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ANALYSIS AND DESIGN OF HIGH BEAM EFFICIENCY
APERTURE ANTENNAS

DISSER TAT ION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

Carl Allan Mentzer, B.E.E., M.S.E.E.

The Ohio State University
1974

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CHAPTER I
INTRODUCTION

With the increasing use of electromagnetic systems comes the increased probability of interference between two or more of these systems. One means of improving the electromagnetic compatibility (E.M.C.) of radiating systems is to reduce the extraneous coherent signal (either deliberate, as in jamming, or accidental) received by the antenna by using an antenna which has a radiation pattern with low sidelobe and backlobe levels. This is also required if one is to develop a low noise (or low temperature) antenna. Here noise signals from directions of arrival other than the desired signal would now be more highly attenuated. The antenna beam efficiency represents a valid figure of merit for a directional antenna's radiation properties. The beam efficiency of an antenna is defined to be the ratio of the power radiated in the main beam region to the total power radiated, or[1]:

\[
B = \frac{\int_{0}^{\theta_1} \int_{0}^{\phi_1} S \sin \theta \, d\theta \, d\phi}{\int_{0}^{2\pi} \int_{0}^{\pi} S \sin \theta \, d\theta \, d\phi}
\]

where \( S \) is the transmitted power density. The angular region near the main beam is usually limited to the region between the first nulls of the pattern. For an antenna such as the corrugated horn which has no definite lobe structure, this is a poor definition. A better definition is the percentage of the power radiated contained in twice the half power beamwidth of the antenna (i.e., \( \theta_1 = 2 \times \text{HPBW} \)). With this definition, it is apparent that a low noise antenna (i.e., one which has a low-susceptibility to interfering noise because of its low sidelobe and backlobe levels) will have a high beam efficiency. The term "high beam efficiency" is better than "low noise" in describing this antenna property since it avoids possible confusion with other system noise properties such as the thermal noise or antenna noise temperature. For communications systems or for radiometer systems, a high beam efficiency antenna is a necessity. In communications systems, two antennas mounted on a tower may interfere substantially with each other (because of the small separation) even though both of the antennas have relatively low sidelobe levels. In radiometer systems, where the radiometer integrates the energy received by the antenna to measure the temperature of an extended surface, it is important to accurately know the radiation pattern (including sidelobe and backlobe levels) to allow the design engineer to evaluate the suitability of the proposed antenna for the intended application.
It is important to realize that the application influences the choice of antenna to be used in a given system. For example, an antenna with a relatively high backlobe when used for an airborne radiometer system looking at the ground is acceptable if the backlobe is "seeing" the cold sky. The same radiometer system under different conditions could be in serious error if this backlobe now "sees" the warm earth or the sun. This simple example illustrates the need for an accurate yet simple technique for obtaining the antenna radiation pattern prior to an expensive construction and measurement program. The effort in this dissertation is focussed on the antenna lobe structure in the two principal planes (the E-plane and the H-plane) and is not directed toward determining the beam efficiency. The comparisons in this dissertation do not require the added sophistication of evaluating the beam efficiency since the relative merits of these antenna types can be established directly from the patterns.

High beam efficiency antennas, i.e., directional antennas with low side and back lobes, have represented a topic of interest for many years. Perhaps the first such antenna was the horn reflector as developed and used extensively throughout the United States by the Bell Telephone System for microwave communications links. This antenna is still undergoing design improvements: Thomas[2] in recent years has developed a system of blinders to reduce an E-plane sidelobe at 90° on the pattern. Other high beam efficiency (low noise) aperture antennas include the corrugated horns[3-9], both pyramidal and conical horns, and the dual mode horn[10]. The horns have similar patterns but differ in the operating bandwidth[11] (a 2:1 bandwidth for the corrugated horn versus only a 10% bandwidth for the dual mode horn). Because of the similarity of the pattern structure, the dual mode horn has not been included in the antennas analyzed in this dissertation. A corrugated dual mode horn has been built and the resulting patterns show extremely low sidelobe levels for a very narrow band of operating frequencies[12]. The operating bandwidth of only a few percent make this horn of limited interest.

The work presented here includes computer programs in the Fortran IV language to compute the complete radiation patterns including sidelobe and backlobe levels of nearly all of the commonly used aperture antennas. The antenna types considered include the conventional pyramidal electromagnetic horn, the pyramidal corrugated horn, the horn-reflector, the corrugated horn-reflector, the offset fed parabola and the cassegrain. The center fed parabola has already been accurately treated[13] and is not included here.

The analysis of the above antenna types use conventional aperture integration techniques[14,15,16] combined with the Geometrical Theory of Diffraction[17] (G.T.D.) and its extensions including equivalent currents[18] and slope wave diffraction[19,20]. The aperture integration is used to obtain the antenna directivity and radiation pattern near the main beam. The G.T.D. is then used to obtain the
balance of the radiation pattern using the principle of stationary phase where applicable and equivalent electric and magnetic line sources when stationary phase does not apply.

Of the aperture antennas considered here, the one least suitable for high beam efficiency (or low noise) applications is the conventional electromagnetic horn. It is included in Chapter II not as an example of a low noise antenna but as a convenient antenna to verify the validity of an analytical technique. The technique used to analyze the H-plane pattern of the horn is that of slope wave diffraction[21]. Both the H-plane and the E-plane patterns of the electromagnetic horn have been previously treated using conventional G.T.D. techniques[22,23]. The H-plane analysis previously used, required a large number of rays to be included and was a tedious programming problem. The new slope diffraction coefficients, discussed in Appendices A and B, allow a simple diffraction analysis of the H-plane edge contribution instead of the previous complex H-plane horn analysis. This earlier analysis required one to account for many reflected rays inside the horn. This work also for the first time combines the equivalent current concepts with the slope diffraction coefficient to allow computation of radiation patterns outside the principal planes of the horn.

The first antenna considered which fits the high beam efficiency (low noise) category is the corrugated horn. This antenna has been established as having low sidelobes and backlobes[24] and for square pyramidal or conical horns, axially symmetric radiation patterns[25, 26,27]. Our present study focusses attention on the pyramidal corrugated horn which, because of the finite length straight edges, is the most complex to analyze. The same general concepts are applicable to the conical corrugated horn, the major difference being that it would not require retaining the equivalent current concepts except for the main beam region and the back lobe region. The balanced hybrid mode used for the analysis of corrugated horns has been based on the assumption that the corrugated walls represent an infinite impedance, forcing the tangential magnetic field to zero. Chapter III examines the properties of the corrugated wall through an integral equation analysis[28]. This analysis examines the surface current density on the walls of the corrugations, the power loss in the corrugations, as well as the fields scattered by the onset of the corrugations as a function of the corrugation density, depth, and shape. Also included here is an analysis of a new shape corrugation, a V-shape, which may find spacecraft applications because of the ability of the surface to fold like a camera bellows. Chapter IV presents a technique to calculate the full radiation pattern of the corrugated horn whereas previously only the main beam region could be predicted from the standard design curves for the conventional horn H-plane patterns[29]. Also included is a set of backlobe level curves as a design aid, and in Appendix E, measured aperture distributions and internal fields for an X-band corrugated horn.
In Chapter V, the horn reflector antenna is considered. This antenna, formed by placing a segment of a paraboloidal reflector over a conventional horn antenna, is widely used in communications systems [30,31,32]. The computer program developed for the analysis of the pyramidal horn reflector should be a valuable design tool since it allows one to examine many possible designs having the same beamwidth. In addition, the program allows one to calculate the radiation patterns which would result from placing the paraboloidal reflector on top of a pyramidal corrugated horn.

Turning to the class of reflector antennas, one usually thinks first of the center fed paraboloid. This antenna is not included in this work since it has been accurately analyzed by Ratnasiri [33]. One of the drawbacks associated with medium beamwidth center fed paraboloidal reflector is the spillover and aperture blockage due to the feed horn and supporting members. The offset fed parabola eliminates the aperture blockage problem by using only a segment of a paraboloidal reflector. This antenna has been analyzed previously by this author and others [34] and the analysis is therefore only summarized in Chapter VI. For small beamwidth applications, the cassegrain or dual reflector is commonly used. This antenna usually has a paraboloidal reflector and a hyperboloidal subreflector. While there has been considerable work done on the design and modification of the cassegrain [35-39], only the main beam region of the pattern has been considered. In Chapter VII, the sidelobe levels are calculated and some design recommendations made.

Finally, the results of these various computations are summarized in the concluding chapter and recommendations are made based on antenna beamwidth, sidelobe characteristics and mechanical structure.
CHAPTER II
PATTERN ANALYSIS OF CONVENTIONAL HORN ANTENNAS

It has previously been established that the pyramidal horn antenna has high sidelobes and backlobes, particularly in the E-plane[40] and as such represents a poor radiometer antenna. However, the H-plane G.T.D. analysis of the pyramidal horn was quite complex for a relatively simple pattern shape and since this analysis represents the starting point for other low side and backlobe structures, a substantial effort is focused on improving the analysis in the present chapter. Thus the goal of this chapter is distinct in that here attention is focused on further developing the G.T.D. solution for the antenna whereas in the remaining chapters, the major goal is to seek and identify the sources of high lobes and as much as is practical to reduce the level of such lobes.

Principal plane radiation patterns of horn antennas have been accurately computed using the Geometrical Theory of Diffraction (G.T.D.) by treating the pyramidal (three dimensional) horn as a sectoral (two dimensional) horn[40,41]. The original G.T. D. study of the horn included the first order diffracted rays shown in Fig. 1. In the E-plane, the direct ray (ray "A" in the E-plane of Fig. 1) accurately approximated the diffraction from the horn-waveguide junction and when combined with the edge rays ("B" and "C") yielded good results except for a discontinuity at the point where ray "B" (or "C") became shadowed by the opposite wall of the horn. Fortunately, this discontinuity is significant only in the vicinity of the shadow boundary where the diffracted fields are maximum. These discontinuities were removed by including higher order diffracted rays (for example, ray "D" as indicated). With each multiply diffracted ray came a new (but fortunately smaller amplitude) discontinuity. By including all the appropriate images, one could calculate with great accuracy the E-plane pattern. The situation in the H-plane however is more complex since the single direct ray from the horn vertex to the H-plane edges is not a good approximation when using conventional G.T.D. since the electric field vanishes along the horn walls. The horn-waveguide junctions are illuminated by a pair of plane waves bouncing back and forth in the waveguide. In this case, there are many significant diffracted rays originating from the horn-waveguide junction. These include singly diffracted rays (rays 1 and 2 in the H-plane of Fig. 1), image rays (rays 3-5), and multiply diffracted rays (rays 6-9). This technique, while yielding good pattern results when the effects of the E-plane edges were included for the backlobe, was clearly very tedious and at

*This G.T.D. solution for the H-plane pattern did not include the diffraction from the E-plane edges near the main beam axis and the solution is in error but neglecting this term does not alter the pattern significantly for reasons to be discussed.
best a difficult bookkeeping task on the computer. For large horns with low flare angles, the situation is particularly tedious. This chapter presents some alternatives to the above procedure which simplify the pattern calculations in the principal planes and in addition allow pattern calculations in any arbitrary plane.

Fig. 1—Original G.T.D. analysis of conventional horn showing only part of the first order diffracted rays which must be included in the principal planes.

The techniques for computing radiation patterns discussed in this chapter include the conventional aperture integration, as obtained by Schelkunoff[42], and the G.T.D. and its extensions. The extensions of the G.T.D. include the equivalent current concept as formulated by Ryan and Peters[43] in terms of conventional diffraction concepts and extends this concept to include a new equivalent current. This equivalent current contains the effect of the diffracted fields when the source (or the observation point) lies on the surface of the wedge for the soft boundary condition. This equivalent current makes use of a new form of the slope diffraction coefficient developed by Keller[6]. This new form of the slope diffraction coefficient, which has no discontinuity across the shadow boundary, has been applied to both two dimensional and three dimensional horn geometries. The aperture integration, the equivalent G.T.D. edge currents and the equivalent slope wave currents are used respectively for the main
beam region, the E-plane edge contribution, and the H-plane edge
collection to the radiation pattern of a three dimensional
pyramidal electromagnetic horn for arbitrary observation points.
For H-plane patterns, the slope diffraction may be applied as
previously done for the E-plane pattern using conventional G.T.D.
(e.g., one direct ray and one diffracted ray from each edge as
indicated in Fig. 1).

The calculations presented here begin with a detailed, complete
H-plane analysis of a conventional horn including the aperture
integration and equivalent currents. Each of the techniques is
detailed and the results shown. Then various approximations are
made and the results compared.

A. Aperture Integration

The radiated field of an aperture antenna may be found in terms
of an integral involving the electric and magnetic field
distributions in the aperture of the antenna. This expression as given by
Ratnasiri[44] or Silver[45] is:

\[ \mathbf{E} = \frac{jk}{4\pi} \frac{e^{-jkR}}{R} \int_A \mathbf{R} \times [\hat{n} \times \mathbf{E}^a] - Z_0 \mathbf{R} \times (\hat{n} \times \mathbf{H}^a) e^{-jk\bar{p}\cdot\hat{R}} dS \]

where
- \( \hat{R} \) is a unit vector in the direction of the observation point,
- \( \bar{p} \) is a vector to the source point,
- \( \hat{n} \) is a unit vector normal to the aperture,
- \( k = \frac{2\pi}{\lambda} \) is the propagation constant,
- \( Z_0 = 120\pi \) is the characteristic impedance,

and \( \mathbf{E}^a = \mathbf{H}^a \) are the electric and magnetic fields in the
aperture of the antenna.

The aperture fields may be adequately represented by the fields
existing over the plane of the aperture inside an infinite horn.
This approximation accounts for the spherical propagation in the
expanding horn geometry but neglects the perturbation of the fields
in the aperture of the horn due to the termination of the horn walls.
The magnetic field in the aperture is approximately given by:

\[ \mathbf{H}^a = \frac{1}{Z_0} (\mathbf{S} \times \mathbf{E}^a) \]
where the impedance in the horn is approximated by $Z_0 = \sqrt{\mu_0/\varepsilon_0}$ and $\hat{S}$ is a unit vector along the radial propagation path in the horn. A very common approximation is to neglect the radial propagation in the horn. This is an excellent approximation for moderate flare angle horns as will be shown later. The electric field inside the infinite pyramidal horn in the coordinate system of Fig. 2 is:

$$E = \hat{x} E_x + \hat{z} E_z$$  \hspace{1cm} (4)

where

$$E_x = F(x,y,z) \cos \frac{\pi y}{a} \cos \theta_x,$$
$$E_z = -F(x,y,z) \cos \frac{\pi y}{a} \sin \theta_x,$$

and

$$F(x,y,z) = e^{jk(\rho \cos \theta - \sqrt{x^2+y^2+(\rho \cos \theta + z)^2})}$$

The parameter, $F$, is a complex amplitude function which accounts for the path length variation from the horn throat to the observation plane (aperture plane). The amplitude of the aperture electric field is indicated in Fig. 2 by the shaded area. Using this expression for the electric field in the horn aperture yields the following aperture fields:

$$E^a = \hat{x} E^a_x + \hat{z} E^a_z$$  \hspace{1cm} (5)

$$\hat{R}^a = \frac{1}{Z_0} [\hat{x} E^a_x \sin \theta_y + \hat{y} (E^a_x \cos \theta_x \cos \phi - E^a_z \sin \phi) + \hat{z} (-E^a_x \sin \theta_y)] .$$  \hspace{1cm} (6)

Equation (6) follows exactly from Eqs. (3) and (5) where

$$\hat{S} = \hat{x} \sin \theta \cos \phi + \hat{y} \sin \phi \sin \theta + \hat{z} \cos \phi$$

and

$$\sin \theta \cos \phi = \sin \theta_x,$$
$$\sin \theta \sin \phi = \sin \theta_y,$$
$$\cos \phi = \cos \theta_x \cos \theta_y.$$
Fig. 2—Horn geometry.
Substituting these fields into the expression for the far field of the horn using:

\begin{align}
\hat{R} &= \hat{x} \sin \theta \cos \phi + \hat{y} \sin \theta \sin \phi + \hat{z} \cos \theta \\
\hat{n} &= \hat{z} \\
\rho \cdot \hat{R} &= x' \sin \theta \cos \phi + y' \sin \theta \sin \phi
\end{align}

yields after considerable manipulation:

\begin{align}
E_\theta &= \frac{-jk}{4\pi} \frac{e^{-jkR}}{R} \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} A_\theta e^{jk \sin \theta (x' \cos \phi + y' \sin \phi)} dx' dy' \\
E_\phi &= \frac{-jk}{4\pi} \frac{e^{-jkR}}{R} \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} A_\phi e^{jk \sin \theta (x' \cos \phi + y' \sin \phi)} dx' dy'
\end{align}

where

\begin{align}
A_\theta &= -E_x^3 \cos \phi (1 + \cos \theta_x \cos \theta_y \cos \phi) + E_z^3 \cos \phi \sin \theta_x + \sin \phi \sin \theta_y \\
A_\phi &= E_x^a \sin \phi (\cos \phi \cos \theta_x + \sin \theta_x) + E_z^a (\sin \theta_y \cos \phi - \sin \theta_x \sin \phi)
\end{align}

These expressions reduce to the standard form presented in Silver\cite{46} or Jordan and Balmain\cite{47} when the radial propagation is neglected (i.e., for $\theta_x = 0 = \theta_y$). In neglecting the radial propagation, the phase distribution of the fields in the aperture must still be included.

Results have been obtained by numerically integrating the expressions of Eqs. \eqref{6a} and \eqref{6b} for a horn with slant length $\rho_H=13.1\lambda$ and $\rho_E=13.77\lambda$ and flare angles $\theta_H=16.6^\circ$ and $\theta_E=17.5^\circ$ in the H-plane and E-plane respectively. These dimensions result in a phase taper...
of 193° in the E-plane and 228° in the H-plane. These results appear in Figs. 3 and 4 for the cases where the effect of the radial propagation is included in the aperture fields and also where it is neglected. Comparing the results, it is clear that neglecting the radial propagation for this horn is certainly justified for this example.

![Graph](image)

**Fig. 3**—H-plane results of numerical aperture integration including the effects of radial propagation inside the horn.
B. G.T.D. Equivalent Currents

An equivalent edge current[48] is a current which, when flowing on an infinitely long wire, yields the same field at an observation point as the diffracted field from the edge of an infinite wedge as illustrated in Fig. 5. Because of the angular dependence of the diffraction coefficient, the equivalent current will have a directional pattern. The diffracted field in terms of the incident plane wave field and the asymptotic form of the diffraction coefficient is[49]:

\[
E^d = E^i \frac{[D(β, ψ, ψ_0, n) \pm D(β, ψ, ψ_0, n)] e^{-jkr}}{\sqrt{r}}
\]

\[
ρ_E = 13.77 \lambda \quad θ_E = 17.5°
\]

\[
ρ_H = 13.1 \lambda \quad θ_H = 16.6° \quad α_0 = 0.761 \lambda
\]

Fig. 4—H-plane results of numerical aperture integration neglecting the effect of radial propagation inside the horn.
where the "+" (or "-") applies for $E^d$ perpendicular (or parallel) to the edge and

$$D(\beta, \phi, n) = e^{-j \frac{\pi}{4} \frac{1}{n} \sin \frac{\pi}{n}} \sqrt{\frac{1}{2\pi k \sin \beta}} \begin{bmatrix} 1 \cos \frac{\pi}{n} - \cos \frac{\phi}{n} \\ 0 \sin \frac{\pi}{n} - \cos \frac{\phi}{n} \end{bmatrix}$$

$$\phi = \psi + \psi_0.$$

Near the shadow boundaries (i.e., for $\psi = \psi_0 \pm \pi$), the asymptotic form of the diffraction coefficient is singular. The diffracted field in terms of the $v_B$ function which is continuous across the shadow boundary is:

$$E^d_{\perp} = E^i_{\perp} [v_B(r, \psi - \psi_0, n) \pm v_B(r, \psi + \psi_0, n)] \frac{1}{\sin \beta}$$

or

$$H^d_{\parallel} = H^i_{\parallel} [v_B(r, \psi - \psi_0, n) \pm v_B(r, \psi + \psi_0, n)] \frac{1}{\sin \beta}$$

Fig. 5---Wedge geometry.
The field radiated by an infinite y-directed magnetic line source is [50]:

$$H_y = -Y_0 k I^m_y e^{\frac{j \pi}{4}} \frac{e^{-jkr}}{2j2\pi k \sqrt{r}}.$$

Equating the line source fields and the asymptotic form of the diffracted field for the hard boundary condition ($E^i$ perpendicular to the edge) yields:

$$I^m_y = \frac{2j}{\sqrt{v_0 k}} G^m_{\psi, \psi_0, n} H^i_{y} \tag{20}$$

where

$$G^m_{\psi, \psi_0, n} = \frac{1}{\sin \beta} \left[ \sin \beta + \frac{1}{\cos \left( \frac{\psi - \psi_0}{n} \right)} + \frac{1}{\cos \left( \frac{\psi + \psi_0}{n} \right)} \right]. \tag{21}$$

The radiated field of the finite length equivalent magnetic line source is [51]

$$E^m = \frac{4k}{4\pi} e^{-jkR} \int_{-a/2}^{a/2} \hat{R} \times \text{T}^m(y') e^{jk \hat{R} \cdot \hat{R}} dy'. \tag{22}$$
where:

\[ \hat{R} = \text{unit vector to the observation point at } R, \theta, \phi, \]
(or \( x, y, z \)),

\[ \mathbf{R}' = \text{vector to the source point at } x', y', z' = 0, \]

and

\[ \mathbf{I}^m(y') = \mathbf{I}^m \mathbf{I}'(y') \] is the equivalent magnetic current.

The resulting line integrals in terms of the \( \hat{\theta} \) and \( \hat{\phi} \) components are:

\[
(23) \quad E_\theta = \frac{-i}{2\lambda} \frac{e^{-jkR}}{R} \int_{-a/2}^{a/2} I^m_{y}(y') \cos \phi \ e^{jk \sin \theta (x' \cos \phi + y' \sin \phi)} \, dy',
\]

\[
(24) \quad E_\phi = \frac{i}{2\lambda} \frac{e^{-jkR}}{R} \int_{-a/2}^{a/2} I^m_{y}(y') \sin \phi \ e^{jk \sin \theta (x' \cos \phi + y' \sin \phi)} \, dy'.
\]

Because of the local nature of the diffracted field, these equivalent currents may be used in the analysis of finite edge, three dimensional geometries such as a pyramidal horn. In the case of the horn, there is a phase distribution along the equivalent edge current because of the phase variation of the incident field due to the variation in path length from the horn vertex to points on the edge. Since both of the \( E \)-plane edges contribute equally to the \( H \)-plane pattern, it is necessary to only perform one numerical integration. In the coordinate system selected, the \( H \)-plane corresponds to the \( \phi = 90^\circ \) plane. Thus the \( E \)-plane edge contribution is:

\[
(25) \quad E_\theta = 0
\]

\[
E_\phi = 2 \left[ \frac{i}{2\lambda} \frac{e^{-jkR}}{R} \cos \theta \int_{-a/2}^{a/2} I^m_{y}(y') e^{-jk y \sin \theta} \, dy' \right]
\]

where

\[
I^m_{y}(y') = \frac{2j}{k} \mathbf{E}_\perp \frac{1}{2 \sin \beta} \left[ \begin{array}{c} 2 \\ -\cos \frac{\psi}{2} \end{array} \right]
\]

\[ \beta = \theta \]

15
\[
\psi = \begin{cases} 
2\pi - \alpha_E & \text{if } \theta > 90^\circ \text{ (rear half space)} \\
\pi - \alpha_E & \text{if } \theta < 90^\circ \text{ (front half space)}
\end{cases}
\]

and

\[
E^i = e^{-jkp_E(1-\cos\alpha_E)} e^{-jk\left(\sqrt{\rho_E^2 + y^2 - \rho_E}\right)}
\]

The contribution of the E-plane edges shown in Fig. 6 to the H-plane radiation pattern in the back direction (E, in the \(\phi=90^\circ\) plane) of the horn already described is shown in Fig. 7. The corresponding lobe in the front half space (\(\theta<90^\circ\)) is taken into account by the aperture integration and not included here.

Fig. 6--Equivalent G.T.D. current of E-plane edge of horn wall.
C. Slope Wave Diffraction Coefficient and Slope Wave Equivalent Currents

As outlined in the introduction to this Chapter, the H-plane pattern analysis of the horn has in the past been particularly tedious because of the large number of diffracted rays which need to be included since the electric field vanishes along the horn wall. Associated with this, is the problem of the diffraction coefficient vanishing for the case where the incident electric field is parallel to the edge (H-plane edge of the horn) and the observation point is on the surface of the wedge (the vertex of the horn walls). Under these conditions, the incident electric field may be related to the tangential magnetic field along the surface of the wedge (horn...
Through reciprocity, the tangential magnetic field on the surface of the wedge (H-plane horn wall) may be related to the radiated electric field parallel to the edge so that the horn may be analyzed under transmit conditions.

In analyzing the diffraction from the H-plane edges of the horn, the starting point is an analysis of the canonical problem: a two dimensional wedge with a plane wave (polarized parallel to the edge) incident on the wedge at an angle $\phi_0$ and the observation point on the surface of the wedge. The incident field at the edge is $E_z^i$ and has the phase referenced to the edge. This wedge with zero wedge angle is one of the horn walls. The fields diffracted along the surface of the wedge are treated in detail in Appendices A and B. Under these conditions, the tangential electric field and normal magnetic field are zero along the surface of the wedge while the tangential magnetic field is non-zero. A ray analysis, similar to that commonly used in G.T.D. analyses, will be used in the slope wave diffraction analysis. The diffracted tangential magnetic field intensity on the surface of the wedge a distance $\rho$ from the edge is (from Appendix A):

$$H_d^d = -\frac{1}{3\mu_0} \frac{1}{\rho} \frac{\partial E_z}{\partial \phi} = \frac{2k}{\omega \mu} E_z^i D_{SW}(\phi_0,n) e^{-jk\rho} \rho^{3/2}$$

where $E_z^i$ is the incident electric field at the edge, and

$$D_{SW}(\phi_0,n) = \frac{e^{j\pi/4} \sin \frac{\pi}{n}}{\sqrt{2\pi n^2 k^{3/2} \sin \beta}} \frac{\sin \phi_0/n}{\left(\cos \frac{\pi}{n} - \cos \frac{\phi_0}{n}\right)^2}$$

is the slope wave diffraction coefficient. On the shadow boundary ($\phi_0=\pi$), this form of the diffraction coefficient is singular. Another form of slope wave diffraction coefficient which has a continuous transition through the shadow region is derived in Appendix B for the diffraction from a half plane. This diffraction coefficient is given by:

$$v_{SW}(\phi_0,\rho) = -\text{sgn}(-\cos \frac{\phi_0}{2}) \sin \psi e^{-jk\rho} \psi e^{-j(\kappa\rho + \frac{n}{4})} \times$$

$$x \left[ e^{jk\rho} (1+\cos \psi) \int_0^\infty e^{-j\tau^2} d\tau + \frac{j}{2\sqrt{k\rho (1+\cos \phi_0)}} \right]$$

$$\sqrt{k\rho (1+\cos \phi_0)}$$
and the diffracted field is given by:

\begin{equation}
H_d^\rho = \frac{2k}{\omega\mu} E_z v_{SW} (\phi_0, \rho).
\end{equation}

For points far from the shadow boundary (i.e., for \(k\rho(1+\cos\psi) \gg 1\)), this form of the slope diffraction coefficient reduces to the previous form, namely:

\begin{equation}
v_{SW}(\phi_0, \rho) \xrightarrow{k\rho(1+\cos\phi_0) \gg 1} \frac{e^{-jk\rho}}{\rho^{3/2}} D_{SW}(\phi_0, 2)
\end{equation}

The diffraction from the H-plane horn walls under transmit conditions is the reciprocal of the problem just outlined. The reciprocity theorem states \cite{52} that the reaction of the fields from source 1 with the sources at point 2 is equal to the reaction of the fields from source 2 with the sources at point 1, or:

\begin{equation}
\int (E_1 \cdot \vec{J}_2 - \vec{H}_1 \cdot \vec{M}_2) dS = \int (E_2 \cdot \vec{J}_1 - \vec{H}_2 \cdot \vec{M}_1) dS.
\end{equation}

For the case of the two dimensional wedge and line sources shown in Fig. 8 used in the horn analysis, this reduces to:

\begin{equation}
E_1 \cdot J_2 = -H_2 \cdot M_1
\end{equation}

where \(\vec{J}_2\) is the electric line source at point 2 which radiates the fields, \(\vec{E}_2, \vec{H}_2\)

and

\(\vec{M}_1\) is the magnetic dipole line source on the surface of the wedge which reacts with the tangential magnetic field of source 2 along the surface of the wedge.

This means that if we can find \(\vec{H}_2\) at point 1 on the surface of the wedge due to an incident electric field \(\vec{E}_2\) (radiated by \(\vec{J}_2\)), then we will know the electric field at point 2, \(\vec{E}_1\), when the magnetic dipole line source, \(\vec{M}_1\), acting (i.e., when the horn is transmitting). The fields \(\vec{E}_1\) and \(\vec{H}_2\) may be separated into geometric (incident and reflected) rays and diffracted rays such that:

\begin{equation}
\vec{E}_1 = \vec{E}_1^g + \vec{E}_1^d
\end{equation}
and

(34) \[ \mathbf{H}_2 = \mathbf{H}^g_2 + \mathbf{H}^d_2. \]

Fig. 8--Wedge and source geometry used in reciprocity argument.

