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The Ohio State University, Ph.D., 1974
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A NORMATIVE MODEL OF NET OPERATING INCOME AND THE
EFFECT OF INFLATION ON THE MARKET VALUE
AND EQUITY INTEREST OF MULTI-FAMILY
DWELLING UNITS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

Joseph Goldsten, B.S., M.S.

The Ohio State University
1974

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I wish to express my appreciation to Professor Ronald L. Racster, Associate Professor of Finance, The Ohio State University, and Chairman of my Dissertation Committee, for his guidance and assistance over the past years. I wish to acknowledge also the valuable suggestions and criticisms of Professor W. Wayne Talarzyk, Professor of Marketing, The Ohio State University.

This research would not have been possible without the cooperation of Mr. Ronald Vukas and Mr. Charles A. Achilles of the Institute of Real Estate Management.

In addition, I wish to thank Professor Frederick J. Nowak, Instructor, Administration, Washington and Lee University, for his invaluable assistance in computer programing.

My grateful appreciation is also due Sandra Vinson who devoted many hours to the typing of the final draft of this dissertation.

Finally, a special debt of thanks is owed my wife, Eve, for countless hours spent typing the many drafts of this dissertation, and more importantly, for her helpful editorial assistance, her constant encouragement, and continuous patience throughout the past years.
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>ii</td>
</tr>
<tr>
<td>VITA</td>
<td>iii</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>vi</td>
</tr>
<tr>
<td>CHAPTER</td>
<td></td>
</tr>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>The Problem</td>
<td>1</td>
</tr>
<tr>
<td>Research Objectives</td>
<td>4</td>
</tr>
<tr>
<td>Benefits</td>
<td>5</td>
</tr>
<tr>
<td>Data and Methodology</td>
<td>7</td>
</tr>
<tr>
<td>Outline of Sections to Follow</td>
<td>8</td>
</tr>
<tr>
<td>II. THEORETICAL AND EMPIRICAL BACKGROUND OF VALUATION</td>
<td>10</td>
</tr>
<tr>
<td>Introduction</td>
<td>10</td>
</tr>
<tr>
<td>Theory of Valuation</td>
<td>10</td>
</tr>
<tr>
<td>Income Property Appraisal</td>
<td>15</td>
</tr>
<tr>
<td>Estimation of Net Operating Income</td>
<td>22</td>
</tr>
<tr>
<td>Summary</td>
<td>26</td>
</tr>
<tr>
<td>III. INFLATION AND ITS IMPACT ON VALUE</td>
<td>27</td>
</tr>
<tr>
<td>Introduction</td>
<td>27</td>
</tr>
<tr>
<td>Economic Theory of Inflation</td>
<td>28</td>
</tr>
<tr>
<td>Effect of Inflation on Market Value</td>
<td>34</td>
</tr>
<tr>
<td>Equity Interest and the Wealth Transfer Effect</td>
<td>40</td>
</tr>
<tr>
<td>Summary</td>
<td>44</td>
</tr>
<tr>
<td>Pragmatic Interpretation of Inflation</td>
<td>46</td>
</tr>
<tr>
<td>Summary</td>
<td>49</td>
</tr>
<tr>
<td>IV. RESEARCH APPROACH</td>
<td>50</td>
</tr>
<tr>
<td>Introduction</td>
<td>50</td>
</tr>
<tr>
<td>Objectives</td>
<td>51</td>
</tr>
<tr>
<td>Research Design</td>
<td>52</td>
</tr>
<tr>
<td>Sources of Data</td>
<td>53</td>
</tr>
</tbody>
</table>

iv
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data Limitations</td>
<td>57</td>
</tr>
<tr>
<td>Variables</td>
<td>60</td>
</tr>
<tr>
<td>Summary</td>
<td>76</td>
</tr>
<tr>
<td>V. METHODOLOGY</td>
<td>78</td>
</tr>
<tr>
<td>Introduction</td>
<td>78</td>
</tr>
<tr>
<td>Net Operating Income Model</td>
<td>78</td>
</tr>
<tr>
<td>Equity Model</td>
<td>89</td>
</tr>
<tr>
<td>Summary</td>
<td>92</td>
</tr>
<tr>
<td>VI. RESULTS</td>
<td>94</td>
</tr>
<tr>
<td>Introduction</td>
<td>94</td>
</tr>
<tr>
<td>Net Operating Income Model</td>
<td>95</td>
</tr>
<tr>
<td>Equity Model</td>
<td>124</td>
</tr>
<tr>
<td>Summary</td>
<td>131</td>
</tr>
<tr>
<td>VII. CONCLUSIONS</td>
<td>134</td>
</tr>
<tr>
<td>Introduction</td>
<td>134</td>
</tr>
<tr>
<td>NOI Model</td>
<td>135</td>
</tr>
<tr>
<td>The Effects of Inflation</td>
<td>138</td>
</tr>
<tr>
<td>Implications</td>
<td>141</td>
</tr>
<tr>
<td>Future Work</td>
<td>144</td>
</tr>
<tr>
<td>APPENDIX</td>
<td></td>
</tr>
<tr>
<td>A. IREM DATA FORM AND INSTRUCTION SHEET</td>
<td>147</td>
</tr>
<tr>
<td>B. SUMMARY LIST OF VARIABLES</td>
<td>154</td>
</tr>
<tr>
<td>C. FACTOR ANALYSIS</td>
<td>157</td>
</tr>
<tr>
<td>D. MODELS--DUMMY VARIABLES</td>
<td>162</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>164</td>
</tr>
</tbody>
</table>
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-1</td>
<td>SUMMARY OF MARKET VALUE ILLUSTRATIONS</td>
<td>40</td>
</tr>
<tr>
<td>3-2</td>
<td>SUMMARY OF EQUITY MODEL ILLUSTRATIONS</td>
<td>45</td>
</tr>
<tr>
<td>5-1</td>
<td>FACTOR LOADING MATRIX</td>
<td>83</td>
</tr>
<tr>
<td>6-1</td>
<td>FACTOR-VARIABLES</td>
<td>96</td>
</tr>
<tr>
<td>6-2</td>
<td>FACTOR ANALYSIS OF INTERNAL VARIABLES</td>
<td>97</td>
</tr>
<tr>
<td>6-3</td>
<td>ROTATED FACTOR LOADINGS DEFINING DIMENSIONS OF INTERNAL VARIABLES AND COMMUNALITIES</td>
<td>98</td>
</tr>
<tr>
<td>6-4</td>
<td>FACTOR ANALYSIS OF EXTERNAL VARIABLES</td>
<td>101</td>
</tr>
<tr>
<td>6-5</td>
<td>ROTATED FACTOR LOADINGS DEFINING DIMENSIONS OF EXTERNAL VARIABLES AND COMMUNALITIES</td>
<td>102</td>
</tr>
<tr>
<td>6-6</td>
<td>FACTOR ANALYSIS OF INTERNAL-EXTERNAL VARIABLES</td>
<td>104</td>
</tr>
<tr>
<td>6-7</td>
<td>ROTATED FACTOR LOADINGS DEFINING DIMENSIONS OF INTERNAL-EXTERNAL VARIABLES AND COMMUNALITIES</td>
<td>105</td>
</tr>
<tr>
<td>6-8</td>
<td>MATRIX OF CORRELATION COEFFICIENTS</td>
<td>111</td>
</tr>
<tr>
<td>6-9</td>
<td>NOI MODEL NUMBER 1</td>
<td>113</td>
</tr>
<tr>
<td>6-10</td>
<td>NOI MODEL NUMBER 2</td>
<td>114</td>
</tr>
<tr>
<td>6-11</td>
<td>NOI MODEL NUMBER 3</td>
<td>115</td>
</tr>
<tr>
<td>6-12</td>
<td>NOI MODEL NUMBER 4</td>
<td>116</td>
</tr>
<tr>
<td>6-13</td>
<td>STABILITY OF COEFFICIENTS</td>
<td>118</td>
</tr>
<tr>
<td>6-14</td>
<td>CONFIDENCE INTERVALS NOI MODEL NUMBER 1</td>
<td>119</td>
</tr>
<tr>
<td>6-15</td>
<td>CONFIDENCE INTERVALS NOI MODEL NUMBER 2</td>
<td>120</td>
</tr>
<tr>
<td>6-16</td>
<td>SENSITIVITY OF NOI MODEL NUMBER 1</td>
<td>121</td>
</tr>
<tr>
<td>Table</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>--------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>6-17</td>
<td>SENSITIVITY OF NOI MODEL NUMBER 2</td>
<td>122</td>
</tr>
<tr>
<td>6-18</td>
<td>AVERAGE MARKET VALUE PER ROOM</td>
<td>125</td>
</tr>
<tr>
<td>6-19</td>
<td>AVERAGE MORTGAGE PER ROOM</td>
<td>126</td>
</tr>
<tr>
<td>6-20</td>
<td>EQUITY MODEL</td>
<td>127</td>
</tr>
</tbody>
</table>
CHAPTER I

INTRODUCTION

The Problem

Financial and economic theory suggest that Net Operating Income, the residual after deduction of expenses and before depreciation and financing charges, is the fundamental source of value, modulated by the capitalization rate. In an economic environment characterized by relatively severe inflation, estimates of value must recognize specifically the impact of inflation. This dissertation is directed at the problem of estimating Net Operating Income\(^1\) over the economic life of a particular class of property, multi-family housing; quantification of the effect of building age on Net Operating Income; and the impact of inflation on Net Operating Income,\(^2\) market value, and equity interest.

The critical investment decisions of developers, investors, and mortgagors, require predictions of value. The ability to estimate NOI and the impact of inflation is crucial to the efficient allocation of resources within the housing market and the economy. If the NOI stream is understated, the capitalized value may be insufficient to justify new

\(^1\)Net Operating Income equals Effective Gross Income minus expenses.

\(^2\)Hereafter, the term Net Operating Income will be represented by the symbol NOI.
construction. If the NOI stream is overstated, the capitalized value may justify the building but may result in misallocation of resources.

**Value**

Four classifications of value are most frequently estimated: (1) market value, (2) the most probable selling price, (3) investment value, and (4) equity interest. Although numerous definitions have been developed for market value, the concept implies "the price the property would bring in a competitive market under all conditions requisite to a fair sale, which would result from negotiations between a buyer and a seller, each acting prudently, with knowledge, and without undue stimulus."³

The most probable selling price, as defined by Ratcliff,⁴ is a more flexible and realistic approach, recognizing value as the most likely selling price under existing market conditions.

Investment value is based on the NOI stream discounted at rates reflecting the individual investor's personal financial circumstances, including taxes, opportunity costs, available financing, and portfolio of investments. Investment value is a highly individualistic and personal concept of value.

Estimates of market value and the most probable selling price


are market derived. The "income approaches," i.e., discounting the NOI stream, are the most common procedures for determining market value and, in many instances, the most probable selling price. The use of Gross Income\(^5\) multipliers, NOI multipliers, and regression techniques is gaining credence based on empirical studies. Value is a direct function of NOI; consequently, where GI multipliers and regression techniques are employed, some norm of NOI is implicit and all valuation techniques explicitly or implicitly include an estimate of NOI.

**Estimating NOI**

Numerous books and articles by recognized authorities define a variety of techniques for estimating and evaluating future NOI streams. Development of quantified models using statistical inference to estimate NOI or to define norms over time, however, is sparse. The quality of estimates employed for purposes of valuation is dependent to a large degree upon personal judgment which may reflect a wide latitude of structure and experience.

Approximately forty years ago, Frederick M. Babcock\(^6\) stated four premises in the form of curves and tables, defining NOI as a function of building age. Premise I assumes that future income is constant and is equivalent to the average stabilized NOI commonly used by appraisers. Premises II and III state that income is an increasing negative function of age. Premise IV assumes a straight line declining

---

\(^5\)Hereafter, the term Gross Income will be represented by the symbol GI.

NOI. As related to Premises II, III, and IV, Babcock states:

No such study has been made in which a significant number of instances were included to draw a general conclusion with respect to the exact shape to give such income curves.7

As there are no published quantified and empirically tested models which can be used as characteristic guides for NOI estimates, the Babcock premises receive little attention in current real estate practice. However, the decline of NOI over the economic life of an apartment building is widely recognized.

Inflation

The accelerating rate of inflation over the past eight years has direct bearing on NOI, market value, and equity interest. The consensus of authoritative practitioners and academicians is that real estate is an inflation hedge.8 The persistent claim that NOI, market value, and equity interest exceed or keep pace with inflation is a primary motive for investing in real estate. Quantified support for this claim with respect to multi-family housing is not evident.

Research Objectives

This research has three primary objectives: One, the development of a normative model of NOI using variables derived from the economic environment; Two, empirical quantification of the effect of

7 Ibid., p. 412.

building age on average NOI; and Three, determination as to whether or not income producing multi-family housing is an inflation hedge.

Due to the large number of unique variables to which each parcel of real estate is subject, the model is not expected to predict NOI for a specific property but rather to predict the average NOI for the class of property. The model defines the effect of building age on NOI. The Babcock premises are quantified and tested for the specific class of property. Measurement of the holding period change in NOI, market value, and equity interest under inflation facilitates testing of the widely-held belief that multi-family housing is an inflation hedge.

Benefits

The primary benefits of this research are derived from adaptation and projection of economic theory to the development and empirical quantification of normative Net Operating Income and equity models, based on aggregate data. The principles of valuation theory are extended to recognize explicitly the effects of inflation and building age in the NOI model and inflation and the wealth transfer effect in the equity model. These models have pragmatic value as they identify a small number of variables conveying a large body of information that is both historically accessible and amenable to estimation by practitioners. As normative models, they provide a structure for analysis not previously available.

The NOI and equity models can be used by appraisers, mortgagors, and policy makers to augment current valuation procedures. Based on
average data, they provide a standard of comparison as well as insights into the effects on NOI and value of key variables such as inflation, building age, capitalization rates, and loan-to-value ratios. Estimates of reversion value by current techniques are, at best, tenuous, but nevertheless, critical to many investment decisions. The models provide a framework for estimating future productivity, critical to reversion value, and serve as a standard for the class of property studied.

The widely accepted theory of real estate valuation defined by Babcock in his classic work, is extended in this research to include empirically based mathematical models. The identification of key variables and their interrelationships in estimating the productivity of income producing property over time, is a significant step forward.

The models define and explain relationships between productivity, value, building age, and inflation which are vital to the development of policy in regard to housing and taxation at both the local and national level. Consequently, normative models are particularly significant. If private interests are to be motivated to construct necessary housing, then housing and tax policy must recognize the effects of inflation, building age, capitalization rates, and loan-to-value ratios on present and future productivity as defined in the models.

This research is unique in that there is little evidence of comparable inquiry. This dissertation should hopefully stimulate similar studies in regard to other classes of property as well as analysis on the basis of market and/or geographical stratification. The development of models that facilitate accurate prediction of NOI and
value over time for specific properties appears to be a likely extension of this research.

**Data and Methodology**

Characteristically, real estate research is constrained by the limited availability of data. As a result, a large percentage of published research is based on the personal files of a limited number of individuals and institutional sources. In general, data limitations have precluded statistical insights into the significance of variables affecting NOI.

This research employs data developed from apartment house operating experience and the economic environment. Data from the years 1962-1971, including forty-four independent variables suggesting causal relationships with variation in NOI, are used in developing a model of NOI. Data from the years 1965-1971 are utilized in demonstrating the effect of inflation on market value and equity interest.

Analysis is confined to low-rise, multi-family dwellings over twenty-five units. Operating experience data defining income and expenses on a per room basis are gathered by the Institute of Real Estate Management. Data defining the economic environment are collected from institutional and governmental sources. Data are aggregated as arithmetic averages for each year by age classification of the building. Sampling is assumed to be random.\(^9\)

Development of the NOI model employs the technique of factor

\(^9\)There is, in fact, no statistical means of establishing randomness.
analysis and stepwise multiple regression. Factor analysis identifies a small number of independent variables defining the essential dimensions of the data. These variables, having high explanatory and predictive power, are introduced into the regression models.

Stepwise multiple regression is used to predict NOI and to identify those independent variables which are statistically significant. The effect of each included variable on NOI is quantified.

A determinate equity model is developed to determine whether or not income producing multi-family housing is an inflation hedge. The model facilitates isolation of the annual holding period change in market value and equity interest as a measure of the impact of inflation.

Outline of Sections to Follow

Chapter II defines the most widely accepted theoretical concepts of value. These economic concepts are discussed in terms of income property appraisal theory and practice, demonstrating the significance of NOI and the capitalization rate in determining value. Emphasis is on NOI as the primary determinant of value and the effects of building age on NOI.

Chapter III includes a brief overview of the economic theory of inflation including the effects of anticipated and unanticipated inflation, real and nominal interest rates, and the wealth transfer effect. These economic concepts are used to demonstrate the theoretical relationships between the wealth transfer effect, NOI, capitalization rate, interest rates, market value, and equity interest within the
framework of real estate markets. A pragmatically oriented determinate equity model is developed based on theoretical constructs. The model is employed subsequently in Chapter VI to determine empirically whether the class of property is an inflation hedge.

Chapter IV includes the specific objectives and goals of the research, discussions of limitations of real estate research, an analysis of sources and constraints of data, and identification of variables.

Chapter V is a discussion of the research design and methodology. The use of factor analysis and stepwise multiple regression is justified as the appropriate modeling technique for predicting NOI and quantifying the impact of included variables. The employment of a determinate equity model is justified as a means of measuring the impact of inflation on property value and equity interest and of demonstrating the wealth transfer effect.

Chapter VI states the results of the factor analysis and defines the NOI model based on empirical data. Statistical significance of variables included in the model is discussed as well as sensitivity analysis and confidence intervals. Using the equity model, holding period change for market value, equity interest, and the wealth transfer effect under inflation are empirically quantified as a means of determining whether the class of property is an inflation hedge.

Chapter VII states the conclusions derived from the models and implications of these conclusions. Proposals are made for new areas of investigation.
CHAPTER II

THEORETICAL AND EMPIRICAL BACKGROUND
OF VALUATION

Introduction

The most widely accepted economic theories of valuation are expressed in terms of securities such as stocks and bonds. These theories provide significant insights into real estate valuation. As methods of real estate valuation are based on economic concepts of value, appraisal techniques must be compatible with economic facts and a by-product of economic thought.

In this chapter, the most widely accepted theories of valuation will be discussed and extended to real estate. The pragmatic implications of theory are stated. Both theoretical and empirical constructs demonstrate the importance of NOI in determining value.

Theory of Valuation

Merton H. Miller and Franco Modigliani conceptualize the essence of valuation theory in what they have described as the "fundamental principle" of valuation.\(^1\) It is based on the assumptions of perfect capital markets, rational behavior, and perfect certainty.\(^2\)


\(^2\)In terms of economic theory, perfect capital markets assume that: (1) transactions of buyers or sellers are not sufficiently
The "fundamental principle" is a theoretical construct defining value as the discounted present value of all future cash flows between the firm and the investor. Cash flows may be stated in terms of dividends or earnings. 3

Under the assumptions the returns will be uniform on all stocks as investors shift from low return to high return stocks until uniformity is attained at equilibrium. At equilibrium the "fundamental principle" must hold and may be expressed as follows:

\[
P_o = \sum_{t=1}^{n} \frac{D_t}{(1+r)^t} + \frac{P_n}{(1+r)^n}
\]

\(P_o\) = price of stock at beginning of holding period
\(P_n\) = expected terminal value at end of holding period
\(D_t\) = expected dividend at end of period \(t\)
\(r\) = constant expected rate of return obtainable on all stocks
\(n\) = holding period 4

large to influence price, (2) information on prices and other investment characteristics is costless and available to all participants in the market, and (3) there are no transactions costs or tax differentials between dividends and capital gains or between distributed and non-distributed profits. Rational behavior assumes that: buyers and sellers have no preference between wealth as cash payments or increase in market value but prefer more wealth to less wealth. Perfect certainty assumes that: each buyer and seller can predict accurately the future profits of each firm. The distinction between stocks and bonds disappears under this assumption; consequently, all investments are expressed as shares of stock.

3 Miller and Modigliani demonstrate that under conditions of certainty, perfect capital markets, and rationality, the Investment Opportunities Approach, the Stream of Earnings Approach, the Stream of Dividends Approach, and the Discounted Cash Flow Approach to valuation are mathematically equivalent.

Although the holding periods of investors vary, the terminal value is based on expectations of future dividends and terminal value accruing to the new owner. The total cash flow to successive investors is the summation of dividends plus a liquidating dividend. Present and/or future cash dividends are the only benefit stockholders receive from their investment. The basis of common stock valuation must be dividends.

Value is not a function of debt financing:

The market value of any firm is independent of its capital structure and is given by capitalizing its expected return at the rate appropriate to its risk class. The debt-to-equity ratio has no effect on value; consequently, value is a direct function of operating earnings discounted at a rate appropriate for the risk class. Thus, NOI is the primary determinant of value modulated by the capitalization rate. The discount rate applied to the NOI stream may increase or decrease value but value will remain so long as there is a positive NOI.

James C. Van Horne modifies the "fundamental principle" to provide a somewhat more pragmatic approach by substituting $k_e$ for $r$. The symbol $k_e$ represents a market determined rate of return required by investors for stocks in a given risk class. The market value is
the present discounted value of expected dividends and market price at
the end of the holding period. The principle may then be expressed as:

\[ P_0 = \sum_{t=1}^{n} \frac{D_t}{(1+k_e)^t} + \frac{P_n}{(1+k_e)^n} \] (2-2)

The expected market value at the end of the holding period, \( P_n \),
is the discounted value of expected dividends beyond the holding
period. In the case of common stock, corporations have an expected
infinite life. The second term of the equation is equal to the
present value of all future dividends from the end of the holding
period to infinity:

\[ P_0 = \sum_{t=1}^{n} \frac{D_t}{(1+k_e)^t} + \sum_{t=n}^{\infty} \frac{D_t}{(1+k_e)^t} \] (2-3)

The two terms of Equation (2-3) may be combined, defining value as the
summation of dividends to infinity discounted at a market determined
rate representing the risk class of the investment.8

\[ P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+k_e)^t} \] (2-4)

8Within the Miller and Modigliani assumptions, the basis of
stock valuation is the expected future dividends including liquidating
dividends. Stocks that pay no dividends can have positive market
value because of expected market price at the end of the holding
period. The holding period terminal value reflects the expectations
of future dividends, regular or liquidating.
The theory of valuation adapted to real estate

Valuation theory can be readily extended to real estate by pursuing the above arguments utilizing a firm having only one asset, income producing real estate. Equations (2-2), (2-3) and (2-4) may be adapted by defining the terms more specifically:

\[ P_o = \sum_{t=1}^{n} \frac{D_t}{(1+k_e)^t} + \frac{P_n}{(1+k_e)^n} \]  
\( (2-5) \)

\[ = \sum_{t=1}^{n} \frac{D_t}{(1+k_e)^t} + \sum_{t=n}^{\infty} \frac{D_t}{(1+k_e)^t} \]  
\( (2-6) \)

\[ = \sum_{t=1}^{\infty} \frac{D_t}{(1+k_e)^t} \]  
\( (2-7) \)

\( P_o \) = value of property at beginning of holding period  
\( P_n \) = expected reversion value at end of holding period  
\( D_t \) = NOI  
\( k_e \) = market determined capitalization rate for the risk class  
\( n \) = holding period

The reversion value at the end of the holding period will be based on expectations of future NOI accruing to the purchaser. The total cash return to successive investors is the summation of cash flows derived from NOI plus a possible liquidating payment. Whether the investment is in real estate or securities, valuation must be based on future cash flows.\(^9\)

\(^9\)Real estate investments with no positive cash flow can have positive market value because of reversion value which is based on future expected cash flows.
Economic theory extended to income producing real estate defines value as the summation of the expected NOI stream discounted at an appropriate capitalization rate. Both theory and experience indicate that the capitalization rate is always finite. In contrast, NOI can vary from positive to negative. When NOI is zero or negative, value has been extinguished. Thus, NOI is the fundamental source of value modulated by the capitalization rate. Consequently, income property appraisal focuses on estimation of NOI and the capitalization rate.

Income Property Appraisal

Methods of income property appraisal, as techniques of estimating value, are based on economic theory. The four commonly used classifications of value, market value, most probable selling price, investment value, and equity interest, are based on the same underlying economic concepts. Ratcliff does not distinguish clearly between market value and the most likely selling price; consequently, this research focuses primarily on market value, investment value, and equity interest.

The difficulties of making good estimates and the impossibility of arriving at precise estimates of property value regardless of the property or purpose of valuation are recognized by Babcock. He indicates why valuation techniques are limited to approximations:

Judgment plays the principal role in the valuation process and the methods of valuation serve only to break down the complex structure and thereby permit the valuator's judgment to play upon

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more phases of the problem and to take into account the manner in
which the various elements of value operate in creating the
values . . . If valuation technique offers only guides by means
of which to control and direct the valuator's judgment, then it is
obvious that there can never be such a thing as absolute valuation.
There is even a more important element in all valuation—an element
which precludes any possibility of absolute valuation . . . Values
are a reflection of probable future utilization of properties and
. . . the valuation process must, of necessity, deal with predic­tions and forecasts.11

The purchase of income producing real estate involves the
expectation of future productivity. The value of the acquired
property is proportional to the quality, magnitude, and timing of
expected future productivity. The means of converting future produc­
tivity into present value is capitalization. The emphasis on
estimation of NOI inherent in estimates of value is summarized by
Babcock:

... the anticipated future productivity is the primary consider­
ation. The capital value is secondary in character and is derived
from it.12

Supply and demand factors

The multitude of interacting supply and demand factors are
significant determinants of Gross Income, NOI, and value. Real estate
markets are local, regional, and national. Geographically defined
markets interact with each other by numerous linkages. These inter­
acting factors of supply and demand affect all income producing
multi-family housing both individually and collectively.

Supply and demand factors affecting apartment investment and

12Ibid., p. 21.
value are characterized by William Kinnard as follows:

**Supply Factors**

1. **Construction**: Volume, number of units, distribution by type of unit, net addition to standing stock
2. **Rents**: Levels, ranges
3. **Tenancy**: Owner occupancy versus rental
4. **Vacancy**: By type of unit, by location, by rent level
5. **Method of Rental**
6. **Competition**: Availability of units, size and variety of individual rental units, price level versus rent level, amenities offered by competitive projects, rates of change
7. **Standing Stock of Housing**: Number of units, type of units, location of units, condition of existing structures, age distribution of existing structures
8. **Construction Costs**: Materials, labor, land, zoning impact

**Demand Factors**

1. **Population**: Number of persons, rates of increase, family size, age distribution, number of families or households, natural increase (live births over deaths), net migration (in or out)
2. **Income**: Personal income, family income, average wage rates, disposable income, purchasing power, income distribution, correlation with population and employment
3. **Labor Force**: Skills, occupations, labor force participation rate
4. **Employment**: Total employment, sources of employment, fluctuations in employment, skill versus job requirements
5. **Industrial Activity**: Market, diversity, stability, sales
6. **Economic Base**: Activity, stability, diversity
7. **Trade**: Consumer expenditures, total sales
8. **Finance**: Personal savings, distribution of accounts, mortgage lending, mortgage holdings
9. **Transportation**: Types, costs, service, highway system
10. **Real Estate Market**: Number and volume of sales, average prices, interest rates
11. **Land Use**: Existing, planned, average value by type of use
12. **City Growth**: Rate of land absorption by type of use, total area of uses, direction of growth
13. **Tax and Assessment Structure**
14. **Geography**: Climate, topography, soil and subsoil
15. **Cultural Facilities**: Education, museums, theaters, churches

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The number and complexity of supply and demand factors severely complicate estimation of NOI and value. Many of the factors are intercorrelated and not quantifiable. A model that recognizes all supply and demand factors is impractical. Much of the development of appraisal methodology has been directed at the problem of identifying and quantifying a sufficient number of variables to represent these factors adequately.

