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THE RUSSELL–LEIBNIZ DEFINITION OF IDENTITY:
SOME PROBLEMS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

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* * * * *

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1974

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CHAPTER I

INTRODUCTION TO THE PROBLEM

This chapter will introduce and provide a background for the problems to which the next four chapters address themselves. This chapter, then, serves as an introduction to the facets of identity and difference that this dissertation will discuss in detail. In general, the aim of this thesis is to provide a defense of Russell's definition of identity.¹ For the Russellian definition has come under attack from various quarters. The purpose of this chapter is to outline several of these attacks on Russell's definition, while the following chapters serve to answer each of the attacks in detail.

In preparation for both the outline of the alleged problems and the defense of Russell's definition, I will, first, introduce Russell's definition and consider some of its important consequences, and how such consequences are related to the identity of indiscernibles as well as some other principles which involve identity. The question about the Russellian definition is over its adequacy.

Since this notion is crucial, the second section of this chapter shall be devoted to a brief discussion of the notion of what it is for a definition to be adequate, as well as some demands that are too restrictive. The last and most important section of this chapter will be an outline of several problems which will be discussed in detail in later chapters.

I. The Definition of Identity

One may introduce the "=" sign into a formalism either as defined or as primitive. If the "=" sign is introduced as a defined sign—in the manner that Russell does—then certain crucial sentences, such as

\[(x)(y)(x\neq y = (\exists f)(fx = fy))\]

are rendered necessary or analytic. They are necessary by virtue of the definition. In a system where the notion of sameness or difference is introduced as a primitive or undefined, similar sentences are not necessary or analytic. Often, however, the introduction of the "=" sign as primitive in a formalism is accompanied by a set of rules and/or axioms. These axioms, such as "(x)(x = x)," are derivable from the Russellian definition. The reluctance to introduce "=" as defined probably stems from a desire to avoid quantifying over predicate variables, thereby avoiding commitment to properties.
Irrespective of whether the "=" sign is introduced as primitive or defined, questions may be raised as to the adequacy of the rules and/or axioms in the one case, and the definition in the other. Since the charges of inadequacy in either case are the same sort of charges, I shall introduce the "=" sign as defined in the Russelian manner. Suppose, then, one does introduce the "=" sign in the manner of Russell, namely,

\[(1) \ x=y=df(f)(fx=fy)\]

The motivation for defining identity in such a fashion is that one often thinks of \(x\) and \(y\) as being the same if, and only if, "they" share all properties in common. Conversely, one thinks of \(x\) and \(y\) as different, if, and only if, one has a property that the other lacks. The range of "\(f\)" in the definition is meant to include at least undefined predicates. It has been the subject of much debate as to whether the "\(f\)" of the definition should range over relational predicates such as "being-to-the-left-of-\(a\)," modal predicates such as "being-necessarily-\(f\)," metalinguistic predicates such as "being-designated-by-'\(a\)'" and predicates which involve identity, such as "\(\ldots=a\)." I shall discuss neither modal predicates nor ones involving identity. I shall take relational predicates as within the range of "\(f\)" of the Russelian definition. There will also be occasion to speak of "metalinguistic" predicates.
To begin with, it follows from the above definition that

\[(2) \ (x)(y) (x=y \equiv (f)(fx=fy))\]

From (2) it follows that both

\[(3) \ (x)(y) (x=y \rightarrow (f)(fx=fy))\]

as well as

\[(4) \ (x)(y) (f)(fx=fy) \rightarrow x=y)\]

Equivalent to (3) and (4) respectively are:

\[(5) \ (x)(y) ((\exists f)(fx=fy) \rightarrow x \neq y)\]

and \[(6) \ (x)(y) (x \neq y \rightarrow (\exists f)(fx=fy))\]

Notice that (2) - (6) all follow from the definition of "\(=\)" and are thus trivially true. Notice, further, that there is a rule which seems to be the metalinguistic counterpart of (3), namely,

\[(7) \ x=y \frac{fx}{fy}\]

In questioning the adequacy of the Russellian definition, the crucial sentences and/or rules under consideration will be (3), (6), and (7), respectively. The sorts of questions raised about the adequacy of (3) and (6), on the one hand, and (7) on the other will differ. For (7) is a rule of inference, while (3) and (6) are sentences. Rules are either valid or invalid, but neither true nor false as sentences are.
Notice that corresponding to sentence (3), which is within the Russellian formalism, there is another sentence, not within the Russellian formalism, as follows:

(3)' If "two" things are one and the same, then "they" share all properties in common.

Further, corresponding to sentence (6), there is the sentence

(6)' For any pair of things, one has a property that the other lacks.

Call (3)' and (6)' the principle of identity (PI) and the identity of indiscernibles (II) respectively. There is also an English counterpart of (7) which will be called the principle of substitutivity (PS).

PS: For all expressions A and B, "A=B" expresses a true proposition if and only if, for all sentences S and S', S' is like S save for containing an occurrence of B where S contains an occurrence of A, then S expresses a true proposition only if S' does also.2

The question over the adequacy of such sentences has not been over the necessity of (3) and (6) but rather over the alleged non-necessity of PI, II, and the alleged invalidity of PS. This discussion will concern itself primarily with arguments for the invalidity of PS and also for the alleged non-necessity of II.

In brief, three topics will be covered. First. It has been argued (against PS) that there are some contexts (some values for "f") which render PS invalid. This would make the rule inadequate. This will be discussed in Chapter II. Second. It has been argued that when the "=" sign in (6) is interpreted to mean the English "being-one-and-the-same-as" then the II is not necessary, and, thus, Russell's definition would be inadequate. This will be the subject of Chapters III and IV. Third. It has been argued that if the II is applied to distinguish kinds of things, it is inadequate. This will be discussed in Chapter V. Before giving some of the above arguments in greater detail, we shall turn to a discussion of the notion of adequacy involved.

II. The Notion of Adequacy

Since the purpose of introducing the "=" sign is to enable one to transcribe the English "being-one-and-the-same-as," the question of the adequacy of the definitions, axioms, or metalinguistic rules for its use is a relative notion, relative to the notion that one hopes the sign will capture or reflect. I shall speak only about the adequacy of definitions, which discussion will apply equally to axioms for "=" as well as metalinguistic rules. Certain illegitimate demands that might be made on one who attempts an analysis of identity will be mentioned.
First, begin with the adequacy of definitions. As I shall employ the term, a definition is neither true nor false, correct or incorrect, but rather purely stipulative. That is, a definition is a metalinguistic sentence indicating an abbreviation. As such, all defined terms are in principle eliminable in terms of their definitions. Definitions provide abbreviations or shorthand ways of saying what one already has the means to express. How, then, are definitions important? They are important because it is often thought that the terms introduced by a definition serve as a transcription of some other terms within English or some other language. If what one considers the relevant features of the English term are captured within the formalism by the definition, then one may say that a proposed definition is in that sense adequate. It would be important because one would not have to introduce into the formalism a primitive sign which one correlated to the ordinary notion one was trying to capture. Thus part of the importance of definitions has to do with the keeping of primitive signs to a minimum. Such a reduction has several benefits. First, one may argue that a system is more elegant if it has fewer undefined signs. More important for the purposes at hand, one may argue that if one is able to reduce one type of talk to another, by introducing a definition and arguing that nothing is lost
by not considering what is defined primitive or unde-

fined, then the reduction may be an important discovery.
Further, if one introduces "=" by definition, then, as
mentioned above, the sentences which are crucial to this
discussion become analytic.

For an example at an "attempt" at reduction, sup-
pose the following definition is proposed:

\[(8) \, \, \, \, [\forall x f(x)] (x) = df f(x)\]

where "x" and "f" are to be taken as variables. Suppose
one is attempting here to eliminate talk about a class in
terms of talk about the property that determines the class.
That is, all class expressions are, on definition (8),
eliminable. As a mere rule for replacement or elimination,
the definition is innocent. One will, however, run into
problems if one supposes that the "\(\forall x f(x)\)" sign in (8) cap-
tures the set-theoretical notion of "the-class-of-f's."
That is, one may wonder if the definition is adequate to
reflect the set-theoretical notion of class. To see this,
introduce two notions of class, \(\text{Cls}_1\) and \(\text{Cls}_2\). \(\text{Cls}_1\) is
the notion introduced by (8) above. \(\text{Cls}_2\) is the set-
theoretical notion. If all of the features of \(\text{Cls}_2\) are
captured by or reflected in \(\text{Cls}_1\), then there is a perfectly
legitimate sense in which one may claim that one has
reduced class talk to attribute talk. If the notions of
\(\text{Cls}_2\) and \(\text{Cls}_1\) differ in any relevant way, then, one has
shown definition (8) inadequate to do the job of replacing class talk with attribute talk. The details surrounding this and related issues will be explored more fully in Chapter V. For the moment, however, notice that while for anything which is a $\text{Cls}_2^0$ it is necessary that,

\[(9) (x)([\hat{x}f(x)](x)= [\hat{x}g(x)](x)) \Rightarrow [\hat{x}f(x)]=[\hat{x}g(x)]\]

where the "^\wedge" signs indicate $\text{Cls}_2$ talk. Notice further that for attributes, it is not held to be necessary that

\[(10) (x)(f(x)=g(x)) \Rightarrow f=g\]

Using (8), one obtains from (10) the following:

\[(11) (x)(([\hat{x}f(x)](x)= [\hat{x}g(x)](x)) \Rightarrow [\hat{x}f(x)]=[\hat{x}g(x)]\]

where the "^\wedge" sign here indicates $\text{Cls}_1$ talk. Remember that (9) is a necessary truth. Since (10) (and so too (11)) is not, one has located a difference. What this shows is that one attempt to define class talk in terms of attribute talk, namely, definition (8), has failed. Differently, (8) is not an adequate definition of $\text{Cls}_2$. This is not to show that all such attempts at reduction of class talk to attribute talk must fail, but only that (8) in particular is inadequate.

How does all of the above apply to a definition similar to Russell's of the "=" sign? As above, "=" is to be introduced in the following manner:

\[(1) x=y=df(f)(fx=fy)\]

As before, it follows from this definition that,
That this consequence follows from the definition is not in doubt. So to claim that it might be synthetic or possibly false would be misleading. The problem is not: is the above consequence analytic or synthetic? For it is straightforwardly analytic. The problem over the consequence arises in the adequacy of the definition, not in the analyticity of (2). Depending on the job one hopes the "=" sign can do, the term could be adequate or inadequate, respectively. Some philosophers, in claiming that (2) is synthetic were really talking about another sentence or pair of such, namely,

(2a) x is identical to y if every property and relation of x is a property and relation of y.

(2b) x is numerically the same as y if every property of x is a property of y.

To claim that Russell's definition of 1) is inadequate might be to claim that sentence (2) and sentence (2a) or (2b) might differ in truth value. For if they can differ in truth value then sentence (2) will be true while sentence (2a) or (2b) false. If one can successfully argue for this latter possibility one has not shown (2) to be synthetic—rather, what has been accomplished is to show that (1) is inadequate as a transcription of either of the English sentences (2a) or (2b). Thus one part of the criteria for adequacy is sameness of truth value between
the ordinary English and the transcription. Some further
discussion of the notion of adequacy involved will be found
in Chapters II through V.

Some criteria for adequacy are too restrictive,
however. Consider again the purposes of the formalism.
Beginning with certain ordinary notions expressed in
English by words such as "difference," "identity,"
"object," "possibility," etc., one then transcribes some
of these into the formalism one has constructed. Or,
depending on one's purposes, one may attempt to ground
them in an ontological analysis. This transcription or
analysis is accomplished in part by a matching of ordinary
English terms with terms in the formalism. There is, of
course, always a question of adequacy of the transcriptions
or analyses. As already mentioned, the next four chapters
can be read as a discussion of the adequacy of the "="
sign in capturing the notion of "being the same (kind of)
thing as." Several of the illegitimate restrictions that
one might put on the adequacy of such transcriptions are
the following.

One cannot demand that a relation of logical
equivalence hold between the ordinary notion and its
transcription. For such relations hold between sentences
within a language. For example, a sentence within a
formalism will not be logically equivalent to a sentence
not within the formalism even if the transcription is adequate. Suppose "this is green" is transcribed as "f(a)." Notice that while "f(a)" is logically equivalent to "¬f(a)," "f(a)" is not logically equivalent to "it is not the case that this is not green." This is not, of course, to say that the two could differ in truth value— but rather to point out that such relations as logical equivalence only hold within a given language.

Further, one cannot claim that a transcription (or analysis) is inadequate because one does not think (or, better, say) the same thing when one thinks of the ordinary notion as when one thinks of the transcription (or analysis). Such a line of argument appears to be at the root of a traditional argument against a physical object being a collection of sense data. When one thinks of riding on a train, one does not think of being supported by sense data. Carried to its extreme, such an argument can only be viewed as a refusal to do analysis. In this context, what one must insist upon is that the mere possession of more than one word (concept, notion) for sameness or difference should not for that reason commit one to more than one analysis. We shall return to this in Chapter IV.

Third, one might think the definition inadequate because it makes the II trivial. For some philosophers
have held that the II should be informative, or interesting, as well as necessary. But if this demand that the II be both non-trivial and necessary comes to demanding that the II be both necessary and not necessary, then it is a fortiori unreasonable. This demand shall be further discussed in Chapter IV.

III. Challenges to the Definition

We turn now to the major task of this chapter, namely, a presentation of the arguments which attempt to show that PS and II are inadequate.

First, how does one challenge PS? The example which I shall discuss is the following:

1) Giorgione = Barbarelli.
2) Giorgione is so-called because of his size.
3) Barbarelli is so-called because of his size.

Obviously, since 1) and 2) are true, yet 3) false, the above is an invalid argument--however, it seems to be one which results from substitution. That is, one obtains 3) by application of PS. Hence, one concludes that PS is not a valid rule. There are several routes around such an example. First one may insist that 2), though true, is covertly metalinguistic, and is equivalent to:

Giorgione is called "Giorgione" because of his size.

If one grants this, the conclusion which then follows will merely be the following true statement:

Barbarelli is called "Giorgione" because of his size.
Thus if one makes the metalinguistic element of 2) explicit, the argument may seem to be harmless. However, one may insist that even though 2) is indeed covertly metalinguistic—it nevertheless is a context true of Giorgione but false of Barbarelli even though Giorgione is Barbarelli. What one may also argue for, as Cartwright does, is that even though one has a context which falsifies PS, nevertheless, that context does not falsify PI.3 As shall be seen, Cartwright argues that the inference contained in the "Giorgione" example is not a counter-example to PI but is a counter-example to PS. For all PS requires, he argues, is that the names substituted be in a true proposition with "=" and the linguistic contexts into which they are substituted be the same. Given that the contexts are the same, he thinks PS is shown to be inadequate. I shall show in Chapter II that a crucial premise in the above argument, namely, that the contexts are the same, is false. If my argument is correct, then the "Giorgione" argument has not been shown to be a counter-example to PS.

How, then, does one challenge the II? One may read the II as claiming that if "two" things have all properties and relations in common, then "they" are the

3Richard Cartwright, "Identity and Substutivity," pp. 119-133.
same thing. Or, alternatively, if one individual is different from another, then they differ in a property. A denial of the II amounts to arguing that it is possible for a difference of individuals not to entail a difference of property. For example, Black argues that in a symmetrical universe, there can be two things with all of their properties and relations in common.\textsuperscript{4} Russell's definition seems to entail that it is impossible for two things to have all properties and relations in common. But consider a world, Black argues, consisting of two spheres placed a mile apart. In this world the spheres share all (one-place) properties (i.e., both are the same weight, have the same color, etc.). Given that we do not have a way of specifying a property one sphere has that the other does not, one cannot introduce names for the spheres. It follows that we cannot specify a relational property. Given that one cannot specify such a property, it follows that II is false in this universe. Therefore, Russell's definition of "=" is inadequate. Against Black I shall argue, first, that there is a perfectly good sense in which one can name the objects in question, and second, that even without names one can refute Black. Black misses the point of the II if he thinks one must be able

\textsuperscript{4}Max Black, "The Identity of Indiscernibles," Mind, LXI (April, 1952), 153-164.
to specify predicates true of the objects in question. His worry about specification gets him involved in irrelevant epistemological issues.

Another challenge to II is the following: one may hold that for any pair of individuals, say a and b, any property could be had by either individual, then it seems to follow that both a and b could have all properties (and relations) in common. If this is possible, then the II is not necessary, and so Russell's definition will have been shown to be inadequate. More explicitly, consider two individuals a and b, and any arbitrarily selected predicate "f." "f(a) \cdot f(b)" is not a contradiction. Thus it is possible for two things to have any property in common. It follows that it is possible for two things to have all properties in common. That is, it is possible that a be different from b and "(f)(f(a)=f(b))" be true. Thus Russell's definition is inadequate. Another argument against the II which will be considered in Chapter IV relies on the notion of individuation. As will be seen, there is more than one notion of individuation— one much stronger than the other. As will also be seen in Chapter IV, even if one accepts the stronger sort of individuation one is still not forced to deny the II. In this context, I shall argue that even if one does accept two notions of difference, that does not by itself show the II possibly false.
Another implicit attack on the II is to be found in the context of a paper on kinds of things. Suppose we use the identity of indiscernibles to distinguish kinds. That is, we are to count kinds C and A as different if $(\exists f)((x)(C x \rightarrow f x) \cdot (y)(A y \rightarrow f y))$. Cartwright,⁵ argues that though classes are extensional (i.e., $f$) this does not by itself do the job for distinguishing classes from attributes for two reasons. First, the very sentence expressing "extensionality" differs in the two cases, viz., the relations differ (member-of vs. instance-of). To claim that classes are extensional while attributes are intensional is not to claim that there is some one property, namely extensionality, that all classes but no attributes possess. Second, even if there were only one property involved, it is still true that some among attributes are extensional, and hence, contrary to the way Cartwright represents Quine, extensionality is not false of all attributes. I shall argue that although Cartwright is most likely correct in his conclusion, his argument fails because he implicitly demands more than is reasonable. As will be seen, Cartwright demands that classes (as a kind) be individuated from attributes (as a kind) in the stronger sense of individuation, which sense is stronger than the II.

CHAPTER II

THE PRINCIPLE OF SUBSTITIVITY

In a recent paper, Richard Cartwright defends what we have called the Principle of Identity (hereafter, PI). Cartwright formulates PI as follows:

PI: if x = y, then every property of x is a property of y.2

Cartwright neither defines nor gives axioms for the "=" sign used above. I think it is fair to read it as the ordinary English one-and-the-same. As it is stated, PI is clearly about objects and the properties they have. Notice further that Cartwright's PI does not reflect that other part of the Identity of Indiscernibles, namely, the claim that if "two" individuals have all properties in common, then "they" are one.

Cartwright wants to distinguish sharply between PI and another principle, what has been called the "Principle of Substitivity" (hereafter, PS), which he formulates as follows:

---

2 Ibid., p. 121.
PS: for all expressions A and B, 'A=B' expresses a true proposition if and only if, for all sentences S and S', if S' is like S save for containing an occurrence of B where S contains an occurrence of A, then S expresses a true proposition only if S' does also.3

It should be noticed that, unlike PI, PS is about language, that is, it is about expressions and their substitution in certain contexts. At first glance this difference may seem trivial. For one may be tempted to claim that PS is merely a metalinguistic counterpart of PI. If one adopts such a view of the difference, one might hold that PI is true iff PS is true. That is, the truth of PI would depend upon the truth of PS and conversely. To adopt such a view might have upsetting consequences. For its acceptance entails that any counter-example to PS is a fortiori a counter-example to PI. Cartwright believes that there are counter-examples to PS. Yet PI seems trivially true and he is at pains to preserve it. So he argues that a counter-example to PS is not necessarily a counter-example to PI.

