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A HIGH STATISTICS STUDY OF $\omega^0$ PRODUCTION

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

Michael Herman Shaevitz, B.S., M.Sc.

* * * * * *

The Ohio State University

1975

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ACKNOWLEDGMENTS

I would like to thank all the people who have made the completion of this project possible.

My two physics advisors, Bill Reay and Noel Stanton, gave me both guidance and help with all aspects of the experiment.

Many physicists contributed to the success of this endeavor including Maris Abolins, Ken Edwards, George Luste, Gary Luxton, Jim Prentice, and Kurt Reibel. I would especially like to thank two collaborators, John Martin for his timely good words and Ron Sidwell for providing extracurricular activities.

My thanks goes to the many students who helped with building and analysis along the way; Paul Brockman, Dennis Legacy, Carmin Zanfino, Hank Pavolny, Gert Hartner and especially Jay Horowitz who lead the way and John Dankowycz who will carry on.

The technical staff has done a very commendable job providing any specialized equipment needed. John Fitch, Chuck Rush, Doug Keithley, and Dan O'Hara of Ohio State and Brian Dodge, Herb Coombs, and Tony Kiang have given both support and friendship throughout the experiment. Technical computing problems were solved by John Heimaster who worried about the details no one else would.

Expert typing of the thesis was provided by Holly Antosz and Arleen Danford expertly handled details at Ohio State.

I thank my parents for encouragement along the way, and, last but not least, my wife Linda who has always been patient and understanding even through the harder moments.
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INTRODUCTION

Charge exchange vector meson production has been studied in the past in an attempt to isolate the effects of single particle exchange. It is thought that these reactions can be described by only a few Regge exchanges due to t-channel quantum number restrictions and that these exchanges can be isolated by appropriate combinations of production density matrix elements. These efforts have previously met with some difficulty both because the experimental measurements are hard and because the theoretical framework is not very rigid. Nevertheless, investigations have proceeded with each new feature of the data being answered by a new theoretical idea. Until recently, the limiting factor in the system was the precision of the data but with the advent of large aperture spectrometers came new high statistics data able to actually guide theoretical studies.

This paper describes a detailed experimental study of $\omega^0(783)$ production via the reaction

$$\pi^- p \rightarrow \omega^0 n$$

(1)
Experimentally, this reaction is very clean; the $\omega^0(783)$ is a narrow resonance with a unique three pion decay signature. The main difficulty is the presence of neutrals in the final state. On the theoretical side, of the known low lying resonances only the $\rho(765)$, $J^{PG} = 1^{--}$, and the $B(1234)$, $J^{PG} = 1^{++}$, have the proper quantum numbers for exchange. An analysis of the decay angular distribution allows a separation of natural and unnatural parity exchange and should give precise information on the $\rho$ and $B$ couplings.

Past experiments have studied reaction (1) using both bubble and spark chamber techniques. The bubble chamber experiments are forced because of missing neutrals to study the charge symmetric reaction using deuterium as a target.

$$\pi^+d + \omega^0(p) \rightarrow (p_s)$$

Large backgrounds under the $\omega$ signal in these experiments result from their poor resolution and inability to detect the $\pi^0$; large deuterium corrections are also necessary in the forward direction. If, as in the only counter experiment, (5) neutron counters are used to detect the recoil particle, then both the accepted solid angle and low-t detection efficiency are severely reduced. The small cross sections for reactions (1) and (2) coupled with the poor sensitivity of the above experiments have produced experiments with less than 800 events total. With so few events, any detailed background investigations or
comparisons with theory are limited. A further problem is contamination from other reactions in the final sample which produces erroneous results when phenomenologically analyzed as reaction (1). The three reactions

$$\pi^+ d \rightarrow \rho^0 p (p_s)$$

$$\pi N \rightarrow \omega^0 \Delta (1232)$$

(3)

and

$$\pi N \rightarrow \rho N^*$$

present particular problems for the above experiments and may be contained in their final data samples.

A compilation of the previous data (1-9) is shown in Figures 1 through 4 along with three model fits. (10) Within the precision of the data, the fits are able to describe the general shapes of the data with the exception of $\rho_{00}$ at very small $t$. The excess of helicity zero $\omega$'s can either be attributed to experimental background problems or a breakdown of the simple $\rho$-$\Delta$ exchange models and is a key motivation for further investigations.

We have undertaken to resolve some of the experimental problems by performing a high statistics measurement of reaction (1) with close attention to problems of bias and background. A description of the apparatus will be given in Chapter I; the details of the data analysis in Chapter II and final results in Chapter III. The features of the data will then be analyzed in terms of a simple $\rho$-$\Delta$ exchange
model and compared to a fit using a popular Regge absorption model in Chapter IV.
Figure 1. Previous Density Matrix Measurements (s-channel)
Figure 2. Previous Density Matrix Measurements (s-channel)
Figure 3. Previous Differential Cross Section Measurements. (Data-Ref.(5) and (4), Curves-Ref. (10))

$\frac{d\sigma}{dt} \left[ \mu b/(\text{GeV/c})^2 \right]$

$\pi^- p \rightarrow \omega^0 n (4.5 \text{ GeV/c})$
$\pi^+ n \rightarrow \omega^0 p (6.95 \text{ GeV/c})$
Figure 4. Helicity Zero Projected Cross Section (s-channel)
(Data - Ref. (4), (5), and (7), Curves - Ref. (10))
CHAPTER I
DESCRIPTION OF THE EXPERIMENT

We have measured $\omega^0$ production at 6 GeV/c for the reaction

$$\pi^- p + \omega^0 n \rightarrow \pi^+ \pi^- \pi^0 \rightarrow \gamma \gamma$$

All $\omega^0$ decay products were detected; the charged particles using a conventional forward spectrometer, the neutrals using a system of spark chambers and lead glass shower detectors. A schematic drawing of the equipment layout is given in Figure 5.

The discussion of apparatus will be divided into five major subsections: beam, charged particle spectrometer, gamma ray shower detector, scintillator anti-counter system, and fast logic.

A. The Beam

The beam used in this experiment was a high momentum secondary pion beam in the EPB II area of the ZGS. Pions produced at $1.5^\circ$ were steered to the experimental area by a two-stage beam transport system. A momentum dispersed focus was produced by the first stage at the position of the counter hodoscope, BH. Each BH counter subtended a momentum bite of $1/8$ FWB at 6 GeV/c.
Figure 5  Experimental Layout
The second stage recombined the momenta and focused the beam on anti-counters, BV1 and BV2, located 120" downstream of the hydrogen target. These counters were used to reject non-interacting beam events.

The characteristics of the beam line are shown in Table 1. During typical running conditions, the spot size was $\frac{1}{4}"$ at the hydrogen target; any beam halo outside of the 1" radius target was removed by a hole anti-counter, BHR+BHL. Normally, the beam intensity was 75,000 particles per pulse, 80% of which passed within the hole anti-counter. Muon contamination was found to be 2% at 6 GeV/c by measuring the amount of beam that traversed three feet of steel. The number of kaons was estimated from published yield curves\(^{(12)}\) for our beam. At this momentum electron contamination is negligible.

Four magnetostrictive readout spark chambers before the hydrogen target determined the slope and intercept of a beam particle to $\pm 0.0001$ radians and $\pm 0.015"$ FWHM respectively.

B. Charged Particle Spectrometer

The charged particle spectrometer consisted of five spark chambers before and after a large aperture magnet. Good acceptance for large-angle tracks was obtained by using a thin magnet, only forty inches deep, and locating the chambers very close to it. A momentum resolution of 9% FWHM was measured for particles of 6 GeV/c which corresponds
TABLE 1

CHARACTERISTICS OF BEAM 8 AT ARGONNE NATIONAL LABORATORY
AS RUN FOR EXPERIMENT 337

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<th>Characteristic</th>
<th>Value</th>
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<tr>
<td>Momentum Range</td>
<td>4.5 to 9 GeV/c</td>
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<tr>
<td>Spot Size at Final Focus</td>
<td>$\frac{1}{2}$&quot; x $\frac{1}{2}$&quot;</td>
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<tr>
<td>Flux at 6 GeV/c per $2 \times 10^{11}$ protons on a 2&quot; Be Target</td>
<td></td>
</tr>
<tr>
<td>$\pi^-$</td>
<td>$10^5$</td>
</tr>
<tr>
<td>$\pi^+$</td>
<td>$3.5 \times 10^5$</td>
</tr>
<tr>
<td>$p$</td>
<td>$2 \times 10^6$</td>
</tr>
<tr>
<td>Production Angle</td>
<td>1.5°</td>
</tr>
<tr>
<td>Momentum Bite</td>
<td>2.5% (1.5&quot; Momentum Slit)</td>
</tr>
<tr>
<td>$\Delta P/P$</td>
<td>0.5% (Using $\frac{1}{4}$&quot; Momentum Hodoscope Elements)</td>
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<td>$\Delta \theta$</td>
<td>.0002</td>
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<tr>
<td>$\Delta x_{INTERCEPT}$</td>
<td>.030&quot; (Beam Spark Chambers: Position Resolution $\Delta x = .02&quot;$ FWHM)</td>
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<td>Focus</td>
<td>120&quot; Downstream of Hydrogen Target</td>
</tr>
<tr>
<td>Beam Contamination (Negative Beam)</td>
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</tr>
<tr>
<td>$\mu^-$</td>
<td>$2 \pm .4%$</td>
</tr>
<tr>
<td>$K$</td>
<td>$.6 \pm .1%&quot;</td>
</tr>
<tr>
<td>Hydrogen Target</td>
<td>2&quot; Diameter x 16&quot; Length</td>
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</tbody>
</table>
to 2.5% for a typical 2 GeV/c pion from $\omega$ decay. Of the ten chambers in the system, two chambers upstream and two downstream were rotated by $45^\circ$ and $15^\circ$ respectively and were used to eliminate ambiguities encountered for events with more than one charged particle track in the spectrometer. Spark positions in the chambers were read out by standard magnetostrictive techniques using a SAC scaler system interfaced to CAMAC. Table 2 gives the detailed characteristics for this system.

Once each ZGS pulse the spectrometer chambers were pulsed and data recorded for a non-interacting beam event. This calibration data was used off-line to calculate the exact spatial position of each chamber.

C. Gamma Ray Detection System

Gamma rays were detected in a system composed of lead glass Cerenkov counters and magnetostrictive readout spark chambers. Incident gamma rays were required to shower in a $1\frac{2}{3}$ radiation length sheet of lead (Probability of each interaction = 63.7) The position of each shower was measured by three closely packed spark chambers; two rotated by $12\frac{2}{3}^\circ$ to remove x-y pair ambiguities. By placing the first chamber within $\frac{1}{4}$" of the lead converter, the tightly collimated gamma-showers resulted in a single magnetostrictive position which was used as the shower conversion center. A test beam measurement of electron showers in a similar apparatus indicated a position resolution of $1/8"$.
TABLE 2

SPECTROMETER CHARACTERISTICS

<table>
<thead>
<tr>
<th>Beam</th>
<th>Rotation</th>
<th>Active Area</th>
<th>Distance from Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>NO</td>
<td>24&quot; x 16&quot;</td>
<td>19&quot;</td>
<td></td>
</tr>
<tr>
<td>45°</td>
<td>28&quot; x 18&quot;</td>
<td>24.5&quot;</td>
<td></td>
</tr>
<tr>
<td>NO</td>
<td>40&quot; x 30&quot;</td>
<td>29.75&quot;</td>
<td></td>
</tr>
<tr>
<td>45°</td>
<td>42&quot; x 36&quot;</td>
<td>35&quot;</td>
<td></td>
</tr>
<tr>
<td>NO</td>
<td>40&quot; x 40&quot;</td>
<td>40.5&quot;</td>
<td></td>
</tr>
<tr>
<td>15°</td>
<td>5' x 7'</td>
<td>91.5&quot;</td>
<td></td>
</tr>
<tr>
<td>15°</td>
<td>5' x 7'</td>
<td>97.5&quot;</td>
<td></td>
</tr>
<tr>
<td>NO</td>
<td>5' x 7'</td>
<td>103.5&quot;</td>
<td></td>
</tr>
<tr>
<td>NO</td>
<td>5' x 7'</td>
<td>109.5&quot;</td>
<td></td>
</tr>
<tr>
<td>NO</td>
<td>5' x 7'</td>
<td>115.5&quot;</td>
<td></td>
</tr>
</tbody>
</table>

MAGNET

<table>
<thead>
<tr>
<th>Type</th>
<th>Picture Frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Bdl</td>
<td>SCM-104</td>
</tr>
<tr>
<td>Central Field</td>
<td>240 kg - inches</td>
</tr>
<tr>
<td>Size</td>
<td>5.9 kgauss</td>
</tr>
<tr>
<td>Center</td>
<td>84&quot; W x 40&quot; H x 40&quot; D</td>
</tr>
<tr>
<td></td>
<td>63&quot; from LH₂ Target</td>
</tr>
</tbody>
</table>

CHAMBER HIGH VOLTAGE

| Method                  | Capacitor Bank Discharge |
|                        | 10 nfarads/5' x 7' Area  |
| Pulse Height           | 6.2 KV                    |
| Risetime               | 150 nsec                  |
| Delay                  | 550 nsec                  |
| Clearing Field         | 75 v DC                   |
|                        | 1 KV - 600 nsec Pulsed    |

Readout

| Method                      | Magnetostrictive |
|                            | S.A.C. Midas, 4 Scalers/Plane |

Resolution

<table>
<thead>
<tr>
<th>Position</th>
<th>0.048&quot; (FWHM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Momentum</td>
<td>9% (FWHM) at 6 GeV/c</td>
</tr>
</tbody>
</table>
FWHM for 1 GeV/c incident electrons. Further investigations using electrons transported to the detector gave similar results. Care was taken to allow the system to record multi-spark events with good efficiency; each spark chamber plane fed electronics which could digitize from eight to twelve sparks and increased storage capacitance was used on these chambers.

A fifty-six element lead glass array positioned immediately downstream of the shower chambers was used to measure gamma ray energies. The blocks were stacked in the symmetrical array shown in Figure 6; a small hole in the array allowed the beam to pass through unaffected. Each block was $7\frac{1}{2}" \times 7\frac{1}{2}"$ on the face and 10 radiation lengths deep ($\approx 12"$) and had a single 5" photomultiplier tube glued to its downstream face. The signal from the photo tube was split in the ratio of 4 to 1; the larger part going to an ADC for digitizing the pulse area and the smaller part to a passive adder of all the block signals. With this arrangement, the ADC readings after small corrections for geometrical losses were proportional to the energy deposited in the block. Unfortunately, this proportionately constant was sensitive to photomultiplier tube gain variations due primarily to changes in the ambient temperatures.

