A COMPARATIVE STUDY OF PATTERNS OF PACING AND REVIEW IN A PRE-CALCULUS MATHEMATICS SEQUENCE FOR COLLEGE FRESHMEN

DISSEPTION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

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* * * * *

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TABLE OF CONTENTS

ACKNOWLEDGMENTS .............................................. ii

VITA ........................................................ iii

LIST OF TABLES .............................................. v

LIST OF FIGURES ............................................. vi

Chapter

I. THE NATURE OF THE PROBLEM. ............................. 1

   Introduction
   Significant of the Problem
   Setting for the Study
   Mathematics Courses at Ohio State
   The CRIMEL Program
   Course Changes for the CRIMEL Program
   Design of the Study
   Analysis of the Study
   Limitations
   Summary
   Format of the Dissertation

II. REVIEW OF RELATED LITERATURE. .......................... 14

   Research Related to Homogeneous Grouping
   Research Related to the CRIMEL Program
   Current Related Research

III. THE STUDY. .............................................. 25

   The Structure of the Courses Used in the Study
     Topics Introduced in Math 159 and 150
     Topics Introduced in Basic Algebra, Math 101
   Methods of Instruction Used in the Courses of the Study
   Test Instruments Used in the Courses
   Preparation for the Study
   The Conduct of the Study
   The Structure of the Study
   Hypotheses
IV. THE RESULTS OF THE STUDY

Comparison of Level III Students
  Comparison of Level III Students by Course
  Comparison of Level III Students by Pace
Comparison of All Students by Course
  Data on Total Course Population
Comparison of Level II Students by Course
Comparison of Level IV and V Students by Course
Comparison of Crimel Students by Pace

V. SUMMARY AND CONCLUSIONS

Restatement of the Problem
  Summary of Results
  Interpretation of the Conclusions
  Implications and Recommendations for Action
  Implications for Further Study

Appendix

A. .............................................. 84
B. ................................................ 87
C. .............................................. 166
D. .............................................. 174

BIBLIOGRAPHY .............................................. 176
LIST OF TABLES

Table                                                                 Page
1. Decision Table Based on Test 1 ................................................... 20
2. Number of Students Per Category of Recommended vs.
   Chosen Pace .................................................................................. 35
3. Level III Students, Pre-Course Data - ANOVA By Course. ................. 43
4. Level III Students, Achievement Data - ANOVA By Course ................. 44
5. Level III Students, Achievement Data - ANOVA By Pace ................... 46
6. Level III Students, Total Points In Course - ANOVA
   by Pace ......................................................................................... 47
7. Number of Students in the Study Population for Each
   Category of Recommended vs. Chosen Pace ................................ 48
8. Distribution of the Course Populations ......................................... 49
9. Placement Levels for Those Who Dropped Either Course ................. 50
10. Comparison of Students Placing Level II - ANOVA
    By Course .................................................................................. 52
11. Comparison of Students Placing Levels IV or V -
    ANOVA By Course ....................................................................... 54
<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Diagram of CRIMEL Pacing Options and the Review - Math 150 Sequence</td>
<td>9</td>
</tr>
<tr>
<td>2. Student Groups Proceeding Faster, Slower or as Recommended by Their Test 1 Scores</td>
<td>64</td>
</tr>
</tbody>
</table>
CHAPTER I

THE NATURE OF THE PROBLEM

Introduction

During the 1970-71 academic year the Department of Mathematics at The Ohio State University began a long term development program designed to allow students to proceed through freshman mathematics at their own pace. As a first step, students in college algebra were given three options after their first regular midterm examination. They could remain in the "regular" course, they could elect to take only sixty percent of the regular course during the quarter (repeat the material already covered and retest before going on), or they could elect to accelerate their pace (and thus start the next course part way through the quarter). The name chosen for this experimental program was Curriculum Revision and Instruction in Mathematics at the Elementary Level (CRIMEL).

The primary purpose of this study was to compare the effects of the CRIMEL program and the traditional program on achievement by students with marginal pre-college preparation in mathematics. The secondary purpose of the study was to prepare an overall comparison of the CRIMEL course and the traditional course on the basis of achievement.
Significance of the Problem

In an era when a higher percentage of college students find mathematics a part of their field of study, mathematicians cannot ignore the student who does not come to college mathematics "well prepared". Alternate methods of working with these students must be considered. This study is concerned with research into one of these alternatives, a program which utilizes flexible pacing. Chapter 2 contains references to other research done in connection with the CRIMEL project and to similar projects at other institutions.

Whether a student with marginal skills should attempt college algebra and trigonometry immediately, with the option of taking two quarters to complete the course if necessary, or should review pre-college algebra first and then proceed with the course in college algebra, is of strong local interest. Placing beginning students into the correct mathematics course at Ohio State University is an important problem because of the large number of students whose entire curriculum depends on which mathematics course they are advised to take. Mistakes, either placing the student beyond his capabilities or beneath them, can be costly for the student, the department, and the university. This study contributes to the body of research on mathematics placement procedures, and on individualized instruction techniques.
Mathematics Courses at Ohio State

In order to provide for a wide range of interests and abilities among Ohio State's large student population and to meet the needs of the many academic departments that require mathematics training for their students, seven sets of beginning courses are offered by the mathematics department.


Placement into one of these sequences is based on preparation in mathematics as measured by scores on two examinations and on high school mathematics courses and grades. The first examination is the mathematics portion of the American College Testing Program Examinations, which is required of all students entering Ohio State. On the basis of the student's score on this first examination (denoted ACTM in this study) he is then given one of two forms of the
Ohio State University Mathematics Placement Test (OSUM). The student's placement level is then determined by his score on this second examination and either, on his ACT Composite Score, or on his high school mathematics courses and grades. There are five levels, Level I being the highest (ready to enter calculus with no further preparation) and Level V being the lowest. Complete information on the placement procedure is given in Appendix A.

Before enrolling in the first course of any sequence (except Math 180) a student must have attained a given placement level or must have completed a remedial algebra course, Math 101. Courses for which a student is eligible at each placement level are listed below.

Level I: Math 117 and 151. (Students placing at this level are given examination credit for Math 150.)

Level II: Math 105, 116, 121, 150.


Level IV: Math 101.

Level V: Students in this group must retake the placement examination after a review of high school algebra and place at a higher level before being eligible for any course except Math 180.

This study is concerned with Math 150 in both the traditional and revised forms. In particular it seeks to re-evaluate placement Level III with respect to the new curriculum.
The CRIMEL Program

The placement procedure and variety of course offerings mentioned above were developed over the years by the Department of Mathematics in an attempt to take into account the background, interest, ability, motivation, and needs of the individual student who comes to the department for instruction. However, prior to the beginning of the CRIMEL program, these efforts were still judged by members of the department to be insufficient to meet the needs of the student. These quotes from Professor John Riner and Robert Fisher two of CRIMEL's creators, indicate the problems which motivated the program.

For several years the department has been aware that it is not desirable to teach large numbers of students in the lock-step manner we now use. Under our present instructional system a class of, say, 1000 students is expected to "master" the material presented in three weeks of instruction, is examined and then is pushed forward into another three weeks of work. This continues for the ten weeks of the quarter. Little attention is given to the person who is not adequately prepared to proceed past the first test in the course or to the bright student who could advance at a much faster pace.

In the past we have not been able to take steps to correct this situation. We have had neither the technical nor the human resources to attack the problem. Recent departmental development indicates that we may soon have the capability to implement an instructional system designed to allow each student to proceed through his education in mathematics at a pace consistent with his background, interests, ability and needs. We estimate that it will take about 10 years to make the system fully operational.

A concept basic to the system is that of the instructional module. Sequences of these modules will replace the present courses. In general a given mathematics course contains several topics that frequently are related to
sequential fashion. A student can master a topic only if he knows the facts and techniques of the preceding topic. An instructional module (IM) would be limited to several related topics. Thus a given course could be decomposed into several IMs. While a course extends over ten weeks an IM would extend, for the average student, over a much shorter period of time, perhaps from 2 to 5 weeks. The content of a course is determined by the amount of material that the average student can learn in a ten week quarter. The IM would have no such limitation. The content would be determined by what an average student needs to know about the topic(s) of the module. The amount of credit given for completing an IM would be determined by the time required by the average student to master the content of the IM.

When the system is fully developed the student working through a particular IM would have at his disposal not only text, workbook and programmed materials (these are the basic items) but instructional video tapes, film strips and computer assisted instruction. At every stage of development we anticipate that adequate tutorial help and counselling will be available to the student.

The interested and talented student should be able to reach his goal in an efficient manner. The average student will probably save no time but, through the individualization of training provided by the system, his personal "weak-spots" should be corrected and his training should be more complete. For the weak student the system should relieve the pressures induced by the "locked-step" approach and the timing of the traditional methods of teaching mathematics. Such a student would be able to obtain a few hours of credit for the knowledge he has gained rather than a failing grade on a larger body of knowledge. (26.1)

Course Changes for the CRIMEL Program

During the initial year of the CRIMEL project the algebra and trigonometry course, normally designated Math 150, a five credit course, was run at three different paces. For registration purposes the course was renamed for the project and was designated Math 159.01, a three credit algebra course, and Math 159.02, a two credit
trigonometry course. Math 151, Differential Calculus, was renumbered 159.03, three credits, and Math 159.04, two credits.

