate \( \theta \) should be zero for the largest taxable income, \( Y \). The net marginal tax rate should be zero, but not the statutory marginal rate. Let us consider why the net marginal tax rate should be zero for the largest possible taxable income, \( Y \). This occurs because there is no additional revenue from raising \( [\theta - C^\theta u_2 - I_y] \) above zero.

We know from equation (61) that at \( Y \), there is no higher-earning ability taxpayers whose tax bill will thereby be raised. On the other hand,
for every possible net marginal tax rate, \( \left[ \theta - C_yu_2 - I_y \right] > 0 \), there is a loss in revenue equal to \( \left[ \theta - C_yu_2 - I_y \right] \frac{dy}{d\theta} \) for each taxpayer, in the class of largest taxable income, per unit of any rise in the net marginal tax rate. We graph the net and statutory rates across the distribution of taxable income for a given value of \( \bar{y} \).

We will see how the structure of marginal tax rates depends on the distribution function of taxable income.
On the other hand, the first-order conditions for maximizing the welfare function with respect to \( \bar{y} \) are independent of the density function of taxable income, \( f(\phi) \). The left-hand side measures the marginal loss in tax collection caused by a small increase in the mean value of \( \bar{y} \). In other words, the cost to the Treasury of an increase in the expected value of tax-avoidance activity is equal to the net marginal tax rate times the taxpayer's reaction in tax-avoidance for a small increase in its mean value. The right-hand side is the decrease in marginal cost of "surveillance", when allowing for a small increase in tax-avoidance rewards.

These two marginal conditions (60) and (61) have to be satisfied simultaneously. We can imagine an iteration process starting by solving (61) for given values of \( \bar{y} \) and then plug in the marginal tax rates obtained in equation (60) until we reach convergence. From the general shape of the statutory tax rates, we can graph the gross tax revenue collected by the government. We observe from equation (61) that an additional dollar of income will be taxed at the marginal tax rate which depends on the proportion of taxpayers with level of taxable income greater or equal to \( y \). The marginal tax rate on the last dollar of a given level of taxable income, \( y_1 \), will be collected from all individuals with an equal or greater taxable income. We illustrate the gross tax function in figure 16.
2.4. Structure of optimal marginal trade-off across taxable incomes.

To gain some insight into the evolution of the marginal trade-off for the different classes of taxable income, we must differentiate equations (60) and (61) with respect to $y$. Before deriving the conditions on the evolution of the marginal tax rates across the distribution of taxable income, we look at the problem in another way to draw information about the sign of an important expression. Aggregate revenue equals the marginal tax rates times the number of taxpayers earning an equal amount or more. Hence, the equivalent form of the maximization problem is:

$$\int_0^N \left[ t(y) - C(\bar{y}(y)) - I(y) \right] dD(y) \text{ subject to } t(0) = 0$$
We only look at the first-order conditions with respect to \( \theta \), in order to get information about the second-order conditions that must be satisfied for a maximum.

The first-order conditions are found by differentiating the following functional:

\[
\mathcal{R}(\theta) = \int_{0}^{N} \left[ \Theta(y) - C_{\theta} \tilde{y}' - I_{y} \right] (1 - F(\theta, \tilde{y}, y)) dy
\] (64)

For an interior maximum, we have:

\[
\frac{\partial \mathcal{R}}{\partial \theta} = 1 - F(\phi) - \left[ \Theta(y) - C_{\theta} \tilde{y}' - I_{y} \right] \frac{\partial \phi}{\partial \theta} = 0; \quad (65)
\]

the same equation as (85), and the second-order conditions are as follows:

---

6) The result in (63) is derivable from integration by parts:

a) \( \int uv'dy = uv(\infty) - uv(0) - \int u'vdy \).

Let \( u = t - C - I \) and \( v = D \), where \( t(0) = 0 \) and \( D(Y) = 1 \). Then the right-hand side of (87) is:

\[
\int (1 - v)u'dy = \int u'dy - \int vu'dy = u(Y) - u(0) - \int vu'dy
\]

\[
= u(Y) - u(0) - u(Y)v(Y) = u(0)v(0) + \int u'vdy \quad \text{by a}
\]

\[
= \int u'vdy \quad \text{since} \ v(Y) = 1 \text{ and } u(0) = 0.
\]
\[ \frac{\partial^2 R}{\partial \Theta^2} = - \left[ 2f \Phi_\Theta + \text{NMAR} \left[ f' \Phi_\Theta + f \Phi_{\Theta\Theta} \right] \right] < 0 \]  

(66)

where \( \text{NMAR} = \Theta - C_{y\theta} \gamma' - I_y \).

Finally, we make an additional assumption on the tax-avoidance cost function: \( b'''(x) = 0 \), that is, we hypothesize a quadratic tax-avoidance cost function. Then, we have the following terms signed:

- \( b' > 0 \), \( b'' > 0 \), \( b''' = 0 \)
- \( C_{\gamma} < 0 \), \( C_{\gamma\gamma} > 0 \), \( I_Y > 0 \), \( I_{YY} > 0 \)
- \( \Phi_\gamma > 0 \), \( \Phi_{\gamma\gamma} > 0 \), \( \Phi_Y = 1 \)
- \( \Phi_{\Theta\Theta} = 0 \), \( \Phi_{\gamma\Theta} = 0 \), \( \Phi_{\Theta\gamma} = 2 \gamma/b'' > 0 \).

Total differentiation of (61) with respect to taxable income, \( y \), gives:

\[
\sigma' = \frac{-\frac{\gamma^2}{\gamma Y} C_{\gamma \gamma} \Phi_\gamma - I_{YY} f \Phi_\Theta + \text{NMAR} \Phi_\Theta + f + \gamma \left[ \text{NMAR} (f' \Phi_\Theta + f \Phi_{\Theta\Theta}) + f \Phi_\gamma \right]}{- \left[ 2f \Phi_\Theta + \text{NMAR} (f' \Phi_\Theta^2 + f \Phi_{\Theta\Theta}) \right]} \tag{67}
\]

Similarly, for equation (84), differentiating with respect to \( y \), we obtain:

\[
\tilde{\gamma}' = \frac{I_{YY} \phi_{\gamma\gamma} - \sigma [\phi_{\gamma\gamma} + (\Theta - I_Y) \phi_{\Theta\Theta}]}{(\Theta - I_Y) \phi_{\gamma\gamma} + C_{\gamma\gamma} + \sigma' \left[ C_{\gamma\gamma} \phi_\Theta + C_{\gamma} \phi_{\Theta\Theta} \right]} \tag{68}
\]

Equations (67) and (68) have to be solved simultaneously to obtain an optimal income-tax structure.

