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The Ohio State University, Ph.D., 1974  
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THE EFFECT OF THREE INSTRUCTIONAL BASES FOR DECIMALS ON COMPUTATION SKILLS OF SEVENTH-GRADE STUDENTS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By
Judith Langdon Bauer, B.S., M.A.

The Ohio State University

1974

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1974
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THE EFFECT OF THREE INSTRUCTIONAL BASES ON DECIMAL COMPUTATION SKILLS OF SEVENTH-GRADE STUDENTS

By

Judith Langdon Bauer

The Ohio State University, 1974

Dr. James K. Duncan, Advisor

The major problem investigated in the study was to determine the effect of the definitional base used for decimals on decimal computational ability. A treatment corresponding to each definitional base was applied. Each treatment was a set of instructional materials on decimals which was prepared by the investigator. Decimals in each set of instructional materials were defined on one of three bases: common fraction equivalents (common fractions), expanded exponential notation (place-value), a point on the rational number line (number line).

A secondary problem which was investigated was to determine the effect of the definitional base used for decimals on concomitant outcomes. The concomitant outcomes analyzed were transfer to metric and conditional reasoning ability.

The three treatments were randomly assigned to three seventh-grade mathematics classes of each of the six Columbus Public School teachers who participated in the study. Inservice was provided for the participating teachers to familiarize them with all three definitional
bases and to solidify their cooperation in maintaining purity of treatment.

A total of 568 students received fifteen class periods of instruction during treatment. The students were pre-tested, posttested, and retention tested on parallel forms of an achievement test of decimal computation constructed by the investigator. An investigator-constructed transfer to metric test and the Cornell Conditional Reasoning Test - Form X were administered after the treatment period.

Analysis of variance for repeated measures indicated that there was a significant gain in achievement on decimal computation between pretest and posttest and between pre-test and retention test (p < 0.001). ANOVA was applied to the criterion measures and indicated that there were no significant differences among the three treatments.

Analyses of the responses to a questionnaire given to the six teachers indicated that teacher preferences relative to the definitional bases shifted over the treatment period, away from common fractions and towards number line. Student preferences as perceived and reported by teachers were congruent with neither those of their teacher nor current mathematics curriculum.

The results of the study indicate that there may be three viable alternative definitional bases for decimals for seventh-grade mathematics students in terms of decimal computational achievement. Further research is needed to determine whether these results are obtained for younger children.
Chapter I
INTRODUCTION

The Problem

The major problem investigated in the study was the effect of the definitional base used for decimals on decimal computational ability. A secondary problem which was investigated was the effect of the definitional base used for decimals on concomitant outcomes. The concomitant outcomes analyzed were transfer to computation in the metric system and conditional reasoning ability.

When a term such as "number" is defined, the child attaches a referent to the defined term. The referent initially may be concrete. As the term becomes more internalized by the child, the referent usually becomes less concrete and more abstract. When the child uses the defined term, as in numerical computation, he often returns to his referent to provide meaning and rationale for what he is doing. This referent might be termed his definitional base.

The term "meaningful instruction" has been used to describe the antithesis of rote learning (6). If the teacher always couches rationale for the development of algorithms in terms of the definitional base, and if this is what the child turns to for his justification of computation, then the definitional base and the resultant rationale specifies the "meaningful" aspect of algorithm instruction.
This investigation sought to determine the effect of the definitional base used for decimals on decimal computational ability. A treatment corresponding to a definitional base for decimals was applied, each treatment being a set of instructional materials prepared by the investigator. Decimals in each set of instructional materials were defined on one of three bases: common fraction equivalents (semi-concrete), expanded exponential notation (abstract), a distance along the rational number line (concrete).

Introduction and Need for the Study

A primary goal for the mathematics education of children is the development of the child's ability to compute. A major portion of the mathematics curriculum for kindergarten through eighth grade is devoted to the attainment of this goal. The results of international, national, and statewide tests (33, 42, 40) indicate that American school children are having difficulty with computation.

One of the areas of computation which has received the least amount of attention by researchers, and one with which school children have much difficulty, is computation with decimals. Research evidence is especially lacking in the area of computational algorithms for decimals. The research which has been conducted regarding decimal computation has been primarily in the area of error analysis. It has been established that division with decimals is more difficult than are the other three arithmetic operations. Several researchers have conducted studies aimed towards finding an effective algorithm for locating the decimal point in the quotient of a decimal division problem.

The error analysis research has focused on analyzing errors within the context of one definitional base for decimals, the common fraction definitional base. Is this
definitional base the most efficient one in terms of achievement, retention, or transfer? These questions have not received sufficient attention. O'Brien (43) studied three approaches to the teaching of decimals. However, in the report of his study he states that his materials dealing with division of decimals may not have been completely adequate. The role of the definitional base used for decimals in the development of decimal computational algorithms needs to be investigated more thoroughly.

Computational ability is a necessary skill for daily functioning in the real world. It has been projected that by the middle of the 1980's the *Système International*, the modern metric system, will be the standard system of measure for the United States. The metric system already is the official system of measure for the majority of countries in the world. In preparation for the anticipated conversion to the metric system, many states, including Delaware, Vermont, Virginia, Minnesota, California, and Michigan, have enacted some form of metric legislation. These states and others have proposed timetables for metric education in the schools which call for the metrification process to begin in the schools within the next two years. The adoption of the metric system as the standard system of measure means that now, more than before, the product of the American school must be proficient in computation with decimals. This ability should probably be developed at an earlier stage in the student's educational career than is being done currently.

The development of computational proficiency with decimals in today's schools is based on extensive work with common fractions. Only after the algorithms for computation with common fractions are developed is attention given to the development of the algorithms for decimals.
The result is that children in American schools do not develop the ability to compute with decimals until the upper elementary, or more usually the junior high school, years. If measurement concepts and skills are to be developed within the metric system, the child ought to be able to compute with decimals in the middle and upper elementary grades. Is it necessary to define decimals in terms of common fractions and to postpone the development of decimal computational algorithms until after the algorithms for common fractions are developed? If not, how else may decimals effectively be defined? These questions have not received much attention to date.

Decimal computational ability is an anticipated outcome of decimal algorithmic learning. It is the specific instructional objective. The teacher who is teaching for decimal computation performs the instructional act and structures the educational environment to foster student attainment of that objective. However, decimal computational ability is not the sole outcome of that instruction. Other outcomes which are a result of that instruction are concomitant outcomes. Transfer is one type of concomitant outcome of instruction which is a result of intentional instructional acts. But many learning outcomes are neither planned for nor a result of intentional acts by the teacher. These outcomes have been called adventitious learning outcomes (24). It is necessary not only to investigate anticipated outcomes of algorithmic learning, but also concomitant outcomes. Does the definitional base on which decimals are defined affect concomitant outcomes of decimal instruction? Does the definitional base affect transfer to metric?

A basic goal in addition to computational ability for the education of children is the development of the child's reasoning abilities. According to Piaget (35), the highest
level of a child's cognitive structure is hypothetical-deductive thought. One basic component of deductive thought is conditional reasoning (16). Since the development of conditional reasoning ability is fundamental to the cognitive development of the child, research attention should be focused upon isolating significant variables in developing conditional reasoning ability and ways in which these variables can be manipulated in order to facilitate the acquisition of this reasoning process. If children do use their experience with a definitional base for decimals as a mechanism of rationale for computational strategies, will their conditional reasoning ability relate to this base?

There are many stimuli acting upon a child during each stage of the development of his cognitive processes. One important stimulus set is provided within his classroom environment. Some elements of this stimulus set are intentional while others are not. Numerous analyses of classroom behavior have been made since the turn of the century. These studies have ranged from analyzing the structure of recitation to establishing category systems descriptive of the verbal behavior in the classroom. While numerous analyses of verbal behavior in the classroom have contained categories for logical language patterns (37, 48, 2, 22, 18), relatively little research has been conducted into the relationship between the classroom environment, especially the nonintentional components, and the development of logical thinking in children. Gregory (24) conducted an initial study of the impact of the verbal environment of the mathematics classroom on the logical ability of adolescents. He found a positive relationship between the rate of teacher utilization of conditional reasoning paradigms in ordinary language and conditional reasoning ability in children. Since there is a positive relationship between one element
of the set of stimuli provided by the classroom environment and conditional reasoning ability, might not other elements also be related positively? Would the base for thinking about the decimal algorithms relate to changes in conditional reasoning ability?

Research in the field of logical development in children has focused primarily on the correlates of these abilities in terms of inherent student characteristics (31) and training in logical thinking to improve logical thinking (34,14). Is conditional reasoning ability a concomitant outcome of algorithmic learning? Does the way in which concepts are defined affect the development of conditional reasoning ability? Is the level of abstraction of a definitional base a significant variable? Many questions relative to the acquisition of conditional reasoning ability remain unanswered.

Objectives

The purpose of this study was 1) to examine the effect of the definitional base being used to teach decimals on the computational ability of students with decimals, 2) to examine the effect of the definitional base for decimals on the ability of students to convert units within the metric system, and 3) to examine the effect of the definitional base for decimals on the conditional reasoning ability of students.

Definition of Terms

1. **Algorithm**: a rule or procedure for solving a mathematical problem.

2. **Common fraction**: a numeral of the form \( \frac{a}{b} \) where \( a \) and \( b \) represent whole numbers and \( b \) is not zero.
3. **Computation**: the process of determining the standard numeral for a number (43).

4. **Concomitant outcome**: a learning outcome which accompanies the anticipated outcome in an incidental way.

5. **Conditional logic**: reasoning in which a condition is given from which a consequent follows.

6. **Conditional reasoning ability**: the ability to apply principles of conditional inference on problems presented on and measured by the *Cornell Conditional Reasoning Test - Form X*.

7. **Decimal**: a numeral for any number expressible in the form \((a \times 10^n) + (b \times 10^{n-1}) + \cdots\), where \(a, b, \ldots\) represent whole numbers and \(n\) represents an integer.

8. **Definitional base**: the concrete, semi-concrete, or abstract referent for a defined term.

9. **Place-value numeration**: a system of numeration in which each number can be expressed as \((a_k \times n^k) + (a_{k-1} \times n^{k-1}) + \cdots + (a_0 \times n^0)\), where \(a_i\) and \(k\) are whole numbers.

10. **Quasi-experiment**: a research activity which is short of being a true experiment in that scheduling of experimental stimuli is not under full control (10).

11. **Real number line**: a line on which every real number can be represented by a point.

**Hypotheses**

The following four hypotheses, stated in null form, were tested in this study.
H₁: The three treatments are equally effective as measured by an achievement test of decimal computation.

H₂: The three treatments are equally effective as measured by a retention test of decimal computation.

H₃: The three treatments are equally effective as measured by a transfer to metric test.

H₄: The three treatments are equally effective as measured by a conditional reasoning test.

Assumptions

The following assumptions were made in conducting this investigation.

1. Extraneous variables in students and teachers, such as sex, intelligence, motivation, mathematical ability, and understandings of language, were distributed randomly across the sample.

2. The instruments used measured that which they purport to measure, i.e. the instruments were valid.

3. The teachers cooperated with the investigator in using only the instructional materials which were distributed to them.

4. The teachers maintained purity of treatment in adhering to the prescribed definitional base.

Limitations of the Study

The following limitations may have affected the outcomes of this investigation.

1. The adherence to the instructional materials and definitional bases may have been influenced by the presence of recording equipment in the classroom.
2. The students being taught may have influenced the method of instruction.
3. The achievement of students may have been influenced by out-of-school assistance or tutoring.
4. The investigator had no control over the previous mathematics training of the students.

Delimitations of the Study

The following are delimitations of the investigation.

1. The study was conducted using six teachers of seventh-grade mathematics in the Columbus Public Schools and the students from three classes of each of these teachers.
2. The study was conducted during the 1973-74 academic year.
3. The only analysis of the tape recordings of the class sessions was to determine the degree of adherence to the prescribed definitional base.
4. No control over the non-school learning environment was provided.

Procedures

Three sets of instructional materials were prepared by the investigator. Each set of materials used a different definitional base for decimals, common fraction, place-value, or number line, and constituted one of the three treatments. Throughout this report Treatment 1 refers to the common fraction definitional base, Treatment 2 refers to the place-value definitional base, and Treatment 3 refers to the number line definitional base. The three treatments are described in greater detail in Chapter III of this report.
The study was conducted in seventh-grade mathematics classes in the Columbus Public Schools. Each teacher who participated in the study received a set of materials for Treatment 1, Treatment 2, and Treatment 3. The treatments were administered over a three-week period, one treatment per class. Thus each teacher administered all three treatments concurrently. Inservice training was given to the teachers before the treatment period to familiarize them with the definitional bases and instructional materials, as well as to solidify their support in maintaining purity of treatment.

The students were pretested prior to treatment on a decimal computation achievement test. On the last day of the treatment period the students were posttested on a decimal computation achievement test. Five weeks after treatment a transfer to metric test was administered, six weeks after treatment a retention decimal computation test was administered, and seven weeks after treatment a conditional reasoning test was administered. The data obtained from these measures were analyzed by means of analysis of variance.