Let us examine his argument. Cartwright wants to show, first, that there is at least one counter-example to PS. He wants to show, second, that this counter-example to PS does not render the PI false. I agree with Cartwright that the argument he chooses is not a counter-example to PI. What I shall argue, however, is that he does not show that

3Ibid., p. 120.
the alleged counter-example challenges the truth of PS. If it does not, then Cartwright has not accomplished what he attempted to, namely, he has not shown that the falsity of PS does not imply falsity of PI. That is, he has not shown that PS can be false while PI true.

Cartwright's supposed counter-example to PS is a traditional one.

A. 1) Giorgione was so-called because of his size.
   2) Giorgione = Barbarelli
   3) Barbarelli was so-called because of his size.

Even though sentences 1) and 2) are true, Cartwright believes that substitution of the expression "Barbarelli" for the expression "Giorgione" in 1) is not truth preserving; that is, 3) is false. Here we have, so Cartwright believes, a counter-example to PS. But does it follow that it is also a counter-example to PI? Cartwright argues that it is not. Although I shall not give his argument in detail here, his claim is, roughly, that the linguistic expression "... is so-called because of his size," while forming a sentence which expresses a true proposition when attached to the expression "Giorgione" and forming a sentence which expresses a false proposition when attached to "Barbarelli," cannot represent or pick out a property that Giorgione has but Barbarelli lacks. If not, then the supposed counter-example to PS is not a counter-example to PI.

I agree with Cartwright in making the distinction he does between PS and PI. I also agree that there may be
some counter-examples to PS which are not counter-examples to PI. I do not, however, agree that Cartwright has shown A to be a counter-example to PS. In what follows, I shall consider two arguments to save PS from Cartwright's attack, the second of which he does not even consider.

The first argument involves elimination of the pronomial "so-called." Consider the following way of rephrasing A.

B. 1) Georgione was called "Giorgione" because of his size.
2) Giorgione = Barbarelli

Substitution of the expression "Barbarelli" for the expression "Giorgione" in 1) yields,

3) Barbarelli was called "Giorgione" because of his size.

Now the conclusion of B is true while the conclusion of A is false. That is, while substitution in A seems to lead us from true premises to a false conclusion, this unhappy occurrence does not repeat itself in B. Cartwright argues against this move. He claims that though B indeed does not count as a counter-example to PS, nonetheless A does. That is, A and B are different examples, one of which does while the does not count as a counter-example to PS. Let us go along with this suggestion of Cartwright's, namely, refuse to count B as the same argument as A. How then, is one to show that A is not, as I have claimed, a counter-example to PS?
Cartwright says of 1) and 3) in A that they are alike, save that 3) contains the name "Barbarelli" where 1) contains the name "Giorgione."\(^4\) And indeed if this were true, Cartwright would have presented us (in A) with a counter-example to PS. However, I shall argue that what Cartwright claims is false. That is, I shall argue that the mere difference in names is not the only difference in the sentences under consideration. Put differently, in order to show that argument A is indeed a counter-example to PS, Cartwright must assume that the expression "... is so-called because of his size" as attached to the expression "Barbarelli" is one and the same as the expression "is so-called because of his size" as attached to the expression "Giorgione." If one can make this assumption, then A is a counter-example to the PS. But can one make this assumption? Or, if one does, what is its justification? For crucial to the idea of substitution is that one begins with a term and an expression that it is imbedded in or attached to. Substitution takes place only when one replaces the term and the expression remains the same. What I shall argue is that the expressions considered above are different expressions. That is, it is false to assume that they are the same. If my argument is successful, then substitution has not taken place in A. If this is correct,

\(^4\)Ibid., p. 122.
then A is not a counter-example to PS. More explicitly, my argument shall be that the proper form of A is not:

\[
\frac{F(a)}{a=b} \quad \frac{F(b)}{}
\]

as would be the case if substitution has occurred. Rather, if one can successfully argue that the above-mentioned expressions are different, then one may schematically represent the proper form of the argument A as:

\[
\frac{F_1(a)}{a=b} \quad \frac{F_2(b)}{}
\]

where the subscripts on the F's represent the differences between the two expressions that is yet to be argued for. Notice that the latter, like A, is an invalid argument—but not one in which substitution has occurred at all—so one could hardly count it as any sort of counter-example to PS.

How does one argue that the expression "... is so-called because of his size" as attached to the name "Giorgione" is different from the expression "... is so-called because of his size" as attached to the name "Barbarelli"? One must have or give some reasonable criteria which will enable us to count the above expressions as two. Such criteria must be applicable to all expressions—indeed, one must argue that they are necessary in order for us to argue about what is to count as
one expression. I suggest two criteria for the sameness of expressions. "Two" expressions are to be counted as one if and only if they fulfill both criteria. The first criterion is the following: if two linguistic expressions are to be counted as the same then they must be tokens of the same geometrical type. Cartwright himself may have something like this in mind when he insists that the sentences "Giorgione was so-called because of his size" and "Giorgione was called 'Giorgione' because of his size" are different sentences. As noted above, it was the former but not the latter that led Cartwright to claim that PS is invalid in at least one instance. If difference in geometrical type is sufficient to enable us to count two expressions as different, is sameness of geometrical type sufficient to enable us to count two expressions as the same? Cartwright does not explore this. Consider the following premises, however:

C. 1) He is not feeling well.
2) He = the King of England.

If one is given the further information that the ill man mentioned in 1) above is different from the man who is kind, one would not even consider concluding that the King is ill—for the simple reason that the "he" in 1) and the "he" in 2) are different words—even though they exhibit the same shape. So similarity of shape of two expressions is not sufficient to enable us to call them one and the
same. Hence the second criterion, namely, that the expressions one is dealing with must mean the same thing. This is not to appeal to some specific theory of meaning or invoking some problematic notion of "having the same meaning." Rather, anyone who argues within the context of counter-examples must implicitly employ such a notion. If he does not, then he can find counter-examples to any logical principle merely by equivocating on the meaning of the expressions or sentences involved.

An objector may claim that my criteria for sameness of expressions is out of place—that ordinarily we count two expressions as the same word merely if they have the same shape. The objector says of C, however, that there are two different uses of the same word. But this is a mere verbal disagreement as to what it is to count as one word. While I claim that we do not draw the conclusion in C because the two shapes are different words, my objector claims that we do not draw the conclusion in C because we have two uses of the same word. In a sense, the difference between us is trivial—we both recognize the importance of distinguishing some difference in the "he" in 1) and the "he" in premise 2).

We are now in a position to answer the question: is the expression ". . . is so-called because of his size" as attached to the expression "Giorgione" the same as the expression ". . . is so-called because of his size" as
attached to the expression "Barbarelli"? With respect to the first criterion the answer is clearly yes. With respect to the second criterion the answer is no. All one must do is note that the expression ". . . is so-called because of his size" as attached to "Giorgione" has the same meaning as ". . . is called 'Giorgione' because of his size" but ". . . is so-called because of his size" as attached to "Barbarelli" does not have this meaning. Just as extra-linguistic items can individuate the two "he"-s in C, so too can linguistic items individuate the two expressions under consideration. That is, the difference in men make the "he"-s different and the difference in names preceding "is so-called because of his size" makes those different expressions. I conclude that A is not a counter-example to PS.

One may argue, however, that there is a clear sense in which all occurrences of ". . . is so-called because of his size" means the same thing. And this of course is true. But the sense of "meaning the same thing" here involved is the sense in which the two different uses of "he" in C above mean the same thing. In one sense, I know the meaning of the pronoun "he" without knowing who is being referred to. That is, I know how to use the word. In another sense of knowing the meaning of a word, I do not know the meaning of an occurrence of "he" unless I know who is being singled out. It is this latter sense that is
crucial. So there are three things to consider. First: the shape of the marks; second: the general rules for use; third: for words like pronouns, the specific object picked out. If one is upset by my calling reference an aspect of meaning and would rather claim that, say, two occurrences of "he" have the same meaning but are used differently, that is perfectly acceptable. Again, this would be a mere verbal disagreement as long as he realized there must be some differences in the two different uses of "he" or "... is so-called because of his size."

What is strange is that Cartwright himself argues to support my view.

... the expression 'so-called' is just the kind of expression that cannot stand on its own. To make sense of sentences in which it occurs, it is necessary to look at the environment—linguistic or otherwise—of the expression ... .

As the "environment" changes, so too does the sense of "... is so-called because of his size." But what does Cartwright intend here by his use of "sense"? Is he referring to the meaning of the expressions involved? If so, then he is close to claiming, as I have, that we are dealing with two different expressions. Differently, if the reference of "so-called" is fixed, that is, if we always take it to mean "called 'Giorgione,'" then 3) in A is no longer false. If the reference of "so-called"

\[5\text{Ibid.}, \ p. \ 124.\]
changes, being "called 'Giorgione'" once and "called 'Barbarelli'" a second time, then the whole expression has changed. For just as one use of "he" is different than another use of "he," the difference depending upon who is being referred to, so too it is a mistake to suppose the sign ". . . is so-called because of his size" can only be one expression. So the correct conclusion to draw is that not all uses of ". . . is so-called because of his size" express the same thing, but are as many expressions as there are different names to which ". . . is so-called because of his size" is attached. Indeed, if one does treat ". . . is so-called because of his size" as one expression, one is led into the difficulties that Cartwright believes we are, but one would be led into such difficulties with any word or phrase whose meaning shifts from premise to conclusion.

An objector may claim, however, that there is a crucial difference between, say, premise 1) of C and premise 1) of A. Given the ordinary rules of use, the objector claims, premise 1) of C can indeed express many propositions—but "Giorgione is so-called because of his size" cannot. And this of course is true. But all this shows is that the rules for "so-called" fix a meaning for ". . . is so-called because of his size" depending upon which name we begin the sentence with. But this is exactly parallel to the case of the pronoun. The pointing rules
during a given use of "he" also fix a meaning for "he" depending upon who is referred to. If I point first at Jones and then at Smith, we have two meanings (or, if you will, uses) of "he." Alternatively, if I begin sentence 1) (in A) with "Giorgione" and in 3) with "Barbarelli," I have two different meanings for "... is so-called because of his size."

The argument has been two-fold. First, if we translate the premises of A into those of B, there is no problem. Second, even without such translation we have shown that it is reasonable to hold that the linguistic expressions differ in 1) and 3) in A. So on either grounds we have not falsified PS. What has been shown is that Cartwright has not provided us with a counter-example to PS and thus not shown by means of A that it is false that PS is true iff PI is. I have not of course argued that there are no counter-examples which falsify PS, only that Cartwright's example does not do the job.
CHAPTER III

SYMMETRICAL UNIVERSES

Is it possible that there be two different individuals with all their properties and relations in common? Must it be the case that for any two individuals there be a property (or relation) of one that is not a property (or relation) of the other? Stated in terms of an Ideal Language, if the sign "=" is to be construed as the English "one-and-the-same," is "(x)(y)(x≠y → (∃f)(fx • ¬fy))" a necessary truth? Put in these terms, these questions raise one aspect of the traditional problem of the Identity of Indiscernibles (hereafter, II). Some time ago, Black, in his article, The Identity of Indiscernibles,¹ attempted to show that the II is not a necessary truth. Black attempts to describe a possible universe in which, so it is claimed, two individuals have all their properties and relations in common. If such a universe is shown to be possible, this would amount to a refutation of the claim that the II is a necessary truth. Differently, Black argues that in his possible world at least "x≠y" is true while "(∃f)(fx • ¬fy)" is false. He does so by asking us to

envision a possible world which is symmetrical. Two spheres with the same weight, color, etc., are placed a mile apart. Every property of the one is claimed to be a property of the other. In this world, Black argues, the II is false.

I propose to show that Black's arguments are mistaken. In doing so, I shall first present another objection to the II which is different from and hence not to be confused with Black's argument. I shall, second, show that one of the individuals in Black's universe differs from the other in a relational property. Third, I shall argue that one possible reason Black may miss the property is that he infers from the fact that his universe is symmetrical that any property of the one object is the property of the other. Differently, there is an ambiguity in the notion of being symmetrical. Black's inference may be based on this confusion. If so, the inference is question begging. Fourth, I shall discuss the principal reason that Black misses the property I suggest, viz., he refuses to let the defender of the II employ logically proper names to name the objects involved. Usually a condition sufficient for the use of a logically proper name is the existence of the object named. Black's universe, containing as it does two objects, thus requires two logically proper names. But Black argues that we cannot use names in this case. I shall argue against this restriction of the use of
names on the grounds that it misrepresents the problem of
the II, making it a problem about what we know, not what
is or could be the case. If my arguments are correct, then
the II will not, at least for Black's reasons, have been
shown to be possibly false. Fifth, I shall argue that the
use of names is not question-begging. Sixth, to substan-
tiate the points made about names, I shall present a brief
argument, without employing names, showing that the II is
true in Black's universe. Seventh, I shall discuss a
sense in which one might claim that, even given the above
arguments, Black's argument might still be thought to have
some point—even though Black himself does not discuss
this. I shall not, in this chapter, be arguing that the
II is necessary, rather my claim is a weaker one, namely,
that it is true in Black's possible world.

I. On Possibility

Consider again the problem with which one begins:
Is it possible for there to be two different objects which
have all their properties and relations in common? The
answer one gives will depend upon one's notions of possi-
bility, difference, what is to count as one thing, as well
as one's notions of property and relation. Focus upon the
first of these: possibility. By employing a combinatorial
notion of possibility, some philosophers have dismissed the
II out of hand.² For suppose the universe consisted of only two things, a and b. Suppose further that there are only two properties, f and g. Given that there are certain rules for that is and what is not a wff., one may form all possible atomic sentences. As it turns out, there are only four, viz., "fa," "fb," "ga," and "gb." Imagining a truth table for these, one sees at once there are sixteen "possible" worlds. In the top row, there will be all "T"-s. Thus the possible world represented by the top row is such that a and b have all properties in common. Such arguments can be extended to cover relations as well, and as many individuals as one needs. One may think that in such a "possible" world the II will be false, for any property (or relation) had by a will be had by b and conversely. By "relations," I do not intend merely the relation of, say, to-the-left-of, but rather having a relational property, say, to-the-left-of-a, which would count as a different property from to-the-left-of-b, given that a is different from b.

For the moment, it will be sufficient to indicate an objection to this argument. One may claim, that this notion of combinatorial possibility is irreducibly

philosophical or inexplicable, and one cannot legitimately arrive at the conclusion that there is a possible world in which the II is false. To begin with notice that this notion of possibility makes it possible that one and the same thing be both black and white. Notice further that the notion of possibility makes the ordinary laws of space and time possibly false. That is, on this notion of possibility, there are some possible worlds in which two things can be in one place, something can be to the right of itself, some event can occur after itself, etc. Some of these possible worlds, if relations are introduced, would seem to violate the ordinary laws of space and time. Such laws are often held to be necessary. These laws are specified in the following fashion. Consider the relation prior-to: it is held to be "necessary" that 

\[(x)(y)(z)((Pxy \land Pyz) \land Pxz).\] 

Such a law will fail in some combinatorially possible world and in that sense maybe held not to be necessary. For there will be a possible world where, say, both "Pab" and "Pbc" will be assigned the truth value true, while "Pac" will be assigned the truth value false. Differently, the statement of the transitivity of prior-to is not a logical truth.

The strength of Black's article can be seen in the light of such an argument. For Black retains the above

\[^{3}\text{Ibid.}, \text{pp. 236-242.}\]
necessity of the ordinary laws of space and time and still constructs a possible world where, he claims, there are two objects which have all properties and relations in common. That is, he retains the necessity of the laws of space and time and still offers a world in which, so he claims, the II is false. Notice that he wants to claim that it is a necessary truth that "different objects must be in different places." That is, his notion of what is possible differs from the merely combinatorial one. In passing, one might note that Black does not need to hold that such spatial laws are "necessary" in some special sense to get his particular example off the ground. In constructing a possible world as Black does, he is not forced to claim that one object necessarily is not to the left of itself—rather, all he need claim for his artificial universe is that the spatial laws that it exhibits are as a matter of fact the same as the spatial laws as the actual universe. Nevertheless, since I believe Black's argument can be refuted within the context of his notion of necessity, no purpose will be served here by discussing whether spatial laws are necessary or not.

A much fuller discussion of possibility and its relation to the II will be presented in the next chapter. For the moment, I should like to mention, merely to

\[^4\text{Max Black, "The Identity of Indiscernibles," p. 158.}\]
separate it from the above discussion, a more traditional notion of possibility. Traditionally philosophers have talked of what is possible in terms of what one can conceive, or what one can think. The idea is that one cannot think of the impossible. Thus the thinkability of a state of affairs becomes the criterion for whether it is possible. The criterion is overtly psychologistic. Even worse, it does not separate the possible from the impossible, in a different sense of possible of course. For obviously we can think of, or perhaps better, state what is impossible. Suppose one says: "Consider a possible universe which contains two things all of the properties and relations of one being properties and relations of the other." This does not in any way establish the possibility of such a universe; merely because one considers it. The possibility of such a universe is to be argued for—not assumed from the outset. In a similar vein, suppose someone, say Schwarz, asks us to consider a world with one individual a, which both has the property F and lacks the property F. There arise two questions. Is it possible to consider or represent this "world"? Is it a possible world? The answer to these questions are yes and no respectively. So these questions are independent of one another. There is no reason to accuse Black of employing such a psychologistic notion of possibility. There is one place, however, where Black seems to come close to
this view of possibility. He states: "I can imagine only what is logically possible." Irrespective of whether this is operative within Black's argument, the point here is that a world is not rendered possible merely by the fact that one can consider it.

II. The Distinguishing Property

Black describes the universe he constructs as follows:

Isn't it logically possible that the universe should have contained nothing but two exactly similar spheres? We might suppose that each was made of chemically pure iron, had a diameter of one mile, that they had the same temperature, colour, and so on, and that nothing else existed. Then every quality and relational characteristic of the one would also be a property of the other. Black claims that such a possibility refutes the II. One who is defending the II, at least against Black's arguments, must ask if it is really true of Black's universe that every property and relation of one object is a property and relation of the other. I suggest it is not. The spheres differ in relational properties.

There are two objects in Black's possible world. This is reflected by the appearance of two logically proper names in a language built to represent perspicuously the possible world that Black envisions. Since a logically

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5 Ibid., p. 156.
6 Ibid.
proper name is the type of symbol used to represent one and only one object, and since there are two objects in Black's universe, then a proper representation of there being two requires the employment of two names, "a" and "b."

Consider the property closer-to-a-than-to-b. This is a relational property had by a and not by b. Having found a property the one has and the other lacks, I conclude Black has failed to refute the II. For he has not provided an example of a possible universe where two things have all properties and relations in common. Notice that Black's opponent is not forced to choose a relational property of a different sort; a property which it is impossible for more than one object to have. I have in mind such alleged properties as numerically-the-same-as-a, which only a can have. A philosopher who chooses such a property to show the II to be true in Black's universe may be thought to be open to the charge of question begging. The property I choose is a relational one but it is a property that could be had by objects other than a. That is, there could be three spheres, two of which had the property of closer-to-a-than-to-b. Thus the relational property chosen to distinguish a from b is not of the alleged question begging variety.

Why would Black object to such properties doing the job of distinguishing one sphere from another?
Consider the properties: closer-to-a-than-to-b and closer-to-b-than-to-a. In an obvious sense these can be considered the same, in that they involve the same relations. They can also be considered different, in that they involve different objects. Being different properties, if \( a \) had the former but not the latter, and \( b \) has the latter but not the former, then either property will distinguish \( a \) from \( b \). But in a symmetrical universe are properties like the above different? One must separate several issues. First. Given that there are two objects, is there a property had by one but not had by the other? If the argument against Black is correct, the answer to this question is yes. Second. Given that there are such properties, does that entail that they are different independently of the objects that they involve? It is important to keep these questions separate, for in the first case we are asking about objects and whether they happen to differ in their properties, while in the second, we are asking about relational properties and how they differ. If it turns out that the only way to distinguish the properties is in terms of the objects that are involved in them, it will not follow that the properties are not two. It shall be argued in the last section of this chapter that indeed the only way to distinguish the properties in question is by appeal to the objects involved in them. But,
as I shall argue in detail, this involves no circularity. For if the objects are two, so too are the properties. One may object and claim that the properties are not known to be two unless one knows that there are two things. The reply is that one knows there are two things because Black said there are.

