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AN EXPERIMENTAL STUDY OF THE TRANSITION FROM
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TRANSLATING SOLID BODIES UNDER
OSCILLATORY MOTION CONDITIONS

DISSERTATION
Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By
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1975

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CHAPTER I

INTRODUCTION

1.1 Friction

Friction is a phenomenon which affects us every day. It can be both beneficial and harmful. For example, it is the friction between a person's shoes and the floor which enables him to walk. The friction between his hand and a pencil enables him to pick the pencil up from a desk and hold it to write.

On the debit side, friction must be overcome in any machine where there is motion between two surfaces. Overcoming this friction can be the chief source of energy usage in a machine, which makes the reduction of friction important in making more efficient use of energy. Unwanted friction can also hinder the smooth and quiet operation of a machine.

1.2 Early History

The history of the study of friction is really fairly recent. Leonardo da Vinci (1452-1510) knew of the first two classical laws of dry friction and wrote about them
in his notebooks [19].* Since da Vinci's work was not widely distributed, his knowledge did not become known until years later. Nothing was reported for many years until G. Amontons (1663-1705) conducted his now famous experiments on friction [1]. Among other things he found that the friction force was directly proportional to the normal force. He also believed that the friction force was dependent upon a complex relationship between normal force, time, and velocity of sliding.

Charles Coulomb (1736-1806) was perhaps the greatest of the early investigators. He did extensive research on the topic [17] and has since had his name become synonymous with dry friction. Coulomb proposed a two-term friction relationship which has only recently been rediscovered [9].

A complete early history of the study of friction may be found in the literature [10], [18]. The history of friction study from the Russian viewpoint may be found in a book by Kragelskii [34].

1.3 Classical Laws

There are three classical laws of dry friction which have been handed down to us from the early researchers

*Numbers in [ ] refer to references listed on page 289.
and which are taught in elementary physics and mechanics courses [5]. The first law states that the friction force is directly proportional to the normal load on the surface. That is,

\[ F = \mu N \]  \hspace{1cm} (1.1)

where

- \( F \) is the friction force
- \( N \) is the normal force
- \( \mu \) is the proportionality constant (the coefficient of friction)

The second law states that the friction force is independent of the area of contact. For example, if the area of contact is doubled, the friction force remains the same as long as the normal force does not change. The third classical law states that the friction force is independent of the velocity of sliding. That is, the friction force will be the same whether the relative velocity between the two surfaces is 0.001 m/sec or 1000 m/sec.

These three laws have been used by engineers to solve practical problems for many years with considerable success. They allow the simple formulation and solution of many problems where friction is involved. However, even with the many successful applications of the classical model, it is known that there are situations where each of these laws is broken to a greater or lesser extent.
It has been shown by Whitehead [53] that the first law is broken by copper sliding on copper at very light loads. His results, given in Figure 1-1, show the coefficient of friction obviously dependent upon the load. Whitehead attributes this to the oxide film which is formed on the surface of the copper when it is exposed to the air. Plastics are known to follow a law of the form [42]:

\[ F = \mu N^a \]  

(1.2)

where

- \( F \) the friction force
- \( N \) the normal force
- \( \mu \) the proportionality constant
- \( a \) a constant exponent not usually equal to 1

If a hard material is coated with a softer material, such as a babbit coated steel, it has been shown by Bowden and Tabor [9] that as long as the softer material remains unbroken, the friction force remains fairly constant even though the normal force is increased. This is attributed to the smaller shear forces needed to move the projections through the softer material.

Exceptions to the second law of friction exist also. Bailey and Courtney-Pratt [2] have shown that for mica
Figure 1-1. Variation of Coefficient of Friction with Load for Copper on Copper.
the friction force is proportional to the area of contact, not independent of it. This is explained by the fact that since mica has molecularly smooth cleavage planes, the real area of contact equals the apparent area of contact. Exceptions to the second law usually occur with very smooth, clean surfaces.

Certain exceptions to the first and second laws have been pointed out, and there are others. Most materials, especially metals, obey the first and second laws fairly closely, however, over the region of usual engineering interest. The third law is a different story. The coefficient of friction is, under many conditions, a function of the velocity of sliding. This leads to the subject of this study and will be discussed more fully in Chapter II.

1.4 More Recent Research

In the last 50 years there has been renewed interest in the study of friction along with lubrication and wear. These three subjects are quite interrelated and because of this a new word has been coined to describe their study--tribology.

Most of this recent work has been pointed in two directions. First, there has been a group whose prime interest has been the study of the actual surface
phenomena which cause friction and wear. This group has been led especially by the contributions of F. P. Bowden and D. Tabor [9], [10] at the Research Laboratory on the Physics and Chemistry of Surfaces at the University of Cambridge. Through the use of such tools as the optical microscope, electron microscope, x-ray, and electron diffraction techniques, and other methods, much information has been gathered on the surface interactions which give rise to friction.

The other main thrust of activity has been in the area of lubrication. Much of this work has been brought about by greater technological need. The extreme conditions of outer space, for example, have required new approaches to lubrication. The high pressures and temperatures of modern machinery, the jet engine for example, have raised interest in finding new and better lubricants.

Several excellent bibliographies dealing with this more recent work are available [8], [11].

This present work does not deal directly with either of these two areas of research, but it is instead an investigation of the frictional behavior of materials under reciprocating sliding conditions with the emphasis on development of useful friction models for use in mechanical system analysis.
2.1 Static vs. Kinetic Friction

The third law of friction states that the coefficient of friction is independent of the velocity of sliding. This law is a less accurate model of real behavior than the first two laws. Elementary physics and mechanics texts [5] distinguish between two coefficients of friction, the so-called static coefficient of friction and the kinetic coefficient of friction. This is the first hint that the coefficient of friction is not a constant with respect to velocity. One value is given for zero velocity and one value is given for any other velocity. This concept of two coefficients of friction is extensively used and values are tabulated in standard handbooks [4].

The ancients knew the concept of static and kinetic friction, for Themistius (390-320 BC) wrote, "Generally it is easier to further the motion of a moving body than to move a body at rest" [48]. Today this concept is still useful to engineers. However, as problems become more complex and the demand for more exact solutions more pressing,
2.2 More Recent Work: Friction vs. Velocity

Beginning with Amontons' observations on friction, researchers have been interested in any possible variation of friction with velocity. As pointed out earlier, Amontons noted a dependence of friction on velocity. Coulomb, who did extensive work on friction, however, came to the conclusion that friction force was independent of velocity. Since that time various researchers have come to differing conclusions. Some have agreed with Coulomb's initial work that friction is independent of velocity. Some have concluded that the coefficient of friction increases with velocity. Others have found a decreasing coefficient of friction with increasing velocity.

Table 2.1 summarizes the results of some of the investigations into the influence of velocity on friction starting with Coulomb in 1785 and ending with Dobrovolsky in 1934. Listed next to the researcher's name is the date of publication, some of the materials investigated, the maximum velocity obtained during the study, and the slope of the friction vs. velocity curve. The divergence of opinion on the slope of the friction-velocity curve is apparent. Two researchers, Conti and Kimball, even found the friction-
Table 2.1

Results of Some Early Investigators

| RESEARCHER       | DATE | MATERIALS                          | MAXIMUM SPEED | SLOPE OF
|                 |      |                                   |               | FRICTION-
|                 |      |                                   |               | VELOCITY CURVE |
| Coulomb [48]    | 1785 | oak, iron, various others         | 2.5 m/sec     | zero         |
|                 |      |                                   | (8.2 ft/sec)  |              |
| Vince [51]      | 1785 | textiles                          | 0.33 m/sec    | positive     |
|                 |      |                                   | (1.1 ft/sec)  |              |
| Rennie [43]     | 1829 | metals and various others         | 2.56 m/sec    | zero         |
|                 |      |                                   | (8.4 ft/sec)  |              |
| Morin [38]      | 1835 | unknown                           | 4 m/sec       | zero         |
|                 |      |                                   | (13 ft/sec)   |              |
| Poiree [41]     | 1852 | rails and wheels                  | 22 m/sec      | negative     |
|                 |      |                                   | (72 ft/sec)   |              |
| Bochet [7]      | 1858 | unknown                           | unknown       | negative     |
| Jenkins & Ewing [30] | 1867 | journals                          | 0.003 m/sec   | zero         |
|                 |      |                                   | (0.01 ft/sec) |              |
| Conti [16]      | 1875 | various                           | unknown       | maximum      |
| Kimball [31]    | 1877 | wood, leather, cast iron, others  | 1.27 m/sec    | maximum      |
|                 |      |                                   | (4.17 ft/sec) |              |
| Galton [26]     | 1878 | railway or brakes                 | 42 m/sec      | negative     |
|                 |      |                                   | (136 ft/sec)  |              |
| Hirn [29]       | 1884 | unknown                           | unknown       | positive     |
| Krumme [35]     | 1929 | cotton on steel                   | unknown       | positive     |
| Dobrovolsky [21] | 1934 | belt drives                       | unknown       | positive     |
velocity curve to increase with increasing velocity, reach a maximum, and then fall.

More recent research has helped to clear up much of the confusion and ambiguity apparent in early studies of the velocity dependence of friction. There appear to be two main types of friction-velocity behavior at low velocities [15]. These are shown in Figure 2-1. The decreasing friction characteristic, curve (a), is most commonly encountered since it is found with plain mineral oil and ordinary bearing surfaces. The increasing friction characteristic, curve (b), is encountered when better boundary lubricants are used such as fatty oils or fatty acids. Un lubricated metals appear to follow the characteristic of curve (a) while unlubricated non-metals appear to follow the characteristic of curve (b).

Forrester [24] studied the effect of velocity on friction over the range 0.01 to 2.26 cm/sec (0.004 to 0.89 in/sec) using several combinations of bearing materials and lubricants. The material pairs studied were babbit on steel, hard steel on bronze, mild steel on mild steel, and cadmium nickel on steel. He found that with both paraffin and engine oil the friction decreased with increasing velocity. He found that with oleic acid (a fatty acid) as the lubricant, the friction/velocity curve had a positive slope. His experiments with non-lubricated metal surfaces also showed a decreasing friction/velocity curve.
Figure 2.1. Types of friction variation with velocity.
Forrester's experiments were of two types: 1) with excess lubricant, and 2) with thin films. He found that thin films of only a few molecules thickness could provide significant lubrication. Thicker films did give better lubricating efficiency, however. Other factors studied were surface finish and run-in.

Bristow [13] studied friction from less than 0.01 to 0.5 cm/sec (0.004 to 0.197 in/sec) but not down to zero velocity (static friction). Materials used in Bristow's studies were hard steel on hard steel, brass on hard steel, bronze on hard steel, and tin on hard steel. Lubricants used were a series of fatty esters and various percentages of oleic acid in mineral oil.

The experiments with the fatty esters consisted of a series of nine esters of various molecular weights used with both steel on steel and bronze on steel. Bristow found that, in general, friction decreased with increasing molecular weight of the ester. The friction-velocity curves had a decreasing friction characteristic.

Mineral oil was compounded with various percentages of oleic acid and run with brass sliding on steel and steel sliding on steel. For low concentrations of oleic acid the friction-velocity characteristic was found to be decreasing. At higher concentrations of oleic acid the friction-velocity characteristic was slightly increasing.
The overall friction fell as the percentage of oleic acid was increased.

Bristow also reported on the effect of temperature on the friction-velocity behavior. He found that the effect of temperature was somewhat complicated. In fact, the two examples given show different results. For tin sliding on hard steel lubricated with ethyl palmitate the coefficient of friction decreased with increasing temperature over almost the entire range of velocities. For steel sliding on hard steel lubricated with a mineral oil the coefficient of friction increased with increasing temperature over almost the entire range of velocities.

Barwell and Milne [3] have given results for friction versus velocity over the range 0.0015 to 0.6 cm/sec (0.0006 to 0.236 in/sec). The purpose of their paper was to discuss some of the problems encountered in trying to define "oiliness" of a lubricant. Temperature, roughness, and hardness were also studied for their effect on the friction-velocity curve. Various combinations of soft steel and hard steel were studied using a number of lubricants—a silicon fluid, a mineral base oil, straight mineral oil, oleic acid, and methyl stearate. These researchers found a falling friction curve with increasing velocity in almost all cases. Friction data with other material combinations is also given but not plotted against velocity.
2.3 Unidirectional Steady State vs. Reciprocating Sliding

Almost all the studies on friction have been done with unidirectional, steady state sliding. That is, the sliding was done at constant speed in one direction only. For example, the three studies of friction-velocity characteristic cited earlier [3], [13], [24] were all done with unidirectional, steady state sliding. In practice, however, not all sliding is unidirectional; much is of a reciprocating nature.

There are at least four differences between reciprocating sliding and unidirectional, steady state sliding. These differences change the conditions of operation enough to warrant separate study--thus lending impetus to the present work. First, repeated traversal takes place. By its very definition reciprocating sliding is a motion back and forth over the same wear track. This differs from the usual unidirectional sliding tests where pains are often taken to ensure that a fresh surface is always in contact. Repeated traversals can cause surface changes such as work hardening, changes in crystallite size; and formation of oxides and carbides [27] that can significantly affect the friction results. When lubricants are used, repeated traversal can cause the lubricant film to be altered or destroyed.
Second, directional effects may be encountered. For example, motion to the left may cause the surface asperities to be deformed in that direction. Then on the return stroke to the right, this orientation of the asperities can result in changes in the friction force. With lubricant films, reorientation may either enhance or diminish the effectiveness of the lubrication. This effect takes place, for example, with molybdenum disulfide films [14].

Third, conditions are dynamically changing. This differs from the method usually used to study friction as a function of velocity. The usual method is to run tests at different constant velocities. A test may involve running different specimens at, say, eight different velocities over a wide range, plotting the friction against these velocities, and fitting a smooth curve to the data. This was done by Barwell and Milne [3] over the range 0.0015 to 0.6 cm/sec (0.0006 to 0.236 in/sec). Reciprocating test conditions with a sinusoidal motion give constantly changing velocities.

Fourth, the velocities pass through zero. Most studies of friction/velocity effects extend down to fairly low velocities but not to zero. For example, Barwell and Milné [3] go down to 0.0015 cm/sec (0.0006 in/sec). Thus static friction is not measured. With reciprocating sliding the velocity goes through zero twice each cycle. This allows the investigator to study static friction as a function of frequency of oscillation.
Very little work has been reported on friction during reciprocating sliding other than in the very low amplitude region of fretting. Halliday and Hirst [28], for example, have reported the lowering of friction in the fretting region.

Wear under conditions of reciprocating sliding has received some attention. One paper of interest is by Ward [52]. In this study he compared wear for reciprocating sliding against wear for unidirectional sliding. His results show a greater wear rate for reciprocating sliding than for the unidirectional case which he attributes to the build-up of wear debris. Friction was not measured, but it seems logical to suspect that if wear rate is affected by the sliding mode then friction force may possibly be affected in some way also.

Friction of a series of eight bronzes sliding against hardened steel under conditions of lubricated reciprocating sliding has been reported by Montgomery [37]. The friction force was not continuously recorded but was taken to be the "maximum instantaneous friction force during a reversal measured with a load cell." Addition of small amounts of fatty acids to the lubricant was found to decrease both friction and wear.

Tamai [47] has reported on reciprocating sliding of a large number of metals. All his tests were run without lubrication. The method used was a so-called friction
pendulum. This consisted of a pendulum, the bearings of which were made up of the pair of friction materials under test. These bearings were in the form of crossed cylinders giving theoretical point contact. The pendulum was set in motion and the decay recorded. The average friction coefficient was then calculated from the decay records. An average coefficient was therefore obtained over a range of amplitude and velocity and instantaneous friction could not be measured. Tamai concluded that the coefficient of friction is lower in reciprocating sliding than in single-traverse for soft metals and about the same for hard metals.

In a paper concerned primarily with the effect of reversal frequency on wear, Evdokimov and Movsesov [23] give some results of interest in the present discussion. They found that during reciprocating sliding the mean frictional force is greater than during unidirectional sliding. Under dry sliding conditions this increase can be from 50 to 80%. For lubricated conditions this increase is 30 to 40% and for lubricants with oleic acid there is almost no increase. The materials used in these tests were steel-steel and steel-bronze. They also found the wear to increase significantly with frequency of reversal.

Some work has also been done with seals (rubber on steel) used during reciprocating sliding [33], [39], [45]. This has application to such devices as hydraulic cylinders.
CHAPTER III

FRICITION TESTING MACHINES

3.1 Previous Machines

Many of the early researchers studying friction used methods which would be considered rather crude by today's standards. Among the methods used were the inclined plane, weights, and springs. Using the inclined plane, a weight is placed on the plane which is tilted until the weight begins to move down the plane. The angle of tilt the plane makes with the horizontal is then measured, and it can easily be shown that the coefficient of friction is given by,

$$\mu = \tan \theta$$

where \( \theta \) is the angle of tilt. This method gives the static coefficient.

Weights are used in a force balancing system that essentially weighs the friction force. Weights are added using a system of pulleys until motion takes place. Knowing the total weight added and the weight of the mass moved, the coefficient of friction is easily measured.

When springs are used to measure friction, the spring is slowly extended until motion takes place. The extension
of the spring is measured and from previous calibration
the static coefficient of friction is obtained.

Friction force and coefficient have also been cal-
culated from other observed phenomena. For example,
friction force can be calculated from the deceleration of
a moving body or the decay of oscillation of a vibrating
system. Friction has even been calculated from the heat
generated by two surfaces as they slide.

All these methods are still used to some extent today.
More sophisticated methods of measuring both friction and
wear have come into use, however. A list of over 100 test
machines has been compiled by the American Society of
Lubrication Engineers' Subcommittee on Wear [6]. This
catalog gives information on each machine listed. Some of
the more commonly used devices are listed in a paper by
Kitchen and Azzam [32]. The authors present a table
comparing the relevant features of each. A list of test
machines frequently used in evaluating solid lubricants is
given in a paper by Devine, et al. [20].

Most of the existing machines are not capable of
reciprocating sliding. For example, only two of the ten
machines listed by Kitchen and Azzam are capable of
oscillating motion, and these two are angular motion. Other
machines are for special purposes such as simulating gears
or cams. These are inappropriate for the present study.
Thus, because of these and other factors such as cost, it
was decided to design and build a new device for the present study.

### 3.2 Measuring System Design Specifications

The problem was to design a device which would reliably measure changing friction forces under conditions of reciprocating sliding in order to obtain data useful in modeling conditions of dry friction and boundary lubrication. In order to achieve this goal certain design decisions had to be made. These included general performance specifications, choice of contact geometry, method of application of normal force, and method of measuring friction force. In the area of performance specifications the parameters of major concern were natural frequency, stiffness, sensitivity, and alignment.

Natural frequency is of importance in the force measurement since we require fast response to the rapidly varying friction forces which arise during reciprocating motion. In the system used to apply the normal load, there is some question as to whether its natural frequency should be high or low. A high frequency loading system has vibration problems but responds rapidly to changes in contour. A low frequency system more closely approximates friction of massive machine members. Stiffness should be as high as possible throughout the test machine. This best
simulates conditions in machinery which are usually made as stiff as practical to minimize deflections. Sensitivity should be as high as possible to allow detection of small forces in low-friction materials. Metal-on-metal coefficients of friction are rarely below \( \mu = 0.04 \) even when good boundary lubricants are used. Plastics can reach \( \mu = 0.02 \) (Teflon). If transition or hydrodynamic lubrication develop, \( \mu = 0.001 \) is possible. In general, sensitivity can only be achieved by sacrificing stiffness. The friction test machine should be designed to insure proper alignment between friction surfaces. This is true especially for line and area contact in order to obtain reproducibility of results.

Three choices are available in the area of contact geometry: point, line, and area. These are only theoretical classifications, of course, since area contact must always be present in order to support the load. Point contact is the most used in test machines because of the simple configuration and good alignment it affords. Line contact is used mainly in devices designed to simulate the friction in gears, cams, and rollers. Area contact creates the most difficult alignment problems but best simulates many practical bearing designs.

Four methods of applying the normal load were considered: springs, hydraulic, pneumatic, and dead weight. The use of springs gives high natural frequency but wear may cause
changes in the normal load. Vibrations can also be troublesome. Hydraulic and pneumatic methods are capable of conveniently applying large loads. For small loads they are relatively complex and friction in the cylinder is hard to account for. Dead weights are simple and potentially the most accurate normal load method. They have low natural frequency when the load is applied directly and can have high natural frequency if levers are used.

Most dynamic force measurements are carried out by transducing the force to an elastic deflection of some flexible member and then converting the small motion to an electrical signal. Several motion-to-voltage transducing schemes were considered for use in measuring the friction force. These included:

1. capacitive.
2. differential transformer.
3. electro-optical.
4. strain gage.

While each of these techniques could probably be developed into a workable scheme for the present application, it was decided to try to develop a relatively simple strain gage method using readily available equipment.

The usual method is to bond the gages to an elastic element and measure the strain in that member caused by the deflection. This requires a fairly high state of stress in the structural element. For the range of friction forces
anticipated this would require thin members and/or large displacement and would give rise to a delicate and compliant system.

Another available form of strain gage is the unbonded strain gage in which the gage wire is not attached to a structural element but is used as a structural element itself. This gives the same advantages as bonded gages: low sensitivity to shock and vibration, static through dynamic response, continuous resolution, excellent linearity, and simplicity of design. An additional advantage is that a stiff system can be constructed without the need for delicate members on which to mount the gages. A major disadvantage is the relatively low output levels. This can be overcome to a certain extent through the use of proper amplification. However, amplification has its own limitations mainly due to noise.

3.3 Design of the Present System

The present measurement system can be thought of as consisting of three parts. First is the actuator system which imparts the required reciprocating motion to the friction pair. Second is the measurement system itself, which consists of the friction pairs, the normal loading device, the friction force measuring device, and the velocity sensing device. Third is the signal conditioning and
recording system consisting of the recorders and filters. A schematic of the overall system is shown in Figure 3-1. A photograph of the system as set up in the laboratory is shown in Figure 3-2.

3.3.1 Actuator System

The actuator system is a high-performance hydraulic servomechanism capable of producing translational motion in response to an input voltage command signal. The heart of the system consists of a Vickers hydraulic cylinder Model WAS-3232-SOSORV-23 having a 3.81 cm (1-1/2 in.) bore and a 12.7 cm (5 in.) stroke. The working pressure is 350 N/cm² (500 psi) which gives a maximum available force of 3900 N (900 lbf), well above the maximum friction force. This high actuating force insures smooth operation since there is only very slight loading by the friction force.