Organization of the Report

In Chapter II literature related to this investigation is reviewed and discussed. Chapter III contains a theoretical and pedagogical description of the three definitional bases. Definitive descriptions of the methodology and analyses of data are contained in Chapters IV and V. The results of the study are discussed and recommendations are made relative to this study and future research in Chapter VI.
Chapter II
Review of the Literature

The major problem investigated in this study was to determine the effect of the definitional base for decimals on the decimal computational ability of students. A secondary problem was to search for relations between the definitional bases for decimals and two concomitant learning outcomes, the ability to reason conditionally and the ability to do metric computation. The implications of research from four research areas is related to this study. These four areas are: computation with decimals, concomitant outcomes of algorithmic learning, conditional reasoning correlates, and classroom analysis. Literature from these four research areas is reviewed in this chapter.

Research related to computation, specifically to computation with decimals, is examined in the first section of this chapter. The research evidence first is examined to determine the extent and nature of the difficulties that students have in computing with decimals. The decimal computation research evidence also is examined to find viable alternatives for decimal instruction.

Decimal computation is facilitated by the generation of computational algorithms. Therefore, the second section of this chapter is devoted to a review of the literature related to algorithmic learning and concomitant outcomes of algorithmic learning. The purpose of this review is to establish links between the development of computational algorithms and the definitional base upon which numbers are defined. The algorithmic learning research also is
examined to find evidence of concomitant outcomes of this type of learning.

One of the major goals of mathematics instruction is a process goal, the development of the child's reasoning abilities. The growth of this process is fostered throughout the education of the student. A basic component of deductive thought is conditional reasoning. Since the development of conditional reasoning ability is fundamental to the cognitive development of the child, it is one of the concomitant outcomes of decimal instruction which was investigated in this study. In the third section of this chapter literature related to correlates of conditional reasoning is summarized to determine the relationship between conditional reasoning ability and instructional variables.

Instructional materials were prepared to administer the three treatments in this study. However, student learning is influenced by the teacher as well as by instructional material. One vehicle for collecting data relative to teacher behavior is classroom analysis. The fourth section of this chapter is a review of literature concerning classroom analysis, especially analysis of the verbal behavior patterns of teachers. The purpose of this review is to establish necessary controls and concerns for the design and conduct of the study.

Decimal Computation

The development of computational ability long has been a fundamental goal of mathematics education. This goal has its roots in the 3-R's of grammar school. Despite what might be termed revolutions in the teaching of mathematics and the availability of electronic calculators, the development of computational ability appropriately remains a fundamental goal. While the Cambridge Conference
Report recommended the replacement of arithmetic drill in the mathematics curriculum, it was made clear in the report that "reasonable proficiency in arithmetic calculation ... is essential to the study of mathematics." (23)

School children have difficulty with decimal computation, even though they demonstrate proficiency with whole number computation. The results of the 1973 Michigan Educational Assessment Program (40) clearly support this statement. The Michigan Assessment test is a criterion-referenced instrument. Of the 45 criteria tested at the seventh-grade level in 1973, 8 were whole-number-computation criteria and 3 were decimal-computation criteria. The percentage of the seventh-grade Michigan population meeting criteria in whole number computation ranged from 62 percent to 94 percent. The percentage meeting decimal computation criteria ranged from 14 percent to 75 percent. Apparently, proficiency in whole number computation does not insure proficiency in decimal computation. This seems to be true even though algorithms for decimal computation rely on the power-of-ten structure of decimals just as do whole-number computational algorithms.

Research in the area of decimal computation is limited in terms of quantity and in implications for mathematics instruction. A compilation and classification of computational research conducted during the period 1900-1972 cites 397 studies. Of these studies, only 26 or approximately 6 percent have either a primary or secondary focus on decimal computation. Only 12 or 3 percent of the cited studies investigated decimal computation as a primary concern. (49) Thus, children are having difficulty with decimal computation, but little research attention is being focused on the problem.

The limited research regarding computation with decimals can be classified into three categories: studies
to determine the relative difficulty of the four basic arithmetic operations with decimals, studies concerned with alternative methods to teaching the placement of the decimal point in a quotient, and studies investigating the relative effectiveness of approaches to instruction in decimal computation.

In 1928, Brueckner (8) administered a decimal computation test to 300 students in grades six through eight. He reported that of the 6610 errors made, 580, 465, 1814, and 3751 of these were made in addition, subtraction, multiplication, and division, respectively. A study by Guiler (29) substantiates the finding by Brueckner that division is the most difficult of the four operations with decimals. He reported that 94 percent of the children he tested had difficulty with division, while 64 percent had difficulty with multiplication, and 33 percent had difficulty with subtraction and addition. In 1941, Grossnickle (26) analyzed pupil difficulties with decimals and cited misplacement of the decimal point as the most prevalent source of error. More recently, Lankford (39) used an interview technique to analyze the nature of computational errors made by seventh-grade students. He found that the nature of computational errors is diverse and that pupils vary widely in computational strategies employed in exercises. However, while his analysis includes computation with common fractions, it does not include computation with decimals. The research related to the relative difficulty of the four arithmetic operations shows that division and multiplication are more difficult than addition and subtraction. However, possible sources of this additional difficulty have not received much research attention. Perhaps the base upon which decimals are defined contributes to this difficulty. However, decimal research is lacking in this area.
Since errors in decimal computation are made most frequently in division and misplacement of the decimal point is the most common source of error, decimal research attention has been focused upon alternative methods of decimal point placement in a quotient. Investigations regarding decimal point placement in a quotient have been conducted by Van Engen (52), Brown (3), Grossnickle (25), and Flournoy (20). The two methods of decimal point placement studied may be termed a subtractive method and a multiplicative method. In the subtractive method, the decimal point is located in the quotient by subtracting the number of decimal places in the divisor from the number of decimal places in the dividend. In the multiplicative method, the divisor is changed to a whole number by multiplying it by some power of ten.

Subtractive Method

\[
\begin{array}{c}
0.4 \big) 0.12 \\
\hline
0.3 \\
1.2 \\
\hline
2 - 1 = 1
\end{array}
\]

There is one decimal place in the quotient.

Multiplicative Method

\[
\begin{array}{c}
0.4 \big) 1.2 \\
\hline
0.3 \\
1.2 \\
\hline
0.4 \times 10 = 4 \\
1.2 \times 10 = 1.2
\end{array}
\]

The evidence gathered in these investigations indicate that accuracy in division of decimals is greater with the subtractive method, but greater understanding is achieved by the multiplicative method. However, this research focuses on alternative algorithms, and does not address
the question of the role of the definitional base for decimals in the relative difficulty of the four arithmetic operations with decimals.

Reports of the relative effectiveness of three approaches to the teaching of decimals have been made by Johnson (36), Faires (17), and O'Brien (43). Johnson reports on a comparison of a numeration approach and a common fraction approach to the teaching of decimals. The numeration approach is similar to the place-value definitional base used in this study. The common fractions approach is similar to the common fractions definitional base used in the study. However, "approach" as used by Johnson and "definitional base" as used in the present study differ. Approaches are rationale provided for algorithm development, while the definitional base is the referent upon which the algorithm is developed. That is, the approaches cited in the Johnson study do not address the question of the role of meaningful instruction in algorithmic learning.

In support of his position that decimals be taught before common fractions, Johnson cites two experiments in which students who were taught decimals prior to fractions evidenced greater speed and accuracy on addition and subtraction exercises with common fractions and whole numbers than those taught the common denominator method with common fractions. However, Johnson does not report on outcomes related to decimal computation or to the operations of multiplication and division. Johnson leaves the questions of the effectiveness of decimal instruction prior to common fraction instruction on decimal computation and the role of definition in instruction unanswered.

Faires conducted a study of 496 fifth-grade students on the relative effectiveness of two instructional sequences: decimals-before-fractions and fractions-before-
decimals. Faires tested the subjects on the four arithmetic operations with whole numbers, fractions, and decimals and reported that decimal computation can precede common fraction computation "with a resulting gain in achievement and with at least as good an understanding of fractional concepts." (17, p. 76) This report, however, does not indicate the ways in which decimals were defined in either instructional sequence. The study seems, like those reported by Johnson, to be more a sequencing study than a study of the development of decimal computation ability.

The O'Brien study is more relevant to the present study. He used three approaches to the teaching of decimals: place-value (or numeration), common fractions, and "learn the rule". He reported that achievement and retention were not as high on the place-value approach as on the other two approaches. He did report, however, that the place-value treatment provided reinforcement of whole number skills. He further found that achievement in division was less than expected in all three treatment groups. He attributed this outcome to a lack of representation in the instructional materials of one type of division problem. O'Brien did not report, however, how decimals were defined in his learn-the-rule treatment. Thus, the role that definition plays in the development of computational ability was not investigated in the O'Brien study.

Research in the area of development of computational ability with decimals is limited in quantity and implications for instruction. The majority of the available literature is concerned with diagnosing the number and types of errors made in computation with decimals. Little attention has been focused on concomitant outcomes of decimal algorithmic learning. No studies were found which
investigated the role of definition (definitional base) of decimals in the development of computational ability.

Algorithmic Learning

While research has been conducted to study the acquisition of computational algorithms and algorithmic learning, the primary focus of this research has been on which of alternative algorithms resulted in greater computational achievement. A significant exception is a study by Brownell and Moser (6) investigating two alternative algorithms for the renaming process in subtraction. They use a $2 \times 2$ design of one variable, equal additions versus borrowing, cross the other, meaningful versus rote instruction. Differential effects were found on the measures of understanding and accuracy. Thus, their recommendation was that the algorithm to be used depends upon the desired outcome. They found that while equal additions resulted in greater understanding, decomposition was as effective as equal additions in terms of accuracy.

One of the most desirable outcomes of algorithmic learning is retention. Many computational studies have examined retention of the algorithm or method under investigation. Within the range of decimal computation studies previously cited, O'Brien (43), for example, reported that the rule approach was more effective than the common fraction approach in retention of decimal computational skills.

The transfer effect of computational algorithmic learning to other cognitive areas of mathematics learning, such as problem solving and logical reasoning, have not received much research attention. Osborne (45) investigated the effect of logical ability as measured by the Hill test on two conceptual bases for whole number subtraction. While his evidence was not conclusive, he did
conjecture that the ability to apply logical reasoning paradigms is a correlate to understanding the subtraction process. The understanding of the subtraction process hinges, in part, on the conceptual base used.

While there have been numerous studies relative to algorithmic learning, especially learning whole number algorithms, few researchers have addressed the question of what outcomes might be expected other than computational proficiency. Specifically, the relationship between algorithmic learning and the development of higher order cognitive processes such as problem solving and deductive reasoning has not been studied.

**Conditional Reasoning Correlates**

The role of conditional reasoning ability in the learning of mathematics remains unclear. Is it a prerequisite for algorithmic learning, a by-product of algorithmic learning, or completely unrelated to algorithmic learning?

There have been numerous studies relative to variables which are correlated with conditional reasoning ability. These variables can be classified in two categories: inherent variables, such as age, sex, and intelligence; and environmental variables, such as the linguistic structure of the classroom and socioeconomic background. A third category, cognitive variables, has not been studied. That children between the ages of six and nine are capable of inferring conditionally has been substantiated by many studies (9, 54, 55, 34, 31, 44). Conditional reasoning ability further seems to be positively correlated with age. Piaget (46), Inhelder and Piaget (35), and Ennis and Paulus (16) report that children of ages eleven to seventeen measure consistently above younger children on tests of conditional reasoning ability. The evidence reported
by Burt (9), Hill (31), and Ennis and Paulus (16) supports the conjecture that there is little correlation between the ability to reason conditionally and the sex of the individual. These studies also support the hypothesis that conditional reasoning ability is correlated with intelligence.

Socioeconomic background seems to have no relationship with conditional reasoning ability according to the studies conducted by Ennis and Paulus. Gregory (24) studied the impact of the verbal environment of mathematics classrooms on seventh-grade students' logical abilities. He examined the relationship between the frequency of utilization of conditional reasoning paradigms of teachers and the conditional reasoning performance of their students. His results indicate that there is a positive relationship between these two variables. This relationship supports the idea that the stimuli provided by the mathematics classroom environment have a significant influence on the ability of adolescents to reason conditionally. Gregory further reports that the mathematical ability and reading ability of the child are correlates of his logical reasoning ability as measured by the Cornell Conditional Reasoning Test.

Research in the area of correlates to conditional reasoning shows that age is related to the ability to infer conditionally. Other factors correlated to logical reasoning are mathematical ability, reading ability, and the frequency of teacher utilization of conditional reasoning paradigms.

**Classroom Analysis**

Many researchers have established category systems for analyzing the behavior which takes place in the classroom. Descriptive studies to classify classroom behavior...
can be divided into two groups: those which classify the nature of the classroom interaction for the purpose of describing the teaching act and those which classify the nature of teacher verbal patterns. Researchers in the area of categorization of classroom interaction to describe the teaching act (19, 48, 13, 2) currently are examining those teacher moves and strategies which characterize instruction in an attempt to establish a theory of instruction.

The patterns of teacher verbalizing have been examined in terms of linguistic structure and cognitive structure (37, 32, 56, 48, 2, 18, 24). Wright and Proctor (56), Smith and Meux (48), and Bellack (2) have shown that conditional reasoning is one cognitive component of teacher verbalization. Conditional inferring is a pattern used more by teachers of mathematics than by teachers of other subjects according to the reports of Smith and Meux and Fey (18). Fey found that conditional statements are uttered more frequently in mathematics classrooms than other subject area classrooms. Gregory (24) found that the frequency of these utterances is positively correlated with student conditional reasoning ability. He reported that this influence overrides the influence of other verbal stimuli acting upon the adolescent.