Normally, students in Math 150 must have been placed in Level II on the basis of college entrance information or they must have passed an algebra review course, Math 101. During registration for Autumn Quarter 1970, those students who displayed marginal skills in algebraic manipulation or their entrance examinations, who were placed in Level III, and who registered before July 28, were allowed to enroll in Math 159.01 and Math 159.02. Students who registered after July 28 and who were Level III were enrolled in Math 101, Basic Algebra, as usual. These students were then able to schedule Math 150 during the Winter Quarter. Thus students enrolled in the experimental course either placed Level II, or placed Level III and were assigned an early registration date by university admissions officials, or placed Level III, IV, or V and already had completed Math 101. Students in the Winter Quarter Math 150 course, which served as the control group taught in the traditional manner, were either placed in Level II by their entrance information, or placed Level III and were assigned a late registration date and completed Math 101 during the Autumn Quarter, or placed Level III, IV or V and completed Math 101.

Design of the Study

All of the students in the experimental course were registered for both Math 159.01 and Math 159.02 and started the course in the
traditional way. At the end of two weeks, Test 1 was given to all students in the course. On the basis of their score on Test 1 they were recommended to attend an "accelerated pace" section, a "regular pace" section, or a "reduced pace" section. Those who were accelerated added Math 159.03 to their schedules. Material was presented at the traditional pace to those in the regular paced section. Those who attended the reduced pace sections dropped Math 159.02 from their Autumn schedules, repeated the material already covered and retook Test 1. They completed the Math 159.02 part of the course during the Winter Quarter. Students in the accelerated and regular sections had their Test 1 scores used as part of their final grade in Math 159.01. Students in the reduced pace sections had only the retake of Test 1 used as part of their final grade.

Figure 1 indicates the timing of the examinations given to the various groups of students. Test 1 refers to a set of 20 multiple choice questions on basic algebraic concepts and skills. Test 2 refers to a set of 20 questions covering the basic concepts and skills of polynomial, exponential and logarithmic functions. Test 3 refers to a set of 20 questions on trigonometric functions and complex numbers. The various forms of these tests are given in Appendix B. These examinations were written by members of the faculty who have been involved in testing in Math 150 for several years. The items chosen for these tests came from item banks built from past examinations. Item analyses of earlier tests given in Math 150 were also
Figure 1.

<table>
<thead>
<tr>
<th>Autumn Quarter</th>
<th>Winter Quarter</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>Math 159.02</td>
</tr>
<tr>
<td>T2</td>
<td>Math 159.03</td>
</tr>
<tr>
<td>Math 159.01</td>
<td>Math 159.04</td>
</tr>
<tr>
<td>Math 159.02</td>
<td>Math 152</td>
</tr>
</tbody>
</table>

Placement Level II
Entering Winter Quarter

Placement Level III

Math 159

T1 repeat

Math 159.02

T2

Math 151

T3

Placement Level III

Late registration date

Control

Placement Level IV

Placement Level II

Entering Winter
considered in choosing items to be used in the various examinations. Item analyses were run on examinations given during the study to supply further data on these instruments. Test evaluation information may also be found in Appendix B.

On the basis of their scores on Test 1 the students in the control course were divided into groups comparable to the groups recommended for the accelerated, regular, and reduced pace groups of the experimental course, but only on paper, for purposes of comparison.

The study was split into two quite different aspects. The first was a comparison of the Level III students who had been placed into either Math 159 or Math 150 on the basis of something unrelated to their mathematical skills or abilities (their date of orientation). The second aspect was a comparison of all the students who enrolled in either course, those who proceeded at the normal rate compared to those who proceeded at either the faster or slower rate provided by CRIMEL. This second aspect of the study is itself divided into two parts. A comparison of the Math 159 students and the Math 150 students, and a comparison of students within the CRIMEL structure who chose the traditional pace to those who chose one of the new paces.

**Analysis of the Study**

The comparisons outlined above were made using a one-way analysis of variance technique. Since students in the CRIMEL course were
recommended for a pace on the basis of their T1 score it was necessary to use T1 as a blocking variable. The one way analysis of variance procedure was run to compare treatment results measured by T2 and T3 for each of the three recommendation levels based on the T1 scores. (For example, a one way analysis was run to compare the groups of students who chose accelerated, regular and reduced paces or were in 150 for whom the regular pace was recommended i.e. those scoring between 16 and 11 on T1, inclusive.) When the analysis of variance indicated significance t-statistics were used for pairwise comparison of the study groups.

Limitations

It should be noted that students enrolling in Math 150 did not represent the entire student population enrolling in their first or second college mathematics course.

As previously stated only students who achieved sufficiently well on their entrance exams were permitted to enroll in Math 150. Some students entering Ohio State score high enough on their entrance exams to receive credit for Math 150 and enter a calculus course immediately. Students who did not score well enough could not enter Math 159.01 or 150 until they had brought their algebraic skills up to a sufficient level to be considered ready for college algebra (by taking Math 101 or an algebra review course offered by another institution).
Students qualified for Math 150 could have chosen to take one of the other two college algebra courses offered at Ohio State. Math 150 is usually taken by students interested in the biological and physical sciences, engineering, agriculture and related fields. Thus the study population was somewhat biased (as compared to all first year math students) by the fact that most of the students recognized some need for their mathematics training.

As in most studies it was necessary to limit the population of the study to those students for which full data was available. This eliminated students who started the course and later withdrew and those students who did not take examinations with the group because of illness or schedule conflicts. Specific data on the numbers of students deleted from the study will be given in Chapter 4.

It was also necessary to write "equivalent forms" of examinations for the four different student groups (accelerated, regular and reduced paces of Math 159, and the traditional Math 150). It must be assumed that had the forms of a test been interchanged among the four groups that the scores for each group would not have been significantly different from those occurring in this study. (For example, if the questions on the accelerated pace Test 2 had appeared on the regular pace Test 2 and vice versa the means for the two groups would not have been significantly different from the means reported for the two groups in Chapter 4.) This assumption is based on the expertise of the instructors who wrote the examinations, and their experience in testing in this course over a number of years. More
will be said about the instrumentation of the courses in Chapter 3.

Summary

This study seeks to answer in part the questions posed below:

1. Of the students who entered Autumn Quarter and placed into Level III on the basis of their ACT and OSUM scores, did those who took Math 101 have a higher level of achievement in Math 150 than those who entered Math 159 directly?

2. Was the performance of students in the CRIMEL program who chose the accelerated pace adversely affected by that choice?

3. Was the performance of those who chose the reduced pace improved by that choice?

Format of the Dissertation

Chapter I presents an overview of the study and its setting. Chapter II contains a review of the literature and its relation to the study and the problem in general. The details of the study, its design and statistical procedures, are given in Chapter III. Chapter IV presents the results of the statistical analysis; and Chapter V contains the summary of the findings, their implications for the experimental program, and their implications for further research.
CHAPTER II

REVIEW OF RELATED LITERATURE

Research Related to Homogeneous Grouping

Although there has been a great deal of research dealing with homogeneous grouping it is of very inconsistent quality. The most consistent result is that it is not the grouping but the different treatments given to different groups that will have a chance of yielding significant results. There are four good reviews of this literature. Day's work in 1964 (6); Goldberg, Passow, and Justman's work in 1966 (14, 1-22); Suydam's reports published in 1972 (33 and 34); and Esposito's paper in 1973 (10) present an up to date comprehensive guide to the better research done in this area and indicate drawbacks in some of the research on homogeneous grouping.

One well controlled study which dealt with pre-calculus mathematics was conducted by Major Wilburn Schrank (31) at the United States Air Force Academy Preparatory School during the 1966-67 academic year. The population of the study consisted of 204 enlisted airmen who were preparing for the Air Force Academy Competitive examinations. For the study they were divided into two equivalent groups, A and B, on the basis of College Entrance Examination Board
mathematics aptitude and achievement tests, Educational Testing Service algebra and plane geometry tests, and their previous academic record. Also for purposes of the study the mathematics course was divided into two sequences. The first sequence covered plane geometry and trigonometry and the second sequence covered algebra. During the first sequence group A was sectioned randomly while group B was sectioned by ability. During the second sequence group A was sectioned by ability while group B was sectioned randomly. The instructors and students were not given information as to how the sectioning was done. All of the sections covered the first sequence material at the regular rate, but during the second sequence all of the superior students were accelerated through the material. They covered the second sequence in sixty-five percent of the normal time.

A mean score was computed for each student for each sequence. The difference between mean scores of groups A and B was found to be significant at the 0.05 level in favor of the random grouping, for the first sequence. But, for the second sequence the difference was found to be significant at the 0.001 level in favor of the ability grouping. Thus it was concluded, "it is not the grouping at all that produces the various effects but rather what is done for the students once the groups have been established." (31, 128)

Research Related to the Crimel Program

The research base for the various aspects of the CRIMEL program is fairly extensive. Research has been reported on the placement
procedures used at Ohio State, the comparative effects of various large lecture techniques on student achievement, and the analysis of multiple choice examinations. In addition to these related studies a pilot study was done during the Winter and Spring of 1970 to determine criteria for placement of students in the CRIMEL project into the three pacing levels. These studies are discussed in some detail below.

Crosswhite (5) established the reliability of the placement procedure for students taking the "D" form of the Ohio State University Mathematics Placement Test. (This test was used to place 1153 of the 1685 students in the CRIMEL study population.) Multiple regression techniques were used to determine criteria for placement of entering students into calculus their first quarter. From the original thirteen independent variables two emerged as valid predictors of achievement and success in first quarter calculus. The multiple score (M.S.)

$$M.S. = 2(D) + QPTS$$

was found to be the best predictor where $D$ is the student's score on the D placement test and $QPTS$ is the number of high school quality points earned as follows. For each semester of high school math a student receives four points for an "A" grade, three points for a "B", two points for a "C", one point for a "D", or zero points for an "E". It was recommended that those students who achieved a Multiple Score of 55 or higher be placed directly into the calculus sequence (Level I). Those students who did not have a high enough
Multiple Score were placed into a college algebra course (Level II).