The hydraulic cylinder is fed through a Vickers Model SA4-03-30-220-14 servo valve driven by a Vickers Model EAPS-A-11 servo amplifier. The servo amplifier responds to the driving voltage of a function generator which produces the appropriate input function and a feedback voltage from a precision film-type potentiometer which measures slide displacement. A Wavetek Model 114 was used to produce the sinusoidal and triangular motions used in the tests. The gaussian random driving signal was produced by a Hewlett Packard 101-3722A Noise Generator.
Figure 3-1. Schematic of Overall Measurement System.
Figure 3.2. Photograph of Test Set-Up in Laboratory.
Hydraulic oil was provided to the actuator system by a Denison Model PA-072-560-R hydraulic pump driven by a 25-horsepower electric motor. The system was equipped with an accumulator which stored the hydraulic oil at high pressure. The use of the accumulator allowed the system to be run for extended periods of time with the pump off, thereby eliminating any vibrations caused by the pump. Approximately 25 strokes of the cylinder were possible before it was necessary to recharge the accumulator by starting the pump.

3.3.2 Measurement System Design

After considering several designs for the friction test machine, the design of Figure 3-3 was finally adopted for further development. This method employs an unbonded strain gage device (1) in Figure 3-3) sensing the deflection of a stiff beam (2) to which the friction force is applied. One member of the friction pair is attached to the underside of this beam and the other member (3) is attached to a dovetail slide (4) which is reciprocated back and forth by the hydraulic cylinder of the actuating system. The transducing beam is attached to a shaft (5) which floats in angular contact ball bearings housed in the supports (6) located at either end of the shaft. Use of the angular contact ball bearings allows the nuts at the ends of the
Figure 3-5. Sketch of Measuring Device.
shaft to be adjusted to eliminate play from the bearings. Deflection of the transducing beam is measured by use of an auxiliary beam (7) mounted on the shaft next to the transducing beam. Deflection is measured relative to this beam using a commercially available deflection transducer. A photograph of the testing device is shown in Figure 3-4.

The transducer chosen was the Statham Universal Transducing Cell. This is an unbonded strain gage type device which has the following characteristics:

- displacement range ± 0.06 mm
- accuracy better than 0.15% FS
- thermal sensitivity shift less than 0.01% FS/°F
- thermal zero shift less than 0.01% FS/°F
- full scale output ± 8 mV/V nominal
- rated excitation 7.5 V max

The weight of the two beams is supported at one end by the shaft and at the other end by the two friction specimens in contact. Thus, the normal force application is of the dead weight type caused by the intrinsic weight of the beams themselves. A normal force of approximately 18 N (4 lbf) is supplied simply because of the design of the measurement device. For normal forces smaller than this amount a counterweight is available, and for larger
Figure 3.4. Photograph of Measuring Device.
normal forces weights can be added to the end of the auxiliary beam. Calibration of the normal force is also done with dead weights by use of a pulley system. The normal force is not monitored continuously during the testing but set to the proper value and assumed constant throughout the testing.

Beam design consisted of choosing dimensions and material to satisfy the conditions of stiffness, sensitivity, natural frequency, and alignment. Final choices were:

\[ \begin{align*}
\ell &= \text{length of beam} = 18.42 \text{ cm (7.25 in.)} \\
b &= \text{width of beam} = 6.35 \text{ cm (2.5 in.)} \\
h &= \text{depth of beam} = 1.375 \text{ cm (0.550 in.)}
\end{align*} \]

The transducing beam was made from steel. For calculation of stiffness, the transducing beam was considered to be a perfect cantilever beam and the cantilever beam deflection equation was used.

\[ \delta = \frac{p\ell^3}{3EI} \]  \hspace{1cm} (3.2)

where \( \delta \) \# deflection, cm

\( p \) \# load, N
Theoretical stiffness is therefore given as,

\[ K = \frac{P}{\delta} = \frac{3EI}{L^3} = 14,400 \text{ N/cm} \ (8,200 \text{ lbf/in}) \]

Theoretical sensitivity can be obtained directly using the transducing cell sensitivity,

\[ \frac{e_o}{\delta} = \frac{8 \text{ mv}}{v} \times \frac{1}{0.06 \text{ mm}} \times \frac{10 \text{ mm}}{\text{ cm}} = \frac{1370 \text{ mv}}{\text{ cm-v}} = \frac{3480 \text{ mv}}{\text{ in-v}} \]

and the theoretical stiffness. Theoretical sensitivity is,

\[ \frac{e_o}{\delta} = \frac{e_o}{\delta} \times \frac{\delta}{P} = \frac{0.0955 \text{ mv}}{\text{ N-volt excitation}} \ (0.435 \text{ mv} \text{ lbf-volt excitation}) \]
The theoretical natural frequency was calculated from the formula [4]:

\[ f = 0.56 \sqrt{\frac{gEI}{wk^3}} \]  

(3.3)

where the symbols are as defined previously except:

- \( f \) = frequency, Hz
- \( g \) = gravitational constant = 980 cm/sec\(^2\)
- \( w \) = linear weight density, N/cm\(^2\)

Substitution of values results in,

\[ f = 315 \text{ Hz} \]

The major alignment problem would stem from torsion of the transducing beam. Twisting of the transducing beam could cause the upper member of the friction pair to tilt in one direction as the lower member slid past and then tilt in the other direction as the lower member slid past on the return stroke. This rocking action if excessive might cause the upper specimen to wear in a rounded manner rather than flat. The angular displacement of a rectangular beam with a torque applied is given by [46]:

\[ \theta = \frac{T\ell}{\beta Gbh^3} \]  

(3.4)

where \( b, h, \) and \( \ell \) are as defined previously and,
\( \theta \) angular displacement, rad

\( T \) torque, N-cm

\( G \) modulus of elasticity in torsion = \( 8.34 \times 10^6 \) N/cm\(^2\)

\( \beta = 0.286 \) for \( \frac{b}{h} = \frac{2.5}{0.55} = 4.5 \)

Since the friction force is applied 3.8 cm (1.5 in.) from the neutral axis as shown in Figure 3-5,

\[ T = 3.8 \, F \]

Therefore,

\[
\frac{\theta}{\beta} = \frac{3.8 \, b}{0.286 \, Gbh^3} = 1.7 \times 10^{-6} \text{ rad/N} \quad (7.6 \times 10^{-6} \text{ rad/lbf})
\]

If it is assumed that the upper friction pair has worn a flat spot 0.16 cm (1/16 in.) across, and that the friction force is 4.45 N (1 lbf), then one side of the upper friction pair would be \( 1.25 \times 10^{-6} \) cm \((0.5 \times 10^{-6} \text{ in.})\) higher than the opposite side if a completely rigid pair were assumed. But the friction pair is crushed together due to the Hertzian deflections. To investigate the size of these deflections in relation to the tilting of the upper friction pairs, assume that the interface is a sphere of radius 5.08 cm (2 in.) resting on a flat. Then the Hertzian
Figure 3-5. Source of Torque on Measuring Beam.
Deflection is given by [43]:

\[ \delta = 3.12 \frac{3 \sqrt{p}}{E R} \]  \( (3.5) \)

where
- \( \delta \) = overall approach of both bodies, cm
- \( P \) = normal load, N
- \( E \) = Young's modulus, N/cm\(^2\)
- \( R \) = radius of sphere, cm

Substituting in the assumed values for \( P \) and \( R \) and further assuming that the friction pair is steel on steel yields:

\[ \delta = 25.6 \times 10^{-6} \text{ cm} \quad (10 \times 10^{-6} \text{ in.}) \]

This value is approximately twenty times the misalignment caused by tilting of the upper specimens. Thus the compression of the friction pair at the interface overcomes any misalignment due to torsion of the transducing beam and the transducing beam can be considered sufficiently stiff.

Velocity was measured using a Trans-Tek Model 112-001 linear velocity transducer. This instrument consists of a coil through which a magnetic core is passed. A voltage is produced proportional to the relative velocity between the core and coil. To sense the relative velocity between the two members of the friction pair the coil (\( \delta \) in Figure 3-3) was mounted on the slide and the core was mounted
at the end of the transducing beam close to the upper friction member using a bracket (9 in Figure 3-3) to hold and align both ends. The core came from the manufacturer with a diameter only slightly smaller than the bore of the coil. To insure that the core did not come in contact with the bore of the coil, the core was ground to a smaller diameter. This was necessary since if the coil and core rubbed additional dry friction would have been produced which would have interfered with the friction being measured.

3.3.3 Signal Conditioning and Recording

Signal conditioning consisted first of all of amplification of the two signals, the friction force signal and the relative velocity signal. The signal from the friction force transducer was amplified using a Dana Model 2210 DC amplifier with gains of 20, 50, 100, 200, 300, 500, and 1000. Gains used during the test were usually 100, 200, or 500. The signal from the velocity transducer was amplified by a Bay Laboratories, Inc. Model 14SV DC amplifier with gains of 1, 2, 5, 10, 20, 50, and 100.

The measurement system picked up some extraneous 60 Hz noise which when amplified became large enough to interfere with the measurements being taken. To eliminate this noise both the force and velocity signals were passed through a Krohn-Hite Model 3202 filter set for low pass and a cutoff
frequency of 50 Hz.

The velocity transducer was a self-generating device requiring no outside power source. The force transducer, however, because it employed a strain gage bridge, needed an outside voltage supply. This was provided by a SRC Division/Moxon Model 14041 DC power supply.

The two signals were observed and recorded using a variety of recording devices. A Tektronix Type 502A dual beam oscilloscope was available for continuous monitoring of the force and velocity signals either as time traces or in x-y mode. For making permanent records of the force and velocity an oscilloscope camera was available but because of the large number of tests run this method was considered too costly. Thus for time plots a Brush Mark 280 strip chart recorder was used. This instrument has two channels for recording both signals simultaneously and sufficiently high frequency response to faithfully reproduce the two signals. For making friction force vs. velocity plots a Brush 500 x-y plotter was used. This instrument can be used to record friction vs. velocity plots directly only up to a reciprocating frequency of about 0.1 Hz. Above that frequency the response is not sufficiently rapid to follow the signals and the signals must be recorded on a tape recorder and played back at a slower speed. A Precision Instrument Model PI 6200 tape recorder was used for this
task. The recorder was operated in FM mode which gives a recorder frequency response down to the DC levels needed to record a signal having a fundamental frequency component as low as 0.01 Hz.

3.4 Static and Dynamic Calibration of the Apparatus

Since the quantity to be measured was a dynamically varying quantity, both static and dynamic calibrations were undertaken. The static calibration consisted simply of applying a force input at the point of the friction force using weights and recording the output. This was done for a wide range of loads increasing and decreasing in both directions since the friction in both directions must be measured. The results are plotted on a calibration curve in Figure 3-6. This procedure produced a measured static sensitivity of,

\[
0.0921 \frac{mV}{N \cdot \text{volt excitation}} \quad (0.410 \frac{mV}{\text{lbf-volt excitation}})
\]

For comparison, the theoretical sensitivity was,

\[
0.0955 \frac{mV}{N \cdot \text{v}} \quad (0.435 \frac{mV}{\text{lbf-v}})
\]

The experimental stiffness is related directly to experimental sensitivity and can be calculated as 14,000 N/cm (7700 lbf/in.).
Figure 3-6. Static Calibration Curve for Force Transducer.
Dynamic calibration consisted of obtaining the frequency response over the frequency range from 1 to 1000 Hz. Only magnitude information was obtained. A small two-pound shaker was used to apply a force at the point where the friction force would be applied. The force was applied through a piezoelectric force transducer of known sensitivity. The input force was measured through this transducer and the output voltage was the output of the strain gage transducer. Output voltage over input force is plotted in Figure 3-7. As can be seen, the response is flat ± 1% to 50 Hz. The natural frequency is shown by the spike at 220 Hz. This corresponds to the calculated frequency of 315 Hz. From the height of the spike at 220 Hz the damping ratio, $\zeta$, was calculated as 0.06. The system appears to be essentially a lightly-damped second order type.

A pulse test to confirm the results of the frequency response test was also run by striking the transducer beam and recording the resultant transient output. From this output the natural frequency was found to be 220 Hz and the damping ratio, computed from the decay of the response, was found to be 0.03.

The velocity transducer was calibrated by the manufacturer but since the magnetic core was ground to a smaller diameter to increase the clearance between the core and the bore of the coil, it was necessary to recalibrate the velocity transducer. This was accomplished by use of an
Figure 3-7. Dynamic Calibration Curve for Force Transducer.

Excitation - 6 volts

Excitation - 6 volts

0.554 mv/N

Frequency, Hz
air dashpot as illustrated in Figure 3-8. The core was sus­
pended from the dashpot which could be adjusted to vary the velocity at which the core moved downward through the coil due to gravity. The velocity was calculated by timing the travel over a fixed distance and simply dividing distance by time. The output of the transducer was monitored for several velocities and an average transducer sensitivity of 85 mv/cm/sec (215 mv/in./sec) calculated. The standard deviation was 2 mv/cm/sec (5 mv/in/sec). The velocity transducer is a zero order instrument.

3.5 Theoretical and Experimental Evaluation of Friction Force Measurement Accuracy

The system characteristics now having been obtained, the next step is to investigate the useful range of the instrument. Every instrument has limitations and in order to evaluate the results of measurements these limitations must be understood.

To do this it is first necessary to investigate the nature and type of input which is to be measured. Because the measurement system is not infinitely stiff, even if the input motion were perfect, the relative motion between the rider and flat would not be perfect. In other words, it is not possible to achieve perfect sinusoidal, triangular, or any other motion between the rider and flat. The reason for this is that the rider and flat will stick together
Figure 3-8. Calibration of Velocity Transducer.
whenever the relative velocity goes through zero. The stick results because the spring force of the beam is insufficient to overcome the static friction between the two specimens and no relative motion can result until this is done. Thus relative displacement, relative velocity, and friction force will not be ideal but will only approach ideal. Figures 3-9 and 3-10 illustrate in a much exaggerated manner what happens for a sinusoidal input and a triangular input.

Since the friction force resists the relative motion and that motion is reciprocating, one might at first glance expect the friction force to be an alternating step function as in Figure 3-11. If this were actually the case, then since there is little damping in the measuring system the response would be as in Figure 3-12. In other words, there would be large overshoots and sustained oscillations. This has not been observed in the actual measurements and the reason is that, as has been shown in Figures 3-9 and 3-10, the friction force acts more like a terminated ramp input than a step input.

As a model of the friction-measuring system, consider Figure 3-13. The model consists of a rider of mass M, sliding on a flat. The rider is constrained relative to ground by a spring of rate K. Dry friction with constant coefficient of friction is assumed to exist between rider and flat. Viscous damping is neglected.
Figure 3-9. Ideal Variation of Displacement, Velocity, Friction Force, and Relative Velocity for Sinusoidal Motion.
Figure 3-10. Ideal Variations of Displacement, Velocity, Friction Force, and Relative Velocity for Triangular Motion.
Figure 3-11. Ideal Square Wave.
Figure 3-12. Response of Undamped Second Order System to an Alternating Square Wave Input.
Figure 3-13. Model of Measurement System.
Assume the flat and rider at rest in the equilibrium position. The friction force is now zero. The flat is then set into motion to the right. The flat and rider are still in contact with zero relative velocity and the friction force balances the force exerted by the spring. The friction force and the spring force balance until the friction reaches its maximum value at which point relative sliding takes place. Thus the friction force varies in a manner more like a terminated ramp than a step function.

Investigating further, assume the displacement of the flat is sinusoidal and is given by the function:

\[ d = \frac{A}{2} \cos (2\pi ft) \]  

where

- \( d \) = displacement, cm
- \( A \) = peak-to-peak amplitude, cm
- \( f \) = frequency of oscillation, cycles/sec
- \( t \) = time, sec

The function chosen is a cosine since the jump in friction force actually takes place at the ends of the strokes in each direction. The cosine function allows study of motion from the end of the stroke for time equal to zero onward.

The following are also defined:

- \( \mu \) = coefficient of friction
- \( N \) = normal force, N
$K$ is spring constant, N/cm

To just initiate the sliding, the spring force must just balance the maximum friction force:

$$K \left( \frac{A}{2} - d \right) = \mu N \quad (3.7)$$

$$\frac{KA}{2} \left[ 1 - \cos (2\pi f t) \right] = \mu N \quad (3.8)$$

$$KA \sin^2 (\pi ft) = \mu N \quad (3.9)$$

Now solving for the time it takes to break the two samples loose:

$$\sin^2 (\pi ft) = \frac{\mu N}{KA} \quad (3.10)$$

$$t = \frac{1}{\pi f} \sin^{-1} \sqrt{\frac{\mu N}{KA}} \quad (3.11)$$

Then the rise time, $T$, is twice the value given by Equation (3.11) since the force must go from negative to zero and then from zero to positive, twice as long.

$$T = \frac{2}{\pi f} \sin^{-1} \sqrt{\frac{\mu N}{KA}} \quad (3.12)$$
Since for the value of the system parameters $\sqrt{\frac{\mu N}{K_A}}$ is small, $\sin^{-1} \sqrt{\frac{\mu N}{K_A}} \approx \sqrt{\frac{\mu N}{K_A}}$ and,

$$T = \frac{2}{\pi \zeta} \sqrt{\frac{\mu N}{K_A}}$$  \hspace{1cm} (3.13)

It has been shown [22] that for a terminated ramp input to a second order system, the transient error at the top of the ramp can be no larger than,

Transient Error < \frac{1}{\omega_n T \sqrt{1-\zeta^2}} e^{-\zeta \omega_n T}  \hspace{1cm} (3.14)

and the steady state error is given by,

SS Error = \frac{2\zeta}{\omega_n T}  \hspace{1cm} (3.15)

where $\omega_n \triangleq$ system natural frequency, rad/sec

$T \triangleq$ rise time, sec

$\zeta \triangleq$ damping ratio

Combining (3.13) and (3.14) results in:

$$\text{Transient Error} < \frac{\pi \zeta}{2 \omega_n \sqrt{\frac{K_A}{\mu N(1-\zeta^2)}}} e^{-\frac{2\zeta \omega_n}{\pi \zeta} \sqrt{\frac{\mu N}{K_A}}}$$  \hspace{1cm} (3.16)
Combining (3.13) and (3.15) gives,

\[
SS \text{ Error} = \frac{n \pi f}{2 \omega_n} \sqrt{\frac{KA}{\mu N}}
\]  

(3.17)

Equation (3.16) gives an estimate of the maximum error in measuring the static coefficient of friction, while equation (3.17) gives an indication of the error in measuring the kinetic coefficient of friction. Both errors are directly proportional to frequency of oscillation of the flat, \( f \), the square root of the peak-to-peak amplitude of that oscillation, \( A \), and inversely proportional to the square root of the friction coefficient, \( \mu \), and the normal load, \( N \). Measurement system accuracy thus deteriorates for larger frequency and amplitude and smaller friction coefficient and normal load.

To gain an insight into what this means in terms of the present system parameters, some values can be substituted into Equations (3.16) and (3.17) which are indicative of the worst extreme of measurement accuracy by virtue of the high values of \( f \) and \( A \), and the low values of \( \mu \) and \( N \):

\[
\begin{align*}
A &= 2.54 \text{ cm (1 in.)} \\
K &= 14,000 \text{ N/cm (7700 lbf/in.)} \\
N &= 4.45 \text{ N (1 lbf)} \\
\mu &= 0.25 \\
\zeta &= 0.03
\end{align*}
\]
\[ f = 1 \text{ Hz} \]
\[ \omega_n = 1380 \text{ rad/sec} \]

Then,

Transient Error < 18%

SS Error = 1.2%

This is indicative of the worst extreme of measurement accuracy predicted by the theory at the maximum frequency considered, 1 Hz.

At 0.1 Hz the maximum transient error is 1% and steady state error 0.12%. At 0.01 Hz maximum transient error is effectively zero and steady state error is 0.012%.

The error predicted for the static friction measurements at high values of oscillation frequency are the largest predicted. The error will not be as large as predicted, however, for a number of reasons.

First, it should be pointed out that the values predicted are maximum values. They represent the upper limit predicted by the simple theory.

Second, input is actually not a terminated ramp but a terminated sine wave. Thus, the jumps back and forth between negative and positive friction actually develop much more smoothly than the theory predicts. This keeps the value of any overshoot down.
Third, the static error predicted here is the error just after the jump has occurred. Since the friction traces have been found to be symmetrical in most cases, the static value at the end of the stroke just before the jump occurs can also be used to measure the static coefficient. This value will have considerably less error because any transient will have died out.

Fourth and most important is the experimental verification. Experiments show the error predicted to be high. One way to experimentally verify the accuracy of the system would have been to raise the natural frequency of the system and compare the results of the present system with the results of the higher frequency system. If the response was the same for both systems, then the present system would be proved adequate by the comparison. Since the system has already been designed for maximum natural frequency, this could not be done, so it was decided to do just the opposite. The system natural frequency was degraded from 220 Hz to 40 Hz by adding mass to the end of the transducing beam. This mass was balanced by a large counterbalance so that the normal force remained the same. Experiments with the two systems showed the responses to be very close.

Figure 3-14 shows the responses of the 220 Hz and the 40 Hz natural frequency systems under the same conditions and using the same friction specimens. The basic shape of
Figure 3-14. Comparison of response of 220 Hz system and the 40 Hz system.
the responses is the same with the static and kinetic friction values being approximately the same. The major difference is that the 40 Hz system has larger amplitude fluctuations in the friction force. It can thus be concluded that the system does measure the friction force adequately over the range 0.01 to 1.0 Hz.

3.6 Damping

Damping in the system as originally designed was low. Damping ratio is $\xi = 0.03$. This did not give much damping to any oscillations which might arise from either measurement transient or relaxation oscillations. The source of measurement transients has been discussed earlier. Relaxation oscillations arise because of the variation of friction force with velocity. An analysis following that of Bristow [13] helps explain this effect.