Two kinds of standards were used to monitor the relative gain drifts between tubes. The light output from a nitrogen laser was transmitted to each block via plastic
### Lead Glass Array

#### Shower Spark Chambers

<table>
<thead>
<tr>
<th>Rotated</th>
<th>Active Area</th>
<th>Distance from Converter</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. 1 - No</td>
<td>5(\times)5(\times)12.5(\times)12.5&quot;</td>
<td>0.5&quot;</td>
</tr>
<tr>
<td>No. 2 - 12.5°</td>
<td>5(\times)5(\times)12.5(\times)12.5&quot;</td>
<td>1.5&quot;</td>
</tr>
<tr>
<td>No. 3 - 12.5°</td>
<td>5(\times)5(\times)12.5(\times)12.5&quot;</td>
<td>2.5&quot;</td>
</tr>
</tbody>
</table>

#### Lead Converter

1.5 R.L. \(\times\) 60" \(\times\) 60"

Position:
140" downstream of hydrogen target

#### Lead Glass Cerenkov Counter Array

- **Size**: 7-1/2" \(\times\) 7-1/2" \(\times\) 12"
- **Type of Glass Boverns Optical**: PEM 62
- **Phototube**: (5" D) EMI
- **Glued Directly to Block**

---

**Figure 6** Gamma Detector
fiber optics and a small button of NaI scintillator containing an Am\textsuperscript{241} alpha source was glued to the center of the upstream face of each block. The laser pulse heights were recorded each beam pulse to monitor short term gain drifts; the alpha pulse heights twice daily to remove drifts over a longer period of time.

Gamma rays from the decays of neutral pions were used as a standard for absolute calibration. For this apparatus, the uncertainty in the shower energy is the dominant contribution to the error in the \( \pi^0 \) mass measurement. A di-gamma mass histogram for each block was made by incrementing the histogram for a given block whenever it was a shower center for one of the two gamma ray showers (See Figure 7).

The ratio of the centroids for these histograms to the actual \( \pi^0 \) mass was then used to correct the tube gains. An absolute calibration was necessary only once for every month's data and changes between consecutive calibrations were less than 5\%.
1) Formation of Histogram for Block #N

A) Calculate Di-Gamma Mass For All Block #N Events

B) Sum Over All Blocks

\[ \sum_{I=1}^{56} \]

C) Find Centroid Of Di-Gamma Mass Distribution And Compare With True \( \Pi^0 \) Mass

2) Correct Tube Gain For Block #N by Ratio

\[ \left( \frac{\Pi^0 \text{ Mass}}{\text{Centroid Mass}} \right)^{1/2} \]

Figure 7. Method For Lead Glass Energy Calibration
D. Anti-Counter System

The anti-counter system reduced the number of events containing extra charged particles or gamma rays. In the forward direction, two sets of picture frame anti-counters (AA1 and AA2 in Figure 5) served to limit the allowed solid angle to that covered by the spark chambers and gamma detector. These counters were made sensitive to gamma rays by covering the upstream side with ¼" lead sheets.

Surrounding the hydrogen target were counters used to reject gamma rays or charged particles recoiling at wide angles. Four alternate layers of 1/8" scintillator and ¼" lead sheets were placed on each of the four lateral sides (Figure 8). The system was made insensitive to single neutron recoils by rejecting events only if two or more of these counters detected a particle. From real data the fraction of the time a neutron showered and set one counter was found to be 5.2% leading to an estimate of .5% for the good recoil neutrons lost by this veto scheme. Charged particles and gamma rays of momentum greater than 75 MeV set at least two counters and were rejected. All target anti-counters were latched and recorded with each event allowing more stringent constraints to be investigated off line.

E. Fast Logic and Trigger

The final state for $\omega^0$ production and decay
Figure 8  Target Anti-counter System
consists of two charged pions, two gamma rays, and a slow recoil neutron. For this experiment, a system of scintillation counters (Table 3) was set up to detect such an event and isolate it from unwanted background. The incident beam signal was a count from each of the three counters, S1, B1, B2, and none from the halo counters BHR and BHL. Two elements of the horizontal hodoscope, H2, and a count in HØ, indicated the presence of two charged particles. Possible gamma ray showers were flagged by a "NO-YES" combination in the paired vertical hodoscope GHF and GHR. Charged particles and gamma rays outside of the spectrometer were rejected by the anti-counter system, as discussed in Section D. This system was insensitive to single recoil neutrons.

The fast logic (Figure 9) initiated an event trigger when:

1. An incident beam particle interacted in the target.
   \[ S1 \cdot B1 \cdot B2 \cdot (BHR + BHL) \cdot (BV1 + BV2) \]

2. At least two charged particles were present after the magnet with at least one leaving the target at an angle greater than 4°.
   \[ HØ (>1) \cdot H2(>2) \]

3. At least two possible gamma showers were present in the gamma detector
   \[ \bar{GHF}_1 \cdot \bar{GHR}_1 (>2) \]
The anti-system indicated no unwanted charged particles or gamma rays.
### Table 3

<table>
<thead>
<tr>
<th>NAME</th>
<th>SIZE</th>
<th>POSITION</th>
<th>PURPOSE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(+ Z Downstream, Z=0 At L\textsubscript{H2} Target)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S1</td>
<td>1/16&quot; x 2&quot; x 1\frac{1}{2}&quot;</td>
<td>-170&quot;</td>
<td>Detect Beam</td>
</tr>
<tr>
<td>B1</td>
<td>1/8&quot; x 3&quot; x 3&quot;</td>
<td>-36&quot;</td>
<td></td>
</tr>
<tr>
<td>B2</td>
<td>1/16&quot; x 2&quot; x 2&quot;</td>
<td>-25&quot;</td>
<td>Anti Beam Halo</td>
</tr>
<tr>
<td>BHR(right)</td>
<td>\frac{1}{4}&quot; x 16&quot; x 24&quot;</td>
<td>+14&quot;</td>
<td>Signals a Charged Particle at an Angle to the Beam &gt; 4°</td>
</tr>
<tr>
<td>BHL(left)</td>
<td>with 1&quot; Hole</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HØ</td>
<td>1/8&quot; x 17&quot; x 24&quot;</td>
<td>+23&quot;</td>
<td>Anti Particles Outside Fiducial Volume</td>
</tr>
<tr>
<td>AA1</td>
<td>Opening 26&quot; x 16&quot;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AA2</td>
<td>&quot; 40&quot; x 40&quot;</td>
<td>+42&quot;</td>
<td></td>
</tr>
<tr>
<td>H2</td>
<td>30 Counters</td>
<td>+122.8</td>
<td>Charged Particle Hodoscope</td>
</tr>
<tr>
<td></td>
<td>Each 1/8&quot; x 4&quot; x 42&quot;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BV1</td>
<td>\frac{1}{4}&quot; x 4&quot; x 4&quot;</td>
<td>+124&quot;</td>
<td>Anti Non-Interacting Beam</td>
</tr>
<tr>
<td>BV2</td>
<td></td>
<td>+129&quot;</td>
<td></td>
</tr>
<tr>
<td>GHP(front)</td>
<td>16 Counters</td>
<td>+140.0</td>
<td>No Gamma Hodoscope</td>
</tr>
<tr>
<td>GHR(\circ ar)</td>
<td>Each 1/4&quot;x7\frac{1}{2}&quot;x30&quot;</td>
<td>+143.5</td>
<td>Yes</td>
</tr>
<tr>
<td>Target</td>
<td>See Figure 8</td>
<td></td>
<td>Anti Extra Particles at Wide Angles from the Target</td>
</tr>
<tr>
<td>Anti-Counters</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 9  Simplified Trigger Diagram
CHAPTER II
DATA ANALYSIS

A. Reconstruction Software

The reconstruction software is basically the same for on-line and off-line analysis, and is composed of four sections:

1. Beam Reconstruction

For each event, the information from the momentum hodoscope and beam spark chambers is used to obtain the momentum, direction, and position of the incident pion upstream of the hydrogen target. Any event with more than one beam track is discarded and multiple hodoscope events are treated as if they have one element set at the average position.

2. Charged Particle Momentum Determination

Charged particle trajectories are reconstructed by searching for sparks within corridors or roads in space; these roads are set up from approximate track locations determined by counters and other chambers. This method has the advantage of reducing the number of trial attempts by limiting the legal combinations. The horizontal hodoscope, H2, provides a starting point for the track search and the rotated UV chambers allow correlations of
tracks obtained in separate plan and elevation views. In each view, a track is accepted if it contains two of the three X or Y planes and one of the two U or V planes. A final set of tracks is found which meet the U-V constraints, match at the magnet center, and have a common intersection with the beam track.

In principle these tracks should be straight lines before and after the magnet, but a large magnetic fringe field causes a slight curvature near the magnet. To eliminate this curvature, the position of each spark along the trial track is corrected using a polynomial fit to the fringe field, together with an approximate value of the particle's momentum. The final tracks obtained in this manner are straight lines corresponding to particle trajectories with the fringe field turned off. These corrected tracks are then used in conjunction with a momentum function to give final values for the momentum. (Appendix E)

Spark chamber efficiency is monitored by participation counters which are incremented when a given chamber contains a spark along the found trajectory. At the end of a data run, the overall single and double track efficiency is calculated from these counters. (Appendix F)

3. Energy and Position Reconstruction for Gamma Ray Showers

The gamma shower software is a searching program with
the ability to isolate tightly collimated gamma showers from charged particles and general background. A flow diagram is given in Figure 10 and a brief description follows.

Shower candidates are found which have energy deposition in a single lead glass block greater than 30 MeV. These candidates are taken as true showers if one x and one y spark is present in either of the first two chambers, and a subsequent chamber has at least one confirming spark. The conversion vertex is set equal to the position of the leading x and y sparks; an average is used for showers with more than one leading spark. To eliminate the ambiguities present when charged particle showers overlap true gammas, any found shower within 5" of an extrapolated charged particle trajectory is deleted. A final value of the shower energy is obtained by adding any energy in surrounding blocks close to the shower vertex and making corrections for losses in the detection system. (Appendix D)

4. Kinematic Analysis

The results of the three previous sections are used to generate four vectors for each of the detected particles and these four-vectors are fit to the hypothesis

\[ \pi^- p \rightarrow \nu^0 n \]

\[ \pi^+ \pi^- \pi^0 \]

\[ \gamma \gamma \]
CONVERT ADC NO.'S TO ENERGY

LOOP OVER BLOCKS $E > E_{\text{threshold}}$

AT LEAST ONE X AND ONE Y SPARK IN FIRST TWG CHAMBERS NEAR THIS BLOCK

NO

YES

AT LEAST ONE CONFIRMING SPARK IN OTHER CHAMBERS

GOOD SHOWER

REMOVE SHOWERS DUE TO CHARGED PIONS

ADD SURROUNDING BLOCKS IF NEEDED

APPLY ENERGY CORRECTIONS

FIND TWO SHOWERS WITH CLOSEST EFFECTIVE MASS

Figure 10  Flow Chart of Gamma Shower Reconstruction
This procedure is briefly described below and given in Figure 11.

Reconstruction problems for low momentum particles and any inefficiencies of detectors near their edges are eliminated by requiring all events to pass the constraints given in Table 4.

The two gamma energies are fit using the neutral pion as a constraint and the fitted gamma four-vectors together with those of the charged particles are used to generate a three-pion effective mass and a nucleon missing mass.

Final four-vectors are found by using a fit to the hypothesis $\pi^- p \rightarrow V^0 n$ with the nucleon missing mass constrained to its nominal value. These four-vectors are made available for on-line binning and plotting further analysis off-line.
ALL EVENTS WITH ONE PLUS AND ONE MINUS CHARGED TRACK AND \( \geq 2 \) GAMMA SHOWERS

- NO
- EVENT PASSES GEOMETRY AND MOMENTUM CUTS
- YES
- EFFECTIVE MASS OF TWO GAMMAS = \( M_{\pi^+} \pm 40 \text{ MeV} \)
- NO
- CONSTRAIN TWO GAMMA ENERGIES TO GIVE \( M_{\pi^+} \)
- NO
- THREE PION MISSING MASS BETWEEN 500-1100 MeV
- NO
- DISCARD EVENT
- YES
- FIT EVENT TO \( \pi^+P \to \gamma^0n \) HYPOTHESIS

GOOD EVENT

Figure 11  Flow Chart of Kinematic Analysis
TABLE 4

KINEMATIC CUTS

**MOMENTUM CUTS**

- Momentum of Each Charged Particle > 400 MeV/c
- Energy of Each Gamma Ray > 300 MeV
- Ratio \(\frac{\text{Slow Gamma Energy}}{\text{Fast Gamma Energy}}\) > 0.2

**GEOMETRICAL CUTS**

- All Charged and Gamma Ray Trajectories Within the Detectors By 1"
- Gamma Ray - Charged Particle Distance at Lead Converter > 5"


B. On Line Data Collection and Analysis

Data collection and on line analysis were performed on a General Automation SPC 16/85 computer interfaced to the experiment through CAMAC. Data acquisition was controlled by a hand-shaking system between the computer and the fast logic. When the computer was ready to accept data a pulse was sent to the fast logic via CAMAC; a LAM was returned when an event was ready for processing. Each event consisted of 260 16-bit words of data and the collection system was capable of recording up to forty events per 600 msec ZGS pulse. All raw data was recorded on magnetic tape for re-analysis off line.

In the 5½ seconds between ZGS pulses, about forty percent of the recorded events were analyzed. The results of these analyzed events provided a continuous performance monitor of the hodoscopes, spark chambers, and lead glass counters. Of particular importance was the ability to monitor the spark chamber efficiency found by the trackfinding and fitting. Plots of the three pion effective mass and nucleon missing mass also provided a check on the number of good \( \omega^0 \) events being recorded. Pulse to pulse calibration data (straight through beam tracks and laser calibration of the lead glass) was not analyzed by the on-line program but was recorded for use off-line.
C. Off-Line Analysis

All recorded events were re-analyzed off-line on either the General Automation machine or at Michigan State University on a CDC 6500. The basic software was the same as used on-line but greater use was made of the calibration data. Information from the recorded non-interacting beam and laser events was used to correct the spectrometer spatial parameters and the lead glass gain drifts. The lead glass ADC pedestals were continuously updated and the neutral pion mass used to control the average gain of lead glass system. By incorporating these monitors, the gamma ray energy resolution was improved by twenty percent and the charged momentum by fifteen.