Time horizon

From a temporal point of view, the relevancy of immediate markets and short-term value fluctuations is questionable. Valuation must reflect investment aspects over a longer period of time although forecasts over extended time periods are neither relevant nor feasible as capitalization methods diminish the significance of long-term projections. In practice, the characteristics of the future NOI stream are difficult to estimate, due primarily, to limitations of comparable data and the absence of a normative model.

Capitalization of income approaches

Capitalization of income appraisal techniques are widely used in estimating the market value of multi-family housing. The appraiser must do the following to provide the necessary inputs: (1) estimate the expected future GI, (2) estimate the expected operating expenses during the period the GI will be received, (3) determine NOI as GI

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14 Babcock, The Valuation of Real Estate, op. cit., p. 3.
less vacancies and delinquent rents, (4) estimate the duration of the NOI stream which may be based on the economic life of improvements or the typical investor holding period, (5) select the market determined capitalization or discount rate applicable to the property, and (6) select the capitalization technique to process NOI in view of the assumptions pertinent to the property. It is clear that the judgmental capacities of the appraiser are critical in all aspects of the estimate.  

Multipliers

Direct conversion ratios or multipliers express the relationship between the selected form of income treated as a perpetuity and the value of the property. Approximations of value based on direct conversion ratios have been widely used by mortgagors, real estate brokers, and appraisers for many years. More recently, the use of GI or NOI multipliers has received increasing attention as a means of determining market value. Ratcliff sums up the current state of the art by defining "market simulation" in terms of multipliers as an acceptable methodology for valuation. Using multipliers, prediction of the most probable selling price reflects the characteristics of buyers and sellers and how market forces of supply and demand interact.

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16 Conversion ratios are referred to as gross income multipliers, effective gross income multipliers, or net operating income multipliers.
in establishing price in a particular submarket.\(^{17}\)

A study of the work of 53 leading appraisers of income properties of all types, ranging in value from $11,000 to $22 million and located in 15 states, indicates that some form of direct conversion ratio is utilized as part of the analysis in 77 per cent of the appraisals.\(^{18}\) Conversion ratios based on GI and NOI are evident in equal frequency. A comparison between the values predicted by the use of conversion ratios and the final appraisal value indicates that the values are identical in 40 per cent of the cases. In 75 per cent of the cases, the difference is less than 3 per cent.

To evaluate the reliability of direct conversion ratios, Ratcliff develops twelve samples containing 14-25 properties with building age up to 10 years where income and selling price are known. Properties are sufficiently similar in each group to be regarded as "comparables" in normal appraisal practice.\(^{19}\) Regression equations are calculated for each sample with the selling price as the dependent variable and GI or NOI as the independent variable. In comparing the predicted prices with the actual selling price, the average deviation

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\(^{17}\) Richard U. Ratcliff, Modern Real Estate Valuation: Theory and Practice (Madison, Wisconsin: Democrat Press, 1965), p. 34. Direct conversion ratios, as commonly used by appraisers, do not recognize explicitly many of the variables imputed in the typical capitalization calculation such as economic life or reversion value.


\(^{19}\) Six of the nine samples obtained from Alameda County, California, include only four family structures and three vary from five to ten units. The size of apartment houses included in the three Chicago area samples is 26 to 80, 12 to 38, and 12 to 98 units.
does not exceed two per cent. The sample data demonstrate that, within the limits of the particular experiment, 80 to 96 per cent of the variation in selling price is explained by NOI or GI.\textsuperscript{20}

The hypothesis that a GI multiplier based on comparable properties can reasonably predict selling price is tested and verified utilizing a sample of 385 actual sales of residential income property in the Vancouver market during 1966, 1967, and 1968.\textsuperscript{21} The sample represents at least 75 per cent of the sales of income properties in the market area during the specified period.

Gross Income and Net Operating Income ratios are an acceptable basis for estimating the most probable selling price over the short run period provided the sample is of reasonable size and includes reasonably comparable properties.\textsuperscript{22} This infers that GI or NOI are primary determinants of value and that a vast amount of information accepted as pertinent to property value is contained in these two variables.

A striking similarity exists between multipliers and Price-Earnings ratios used in security analysis.\textsuperscript{23} Price-Earnings ratios of securities are generally interpreted to define the amount an investor

\textsuperscript{20}Ibid., pp. 44-48.


\textsuperscript{22}Ibid., p. 271.

is willing to pay for one dollar of current earnings. Security analysts have long realized that differences between the Price-Earnings ratios of securities are primarily attributable to differences in expectations of future earnings. The purchaser of securities is interested in "the proper relationship between present price and expected future earnings." Most assuredly, the purchaser of income property has the same interest.

All estimates of value based on the capitalization of income approaches explicitly or implicitly capitalize the estimated future NOI stream. GI without NOI has no economic value. Where multipliers are used, there is an assumed normative future stream of NOI that characterizes the class of property in the sub market.

**Estimation of Net Operating Income**

Fundamental characteristics of the NOI stream that must be considered in estimating value are the trend, the probable and possible fluctuation about the trend, and the duration of the expected income. Expected income from land is a perpetuity in that land is indestructible and will always have value based on future income. The economic life of a building is determined by expected NOI with building value approaching zero as NOI approaches zero.

**Depreciation**

NOI and property value decline through depreciation expressed

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as physical deterioration, functional obsolescence, and economic obsolescence. Physical deterioration is usually associated with use, wear and tear. This is generally a reflection of building quality and maintenance. Functional obsolescence, usually the most significant facet of depreciation, is associated with decreased utility, inadequacy, and architectural plan, style, and design. Economic obsolescence results primarily from environmental conditions such as neighborhood change, public transportation, real estate taxes, population patterns, declining local economy, and supply of competitive buildings.  

Babcock premises

Babcock developed a series of four premises defining different trends of NOI over time. These premises are expressed in the form of tables and curves described as "being selected empirically." Undoubtedly, the curves reflect considerable expertise although there is no supporting data.

Premise I assumes constant NOI occurring in equal annual installments. Premise II assumes annual installments with successive income installments declining to zero but being proportional to the annual amounts found in the value curve corresponding to a level annual annuity of 10 per cent. Premise III assumes annual NOI installments declining to zero being proportional to the annual amounts found in the


value curve corresponding to a level annual annuity at three and one-half per cent. Premise IV assumes NOI to occur in annual installments declining in equal amounts to zero.27

The income curve of each property is unique, reflecting all the associated variables. The relative decline of NOI over time is generally accepted but there are no quantified statistically determined models available to define the characteristic decline.

**NOI estimate methodology**

The typical procedure for estimating NOI is to estimate Effective Gross Income (EGI), GI less vacancies and delinquent rents. Operating expenses are then estimated and subtracted from EGI leaving the anticipated NOI for the year. It is customary to treat EGI and expenses as if they were collected and paid at the end of the yearly period. For the particular class of multi-family dwelling units being studied, one year leases are most common.

Expenses include all items necessary to maintain services such as total payroll expenses, utilities, management, painting and decorating, supplies, insurance, and real estate taxes. Expenses do not include depreciation or charges attributable to the financial structure.

The American Institute of Real Estate Appraisers suggests three steps for assembling and processing income data to be utilized in the income approach:

27Ibid., pp. 412-414.
1. Obtain the rent schedules and the percentage of occupancy for the subject property and for comparable properties for the current year and for several past years. This information provides gross rental data and the trend in rentals and occupancy. This data is then related and adjusted by the comparative method to ascertain the estimate of gross income which the subject property should produce to attract investors in the market.

2. Obtain expense data such as taxes, insurance, and operating costs being paid by the subject property and by comparable properties. The trend in these expenses is also necessary.

3. Estimate the remaining useful economic life of the building to establish the probable duration of its income.28

Source of data

For purposes of valuation, a primary source of data for preparing forecasts is the history of the property to be appraised. This data must be evaluated critically to determine if experience is consistent with similar properties. Where experience is consistent, average annual figures from the property should be employed. Where experience deviates significantly from comparables, adjustments must be made. Comparative information is generally available from sources such as the appraiser's files or from personal contacts with owners, appraisers, and brokers.

Published studies are employed frequently as guides for comparisons and estimates of future trends. These guides provide standards of performance generally based on large market segments. The Apartment Building Income—Expense Analysis, a compilation and

analysis of actual incomes and expenses experienced in apartment building operation, published by the Institution of Real Estate Management of the National Association of Real Estate Boards, is believed to be the most widely used authoritative study.

Summary

Valuation theory and appraisal practice demonstrate that the future NOI stream is a critical determinant of value. The use of multipliers, a capitalization of income approach, inherently assumes a future stream of NOI which characterizes the class of property. The ability to estimate the magnitude and characteristics of the future stream of NOI is critical to all income approaches.

The large number of variables affecting NOI has hampered the development of predictive NOI models. Through this research, a normative predictive NOI model is developed for a specific class of property.

From both a theoretical and pragmatic point of view, multipliers are justified as acceptable valuation techniques. In subsequent chapters multiplier valuation models are used to determine change in market value and equity value.
CHAPTER III

INFLATION AND ITS IMPACT ON VALUE

Introduction

The American economy has generally demonstrated a continuing inflationary trend since the early years of the 19th century, although some periods of severe deflation have been evident. Since 1965, the rise in prices has accelerated causing inflation to become one of the dominant problems of the United States economy.\(^1\) Inflation is characterized by a widely diffused rise in prices. In recent years, the inflationary trend has persisted and even accelerated through a period of mild recession.

Due to historically low rates of inflation and the assumption that property value keeps pace with inflation, theoretical and pragmatic constructs of value generally do not explicitly recognize inflation. The relatively high rates of inflation characterizing the economy in recent years suggest that estimates of value must demonstrate quantitative cognizance of inflation.

The inflationary effect on NOI, the capitalization rate, and debt, in the case of leveraged property, must be recognized specifically in valuation models. NOI and capitalization rate (CR) for a

specific property may rise or fall as a function of many variables including supply and demand factors, management, and economic age. Consequently, the effects of inflation are difficult to isolate and to measure.

The economic theory of inflation can be adapted to the development of determinate multiplier valuation models of unencumbered property or the equity interest in encumbered property. These models are used to evaluate the widely accepted belief that income producing property and in particular, apartment houses, are an inflation hedge. For purposes of this research, the term "inflation hedge" as commonly used, means preservation of purchasing power.

**Economic Theory of Inflation**

Economic theory defines inflation as a tax on money and a means

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3. The definition of the term "inflation hedge" is ambiguous and subject to individual interpretation. For example, an investment that rises in value over a non-inflationary or deflationary period may represent a reasonable return for the level of risk exposure. If the same rate of return, based on current dollars, occurs over an inflationary period and the rate of inflation equals the rate of return, some would interpret the investment as an inflation hedge—value has risen at the inflationary rate. In terms of constant dollars, the investment has a zero rate of return and in terms of opportunity costs could be viewed as a loss. A more logical definition of inflation hedge is that the investment value, expressed in constant dollars, must increase at a rate commensurate with the risk class and other qualities of the investment. Under conditions of inflation, return on investment must be viewed in terms of constant dollars. For example, an investment in which the value expressed in constant dollars does not change over an inflationary period has a zero rate of return. Therefore, the use of constant dollars neutralizes the impact of inflation and facilitates the expression of return independent of inflation.
of redistributing wealth from creditors to debtors. Inflation is a rise in the nominal price of goods and services resulting in a higher price for a given level of utility. Governmental indices are widely used as a recognized measure of inflation. The Consumer Price Index and the Wholesale Price Index are considered to be reliable indices of the relative level of inflation or deflation in various segments of the economy.

Prices must be viewed in relationship to the exchange value for goods--purchasing power. To facilitate comparison between price levels of different time periods, prices or value can be stated in "constant dollars" reflecting the price level and purchasing power for a base period. The base period has an index value of 100. The term "current dollars" refers to prices or value expressed in terms of the period in which they are recorded.

**Anticipated and unanticipated inflation**

Inflation results in the loss of wealth to holders of money. While inflation can mean a transfer of wealth from creditors to debtors, this is not necessarily the case. The critical distinction in terms of wealth transfer is between anticipated and unanticipated inflation.

Inflation has two critical dimensions--timing and degree of change in prices. Under fully anticipated inflation, the rate of price change and timing is accurately estimated and recognized in transactions. Unanticipated inflation occurs when the rate and timing of inflation are not fully anticipated.
Anticipated and unanticipated inflation can be seen in terms of interest rates as defined by Irving Fisher:

The theoretical relation existing between interest and appreciation implies, then, that the rate of interest is always relative to the standard in which it is expressed. The fact that interest expressed in money is high, say 15 percent, might conceivably indicate merely that general prices are expected to rise (i.e., money depreciate) at the rate of 10 percent, and that the rate of interest expressed in terms of goods is not high, but only about 5 percent . . . . . . . The money rate and the real rate are normally identical; that is, they will, as has been said, be the same when the purchasing power of the dollar in terms of the cost of living is constant and stable. When the cost of living is not stable, the rate of interest takes the appreciation and depreciation into account to some extent, but only slightly, and in general, indirectly. That is, when prices are rising, the rate of interest tends to be high, but not so high as it should be to compensate for the rise, and when prices are falling, the rate of interest tends to be low, but not so low as it should be to compensate for the fall. 4

"Real" and nominal interest rates

The "real" ex post interest rate on an investment will reflect change in purchasing power and can be either higher or lower than the nominal rate. This is demonstrated by a 10 year bond with a 7 per cent nominal rate and an inflationary rate of 4 per cent. Although the nominal rate of return is 7 per cent, the "real" rate of return is only 3 per cent. If neither inflation nor deflation is present, "real" and nominal rates are equal. The ex ante nominal interest rate depends upon anticipated inflation. For example, assume a 3 per cent annual inflation anticipated over a loan contract with the "real" rate of

interest at 4 per cent. The nominal rate would be 7 per cent.\(^5\)

The degree of inflation normally varies over the life of an investment. For example, if an anticipated inflationary rate of 5 per cent was expected to decline to 2 per cent over a period of years, the anticipated inflationary rate for a one year investment would be much higher than the ten year investment. The nominal rate of interest on a ten year loan would be less than the nominal rate of interest on a one year loan.\(^6\)

If inflation is fully anticipated, the nominal rate of interest would then include the "real" rate, plus the rate of inflation. The consensus is that inflation is rarely fully anticipated. Unanticipated inflation, in the extreme, can result in negative interest rates. For example, with "real" interest rates at 5 per cent, and an \textit{ex post} inflation rate of 5 per cent, a nominal interest rate of 10 per cent would have represented fully anticipated inflation. Assuming that inflation was anticipated to the degree that the nominal rate was 10 per cent and the rate of inflation was 10 per cent, the "real" rate of interest would be zero. Carrying the illustration to a higher level of inflation, 15 per cent, the "real" rate of interest would be minus 5 per cent. It is not difficult to find illustrations of negative interest.


\(^6\)Ibid., p. 73.
Wealth transfer

Unanticipated inflation results in wealth being transferred from owners of monetary assets to owners of monetary liabilities. Debtors do not gain from fully anticipated inflation but do gain from unanticipated inflation because of the two types of assets—monetary and real. Monetary assets may be defined as future claims to fixed amounts of money usually bearing fixed interest rates. The amount of a contractual claim is independent of inflation or deflation. Cash, time deposits, mortgages, and bonds are typical examples of monetary assets that define a fixed claim. For every monetary asset, there exists a monetary liability, the debtor's side of the same contract. A house, car, hour of labor, or apartment house are real assets which do not represent claims to fixed amounts of money. If the price of an obligation to deliver goods or services can change, it is a real liability.\(^7\) The transfer of wealth to the net monetary debtor, in current dollars, under conditions of unanticipated inflation is illustrated as follows:

\[
\text{equity} = \text{net wealth} \quad (3-1)
\]

\[
\text{equity} = \text{assets} - \text{liabilities} \quad (3-2)
\]

Liabilities and assets may be classified as real or monetary as defined above.

equity = (real assets + monetary assets)  
     - (real liabilities + monetary liabilities)   (3-3)

equity = (real assets - real liabilities)  
     + (monetary assets - monetary liabilities)    (3-4)

equity = net real assets + net monetary assets    (3-5)

Assume a firm has a real asset with a market value of $100.00 at the beginning of the holding period and no real liabilities. The firm has no monetary assets but has a monetary liability of $75.00 in the form of debt. (Net monetary assets are negative.)

$25.00 = ($100.00 - 0.00) + (0.00 - $75.00)
$25.00 = $100.00 - 75.00

If the general price level rises by a ratio of $Z$, over the holding period, theoretically, all real assets and real liabilities will rise by the same ratio.

\[ \text{new equity} = Z(\text{net real assets}) + (\text{net monetary assets}) \]  (3-6)

If the price level rises 10 per cent, the general price level ratio, $Z$, will be 1.10.

$35.00 = 1.10(100.00) - 75.00$

Ten per cent inflation results in a 40 per cent increase in equity. The increase in equity is derived from the increase in asset value expressed in current dollars.

If there are positive net real assets, equity will rise as the price level rises. Net real assets must equal equity for equity
to rise as much as the increase in price level. If net real assets exceed equity, equity can increase more than the rise in price level, as illustrated above. Net monetary assets are fixed; consequently, are not affected by the rise in prices. The wealth transfer effect in current dollars is due to the change in market value of the asset.

**Effect of Inflation on Market Value**

Direct conversion ratios or multipliers, as defined by Ratcliff, are a market simulation and consequently an acceptable valuation methodology. This is a pragmatic methodology using observed market data derived from comparable property. Although multiplier models are treated mathematically as a perpetuity, they are not perpetuities. Conceptualization as a perpetuity, particularly under inflation, leads to erroneous conclusions.

Theoretically, value defined by Equation (2-7) is equal to value defined by a multiplier model:

$$P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+k_e)^t} = \text{NOI(Multiplier)} = \frac{\text{NOI}}{CR}$$

The reciprocal of a multiplier is considered an overall capitalization rate, CR.

Equation (2-7) explicitly recognizes the expected $D_t$ and $k_e$. The multiplier model implicitly recognizes that NOI changes over time. CR and $k_e$ are normally not equal. Multiplier models, as a market

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8Ibid., pp. 674-675.
simulation, based on comparable property, subsume all variables critical to valuation. Consequently, income property value, expressed in nominal or current dollars, is defined as:

\[ C = f(\text{NOI}, CR) \]  
\[ C = \frac{\text{NOI}}{\text{CR}} \]  

\( C \) = Market Value in current dollars  
\( \text{NOI} \) = Net Operating Income  
\( \text{CR} \) = Capitalization Rate

The capitalization rate is the sum of two components—return on investment and recapture of investment. "Return on" may be viewed as an interest rate reflecting risk, liquidity, risk-free interest rate, and the general conditions prevailing in the money markets. "Return of" provides for recapture of investment at a rate compatible with the economic life of the property.

Recognizing the components of the capitalization rate, value is defined as:

\[ C = f(\text{NOI}, I, R) \]  
\[ C = \frac{\text{NOI}}{I+R} \]  

\( I \) = rate of return on investment  
\( R \) = rate of recapture of investment

Inflation is generally assumed to increase the NOI, the capitalization rate, and the required rate of return on investment. NOI and return on investment (I) can increase at different rates and at rates
other than the inflationary rate. The recapture rate (R) is not affected by inflation; consequently, with fully anticipated inflation, the capitalization rate cannot increase at the inflationary rate.\(^9\)

The recapture rate is typically a small portion of the capitalization rate.

Recognizing the effects of inflation over a single annual holding period and the different rates of change of NOI and return on investment (I), the new value at the end of the holding period may be expressed:

\[
C = f(\text{NOI}, I, R, p, i) \tag{3-11}
\]

\[
C = 
\frac{(1+p)\text{NOI}}{(1+i)I+R} \tag{3-12}
\]

\(i\) = rate of increase in return on investment over the holding period
\(p\) = rate of increase in NOI over the holding period\(^10\)

Simple numerical examples are used to illustrate the impact of inflation. For ease of illustration, the R component of the CR is arbitrarily selected. The market value at the beginning of the holding period expressed in current dollars is:

\[
C = \frac{10.00}{.08+.02} = 100.00
\]

\[
\text{NOI} = 10.00
\]

\[
I = .08
\]

\[
R = .02
\]

\(^9\)Depending on the method of capitalization, the recapture rate can decrease with inflation. Where the recapture rate is based on a sinking fund, the rise in interest rates accompanying inflation will reduce the sinking fund requirement.

\(^10\)Using ex post data, the terms \((1+p)\text{NOI}\) and \((1+i)I+R\) may be observed as NOI and CR respectively at the end of the holding period.
Assume an inflationary rate of 10 per cent over the holding period is fully anticipated; consequently, the NOI and required return on an investment increases 10 per cent. The recapture rate is not affected. The impact of inflation will increase value:

\[
C = \frac{(1+.10)(10.00)}{(1+.10)(.08+.02)} = 101.85
\]

Under conditions of fully anticipated inflation, value expressed in current dollars increases $1.85 or 1.85 per cent, substantially less than the inflationary rate of 10 per cent. Consequently, the property is not an inflation hedge. Value increases because the rate of recapture of investment did not increase with inflation.

Illustrating the extreme conditions, assume that the increase in NOI fully anticipates inflation but that inflation is fully unrecognized in terms of the required return on investment:

\[
C = \frac{(1+.10)(10.00)}{.08+.02} = 110.00
\]

Value has increased $10.00, a 10 per cent increase which equals the rate of inflation. The property is by definition an inflation hedge.

In the other extreme, assume that NOI does not increase, but the required rate of return on investment fully anticipates inflation:

\[
C = \frac{10.00}{(1+.10)(.08+.02)} = 92.59
\]

Value has decreased $7.41 or minus 7.41 per cent while the general
price level has increased 10 per cent.

Market value expressed in current dollars can be illusory in terms of an inflation hedge. Value expressed in constant dollars facilitates a direct interpretation of the effect of inflation. If the value expressed in constant dollars remains constant or increases over the holding period, then the property is, by definition, an inflation hedge.

Market value expressed in constant dollars ($V$) may be derived from market value in current dollars ($C$) by deflating the current dollar amounts using a suitable index of inflation ($P$):

$$V = f(P, C)$$  \hspace{1cm} (3-13)

$$V = \frac{1}{P} \cdot C$$  \hspace{1cm} (3-14)

Substituting Equation (3-11) for $C$:

$$V = f(NOI, I, R, p, i, P)$$  \hspace{1cm} (3-15)

Substituting Equation (3-12) for $C$:

$$V = \frac{1}{P} \left( \frac{(1+p)NOI}{(1+i)I+R} \right)$$  \hspace{1cm} (3-16)

Equation (3-16) specifically recognizes the effects on market value of the rate of inflation, change in NOI, and change in I over the holding period.

The possible inflationary effects on the variables can result
in a continuum of property value over an extended range. Different segments of the economy are subject to different rates of inflation. It is possible for the rate of change of NOI and I to deviate significantly from the rate of change in the price index. The mathematical relationships expressed in the valuation models, Equations (3-12) and (3-16), indicate that, ceteris paribus, for a property to be an inflation hedge, NOI must increase at least at the inflationary rate when the capitalization rate remains constant. If the capitalization rate increases with inflation, then the rate of increase in NOI must exceed the inflationary rate.

Using Equation (3-16), market values as determined in the previous illustrations may be expressed in constant dollars. Table 3-1 facilitates a comparison between market values expressed in current and constant dollars.

With an inflation hedge defined in terms of preservation of purchasing power, Table 3-1 demonstrates the benefits of expressing value and holding period change in constant dollars. If the holding period change in market value is zero or positive, the property is an inflation hedge.

11 The consensus is that the capitalization rate rises with inflation but at a lower rate.
TABLE 3-1
SUMMARY OF MARKET VALUE ILLUSTRATIONS\(^a\)

<table>
<thead>
<tr>
<th>(V)</th>
<th>Change in (V)</th>
<th>(C)</th>
<th>Change in (C)</th>
<th>(P)</th>
<th>(p)</th>
<th>(i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100.00</td>
<td>0.00%</td>
<td>$100.00</td>
<td>0.00%</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>92.59</td>
<td>-7.41</td>
<td>101.85</td>
<td>+1.85</td>
<td>1.10</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>100.00</td>
<td>0.00</td>
<td>110.00</td>
<td>+10.00</td>
<td>1.10</td>
<td>0.10</td>
<td>0.00</td>
</tr>
<tr>
<td>84.18</td>
<td>-15.83</td>
<td>92.59</td>
<td>-7.41</td>
<td>1.10</td>
<td>0.00</td>
<td>0.10</td>
</tr>
</tbody>
</table>

\(^a\)Rate of return on investment \((I)\) = .08
Rate of recapture of investment \((R)\) = .02
Net Operating Income at beginning of holding period = $10.00

\(^b\)Change over the holding period

Equity Interest and the Wealth Transfer Effect

A brief discussion of equity interest is crucial to the understanding of the impact of inflation on real estate investment. The inflationary effect on property value and the wealth transfer effect on equity interest are often confused.

The logical extension of the wealth transfer effect to income property suggests that in the vast majority of instances equity should rise more rapidly than the indices of inflation. Real estate investments are usually highly leveraged exceeding the debt-to-equity ratio of manufacturing firms. A typical apartment house investment has 25 per cent equity, 75 per cent monetary liabilities in the form of
mortgages and no significant monetary assets or real liabilities.\textsuperscript{12}

Equity value is a function of many variables including:

1. Security of the principal invested
2. Rate of return on the principal invested
3. Certainty of the returns
4. Possibility of capital appreciation
5. Relative ease of liquidation of the investment
6. Relative burden of managing the investment
7. Possibility of obtaining favorable financing or refinancing
8. Neighborhood trends
9. Possibility of producing income-tax-free income\textsuperscript{13}

These qualities are recognized implicitly in the capitalization rate and in equity interest, as equity interest is an inverse function of the capitalization rate.

Previously, equity has been defined as the sum of net real assets and net monetary assets. In more pragmatic terms, equity in constant dollars ($E$) may be expressed as market value ($V$) minus debt expressed in constant dollars ($M/P$).

$$E = f(V, \frac{M}{P})$$  \hspace{1cm} (3-17)

$$M = \text{debt in current dollars}$$

$$E = V - \frac{M}{P}$$  \hspace{1cm} (3-18)

Substituting Equation (3-15) for $V$:

$$E = f(NOI, I, R, p, i, P, M)$$  \hspace{1cm} (3-19)


\textsuperscript{13}Kahn, Case, and Schimmel, Real Estate Appraisal and Investment, op. cit., p. 123.
Substituting Equation (3-16) for V:

\[ E = \frac{1}{P} \frac{(1+p)\text{NOI}}{(1+i)I+R} - \frac{M}{P} \]  
(3-20)

The same assumptions and data used in the preceding section are used in the illustrations that follow. Indebtedness (M) is assumed to be constant over the holding period.

