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DETERMINATION OF THE DECAY PARAMETERS
OF RESONANT STATES

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the
Degree Doctor of Philosophy in the Graduate School of
The Ohio State University

By

Nicholaos Tsoupas, B.S., M.S.

* * * * *

The Ohio State University

1974

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DEDICATION

To my parents and teacher
ACKNOWLEDGEMENTS

I wish to express my sincere thanks to Professor H. J. Hausman, my adviser, whose continuous instructions, advise and stimulating discussion made the undertaking of this project an instructive pleasure and experience.

I would like also to thank Dr. N. L. Gearhart for his collaboration; Dr. R. G. Seyler for his valuable advise and guidance in the theoretical part of this work; R. L. Johnson, our machine shop supervisor, who made me familiar with the machinery of the shop; Dr. J. F. Morgan for his discussions about some theoretical aspects of this work; G. H. Terry for helping collect the experimental data; and D. Alexander for his assistance in the electronics of the experiment.

I can find no words to thank Dr. L. A. Alexander for his patience and great help in the computer work from the beginning of this project. A lot of thanks go to Dr. W. S. McEver not only because his black and white pictures gave a beautiful color in this dissertation but also because of the long and interesting discussions about important matters of nuclear physics and life!

I am indebted to Drs. H. J. Hausman, R. G. Arns, and S. L. Blatt for the important suggestions and corrections of the present dissertation. Miss Sherrie Shadoan deserves my thanks for the fine job she did in typing the final draft.
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>ii</td>
</tr>
<tr>
<td>VITA</td>
<td>iii</td>
</tr>
<tr>
<td>LIST OF ILLUSTRATIONS</td>
<td>vi</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>ix</td>
</tr>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>A. Nuclear Reaction Models</td>
<td>1</td>
</tr>
<tr>
<td>B. Nuclear Structure Models</td>
<td>7</td>
</tr>
<tr>
<td>B1. Weak Coupling Model</td>
<td>9</td>
</tr>
<tr>
<td>B2. Strong Coupling Model</td>
<td>11</td>
</tr>
<tr>
<td>C. Partial Decay Proton Widths of $^{29}$P Nucleus</td>
<td>14</td>
</tr>
<tr>
<td>II. EXPERIMENTAL PROCEDURE</td>
<td>15</td>
</tr>
<tr>
<td>A. Angular Distributions</td>
<td>15</td>
</tr>
<tr>
<td>B. Goldfarb-Seyler Angular Correlation</td>
<td>25</td>
</tr>
<tr>
<td>C. Spin-Flip Angular Correlations</td>
<td>32</td>
</tr>
<tr>
<td>III. ANALYSIS OF THE EXPERIMENTAL DATA</td>
<td>38</td>
</tr>
<tr>
<td>A. Angular Distribution</td>
<td>38</td>
</tr>
<tr>
<td>B. Angular Correlation in the Spin-Flip Geometry</td>
<td>46</td>
</tr>
<tr>
<td>C. Angular Correlation in the Goldfarb-Seyler Geometry</td>
<td>50</td>
</tr>
<tr>
<td>D. Resonance Analysis Program for the Determination of the Partial Decay Proton Widths</td>
<td>55</td>
</tr>
<tr>
<td>E. Discussion of Results</td>
<td>80</td>
</tr>
<tr>
<td>E1. Discussion of Results</td>
<td>80</td>
</tr>
<tr>
<td>E2. The 3.10 MeV Resonance</td>
<td>82</td>
</tr>
<tr>
<td>E3. The 3.57 MeV Resonance</td>
<td>82</td>
</tr>
<tr>
<td>E4. The 4.23 MeV Resonance</td>
<td>83</td>
</tr>
<tr>
<td>E5. The 4.43 MeV Resonance</td>
<td>83</td>
</tr>
</tbody>
</table>
TABLE OF CONTENTS (continued)

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV. CONCLUSIONS</td>
<td>85</td>
</tr>
<tr>
<td>V. SUMMARY</td>
<td>92</td>
</tr>
<tr>
<td>APPENDIX</td>
<td>94</td>
</tr>
<tr>
<td>I. DERIVATION OF THE THEORETICAL EXPRESSION OF THE ANGULAR DISTRIBUTION OF SCATTERED PARTICLES</td>
<td>94</td>
</tr>
<tr>
<td>II. DERIVATION OF THE THEORETICAL EXPRESSION OF THE TRIPLE ANGULAR CORRELATION</td>
<td>105</td>
</tr>
<tr>
<td>III. SPECIAL GEOMETRIES USED IN ANGULAR CORRELATION</td>
<td>113</td>
</tr>
<tr>
<td>IV. TARGET PREPARATION</td>
<td>123</td>
</tr>
<tr>
<td>1. (^{28})Si Target on Carbon Backing</td>
<td>123</td>
</tr>
<tr>
<td>2. (^{28})Si Self-Supported Target</td>
<td>123</td>
</tr>
<tr>
<td>3. (^{24})Mg Self-Supported Target</td>
<td>124</td>
</tr>
<tr>
<td>4. (^{176})Au Target Self-Supported</td>
<td>124</td>
</tr>
<tr>
<td>V. ALIGNMENT OF THE COLLIMATOR HOLDERS</td>
<td>125</td>
</tr>
<tr>
<td>VI. SOLID ANGLE MEASUREMENT</td>
<td>126</td>
</tr>
<tr>
<td>VII. ELECTRONICS ARRANGMENT</td>
<td>128</td>
</tr>
<tr>
<td>a) Angular Distribution</td>
<td>128</td>
</tr>
<tr>
<td>b) Angular Correlation in Spin-Flip Geometry</td>
<td>128</td>
</tr>
<tr>
<td>VIII. CONVERSION OF EXPERIMENTAL DATA INTO ABSOLUTE CROSS SECTION</td>
<td>136</td>
</tr>
<tr>
<td>IX. CORRECTION FOR THE FINITE SIZE OF THE (\gamma)-RAY DETECTORS</td>
<td>138</td>
</tr>
<tr>
<td>X. PROGRAMS USED FOR THE ANALYSIS OF THE EXPERIMENTAL DATA</td>
<td>140</td>
</tr>
<tr>
<td>A. SOUTH</td>
<td>140</td>
</tr>
<tr>
<td>B. DURA1</td>
<td>140</td>
</tr>
<tr>
<td>C. ADANA</td>
<td>140</td>
</tr>
<tr>
<td>D. SFANA</td>
<td>143</td>
</tr>
<tr>
<td>E. Program for Resonance Analysis</td>
<td>143</td>
</tr>
</tbody>
</table>
LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Schematic diagram for the formation and decay of a compound nucleus into different channels</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>Nuclear energy levels and radiations involved in a particular CN formation and decay</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>Schematic diagram of the reaction $^{28}$Si(p,p',γ) for the study of the resonant states of $^{29}$P</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>The 17&quot; &quot;ORTEC Model 600&quot; scattering chamber</td>
<td>18</td>
</tr>
<tr>
<td>5</td>
<td>Solid state detector mounting and collimation arrangement</td>
<td>21</td>
</tr>
<tr>
<td>6</td>
<td>Top view of interior of the scattering chamber (The arrangement of the detectors is shown)</td>
<td>23</td>
</tr>
<tr>
<td>7A</td>
<td>The proton spectrum of the $^{28}$Si + p reaction. The peaks from left to right are $^{28}$Si inelastic and $^{12}$C, $^{16}$O $^{28}$Si elastics (The $^{12}$C, and $^{16}$O peaks are due to the carbon backing of the $^{28}$SiO$_2$ target and the O$_2$ contained in the SiO$_2$ compound)</td>
<td>28</td>
</tr>
<tr>
<td>7B</td>
<td>The same as in Fig. A but only the $^{28}$Si inelastic peak allowed from the single channel analyzer to appear in the spectrum</td>
<td>30</td>
</tr>
<tr>
<td>8</td>
<td>The ORTEC scattering chamber with the NaI(Tl) γ-ray detector mounted on top for the performance of the angular correlation in the spin-flip geometry</td>
<td>33</td>
</tr>
<tr>
<td>9A</td>
<td>The p'-γ coincidence γ-ray spectrum of the angular correlation in the spin-flip geometry</td>
<td>36</td>
</tr>
<tr>
<td>9B</td>
<td>The corresponding γ-ray spectrum to Fig. 9A accidental p'-γ coincidence</td>
<td>36</td>
</tr>
<tr>
<td>10</td>
<td>Experimental points of an angular distribution of inelastically scattered protons from $^{28}$Si (The residual $^{28}$Si nucleus is left in its 2$^+$ first excited state.)</td>
<td>40</td>
</tr>
</tbody>
</table>
### LIST OF ILLUSTRATIONS (continued)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>11. Angular distributions of inelastically scattered protons from $^{28}\text{Si}$ over 3.34 MeV resonance of $^{29}\text{P}$. The least squares fit curves, calculated in terms of the first six Legendre polynomials, are shown</td>
<td>42</td>
</tr>
<tr>
<td>12. The plot of the expansion coefficients $A_0$ to $A_5$, versus energy of the Angular distributions</td>
<td>44</td>
</tr>
<tr>
<td>13. Experimental points of the $(p'-\gamma)$ angular correlation, in the spin-flip geometry, from the reaction $^{28}\text{Si}(p,p',\gamma)^{28}\text{Si}$</td>
<td>48</td>
</tr>
<tr>
<td>14. The least squares fit curve in terms of the $\cos(\kappa\phi)$ ($\kappa = 0, 1, 2, 3, 4, 5$) functions of the angular correlation in the spin-flip geometry</td>
<td>51</td>
</tr>
<tr>
<td>15. Plot of the expansion coefficient $A_6$ to $A_5$ versus energy of the angular correlations in the spin-flip geometry</td>
<td>53</td>
</tr>
<tr>
<td>16. The expansion coefficients $A_0, A_2, A_4$ versus energy, of the angular correlations in the Goldfarb-Seyler geometry</td>
<td>56</td>
</tr>
<tr>
<td>17A. The fit of the theoretically calculated coefficients $A_0, A_1, A_2$ of the angular distributions to the experimentally determined coefficients, over the energy region 2.9 to 3.8 MeV (The energy is the center of mass proton energy)</td>
<td>61</td>
</tr>
<tr>
<td>17B. Same as in Fig. 17A but for the coefficients $A_0, A_2, A_3, A_4$ (The theoretical $A_5$ coefficient is zero and it is not included in the figure.)</td>
<td>63</td>
</tr>
<tr>
<td>17C. Same as in Fig. 17A but for the energy region 3.7 to 4.5 MeV</td>
<td>65</td>
</tr>
<tr>
<td>17D. Same as in Fig. 17A but for the coefficients $A_0, A_3, A_4, A_5$ and energy region 3.7 to 4.5 MeV</td>
<td>67</td>
</tr>
<tr>
<td>18A. The fit of the theoretically calculated coefficients $A_0, A_2, A_4$ of the angular correlations in the Goldfarb-Seyler geometry to the experimentally determined coefficients over the energy region 2.9 to 3.8 MeV. (The energy is the center of mass proton energy.)</td>
<td>69</td>
</tr>
<tr>
<td>18B. Same as in Fig. 18A but for the energy region 3.7 to 4.5 MeV</td>
<td>70</td>
</tr>
<tr>
<td>19A. The fit of the theoretically calculated coefficients to $A_0, A_1, A_2$ of the angular distribution to the experimentally determined coefficients, over the energy region 2.9 to 3.8 MeV (The energy is the center of mass proton energy.)</td>
<td>72</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>19B</td>
<td>Same as in Fig. 19A but for the coefficients $A_0$, $A_3$, $A_4$, $A_5$</td>
</tr>
<tr>
<td></td>
<td>Page: 74</td>
</tr>
<tr>
<td>19C</td>
<td>Same as in Fig. 19A but in the energy region 3.7 to 4.5 MeV</td>
</tr>
<tr>
<td></td>
<td>Page: 76</td>
</tr>
<tr>
<td>19D</td>
<td>Same as in Fig. 19A but for the coefficients $A_0$, $A_3$, $A_4$, $A_5$ and the energy region 3.7 to 4.5 MeV</td>
</tr>
<tr>
<td></td>
<td>Page: 78</td>
</tr>
<tr>
<td>A1</td>
<td>Schematic diagram of a $x(p,p',y)x$ reaction with a compound nucleus formation</td>
</tr>
<tr>
<td></td>
<td>Page: 95</td>
</tr>
<tr>
<td>A2</td>
<td>Orientation of the coordinate system and arrangement of detectors for the angular correlation in the Goldfarb-Seyler geometry</td>
</tr>
<tr>
<td></td>
<td>Page: 114</td>
</tr>
<tr>
<td>A3</td>
<td>Orientation of the coordinate system and arrangement of detectors for the study of the angular correlation in the spin-flip geometry</td>
</tr>
<tr>
<td></td>
<td>Page: 119</td>
</tr>
<tr>
<td>A4</td>
<td>Arrangement of electronics for the angular distribution of protons</td>
</tr>
<tr>
<td></td>
<td>Page: 129</td>
</tr>
<tr>
<td>A5</td>
<td>Arrangement of electronics for the $p'-\gamma$ angular correlation in the spin-flip geometry</td>
</tr>
<tr>
<td></td>
<td>Page: 131</td>
</tr>
</tbody>
</table>
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Numerical values of solid angles subtended by the proton detectors. (Each detector takes the name of the slot it occupies inside the ORTEC scattering chamber.)</td>
<td>26</td>
</tr>
<tr>
<td>2. Outputs of the Logic Box for certain input signals. The input signals are shown in column 1 (The x mark means that a signal appears on the output indicated at the top of the column.)</td>
<td>134</td>
</tr>
<tr>
<td>3. The partial decay proton widths and phases of nine of the resonant states in $^{29}$P. The experimentally determined decay proton widths and phases for nine of the resonances involved in the calculations are shown in the last eighteen columns of the table</td>
<td>59</td>
</tr>
<tr>
<td>4. The reduced widths of six of the resonant states in $^{29}$P from excitation energies 5.7 to 7.1 MeV. The reduced widths which were calculated using the experimental partial decay proton widths of Table 3 are shown in the last ten columns</td>
<td>88</td>
</tr>
<tr>
<td>5. The wave functions of six of the resonant states in $^{29}$P as calculated using the simplified model discussed in Chapter IV</td>
<td>90</td>
</tr>
</tbody>
</table>
CHAPTER I
INTRODUCTION

A. Nuclear Reaction Models

There are two extreme models which have had considerable success in describing nuclear reactions. They are known as the Compound Nucleus\(^1\) (CN) model and the Direct-Reaction\(^2\) (DI) model. Basically they differ from one another in two ways: a) the typical time of interaction and; b) the method by which the energy of the incident particle is shared by the nucleons of the target nucleus. In DI the time of interaction is of the order of the time it takes the incident particle to travel the nuclear diameter, and this is of the order of \(10^{-22}\) sec. In DI there are some principle mechanisms in which the incident particles interact with the target nucleus, (pick up reactions, stripping reactions, etc.) but no intermediate state is formed. The time of interaction is of the order (of the time it takes the incident particle to travel the nuclear diameter) \(10^{-22}\) sec.

In a CN reaction mechanism the incident particle interacts with the nucleus and forms a metastable intermediate state with definite symmetry properties (spin and parity). The target nucleus-particle system is called a compound nucleus and has a lifetime of the order \(10^{-17}\) sec. Since the formed states are not bound states the stability of the CN is due to Coulomb or centrifugal barriers or both. The CN decays by means of
different channels Fig. 1 and the decay process is independent of the way the CN was formed. If the energy of the incident particle is appropriate, the cross section for the CN formation is very large, and for this reason the yield of the outgoing particles exhibits large maxima, called resonances.

The nuclear reaction model which we feel best describes the interaction under study is the CN model. Consequently, we shall discuss this reaction in more detail.