The geometric fields alone would satisfy the reciprocity statement in Eq. (32) since these fields are the fields of the sources above an infinite conducting plane. Since the system of equations is linear, the diffracted components of the fields must also satisfy Eq. (32):

(35) \[ E^d_{1z} J_{2z} = -H^d_{2\rho} M_1. \]

In the case of the horn, the field of interest is \( E^d_1 \) when source 1 is acting. This is given by:

(36) \[ E^d_{1z} = -H^d_{2\rho} \frac{M_1 x}{J_{2z}} \]

where

\( H^d_{2\rho} \) is given by Eq. (17) to be:
The incident electric field in (37) is related to the electric line source \( J_2 = \dot{Z} I_{2z} \delta(\rho-p_2) \delta(\phi-\phi_2) \) by:

\[
E^i_{2z} = Z_0 k I_{2z} \frac{e^{j\pi/4}}{2\sqrt{2\pi k}} \frac{e^{-jk\rho_2}}{\sqrt{\rho_2}}.
\]

Substituting Eqs. (37) and (38) into Eq. (36) yields:

\[
E^d_{1z} = -M_1 x_k \left( \frac{1}{2} \right)^{\frac{1}{2}} e^{j\pi/4} \frac{e^{-jk\rho_2}}{\sqrt{\rho_2}} D_{sw}(\phi_2, \rho) e^{3j/2}.
\]

In this form, we still need to know the magnetic dipole line source strength, \( M_1 \). As an alternative, a knowledge of the magnetic field along the surface, in terms of \( M_1 \), is adequate. The source:

\[
\vec{M}_1 = \hat{z} K_1 x \delta(\rho-p_1) \delta(\phi-\pi)
\]

radiates in free space the electric field:

\[
E_1 = \hat{z} \frac{k}{4} K_1 x H_1^{(2)}(k|\rho-p_1|) \sin \phi.
\]

The associated magnetic field along the surface of the wedge when \( \vec{M}_1 \) is located on the wedge is given by:

\[
\hat{H}_1 = -\frac{2}{3j \omega} \hat{v} \times E_1 = \hat{\rho} H_{\rho} + \hat{\phi} H_{\phi}.
\]

Of particular interest is \( H_{\rho} \) incident on the edge of the wedge, i.e.,

\[
H^i_{1\rho}(o, o) = H_{1\rho}(\rho, \phi)\bigg|_{\rho=0} = -\frac{2}{3j \omega} \left\{ \frac{k}{4j} K_1 x \frac{1}{\rho} \frac{3}{\phi} \right\} \left[ H_1^{(2)}(k|\rho-p_1|) \sin \phi \right]_{\rho=0} \phi=0.
\]
Solving Eq. (44) for $M_{1x}$, using the asymptotic approximation for the Hankel function, and substituting into Eq. (39) yields:

$$E_{1z}^{d} = jkZ_{0}H_{1p}^{i} D_{sw}(\phi_{2},\eta) e^{-\frac{-jkp_{2}}{\sqrt{p_{2}}}}$$

where $H_{1p}^{i}$ is the total (incident plus reflected) tangential magnetic field incident on the edge of the wedge. If, instead of using the plane wave form of the slope wave diffraction coefficient given in Eqs. (26) and (27), we had used the form given in Eqs. (28) and (29), the reciprocity theorem yields:

$$E_{1z}^{d} = jkZ_{0}H_{p}(\rho_{1},0) v_{sw}(\phi_{2},\rho_{1}) e^{-\frac{-jkp_{2}}{\sqrt{p_{2}}}}$$

where $H_{p}(\rho_{1},0) = H_{p}^{i}(0,0)e^{\frac{jkp_{1}}{\rho_{1}}/3/2}$ is now the incident tangential magnetic field some distance $\rho_{1}$ away from the edge. The $e^{-\frac{-jkp_{2}}{\sqrt{p_{2}}}}$ terms in Eqs. (45) and (46) show the cylindrical wave nature of the slope diffracted fields. When applying one of the above two dimensional formulations to a pyramidal horn antenna, it may be necessary to formulate equivalent edge currents in terms of the slope diffraction coefficient in order to account for the finite length of the edges of the H-plane walls. These slope wave equivalent edge currents are discussed next.

The slope wave equivalent edge current is a current which, when flowing on an infinite wire, yields the same field at an observation point as the slope wave diffraction coefficient for an infinite wedge. The equivalent edge current is obtained by equating the diffracted field and the field of the wire at the observation point. For the three dimensional geometry, the equivalent edge current is then integrated along the finite length of the edge (which in this case is the edge of the H-plane wall of the horn) to obtain the slope diffracted field for the finite edge.

Referring to Harrington[53], the far zone field of an infinite $x$-directed electric line source is:
\( E_x = -Z_0 k I_x e^{j \frac{\pi}{4}} \frac{e^{j k r}}{2 \sqrt{2 \pi k}} e^{-j k r} \)

where 
\( Z_0 \) = characteristic impedance of free space,
\( k = \frac{2\pi}{\lambda} \) where \( \lambda \) = wavelength
\( I_x \) is the electric current.

The diffracted electric field due to a tangential magnetic field incident on the edge along the surface of the wedge is given by Eq. (45) or by Eq. (46). The form in Eq. (45) will be used in this part of the discussion.

Equating these fields yields the equivalent current:

\( I_x = -2\sqrt{2 \pi k} e^{j \frac{\pi}{4}} H_{1p} D_{sw}(\phi_0, n) \).

This equivalent current when flowing on the edge of the horn will have both amplitude variation (due to the angular dependence of the diffraction coefficient) and phase variation (due to the changing path length from the horn vertex to points along the edge). The line source shown in Fig. 9 is used in the horn analysis. The far field of this finite length electric line source is given by[54]:

\[
\Phi = \frac{jkZ_0}{4\pi} e^{j k R} \frac{e^{-j k r}}{R} \int_{-b/2}^{b/2} \hat{R} \times [\hat{R} \times I_x(x') e^{j k R}] e^{j k R'} \cdot R \, dx'
\]

where 
\( \hat{R} \) = unit vector to observation point at \( R, \phi, (or x,y,z) \),
\( R' \) = vector to source point at \( x',y',z' \),
and
\( I_x(x') \) is the equivalent electric current at \( x' \) on the wire.

The resulting integrals to be evaluated are:

\[
E_\theta = \frac{-jkZ_0}{4\pi} e^{j k R} \frac{e^{-j k r}}{R} \cos \phi \cos \phi \int_{-b/2}^{b/2} I_x(x') e^{j k \sin \theta(x' \cos \phi + y' \sin \phi)} dx'
\]

\[
E_\phi = \frac{jkZ_0}{4\pi} e^{j k R} \frac{e^{-j k r}}{R} \sin \phi \int_{-b/2}^{b/2} I_x(x') e^{j k \sin \phi(x' \cos \phi + y' \sin \phi)} dx'.
\]
In the H-plane (\(\phi=90^\circ\)) these integrals reduce to a pair of integrals, one line integral for each H-plane edge, of the form:

\[
\begin{align*}
E_\theta &= 0 \\
E_\phi &= \frac{jZ_0}{2\lambda} \frac{e^{-jkR}}{R} \int_{-b/2}^{b/2} I^e(x') e^{jk \frac{a}{2} \sin \theta} dx'
\end{align*}
\]

where

\[
i^e(x') = -2\sqrt{2\pi} e^{j \frac{\pi}{4}} H_0^1 D_{SW}(\pi-a_H + \theta, 2)
\]

and

\[
H_0^1 = -\frac{1}{j\omega \mu \rho} (v \times E_{ap})_{\text{edge}}
\]

\[
= \frac{1}{Z_0} E_0 \frac{\pi}{jka} e^{-jk\rho_H (1-\cos\alpha_H)} e^{-jk(\sqrt{\rho_H^2+x'^2}-\rho_H)}
\]

The H-plane contribution (\(E_\phi\) in the \(\phi=90^\circ\) plane) to the H-plane radiation pattern of the horn already described is shown in Fig. 10.
Fig. 10--H-plane pattern of equivalent slope wave currents on finite length H-plane horn walls.

\[ \rho_e = 13.77 \lambda \quad \theta_e = 17.5^\circ \]

\[ \rho_h = 13.1 \lambda \quad \theta_h = 16.6^\circ \quad a_0 = 0.761 \lambda \]
The solution is not valid near the shadow boundary (at 16.6°) since this form of the slope diffraction coefficient is singular for this angle. Numerical results are shown for 21° < θ < 180°. A form which is continuous across the shadow boundary is used in a similar horn analysis and discussed later. This shadow boundary singularity presents no problem in the present case since the aperture integration will be used to obtain the pattern in the vicinity of the shadow boundary.

The composite computed H-plane pattern, shown in Fig. 11, is formed by using the aperture integration (Eqs. (10) and (11) or Fig. 4) until it reaches the same level as the H-plane edge diffracted field (Eqs. (50) and (51) or Fig. 10). The E-plane edge diffracted field (Eqs. (23) and (24) or Fig. 7) is then added to the pattern. This figure also shows the measured pattern from Ref. [22]. Notice that the agreement is excellent for angles less than 90°. Beyond 90°, the calculations predict the general pattern level and shape. The fine lobe structure between 90° and 120° of the measured curve is probably measurement error. The computer program, discussed in Appendix F, may be made completely general for any arbitrary observation point (θ, φ) if the proper shadowing is included. In examining the above results, the first thing one might ask is whether it is really necessary to perform the aperture and line integrations. This is perhaps best answered by looking at the simpler analyses and then commenting on each. The simplest analysis would be to consider point diffraction from each edge of the horn plus a direct geometrical optics ray through the horn aperture for observation points inside the angular region defined by the extension of the horn walls (i.e., θ < the flare angle). The E-plane edges would use the G.T.D. diffraction coefficient of Eq. (15) or (18) and the H-plane edges would use the slope diffraction coefficient of Eq. (27) or (28). The component of the direct or geometric ray parallel to the H-plane edge is given by

\[
E_{\parallel} = \begin{cases} 
\cos \left( \frac{\pi}{2} \rho \cos \theta \tan \theta \right) \frac{e^{-jkr}}{r} & \text{for } \theta_H < \theta_H^* \\
0 & \text{for } \theta_H^* < \theta_H
\end{cases}
\]

The slope wave diffracted electric field parallel to the H-plane edges of the horn at the point (r, θ) is given by

\[
E_{\parallel} = jk Z_0 \rho \frac{e^{-jkr}}{r} \left| D_{sw}(\pi - \alpha_H^\theta_H n) \right| \text{for } -90 < \theta_H < 180 + \alpha_H
\]

\[
+ D_{sw}(\pi - \alpha_H - \theta_H^* n) e^{-j2k\rho_H \sin \theta_H \sin \theta_H} \text{ for } -180 - \alpha_H < \theta_H < 90
\]
Fig. 11—Computed and measured H-plane patterns of a pyramidal horn.

dB BELOW MAIN BEAM MAXIMUM

APERTURE INTEGRATION

H-PLANE EDGE CONTRIBUTION

E-PLANE EDGE CONTRIBUTION

ϕ = 13.77°, θ = 17.5°

ϕ = 13.1°, θ = 16.6°, ϕ = 0.761°
where $H_i$ is incident tangential magnetic field intensity at the
H-plane edge of the horn, $\alpha_H$ is half of the horn flare angle, $\phi_H$
is the slant length of the H-plane horn wall, $\theta_1$ is the polar
angle measured in the H-plane, and $D_{SW}(\phi_0,n)$ is given by Eq. (27).
The total fields are given by the sum of Eqs. (53) and (54) along
with a term for the diffracted fields from the E-plane edges when
required according to the concepts of G.T.D. The slope diffraction
coefficient in Eq. (27) has been used along with the E-plane edge
contribution using the G.T.D. fields of the form of Eqs. (16), (17),
and (18) to obtain the results shown in Fig. 12. Observe that on
the shadow boundary ($\theta=16.6^\circ$) the solution is singular. The next
step in complexity is to use the slope diffraction coefficient of
equation (28) (instead of the form in Eq. (27)) which involves a
Fresnel integral and is continuous through the shadow region.
Using this form, the resulting pattern is shown in Fig. 13. While
this result is acceptable, the backlobe agreement with measurement
is not very good (c.f., Fig. 11). The backlobe level has been
predicted but for angles between 120$^\circ$ and 160$^\circ$ on the pattern
the agreement is fairly poor. This may be overcome by using the
equivalent current concepts discussed previously. When this is done,
the results, shown in Fig. 14, are nearly identical to the previous
results (Fig. 11) which agree very well with the measurement. It
appears that to predict only the backlobe level, only the four edge
diffracted rays need to be considered but in order to predict the
backlobe pattern accurately, the equivalent edge currents on the
finite edges must be included in the analysis. If one demands
even more accurate results, higher order diffractions may be in­
cluded in the analysis but in view of the already excellent agree­
ment, this complication seems unnecessary. A slightly different
approach to the horn problem would be to only calculate the dif­
fracted fields for the sidelobe and backlobe levels and then combine
this with the aperture fields from standard horn design curves as
given in Jasik[55]. This eliminates the need to perform the
aperture integration for cases where the single direct geometrical
optics ray fails to predict the fields of the aperture integration.
The aperture fields and diffracted fields may be combined, as
before, by using the aperture fields (either from design curves or
actual integration) until the functional form of the two patterns
match.

The gain of the horn may also be estimated from the G.T.D.
analysis or the aperture integration. The on-axis field intensities,
directly related to the gain of the horn, obtained from each of the
previously described methods are shown below. These values are in
dB, normalized to the maximum electric field in the aperture
(assumed to be unity) with the $e^{-jkR/R}$ normalized out.
Fig. 12—Computed H-plane horn pattern using 1 diffracted ray from each edge (when not shadowed) plus a geometrical ray through the horn aperture. The slope diffraction coefficient used here is singular at the shadow boundary (16.6°).
Fig. 13—Computed H-plane horn pattern using 1 diffracted ray from each edge (when not shadowed) plus a geometrical optics ray through the horn aperture. The slope diffraction coefficient is the form in Appendix B which is continuous.
Fig. 14—Computed H-plane horn pattern using equivalent line sources on the finite horn edges and a geometrical optics ray through the horn aperture.
Technique & On Axis Field
--- & ---
Equivalent Currents + Direct Ray & 24.0 dB
Cylindrical Wave Point Diffraction + Direct Ray & 23.41 dB
Plane Wave Point Diffraction + Direct Ray & 23.38 dB
Aperture Integration & 22.76 dB

Each of these techniques makes a different approximation but the results show only a slight variation in the on axis fields. Since the three G.T.D. methods agree most closely and since the aperture integration improperly models the effects of edge diffraction, the actual on axis field intensity is most likely near 23.5 dB. To obtain a better estimate of the on axis field from the aperture integration, an extended aperture should be used in the integration to properly include the effects of edge diffraction.

D. Conclusions

The question of why valid results have been obtained in the past studies using the G.T.D. analysis appears to be appropriate at this time. In the region of the main beam, the geometrical optics ray contributes the largest field and the rays diffracted by the E-plane edges are the most significant edge diffracted fields. The H-plane diffracted field is a second order contribution. Thus the E-plane pattern could be computed with good accuracy using the conventional G.T.D. analysis. The H-plane pattern, however, requires that contributions from all four edges be included. The fields diffracted from the E-plane edges have the same functional dependence as the geometrical optics ray in the H-plane as a function of aspect angle. Thus, ignoring the presence of the E-plane edges when computing the H-plane pattern would yield a possible error on the order of several dB which would not be detectable from the shape of the main beam region of the pattern. The error, however, would be detected in the level of the E-plane edge contribution to the back lobe of the H-plane pattern.

The results of this chapter demonstrate that the slope wave diffraction techniques developed here and in Appendices A and B are extremely valuable tools in analyzing the conventional horn antenna. In addition, as will be shown in Chapters IV and V, it is useful in the analysis of the corrugated horn and the horn reflector. The technique described here, combined with the conventional G.T.D. analysis may be used to analyze the diffraction from an arbitrary aperture distribution through a Fourier expansion of the aperture fields into sine and cosine terms. The diffraction from the cosine terms could be found from conventional G.T.D. techniques and the sine terms from the slope wave techniques.
CHAPTER III
PROPERTIES OF CUT-OFF CORRUGATED SURFACES*

The corrugated horn has been established as an antenna with low side and back lobes, rotationally symmetric patterns (for square pyramidal and conical horn shapes), and broad-band performance[56-63]. These properties make this horn useful for many applications; including the one presently under study, i.e., as a high beam efficiency antenna. A ray optics model of the dominant radiation mechanisms of conventional horn antennas is shown in Fig. 15. In the corrugated horn (shown in profile in Fig. 16) the corrugated surface (with capacitive surface impedance in the E-plane of the horn) serves to reduce or eliminate the fields associated with ray "a" of Fig. 15. This in turn reduces or eliminates the usual high E-plane sidelobes. The influence of the corrugation shape and density on the scattering from the onset of the corrugations and on the losses in the horn has been ignored in previous work. The parametric study reported here attempts to generate further understanding of the operating principles of the corrugated horn and to establish design criteria for the construction of practical corrugated horn geometries through a detailed analysis of the groundplane corrugated surface junction. The analysis includes studying the surface currents flowing on the corrugations and the loss in the corrugations as well as the scattering from the onset of the corrugations. The usual corrugated horn requirements are for 8 or more corrugations per wavelength and a corrugation depth "d" between 0.25λ and 0.5λ.** In addition, it is usually specified that the tooth thickness ("t" in Fig. 16) should be much less than the corrugation width ("W" in Fig. 16). In order to study the effects of varying these parameters, an integral equation solution was used to find the radiated fields and the surface currents associated with the groundplane corrugated surface models of Figs. 17 and 18. A brief description of the integral formulation is included in the next section and the results are presented in the following section. A solution for the complete corrugated horn radiation pattern excited by only one mode is given in the next Chapter.

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**This depth would be in free space wavelengths for the parallel plate waveguide geometry of the corrugations. If the sides of the slot are terminated in a conducting plane, then these electrical lengths would correspond to those of the TE_{10} waveguide mode. Thus, some improvement in the operating bandwidth of the pyramidal corrugated horn could be achieved if the depth of the first few corrugations are appropriately tapered.
Fig. 15—Conventional horn geometry.

Fig. 16—Corrugated horn geometry.
Fig. 17—Rectangular corrugated surface model used to find the scattered fields and surface currents associated with a groundplane-corrugated surface junction.

Fig. 18—V-shape corrugated surface model used to find the scattered fields and surface currents associated with a groundplane-corrugated surface junction.
The groundplane-corrugated surface models shown in Figs. 17 and 18 were chosen for their similarity to one wall of a sectoral corrugated horn. In the corrugated horn, the illumination of the corrugations is by a cylindrical wave due to the diffraction from the horn-waveguide junction. In the model used, the magnetic line source pair provides a similar cylindrical wave illumination while at the same time allows placing a null in the far field in the direction of the adjacent edge of the groundplane. Then one need not match over the entire groundplane but only over the illuminated part, thus saving the limited number of match points for the corrugated surface side of the model. The remaining match points are divided among the 20 corrugations. For all corrugation depths where cut-off operation is obtained (i.e., \(0.25 < d/\lambda < 0.5\) for the square corrugation), the energy is forced away from the corrugated surface. Since the energy is forced off the corrugations, the corrugated surface matching points may be terminated without affecting the currents in the corrugations near the junction. Thus the surface model need not be closed. This conclusion was verified by finding the currents and scattered fields associated with the open model shown and a similar closed model and observing no significant differences. Therefore, this finite model is a good approximation to an infinite groundplane corrugated surface junction as long as the corrugated surface is operated in the cut-off mode.

A. Integral Equation Formulation

The surface currents on the corrugations and the scattering by the groundplane-corrugated surface junction of Figs. 17 and 18 were found using the H-field formulation for the transverse electric field case discussed by Harrington[64]. For this case, there is only a z component of magnetic field, \(\mathbf{H}\), and a tangential component of surface current, \(J(\mathbf{n} \times \mathbf{H})\), where \(\mathbf{n}\) is the unit normal. At any point, the total magnetic field, \(H_z\), is the sum of the incident and scattered magnetic fields, \(H_z^I\) and \(H_z^S\) respectively. The scattered field is related to its source, the surface current, by

\[
H_z^S = \hat{U}_z \cdot \nabla \times \int J G \, d\mathbf{x}'
\]

where \(\mathbf{J} = J \, d\mathbf{x}' = [-H_z^I] \times d\mathbf{x}'\) when \(H_z\) is evaluated on \(C^+\) (just outside the contour \(C\) where the surface current, \(\mathbf{J}\), flows - the interior of \(C\) lies on the left side of \(d\mathbf{x}'\)).

and \(G\) is the two-dimensional Green's function

\[
G(\mathbf{r}, \mathbf{r}') = \frac{1}{4\pi} H_0^{(2)}(k|\mathbf{r}-\mathbf{r}'|).
\]
The resulting integral equation

\[
J = -[H_z^i + \hat{U}_z \cdot \nabla x \int_{\gamma} JG \, d\vec{n}']_{C+}
\]

is solved for the surface current, \( J \), by point matching using pulse basis functions. This integral equation reduces to the matrix equation

\[
[x_{mn}] [f_n] = [g_m]
\]

where

\[
[x_{mn}] = \frac{i}{\eta_0} \Delta C_n (\hat{n} \cdot \vec{R}) H_{1}^{(2)} (k |\vec{p}_m - \vec{p}_n|)
\]

= matrix of coupling coefficients between the \( m \)th and the \( n \)th segments at \( \rho_m \) and \( \rho_n \) on the surface contour.

\[
\hat{R} = \frac{\vec{p}_m - \vec{p}_n}{|\vec{p}_m - \vec{p}_n|} = \text{unit vector between } n\text{th and } m\text{th points},
\]

\[
\Delta C_n = J_n \, d\vec{n} = \text{current moment at the } n\text{th point},
\]

\[
[f_n] = \text{column vector of unknown surface current on the } n\text{th segment at the points } (x_n, y_n) \text{ on the surface contour},
\]

and

\[
[g_m] = \text{column vector of the incident field } (= -H_z^i) \text{ at the points } (x_m, y_m) \text{ on the surface contour}.
\]

This matrix equation has been solved for the surfaces shown in Figs. 17 and 18 using a Crout\cite{65} matrix inversion subroutine. Other more powerful inversion subroutines which included pivoting and iterative improvement were used for comparison but provided no improvement beyond the 3 to 5 place accuracy obtained from Crout's method. These other methods were not used regularly because they required nearly twice the storage since both the matrix and its inverse had to be stored. The subroutine listing for the Crout method is included in Appendix C along with the rest of the computer program.
B. Results for Square Corrugations

The parameters of the square corrugations of Fig. 17 which were varied in this study include the corrugation depth, the corrugation density and the corrugation shape. The notation used to relate these parameters to the model and the range of values considered for these parameters are shown in Table 1.

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>NOTATION in Fig. 17</th>
<th>VALUES CONSIDERED</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corrugation depth (in wavelengths)</td>
<td>d/λ</td>
<td>0.25, 0.3125, 0.375, 0.4375, 0.5</td>
</tr>
<tr>
<td>Corrugation density (in corrugations per wavelength)</td>
<td>NCOR-1/TC or N/λ</td>
<td>4, 6, 8, 10, 12</td>
</tr>
<tr>
<td>Corrugation shape (ratio of corrugation gap width to corrugation period)</td>
<td>W/TC</td>
<td>0.5, 0.6, 0.7, 0.75, 0.8, 0.9</td>
</tr>
</tbody>
</table>

Many combinations of these parameters were studied to ascertain the influence of each on properties such as the scattering from the groundplane-corrugated surface junction, the surface current flowing on the corrugations and also the power loss in the corrugations.

Because of computer storage limitations, only 200 matching points could be used for each surface. These points were divided to allow for at least 8 matching points per wavelength along the surface. This matching point density is adequate if the fields in the corrugations are primarily those of the TEM mode. The verification of the dominant status of the TEM mode in the corrugations is contained in the higher order mode study included in Appendix D of this report.

Of the 200 matching points available, ten were used on the ground-plane side of the model of Fig. 17. The locations of these points are indicated in Fig. 19. The physical optics current of an infinite groundplane is also shown in amplitude and phase in Fig. 19. Also shown in Fig. 19 by the dashed lines are the amplitude and phase of the surface current which exists at the midpoints of the surface segments. This demonstrates that the current model used in the calculations is an accurate model of the true surface current.
Fig. 19--Surface currents near the line sources used to model the groundplane-corrugated surface junction.
The fields associated with the groundplane-corrugated surface junction are shown in Fig. 20 for corrugation depths from 0.25λ to 0.4375λ. The results shown are for a corrugation density of 8 corrugations per wavelength (i.e., corrugation period = TC = λ/8) and for a corrugation shape ratio (ratio of corrugation gap width to corrugation period) of W/TC = 0.75. The patterns shown in Fig. 20 are the radiation zone total and scattered magnetic field patterns produced by the magnetic line sources of amplitude l/z₀ and the corrugations. The scattered field phase is also shown in Fig. 20. Notice that the direction of maximum radiation intensity varies from about 65° above the corrugated surface for the 0.25λ depth to about 35° for the 0.4375λ depth. All of the other corrugation densities and corrugation profiles showed essentially the same patterns at the same corrugation depths and are therefore not included. The smoothly varying phase pattern indicates that the phase center of the scattered field is nearly at the origin of the coordinate system (the junction between the groundplane and the corrugations).

This range of angles for this scattered field maximum explains the peculiar behavior of the small corrugated horn discussed previously[66]. This horn had a pattern which showed a slight frequency dependence. However, as the end of this horn was cut off, the pattern of the remaining corrugated horn became quite frequency dependent. The geometry is shown in Fig. 16. For the original corrugated horn, the scattered field maximum illuminated the opposite wall of the horn and weakly illuminated the aperture edge (the aperture edge was at about 43°). However, after the modification, the edge of the horn occurred at an angle of 50° and thus was more strongly illuminated. The fields diffracted by this edge phased destructively and constructively as a function of frequency with the desired radiated field and thus caused the frequency dependent patterns. The magnetic field intensity associated with the rays diffracted by the corrugated surface junction may be expressed in the form

\[
H^D = H^1 e^{-jkr} / \sqrt{r} \ F(\theta)
\]

where \(H^1\) is the magnetic field incident on the junction, \(r\) is the range as measured from the junction, \(\theta\) is the angle shown in Fig. 17, and \(F(\theta)\) is the complex pattern factors for the scattered fields shown in Fig. 20.

It is noted that the phase is a slowly varying function of \(\theta\) and thus presents no difficulty for the application of GTD. This information is now sufficient to compute the diffracted fields associated with ray "b" of Fig. 16. The far field of the ray in the direction of the
Fig. 20—Radiation pattern associated with a groundplane-corrugated surface junction illuminated by a cylindrical wave:
   a) total fields, b) scattered field amplitude, and c) scattered field phase.
horn edge may be readily obtained from the preceding equation. Next the surface discontinuity would be replaced by a line current whose fields have that magnitude. This line current would then illuminate the 90° corner at the end of the horn wall and the diffracted fields could now be computed using the techniques of the Geometrical Theory of Diffraction. If the fields in the deep shadow are to be computed, then the diffraction from the thick wall would be computed by representing it as a pair of 90° wedges[67]. While these computations have not been carried to completion at this time, they represent a straightforward implementation of accepted G.T.D. practices and should yield accurate results. The major goal at this time is to establish the condition for which this edge diffraction is negligible and this would occur when the angle to the opposite edge is less than 20°-25°.

As mentioned earlier, another property of the corrugated surface which is of interest is the rate of decay of the surface current flowing on the corrugation walls. The decay in the amplitude of the surface current is due to the energy being forced away from the corrugations and not caused by power loss in the conductor. The loss will be discussed later. Figure 21 shows the normalized surface current versus corrugation number for a surface with 8 corrugations per wavelength and a profile ratio of W/TC = 0.75 at several corrugation depths. The surface current plotted is the current which exists at the bottom of each of the 20 corrugations (point "B" in the insert) normalized with respect to the surface current which exists at the same X-coordinate on an infinite groundplane with the same sources acting (point "A" in the insert). This normalization removes the range dependence of the incident cylindrical fields and also selects the maximum current which exists on the surface of the corrugations.

Since to a good approximation only a TEM mode exists in the corrugation, the currents on the teeth at any point are given by

\[ |J_y| = J_s \cos \beta (d + y) \]

where \( y \) as shown in Fig. 17 assumes negative values and \( \beta = \frac{2\pi}{\lambda_0} \) is the propagation constant of a TEM wave in the parallel plate waveguide region. This has also been established by all of the computations that have been made.

Notice that the most rapid decay is obtained for the 0.25\( \lambda \) case and as expected no decay occurs for the 0.5\( \lambda \) case. The variations of the current near the end of the surface are caused by fields reflected from the termination of the structure and may be ignored. This property (the decay of the surface current) has been examined for other corrugation densities and profiles. Figures 22 and 23 show the surface currents (normalized as above) existing on 0.25\( \lambda \) and 0.375\( \lambda \) deep corrugations for various corrugation densities.
Fig. 21—Decay of surface current on a corrugated surface due to the energy being forced away from the corrugations.
Fig. 22—Decay of surface currents on 0.25a deep corrugations as a function of corrugation density.
Fig. 23--Decay of surface currents on 0.375\(\lambda\) deep corrugations as a function of corrugation density.
These results are plotted as a function of the X-coordinate measured in wavelengths from the onset of the corrugations. Corrugation densities ranged from 4 to 12 corrugations per wavelength and the results for each corrugation depth showed essentially no dependence on the corrugation density. The dependence of the surface current on the corrugation profile was also investigated. The corrugation profile was specified by the ratio of the corrugation gap width (W) to the corrugation period (TC) and the corrugation density was 8 corrugations per wavelength. The results are shown in Figs. 24 and 25 for 0.25a and 0.375a deep corrugations, respectively. The range of W/TC ratios considered was from 0.5 (thick metal vane between corrugations) to 0.9 (very thin metal vane). Beyond approximately 1/2λ from the onset of the corrugations, the decay of the surface current per wavelength is nearly constant (∼6 dB/λ at d/λ = 0.25 and ∼4 dB/λ at d/λ = 0.375). Thus in a practical situation, one could use very thin vanes near the onset of the corrugations and thicker vanes (which are easier to construct) further from the onset of the corrugations. These curves then establish the length of corrugated horn required to reduce the edge diffracted fields. Since the current density at the top of the teeth is almost zero (see Appendix D) only a few teeth would be required here. For the 3/8a depth, the currents at the top of the tooth would be 3 dB below those at the bottom. Thus the illumination of the distant end of the horn would be −15 dB for x > 2λ for W/TC > 0.7. Thus the fields of the diffracted ray ("a" of Fig. 15) that phase constructively and destructively with the geometrical optics fields are reduced to a negligible value with a surface that is two wavelengths in extent.

Another property of the corrugated surface which was investigated is the power loss in the corrugations (both the total loss and also the location of regions where the maximum loss occurs). In an effort to make the results more general, the corrugated-surface loss is normalized with respect to the loss in an equal size segment of an infinite groundplane of the same material. The results shown in Fig. 26 show the normalized loss in dB versus corrugation number "N" for various corrugation depths. The loss in the corrugated surface is the sum of the loss in each corrugation from the onset of the corrugations (at the origin of the coordinate system) up to and including the loss in the Nth corrugation. The loss in the groundplane (used for normalization) is the loss in the region from the origin up to the X-coordinate of the end of the Nth corrugation. Notice that most of the loss occurs in the first few corrugations and also that it appears that all of the curves (except the 0.5λ deep case) would cross the 0 dB line (equal loss in corrugations and groundplane) after a reasonable number of corrugations (the 0.3125λ and 0.375λ deep cases cross in fewer than 20 corrugations). The power loss is also dependent upon the corrugation density and shape. Figures 27 and 28 show the power loss in 0.25a and 0.375a deep corrugations for various corrugation densities. This loss is the loss in the corrugations from the origin to "X" normalized with respect to the
Fig. 24--Decay of surface currents on 0.25\(\lambda\) deep corrugations as a function of corrugation shape.
Fig. 25—Decay of surface currents on 0.375\(\lambda\) deep corrugations as a function of corrugation shape.
Fig. 26—Power loss in a corrugated surface for various corrugation depths.