An analyzed event buffer for all events with at least two charged particles was recorded on magnetic tape for further analysis at a later time. Table 5 gives the statistics for the different categories of recorded events.
TABLE 5
BREAKDOWN OF RECORDED EVENTS

<table>
<thead>
<tr>
<th>TYPE</th>
<th>PERCENTAGE (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibration Events</td>
<td>4.5</td>
</tr>
<tr>
<td>Double Beam Tracks</td>
<td>19.2</td>
</tr>
<tr>
<td>Single-Prong Events</td>
<td>42.5</td>
</tr>
<tr>
<td>Three-Prong Events or Two-Prong Events with the Same Charge</td>
<td>9.7</td>
</tr>
<tr>
<td>Good Two-Prong Events</td>
<td>24.1</td>
</tr>
<tr>
<td>Without Two Gamma Showers</td>
<td>15.8</td>
</tr>
<tr>
<td>Two Gamma Showers Not From a ( \pi^0 ) Decay</td>
<td>4.4</td>
</tr>
<tr>
<td>&quot;Good&quot; Events</td>
<td>3.9</td>
</tr>
<tr>
<td>(Digamma Mass Between 100 and 170 MeV)</td>
<td></td>
</tr>
</tbody>
</table>
1. Cross Section and Density Matrix

After the initial analysis, each event can be described by the following kinematic variables:

\[ M_{2\gamma} \] = Digamma effective mass
\[ M_{3\pi} \] = Tri-pion effective mass
\[ M_{\text{RECOIL}} \] = Tri-pion (nucleon) missing mass
\[ t \] = Four-momentum transfer squared
\[ \theta,\phi \] = Angles of the tri-pion decay plane normal in a specified frame

In this paper, results will be presented for \( \theta \) and \( \phi \) measured in the two three-pion rest frames. (Figure 12)

**s-Channel Helicity Frame**

The \( z \)-axis is anti-parallel to the recoil neutron direction in the tri-pion rest frame

**t-Channel (Gottfried-Jackson) Frame**

The \( z \)-axis is along the incident beam direction in the tri-pion rest frame.

For both frames, the \( y \)-axis is along the normal to the production plane, and the \( x \)-axis is chosen to give a right handed coordinate system.

The other variables, for example Dalitz plot position, production angle, etc., are made available for correlation studies but are not used to obtain cross sections.

With the above definitions, the differential cross section is related to the observed data by
$\hat{n}$ = 3-pion decay plane normal

$\hat{n} = \frac{\vec{p}_{\pi^+} \times \vec{p}_{\pi^-}}{|\vec{p}_{\pi^+} \times \vec{p}_{\pi^-}|}$

Production normal

$\hat{n} = \frac{\vec{p}_{\text{\pi in}} \times \vec{p}_\omega}{|\vec{p}_{\text{\pi in}} \times \vec{p}_\omega|}$

Figure 12 $\omega$ Decay Frame Definitions
\[
\frac{d\sigma}{dt} = \sum_{\theta, \phi} \frac{Y(t, \theta, \phi)}{I \cdot N \cdot \Delta t \cdot A(t, \theta, \phi) \cdot C}
\]  

(1)

\[Y(t, \theta, \phi) = \text{Number of good events within a } (t, \theta, \phi) \text{ bin}\]

\[I = \text{Total incident beam}\]

\[N = \text{Number of protons in the liquid hydrogen target}\]

\[\Delta t = t \text{ bin width}\]

\[A(t, \theta, \phi) = \text{Acceptance for this } (t, \theta, \phi) \text{ bin from the Monte Carlo simulation (Appendix B)}\]

\[C = \text{Overall normalization corrections}\]

A description of the procedure for defining "good" events will be deferred until Chapter III, Section A; it suffices to say here that cuts are made on \(M_{2\gamma}, M_{3\pi}\), and \(M^{\text{RECOIL}}\). In our analysis we have used forty bins for \(\cos \theta\) and thirty-six bins for \(\phi\).

The density matrix elements (dm\(\varepsilon\)'s) are related to the angular distribution by expression (13) in Appendix A. (It is assumed that the constant s-wave background has been eliminated by a linear subtraction) Each dm\(\varepsilon\) can be obtained by projecting the angular variation governed by the given element. Again the acceptances are used to relate the true (weighted) distribution to the measured yield. With the definition,

\[W(t, \theta, \phi) = \frac{Y(t, \theta, \phi)}{A(t, \theta, \phi)}\]  

(2)
the moment of the function \( f(\theta,\phi) \) is given by

\[
\langle f(\theta,\phi) \rangle = \sum_{\theta, \phi \text{ Bins}} \frac{f(\theta,\phi) W(t,\theta,\phi)}{N}
\]  

where

\[
N = \text{Total Weighted Events} = \sum_{\theta, \phi \text{ Bins}} W(t,\theta,\phi)
\]

The statistical error in the moment, \( \langle f(\theta,\phi) \rangle \) is given by

\[
\delta\langle f(\theta,\phi) \rangle = \sqrt{\frac{\langle (f(\theta,\phi))^2 \rangle - \langle f(\theta,\phi) \rangle^2}{N}}
\]  

The function, \( f(\theta,\phi) \), related to each dme is given in Appendix C together with its connection to the \( Y^L_M \) moments.

Spin projected cross sections which are related to the production amplitudes (Appendix A) are calculated from the dme's using the relation

\[
\sigma_{ij} = \rho_{ij} \frac{d\sigma}{dt}
\]

and have statistical error

\[
\delta^2 \sigma_{ij} = (\delta\rho_{ij} \frac{d\sigma}{dt})^2 + (\rho_{ij} \delta\frac{d\sigma}{dt})^2
\]

2. Corrections and Overall Normalization

In expression (1) for the differential cross section, \( A(t,\theta,\phi) \) represents the corrections dependent on kinematic variables. These corrections are calculated by a simulation
program (Appendix B) that removes events which fail the geometry cuts, have pions decaying in flight or particles converting in the hydrogen target.

The time dependent parts of the C corrections are handled by the reconstruction software. Spark chamber efficiency for both the charged and gamma detectors are derived from the individual participation ratios (Appendix F). Events with two or more beam tracks are rejected and the total beam corrected accordingly. Good events rejected by random coincidences with the anti-counter system are continuously monitored during data collection; these effects are removed on an hourly run basis during off-line analysis.

A list of the time-independent corrections is given in Table 6 with comments on entries of special interest given below:

Software Reconstruction Inefficiency

The track reconstruction software losses have been estimated by hand scanning storage oscilloscope displays of 1600 events. Only two-track events were examined and all so-called "bad events" were re-analyzed by hand on an enlarged position scale. With this method the inefficiency was measured to be 2.4 ± .8% including a systematic error estimate.

The losses due to inefficient gamma shower finding software were particularly difficult to measure because of
**TABLE 6**

**SUMMARY OF CORRECTIONS**

<table>
<thead>
<tr>
<th>Event Loss</th>
<th>Constant Corrections</th>
<th>Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Either γ-Ray Fails to Convert In Lead Converter (1.6RL)</td>
<td>.550</td>
<td>.024</td>
</tr>
<tr>
<td>Beam Contamination (Muons, .02) (Kaons, .006)</td>
<td>.026 ± .006</td>
<td></td>
</tr>
<tr>
<td>Beam Attenuation in Target and Upstream Material</td>
<td>.021 ± .003</td>
<td></td>
</tr>
<tr>
<td>Final π⁺ or π⁻ Interacts in Detector</td>
<td>.040 ± .004</td>
<td></td>
</tr>
<tr>
<td>Either δ-ray Converts Upstream of Lead Converter</td>
<td>.098 ± .009</td>
<td></td>
</tr>
<tr>
<td>Associated γ-ray Vetos Events</td>
<td>Less than .001</td>
<td></td>
</tr>
<tr>
<td>Gamma Hodoscope Requirement Rejects a Good Event</td>
<td>.122 ± .006</td>
<td></td>
</tr>
<tr>
<td>Scintillation Counter Inefficiency, Cracks in Hodoscopes, etc.</td>
<td>.032 ± .008</td>
<td></td>
</tr>
<tr>
<td>Software Reconstruction Failure</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spectrometer</td>
<td>.024 ± .008</td>
<td></td>
</tr>
<tr>
<td>Gamma Shower Detector</td>
<td>.020 ± .020</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Event Loss</th>
<th>Good Events Lost by Mass Cuts</th>
<th>Typical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>M₂γ</td>
<td>100 to 170 MeV</td>
<td>.018 ± .005</td>
</tr>
<tr>
<td>M₃π</td>
<td>740 to 830 MeV</td>
<td>.070 ± .010</td>
</tr>
<tr>
<td>MRECOIL</td>
<td>500 to 1100 Events</td>
<td>.126 ± .020</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Event Loss</th>
<th>Events with Undetected Extra Particles</th>
<th>Typical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spectrometer</td>
<td>- .038 ± .005</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Event Loss</th>
<th>Branching Ratio For ω⁻ Decaying to other Final States</th>
<th>Typical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spectrometer</td>
<td>.10 ± .006</td>
<td></td>
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</tbody>
</table>

**CORRECTIONS CALCULATED BY RECONSTRUCTION SOFTWARE**

<table>
<thead>
<tr>
<th>Event Loss</th>
<th>Typical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Second Beam Particle Arriving within the Live Time of Beam Chambers</td>
<td>.20</td>
</tr>
<tr>
<td>Partial Chamber Inefficiency</td>
<td>Spectrometer .13</td>
</tr>
<tr>
<td></td>
<td>Gamma Detector .02</td>
</tr>
<tr>
<td>Random Veto Coincident with Good Event</td>
<td>.03</td>
</tr>
</tbody>
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**CORRECTIONS HANDLED BY MONTE CARLO SIMULATION**

<table>
<thead>
<tr>
<th>Event Loss</th>
<th>Average Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry of Detector System</td>
<td>.116</td>
</tr>
<tr>
<td>Charged Particles and γ-Rays within 5&quot; at Lead Converter</td>
<td></td>
</tr>
<tr>
<td>Charged Pion Decays in Flight</td>
<td></td>
</tr>
<tr>
<td>Interactions and Conversion of Outgoing Particles in the Hydrogen Target</td>
<td></td>
</tr>
</tbody>
</table>
the rotated spark chambers in the system. An interactive program was written to display the lead glass energy and expanded spark chamber information for operator determined positions in the array. By hand scanning 1000 events, the losses were found to be $2 \pm 2\%$.

**Gamma Hodoscope Trigger Rejects a Good Event**

Approximately one tenth of the data recorded were taken with a weaker gamma trigger requiring only one or more "NO-YES" combination in the gamma hodoscope. By examining the $\omega$ events, we are convinced that this sample, in effect, contains all produced $\omega$ events. A "NO-YES" combination is observed at a gamma position 86% of the time and an extra "NO-YES" pair from shower backscatter is present in 76% of the events. These two percentages give an $\omega$ triggering efficiency of greater than 99% for the weak trigger.

An examination of the weak trigger data sample shows that 88% of these events contain at least two "NO-YES" combinations, the standard gamma ray trigger. If the missing 12% has the same $t$, $\cos \theta$ and $\phi$ distribution as the main sample, a simple multiplicative correction can be made. Figure 13 shows a comparison of these distributions for the two samples; the agreement is quite good and indicates that the simple correction will introduce no strong biases.

**Gamma Ray Conversion Efficiency**

The gamma ray conversion efficiency is highly dependent on the effective cutoff energy for observed secondary
Figure 13 Comparison of Weak and Strong Gamma Trigger
electrons. For detector, this cutoff energy is very difficult to measure being dependent on the position of conversion in the lead and the secondary electron's angle and energy. An upper limit can be set on the conversion probability by using the theoretical pair production cross section of 66.6% for 1.5 radiation lengths. Published monte carlo results predict a value of 62.9% for a 10 MeV cutoff. We believe that the parameters of our system lie between these two extremes and in our normalization use the average with a systematic error covering the two limits.

3. Resolution Studies

Non-interacting beam events provide a means to measure the charged particle momentum resolution. The momentum error is found to be 9% (FWHM) at 6GeV/c corresponding to a spark chamber position error of .040" (FWHM) for each spark on the trajectory. For typical two track events an average position error of .050" is observed. The total error at other momenta can then be approximated by combining a constant position error of .050" with a multiple scattering error of 2.4% (FWHM); the latter figure derived from the mass seen by particles traversing the spectrometer

\[
\frac{\Delta P}{P} = ((.024)^2 + (\frac{.11P}{6\text{ GeV/c}})^2)^{\frac{1}{2}}
\]

(FWHM)

The error in the charged particle angle measurement was dominated by the multiple scattering in the hydrogen target
with
\[ \Delta \theta \text{ (FWHM)} = \frac{0.036}{P} \left( \frac{\text{Radiation Length of Material Traversed in Exiting the Target}}{2} \right)^{\frac{1}{2}} \]  \hspace{1cm} (2)

For symmetric decays of the $\pi^0$, the fractional error in the calculated di-gamma mass is directly related to that of the measured gamma energies.

\[ \frac{2}{M_{Y_1 Y_2}} = k E_1 E_2 \]  \hspace{1cm} (3)

\[ 2 \frac{\delta M_{Y_1 Y_2}}{M_{Y_1 Y_2}} = \left[ \left( \frac{\delta E_1}{E_1} \right)^2 + \left( \frac{\delta E_2}{E_2} \right)^2 \right]^{\frac{1}{2}} \]  \hspace{1cm} (4)

for $E_1 = E_2$

\[ \frac{\delta M_{Y_1 Y_2}}{M_{Y_1 Y_2}} = \frac{1}{\sqrt{2}} \frac{\delta E_1}{E_1} \]  \hspace{1cm} (5)

In Figure 14 are shown plots of the di-gamma mass spectrum for various bins of gamma energy. From the widths of the peaks in each of these plots, we derive the energy resolution using equation (5). Also shown is a plot of the energy resolution versus energy; as expected from photon statistics, the resolution is linear in the variable $E_1^{\frac{1}{2}}$. A fit to this curve then gives

\[ \frac{\Delta E_Y}{E_Y} \text{ (FWHM)} = 0.055 + 0.175/ \sqrt{E} \]
Figure 14: Gamma Shower Energy Resolution
A measurement of the shower position resolution can be made by tracking electrons of various energies through the spectrometer to the lead converter. The position error is then calculated by comparing the found shower position with the position determined by the spectrometer. Figure 15 shows the results of this test; a resolution of .3" (FWHM) is found to hold for all energies from 800 to 1500 MeV.