Suppose an incident particle with spin $s_1$ and orbital angular momenta $\ell_1$, interacts with a nucleus $^{A_Z}_A$ being in a state of spin $a$ and parity $P$ and both form a CN with spin $b$ and parity $P$, Fig. 2. This state of the CN is also called a resonance state, and is characterized by the resonance energy $E_0$ and a total width $\Gamma = \frac{h}{\tau}$, where $\tau$ is the lifetime of the CN state. The CN can then decay by emitting a particle having spin $s_2$ an orbital angular momentum $\ell_2$ to a residual nucleus $^{A_{Z_2}}_{Z_2}B$ having spin $c$. (We consider only a two-body process.) For a certain CN state $b$, outgoing particle, and final nuclear state $c$, the conservation of angular momentum and parity allows only a few particular sets of $s_2$, $\ell_2$ and $j_2$ quantum numbers by which the outgoing particle can be characterized. Each set of the above quantum numbers corresponds to a so-called open channel and every open channel has a decay amplitude $g(b \rightarrow c, \ell, j)$ which is normalized so that:

$$|g(b \rightarrow c, \ell, j)|^2 = \Gamma(b \rightarrow c, \ell, j)$$

$\Gamma(b \rightarrow c\ell j)$ is the partial decay width and gives the probability per unit time (multiplied by $\hbar$) for decay into the particular open channel. The total width of the state is the sum of the partial widths.
Fig. 1 Schematic diagram for the formation and decay of a compound nucleus into different channels.
$A + P \rightarrow CN^*$

$A + P$ (Compound elastic)

$A^* + P'$ (Compound inelastic)

$X + a$ (Reaction)
Fig. 2 Nuclear energy levels and radiations involved in a particular CN formation and decay.
\[ \Gamma = \sum_{\ell j} \Gamma(b \rightarrow c\ell j). \]

The determination of the partial decay widths and the relative phases of the resonance states of \(^{29}\text{P}\) is one of the main purposes of this experiment which will be discussed later.

The definition of the relative phases is given in the Appendix in the derivation of the theoretical expression of the angular distribution. Nuclear structure models which might use the experimental values of the partial decay widths for comparison are discussed next.

B Nuclear Structure Models

The nuclear models which have had the best success in describing the properties of the low-lying bound states of \(^{29}\text{P}\) are discussed in this section.

Strong evidence that the nucleons inside the nucleus may be considered as occupying states having characteristics of particles moving independently in an averaged nuclear field (like the electrons under the influence of the coulomb field of the nucleus) has led to the development of a nuclear shell model\(^3\). Although different theoretical expressions for the averaged nuclear field have been used and although the \(l \cdot s\) (spin-orbit) interaction has also been taken into account, it was not possible for this model to explain all the regularities of the nuclear levels observed in nuclei. Moreover, the large quadrupole moment observed in many nuclear states as well as the short lifetimes of states decaying by electric quadrupole radiation could not be explained by the simple shell model.

The reasons that the nuclear shell model does not provide a complete
explanation of the structure of the nuclei is probably due to the fact that there are essential differences between atomic and nuclear structures arising from the fact that the nuclear field is generated by the nucleons themselves, while the atomic field is mainly due to the attraction of the central nucleus. Another basic difference is that the atomic field can be treated to a first approximation as static quantity (this is due to the large nuclear mass as compared with the electrons) and collective oscillations because of the mutual interaction of the electrons is a second order effect. But in the nuclear case long-range terms in the field (which gives rise to the collective oscillations of the nucleus as a whole) must play an important role. This is supported by the fact that the nuclear structure model which takes into account the collective oscillations of the nucleus (collective model) explains the observed large quadrupole moments and short lifetimes of nuclear states mentioned above. The collective oscillations of the nucleus are similar to those a droplet exhibit because of the mutual attraction of its constituent particles. A brief discussion of this model is given in the next section. The collective model is the basis of the unified model which, like the shell model has been applied for the explanation of the low-lying energy states of $^{19}$P. The unified model considers the nucleus as being composed of a core nucleus whose collective oscillations are coupled to the individual particles excitations. The outline of two limited cases of the unified model, namely the weak coupling model (where the core-particle interaction is weak and the strong coupling model, where the core-particle interaction is strong) is given in the next two sections. Further details about these two particular models are given in Ref. 10.
B1. Weak Coupling Model

The main assumption made in this model is that the frequencies $\omega_p$ which correspond to the single particle excitations are large compared with the frequencies $\omega_c$ of the collective excitations of the core. Under this assumption the Hamiltonian of the system core-single particles can be written

$$H = H_s(\alpha_{\lambda\mu}) + H_p(\lambda) + H_{\text{int}}(x,\alpha_{\lambda\mu})$$

where:

$$H(\alpha_{\lambda\mu}) = \sum_{\lambda\mu} \{ \frac{1}{2}B_{\lambda} | \alpha_{\lambda\mu}|^2 + \frac{1}{2}C |\alpha_{\lambda\mu}|^2 \}.$$ 

is the part of the Hamiltonian which describes the collective oscillations of the core and $\alpha_{\lambda\mu}$ are the collective coordinates which physically represent the displacement of the surface from the equilibrium shape, which is supposed to be a sphere. Mathematically the $\alpha_{\lambda\mu}$ are the expansion coefficients of the spherical harmonics $Y^\mu_\lambda(\theta,\phi)$ which are necessary to describe the deformed nuclear surface\(^5\) which is described by the equation

$$R(\theta,\phi) = R_0 [1 + \sum_{\lambda\mu} \alpha_{\lambda\mu} Y^\mu_\lambda(\theta,\phi)]$$

from which

$$\alpha_{\lambda\mu} = \frac{4\pi}{3A} \sum_{p=1}^{A} \frac{r_p}{R^2} \sum_{\lambda\mu} \langle Y^\mu_\lambda | \phi_p \rangle \langle \phi_p | Y^\mu_\lambda \rangle$$

The index $p$ corresponds to the individual particle coordinates. $H_p(x)$ is the Hamiltonian of the particle moving in a spherical nucleus and $x$ represents the coordinates and spins of all the particles.

$H_{\text{int}}$ is the interaction Hamiltonian between the particles and the deformed nucleus and is given by
10

\[ H_{\text{int}} = - \sum_{p=1}^{A} k(r_p) \sum_{\lambda \mu} \alpha_{\lambda \mu} \gamma_{\lambda}^{\mu}(\theta_p, \phi_p) \]

\( k(r_p) \) has the form of a delta function at the nuclear surface because of the sharp nuclear boundary assumption.

The frequencies of the core oscillations are given by:

\[ \omega_{\lambda} = \sqrt{\frac{C_{\lambda}}{B_{\lambda}}} \]

The above unified model can be simplified by assuming only one single particle is coupled to an oscillating core having stable structure i.e., no single particle excitations. For spherical nuclear cores which undergo small deformations, the coupling is often assumed to be sufficiently weak so that the particle motion and the core oscillations can be treated as being the unperturbed Hamiltonian and the interaction Hamiltonian plays the role of the perturbation. Also in weak coupling the particle motion can be treated independently of the core oscillations. Thus the unperturbed wave function which corresponds to the Hamiltonian \( H = H_S + H_p \) can be written as

\[ |I'M\rangle = \sum_{jNR} |j\rangle |NR\rangle (j;NR;IM|IM) \]

\( I \) and \( M \) are the total spin and its projection respectively of the coupled system, \( |j\rangle \) is the single particle wave function, and \( j \) is the total angular momentum of the single particle which is calculated by assuming that the single particles occupy one of the available orbits as they are given from the shell model theory. \( |NR\rangle \) is the wave function due to the core oscillations, \( N \) is the number of phonons, and \( R \) is the surface angular momentum of the core. \( (j;NR;IM|IM) \) is the Clebsh Gordon coefficient resulting from the coupling of the single particle angular momenta with the angular momenta of the core. The energy levels are calculated by diagonalizing the matrix \( (IM|H_S + H_p + H_{\text{int}}|I'M') \). Also, Satchler\(^{11}\)
has derived an expression for the reduced width $\gamma_{j\ell}^2$ in the weak-coupling model. The reduced width measures the degree to which the compound nucleus with spin I can be regarded as made up of a nucleus with spin $I_0$ and a nucleon in an orbit with $j$ and $\ell$ quantum numbers.

If the initial nucleus is an even-even core nucleus, then $I_0 = R + j$ and the expression given by Satchler reduces to

$$\gamma_{j\ell}^2 = \frac{(2R + 1)(2j + 1)}{2} W(R0; Ij, Rj) \gamma_0^2(j\ell)$$

(1)

where $\gamma_0^2(j\ell)$ is the single particle reduced width. The theoretical formula which relates the reduced widths $\gamma_{j\ell}$ to the experimental decay widths $\Gamma_{j\ell}$ will be given after the discussion about the strong coupling model.

B2. Strong Coupling Model

If the core nucleus acquires a large deformation, then the core-particle coupling becomes strong and the interaction term is too large to be treated as a perturbation. The Hamiltonian for the strong coupling takes into account the oscillation of the nucleus about its deformed equilibrium position, the rotational motion of the nucleus, and the single particle motion. A detailed discussion of the strong coupling Hamiltonian and its solution is given in Ref. 8. The strong coupling wave function has the form

$$\psi_{MK} = \frac{(-1)^j}{16\pi^2} \phi_{n\beta \gamma}(\beta, \gamma) \left\{ D_{MK}^{I}(\theta_0) \sum c_j \chi_j^{\ell} + D_{MK}^{I}(\theta_0) \sum (-c_j) \chi_j^{\ell} \right\}$$

where $I$ is the spin of the compound nucleus, $j$ is the total spin of the single particle, $K, M$ are the projections of the spin $I$ on the symmetry
axis of the nucleus and the z axis respectively, \( \Omega \) is the projection of \( j \) on the symmetry axis of the nucleus, \( \phi_{n\beta, n\gamma} \) is the wave function describing the rotational motion of the nucleus and it is identical to the function which describes the motion of a symmetric top, and \( \chi_0^j \) describes the motion of the particle with respect to the deformed nucleus.

The diagonal matrix elements \( \langle \psi_{MK}^I | H | \psi_{MK}^I \rangle \) contain a term due to pure vibrations and a cross term due to the rotation-vibration coupling. The off-diagonal elements contain the rotation-vibration coupling. The theoretical expression of the reduced widths \( \gamma_{j\ell} \) in the strong coupling model was derived by Satchler\(^{11}\). For the case where a single particle with \( j, \ell \) quantum numbers is coupled to a core nucleus the quantity \( \gamma_{j\ell}^2 \) is given by

\[
\gamma_{j\ell}^2 = g \frac{2R + 1}{2I + 1} (IK|R,j,\Omega)(\phi_\ell|\phi_1) C_{j\ell}(\Omega) \gamma_0^2(j\ell)
\]

where \( g \) is a numerical constant, \( (\phi_\ell|\phi_1) \) is an overlap integral of the initial and final vibrational wave functions and

\[
C_{j\ell}(\Omega) = \sum_{\Lambda, \Sigma} C(\ell \downarrow j; \Lambda \Sigma) a_{n\ell \Sigma}(\Omega) .
\]

The quantities \( a_{n\ell \Sigma} \) have been calculated by Nilson\(^{13}\). A criterion has been suggested by Bohr and Mottelson\(^{14}\) for nuclei with an even-even core as to whether the particle-core coupling is strong or weak. The criterion states that if the \( 2^+ \) first excited state has an excitation energy less than the critical value \( E(e) = \frac{32h^2}{(MAR_0^2)} \) the rotational spectrum is more probable, consequently the strong coupling is valid, otherwise the weak coupling is valid. In the above formula of the critical value \( A \) is
the nuclear mass number, \( M \) is the nucleonic mass, \( R = 1.4A^{1/3} \) Fermis.
For the nucleus \(^{28}\text{Si}\) the critical value is calculated to be \( E(2) = 2.22 \) MeV and the excitation energy of the first \( 2^+ \) excited state is \( E = 1.77 \) MeV. Thus the strong coupling is more favorable.

The nuclear structure models discussed above calculate the energies, spins, and parities of the energy levels for nuclei which consist of a core nucleus plus a particle interacting with the core. Such calculated values can be compared with the experimental one so that a test about the validity of the models is possible. An additional test is provided by means of the reduced widths which are given by the equations (1) or (2).

The reduced widths \( \gamma_{j\ell}^2 \) are related to the experimentally measured partial decay widths \( \Gamma_{j\ell} \) through the formula

\[
\Gamma'_{j\ell} = \frac{2p_{\ell} \gamma_{j\ell}^2}{1 + \sum_{j\ell} \gamma_{j\ell}^2 \gamma_{j\ell}^2}
\]

where \( p_{\ell} \) is the penetrability of the Coulomb barrier and is given by:

\[
p_{\ell} = \frac{a}{\mathcal{A}_{\ell}(F^2_{\ell} + G^2_{\ell})} = \frac{F_{\ell}}{A_{\ell}^2}, \quad A_{\ell}^2 = F^2_{\ell} + G^2_{\ell},
\]

\( a \) is the radius of the target nucleus, and \( F_{\ell} \) and \( G_{\ell} \) are the regular and irregular solutions of the Coulomb wave equations respectively.

\[
S_{\ell} = \frac{P}{2A_{\ell}^2} \left. \frac{d}{d\epsilon} (A_{\ell}^2) \right|_{\text{at radius of target nucleus}},
\]

the dot on the quantity \( S_{\ell} \) means the derivative of \( S_{\ell} \) with respect to the energy at \( E = E_{\ell} = (\text{Resonance energy}) \).
C. Partial Decay Proton Widths of \(^{29}\text{P}\) Nucleus

In this experiment the partial decay proton widths of some of the resonance of \(^{29}\text{P}\) from 5.7 to 7.7 MeV excitation energies are studied.

According to the model discussed in this chapter, the nucleus of \(^{29}\text{P}\) can be considered as being composed of a \(^{28}\text{Si}\) core at the \(2^+\) first excited state or ground state coupled to a proton with spin \(j\) and orbital angular momenta \(\ell\). The degree of such coupling is characterized with the partial decay width \(\Gamma_{j\ell}\) of the outgoing proton (having \(j\), \(\ell\) quantum numbers) which results from the decay of \(^{29}\text{P}\) to the \(2^+\) first excited state or ground state of \(^{28}\text{Si}\). (In Fig. 3 the formation of the CN of \(^{29}\text{P}\) and its decay is shown.) The present dissertation does not include the comparison of the experimentally determined partial proton decay widths with the corresponding values derived from the nuclear structure models discussed above. Such comparisons, but for low lying energy levels of \(^{29}\text{P}\), are given in Ref. 16.
II. EXPERIMENTAL PROCEDURE

In general we have found that the number of partial decay widths and relative phases needed to describe the resonance decays in $^{28}\text{p}$ are greater than the number of experimental parameters which can be obtained from a single type of experiment, such as inelastic proton angular distributions. Consequently, we decided to perform three types of experiments, whose results (cross-sections) depend upon the same resonant widths, in order to have sufficient experimental parameters to uniquely determine the resonant decay widths. The experiments chosen were: a) The angular distributions of the inelastically scattered protons from the reaction $^{28}\text{Si}(p,p')^{28}\text{Si}$ ($2^+$ first excited state) Fig. 3; b) Angular correlations of the inelastically scattered protons in coincidence with the emitted $\gamma$-ray from the reaction $^{28}\text{Si}(p,p',\gamma)^{28}\text{Si}$ in the Goldfarb and Seyler$^{17}$ geometry; c) Angular correlation of the same radiations as in (b) but in the spin flip geometry$^{18}$). A detailed discussion for each one of the above cases will be given below.

A. Angular Distributions

The angular distributions of the inelastic protons were measured inside a 17" "ORTEC model 600"$^{19}$ scattering chamber (Fig. 4) placed at the end of the 15° N beam line of the O.S.U. Van de Graaff accelerator. A $^{28}\text{Si}$ target 35 $\mu$g/cm$^2$ thick deposited on a carbon backing (see Appendix
Fig. 3  Schematic diagram of the reaction $^{28}\text{Si}(p, p'\gamma)^{28}\text{Si}$ for the study of the resonant states of $^{29}\text{p}$. 
Fig. 4 The 17" "ORTEC Model 600" scattering chamber.
under target preparation) was placed at the center of the chamber. The target was oriented at an angle of 72° with respect to the incident beam direction. Six solid state detectors were used for the angular distribution experiment. Each detector was placed at the end of a cylindrical "collimator holder" (Fig. 5). The collimator holders were mounted on the "collimator holder stands" which were properly inserted inside radial slots at the base of the scattering chamber. The purpose of the scattering and antiscattering collimators, which are mounted on the collimator holder was to define the solid angle and reduce the background radiation, respectively. A top view of the scattering chamber is shown in Fig. 6. The use of the six detectors make it possible to collect information from six different angles at the same time. Since the base of the scattering chamber can be rotated about a vertical axis through its center, information for six other scattering angles can be collected for every angular position of the scattering chamber base. Two different positions of the scattering chamber base were selected so that data could be collected from 12 angles between 20° and 165°. In general, it was not possible to use the data of the 20° scattering angle because of the large count rate in the detector.