For the limiting case of the largest taxable income, we know that the net marginal tax rate vanishes and from (60), we get:
or \[
\bar{\gamma}' = - \frac{1 + \phi_0}{\phi_{\bar{\gamma}}} \tag{69}
\]

Thus, equation (67) reduces to:

\[
\theta' = \left( \bar{\gamma}' \frac{2 C_{\bar{\gamma}Y}}{\phi_{\bar{\gamma}}} - \frac{I_{YY} \phi_0}{\phi_{\bar{\gamma}}} + \bar{\gamma} \phi_{\bar{\gamma}} + 1 \right) / -2 \phi_0 \tag{70}
\]

and substituting \(\bar{\gamma}' = - \frac{1 + \phi_0}{\phi_{\bar{\gamma}}}\) in (70), we obtain:

\[
\theta' = \frac{2 C_{\bar{\gamma}Y}}{\phi_{\bar{\gamma}}} + \frac{I_{YY}}{\phi_{\bar{\gamma}}} > 0 \tag{71}
\]

and \(\bar{\gamma}'\) is clearly negative from (69).

For the general case, \(y < \bar{Y}\), additional specifications are needed because no easy solution of this simultaneous system exists. Assuming a constant level of the "surveillance" variable, \(\bar{\gamma}' = 0\), gives:

\[
f(1 - I_{YY} \phi_0) + K\phi_0' \phi_0 = \frac{2r(3g + WMRf'0^2)}{\left[ 2r \phi_0 + NMRf' \phi_0 \right]} \tag{72}
\]

From (66) the denominator is negative and the numerator is positive unless \(- I_{YY} \phi_0 + NMRf' \phi_0\) is large enough to outweigh the positive term \(f\). We observe that marginal tax rates depend crucially on the form on the distribution earning ability. Marginal tax rates can increase in certain range if \(f'\) is negative and large enough in absolute value.

We have studied the impact of tax-avoidance activity when there are a distribution of earning ability and social inequality-aversion. We have abstracted from the full work-leisure choice in optimal income taxation, because tax-avoidance activity is complementary in its effects. The work-leisure choice has been analyzed by Mirrlees in [16]. In both
cases, taxpayer's reaction requires less progressivity and halts in income redistribution short of complete equality in a society anxious to achieve efficient allocation of resources.
MACROECONOMIC EFFECTS OF TAX AVOIDANCE

Introduction

In most discussions of the effects of fiscal and monetary policy, the authors overlook the impact of tax avoidance on fiscal-policy actions. From our microeconomic model of tax avoidance, we are able to predict the qualitative effects of changes in tax rates or income level on the amount of tax-avoidance activity. We observed that the effects were positive. The value of the impact is used to derive the incidence of tax avoidance on the macroeconomic effects of discretionary policy actions. Tax avoidance erodes the tax base and greater increases in tax rates are required to levy a given increase in taxes. In addition, built-in flexibility is also qualitatively altered by the marginal reaction of taxpayers to changes in their level of income.

In section 2, we endogenize the labor supply and drop the assumption of a constant price level to analyze cases in which "restrictive" policy actions can generate inflationary tendency.

Section 1

A macroeconomic model
To study the effects of tax avoidance at the macroeconomic level, we use an IS-LM framework. The model includes three markets: the commodity-market, the money-market and the bond-market.

Intrasector claims may exist, such as bank deposits and corporate securities, but they cancel out in the consolidated equations. The model excludes any assets (except the monetary base and government bonds) that express claims between the private and government sectors that do not cancel out when each sector is consolidated. We assume the price level $P$ is predetermined and constant. Finally, we introduce explicitly the government budget constraint into the model. Recognizing the random element in tax collection, we express $T$ as an expected value. We have demonstrated, in section 1.2. of chapter II, why the government can ignore risk and use the expected value in its computations. If we assume that the Treasury ignores tax-avoidance activity, it would be forced to finance part of the originally equilibrated budget change by an open-market operation (issuing new bonds, or new money) to cover the extend to which budgeted financing falls short because of tax avoidance. More generally, the government budget should include projections of probable erosion in the base due to tax avoidance.

The IS-LM model to be considered is the following:

1. $Y = E + G$ the commodity-market equilibrium
2. $E = E(Y_d, r, W)$ the expenditure function including tax-avoidance expenditures.
3. $G = G$ government spending including interest payments on outstanding government bonds.
4. $Y_d = Y - \Theta(Y - \bar{X})$ disposable income.
5. \( T = \theta(Y - \bar{X}) \) the tax function.

6. \( M^d = L(Y, r, W) \) the money demand.

7. \( M^s = M_o \) the money supply.

8. \( M^d = M^s \) money-market equilibrium.

9. \( W = K + M_o + \delta cB/r \) the definition of private wealth.

10. \( G - T = \Delta M_o + c \Delta B/r \) the government's budget constraint.

The notation used in equations (1) through (10) is as follows:

- \( Y \) = GNP.
- \( E \) = expenditure function of the private sector.
- \( G \) = government spending.
- \( Y_d \) = disposable income.
- \( T \) = total expected tax collection.
- \( M^d \) = demand for money.
- \( M^s \) = supply of money.
- \( W \) = private wealth.
- \( \theta \) = a flat effective tax rate.
- \( K \) = physical stock of capital.
- \( B \) = number of government bonds (assumed to be perpetuities).
- \( c \) = coupon rate on government bonds.
- \( \delta \) = partial discount of future tax liabilities.
- \( r \) = the interest rate.

A few points of interpretation should be noted. First, the restraint shows that budget deficits or surplus must be offset by changes in the monetary base or borrowing from the private sector. We assume
partial discount of future tax liabilities on the part of the private sector. Second, the money stock does not include explicitly the banking sector. We hypothesize that the banking and non-banking sector are consolidated into a single private sector, then, banks' deposit liabilities cancel out against non-bank holdings of deposits; leaving the net holdings of the private sector equal to the government money and government bonds. Third, the Treasury and the Central bank are consolidated in a single government sector. Thus, combined outstanding monetary liabilities (currency in circulation and central bank deposits) constitute the monetary base held by the private sector including banks.