There are two related points to be made in this connection. First, one may think that what the II states is that difference in property grounds (in some ontological sense perhaps) the difference in object, and thus one would be involved in circularity. But this would be to misconstrue the question. For the problem, as Black puts it, is merely the following: Given that there are two things, is there a property had by one but not the other in a symmetrical universe with two objects? Second, notice that if someone takes the II as a decision procedure for discovering how many things there are in Black's universe, he might raise the above objection, that is, he might think (quite correctly) that he cannot find a unique property for each object until he knows that there are two objects. But, again, this is to misconstrue the whole issue. For Black does not ask you for a method by means of which one could count the things in his universe. Rather, he tells you that there are two of them. The question over the II is: Given that there are two objects, is there a property the one has that the other has not?
III. Weak and Strong Symmetry

Black is attempting to show that it is possible for two different things to have all their properties and relations in common. Since his attempted refutation of the II rests upon an example of a possible universe, his argument should not be the following:

1) There are two objects in Black's possible universe.

2) Black's universe is symmetrical.

Hence

3) The two objects have all their qualities and relations in common.

and

4) It is possible for two objects to have all properties and relations in common.

Jointly, 1) and 3) entail 4). But the defender of the II will object. For on his grounds, if 1) is true, then 3) cannot be true, i.e., there must be a distinguishing property. Thus the defender of the II may grant Black his description of the possible universe, namely 1) and 2), but insist that such a description by itself does not establish 3) (and thus 4). Black, however, talks as if he has shown 3) to be true. Rather, what he has done is to describe a possible universe. That is, he claims both 1) and 2) to be true. The truth of 3) does not follow from that of 1) and 2). One may confuse 2) with 3) and thus suppose that if 2) is true, so too is 3). Notice that Black claims the spheres to be
... so symmetrically placed with respect to one another that neither has any quality or character the other does not also have.\(^7\)

This I take to be the move from 2) above to 3). From the universe being symmetrical, Black infers that they have all properties and relations in common. He seems to take "being symmetrical" in such a way that it entails "having all properties and relations in common." But such an entailment rests upon a confusion of two senses of "symmetrical." "Two" objects may be said to be symmetrical if they have all properties and relations in common. If this sort of symmetry holds, call the objects completely symmetrical. Two objects may be said to be symmetrical if they have all properties and relations of the same kind in common. What I have in mind is the following: Both objects in Black's universe have properties of the kind: being-closer-to-x-than-to-y. Hence they are in the latter sense symmetrical. If objects are symmetrical in this sense, call them weakly symmetrical. However, it does not follow from two objects being weakly symmetrical that they are completely symmetrical. For while it is true of a that it is closer to a than to b, it is not true of b that it is closer to a than to b. Thus, though their relations are of the same kind, their instances, as it were, are different. There are two equivalent ways of viewing

\(^7\)Ibid.
this matter. One may point out the differences between the senses of "symmetrical" as above. Or one may merely note that "being symmetrical" in Black's universe is weaker than "having all properties and relations in common." In neither case is 3) shown to follow from 2). Only the important sense of "being-symmetrical," namely, being completely symmetrical, entails the denial of the II, however. And in this sense Black's universe is not symmetrical. Of course, Black may agree and argue that he has not shown the II to be false in his universe. Rather, all his argument amounts to is an attempt to show that the defender of the II cannot show it to be true. This, of course, is a much weaker claim. The remainder of this chapter will consist of an argument to the effect that even the weaker claim is mistaken, namely, the defender of the II can show it to be true in Black's possible world.

IV. Logically Proper Names

The heart of Black's paper consists in his refusal to let the defender of the II employ logically proper names. Before considering his reasons for such a refusal, let us consider the results. Remember that the distinguishing property chosen was: closer-to-a-than-to-b. It is important to note that the above property is one represented by a predicate with proper names imbedded in it. Suppose one
attempts to represent the universe Black asks us to imagine. Suppose, further, that for the moment, proper names are not employed. What is readily seen is that if proper names are not used, it will not be possible to represent the above property which distinguishes one sphere from another. Notice that it does not follow from the fact that certain restrictions are placed upon the way one represents objects and hence properties involving them, that those properties one cannot represent do not exist. Either one employs proper names or not. If one does, then one can represent the above distinguishing property. Thus, by itself, the restriction on names does not refute the II. What one may conclude is that a language without names is just not rich enough to represent the distinguishing characteristics. What Black concludes is that there are no distinguishing characteristics. But the latter conclusion does not follow from the former.

Black's refusal to let his opponent use names is related to the ambiguity between the two notions of symmetry mentioned above. For without names the only way to distinguish one relational property from another will be in terms of the relation involved; e.g., being-a-mile-from-x is different from being-closer-to-x-than-to-y. Since both of the spheres have such relations and since Black places a restriction on the use of names, he may conclude that if
his universe is weakly symmetrical, then it is also completely symmetrical. Thus, if one does not use names, Black may feel the move from 2) to 3) in the argument in Section III is valid. But this is mistaken, for all one can properly conclude is that if one does not use logically proper names to represent such a universe and if that universe is weakly symmetrical, then if there is a property difference between the two objects, one cannot represent it. One should not conclude that, if one does not use names, then there is no property differences between the objects. Thus the proper conclusion is one about language, not about the universe that the language is about. This by itself does not establish the truth of the II in Black's universe, rather it shows that Black's challenge to it does not show it to be false.

Turn now to Black's reason for the crucial refusal to let the defender of the II employ logically proper names.

I know how to take a book off a shelf, but I don't know how to identify one of two spheres supposed to be alone in space and so symmetrically placed with respect to each other that neither has any quality or character the other does not also have.\(^7\)

You talk as if naming an object and then thinking about it were the easiest thing in the world. But it isn't so easy. Suppose I tell you to

\(^7\)Ibid.
name any spider in my garden: if you can catch one first or describe one uniquely you can "name" it easily enough. But you can't pick one out let alone "name" it, by just thinking.

To place in perspective the sort of arguments Black invokes, return to an aspect of the II that we have been considering. The question with which we began was: Is it possible that there be two different things such that all of the properties and relations of the one are also properties and relations of the other? The problem is about what there is, or at least what there could be. The problem is not about what one is acquainted with. The problem is not how to identify what there is or could be. The problem is not what one must know (a unique description perhaps?) in order to apply a name. I shall here take it as axiomatic that what there is or could be is independent of whether I know it, can identify it, or know a unique description of it. To take the II to involve such questions is to introduce epistemological issues where they are irrelevant. In reply to Black it can be argued that when there are (said to be) two things, then that fact can be represented by employing two signs. Such a representation is independent of how I came to know that there are two. Note that this fact is the one with which Black begins.

9Ibid., p. 157.
In the passages quoted above Black makes several demands. In order to name the two objects, he demands that one must either perceive them or somehow know a unique description of them. Let us examine the first of these claims. One cannot name what one cannot perceive. If this is Black's reason for his restriction on names, then we cannot name the spheres, since by supposition we are not a part of his fanciful world. But surely he would not want to rest the argument on these considerations, since they apply to any object in any universe (symmetrical or not) that one cannot perceive. Suppose that one was in Black's world. Or, alternatively, suppose our world were symmetrical. Could one name the spheres, that is, call one of them "a" and "b." It seems clear that one could. So if one perceived them one could give them names. Or perhaps Black thinks that if one was in his universe, one's presence would destroy its symmetry. And indeed, it would, but only in the sense that if we introduce another thing (be it a perceiver or not) into any universe we establish a whole new set of relations. But this claim is much broader than Black's original restriction. For now Black is merely denying that we can represent by the use of names, any (symmetrical or non-symmetrical) allegedly possible universe that does not contain perceivers and hence namers and names. But now all his claim comes to is the truism that without namers there can be no names.
In satisfying the truism several issues must be kept separate. First. What is the principle condition that must obtain to permit the introduction of a logically proper name into a language which represents or reflects our world? Second. What are some of the conditions that must obtain to permit the introduction of names to represent what someone claims is a "possible" world? Third. What are the ways and methods of introducing names? Fourth. Given that names have been introduced for the objects, does each of the objects have a unique description? It will be argued that only the last question involves the truth of the II.

Consider now each of these questions in turn. First. One condition that must be met for the introduction of a name into a language that represents our world is that there be an object to which it refers. To see this, recall the distinction between interpreted and uninterpreted signs. An interpreted sign (a name) is one that is coordinated with an object. Such coordination cannot occur without the existence of both the sign and the object. Thus a requirement for the introduction of a name is the existence of the object to which it would refer. Notice first that the statement of this requirement does not have, as part of it, the further requirement that the object named have a unique description. This particular
requirement for a sign's being a name thus begs no questions concerning the necessity or even the truth of the II. It merely states the traditional requirement placed upon a sign being a name, i.e., it must refer, or be coordinated to an object. Notice, second, that any such coordination requires a coordinator. This satisfies Black's truism, that is, without namers, there can be no names.

Second. One may claim that it is impossible to introduce names to refer to the objects in Black's universe, for by assumption this universe is merely possible and the spheres Black talks about do not actually exist. Perhaps one may conclude the requirement for introducing names cannot be met given that one only introduces proper names when there are objects to be named. And of course this would be correct. But there is another sense of "name" or, more generally, "language" that one can employ. One can distinguish between at least two uses of language to represent worlds. One such use involves certain objects in the actual world, introducing names for the objects and predicate signs for the properties. In this use of names one attaches signs (names) to objects which exist. Such attaching cannot occur without an attacher. Or, satisfying the truism again, without namers there are no names. Another use of languages is to provide a model of (supposedly) possible worlds that do not exist. Since the
making of models requires a modeler, this use of names again presupposes a namer. But this latter model is a model in a different sense. For one is not, in representing such worlds, coordinating a name with an existent object as above. Rather the use of names, say "a" and "b," must be merely taken to represent the fact that there are said to be two objects in such a universe. These uses are similar in that in both cases the language serves as a model for a world. But they are different in that in the former case we begin with our world and construct a model to represent it. While in the latter case, one begins with the claim that there are two objects and build a model to reflect that claim. Differently, there are two ways one may use language as a model. One may build a model to reflect the world the way it is. Or, one may build a model to reflect certain claims about the way the world is or some world could be. In the former case one begins with the world and constructs (hopefully) an adequate model to represent it. In the latter case one begins with what someone says is or could be the case, and constructs a model to capture what that person says. Names are being employed here in this latter sense, that is, reflecting Black's claim that there are two objects. If one is clear on the difference between these two senses in which language is used as a model, then it is also
clear that as long as we stick to the latter sense of using language (and hence names) there should be no problem resulting from the fact that this supposed world does not exist; we are not coordinating the signs "a" and "b" to two different objects as the table in front of me and the cigarette on it. Rather, the use of "a" and "b" merely serves to reflect what Black says, namely, that there are two things.

Third. Given that one is going to assign a name to every object, there is the further issue of how we go about introducing such names. That is, what are the ways of introducing names into a language? This issue is important because Black may be requiring that the namer know a unique description of the object before introducing a name for it. For often when names are introduced, they are done so by rules such as: let "a" designate (\(\exists x\)Fx, where "F" is some predicate term that happens to pick out one and only one object. If "F" either picks out more than one object or picks out none, then the rule fails to introduce "a" as a name, given the criterion for the introduction of names. Or one may introduce names by some ostensive gesture, either by pointing or saying that "a" designates this, where it is clear from the context what object the gesture or "this" designates. Suppose however, one has a universe, slightly different from Black's,
consisting of two objects such that every property and relation of one is a property or relation of the other, including spatial and temporal position. That is, drop the notion that two things cannot be in the same place at the same time. Obviously one cannot point to one to distinguish it from the other. Neither can one use either the other way of introducing names, since by supposition there are no properties had uniquely by either object. Is one then to claim that one cannot introduce names to represent our supposed universe? To claim that one cannot is a mistake. For although one cannot successfully formulate such rules as: let "a" represent the object which is F and "b" represent the object which is G— nonetheless we can introduce our names as follows: let "a" and "b" represent the two objects which share all properties and relations in common. Here what one is doing is using names to reflect the supposition that there are two things. If one asks: Which one does "a" name?—and is asking for a property that a has that b does not, he cannot, of course, be answered, since by supposition they share all properties and relations in common. In the case of Black's universe, where the objects do differ in spatial position, our introduction of "a" and "b" merely serves to reflect his claim that there are two things.
Fourth, neither the criterion for the introduction of names (that there be an object) nor the ways of introducing names into a language have any bearing on the truth of the II within Black's universe. For the use of "a" and "b" reflects what Black says. Their introduction into a language in no way guarantees that the world they represent is a possible world. One of the ways the question of the truth of the II in Black's world comes up is in the following manner: Given that we have names for all the objects in question, does each of these objects named have a unique description? It is this question which involves the Identity of Indiscernibles. Neither the question about the criteria for names nor the question about their introduction involves the Identity of Indiscernibles. That is, one first represents the world and then asks whether the world one has represented is a possible one. If one is using language to represent the actual world then there is no question whether the world that the language represents is possible or not. If, on the other hand, one is using language to reflect what someone says then it is clear that our language may represent an impossibility. And, supposing that one has introduced names for Black's two spheres, one can say that a has the unique property closer-to-a-than-to-b while b does not.
V. Other Distinguishing Properties

Perhaps Black thinks that the use of names to represent the objects and hence the different relational properties might be question begging. That is, perhaps he assumes that all relational properties involving names are question begging, in the sense that they imply that the II is a "necessary" truth. But to see that this is not the case consider the following three properties:

(1) being-named-by-"a", (2) being-numerically-the-same-as-a, (3) being-closer-to-a-than-to-b. Property (1) is a metalinguistic property and may not be considered a property of things at all. But let us consider it such for the sake of argument. Some would prefer, in place of property (2), the property of being-identical-to-a, since identity is a sign in the object language while being-numerically-the-same is not. But if the II is not necessary, then two things could be identical (in the sense of sharing properties and relations) yet not numerically the same, so we shall use (2). Both relational properties (1) and (2) are question begging in the sense that they provide a unique property for the object a, thus proving the II or at least that aspect of it which implies that every object has a unique property or set of such. Both properties are true of one and only one object, assuming that one is operating within a context in which each thing has
one and only one name, and each name names a different thing. Black wishes to exclude these properties for he may feel that they make the II trivial. I shall return to the charge of "triviality" in the next chapter. The point here is that property (3) is a non-question begging property in the sense that it could be a property of more than one thing even though in Black's universe in fact it is not. The use of two names in representing Black's universe reflects only his claim that there are two things. So there is nothing question begging about the employment of names for spheres and hence for the relations involving them.

Black might feel that the use of a logically proper name relies on a notion of difference which is not a difference in property. For the question at issue is, in effect: is the "=" sign, as defined by Russell, adequate to capture our ordinary notion of sameness. Russell's definition, remember, was "x=y df(f)(fx=fy)." Black is claiming that in his possible universe one sphere is identical to (as above defined) the other yet they are two. Thus Russell's definition, if Black were right, would not be adequate as a transcription of numerical sameness. The philosopher who holds the II as necessary claims that Russell's definition is adequate to capture or reflect the notion of one-and-the-same. Often the
philosopher who seeks to refute the II bases his claim on the idea that there must be a different notion of difference—different, that is, from non-identity. This has been traditionally expressed by "numerical difference." This notion will be more fully explored in Chapter IV. Black might think that the use of proper names presupposes that the notion of numerical difference is different from that of non-identity and thus should not be used by one who seeks to show that all difference in objects amounts to is difference in property. Otherwise the defender of the II would be employing a different notion of difference than non-identity. But if this is Black's argument against the use of names, it is mistaken. Black seeks to show the II is false. To do so, he gives an example of a possible universe. He says two things about this universe: (a) it is symmetrical, and (b) there are two objects. Does his claim that there are two carry with it any claim about what it is to be two or different? If it did, then he would be begging questions. Holding or claiming there are two things, entails, by itself, no theory about what it is to be two, for that is what is at issue. That there are two is where one begins. With this in mind, Black's objection can be reconstructed as: "You want to show that (in my universe) the two different objects differ in a property or relation. The names of
the properties you have given contain the names of things. But then the names are a function of the difference between the two with which we began. You have not explained that difference." The answer to Black is two-fold. First, he may be requiring that one show the two to be different (i.e., two). But this is what he has granted. It is the fact with which one begins. Second, he may be thinking that the use of proper names reflects or entails a philosophical position about what it is to be one thing. I have in my argument against Black been employing two logically proper names to reflect Black's claim with which he begins, namely, that there are two objects. I have shown that they differ in a property by providing a context true of the one and false of the other.

VI. On Showing the II True

It has been argued that Black's restriction on the use of proper names in his symmetrical universe is misguided. That is, I have urged that introducing "a" and "b" to represent the two objects is neither question begging, nor somehow circular. To further show that the use of names is innocent with respect to Black's universe, consider the following argument proposed by Black's opponent and Black's reply:

Let me try to make my point without using names. Each of the spheres will surely differ from the other in being at some distance from that other
one, but at no distance from itself—that is to say, it will bear at least one relation to itself—being at no distance from, or being in the same place as—that it does not bear to the other. And this will serve to distinguish it from the other.

Not at all. Each will have the relational characteristic being at a distance of two miles, say, from the center of a sphere one mile in diameter, etc. And each will have the relational characteristic (if you want to call it that) of being in the same place as itself. The two are alike in this respect as in all others.10

Black, then, claims that each of the objects in question will have all the same characteristics, both relational and non-relational. He is demonstrably mistaken.

Consider the following two propositions:

(A) To show the II true (in Black's possible world), one must construct a predicate pair \((f,-f)\), each of which is true of one of the objects.

(B) To show the II true in Black's possible world, all one need do is show the following to be true: the objects in Black's world differ in a property.

Notice that while (A) entails (B), the converse is not true. For, as will be shown below, one can demonstrate that the objects differ in a property without being forced to specify some predicate pair \((f,-f)\) each of which is true of one of the objects. One may view Black's argument as the following:

1) He asserts (A).
2) He claims that if one cannot name the objects one cannot construct the predicate pair.

10Ibid.
3) He claims that one cannot name the objects.
4) It follows that one cannot construct the predicate pair and hence cannot show the II true.

What this chapter has done so far is to argue that 3) is false. Differently, it has been urged that irrespective of whether one knows which objects the names represent, one can still introduce names. But Black's argument can be shown wrong without employing names.

The idea is simple. What one does is grant Black 3) but deny that (A) is the only method to show the II true. In effect, one relies on (B). The argument's strategy is the following: one describes Black's universe without employing names and derives a contradiction. Consider what Black says.

a) there are two things.
 b) they share all properties and relations in common.
 c) spatial laws in Black's world are the ordinary ones.

Represent these claims as:

a)' (3x)(3y)(x\neq y \cdot (z)(z=y \lor z=x))
b)' (x)(y)(f)(fx=fy)
c)' (x)(y)(x\neq y \implies Rxxy)
    (x)\neg Rxxx

where c)' reads, firstly, that everything is closer to itself than anything else, and, secondly, nothing is closer to itself than itself. The derivation of the contradiction proceeds as follows:
1) \((\exists x)(\exists y)(x \neq y \cdot (z)(z = yv z = x))\)
2) \((x)(y)(f)(fx = fy)\)
3) \((x)(y)(x \neq y \cdot Rxxxy)\)
4) \((x) - Rxxx\)
5) \((\exists y)(x \neq y \cdot (z)(z = yv z = x))\)
6) \(x \neq y \cdot (z)(z = yv z = x)\)
7) \(x \neq y\)
8) \((y)(f)(fx = fy)\)
9) \((f)(fx = fy)\)
10) \(Rxxx = Rxy\)
11) \((Rxxxy \cdot (Rxxx \cdot Rxxx))\)
12) \(Rxxxy \Rightarrow Rxxx\)
13) \(-Rxxx\)
14) \((y)(x \neq y \cdot Rxxxy)\)
15) \(x \neq y \cdot Rxxxy\)
16) \(Rxy\)
17) \(Rxxx\)
18) \(Rxxx \cdot -Rxxx\)
19) \((\exists x)(Rxxx \cdot -Rxxx)\)

Given 19), at least one of a)'', b)'' and c)'' has to be false and one must be true. Since Black insists on a)'' and c)'' while b)'' is the one in question, one concludes it to be false. If this is so, then the following follows:

\((\exists x)(\exists y)(\exists f)(fx \neq fy)\)

That is, the objects differ in some property or other.