An estimate of the error in the various derived quantities of an ω event is then obtained using these parameterizations in conjunction with a monte carlo simulation program. The predicted distributions for the neutral pion, neutron, and omega mass along with the t resolution are given in Figure 16.
Figure 15  Electron Shower Position Resolution
Figure 16 Monte Carlo Simulation of Experimental Errors
CHAPTER III
RESULTS OF THE EXPERIMENT

A. Selection of the Final Event Sample

As stated previously, good events are defined in terms of requirements on the derived quantities $M_{2\gamma}$, $M_{\text{RECOIL}}$, and $M_{3\pi}$. (In all cases, the recoil and the three pion mass are found using a fit of the di-gamma mass to a neutral pion; the three pion mass contains, in addition, a kinematic fit with the neutron mass as a constraint) This section describes the method we have used to determine these mass limits and how the limits affect the final results.

A simple di-gamma mass cut of 100 to 170 MeV is made to isolate events coming from neutral pions; the number of good events outside this cut has been measured from the mass shape to be $(1.8 \pm 5)\%$. (Figure 17)

The broad neutron peak in the missing mass spectrum (Figure 18) extends beyond the mass region where extra pion background may exist. This background is likely to be caused in part by $\omega^0 - \Delta(1230)$ associated production and can only be eliminated by a tight cut on the missing mass. By using the lower half of the nucleon peak to approximate the upper half, we can separate the true neutron signal from the background. With this technique, we estimate that
Figure 17. Di-Gamma Effective Mass
Figure 18. Neutron Missing Mass
a missing mass cut of 500 to 1100 MeV will reduce the background to below 4% and only remove 12.6% of the true neutron recoils. Figure 19 shows missing mass plots for three $t'$ bins; the contamination and good event loss is seen to be independent of $t$ and is connected by a simple normalization factor of 12.6%. In Chapter III, Section B, a comparison will be given between density matrix elements derived from events in the upper and lower half of the neutron peak; this comparison should reveal the effects of any residual contamination.

A three pion mass histogram for events passing the above two constraints is shown in Figure 20; clear $\eta(556)$ and $\omega^0(783)$ signals are seen over a flat background. The events in the $\omega^0$ peak have a Dalitz plot and density consistent with a $J^P = 1^-$ particle (Figure 21) and show no region of strong depopulation from experimental biases. As a function of $t'$, the $\omega^0$ line shape stays constant (Figure 22) with a width of 23 MeV (FWHM).

In order to remove any non-resonant background from the final sample, we subtract from the central $\omega$ region events in two control regions (Figure 23); this subtraction is done on each $(t, \theta, \phi)$ bin separately. The size and position of the three regions are found by examining the spin structure of the decay angular distribution as a

*In this paper, $t'$ is defined as $t' = |t - t_{\text{MIN}}|$. 
Figure 19. Neutron Missing Mass
Figure 20. Tri-Pion Effective Mass
Figure 21. Dalitz Plot Density for Final Data Sample
Figure 22. Tri-Pion Effective Mass
THREE PION EFFECTIVE MASS

\[
\text{WIDTH (LOW CONTROL)} + \text{WIDTH (HIGH CONTROL)} = \text{WIDTH (CENTRAL)}
\]

\[
M_H - M_W = M_W - M_L
\]

Figure 23 Background Subtraction Regions
function of three pion mass to see where the $\omega^0$ signal disappears.

Spherical harmonic moments are particularly useful for this analysis because of their ability to isolate contributions from given spin waves. For example, a single angular momentum state, $\ell$, will generate $Y^L_M$ moments only up through $L = 2\ell$; even $L$ moments have mainly pure spin parts and odd moments show interference between spins. For a spin-one $\omega^0$ produced over an s-wave background only moments with $L \leq 2$ are expected; for pure s-wave all moments but $Y^0_0$ are zero.

Figures 24 through 32 show plots of various spherical harmonic moments versus three pion mass. Results are given for three $t'$ bins.

- $0.0 < t' < 0.05$ Figures 24, 25
- $0.05 < t' < 0.30$ Figures 26 - 28
- $0.30 < t' < 1.2$ Figures 29 - 31

In all plots, $Y^L_M$ refers to the value of the moment times the differential cross section.

$$Y^L_M = \langle Y^L_M \rangle \frac{d\sigma}{dt\,dM_{3\pi}}$$

See Appendix C

in $\mu b/(GeV/c)^2(MeV)$

The angular distributions used to generate these plots have been corrected for experimental bias by the monte carlo
Figure 24. $Y^L_M$ Moments for $0.\lt t' \lt .05$ (Gev/c)$^2$
Figure 25. $Y_M^L$ Moments for $0. < t' < 0.05$ (Gev/c)$^2$
Figure 26. $Y_{LM}^1$ Moments for $0.05 < t^< 0.3$ (GeV/c)$^2$
Figure 27. $\gamma^L_M$ Moments for $0.05 < t' < 0.3$ (GeV/c)$^2$
Figure 28. $Y^L_M$ Moments for $0.05 < t' < 0.3 \text{ (GeV/c)}^2$
Figure 29. $\gamma^L_M$ Moments for $0.3 < t' < 1.2 \text{ (GeV/c)}^2$
Figure 30. $\gamma^L_M$ Moments for $0.3 < t' < 1.2$ (GeV/c)$^2$
Figure 31. $\gamma_M^L$ Moments for $0.3 < t' < 1.2 \text{ (GeV/c)}^2$
Figure 32. \( Y_0^4 \) Moments
generated acceptances. A uniform Dalitz plot is assumed except in the $\omega^0$ region (700 to 860 MeV) where a Dalitz plot corresponding to a $J^P = 1^-$ matrix element is employed. Moments are calculated by the method of Chapter II, Section C, and no systematic errors are included.

For the $t'$ regions above .05 (GeV/c)$^2$, all moments examined are consistently small except the five $L = 2$ and $L = 1$ moments corresponding to the $\omega^0$. The $\omega^0$ peaks in the three $L = 2$ moments suggest a central region defined as 740 to 830 MeV and the non-resonant background is shown to be flat below 730 and above 840 MeV. The small $L=3$ and $L=4$ moments in the $\omega^0$ region are probably caused by interference with other high order spin waves and by slight errors in the calculated acceptance. These "illegal" moments are very small in the $\omega^0$ region (Table 7) with the largest being two percent of the cross section. We find that these moments can be increased slightly by changing the kinematic cuts of Table 4 but that the final density matrix elements are unaffected.

Below $t'$ of .05 (GeV/c)$^2$, an enhancement appears in the cross section and $Y_0^2$ moment just above the $\omega$ mass. A study of this mass region shows a clear $\rho^-$ signal coupled to a $n - \pi^+$ mass spectrum with a peak near 1400 MeV. (Figures 33 and 35). We believe this background to come from associated $\rho^- N^*$ production with the $N^*$ decay pion
Figure 33. $M_{\pi^-\pi^0}$ vs. $M_{3\pi}$
going forward in the laboratory. The strong peripheral \( t' \) dependence is illustrated in Figure 34 which shows this background to disappear beyond \( t' = 0.08 \) (GeV/c). The lack of similar events with an associated \( \rho^0 \) is caused by the low energy cutoff imposed on the detected gamma rays and low mass \( n^- \pi^+ \) events, for example \( \Delta(1230) \), are eliminated by the charged pion momentum cut.

These \( \rho^- - N^* \) events must be removed from the high mass control region before a proper background subtraction can be performed. To accomplish this result, we use a simple cut on the negative dipion mass to be below 600 MeV for \( t' < 0.08 \) (GeV/c)\(^2\) (Figure 35).

Figures 36 through 38 show the cross section and moments after this cut is applied; again using the \( \omega^0 \) peaks to define the central region, we obtain the final region definitions.

<table>
<thead>
<tr>
<th>Control</th>
<th>Mass Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Control</td>
<td>680 - 725 MeV</td>
</tr>
<tr>
<td>Central</td>
<td>740 - 830 MeV</td>
</tr>
<tr>
<td>High Control</td>
<td>845 - 890 MeV</td>
</tr>
</tbody>
</table>

It should be noted that an analysis without the removal of the \( \rho^- N^* \) background could make large errors in the final cross sections and density matrix elements at small momentum transfer. These background events have a peak of opposite sign to the events in the \( \gamma^2_o \) moment; an incorrect subtraction will tend to increase the \( \gamma^2_o \) moment.
Figure 34. Scatter Plot of $t'$ vs. $M_{3\pi}$
\[ 840 < M_{3\pi} < 950 \text{ (MeV)} \]
\[ 0 < t' < 0.08 \text{ (GeV/c)}^2 \]

**Figure 35** $\rho^{-} \rightarrow N^{*}$ Background
Figure 36. $\gamma_M^L$ Moments for $0.0 < t' < 0.05$ (GeV/c)$^2$
$\rho$- Events Removed
Figure 37. $\gamma_M^L$ Moments for $0. < t' < 0.05$ (GeV/c)$^2$
\(\rho\)-Events Removed
Figure 38. $\gamma^L_M$ Moments for $0.<t'<0.05$ (GeV/c)$^2$
\rho- Events Removed
and give a larger value to the density matrix element, $\rho_{oo}$. For the $t'$ region from 0 to .02(GeV/c) $\rho_{oo}$ changes from .73 to .60 after applying the correct subtraction; results for higher $t'$ regions are not changed.

B. Density Matrix Elements

From the measured decay angular distribution with the constant s-wave background removed, we can extract six of the ten numbers needed to completely describe the density matrix. (The missing numbers are the imaginary parts of the off diagonal elements) The errors in the following dme plots are purely statistical; an indication of the size of our systematic errors can be drawn from the non-s and p wave moments given in Table 7.

As a check on our moment method for finding dme's, we have used the program on monte carlo generated distributions with various amounts of s and p wave. In all cases, the calculated moments and errors agreed with the values from which the data was generated. The results of the moment program were also compared to those of a least-squares fit; the two results were the same when a large input sample (20,000 events) was used.

Small changes in the central and control regions did not affect the dme's except for the two s-wave interference terms; the s-wave terms change when an asymmetric central region is used. These changes are expected and come from the Breit-Wigner phase of the $\omega$ changing through its mass
### TABLE 7
A COMPARISON OF "LEGAL" AND "ILLEGAL" MOMENTS

\[ Y_M \, \frac{d\sigma}{dt} \, \text{In The } \omega \, \text{Region} \]

\[ 740 < M_{3\pi} < 830 \, \text{MeV} \]

<table>
<thead>
<tr>
<th>( t ) Range ((\text{GeV/c})^2)</th>
<th>( \frac{d\sigma}{dt} ) (\mu\text{b}/(\text{GeV/c})^2)</th>
<th>( \frac{d\sigma}{dt} ) (\mu\text{b}/(\text{GeV/c})^2)</th>
<th>( Y_0^2 )</th>
<th>( Y_2^3 )</th>
<th>( Y_2^3 )</th>
<th>( Y_3^3 )</th>
<th>( Y_4^3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0 - .05 )</td>
<td>114.1</td>
<td>28.5</td>
<td>2.7</td>
<td>1.4</td>
<td>-2.0</td>
<td>-.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \pm 2.4 )</td>
<td>( \pm 1.3 )</td>
<td>( \pm 1.2 )</td>
<td>( \pm .8 )</td>
<td>( \pm .8 )</td>
<td>( \pm 1.2 )</td>
<td></td>
</tr>
<tr>
<td>( .05 -.3 )</td>
<td>174.9</td>
<td>49.6</td>
<td>1.3</td>
<td>.6</td>
<td>.2</td>
<td>3.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \pm 1.5 )</td>
<td>( \pm .6 )</td>
<td>( \pm .6 )</td>
<td>( \pm .4 )</td>
<td>( \pm .5 )</td>
<td>( \pm .6 )</td>
<td></td>
</tr>
<tr>
<td>( .3 - 1.2 )</td>
<td>39.60</td>
<td>-6.21</td>
<td>.43</td>
<td>-.23</td>
<td>.14</td>
<td>-.36</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \pm .50 )</td>
<td>( \pm .14 )</td>
<td>( \pm .14 )</td>
<td>( \pm .11 )</td>
<td>( \pm .12 )</td>
<td>( \pm .15 )</td>
<td></td>
</tr>
</tbody>
</table>

All moments are \( Y_M^L \, \frac{d\sigma}{dt} \) in units of \( \mu\text{b}/(\text{GeV/c})^2 \)
range.

To check whether a contamination of events with an extra $\pi^0$ in the final state will affect our results, we have compared the dme's extracted from data with different missing mass cuts. (Figures 39 and 40); no large differences are apparent.

Our final s-channel density matrix elements are shown in Figures 41 through 43 and listed in Tables 8&9; the t-channel elements are in Figures 44 through 46 and Tables 10 & 11. The data show an unexpected spike in the unnatural parity, helicity zero element, $\rho_{oo'}$, at small $t'$ and an overall dominance at large $t$ by $\rho^+$, natural parity exchange. Previous investigations of the reaction have hints of this behavior although limited by statistics and are in general agreement with our results (Figure 47).