Fourth, the government budget constraint places restriction on the four variables: $G$, $\Theta$, $M^S$, and $B$, among which the authorities choose. Once any three of these have been set, the fourth one is determined endogenously by the requirement (10). Fifth, the private-sector budget restraint does not include the tax-avoidance expenditures, because for our purpose they are considered as regular expenditures out of disposable income. Tax-avoidance expenditures are included in the private-sector expenditure function. The impact of tax-avoidance on fiscal policy comes from the effects of tax avoidance on the amount of taxes collected. The individual cost is of no concern for the Treasury.

Section 2

Comparative-static analysis of discretionary policy measures

The effects of any change in policy variables upon the system can be found by standard comparative-static analysis. First, we neglect the
budget constraint and derive the comparative-static results for changes in government expenditures, $G$, tax rate, $\Theta$, money supply, $M_0$, and government bonds, $B$.

Equations (1) and (9) can be solved for $Y$, and $r$. Taking the total differential of this reduced-form system, we get the following system in two equations:

$$
\begin{align*}
& \left[ 1 - E_c(1 - \Theta(1 - \tilde{X}_Y)) + -(E_r - E_r^d)cB/r^2 \right] \quad \frac{dY}{dr} \\
& \left[ L_Y - L_r^d cB/r^2 \right] \\
& \left[ -E_c(Y - \tilde{X})d\Theta + F_c \Theta \tilde{X}_\Theta d\Theta + dG \right] \\
& \left[ dM_0 \right]
\end{align*}
$$

The correspondence principle imposes some restrictions on the values of the model's parameters. We assume the following usual sign restrictions:

$$
0 < E_c < 1, \quad E_r < 0, \quad E_r^d > 0, \quad L_Y > 0, \\
L_r < 0, \quad L_r^d > 0, \quad \text{and} \quad 0 < \Theta < 1,
$$

and there are adding-up restrictions, e.g., $E_W + I_W + E_W^d = 1$.

These values ensure a positive multiplier $1/\Delta > 0$.

$\Delta = 1 - E_c(1 - \Theta) + M_Y$ refers to the denominator of the comparative-static results without tax-avoidance activity.

$B = (E_r - E_r^d) cB/r^2)/(L_r - L_r^d cB/r^2)$ represents the ratio of the
effects of an increase in the interest rate in commodity and the money-market. The model includes the wealth effects. \( \Delta \chi_{Y} \) refers to the case including tax avoidance. The effects of changes in income and tax rate on the level of tax-avoidance activity were developed in our micro-economic model of tax avoidance and we proved that \( \chi_{Y} > 0 \), and \( \chi_{\theta} > 0 \).

2.1. Comparative-static without the budget constraint.

Table 5 presents the first-period qualitative impact of an increase in government purchases ignoring the government budget constraint. The presence of tax-avoidance activity and a flat tax schedule shows a larger impact of government spending. The reason lies in the fact that an increase in income induces people to engage in more tax-avoidance activity caused by the income effects, which in turn reduces the effective tax rate.
First-period impact multiplier, \( \frac{dy}{dg} \) (rev. purchases), for aggregate demand for the model ignoring the government budget constraint:

a) No tax avoidance:

\[
\frac{dy}{dg} = \frac{1}{1 - E_c (1 - \theta) + BL_Y} > 0
\]

b) With tax avoidance and a flat tax schedule:

\[
\frac{dy}{dg/X_Y} = \frac{1}{1 - E_c (1 - \theta (1 - \delta X_Y)) + BL_Y} > 0
\]

\[
\frac{dy}{dg} < \frac{dy}{dg/X_Y}
\]

always

\[
B = \frac{E_c - E_c \delta c E / r^2}{L_c - L_c \delta c E / r^2}
\]
TABLE 6

First-period impact multiplier, \( \frac{dY}{d\Theta} \) (tax rate), for aggregate demand for the model ignoring the government budget constraint.

a) No tax avoidance:

\[
\frac{dY}{d\Theta} = \frac{-E_cY}{1 - E_c(1 - \Theta) + BL_Y} < 0
\]

b) With tax avoidance and a flat tax schedule:

\[
\frac{dY}{d\Theta}/X,Y,\Theta = \frac{-E_c(X - \bar{Y}X) + E_c\Theta \bar{Y} X}{1 - E_c(1 - \Theta(1 - \bar{Y}X)) + BL_Y} < 0
\]

\[
\frac{dY}{d\Theta} \leq \left| \frac{dY}{d\Theta}/X,Y,\Theta \right| \quad \text{or} \quad \left| \frac{dY}{d\Theta} \right| \leq \left| \frac{dY}{d\Theta}/X,Y,\Theta \right|
\]

\[
\Delta \equiv \frac{\varepsilon_X}{\Theta} \rightarrow \left[ 1 + \frac{\varepsilon_Y}{\Theta} \right] \frac{\Delta}{\Theta E_c} = \left[ 1 + \frac{\varepsilon_X}{\Theta} \right] \frac{Y}{\Theta} \left[ -\frac{dY}{d\Theta} \right]^{-1}
\]

1) \( \Delta = 1 - E_c(1 - \Theta) + BL_Y \)

2) \( \varepsilon_X = \frac{\delta X}{\delta Y} \frac{Y}{X}, \quad \varepsilon_Y = \frac{\delta X}{\delta \Theta} \frac{\Theta}{X} \)
Table 6 shows the first-period impact of changes in the tax rate. In our case of constant price level the policy is clearly deflationary, but with the existence of tax-avoidance activity, we observe a reduction in the absolute impact. This is the case, if the elasticity of tax-avoidance activity with respect to income and tax rate satisfies the following relationship:

$$
\xi_Y^X < \left[ 1 + \xi_{\theta}^X \right] \frac{Y}{\theta} \left[ - \frac{\partial Y}{\partial \theta} \right]; \quad (12)
$$

that is, if the income elasticity of tax avoidance is smaller than a weighted sum of one plus the price elasticity of tax-avoidance activity. The later expression can be interpreted as the effect of change in the tax rate on tax-avoidance activity.