But notice the "f" in the above sentence is a variable, so one has not given some pair of predicates (the (A) requirement), rather one has shown the above sentence to be true, that is, one has satisfied (B).

There are several things to notice about this argument. First, the "=" sign needs to be taken as primitive and not defined as Russell does. For if so defined, one gets a contradiction from a)'' and b)'' alone. Second, notice that nothing in the argument makes reference to
Black's universe being symmetrical. The argument works in any universe for any two spatial things which are claimed to share all properties and relations and when spatial law's are ordinary. This is reflected by the fact that in deriving the contradiction one does not rely on the fact that there are only two, i.e., the phrase "\((z)(z=xvz=y)\)" is superfluous to the argument.

In effect, Black reads the problem of the II as follows: Given that there are two objects, can one specify a predicate true of one but false of the other? And given his argument against names, Black argues that one cannot specify such a predicate. All of this both involves Black in irrelevant epistemological issues as well as committing him to the (A) method above of showing the II true. The above argument shows that the two objects in Black's world differ in a property and hence the II is true in Black's universe. Further, the above argument also shows that the specification of some predicate is not the only way to demonstrate the truth of the II.

Perhaps the above way of putting the argument reflects the sense in which names are and the sense in which names are not innocent. If one is merely reflecting, by the use of two different signs, that there are two objects, their use is innocent. If, by the use of names, one supposes one can say which objects are named
"a" and "b" respectively, in the sense of giving a description of them independent of the use of names, one is mistaken and the use of names not so innocent.

VII. The Property Difference

The question Black has raised is the following: Given that there are two objects, is there a context true of one but not of the other, or differently, a property had by one but not the other? It has been argued that in Black's universe there is. But there is also a different yet closely related question. Given that the predicates are different, can one determine that they are different without appealing to the names imbedded in them? Note that this latter question is one about the contexts true of or properties had by objects, not the objects with which we began. Since Black grants us that there are two objects, one shows that there are two different properties, one had by a, and the other by b. If to distinguish the properties in question one needs to appeal to the objects in question being different, one may feel that the defender of the Russell-Leibniz notion of identity has failed. But has he? It all depends on how one views Black's attack. The II can be thought to be inadequate for at least two reasons, one stronger than the other. The strongest reason is a demonstration that it is possible for two things to share all properties and relations in common. This is
what Black attempted and where he failed. A weaker reason might be the following: suppose in some possible world the II is true one must employ a different notion of difference than non-identity. This would show merely that Russell's definition of "=" was not the only sort of sameness, not that the two objects in question failed to have all their properties and relations in common. If the exhibiting of the inadequacy of the Russellian definition rests upon forcing one to use a notion of difference different from non-identity, then Black may believe he has succeeded. Indeed, Black may believe the different notion of difference involved is numerical difference, and forcing one to use this notion amounts to forcing one to give up the Russell-Leibniz notion of identity. But this line of thought is confused. The notions of identity (as defined), and numerical sameness are attempted analyses of the commonsensical notion of "being the same as." If Black thinks the use of names forces one to employ a different notion of identity, in a sense he is correct. But that does not show that one must employ the philosophical notion of numerical difference. Rather, it is the commonsensical notion of difference that one employs in refuting Black. And identity, as defined by Russell, may well be its analysis. The sense in which what is analysed is different from the analysis of it is that one does not think of one when thinking of the other. To demand the notions be the same in this sense
is to give up analysis. I shall return to this topic in Chapter IV.

What has been shown is that Black's attack on the II fails. It fails for a number of reasons. First, merely considering Black's world does not establish that it is possible, and as will be seen in Chapter IV, it is always open to the defender of the II to claim that there must be a property difference, or that there are not two things. Second, supposing that there are two things and employing proper names, it has been shown that there are properties in Black's universe which distinguish the objects. Third, it has been argued that the proper conclusion to be drawn about Black's restrictions on naming is not necessarily that the II is false, but that, perhaps one cannot even represent the universe he asks one to imagine. Fourth, if one views naming as involving knowing which objects one is attaching a name to, then one will overlook the possibility of using names as a model to reflect what Black says. Fifth, it was argued that the use of names is not question begging. The use of names in this chapter is not question begging in that it does not imply any philosophical position about what it is to be one thing. Indeed, an argument against Black was presented that did not involve the use of names. Black's argument against the II has been shown to have failed. But the possibility has been left
open that one may argue that the use of proper names involves some other notion of difference than non-identity. What is important to note in this context, is that if this is so, Black himself must use a different notion of difference other than non-identity as defined by Russell to get his example off the ground. That is, he does not begin his example as: "consider an object which is F and another object which is not-F," but rather he begins with "consider **two** objects which are F."
CHAPTER IV

THE NECESSITY OF THE II

In the last chapter, we saw Black's attempt to show the possible falsity of the identity of indiscernibles fails. That is, we saw Black's attempt to establish the following does not succeed:

It is possible for two things to have all properties and relations in common.

What was concluded was, even if one grants Black his views about names, that while one could not construct a predicate which one object had while the other did not, nonetheless, given Black's views about space, the two objects had to differ in a relational property. So the II is true in Black's universe. But by showing Black's attack to be a failure, one does not thereby necessarily secure the II. One must defend it against other attacks. In this chapter I should like to do just that, namely, to present a further set of considerations that have been advanced in the attempt to answer the question over the adequacy of Russell's definition of "=," or alternatively, the necessity of the identity of indiscernibles. These are the sort of considerations that have led some philosophers to draw a distinction between numerical and qualitative difference.
Two things are qualitatively different if, and only if, one has a quality or relation that the other lacks. But two things can be numerically different even if they are qualitatively the same; that is, even if they have all properties and relations in common. The notion of difference typically associated with Russell's definition of sameness is that of qualitative difference. In passing, one should note that the notion of qualitative difference above expressed relates to individuals which differ in a quality (or relational property). It is not the notion that two qualities are themselves different, but rather refers to the individuals which have the qualities.

In this chapter, what I shall do is consider some distinctions and arguments that must be made in any discussion of the II or the adequacy of Russellian definition. To begin with, there will be a discussion of some of the demands put upon the defender of the II. These demands are that the II be proven and that it be an interesting principle as well as a necessary one. I shall argue that these demands are illegitimate. Second. Often the opponent of the II thinks that his job is to establish that there is more than one sense of difference. Crucial to this chapter will be the several senses in which one can claim to have shown there to be different senses of difference. Third. Turning to an argument which claims to establish that it
is possible for there to be two things such that they share all properties and relations in common, I shall argue that the argument either assumes what it intends to establish or is simply invalid. Fourth. Often Russell's definition is claimed to be inadequate because properties, being sharable, cannot individuate. I shall discuss several senses of individuation, and how these are related to the II. Fifth. One traditional reason for holding there to be numerical difference in addition to qualitative difference has been an adoption of a metaphysics of substance and attribute, where the function of the substance is to account for the possibility of two things not differing in a quality or relation. What I shall discuss in this section are the a priori features of substances (pure individuators, substrata, bare particulars).

I. Unreasonable Demands

A three-fold demand is often put on the defender of the II. Often the opponent of the II demands that (1) the II be proven, (2) the II be necessary, and (3) the II be interesting. In this section I shall argue, first, that (1) is unreasonable, and, second, that the conjunction of (2) and (3) places a much too stringent demand upon the defender of the II.

First, should the II have to be proven? The defender of the II is in a position something like the
following: If he merely adopts Russell's definition of 
"=", holding it to be the only notion of sameness, then
he will be charged with question begging. For he has
failed to argue for the definition's adequacy. If he
leaves the "=" sign undefined, or primitive, then the II
is not necessary. Perhaps one should insist that defending
the II is not proving it in some deductive fashion—rather
it is arguing indirectly that Russell's definition is
adequate, namely, by showing all alternate positions to
be either mistaken or subject to the same supposed dif-
ficulties that the II is. For clearly there is a differ-
ence between proving a statement in the sense of giving a
proof that it is true, or perhaps, necessarily true; and
in the sense of giving an argument that the definition is
adequate. If the former sort of proof is demanded, then
if it is reasonable to ask the defender of the II for
proof, is it not just as reasonable to ask the ontologist
who holds substrata to show that no one substance can be
in two ordinary objects? For it is within the context of
such an ontology that the II is held to be possibly false.
This question will be explored in the last section.

Second, some have demanded that the II be both an
interesting as well as a necessary truth. For example,
Black argues against his opponent using such "properties"
as being-different-from-a in order to show that the II is
necessary. His argument amounts to objecting that if such properties are used then the II is trivially true. But what "trivially true" amounts to in the context of Black's argument is "necessary." Black says to his opponent:

\[ \ldots \text{you are merely redescribing the hypothesis that } a \text{ and } b \text{ are different by calling it a case of "difference of properties". \ldots your famous principle reduces to the truism that different things are different. How true! And how uninteresting!} \]

For Black to call such necessary truths "trivial" may amount to a restatement of the idea that necessary truths tell us nothing about the world. But the very question at issue is whether the II is or is not a necessary truth. For Black the II is not necessary since he believes his symmetrical universe refutes it. Black also wants the II to be "interesting." But suppose one means by an "interesting" principle, one which says something about the world. Suppose one holds, further, that no necessary truths tell us anything about the world. If one also holds that the II should be both "interesting" and "necessary" then the demands amount to asking the defender of the II to hold it to be both necessary and not necessary. If there is more than one sense of "necessity" available, then the demand may not be too stringent, for it may

\[ ^{1}\text{Max Black, "The Identity of Indiscernibles," Mind, LXI, No. 242 (April, 1952), 155.} \]
amount to a demand that the II be necessary in one sense yet not necessary in another. However, if by necessary one means only "logically necessary," then the demand is impossible to meet.

II. Different Senses of Difference

In this section I shall distinguish two issues. One issue has to do with whether besides the notion of qualitative difference, there is another notion of difference which is legitimate; that of numerical difference. The other issue has to do with whether the Russellian definition of qualitative difference is an adequate one. And notice that there are two ways in which it could be inadequate. Russell's definition may fail to capture the notion of qualitative difference. But, in addition, if there is also a legitimate notion of numerical difference, Russell's definition cannot be taken as a definition of difference as such, but must be supplemented by another notion of difference. One may put the above points in the following way: are there arguments which force one to adopt two sorts of difference, namely, qualitative and numerical? For if there are no cogent arguments forcing one to adopt a sort of difference different from that expressed by the definition of "=," then Russell's definition will suffice. But notice that this does not imply
that if there are arguments forcing one to adopt two sorts of difference, then Russell's definition does not suffice. So the second issue is the following: suppose one had located reasons for introducing more than one kind of difference, will this render the Russellian definition inadequate? Of course, if one insists that the criterion for adequacy of the Russellian definition be that it be the only notion of difference or sameness, then if the answer to the first question is yes, one need not raise the second question. But, as I hope to show both in this and the last section of this chapter, such insistence is unreasonable. In effect, this amounts to arguing that there are some senses in which employing two notions of difference does not by itself render Russell's definition inadequate.

If a philosopher claims that there are two notions of difference (or two kinds of difference), one must inquire into exactly what sense of "two notions" the philosopher has in mind. For there are at least two distinct senses of having two notions of difference. One is the sense in which we begin with some ordinary notion and subject it to analysis. The ordinary notion is what one may call the pre-analytic notion. The post-analytic notion will be different if only in the sense that in thinking of the ordinary notion one is not necessarily thinking about its
analysis (or conversely). One might conclude that since these two notions (i.e., the pre- and post-analytic notions) are different, the II is possibly false. For the II is the following: for any two things, they differ in a quality. If one reads the antecedent of this as the pre-analytic notion, and the consequent as the post-analytic notion, then of course there will be two notions of difference. This sort of duality, however, will be found in any analysis.

The other sense of a different sense of difference is the one in which one's analysis forces one to have two differing notions of difference as the end result of the analysis of the ordinary notion or notions. These two notions of differing notions of difference must be kept distinct. For if they are confused, then of course Russell's definition would seem inadequate—for here the adequacy of an analysis would entail having one and the same notion in both what is being analyzed as well as its analysis. Since this is not the case with Russell's definition, or, for that matter, with any analysis, the definition would seem inadequate. But any analysis is inadequate in this sense. The point I am making is that an analysis cannot be criticized for being an analysis. This is sometimes called the paradox of analysis. The point here is that the criticism is of the business of giving analyses
in general, as opposed to a criticism of the particular analysis in question, namely, the Russellian notion of sameness.

What the opponent of the II must show is, first, that there are two sorts of difference (as the end result of analysis), and, second, that this entails that the II is possibly false, or, what amounts to the same thing, that Russell's definition is inadequate.

III. Arguments Attempting to Establish the Non-Necessity of the II

It might be thought that one needs a distinct notion of numerical difference to account for the possibility of there being two objects sharing all properties and relations. It seems that one would. However, this is to assume that there is such a possibility. But the question of whether or not there is such a possibility is the very question at issue. The opponent of the II might retort that the question at issue is not whether there is such a possibility but rather the claim that there is not such a possibility. But the two cases are clearly different. For the proponent of the II can provide examples of situations where two objects are qualitatively different. The opponent of the II must argue (not provide an example) that it is possible for two objects to share all properties.

There have been, however, a number of arguments designed to show that this possibility exists. I shall
discuss several of these at some length. These are arguments to establish:

It is possible for there to be two things which share all properties and relations in common.

One argument for the above proceeds as follows:

1) Consider any two objects, a and b.
2) Consider any property f, suppose it to be the only property.
3) Both \( f(a) \) and \( f(b) \) are possible; so too is their conjunction, "\( f(a) \cdot f(b) \)" and, since f is the only property, so too is "\( a=b \)."
4) It follows that it is possible for two objects to share all properties and relations.

The conclusion announces that an object may be different from another yet they may be identical in the sense that they share all properties and relations.

But the defender of the II will object. He will ask: exactly what is the notion of two involved in premise 1)? Is it the notion of numerical difference, or the notion of differing in a property (i.e., non-identity), or the ordinary (i.e., pre-analytic) notion of two? If an analysis has been given to the notion, then there are two alternative analyses.

First, suppose one interprets the notion of two as "numerically different." Then the first premise reads:

Consider two numerically different things, a and b. The notion of numerical difference here is such that if a is numerically different from b, then it follows that a and b could share all properties and relations in common. Hence, the conclusion follows from premise 1) and the notion of numerical difference alone. But, interpreted in this manner, the argument establishes that there is numerical difference and hence the objects could have all properties and relations in common merely by assuming it.

Second. Suppose one interprets the notion of two in a different way. That is, one interprets the notion of two to be such that a and b differ in at least one property. Looked at in this manner the argument is invalid. To see how this is so, suppose G to be the only property. Then a and b will differ in G, or, more explicitly, the true sentences will be "Ga" and "Gb." One might then claim that while -Gb, nonetheless it is possible that Gb (i.e., "Gb" is not a contradiction). Since a is G (and hence could be G) and b could be G one may conclude that it is possible for a and b to be two yet share all properties (namely, G) in common. It is this last move which is invalid. In effect, one argues from "p" or "possibly p" and "possibly q" to "it is possible that p and q." The invalidity becomes strikingly apparent if one considers the actual structure of the argument:
1) \( a \neq b \), i.e., \( a \) and \( b \) are two.

2) a) \( Ga \rightarrow Gb \)
   b) \( \Diamond Gb \), (i.e., "\( Gb \)" is not a contradiction)

3) \( \Diamond (Ga \land Gb) \), (i.e., "\( Ga \cdot Gb \)" is not a contradiction) 
   and since \( G \) is the only property, \( \Diamond (a \equiv b) \)

4) \( \Diamond (a \neq b \land a \equiv b) \)

Both the move from 1) and 2) to 3) and the move from 1), 2) and 3) to 4) are invalid. Both are invalid because of an expansion of the scope of possibility. One may argue, however, that even though the step to 3) is invalid, nonetheless 3) is true, i.e., "\( Ga \cdot Gb \)" is not a contradiction. Even granting this, the crucial "possibility" to argue for is 4) and not 3). And since 4) involves a straightforward contradiction, it is false that it is possible. Thus, reading the notion of "two" as "differing in a property" (non-identical in the Russellian sense), the conclusion of the argument amounts to: it is possible that \( a \) and \( b \) both differ and not differ in a property.

What does follow from 1) and 3) is that if \( a \) differs in a property from \( b \) it is possible that they do not, that is,

\[ a \neq b \Rightarrow \Diamond (a \equiv b) \]

What does not follow is:

\[ \Diamond (a \neq b \land a \equiv b) \]

In the conclusion that follows, the scope of possibility is the narrower one. So, while one can conclude that both
a and b are two (differ in a property) and could be one (fail to differ in a property), one cannot conclude that a and b could be both two (differ in some property), as well as one (not differ in a property). What the above shows is that within the context of the argument is that if the notion of two objects is taken to be that of numerical difference then one is begging the very question at issue. On the other hand, if the notion of two objects is taken to involve Russell's definition of "=," then the conclusion just does not follow.

Notice that the notion of possible that has been employed is the notion of not being a contradiction. There is, however, another crucial notion of possibility that is important in this context, which notion must not be confused with that of not being a contradiction. This different notion of possibility is related to the status of interpretation rules for names in a symbolic language built with a stipulation that different names will designate different things. That is, there will be a one-one correlation between names and things. But notice that such a rule has two parts. One part is that no one name will designate two things. This is essential in order to avoid equivocation as was shown in Chapter II. The other part is that no two names will designate one thing. Though perhaps useful, this part of the rule is not
essential. So another notion of possible relevant to this discussion is the following: when it is claimed that it is possible that a is identical with b, one may simply intend that "a" and "b" could stand for the same individual.

The above distinction is crucial. For notice that one may agree that if the notion of two in the above argument is numerical difference it is question begging, and if the notion is non-identity the argument is invalid. One may still argue that the notion of two is the ordinary (pre-analytic) notion, and that the fact that \(fa \cdot fb\) is not a contradiction establishes that it is possible for there to be two things with all their properties in common (where \(f\) is the only property). For "a" and "b" are tokens of different types. Given the rule one name-one thing it would seem possible that \(a\) and \(b\) be two and yet identical. Thus it seems the opponent of the II can get the desired conclusion.

But the defender of the II will insist that the conclusion does not follow. For the opponent of the II has to rely on the other notion of possible as well. The proponent of the II will claim that the rule one name-one thing is an interpretation procedure for the actual world. To insist that it holds for all possible worlds is, in effect, to insist that there is a possibility of two things having all properties and relations in common. But this
is the very question at issue. Furthermore, to hold that the interpretation rule holds for all possible worlds is another way of saying that the rule is somehow necessary. But surely this is false, for "a" and "b" could stand for the same item. And as mentioned above, this is surely an alternate way to read "it is possible that a=b," namely, "'a' and 'b' could stand for the same thing." Differently put, one cannot build by stipulation into a rule that there are such possibilities because that is not a matter of stipulation.