RELATIVE LOSS (dB) = 10 \log \left( \frac{\int_{0}^{X_n} |J_s| d\ell \text{ OVER CORRUG}}{\int_{0}^{X_n} |J_s|^2 d\ell \text{ OVER GROUNDPLANE}} \right)

- \frac{n^2}{\alpha n} + \frac{1}{\alpha n} = \frac{N^2}{\alpha n} = \frac{\alpha n}{\alpha n}
- \frac{n^2}{\alpha n} + \frac{1}{\alpha n} = \frac{N^2}{\alpha n} = \frac{\alpha n}{\alpha n}
- \frac{n^2}{\alpha n} + \frac{1}{\alpha n} = \frac{N^2}{\alpha n} = \frac{\alpha n}{\alpha n}
- \frac{n^2}{\alpha n} + \frac{1}{\alpha n} = \frac{N^2}{\alpha n} = \frac{\alpha n}{\alpha n}
- \frac{n^2}{\alpha n} + \frac{1}{\alpha n} = \frac{N^2}{\alpha n} = \frac{\alpha n}{\alpha n}
Fig. 27—Power loss in 0.25λ deep corrugations from the origin to "X" (normalized with respect to the loss in a finite segment of an infinite groundplane from the origin to "x") for various corrugation densities.
Fig. 28—Power loss in 0.375\(\lambda\) deep corrugations from the origin to "X" (normalized with respect to the loss in a finite segment of an infinite groundplane from the origin to "x") for various corrugation densities.
loss in the same length segment of an infinite groundplane. Notice that the lower densities are less lossy than the high density surfaces. This result is reasonable in view of the fact that the surface current decay per wavelength is nearly independent of the corrugation density (c.f., Figs. 22 and 23). Thus the higher corrugation density with its greater surface area should be more lossy. The influence of the corrugation shape on the power loss may be seen in Fig. 29. Shown here is the loss in 20 corrugations (from x = 0 to x = 2.5λ) normalized with respect to a 2.5λ segment of an infinite groundplane versus the corrugation depth (D/λ) for various W/TC ratios. The W/TC = 0.5 case corresponds to a thick metal vane between corrugations while the W/TC = 0.9 case corresponds to a very thin vane. Notice that the thin vanes have lower loss than the thick vanes and that there exists a range of depths over which the loss is minimized.

The general conclusion which might be drawn at this point is that corrugation density and tooth shape can be selected to optimize the performance of a corrugated horn by a) choosing as few corrugations as four/wavelength, b) by making the vanes as thin as is practical. However, there is one final parameter that must be considered. The impedance of the horn would be related to the magnitude of the wave reflected back toward the source. The computer programs developed here are not well designed to evaluate this reflection coefficient. However, they can be used to obtain an indication of this parameter. The current on the groundplane region of Fig. 17 can be expressed in the following form:

\[ \mathbf{J} = \mathbf{J}_s + \mathbf{J}_1 + \mathbf{J}_2 \]

where \( \mathbf{J}_s \) is the current density of the source when placed on an infinite conducting plane, \( \mathbf{J}_1 \) is the current density associated with the wave reflected by the onset of the corrugations, and \( \mathbf{J}_2 \) is the current density associated with the waves reflected by the end of the groundplane.

Since the two line sources are phased to give zero far field in the \( \theta = \pi \) direction, and since the diffraction coefficient associated with the back scattered field at this edge is small, \( \mathbf{J}_2 \) may be neglected. When the observation position is sufficiently removed from the edge, the current density \( \mathbf{J}_1 \) will decay as \( 1/\sqrt{r} \) since it is caused by the radiation of a line source. Thus the equation for the surface current has the form:

\[ \mathbf{J} = \mathbf{J}_s + 2\pi \times \frac{H_0}{\sqrt{r}} \]

where \( r \) is the distance on the groundplane from the onset of corrugations. Since \( \mathbf{J}_s \) is known exactly, \( \mathbf{J}_1 \) can be found with reasonable accuracy. Its representation as \( 2H_0 \) is, of course, only approximate.
Fig. 29--Power loss in 20 corrugations versus corrugation depth for various corrugation profiles.

A plot of $|H_0|$ versus the corrugation depth is shown in Fig. 30 for the $W/TC = 0.75$, 8 corrugations per wavelength case. As previously discussed, the $\lambda/4$ depth has a large reflected wave associated with it. The reflection from the junction also depends on the tooth shape and density. Figures 31 and 32 show $|H_0|$ versus corrugation density and profile ratio respectively. Notice that at both $\lambda/4$ and $3\lambda/8$ depths, the higher corrugation density and thick teeth ($W/TC = 0.5$) have lower reflections associated with them. The $W/TC = 0.9$ points in Fig. 32 are shown with a dashed line as some difficulties were
noticed for cases where matching segments were very close together. This is probably the case for the \( \frac{W}{TC} = 0.9 \) data.

A general observation is that the impedance as a function of corrugation density would require as many corrugations as possible in order that the VSWR be maintained low over the largest possible bandwidth.

Now we arrive at the optimum design of the corrugated surface in terms of expense, and operation. There should be from 8 to 12 corrugations over the first wavelength. These teeth should be as thin as is possible for low loss. Then the tooth density should be tapered to 4 per wavelength and the tooth width should be increased to \( \frac{W}{TC} = 0.5 \). In addition, the first few teeth should be plated to further reduce losses. Also, for low loss, the waveguide and throat region of the horn should be plated regardless of whether the horn is corrugated or not.
Fig. 31—$|H_0|$ versus corrugation density.

$\text{CORRUGATION DENSITY } = \frac{1}{\text{TC}}$
(CORRUGATIONS PER WAVELENGTH)

$d = \frac{\lambda}{4}$

$\frac{w}{TC} = 0.75$

$d = \frac{3\lambda}{8}$

Fig. 32—$|H_0|$ versus corrugation shape.

$d = \frac{\lambda}{8}$
However, if loss is not an important consideration, then the reflection coefficient and consequently the horn VSWR should be minimized by increasing the corrugation density to at least 8 corrugations per wavelength. The reflection coefficient can also be reduced by initiating the corrugation at a point further removed from the horn waveguide junction. However, one must be certain to initiate the corrugations before a second mode could be excited in the horn (Ref. [5]).

C. Results for "V" Corrugations

The study of the vee corrugations shown in Fig. 18 was undertaken because there are situations where the square shape of the previous section is impractical or unsuitable. The thin vane of the square corrugation is impractical at frequencies much above 12 GHz because of the difficulty encountered in machining or extruding the corrugations. The difficulty should be eliminated with the V-shape corrugation. Also, the square shape is not as suitable as the V-shape for an application where the surface needs to fold or collapse in order to take up less space. Using an asymmetric V-shape, an unfurlable corrugated horn antenna which would fold like a camera bellows is practical. The corrugated surfaces considered in this section have symmetric V-shape corrugations defined by the corrugation depth, $d/\lambda$, and the corrugation density, $NCOR$ (in corrugations per wavelength). The range of parameters presented for the V-shape is not as complete as for the square shape because of some problems encountered in calculating the surface currents for the deep corrugations and for high corrugation densities ($NCOR>8$). These cases will not be reported here. It is believed that for these cases, the coupling coefficient between adjacent segments at the top or bottom of each corrugation is erroneous because of the very small distance between the matching points. A sufficient number of cases are available to indicate that the V-shape does operate as a cut-off corrugated surface for corrugation depths from 0.3125 to about 0.625. A corrugated horn using V-shape corrugations has been built and tested to verify this work[68]. Cut-off operation of a corrugated surface requires a capacitive surface impedance. That the surface impedance of the vee corrugation is capacitive over most of this range of depths has been verified using Harrington's results for the fields in a wedge shape waveguide with radial propagation[69]. The impedance looking into the wedge waveguide is:

$$Z = jZ_0 \frac{J_1(kp)}{J_0(kp)}$$

where $p$ is the radial distance from the bottom of the corrugation. For the deep, narrow corrugations being considered, this yields an operating range of approximately $0.382 < d/\lambda < 0.605$. This operating band is narrower than the computer calculations indicated but then the wedge radial waveguide is only an approximate model for the V-shape corrugations. The results of the computer calculations are discussed next.
Figure 33 shows: a) the groundplane matching points, b) the surface current magnitude, and c) the surface current phase in the source region. Figure 33 also shows the physical optics current (on an infinite groundplane) and the approximate current representation for the computer selected match points. Both the amplitude and phase are accurately modeled for this point selection.

The properties of the "V" corrugations which were studied include the radiation patterns of the surface model, the surface currents in the corrugations, and the power loss in the corrugations. Figure 34 shows the radiation pattern (total magnetic field) of the surface model shown in the insert and Fig. 18 for several vee corrugation depths. As with the square corrugations, the radiation pattern is nearly independent of the corrugation density (i.e., the pattern at a given depth is nearly identical for the 4, 6, or 8 corrugations per wavelength cases) and the direction of maximum intensity decreases with increasing corrugation depth.

Other similarities between the V-shape and the square corrugations are found in the surface current decay and power loss. Figures 35, 36, and 37 show the surface current (in dB) which exists near the bottom of the vee corrugations normalized with respect to the surface current which exists at the same "X" coordinate on an infinite groundplane with the same sources acting. Figure 35 shows this current versus the distance (in wavelengths) from the onset of the corrugations for two corrugation depths near the low end of the operating band. At the lowest frequency in the cut-off band (d/\lambda = 0.3125), the surface current decay along the vee corrugated surface is nearly identical to that of the square corrugations at the lowest frequency in its band (c.f., d/\lambda = 0.25 in Fig. 21). As was the case for the square corrugations, the surface current decay is most rapid at the low end of the frequency band. Figures 36 and 37 show similar current curves for various corrugation densities at depths of 0.375\lambda and 0.5\lambda respectively. Notice that near the lower end of the operating band the surface current decay is independent of the number of corrugations per wavelength while near the upper end of the band there is some dependence on the corrugation density.

Figures 38 and 39 show the relative power loss in the vee corrugations. The loss which is plotted is the loss in the corrugations from the origin to the "X" coordinate normalized with respect to the loss in the same length segment of an infinite groundplane. Figure 38 shows the loss at the two lowest frequencies for the 8 corrugations per wavelength case. The loss here is higher than the square corrugations because of the higher surface current near the point of the vee and also because of the greater arc length in the V-shape surface. Figure 39 shows the loss at d/\lambda = 0.375 for 4, 6, and 8 vee corrugations per wavelength.
Fig. 33—Matching point location and surface currents near the line sources used to model the groundplane-corrugated surface junction.
Fig. 34—Radiation pattern of a groundplane-"V" corrugation junction illuminated by a cylindrical wave.

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Fig. 35—Relative surface current on vee corrugations versus distance from the onset of the corrugations.
Fig. 36--Relative surface current on 0.375\(\lambda\) deep vee corrugations versus the distance from the onset of the corrugations for several corrugation densities.
Fig. 37—Relative surface current on 0.5λ deep vee corrugations versus the distance from the onset of the corrugations for several corrugation densities.
Fig. 38—Relative power loss in vee corrugations from origin to "X" (normalized with respect to the loss in O-X segment of infinite groundplane) for two corrugation depths.
Fig. 39—Relative power loss in 0.375 deep vee corrugations from origin to "X" normalized with respect to the loss in 0→X segment of infinite groundplane) for three densities.
As with the square corrugations, the lower corrugation density has the lower loss. Because of the higher current (and consequently higher loss), if one is to use the V-shape corrugations in a situation where low loss is important something such as polishing or silver plating the first few corrugations should be considered. For an unfurlable horn, the first part of a horn might be rigid and use square corrugations to obtain lower loss while the remainder of the horn could be an unfurlable geometry using triangular teeth. Also, because the current in the corrugations becomes very small after 4 or 5 corrugations the loss in the corrugations will eventually become smaller than the loss in the groundplane.

D. Discussion of Corrugated Surface Properties

A parametric study of square and vee corrugations has shown that both shapes operate as cut-off corrugated surfaces over substantial bandwidths with relatively low loss. The study has also shown that the rate of decay of surface current is nearly independent of the corrugation density (the number of corrugations per wavelength) and that the lower corrugation densities (4 or 6 corrugations per wavelength) are desirable because of the lower loss. A significant part of the loss is confined to the first few corrugations and should allow one to treat this region with special care such as silver plating or special polishing of the surface when using corrugated surfaces in situations where low loss is important. In any event, the loss in the walls for most practical corrugated horns would be as low as (if not lower than) the loss in the same shape and size conducting wall of the same material with the same surface finish. The results also indicate that the VSWR of the horn will depend on the corrugation shape. For the groundplane-corrugated surface junction considered, the VSWR decreased with increasing corrugation density and with thinner teeth between the corrugations. Since the loss increases with increased corrugation density, some compromise is required. Six to eight corrugations per wavelength near the onset of the corrugations and then decreasing the density to two to four per wavelength should be adequate. Again, since the loss was lower and the VSWR higher for the very thin teeth, some compromise is also indicated. A corrugation width to period ratio of $W/TC = 0.75$ should be a good compromise value if one wants to avoid the complexity of changing the corrugation shape along the corrugated wall.

While this work has yielded considerable insight about corrugated surface design, there remain several sets of important design data that can be obtained with a similar approach. For example, the sources on the groundplane should be moved off the groundplane and the corrugated surface performance analyzed for various source locations. This would simulate the fields incident on the corrugations due to the diffraction from the horn-waveguide junction and the onset of the corrugations on the opposite wall of the horn. Also, the groundplane should include a corner similar to the horn-waveguide junction.
Ideally, the entire horn should be considered. This has been programmed using the integral equation approach but is of limited value because of the extremely small horn which can be handled due to computer storage limitations. Using image techniques and other procedures currently being developed at the ElectroScience Laboratory, it should be possible to consider 10 or more corrugations in this model for VSWR computations. This number of corrugations should provide adequate reduction of the surface current at the top of the last corrugation to obtain valid calculations for realistic horn throat geometries. For the corrugated horn pattern, a diffraction coefficient could be associated with the scattering from the onset of the corrugations using the integral equation approach. Then, using the techniques of the Geometrical Theory of Diffraction, one could proceed to obtain full horn patterns including the back lobe radiation. An alternate approach is discussed in the next chapter where the pattern is computed using a knowledge of the fields in the horn aperture.
CHAPTER IV
PATTERN ANALYSIS OF CORRUGATED HORN ANTENNAS*

In general, the near axis E-plane radiation pattern of a pyramidal corrugated horn may be adequately predicted from standard design curves[70] established for the H-plane patterns of conventional horn geometries. This method, however, fails to predict the far-out sidelobe and backlobe radiation levels. The work presented here uses a knowledge of the aperture fields to predict the pattern using aperture integration and diffraction theory. The assumptions made concerning the aperture fields were verified by probing the internal fields and aperture fields of an X-band corrugated horn. The results of this field probing are contained in Appendix E. The method of solution used in this Chapter parallels that given in Chapter II. Specifically, the pattern in the main beam region is computed using conventional aperture integration procedures, the contribution of the H-plane edges is found using precisely the analysis given in Chapter II, and the contribution of the E-plane edges is found by use of duality with respect to the H-plane analysis of the conventional horn.

A. Horn Geometry and Model

The corrugated horn is formed by replacing the conventional E-plane horn walls by impedance walls which force the tangential magnetic field to zero along the wall. (The horn geometry is shown in Fig. 40.) The capacitive corrugated surface does this at the frequency where the corrugations are a quarter wavelength deep. At frequencies where the corrugation depth ("d" in side view of Fig. 40) is greater than \lambda/4 but less than \lambda/2 (actually (2n+1) \lambda/4 < d < (2n+2) \lambda/4) the tangential magnetic field is reduced. For example, the fields of ray "a" in Fig. 15 of the preceding chapter are reduced by approximately 15 dB at a distance of 2\lambda (c.f., Fig. 73). The effect of this is to modify the uniform electric field distribution in the E-plane of the waveguide to the cosine distribution in the horn aperture when the horn is properly designed so that no higher order modes are excited at the point where the corrugations are started in the horn. The measurements discussed in Appendix E show that this transition takes place near the onset of the corrugations. By the fifth corrugation, the cosine distribution is established. This information has been used to establish the horn model shown in Fig. 41. In this model, the aperture electric field is assumed to be:

\[
\bar{E}^a = \hat{y} H_0 \cos \frac{\pi x}{b} \cos \frac{\pi y}{a} e^{jk(\rho_H \cos \theta_H - \sqrt{x^2 + y^2 + (\rho_H \cos \theta_H + z)^2})}
\]

*This work was supported in part through National Science Foundation Grant GK-40489.
SIDE VIEW OF CORRUGATED HORN

TOP VIEW OF CORRUGATED HORN

Fig. 40--Corrugated horn geometry.
Fig. 41—Coordinate system and horn model.
The radiation pattern near the main beam is computed using the aperture integration method discussed in Chapter II. The expressions which are numerically integrated to obtain the main beam portion of the corrugated horn pattern are:

\[
E^A_\phi = \frac{jk}{4\pi} \frac{e^{-jkR}}{R} \int_{-b/2}^{b/2} \int_{-a/2}^{a/2} E^a_{x'}(x', y') \cos \phi (1 + \cos \theta) \cdot \\
\cdot e^{jk \sin \theta (x' \cos \phi + y' \sin \phi)} \, dx' \, dy'
\]

\[
E^A_\theta = \frac{jk}{4\pi} \frac{e^{-jkR}}{R} \int_{-b/2}^{b/2} \int_{-a/2}^{a/2} E^a_{x'}(x', y') \sin \phi (1 + \cos \theta) \cdot \\
\cdot e^{jk \sin \theta (x' \cos \phi + y' \sin \phi)} \, dx' \, dy'
\]

where \( E^a_x = Z_0 H_0 \cos \frac{\pi x}{b} \cos \frac{\pi y}{a} e^{jk(\rho H \cos \phi R + \sqrt{x'^2 + y'^2 + (\rho H \cos \phi + z)^2})} \)

In this form, the radial propagation inside the horn has been neglected but as discussed in Chapter II, this is an excellent approximation. The far side lobes and back lobes are computed from the equivalent line sources shown in Fig. 41. The line source amplitudes are determined as discussed in Chapter II in terms of the diffracted fields. In this case, since the fields at all four walls go to zero, the slope diffraction formulation is used for all four of the equivalent currents flowing on the aperture edges. The H-plane edge contribution to the radiation pattern is given by:

\[
E^H_\phi = \frac{-jkZ_0}{4\pi} \frac{e^{-jkR}}{R} \cos \theta \cos \phi (I_1 + I_2)
\]

\[
E^H_\theta = \frac{jkZ_0}{4\pi} \frac{e^{-jkR}}{R} \sin \phi (I_1 + I_2)
\]

where

\[
I_1 = \int_{-b/2}^{b/2} I_{x1}(x') e^{jk \sin \theta (x' \cos \phi + \frac{a}{2} \sin \phi)} \, dx'
\]

\[
I_2 = \int_{-b/2}^{b/2} I_{x2}(x') e^{jk \sin \theta (x' \cos \phi - \frac{a}{2} \sin \phi)} \, dx'
\]
\[ I_{x1}^e(x') = \hat{x}-\text{directed equivalent electric current on H-plane edge at } y = a/2 \]
\[ = -2\sqrt{2\pi k} e^{j\pi/4} H_0^1 D_{SW}(\pi - \alpha_H + \phi, 2), \]

\[ I_{x2}^e(x') = \hat{x}-\text{directed equivalent electric current on H-plane edge } y = -a/2 \]
\[ = -2\sqrt{2\pi k} e^{j\pi/4} H_0^1 D_{SW}(\pi - \alpha_H - \phi, 2), \]

\[ H_0^1 = -\frac{1}{j\omega \mu} \hat{\nu} \cdot (\nabla \times \mathbf{E}) |_{\text{Edge}}, \]

and \( D_{SW}(\phi, 2) \) is given by Appendix A and \( H^a \) by Eq. (63). The equivalent line sources used for the thick edge in the E-plane of the horn utilize the standard G.T.D. formulation as well as the slope wave diffraction analysis. The approximations made for the thick wedge with an impedance wall are discussed next.

B. Diffraction from the Thick Edge Formed by the Impedance (Corrugated) Surface and the Conducting Outer Wall of the Horn

The fields diffracted from the E-plane walls of the corrugated horn model are diffracted by a thick wedge with impedance boundary conditions as assumed in Fig. 42. The \( Z_s=0 \) conditions on the end and one side of the wedge force the tangential electric field to zero on the surface. The capacitive surface impedance on the side where the fields are incident forces the tangential magnetic field (normal electric field) to be small along this face of the wedge. Since no diffraction coefficient exists at this time for this type of impedance wedge, an approximate wedge model was formulated. The model selected is shown in Fig. 45. The equivalent line source amplitude is formulated in terms of the fields diffracted from edge "A" (in Fig. 42) toward edge "B" (in Fig. 42) as though the edge were an infinitely thin wedge with \( Z_s=j\omega \) on both sides as shown in Fig. 43. This wedge is the dual of the half plane considered in the slope wave diffraction of Chapter II. The line source of Fig. 44 produces the same field at point "B" as is diffracted from the edge of Fig. 43 to Point "B". Because the diffraction is directional, the equivalent line source must also be directional. Also, since the diffracted fields depend on the wedge angle, it is not surprising that the fields at point "B" in Fig. 45 differ from the fields at point "B" in Fig. 42. To obtain an estimate of the relative levels of these fields, a similar problem was considered. The exact G.T.D.
fields using the self-consistent method for the thick wedge of Fig. 46 were compared to the approximate G.T.D. fields obtained using an equivalent current approach similar to what has just been described and the results are shown in Fig. 47. The 3.8 dB difference in diffracted field level is due to the difference in the diffraction in the 270° direction between a 0° wedge angle (used in formulating the equivalent current) and the diffraction from a 90° wedge. Since a similar difference would be expected to occur for the impedance wedge of Fig. 43, the equivalent current of Fig. 44 was scaled by 3.8 dB. The field diffracted at \((r,\alpha)\) by the two dimensional thick edge of Fig. 42 is given by:

\[
E_d^\perp = 1.55(K(\alpha)E_{sw}(\alpha)e^{jk\alpha} \sin\alpha + 2E_{sw}(270^\circ)v_B(\alpha-270^\circ,d,2)) \cdot \frac{e^{-jk_r}}{\sqrt{r}}
\]

where \(\alpha\) is measured from the impedance surface side of the wedge

\[
K(\alpha) = \begin{cases} 
2 & \text{if } 180^\circ < \alpha < 270^\circ \\
0 & \text{otherwise}.
\end{cases}
\]

\(E_{sw}(\alpha)\) = the slope wave diffracted field in the direction measured from the \(Z_s = j\infty\) surface of the thick edge,
Fig. 43--Slope diffraction from impedance wedge.

Fig. 44--Equivalent, directional line source.
Fig. 45—Thick edge equivalent to corrugated horn wall.

Fig. 46—Conducting thick edge.
Fig. 47—Comparison of exact G.T.D. and approximate G.T.D. thick edge diffracted fields (edge thickness = 0.5\lambda).
\[ E_{SW}(\alpha) = Z_0 H^d = jk \, Z_0 \left(-E^i_p\right) D_{SW}(\alpha, 2), \]

\[ E^i_p \sim \frac{1}{jw_0 \left( \frac{1}{\rho} \frac{\partial H^i}{\partial n} \right)} \] normal derivative of the vanishing incident tangential magnetic field along the edge (from Eq. (63)),

\[ D_{SW} = \text{slope wave diffraction coefficient of Chapter II} \]

\[ v_B = \text{G.T.D. diffraction coefficient of Eq. (156), Appendix B.} \]

The finite length of the E-plane horn edges may be taken into account by equivalent currents as previously formulated for the H-plane edges. In this case there will be four equivalent currents, one for each edge of each wall. In the E-plane (\(\phi=0\)), the diffracted field from the E-plane edges is:

\[ E^E_\theta = \frac{jk}{4\pi} \frac{e^{-jKR}}{R} \int_{-a/2}^{a/2} (I_1(\phi) + I_2(\phi) + I_3(\phi) + I_4(\phi)) dy', \]

where

\[ I_1(\phi) = \text{magnetic current on inside edge at } x = b/2 \]

\[ = 2 \lambda e^{j \frac{\pi}{4}} K(\pi-\alpha_E + \phi) \frac{\partial H^i_v}{\partial n} D_{SW}(\pi-\alpha_E + \phi, 2) e^{-jk\sqrt{\rho_E^2 + y'^2 - \rho_n}} \cdot e^{jk \sin \phi \left(\frac{b}{2} \cos \phi + y' \sin \phi\right)} \]

\[ I_2(\phi) = \text{magnetic current on outside edge at } x = b/2 + d \]

\[ = I_1(270^\circ) v_B(90-\alpha_E + \phi, \frac{3}{2}) e^{jk\left(\frac{b}{2} + d \cos \alpha_E\right) \cdot \sin \phi \cos \phi - d \sin \alpha_E \cos \phi} \]
I_3 = magnetic current on inside edge at x = -b/2

\[ I_3 = 2 \lambda e^{-j \frac{\pi}{4}} K(\pi-\alpha_E-\theta) \frac{3H_y^i}{3n} D_{SW}(\pi-\alpha_E-\theta,2) e^{-jk(\sqrt{\rho_x^2+y^2}-\rho_E)} \cdot jk \sin \left( \frac{b}{2} \cos \phi + y \sin \phi \right) \]

I_4 = magnetic current on outside edge at x = -(b/2 + d)

\[ I_4 = I_3(270^\circ) v_B(90-\alpha_E-\theta,d,\frac{3}{2}) e^{-jk\left(\frac{b}{2} + d \cos \alpha_E\right)} \cdot \sin \epsilon \cos \phi - d \sin \alpha_E \cos \epsilon \]

\[ \alpha_E = E-plane flare angle of horn. \]

This model yields calculated radiation patterns which agree very well with measured patterns. Both calculated and measured results appear in the next section.

C. Radiation Patterns and Backlobe Levels

Radiation patterns using the formulation discussed have been calculated and are compared with measured E-plane corrugated horn patterns in Figs. 48 and 49. The pertinent horn dimensions are:

<table>
<thead>
<tr>
<th>TABLE 2</th>
<th>CORRUGATED HORN DIMENSIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aperture Size</td>
<td>Fig. 48</td>
</tr>
<tr>
<td>Slant length---E-plane</td>
<td>8.2(\lambda) x 8.2(\lambda)</td>
</tr>
<tr>
<td>H-plane</td>
<td>42.21 cm</td>
</tr>
<tr>
<td>Flare angle---E-plane</td>
<td>17°</td>
</tr>
<tr>
<td>H-plane</td>
<td>15.5°</td>
</tr>
<tr>
<td>Wall thickness</td>
<td>0.53(\lambda)</td>
</tr>
<tr>
<td>Operating Frequency</td>
<td>10.0 GHz</td>
</tr>
</tbody>
</table>

Observe the excellent agreement between the measured and calculated results. The calculated results for the pyramidal horn ignore the secondary diffraction from the edges formed by the outside of the horn. This probably accounts for the slight disagreement for angles between 100° and 165°. Figure 50 shows the calculated backlobe level versus horn slant length for rectangular and square aperture horns. The
H-plane slant length and flare angle have been adjusted to provide a common point of intersection for the E-plane and H-plane walls at the horn apex. Several measured backlobe levels have been used to verify these calculated levels.

D. Conclusions

This chapter demonstrates the techniques necessary to calculate corrugated horn E-plane sidelobe and backlobe levels. Principal plane patterns could have been obtained using the stationary phase approximation instead of the line sources on the horn edges. Previous pattern prediction has been through conventional H-plane design curves and has been limited to the main beam region. The techniques described here accurately predict the complete corrugated horn radiation pattern. This antenna, with its low sidelobe and backlobe levels is ideally suited for medium to broad beam low noise applications (beamwidths greater than approximately 5 degrees) and as a feed antenna for narrow-beam low noise antennas using reflectors (i.e., the corrugated horn reflector, the offset fed parabolic reflector, the center fed parabola and the cassegrain). In addition, the corrugated pyramidal horn may be constructed using corrugated surfaces on all four horn walls so that the horn will handle arbitrary polarization. Because of the axially symmetric radiation pattern of the square pyramidal corrugated horn, it has excellent circular polarization properties. The conical corrugated horn has similar radiation properties and even better polarization properties[71].
Fig. 48--E-plane radiation pattern of 8.2λ x 8.2λ X-band corrugated horn.
Fig. 49—E-plane radiation pattern of 2.96λ × 2.96λ X-band corrugated horn.
Fig. 50—Ratio of front lobe level to back lobe level (dB) for corrugated horns with a) Rectangular aperture $A=0.5 \times B$
b) Square aperture $A=B$. 
The horn-reflector antenna, commonly used in communications systems where low sidelobe levels are required, is a combination of an electromagnetic horn and a reflector which is a section of a paraboloid of revolution. The horn-reflectors considered here have horns of pyramidal cross section which may have conventional or corrugated E-plane walls. From the experience with conventional and corrugated horn geometries, the pattern of the corrugated horn-reflector should have sidelobe levels significantly lower than the levels achieved with conventional horn-reflectors. The reduced sidelobe levels would make the corrugated horn-reflector very desirable for use as a radiometer antenna. Both conventional and corrugated horn-reflectors are considered in this analysis although no measured corrugated horn-reflector patterns are yet available.

The radiation patterns for both the corrugated and conventional horn-reflector are computed using the aperture integration and equivalent current concepts discussed earlier. The usual stationary phase approximations do not apply here and thus the aperture integration must be used near the main beam. The aperture fields are obtained from geometrical optics by tracing rays back into the horn which has an assumed cosine amplitude distribution in the H-plane and a uniform (cosine) distribution in the E-plane of the conventional (corrugated) horn-reflector. The cosine/uniform distribution of the conventional horn is simply the TE₀₁ mode expanding inside the horn from the horn apex. The cosine/cosine distribution of the corrugated horn has been experimentally verified and is discussed in the previous chapter and in Appendix E. This ray tracing from the horn-reflector aperture back into the horn neglects the depolarization associated with the reflection from the parabolic reflector.