As a model, take the system of Figure 3-15. This is a spring-mass-damper system with dry friction. The dry friction is taken to depend upon velocity as in Figure 3-16. Thus the equation of motion is given as,

$$M\ddot{x} + B\dot{x} + Kx = +F(v)$$  \hspace{1cm} (3.18)

where $v$ is the velocity of the flat relative to the mass $M$. The plus sign is necessary on the right side of the equation since the friction force opposes the relative velocity.
Figure 3-15. A Spring-Mass-Damper System with Dry Friction.
Figure 3-16. Friction Force Variation with Velocity.
For the purposes of analysis assume that the flat is sliding to the right with constant velocity \( V \). Then the relative velocity is \( \dot{v} = V - \dot{x} \), and the equation of motion becomes,

\[
M\ddot{x} + B\dot{x} + Kx = F(V - \dot{x}) \quad (3.19)
\]

If the system is in a stable equilibrium, then the deflection, \( x \), of the mass will be \( x = \frac{F(V)}{K} \). To study the stability of this operating point we can add a small perturbation, \( x = \frac{F(V)}{K} + \epsilon \). The equation of motion becomes,

\[
M\ddot{\epsilon} + B\dot{\epsilon} + K\epsilon = F(V - \epsilon) - F(V) \quad (3.20)
\]

Expanding \( F(V - \epsilon) \) in a Taylor's series gives,

\[
F(V - \epsilon) = F(V) - \epsilon F'(V) + \frac{\epsilon^2}{2} F''(V) - \ldots \quad (3.21)
\]

Substituting this back into the equation of motion and dropping higher order terms gives:

\[
M\ddot{\epsilon} + [B + F'(V)] \dot{\epsilon} + K\epsilon = 0 \quad (3.22)
\]

This is the equation of motion of the induced oscillations. In this equation the slope of the friction force
vs. velocity curve appears as part of the damping coefficient along with B. If the friction force vs. velocity curve is as in Figure 3-16, then this derivative is negative. This is the same as adding negative damping to the system and the perturbations can become unstable if \( B + F'(V) \) becomes negative. Thus relaxation oscillations can arise from a negative slope on the friction/velocity curve. This analysis shows how oscillations can arise from within the measuring system itself rather than being caused by an externally generated disturbing input such as vibrations arising in the slide or the hydraulic cylinder.

A viscous damper was designed to check the effect of adding damping to the system. This damper was a shear type device as shown in the sketch of Figure 3-17. It was mounted between the transducing beam and the reference beam as in the photograph of Figure 3-18. This design was chosen to eliminate any dry friction which might produce a dead zone in the measuring device. The damper designed produced a measured damping ratio of \( \xi = 0.2 \) when attached to the measuring system. No dry friction was detectable after proper alignment.

Some measurements were made with and without the damper to experimentally evaluate the effectiveness of the system with added damping. This experimental procedure showed that addition of the damper caused the static friction measurement to be depressed. For this reason use of the
Figure 3-17. Schematic of Shear Type Damper.
Figure 3.18. Photograph of Damper in Place Between the Two Beams.
damper was abandoned. Details of the design and study of the damper are given in the appendix.

It was suggested that the dry friction between the rider and flat adds damping to the system. If a spring-mass system with dry friction as in Figure 3-19 is set into motion from some initial displacement, the response will be as in Figure 3-20. The envelope is straight line. This can be compared to viscous damping by defining an equivalent viscous damper [50]:

\[
B_{eq} = \frac{4\mu N}{\pi \omega_n X}
\]

(3.23)

where

- \(B_{eq}\) \(\triangleq\) equivalent damping coefficient, \(\frac{N\text{-sec}}{cm}\)
- \(\mu\) \(\triangleq\) coefficient of friction, assumed constant
- \(N\) \(\triangleq\) normal load, N
- \(\omega_n\) \(\triangleq\) frequency of oscillation, Hz
- \(X\) \(\triangleq\) amplitude of oscillations, cm

To obtain an equivalent damping ratio,

\[
\xi_{eq} = \frac{B_{eq}}{2\omega_n N} = \frac{2\mu N}{\pi XX}
\]

(3.24)

Note that the damping ratio is inversely proportional to the amplitude of oscillations. Thus smaller motions would
Figure 3-19. Spring-Mass System with Dry Friction.
Figure 3-20. Decay of Spring-Mass System Motion with Dry Friction.
be more rapidly damped.

Substituting some values representative of the present system:

\[ \mu = 0.25 \]
\[ N = 4.45 \text{ N (1 lbf)} \]
\[ K = 14000 \text{ N/cm (7700 lbf/in.)} \]
\[ X = 0.000254 \text{ cm (0.0001 in.)} \]

then \[ \xi_{eq} = 0.65 \]

This analysis would appear to indicate that substantial damping is induced by the dry friction being measured. This is not true, however. The flaw in the theory is that the analysis here is for free vibrations. This assumes that friction force changes direction because relative velocity changes direction. In the actual measurement system the relative velocity is for most of the stroke in one direction and thus the friction force is usually in one direction. The dry friction thus does not add to the damping of the system except near zero velocity. The small amount of damping in the system comes from internal damping in the transducing beam, from damping in the strain gage deflection measuring device, and from the movement of the beam through the air. Additional damping is provided when the velocity transducer is used due to the magnetic coupling between the coil and the magnetic core.
4.1 Test Conditions

There are a number of factors that can affect frictional behavior. Among these factors are,

1. specific material combinations
2. lubricant
3. load
4. area of contact
5. velocity
6. crystal structure and orientation
7. hardness
8. surface films, oxides, nitrides, carbides, etc.
9. surface roughness
10. atmospheric composition
11. humidity
12. curvature of mating surfaces
13. temperature
14. type of motion (rolling, twisting, reciprocating, etc.)
15. time
16. bulk material properties
17. electrical current passing through interface
18. residual stresses
19. vibrations

Some of these are of course more important than others. An attempt was made to hold constant those factors which could be controlled. Others were monitored.

4.1.1 Contact Geometry

There are three possible theoretical contact geometries: point, line, and area. Point contact was chosen for the present study for two reasons: the inherently good alignment afforded and the fact that this contact geometry helps ensure conditions of boundary lubrication. The specimens consist of pins with spherical ends which slide on flats. Thus the initial contact is theoretically point but of course since no load can be carried without an area over which it is applied, there is actually an initial area of contact due to Hertzian deformation. This area increases as the specimens are worn during the rubbing process. The area and contact conditions therefore change with time of sliding.

4.1.2 Boundary Lubrication

When tests are being run with a liquid lubricant, it is necessary to ensure that only boundary lubrication
conditions existed. Theoretical reasons for believing boundary lubrication conditions were present are the high contact stress and the low velocity. High contact stress occurred because of the point contact. Low velocity occurred because of the low frequency of oscillation.

The effect of velocity and contact stress on lubrication conditions might best be explained in terms of the familiar $\frac{ZN}{P}$ curve. This is a curve relating coefficient of friction, $\mu$, with the dimensionless number $\frac{ZN}{P}$. It is usually seen for journal bearings but similar curves exist for other sliding conditions. An example of a $\frac{ZN}{P}$ curve is shown in Figure 4-1. The symbols in the dimensionless number are,

- $Z \Delta$ viscosity
- $N \Delta$ speed
- $P \Delta$ pressure

In terms of this curve, high contact stress and low velocity place the operating condition well to the left on the $\frac{ZN}{P}$ curve. This is the region of boundary lubrication. In this region a hydrodynamic wedge cannot form.

Experimental verifications of boundary lubrication are the high values of the friction coefficient obtained during tests and the uniform wear of the flat test specimens.

When friction tests were run, values of friction coefficient greater than 0.1 were measured for the sliding of metals. If hydrodynamic conditions had developed these
Figure 4-1. A Typical $\frac{Z_N}{p}$ Curve.
values would be much smaller, of the order of 0.01 or less. Thus one can conclude that no complete hydrodynamic film developed.

The observation that the wear track is uniform also supports this conclusion. If hydrodynamic lubrication had developed to any extent during the middle of the stroke where the velocity is highest, then wear would be lighter in the middle of the flat specimen than at the ends. This type of wear was not observed in any case.

4.1.3 Contact Stress

The Hertzian formula for maximum compressive stress for a sphere on a flat plate is given as [44]:

$$\text{Max. } S_c = 0.638 \sqrt[3]{\frac{P}{D^2 \left[ \frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2} \right]}}$$

(4.1)

where

- $P$ = normal load, N
- $D$ = diameter of sphere, cm
- $v_1$, $v_2$ = Poisson's ratio for bodies 1 and 2
- $E_1$, $E_2$ = modulus of elasticity for bodies 1 and 2, N/cm$^2$

Originally, friction tests were being run with a 1/8-inch spherical radius on the pin. It was found that this
caused too rapid a rate of wear, making it difficult to get frequency data which did not vary greatly with time. It was decided that the parameter of interest was the contact stress and that to increase the wear life the contact stress would have to be reduced.

Since the contact stress equation is of the form:

$$\text{Max. } S_c = K \frac{P^{1/3}}{D^{2/3}}$$  \hspace{1cm} (4.2)

the contact stress could be reduced by either reducing $P$ or increasing $D$. Reduction of $P$ did not appear attractive for two reasons: first, it would reduce the sensitivity of the friction force measurements since the transducer is designed to measure $\mu P$. Second, reduction of $P$ does not reduce $S$ as much as does increase of $D$. This is because $P$ has a 1/3 power and $D$ a 2/3 power. It was, therefore, decided to increase $D$.

Plotted in Figure 4-2 is a graph of the reduction of contact stress from the value with 1/8-inch radius pin against pin diameter for larger pins. As can be seen, there is a diminishing return on increasing the diameter. Weighed against this is the need to keep the contact stress high enough so as to ensure boundary lubrication. In view of these two trade-offs it was decided that a spherical diameter of 4 inches on the end of the pin was the best compromise. This gives a contact stress only 16% of the
Figure 4-2. Reduction of Contact Stress with Change in Spherical Diameter of Pin.
contact stress with the original 1/4-inch diameter spherical radius.

4.1.4 Vibrations

Vibrations in the system can influence the friction measurements significantly. Small amplitude, high frequency oscillations are often added to systems containing dry friction in order to reduce the coefficient of friction. This is known as dither [49].

In the present friction measurement system, it was found that when the hydraulic pump was running the vibrations transmitted to the measuring device were severe enough to affect the measurements. To bypass this problem the pump was run only intermittently. The rest of the time the hydraulic cylinder used to impart the oscillating motion was run from fluid stored under pressure in an accumulator. The accumulator held enough hydraulic fluid to run the system for about four minutes. At the end of that time the pump was restarted and run until the accumulator was recharged. This procedure eliminated the interfering vibrations except for short bursts.

The four minutes was long enough for the friction force to stabilize. This was necessary since in some cases the static friction remained at a low level for a period of time after the disturbing vibrations had been eliminated and
built up slowly until a maximum was reached. Measurements were taken after the friction traces reached this steady state condition.

4.1.5 Temperature and Humidity

Room temperature and humidity were monitored during the tests. Temperatures during the tests ranged from 20 to 26°C (68 to 79°F). In this range the temperature would have little effect on the friction measurements. The bulk temperature of the friction specimens would be very close to room temperature. The specimens are connected to large masses of metal which act as good heat sinks, and little frictional heating exists because of the relatively small forces and rubbing velocities being generated. Of course, at the interface of the friction pair there may be flash temperatures which are much higher than the surrounding room temperature. These flash temperatures occur only over a small area at a junction and are short lived.

Humidity is one of the variables which can affect the coefficient of friction. This effect is very slight for metals rubbing either unlubricated or with a liquid lubricant. It is possibly more important for a material such as nylon which can absorb moisture. Humidity can be critical when dry lubricants such as molybdenum disulfide or graphite are used. Graphite, for example, which is a good lubricant at high or normal relative humidity, actually fails as a
lubricant at low relative humidity, say at 6% relative humidity. This was first noted in high altitude aircraft (above 20,000 ft.) when the carbon brushes in the motors and generators failed rapidly in the thin, dry atmosphere [14]. Molybdenum disulfide behaves in the opposite manner since it lubricates better at low relative humidity. In one test [40] an increase in relative humidity from 15% to 70% caused the coefficient of friction to increase from 0.05 to 0.38.

Humidity was monitored using a sling psychrometer. Wet and dry bulb temperatures being taken, the relative humidity was calculated. These values ranged from 38-70% relative humidity during the testing. During the testing of the molybdenum disulfide coating the relative humidity range was 50-60%.

4.1.6 Hardness

Hardness can have a major influence on the coefficient of friction; therefore hardness measurements were taken for the materials used. Table 4.1 lists the hardness for the materials investigated in this study.

4.1.7 Surface Roughness

Surface roughness can also affect the friction coefficient measurements. This quantity is difficult to control accurately. An attempt was made to keep the surface conditions as uniform as possible; however there was some variation. Table 4.2 lists the RMS values of surface roughness of the
Table 4.1: Hardness

<table>
<thead>
<tr>
<th>Material</th>
<th>Hardness</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1020 steel</td>
<td>89-91</td>
<td>Rockwell B</td>
</tr>
<tr>
<td>hard steel</td>
<td>58-63</td>
<td>Rockwell C</td>
</tr>
<tr>
<td>brass</td>
<td>73-76</td>
<td>Rockwell B</td>
</tr>
<tr>
<td>oilite bronze</td>
<td>not measured</td>
<td></td>
</tr>
<tr>
<td>nylon</td>
<td>75</td>
<td>Shore D</td>
</tr>
<tr>
<td>glass</td>
<td>not measured</td>
<td></td>
</tr>
<tr>
<td>paper</td>
<td>not measured</td>
<td></td>
</tr>
<tr>
<td>brake material</td>
<td>not measured</td>
<td></td>
</tr>
</tbody>
</table>
Table 4.2: Roughness
(μ in. rms)

<table>
<thead>
<tr>
<th>Material</th>
<th>Parallel</th>
<th>Perpendicular</th>
</tr>
</thead>
<tbody>
<tr>
<td>1020 steel</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>hardened steel</td>
<td>&lt;4</td>
<td>8</td>
</tr>
<tr>
<td>brass</td>
<td>4</td>
<td>13</td>
</tr>
<tr>
<td>oilite</td>
<td>not measured</td>
<td></td>
</tr>
<tr>
<td>nylon</td>
<td>not measured</td>
<td></td>
</tr>
<tr>
<td>glass</td>
<td>not measured</td>
<td></td>
</tr>
<tr>
<td>paper</td>
<td>not measured</td>
<td></td>
</tr>
<tr>
<td>brake material</td>
<td>not measured</td>
<td></td>
</tr>
</tbody>
</table>
flats for some of the materials used in this study. Values are given both parallel to and perpendicular to the grain of the surface. The oscillatory motion was made parallel to the grain. Pins were polished to a high degree on a lathe. The directionality is therefore circular. RMS roughness is less than 4 μ in. for the pins. Measurements were made with a Brush Surface Analyzer consisting of a Model BL 905 amplifier connected to a Model BL 902 recorder.

4.2 Specimen Preparation

Surface preparation is the single most important influence on friction measurements [8]. With this in mind, considerable care was taken in handling and preparing the test specimens. It is impossible for a surface to be completely free of surface contaminants. If such a surface did exist, then the moment it was exposed to the atmosphere, gases from the atmosphere would be absorbed and, in those cases where such reactions can take place, oxides, nitrides, and other chemical compounds would form. The complete elimination of these contaminants is thus impossible.

There are two classes of contaminants which must be controlled or eliminated [36]. The first consists of the oxides, nitrides, and absorbed films of gases. The second includes fats, oils, greases, and other organic compounds. Control of each of these classes of contaminants requires a different procedure. Because of the wide range of materials
investigated, surface preparation varied. However, where it was possible to do so abrasion was used to reduce absorbed gases, oxides, and other reaction products; and solvents were used to eliminate organic compounds.

The procedure for the steel specimens was,
1. grind surface.
2. abrade with crocus cloth, 4/0 emery paper, and 4/0 steel wool.
3. wash with reagent grade isopropyl alcohol and thiophene-free benzene.
4. rinse with isopropyl alcohol then with thiophene-free benzene.

The procedure for the brass specimens was similar to that for steel except that the specimens were initially milled instead of ground. The nylon specimens had milled surfaces and were washed with water. Glass specimens were washed with reagent grade isopropyl alcohol and thiophene-free benzene. The oilite and paper were tested with no surface preparation. The surface of the brake material was slightly abraded to eliminate any directionality due to run-in.

All specimens were prepared immediately prior to testing. Testing commenced no more than one half hour after the specimens were prepared.
4.3 Friction Variations Due to Variations in Contour

The possibility that some variations in friction force could be due to variations in surface contour was investigated. These variations could come from two sources: initial variations and variations due to wear.

4.3.1 Effect of Non-Parallel Flat

To investigate the effect of a flat which is not parallel with the direction of motion, the model of Figure 4-3 was considered. The surface of the flat is skewed at an angle $\theta$ to the direction of motion. Motion is in the $x$ direction and because of the nonparallel surface the rider will move in the $y$ direction.

For sinusoidal motion,

$$x = \frac{A}{2} \sin (2\pi ft)$$ \hspace{1cm} (4.3)

$$y = x \tan \theta$$

$$y = \frac{A}{2} \tan \theta \sin (2\pi ft) \hspace{1cm} (4.4)$$

The vertical acceleration of the rider is,

$$\ddot{y} = -2Am^2f^2 \tan \theta \sin (2\pi ft) \hspace{1cm} (4.5)$$
Figure 4-3. Model for the Effect of a Non-Parallel Flat.
The effective mass of the beam assembly is 2.04 kg (4.5 lbm). Thus using Newton's law, the change in normal force \( \Delta N \) due to vertical acceleration is,

\[
\Delta N = 0.041 \pi^2 f^2 \tan \theta \sin (2\pi ft) \, \text{N} \quad (4.6)
\]

and

maximum \( \Delta N = 0.041 \pi^2 f^2 \tan \theta \, \text{N} \quad (4.7) \)

To get a feel for the numbers involved, substitute the maximum values:

\[ A = 5.08 \text{ cm (2.0 in.)} \]

and \( f = 1 \text{ Hz} \)

Then for a 1\% change in normal load which would mean a 1\% change in friction force \( \Delta N = 0.0445 \text{ N (0.01 lbf)} \).

\[ \theta = \tan^{-1} 0.0218 \]

\[ \theta = 0.0218 \text{ radians or 1.2 degrees} \]

This is equivalent to a variation in thickness of 0.056 cm per cm of length (0.022 in./in.). This is considerably more than the 0.00254 cm/cm (0.001 in./in.) estimated to be the maximum possible. Thus friction measurements are not significantly affected by the small amount of non-parallelism possible.
4.3.2 Effect of Surface Waviness

To investigate the effect of surface waviness on the friction measurements consider the model of Figure 4-4. The rider slides in the x direction on a surface whose waviness imparts a sinusoidal vertical motion to the rider. Sinusoidal horizontal motion is assumed.

The equation for the motion the lower surface imparts to the rider can be expressed as:

\[ y = B \sin \left( \frac{2\pi x}{\lambda} + \theta \right) \]  \hspace{1cm} (4.8)

Displacement in the x direction is,

\[ x = \frac{A}{2} \cos (2\pi ft) \]  \hspace{1cm} (4.9)

so that,

\[ y = B \sin \left[ \frac{A\pi}{\lambda} \cos (2\pi ft) + \theta \right] \]  \hspace{1cm} (4.10)

Vertical acceleration is then,

\[ \ddot{y} = - \frac{4AB\pi^3 f^2}{\lambda^3} \cos 2\pi ft \cos \left[ \frac{A\pi}{\lambda} \cos (2\pi ft) + \theta \right] \]

\[ - \frac{4A^2B\pi^4 f^2}{\lambda^2} \sin 2\pi ft \sin \left[ \frac{A\pi}{\lambda} \cos (2\pi ft) + \theta \right] \]  \hspace{1cm} (4.11)
Figure 4-4. Model for the Effect of Surface Waviness
The maximum possible vertical acceleration is,

$$\text{max } \ddot{y} = \frac{4AB\pi^3f^2}{\lambda} \left(1 + \frac{A\pi}{\lambda}\right) \tag{4.12}$$

Using an effective mass of 2.04 kg (4.5 lbm) the maximum possible change in vertical force $\Delta N$ is,

$$\text{max } \Delta N = 0.082 \frac{AB\pi^3f^2}{\lambda} \left(1 + \frac{A\pi}{\lambda}\right), N \tag{4.13}$$

Maximum values of $A$ and $f$ are,

$$A = 5.08 \text{ cm (2 in.)}$$
$$f = 1 \text{ Hz}$$

giving,

$$\text{max } \Delta N = \frac{12.85 B}{\lambda} \left(1 + \frac{15.95}{\lambda}\right), N \tag{4.14}$$

As an example, suppose that because of the sinusoidal motion of the rider, the flat wore in a sinusoidal manner; then,

$$A = \lambda$$
and \[ \text{max } \Delta N = 0.082 B \pi^3 f^2 (\pi + 1), \ N \]

For the maximum frequency, \( f = 1 \) cycle/sec, and a normal force variation of 1%, \( \text{max } \Delta N = 0.0445 \ N \ (0.01 \ \text{lbf}). \)

\[ B = 0.00424 \ \text{cm} \ (0.00167 \ \text{in.}) \]

Thus the flat would have to be worn to a depth 0.008 cm (0.003 in.) deeper in the middle than at the ends of the wear track. This is a significant amount of wear and was not observed. The wear was in fact observed to be uniform. Because of the care taken in preparation, initial surface waviness should be no problem.

4.4 Planning

In planning the testing considerable flexibility was available because of the use of the hydraulic servomechanism to produce the reciprocating motion. The versatility of this device allows a wide range of input motion waveforms, amplitudes of motion, and frequencies of reciprocation. The design of the measurement system allowed the normal load to be changed.

The basic input motion chosen was sinusoidal motion. This choice was made because the motion varied smoothly over a range of velocities. With sinusoidal motion as the
principal input waveform, the velocity could be changed by changing either the frequency of reciprocation or the amplitude of the motion. It was decided to run most of the tests by varying the frequency since this variable could be changed over a larger range than the amplitude. The frequency ranged from 0.01 Hz to 1.0 Hz. The lower limit is arbitrarily chosen and the upper limit is limited by the response of the measurement system. This is a range of 100 to 1. The range of possible amplitudes was from 0.25 cm (0.1 in.) to 5.08 cm (2.0 in.), a range of 20 to 1. The lower limit is limited by the response of the system and the upper limit by the stroke of the actuating cylinder. The amplitude was held at 2.54 cm (1.0 in.) peak-to-peak for all the tests where the frequency was changed. This value was chosen because it is arbitrarily in the middle of the range of amplitudes.