C. Cross Section

From the event yield in the final sample and the corrections of Table 6, we have calculated a total forward cross section for $\omega^0$ production of

$$\sigma_T = 97.10 \pm 6.50 \ \mu b$$

The error presented includes an estimate of our systematic errors which are dominated by those of the gamma conversion probability and the event loss in cutting out the tails of the various mass distributions. With our cross section
Figure 39. Density Matrix Elements (s-channel)
For $500 < M_{\text{recoil}} < 940$ MeV
Figure 40. Density Matrix Elements (s-channel)
For $940 < M_{\text{RECOIL}} < 1100$ MeV
Figure 41. Density Matrix Elements (s-channel)
Figure 42. Density Matrix Elements (s-channel)
Figure 43. Density Matrix Elements (s-channel)
<table>
<thead>
<tr>
<th>1 (eV/c)</th>
<th>RHUL0</th>
<th>RHU0+</th>
<th>RHU0-</th>
<th>RHUL0</th>
<th>DSIG/01</th>
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<td>0.00</td>
<td>0.02</td>
<td>0.04</td>
<td>0.04</td>
<td>0.02</td>
<td>0.12</td>
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<td>0.02</td>
<td>0.05</td>
<td>0.06</td>
<td>0.04</td>
<td>0.03</td>
<td>0.13</td>
</tr>
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<td>0.08</td>
<td>0.11</td>
<td>0.09</td>
<td>0.08</td>
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<td>0.15</td>
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<td>0.38</td>
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<td>0.53</td>
<td>0.35</td>
<td>0.18</td>
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<td>0.80</td>
<td>0.79</td>
<td>0.56</td>
<td>0.22</td>
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<td>1.07</td>
<td>1.06</td>
<td>0.85</td>
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<td>1.67</td>
<td>1.22</td>
<td>0.30</td>
</tr>
<tr>
<td>0.60</td>
<td>1.68</td>
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<td>2.11</td>
<td>1.56</td>
<td>0.32</td>
</tr>
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<td>3.04</td>
<td>2.17</td>
<td>0.34</td>
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<td>1.00</td>
<td>2.96</td>
<td>3.77</td>
<td>3.76</td>
<td>2.88</td>
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</tr>
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<td>3.49</td>
<td>4.39</td>
<td>4.38</td>
<td>3.65</td>
<td>0.38</td>
</tr>
<tr>
<td>1.50</td>
<td>4.24</td>
<td>5.30</td>
<td>5.29</td>
<td>4.56</td>
<td>0.40</td>
</tr>
<tr>
<td>2.00</td>
<td>5.54</td>
<td>6.83</td>
<td>6.82</td>
<td>5.83</td>
<td>0.42</td>
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<td>8.47</td>
<td>8.46</td>
<td>7.32</td>
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<td>10.11</td>
<td>10.10</td>
<td>9.04</td>
<td>0.46</td>
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<tr>
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Figure 44. Density Matrix Elements (t-channel)
Figure 45. Density Matrix Elements (t-channel)
Figure 46. Density Matrix Elements (t-channel)
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### TABLE 11

#### 1-CHANNEL HELICITY FRAME

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</tr>
<tr>
<td>0.46</td>
<td>0.081/-0.035</td>
<td>0.030/-0.015</td>
<td>-0.012/-0.029</td>
<td>41.8/-4.4</td>
</tr>
<tr>
<td>0.48</td>
<td>0.041/-0.030</td>
<td>0.011/-0.022</td>
<td>-0.024/-0.036</td>
<td>32.4/-4.1</td>
</tr>
<tr>
<td>0.50</td>
<td>0.026/-0.022</td>
<td>-0.027/-0.019</td>
<td>0.005/-0.031</td>
<td>24.0/-4.1</td>
</tr>
<tr>
<td>0.52</td>
<td>0.019/-0.052</td>
<td>-0.000/-0.025</td>
<td>-0.007/-0.046</td>
<td>15.1/-4.1</td>
</tr>
<tr>
<td>0.54</td>
<td>0.007/-0.071</td>
<td>0.001/-0.054</td>
<td>-0.015/-0.051</td>
<td>12.7/-4.3</td>
</tr>
<tr>
<td>0.56</td>
<td>0.061/-0.050</td>
<td>-0.033/-0.027</td>
<td>-0.027/-0.048</td>
<td>9.8/-4.9</td>
</tr>
</tbody>
</table>
Figure 47. Comparison of Density Matrix Elements From This Experiment (Crosses) and Ref. (7) (Open Circles)
and the 4 GeV/c cross sections\(^{(1)}\) as strong constraints, all data from 2 to 48 GeV/c are consistent with the parameterization

\[
s_{\text{TOTAL}} \propto p^{-2.38} \quad \text{(Figure 48)}
\]

The resulting differential cross section is shown in Figure 49; (also Table 8) there is a strong turnover in the forward direction and smooth falloff for \(t'\) beyond \(.2\text{(GeV/c)}^2\). A fit of the behavior to the form \(A t e^{bt}\) gives \(b = -6.5 \pm .2(\text{GeV/c})^{-2}\). Comparing our results with those of Reference (7), we conclude that the data have similar shapes with a slight difference in overall normalization. (Figure 50)

Spin projected cross sections, \(\rho_{ij} \frac{d\sigma}{dt}\), are given in Figures 51 through 54 for both \(s\) and \(t\)-channel helicity frames. It is interesting to note that for the projection \(\rho_{oo} \frac{d\sigma}{dt}\), the rise in \(\rho_{oo}\) at small \(t'\) conspires with the fall of the cross section to produce a flat behavior. As seen in the density matrix elements, the major part of the cross section is dominated by natural parity exchange.
Figure 48  Total Cross Section vs. $P_{\text{Beam}}$

$\sigma_{\text{TOTAL}} \propto P_{\text{LAB}}^{2.38}$
Figure 49. Differential Cross Section
Figure 50. Comparison of the Differential Cross Section From This Experiment (Crosses) and Anderson, et al. (Open Circles)
Figure 51. Spin Projected Cross Sections (s-channel)
Figure 52. Spin Projected Cross Sections (s-channel)
Figure 53. Spin Projected Cross Sections (t-channel)
Figure 54. Spin Projected Cross Sections (t-channel)
CHAPTER IV
PHENOMENOLOGY

General Remarks

Current models for vector meson production rely heavily on unproven theoretical restrictions to reduce the number of free parameters. For the case at hand, exchange degeneracy, SU(3), and information from \( \rho - \omega \) interference measurements are used to relate the three reactions:

\[
\begin{align*}
(1) \quad & \pi N \rightarrow \omega^0 N \\
(2) \quad & \pi N \rightarrow \rho^0 N \\
(3) \quad & KN \rightarrow K^*(890)N
\end{align*}
\]

We choose to examine our data and those of others using a description in terms of \( B \) and \( \rho \) exchange for \( \omega^0 \) production coupled with \( \pi \) and \( A_2 \) for \( \rho^0 \) production. Our choice is motivated by present work being done by one of the authors of Reference 10, R. D. Field. In an attempt to keep this prescription simple, absorptive corrections are only considered to be important for the evasive s-channel amplitude with net helicity flip zero. These corrections allow the amplitude which would normally be zero at \( t' = 0 \) to take on a finite value. The choice of this absorption scheme is motivated by the behavior observed in other reactions.
(for example $\pi N \rightarrow \pi N^{(16)}$) where model independent analysis is available.

We assume that the helicity amplitudes are described by pure Regge poles with WSNZ (Wrong-Signature-Nonsense-Zeros) and associated cuts for each pole which are parameterized like the pole but without the restrictions at $t' = 0$. The phase of the cut is taken to be near $180^\circ$ from the pole phase.

Using the results of Appendix A, the observable cross sections can be written in terms of the exchange amplitudes as follows:

$$\rho_{oo} \frac{d\sigma}{dt} = \left| U_{++}^o \right|^2 + \left| U_{+-}^o \right|^2 = \left| P_o \right|^2 \quad (4)$$

$$\left( \rho_{ll}^{11} - \rho_{ll}^{11} \right) \frac{d\sigma}{dt} = \left| U_{++}^1 \right|^2 + \left| U_{+-}^1 \right|^2 = \left| P_- \right|^2 \quad (5)$$

$$\left( \rho_{ll}^{11} + \rho_{ll}^{11} \right) \frac{d\sigma}{dt} = \left| N_{++}^1 \right|^2 + \left| N_{+-}^1 \right|^2 = \left| P_+ \right|^2 \quad (6)$$

$$\sqrt{2} \, \text{Rep}_{10} \frac{d\sigma}{dt} = \text{Re} \left( U_{++}^1 U_{++}^{*1} + U_{+-}^1 U_{+-}^{*1} \right) \quad (7)$$

$$\quad = \left| P_o \right| \left| P_- \right| \cos \Delta$$

where

- $U$ refers to unnatural parity exchange
- $N$ refers to natural parity exchange

$$U_{\alpha\beta}^1 = \frac{1}{\sqrt{2}} \left( M_{\alpha\beta}^1 - M_{\alpha\beta}^{-1} \right) \quad (8)$$

$$N_{\alpha\beta}^1 = \frac{1}{\sqrt{2}} \left( M_{\alpha\beta}^1 + M_{\alpha\beta}^{-1} \right) \quad (9)$$
For the simple (ρ-B), (π- A₂) model, the unnatural parity exchange non flip amplitudes vanish identically and the other amplitudes are given as follows:

\[ \omega - \text{Production} \]
\[ U_{+-}^0 = B_{+-}^0 \quad U_{++}^0 = 0 \]
\[ U_{+-}^+ = B_{+-}^1 + C_{+-} \quad U_{++}^+ = 0 \]
\[ N_{+-} = \rho_{+-} + C_{+-} \quad N_{++} = \rho_{++} \]  \hspace{1cm} (10)

\[ \rho - \text{Production} \]
\[ U_{+-}^0 = \pi_{+-}^0 \quad U_{++}^0 = 0 \]
\[ U_{+-}^+ = \pi_{+-}^1 + C_{+-} \quad U_{++}^1 = 0 \]
\[ N_{+-} = A_{2+-} + C_{+-} \quad N_{++} = A_{2++} \]  \hspace{1cm} (11)

where \( \pi, B, \rho, A_2 \), are the Regge Pole contributions and \( C \) is the combined \( \pi-A_2 \) or \( \rho-B \) cut.

A. Features of the Data

Before presenting the results of a full model fit to our data, we would like to describe some basic features of the cross sections and their explanation in the context of the simple ρ-β exchange model.
1) Large Helicity Zero $\omega$ production at small $t'$

The small $t$ enhancement of zero helicity $\omega$'s is contrary to the simple description given above; the $B$ exchange amplitude, $U_{+-}^0$, must be zero at $t' = 0$ from angular momentum conservation. Absorption should affect this amplitude which has a net helicity flip of one, very little and the usual schemes would not change the turnover at $t' = 0$. In the framework of current ideas, one explanation has been proposed\(^{(11)}\) in terms of a new Regge exchange. This exchange would have the quantum numbers $J^{PC} = 2^{-+}$ and could be the exchange degenerate partner of the elusive $A_1$. The inclusion of this exchange is somewhat displeasing because of its complications to the simple model and the lack of a similar exchange in $\rho$ production. The latter objection can be partially dispelled by examining the different signature zero structure of the exchanges in $\omega^0$ and $\rho^0$ production. In $\pi N \to \rho N$, the $A_1$ contribution would have a signature zero at $\alpha(t) = 0$ which is near the pion exchange pole and would be overshadowed by the pion exchange contribution. On the other hand, in $\omega^0$ production, the new exchange would not have a zero at $\alpha(t) = 0$ whereas the $B$ exchange does vanish as $t'$ goes to zero. Thus, $B$ exchange cannot mask the contribution from the new exchange in the forward direction.
Other explanations of the non-turnover of $\rho_{oo} \, d\sigma/dt$ in terms of absorptive effects also have bad features. As stated before, conservation of angular momentum demands that amplitudes with a net helicity flip go to zero in the forward direction and leads to non-zero contributions only from net non-flip amplitudes. To produce a sizeable helicity non-flip contribution at small $t$, without introducing a new particle exchange, requires either the absorption corrections to change the helicity structure of amplitudes or the coupling of the $B$ to s-channel non-flip. Indeed, a pure $B$ exchange Regge pole in the $t$-channel can couple to non-flip in the s-channel but only at very small $t'$. 

\[ B_{\text{non flip}}: B_{++}^O = \sqrt{\frac{t}{t'}} B_{+-}^O : B_{\text{flip}} \]  

(12) 

Possibly the absorptive corrections could make the above $B_{++}^O$ very large near the forward direction; although this is impossible in the context of current models.

Absorption that changes helicity would allow the large natural parity exchange to contribute to the $U_{++}^O$ amplitude. If this is true, one would expect the $U_{++}^O$ amplitude to fall very steeply with energy. Previous data show no such tendency and preliminary data from this experiment at 8.5 GeV/c indicate little change in the shape of $\rho_{oo} \, d\sigma/dt$ at small $t$.

We tend to agree with the new exchange model, called
"Z" by Reference (11). An unambiguous, model-independent explanation must necessarily wait for new higher energy and polarized target data. For the rest of this paper we will assume this effect can be parameterized if not explained by the "Z" exchange prescription.

2) Absence of WSNZ in $\rho^+$ $d\sigma/dt$ and Absorption

The WSNZ for the $\rho$ trajectory at $t = .6$ GeV/c has been substantiated in other reactions, for example $\pi^- p \rightarrow \pi^0 n$. The reason for its absence here is typically given in terms of the B cut filling in the zero. At first glance, this may seem to be in conflict with the strong turnover in the forward direction where a large cut term should be most apparent; for example in charge exchange $\rho^0$ production, the peak in $\rho^+$ at $t = 0$ is thought to be due mostly to absorption. Resorting to exchange degeneracy to relate $\rho$ and $\omega$ production, a qualitative picture (which has the above properties) can be given as follows:

First the $\rho$ data is fit to a model with $\pi$ and $A_2$ exchange with their associated cuts.
\[
\frac{A_{\pi^- p + \omega_n}}{\pi \text{ exchange}} = i \tan (\frac{1}{2} \pi \alpha_n(t))
\]
\[
\frac{A_{\pi^- p + \rho^0_n}}{A_2 \text{ exchange}} = i \tan (\frac{1}{2} \pi \alpha_n(t))
\]
\[
\alpha_n(t) = -0.02 + 0.82t
\]
\[
\alpha_n(0) < 0
\] (13)

and

\[
\frac{A_{\pi^- p + \omega_n}}{\pi \text{ exchange}} = i \tan (\frac{1}{2} \pi \alpha_n(t))
\]
\[
\frac{A_{\pi^- p + \rho^0_n}}{A_2 \text{ exchange}} = i \tan (\frac{1}{2} \pi \alpha_n(t))
\]
\[
\alpha_n(t) = 0.5 + 0.82t
\]
\[
\alpha_n(0) > 0
\] (14)

we obtain the \(\omega^0\) phases given below

In this case, the two cuts add destructively producing a small sum which manifests itself as a turnover in \(\rho^+\) at small momentum transfer. Away from \(t' = 0\), the \(B\) cut remains large and at \(t' = 0.6\) partially fills in the \(\rho\)-pole WSNZ.
3) \textbf{Re }\rho_{10} \text{ and Unnatural Parity Amplitudes}

Excluding the forward direction where the unnatural parity non-flip amplitudes are large, the element \textbf{Re }\rho_{10} gives an estimate of the relative $P^0$ - $P^-$ phase.

If as discussed in (2), the cut contribution for $t > .3(\text{GeV/c})^2$ is predominantly $180^\circ$ out of phase with the $B_{\text{pole}}$ term, one expects $\cos \Delta$ to be near one, commonly called phase coherence. The data show this not to be the case (Fig. 55); the ratio of \textbf{Re }\rho_{10} to its value with $\cos \Delta = 1$ is near .5. This behavior presents problems for simple models which fill in the $\rho$ trajectory WSNZ with a large $B_{\text{cut}}$. 
Figure 55. \( \cos(\Delta) = \sqrt{2} \text{Re} \rho_{10} / (\rho_{00} \cdot \rho_{-})^{\frac{1}{2}} \) Where \( \Delta \) is the Phase Difference Between \( P_0 \) and \( P_- \).
4) **Energy Dependence**

Although the previous data at other energies contain much lower statistics, it is still interesting to examine our results in conjunction with these experiments. The energy dependence of the total cross section shows a simple power law dependence from beam momenta of 3 to 40 GeV/c. (Figure 48)

\[
\sigma \propto \frac{P_{LAB}}{T}^{2.38 \pm 0.20}
\]

Since the total center of mass energy is proportional to \( P_{LAB} \), this equation leads to an effective \( \alpha \) at the average \( t \) for production.

\[
\alpha_{EFF}^{AVG} = -2.38 \pm 0.20
\]

\[
\alpha_{EFF}^{t_{avg}} = -0.24 \pm 0.10
\]

Our differential cross section is described quite well by the expression

\[
\frac{d\sigma}{dt} = 4103 t' e^{-6.5 t'} \mu b/(GeV/c)^2.
\]

which gives an average \( t' \) of .15 (GeV/c)^2.