An important part of the preparation of the experimental set up was the alignment of the detector collimators and the measurement of the solid angles subtended by the detector collimators. The measurement of the solid angles defined by the scattering collimators was done in two ways: Geometrically and by Rutherford scattering. See Appendix (Solid Angle Measurement) for more details. Both measurements gave the same results within the experimental errors. The results from the
Fig. 5  Solid state detector mounting and collimation arrangement.
Particle detector

Scattering Collimator

Antiscattering Collimator

Detector stand

Collimator holder

Collimator holder stand

Base of scattering chamber
Fig. 6  Top view of the interior of the scattering chamber.

(The arrangement of the detectors is shown.)
measurement of the solid angles are shown in Table 1.

With the arrangement of the electronics it was possible to collect six different proton spectra at the same time. The signals for three of the proton detectors (the ones in the forward quadrants) were examined for pile-up and the spectra were corrected for the pile-up. The count rate for the three proton detectors located in the backward quadrants, was low enough so that pile-up correction was not necessary. The schematic of the electronics for the data collection of the angular distributions as well as a brief discussion is given in the Appendix (Electronic Arrangement).

The spectrum of the scattered particles from each detector was stored in 512 channels of our computer\textsuperscript{33}) and then on magnetic disks. The spectrum of the protons scattered from $^{28}\text{Si}$ is shown in Figs. 7a and 7b. Figure 7a is a complete spectrum of the reaction showing the elastic peaks of $^{28}\text{Si}$, $^{16}\text{O}$, and $^{12}\text{C}$ as well as the inelastic peak corresponding to protons leaving the residual nucleus $^{28}\text{Si}$ in its first excited $2^+$ state. The presence of the $^{16}\text{O}$ in the target was due to the fact that the $^{28}\text{Si}$ which was evaporated on the carbon foil was in the form of SiO$_2$. Figure 7b is the proton spectrum gated so that only the inelastic peak is present.

8. Goldfarb-Seyler Angular Correlations

The $(p',\gamma)$ angular correlations in the Goldfarb-Seyler\textsuperscript{17}) geometry for the reaction $^{28}\text{Si}(p,p',\gamma)^{28}\text{Si}$ were measured in this laboratory by Gearhart et al.\textsuperscript{20}) For this particular geometry the gamma rays, in coincidence with the inelastically scattered protons leading to the $2^+$ first excited state in $^{28}\text{Si}$, were detected in a plane perpendicular to
Table 1  Numerical values of solid angles subtended by the proton detectors. (Each detector takes the name of the slot it occupies inside the ORTEC scattering chamber.)
<table>
<thead>
<tr>
<th>SOLID STATE DET.</th>
<th>15°</th>
<th>60°</th>
<th>120°</th>
<th>190°</th>
<th>240°</th>
<th>300°</th>
</tr>
</thead>
<tbody>
<tr>
<td>SOLID ANGLE MSR</td>
<td>1.14</td>
<td>4.37</td>
<td>4.53</td>
<td>1.30</td>
<td>4.52</td>
<td>4.37</td>
</tr>
</tbody>
</table>
Fig. 7A  The proton spectrum of the $^{28}\text{Si} + p$ reaction. The peaks from left to right are $^{28}\text{Si}$ inelastic and $^{12}\text{C}$, $^{16}\text{O}$ $^{28}\text{Si}$ elastics. (The $^{12}\text{C}$, and $^{16}\text{O}$ peaks are due to the carbon backing of the $^{28}\text{SiO}_2$ target and the $\text{O}_2$ contained in the $\text{SiO}_2$ compound.)
Fig. 7B  The same as in Fig. A but only the $^{28}\text{Si}$ inelastic peak allowed from the single channel analyzer to appear in the spectrum.
the outgoing proton direction. In a coordinate system where the outgoing proton direction is taken as the z-axis, the gamma-ray detectors moved in the plane $\Theta_{\gamma} = \frac{\pi}{2}, \phi_{\gamma}$ variable. Figure A2 in the Appendix shows the arrangement of the detectors for the Goldfarb-Seyler geometry correlation. Details about the experimental set up, electronics and collection of experimental data are given in ref. 20.

C. **Spin-Flip Angular Correlations**

Spin-flip angular correlations$^{18}$ were measured in the 17'' Ortec chamber which had been modified so that a 4'' x 5'' NaI(Tl) detector could be mounted on the chamber. In this particular correlation geometry, the $z$-axis is assumed to be along the direction of the emitted gamma rays; the coincident protons being detected in a plane perpendicular to the $z$-direction.

The arrangement of the solid state detectors and the values of the solid angles of the proton detectors were identical to those of the angular distribution experiments. The $^{28}$Si target with carbon backing had a 60 $\mu$g/cm$^2$ thickness and was placed making an angle $72^\circ$ with the incident beam. The 4'' x 5'' NaI(Tl) $\gamma$-ray detector was placed outside the Ortec chamber at a distance 16 cm above the target as shown in Fig. 8. More details about this geometry are given in Appendix (Special Geometries in the Angular Correlations). The electronics arrangement (see Appendix, Electronics Arrangement) was such that the collection of six $\gamma$-ray spectra at the same time was possible.

Each $\gamma$-ray spectrum includes the $\gamma$-rays in coincidence with inelastically scattered protons at a certain angle, and was stored in our
Fig. 8  The ORTEC scattering chamber with the NaI(Tl) γ-ray detector mounted on top for the performance of the angular correlation in the spin-flip geometry.
IBM 1800 computer\textsuperscript{33}) in 1024 energy channels; the first 512 channels being the spectrum of the true + accidentals coincidences and the second 512 channels only the accidentals coincidences. The Figures 9A and 9B show the $p-\gamma_T$ and $p-\gamma_A$ spectra for a particular proton detector angle.
Fig. 9A  The $p'-\gamma$ coincidence $\gamma$-ray spectrum of the angular correlation in the spin flip geometry.

Fig. 9B  The corresponding to Fig. 9A accidental $p'-\gamma$ coincidence $\gamma$-ray spectrum.
III. ANALYSIS OF THE EXPERIMENTAL DATA

A. Angular Distribution

As was mentioned in the introduction the partial decay widths and the relative phases are involved in the expansion coefficients $A_k$ of the angular distribution $d\sigma = \sum_k A_k(E)P_k(\cos \theta)$, where $P_k(\cos \theta)$ are the Legendre polynomials of order $k$, and $\theta$ are the scattering angles. The explicit expression of the coefficients $A_k(E)$ in terms of the partial decay widths and relative phases is given by formula A23 in the Appendix. It is the purpose of these measurements to compare the theoretically calculated expansion coefficients with the experimentally measured coefficients.

The analysis of the experimental data consists in the determination of the experimental coefficients $A_k$. At every incident proton energy the spectrum of the scattered protons was collected from 11 different angles. A typical proton spectrum is shown in Fig. 7a. Each spectrum was stored in 512 channels in the memory of the computer and then into the on-line disk. For every spectrum, the number of the inelastically scattered protons (area under the peak of interest) leading to the $2^+$ first excited states of $^{28}$Si (Fig. 3) was found by making use of the program SOUTH. To make the analysis faster, the program SOUTH was slightly modified to punch on a card the scattering angle, the area of
the peak of interest, the error in the area, the correction factors which take into account the pulses rejected because of pile-up, and the dead time of PACE\(^{23}\) system. Thus for every run corresponding to a certain incident proton energy a set of 11 cards was collected. This set of cards together with the energy information was used as input data for the program ADANA\(^{35}\) (Angular Distribution Analysis).

In this program the scattering angles were converted from Lab. System to C. M. System and the number of counts were converted to number of counts/msr. Next the plot of the angular distribution (number of counts/msr. versus scattering angle) was displayed on the screen (Fig. 10). The program then calculated a least squares fit to the experimental points in terms of the first five Legendre polynomials and the expansion coefficients \(A_0\) to \(A_5\) were printed out and also stored on the off-line disk, together with the corresponding energy. The least squares calculated curve was displayed on top of the angular distribution for visual comparison. In Figure 11 five angular distributions taken in the vicinity of the \(E_p = 3.35\) MeV \(^{29}\)P resonance are shown. The least squares fit curves of the above distributions are also shown in Fig. 11. The program continues by plotting all of the coefficients calculated from the angular distributions as a function of energy (Fig. 12).

In Fig. 12 the energy is the CM energy and the value of coefficients is given in units of \(\text{mb/sr}\). The formula for the conversion of the number of counts into absolute cross-section is given by:

\[
\frac{d\sigma(\Theta)}{d\Omega_p} = \frac{0.266 \times (\# \text{ of counts}) \times TM}{q \times t \times d\Omega_p} \quad \text{in (mb/sr)}
\]
Fig. 10 Experimental points of an angular distribution of inelastically scattered proton from $^{28}\text{Si}$ nuclei (The residual $^{28}\text{Si}$ nucleus is left in its $2^+$ first excited state.)
Fig. 11 Angular distribution of inelastically scattered protons from $^{28}\text{Si}$ over the 3.34 MeV resonance of $^{29}\text{P}$. The least squares fit curves, calculated in terms of the first six Legendre polynomials, are shown.
Fig. 12  The plot of the expansion coefficients $A_0$ to $A_5$, versus energy of the angular distributions.
where, TM is the mass number of the target nucleus, q is the collected charge in μCb, t is the target thickness in μg/cm², and dΩp is the solid angle in msr. Derivation of the above formula is given in the Appendix (Conversion of Experimental Data into Absolute Cross Section).

Finally, the CM energy and the coefficients corresponding to this energy are punched on cards. (The coefficients were converted into absolute cross sections.) This deck of cards is the input data of the program for the determination of the partial widths and phases. More discussion about this program will be given shortly.

B. Angular Correlation in the Spin-Flip Geometry

The angular correlation in the spin-flip geometry can be expressed as

\[ \frac{d^2\sigma}{dΩ_p dΩ_γ} = \sum_k A_k(E) \cos(kφ) \]

where \( φ \) is the angle between the inelastically scattered proton direction and the beam direction.

An explicit expression of the coefficients \( A_k(E) \) in terms of the partial decay widths and phases can be seen in equations A46, A48, and A51 in the Appendix. The coefficients \( A_k(E) \) (k = 1 to 5) are another set of five independent coefficients which contain the same set of parameters as the angular distribution coefficients. The analysis of the experimental data for the calculation of the experimental coefficients \( A_k(E) \) goes as follows.

At every incident proton energy the spectrum of the γ-rays emitted perpendicular to the reaction plane in coincidence with the inelastically
scattered protons at a certain angle in the reaction plane was stored in 512 channels (the spectrum includes true coincidences plus accidentals coincidences). In the next 512 channels the spectrum of the accidental coincidences was stored.

The electronics arrangement allowed six such different spectra of 1024 channels each to be collected at the same time, and then stored on disk. For the analysis of the data the program DURAl was called and the spectrum of true coincidences [(trues + accidentals) - (accidentals)] was calculated and then displayed on the screen. With the same program the total number of true coincidences under the photopeak was found (no background was subtracted) and punched on cards together with the corresponding angles of the proton detector, the uncertainty in the number of coincidences, and the pile-up correction for γ-rays and protons. Thus for every run at a given incident proton energy a set of 11 cards was collected. This set of cards together with the energy information was used as input data in the program SFANA (Spin-Flip Analysis) for the calculation of the coefficients.

From this point the analysis proceeded as described in the analysis of the proton angular distributions. Thus the scattering angles were converted to CM system and the number of counts to number of counts/msr. Then the experimental angular correlations (number of coincidences/msr. versus C.M. proton scattering angle) was plotted on the screen (Fig. 13). (The efficiency of the γ-ray detector and its solid angle, were common to every correlation and will be taken into account only in the conversion of the coefficients in the absolute cross section.)
Fig. 13  Experimental points of the \((p' - \gamma)\) angular correlation in the spin flip geometry, from the reaction \(^{28}\text{Si}(p, p', \gamma)^{28}\text{Si}\)
A least squares fit to the experimental points in terms of the cos (κφ) functions (k = 0, 1, 3, 4, 5) calculated the coefficients A₀ to A₅, printed them out, and stored them onto a disk together with the corresponding energy. The least squares curve was displayed on top of the experimental points for comparison. In Fig. 14 the experimental angular correlation together with the calculated least squares fit curve is shown. The plot of the coefficients from A₀ to A₅ is shown in Fig. 15. The formula which is used by the program to convert the coefficients into absolute cross section (mb/(sr)²) is given by

\[ \frac{d^2\sigma}{d\Omega_y d\Omega_p} = 0.266 \times 10^3 \left( \frac{\text{# of coinc.}}{q \cdot t \cdot d\Gamma_p \cdot \epsilon_y \cdot d\Omega_y} \right) (\text{mb/(sr)}^2); \]

\( d\Omega_y = 323.0 \text{ msr} \) is the solid angle and \( \epsilon_y = 0.2 \) the efficiency of the \( \gamma \)-ray detector for a \( \gamma \)-ray with energy \( E = 1.8 \text{ MeV} \) and was found by using a calibrated \(^{60}\text{Y}\) source. The rest of the quantities are the same as those in the formula for the conversion of the angular distribution coefficients into absolute cross section. The converted coefficients and their corresponding C.M. energy are punched on cards. This deck of cards was then used as input data for the program of the resonance analysis for the determination of the partial decay widths and phases.

C. Angular-Correlation in the Goldfarb-Seyler Geometry

Proton-gamma angular correlation in the Goldfarb-Seyler geometry had previously been measured by Gearhart et al.\(^{21}\) at this laboratory. These measurements were performed in order to determine the spins of the resonance states in \(^{29}\text{P}\) and only relative cross sections were measured.
Fig. 14  The least squares fit curve in terms of the $\cos(k\phi)$

$(k = 0, 1, 2, 3, 4, 5)$ functions of the angular correlation

in the spin flip geometry.
Fig. 15  Plot of the expansion coefficients $A_0$ to $A_5$ versus energy of the angular correlations in the spin flip geometry.
In order to utilize this data, (along with the proton angular distribution and spin-flip correlations) to help determine the partial decay proton width and phases, it was necessary to make corrections in the coefficients for the finite size of the solid angle of the gamma-ray detectors. See Appendix (Correction for the Finite Size of γ-ray Detectors Solid Angle).

The formula

\[
\frac{d^2\sigma}{d\Omega_p d\Omega_\gamma} = 0.266 \times 10^3 \frac{(\text{# of coinc. counts}) \cdot \text{TM}}{q \cdot t \cdot d\Omega_p \varepsilon d\Omega_\gamma}
\]

was used for the conversion of the coefficients into absolute cross section. The proton solid angle was measured as \(d\Omega_p = 7.0\) msr.

\[\varepsilon_\gamma = 0.2\]

\[t = 40\ \mu g/cm^2\]

\[d\Omega_\gamma = 229.0\ \text{msr.}\]

The converted coefficients and their corresponding C.M. energy were punched on cards to be used as input data for the Resonance Analysis program. In Fig. 16 the expansion of the coefficients \(A_0, A_2, A_4\) in the Goldfarb-Seyler geometry are shown.

D. Resonance Analysis Program for the Determination of the Partial Decay Proton Width

The resonance analysis program calculated the value of the theoretical expansion coefficients \(A_k\) versus energy and then plots these values on top of the experimentally determined expansion coefficients \(A_k\), so that visual comparison of the theoretically calculated curve to the experimental points is possible. (See Appendix "Resonance Analysis Program: for more details.")
Fig. 16 The expansion coefficients $A_0$, $A_2$, $A_4$, versus energy,
of the angular correlations in the Goldfarb-Seyler geometry.
The experimentally determined expansion coefficients $A_k$ are first plotted on the computer display screen over a chosen energy interval. Then reasonable values (a trial set) of the partial decay widths and phases are chosen by the experimenter and provided to the computer via the typewriter. The computer then calculates the theoretical values for the coefficients on the same energy interval and displays the theoretical coefficients as an overlay to the experimental coefficients so that visual comparison can be made. There is an option in the program to provide coefficients for the case of interfering or non-interfering resonances.