The horn-reflector geometry, while complex to analyze, may be described in terms of relatively few parameters. Figure 51 shows the coordinate system used in the calculations and some of the important parameters. The horn H-plane walls are planes containing the X-axis. The horn axis is assumed to be in the Y-Z plane. The E-plane walls are contained in planes at angles $\phi$ from this Y-Z plane. The horn axis is at an angle $\beta$ from the reflector axis (the Z-axis, see Fig. 52). Because it is possible to maintain the aperture height (WH of Fig. 52) and width (WE at the aperture center) by changing the flare angles and the angle between the horn axis and the reflector axis, it is possible to obtain the same antenna beamwidth with a variety of aperture shapes even though the same focal length paraboloid is used. Figures 53 and 54 illustrate one of the problems associated with cases where the angle between the horn axis and
Fig. 51--The horn reflector antenna.
Fig. 52--Illustration of some horn reflector shapes possible by changing the angle between the horn axis and the reflector axis while maintaining the aperture width in the E-plane (WE) and H-plane (WH).
reflector axis is not 90°. These figures show typical aperture shapes for angles where \( \beta > 90° \) and \( \beta < 90° \). The common horn-reflector geometry with \( \beta = 90° \) eliminates the need for the slightly wider aperture width associated with the \( \beta > 90° \) case of Fig. 53 or the slightly narrower illuminated region of the \( \beta < 90° \) case of Fig. 54. Because of the limited interest in the large antenna resulting from having \( \beta < 90° \), only \( \beta > 90° \) will be considered here. The \( \beta > 90° \) horn-reflector is of particular interest because of the considerable reduction in length. Several radiation patterns for 5° half power beamwidth horn-reflectors using 10\( \lambda \) and 20\( \lambda \) focal length paraboloids have been calculated and are presented later in this chapter.

A. Analytical Model

The coordinate system used in the horn reflector analysis is shown in Fig. 55. The electric field in the aperture is assumed to be X-polarized and is determined by ray tracing into the horn portion of the horn-reflector. The other polarization, namely \( E^y = y E_y \), was not considered because of the higher sidelobe levels which would result. The H-plane pattern for the other polarization would be the same as the corrugated horn-reflector E-plane pattern which will be shown later. The observation point in the far zone of the horn-reflector is at the point \( (R, \theta, \phi) \) and the computed fields at this point are \( E_0 \) and \( E_\phi \). The E-plane pattern of the horn-reflector will be the \( E_\theta \) pattern in the \( \phi = 0° \) plane (X-Z plane), while the H-plane pattern will be the \( E_\phi \) pattern in the \( \phi = 90° \) plane (Y-Z plane). These patterns are computed using aperture integration techniques for the main beam region and diffraction techniques for the remainder of the pattern.

In the vicinity of the main beam (i.e., for \( \phi \) small), the fields are computed by aperture integration over the shaded area of Fig. 56. The upper and lower aperture edges are determined by the intersection of the front and rear H-plane walls of the horn with the paraboloid. The horn axis is assumed to be in the Y-Z plane with its apex at the focus of the paraboloid and at an angle \( \beta \) from the reflector axis. The sides of the aperture are determined by projecting the locus of the intersection of the E-plane walls with the paraboloid forward (in the +Z direction) to the plane of the front H-plane wall (i.e., a plane at an angle \( \beta - \alpha_H \) from the reflector axis). The aperture integration is performed over the plane of the front H-plane wall. The radiated electric field is given in Eq. (2) in Chapter II and repeated here for convenience:

\[
E = \frac{-i k}{4 \pi} \frac{e^{-jkR}}{R} \int_A \hat{R} \times [\hat{n} \times E^a - z_0 \hat{R} \times (\hat{n} \times \hat{R}^a)] e^{jk\phi \hat{R} \cdot \hat{R}} dS
\]
Fig. 53--Illustration of the aperture shape for horn axis to reflector axis angles greater than 90°.

Fig. 54--Illustration of the aperture shape for horn axis to reflector axis angles less than 90°.
where \( \hat{n} \) is the normal to the aperture surface,
\[
\hat{n} = \hat{z} \sin(\beta - \alpha_H) - \hat{y} \cos(\beta - \alpha_H),
\]
\( \hat{R} \) is the unit vector to the observation point,
\[
\hat{R} = \hat{x} \sin \phi + \hat{y} \sin \phi + \hat{z} \cos \phi,
\]
\( \rho \) is the vector to the source point in the aperture plane,
\[
\rho = x x' + y y' + z z',
\]
and \( E^a \) and \( H^a \) are the electric and magnetic fields in the aperture of the antenna.

It is observed that the factor \( \rho' \cdot \hat{R} \) produces a linear phase taper in the aperture plane (plane of the H-plane wall). The initial solution projected the reflected rays from the paraboloid to a constant phase plane (a \( z = \text{constant} \) plane in this case). The results differed from those obtained by integrating over the actual aperture plane and further were inconsistent with the edge diffracted fields. The aperture fields are a function of the distribution in the horn. If a cosine distribution is assumed in the horn, a slightly distorted cosine function will appear in the aperture plane due to the varying distance from the horn apex (at the focus of the paraboloid) to the reflector surface. The assumption that the magnetic field in the aperture is related to the electric field in the aperture by:
(70) \[ \mathbf{E}^0 = \frac{1}{\gamma_0} (\hat{y} \times \mathbf{E}^a) \]

where

\[ \mathbf{E}^a = \hat{x} A(x,y) \] (A(x,y) is discussed in Appendix F).

is a reasonable approximation since the wave has a uniform phase front after reflection and is a finite segment of an inhomogeneous plane wave. There is also a small y-component of \( \mathbf{E} \) in the aperture since the walls of the horn-reflector are not contained in planes parallel to the Y-Z plane. This component has been neglected in this aperture integration since this would produce only a minor deviation near the main beam and the remaining lobes are obtained using G.T.D. techniques. The function \( A(x,y) \) is a complex function of position in the aperture and accounts for the path length phase and amplitude variations over the aperture. After considerable manipulation, the integral in (2) reduces to:

(71) \[ \mathbf{E} = \delta \mathbf{E}_\theta + \hat{\phi} \mathbf{E}_\phi = \frac{-jk}{4\pi} e^{-jkR} \int_A [\delta A_\theta + \hat{\phi} A_\phi] e^{-jk\hat{p}' \cdot \hat{R}} dS \]

where

\[ A_\theta = A(x,y)[-\cos(\beta' - \alpha_H)\cos \phi(1 + \cos \theta)], \]
\[ A_\phi = A(x,y)[\sin(\beta' - \alpha_H)\sin \theta + \cos(\beta' - \alpha_H)\sin \phi(1 + \cos \theta)], \]
\[ \hat{p}' \cdot \hat{R} = x' \sin \theta \cos \phi + y' \sin \theta \sin \phi + z' \cos \phi, \]

and

\( \beta' = \beta - 90^\circ. \)

This form has been programmed and used to obtain the main beam patterns reported later in this chapter.

Radiation patterns for regions other than near the main beam have been obtained using diffraction techniques which include wedge diffraction, slope diffraction from a half plane, and equivalent current concepts. These techniques have been discussed in other sections of this work and will not be repeated in detail here. The diffraction from the finite edges of the horn-reflector is calculated using equivalent magnetic currents on the E-plane aperture edges and equivalent electric currents on the H-plane aperture edges as indicated in Fig. 56. The fields radiated by an arbitrary electric current are given by:[72]
EQUIVALENT MAGNETIC LINE SOURCES

EQUIVALENT ELECTRIC LINE SOURCES

EQUIVALENT APERTURE CURRENTS IN THE PLANE OF THE H-PLANE WALL OF THE HORN

Fig. 56—Horn reflector model.

\[ \mathbf{E}^e = \frac{jkZ_0}{4\pi} \frac{e^{-jkR}}{R} \int_L e^{jk\hat{\rho} \cdot \hat{R}} \hat{R} \times (\hat{R} \times \mathbf{E}^e) dx'(x',y',z') \]

where \( \mathbf{E}^e = \frac{i}{k} \mathbf{I}^e(x',y',z') \) is the equivalent electric current directed along the tangent to the H-plane edge at the point \((x',y',z')\),

\[ = x i^e_x + y i^e_y + z i^e_z \]

and the other variables are the same as in Eq. (2). For the lower edge, there is only the \( I^e_x \) component since this edge is parallel to the x-axis for the geometry of Fig. 55. However, because of the curvature in the upper edge, there will also be both \( I^e_x \) and \( I^e_y \) components of current. Only the electric current is used. This is a slight contradiction with the assumption made previously for the aperture field distribution but the more correct value is used here (to satisfy the boundary conditions of the horn-reflector). The integration is along the locus of the upper and lower edges of the reflector. This equation becomes:

\[ \mathbf{E}^e = \hat{\mathbf{e}}_\theta E^e_\theta + \hat{\mathbf{e}}_\phi E^e_\phi = \frac{jkZ_0}{4\pi} \frac{e^{-jkR}}{R} \int_L e^{jk\hat{\rho} \cdot \hat{R}} [\delta A^e_\theta + \delta A^e_\phi] dx' \]
where

\begin{equation}
A_{\theta}^e = -i_x e \cos \phi \cos \theta - i_y e \cos \phi \sin \theta + i_z e \sin \phi,
\end{equation}

and

\begin{equation}
A_{\phi}^e = i_x e \sin \phi - i_y e \cos \phi.
\end{equation}

The fields radiated by an arbitrary magnetic current element are similarly given by:

\begin{equation}
\mathbf{E}^m = \Theta E_{\theta}^m + \Phi E_{\phi}^m = \frac{jk}{4\pi} \frac{e^{-jkR}}{R} \int_{L} e^{jk\hat{\rho} \cdot \hat{R}} [\mathbf{R} \times \mathbf{I}^m] d\xi,
\end{equation}

where \( \mathbf{I}^m = \hat{\mathbf{I}}^m(x',y',z') \) is the equivalent magnetic current directed along the tangent to the \( E \)-plane edge at the point \( (x',y',z') \),

\[ = \hat{x} I_x^m + \hat{y} I_y^m + \hat{z} I_z^m, \]

and the other variables are the same as in Eq. (2). Only the magnetic currents are retained corresponding to the hard boundary condition due to the presence of the horn walls. This equation reduces to:

\begin{equation}
\mathbf{E}^m = \frac{jk}{4\pi} \frac{e^{-jkR}}{R} \int_{L} e^{jk\hat{\rho} \cdot \hat{R}} [\Theta A_{\theta}^m + \Phi A_{\phi}^m] d\xi,
\end{equation}

where

\begin{equation}
A_{\theta}^m = i_x^m \sin \phi - i_y^m \cos \phi,
\end{equation}

and

\begin{equation}
A_{\phi}^m = i_x^m \cos \theta \cos \phi + i_y^m \cos \theta \sin \phi - i_z^m \sin \theta.
\end{equation}

When the proper equivalent currents are used in Eqs. (74), (75), (78) and (79), the line integrals indicated in Eqs. (73) and (77) will accurately describe the diffracted fields. Thus four line integrals, one for each of the aperture edges, need to be evaluated. The equivalent currents are evaluated using a combination of two dimensional wedge diffraction and slope diffraction at each point of the aperture edge.
The equivalent electric current on the bottom H-plane edge is found, as before, by equating the fields of an electric line source to the fields slope diffracted by a half plane. This means that for the three dimensional case, the three dimensional nature of the incident field is included but that the diffraction for the equivalent current is handled on a two dimensional basis. The line integration along the finite edge subsequently includes the three dimensional behavior of the diffracted fields. Because of the vanishing electric field incident on the edge, the diffraction from this edge is treated by slope wave diffraction as done in Chapter II for the horn. This diffraction coefficient relates the incident tangential magnetic field to the diffracted electric field. The tangential magnetic field incident on the bottom edge at the point \((x', y', x')\) is given by:

\[
H^\mu_\rho|\text{edge} = \frac{1}{jk\rho^2} \frac{\pi}{2\rho \tan \alpha_H} E_0 \frac{\rho \rho_c}{\rho} e^{jk(\rho_T-\rho)}
\]

where 

\[
\rho = \sqrt{(x')^2+(y')^2+(z')^2} = \text{distance from horn apex to point of diffraction on the bottom edge},
\]

\[
E_0 = \text{electric field intensity in the } y=y_c \text{ plane at the aperture center (assumed unity at } x=0, y=y_c),
\]

\[
\rho_c = \text{distance from the horn apex to the point on the reflector surface which reflects the ray passing through the aperture center},
\]

\[
\rho_T = \text{the distance from the horn apex to the top edge in the } y-z \text{ plane},
\]

\[
e^{jk(\rho_T-\rho)} \text{ is the phase of the incident field at the lower edge relative to the incident field phase reference at the top edge.}
\]

This field is incident obliquely on the lower edge: the obliquity angle is:

\[
\beta_0 = \frac{\pi}{2} - \tan^{-1}\left(\frac{x'}{\sqrt{(y')^2+(z')^2}}\right).
\]

From Chapter II, the slope wave diffracted field at a distance \(\rho_2\) with incident field \(H^\mu_\rho\) a distance \(\rho\) from the edge is given by:

91
The \( v_{sw} \) form of the slope wave diffraction coefficient has been used since it allows one to evaluate the fields on the shadow-boundary and for the horn-reflector, the shadow boundary occurs in a region where other techniques are invalid. In addition, it is important that this form be used since the upper edge of the horn-reflector occurs on this shadow boundary and one needs this form to find the fields incident on this edge. In terms of the incident field at the edge:

\[
E^d = \frac{jkZ_0}{\sin \beta_0} \ H_p^i(\rho,0) \ v_{sw}(\phi_2,\rho) \ e^{\frac{-jk\rho_2}{\sqrt{\rho_2}}}
\]

From Eq. (47), the fields of the x-directed electric line source are:

\[
E_x = -Z_0 k I_x^e \ e^{\frac{j \pi \rho}{2\sqrt{2} \pi k}} \ e^{\frac{-jk\rho_2}{\sqrt{\rho_2}}}
\]

Equating the fields of Eqs. (82) and (47) yields:

\[
I^e = \hat{x} \left[ \frac{-2\sqrt{\lambda}}{Z_0} \ e^{-j \frac{\pi \rho}{2}} \right] \ H_p^i(0,0) \ e^{jk\rho_2} \ e^{\frac{3}{2} \sqrt{\rho_2}} \ v_{sw}(\phi_2,\rho).
\]
Now substituting the value of $H^i(0,0)$ as given by Eq. (80) yields

$$
\frac{I_x^e}{2} = \frac{2\sqrt{2}}{2}\ e^{-\frac{j\pi}{4}} \frac{1}{\sin \beta_0} \frac{\pi}{2\rho \tan \alpha_H} \ E_0 \ \frac{\rho e}{\rho} \ e^{jk\rho_T} \ \rho^{3/2} \ .
$$

where

$$\phi_2 = \pi + \beta - \alpha_H - \phi'$$

for the lower edge of the horn-reflector aperture,

and

$$\phi'$$ is the projection of the angle to the observation point onto the plane normal to the edge at the point of diffraction.

Following a similar procedure, one may find an equivalent slope wave current on the top edge due to the field radiated directly from the horn apex. Since this field is not incident at grazing incidence on the top edge, there will be two terms in the slope diffracted field, one for the incident shadow boundary and one for the reflection boundary. In addition, there is also an equivalent (non slope wave) GTD current on the top edge where the source is the slope wave diffracted fields from the lower edge.* These currents are given by:

$$
\frac{\rho e}{\rho} \ \frac{1}{\sin \beta_1} \ \frac{1}{\sin \beta_0} \ \frac{2\pi}{2\rho \tan \alpha_H} \ E_0 \ \frac{\rho e}{\rho} \ e^{jk(\rho + \rho' - \rho)} \ \rho^{3/2} \ .
$$

*There is also an additional slope wave diffracted field associated with the fields reflected from the lower portion of the paraboloid, incident on the lower edge of the aperture. The shadow boundary for this term lies in the direction of the main beam and the effects of this diffraction are negligible except in the vicinity of the main beam. Since the aperture integration is used for this portion of the pattern, this term may be neglected.
where
\[ p = \sqrt{(x')^2 + (y')^2 + (z')^2} \]

\( p' \) is the distance from the horn apex to the point of diffraction on the bottom edge which illuminates the top edge and is subsequently diffracted.

\[ \beta_0 = \text{obliquity angle for the bottom edge}, \]
\[ \beta_1 = \text{obliquity angle for the top edge}, \]

\( \psi \) and \( \psi_0 \) are the angles, in the plane normal to the diffracting edge, to the observation point and source points respectively.

The equivalent magnetic line source currents on the H-plane walls are determined in terms of conventional diffraction theory for a uniform E-plane field distribution in the horn or in terms of the slope diffraction theory for the cosine E-plane field distribution (corrugated E-plane walls). For the uniform field distribution, the current is:

\[
I^m = \hat{t} \frac{2j}{k} E^i \frac{1}{2 \sin(\beta_H)} \left[ \frac{2}{-\cos \psi} \right]
\]

where
\[ \hat{t} \] is the unit tangent vector along the E-plane edge,
\[ E^i \] is the incident electric field,

and
\[ \psi \] is the angle to the observation point measured in the plane normal to the edge.

A similar equivalent current accounts for the diffraction from each of the E-plane edges. For the case where the E-plane walls are corrugated, the equivalent magnetic currents have the form obtained by equating the fields radiated by a magnetic line source (Eq. (19) of Chapter II) and the fields slope diffracted by an edge where the normal component of incident magnetic field is vanishing (this case is the dual of the cases considered in Appendices A and B and is similar to the corrugated horn analysis of Chapter IV). The far zone slope diffracted field associated with the vanishing tangential component of incident magnetic field is given by the \( E_{SW} \) term of Eq. (68), i.e.,

\[
E_{SW} = Z_0 H^d_{||} = jk Z_0 (-E^i_{\rho}) D_{SW}(\theta_2) e^{-jkr} \frac{e^{-j\theta}}{\sqrt{r}}
\]
where \( E_p = \frac{1}{j\omega} \frac{1}{p} \left( \frac{3H_i}{\lambda n} \right) \) is the field incident on the edge which is proportional to the normal derivative of the vanishing magnetic field,

\[ D_{sw}(\theta,2) \]

is the plane wave form of the slope diffraction coefficient given in Appendix A.

Equating this field to Eq. (19) yields the equivalent magnetic current:

\[ I^m = -\hat{E}_E 2\sqrt{\lambda} \frac{\partial H_i}{\partial n} D_{sw}(\psi,2) \]

where \( \psi \) is as defined above.

\( D_{sw}(\psi,2) \) is the plane wave form of the slope wave diffraction coefficient for the half plane \( n=2 \) given in Appendix A.

The \( D_{sw} \) form of the slope wave diffraction coefficient has been used even though the function is singular on the shadow-boundary since the incident wavefront is planar (i.e., source at infinity) after reflection from the paraboloidal surface. This model for the horn-reflector with corrugated E-plane walls neglects the scattering from the end of the corrugated horn wall and assumes that the co-sinusoidal field distribution in the corrugated horn E-plane is projected to the horn-reflector aperture after reflection from the paraboloid. These approximations should be valid in view of the attenuation of the fields along the corrugated wall (see the calculated results of Chapter III and the measured aperture field distribution shown in Appendix E). These equivalent currents and the aperture integration have been used to obtain the horn-reflector patterns shown in the next section.

B. Radiation Patterns

This section presents numerous calculated horn-reflector patterns along with a measured pattern and some additional computations to demonstrate the validity of the calculated results. In a recent paper [73], Thomas presented a measured and calculated pattern for a horn reflector which had a 14.5° E-plane flare angle, 15° H-plane flare angle and a horn axis to reflector axis angle of 90°. At 3.74 GHz, the aperture width was 32.4\( \lambda \), and the length of the E-plane aperture edge was 32.4\( \lambda \). The reflector focal length was 29.2\( \lambda \). The envelope of his measured pattern and the computed pattern using the techniques just discussed are shown in Fig. 57 for the E-plane. Figure 58 shows the computed H-plane pattern. No measurements were available for the H-plane. Thomas'
Fig. 57—Comparison of measured and calculated E-plane horn-reflector patterns.
Fig. 58--Computed H-plane pattern of horn-reflector.
goal was reduction of the lobe near 90° on the E-plane pattern through the use of blinders. As will be shown later in this section, this can be much more effectively done through the use of corrugated E-plane horn walls.

Calculated radiation patterns for several additional horn-reflectors are included in this section. Both E-plane and H-plane patterns are included for each of the 8 horns listed in Table 3. The other half of the E-plane is the same as the one shown because of the symmetry of the horn-reflectors in this plane. The H-plane pattern is not symmetric but the half with all the interesting features has been shown. In general, the other half of the H-plane pattern will have much lower side-lobe levels than shown in this half of the pattern. The E-plane walls of horns 1 through 5 are conventional conducting walls while horns 6 to 8 have corrugated walls. In each case, the horn flare angles have been adjusted to maintain the 5° half power beamwidth of each antenna. This means that the aperture of the horn reflector with corrugated walls will be wider than the horn-reflector with conventional walls. Each of the radiation patterns includes a scale drawing of the antenna profile. The radiation patterns for horn-reflector number 1 appear in Fig. 59. The solid line is the E-plane pattern (Eₚ in the φ=0 plane) while the dashed line is the H-plane pattern (Eₜ in the φ=90° plane). The H-plane lobe near 80° is due to the "spillover" of the energy from the horn which does not strike the reflector. The level of this lobe may be predicted using standard H-plane horn pattern curves and horn gain equations.[74] For horn-reflector number 1, the pattern level for the horn alone is -14.4 dB in the direction along the wall of the horn. The gain of the horn is 27.6 dB while the gain of the horn-reflector is 31.6 dB. Using this information, combined with a 6 dB reduction for the diffraction from the top edge (based on a 2 dimensional analysis), one concludes that the "spillover" lobe should be at a level approximately given by -14.4 dB - (31.6 dB - 27.6 dB) - 6 dB or -24.4 dB. This calculation is in reasonable agreement with the calculated pattern. Extending the upper edge of the reflector would redirect this lobe in the general direction of the main beam. An extension corresponding to a 10° angular segment (center of curvature at the lower edge) would shift this lobe slightly from the main beam maximum and reduce its maximum by about 20 dB. The H-plane lobe at 160° is due to the diffraction from the E-plane edges of the horn aperture. This lobe plus much of the energy in the 30° to 180° portion of the E-plane can be significantly reduced by replacing the conducting horn walls by corrugated walls. This has been done in antenna number 6 and the resulting patterns appear in Fig. 64. One justifiable objection to the horn reflector is its size. The horn reflectors just mentioned are over 45 wavelengths long. As mentioned earlier, a substantial reduction in length may be realized by directing the horn into the reflector (i.e., make β > 90°). Horn-reflectors number 2 and 3 show the patterns obtained with a 20λ focal length reflector for horn axis
to reflector axis angles of 120° and 140° respectively (patterns in Figs. 60 and 61 respectively). Note the size reduction illustrated by the scale drawings of the antennas and the improved E-plane patterns. The H-plane patterns still have the "spillover" lobe (now at 120° - \( \alpha_H = 105.3° \) for horn 2 and at 140° - \( \alpha_H = 122.8° \) for horn 3) and E-plane edge lobe although this lobe is now beyond 180° on the patterns. The indicated lobe on the H-plane pattern is in error due to the singular nature of the plane wave diffraction coefficient on the shadow boundary. The dots indicate the anticipated pattern in this direction. Similar patterns result when a shorter focal length reflector is used. Figures 62 and 63 show the radiation patterns for horn-reflectors 4 and 5 with a 10\( \lambda \) focal length reflector. Here the "spillover" lobe is lower since the gain difference between the horn and the horn-reflectors increases for the smaller horn. Figures 65 and 66 show the radiation patterns obtained for horn-reflectors number 7 and 8 with corrugated E-plane walls. These two horn-reflectors are comparable in design, but superior in performance, to antennas 2 and 4 respectively.

C. Conclusions

In reviewing the results presented here for the various horn-reflectors geometries, some general conclusions may be drawn about this antenna type. For low noise applications, the small horn-reflectors (i.e., one with \( \beta > 90° \)) with corrugated E-plane walls and an extended H-plane edge is the best horn-reflectors geometry. The corrugated walls greatly reduce the lobe levels in the E-plane and the extended H-plane edge would reduce the "spillover" lobe over the top of the reflector. The additional suggestion for having the angle from the reflector axis to the horn axis (\( \beta \)) greater than 90° greatly reduces the size of the antenna and would justify using the horn-reflectors instead of the corrugated horn alone. The usual \( \beta = 90° \) case appears to be the worse design in terms of the "spillover" lobe levels and increased gain due to the reflector. The gain of the horn reflector for the \( \beta = 90° \) case does not seem to justify the complication of using a reflector on top of the horn. For the \( \beta = 90° \) cases considered the gain of the horn alone was only 3 or 4 dB lower than the gain of the horn-reflectors. The \( \beta > 90° \) designs are also better since the equivalent horn has a lower gain thus reducing the relative level of the "spillover" lobe in the H-plane.
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Fig. 59—Radiation patterns of horn-reflector number 1 (Table 3).
Fig. 60—Radiation patterns of horn-reflector number 2 (Table 3).
Fig. 61—Radiation patterns of horn-reflector number 3 (Table 3).
Fig. 62—Radiation patterns of horn reflector number 4 (Table 3).
Fig. 63—Radiation patterns of horn-reflector number 5 (Table 3).
Fig. 64—Radiation patterns of corrugated horn reflector number 6 (Table 3).
Fig. 65—Radiation patterns of corrugated horn reflector number 7 (Table 3).
Fig. 66—Radiation patterns of corrugated horn-reflector number 8 (Table 3).
CHAPTER VI
SUMMARY OF THE PATTERN ANALYSIS OF AN
OFFSET FED PARABOLIC REFLECTOR ANTENNA

The radiation pattern of an offset fed parabolic reflector has been analyzed by Dr. P. H. Pathak and this author under the supervision of Professor L. Peters, Jr. [75] and will not be treated in detail here.* The techniques used in the analysis of this antenna are essentially the same ones used in the analysis of the other antenna types included in this work and are therefore only briefly summarized in this section.

A. Antenna Geometry

The offset fed parabola, because of the lack of aperture blockage, has radiation properties which make it suitable for radiometer applications when properly designed. In practice, the feed horn phase center is located at the focus of the reflector. The reflector is a segment of a paraboloid described by passing a plane at an angle, \( \theta_1 \), from the reflector axis through the paraboloid. The reflector geometry as shown in Fig. 67a is described by the following parameters:

\[ F = \text{focal length of the reflector} \]
\[ \theta_1 = \text{angle between reflector axis and feed assembly axis} \]
\[ \theta_2 = \text{angle between reflector axis and plane of reflector rim.} \]

In the unprimed coordinate system of Fig. 67b, the paraboloid has an equation:

\[ (90) \ X^2 + Y^2 = 4F (Z + F) \]

and the equation of a plane making an angle \( \theta_2 \) with the \( Y-Z \) plane and passing through the point \( (AF, 0, -F) \) is:

\[ (91) \ X - AF = (Z+F) \tan \theta_2. \]

The rim of the C-band reflector is described by the intersection of the paraboloid and the plane. The major and minor diameters of the elliptic rim may be found by describing the paraboloid and the plane

*This work was supported in part by Contract N00178-71-C-0264 with U. S. Naval Weapons Laboratory, Dahlgren, Virginia, monitored by Mr. Joseph Halberstein.
in a rim centered coordinate system, and comparing the equation of the intersection of these two surfaces to the equation of an ellipse:

\[
\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1. \tag{92}
\]

This results in:

\[
(93) \quad A = \sqrt{\frac{4F \cos \theta_2}{\sin^3 \theta_2} \left( \frac{F \cos \theta_2}{\sin \theta_2} + AF \right)}
\]

and

\[
(94) \quad B = A \sin \theta_2.
\]

With these values known, it is then possible to express the coordinates of any rim point in polar coordinates as:
(95) \[ \mathbf{r} = \rho(\phi_r) \left[ \cos \phi_r \hat{x}' + \sin \phi_r \hat{y}' \right] \]

where

(96) \[ \rho(\phi_r) = \frac{A(B)}{\sqrt{A^2 \sin \phi_r + B^2 \cos^2 \phi_r}}. \]

The geometry presented here is intended to give the reader some idea of the important parameters used in the analysis and additional geometric relations may be found in Reference 34.

The main beam and first few sidelobes of the far zone antenna pattern are computed using aperture integration techniques described in the next section. The integration over the focal plane of the reflector makes use of an illumination function \( G(\theta_f, \phi_f) \) which approximates the feed horn patterns measured at the ElectroScience Laboratory. Beyond the first few sidelobes, the far zone fields are calculated using the geometrical theory of diffraction (G.T.D) to compute the diffracted fields. In the G.T.D. analysis, the influence of the dielectric which covers the edge is neglected and the edge is assumed to be sharp and perfectly conducting. Spillover pattern functions which approximate the measured horn patterns are used for the feed horn spillover in the elevation, 45°, and azimuth planes.