Using Equation (3-18), equity interest at the beginning of the holding period is:

\[ E = 100.00 - \frac{75.00}{1.00} = 25.00 \]

\[ V = 100.00 \quad M = 75.00 \quad P = 1.00 \]

Using Equation (3-20), assume a 10 percent inflation occurs over the holding period. The inflation is fully anticipated by a rise in the NOI but fully unanticipated in the capitalization rate. The new equity is:

\[ E = \frac{1}{1.10} \frac{(1+.10)10.00}{.08 + .02} - \frac{75.00}{1.10} \]

\[ P = 1.10 \]

\[ E = 100.00 - 68.18 = 31.82 \]

Equity in constant dollars has increased $6.82 or 27.3 per cent. Asset value, the first term of the equation, has not changed. The gain in equity resulting from the effects of inflation is fully attributable to the reduction in debt, the second term of the equation, expressed in constant dollars. Indebtedness in terms of real dollars has been reduced 9.09 per cent by a 10 per cent rise in the general
price level. Expressed in constant dollars, creditors have sustained a decrease in wealth of $6.82 while the debtor's wealth has increased $6.82. This is a direct transfer of wealth attributable to inflation.

A 27.3 per cent rise in equity versus a 10 per cent inflationary rate indicates that although property value may not be an inflation hedge, an equity interest can be an inflation hedge. If there were no indebtedness, the equity interest would not have changed. The 10 per cent loss by the creditors represents a 27.3 per cent gain to equity and is the result of leverage under inflation.

If NOI and the rate of return on investment demanded by investors (I) fully anticipates inflation, property value and equity expressed in constant dollars will decrease:

\[
E = \frac{1}{1.10} \frac{(1+.10)10.00}{(1+.10)0.08+.02} - \frac{75.00}{1.10} \]

\[
P = 1.10
\]

\[
i = .10
\]

\[
E = 92.59 - 68.18 = 24.41
\]

The wealth transfer effect represented by the decrease in debt is $6.82 but the market value of the building decreased $7.41 resulting in an equity decline of $.59 or minus 2.63 per cent. The equity interest does not represent an inflation hedge.

The most negative circumstance in terms of the equity interest would occur when (I) increases at the inflationary rate but NOI does not increase:

\[
E = \frac{1}{1.10} \frac{10.00}{(1+.10)0.08+.02} - \frac{75.00}{1.10} \]

\[
P = 1.10
\]

\[
i = .10
\]

\[
E = 84.17 - 68.18 = 16.00
\]
Equity has decreased $9.00 or minus 36 per cent. The $15.83 decline in property value has more than offset the benefit of the wealth transfer effect, $6.82.

The debt-to-asset ratio determines the magnitude of the wealth transfer effect for a given rate of inflation. The wealth transfer effect is a positive function of the inflationary rate over the holding period and the magnitude of the debt (usually mortgages) expressed in constant dollars.

Table 3-2 summarizes the above illustrations plus the effect of a .50 and .90 debt-to-asset ratio. The selected samples represent extreme points along a continuum. For the given conditions, an increase in the debt-to-asset ratio significantly increases the holding period change in equity interest. The change in M/P is the wealth transfer effect.

Summary

The equity model (Equation 3-20) explicitly recognizes in constant dollars the inflationary impact on property value and equity interest and the wealth transfer effect. These three dimensions are critical in assessing income property as an inflation hedge. The model is empirically useful as it is compatible with available historical data and within the scope of estimation by experienced appraisers. It is this model that is used to determine empirically if the class of property studied is, on average, an inflation hedge.
TABLE 3-2

SUMMARY OF EQUITY MODEL ILLUSTRATIONS\(^a\)
(Constant Dollars)

<table>
<thead>
<tr>
<th>E</th>
<th>Change (^b) in E</th>
<th>V</th>
<th>Change (^b) in V</th>
<th>M/P</th>
<th>Change (^b) in M/P</th>
<th>P</th>
<th>p</th>
<th>i</th>
<th>Debt to Asset Ratio</th>
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<tr>
<td>$10.00</td>
<td>0.00%</td>
<td>$100.00</td>
<td>0.00%</td>
<td>$90.00</td>
<td>0.00%</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
<td>.90</td>
</tr>
<tr>
<td>10.77</td>
<td>+7.70</td>
<td>92.59</td>
<td>-7.41</td>
<td>81.82</td>
<td>-9.09</td>
<td>1.10</td>
<td>0.10</td>
<td>0.10</td>
<td>.90</td>
</tr>
<tr>
<td>18.18</td>
<td>+81.80</td>
<td>100.00</td>
<td>0.00</td>
<td>81.82</td>
<td>-9.09</td>
<td>1.10</td>
<td>0.10</td>
<td>0.00</td>
<td>.90</td>
</tr>
<tr>
<td>2.36</td>
<td>-76.40</td>
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<td>-15.83</td>
<td>81.82</td>
<td>-9.09</td>
<td>1.10</td>
<td>0.00</td>
<td>0.10</td>
<td>.90</td>
</tr>
<tr>
<td>25.00</td>
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<td>100.00</td>
<td>0.00</td>
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<td>0.00</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
<td>.75</td>
</tr>
<tr>
<td>24.41</td>
<td>-2.36</td>
<td>92.59</td>
<td>-7.41</td>
<td>68.18</td>
<td>-9.09</td>
<td>1.10</td>
<td>0.10</td>
<td>0.10</td>
<td>.75</td>
</tr>
<tr>
<td>31.82</td>
<td>+27.30</td>
<td>100.00</td>
<td>0.00</td>
<td>68.18</td>
<td>-9.09</td>
<td>1.10</td>
<td>0.00</td>
<td>0.10</td>
<td>.75</td>
</tr>
<tr>
<td>16.00</td>
<td>-36.00</td>
<td>84.17</td>
<td>-15.83</td>
<td>68.18</td>
<td>-9.09</td>
<td>1.10</td>
<td>0.00</td>
<td>0.10</td>
<td>.75</td>
</tr>
<tr>
<td>50.00</td>
<td>0.00</td>
<td>100.00</td>
<td>0.00</td>
<td>50.00</td>
<td>0.00</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
<td>.50</td>
</tr>
<tr>
<td>47.14</td>
<td>-5.70</td>
<td>92.59</td>
<td>-7.41</td>
<td>45.45</td>
<td>-9.09</td>
<td>1.10</td>
<td>0.10</td>
<td>0.10</td>
<td>.50</td>
</tr>
<tr>
<td>54.55</td>
<td>+9.10</td>
<td>100.00</td>
<td>0.00</td>
<td>45.45</td>
<td>-9.09</td>
<td>1.10</td>
<td>0.10</td>
<td>0.00</td>
<td>.50</td>
</tr>
<tr>
<td>38.73</td>
<td>-22.54</td>
<td>84.18</td>
<td>-15.83</td>
<td>45.45</td>
<td>-9.09</td>
<td>1.10</td>
<td>0.00</td>
<td>0.10</td>
<td>.50</td>
</tr>
</tbody>
</table>

\(^a\)Rate of return on investment \((I) = .08\), rate of recapture of investment \((R) = .02\), Net Operating Income at beginning of holding period = $10.00

\(^b\)Change over the holding period
Pragmatic Interpretation of Inflation

Under conditions of accelerating inflation, ownership of income property is widely accepted as an inflation hedge. This is based on the assumption that income and value rise at least at the inflationary rate. Discussion of change in income and value during an inflationary period tends to understate the significance of the less quantifiable factors of supply and demand as well as direct correlates of inflation such as capitalization rates and monetary and fiscal policy. Mindful of the fact that there have been periods of severe deflation of real estate values, the inflationary hedge concept appears to be based on the general belief that rents will keep pace with inflation. 14

According to Alfred Ring, 15 the appraiser need not be concerned with inflation so long as value is determined on a cash basis on unencumbered property in current dollars. However, under conditions of long term leases, rent control, or where property is subject to a mortgage and a split interest is to be estimated, the appraiser must consider inflation. In general, the assumption is that property income will move in direct relation to inflationary trends.

Ring asserts that few investments have equalled the inflationary protection afforded by improved real estate. The steep inflationary period between 1964 and 1970 is used to support the

14 W. R. Beaton, Real Estate Investment, op. cit., p. 16.

claim that apartments, shopping centers, and selected commercial properties, have increased in value in terms of constant dollars. The rise in real value is attributed to rapidly increasing costs of construction and the rising cost of money which reached statutory usury levels during this period. Ring summarizes his observations on inflation:

In the absence of rent and price control, real property, like a ship upon the ocean waters, floats above its purchasing power constant dollar line irrespective of depth or rise in the level of prices. It is this ability to uphold its purchasing power integrity that has in recent years popularized the demand for shares of real estate trusts and syndicates.\(^{16}\)

Arthur Weimar, Homer Hoyt, and George F. Bloom\(^{17}\) contend that the long historical trend of gradual inflation in the United States economy has been most important in stimulating real estate investment. With the exception of property poorly located, improperly designed, or subject to changes in the internal structure of land uses, real estate values, as measured in "real" dollars, have been maintained.

There is substantial evidence that over the past fifty years inflation in the United States has not been fully anticipated. In terms of bonds, notes, long-term leases, and mortgages, a transfer of wealth is evident from net monetary creditors to net monetary debtors.\(^{18}\)


\(^{18}\)Alchian and Allen, University Economics, op. cit., p. 679.
The consensus of authoritative writers is that real estate investment is an excellent inflation hedge with very few limiting conditions. The primary limitation is property subject to fixed payment leases extending over a long period of time.

By means of interviews with institutional investors and real estate developers, as well as a survey of periodical and trade journal articles, Maurice Kilbridge, Robert O'Block, and Paul Teplitz studied the most common methodologies used to determine investment value and investment rate of return. Analysis of the eight techniques commonly used indicates that the impact of inflation is either ignored or it is implicitly assumed that income will rise in proportion to the rise in prices.

Virtually no quantified analysis is available to support the contention that GI, NOI, and the value of real estate rise in step with inflation. In fact, some contend that real estate investment is not a desirable hedge: "For many years, Dr. Herbert B. Dorau, a noted urban-land economist, has been showing that large amounts of realty, not only do not advance in value faster than the loss of the dollar, but in many cases they lag behind many other commodities in price rises during inflationary eras."  

Gross Income has been the primary focus of analysis of income

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19 Maurice D. Kilbridge, Robert P. O'Block, and Paul V. Teplitz, Urban Analysis, Division of Research, Graduate School of Business Administration (Boston, Mass.: Harvard University, 1970), p. 100.

20 Kahn, Case, and Schimmel, op. cit., p. 313.
property as an inflation hedge. The primary determinants of value, NOI and the capitalization rate, have not been given adequate emphasis. The different effects of inflation on market value and equity interest have not been analyzed adequately. Based on the debt-to-equity ratio and the rate of inflation over the holding period, property may not be an inflation hedge as indicated by market value, but may be an inflation hedge as judged by equity interest.

**Summary**

The concepts of value defined in Chapter II have been extended to include the effects of inflation. Based on economic theory an equity valuation model is developed which is used to evaluate empirically the effects of inflation on equity interest and market value. The model recognizes that the effect of inflation on NOI and CR can be different as well as that the effect of inflation on equity interest is a function of market value and the debt-to-equity ratio.

Equity interest, market value, and debt expressed in constant dollars in the model facilitate a simple direct determination if the property is an inflation hedge as determined by the holding period change. In subsequent chapters, the model is used to determine empirically whether or not the class of property under study is an inflation hedge.
CHAPTER IV
RESEARCH APPROACH

Introduction

The characteristic limitations of real estate data are inherent in this research. While these limitations are well known to practitioners and researchers, they do not necessarily impugn the benefits of past or ongoing studies. This research must be viewed within the framework of these constraints.

The biases and restrictions associated with available data impose limits upon the statement and testing of generalizations. The defined populations typically include only the cases used in the analysis with generalizations extended to a broader population on an intuitively logical basis. Utilization of advanced sampling and statistical techniques is hampered by the paucity of data. The absence of organized markets and widely published details of transactions coupled with the diversified type of ownership places much of the financial data beyond the researcher's grasp. Sensitivity to age, type of building structure, location, financial structure, and quality of management, are frequently not recognized by the data. Unsystematic observations and limited reliability are often the case. Few researchers contend that their sample is a statistical representation of a specifically definable population. The cost of data acquisition is significantly greater than in other areas of investment.
There is great hesitancy on the part of investors, managers, and private owners of income-producing real estate to divulge pertinent financial information. Often, where information has been made available, it is has been under the condition that the source and specific location of properties remain confidential or anonymous.

This research (as well as most real estate research) assumes that aggregation of individual cases tends to ameliorate the effect of differences between properties. Of particular interest is the impact of varying managerial practices on maintenance, repair, and replacement as related to operating expenses and NOI. A wide variety of managerial policies consonant with the objectives of the various owners is evident. Expenses can be moved from year to year based on managerial decision. It is assumed that the impact of managerial policy is neutralized in aggregated data; i.e., properties demonstrating a high level of managerial proficiency are offset by a sufficient number of poorly managed properties resulting in a representative average.

Objectives

This research, directed at estimating NOI and the impact of inflation on multi-family housing units, has three primary objectives:

One, the development of an explanatory and predictive model of Net Operating Income based on operating experience and the economic environment. Using readily available data, the testable inferential model will emphasize a parsimony of statistically independent variables. For purposes of prediction, the included variables must be amenable to estimation by practitioners. The model will define a norm which can be
used to approximate individual properties.

Two, quantification of the effect of building age on Net Operating Income. The NOI model is used to define change in NOI as a function of building age and to quantify empirically the Babcock premises.

Three, determination as to whether this class of property is an inflation hedge based on change in market value, equity interest, and NOI as a function of inflation.

The objectives must be viewed in terms of a norm for the class of property from which the data are derived. Generalizations about other classes of investment property are not intended.

As operating data used in developing the NOI model are representative averages, the objectives are not stated in the form of hypotheses. The underlying distribution of the data cannot be measured. Where possible, rigorous statistical methodology is employed. The equity model is a determinate model based on averages and is not amenable to statistical analysis.

Research Design

Viewed in proper perspective, models are abstractions of reality rather than precise representations. They must, however, be sufficiently representative of reality to facilitate analysis and conclusions applicable to the real world.

To meet the objectives of this research, two models are constructed. A probabilistic model explains and predicts NOI and quantifies the effects of building age and inflation on NOI. A
determinate model is employed to demonstrate that the class of property is or is not an inflation hedge as evidenced by the change in market value and equity interest expressed in constant dollars.

The research design includes the following:

1. **Definition of the Problem** -- The general problem and specific objectives have been stated including the associated theoretical and empirical background.
2. **Identification of Variables** -- Relevant variables are selected suggesting causal relationships based on preliminary analysis of available data and background.
3. **Formulation of Preliminary Models** -- Based on objectives, accepted theory, and preliminary analysis of data, the general form of the models is defined.
4. **Selection of Methodology** -- The modeling methodology is justified in terms of the objectives and the available data.
5. **Estimation of the Parameters** -- Using the appropriate methodology, the constants and parameters are defined.
6. **Acceptance or Rejection of the Model** -- Based on tests of statistical significance, stability of parameters over time, confidence intervals, and sensitivity analysis, the ability of the NOI model to meet the objectives is assessed.

**Sources of Data**

Data utilized in this research are derived from the collection efforts of the United States Department of Commerce, the United States Department of Housing and Urban Development, the American Life
Insurance Association, and the Institute of Real Estate Management (IREM) of the National Association of Real Estate Boards.

The Institute of Real Estate Management collects data for publication of the Annual Apartment Building Income-Expense Analysis, a compilation and analysis of operating data. With successive enlargements in scope, information has been published annually for the past 17 years. The number of buildings represented in the survey has increased from 146 in 1956 to 2,161 in 1971, representing 211,827 apartments and 837,729 rooms. The survey includes multi-family dwelling units having more than 12 units per building with less than 20 per cent of the building in office or store occupancy. Building data are grouped into four classifications: (1) high-rise elevator buildings of four stories or more, (2) low-rise buildings under twenty-five units, three stories or less, (3) low-rise buildings twenty-five units or more, three stories or less, and (4) single management garden apartment buildings located on a sizeable landscaped lot.\footnote{1}{Apartment Building Income-Expense Analysis (Chicago, Ill.: Institute of Real Estate Management of the National Association of Real Estate Boards, 1963-1970).}

In addition to serving as a source of data, the Annual Apartment Building Income-Expense Analysis serves as a guide, a standard of comparison, and a basis for decision-making by counselors, appraisers, brokers, mortgagors, and property managers. Increasing use of this publication is evident.
IREM Data Collection

Data are collected from a file of past contributors, who own or manage property, accounting firms, banks, and insurance companies. The contributors may or may not be members of the Institute of Real Estate Management. The Institute states that every effort is made to obtain a sample representative of the population of the four types of apartments analyzed. Approximately two-thirds of the buildings from the previous year report the following year. With the total number of buildings increasing each year, the number of repeat buildings in each year is somewhat less than two-thirds.²

Due to the confidential nature of the data, examination of individual data forms is not permitted. The data form in combination with attached instructions facilitates collection of standardized data. (See Appendix A for the data form, "The Apartment Building Income-Expense Analysis," and instruction sheet, "How to Fill Out Your Apartment Building Income-Expense Analysis," used for collection of operating results for the year 1971.)

Detailed information relevant to income and expenses exclusive of mortgage payments and depreciation is reported on the basis of dollars per room. A standardized chart of accounts has been used for the past ten years. Data collected prior to 1962 are not compatible with subsequent data owing to the difference in the chart of accounts.

²This information is based on personal interviews with the Director of Administrative Services, Division of the Institute of Real Estate Management, May 17th, 1972.
Each contributor receives a copy of the publication plus an individual computer print-out on his building permitting a comparison with national, regional, and metropolitan averages.

The Institute analyzes each individual report and where the information appears to be inconsistent contacts the source to verify or to revise the report. Based on detailed knowledge of individual reports, the Institute claims that the sample is not biased in terms of the quality of management. Data collected by the Institute represents the largest sample of its type collected in a systematic manner. However, statistical justification that the data are representative or a random sampling of the four apartment populations, is not possible.\(^3\)

Data for the four types of buildings are available for national, regional, and metropolitan areas as well as age classifications of the structure for the years 1960 through 1971. The most consistent series is based on dollars per room \textit{per annum} extending from 1962 through 1971; consequently, it is utilized in this research.

Other data sources

Publications of the United States Department of Commerce and the Department of Housing and Urban Development provide data defining the economic environment. The Department of Commerce sources include: \textit{Survey of Current Business}, \textit{Business Statistics}, and \textit{Statistical

\(^3\)Ibid.
The American Life Insurance Association, sponsored by major life insurance companies, provides economic and investment research to assist the investment activities of life insurance companies. This research utilizes market-derived data on the capitalization rates and loan-to-value ratios provided by the Association.

**Data Limitations**

The Institute of Real Estate Management aggregates and publishes apartment house data in the statistical format of arithmetic averages only. Aggregation of data severely diminishes the statistical viability, forcing assumptions about the population. Statistical measures such as standard deviation, variance, and characteristics of the underlying distribution are not available.

Data are stratified into five age group classifications: (1) buildings erected since 1961, (2) from 1946-1960, (3) from 1931-1945, (4) from 1921-1930, and (5) from 1920 or before. Sample sizes from which each average is determined, are not the same. Gross sample size for low-rise buildings over 24 units varies from a minimum of 83 to a maximum of 329 buildings per year. The number of apartments included in this sample varies from a minimum of 3,699 to a maximum of 20,887 per year. The number of rooms, the basis upon which the data

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4 See specific categories under individual variables.
5 Apartment Building Income-Expense Analysis, op. cit., passim.
are reported, varies from 11,976 to 70,915.\textsuperscript{6}

In all instances, averages are a summation of raw data and not averages of averages. Variances of the raw data from which averages are developed will be larger than variances derived from averages. Use of averages provides considerable smoothing of the data; consequently, individual properties may vary beyond the limits evident in models based on averages.

By virtue of the uniquely large body of data included in the underlying sample, the models and analysis, based on averages, facilitate insights that have not been previously quantified or subjected to statistical interpretation. Quantification of relationships between variables is a significant advance over published work previously limited to personal experience or average annual data subjected to intuitive interpretation. Data limitations constrain the statistical power of the analysis; consequently, objectives are not stated in the form of testable hypotheses.

Assumptions

Due to the unknown statistical characteristics of the raw data, assumptions must be made to facilitate statistical analysis. As the modeling technique involves multiple regression, knowledge of the underlying distribution is important in evaluating results. This research assumes that all data are normally distributed about the mean. Consequently, arithmetic means are assumed to be the central

\textsuperscript{6}Ibid., 1962 through 1971.
value of a normally distributed representative sample.

The work of Recht and Lowenstein\(^7\) is used to justify the use of arithmetic averages as being representative of the sample. They demonstrated that returns expressed as GI divided by market value or sale price and NOI divided by market value or sale price were normally distributed about the mean. The data collected over the period of 1963 and 1964 were based on a sample of 370 multi-family residential transactions, representative of all types of apartment units in the city of San Francisco.

The study, which appears to be the largest sample from a metropolitan area subjected to analysis in terms of distribution of returns, is summarized as follows:

We found that the data for a given apartment type and time period assumed a "normal" distribution around the mean, thus supporting the use of average rates of return as a valid indicator of market values for investment and appraisal purposes.\(^8\)

Although the IREM states that the sample is random, there is no statistical basis for testing the random qualities of the underlying data. This research assumes that the data are random and representative of a national sample for the year in which they were collected. While the raw data sample size varies significantly, it is assumed that each average is representative of the defined population.


\(^8\)Ibid., p. 248.
Variables

Data are classified into two general categories: (1) internal variables derived from the Institute of Real Estate Management defining income and expenses and (2) external variables characterizing the general economic and business environment. Both internal and external data represent annual averages for the years 1962-1971. As collected, the internal variables are stratified by age classification of buildings.

Internal variables

Internal variables, $X_2$, through $X_{28}$, are expressed as averages. Variables $X_2$ through $X_{23}$ are expressed in dollars per room per year. The number of rooms is enumerated in the manner typical for the industry; a living room, dining room, bedroom, and kitchen are considered four rooms. In many apartments, particularly those built in recent years, the functions of the dining room and kitchen are not clearly delineated. If one room serves the functions of living room and dining room and is less than 260 sq. ft. it is counted as one room. If the room exceeds 260 sq. ft. it is counted as one and a half rooms. A breakfast room more than 100 sq. ft. is counted as a separate room even if there is not a separate dining room.\footnote{Apartment Building Income-Expense Analysis, op. cit., p. 138.}

If the kitchen is a separate walk-in room then it is counted as a full room irrespective of size. A combination kitchen-dining room is treated as one room if the area does not exceed 105 sq. ft. If the
room contains 105-140 sq. ft., it is counted as one and a half rooms, and over 140 sq. ft., two rooms. If the kitchen is an integral part of the living room (such as the pullman-type kitchen) it is counted as a half room. Other rooms such as bathrooms, porches, halls, and closets, are not counted.\(^{10}\)

The internal data utilized in this research are derived solely from low-rise buildings containing 25 or more apartments. Both elevator and walk-up buildings of three stories or less are included. The data include only unfurnished apartments meaning that a minimum of 80 per cent of the apartments in the building are unfurnished. Office or store occupancy is included in the data providing that it does not represent more than 20 per cent of the rentable area. Rentable area, expressed in square feet, includes all space for which income is received such as apartment units, stores, offices, and garages.\(^{11}\) (See Appendix B for a summary listing of variables.)

**Variable \(X_1\)**

This variable simply identifies each included year beginning with 1962 as 1 and increasing to 10 for the year 1971. The variable identifies the year in which the data were gathered and facilitates simple trend line analysis used to gain insights into data trends in model building.

\(^{10}\)Ibid.

\(^{11}\)Ibid.
Income variables

\( X_2 \) -- Gross Possible Rental

This is the gross possible rental income obtainable from the property on the basis of existing rent schedules. It is the summation of apartment rental, garages and parking, and stores and offices. Apartment rentals include all rents that could have been collected under conditions of 100 per cent occupancy including employee apartments. Garages and parking include income derived from separate charges for use of a parking area or garage. Where parking facilities are included in the apartment rent, the apartment rental is reduced by the amount applicable to parking facilities. Income from stores and offices is reported on the basis of 100 per cent occupancy. This variable includes income derived from rentable area only and does not include other services. This is contract rental and should not be confused with current economic rental sometimes designated as "reasonable gross income."

\( X_3 \) -- Miscellaneous Other Income

This variable includes income derived from services sold to tenants such as maid service, laundry and vending machines, air conditioning, telephones, and utilities where not included in the rent.

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12 Variables \( X_2 \) through \( X_6 \) are derived from Apartment Building Income-Expense Analysis.

Also included is income from special items such as signs placed on the building.

Although many buildings show no miscellaneous other income, the trend of recent years is an increase in this account.

\( X_4 \) -- Gross Possible Total Income

This is the total income derived from apartment rentals, garages and parking, stores and offices, and miscellaneous other income. In essence, this is the Gross Income derived from the property based on existing rent schedules at full occupancy plus miscellaneous other income. This is sometimes defined as "actual gross income." It is this income that is utilized in the Gross Income multipliers.

\( X_5 \) -- Vacancies and Delinquent Rents

For the purposes of this research, vacancies and delinquent rents are defined as the difference between Gross Income and total actual collections (Effective Gross Income).

\( X_6 \) -- Effective Gross Income

This is the total of actual collections resulting from apartment rentals, garages and parking, stores and offices, and miscellaneous other income. This variable is a fundamental determinant of Net Operating Income as NOI for any period is EGI minus the total of expenses.

The income variables listed above are selected as inputs to the modeling procedure because they reflect the general economic conditions, supply and demand factors, economic age of the building, depreciation,
and the quality of management. In an accounting sense, these variables are directly additive in determining the NOI; consequently, they are most significant in the determination of value. As previously discussed, these incomes are used in GI multipliers to estimate market value.

**Expense variables**

\[ X_{7} \] -- Total Payroll Expenses

Included in this variable are the salary expenses for janitors, doormen, maids, elevator operators, telephone switchboard operators, and maintenance personnel, and associated payroll taxes and welfare benefits. The market value of facilities such as apartments provided for personnel living on the premises are included in this account. (Other salary items such as wages for administrative personnel and decorators are listed under variables \( X_{13} \), Other Administrative Expenses, and \( X_{14} \), Painting and Decorating.)

\[ X_{8} \] -- Electricity

This variable includes the total of electrical bills excluding electrical heat (which is reported under Variable \( X_{11} \), Heating Fuel). Electrical charges to tenants, appearing in Variable \( X_{3} \), Miscellaneous Other Income, are not deducted from this account. Primarily, this variable represents electrical costs for public and tenant areas, elevators, air conditioning, and other related services.

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\(^{14}\) Variables \( X_{7} \) through \( X_{22} \) are derived from *Apartment Building Income-Expense Analysis*. 
$X_g$ -- Gas

The cost of gas for cooking, hot water, swimming pools, and air conditioning are included in this variable. (Gas used for heating is included in Variable $X_{11}$, Heating Fuel.) Where gas bills for heating and utilities are combined, the lowest summer month is multiplied by twelve as a means of determining this variable. The remainder is assumed to be for heat.

$X_{10}$ -- Water

This variable contains costs for water irrespective of the use and for sewerage where water bills include sewerage charges.

$X_{11}$ -- Heating Fuel

This variable includes costs of heating fuel—coal, gas, oil, or electricity but does not include ancillary costs such as ash removal or maintenance of the heating system.