Paul (25) as a part of his study of the effects of high school calculus courses on achievement in college calculus reaffirmed the earlier work by Crosswhite. Using similar techniques he arrived at the same equation for predicting success although the 'best' Multiple Score for separating Level I and Level II was found to be 56.

Waits (35) conducted a study which compared the effect of three different large group instructional techniques on achievement in mathematics. The study was conducted at The Ohio State University using the 971 students enrolled in Math 117 during Winter Quarter, 1967. Math 117 is the second course in the sequence Math 116-117, and is a one quarter survey of calculus for non-science majors. The three large lecture techniques used were (1) live, large lecture — recitation (LLL-R) method (three lectures per week in a large lecture hall with recitation sessions two periods per week in small classrooms), (2) the daily television - daily recitation (DTV-R) method (daily televised lectures for part of the class period, in addition to daily recitations, in small, television equipped classrooms), (3) the televised large lecture - recitation (TVLL-R) method (the LLL-R method with the lectures presented via television). The course was offered five times during the day and the teaching methods were distributed as follows: the LLL-R method was used at noon, the DTV-R method was used at 10 and 3, and the TVLL-R method was used at
8 and 1. The lecturer in all cases was, the investigator. All students took common exams given in the evening four times during the quarter. An analysis of covariance model was used which accounted for variance in the examination scores by including an effect for method, an effect for recitation instruction and by making certain covariate background adjustments. It was found that at the .01 level of significance the achievement effects for the different methods were significantly different. The LLL-R method resulted in higher achievement scores than either TV lecture method. However, there was no significant difference in effects between the two television methods.

During the study of the effects of CRIMEL on student achievement two different television lecture methods were used. In the Autumn a method similar to the TVLL-R method was used. Although the students were in a small classroom setting when viewing television, the lectures were for the full class time every other day. During the Winter Quarter the DTV-R method was used. From the results of Waits' study there was no reason to expect this difference in instructional methods to contribute significant differences in achievement.

To prepare for the implementation of CRIMEL on a wide scale in the Autumn, 1970 a pilot project was run in the Spring, 1970 using the students in Math 116 as the study group. This study was done by David G. Mader (19). Many of the decisions on recommending pacing levels to CRIMEL students were made on the basis of the experience gained during this pilot project. Information was collected on the
students in Math 116 during the Winter, 1970, which served as control data during the Spring pilot project. Pre-course data, midterm and final exam scores were recorded for each student and analyzed to determine if pre-course or first midterm scores could predict student achievement in the course (as measured by total points earned on midterms and final). Using multiple correlation techniques it was found that pre-course data had a correlation of from 0.055 to 0.388 with achievement in Math 116 (depending on which types of pre-course data were available for a given group of students). When the first midterm score was added to the list of predictors the correlation factors ranged from 0.688 to 0.803. Because of the variation in pre-course data available on students in the course and the high correlation between first midterm scores and achievement, it was decided to study the possibility of predicting achievement on the basis of only the first midterm grade. In fact, since it would not be necessary to predict all students' total point scores, but rather to identify superior and deficient students for the accelerated and reduced pace sections of CRIMEL, a somewhat different predictive scheme was considered. Students were identified as superior if their final grade in Math 116 was an A or B, students were identified as deficient if their final score in Math 116 was a D or E (failing). By using only the first midterm score the student was predicted to be in one of four categories with respect to achievement based on total points. Those four categories were (1) superior (grade A or B in Math 116),
(2) not deficient (grade A, B or C), (3) not superior (grade C, D or E), (4) deficient (grade D or E). The percent of correct predictions made in this way are given in the table below (19, 43).

**TABLE 1**

**DECISION TABLE BASED ON TEST I**

<table>
<thead>
<tr>
<th>TEST I RANGE</th>
<th>PREDICTED PERFORMANCE</th>
<th>% CORRECT PREDICTIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>22 - 25</td>
<td>SUPERIOR</td>
<td>89</td>
</tr>
<tr>
<td>17 - 21</td>
<td>NOT DEFICIENT</td>
<td>94</td>
</tr>
<tr>
<td>13 - 16</td>
<td>NOT SUPERIOR</td>
<td>98</td>
</tr>
<tr>
<td>0 - 12</td>
<td>DEFICIENT</td>
<td>85</td>
</tr>
</tbody>
</table>

From the analysis of the Winter Quarter data, three selection criteria were developed to use in assigning students in the CRIMEL pilot project to the three paces of the course. These three strategies were compared during the Spring. The first strategy used both pre-course data and the first midterm to decide which pace the student was assigned to; the second strategy used only the first midterm to decide which pace a student was assigned to; the third strategy used both the first midterm and student choice to decide which pace a student was assigned to. At the end of the quarter the strategies were evaluated on the basis of percent of students correctly placed by the strategy i.e. how often the strategy placed a student in a pace he completed. The results were suspect since the quarter was interrupted by riots which closed the school for over a week and seriously disrupted the continuity of instruction. However, the strategies were 'scored' as follows: Strategy I placed 80% of its students correctly, Strategy II placed 78% of its students correctly, Strategy III placed 86% of its students correctly. Because of the disruptions this study of the three strategies was replicated in the Autumn when CRIMEL was officially begun, but only in the Math 115 course. The Math 120 and 159 courses used Strategy III because of its simplicity and since it had not been shown to be worse than the more complicated Strategy I. In addition to comparing the three strategies for selection of students for the various paces, Mader looked at the overall comparison of the CRIMEL students in the Autumn Quarter Math 115 course and the Winter Quarter Math 116 course which had
served as the control group for the pilot study. His study revealed that the students who were recommended for the accelerated pace on the basis of Test 1 during the Autumn Quarter CRIMEL course had significantly lower total point scores than the corresponding group in the Winter Quarter (Math 116) course. Those who were recommended for the regular pace in the CRIMEL course showed no significant differences from the corresponding Winter group. Those recommended for reduced pace in the CRIMEL course had significantly higher total point scores than the corresponding Winter group. The three selection strategies placed in the same relative positions during the Autumn that had placed in the Spring pilot project with respect to 'correct' student placement. Strategy I placed 74% correctly, Strategy II placed 71% correctly, and Strategy III placed 77% correctly.

During the pilot project Bashford (2) attempted to determine the relationships between the organizational innovations in CRIMEL, the student's causal attributions and student achievement in the course. Attribution of causality was measured on a scale which ranged from external (causality beyond the control of the subject) to internal (causality within the control of the subject). Differences between the pretest and post-test scale measures were noted. No highly significant differences were found, but there was a shift toward the internal end of the scale on the part of the accelerated and reduced pace students. Attitudes of both students and teachers were quite favorable with respect to both the placement of students into the
pacing levels and the treatments within the paces. These attitudes were measured in controlled interviews. Reaction was most favorable from the students in the accelerated and reduced pace sections. Students in the regular sections felt that the homogeneity of their class was the best aspect of the project.

Current Related Research

Certainly Ohio State is not the only school to recognize the importance of individualizing instruction in college mathematics. Many schools are studying the uses of audio-visual aids, course modules, computer assisted instruction, flexible testing, etc. in assisting students to succeed in their mathematics training. The University of California at San Diego entered into an extensive program of flexible testing during the 1971-72 academic year. Their program included all the courses in the undergraduate program. Although a formal report is not available, an informal summary of their experiences was given to other schools interested in studying this technique (41). Nunney (23) reports on current uses of audio-visual techniques in his paper of 1972. Ohlemeier (24) has written a summary of "mini" math courses offered at Barton County Junior College. This closely resembles the type of module which is the aim of the CRIMEL program in future years, and allows flexible pacing and testing. Hillsborough Junior College (32) initiated an individualized instructional program in 1970 which utilized diagnostic tests and individual counseling for each student as he proceeded through the various algebra courses until he had completed previously
agreed upon objectives at prearranged proficiency levels. As more schools gain access to computers capable of handling computer assisted instruction more individual help will be available through programs such as the one reported by Rockhill from The State University of New York, State University College at Brockport (30). Here the computer was used to administer and evaluate problems with which each student needed additional help, the computer then gave the student a printout of instructional aids related to his individual needs.

These studies give some indication of the scope of the research being done on beginning college mathematics instructional techniques. There are many more worthwhile studies of this nature being conducted in the public schools but they are somewhat outside the scope of this study. The body of literature must get much larger before any broad generalizations about the success of particular techniques of individualization may be stated with authority, but evidence is mounting that with these new techniques many more students may eventually be able to successfully complete college algebra than would be expected to do so under the usual college curriculum.
CHAPTER III
THE STUDY

When the CRIMEL program was originated the aim of the project was to improve student achievement in freshman mathematics courses by allowing more flexibility in instructional techniques than had been available in the past. During the program's initial year this flexibility was limited largely to variations in the time allowed for students to master the material, e.g. the three pacing levels. Thus the aim of this study was to consider the effects of the various pacing levels on student achievement.

It is always difficult to test new educational programs on human subjects. Randomly assigning students either to an experimental or to a control group and offering privileges only to one of the groups would appear very unfair to the students and furthermore could not be enforced in most situations at the college level since the students may choose to withdraw from the course at will. In this instance, the decision to limit the program to part of the student population was made because of budget limitations. The experimental program could be funded for only one quarter. In order to maximize exposure to the new program the CRIMEL course was offered to all students taking algebra and trigonometry during the Autumn Quarter, the quarter in which this course has its largest audience. During the subsequent Winter and
Spring Quarters the course was offered under the traditional format. For this study measures were taken on students in the CRIMEL course Autumn and Winter (reduced pace only during Winter) and the Winter Quarter traditional course, Math 150.