From other experiments (11), the \( \rho \) trajectory is found to be

\[
\alpha_{\rho}(t) = .5 + .82t
\]

\[
\alpha_{\rho}(-.15) = .38
\]
The observed energy dependence, thus, indicates that \( \omega \) production cannot be dominated by \( \rho \) exchange.

A previous model\(^{12}\) incorporating another natural parity Regge exchange called the \( \rho' \) is able to explain this steep energy fall off by giving the new exchange a trajectory similar to the pion; the authors state that this new exchange may just be a convenient parameterization of absorption effects.

For our prescription, this \( s \)-dependence is a natural consequence of the very large \( \bar{B} \)-cut contribution to \( \rho^+ \frac{d\sigma}{dt} \). Because \( \rho \) exchange dips in the forward direction and has a zero at \( t' = .6 \text{(GeV/c)} \), the major portion of the cross section is provided by \( B \) exchange and its associated cut. This model would predict the cross section to fall with an effective trajectory close to the \( B \).

\[
\alpha_{B}(t) = -.02 + .82t
\]

\[
\alpha_{B(-.15)} = -.14
\]

Thus, the value of the \( B \) effective trajectory is in good agreement with the observed energy dependence.

5. \( SU(3) \) Relations and \( \rho-\omega \) Interference

A comparison of the phase relations derived from \( SU(3) \) and from \( \rho-\omega \) interference provides a check on our understanding of production mechanisms. One can derive two \( SU(3) \) relations relating \( \rho, \omega, \) and \( K^*(890) \) production
assuming ideal $\omega-\rho$ mixing.

$$A(K^+ n \to K^{*0} p) = \frac{(A(\rho) + A(\omega))}{\sqrt{2}} \quad (20)$$

$$A(K^- p \to K^{*0} n) = \frac{(A(\rho) - A(\omega))}{\sqrt{2}} \quad (21)$$

Transforming these to relations among the projected cross sections gives

$$\rho_{ij} \frac{d\sigma}{dt} = \sigma_{ij}(K^{*0}) \quad (22)$$

and

$$\sigma_{ij}(K^{*0}) = \frac{1}{2} (\sigma_{ij}(\rho) + \sigma_{ij}(\omega) + 2(\sigma_{ij}(\rho)\sigma_{ij}(\omega)))^{\frac{1}{2}} \xi \cos (\chi_\rho - \chi_\omega)$$

$$\sigma_{ij}(K^{*0}) = \frac{1}{2} (\sigma_{ij}(\rho) + \sigma_{ij}(\omega) - 2(\sigma_{ij}(\rho)\sigma_{ij}(\omega)))^{\frac{1}{2}} \xi \cos (\chi_\rho - \chi_\omega) \quad (23)$$

where

$$\xi = \text{nuclear spin coherence between } \rho \text{ and } \omega$$

production amplitudes

$$\chi_{\rho,\omega} = \text{Production amplitude phase}$$

As indicated above, the cross sections for $\rho$ and $\omega$ production could have different helicity flip and non-flip contributions which is parameterized by the coherence factor, $\xi$.

Combining our data with that of Ref. 15, we obtain the phase relations given in Figure 56 for 6 GeV/c production at $t' = .15(\text{GeV/c})$. Each vector corresponds to the magnitude of a given amplitude and the angular relationships reflect the relative phase differences among the four reactions.
These phases are chosen such that (20) and (21) are satisfied.

The relative $\rho - \omega$ phase is also available from $\rho - \omega$ interference measurements; on each plot we indicate this phase by the double arrow. If the two measurements, SU(3) and $\rho - \omega$ interference, are consistent, the arrow representing $\omega$ production should point towards the double arrow. For all projections this is the case; one concludes that even with all the diverse cut effects coming into $\omega$ and $\rho$ production, SU(3) predictions are still valid.
B. Comparison with a Recent Theoretical Model

To illustrate the detailed characteristics of the model described in Section A, we present the results of fit to our data. (Figures 57 through 60) These results are from R. D. Field and are part of an overall fit he has done trying to explain \( \omega^0, \rho^0, \) and \( K^*(890) \) production.\(^{(30)}\) The contribution to each amplitude is given in expression (10) and (11) with the addition of "\( Z \)" type exchange to unnatural-parity amplitudes with helicity non-flip at the nucleon vertex.

For \( t' \) less than 1.0(\text{GeV/c})^2, the agreement between the model and data is quite good. Additional tests are also necessary to check other features of the model.

One of these tests is the overall energy dependence of the total cross section. Regge pole exchange models predict the energy dependence of pure exchange amplitudes but not specifically that of cut contributions. For this model the cuts are given a dependence similar to the \( B \)-pole. In Figure (61), the model prediction deviates from the observed cross section at high momentum; this behavior is caused by the slow energy dependence of \( \rho \)-exchange showing itself at large energies. The disagreement could be from a relative normalization error in the data or a problem with this model; future data from NAL at 100 GeV/c should determine which is at fault.
AMPLITUDE VECTOR DIAGRAMS

(K°° AND K°° ARE MULTIPLIED BY $\sqrt{2}$

$P_0$

(UNNATURAL PARITY)
HELICITY ZERO)

$K°°$

$p^+ω$ INTERFERENCE MEASUREMENT

Su (3) PREDICTION

$P^+_\perp$

(NATURAL PARITY)

$0.08 < t' < 0.2 \text{ (GeV/c)}^2$

$P^_-$

(UNNATURAL PARITY - HELICITY ONE)

Figure 56 Comparison of Relative $p - ω$ Phase from Su(3) and Interference Measurements
Figure 57. Results of a Model Fit by R. Field to $d\sigma/dt$. 

$dsig/dt$ ($\mu b/(\text{GeV/c})^2$) 

log scale
Figure 58. Results of a Model Fit by R. Field to $\rho_{oo} \, d\sigma/dt$. 

$\frac{d\sigma}{dt} (\text{fb/GeV/c})$

"B" EXCHANGE
$(\approx t e^{-7t})$

"Z" EXCHANGE
$(\approx e^{-20t})$
Figure 59. Results of a Model Fit by R. Field to $\rho^+ \, d\sigma/dt$. 

\[ \frac{d\sigma}{dt} \text{(mb/(GeV/c)^2)} \]
Figure 60. Results of a Model Fit by R. Field to \( \rho^- \) d\( \sigma \)/d\( t \).
Figure 61 Total Cross Section vs. $P_{\text{Beam}}$
Another test of the model involves the relative $\rho-\omega$ production phase. By fitting $\omega$ and $\rho$ production data separately, the model is able to predict this phase. A comparison of this prediction to data from $\rho-\omega$ interference$^{28}$ shows general agreement. (Figure 62)

In general, this model can match the present data on vector meson production but for complete confirmation further polarization experiments are necessary.

C. Conclusion

This experiment has succeeded in isolating a very large sample of produced $\omega^0$ events with very little background contamination; any experimental biases in the data have been thoroughly studied. By measuring in detail the spin projected cross sections for the reaction, the experiment has provided information on possible exchange contributions. A strong contribution from the helicity-zero, nucleon nonflip amplitude was found to be necessary to explain the small $t$ behavior of $\rho_{00}$ $d\sigma/dt$. No large dip was observed in the natural parity cross section at $t' = 0.6(\text{GeV/c})^2$. The detail of this experiment when coupled to similar measurements of $\rho^0$ and $K^*(890)$ production has been shown to strongly constrain any theoretical model which tries to explain vector meson production.

From these results, it is clear that future studies of this reaction will continue to provide interesting physics.
Figure 62. Relative $\rho - \omega$ Production Phase From Ref. (28). (Open Circles) (Curve is result of fit to $\rho$ and $\omega$ production separately)
A current experiment at Argonne will measure $\rho - \omega$ interference in the three pion mode by combining the data of this experiment with that of $\pi^+ n \rightarrow \omega \rho$. Results of other high statistics $\omega^0$ production experiments at $8.5^{(30)}$ and $4^{(1)}$ (GeV/c) will be available shortly and should provide detailed information on energy dependence. Current polarization studies of $\rho^0$ production$^{(31)}$ have shown unexpected effects suggesting that similar $\omega^0$ measurements will also be very fruitful.

At this point in time, our knowledge of strong interaction dynamics is very incomplete; hopefully, continued study of charge-exchange vector meson production will lead us to some new knowledge of high-energy particle production.
APPENDIX A

GENERAL THEORY

The pionic production of vector mesons

\[ \pi + N \rightarrow V + N' \]

\[ J^P \ (0-) \ (\frac{1}{2}^+) \ (1-) \ (\frac{1}{2}^+) \quad (0) \]

is described by twelve complex s-channel helicity amplitudes, \( M_{\text{PRODUCTION}}^{\pi N N'} \). These amplitudes are related to the probabilities of producing the vector mesons in different helicity states. The subsequent decay of the vector meson is also dependent on the helicity of the decaying particle and can be used to extract combinations of the production amplitudes.

The differential cross section for the combined production and decay process is given as follows:

\[
\frac{d\sigma}{d\Omega dt} = \frac{1}{128\pi^2 s} \sum_{\lambda_N \lambda_N'} \lambda_V \lambda_N \lambda_N' M_{\text{PRODUCTION}}^{\pi N N'}(s,t) M_{\text{DECAY}}^{V \rightarrow (\pi N)}(s,t) \left( \lambda_V \lambda_N \lambda_N' \right)^2
\]

\[
= \frac{1}{128\pi^2 s} \sum_{\lambda_V \lambda_N} M_{\text{DECAY}}^{V \rightarrow (\pi N)}(s,t) \rho_{\lambda_V \lambda_N}(s,t)
\]

where \( d\Omega \) refers to the decay plane normal orientation.

Using the relation

\[
\rho_{\lambda_V \lambda'_V} = \rho_{\lambda_V \lambda'_V} / \text{Tr} \left( \rho_{\lambda_V \lambda'_V} \right)
\]

(2)
to normalize the density matrix, the spin projected cross sections are then given by

$$\rho V V f \lambda \lambda V V N N \left( \frac{d\sigma}{dt} \right) = \frac{1}{2} \sum_{\lambda \lambda N N}^{\text{Production}} M_{V N N N} (s, t) \frac{M_{V N N N} (s, t)}{r}$$

Decay of a Particle into three pions

In expression (1) the decay amplitude $M (m \Omega)$ describes the decay distribution for a particle $V$ with helicity $\lambda_V$. Specifically, this is a decay into three pions. The formalism for the description of such a decay is given below $^{(21)}$ and is very similar to that for two-particle decays. Important differences between the two and three body decays will be noted.

In a given rest frame of the decaying particle, the three-particle system with arbitrary orientation can be described by

$$|a, \beta, \gamma; E_1, \lambda_1 >$$

where $a, \beta, \gamma$ are the Euler angles describing the orientation of the decay plane (For two particle states these angles describe the vector direction of one of the particles)

$E_1, \lambda_1$ are the momentum and helicity of each of the particles. The direction of each particle within the plane is uniquely determined by the $E_1$'s.

A state of definite angular momentum can be obtained from these states by using the D-functions.
where

\[ m = \text{component of spin, } j, \text{ along the } z \text{ axis} \]

\[ k = \text{component of spin, } j, \text{ along the decay plane normal} \]

For two body states \( k = \lambda_1 - \lambda_2 \) but for a three particle system \( k \) is not determined.

The parity transformation properties of the states are easily obtained from the above definition and yields

\[ p |E_i \lambda_i; jmk\rangle = n_1 n_2 n_3 (-1)^{S_1 + S_2 + S_3 + k} |E_i - \lambda_i; jmk\rangle \]

where \( n_i \) is the parity of particle "i"

\( S_i \) is the spin of particle "i"

The decay of a particle with definite spin parity \( J^n \) and mass into these particles is then given by

\[ A_m (E_i \lambda_i; \alpha \beta \gamma) = \langle E_i \lambda_i \alpha \beta \gamma | M | JM \rangle \]

\[ = \sqrt{\frac{2J+1}{8\pi^2}} \sum_k F_k^j (E_i \lambda_i) D_{mk}^j (\alpha \beta \gamma) \]

with

\[ F_k^j (E_i \lambda_i) = \langle E_i \lambda_i \ Jmk | M^{\text{DECAY}} | Jm \rangle \]

and \( M^{\text{DECAY}} \) is the decay operator

Since the decay operator, \( M^{\text{DECAY}} \) is rotationally invariant
the decay amplitudes $F$ depend only on the rotational invariants, $E_i$, $\lambda_i$, $J$ and $k$.

For parity conserving decays the $F$'s are related by

$$F^J_k(E_i, \lambda_i) = \eta J \eta \eta_2 \eta_3 (-1)^{S_1 + S_2 + S_3 + k} F^J_{E_i - \lambda_i}$$

(6)

For the decay of $J$ into three pions the above expression leads to the relation

$$F^J_k(E_i) = -\eta(-1)^k F^J_{-k}(E_i)$$

(7)

Depending on the actual spin parity of the decaying resonance, this expression can be used to limit $k$ to specific values.

For example

<table>
<thead>
<tr>
<th>$J^P$</th>
<th>$k$</th>
<th>$J^P$</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0+</td>
<td>Not Allowed</td>
<td>1-</td>
<td>$k=0$</td>
</tr>
<tr>
<td>0-</td>
<td>$k=0$</td>
<td>1+</td>
<td>$k=\pm 1$</td>
</tr>
</tbody>
</table>

General Expression for the Angular Distribution

A description of the reaction in expression (0) can now be described in terms of production and decay amplitudes as

$$\frac{d\sigma}{d\Omega d\Omega dE_1 dE_2} \sim \frac{2J + 1}{8 \pi^2} \sum M M' M_{k' k} \rho^J_{\lambda_i} D^*_{M' M} D_{M' k' k} \frac{F^J_k F^J_{k'}}{\lambda_i}$$

(3)
where \( \rho_{\text{MM}'}^{J} \) is the production density matrix in expression (3).