It was desired to obtain a unique set of input decay widths and phases for the expansion coefficients for three different experimental geometries. Since it was possible to obtain a good visual fit to the coefficients of one geometry with a particular set of trial values but a poor fit to the coefficient of another geometry with the same trial values, it was necessary to compare the visual fits to the coefficients of the three different geometries in-turn. A considerable amount of reiteration was required to obtain a single set of decay widths and phases which met the requirement of a best visual fit to all of the coefficients. Such a set of parameters is shown in Table 3. It is well to point out that we are unable to say that these are a unique set of parameters. In this table there are listed the resonances which are involved in the calculations in order to obtain the fits shown in Fig. 17 (a,b,c,d), 18 (a,b), and 19 (a,b,c,d). The partial decay proton widths and relative phases are also shown in Table 3. In the first and second column are the resonance energies in lab system and C.M. system for the resonances involved in the fit. The third and fourth columns contain the spin and parities of the resonances. The
Table 3  The partial decay proton widths and phases of nine of the resonant states in $^{29}$P. The experimentally determined decay proton widths and phases for nine of the resonances involved in the calculations are shown in the last eighteen columns of the table.
Fig. 17A The fit of the theoretically calculated coefficients $A_0$, $A_1$, $A_2$ of the angular distributions to the experimentally determined coefficients, over the energy region 2.9 to 3.8 MeV. (The energy is the center of mass proton energy.)
Fig. 17B  Same as in Fig. 17A but for the coefficients $A_0$, $A_2$, $A_3$, $A_4$.
(The theoretical $A_5$ coefficient is zero and it is not included in the figure).
Fig. 17C  Same as in Fig. 17A but for the energy region 3.7 to 4.5 MeV
Fig. 17D  Same as in Fig. 1/A but for the coefficients $A_0, A_3, A_4, A_5$ and energy region 3.7 to 4.5 MeV.
Fig. 18A. The fit of the theoretically calculated coefficients $A_0$, $A_2$, and $A_4$ of the angular correlations in the Goldfarb-Seyler geometry to the experimentally determined coefficients over the energy region 2.9 to 3.8 MeV. (The energy is the center of mass proton energy.)
Fig. 18B    Same as in Fig. 18A but for the energy region 3.7 to 4.5 MeV.
Fig. 19A  The fit of the theoretically calculated coefficients to $A_0$, $A_1$, $A_2$ of the angular distribution to the experimentally determined coefficients, over the energy region 2.9 to 3.8 MeV (The energy is the center of the mass proton energy.).
Fig. 19B  Same as in Fig. 19A but for the coefficients $A_0$, $A_3$, $A_4$, $A_5$. 
Fig. 19C  Same as in Fig. 19A but in the energy region 3.7 to 4.5 MeV.
Fig. 19D  Same as in Fig. 19A but for the coefficients $A_0, A_3, A_4, A_5$ and the energy region 3.7 to 4.5 MeV.
total width $\Gamma$ and the elastic width $\Gamma_p$ of every resonance are shown in the fifth and sixth columns. $\Gamma$ and $\Gamma_p$ were determined from analysis of the elastically scattered proton data$^{27,28}$. The total inelastic width $\Gamma_p'$ for every resonance is shown in the eighth column ($\Gamma_p' = \Gamma - \Gamma_p$). (No other particle channels are open and $\Gamma_\gamma$ is negligible.)

In the rest of the columns are the values of the partial decay proton widths and relative phases which give the visual fits shown in Fig. 17, 18, and 19. In the next section a discussion of the results for every resonance will be given.

E. Discussion of Results

The spins of the resonance states in $^{29}\text{P}$ from 3.0 to 5.2 MeV proton bombarding energy have been determined at our lab by N.L. Gearhart et al.$^{21}$ through a $^{28}\text{Si}(p,p',\gamma)^{28}\text{Si}$ reaction by studying the shape of the expansion coefficients of the $p', \gamma$ angular correlation, versus energy, in the Goldfarb-Seyler geometry$^{17}$. Vorona et al.$^{27}$ and Belote et al.$^{28}$ have also studied some resonance states in $^{29}\text{P}$ mainly through the analysis of elastically scattered proton data. From this analysis the total widths, $\Gamma$, and the elastic widths, $\Gamma_p$, for the resonances were determined. The numerical values of $\Gamma$ and $\Gamma_p$ for the different resonance states were taken from the work of Belote et al.$^{28}$ Each individual resonance will be discussed below.

E1. The 3.10 MeV Resonance

The spin of this state was determined from N. L. Gearhart et al.$^{21}$ to be $5/2$ or greater in agreement with the assignments made in Ref. 27.
and 28. However, recently an experiment was performed at our laboratory by W. S. McEver et al.\(^{29}\) on the elastic scattering of polarized protons over this resonance which showed that the spin of the resonance was 7/2 with a negative parity. Although it was possible for us to obtain a reasonable fit to this resonance assuming either a spin and parity of 5/2\(^{-}\) or 7/2\(^{-}\), we chose to use the value of 7/2\(^{-}\) determined by the polarized beam measurements since these measurements are more definitive than the techniques used in Refs. 27 and 28. With this assignment it was possible to obtain a good fit for the coefficients with the parameters shown in Table III.

In Ref. 28 it is stated that no good fit of the single angular distribution they measured at the peak of the resonance could be obtained by using spin 7/2\(^{-}\) and giving the total inelastic decay strength to the \(\Gamma_{1,3/2}\) partial width. Possible reasons for this might be the fact that no interference was taken into account, or the failure of the assumption that the decay takes place through the lowest possible outgoing \(\ell\)-wave. Our results show that no only does this resonance interfere with the 3/2\(^{+}\) resonance at 3.71 MeV and the 1/2\(^{+}\) resonance at 3.98 MeV, but also that both the \(\ell = 1\) and \(\ell = 3\) decay channels contribute to the cross section.

Recently N. A. Detorie et al.\(^{31}\) studied the resonance state in \(^{29}\)P excited in a \(^{32}\)S(p,a)\(^{29}\)P reaction. In this study they claim that an additional resonance exists at 3.075 MeV. We searched this region by taking the excitation function of the inelastically scattered protons from the \(^{28}\)S(p,p')\(^{28}\)Si\(^{+}\) reaction with a thin target and small energy intervals but could not find any indication of the existence of this state in our reaction channel.
E2. The 3.33 MeV Resonance

Using the value $3/2^+$ for the spin and parity of this resonance it was possible to obtain a fit for the coefficients by giving nearly all the strength of the decay width to the $\Gamma_{1/2}$ partial decay width. This resonance interferes through this decay channel with both the 3.571 $3/2^-$ resonance and the 2.88 MeV $1/2^-$ resonance. This interference appears in the $A_1$ and $A_3$ coefficients of both the angular distributions and spin-flip correlation results. To obtain the $A_3$ coefficient of the angular distribution and angular correlation in the spin-flip geometry, a small fraction of the total partial decay inelastic width was given to the $\Gamma_{5/2}$ partial decay width.

E3. The 3.57 and 3.71 MeV Doublet

From analysis of elastic and inelastic scattering data Vorona et al.\textsuperscript{27} assigned the spins and parities to be $3/2^-$ for the 3.57 MeV resonance state and $1/2^-$ for 3.71 MeV resonance state. From the same type of experiment Belote et al.\textsuperscript{28} concluded that the values for the resonances were $3/2^-$ and $3/2^+$ respectively. The contradiction in the spin and parities of 3.71 MeV resonance was resolved by N. L. Gearhart et al.\textsuperscript{21} from analysis of $p'-\gamma$ angular correlation in the Goldfarb-Seyler geometry. The assignment of $3/2^+$ was made for this resonance. The values of the partial decay width of these two resonances are shown in Table 3 and the fit to the coefficients is shown in Figs. 17, 18, and 19. It is noticeable that the $A_1$ coefficient in the angular distribution is mainly
due to the interference of this doublet. This supports the given assignment of the opposite parity of the states. We can say conclusively that the decay of the 3.57 MeV resonance to the $2^+$ first excited state of $^{28}$Si happens by means of an $\ell = 1$ wave and the decay of the 3.71 MeV resonance by means of an $\ell = 0$ and $\ell = 2$ wave.

E4. The 4.23 MeV Resonance

The $5/2^+$ assignment made by N. L. Gearhart et al. for the spin of this state was used in the present calculations. The $A_1$ and $A_3$ coefficients of the angular distribution and angular correlation in the spin-flip geometry, Fig. 12 and 15 are due to the interference of this resonance with the 4.43 MeV $3/2^-$ resonance. The even coefficients of all geometries do contain some interference with the 3.84 MeV $1/2^+$ resonance as well as with the 5.4 MeV $1/2^+$ resonance. To obtain the fits shown in Figs. 17, 18, and 19 we gave relatively large values to the partial decay amplitudes corresponding to the waves $\ell = 0$, $\ell = 2$, and $\ell = 4$.

E5. The 4.43 MeV Resonance

From analysis of angular correlation measurements in the Goldfarb-Seyler geometry the spin of this resonance was determined to be $3/2$. Since the 4.43 MeV resonance is only very weakly excited in the elastic channel, it appears that an $\ell$-value assignment based upon the elastic scattering data is not feasible. From the shape of the $A_1$ coefficients of the angular distribution and correlation in the spin-flip geometry, it turns out that the strong $A_1$ coefficient of the 4.23 MeV resonance can be reproduced from the theoretical data by assuming interference of the 4.23 MeV resonance with the 4.43 MeV resonance if we assume negative parity for
this state. Also the strong $A_1$ coefficient of the angular distribution and the angular correlation is due to interference with the 3.98 MeV $1/2^+$ resonance and the 5.4 MeV $1/2^+$ resonance. (This last resonance was observed by M. W. Brenner et al.\textsuperscript{30})

Although it was possible to obtain a good fit for the coefficient of angular distributions and angular correlation in the spin-flip geometry by giving the total strength of the inelastic decay width to the $\Gamma_{1,1/2}$ partial decay width, some small strength was given to the $\Gamma_{1,3/2}$, $\Gamma_{3,5/2}$ partial decay width in order to obtain a better fit for the $A_2$ coefficient of the angular correlation in the Goldfarb-Seyler geometry.
CHAPTER IV
CONCLUSIONS

In this experiment we investigated some of the properties of the resonance states in $^{29}$P from excitation energies of 5.7 to 7.7 MeV and determined the partial decay proton widths $\Gamma_{\ell, j}$ for the resonant states. The entire analysis of the experimental data was done under the assumption that only compound nucleus reactions take place; i.e. we assume that there is no direct background. Utilizing the compound nucleus theory we have been able to extract essentially all of the nuclear information regarding the structure of these resonances, i.e., the energy, spin, parity, total width, partial decay widths and relative phases.

With all of the necessary experimental information on the properties of those resonances, it should be possible to describe the wave function of the resonances within the framework of a model. We base our analogies upon the nuclear structure model in which the $^{29}$P nucleus is assumed to consist of a core nucleus* of $^{28}$Si coupled to a single proton having certain $\ell$ and $j$ values. We take into consideration the fact that the decay of $^{29}$P happens only through proton emission, at least for the energy region of $^{29}$P under study. (By comparison, the Q values for (p,n), (p,α) and (p,d) reactions are much higher than for the (p,p') reaction.) The residual

* By "core-nucleus" we mean a nucleus whose single particle excitations are large compared to collective oscillations.
nucleus is the $^{28}$Si nucleus, either in its ground state or in its first $2^+$ excited state.

The wave function $\psi_\lambda$ which describes a resonant state $\lambda$ is assumed to be linear combination of single particle proton shell model state, $p(\ell,j)$ coupled to the ground state $^{28}$Si(p$^+$) and the first excited state $^{28}$Si(2+) of $^{28}$Si nucleus.

$$\psi_\lambda (J^\pi) = \sum_{\ell, j} \alpha_{\ell,j}^\lambda [p(\ell,j) \otimes 28Si(p^+)] + \sum_{\ell, j} \alpha_{\ell,j}^{28Si} [p(\ell,j) \otimes 28Si(2^+)]$$

The coefficients $\alpha_{\ell,j}^\lambda$ are the parentage coefficient and they are proportional to the reduced partial decay proton widths $\gamma_{\ell,j}$.

The parenthesis which follows the coefficients $\alpha_{\ell,j}$ includes the wave functions of the $^{28}$Si core nucleus and of the single particle as well as the geometry coefficients which results from the coupling of the angular momenta associated with the above wave functions.

The fact that the coefficient $\alpha_{\ell,j}$ is proportional to the reduced width comes out of the definition of the reduced width which is taken from Lane$^{32}$:

$$\gamma_{\ell,j} = \frac{\hbar^2}{2m_c a_c} \int [p(\ell,j) \otimes 28Si(p^+)]^* \psi_\lambda (J^\pi) \, d^4S$$

The orthogonality of the $[p(\ell,j) \otimes 28Si(I)]$ implies that $\alpha_{\ell,j}$ must be proportional to $\gamma_{\ell,j}$. In this model $\gamma_{\ell,j}$ is the degree we can consider the compound state $\psi_\lambda (J^\pi)$ as being composed of $^{28}$Si core nucleus coupled to a proton with certain $\ell$ and $j$ values. The experimental reduced decay proton widths are calculated from the experimentally determined partial decay proton widths, taking into account the penetrability of the core
nucleus. The formula which relates the reduced widths to the partial decay widths is given by

\[ \Gamma_{\ell j} = \frac{2 \rho \gamma_{\ell j}^2}{1 + \sum_{\ell_j} \gamma_{\ell j}^2 S_{\ell}} \]

The meaning of the different symbols are given in Chapter I at the end of the discussion about the strong coupling model.

The term \( \sum_{\ell} \gamma_{\ell j}^2 S_{\ell} \) is small compared to unity if the value \( \gamma_{\ell j}^2 = \frac{\Gamma_{\ell j}}{2 \rho_{\ell}} \) is small compared to the value \( 0.3 \frac{\hbar^2}{\mu a^2} \) where \( \mu \) is the reduced mass of the residual nucleus and proton and \( a \) is the radius of the residual nucleus.

The values of the reduced widths \( \gamma_{\ell j}^2 \) were calculated using the formula

\[ \gamma_{\ell j}^2 = \frac{\Gamma_{\ell j}}{2 \rho_{\ell}} \]

since the reduced widths for the states studied are smaller than the value of \( 0.3 \frac{\hbar^2}{\mu a^2} = 0.4 \text{ MeV} \).

The penetrabilities \( P_{\ell} \) of the coulomb barrier for the different \( \ell \)-values and at the energy region \( E = 4.0 \text{ MeV} \) were calculated using the graphs of \( P_{\ell} \) versus energy, of Ref. 12. The values of \( \gamma_{\ell j}^2 \), for the different resonance states are shown in Table 4. As an example of how we can write the wave function of a resonant state \( \ell \)-value using the numerical values of the reduced widths \( \gamma_{\ell j} \) of Table 4 we examine the 3.1 MeV resonance.

