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A THEORY OF PREFERENCE-BASED CHOICE:
ITS EMPIRICAL IMPLICATIONS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

Paul John Strand, B.A., A.M.

* * * *

The Ohio State University

1975.

Reading Committee:
Stuart Thorson
C. Richard Hofstetter
Aage Clausen
C. Richard Hofstetter

Approved By

Department of Political Science
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VITA

April 7, 1949 .......... Born - Grand Forks, North Dakota

1971 .................. B.A., Macalester College, St. Paul, Minnesota

1971-1972 .......... Teaching Associate, Department of Political Science, University of Illinois- Chicago Circle Campus, Chicago, Illinois


1972-1975 .......... Research Associate, Department of Political Science, The Ohio State University, Columbus, Ohio

FIELDS OF STUDY

Major Field: Formal Political Theory

Individual Choice, Professor Stuart Thorson

Social Choice, Professor Stuart Thorson
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I. INTRODUCTION

In this chapter, the subject of the investigation is introduced, the approach that will be taken with regard to the subject is described, the importance of the subject is illustrated, and, lastly, the contents of the remaining chapters is reviewed. The format of this chapter will also provide an introduction to the format that will be used in the remaining sections of the dissertation.

1. THE SUBJECT

I will investigate, in this dissertation, the notion of individual choice. The situations with which I am concerned can be thought to include those in which an individual is faced with a number of alternatives over which he can select one or more. These situations could, for example, be committee meetings in which various policies are being considered for adoption. Or, they could be elections in which various candidates are being considered for support. I am not concerned, here, with the specific alternatives that are selected. Rather, I am concerned with the manner in which selections are made. And, this manner will not be assumed to be affected by changes that might occur in the content of a choice situation.

2. THE APPROACH

I will be using a formal approach in the investigation of choice. Consequently, a major part of the dissertation will involve the con-
sideration of artificial languages. These languages will be considered in so far as they pertain to the description of individual choice. English will be used to "talk about" the artificial languages. These languages, however, (rather than English) will be used to "talk about" choice.

The formal approach is by no means novel. This approach is very much in the tradition of Hicks (1956), Samuelson (1938), Uzawa (1959), Arrow (1959), and Fishburn (1974). Unfortunately, formal investigations of individual choice have seldom been conducted by political scientists. They have typically been conducted by economists. Nevertheless, Downs (1957) and Buchanan and Tullock (1962) have greatly influenced the work of political scientists such as Black (1958), and Riker and Ordeshook (1973).

3. THE IMPORTANCE OF CHOICE

The notion of choice is at the foundation of both normative and descriptive political science. Consider, for example, democratic theory. Here, the relationship between social choices and individual choices is central. And, as Arrow (1963) has indicated, the manner in which individual choice is described is very important in the evaluation of procedures for determining social choices. In fact, much of the literature on individual choice focuses directly on the consequences that individual choice has on social choice (the major works include Arrow, 1963, Fishburn, 1973, Sen, 1970, Pattanaik, 1968, and Murakami, 1968). I will, for the most part, ignore these types of consequences. My major concern is with the description of individ-
ual choice.

The importance of individual choice in descriptive political science is best exemplified in survey research. The survey, as I shall argue, is a choice experiment that is used to access individual preferences. So, to the extent that the evidence used in descriptive political science is based on the results of surveys, choice is, here, absolutely central. Moreover, if different descriptions of individual choice have empirical consequences in terms of the conclusions that can be drawn from a survey, then the notion of choice is not only central, its investigation is necessary.

4. A PREVIEW

Because there are numerous languages, both available and constructable, and because languages can differ in very significant ways, I will begin, in chapter 2, by discussing the notion of language. The problems of interpretation and truth-assessment will be used, in this regard, to specify the properties that a language should exhibit before being considered for use in the expression of a scientific theory. Both syntactic and semantic properties will be discussed. Since ordinary languages (i.e., English, French, German, etc.) will be shown not to exhibit these properties, a formal language will be constructed for the description of individual choice.

In chapter 3, I trace the developments that have occurred with regard to the formal investigation of choice. I will point out, at this time, those areas in which consensus has not been reached. Also, in chapter 3, I introduce and discuss the notion of rationality. I
stress, in this discussion, that the term "rationality" is not to be used for the classification of individuals. Rather, it is to be used as a description of all preference-based choices. Chapter 3 concludes with a formal theory of individual choice. Considerations in constructing the theory are based partly on intuition and partly on (where available) empirical evidence. The criterion for these considerations can be simply described: plausibility. The theory is constructed on the principle of plausibility rather than performance.

The theory that is constructed in chapter 3 is applied in chapter 4. The context of this application is the social survey. The survey is analyzed from the perspective of the theory. The intent is to determine that which must be true of choice situations motivated by a survey in order for a political scientist to make inferences from its results. I will, in the process, provide a formal analysis of measurement under conditions where that which is measured does not have sufficient structure to accept a numerical representation. This, I argue, should be of substantial interest to a political scientist. For, it is doubtful that a political scientist's subject matter is often amenable to numerical representation. The fact that this subject matter is not amenable to numerical representation does not prohibit, as I shall point out, the possibility of rigorous measurement.

Chapter 5 is the final chapter. I summarize, here, that which has been done in the previous chapters. Hopefully, chapter 5 will provide a brief but useful account of some of the major arguments presented in this dissertation.
II. SCIENTIFIC THEORIES

This chapter focuses, first, on the structural and interpretative properties of languages. The focus will provide a perspective from which to consider the notion of a scientific theory. A scientific theory will, here, be a language that is required to satisfy certain structural and interpretative conditions. After describing these conditions, I will begin to describe the scientific theory that is used for a description of individual choice behavior.

Unfortunately, the topics considered in this chapter are interrelated and, as a result, cannot be ordered for expository convenience. Thus, I will give a brief account of that which will follow. First of all, a distinction is made between languages and the objects that languages are used to describe (sections 1 and 2). Having made this distinction, I will be in a position to consider the meaning of the phrase "true sentence" (section 3). The notion of a true sentence will be linked to the compatibility between a given language and a given set of objects. Whereas some languages will be appropriate for the description of some objects (the sentences of these languages will be true when interpreted by these objects), other languages will not. Section 4 provides a precise treatment of the notion of compatibility. After considering this notion, I will focus on a certain class of objects (section 5). These are sometimes referred to as phenomena and are sometimes referred to as empirical objects. Finally, the results of sections one through five are brought to bear on the notion of a scientific theory (section 6). In the last section, the preliminaries for a theory of individual choice are laid out. This theory will be
developed in the following chapter.

1. LANGUAGE

A description of languages should be both abstract and general. Numerous languages exist, and the domain of the discussion should not arbitrarily exclude any one of them. English is an obvious candidate for inclusion. But, so are Fortran and the Predicate Calculus. Each is a language, each is used in political investigations, and each (as shall be shown) is significantly different.

1.1 The Components of a Language

The first step in understanding a language will involve a specification of its components:

Given some finite set of symbols \( V \) (the vocabulary), a language \( L \) over \( V \) is a set of finite strings of symbols drawn from \( V \). Although \( V \) is finite by definition, \( L \) may be finite or infinite. For example, if \( V \) is the set \( \{a, b, c\} \) then the set of strings \( \{abc, acb, bac, bca, cab, cba\} \) is a finite language over \( V \). The set of strings of the form \( \{abc, abbc, abbbc, \ldots\} \) is an infinite language over \( V \). The set \( \{a, b, c\}, i.e., \( V \) itself, is counted as a language over \( V \) by the definition above. ... The strings in a language are known as sentences of \( L \). (Kimball, 1973, p. 1)

A language is, very simply, a set of sentences constructed from a vocabulary. English, according to this definition, is a set of sentences constructed from the English vocabulary \( (a, b, c, \ldots) \). I should point out, however, that the investigation of components has tended to exclude ordinary languages (i.e., English, French, German, etc.). It has tended, instead, to focus on artificial or formal languages (i.e., Fortran, the Predicate Calculus, etc.). The latter, unlike
the former, have been amenable to description.

1.2 Languages and Grammars

Having specified the components of a language, I can begin to specify manners in which one language might differ from another. One such manner involves the recognition of vocabulary strings that are sentences and vocabulary strings that are not. A sentence is a vocabulary string that, given a language, is in the set of vocabulary strings that constitute the language. It is obvious, from the previous definition, that a language need not contain all the strings that are constructable from its vocabulary. "The man ran." is an example of an English sentence. "Ran man the." is an example of a string that, although constructed from the English vocabulary, is not in the English language. Now, given a language, one can ask whether or not there is a procedure for distinguishing strings that are sentences from strings that are not. Languages will differ on the basis of a procedure's existence.

A procedure for recognizing sentences is provided by a grammar. A grammar is:

...some finite specification of the sentences of L. If L is a finite language, then L itself can be considered as a grammar of L; we might call this the list grammar of L. If L is an infinite language, however, the grammar of L must be some means of specifying the constituency of L other than a list, since grammars are finite objects. (Kimball, 1973, p. 2)

Suppose, for example, we consider a language \( L' = \{01, 0011, 000111, \ldots \} \) over a vocabulary \( V = \{0, 1, \} \). A grammar of \( L' \) might consist of the rules \( R_1: S \rightarrow 0S1 \) and \( R_2: S \rightarrow 01 \), where \( S \) refers to a start symbol" (Hopcroft
& Ullman, 1969, p. 10). If $R_1$ were applied $k$ times ($Osk^n$), and $R_2$ once, it would be possible to generate any sentence in $L'$. If a language has a known grammar, the grammar will provide a procedure for distinguishing strings that are in the language (sentences) from strings that are not. Any string that could be generated by the rules of the grammar would be in the language. The string '0100' could not, for example, have been generated by the grammar specified in the earlier example. This would be an indication that the string is not in $L'$. A procedure for recognition will not exist for a language without a known grammar.

The notion of a grammar is useful in establishing the understandability of a language (Hopcroft & Ullman, 1969, p. 8). A language that does not have a known grammar is not as understandable as is a language that has a known grammar. "Understandable" refers, here, to an insured ability to determine whether or not a given vocabulary string is also a sentence. If a language has a known grammar, the grammar will insure the status of any vocabulary string. In fact, it may be possible to construct a machine for distinguishing the sentences of such a language (Hopcroft & Ullman, 1969, p. 9). If, on the other hand, a language does not have a known grammar, the status of a string may not be amenable to recognition by a machine. English is not, in this sense, as understandable as Fortran or the Predicate Calculus. It, unlike the others, does not have a known grammar (Hopcroft & Ullman, 1969, p. 8). There are vocabulary strings that are neither obviously English sentences nor obviously not (i.e., "The house ate sixty-two."). Indeed, the lack of a known grammar distinguishes the set of artificial from
the set of ordinary languages.

The obvious question, from a social scientist's perspective is: Is there any advantage attending the use of languages that are understandable in the sense that they have known grammars? If not, there is an overriding advantage attending the use of ordinary languages. Ordinary languages are, if not understandable, familiar. Unfortunately, this question will not admit an easy answer. In fact, the question will not be answerable prior to a discussion of the conditions that a scientist's language will be required to satisfy. It may be the case that, if we wish to require that this language exhibit certain properties, we must also require that it be of a certain type (i.e., perhaps having a known grammar is a condition that is implied by some other requirement). Implications of this sort are considered throughout this chapter.

2. RELATIONAL STRUCTURES

In the previous section, no mention was made of the interpretation, or, semantics, of a language. Only the structure, or syntax, was considered. It will be necessary, here, to distinguish the two; it will be necessary to distinguish a language from the objects that a language might describe (see, in this regard, section 4). If the motivation for this dissertation were mathematical rather than scientific, I would not be concerned with the description of particular objects (I would, however, maintain the distinction). Whereas a mathematician's investigations focus on the attributes of languages (see Margaris, 1967, chapter 3), a scientist's investigations must focus
on both the attributes of languages and the compatibility between a particular language and a particular set of objects (see, in this regard, section 4). Particular objects, although unimportant to mathematicians, are important to scientists. In this section, the notion of a relational structure is introduced. It is this notion that allows the distinction between languages and objects. For, it is a relational structure that is said to provide an interpretation for a language.

2.1 The Components of a Relational Structure

The discussion of a relational structure will be based on Tarski (1954) and Pfanzagl (1968). Briefly, a relational structure \( S \) is a set of objects \( O = \{ o_1, o_2, o_3, \ldots, o_n \} \) together with a set \( I \) of relations \( R^m \subseteq O(1) \times O(2) \times O(3) \times \ldots \times O(m) \) of order \( m \in M \) (where \( M = \{1, 2, 3, \ldots, n\} \) defined on these objects. A relational structure is denoted in the following manner: \( S = \langle O; R^m \rangle \). When the order and the relation in question is unambiguous, both the superscript and the subscript will be deleted (e.g., \( S = \langle O; R \rangle \)). Consider, as an illustration, the set \( A = \{1, 2, 3\} \) and the mathematical relation "greater than" (\( > \)). The relational structure \( S' \) denoted by the set \( I \) and the relation \( > \) is: \( S' = \langle A; \rangle = \langle \{1, 2, 3\}; \{ (2,1), (3,1), (3,2) \} \rangle \). An element \((x,y)\) will be in \( > \) if and only if \( x \in A \) is greater than \( y \in A \). For a politically interesting illustration, consider a set of individuals \( B = \{ Mr A, Mr B, Mr C \} \) and a relation "is more participative in politics than" (MP). A relational structure \( S'' \) denoted by the set \( B \) and the relation MP might be: \( S'' = \langle B; MP \rangle = \langle \{ Mr A, Mr B, Mr C \}; \{ \} \rangle \).
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\{ (Mr A, Mr b), (Mr B, Mr C), (Mr A, Mr C) \}. Here, \((x, y) \) will be in MP
if and only if \(x \in B \) is "more participative than" \(y \in B \). In both of
these examples, the relational structure is simply a set of objects
together with a relation defined on these objects. In one case, the
objects and relations are numerical, and in the other, empirical (see
section 5 for a precise treatment of empirical). Now, with the defini­
tions of language and relational structure, I am in a position to
consider the issue of interpretation. Languages will not, in this
regard, be considered to be automatically interpreted. 5

2.2 Interpretation

The symbols in the vocabulary of a language can be classified
according to whether they are (i) object-symbols, (ii) relation-sym­
bols, (iii) variable symbols, (iv) connectives, or (v) quantifiers
(Margaris, 1967; Thorson, 1972). Now, a relational structure \((S) \) is
said to be an interpretation of a language \((L) \) if and only if there
exists a one-to-one function from the objects of \(S \) into the object­
symbols in \(L \) and from the relations in \(S \) to the relation-symbols in
\(L \) (for a discussion of such functions, see Pfanzagl, 1968).

Consider an illustration. \(L' \) is a language that consists of
the following two sentences:

\[
L': \quad S1: \forall x_i \in X((x_i, x_j) \notin G) \\
S2: \forall x_i, x_j, x_k \in X((x_i, x_j) \notin G \land (x_j, x_k) \notin G \rightarrow (x_i, x_k) \notin G)
\]

\(S' = <B, TT> \) is a relational structure wherein \(B \) is a set of buildings
and \(TT \), an 'is taller than' relation. At the moment, \(L' \) is uninter­
preted with respect to \(S' \). So, suppose I construct a function \(\alpha \)
that maps the object-symbols in \(X \) \((x_i, x_j, \text{ etc.}) \) into the objects in
B (b₁, b₂, etc.) and the relation-symbol G into the relation TT. Through α, S′ interprets L′. Its sentences would, under α, convey the following information about \( \langle B; TT \rangle \):

(i) No building in B is taller than itself.

(ii) For any three buildings in B, if one building is taller than a second and a second taller than a third, then the first is taller than the third.

In summary, a relational structure is a set of objects together with a set of relations defined on these objects. A relational structure is said to interpret a language when there exists a function from the object-symbols in a language into the objects in the relational structure and from the relation-symbols in a language into the relations in the relational structure. When no such function is specified, a language is said to be uninterpreted.

3. TRUTH

It will be necessary, in subsequent sections, to consider the truth of a language’s sentences. So, in order to avoid confusion as to what I mean by the phrase "true sentence," I define the notion of truth. The following discussion is based on Tarski (1956).

3.1 The Semantic Issue

There are two major aspects involved in the notion of truth. One is methodological, and the other, semantic (Thorson, 1972, p. 48). The methodological aspect focuses on the construction or adoption of a convention for assessing the truth of a sentence. And, the semantic aspect focuses on the meaning of truth. The term "empirical," for
example, suggests a perspective from which all acceptable methodological conventions must (either directly or indirectly) be based on a systematic consultation with sensory experience (Nagel & Brandt, 1965). It is the attributes of these conventions that concern methodologists; the problems of systematic consultation are by no means trivial. The present concern does not, however, rest with the methodological aspect of truth. Rather, it rests with the definition, or semantic aspect, of truth. The investigation will focus on the properties that will render a definition "adequate." This investigation is clearly appropriate. For, without an adequate definition of truth, the question of assessment is superfluous (see, in this regard, section 7). We would not, for example, want a definition to allow a single sentence to be true and false simultaneously. We would, on the other hand, want a definition to require that every sentence be either true or false.

3.2 Preliminaries to the Definition of Truth

For reasons that will be discussed later (see section 7.3), an adequate definition of truth requires a distinction between an object-language and a meta-language. An object-language contains the sentences on which "truth" is being predicated; it is the "object of our investigations" (Tarski, 1956, p. 167). A meta-language is a language that contains, in addition to the names of the sentences in an object-language, logical expressions and expressions such as "truth," "true," or "true sentence." A meta-language is a language that is used for the purpose of formulating a definition of truth in an object-language.
This is the only means, according to Tarski, of avoiding the "insuper­able difficulties" that have persisted in the numerous attempts to give precise definition to the term (reasons for these difficulties will be discussed in section 7.3).  

Consider, as an illustration, a case in which English is used as a meta-language for formulating a definition of truth in a Boolean algebra. We might, in this regard, construct English sentences of the following form:

"'x' is true if and only if P."

In an actual sentence, "x" would be replaced with an English name for a Boolean algebra sentence, and "P" would be replaced with the English translation for that same Boolean algebra sentence. The sentence:

"The sentence 'x e \{x,y\} ' is true if and only if x is in the set \{x,y\} ." is an example of an actual sentence of this form. In this sentence, one language is used for the purpose of considering the truth of a sentence in another.

3.3 The Definition of Truth

According to Tarski (1956), an adequate definition of truth should be capable of expressing the following notion:

a true sentence is one which says that a state of affairs is so and so, and the state of affairs indeed is so and so. (p.155)

Intuitively, a true sentence is one which corresponds with the facts. Consider, in this regard, the example used in the previous paragraph:

"The sentence 'x e \{x,y\} ' is true if and only if x is in the set \{x,y\} ." The problem of determining whether or not x is in the set
\{x,y\} is methodological and might be handled in one of numerous ways. But, regardless of how it is handled, the sentence does express that which Tarski requires of an adequate definition. The symbol "Tr" is used, here, to denote the set of true sentences:

Convention T: A formally correct definition of the symbol "Tr" formulated in the meta-language, will be called an adequate definition of truth if it has the following consequences:

(i) all sentences which are obtained from the expression "x \in Tr if and only if p" by substituting for the symbol 'x' a structural-descriptive name of any sentence of the language in question and for the symbol 'p' the expression which forms the translation of this sentence into the meta-language;

(ii) the sentence "for any x, if x \in Tr then x \in S" (in other words, "Tr \subseteq S"). (Tarski, 1956, p. 188).

Part (i) has already been discussed. Part (ii) requires that the set of true sentences in some language L be a subset of the set of sentences in L. Unfortunately, as Tarski points out, a direct application of this convention will not guarantee an adequate definition of truth. It would be impossible, in languages with an infinite number of sentences, to complete an adequate definition. For, such a definition would require the completion of an infinite sentence (see Tarski, 1956, p. 189, in this regard).

In order to circumvent this dilemma, Tarski introduces the notion of satisfaction. The notion of satisfaction is not, however, restricted to sentences (as is the notion of truth). It applies, more generally, to sentential functions. These, unlike sentences, are allowed to have free variables. A sentence, in this regard, is a sentential function
in which all free variables have been replaced with objects. Now, to introduce the notion of satisfaction, Tarski uses the following phrase:

for all $a$, $a$ satisfies the sentential function '$x$' if and only if $p$. (p. 190)

Suppose the sentential function in question ('$x$') has only one free variable. If "$p" is replaced with a meta-language translation for the sentential function '$x$' (after the free variable in '$x$' is replaced with an object-symbol) and '$x$' with the meta-language name for '$x$' the result would be a phrase such as:

for all $a$, $a$ satisfies the sentential function '$x \in \{1,2\}$' if and only if $a$ is a subset of $\{1,2\}$.

In this particular case, the only objects that would satisfy the sentential function '$x \in \{1,2\}$' would be $\{1\}$, $\{2\}$, and $\{1,2\}$. If $a$ were anything else, $a$ would not satisfy the function.

A recursive method can, with this base, be used to generate a notion of satisfaction for any sentential function. This method would produce the following general definition of satisfaction:

The sequence $f$ of objects satisfies the sentential function '$x$' if and only if $f$ is an infinite sequence and $p$.

An infinite sequence of objects can be applied to any sentential function if the only considered objects are those that correspond with free variables. Suppose, for example, that a sentential function has $n$ terms, $m$ of which are free variables. If these variables appear as the first $m$ terms in the function, then an infinite sequence $f$ satisfies this function if and only if the first $m$ objects in $f$ satisfy the function. In the previous example, the sequences ($\{1^2\}, \ldots$),
(\{2^2, \ldots\}) and (\{1,2^2, \ldots\}) would satisfy the sentential function 'x ∈ \{1,2^2\}'.

It is now possible to consider Tarski's definition of truth:

**Definition 2.3.1:**

\[
x \text{ is a true sentence- in symbols } x \in \text{ Tr- if and only if } x \in S \\
\text{and every infinite sequence of objects satisfies } x.
\]

(\text{p. 195})

We know that, for a sentence, satisfaction cannot depend upon the properties of the objects in a sequence. For, satisfaction only involves a consideration of those objects that correspond to free variables in a function. And, because a sentence is a function that contains no free variables, no objects are considered. Thus, satisfaction cannot depend upon the properties of particular objects. Only two possibilities remain. Either every sequence of objects will satisfy a sentence, or none will. Any other possibility would imply that the properties of particular objects are important. With Definition 2.3.1, we have an adequate definition of truth. It might be useful to point out, in conclusion, that the objects used to determine the truth of a sentence are obtained from the relational structure that interprets the language containing the sentence. Truth is not defined on the sentences of uninterpreted languages.