$X_{12}$ -- Management

Included in this variable are salaries of management personnel paid directly by the building owner plus agency fees. Leasing or rental fees paid in addition to management fees plus supervisory charges paid for alterations to a managing agent or management personnel are included.

This variable is particularly significant as incomes and expenses may vary as a function of the quality of management. There is an accelerating trend towards professionalism in property management, i.e., the Certified Property Manager. The belief that the
quality of management substantially affects the NOI over the life of
the property has obvious implications as to property value.

$X_{13}$ -- Other Administrative Expenses

Costs derived from dues and professional organizations, architectural or professional engineers' fees, attorneys' and auditors' fees, telephone and building office expenses, and office supplies are included. Many buildings have an office located on the premises.

$X_{14}$ -- Painting and Decorating

This variable includes all costs associated with painting and decorating the interior of the building including personnel on the building payroll. (Exterior painting is included in Variable $X_{15}$, Maintenance and Repairs.)

$X_{15}$ -- Maintenance and Repairs

Included in this variable are all maintenance and repairs for exterior and interior. Typical costs would include landscaping, exterior painting, air conditioning service contracts, fire extinguishers, and all repairs and replacements not considered a capital expenditure.

$X_{16}$ -- Supplies

This variable includes uniforms for employees, janitorial supplies, and other consumable supplies not included under painting, decorating, and maintenance or repair.
$X_{17}$ -- Services

Costs derived from outside services such as window washing, exterminating, rubbish removal, and television antenna services, are included. Costs associated with painting, decorating, maintenance, or repairs, are excluded.

$X_{18}$ -- Miscellaneous Operating Expenses

This variable includes expenses which do not fall in other categories, i.e., casualty losses not covered by insurance, signs for parking lots, and name plates for doors.

$X_{19}$ -- Insurance

This variable includes all annual insurance costs except mortgage insurance or fringe benefits for employees. Many policies extend for a period longer than one year; consequently, a pro rata annual cost is included.

$X_{20}$ -- Real Estate Taxes

Included in this variable are state and local real estate taxes plus non capitalized assessments.

$X_{21}$ -- Other Taxes

This variable includes taxes other than real estate plus fees and permits. Typical taxes included are personal property taxes, franchise taxes, and sign permit fees. Income taxes of any type are excluded from this variable.
\( X_{22} \) -- Total Expenses

Included in this variable are all expenses associated with the operation of the building excluding ground rent, mortgage interest or amortization, income taxes, and depreciation. The total expenses are a summation of variables \( X_7 \) through \( X_{21} \).

Expenses reflect the general conditions of the economy, the quality of management, and level of services offered. As NOI is the residual of EGI minus expenses, estimation of expenses is critical in determining NOI and consequently, property value. Of the variables subject to managerial control, the management influence on NOI may be expressed most strongly through control of expenses. Any estimate of NOI must reflect future expenses.

Performance variables\(^{15}\)

\( X_{23} \) -- Net Operating Income

Net Operating Income is Effective Gross Income, \((X_g)\) less Total Expenses, \((X_{22})\). This is a measure of productivity of the property.

\( X_{24} \) -- Turnover Rate

This variable is expressed as a percentage representing the number of new tenants during the year. Turnover rates are reported one full year after initial occupancy. Turnover rate data were not available by age classification; consequently, the turnover rates are

\[^{15}\text{Variables } X_{23} \text{ and } X_{24}, \text{ Ibid.}\]
averages representing all age groups. Obvious increases in expenses are associated with a high turnover rate.

Operating ratio variables

\( X_{25} \) -- Operating Ratio A

This variable is the Total Expenses (\( X_{22} \)) divided by GI (\( X_4 \)).

\( X_{26} \) -- Operating Ratio B

This variable is the Total Expenses (\( X_{22} \)) divided by Effective Gross Income (\( X_6 \)).

\( X_{27} \) -- Net Income Ratio A

This ratio is NOI (\( X_{23} \)) divided by GI (\( X_4 \)).

\( X_{28} \) -- Net Income Ratio B

This variable is NOI (\( X_{23} \)) divided by EGI (\( X_6 \)).

Variables \( X_{25} \) through \( X_{28} \) are performance ratios reflecting many of the variables included in this research as well as supply and demand factors and managerial performance. In recent years these ratios reflect a deteriorating trend. Since these are derivative ratios, they are difficult to estimate directly. However, they are widely used as measures of performance (particularly managerial performance).

\( X_{30} \) -- Average Age of Building

The institute of Real Estate Management classifies buildings

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\(^{16}\) Variables \( X_{25} \) through \( X_{28} \), Ibid.

\(^{17}\) Ibid.
into five age groups: (1) buildings erected after 1961, (2) between 1946-1960, (3) between 1931-1945, (4) between 1921-1930, and (5) before 1920. Transformation of the data to average age expressed in years is a more sensitive form. For example, buildings erected between 1946-1960 were on the average erected in 1953. When data were collected in 1962, the average age of the buildings erected in 1953 was 9 years. By 1971 the average age had increased to 18 years. Consequently, by means of this simple transposition the annual increase in age is explicitly recognized. This could not be done for the buildings erected before 1920. The assumption was made that all buildings erected before 1920 were on the average erected in the year 1910. Consequently, an average age could be calculated for each year in which the data were collected. Although the assumption of the year 1910 is somewhat arbitrary, the information contained in the classification approach is not lost. Utilizing average age over the ten year period in which data were collected, a continuum of building age from six months to 61 years is established. In this form the age variable explicitly recognizes the change in age over the period in which the data were recorded.  

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18 Models were developed and tested utilizing the age classifications reported by the IREM as dummy variables. The predictive and explanatory power of the models was unsatisfactory. Further discussion of the use of dummy variables is found in Appendix D, Models--Dummy Variables.
External variables

$X_{31}$ -- Consumer Price Index\(^{19}\)

This variable is a general index of inflation and a measure of the purchasing power of the dollar. The index is expressed in terms of the base period of 1967 and is used in this research primarily as a deflator to express constant dollars. This is a broadly based price index reported by the government.

$X_{32}$ -- Wholesale Price Index\(^{20}\)

This variable indicates the change of prices of commodities from the base year of 1967. The purchasing power of the dollar vis-à-vis commodities is defined by this index.

$X_{33}$ -- Construction Cost Index\(^{21}\)

This is an index of the purchasing power of the dollar in goods and services used in construction as gathered by the Department of Commerce.

$X_{34}$ -- Risk-Free Interest Rate\(^{22}\)

The risk-free interest rate is expressed in terms of new issues of three month Treasury Bills. This rate is an indicator of the


\(^{20}\)Ibid., p. 43.

\(^{21}\)Ibid., p. 52.

\(^{22}\)Ibid., p. 91.
general conditions in the money markets (and in particular short-term money markets) and usually reflects current monetary policy and the availability of mortgage money.

$X_{35}$ -- Home Mortgage Rates$^{23}$

This variable states the current home mortgage rate for purchases of new homes. It reflects the general conditions of mortgage markets in terms of the supply and demand for funds. These rates usually reflect monetary policy.

$X_{36}$ -- Yield on Corporate Bonds$^{24}$

This variable is the average yield on corporate bonds reflecting the general conditions of long-term money markets. Corporate bonds and mortgages may be viewed as competing alternative investments.

$X_{37}$ -- Dividend Yield on Common Stocks$^{25}$

Investors' expectations are reflected in this variable in that investors apply a capitalization rate to expected earnings or dividends. This particular variable is viewed as an index of investor psychology.

$X_{38}$ -- Personal Income Current Dollars$^{26}$

This variable defines the purchasing power available to the public and is assumed to be indicative of demand factors in real estate

$^{23}$Ibid.
$^{24}$Ibid., p. 105.
$^{25}$Ibid., p. 106.
$^{26}$Ibid., p. 7.
markets.

\( X_{39} \) -- Personal Income Constant Dollars\(^{27}\)

Income is expressed in constant dollars of 1967 and is assumed to be indicative of the demand for housing. The use of constant dollars tends to minimize the illusory effects of current dollars.

\( X_{40} \) -- National Rental Vacancy Rates\(^{28}\)

This variable indicates rental property vacancy throughout the country and is assumed to be an indicator of the balance of supply and demand for rental property.

\( X_{41} \) -- Number of New Private Multi-Family Units Started Annually\(^{29}\)

This variable is assumed to be indicative of the supply of apartments relating to current and future pricing.

\( X_{42} \) -- Consumer Price Index Less Shelter\(^{30}\)

This variable is a general index of inflation expressed in the base period of 1967 excluding the shelter component.

\( X_{43} \) -- Boeckh Construction Price Index (Apts., Hotels, Office Bldgs.)\(^{31}\)

This variable is the Boeckh index of apartments, hotels, and office buildings.

\(^{27}\)Ibid.


\(^{29}\)Ibid., p. 681.


\(^{31}\)Ibid., p. 53.
office buildings. It indicates the relative price level of goods and services required for construction and is assumed to affect the market value of new and existing property as suggested in the "cost approach."

\[ X_{44} \] -- Boeckh Construction Price Index (Residential)\(^{32}\)

This is a Boeckh residential index indicating the relative prices for goods and services utilized in home construction. It is assumed that this index is indicative of the rise in price of single family homes indirectly affecting the supply and demand for apartments.

\[ X_{45} \] -- Disposable Personal Income in Current Dollars\(^{33}\)

This variable is expressed in current dollars and is indicative of the general state of the economy and the ability of the public to consume. Rise in disposable personal income is assumed to be indicative of increased demand for apartments.

\[ X_{46} \] -- Disposable Personal Income in Constant Dollars\(^{34}\)

This variable is expressed in constant dollars using 1967 as a base year. This index is assumed to be the most accurate measure of the public's capacity to purchase goods and services; consequently, directly affects the demand for housing.

\(^{32}\)Ibid.

\(^{33}\)Ibid., p. 7.

\(^{34}\)Ibid.
$X_{47}$ -- Average Capitalization Rate\(^{35}\)

The capitalization rate, a market determined rate, is derived by dividing the NOI by the estimated market value for apartment buildings conventionally financed. This is a weighted national average computed from published quarterly averages.

$X_{48}$ -- Average Loan-to-Value Ratio\(^{36}\)

This ratio is computed for each loan by dividing the loan sum by the estimated market value for apartment buildings conventionally financed. The variable is a weighted annual average computed from published quarterly averages.

Variables $X_{47}$ and $X_{48}$ were obtained from the American Life Insurance Association. Data reflect the loan activity of major life insurance companies for conventionally financed apartment buildings. Data are available for the years 1966-1971 representing an annual number of loan placements of 313 to 1267. All types of apartment houses are included. The expertise available to the major insurance companies suggests that the capitalization rates and loan-to-value ratios are representative of the market for conventionally financed buildings of the type acceptable to insurance companies.

For the purposes of determining the impact of inflation on property value and equity interest, it is assumed that the data are


\(^{36}\)Ibid.
representative of a national sample. The relative change in the data over an inflationary period is more significant than the absolute level.

The price index variables, $X_{31}, X_{32}, X_{33}, X_{42}, X_{43},$ and $X_{44},$ define the rise in price level in different segments of the economy assumed to influence apartment house incomes and expenses and, consequently, NOI.

The construction indices, Variables $X_{33}, X_{43},$ and $X_{44},$ are assumed to be indicative of the inflationary effect on the value of new and "in place" housing. Consequently, to maintain an acceptable rate of return, NOI must reflect the rises in these indices.

Investment yield variables, $X_{34}, X_{35}, X_{36},$ and $X_{37},$ define conditions in the money markets and reflect monetary policy. If property value is to be maintained, NOI must rise with these variables. If NOI does not rise, property value must fall to maintain the relative yield required for alternative investments.

The personal income variables, $X_{38}, X_{39}, X_{45},$ and $X_{46},$ are assumed to be indicators of the demand side of the housing market. Rising income is assumed to indicate a demand for more and higher quality housing and to impart a positive bias to rents and consequently to NOI.

**Summary**

The theoretical and pragmatic facets of the general problem of valuation, including valuation under inflation and valuation of income property as an inflation hedge, have been expanded upon in the preceding chapters. Three specific objectives central to the general problem are
defined: (1) the development of an explanatory and predictive model of aggregate NOI using variables derived from operating data and the economic environment, (2) empirical quantification of the effect of building age on aggregate NOI, and (3) determination as to whether the class of property is an inflation hedge. A six step research design is employed to attain the objectives.

Both the objectives and the research design must be viewed from the perspective of data limitations characterizing valuation-oriented real estate research. Using aggregate data, quantitative analysis necessitates assumptions about characteristics of the underlying data. These assumptions limit the statistical power of the analysis; consequently, objectives are not stated in the form of testable hypotheses. Recognizing these limitations, this research represents a significant contribution to the estimation of value, particularly, as a function of property age and inflation.

A large number of variables ($X_2$-$X_{46}$) suggesting a causal relationship to the level and change of NOI are used in developing the NOI model. Many of these variables contain overlapping information. The methodology employed in this research identifies a small number of significant independent variables that can be estimated by the practitioner.
CHAPTER V

METHODOLOGY

Introduction

This chapter includes the formulation of the models and the specific methodology used to develop and to accept or reject quantified models. Two separate and distinct models are developed, the probabilistic NOI model and the determinate equity model. These models embody the theoretical and pragmatic background covered in previous chapters as well as insights derived from preliminary data analysis, including graphical analysis and simple regression. The modeling techniques utilized are not original; they are adaptations for the purposes of this research and the particular data characteristics. Consequently, discussion of factor analysis and stepwise multiple regression is directed primarily at justification of the selected methodology and the problems of application.

Net Operating Income Model

The rationale of the methodology used in developing the NOI model is summarized succinctly by William F. Massy:

Knowledge of a cause and effect link implies that we understand a relationship between two or more variables. We must search for independent or explanatory variables that (1) account for a substantial fraction of the behavior of the dependent variable under
study, and (2) appear reasonable in terms of our subjective understanding of the problem.\(^1\)

The probabilistic NOI model is used for purposes of prediction and explanation. The predictive qualities of the model are based on the included variables accounting for a high percentage of the variation in NOI. The statistical independence of variables facilitates quantification of the explanatory power of each variable. The empirical relationship between building age, \(X_{30}\), and NOI is defined, thus quantifying the Babcock premises. The effect of inflation on NOI is quantified.

The model must be amenable to acceptance or rejection based on tests of statistical significance, stability of parameters over time, and sensitivity analysis. In order for it to have pragmatic value, the model must embody a small number of variables that are historically accessible and easily estimated by the practitioner.

**Formulation of preliminary model**

Based on the preceding discussions of relevant theoretical and empirical background as well as objectives of the model, NOI may be defined as:

\[
NOI = f(\text{Internal Variables, External Variables})
\]  

Specifically, NOI is a function of 44 variables:

\[ x_{23} = f(x_1, \ldots, x_{22}, x_{24}, \ldots, x_{28}, x_{30}, \ldots, x_{46}) \]  

The development of an explanatory and predictive model necessitates an understanding of the underlying dimensions and compression of the data into a more meaningful and manageable form. Factor analysis is the appropriate methodology. A relatively small number of variables representing the dimensions of the data is introduced into a multiple regression.

**Factor analysis**

Factor analysis is a group of techniques originally developed to analyze psychological data and now widely used in administrative and social sciences. It is a complementary technique to other multivariate methods.

The techniques used in this research are based on the extensive work of Harman\(^2\) and Rummel,\(^3\) consequently, the following discussion is brief. A more detailed discussion of the principal-factor method, varimax rotation, and the computer program-factor analysis is located in Appendix C--Factor Analysis.


Harman describes factor analysis as follows:

The principal concern of factor analysis is the resolution of a set of variables linearly in terms of (usually) a small number of categories or "factors." This resolution can be accomplished by the analysis of the correlations among the variables. A satisfactory solution will yield factors which convey all of the essential information of the original set of variables. Thus, the chief aim is to attain scientific parsimony or economy of description. A principal objective of factor analysis is to attain a parsimonious description of observed data.

Massy lists four ways in which factor analysis may be useful:

(1) Separation and analysis of distinct dimensions that are latent in a larger set of variables. (2) Separation and analysis of distinctly different groups of people which exist in a larger population. (3) Identification of certain likely variables for subsequent regression or discriminant analysis, from among a much larger set of potential independent variables. (4) Summarization of the common parts of a set of explanatory variables into a smaller number of new variables which can be used in regression or discriminant analysis.

In developing the NOI model, factor analysis is used to identify the underlying dimensions of the data and to facilitate selection of a small number of variables which convey the essential information inherent in the 44 variables. The use of rotational procedures identifies independent variables possessing high explanatory power and minimal correlation, thus reducing significantly the problem of multicollinearity in the subsequent regression model. Variables that can be estimated by the practitioner are selected for introduction into the regression model. Parsimony of variables for the development of a

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4Harman, Modern Factor Analysis, op. cit., p. 4.
5Ibid., p. 5.
pragmatically useful regression model is thus accomplished through factor analysis.

The following terms are critical to the discussion and interpretation of factor analysis:

Factor: All variables are considered dependent and a function of underlying factors. Consequently, each factor may be viewed as a function of the variables. Thus, a factor is defined as a linear combination of observed variables.

\[ F = a_1X_1 + a_2X_2 + a_3X_3 \ldots + a_nX_n \]  

(5-3)

\[ F = \text{factor} \]
\[ a = \text{coefficient} \]
\[ X = \text{variable} \]

Factor Score: A linear combination of observed variables representing a single case

\[ F_i = a_1X_{1i} + a_2X_{2i} + a_3X_{3i} + \ldots + a_nX_{ni} \]

Factor Loading: The correlation between the factor score and variable

Eigenvalue: The sum of the squares of the loadings of each factor

Communality: The explained variance of each variable summarized across factors

Factor Loading Matrix: The matrix displays the results of the factor analysis in a form which facilitates selection of variables to be introduced into the regression model as illustrated in Table 5-1.

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8 Ibid., p. 214.
9 Ibid. 10 Ibid. 11 Ibid., p. 215.
### TABLE 5-1
FACTOR LOADING MATRIX

<table>
<thead>
<tr>
<th>Variable</th>
<th>A</th>
<th>B</th>
<th>( h^2 ) (Communality)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.80</td>
<td>.40</td>
<td>.80</td>
</tr>
<tr>
<td>2</td>
<td>.70</td>
<td>.70</td>
<td>.98</td>
</tr>
<tr>
<td>3</td>
<td>.60</td>
<td>.60</td>
<td>.72</td>
</tr>
<tr>
<td>4</td>
<td>.50</td>
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<td>.50</td>
</tr>
<tr>
<td>5</td>
<td>.40</td>
<td>.40</td>
<td>.32</td>
</tr>
<tr>
<td>eigenvalue</td>
<td>1.90</td>
<td>1.42</td>
<td>3.32</td>
</tr>
</tbody>
</table>

In a factor loading matrix, with columns representing factors and rows representing variables, the eigenvalues are the sum of the squares in each column, and the communalities are the sum of the squares of factor loadings across rows.

Factor analysis can employ a number of procedures. The analysis used in this research is R-type factor analysis based on correlation among variables. The procedures are the principal-factor method and varimax rotation. The former extracts factors which are independent and orthogonal, defining underlying dimensions of the data. Factors are extracted until the eigenvalue of the last extracted factor approaches one.\(^\text{12}\)

This method does not usually provide the desired parsimony of data nor clear delineation of related variables defining the underlying dimensions of the factors. Highly loaded variables in a given factor

\(^{12}\text{Rummel, Applied Factor Analysis, op. cit., pp. 357-367, passim.}\)
are necessary to define the dimensions of the factor.\textsuperscript{13} The use of varimax rotation increases factor loadings on certain variables while preserving independence between factors.\textsuperscript{14} The communality is not affected by rotation.\textsuperscript{15}

Based on causal relationships, correlation between variables, and the practitioner's capacity to estimate the variables, highly loaded variables are selected for introduction into the regression model. Computations are performed on an IBM 1130 computer utilizing the 1130 Statistical System (1130-CA-06X).\textsuperscript{16} (For more detailed discussion of principal-factor method, varimax rotation, and the computer program-factor analysis, see Appendix C -- Factor Analysis.)

**Multiple regression**

The selection of an appropriate multivariate technique for the development of the NOI model is based on three judgments relative to the nature and use of data identified through factor analysis, as stated by Sheth:

\begin{quote}
... (1) Are some of the variables dependent upon others, therefore requiring special treatment? (2) If yes, how many are to be treated as dependent in a single analysis? and (3) What are the presumed properties of the data? Specifically, are the data
\end{quote}

\textsuperscript{13}Ibid., pp. 372-375.


qualitative (nonmetric) ... or quantitative (metric) ... . The technique to be utilized will depend upon the answers to these questions.\textsuperscript{17}

Sheth states that multiple regression is an appropriate method when there is a single metric dependent variable assumed to be a function of other independent variables. The objective is to predict the variability or the level of the dependent variable based on covariance with the independent variables. Least squares is the fundamental statistical process.\textsuperscript{18}

Regression is the appropriate method as each of the requirements outlined by Sheth is met. Examination of the data suggests that the relationships between variables are linear or can be transformed into linear relationships;\textsuperscript{19} consequently, a linear regression model is developed. The general form of the model is:

\[ \text{NOI} = a + b_1 X_1 + b_2 X_2 \ldots + b_n X_n + e \]  \hfill (5-4)

\begin{align*}
  a &= \text{constant} \\
  b_i &= \text{regression coefficient} \\
  e &= \text{error term}
\end{align*}


\textsuperscript{18}Ibid.

\textsuperscript{19}Building age data \((X_{30})\) as collected by the IREM are classified into five age groups. As defined in Chapter IV, this variable is transformed to represent average building age. Models are developed, some using the transformed variable and others using dummy variables to represent the age classification. Four dummy variables are required in the regression model to represent the five age classifications. Models including dummy variables were subsequently discarded as they did not prove to be acceptable. Consequently, they are not discussed.
In view of the extensive use of regression techniques, discussion of basic theory and methodology is not necessary. The assumptions that underlie the use of regression methods for purposes of prediction and explanation must be discussed in terms of the data.

Assumption One -- The relationship between the dependent and the independent variable is linear.

Linearity of the data included in the model is evaluated through analysis of residuals.

Assumption Two -- The error term is random and normally distributed about the regression line with a mean of zero.

The distribution of error terms about the regression line must be homoscedastic. Some degree of heteroscedasticity is anticipated. This does not bias predictions or estimates of explanatory power.\(^\text{20}\) The effect is to reduce efficiency and to increase the standard error of the estimate. Acceptance of the model depends upon analysis of residuals.

Assumption Three -- The excluded variables have little effect on the dependent variable and low correlation with the independent variables.

If this assumption is violated, the error term will not be randomly distributed. Many variables included in the model are assumed to be correlated with supply and demand factors. This does not bias prediction or explanation as the validity of the causal

relationships is dependent upon the excluded variables (many unquantifiable) and the effect is included in the prediction.

Assumption Four -- There is no exact linear relationship between the independent variables.

Multicollinearity results in inaccurate and unstable estimates of regression coefficients; consequently, the explanatory power of included variables cannot be interpreted. This does not bias the predictive qualities of the model. Factor analysis and transformation of variables are used to minimize colinearity. Variables introduced into the regression expressed in current dollars are transformed to constant dollars, using the Consumer Price Index, \( X_{31} \), as a deflator. The transformed variables are ratios; consequently, correlation between independent variables is reduced.

**Computer program-regression**

Computations of the stepwise linear regression program are performed on an IBM 1130 computer using the 1130 Statistical System (1130-CA-06X). Transformation options are individually programmed. The output from the program includes:

1. high and low value of each variable
2. mean of each variable
3. standard deviation of each variable
4. sample variance of each variable
5. matrix of correlation coefficients
6. residual standard deviation
7. standard error of the mean of the predicted dependent variable
8. multiple correlation coefficient
9. square of the multiple correlation coefficient = sum of the squares due to regression divided by adjusted total sum of the squares
10. degrees of freedom
Acceptance of model

The model is evaluated for acceptance on the basis of the square of the multiple correlation coefficient, analysis of variance, standard error of the estimate, and tests of statistical significance of the regression as well as beta and regression coefficients. The residual plots are examined for discernible patterns suggesting a lack of fit.

An acceptable model demonstrates pragmatic as well as theoretical relationships. The coefficients must be reasonable in terms of sign and magnitude. The equation must be plausible in light of experience and the included variables. The model is evaluated in terms of predictability and the explanatory power of each variable.

In assessing the explanatory power of variables, confidence intervals provide readily discernible limits as to the effects of a given variable. Confidence intervals are determined for the regression and beta coefficients. The confidence interval for the scatter of the actual observations about the estimated regression line is computed from the standard error of the estimate. 22

21 IBM Application Program, op. cit., pp. 7-8.

The practical value of a model is dependent upon its sensitivity to sampling error associated with the independent variables. Sensitivity of Net Operating Income to change in each variable is evaluated in terms of the ease with which the variable can be estimated. If an independent variable with a high explanatory power must be estimated with undue precision, the value of the model is jeopardized.

**Verification of the model**

Data beyond that included in the sample are not available for verification of the model. As the model is developed from data gathered over a ten year period, verification is based on stability of regression and beta coefficients and the square of the multiple regression coefficient derived from shorter time spans\(^ {23} \) plus the prediction of NOI using a holdout sample.

Using the independent variables introduced into the NOI model, stability of regression coefficients is evaluated by developing two additional NOI models based on data collected in the years 1962-1966 and 1962-1968. The data were partitioned based on the abnormally high rate of inflation after 1966. The model developed from the 1962-1968 data is used to predict NOI for the years 1969, 1970, and 1971 treated as holdout samples.

**Equity Model**

The purpose of the equity model is to determine if the class of property studied has been an inflation hedge over the years 1966-1971,

a period of accelerating inflation. The model is used to measure in constant dollars the impact of inflation on market value, equity interest, and the wealth transfer effect. The only predictive qualities associated with the model are derived from the assumption that the future is like the past.

The general form of the determinate model is the same as that developed in Chapter III under the Section titled Equity and the Wealth Transfer Effect, Equation (3-20):

\[
E = \frac{1}{P} \frac{(1+p)NOI}{(1+i)} (L) \frac{M}{P}
\]

The indebtedness (M), mortgage, may be expressed as the product of the market value in current dollars (C) and the loan-to-value ratio (L):

\[
M = C(L)
\]

Substituting Equation (3-12) for market value (C):

\[
M = \frac{(1+p)NOI}{(1+i)} L
\]

Substituting Equation (5-5) for M in Equation (3-20):

\[
E = \frac{1}{P} \frac{(1+p)NOI}{(1+i)} (1+p)NOI L - \frac{(1+p)NOI L}{P}
\]

This is the general form of the equity model expressed in constant dollars. The model is adapted to the data through the following
substitutions:

\[ P = X_{31}, \text{ Consumer Price Index} \]
\[ (1+p)\text{NOI} = X_{23}, \text{ Net Operating Income in current dollars} \]
\[ (1+i)\text{I+R} = X_{47}, \text{ Average Capitalization Rate} \]
\[ L = X_{48}, \text{ Average Loan to Value Ratio} \]

The model stated in terms of the above variables is:

\[ E = \frac{1}{X_{31}} \frac{X_{23}}{X_{47}} - \frac{X_{23}}{X_{47}} \frac{X_{48}}{X_{31}} \]  \hspace{1cm} (5-7)

The first term of the model is market value, and the second term is the indebtedness, mortgage. Both terms are expressed in constant dollars. By rearranging the terms, the model can be simplified:

\[ E = \frac{1}{P} \frac{(1+p)\text{NOI}}{(1+i)\text{I+R}} (1-L) \]  \hspace{1cm} (5-8)

Substituting variables:

\[ E = \frac{X_{23}}{X_{31}X_{47}} (1-X_{48}) \]  \hspace{1cm} (5-9)

A one year holding period is assumed. Consequently, the effect of mortgage amortization is minimized. The error introduced by assumption of a constant level of debt magnifies the wealth transfer effect as illustrated in Table 3-2. Analysis based on a one year holding period does not mean that the property must be sold but rather that current values are recognized. Zero transactions costs are assumed.