The modification of teacher verbal behavior also has been investigated. Gage, Runkel, and Chatterjee (21) and Tuckman and Oliver (51) cite evidence that teacher verbal behavior can be modified by the use of detailed feedback. Tuckman, McCall, and Hyman (50) found that teacher verbal behavior can be modified by invoking a discrepancy between his observed behavior and his own self-perception.

The research evidence suggests that the verbal behavior patterns of teachers are a function of the models with
which they are provided. The verbal behavior patterns of teachers in terms of conditional reasoning paradigms is correlated with the conditional reasoning ability of their students.

Summary of Research

Little research attention has been focused on the development of computational algorithms for decimals. The decimal computational research has shown that division and multiplication are more difficult than subtraction and addition. A few decimal studies have been conducted to investigate the question of whether common fraction instruction should precede or follow decimal instruction. Some studies indicate that there are at least two viable rationales for the development of the decimal computational algorithms. These rationales are common fraction equivalents and base ten numeration. However, decimals must be defined on some definitional base. The effects of using common fraction equivalents and base ten numeration as the definitional referents for decimals have not been investigated.

Concomitant outcomes of decimal computational development other than retention have not been studied. Of particular relevance to the present study is the process outcome, conditional reasoning ability. Correlates to conditional reasoning ability have been investigated. Age, mathematical ability, reading ability, and frequency of teacher utilization of the language of conditional logic have been found to be positively related to the ability to infer conditionally. The role played by the conceptual-perceptual base in which the learner is operating on the development of conditional reasoning ability has not been investigated.
The conditional reasoning research suggests that in the present study the frequency of conditional reasoning paradigms presented the learner should be controlled. Classroom analysis research further suggests that teacher verbal behavior is shaped by the verbal models provided the teacher. This research evidence indicates that in this study the frequency of the language of conditional logic in the instructional materials should be constant across treatments.

A synthesis of the four research areas reviewed is in its beginning stages. This synthesis needs to be further developed and expanded to include the discipline structure and perceptual base underlying algorithmic learning.
Chapter III

The Treatments

The effects of three treatments were compared in this study. Each treatment utilized one of three definitional bases for decimals: common fractions, place-value, or number line. Since only one of the three definitional bases, common fractions, is prevalent in current mathematics textbooks, three sets of instructional materials, each representing one of the three definitional bases, were prepared by the investigator. Each set of instructional materials was designed to provide meaningful instruction utilizing one of the three definitional bases. Each definitional base and the corresponding development of the computational algorithms for decimals is described in this chapter.

Treatment I - Common Fractions

A decimal in Treatment I was defined in terms of common fraction equivalents. Since any terminating decimal is a rational number, it may be written as $\frac{a}{b}$ where $a$ and $b$ are integers and $b$ is not zero. Further, because of the base ten structure of the Hindu-Arabic numeration system it is possible to name each terminating decimal as $\frac{c}{10^d}$ where $c$ is an integer and $d$ is a whole number.

Using this definition, the following are equivalent.

\[
.1 = \frac{1}{10} \\
.3 = \frac{3}{10} \\
.01 = \frac{1}{100} \\
.19 = \frac{19}{100}
\]
Students in Treatment I named the decimal places in terms of their common fraction values. The students also were given practice in naming decimal and common fraction equivalents.

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</tr>
</tbody>
</table>

If two quantities are mathematically equivalent, then one may be substituted for the other in any mathematical expression without changing the value of the expression.

\[.3 = \frac{3}{10} \text{ and } .1 = \frac{1}{10} \Rightarrow .3 + .1 = \frac{3}{10} + \frac{1}{10}\]

\[= .3 + .1 = \frac{3 + 1}{10}\]

\[= .3 + .1 = \frac{4}{10}\]

\[.3 + .1 = \frac{4}{10} \text{ and } \frac{4}{10} = .4 \Rightarrow .3 + .1 = .4\]
Using common fraction equivalents the usual algorithms for addition and subtraction were generated: To add decimals, line up the decimal points in the addends and in the sum; To subtract a decimal, line up the decimal points in the minuend, subtrahend, and difference.

\[
\begin{array}{c}
3 \\
+ 1 \\
\downarrow
\end{array}
\quad
\begin{array}{c}
1.4 \\
- 0.6 \\
\downarrow
\end{array}
\quad
\begin{array}{c}
0.8
\end{array}
\]

The substitution principle and common fraction equivalents were used to develop the multiplication algorithm: To multiply decimals, point off the same number of decimal places in the product as the sum of the number of decimal places in the factors.

\[
.5 = \frac{5}{10} \quad \text{and} \quad .3 = \frac{3}{10} \quad \Rightarrow \quad .5 \times .3 = \frac{5}{10} \times \frac{3}{10}
\]

\[
= .5 \times .3 = \frac{5 \times 3}{10 \times 10}
\]

\[
= .5 \times .3 = \frac{15}{100}
\]

\[
.5 \times .3 = \frac{15}{100} \quad \text{and} \quad \frac{15}{100} = .15 \quad \Rightarrow \quad .5 \times .3 = .15
\]

\[
\begin{array}{c}
.5
\end{array}
\times
\begin{array}{c}
.3
\end{array}
\]

\[
.15
\]
The division algorithm was developed in two steps. First, division of a decimal by a whole number was examined.

\[
.12 = \frac{12}{100} \Rightarrow .12 \div 3 = \frac{12}{100} \div 3 \\
= .12 \div 3 = \frac{12}{100} \times \frac{1}{3} \\
= .12 \div 3 = \frac{12}{300} \\
= .12 \div 3 = \frac{4}{100}
\]

\[
.12 \div 3 = \frac{4}{100} \quad \text{and} \quad \frac{4}{100} = .04 \Rightarrow .12 \div 3 = .04
\]

If the algorithm for division of a decimal by a whole number is used, the solution of the preceding problem appears as follows.

\[
3) \underline{12} \\
\underline{12}
\]

That is, to divide a decimal by a whole number, place the decimal point in the quotient directly above the decimal point in the dividend.

The algorithm used for dividing by a decimal was to change the division problem to an equivalent problem in which the divisor is a whole number.

\[
.20 \div .5 = \frac{20}{5} \Rightarrow .20 \div .5 = \frac{20 \times 10}{5 \times 10} \\
= .20 \div .5 = \frac{2.0}{5} \\
= .20 \div .5 = 2.0 \div 5 \\
= .20 \div .5 = .4
\]
If the problem is restated mentally the work may appear as follows.

\[
\begin{array}{c}
30 \\
\hline
20 \\
\end{array}
\]

To summarize, in Treatment I a decimal was defined as a rational number with common fraction equivalents one of which is \( \frac{a}{10^b} \) where \( a \) is an integer and \( b \) is a whole number. All algorithms were developed by appeal to the definition. The students were encouraged to think of a decimal as another way of writing a common fraction. A sample of the developmental material and a student practice sheet for Treatment I are contained in Appendix 1.

Treatment II - Place-value

A decimal in Treatment II was defined in terms of expanded notation utilizing place value. Any numeral in the Hindu-Arabic numeration system can be written in expanded notation using powers of ten to represent the value of each place. If \( a, b, c, \) and \( d \) are whole numbers, then \( abcd \) is equivalent to \( (a \times 10^3) + (b \times 10^2) + (c \times 10^1) + (d \times 10^0) \).

\[4356 = (4 \times 10^3) + (3 \times 10^2) + (5 \times 10^1) + (6 \times 10^0)\]

Likewise, the decimal numeral \( .6521 \) can be represented using expanded notation.

\[.6521 = (6 \times 10^{-1}) + (5 \times 10^{-2}) + (2 \times 10^{-3}) + (1 \times 10^{-4})\]

Since not all students in the study were familiar with negative integers and negative exponents, a different symbolism was used in Treatment II instructional materials.

\[.6521 = (6 \times 10^{-1R}) + (5 \times 10^{2R}) + (2 \times 10^{3R}) + (1 \times 10^{4R})\]

In this notation an exponent of \( aR \) meant the \( a \)th place to the right of the decimal point. A decimal then in
Treatment II was defined in terms of expanded positional notation. By definition, the following are equivalent.

\[ .1 = 1 \times 10^{1R} \]
\[ .3 = 3 \times 10^{1R} \]
\[ .01 = (0 \times 10^{1R}) + (1 \times 10^{2R}) \]
\[ .19 = (1 \times 10^{1R}) + (9 \times 10^{2R}) \]

The students in Treatment II named the decimal places in terms of place-value equivalents.

\[
\begin{array}{cccccccc}
9 & 8 & \cdot & 7 & 6 & 5 & 4 & 3 & 2 \\
\downarrow & & & & & & & & \\
tenths & 10^{1R} & & & & & & \\
\downarrow & & & & & & & & \\
hundredths & 10^{2R} & & & & & & \\
\downarrow & & & & & & & & \\
thousandths & 10^{3R} & & & & & & \\
\downarrow & & & & & & & & \\
ten-thousandths & 10^{4R} & & & & & & \\
\downarrow & & & & & & & & \\
hundred-thousandths & 10^{5R} & & & & & & \\
\downarrow & & & & & & & & \\
millionths & 10^{6R} & & & & & & \\
\end{array}
\]

Defining decimals in this way provides a natural extension of the whole number definitions. Thus, algorithms for decimal computation were treated as extensions of the whole number algorithms.
.3 = 3 \times 10^{1_R} \text{ and } .1 = 1 \times 10^{1_R} \Rightarrow .3 + .1 = (3 \times 10^{1_R}) + (1 \times 10^{1_R})

\Rightarrow .3 + .1 = (3 + 1) \times 10^{1_R}

\Rightarrow .3 + .1 = 4 \times 10^{1_R}

.3 + .1 = 4 \times 10^{1_R} \text{ and } 4 \times 10^{1_R} = .4 \Rightarrow .3 + .1 = .4

This definition for decimals, the substitution principle, and the distributive property of multiplication over addition, led to the same algorithms for addition and subtraction of decimals as those developed in Treatment I.

\[
\begin{array}{c}
.3 \\
+ 1 \\
\hline
\frac{4}{8}
\end{array}
\quad
\begin{array}{c}
1.4 \\
- 6 \\
\hline
\frac{8}{8}
\end{array}
\]

The use of expanded notation, the substitution principle, the distributive property of multiplication over addition, the commutative property of multiplication, and the associative property of multiplication led to the same algorithm for multiplication of decimals as that developed in Treatment I.

.5 = 5 \times 10^{1_R} \text{ and } .3 = 3 \times 10^{1_R} \Rightarrow .5 \times .3 = (5 \times 10^{1_R}) \times (3 \times 10^{1_R})

\Rightarrow .5 \times .3 = (5 \times 3) \times (10^{1_R} \times 10^{1_R})

\Rightarrow .5 \times .3 = 15 \times 10^{2_R}

.5 \times .3 = 15 \times 10^{2_R} \text{ and } 15 \times 10^{2_R} = .15 \Rightarrow .5 \times .3 = .15

.5 \\
\times .3 \\
\hline
.15
In Treatment II, as in Treatment I, the division algorithm was developed in two steps. The algorithm for dividing a decimal by a whole number was generated by examining the problem using expanded notation and the substitution principle.

\[ .12 = (1 \times 10^{1R}) + (2 \times 10^{2R}) \Rightarrow .12 = 12 \times 10^{2R} \]

\[ = .12 \div 3 = \frac{12 \times 10^{2R}}{3} \]

\[ = .12 \div 3 = 4 \times 10^{2R} \]

\[ .12 \div 3 = 4 \times 10^{2R} \quad \text{and} \quad 4 \times 10^{2R} = .04 \Rightarrow .12 \div 3 = .04 \]

\[
\begin{array}{r}
3 & \left\uparrow \begin{array}{r}
04 \\
\end{array} \\
\hline
12
\end{array}
\]

The algorithm used to divide by a decimal was to change the division expression to an equivalent expression in which the divisor is a whole number.

\[ .20 = 20 \times 10^{2R} \quad \text{and} \quad .5 = 5 \times 10^{1R} \Rightarrow .20 \div .5 = \frac{20 \times 10^{2R}}{5 \times 10^{1R}} \]

\[ = .20 \div .5 = \frac{20 \times 10^{2R} \times 10}{5 \times 10^{1R} \times 10} \]

\[ = .20 \div .5 = \frac{20 \times 10^{1R}}{5 \times 10^{0}} \]

\[ = .20 \div .5 = \frac{20 \times 10^{1R}}{5} \]

\[ = .20 \div .5 = 2.0 \div 5 \]
If the algorithm were used, the work might appear as follows.

\[
\begin{array}{c}
52) 230 \\
\hline
20
\end{array}
\]

The algorithms used for computation with decimals in Treatment II were the same as those in Treatment I. However, the way in which decimals were defined and hence the rationale provided for each algorithm was different. In Treatment II, the students defined a decimal in terms of its equivalent expanded positional notation. A sample of the developmental material and a student practice sheet for Treatment II are contained in Appendix 1.