4. MODELS

The notion of a model will be derived from a mathematical foundation (Margaris, 1967; Tarski, 1953) and will be introduced through the notions of relational structures (section 2), interpretation
(section 2), and truth (section 3). To review, a relational structure (S) is a set of objects (O) together with a set of relations (R^n) defined on these objects. A relational structure with a single unambiguous relation will be denoted in the following manner: S = \langle O; R \rangle. An interpretation is a function that maps the objects of a relational structure into the object-symbols of a language and the relations of the relational structure into the relation-symbols of the language. Finally, a sentence is said to be true if and only if it is satisfied by all its objects.

Now, consider a language L which is composed of the following sentences:

L:

S1: \( \forall a, b, c \in A ((a, b) \in R \land (b, c) \in R \rightarrow (a, c) \in R) \)
S2: \( \forall a, b \in A ((a, b) \in R \lor (b, a) \in R) \)

And, consider two relational structures, RS_1 = \langle B; ATA \rangle and RS_2 = \langle M; FO \rangle, where RS_1 is a set of buildings (B) under the relation "is at least as tall as" (ATA) and, RS_2, a set of all living males (M) under the relation "is the father of" (FO). In order to motivate the notion of a model, I ask the following question: Can L be used to describe either RS_1 or RS_2? First, consider RS_1. In order to determine whether or not the sentences of L offer a description of RS_1, L must be interpreted with RS_1. So, consider a mapping from the objects in B into the object-symbols in A (i.e., suppose A is interpreted as a set of buildings) and from the relation ATA into the relation-symbol R (i.e., suppose "(a, b) \in R" is interpreted to mean that building a is at least as tall as building b). Now, S1 and S2 would, under this interpretation, provide the following information.
about $\langle B; \text{ATA} \rangle$:

(i) For any three buildings $b_1$, $b_2$, and $b_3$, if $b_1$ is at least as tall as $b_2$ and if $b_2$ is at least as tall as $b_3$, then $b_1$ is at least as tall as $b_3$.

(ii) For any two buildings $b_1$ and $b_2$, either $b_1$ is at least as tall as $b_2$ or $b_2$ is at least as tall as $b_1$.

Do $S_1$ and $S_2$ describe $\langle B; \text{ATA} \rangle$ under this interpretation? One would have to agree that they do. Height is generally thought to satisfy the conditions expressed in (i) and (ii).

Now, consider $RS_2$. In order to interpret $L$ with $RS_2$, a mapping must be constructed from the objects in $M$ into the object-symbols in $A$ (so, suppose $A$ is interpreted as the set of living males) and from the relation $FO$ into the relation-symbol $R$ (so, suppose "$(a,b) \in R$" is interpreted to mean that $a$ is the father of $b$). $S_1$ and $S_2$ would, under this interpretation, provide the following information about $\langle M; FO \rangle$:

(i) For any three males $a$, $b$, and $c$, if $a$ is the father of $b$ and if $b$ is the father of $c$, then $a$ is the father of $c$.

(ii) For any two males $a$ and $b$, either $a$ is the father of $b$ or $b$ is the father of $a$.

Do $S_1$ and $S_2$ describe $\langle M; FO \rangle$ under this interpretation? No, they do not. First of all, it need not be the case that, given two males, one is the father of the other. Secondly, it simply cannot be the case that one's grandfather is also one's father. These examples indicate that sentences appropriately interpreted by one relational structure need not be appropriately interpreted by another. They also indicate that a sentence which is true under one interpretation
need not be true under all interpretations. This is precisely why it is so important to distinguish a language from that which a language describes. In many cases, a relational structure will be considered to have properties that are independent of the language that might be used to describe it. Furthermore, we may not want to violate these properties through the use of a language. In these cases, it will be necessary to consider the compatibility between language and relational structure.

It is now possible to define, precisely, the notion of a model (see Margaris, 1967 or Thorson, 1972 for similar definitions):

Definition 2.4.1:

A model \( (M) \) is an interpretation for a language \( (L) \) such that every sentence in \( L \) is true under \( M \).

We know, from the previous section, that a sentence is true if and only if it is satisfied by every sequence of objects. Thus, if the sentences of a language are satisfied by the sequences supplied by the relational structure that interprets the language, this relational structure is said to be a model of the language. A major part of scientific inquiry involves the construction of languages that will accept, as models, interesting (in our case, politically relevant) relational structures. Part of this problem involves a specification of relational structures. We may not, as yet, have a useful specification of a politically relevant relational structure. What relations should be considered in such a structure? And, what properties do the relations satisfy? These are open and important questions. At present, a major part of our investigations must focus on the con-
struction of languages that will not violate our intuition as to the properties that politically relevant relational structures satisfy. For, we do not always have convincing evidence about the nature of these structures. In addition, the investigations must focus on the properties of the languages that are used to describe relational structures. For, although the definition of a model does not exclude the possibility of considering the interpretation of ordinary languages, this consideration would have both unintended and undesirable results. These results derive from the relationship between the properties of a language and the properties of Tarski's definition of truth. For reasons that will be specified shortly (see section 7), Definition 2.4.1 will only be useful if 'L' is not allowed to be an ordinary language.

5. THE TREATMENT OF "PHENOMENA"

Before discussing the conditions that a scientific theory will be required to satisfy, it will be useful to consider, very briefly, the term "phenomena." This term appears repeatedly in subsequent discussions. And, unless there is some understanding as to how the term is being used, the subjects of these discussions may become either confused or unattended.

I make no pretense, here, of being able to sort out and solve the complex philosophical problems that have been raised with regard to the use of the term. This is not my intent. I will, however, provide a statement as to how the term will be used in this dissertation. The interested reader is referred to Russell (1914), Popper
(1972), Margenau (1962), and Nagel and Brandt (1965) for further discussion.

5.1 Empirical Relational Structures

Before giving a precise definition to "phenomena" I will consider a special class of relational structures—empirical relational structures (see Pfanzagl, 1968). In colloquial contexts, the term "empirical" refers to statements or investigations that have sensory components (Nagel & Brandt, 1965, p. 179). An empirical investigation is an investigation that employs observations, and the truth of empirical sentences is contingent upon the characteristics of the relational structures that interpret them. Accordingly, sentences that are only about the integers (1, 2, 3, etc.) and mathematical relations (> , ≥ , etc.) are not empirical sentences. The truth of '2 > 1', for example, is not contingent upon inherent properties of the structure <I; > > (where 'I' is the set of integers and, ' > ', the relation 'greater than'). It is only contingent upon the language which interprets it. On the other hand, the truth of 'Bill is taller than Joe' is contingent upon, not only the language which interprets the structure <I; TT> (where I is a set of individuals and, TT, the relation 'is taller than'), but also upon the properties of <I; TT> itself. The latter is an empirical relational structure and the former is not. An empirical relational structure has inherent properties that are independent of any language that might be used to describe it. Other relational structures do not have this type of property.

Although I adopt the colloquial position on the association be-
tween the empirical and the observable, I do not require that the inherent properties of empirical relational structures be observable. I require only that these properties have observable aspects or implications. The objects or relations need not, themselves, be observable. So, although some aspects of an empirical relational structure will be observable, the objects and relations may be "irreducible constructions" (Russell, 1914).

Consider, in this regard, a relational structure \( S \) containing a set of individuals \( I \) under the uniary relation "voted in the last Presidential election" \( \text{VPE} \); \( S = \langle I; \text{VPE} \rangle \). Clearly, this is an empirical relational structure. Yet, it is not the observable aspects of individuals' votes that are of interest to a political scientist. Whether an individual, in voting, pulled a lever or marked a ballot is irrelevant. Furthermore, it is by no means clear that an investigation of \( \langle I; \text{VPE} \rangle \) can (at some future level of scientific development) be "reduced" to an investigation of the observable activities on which voting may be contingent (see Searle, 1969 on the difference between constitutive and regulatory laws). \( \langle I; \text{VPE} \rangle \) provides an excellent example of a relational structure that, although an "irreducible construction" in the sense of being non-derivable from observable activities, is empirical. Thus, an empirical relational structure is a relational structure with inherent properties that, although not necessarily observable, are independent of any language that might be used for its description.
5.2 Phenomena

The discussion of phenomena will follow the discussion of empirical relational structures. Recall, from the previous paragraph, the structure $S = \langle I; VPE \rangle$. This structure, although empirical, is not directly linked to sensory experience; it is an abstraction. Yet, the structure is assumed to have properties of its own. It is these properties we would not want to violate through the selection (for the purpose of description) of a particular language (see, in this regard, section 4). These properties, rather than being abstracted from a particular language, are abstracted from phenomena that are associated with the structure itself (i.e., the pulling of levers, the marking of ballots, the presence of candidates, an election, etc.). It is from phenomena that empirical relational structures are abstracted. Herein lies the distinction between scientific and mathematical investigations. In the latter, the properties of a relational structure are not abstracted from phenomena. They are abstracted from a specific language. Thus, the structure $\langle I; > \rangle$, where $I$ is a set of integers and $>$, the relation 'greater than', can be used to denote a specific mathematical language. $\langle I; VPE \rangle$, on the other hand, cannot (as of yet) be used to denote a specific language. It can, however, be used to denote certain phenomena.

Unfortunately, this discussion does not give any indication as to whether or not it is useful to consider one or another empirical relational structure. This, however, involves the nature of abstraction; it involves the linkage between empirical relational structures and phenomena. It does not involve only the notion of phenomena. One
of our major problems is, at present, the discovery of useful linkages. As Hanson (1958) and Popper (1972) point out, the danger lies in the mistaken impression that empirical relational structures are not abstractions but are directly linked to sensory experience. This is, as the discussion of $\langle I; VPE \rangle$ illustrates, an untenable position.

It should be noted, in conclusion, that this discussion renders measurement a very important problem. Because empirical relational structures need not be directly observable, the manner in which they are measured can be consequential in terms of the conclusions that are drawn from measurement. Measurement, like the decision on abstraction, presents an extremely complex problem (see, in this regard, chapter 4).

6. SCIENTIFIC THEORIES

How does a scientist describe the empirical relational structures in which he is interested? He uses, for this purpose, a language (Beth, 1959). This language will, among other things, replace momentary observations with durable and accessible representations. I now examine some conditions that such a language might be required to satisfy. These conditions are considered neither necessary nor sufficient for a language to be a scientific theory. Yet, if one intends to perform (with a scientific theory) certain tasks in certain ways, then the theory will have to be a language that satisfies certain conditions. The selection of the conditions will be shown to be the result of demands that the aspirations of a scientist place on the language he uses. Three conditions will be considered:
6.1 A Reasonable Theory of Meaning

An essential function of scientific theories is the provision of socially relevant and dependable information about objects which are observable by man, on the understanding that the dependability of this information is due to its being backed up by the outcome of other observations carried out by human investigators. In other words, scientific theories provide us with knowledge of things observable because that is what socially relevant and reliable information about such things comes to. (Mehlberg, 1962, p. 278)

Scientific theories can provide this information, in part, because they describe relational structures which have inherent properties. A language that does not describe such structures cannot provide information about "things observable." It cannot, as a result, be a scientific theory. I am concerned, here, not only with the properties of a language, but with the relationship between a language and a relational structure that interprets it. This concern is suggested by the first condition, for the problem of meaning involves the relationship between a language and a relational structure. Thus, the condition that a scientific theory admit a "reasonable theory of meaning" (Beth, 1964) is a condition that focuses on the relationship between a language and the structure it describes. There are, in this regard, numerous possibilities for error. One such error can
result from ambiguous meaning, and another, from unrestricted express-
ability. These errors can only be avoided in a language that admits 
a reasonable theory of meaning.

6.1.1 Ambiguous Meaning

The error of ambiguous meaning will occur, as the phrase suggests, 
when a relational structure is described by ambiguous sentences. The 
use of ambiguous sentences will prohibit a scientist from retaining 
any reliable information as to the structure that is described. This 
is why we must require that the meaning of a scientific theory's sen-
tences be sufficiently clear to allow a scientist to specify, from 
the sentences alone, that which the sentences describe. If this is 
not possible, the sentences provide neither "reliable" nor "depend-
able" information about this structure.

Consider, in this regard, the following sentence:

"The lady is a light housekeeper."

What structure does this sentence describe? Does it describe an 
element from the relational structure \( \langle F; LH \rangle \) where \( F \) is a set of 
females and \( LH \) a unary relation "is a light housekeeper?" Does it 
describe an element from the relational structure \( \langle HK; LT \rangle \) where 
\( HK \) is a set of housekeepers and \( LT \) a binary relation "weighs less 
than?" Or, does it describe an element from the relational structure 
\( \langle HW; BT \rangle \) where \( HW \) is a set of housewives and \( BT \) a binary relation 
"is a worse housekeeper than?" Each is a distinct structure, yet 
each is appropriately described by this single sentence. It would 
not be possible, given only the sentence, to unambiguously specify
the structure the sentence describes. Unfortunately, if a sentence can have multiple meanings, it can also have no assurable meaning.

The problem of ambiguous meaning raises serious doubts as to the appropriateness of using ordinary languages as scientific theories. Their sentences often have multiple or ambiguous meanings. One might, of course, attempt to salvage ordinary languages by stipulating a usable subset of an ordinary language and by restricting the meaning of relevant terms. This strategy has, in fact, been applied to a set of terms that include "power" (Dahl, 1963) and "alienation" (Seeman, 1959). It has not, however, been successful. Political scientists have yet to agree on the structure that is "appropriately" described by sentences containing the terms "power" or "alienation." This failure may be instructive. Perhaps the difficulty associated with ambiguous meaning results from the fact that, with ordinary languages, one cannot deal with uninterpreted sentences. They do not exist. In such languages, the use of a sentence may give the impression that reference is being made to a specific relational structure when, in fact, no such structure has been specified. And, as with the earlier example, numerous structures could provide plausible interpretations for a single sentence. Thus, if stipulation is considered as a strategy for salvaging ordinary languages, one must begin by stipulating a relational structure rather than a usable subset and the meaning of terms.

The inseparability of syntax and semantics is, perhaps, why there are no known grammars for ordinary languages. That which makes a string of symbols a sentence may be inextricably tied to the semantics
of the language. This is one of the major advantages of artificial languages. Because the semantics of these languages is artificially established after the syntax has been specified, a sentence cannot give the impression of describing when it is not.

6.1.2 Unrestricted Expressability

One of the properties (in addition to the lack of a specified grammar) that distinguishes ordinary languages from artificial languages is their different "means of expression" (Beth, 1964). Artificial languages, unlike ordinary languages, are subject to specific constraints. One can, for example, construct the following sentence in English: "The vocabulary string \(^x \in \mathcal{L}, y \notin \mathcal{L}\) is a sentence in Boolean algebra." It is not possible, however, to construct a sentence in Boolean algebra that describes a sentence in English. One can "talk about" Boolean algebra in English but one cannot "talk about" English in Boolean algebra. In addition, English sentences can predicate properties to other English sentences: "The sentence 'The man ran.' is a true sentence." This, too, is impossible in those artificial languages that have been considered here (see Carnap, 1967, for further examples). These limitations might, at first glance, be seen as evidence that favors the use of ordinary languages. If one language contains a wider range of sentences than another, would it not be more useful than the other? Before this question can be answered, we must determine whether or not the greater descriptive capability of ordinary languages adversely affects their ability to admit a reasonable theory of meaning.

Downs (1957) provides an excellent example of that which can
occur as a result of an unrestricted expressability. After concluding, on the basis of his "theory," that the rational voter will not vote, Downs explains the anomalous observations of voting by referring to the "extrinsic" or "socially valuable" characteristics of the vote. But, as Moon (1975) points out:

Not only are these actors supposed to be self-regarding and, therefore, insensitive to the effects of their actions on the well-being of others, but they are also supposed to calculate the benefits of their actions in terms of their consequences; they are assumed not to govern their behavior in accordance with general rules. Indeed, the point of the theory is to explain behavior without reference to such notions. (p. 123)

Downs attempted to modify his "theory" in order to explain its obvious miscalculations. In doing so, he undermined the basic premise of the original "theory" (i.e., the premise of instrumental rationality under which an individual acts only in his own self-interest). If Downs had chosen to use a language in which it is possible to distinguish between vocabulary strings that are and are not sentences, this problem may not have occurred. For, it is likely that, in light of his stated intentions, he would not have allowed vocabulary strings dealing with "social value" and "long-run benefits" to be sentences in an artificial language. These are precisely the types of sentences he, with his "theory," was attempting to replace.

The problem that results from an inability to distinguish a language from a relational structure is evidenced, again, with this example. Because English is not a language that is initially uninterpreted, it was not incumbent upon Downs to seriously consider the nature of the empirical relational structure he chose to investigate. Had he
done so, he may have decided to exclude, from that structure, relations such as "is more socially valuable than" or "is more extrinsically pleasing than." He chose to consider a relational structure containing the relation "offers more utility income than" (see Downs, 1957, chapters 2 and 3). It seems, in this case, that unrestricted expressability tended to foster a confused investigation. Downs sacrificed rigor for an apparent richness.

The problems of ambiguous meaning and unrestricted expressability provide strong incentives for the use of artificial languages and strong disincentives for the use of ordinary languages. There are, however, additional reasons why the latter should not be used as scientific theories.

6.2 A Specifiable Rule of Inference

Suppose it is possible to exhibit a scientific language that admits a reasonable theory of meaning. At this point, the language need be nothing more than a set of sentences unambiguously interpreted by a specified empirical relational structure. So, how will this set of sentences change as the scientific community becomes more successful and sophisticated in its observational techniques? Clearly, the set must grow. It must grow in order to represent the new and different observations that are being made. This growth can lead to a considerable problem:

Most science starts out as investigation of properties of obviously important features of the real world. It is wholly empirical in tone, inductive in method, and commonsensical in organization. As the knowledge of properties increases, however, it becomes increasingly
difficult both to organize knowledge and to plan further investigation. The sheer bulk of the sentences and the relationships among them discourage further inductive work and makes it hard to evaluate the significance of new discoveries. (Riker & Ordeshook, 1973, p. xi)

One might, in this regard, be inclined to require of a scientific theory that it do more than admit a reasonable theory of meaning. Perhaps a scientific theory should also be deductively organized.

If a language is deductively organized, it will be possible to "move about" within the language so as to arrive at descriptive sentences without having consulted sensory experience. It will be possible to generate sentences, not only from observations, but from other sentences in the language. However, in order to benefit from the "movement" that a deductively organized language offers, we must resolve some important issues. One such issue concerns the nature of the conditions under which movement is allowed. The decision as to when and how one moves about within a language is made through the acceptance of a rule of inference (Beth, 1964). A rule of inference justifies and instructs the movement from one set of sentences to another. And, it does so without reference to the structure that the sentences describe. A commonly used rule of inference is modus ponens. Beth (1964) characterizes this rule in the following manner: "From premises \( U \rightarrow V \) and \( U \), infer the conclusion \( V \)." To paraphrase Beth: From sentences of the form "\( U \rightarrow V \)" and "\( U \)" move to a sentence of the form "\( V \)," regardless both of the nature of "\( U \)" and "\( V \)" and the observation of the phenomenon that "\( V \)" represents. A language in which 1) this type of movement is possible, and 2) all the strings that result from the application of the rule of inference are sentences
in the language, is said to be deductively closed (Tarski, 1956). In order to describe the impact of a rule of inference, consider, again, the notion of truth (see section 3). Suppose, in this regard, that a scientific theory is a language that can contain only true sentences. Given an accepted methodology for judging the truth of a sentence, this methodology can be applied in order to determine its truth-value (i.e., it can be applied in order to determine whether or not the sentence is in the theory). The membership of the scientific theory can thus be determined by repeated applications of a methodology. Alternatively, if the theory (i) accepts modus ponens as a rule of inference, and (ii) contains the sentences "U→V" and "U", then it will be possible to obtain the truth value of any sentence "V" without consulting a methodology. "V" can be considered both true and in the theory. A rule of inference reduces, in effect, the empirical requirements of a scientific theory. It will be possible to know (if the rule is appropriate) whether or not a sentence is true without consulting a methodology. If the truth of a sentence is obtained in this manner, the sentence is often referred to as a prediction (Rudner, 1966; Hempel & Oppenheim, 1948), and the process through which it is obtained, a proof (Margaris, 1967) or explanation (Hempel & Oppenheim, 1948). Some would argue, as do both Rudner and Hempel and Oppenheim, that prediction is a major function of scientific theories.

Again, ordinary languages suffer under this requirement. Not only do they not admit a reasonable theory of meaning, they do not lend themselves to the specification of a rule of inference.
The main advantage of the introduction of a formalized language $L$ consists in the fact that it creates the possibility of giving a precise and exhaustive statement of the rule of inference which can be applied in proofs within the domain of science for which $L$ is meant to provide a means of expression. This statement can be given in terms referring exclusively to the "typographical" (formal or syntactic) structure of the expressions (formulas, sentences) which appear as premises or as conclusions in an inference, and hence without any reference to their meaning or their truth. (Beth, 1964, p. 71)

One might, however, consider salvaging ordinary languages by restricting the types of inferences that are allowable. Unfortunately, as we shall see in the following section, this will not be sufficient to avoid a negative result.

6.3 Consistency

Given the requirement that a scientific theory have a specifiable rule of inference, there should be some assurance that the application of this rule will not lead to impossible or counter-intuitive results. It is for this reason that the results of movements justified by a rule of inference are required to satisfy a certain condition.

Recall, from the previous section, the suggestion that the sentences of a scientific theory should be true. Now, it would certainly be inadvisable to have the application of a rule of inference result in the simultaneous predications of true and false to a single sentence. This possibility is eliminated by the requirement that a scientific theory be consistent:

$L$ is said to be consistent, if and only if, there is no formula $U$, such that both $U$ and its negation $\overline{U}$ are provable in $L$. (Beth, 1959, p. 71)

This condition prohibits situations in which both a sentence and its
negation are true; it prohibits a sentence from being both true and false.

Unfortunately, ordinary languages do not meet this requirement. They are sufficiently complex to generate the semantic paradoxes. Rogers (1964) characterizes a semantic paradox in the following example:

Consider the following sentence:

'The sentence on page 84, line 1, is not true.'

Clearly, this sentence is true if and only if the sentence on page 84, line 1, is not true. But the sentence on page 84, line 1, is just the sentence itself. Thus we conclude that the sentence is true if and only if it is not true; that is, that it is both true and false. But, this is a contradiction. Yet it seems that every step in the above argument is an admissible step within the logic of the ordinary English language. Until the language is more precisely specified than it has been heretofore, the conclusion seems inescapable that anyone who wishes to develop semantical theory within it is committed to this type of a contradiction. (p. 84)

Rogers has shown, with this example, that it is impossible to predicate "true" in an unambiguous manner if the domain of the predications is the English sentences. Tarski (1956) shows, with similar examples, that such paradoxes can be constructed in any language if:

(i) the language contains sentences of the form "P is true if and only if x", where 'P' is a sentence in the language, and where 'x' is the name of that sentence (i.e., "John Doe is dead" is true if and only if John Doe is dead), and if

(ii) in the language the ordinary laws of logic hold.