As we mentioned earlier the wave functions which describe the \( ^{29}\text{p} \) states in this model are given by

\[ \psi_{\gamma}(\ell) = \sum_{\ell,j} \alpha_{\ell,j} \left[ P(\ell,j) e^{i \beta_{\ell,j}} S_{\ell}^{(\ell)} + \sum_{\ell_j} \alpha_{\ell_j} \left[ P(\ell_j) e^{i \beta_{\ell_j}} S_{\ell_j}^{(\ell)} \right] \right] \]
Table 4

The reduced widths of six of the resonant states in $^{29}$P from excitation energies 5.7 to 7.1 MeV. The reduced widths which were calculated using the experimental partial decay proton widths of Table 3 are shown in the last ten columns.
<table>
<thead>
<tr>
<th>$E_p$ (MeV)</th>
<th>$E_x^{29p}$ (MeV)</th>
<th>Elastic $\gamma^2_{j}$ (keV)</th>
<th>$\gamma^2_{s1/2}$ (keV)</th>
<th>$\gamma^2_{p1/2}$ (keV)</th>
<th>$\gamma^2_{p3/2}$ (keV)</th>
<th>$\gamma^2_{d3/2}$ (keV)</th>
<th>$\gamma^2_{d5/2}$ (keV)</th>
<th>$\gamma^2_{f5/2}$ (keV)</th>
<th>$\gamma^2_{f7/2}$ (keV)</th>
<th>$\gamma^2_{g7/2}$ (keV)</th>
<th>$\gamma^2_{g9/2}$ (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.10</td>
<td>5.73</td>
<td>42.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.2</td>
<td>2.65</td>
<td></td>
</tr>
<tr>
<td>3.33</td>
<td>5.96</td>
<td>7.7</td>
<td>0.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.1</td>
<td>0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.57</td>
<td>6.19</td>
<td>16.0</td>
<td></td>
<td>8.0</td>
<td></td>
<td></td>
<td></td>
<td>54.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.71</td>
<td>6.32</td>
<td>21.6</td>
<td>27.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.2</td>
<td>1.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.23</td>
<td>6.82</td>
<td>4.8</td>
<td>1.75</td>
<td></td>
<td>0.72</td>
<td>0.48</td>
<td></td>
<td>2.1</td>
<td>10.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.43</td>
<td>7.02</td>
<td>24.0</td>
<td>53.0</td>
<td>2.4</td>
<td></td>
<td></td>
<td></td>
<td>11.0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 5  The wave functions of six of the resonant states in $^{29}\text{P}$ as calculated using the simplified model discussed in Chapter IV.
<table>
<thead>
<tr>
<th>$E_p$ (MeV)</th>
<th>$E_e$ ($^3P_2$) (MeV)</th>
<th>Spin</th>
<th>Parity</th>
<th>Allowed $L$-wave Elastic Channel</th>
<th>Allowed $L$-wave Inelastic Channel</th>
<th>Wave Function</th>
<th>$\Psi(J^P)dw$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.19</td>
<td>5.737</td>
<td>7/2</td>
<td>$-3^-$</td>
<td>0, 2, 4</td>
<td>s, d, g</td>
<td>$2.7 { p(d_3) e^{2S(3)} }$ - $0.9 { p(d_{3/2}) e^{2S(5/2)} }$ - $0.3 { p(d_{3/2}) e^{2S(5/2)} }$</td>
<td></td>
</tr>
<tr>
<td>3.57</td>
<td>6.191</td>
<td>3/2</td>
<td>$-1^-$</td>
<td>0, 2</td>
<td>p, f</td>
<td>$4.6 { p(d_3) e^{2S(3)} }$ + $2.8 { p(d_{3/2}) e^{2S(5/2)} }$ + $7.4 { p(d_{3/2}) e^{2S(5/2)} }$</td>
<td></td>
</tr>
<tr>
<td>3.71</td>
<td>6.376</td>
<td>3/2</td>
<td>$-2^-$</td>
<td>0, 2</td>
<td>s, d, g</td>
<td>$4.5 { p(d_3) e^{2S(3)} }$ + $5.1 { p(d_{3/2}) e^{2S(5/2)} }$ + $1.1 { p(d_{3/2}) e^{2S(5/2)} }$</td>
<td></td>
</tr>
<tr>
<td>4.25</td>
<td>6.828</td>
<td>3/2</td>
<td>$-2^-$</td>
<td>0, 2</td>
<td>s, d, g</td>
<td>$2.2 { p(d_3) e^{2S(3)} }$ + $1.3 { p(d_{3/2}) e^{2S(5/2)} }$ + $0.8 { p(d_{3/2}) e^{2S(5/2)} }$</td>
<td></td>
</tr>
<tr>
<td>4.45</td>
<td>7.021</td>
<td>3/2</td>
<td>$-1^-$</td>
<td>0, 2</td>
<td>p, f</td>
<td>$4.9 { p(d_3) e^{2S(3)} }$ + $1.5 { p(d_{3/2}) e^{2S(5/2)} }$ + $3.3 { p(d_{3/2}) e^{2S(5/2)} }$</td>
<td></td>
</tr>
</tbody>
</table>
since

\[ \alpha \ell_{i,j} \propto \gamma \ell_{i,j} \]

we can write the wave function \( \psi_\lambda(J^\pi) \) as

\[ \psi_\lambda(J^\pi) \propto \sum_{\ell_{i,j}} \gamma_{\ell_{i,j}} [P(\ell_{i,j}) \Theta^z S_i(\ell)] + \sum_{\ell_{i,j}} \alpha_{\ell_{i,j}} [P(\ell_{i,j}) \Theta^z S_i(\ell)] \]

From Table 4 we get the values of \( \gamma_{\ell_{i,j}} \) for the 3.1 MeV resonance and we substitute them in to the above equation.

\[ \psi_\lambda(\frac{1}{2}^-) \propto 6.5 [P(1,\frac{1}{2}) \Theta^z S_i(0^+)] + 2.7 [P(1,\frac{1}{2}) \Theta^z S_i(1^+)] + \]

\[ + 1.0 [P(1,\frac{1}{2}) \Theta^z S_i(2^+)] + 1.3 [P(1,\frac{1}{2}) \Theta^z S_i(3^+)] \]

The wave functions for the rest of the resonances are tabulated in Table 5.

The description of the resonant states of \(^{29}\text{P}\) in terms of these wave functions may not be a complete and accurate description because the model used for the derivation of the wave function is very simplified.
CHAPTER V
SUMMARY

With the set of experiments we performed and which are described in this dissertation we were able to determine the partial decay proton widths ($\Gamma_{j}$) and relative phases ($\phi_{j}$) associated with the protons emitted from the decay of $^{29}$P resonant states.

We found that a single type of experiment (for example the angular correlations in the Goldfarb-Seyler geometry) cannot provide a unique solution to our problem. In other words a different set of values of $\Gamma_{j}$ and $\phi_{j}$ could fit the experimental data equally well. A unique solution of the problems emerges only if we impose additional restrictions on the parameters $\Gamma_{j}$ and $\phi_{j}$, i.e., that these parameters fit the experimental data of different and independent experiments. Although the determined values of $\Gamma_{j}$ and $\phi_{j}$ (Table 3) which fit the experimental data of the three present experiments appear to be unique, we are not able to guarantee that they represent a unique solution.

By means of these techniques we were able to determine not only the outgoing proton $\ell$-waves (those which could escape the Coulomb barrier), resulting from the decay of the compound nucleus $^{29}$P but also the $j$ value. Previous attempts$^{28}$ at determining the partial decay proton widths from the results of a single angular distribution could provide information
(of doubtful validity) only for one outgoing proton $\ell$-wave.

The wave functions reported in Table 5 are based on single core-excitation model. The actual situation is surely more complicated.

The Coulomb barrier is responsible for the fact that we cannot derive information for proton waves with large $\ell$-value (assuming these values are allowed from conservation laws). For neutrons the situation is different and our technique may well have applicability for the determination of partial decay widths of neutrons emitted in nuclear reactions. This is a general comment which takes into consideration only the analysis of experimental data and not the difficulty of performing experiments of the same type we performed but with neutrons instead of protons.
APPENDIX

I. DERIVATION OF THE THEORETICAL EXPRESSION OF THE ANGULAR DISTRIBUTION OF SCATTERED PARTICLES

The derivation of the angular distribution of scattered particles with spin $s$ from a target nucleus with spin $a$ is given below. An intermediate CN state with spin $b$ and parity $P$ is assumed to be formed during the reaction. The CN state decays to another nucleus having spin $c$ by emission of a particle (see Fig. A1). ($L$ and $\ell$ are symbols for the total and orbital angular momentum of the incident or outgoing particles.

The statistical and efficiency tensor formalism is used for the derivation of angular correlation expression. More details about angular distributions and correlations are given in Ref. 26.

The probability of detection of radiations by an arrangement of detectors is given by

$$W = \sum_{b, b', a, a'} \rho_{k_{a, k_{a'}}(b, b')(a, a')} e_{k_{b, k_{b'}}(b, b')(a, a')}$$

$\rho_{kbkb}$ and $e_{kbkb}$ are the statistical and efficiency tensors, respectively. B represents the set of quantum numbers $(a, L_1, L_2, L_2, c)$. The statistical tensor is written in terms of the density matrix as:
Fig. A1 Schematic diagram of a $X(p,p',\gamma)$ reaction with a CN formation.
Making use of the Wigner Eckart Theorem, we can factor the energy dependent matrix elements.

\[ p_{k'kb'}(b_{B}b'B') = \sum_{b'} (-1)^{b'_c} \langle b'_c b'_c | k'kbk \rangle \langle b_{B'} | b_{B'} \rangle \]  
(A1)

\[ \langle b'_{B'} | b'_{B'} \rangle = \sum_{BcB'c} \langle b_{B} | b_{B} \rangle \langle b_{B} | b_{B} \rangle \langle b_{B'} | b_{B'} \rangle \langle b_{B'} | b_{B'} \rangle \]  
(A2)

The energy independent part of (A2) is written as:

\[ \langle b'_{B'} | b'_{B'} \rangle = \sum_{\omega, \Lambda, t, L} \langle a_{\omega} | a_{\omega} \rangle \langle \lambda, \Lambda | \lambda, \Lambda \rangle \langle b_{t} | a_{\lambda} \times L, \Lambda \rangle \langle a_{\lambda} \times L, \Lambda | b_{L} \rangle \]  
(A3)

\[ \langle a_{\omega} | a_{\omega} \rangle = \sum_{k, k, a} (-1)^{\omega} \langle a_{\omega} | a_{\omega} \rangle \langle k, k, a | k, k, a \rangle \]  
(A4)

\[ \langle \lambda, \Lambda | \lambda, \Lambda \rangle = \sum_{\lambda} (-1)^{\lambda} \langle \lambda, \Lambda | \lambda, \Lambda \rangle \langle L_{\lambda}, L_{\Lambda} \times L_{\lambda}, L_{\Lambda} | L_{\lambda}, L_{\Lambda} \rangle \]  
(A5)

The script \( L \) represents the quantum numbers \( \ell \) and \( \Lambda \). By making the proper substitutions, the eqs. (A1) to (A5) give
Using the properties of 9-j symbols

\[ f_{kbk'}(bB, b'B') = \sum_{k,k_c,B_c} \hat{b}_{k_{c}k'a} \hat{b'}_{k'a} \left( \begin{array}{ccc} a & a' & k_a \\ b & b' & k_b \\ L, L' & k_c \\ L' & k_{c'} & k_{c'} \\ L'' & k'' & k'' \\ L'' & k_{c''} & k_{c''} \end{array} \right) \]

(A6)

By reducing the efficiency tensor \( \epsilon_{kk'b'}(bB, b'B') \) in terms of the product of the efficiency tensor of state \( c \) and the efficiency tensor of radiation \( L_2 \),

\[ \epsilon_{kk'b'}(bB, b'B') = \sum_{k_{c1}k_{c2}} \hat{b}_{k_{c1}k_{c2}k_{c3}} \left( \begin{array}{ccc} c & c' & k_{c1} \\ b & b' & k_{c2} \\ L_2 & L_2' & k_{c3} \end{array} \right) \]

(A7)

Substitution of (A6) and (A7) into the distribution function \( W \), gives

\[ W \propto \sum_{bb'B'B'} \hat{b}_{k_{b}k'_{b}k'_{b}} \left( \begin{array}{ccc} a & a' & k_a \\ b & b' & k_b \\ L, L' & k_c \\ L' & k_{c'} & k_{c'} \\ L'' & k'' & k'' \\ L'' & k_{c''} & k_{c''} \end{array} \right) \]

(A8)
For well-defined states $a$ and $c$

$$\hat{\chi}_a(\omega, a') = \frac{1}{\mathcal{A}} \hat{\omega}_{a a'} \hat{\chi}_{a a'} \hat{\omega}_{a a'}$$

and the Clebsch-Gordon coefficients

$$\langle k_{a a_1} k_{b b_1} \mid h_{a b} \rangle = \langle c c_1 k_{a_2 b_2} \rangle = \begin{cases} 1 & \text{if } k_1 = k_2 = k \\
0 & \text{otherwise} \end{cases}$$

Because of the (A9) and (A10) the distribution function (A8) becomes

$$W \propto \sum_{k k_1} \sum_{h b} \sum_{m k} \left\{ a a c \right\} \hat{\omega}_{a a'} \hat{\chi}_{a a'} \hat{\omega}_{a a'} \left\{ c c c \right\}$$

$$W(\omega, a, a', c, c') = \left\{ a a c \right\} \hat{\omega}_{a a'} \hat{\chi}_{a a'} \hat{\omega}_{a a'} \left\{ c c c \right\} \left\{ a a c \right\}$$

But since

$$\left\{ a a c \right\} \hat{\omega}_{a a'} \hat{\chi}_{a a'} \hat{\omega}_{a a'} \left\{ c c c \right\} = \frac{\delta_{a, a'}}{\mathcal{A}} W(h b', L, L'; k a)$$

(A11) becomes

$$W \propto \sum_{k k_1} \sum_{h b} \sum_{m k} \left\{ a a c \right\} \hat{\omega}_{a a'} \hat{\chi}_{a a'} \hat{\omega}_{a a'} \left\{ c c c \right\} \left\{ a a c \right\}$$

$$W(h b', L, L'; k a) \left\{ a a c \right\} \hat{\omega}_{a a'} \hat{\chi}_{a a'} \hat{\omega}_{a a'} \left\{ c c c \right\} \left\{ a a c \right\}$$

(A12)
The statistical and efficiency tensors can be written as:

\[ P_{kk}(x, y) = \sum_{k'} c_{kk'}(x, y) \Delta^k_{kk'}(R) \]

\[ E^k_{kk}(x, y) = \sum_{k''} c^*_{kk''}(x, y) D_{kk''}(R) \]

Where \( D \) is the symbols for the rotation matrix. Making use of (A13)

\[ P_{kk}(x, y') E^k_{kk}(y, y') = \sum_{k'} c_{kk'}(x, y') c^*_{kk'}(y, y') D^k_{kk'}(R) D^*_{kk'}(R) = \]

\[ = \sum_{k' k''} c_{kk'}(x, y') c^*_{kk''}(y, y') D^k_{kk'}(R) D^*_{kk''}(R) = \sum_{k' k''} c_{kk'} c^*_{kk''} D^k_{kk'}(R) \]

(A14)

For unpolarized radiation \( C_{kk} = 0 \) if \( k \neq 0 \). Substitution of (A14) into (A12) gives

\[ W(b, l, l', k_0) = \sum_{k} <c_{zz} b|0|a_z b> <a_z b|0|a_z b> \]

Use was made of \( D_{00} = P_k(\cos \theta) \) where \( P_k \) is the Legendre Polynomial of \( k \) order.