To put the above point in a different fashion, consider the following two possible worlds:

1) \( Fa \land Fb \)
2) \( Fa \land \neg Fb \)

(The notion of possible world here is merely a shorthand way of speaking of the well-formed combinations of letters that are used in describing the actual world.) Suppose \( F \) to be the only property. In possible world 2) are there one or two individuals? The answer here is clear. There must be two, for one the assumption that there is only one individual, one reaches a contradiction. In possible world 1) are there one or two things? This question is crucial, for if there are two, then it is possible for there to be two things, namely \( a \) and \( b \), with all properties in common, namely \( F \). One may reply that there is only one thing that
could be named by "a" and "b." Such a reply amounts to a
decision to take any difference of individuals to entail
difference of properties. Since there is no property dif­
ference a and b are the same individual. In this reply,
the II is not argued for, but assumed. Or, alternatively,
one may reply that, since there are two tokens of differ­
ent types in 1), namely "a" and "b," there must be two
things. Such a reply amounts to a decision that it is not
necessary for any two things to share all properties and
relations in common. Rather, its negation (for some pos­

sible world) is assumed.

Another way of stating the above is the following:
Notice that the argument from the possibility of "Fa·Fb"
(where again F is the only property) to the possibility
that there be two objects, a and b, which share all prop­
eries in common (namely, F) also either depends upon an
enlargement of the scope of possibility, or it is not
sound. To see this, consider the following two arguments:

A₁
1) it is possible that "a" and "b" designate
different objects.
2) "Fa·Fb" is possible
3) it is possible that a be different from b
yet have all the same properties.

A₂
1) it is necessary that "a" and "b" designate
different objects.
2) "Fa·Fb" is possible
3) it is possible that a be different from b
yet have all the same properties

With respect to premise 1) and 2) of arguments A₁ and A₂
above, the defender of the II might insist that these
notions of possibility are different, and that, since they are different, the respective conclusions do not follow. But even on the supposition that the notions of possibility are the same in all cases, in $A_1$ the conclusion does not follow, while in $A_2$ the argument is either not sound or question begging. To see how $A_1$ is invalid, let "q" be "'a' and 'b' designate different objects," let "p" be "'Fa•Fb' is possible." In the conclusion of $A_1$, the scope of possible is illegitimately enlarged. One moves invalidly from "possibly q" and "possibly p" to "possibly p•q." With respect to $A_2$ above, again on the assumption that the notion of possible is the same throughout, the argument is not invalid, but the defender of the II may insist that premise 1) is false. It may be argued that if 1) is made a rule then it is necessary. There is a reply to this response. The defender of the II will point out that it is one thing to begin with several commonsensical notions and argue for the possibility of two things having all properties in common—but it is another thing to stipulate that there are such possibilities. But that is now what is being done. To conclude, $A_1$ is invalid while $A_2$ is either not sound or question begging.

Notice that the above arguments do not establish (prove) that it is not possible for there to be two things with all properties and relation in common. Rather what
it shows is that the arguments given to show that it is possible do not work. For the question remains: is it possible for there to be two things with all properties and relations in common? But it remains only a question, not something that has been shown.

IV. Differing Senses of Individuation

Consider an argument for numerical difference that runs as follows:

1) all properties (attributes, characteristics, universals, predicates) are sharable.

2) So no one property is necessarily had by only one thing.

3) For something to individuate an object, it must be necessarily unique to that object in the sense that it could not be a constituent of another object.

4) No property is necessarily unique to any object.

5) It follows that no property individuates, from the "sharability" of properties and the notion of what it is to individuate.

6) So every object needs some constituent other than a property which will be necessarily unique to it. Substrata are entities which fulfill exactly this role. "Two" substrata are numerically different from each other in the sense that they could have all properties and relations in common.

In this section I should like to argue that premise 3) in the above argument is much too strong a condition for individuation. In the section following I shall argue further that even if one adopts this requirement that
individuators must be necessarily unique, one is still not forced to give up the II. This will amount to an attack on premise 1), namely, the notion that all properties are sharable.

In this section three what I should like to argue is that there is a sense of individuate different from 3) above, which sense is crucial to any discussion of the II. For the demand that an entity's being necessarily unique to an object in order to individuate is too strong. To see that 3) is not the only notion of what it is to individuate, consider the following two notions of individuation.

(1) two objects are individuated if they each have constituents which are necessarily unique to them.

(2) two objects are individuated if their respective constituents necessitate that there is more than one object.

These two conditions are distinct. For notice that while one might hold that no one predicate is necessarily had by only one object, it is not the case that no pair of predicates necessitate there being more than one object. For example, the exemplification of the predicate pair \((f, -f)\) necessitate there being more than one object.

(condition (1)) as long as one grants premise 1). Notice that if two objects contain entities which are necessarily unique to them, then it follows that they contain entities which necessitate there being more than one. The converse of this is not true, however. As will be seen in the next chapter which discusses the difference between classes and attributes, it is relatively easy to confuse these senses of individuation and shift from (2) to (1)—even though (1) is not entailed by (2).

Notice that condition (1) is identical with premise 3) in the above argument. If one agrees that properties are sharable (premise 1)) and one holds to condition (1), then it follows that to individuate one requires the introduction of some entity which is both not a property and necessarily unique to an object. If, on the other hand, one agrees with premise 1) but holds the sense of individuation expressed in condition (2) above, then one is not forced to introduce individuators.

The opponent of the II may claim that "all you have shown is that some pairs of objects may have constituents (namely, the (f,-f) pair) which necessitate that there be two objects. The II makes a much stronger claim: it holds that for every pair of objects there is a property difference. You have not shown that to be the case." The reply is that the above argument was merely to show
that it does not follow from the sharability of properties alone that one needs substrata. One needs further premises. Further, as mentioned above, the demand that one prove the II is unreasonable—but that is what the opponent of the II is now demanding.

How then, does one decide whether the II is necessary or not? It looks as if the opponent of the II decides the issue by adopting both premise 1) and condition (1) or, what is the same, premise 3), and is thus forced to introduce some special sort of constituent to do the job of individuation. On the other hand, the proponent of the II may adopt premise 1) but reject condition (1), that is, premise 3). He will then hold one of two positions. He may claim that unless it is successfully argued that there is a possibility of two objects having all properties and relations in common, he will not be forced to introduce an individuator to solve the problem of individuation. Or, he may claim that the problem of individuation need not be solved across all possible worlds. One way of arguing that the problem need not be solved across all possible worlds is by arguing that even the introduction of substrata rests on certain "necessary" truths which are not themselves logically necessary. To that we now turn.
What was shown in Section IV was that if one holds to a sense of individuate other than that expressed by premise 3), one is not forced to introduce substrata as individuators. For one is forced to introduce substrata, hence denying the necessity of the II, only if one accepts both premise 1) and premise 3). So the defender of the II has, in effect, two ways to argue that the argument is not sound. The first of these ways was explored in the last section, namely, an argument to the effect that one need not endorse premise 3). What I shall do in this section is discuss several motives for claiming that for an entity to individuate an object it must be necessarily unique to that object. It will be shown that they all involve a misunderstanding of the II. I hope to show, further, that even granting the need to find an entity which is necessarily unique to an object, one is not forced to accept substrata.

The II states that for any two objects there is a property difference. There is an ambiguity in the notion of property difference that needs to be explored—for it is crucial to any discussion of the II. Consider two objects a and b, where a is red and b is green. Red and green are different properties. In this sense, there is a property difference between a and b, since a is red and
b is green. But important to note is the simple fact that while a is red, b is not red. This is an entirely distinct sense of property difference. It is this latter sense of property difference which is intended by the II. Stated explicitly, the II claims that for any two objects, one has a property the other lacks.

Related to the above distinction is the following alleged problem concerning Russell's definition. Russell's definition holds that difference of individuals is a function of property difference. But difference of property is, in part, a function of difference of individuals. That is, suppose a is f and b is g. Then, given that f and g are different properties, a and b will be different. But the sameness or difference of f and g depends upon difference of their extensions, namely, a and b. Thus one seems to be caught in a circle. The circle is broken if one realizes the ambiguity of the notion of property difference. For notice I claimed above that a is f and b is g. This is one sort of property difference. If I had claimed that a is f and b is not f then the alleged problem disappears. For the question as to whether the extensions of the predicates "f" and "¬f" are or could be the same or different does not arise. It is this second sort of property difference that is relevant to the II.

Recall the previous chapter where Black argues that his possible symmetrical universe refutes the II.
His chief argument was against the use of names to name the objects in question. Perhaps one motive for his arguing against the use of names relates to the above distinction. For consider the following properties: being-closer-to-a-than-to-b and being-closer-to-b-than-to-a. One may view Black as asking how one knows these properties to be different. One reply is that, given that a is different from b, the relational properties in question will be different. The defender of the II should point out, however, that if Black is worried about this property difference, he is not talking about the II. For the proper predicate pair relevant to the II is not the above but rather "... being-closer-to-a-than-to-b" and "... not-being-closer-to-a-than-to-b." The question then becomes: are each of these predicates true of at least one object?

Another motive for accepting that an entity be unique to an object in order to distinguish one thing from another is the following: Consider three objects, a, b, and c. Suppose a to be f and g, b to be g and h, and c to be f and h. Consider the first two, a and b. The property f distinguishes a from b since a is f while b is not f. However, f does not distinguish a from c, since both a is f and c is f. One may consider this a failure of the II, since there is no one (non-complex) property which
distinguishes a from b and c, or, alternatively, b from c and a, and c from a and b. This is a mistaken reading of the II which has serious consequences. Consider the following:

1) The II entails that there is one (non-complex) property that distinguishes each object from every other object.

2) It is not the case that there is always one (non-complex) property which distinguishes each object from every other object.

3) It follows that the II is false.

This argument is valid but premise 1) is false. To see how this is so, recall the symbolization of the II:

\[(x)(y)(\exists f)(x \neq y \rightarrow (fx \neq fy))\]
or, its equivalent

\[(x)(y)(x \neq y \rightarrow (\exists f)(fx \neq fy))\]

where the values of "f" are properties. Premise 1), however, represents the II as stating:

\[(x)(\exists f)(y)(x \neq y \rightarrow (fx \neq fy))\]

This involves a "quantifier shift," and while the latter version entails the former, the entailment does not go the other way. In the case of a alone, the two versions read:

\[(y)(a \neq y \rightarrow (\exists f)(fa \neq fy))\] and

\[(\exists f)(y)(a \neq y \rightarrow (fa \neq fy))\]

If one interprets the II in the latter fashion, one may begin looking for one (non-complex) entity to distinguish a from every other item. Clearly, (non-complex) properties
will not do the job as long as one holds that properties are sharable. This may lead one to adopt some special entity which serves to distinguish each object from every other object. One might adopt substances (substrata, bare particulars) to do this job. But the defender of the II would insist that the II takes things pair by pair—and the question in all cases is: is there a property one has that the other lacks? There is no requirement that the property that distinguishes, say, $a$ from $b$, also be the same property as that which distinguishes $a$ from $c$. Notice that if one embraces complex properties or predicates, then, assuming that the II is true, there will always be one (complex) property distinguishing any one thing from everything else. Consider the above example again: $a$ is both $f$ and $g$. This fact distinguishes $a$ from $b$ (which is not $f$) as well as from $c$ (which is not $g$). Thus the "quantifier shift" is innocent as long as one is speaking of such complex properties. But the opponent of the II might have an objection here as well. He might claim that to individuate one requires one simple entity, i.e., complex properties would not suffice to do the job.

Another reason that may lead one to conclude that the II is not necessary is a confusion between the following questions. First. Is the II true? Second. Is the II a necessary truth? These questions can become confused
such that one way misread the II and demand entities which are necessarily unique in order to individuate. Consider \( a \) and \( b \), where \( a \) is \( f \) and \( g \), while \( b \) is \( f \). Suppose a philosopher makes a list or forms a set of these properties, namely, \((f,g)\). The following question arises: can one determine from the list or set alone whether there are one or two items in the original situation? Since one cannot, one may conclude that the II is possibly false.

But the defender of the II will object. To begin with, he will claim that making one list for the entire situation is misreading the II. Rather, one should construct two lists, one for \( a \) and one for \( b \). The list or set for \( a \) will be \((f,g)\), while that for \( b \) will be \((f)\). In this case it is clear that one of the items, namely \( a \), has a property, namely \( g \), which \( b \) lacks. So with respect to \( a \) and \( b \) at least, the II is true. It is a further question whether \( b \) could have been \( g \), and thus the sets for both \( a \) and \( b \) would be the same. Indeed, this sort of possibility has already been explored in the previous two sections.

The point here is that the construction of one list for the entire situation is a misreading of the II. For the II asks: given that there are two objects, is there a property that one has which the other does not? So in considering the II, one has to speak of two lists or sets, one per object. The II does not ask: given one
set of properties, can one determine whether the original situation involved one or two objects? For if one is asking this question, one is not asking how to distinguish one object from another, but rather one is asking how to distinguish one situation, namely, a situation where there are two things, from another, namely, a situation where there is only one. But this is to fuse the question of whether the II is true with whether the II is necessary. For there cannot be both two as well as one thing. So at least one of the situations is only possible. In effect, as long as one sticks with only one list or set, one is asking a different sort of question, namely, that an actual situation (where there are two things) be individuated from a possible one (where there is only one thing).

One may think that if one claims that there "exist" negative properties, or forms his lists or sets out of positive and negative predicates true of the objects in question, the above remarks no longer apply. For the list or set for a and b will now be (f, g, -g). Now one can determine that there is more than one thing, given that all the items in the list are exemplified and that no one thing can both have and not have a property. But even granting this, namely, that one can determine whether there are one or two things involved, is still to misconstrue the II. To see how this is so, consider the following three sentences:
(1) \((x)(y)(x\neq y \rightarrow (\exists f)(fx \neq fy))\)

(2) \((x)(y)(\exists f)(fx \neq fy) \rightarrow x \neq y\)

(3) \((x)(y)(x \neq y \equiv (\exists f)(fx \neq fy))\)

The last entails both (1) and (2). (1) is the symbolization of the II. (2) is the symbolization of the Principle of Identity (PI). The business of "determining" from one set of properties or predicates (including the negative ones) whether there are one or two things clearly corresponds to PI but not to II.

The points made above about the ambiguity of the notion of property difference and the formation of one list relate closely to some earlier points made about the rule one name-one thing. Notice that while a philosopher may hold that it is not necessary that \("(x)-(Pedx=Greenx),"\) he may hold that the rule one name-one thing is somehow necessary. But consider the following predicate pair: (being-designated-by-"a", being-designated-by-"b"). Is this like the (Red,Green) pair or the (Red,not-Red) pair? It is clearly not the latter, since that would involve the pair (being-designated-by-"a", not being-designated-by-"a"). So the ambiguity inherent in the notion of a property difference is also present in having a different name. One might be referring to a difference in names, e.g., "a" is different from "b," or one might be referring to something having a name that something else does not.
Notice that this calls the alleged necessity of the one name-one thing rule into question for the same sort of reason the philosophers who have held it called the alleged non-necessity of "(x)-(Redx=Greenx)" into question. That is, the following sentence might be held not to be necessary: (x)-(x is designated by "a"*x is designated by "b"). One may argue that the sentence is necessary because it reflects a rule which is a stipulation. But if one can do this with the metalinguistic predicates, that is, claim that being-designated-by-"a" is incompatible with being-designated-by-"b" by stipulation, I see no reason one cannot perform the same sort of feat with the properties Red and Green.

The above arguments given all point out various misreadings of the II. These misreadings explain why one might want an item to be necessarily unique to each object in order to individuate it. To begin with, if one confuses the two notions of property difference one may be led to believe that Russell's definition (in combination with a definition of "=" for properties which involves their extensions) is circular and hence one requires a different sense of difference. Second, the belief that one item should distinguish any one thing from every other thing (i.e., the quantifier shift) may lead one to posit entities which are necessarily unique to objects. Third,
From the fact that one cannot determine from one list or set of (positive) properties whether there are one or two things in the original situation may lead one to introduce special entities which will enable one to determine how many things there are. If these are the motives for introducing substrata, they involve a misreading of the II. Thus the defender of the II does not need entities which are necessarily unique to an object, at least not for the above reasons. What I shall argue in the remainder of this section is that even if the demand that an item be necessarily unique to an object be accepted, one is still not forced to adopt substrata.

Assume a miniature universe of two white squares, a and b, one to the left of the other. Let "W" stand for the property white, "S" for the property square, and "L" for the relation being-to-the-left-of. The atomic sentences describing this universe are: Wa, Sa, Wb, Sb, Lab. In the context of two different ontologies, I shall show that, even granting that one needs an entity which is necessarily unique to an object, one is still not forced to introduce substrata to individuate. One ontology (A) grounds the difference between a and b in a constituent which is a quality or relation. The other ontology (B) grounds the difference of a and b in a constituent of the objects in question which is not a property (quality,
attribute, universal) but rather is a substance (substratum, bare particular). One supposed difference between these two ontologies is that from one list of constituents of a and b an ontologist with a (B) ontology can determine whether there is more than one thing, while an (A) ontologist cannot. To see how this is so, consider the following two lists of constituents, one for each of the respective ontologies:

\[
\begin{array}{cc}
A & B \\
\text{W} & \text{W} \\
S & S \\
L & L \\
a_1 & b_1
\end{array}
\]

where \(a_1\) and \(b_1\) are the individuators in a and b respectively. From list (B) one can then determine that there are two things. Can one determine that there are two things from list (A)? From list (A) one knows that something is white, that something is square, and that something (or other) is to the left of something (or other). Obviously it is the relation which is crucial. Equally obviously, if we assume that "(x)-Lxx," that is, that nothing can be to the left of itself, and Russell's definition of ",", we can deduce that there are two things. But an objector will say that it is possible for something to be to the left of itself, that is, "(x)-Lxx" is not a logical truth, so one cannot conclude from the set or list \((W, S, L)\) that there are two things. An ontologist
of the (A) variety might take another tack and insist that it is not "L" that belongs on the list or in the set, but rather \( L_b \), where \textit{being-to-the-left-of-b} is considered a property of an object, i.e., it is counted as one of the values of the variable "f" in the Russellian definition. But this different set is open to the same objection as was the original one—for it is possible (i.e., not a logical contradiction) that both objects exemplify \( L_b \).

All of these arguments involve the misrepresentation of the II discussed earlier in this section.

There are, however, reasons to suppose that in order to determine how many things there are one need not introduce substrata. Remember that in our miniature universe there are two objects—that is a fact—an ordinary fact—from which we begin analysis. There are (at least) three ways to ground, i.e., three analyses we can give to there being two objects. One may claim that the two objects are non-identical in the Russellian sense. Or may say that there is a special relation—\textit{numerical difference}—which holds between the objects. Or one may claim that each ordinary object would be said to contain or have as a constituent one special object, namely, an individuator. These latter two alternatives both involve the concept of numerical difference. The difference between them is that the latter uses entities which themselves are numerically
different, while the former uses the relation of numerical difference which holds between the objects. Consider then, the following two lists:

\[
\begin{array}{ll}
\text{A'} & \text{B} \\
W & W \\
S & S \\
L & L \\
D & a_1 \\
& b_1
\end{array}
\]

where again \(a_1\) and \(b_1\) are the individuators in \(a\) and \(b\). What is important to point out is that now one can determine from list A' that there is more than one object. The grounds for such a determination is the presence on the list of "D." One may reply that "D" is no more useful than was "L"—but I shall argue that this is not the case. Consider again the B list. How does one determine that there are two things? Because, the B ontologist holds, individuators are the sort of thing that cannot be constituents of two ordinary objects. That is just the kind of thing they are. Important to note is that, without stating what sort of items individuators are, one cannot determine from list B that there are two ordinary things. The A' list allows one to determine that there is more than one ordinary object in an exactly similar fashion—namely, D is not exemplified if there is only one object; that is, D is the sort or kind of relation that requires there be more than one thing for its exemplification.
Differently, "(x)-Dxx" is involved in the very notion of object, just as in the other ontology, every object required that it contain or have as a constituent a different individuator. In the one case you have talk of a sort or entity—in the other case one speaks of some necessary a priori truth, namely, "(x)-Dxx." Notice that the predicate "not-being-different-from-a" is true of only a. In effect it is a predicate which satisfies the stronger sort of individuation, i.e., it is necessarily unique to a. Notice that if one has such predicates, one is, in effect, denying premise 1) in Section IV's argument. For such predicates are no more sharable than are bare particulars.