The model is developed from widely accepted determinate

\[ 24 \text{Variables } X_{31}, X_{23}, X_{47}, \text{ and } X_{48}, \text{ and } E \text{ are expressed as averages over the twelve month holding period.} \]
valuation models. The significance of the computed results is dependent upon the sampling error which is not quantified. The basis on which the data were gathered is defined in Chapter IV.

**Summary**

Based on the theoretical and pragmatic background, the methodology for developing a probabilistic NOI model and a determinate equity model is defined and justified. The methodology reflects data limitations characterizing real estate research.

The rationale of the NOI model is based on recognition of theory and experience in the identification of independent variables accounting for a high percentage of the variation in NOI. Factor analysis, principal-factor method and varimax rotation, is used to identify independent variables defining the underlying dimensions of the data for introduction into the regression model. The model defines the relationship between the independent variables and the dependent variable, NOI. Demonstrating both predictive and explanatory power, the model quantifies the effect of each independent variable.

The criteria for acceptance of the model are: (1) plausibility in terms of theory and experience, (2) explanation of a high percentage of the variation in NOI, (3) statistical significance in terms of included variables, and (4) accommodation of the data as indicated by the residual plots. For purposes of verification, two additional regression models are developed, utilizing data from the years 1962-1966, and 1962-1968 and the independent variables introduced into the NOI model. Stability of regression coefficients across all models as
well as prediction of NOI for the years 1969, 1970, and 1971 (using the model based on 1962-1968 data) is interpreted to be sufficient verification.

The equity model is used to measure the change in equity interest, market value, and the wealth transfer effect under inflation. The annual holding period changes, (expressed in constant dollars) over a period of relatively high inflation (1967-1971), assess the validity of the belief that the value of income producing property will rise or keep pace with inflation. The model developed in Chapter III, Equation (3-20) is expressed in pragmatic terms using NOI, Consumer Price Index, average capitalization rate, and average loan-to-value ratio. Predictive qualities are not associated with the model.
CHAPTER VI

RESULTS

Introduction

The results of the development of the NOI and equity models are defined in this chapter. The models are quantified and results discussed within the framework of the methodology and objectives at each major step of the development. Conclusions and implications are discussed in Chapter VII.

The objective of developing a predictive and explanatory model of NOI that quantifies the effect of inflation and building age on NOI is accomplished through factor analysis and stepwise multiple regression. Factor analysis identifies a small number of variables that can be estimated by the practitioner for introduction into the regression. Stepwise multiple regression retains only those variables which are statistically significant in explaining the variation in NOI. The regression quantifies the relationship between the included variables and NOI providing a predictive and explanatory model.

Four models are developed. Models 1 and 2 are directed at the objectives and Models 3 and 4 provide verification of Models 1 and 2. Models 1 and 2 are accepted on the basis of the square of the multiple correlation coefficient, tests of statistical significance of the regression and regression coefficients, residual plots, and plausibility of the model.
The equity model is used to evaluate the impact of inflation on equity interest and market value recognizing the wealth transfer effect associated with encumbered property under inflation. Each of the components of the model, equity interest, market value, and mortgage, is analyzed in terms of the inflationary impact on the holding period change. The summation of holding period changes over a period of relatively high inflation, 1966-1970, determines if multi-family dwelling units are an inflation hedge.

**Net Operating Income Model**

**Factor analysis**

Factor analysis resolves the variables into five factors conveying the underlying dimensions of the data. Variables displaying high factor loadings, the correlation between the factor score and the variable, conveying the essential dimensions of the factor, are selected for introduction into the multiple regression. Six variables representing the five factors are displayed in Table 6-1.

As computer capacity is limited to 30 variables, factor analysis is executed in three steps: (1) analysis of internal variables derived from building operating data, (2) analysis of external variables derived from the economic environment, and (3) analysis of internal-external variables derived from steps (1) and (2). The procedures used in each step are the principal-factor method and varimax rotation.

Interpretation of the factor analysis is based on the varimax rotated factor loading matrix. Discussion is confined to the salient points of the analysis—correlations, communalities, and factor loadings.
In all instances, factors are extracted until an eigenvalue of 1.00 is approached.

**TABLE 6-1**

**FACTOR-VARIABLES**

<table>
<thead>
<tr>
<th>Factor No.</th>
<th>Factor Dimension</th>
<th>Variable Selected</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Age-Performance</td>
<td>$X_{30}$, Average Age of Building</td>
</tr>
<tr>
<td>2</td>
<td>Management-Inflation</td>
<td>$X_{12}$, Management</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$X_{42}$, Consumer Price Index Less Shelter</td>
</tr>
<tr>
<td>3</td>
<td>New Units Started</td>
<td>$X_{41}$, Number of New Private Multi-Family Units Started Annually</td>
</tr>
<tr>
<td></td>
<td>Annually</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Miscellaneous Other Income</td>
<td>$X_{31}$, Miscellaneous Other Income</td>
</tr>
<tr>
<td>5</td>
<td>Vacancies and Delinquent Rents</td>
<td>$X_{5}$, Vacancies and Delinquent Rents</td>
</tr>
</tbody>
</table>

Analysis of the internal variables, $X_2 - X_{28}$ and $X_{30}$, is summarized in Table 6-2 and Table 6-3. Four factors explain 85.65 per cent of the total variance of the data. Rotated factors 1 and 2 explain 71.74 per cent of the variance. Signs preceding the factor loadings indicate the direction of the relationships between the factor and the variable.

The analysis suggests that three dimensions explaining 80.54 per cent of the total variance of the data can be used to identify independent variables to be introduced into the regression models.
Identification and labeling of the dimensions of each factor are based on factor loadings.

**TABLE 6-2**

**FACTOR ANALYSIS OF INTERNAL VARIABLES**

<table>
<thead>
<tr>
<th>1. Number of factors extracted: 4 a</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. Per cent of total variance of data explained by the four factors: 85.65</td>
</tr>
<tr>
<td>3. Per cent of total variance of data explained by the rotated factors:</td>
</tr>
<tr>
<td>Factor 1</td>
</tr>
<tr>
<td>Factor 2</td>
</tr>
<tr>
<td>Factor 3</td>
</tr>
<tr>
<td>Factor 4</td>
</tr>
</tbody>
</table>

aFactors extracted until the eigenvalue approaches 1.00.

Factor 1 is highly loaded on variables relating to performance ratios, $X_{25}$, $X_{26}$, $X_{27}$, $X_{28}$; Net Operating Income, $X_{23}$; and Average Age of Building, $X_{30}$. This factor loads heavily on financial measures of performance and Average Age of Building. The dimension defined by the factor is identified and labeled as Performance. Variables are highly correlated confirming that performance is a function of building age as suggested by theory and practice. The high factor loadings and communalities indicate high explanatory power of the variables. The dimension of the factor can be represented effectively in a regression model by variable $X_{30}$, Average Age of Building, as an independent variable.
<table>
<thead>
<tr>
<th>Variables</th>
<th>Factor Loadings</th>
<th>Communality</th>
</tr>
</thead>
<tbody>
<tr>
<td>X23</td>
<td>Net Operating Income</td>
<td>.965</td>
</tr>
<tr>
<td>X25</td>
<td>Operating Ratio A</td>
<td>.983</td>
</tr>
<tr>
<td>X26</td>
<td>Operating Ratio B</td>
<td>.982</td>
</tr>
<tr>
<td>X27</td>
<td>Net Income Ratio A</td>
<td>.947</td>
</tr>
<tr>
<td>X28</td>
<td>Net Income Ratio B</td>
<td>.982</td>
</tr>
<tr>
<td>X30</td>
<td>Average Age of Building</td>
<td>.914</td>
</tr>
<tr>
<td>X2</td>
<td>Gross Possible Rental</td>
<td>.989</td>
</tr>
<tr>
<td>X4</td>
<td>Gross Possible Total Income</td>
<td>.988</td>
</tr>
<tr>
<td>X6</td>
<td>Effective Gross Income</td>
<td>.989</td>
</tr>
<tr>
<td>X12</td>
<td>Management</td>
<td>.783</td>
</tr>
<tr>
<td>X20</td>
<td>Real Estate Taxes</td>
<td>.871</td>
</tr>
<tr>
<td>X22</td>
<td>Total Expenses</td>
<td>.994</td>
</tr>
<tr>
<td>X3</td>
<td>Miscellaneous Other Income</td>
<td>.863</td>
</tr>
<tr>
<td>X18</td>
<td>Miscellaneous Operating Expenses</td>
<td>.449</td>
</tr>
<tr>
<td>No.</td>
<td>Variables</td>
<td>Name</td>
</tr>
<tr>
<td>-----</td>
<td>---------------------------------</td>
<td>-----------------------</td>
</tr>
<tr>
<td>X5</td>
<td>Vacancies and Delinquent Rents</td>
<td>.694</td>
</tr>
<tr>
<td>X7</td>
<td>Total Payroll Expense</td>
<td>.788</td>
</tr>
<tr>
<td>X8</td>
<td>Electricity</td>
<td>.659</td>
</tr>
<tr>
<td>X9</td>
<td>Gas</td>
<td>.660</td>
</tr>
<tr>
<td>X10</td>
<td>Water</td>
<td>.653</td>
</tr>
<tr>
<td>X11</td>
<td>Heating Fuel</td>
<td>.774</td>
</tr>
<tr>
<td>X13</td>
<td>Other Administrative Expenses</td>
<td>.295</td>
</tr>
<tr>
<td>X14</td>
<td>Painting and Decorating</td>
<td>.744</td>
</tr>
<tr>
<td>X15</td>
<td>Maintenance and Repairs</td>
<td>.840</td>
</tr>
<tr>
<td>X16</td>
<td>Supplies</td>
<td>.222</td>
</tr>
<tr>
<td>X17</td>
<td>Services</td>
<td>.516</td>
</tr>
<tr>
<td>X19</td>
<td>Insurance</td>
<td>.701</td>
</tr>
<tr>
<td>X21</td>
<td>Other Taxes</td>
<td>.588</td>
</tr>
<tr>
<td>X24</td>
<td>Turnover Rate</td>
<td>.623</td>
</tr>
</tbody>
</table>

Negative signs indicate direction of the relationship between the factor and the variable.

Variables X5, X7, X8, X9, X10, X11, X13, X14, X15, X16, X17, X19, X21, and X24 are not included in a factor dimension because of the relatively low factor loadings with all factors.
Factor 2 is highly loaded on variables $X_2$, $X_4$, $X_6$, $X_{12}$, $X_{20}$, and $X_{22}$, suggesting an underlying dimension relating to income, expenses, and management. Ceteris paribus, both incomes and expenses are anticipated to be functions of management. Expenses are not highly correlated with incomes, yet both incomes and expenses are relatively highly correlated with management, confirming the expected relationship. On this basis, the factor is labeled Management. Both communalities and loadings are high, indicating high explanatory power. The variable $X_{12}$, Management, conveys a high percentage of the informational content of factor 2 when used as an independent variable in a regression model.

Factor 3 has one highly loaded variable, $X_3$, Miscellaneous Other Income. The combination of high loadings and high communalities suggests that the variable does represent a dimension of the data. This factor explains only 8.80 per cent of the total variance.

The most highly loaded variable in factor 4 is $X_{18}$, Miscellaneous Operating Expenses. Both the factor loading and communality are relatively low as is the percentage of variance explained by the factor. The significance of this variable in a regression model is questionable.

The remaining variables have low factor loadings and with few exceptions exhibit low communalities. These variables do not suggest underlying dimensions of the data. As the factor loadings for each variable reflect all the variables included in the analysis, variables cannot easily be discarded as their factor loadings might change when analyzed in

---

combination with external variables. Factor analysis of both internal and external variables must be given consideration in determining which variables enter the internal-external analysis.

**External factor analysis**

Factor analysis of the external variables $X_{31}-X_{46}$, is summarized in Table 6-4 and Table 6-5.

**TABLE 6-4**

FACTOR ANALYSIS OF EXTERNAL VARIABLES

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Number of factors extracted:</td>
<td>2&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>2. Per cent of total variance of data explained by the two factors:</td>
<td>97.20</td>
</tr>
<tr>
<td>3. Per cent of total variance of data explained by the rotated factors:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Factor 1</td>
</tr>
<tr>
<td></td>
<td>Factor 2</td>
</tr>
</tbody>
</table>

<sup>a</sup>Factors extracted until the eigenvalue approaches one

A very high percentage of the total variance of data, 97.20 per cent, is explained by two factors. Most of the variance is explained by factor 1 with factor 2 explaining 11.16 per cent of the total variance.

Factor 1 displays very high loadings on all variables except $X_{37}$ and $X_{41}$. The highly loaded variables displaying high communalities and correlations are primarily measures of inflation and, to a lesser extent, growth of the economy in terms of personal income and disposable income. The primary dimension defined by factor 1 is interpreted to be Inflation.
TABLE 6-5

ROTATED FACTOR LOADINGS DEFINING DIMENSIONS OF EXTERNAL VARIABLES AND COMMUNALITIES

<table>
<thead>
<tr>
<th>Variable No.</th>
<th>Name</th>
<th>Communality</th>
<th>Factor Loadings</th>
</tr>
</thead>
<tbody>
<tr>
<td>X31</td>
<td>Consumer Price Index</td>
<td>.996</td>
<td>.998</td>
</tr>
<tr>
<td>X32</td>
<td>Wholesale Price Index</td>
<td>.984</td>
<td>.990</td>
</tr>
<tr>
<td>X33</td>
<td>Construction Cost Index</td>
<td>.995</td>
<td>.998</td>
</tr>
<tr>
<td>X34</td>
<td>Risk-Free Interest Rate</td>
<td>.871</td>
<td>.712</td>
</tr>
<tr>
<td>X35</td>
<td>Home Mortgage Rates</td>
<td>.937</td>
<td>.934</td>
</tr>
<tr>
<td>X36</td>
<td>Yield on Corporate Bonds</td>
<td>.984</td>
<td>.970</td>
</tr>
<tr>
<td>X38</td>
<td>Personal Income Current Dollars</td>
<td>.992</td>
<td>.991</td>
</tr>
<tr>
<td>X39</td>
<td>Personal Income Constant Dollars</td>
<td>.929</td>
<td>.941</td>
</tr>
<tr>
<td>X40</td>
<td>National Rental Vacancy Rates</td>
<td>.897</td>
<td>-.942</td>
</tr>
<tr>
<td>X42</td>
<td>Consumer Price Index Less Shelter</td>
<td>.997</td>
<td>.998</td>
</tr>
<tr>
<td>X43</td>
<td>Boeckh Construction Price Index (Apts., Hotels, Office Bldgs.)</td>
<td>.998</td>
<td>.999</td>
</tr>
<tr>
<td>X44</td>
<td>Boeckh Construction Price Index (Residential)</td>
<td>.998</td>
<td>.999</td>
</tr>
<tr>
<td>X45</td>
<td>Disposable Personal Income Current Dollars</td>
<td>.963</td>
<td>.977</td>
</tr>
<tr>
<td>X46</td>
<td>Disposable Income Constant Dollars</td>
<td>.867</td>
<td>.915</td>
</tr>
<tr>
<td>X41</td>
<td>Number of New Private Multi-Family Units Started Annually</td>
<td>.955</td>
<td>.846</td>
</tr>
<tr>
<td>X37b</td>
<td>Dividend Yield on Common Stocks</td>
<td>.718</td>
<td>.528</td>
</tr>
</tbody>
</table>

Negative signs indicate direction of the relationship between the factor and the variable.

Not included in a factor dimension because of relatively low factor loadings.
Any of these variables can be used to represent the underlying dimension.

The factor loading of National Rental Vacancy Rate, $X_{40}$, is negative, indicating that the variable is negatively correlated with the factor. This is consistent with the expected relationship between rental vacancies and inflation and income. Rising incomes and rates of inflation suggest historically the rising phase of the business cycle which tends to be accompanied by a decrease in rental vacancies. Also, $X_{40}$ is highly negatively correlated with all external variables.

The Consumer Price Index Less Shelter, $X_{42}$, is selected as the variable included in the internal-external factor analysis representing the dimension Inflation. The factor loading of .998 and the communality of .997 are extremely high as is the correlation with the other highly loaded factors. As an independent variable utilized to predict NOI, it is a particularly desirable index of inflation because the housing component has been removed.

Factor 2 is highly loaded on only one variable, Number of New Private Multi-Family Units Started Annually, $X_{41}$. Both the communality and factor loading are high. Factor 2 defines a dimension of housing supply; consequently, the factor is identified as Supply. The variable $X_{41}$ is included in the analysis of internal and external variables.

The Dividend Yield on Common Stocks, $X_{37}$, has a loading of .528 in factor 1 and .663 in factor 2. The variable cannot be associated with the underlying dimensions defined by the two factors but suggests a third dimension which is not readily definable using varimax rotation.
This indicates the desirability of testing the statistical significance of the variable in the regression model.

**Internal-external factor analysis**

The internal-external factor analysis includes 28 of the variables in the original internal analysis plus variables $X_{41}$ and $X_{42}$ from the external analysis. Variables selected to represent the dimensions identified in this data are introduced into the regression model.

Factor analysis of the internal-external variables, $X_2$-$X_{28}$, $X_{30}$, $X_{41}$, and $X_{42}$ is summarized in Table 6-6 and Table 6-7.

**TABLE 6-6**

**FACTOR ANALYSIS OF INTERNAL-EXTERNAL VARIABLES**

<table>
<thead>
<tr>
<th>1. Number of factors extracted:</th>
<th>5 $^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. Per cent of total variance of data explained by the five factors:</td>
<td>87.28</td>
</tr>
<tr>
<td>3. Per cent of total variance of data explained by the rotated factors:</td>
<td></td>
</tr>
<tr>
<td>Factor 1</td>
<td>35.89</td>
</tr>
<tr>
<td>Factor 2</td>
<td>31.64</td>
</tr>
<tr>
<td>Factor 3</td>
<td>6.37</td>
</tr>
<tr>
<td>Factor 4</td>
<td>7.74</td>
</tr>
<tr>
<td>Factor 5</td>
<td>5.69</td>
</tr>
</tbody>
</table>

$^a$Factors extracted until the eigenvalue approaches 1.00.

Five factors explain 87.28 per cent of the variance of the data. Factors 1 and 2 of the rotated matrix account for 67.23 per cent of the variance
TABLE 6-7

ROTATED FACTOR LOADINGS DEFINING DIMENSIONS OF INTERNAL-EXTERNAL VARIABLES AND COMMUNALITIES

<table>
<thead>
<tr>
<th>Variable No.</th>
<th>Name</th>
<th>Communality</th>
<th>Factor Loadings</th>
</tr>
</thead>
<tbody>
<tr>
<td>X23</td>
<td>Net Operating Income</td>
<td>.962</td>
<td>.924</td>
</tr>
<tr>
<td>X25</td>
<td>Operating Ratio A</td>
<td>.983</td>
<td>-.967</td>
</tr>
<tr>
<td>X26</td>
<td>Operating Ratio B</td>
<td>.985</td>
<td>-.975</td>
</tr>
<tr>
<td>X27</td>
<td>Net Income Ratio A</td>
<td>.962</td>
<td>.962</td>
</tr>
<tr>
<td>X28</td>
<td>Net Income Ratio B</td>
<td>.985</td>
<td>.979</td>
</tr>
<tr>
<td>X30</td>
<td>Average Age of Building</td>
<td>.902</td>
<td>-.942</td>
</tr>
<tr>
<td>X2</td>
<td>Gross Possible Rental</td>
<td>.992</td>
<td>.851</td>
</tr>
<tr>
<td>X4</td>
<td>Gross Possible Total Income</td>
<td>.994</td>
<td>.850</td>
</tr>
<tr>
<td>X5</td>
<td>Effective Gross Income</td>
<td>.996</td>
<td>.897</td>
</tr>
<tr>
<td>X12</td>
<td>Management</td>
<td>.776</td>
<td>.814</td>
</tr>
<tr>
<td>X20</td>
<td>Real Estate Taxes</td>
<td>.870</td>
<td>.905</td>
</tr>
<tr>
<td>X22</td>
<td>Total Expenses</td>
<td>.977</td>
<td>.891</td>
</tr>
<tr>
<td>X42</td>
<td>Consumer Price Index Less Shelter</td>
<td>.927</td>
<td>.862</td>
</tr>
<tr>
<td>X41</td>
<td>Number of New Private Multi-Family Units Started Annually</td>
<td>.601</td>
<td>.691</td>
</tr>
</tbody>
</table>
TABLE 6-7--Continued

| No. | Variable Name                      | Communality | Factor Loadings
<table>
<thead>
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<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>X3</td>
<td>Miscellaneous Other Income</td>
<td>.875</td>
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<tr>
<td>X5</td>
<td>Vacancies and Delinquent Rents</td>
<td>.744</td>
<td></td>
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<tr>
<td>X13</td>
<td>Other Administrative Expenses</td>
<td>.477</td>
<td></td>
</tr>
<tr>
<td>X18</td>
<td>Miscellaneous Operating Expenses</td>
<td>.510</td>
<td></td>
</tr>
<tr>
<td>X7</td>
<td>Total Payroll Expense</td>
<td>.829</td>
<td></td>
</tr>
<tr>
<td>X8</td>
<td>Electricity</td>
<td>.656</td>
<td></td>
</tr>
<tr>
<td>X9</td>
<td>Gas</td>
<td>.727</td>
<td></td>
</tr>
<tr>
<td>X10</td>
<td>Water</td>
<td>.689</td>
<td></td>
</tr>
<tr>
<td>X11</td>
<td>Heating Fuel</td>
<td>.826</td>
<td></td>
</tr>
<tr>
<td>X14</td>
<td>Painting and Decorating</td>
<td>.780</td>
<td></td>
</tr>
<tr>
<td>X15</td>
<td>Maintenance and Repairs</td>
<td>.845</td>
<td></td>
</tr>
<tr>
<td>X16</td>
<td>Supplies</td>
<td>.383</td>
<td></td>
</tr>
<tr>
<td>X17</td>
<td>Services</td>
<td>.533</td>
<td></td>
</tr>
<tr>
<td>X19</td>
<td>Insurance</td>
<td>.748</td>
<td></td>
</tr>
<tr>
<td>X21</td>
<td>Other Taxes</td>
<td>.654</td>
<td></td>
</tr>
<tr>
<td>X24</td>
<td>Turnover Rate</td>
<td>.642</td>
<td></td>
</tr>
</tbody>
</table>

aNegative signs indicate direction of the relationship between the factor and the variable.

bVariables X7, X8, X9, X10, X11, X14, X15, X16, X17, X19, X21, and X24 are not included in a factor dimension because of the relatively low factor loadings.
of the data.

Factor 1 is highly loaded on variables $X_{23}$, $X_{25}$, $X_{26}$, $X_{27}$, $X_{28}$, and $X_{30}$. These variables displaying high communality, correlation, and factor loadings, are those same variables which are highly loaded in factor 1, Performance, in the analysis of internal variables. The signs are also identical, representing the direction of the relationship between the factor and the variable. The underlying dimension of this factor is Performance as is manifest through NOI and operating ratios. It is assumed that performance is a function of age as defined by Babcock. The high correlation between building age and performance measures supports the assumption. Age is the most accessible of the highly loaded variables and directly answers the objective to measure the effect of age on NOI. Average Age of Building, $X_{30}$, is introduced into the regression model as an independent variable defining the dimension of Performance as a function of age.

Factor 2 is highly loaded on variables $X_{2}$, $X_{4}$, $X_{6}$, $X_{12}$, $X_{20}$, $X_{22}$, and $X_{42}$. The highly loaded variables included in this factor are the same as those in factor 2 of the analysis of internal variables with the addition of the Consumer Price Index Less Shelter, $X_{42}$. The factor loadings and communalities are relatively high with the communality of Management, $X_{12}$, being somewhat less than the other variables. Incomes, $X_{2}$, $X_{4}$, and $X_{6}$; and expenses, $X_{20}$, and $X_{22}$; are to a degree a function of management. As this function is to increase incomes and decrease expenses, a positive correlation between incomes and management and

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Babcock, The Valuation of Real Estate, op. cit., Chapter 27, pp. 412-414.
negative correlation between expenses and management is anticipated. Incomes and expenses and the cost of management rise with inflation. Varimax rotation was not successful in separating management and inflation; consequently, the direction (sign) of the relationship cannot be easily interpreted. Variables loading high on factor 2 display relatively high correlations. Consequently, the factor is identified as Management-Inflation. Variables \(X_{12}\) and \(X_{42}\) are introduced into the regression models representing the underlying dimensions of Management-Inflation.

Factor 3 explains 6.37 per cent of the variance and has only one variable loaded significantly—Number of New Private Multi-Family Units Started Annually, \(X_{41}\). Compared to factors 1 and 2, the loading on this variable is relatively low, .691. The communality is quite low, .601 indicating that variables have not been included in this analysis that can explain adequately the variation in \(X_{41}\). The variable is introduced into the regression models but the low communality and factor loading suggest that the variable may not be significant in a stepwise regression.

Factor 4 is highly loaded on Miscellaneous Other Income, \(X_3\), and the communality of this variable is relatively high. This factor explains 7.74 per cent of the total variance of the data. The variable is the most highly loaded variable in factor 3 of the internal analysis. The factor is identified as Miscellaneous Other Income and the variable \(X_3\) is introduced into the regression.

Factor 5 is not highly loaded on any variable as compared to factors 1-4. The highest loadings are on variables \(X_5\), \(X_{13}\), and \(X_{18}\).
The communalities are relatively low with Vacancies and Delinquent Rents, $X_5$, being significantly higher. The underlying dimension of this factor is not clearly defined as it is difficult to identify a strong relationship between $X_5$, Other Administrative Expenses, $X_{13}$, and Miscellaneous Operating Expenses, $X_{18}$. The factor is identified in terms of Vacancies and Delinquent Rents based upon the higher communality of variable $X_5$. Variable $X_5$ is introduced into the regression models as most practitioners have considerably more experience in estimating this variable than variables $X_{13}$ and $X_{18}$.

The remaining variables, $X_7$, $X_8$, $X_9$, $X_{10}$, $X_{11}$, $X_{14}$, $X_{15}$, $X_{16}$, $X_{17}$, $X_{19}$, $X_{21}$, and $X_{24}$, are not highly loaded in any of the five factors. Consequently, they are not utilized in the development of the regression models.

Factor analysis of the internal-external variables has identified Performance, Management-Inflation, New Units Started Annually, Miscellaneous Other Income, and Vacancies and Delinquent Rents as the underlying dimensions of the data. Based on factor loadings, communalities, and correlations, $X_3$, $X_5$, $X_{12}$, $X_{30}$, $X_{41}$, and $X_{42}$, are identified as variables possessing a high degree of explanatory power and are introduced into the regression models. Parsimony of variables has been attained. Six variables convey the essential information contained in forty-four variables.