For absorbed radiation of spin \( s \) particle

\[ C_{kk} = \frac{\hat{\epsilon} \hat{l} (c, c')}{4\pi} (L \cdot k) \cdot \hat{\ell} \cdot k \]

(A16)

For emitted radiation of spin \( s \) particle

\[ E^k_{k} = \frac{\hat{\epsilon} \hat{l} (c, c')}{4\pi} (L \cdot k) \cdot \hat{\ell} \cdot k \]

(A17)
Substituting (A16) and (A17) into A15

\[ W(\alpha) \sum_{k} \sum_{\xi} \frac{\hat{b}^2 \hat{b}^2}{(4\pi)^\frac{1}{2}} (-)^{a+b-1} \hat{L}_1 \hat{L}_1 \hat{L}_2 \hat{L}_2 \hat{L}_3 \hat{L}_3 \]

(A18)

\[ (l, 0, l', 0 \mid k0) W(l, l', l, l' \mid ks) (l_2, 0, l_2 \mid k0) W(l_2, l_2, l_2 \mid ks) \]

\[ W(b b' l, l' \mid k a) W(b b' l_2, l_2 \mid k c) \]

Multiplication of \( W(\Theta) \) by an appropriate constant will convert the angular distribution expression into absolute cross section

\[ \frac{d\sigma(\Theta)}{d\Omega} = \text{const} \ W(\Theta) \]  

(A19)

Integration of (A19) over all solid angles gives:

\[ \sigma = \text{const} \sum_{k} \sum_{\xi} \frac{\hat{b}^2 \hat{b}^2}{(4\pi)^\frac{1}{2}} (-)^{a+b-1} \hat{L}_1 \hat{L}_1 \hat{L}_2 \hat{L}_2 \hat{L}_3 \hat{L}_3 \]

\[ (l, 0, l', 0 \mid k0) W(l, l', l, l' \mid ks) (l_2, 0, l_2 \mid k0) W(l_2, l_2, l_2 \mid ks) \]

\[ W(b b' l, l' \mid k a) W(b b' l_2, l_2 \mid k c) \]

For \( k = 0 \)

\[ (l, 0, l', 0 \mid 00) = \frac{1}{l_1} \quad \text{and} \quad l_1 = l'_1 \]

\[ W(l, l', l, l' \mid 0s) = \frac{1}{l_{11}} \quad \text{and} \quad l_1 = l' \]

(A21)
Substituting expressions (A21) into (A20)

\[ \sigma = \frac{\text{const}}{(4\pi)} \sum_{\mathbf{A}_1} \frac{\mathbf{b}^2}{\mathbf{A}_1^2 \hbar^2} | \langle \mathbf{A}_2, \mathbf{b} | 0 | \mathbf{A}_2, \mathbf{b} \rangle |^2 \]  

(A22)

Comparing (A22) with expression of total cross section for resonance scattering

\[ \sigma = \pi \sum_{\mathbf{A}_1} \frac{\mathbf{b}^2}{\mathbf{A}_1^2 \hbar^2} | \langle \mathbf{A}_2, \mathbf{b} | 0 | \mathbf{A}_2, \mathbf{b} \rangle |^2 \]

the multiplication constant turns out to be

\[ \text{const} = (2\pi)^2 \]

Thus the theoretical expression for the angular distribution function for resonance reaction is given by

\[ W(\theta) = \sum_{k_1 k_2 \ldots} \frac{(2\pi)^2}{(4\pi)^2 \hbar^2 \mathbf{A}_1} | \langle \mathbf{A}_2, \mathbf{b}, \ldots | 0 | \mathbf{A}_2, \mathbf{b}, \ldots \rangle |^2 \]

(A23)

Thus the theoretical expression for the angular distribution function for resonance reaction is given by

\[ W(\theta) = \sum_{k_1 k_2 \ldots} \frac{(2\pi)^2}{(4\pi)^2 \hbar^2 \mathbf{A}_1} | \langle \mathbf{A}_2, \mathbf{b}, \ldots | 0 | \mathbf{A}_2, \mathbf{b}, \ldots \rangle |^2 \]

(A23)

The symbol \( \chi \) is the reduced wavelength for the system projectile-target

\[ \chi = \frac{\hbar}{2mE_{cm}} \]
where \( \mu \) is the reduced mass of projectile-target and \( E_{cm} \) is the center-of-mass energy of the projectile. For proton as a projectile and \(^{28}\text{Si}\) as a target, the value of the reduced wave length is given by

\[
\lambda = \frac{\hbar}{\sqrt{A \cdot \mu \cdot E_{cm}}} = \frac{c_s}{E(k_{es})} = \frac{\sqrt{A - \left(\frac{A}{2} - 1\right) \cdot (\text{sample mass})}}{c} \cdot E(k_{es})
\]

For computational purposes, the angular distribution expression (A23) is written as the product of two parts: one which depends upon the energy and the type of interaction, and another part which depends upon the geometry. Thus

\[
\frac{d\sigma}{d\Omega} = \sum_{k_{es}} B_k \left( b_{iL_1L_1L_2L_2} \right) \left( c_{iL_1^*L_1^*L_1^*L_1^*} \right) \left( c_{sL_2L_2} \right) \left( c_{sL_2L_2} \right) \left( \text{angular distribution} \right)
\]

In the computer program for the resonance analysis, the coefficient \( B_k \) is under the name geometry coefficient of angular distributions. For spin \( s = \frac{1}{2} \) particles, the following relation is valid

\[
\hat{L} \cdot \hat{c} \left( c \mid k \right) = \left( L_{\frac{1}{2}} \mid L - \frac{1}{2} \mid k \right)
\]

Substitution of (A27) into (A26) gives

\[
A_k = \frac{c_s}{4 \pi \hbar^3 \gamma} \left( -e^{i\theta} \right) \left( L_{\frac{1}{2}} \mid L_{\frac{1}{2}} \mid k \right) \left( L_{\frac{1}{2}} \mid L_{\frac{1}{2}} \mid k \right) \left( \text{angular distribution} \right)
\]

Formula (A28) is equivalent to formula (A26) but easier for computational
calculations. When using either formula (A26) or (A28) for computer calculations, parity conservation must be built into the program which calculates their numerical values.

For the special case of target nucleus with \( a = 0^+ \), final nucleus with \( c = 2^+ \) and proton as a projectile, the total angular momenta of the proton must be equal to the spin of the CN nucleus \( L_1 = b, L'_1 = b' \). Thus the expression for the geometry coefficients of the angular distribution becomes

\[
B_k = \frac{c_i}{8} \sum_{l,l',l''} \frac{\hat{b}}{\hat{b}'} L_2 \hat{l}_2 \hat{b}_2 \hat{l}_2 \hat{b}_2 (-)^{\hat{b}'-1} (l, c, l', c' | k, 0) W(l, l', b', b'; k_2) W(l, l', l_2 L_2; k_2) W(b'b'b'; k_2)
\]

and

\[
A_k = \frac{c_i}{8} (-)^{\hat{b}'-1} \hat{b} \hat{b} L_2 \hat{l}_2 \hat{l}_2 (b' b', b', b' | k, 0) (l_2 L_2 - l_2 | k, 0) W(b'b'b'b'; k_0) W(b'b L_2 L_2; k_2)
\]
II. DERIVATION OF THE THEORETICAL EXPRESSION OF THE TRIPLE ANGULAR CORRELATION

For this triple angular correlation an incident radiation whose direction is fixed interacts with the nucleus which emits two successive radiations, $R_1$, and $R_2$. The relative probability $W(\theta)$ that the radiation $R_1$ will be emitted at an angle $\theta$ with respect to $R_2$ will be derived below.

The three radiations in this particular experiment are, (see Fig. A1), the incoming radiation which consists of protons, the inelastic proton which results from the decay of the CN to the state $c$, and the $\gamma$-ray emitted from the decay of the nucleus being on a state $c$ to another state $d$.

Using the statistical and efficiency tensor formalism the correlation function is written

$$W = \sum_{k_b k_b'} f_{k_b k_b'}(b B, b' B') \hat{C}^*_{k_b k_b}(h B, h B')$$ (A30)

Following the derivation procedure which leads to the expression (A6) for the statistical tensor. This procedure gives again

$$f_{k_b k_b'}(b B, b' B') = \sum_{k_a k_a'} \hat{a}_a \hat{a}'_{a'} \left\{ \begin{array}{ccc} a & a' & k_a \\ L. L. & L. & k_a \end{array} \right\} (l_b a_l k_a l_{b'} a'_{l'} k_{a'})$$ (A31)

$$\text{Br} \ f_{l_b a_l a'_{l'} a'_{l'}} \langle l_b a_l (n, l) | R_b \rangle \langle c B | b B' \rangle \langle h B | h B' \rangle$$
the efficiency tensor is written as

$$E_{k,b,k,b'} = \sum_{k_h} h^b h^c \left\{ \begin{array}{c} c' \cr c \end{array} \right\} \left\{ \begin{array}{c} \alpha \cr \beta \end{array} \right\} \left| k_h b k'_h \right|$$  \hspace{1cm} (A32)

$$E_{k,k',c,c'} = \sum_{k_h} h^c h^{c'} \left\{ \begin{array}{c} c' \cr c \end{array} \right\} \left\{ \begin{array}{c} L_1 L_2 L_3 \cr k_h k_h k_h \end{array} \right\} \left| k_h k_h k_h \right|$$  \hspace{1cm} (A33)

Now substituting (A33) into (A32) then (A31) and (A32) into (A30), the correlation function becomes

$$W = \sum_{k_h} h^a h^b \left\{ \begin{array}{c} c' \cr c \end{array} \right\} \left\{ \begin{array}{c} L_1 L_2 L_3 \cr h_h h_h h_h \end{array} \right\} \left| k_h k_h k_h \right|$$  \hspace{1cm} (A34)

$$\langle d\gamma_3 c' \gamma_2 b | a, a', b \rangle < d\gamma_3 c' \gamma_2 b | 0' | a, a', b' \rangle$$

The summation goes over $b, b', B, B' k_1 k_2 k_3 k_a k_b k_c k' d k_1 k_2 k_3 k_a k_b k_c k' d$

For well-defined states $a$ and $d$ we get

$$\delta_{k_a h_a(a, a')} = \delta_{a' h_a}$$  \hspace{1cm} (A35)
The following relation also holds

\[
\begin{bmatrix}
\alpha & \alpha & c \\
L_1 & L_2 & k
\end{bmatrix} = \frac{e^{-\alpha k} - b - L_1}{\alpha k} \mathcal{W}(b' L_1 L_2; 1, k, a)
\]  

(A37)

Making use of (A35), (A36), (A37) the formula (A34) becomes

\[
\mathcal{W} = (2\pi)^2 \sum \frac{\hat{\mathcal{B}}^{(k)} b^*}{\alpha^2 k_1} c^{-\alpha k} - b - L_1 \mathcal{W}(b' L_1 L_2; 1, a, k)
\]

(A38)

By making use of \(\rho_{kk'} = \sum_{k'} C_{kk'} D_{kk'}(R, 1)\) and the fact that \(C_{kk'} = 0\) for \(k' \neq 0\) if the incident radiation is unpolarized

\[
f_{kk} = C_{kk} D_{kk}(R, 1) = C_{k0} \frac{\sqrt{\pi}}{\alpha k} \mathcal{W}^{k*}
\]

(A39)

Substitution of (A39) into (A38) gives

\[
\mathcal{W} = (2\pi)^2 \sum \frac{\hat{\mathcal{B}}^{(k)} b^*}{\alpha^2 k_1} c^{-\alpha k} - b - L_1 \mathcal{W}(b' L_1 L_2; 1, a, k)
\]

(A40)
By setting $\kappa_2 = -\kappa'_2$

\[
(k_{1'}, k_{2'}, k_{2'}') = (k_{1'}, k_{2}, k_{2})/(k_{3}, k_{3}) \quad (\sim) \quad k_{2'} - k_{3}
\]

and

\[
\gamma_{k_2'}(\nu_1) = (-)^{k_2} \gamma_{k_2'}
\]

The correlation function becomes

\[
W = (2\pi \hbar)^4 \sum_{\alpha_i} \frac{\hbar^2 c}{\tilde{c}^2} \exp (a - b - L - \kappa - c' - L_2) W(b', L_2'; k, \alpha)
\]

\[
W(c, c' \mid L_3, k_3')\left\{ L_2' L'_2 \right\} (k_{1'}, k_{2} - l_1, k_{2}) \alpha_i(c, c', k_{2})
\]

\[
C_{k_0}(\hat{\nu}_2, \hat{\nu}_2') C_{k_0}(\hat{\nu}_2, \hat{\nu}_2') (2\pi)^2 \gamma_{(O_1)} \gamma_{(O_2)} \gamma_{(O_2')}
\]

\[
< \hat{c}_{2} c_{2} \mid (\hat{\omega}_{2}, \hat{\nu}_{2}) \rangle \langle \hat{c}_{2'} c_{2'}' \mid (\hat{\omega}_{2}, \hat{\nu}_{2}) \rangle
\]

By making the following substitutions

\[
C_{k_0}(\hat{\nu}_2, \hat{\nu}_2') = (-)^{k_0} \frac{L_1 + k_0 - \hat{\nu}_1 \hat{\nu}_2}{4\pi S} (k_{0}, \alpha \mid c, c') W(k_{0}, \alpha \mid L_1, L_2', k_{2})
\]

(for absorption of spin s particles)

\[
C_{k_0}(\hat{\nu}_2, \hat{\nu}_2') = (-)^{k_0} \frac{L_1 + k_0 - \hat{\nu}_1 \hat{\nu}_2}{4\pi S} (k_{0}, \alpha \mid c, c') W(k_{0}, \alpha \mid L_1, L_2', k_{2})
\]

(for emission of particles with spin s) and

\[
C_{k_0}(\hat{\nu}_2, \hat{\nu}_2') = (-)^{k_0} \frac{L_1 + k_0 - \hat{\nu}_1 \hat{\nu}_2}{4\pi S} (k_{0}, \alpha \mid c, c') W(k_{0}, \alpha \mid L_1, L_2', k_{2})
\]

(for $\gamma$-ray emission.)
the correlation function \((A41)\) becomes

\[
W(\Omega_1, \Omega_2, \Omega_3) = (2\pi)^2 \sum_{L, r, l} \frac{b b' c c'}{e^{i r y}} (-1)^{L_2 + L_3 + L - r} \left\{ \begin{array}{c} c' c \cr l' l \cr L_2 L_3 L \end{array} \right\} (k_1 k_2 k_3) W(b b' L_1 L_2 L_3) W(c c' L_1 L_2 L_3) \left\{ \begin{array}{c} L_1 L_2 L_3 \
 b' b \cr L \end{array} \right\} (k_1 k_2 k_3)
\]

\[(A42)\]

To find the numerical values of the constant \(|(d|^3|c)|^2\), the integral overall \(\gamma\)-rays emitted in a solid angle \(4\pi\) is evaluated and then compared to the angular distribution theoretical expression.

\[
W(\Omega_1, \Omega_2, \Omega_3) = \int W(\Omega_1, \Omega_2, \Omega_3) d\Omega_3
\]

\[(A43)\]

The integration of the spherical harmonic \(Y_{k_3}^{k_3}(\Omega_3)\) which is involved in the correlation function gives

\[
\int Y_{k_3}^{k_3}(\Omega_3) d\Omega_3 = \sqrt{\frac{4\pi}{k_3}} \delta_{k_3 0}
\]

Also for \(k_3 = 0, k_3 = 0\)

\[
\left\langle L, k, k, L, | c c \right\rangle = \frac{c_{L, k, k}}{e^{i r y}} \Rightarrow k = L, k = -L
\]

\[
\sum_{k_3} Y_{k_3}^{k_3} Y_{k_3}^{k_3} = \frac{1}{4\pi} \sum_{k_3} D_{k_3}^{k_3}(\Omega_3) \bar{D}_{k_3}(\Omega_3) = \frac{1}{4\pi} \sum_{k_3} \bar{D}_{k_3}^{k_3}(\Omega_3) \bar{D}_{k_3}(\Omega_3) = \frac{1}{4\pi} \sum_{k_3} \bar{D}_{k_3}^{k_3}(\Omega_3) \bar{D}_{k_3}(\Omega_3)
\]
\[ \langle L_3, L_3-1 | cc \rangle = \frac{C}{L_3!} \]

\[ \mathcal{W}(cc L_3 L_3'; cl) = \frac{C}{L_3!} \]

\[ \left\{ \begin{array}{c}
C \left[ \frac{1}{L_3} \right] \\
\mathcal{W}(bb'L_2 L_2'; cc) \\
C \left[ \frac{1}{L_2} \right]
\end{array} \right. \]

The integral (A43) of the correlation function is given by using all the expressions following (A43).

\[ \mathcal{W}(c) = \sum_{n=1}^{\infty} \frac{Z^n}{L_3^n} \left( \begin{array}{c}
\frac{1}{L_3} \\
\mathcal{W}(bb'L_2 L_2') \\
\frac{1}{L_2}
\end{array} \right) \]

Comparing (A44) with eq. (A18)

\[ \left( d | \bar{z}_3 | c \right)^2 = 1 \]

and the angular correlation expression is given by

\[ \frac{d^2 \mathcal{W}}{d \Omega_{\bar{z}_3} d \Omega_{c}} = \sum_{L_3, L_3'} \frac{4 \pi^3 \delta^3 \mathcal{W}(cc'; bb'')} {L_3 L_3' L_2 L_2'} \]

\[ \langle \bar{z}_3, L_3, \bar{l}_3 | bb' \rangle \mathcal{W}(bb'L_2 L_2'; cc) \mathcal{W}(cc'L_3 L_3'; l_3) \left( \begin{array}{c}
\langle c' \bar{l}_3 | L_3 \rangle \\
\langle c' \bar{l}_3 | l_3 \rangle
\end{array} \right) \]

\[ \mathcal{W}(bb'L_2 L_2'; cc) \mathcal{W}(cc'L_3 L_3'; l_3) \left( \begin{array}{c}
\langle c' \bar{l}_3 | L_3 \rangle \\
\langle c' \bar{l}_3 | l_3 \rangle
\end{array} \right) \]
The matrix element \( \langle c \sigma_2 b | 0 | a \sigma_1 b \rangle \) describes the interaction of the incident particles with the target nucleus. The energies of the incident particles used are low enough so that the interaction is classified to the Compound Nucleus Reactions.