One may claim that, even if in some sense "(x)-Dxx" is some a priori truth, so too is the claim that individuators are not sharable. From this aspect again, there seems no basis for decision between A' and B. But from another point of view, A' is preferable to B. For while both not-being-different-from-a as well as a₁ are both necessarily unique to a, the former and not the latter is an instance of the "f" variable in the Russelian definition of "." Thus in each case we have two kinds of difference. In ontology A', one has the relation D and non-identity; in ontology B, one has substrata and non-identity, but the one on the property level, i.e.,
not-being-different-from-a renders the II "necessarily" true, while the one which involves individuators does not. Thus having two senses of difference need not render the II possibly false. That is, \( xDy>(\exists f)(fx\cdot fy) \) might be necessarily true given the sort of entity that D is. By "necessary" here one means not logically necessary but some sort of a priori necessity--the same sort that the opponents of the II use when they claim individuators are not sharable. Thus, even granting that to individuate an entity must be necessarily unique to an object, the adequacy of the Russellian definition may be thought to rest upon a priori grounds, namely, that everything is the same as, or perhaps, nothing is different from, itself while nothing is the same as anything else. But the arguments considered in this section against the II also rest on the same sort of grounds. Differently, the very notion of an entity's being necessarily unique does not involve a logical necessity. One may speak of a property of a, viz., not-being-different-from-a, or one may speak of a₁, the substrata in a. And if one employs the former, the sort of "necessity" established for the II is not a logical necessity. If one employs the latter, the sort of alleged "non-necessity" of the II is also not a logical non-necessity. One may point out a difference, namely, \( (x)-Dxx \) is statable, while the notion that substrata
are not sharable is not, perhaps it only shows itself. But this in effect amounts to putting truths believed to be necessary into the coordination procedure of $a_1$ to $a$ and $b_1$ to $b$.

This connects with a point made earlier in this chapter where it was argued that it is unreasonable to ask the defender of the II that the II be proven necessarily true. In effect, the establishment of the II, even when one accepts the demand that an entity must be necessarily unique to an object to individuate, rests upon the same sort of considerations that the establishment of its alleged non-necessity does. In each case we have an entity which is unique to an object. However, just as it is not logically necessary that "(x)-Dxx," so too is the claim that individuators are not sharable not logically necessary. In effect, it is unreasonable to ask either proponent or opponent to prove their claims. Both must argue indirectly, showing that the other position involves consequences which are not acceptable.

The arguments for the inadequacy of the Russellian definition have been shown to be question begging, invalid, not sound, or lastly, themselves resting on some alleged a priori truth.
CHAPTER V

THE IDENTITY OF INDISCERNIBLES AND KINDS OF THINGS

Many philosophers have held that classes are different sorts of things than attributes. Cartwright,\(^1\) taking Quine as representative of these philosophers, has recently attempted to show that drawing a distinction between classes and attributes is not as straight-forward as one might first suppose. To so argue, one must, of course, be perfectly clear about what it is for two kinds of things to be different. Although in agreement with respect to Cartwright's general result, I disagree with some of the arguments he employs in obtaining that result. This chapter will then have three parts. The first will be a general discussion of several of the ways one can formulate a notion of difference between kinds of things. In the second, there will be an examination of several of Cartwright's arguments which are mistaken. For the third, I shall propose an alternative to Cartwright's way of viewing the situation.

I. Criteria of Difference

It is widely held that if two individuals differ then they differ in some property. That is, given two individuals a and b, then there is some property had by a and not by b. It is this difference in property that makes them two. As has been seen, there is considerable disagreement with this view, but this disagreement with respect to individuals is outside the scope of this chapter. Granted, then, that one distinguishes arbitrarily selected individuals by this method, how can one show that two kinds of things are different? What one might do is show that two kinds are mutually exclusive. One way to do this is to find some property, F, which all members of one kind have, but no members of the other kind have. The finding of such a property guarantees at least two kinds of things. It guarantees that neither kind will be a subkind of the other (as horses are a sub-kind of mammal); it also guarantees that no thing will be a member of both kinds, that is, that the kinds will not overlap. In this connection let us formulate several possible versions of this criterion.

I. If kind A differs from kind C, then for some F, it is true of all A's that F but false of all C's that F.

One might read I in the way that would distinguish, say, horses from cows. But it may be thought that this leads
to trouble. For consider two sorts C and A. If I distinguishes them only as a matter of fact, i.e., if the instantiation of I is not a necessary truth, one may ask whether an adequate distinction between Cs and As has been made. One may claim that as long as it is possible that something could be both, the distinction has not been made. Cartwright himself exploits this objection to I, as shall be seen in more detail later. Suppose then one formulates a new and more stringent criterion of difference:

II. If kind A differs from kind C, then for some F, it is necessary that all As are F, and necessary that F is false of all Cs.

In passing, notice that this sort of objection to I above is exactly parallel to the objections raised against the identity of indiscernibles in the last chapter. The only difference is that here one is speaking of kinds of things, whereas in the last chapter the arguments dealt with individuals.

To continue, if one uses I (or II) to single out Cs from As, it may be objected that all F really does is single out Cs from non-Cs, as opposed to Cs from As. That is, there is an objection one can make to I or II which suggests a wholly different criterion for distinguishing kinds. I or II, it may be said, simply sorts out Cs from non-Cs, provided there are more than two sorts in the range of the variables. Even if it is (necessarily) true that
cows have $F$, and (necessarily) false that horses do, it might also be (necessarily) false that dogs do. So one has not distinguished cows from horses, but cows from non-cows. In order to answer this objection, one may formulate a criterion as:

III. If kind $A$ differs from kind $C$, then for some $F$, it is true of all and only $A$s that $F$, and false of all and only $C$s that $F$.

Notice again that this sort of objection corresponds to the two senses of individuation discussed in the last chapter. In effect, the new criterion of difference of kinds asks for a property which is unique to each of the kinds. One may therefore object that this criterion is too strong. Perhaps another weaker version of III would be:

IV. If kind $A$ differs from kind $C$, then for some $F$ and some $G$, all and only $A$'s are $F$, all and only $C$'s are $G$, and $F$ and $G$ are incompatible.

Notice that the above criteria for difference between kinds assumes that the two kinds are both members of the same category—by this all I mean is that the application of the predicate which does the distinguishing in all cases makes sense, that is, it is well-formed. Let us explore this.

How does one show, say, that individuals are different from properties? Individuals and properties are, for the sake of this discussion, kinds of things. Does
one find one property such that all individuals have it but no properties do? Or, does one find a context true of all individuals but not true of all properties? There is an ambiguity here which needs to be exposed. On the level of, say, individuals, to say a predicate is not true of something is equivalent to claiming that the predicate is false of it. So, if the predicate "F" is true of a but not true of b, one has found a difference between a and b. But suppose one finds a property which is true of all individuals— and not true of properties. Has one then made a distinction? The answer is not clear.

If, to make the distinction or show the difference, one must find a predicate true of the individuals and not true of the properties, one has succeeded. If, however, one must find a predicate true of all the individuals but false of all the properties one may have failed, since one might hold that no property true of any individuals can be either true or false of any property. This reflects an ambiguity in the notion of not being true of something. Some sentence may be ruled as not-true if it is false, or it may be counted as not-true if it is nonsense, or not well-formed.

Thus at least one sort of difference between kinds is not to be reflected in some predicate true of one but false of the other. Indeed, the most interesting divisions
between entities not only is not, but cannot be expressed by a difference of predicate. I have in mind the differences between, say, individuals, properties, propositional entities, and the like. The classic way of showing these types of things to be different stems from Russell. Assuming a predicate can be predicated of itself, one arrives at a paradox. So one makes a distinction. Individuals and properties are not different because of a difference of property, but rather, because, if they were represented by the same sort of sign, the results are paradoxical. Thus, within a category of thing, we may say that difference of kind is reflected in difference of property. But between categories of things, such as individuals, properties, and propositional entities, difference of kind is not so reflected. Let us make this new notion of difference between kinds explicit.

V. If kind A differs from kind C, then there is some F which is applicable to (either true or false of) all A's but not applicable to (neither true nor false of) any C's.

Contrast V with I. Now as a criterion for distinguishing natural kinds of things, e.g., horses and cows, I might do. But one wonders whether it will do for those metaphysical distinctions between the sorts of entities that philosophers often discuss. For example, in distinguishing between individuals and attributes, it would be misleading to say that what Russell showed was that, whereas it is
true of all attributes of the first logical type that they may be instanced by individuals, it is false of all individuals of the lowest logical type that they may be instanced by individuals. For it is not just false that individuals may instance themselves, but a kind of nonsense. However, by means of V one can distinguish individuals from properties of individuals. For take any property had by any individuals; it is either true or false of each individual, but it is neither true nor false of any property of any individual. When Russell points out that no predicative function takes itself as a value, he is not affirming the claim "(f)-f(f)." Cartwright, in his article directed against Quine, does not even consider V as a means of distinguishing kinds of things. Obviously, though, this has been one of the classic ways of drawing distinctions between kinds.

Are classes and attributes two categories? Or are they two kinds within a category? As it turns out, the class-attribute case fits none of the criteria for difference outlined above. The distinction between them, however, is made in the classic fashion. Assume every attribute is the class it determines. But then, since two attributes can be coextensive but two classes cannot, we contradict our assumption. What one has shown is that not every attribute need be a class. This, of course, leaves
open the alternative that some among attributes are classes. And if this were the case then we would not find a property that all of the one has and none of the others have. Expansion of these and other points will be done in the light of certain criticisms of Cartwright's paper against Quine.

It shall be shown that, unnoticed by him, Cartwright shifts his criterion for distinguishing kinds from I to III (or IV). Furthermore, he seems to implicitly argue for the impossibility of meeting such a set of criteria. Thus he despairs at making the distinction between classes and attributes. And though he does not mention it, the result is general as long as we read I and III in terms of variables. Given the reading Cartwright gives to I, which extends to III, one could, if convinced by Cartwright's arguments, despair at distinguishing any two sorts criterially. But then, as mentioned above, many of his arguments then become mere variants of the arguments against the identity of indiscernibles given in the last chapter. Interestingly, Cartwright's own position is that the II will suffice to distinguish kinds of things, but his arguments amount to an implicit rejection of the II.
II. Cartwright's Argument

What then, is Cartwright's own explicit criterion of difference? He tells us the following:

... to say how two sorts of things differ, ... it is necessary to say something true of the form, whereas it is true of all C's that ..., it is true of no A's that ... .²

An ambiguity must be pointed out immediately. What is the force of "it is true of no A's that ..."? One may read it as: "it is false of all A's that ...," or, alternatively, "it is neither true nor false of A's that ...." That is, the context represented by "..." would not be true of any A's. Thus, given the above quote alone, one cannot decide whether Cartwright intends I or V. Cartwright makes it clear in another context that he means the former. He claims that "what is said to be true of all classes is simply not what is said to be false of all attributes."³ Cartwright's criterion, then, seems to correspond to I (or II) and not to V. Further, Cartwright's explicit criterion of difference of kinds does not match IV above. One is thus asked to find some property, say F, such that all C's are F, and no A's are F, or conversely. Understanding this criterion is important for, as shall be shown later, Cartwright subtly shifts to

²Ibid., p. 232.
³Ibid., p. 241.
a quite different and more stringent one. Differently, Cartwright either shifts to III (or IV) or perhaps he is inaccurate in his original statement of what is to count as a difference of kinds. To see how this occurs, first turn to his argument against Quine.

Cartwright takes Quine as the representative of those many philosophers who have tried to provide F to distinguish classes from attributes. This F is the identity condition for the two sorts. Quine claims that "Classes are identical when they have the same members" while "attributes may be distinct even though present in all and only the same things." Cartwright rephrases this as

(1) if y is any class having the same members as x, then y is identical to x.\(^5\)

(2) if y is any attribute present in the same things as x, then y is identical to x.\(^6\)

Cartwright reads Quine as claiming that (1) is true of all classes while (2) is false of all attributes. Since (1) is not the same as (2), (1) and (2) have not, so Cartwright argues, shown a difference between the two kinds. In passing, one may wonder if (2) is really the feature Quine wishes to point out about attributes. Are not logically equivalent predicates, those which necessarily have the


\(^5\)Cartwright, p. 233.

\(^6\)Ibid.
same extension, to count as denoting the same property. If the variables $x$ and $y$ in (2) can range over the same attribute, then is Quine claiming that no attribute is self-identical? It is certainly arguable that (2) does not accurately represent the modality in the phrase "attributés may be distinct," contained in Quine's statement about attributes. I shall return to this problem in Section III; for now I will go along with Cartwright, as he casts about for repairs to (his reading of) Quine's view. For despite the above note about Cartwright's formulation of Quine's view, Cartwright is correct in claiming that Quine has not fulfilled I. That is, Quine has not given one property that all classes but no attributes have. Putting the same point in the context of the discussion in the last chapter, Quine has given us a property difference—but the sort of property difference he has provided is like the $(f, g)$ pair, as opposed to the $(f, -f)$ pair. But it is this latter pair that Cartwright's own explicit criterion concerns.

Cartwright notes that the failure of (1) and (2) to satisfy I is independent of the identity conditions of classes and attributes. That is, even if all properties were identical when present in all and only the same things, i.e., if (2) were true of all attributes, this

\[\text{Ibid.}\]
would not show the kinds to be the same. One way to view
the failure is to notice that (1) and (2) are still dif-
ferent, the difference now to be found in the words
"class" imbedded in (1) and "attribute" imbedded in (2).
So Cartwright suggests that we suppress these notions and
come up with a more neutral sentence which, hopefully,
will provide one with a difference. This new sentence he
expresses as:

(3) if \( y \) is anything having the same members as
\( x \), then \( y \) is identical with \( x \).\(^8\)

Cartwright argues against the adequacy of (3) as follows:

Attributes have members or they do not. As already
noted, their membership is in either case the mem-
bership of some class: some non-empty class in the
former case, the empty class in the latter. So if
every class satisfies (3), every attribute is iden-
tical with some class and thus after all itself
satisfies (3). The fact is that it is curious to
resort to (3) in the attempt to distinguish classes
from objects of any sort. For, if every class
satisfies (3), there simply are no non-classes at
all; hence no objects from which classes need to
be distinguished.\(^9\)

This is Cartwright's argument against (3). First examine
the conclusion. Cartwright claims that "if every class
satisfies (3), there simply are no non-classes," that is,
everything is a class. Do all classes satisfy (3)? If
they satisfy (1), then they satisfy (3). It must be
granted that they satisfy (1). It follows that, if

\(^8\)Ibid.

\(^9\)Ibid., p. 234.
Cartwright's argument is correct, there are no non-classes. If this is the correct reading, it is curious that Cartwright should bother with the other arguments in his paper at all. For, if correct, he has established that there is only one sort of thing, namely, classes. Differently, if one may assume that some things are non-classes, it follows that not every class satisfies (3). But if this is so, then not every class satisfies (1) either. This is surely unsatisfactory. There is, however, another way to read Cartwright's conclusion. Suppose one characterizes the notion of class and non-class by satisfaction or non-satisfaction of (3). Then given that his earlier argument already established that everything satisfies (3), his conclusion reads as follows: if everything satisfying (3) satisfies (3), then nothing fails to satisfy (3). This way of viewing the argument makes the only interesting part of the argument Cartwright's "proof" that everything does satisfy (3), i.e., the consequent of the conclusion.

Let us then examine the argument carefully. The idea is, roughly, that given that any entities either have or have not members, their membership will be that of some class and hence they will be identical to that class. But how does one know this to be true? Consider some entity, say $\emptyset$, that has as members $a$ and $b$. There is of course a class which has members $a$ and $b$. But how does one know
that \( \emptyset = (a, b) \) is true? One may attempt a reply by pointing out that they are identical simply as a result of the application of (3). And indeed, if one knew (3) to be true of everything, then this reply would be in order. But one does not know (3) to be true of everything. But if one does not, then one cannot make the inference from "\( \emptyset \) has the same members as \( (a, b) \)," to "\( \emptyset = (a, b) \)." This move is, as above noted, crucial to Cartwright's argument. Cartwright, as will be argued in more detail, must assume (3) to be true of everything to obtain the conclusion.

Furthermore, the argument seems to trade on an equivocation on the notion of "having the same members." Does it really make sense to say that the membership of an individual, if it has no members, is the membership of, i.e., has the same members as, the null class? The suspicion of equivocation is correct as will be shown. However, even disregarding the equivocation, it will be shown in some detail that Cartwright's general argument against (3) is incorrect. Let us first try to restate the argument as clearly as possible. Let "\( M(x) \)" be "\( x \) has members"; "\( \text{Cls}(x) \)," "\( x \) is a class"; and "\( (3)(x) \)," "\( x \) satisfies (3)."

\begin{align*}
1) & \quad (x)(M(x) \lor \neg M(x)) & \text{Assumption} \\
2) & \quad (x)(M(x) \lor (y)(\text{Cls}(y) \cdot (z)(z \in x = z \in y))) & \text{Assumption} \\
3) & \quad (x)(\neg M(x) \lor (y)(\text{Cls}(y) \cdot (z)(z \in x = z \in y))) & \text{Assumption} \\
4) & \quad (x)(\text{Cls}(x) = (3)(x)) & \text{Assumption}
\end{align*}
5) \((x)(M(x) \supset (\exists y)(x = y \cdot Cls(y)))\)
6) \((x)(\neg M(x) \supset (\exists y)(x = y \cdot Cls(y)))\)
7) \((x)(M(x) \supset Cls(x))\)
8) \((x)(\neg M(x) \supset Cls(x))\)
9) \((x)(Cls(x))\)

This is Cartwright's reductio of the claim that (3) can be used to distinguish classes from attributes. For if everything is a class, then everything is picked out by "Cls," and there is nothing to distinguish classes from. Is everything a class? Doubtless Cartwright would argue yes, if (3) is the characterization of classes. I shall now show that he is mistaken.

Consider 1). It seems an innocent tautology. But is it so innocent? Is it true, for example, of individuals? Certainly every class satisfies it: every class either has members or is the null class and has no members. But if one claims that an individual is not a class, is this equivalent to saying it has no members? I shall return to this point later.

2) and 3) embody Cartwright's claim that, given 1), the membership of an entity is the membership of some class. 5) and 6) follow, presumably, from at least 2) and 3), respectively. They state, as he does, that an entity is identical with the class which has the same membership as it does. Take 6). Presumably what makes it
true is that since \( x \) does not have members, and the null class does not have either, \( x \) is identical with null class. This is an appeal to the satisfaction of (3) by everything. But this is the very point at issue. Suppose \( x \) is an individual which by assumption has no members, and let \( y \) be the null class. Then it might seem that \( x \) is identical with \( y \) according to (3). But how do we know that (3) holds in this case? Doubtless (3) holds for classes, or so it is assumed in 4), but does it hold for anything that has members or not? In fact it is easy to show that (3) cannot be true for individuals, unless there is but one entity that has no members. For suppose that \( a \) and \( b \) are two different individuals, each with no members. In (3), let \( x = a \) and \( y \) the null class, and assume (3) is satisfied. Then let \( x = b \), and \( y \) the null class, and again assume that (3) is satisfied. It follows that \( a \) is identical with \( b \). Thus if (3) holds when individuals have no members, there can be but one entity having no members if one reads the "=" sign as the English "one and the same" as Cartwright seems to. Thus (3) is true for entities having no members only if there is but one entity having no members. This one should take as a reductio of the claim that (3) is true when \( x \) is an entity having no members. Thus, given 4), and the above reductio, it follows that there is at least one thing that is not a class.
The reductio just given is on the assumption that (3) is used to justify 6). As has been pointed out, since (3) is not known to be true of everything, it cannot be used to justify 6). Of course, if one believed that (3) is axiomatic for the membership relation one might conclude that everything that has members or does not must be identical with anything else that has the same members. However, if (3) is such an axiom for membership, then it is true for every value of x and y. But this is not the case if there is more than one entity which has no members. One way to maintain that (3) is an axiom for membership is to restrict the range of variables to classes. That is, (3) would be an axiom only for classes. But this would, in effect, turn (3) into (1). There is another alternative, of course. One could point out that the claim that individuals have no members is not to be taken in the same sense as the claim that the null class has no members, i.e., to claim that the phrase "has no members" is ambiguous. This, of course, would throw doubt on 1). Cartwright might object that he is not taking (3) as an axiom for membership. Rather, he might say, his claim is that since x, which is, say, an individual, is identical with some class, and it is assumed that (3) is true for classes, (3) is true of x. But then he is not basing 6) on the satisfaction of (3). So what does justify 6)? This is, of course, the question with which we began.
There is one other alternative: the very definition of the null class allows Cartwright to claim that if \( x \) has no members, then it is identical with the null class. Let the null class be defined as follows: "\( x \) is the null class" means "\( x \) has no members." Assume two different individuals, \( a \) and \( b \), to satisfy the definition, which the null class also satisfies. Then \( a \) is identical with \( b \). To avoid this conclusion we must assume that if individuals have no members, there is only one entity that has no members. Again, this should be taken as a reductio, since there is more than one individual. However, the reductio might push one to a different conclusion, namely, that something is wrong with the above definition of the null class. If one tries an alternative, "\( x \) is the null class" means "\( x \) is a class with no members," then of course no non-class satisfies it. This latter definition cannot be used to prove that an individual is identical with a class. That must be assumed. And this shows something important about Cartwright's claim that an entity has members or it does not. For not having members means something different in the case of classes than in the case of individuals. If it did not, one could use the definitions just given without difficulty. And if it is questionable to say of an individual, as one says of the null class, that it has no members, it also becomes
questionable that one can say of an individual, in the same sense as we say of a class, that it does have members. It is questionable that either predication makes sense of non-classes.