2.2. Comparative statics with the government budget constraint.

In table 7, we compare first-period effects of selected monetary-fiscal policies upon aggregate demand $Y$. Note that policies A and B can be regarded as a set of basic policies; in the sense that any monetary-fiscal policy can be expressed as a linear combination of policies A and B. For example, D is equivalent to a combination of a positive dose of policy B ($d_3 = 1$) and a negative dose of policy A, being large enough to undo the change in money supply caused by policy B. Here the budget constraint is explicitly taken into account and the model is in an initial equilibrium position. Thus, we have no change in the previous period: $\Delta M_{-1} = \Delta B_{-1} = 0$. The solution of the first-period effects of any policy is found by solving equations (1) to (10). Now the
First-period effects of selected monetary-fiscal policies for aggregate demand $Y$, and the endogenous variable of the budget constraint.

Policy

A/ Cut in tax rate offset by induced taxes and money supply:

$$\frac{dY}{Yd\theta} = -1$$
$$\frac{dG}{d\theta} = dB = 0.$$  

a) No tax avoidance:

$$\frac{dY}{Yd\theta} = \frac{E_C + E_y + B(1 - L_y)}{1 - E_C(1 - \theta) + B(L_y + \theta) + \Theta(E_w - BL_y)} > 0$$

$$\frac{dM}{Yd\theta} = \frac{1 - E_C + BL_y}{1 - E_C(1 - \theta) + B(L_y + \theta) + \Theta(E_w - BL_y)} > 0$$

b) With tax avoidance and a flat tax schedule:

$$\frac{dY}{Yd\theta/X_y,X_{\theta}} = \frac{(E_C + E_w + B(1 - L_w))(1 - (\overline{\delta}/Y)(X + \Theta X_{\theta}))}{1 - E_C + (E_C + E_w)\Theta(1 - \overline{\delta}X_y) + BL_y - \Theta(1 - X_y)B(L_w - 1)} > 0$$

$$\frac{dM}{Yd\theta/X_y,X_{\theta}} = \frac{(1 - E_C + BL_y)(1 - (\overline{\delta}/Y)(X + \Theta X_{\theta}))}{1 - E_C + (E_C + E_w)\Theta(1 - \overline{\delta}X_y) + BL_y - \Theta(1 - \overline{\delta}X_y)B(L_w - 1)} > 0$$

$$\frac{dY}{Yd\theta} \gg \frac{dY}{Yd\theta/X_y,X_{\theta}} \quad \frac{dM}{Yd\theta} \gg \frac{dM}{Yd\theta/X_y,X_{\theta}}$$
as \( \xi_Y^X \leq (dY/d\Theta)^{-1}(1/\Theta)(1 + \xi_Y^X) \)

B/ Rise in purchases financed by induced taxes and money supply:

\[ dG = 1, \quad d\Theta = dB = 0. \]

a) No tax avoidance:

\[
\frac{dY}{dG} = \frac{1 + E_C + B(1 - I_W)}{1 - E_C(1 - \Theta) + B(1 - \Theta) + E_C(1 - \Theta) + B(1 - \Theta)} > 0
\]

\[
\frac{dM}{dG} = \frac{(1 - \Theta)(1 - E_C) + BL_Y}{1 - E_C(1 - \Theta) + B(L_Y + \Theta) + E_C(1 - \Theta) + B(L_Y + \Theta)} > 0
\]

b) With tax avoidance and a flat tax schedule:

\[
\frac{dY}{dG} = \frac{1 + E_C + B(1 - I_W)}{1 - E_C + (E_C + E_W)\Theta(1 - 8X_Y) + BL_Y - \Theta(1 - 8X_Y)B(I_W - 1)} > 0
\]

\[
\frac{dM}{dG} = \frac{(1 - \Theta)(1 - 8X_Y))(1 - E_C) + BL_Y}{1 - E_C + (E_C + E_W)\Theta(1 - 8X_Y) + BL_Y - \Theta(1 - 8X_Y)B(I_W - 1)} > 0
\]

\[
\frac{dY}{dG} < \frac{dY}{dG/X_Y}, \quad \frac{dM}{dG/X_Y} > \frac{dM}{dG}
\]

C/ Rise in purchases financed entirely by taxes:

\[ dG = 1, \quad dB = dB/r = 0 \]

\[ dG - dT = 0. \]

a) No tax avoidance:
b) With tax avoidance and a flat tax schedule:

\[
\frac{dY}{dG} = \frac{1 - E_C}{1 - E_C + BL_Y} > 0 \quad \frac{d\Theta}{dG} = \frac{(1 - \Theta)(1 - E_C) + BL_Y}{(1 - E_C + BL_Y)Y} > 0
\]

\[
\frac{dY}{dG/Y,\Theta} = \frac{1 - E_C}{1 - E_C + BL_Y} > 0
\]

\[
\frac{d\Theta}{dG/Y,\Theta} = \frac{[1 - (1 - \tilde{\delta}X_Y)](1 - E_C) + BL_Y}{(1 - E_C + BL_Y)(Y - \tilde{\delta}X - \Theta\tilde{\delta}X_Y)} > 0
\]

\[
\frac{dY}{dG} = \frac{dY}{dG/Y,\Theta} \quad \frac{d\Theta}{dG} < \frac{d\Theta}{dG/Y,\Theta}
\]

D/ Rise in purchases financed entirely by printing money:

\[
dG = 1, \quad dM = 1
\]

\[
dB = 0.
\]

a) No tax avoidance:

\[
\frac{dY}{dG} = \frac{1 + B}{1 - E_C + BL_Y} > 0 \quad \frac{d\Theta}{dG} = \frac{-\Theta(1 + B)}{(1 - E_C + BL_Y)Y} < 0
\]

b) With tax avoidance and a flat tax schedule:

\[
\frac{dY}{dG/Y,\Theta} = \frac{1 + B}{1 - E_C + BL_Y} > 0
\]

\[
\frac{d\Theta}{dG/Y,\Theta} = \frac{-\Theta(1 - \tilde{\delta}X_Y)(1 + B)}{(1 - E_C + BL_Y)(Y - \tilde{\delta}X - \Theta\tilde{\delta}X_Y)} < 0
\]

\[
\frac{dY}{dG} = \frac{dY}{dG/Y,\Theta} \quad \frac{d\Theta}{dG} \quad \frac{d\Theta}{dG/Y,\Theta}
\]

as \( \xi_Y^X \leq 1 + \xi_\Theta^X \)
E/ Rise in purchases financed entirely by bonds:

\[ \frac{dG}{dM} \overset{c dB/r = 1}{=} \frac{dM}{c dB/r = 1} \]
\[ dT = 0 \]