The Structure of the Courses Used in the Study

Topics Introduced in Math 159 and Math 150

The following is a list of topics occurring in Math 159.01 and Math 159.02 and Math 150 in the order of their appearance. Tests 1, 2 and 3 are shown in their respective positions. The tests are not "cumulative" in that each covers only material introduced after the previous test.

Math 159.01

Real Numbers

Sets, Number Lines, Intervals

Basic Properties of Real Numbers

Exponents and Radicals

Order Axioms

Solving Inequalities

Absolute Value

Test 1
Functions

Basic Concepts
Coordinate Plane, \( R \), Graphs
Polynomial Functions
Lines and Mappings
Graphing
Inverse Functions
Exponential and Logarithmic Functions

Math 159.02

The Trigonometric Functions
The Trigonometric Point
The Trigonometric Functions
Tables
Graphs of the Trigonometric Functions
Addition Formulas
Trigonometric Identities
The Equation \( y = A \sin(ax + b) \)
Angles, Right-Triangle Trigonometry
The Law of Sines; Cosines.

The Complex Numbers
Complex Numbers, The Conjugate
Graphical Representation of the Complex Numbers
Roots of Complex Numbers
Inverse Functions

Inverse Functions for the Trigonometric Functions

Trigonometric Equations

Test 3

Topics Introduced in Basic Algebra, Math 101

The following is a list of topics occurring in Math 101 in the order of their appearance.

Sets, Operations on Sets, Cartesian Products.
Fundamental Laws of Numbers
Special Sets: Naturals, Integers, Rationals, Reals, Complexes.
Coordinate Systems for Graphing.
Linear Equations and Inequalities.
Absolute Value Equations and Inequalities in One Variable.
Systems of Linear Equations.
Functions.
Polynomials.
Factoring Polynomials.
Division of Polynomials.
Rational Expressions.
Quadratic Equations and Functions.
Fractional Equations and Functions.
Inverse Functions.
Exponential and Logarithmic Functions.
Methods of Instruction Used in the Courses of the Study

In Math 159 all students started the course with heterogeneous groupings and with television used for the lectures. They viewed these lectures every other weekday in classrooms seating 30 to 40 students. On alternate days they met in these rooms with a teaching assistant for recitation. After the first test they were homogeneously grouped so that at each hour there were separate classes proceeding at each of the pacing levels. The regular and reduced pace sections continued utilizing television lectures alternating with recitations. The accelerated sections were taught by individual instructors without television lectures. All classes continued to meet five days each week even though the accelerated students were earning eight hours credit, while the reduced pace students were earning three hours credit. At a given pacing level all sections followed a set syllabus. These syllabi differed between pacing levels, naturally, in the speed with which topics were introduced and in the number and variety of problems assigned. For a given topic one might find the accelerated pace section spending half an hour discussing the more difficult problems in the assignment, then moving on to the next topic; the regular sections could spend part of one day introducing the topic and discussing some of the assigned problems and part of another day going over the more difficult problems; the reduced pace sections could spend a least one full day going over the topic and the elementary assigned problems and half of another day
working on some of the more difficult problems.

One note, which must be introduced, concerns difficulties encountered during the Math 159.02 reduced pace course. The department intended to have the reduced pace course meet three days each week during the Winter Quarter, two days weekly scheduled as class meetings with the third day providing an optional problem section. Thus 30 days of instruction were planned for presenting material normally presented in 20 days. Because of the days on which the school holidays fell and due to unexpected difficulties in scheduling more than two class meetings per week, students in reduced pace Math 159.02 had at most one more day of instruction than those in the regular pace, and approximately half of the reduced pace students had three days less instruction than the regular pace students. The T3 scores for the reduced pace cannot therefore be considered in the same light as the reduced pace T2 scores for the purpose of reflecting additional classroom instruction for each topic. By contrast, in Math 159.01 the reduced pace students received 50 days instruction on the same topics covered by the regular pace students in 30 days. Although the number of instructional hours per week was quite different for the reduced pace as compared to the regular pace, the total number of instructional hours for the course did not change enough to allow a different pace to be truly established. The reduced pace Math 159.02 resembled the regular pace Math 159.02 in total instructional hours but with longer and more frequent weekends. This problem affects
the interpretation of all the comparisons made which include the reduced pace course.

In Math 150 all the students had television lectures during the entire quarter. A slightly different schedule was used to divide the classroom time between lecture and recitation. On each day, the first 20 minutes of class were devoted to recitation and the remaining 28 minutes to the television lecture. Again the 30 to 40 student class was the daily setting. While this difference in lecture-recitation style might be thought to affect the results of the study there is good reason to believe that the effect is not significant. Waits (35), as indicated in Chapter 2, has shown that there is no reason to expect significant differences in achievement between classes which vary only in this respect. Except for this difference in the television lecture schedule, the Winter Quarter Math 150 was very close to the regular Math 159 course in the method of instruction, the syllabus, the timing of the examinations within the quarter and the problem assignments.

Test Instruments Used in the Courses

All of the examinations used in the study consisted of multiple choice items. Students recorded their answers on IBM scanning sheets which were then graded by an IBM light scanning device which made a computer card for each test sheet with the student's name, student number, section number, score and responses to each individual
question. The computer deck thus generated was used to prepare an item analysis of each examination. These examinations and summaries of their item analyses are given in Appendix B. Reliability as measured by the Kuder-Richardson 20 ranged from .541 to .844 with most of these reliability coefficients falling in the .650 to .700 range.

The three tests given to accelerated pace students define the sets of items which make up the in-course achievement measures, T1, T2 and T3. The tests given to other paces of the CRIMEL course and the Math 150 course contain sets of items equivalent to these 20 question tests. Figure 1 indicates when these items appear on examinations, a more detailed account appears below in the section on the conduct of the study. List of the equivalent sets of questions appear in Appendix C.

Preparation for the Study

In 1970, during Winter and Spring Quarters, David G. Mader conducted a pilot study for the CRIMEL project using students in the Math 116-117 sequence. His main purpose in conducting this pilot program was to develop strategies for placing students into the various paces of the CRIMEL program. The literature at that time indicated that the best strategies were multiple-stage placement strategies and that the best of these included an in-course measure taken after some course work. Mader added the first midterm score in
Math 116 to the two stage placement procedure already in effect at The Ohio State University and attempted to use these as predictors of success in the course as measured by the students total points on all tests. Where high school quality points were available he found that correlation with total points to be .209. Where ACTM scores were available (these are often not available for transfer students from other colleges) he found their correlation with total points in the course to be about .2 also. For those students who placed into Math 116 by placing Level II or III he found the correlation between OSUM scores and total points in the course to be about .34. The best predictor by far however, was their first in-course examination score. The correlation between Test 1 score and total points in the course was calculated for each mathematics placement category and these coefficients ranged from .687 to .800. Using multiple regression increased the correlation to range from .688 to .803. For this reason it was decided that recommendations for various pacing levels would be made on the basis of the student's Test 1 score for the first year of the CRIMEL project.

The Conduct of the Study

For each student in Math 159 and Math 150, pre-course data were collected including sex, high school quality points (QPTS) (explained in Appendix A), raw score of the mathematics section of the American College Testing Program Examinations (ACTM), raw score on the Ohio
State University Mathematics Test (OSUM) and the quarter and year in which the student entered Ohio State.

During the course, achievement was measured by the three twenty-question tests, T1, T2 and T3. The first midterm examination (T1) was used by the CRIMEL classes to assign students to the three pacing levels. Students with Test 1 scores greater than or equal to 17 (of a possible 20) were recommended for the accelerated pace; students with scores less than 17 but greater than or equal to 10 were recommended for the regular pace; students with scores less than 10 were recommended for reduced pace. The individual could accept this recommendation or request an assignment one pace faster or slower.

The possible combinations of recommendation and choice are given in Table 2. For the students in Math 150 Winter Quarter their "recommendation" on the basis of Test 1 had no bearing on the conduct of their class, so that, in Figure B, they are indicated in a single choice of pace, namely the traditional pace of the course. Table 2 also shows the number of students who fell into each category of recommended and chosen pace.

All the students except those choosing the reduced pace had half of their final grade in Math 159.01 determined by this Test 1 score. Those who chose reduced pace were in classes which reviewed and were retested over the beginning material. Thus, their original Test 1 score was not used to help determine their Math 159.01 grade.

The test designated Test 2 was administered in different ways to the different paced groups. The accelerated and regular students took
TABLE 2

NUMBER OF STUDENTS PER CATEGORY
OF RECOMMENDED VS. CHOSEN PACE

Chosen Pace
Math 159 Math 150

<table>
<thead>
<tr>
<th>Recommended Pace</th>
<th>ACC</th>
<th>REG</th>
<th>RED</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACC</td>
<td>129</td>
<td>90</td>
<td>1</td>
</tr>
<tr>
<td>Group 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group 3</td>
<td>41</td>
<td>772</td>
<td>85</td>
</tr>
<tr>
<td>Group 4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group 5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group 6</td>
<td>0</td>
<td>63</td>
<td>146</td>
</tr>
<tr>
<td>Group 7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group 8</td>
<td></td>
<td></td>
<td>194</td>
</tr>
<tr>
<td>Group 9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group 10</td>
<td></td>
<td></td>
<td>156</td>
</tr>
</tbody>
</table>

equivalent one hour tests at different times during the quarter, the reduced pace students took Test 2 as 20 questions of a 38 question exam in which the other 18 questions were from Test 1. Two versions of this 38 question exam were given. All students in the reduced pace were required to take the first version, the second was given as an optional final exam. Achievement on Test 2 for reduced pace students was taken to be the last score earned on the twenty Test 2 questions which appeared on each version. Test 3 was administered to the accelerated pace students as a one hour final exam in Math 159.02. In the regular pace classes, 5 of the questions appeared on a midterm and the other 15 questions appeared as part of their final exam in Math 159.02. The reduced
pace students were given Test 3 as part of their final examination in Math 159.02 during the Winter Quarter.