B. Aperture Integration

The main beam and first few sidelobes of the far field antenna pattern are calculated using a Simpson integration over the X-Y coordinates of the reflector focal plane. The reflector, the plane of integration and the coordinates of the observation point are shown in Fig. 68. The far zone electric field due to both electric and magnetic current sources is [76]:

(97) \[ E(\mathbf{R}) = \frac{+jZ_0}{4\pi} \frac{e^{-jkR}}{R} \int_{\text{source}} e^{jkr} \mathbf{r} \times (\mathbf{r} \times \mathbf{d} \mathbf{E}(\mathbf{R}')) + \]

\[ + Y_o \mathbf{r} \times \mathbf{d} \mathbf{m}(\mathbf{R}'); \]

where \( \mathbf{R} = R \hat{R} \) is vector to field point

\( \mathbf{R}' = R' \hat{R}' \) is vector to source point

\( \mathbf{d} \mathbf{E}(\mathbf{R}') = J_S(\mathbf{R}')dS = \text{electric current moment} \)

\( \mathbf{d} \mathbf{m}(\mathbf{R}') = K_S(\mathbf{R}')dS = \text{magnetic current moment} \).
Thus, by duality, the far zone magnetic field may be expressed by:

\[(98) \mathcal{H}(\mathbf{R}) = + \frac{jk}{4\pi} \mathcal{V}_0 e^{-jkR} \int_{\text{source}} e^{jkR'} \mathbf{\hat{R}} \times (\mathbf{\hat{R}} \times \mathbf{K}_S(\mathbf{R}')) - \\
-2 \mathcal{Z}_0 \mathbf{\hat{R}} \times \mathcal{J}_S(\mathbf{R}')) dS'\]

where

\[\mathbf{K}_S = \text{magnetic surface current} = \mathbf{E} \times \mathbf{n}\]
\[\mathcal{J}_S = \text{electric surface current} = \mathbf{n} \times \mathbf{H}\]

In general, the surface of integration \(dS'\) must completely enclose the antenna but for the main beam and first few sidelobes, some important simplifications occur. Over the back surface of the metallic reflector, the magnetic surface current, \(\mathbf{K}_S = \mathbf{E} \times \mathbf{n}\), is zero and the electric surface current, \(\mathcal{J}_S = \mathbf{n} \times \mathbf{H}\), is very small. Because of this, the rear surface of the reflector may be neglected in the integration. Another simplification occurs because the observation point is near the reflector axis (i.e., \(\mathbf{R} \sim \mathbf{Z}\)) and \(\mathbf{R} \times \mathbf{K}_S\) and \(\mathbf{R} \times \mathcal{J}_S\) over the region between the focal plane and the reflector...
rim are nearly zero and may be neglected in the integration over $S'$. Thus only the focal plane fields need to be considered in the near-axis far field computations and the expression for the far zone magnetic field becomes

$$H = \frac{jk}{4\pi} \frac{e^{-jkR}}{R} \int_{\text{focal plane}} e^{-jkR'} \cdot R x (\hat{n} \times \overline{H}^a) + Y_o \cdot (\hat{R} x (\overline{E}^a \times \hat{n})) dS'$$

where $\overline{E}^a$ and $\overline{H}^a$ are the electric and magnetic fields in the aperture (focal) plane. The electric and magnetic fields in the focal plane are related by:

$$\overline{E}^a = z_0 \cdot \overline{H}^a \times \hat{z}$$

since the focal plane is in the far field of the feed horn and since the focal plane is an equiphase surface. Thus the far zone magnetic field is:

$$H = \frac{jk}{4\pi} \frac{e^{-jkR}}{R} \int_{\text{focal plane}} e^{jkR'} \cdot R x (\hat{n} \times \overline{H}^a) + \hat{R} x [\hat{R} x ((\overline{H}^a x \hat{z}) \times \hat{n})] dS'$$

where

$$\hat{R} = \hat{x} \sin \phi \cos \theta + \hat{y} \sin \phi \sin \theta + \hat{z} \cos \phi$$

$$\hat{n} = \hat{z}$$

and

$$\overline{H}^a = H^a \hat{x}.$$ 

Using the above expressions for $\hat{R}$, $\hat{n}$ and $\overline{H}^a$, the integrand reduces to:
\[ e^{jkR'} \hat{R} \hat{H}_x = e^{jkR'} (\cos \phi \cos \phi' + \sin \phi \sin \phi') \]

For angles near the main beam, the \( \sin \phi \) and \( \sin^2 \phi \) terms are small and may be neglected while the \( \cos \phi \) and \( \cos^2 \phi \) terms are approximately 1. The phase term yields:

\[ e^{jkR'} \hat{R} = e^{jkR'} \sin \phi (\cos \phi \cos \phi' + \sin \phi \sin \phi') = e^{jkR'} \sin \phi \cos(\phi - \phi'). \]

Thus the main beam magnetic field may be found from:

\[
(101) \quad \hat{H} = \int_{\text{focal plane}} \frac{e^{jkR}}{2\pi R} \sin \phi \cos(\phi - \phi') \hat{H}_x \, ds'.
\]

The aperture fields used in the aperture integration were obtained by a functional representation of the measured feed horn patterns. The feed horn patterns for the mid-C-band frequency used throughout this section appear in Fig. 69. These patterns (and others in Ref. 77) were measured azimuth patterns for constant elevation angles. The 5 dB contours were drawn in by hand.

Antenna patterns calculated using the aperture integration appear in Figs. 70 through 72. These figures show the sum pattern in the elevation, 45° and azimuth planes respectively at a mid-C-band frequency. The calculated 3 dB beamwidths in azimuth and elevation show excellent agreement with the measured values (1.75° calculated versus approximately 1.8° measured). The calculated first sidelobe levels of -28 dB in azimuth and -30 dB in elevation are somewhat lower than the measured values which are -25 dB in the azimuth plane and -25 dB above the reflector axis and -21.4 dB below the reflector axis in the elevation plane. This asymmetry and increased sidelobe level are probably due to the presence of the dielectric layer in front of the C-band metallic reflector. Additional comments on the influence of this layer appear later in this section.
Fig. 69--Measured feed patterns for $E$ parallel to azimuth plane.
Fig. 70--Elevation (H-plane) sum pattern at a mid C-band frequency calculated using aperture integration over the reflector focal plane.
Fig. 71—45° plane sum pattern at a mid C-band frequency calculated using aperture integration over the reflector focal plane.
Fig. 72—Azimuth (E-plane) sum pattern at a mid C-band frequency calculated using aperture integration over the reflector focal plane.
C. Diffracted Field Analysis and Results

The analysis presented in the previous chapter is inadequate for computing radiated fields for angles sufficiently removed from the beam axis. There are for this case two common types of far out lobes and these are the spillover lobes and the back lobes. These may both be treated by means of the Geometrical Theory of Diffraction[78].

The diffracted field in terms of the incident field and the dyadic diffraction coefficient $\mathbf{D}_\mathbf{A}$ of Reference 79 is:

\[
\mathbf{H}_\mathbf{d}(P) = \mathbf{H}_\mathbf{i}(Q) \cdot \mathbf{D}_\mathbf{A} F(S) e^{-ikS},
\]

where $F(S)$ denotes the spatial attenuation of the diffracted field over a distance "S" along the ray path to the observation point. Similarly, $e^{-ikS}$ is the phase delay (for an $e^{i\omega t}$ time dependence, where $\omega$ = angular frequency of the wave, and $k$ = free space wave number along the path).

Alternatively, the diffracted field may be expressed in a convenient matrix representation as:

\[
\begin{bmatrix}
\mathbf{H}_\mathbf{i}(P) \\
\mathbf{H}_\mathbf{d}(P)
\end{bmatrix} =
\begin{bmatrix}
-D_h & 0 \\
0 & -D_s
\end{bmatrix}
\begin{bmatrix}
\mathbf{H}_\mathbf{i}(Q) \\
\mathbf{H}_\mathbf{d}(Q)
\end{bmatrix} F(S) e^{-ikS}.
\]

$F(S)$ is explicitly given by [79]

\[
F(S) = \frac{\rho_c}{\sqrt{S(\rho_c^2+S^2)}} \approx \frac{\rho_c}{\sqrt{S}}, \text{ if } \rho_c << S
\]

where $\rho_c$ is the distance to the caustic. (The condition $\rho_c << S$ is true in the far-zone of the reflector.)

The diffraction coefficients $D_s$ and $D_h$ are the acoustic soft and hard wedge diffraction coefficients, respectively. The interior wedge angle is defined as $(2-n)$; $n = 2$ for a half plane. $D_s$ and $D_h$ are given by [79]

\[
D_s = \frac{1}{\sin\beta_0} \left[ (d^+(\beta^-,n)F[\kappa a^+(\beta^-)] + d^-(\beta^-,n)F[\kappa a^-(\beta^-)] \right.
\]

\[
+ \left. (d^+(\beta^+,n)F[\kappa a^+(\beta^+)] + d^-(\beta^+,n)F[\kappa a^-(\beta^+)] \right],
\]

where
The parameters which appear in $F[\kappa a^- (\beta)]$ are defined below

\begin{equation}
\kappa^+ (\beta) = 1 + \cos(-\beta + 2nN^- \pi)
\end{equation}

in which $N^-$ is the positive or negative integer or zero, which most nearly satisfies the equations

\begin{equation}
\begin{cases}
2\pi N^- - \beta = -\pi, \\
2\pi N^+ - \beta = \pi.
\end{cases}
\end{equation}

$\kappa = kL$ is the largeness parameter in the asymptotic evaluation of the pertinent integrals involved in the formulation of the dyadic diffraction coefficient. The quantity $L$ (appearing in $\kappa = kL$) may be viewed as a distance parameter which depends upon the type of edge illumination.

We have thus described (briefly) the analysis for calculating the edge diffracted fields of the reflector antenna. For the elliptically shaped edge of this reflector antenna, there are always two diametrically opposite points on the edge (except at the caustic) which give rise to diffracted rays in the elevation, azimuth and the 45° planes, respectively. As mentioned earlier, the ray directly radiated by the feed (spillover) must be added to the diffracted rays whenever the feed rays are not shadowed by the reflector. The diffracted rays properly combine with the feed ray at the shadow boundaries to produce a total field which is finite and continuous across the shadow boundaries. The latter is due to the Fresnel functions occurring in the expression for the wedge diffraction coefficients ($D_s$ and $D_h$) of Eq. (103). In addition to the two edge diffracted rays, it is possible to have the rays arriving at P which are produced by the reflection of edge diffracted rays from the concave surface of the reflector. These diffracted reflected rays are not included in our analysis since
these are of negligible amplitude in the H-plane because the electric field is polarized parallel to the edge of the reflector, and the reflector surface is very nearly flat; in the E-plane these rays are again not included as the dominant contribution still comes from the diffracted electric field component which is parallel to the edge. We note that each edge diffracted ray is shadowed in a small region behind the reflector as indicated in Fig. 73. Since the reflector is very nearly flat, this shadowed region for the edge diffracted rays is extremely small. The edge diffracted ray along the shadow boundary gives rise to surface rays within the shadow region. However, these fields are negligible for the reflector under investigation and their inclusion is not necessary as is verified by the calculated results which do not show any discontinuities at the shadow boundaries of the edge diffracted rays.

![Diagram of diffracted ray shadow region](image)

**Fig. 73**--Illustration of diffracted ray shadow region.

The elliptic edge of the reflector gives rise to a focusing effect behind the reflector due to the formation of a caustic of the diffracted rays. However, of the three planes considered, only the elevation plane (H-plane) contains the caustic. Figure 74 illustrates the caustic direction. Every point on the edge contributes to the diffracted field in the caustic direction so that the ordinary GTD analysis based on just two edge diffracted rays is invalid in the vicinity of the caustic; in addition the GTD result
tends to become singular on the caustic. It is obvious that a smooth transition is required from the solution at the caustic to the GTD solution in some region close to the caustic. Even though GTD fails at caustics, it may be easily modified to analyse the field behavior on and in the neighborhood of the caustic. We use equivalent edge currents to evaluate the field in the vicinity of the caustic; the equivalent edge currents are determined as previously discussed from the GTD representation [80].

![Diagram](image)

\[ \theta_c = \text{CAUSTIC DIRECTION} = 360^\circ - 2\theta_2 \]

\[ \text{IN THE ELEVATION PLANE} \]

Fig. 74--Illustration of diffracted rays in the direction of the caustic.

Antenna patterns calculated using the theory outlined in this section and the computer programs presented in Reference 34 appear in Figs. 75, 77, and 78 for elevation, 45°, and azimuth scan at a mid-C-band frequency. The geometry of the antenna used in the calculations is described in the parameters presented in the introduction to this section. These parameters are:

\[ \theta_1 = 51.5^\circ \]

\[ \theta_2 = 68.33^\circ \]

\[ F = 129.54 \text{ cm} = 51 \text{ inches} \]

\[ A = 116.23 \text{ cm} = 45.75 \text{ inches} \]
Some measured antenna patterns were available for an offset fed parabola with a dual frequency 51" focal length reflector and a multi-horn monopulse feed assembly. The C-band reflector is the antenna of interest and is situated slightly above and behind an X-band reflector made of a wire grid embedded in a dielectric skin. The X-band reflector operates with vertical polarization and the C-band with horizontal polarization. Because of the polarization difference, the X-band grid has little or no influence on the C-band antenna patterns. The dielectric material around the grid and the honeycomb material which supports the X-band grid are neglected in the computed results. Measured results obtained by Naval Weapons Laboratory on this antenna appear in Figs. 76 and 79 for elevation and azimuth scan. The calculated results have been plotted to the same scale as the measured results for easy comparison. Calculations were performed in 0.2 degree increments, then plotted by the computer on a Calcomp plotter. The aperture integration results were added separately as was the caustic correction term in the elevation plane pattern. Comparing the measured and calculated results quickly shows that all the pattern trends are predicted by the calculations but that at the low levels (-40 dB to -60 dB) the measured results are some 5 to 15 dB higher than calculated. There are several possible explanations for this discrepancy; 1) noise in the pattern recording equipment preventing accurate measurement at these low levels, 2) scattering by obstacles on or around the pattern measuring range, or 3) the model used in the calculations did not exactly match the antenna used in the measurement program. Commenting further on item 2 above a large parabolic reflector on another building near the pattern range at N.R.I. is known to have caused the large lobes recorded near 180° on the measured patterns. Other buildings and a roof-top railing may have caused some other measurement problems. On item 3; since the reflector was not available to the author for detailed physical measurements, calculations were based on the best available dimensions which at times were measured from approximately scaled sketches supplied by N.W.L. Better agreement could certainly be obtained with more accurate information on the physical dimensions of the antenna for one particular case. For example, the spillover lobe at -98° on the elevation pattern of Fig. 71 is 4 dB higher than measured but only a 6 degree change in the feed assembly positioning would bring this lobe into agreement with the measurement. The feed position is determined by the angle 01 shown in Fig. 67 and was measured from a sketch supplied by N.W.L. which later turned out to be not to scale. In spite of these problems, the agreement between the measured and calculated values is fairly good. Near the main beam, the measured sidelobe levels are higher than calculated but this is almost certainly due to the presence of the dielectric layers and the honeycomb support which were in front of the metallic reflector during the measurements. Theoretical treatment of the reflection from the dielectric is very difficult because of the anisotropic properties of the honeycomb. The feed horn
is normally directed at the center of the reflector, thus causing nearly equal spillover lobes above and below the reflector. The measured patterns however were for a case where the feed was directed considerably below the reflector center. The lower (upper) edge of the reflector occurred at 27.5° (57.3°) on the feed horn elevation pattern of Fig. 69 resulting in a lower (upper) edge illumination of -6.5 dB (-23.5 dB). Redirecting the feed to the center would result in edges at ± 42.4° on the horn pattern and levels of -13 dB at both edges. This would reduce the 98° spillover lobe in the H-plane by 6.5 dB while only slightly increasing the spillover lobe on the top edge (near 180° on the E-plane and H-plane patterns). In addition to redirecting the horn, the lobes at ±90° in the E-plane may be reduced substantially by the use of absorber covered blinders on the feed horn. These lobes are the 90° lobes on the feed horn pattern of Fig. 69. A reduction of those lobes by 15 dB has been demonstrated[81] through the use of these blinders.

D. Conclusions

The complete pattern of an offset fed parabolic reflector antenna has been calculated using singly-diffracted rays of the geometrical theory of diffraction (G.T.D.) with integral corrections near the main beam and in the caustic direction. The G.T.D. solution has several advantages over other solutions since it is obtained in the form of simple functions and its computation cost is low. Also, since it is directly related to the radiation mechanism of the antenna, the G.T.D. solution can readily be extended or modified for use as a design tool. For aperture antennas where the aperture blockage is not significant (which should certainly be the case with the offset feed) the agreement between the calculations and measurements is usually excellent. For this particular antenna, the agreement is only fair since the pattern shapes are predicted but the measured pattern levels are higher than the calculated levels. The calculated values are probably more accurate than the measured values for an antenna of this size because of the very low levels being measured. Better agreement should be obtained if the measurements are repeated on a better pattern range and/or if calculations are repeated with more accurate antenna dimensions for input parameters. In considering this type of antenna for high beam efficiency (low noise) applications, significant improvement can be made in the pattern by slight design changes. For example, use of a corrugated feed horn would greatly reduce the E-plane spillover lobes between 60° and 120° on Fig. 78. Also, redirecting the feed so that the horn axis was directed at the center of the reflector would reduce the spillover lobe at 90° on the H-plane pattern. With these modifications, the offset fed parabola can be made a low noise antenna and would probably be the best design for antennas which have beamwidths on the order of 1 or 2 degrees.
Fig. 75—Elevation plane (H-plane) patterns at a mid C-band frequency calculated using G.T.D. with aperture integration ($-10^\circ \leq \theta \leq 10^\circ$) and caustic correction ($\theta \sim -135^\circ$).
Fig. 76—Measured elevation plane (H-plane) pattern at a mid C-band frequency.
Fig. 77—45° plane pattern at a mid C-band frequency calculated using G.T.D. and aperture integration ($-10^\circ \leq \theta \leq 10^\circ$).
Fig. 78—Azimuth plane (E-plane) pattern at a mid C-band frequency calculated using G.T.D. with aperture integration ($-10^\circ \leq \theta \leq 10^\circ$).
Fig. 79--Measured azimuth plane (E-plane) pattern at a mid C-band frequency.
The cassegrain, or dual-reflector antenna, is of interest primarily for narrow beam applications since the aperture blockage due to the subreflector is less serious in the high gain, narrow beam antenna. While there has been considerable work done on the effects of reflector shaping [82], polarization characteristics [83], efficiency [84], and general design [85,86], there has been relatively little published about the far-out sidelobe levels associated with the cassegrain. Since the interest here is in high beam efficiency antennas, this is one of the most important parameters. This effort will not examine one specific design which may or may not be somebody's "optimum" design, but rather, we will attempt to establish some general design criteria for low noise cassegrain antenna design by examining the radiation patterns of a typical cassegrain antenna. In addition, the computer programs developed here may be of interest in calculating the sidelobe levels of a particular antenna since it is extremely difficult if not impossible to accurately measure the far-out sidelobe levels of large aperture antennas.

A. Analytical Model

The radiation pattern analysis of the cassegrain is divided into two parts, the aperture integration for observation points near the main beam, and the Geometrical Theory of Diffraction for the sidelobe and backlobe region. The geometry of the antenna, shown in Fig. 80, is the most common cassegrain design, namely a paraboloidal main reflector (focal length = $F$, diameter = $D_p$) and a hyperboloidal subreflector (focal length = $F'$, diameter = $D_h$). One focus of the hyperboloid coincides with the focus of the paraboloid (located at $z = F$). The feed antenna, located at the other focus of the hyperboloid (at $z = F - 2F'$), is assumed to have an axially symmetric pattern. The field of the feed at a distance $S$ from the feed is given by:

$$F(\theta, s) = \begin{cases} 
(0.00316 + \cos^m \theta) \frac{e^{-jks}}{s} & \text{for } \theta < 90^\circ \\
0.00316 \frac{e^{-jks}}{s} & \text{for } \theta \geq 90^\circ ,
\end{cases}$$

where $m$ is chosen to provide the desired amplitude taper over the subreflector. The feed polarization is assumed to be such that the electric field is always parallel to the x-z plane. This feed pattern is used in determining the aperture fields used in the aperture integration and also in determining the spillover fields. This pattern function with a -50 dB backlobe level is very realistic for a feed antenna such as a corrugated horn.
Fig. 80—Geometry of cassegrain or dual-reflector antenna employing a paraboloidal reflector and a hyperboloidal subreflector.

B. Aperture Integration

The equation for the aperture integration was given in Eq. (2), Chapter II and is repeated here for convenience:

\[ E = -\frac{\mathbf{k}}{4\pi} \frac{e^{-jkR}}{R} \iint_A \hat{R} \times [\hat{n} \times E^a - Z_o \hat{R} \times (\hat{n} \times \mathbf{R}^a)] e^{jk\hat{p} \cdot \mathbf{R}} \, ds \]

where as before:

\( \hat{R} \) is a unit vector in the direction of the observation point,
\( \mathbf{p} \) is a vector to the source point,
\( \hat{n} \) is a unit vector normal to the aperture, 
\( k = \frac{2\pi}{\lambda} \) is the propagation constant,  
\( Z_0 = 120\pi \) is the characteristic impedance,  
\( E^a \) and \( H^a \) are the electric and magnetic fields in the aperture of the antenna.

The electric and magnetic fields in the aperture are assumed to be the geometrical optics fields from the subreflector and are related by Eq. (3) where the direction of propagation \( S \) is now \( \hat{n} \):

\[
(3) \quad H^a = \frac{1}{Z_0} \hat{n} \times E^a.
\]

This is a good approximation since the fields in the aperture plane are a finite segment of a nonhomogeneous plane wave. The radiated electric field in the vicinity of the main beam is given by:

\[
(111) \quad E = -\frac{jk}{2\pi} \frac{e^{-jkR}}{R} \int_A E^a (\hat{\phi} \sin \phi - \hat{\theta} \cos \phi) (1 + \cos \theta) \cdot
\]

\[
e^{jk(x' \sin \phi + y' \sin \theta \sin \phi + z' \cos \theta)} dx' \, dy'.
\]

This form neglects the small \( \hat{z} \) component in the aperture (a term proportional to \( \sin \theta \) which is small for the main beam region) and the cross polarized component (a \( y \) component due to the reflections from the hyperboloid and paraboloid). The most significant cross polarized component would arise from the scattering from the subreflector supports. This contribution would also be small and the supports have been neglected in this analysis. The aperture fields are found via ray optics. The pertinent rays propagate from the feed antenna, are reflected from the hyperboloidal subreflector to the paraboloidal reflector and then through the aperture plane. The numerical integration is performed over the aperture plane defined by the rim of the paraboloid (i.e., a circle of diameter \( D_p \) in Fig. 81). The aperture blockage due to the subreflector is treated by assuming an aperture distribution obtained from the negative value of the incident field taken over the shadow region of the hyperboloidal subreflector. In this case, the planar aperture is a disc of diameter \( D_h \) in the plane of the rim of the hyperboloid as shown in Fig. 81. The blockage caused by the subreflector supports has been neglected. These two sets of fields obtained by the aperture integrations include inherently the reflected diffracted fields from the aperture rim (i.e., those diffracted fields associated with the reflection shadow boundary of the paraboloid), and the incident diffracted fields from the subreflector rim (i.e., those diffracted fields associated with the incident shadow boundary of the plane wave.
diffraction from the rim of the hyperboloid). Since these components are the dominant diffracted fields in the main beam, the diffracted fields need not be included in evaluating the main beam fields. However, for angles near the feed spillover shadow boundary (at $\theta = \theta_H$ in Fig. 82), the diffracted fields need to be included. The aperture integrations for the main aperture and the subreflector aperture (aperture blockage) have been combined with the feed diffraction from the subreflector rim and the spillover fields to obtain the radiation pattern for angles near the main beam. When the diffracted field contribution blends smoothly with the aperture integration results, the aperture integrations are stopped and the radiated fields found from the G.T.D.

Fig. 81--Aperture integration approximation: aperture blockage approximated by a flat disk the same diameter as the hyperboloid, $D_H$, located in the plane of the hyperboloid rim.
Fig. 82—Shadow boundaries and distances associated with the cassegrain geometry:
- Direct feed spillover occurs for $\beta_H < \theta < \beta_P$
- Reflected spillover occurs for $\alpha_H < \theta < \alpha_P$.
C. Diffraction Analysis

A fairly simple two point diffraction analysis for each of the rims should predict the far-out sidelobe levels except near the rear axial caustic of the main reflector. In the vicinity of this caustic, the equivalent current concepts are used. Away from the caustic, the diffracted fields from the rim of the paraboloid which may be significant include: 1) the diffraction of the direct illumination from the feed (this field should be very small for a good feed design), 2) the diffraction of the reflected fields from the hyperboloid (if this energy strikes the paraboloid rim), 3) the diffraction of the diffracted energy from the rim of the hyperboloid to the rim of the paraboloid, and 4) the diffraction of the diffracted energy from the opposite edge of the rim of the paraboloid. The significant diffracted fields from the hyperboloid include: 1) the diffraction of the direct illumination from the feed, 2) the diffraction of the fields from the feed, reflected from the hyperboloid, and subsequently reflected from the paraboloid, and 3) the diffraction of the diffracted energy from the opposite edge of the rim of the hyperboloid. The blockage of the diffraction from the rim of the paraboloid by the subreflector is neglected since in general the subreflector is small and this term would only slightly affect a small region of the pattern.

In much of the work to follow, many of the parameters are evaluated by a knowledge of the reflection mechanisms involved. This rather special geometry makes many direct interpretations possible. The two major factors which allow these interpretations involve the focusing action of the paraboloid and the hyperboloid. The focusing action of the paraboloid is such that the fields of a point source at the focus of the paraboloid are reflected from the surface as though the image source was at infinity. The focusing action of the hyperboloid is such that the fields of a point source at one focus of the hyperboloid are reflected from the surface as though the image source was a point source at the other focus. While all of the various distances to be computed could have been obtained from differential geometry and the relations discussed in Appendix 6, the approach used here is less prone to error and lends more physical insight into the involved processes in this analysis.

The diffraction analysis for the paraboloid and hyperboloid rims has been patterned after the work described in Chapter VI for the offset fed parabola. The diffraction by curved edges has been treated in detail by Kouyoumjian and Pathak [87]. For the cassegrain, the equations for the diffraction from the curved edges reduce to:

\[
\begin{bmatrix}
E^d_{\|} \\ E^d_{\perp}
\end{bmatrix} =
\begin{bmatrix}
D_S & 0 \\
0 & \text{D_H}
\end{bmatrix}
\begin{bmatrix}
E^i_{\|} \\ E^i_{\perp}
\end{bmatrix}
\frac{\rho_c}{\sqrt{s(\rho_c + s)}} e^{-js}
\]

(112)
where \( E_\parallel^d \) and \( E_\parallel^i \) are the diffracted and incident electric fields parallel to the diffracting edge,

\( E_\perp^d \) and \( E_\perp^i \) are the diffracted and incident electric fields perpendicular to the diffracting edge,

\[
\sqrt{\frac{\rho_c}{s(\rho_c + s)}}
\]

is the geometrical optics spreading factor for the diffracted field,

\( \rho_c \)

is the distance from the diffracting edge to the caustic of the curved edge (as given in Eq. (113),

\( s \)

is the distance from the point of diffraction to the observation point,

and \( D_S \) and \( D_H \) = the diffraction coefficients for the soft and hard boundary conditions given in Eqs. (105)-(109) of Chapter VI or in Ref. 87.

The caustic distance, \( \rho_c \), is given by Kouyoumjian [88] to be:

\[
(133) \quad \frac{1}{\rho_c} = \frac{1}{\rho_e} - \frac{\hat{n}_e \cdot (\hat{s}' - \hat{s})}{a \sin^2 \beta_0}
\]

where \( \rho_e \) = the radius of curvature of the incident field in the plane containing the incident ray and the tangent to the edge,

\( \hat{n}_e \) = the outward directed unit normal vector to the edge at the point of diffraction,

\( \hat{s}' \) and \( \hat{s} \) are the unit vectors along the incident ray and diffracted ray respectively,

\( a \) is the radius of curvature of the edge at the point of diffraction,

and \( \beta_0 \) is the angle between the edge tangent and the incident ray at the point of diffraction.

Because of the circular symmetry of the system, all the incident and diffracted rays of the cassegrain will be incident normally on the edge. Thus \( \beta_0 \) will always be 90°. For diffraction from curved edges, the distance parameter \( L \) (in \( k = kL \) of Eqs. (105)-(108)) to be substituted in the diffraction coefficients \( D_S \) and \( D_H \) depends on the type of illumination incident on the edge as well as the principal radii of curvature of the edge at the point of diffraction. In general, there will be two \( L \)'s, an \( L \) associated with the incident shadow boundary and an \( L' \) associated with the reflection shadow.
boundary. In the most general form, Kouyoumjian [89] gives $L$ to be:

$$
L = \frac{s(p_e+s)p_1p_2\sin^2\beta_0}{p_e(p_1+s)(p_2+s)}
$$

where $\beta_0$ is as defined after Eq. (113),

$p_e$ is the radius of curvature of the incident (reflected) wavefront for $L^i$ ($L^r$) taken in the plane containing the incident ray and the edge tangent at the point of diffraction,

$p_1$ and $p_2$ are the principal radii of curvature of the incident (reflected) wavefront for $L^i$ ($L^r$),

and $s$ is the distance to the observation point from the point of diffraction.

Because of the symmetry of the cassegrain, the plane of incidence (the plane containing the incident ray and the surface normal) always coincides with a principal plane of the surface at the point of diffraction. This means that $p_e$ will always equal $p_1$ or $p_2$. As an alternative to this argument, the use of a feed with a spherical wavefront ensures that $p_e^i = p_1^i = p_2^i$ and the use of the hyperboloid (which reflects a spherical wavefront) ensures that $p_e^r = p_1^r = p_2^r$.

Thus for the analysis of the cassegrain,

$$
L = \frac{sp_1}{p_1+s}
$$

where $\beta_0 = 90^\circ$ has been included.

For observation points in the far field (i.e., for $s >> p_1$) this further reduces to:

$$
L = p_1^i
$$

It should be pointed out that in general $L^i$ and $L^r$ will differ since the distance to the caustic of the reflected field will not be the same as the source distance for a curved surface (i.e., the curvature of the reflected wavefront is not equal to the curvature of the incident wavefront for reflection from a curved surface). It is further noted that the form of the diffraction coefficient associated with the incident field is unchanged by the curvature of the edge whereas the diffraction coefficient associated with the reflected field is dependent upon the caustic distance associated with the reflected field. This simple concept may be applied to diffraction computations when using the asymptotic format of Keller for the far field or the form given by Pauli which has been used earlier in this dissertation.
D. Diffraction from the Cassegrain Subreflector

The fields diffracted from the hyperbola rim to the point \((s, \theta, \phi)\) include a term from the direct feed illumination (with a spherical phase front), a term associated with the reflected field from the parabola (with a planar wavefront), and the multiply diffracted terms. The diffraction of the direct feed illumination from the upper edge of the hyperbola rim of Fig. 82 is given by

\[
\begin{bmatrix}
E^d_H \\
E^d_L
\end{bmatrix} =
\begin{bmatrix}
D_s & 0 \\
0 & D_H
\end{bmatrix}
\begin{bmatrix}
\sin \phi \\
\cos \phi
\end{bmatrix}
F_x(\beta_H, R_1)e^{jk(z_H \cos \theta + r_H \sin \theta) \frac{\rho_c}{s}} e^{-jks}
\]

where

- \(F_x(\beta_H, R_1)\) is the incident field at the hyperboloid rim due to the feed illumination (given by Eq. (110)),
- \(R_1 = \frac{r_H}{\sin \beta_H}\) is the distance from the feed horn to the hyperboloid rim (shown in Fig. 82),
- \(\sqrt{\frac{\rho_c}{s}} = \sqrt{\frac{\rho_c}{s(\rho_c + s)}}\) is the geometrical optics spreading factor,
- \(\rho_c = \frac{r_H}{\sin \theta}\) is the caustic distance (from Eq. (113)) associated with the circular rim (radius = \(r_H\)) of the hyperboloid,
- \(j k(z_H \cos \theta + r_H \sin \theta)\) shifts the phase of the diffracted field from the point of diffraction to the origin of the coordinate system.

The distance parameters for this diffracted term are:

\[(118) \quad L^i = R_1 = \frac{r_H}{\sin \beta_H},\]

where \(R_1\) is as above and

\[(119) \quad L^r = R_2 = \frac{r_H}{\sin \alpha_H},\]
where $R_2$ is the distance from the focus to the hyperboloid rim as shown in Fig. 82.