Since the maximum velocity of reciprocation could be changed by varying the amplitude as well as the frequency, it was decided to conduct a select number of tests holding the frequency of reciprocation constant and varying the amplitude. In these tests the frequency was held constant at 0.1 Hz. This represents a frequency in the center of the available range. Four different material/lubricant combinations were chosen to be representative of different lubricant combinations. Materials chosen were nylon and mild steel because of their availability and wide use.
Lubricant conditions were chosen to be representative of unlubricated sliding, sliding with a "poor" boundary lubricant, and sliding with a "good" boundary lubricant.

Besides the tests run with the sinusoidal input motion, a few tests were run with other input motions to investigate the result of changing motion. A few tests were run with a triangular input motion since this motion produces constant velocity in each direction. The constant velocity helped point out that the frictional variations were mainly caused by velocity changes and were not caused by any other effect such as surface changes at the beginning and end of the stroke where the velocity was slower. Random motion inputs were investigated to make certain that the periodic nature of the sinusoidal input motions did not give rise to improper conclusions since only one quantity, amplitude, or frequency could be varied at any one time. With random excitation both amplitude and frequency are changing continuously.

Some tests were run with increased load to determine the effect changing the normal force had on the results. The load was changed from the standard load of 4.45 N (1.0 lbf) to 44.5 N (10.0 lbf). Tests were also run to determine the repeatability of the results.

Before any of the above testing was started, tests were run to determine the effect of test duration on the coefficient of friction. Test results here display
coefficient of friction vs. number of cycles of reciprocation. The tests were run at 0.1 Hz frequency and 2.54 cm (1 in.) peak-to-peak amplitude.
CHAPTER V

TEST RESULTS

5.1 Introduction

This chapter presents the results of a series of tests to investigate frictional behavior during reciprocating sliding. These tests were run using various combinations of motions, lubricants, materials, etc. The results can be categorized as taking on two general forms. These two forms are shown for sinusoidal motion in the friction force/time traces of Figure 5-1. The first type, characterized by the upper curve, has high values of the coefficient of friction at low velocity and lower coefficient at higher velocity. The second type, characterized by the lower curve, has low friction coefficient at low velocity and higher friction coefficient at higher velocity.

In order to talk about the friction traces more conveniently, a static and a kinetic coefficient of friction were defined. The static coefficient was defined as the value of the friction force divided by the normal load when the velocity passes through zero. When friction force varies with velocity, giving kinetic friction coefficient
Figure 5-1. Two Types of Friction Force vs. Time Curve
as a single number does not, of course, completely describe the behavior. For convenience in making certain comparisons, however, the kinetic coefficient was defined as the value of the friction force divided by the normal load when the velocity is a maximum. Thus, the kinetic coefficient as defined here is actually the maximum coefficient of friction for the particular range of velocities covered when the response is as shown in the lower half of Figure 5-1 and the minimum coefficient of friction when the response is as shown in the upper half of Figure 5-1.

This definition is useful since the coefficient of friction is not a constant value but changes with velocity. It should be apparent that the kinetic coefficient will change value as the amplitude and frequency of the reciprocating motion changes because the maximum velocity changes. It should also be noted that these definitions make it possible for the static coefficient to be smaller than the kinetic coefficient. This would occur where friction traces of the second type are obtained.

The variation in friction force investigated here is due primarily to the velocity variation during the reciprocating motion. This is seen more clearly when the friction output is presented on force vs. velocity plots instead of on time traces. Examples of this are shown in Figures 5-2 and 5-3. Figure 5-2 shows friction results of the first type and Figure 5-3 shows results of the second type.
The first plot of Figure 5-2 gives the friction force vs. velocity behavior during reciprocating motion for mild steel sliding on mild steel and lubricated with a simple mineral oil. In this case the friction force starts out at a relatively high level and falls off as the velocity increases in a smooth exponential-appearing manner. The decrease in the friction force is rapid at low velocity and slows down as the velocity increases.

The first plot of Figure 5-3 gives the friction force vs. velocity behavior during reciprocating motion for mild steel sliding on nylon with no lubricant. This is an example of the second general classification of behavior where the friction force starts off at a relatively low value for zero velocity and increases as the velocity increases. The increase in this case appears to be of the exponential type also, increasing relatively rapidly at low velocity and more slowly as the velocity increases. It will be noted that there is a peculiarity on the rise in the force vs. velocity plot as the friction force changed direction at zero velocity. There is a very rapid rise in the value of the friction force with a very small change in velocity from zero. This behavior was also observed for steel on steel lubricated with the oleic acid in mineral oil mixture. It did not occur on friction force vs. velocity plots of the decreasing type. In those cases where this rapid rise occurred, the static coefficient was taken to
be the value where the friction/velocity curve resumed the smoother behavior (+ a in Figure 5-3).

This peculiarity could be a spurious effect resulting from a measurement system problem, but this is not probable since the behavior was not observed for any of the plots of the decreasing type friction character nor for all the plots of the increasing type. Rather, it is more likely that there actually is a rapid rise in friction force as the two friction specimens first slip relative to each other. This might be caused, for example, by the spherical tipped pin sliding out of a depression at the end of the stroke caused by hertzian deformation of the flat.

The other plots in Figures 5-2 and 5-3 illustrate the velocity behavior of the other materials investigated in this study. As can be seen, their behavior can be classified into one of the two general classes. The only exception is the behavior of steel on brass lubricated with oleic acid. In this case the behavior is a hybrid between the two general classifications with high static coefficient rapidly falling off and then rising again slowly. This particular material/lubricant combination was listed with the increasing classification because the slow increase in kinetic coefficient was thought to be the more significant of the two effects.

5.2 Variations with Number of Cycles

Investigations were conducted with each of the material combinations and lubricant conditions to determine how the
Figure 5-2. Friction Force vs. Velocity Curve of the Decreasing Type.
Figure 5-2 (cont.). Friction Force vs. Velocity Curve of the Decreasing Type.
Figure 5-2 (cont.). Friction Force vs. Velocity Curve of the Decreasing Type.
Steel on Nylon
No Lubrication

Figure 5-3. Friction Force vs. Velocity Curve of the Increasing Type.
Figure 5-3 (cont.). Friction Force vs. Velocity Curve of the Increasing Type.
Figure 5-3 (cont.). Friction Force vs. Velocity Curve of the Increasing Type.
friction changed with the number of oscillatory cycles performed. This information was obtained by continuously monitoring the friction force while the two samples were reciprocated relative to each other for 500 cycles at 0.1 Hz frequency and 2.54 cm (1 in.) peak-to-peak amplitude. This represents a time of oscillation of about an hour and half. The normal load was 4.45 N (1 lbf).

These tests were performed for two primary reasons. First, they were undertaken to discover a suitable region in which to perform the studies of friction as the frequency of oscillation changed. Since the contact conditions continuously change as sliding takes place, the friction force will change with the number of oscillations. These changes may be due to the breakdown or build-up of surface films or the change in contact geometry due to wear. By investigating the variation of friction with the number of cycles, it is possible to pick a region where the effect of the number of cycles is minimal. For example, it may be found in a particular case that there is very little change in friction after 100 cycles. Then, before starting tests of friction vs. frequency, the specimens would be "run-in" for 100 cycles.

The second reason for studying variations of friction with the number of cycles is for the information it gives on frictional behavior. For example, the breakdown of lubricant films may be observed. Or, the erratic nature of the frictional behavior may be pointed out. In other words, variation of
friction with the number of cycles is interesting for its own sake.

The data is presented by plotting coefficient of friction as a function of number of cycles of oscillation. Both static and kinetic coefficient (as earlier defined) are plotted on a linear scale. Number of cycles of oscillation is plotted on a log scale.

5.2.1 Mild Steel on Mild Steel

Tests were run with 1020 cold rolled steel under three conditions of lubrication: no lubricant, mineral oil, and an oleic acid-mineral oil mixture. This is representative of conditions of dry friction, a poor boundary lubricant, and a good boundary lubricant.

The test with no lubricant, Figure 5-4, shows a static coefficient higher than the kinetic, the static being 0.60 and the kinetic being 0.40 to 0.50. The static coefficient remains fairly constant as the number of cycles increases, but kinetic coefficient shows a slow rise with increasing cycling.

The mineral oil lubricated tests, Figure 5-5, also show a higher static coefficient than kinetic coefficient. In this case the values of the coefficients are initially lower than in the dry case. The static coefficient starts at 0.32 and rises to 0.65 as the test proceeds. The kinetic coefficient remains constant at 0.2 until about 300 cycles when there is a sudden jump in its value up to 0.5. These
Figure 5-4. Variation of Friction with Number of Cycles of Reciprocation.
Coefficient of friction

Steel on Steel
Mineral Oil

\[ \mu_k \] (velocity = 0.8 cm/sec)

\[ \mu_s \]

Figure 5-5. Variation of Friction with Number of Cycles of Reciprocation.
final values are about the same as those obtained with no lubricant.

In the oleic acid lubricated test, Figure 5-6, the static coefficient is initially lower than the kinetic coefficient. Both values are fairly close together, however, static coefficient starting at about 0.11 and kinetic at about 0.12. The coefficients are fairly well behaved until 200 cycles at which point the static coefficient climbs above the kinetic. This represents a breakdown in the "good" lubricating properties of the boundary film.

5.2.2 Hard Steel on Hard Steel

The tests were run using oil hardening tool steel for flats and drill rod for pins. Both were hardened to a Rockwell C value of over 60. Four lubrication conditions were investigated: dry, mineral oil, oleic acid and mineral oil mixture, and a bonded film of molybdenum disulfide.

The dry tests, Figure 5-7, did not show easily distinguishable static and dynamic friction coefficients. The friction traces were erratic as shown in the friction-time trace of Figure 5-8. For this reason only the static coefficient is given. This coefficient starts at 0.3 on the first cycle but quickly rises to 0.6 on the second cycle and finally levels out at about 0.9. The tests with mineral
Figure 5-6. Variation of Friction with Number of Cycles of Reciprocation.
Figure 5-7. Variation of Friction with Number of Cycles of Reciprocation.
Figure 5-8. Friction-Time Trace for Hard Steel on Hard Steel, $A = 1$ in. (2.54 cm) peak-to-peak, $f = 0.1$ Hz.
oil as a lubricant, Figure 5-9, have a static coefficient larger than the kinetic coefficient. The static coefficient starts at 0.5 and goes to 0.3 after 500 cycles. The kinetic coefficient starts at 0.3 and goes to 0.2 after 500 cycles. With oleic acid and mineral oil as the lubricant, Figure 5-10, the static coefficient is lower than the kinetic coefficient after an initial 10 cycles. After this initial "run-in" period, the friction is well behaved. The static coefficient has a value of about 0.11. The kinetic coefficient is very close at 0.12.

A fourth test was run with a sprayed-on, bonded film of MoS$_2$ as the lubricant, Figure 5-11. The product used was Molykote M-8800. This is applied in a thickness of about .0005 in. The results show a static coefficient higher than the kinetic coefficient. The static coefficient is constant at 0.4 until about 50 to 100 cycles at which point the value falls to 0.32. The kinetic coefficient is initially 0.3 falling to 0.22 after 100 cycles. This fall in the friction force occurs as the result of the binder being worn from the bonded film. At about 100 cycles the 0.0005 thick film is worn through. A burnished film of MoS$_2$ is left however which continues to lubricate the friction pair.

5.2.3 Steel on Nylon

The tests with steel pins run on nylon flats, Figure 5-12,
Figure 5-9. Variation of Friction with Number of Cycles of Reciprocation.
Figure 5-10. Variation of Friction with Number of Cycles of Reciprocation.
Figure 5-11, Variation of Friction with Number of Cycles of Reciprocation.
Figure 5-12. Variation of Friction with Number of Cycles of Reciprocation.
show a static coefficient lower than the kinetic coefficient. The combination is very well behaved. The static coefficient is a constant 0.05 for the entire 500 cycles. The kinetic coefficient is approximately 0.075 for the entire 500 cycles. These tests were run without any lubricant.

5.2.4 Steel on Brass

Tests were run with mild steel pins reciprocated on brass flats. Three different lubrication conditions were investigated: no lubricant, mineral oil, and oleic acid mixed with mineral oil.

The unlubricated situation, Figure 5-13, has a static coefficient higher than the kinetic coefficient. The static coefficient starts at 0.38 and rises slowly with increasing cycles to a value of 0.48. The kinetic coefficient remains constant at about 0.32 for the 500 cycles. With mineral oil as the lubricant, Figure 5-14, the static coefficient is again larger than the kinetic coefficient. The static coefficient starts at 0.18 and increases continuously to 0.32. The kinetic coefficient starts at 0.12 and increases to 0.18. The first cycle of the oleic acid lubricated situation, Figure 5-15, has a higher kinetic than static coefficient. After the first cycle the static coefficient becomes larger than the kinetic. Both coefficients start at about 0.14. By the 500th cycle the static coefficient is
Figure 5-13. Variation of Friction with Number of Cycles of Reciprocation.

- $\mu_k$ (velocity = 0.8 cm/sec)  
- $\mu_s$  
  Steel on Brass  
  No Lubricant
Figure 5-14. Variation of Friction with Number of Cycles of Reciprocation.
Figure 5-15. Variation of Friction with Number of Cycles of Reciprocation.
0.18 and the kinetic coefficient is 0.14.

5.2.5 Steel on Oilite Bronze

Steel pins were reciprocated on Oilite bronze flats, Figure 5-16, to study this common bearing material. The sintered bronze was impregnated with an SAE 30 oil at the factory and no additional oil was added to the surface. The test shows that for the first 10 cycles the static and kinetic coefficients are the same. By the 20th cycle the static coefficient had become larger than the kinetic coefficient and remained so through the 500th cycle. The coefficients start at 0.15. By the 500th cycle the static value is 0.22 and the kinetic value is 0.17.

5.2.6 Steel on Glass

Steel pins were reciprocated on glass with oleic acid in mineral oil as the lubricant, Figure 5-17. The glass used was microscope slides cemented to a back-up plate of steel. In this test the static coefficient started out lower than the kinetic coefficient. The static coefficient started at 0.13 and the kinetic at 0.14. After 200 cycles the static coefficient became larger than the kinetic with the static being about 0.14 and the kinetic about 0.12.
Figure 5-16. Variation of Friction with Number of Cycles of Reciprocation.

Steel on Oilite Bronze
No Lubricant

- $\mu_K$ (velocity = 0.8 cm/sec)
- $\mu_S$
Figure 5-17. Variation of Friction with Number of Cycles of Reciprocation.
5.2.7 **Hard Steel on Brake Material**

Hardened steel pins were reciprocated on an asbestos-based brake material flat, Figure 5-18. This flat was cut from a disk brake pad taken from a Ford Motor Company Thunderbird. No lubricant was used. The static coefficient was lower than the kinetic, being 0.13. The higher kinetic coefficient started at 0.15 and rose to 0.17 by the end of 500 cycles.

5.2.8 **Steel on Paper**

Steel pins were run on paper specimens constructed from computer cards pasted on steel backing, Figure 5-19. No lubricant was present. The static coefficient was lower than the kinetic. The static coefficient was 0.19 and the kinetic coefficient 0.21. The test shows a slight rise beginning at 500 cycles.

5.3 **Some Interpretations of the \( \mu \) vs. Number of Cycles Tests**

The tests to determine the changes in coefficient of friction with number of cycles point up some interesting results.

5.3.1 **Effectiveness of Lubricants**

Four conditions of lubrication were investigated: no
Figure 5-18. Variation of Friction with Number of Cycles of Reciprocation.
Figure 5-19. Variation of Friction with Number of Cycles of Reciprocation.

- $\mu_k$ (velocity = 0.8 cm/sec)
- $\mu_s$

Steel on Paper
No Lubricant

Number of Cycles, N
lubrication, a straight mineral oil, a mixture of oleic acid in mineral oil, and a bonded dry film lubricant. The straight mineral oil was chosen to be representative of a so-called "poor" boundary lubricant. The oleic acid-mineral oil mixture was chosen to be representative of a so-called "good" boundary lubricant. The bonded dry film lubricant consisted of MoS$_2$ with an air drying organic binder.

The unlubricated tests of the metals show the highest values of the coefficients of friction. The most erratic frictional behavior is also noted under these conditions. When mineral oil was introduced as a lubricant, the erratic behavior disappeared and the values of the coefficients of friction decreased. Addition of oleic acid to the mineral oil caused a further reduction in the coefficients of friction. Use of the bonded film lubricant lowered the coefficient of friction somewhat, but not nearly as much as the oleic acid-mineral oil mixture.

Among all the material combinations considered, the metal on metal friction pairs produced values of the coefficients of friction much higher than the metal on non-metal friction pairs. The hard steel on hard steel pair produced the highest value with a value of 1.0. The static and kinetic coefficients were indistinguishable. Mild steel on mild steel produced a static coefficient of 0.6 and a
kinetic coefficient of 0.5. There was a sizable amount of wear debris generated and it may be that the presence of this debris caused the mild steel to produce lower values of friction than the hardened steel pair which showed little wear. The steel on brass pair produced a static coefficient of 0.5 and a kinetic coefficient of 0.35.

The three metal on metal pairs all have static coefficients higher than the kinetic coefficients. On the other hand, the steel on non-metal pairs have static coefficients lower than their respective kinetic coefficients. The values of these coefficients are lower than the metal-metal pairs. Steel on nylon pair had a static value of 0.05 and a kinetic value of 0.075. This combination had the lowest friction coefficients of all the combinations tested whether lubricated or un lubricated. Steel on brake material had a static coefficient of 0.13 and a kinetic coefficient of 0.17. The steel on paper had a static coefficient of 0.19 and a kinetic coefficient of 0.21.

Steel on oilite had a higher value of static coefficient than kinetic coefficient. The static was 0.22 and the kinetic 0.17. No lubricant was added to the oilite. However, it is actually lubricated since it is impregnated with SAE 30 oil. This oil is squeezed out by the contact pressure and by capillary action. The oil can actually be seen transferred to the pin after the test.
Mineral oil was introduced as a lubricant for the three metal-metal combinations tested. The mineral oil did reduce the coefficients somewhat. It also reduced or eliminated the erratic frictional behavior present with some of the dry tests.

The mild steel on mild steel tests showed an initial value of the static coefficient of about 0.5. The kinetic coefficient was lower at 0.2. After 500 cycles, however, the coefficients of friction had climbed to values approximately equal to those obtained in the dry tests. The static coefficient after 500 cycles was 0.65 and the kinetic coefficient was 0.55.

For the hard steel on hard steel tests, the improvement was significant. Separate static and kinetic coefficients were recognizable where they had not been for the dry tests. The static coefficient was 0.35. The kinetic coefficient was lower at 0.20. This is much lower than the dry value of 1.0.

Steel on brass friction coefficients were also reduced. The static friction coefficient was reduced from 0.50 dry to 0.30 with mineral oil. The kinetic coefficient was reduced from 0.35 to 0.18 by the use of mineral oil.

Use of mineral oil as a lubricant reduced all the friction coefficients. However, the values were still higher than the steel on non-metals. Also, the static coefficient
was still higher than the kinetic.

Oleic acid was added to the mineral oil to investigate the effect of a so-called "good" boundary lubricant. This lubricant combination was used with the three metal on metal pairs. Results showed significant improvement over the mineral oil lubricated condition. Friction coefficients dropped even lower.

Both steel pairs gave similar results. The mild steel on mild steel and the hardened steel on hardened steel both had lower static coefficients than kinetic coefficients. The static coefficient was 0.11 and the kinetic coefficient was 0.12 in both cases. The mild steel pair deviated from this behavior after about 200 cycles. At this point the static coefficient rose to a value of 0.14 while the kinetic coefficient remained at 0.12. The hardened tool steel did not show this transition.

Tests of the steel on brass combination revealed a higher static coefficient than kinetic coefficient. The static coefficient was 0.18 and the kinetic coefficient was 0.13. Thus the oleic acid appears to be a more effective lubricant for the steel-steel pairs than the steel-brass pair.

Oleic acid mixed with mineral oil was also used as the lubricant when steel pins were tested on glass. This combination produced a static coefficient of 0.13 and a higher kinetic coefficient of 0.14. After 200 cycles the
static coefficient became 0.14 and the kinetic 0.13.

A bonded film MoS$_2$ coating was used on one material combination to investigate this lubricant. The friction pair was hard steel on hard steel. Results showed a static coefficient of 0.40 changing to a value of 0.35 after 100 cycles. The kinetic coefficient was lower, starting at 0.30 and dropping to 0.22 after 100 cycles. These values of friction coefficient are relatively high, being no better than those obtained when mineral oil is used as the lubricant.

5.3.2 Some Results and Conclusions

A. The unlubricated metals are poor friction pairs in terms of having high coefficients of friction.

B. Static coefficients for the unlubricated metals are higher than the kinetic coefficients.

C. The hard steel gives more erratic frictional behavior than the softer mild steel. This is consistent with the observation by others that harder metals give more erratic behavior and results that are less reproducible.

D. The build-up of wear particles with the softer metals may be helpful in reducing friction by reducing adhesion. The two friction specimens would roll on the wear particles, acting as a lubricant of sorts.
E. The unlubricated metal on non-metal friction pairs give good results in terms of giving low values of the friction coefficients. It might be said that the non-metals tested have good self-lubricating properties in terms of friction coefficient.

F. Friction pairs made up of two different materials give lower values of the coefficients of friction than friction materials made up of two like materials (cf. mild steel on mild steel and hard steel on hard steel). For example, even the steel on brass pair has lower values of friction coefficients than the two like material pairs tested. This is consistent with the well-known design rules that two differing materials are best for friction pairs.

G. The steel on brake material had a static coefficient of friction lower than the kinetic coefficient. This means less chatter and brake squeal than if kinetic coefficient were less than static. This is due to the stabilizing effect of the increasing friction characteristic. An explanation for this is given in the section on damping in Chapter III.

H. Use of mineral oil as a lubricant gives lower values of friction coefficient and gives less erratic frictional behavior.

I. The static coefficient is larger than the kinetic coefficient when mineral oil is used as the lubricant with metal on metal pairs.
J. Mineral oil does not give the low values of friction coefficient obtained with the metal-non-metal pairs or when oleic acid, a "good" boundary lubricant, is used as the lubricant.

K. Mild steel on mild steel lubricated with mineral oil shows a sharp rise in friction coefficient after 200 cycles. This may be due to wear particle build-up.

L. Use of the oleic acid-mineral oil combination gives the lowest friction coefficients to the metal-on-metal friction pairs.