The matrix element due to the compound nucleus reaction can be expressed as:

\[
\langle c \sigma_2 b | 0 | a \sigma_1 b \rangle = \frac{\sqrt{\Gamma_{L_1} \Gamma_{L_2}} \sqrt{\Gamma_{L_1} \Gamma_{L_2}} \frac{i \phi_{L_1} \phi_{L_2}}{(E - E_0) + i \frac{\Gamma}{2}}}{E - E}
\]

The above expression is the Breit-Wigner formula for the resonance reactions. In both expressions of angular distribution and angular correlation, the product \( \langle c \sigma_2 b | 0 | a \sigma_1 b \rangle \langle c' \sigma_2 b' | 0 | a \sigma_1 b' \rangle \) of the matrix elements appears. The real part of the above product is needed and this is calculated below.
The express (A46) is written this way for computational purposes. The program for resonance analysis which is discussed in Chapter X makes use of this expression.
III. SPECIAL GEOMETRIES USED IN THE ANGULAR CORRELATION

The general expression of the triple angular correlation can take different forms which depend upon the orientation of the coordinate system with respect to the directions of the radiations. (The word "geometry", which is seen below, is referring to the orientation of the coordinate system relative to the radiation.)

Two different types of geometries were used in this experiment, the Goldfarb-Seyler Geometry\(^\text{17}\)\(^\text{17}\) and the spin-flip geometry\(^\text{18}\)\(^\text{18}\). A brief discussion about the two geometries will be given and the form of the angular correlation expressions for each of these geometries will be written down.

In the Goldfarb-Seyler geometry the x-axis is defined along the incident beam direction. Figure A2, the z-axis is chosen on the reaction plane and for experimental convenience the reaction plane which coincides with the perpendicular is chosen. The y-axis then lies on the horizontal plane.

The proton detector is placed along the z-axis, having an angular coordinate \(\theta_2 = 0^\circ\) and since \(\phi_2\) is not defined, the projection quantum number \(\kappa_2\) of the spherical harmonic \(Y_{k_2}^{\kappa_2}(\theta_2, \phi_2)\) must be \(\kappa_2 = 0\). The Clebsch-Gordon coefficient \(\langle k_1 \kappa_1 \kappa_2 | k_3 \kappa_3 \rangle\) in (A45) expression give \(\kappa_1 = \kappa_3 = \kappa\) and the spherical harmonic \(Y_{k_2}^0(0^\circ) = \frac{k_2}{(4\pi)^{1/2}}\).
Fig. A2  Orientation of the coordinate system and arrangement of detectors for the angular correlation in the Goldfarb-Seyler geometry.
Proton detector

Target

Beam direction

Proton detector

\( \gamma \)-ray detector

\( \phi \)
The angular coordinates of the incident beam direction are

\[(\theta_1, \phi_1) = (90^\circ, 0^\circ)\]

and the corresponding spherical harmonic

\[Y_{k_2}^{*}(90^\circ, 0^\circ) = P_k^*(\cos 90^\circ) = P(k_1, \kappa)\]

The \(\gamma\)-ray which is emitted on the \(x-y\) plane and has coordinates \((\theta_3, \phi_3) = (90^\circ, \phi_3)\) and the spherical harmonic \(Y_{k_3}^{*}(\theta_3, \phi_3)\) because

\[Y_{k_3}^{*}(90^\circ, \phi_3) = P(k_3, \kappa)e^{i\kappa\phi_3}.\]

The real part of \(Y_{k_1}^{*}(\Omega_1)Y_{k_2}^{*}(\Omega_2)Y_{k_3}^{*}(\Omega_3)\) in this geometry becomes

\[\text{Re} \left[ Y_{k_1}^{*}(\Omega_1)Y_{k_2}^{*}(\Omega_2)Y_{k_3}^{*}(\Omega_3) \right] = \frac{1}{4\sqrt{2}} P(k_1, \kappa) P(k_2, \lambda_2) \cos(k_3 \phi_3)\]

There is a restriction for the values which \(k_3\) takes, this restriction is not included in the correlation function but use of it is made here. If \(k_3 = \text{odd}\) then the correlation function \(W = 0\). Also the type of function \(P(k, \kappa)\) is such that \(P(k, \kappa) = 0\) if \(\kappa - k = \text{odd}\). For nonzero values of the correlation function the following conditions must be satisfied, \(k_3 = \text{even}, \kappa = \text{even}, k_1 = \text{even}\) and the conservation of parity yields \(k_2 = \text{even}\).

(The constant \(\lambda^2 = 0.215 \text{ barns} \frac{\text{E(keV)}}{\text{E(keV)}} = \frac{C_1}{\text{E(keV)}}\).

In the particular case where \(^{28}\text{Si}\) is bombarded by protons, \(a = 0^+,\) \(c = c' = 2^+,\) \(d = 0^+,\) the angular momentum conservation gives

\[L_1 = b \quad L'_1 = b' \quad L_3 = L'_3 = c\]
and eq. (A45) is written as

\[
\frac{d^2 \alpha}{d\Omega_\alpha d\Omega_\beta} = \sum \frac{c_1 c_2}{4 \hat{a}_1^2 \hat{a}_2^2 \hat{b}_1^2 \hat{b}_2^2} (-)^{i-k} W_h(k, h) W_h(k, h) \langle \hat{c}_1 \hat{c}_2 | \hat{h}_1 \hat{h}_2 \rangle_{l, k, c} W_h(k, h) \langle \hat{c}_1 \hat{c}_2 | \hat{h}_1 \hat{h}_2 \rangle_{l, k, c} W_h(k, h) \langle \hat{c}_1 \hat{c}_2 | \hat{h}_1 \hat{h}_2 \rangle_{l, k, c} W_h(k, h) \langle \hat{c}_1 \hat{c}_2 | \hat{h}_1 \hat{h}_2 \rangle_{l, k, c}
\]

where the summation goes over \( \kappa, b, b', j_1, j_2, j_2 \). For computational purposes the correlation function is written as:

\[
W(\Omega_1, \Omega_2, \Omega_3) = \sum \mathcal{B}_k \langle \hat{c}_1 \hat{c}_2 \hat{h}_1 \hat{h}_2 \hat{c}_3 \hat{c}_4 \hat{h}_3 \hat{h}_4 \rangle_{l, k, c} \mathcal{E}(\Omega_1, \Omega_2, \Omega_3)
\]

(A48)

The summation goes over \( \kappa, b', b, j_1, j_1, j_2, j_2 \), and:

\[
\mathcal{B}_k = \sum \frac{c_1 c_2 c_3 c_4}{4 \hat{a}_1^2 \hat{a}_2^2 \hat{b}_1^2 \hat{b}_2^2} (-)^{k-c} \mathcal{E}(\Omega_1, \Omega_2, \Omega_3)
\]

(A49)

\[
\mathcal{E}(\Omega_1, \Omega_2, \Omega_3) = \langle l_1 \hat{c}_1 \hat{h}_1 | l_2 \hat{c}_2 \hat{h}_2 \rangle_{l, k, c} \mathcal{W}(l_1 l_2 l_3 l_4 l_5 l_6 l_7 l_8 l_9 l_{10} l_{11} l_{12} l_{13} l_{14} l_{15}) \langle \hat{c}_1 \hat{c}_2 \hat{h}_1 \hat{h}_2 | l_1 l_2 l_3 l_4 l_5 l_6 l_7 l_8 l_9 l_{10} l_{11} l_{12} l_{13} l_{14} l_{15} \rangle
\]

The summation goes over \( l_1, l_1', l_2, l_2', k_1, k_2, k_3 \). The coefficients \( \mathcal{B}_k \) are under the name geometry coefficients of the Goldfarb-Seyler geometry in
program used for the resonance analysis. For spin \( s = \frac{1}{2} \) particle relation (A27) is valid. By substituting (A27) into (A49) another equivalent expression to (A49) results

\[
A_k = \sum_{L_1 L_2 L_3} \frac{c_1 c^*}{4 a_s^2 s^2} \left( \hat{L}_1 \hat{L}_2 \hat{L}_3 \right) \left( \delta_L + \frac{k_1^2 L_1 L_2 L_3}{l} \right) W(b b' b'') \left( l'' l'' l'' l'' \right) \left( b'' b'' b'' b'' \right) \]

(A50)

The \( A_k \) expression is simpler than the \( B_k \) and faster for the computer to calculate \( A_k \). The constant \( c_1 c^* = 0.675 \times 10^6 \text{ mBcm}^2/(\text{sr})^2 \).

(If the x-axis of the coordinate system is chosen to be perpendicular to the beam direction, the coefficients \( A_k \) and \( B_k \) must be multiplied by the factor \((-)^{\kappa/2}\).

In the spin-flip geometry\(^{18}\) the x-axis is along the direction of the incident beam Fig. A3, the y-axis lies on the reaction plane which is chosen to coincide with the horizontal plane and the z-axis is along the perpendicular to the reaction plane. The \( \gamma \)-ray detector detects \( \gamma \)-rays emitted along the z-axis, in coincidence with the protons scattered in the reaction plane. Thus the \( \gamma \)-ray detector is placed along the z-axis, and the angular coordinates of the \( \gamma \)-rays emitted along the z-axis are:

\[
\theta_3 = 90^\circ; \phi_3 = \text{undefined, thus } \kappa_3 = 0 \text{ and the spherical harmonic } \gamma_{k_3}^{K_3}(90^\circ \kappa_3 = 0) = \frac{\hat{k}_3}{(4\pi)^{1/2}} . \]

The Clebsch-Gordon coefficient \( (k_1 k_2 k_2 | k_3 k_3) \) with \( k_3 = 0 \) gives \( k_1 = -k_2 = \kappa \). The angular coordinates of the incident
Fig. A3  Orientation of the coordinate system and arrangement of detectors for the study of the angular correlation in the spin flip geometry.
\( y \)-ray detector

Incident beam

Proton detector
beam are $\theta_1 = 90^0$, $\phi_1 = 0^0$ and the spherical harmonic becomes

$$\gamma_{k_1}^{k_1^*}(q_0, c) = P_{k_1}^{k_1^*}(c, c) = P(k_1, k_1)$$

The angular coordinates of the outgoing proton are: $\theta_2 = 90^0$, $\phi_2 = \phi$ and the spherical harmonic becomes

$$\gamma_{k_2}^{k_2^*}(q_0, \phi) = P(-k_2 - 1) e^{i\phi}$$

and the real part of the expression

$$\text{Re} \left[ \gamma_{k_1}^{k_1^*}(q_0, c) \gamma_{k_2}^{k_2^*}(q_0, \phi) \right] = \frac{1}{(4\pi)^{k_1 + k_2}} P(k_1, k_1) P(-k_2 - 1, k_2)$$

The geometry coefficient for the spin-flip geometry becomes

$$B_k(b, b', L, L') = \sum \frac{c_1 c_2^*}{4 \epsilon^2} (-1)^{b' - L'} W(b b' k_1 k_2 L L') \frac{c c_1 c_2}{b b' b' b''} W(c c_1 c_2 k_1 k_2 L L')$$

(A51)

and the simplified formula which holds for spin $\frac{1}{2}$ particles is:

$$\mathcal{A}(b b' l'' l''', \epsilon) = \frac{c_1 c_2^*}{4 \epsilon^2} \sum_{b, b' L, L'} c(-1)^{b' - L'} b b' L L' W(b b' k_1 k_2 L L')$$

W(c c_1 c_2 k_1 k_2 L L') \frac{c c_1 c_2}{b b' b' b''} \frac{c c_1 c_2}{b b' b' b''} W(c c_1 c_2 k_1 k_2 L L')$$

(A52)

where $c_1 c_2^* = 0.675 \times 10^6 \text{mb} / (\text{sr})^2$, $k_1$ takes even values only.
$k_1$, $k_2$ and $\kappa$ are either all even if the parities of the states are the same or all odd if the parities of opposite.
IV. TARGET PREPARATION

1) \(^{29}\text{Si} \text{ Target on Carbon Backing}

The material used for evaporation was SiO\(_2\) and was placed in the form of small pieces inside a carbon crucible. The electron gun evaporation technique was used and the voltage between filament and crucible was about 1.5 keV. The filament current was 0.4 Amp and the current between filament and crucible ran from 100 to 200 mA.

The SiO\(_2\) vapors were collected on 3 x 1 inches glass slides covered with carbon backing\(^{22}\) of thickness 10 to 20 \(\mu\text{g/cm}^2\). The slides were at a distance of six inches from the carbon crucible. The time for evaporation was about 30 minutes.

2) \(^{28}\text{Si} \text{ Self-Supported Target}

To deposit a release agent, the glass slide was immersed in a water solution of "Tide" and then let dry.

For the deposition of the \(^{28}\text{Si} \text{ material, the glass slide was positioned at a distance of 25 to 30 cm from the carbon crucible which was full of }^{28}\text{SiO}_2\). The evaporation of the \(^{28}\text{SiO}_2\) lasted for about one hour. The voltage between crucible and filament was approximately 1700 V and the current 200 \(\mu\text{A}\). If during the release of the \(^{28}\text{Si} \text{ target from the glass slide the film does not float on the water, the glass slide is immersed partially in the water until the water slowly gets between the film and the glass. At this point a small push with}
a sharp object helps to separate the film from the glass surface.

3) $^{24}$Mg Self-Supported Target

For alignment of the electronics and especially for the determination of the coincidence delay curves a $^{24}$Mg target was used. The glass slides were immersed in "Tepol 610" with some water in it, so that a uniform layer was formed on the slides.

A clean $^{24}$Mg wire was cut into small pieces and then placed into a tantalum boat, above which and at a distance of 5", the glass slides were placed. The current through the tantalum boat is increased slowly up to 17-18 Amps. A sudden increase of current causes the tantalum pieces to pop out of the boat.

To float the film, acetone is used.

4) $^{176}$Au Self-Supported Target

The glass slides are immersed into a 15% p.v. "Tepol 610" water solution. About 0.02 gr of gold were placed inside a tantalum-molybdenum boat and the glass slides at a distance of 25-30 cm apart.

The current through the boat is about 35 A. The above quantity of gold makes a gold foil with a thickness less than 200 $\mu g/cm^2$. 
V. ALIGNMENT OF THE COLLIMATOR HOLDERS

A telescope was placed at a distance of 4 m from the center of the scattering chamber. The Faraday cup of the chamber was removed so that the telescope can see inside the chamber. Next the axis of the telescope was aligned with the axis of the beam which was defined by the beam collimators. (See Fig. 4)

By rotating the base of the scattering chamber each cylindrical collimator holder was brought to coincide with the axis of the telescope as much as possible. Then by properly shimming the collimator holders a precise alignment was done. In other words, the center of the beam collimators and the center of both the scattering and antiscattering collimators were on the same axis, the axis of the beam.
VI. SOLID ANGLE MEASUREMENT

a) The geometrical measurement was done by using the formula which defines the solid angle

\[ d\Omega = \frac{\Delta s}{R^2} \]

where

\( \Delta s = \) Area of the hole of the scattering collimator.

\( R = \) Distance between the center of the scattering chamber and the center of the scattering collimator

b) The Rutherford scattering of 2 MeV protons from a self-supported Au target (see target preparation in Appendix) was performed and the number of protons scattered into every scattering collimator was measured.

The analysis went as follows. The Rutherford formula is given by:

\[
\frac{d\sigma}{d\Omega} = \frac{4\pi q_0^2 (Z_1 Z_2 e^2)}{E} \left[ \frac{1}{\sin \frac{\theta}{2}} \right]^2
\]

(A53)

where

\( Z_1 = \) Atomic number of incident particles

\( Z_2 = \) Atomic number of target nucleus

\( E = \) Energy of the incident beam in MeV

\( \theta = \) The scattering angle

The experimental definition of the scattering cross section is given by:

\[
\frac{d\sigma}{d\Omega} = 2.65 \times 10^7 \frac{N_Y}{Y \Omega}
\]

(A54)
where:

- $Y = \text{Number of scattered particles}$
- $t = \text{target thickness in } \mu\text{g/cm}^2$
- $d\Omega = \text{solid angle in msr}$
- $I = \text{collected charge in } \mu\text{C}$.