Analysis of the external variables suggests a dimension associated with Dividend Yield on Common Stocks, $X_{37}$, which cannot be interpreted through varimax rotation. The variable is introduced into
the regression models to determine if the variation in NOI explained by the models is increased as well as to test the statistical significance of the variable in the regression.

The variables selected for introduction into the regression models demonstrate a relatively high degree of statistical independence, minimizing the problem of multicolinearity associated with explanatory models. Table 6-8 displays the correlation coefficients for the dependent and selected independent variables plus variable X₃. Correlations between the selected independent variables are relatively low with the exception of the correlation between variables X₅/X₃₁ and X₃₀ which is -.661.

In several instances, the factor analysis has not provided clear identification of the underlying dimensions. However, those variables which have been selected to represent the factors are statistically justifiable and suggest logically causal relationships to NOI. In addition, these variables are historically accessible or well within the feasibility of professional estimates.

Multiple regression-NOI models

Using the variables identified and selected on the basis of factor analysis, four NOI models are developed and evaluated. The objectives of this research, the development of a predictive and

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³To minimize multicolinearity, all variables expressed in current dollars, X₃, X₅, X₁₂, and X₂₃, are transformed to constant dollars in the regression models. In order to depict clearly the relationships between variables as they are used in the regression models, the correlation coefficients in Table 6-8 are based on transformed variables.
### TABLE 6-8

**MATRIX OF CORRELATION COEFFICIENTS**

<table>
<thead>
<tr>
<th>Variable</th>
<th>$X_3/X_{31}$</th>
<th>$X_5/X_{31}$</th>
<th>$X_{12}/X_{31}$</th>
<th>$X_{23}/X_{31}$</th>
<th>$X_{30}$</th>
<th>$X_{37}$</th>
<th>$X_{41}$</th>
<th>$X_{42}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_3/X_{31}$</td>
<td>1.000</td>
<td>0.392</td>
<td>0.362</td>
<td>0.467</td>
<td>-0.327</td>
<td>0.062</td>
<td>-0.036</td>
<td>-0.056</td>
</tr>
<tr>
<td>$X_5/X_{31}$</td>
<td>0.392</td>
<td>1.000</td>
<td>-0.081</td>
<td>0.480</td>
<td>-0.661</td>
<td>-0.064</td>
<td>0.090</td>
<td>-0.105</td>
</tr>
<tr>
<td>$X_{12}/X_{31}$</td>
<td>0.362</td>
<td>-0.081</td>
<td>1.000</td>
<td>0.252</td>
<td>0.047</td>
<td>0.340</td>
<td>0.098</td>
<td>0.389</td>
</tr>
<tr>
<td>$X_{23}/X_{31}$</td>
<td>0.467</td>
<td>-0.480</td>
<td>0.252</td>
<td>1.000</td>
<td>-0.876</td>
<td>0.612</td>
<td>-0.123</td>
<td>-0.120</td>
</tr>
<tr>
<td>$X_{30}$</td>
<td>-0.327</td>
<td>-0.661</td>
<td>0.047</td>
<td>-0.876</td>
<td>1.000</td>
<td>0.070</td>
<td>0.042</td>
<td>0.129</td>
</tr>
<tr>
<td>$X_{37}$</td>
<td>0.062</td>
<td>-0.064</td>
<td>0.340</td>
<td>0.061</td>
<td>0.070</td>
<td>1.000</td>
<td>-0.224</td>
<td>0.566</td>
</tr>
<tr>
<td>$X_{41}$</td>
<td>-0.036</td>
<td>0.090</td>
<td>0.098</td>
<td>-0.123</td>
<td>0.042</td>
<td>-0.224</td>
<td>1.000</td>
<td>0.473</td>
</tr>
<tr>
<td>$X_{42}$</td>
<td>-0.056</td>
<td>-0.105</td>
<td>0.389</td>
<td>-0.120</td>
<td>0.129</td>
<td>0.566</td>
<td>0.473</td>
<td>1.000</td>
</tr>
</tbody>
</table>
explanatory model of NOI and quantification of the effects of building age and inflation on NOI, are met by Models 1 and 2, Table 6-9 and Table 6-10. Models 3 and 4, Table 6-11 and Table 6-12, are developed for use in verification of Models 1 and 2.

Model 1 is the preferred model with independent variables $X_3$, $X_5$, $X_{12}$, $X_{30}$, $X_{41}$, and $X_{42}$ introduced into the stepwise multiple regressions. Factor analysis suggests the significance of $X_{37}$, Dividend Yield on Common Stocks, but does not facilitate interpretation of the variable in terms of a dimension of the data. An alternative, Model 2, is developed using the same input variables as in Model 1 plus $X_{37}$ to determine if the predictive and explanatory qualities of the model are enhanced. Models 1 and 2 are based on data collected over the years 1962-1971, a total of 50 cases. Models using dummy variables to represent building age were evaluated and found unacceptable. (See Appendix D)

In order to minimize multicollinearity, as well as to express variables in a form more amenable to estimation, all variables expressed in current dollars are transformed to constant dollars by dividing by the Consumer Price Index, $X_{31}$. In each of the four models, analysis of the plots of residuals indicates a quadratic relationship with Average Age of Building, $X_{30}$. The quadratic function $a + bx + cx^2$, is transformed to a linear relationship by creating a variable, $X_{50}$, where $(X_{30})^2 = X_{50}$. The Average Age of Building is represented by the linear form, $b_{30}X_{30} + b_{50}X_{50}$. No discernible pattern is evident in the residuals after these transformations.
TABLE 6-9

NOI MODEL NUMBER 1

\[
\frac{X_{3.23}}{X_{3.31}} = a + b_3 \frac{X_3}{X_{3.31}} + b_5 \frac{X_5}{X_{3.31}} + b_{12} \frac{X_{12}}{X_{3.31}} + [b_{30}X_{3.30} + b_{50}X_{5.50}] + b_{42}X_{42}
\]

\[X_{5.50} = x_{3.30}^2\]

Years: 1962-1971
Number cases: 50
Multiple \(R^2 = .9146\)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(b_3)</td>
<td>0.8186</td>
<td>0.3053</td>
<td>0.1504</td>
<td>0.0561</td>
<td>2.6809</td>
<td>0.01</td>
</tr>
<tr>
<td>(b_5)</td>
<td>-1.2597</td>
<td>0.2866</td>
<td>-0.3350</td>
<td>0.0762</td>
<td>-4.3963</td>
<td>0.01</td>
</tr>
<tr>
<td>(b_{12})</td>
<td>3.4828</td>
<td>0.8706</td>
<td>0.2302</td>
<td>0.0575</td>
<td>4.0034</td>
<td>0.01</td>
</tr>
<tr>
<td>(b_{30})</td>
<td>-3.5824</td>
<td>0.4902</td>
<td>-1.7090</td>
<td>0.2338</td>
<td>-7.3096</td>
<td>0.01</td>
</tr>
<tr>
<td>(b_{42})</td>
<td>-0.4632</td>
<td>0.2154</td>
<td>-0.1075</td>
<td>0.0500</td>
<td>-2.1500</td>
<td>0.01</td>
</tr>
<tr>
<td>(b_{50})</td>
<td>0.0235</td>
<td>0.0071</td>
<td>0.6881</td>
<td>0.2083</td>
<td>3.3034</td>
<td>0.01</td>
</tr>
</tbody>
</table>

\(a = 233.5549\)
\(F\) ratio = \(\frac{\text{mean square of regression}}{\text{mean square of error}} = 76.789\) (significant at the 0.01 level)
Standard error of the estimate = 12.7489

Correlation Matrix

<table>
<thead>
<tr>
<th>Variable</th>
<th>(X_3/X_{3.31})</th>
<th>(X_5/X_{3.31})</th>
<th>(X_{12}/X_{3.31})</th>
<th>(X_{3.30})</th>
<th>(X_{42})</th>
<th>(X_{5.50})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(X_3/X_{3.31})</td>
<td>1.000</td>
<td>0.392</td>
<td>0.362</td>
<td>-0.327</td>
<td>-0.056</td>
<td>-0.291</td>
</tr>
<tr>
<td>(X_5/X_{3.31})</td>
<td>0.392</td>
<td>1.000</td>
<td>-0.081</td>
<td>-0.661</td>
<td>-0.105</td>
<td>-0.532</td>
</tr>
<tr>
<td>(X_{12}/X_{3.31})</td>
<td>0.362</td>
<td>-0.081</td>
<td>1.000</td>
<td>0.047</td>
<td>0.389</td>
<td>0.091</td>
</tr>
<tr>
<td>(X_{3.30})</td>
<td>-0.327</td>
<td>-0.661</td>
<td>0.047</td>
<td>1.000</td>
<td>0.129</td>
<td>0.964</td>
</tr>
<tr>
<td>(X_{42})</td>
<td>-0.056</td>
<td>-0.105</td>
<td>0.389</td>
<td>0.129</td>
<td>1.000</td>
<td>0.122</td>
</tr>
<tr>
<td>(X_{5.50})</td>
<td>-0.291</td>
<td>-0.532</td>
<td>0.091</td>
<td>0.964</td>
<td>0.133</td>
<td>1.000</td>
</tr>
</tbody>
</table>
TABLE 6-10
NOI MODEL NUMBER 2

\[
\frac{X_{23}}{X_{31}} = a + b_3 \frac{X_3}{X_{31}} + b_5 \frac{X_5}{X_{31}} + b_{12} \frac{X_{12}}{X_{31}}
\]

\[
+ [b_{30}X_{30} + b_{50}X_{50}] + b_{37}X_{37} + b_{42}X_{42}
\]

\[X_{50} = X_{30}^2\]

Years: 1962-1971
Number cases: 50
\[R^2 = .9251\]

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>b_3</td>
<td>0.7806</td>
<td>0.2897</td>
<td>0.1434</td>
<td>0.0532</td>
<td>2.6954</td>
<td>0.01</td>
</tr>
<tr>
<td>b_5</td>
<td>-1.2579</td>
<td>0.2716</td>
<td>-0.3345</td>
<td>0.0722</td>
<td>-4.6329</td>
<td>0.01</td>
</tr>
<tr>
<td>b_{12}</td>
<td>3.2537</td>
<td>0.8302</td>
<td>0.2151</td>
<td>0.0548</td>
<td>3.9251</td>
<td>0.01</td>
</tr>
<tr>
<td>b_{30}</td>
<td>-3.6075</td>
<td>0.4646</td>
<td>-1.7210</td>
<td>0.2216</td>
<td>-7.7662</td>
<td>0.01</td>
</tr>
<tr>
<td>b_{37}</td>
<td>19.3910</td>
<td>7.9806</td>
<td>0.1262</td>
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<td>0.01</td>
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<tr>
<td>b_{42}</td>
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<td>0.2351</td>
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<td>-3.1816</td>
<td>0.01</td>
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<tr>
<td>b_{50}</td>
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<td>0.0067</td>
<td>0.6989</td>
<td>0.1974</td>
<td>3.5405</td>
<td>0.01</td>
</tr>
</tbody>
</table>

\[a = 202.0829\]
\[F \text{ ratio} = \frac{\text{mean square of regression}}{\text{mean square of error}} = 74.168\]
(significant at the 0.01 level)
Standard error of the estimate = 12.0787

Correlation Matrix

<table>
<thead>
<tr>
<th>Variable</th>
<th>(X_3/X_{31})</th>
<th>(X_5/X_{31})</th>
<th>(X_{12}/X_{31})</th>
<th>(X_{30})</th>
<th>(X_{37})</th>
<th>(X_{42})</th>
<th>(X_{50})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(X_3/X_{31})</td>
<td>1.000</td>
<td>0.392</td>
<td>0.362</td>
<td>-0.327</td>
<td>0.062</td>
<td>-0.056</td>
<td>-0.291</td>
</tr>
<tr>
<td>(X_5/X_{31})</td>
<td>0.392</td>
<td>1.000</td>
<td>-0.081</td>
<td>-0.661</td>
<td>-0.064</td>
<td>-0.105</td>
<td>-0.532</td>
</tr>
<tr>
<td>(X_{12}/X_{31})</td>
<td>0.362</td>
<td>-0.081</td>
<td>1.000</td>
<td>0.047</td>
<td>0.340</td>
<td>0.389</td>
<td>0.091</td>
</tr>
<tr>
<td>(X_{30})</td>
<td>-0.327</td>
<td>-0.661</td>
<td>-0.047</td>
<td>1.000</td>
<td>0.070</td>
<td>0.129</td>
<td>0.964</td>
</tr>
<tr>
<td>(X_{37})</td>
<td>0.062</td>
<td>-0.064</td>
<td>0.340</td>
<td>0.070</td>
<td>1.000</td>
<td>0.566</td>
<td>0.073</td>
</tr>
<tr>
<td>(X_{42})</td>
<td>-0.056</td>
<td>-0.105</td>
<td>0.389</td>
<td>0.129</td>
<td>0.566</td>
<td>1.000</td>
<td>0.133</td>
</tr>
<tr>
<td>(X_{50})</td>
<td>-0.291</td>
<td>-0.532</td>
<td>0.091</td>
<td>0.964</td>
<td>0.073</td>
<td>0.133</td>
<td>1.000</td>
</tr>
</tbody>
</table>
### TABLE 6-11

**NOI MODEL NUMBER 3**

\[
\frac{X_{23}}{X_{31}} = a + b_{3} \frac{X_{3}}{X_{31}} + b_{5} \frac{X_{5}}{X_{31}} + b_{12} \frac{X_{12}}{X_{31}} + [b_{30} X_{30} + b_{50} X_{50}] + b_{42} X_{42}
\]

\[X_{50} = X_{30}\]

Years: 1962-1968  
Number cases: 35  
Multiple \( R^2 = .9236 \)

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>(b_{3})</td>
<td>0.8343</td>
<td>0.3156</td>
<td>0.1785</td>
<td>0.0675</td>
<td>2.6444</td>
<td>0.01</td>
</tr>
<tr>
<td>(b_{5})</td>
<td>-1.4555</td>
<td>0.3299</td>
<td>-0.4034</td>
<td>0.0914</td>
<td>-4.4135</td>
<td>0.01</td>
</tr>
<tr>
<td>(b_{12})</td>
<td>3.3946</td>
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<td>0.2153</td>
<td>0.0627</td>
<td>3.4338</td>
<td>0.01</td>
</tr>
<tr>
<td>(b_{30})</td>
<td>-3.5048</td>
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<td>0.2728</td>
<td>-5.9435</td>
<td>0.01</td>
</tr>
<tr>
<td>(b_{42})</td>
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<td>0.5204</td>
<td>-0.1245</td>
<td>0.0555</td>
<td>-2.2432</td>
<td>0.02</td>
</tr>
<tr>
<td>(b_{50})</td>
<td>0.0212</td>
<td>0.0088</td>
<td>0.5776</td>
<td>0.2416</td>
<td>2.3907</td>
<td>0.02</td>
</tr>
</tbody>
</table>

\(a = 305.8181\)

\(F\) ratio = \(
\frac{\text{mean square of regression}}{\text{mean square of error}} = 56.478
\)

(significant at the 0.01 level)

Standard error of the estimate = 12.6925

**Correlation Matrix**

<table>
<thead>
<tr>
<th>Variable</th>
<th>(X_{3}/X_{31})</th>
<th>(X_{5}/X_{31})</th>
<th>(X_{12}/X_{31})</th>
<th>(X_{30})</th>
<th>(X_{42})</th>
<th>(X_{50})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(X_{3}/X_{31})</td>
<td>1.000</td>
<td>0.396</td>
<td>0.443</td>
<td>-0.339</td>
<td>-0.071</td>
<td>-0.320</td>
</tr>
<tr>
<td>(X_{5}/X_{31})</td>
<td>0.396</td>
<td>1.000</td>
<td>-0.056</td>
<td>-0.655</td>
<td>-0.206</td>
<td>-0.520</td>
</tr>
<tr>
<td>(X_{12}/X_{31})</td>
<td>0.443</td>
<td>-0.056</td>
<td>1.000</td>
<td>-0.033</td>
<td>0.221</td>
<td>-0.039</td>
</tr>
<tr>
<td>(X_{30})</td>
<td>-0.339</td>
<td>-0.655</td>
<td>-0.033</td>
<td>1.000</td>
<td>0.092</td>
<td>0.964</td>
</tr>
<tr>
<td>(X_{42})</td>
<td>-0.071</td>
<td>-0.206</td>
<td>0.221</td>
<td>0.092</td>
<td>1.000</td>
<td>0.094</td>
</tr>
<tr>
<td>(X_{50})</td>
<td>-0.320</td>
<td>-0.520</td>
<td>-0.039</td>
<td>0.964</td>
<td>0.094</td>
<td>1.000</td>
</tr>
</tbody>
</table>
TABLE 6-12
NOI MODEL NUMBER 4

\[
\frac{x_{23}}{x_{31}} = a + b_3 \frac{x_3}{x_{31}} + b_5 \frac{x_5}{x_{31}} + b_{12} \frac{x_{12}}{x_{31}}
+ [b_{30}x_{30} + b_{50}x_{50}]
\]

\[x_{50} = x_{30}^2\]

Years: 1962-1966
Number cases: 25
Multiple \(R^2 = .9195\)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(b_3)</td>
<td>0.8372</td>
<td>0.3856</td>
<td>0.1907</td>
<td>0.0878</td>
<td>2.1719</td>
<td>0.02</td>
</tr>
<tr>
<td>(b_5)</td>
<td>-1.5231</td>
<td>0.3722</td>
<td>-0.4493</td>
<td>0.1098</td>
<td>4.0919</td>
<td>0.01</td>
</tr>
<tr>
<td>(b_{12})</td>
<td>4.2209</td>
<td>1.6702</td>
<td>0.2212</td>
<td>0.0875</td>
<td>2.5280</td>
<td>0.01</td>
</tr>
<tr>
<td>(b_{30})</td>
<td>-3.6334</td>
<td>0.7617</td>
<td>-1.6141</td>
<td>0.3384</td>
<td>4.7597</td>
<td>0.01</td>
</tr>
<tr>
<td>(b_{50})</td>
<td>0.0209</td>
<td>0.0120</td>
<td>0.5332</td>
<td>0.3062</td>
<td>1.7413</td>
<td>0.05</td>
</tr>
</tbody>
</table>

\(a = 187.5330\)

\(F\) ratio = \(\frac{\text{mean square of regression}}{\text{mean square of error}} = 43.419\)
(significant at the 0.01 level)
Standard error of the estimate = 13.7963

Correlation Matrix

<table>
<thead>
<tr>
<th>Variable</th>
<th>(x_3/x_{31})</th>
<th>(x_5/x_{31})</th>
<th>(x_{12}/x_{31})</th>
<th>(x_{30})</th>
<th>(x_{50})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_3/x_{31})</td>
<td>1.000</td>
<td>0.367</td>
<td>0.499</td>
<td>-0.272</td>
<td>-0.277</td>
</tr>
<tr>
<td>(x_5/x_{31})</td>
<td>0.367</td>
<td>1.000</td>
<td>-0.153</td>
<td>-0.654</td>
<td>-0.524</td>
</tr>
<tr>
<td>(x_{12}/x_{31})</td>
<td>0.499</td>
<td>-0.153</td>
<td>1.000</td>
<td>-0.009</td>
<td>-0.121</td>
</tr>
<tr>
<td>(x_{30})</td>
<td>-0.272</td>
<td>-0.654</td>
<td>-0.009</td>
<td>1.000</td>
<td>0.963</td>
</tr>
<tr>
<td>(x_{50})</td>
<td>-0.277</td>
<td>-0.524</td>
<td>-0.121</td>
<td>0.963</td>
<td>1.000</td>
</tr>
</tbody>
</table>
As indicated by the factor analysis, variable $X_{41}$, representing factor 3, New Units Started Annually, is introduced into each of the models. Using an $F$ value of 2.00 to determine if an independent variable will enter or be removed from the regression, $X_{41}$ with a low communality of .601 was not statistically significant. Consequently, it does not appear in any of the models.

The difference in regression coefficients $b_{42}$ between Models 1 and 2 reflects the inclusion of variable $X_{37}$, Dividend Yield on Common Stocks, in Model 2. This variable has a relatively high correlation with $X_{42}$, Consumer Price Index Less Shelter, suggesting that the coefficients $b_{37}$ and $b_{42}$ may not be stable or reliable in terms of explanatory power when both variables are included in the model. The slightly higher predictive capacity, indicated by the multiple correlation coefficient of Model 2, is not jeopardized.

Acceptance of NOI model

Acceptance of the NOI model is based on the model explaining a high percentage of the variation in NOI, explanatory power of each independent variable, sensitivity analysis, residual plots, and plausibility of the model.

The multiple $R^2$ of all models exceeds .914 indicating that a high percentage of the variability of NOI, expressed in constant dollars, is explained by the independent variables. The regressions are statistically significant at the 0.01 level as measured by the $F$ ratio. All
coefficients in Models 1 and 2 are significant at the 0.01 level.\footnote{In model 3, Table 6-11, the coefficients are significant at the 0.01 level with the exception of $b_{42}$ and $b_{50}$ which are significant at the 0.02 level. In model 4, Table 6-12, the coefficients are significant at the 0.01 level with the exception of $b_{3}$ which is significant at the 0.02 level and $b_{50}$ which is significant at the 0.05 level.}

The explanatory power of the independent variables is reliable based on the stability of regression coefficients, Table 6-13; the relatively low correlation between independent variables; the high multiple $R^2$; and the statistical significance of the coefficients and the regression. The relatively low correlation between independent variables, Tables 6-9 and 6-10, indicates that multicollinearity is minimal.

**TABLE 6-13**

**STABILITY OF COEFFICIENTS**

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Regression Coefficients--Model Number 1 2 3 4</th>
<th>Coefficient Deviation$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_3$</td>
<td>0.8186 0.7806 0.8343 0.8372</td>
<td>4.02%</td>
</tr>
<tr>
<td>$b_5$</td>
<td>-1.2597 -1.2579 -1.4555 -1.5231</td>
<td>10.85</td>
</tr>
<tr>
<td>$b_{12}$</td>
<td>3.4828 3.2537 3.3946 4.2209</td>
<td>17.63</td>
</tr>
<tr>
<td>$b_{30}$</td>
<td>-3.5824 -3.6075 -3.5048 -3.6334</td>
<td>2.15</td>
</tr>
<tr>
<td>$b_{37}$</td>
<td>----- 19.3910 ----- -----</td>
<td>-----</td>
</tr>
<tr>
<td>$b_{42}$</td>
<td>-0.4632 -0.7470 -1.1667 -----</td>
<td>47.25</td>
</tr>
<tr>
<td>$b_{50}$</td>
<td>0.0235 0.0238 0.0212 0.0209</td>
<td>6.37</td>
</tr>
</tbody>
</table>

$^a$Maximum deviation from the mean of the coefficients in each model.
The sign of the regression coefficient within a .9544 confidence interval is invariant between Models 1 and 2, Tables 6-14 and 6-15. The confidence interval at the .9544 level for the standard error of the estimate is ±15.78 per cent and ±14.96 per cent of the mean NOI (in constant dollars) for Models 1 and 2 respectively.

**TABLE 6-14**

**CONFIDENCE INTERVALS NOI MODEL NUMBER 1**

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Regression Coefficient</th>
<th>Confidence Interval .9544 Probability$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_3$</td>
<td>0.8186</td>
<td>0.2080 . . . . . . . . . 1.4292</td>
</tr>
<tr>
<td>$b_5$</td>
<td>-1.2597</td>
<td>-1.8329 . . . . . . . . . -0.6865</td>
</tr>
<tr>
<td>$b_{12}$</td>
<td>3.4828</td>
<td>1.7416 . . . . . . . . . 5.2240</td>
</tr>
<tr>
<td>$b_{30}$</td>
<td>-3.5824</td>
<td>-4.5628 . . . . . . . . . -2.6020</td>
</tr>
<tr>
<td>$b_{42}$</td>
<td>-0.4632</td>
<td>-0.8940 . . . . . . . . . -0.0324</td>
</tr>
<tr>
<td>$b_{50}$</td>
<td>0.0235</td>
<td>0.0093 . . . . . . . . . 0.0377</td>
</tr>
</tbody>
</table>

$^a$Confidence interval .9544 probability for the standard error of the estimate . . . . . . ±25.4978.

The sensitivity of change in NOI to a unit change in a variable and to a 10 per cent deviation from the variable mean is displayed in Table 6-16 and Table 6-17 for Models 1 and 2. A ten per cent error in estimating a variable has no more than a 3.99 per cent effect on NOI in Model 1 and a 4.72 per cent effect in Model 2 based on the average NOI. As the building age is known or can be predicted precisely, sensitivity
does not suggest error but rate of change of NOI at the average building age of 28 years.  

**TABLE 6-15**  
CONFIDENCE INTERVALS NOI MODEL NUMBER 2

<table>
<thead>
<tr>
<th>Regression Coefficients</th>
<th>Coefficient</th>
<th>Regression Coefficient</th>
<th>Confidence Interval .9544 Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>b₃</td>
<td>0.7860</td>
<td>0.2066 ... 1.3654</td>
</tr>
<tr>
<td></td>
<td>b₅</td>
<td>-1.2579</td>
<td>-1.8011 ... -0.7147</td>
</tr>
<tr>
<td></td>
<td>b₁₂</td>
<td>3.2537</td>
<td>1.5933 ... 4.9141</td>
</tr>
<tr>
<td></td>
<td>b₃₀</td>
<td>-3.6075</td>
<td>-4.5367 ... -2.6783</td>
</tr>
<tr>
<td></td>
<td>b₃⁷</td>
<td>19.3910</td>
<td>3.4298 ... 35.3522</td>
</tr>
<tr>
<td></td>
<td>b₄₂</td>
<td>-0.7470</td>
<td>-1.2172 ... -0.2768</td>
</tr>
<tr>
<td></td>
<td>b₅₀</td>
<td>0.0238</td>
<td>0.0104 ... 0.0372</td>
</tr>
</tbody>
</table>

The overall residual plots suggest normal distributions. The plot of predicted NOI versus the residuals approximates a horizontal band indicating homoscedasticity and the inclusion of necessary independent variables in the model. The plot of independent variables versus residuals approximates a horizontal band, suggesting that an adequate number of independent variables has been included, that variance is relatively constant, and that variables are linear.

5The effect of building age on NOI, ceteris paribus, is a quadratic relationship: \([-3.5824 X_{30} + 0.0235 X_{30}^2]\) for Model 1 and \([-3.6075 X_{30}^2 + 0.0238 X_{30}\)] for model 2.