A similar term occurs for the diffraction from the bottom edge of the hyperboloid. The differences which occur are in the angle from the rim tangent plane to the observation point (used in the diffraction coefficient) and in the phase term relating the phase of the diffracted field to the origin of the coordinate system. The diffraction of the nonhomogeneous plane wave (feed radiation reflected from the hyperboloid and the paraboloid) by the top edge of the hyperboloid rim is given by:

$$\begin{bmatrix} E^d_\parallel \\ E^d_\perp \end{bmatrix} = \begin{bmatrix} D_s & 0 \\ 0 & D_H \end{bmatrix} \begin{bmatrix} \sin \phi \\ \cos \phi \end{bmatrix} E^i_x e^{jK(z_H \cos \theta + r_H \sin \theta)\sqrt{\rho_C}s} e^{-jks},$$

where $E^i_x$ = the incident field on the hyperboloid rim after reflection from the paraboloidal surface and the hyperboloidal surface,

and $\rho_C = r_H / \sin \theta$ from Eq. (113).

The $L^1$ distance parameter is made large for this case since the wavefront is a nonhomogeneous plane wave. The $L^1$ distance parameter is found in terms of the radius of curvature of the reflected wavefront which is related to the radius of curvature of the hyperboloid at the point of diffraction. The surface radius of curvature needed is that in the plane containing the diffracted ray and the edge normal. This value of the surface curvature may be found using differential geometry but since the reflected wavefront is known for the special case where the source is at one focus of the hyperboloid, we can use this to obtain the desired radius of curvature, $R_c$, from:

$$\frac{1}{R_2} = \frac{1}{R_1} + \frac{2}{R_c \cos \theta_{os}}$$

where $R_1$ is the source distance = $r_H / \sin \theta_H$

$R_2$ is the image distance = $r_H / \sin \alpha_H$

$\theta_{os}$ is the angle between the surface normal and the unit vector to the source point at the point of diffraction.

Thus

$$R_c = \frac{2 R_1 R_2}{\cos \theta_{os}(R_1-R_2)} .$$

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Using this radius of curvature for the surface, one obtains the radius of curvature of the wavefront for the plane wave incident on the hyperboloid from:

\[
\rho_r = \frac{1}{s'} + \frac{2}{R_c \cos \theta_{op}}
\]

where 
- \( s' \) = source distance (infinite)
- \( R_c \) = as above,
- \( \theta_{op} \) = the angle between the surface normal and the unit vector to the source point at the point of diffraction.

Thus the \( L^r \) distance parameter is:

\[
L^r = \rho_r = \frac{R_1 R_2}{R_1 - R_2} \frac{\cos \theta_{op}}{\cos \theta_{os}}
\]

where \( R_1 \) and \( R_2 \) are the previously defined source and image distances.

Another similar term occurs for the bottom edge of the hyperboloid. In addition to these terms, higher order diffraction across the hyperboloid must be included. The doubly diffracted fields from the hyperboloid are given by:

\[
\begin{bmatrix}
E_{H}^{dd} \\
E_{L}^{dd}
\end{bmatrix} =
\begin{bmatrix}
D_s & 0 \\
0 & D_H
\end{bmatrix}
\begin{bmatrix}
E_{H}^1 + E_{H}^2 \\
E_{L}^1 + E_{L}^2
\end{bmatrix} e^{jk(z_H \cos \theta + r_H \sin \theta) \sqrt{\frac{\rho_c}{s}} e^{-jks}}
\]

where 
- \( \rho_c = \frac{r_H}{\sin \theta} \) (from Eq. (113)),
- \( L^i = 2r_H \),

and

\[
L^r = \rho_1 = \frac{2r_H R_1 R_2}{R_1 R_2 - 2r_H |R_1 - R_2| \cos \theta_S \cos \theta_p}
\]
The feed fields diffracted from the opposite edge of the hyperboloid are the $E_{\|}$ and $E_{\perp}$ incident fields above. These fields are given by:

$$
\begin{bmatrix}
E_{\|}^1 \\
E_{\perp}^1
\end{bmatrix} =
\begin{bmatrix}
D_s & 0 \\
0 & D_H
\end{bmatrix}
\begin{bmatrix}
\sin \phi \\
\cos \phi
\end{bmatrix}
F_x (\theta_H, R_I) \frac{1}{\sqrt{2r_H}} \ e^{-jk2r_H}
$$

where $F_x (\theta_H, R_I)$ is the incident feed field at the bottom edge (from Eq. (110)), and

$$
\frac{1}{\sqrt{2r_H}} \ e^{-jk2r_H}
$$

shows the spreading and phase delay from the bottom to the top edge of the hyperboloid (the "j" is necessary since these rays pass through the axial caustic),

with

$$
L^t = R_I = r_H / \sin \theta_H,
$$

$$
L^r = R_2 = r_H / \sin \alpha_H,
$$

The plane wave fields (from the paraboloid), diffracted by the hyperboloid, incident on the opposite edge of the hyperboloid are given by:

$$
\begin{bmatrix}
E_{\|}^2 \\
E_{\perp}^2
\end{bmatrix} =
\begin{bmatrix}
D_s & 0 \\
0 & D_H
\end{bmatrix}
\begin{bmatrix}
\sin \phi \\
\cos \phi
\end{bmatrix}
E_x^i \frac{1}{\sqrt{2r_H}} \ e^{-jk2r_H}
$$

where $E_x^i$ is the amplitude of the nonhomogeneous plane wave incident on the hyperboloid rim (feed field after reflection from the hyperboloid and paraboloid),

with $L^i$ large (planar wavefront incident),

and $L^r$ as in Eq. (124).

The diffraction from the hyperboloid is given by the sum of Eqs. (117), (120), and (125) plus similar terms for the diffraction from the opposite edge.
E. Diffraction from the Cassegrain Reflector

The diffracted fields from the rim of the paraboloid may include terms for the direct feed illumination of the rim (this term should be insignificant for a good feed design), terms for the reflected fields from the hyperboloid (if \( \alpha_H < \alpha_P \) in Fig. 82), and the multiply diffracted terms for the illumination of the paraboloid rim due to the diffraction from the hyperboloid rim. For low noise design, the feed should have low sidelobe and backlobe levels. If this is not the case, the far out sidelobes of the feed will be reflected by the paraboloid and result in poor antenna performance because of the excessive spillover. For the feed chosen (pattern function given in Eq. (110)) and indeed for any good feed design, the diffraction of the feed fields by the rim of the paraboloid would be insignificant. For this reason, this term has not been included in the analysis.

The paraboloid rim is illuminated by the feed radiation reflected from the hyperboloid if \( \alpha_H < \alpha_P \) (see Fig. 82). This component of the diffracted fields from the rim of the paraboloid is given by:

\[
\begin{bmatrix}
E_{d||} \\
E_{d\perp}
\end{bmatrix} =
\begin{bmatrix}
D_S & 0 \\
0 & D_H
\end{bmatrix}
\begin{bmatrix}
E_{r||} \\
E_{r\perp}
\end{bmatrix} e^{jk(z_p \cos \theta + r_p \sin \theta) \frac{\rho_c}{s}} e^{-jks}
\]

where \( E_{r||} \) and \( E_{r\perp} \) are the components of the incident field reflected from the hyperboloid,

\( \rho_c = \frac{r_p}{\sin \theta} \) (from Eq. (113)),

with \( L^r = R^r = \frac{r_p}{\sin \alpha_p} \) (shown in Fig. 82),

and \( L^r \) is assumed large since this reflected energy from the paraboloid is nearly collimated.

The doubly diffracted fields from the paraboloid rim are due to incident fields which have been diffracted by the hyperbola rim and the opposite rim of the paraboloid. These fields are given by:

\[
\begin{bmatrix}
E_{dd||} \\
E_{dd\perp}
\end{bmatrix} =
\begin{bmatrix}
D_S & 0 \\
0 & D_H
\end{bmatrix}
\begin{bmatrix}
E_{d||} + E_{r||} + E_{r2||} + E_{r3||} \\
E_{d\perp} + E_{r\perp} + E_{r2\perp} + E_{r3\perp}
\end{bmatrix} e^{jk(z_p \cos \theta + r_p \sin \theta) \frac{\rho_c}{s}} e^{-jks}
\]

where \( E_{d||} \) and \( E_{d\perp} \) are the incident fields due to the diffraction of the feed fields by the hyperboloid rim,
$E^2_{ii}$ and $E^2_L$ are the incident fields due to the diffraction of the nonhomogeneous plane wave by the hyperboloid rim,

$E^3_{ii}$ and $E^3_L$ are the incident fields due to the diffraction from the opposite edge of the paraboloid,

$\rho_c = r_p/\sin \theta = \text{the caustic distance from Eq. (113) for the paraboloid rim (of radius } r_p).$

The distance parameters used in $D_S$ and $D_H$ in Eq. (129) are different for each of the three portions of the doubly diffracted terms. In addition, if the second edge is on a shadow boundary of the diffraction from the first edge, the wavefront is complex and cannot be treated at this time. This is the situation for the doubly diffracted term associated with the feed illumination on the hyperbola for standard cassegrain designs. This term will usually lead to a discontinuity on the shadow boundary at $\theta = \gamma$ (in Fig. 82) but is included anyhow. For the $E_{ii}$ and $E_L$ terms, the $L^i$ distance parameter for Eq. (129) is:

(130) \[ L^i = R^i \]

where $R^i = \text{the distance between the hyperboloid rim and the paraboloid rim (shown in Fig. 82).}$

The parameter $L^i$ is assumed large since the reflection from the paraboloid will have an almost planar wavefront for this term. The feed fields diffracted by the hyperboloid rim and then incident on the paraboloid rim are given by:

(131) \[
\begin{bmatrix}
E^1_{ii} \\
E^1_L
\end{bmatrix} =
\begin{bmatrix}
D_S & 0 \\
0 & D_H
\end{bmatrix}
\begin{bmatrix}
\sin \phi \\
\cos \phi
\end{bmatrix}
F(\beta_H, R_1) \left[ \frac{R_2^2}{R_4^2} \right] e^{-jkR_4} \\

\text{where } F(\beta_H, R_1) \text{ is the feed field at the hyperboloid rim (from Eq. (110)),}

\text{with } L^i = \frac{R_1 R_4}{R_1 + R_4} \text{ (} \rho_1 \text{ and } s \text{ in Eq. (115) equal to } R_1 \text{ and } R_4),

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and $L^r = \frac{R_2 R_4}{R_2 + R_4}$ ($\rho_1$ and $s$ in Eq. (115) equal to $R_2$ and $R_4$).

The distance parameters for Eq. (129) associated with the $E^2_{II}$ and $E^2_{I}$ terms are:

\begin{equation}
L_2^I = R_4
\end{equation}

and

\begin{equation}
L_2^r \text{ is assumed large (as was done for } L_1^r).\end{equation}

The plane wave fields diffracted from the hyperboloid rim and then incident on the paraboloid are given by:

\begin{equation}
\begin{bmatrix}
E_{II}^2
\end{bmatrix} =
\begin{bmatrix}
D_s & 0 \\
0 & D_H
\end{bmatrix}
\begin{bmatrix}
\sin \phi \\
\cos \phi
\end{bmatrix}
E_{X}^I \sqrt{\frac{R_2^4}{R_4(R_2^2 + R_4^2)}} e^{-jkR_4}
\end{equation}

where $E_{X}^I$ is as described following Eq. (120),

with $L^I = R_4$ ($\rho_1$ \rightarrow \infty in Eq. (115) with $s$ equal to $R_4$),

and $L^r = \frac{R_4 R_1^2}{R_2 R_4^2}$ ($\rho_1$ and $s$ in Eq. (115) equal to $R_2$ and $R_4$).

The $L^I_3$ distance parameter for Eq. (129) associated with $E_{II}^3$ and $E_{I}^3$ given by:

\begin{equation}
L_3^I = 2r_p
\end{equation}

and

\begin{equation}
L_3^r \text{ is assumed to be } 2r_p.
\end{equation}

The above $L^r$ should be found from differential geometry but since the reflection shadow boundary is near the main beam, the affect of this part of the diffraction is included in the aperture integration as previously discussed. Thus $L_3^r$ is relatively unimportant. The fields diffracted from the bottom of the paraboloid and then incident on the top are given by:
where \( E_{\|} \) and \( E_{\perp} \) are the doubly diffracted terms of Eq. (129) evaluated in the direction of the opposite edge, and \( J \frac{1}{\sqrt{2r_p}} \) is the geometric spreading factor from the bottom edge through the axial caustic to the top edge.

These diffracted fields combined with similar terms for the diffraction from the bottom edge have been used to obtain the fields diffracted from the paraboloid.

The two-point diffraction analysis just described is adequate for the diffraction pattern of the paraboloidal rim except in the caustic directions. The diffraction from the paraboloid has two caustics, one in the direction of the main beam which is accounted for in the aperture integration, and another in the rear axial direction which must be accounted for by equivalent current concepts. At each point on the rim, both electric and magnetic equivalent currents are computed from a two dimensional analysis. These currents are evaluated by equating the diffracted fields in the direction of the caustic to the fields radiated by electric and magnetic line sources. The fields radiated in the direction of the caustic and near the caustic are then found by performing a line integration around the rim. The fields radiated by the electric and magnetic line sources were given earlier and are repeated here for convenience:

\[
\begin{align*}
    (137) \quad & \begin{bmatrix} E_{\|}^3 \\ E_{\perp}^3 \\ E_{\perp}^1 \end{bmatrix} = \begin{bmatrix} D_s & 0 \\ 0 & D_H \end{bmatrix} \begin{bmatrix} E_{\|}^1 \\ 1 \\ E_{\perp}^1 \end{bmatrix} \frac{j}{\sqrt{2r_p}} e^{-jk2r_p} \\
    \text{where} \quad & E_{\|}^1 \text{ and } E_{\perp}^1 \text{ are the doubly diffracted terms of Eq. (129)} \text{ evaluated in the direction of the opposite edge,}
\end{align*}
\]

\[
\begin{align*}
    (74) \quad & A_{\theta}^e = -I_x^e \cos \phi \cos \theta + I_y^e \cos \theta \sin \phi + I_z^e \sin \theta \\
    (75) \quad & A_{\phi}^e = I_x^e \sin \phi - I_y^e \cos \phi \\
\end{align*}
\]

(73) \[ E^e = \frac{jkZ_0}{4\pi} \frac{e^{-jkR}}{R} \int_L e^{jk\hat{\rho} \cdot \hat{R}} [\hat{\rho} A_{\theta}^e + \hat{\phi} A_{\phi}^e] d\ell'(x',y',z') \]

where

\[
\begin{align*}
    (73) \quad & E^e = \frac{jkZ_0}{4\pi} \frac{e^{-jkR}}{R} \int_L e^{jk\hat{\rho} \cdot \hat{R}} [\hat{\rho} A_{\theta}^e + \hat{\phi} A_{\phi}^e] d\ell'(x',y',z') \\
    \text{where} \quad & \rho' \cdot \hat{R} = x' \sin \theta \cos \phi + y' \sin \theta \sin \phi + z' \cos \theta \\
    \text{and} \quad & A_{\theta}^e = -I_x^e \cos \phi \cos \theta + I_y^e \cos \theta \sin \phi + I_z^e \sin \theta \\
    \text{and} \quad & A_{\phi}^e = I_x^e \sin \phi - I_y^e \cos \phi
\end{align*}
\]
(77) \[ \vec{E}^m = \frac{jk}{4\pi} \frac{e^{-jkR}}{R} \int_{\Omega} e^{j\vec{k} \cdot \vec{R}} \left[ \hat{\phi} A^m_\phi + \hat{\rho} A^m_\rho \right] d\xi. \]

where

\[ A^m_\phi = I_x^m \sin \phi - I_y^m \cos \phi \]

\[ A^m_\rho = I_x^m \cos \phi \cos \rho + I_y^m \cos \phi \sin \rho - I_z^m \sin \rho. \]

The equivalent electric and magnetic currents are given by:

(138) \[ \vec{T}^e = 2 \frac{2\sqrt{\lambda}}{\lambda} e^{-j \pi \frac{2}{4}} \hat{\rho} D_S = \hat{x} I_x^e + \hat{y} I_y^e \]

(139) \[ \vec{T}^m = 2 \frac{2\sqrt{\lambda}}{\lambda} e^{-j \pi \frac{2}{4}} \hat{\rho} D_H = \hat{x} I_x^m + \hat{y} I_y^m \]

where \( \hat{\tau} \) is the unit tangent vector along the rim (\( \hat{\tau} = \hat{\rho} \) for the circular rim) = \(-\hat{x} \sin \phi' + \hat{y} \cos \phi' \).

\( \phi' \) is the angle from the x-axis to the source point on the rim.

\( E_{\parallel}^i \) and \( E_{\perp}^i \) are the components of incident electric field parallel and perpendicular to the rim (values are given by the sum of Eqs. (131), (134) and (137)),

and

\( D_S, D_H \) are the acoustic soft and hard diffraction coefficients (with \( L^1 = R^4 \) and \( L^r \) large).

The incident electric field has throughout been assumed to be polarized parallel to the x-z plane and is actually a sum of 3 terms, one term for the reflected field from the hyperbola, one for the feed diffraction from the hyperbola, and one for the diffraction by the hyperboloid rim of the energy reflected from the paraboloid. These fields have been used in the preceding equations to obtain via a numerical integration the backlobe region of the following patterns.
F. Radiation Patterns

Because of the number of parameters which affect the radiation pattern of the cassegrain, a detailed parametric study using the computer program developed from the preceding equations has not been undertaken. A set of radiation patterns has been included for a medium beamwidth cassegrain to illustrate the radiation characteristics. The antenna chosen has a beamwidth similar to the other antennas included in this work and provides a basis for comparing the antennas. The pattern, shown in Fig. 83, demonstrates the main drawback of the cassegrain for small to medium beamwidth applications, namely aperture blockage. This antenna, with a 20λ diameter reflector, has a subreflector of 2.67λ diameter. With this small subreflector (D_h = 0.133 D_o) the feed must be moved closer to the hyperboloidal subreflector (i.e., F' must be made smaller) to be able to maintain a reasonable size feed horn which will provide the desired taper over the subreflector. For the case under consideration, the paraboloid has a focal length of 8.0λ while the hyperboloid focal length is 2.46λ. The feed pattern is assumed to provide a 15 dB taper over the subreflector. With this geometry, the direct radiation from the feed contributes to the pattern for angles between 17.5° and 89.8° (β_h < θ < α_p). The eccentricity of the subreflector is 1.67 and results in maximum aperture efficiency for the given taper. This means that the angle of the shadow boundary of the reflected feed energy from the hyperboloid (at θ = α_h in Fig. 82) is equal to the angle to the paraboloid rim (at θ = α_p). It is this geometry (with α_h = α_p) that leads to an analytical problem associated with evaluating a doubly diffracted field from an edge which lies on the shadow boundary of fields from the first edge. This accounts for the steep slope on the pattern in the vicinity of the shadow boundary at θ = 117° on the pattern of Fig. 83. The reason that this problem cannot be eliminated is that the wavefront incident on the second edge is not well defined on the shadow boundary. Having the eccentricity so that α_h < α_p results in excessive spillover of the reflected feed energy from the hyperboloid. Conversely, having the eccentricity so that α_h > α_p requires using a larger reflector to obtain the same beamwidth since not all of the reflector is illuminated. If the larger reflector can be tolerated, using α_h > α_p reduces the reflector rim illumination and subsequently reduces the pattern level beyond θ = 90°. The results shown in Fig. 83 do not differ significantly from the calculated and measured results obtained by Ratnasiri [13] for a center fed parabola of approximately the same dimensions with an open ended waveguide feed. Even with this small subreflector, the cassegrain pattern is inferior to the offset fed parabola or horn-reflector for this beamwidth. For extremely large aperture (very narrow beamwidth) antennas, the cassegrain is probably mechanically simpler to build and use than any other choice. Radiation patterns for antennas with a very small beamwidth (less than 1°) have not been included because of the computer time required for the aperture integration portion of the pattern. This class of antenna could however be analyzed using the same computer program.
Fig. 83—Radiation patterns of a cassegrain with $D_p=20\lambda$, $F=8\lambda$, $D_h=2.67\lambda$, $F'=2.46\lambda$ and 15 dB feed taper over subreflector. For this geometry, direct feed spillover occurs for $17.5^\circ < \theta < 89.8^\circ$ and no reflected feed spillover occurs.
G. Conclusions

Cassegrain or dual-reflector antennas for low noise applications should be designed for low aperture blockage and minimum spillover of the feed energy. This means using a small subreflector diameter (hyperboloid diameter equal to 0.14 to 0.1 times the reflector diameter) and using a hyperboloid with a focal length less than half the paraboloid focal length. In addition, for low feed spillover, the feed should provide a 15 to 20 dB taper over the subreflector and have very low sidelobe and backlobe levels. (In general the corrugated horn fits these conditions.) Since the horn blocks some of the energy reflected from the hyperboloid, the hyperboloid focal length should be adjusted so that the shadow of the feed blockage on the main reflector is the same as the shadow of the subreflector. This should provide minimum aperture blockage. To eliminate the spillover of reflected energy from the subreflector, the eccentricity of the subreflector should be chosen so that the shadow boundary for this reflection (at \( \theta = \alpha_H \) in Fig. 82) falls inside the main reflector (i.e., \( \alpha_H > \alpha_p \)). Since the main reflector is inefficiently used if \( \alpha_H > \alpha_p \), the eccentricity should be chosen so that \( \alpha_H = \alpha_p \) as done for the antenna whose pattern was shown in Fig. 83.
Computer programs have been developed which allow the antenna or systems designer to quickly and inexpensively evaluate the suitability of numerous aperture antennas for low noise applications. Low noise applications require antennas with a high beam efficiency. To attain a high beam efficiency, an antenna must confine most of the radiated energy to the main beam region and have low sidelobe and backlobe levels. The programs developed here do not specifically evaluate the beam efficiency but they do allow one to compare sidelobe levels of several directive aperture antennas. The antennas considered include the pyramidal electromagnetic horn, the pyramidal corrugated horn, the horn-reflector, the corrugated horn-reflector, the offset fed parabola, and the cassegrain or dual-reflector antenna. The basic approach to the analysis of these antennas was to employ the well known aperture integration techniques to obtain the main beam portion of the pattern and then to use the Geometrical Theory of Diffraction (G.T.D.) and its extensions to obtain the balance of the radiation pattern. In addition to analyzing the radiation patterns of these antennas, the G.T.D. has been advanced through the development of a new form of slope wave diffraction coefficient which is continuous across the shadow boundary and through the formulation of an equivalent current based on the slope wave diffraction coefficient.

As previously indicated, the choice of low noise antenna depends on the application. Often, funds are unavailable for an extensive antenna construction and measurement program. It is here that the computer analysis is invaluable. As a result of developing the computer programs to analyze these antennas, some general conclusions may be drawn based on the beamwidth desired for the antenna. In general, the conventional electromagnetic horn with its high sidelobes and backlobe is unsuitable for low noise applications. For medium beamwidth applications where the antenna half power beamwidth is greater than approximately 10°, the corrugated horn with its low sidelobes and backlobe is the most suitable low noise antenna. This antenna would also be the best choice for a feed antenna for any of the reflector systems which one would need to consider for narrow beamwidth applications. In the beamwidth range from about 2° to 10°, the corrugated horn-reflector would be the best low noise antenna, not having the spillover associated with the offset fed parabola (which would be the second best design). For very narrow beam applications (less than 2°) where one considers construction problems which may be encountered with such a large horn-reflector, the family of reflector antennas must be considered. Here the offset fed parabola with a corrugated horn feed is probably the best choice since it would have low spill-over (if properly designed) and it does not have the problems associated with the aperture blockage of the center fed and cassegrain designs.
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[85] Galindo, op. cit.

[86] Williams, op. cit.

[88] Ibid.

[89] Ibid.


In this section, we are interested in finding the fields diffracted along the surface of a wedge for the case where the incident field is polarized along the edge of the wedge. Since the usual diffraction coefficient is zero for this case, we will examine the diffraction at some point \((\rho, \phi)\) off the wedge surface as shown in Fig. 84, then find the magnetic field associated with the diffracted electric field through Maxwell's equations, and finally allow the angle to the observation point to go to zero. In this manner one finds the form of the slope diffraction coefficient first obtained by Keller.[17]

Fig. 84—Wedge geometry for slope diffraction derivation.

In this analysis, we will assume that a plane wave, polarized parallel to the edge (normal to the page in Fig. 84), is incident on the wedge from some angle \(\phi_0\), i.e.,

\[
E^i = \hat{z} E_0.
\]

Under these conditions, the total electric field at the observation point \((\rho, \phi)\) is given by

\[
E_z = E_0 \left[ v^*(\rho_0, \phi_0) - v^*(\rho, \phi, \phi_0) \right]
+ \frac{e^{-jk\rho}}{\sqrt{\rho}} \left[ D(\phi_0, \hat{n}) - D(\phi, \phi_0, \hat{n}) \right]
\]
where

\[ v^*(\rho, \psi) = \begin{cases} \ e^{jk\rho \cos \psi} & \text{if } -\pi \leq \psi \leq \pi \\ 0 & \text{otherwise} \end{cases} \]

\[ \psi = \phi \pm \phi_0 \]

and

\[ D(\psi, n) = \frac{e^{-j \frac{\pi}{4}} \sin \frac{\pi}{n}}{\sqrt{2\pi k}} \left( \frac{1}{\cos \frac{\pi}{n} - \cos \frac{\psi}{n}} \right) \]

In Eq. (141), the \( v^* \) terms represent the geometric fields: the first \( v^* \) term is an incident plane wave field at \((\rho, \phi)\) with phase reference at the edge of the wedge while the second \( v^* \) term is the reflected field. The third and fourth terms, due are the cylindrically diffracted fields caused by the discontinuity in the incident and reflected geometric fields respectively. The magnetic field at this same point may be found from Maxwell's curl equation:

\[ \mathbf{H} = -\frac{1}{\omega \mu} \mathbf{V} \times \mathbf{E} \]

which in cylindrical coordinates becomes:

\[ \mathbf{H} = -\frac{1}{\omega \mu} \left[ \hat{\rho} \frac{1}{\rho} \frac{\partial E_z}{\partial \phi} - \hat{\phi} \frac{\partial E_z}{\partial \rho} \right] = \hat{\rho} H_\rho + \hat{\phi} H_\phi . \]

Substituting Eq. (141) into Eq. (145) yields:

\[ H_\rho = -\frac{1}{\omega \mu} E_0 \left\{ -jk \sin(\phi - \phi_0) v^*(\rho, \phi - \phi_0) + jk \sin(\phi + \phi_0) v^*(\rho, \phi + \phi_0) \right. \]

\[ + \frac{e^{-jk\rho}}{\rho^{3/2}} \left[ \frac{\partial D(\phi - \phi_0, n)}{\partial \phi} - \frac{\partial D(\phi + \phi_0, n)}{\partial \phi} \right] \]

and

\[ H_\phi = \frac{1}{\omega \mu} E_0 \left\{ jk \cos(\phi - \phi_0) v^*(\rho, \phi - \phi_0) - jk \cos(\phi + \phi_0) v^*(\rho, \phi + \phi_0) \right. \]

\[ + \frac{a}{\rho} \left( \frac{e^{-jk\rho}}{\sqrt{\rho}} \right) \left[ D(\phi - \phi_0, n) - D(\phi + \phi_0, n) \right] \]
For observation points on the surface of the wedge \((\phi=0)\), these equations become:

\[
(148) \quad H_p = \frac{1}{j\omega} E_o \left\{ -2jk \sin \phi_o \nabla \times (\rho, \phi_o) \right. \\
- \frac{e^{-jk\rho}}{\rho^{3/2}} \left[ \frac{\partial D(\phi-\phi_o, \rho)}{\partial \phi} - \frac{\partial D(\phi+\phi_o, \rho)}{\partial \phi} \right] \bigg|_{\phi=0} \right. \\

(149) \quad H_\phi = \frac{1}{j\omega} E_o \left\{ 0 + \frac{\partial}{\partial \rho} \left( \frac{e^{-jk\rho}}{\rho} \right) \right\} [0] \equiv 0
\]

Consider now the terms in Eq. (148) of the form:

\[
(150) \quad \frac{\partial D(\phi+\phi_o, \rho)}{\partial \phi} = \frac{\partial}{\partial \phi} \left[ -\frac{j \frac{\pi}{4}}{\sqrt{2\pi k}} \frac{\frac{1}{n} \sin \frac{\pi}{n}}{\cos \frac{\pi}{n} - \cos \left( \frac{\phi+\phi_o}{n} \right)} \right] \\
= -\frac{j \frac{\pi}{4}}{n\sqrt{2\pi k}} \sin \frac{\pi}{n} \frac{-\frac{1}{n} \sin \left( \frac{\phi+\phi_o}{n} \right)}{\left[ \cos \frac{\pi}{n} - \cos \left( \frac{\phi+\phi_o}{n} \right) \right]^2}.
\]

Thus,

\[
(151) \quad \left[ \frac{\partial D(\phi-\phi_o, \rho)}{\partial \phi} - \frac{\partial D(\phi+\phi_o, \rho)}{\partial \phi} \right] \bigg|_{\phi=0} = \frac{-j \frac{\pi}{4}}{n^2\sqrt{2\pi k}} \sin \frac{\pi}{n} x
\]

\[
\left[ \frac{-\sin \left( \frac{\phi-\phi_o}{n} \right)}{\left[ \cos \frac{\pi}{n} - \cos \left( \frac{\phi-\phi_o}{n} \right) \right]^2} + \frac{\sin \left( \frac{\phi+\phi_o}{n} \right)}{\left[ \cos \frac{\pi}{n} - \cos \left( \frac{\phi+\phi_o}{n} \right) \right]^2} \right] \bigg|_{\phi=0}
\]

Using this result in Eq. (148), the resultant tangential magnetic field along the surface of the wedge is given by:
\[ H_p(\rho, \phi=0) = \frac{k}{\omega \mu} E_o \left( -2 \sin \phi_0 v^*(\rho, \phi_0) - \frac{e^{-j k \rho}}{\rho^{3/2}} [2 D_{sw}(\phi_0, n)] \right) \]

where

\[ D_{sw}(\phi_0, n) = \text{the slope wave diffraction coefficient} \]

\[ = \frac{e^{j \pi/4}}{n^2 \sqrt{2\pi} k^{3/2}} \sin \frac{\pi}{n} \frac{\sin \frac{\phi_0}{n}}{\left[ \cos \frac{\pi}{n} - \cos \frac{\phi_0}{n} \right]^2} \]