Combining (A53) and (A54)

$$t = \text{const} \frac{Y}{d\Omega \left[ \frac{1}{d\Omega \frac{d\Omega}{\Delta}} \right]^A}.$$

By substituting the value of the scattered protons $Y$, the angle $\theta$, and the corresponding value $d\Omega$ as measured by the geometrical method, the target thickness $t$ was found to be the same within 0.5% error for all solid angles.
VII. ELECTRONICS ARRANGEMENT

a) Angular Distribution

In Fig. A4 the electronics set up for one of the six solid state detectors is shown. The electronic modules, on the left of the Research amplifier module, are the fast electronics and discriminate between piled up and unpiled up signals. The piled up signals are rejected so that only unpiled up signals appear at the output of the Linear Gate (on the right of the Research Amplifier, Fig. A4). The single channel analyzer allows signals with certain energies to enter the stretcher of the PACE system. Thus the total spectrum or any part of the spectrum can be chosen to be transmitted into the stretcher.

The signals in the stretchers are digitized from the ADC of the PACE system and are sent into the computer by means of the PACE system interface. The spectrum of each detector was stored in 512 channels.

b) Angular Correlation in Spin-Flip Geometry

In Fig. A5 the schematic of the electronics is shown. Since there are six solid state detectors for the detection of the protons, there are also six identical arrangements of electronic modules associated with each solid state detector like the one shown in Fig. A5. The input of the OR GATE module shown in this figure accepts signals which are outputs of the S.C.A. (Fig. A5). For every such signal the OR GATE gives an output.
Fig. A4 Arrangement of electronics for the angular distribution of protons.
Fig. A5  Arrangement of electronics for the \((p'-\gamma)\) angular correlation in the spin flip geometry.
The fast electronics reject the piled up signals, and for every unpiled up proton or γ-ray a fast negative signal is generated and goes as input into the Logic Box. The Logic Box gives output according to the logic shown in Table 2. In this table, the left column has the possible proton-γ-rays or proton-accidental-γ-rays coincidences. The columns 1, 2, 4, 8, OR are the outputs. The x marks means output signal from the output shown at the top of the column.

Output signal from the OR OUTPUT of the Logic Box means p-γ or p-Aγ coincidence. Which particular proton came in coincidence with the γ-ray Aγ is indicated by the combination of output signals as shown in Table 2. The Gate and Delay Generators (G.D.G.) shown in Fig. A5, change the output fast signals from the Logic Box to slow signals.

If the p-γ coincidence occurs with a proton having certain energy as specified by the single channel analyzer, then the inputs in the universal coincidence module are in time coincidence and an output (event) occurs. Thus an event signal means coincidence of a proton having certain energy with a γ-ray. The event signal opens the gate for the routing signals to be accepted so that identification of the particular proton which came in coincidence with γ is possible.

The Event signal also opens the Gate of the PACE system so that the stretcher of the PACE system accepts the γ-ray which came in coincidence with the proton. The Analog to Digital Converter (ADC) of the PACE system digitizes the γ-ray and stores it into one of the 6 x 1024 channels.

As we can see in Table 2, there are six groups of 1024 channels for the storage of the p-γ and p-Aγ coincidences as shown in Table 2. In the first 512 channels of each group the p-γ coincidences are stored and in the second 512 channels the p-Aγ coincidences.
Table 2 Outputs of the Logic Box for certain input signals. The input signals are shown in column 1. (The x mark means that a signal appears on the output indicated at the top of the column.)
<table>
<thead>
<tr>
<th>INPUT SIGNALS</th>
<th>OUTPUTS</th>
<th>CHANNELS FOR STORAGE</th>
<th>GROUP OF 512 CHANNELS</th>
<th>GROUP OF 1024 CHANNELS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 2 4 8 OR</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma + p_1$</td>
<td>x</td>
<td>512</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$\alpha \gamma + p_1$</td>
<td>x</td>
<td>x</td>
<td>512 2</td>
<td></td>
</tr>
<tr>
<td>$\gamma + p_2$</td>
<td>x</td>
<td>x</td>
<td>512 3</td>
<td></td>
</tr>
<tr>
<td>$\alpha \gamma + p_2$</td>
<td>x</td>
<td>x</td>
<td>512 4</td>
<td></td>
</tr>
<tr>
<td>$\gamma + p_3$</td>
<td>x</td>
<td>x</td>
<td>512 5</td>
<td></td>
</tr>
<tr>
<td>$\alpha \gamma + p_3$</td>
<td>x</td>
<td>x</td>
<td>512 6</td>
<td></td>
</tr>
<tr>
<td>$\gamma + p_4$</td>
<td>x</td>
<td>x</td>
<td>512 7</td>
<td></td>
</tr>
<tr>
<td>$\alpha \gamma + p_4$</td>
<td>x</td>
<td>x</td>
<td>512 8</td>
<td></td>
</tr>
<tr>
<td>$\gamma + p_5$</td>
<td>x</td>
<td>x</td>
<td>512 9</td>
<td></td>
</tr>
<tr>
<td>$\alpha \gamma + p_5$</td>
<td>x</td>
<td>x</td>
<td>512 10</td>
<td></td>
</tr>
<tr>
<td>$\gamma + p_6$</td>
<td>x</td>
<td>x</td>
<td>512 11</td>
<td></td>
</tr>
<tr>
<td>$\alpha \gamma + p_6$</td>
<td>x</td>
<td>x</td>
<td>512 12</td>
<td></td>
</tr>
</tbody>
</table>
VIII. CONVERSION OF EXPERIMENTAL DATA INTO ABSOLUTE CROSS SECTION

The differential cross section for a particular reaction to occur during the bombardment of a nucleus by protons is given by:

\[
\frac{d\sigma}{d\Omega_p} = \frac{\# \text{ of scattered particles in an angle } \theta \text{ w.r.t. the beam}}{\# \text{ of incident protons} \times \# \text{ of target nuclei}} d\Omega_p
\]

But

\[
(\# \text{ of incident protons}) = (\# \text{ of } \mu \text{Cb}) \times (\# \text{ of protons/} \mu \text{Cb}) = q \times 0.624 \times 10^{13}
\]

\[
\# \text{ of target nuclei/cm}^2 = \frac{N_{AV}}{TM} (\rho dx)
\]

\[
= \frac{N_{AV}}{TM} \times 10^{-6} (\rho dx) = 6.023 \times 10^{17} (\rho dx) = 6.023 \times 10^{17} t
\]

where \(t\) is \(\mu g/cm^2\).

Thus:

\[
\frac{d\sigma}{d\Omega_p} = \frac{\# \text{ of scattered particles in an angle } \theta \text{ w.r.t. the beam}}{0.624 \times 10^{13} q \times 6.023 \times 10^{17} \times 10^{-3} d\Omega_p}
\]

The units of the above expression are \(cm^2/sr\). To convert the units in \(mb/sr\) we multiply by \(10^{-24} \times 10^{3}\).
Thus

\[ \frac{d\sigma(\theta_1)}{d\Omega_p} = \frac{0.266(\pi s/cm^2 \times \text{atangle}(\theta_1)) \times TM}{q \cdot t \cdot d\Omega_p} \]

where \( q \) in \( \mu \text{cm}^2 \), \( t \) in \( \mu \text{g/cm}^2 \), \( d\Omega_p \) in \( \text{msr} \) and \( TM \) is the mass number of the nucleus.

In the case of a reaction in which a particle and a \( \gamma \)-ray appears, after the bombardment of the target nucleus by a proton, the differential cross section to detect the emitted \( \gamma \)-ray and proton at two particular directions is given by

\[ \frac{d^2\sigma}{d\Omega_p d\Omega_\gamma} = 0.266 \times 10^3 \frac{(\text{# of } X-P \text{ events})}{q \cdot t \cdot d\Omega_p e_\gamma d\Omega_\gamma} \]

where \( q \) is \( \mu \text{cm}^2 \), \( t \) is in \( \mu \text{g/cm}^2 \), \( d\Omega_p, d\Omega_\gamma \) are in \( \text{msc} \), \( e_\gamma \) is the efficiency of the \( \gamma \)-ray detector. The derivation of the above formula is similar to the derivation of the differential cross section for an angular distribution.
IX. CORRECTIONS FOR THE FINITE SIZE OF THE $\gamma$-RAY DETECTORS

If the $\gamma$-ray detector was a point detector the any angular distribution of correlation of radiations detected by this detector could have the form

$$W(t) = \sum \alpha_\ell P_\ell (\cos \theta)$$

Because of the finite size of the detector the correlation would look like

$$W'(t) = \sum \alpha'_\ell P_\ell (\cos \theta)$$

R. G. Arns et al.\textsuperscript{24)} have calculated theoretical correction factors $b_\ell$ to account for the finite size of the detectors, namely: $\alpha'_\ell = \alpha_\ell b_\ell$. At our laboratory numerical values of the correction factor $b_\ell$, for various $\gamma$-ray energies and various distances from the source have been measured\textsuperscript{25}). Now suppose that the correction is expanded in the form

$$W' = \sum B'_\ell \cos^2 (\theta)$$

(A60)

The experimental coefficients $B'_\ell$ can be corrected for the finite size of the $\gamma$-size of the $\gamma$-ray detectors, using the correction factors $b_\ell$.

The procedure is the following:

$$W(t) = \sum \alpha_\ell P_\ell (\cos \theta) = \alpha'_c + \frac{1}{4} \alpha'_2 + \frac{9}{64} \alpha'_4 + \frac{25}{126} \cos^2 \theta$$

$$= (\alpha'_c + \frac{1}{4} \alpha'_2 + \frac{9}{64} \alpha'_4) \cos^2 \theta + \frac{25}{126} \left( \frac{35}{64} \alpha'_4 \cos^2 \theta \right)$$
In the above formula the Legendre Polynomials $P_L$ were expressed in terms of $\cos k\phi$ functions. Comparing the above expression with (A60) we get

$$B_0' = A_0' + \frac{1}{4} A_2' + \frac{9}{64} A_4'$$

$$B_2' = \frac{3}{4} A_2' + \frac{5}{16} A_4'$$

$$B_4' = \frac{35}{64} A_4'$$

Solving for $A''s$ in terms of $B''s$ we get

$$A_0'' = B_0' - \frac{1}{3} B_2' - \frac{1}{15} B_4'$$

$$A_2'' = \frac{4}{3} B_2' - \frac{16}{21} B_4'$$

$$A_4'' = \frac{64}{35} B_4'$$

The expressions (A62) give the $A''s$ in terms of the known quantities $B''s$. Then we correct the quantities $A''s$ with the known correction factors, and we get the corrected values of $A's$. The corrected coefficients $B's$ are given in terms of $A's$ by the relations similar to (A61) but the primes are dropped. (The unprimed quantities represent the corrected coefficients.)
X. PROGRAMS USED FOR THE ANALYSIS OF

THE EXPERIMENTAL DATA

The following programs were used for the analysis of the experimental results.

A. SOUTH

With this program\textsuperscript{34} the number of the inelastically scattered protons was determined (area of the peak of interest). The program was modified to punch on a card the calculated area, the statistical error, the scattering angle, and corrections for the piled-up pulses and dead time of the PACE system. Further details are described in Ref. 34.

B. DURAT

With this program\textsuperscript{34} the number of coincidences was calculated (area of the photopeak). The program was modified to punch on a card the calculated area of the photopeak, the statistical error extracted for the photopeak, the proton scattering angle, and the pile-up corrections. More details are given in Ref. 34.

C. ADANA

The program name ADANA is an acronym for "Angular Distributions Analysis"; the purpose of this program is to expand the angular
distribution in terms of the Legendre polynomial (the first six Legendre polynomials were chosen) and to calculate the expansion coefficients (the least squares fit method was used). A detailed description of the program follows.

1. **ADANA**
   This program reads two cards. The first determines where the angular distribution must be plotted (plotter, screen) and the second determines the minimum percentage error of the number of counts. Link (DOKC1).

2. **DOKC1**
   Reads the data cards of the angular distribution (angle, number of counts, error pile-up corrections) Then the program calls the subroutine LABCM which divides the number of counts by the solid angle of the detectors and makes center-of-mass corrections for the energy and scattering angle. This subroutine can be skipped by using a toggle switch.
   
   a. **CALLCORCT.** This subroutine assigns a minimum percentage error to the number of counts only if the initial error is smaller than a chosen minimum error.
   
   b. **CALL NLLS.** This subroutine calculates the expansion coefficient by the least squares fit program. The subroutine SUBRT which is called by the NLLS determines the functions in terms of which the expansion is made. Different sets of toggle switches link to the following programs.
3. **WROUT**

This program stores on disk the energy at which the angular distribution was taken and the calculated coefficients. It is optional for the coefficient to be printed out. There is the option to link to DOKC1 for analysis of another angular distribution or to link to DRAW1 or DRAW2.

4. **DRAW1**

(This program can also be linked directly from DOKC1 skipping WROUT.) It draws the experimental points of angular distribution and then plots the least squares fit curve. DRAW1 links to DOKC1 for another analysis of the angular distribution.

5. **DRAW2**

(This program can also be linked directly from DOKC1 by skipping WROUT.) It is the same as DRAW1 but it draws, on the same set of axis, as many angular distributions as are designed. It is particularly useful for the plotter. DRAW2 links back to DOKC1 for analysis of another distribution.

After the analyses of all the angular distributions are finished and the coefficients are stored on disk the program DOKC1 links to PCAR which can also be in queue.

6. **PCAR**

This program reads the energy and the coefficients from disk, converts the coefficients to absolute cross sections and then punches the energy and coefficients on cards.

7. **PLOCO + Link (PLOTO)**

This program reads the coefficients from disk and plots them on the plotter or screen. It may also be
D. SFANA

The name of the program is an acronym for "Spin-Flip Analysis". This program is identical to the program ADANA discussed earlier. The major difference is that the subroutine SUBRI contains $\cos (\kappa \phi)$ functions, as a result the expansion of the correlation is done in terms of these functions. Other small differences of the program SFANA with the program ADANA is the FORMAT of the input data.

E. Program for Resonance Analysis

This program was explained earlier. Since this program is written in extended precision none of its constituent programs can be queued and the process interrupt cannot be used. To call any of the programs we execute the program MASTER (which is already stored on the disk) and by setting the proper toggle switches we press ENTER to bring the desired program. A detailed discussion will be given here for each of its constituent programs.

1. RDATA  This program read the experimental data and theoretical data. The experimental data are the coefficients and the errors (in units of absolute cross section). The computer reads the data and stores them on disk. The Theoretical data are the expressions A26 for angular distribution, A49 for the Goldfarb-Seyler geometry, and A51 for the spin-flip geometry. (These coefficients
were calculated by means of different programs and then punched on cards. The subroutines for the 9-j symbols, W-coefficients, and C. G. coefficients, exist in our Van de Graaff laboratory. Since the calculation of the theoretical data is trivial no discussion about these programs will be given.) The theoretical data are read from the computer and then stored on disk.

2. PRARM  This program reads the resonance parameters (partial decay widths, phases, energy spin, and parity) and stores them on disk.

3. RANGE  With this program we select the energy region, of the experimental coefficients, in which we wish to make the theoretical fits.

4. PDATA + Link (PDATC)  These programs display on the screen the coefficients (we can select the coefficients to be plotted) versus energy, over the energy region chosen by the program RANGE. This program links to program CALC or CALC1.

5. CALC  The program CALC calculates the theoretical value of the coefficients versus energy using a Breit-Wigner formalism and includes interference between the resonances. The calculated values are stored on disk. The program links to PCALC.

6. CALC1  The program CALC1 is the same program as CALC but no
interference between the resonances is included. Links to PCALC.

7. PCALC  
The program PCALC plots the theoretical values of the coefficients (as calculated from CALC or CALC1) versus energy on top of the already plotted experimental coefficients.

8. CPARM  
With this program we can change any of the resonance parameters.

9. WPARM  
This program writes the resonance parameter on the typewriter.

10. PPARM  
This program punches the resonance parameter on cards.
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