---

a Confidence interval .9544 probability for the standard error of the estimate . . . . . . . . ±24.1574.
### TABLE 6-16
SENSITIVITY OF NOI MODEL NUMBER 1

#### Sensitivity of NOI to a Unit Change of a Variable

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit Change of Variable</th>
<th>Change in NOI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_3 / X_{31}$</td>
<td>$1.00$</td>
<td>$0.82$</td>
</tr>
<tr>
<td>$X_5 / X_{31}$</td>
<td>$1.00$</td>
<td>$-1.26$</td>
</tr>
<tr>
<td>$X_{12} / X_{31}$</td>
<td>$1.00$</td>
<td>$3.48$</td>
</tr>
<tr>
<td>$[X_{30} + X_{50}]^a$</td>
<td>$1.00 \text{ year}$</td>
<td>$-2.20$</td>
</tr>
<tr>
<td>$X_{42}$</td>
<td>$1.00 \text{ index}$</td>
<td>$-0.46$</td>
</tr>
</tbody>
</table>

#### Sensitivity of NOI to Change of a Variable Based on Average Data

<table>
<thead>
<tr>
<th>Variable</th>
<th>Variable Average</th>
<th>Per cent Change of Variable</th>
<th>Per cent Change in NOI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_3 / X_{31}$</td>
<td>$$5.11$</td>
<td>$10.00$</td>
<td>$0.26$</td>
</tr>
<tr>
<td>$X_5 / X_{31}$</td>
<td>$15.62$</td>
<td>$10.00$</td>
<td>$1.22$</td>
</tr>
<tr>
<td>$X_{12} / X_{31}$</td>
<td>$18.51$</td>
<td>$10.00$</td>
<td>$3.99$</td>
</tr>
<tr>
<td>$[X_{30} + X_{50}]^a$</td>
<td>$28 \text{ years}$</td>
<td>$1.00 \text{ year}$</td>
<td>$1.36$</td>
</tr>
<tr>
<td>$X_{42}$</td>
<td>$101.49 \text{ index}$</td>
<td>$10.00$</td>
<td>$2.89$</td>
</tr>
</tbody>
</table>

^aBased on average building age of 28 years. Change in NOI is a quadratic function of age.

^bBased on average annual NOI of 161.38 per room.
### TABLE 6-17
SENSITIVITY OF NOI MODEL NUMBER 2

#### Sensitivity of NOI to a Unit Change of a Variable

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit Change of Variable</th>
<th>Change in NOI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_3 / X_{31}$</td>
<td>$1.00$</td>
<td>$0.78$</td>
</tr>
<tr>
<td>$X_5 / X_{31}$</td>
<td>$1.00$</td>
<td>$-1.26$</td>
</tr>
<tr>
<td>$X_{12} / X_{31}$</td>
<td>$1.00$</td>
<td>$3.25$</td>
</tr>
<tr>
<td>$[X_{30} + X_{50}]^a$</td>
<td>$1.00$ year</td>
<td>$-2.25$</td>
</tr>
<tr>
<td>$X_{37}$</td>
<td>$1.00$ per cent</td>
<td>$19.39$</td>
</tr>
<tr>
<td>$X_{42}$</td>
<td>$1.00$ index</td>
<td>$-0.75$</td>
</tr>
</tbody>
</table>

#### Sensitivity of NOI to Change of a Variable Based on Average Data

<table>
<thead>
<tr>
<th>Variable</th>
<th>Variable Average</th>
<th>Per cent Change of Variable</th>
<th>Per cent Change in NOI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_3 / X_{31}$</td>
<td>$5.11$</td>
<td>10.00</td>
<td>.25</td>
</tr>
<tr>
<td>$X_5 / X_{31}$</td>
<td>15.62</td>
<td>10.00</td>
<td>1.22</td>
</tr>
<tr>
<td>$X_{12} / X_{31}$</td>
<td>18.51</td>
<td>10.00</td>
<td>3.73</td>
</tr>
<tr>
<td>$[X_{30} + X_{50}]^a$</td>
<td>28 years</td>
<td>1.00 year</td>
<td>1.39</td>
</tr>
<tr>
<td>$X_{37}$</td>
<td>3.35 per cent</td>
<td>10.00</td>
<td>4.03</td>
</tr>
<tr>
<td>$X_{42}$</td>
<td>101.49 index</td>
<td>10.00</td>
<td>4.72</td>
</tr>
</tbody>
</table>

^aBased on average building age of 28 years. Change in NOI is a quadratic function of age.

^bBased on average annual NOI of 161.38 per room.
Verification of the NOI model

Verification of the models is based on stability of regression coefficients over models and time, high coefficients of multiple correlation in all models, and prediction of NOI for 1969, 1970, and 1971, using Model 3. The stability of regression coefficients, based on fitting models to data over different time periods, is a primary means of model verification. Models 3 and 4 serve this function. Table 6-13 displays the regression coefficients and the maximum percent deviation from the mean of the respective coefficients in each model. Coefficient $b_{37}$ was not introduced into Model 1 and is not statistically significant in Models 3 and 4. With the exception of $b_{42}$, the coefficients are relatively stable over different time periods.

Variable $X_{42}$, Consumer Price Index Less Shelter, was not statistically significant in Model 4 based on the years 1962-1966. During this period, the rate of inflation was quite low; consequently, inflation was not a significant determinant of NOI. The unanticipated accelerating inflation in the years 1966, 1967, and 1968 may explain why the inflation coefficient, $b_{42}$, is higher in Model 3 than in Models 1 and 2. Inflation was increasingly anticipated in 1969, 1970, and 1971, reducing the negative effect of the coefficient.

Verification of the model includes the estimation of NOI for the years 1969, 1970, and 1971 using Model 3.\textsuperscript{6} Fifteen estimates are

\textsuperscript{6}Estimates utilize the mean ages of the five age classifications identified in the IREM data sample.
calculated with one estimate exceeding two standard errors of the estimate. As the estimates are biased low, the dispersion of the estimates is less than ±2 standard errors of the estimate.

**Equity Model**

The equity model is used to determine if the class of property is an inflation hedge as measured by the annual holding period changes in equity interest and market value expressed in constant dollars. The equity model derived in Chapter V:

\[
E = \frac{1}{P} \frac{(1+p) \text{NOI}}{(1+i)I+R} - \frac{(1+p) \text{NOI}}{(1+i)I+R} \frac{1}{P}
\]

and expressed in terms of the data:

\[
E = \frac{1}{X_{31}} X_{23} X_{47} - \frac{X_{23} X_{48}}{X_{47} X_{31}}
\]

is quantified and analyzed in terms of the two components:

\[
\text{Market Value} = \frac{X_{23}}{X_{31} X_{47}}
\]

and

\[
\text{Mortgage} = \frac{X_{23} X_{48}}{X_{31} X_{47}}
\]

The results are tabulated in Table 6-18, Average Value Per Room; Table 6-19, Average Mortgage Per Room; and Table 6-20, Equity Model. Data used in quantifying the equity model are recorded in Tables 6-18 and 6-19.
TABLE 6-18

AVERAGE MARKET VALUE PER ROOM (CONSTANT DOLLARS)

\[
\text{Market value} = \frac{X_{23}}{X_{31}X_{47}}
\]

Years: 1966-1970
Holding period: 12 months

<table>
<thead>
<tr>
<th>Year(^a)</th>
<th>NOI (X_{23})</th>
<th>CPI (X_{31})</th>
<th>Cap Rate (X_{47})</th>
<th>Market Value</th>
<th>Holding Per.Chg.</th>
<th>Holding Per.Chg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1966</td>
<td>$159.99</td>
<td>97.20</td>
<td>8.40 %</td>
<td>$1959.51</td>
<td>$-82.53</td>
<td>-4.21 %</td>
</tr>
<tr>
<td>1967</td>
<td>$161.42</td>
<td>100.00</td>
<td>8.60</td>
<td>$1876.98</td>
<td>$+132.50</td>
<td>-7.06</td>
</tr>
<tr>
<td>1968</td>
<td>$161.78</td>
<td>104.20</td>
<td>8.90</td>
<td>$1744.48</td>
<td>$-20.55</td>
<td>-1.18</td>
</tr>
<tr>
<td>1969</td>
<td>$180.77</td>
<td>109.80</td>
<td>9.55</td>
<td>$1723.95</td>
<td>$100.56</td>
<td>5.83</td>
</tr>
<tr>
<td>1970</td>
<td>$224.92</td>
<td>116.30</td>
<td>10.60</td>
<td>$1824.49</td>
<td>$-3.53</td>
<td>-0.19</td>
</tr>
<tr>
<td>1971</td>
<td>$215.36</td>
<td>121.30</td>
<td>9.75</td>
<td>$1820.96</td>
<td>----</td>
<td>----</td>
</tr>
</tbody>
</table>

\(^a\)Year begins July 1
### TABLE 6-19

**AVERAGE MORTGAGE PER ROOM (CONSTANT DOLLARS)**

Mortgage = \( \frac{X_{23} \times X_{48}}{X_{31} \times X_{47}} \)

Years: 1966-1970

Holding period: 12 months

<table>
<thead>
<tr>
<th>Year</th>
<th>NOI (X_{23})</th>
<th>CPI (X_{31})</th>
<th>Cap Rate (X_{47})</th>
<th>Loan/Val (X_{48})</th>
<th>Mortgage</th>
<th>Holding(^b) Per.Chg.</th>
<th>Holding(^b) Per.Chg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1966</td>
<td>$159.99</td>
<td>97.20</td>
<td>8.40 %</td>
<td>0.7030</td>
<td>$1371.53</td>
<td>$-38.57</td>
<td>-2.80 %</td>
</tr>
<tr>
<td>1967</td>
<td>161.42</td>
<td>100.00</td>
<td>8.60</td>
<td>0.7125</td>
<td>1337.35</td>
<td>-53.91</td>
<td>-4.03</td>
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<tr>
<td>1968</td>
<td>161.78</td>
<td>104.20</td>
<td>8.90</td>
<td>0.7275</td>
<td>1269.11</td>
<td>-64.73</td>
<td>-5.10</td>
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<tr>
<td>1969</td>
<td>180.77</td>
<td>109.80</td>
<td>9.55</td>
<td>0.7415</td>
<td>1278.30</td>
<td>-71.45</td>
<td>-5.59</td>
</tr>
<tr>
<td>1970</td>
<td>224.92</td>
<td>116.30</td>
<td>10.60</td>
<td>0.7430</td>
<td>1358.59</td>
<td>-55.87</td>
<td>-4.12</td>
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<tr>
<td>1971</td>
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<td>121.30</td>
<td>---</td>
<td>----</td>
<td>---</td>
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<td>---</td>
</tr>
</tbody>
</table>

\(^a\) Year begins July 1

\(^b\) Holding period change excludes amortization of mortgage.
TABLE 6-20

EQUITY MODEL (CONSTANT DOLLARS PER ROOM)

Equity = \frac{1}{X_{31}X_{47}} \frac{X_{23}}{X_{47}} - \frac{X_{23}X_{48}}{X_{47}X_{31}}

Years: 1966-1970
Holding Period: 12 months

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
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</thead>
<tbody>
<tr>
<td>1966</td>
<td>$581.95</td>
<td>$-43.95</td>
<td>-7.55%</td>
<td>$-82.53</td>
<td>$-38.57</td>
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<td>1967</td>
<td>$539.63</td>
<td>-78.59</td>
<td>-14.56</td>
<td>-132.50</td>
<td>-53.91</td>
</tr>
<tr>
<td>1968</td>
<td>$475.37</td>
<td>44.18</td>
<td>9.29</td>
<td>-20.55</td>
<td>-64.73</td>
</tr>
<tr>
<td>1969</td>
<td>$445.63</td>
<td>172.01</td>
<td>38.60</td>
<td>100.56</td>
<td>-71.45</td>
</tr>
<tr>
<td>1970</td>
<td>$468.90</td>
<td>52.34</td>
<td>11.16</td>
<td>-3.53</td>
<td>-55.87</td>
</tr>
</tbody>
</table>

a Year begins July 1
b Holding period change excludes amortization of mortgage.

The Average Capitalization Rate, $X_{47}$, and the Average Loan-To-Value Ratio, $X_{48}$, are not available on a consistent basis for the years preceding 1966; consequently, the equity model is quantified for only five years, 1966-1970. The rate of inflation between 1966 and 1970 averaged 4.53 per cent, appreciably higher than the 1.64 per cent for the period 1962-1965; thus, the period is appropriate for evaluating the impact of inflation.

The sample data expressed as averages used in quantifying the model are not the averages used in the NOI model. Data are averages of all raw data collected in a given year. This is the same raw data that is stratified by age classification in the NOI model. The averages
used in the equity model do not explicitly recognize age but project a weighted average of the total sample. The annual holding period begins July 1 of the year indicated.

Market value per room

Market value in constant dollars, Table 6-18, is directly proportional to NOI, $x_{23}$, and inversely proportional to the Consumer Price Index, $x_{31}$, and the market determined Average Capitalization Rate, $x_{47}$. The holding period change in market value is the difference between the initial and terminal value over a twelve month period. The market value at the end of the holding period is the beginning market value for the next holding period.

In the years 1966-1970, market value declined in four out of the five holding periods with a net decline of $-138.55$ or -$7.07$ per cent. Between 1966 and 1968, the net decline in market value was -$12.02$ per cent with the capitalization rate rising from $8.4$ per cent to $8.9$ per cent. In 1966 and 1967, the rate of increase in NOI was less than the rise in the Consumer Price Index. In 1970, the NOI decreased while the Consumer Price Index increased $4.30$ per cent.

The holding period change is not closely correlated with the Consumer Price Index or the Capitalization Rate. This may reflect unanticipated inflation in terms of NOI and/or the effect of monetary policy on the capitalization rate. Due to the dispersion of the data and the limited span of years, trends are difficult to perceive and statistical analysis is not appropriate.
Based on the holding period changes property, on average, decreased in value over the inflationary period studied.

**Mortgage per room**

The average mortgage at the beginning of the holding period is directly proportional to the market value of the property and the loan-to-value ratio, Table 6-19. The model assumes no amortization with the holding period change in the mortgage attributable solely to the change in the Consumer Price Index, \( X_{31} \). The average mortgage declined in the years 1966-1968, reflecting declining market value and small increases in the loan-to-value ratio. Although average market value decreased in 1969, the average mortgage increased because of the rise in the loan-to-value ratio. In 1970, the average mortgage increased due to the rise in market value of property. The loan-to-value ratios were virtually constant for the years 1969 and 1970.

The size of the mortgage, as defined by the loan-to-value ratio, and the rate of inflation, determine the annual holding period change for the mortgage expressed in constant dollars. The holding period change expressed as a percentage is equal to the rate of inflation and is independent of the loan-to-value ratio. The holding period change in equity is a negative function of the holding period change in the mortgage. In all five periods the holding period change was negative, varying from a minimum of -2.80 per cent to a maximum decline of -5.59 per cent.
The equity model

The equity model, Table 6-20, is used to define equity at the beginning of each holding period and the holding period change expressed both in dollars and as a percentage of the equity at the beginning of the period. The table also includes the holding period change for market value and the mortgage as defined in Table 6-18 and Table 6-19.

The change in equity over the holding period is the sum of the change in market value and the wealth transfer effect attributable to the mortgage. A negative change in the mortgage represents an increase in equity.

The decrease in equity interest over the five years compared to the initial equity in 1966 is significantly greater than the comparable change in market value. This is attributed to the increase in loan-to-value ratio from .703 to .743 over the five holding periods.

Holding period change in equity was negative in 1966 and 1967 and positive in 1968, 1969, and 1970. These changes appear extremely volatile in that they vary from -7.55 per cent to +38.60 per cent. This reflects the high degree of leverage evidenced by the loan-to-value ratios.

The equity holding period change of +38.60 per cent in 1969 was coincident with the highest rate of inflation experienced in the five years studied, 5.92 per cent. This rapid rate of inflation appears to have been anticipated by property managers as the NOI increased 24.42 per cent, fully offsetting the rise in capitalization rate and the rate of inflation. The holding period change in equity of $172.01 is the
result of an increase in market value of $100.56 plus the wealth transfer effect of $71.45. During 1969, the loan-to-value ratio approximated the maximum of the five year period. The effects of leverage are clearly apparent.

The average equity holding period change is 7.39 per cent; however, two out of the three periods indicate a significantly negative change. In four out of the five periods, the market value change was negative. In 1968 and 1970, the wealth transfer effect was sufficient to offset the decline in market value resulting in a positive change for equity. Only in 1966 and 1967 did the decline in market value exceed the wealth transfer effect resulting in a negative change for equity.

Although two of the three holding period changes are negative, equity interest did, on average, increase, suggesting that equity interest is an inflation hedge. The positive equity holding period changes are attributable primarily to the wealth transfer effect, a function of the loan-to-value ratio and the rate of inflation.

Summary

Factor analysis identifies five underlying dimensions of the data: (1) Performance, (2) Management-Inflation, (3) New Units Started Annually, (4) Miscellaneous Other Income, and (5) Vacancies and Delinquent Rents. Selected variables derived from factor analysis—Average Age of Building, $X_{30}$; Management, $X_{12}$; Consumer Price Index Less Shelter, $X_{42}$; Number of New Private Multi-Family Units Started Annually, $X_{41}$; Miscellaneous Other Income, $X_3$; and Vacancies and Delinquent Rents, $X_5$, are introduced into the multiple regressions.
Dividend Yield on Common Stocks, $X_{37}$, suggests an undefined dimension of the data and is introduced into Models 2, 3, and 4.

Models 1 and 2 are developed as predictive and explanatory models of NOI defining the effects of inflation and building age on NOI. Although Model 2 has a slightly higher coefficient of multiple correlation, Model 1 is preferred. Model 2 includes Variable $X_{37}$ which is difficult for the practitioner to estimate. Also, the variable adversely affects interpretation of the impact of inflation. Models 1 and 2 are acceptable on the basis of explanation of a high percentage of the variation in NOI, sensitivity analysis, residual plots, and plausibility.

Model 3 (1962-1968) and Model 4 (1962-1966) are developed to facilitate verification of Models 1 and 2 in terms of stability of regression coefficients over time and across all models, high coefficients of multiple correlation in all models, and prediction of NOI for 1969, 1970, and 1971, using Model 3.

Both Models 1 and 2 are acceptable and verified predictive models of NOI. Model 1, which is preferred, provides an acceptable and verified quantification of the effects of inflation and building age on NOI.

The equity model serves as the vehicle for evaluating the effect of inflation on market value and on equity interest for encumbered property. The impact of inflation is measured by the annual holding period changes over the years 1966-1970. The effect of inflation on the components of market value, equity interest, and the wealth transfer
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effect, is evaluated independently. Equity interest is an inverse function of The Consumer Price Index, $X_{31}$, and Average Capitalization Rate, $X_{47}$, and a positive function of NOI, $X_{23}$, and Average Loan-to-Value Ratio, $X_{48}$. In four out of the five holding periods, equity interest increases suggesting that equity interest is an inflation hedge.
CHAPTER VII

CONCLUSIONS

Introduction

This chapter states conclusions, implications, and proposals for future areas of investigation.

The NOI Model 1:

\[
\frac{X_{23}}{X_{31}} = 233.5549 \cdot \frac{X_3}{X_{31}} - 1.2597 \cdot \frac{X_5}{X_{31}} + 3.4828 \cdot \frac{X_{12}}{X_{31}}
+ [-3.5824 \cdot X_{30} + 0.0235 \cdot X_{30}^2] - 0.4632 \cdot X_{42}
\]

is a valid, predictive, and explanatory model of average NOI based on data collected over the years 1962-1971. The variables derived from operating experience and the economic environment are historically accessible and amenable to estimation by the practitioner.

The effect of building age on NOI is a quadratic relationship as defined in Model 1. NOI expressed in constant dollars is a decreasing function of building age.

The NOI Model clearly defines a negative relationship between the Consumer Price Index Less Shelter, \(X_{42}\), and NOI, expressed in constant dollars. This refutes the traditional view that NOI increases at a rate that equals or exceeds the rate of inflation.

Based on market value, unencumbered property is not an inflation
hedge. Market value, expressed in constant dollars, decreased in four out of five years between 1966 and 1971. The widely held belief that market value of this class of property will at least keep pace with the rate of inflation is rejected.

As defined in the equity model:

\[ E = \frac{1}{X_{31} X_{47}} X_{23} X_{48} \]
\[ \text{Market Value} = \frac{1}{X_{31} X_{47}} X_{23} \]
\[ \text{Mortgage} = \frac{X_{23} X_{48}}{X_{31} X_{47}} \]

equity interest appears to be an inflation hedge based on a positive annual holding period change in three of the five years between 1966 and 1970. This conclusion must be qualified, in light of the two negative holding period changes in the years 1966 and 1967.

**NOI Model**

Net Operating Income, expressed in constant dollars per room may be predicted and explained as a function of five independent variables: (1) Miscellaneous Other Income, \( X_3 \), (2) Vacancies and Delinquent Rents, \( X_5 \), (3) Management, \( X_{12} \), (4) Average Age of Building, \( X_{30} \), and (5) Consumer Price Index Less Shelter, \( X_{42} \). The regression coefficients in Model 1 quantify the relationship between the independent variables and NOI. The subjective understanding of causal relationships between
NOI and the independent variables is confirmed through statistical association. Nevertheless, NOI is a function of a large number of variables not included in the model. The included variables convey necessary information for the prediction and explanation of a high percentage of variation in NOI.

The predictive and explanatory qualities of the model are valid only under the assumption that the future approximates the period represented by the sample data. The high percentage of the variation in NOI, 91.6 per cent, explained by the model and the absence of discernible patterns in the residual plot lends credence to the model for purposes of prediction. Confidence in the explanatory power of the independent variables is based on causal relationships, minimization of multicolinearity and stability of coefficients over different time periods.

Expansion of revenue-producing tenant services is a desirable policy to effect increase in NOI and market value. The regression coefficient \( b_3 \), (0.8186) indicates that a one dollar increase in Miscellaneous Other Income, \( X_3 \), will result in a $0.82 increase in NOI. As expenses are associated with production of Miscellaneous Other Income, a coefficient of less than one is anticipated.

The decrease in NOI is larger than the decrease in Effective Gross Income associated with Vacancies and Delinquent Rents, \( X_5 \). The regression coefficient \( b_5 \) (-1.2597) defines a decrease in NOI greater than the decline in EGI. Two factors contribute to this relationship: (1) Vacancies and Delinquent Rents are directly related to costs
associated with acquisition of tenants and collection expenses, and (2) other costs may rise, such as required services for a vacant apartment. Consequently, NOI will fall faster than EGI, emphasizing the necessity of minimizing vacancies and delinquent rents.

Assuming the compensation of management is directly proportional to the quality and creativity of services provided, a policy of increasing investment in management can increase NOI and market value. The regression coefficient \( b_{12} \), defines a positive relationship between Management, \( X_{12} \), and NOI. Each dollar spent on management results in a $3.48 increase in NOI. The law of diminishing returns suggests some undefined limit.

**NOI--building age**

Net Operating Income decreases as a quadratic function of building age:  \(^1\)

\[
[-3.5824 X_{30} + 0.0235 X_{30}^2]
\]

The decline in NOI is a decreasing function of age defining the normative characteristic of depreciation.

The relationships defined in the Babcock premises are empirically quantified. Premise I, assuming constant NOI occurring in equal annual installments, is rejected. The direction of Premises II, III, and IV, is confirmed. However, where Babcock defines the decline of NOI as straight line in Premise IV and an increasing function of age in Premise

\(^1\)The sign of the quadratic function remains negative from 1 to 157.44 years, well beyond the pertinent range.
II and III, this research defines the decline as a decreasing function of age.\textsuperscript{2}

The quadratic relationship between building age and NOI is suspect when applied to buildings in the extreme age limit of the sample, beyond 50 years. There is no representation of razed buildings or capital investment which can extend the economic life. NOI of older buildings may represent significant capital improvement. This distortion permeates all of the data to some degree but is logically more significant in older buildings.

The Effects of Inflation

Market value

Unencumbered property is not an inflation hedge as the market value holding period change in four of the five years between 1966 and 1970 is negative. During this period, the capitalization rate and the Consumer Price Index rose at an increasing rate. Recognizing that the capitalization rate is a complex variable subject to the influence of many factors other than inflation, it appears, nevertheless, to have fully anticipated inflation in all years except 1966. The NOI fully anticipates inflation in the year 1969 only. If unencumbered property is an inflation hedge, NOI must rise at or above the sum of the rate of inflation and the increase in the capitalization rate. This occurred only in one year, 1969.

\textsuperscript{2} Differences in the rates of decline may be attributable to the derivation of Babcock's premises. Premises II and III are derived from level annuity tables.
Wealth transfer effect

The wealth transfer effect, occurring over an annual holding period for the average mortgage (excluding amortization), is a positive function of inflation and the loan-to-value ratio resulting in the transfer of wealth from the mortgagor to the mortgagee. Consequently, a prudent policy for all property owners under inflationary conditions is to maximize the debt. For encumbered property, this effect can offset the decline in market value associated with inflation. The wealth transfer effect, expressed as a percentage of the mortgage, is equal to the inflation rate over the holding period.

Equity interest

Investors seeking an inflation hedge should acquire an equity interest only, maximizing the loan-to-value ratio. For encumbered property, the holding period change for equity is positive in three of five years studied. The summation of annual holding period changes over the five years is significantly positive; consequently, equity interest appears to be an inflation hedge. However, with two of the five years displaying negative holding period changes, conclusive statements are not justified.

---

3 The model does not recognize the benefits accruing to equity through amortization; consequently, the annual holding period change in equity value tends to be understated. Since the sample includes averages of all buildings reported each year, the percentage of newer buildings included in the population increases; thus, in each succeeding sample, the average age of buildings is biased downward. This results in a positive bias of NOI and equity holding period change.

4 Decline in equity value in the years 1966-1970 is attributable to the increase in the loan-to-value ratio from .7030 to .7415 combined
As the equity model is determinate and does not facilitate acceptance or rejection on a statistical basis, confidence intervals cannot be determined. In view of the large body of data utilized in deriving averages in the sample, the results cannot be dismissed lightly.

NOI—Inflation

Contrary to the general belief, NOI does not keep pace with inflation. The regression coefficient $b_{42} (-0.4632)$ in NOI Model 1 defines a negative relationship between the Consumer Price Index Less Shelter, $X_{42}^s$, and NOI. Recognizing NOI as a primary determinant of value, ceteris paribus, the negative coefficient implies that unencumbered property is not an inflation hedge. Market value holding period change in the equity model supports this implication in four of the five years studied.

While change in the capitalization rate can overcome the effect of inflation, capitalization rates, as a general rule, tend to anticipate inflation. This tendency has been evident in the trend of capitalization rates during a period of accelerating inflation since 1965. Year to year aberrations about the trend line are functions of monetary policy and supply and demand factors in capital markets. In most instances, the decline in NOI associated with inflation will not be offset by a declining capitalization rate; consequently, NOI does not rise

with a decrease in market value. The equity holding period change is negative for 1966 and 1967, years in which NOI did not fully anticipate the accelerating inflationary rate. The resulting decreases in market value were not completely offset by the wealth transfer effect. Recognizing the persistence of relatively high inflationary rates, NOI increased sharply in 1968 and 1969, more than offsetting the rise in capitalization rates.
sufficiently to maintain market value expressed in constant dollars.\textsuperscript{5}

**Implications**

The benefits derived from this research, as stated in the introduction to Chapter I, are the adaptation and projection of economic theory to the development and empirical quantification of normative NOI and equity models, based on data expressed as averages. The models provide a framework of analysis not previously available with significant implications for theoreticians, investors, appraisers, property managers, mortgagors, and those establishing housing policy at all levels.

This research adds an empirical dimension to Babcock's theoretical concept of real estate valuation with the development of a mathematical model defining change in productivity over time. Identification of key variables and in particular, the inclusion of inflation, represents a significant step forward in the valuation process.