a) No tax avoidance:

\[ \frac{dY}{dG} = \frac{1}{1 - \frac{E}{C} + B L_Y} > 0 \]
\[ \frac{d\Theta}{dG} = \frac{\Theta}{(1 - \frac{E}{C} + B L_Y)Y} < 0 \]

b) With tax avoidance and a flat tax schedule:

\[ \frac{dY}{dG}/X_Y, X_\Theta \overset{1 - \frac{E}{C} + B L_Y}{=} > 0 \]
\[ \frac{d\Theta}{dG}/X_Y, X_\Theta = \frac{-\Theta(1 - \frac{E}{C} + B L_Y)}{(1 - \frac{E}{C} + B L_Y)} < 0 \]

\[ \frac{dY}{dG} = \left| \frac{d\Theta}{dG} \right| < \left| \frac{d\Theta}{dG}/X_Y, X_\Theta \right| \]

as \( \xi/Y \overset{1 + \Theta}{=} 1 + \Theta \).

Reduced-form system has three endogenous variables. The third endogenous variable is generated by the government budget constraint. Finally, in table 7, we deal with a flat tax schedule.

For policy A, we can see tax cut costing one unit of revenue at the initial equilibrium level of income; no changes in government purchases and bonds issue; implies an increase in income and an endogenous increment in the money supply. In table 7, we always give the changes in endogenous variables: a), without tax avoidance, and b), with tax avoidance. Then, we compare the direction and the relative size of the effects relative to the change that would occur in the absence
of tax-avoidance activity. The conditions that must be satisfied by the marginal reaction of tax-avoidance activity are given in elasticity terms. For example, in policy A, the first-period impact of a tax cut is smaller with tax avoidance than without it, if the income elasticity of tax-avoidance activity, \( \xi^X \), is smaller than one plus the price elasticity of tax avoidance, \( \xi^X \), weighted by the inverse of the tax rate times the inverse of the impact of the tax cut without tax avoidance. A tax cut decreases the amount of tax avoidance, however, the increase in income induces a rise in tax-avoidance activity, producing a larger impact on change in national income. Therefore, the first-period impact is smaller with tax avoidance as long as the income effect is dominated:

\[
\xi^X_Y < (\frac{dY}{Y} \cdot \frac{d\theta}{\theta})^{-1} (1 / \theta) (1 + \xi^X). \quad (13)
\]

Under policy B, a rise in government purchases, \( d\Omega = 1 \), with no changes in tax rate and government debt, shows that tax avoidance always induces a larger increase in income and also requires a greater change in the money supply. The larger level of tax avoidance induced by higher income increases the impact of fiscal policy if the money supply provides the support for it.

The impact of other policy-mixes can be read in table 7. For example, policy C, featuring a rise in government purchases financed entirely by taxes implies a greater change in tax rate under tax avoidance than would take place in the absence of this activity. These results show how the government can be misled by ignoring the effects of tax avoidance. If policymakers were unaware of this phenomenon,
they would underreact or overdo policy changes and subsequently find themselves having to act on policy variables such as money supply or open-market operations to finance unexpected disequilibrium.

Section 3

Can an increase in tax rate be inflationary?

In the previous section, we assumed a fixed price level and the economy was not at full employment. We studied the impact of fiscal policy on the national income level with and without explicitly recognizing the government budget constraint. Now, we should remove the assumption of fixed price level and introduce explicitly the labor-market.

Tax-avoidance activity creates losses for productive labor supply to the extent that individuals spend resources in cutting their tax bill. They are social deadweight losses. Income tax also alters the trade-off between market and non-market income, such as self-employment activities (more households services, transform or repaint his home). The existence of these trade-offs shifts the labor supply or more generally, the labor supply is not only function of the after-tax wage rate (assuming no money illusion), but depends on the level of time spent on tax-avoidance information and/or non-market activities captured by $I(\Theta, Y)$. The taxpayer finds it worthwhile to enter into tax-free activities. The taxpayer reallocates his time to seek information about tax-avoidance strategies up to the point where the marginal productivity
of such activity equals his after-tax market wage rate. The microeconomic foundations of the impact of tax avoidance on the endogenized labor supply are analyzed in section 4 of chapter 1.

We assume taxpayers are free of money illusion. The labor supply is therefore a function of after-tax real wage rate and of the level of non-market activities and time spent on tax-avoidance information. We write the supply of labor as follows:

\[ N^s = N \left[ (1 - \Theta)w, I(\Theta, Y) \right] \] (14)

with

\[ N_1 = \frac{N}{(1 - \Theta)w} > 0, \text{ and } N_2 = \frac{\delta N}{\delta I} < 0. \] (15)

The supply of aggregate output \( Y \) is function of the supply of labor \( N \), offered in the labor-market. The stock of capital is held constant, since we are interested in short-run fiscal policy. We assume perfect competition by the demanders of labor, so that the aggregate marginal productivity is equated to the real wage rate:

\[ f'(N) = \frac{w}{P} = w \] (16)

where the production function satisfies the usual properties \( f'(N) > 0, \) and \( f''(N) < 0 \). It follows from the labor-market that the level of labor services exchanged is determined:

\[ N = N \left[ (1 - \Theta) f'(N), I(\Theta, f(N)) \right] \] (17)

taking account of equations (14), (16) and the aggregate output function:

\[ Y = f(N) \] (18)
To find the effects of an increase in the effective tax rate, we differentiate (18) with respect to

$$\left[1 - (1 - \Theta)f^m N_1 - I_y f' N_2 \right] \frac{dN}{d\Theta} = \left[-N_1 f' + N_2 \Theta \right] \frac{d\Theta}{d\Theta}$$

and rearranging, we obtain:

$$\frac{dN}{d\Theta} = \frac{-N_1 f' + N_2 \Theta}{l - (1 - \Theta)f^m N_1 - I_y f' N_2} < 0. \quad (19)$$

The numerator being negative ($l_{\Theta} > 0$), and the denominator being positive ($I_y > 0$), equation (19) is negative.

We want to determine under which circumstances an increase in the effective tax rate might be inflationary in the presence of tax-avoidance activity. In such a case, we have results opposite to what we would expect from such a fiscal-policy action.