M. Oleic acid gave lower static coefficients than kinetic coefficients except for the steel on brass combination.

N. Breakdown of the oleic acid lubricant films takes the form of the static coefficient becoming higher than the kinetic coefficient. This change is not great and the coefficients of friction remain low relative to the dry or mineral oil lubricated conditions.

O. Values of both static and kinetic coefficients are the same for both mild steel on mild steel and hard steel on hard steel lubricated with the oleic acid-mineral oil mixture. The softer steel does undergo a transition to a higher static coefficient, however. This is possibly due to surface wear.

P. When the bonded molybdenum disulfide lubricant is used, the transition from higher to lower friction coefficients
is due to the wearing away of the binder. The MoS$_2$ is then burnished into the surface. The relatively high values of the friction coefficients are consistent with the fact that for MoS$_2$ the coefficients of friction are higher in reciprocating sliding than in unidirectional sliding.

5.4 Results of the Friction vs. Frequency Tests

In this section the results of the tests to investigate friction force changes as the frequency of reciprocation was varied are given. Changing the frequency of the sinusoidal motion directly changes the maximum value of the sinusoidally varying velocity. Since for convenience the value of the coefficient of friction at the maximum velocity has been called $\mu_K$, the kinetic friction coefficient, any change in $\mu_K$ as frequency is changed might be explained as strictly a velocity effect, or there might be, in addition, an intrinsic frequency effect. (In Section 5.6 it is shown that, for the most part, changes in $\mu_K$ are explainable strictly on the basis of velocity.) For these tests the frequency of oscillation was varied from 0.01 to 1.0 Hz, at a peak-to-peak amplitude of 2.54 cm (1 in.). This translates into a maximum velocity during each cycle of 0.08 to 8.0 cm/sec (0.03 to 3 in./sec). The values of the maximum velocity are given on the horizontal axis of the graphs presented in this section to emphasize the known dependence of $\mu_K$ on velocity.
The value of the static coefficient of friction, $\mu_s$, does not depend upon velocity of course, since it is defined at zero velocity only. Therefore, any changes in the values of $\mu_s$ as the frequency of reciprocation is changed are much more interesting and require further explanation. This explanation is given later in terms of elastic effects present in all real mechanical systems.

In the graphs presented in this section, both $\mu_s$ and $\mu_k$ are plotted. Both values are obtained at each of the frequencies run, 0.01, 0.02, 0.05, 0.1, 0.2, 0.5, and 1.0 Hz. The results are plotted on semi-log plots with the friction coefficients on the linear scale and the frequency (or the maximum velocity) on the log scale. The frequencies correspond to maximum velocities of 0.08, 0.16, 0.4, 0.8, 1.6, 4.0, and 8.0 cm/sec (0.03, 0.6, 0.15, 0.3, 0.6, 1.5, and 3.0 in./sec).

In order to run the tests in such a way that any variation in the friction coefficients due to the number of cycles of reciprocation run was a minimum, two precautions were taken. First, the data on friction coefficients as a function of the number of cycles (given in Section 5.2) was consulted to pick a region in which the friction coefficients changed very little as further cycles of reciprocation were run. These regions were used for the present tests. Second, the frequencies (or maximum velocities) were run in both directions starting at the lowest frequency (smallest maximum velocity), proceeding to the highest
frequency (largest maximum velocity) and then back to the
lowest frequency (smallest maximum velocity). Thus, the
range of frequencies (maximum velocities) was traversed
in both directions and there are two points plotted for
each friction coefficient at each frequency (maximum veloc-
ity). In two cases (Figures 5-22, 5-23, 5-44, and 5-45) the
points fell far enough apart that it was decided to go
through the frequency (maximum velocity) range a third time.
In these two cases there are three points plotted for each
friction coefficient at each frequency (maximum velocity).

5.4.1 Mild Steel on Mild Steel

The frequency variation tests for mild steel on mild
steel were run with three different lubricant conditions.
These were dry, with mineral oil, and with the oleic acid-
mineral oil mixture. The un lubricated tests, Figures 5-20
and 5-21, showed a static coefficient higher than the
kinetic. Both the static and the kinetic coefficients
decreased with increasing frequency (velocity). When
mineral oil was introduced as a lubricant, Figures 5-22 and
5-23, the static coefficient stayed higher than the kinetic.
Both friction coefficients decreased with increasing
frequency (velocity). With the oleic acid-mineral oil
combination being used as lubricant, Figures 5-24 and 5-25,
the static coefficient dropped below the kinetic values.
Figure 5-20. Variation of Static Coefficient of Friction with Frequency of Reciprocation.

Steel on Steel
No Lubrication
Amplitude = + 1.27 cm (0.5 in.)
Figure 5-21. Variation of Kinetic Coefficient of Friction with Velocity.

Steel on Steel
No Lubrication
Amplitude = ± 1.27 cm (0.5 in.)

Velocity, cm/sec (Frequency, Hz)
Steel on Steel
Mineral Oil
Amplitude = ± 1.27 cm (0.5 in.)

Figure 5-22. Variation of Static Coefficient of Friction with Frequency of Reciprocation.
Steel on Steel

Mineral Oil

Amplitude = ± 1.27 cm/(0.5 in.)

Figure 5-25. Variation of Kinetic Coefficient of Friction with Velocity.
Figure 5-24. Variation of Static Coefficient of Friction with Frequency of Reciprocation.

Steel on Steel
Oleic Acid
Amplitude = ± 1.27 cm (0.5 in.)
Kinetic Coefficient of Friction

Steel on Steel
Oleic Acid
Amplitude = ± 1.27 cm (0.5 in.)

Figure 5-25. Variation of Kinetic Coefficient of Friction with Velocity.
Also, both coefficients increased with increasing frequency (velocity).

5.4.2 Hard Steel on Hard Steel

Hard steel on hard steel was tested under four lubrication conditions—dry, with mineral oil, with the oleic acid in mineral oil mixture, and with the bonded film of molybdenum disulfide. The unlubricated conditions, Figures 5-26 and 5-27, showed a higher static than kinetic friction coefficient. There was little change in values with frequency (velocity). The plot of friction coefficients against frequency of oscillation (velocity) is flat. The test run with mineral oil as a lubricant, Figures 5-28 and 5-29, showed that both the static and kinetic coefficients decreased with increasing frequency (velocity). The static coefficient was larger than the kinetic coefficient. With the oleic acid mixture as the lubricant, Figures 5-30 and 5-31, the static coefficient was lower than the kinetic. The static coefficient increased slightly with increasing frequency and the kinetic coefficient stayed constant as the velocity increased. When the bonded film MoS₂ lubricant was introduced, Figures 5-32 and 5-33, the static coefficient was higher than the kinetic. The values of the coefficients were independent of the frequency of reciprocation (velocity).
Figure 5-26. Variation of Static Coefficient of Friction with Frequency of Reciprocation.
Figure 5-27. Variation of Kinetic Coefficient of Friction with Velocity.

Hard Steel on Hard Steel
No Lubrication
Amplitude = ± 1.27 cm (0.5 in.)
Figure 5-28. Variation of Static Coefficient of Friction with Frequency of Reciprocation.
Figure 5-29. Variation of Kinetic Coefficient of Friction with Velocity.

Hard Steel on Hard Steel
Mineral Oil
Amplitude = ± 1.27 cm (0.5 in.)
Figure 5-30. Variation of Static Coefficient of Friction with Frequency of Reciprocation.

Hard Steel on Hard Steel
Oleic Acid
Amplitude = ± 1.27 cm (0.5 in.)
Figure 5-31. Variation of Kinetic Coefficient of Friction with Velocity.
Figure 5-32. Variation of Static Coefficient of Friction with Frequency of Reciprocation.

- **Hard Steel on Hard Steel**
- Molybdenum M-8800
- Amplitude = ± 1.27 cm (0.5 in.)
Hard Steel on Hard Steel
Molykote M-8800
Amplitude = ± 1.27 cm (0.5 in.)

Figure 5-33. Variation of Kinetic Coefficient of Friction with Velocity.
5.4.3 Steel on Nylon

The steel on nylon test, Figures 5-34 and 5-35, showed a static coefficient lower than the kinetic. The coefficients both increased with increasing frequency of reciprocation (velocity).

5.4.4 Steel on Brass

The steel on brass tests were run under three conditions of lubrication: unlubricated, lubricated with mineral oil, and lubricated with the oleic acid in mineral oil mixture. The unlubricated test, Figures 5-36 and 5-37, showed a static coefficient higher than the kinetic. The coefficients were virtually independent of oscillation frequency (velocity). When mineral oil is used as the lubricant, Figures 5-38 and 5-39, the static coefficient stayed at a higher value than the kinetic. The coefficients both decreased with increasing frequency (velocity). With oleic acid as lubricant, Figures 5-40 and 5-41, the static coefficient was still larger than the kinetic. The static coefficient was independent of the frequency and the kinetic coefficient increased with increasing velocity.

5.4.5 Hard Steel on Oilite Bronze

Hard steel on Oilite bronze, Figures 5-42 and 5-43, displayed a higher static than kinetic coefficient. The static coefficient was independent of the frequency. The
Steel on Nylon
No Lubrication
Amplitude = ± 1.27 cm (0.5 in.)

Figure 5-34. Variation of Static Coefficient of Friction with Frequency of Reciprocation.
Kinetic Coefficient of Friction
Steel on Nylon
No Lubrication
Amplitude = ± 1.27 cm (0.5 in.)

Figure 5-35. Variation of Kinetic Coefficient of Friction with Velocity.
Figure 5-36. Variation of Static Coefficient of Friction with Frequency of Reciprocation.

Steel on Brass
No Lubrication
Amplitude = ± 1.27 cm (0.5 in.)
Figure 5-37. Variation of Kinetic Coefficient of Friction with Velocity.

Steel on Brass
No Lubrication
Amplitude = ± 1.27 cm (0.5 in.)
Steel on Brass
Mineral Oil
Amplitude = ± 1.27 cm (0.5 in.)

Figure 5-38. Variation of Static Coefficient of Friction with Frequency of Reciprocation.
Steel on Brass
Mineral Oil
Amplitude = ± 1.2 cm (0.5 in.)

Figure 5-39. Variation of Kinetic Coefficient of Friction with Velocity.
Steel on Brass
Oleic Acid
Amplitude = ± 1.27 cm (0.5 in.)

Figure 5-40. Variation of Static Coefficient of Friction with Frequency of Reciprocation.
Figure 5-41. Variation of Kinetic Coefficient of Friction with Velocity.

Steel on Brass

Oleic Acid

Amplitude = ± 1.27 cm (0.5 in.)

Velocity, cm/sec (Frequency, Hz)
Figure 5-42. Variation of Static Coefficient of Friction with Frequency of Reciprocation.

Hard Steel on Oilite Bronze

SAE 50

Amplitude = ± 1.27 cm (0.5 in.)
Figure 5-43. Variation of Kinetic Coefficient of Friction with Velocity.
kinetic coefficient decreased with increasing velocity.

5.4.6 **Steel on Glass**

The results for the steel on glass test, Figures 5-44 and 5-45, showed a higher static than kinetic coefficient. Both the static and kinetic coefficients of friction decreased with increasing frequency of reciprocation (velocity).

5.4.7 **Hard Steel on Brake Material**

Hard steel on brake material, Figures 5-46 and 5-47, showed a static coefficient lower than the kinetic coefficient. Both the static and kinetic coefficients increased with increasing frequency of reciprocation (velocity).

5.4.8 **Steel on Paper**

The test results for steel on paper, Figures 5-48 and 5-49, showed a lower static than kinetic friction coefficient. Both the static and kinetic coefficients increased with increasing frequency of reciprocation (velocity).

5.5 **Some Observations on the Variation of Friction as Frequency Changed**

Two general observations can be made on the results of the tests run to measure the friction variation as the frequency of the sinusoidal reciprocation was changed. The static coefficient of friction, \( \mu_s \), is not necessarily a constant but can depend upon the frequency at which the
Figure 5-44. Variation of Static Coefficient of Friction with Frequency of Reciprocation.
Steel on Glass

Oleic Acid

Amplitude = ± 1.27 cm (0.5 in.)

Figure 5-45. Variation of Kinetic Coefficient of Friction with Velocity.
Figure 5-46. Variation of Static Coefficient of Friction with Frequency of Reciprocation.

- Hard Steel on Brake Material
- No Lubricant
- Amplitude = \( \pm 1.27 \) cm (0.5 in.)
Figure 5-47. Variation of Kinetic Coefficient of Friction with Frequency of Reciprocation.
Figure 5.48. Variation of Static Coefficient of Friction with Frequency of Reciprocation.

Steel on Paper
No Lubricant
Amplitude = ± 1.27 cm (0.5 in.)
Steel on Paper
No Lubricant
Amplitude = ± 1.27 cm (0.5 in.)

Figure 5-49. Variation of Kinetic Coefficient of Friction with Velocity.
two elements of the friction pair are reciprocated relative to each other. This phenomenon is believed to be explainable in terms of system elasticity; possible explanations for this behavior are given in Section 6.1.2 in terms of the most generally accepted theory of friction.

While each point on the curves obtained for the kinetic coefficient of friction was obtained at a different frequency, the variations in the kinetic coefficient are best explained as purely a velocity effect. [Evidence for this conclusion is given in the next section (5.6).] This is because the kinetic coefficient was defined as the value of the friction coefficient at the highest velocity reached during an individual test and changing the frequency of reciprocation directly changes the maximum velocity reached for each point on the curves. For this reason the kinetic friction coefficient, $\mu_k$, was plotted as a function of velocity.

The unlubricated steel on steel test showed a definite negative slope to the friction vs. frequency (velocity) curves, while both the hard steel on hard steel and the steel on brass tests showed basically flat curves. This may be due to the formation of hard oxide wear debris. The hard steel wears very little and the brass, although it does wear, does not form the hard oxide wear products.

The three unlubricated metal on non-metal pairs all showed positive slope on the friction coefficient vs.
frequency (velocity) curves. The largest positive slope occurred with the hard steel on brake material pair. The steel on paper pair had the next largest positive slope followed by the steel on nylon pair.

The metals lubricated by mineral oil all showed a significant negative slope to their friction coefficient vs. frequency (velocity) curves. Mineral oil lubricated mild steel on mild steel had the largest negative slope. Steel on brass had the next largest followed by the hard steel on hard steel pair.

With the oleic acid in mineral oil mixture used as the lubricant, the metals tested all showed a slight positive slope to the friction coefficient vs. frequency (velocity) curves. This was true even for the steel on brass where the static coefficient was larger than the kinetic.

Use of the oleic acid mixture does not insure a positive slope to the friction vs. frequency curve as can be seen by the steel on glass test. In this test there was a definite negative slope.

When the bonded molybdenum disulfide coating was used on the hardened steel specimens, the coefficients appeared independent of frequency (velocity) having zero slope.

The steel on oilite test results showed a static coefficient independent of frequency and a kinetic coefficient with a slightly negative slope with increasing velocity.
Almost all of the data presented on the plots is fitted very well by a straight line on the semi-log plots. Most of the deviations can be attributed to the normal scatter of points expected in any friction test.

5.6 Use of Both Amplitude Changes and Frequency Changes to Prove Basic Dependence of Kinetic Coefficient on Velocity

We suspect that the variations in the friction force observed when sinusoidal motions are used are basically due to the changing velocity conditions. By changing the frequency of reciprocation while holding the amplitude constant, the maximum value of the sinusoidally-changing velocity could be changed. This was investigated in Sections 5.4 and 5.5. The velocity conditions can also be changed by varying the amplitude of the sinusoidal motion while holding the frequency of reciprocation constant. By comparing these two kinds of results, we may be able to determine whether there is an intrinsic frequency effect or not.

The tests were run at peak-to-peak amplitudes of 0.25, 0.5, 1.25, 2.5, and 5.0 cm (0.1, 0.2, 0.5, 1.0, and 2.0 in.). The frequency of reciprocation chosen was 0.1 Hz because it was the midpoint value for the test of variation of friction coefficient with frequency of reciprocation. These conditions give corresponding maximum velocities of 0.08, 0.16, 0.4, 0.8, and 1.6 cm/sec (0.03, 0.06, 0.15, 0.3, and 0.6 in./sec). Testing started with the largest amplitude
proceeding to the smallest amplitude and back again. This procedure produced two points for each value of amplitude (or velocity) run. This was done to negate the effects of wear on the results of the tests. The data is presented by plotting the results on semi-log plots. For the static coefficient, which is the friction coefficient for velocity equal zero, the static coefficient is plotted on the linear scale and the amplitude on the log scale. For the kinetic coefficient, which depends upon the value of the velocity, the kinetic coefficient is plotted on the linear scale and the corresponding maximum velocity is plotted on a log scale.

Four material-lubricant conditions were investigated in order to be representative. These were steel on nylon unlubricated, steel on steel unlubricated, steel on steel lubricated with mineral oil, and steel on steel lubricated with the oleic acid-mineral oil mixture.

5.6.1 Steel on Nylon

The steel on nylon test, Figures 5-50 and 5-51, shows a static coefficient lower than the kinetic. Both coefficients increase with increasing amplitude of reciprocation (velocity).

5.6.2 Steel on Steel, Unlubricated

The unlubricated steel on steel test, Figures 5-52 and 5-53, shows a static coefficient higher than the kinetic. There is a decrease in both the static and
Steel on Nylon
No Lubricant
Frequency - 0.1 Hz

Figure 5-50. Variation of Static Coefficient of Friction with Amplitude of Reciprocation.
Figure 5-51. Variation of Kinetic Coefficient of Friction with Velocity.

Steel on Nylon
No Lubricant
Frequency = 0.1 Hz
Figure 5-52. Variation of Static Coefficient of Friction with Amplitude of Reciprocation.
Figure 5-55. Variation of Kinetic Coefficient of Friction with Velocity.
Steel on Steel
Mineral Oil
Frequency = 0.1 Hz

Figure 5-54. Variation of Static Coefficient of Friction with Amplitude of Reciprocation.
Figure 5-55. Variation of Kinetic Coefficient of Friction with Velocity.

Steel on Steel
Mineral Oil
Frequency = 0.1 Hz
Steel on Steel
Oleic Acid
Frequency = 0.1 Hz

Figure 5-56. Variation of Static Coefficient of Friction with Amplitude of Reciprocation.
Figure 5-57. Variation of Kinetic Coefficient of Friction with Velocity.
kinetic friction coefficients with increasing amplitude (velocity).

5.6.3 Steel on Steel, Mineral Oil

The mineral oil lubricated test, Figures 5-54 and 5-55, shows a static coefficient larger than the kinetic. Both the static and kinetic coefficients decrease with increasing amplitude (velocity).

5.6.4 Steel on Steel, Oleic Acid

When the oleic acid-mineral oil mixture was added as a lubricant, Figures 5-56 and 5-57, the static coefficient dropped to a value lower than the kinetic. Both coefficients increased with increases in amplitude (velocity).

5.6.5 Observations

Results of the testing to study variations in the coefficient of friction as the amplitude of the sinusoidal motion changed are very similar to the results of the tests to study changes in the friction coefficient as frequency of reciprocation was changed. That is, if increasing the frequency of reciprocation caused the friction coefficients to increase, then increasing the amplitude of the motion also caused the friction coefficients to increase. Both
the static and the kinetic coefficients of friction displayed the same behavior for amplitude changes and frequency changes. This fact supports the assertion that the kinetic coefficient of friction is basically a velocity-dependent phenomenon. At the same velocities the kinetic coefficient is approximately equal. Table 5.1 shows a comparison of kinetic coefficient values at the same velocity but at different amplitudes and frequencies. In other words, the velocity was held constant when amplitude was increased by decreasing frequency. Two values of velocity are tabulated 0.08 cm/sec (0.03 in./sec) and 1.6 cm/sec (0.6 in./sec). As can be seen, the values are almost equal except for steel sliding on nylon at the lower velocity where the percentage difference is large but the absolute value of the difference is small, being only 0.01. The only other discrepancy is with mild steel sliding on mild steel and lubricated with mineral oil. At the lower velocity, the two values tabulated do not agree well, one value being 0.32 and the other 0.48. This unpredictable behavior of steel on steel lubricated with mineral oil shows up throughout this study, for example in the random motion tests of Section 5.8.

It should be noted that the static coefficient of friction varied as the amplitude of the motion changed just as it did when the frequency of the motion changed.
TABLE 5.1

Comparison of Some Kinetic Friction Coefficient Values at the Same Velocities for Different Values of Amplitude and Frequency

<table>
<thead>
<tr>
<th></th>
<th>Velocity</th>
<th>Steel on Nylon</th>
<th>Steel on Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.08 cm/sec (0.03 in./sec)</td>
<td>1.6 cm/sec (0.6 in./sec)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A=2.5 cm (1 in.) f=0.01 Hz</td>
<td>A=0.25 cm (0.1 in.) f=0.1 Hz</td>
<td></td>
</tr>
<tr>
<td>A=2.5 cm (1 in.) f=0.01 Hz</td>
<td>0.025</td>
<td>0.035</td>
<td>0.046</td>
</tr>
<tr>
<td>A=2.5 cm (1 in.) f=0.2 Hz</td>
<td>0.046</td>
<td>0.045</td>
<td></td>
</tr>
<tr>
<td>A=5.0 cm (2 in.) f=0.1 Hz</td>
<td>0.105</td>
<td>0.105</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Steel on Steel

<table>
<thead>
<tr>
<th></th>
<th>No Lubrication</th>
<th>Mineral Oil</th>
<th>Oleic Acid</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.52</td>
<td>0.32</td>
<td>0.105</td>
</tr>
<tr>
<td></td>
<td>0.50</td>
<td>0.48</td>
<td>0.105</td>
</tr>
<tr>
<td></td>
<td>0.44</td>
<td>0.22</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>0.44</td>
<td>0.24</td>
<td>0.115</td>
</tr>
</tbody>
</table>
There must be some underlying phenomenon which causes the static coefficient to change with both amplitude and frequency of motion. Section 6.1.2 explores some possible explanations.

5.7 Triangular Waveform Motion

It is possible to reciprocate with other motions than the sinusoidal motion used up to this point. The sinusoidal motion was chosen because of the smooth variation in velocity. One other possible input motion is a triangular motion which gives a constant velocity in each direction.

To compare the friction traces produced by triangular displacement inputs and sinusoidal inputs, these inputs were applied to the same test specimens under the same conditions. Only the input waveform was changed. All other parameters were held constant.