The Math 150 course during Winter Quarter was designed to correspond to the regular pace Math 159 of the CRIMEL program. Topics were presented in the same order and manner. Tests were administered in the same format and at the same time intervals as those of the regular pace. (See Figure 1, Chapter 1).

The comparisons between Math 159 and Math 150 which were made complicated by the obvious differences in initial achievement. The TI scores for Math 150 were skewed downward further than anticipated. Math 159 students recommendations were distributed 16.5% accelerated, 67.7% regular, 15.8% reduced pace. In contrast Math 150 students recommendations were distributed 2% accelerated, 54.3% regular and 43.7% reduced pace. The overall comparisons among the various paces within CRIMEL may be more informative.

The Structure of the Study

Since the Level III students were placed into either Math 159 or Math 150 on the basis of their date of orientation, which has no clearly definable relationship to their mathematics background or ability, for purposes of this study they were considered to be randomly placed into the two courses. To check for differences in background ACTM and OSUM scores and high school quality points (QPTS) were used. Since no differences in background were expected, the
criteria were met for use of a t-test to compare achievement scores in the two courses. The analysis of variance procedure was used to compare achievement between the various paces of CRIMEL chosen by the Level III students and the traditional Math 150.

The second part of the study was an overall comparison of the CRIMEL results in achievement to the results in Math 150. Here it was necessary to realize that placement into these two courses was anything but random. Students who were interested in the sciences and related mathematics, who had good high school backgrounds and were likely to score Level II were eligible for Math 150 their first quarter in college. Students who had less interest and ability in mathematics were likely to score below Level II and, if their major field required Math 150, they would enroll in Math 101 before enrolling in Math 150. Since most students enter Autumn Quarter it was expected that the mathematics background of the CRIMEL students would be somewhat different from that of the Math 150 students. It was recognized that it might be necessary to control for background in order to make valid comparisons of achievement in the courses. The most valid way available to block for initial differences in students entering the two courses was to use placement level. Students who had placed Level II were eligible to enter either course, students who had placed Level III were already being considered in part one of the study, and students who had placed Level IV or V were not eligible for either course until they had completed a remedial course. Thus, all comparisons between these two courses were made
within a placement level category:

1) students judged ready for college algebra (Level II),
2) students judged marginally ready for college algebra (Level III),
3) students judged not ready for college algebra. (Level IV or V).

Within each category listed above and within each recommendation level a t-test was used to compare achievement as measured by T2 and T3.

The third part of the study was an overall comparison of the new paces available within the CRIMEL program to the regular pace within the CRIMEL program. Here again the question was how a departure from the 'norm' affects the students who choose to do so. The comparison was made, within a recommendation level, between those who chose a new pace and those who chose the normal pace.

In this third part of the study an ANOVA procedure was run for each recommendation level to test for differences in ACTM scores, high school quality points, T1, T2 and T3 scores among the groups of students in the various paces.

From Table 2 on page 35 the comparisons of interest in the last two parts of the study can be delineated, as follows:

1. How did Groups 1 and 2 compare to Group 8?
2. How did Groups 3, 4 and 5 compare to Group 9?
3. How did Groups six and seven compare to Group 10?
Internal to the above we wish to pay particular attention to:

i) How did Group 1 compare to Group 27?

ii) How did Group 3 compare to Group 4?

iii) How did Group 5 compare to Group 4?

iv) How did Group 7 compare to Group 6?

That is, how did students who tried the non regular paces compare to those, in the same beginning class, who chose to remain in the regular or traditional pace?

In addition to these questions it would be interesting to pursue some others.

1) How do Groups 1, 4 and 7 compare with each other?

2) How do Groups $1 + 4 + 7$, $2 + 5$ and $3 + 6$ compare with each other? In particular how do $1 + 4$ and $2 + 5$ compare, how do $4 + 7$ and $3 + 6$ compare?

With these questions in mind the specific hypotheses to be tested were formulated.

**Hypotheses**

The aim of the study was to test the following hypotheses.

$H_0^1$: Among students entering Autumn Quarter placing in level 3, there is no significant difference in achievement on Test 2 and Test 3 between those entering the experimental course and those enrolling in Math 101 and then in Math 150, the control course.
$H_0^2$: There is no significant difference in achievement on Test 2 and Test 3 between the accelerated group in the experimental course and the student group designated accelerated by Test 1 in the control course.

$H_0^3$: There is no significant difference in achievement on Test 2 and Test 3 between the regular group in the experimental course and the students designated regular by Test 1 in the control course.

$H_0^4$: There is no significant difference in achievement on Test 2 and Test 3 between the reduced pace group of the experimental course and the student group designated reduced pace in the control course.

$H_0^5$: Among students in the experimental course who were recommended for the accelerated pace, there is no significant difference in achievement on Test 2 and Test 3 between those who went at the accelerated pace and those who went at the regular pace.

$H_0^6$: Among students in the experimental course who were recommended for the regular pace, there is no significant difference on Test 2 and Test 3 between those who went at the accelerated pace and those who went at the reduced pace.

$H_0^7$: Among students in the experimental course who were recommended for the reduced pace, there is no significant difference on Test 2 and Test 3 between those who went at the regular pace and those who went at the reduced pace.
Among students in the experimental course, there is no significant difference in achievement as measured by Test 2 and Test 3 between those who chose a pace faster than their recommended pace, those who chose a pace slower than their recommended pace, and those who chose the recommended pace.

In terms of the null hypotheses stated above, the experimental course could be considered successful if $H_0^2$ and $H_0^5$ were rejected and the others were either not rejected or were rejected in favor of the experimental course. In that case the answer to the questions posed in Chapter 1 would be that 1) the Level III students did as well in Math 159 alone as they did in the Math 101-150 sequence 2) the accelerated students were not forced to do 'average work' by going at a faster pace, and 3) the regular and reduced pace students were not hurt by this new course, in fact, the students who did badly on the first midterm in the course did as well on Tests 2 and 3 as the students who did average work on their first midterm, and thus were probably helped by the new methods.
CHAPTER IV
THE RESULTS OF THE STUDY

Since the main thrust of this study was to compare the relative progress of students in the Math 159 and Math 150 courses who entered Autumn and placed Level III, this will be considered first. In the second part of the chapter the overall achievement of students in the two courses is discussed. The analysis of variance procedure was used to detect differences in both background and achievement scores. In all cases the fundamental level of significance was 0.05.

Comparison of Level III Students

Comparison of Level III Students by Course

Of the 1327 students in Math 159 for whom complete achievement data was available, 93 were determined to be Level III and to have entered Autumn Quarter; of the 357 students with complete data sets in Math 150, 79 were determined to be Level III, to have entered Autumn Quarter, and to have taken Math 101 in the Autumn.

Table 3 displays the course means of these Level III students on the American College Testing Program's Mathematics Examination (ACTM), The Ohio State University Mathematics Examination "B" Test (OSUM), and the quality points they earned in high school (QPTS). Since these
students entered Autumn Quarter their ACTM scores and high school quality points would, for the large majority, reflect their standing in mathematics at the end of their senior year in high school (the 1969-70 school year). The OSUM scores were earned on an examination taken during late summer just before the start of their first course at Ohio State.

TABLE 3

LEVEL III STUDENTS
PRE-COURSE DATA - ANOVA BY COURSE

<table>
<thead>
<tr>
<th>Variable</th>
<th>Course</th>
<th>t and F tests</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Math 159</td>
<td>Math 150</td>
</tr>
<tr>
<td>ACTM</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>21.037</td>
<td>21.419</td>
</tr>
<tr>
<td>SD</td>
<td>2.799</td>
<td>2.273</td>
</tr>
<tr>
<td>OSUM</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>23.430</td>
<td>23.418</td>
</tr>
<tr>
<td>SD</td>
<td>3.150</td>
<td>3.112</td>
</tr>
<tr>
<td>QPTS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>19.011</td>
<td>17.816</td>
</tr>
<tr>
<td>SD</td>
<td>6.089</td>
<td>5.124</td>
</tr>
</tbody>
</table>

The results of the analysis of variance (ANOVA) procedures show no significant differences at the .05 level on any of these pre-course measures. Thus there was no reason to reject the equivalence of these groups when they entered Ohio State, Autumn Quarter, 1970.
The next measure taken was Test 1. This test was administered two weeks after the beginning of the respective courses (two weeks into Autumn for the Math 159 course, two weeks into Winter for the Math 150 course). The Math 150 students took Test 1 after Math 101 and two weeks of Math 150. By contrast the Math 159 students took Test 1 after only two weeks of Math 159 as their entire college mathematics experience.

Table 4 shows the results of the ANOVA procedure for Tests 1, 2, and 3.