Thus, if a scientist is unwilling to reject the use of a rule of inference, he must reject the use of ordinary languages for the expression of scientific theories. These languages, because of their greater "means of expression," will not satisfy the condition of consistency.

It was suggested earlier (section 4) that a satisfactory defini-
tion of truth requires two languages. The paradox gives us an indication why this is necessary. First, according to Rogers (1964), one must:

...make a distinction between an object-language and a meta-language. The language being formalized, and thus being talked about, is called the object-language; the language within which we formalize this language, that is, the language we use for this purpose, is called the meta-language. Let us call the object-language "L" and the meta-language "M." In order to formalize L, within M we must first lay down rules which determine the syntax of L; and then rules which determine the semantics of L, and thus provide an interpretation of L. As the first of our syntactical rules, we give a full specification of the primitive signs of L, that is, its alphabet and primitive signs, ... Then a formation rule is introduced for the purpose of defining which combinations of these primitive signs of L are accepted as well-formed formulas of L. (p. 84)

Once this distinction has been made, it is easy to understand how Rogers was able to construct the paradox. Because the sentence in question contained the term "true" it must have been a sentence in a meta-language (see section 4). And, it must be considered to be predicing truth to an object-language sentence. Yet, there is no object language sentence on page 84. Thus, the meta-language sentence "is irrelevant but not contradictory" (Thorson, 1972, p. 43).

6.4 A Scientific Theory

It is now possible to describe, precisely, what kind of a language a scientific theory must be if it is to satisfy all the conditions described in this chapter. First of all, it must be a language that is built both from a specified vocabulary and on the basis of a known grammar. Secondly, it must be a language that contains a finite set of initial sentences (the axioms). These sentences will be used
to generate additional sentences (theorems). Thirdly, it must have a
specified rule of inference; movement from one sentence to another must
be unambiguous and unrelated to semantics. Thus, forthly, the lan-
guage must be closed under deduction. Lastly, a scientific theory must
be interpretable by a specified empirical relational structure such
that this structure renders the sentences of the language "true." The
theory must, however, be "detachable" from this structure. A language
that has these properties can satisfy the conditions that, here, it has
been suggested a scientific theory should satisfy. Unfortunately, or-
dinary languages do not have these properties.

7. PRELIMINARIES FOR A THEORY OF CHOICE: SET THEORY

Set theory will provide a foundation for the language in which
individual choice will be described. A brief introduction to set
theory will be given here. The interested reader is referred to
Debreu (1959) or to Lipschutz (1966).

Definition 2.5.1: Let a set be any well-defined
collection of objects

Sets will be denoted by upper-case letters (i.e., A_1, A_2, A_3, ... A_n,
B_1, B_2, B_3, ... B_n, etc.). Objects will be denoted by lower case letters
(a_1, a_2, a_3, ... a_n, ... b_1, b_2, ... b_n, etc.). The empty set and univer-
sal set will both have fixed contents and will be denoted by the sym-
bols {empty} and U, respectively. The empty set is the set that contains
no objects and the universal set is the set from which all other sets
are derived. A specification of the objects in a universal set should,
for any investigation, be possible. The content of this set establishes
the boundaries of an investigation.
One of two methods will be used to specify the content of a set.

Examples of these methods are as follows:

(i) \( A = \{0,1,2,3,4\} \)

(ii) \( A = \{x \mid x \in \mathbb{N} \land 0 < x < 5\} \)

The first sentence reads: The set \( A \) is the collection of objects \( 1,2,3,4 \). The latter sentence reads: The set \( A \) is the collection of objects \( "x" \) such that \( "x" \) is any integer \( (I) \) greater than 0 and less than 5. Each of these sentences conveys the same information. Membership in a set will be denoted by the symbol "\( \in \)". The sentence "\( a \in A \)" reads: \( a \) is a member of the set \( A \). The negation of the sentence "\( a \in A \)" is written "\( a \notin A \)" and is read: \( a \) is not a member of \( A \).

Now, will introduce a few additional members of the set theory vocabulary.

**Definition 2.5.2:** If, for any sets \( A \) and \( B \), \( x \in A \Rightarrow x \in B \), then \( A \) is said to be a subset of \( B \) (written \( A \subseteq B \)). If there exists an \( x \in B \) such that \( x \notin A \), then \( A \) is said to be a proper subset of \( B \) (written \( A \subset B \)).

**Definition 2.5.3:** If \( C = \{x \mid x \in A \land x \in B\} \), then \( C \) is said to be the union of the sets \( A \) and \( B \) (written \( C = A \cup B \)).

**Definition 2.5.4:** If \( C = \{x \mid x \in A \lor x \in B\} \), then \( C \) is said to be the intersection of the sets \( A \) and \( B \) (written \( C = A \cap B \)).

**Definition 2.5.5:** If \( C = \{x \mid x \in A \land \neg \neg x \in B\} \), then \( C \) is said to be the complement of \( A \) (written \( A' \)).

**Definition 2.5.6:** If \( C = \{(a_1, a_2, \ldots, a_n) \mid a_1 \in A_1, a_2 \in A_2, \ldots, a_n \in A_n\} \), then \( C \) is said to be the set of ordered \( n \)-tuples of \( A_1, A_2, \ldots, A_n \) (written \( C = A_1 \times A_2 \times \ldots \times A_n \)).
Definition 2.5.7: If $C$ is a subset of $A_1 \times A_2 \ldots \times A_n$, then $C$ is an $n$-ary relation on $\bar{A_1} \times \bar{A_2} \ldots \times \bar{A_n}$ (written $C \subseteq \bar{A_1} \times \bar{A_2} \ldots \times \bar{A_n}$).

Definition 2.5.8: If $C \subseteq A \times B$, and if, for all $a \in A$ and $b_1, b_2 \in B$, $(a, b_1) \in C \wedge (a, b_2) \in C \Rightarrow b_1 = b_2$, then $C$ is a function from $A$ to $B$ (written $C : A \rightarrow B$).

The notion of a relation (esp. a binary relation) is central to the discussion of choice. For this reason, I will focus on and define some common binary relations. These definitions are from Fishburn (1973, chapter 7). First, some simple definitions. A binary relation $R$ on $X \times X$ is:

Definition 2.5.9: reflexive if and only if $(x, x) \in R$ for all $x \in X$.

Definition 2.5.10: irreflexive if and only if $(x, x) \notin R$ for all $x \in X$.

Definition 2.5.11: symmetric if and only if $(x, y) \in R \Rightarrow (y, x) \in R$ for all $x, y \in X$.

Definition 2.5.12: asymmetric if and only if $(x, y) \in R \Rightarrow (y, x) \notin R$ for all $x, y \in X$.

Definition 2.5.13: connected if and only if $(x, y) \in R \vee (y, x) \in R$ for all $x, y \in X$.

Definition 2.5.14: weakly connected if and only if $x \neq y \Rightarrow (x, y) \in R \vee (y, x) \in R$ for all $x, y \in X$.

Definition 2.5.15: transitive if and only if $(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R$ for all $x, y, z \in X$.

Definition 2.5.16: negatively transitive if and only if $(x, y) \notin R$ and $(y, z) \notin R \Rightarrow (x, z) \notin R$ for all $x, y, z \in X$.

Some of the more complicated but interesting binary relations include the following. A binary relation $R$ on $X \times X$ is a:

Definition 2.5.17: suborder if and only if there exists no sequence $(a_1, a_2), (a_2, a_3), \ldots, (a_{n-1}, a_n) \in R$ such that $(a_n, a_1) \in R$. 

Definition 2.5.18: total order if and only if $R$ is a suborder and $x \neq y \Rightarrow (x, y) \in R \vee (y, x) \in R$ for all $x, y \in X$.
Definition 2.5.18: **strict partial order** if and only if \( R \) is irreflexive and transitive

Definition 2.5.19: **weak order** if and only if \( R \) is asymmetric and negatively transitive

Definition 2.5.20: **linear order** if and only if \( R \) is irreflexive, transitive and connected

The axioms for the theory of sets include (Margaris, 1987):

\[
\begin{align*}
A_1: \quad (A \cup B) \cup C & = A \cup (B \cup C) \\
A_2: \quad (A \cap B) \cap C & = A \cap (B \cap C) \\
A_3: \quad A \cup B & = B \cup A \\
A_4: \quad A \cap B & = B \cap A \\
A_5: \quad A \cup (B \cap C) & = (A \cup B) \cap (A \cup C) \\
A_6: \quad A \cap (B \cup C) & = (A \cap B) \cup (A \cap C) \\
A_7: \quad A = \emptyset \cup A \\
A_8: \quad A \cup U = A \\
A_9: \quad A \cup A' = U \\
A_{10}: \quad A \cap A' = \emptyset
\end{align*}
\]

These axioms will be used in the presentation of the theory of individual choice. Additional axioms will be given when the theory is developed and discussed.
1. I will not define a scientific theory as a language that meets certain requirements. Rather, I will posit certain reasonable demands that a scientist might impose on a language and show that, as a result of these demands, a scientific theory must be a language that meets certain conditions. One could always disagree as to the appropriateness of the demands I posit.

2. Hopcroft and Ullman (1969, p. 10) refer to these rules as productions. Accordingly, they consider grammars to be sets containing variables, terminals, productives, and start symbols. These are generative grammars that cannot easily be thought to apply to ordinary languages.

3. See Pfangagli (1968, p. 18) for a discussion of the order of a relation.

4. I am not concerned, here, with the application of a methodology for determining whether or not one individual is more participative than another. I am only interested in a relational structure that is often considered to have political relevance. \langle B;MP \rangle is, in this regard, a good example.

5. Again, it is difficult to think of an ordinary language as an uninterpreted language. This is not true of artificial languages. Yet, the desirability of being able to make the distinction will, as shall be shown, be considerable.

6. These difficulties are described in Tarski (1956, p. 165).

7. A sequence \( \xi \) of a set of objects \( O \) is a member of the set \( O^* = O(1) \times O(2) \cdots \).

8. A language \( (L) \) is deductively closed if "\( U \Rightarrow V \) \( \in \) L and "\( U \) \( \in \) L implies "\( V \) \( \in \) L."
III. A THEORY OF INDIVIDUAL CHOICE

This chapter begins with discussions of choice (section 1) and rationality (section 2). These discussions are followed by the presentation of a formal theory of individual choice (section 3).

Contemporary investigations of choice are discussed in section 1. This discussion will require the introduction of two notions: utility and preference. Utility is sometimes used as a name for that which governs choice, and is sometimes used as a name for measures of preference. Preference is always used as a name for that which governs choice. Preference, however, is not assumed to have the same properties as utility (when the latter is used for the same purpose). We will see, in this section, that there has been a tendency, in individual choice research, to relax the properties that the perception and measurement of preference have been assumed to satisfy.

"Rationality" is discussed in section 2. This notion has generated considerable confusion in the investigation of choice. In the present context, "rationality" will refer to a relationship between preference and choice. Other applications of the term will not be considered appropriate. Three inappropriate applications will be discussed at length. One associates rationality with the pursuit of a goal (Downs, 1957; Riker & Ordeshook, 1973; Buchanan & Tullock, 1962). One associates it with the classification of individuals (Downs, 1957; Riker, 1962). And, one associates it with the existence of certain preferences (Hansson, 1968; Jamison & Lau, 1973; Herzberger, 1973).
Arguments against the use of each of these applications will be considered.

Finally, in section 3, a formal theory of individual choice is presented. Decisions made with regard to the formalization will be based on arguments developed in sections 1 and 2. Where this theory differs from others, the differences and their consequences will be noted.

1. CHOICE

The importance of choice is, for economics, obvious. Individuals choose, and their choices affect the performance of economic systems. Thus, it is not surprising that the contemporary investigation of choice originates in economics. Choice provides a specific foundation for a theory of commodity value (Hicks & Allen, 1934; Armstrong, 1939) and a general foundation for a theory of economic behavior (Von Neumann & Morgenstern, 1947). Nevertheless, the importance of choice is also obvious for political science. Rulers and rules are chosen in every political regime. Although choices may involve different sets of individuals, they are made. Likewise, committees, legislatures, and parliaments choose. And, the choices of these bodies are determined, in various ways, by the choices of their members (see Black, 1958 for a discussion of these points). In addition, the choice experiment (survey) provides a major source of evidence for empirical political science. Thus, choice is both a subject and a means of political research.

Some economists believe that no economic theory can be developed
without an adequate understanding of individual choice (Hicks, 1956, Chapter 1). Others believe that the investigation of "fundamentals" or "foundations" is unnecessary (see, in this regard, Houthakker, 1961, p. 707). To argue, from a political scientist's perspective, for one position or the other is not my intent. Indeed, it is doubtful that either position would be tenable for every political analysis. There are some analyses, however, for which choice is of obvious importance (see, for example, Downs, 1957; Riker & Ordeshook, 1973; Buchanan & Tullock, 1962). In these cases, interest in the notion of individual choice has been greatly stimulated by the work of Kenneth Arrow (1951). According to Arrow's theorem, it is generally impossible to calculate the relative social values of three or more commodities if the calculations are to be based on the relative values that individuals place on these commodities. Given the theorem, a careful investigation of individual choice becomes a very important part of both welfare economics and political theory. What conditions can one reasonably expect a social choice (either economic or political) to satisfy? This question cannot be answered without first considering the manner in which individuals both calculate the value of commodities (again, either economic or political) and choose. Perhaps Arrow's description of this manner is implausible. And, perhaps the predication of "desirable" to various social choices will depend upon the acceptance of a particular description. Both of these issues are central in the investigation of the problems of social choice. However, it is only the former issue that is addressed in this dissertation. What, in this regard, constitutes a "plausible" theory of individual choice?
This section provides a description and an evaluation of various attempts to answer this question. It also serves as an introduction for the presentation of a theory.

1.1 From Utility to Preference

An early and important perspective on the notion of choice is provided by Armstrong (1939):

For the purpose of formulating a theory of value the economist assumes that an individual can at any moment be regarded as confronted with a number of alternatives between which he must choose and that his choice is determinate, i.e., if we knew all the relevant circumstances we could determine which alternative is chosen. (p. 53)

The important question is, of course: How does an individual choose? To answer this question, economists answered a related question: What is the result of choice? Their answer was, initially, a change in utility (see Hicks, 1956, p. 5, or Majumdar, 1958, Chap. 1, for a discussion of this point). Utility is a commodity that was assumed to be considered for the purpose of choice. It was assumed that an individual would choose alternatives with "greater utility" over those with "lesser utility."¹ Disagreements concerning the nature of perception and measurement have persisted since the concept's introduction. I will, here, describe some of the major positions that have been taken with regard to these issues.

Under cardinal theories, utility is the name of that which is assumed to govern choice. And, it is also assumed to be amenable to cardinal measurement (Robertson, 1952). If, under this assumption, \( \phi \) were a measure of an individual's utility, then, for all transforma-
tions $t'$, if $t'(\phi) = x\cdot\phi$ (where $x$ is any positive real number), $t'(\phi)$
and $\phi$ would provide the same measure of the individual's utility. Fur­
thermore, all other transformations on $\phi$ would not be admissible; they
would provide different measures of the individual's utility (see Suppes
& Zinnes, 1963, p. 11, for a further discussion of this point). Ac­
cordingly, if an individual is said to enjoy five utiles with one choice
and ten utiles with another (see Luce & Raiffa, 1957 for a discussion
of the term "utiles), he can be said to enjoy twice as much utility as
a result of making one choice rather than the other.

A distinguishing feature of cardinal theories is their implied
position on perception (Majumdar, 1958). If utility is cardinally
measurable, it must be assumed to be something that is absolutely
perceivable. For, if a measure of a state of utility can be assumed
to satisfy the property described in the previous paragraph, that state
must also (by an individual) be identifiable "without reference to any
other states" (Majumdar, 1958, p. 39). Comparison must be assumed to
be unnecessary. There are a number of criticisms that can be directed
against the cardinal theories. These criticisms have resulted in both
a gradual erosion of the measurement assumption and a basic reconsid­
eration of the notion of utility. A discussion of the criticisms will
be followed by a discussion of the reconsideration.

In the first place, an assumption of absolute perception must
precede an assumption of cardinal measurement. For, if perception is
not absolute, measurement cannot be cardinal. The perception assump­
tion is not, however, sufficient to imply the measurement assumption.
A simple example will illustrate this point. Celcius and Fahrenheit
thermometers both measure temperature. And, although temperature is
assumed to be something that is absolute, neither the Celsius nor the
Fahrenheit thermometers provides a cardinal measure of temperature.
There is no reason to suppose that measures of utility are not subject
to this same situation. Although it may be reasonable to assume that
utility is something that is absolutely perceivable, it may not be
reasonable to assume that it is cardinally measurable. It is therefore
incumbent upon the cardinalist to argue for, in addition to his
position on perception, its implied position on measurement. This
argument must, furthermore, be made independently of the perception
argument.

Perhaps these arguments are constructable in situations where the
objects of choice are both quantifiable and divisible (i.e., money).
But, if these are the only situations that can be described by cardinal
theories, the theories would eliminate, for a political scientist, the
description of many (if not all) interesting choice situations.

In the second place, the notion of cardinal measurement, if rea-
sonable, is of little use for either an economist or a political sci-
entist. The economist is concerned with choices given limited re-
sources, and the political scientist is concerned with choices over
finite, mutually exclusive, and indivisible policies.

There is no particular sense in saying, for exam-
ple, that an individual has attained some (specifiable
or unspecifiable degree of) satisfaction in a situ-
ation unless we have in mind some other situation in
which he would obtain more (or less) satisfaction.
(Majumdar, 1958, p. 22)

Thus, even if the assumption of absolute perception were reasonable,
the constraints of economic and political situations dictate the use of comparative measures.

The most convincing criticisms of cardinal theories have been directed against their excessive restrictions (Luce & Raiffa, 1957, p. 16). Whereas they must assume that individual choice is based on the absolute perception of, in this case, utility, they cannot provide any more information with regard to individual choice than can theories based on the less restrictive assumption of comparative perception. It is for this reason, as Hicks and Allen (1934) point out, that the notion of utility should be replaced with that of preference. Preference refers, here, to an ordinal scale that is assumed to govern individual choice. Theories based on preference (rather than utility) do, however, adopt quite different positions on the issues of perception and measurement. They assume, with regard to the issue of perception, that calculations are comparative. They also assume, with regard to the issue of measurement, that preference is only amenable to ordinal measurement. Thus, if \( \phi \) were a measure of an individual's preferences, then, for all transformations \( t' \), if \( t' \) satisfies:

\[ \phi(x) > \phi(y) \rightarrow t'((\phi(x)) > t'((\phi(y))) \]  (where \( x \) and \( y \) are alternatives), \( t'(\phi) \) and \( \phi \) would provide the same measure of the individual's preferences (see Suppes & Zinnes, 1963, p. 12, in this regard). The properties that are assumed to be satisfied by a measure of preferences are considerably weaker than those that are assumed to be satisfied by a measure of utility. Measures that would differ under cardinal theories would be the same under ordinal theories.
In a somewhat different treatment of these issues, it is only preference that is assumed to govern individual choice. Utility, in this case, is a name that refers only to measures of preference. It never refers to a stronger version of the preference assumption (see, for example, Luce & Raiffa, 1957, Chap. 2). Accordingly, cardinal theorists would be said to be concerned, not only with the existence of preference, but with the magnitude of preference. Their measures, because they would have to account for magnitude, would be assumed to have cardinal properties. Ordinal theorists, on the other hand, would not be concerned with either the perception or the measurement of magnitude. Their measures would not, therefore, be assumed to have cardinal properties. This manner of treatment is, perhaps, more illustrative than the former. It makes a clear distinction between a measure (utility) and that which is measured (preference). Furthermore, it suggests that a determination as to the properties that a measure can be assumed to exhibit must be based on a specification of the properties that, in this case, preference can be assumed to exhibit. And, as we shall see, the relaxation from a cardinal to an ordinal assumption of perception is by no means the final step in the development of a "plausible" theory of individual choice. Thus, utility will, in the remainder of this discussion, refer to measures of preference.

1.2 Ordinal Theories: From Strong to Weak Indifference

The gradual relaxation of preference (see Houthakker, 1961, or Herzberger, 1973) begins with an ordinal theory due to Hicks and Allen (Hicks & Allen, 1934). They reject, in this theory, the assumption that preference is absolutely perceivable and the assumption that
utility is a cardinal measure. They, as does Hicks (1956), maintain
that "the special properties of a cardinal index are irrelevant to the
econometric theory of consumers' behavior" (p. 9). The foundation of
their theory is an assumption that individuals can, given two alterna­
tives, compare the alternatives and determine whether or not they
either prefer one to the other or are indifferent between the two.
They are not concerned with the magnitude of preference, but with the
existence of preference. Both the perception assumption and the mea­
surement assumption have, with respect to the cardinal theorists, been
relaxed.

The substance of the Hicks-Allen theory is provided through three
additional assumptions. First, preference is assumed to be a transi­
tive relation (Hicks, 1956, p. 24). Second, indifference is assumed
to be a transitive relation (Hicks, 1956, p. 25). And third, for any
pair of alternatives, an individual is assumed to either prefer one
to the other or be indifferent between the two (Hicks, 1956, p. 25).
These assumptions are sufficient for what is commonly labeled "indif­
ference curve analysis" (Majumdar, 1958; Hicks, 1956). In this type
of analysis, the relations of preference and indifference are used for
placing alternatives in "indifference sets" that are ordered by a
preference relation. An individual's preferences are, with indiffer­
ence curve analysis, represented spatially. Figure 1 provides an exam­
ple of a spatial representation. An "x," in this figure, represents an
alternative. Figure 1 indicates that, for those alternatives that are

labeled, E is preferred to A, B, C, and D. C is preferred to A, B, and
Fig. 1: A spatial representation of preference using indifference curves.
D. And, B is preferred to A and D. If two alternatives (e.g., A and D) are on the same curve, the individual is indifferent between them and will be willing to substitute the one alternative for the other. Further assumptions concerning the relationship between different individual's indifference curves are frequently made in research on social choice or measurement (see Wendell & Thorson, 1974; Plott, 1967; Hinich, Ledyard, Ordeshook, 1964 and Coombs, 1964, in this regard).

I am not, at this point, concerned with these assumptions. I am only concerned with the construction of a theory that is capable of describing individual choice.