**Treatment III - Number Line**

A decimal in Treatment III was defined in terms of its location on a rational number line.

\[
\begin{array}{c}
0.1 \quad 0.19 \quad 0.3 \\
\hline
0 \quad \frac{3}{10} \quad \frac{1}{10} \quad 1
\end{array}
\]

The students in Treatment III were given practice in naming a point on a rational number line by a decimal and in locating a decimal on a rational number line. To develop the algorithms for addition and subtraction of decimals, a rational number line and the concept of directed distance were used.

\[
\begin{array}{c}
0.3 + 0.1 = 0.4 \\
\hline
0 \quad \frac{3}{10} \quad \frac{1}{10} \quad 1
\end{array}
\]

A "jump" of three tenths followed by a "jump" of one tenth is equivalent to a "jump" of four tenths.
After using a number line for addition and subtraction, the students then generated the usual algorithms for addition and subtraction of decimals.

\[
\begin{array}{c}
\text{.3} \\
+ \text{.1} \\
\hline \\
\text{.4}
\end{array}
\quad
\begin{array}{c}
\text{1.4} \\
- \text{.6} \\
\hline \\
\text{.8}
\end{array}
\]

Two perpendicular rational number lines were used to generate the multiplication algorithms. Since each square of the following grid represents .01, the product of .5 and .3 is .15.
The division algorithm was developed by first examining division of a decimal by a whole number. The students were to interpret a division statement such as $0.8 \div 2 = n$ as the equivalent statement $n \times 2 = 0.8$.

Thus $0.8 \div 2 = 0.4$.

This procedure generated the same algorithm as that generated in Treatment I and II.

\[
\begin{array}{c}
2 \div 0.8 \\
\hline
4
\end{array}
\]

Division by a decimal was accomplished by changing the division expression to an equivalent expression in which the divisor is a whole number.

The algorithms used for decimal computation in Treatment III were the same as those in Treatments I and II. However, in Treatment III a decimal was defined as a location on a rational number line. Rational number lines then provided the rationale for the algorithms which were developed. While the definitional bases used in Treatments I and II are somewhat standard in mathematics education, the one used in Treatment III has not been used in developing
decimal concepts and computational algorithms for decimals. A sample of the developmental material and a student practice sheet for Treatment III are contained in Appendix 1.

Each of these three treatments was administered in six classes of seventh-grade mathematics. The length of instructional time devoted to each set of instructional materials was fifteen thirty-minute class periods. A more specific description of the design of the study, experimental conditions, and data collected are contained in Chapter IV of this report.

The rationale for each definitional base and the contents of each set of instructional materials were explained to the participating teachers in an inservice setting. The inservice procedure is explicated further in Chapter IV.
Chapter IV
Methodology

Selection of the Sample

The study was conducted in eighteen seventh-grade mathematics classes of the Columbus Public School System in Columbus, Ohio. The criteria for selection required a large sampling frame. The Columbus system was chosen because it is the largest public school system in the central Ohio area.

Six teachers of mathematics were selected for participation in the study. One selection criterion was that each teacher must be teaching at least three seventh-grade mathematics classes. Three classes of the six teachers were selected for participation in the study, for a total of eighteen classroom units.

The seventh-grade level was chosen for the following reasons:

1. A preliminary survey indicated that the seventh-grade level is the first level in which all four basic arithmetic operations are treated for decimals.

2. Mathematics instruction in the seventh grade is conducted in a departmental structure as opposed to the self-contained structure utilized in kindergarten through grade six.

3. One of the concomitant outcomes of algorithmic learning to be investigated was conditional
reasoning ability. Research evidence gathered by Piaget and others indicates that the majority of children at age twelve are capable of abstract conditional inference.

Description of the Sample

Six teachers, four women and two men, participated in the study. Three classes for the six teachers were selected for study.

The classes were selected on the basis of meeting scheduling requirements for observation and audio tape recording by the investigator. The school system has classes divided into three categories on the basis of student intelligence, reading ability, mathematical ability, and teacher recommendation. These categories are honors, regular and modified. Seventeen classes were regular classes. The remaining class was an honors class. A test for homogeneity of variance indicated that the inclusion of this honors class in the sample did not affect significantly the homogeneity of the sample.

In all, 568 students in the eighteen classes participated in the study. In the statistical analysis the data for some pupils were eliminated for one of the following reasons.

1. Complete data were not available.
2. The pupil had transferred into or out of the class during the period of treatment.
3. The pupil was repeating seventh-grade mathematics.

The Design

Research evidence suggests that the teacher variable is a strong influence in classroom instruction. The design used in the study was selected to compensate for teacher differences. The unit of analysis was the class mean on
each criterion measure used in the study, as classes rather than individuals were assigned randomly to treatment.

**Experimental Design**

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<th>C₂</th>
<th>C₃</th>
<th>M</th>
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<td>B₁</td>
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<tr>
<td>A₃</td>
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<td>A₄</td>
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<td>A₅</td>
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<td>B₃</td>
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<td>A₆</td>
<td>B₁</td>
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<td></td>
<td>B₂</td>
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<tr>
<td></td>
<td>B₃</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>A₇</td>
<td>B₁</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>B₂</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( A_i = \text{Teachers ( } i = 1, \ldots, 6) \)

\( B_j = \text{Treatments ( } j = 1, 2, 3) \)

\( C_k = \text{Computational Measure ( } k = 1, 2, 3) \)

\( M = \text{Transfer to Metric Measure} \)

\( L = \text{Conditional Reasoning Ability Measure} \)
Procedural Steps

Six teachers representing four junior high schools were selected for participation in the study after central office approval was obtained. The principal and mathematics department chairman in each of the four schools then received a copy of the research proposal abstract. After the principals and department chairmen had had an opportunity to examine and approve the proposal, a meeting was held to gain building cooperation. Those in attendance at each meeting included the principal, department chairman, one or two mathematics teachers who were teaching at least three seventh-grade mathematics classes, and the investigator.

After building and teacher approvals for participation were obtained, the investigator met with the participating teachers to discuss curricular considerations as well as the school calendar. Recommendations for selection of the optimal starting and termination dates for the study were then obtained from the participating teachers. Following the selection of the starting date for the study, each teacher received a complete set of the instructional materials that had been prepared by the investigator for each treatment. The materials for each treatment consisted of two components, a teacher's manual and student practice sheets. The teacher's manual contained fifteen lessons for each treatment. Each lesson consisted of the performance objectives for the lesson and a conceptual development with instructional examples. Each treatment was designed for fifteen days' duration, one lesson per day.

As at least one of the three conceptual developments was unfamiliar to the teachers, the teachers then had an opportunity to study the materials. Inservice training sessions were held in each building to describe the rationale for each definitional base, discuss methods of
instruction, and answer teacher questions relative to the treatments as well as procedures. In the inservice sessions the importance of keeping each treatment pure was stressed. The teachers were encouraged to use whatever instructional strategy and additional materials they wished for instruction, so long as the treatments were not contaminated. Much time was spent in each session discussing particular strategies and aides and their relationship with each definitional base.

A pretest on decimal computational ability was administered to each student at the beginning of the treatment period. The decimal pretest, developed by the investigator, consisted of twenty-four items. A Kuder-Richardson-20 reliability coefficient of 0.81 for the pretest was obtained. The data for establishing the KR-20 reliability coefficient was gathered by administering the test to two eighth-grade mathematics classes in one of the participating schools. Content and curricular validity were judged by a jury of mathematics educators and a mathematics textbook editor. The jury was comprised of Alan R. Osborne, Professor of Mathematics Education, The Ohio State University; F. Joe Crosswhite, Professor of Mathematics Education, The Ohio State University; and J. Conrad White, Managing Editor, Mathematics, Charles E. Merrill Publishing Company. The judges agreed unanimously as to the content and curricular validity of the test. A copy of this criterion measure is contained in Appendix 2.

The treatment was administered during a period of fifteen school days. Each of the fifteen lessons was presented in a thirty-five minute class period. In order to maintain control over several possible confounding variables the following rules were followed by the participating teachers.
1. The teachers were to provide no out-of-class student assistance.

2. The teachers were to make no homework assignments.

3. The teachers were to ask their students not to seek help from parents, siblings, or peers.

4. The teachers were not to use the regular mathematics textbook.

5. The teachers were to request that their students not use the regular mathematics textbook.

In an effort to further control for contamination of treatments, three class sessions for each classroom unit were tape recorded during treatment. The dates on which the recordings were to be made were not announced to the teachers in advance. The tape recordings subsequently were used to determine whether the teachers maintained purity of treatment.

On the fifteenth day of the treatment period the decimal computation test that was used as a pretest was readministered to determine student achievement in decimal computation. Since the same test was used as a pretest and a posttest of decimal computational ability, the teachers were asked not to return the tests to the students and not to review the tests with the students after either administration.

Three weeks after the treatment period the participating teachers were asked to respond to a questionnaire. The purpose of this questionnaire was to ascertain teacher preference with respect to treatments both prior to and following treatments. The teachers also were asked to indicate pupil receptivity relative to the three treatments. A copy of this questionnaire is given in Appendix 3.
Five weeks after the administration of the decimal posttest a transfer to metric criterion measure (See Appendix 4) was administered. A Kuder-Richardson-20 reliability coefficient of 0.83 was obtained for this test. Content validity for this measure was established by the previously mentioned jury.

Six weeks after the posttest was administered a decimal computation retention test was given to all participating students. This test was parallel in form to the pretest and posttest. During the six-week period between the administrations of the posttest and retention test the students received no additional decimal instruction. A copy of this test is given in Appendix 5.

Seven weeks after completion of the treatment the Cornell Conditional Reasoning Test - Form X was administered to all participating students. This test is composed of three dimensions: (1) concrete-natural items, dealing with concrete things and containing statements of neutral truth value; (2) unfamiliar items dealing with abstract content; and (3) affective items dealing with emotionally loaded content. A coefficient of equivalence of 0.90 and a coefficient of stability of 0.80 has been established for seventh-grade subjects (14). It also was reported by Gregory (24) to have both high content validity and high construct validity. A copy of this test is given in Appendix 6.

Analysis of Data

Test papers were hand scored by the investigator. Class means were calculated for each criterion measure. The data for class means were keypunched on IBM cards for computer analysis.

The hypotheses of no significant differences on each criterion measure with respect to treatment were analyzed
using one-way analyses of variance. A more detailed description of the statistical procedures for data analysis as well as the results of the analyses are in Chapter V of this report.
Chapter V

ANALYSIS OF DATA

The hypotheses were tested using analyses of variance, to analyze the data gathered in this investigation. A statement of the results of the analyses of variance and a discussion of the results are contained in this chapter.

Description of Analysis of Variance

According to Kerlinger (38) analysis of variance is "a method of identifying, breaking down, and testing for statistical significance the differences between variances that come from different sources of variation. That is, a dependent variable has a total amount of variance, some of which is due to the experimental treatment, some to error, and some to other causes."

Assumptions Underlying Inferences Using Analysis of Variance

Given sample data, there are three assumptions which must be made before analysis of variance may be used to make inferences regarding the population. These assumptions are as follows:

1. For each treatment population \( j \), the distribution of \( e_{ij} \) is assumed normal . . .

2. For each population \( j \), the distribution of \( e_{ij} \) has a variance \( s_e^2 \), which is assumed to be the same for each treatment population.
3. The errors associated with any pair of observations are assumed to be independent. (53)

The first assumption states that the random error is normally distributed in each treatment population. This statement actually asserts that each population has a normal distribution of scores, \( x_{ij} \). According to Hays "... inferences made about means that are valid in the case of normal populations are also valid even when the norms of the population distributions depart considerably from normal, provided that the \( n \) in each sample is relatively large." (30)

The second assumption asserts that the error variance, \( s^2_e \), is the same for all treatment groups. This assumption may be violated without risk, provided that the number of units of analysis in the treatment groups are the same. (30)

The third assumption states that the errors associated with any pair of observations are independent. In order to draw valid conclusions from the F test in an analysis of variance, the experimenter must ensure that all observed scores are based upon independent observations. (30)

As the assumptions underlying analysis of variance were met, this procedure was used for data analyses.

Results

Four hypotheses were tested in this investigation by means of analysis of variance. These hypotheses, stated in null form, are:

\( H_1: \) The three treatments are equally effective as measured by an achievement test of decimal computation \( (\mu_1 = \mu_2 = \mu_3) \).

\( H_2: \) The three treatments are equally effective as measured by a retention test of decimal computation \( (\mu_1' = \mu_2' = \mu_3') \).
$H_3$: The three treatments are equally effective as measured by a transfer to metric test 
$(u_1'' = u_2'' = u_3'')$.

$H_4$: The three treatments are equally effective as measured by a conditional reasoning test 
$(u_1''' = u_2''' = u_3''')$.

The 0.05 probability level was established before the investigation to test for the validity of each of the four hypotheses. The data were analyzed using the MANOVA (Clyde Multivariate Analysis of Variance) computer program.

The class mean IQ scores were subjected to a test for homogeneity of variance to justify the use of analysis of variance in this investigation. The class mean intelligence scores as obtained by the California Test of Mental Maturity are shown in Table 1.
Table 1. Class Mean Intelligence Scores by Treatment*

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Treatment 1</th>
<th>Treatment 2</th>
<th>Treatment 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
</tr>
<tr>
<td>1</td>
<td>53.52</td>
<td>8.15</td>
<td>65.66</td>
</tr>
<tr>
<td>2</td>
<td>54.05</td>
<td>6.49</td>
<td>56.37</td>
</tr>
<tr>
<td>3</td>
<td>49.70</td>
<td>9.49</td>
<td>50.32</td>
</tr>
<tr>
<td>4</td>
<td>56.24</td>
<td>6.93</td>
<td>53.34</td>
</tr>
<tr>
<td>5</td>
<td>55.74</td>
<td>8.82</td>
<td>58.56</td>
</tr>
<tr>
<td>6</td>
<td>60.56</td>
<td>8.45</td>
<td>58.48</td>
</tr>
</tbody>
</table>

Mean | 54.968 | 57.122 | 54.812 |
SD  | 3.270 | 5.253 | 3.277 |

*T₁: Common fractions
T₂: Place-value
T₃: Number line
The results of Levene's Test for homogeneity of variance for intelligence scores are given in Table 2.