In Eq. (152), the first term:

\[ -2 \frac{k}{\omega \mu} E_o \sin \phi_0 v^*(\rho, \phi_0) = \hat{\rho} \cdot (H^l_\phi + H^r_\phi) \]

is the physical optics portion of the tangential magnetic field along the surface of the wedge. Also, it is clear from the \( e^{-j k \rho/\rho^{3/2}} \) coefficient on the diffracted portion of Eq. (152) that the slope diffracted term is a higher order correction term for usual edge diffracted fields of the Geometrical Theory of Diffraction.
APPENDIX B

CYLINDRICAL WAVE FORM OF THE SLOPE DIFFRACTION COEFFICIENT

In Appendix A, a plane wave slope diffraction coefficient was obtained to evaluate the fields on the surface of a wedge when the wave is polarized parallel to the edge. This diffraction coefficient is singular for an angle of diffraction of considerable interest, e.g., along the shadow boundary. Rudduck and Wu [Ref. 20] have found a form of the slope diffraction coefficient from the eigenvalue form of the diffraction coefficient. This series form is valid for source points near the diffracting edge, requiring that more terms in the infinite series be retained as the distance from the edge is increased. In this Appendix, another form is evaluated for all diffraction angles by using the Fresnel integral format for the diffracted fields for an angle of incidence, $\phi_o$, as shown in Fig. 85. For the wedge geometry, the total electric field at the point $(\rho, \phi)$ is given by:

\[ (154) \quad E = \hat{z} E_o \left\{ v^*(\rho, \phi - \phi_o) - v^*(\rho, \phi + \phi_o) \right\} + v_B(\rho, \phi - \phi_o, n) - v_B(\rho, \phi + \phi_o, n) \]

where

\[ (155) \quad v^*(\rho, \psi) = \begin{cases} e^{jk\rho} \cos \psi & \text{if } -\pi \leq \psi \leq \pi \\ 0 & \text{otherwise} \end{cases} \]

\[ (156) \quad v_B(\rho, \psi, n) = \frac{2e^{j\frac{\pi}{4}}}{\sqrt{\pi}} \frac{1}{n} \sin \frac{\pi}{n} \frac{1}{\cos^\frac{\pi}{n} - \cos^\frac{\psi}{n}} \frac{\cos^\frac{\psi}{n}}{e^{jk\rho} \cos \psi} \]

\[ x \int_{-\infty}^{\infty} e^{-j\frac{2}{\tau^2}} d\tau \]

and $n = \text{wedge angle (} n = 2 \text{ for the half plane)}$. This solution is approximate for the arbitrary wedge angle but is exact for the special case of the half plane. For this reason and because the antenna edges of interest can be represented by the thin edge, we will restrict our solution to the case of the half plane ($n = 2$ in Eq. (156)). While Eq. (154) accurately describes the diffraction from the half plane, it fails to predict the higher order
diffracted fields along the surface of the wedge which for this polarization become important since the geometrical optics and first order diffracted fields vanish (as they must to satisfy the boundary conditions). This is the case which is of interest for application to the various horn geometries. To obtain these higher order diffracted fields, we first find the magnetic field associated with the electric field at the point \((\alpha, \phi)\) given by Eq. (154) and then allow the angle to the observation point \((\phi)\) to go to zero. Maxwell's curl equation yields:

\[
\begin{align*}
\mathbf{H} &= -\frac{1}{j\omega \mu} \nabla \times \mathbf{E} = -\frac{1}{j\omega \mu} \left[ \frac{1}{\rho} \frac{\partial E_z}{\partial \phi} - \hat{\phi} \frac{\partial E_z}{\partial \rho} \right] \\
&= \hat{\rho} H_\rho + \hat{\phi} H_\phi.
\end{align*}
\]

Substituting Eq. (154) into the above results in:

\[
\begin{align*}
H_\rho &= -\frac{1}{j\omega \mu} \frac{E_\phi}{\rho} \left\{ \frac{\partial v^*(\rho, \phi, \phi_0)}{\partial \phi} - \frac{\partial v^*(\rho, \phi + \phi_0)}{\partial \phi} + \frac{\partial v_B(\rho, \phi + \phi_0, 2)}{\partial \phi} - \frac{\partial v_B(\rho, \phi, \phi_0, 2)}{\partial \phi} \right\}
\end{align*}
\]

and
The terms in Eqs. (158) and (159) associated with the geometrical optics incident and reflected fields are given by:

\[
\frac{3V^*(p, \phi_0)}{\partial \phi} = -j k \sin(\phi^+ - \phi_0) V^*(p, \phi^+- \phi_0) ,
\]

and

\[
\frac{3V^*(p, \phi^+ \phi_0)}{\partial \rho} = j k \cos(\phi^+ - \phi_0) V^*(p, \phi^+ \phi_0) ,
\]
as was mentioned in Appendix A. The terms associated with the diffracted fields may be found explicitly in terms of the derivatives of the \(V_B\) diffraction coefficient which for the half plane is:

\[
V_B(p, \psi, \phi) = \frac{e^{i \pi/4}}{\sqrt{\pi}} \text{sgn}(-\cos \frac{\psi}{2}) e^{jk\rho \cos \psi} \int_{-\infty}^{\infty} e^{-j\tau^2} d\tau
\]

where

\[
\text{sgn}(x) = \begin{cases} 
+1 & \text{if } x > 0 \\
-1 & \text{if } x < 0
\end{cases}
\]

and

\[
\psi = \phi^+ \phi_0 .
\]
The partial derivatives of the diffraction coefficient are:

\[
\frac{3V_B(p, \psi, \phi)}{\partial \phi} = -\text{sgn}(-\cos \frac{\psi}{2}) \frac{e^{-j(k - \pi/4)}}{\sqrt{\pi}} jk\rho \sin \psi \times \begin{cases} 
e^{jk\rho(1+\cos\psi)} \int_{-\infty}^{\infty} e^{-j\tau^2} d\tau + \frac{j}{2\sqrt{k\rho(1+\cos\psi)}} \end{cases}
\]

and
Substituting Eqs. (160), (161), (163) and (164) into Eqs. (158) and (159) with the observation point on the surface of the half plane (i.e., \( \phi = 0 \)) yields:

\[
H_p(p,0) = -\frac{1}{j\omega} E_0 \left\{ 2jk \sin \phi_0 v^*(p,\phi_0) + 2jk \operatorname{sgn}(\cos \frac{\phi_0}{2}) \sin \phi_0 x \right. \\
\left. \times e^{jk\rho(1+\cos\phi)} \int_{-\infty}^{\infty} e^{-j\tau^2} d\tau + \frac{j}{2\sqrt{k\rho(1+\cos\phi_0)}} \right\}
\]

and

\[
H_\phi(p,0) = \frac{1}{j\omega} E_0 \{ 0 + 0 \} = 0
\]

Equation (165) may be rewritten:

\[
H_p(p,0) = \frac{k}{\omega \mu} E_0 \{ 2v^*_s(p,\phi_0) + 2v^*_w(p,\phi_0) \}
\]

where

\[
v^*_s(p,\phi_0) = -\sin \phi \ v^*(p,\phi_0)
\]
(169) \[ v_{SW}(\rho, \phi_0) = -\sin \phi_0 \text{sgn} \left( -\cos \frac{\phi_0}{2} \right) e^{-j \left( \frac{kp - \pi}{4} \right)} \]

\[ x \left\{ e^{jk_\rho(1+\cos \phi_0)} \int_{-\infty}^{\infty} e^{-j\tau^2} d\tau + \frac{j}{2j_k^2(1+\cos \phi_0)} \right\} \]

The first term of Eq. (167) corresponds to the components of incident and reflected magnetic field along the wedge. The slope wave diffracted field along the surface given by the \( v_{SW} \) term should be a higher order diffraction term (i.e., one with a \( 1/\rho^{3/2} \) range dependence for \( \sqrt{k\rho(1+\cos \phi)} >> 1 \)) as found in Appendix A. That this is truly the case, may be shown by integrating the Fresnel integral in \( v_{SW} \) by parts. In the integral:

(170) \[ I = e^{jk_\rho(1+\cos \phi_0)} \int_{-\infty}^{\infty} e^{-j\tau^2} d\tau, \]

let "a" denote the term \( k\rho(1+\cos \phi) \) and then perform a change of variables letting \( x = \tau^2 \):

(171) \[ I = e^{j\alpha} \int_{-\infty}^{\infty} e^{-jx} \frac{dx}{\sqrt{a}} d\tau \]

\[ = e^{j\alpha} \int_{-\infty}^{\infty} e^{-jx} \frac{dx}{\sqrt{x}} \frac{dx}{d\tau} \]

Integrating this by parts yields:

(172) \[ I = -\frac{j}{2j\alpha} + \frac{j e^{j\alpha}}{4} \int_{-\infty}^{\infty} \frac{e^{-jx}}{x^{3/2}} dx \]

Integrating by parts a second time yields:
Substituting this expression into Eq. (169) shows that the terms of order $1/\sqrt{\rho}$ cancel leaving the $1/\rho^{3/2}$ term dominant. Equation (169) for the slope diffraction coefficient is left in terms of the Fresnel integral since it is easily evaluated on a digital computer.

The value of the slope diffraction coefficient of Eq. (169) is plotted in amplitude (in dB) and phase in Fig. 86 for several observation distances, $R$ from 1 to 50 wavelengths, versus the angle of incidence $\phi_0$. 

(173) \[ I = \frac{-j}{2ja} + \frac{1}{4a^{3/2}} - \frac{3}{8} e^{ja} \int_{-\infty}^{\infty} \frac{e^{-jx}}{x^{5/2}} \, dx \]

\[ = \frac{-j}{2\sqrt{k_0(1+\cos\phi_0)}} + \frac{1}{4(k_0(1+\cos\phi_0))^{3/2}} \]

\[ - \frac{3}{8} e^{jk_0(1+\cos\phi_0)} \int_{-\infty}^{\infty} \frac{2e^{-j\tau^2}}{\tau^4} \, d\tau \]
Fig. 86--Tangential magnetic field amplitude and phase versus the distance "R" from the diffracting edge for a unit amplitude plane wave incident on a half plane at an angle "\( \phi_0 \)" from the surface of the wedge.
APPENDIX C
COMPUTER PROGRAM FOR CORRUGATED SURFACE ANALYSIS

The main computer program and the subroutines required to solve
the set of equations obtained from the integral equation of Chapter III
by point matching will now be briefly discussed. The main program
listing is at the end of this section. The input parameters to the
main program are contained on two cards: the first specifies the
corrugation shape, and the second the groundplane and corrugated
surface dimensions. The data on the first card is grouped into five
groups of (F8.3,I2) fields. Each group locates a corner (in F8.3
field) and specifies the number of segments (in I2 field) to be placed
on the line from the previous point to the current point. Figure 87
shows a typical period or cell from a corrugated surface of period

\[ TC = \frac{1}{N_{COR}} \]

where \( N_{COR} \) is the corrugation density (in corrugations per
wavelength). The \( X_1P \) to \( X_5P \) are expressed as ratios normalized to the
corrugation period \( TC \) with \( X_5P = TC \). The heights of the corners are
assumed to be either \( H = 0 \) or \( H = -HC \) as shown. Additional information
on these parameters is contained in the discussion of the VSURDV sub-
routine. The remaining input parameters are shown in Fig. 88 and
listed below:

- **EPL** locates the end point on the left or groundplane side in cm.,
- **NGP** specifies the number of matching segments to be placed on the
groundplane side of the origin,
- **EPR** locates the end point on the right or corrugated surface side
of the origin (in cm.),
- **WE** is the electrical wavelength (in cm.),
- **HCWE** is the height of the corrugations in wavelengths,
- **NCOR** is the number of corrugations per wavelength, and
- **KSET** is an identifier for the set of data and is not used in
any calculations.

Also shown in Fig. 88 as a broken line is the path used for closing the
body to check the validity of the open surface model. The body is
closed by setting the last calling parameter in the CALL VSURDV (line
36 of the main program listing) to 1 rather than 0 (open body). The
match points on the groundplane and corrugated surface are determined
by calling the GDPLDV and VSURDV subroutines in this order (lines 35 and
36 in the listing). These subroutines are discussed later. The output
includes a list of the "X" and "H" coordinates of the match points, the
direction of the surface normals, and the length of each segment as
determined by these subroutines (line 39). Also listed as output is
the incident field on each segment (line 74) and after the matrix in-
version, the surface current on each segment (line 82). Also plotted
are the magnitude (line 88) and phase (line 93) of the surface current
versus segment number and the radiation zone magnetic field pattern
versus the observation angle (line 122). In addition to these plots
Fig. 87—Typical corrugation showing the coordinates of the endpoints of the straight line segments forming the corrugation.

Fig. 88—Corrugated surface model showing the closed body (dashed line) used to check the validity of the open surface model.

the magnitude and phase of the scattered field and the total field are listed versus $e$ (line 12). Finally, the relative power loss is summed and listed versus the segment number (line 136). The Fortran listing follows.
1 C  TE CASE  GAUSSIAN INTEGRATION USED TO FILL IN MATRIX
2 C  TE CASE  GAUSSIAN INTEGRATION USED TO FILL IN MATRIX
3 COMPLEX HINC-HTC-HFORM
4 COMPLEX F5S,SCO,SNS,SST
5 COMPLEX S5T,CTC-STST=FIN+HAN2+DJC+STS
6 FORMAL /GOOD/ DJC
7 N1,NI,STST
8 FORMAL /PAWG/ DJZ
9 COUNTER/MAjIV/X(230)=H(230)+TH(230)+SEG(230)+DEL+HC+G+EPL+EPR+WE
10 COMPLEX FINC(230)+C(230)+E(230)
11 NTH=230
12 CE IS THE ELECTRICAL WAVELENGTH
13 C SOURCE LOCATIONS
14 N1=4.0
15 N2=-6.0
16 READ(5,1) X1P+M1,X2P+M2,X3P+M3,X4P+N4,X5P+N5
17 1 FORMAT(0.3,12)
18 READ(5,2) ELHGP+EMP+HFHC+HCOR+KSET
19 2 FORMAT(F10.4,E15.15,2S15.15)
20 WRITE(6,997) EP,EPR+HFHC+HCOR+HGP
21 997 FORMAT('GROUNDPLANE FROM '+F10.4+' CM TO ORIGIN, VEE CORRUGATION
22 FROM ORIGIN TO '+F10.4+' CM/VEE WAVELENGTH',F10.4,CORRUG
23 NATION HEIGHTS',F10.4,WAVELENGTHS',/15,CORRUGATIONS
24 4PLR WAVELENGTH/TH/IS/15,9 POINTS ON GROUNDPLANE')
25 P1=3.14159
26 P2=PI/2.
27 RONG=180./PI
28 TC=VE/ICOR
29 X1=X1P*TC
30 X2=X2P*TC
31 X3=X3P*TC
32 X4=X4P*TC
33 X5=X5P*TC
34 HC=HCW+WE
35 WRITE(6,20)
36 CALL GISPLDV(NGP,MP)
37 CALL VSURDV(X1,X2,X3,X4,TC,M1,N2,M3,N4,N5,MP,0)
38 NMP=1
39 WRITE(6,21)
40 21 FORMAT('15,4F15.4')
41 WRITE(6,18) N
42 18 FORMAT('N=',I5,' EXCEEDS DIMENSION OF C')
43 CALL EXIT
44 19 CONTINUE
45 10 CONTINUE
46 9B 3661,CR,IC=CO11(R,IC)
47 C THIS COMPLETES THE FILLING OF THE MATRIX
48 WRITE (6,1222) N+WE
49 1222 FORMAT(XH N=134,N WE='E15.8')
50 C THIS FINDS THE INCIDENT FIELD ON THE TH SEGMENT
51 NO 455 N=1
52 455 NO 455 IC=1
53 3661 C11,R,IC=CO11(R,IC)
54 C THE SIGN ON THE INCIDENT FIELD HAS BEEN ADJUSTED TO AGREE WITH
55 C THE INTEGRAL EQUATION
66 RHO=SORT(X*YG*HG*HG)  
67 PHI=ATAN2(HG,XG)*RDCG  
68 CALL WILLUM(RHO,PHI,HNC,WE)  
69 FNC(NJ)=HNC  
70 E(NJ)=FNC(NJ)  
71 CONTINUE  
72 WRITE(6,2947)  
73 2947 FORMAT(5H+*10X+1GH INCIDENT FIELDS )  
74 WRITE(6,2948) (NJ,FNC(NJ)),NJ=1+H)  
75 2948 FORMAT(15+15.0+4H +J +E15.0)  
76 CALL CROUT2(C,F,H,NDI)  
77 WRITE(7,9490) (SETM=M(X(M)+H(M)+TNM1)SEGM(M),F(K)+M+1)  
78 9300 FORMAT(15+4F15.0+2E15.0)  
79 NO 554 IKUR=1,N  
80 AAF=CARSF(1KUR))  
81 ANF=57.296*ATAN2(AIMAG(F(1KUR)),REAL(F(1KUR)))  
82 554 WRITE(6,551)IKUR+AAF+ANF  
83 553 FORMAT (*+F(*+14,+E15.0* AT ANGLE=+E15.0)  
84 NO 9553 IKUR=1,H  
85 IND=IKUR+1  
86 Y(I)=CABS(F(IKUR))  
87 XRRO=FLOAT(IKUR)  
88 9553 CALL PLOT(XRRO,Y+1+IND+5.0+0,0)  
89 NO 9544 IKUR=1,N  
90 IND=IKUR+1  
91 Y(I)=57.296*ATAN2(AIMAG(F(IKUR)),REAL(F(IKUR)))  
92 XRRO=FLOAT(IKUR)  
93 9554 CALL PLOT(XRRO,Y+1+IND+100,0+100,0)  
94 RH0=200.  
95 WDNRM=2.*CPLX(0.+2.*PI*RHO/WC))/SQRT(RHO)  
96 WRITE(6,311)  
97 311 FORMAT(*1H+1X+16RELATIVE H FIELD+33X+13HTOTAL H FIELD /2X+9HMA
98 3 ANGLE+7X+2HANGLE+7X+ANG+1X+9X+2HANG+7X+2H0R,6X+9HMA
99 3 SHANGLE)  
100 NO 317 JNX=1,100  
101 THS=0.1745329*FLOAT(JNX)  
102 T=CMPLX(0.+0.*N)  
103 NO 310 I=1,N  
104 YN=0.500*(X(I)+X(I+1))  
105 WM=0.500*(H(I)+H(I+1))  
106 THN=TN(I)  
107 310 T=T+((F(I)+EXP(0.*G*(((XN*COS(THS))+COS(THS)))I)+H*N*SIN(THS))))  
108 2 *COS(THS)+*SEGM(I))  
109 C  
110 TE=STS  
111 CH=CAUS(T)  
112 DB=20.+AIOC101(CM)  
113 FANG=57.296*ATAN2(AIMAG(T),REAL(T))  
114 THSD=THS+57.296  
115 ABCS(JNX)=CM  
116 CALL WILLUM(2ANN,THSD,HINC,WE)  
117 HCT=T+HINC/NDNRM  
118 WM=CAFS(HCT)  
119 HTR=ROC*ATAN2(AIMAG(HCT)+REAL(HCT))  
120 HTD=20.+ALOC101(HMT)  
121 HTOT(JNX)=HTM  
122 317 WRITE(6,312) CMANG,DX,THSD+HD+HM+HTA  
123 312 FORMAT(E15.0+2F10.4+4X+F10.4+2X+F10.4+2X+F10.4+2X+F10.4+2X+F10.4+2X+F10.4)  
124 NO 950N JC=1,100  
125 Y(I)=ANFS(JC)  
126 Y(J)=HTOT(JC)  
127 UN=FLOAT(JC)  
128 IND=JC+1  
129 9500 CALL PLOT(U+Y+1+IND+100,0+0,0)  
130 PLOSS=0,0

171
WRITE(6,9994)
9994 FORMAT(' POWER LOSS PER UNIT RESISTIVITY/* M*.5X,'PLOSS*)
9999 CONTINUE
9999 CALL EXIT
FND
Fig. 89--Illustration of some of the possible corrugation shapes available by changing the subroutine parameters X1-X5.
SUBROUTINE GDPLDV

Subprogram GDPLDV is the groundplane surface division subroutine which is designed to symmetrically place 8 short segments around the source pair and divide the remaining segments along the rest of the groundplane. In use, this subprogram is called only once with the calling parameter NGP set to the number of matching segments one desires on the groundplane side of the origin. The 8 segments around the sources each have length equal to 0.25 x the distance between the sources. These segments are placed two to the left of the left source, four between the sources, and two to the right of the right source. The remaining NGP-8 segments are placed with at least one segment between the left endpoint and the symmetrically placed segments and the rest of the segments between the right of the symmetrically placed segments and the origin. The numbering of the endpoints is from the left endpoint and is indexed by variable MP. The coordinate of the points are stored in \([X(MP),Y(MP)]\) and the surface normals and segment lengths in TN (MP) and SEGL(MP) respectively. These variables are shared with the main program and other subroutines through COMMON/MINDV/. The subroutine listing follows.
SUBROUTINE GDPOLOV(NGP,MP)

COMMON /MAIM-V/ X(230),H(230),SCPL(230),DEL+HC+6+CPL+EPR+W
COMMON /HC+6+DZ1/ RZ2

NCL1=ANSE(EPL+/NGP=41)

NCL2=ABS((DZ2-DZ1)/2.0)

X1=DZ2-NCL2

X2=DZ2+NCL2

X3=DZ1-NCL2

X4=DZ1+NCL2

WHITE(6+20) DEL1+DEL2+X1,X2,X3,X4

FORMAT(6F12.6)

TU=ANSE(EPL-X1)

U1=FIX(N/100)

IF(N1 1,1+2

NCL=TD/N1

NO 3 I=1,N1

T(I)=EPL+(I-1)*DEL

SEGL(I)=DEL

MP=I

GO TO 4

MP=I

X(MP)=EPL

SEGL(MP)=TD

JJ=MP

DEL=DEL2/2.0

NO 5 I=1,N1

J=JJ+I

T(J)=X1+(I-1)*DEL

SEGL(J)=DEL

MP=J

J=MP

MP=NGP-MP

IF(N1 6,6,7

NCL=ABS(X4)/N1

NO 9 I=1,N1

J=JJ+I

T(J)=X4+(I-1)*DEL

SEGL(J)=DEL

MP=J

NO 10 I=1,MP

TN(I)=1,57079

I(2)=0.0

RETURN

WHITE(6+21)

FORMAT('POOR POINT SELECTION')

GO TO 8

END
The subroutine VSURDV performs the surface division on arbitrary shape vee corrugations. Because of the periodicity of the corrugated surface, only one corrugation need be specified in detail and all the remaining corrugations are images. Figure 87 shows a period of the corrugated surface with the corner locations specified by the input parameters X1P to X5P. The input to this subroutine requires the corners be located in cm rather than normalized to the corrugation period. Lines 28 to 32 of the main program remove this normalization. The segment endpoint numbering begins at "MP" at the origin and continues to the right endpoint. If the calling parameter KSURF is 1 (or 0) the surface will be closed (or open) and the numbering continued along the dotted path of Fig. 88 (or the numbering terminated). Some shapes which can be handled are shown in Fig. 89 and include the symmetric V-shape (case 1) and the square corrugation (case 4) as well as asymmetric and flat bottom vee corrugations. The subroutine listing follows.

```fortran
1 SUBROUTINE VSURDV(X1,X2,X3,X4,X5,X1N,X2N,X3N,X4N,X5N,MP,KSURF)
2 IF KSURF=1, SURFACE IS CLOSED
3 IF KSURF=0, SURFACE IS OPEN
4 COMMON/MAINOV/X1(230),X2(230),X3(230),X4(230),X5(230),N1(230),N2(230),N3(230)
5 DATA PI,P12/3.1415926,1.5707963/
6 M=MP
7 J=MP
8 IF(N1).EQ.2,1,N1
9 1 NEL=X1/X1
10 DO 10 I=1,N1
11 JJ=J+I
12 X(J)=X(J)+DEL
13 HT=(X(J)+X(J+1))/2
14 TN(J)=PI2
15 REGL(J)=DEL
16 MP=J
17 CONTINUE
18 XT=X(MP)+DEL
19 DX=(X2-X1)/N2
20 DH=HC/N2
21 NEL=SART(DX*DX+DH*DH)
22 T=ATAN2(DX,0H)
23 J=MP
24 NO 20 I=1,N2
25 J=J+1
26 X(J)=XT+DX*FLOAT(I-1)
27 H(J)=D*FLOAT(I-1)
28 TN(J)=T
29 REGL(J)=DEL
30 MP=J
31 CONTINUE
32 XT=X(MP)+DX
33 J=MP
34 IF(N3).EQ.3,3,N3
35 3 NEL=(X3-X2)/N3
36 NO 30 I=1,N3
37 J=J+1
38 X(J)=XT+DEL*FLOAT(I-1)
39 H(J)=HC
40 TN(J)=PI2
41 REGL(J)=DEL
42 MP=J
43 CONTINUE
```
XT=X(MP)+DEL

NX=(X4-X3)/4

NH=HC/N

DEL=SGRT(DX*DX+DI*DH)

T=tanh(0.7077*ATAN2(CH,DX))

JW=MP

DO 40 I=1,N4

J=JU+1

X(J)=XT+DX*FLOAT(I-1)

H(J)=HC*DEL*FLOAT(I-1)

TN(J)=T

SEG(J)=DEL

MP=J

CONTINUE

50

XT=X(MP)+DX

JW=MP

TF(N5)=6+6.5

DEL=(TC-X4)/N5

DO 60 I=1,N5

JU=JU+1

X(J)=XT+DEL*FLOAT(I-1)

H(J)=0.0

TN(J)=PI2

SEG(J)=DEL

MP=J

CONTINUE

60

NTCELL=IFIX((EPR*0.1)/TC)

NCELL=N1+N2+N3+N4+N5

DO 60 N=2,NTCELL

J=J+I

X(J)=X(K)+TC*(N-1)

H(J)=H(K)

TN(J)=TN(K)

SEG(J)=SEG(K)

MP=J

CONTINUE

62

IF(KSURF) 7,7+3

MP=MP+1

X(MP)=NTCELL*TC

H(MPI)=0.0

WRITE(6,+62)

7,7 FORMAT(/ ' OPEN SURFACE'/)

RETURN

8

XMAX=NTCELL*TC

JU=MP

JW=MP

HP=HC*0.1

N=IFIX((HP*0.1)/WE)

DEL=HP/N

DO 70 I=1,N

J=JU+1

X(J)=XMAX

H(J)=0.0

TN(J)=0.0

SEG(J)=DEL

MP=J

CONTINUE

70

K=IFIX((XMAX-EPL)*K/WE)

DEL=(XMAX-EPL)/K

DO 100 I=1,K

J=J+I

X(J)=XMAX-(I-1)*DEL

H(J)=HP

TN(J)==PI2

100

CONTINUE

100
SEG(0) = DEL
MP = J
JJ = MP
K = I.FIX(HP = 0./WE)
DEL = HP/N
NO 90, J = 1, N
J = JJ + 1
X(J) = EPL
H(J) = HP + (I - 1) * DEL
TN(J) = PI
SEG(J) = DEL
MP = J
MP = MP + 1
X(MP) = EPL
H(MP) = 0.0
TN(MP) = 0.0
SEG(MP) = 0.0
WRITE(6, 91)
FORMAT(/' CLOSED SURFACE'/)
RETURN
FNO
SUBROUTINE CO

Subroutine CO generates the coupling coefficients for the "C" matrix of section II. The subroutine finds the magnetic field at segment "MR" due to a current on segment "MC" by performing a 5-point Gaussian integration along the length of segment "MC". The program listing follows.

```
1 COMPLEX FUNCTION CO(MR,MC)
2   THIS GIVES THE OLD MATRIX COEFFICIENTS
3   COMMON/MAINV/ X(230)+H(230)+T(230)+SGL(230)+DEL*HC+OPL+ERP*KE
4   COMMON/DG/ OJC
5   COMMON/AHAN21
6   DATA GU1, GU2, GU3, GU4, GU5, GW1, GW2, GW3, GW4, GW5/ -0.9061798, 0.5384693
7       2.000000, 0.3889786, 0.9061798, 0.5384693, 0.3889786
8       3.0, 2.366068
9   TF(MR+1,MC) GO TO 100
10  CO=COMPLX(0.500,0.0)
11  NO TO 200
12  100 CONTINUE
13   DEL=SGL(MC)
14   DEL2=DEL/2.
15   XM=0.500*(X(MR)+X(MR+1))
16   XMNM=0.500*(H(MR)+H(MR+1))
17   FP=FL(YC)
18   DVDFEP=-(LPU+EPL)/2.0
19   NVM=(LPU+EPL)/2.0
20   YU5=GUS+DVDFEP+DVS*EP
21   YU1=GUS+DVDFEP+DVS*EP
22   YU2=GUS+DVDFEP+DVS*EP
23   YU3=GUS+DVDFEP+DVS*EP
24   YU9=GUS+DVDFEP+DVS*EP
25   YUH=GUS*(X(MC+1)2
26   NLM=(H(MC)+H(MC+1))/2.0
27   NLM=(H(MC)+H(MC+1))/2.0
28   NLM=(H(MC)+H(MC))/2.0
29   NLM=(H(MC)+H(MC))/2.0
30   HUXI=GUS*DELH*NHM
31   HUX2=GUS*DELH*NHM
32   HUX3=GUS*DELH*NHM
33   HUX4=GUS*DELH*NHM
34   HUX5=GUS*DELH*NHM
35   THN=TAN(MC)
36   XR=XMNM-XU1
37   YR=HUXM-HUX1
38   R1DN=(XR+COS(THN)+YR*SIN(THN))/SORT(XR*XR+YR*YR)
39   XR=XMNM-XU2
40   YR=HUXM-HUX2
41   R2DN=(XR+COS(THN)+YR*SIN(THN))/SORT(XR*XR+YR*YR)
42   XR=XMNM-XU3
43   YR=HUXM-HUX3
44   R3DN=(XR+COS(THN)+YR*SIN(THN))/SORT(XR*XR+YR*YR)
45   XR=XMNM-XU4
46   YR=HUXM-HUX4
47   R4DN=(XR+COS(THN)+YR*SIN(THN))/SORT(XR*XR+YR*YR)
48   XR=XMNM-XU5
49   YR=HUXM-HUX5
50   R5DN=(XR+COS(THN)+YR*SIN(THN))/SORT(XR*XR+YR*YR)
51   FACTOR=SORT(DVDFEP+DVDFEP+DELH*DELH)
52   CO=FACTOR*(
53     2*(GW1*AHAN21*G*SORT(((XU1-XMNM)**2+((HUX1-HUXM)**2))*R1DN)
54     2*(GW2*AHAN21*G*SORT(((XU2-XMNM)**2+((HUX2-HUXM)**2))*R2DN)
55   )
```

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56 2 * GW3 * AHAH21 6 * SORT ( (XU5 - XXY) ** 2 ) + ( (XU5 - XXY) ** 2 ) * R3DN)
57 2 * GW4 * AHAH21 6 * SORT ( (XU4 - XXY) ** 2 ) + ( (XU4 - XXY) ** 2 ) * R4DN)
58 2 * GW5 * AHAH21 6 * SORT ( (XU5 - XXM) ** 2 ) + ( (XU5 - XXM) ** 2 ) * R5DN)
59 CO = C0 * IJC
60 200 CONTINUE
61 RETURN
62 FND
SUBROUTINE AHN21

Subroutine AHN21 is a double precision Hankel function of type 2 and order 1. The function is generated using the polynomial approximations for $J_1(X)$ and $Y_1(X)$ presented in Abramowitz[90]. The program listing follows.