Specimen: hard steel on hard steel
Lubricant: mineral oil
Amplitude: 2.54 cm (1 in.) p-p
Frequency: 0.1 Hz
Normal Load: 4.45 N (1 lbf)

The results are shown as friction-time plots in Figures 5-58 and 5-59. It should be noted that the friction peaks at the beginning and end of the stroke which are characteristic of the sinusoidal input are not
Figure 5-58. Friction Force vs. Time Trace for a Triangular Input Motion.
Figure 5-59. Friction Force vs. Time Trace for a Sinusoidal Input. Same Specimen as Figure 5-39.
present for the triangular input. The smooth change in velocity at the end of each stroke which is characteristic of the sinusoidal input does not take place with the triangular input. Instead, there is a rapid change in velocity from positive to negative. The velocity cannot change instantaneously but changes so rapidly that any friction force variation in this region cannot be followed by the measurement system.

It should also be pointed out that the friction coefficient is slightly lower for the triangular input than for the sinusoidal input at all velocities. For the triangular input the coefficient is a constant 0.16. For the sinusoidal input the coefficient varies from a minimum of 0.18 to a maximum of 0.23, the minimum occurring at the center of the stroke and the maximum at the end.

The constant velocity for the triangular input is lower than the maximum velocity of the sinusoidal input. In fact,

\[
v_{\text{triangular}} = \frac{\max v_{\text{sinusoidal}}}{\sqrt{2}}
\]

(5.1)

Since the friction/velocity curves previously derived for hard steel on hard steel lubricated with mineral oil showed the kinetic friction coefficient decreasing with increasing velocity, it would be expected that the friction
coefficient obtained with triangular motion would be larger than the minimum friction coefficient obtained with the sinusoidal motion. Referring to Figure 5-29, a friction coefficient of 0.16 would be expected for the triangular motion and a minimum friction coefficient of 0.155 for the sinusoidal motion. The value obtained for the triangular motion agrees with the expected value. The minimum value of the friction coefficient obtained with the sinusoidal motion does not agree well with the expected value. This is not too surprising, however, since it has been seen earlier (Table 5.1, for example) that results are not highly repeatable when mineral oil is used as a lubricant. (In the next section this deviation from a strict velocity dependence is encountered again for mild steel on mild steel lubricated with mineral oil using random motion.)

5.8 Random Waveform Motion

Use of a periodic motion such as sinusoidal motion as the driving motion for friction tests of necessity gives periodic friction-time curves. The effect of certain changes such as amplitude and frequency can then be studied by varying them individually. This approach allows the investigator to look at only one variation at a time. Random driving motion allows some different insight into
the frictional variations. With random motion both the amplitude and the frequency of the motion change continuously and at the same time. This allows observation of the frictional variation under even less restraint than one has when the sinusoidal motion is used.

For the random motion tests four representative specimen/lubricant pairs were used. These were steel on nylon unlubricated, steel on steel unlubricated, steel on steel lubricated with mineral oil, and steel on steel lubricated with the oleic acid-mineral oil mixture.

The random motion used had a gaussian amplitude distribution and a power spectral density as shown in Figure 5-60 with a bandwidth of 0.15 Hz. An example of the displacement-time history is shown in Figure 5-61. Each test was run for 10 passes through zero velocity.

The force vs. velocity plots obtained and shown in Figures 5-62, 5-63, 5-64, and 5-65 show that the friction force is closely velocity dependent except in the case of the mineral oil lubricated steel on steel. In that case the static coefficient varies from 0.35 to 0.70, depending on how fast the motion goes through the zero velocity point. When the zero velocity point is traversed rapidly, the static coefficient is low and when the traversal is slow the static coefficient is high. The kinetic coefficient likewise varies over a wide range at each velocity.
Figure 5-60. Power Spectral Density of Random Motion Input.
Figure 5-61. Typical Displacement-Time Plot for the Random Motion.
Figure 5-62. Friction Coefficient vs. Velocity for Random Motion Input. Steel on Nylon. No Lubricant.
Figure 5-63. Friction Coefficient vs. Velocity for Random Motion Input. Steel on Steel. No Lubricant.
Figure 5-64. Friction Coefficient vs. Velocity for Random Motion Input. Steel on Steel. Mineral Oil.
Figure 5-65. Friction Force vs. Velocity for Random Motion Input. Steel on Steel. Oleic Acid.
The other three tests showed some variation in friction force at each velocity but not nearly so much as with the mineral oil lubricated steel on steel. This is because in these cases the static coefficient vs. frequency and amplitude curves are rather flat.

One would conclude, therefore, that the friction variations noted in these tests are due largely to a dependence of the friction force on the velocity of sliding. There are exceptions, however, as is shown by the steel on steel lubricated with mineral oil results.

5.9 Repeatability

The problem of repeatability has two aspects which must be considered: first, repeatability from cycle to cycle; and second, repeatability from test to test.

5.9.1 Repeatability from Cycle to Cycle

By repeatability from cycle to cycle is meant the variations the friction force undergoes from one cycle to the next in a particular test run. For example, how does the friction force vary from cycle 167 to cycle 168? This tells something about the randomness of the friction trace. If the small variations differ greatly from cycle to cycle, then these variations are truly random. If, on
the other hand, one can detect a certain repeatability from cycle to cycle, then these variations are not truly random but systematic in nature. The fluctuations may be caused by some random conditions spread over the surface of the test specimens, but since the surface is traversed periodically the fluctuations will be periodic with a period equal to the period of traversal.

On a short-term basis, the friction variations do appear to be systematic in most cases. The variations do not appear to be systematic where there is severe frictional behavior as with unlubricated hard steel on hard steel. An example is given using hard steel on hard steel lubricated with mineral oil. The results are shown in Figures 5-66 and 5-67. There is a clear systematic nature to the small friction fluctuations.

The problem of changes in the friction on a long-term basis is dealt with in Section 5.2. Over the long term the friction force changes due to wear and surface changes.

Along with any systematic randomness comes the question of directionality. Suppose a repeatable fluctuation is noted from cycle to cycle as the pin moves to the right relative to the flat. Will this fluctuation also be seen as the pin moves to the left relative to the flat? The answer to this question discloses some more about the nature of these fluctuations.
To study this possibility, it is necessary only to compare the friction trace of left-to-right to the trace of right-to-left. This is done in Figures 5-68 and 5-69. The arrow indicates whether the trace is left to right or right to left.

As can be seen, there is indeed a variation of friction force with position that is independent of the direction of motion. The various fluctuations which at first appear random in fact depend upon the position of the rider on the flat and do not depend upon the direction of motion. The example is hard steel on hard steel lubricated with mineral oil.

To investigate this variation of friction with position further, tests were run at different amplitudes of motion. The time traces obtained during those tests were then superimposed so that comparison could be made more easily. Runs were made with the following conditions:

1. \( A = 2.54 \text{ cm (1 in.)} \), \( f = 0.1 \text{ Hz} \)
2. \( A = 1.27 \text{ cm (0.5 in.)}, f = 0.1 \text{ Hz} \)
3. \( A = 1.27 \text{ cm (0.5 in.)}, f = 0.2 \text{ Hz} \)

Chart speeds were varied so that direct comparisons could be made. All other variables are constant.

Figures 5-70 and 5-71 show the trace of run 1 for \( A = 2.54 \text{ cm (1 in.)} \) with the runs for \( A = 1.27 \text{ cm (0.5 in.)} \) overlaying. Since only the amplitude of motion was changed for run 2, the velocity is reduced to half its
Figure 5-66. Frictional Variations for Hard Steel on Hard Steel. Lubricated with Mineral Oil. $A = 1.27$ cm (0.5 in.), $f = 0.1$ Hz.
Figure 5-67. Frictional Variations for Hard Steel on Hard Steel. Lubricated with Mineral Oil. \( A = 1.27 \text{ cm (0.5 in.)}, f = 0.2 \text{ Hz}. \)
Figure 5-68. Comparison of Friction Force from Right to Left with the Force from Left to Right.
Figure 5-69. Velocity Variations for Comparison of Friction Force for Motion in Both Directions (Cf. Figure 5-68).
value with $A = 2.54$ cm (1 in.). With $A = 1.27$ cm (0.5 in.) the distance traveled corresponds to the middle half of the travel for $A = 2.54$ cm (1 in.). Thus, comparison is made with the middle half of the friction and velocity traces. This comparison is shown in Figure 5-70. It should be noted that the fluctuations in both the friction and velocity traces correspond remarkably well, indicating the position dependence. The magnitude of the fluctuations is larger for $A = 1.27$ cm (1 in.) and the fluctuations are obscured at the ends of the stroke where the friction force goes up.

In Run 3 both the amplitude and the frequency were changed. The amplitude was reduced and the frequency increased so as to keep the maximum velocity the same as in Run 1. Comparison of Runs 1 and 3 is given in Figure 5-71. Again there is a marked correlation in the fluctuations with position except at the ends of the stroke where the increase in friction force obscures the variations. In this case, the fluctuations in friction force are the same size for both runs. This is especially true at the center of the stroke where the velocities are the same. Thus the size of the variations is directly attributable to the magnitude of the velocity with lower velocities giving larger fluctuations.
Figure 5-70. Comparison of Friction Force with Position, Runs 1 and 2.
Figure 5-71. Comparison of Friction Force with Position, Runs 1 and 3.
5.9.2 Repeatability from Test to Test

While repeatability from cycle to cycle is of interest, more important is repeatability from test to test. Suppose a test is run on day A giving certain results. Then it would be hoped that the same results would be achieved with another set of samples on day B. This is indeed a difficult problem as can be seen by the wide range of friction coefficients that are quoted in the literature. There may be significant changes in the friction measured from day to day even on the same machine. The causes of this problem are as many as the causes of changes in friction coefficient. However, the major cause is in the surface condition of the samples. This is the reason so much time was put into their preparation. This problem is discussed in Section 4.2.

A consultation with the literature will show that it is not unusual to obtain results from friction tests which vary by 25% or more from test to test. With that in mind, it is instructive to examine the results of the present tests in order to study the reproducibility of the results. To do that we might look at three things: 1) scatter of data within a test; 2) comparison of the different types of tests run such as frequency variation and amplitude variation; and 3) the same type test run on different samples.
To investigate the scatter within individual tests we must look at the frequency and amplitude tests. These tests were run in both directions so that there is more than one point for each frequency or amplitude. The scatter of the data points at each of these frequencies or amplitudes therefore gives a measure of the reproducibility. In most cases this scatter is very slight with the data points lying side by side or on top of each other. In a few cases, however, there is considerable scatter. In particular with mineral oil as the lubricant, Figures 5-22, 5-23, 5-44, and 5-45, the steel on steel possessed considerable scatter in both the frequency and amplitude tests. The scatter was so great in the frequency test that the range of frequencies was traversed three times instead of two to give more data over which to average. The steel-on-brass lubricated with mineral oil, Figure 5-39, and the steel on glass lubricated with oleic acid in mineral oil, Figures 5-44 and 5-45, also gave a wide degree of scatter.

It is also possible to investigate reproducibility by comparing results obtained in each of the different types of tests run. These were studies of variation of coefficient of friction with number of cycles (N), with frequency (f), and with amplitude (A). In addition, data can be extrapolated from the random motion tests. All four of these tests were run for four different materials/lubricant
combinations: steel on nylon unlubricated, steel on steel unlubricated, steel on steel lubricated with mineral oil, and steel on steel lubricated with the oleic acid mixture. In order to make the comparison on an equal basis, that is, the same amplitude and frequency for the static coefficient of friction and the same velocity for the kinetic coefficient of friction, values of static and kinetic coefficient were tabulated for $f = 0.1 \text{ Hz}$ and $A = 2.54 \text{ cm (1 in.)}$. These values represent the midpoint of the range of values covered in the previous tests and are the values used to run the variation in friction coefficient with number of cycles tests. This means the values obtained for the kinetic coefficient of friction were for a velocity of $0.8 \text{ cm/sec (0.3 in./sec)}$. The values of the static and kinetic coefficients of friction for these conditions are shown in Table 5.2. The three metal-on-metal pairs show very good correlation between the data from the four sources. It was not possible to extrapolate an approximate value from the random test of steel on steel lubricated with mineral oil, however, because of the nature of that plot given in Section 5.8.

The largest variations in friction coefficient from test to test occurred with the steel on nylon tests. Here the static coefficient ranged from 0.0275 to 0.05 and the kinetic coefficient from 0.04 to 0.075. These are large
Table 5.2
Comparison of Friction Coefficients Obtained from Different Tests

<table>
<thead>
<tr>
<th></th>
<th>$\mu_s$</th>
<th>$\mu_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>($v=0.8$ cm/sec)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Steel on Nylon</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f$</td>
<td>.0275</td>
<td>.04</td>
</tr>
<tr>
<td>$N$</td>
<td>.05</td>
<td>.075</td>
</tr>
<tr>
<td>$A$</td>
<td>.035</td>
<td>.045</td>
</tr>
<tr>
<td>random</td>
<td>.0275</td>
<td>.05</td>
</tr>
<tr>
<td><strong>Steel on Steel--Dry</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f$</td>
<td>.56</td>
<td>.45</td>
</tr>
<tr>
<td>$N$</td>
<td>.54</td>
<td>.42</td>
</tr>
<tr>
<td>$A$</td>
<td>.56</td>
<td>.45</td>
</tr>
<tr>
<td>random</td>
<td>.58</td>
<td>.40</td>
</tr>
<tr>
<td><strong>Steel on Steel--Mineral Oil</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f$</td>
<td>.52</td>
<td>.24</td>
</tr>
<tr>
<td>$N$</td>
<td>.56</td>
<td>.20</td>
</tr>
<tr>
<td>$A$</td>
<td>.50</td>
<td>.24</td>
</tr>
<tr>
<td>random</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td><strong>Steel on Steel--Oleic Acid</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f$</td>
<td>.10</td>
<td>.115</td>
</tr>
<tr>
<td>$N$</td>
<td>.10</td>
<td>.115</td>
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<tr>
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<td>.115</td>
</tr>
<tr>
<td>random</td>
<td>.09</td>
<td>.115</td>
</tr>
</tbody>
</table>

*Data could not be extrapolated.*
percentage variations but in terms of magnitude they are small because of the small value of the coefficients.

The third place to look for evidence concerning reproducibility of results comes from looking at two tests of the same type run with two different friction pairs. This was done with several samples comparing both coefficient of friction vs. number of cycles tests and the coefficient of friction vs. frequency of reciprocation tests.

Two examples of repeatability from test to test are given in Figure 5-72 and 5-73 for coefficient of friction vs. number of cycles. Figure 5-72 compares two tests run using steel on brass lubricated with mineral oil. As can be seen by the figure, both the static and the kinetic coefficients of friction started off at lower values in the second test than in the first. After 10 cycles, however, both tests agree rather well. The two kinetic coefficients agree more closely than do the two static coefficients. This is representative in that the kinetic coefficients tend to agree more closely than do the static in most cases observed. The static coefficient is usually more widely scattered.

Figure 5-73 shows the results of two steel-on-steel lubricated with mineral oil tests for the variation of friction coefficient with number of cycles. This particular combination is presented here because of the large jump in
Figure 5-72. Variation of Friction with Number of Cycles of Reciprocation.
Figure 5-73. Variation of Friction with Number of Cycles of Reciprocation.
the kinetic coefficient of friction after about 300 cycles observed in the original tests. As can be seen from the figure, both tests are extremely close in their results. Both the static and kinetic coefficients lie together over most of the 500 cycles. The large jump in the value of kinetic coefficient which was observed in the first test is not observed in the second test. Perhaps this jump was due to some peculiarity of the particular specimen (such as an inclusion in the metal). If, on the other hand, it is typical, it may happen over a range of cycles and thus might not be found unless a larger number of cycles were run.

Repeatability from test to test for the variation of coefficient of friction with frequency of reciprocation was investigated using four material-lubricant configurations: steel on brass lubricated with mineral oil, steel on steel unlubricated, steel on steel lubricated with mineral oil, and steel on steel lubricated with the oleic acid in mineral oil mixture. Figure 5-74 shows the results of the steel on brass tests. It can be seen that the values of the coefficients agree well from test to test for this case.

The results for the steel on steel tests with the three differing lubrication conditions are given in Figures 5-75, 5-76, and 5-77. The unlubricated condition compared in Figure 5-75 shows good agreement from test to test with no more than 10% variation in values between the two tests. With mineral oil as lubricant, Figure 5-76, the kinetic
Steel on Brass

Mineral Oil

--- Present Test

----- Previous Test, Figures 5-38 and 5-39

Amplitude = ± 1.27 cm (0.5 in.)

Figure 5-74. Variation of Friction with Frequency of Reciprocation (Velocity).
Steel on Steel
No Lubricant

--- Present Test

--- Previous Test, Figures 5-20 and 5-21

Amplitude = ± 1.27 cm (0.5 in.)

 Figure 5-75. Variation of Friction with Frequency of Reciprocation (Velocity)
Steel on Steel

Mineral Oil

Present Test

Previous Test, Figures 5-22 and 5-25

Amplitude = ± 1.27 cm (0.5 in.)

Variation of Friction with Frequency of Reciprocation

Figure 5-76. Variation of Friction with Frequency of Reciprocation (Velocity).
Steel on Steel
Oleic Acid

- Present Test
- Previous Test, Figures 5-24 and 5-25

Amplitude = ± 1.27 cm (0.5 in.)

Figure 5-77. Variation of Friction with Frequency of Reciprocation (Velocity).
coefficients differ in value from test to test by less than 10%. The static coefficients differ more widely as might be expected from the results of other tests with this material-lubricant combination. The results with oleic acid as the lubricant, Figure 5-77, also show the good agreement demonstrated in the other repeatability tests.

Results are generally repeatable from test to test within 20%. The same slopes of the friction vs. frequency curves are noted. The only exceptions shown here were the size of the jump in kinetic coefficient for the mineral oil lubricated steel on steel case and the variation in static coefficient with frequency for the mineral oil lubricated steel on steel. Even these, however, were not large variations.

5.10 Normal Force Variation

An attempt was made to study changes in the coefficient of friction vs. frequency of reciprocation curves using mild steel on mild steel under three conditions of lubrication: no lubricant, mineral oil, and the oleic acid in mineral oil mixture. The normal force was increased from the 4.45 N (1 lbf) used on the previous tests to 44.5 N (10 lbf). The pins had spherical ends of 4 in. diameter as in the previous tests and all other conditions were the same. When the unlubricated steel on steel test was run, the pin and flat quickly wore and the surfaces became
galled. Friction force was so erratic that no measurements were possible. The coefficient of friction would be a certain value on one cycle and the next cycle it would increase by two to three hundred percent. Thus no data was obtainable for dry lubrication conditions and 44.5 N (10 lbf) load.

When mineral oil was applied to the surfaces as lubricant, measurements once again became possible. Figure 5-78 shows the results of the test with 44.5 N (10 lbf) normal load as well as the results of the previous test with the 4.45 N (1 lbf) normal load. As can be seen, the kinetic coefficients of friction agree quite closely, being less than 10% different. The static coefficients differ greatly, however. With the 44.5 N (10 lbf) normal force the static coefficient is much lower than with the 4.45 N (1 lbf) normal force. This could be due to the presence of more wear debris with the higher load.

Figure 5-79 shows the results of the test with oleic acid as the lubricant. In this case with the 44.5 N (10 lbf) load, the static coefficient becomes larger than the kinetic coefficient and the slope of both the static and kinetic coefficient vs. frequency of reciprocation curves becomes negative. Both these characteristics are opposite the results obtained with 4.45 N (1 lbf) normal load, where the static coefficient was lower than the kinetic and the slopes were both positive. This difference
Figure 5-78. Coefficient of Friction as a Function of Frequency of Reciprocation (Velocity) for a Normal Load of 44.5 N (10 lbf).
Figure 5-79. Coefficient of Friction as a Function of Frequency of Reciprocation (Velocity) for a Normal Load of 44.5N (10 lbf).
in behavior is possibly caused by a breakdown of the lubricating film due to the higher contact pressure.
DISCUSSION OF RESULTS

6.1 Explanations for Observed Behavior

It is difficult to explain in any general way why the friction increases or decreases with increasing velocity. Friction is a surface phenomenon and it is difficult to observe the surface while sliding is taking place. About the best that can be done is to hypothesize on the basis of what is known about the surfaces involved and the currently accepted theory of friction.

In order to do this, it is first necessary to understand the surface interactions involved. A surface, no matter how smooth it may appear, is made up of peaks and valleys. These peaks are known as the surface asperities. If we place two surfaces into contact, therefore, they do not touch over the entire area but only over some small fraction of this area of contact. This is illustrated in Figure 6-1. If a simple lubricant such as a straight mineral oil is added to the surface, this lubricant will fill up the voids between the two surfaces caused by the
Figure 6-1. Two Friction Surfaces in Contact without Lubricant
asperities; this is illustrated in Figure 6-2. Now, if the lubricant used is a polar compound, then the molecules of the lubricant attach themselves to the surface and form a protective film between the two friction surfaces. Figure 6-3 illustrates this situation.

The adhesion theory of friction says that when the two surfaces come together at the touching asperities, junctions form. These junctions must then be broken before the surfaces can slide relative to each other. The force necessary to break these junctions is then the friction force. The friction force increases with increasing load, since an increase in load will cause more asperities to come into contact and it will be necessary to break more junctions to cause sliding.

6.1.1 Reasons for Changes in $\mu_k$ with Velocity

One reason for changes in $\mu_k$ with velocity is related to the rate of shear of the individual junctions. The junctions may break at different values of force depending on how rapidly they are loaded. This may be understood better by comparison with the simple laboratory tensile test. In that test, it is known that the test specimen will break at different values depending on how rapidly the load is applied. In some cases, for example, a ductile
Figure 6-2. Two Friction Surfaces in Contact in the Presence of a Simple Liquid Lubricant. The Lubricant Fills the Void between the Surfaces.
Figure 6-3. Two Friction Surfaces in Contact in the Presence of a Polar Lubricant. The Molecules are Attached to the Surfaces.
fracture will be obtained if the load is applied slowly and a brittle fracture will be obtained if the load is applied rapidly.

When velocity is low the junctions are sheared slowly, and when velocity is high the junctions are sheared more rapidly. Because of this rate of shear, materials which creep show increasing $\mu$ vs. velocity curves, as did the results of the steel on nylon tests. If the junction fracture is brittle, then a decreasing $\mu$ vs. velocity curve would be expected. This could be part of the cause of the decreasing results for unlubricated steel on steel.