Table 2. Levene's Test for Homogeneity of Variance on Intelligence Scores

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>ms</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Groups</td>
<td>2</td>
<td>7.287</td>
<td>3.643</td>
<td>0.65</td>
</tr>
<tr>
<td>Within Groups</td>
<td>15</td>
<td>84.563</td>
<td>5.638</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>17</td>
<td>91.850</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The F value obtained in the test for homogeneity of variance indicates that the three treatment groups were homogeneous with respect to intelligence.

$H_1$: The three treatments are equally effective as measured by an achievement test of decimal computation.

$H_2$: The three treatments are equally effective as measured by a retention test of decimal computation.
Hypotheses 1 and 2 were tested using repeated measures analysis of variance on pretest, posttest, and retention mean test scores for decimal computation.

The mean test scores for the decimal computation pretest, posttest, and retention test are shown in Tables 3, 4, 5, and 6.

Table 3. Mean Decimal Computation Test Scores*
For Treatment 1.

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Pretest</th>
<th>Posttest</th>
<th>Retention Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.7</td>
<td>18.5</td>
<td>18.4</td>
</tr>
<tr>
<td>2</td>
<td>7.1</td>
<td>14.8</td>
<td>14.7</td>
</tr>
<tr>
<td>3</td>
<td>7.5</td>
<td>14.1</td>
<td>13.8</td>
</tr>
<tr>
<td>4</td>
<td>9.1</td>
<td>14.9</td>
<td>14.9</td>
</tr>
<tr>
<td>5</td>
<td>10.9</td>
<td>17.7</td>
<td>19.0</td>
</tr>
<tr>
<td>6</td>
<td>11.9</td>
<td>17.5</td>
<td>19.0</td>
</tr>
<tr>
<td>Mean</td>
<td>9.0</td>
<td>16.3</td>
<td>16.6</td>
</tr>
<tr>
<td>SD</td>
<td>1.81</td>
<td>1.70</td>
<td>2.20</td>
</tr>
</tbody>
</table>

*Maximum possible score = 24
Table 4. Mean Decimal Computation Test Scores* For Treatment 2.

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Pretest</th>
<th>Posttest</th>
<th>Retention Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15.1</td>
<td>21.5</td>
<td>22.1</td>
</tr>
<tr>
<td>2</td>
<td>7.7</td>
<td>18.3</td>
<td>18.1</td>
</tr>
<tr>
<td>3</td>
<td>9.2</td>
<td>14.6</td>
<td>14.3</td>
</tr>
<tr>
<td>4</td>
<td>9.0</td>
<td>13.8</td>
<td>12.8</td>
</tr>
<tr>
<td>5</td>
<td>10.6</td>
<td>18.4</td>
<td>17.8</td>
</tr>
<tr>
<td>6</td>
<td>12.6</td>
<td>16.8</td>
<td>18.4</td>
</tr>
<tr>
<td>Mean</td>
<td>10.7</td>
<td>17.2</td>
<td>17.3</td>
</tr>
<tr>
<td>SD</td>
<td>2.49</td>
<td>2.57</td>
<td>3.01</td>
</tr>
</tbody>
</table>

*Maximum possible score = 24
Table 5. Mean Decimal Computation Test Scores* For Treatment 3.

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Pretest</th>
<th>Posttest</th>
<th>Retention Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10.6</td>
<td>18.8</td>
<td>19.5</td>
</tr>
<tr>
<td>2</td>
<td>7.2</td>
<td>16.7</td>
<td>17.8</td>
</tr>
<tr>
<td>3</td>
<td>7.6</td>
<td>14.8</td>
<td>13.8</td>
</tr>
<tr>
<td>4</td>
<td>7.0</td>
<td>12.4</td>
<td>11.9</td>
</tr>
<tr>
<td>5</td>
<td>10.9</td>
<td>18.0</td>
<td>17.3</td>
</tr>
<tr>
<td>6</td>
<td>11.0</td>
<td>17.6</td>
<td>17.8</td>
</tr>
</tbody>
</table>

| Mean    | 9.1     | 16.4     | 16.4           |
| SD      | 1.80    | 2.18     | 2.62           |

*Maximum possible score = 24
Table 6. Mean Decimal Computation Test Scores* By Treatments

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Pretest</th>
<th>Posttest</th>
<th>Retention Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.0</td>
<td>16.3</td>
<td>16.5</td>
</tr>
<tr>
<td>2</td>
<td>10.7</td>
<td>17.2</td>
<td>17.3</td>
</tr>
<tr>
<td>3</td>
<td>9.1</td>
<td>16.4</td>
<td>16.4</td>
</tr>
</tbody>
</table>

| Mean      | 9.6     | 16.6     | 16.7           |
| SD        | .88     | .04      | .04            |

*Maximum possible score = 24

Table 7 contains the results of the repeated measures analysis of variance on decimal computation scores.

Table 7. Analysis of Variance Results For Decimal Computation

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>ms</th>
<th>F</th>
<th>p less than</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Groups</td>
<td>2</td>
<td>15.264</td>
<td>7.632</td>
<td>0.475</td>
<td>0.631</td>
</tr>
<tr>
<td>Within Groups</td>
<td>15</td>
<td>240.911</td>
<td>16.061</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Between Measures</td>
<td>2</td>
<td>600.264</td>
<td>300.132</td>
<td>210.881</td>
<td>*0.001</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1.866</td>
<td>0.467</td>
<td>0.328</td>
<td>0.857</td>
</tr>
<tr>
<td>Within Measures</td>
<td>30</td>
<td>42.697</td>
<td>1.423</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Significance for repeated measures only.
The analysis of decimal computation resulted in a failure to reject Hypotheses 1 and 2. The analysis did indicate a significant gain in decimal computational ability across the three treatment groups.

H₃: The three treatments are equally effective as measured by a transfer to metric test.

Hypothesis 3 was tested using analysis of variance on mean scores on a transfer to metric test. Table 8 contains the data used for this analysis.

Table 8. Mean Metric Scores* by Treatment

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Treatment 1</th>
<th>Treatment 2</th>
<th>Treatment 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.7</td>
<td>9.8</td>
<td>5.7</td>
</tr>
<tr>
<td>2</td>
<td>3.8</td>
<td>3.5</td>
<td>4.6</td>
</tr>
<tr>
<td>3</td>
<td>8.4</td>
<td>9.0</td>
<td>7.9</td>
</tr>
<tr>
<td>4</td>
<td>4.6</td>
<td>3.4</td>
<td>5.2</td>
</tr>
<tr>
<td>5</td>
<td>8.7</td>
<td>7.1</td>
<td>7.1</td>
</tr>
<tr>
<td>6</td>
<td>8.9</td>
<td>7.2</td>
<td>6.6</td>
</tr>
</tbody>
</table>

| Mean    | 6.350       | 6.667       | 6.183       |
| SD      | 2.562       | 2.699       | 1.238       |

*Maximum possible score = 15
The results of the analysis of variance are shown in Table 9.

Table 9. Analysis of Variance Results for Transfer to Metric

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>ms</th>
<th>F</th>
<th>p less than</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Groups</td>
<td>2</td>
<td>0.723</td>
<td>0.362</td>
<td>0.071</td>
<td>0.932</td>
</tr>
<tr>
<td>Within Groups</td>
<td>15</td>
<td>76.917</td>
<td>5.128</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The analysis of variance for transfer to metric data resulted in a failure to reject Hypothesis 3.

H₄: The three treatments are equally effective as measured by a conditional reasoning test.

The data for the class mean scores on the Cornell Conditional Reasoning Test - Form X are given in Table 10.
Table 10. Mean Scores on The Cornell Conditional Reasoning Test* by Treatment

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Treatment 1</th>
<th>Treatment 2</th>
<th>Treatment 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>29.4</td>
<td>45.7</td>
<td>39.2</td>
</tr>
<tr>
<td>2</td>
<td>37.5</td>
<td>37.7</td>
<td>41.2</td>
</tr>
<tr>
<td>3</td>
<td>35.0</td>
<td>34.8</td>
<td>29.6</td>
</tr>
<tr>
<td>4 **</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>39.8</td>
<td>38.6</td>
<td>37.2</td>
</tr>
<tr>
<td>6</td>
<td>37.1</td>
<td>36.2</td>
<td>39.7</td>
</tr>
</tbody>
</table>

Mean 35.760 38.600 37.380
SD 3.942 4.226 4.578

*Maximum possible score = 78
** Data not available.

The results of the analysis of variance on these data are given in Table 11.
Table 11. Analysis of Variance Results for Conditional Reasoning Scores

<table>
<thead>
<tr>
<th>Sources</th>
<th>df</th>
<th>SS</th>
<th>ms</th>
<th>F</th>
<th>p less than</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Groups</td>
<td>2</td>
<td>20.298</td>
<td>10.149</td>
<td>0.560</td>
<td>0.585</td>
</tr>
<tr>
<td>Within Groups</td>
<td>12</td>
<td>217.440</td>
<td>18.120</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The analysis of variance on conditional reasoning data resulted in a failure to reject Hypothesis 4.

While the classes were homogeneous with respect to intelligence, the presence of one honors class in the first cell for Treatment 2, led to the question as to how results would differ if analysis of covariance was used.

Therefore the decimal computation data were submitted to a post hoc analysis of covariance, with intelligence score as covariate. The results of the analysis of covariance are shown in Table 12.
The analysis of covariance on decimal computation data resulted in a failure to reject Hypothesis 1.

**Discussion**

The results of the analyses of the data collected in this investigation indicate that the three treatments were equally effective as measured by achievement tests in decimal computation, a transfer to metric test, and a conditional reasoning test. Each treatment did, however, result in significant gains in decimal computational ability from the pretest measure ($p < 0.001$). The gains also were significant from the pretest measure to the retention test measure. Thus each treatment was effective in promoting growth in the ability to compute with decimals.

Cognitive outcomes were the major outcomes investigated in this study. However, in light of the lack of significant differences across treatments, information also was gathered to look for affective outcomes in terms of teacher and student preferences. This information was
obtained by means of a questionnaire which was sent to each teacher who participated in the study. The teachers were asked to rank-order the three treatments in terms of their preference both prior to the study and after the study. The teachers also were asked to indicate the relative receptivity of their students to the three treatments. A copy of this questionnaire is contained in Appendix 3 of this report.

A summary of the data gathered from this questionnaire is shown in Table 13.

Table 13. Mean Rank of the Three Treatments

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Teacher Ranking</th>
<th></th>
<th>Student Receptivity as Reported by Teachers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Prior to Study</td>
<td>After Study</td>
<td></td>
</tr>
<tr>
<td>Fractions</td>
<td>1.2</td>
<td>1.6</td>
<td>2.0</td>
</tr>
<tr>
<td>Place-value</td>
<td>1.8</td>
<td>1.8</td>
<td>1.4</td>
</tr>
<tr>
<td>Number Line</td>
<td>3.0</td>
<td>2.6</td>
<td>2.4</td>
</tr>
</tbody>
</table>

1: Highest  2: Middle  3: Lowest

These data indicate that with one exception the order of teacher preference prior to treatment was 1. fractions, 2. place-value, and 3. number line. This same order held after treatment. However, three teachers changed their ranking after treatment. The shift in preference was away
from the common fraction definitional base, which is the
dominant definitional base used in contemporary curricular
materials.

While the teachers slightly favored the common fraction
definitional base, the student receptivity ranking placed
the place-value definitional base as the most preferred.
The student order of receptivity was 1. place-value,
2. common fraction, and 3. number line.

The place-value and number line definitional bases
seem to be as effective as the common fraction definitional
base in terms of the cognitive outcomes investigated. In
terms of teacher preference, the common fractions defini-
tional base was only slightly preferred to the place-value
definitional base. The students, however, preferred the
place-value definitional base to the common fraction def-
initional base. The number line definitional base was the
least preferred by both teachers and students.
Chapter VI
Conclusions and Recommendations

Conclusions

The major problem investigated in this study was to determine the effect of the definitional base for decimals on decimal computational ability of students. The following hypotheses were tested to determine this effect.

$H_1$: The three treatments are equally effective as measured by an achievement test of decimal computation.

$H_2$: The three treatments are equally effective as measured by a retention test of decimal computation.

These hypotheses were tested using analysis of variance. Analysis of the data obtained on the decimal computation measures resulted in failure to reject either hypothesis. Thus, the three treatments seemed to be equally effective in terms of decimal computational ability.

A secondary question, especially in educational research, is were the three treatments significantly effective? The data analysis for repeated measures indicated that all three treatments were effective at the 0.001 confidence level. This significance applied to both the posttest measure and the retention test measure. Thus, the three treatments were equally effective as measured by achievement tests of decimal computation. The effect of all three treatments was significant.
Another problem investigated in this study was to determine the effect of the definitional base for decimals on concomitant outcomes. The following hypotheses were tested relative to these effects.

$H_3$: The three treatments are equally effective as measured by a transfer to metric test.

$H_4$: The three treatments are equally effective as measured by a test of conditional reasoning ability.