The substitutability of indifferent alternatives illustrates a very important and consequential point. Indifference is, under the Hicks-Allen theory, based on compensation (Majumdar, 1958). If an individual is indifferent between two alternatives, he must be willing to substitute the one for the other. Two arguments have been raised against this assumption of "strong" indifference. The first is based on an individual's limited capability for discrimination (Armstrong, 1939) and the second, on the representation's implied assumption concerning the nature of offered alternatives (Aumann, 1962).

Indifference, according to Armstrong, should not be based on compensation. It should be based on approximation. Indifference does not necessarily indicate an absence of preference. For, if an individual cannot perceive a difference between alternatives, the alternatives will be considered indifferent. Thus, it is only the lack of a perception, not the lack of a preference, that can be inferred from an observation of indifference. An individual would, for example, indi-
cate indifference between two cups of coffee if one of the cups contained one grain of sugar, and the other, two. And, an individual would also be indifferent between two cups with two and three grains, or between two cups with three and four grains, and so on. Yet, an individual would presumably not be indifferent between two cups with one and 250 grains. Armstrong would argue that an individual would actually have a preference for one of two cups if one cup has $n$ grains and the other $n+1$. The preference is merely not perceivable. Similarly, an individual might be indifferent between a Chevrolet for $3500 and a Ford for $3720, or between the same Chevrolet and a Ford for five dollars more. Yet, the individual would presumably not be indifferent between a Ford for $3720 and the same Ford for five dollars more. In both cases, an individual's limited perception would prohibit the recognition of an actual preference. The major consequence of the approximation position is the forfeiture of the assumption of transitive indifference (as indicated in both examples).

I will, in the following section, consider a counter argument to the Armstrong position. I will argue that a theory of individual choice should be based, not on posited preference, but on perceived preference. This, because we are interested, with the investigation of individual choice, in the consequence of preference. And, an individual would presumably choose in the same manner if he were either indifferent between two alternatives or had a non-perceivable preference for one or the other.

The second argument against the compensation or "strong" notion of indifference is based on the notion's implied assumption concerning
the comparability of alternatives. Consider, in this regard, an example by Majumdar (1958):

Let us suppose that an individual is faced with two alternative welfare situations (however defined), X and Y. X may be as commonplace as the consumption of a cup of coffee, and Y the purchase of a copy of the Times, ... It would perhaps be quite permissible to assume that the individual finds his welfare to be greater or less according as he gets more or less of X, other things remaining the same (provided of course that X constitutes the source of satisfaction). Similarly, it may be assumed that he finds his welfare position improves or deteriorates according as he has more or less of Y. ... But that, unfortunately, is as far as we can go. We cannot, for example, take it for granted from the above that the individual could or would state a relation of preference (or indifference) between X and Y. It is conceivable that the two kinds of enjoyment were just not comparable. (p. 6)

With this example, we might be willing to assert that utility, as a result of certain choices, "neither increases nor decreases nor remains the same" (Majumdar, 1958, p. 7). And, if we are willing to say this, we cannot accept a theory in which indifference is assumed to be based on compensation. A closely related argument concerns the multiplicity of criteria on which preference is based. Politically interesting investigations of choice will frequently involve alternatives that do not comprise (either directly or indirectly) different amounts of a single commodity (i.e., money). It may be the case that preferences over these alternatives are determined on the basis of multiple criteria. Suppose, by way of example, that preferences are based on two criteria, X and Y. If an individual always prefers more to less of both X and Y, then an alternative providing seven units of X and seven units of Y would be preferred to an alternative providing five units of X and five units of Y. But, would an individual prefer the seven X and seven Y
alternative to an alternative providing nine units of X and five units of Y? He may well be indifferent. He may also be indifferent between the seven X - seven Y alternative and an eight X - five Y alternative. Yet, the individual would not be indifferent between the nine X - five Y alternative and the eight X - five Y alternative. Multiple criteria provide a very reasonable argument against the "strong" (compensation or transitive) assumption of indifference. In the case of alternative policies, two reasonable political criteria might be the level of involvement and the time for implementation.

1.3 The Meaning of Preference

What we have seen, with regard to the assumption of preference, is a series of moves. These moves have taken us from a cardinal assumption to a transitive indifference assumption and from a transitive indifference assumption to an intransitive indifference assumption. The tendency is clearly toward a relaxation of the properties that an individual's preferences are assumed to exhibit. This relaxation has not ended with the representation of indifference. In a well-known experiment by May (1953), it was determined that preferences, if based on multiple criteria, cannot be assumed to be transitive. The experiment involved the hypothetical choice of a wife. The subjects were asked to choose on the basis of wealth, beauty, and intelligence. They could not always order the alternatives in a transitive manner. With this result, the fundamental assumption of transitive preference becomes tenuous. Additional arguments against transitive preference can be found in Coombs (1953) and Tversky (1969). These arguments raise
some rather serious questions as to the relationship between the properties that preferences can be assumed to exhibit and the possibility of using information about choices to obtain information about preferences (i.e., can the choice of one alternative over another be assumed to indicate a preference of that alternative over the other?). They also raise questions as to the degree to which the investigation of individual choice should be based on the assumption of certain preferences. I will consider and attempt to answer these questions in the following two sections.

2. RATIONALITY

It is very easy, in the context of individual choice, to either overestimate or miscalculate the importance of the term "rationality." Because either mistake can be consequential, it is necessary that the choice theorists' uses of the term be investigated. The focus will be primarily economic. However, uses that are either due to political scientists or motivated by the investigation of political phenomena will also be considered.

"Rationality" will only be considered in a stipulative manner. I hope to indicate how the term both is and can best be used in investigations of individual choice. These indications neither do nor should have any bearing on uses that appear in other contexts. I am only interested in uses that are appropriate in the context of individual choice investigations.
2.1 Perceived vs. Goal Directed Preference

It is obvious, from the preceding sections, that investigations of choice have been closely associated with the notion of preference. Preference is assumed to provide the motivation for choice. A reasonable question to ask, given this assumption is: How are preference calculations made? Or, can preference calculations be assumed to be "efficient" or in the "best interests" of the individual involved? The way in which these questions are answered will be consequential in the development of a theory of individual choice. I assume, in response to the first question, that the individual makes the calculations. The individual is assumed to be the only definitive judge of his preference. I assume, in response to the second question, that preference calculations cannot be judged according to whether or not they are efficient or in the best interests of the individual. I intend, here, to give reasons for these answers.

I am not, either in the investigation of individual choice or in the use of the term "rationality," interested in a "reified" or "objective" notion of preference. I am interested in that which occurs as a consequence of an individual's preference calculations (Luce & Suppes, 1965, also advocate this approach). It is for this reason that this investigation of choice will be based on an individual's perception of preference; this perception can stand independently of any "best interest" or "efficiency" estimates to which they might be subjected (see Luce & Raiffa, 1957 for a further discussion of this point). The use of such estimates involves issues that attend the construction, not the consequence, of preference. And, as we shall
see, investigations concerning the consequence of preferences can be carried out independently of investigations concerning the construction of preferences. Identical preferences will be assumed to have identical consequences, regardless of the manner of construction.

Suppose, by way of example, that individuals are given puzzles, each involving numerous series of choices. And, suppose that, for each puzzle, one series leads to a pay-off or solution. It would, in this case, be a wholly inappropriate use of "rational" if the term were withheld or attached according to an individual's ability to make the choices that lead to a solution. The phrase "a theory of rational choice" will simply refer to a theory of choice in which preferences are assumed to be given and in which preferences are assumed to determine choice. Issues concerning the formation or appropriateness of preferences will not be addressed. A rational choice theory will only be concerned with the translation of preferences into choices. Thus, I will assume that the relative preference of an alternative for an individual is precisely what the individual perceives the relative preference of the alternative to be (see also Majumdar, 1958). It would be inappropriate, in this context, to posit "appropriate" preferences and judge the rationality of an individual or his choices on the basis of these posited preferences. This is not the intent of a rationality assumption.

If the present intent of the rationality assumption is mistaken, the investigation of individual choice can easily degenerate into a typological investigation in which the question: "Is 'x' rational?" is used for classification. This type of investigation may, at first,
seem appropriate. But, if one reconsiders the purpose of a choice investigation, he must conclude that it is for the classification of alternatives, not individuals. Individual choice theories are, here, developed for the purpose of predicting or explaining choices. There is no reason to suspect that a classification of individuals as "rational" or "irrational" will be of any help whatsoever in this effort. It is for this reason that the individual's perception of preference is the only material that will be used in the forthcoming theory of individual choice. To suggest that a theory of rational choice is a theory about the choices of "rational" individuals (rather than a general theory about preference-based choice) has, if to be rational means to satisfy the conditions of the theory, little if any merit.

Unfortunately, the perceived preference approach is not used in every investigation of individual choice. Riker (1962) claims that the reliance on nothing but an individual's perception "weakens the condition to the point that all choices resulting in action are said to be rational." He goes on to say that the condition must be stated in such a way as to be "more than a tautology" (p. 18). The approach represented by these comments is clearly at odds with the perceived preference approach. Typically, those who adopt this approach (Riker, 1962; Downs, 1957; Buchanan & Tullock, 1962; Riker & Ordeshook, 1973) will posit the existence of a goal and evaluate the rationality of an individual's choices from the perspective of the posited goal. For Downs, the goal is the selection of a government. Choice is only rational if directed toward this end. Although somewhat more flexible
than Downs in their treatment, Riker and Ordeshook also consider goals to be a very important part of rationality. According to them, a researcher can posit a goal and obtain, from it, an appropriate set of (transitive) preferences. Or, a researcher can posit a set of (transitive) preferences and obtain, from it, a goal. In both cases, the relationship between rationality and goal is clearly specified. For Buchanan and Tullock, rational choice also requires the specification of a goal:

Rational action requires the acceptance of some end and also the ability to choose the alternatives which will lead toward goal achievement. (p. 36)

The question is: Are the perceived preference and the goal directed preference approaches significantly different? And, if so, how can the difference be reconciled?

Consider, in order to answer the above questions, another: Where (in individual choice theories) can the term "rational" be applied? Obviously, there is no "right" or "wrong" application. But, there may be useful (and other) applications. So, what are the various possibilities for application that attend the goal directed approach? First of all, suppose that an individual does not have an appropriate goal and chooses in accord with his (inappropriate) preferences. The individual would be classified as "irrational." Secondly, suppose an individual has an appropriate goal and does not choose in accord with his (appropriate) preferences. Again, the individual would be classified as "irrational." Thus, an individual could be classified as irrational, not only because of a misassociation between preference and choice, but also because of a misassociation between externally defined appropriate preferences (preference...
ences that lead to choices that, in turn, lead to "solutions") and perceived appropriate preferences. It would seem, however, that questions as to what it means for an individual to "understand a situation" (construct appropriate preferences) are quite different from questions as to what it means for an individual to "choose in accord with his understanding of a situation" (choose in accord with his preferences). Unfortunately, these questions are indistinguishable under the goal directed preference approach. They are not, however, indistinguishable under the perceived preference approach. Accordingly, the perceived preference approach will be adopted in this effort to determine what it means for an individual to "choose in accord with his understanding of a situation."

"Rational" in the present context, will be used to refer to a theory about preference-based choices (i.e., "'T' is a rational choice theory."). It will also be used as a label in the classification of alternatives (i.e., "Alternative "a" is a rational choice and alternative "b" is not."). "Rational" will not be used as a label for the classification of individuals. If applied to individuals, the term is applied inappropriately. I will assume that all individuals make rational choices in the same sense as physicists assume that all moving bodies have a force equal to the product of their mass and acceleration. The important task, of course, is the development of a theory that will describe individual choice. I will ask, as do Luce and Suppes (1964), "whether a theory seems to describe behavior, not whether it characterizes a rational man" (p. 253).
2.2 Correspondence

It was determined, in the previous section, that it is unnecessary to assume that individuals make choices that are consistent with maximum utility. Rather, it is only necessary to assume that individuals make choices that are consistent with their perceptions of maximum utility. It must be assumed, in this regard, that there is a correspondence between preference and choice. The assumption of a correspondence is, in large part, the substance of a rationality assumption. We must assume that individuals can, in some sense, bring alternatives under a category of "greater or less" (Robertson, 1952) and that choices will reflect the categorization. Hicks (1956) refers to this assumption as the "preference hypothesis" (p. 16). He uses this hypothesis in order to answer the question as to how the consumer behaves:

In order to get any answer to this question, we have to make assumption about the principles governing his behavior. The assumption of behavior according to a scale of preference comes in here as the simplest hypothesis, not necessarily the only hypothesis, but the one which, initially at least, seems to be the most sensible hypothesis to try. (p. 17)

Luce and Raiffa (1957) refer to this assumption as the "postulate of rational behavior." They postulate that:

of two alternatives which give rise to outcomes, a player will choose the one which yields the more preferred outcome, or, more precisely, in terms of the utility function he will attempt to maximize expected utility. (p. 55).

Luce and Raiffa then suggest that this postulate only describes the word "preference" and warn against attempts to identify utility with "some objective measure" which will certainly lead to the conclusion that "people are not generally rational" (p. 55). This type of conclu-
sion, they claim, is "irrelevant." Their position is obviously very similar to the position taken in the previous section.

Progress has certainly been made (in a specification of the meaning of "rational") through an identification of rationality with a correspondence between preference and choice. I have yet, however, to consider the exact specification of the correspondence. This specification is usually given in the form of a rational choice function (Uzawa, 1959) or a choice function (Arrow, 1959; Wilson, 1970; Fishburn, 1973, 1974; Jamison & Lau, 1973). Justification for the adoption of one or another of these functions is usually based (in part) on the assumption that preferences satisfy certain properties. Methods for evaluating these functions will be given in sections 2.4 and 3. It is in section 3 that the theory of individual choice is formalized. It would be difficult, without a knowledge of the material contained in section 3, to attempt either a description or an evaluation of the various choice functions. At this point, it is sufficient to note that "rationality" is a term that denotes preference-based choice. Other, non-preference-based, choice situations will also be introduced and discussed in section 3.

2.3 Falsifiability

I have assumed, throughout this discussion, that preferences are unobservable. Only the consequences of preferences will, through situations of choice, be assumed to be observable. Given this assumption and the assumption that an individual's preferences need be no more than that which the individual perceives them to be, there may be some
doubt as to the usefulness of such a preference-based choice theory. I have, in fact, considered one of the forms that this doubt has taken (Riker, 1962). Clearly, if every possible choice can be "rationalized" by a theory of individual choice, the theory will provide little or no information about that choice. We would know as much about individual choice (in terms of being able to predict it) without the theory as we would with it. Thus, I will require that a theory of choice not be capable of rationalizing every possible choice. This condition is consistent with Majumdar (1958), Richter (1971), and Hicks (1956).

The requirement of falsifiability is considered at this point, not only because of its importance in the construction of a theory of choice, but also because it is easily misunderstood. A requirement that all choices not be capable of being rationalized is very different from a requirement that not all individuals be determined to be "rational." This, however, is a frequent extension of the falsifiability condition (Downs, 1957; Buchanan & Tullock, 1962). Because it is reasonable to require that a rational choice theory be general in the sense of being descriptive of the behavior of all individuals, it is not reasonable to allow this extension. Thus, as was indicated earlier, to apply the term "rational" to individuals is inappropriate. This does not, however, have any bearing at all on the requirement of falsifiability. There should, in all situations of choice, exist an alternative such that the choice of that alternative is not "rationalizable."

2.4 Further Modifications in the Meaning of "Rational"

I have argued, to this point, that "rationality" refers to a
relationship between preference and choice. I have also argued that goal directed restrictions cannot be appropriately applied in the investigation of choice. Unfortunately, such restrictions have frequently found their way into politically interesting analyses of choice (i.e., Downs, 1957; Riker & Ordeshook, 1973; Buchanan & Tullock, 1962). These restrictions have not served to clarify issues attending the construction of a theory that will accept choice as a model. Other investigations of choice have not, however, been immune from obfuscating restrictions.

Economic investigations of choice are frequently based, not on preferences, but on certain preferences. These certain preferences are often referred to as rational preferences (Jamison & Lau, 1973; Herzberger, 1973). The various properties that rational preferences are assumed to satisfy correspond to the properties discussed in the previous section on choice (e.g., transitivity of preference, transitivity of indifference, connectedness, etc.). I do not intend to reconsider these properties at this time. I will, however, suggest that the notion of "rational" preference is misleading. It suggests, not only that individuals must display a certain relationship between preference and choice, but that they must also exhibit certain preferences. If they do not do so, they are not considered "rational." As Riker and Ordeshook (1973) put it: "These are (transitivity of preference, transitivity of indifference, and connectedness) the ideals for behavior which about everybody accepts in the abstract." I must, however, disagree. This type of argument leads to the same problem as does the predication of a goal directed preference. The question as to whether
or not an individual's preferences for alternatives will satisfy certain properties is an empirical question. Thus, it is on empirical grounds that these "rational" preference properties must be evaluated. Whether or not everybody agrees that the properties are desirable is quite another issue. If an individual's preferences do not satisfy "rational" properties, then the assumption (not the preferences) is unwarranted. I will consider, in the presentation of the formal theory, the motivations for (section 3.3), and the plausibility of (section 3.1), various "rational preference" properties. But, in no case will these properties be considered fundamental to a theory of individual choice.

3. A THEORY OF INDIVIDUAL CHOICE

With the background that has been provided by the discussions of choice, preference, and rationality, I will develop a theory of individual choice. In section 3.1 I will define and discuss the elements of a choice situation. I will distinguish, in section 3.2, various types of choice situations. Only certain situations will be assumed to be preference based. In section 3.3 I will introduce the notion of rational choice. Rational choice will be described through a property (a rational choice property) that choice functions will be required to exhibit. In section 3.4 I will evaluate some common rational choice properties. And, in the final section I will consider the motivation for some of the properties that have not been labeled appropriate.
3.1 The Elements of a Choice Situation

I will, in this section, define and discuss some of the elements of the theory of choice. Specifically, I will define and discuss those elements that represent elements of a choice situation.

Definition 3.1.1: let an agent set be some arbitrary set \( I \)

I will represent a set of agents. These agents may, for example, be legislators, committee members, or voters. The agents may, on occasion, be faced with a situation of choice (Armstrong, 1939).

Definition 3.1.2: let a choice space be some arbitrary set \( X \) together with a family \( \mathcal{X} \) of finite subsets of \( X \)

\( X \) will represent a set of alternatives. These alternatives may appear in a set presented to an agent for choice. \( \mathcal{X} \) will represent the family of such sets. The elements of \( \mathcal{X} \) \( (Y_1, Y_2, Y_3, \ldots, Y_n) \) are referred to as presentation sets. Alternatives may, for example, be competing policies, candidates, survey item response categories, etc. Presentation sets may, for example, be sets of offered policies (agendas), sets of offered candidates (elections), etc.

Because the alternatives in a situation of choice are represented by elements of \( \mathcal{X} \), \( \mathcal{X} \) will be required to satisfy the following condition:

CF1: \( \mathcal{X} \) must contain all non-empty subset of \( X \)

To suggest that \( \mathcal{X} \) need not contain all non-empty subsets of \( X \) is to suggest that one combination of alternatives can be a presentation set and another, not. However, the notions of an alternative and a presentation set have not been given sufficient structure to warrant this type
of classification. Each is, by design, too general. It would be inappropriate to allow a theory of choice to prohibit any alternative from being in a presentation set with any other alternative. For, if this is allowed, there can be no assurance that a theory is capable of describing all choices, regardless of presentation set content. Moreover, without CFL, it would not be possible to require that a theory of individual choice be considered "plausible" by virtue of its description of observed choices. It might, in the absence of CFL, be considered "plausible" by virtue of an arbitrarily restricted domain.\(^3\)

CFL is, in the construction of some theories of individual choice (Arrow, 1959; Fishburn, 1974; Herzberger, 1973), a requirement. In others (Richter, 1971; Wilson, 1970), it is not. I, however, believe it is necessary in order to safeguard a theory of rational choice from a "gerrymandered satisfaction of its requirements" (Herzberger, 1973, p. 191). This can only be accomplished if, according to Herzberger, the theory is formulated "in a framework of fully extended choice functions." And, by Herzberger's definition, a choice function is extended if and only if its domain includes all finite subsets of \(X\).

Definition 3.1.3: let a preference space be some arbitrary set \(G\) together with a family \(\mathcal{H}\) of subsets of \(G\)

\(G\) will represent preferences, and \(\mathcal{H}\), combinations of preferences, or preference sets. These sets \((P_1, P_2, P_3 \ldots P_n)\) will, under certain conditions, determine choices. The notion of preference was the subject of considerable discussion in sections 1 and 2. Consider, for the purpose of justifying the present notion of preference, the conditions that will be required of \(G\) and \(\mathcal{H}\).
CF2: \( G \) must be the cross-product of \( X \times X \) (\( C = X \times X \))

CF3: \( \mathcal{B} \) must contain all asymmetric subsets of \( G \)
\[ (\mathcal{B} = \{ P_1 \mid P_1 \subseteq G \wedge ((x,y) \in P_1 \rightarrow (y,x) \notin P_1) \}) \]

CF2 and CF3 each do two things. CF2 carries an assumption that binary relations provide an appropriate representation for the notion of preference. It then insures the availability of all such relations. CF3 carries an assumption that asymmetric sets of binary relations provide an appropriate representation for the notion of a preference set. It then insures the availability of all such sets. Justification for CF2 and CF3 must involve appeals to both intuition and to the history of the investigation of choice.

Despite a lack of consensus as to the properties that preference is assumed to satisfy, there are certain areas in which agreement exists. First of all, the domain of preference is always assumed to be alternatives. Individuals prefer certain alternatives over others. Preference describes this relation. Secondly, a preference is not assumed to be affected by changes in a choice situation. An individual's preference for one policy or candidate over another is assumed to remain unchanged regardless of the agenda or election in which the candidates or policies might appear. Choices may change as presentation sets change, but preferences will not. Lastly, if an agent is said to prefer one alternative to another then the agent cannot be said to simultaneously prefer the other to the one. Aside from these three conditions, there may be little else that choice theorists would agree to associate with the notion of preference.

CF2 associates preferences with alternatives. It also protects
preference from being affected by choice situations. If preference had been defined on presentation sets, or if preference had been considered to be a higher order relation, the protection could not have been given. CF3 prohibits an individual from simultaneously preferring one alternative to another and the other to the one. Such preferences would not satisfy the asymmetry condition.