Perhaps (3) plays no logical role in Cartwright's argument that everything is a class. The crucial steps in his argument are 1), 5), and 6). For, if 5) and 6) are true, there is of course nothing to distinguish classes from. The question is, how does he justify 5) and 6)? There is, however, a different way to take Cartwright's argument. Suppose that anything that has members or does not have members is a class, since membership is only predicable of classes. Now, since not everything is a class, it follows that it does not make sense to predicate having members or not having members of some entities. But this means that anyone who cites non-satisfaction of (1) or (3) as a criterion for non-classes is in trouble. For it does not make sense to say, for example, that (3) is false of attributes if membership does not properly apply to them at all.

However, there is a reply to this argument. For the notion of non-satisfaction of an open sentence is ambiguous. If (3) is not satisfied by any attributes, does this mean that there is some attribute x which has the same members as y and is not identical with it? That
is, does it mean that (3) is false of all attributes? Or does it mean that (3) does not apply to attributes, i.e., that they are not in the range of its variables? Remember that the same sort of ambiguity pervades I. To say that As are not F might mean that F is false of all As. But it might also mean that neither "F" nor "not-F" is true of As. As mentioned earlier, there is precedence for such claims in distinguishing kinds. Cartwright seems to treat the claim that an entity does not satisfy an open sentence as equivalent to the claim that the sentence is false of that entity.

It has been shown that one way to read Cartwright is that he assumes (3) to be true of all objects. It turns out that, in the context of another discussion, he denies that something equivalent to (3) will be true of everything. In the context of talking about the relation of membership, Cartwright notes, quite correctly, that it is possible that the relation of membership does not satisfy

\[(7) \ (x)(y)((x)(xRx xRy) x=y)^{10}\]

Since "x" and "y" are taken to range over all objects, (7) is merely a closed version of (3). In speaking about (7), Cartwright argues:

\[\text{Ibid.}, \ p. \ 237.\]
But of course doubt can be raised whether membership does satisfy (7). It does not if there is more than one memberless object.11

Presumably what Cartwright has in mind is something like the following: Take any two individuals, a and b. Assume that they have no members. Since they are two, but there is only one null class, (7) is false. But if (7) is false, how can (3) be true of everything? This is, of course, the same argument that was employed earlier against Cartwright's argument against (3).

Before passing to some of Cartwright's other arguments, it should be noted that he uses an argument against (1) similar in some important respects the one against (3).

Wherein lies the confidence that no attribute satisfies (1)? Well, if some attribute were to satisfy (1), its membership—empty or otherwise—would presumably coincide with that of some class and hence it would be identical with that class; but no attribute is a class. Thus, we agree that no attribute satisfies (1) only because we are antecedently prepared to agree that no attribute is a class.12

Notice that he claims that if an attribute were to satisfy (1), then it would be identical to a class because its membership coincides with that of some class. This sounds like an appeal just to the claim that if x and y have the same members, then they are identical, i.e., another appeal to (3). But (1) also contains the predicate "is a class,"

11Ibid.

12Ibid., p. 233.
and presumably \( x \) and \( y \) must both satisfy it as well as being identical when they have the same members. Thus is (1) is true of \( x \), then \( x \) is a class and is identical with any other class having the same members. Cartwright goes on to say that we are only prepared to deny an attribute satisfies (1) on the antecedent grounds that an attribute is not a class. This must mean that no attribute, it is assumed, satisfies the predicate "is a class" in (1) for, if it did, it would satisfy the rest of (1). This sheds light on his argument against (3). For (3) is like (1) except that the predicate "is a class" has been dropped. What Cartwright does is to show, he thinks, that every attribute does satisfy the predicate "is a class," hence automatically satisfies (3), or, what comes to the same now, (1). But one cannot use (3) to show that every attribute satisfies "is a class." (3) would only be satisfied by an entity if one could agree antecedently that the entity is a class. One cannot use (3) to show that an entity obeys (3) unless one tacitly assumes that (3) is true of everything.

The argument against Cartwright has not established that attributes do not satisfy (3). Rather, what has been shown is that it is false that attributes must satisfy (3). It is still open to argument that they in fact do or, at least, that some of them do. Such an argument shall
be given in Section III. In other words, Cartwright's question about (1)—wherein lies our confidence that no attribute satisfies (1)?—is open against (3) as well. It is just not open to claim they must satisfy (3), at least not the way Cartwright tries to prove it.\textsuperscript{13}

Cartwright casts about for other attempts at repair. That is, he attempts to find another open sentence which will be satisfied by all classes but no attributes—thus distinguishing the two kinds. He asks us to consider the following:

\begin{enumerate}
    \item if \( y \) is congruent with \( x \), then \( x=y \)\textsuperscript{14}
\end{enumerate}

which is the definitional expansion of:

\begin{enumerate}
    \item if \( y \) is anything having the same members or present in the same thing as \( x \), then \( y \) is identical with \( x \).\textsuperscript{15}
\end{enumerate}

\textsuperscript{13}Perhaps what Cartwright is suggesting is the following. Represent (3) as:

\begin{enumerate}
    \item (3)\textsuperscript{' } (x)(xRy\iff x=y)
\end{enumerate}

where "R" is "has the same members as." Assume this to be a characterization of all classes. That is, any class, when plugged into the "y" place will make (3)' true. Assume, further, that "x" literally ranges over everything (classes, attributes, individuals, propositional entities, or what have you). Grant Cartwright the premise "(x)(Mxy=Mx)" where again "x" ranges over everything. Then his conclusion, namely, that if all classes satisfy (3)', then nothing fails to be a class, follows. Doubt is cast, however, on both the practice of allowing "x" to range over everything, as well as the notion of "having no members" to be found in the premise above referred to.

\textsuperscript{14}Cartwright, p. 234.

\textsuperscript{15}Ibid.
Cartwright has two arguments against (5) above. One of them is the same argument as the one against (3), which has been refuted. The other is the following:

Notice, in the first place, that this respect in which classes differ from attributes is one in which non-attributes generally differ from attributes. For suppose x to be any non-attribute. Then x is a class or not. If the former, then x evidently satisfies (4); and consequently x vacuously satisfies (4). So (4) is satisfied by every non-attribute whatever. But then it seems odd to cite satisfaction of (4) as the way in which classes are to be distinguished from attributes.16

A little reflection will show that Cartwright is no longer employing I but has shifted to a much stronger criterion, namely III. There is no indication in the text that he is aware of the shift. This sort of shift is exactly the one noted in the last chapter, where two senses of individuation were distinguished. The first was simply the requirement that one individual have a property the other did not. The second was the stronger requirement that what individuated one thing from another had to be unique to the objects in question. But that is now what Cartwright is demanding, the difference being that he is speaking in the context of difference of kinds of things. Put differently, Cartwright begins by arguing that a property true of one kind but false of another (viz., criterion I) will do the

16 Ibid.
job of distinguishing the two kinds, class and attribute. In arguing against (5) he talks as if he has found such a property but goes on to argue that it will not do. To use a different example, suppose one wants to distinguish, say, horses from cows. Then (by I) one cites a property that all cows have but no horses have. At this point Cartwright may reply: "true, that distinguishes cows from non-cows, but one wants to distinguish cows from horses." Something has surely gone wrong. Perhaps one way to see what has happened is to reformulate Cartwright's original argument against Quine. Assume Cartwright to have shifted to or implicitly accepted III (or IV) as a way of distinguishing kinds. Then his argument comes to: Quine has provided one with a property that all classes have and a different property that all attributes have but failed to demonstrate their incompatibility. But, as shall be seen, this criterion is too strong. The shift of criterion has been from I to III (or IV). Criterion IV needs considerable discussion. In fact, it is questionable whether its conditions can be satisfied by any two kinds.

Suppose that one finds a respect F which all and only Cs have. All other kinds will be non-Fs. Thus, if there are more kinds than Cs and As, they too will be non-Fs, and the second part of IV will not be satisfied.
In order to satisfy Cartwright if there are more than two kinds, one would have to find a property G that all and only As have, and show that G is incompatible with F. This last condition is crucial. Without it, one would simply be back to Quine's position with (1) and (2), and Cartwright's criticism that what is said to be true of As is not what is said to be false of Cs. That is, one has to show that what is said to be true of Cs and what is said to be true of As cannot both be true of anything. This, remember, is criterion IV. It is the last condition, that of establishing incompatibility, that is the cause of the difficulty in fulfilling IV. Let us explore some ways it might be met.

**First.** G is logically equivalent to not-F (or F to not-G). But what Cartwright explicitly wants, to repeat, is not just a way of distinguishing Cs from non-Cs (or As from non-As), but Cs from As. But rather obviously, I does not give him this.

**Second.** G implies not F, but not conversely. The problem with this interpretation of the third condition of IV is in showing that G entails not F. Now if G logically entails but is not logically entailed by not F, G must be a function of not F and, say, Ø. Define "G" as "Ø and not F." If there is anything that is G, then it is both Ø and not F. But how does one know that there is anything that
is G? If "G" is true of something, then that thing is Ø and not F, but unfortunately the very definition of "G" begs the question. For it assumes that an A is not F, and this begs the question.

Assume, for example, that Ø alone picks out all and only As. We must now show, not assume, that Øs are also not F. But unless G is also defined (in the same manner as G) the statement that Ø implies not F is at best merely contingent. Thus, though it might be a fact that all Øs are not F, it would be logically possible that this not be so. If Ø is defined, the same problems again arise. Take another case. Suppose Ø picks out all and only As and some Cs. Again, the intersection of Ø and not-F picks out all and only As. But again one must ask how one can show that there are Øs which are not F. For the assumptions here are compatible with the claim that all and only Cs are Ø.

The problems with satisfying criterion IV then are either showing that F is incompatible with G, or given that F and G are incompatible, showing that there is a G. Rather obviously, if G is not a function of not-F, one cannot show the incompatibility. If it is, the problems just shown arise.

The interplay between I and IV provides a key to much of Cartwright's argument. Consider again (1) and
(2). Cartwright points out that what is said to be true of all classes is simply not what is said to be false of all attributes. Cartwright views Quine as having unsuccessfully attempted to fulfill I. But another way to see Cartwright's point is to view Quine as having tried to fulfill the first two conditions of IV, without having fulfilled the third. Cartwright's attempts at repairs amount to constructing a sentence (5), which is said to be true of all classes and false of all attributes, thus fulfilling I. Then he raises two sorts of objections. One is that even if I is fulfilled, (5) does not separate Cs and As, because the property it attributes to Cs picks out too many kinds. This in effect returns one to criterion IV. The other sort of objection is that one must show that the As do not fulfill (5) or, to put it another way, one must show that I is fulfilled. In fact, Cartwright tries to argue that not only is it the case that one cannot show that attributes do not fulfill (5) but rather, that one can show they do fulfill (5). Now if one returns to IV one gets the same sort of argument. To show that G is incompatible with F, or to show there are any Gs, (if G is defined in the way explained above), comes to showing that As are not F.
Cartwright's second argument against (5) is essentially the same one he employed against (3). The argument against (3) has been disposed of. Nonetheless, even if one assumes it is correct, it can be shown that Cartwright cannot consistently employ both arguments. For notice that Cartwright's claim that (5) is vacuously satisfied by anything that is a non-attribute and a non-class, e.g., an individual x, conflicts with his other argument against (5) and (3). For to say some x vacuously satisfies (5) means that there is nothing that has the same members as x. In his other argument against (5) Cartwright assumes that everything has members or does not, and thus that everything has the same membership as some class; thus in this case there is always something that has the same members as x. This conflict brings out the ambiguity of the notion of having no members. In one case an individual which is assumed to have no members does not have the membership of the null class— or else it would have satisfied (5) non-vacuously. In the other case, an individual which is assumed to have no members is said to have the membership of some class, namely, the null class, is said to satisfy (5) non-vacuously. It also follows that if "having the same members" means the same in this new argument as in his other argument, vacuous satisfaction becomes nonsense for another reason. For if it is true

17Ibid.
that everything has members or does not, how could any-
thing vacuously satisfy such a function? If no individual
is present in or has the same members as anything, then
individuals must be ruled out of the range of the variables
in (5). The only way, then, to claim that an individual
vacuously satisfied (5) is to give up the assumption that
everything has members or does not, at least the way Cart-
wright interprets this assumption.

The shift between I, III, and IV, shows itself
again in his discussion of class and attribute theory.
What emerges from his discussion is not merely that no one
who has tried to distinguish classes from attributes has
succeeded, but something much stronger: I and IV are
inadequate for showing difference between kinds.

Cartwright claims that what he calls class theory
diverges from attribute theory in that the former and not
the latter contains an axiom of extensionality. Yet he
claims that this is merely a difference in the theories,
and that he sees "no coherent way in which it can be sup-
posed to carry over to the objects dealt with in the
theories."\(^{18}\) What he means is not clear. Let us try
some interpretations.

**First.** Cartwright may be entertaining the possi-
ibility that one or another or both theories might be false.

Perhaps it is false that classes have members, or that they are identical when their members are. To determine the truth or falsity of the theories assumes they have been interpreted. However, Cartwright's discussion implies that he is treating the theories as uninterpreted. Thus it is much more likely that he is asking for the interpretation rules for the theories and not, given that the theories have been interpreted, whether they are true.

Second. Assume the two theories are uninterpreted. Each contains the sign "¢." Assume that in the first, one interprets "¢" into the relation of member, in the second, into the relation instance-of. What is the universe of discourse, the values of the variables, for each theory? Presumably part of the universe of discourse for each theory will consist of individuals. But why could not the other values be precisely the same for each theory? That is, why could not an attribute, or a class, stand in two different relations to the same individuals? One would only be prepared to say that the values of the variables in the two theories diverged if one were prepared to say that classes and attributes were distinct kinds. But obviously, this would rest on some prior decision about their difference. The claim that an attribute could not stand in the member relation with some individual would rest on the logically prior claim that no attribute has members.
To see the above points in a different light, recall that Cartwright's implicit criterion of difference, IV, asks for two properties, $F$ and $G$, such that all and only $Cs$ are $F$, and all and only $As$ are $G$. We have seen that unless $F$ and $G$ can be shown to be logically incompatible, IV cannot be used to show that $Cs$ and $As$ are different. If one considers class theory and attribute theory as presenting two different properties, $F$ and $G$, then the same point applies again. The fact that class theory, $F$, is different from attribute theory, $G$, does not show that the same entity may not be both $F$ and $G$.

**Third.** Suppose that attribute theory and class theory did not diverge with respect to the given axiom of extensionality. The fact that both theories contain the same signs is really quite irrelevant to this assumption. One can consider two formal theories the same when there is an isomorphism between them such that one is a mere rewrite of the other. One might conclude from the second interpretation, above, that Cartwright is implying that if class and attribute theory were isomorphic in the way described, one would be justified in claiming that classes and attributes were the same. It is doubtful that he is saying this, for the following reasons: (1) If class and attribute theory were isomorphic, then in effect there would be but one formal theory. But a formal
theory may have many interpretations. One interpretation might yield properties true of classes, the other, properties true of attributes. Again one would be left with a question about the relation between these two sets of properties. That is, if $F$ is true of entities $x$ and $G$ true of entities $y$, one still has not shown that the values of $x$ and $y$ do not overlap. To do that, $F$ and $G$ would have to be shown to be logically incompatible. And further, in what way, by what interpretation rule, does one specify the properties $F$ and $G$? On what grounds does one make this decision? I shall return to this point in Section III.

(2) The fact that the formal theories are isomorphic does not in itself show that there is no difference between classes and attributes, even brushing aside the first objection above. For there is no prima facie reason why classes and attributes might not have many properties in common, yet still be different kinds. If it is assumed they are different kinds and the identity of indiscernibles for kinds is granted (viz., I), it follows that at least one of the isomorphic formal theories was incomplete. In that case, Cartwright's objection that the theories do not provide a difference would still stand.

The third interpretation is, of course, contrary to the case that Cartwright actually considers. For he makes the assumption that his antagonist makes, that class
and attribute theory diverge. And here, one would think, criterion I would help one solve the problem of distinguishing classes from attributes. But it should be clear by now that this is wrong. Cartwright implicitly argues that I cannot be used to establish difference in kind, not just between $C_s$ and $A_s$, but between any two kinds. And this is serious, because it certainly seems that I is merely a way of stating the identity of indiscernibles between kinds, which is that two entities are different if one has a property which the other does not have, and this is what I amounts to, except that it is stated in terms of kinds.

Suppose, for example, one tries to distinguish $C_s$ from $A_s$ by giving a property $D$ that all $C_s$ have and no $A_s$ have. Cartwright has two objections. First: How does one know that no $A_s$ have $D$? Second: How does one know that only $C_s$ are $D$, and only $A_s$ are not $D$? Now if one tries to answer the first objection by providing some property $G$ which $A_s$ have, and trying to show that $G$ implies not $D$, one runs into all the difficulties explored above. But even if one avoids this, one is still left with Cartwright's second objection, the new criterion of difference, IV. There is a sense in which IV is stronger than I, the identity of indiscernibles for kinds. For the identity of indiscernibles merely asks for a property which one
entity has and the other does not, in order to distinguish them. It does not require that the property be unique, or that its absence be unique, to the entities one wishes to distinguish.

Now one might object and, indeed, Cartwright himself might object, that his position has been badly misrepresented. After all, all that he has tried to show is that none of the proposals to distinguish classes from attributes meet I or, later, IV. The point here is that the proposals fail not because they do not meet the criteria but because the criteria cannot be met. Cartwright's paper might just as well have been directed against an alleged distinction between horses and cows, as between classes and attributes. I and IV are, after all, stated in terms of variables. Thus all the same arguments apply. If one points to a property D which all cows have and no horses have, one may ask first how one can show horses do not have it and second, how one can show that other things besides cows do not have D, and others besides horses, not-D. If other things besides cows are D, say lions, then one has not distinguished horses from cows; if other things besides horses are not-D, say elephants, then one has not distinguished cows from horses. And so on. Of course there is one crucial difference. In the case of horses and cows, one may use perception to "see" the
difference. Whether one can use this way out in the case of classes and attributes will be discussed in the next section.