It should be pointed out that the junctions may possess properties which are different from the bulk properties of the material because of their small size. Consider, for example, that music wire is hard but in small sizes it is ductile. Thus, when we think of junctions we cannot think of them the same way we consider a large volume of material.

A second reason for changes in $\mu_k$ with velocity is related to the formation of the adhesion junctions. Formation of full strength junctions takes time. At higher velocities the asperities are in contact for shorter lengths of time. Therefore, there is less time to establish junctions or for these junctions to develop full strength. The two surfaces also have more time to conform to each other at lower velocity. This effect would cause
decreasing friction force as velocity increased.

The third reason that can be given for the variation of $\mu_k$ with velocity concerns the proposition that there is some sort of transition from boundary lubrication to fluid film lubrication. This is not to say that a full fluid film develops. If this were so, then the friction coefficient would fall to a very low value. Rather, because the fluid fills the voids between the two surfaces as is shown in Figure 6-2, it is possible that small isolated hydrodynamic wedges may form, supporting part of the normal load. More of these wedges would be formed the higher the velocity, and the friction force would, therefore, diminish with increasing velocity.

A fourth possible reason for the change in $\mu_k$ with velocity concerns the boundary film. It is possible that the boundary film may be disturbed by the sliding process; then at higher velocities the disturbances would be even more pronounced. At higher velocities the film would have less time to repair itself or reform on the surface. Thus, in these cases one would expect the friction force to increase with increasing velocity. This may be the main reason for the increasing friction coefficient when oleic acid is used as a lubricant.

It is probably that all these effects and perhaps
others come into play to a certain extent for every material pair/lubricant configuration. One or more effects predominate, however, because of the make up of the materials in contact and the nature of the lubricant present. Not all materials creep the way the nylon or the phenolic and asbestos brake material would be expected to creep. The mineral oil does not form a film such as is formed by the oleic acid, and thus there would be no film to be disturbed by higher velocity motion. Junctions can be expected to form more readily when mineral oil is used as the lubricant than when oleic acid is included in the lubricant since the oleic acid forms a protective layer to keep the two surfaces apart. The effect which predominates will determine the form taken by the kinetic friction coefficient vs. velocity curve.

6.1.2 Reasons for Changes in $\mu_s$

It is believed that most of the changes in $\mu_s$ observed in these tests can be explained in terms of phenomena which arise because of the elasticity of the mechanical system. This is not to say that there is no real change in the static coefficient of friction and that the changes in $\mu_s$ observed are merely because of some problem in measurement. Any real mechanical system will have a certain amount of elasticity. No real system can be made infinitely stiff.
because all real materials deform under load. In the friction-measuring device some elasticity was intentionally built in since the force measurements were actually made by transducing the force to a deflection and then transducing the deflection to a voltage using the strain gages. Because of this elasticity, the two friction specimens stay in contact for a small length of time during each reversal of direction when the velocity is zero. All mechanical systems will show a similar phenomenon because of their elasticity. In this case, the time of contact is the same as the rise time discussed in Section 2.5 and is given by:

\[
\text{Time of Stick} = \frac{2}{\pi f} \sqrt{\frac{\mu N}{AK}} \quad (6.1)
\]

To get a feel for the magnitudes of the times involved, consider some typical values:

- \( N = 4.45 \text{ N (1 lbf)} \)
- \( \mu = 0.2 \)
- \( A = 2.54 \text{ cm (1 in.)} \)
- \( K = 14,000 \text{ N/cm (7700 lbf/in.)} \)

and \( f = 0.01 \text{ Hz and 1 Hz}. \)

Then,

\[
\text{Time of Stick} = 0.325 \text{ sec for } f = 0.01 \text{ Hz} = 0.00325 \text{ sec for } f = 1.0 \text{ Hz}.
\]
One possible reason for the changes in the values of $\mu_s$, then, relates to the time of stick. These changes can come from two sources. First, the time of stick influences the formation of junctions. This is the same effect which could influence $\mu_k$. The junctions take time to become full strength. Thus, as the time of stick changes, the strength of the junctions change. Secondly, being in contact for a longer period of time could give the asperities time to squeeze out any lubricant or to penetrate a film of contaminant. Thus, one would expect higher values of $\mu_s$ for longer times of stick. As is seen from Equation (6.1), time of stick decreases both as frequency and amplitude increase. Thus, changes in both frequency and amplitude can cause changes in $\mu_s$ if the materials/lubricant combination is indeed sensitive to the time of stick.

A second possible source of change in $\mu_s$ stems from the rate of application of the load. When the two friction specimens are in contact at zero relative velocity, they are being loaded in order to cause slip. The rate of application could cause changes in $\mu_s$. The rate of application of the force at slip is given by,

$$\frac{dF}{dt} = K \frac{dD}{dt} \bigg|_{t=\Delta}$$  \hspace{1cm} (6.2)
where \( \Delta \) is the time at which slip occurs
and
\[ D = \frac{\Delta}{2} \cos 2\pi ft. \]

So,
\[ \frac{dD}{dt} = -\pi f A \sin 2\pi ft \quad (6.3) \]

\[ \frac{dF}{dt} = -K\pi f A \sin 2\pi ft \quad (6.4) \]

and
\[ \Delta = \frac{1}{\pi f} \sin^{-1} \sqrt{\frac{\mu N}{KA}} \quad (6.5) \]

Equation (6.5) is the same as Equation (3.11) and is actually one-half the time of stick.

Then,
\[ \frac{dF}{dt} = -K\pi f A \sin [2\sin^{-1} \sqrt{\frac{\mu N}{KA}}] \quad (6.6) \]

Now, since \( \Delta K >> \mu N \),
\[ \frac{dF}{dt} = -2\pi f \sqrt{\mu NAK} \quad (6.7) \]

If the same typical values substituted in Equation (6.1) are used, then:
\[ \frac{dF}{dt} = 1.2 \text{ N/sec for } f = 0.01 \text{ Hz} \]
\[ = 120 \text{ N/sec for } f = 1.0 \text{ Hz} \]
The rate of application of the load could affect the static coefficient in the same ways as were discussed in regard to the kinetic coefficient.

A third possible source of change in $\mu_s$ has to do with creep. Creep could cause junction size to increase. These larger junctions would then present more area which would have to be sheared and thus an increase in $\mu_s$ would occur.

A fourth possible mechanism which could explain the increases in $\mu_s$ is a loading phenomenon. As the tangential load is applied, the junctions are bent over and distorted. Thus, the junctions would conform more closely and present more area which would have to be sheared. This would cause an increase in the static coefficient.

Three of the four possible explanations presented, the first, third, and fourth, would be reasons for a decrease in the static coefficient as the frequency or amplitude increased. The second possible explanation, rate of application of load, could give either increasing or decreasing values of $\mu_s$ as the frequency and amplitude changed, depending on whether the materials in contact showed the tendency to creep. If there were creep at the friction interface then the static coefficient would tend to increase for increasing frequency or amplitude. This type behavior was noted for the nylon and the phenolic
and asbestos brake material, two materials which could be expected to creep. Materials which do not creep would show a decreasing behavior. It might be pointed out that when oleic acid was used as the lubricant, the static coefficient of friction increased as frequency and amplitude increased. It may be that the oleic acid film allows creep at the friction interface.

It should also be pointed out that the static friction coefficient was not sensitive to frequency and amplitude in all cases (c.f., Figures 5-26, 5-32, 5-36, 5-40, and 5-42).

6.2 Modeling of Friction Behavior in Mechanical System

Analysis

The test data acquired in Chapter V was all presented in graphical form by plotting the changes in the friction coefficient against either frequency, amplitude, or velocity on semi-log graphs. These plots were all fitted closely by a straight line, and thus it is possible to present empirical formulas to represent this data. These formulas will take the forms:

\[ \mu_s = m \log f + a \]  \hspace{1cm} (6.8)

\[ \mu_s = n \log A + b \]  \hspace{1cm} (6.9)
\[ \mu_k = p \log v + c \]  

(6.10)

where 
- \( f \) = frequency of reciprocation, Hz
- \( \Lambda \) = peak-to-peak amplitude, cm
- \( v \) = relative velocity, cm/sec
- \( a, b, c, m, n, p \) = constants.

To fit the data of Chapter V to these formulas, the notation of Figures 6-4, 6-5, and 6-6 was used. Using this notation Tables 6.1 and 6.2 were generated. Table 6.1 was generated from the results of the frequency tests and Table 6.2 from the results of the amplitude tests. These formulas present the data in a manner which is useful if a system is undergoing sinusoidal reciprocating motion in the region of interest.

The model of Equations (6.8), (6.9), and (6.10) suffers in a very important way, however, when the motion is not of the prescribed type. It does not allow for the smooth transition from the static to the kinetic coefficient of friction. As a matter of fact, a quick look at Equation (6.10) shows that as the velocity grows very small the kinetic coefficient is predicted as going to infinity. This is clearly a problem, and a more useful model must be developed which avoids this problem in those cases where the motion is not of the sinusoidal type.
Figure 6-4. Notation Used for Static Coefficient of Friction as a Function of Frequency of Reciprocation Data.

\[ \mu_s = m \log f + a \]

slope = \( m \)
Figure 6-5. Notation Used for Static Coefficient of Friction as a Function of Peak-to-Peak Amplitude of Reciprocation Data.

\[ \mu_s = n \log A + b \]

\[ \text{slope} = n \]
Figure 6-6. Notation Used for the Kinetic Coefficient of Friction as a Function of Velocity Data.
### Table 6.1

Empirical Constants from Frequency Tests

\[ \mu_S = m \log f + a, \mu_K = p \log v + c \]

<table>
<thead>
<tr>
<th></th>
<th>( \mu_S )</th>
<th>( \mu_K )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( a )</td>
<td>( m )</td>
</tr>
<tr>
<td>Steel on Steel</td>
<td></td>
<td></td>
</tr>
<tr>
<td>unlubricated</td>
<td>0.46</td>
<td>-0.10</td>
</tr>
<tr>
<td>mineral oil</td>
<td>0.38</td>
<td>-0.14</td>
</tr>
<tr>
<td>oleic acid</td>
<td>0.115</td>
<td>+0.012</td>
</tr>
<tr>
<td>Hard Steel on Hard Steel</td>
<td></td>
<td></td>
</tr>
<tr>
<td>unlubricated</td>
<td>0.90</td>
<td>+0.01</td>
</tr>
<tr>
<td>mineral oil</td>
<td>0.19</td>
<td>-0.04</td>
</tr>
<tr>
<td>oleic acid</td>
<td>0.10</td>
<td>+0.008</td>
</tr>
<tr>
<td>MoS₂</td>
<td>0.33</td>
<td>0</td>
</tr>
<tr>
<td>Steel on Nylon</td>
<td>0.04</td>
<td>+0.015</td>
</tr>
<tr>
<td>Steel on Brass</td>
<td></td>
<td></td>
</tr>
<tr>
<td>unlubricated</td>
<td>0.45</td>
<td>-0.015</td>
</tr>
<tr>
<td>mineral oil</td>
<td>0.43</td>
<td>-0.08</td>
</tr>
<tr>
<td>oleic acid</td>
<td>0.19</td>
<td>0</td>
</tr>
<tr>
<td>Steel on Oilite</td>
<td>0.18</td>
<td>0</td>
</tr>
<tr>
<td>Steel on Glass</td>
<td></td>
<td></td>
</tr>
<tr>
<td>oleic acid</td>
<td>0.16</td>
<td>-0.02</td>
</tr>
<tr>
<td>Hard Steel on Brake Material</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.17</td>
<td>+0.04</td>
</tr>
<tr>
<td>Steel on Paper</td>
<td>0.22</td>
<td>+0.02</td>
</tr>
</tbody>
</table>
TABLE 6.2

Empirical Constants from Amplitude Tests

\[ \mu_s = n \log A + b, \ \mu_k = p \log v + c \]

<table>
<thead>
<tr>
<th></th>
<th>( \mu_s )</th>
<th></th>
<th>( \mu_k )</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( b )</td>
<td>( n )</td>
<td>( c )</td>
<td>( p )</td>
</tr>
<tr>
<td>Steel on Nylon</td>
<td>0.034</td>
<td>+0.002</td>
<td>0.043</td>
<td>+0.008</td>
</tr>
<tr>
<td>Steel on Steel</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>dry</td>
<td>0.59</td>
<td>-0.054</td>
<td>0.42</td>
<td>-0.045</td>
</tr>
<tr>
<td>mineral oil</td>
<td>0.63</td>
<td>-0.18</td>
<td>0.28</td>
<td>-0.18</td>
</tr>
<tr>
<td>oleic acid</td>
<td>0.0935</td>
<td>+0.006</td>
<td>0.113</td>
<td>+0.008</td>
</tr>
</tbody>
</table>
Figures 5-2 and 5-3 show the form of the friction force vs. velocity curves obtained during the testing. An inspection of these curves plus those introduced in Section 5.8 on random motion suggests that the friction coefficient might be represented as a constant plus an exponentially varying component dependent upon the velocity.

Figure 6-7 illustrates the composition of this model. In equation form,

\[ \mu(v) = \mu_m + (\mu_s - \mu_m) e^{-v/v_0} \quad (6.11) \]

or

\[ \mu(v) = \mu_m (1 - e^{-v/v_0}) + \mu_s e^{-v/v_0} \quad (6.12) \]

where

- \( v \) relative velocity, cm/sec
- \( \mu(v) \) the instantaneous value of the friction coefficient
- \( \mu_s \) static coefficient, constant or variable
- \( \mu_m \) the asymptotic value of the friction coefficient at high velocity
- \( v_0 \) velocity constant, cm/sec

In those instances where the static coefficient of friction varies only slightly, it may be assumed constant. For example, steel on nylon unlubricated, steel on steel unlubricated, and steel on steel lubricated with oleic acid have behavior for which the static coefficient of
Figure 6-7. Model of Friction Coefficient as a Function of Velocity.
friction can be assumed constant. The behavior under random motion, Figures 5-62, 5-63, and 5-65, points this out. In general, those materials/lubricant combinations for which the static coefficient vs. frequency and amplitude curves were rather flat can be assumed to have constant static coefficient for purposes of Equations (6.11) and (6.12). Steel on steel lubricated with mineral oil is an example of a materials/lubricant combination for which the static coefficient of friction does not stay relatively constant, as can be seen by Figure 5-64.

Equation (6.11) gives the friction coefficient for positive velocities. The actual sign of the friction force will depend upon the direction of motion in a specific system. Also, when $v = 0$ this model gives $\mu(0) = \mu_s$. The actual value of $\mu(0)$ will be determined from the conditions of the particular problem under consideration. An example is given in the next section which illustrates this.

For the friction model to follow the exponential type of relationship expressed by Equation (6.11), the values given in Table 6.3 must be met. This table gives information which can be used to determine the velocity constant, $v_o$, if a plot of friction coefficient vs. velocity is available. The velocity constant, $v_o$, is the value of the velocity at the point where the varying component of the friction
Table 6.3

Relationships for Exponential Behavior

<table>
<thead>
<tr>
<th>$\frac{v}{v_0}$</th>
<th>$\frac{\mu - \mu_m}{\mu_s - \mu_m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>.368</td>
</tr>
<tr>
<td>2</td>
<td>.135</td>
</tr>
<tr>
<td>3</td>
<td>.050</td>
</tr>
<tr>
<td>4</td>
<td>.018</td>
</tr>
<tr>
<td>$\infty$</td>
<td>0</td>
</tr>
</tbody>
</table>
coefficient has dropped to 36.8% of the total varying component. For example, if the static coefficient is 0.60 and the coefficient of friction at a velocity high enough that its variation with velocity is slight is 0.40, then the varying component of the friction coefficient is equal to 0.20. The velocity constant will then be the velocity at which the friction coefficient is equal to 0.40, the constant component, plus 36.8% of 0.20. In this example that would be 0.47 \[0.40 + 0.368 (0.20)\].

This method was used to determine the velocity constant from the four plots of friction coefficient vs. velocity obtained using random motion, Figures 5-62 through 5-65. To see how well the model fits these results, the four plots are reproduced in Figures 6-8 and 6-11 with a plot of the values of the friction coefficient as determined by the present model superimposed. These plots show how well the model fits the results of the tests. For the steel on nylon, the steel on steel unlubricated, and the steel on steel lubricated with oleic acid, the correlation is very good even for the static coefficient of friction. This is because the static coefficient varies only slightly as the frequency is increased. For the steel on steel lubricated with mineral oil the agreement between model and experiment is not quite so good. In this case, because the static coefficient does vary quite a bit, the
model and experiment do not agree well at lower velocities. As the velocity increases to the order of \( v_0 \), this agreement becomes much better, and even in this case the model may be used to approximate the behavior of this materials/lubricant combination with some combination that useful results will be obtained.

The values of the velocity constants, \( v_0 \), obtained by this fitting process are given in Table 6.4. In this table the first four values were obtained by fitting the model to the random motion data. Random motion tests were not run for the other materials/lubricant combinations, and the values of \( v_0 \) given for these combinations were obtained by fitting the model to the results of the tests run with varying frequency. One value, that for steel on brass lubricated with oleic acid, is not given because the behavior of this combination does not fit the behavior described by the model (cf., Figure 5-3). Since the effects of system elasticity (and, for that matter, many other parameters) on numerical values of friction coefficients are presently not clear, one should not consider the models proposed in this section as being universally applicable.

6.3 Examples of Application of the Model

In this section two simple examples of application of the proposed model to system analysis are given. A digital simulation technique using the IBM System/360 Continuous System Modeling Program (CSMP) is used to illustrate the
Figure 6-8. Result of Fitting the Friction Model to the Random Motion Data for Steel on Nylon with no Lubricant.
Figure 6-9. Result of Fitting the Friction Model to the Random Motion Data for Steel on Steel with no Lubricant.
Figure 6-10. Result of Fitting the Friction Model to the Random Motion Data for Steel on Steel with Mineral Oil.
Figure 6-11. Result of Fitting the Friction Model to the Random Motion Data for Steel on Steel with Oleic Acid.
### Table 6.4

**Velocity Constants**

<table>
<thead>
<tr>
<th>Materials/Lubricant Combinations</th>
<th>( v_0 ) (cm/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel on Nylon</td>
<td>0.10</td>
</tr>
<tr>
<td>Steel on Steel</td>
<td></td>
</tr>
<tr>
<td>no lubricant</td>
<td>0.18</td>
</tr>
<tr>
<td>mineral oil</td>
<td>0.20</td>
</tr>
<tr>
<td>oleic acid</td>
<td>0.04</td>
</tr>
<tr>
<td>Hard Steel on Hard Steel</td>
<td></td>
</tr>
<tr>
<td>no lubricant</td>
<td>0.01</td>
</tr>
<tr>
<td>mineral oil</td>
<td>0.29</td>
</tr>
<tr>
<td>oleic acid</td>
<td>0.09</td>
</tr>
<tr>
<td>MoS₂</td>
<td>0.01</td>
</tr>
<tr>
<td>Steel on Brass</td>
<td></td>
</tr>
<tr>
<td>no lubricant</td>
<td>0.01</td>
</tr>
<tr>
<td>mineral oil</td>
<td>0.09</td>
</tr>
<tr>
<td>oleic acid</td>
<td>-</td>
</tr>
<tr>
<td>Steel on Oilite Bronze</td>
<td>0.14</td>
</tr>
<tr>
<td>Steel on Glass - oleic acid</td>
<td>0.18</td>
</tr>
<tr>
<td>Hard Steel on Brake Material</td>
<td>0.85</td>
</tr>
<tr>
<td>Steel on Paper</td>
<td>1.20</td>
</tr>
</tbody>
</table>

(1) Results obtained from random motion tests.

(2) Results obtained from frequency tests.

(3) Behavior of steel on brass did not fit model.
model's application. This is IBM program number 360A-CX-16X [54], [55]. It is an application-oriented language designed to simulate continuous systems.

6.3.1 Example 1

The system for the first example is shown in Figure 6-11. This is a spring-mass system without viscous friction, but with dry friction, consisting of a mass \( m \) connected to ground by a spring of rate \( K \). The coefficient of friction is a function of velocity as given by Equation (6.11), and the values of \( \mu_m, \mu_s, \) and \( v_0 \) are those obtained for mild steel on mild steel run dry. The normal force between the mass \( m \) and the ground is \( N \).

The equation of motion for this system is,

\[
m\ddot{x} + N\mu(\dot{x}) + Kx = 0
\]  

(6.13)

where

\[
\mu(\dot{x}) = \mu_m + (\mu_s - \mu_m) e^{-\dot{x}/v_0}
\]  

(6.14)

This system is solved using the CSMP-program of Figure 6-13. In this program,

\[
\begin{align*}
XDD &= \ddot{x} \\
XD &= \dot{x} \\
X &= x \\
MU &= \mu(\dot{x})
\end{align*}
\]
Normal Force = N

Figure 6-12. System for Example 1.
TITLE EXAMPLE 1 USE OF FRICTION MODEL
ADU=-(N/MJvAU-(X/M)xA
XU=INITIALCXI.XAU)
X=INITIALCXX.X)
ILCN
ICAY=0.0,ICX=0.1
PARAM
(A=1.0,U=2.0,NA=500.0
V=ABS(XAU)
MUABS=MUK+(MUS-MUK)*EXP(-V/10)
A=INSM(XAU,-1.0,1.0)
MU=MUABS*
PARAM
MUS=0.5,4,421,4O=0.07
TLLER DELT=0.001,DT1TLET=5.055,T^=4.0
PARAM X,NA
END
STOP

Figure 6-13. CSMP Program for Example 1 with Velocity Dependent Friction Coefficient.
The system was solved for initial conditions $\dot{x}(0) = 0$ and $x(0) = 0.1$. System response is plotted in Figure 6-14. Also plotted in Figure 6-14 is the response of the system with velocity independent coefficient of friction. In this case, $\mu = \mu_m = 0.42$ with all the other system parameters the same as for the velocity-dependent case. The CSMP program for the velocity-independent friction coefficient is given in Figure 6-15.