The requirement that all preferences and all asymmetric combinations of preferences be available will insure that a theory of choice not be restricted by the content of an agent's preference set. If an individual were asked to choose, from a set of policies, those he liked the most, would not the choice be, by construction, preference based? I believe that it would, regardless of the nature of the individual's preferences. It is for this reason that all preferences and all asymmetric combinations of preferences be allowable. The single asymmetry condition is not, however, in agreement with a vast majority of the individual choice theorists (see Jamison & Lau, 1973, in this regard). But, I feel that there is only one justifiable reason for further restricting the content of a preference set. If, in this regard, it can be argued that agents' preference sets satisfy conditions over and above asymmetry, then an assumption that preference sets do satisfy these conditions is not inconsistent with the condition that a rational choice theory be general (i.e., that it not be restricted to the existence of certain preferences). However, the available evidence is not, in this regard, convincing. Thus, any asymmetric combination of preferences will be considered a possible preference set.

The remaining element of a choice situation is the choice.
Definition 3.1.4: let a choice function be any function from $X \times Y$ into $X$

A choice function will generate sets of equally adequate alternatives (Chernoff, 1954). These sets will be referred to as choice sets. Bear in mind, however, that a choice set need not correspond to the set of alternatives an individual will select when faced with a choice situation. Selection is often relativized by budget or by law. In an election, for example, an individual is typically not allowed to vote for more than one candidate. Rather, if the individual votes, he must select one candidate from a set that may, for him, contain a number of equally adequate candidates. And, with regard to the purchase of automobiles, financial considerations would force most individuals to select one automobile from a set that might contain a number of equally adequate automobiles. Choice, unlike selection, is not assumed to be relativized by such factors. This distinction must be kept in mind, as rational choice theories do not deal with selection. They deal only with choice (see Richter, 1971 or Majumdar, 1958 for a discussion of this point). Unfortunately, limitations are often imposed at situations of choice. This renders the observability of choice sets a difficult problem.

A condition that all choice functions must satisfy is described in $CF_4$. With this condition, the theory's description of a choice situation will be complete.

$CF_4$: If $C$ is a choice function, then for each $(Y_i, P_j) \in X \times Y$, $C(Y_i, P_j)$ must be a non-empty subset of $Y_i$

A choice function must assign, for every presentation and preference set pair, a choice set that is non-empty. If a choice function is not
required to satisfy $\mathcal{C}F^4$, then it will be necessary to accept one of two additional assumptions about the nature of choice. It must either be assumed that certain elements from $\mathcal{X}\times\mathcal{Y}$ will not occur, or that a choosing agent can, on occasion, be incapable of choosing. With regard to the first assumption, neither the presentation set nor the preference set nor the choice function has been given sufficient structure to warrant a classification of elements from $\mathcal{X}\times\mathcal{Y}$. There is no reason to believe that certain elements will occur while others will not. Thus, it must be assumed that any element from $\mathcal{X}\times\mathcal{Y}$ can occur. This leaves the second assumption. And, if a choice theory allows agents to be incapable of choice, the theory would certainly not provide a very reasonable description of individual choice. For, it would not make sense to suggest that an individual cannot, given any set of alternatives, choose. Perhaps an unoffered alternative would, if offered, be preferred to (and chosen over) all offered alternatives. But, an individual would still be able to choose among alternatives that are offered. The worst of all possible situations would be one in which no offered alternative is preferred to any other offered alternative. Yet, choice (or in some cases selection) would still not be impossible.

There is general agreement among choice theorists as to the importance of the requirement that a choice set be non-empty (Arrow, 1959; Fishburn, 1974; Uzawa, 1959; Hicks, 1956; Jamison & Lau, 1973; Richter, 1971; Sen, 1973). As Sen (1973) points out:

This point can be illustrated with a variation of the classic story of Buridan's ass. This ass, as we all know, could not make up its mind between two haystacks; it liked both very much but could not decide which one was better. Being unable to choose this
dithering animal ultimately died of starvation. The dilemma of the ass is easy to understand, but it is possible that it would have been better off by choosing either of the haystacks rather than nothing at all. (p. 243)

This example indicates two things. First, it points out the difference between the existence of preference and the problem of choice. But, it also indicates that selection can be difficult. I am not, however, interested in the difficulty of selection. I am interested in the construction of choice sets given presentation and preference sets.

In summary, there are four axioms that will be included in the theory of choice. These axioms restate the conditions described in CF1 through CF4:

\[ A_1: \forall y \exists x (y \leq x \rightarrow y \in \mathcal{X}) \quad \text{(UNRESTRICTED PRESENTATION)} \]
\[ A_2: \forall p \exists g (p \leq g \land ((x, y) \in \mathcal{P} \rightarrow (y, x) \notin \mathcal{P} \land \mathcal{P} \subseteq \mathcal{Y}) \) \quad \text{(PREFERENCE)} \] 
\[ A_3: \forall (x, y) \exists x (x, y) \in \mathcal{X} \land (x, y) \in \mathcal{X} \rightarrow (x, y) \in \mathcal{G} \) \quad \text{(UNRESTRICTED PREFERENCE)} \]
\[ A_4: \forall (y_1, p_1) \exists \mathcal{P} (y_1 \in \mathcal{P} \land \mathcal{P} \subseteq \mathcal{Y} \land (c(y_1, p_1) \subseteq y_1 \neq \emptyset) \quad \text{(NON-VACUOUS CHOICE)} \]

A theory of choice should not restrict the content of presentation sets (UNRESTRICTED PRESENTATION), it should be based on asymmetric preference sets (PREFERENCE), it should not restrict the relative preferability of alternatives (UNRESTRICTED PREFERENCE), and it should indicate a non-vacuous choice in all possible choice situations (NON-VACUOUS CHOICE). These axioms will provide the foundation for a theory of choice.

Before turning to the notion of rational choice, I will consider various types of choice functions. These types are described in the following section.
3.2 Judgment vs. Valuation

It is obvious, from CF4, that choice must be affected by changes in \( \mathcal{X} \). If a presentation set changes, a choice set must be allowed to change. For, the latter is required to be a subset of the former. Choice need not, however, be affected by changes in \( \mathcal{Y} \). If a presentation set remains unchanged, a choice set can also remain unchanged, despite any changes that might occur with regard to the content of a preference set. This observation suggests a classification:

Definition 3.2.1: If \( C \) is a choice function in which the choice set is only relativized by \( \chi \), then \( C \) will be referred to as a judgment function (J)

Definition 3.2.2: If \( C \) is a choice function in which the choice set is relativized by both \( \chi \) and \( \mathcal{B} \), then \( C \) will be referred to as a valuation function (V)

A similar classification is suggested by Pfanzagl (1968, p. 66). The first of these two sets of functions would describe a choice situation in which agents were faced with a choice of one or more "tallest" from a set of buildings or one or more "loudest" from a set of sounds, etc. Presumably, the choice sets that attend these situations would remain the same if the agents' preferences for heights or sounds were changed. J would also describe choice situations in which agents are asked to reveal personal attributes such as sex, age, or income. Again, choices in such situations would be assumed to be unaffected by changes in preferences for age, sex, etc.

The second class of choice functions will describe another type of choice situation. A valuation function will be assumed to determine choices in situations where agents are faced with the problem of
choosing favored policies or providing opinions on requested topics, etc. In these cases, choices would be assumed to be affected by changes in preferences. Differences between the two classes of choice functions will provide a foundation for the introduction of rational choice properties. I turn, now, to these properties.

3.3 Rational Choice

None of the previous definitions or conditions dealt specifically with the notion of rational choice. They dealt, more generally, with choice. I will, at this point, introduce the notion of rational choice. The material discussed in sections 2, 3.1, and 3.2 will be used to determine in what manner and under what conditions the term "rational" can be predicated.

It is obvious, from definition 3.1.4, that numerous functions will satisfy the conditions of a choice function. Thus, if we want a theory of individual choice to describe individual choice, we must have some idea as to which of these functions an individual uses. This will require, in addition to the four axioms given in section 3.1, an axiom that indicates the exact manner in which choice proceeds from preference. The notion of "rational choice" suggests, in this regard, the use of a certain type of choice function. It suggests the use of a choice function that will satisfy requirements above and beyond those that make the function a choice function. Definition 3.3.1 establishes the nature of these requirements.

Definition 3.3.1: \( \mathfrak{R} \) is a rational choice property on choice functions provided its domain is the set \( V \) of valuation functions and provided there exists one and only one \( v \in V \) that satisfies \( \mathfrak{R} \).
This definition does four things. First of all, it associates "rational" with preference-based choice. Although not every choice situation will be assumed to be in the domain of a rational choice property, its domain will be assumed to include all situations that are affected by changes in preference. Secondly, the definition requires that the notion of rational choice be non-vacuous. There must exist a choice function such that the function is capable of satisfying the conditions of a rational choice property. This, of course, implies that there must exist a choice set that satisfies 5 for every preference-based choice situation. If a rational choice theory is a theory that is designed to represent preference-based choice, then the theory should indicate a choice in every preference-based choice situation. For, according to 3.1.4, choice is always assumed to be possible. Thirdly, it requires that the notion of rational choice be specific in its representation of the relationship between preference and choice. There can never, in this regard, be more than one choice set capable of satisfying the conditions of a rational choice property. Because an individual is assumed to construct one, and only one, choice set in any choice situation, a rational choice property should not allow, or, "rationalize" more than one choice set. Accordingly, there can only exist one choice function in V that will satisfy the conditions of a rational choice property. Finally, the definition insures that the predication of "rational" be based solely on the relationship between preference and choice. Because the property does not allow for the subscripting of a choice function, different agents with identical preferences will be assumed to construct the same choice sets if faced
with the same preference-based choice situation. It may be the case that no rational choice property can satisfy this requirement when the set of agents is interpreted as individuals. If this is the case, then different individuals might be said to satisfy different rational choice properties. Nevertheless, the most desirable situation would be one in which the choices of all agents satisfy the same rational choice property. To a priori allow agent variation within a single rational choice property would be to confuse two issues, one dealing with the nature of preference-based choice and another dealing with the nature of individual difference.

Consider, for illustrative purposes, a few rational choice property candidates:

(i) \( C(Y_1, P_j) = \{ y \in Y_1 \} \)

(ii) \( C(Y_1, P_j) = \{ x | x \in Y_1 \land \exists y \in Y_1 ((x, y) \in P_j) \} \)

(iii) \( C(Y_1, P_j) \subseteq Y_1 \)

Can (i) be a rational choice property? No, it cannot. If a choice function satisfies (i), it cannot be affected by changes in preference. The choice set would be fixed, given some presentation set, regardless of the context of a preference set. Thus, a choice function that satisfies (i) cannot be a valuation function. Can (ii) be a rational choice property? If a function satisfies (ii), it will obviously be affected by preference. So, we need to know whether or not a function that satisfies this property can, in fact, be a choice function. Consider, in this regard, the pair \( (x, y_2, \emptyset) \) in \( X \times Y \). What will the choice set contain if the function that generates it must satisfy (ii)? It will be empty. Thus, because \( Y \) is allowed to contain empty
preference sets (see the UNRESTRICTED PREFERENCE axiom), (ii) cannot be a rational choice property, for a function that satisfies (ii) cannot be a choice function. Lastly, can (iii) be a rational choice property? A function can satisfy (iii) and be affected by preference. Also, a function need not produce an empty choice set in order to satisfy (iii); a choice function can satisfy (iii). However, there can be more than one function that will satisfy (iii). Suppose, for example, that one choice function indicates that the choice set for the pair \( \{x, y, z\} \), \( \{(x, y), (z, y)\} \) is \( \{x\} \) and that another indicates it is \( \{x, z\} \). Both functions satisfy (iii) and choice under both functions can be affected by changes in the content of a preference set. Thus, (iii) cannot be a rational choice property because it does not satisfy the conditions of the definition. It is not strong enough to prohibit more than one \( v \in V \) from satisfying its conditions.

Given the definition of a rational choice property and these three examples, consider some plausible rational choice property candidates. Two of these candidates have frequently appeared in the individual choice literature (see Herzberger, 1973 or Jamison & Lau, 1973 for a discussion of these candidates). The other has not.

\[ \begin{align*}
\text{RC1: } & C(Y_1, P_j) = \{x | x \in Y_1 \land \forall y \in Y_1 ((x, y) \notin P_j) \} \\
\text{RC2: } & C(Y_1, P_j) = \{x | x \in Y_1 \land \forall y \in Y_1 ((y, x) \notin P_j) \} \\
\text{RC3: } & C(Y_1, P_j) = \{x | x \in Y_1 \land \forall y \in Y_1 (\alpha(x) \geq \alpha(y)) \} \\
\end{align*} \]

where \( \alpha(a) = \# \{ x | x \in Y_1 \land (x, a) \notin P_j \} \)

According to RC1, an agent will, when faced with a choice situation, choose those offered alternatives that are strictly preferred to all other offered alternatives. According to RC2, an agent will choose those offered alternatives that are never dispreferred to other offered
alternatives. According to RC3, an agent will, when faced with a choice situation, choose those offered alternatives that are least frequently dispreferred to other offered alternatives. The question is: Under what conditions will one or the other of these properties satisfy the conditions of a rational choice property?

Consider, first of all, RC1. Does an assumption that RC1 is a rational choice property imply other assumptions about the nature of preference? Unfortunately, it does. Each preference set in $\mathcal{Y}$ must satisfy the following conditions if RC1 is to be considered a rational choice property.

- **CF6**: \( \forall P_i \in \mathcal{Y} ((x,y) \in P_i \land (y,x) \in P_i) \) (\( P_i \) must be connected)
- **CF7**: \( \forall P_i \in \mathcal{Y} ((x,y) \in P_i \land (y,z) \in P_i \rightarrow (x,z) \in P_i) \) (\( P_i \) must be transitive)

**Theorem 3.3.1**: If RC1 is a rational choice property, then, for all \( P_i \in \mathcal{Y} \), \( P_i \) must be transitive and connected.

**Proof**: (i) Suppose, for some \( P_i \in \mathcal{Y} \), \( P_i = \{(x,y), (y,z), (x,z)\} \). If some \( v' \in V \) satisfies RC1, then \( v'((x,y), (y,z), (x,z)) \neq \emptyset \). But, this is impossible, for \( v' \) is a choice function. Thus, \( P_i \) must be connected.

(ii) Suppose, for some \( P_i \in \mathcal{Y} \), \( P_i = \{(x,y), (y,z), (z,x)\} \). If some \( v' \in V \) satisfies RC1, then \( v'((x,y), (y,z), (z,x)) \neq \emptyset \). Again, this is impossible, for \( v' \) is a choice function. Thus, \( P_i \) must be transitive.

If RC1 is assumed to be a rational choice property, and if each \( P_i \in \mathcal{Y} \) does not satisfy CF6 and CF7, then either (i) not all sets of alternatives in \( X \) can be presentation sets (i.e., alternatives for which the corresponding preference sets are either intransitive or non-connected cannot be presentation sets), (ii) certain collections of alternatives can be presentation sets on some occasions but not presentation sets on others (i.e., they can only be presentation sets when the agents
involved have transitive and connected preference sets over them), or (iii) there can exist a choice situation for which there is no rational choice. Each possibility would, of course, entail a relaxation of the axioms discussed earlier (see section 3.1). Arguments against (i) have already been stated. The notion of an alternative does not have sufficient structure to allow this possibility. With regard to (ii), it has been argued that the determination of a preference-based status should (for any choice situation) be made independently of an agent's preferences. If (ii) is allowed, a choice would presumably be assumed to be preference-based when an individual has "appropriate" preferences, and would not be assumed to be preference based at other times. This, however, is inconsistent with the position that a rational choice property should describe choice in all situations that are assumed to be affected by the content of preference sets (see definitions 3.2.2 and 3.3.1). For, according to 3.2.2, if a choice situation is preference based given one preference set, it is preference based given any preference set. Thus, (ii) cannot be allowed. Lastly, (iii) cannot be allowed because it is assumed that individuals can choose over any set of alternatives. This leaves only one possibility. It must be argued, on empirical grounds, that CF6 and CF7 are plausible. This would, as was indicated earlier (see section 1) be very difficult. Aumann (1962), Majumdar (1958), Hicks (1956), and Luce (1956) provide strong arguments against the assumption of connectedness. Tversky (1969), May (1953), and Coombs (1958) provide arguments and evidence against the assumption of transitivity.

Now, consider RC2. Does an assumption that RC2 is a rational
choice property imply any further assumptions about the content of $\mathcal{U}$? Again, it does. Each $P_i$ in $\mathcal{Y}$ must be assured to satisfy the following condition if RC2 is to be considered a rational choice property.

$$
\forall P_i \in \mathcal{Y} (\forall x_0, x_1, \ldots, x_n \in x((x_0, x_1), (x_1, x_2) \ldots (x_{n-1}, x_n) \in P_i \Rightarrow (x_n, x_1) \not\in P_i) \text{ (P must be acyclic)}
$$

Theorem 3.3.2: If RC2 is a rational choice property, then, for all $P_i \in \mathcal{Y}$, $P_i$ must be acyclic.

Proof: Suppose, for some $P_i \in \mathcal{Y}$, $P_i = \{ (x, y), (y, z), (z, x) \}$. If some $v' \in V$ satisfies RC2, then $v'(\{ x, y, z \}, P_i) = \emptyset$. But, this is impossible, for $v'$ is a choice function. Thus, $P_i$ must be acyclic.

Again, if RC2 is considered to be a rational choice property and if each $P_i \in \mathcal{Y}$ does not satisfy CF8, then at least one of the conditions described in the previous paragraph must be allowed. But, since each of these conditions has been ruled out, the only question is: Is CF8 plausible? There are a number of reasons why the condition is not plausible. The first involves the dimensionality of alternatives. In the May experiment (May, 1954), it was discovered that cycles would appear when preferences were based on multiple criteria. The Coombs (1953) and Tversky (1969) pieces cited in a previous paragraph also provide arguments against the plausibility of an acyclic assumption. These results would be consistent with Arrow's impossibility theorem (Arrow, 1962). Arrow proved that a cyclic social preference could result from certain combinations of individual preferences. In an argument against an acyclic condition, the Arrow result would simply be "transferred" from the social to the individual level. This, in effect, is what the May experiment does.

Riker and Ordeshook (1973) advocate the inadmissibility of
cyclicity. They argue that the experimental situations in which it is observed are of little interest to the subjects. And, cyclicity can be expected in such situations. It would seem, here, that the Riker and Ordeshook argument could also be used to advocate cyclicity. We know, from empirical studies, that politics is not salient for individuals (Erickson & Luttbeg, 1973). Thus, would we not expect cyclic preferences to occur in political situations? This would certainly seem plausible. And, if cyclic preferences are plausible, they cannot be disallowed. This would be inconsistent with the position that a theory of rational choice should be based on perceived preference (see section 2). Thus, there is substantial doubt as to the plausibility of CP8 and as to the acceptability of RC2.

Finally, consider RC3. Does an assumption that RC3 is a rational choice property imply other assumptions about the nature of preference? No, it does not. We know, from the statement of RC3, that:

\[ \forall p \in \mathcal{A} \forall y_1 \in X \forall x \in Y_j (0 \leq \alpha(x) \leq \# \{y_j\} ). \]

We also know, from number theory (see Margaris, 1967, chapter 3), that "\leq" induces a weak order on the integers. Now, because the set of maximal elements \( \{ x | x \in S \wedge \forall y \in S (x \geq y) \} \) will not be empty if the set from which it is derived is a weakly ordered finite set (see McCoy, 1972, chapter 3 or Pervin, 1964, chapter 2), a function that is required to satisfy RC3 can be a choice function without requiring that \( \mathcal{A} \) satisfy any additional conditions (i.e., the choice set can always be non-empty). Thus, RC3 can be considered to be a rational choice property without either strengthening the PREFERENCE axiom or altering one or more of the other axioms.
3.4: Evaluating the Rational Choice Properties

It has been established, in the previous section, that RC3 (unlike the other rational choice properties) is not inconsistent with the axioms specified in section 3.1. The plausibility of RC3 has yet, however, to be considered. This consideration will rely on the arguments of numerous individual choice theorists.

There is near unanimity with regard to an assumption that individuals will, from a set of offered alternatives, choose the most preferred alternatives if such alternatives exist. This is the basis of Robertson's assumption that individuals can bring alternatives under a category of "greater or less" and choose in a manner that reflects the categorization (Robertson, 1952). In fact, this is the basis of both RC1 and RC2. If the preferences over a set of offered alternatives are such that the intransitivities or cycles are avoided, then it is assumed that an individual will choose, from this offered set, the "maximal" alternatives. RC1 and RC2 reflect this assumption. They differ in that, under RC1, the set of maximal alternatives will only contain a single alternative (see Debreu, 1959 for this result). This set can, under RC2, contain any number of alternatives. Thus, both of these properties seem reasonable in this regard.

However, neither RC1 nor RC2 differ, in this regard, from RC3. For, under RC3, if a non-empty maximal set exists, this set must be the choice set. Suppose, for example, that $x_i$ is a maximal alternative in a set $Y_i$. By RC3, $\preceq(x_i) = \# \{ Y_i^2, Y_i^3 \}$. And, as was noted earlier, for any alternative $x_j$ in $Y_i$, $0 \leq \preceq(x_j) \leq \# \{ Y_i^2, Y_i^3 \}$. Thus, if $x_i$ is a maximal alternative, there can exist no other alternative $x_j$ such that
\(\alpha(x_j) > \alpha(x_i) \geq \alpha(x_j)\) for all \(x_j\) in \(Y_i\). Thus, by RC3, a maximal alternative in \(Y_i\) must be in the choice set of \(Y_i\). Now, suppose some alternative \(x_j\) in \(Y_i\) is not a maximal alternative. If so, then there must exist an \(x_k\) in \(Y_i\) such that \((x_k, x_j)\) is in \(P_j\). Thus, \(\alpha(x_j) \leq \#(Y_i)\). So, if a \(Y_i\) contains maximal alternatives, these will (by RC3) be the only alternatives in the choice set of \(Y_i\).

One major difference between RC3 and the other properties is that it does not require, in addition to the choice of maximal alternatives, the existence of maximal alternatives. Again, this requirement confuses two issues. One issue involves the nature of the relationship between preference and choice, and the other, the construction of specific preference sets. This, as was argued in section 2, is both obfuscating and inappropriate. Where RC3 and any of the other properties indicate a non-empty choice set, the indications will be identical. Thus, RC3 is as specific as the other properties, is not obfuscating, and is less restrictive. The evidence in favor of RC3 is substantial.

3.5 The Motivation for Restrictions

One might ask, given the results of the previous section, why there is so much interest in the restrictive rational choice properties (RC1 and RC2). Why are the choice functions in theories of rational choice required to satisfy these properties rather than RC3? Consider, for the purpose of answering this question, the issue of performance. What is a theory of rational choice designed to do? It is designed to explain individual choice behavior. Two criteria have,
in this regard, been used for the evaluation of rational choice theo-
ries (see Jamison & Lau, 1973). One criterion deals with the nature
and existence of a function from an \( \langle \text{ALTERNATIVE}; \text{PREFERENCE} \rangle \) empiri-
cal structure into an \( \langle \text{INTEGERS}; \text{GREATER THAN} \rangle \) numerical structure
(the criterion of numerical representation). The question, here, is:
Under what conditions will such a function exist, and what conditions
can the function satisfy? The second criterion deals with the nature
of the implied restrictions that a theory of choice can place on in-
dividual demand, or, choice (the criterion of prediction). Table 1
(see Jamison & Lau, 1973, p. 901), indicates the major contributors
in each of these areas.