These hypotheses were tested using analysis of variance. Analyses of the data obtained on the metric and conditional reasoning measures resulted in failure to reject either hypothesis. Thus, the three treatments seemed to be equally effective in terms of transfer to metric and conditional reasoning outcomes.

While hypotheses relative to preferences for definitional base were not submitted to statistical analysis, an effort was made to gather information to serve as an indicator of affective outcomes. The information gathered from the teacher questionnaire indicated some interesting results. The preference of teachers for the common fraction definitional base prior to treatment coincided with the definitional base used in the majority of mathematics textbooks and in the textbook used in the seventh-grade mathematics classes of the Columbus Public Schools. After the treatment period this preference remained, but it was not as pronounced as at pre-treatment. The student preference, as reported by the teachers, did not coincide with that of the teachers. The students preferred the place-value definitional base over the other two.
Implications

The results of this study have implications both in terms of mathematics curriculum and teacher attitudes. While the three treatments were equally effective in terms of the cognitive outcomes which were investigated, the three treatments differed on preferential criteria. If each of several definitional bases for a concept is effective for learning, then perhaps the definitional base should be chosen which is most compatible with teacher and student preferences.

Another curricular implication of the results of this study is in the area of sequencing. The students in this study who defined decimals in terms of their common fraction equivalents did not achieve better than those who used another definitional base. This outcome was obtained despite the fact that each treatment class had just completed a unit on common fractions. It could therefore be argued that due to factors of history and immediacy the common fractions definitional base would be expected to be more effective than either place-value or number line. However, while the differences were not significant, the scores on the criterion measures for the common fraction treatment group were consistently below those of the other two treatment groups.

It may be the case that the computational algorithms for common fractions need not be developed prior to those for decimals. It may be that the development of computational algorithms for decimals can precede those for common fractions, or they may be developed concurrently. It is desirable for skill in decimal computation to be developed by the upper elementary years for fostering measurement skills in the metric system. The results of this study indicate that this may be a viable curricular alternative.
A somewhat related curricular implication of the results of this study is in the area of instructional materials development. In the majority of mathematics textbooks and materials which deal with decimal concept formation, decimals are defined in terms of common fraction equivalents. If other definitional bases are as effective as the common fraction definitional base, then textbooks and materials should reflect these alternatives. Further, if the curriculum were to be restructured so that the development of decimal computational algorithms preceded the development of common fraction computational algorithms, then major revision would have to be done on elementary mathematics textbooks and materials.

Prior to the study, the participating teachers expressed a feeling of uneasiness about their ability to maintain purity of treatments. They predicated these feelings on the fact that they were unfamiliar with any other definitional base for decimals than the common fraction definitional base. For this reason, inservice training was provided for these teachers. The preferential results reported in this study well may reflect a feeling of greater confidence in teaching unfamiliar material. Whatever the cause, there is a need to examine the mathematics methods component of preservice teacher education programs and ensure that alternative definitional bases for concepts, where appropriate, are investigated.

The results of this study indicate that teacher preferences for instruction can be modified. While a halo effect may have been operating, teacher preferences for the three definitional bases did shift after they had worked in the three bases for a three-week period.
Suggestions for Improving and Extending the Study

In retrospect, there are several improvements and extensions which may be suggested for this study. These suggestions fall into the categories of data analysis, instructional materials, and inservice training.

The results of the analysis of variance on the data obtained from the tests of decimal computation suggest that the three definitional bases were equally effective. If the test data were to be subjected to an item analysis, additional information regarding treatment effects could be obtained. It would be possible with item analysis information to detect differences, if they exist, in effectiveness of the treatments with respect to each of the four arithmetic operations tested, addition, subtraction, multiplication, and division. It may be that one definitional base is more effective for addition and subtraction, and another is more effective for division. This study did not deal with that problem.

From discussions with students and teachers both during and after treatment, it was apparent that the instructional materials for the number line definitional base should have included more number line models for students to use. Both teachers and students expressed frustration with having to draw so many number lines. If more models had been provided, this frustration could have been eased and more time spent on investigating underlying principles. The discussions and observations also led to a recommendation that the development of the algorithm for decimal division should have been more deliberate and a greater number of examples should have been provided.

The inservice component was a very important component of this study. It was during this phase that teacher cooperation was gained and feelings of uneasiness were
lessened. It was not possible, however, in the time allocated for inservice training to work completely through each set of materials. In light of the importance of inservice training to the success of the investigation, the study would have been improved if a greater amount of time had been allocated for this activity.

Suggestions for Further Research

There has been limited research in the area of the development of computational algorithms for decimals. This investigation and its accompanying results suggest the need for further research in this area.

The three definitional bases which were investigated were equally effective for seventh-grade mathematics students. These students all had extensive experience with common fractions and common fraction computation prior to the treatment period. The extent to which this experience influenced the results of the study is not known. Therefore, there is a need to replicate this study with students in lower grades who have had less, or no, experience with common fractions.

In this study only two concomitant outcomes, transfer to metric and conditional reasoning ability, of decimal instruction were investigated. The study should be replicated with additional concomitant outcomes, such as divergent thinking or problem-solving. Since little is known about concomitant outcomes of algorithmic learning, algorithmic learning studies should address these questions.

Five of the six teachers in this study had prior teaching experience in seventh-grade mathematics and had taught decimals using the common fraction definitional base. They, however, had no experience with the place-value or number line definitional base. The extent to which this differential in experience with the treatments influenced
the outcomes of the investigation is not known. The study should be replicated with these same teachers as they now have experience in using each definitional base for decimal instruction.

Several interesting questions are raised by this investigation: Given alternative definitional bases for decimals, for which one will a student opt if given free choice? Will he choose consistently or will his choice vary with the situation? Does a student return to a definitional base when confronted with a problem situation or does he search for an algorithm? These questions apply to definition in general and are not specific to decimals. However, the area of decimal algorithmic learning is a viable setting for research related to these questions.

The study by Gregory (24) indicated that there was a positive relationship between conditional reasoning ability of students and frequency of teacher utilization of conditional reasoning paradigms in ordinary language. However, he did not investigate factors which may influence this frequency. Before a cause and effect relationship can be established between frequency of teacher utilization of the language of conditional logic and student conditional reasoning ability, a method for manipulating the frequency variable must be found. Does the context within which a teacher is operating affect this frequency? The frequency of conditional statements in the instructional materials prepared for this study was controlled. The number of conditional statements was the same across the three treatments. Tape recordings of each teacher in each class were made both prior to and during treatment. Thus the tapes contain both baseline and experimental data. The logical next step is to analyze these recordings in terms of frequency of teacher utilization of conditional reasoning
paradigms during instruction. This analysis could complete a link in the chain of research necessary to establish the causal relationship between teacher utilization of the language of conditional logic and conditional reasoning ability.
Appendix 1
Instructional Materials
Lesson 3

UNIT OBJECTIVE: Addition of Decimals

PERFORMANCE OBJECTIVES:

The student will be able to find the sum of a whole number and a decimal.

The student will be able to find the sum of two like decimals.

DEVELOPMENT:

What is the sum of 3 and .1?

If \( \frac{1}{10} \), then \( 3 + \frac{1}{10} = \frac{31}{10} = 3.1 \).

Thus, \( 3 + .1 = 3.1 \).

What is the sum of .5 and .4?

If \( \frac{5}{10} \) and \( \frac{4}{10} \), then \( .5 + .4 = \frac{5}{10} + \frac{4}{10} = \frac{9}{10} = .9 \).

Thus, \( .5 + .4 = .9 \).

What is the sum of .16 and .31?

If \( \frac{16}{100} \) and \( \frac{31}{100} \), then \( .16 + .31 = \frac{16}{100} + \frac{31}{100} = \frac{47}{100} = .47 \).

Thus, \( .16 + .31 = .47 \).

What is the sum of .05 and .06?

If \( \frac{5}{100} \) and \( \frac{6}{100} \), then \( .05 + .06 = \frac{5}{100} + \frac{6}{100} = \frac{11}{100} = .11 \).

Thus, \( .05 + .06 = .11 \).
Lesson 3

Write each decimal as a fraction with a power of 10 in the denominator.
1) .6 = __________ 2) .1 = __________ 3) .9 = __________ 4) .01 = __________
5) .12 = __________ 6) .07 = __________ 7) .23 = __________ 8) .009 = __________
9) .013 = __________ 10) .175 = __________

Write each fraction as a decimal.
11) \( \frac{7}{10} = \) __________ 12) \( \frac{1}{10} = \) __________ 13) \( \frac{4}{10} = \) __________ 14) \( \frac{3}{100} = \) __________
15) \( \frac{17}{100} = \) __________ 16) \( \frac{99}{100} = \) __________ 17) \( \frac{1}{100} = \) __________ 18) \( \frac{1}{1000} = \) __________
19) \( \frac{12}{1000} = \) __________ 20) \( \frac{213}{1000} = \) __________

Place a numeral in each blank to make a true statement.
21) \( 5 + .2 = 5 + \) __________
22) \( 7 + 13 = 13 + \) __________
23) \( 15 + .09 = 15 + \) __________
24) \( 22 + .12 = 22 + \) __________
25) \( 3 + .003 = 3 + \) __________
26) \( 13 + .4 = \frac{134}{10} + \) __________
27) \( .5 + .1 = \) __________
28) \( 12 + .25 = \frac{1225}{100} + \) __________
29) \( .11 + .51 = \) __________
30) \( .59 + .52 = \) __________
31) \( .026 + .351 = \) __________
32) \( .9 + .4 = \) __________
Place a word in each blank to make a true statement.

33) tenths + tenths = ____________
34) hundredths + hundredths = ____________
35) thousandths + thousandths = ____________

36) John shows the following work for 2 + .007. Is he correct? Why or why not?
If 2 = \( \frac{2000}{1000} \) and .007 = \( \frac{7}{1000} \), then
\[
2 + .007 = \frac{2000}{1000} + \frac{7}{1000} = \frac{2000 + 7}{1000} = \frac{2007}{1000} = 2.007
\]
Thus, 2 + .007 = 2.007
UNIT OBJECTIVE: Addition of Decimals

PERFORMANCE OBJECTIVES:

The student will be able to state the algorithm for adding like decimals.

The student will be able to add like decimals using the addition algorithms.

DEVELOPMENT:

If \( \frac{5}{10} + \frac{5}{10} \), then

\[ \frac{5}{10} + \frac{5}{10} = \frac{10}{10} = 1 \frac{1}{10} \]

If \( 0.17 + 0.68 \), then

\[ 0.17 + 0.68 = 0.85 \]

To add decimals which have the same number of places to the right of the decimal point, line up the decimal points in the addenda then add and line up the decimal point in the sum.
Add without changing decimals to common fractions.

<table>
<thead>
<tr>
<th>Addition</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75 + 0.0082</td>
</tr>
<tr>
<td>18.39 + 169.01</td>
</tr>
<tr>
<td>738.01</td>
</tr>
<tr>
<td>247.00</td>
</tr>
<tr>
<td>73.05</td>
</tr>
</tbody>
</table>

Name _______________________

Lesson 4

74
Place a word or numeral in each blank to make a true statement.

26) tenths + tenths = ________________
27) hundredths + hundredths = ________________
28) thousandths + thousandths = ________________

29) To add decimals which have the same number of decimal places, line up the _____ in the addends, _____, and then line up the decimal point in the _____.

30) .5 + .4 = .4 + ____ = ___. Therefore, addition of decimals is ________.

31) .3 + (.7 + .4) = ____ and (.3 + .7) + .4 = _____. Therefore, addition of decimals is ________.

32) 0 + .5 = .5 + 0 = ____.

33) How would you add .5 and .06?
UNIT OBJECTIVE: Addition of Decimals

PERFORMANCE OBJECTIVES:

The student will be able to state the algorithm for adding unlike decimals.

The student will be able to find the sum of two unlike decimals using the addition algorithm.

The student will be able to find the sum of two decimals when it is necessary to "carry across" the decimal point.

DEVELOPMENT:

In the last lesson you found that a short way to add like decimals is to line up the decimal points in the addends and the sum. Do you think that this method will work for finding the sum of .3 and .04?

If you think of .3 as 3/10 and .04 as 4/100, then

\[ .3 + .04 = \frac{3}{10} + \frac{4}{100} \]

\[ = \frac{30 + 4}{100} \]

\[ = \frac{34}{100} \]

\[ = .34 \]

If you try to use the rule for the same problem, then your work looks like this.

\[
\begin{array}{c}
\frac{30}{+104} \\
\hline
\frac{34}{-}
\end{array}
\]

Thus, the rule seems to work for unlike decimals.

Study some more examples to verify the rule.
Examples:

1. Find the sum of .1 and .16.

   If .1 = $\frac{1}{10}$ and .16 = $\frac{16}{100}$, then
   
   \[ .1 + .16 = \frac{1}{10} + \frac{16}{100} = \frac{10}{100} + \frac{16}{100} = \frac{26}{100} = .26 \]

   \[ \begin{array}{c}
   + \\
   \hline
   10 \\
   \hline
   16 \\
   \hline
   26
   \end{array} \]

2. Find the sum of .53 and .4.

   If .53 = $\frac{53}{100}$ and .4 = $\frac{4}{10}$, then
   
   \[ .53 + .4 = \frac{53}{100} + \frac{4}{10} = \frac{53}{100} + \frac{40}{100} = \frac{93}{100} = .93 \]

   \[ \begin{array}{c}
   + \\
   \hline
   53 \\
   \hline
   40 \\
   \hline
   93
   \end{array} \]

Notice how easy it is to find a "common denominator" for decimals.
Lesson 5

Place a word or numeral in each blank to make a true statement.