III. A Distinction Between Classes and Attributes

In this section an acceptable distinction between classes and attributes shall be argued for. Specifically, it shall be shown that in the finite case at least, all classes are attributes but not conversely. Cartwright himself suggests this as a possibility but he does not attribute awareness of this possibility to Quine. He seems to think that Quine and others have meant to distinguish all classes from all attributes. This is why his explicit criterion of difference between kinds is I. It shall be shown that he is mistaken and that a plausible reading of Quine, as opposed to Cartwright's own misreading, shows Quine quite aware of the possibility that all classes are attributes. Finally, some difficulties in the interpretation of class and attribute theory shall be discussed.

Before proceeding to the argument to distinguish classes from attributes, one must make clear both an underlying assumption and the exact question to be answered. The question, which is the traditional one over classes and attributes, is whether or not an attribute is identical with the class that it determines. On the assumption and the question some comments are in order.
First. As has been already pointed out, Cartwright sometimes seems to take the question over classes and attributes differently than above. He sometimes argues against attempts to distinguish them in a peculiar way: He points out that it is possible that an attribute might be identical with some class other than the one which it is said to determine. It is not even clear what this means. Cartwright, in fact, never does state exactly what the question he is considering is. It is clear that those philosophers who have raised the question and given answers to it, e.g., Russell, Quine, are trying to answer the question as I have stated it.

Second. The assumption that has been made seems uncontroversial. But that every class has an attribute which determines it is controversial and it certainly appears that if every class does not have a determining attribute, the point of raising the traditional question may seem to be lost. For the claim that every class is determined by an attribute has been called into question. In certain cases, e.g., that of the real numbers, there are classes for which no attribute can be specified. I shall not question that the attribute cannot be specified; whether this shows that there is no attribute which

determines the class is another matter. Remember the chapter on Black's symmetrical universe, where, it was shown that even if one grants that the properties cannot be specified, the objects in question still differ in a property. However, in order to avoid these worries, one can proceed on the assumption that one is dealing with the finite case, where it is clear that every class has a specifiable determining attribute. As will be seen, the argument to distinguish a class from the attribute which determines it needs only the weak assumption that every attribute determines a class. The assumption of the finite case is concerned with issues related to such worries which shall be discussed after the distinction has been secured.

Is an attribute identical to the class it determines? The answer is: not necessarily in all cases. This I shall now show. Consider the law of extensionality for classes, (l). What does (l) tell us? The claim is that if x and y have the same members, they are identical. What does "identical" mean? Assume that "identity" is not defined in terms of the membership relation, but rather, that it means that whatever is true of x is true of y, and conversely. Assume now that a given attribute is identical to the class it determines. Suppose, for example, that x is the attribute-class (since they are
identical, by assumption) red (a specific shade), and \( y \) is the attribute-class square. Assume further that red and square are coextensive. If one assumes that (1) applies—since, after all, red and square are classes—then one must conclude from the fact that red and square have the same members that red is identical to square and hence everything true of red is true of square. But since red and square are also attributes, this is false. It is false because red is a color and square is not. The point is that one has provided a reductio argument to show that not every attribute is identical with the class it determines. For, in the case in question, the assumption that every attribute is identical with the class it determines yields a contradiction. From the fact that \( x \) and \( y \) have the same members, one can deduce in this case that they are identical. Thus one can deduce that "Square is a color" is true. But "Square is a color" is false. Given that we do not wish to give up the truth of (1) for classes, and that we also do not want to give up the truth that square is not a color, we must conclude that at least one attribute cannot be identical to the class it determines.

If this characterization of the distinction between an attribute and the class it determines is correct, then given the assumption that red and square are coextensive,
all that one has shown is that there is at least one attribute which is not identical with the class it determines. It might still be the case and, indeed, I shall later argue that in the finite case it is true that, at least some classes are attributes.

Cartwright might very well be unhappy with these conclusions. Undoubtedly, he would say that one is not making the distinction that Quine was trying to make, between all classes and all attributes. Indeed, he takes this as the problem from the beginning, as I shows. Now it may be the case that Quine and others, whom Cartwright unfortunately does not specify, have used (1) and (2) in an attempt to show there is a property that all classes and no attributes have, and conversely. But it is not at all clear that this is so. Rather, Cartwright has misread the whole issue and has, throughout most of his paper, attacked a straw man. Showing this will also reinforce the above distinction between classes and attributes. Since the argument is rather complex, a brief summary is in order. I shall show that (2) is not a plausible interpretation of what Quine says is false of attributes. (2) itself is ambiguous. On one reading of it, the claim that it is false seems patently absurd. Another reading of it is at least compatible with what is most likely the correct interpretation of Quine. That is, it is compatible
with the above claim—which perhaps Quine, too, is at
least entertaining—that all classes (in the finite case)
are attributes. Surprisingly, Cartwright himself enter-
tains the possibility that at least some classes are
attributes. Thus he is either, in his own understanding
of his translation of Quine in (2), failing to see that
(2) can be read as compatible with this possibility, or,
seeing it, claiming that Quine does not. For if Cart­
wright had seen that there is a reading of (2) that leaves
open the possibility that some classes are attributes and
he had believed Quine saw this, why a criterion of differ­
ence, viz., I, in which a complete divorce of classes from
attributes is always the question at issue?

There is a long tradition, stemming from Russell,
behind the claim that all classes are attributes. Russell,
like Quine after him, says that one cannot in every case
identify an attribute with the class it determines, because
of the possibility that two attributes determine the same
class. He goes on to say that certain attributes share all
the properties of classes, and in effect claims that one
can treat all classes as a certain sort of attribute,
though not conversely.\textsuperscript{20} Cartwright, in two of his

\textsuperscript{20}Bertrand Russell, \textit{Introduction to Mathematical
184ff. What Russell says is that we need not commit our­selves as to the existence or non-existence of classes,
since we can find a definition of class terms in terms of
attribute expressions. I shall take up the notion of
reduction later in the chapter.
arguments—one of which will be discussed presently—leaves open exactly the same possibility. What is puzzling is that Cartwright did not see that this possibility—that all classes are attributes but not conversely—is the very sort of distinction that Quine was attempting to make.

Quine says that attributes may be distinct even though present in all and only the same things. Cartwright, in his translation of this statement (2), loses the modality; he says that no attribute satisfies (2). The following formulation of Quine's view is more accurate, for reasons to be given presently:

\[(A) \text{ For every attribute } x \text{ it is possible that there is at least one attribute } y \text{ coextensive with } x \text{ but non-identical with } x.\]

Notice that (A) leaves open the following possibilities: (a) a given attribute x could have all coextensive attributes identical with it. (b) A given attribute x could have some attributes coextensive with it and non-identical with it, and could have some coextensive attributes identical with it. This reading of Quine interprets the phrase "may be distinct," as part of what Quine is saying about the identity condition of attributes. But there is another way to take the modality. It might register Quine's hesitation as to whether all attributes were extensional, i.e., identical with all coextensive
attributes, or not. The trouble is that the second alterna-

tive is unclear. It could mean:

(B) No attribute $x$ is identical with any coextensive attribute $y$.

(C) For every attribute $x$ there is a coextensive and non-identical attribute $y$.

(D) There is at least one attribute $x$ that is coextensive with a non-identical attribute $y$.

Compare (B) with Cartwright's denial of (2). (2) says that if $y$ is any attribute present in the same things as $x$, then $y$ is identical with $x$. In claiming that (2) is false of all attributes, Cartwright might mean (B). But he also might mean (C). It is doubtful that he would take (D) as an alternative, since it does not fit with (I); it does not give a property of all attributes. Consider now (B) and (C) to see how they compare with (A), which seems to be the correct interpretation of Quine.

Consider (B). It is patently false, since there are logically equivalent attributes which are identical. Of course it is true that every attribute has coextensive and logically equivalent attributes which are identical with it. Suppose that there were only one attribute. Presumably if it is present in anything it is at least present in the same things as itself. But (B) states that if this is the case, it is not self-identical. But
this is disastrous. Such reasoning can be extended to all attributes.

Ruling out (B) as a plausible interpretation of Quine, turn now to (C). (C) makes a very dubious existence claim. It certainly is not clear that every attribute is coextensive with another, non-identical attribute. Now (C) is compatible with one interpretation of Quine, namely, (A). For (C) as well as (A) does leave open the possibility that some classes are attributes. How? It is true that, at first glance (C) seems incompatible with this possibility, for every class is identical with every coextensive class, yet according to (C) no attribute is identical with every attribute coextensive with it. However, (C) leaves open the possibility, as does (A), that an attribute is identical with some coextensive attributes. And, as shall be explained in more detail below, all that one needs to support the claim that some classes are attributes is that some attributes are identical with some of their coextensive attributes. (C), however, is not equivalent to (A), because it makes the strong claim that every attribute is paired with a coextensive, non-identical attribute, whereas (A) merely leaves open the possibility that this occurs. It is doubtful that Quine meant to make an existence claim like (C).

It is questionable whether Cartwright sees the possibility that (C) leaves open, that some classes are
attributes. More accurately, though he may see that some classes may be attributes, he may not see that (C) is compatible with this view. For if he did, why does he not give Quine credit for seeing it, too. After all, there is a long tradition of considering the possibility that some classes are attributes. Notice that in his argument Cartwright first quotes Quine as saying that the law of extensionality is not considered to extend to attributes, then represents Quine as saying that two attributes with the same extension "may well be distinct." Interestingly, Cartwright here restores the modality that he had dropped from (2), even though Quine later drops the modality. It is from the fact that attributes might be distinct even when coextensive that Cartwright does go on to show that some classes may still be attributes, though not all attributes must be classes. Again it is puzzling as to why he does not attribute such awareness of Quine, especially given the argument cited above, in which he has read Quine more accurately than his original formulation of Quine's view, namely (2), would indicate.

The upshot of this is that Cartwright thinks, from what one can tell, that (C)—if (C) is his interpretation of Quine—is meant by Quine to be incompatible with the claim that some classes are attributes. Cartwright must be assuming that Quine did not see that (C) is compatible
with the claim that some Cs are As. Otherwise, why does Cartwright bother with criteria I or IV? For one needs them to show that no class is an attribute. Cartwright, in other words, is reading Quine as saying that no class is an attribute; thus Cartwright, if he is indeed attributing (C) to Quine, must either be accusing Quine of not seeing that (C) leaves open the possibility that some classes are attributes, or else Cartwright does not see the possibility. But if the "other" philosophers who have tried to distinguish classes from attributes—whom Cartwright refers to but does not name—include Russell, then it is simply false that none of them saw the possibility. On the contrary, Russell exploited it.

One must conclude that Cartwright is attacking a straw man in much of his paper. Differently, the very way in which he formulates the problem of distinguishing classes and attributes leads him away from a proper reading of Quine. Cartwright seems to think that in the past philosophers have considered it an all or none matter. If one puts the question as above, namely, "Is an attribute identical with the class it determines?" one focuses attention on the possibilities more clearly.

In the above, use has been made of the claim that, though there is at least one attribute that is not identical with the class it determines, some classes may be
attributes. In another context, Cartwright himself raises such a possibility in an interesting analogy. If \( x \) is the father of \( z \), and \( y \) is the father of \( z \), then \( x \) and \( y \) are identical. Yet, though all fathers are parents, if \( x \) is the parent of \( z \), and \( y \) is the parent of \( z \), \( x \), and \( y \) may be non-identical. To carry over the analogy, if \( x \) and \( y \) are classes with the same members, then \( x \) and \( y \) are identical. Why could not \( x \) and \( y \), which are classes, also be attributes? One would thus have the situation in which all classes are attributes, though not all attributes are classes. The argument above establishes the latter without ruling out the former.

To give a different analogy, one may be in a position like DesCartes, wondering if he might not always be dreaming. Even if this were true, it would still be necessary, within the dream, to distinguish dreams from waking life, in a narrower sense of "dream." Switching back to the class-attribute case, one must distinguish a wide and a narrow use of "attribute," attributes\(_1\) and attributes\(_2\) respectively. Attributes\(_2\) will be all attributes\(_1\) that are not classes. But what subclass of attributes\(_1\) has all the properties that classes must have? These requirements include at least extensionality, and that for every set of individuals, there must be one

\[21\] Cartwright, "Classes and Attributes," pp. 239-240.
class, i.e., one attribute. Professor Bergmann has recently suggested such a scheme. By a familiar device, one defines all the unit classes, e.g., let \( \emptyset(x) \) be defined by \( x=a \), where \( a \) is an individual. Then one easily gets all classes. Any defined logical operation will yield a predicative function which is logically equivalent to one of the defined function. Hence extensionality holds among the classes. There may indeed be attributes which are coextensive with a member of the subclass but, since the statement of their coextensiveness is not a necessary truth, these attributes are not considered classes. If the statement of their coextensiveness were a necessary truth, these attributes would be in the subclass. This of course does not rule out statements of coextensiveness which are necessary among attributes not in the subclass; for example, if \( F(x) \) is some predicate function, \( \neg F(x) \) is logically equivalent to and hence identical with it. But to put both in the subclass of classes would falsify extensionality. Nor can one use extensionality to pick out just those attributes which are to be called classes, since it may be the case that every attribute can be paired with a coextensive and non-identical attribute. Thus the subclass of classes

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must be picked out in some other way, though it will be true that extensionality holds within the subclass.

It should be clear that this view of classes, that all classes are attributes but not conversely, is not one which Quine, if understood correctly, would now accept.\(^\text{23}\)

For one thing, Quine believes that the notion of class is clearer than that of attribute because the identity conditions for classes are clear, whereas it is not clear what the identity conditions for attributes are. About this I shall have more to say below. At any rate, the claim that all classes are attributes seems only to be applicable in the finite case where, for every collection of individuals, one can in fact specify an attribute which determines it. But in the infinite case it is at least questionable that there is an attribute for every class. Quine, in fact, claims there are more classes than attributes, and thus could not accept the claim that classes are a subclass of attributes. If Quine is correct, the question that has been considered in this section, namely, whether an attribute is identical with the class it determines, might at first glance seem superfluous. The appearance is somewhat deceptive. For even if there are more classes than attributes, it might still be the case that some classes are attributes, and some are not, and

further, that some attributes are not classes. However, one must point out that such a strange view would probably involve its proponent in the claim that those classes which are attributes stand in both the member and instance relation to individuals, a claim which one does not need to make if all classes are attributes. For in the latter case, speaking of the membership relation would serve no purpose, as shall be discussed below, while in the former case, it would be difficult to see what classes which were not and classes which were attributes had in common, if not the membership relation.

Furthermore, it is certainly not clear that Quine is correct in his contention that there are more classes than attributes. For one thing, Quine argues that attributes for certain classes, e.g., those that constitute the real numbers, are not specifiable; but does this entail that there are no such attributes? For another, it is not clear that the claim that there are some classes, e.g., classes of individuals, entails that there are those classes which, so to speak, outrun the attributes. Third, the claim that there are more classes than attributes raises the question as to what a class is. As shall be discussed presently, the fact that their identity conditions are clearer than that of attributes is a weak justification for according them existence. For to what do the
identity conditions apply? Attributes may not have identity conditions that are clear, but that at least some attributes are perceived seems uncontroversial. Does one perceive classes? To get some perspective on this question, return to Bergmann's view in the finite case.

In the finite case there would be, on a view like Bergmann's, no sense in distinguishing classes from all attributes. This entails that one no longer needs to speak of a special relation of membership that classes have to their members. One can speak instead of the instance relation that attributes which are classes have to their instances. But does this mean that there is no distinction one can draw between classes and attributes? The case is not that clear. For all Bergmann has done is show that the class calculus he constructed in the finite case, and a certain part of the attribute\textsubscript{1} theory, are isomorphic. But this does not show that there might not be more than one interpretation of what amounts to one formal theory. As Goodman once pointed out, to show that the relations between gorillas and the relations between intersecting lines are isomorphic, does not show that gorillas are intersecting lines. The problem, in other words, is this: How can one justify Bergmann's claim, the one which has been accepted so far, that classes are a subclass of attributes\textsubscript{1}?
Now certainly Bergmann can in a perfectly reasonable way justify his contention that there is no sense in distinguishing classes from attributes. Given that one should not multiply kinds of things unnecessarily, there is no sense in claiming there are two kinds when one can get by with one. This is a position which no one would take if they did not feel, from the first, that it is not clear what classes are. But what does this mean? In Bergmann's case it means that, unlike the case of individuals and attributes, classes are not objects of perception. But of course one cannot get by without classes. Now since the objects Bergmann speaks of must be kinds of things with which one is perceptually acquainted, the preservation of the truth that there are classes takes a reductive form. Some attributes have at least all the formal properties that classes are said to have, and that is all that one needs for the reduction. Or, to put the point another way, nothing essential is lost in the reduction, since one cannot specify what other properties classes have besides these formal ones which attributes also have.

Bergmann is not alone in this belief that on perceptual grounds classes should not constitute some ultimate ontological category. Quine once thought something similar; he dismissed classes on the grounds that he did
not believe in abstract entities. Now he has changed his mind about classes. In fact, in Set Theory and Its Logic, he claims that the notion of class is clearer than that of attribute because, as has already been brought out, the identity conditions for classes are clearer. This seems to be a strange position indeed. Surely there is at least one commonsensical notion of attribute which is clearer than that of class. And presently, the reasons for this view will be given. The point now is that Quine, relying on a formal, theoretical notion of attribute and class, concludes that the notion of class is clearer than that of attribute. Here one must agree with Cartwright, that it is not at all clear that such formal notions will do to distinguish classes from attributes. The question is, how does one interpret such a formal theory? For the identity conditions for classes are clear, only if one can specify what sort of thing they applied to.

Why is there at least one commonsensical notion of attribute which is clearer than that of class? Seeing a shade of color, one can recognize it again. What one sees and what one recognizes is the attribute. Or, one can distinguish one attribute from another, e.g., a shade of red from another shade, or from a shape. Does one make this distinction perceptually on the basis of properties

\(^{24}\text{Ibid.}, \ p. \ 2.\)
of the distinguished attributes? It can be argued, that it is not necessarily the case that attributes are distinguished by their properties, but rather to use traditional terms, are merely qualitatively distinct. This stands in direct contrast to Quine, who seems to think that existence should only be accorded those entities which have clear identity conditions. This is one of his reasons for denying existence to attributes. But, perhaps not all difference is criterial. Returning to the issue of perception, when one perceives a shade of color, one may recognize it as such without perceiving all the individuals which have it. However, since classes are extensional, it seems that to perceive them, we would have to perceive their members. This is one reason for claiming that one does not perceive a class when one perceives its determining attribute. It of course does not show that one does not perceive at least some classes. Russell, however, made it clear that the problem over perceiving classes was not just one of the spatial or temporal inaccessibility of their members. Consider, for example, the class of red things. Russell points out that the class is not the heap of things belonging to the class, not the individuals, not the attribute red.\footnote{Russell, Introduction to Mathematical Philosophy, pp. 182-184.} The point is that Russell exhausts
those categories of entities he finds to be commonsensical in the above sense, i.e., categories of objects perceived. He then is puzzled as to what a class could possibly be.

He had an answer. Given that one does perceive attributes, the notion of attribute is clearer than that of class. Russell then tried to "reduce" classes to attributes. That is, he tried to find a subclass of attributes that had the same formal properties as classes. Now it may be said of Russell, and Bergmann, too, that the attributes they reduce classes to are highly complex, in the sense of being represented by defined predicate expressions, and hence no clearer perceptually than classes. But their view could (whether or not they wish to) be defended by claiming that the notion of a complex attribute depends on the notion of a simple one, and simple attributes are perceived. By "depends" one means only that knowledge of the existence of a category of attributes is, for Russell and Bergmann, dependent upon the fact that we perceive some attributes. This is in clear contrast to a view like Quine's, where the notion of attribute seems purely formal. Neither Russell's nor Bergmann's is. One understands the notion of attribute, in the final analysis, because one perceives attributes. It is still open, of course, to claim that classes are both different from attributes and perceived. But this is far from obvious, and surely it
would take argument. But the very fact that the case
would have to be argued at all makes it suspicious that
they are perceived.
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