I will consider only briefly the criterion of numerical representa-
tion. The motivation for this criterion is straightforward. If it
is possible to describe the \( \langle \text{ALTERNATIVE}; \text{PREFERENCE} \rangle \) empirical
structure (this structure will hereafter be denoted by: \( \langle X;\mathcal{Y} \rangle \)) with
the theory that is used to describe the \( \langle \text{INTEGERS}; \text{GREATER THAN} \rangle \)
umerical structure (this structure will hereafter be denoted by:
\( \langle I;\gg \rangle \)), it will be possible to take advantage of the extensive
knowledge that attends this theory. One would simply interpret the
theory with the elements in \( \langle X;\mathcal{Y} \rangle \) and would, as a result, have all of
the theory’s known theorems at his disposal. So, under what conditions
will there exist a function from \( \langle X;\mathcal{Y} \rangle \) into \( \langle I;\gg \rangle \) that will pre-
serve the information contained in \( \langle Y;\mathcal{Y} \rangle \) ? And, how much of this
information will the function preserve (see Pfanzagl, 1968 for a fur-
ther discussion of such functions)?
# TABLE 1

PREFERENCE SETS, NUMERICAL REPRESENTATIONS 
AND RESTRICTIONS ON CHOICE: 
MAJOR CONTRIBUTORS

<table>
<thead>
<tr>
<th>Preference Set Requirement</th>
<th>Numerical Representation</th>
<th>Restrictions on Choice</th>
</tr>
</thead>
<tbody>
<tr>
<td>linear order</td>
<td>Debreu (1954)</td>
<td>Sen (1971)</td>
</tr>
<tr>
<td></td>
<td>Chipman (1960)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Suppes &amp; Zinnes (1963)</td>
<td></td>
</tr>
<tr>
<td>weak order</td>
<td>Debreu (1954)</td>
<td>Houthakker (1950)</td>
</tr>
<tr>
<td></td>
<td>Chipman (1960)</td>
<td>Arrow (1959)</td>
</tr>
<tr>
<td></td>
<td>Suppes &amp; Zinnes (1963)</td>
<td>Uzawa (1960)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Samuelson (1938)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Richter (1966)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Jamison &amp; Lau (1973)</td>
</tr>
</tbody>
</table>
The known representation theorems are due to Fishburn (1973).

Two of these theorems are relevant to the present discussion:

**Theorem 3.5.1:** CF6 and CF7 hold if and only if there exists a function \( n \) from \( X; \mathcal{U} \) into \( I; \succ \) such that:

1. \( \forall P_1 \in \mathcal{U} \ ( (a, b) \in P_1 \iff n(a) > n(b) \)
2. \( \forall P_1 \in \mathcal{U} \ ( n(a) \not> n(b) \land n(b) \not> n(a) \iff a = b \)

**Theorem 3.5.2:** CF8 holds if and only if there exists a function \( n \) from \( X; \mathcal{U} \) into \( I; \succ \) such that:

1. \( \forall P_1 \in \mathcal{U} \ ( (a, b) \in P_1 \iff n(a) > n(b) \)

If (and only if) CF6 and CF7 hold (i.e., if RC1 is an acceptable rational choice property), will it be possible to construct a function \( n \) from \( X; \mathcal{U} \) into \( I; \succ \) such that one alternative \( x \) will be preferred to another alternative \( y \) if and only if the image of the one under \( n \) \((n(x))\) is greater than the image of the other under \( n \) \((n(y))\). Furthermore, if neither \( n(x) > n(y) \) nor \( n(y) > n(x) \), then alternative \( x \) is alternative \( y \) (i.e., \( x \) and \( y \) must be one and the same alternative).

If (and only if) CF8 holds (i.e., if RC2 is an acceptable rational choice property), will it be possible to construct a function \( n \) from \( X; \mathcal{U} \) into \( I; \succ \) such that, if one alternative \( x \) is preferred to another \( y \), then the image of the one under \( n \) \((n(x))\) will be greater than the image of the other under \( n \) \((n(y))\). Unfortunately, if \( n(x) > n(y) \), it need not be the case that \( (x, y) \in P_1 \). One alternative \( x \) may be indifferent to another \( y \), yet the image of the one \((n(x))\) greater than the image of the other \((n(y))\). Obviously, the function that is constructable under CF6 preserves less information than does the function that is constructable under CF6 and CF7. It
will, however, preserve some information. The function would indicate
that \( n(x) \geq n(y) \) if and only if \((x, y) \in P_1\). Thus, under \( CP6 \) and \( CP7 \) to-
gether, and under \( CP3 \) alone, there will exist a function from \( <X; \mathcal{U}> \)
into \( <I; \iota> \) that will preserve the information in \( <X; \mathcal{U}> \).

The latter representation theorem not only indicates the existence
of an information preserving function for \( CP8 \). It also indicates that
no information preserving function can be constructed from \( <X; \mathcal{U}> \)
into \( <I; \iota> \) unless \( CP8 \) holds. Thus, for any function from \( <X; \mathcal{U}> \)
into \( <I; \iota> \) that is constructed under \( RC3 \), it need not be the case
that either (i) \( n(x) > n(y) \rightarrow (x, y) \in P_1 \), or (ii) \( (x, y) \in P_1 \rightarrow n(x) > n(y) \).
It is not possible, with \( RC3 \), to obtain a numerical representation of
the choice problem.

The criterion of prediction is perhaps the only criterion that
need be used for the evaluation of rational choice theories. If a
rational choice theory can predict individual demand (choice), then
application of the numerical representation criterion is unnecessary.
The theory itself provides a representation of the individual choice
process. Whether or not the representation is specific and numerical
is irrelevant. Thus, the criterion of prediction, if not the only
criterion that need be used to evaluate rational choice theories, is
by far the more important of the two.

Actually, the criterion of prediction has two components. The
first concerns the existence of an "irrational" choice (Richter, 1971)
and the second concerns the relationship between choices over present-
ation sets that are included in one another (Arrow, 1959; Uzawa, 1960;
Fishburn, 1973). The question, with regard to the first component, is:
Can a rational choice theory rationalize any choice? The answer must be: no. Suppose, for example, that a series of individual choices is observed. Will this series, together with a theory of rational choice, provide any information as to the occurrence or non-occurrence of subsequent choices? They must if the first component of the prediction criterion is to be satisfied. There must, given this component, exist a choice set that cannot be "rationalized." Fortunately, none of RC1, RC2, or RC3 fail to satisfy this component.

Theorem 3.5.3: Given A1 through A4, there will exist an irrational choice if any of RC1, RC2, or RC3 holds.

Proof: Suppose that, for the sets \{a, b\}, \{b, c\}, \{a, c\} in \mathcal{P} comprising \mathcal{H}, C({a, b\}, \cdot) = \{a\}, C({b, c\}, \cdot) = \{b\}, and C({a, c\}, \cdot) = \{a\}. According to RC1, RC2, and RC3, if C({x, y\}, \cdot) = \{x\}, then \((x, y) \in P\). For, if neither \((x, y)\) nor \((y, x)\) \in P, C({x, y\}, P) = \{x\}. And, if only \((y, x)\) \in P, C({x, y\}, P) = \{y\}. Thus, \((a, b), (b, c), (a, c) \in P\). And, by RC1, RC2, and RC3, C({a, b\}, P) \neq \{a\}, \{b\}, \{c\}. There exists an irrational choice.

All of the rational choice properties introduced in section 3.3 satisfy the first component of the prediction criterion.

The second component of the prediction criterion is related to, but more general than, the first. It concerns the relationship between the choice sets of two presentation sets \(Y_i\) and \(Y_j\) when one is a subset of the other. The motivation for this component is usually expressed in the following manner: "Should an alternative \(y\) that is in the choice set from (a presentation set) \(Z\) also be in the choice set from every subset of \(Z\) that contains \(y\)" (Fishburn, 1973, p. 195)? Whether or not this condition is satisfied depends, of course, on whether or not a rational choice theory implies that it be satisfied. These, and similar questions, are designed to assess the impact of a
rational choice theory on our ability to predict the choices of an individual. If, in the Fishburn case, it is known that an individual has chosen an alternative from one set, will it be possible to predict that the individual will choose that alternative in all subsets containing it? Consider, in this regard, the following predictive conditions:

Condition (i) \( \forall Y_1, Y_j \subseteq X \ (Y_1 \subseteq Y_j \land Y_1 \cap c(Y_j) \neq \emptyset \rightarrow c(Y_1) = Y_1 \cap c(Y_j) \)  

Condition (ii) \( \forall Y_1, Y_j \subseteq X \ (Y_1 \subseteq Y_j \land Y_1 \cap c(Y_j) \neq \emptyset \rightarrow Y_1 \cap c(Y_j) \subseteq c(Y_1) \)  

Condition (i) is due to Arrow (1959). It requires that, "if some alternatives are chosen out of a set \( Y \) and then the range of alternatives is narrowed to \( X \) but still contains some previously chosen alternatives, no previously unchosen alternatives become chosen and no previously chosen alternatives become unchosen." (Arrow, 1959, p. 123) 

Condition (ii) is due to Uzawa (1956) and to Chernoff (1954). It is weaker than Condition (i). Although, under the same conditions as Arrow describes, it requires that chosen alternatives remain chosen, it does not require that unchosen alternatives remain unchosen.

The following theorems are due to Fishburn (1973).

Theorem 3.5.4: If CF6 and CF7 hold, then RC1 implies Condition (i)

Theorem 3.5.5: If CF8 holds, then RC2 implies Condition (ii)

Thus, RC1 and RC2 do place restrictions on choices that are made in presentation sets that are contained in one or the other. In doing so, they supply the investigation of individual choice with a known predictive efficacy. Unfortunately, as the following example indicates, RC3 does not supply this same efficacy. Suppose

\[ P_1 = \{(x,y),(y,z),(y,w),(z,x),(w,x),(z,w)\} \]  

In this case,
Thus, under RC3, an unchosen alternative can become chosen and a chosen alternative unchosen as a presentation set is contracted. The more important question, however, is whether or not RC1 or RC2 are "plausible" rational choice properties in the sense of being descriptive of preference based choice. This investigation began with an inquiry as to what constitutes an acceptable theory of rational choice. Most investigations, however, begin with an inquiry as to the properties of a proposed theory (i.e., Condition (i) or Condition (ii)). Such investigations are, for the most part, question-begging. They fail to address the issue of plausibility. One should begin with a theory that can be considered plausible and then investigate its consequences. This is what I have attempted to do. I will, in the following chapter, investigate the consequences of this theory.
NOTES: CHAPTER III

1. The assumption that alternatives with "greater utility" will be chosen over alternatives with "lesser utility" is based, in part, on an assumption of individual rationality. This assumption is the subject of the following section.

2. Hicks (1956, Chap. 2) eventually abandons the "geometric" method of describing preference through indifference curve analysis. He argues that the notion is misleading in all but the simplest cases.

3. Theorem 4 in Richter (1971, p. 34) provides an excellent example of this problem. The theorem would not be constructable if the domain of choice were not allowed to be restricted.

4. See Hansson (1968a, 1968b) for a further discussion of the notion of preference.

5. See, for example, the earlier references to May (1954) and Tversky (1969) in section 2.

6. I will return to the problem of selection in Chapter 4.
IV. The Survey as Choice Experiment

I will investigate, in this chapter, the foundations of survey research. This investigation will rely heavily on the results of the previous chapters. The survey will be viewed as a choice experiment that is designed to access preference sets (see, in this regard, Bush, Galanter, & Luce, 1963, p. 87). The problems of access will be discussed in the context of the theory described in Chapter 3 (section 3).

Before the empirical implications of the theory can be investigated, it will be necessary to consider the notions of measurement and faithful representation (see Kantz, Luce, & Suppes, 1970; or Suppes & Zinnes, 1963). For, it is with reference to these notions that the specification of a theory is paramount. It would be impossible, without this specification, to determine whether or not a survey-based measurement structure can faithfully represent an (as-of-yet unspecified) empirical relational structure (see Chapter 2, section 4 for a discussion of empirical relational structures).

Finally, the present prerequisites for the measurement of certain empirical relational structures are compared with other prerequisites (see, in this regard, Coombs, 1964, Chapter 1). The present prerequisites will be seen to be much weaker than those found elsewhere.

1. FAITHFUL REPRESENTATION

This section focuses on the distinction between empirical relational and measurement structures. The discussion will be based, in
part, on the foundations laid in the second and fourth sections of Chapter 2.

1.1 Empirical Relational Structures and Measurement Structures

Recall, from Chapter 2 (section 4), that the elements in empirical relational structures are not directly linked to sensory experience. Rather, they are abstracted from phenomena that are linked to sensory experience. This, I have argued, poses some difficult problems with regard to measurement. Measurement refers, in this case, to the construction of accessible representations of empirical relational structures. These representations will be referred to as measurement structures. Measurement structures are constructed for the purpose of representing empirical relational structures.

Because empirical relational structures are not directly linked to sensory experience, there will always be questions as to the correspondence between the content of a measurement structure and the content of an empirical relational structure it represents. It is in this context that the notion of faithful representation will be considered. Before introducing this notion, I will introduce some standard conventions. Consider the following general structures:

ES = \langle EO; ER \rangle and MS = \langle MO; MR \rangle. ES will denote an arbitrary empirical relational structure and MS, an arbitrary measurement structure. EO will denote a set \{e_0, e_1, e_2, \ldots, e_n\} of objects from ES and MO will denote a set \{m_0, m_1, m_2, \ldots, m_m\} of objects from MS. Finally, ER will denote a relation from ES and MR will denote a relation from MS.
1.2 Complete Representation

There are a number of possibilities with regard to the representation of ES by MS. MS might, for example, be required to be a complete representation:

COMPLETE REPRESENTATION (CR): MS is a complete representation of ES if and only if there exists a function \( \phi: ES \rightarrow MS \) such that:

(i) \( \forall e_0 \in EO \exists m_0 \in MO (m_0 = \phi(e_0)) \), and

(ii) \( (\phi(e_0), \phi(e_0)) \in MR \leftrightarrow (e_0, e_0) \in ER \)

In a complete representation it must be the case that, for every empirical object, there exists a measurement object that represents it. This measurement object will be referred to as the image of the empirical object. It must also be the case that 1) for every element in an empirical relation, the corresponding element (the element in MO x MO that contains the image of each empirical object) must be in the measurement relation, and 2) for every element in EO x EO that is not in an empirical relation, the corresponding element in MO x MO must not be in the measurement relation. In other words, if an element \((e_0, e_0)\) appears in a relation in an empirical structure (ES), it must be associated with an element \((\phi(e_0), \phi(e_0))\) that appears in a relation in a measurement structure that represents ES. And, if \((e_0, e_0)\) does not appear in a relation in an empirical structure (ES), it must not be associated with an element \((\phi(e_0), \phi(e_0))\) that does appear in a relation in a measurement structure that represents ES.

This very restrictive type of representation raises two problems. First, a measurement structure is seldom capable of recognizing extremely small empirical differences. Suppose that ES is a set of days (D) to-
gether with the relation "is hotter than" (HT). And, suppose that MS is a set of thermometer values (V) under a relation "is greater than" (GT). Between any two thermometer values there will exist an infinite number of temperatures. One day could be hotter than a second \((d_1, d_j) \in HT\) without having a greater thermometer value \((\phi(d_1), \phi(d_j)) \notin GT\). A thermometer-type measurement structure will seldom provide a complete representation of a \(<\text{DAYS}; \text{HOTTER THAN}>\) empirical structure. Yet, the representation it does provide is often considered useful.

A complete representation is also subject to a problem of excessiveness. A scientist may neither need nor want a complete representation. Suppose, for example, that a scientist is using a measurement structure containing the set of positive real numbers together with the relation "is greater than" to represent an empirical structure containing a set of individuals together with the relation "is older than." If the measurement structure were required to be a complete representation of the empirical structure, the scientist would be forced to obtain information on the year, month, day, hour, minute, second, etc., of each individual's birth. And, even this may be insufficient. For, complete representation would require that a measurement structure both perceive and indicate any difference. This example illustrates a very important point. The amount of information that a complete representation conveys is seldom either useful or desirable. For a social scientist, yearly differences in age are often thought to provide sufficient information. If there is less than a year's difference between the ages of two individuals, neither
individual is considered older than the other.

1.3 Faithful Representation

A **faithful representation** is somewhat less restrictive than a complete representation:

**FAITHFUL REPRESENTATION (FR):** MS is a faithful representation of ES if and only if there exists a function \( \phi : ES \rightarrow MS \) such that:

1. \( \forall e_0 \in EO \exists m_0 \in MO (m_0 = \phi(e_0)) \), and
2. \( (\phi(e_0), \phi(e_0)) \in MR \rightarrow (e_0, e_0) \in ER \)

Both faithful and complete representation require that each empirical object have an image in the set of measurement objects that is used for representation. And, both require that, if an element \((e_0, e_0)\) does not appear in a relation in an empirical structure \((ES)\), it must not be associated with an element \((\phi(e_0), \phi(e_0))\) that does appear in a relation in a measurement structure that represents \(ES\). A faithful representation does not, however, require that elements in empirical relations have corresponding elements in measurement relations. This property is somewhat more reasonable than the complete representation property in that it allows the empirical structure to contain unaccessed information. Unfortunately, the faithful representation property can be satisfied by a measurement structure that conveys no information. If the relation in MS is empty, then each element that appears in this relation will also appear in the empirical relation it represents. Consider, as an example, the use of a hospital thermometer for a determination of the relative temperatures of samples of liquid oxygen and liquid hydrogen. Because a hospital thermometer
cannot record the extremely low temperatures of these liquids, MR will be empty yet MS will be a faithful representation of this particular empirical structure. An uninformative but faithful representation would also result from the use of an item about Presidential voting if applied for the purpose of determining the political efficacy of fourth graders. Fourth graders do not vote. Again, MR would be empty yet MS could be a faithful representation of ES. This example suggests that, for any empirical relational structure, there is a range of possible faithful representations. The range is bounded on one end by a complete representation, and at the other, by what might be called a vacuous representation (VR). Between these bounds lies a set of what might be called non-vacuous faithful representations (NVFR). One should, in most cases, seek a representation from this set.

Before I apply the notion of faithful representation to the problems of political measurement, I will summarize the results of the present discussion. The relationship between type of representation and type of information conveyed is displayed in Figure 2.

\[\text{INSERT FIGURE 2 ABOUT HERE}\]

The vertical line locates the vacuous representation. No information is conveyed if a measurement structure is a vacuous representation of an empirical structure. All faithful representations lie to the right of the vertical line. These representations will only convey information that is correct. And, they will convey at least some information about the structures they represent. This information will, in the limiting case, be complete. The non-faithful representations (NFR)
<table>
<thead>
<tr>
<th>TYPE OF INFORMATION:</th>
<th>MIXED</th>
<th>NONE</th>
<th>COMPLETE</th>
</tr>
</thead>
<tbody>
<tr>
<td>TYPE OF REPRESENTATION:</td>
<td>NFR</td>
<td>VR</td>
<td>NVFR</td>
</tr>
</tbody>
</table>

Fig. 2: Relationship between type of representation and type of information conveyed by that representation.
lie to the left of the vertical line. These representations provide incorrect information about the structures they represent. All the information they provide will not be incorrect, but some of it must be incorrect. It is the non-faithful representations that one should, in measurement, seek to avoid.

2. POLITICAL MEASUREMENT

The discussion has, to this point, been quite general. It has not dealt specifically with the problems and procedures of political measurement. I will now describe the components of that measurement. This description cannot, of course, cover all forms of political measurement. Thus, its domain will be restricted to survey research. The major components of survey research are a world and a survey. A world is a set of empirical relational structures and a survey is an instrument that is used to construct a set of measurement structures. I will describe, in this section, the components of a world and the components of a survey.

2.1 The World

The world, for a political scientist who does survey research, contains a set of individuals. These individuals have a number of salient characteristics. Included among these characteristics are: level of political efficacy, party identification, level of political trust, level of political participation, etc. Characteristics are important in that they define characteristic relations \((CR_i)\) such as "more efficacious than" and "more participative than." One individual
could, for example, be more efficacious than another. Or, one could be more participative than another. I will assume, throughout this discussion, that these relations are asymmetric. This is not an unlikely assumption, for many important political relations are implicitly assumed to satisfy asymmetry.

An empirical structure, for a political scientist, is a set of individuals (I) together with a characteristic relation (CR_i). For any set I, there may be as many empirical relational structures as there are characteristic relations. The goal of a political scientist is, in part, to discover the linkages between these structures. If, for example, one individual is more efficacious than another, will this individual also tend to be more participative than the other? Consider, as an illustration, an article by Converse, et al. (1969). In this article, attitudes toward Vietnam involvement, civil rights, social welfare, etc. are used for constructing characteristic relations. Individuals are determined to be more hawkish than others, more pro-civil rights than others, more pro-social welfare than others, etc. Finally, the contents of characteristic relations are correlated. The correlations are obtained for the purpose of determining whether or not those who are more hawkish than others are also more pro-civil rights than others, etc. This type of research is at the foundation of a major part of contemporary political inquiry. It requires, at a minimum, an assumption that the investigation of these I; CR_i empirical structures will be fruitful for obtaining information about the world (see, in this regard, the discussion of phenomena in Chapter 2). I will not investigate this assumption. Rather, I
will investigate the assumption that the content of $<I;CR_1>$ empirical relational structures is knowable. It is this assumption that involves the notion of measurement. And, an assumption that we can access the content of $<I;CR_1>$ empirical relational structures is crucial to the type of research that the Converse, et al. article exemplifies.

2.2 The Survey

Empirical structures, I have argued, are accessed through measurement structures. In survey research, the measurement structures are obtained from surveys. These surveys will, here, be considered as special types of choice experiments (Bush, Galanter, & Luce, 1963). This consideration is, perhaps, uncommon. Yet, (as will be shown) the elements of a survey coincide with the elements of a choice experiment. A description of these elements will be based on Bush, et al. (1963). They present, in this article, a lucid and rigorous classification of choice experiments.