1) To add two decimals, line up the _________ ________ in the _________ and in the _________.

2) Since the common denominator for tenths and hundredths is _______, the sum of tenths and hundredths is ________.

3) Study the two solutions to the problem 2.87 + 3.62. Which one is correct? Use common fractions to prove your answer.

\[ \begin{array}{c c c c}
& 2.87 & + & 3.62 \\
\hline
& & & 6.49 \\
\end{array} \]

4) Place a numeral in each blank to make a true statement.

If \( \frac{6}{10} \) and \( \frac{17}{100} \), then

\( \frac{6}{10} + \frac{17}{100} = \frac{17}{100} = \) _______.  

Name ________________________
Find each sum. Use the short method. If you wish
to check your answers, use common fractions.

5) $0.45 + 0.23 = $ ______  6) $1.8 + 2.6 = $ ______  7) $0.09 + .42\frac{1}{4} = $ ______

8) $3.9 + 4.13 = $ ______  9) $0.0018 + .007 = $ ______  10) $0.91 + .73 = $ ______

11) $3.6 + 2.3\frac{4}{5} = $ ______  12) $77.5 + 7.32 = $ ______  13) $7 + .06 = $ ______

14) $0.01 + .001 = $ ______  15) $1 + .05 = $ ______  16) $3.33 + 6.47 = $ ______

17) $0.00389 + .00363 = $ ______  18) $18.6 + 7 = $ ______
19) 17 + 3.6 = 
20) 7.07 + 1.48 = 

21) 438.7 + 29.68 = 
22) 4.395 + 170.9 = 

23) 62.379 + 42.631 = 
24) 2.7851 + 49.907 = 

25) 2000.3 + 4.83 = 
26) 72.96 + 143.2 = 

27) 14.5 + 200.01 = 
28) .716 + 5.82 = 

Place a numeral or word in each blank to make a true statement.

29) If \( .23 + .6 = .6 + \_\), then addition of decimals is commutative.

30) If \( .4 + (.72 + .98) = (.4 + .72) + .98 \), then addition of decimals is ____________.

31) Is addition of decimals associative and commutative?

Use Exercises 29 and 30 to prove your answer.

Find the errors in each problem.

32) \( .5 + .07 = \_ .12 \_ \)
33) \( .5 + .07 = \_ .12 \_ \)
34) \( 4 + .12 = \_ .16 \_

\[
\begin{array}{c}
.5 \\
4.07 \\
1.2 \\
.5 \\
+.07 \\
+.12 \\
4 \\
+.12 \\
.16
\end{array}
\]
Lesson 3

UNIT OBJECTIVE: Addition of Decimals

PERFORMANCE OBJECTIVES:

The student will be able to find the sum of a whole number and a decimal.

The student will be able to find the sum of two like decimals.

DEVELOPMENT:

What is the sum of 3 and .1?

If .1 = 1\times 10^{-1} and 3 = 3\times 10^0, then

\[
3 + .1 = 3 \times 10^0 + 1 \times 10^{-1} = 3.1
\]

Thus, 3 + .1 = 3.1.

What is the sum of .5 and .4?

If .5 = 5\times 10^{-1} and .4 = 4\times 10^{-1}, then

\[
.5 + .4 = 5\times 10^{-1} + 4\times 10^{-1} = (5+4)\times 10^{-1} = 9 \times 10^{-1} = .9
\]

by the Distributive Property

Thus, .5 + .4 = .9.

What is the sum of .16 and .31?

If .16 = 1\times 10^{-1} + 6\times 10^{-2} and .31 = 3\times 10^{-1} + 1\times 10^{-2}, then

\[
.16 + .31 = 1\times 10^{-1} + 6\times 10^{-2} + 3\times 10^{-1} + 1\times 10^{-2} = (1+3)\times 10^{-1} + (6+1)\times 10^{-2} = 4 \times 10^{-1} + 7 \times 10^{-2} = .47
\]

Thus, .16 + .31 = .47.

What is the sum of .05 and .06?

If .05 = 5\times 10^{-2} and .06 = 6\times 10^{-2}, then

\[
.05 + .06 = 5\times 10^{-2} + 6\times 10^{-2} + 0\times 10^{-1} + 6 \times 10^{-2} = (5+6)\times 10^{-2} = 11 \times 10^{-2} = 1 \times 10^{-1} + 1 \times 10^{-2} = .11
\]

Thus, .05 + .06 = .11.
Lesson 3

Write each decimal in expanded notation.

1) .6 = ____________________________
2) .9 = ____________________________
3) .3 = ____________________________
4) .01 = ____________________________
5) .12 = ____________________________
6) .07 = ____________________________
7) .23 = ____________________________
8) .009 = ____________________________
9) .013 = ____________________________
10) .175 = ____________________________

Write each numeral in expanded notation as a decimal.

11) 7x10⁻¹ = _____________
12) 1x10⁻² = _____________
13) 4x10⁻¹ = _____________
14) 3x10⁻² = _____________
15) 1x10⁻¹ + 7x10⁻² = _____________
16) 9x10⁻² + 9x10⁻³ = _____________
17) 1x10⁻² = _____________
18) 1x10⁻³ = _____________
19) 1x10⁻² + 2x10⁻³ = _____________
20) 723x10⁻³ = _____________
Place a numeral in each blank to make a true statement.

21) \(5 + 2 \times 5x \quad + \quad 2x \quad + \quad x\)

22) \(7 + 13 \times 7x \quad + \quad 1x \quad + \quad 3x \quad + \quad 5x\)

23) \(15 + 0.9 \times 1x \quad + \quad 5x \quad + \quad 10x \quad + \quad 9x \quad + \quad x\)

24) \(2.2 + 0.42 \times 2x \quad + \quad 2y \quad + \quad 4x \quad + \quad 2y \quad + \quad 2x\)

25) \(34.003 \times 3x \quad + \quad 3x\)

26) \(3 + 4 \times 3x \quad + \quad 4x \quad = \quad (3 + 4) \times x\)

27) \(5 + 1 \times 5x \quad + \quad 1x \quad = \quad (5 + 1) \times x\)

28) \(12 + 25 \times 1x \quad + \quad 2x \quad + \quad 2y \quad + \quad 5x \quad + \quad (1 + 2) \times x + (2 + 5) \times x\)

29) \(11 + 51 \times 1x \quad + \quad 8x \quad + \quad 5x \quad + \quad 11 \quad + \quad (1 + 5) \times x \quad + \quad (1 + 1) \times x\)

30) \(59 + 32 \times (5 + 3) \times x \quad + \quad (9 + 2) \times x\)

31) \(0.036 \times 0.351 \times (6 + 3) \times x \quad + \quad (2 + 5) \times x \quad + \quad (6 + 1) \times x\)

32) \(9 + 4 \times (9 + 4) \times x \quad + \quad \times 10^6 \quad + \quad \times 10^6\)

Place a word in each blank to make a true statement.

33) Tenths + Tenths = 

34) Hundredths + Hundredths = 

35) Thousandths + Thousandths = 

36) John shows the following work for \(2 + .007\). Is he correct? Why or why not?

If \(2 = 2000 \times 10^{-3R}\) and \(.007 = 7 \times 10^{-3R}\), then

\[2 + .007 = (2000 + 7) \times 10^{-3R} = 2007 \times 10^{-3R} = 2.007\]

Thus, \(2 + .007 = 2.007\).
Lesson 4

UNIT OBJECTIVE: Addition of Decimals

PERFORMANCE OBJECTIVES:

The student will be able to state the algorithm for adding like decimals.

The student will be able to add like decimals using the addition algorithm.

DEVELOPMENT:

If \( .5 = 5 \times 10^{-1} \) and \( .4 = 4 \times 10^{-1} \), then

\[
.5 + .4 = (5 + 4) \times 10^{-1} = 9 \times 10^{-1} = .9
\]

If \( .17 = 1 \times 10^{-1} + 7 \times 10^{-2} \) and \( .68 = 6 \times 10^{-1} + 8 \times 10^{-2} \), then

\[
.17 + .68 = (1 + 6) \times 10^{-1} + (7 + 8) \times 10^{-2} = 7 \times 10^{-1} + 15 \times 10^{-2} = 8 \times 10^{-1} + 5 \times 10^{-2} = .85
\]

If \( .6 = 6 \times 10^{-1} \) and \( .7 = 7 \times 10^{-1} \), then

\[
.6 + .7 = 6 \times 10^{-1} + 7 \times 10^{-1} = (6 + 7) \times 10^{-1} = 13 \times 10^{-1} = 1 \times 10^0 + 3 \times 10^{-1} = 1.3
\]

To add decimals which have the same number of places to the right of the decimal point, line up the decimal points in the addends, then add and line up the decimal point in the sum.
Lesson 4

Add without using expanded notation.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.3</td>
<td>2</td>
<td>.7</td>
<td>3</td>
</tr>
<tr>
<td>+</td>
<td>.5</td>
<td>+</td>
<td>.3</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>1.1</td>
<td>+</td>
<td>.9</td>
<td>+</td>
</tr>
</tbody>
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Place a word or numeral in each blank to make a true statement.

26) tenths + tenths =
27) hundredths + hundredths =
28) thousandths + thousandths =
29) To add decimals which have the same number of decimal places, line up the ____________ in the addends, ____________, and then line up the decimal point in the ____________.
30) .5 + .4 + __ = __. Therefore, addition of decimals is ____________.
31) .3 + (.7 + .4) = __ and (.3 + .7) + .4 = __. Therefore, addition of decimals is ____________.
32) 0 + .5 = .5 + 0 = __.
33) How would you add .5 and .06?
Lesson 5

UNIT OBJECTIVE: Addition of Decimals

PERFORMANCE OBJECTIVES:

- The student will be able to state the algorithm for adding unlike decimals.
- The student will be able to find the sum of two unlike decimals using the addition algorithm.
- The student will be able to find the sum of two decimals when it is necessary to "carry across" the decimal point.

DEVELOPMENT:

In the last lesson you found that a short way to add like decimals is to line up the decimal points in the addends and the sum. Do you think that this method will work for finding the sum of .3 and .04? If you think of .3 as $3 \times 10^{-1}$ and .04 as $0 \times 10^{-1} + 4 \times 10^{-2}$, then

\[
.3 + .04 = 3 \times 10^{-1} + 0 \times 10^{-1} + 0 \times 10^{-1} + 4 \times 10^{-2} \\
= (3+0) \times 10^{-1} + (0+4) \times 10^{-2} \\
= 3 \times 10^{-1} + 4 \times 10^{-2} \\
= .34
\]

If you try to use the rule for the same problem, then your work looks like this.

\[
\begin{array}{c}
30 \\
\hline
4 \\
\hline
34
\end{array}
\]

Thus, the rule seems to work for unlike decimals.

Study some more examples to verify the rule.

Examples:

1. Find the sum of .1 and .16.

   \[
   .1 + .16 = 1 \times 10^{-1} + 1 \times 10^{-1} + 6 \times 10^{-2} \\
   = (1+1) \times 10^{-1} + 6 \times 10^{-2} \\
   = 2 \times 10^{-1} + 6 \times 10^{-2} \\
   = .26
   \]

\[
\begin{array}{c}
10 \\
\hline
4 \\
\hline
14
\end{array}
\]


2. Find the sum of .53 and .4.

If \( .53 \times 10^1 + 3 \times 10^0 \) and \( .4 \times 10^0 \), then

\[
.53 + .4 = .53 \times 10^1 + 3 \times 10^0 + .4 \times 10^0 = (5.3) \times 10^0.
\]

\[
\begin{array}{c}
.53 \\
+ .40 \\
\hline
.93
\end{array}
\]

Notice how easy it is to change unlike decimals to like decimals.
Lesson 5

1) Place a word or numeral in each blank to make a true statement.

To add two decimals, line up the ____________ and in the ____________.

2) Study the two solutions to the problem $2.87 + 3.62$. Which one is correct? Use expanded notation to prove your answer.

   a) $2.87$
   b) $2.17$
   + $3.62$
   + $3.62$
   $5.49$
   $6.49$

3) Place a numeral in each blank to make a true statement.

If $.6 = 6 \times ____$ and $.17 = 1 \times ____ + 7 \times ____$, then

$.6 + .17 = (6+1) \times ____ + 7 \times ____ = ____$

Find each sum. Use the short method. If you wish to check your answers, then use expanded notation.

4) $.45 + .23 = ____$
5) $1.8 + 2.6 = ____$
6) $.09 + .024 = ____$

7) $3.8 + 4.13 = ____$
8) $.0012 + .007 = ____$
9) $.98 + .73 = ____
10) 3.4 + 2.34 = ___
11) 77.5 + 7.32 = ___
12) 7 + .06 = ___

13) .01 + .001 = ___
14) .1 + .05 = ___
15) 3.33 + 4.67 = ___

16) .00389 + .00263 = ___
17) 41.4 + 7 = ___
18) 17 + 3.6 = ___

19) 7.07 + 1.43 = ___
20) 438.7 + 29.61 = ___

21) 4.395 + 170.9 = ___
22) 62.379 + 42.631 = ___

23) 2.7831 + 49.707 = ___
24) 200.3 + 4.83 = ___
25) $48.96 + 143.2 = $ 
26) $14.5 \times 200.01 = $ 

27) $0.716 + 3.22 = $ 

Place a numeral or word in each blank to make a true statement.