If an integration over \( dy, dE_1, dE_2 \) and \( dm \) is performed the expression obtained gives the angular distribution of the decay plane normal

\[
I(\theta, \phi) = \frac{2J + 1}{4\pi} \sum_{\text{MM}'} \rho_{\text{MM}'}^{J} D_{\text{MK}}^{J} (\theta, \phi, \theta_0) D_{\text{M'K}}^{J} (\phi, \theta, \theta_0) g_k
\]

where

\[
g_k^J = \int dm \, dE_1 \, dE_2 \sum_{\lambda_i} |F_k^J|^2.
\]

This distribution is properly normalized by demanding

\[
\sum_M \rho_{\text{MM}}^{J} = 1 \quad \text{and} \quad \sum_k g_k^J = 1
\]

For the special case of the decay, \( J^P = 1^- \), the parity relation (7) restricts \( k = 0 \) and, thus, the angular distribution is given by (22)

\[
I(\theta, \phi) = \frac{3}{4\pi} \sum_{\text{MM}'} \rho_{\text{MM}'}^{J} D_{\text{MO}}^{J} (\phi, \theta, \theta_0) D_{\text{M'O}}^{J} (\phi, \theta, \theta_0)
\]

This expression is very similar to the expression for a two-particle decay, i.e. \( \rho + 2\pi \). In fact, because of the limitations on \( k \) the usual two-particle parity conservation symmetry also holds here.
\[ I(\theta, \phi) = I(\pi-\theta, \pi + \phi) \]

In general, for three-body decays parity conservation does not lead to additional symmetries in the angular distributions.

**S-Wave Background Effects**

In an actual experiment the \( \omega \) is produced over a background consisting of phase space and other resonances. If we assume that there exists an s-wave background then the angular distribution can be parameterized in terms of a composite density matrix

\[ \rho_{JJ', MM'} = \sum_{\lambda N' \lambda N} M^*_{\lambda N' \lambda N} M_{\lambda N' \lambda N}' \]

with the restrictions from parity, Hermiticity and normalization being

\[ \rho_{JJ', MM'} = \rho_{M'M} \quad (\text{Hermitian}) \]
\[ \rho_{OO} + \rho_{11} + \rho_{11} + \rho_{-1-1} = 1 \]

Then the angular distribution is

\[ \rho_{JJ'} = \eta_{J} \eta_{J'} (-1)^{J-J'} (-1)^{M-M'} \rho_{J} \]

Then the angular distribution is
The Hamiltonian for the system can be expressed as:

\[ H(\theta, \phi) = \frac{1}{4\pi} \left( 3(\rho_{11} \sin^2 \theta + \rho_{11} \cos^2 \theta) - 2\sqrt{2} \text{Re} \rho_{10} \sin \theta \cos \phi \right. \\
- \rho_{1-1} \sin^2 \theta \cos 2\phi + \rho_{\infty}^{10} + 2\sqrt{3} \text{Re} \rho_{\infty}^{10} \cos \theta \\
- 2\sqrt{6} \text{Re} \rho_{10}^{10} \sin \theta \cos \phi) \\
= \frac{1}{4\pi} \left( 1 + (\rho_{11}^{10} - \rho_{11}^{11}) (3\cos^2 \theta - 1) + 2\sqrt{3} \text{Re} \rho_{\infty}^{10} \cos \theta \\
- 3\sqrt{2} \text{Re} \rho_{10}^{11} \sin \theta \cos \phi - 2\sqrt{6} \text{Re} \rho_{10}^{10} \sin \theta \cos \phi - 3\rho_{1-1}^{11} \sin^2 \theta \cos 2\phi \right)
\]

with \( W(\theta, \phi) \, d\Omega = 1 \)

\[ W(\theta, \phi) = \frac{\rho_{\infty}^{10}}{4\pi} + \frac{3}{4\pi} \left( \rho_{11}^{10} \cos^2 \theta + (\rho_{11}^{11} + \rho_{1-1}^{11}) \sin^2 \theta \sin^2 \phi \right. \\
+ (\rho_{11}^{11} - \rho_{1-1}^{11}) \sin^2 \theta \cos^2 \phi - \sqrt{2} \text{Re} \rho_{10}^{11} \sin^2 \theta \cos \phi) \\
+ (\text{interference terms}) \]

Restrictions on Amplitudes From Particle Exchange

If the helicity amplitudes are derived from models with particle or Regge pole exchange in the t channel, then the conservation laws lead to relations among the amplitudes.

For an exchange of a particle (or Regge pole) with parity \( \eta \), G-parity \( g \), isospin \( I \) and signature \( \xi, (-1)^J \); the following relations hold:

Parity Conservation at the Baryon Vertex

\[ M_{\lambda \lambda \lambda}^{\lambda' N' N} = \eta \xi (-1)^{\lambda' - \lambda} M_{\lambda \lambda N'}^{\lambda' N} - \lambda_N \]

\[(14)\]
Parity Conservation at the Meson Vertex

\[ M\lambda_V\lambda_N',\lambda_N = -\eta\xi(-1)^{\lambda_V} M\lambda_V\lambda_N',\lambda_N \]

or

\[ \rho\lambda_V\lambda_V' = (-1)^{\lambda_V} \lambda_V' \rho\lambda_V - \lambda_V' . \quad (15) \]

G-Parity Conservation at the Baryon Vertex

\[ M\lambda_V\lambda_N',\lambda_N = -g\eta(-1)^{I} (-1)^{\lambda_N} M\lambda_V\lambda_N',\lambda_N . \quad (16) \]

Since the product \( \eta\xi \) is important for the parity relations, it is common to split the helicity amplitudes into parts that have contributions from natural \((\eta\xi = +1)\) or unnatural \((\eta\xi = -1)\) parity exchange.

Let

\[ N_{1\alpha\beta} = 1/\sqrt{2}(M_{1\alpha\beta} + M_{-1\alpha\beta}) \quad (17) \]

Natural Parity Exchange

\[ U_{1\alpha\beta} = 1/\sqrt{2}(M_{1\alpha\beta} - M_{-1\alpha\beta}) \quad (18) \]

Unnatural parity Exchange

Then the parity relations above give

\[ N_{1\alpha\beta} = 0 \text{ for } \eta\xi = -1 \]

\[ U_{1\alpha\beta} = 0 \text{ for } \eta\xi = +1 . \quad (18) \]
Because of parity conservation at the meson vertex, only unnatural parity exchange can contribute to $\lambda_V = 0$ amplitudes. Therefore

$$U_{0\alpha\beta} = M_{0\alpha\beta}$$ (19)

The spin projected cross sections are then expressed in terms of these amplitudes as

$$\rho_{o\beta} \frac{d\sigma}{dt} = K \left| U_{o++} \right|^2 + K(-t/S_o) \left| U_{o+-} \right|^2$$ (20)

$$\rho^+ \frac{d\sigma}{dt} = (\rho_{11} + \rho_{1-1}) \frac{d\sigma}{dt} = K \left[ (t/S_o)^2 \left| N_{++} \right|^2 + (-t/S_o) \left| N_{+-} \right|^2 \right]$$ (21)

$$\rho^- \frac{d\sigma}{dt} = (\rho_{11} - \rho_{1-1}) \frac{d\sigma}{dt} = K \left[ (t/S_o)^2 \left| U_{++} \right|^2 + (-t/S_o) \left| U_{+-} \right|^2 \right]$$ (22)

$$\sqrt{2} \text{Re}(\rho_{10}) \frac{d\sigma}{dt} = K \left[ (-t/S_o)^{3/2} \text{Re}(U_{+-}^1 U_{++}^0) + (-t/S_o)^{1/2} \text{Re}(U_{++}^1 U_{+-}^0) \right]$$ (23)

where $K$ and $S_o$ are normalization constants, the $t$ dependence is calculated by assuming simple Regge pole exchange.

It is interesting to note at this point that amplitudes of different naturality do not interfere with each other. Thus, the matrix element $\text{Re}(\rho_{10})$ only has contributions from unnatural parity exchange. The $s$-wave background described previously must have parity minus being a zero spin state of three $0^-$ particles. Thus, a particle exchange description of the $s$-wave production process leads to the following parity conservation relations:
The second equation clearly shows the s-wave amplitudes to be from only natural parity exchange and, therefore, they only interfere with other natural parity amplitudes. Thus, the matrix element $\rho^{10}_{oo}$ must vanish since $M_{0\alpha\beta}$ is all unnatural exchange and $\rho^{10}_{oo}$ is only an interference of the s-wave with $N_{\pm l \alpha \beta}$ amplitudes. With these limits from parity conservation and particle exchange, a simplified density matrix can be written which separates the system into different naturality states. (23)

$$
\rho_{ss} \quad \rho_{ls} \quad 0 \quad 0 \\
\rho_{ls}^* \quad \rho^+ \quad 0 \quad 0 \\
0 \quad 0 \quad \rho_{oo} \quad \rho^{10} \\
0 \quad 0 \quad \rho^{10} \quad \rho^-
$$

If all the interference terms were zero, which is nearly the case for the actual data, this matrix would be diagonal and the eigenvalues would be as shown.
Using the relations (14), (15) and (16), the allowed contributions to each amplitude from known meson resonances can be determined. As stated previously no helicity zero natural parity amplitudes are allowed. Of all the known mesons, no candidate can be found to contribute to the non-flip unnatural parity amplitudes. The required particle would have spin, parity and $g$ parity $2^{-+}$ and would be the exchange degenerate partner of the $A_1$. Listed below are the various amplitudes with their respective quantum number and exchange particle requirements.

<table>
<thead>
<tr>
<th>$N^{0+}$</th>
<th>$\eta^\xi$</th>
<th>$\eta^\eta$</th>
<th>$\eta$</th>
<th>$g$</th>
<th>Known Particles</th>
<th>Sp $\text{Parity, G-Parity}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N^{0-}$</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>Not allowed by parity conservation</td>
<td>$1^{-+}$</td>
</tr>
<tr>
<td>$N^{1+}$</td>
<td>+1</td>
<td>-1</td>
<td>-1</td>
<td>+1</td>
<td>$\rho$ $\rho'$ $g$</td>
<td>$1^{-+}$</td>
</tr>
<tr>
<td>$N^{1-}$</td>
<td>+1</td>
<td>-1</td>
<td>-1</td>
<td>+1</td>
<td>$\rho$ $\rho'$ $g$</td>
<td>$1^{-+}$</td>
</tr>
</tbody>
</table>

| $U^{0+}$ | -1       | +1       | +1    | +1  | B               | $1^{++}$         |
| $U^{1+}$ | -1       | +1       | +1    | +1  | B               | $1^{++}$         |
| $U^{1-}$ | -1       | -1       | -1    | +1  | None            | $2^{-+}$         |
| $U^{2+}$ | -1       | -1       | -1    | +1  | None            | $2^{-+}$         |

**Dalitz Plot Description for a Three Pion Decay**

In expression (8) the decay amplitude, $F_J^k$, is a function of $E_1$ and $E_2$. This amplitude describes the decay process within the decay plane and depends only on the rest
mass, \( M_{3\pi} \), total spin, \( J \), and parity \( P \) of the three pion system. In general, there could also be a dependence on the angle within the plane but this distribution is isotropic for decays into three pions.\(^{(21)}\)

For the \( \omega \) decay, \( J^P = 1^- \) and isotopic spin 1, the simplest decay amplitude squared is

\[
F_{1^-}^{1^-}(E_1, E_2, E_3) = |\vec{P}_1 \times \vec{P}_2|^2 \tag{26}
\]

If one defines the radial Dalitz plot density as

\[
\delta = \frac{|\vec{P}_1 \times \vec{P}_2|^2}{|\vec{P}_1 \times \vec{P}_2|^2} \tag{27}
\]

with \( \delta = 0 \) at the Dalitz plot edge

\( = 1 \) at the center

then the distribution above of \( \omega \) events on the Dalitz plot will be

\[
\frac{dN}{d\sigma} = \text{a linear function peaking at} \tag{28}
\]
the center of the Dalitz plot.

For the \( s \)-wave background under the \( \omega \) the distribution is dependent on whether the three pions are in an isospin zero or one state.\(^{(24)}\) The two distributions of events are given by

\[
s\text{-wave} \quad I = 0 \quad F(0^-) \quad (E_1E_2E_3) = (S_1 - S_2)(S_2 - S_3)(S_3 - S_1) \tag{29}
\]

\[
s\text{-wave} \quad I = 0 \quad F(0^-) \quad (E_1E_2E_3) = 1
\]
with \( S_i = E_i - M_{3\pi} / 3 \). For these two cases the Dalitz radial densities are very different. The I-spin zero distribution peaks at the edge and drops to zero rapidly in the center, the I-spin one is uniform across the complete Dalitz plot.

Thus, the distribution of events as a function of \( \delta \) gives a strong indication of what actual spin states are in the data sample.
APPENDIX B
ACCEPTANCE CALCULATION

In order to determine the true decay distributions and normalized cross section for the recorded data, the geometric acceptance of the detection and triggering apparatus must first be removed. For the final analysis, each event is described by five kinematic variables: the incident beam momentum, the four-momentum transfer squared, the three pion effective mass, $M_{3\pi}$, and the decay angles, $\theta$ and $\phi$. The acceptance is calculated as a function of these variables.

A monte carlo simulation program produces events of the form

$$\pi^- p + Vn \rightarrow \pi^+ \pi^- \pi^0 \rightarrow \gamma\gamma$$

An option provides for two distributions of $V$ decays; one for the decay of a $J^P = 1^-$ particle and the other for Lorentz invariant phase space. The $1^-$ acceptance is used for the $\omega^0$ region and the phase space for the $s$-wave background.
Parameterizing the decay within the three pion decay plane in terms of the di-gamma effective mass, $M_{\pi^+\pi^-}$ and decay angle, $\mu$, the distributions for the two cases are $^{(25,26)}$

$$J^P = 1^-$$

$$\frac{dN}{dM_{\pi^+\pi^-}} = (F(M_{\pi^+\pi^-}, M_{\pi^0}) (M_{\pi^+\pi^-}, M_{\pi^0}, M_{\pi^0}))^3$$

$$\frac{dN}{d\mu} = 1 - \mu^2$$

LIPS

$$\frac{dN}{dM_{\pi^+\pi^-}} = (F(M_{\pi^+\pi^-}, M_{\pi^0}) F(M_{\pi^+\pi^-}, M_{\pi^0}, M_{\pi^0}))$$

$$\frac{dN}{d\mu} = 1$$

where

$$F(x,y,z) = \frac{(x^2 - (y+z)^2)^{\frac{1}{2}}(x^2 - (y-z)^2)^{\frac{1}{2}}}{2x}$$

After a complete event is generated and transformed to the laboratory system, the various geometric and kinematic constraints that represent the detection and trigger system are checked; events failing these cuts are deleted. The detection constraints are given in Table 4 and correspond to those applied to the actual data. Trigger constraints consist of

a) One particle outside the 1" hole in H0,

b) each charged particle in a unique H2 element.