We relate the labor market to the aggregate output level, the later being an endogenous variable of the IS-LM model. We rewrite the two equations of the preceding section, IS-LM model without wealth effects for simplification, but with a flexible price variable, and the production function, as follows:

$$f(N) = \tilde{E} \left[ f(N) - \Theta(f(N) - \bar{X}), r \right] + G \quad (20)$$

$$\frac{M}{P} = L(f(N), r). \quad (21)$$

Differentiating the system totally with respect to:

$$f' \left[1 - E_c(1 - \Theta) \right] \frac{dN}{d\Theta} = -E_c(Y - \bar{X}) + E_p \frac{dr}{d\Theta} \quad (22)$$

$$-\frac{M}{P} \frac{1}{P} \frac{dP}{d\Theta} + L_y f' \frac{dN}{d\Theta} + l_r \frac{dr}{d\Theta} \quad (23)$$
and solving equation (22) for the change in interest rate, we obtain:

\[
\frac{dr}{d\theta} = \frac{f'}{E_r} \left[1 - E_C(1 - \Theta)\right] \frac{dN}{d\theta} + \frac{E_C}{E_r} \left(\gamma - \gamma X\right) - \frac{E_C}{E_r} \theta \gamma X \Theta
\]

\[
= \frac{f'}{E_r} \left[1 - E_C(1 - \Theta)\right] \frac{dN}{d\theta} + \frac{E_C}{E_r} \gamma - \frac{E_C}{E_r} \gamma X(1 + \gamma X) \tag{24} \]

Substituting the value of \(\frac{dr}{d\theta}\) into equation (23), we get the change in price level as a function of \(\frac{dN}{d\theta}\):

\[
- \frac{M}{P} \frac{1}{P} \frac{dP}{d\theta} = \frac{f'}{E_r} \left[1 - E_C(1 - \Theta) + \frac{L_v}{L_r} E_r\right] \frac{dN}{d\theta}
\]

\[
+ \frac{E_C}{f'} - \frac{E_C}{E_r} \gamma X(1 + \gamma X), \tag{25} \]

but \(1 - E_C(1 - \Theta) + \frac{L_v}{L_r} E_r = \Delta\) in the original IS-LM model. We can express the rate of change of the price level in the following terms:

\[
\frac{1}{P} \frac{dP}{d\theta} = - \frac{P}{E_r} \frac{L_v}{L_r} f' \left[\Delta \frac{dN}{d\theta} + \frac{E_C N}{\alpha} - \frac{E_C}{\alpha} \frac{\gamma X}{Y} N(1 + \gamma X)\right] \tag{26} \]

where \(\alpha = f'N/P\) is the labor share.

The condition for \(\frac{1}{P} \frac{dP}{d\theta}\) to be positive is that:

\[
\frac{1}{N} \frac{dN}{d\theta} + E_C \frac{N}{\alpha \Delta} - \frac{E_C}{\alpha \Delta} \frac{\gamma X}{Y} N(1 + \gamma X) < 0
\]

which, since \(E_C/\Delta = m'\) is the tax multiplier, can be written:

\[
\frac{m'}{\alpha} - \frac{m'}{\alpha} \frac{\gamma X}{Y} (1 + \gamma X) < - \frac{1}{N} \frac{dN}{d\theta} \tag{27} \]

We can express the effect on the labor force of changes in the tax rate in elasticity terms:

a) No tax avoidance:
\[ - \frac{1}{N} \frac{dN}{d\Theta} = \frac{N_1 f'/N}{1 - (1 - \Theta)f''} = \frac{\varepsilon^s/(1 - \Theta)}{1 - \varepsilon^s/\varepsilon^d} > 0 \]  

where \( \varepsilon^s = \frac{N_1 (1 - \Theta)f'}{N} > 0 \), and \( \varepsilon^d = \frac{f'}{f'} N < 0 \)

b) With tax avoidance:

\[ - \frac{1}{N} \frac{dN}{d\Theta / TA} = \frac{\varepsilon^s/(1 - \Theta) - \varepsilon^s \varepsilon^I \varepsilon^I / \Theta}{1 - \varepsilon^s / \varepsilon^d - \varepsilon^s / \varepsilon^I Y} > 0 \]  

The impact of tax-rate change on the labor market is greater with tax avoidance if the following condition is satisfied:

\[ \varepsilon^I > \left( - \frac{1}{N} \frac{dN}{d\Theta} \right) \varepsilon^I > 0 \]  

In the case of no tax-avoidance activity, we have inflationary effect of a tax increase if:

\[ \frac{m'}{\alpha} < - \frac{1}{N} \frac{dN}{d\Theta} \]  

with tax avoidance, we have from (27):

\[ \frac{m'}{\alpha} - \frac{m'}{\alpha} \varepsilon^X Y (1 + \varepsilon^X) < - \frac{1}{N} \frac{dN}{d\Theta / TA} \]  

The left-hand side is smaller with tax-avoidance activity. In addition, the right-hand side is greater if the condition (30) on tax-avoidance elasticities is satisfied. We conclude that the possibility of perverse inflationary effects of a tax increase has a greater probability to occur in presence of tax avoidance since the conditions are less stringent.
7) This result has been first derived, when no tax avoidance is allowed, by Blinder [8]
SUMMARY, CONCLUSIONS AND POSSIBLE EXTENSIONS

In Chapter 1, we develop a microeconomic approach to tax-avoidance activity. Taxpayers are able to achieve tax savings by spending resources on tax-avoidance information and carrying out tax savings strategy. In an uncertainty framework, an optimal tax-avoidance tradeoff exists for an expected-utility maximizer. The unconstrained comparative-static results give us the impact on the tax-avoidance risky inputs of changes in the exogenous variables of the model. Then, the expected revenue-compensated change in the degree of tax progression states that tax-avoidance activity increases when a more progressive tax structure is introduced if plausible condition (86) is satisfied. The result has important policy implications. It states that an increase in the degree of tax progression producing the same amount of expected tax liability will induce more tax-avoidance activity. A tax reform aimed at introducing vertical equity should fully discount the efficiency costs against the social benefits of a more equitable after-tax-income distribution.

In a certainty framework, section 4 of Chapter 1 addresses the question of the impact of tax-avoidance on work effort. A utility maximizer subject to time and income constraints will decrease his work effort and increase tax-avoiding time if condition (21) is satisfied. The theorem states an opposite relationship between working time and tax-avoiding time. Tax-avoiding time limits realizable progression in income taxation.