28) If $0.23 + .6 = .6 + $, then addition of decimals is commutative.

29) If $(.4 + (.72 + .98)) = (.4 + .72) + .98$, then addition of decimals is __________.

30) Is addition of decimals associative and commutative?

Use Exercises 28 and 29 to prove your answers.

Find the errors in each problem.

31) $.5 + .07 = .12$  
32) $.5 + .07 = .12$  
33) $.4 + .12 = .16$

\[ \frac{.5}{.5} \]  
\[ +.07 \]  
\[ \frac{1.2}{.12} \]  

\[ \frac{.5}{.5} \]  
\[ +.07 \]  
\[ +.12 \]  
\[ .16 \]
UNIT OBJECTIVE: Addition of Decimals

PERFORMANCE OBJECTIVES:

The student will be able to find the sum of a whole number and a decimal on the number line.

The student will be able to find the sum of two like decimals on the number line.

DEVELOPMENT:

What is the sum of \(3\) and \(0.1\)? If \(0.1\) is one tenth unit on the number line, then \(3 + 0.1\) is shown as follows.

A Jump of 3 units followed by a jump of one tenth unit ends at 3.1. Thus, \(3 + 0.1 = 3.1\).

What is the sum of \(0.5\) and \(0.4\)? If \(0.5\) is five tenth units and \(0.4\) is four tenth units, then \(0.5 + 0.4\) is shown as follows.

A Jump of five tenths followed by a jump of four tenths is the same as a jump of nine tenths. Thus, \(0.5 + 0.4 = 0.9\).

What is \(0.05 + 0.06\)? If \(0.05\) is five hundredths and \(0.06\) is six hundredths, then \(0.05 + 0.06\) is shown as follows.

A Jump of five hundredths followed by a jump of six hundredths is the same as a jump of eleven hundredths. Thus, \(0.05 + 0.06 = 0.11\).
Lesson 3

Write the decimal name for each point on the number line.

Locate each decimal on the number line.

- 0.6
- 0.8
- 0.01
- 3.12
- 5.77
- 4.23
- 7.09
- 2.96
- 9.99
- 10.9
Show each addition on the number line.

21) $5 + 2 = $  

22) $13 + 7 = $  

23) $15 + 0.9 = $  

24) $22 + 4.2 = $  

25) $3 + 0.3 = $  

26) $3 + 4 = $  

27) $5 + 1 = $  

28) $12 + 2.5 = $  

29) $18 + 51 = $  

30) $59 + 32 = $  

31) $26 + 35 = $  

32) $9 + 4 = $
Place a word in each blank to make a true statement.

33) tenths + tenths =

34) hundredths + hundredths =

35) thousandths + thousandths =

36) John added .12 + .25 as follows.

\[ \begin{array}{c}
0 & 0.1 & 0.2 & 0.3 & 0.4 \\
\hline
0 & 0.1 & 0.2 & 0.3 & 0.4 \\
\end{array} \]

Naty added .12 + .25 as follows.

\[ \begin{array}{c}
0 & 0.1 & 0.2 & 0.3 & 0.4 \\
\hline
0 & 0.1 & 0.2 & 0.3 & 0.4 \\
\end{array} \]

Are they both correct? Why or why not?

37) Try adding .59 + .32 using John's method.
Lesson 1

UNIT OBJECTIVE: Addition of Decimals

PERFORMANCE OBJECTIVES:

The student will be able to state the algorithm for adding like decimals.

The student will be able to add like decimals using the addition algorithm.

DEVELOPMENT:

If .5 is five tenths and .4 is four tenths, then

\[
\begin{align*}
& \quad \text{.5} \\
+ & \quad \text{.4} \\
\hline
& \quad \text{.9}
\end{align*}
\]

This is true because a jump of five tenths followed by a jump of four tenths is the same as a jump of nine tenths.

If .17 is seventeen hundredths and .68 is sixty-eight hundredths, then

\[
\begin{align*}
& \quad \text{.17} \\
+ & \quad \text{.68} \\
\hline
& \quad \text{.85}
\end{align*}
\]

This is true because a jump of seventeen hundredths followed by a jump of sixty-eight hundredths is the same as a jump of eighty-five hundredths.

If .6 is six tenths and .7 is seven tenths then

\[
\begin{align*}
& \quad \text{.6} \\
+ & \quad \text{.7} \\
\hline
& \quad \text{1.3}
\end{align*}
\]

This is true because a jump of six tenths followed by a jump of seven tenths is the same as a jump of thirteen tenths or one and three tenths.

To add decimals which have the same number of places to the right of the decimal point, line up the decimal points in the addends, then add and line up the decimal point in the sum.
Lesson 41

Add:

1) 0.3 + 0.5 = 0.8
2) 0.23 + 0.5 = 0.73
3) 0.01 + 0.04 = 0.05
4) 0.3 + 0.5 = 0.8
5) 0.67 + 0.31 = 0.98
6) 0.2 + 0.47 = 0.67
7) 0.032 + 0.432 = 0.464
8) 0.452 + 0.2019 = 0.6539
9) 0.953 + 0.007 = 0.960
10) 0.7 + 0.1742 = 0.8742
11) 0.17002 + 0.82998 = 0.9999
Place a word or numeral in each blank to make a true statement.

26) tenths + tenths = __________________________
27) hundredths + hundredths = ______________________
28) thousandths + thousandths = ______________________
29) To add decimals which have the same number of decimal places, line up the _______ _______ in the addends, _______ and line up the decimal point in the.

30) .5 + .4 + .4 + ___. Therefore, addition of decimals is _____________________.
31) .3 + (.7 + .4) = ___ and (.8 + .7) + .4 + ___. Therefore, addition of decimals is _____________________.
32) 0 + .5 + .5 + 0 = ___.
33) How would you add .5 and .06?
UNIT OBJECTIVE: Addition of Decimals

PERFORMANCE OBJECTIVES:

The student will be able to state the algorithm for adding unlike decimals.

The student will be able to find the sum of two unlike decimals using the addition algorithm.

The student will be able to find the sum of two decimals when it is necessary to "carry across" the decimal point using the addition algorithm.

DEVELOPMENT:

In the last lesson you found that a short way to add decimals is to line up the decimal points in the addends and the sum. Do you think that this method will work for finding the sum of .3 and .04?

If you locate .3 on the number line, you find that it is the same point as .30. Then, a jump of thirty hundredths followed by a jump of four hundredths is the same as a jump of thirty-four hundredths.

\[
\begin{array}{c}
\text{.30} \\
+ \text{.04} \\
\hline
\text{.34}
\end{array}
\]

Thus, the rule seems to work for unlike decimals.

Study some more examples to verify the rule.

Examples:

Find the sum of .1 and .16.
If you locate .1 on the number line, you find that .1 is the same point as .10. A jump of ten hundredths followed by a jump of sixteen hundredths is the same as a jump of twenty-six hundredths.

Find the sum of .53 and .4.

Make a jump of fifty-three hundredths. Now if you make a jump of forty hundredths, it is the same as a jump of four tenths. A jump of fifty-three hundredths followed by a jump of forty hundredths is the same as a jump of ninety-three hundredths.

\[
\begin{array}{c}
53 \\
+ 40 \\
\hline
93
\end{array}
\]
Lesson 5

Name__________________________

Place a word or numeral in each blank to make a true statement.

1) To add two decimals, line up the___________________
in the___________ and in the___________

2) Since a jump of 1 tenth is the same as a jump of
10 hundredths, the sum of tenths and hundredths is

3) Study the two solutions to the problem 2.87 + 3.62.
Which one is correct? Use a number line to prove
your answer.

\[
\begin{array}{c}
\text{① 2.87} \\
+ 3.62 \\
\hline
5.49
\end{array}
\hspace{1cm}
\begin{array}{c}
\text{② 2.87} \\
+ 3.62 \\
\hline
6.49
\end{array}
\]

4) Place a numeral or word in each blank to make a
true statement.

If .6 = ______ hundredths and .17 = 17 ________,
then .6 + .17 = ______ hundredths + 17 ________ =
____ hundredths = ___.
Find each sum. Use the short method. If you wish to check your answers, then use a number line.

5) \( .45 + .23 = \)  
6) \( 1.8 + 2.6 = \)  
7) \( .09 + .04 = \)

8) \( .38 + .13 = \)  
9) \( .0018 + .007 = \)  
10) \( .98 + .73 = \)

11) \( 3.6 + 2.34 = \)  
12) \( 77.5 + 7.32 = \)  
13) \( .7 + .06 = \)

14) \( .01 + .001 = \)  
15) \( .1 + .05 = \)  
16) \( 3.53 + 6.67 = \)

17) \( .00389 + .00163 = \)  
18) \( .416 + 7 = \)  
19) \( 1.7 + 3.6 = \)

20) \( 7.07 + 1.43 = \)  
21) \( 432.7 + 29.68 = \)  
22) \( 4395 + 170.9 = \)
28) 62.379 + 42.631 = ________  29) 2.7131 + 49.907 = ________

25) 2.000.3 + .83 = ________  26) 79.36 + 143.2 = ________

27) 14.5 + 200.01 = ________  28) .716 + 3.82 = ________

Place a numeral or word in each blank to make a true statement.

29) If .23 + .6 = .6 + ________, then addition of decimals is commutative.

30) If .4 + (.72 + .91) = (.4 + .72) + .91, then addition of decimals is ________.

31) Is addition of decimals associative and commutative?

Use Exercises 29 and 30 to prove your answer.
Appendix 2

Decimal Computation Test

Add or subtract as indicated.

1) \(138.35 + 2.53.42\)  
2) \(638.32 + 182.08 + 24.19\)

3) \(2.3407 + 0.039 + 1.753 + 3.4619\)  
4) \(1.4 + 3.8 + 3.7\)

5) \(2.05 + 1.06 + 4.69 + 0.08\)  
6) \(253.28 - 947.93\)

7) \(5 - 1.43\)  
8) \(3.56 - .9\)

9) \(4.9 - 807\)  
10) \(3.4 - 60.56\)

Multiply or divide as indicated.

11) \(.58 \times .6\)  
12) \(.147 \times .03\)

13) \(.31416 \times .75\)  
14) \(.05 \times .01\)

15) \(75 \times .48\)  
16) \(5124 \div 6\)

17) \(108.72 \div 1.2\)  
18) \(2.4472 \div .56\)

19) \(112 \div .014\)  
20) \(.60924 \div .231\)

21) \(53.75 \div .125\)

Which of the symbols \(<, =, >\) or \(\neq\) should be placed in each blank to make a true statement?

22) \(.5 \_ .05\)  
23) \(.9 \_ .90\)

24) Every second that a ball falls through the air its speed increases at the rate of 32.2 feet per second. How fast is the ball traveling after it has fallen for 4 seconds?
Appendix 3
Teacher Questionnaire

1. I will be teaching (or have already taught) the metric system to the classes used in the study before March 14.
   ___Yes   ___No

2. Before the study I would have ranked the three treatments in order of my preference (1-highest, 2-medium, 3-lowest):
   ___Fractions  ___Place-value  ___Number line

3. After the study I rate the three treatments in order of my preference:
   ___Fractions  ___Place-value  ___Number line

4. My students seemed to prefer the treatments in this order (highest to lowest): 1. ____________________
   2.___________________  3. _______________

5. General Comments:

__________________________________________________________________________
__________________________________________________________________________
__________________________________________________________________________
__________________________________________________________________________
__________________________________________________________________________

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Appendix 4

Transfer to Metric Test

In the metric system the basic unit of length is the METER (m). The basic unit of weight is the GRAM (g). The basic unit of volume is the LITER (L). Prefixes are used with each of the basic units to form other units. For example, another unit of length is the decimeter (dm) which is one tenth meter (.1 m). The prefix deci- means one tenth.

- milli- means one thousandth
- centi- means one hundredth
- deci- means one tenth
- deka- means ten
- hecto- means one hundred

Place a numeral in each blank to make a true statement.

1. 1 m = ___ dm  2. 1 m = ___ cm  3. 1 m = ___ km
4. 1 cm = ___ m  5. 1 mm = ___ m  6. 1 dm = ___ cm
7. 1 km = ___ cm  8. 75 cm = ___ m  9. 13 m = ___ cm
10. 27 cm = ___ m  11. 1 kg = ___ g  12. 1 g = ___ mg
13. 1 mg = ___ g  14. 1 mL = ___ L  15. 1 L = ___ mL
Appendix 5

Decimal Computation Retention Test

Add or subtract as indicated.

1) 251.36 + 436.43  2) 731.29 + 103.45 + 16.98
3) 6.4512 + .0057 + 2.257 + 9.112  4) 2.6 + 9.6 + .52
5) 8.16 + .237 + 6.31 + .06  6) .95319 - .94218
7) 10 - 2.75  8) 4.52 - .7
9) 4.9 - .708  10) 2.1 - .0072

Multiply or divide as indicated.

11) .67 x .4  12) .324 x .05
13) 3.1416 x .84  14) .07 x .02
15) 75 x .57  16) 5.124 ÷ 3
17) 81.44 ÷ 1.1  18) 1.2236 ÷ .28
19) 11.2 ÷ .014  20) .00693 ÷ .231
21) 87.5 ÷ .025

Place <, >, or = in each blank to make a true statement.

22) .70 _____ .7  23) .03 _____ .9

24. There are 39.37 inches in a meter. If Robert is 1.8 meters tall, how tall is he in inches?
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PLEASE NOTE:

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