Events are also removed statistically as follows:
a) **Interactions and conversions in the target.**

Probability of interaction for charged particles

\[ (\pi^+ \pi^-) \]

= Path length in the hydrogen target \( \times 0.00325 \) (This assumes an interaction cross section of 30 mb)

Probability of conversion for gamma rays

= Path length in hydrogen \( \times 0.00198 \) (This assumes a 0.69 cm radiation length)

b) **Decays of charged pions in flight.**

Probability of decay =

\[ \frac{\text{Distance from target to last spark chamber}}{\beta \times 307.2} \]

An acceptance for each \((\cos \theta, \phi)\) bin is then calculated by dividing the number of events surviving by the total number generated. An average of one million events was generated for seven \( t' \)-bins between 0 and 1.2 (GeV/c)^2; a linear interpolation provided acceptances at other \( t' \) values.

A comparison of the monte carlo results with observed data shows the computer predictions to be in agreement with actual data distributions. (Figure 64) In Figure 63, we present the predicted acceptance as a function of \( t, \cos \theta, \) and \( \phi \); the curves indicate this acceptance to be smoothly varying with sizeable values in all kinematic regions.
Figure 63  Average Geometric Acceptance
Figure 64  Comparison of Monte Carlo and Actual Data Distributions
APPENDIX C

DME's, $Y^L_M$'s AND MOMENTS

For an angular distribution composed of only $s$ and $p$ waves, with the constant $s$-wave part removed, the production density matrix elements are related to the $(\theta, \phi)$ moments as follows:

\[
\rho_{\infty} = \frac{5}{2} \langle \cos^2 \theta \rangle - \frac{1}{2}
\]

\[
\rho_{11} = \frac{3}{4} - \frac{5}{4} \langle \cos^2 \theta \rangle
\]

\[
\rho_{-1} = -\frac{5}{4} \langle \sin^2 \theta \cos 2\phi \rangle
\]

\[
\rho^+ = \rho_{11} + \rho_{-1} = \langle (3/4 - 5/4 \cos^2 \theta) (3 - 4 \cos^2 \phi) \rangle
\]

\[
\rho^- = \rho_{11} - \rho_{-1} = \langle (3/4 - 5/4 \cos^2 \theta) (4 \cos^2 \phi - 1) \rangle
\]

\[
\text{Re} \rho_{10} = -\frac{5}{4} \sqrt{2} \langle \sin^2 \theta \cos \phi \rangle
\]

\[
\text{Re} \rho_{30} = \frac{\sqrt{3}}{2} \langle \cos \theta \rangle
\]

\[
\text{Re} \rho_{31} = -\frac{\sqrt{3}}{2} \sqrt{2} \langle \sin \theta \cos \phi \rangle
\]

For any distribution, the $< Y^L_M >$ moments are defined as:
\[
\sqrt{4\pi} \left< Y_1^0 \right> = \sqrt{3} \left< \cos \theta \right>
\]
\[
\sqrt{4\pi} \left< Y_1^1 \right> = -\sqrt{3/2} \left< \sin \theta \cos \phi \right>
\]
\[
\sqrt{4\pi} \left< Y_2^0 \right> = \sqrt{5/2} < 3 \cos^2 \theta - 1 >
\]
\[
\sqrt{4\pi} \left< Y_2^1 \right> = \sqrt{15/2} < \sin \theta \cos \theta \cos \phi >
\]
\[
\sqrt{4\pi} \left< Y_2^2 \right> = \sqrt{15/8} < \sin^2 \theta \cos^2 \phi >
\]
\[
\sqrt{4\pi} \left< Y_3^0 \right> = \sqrt{7/2} < \cos \theta (5 \cos^2 \theta - 3) >
\]
\[
\sqrt{4\pi} \left< Y_3^1 \right> = -\sqrt{21/4} < \sin \theta (5 \cos^2 \theta - 1) \cos \phi >
\]
\[
\sqrt{4\pi} \left< Y_3^2 \right> = \sqrt{105/8} < \cos \theta (\sin^2 \theta) \cos^2 \phi >
\]
\[
\sqrt{4\pi} \left< Y_3^3 \right> = -\sqrt{35/4} < \sin^2 \theta \cos^3 \phi >
\]
\[
\sqrt{4\pi} \left< Y_4^0 \right> = \sqrt{9/8} < 35 \cos^4 \theta - 30 \cos^2 \theta + 3 >
\]

The moments are defined as

\[
f(\theta, \phi) = \sum_{i} f(\theta_i, \phi_i) W(t, \theta_i, \phi_i)
\]

\[
N_{\text{TOTAL}}
\]

For s and p wave only

\[
| = 2.\rho_{\text{os}}
\]

\[
| = 2.\rho_{1s}
\]

\[
| = 0.894 (\rho_{\infty} - \rho_{11})
\]

\[
| = 1.549 (\rho_{10})
\]

\[
| = -1.095 \rho_{1-1}
\]

\[
| = 0
\]
APPENDIX D

GAMMA SHOWER ENERGY CORRECTIONS

We have investigated the energy calibration for observed gamma showers and find the same correlations exist between the error in energy and shower position or size. In this study, the neutral pion mass is used as a standard being that the error in the digamma mass is related to the gamma energy errors.

\[ M_{\gamma_1\gamma_{2}} = 2 E_{\gamma_{1}} E_{\gamma_{2}} (1 - \cos \theta_{\gamma_{12}}) \]

\[ \delta \frac{M_{\gamma_1\gamma_{2}}}{M_{\gamma_{1}\gamma_{2}}} = \left[ \left( \frac{\delta E_{\gamma_{1}}}{E_{\gamma_{1}}} \right)^2 + \left( \frac{\delta E_{\gamma_{2}}}{E_{\gamma_{2}}} \right)^2 \right]^{1/2} \]

The corrections to the shower energy can then be obtained empirically by plotting the two gamma mass versus the parameters of a shower. With this method, we obtain the following corrections:

(Corrections are listed in order of application)

(1) Offset \[ E = E + 80 \text{ MeV} \]
(2) Energy Loss \[ E = E + 3.505 (E-147.3 \text{ MeV})^{1/2} + 5.46 \text{ MeV} \]
in lead converter (From Ref.27)

(3) Fraction of Energy in Leading Block (=FEF) \[ E = E \times 1.05 \text{ if FEF } < 1. \]
\[ E = E(1 + 0.2(1 - \text{FEF})) \]

(4) Edge Correction \[ E = E(1 + 0.003562\delta^2) \]
(\(\delta = \text{Distance to an edge in inches}\))

(5) Extra Energy Loss \[ E = E - 0.05 E^2 \]
Correction

We assume the energy dependent corrections are caused by energy loss in the lead converter and light loss around the photomultiplier tube. The geometric corrections, (3) and (4), reflect losses when a shower crosses the junction between two blocks. (There is a .04" magnetic shield between the blocks) A very crude test of two similar detectors in an electron beam gave results in agreement with the above corrections.
APPENDIX E
MOMENTUM FUNCTION

The typical method for determining the momentum of a charged particle in a magnetic field consists of trying to match the given trajectory by numerically integrating the equations of motion. This method is very costly in both time and computer storage and makes track reconstruction impossible on a small computer. We have developed a method whereby a polynomial of track parameters gives the particle momentum directly.

The polynomial is basically an approximation to the true \( \int \vec{B} \cdot d\vec{l} \) and is found by fitting monte carlo generated events that have been tracked through the magnet by numerical integration. The trajectory parameters for the function are shown in Figure 65. Fourteen terms are used in the polynomial (Table 12); each term is divided by an expression for the curvature of a track in a general magnetic field, \( \text{DEN} \). For trajectories with small vertical bends

\[
\text{DEN} = \sin(\theta_{\text{IN}}) - \sin(\theta_{\text{OUT}});
\]
for the general case, corrections are made for the bending of the track in the vertical plane.

The magnet aperture is divided into nine zones (Figure 65) and for each zone there are four sets of coefficients, two for momentum greater than 1 GeV/c positive and negative charge and two for below 1 GeV/c positive and negative. For typical 2 (GeV/c) pions, the error introduced by the function is 2% FWB, much less than the measurement errors; the error introduced for each zone is given below

<table>
<thead>
<tr>
<th>Zone No.</th>
<th>0.3&lt;P&lt;1 GeV/c</th>
<th>1&lt;P&lt;5 GeV/c</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Error %</td>
<td>(FWB)</td>
</tr>
<tr>
<td>Center</td>
<td>2.6</td>
<td>1.6</td>
</tr>
<tr>
<td>Right of Left Center</td>
<td>2.6</td>
<td>1.6</td>
</tr>
<tr>
<td>Up or Down Center</td>
<td>2.6</td>
<td>1.6</td>
</tr>
<tr>
<td>Corners</td>
<td>3.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Momentum Function Terms</td>
<td>Typical Coefficient</td>
<td></td>
</tr>
<tr>
<td>-------------------------</td>
<td>---------------------</td>
<td></td>
</tr>
<tr>
<td>(1) 1./DEN</td>
<td>.1878</td>
<td></td>
</tr>
<tr>
<td>(2) YMP^2/DEN</td>
<td>1.9 x 10^-4</td>
<td></td>
</tr>
<tr>
<td>(3) XMP^2 YMP^2 / DEN</td>
<td>4.1 x 10^-7</td>
<td></td>
</tr>
<tr>
<td>(4) YMP^2 YOUT^2/DEN</td>
<td>8.8 x 10^-7</td>
<td></td>
</tr>
<tr>
<td>(5) SLYIN^2 YMP^2/(1 + SLXIN^2) DEN</td>
<td>-2.73 x 10^-3</td>
<td></td>
</tr>
<tr>
<td>(6) XOUT^4 /DEN</td>
<td>5.43 x 10^-4</td>
<td></td>
</tr>
<tr>
<td>(7) XMP^2/DEN</td>
<td>-1.1 x 10^-4</td>
<td></td>
</tr>
<tr>
<td>(8) XIN + XOUT/DEN</td>
<td>-9.68 x 10^-4</td>
<td></td>
</tr>
<tr>
<td>(9) XOUT^6 /DEN</td>
<td>5.43 x 10^-4</td>
<td></td>
</tr>
<tr>
<td>(10) YMP^2 XIN^2/DEN</td>
<td>1.41 x 10^-8</td>
<td></td>
</tr>
<tr>
<td>(11) YMP^4 / DEN</td>
<td>-1.46 x 10^-6</td>
<td></td>
</tr>
<tr>
<td>(12) YIN^2/DEN</td>
<td>+1.2 x 10^-4</td>
<td></td>
</tr>
<tr>
<td>(13) XMP^4/DEN</td>
<td>-1.8 x 10^-2</td>
<td></td>
</tr>
<tr>
<td>(14) XIN^6/DEN</td>
<td>1.8 x 10^-9</td>
<td></td>
</tr>
</tbody>
</table>
Figure 65  Momentum Function Parameters
APPENDIX F

CHAMBER EFFICIENCY CALCULATION

Spectrometer Chambers

We obtain the efficiency of each spectrometer plane from its participation in found two track events. The track-finding criteria demands that each of the four views contain at least one spark in the two U-V planes and two sparks in the other three planes. With this criteria, the efficiency problem can be broken into two parts in each of the four views.

Let

1. $\varepsilon_i = \text{Efficiency of chamber plane } i = \frac{\text{Number of times this plane contained a spark}}{\text{Number of tracks through the chamber}}$

2. $P_i = \text{Participation ratio of chamber plane } i \text{ for found event} = \frac{\text{Number of times plane } i \text{ fired in a found event}}{\text{Number of found events}}$

U-V Case (At least one of two)

Given two planes 1 and 2, the relations are

3. Probability of at least one of the two planes to fire

$$= \varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2 = P(> 1)$$

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(4) $P_1 = \frac{\varepsilon_1}{P(\geq 1)}$

(5) $P_2 = \frac{\varepsilon_2}{P(\geq 1)}$

Solving (4) and (5) for the $\varepsilon$'s gives

\[
\varepsilon_1 = \frac{P_1 + P_2 - 1}{P_2} \quad \varepsilon_2 = \frac{P_1 + P_2 - 1}{P_1}
\]

**x-y Case** (At least two of three)

The relations for three planes, (1), (2), and (3) are

(7) Probability of at least two of three planes to fire

$= \varepsilon_1 \varepsilon_2 + \varepsilon_1 \varepsilon_3 + \varepsilon_2 \varepsilon_3 - 2\varepsilon_1 \varepsilon_2 \varepsilon_3 = P(\geq 2)$

(8) $P_1 = \frac{\varepsilon_1 (\varepsilon_2 + \varepsilon_3 - \varepsilon_2 \varepsilon_3)}{P(\geq 2)}$

(9) $P_2 = \frac{\varepsilon_2 (\varepsilon_1 + \varepsilon_3 - \varepsilon_1 \varepsilon_3)}{P(\geq 2)}$

(10) $P_3 = \frac{\varepsilon_3 (\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2)}{P(\geq 2)}$

Again solving (8), (9) and (10) for the efficiencies

(11) $\varepsilon_1 = \frac{P_1 + P_2 + P_3 - 2}{P_2 - P_3 - 1}$

(12) $\varepsilon_2 = \frac{P_1 + P_2 + P_3 - 2}{P_1 + P_3 - 1}$

(13) $\varepsilon_3 = \frac{P_1 + P_2 + P_3 - 2}{P_1 + P_2 - 1}$

Once each chamber plane efficiency is known, the overall two track efficiency can be calculated.
(14) Efficiency for detecting and finding two tracks

\[ \begin{align*}
&= P(>1) (\text{Upstream } X \text{ U-V}) P(>2) (\text{Upstream } X) \\
&\qquad \times (\text{Downstream } X) \\
&\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad #
Similar relations hold for the $y$ plane efficiencies.

We wish to solve for the $\varepsilon_i$'s in terms of the measured $P_i$'s. An analytic solution of (15) - (18) is not possible so we resort to an iterative procedure. The expressions on the R.H.S. of (16) through (18) are evaluated first with the $P_i$'s substituted for the $\varepsilon_i$'s and the equations are solved for a new set of $\varepsilon_i$'s. These new $\varepsilon_i$'s are substituted into the RHS again and so forth. Only two or three iterations are needed to obtain stable results. The results are then substituted into in (15) to obtain the final single shower efficiency; the square of this efficiency, the two shower efficiency, is used for the final cross section normalization.
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