Chapter 2 investigates the optimal governmental attitude towards tax-avoidance activity. In a general-equilibrium framework, we determine the optimal tax rate and "surveillance" parameter. We also derive a
measure of the distortion effect of tax avoidance, requiring a deviation from the Conventional Rule for provision of public goods. In section 2 of Chapter 2, we analyze the problem of optimal income tax when distributional aspects of income taxation are considered. The government's problem is to maximize a social welfare criterion (we used the maximin criterion) given a distribution of earning ability and allowing for tax-avoidance activity by taxpayers. The major result is to show that the effective marginal tax rate should be zero (as in Phelps article [19], but statutory marginal rates should allow for the coverage of costs of raising government funds introduced by tax avoidance.

Finally in Chapter 2, the analysis uncovers the consequences of tax avoidance on fiscal policy. For example, a rise in purchases financed by induced taxes and an increase in money supply implies a larger increase in GDP accompanied with a larger increase in the money supply than in the absence of tax avoidance. Tax avoidance increases the uncertainty of fiscal and monetary policy and can lead to a more volatile business cycle rather than helping the economy to move towards the optimal long-run growth path.

In our concluding comments, we would like to discuss some potential extensions of our analysis of tax-avoidance activity.

In our work, we have analyzed the impact of tax-avoidance activity on taxpayers and the government tradeoffs. We derived these tradeoffs involving the type of mean-reducing tax-avoidance activity. One further extension could investigate the effects of variance-reducing tax-avoidance activity. We already noted, in section 1 of Chapter 1, that type-A uncertainty was variance-reducing with an increase in the amount of tax-
avoidance inputs. In this case, a negative risk premium by taxpayers is optimal. This type-A uncertainty did not fully capture the nature of risk at stage with governmental attitude towards tax-avoidance activity. Taxpayers in higher tax brackets have a higher probability to be audited; type-B uncertainty. A new framework is needed to analyze investment in variance-reducing tax-avoidance activity.

In our dissertation, we measured the efficiency costs of tax avoidance and its effect on tax progression. One neglected aspect in our work has been the dynamic implications of tax avoidance. To study the dynamics of tax avoidance, we should take account of the conceptual framework of theory of tax reform presented by Feldstein (On the Theory of Tax Reform by M. Feldstein in Harvard University Discussion Paper Number 408, April 1975). We have used the concept of changes in tax laws, attitude of examiners, and tax courts as exogenously given to the taxpayers. However, in dynamics, these changes are endogenous.

In a multiperiod framework, the government can introduce tax reforms to achieve some increase in social welfare criterion. However, there are tradeoffs involved in tax changes. There are costs of horizontal inequity generated by tax reforms. The principle of horizontal equity is defined by Feldstein as follows: "If two individuals would be equally well-off (have the same level of utility) in the absence of taxation, they should also be equally well-off if there is a tax". Any permanent existing tax structure will not violate horizontal equity if all individuals are free to choose their activities and assuming equal tastes and a single source of income. The existence of tax avoidance will reduce the progressivity of tax structure, but does not introduce horizontal inequity in case of
permanent tax structure. Tax avoidance introduces inefficiency to the extent that tax-avoidance activity uses resources having no payoff in themselves except of achieving tax savings. Again, even if the yield on tax-exempt bonds is too low to induce investment in middle income taxpayer, it does not alter horizontal equity if the opportunity is open to everyone with high income. If all taxpayers are aware of the opportunity, tax-avoidance activity reduces tax progression but is not a source of horizontal inequity. Horizontal inequity is introduced by tax reforms. Tax reforms are changes in the existing tax structure.

In practice, tax reform is piecemeal and dynamic in contrast to the one-and-for-always character of tax design. For example, elimination of the depletion allowance, or tax-exemption of interest payments on municipal bonds. The elimination of the latter would cause a loss to the current owners of such bonds and cause some horizontal inequity. Individuals make commitments based on the existing tax laws and when a tax reform comes along, individuals who were equally well-off before the tax reform are not equally well-off after the reform. Optimal tax reform theory requires balancing these horizontal inequities against the increase in general utilitarian criterion (in terms of income redistribution) and also the benefits of reduction in inefficiency introduced by tax-avoidance activity if tax reform reduced tax-avoidance incentives and also reduced tax-avoidance activity because of uncertainty created by tax reforms. The recognition of this unexplored area of optimal taxation involving dynamic tradeoffs and stability problems opens the field for further research.
APPENDIX 1

To determine the sign of $EU_{CC}(\Theta \gamma X_I - q)$, we must refer to the Arrow [2] and Pratt [21] measure of the taxpayer's risk-attitude:

$$R_A(C) = - \frac{U_{CC}}{U_C}$$

where $R_A$ is commonly known as the measure of absolute risk aversion. We define a taxpayer as risk-averse, risk-neutral and risk-lover if $R_A(C)$ is respectively positive, zero or negative. For a risk-averse taxpayer, Arrow suggested that $R_A(C)$ is decreasing in $C$, i.e., $R_A'(C) < 0$. This assumption of decreasing absolute risk aversion is supposed to hold throughout our developments.

Following the line of reasoning suggested by Sandmo, [27] we must show that under the hypothesis of decreasing absolute risk aversion:

$$EU_{CC}(\Theta \gamma X_I - q) \geq 0.$$ 

Let $\bar{C}$ be the level of consumption when the random variable takes on a particular value: $\gamma = q/\Theta X_I$. If $R_A > 0$ and $R_A' < 0$, then:

$$- \frac{U_{CC}}{U_C} = R_A(C) \leq R_A(\bar{C}) \text{ for } \gamma \geq q/\Theta X_I$$

where $R_A(\bar{C})$ is a given number.

Multiplying both sides by $- U_C(\Theta \gamma X_I - q)$, we obtain:

$$U_{CC}(\Theta \gamma X_I - q) \geq R_A(C)U_C(\Theta \gamma X_I - q)$$

for all $\gamma$, because by the same reasoning as before, for $\gamma \leq q/\Theta X_I$,
the first inequality is reversed and multiplying by a positive number, 
\(- U_c(\theta y X_I - q)\), doesn't reverse the inequality and we are back to the
same inequality.