**TABLE 4**

LEVEL III STUDENTS

Achievement Data - ANOVA by Course

<table>
<thead>
<tr>
<th>Variable</th>
<th>Course</th>
<th>Variable</th>
<th>Course</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Math 159</td>
<td>Math 150</td>
<td>t and F tests</td>
</tr>
<tr>
<td>T1 M</td>
<td>10.860</td>
<td>11.557</td>
<td>t = 1.6670</td>
</tr>
<tr>
<td>SD</td>
<td>2.952</td>
<td>2.432</td>
<td>F = 2.7789</td>
</tr>
<tr>
<td>T2 M</td>
<td>12.484</td>
<td>13.241</td>
<td>t = 1.5241</td>
</tr>
<tr>
<td>SD</td>
<td>2.873</td>
<td>3.635</td>
<td>F = 2.3228</td>
</tr>
<tr>
<td>T3 M</td>
<td>8.742</td>
<td>9.139</td>
<td>t = 0.8411</td>
</tr>
<tr>
<td>SD</td>
<td>3.220</td>
<td>2.925</td>
<td>F = 0.7074</td>
</tr>
</tbody>
</table>

N = 93  N = 79
Note that although the T1, T2 and T3 means were higher for the Math 150 course (those who had completed Math 101) than for the Math 159 course, they were not significantly higher. The lack of significant difference in the T1 scores is particularly interesting since for the Math 159 students Test 1 came after 2 weeks of instruction and for the Math 150 students it came after 12 weeks of instruction, and, in addition most of the Math 101 course was spent on algebra which was the basis of Test 1.

Since no significant differences appear in these comparisons hypothesis $H_0$ was not rejected. It seems reasonable to look at other detectable differences which might determine whether it would be advantageous for Level III students to take Math 101 before Math 150.

Both the Math 101-150 sequences and the reduced pace of Math 159 take two quarters to complete, but if a Level III student completed the Math 159 course satisfactorily at the regular pace then this could save the student time.

**Comparison of Level III Students by Pace**

Table 5 shows the pace at which these students completed the 159 course and how these groups compare on the achievement tests. The ANOVA procedure was used to determine whether there were significant differences in student achievement in the regular pace, reduced pace and traditional courses. One student completed Math 159 at the accelerated pace, which of course does not constitute a valid cell for
TABLE 5

LEVEL III STUDENTS

ACHIEVEMENT DATA - ANOVA BY PACE

<table>
<thead>
<tr>
<th>Course and Pace</th>
<th>159</th>
<th>159</th>
<th>159</th>
<th>150</th>
<th>F Test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ACC</td>
<td>REG</td>
<td>RED</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>N=1</td>
<td>N=41</td>
<td>N=51</td>
<td>N=79</td>
<td>(2.170)</td>
</tr>
<tr>
<td>T1</td>
<td>17.000</td>
<td>12.775</td>
<td>9.118</td>
<td>11.557</td>
<td>30.8162*</td>
</tr>
<tr>
<td>SD</td>
<td>1.804</td>
<td>2.463</td>
<td>2.432</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>16.000</td>
<td>11.550</td>
<td>12.922</td>
<td>13.241</td>
<td>2.4856</td>
</tr>
<tr>
<td>T2</td>
<td>2.316</td>
<td>3.199</td>
<td>3.635</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SD</td>
<td>3.095</td>
<td>3.109</td>
<td>2.925</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>14.000</td>
<td>9.250</td>
<td>8.118</td>
<td>9.139</td>
<td>2.2106</td>
</tr>
<tr>
<td>T3</td>
<td>3.109</td>
<td>2.925</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a ACC is not included in the ANOVA procedure.

* Significant at the .01 level.

the ANOVA. The table shows significant differences in the Test 1 scores. This is to be expected since the students were counseled into the reduced pace section if their Test 1 score was 10 or below. Although some students with scores above 10 did finish in the reduced pace section, and some students with a score of 10 were allowed to continue in the regular section, their numbers were not sufficient to offset the bias created when the reduced pace sections were formed. The dependent variables of the experimental design, T2 and T3, show
no significant differences in achievement. Thus, again we have no reason to differentiate between these courses on the basis of achievement. Of the 93 Level III students who were allowed to go directly into the Math 159 course 42 finished one quarter ahead of the normal 2 quarters required.

The course grades are another factor which may be considered in deciding whether Math 101 should be required of these students. For students in regular pace Math 159 and Math 150 the total point score is simply $T_1 + T_2 + T_3$, but for the reduced pace Math 159 the total point score is $T_1' + T_2 + T_3$ where $T_1'$ is their score on the second version of Test 1 which was given after a review of the first part of the course material. By using this total point score as an estimate of their final average grade in the course, a check for differences among the three groups was again made. Table 6 shows the ANOVA results.

| TABLE 6 |
|---|---|---|---|
| **LEVEL III STUDENTS** |  |
| TOTAL POINTS IN COURSE - ANOVA BY PACE |  |
|  | 159 | 150 | F |
| Reg Red | 159 | 150 |  |
| N = 41 N = 51 | N = 79 | (2.170) |
| Total Points M | 33.875 | 33.952 | 33.493 |
| SD | 5.219 | 7.261 | 6.813 | 0.0897 |
Comparison of All Students by Course

Table 7 shows the distribution of the students with complete data sets among the possible combinations of assignment and choice of pace. As expected, there were not sufficient numbers of students in the reduced-accelerated pace or accelerated-reduced pace combinations to include them in the study. However, the extremely small number of students recommended for the accelerated pace within the Math 150 course was not anticipated. Since seven students are not sufficient to constitute a valid cell in the ANOVA procedure the design of the analysis was altered to exclude the accelerated-150 cell and hypothesis $H_0^2$ was dropped from the study.

**TABLE 7**

NUMBER OF STUDENTS IN THE STUDY POPULATION FOR EACH CATEGORY OF RECOMMENDED VS. CHOSEN PACE

<table>
<thead>
<tr>
<th>Chosen Pace</th>
<th>Math 159</th>
<th>RED</th>
<th>Math 150</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACC</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cell 1</td>
<td>129</td>
<td>90</td>
<td>-</td>
</tr>
<tr>
<td>Cell 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>REG</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cell 3</td>
<td>41</td>
<td>772</td>
<td>85</td>
</tr>
<tr>
<td>Cell 4</td>
<td></td>
<td></td>
<td>194</td>
</tr>
<tr>
<td>Cell 5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RED</td>
<td></td>
<td>63</td>
<td>146</td>
</tr>
<tr>
<td>Cell 6</td>
<td></td>
<td></td>
<td>156</td>
</tr>
<tr>
<td>Cell 7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cell 8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cell 9</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Data on the Total Course Population

The total number of students receiving some grade in Math 159.01 during Autumn Quarter, 1970 was 2258. Of these 213 finished Math 159.01 at the accelerated pace, 1487 finished at the regular pace and 558 finished at the reduced pace. The following table shows the relationship of these populations to the corresponding study populations.

TABLE 8

DISTRIBUTION OF THE COURSE POPULATIONS

<table>
<thead>
<tr>
<th>Math 159 Finished In Math 150 Finished In Math 150 Study</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math 159.01 Study</td>
</tr>
<tr>
<td>----------------------------------------------------------</td>
</tr>
<tr>
<td>Acc - 213 170</td>
</tr>
<tr>
<td>Chosen</td>
</tr>
<tr>
<td>Reg - 1487 925</td>
</tr>
<tr>
<td>Pace</td>
</tr>
<tr>
<td>Red - 558 232</td>
</tr>
<tr>
<td>Drop-Withdrew</td>
</tr>
<tr>
<td>97</td>
</tr>
<tr>
<td>{52 Reg 45 Red}</td>
</tr>
</tbody>
</table>

For 51 of the 97 students who dropped Math 159.01 there were TI scores available; the mean of these scores was 10.24. There were TI
scores available for 77 of the 136 students who dropped Math 150; the mean of these scores was 8.22.

The table below shows the distribution of the available placement level information for the students who dropped the courses.

TABLE 9

<table>
<thead>
<tr>
<th></th>
<th>Math 159</th>
<th>Math 150</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level II</td>
<td>39</td>
<td>25</td>
</tr>
<tr>
<td>Level III</td>
<td>12</td>
<td>15*</td>
</tr>
<tr>
<td>Levels IV and V</td>
<td>11</td>
<td>33</td>
</tr>
<tr>
<td>No Data</td>
<td>35</td>
<td>64</td>
</tr>
</tbody>
</table>

* 14 of these students entered Autumn, all 12 of those in 159 entered Autumn.

Comparison of Level II Students by Course

Students who placed Level II were eligible to enroll in Math 150 (or 159) as their first mathematics course at Ohio State. In this study 930 of the students in Math 159 and 103 of those in Math 150 were Level II and entered Ohio State either Autumn or Winter Quarter.
Of the Level II students \( \frac{1}{4} \) of the Math 159 students were recommended for the accelerated pace, and two of the Math 150 would have been so recommended. Since \( H^2_0 \) was dropped from the study these students were not considered in this part of the study. There were sufficient numbers of students in each course at the other recommendation levels to make valid statistical comparisons using the one-way ANOVA procedure. Here the interest was in the overall results of allowing pacing vs. not allowing pacing. Note that the T2 and T3 scores of the Math 159 students were measures of their achievement in their chosen pace. Table 10 shows the results of these comparisons for each recommended level for both background and achievement data.

Since a student may have been assigned to Level II by either of the two OSUM examinations and the two tests did not have comparable raw scores, the OSUM scores were not included for this group. In considering the background data it would be well to remember that all of the Math 159 students in this population entered Autumn Quarter and went into their mathematics course immediately while of the Math 150 students, 53 entered Autumn and did not take a mathematics course their first quarter, and 50 entered Winter Quarter and entered Math 150 immediately. For 53 of the Math 150 students there existed a time lag of one quarter between the gathering of the background measures and the first achievement measure. This did not exist for the other students in this study group.