According to Bush, et al., "a large part of the naturalistic analysis and an even larger part of the successful experimental analysis of human and animal behavior involves the notion of choice" (1963, p. 79). Accordingly, a choice experiment is an experiment "in which there is a set of two or more empirically defined alternative responses from which the subject chooses just one whenever he is given the appropriate opportunity to do so" (1963, p. 79). This type of experiment contains the following elements:

1. an ordered set $I_N = \{1, 2, 3 \ldots n, \ldots N\}$ of trials
2. a set $S = \{s_1, s_2, \ldots s_p\}$ of stimulus presentations
(iii) a set \( R = \{ r_1, r_2, \ldots, r_q \} \) of responses

(iv) a set \( O = \{ o_1, o_2, \ldots, o_m \} \) of outcomes

(v) a function \( \sigma : I_N \rightarrow S \) (the presentation schedule)

(vi) a function \( \rho : I_N \rightarrow R \) (the response data)

(vii) a function \( \beta : S \times R \rightarrow O \) (the outcome function)

In the case of survey research, \( S \) is a set of items. \( \sigma \) is a function that sequences these items. It is usually constructed so as to avoid response set bias (Block, 1965; Bentler & Jackson, 1972). The sequencing of items has been determined to be very important in this regard. \( R \), in survey research, is a set of responses. \( \rho \) is a function that associates each response with a corresponding item. Lastly, \( O \) is a set of possible outcomes. Whereas, in psychological choice experiments, these outcomes might be sums of money, the statements "right" or "wrong," etc., they are, in survey research, somewhat different. Outcomes are typically codes that are associated with obtainable responses. \( \beta \), in this case, is a function that assigns a response code to each item and response pair.

Consider, again, the work of Converse, et al. (1969). Their evidence is based on the response data obtained from the use of items such as:

"Which of the following do you think we should do in Vietnam?"

1. pull out
2. do the same
3. take a stronger stand
4. don't know

The item is a member of the set \( S \). The responses are members of \( R \). And, the values that precede them are members of \( O \). \( \sigma \) locates this
item somewhere in a survey, \( p \) associates a response with the item, and \( p \) associates a value with that response (e.g., a "pull out" response would receive an outcome of 1). This item, with these responses, is assumed to provide information that can be used to determine the content of a "more hawkish than" relation. It is in this capacity that the item has a significance. It is used to construct a measurement structure that, in turn, is used to access an \( \langle I; CR_1 \rangle \) empirical structure. Before describing the measurement structures that are constructed from this type of item, I will consider a rationalization of the survey. The rationalization will justify the intended use of the survey and will provide a foundation for a description of the resulting measurement structures.

3. A RATIONALIZATION OF THE CHOICE EXPERIMENT

Unfortunately, the content of a measurement structure need not be related to the content of a corresponding empirical structure. Indeed, it is for this reason that the various representation properties have been considered. In the case of survey research, individuals can choose in a manner that disguises actual attitudes (DeFleur & Westie, 1963; Wicker, 1969), they can choose randomly (Converse, 1970), or they can choose on the basis of an item's structure rather than an item's content (Block, 1971; Bentler & Jackson, 1972). Each of these possibilities must be excluded. If individuals choose in any one of the above manners, their choices will not produce measurement structures that can be considered faithful representations of \( \langle I; CR_1 \rangle \) empirical structures. If, for example, an individual chooses the
"take a stronger stand" or any other response on the Vietnam item, we must assume that the choice can be used to access the content of the empirical structure containing the "more hawkish than" characteristic relation. It is for this reason that the choice experiment must be rationalized. A rationalization will explicitly exclude all manners of choice that are not thought to be associated with the faithful representations of \( \langle I; CR_1 \rangle \) empirical structures. I, of course, have chosen a rationalization that is based on the economic and psychological investigation of choice and that was described in the previous chapter. Indeed, I believe that this is by far most reasonable rationalization available.

3.1 Preference Based Choice

In general, there are two types of items that appear in a survey. The first type requires, for response, the use of a judgment function, and the other, the use of a valuation function (see Chapter 3, section 3 for a discussion of these choice functions). The first type includes items about age, sex, place of birth, candidate voted for, etc. The second type includes items for which responses are assumed to be relativized by preference. The Vietnam item cited earlier is a good example of this type. I will, throughout this chapter, only be considering the foundations for the items that require the use of valuation functions. For, it is this type of item that is used to access the content of empirical structures containing characteristic relations. Thus, if the choices induced by these items provide appropriate information, they must be determined by an individual's preferences. If,
for example, an individual choses a pro-social welfare response on an item about social welfare, it must be assumed that the choice is based on the individual's preferences with regard to the responses that are offered on that item. Indeed, it must be assumed that, if the choice had been different, the preferences would have been different. Our use of survey responses is very much dependent on the assumption that preferences determine choices on these items.

Each item \( s_i \in S \) is an \( n \)-tuple of possible outcomes (i.e., pull out of Vietnam, get more involved in Vietnam, etc.). A response is simply a choice of one from this set of outcomes. If an \( s_i \in S = \{ 0^1, 0^2, 0^3, \ldots, 0^n \} \) (if the item has \( n \) possible responses), and if the corresponding response is the \( r \)-th member of this set (\( r \leq n \)), then \( \beta(s_i, r) = 0^r \); the outcome function simply associates the outcome with the chosen response. This is the type of choice experiment that appears in a survey (in addition, of course, to the type exemplified by those items requiring only a judgment function for response).^5

I will now review (from Chapter 3), the elements of a preference-based choice assumption.

(i) the Agent set \( I = \{ 1, 2, 3 \ldots n \} \)
(ii) the Alternative set \( X = \{ x_1, x_2, x_3 \ldots x_n \} \)
(iii) the Presentation set \( Y_i \subseteq X \)
(iv) the Preference set \( P_j \subseteq X \times X \) such that \( P_j \) is asymmetric
(v) the set of all Preference sets \( \mathcal{P} = \{ P_k | P_k \subseteq X \times X \text{ asymmetric} \} \)

To these, I will add an element:

(vi) the \( Y_i \)-restricted Preference set \( \overline{P_j} = P_j \cap (Y_i \times Y_i) \)

The first five elements are fully described in Chapter 3. \( \overline{P_j} \) is a sub-
set of a preference set $P_j$. This subset contains the elements of $P_j$ that will be assumed to be important in determining a choice set given an item with a set $Y_i$ of responses. Also, when it is necessary to indicate the preference set of some particular $i \in I$, this will be denoted in the following manner: $P(i)$.

3.2 The Choice Function

Recall, from Chapter 3, that the assumption of preference-based choice requires an exact specification of the manner in which preferences determine choices. This manner is specified by a rational choice property. I will, in this regard, use RC3.

$$RC3: \mathcal{C}(Y_i, P_j) = \{x | x \in Y_i \land \forall y \in Y_i (\alpha(x) \geq \alpha(y)) \}$$

where $\alpha(a) = \# \{y | y \in Y_i \land (y, a) \notin P_j \}$

This requirement assumes that an individual will, when faced with a presentation set, choose those alternatives that are least frequently dis-preferred to other offered alternatives (see Chapter 3, section 3 for a justification of this requirement).

The set that a choice function produces is called a choice set. Recall, from Chapter 3, that a choice set is very difficult to observe. The nature of choice is very often restricted by situations of choice (i.e., elections, purchasing, etc.). It is selection sets that are observed in these situations. Choice sets are not observed. Bush, et al. (1963) fail to make this distinction in their description of choice experiments that are designed to discover preference (p. 82). Nevertheless, as will be shown, restrictions can have a very negative effect on the construction of faithful survey-based measurement structures. I will, in the next few sections, analyze the survey as though
choice sets were being observed. I will then return to the problems of restriction and selection in section 9.

3.3 Characteristic and Preference Relations

Now, with one last assumption, the preference-based rationalization of the choice experiment will be complete: A pair of individuals will be contained in a characteristic relation if the contents of their preference sets are sufficiently different (the meaning of "sufficiently" must, of course, be specified by the researcher). According to this assumption, the content of characteristic relations is associated with certain combinations of individual preference sets. Without this assumption, the survey could not be used to access the content of characteristic relations. There would be no linkage between characteristic relations and choice sets. Preference sets provide that linkage.

In summary, the rationalization of the choice experiment is based on three assumptions:

ASSUMPTION 1: preference sets determine choice sets

ASSUMPTION 2: a specifiable choice function translates preference sets into choice sets

ASSUMPTION 3: a pair of individuals will be contained in a characteristic relation if their preference sets are sufficiently different.

Figure 3 provides an illustration of the process that these assumptions describe. The assumption of this process rationalizes, for the purpose of accessing the content of characteristic relations, the use of surveys.

The content of characteristic relations is associated with differences
Fig. 3: An illustration of the process that leads from characteristic relation membership to choice.
in preference sets and differences in preference sets are associated with differences in choice sets. Through the assumption of this process, that which leads to characteristic relation membership also leads to different choice sets (given a judicially selected item). Now, having offered a rationalization of the choice experiment, I will return to a discussion of the measurement structures that are constructed from a survey.

4. MEASUREMENT STRUCTURES AND $\Phi$

As I have indicated, surveys are used to construct the measurement structures that, in turn, are used to access $\langle I; CR_i \rangle$ empirical structures. I will begin to describe, in this section, the content of these measurement structures. I will also describe the content of the mappings ($\phi$) from $\langle I; CR_i \rangle$ empirical structures into survey-based measurement structures. The goal, here, is to insure that the mappings satisfy the property of faithful representation. If, for example, we use the responses to an item about Vietnam involvement for the purpose of obtaining information about an $\langle \text{INDIVIDUALS IS MORE HAWKISH THAN} \rangle$ empirical structure, we would want to insure that this information can be used so as to represent the structure in a faithful manner.

4.1 Measurement Structures

Individuals are confronted with a survey for the purpose of obtaining information about their choice sets. These choice sets are the objects of survey-based measurement structures. Why are they inter-
esting? The assumptions used in the rationalization of the survey will help answer this question. We know, from these assumptions, that membership in characteristic relations is associated with differences in choice sets. Thus, if our ultimate goal is to access the content of characteristic relations, we will most likely be interested in choice sets for the differences, across a set of individuals, that they reveal. The application of a survey will only be interesting if difference choice sets are obtained. These differences, if obtained, will be recorded in a measurement relation. A measurement relation is a relation that, in this case, is defined on a set of choice sets. Thus, if an item contains the set \( R \) of responses, the measurement structure associated with this item will contain the set \( 2^R \setminus \emptyset \) (the set of all non-empty subsets of \( R \), written \( R^* \)) of choice sets together with the relation \( MR \) defined on these choice sets (\( MR \subseteq R^* \times R^* \)). This structure will be represented in the following manner: \( MS = \langle R^*; MR \rangle \). I will assume, throughout this discussion, that measurement relations are asymmetric. This assumption is motivated by the assumed asymmetry of empirical relations (see section 2). A specification of the content of \( MR \) will play a central role in determining what is asserted to be the content of a \( CR_i \). I will discuss a procedure for specifying the content of \( MR \) in a later section. At this point, a general description of a measurement structure will suffice.

4.2 The Function

Having described the empirical and measurement structures, I can begin to describe the content of \( \phi \). \( \phi \) is a function from an empirical into a measurement structure. It will be a set of pairs
\((\{1,A\},(2,B),(3,C),(4,A)\ldots\})\) such that the first element of each pair will be a member of the agent set, and the second element, a member of \(R^*\). In survey research, the pair \((j,Z)\) will be in \(\phi\) if and only if \(j \in I\) and \(C(R,P(j)) = Z\). In other words, \((j,Z)\) will be in \(\phi\) if and only if, given a choice problem with the set \(R\) of responses, \(j\)'s choice set is \(Z \subseteq R\). This assumption eliminates the possibility of capricious or masked choice. It would be impossible, under this assumption, for an individual to choose a pro-busing response yet hold an anti-busing position. I have explicitly assumed that preference sets determine choices.\(^6\)

To insure that \(MS\) is a faithful representation of \(ES\), it must be the case that:

\[\forall A,B \in R^* \forall i,j \in I((i,A),(j,B) \in \phi \wedge (A,B) \in MR \rightarrow (i,j) \in CR_k)\]

If any pair of individuals has choice sets that are an element of the measurement relation in \(MS\), this pair of individuals must be an element of the characteristic relation in the empirical structure that \(MS\) represents. The assurance of a faithful representation will depend upon an assurance that this condition is met. This assurance is the subject of the following section.

5. THE USE OF SURVEY DATA

I have assumed, throughout the previous discussion, that neither the preference set nor the characteristic relation is observable. Indeed, if characteristic relations were observable, none of the assumptions that were made for the purpose of accessing their content would be required. One could, if interested, simply design an exper-
iment and directly observe characteristic relations such as "more hawkish than" or "more supportive than." If a preference set were observable, neither Assumption 1 (preference sets over offered responses determine choices) or Assumption 2 (a specific choice function translates preference sets into choices) would be required. One could simply observe the content of preference sets and use Assumption 3 (if a pair of individuals has sufficiently different preference sets they will be in a characteristic relation) to access the content of characteristic relations. But, because neither the characteristic relation nor the preference set is observable, all three assumptions are required. These assumptions provide a linkage between the unobservable characteristic relation to the observable choice. This linkage rationalizes the use of a survey. It does not, unfortunately, guarantee its utility. In order to answer the question of utility, we must ask some informed questions about the measurement properties of a survey.

5.1 Reversing the Process

The rationalizing assumptions take us from unobservable characteristic relations to observable choice sets; they do not guarantee a return. I have yet to show that it is possible to move from observed choice sets to unobservable preference sets, and from these preference sets to unobservable characteristic relations. And, it is this sequence of moves that must be made if the survey is to be used to construct a measurement structure (see Sen, 1973; p. 245, for a similar requirement). Thus, evaluating the possibility of such
moves is necessary to an evaluation of the survey's utility. Unfortunately, a data set that is obtained from a survey will only provide the results of an extremely complex process. It will indicate what individuals chose when given items about social welfare, busing, Vietnam involvement, etc. It will not give direct information about characteristic relations such as "more hawkish than" or "more supportive than." Before these relations can be accessed, additional assumptions must be made. I will show, using the theory of choice presented in Chapter 3, that the movement from observed choice set to characteristic relation can be executed faithfully, but only under certain conditions. Among the questions that are addressed in this section are: What must be specified in the move from choice set to characteristic relation? And, how are measurement relations constructed so as to render a measurement structure a faithful representation of an empirical structure?

Figure 4 provides an illustration of the information that must be specified before a choice experiment can be used for accessing the content of a characteristic relation. C' is a set containing information that is needed for movement from choice set to preference set. It, for example, will indicate how to get from choice sets on a busing item to preference sets with regard to the busing issue. Γ' is a set containing information that is needed for movement from preference set to characteristic relation. Assuming it is possible to move from choice sets on a busing item to preference sets on a busing issue, these preference sets must be useful for placing individuals in the
Fig. 4: An illustration of the type of information that must be specified before the survey can be used to access the content of characteristic relations.
appropriate characteristic relation (i.e., 'is more pro-integration than'). \( \Gamma \) will insure this use. If the contents of both \( C' \) and \( \Gamma \) can be specified without requiring more than the theory of choice allows, and without violating the conditions of a faithful representation, the task will be complete. The problem of moving from choice sets back to characteristic relations will have been solved. Because the specification of \( \Gamma \) will be easier than the specification of \( C' \), I will consider \( \Gamma \) first.

5.2 Moving from Preference Sets to Characteristic Relations

Consider a paraphrase of Assumption 3: If a pair of individuals has sufficiently different preference sets, this pair will be in a characteristic relation. It is this assumption that motivates the construction of \( \Gamma \). Without it, there would be no justification for using preference sets to access characteristic relations. Naturally, one would expect this assumption to be useful in a specification of \( \Gamma \). And, it does suggest that \( \Gamma \) must contain a set of "sufficiently different" preference sets. So, \( \Gamma \) will be defined on \( \mathcal{H} \times \mathcal{J} : \mathcal{H} \subseteq \mathcal{J} \times \mathcal{J} \). \( \Gamma \), like \( CR_1 \) and \( MR \), will be assumed to be asymmetric. \( Y_1 \)-restricted \( \Gamma (\bar{\Gamma}) \) will be the set that contains the preference set pairs that are in \( \Gamma \) and that remain different when each of the preference sets is \( \Gamma \) \( Y_1 \)-restricted. The preference sets contained in \( \bar{\Gamma} \) are the only preference sets in \( \Gamma \) that can be accessed by an item in which \( R= Y_1 \). \( \bar{\Gamma} \) contains, as elements, pairs of preference sets. But, we would not want any arbitrary pair of preference sets to be an element of \( \bar{\Gamma} \). We would only want those that, for some characteristic relation, are
"sufficiently different" to imply membership in a CR. For this purpose, I impose the following general condition on the content of $\Gamma$.

**CONDITION 1:** $(P(i), P(j)) \in \Gamma \Rightarrow (i, j) \in CR_k$

According to Condition 1, if a pair of preference sets is in $\Gamma$, then every pair of individuals with these preference sets must be in the corresponding characteristic relation. If $\Gamma$ can be specified so as to satisfy Condition 1, we will have one of the two pieces of information that must be provided if the choice experiment is to be used to access characteristic relations.

Decisions as to which preference set pairs are to be in $\Gamma$ are, of course, made by a scientist. The choice experiment does not allow the testing of these decisions. Each has the status of an assumption. Indeed, it is on such assumptions that the value of a survey rests.

These assumptions can be expressed in the following manner: If two individuals have significantly different preference sets with regard to the responses on a judiciously selected item, one will be "more efficacious than" the other or "more supportive than" the other, etc. Notice, however, that Condition 1 does not require $\Gamma$ duplicate, with preference set pairs, the content of a characteristic relation. Individuals may stand in a characteristic relation yet their preference sets may not be "significantly different" (see section 1 for a discussion of the disadvantages of complete representation). Condition 1 provides flexibility in the specification of $\Gamma$. It only requires that the relational structure $<\mathcal{D}; \Gamma>$ be a faithful representation of the empirical structure $<I; CR_k>$. Condition 1 provides an adequate specification of $\Gamma$. I now turn to the specification of $C'$. 
5.3 Moving from Choice Sets to Preference Sets

Recall, from section 2, that preference sets are asymmetric binary relations on $X$. These sets produce the choice sets that are observed in surveys. The linkage between preference sets and choice sets is specified in a choice function. This function allows us to move from a knowledge of presentation and $Y_1$-restricted preference sets to a knowledge of subsequent choice sets. We must now consider a function ($C'$) that will allow us to move from a knowledge of presentation and choice sets to a knowledge of $Y_1$-restricted preference sets. $^8$

$C'$ will be a function of the form: $C': C(Y_1,P_j) \rightarrow \mathcal{Y} \cap (Y_1,Y_1)$. $Y_1$ is a known presentation set, $C(Y_1,P_j)$ a known choice set, and $P_j$ an unknown preference set. $C'$ must generate, from this information, the content of a $Y_1$-restricted $P$. Suppose, in this regard, that $C'$ is required to satisfy the following property:

$P_I$: $C'(C(Y_1,P_j)) = \bar{P}_j$

If $C'$ can satisfy $P_I$, it will be able to generate the antecedent $Y_1$-restricted preference set for every presentation and choice set pair. It will also allow me to conclude this investigation of the survey's measurement properties by specifying a procedure for constructing the relation in $\langle R^*; MR \rangle$. Before commenting on the possibility of $C'$ satisfying $P_I$, I will consider the specification of $MR$.

5.4 The Content of $MR$

$MR$ will be required to satisfy the following property:

\textbf{Condition 2:} $(A,B) \in MR \iff$

\begin{enumerate}
  \item $A,B \in R$, \\
  \item $(A,F_1),(B,F_j) \in C'$, and \\
  \item $(P_1,P_j) \in \bar{C}'$. 
\end{enumerate}
A pair of sets can be an element in $\mathcal{MR}$ if and only if 1) the sets are choice sets for some set $\mathcal{R}$ of responses, and 2) the $Y_i$-restricted preference sets that generate these choice sets are an element in $\mathcal{I}$. It is easy to verify that, with Condition 1 and $\mathcal{Pl}$, Condition 2 is sufficient to render $\langle \mathcal{R}^*; \mathcal{MR} \rangle$ a faithful representation of $\langle \mathcal{I}; \mathcal{CR}_i \rangle$. If it is possible to obtain the content of antecedent $Y_i$-restricted preference sets, it will be possible to faithfully represent empirical structures with measurement structures that are derived from surveys, regardless of the content of $\mathcal{I}$. Unfortunately, if it is not possible to obtain the content of antecedent $Y_i$-restricted preference sets from an observation of choice sets, it will be necessary to insure that a certain relationship exists between indistinguishably different preference sets and meaningfully different preference sets, where each is defined as follows:

**DEFINITION 4.5.1:** $P_i$ and $P_j$ are said to be indistinguishably different preference sets if and only if

$$C'(c(Y_i, P_i)) = C'(c(Y_j, P_j))$$

**DEFINITION 4.5.2:** $P_i$ and $P_j$ are said to be meaningfully different preference sets if and only if

$$(P_i, P_j) \in \mathcal{I}, (P_i', P_j) \in \mathcal{I}, \text{ and } P_a, P_a' \text{ are indistinguishably different}$$

I will discuss the exact nature of this relationship in a later section. At present, $\mathcal{Pl}$ renders the discussion unnecessary. It guarantees that, if $Y_i$-restricted preference sets are not identical, they are not indistinguishably different. Any difference is, if $C'$ satisfies $\mathcal{Pl}$, distinguishable. So, I will now consider the question of whether or not it is possible, given the previously discussed rationalization of the survey, to insure that a specification of $C'$ will satisfy $\mathcal{Pl}$. 
6. THE SPECIFICATION OF C'

Suppose that, under RC3, different $Y_i$-restricted preference sets could result in identical choice sets. Suppose, in other words, that two individuals could have different $Y_i$-restricted preference sets yet give identical responses on a survey item. It would not be possible, under these conditions, to insure that a specification of $C'$ satisfies Pl without assuming a direct knowledge of individual preference sets (something which has been explicitly assumed to be impossible). It would not be possible to distinguish one preference set from another (different) preference set. Thus, if it is possible to specify the conditions under which a single choice set can be produced by different $Y_i$-restricted preference sets, it will be possible to obtain some needed information. We will know that, under these conditions, it will be impossible to insure that a specification of $C'$ will satisfy Pl. So, when will these conditions obtain?

6.1 The Size of R and the Specification of C'

Suppose we begin by fixing the size of a set of responses. We know the number of different choice sets that attend this set, and we know (from the definition of a function) that there must be at least as many different $Y_i$-restricted preference sets as there are different choice sets. For, each choice set must be associated with a different preference set. The important question is: What will occur if there are more $Y_i$-restricted preference sets than there are different choice sets? If, for some set of responses, the number of $Y_i$-restricted preference sets is greater than the number of choice sets,
there must be duplication. Different $Y_i$-restricted preference sets must be associated with a single choice set.