\textsuperscript{5}The regression coefficient $b_{42}$ reflects the degree of anticipated or unanticipated inflation by property managers. Variable $X_{42}$, (1962-1966), did not enter NOI Model 4 as it was not statistically significant. The average rate of inflation from 1962-1966 was less than 1.62 per cent per year. During the years 1966, 1967, and 1968, the rate of inflation accelerated to 5.37 per cent reflecting a degree of unanticipated inflation by property managers. The persistence of inflation was anticipated increasingly by property managers. Consequently, the regression coefficient in Model 1 decreased to $-0.4632$. The negative sign indicates that inflation was still not fully anticipated. NOI did not rise at the inflationary rate in the years 1966-1967. This confirms the larger negative regression coefficient $b_{42}$ (-1.1667) evidenced in NOI Model 3 (1962-1968) as contrasted to $b_{42}$ (-0.4632) in NOI Model 1 (1962-1971). Recognition by property managers of the persistent nature of inflation may account in part for the sharp increase in NOI in 1968 and 1969 and the decrease in $b_{42}$ to $-0.4632$ in NOI Model 1.
Of primary interest to the theoretician is the extension of classical theories of value to the explicit recognition of inflation as a primary determinant of value. Productivity and value in the models are expressed in constant dollars, facilitating direct assessment of change in value and the wealth transfer effect.

The NOI model provides a unique measure of the degree of anticipated or unanticipated inflation evident in NOI. A simple deflated comparison of NOI over time does not recognize the decrease in utility associated with increasing building age. The sign and magnitude of the regression coefficient of Consumer Price Index Less Shelter defines the degree of anticipated or unanticipated inflation suggesting a unique dimension of economic and managerial performance.

The models identifying a small number of key variables which are easily estimated, provide a framework for estimating reversion value as a function of future productivity, which is critical to many investment decisions.

Concepts of portfolio management such as maximizing the value of combined assets, necessitate prediction of NOI and value over time. In this context, the models are particularly useful to the investor.

Appraisers, mortgagors, investors, and policy makers can employ the NOI and equity models to augment and to improve current valuation techniques. The models, based on average data, provide a norm or standard of comparison as well as identification and insights into the effects of key variables such as capitalization rates, inflation, loan-to-value ratios, and vacancies and delinquent rents.
In contrast to the rule of thumb methods used for crude approximations, utilization of the models by investors and mortgagors for estimates of future NOI streams and value applied to individual properties, represents a significant advantage.

From both the mortgagor's and mortgagee's point of view, the repayment schedule of a mortgage should, ideally, reflect the productivity of the property. The normative productivity over time defined by the NOI model suggests a new basis for specifying the mortgage payment schedule. This concept might also be extended as a basis for taxation.

In contrast to previous work, this research has defined NOI as a decreasing function of building age suggesting that the intrinsic value of older buildings may be greater than has been generally recognized and that higher loan-to-value ratios are justified.

The advent of price control intensifies the importance of insights derived from the NOI and equity models. If adequate privately owned rental housing is to be available in the future, then housing and tax policies must reflect acceptable growth in NOI and equity interest or the need for some form of government subsidization. These models can provide information critical to the decision processes of mortgagors and purchasers of income property as well as necessary inputs for the formulation of government policy.

The results, conclusions, and implications of this research should stimulate future research of a more definitive nature. The potential benefits of this type of research will hopefully stimulate the free exchange of data that is requisite to a more definitive study.
**Future Work**

This research suggests four primary areas of future work:

1. the updating of the models,
2. the development of models for other types of property,
3. the development of similar models with sample data from individual properties, facilitating a more rigorous statistical analysis and testing of hypotheses, and
4. the development and expansion of an inferential equity model.

Model development and subsequent analysis employed in this research are based on the economic environment and conditions prevailing in real estate markets over the period in which the samples were obtained. The volatile environment experienced in recent years accentuates the need to validate and/or update the models based on the most recent data.

This research has confined itself to a particular classification of property, multi-family dwelling units. Similar modeling techniques could be applied fruitfully to other classes of property, identifying characteristic differences. The development of a general model applicable to all types of income-producing property is an interesting possibility for future work.

The primary limitation of the research has been the use of aggregate data in the form of annual averages. A more forceful statistical analysis including the formal statement and testing of hypotheses can be achieved if data are obtained on the basis of individual properties. Such data would facilitate development of coefficients for different geographical areas and insure a considerably higher degree of confidence.
in the explanatory and predictive qualities of the models.

The development of a more complex and definitive inferential equity model, using the methodology of the NOI model developed in this research would be beneficial in predicting and explaining change in both equity interest and market value and in testing associated hypotheses.
APPENDIX A

IREM DATA FORM AND INSTRUCTION SHEET
**APARTMENT BUILDING INCOME-EXPENSE ANALYSIS**

**INSTITUTE OF REAL ESTATE MANAGEMENT**

155 East Superior Street • Chicago, Ill. 60611 • Phone: (312) 664-9700

**Operating Results for the Year 1971**

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<th><strong>EXPENSES</strong></th>
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<tr>
<td>1</td>
<td>Apartment Rentals at 100% (include value of employees' apt.)</td>
</tr>
<tr>
<td>2</td>
<td>Garages and Parking</td>
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<td>A</td>
<td>Enclosed Garages</td>
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<td>B</td>
<td>Covered Carport</td>
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<tr>
<td>C</td>
<td>Open Lot Parking</td>
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<td><strong>TOTAL GARAGES AND/OR PARKING</strong></td>
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<td>3</td>
<td>Stores at 100% occupancy</td>
</tr>
<tr>
<td>4</td>
<td>Offices at 100% occupancy</td>
</tr>
<tr>
<td>5</td>
<td><strong>GROSS POSSIBLE RENTAL INCOME</strong></td>
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<tr>
<td>6</td>
<td>Miscellaneous (electricity, maid service, laundry, etc.)</td>
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<tr>
<td>7</td>
<td><strong>GROSS POSSIBLE TOTAL INCOME</strong></td>
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<tr>
<td>8</td>
<td>Loss Due to Vacancies and Delinquent Rents</td>
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<tr>
<td>9</td>
<td><strong>TOTAL ACTUAL COLLECTIONS</strong></td>
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<td><strong>BUILDING INFORMATION</strong></td>
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</tr>
<tr>
<td></td>
<td><strong>PHONE NUMBER OF PERSON TO CONTACT:</strong></td>
</tr>
<tr>
<td></td>
<td><strong>AREA:</strong></td>
</tr>
<tr>
<td></td>
<td><strong>NUMBER OF NEW TENANTS DURING THE YEAR</strong></td>
</tr>
</tbody>
</table>

**Please answer all questions below:**

A. Have you changed your type of heating system during 1971? [ ] Yes [ ] No

B. Have you incurred extraordinary expenses for painting & decorating (interior ONLY) during 1971? [ ] Yes [ ] No

C. Have you incurred extraordinary expenses for maintenance & repairs (interior AND exterior) during 1971? [ ] Yes [ ] No

D. Have your premiums for insurance been substantially adjusted during 1971? [ ] Yes [ ] No

E. Have your real estate taxes been substantially adjusted during 1971? [ ] Yes [ ] No

F. Did this building have 12 full months of operation during 1971? [ ] Yes [ ] No

G. Name of local IREM Chapter: ____________________

Please see other side RETURN THIS COPY

This form for previously reported buildings only
**Type of Building** (check one only)
- (1) High-Rise Elevator
- (2) Low-Rise under 25 units
- (3) Low-Rise 25 or more units
- (4) Garden type

**Type of Ownership** (check one only)
- (1) Investor
- (2) F.H.A. Owned
- (3) Co-operative
- (4) Condominium

**Location** (check one only)
- (1) City
- (2) Suburban

**Age of Building** (check one only)
- (1) 1961-to date
- (2) 1946-1960
- (3) 1931-1945
- (4) 1921-1930
- (5) 1920 or before

**Heating Fuel** included in rent (check one only)
- (1) Coal
- (2) Oil
- (3) Gas
- (4) Combination
- (5) Electricity
- (6) Purchased Steam

**List Number of APTS. by Room Count** (Do Not Include Stores or Offices)

<table>
<thead>
<tr>
<th>NO. OF ROOMS</th>
<th>NO. OF UNFURNISHED APTS.</th>
<th>NO. OF Furnished APTS.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
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<tr>
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<td>2½</td>
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<td>3</td>
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<td>3½</td>
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<td>4½</td>
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<td>5</td>
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<td>5½</td>
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<td>6</td>
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<tr>
<td>7</td>
<td></td>
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</tr>
</tbody>
</table>

**Floor Areas in Sq. Ft.** (Fill In)

- Gross Floor Area of Entire Bldg. __________ Sq. Ft.
- Total Rentable Floor Area of:
  - (A) Apartments __________ Sq. Ft.
  - (B) Stores __________ Sq. Ft.
  - (C) Offices __________ Sq. Ft.
  - (D) Other __________ Sq. Ft.
- Total Rentable Area __________ Sq. Ft.

**Number of Parking Spaces** (Fill In)

- (A) Enclosed Garages __________
- (B) Covered Carport __________
- (C) Open Lot Parking __________

**Total Spaces** (A+B+C, ABOVE) __________

**Type of Financing**
- (1) Conventional
- (2) F.H.A. Financed

Section # __________
F.H.A Bldg. Type __________
HOW TO FILL OUT YOUR APARTMENT BUILDING INCOME-EXPENSE ANALYSIS DATA FORM

DO NOT REPORT ANY BUILDING OR GROUP OF BUILDINGS WITH LESS THAN A TOTAL OF 12 RESIDENTIAL UNITS

DO NOT REPORT ANY BUILDING IF OFFICE OR STORE OCCUPANCY REPRESENTS MORE THAN 20 PERCENT OF THE TOTAL RENTABLE AREA.

FRONT SIDE OPERATING FIGURES

These explanations of the various captions should help you complete your report with the minimum of difficulty. If you have any doubt where any expense item belongs, check it against these explanatory notes. The number beside each caption below refers to the item number on the reporting form on the left hand margin of the page.

INCOME

(Round Out To Nearest Dollar - Do Not Use Cents)

NOTE: FOR ITEMS 1 THROUGH 9 BELOW BE SURE TO USE THE RENTAL RATES WHICH WERE IN EFFECT THE YEAR FOR WHICH INCOME IS BEING REQUESTED.

1. Apartment Rentals: This figure should contain all apartment rents which could have been collected, including employee apartments, if 100% of your building had been occupied.

2. Garages and parking: If there is a separate charge made for use of garage or parking area, report this in the appropriate category on either Line 2a, 2b or 2c. Then place the total of these in the space provided. If you include use of garage or parking area in the apartment rent, reduce the apartment rental total on Line 1 by the portion applicable to garages and parking and place this portion in the appropriate category on Line 2.

3.-4. Stores and offices: Show the rental income you could have received from stores on Line 3 and offices on Line 4 if both of these had been 100% occupied. If not applicable to your building omit these lines.

5. Gross possible rental income: This is the total of Lines 1, 2, 3, and 4.

6. Miscellaneous other income: Report here all the income from gas and electricity sold by the building to tenants, maid service, commissions from telephones, laundry and vending machines, signs on the building and air-conditioning charges. Do NOT include interest or dividend income.

7. Gross Possible Total Income: This is the total of lines 5 and 6.

8. Less Vacancies & Delinquent Rents: Temporarily skip this line. Fill in line 9 first, then come back to this line.
9. Total actual collections: On Line 9 show what you actually collected from all sources indicated on Lines 1 through 6 (including the rental value of apartments given to employees as part of their compensation), then subtract Line 9 from Line 7 (gross possible total income) and enter the difference on Line 8 as vacancies and delinquent rents.

EXPENSES

(Round Out To Nearest Dollar - Do Not Use Cents)

10. Total Payroll: Fill in amount paid janitors, doormen, maids, elevator operators, telephone switchboard operators, maintenance personnel, including market rental value of apartment, and payroll taxes and welfare benefits. Wages of administrative personnel and of painters shall be included in Nos. 16 & 18 below.

11. Supplies: On this line show all janitorial supplies, light bulbs, uniforms for employees and other such supplies which do not belong under painting, decorating, maintenance or repairs.

12. Electricity: This is a total figure and includes 100 per cent of your electrical expenses even if you bill some of it back to your tenants (electricity income is provided for above under miscellaneous income). It includes electricity for tenant and public areas, air-conditioning, elevators, laundry and other related purposes. If your building is electrically heated, do NOT report this here but on Line 15 below.

13. Water: Show all water costs including, if applicable in your community, sewerage charges. If other utility charges are billed to the water bill, give an estimate of each charge in its appropriate category.

14. Gas: Show here the cost of gas for utilities, i.e., cooking, air-conditioning, hot water, swimming pools, etc. Do NOT report the gas used for heating the building. This is reported on Line 15 below. If you receive a combined gas bill for heating and utilities, take your gas bill for the lowest summer month and multiply by 12 to determine what this item totals.

15. Heating Fuel: This figure should represent the cost of heating your building whether you use coal, gas, oil, electricity or any other fuel. Do NOT include the costs of ash removal or cost of gas, electricity, etc., used for cooking or hot water.

16. Management Fees: This figure should represent the agency fee and the salaries of personnel in the management office paid directly by the building owner. Also include leasing or rental fees paid in addition to management fees and any alteration supervisory charges paid by the owner to a managing agent. If renting, leasing, or renewal is charged, please show as a separate item.

17. Other Administrative Costs: This category includes the cost of all advertising, legal and auditing fees, dues in professional organizations, architectural or professional engineer's fees, telephone and building office expenses and office supplies paid by the owner.

18. Painting & Decorating (interior): Include on this line the cost of all contracted labor, decorators on building payroll, and all materials and supplies used in the decorating of the interior of the building. Paint, wall-paper, brushes, wall-washing, and similar items belong in this category. Exterior painting should be included in maintenance and repairs on Line 19.

19. Maintenance & Repairs: This category is to account for all items of general maintenance and repairs, both interior and exterior. This includes landscaping costs, exterior painting or cleaning, elevator maintenance contracts, boiler inspection and repair contracts, air-conditioning service contracts; parts, small hand tools, fire extinguishers; plumbing, electrical, plastering, masonry, carpentry, heating, roofing or tuck pointing contractor's services unless such bills properly constitute a capital expenditure. Replacement of floor coverings, draperies, furnishings or light fixtures if not a capital expenditure also belongs in this category.
20. **Services:** This covers such contracted outside services as window washing, lobby directory, exterminating, rubbish removal, TV antenna service, but NOT services chargeable against the painting, decorating, maintenance or repair categories.

21. **Miscellaneous Operating Expenses:** This category is designed to include operating costs which do not fit under any other caption. Among such items might be damage to property of others not covered by insurance, directional signs, door lettering, etc. DO NOT use this category if any other caption may be used.

22. **Insurance:** Include all one year charges for fire, liability, compensation, theft, boiler explosion, rent fidelity bonds and all insurance premiums except those paid to FHA for mortgage insurance or employee benefit plans. If the building's policies are paid on a three-year or five-year basis, pro-rate and include only one year's cost. If your buildings are under a favorable "blanket insurance coverage" at low rates, insert the typical market value rates.

23. **Real Estate Taxes:** This includes all local or state real estate taxes as well as any non-capitalized assessments. If your office does not pay this item, please ascertain figure from owner or County Treasurer's Office.

24. **Other Taxes, Fees, & Permits:** Show on this line any personal property taxes applicable to the building, franchise taxes, sign permit fees or any other tax necessary to the operation of the building. DO NOT INCLUDE any local, state, or federal income taxes paid by the ownership on profits derived from the operations. DO NOT INCLUDE UNDER ANY EXPENSE CATEGORY SUCH ITEMS AS GROUND RENT, MORTGAGE INTEREST OR AMORTIZATION, DEPRECIATION OR INCOME TAXES. THESE ARE NOT REFLECTIVE OF OPERATING EXPENSES.

25. **Total All Expenses:** Make sure the figures on lines 10 through 24 add up correctly to the figure shown on line 25.

26. **Net Operating Income:** Subtract the figure on line 25 from that shown on line 9 and show the difference on line 26.

27. **Turnover:** The number of new tenants that came into the building during the year. If the building is new, the Turnover would begin after one full year of occupancy.

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**BACK SIDE**

### REQUIRED BUILDING DATA

**TYPE OF BUILDING**

**High-Rise Elevator Buildings:** This group is confined to elevator buildings which are four stories or more in height.

**Low Rise (under 25 units):** Includes walk-up buildings and elevator buildings 3 stories or less.

**Low-Rise (25 units or more):** Includes walk-up buildings and elevator buildings 3 stories or less.

**Garden-Type:** We consider this to be a group of low-rise apartment buildings situated on a sizeable landscaped plot, under one management.

**Square Foot Data:** You are urged to supply the square foot data requested. This information will permit our report to show income and operating costs per sq. ft. as well as on a room count basis. If you are unable to report square foot data this year, we will still accept your report.

**Gross Floor Area Of Entire Building:** Show here the floor area of all space (outside wall to outside wall) including living areas, corridors, lobby, stores, offices, garages within the building, basement and public areas on all floors. Do NOT include court areas.
Rentable Floor Area: Determine the total sq. ft. area WITHIN all individual apartment units, stores, offices, etc.; and report these in the appropriate space provided.

NUMBER OF ROOMS

For uniform room count reporting, these principles are suggested: Rooms are counted the same as is usual in the industry (a living room - dining room - bedroom and kitchen are 4 rooms) -- with the following refinements:

**Dining Room:**
- If dining room is combined with living room, count as one room, if less than 260 sq. feet - but 1\(\frac{1}{2}\) rooms if more than 260 sq. feet.
- Count breakfast room as a room, if it is more than 100 sq. feet, regardless of whether there is a separate dining room.

**Kitchen:**
- If a regular, separate walk-in kitchen, count as a full room.
- If a combined dining-kitchen area exists, count as follows:
  - Total area is 105 sq. feet or less........1 room
  - Total area is 105 to 140 sq. feet........1\(\frac{1}{2}\) rooms
  - Total area is over 140 sq. feet...........2 rooms
- If Pullman - or actually an integral part of living room, or so situated as to make most of the equipment visible in the living room, count as half room.

**Other:**
- Bathrooms, porches, halls, closets, etc. are not counted.

NUMBER OF APARTMENTS BY ROOM COUNT

**Furnished Apartments:** If 80 per cent or more of the apartments in the building which you are reporting are furnished, report the entire building as furnished.

**Unfurnished Apartments:** If 80 per cent or more of the apartments in the building which you are reporting are unfurnished, report the entire building as unfurnished.

If the building you are reporting is not within the ratios specified above for furnished or unfurnished do NOT report such building. It will improperly affect the interpretation of statistical data. Choose another building to report instead.

If you have any questions as to the method of filling out the form, please do not hesitate to write or call the Experience Exchange Committee, Institute of Real Estate Management, 155 East Superior St., Chicago, Illinois 60611. Phone (312) 664-9700.

Start Now. It will take much less time than you think. Send in reports on all your buildings. Additional copies of the data form are available on request, or you can reproduce the data form on a copying machine in any quantity necessary to report all your buildings. Encourage others in your city to send in their Apartment Building statistics for a more meaningful sample.

EXPERIENCE EXCHANGE COMMITTEE
APPENDIX B

SUMMARY LIST OF VARIABLES
SUMMARY OF VARIABLES

$X_1$ -- ANNUAL SEQUENTIAL REPORTING PERIOD (1960 = 1)

$X_2$ -- GROSS POSSIBLE RENTAL

$X_3$ -- MISCELLANEOUS OTHER INCOME (OTHER THAN RENTAL SPACE)

$X_4$ -- GROSS POSSIBLE TOTAL INCOME

$X_5$ -- VACANCIES AND DELINQUENT RENTS

$X_6$ -- EFFECTIVE GROSS INCOME (EGI)

$X_7$ -- TOTAL PAYROLL EXPENSE

$X_8$ -- ELECTRICITY

$X_9$ -- GAS (EXCLUDING HEATING FUEL)

$X_{10}$ -- WATER

$X_{11}$ -- HEATING FUEL

$X_{12}$ -- MANAGEMENT

$X_{13}$ -- OTHER ADMINISTRATIVE EXPENSES

$X_{14}$ -- PAINTING AND DECORATING (EXTERIOR ONLY)

$X_{15}$ -- MAINTENANCE AND REPAIRS (INTERIOR AND EXTERIOR)

$X_{16}$ -- SUPPLIES

$X_{17}$ -- SERVICES

$X_{18}$ -- MISCELLANEOUS OPERATING EXPENSES

$X_{19}$ -- INSURANCE

$X_{20}$ -- REAL ESTATE TAXES

$X_{21}$ -- OTHER TAXES

$X_{22}$ -- TOTAL EXPENSES

$X_{23}$ -- NET OPERATING INCOME

$X_{24}$ -- TURNOVER RATE (PERCENTAGE, ALL AREAS AND AGES)
\( x_{25} \) -- Operating Ratio A \( \left( \frac{x_{22}}{x_4} \right) \)

\( x_{26} \) -- Operating Ratio B \( \left( \frac{x_{22}}{x_6} \right) \)

\( x_{27} \) -- Net Income Ratio A \( \left( \frac{x_{23}}{x_4} \right) \)

\( x_{28} \) -- Net Income Ratio B \( \left( \frac{x_{23}}{x_6} \right) \)

\( x_{30} \) -- Average Age of Building

\( x_{31} \) -- Consumer Price Index (Base 1967)

\( x_{32} \) -- Wholesale Price Index (Base 1967)

\( x_{33} \) -- Construction Cost Index (Department of Commerce)

\( x_{34} \) -- Risk-Free Interest Rate (New Issues 3 Month Treasury Bills)

\( x_{35} \) -- Home Mortgage Rates (New Home Purchases)

\( x_{36} \) -- Yield on Corporate Bonds (Average)

\( x_{37} \) -- Dividend Yield on Common Stocks

\( x_{38} \) -- Personal Income Current Dollars

\( x_{39} \) -- Personal Income Constant Dollars

\( x_{40} \) -- National Rental Vacancy Rates (Percentage)

\( x_{41} \) -- Number of New Private Multi-Family Units Started Annually

\( x_{42} \) -- Consumer Price Index Less Shelter (Base 1967)

\( x_{43} \) -- Boeckh Construction Price Index (Apts., Hotels, Office Bldgs.)

\( x_{44} \) -- Boeckh Construction Price Index (Residential)

\( x_{45} \) -- Disposable Personal Income Current Dollars

\( x_{46} \) -- Disposable Personal Income Constant Dollars (1967 = 100)

\( x_{47} \) -- Average Capitalization Rate

\( x_{48} \) -- Average Loan to Value Ratio
The principal-factor method is an adaptation of principal component analysis. Both methods are based on analysis of the correlation matrix with "1's" placed in the diagonal for the principal-factor method. The differences between the two methods is in the variance that is analyzed.

The principal-factor method is superior because the communality of a variable can be no more than the sum of the variances explained by the factors. In contrast, the principal component method will include both factor and error variance. The difficulty in the principal-factor method is that the communalities for each variable must be estimated. The square of the multiple correlation coefficient between the variable and all other variables is used as an estimate of the communality.

The principal-factor method extracts factors that are independent and orthogonal. The extracted factors define the independent sources of variance inherent in the data matrix. As this method extracts factors in decreasing magnitude of variance, it is possible to extract as many factors as there are variables. Trivial factors can be extracted. The selection of the number of factors is critical in terms of distortion of the rotated solution.

1Harman, Modern Factor Analysis, op. cit., pp. 135-137.
Considerable disagreement is evidenced as to the means of determining the number of factors to be extracted. There is no single scientifically justified criterion to determine when to stop factoring. Based on the work of Guttman, Kaiser, and Cattell, R. J. Rummel states that factors should be extracted until an eigenvalue of 1 is reached.  

**Varimax rotation**

The principal-factor method defines the underlying dimensionality of the data. Although the factors are orthogonal, this method does not usually provide the economy of data that is desired. Typically, the first factor will display high loadings on many of the variables and/or many of the variables may be highly loaded on more than one factor. In geometric terms, the first factor may be located between clusters of interrelated variables. Without rotation, related variables, i.e., clusters, cannot be identified through loadings in the factors. Statistically independent variables representing the underlying data cannot be identified.  

Interpretation of factor analysis is most easily accomplished when related variables are highly loaded on a single factor. As statistical independence of factors and variables is desired, interpretation can be improved by varimax rotation. The procedure provides a new set of orthogonal axes and a new factor structure but does not

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5 Ibid., pp. 372-375.

change the communality. The variance is redistributed across factors; consequently, the eigenvalues change. The correlation between highly loaded variables and a factor suggests the underlying structure and facilitates the identification of the factor in terms of its informational content. Variables highly loaded in their respective factors contain a high percentage of the information represented by the factor. For example, assuming five factors, it is practical to identify five variables, one from each factor, that will represent a high percentage of the information contained in the original variables. These five variables are statistically independent of each other; consequently, when introduced into a regression equation, they help to minimize the problem of multicollinearity. Where a number of variables are highly loaded in a given rotated factor, that variable which is most readily available or most easily estimated, can be selected.

**Computer program-factor analysis**

Factor analysis for this research is performed on an IBM 1130 computer, utilizing the 1130 Statistical System (1130-CA-06X). The computational procedure includes the principal-factor method and varimax rotation. The printout includes:

1. high and low value of each variable
2. means, standard deviation, and variance of each variable
3. matrix of correlation coefficients
4. trace (total explained variance in data)
5. characteristic roots (the variance explained by each factor)

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8IBM Application Program, op. cit.
6. cumulative per cent of trace (cumulative per cent of variance explained by each factor)
7. unrotated factor loading matrix
8. communalities
9. varimax rotated factor loading matrix

As the factor loadings for each variable reflect all of the 44 variables included in the analysis, a three step procedure is necessary as the capacity of the computer is limited to 30 variables. The 29 internal variables and 16 external variables are factor analyzed in two groups. Those variables having low loadings in all factors are deleted. The remaining internal and external variables are combined into a single matrix which is factor analyzed. The rationale for this procedure is that the highly loaded internal variables may directly reflect the general economic conditions defined by the external variables; consequently, multicolinearity could persist unless the variables are analyzed in one group. Variables identified for introduction into the regression model are selected on the basis of causal relationships, factor loading, correlation between variables both within the factor and across factors, eigenvalue of the factor, and the ease with which the variable can be quantified or estimated.

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APPENDIX D
MODELS—DUMMY VARIABLES
Models are developed using dummy variables to represent building age by group classification: 1961-1971, 1946-1960, 1931-1945, 1921-1930, and 1920 or before. The coefficients $b_3, b_5, b_{12},$ and $b_{37}$ deviate a maximum of 12 per cent from Model 2. NOI is a quadratic function of inflation. The use of dummy variables causes age to become a step function with differences as high as $40.00$ per room between steps. The step function does not represent a realistic concept of the impact of building age on NOI. Models using dummy variables are fitted to the 1962-1968 data. Estimates for each age group for the years 1969, 1970, and 1971 are computed with 33 per cent being more than two standard errors of the estimate away from the true NOI. Further investigation of models using dummy variables was terminated.
BIBLIOGRAPHY

Books


Kilbridge, Maurice D.; O'Block, Robert P.; and Teplitz, Paul V. *Urban Analysis*. Division of Research, Graduate School of Business Administration. Boston: Harvard University, 1970.


Periodicals


Jancsek, Joel P. "Property Tax Assessment Applications of Multiple Regression Analysis." Center for Real Estate and Urban Economics; University of California, 1972.


Smith, Halbert C. "Investment Analysis in Appraising." Real Estate Appraiser, XXXIII, Number 9 (September, 1967).