There were no differences in background data for either recommendation level that were significant at the .05 level. For
### TABLE 10

COMPARISON OF STUDENTS PLACING LEVEL II
ANOVA BY COURSE

<table>
<thead>
<tr>
<th></th>
<th>Course 159</th>
<th>Course 150</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>28.013</td>
<td>27.475</td>
<td>2.920</td>
</tr>
<tr>
<td>REG M</td>
<td>2.354</td>
<td>2.046</td>
<td></td>
</tr>
<tr>
<td>RED SD</td>
<td>1.980</td>
<td>1.702</td>
<td>0.0262</td>
</tr>
<tr>
<td></td>
<td>23.722</td>
<td>23.552</td>
<td>0.071</td>
</tr>
<tr>
<td>REG M</td>
<td>5.541</td>
<td>5.644</td>
<td></td>
</tr>
<tr>
<td>RED SD</td>
<td>5.966</td>
<td>6.046</td>
<td>0.768</td>
</tr>
<tr>
<td></td>
<td>13.791</td>
<td>12.970</td>
<td>16.180**</td>
</tr>
<tr>
<td>REG M</td>
<td>1.594</td>
<td>1.557</td>
<td></td>
</tr>
<tr>
<td>RED SD</td>
<td>1.497</td>
<td>1.218</td>
<td>0.012</td>
</tr>
<tr>
<td></td>
<td>13.673</td>
<td>15.925</td>
<td>39.837**</td>
</tr>
<tr>
<td>REG M</td>
<td>2.809</td>
<td>2.482</td>
<td></td>
</tr>
<tr>
<td>RED SD</td>
<td>2.771</td>
<td>3.367</td>
<td>0.1002</td>
</tr>
<tr>
<td></td>
<td>10.618</td>
<td>11.866</td>
<td>9.361**</td>
</tr>
<tr>
<td>REG M</td>
<td>3.198</td>
<td>3.104</td>
<td></td>
</tr>
<tr>
<td>RED SD</td>
<td>3.364</td>
<td>3.047</td>
<td>0.001</td>
</tr>
</tbody>
</table>

+ Significant at the .05 level
++ Significant at the .01 level
those recommended for regular pace there were, however, significant
differences at the .001 level for all three achievement measures. The
Math 159 course was significantly better on T1, but significantly
worse on both T2 and T3. For those recommended for reduced pace
there were no significant differences on any of the achievement
scores.

Comparison of Level IV and V Students by Course

Students who placed either Level IV or Level V were required to
take some review work in high school algebra before entering Math 150
(159). For the Level IV students this meant taking Math 101 before
enrolling in Math 150 (159). For most of the Level V students this
meant taking a review course offered by the local school system
(taught on campus) and then taking Math 101 before enrolling in
Math 150 (159). While it is possible for Level V students to go
directly from the review course to Math 150 (159) only about 4% of
them qualify to do so. Since the background data for these students
cannot be considered current in terms of time or recent training, it
was not considered in comparing these student groups.

Table 11 shows the results of the one-way ANOVA procedures for
each recommendation level for the achievement measures T1, T2 and T3.
For the students recommended for regular pace there were differences
at the .001 level on T1 favoring the 159 course, but no significant
differences on T2 or T3. For the students recommended for reduced
TABLE 11
COMPARISON OF STUDENTS PLACING LEVELS IV OR V
ANOVA BY COURSE

<table>
<thead>
<tr>
<th>Rec</th>
<th>Course</th>
<th>T1</th>
<th>T2</th>
<th>T3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pace</td>
<td>159</td>
<td>150</td>
<td>F</td>
<td></td>
</tr>
<tr>
<td>REG</td>
<td>M</td>
<td>13.732</td>
<td>12.182</td>
<td>25.4007++</td>
</tr>
<tr>
<td>SD</td>
<td>1.533</td>
<td>1.299</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RED</td>
<td>M</td>
<td>9.167</td>
<td>8.467</td>
<td>2.5308</td>
</tr>
<tr>
<td>SD</td>
<td>1.030</td>
<td>1.467</td>
<td></td>
<td></td>
</tr>
<tr>
<td>REG</td>
<td>M</td>
<td>12.610</td>
<td>12.864</td>
<td>0.1294</td>
</tr>
<tr>
<td>SD</td>
<td>3.456</td>
<td>3.047</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RED</td>
<td>M</td>
<td>12.917</td>
<td>11.333</td>
<td>2.3664</td>
</tr>
<tr>
<td>SD</td>
<td>2.875</td>
<td>3.371</td>
<td></td>
<td></td>
</tr>
<tr>
<td>REG</td>
<td>M</td>
<td>9.000</td>
<td>8.409</td>
<td>0.6460</td>
</tr>
<tr>
<td>SD</td>
<td>3.633</td>
<td>3.142</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RED</td>
<td>M</td>
<td>8.250</td>
<td>7.987</td>
<td>0.1159</td>
</tr>
<tr>
<td>SD</td>
<td>2.491</td>
<td>2.491</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

REG | N = 41 | N = 44 |
RED | N = 12 | N = 75 |

+ Significant at the .05 level
++ Significant at the .01 level
pace there were no significant differences on any of the three measures. These results for the reduced pace recommendation level are presented for completeness, but because there were only 12 students in Math 159 in this category little or no significance can be attached to them.

Hypothesis \( H_0^3 \) which stated that there would be no differences in student achievement between Math 159 and Math 150 for students recommended for the regular pace, was rejected for T2 and T3 for Level II students, but not rejected for T2 or T3 for Level IV and V students.

Hypothesis \( H_0^4 \) which stated that there would be no differences in student achievement between Math 159 and Math 150 for students recommended for the reduced pace, was not rejected for T2 or T3 for either Level II or Level IV and V students.

The interpretation of these results is very difficult because of the confounding variables which could not be controlled. Two of these in particular are the emphasis on T1 in the CRIMEL course which was not present in Math 150 and the dropout rates for these two courses. Since the students in CRIMEL were told that their Test 1 score would determine their recommended pace in 159, but Test 1 was simply the first midterm in Math 150 the type and amount of preparation for this test may have differed considerably. As has been mentioned before many of the students in Math 159 Autumn Quarter were in curricula which depend on mathematics training rather heavily so the advantages of speeding up and the corresponding disadvantages
(to their scheduling of other courses) of slowing down were not lost on these students. If the Math 150 students did not try as hard to score well on Test 1 this might lower their score on Test 1 by one or two points (or more) and thus increase the average ability of the regular and reduced pace groups relative to the corresponding 159 groups. Since the other two tests were then left to determine their grades more time and effort spent studying for Test 2 and Test 3 may have biased these comparisons between the course groups.

The second of the problems confounding these results was the difference in drop rates between the two courses. While 97 of the CRIMEL students dropped out of both Math 159.01 and Math 159.02 and 2258 students received a final grade in Math 159.01, 136 of the Math 150 students dropped that course during the Winter Quarter and 430 received final grades. The fact that almost one in four Math 150 students dropped the course may indicate that those students who would have gone reduced pace had it been available were forced to drop the course to avoid failing it. They were not then available for the study and this would bias the results on T2 and T3 in favor of the Math 150 students since the students left in that course had in effect elected the regular pace. Certainly there would have been some counter to this because it would have been easier to justify dropping two hours credit from a schedule than five hours credit, but it would be doubtful whether this added motivation to stay in the course could overcome the fear of a low grade to any very large extent.
Comparison of CRIMEL Students by Pace

Within the CRIMEL course, for each pace recommendation there were students who chose to follow the recommendation and those who chose not to. There were students in each recommendation level who chose the regular or traditional pace. Thus, for each recommendation level the comparison could be made between those students who took one of the new opportunities (accelerated or reduced pace) and those who took the traditional course (regular pace). Since all students in these comparisons began the course Autumn Quarter and the majority of the students entered Math 159 without any prior college mathematics (only 35 of the 1327 CRIMEL students in the study had previous college math), the background data as well as T1 can be used to compare initial differences between these groups. Tables 12, 13 and 14 give the breakdown of background and achievement data controlling for recommended pace, and the ANOVA results for each item. Significant differences occur within 14 or the 15 comparisons made. The only comparison which did not yield significant differences was the comparison of T2 scores for students recommended for reduced pace. Thus, hypotheses $H_0^5$, and $H_0^6$ were rejected for T2 and T3 and $H_0^7$ was rejected for T3 but not T2.
Students who were recommended for accelerated pace and chose accelerated pace had significantly better scores on every measure.

On the average the stronger students evidently decided to accept the challenge of acceleration. Notice that the accelerated students have a higher average on T2 and T3 of 2.1 and 1.6 points respectively,
while their original advantage (on T1) was 0.8 points.