6.3.2 Example 2

In Example 1, the coefficient of friction at zero velocity was either plus or minus $\mu_s$. This was because there is an implicit assumption of rigidity of the ground and the mass $m$. In Example 2, the surface against which the mass $m$ develops dry friction is not rigid but imparts motion to the spring mass system by reciprocating back and forth. Thus, the friction coefficient at zero velocity can take on any value between 0 and $\mu_s$ necessary to balance the spring force. This system is illustrated in Figure 6-16.
Figure 6-14. System Response for Example 1.
Figure 6-15. CSMP Program for Example 1 with Velocity Independent Friction Coefficient.
Normal Force = N

Figure 6-16. System for Example 2.
This system is the same as that shown in Figure 6-12 except for the reciprocating input. The equation of motion for this system is,

\[ m\ddot{x} + N\mu (\dot{x} - \dot{y}) + kx = 0 \]  \hspace{1cm} (6.15)

where \( \mu(\dot{x} - \dot{y}) = \mu_m + (\mu_s - \mu_m) e^{-(\dot{x} - \dot{y})/\nu_0} \)  \hspace{1cm} (6.16)

The forcing function is,

\[ \dot{y} = 0.1 \sin (wt) \]  \hspace{1cm} (6.17)

The CSMP program for this example is given in Figure 6-17. In this program symbols and values are the same as for Example 1 except that:

\[ m = 40.0 \]

and \( \Omega = \omega = 6.0 \)

System response is plotted in Figure 6-18. Also plotted in Figure 6-18 is the system response for a velocity independent friction coefficient. The CSMP program for the velocity independent case is given in Figure 6-19.
Figure 6-17. CSMP Program for Example 2 with Velocity Dependent Friction Coefficient.
Displacement

Figure 6-18. System Response for Example 2.
Figure 6-19. CSMP Program for Example 2 with Velocity Independent Friction Coefficient.
6.4 **Comparison of Results with Those of Other Researchers**

It is of interest to compare the results of the present study with some of the results of earlier researchers. In the pursuit of this goal, two aspects were examined. First, how did the values obtained in these tests, which were run under reciprocating sliding conditions, compare with the results of other researchers who conducted their tests under unidirectional steady-state conditions? Secondly, how does the model of friction coefficient with changing velocity proposed here compare with models previously proposed?

In answering the first question, the present values of the friction coefficients were compared with those of Forrester [24], Bristow [13], and Barwell and Milne [3]. These three studies were discussed in Section 2.2. It should be pointed out that these three studies did not investigate friction at zero velocity. In other words, the values of the static coefficient were not obtained. The minimum velocities investigated were 0.01 cm/sec (0.004 in./sec) for Forrester, 0.01 cm/sec (0.004 in./sec) for Bristow, and 0.0015 cm/sec (0.0006 in./sec) for Barwell and Milne. While the values of the friction coefficient at these velocities are not the static coefficient values, for comparison purposes the values at these velocities are compared
to the static values obtained in the present study.

The comparison of the values obtained in the present study with previous unidirectional results is shown in Table 6.5. This table lists five different material/lubricant combinations which can be compared among the four studies. Not all cases are directly comparable; and in those cases where the comparison is questionable, the variation from the present tests is noted. As an example, for steel on steel lubricated with mineral oil, Forrester states that he ran steel on steel lubricated with "engine oil." Since this lubricant may contain unknown additives, it is noted in Table 6.5. The table lists values of the friction coefficient for the lowest velocity in each respective test, \( \mu_s \), and the value for the highest velocity, \( \mu_k \). At the highest velocity \( \mu_k \) can be thought of as the kinetic coefficient since friction coefficient changes very little with velocity at these values of velocity. Also, \( \mu_s \) is the value of the static coefficient in the present study and the value of the coefficient of friction for the smallest velocity for the other studies.

Table 6.5 shows that the values of the kinetic coefficient are very close for all four studies. The values of the static coefficients obtained in the present tests are all higher than the values of friction coefficient at minimum velocity obtained by the previous investigators.
Table 6.5
Comparison of Present Results with Some Previous Results

<table>
<thead>
<tr>
<th></th>
<th>$\mu_s$</th>
<th>$\mu_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Brass-Steel, Oleic Acid</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bristow</td>
<td>(0.09)</td>
<td>0.11</td>
</tr>
<tr>
<td>Present Study</td>
<td>0.19</td>
<td>0.15</td>
</tr>
<tr>
<td><strong>Steel-Steel, Dry</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Forrester</td>
<td>(0.4)</td>
<td>0.4</td>
</tr>
<tr>
<td>Present Study</td>
<td>0.66</td>
<td>0.38</td>
</tr>
<tr>
<td><strong>Steel-Steel, Mineral Oil</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bristow (steel on hard steel)</td>
<td>(0.30)</td>
<td>0.15</td>
</tr>
<tr>
<td>Forrester (engine oil)</td>
<td>(0.22)</td>
<td>0.16</td>
</tr>
<tr>
<td>Barwell &amp; Milne</td>
<td>(0.22)</td>
<td>0.10</td>
</tr>
<tr>
<td>(mineral base oil)</td>
<td>(0.18)</td>
<td></td>
</tr>
<tr>
<td>Barwell &amp; Milne</td>
<td></td>
<td>--</td>
</tr>
<tr>
<td>(refined paraffin oil)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Present Study</td>
<td>0.65</td>
<td>0.20</td>
</tr>
<tr>
<td><strong>Steel-Steel, Oleic Acid</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bristow</td>
<td>(0.20)</td>
<td>0.13</td>
</tr>
<tr>
<td>Forrester</td>
<td>(0.11)</td>
<td>0.12</td>
</tr>
<tr>
<td>Barwell &amp; Milne</td>
<td>(0.15)</td>
<td>--</td>
</tr>
<tr>
<td>Present Study</td>
<td>0.09</td>
<td>0.125</td>
</tr>
<tr>
<td><strong>Hard Steel-Hard Steel, Mineral Oil</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Barwell &amp; Milne</td>
<td>(0.15)</td>
<td>0.10</td>
</tr>
<tr>
<td>(mineral base oil)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Present Study</td>
<td>0.26</td>
<td>0.12</td>
</tr>
</tbody>
</table>

*Those values of $\mu_s$ shown in ( ) are not true static coefficients but values of friction coefficient obtained at the minimum velocity studied. See text.*
by a wide margin except for steel on steel lubricated with oleic acid. These higher values are not surprising since even a small velocity will give results much different from static.

The second aspect to compare is the presently proposed friction-velocity model with some previously proposed. Table 6.6 shows three previously proposed models of friction coefficient as functions of velocity along with the presently proposed model. Also listed in Table 6.6 are the values obtained from each model for the two extremes of velocity, 0 and \( \infty \). The present model and that of Bochet both give the correct values at these two extremes of velocity. The model of Franke gives the correct value for \( v = 0 \) but the friction coefficient tends to zero as the velocity increases. The model of Barwell and Milne is even worse, going to zero or infinity at both zero and high velocity, depending upon the value of the constant \( \gamma \).

Since Bochet's model gives results similar to the present model for extremes of velocity, it is of interest to investigate further. If the term \( 1/(1+av) \) in Bochet's formula is expanded in series form, one obtains:

\[
\frac{1}{1+av} = 1 - av + (av)^2 - (av)^3 + \ldots
\]  

(6.18')

Expanding the \( e^{-v/v_0} \) term from the present model gives:
Table 6.6

Friction vs. Velocity Models

<table>
<thead>
<tr>
<th>Formula</th>
<th>Investigator</th>
<th>Value of $\mu$ at $v = 0$</th>
<th>Value of $\mu$ at $v = \infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu = \mu_m + \frac{\mu_s - \mu_m}{1 + av}$</td>
<td>Bochet [7]</td>
<td>$\mu_s$</td>
<td>$\mu_m$</td>
</tr>
<tr>
<td>$\mu = \mu_s e^{-cv}$</td>
<td>Franke [25]</td>
<td>$\mu_s$</td>
<td>0</td>
</tr>
<tr>
<td>$\mu = \mu_o \left(\frac{V}{V_o}\right)^{\gamma}$</td>
<td>Barwell and Milne [3]</td>
<td>0 or $\infty$</td>
<td>$\infty$ or 0</td>
</tr>
<tr>
<td>$\mu = \mu_m + (\mu_s - \mu_m)e^{-v/V_o}$</td>
<td>Present</td>
<td>$\mu_s$</td>
<td>$\mu_m$</td>
</tr>
</tbody>
</table>
7.1 Evaluation of Results

The results of this study clearly show that under conditions of linear reciprocating motion the frictional behavior does not follow the classical description. That is, the coefficient of friction does not remain constant, in general, as the velocity varies in the region investigated in this study. This is a violation of the classical third law of friction first proposed by Coulomb. These results support the trends obtained in the unidirectional tests of Forrester [24], Barwell and Milne [3], and Bristow [13]. Where the unidirectional tests predict a falling friction coefficient with increasing velocity, the reciprocating tests also showed a falling behavior, or vice versa.

A comparison of the model developed by Barwell and Milne [3] for unidirectional linear motion,

$$\mu = \mu_0 \left(\frac{v}{v_0}\right)^\gamma$$

(6.20)
where \( \mu \) = coefficient of friction \\
\( \mu_o \) = reference coefficient of friction \\
\( v_o \) = reference velocity \\
\( \gamma \) = a constant

with that obtained from the curve fits of the results of the frequency and amplitude tests, Equation (6.10) shows that the same general behavior is found. It should be pointed out, however, that even though these two models show the same general behavior, they are not identical. Equation (6.20) plots as a straight line on log-log paper, while Equation (6.10) plots as a straight line on semi-log paper. Thus, it would appear that there is some difference in the velocity dependence of the friction coefficient depending upon the mode of the sliding, steady state unidirectional, or reciprocating.

Neither the model of Equation (6.10) nor Equation (6.20) is suitable for completely describing the frictional behavior where the velocity goes through zero since unrealistic values of the friction coefficient are predicted at zero velocity with these two models. For this situation a new model is proposed consisting of a constant component plus an exponentially varying component [Equation (6.11)]:
\[ \mu = \mu_m + (\mu_s - \mu_m) e^{-v/v_0} \] (6.21)

This expression is appropriate for use in the modeling of dynamic systems where there is reciprocating sliding with dry friction or boundary lubrication because it gives a finite value of \( \mu \) for \( v=0 \) and a finite value of \( \mu \) as \( v \) grows very large. Thus this model more realistically represents the observed behavior of friction under linear reciprocating motion than those previously proposed. This model fits the behavior of all the material/lubricant combinations investigated in this study except steel on brass lubricated with the oleic acid and mineral oil mixture (see Figure 5-3).

Besides the variations in the coefficient of friction with velocity, this study has shown that the static coefficient of friction also is not always constant. In over half the cases studied in this investigation, the static coefficient was constant or varied only slightly as the frequency or amplitude of the motion was changed. In those cases where the static coefficient varied as either frequency or amplitude was changed, the variation followed the trend of the variation of the kinetic friction coefficient. That is, if the kinetic coefficient decreased with increasing velocity, then the static coefficient decreased with increasing frequency or amplitude. Or, if
the kinetic coefficient increased with increasing velocity, then the static coefficient increased with increasing frequency or amplitude. In no case did the variations in the static and kinetic coefficients show different trends. It is reasonable to conclude, therefore, that the same properties of the interface contribute to the variation in both the static and kinetic coefficients. A single cause was not found for the variation in the static friction coefficient. A number of possible causes were suggested, and it is probable that each comes into play under certain conditions with one or more cause giving the predominant effect.

The lack of a clear-cut correlation between some variable which is measurable under conditions of random motion and the value of the static coefficient of friction is a weakness in the theory developed here. It does not completely eliminate the usefulness of the model, however. The model is still useful when the motion is of the reciprocating linear sinusoidal type for which the frequency and amplitude variation tests were run. Also, for those materials/lubricant combinations for which the static friction coefficient is constant or varies only slightly with frequency and amplitude variations (the majority of the cases studied), the static coefficient can be taken to be constant. In those cases where the static coefficient
is known to vary significantly, use of the model developed here with an average assumed constant value for the static coefficient can be useful as it is still an improvement over the classical third law of friction. Future study into the causes of the variation in the static coefficient will further improve the usefulness of the model.

It should be pointed out that the results presented here are for a certain set of conditions. It would be impossible to reproduce all the conditions which could possibly be encountered since there are so many variables which can affect the coefficient of friction. Section 4.1 gives a partial list of the factors which can affect the friction coefficient, and this list contains nineteen items. The one parameter which was investigated to determine if changes in its value could cause changes in the frictional behavior was the load. Changing the load showed that changes in this variable could cause substantial changes in the frictional behavior. Changes in any of the other many variables which can affect friction may cause changes in the frictional behavior also.

It should be pointed out that the number of tests run for a given set of conditions was not sufficient to define the results statistically. At the beginning of the testing it was decided that testing time would be better spent investigating a number of material/lubricant combinations rather than a relatively few combinations in detail.
Since it seemed unlikely that the conditions investigated would be reproduced exactly in practice, it was decided that it was better to identify trends over a large number of conditions rather than to obtain statistically significant results for a relatively few conditions. For this reason no statistical tolerances are given in this study.

One of the attractive attributes of a reciprocating motion test is that changes in the frictional behavior can be studied as time goes by and the two mating surfaces wear. This is difficult when unidirectional linear tests are run since the surfaces would have to be separated on the return stroke. The friction coefficient vs. number of cycles tests conducted in this study do just that. They show how the behavior of the friction coefficients change as the surfaces wear. These tests show that the velocity variation of the friction coefficient is not a phenomenon observed only with clear fresh surfaces but is present even after considerable changes have taken place in the surface. The velocity variation does vary slightly with wear, however.

Two examples were given in Section 6.3 to show the application of the model developed in this study. These examples showed that the new model can give results different from those obtained by using the classical constant coefficient model. This new model is most useful and applicable when the motion takes place at low velocity since the region of maximum variation in the coefficient
of friction with velocity is at low velocity. When the reciprocating motion reaches high velocities for a large percentage of the time, the difference between the results of the classical model and the new model diminishes. This is because there is only a small difference in the energy dissipated between the classical model and the new model when the velocity is high for the majority of the time.

There was one exception to the general exponential behavior displayed in this study. That was observed with mild steel on brass lubricated with oleic acid. This combination displayed a hybrid behavior with a high static coefficient of friction dropping to a low values and then increasing with increasing velocity. Other than this one exception, the results were consistent with the behavior described here.

7.2 Future Study

This study has brought up several areas where further study would be of interest. One of the more interesting observations of the present study has been the variations in the values of the static coefficient of friction which were observed during the frequency, amplitude, and random motion tests. Several possible explanations for this behavior were suggested, but unlike the variation in the kinetic coefficient of friction which could be explained
as a velocity effect, the variation of static coefficient of friction could not be directly attributable to any quantity measured in the tests. This phenomenon should be investigated further with a view toward asserting its causes in a form suitable for predicting values of static coefficient for any random motion. Since it is suspected that the stiffness of the mechanical system plays a major role, further study should make arrangements for changing this stiffness. Also, the relative acceleration between the two friction specimens should be measured besides the relative velocity, since it is suspected that this quantity may play some role in the changes in the static coefficient. It would be impossible to differentiate the relative velocity to obtain the acceleration since the velocity signal is much too noisy. The natural frequency of the measurement system constructed for this future study should be considerably higher than the 220 Hz of the present system in order to more accurately measure the static friction coefficients.

A second area of possible future investigations is the type of contact at the friction interface. There are three possible theoretical contact conditions: point, line, and area. The present study employed the theoretical point contact of a spherical-ended pin on a flat. This is only a theoretical classification, however, since hertzian deflection and wear at the interface produce area contact
in actuality. Because of this contact configuration the actual area of contact is difficult to determine. A further avenue of study would be to study linear reciprocating motion with theoretical area contact. If this were done the pressures at the interface could be measured and controlled and frictional behavior studied as a function of the interface pressure.

Several attempts were made with the present measuring device to run samples with theoretical area contact. These attempts failed because of the alignment problems involved. Since both friction samples are held rigidly by the present measuring device, the two mating surfaces had to be perfectly parallel to develop area contact. The pin was made with a flat end instead of the spherical end and the surfaces were lapped after being locked in place. This method failed to produce total contact over the entire area. The largest area of contact obtained was approximately 75% of that possible. Study of the coefficient of friction with area changes would require a different measurement scheme to ensure that area contact was maintained.

If further tests are conducted with other materials, it is possible that random motion testing may be the most fruitful approach. The random motion test gives the friction vs. velocity information and indicates those material/lubricant combinations for which the static
coefficient of friction varies significantly. For example, in the four combinations run in this study, three were shown in the random motion tests to have static coefficients which did not vary greatly, while one test gave results where the static coefficient of friction did vary significantly. Random motion thus presents a method for quickly determining the dynamic behavior of the coefficient of friction.
APPENDIX

MEASUREMENT SYSTEM WITH ADDED DAMPER
Appendix: Measurement System with Added Damper

The measurement system as constructed contained little inherent damping. The frequency response test conducted to define the dynamic characteristics of the measurement system, Figure 3-7, showed a relatively large peak at the natural frequency of the system, 220 Hz. From the height of this peak the damping was calculated as being only 6% of critical damping. This is only a small amount of damping and it was decided to attempt to increase the damping by adding a damper to the system in an attempt to reduce the peak at 220 Hz and improve the system response.

The design conceived is illustrated in the sketch of Figure 3-17. This is a shear-type device which develops its damping force through the shear of a liquid layer trapped between two plates which move relative to each other in close proximity. Thus, the damping is purely viscous-type damping and the plates are not allowed to contact each other. This is advantageous since if the plates were to touch, Coulomb friction would be produced and the measuring system would develop a dead zone at small force levels. The Coulomb friction would certainly cause measurement inaccuracies since it would be impossible to
distinguish between the friction in the measurement device itself and the friction being measured.

In order to determine the amount of damping needed, the distributed-mass cantilever measurement beam was modeled as an equivalent spring-mass-damper lumped-parameter system. As an unforced spring-mass-damper system the equation of motion could be written as,

\[ M \ddot{x} + B \dot{x} + Kx = 0 \quad (A.1) \]

or

\[ \ddot{x} + 2\xi\omega_n \dot{x} + \omega_n^2 x = 0 \quad (A.2) \]

The spring rate, \( K \), and the natural frequency, \( \omega_n \), of the beam were known:

\[ K = 14,000 \text{ N/cm (7700 lbf/in.)} \]
\[ \omega_n = 220 \times 2\pi \text{ rad/sec} = 1380 \text{ rad/sec} \]

An equivalent mass, \( M \), could be calculated from,

\[ \omega_n^2 = \frac{K}{M} \quad (A.3) \]

or

\[ M = \frac{K}{\omega_n^2} = \frac{14,000}{(1380)^2} = 0.0071 \frac{\text{N-sec}^2}{\text{cm}} \quad (0.00405 \frac{\text{lbf-sec}^2}{\text{in.}}) \]
If a damping factor, $\xi = 0.5$, is assumed, then the equivalent coefficient of viscous damping, $B$, can be calculated from,

$$2\xi \omega_n = \frac{B}{M},$$  \hspace{1cm} (A.4)

$$B = 2\xi \omega_n M = 2(0.5)(1380)(0.0071)$$

$$= 9.80 \frac{\text{N-sec}}{\text{cm}} (5.60 \frac{\text{lbf-sec}}{\text{in.}})$$

This damping was to be supplied by the shear of the fluid in the damper. The fluid used was a silicone material with the following properties:

$$\rho = \text{density} = 0.97 \frac{\text{gm}}{\text{cm}^3} (0.035 \frac{\text{lbf}}{\text{in}^3})$$

$$\nu = \text{kinematic viscosity}$$

$$= 125 \text{ stokes} (19.4 \frac{\text{in}^2}{\text{sec}})$$

The viscosity is therefore,

$$\mu = \nu \rho = 125 \times 0.97 = 121.0 \text{ poise} (0.00175 \frac{\text{lbf-sec}}{\text{in}^2})$$
Since the fluid is being sheared only, the governing equation is Newton's law of viscosity:

\[
\tau = \frac{F}{A} = \mu \frac{\Delta u}{\Delta y}
\]

where

- \( \tau \) = shear stress, N/cm
- \( F \) = force, N
- \( A \) = area of fluid being sheared, cm
- \( \mu \) = viscosity, poise
- \( \Delta u \) = change in velocity, cm/sec
- \( \Delta y \) = thickness of fluid between the two plates, cm

If a thickness, \( \Delta y = 0.010 \) cm (0.004 in.), is assumed then the necessary area to obtain the required damping, \( B \), can be calculated since,

\[
B = \frac{F}{\Delta u} = \frac{\mu A}{\Delta y}
\]

or

\[
\frac{BAy}{\mu} = \frac{9.80 \times 0.010 \times 10^5}{121} = 81 \text{ cm}^2 (12.5 \text{ in}^2)
\]

A damper was constructed using the design of Figure 3-17 and dimensioned to fit available space. The design consisted of a slot 0.040 cm (0.016 in.) wide in which a "flag" 0.020 cm (0.008 in.) thick moved. This gave a film of fluid on each side 0.010 cm (0.004 in.) thick. The area being sheared was 29.0 cm\(^2\) (4.5 in.\(^2\)), half
on each side of the "flag." This is 36% of the area predicted as being necessary to produce $\xi = 0.5$ by the lumped parameter spring-mass-damper model and should thus produce a useful increase in damping. The damper was placed between the measuring beam and the auxiliary beam. The placement of the damper is shown in the photograph of Figure 3-18.

With the damper in place, the damping factor, $\xi$, was determined by applying a step input of force to the measuring system and recording the decay of the output. The damping factor was then calculated from the magnitude of the force input and the overshoot of the step response. The value obtained was $\xi = 0.2$ (compared to theoretically predicted 0.18).

To test the effectiveness of the added damping, a series of tests was run with a steel on steel friction pair lubricated with mineral oil. The tests were conducted with the damper in place and then the damper was removed and the tests were conducted again with the basic undamped measurement system. The test samples and lubricant were undisturbed between tests so that identical conditions existed at the interface of the friction pair. The result of this test series is illustrated in Figure A-1 for a load of 4.45 N (1 lbf), a peak-to-peak amplitude of 2.54 cm (1 in.), and a frequency of reciprocation of 1 Hz.
Figure A-1 shows both the damped and undamped system response. The damper did little to improve the system response and in fact caused the friction force peaks at the beginning and end of the strokes to be measured lower than the undamped case. The values of friction force in the center of the cycle, that is at maximum velocity, were the same, however. Thus the damper was not a beneficial addition to the measurement device and was not employed in any further tests.
Figure A-1. Comparison of a Damped and an Undamped System Response for Steel on Steel Lubricated with Mineral Oil. $\lambda = 2.54$ cm (1 in.), $f = 1.0$ Hz.


Bibliography (continued)


55. System/360 Continuous System Modeling Program, Application Description, IBM publication, H20-0240-1.