1 FUNCTION AHN21(X)
2 THIS IS THE HANKEL FUNCTION OF TYPE 2 AND ORDER 1
3 DOUBLE PRECISION XD,X,XA,AL,AL1,AL2,A1,A2,A3,A4,A5,A6,ALJ1
4 2TX,XA,AL1,AL2,A1,A2,A3,A4,A5,ALJ1,ALJ2,ALJ3,ALJ4,ALJ5,ALJ6
5 COMPLEX AHN21
6 RX=DBLE(X)
7 IF (X. GT. 3.0) GO TO 200
8 X=RX*#X/9.0*0.0
9 A1=0.317611E+03*1.10000004*#X
10 A2=0.00443150E+00+0.01*#X
11 A3=0.39542900E+00+0.02*#X
12 A4=0.21093531E+00+0.03*#X
13 A5=0.56249905E+00+0.04*#X
14 A6=0.50000000E+00+0.05*#X
15 ALJ1=ALE*#X
16 ALE=0.04099760E+00+0.0278730E+00*#X
17 A2=0.31239510E+00+0.04*#X
18 A3=0.31648700E+00+0.06*#X
19 A4=0.31627900E+00+0.08*#X
20 A5=0.31209100E+00+0.10*#X
21 A6=0.63661900E+00+0.12*#X
22 AHN1=(ALJ1+ALJ2)*LOG(#X/2.0)*0.63661977
23 AHN2=COMPLEX(SNLH(AHJ1)+SNHGL(AHJ1))
24 NO TO 300
25 200 TUX=5.0/#X
26 A1=0.01136553E+00+0.0002033*#TUX
27 A2=0.00249511E+00+0.01#TUX
28 A3=0.00177100E+00+0.02*#TUX
29 A4=0.01659667E+00+0.04*#TUX
30 A5=0.1560-0.05+0.04*#TUX
31 A6=0.77984560E+00+0.05*#TUX
32 T1=0.00079244E+00-0.00021660E+00*#TDX
33 T2=0.00774340E+00+0.01*#TDX
34 T3=0.0066376979E+00+0.02*#TDX
35 T4=0.00095690E+00+0.03*#TDX
36 T5=0.12499612E+00+0.04*#TDX
37 T6=0.12499612E+00+0.05*#TDX
38 T7=TX+T6
39 N3X=0.50000000E+00+0.06*#TDX
40 AHN2=COMPLEX(SNLX+DCOS(T7)+-SNGLX+DSIN(T7))
41 300 CONTINUE
42 RETURN
43 END

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SUBROUTINE CROUTN

The nonsymmetric matrix inversion subroutine solves a system of simultaneous linear equations with complex coefficients using the method developed by P.D. Crout[91]. The set of equations are related to the calling parameters shown in the following listing by

\[ [C] [X] = [F] \]

where
- \([C]\) is the array of complex coupling coefficients
- \([X]\) is the column vector of length \(N\) of unknowns,
- \([F]\) is the column vector of length \(N\) of known variables

before inversion; after inversion, the array contains the solution vector.

NDIM is the dimension of \([C]\) and \([F]\) in the subroutines;
NDIM \(\neq N\).
SUBROUTINE CROUTIC(F(J)MUM)
COMPLEX C(JMUJDK),F(JMUMUN),S
C NONSYMMETRIC CROUT
C FIRST COLUMN OK.
C TO GET THE FIRST ROW
DO 10 J=2,N
  C(1,1)=(C(1,1))/C(1,1)
10 C(1,JS1J)/C(1,1)
DO 11 K=2,N
  KNO=K-1
11 KPO=K+1
DO 12 J=1,NM0
  S=CMPLX(0.0,0.0)
12 S=CMPLX(0.0,0.0)
DO 13 K=1,NM0
  S=S+C(K,1)*C(J,1)
13 S=S+C(K,1)*C(J,1)
DO 14 J=1,NM0
  S=S+C(J,1)*C(J,1)
14 S=S+C(J,1)*C(J,1)
S=S+C(K,1)
DO 15 J=1,NM0
  S=S+C(J,1)
15 S=S+C(J,1)
CONTINUE
CONTINUE
C TO GET ELEMENTS IN COLUMN K BELOW ROW K
IF (KPO.GT.M) GO TO 17
DO 16 I=1,KM0
  S=S+C(K,1)*C(J,1)
16 S=S+C(K,1)*C(J,1)
CONTINUE
CONTINUE
C TO GET ELEMENTS IN ROW K TO THE RIGHT OF COLUMN K
DO 17 J=1,NM0
  S=S+C(K,1)*C(J,1)
17 S=S+C(K,1)*C(J,1)
CONTINUE
CONTINUE
C THIS ENDS THE MATRIX FACTORIZATION
C THIS BEGINS THE BACK SUBSTITUTION
C CONVERSION OF SOURCE SIDE
F(J)=F(J)/C(1,1)
DO 18 I=2,N
  S=CMPLX(0.0,0.0)
18 S=CMPLX(0.0,0.0)
DO 19 J=1,NM0
  IJ=1
19 IJ=1
DO 20 K=1,NM0
  IK=1
20 IK=1
DO 21 J=1,NM0
  SI=S+C(IJ,1)*C(IJ,1)
21 SI=S+C(IJ,1)*C(IJ,1)
F(IJ)=F(IJ)-S/C(IJ,1)
22 F(IJ)=F(IJ)-S/C(IJ,1)
CONTINUE
CONTINUE
C NOW FOR FINAL BACK SUBSTITUTION
NM0=N-1
DO 40 I=1,NM0
  J=I+1
40 J=I+1
DO 41 J=1,NM0
  S=S+C(K,1)*C(J,1)
41 S=S+C(K,1)*C(J,1)
F(K)=F(K)-S
42 F(K)=F(K)-S
RETURN
END
SUBROUTINE WILLUM

Subroutine WILLUM is the subprogram used to generate the incident fields at RHO, PHI from a magnetic line source pair located at DZ1 and DZ2. The line source amplitudes are 1/Zq and phased to produce a cardioid pattern directed along θ = 0 (toward the corrugations). The Fortran listing follows.

1 SUBROUTINE WILLUM(RHO,PHI,HINC,WE)
2 COMMON /HAKIGD/ DZ1,DZ2
3 C THIS SUBROUTINE CALCULATES THE NEAR FIELDS AT RHO,PHI RADIATED BY A PAIR
4 C WIRES SEPARATED BY D AND H ABOVE ORIGIN WITH MAGNETIC CURRENTS FK1 AND FK
5 C COMPLEX HINC*FK1*FK2*JN(2*61)+J
6 COMPLEX ARG
7 PI=3.14159
8 NGR=PI/180.
9 FK=2.*PI/H
10 FK1=(1.,0.)
11 J=(0.,1.)
12 FK2=J
13 H=0.
14 NZ1=4.
15 NZ2=6.
16 RH01=SQRT(DZ1*NZ1+H*H)
17 RH02=SQRT(DZ2*NZ2+H*H)
18 PHI1=ATAN2(H,DZ1)
19 PHI2=ATAN2(H,DZ2)
20 RH01P=SQRT(ABS(RH0*RH0*RH01*RH01-2.*RH0*RH0*COS(PHI-PHI*DEGR)))
21 RH02P=SQRT(ABS(RH0*RHO*RH02*RH02-2.*RH0*RHO2*COS(PHI-PHI*DEGR)))
22 ARG=COMPLEX(FK*RHO1P*0.)
23 IF (CABS(ARG).GE.0.01) GO TO 1
24 HINC=FK1
25 GO TO 2
26 CALL BESSEL(ARG,JN)
27 HINC=FK1*JN(1.5)
28 CONTINUE
29 ARG=COMPLEX(FK*RHO2P*0.)
30 IF (CABS(ARG).GE.0.01) GO TO 3
31 HINC=FK2
32 GO TO 4
33 CALL BESSEL(ARG,JN)
34 HINC=HINC+FK2*JN(1.5)
35 CONTINUE
36 C Z0=377. SCALE FACTOR REMOVED
37 HINC=HINC*FK/4.
38 RETURN
39 END
**SUBROUTINE BESSEL**

BESSEL generates $J_0(x)$, $N_0(x)$, $J_1(x)$, $N_1(x)$, $H_0^{(2)}(x)$, $H_1^{(2)}(x)$ and their derivatives using a power series for small complex arguments, $|Z| < 12$, and the asymptotic forms for $|Z| > 12$. These variables are stored in array JN and returned as a calling parameter. The Fortran listing follows.

```fortran
1  SUBROUTINE BESSEL(Z)JN)
2    COMPLEX JNI2,61.Z,TERMJ,TERMZ24,T1,T2,T3
3    JNI(1,1)=J0(Z)
4    JNI(2,1)=J1(Z)
5    JNI(2,2)=N0(Z)
6    JNI(2,3)=N1(Z)
7    JNI(2,4)=N0(Z)
8    JNI(2,5)=N1(Z)
9    JNI(2,6)=N0(Z)
10   JNI(2,7)=N1(Z)
11   JNI(2,8)=H0(2)(Z)
12   JNI(2,9)=H1(2)(Z)
13   JNI(2,10)=H0(2)(Z)
14   JNI(2,11)=H1(2)(Z)
15   PI=3.14159
16   IF(CAS(2,16.0)) Go To 10
17   FACTOR=0.0
18   TERMJ=0.0
19   M24=-0.2*Z*Z
20   TERMJ=TERMJ*M24/FLOAT(M*(M+1))
21   NO 1 NP=1,2
22   N=NP-1
23   JNI(NP+1)=TERMJ
24   M=0
25   NP=NP+1
26   TERMJ=TERMJ*M24/FLOAT(M*(M+1))
27   JNI(NP+1)=JNI(NP+1)+TERMJ
28   IF(NP=1) Go To 3
29   FACTOR=FACTOR+1.0/FLOAT(M)
30   TERMJ=TERMJ+TERMJ*FACTOR
31   Go To 1
32   IF(FACTOR.GT.1.0E-10) Go To 2
33   1   TERMJ=TERMJ+TERMJ*(M+1)
34   JNI(1,2)=(2.0*3.1415927)+(0.5772157+CLOR(0.5*Z))
35  JNI(1,1)=JNI(1,1)-TERMJ
36   JNI(1,2)=JNI(1,2)-Z/JN(1,1)
37   JNI(1,3)=JNI(2,1)
38   JNI(2,3)=JNI(1,1)-JNI(2,1)/Z
39   JNI(1,4)=JNI(2,2)
40   JNI(2,4)=JNI(1,2)-JNI(2,2)/Z
41   JNI(1,5)=JNI(1,1)-Z/JN(1,2)
42   JNI(2,5)=JNI(2,1)/(Z+1.0)
43   JNI(1,6)=JNI(1,3)-Z/JN(1,4)
44   JNI(2,6)=JNI(2,3)-Z/JN(2,4)
45   RETURN
46   2   C ASYMPTOTIC FORMS USFD FOR ARGUMENTS GREATER THAN 12.
47   3   IN T1=Z -CMPLX(P1/4+0.1)
48   4   T2=Z -CMPLX(3.4PI/4+0.1)
49   5   T3=CSQRT(T2/PI*Z)
50   6   JNI(1,1)=T3*CCOS(T1)
51   7   JNI(2,1)=T3*CCOS(T1)
52   8   JNI(1,2)=T3*CSIN(T1)
53   9   JNI(2,2)=T3*CSIN(T1)
54   10  Go To 11
55   11  END
```
SUBROUTINE PLOT

PLOT is a line printer plotting subroutine which plots up to 10 different curves (stored in the Y array) on the same set of coordinates. The value "X" is plotted along the page while the "Y" array is plotted between YMIN and YMAX across a line on the page with Y(1) plotted as a "#", Y(2) plotted as ".", etc. A call with "IND" = 0 places the Y scale across the page and for IND equal to a multiple of 10, places tic marks across the page. The PLOT subroutine must be called in a "DO" loop to plot an array of data. The Fortran listing follows.

```fortran
1 SUBROUTINE PLOT(X, Y, IND, YMAX, YMIN)
2 DIMENSION X(119), Y(10), MARK(10)
3 DATA MARK(1), MARK(2), MARK(3), MARK(4), MARK(5), MARK(6), MARK(7), MARK(8),
4 MARK(9), MARK(10) / 1, 1, 1, 1, 1, 1, 1, 1, 1, 1 /
5 DATA INLANK, NOPT, DPLUS, DH, DHI, DII /
6 TF(IND) = 1, 1
7 1 WRITE(6, 5)
8 FORMAT(3X, 25X, 4B10)
9 FORMAT(3X, 3B10)
10 M(J) = MARK(IND)
11 NCOUNT = 0
12 SCALE = 100.0 / (YMAX - YMIN)
13 LLL = (YMIN * SCALE + 1.5)
14 N8BJ = 1, 6
15 R = 0
16 N39J = 9, 119
17 YLABEL(J) = MARK(J)
18 WRITE(6, 29) (Y(J), J = 1, 119)
19 5 FORMAT(3X, 1PE9, S9/)
20 GOTO 132
21 NCOUNT = NCOUNT + 1
22 N99J = 1, 119
23 M(J) = INLANK
24 TF(LLL - 11 * ANDLLL - LLL = 110) MLLL = MARK(10)
25 TF(NCOUNT - 10) = 133, 133
26 132 DB9J = 11, 114, 20
27 89 M(J) = DPLUS
28 133 NO20J = 1, 4
29 L(J) = (Y(J) - YMIN) * SCALE + 0.5
30 TF(LLL = 14, 17, 17
31 14 TF(L + 10) = 15, 16, 16
32 15 M(J) = NOPT
33 GOTO 20
34 LL = L + 11
35 MLLL = MARK(J)
36 GOTO 20
37 17 TF(L = 108) = 18, 19, 19
38 18 LL = L + 11
39 MLLL = MARK(J)
40 GOTO 20
41 19 M(119) = NOPT
42 20 CONTINUE
43 IF(NCOUNT - 10) = 21, 25, 21
44 21 WRITE(6, 24) (M(J), J = 1, 119)
45 24 FORMAT(1X, 119A1)
46 GOTO 20
47 25 WRITE(6, 26) (X(J), M(J), J = 9, 119)
48 26 FORMAT(1X, F8.3, 119A1)
49 NCOUNT = 0
50 CONTINUE
51 RETURN
52 END
```
APPENDIX D
HIGHER ORDER MODES IN A SQUARE CORRUGATION

Because of the limited number of matching points available, only four segments were used on each of the vertical walls of each of the corrugations. Over the range of corrugation depths considered, this meant that 8 to 16 matching points per wavelength were used. While this density is adequate to represent the TEM mode in the square corrugation, any appreciable amplitude higher order mode would either not be observed or could introduce errors in the computations. Thus it was necessary to verify that any higher order modes were of small amplitude and only over a limited portion of the corrugation. To do this, the closed surface of Fig. 90 was used with the computer program of Appendix C. The model included a single corrugation illuminated by a magnetic line source located 0.5λ (4 cm) from the corrugation.

![Diagram of higher order modes](image)

Fig. 90--Surface model used to study higher order modes in a single square corrugation.

The 30 segments per vertical wall in the corrugation yields 60 to 240 matching points per wavelength over the range of depths considered (0.125λ ≤ d/λ ≤ 0.5). The relative surface current which exists at each point on the surface of the corrugation is shown on the plots in Figs. 91 to 94 for corrugation depths of 0.125λ, 0.25λ, 0.375λ and 0.499λ respectively. On these plots, the surface current amplitude, normalized to the 1/Z₀ amplitude magnetic line source, is plotted along the corrugation wall where the current is flowing. The amplitude function cos 2π(d-H)/λ corresponding to that of the TEM mode whose maximum is set equal to the maximum current computed in the corrugation is also shown. Notice that in each case the calculated current (the series of steps) very nearly fits the sinusoidal distribution anticipated for the TEM mode in the corrugation.
Also, the computed currents on the two walls at the same depth differ only slightly. Thus the higher order modes are of low amplitude and only over a small region of the corrugation. The loss associated with higher order modes is negligible.

Fig. 91—Relative surface current existing on the walls of a square corrugation at a corrugation depth of $d/\lambda = 0.125$. 
Fig. 92—Relative surface current existing on the walls of a square corrugation at a corrugation depth of $d/\lambda = 0.250$. 
Fig. 93--Relative surface current existing on the walls of a square corrugation at a corrugation depth of $d/\lambda = 0.375$. 

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Fig. 94--Relative surface current existing on the walls of a square corrugation at a corrugation depth of $d/\lambda = 0.499$. 
APPENDIX E

APERTURE FIELDS AND INTERNAL FIELDS OF A CORRUGATED HORN

The assumed cosine/cosine aperture fields used in Chapter IV to compute the corrugated horn radiation patterns were verified by a series of measurements discussed here. The measurement system, shown in Fig. 95, is a nulled radar system capable of amplitude and phase measurement. The probing was performed by moving a small sphere across the aperture on a very fine string. The H, E, and diagonal plane results appear in Figs. 96, 97, and 98 respectively. The calculated amplitude and phase assume cosine amplitude and spherical phase front with source at the apex of the horn walls. The results indicate a very nearly spherical phase front with the cosine amplitude distribution in both the E-plane and the H-plane.

![Fig. 95 -- Aperture probing system.](image)

In an effort to establish the nature of the corrugated horn fields, measurements were performed to probe the internal fields of the horn. This probing was done by drilling small holes through the bottom of the corrugation (the outside wall of the horn) and passing a small string from one E-plane wall to the other. The probe was a small dot of conducting silver paint on this string. The horn cross section is shown in Fig. 99. This figure also shows the various paths used to probe the fields. The results of these measurements are shown in Fig.100. Observe that the field between the horn walls at the first corrugation (Fig.100a) is essentially uniform. The small "C" and "E" on the horizontal scale indicate the top of the corrugation and the edge of the horn wall respectively. By the fifth corrugation...
(Fig. 96d) the cosine amplitude distribution is established and by the fifteenth corrugation (1/3 of the way down the horn) the spherical phase front is established (the phase was measured but has not been shown here).

Fig. 96—Corrugated horn aperture distribution in H-plane.
Fig. 97--Corrugated horn aperture distribution in E-plane.
Fig. 98—Corrugated horn aperture distribution on diagonal.
Fig. 99—Cross section of corrugated horn used for field probing.

CORRUGATION DEPTH = 1.08 cm
F = 8.7 GHz
Fig. 100—Internal field distribution of X-band corrugated horn at various points inside the horn. Operating frequency = 8.7 GHz.
a) at first corrugation, b) at second corrugation, c) at third corrugation, d) at fifth corrugation, e) at eighth corrugation, f) at fifteenth corrugation. The "C" indicates the top of each corrugation on the horizontal scale while the "E" indicates the bottom or edge of the horn wall.
In the study of the horn-reflector, it was desirable that the approximate beamwidth be specified before numerical results were obtained. The manner in which the desired beamwidth was translated to a specific antenna geometry is outlined in this appendix. In addition, some other useful geometric relations are presented which specify the aperture edges. The E-plane and H-plane aperture widths (\(WE\) and \(WH\) in Fig. 48) are found from:

\[
\begin{align*}
WE &= \begin{cases} 
50.8\lambda/BWE & \text{if the E-plane walls are conducting} \\
68.8\lambda/BWE & \text{if the E-plane walls are corrugated}
\end{cases} \\
WH &= 68.8\lambda/BWH,
\end{align*}
\]

where \(BWE\) and \(BWH\) are the desired half power beamwidths in degrees, \(\lambda\) is the wavelength.

Since \(WH\) is the projected H-plane aperture height,

\[
WH = \text{YU} - \text{YL}
\]

where \(\text{YU}\) and \(\text{YL}\) are the y-coordinates of the upper and lower edges as shown in Fig. 51.

Using this information, we first need the H-plane half flare angle \((\alpha_H\) in Fig. 47). The intersection of the paraboloid, whose equation is

\[
x^2 + y^2 = 4f(z+f),
\]

with the front H-plane wall of the horn (at \(\theta = \beta - \alpha_H\) as shown in Fig. 51) determines the upper edge of the aperture (\(\text{YU}\)) for \(x = 0\):

\[
\text{YU} = 2f \frac{\cos(\beta - \alpha_H) + 1}{\sin(\beta - \alpha_H)}.
\]

Similarly, the lower edge is specified by the intersection of the paraboloid and the H-plane wall at \(\theta = \beta + \alpha_H\) and is given by:
The resulting equation for $\alpha_H$ is:

$$\frac{\delta f}{W_H} \sin \alpha_H (\cos \alpha_H + \cos \beta) - \cos 2\alpha_H = -\cos 2\beta$$

and has been solved by an iterative technique. The $z$-coordinate of the upper and lower edges in the $x=0$ plane as shown in Fig. 51 are:

$$(181) \quad z_U = \frac{Y_U}{\tan(\beta - \alpha_H)}$$

The $E$-plane half flare angle is then found from:

$$(182) \quad \alpha_E = \tan^{-1} \left( \frac{WE \cos(\beta-90^\circ)}{2YM} \right)$$

where $YM = \frac{1}{2} (Y_U + Y_L)$.

This was found by projecting (in the $y = YM$ plane) the required $E$-plane width $WE$ at the aperture center (at $y = YM = 1/2 \ (Y_U + Y_L)$) back to the plane containing the $x$ axis and the horn axis (at an angle $\beta$ from the reflector axis) and evaluating the half flare angle.

In performing the numerical aperture integration in the $\phi = \beta - \alpha_H$ plane, the $y$-coordinate was chosen and the integration performed along $x$ between the aperture edges at $x_e$. The location of the edge in the $y = Y$ plane is determined by the intersection of the $E$-plane wall and the paraboloid projected to the $\beta - \alpha_H$ plane. The $x$ and $z$ coordinates of this intersection are:

$$(183) \quad x_e = -[Y \cos(\beta-90^\circ) - z_e \sin(\beta-90^\circ)] \tan \alpha_E,$$

and

$$z_e = Y \frac{\cos(\beta-90^\circ)}{\sin(\beta-90^\circ)} + \frac{2f}{\sin^2(\beta-90^\circ) \tan^2 \alpha_E} \sqrt{x(1+\tan^2 \alpha_E [\sin^2(\beta-90^\circ) + \frac{Y}{f} \sin(\beta-90^\circ) \cos(\beta-90^\circ) - \frac{Y^2}{4f} \sin^2(\beta-90^\circ)])}}.$$
For the special case where $\beta = 90^\circ$

(185) \[ x_e = y \tan \alpha_E, \]

and

(186) \[ z_e = \frac{1}{4f} \left[ y^2 (1 + \tan^2 \alpha_E) - 4f^2 \right]. \]

Another point which is important in the aperture integration is the intersection of the E-plane wall with the H-plane wall and the paraboloid since this point specifies the junction of the straight E-plane wall and the curved upper H-plane wall. The coordinates of this point are

\[
y_B = \frac{2f}{\tan(\beta - \alpha_H)} + 2f \frac{1}{\sqrt{\tan^2(\beta - \alpha_H)}} + 1 + \tan^2 \alpha_E \left[ \cos(\beta - 90^\circ) - \frac{\sin(\beta - 90^\circ)}{\tan(\beta - \alpha_H)} \right]^2 \\
1 + \tan^2 \alpha_E \left[ \cos(\beta - 90^\circ) - \sin(\beta - 90^\circ) \right]^2,
\]

(187)

(188) \[ z_B = y_B / \tan(\beta - \alpha_H), \]

and

(189) \[ x_B = \tan \alpha_E [y_B \cos(\beta - 90^\circ) - z_B \sin(\beta - 90^\circ)]. \]

The coordinates of the curved upper H-plane edge are found from the intersection of the H-plane wall at $\beta - \alpha_H$ and the paraboloid to be:

(190) \[ x_e = \pm \sqrt{4f \left( \frac{y}{\tan(\beta - \alpha_H)} + f \right) - y^2}, \]

(191) \[ z_e = \frac{y}{\tan(\beta - \alpha_H)}. \]

As is clear by now, not all the details have been included but hopefully enough has been given to provide some insight into the geometrical relations for the horn-reflector.
As discussed in Chapter V, the aperture fields are found via ray tracing from the horn to the aperture plane. If we let \((x_a, y_a, z_a)\) denote the coordinate in the aperture plane and \((x_a, y_a, z_p)\) denote the coordinates of the point of reflection from the paraboloid, then the aperture fields are given by:

\[
E_x \approx F(y_E, y_H) \frac{R_0}{R_1} e^{jk(R_0 - R_1 - (z_a - z_p))}
\]

where

- \(R_0\) is the distance from the horn apex to the point on the reflector where the horn axis intersects the paraboloid (unit amplitude fields are assumed here),
- \(R_1\) is the distance from the horn apex to the point of reflection on the paraboloid at \((x_a, y_a, z_p)\),
- \(F(y_E, y_H)\) is the field distribution inside the horn at angles \(y_E\) and \(y_H\) from the horn axis in the E-plane and H-plane respectively.

The horn field distribution depends upon the type of horn walls: cosine distribution in the H-plane; uniform distribution in the E-plane for conventional walls or cosine distribution for corrugated walls.
APPENDIX G
GEOMETRIC RELATIONS FOR THE CASSEGRAIN

In the coordinate system of Fig. 80, the equation of the surface of the main reflector is:

\[(192) \quad x^2 + y^2 = 4Fz\]

where \(F\) is the focal length of the paraboloid.

The equation of the subreflector is:

\[(193) \quad x^2 + y^2 = (e^2-1)[(z-\Delta F)^2 - a^2]\]

where \(e\) = the eccentricity of the hyperboloid,

\[\Delta F = F - F' = \text{difference in the focal lengths of the paraboloid and hyperboloid (as shown in Fig. 80)},\]

and

\[a = eF' = \text{product of the hyperboloid eccentricity and focal length.}\]

From Eq. (192), the rim of the paraboloid of diameter \(D_p\) will be continued in the plane

\[(194) \quad z_{rp} = \frac{1}{4F} \left( \frac{D_p}{2} \right)^2\]

and the rim of the hyperboloid of diameter \(D_h\) will lie in the plane:

\[(195) \quad z_{rh} = \Delta F + a \sqrt{\frac{\left( \frac{D_h}{2} \right)^2 + (F')^2 - a^2}{(F')^2 - a^2}}\]

The angle shown in Fig. 82 from the feed to the rims and from the focus to the rims are given by:

\[(196) \quad \beta_h = \tan^{-1} \left( \frac{D_h/2}{z_{rh} - (F - 2F')} \right)\]
\[ \theta_p = \tan^{-1}\left( \frac{D_p/2}{z_{rp} - (F - 2r^2)} \right), \]

\[ \alpha_H = \frac{\pi}{2} + \tan^{-1}\left( \frac{F - z_{rh}}{(D_p/2)} \right), \]

and

\[ \alpha_p = \frac{\pi}{2} + \tan^{-1}\left( \frac{F - z_{rp}}{(D_p/2)} \right). \]

These angles are used in determining the shadow boundaries of the spillover as discussed in Chapter VII.

As indicated in Chapter VII, the numerical aperture integration is performed in the aperture plane (i.e., in the \( z = z_{rp} \) plane). For each point \((x, y, z_{rp})\) in the aperture, an inverse ray tracing must be performed to find the point of reflection on the paraboloid, the point of reflection on hyperboloid, and then the angle from the reflector (and feed horn) axis to the point of reflection on the hyperboloid. This must be done so that the proper feed illumination is obtained in the aperture. Given the coordinates in the aperture, the illumination is given by

\[ E_x = F(\theta_f) \frac{R_2 e^{-jk(R_1 + R_3 + R_4)}}{R_1(R_2 + R_3)}, \]

where

\( R_1 = \) the distance from the feed to the point of reflection on the hyperboloid,

\( R_2 = \) the distance from the focus to the point of reflection on the hyperboloid,

\( R_3 = \) the distance between the points of reflection on the hyperboloid and on the paraboloid,

\( R_4 = \) the distance between the aperture plane and the point of reflection on the paraboloid,

\( \theta_f = \) the angle from the feed axis to the point of reflection on the hyperboloid,
and
\[ F(\theta_f) = \text{the feed pattern function given in Eq. (110).} \]

The feed angle, \( \theta_f \), is given by:

\[ (201) \quad \theta_f = \tan^{-1} \left( \frac{\rho_h}{z_h - (F - 2F')} \right) \]

where the point of reflection on the hyperboloid is \((\rho_h, z_h)\). The value of \( z_h \) is found by solving:

\[ (202) \quad z_h^2 \left[ \frac{\rho_p^2}{(F - z_p)^2} - \left( \frac{F'}{a^2} - 1 \right) \right] + z_h \left[ 2\Delta F \left( \frac{(F')^2}{a^2} - 1 \right) - \frac{2F_p^2}{(F - z_p)^2} \right] \]
\[ + \frac{F_p^2}{(F - z_p)^2} - (\Delta F)^2 \left( \left( \frac{F'}{a} \right)^2 - 1 \right) + (F')^2 - a^2 = 0 \]

for \( z_h \) where \((\rho_p, z_p)\) are the coordinates of the point of reflection from the parabola,

\[ \rho_p = \sqrt{x^2 + y^2}, \]
\[ z_p = \frac{\rho_p}{4F}. \]

The value of \( \rho_h \) associated with \( z_h \) from the solution of the above equation is:

\[ (203) \quad \rho_h = \rho_p \left( \frac{F - z_h}{F - z_p} \right). \]

One last bit of geometry needed for the diffraction analysis are the angles between the reflector axis and the plane tangent to the paraboloid and hyperboloid rims. The angle to the paraboloid rim tangent plane is given by:
\[
\alpha_{\tan p} = \tan^{-1}\left( \frac{F}{\sqrt{z_{rp}}} \right)
\]

and the angle to the hyperboloid rim tangent plane is:

\[
\alpha_{\tan H} = \tan^{-1}\left\{ \sqrt{e^2 - 1} \frac{z_{rh} - \Delta F}{\left[ (z_{rh} - \Delta F)^2 - a^2 \right]^{1/2}} \right\}
\]