Taking the expectation of both sides:

\[ EU_{cc}(\theta y X_I - q) \geq F_A(C)EU_{c}(\theta y X_I - q) = 0 \]

by the first-order conditions. Thus,

\[ EU_{cc}(\theta y X_I - q) \geq 0 \quad \text{Al(l)} \]
In this technical appendix, we derive the sign of $EU_C(\Theta X_1(Y - \bar{y}))$ and similarly of $EU_C(\Theta X_p(Y - \bar{y}))$. We want to establish that under the assumption of absolute risk aversion, $R_A(C) = -U_C'/U_C > 0$, which implies $U_{CC} < 0$, we have $EU_C(\Theta X_1(Y - \bar{y}))$ negative.

We start from the definition of net income available for consumption or investment defined by (10), where $Y$ stands for a particular realization of the random variable:

$$C = Y - \Theta(Y - \bar{y}) - qI - P.$$  

$A2(1)$

Taking the expected value of this expression, we get:

$$E(C) = Y - \Theta(Y - \bar{y}) - wI - P,$$

which can be rewritten as:

$$E(C) - Y + \Theta(Y - \bar{y})X = -(qI + P)$$

and substituted into $AII(1)$ gives:

$$C = E(C) + \Theta X(Y - \bar{y}).$$

Assuming a risk-averse taxpayer, one obtains:

$$U_C(C) \leq U_C(E(C)) \quad \text{for} \quad Y > \bar{y}$$

multiplying through by $\Theta X_1(Y - \bar{y})$, we get:

$$U_C(C)\Theta X_1(Y - \bar{y}) \leq U_C(E(C))\Theta X_1(Y - \bar{y}) \quad A2(2)$$
Furthermore, for $\gamma \leq \bar{\gamma}$

$$U_C(\gamma) > U_C(E(C))$$

and multiplying by $\Theta X_I(\gamma - \bar{\gamma})$, which is negative, we get AII(2) for all $\gamma$.

Taking the expectation and noting that $U_C(E(C))$ is a given number, we obtain:

$$EU_C(\gamma) \Theta X_I(\gamma - \bar{\gamma}) \leq EU_C(E(C)) \Theta X_I(\gamma - \bar{\gamma}) = 0$$

or

$$EU_C(\gamma) \Theta X_I(\gamma - \bar{\gamma}) \leq 0 \quad A2(3)$$

It is not intuitively easy to provide an economic interpretation for this result. Further development helps. From the first-order conditions, we have:

$$EU_C \Theta X_I = EU_C \gamma \quad A2(4)$$

$$EU_C \Theta Y X_P = EU_C - mEU_P. \quad A2(5)$$

Subtracting $EU_C \Theta \bar{\gamma} X_I$ from equation A2(4) and using A2(3), one obtains:

$$EU_C \Theta X_I(\gamma - \bar{\gamma}) = EU_C(\gamma - \Theta \bar{\gamma} X_I) \leq 0 \quad A2(6)$$

The expected marginal utility of consumption being always positive implies:

$$q \leq \Theta \bar{\gamma} X_I \quad A2(7)$$
For equilibrium under uncertainty the expected marginal value productivity of the input must exceed its price. This contrasts with the equilibrium under certainty, which requires equality between the marginal value productivity of each factor and its price. Assuming a concave transformation function, the optimal quantity demanded of each input is lower in an uncertainty environment than in a certainty one. It follows that the level of tax-avoidance activity is also lowered by the existence of uncertainty.

Similarly, for the second class of expenditures, subtracting \( EU_C \theta \tilde{Y} X_p \) from equation A2(5), we get:

\[
EU_C \theta \tilde{Y} X_p (Y - \tilde{Y}) = EU_C (1 - \theta \tilde{Y} X_p) - \mu EU_p \leq 0 \quad A2(8)
\]

In this case, we don't necessarily have \( 1 \leq \theta \tilde{Y} X_p \) because \( \mu EU_p \) is positive. When the marginal conditions not only include the direct rewards, but also take account of ancillary rewards produced by this class of expenditures, the result is easily understandable.
To determine the sign of $EU_{cc} (\Theta \gamma X_I - q) (Y - \bar{\gamma})$, we rewrite the expression in the following way:

$$E \left[ U_{cc} (\Theta \gamma X_I - q) (Y - \bar{\gamma}) \right] = E \left\{ U_{cc} (\Theta \gamma X_I - q) \left[ (Y - q/\Theta X_I) + q/\Theta X_I - \bar{\gamma} \right] \right\}$$

$$= EU_{cc} (\Theta \gamma X_I - q)^2 + EU_{cc} (\Theta \gamma X_I - q) (q/\Theta X_I - \bar{\gamma}). \quad A3(1)$$

We already know from the interpretation of the second-order conditions that $EU_{cc} (\Theta \gamma X_I - q)^2$ represents the negative income effect. The second term of the expression is also negative, because we proved in A1(1) that $EU_{cc} (\Theta \gamma X_I - q) > 0$ and we know from A2(7) that under risk-averse behavior $q/\Theta X_I \leq \bar{\gamma}$. Consequently, the expression is negative:

$$EU_{cc} (\Theta \gamma X_I - q) (Y - \bar{\gamma}) \leq 0 \quad A3(2)$$

and tells us that under the assumption of decreasing absolute risk aversion ($U_{cc} > U_{cc}^2 / U_{cc}$) an increase in riskiness, that is the taxpayer can end up on the tails of the probability distribution with a higher frequency inducing the taxpayer to ask for a larger economic rewards on his tax-avoidance inputs.

Using the same technique as before, we derive the sign of

$$EU_{cc} (\Theta X(y - \bar{\gamma}) = EU_{cc} (mEU_P/EU_C) (Y - \bar{\gamma}).$$

Adding and subtracting $q/\Theta X_I$, we rewrite the expression as follows:

$$EU_{cc} (\Theta X(y - \bar{\gamma}) = g \Theta X \{ U_{cc}(Y - q/\Theta X_I) + (q/\Theta X_I - \bar{\gamma}) \}$$
\[ = g(x/X_1)EU_{CC}(\Theta Y_1 - q) + g(x/X_1)EU_{CC}(q - \Theta Y_1). \]

Again, the first RHS term is a weighted positive effect caused by a decrease in the required risk-premium induced by an income increment. The second term is a weighted positive effect derived from $A2(7)$ and $U_{CC} < 0$. We conclude that:

\[ EU_{CC} \Theta X(\bar{y} - \bar{y}) > 0 \quad A2(3) \]

The increase in riskiness induces a positive weighted ($r = mEU_B/EU_C$) substitution effect towards the class of input expenditures producing ancillary rewards.
BIBLIOGRAPHY