The original question can now be answered. It will be impossible to insure that a specification of $C'$ will satisfy $P_l$ if, for the presentation set in question, the number of $Y_i$-restricted preference sets is greater than the number of choice sets. However, as will be shown, this condition is frequently met. Indeed, as the cardinality of $Y_i$ increases, the difference between the number of $Y_i$-restricted preference sets and the number of choice sets also increases. The figures are displayed in Table 2. In the case of the four-alternative Vietnam item (see section 2), there are 729 different $Y_i$-restricted preference sets and only 15 different choice sets. In general, if a presentation set contains more than two alternatives it will be impossible to insure that a specification of $C'$ will satisfy $P_l$. It is only in the two-alternative presentation set that the number of $Y_i$-restricted preference sets does not exceed the number of different choice sets.

This result poses a serious problem with regard to the utility of a survey. The cardinality of $R$ must either be restricted, or, $P_l$ replaced with an appropriate and satisfiable requirement. There are advantages and disadvantages attending both the restriction of $R$ and the replacement of $P_l$. A major advantage of two-alternative sets has already been cited. It is possible, with these sets, to obtain measurement structures that are faithful representations of empirical structures without restricting the content of $\Gamma$. This can be done
**TABLE 2**

RELATIONSHIP BETWEEN THE NUMBER OF ALTERNATIVES IN A PRESENTATION SET, THE NUMBER OF DIFFERENT CHOICE SETS ATTENDING THAT PRESENTATION SET, AND THE NUMBER OF DIFFERENT $Y_1$-RESTRICTED PREFERENCE RELATIONS THAT CAN BE DEFINED ON THAT PRESENTATION SET

<table>
<thead>
<tr>
<th>Number of Alternatives</th>
<th>Number of Choice Sets</th>
<th>Number of $Y_1$-Restricted Preference Sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>27</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td>729</td>
</tr>
<tr>
<td>5</td>
<td>31</td>
<td>$5.90 	imes 10^4$</td>
</tr>
<tr>
<td>6</td>
<td>63</td>
<td>$1.43 	imes 10^7$ (a)</td>
</tr>
<tr>
<td>7</td>
<td>127</td>
<td>$1.05 	imes 10^{10}$ (a)</td>
</tr>
<tr>
<td>8</td>
<td>255</td>
<td>$2.29 	imes 10^{13}$ (a)</td>
</tr>
</tbody>
</table>

(a) approximate
because, in a two-alternative presentation set, it is possible to insure that a specification of $C'$ will satisfy $P_1$. And, as has been shown, an arbitrary $\Gamma$ implies that $C'$ satisfy $P_1$. I will now investigate the disadvantages of two-alternative presentation sets. This investigation will be followed by an investigation of the conditions that justify the use of several-alternative presentation sets.

7. A DISADVANTAGE OF THE TWO ALTERNATIVE PRESENTATION SET

Consider, for a moment, an arbitrary $<I; CR_I>$ with unknown but fixed content. I have assumed that a knowledge of individual preference sets will be useful in discovering this content. Preference sets provide material for placing individuals in $CR_I$. Significantly different preference sets (those that are, in accord with $C_1$, different enough to imply that the individuals possessing them be in $CR_I$) are specified in $\Gamma$. Now, in order to obtain (from a survey) as much information as possible about the content of $CR_I$, would it not make sense to consider the relationship between type of item and amount of information that can be obtained (about the content of $CR_I$) through the use of such an item? It would seem so.

7.1 Obtainable Information and the Size of $\Gamma$

$\Gamma$ contains pairs of preference sets that are significantly different in the sense that they imply characteristic relation membership. Now, suppose that items with two-alternative presentation sets are used for obtaining a knowledge of individual preference sets. What is the maximum number of different preference sets that can be revealed as a
result of this item's use? An individual can only exhibit one of three possible choice sets (see Table 3). Thus, an individual can only be assigned one of three different \( Y_1 \)-restricted preference sets. With only three different \( Y_1 \)-restricted preference sets, what is the maximum number of elements that \( \Gamma \) can contain? Regardless of the number of elements in \( \Gamma \), \( \Gamma \) can contain at most three elements (recall that \( \Gamma \) is assumed to be asymmetric). This number, if compared to the numbers of elements \( \Gamma \) could contain if items have several alternatives, is severely limited. The appropriate comparisons are given in Table 3. One can see, from this table, that an increase in the number of alternatives that an item contains will result in an increase in the number of elements that \( \Gamma \) can contain. A move from a two-alternative to a three-alternative item will double the number.

Now, because the maximum number of elements in \( \Gamma \) places an upper bound on the amount of information that can be obtained as to the content of a \( CR_1 \), there is a strong incentive to use items with several-alternatives. The limited information that two-alternative items convey is a major disadvantage. A question remains, however, as to how items with several-alternatives can be used to construct measurement structures that are faithful representation of \( <I;CR_1> \) empirical structures. This question is addressed in the following section.

8. A JUSTIFICATION OF THE SEVERAL-ALTERNATIVE CHOICE PROBLEM

I have, to this point, discussed the possibility of moving from a knowledge of presentation sets and choice sets to a knowledge of
TABLE 3

RELATIONSHIP BETWEEN THE NUMBER OF ALTERNATIVES IN THE CHOICE PROBLEM AND THE MAXIMUM NUMBER OF ELEMENTS THAT \( \bar{F} \) CAN CONTAIN

<table>
<thead>
<tr>
<th>Number of Alternatives in the Choice Problem</th>
<th>Maximum Number of Elements in ( \bar{F} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>6</td>
<td>21</td>
</tr>
<tr>
<td>7</td>
<td>28</td>
</tr>
<tr>
<td>8</td>
<td>36</td>
</tr>
</tbody>
</table>
antecedent $Y_1$-restricted preference sets. I considered this possibility without reference to the content of $\Gamma$. There was a reason for this approach. If it had been possible to move from presentation and choice sets to antecedent $Y_1$-restricted preference sets, it would have been possible to construct faithful representations of our empirical structures given items with an arbitrarily large number of alternatives and $\Gamma$'s with arbitrary contents. Unfortunately, it was determined that the move is only possible when items contain two alternatives. I will now investigate the possibility of insuring faithful representations given the impossibility of obtaining antecedent $Y_1$-restricted preference sets from observed presentation and choice sets. This will, of course, require that PI be relaxed. If the investigation is successful, it will be possible to obtain faithful representations of $\langle I; CR_1 \rangle$ empirical structures with the use of several-alternative items.

8.1 A Relaxation of PI

Consider a property that is somewhat weaker than PI:

$$P2: C'(C(Y_i, P_j)) = \overline{a} \rightarrow C(Y_i, P_j) = C(Y_i, a)$$

P2 requires that $C'$ generate, from a known presentation and choice pair, a preference set that could have generated the choice set. P2, unlike PI, does not require $C'$ generate the antecedent $Y_1$-restricted preference set. A major advantage of P2 over PI is the satisfiability of the former. Whereas it is possible to insure that a specification of $C'$ will satisfy P2 (regardless of presentation set size), it is not possible to make a similar claim with respect to PI. The im-
important question is: Under what conditions will the satisfaction of P2 be sufficient to insure the faithful representation of \( \langle I; CR_4 \rangle \) empirical structures? An answer to this question will require a further investigation of \( \Gamma \).

The previous investigation indicated that it is impossible to insure faithful representations of empirical structures if the content of \( \Gamma \) and the size of a presentation set are allowed to remain arbitrary. For faithful representation, an arbitrary \( \Gamma \) implies that any two different preference sets not be indistinguishably different (see, in this regard, definition 4.5.1). But, in order to insure that any two different preference sets not be indistinguishably different, one must insure that a specification of \( C^* \) will satisfy P1. And, as I have shown, this is generally impossible. In order to obtain the additional information that a several-alternative presentation set can provide, P1 must be relaxed. The weaker P2 is not unduly restrictive in that its satisfaction can be guaranteed for items containing several-alternatives. Unfortunately, with P2, Condition 1 and Condition 2 will not insure faithful representation. For this, it will be necessary to add a condition that will restrict the content of \( \Gamma \).

8.2 The Restriction of \( \Gamma \)

The nature of the required restriction is suggested in the following example:
In this example, a portion of the content of an \( <I;CR_1> \) empirical structure is specified (i). The responses associated with an item that is used to access this \( <I;CR_1> \) are given in (ii). A portion of the content of \( \overline{P} \) is given in (iii) and (iv). Some individual \( Y_i \)-restricted preference sets are given in (v). With these, the content of \( \phi \) can be specified (vi). Now, is it possible to insure a faithful representation of this \( <I;CR_1> \) given Condition 2, \( P_2 \), the preference sets in (v) and the \( \overline{P} \) in (iv)? Unfortunately, it is not. First, suppose that \( C^*\{a_3\}, \cdot \rangle = P_{m} \) and \( C^*\{b_3\}, \cdot \rangle = P_{g} \). If so, \( (\{a_3\}, \{b_3\} ) \in MR \) (by Condition 2). But, then \( (\phi(3), \phi(2)) \in MR \) and \( (3,2) \notin CR_1 \). Thus, \( <R^*;MR> \) is not a faithful representation of \( <I;CR_1> \). Suppose that \( C^*\{a_3\}, \cdot \rangle = P_{m} \) and \( C^*\{b_3\}, \cdot \rangle = P_{g} \). If so, \( (\{b_3\}, \{a_3\} ) \in MR \). But, then \( (\phi(2), \phi(1)) \in MR \) and \( (2,1) \notin CR_1 \). Thus, there is no possible combination of assignments for \( C' \) that will ensure that \( <R^*;MR> \) is a faithful representation of \( <I;CR_1> \).
The type of problem this example poses can be prevented. Consider, in this regard, the following definition:

DEFINITION 4.8.1: \( \mathcal{P}^Z = \{ \mathcal{P}_1 | C(Y_j, P_1) = Z \} \)

\( \mathcal{P}^Z \) is a set of \( Y \)-restricted preference sets. These sets are, under \( P^2 \), indistinguishably different. Sets such as these must be prohibited from creating situations similar to the one described in the previous example. Consider, in this regard, Condition 3:

CONDITION 3: \( \forall Z, Z'((\mathcal{P}_1 \in \mathcal{P}^Z, P_j \in \mathcal{P}^Z') \in \overline{\mathcal{F}} \rightarrow \forall \mathcal{P}_k \in \mathcal{P}^Z \forall \mathcal{P}_m \in \mathcal{P}^Z (\mathcal{P}_k, \mathcal{P}_m) \in \overline{\mathcal{F}}) \)

Condition 3 requires that, for any pair of \( Y \)-restricted preference sets in \( \overline{\mathcal{F}} \), if one or both preference sets in the pair is replaced with an indistinguishably different preference set, the resulting pair must also be in \( \overline{\mathcal{F}} \). Condition 3 creates a set of equivalence classes over the set \( \mathcal{F} \) (see Pfangagl, 1968, Chapter 2; Fishburn, 1973, Chapter 7; or Suppes & Zinnes, 1963, p. 24, for a discussion of equivalence classes). Membership in these classes is determined by the content of choice sets. If two preference sets in \( \mathcal{F} \) produce identical choice sets for some \( Y \), then these two preference sets will be in the same set \( \mathcal{P}^Z \) for that \( Y \). This condition, together with Condition 2 and \( P^2 \), is sufficient to insure that an \( \langle R^*, MR \rangle \) measurement structure provides a faithful representation of an \( \langle I; CR_i \rangle \) empirical structures.

9. RESTRICTING THE CONTENT OF CHOICE SETS

Recall, from Chapter 3, that choice sets are seldom observable. This, unfortunately, is due to an often unavoidable placement of restrictions on the content of choice sets (see, in this regard, Chapter
3, section 3). Recall, further, that discussions in this chapter have been developed as though choice sets were observable. This was done for the purpose of generality. For, if a technique is successful in allowing the observation of choice sets, the present discussions will stand. Measurement problems that are specific to the restriction of choice set content can then (as they will in this section) be discussed as special cases of the more general problems. The major question is: Will the limited successes (Condition 1, P1, Condition 2, and an item with two alternatives are sufficient to insure faithful representation, as are Condition 1, Condition 2, P2, and Condition 3) obtained from an assumption that choice sets are observable remain when the assumption is relaxed and the content of choice sets is restricted?

9.1 Selection vs. Choice

The numbers of choice sets associated with items of various sizes are given in Table 2. We know, however, that surveys do not allow all of these choice sets to be revealed. Typically, individuals are required to select a single response from an offered set (see, in this regard, Pfangagl, 1968; or Bush, Galanter, and Luce, 1963). Thus, with surveys, one must construct measurement structures from selection sets (written $S(Y_i, P_j)$). The cardinality and content of these sets will vary with the nature of the imposed restrictions. The first question then becomes: What is the relationship between selection and choice sets. Often items, a selection set is simply assumed to be a single element subset of an antecedent choice set (Richter, 1971, p. 31).
However, as Farquharson (1969) has indicated, this is not always a warranted assumption. An individual may, in some cases, be more likely to receive an alternative in his choice set if he selects an alternative that is not in his choice set. Although the Farquharson examples are powerful, I will assume that, in a survey, individuals will select from their choice sets. Thus, I will impose the following restriction on the content of selection sets:

\[ R_1: S(Y_i, P_j) \subseteq C(Y_i, P_j) \]

This restriction is assumed to apply to all items that require the selection of a single alternative.

The major concern must, of course, rest with the condition of faithful representation. Will the insurance that an \( \langle R^*, MR \rangle \) measurement structure is a faithful representation of an \( \langle I; CR_1 \rangle \) empirical structure suffer from a restriction \( (R_1) \) that is inherent in the use of surveys? Unfortunately, it will. Consider, in this regard, a situation in which \( P(i) = P(j) \) and \( S(Y_k, P(i)) \neq S(Y_k, P(j)) \). It would not be possible, in this case, to specify \( C' \) in accordance with either \( P_1 \) or \( P_2 \), regardless of the number of alternatives an item contains. It will even be impossible to insure faithful representations through the use of items with two-alternatives.

9.2 Faithful Representations via Forced Choice Problems

The consequences of restricting choice could, of course, be ignored if choice sets could always be assumed to be singletons. Consider, in this regard, the following property:

\[ P_3: \forall Y_j, \exists x \in I(\# C(Y_j, P(1)) = 1) \]
With this property, Condition 1, P2, Condition 2, Condition 3, and R1 will insure that survey-based measurement structures can be faithful representations of \(<I; CR_1>\) empirical structures.

How can the use of P3 be justified? It can be justified on the basis of plausibility or on the basis of a low expected cost. P3 will only be considered plausible if, when interpreted by an \(<\text{ALTERNATIVE}; \text{PREFERENCE}>\) empirical relational structure, it is considered to be true (see, in this regard, Chapter 2, section 3). And, P3 will only be considered to have a low expected cost if the probability of violating faithful representation as a result of violating this assumption is low. I will consider plausibility first. For, if P3 is determined to be plausible, no expected costs will be incurred as a result of its use. This will be the case, because P3 will only be considered plausible if there is thought to be no chance of its violation.

It is obvious that P3 can only be considered plausible if the content of preference sets can always be assumed to be such that it is satisfied. Recall (from Chapter 3, section 3), in this regard, that choice sets will only contain singletons if CF6 and CF7 can be assumed. Thus, if P3 is to be considered plausible, CF6 and CF7 must also be considered plausible. But, arguments against each of these conditions were given in Chapter 3. Thus, rather than review them here, I will simply state that because the use of P3 requires that these properties be plausible, it cannot be considered a plausible property.

9.3 The Expected Cost of CF6 and CF7

To calculate the expected cost of CF6 and CF7, I will consider the probability of violating faithful representation as a result of
violating one of these assumptions. Suppose, in this regard, that
the selection sets in a survey are assumed to satisfy R1. Suppose,
进一步，假设 CF6 和 CF7 被假设。问题在于，如果 CF6 和
CF7 不是可能的，那么它们将被违反的概率是多少？为了回答这个问题，我将假设一个偏好
集合不比另一个更可能出现。有了这个假设，我可以比较（1）没有 CF6 和 CF7 存在的偏好
集合的数量，以及（2）在假设 CF6 和 CF7 存在的情况下，可以假设存在多少偏好
集合。

百分比的减少从（1）到（2）将作为违反 CF6 和 CF7 可能性
的指标。这也将作为违反忠实代表性的概率，作为这些假设的结果。因为，如果它们被违
反，则忠实代表性的保证就无法实现。

表 4 包含了各种项目的百分比减少信息。从表中可以明
显看出，这些假设具有非常高的预期成本。例如，如果一项对象
包含四个或更多的替代方案，违反 CF6 和 CF7（如果它们不是可能的）的概率
几乎为 1.0。因此，必须得出结论，尽管这些假设的预期成本由
于对象包含的替代方案数量而受到影响，它们仍然具有普遍
高的预期成本。

10. CONCLUSION

我已表明，我提出的个别选择理论外

### TABLE 4

RELATIONSHIP BETWEEN NUMBER OF PREFERENCE RELATIONS THAT SATISFY ASYMMETRY AND NUMBER OF PREFERENCE RELATIONS THAT SATISFY ASYMMETRY, TRANSITIVITY, AND CONNECTEDNESS, FOR VARIOUS CHOICE PROBLEM SIZES

<table>
<thead>
<tr>
<th>Size of Presentation Set</th>
<th>Number of Preference Sets that Satisfy Asymmetry</th>
<th>Number of Preference Sets That Satisfy Asymmetry, Transitivity, and Connectedness</th>
<th>Percentage Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>2</td>
<td>33%</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
<td>6</td>
<td>78%</td>
</tr>
<tr>
<td>4</td>
<td>729</td>
<td>24</td>
<td>97%</td>
</tr>
<tr>
<td>5</td>
<td>59,049</td>
<td>120</td>
<td>99.8%</td>
</tr>
<tr>
<td>6</td>
<td>14,348,907</td>
<td>720</td>
<td>99.9%</td>
</tr>
</tbody>
</table>
Chapter 3 does, indeed, have empirical significance. I have used it to point out the problems that can arise when a survey is used for generating measurement structures. The theory has also been instructive in terms of the design of a survey. Unless it is possible to provide evidence that preference sets over the alternatives contained in an item satisfy CP6 and CP7 then the use of items that either (1) contain over two alternatives, or (2) restrict choice, is unjustifiable.

The discussions contained in this chapter also illustrate that measurement is possible when empirical structures do not accept numerical representations. The measurement of preference has, in the past, been based on an assumption that preference sets do satisfy conditions that will allow numerical representation (see, in this regard, Coombs, 1964, Chapter 1). This, as I have shown, is unduly restrictive. The focus should be on faithful, not necessarily numerical, representation.
1. I will only be dealing with relational structures that contain a single relation. Thus, the relational component of $\Phi$ will be assumed (i.e., $\Phi(ER) = MR$).

2. See Chapter 2, section 4 for a discussion of asymmetry.

3. The use of $\langle \text{INTEGERS}; \text{GREATER THAN} \rangle$ numerical structures implies that the corresponding empirical structures satisfy asymmetry.

4. The rationalization of a choice experiment is seldom discussed. I am not, at this point, aware of any other rationalization for a survey.

5. Bush, et al. (1963, p. 82) describe the choice experiment that is designed to obtain information about preference sets in a similar manner.

6. Keep in mind that this discussion is only dealing with items that are assumed to involve valuation functions.

7. Keep in mind that this discussion is assuming that choice sets are observable.

8. Although it might be possible to require that $C'$ only be a relation, this would allow the assignment of different preference sets given a single choice set. And, this type of assignment is unacceptable.

9. This number is $2^n - 1$, where $n$ is the number of alternatives in a presentation set.

10. $P_1$ and $P_2$ will only be equivalent if the number of different $Y_i$-restricted preference sets does not exceed the number of different choice sets.
V. CONCLUSION

I will, in this final chapter, briefly summarize some of the steps that have been taken in this dissertation. This summary will not include the formal components of the previous chapters. It will only describe some of the major points that have been made.

1. THE PROBLEM OF LANGUAGE SELECTION

Two very important and consequential points were made in Chapter 2. First of all, it is necessary to distinguish a relational structure from a language that might be used for the description of that structure. And secondly, because languages can differ significantly, it is necessary (when dealing with empirical structures) to consider the compatibility between a given structure and a given language. Whereas some languages are entirely appropriate for the description of certain structures, others are not.

I argue, in Chapter 2, that certain languages are better than others with regard to the expression of scientific theories. The better languages must accept a reasonable theory of meaning, have a specifiable rule of inference, and be consistent. Ordinary languages cannot satisfy these conditions. Thus, they are not very useful for the expression of scientific theories. One of the major problems with ordinary languages is the inseparability of syntax and semantics. These languages tend to give the impression that their sentences are actually describing some empirical structure when, in fact, no such structure has been specified. It is for this and other reasons that
formal languages are the only languages that are considered for the expression of a theory of individual choice. Ordinary languages are used to "talk about" these formal languages, but the formal languages are used to "talk about" choice.

2. THE DESCRIPTION OF PREFERENCE BASED CHOICE

Most formal investigations of preference-based choice have been conducted by economists. Their selection of a language for the description of choice has typically been motivated by 1) a language's impact on the predictability of an individual's demand, and 2) a language's consequences in terms of the numerical representation of an empirical structure. The former is an empirical motivation and the latter is for convenience. Much is known about the languages that describe numerical structures. The motivation for the theory presented in Chapter 3 is plausibility. If there is no justification for the restrictions that a language will impose on the nature of choice, this language is not used for the description of choice. The product of this process is a language that is meant to describe all preference-based choices, regardless of the content of an individual's preference set. I note, in conclusion, that the languages used by economists are often special cases of the language I construct. If that which the economists' languages require obtains, the implications of these languages for individual demand will coincide with the implications of the language I use.
3. THE LOGIC OF SURVEY RESEARCH

As Richter (1971) points out, one of the most important properties of a theory of individual choice is its ability to tell us something (non-trivial) about choice. I show, in Chapter 4, that the selected theory does, indeed, have empirical consequences. And, these consequences bear on that which political scientists do. With regard to the conduct of survey research, the theory suggests the types of assumptions that must be made in order for this research to be useful. It also provides information as to the conditions that must be met before the measurement structures constructed from the results of surveys will be faithful representations of the empirical structures in which the political scientist is interested. In doing so, the theory suggests ways in which a survey might be conducted so as to reduce the possibility of measurement error. Another important result of Chapter 4 is the illustration that numerical representations of empirical structures is not required for precise measurement. And, because the selected theory of individual choice does not allow numerical representation, this result is very important.
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