TABLE 13

STUDENTS RECOMMENDED FOR REGULAR PACE
ANOVA BY CHOSEN PACE

<table>
<thead>
<tr>
<th></th>
<th>ACC</th>
<th>REG</th>
<th>RED</th>
<th>F, t</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACTM</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>28.432</td>
<td>27.538</td>
<td>24.673</td>
<td>27.4885++</td>
</tr>
<tr>
<td>SD</td>
<td>2.292</td>
<td>3.151</td>
<td>3.688</td>
<td></td>
</tr>
<tr>
<td>QPTS</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>27.415</td>
<td>23.450</td>
<td>19.495</td>
<td>27.6812++</td>
</tr>
<tr>
<td>SD</td>
<td>6.269</td>
<td>5.597</td>
<td>5.588</td>
<td></td>
</tr>
<tr>
<td>T1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>15.610</td>
<td>13.789</td>
<td>12.200</td>
<td>76.2987++</td>
</tr>
<tr>
<td>SD</td>
<td>0.703</td>
<td>1.573</td>
<td>1.111</td>
<td></td>
</tr>
<tr>
<td>T2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>16.293</td>
<td>13.206</td>
<td>13.918</td>
<td>24.8107++</td>
</tr>
<tr>
<td>SD</td>
<td>1.952</td>
<td>2.854</td>
<td>2.863</td>
<td></td>
</tr>
<tr>
<td>T3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>12.512</td>
<td>10.363</td>
<td>8.706</td>
<td>19.6393++</td>
</tr>
<tr>
<td>SD</td>
<td>2.721</td>
<td>3.348</td>
<td>2.720</td>
<td></td>
</tr>
</tbody>
</table>

++ Significant at the .01 level.

Students who were recommended for regular pace also showed significant differences between chosen paces on every measure. The comparisons of interest here are those between the accelerated and regular paces and between the reduced pace and regular paces. Those
who chose the accelerated pace show higher averages than those who chose regular pace on T1, T2 and T3 of 1.9, 3.0 and 2.2 points respectively. Those students who chose reduced pace showed significantly worse scores on both background measures and T1, but had a better average on T2 than those who chose regular pace. On T3 the reduced pace again showed a lower average.

<table>
<thead>
<tr>
<th></th>
<th>ACC M</th>
<th>REG M</th>
<th>RED M</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACTM</td>
<td>26.377</td>
<td>26.222</td>
<td></td>
<td>4.3455^</td>
</tr>
<tr>
<td>SD</td>
<td>3.904</td>
<td>3.448</td>
<td></td>
<td></td>
</tr>
<tr>
<td>QPTS</td>
<td>22.517</td>
<td>19.500</td>
<td></td>
<td>10.4675^^</td>
</tr>
<tr>
<td>SD</td>
<td>5.322</td>
<td>6.163</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T1</td>
<td>9.460</td>
<td>8.548</td>
<td></td>
<td>16.4215^^</td>
</tr>
<tr>
<td>SD</td>
<td>1.512</td>
<td>1.486</td>
<td></td>
<td>4.0523</td>
</tr>
<tr>
<td>T2</td>
<td>12.952</td>
<td>12.452</td>
<td></td>
<td>1.3799</td>
</tr>
<tr>
<td>SD</td>
<td>2.151</td>
<td>3.070</td>
<td></td>
<td>1.1747</td>
</tr>
<tr>
<td>T3</td>
<td>9.079</td>
<td>7.856</td>
<td></td>
<td>6.6994^</td>
</tr>
<tr>
<td>SD</td>
<td>3.071</td>
<td>3.162</td>
<td></td>
<td>2.5883</td>
</tr>
</tbody>
</table>

^ Significant at the .05 level

^^ Significant at the .01 level
For students who were recommended for the reduced pace significant differences were found in both background measures and T1. Again the students who chose the faster pace were on the average stronger according to the pre-treatment measures. No significant differences between the two groups were found on T2. For those who chose reduced pace this T2 score represents the measure taken at the end of the Autumn Quarter Math 159.01 course, i.e. at the end of that part of the course which offered more in-class instruction time for each topic. The T3 score represents (for the reduced pace group) the final achievement measure in the Winter Quarter Math 159.02 course. Recall that in this course while there was more study time between lessons there was not more in-class instruction time for most of the students. Thus, although T2 and T3 both represent the reduced pace, they represent two different types of reduced pace.

To gain some insight into the nature of the differences measured, Table 15 shows all of the in-course measures of the three previous tables represented by using the t-statistic as a standardized difference of means. In each case the t-statistic represents a standardized mean for the new pace as compared to the mean of the normal pace, the ratio of the difference in the means to the estimate of the standard error of the population making up that recommendation level. Where the t value is significantly different from zero this is indicated as well. For example, in the first block the T1, accelerated entry (6.4231) is the t-statistic calculated for the difference
**TABLE 15**  
**SUMMARY COMPARISONS OF ACHIEVEMENT USING t-STATISTICS**  
**AS STANDARDIZED DIFFERENCE OF MEANS**

**COMPARISON OF STUDENTS RECOMMENDED FOR ACCELERATED PACE**

<table>
<thead>
<tr>
<th></th>
<th>Chosen Pace</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ACC</strong></td>
<td>REG</td>
<td>RED</td>
</tr>
<tr>
<td>T1</td>
<td>6.4231**++</td>
<td></td>
</tr>
<tr>
<td>T2</td>
<td>7.1754**++</td>
<td></td>
</tr>
<tr>
<td>T3</td>
<td>3.8344**++</td>
<td></td>
</tr>
</tbody>
</table>

**COMPARISON OF STUDENTS RECOMMENDED FOR REGULAR PACE**

<table>
<thead>
<tr>
<th></th>
<th>Chosen Pace</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ACC</strong></td>
<td>REG</td>
<td>RED</td>
</tr>
<tr>
<td>T1</td>
<td>7.3690**++</td>
<td>-9.0621**++</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.4299)**++</td>
</tr>
<tr>
<td>T2</td>
<td>6.8397**++</td>
<td>2.1811+</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.4036**++</td>
</tr>
<tr>
<td>T3</td>
<td>4.0394**++</td>
<td></td>
</tr>
</tbody>
</table>

**COMPARISON OF STUDENTS RECOMMENDED FOR REDUCED PACE**

<table>
<thead>
<tr>
<th></th>
<th>Chosen Pace</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ACC</strong></td>
<td>REG</td>
<td>RED</td>
</tr>
<tr>
<td>T1</td>
<td></td>
<td>-4.0523**++</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(10.6541)**++</td>
</tr>
<tr>
<td>T2</td>
<td></td>
<td>-1.1747**++</td>
</tr>
<tr>
<td>T3</td>
<td></td>
<td>-2.5883</td>
</tr>
</tbody>
</table>

* + Significant at the .05 level  
**++** Significant at the .01 level
on T1 between the accelerated and regular pace students. This table
gives a summary of the relative standings of the paces on each in-
course measure for each recommendation level. The numbers appearing
in parentheses in the second and third blocks for those who chose
reduced pace are the values of t for t-tests comparing T1' scores
(the second T1 test taken after a review) for the reduced pace group
and T1 scores for the corresponding regular pace group.

Note that, for both accelerated and regular recommendation
levels, the accelerated students maintain a respectable difference
in average score over the corresponding students who chose regular
pace, although the difference did diminish. The differences between
the regular and reduced pace students do not follow the same trend.
For both the regular and reduced pace recommendation levels the T2
average for the reduced pace rivals that of the regular pace. In
the regular recommendation block, notice that the T2 average for the
reduced pace students is significantly better than that of the regular
pace students. In the reduced pace recommendation block the
t-statistic for the differences in T2 scores was not significant. In
both of the regular pace - reduced pace comparisons for T3 the regular
pace students had a significantly better average, although the
differences were not as marked of T3 as of T1. Again recall the
different types of reduced pace that these two measures (T2 and T3)
represent. The part of reduced pace which offered more in-class
instruction seems to definitely have an advantage over the part that
offered only more study time between classes as measured by the shifts in relative standing to the regular groups shown in this table. The ability of the reduced pace students to repeat T1 had a tremendous effect on their final grades. The relative standing of T1 was the most obvious of all the effects of the reduced pace sections.

Hypothesis 8 considers the differences observed when the population of the CRIMEL course is broken into groups by whether their decision was to accept the department recommendations or choose an alternative of their own. Using the diagram in Table 2 the comparisons are indicated in Figure 2 below.

![Diagram](image)

Figure 2. Student groups proceeding faster, slower or as recommended by their Test 1 scores.
The initial measure in the course, $T_1$, distributed unevenly since $T_1$ scores of the Slower group would all be greater than 10 and the Faster group would have all $T_1$ scores below 17. Table 16 presents the ANOVA results for comparisons of these three groups based on both the background and in class measures.

TABLE 16

<table>
<thead>
<tr>
<th></th>
<th>Faster N=104</th>
<th>Same N=1047</th>
<th>Slower N=174</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACTM</td>
<td>M 27.153</td>
<td>27.479</td>
<td>26.536</td>
<td>5.7515+</td>
</tr>
<tr>
<td></td>
<td>SD 3.518</td>
<td>3.298</td>
<td>3.769</td>
<td></td>
</tr>
<tr>
<td>QPTS</td>
<td>M 24.545</td>
<td>23.406</td>
<td>22.759</td>
<td>2.6462</td>
</tr>
<tr>
<td></td>
<td>SD 6.196</td>
<td>5.963</td>
<td>6.647</td>
<td></td>
</tr>
<tr>
<td>$T_1$</td>
<td>M 11.885</td>
<td>13.589</td>
<td>14.846</td>
<td>34.3403++</td>
</tr>
<tr>
<td></td>
<td>SD 3.269</td>
<td>2.887</td>
<td>2.732</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SD 2.637</td>
<td>3.064</td>
<td>2.753</td>
<td></td>
</tr>
<tr>
<td>$T_3$</td>
<td>M 10.433</td>
<td>10.401</td>
<td>10.360</td>
<td>0.0160</td>
</tr>
<tr>
<td></td>
<td>SD 3.375</td>
<td>3.556</td>
<td>3.488</td>
<td></td>
</tr>
</tbody>
</table>

+ Significant at the .05 level
++ Significant at the .01 level

The background data comparisons show as might be expected that there is a tendency for students to choose a pace which reflects somewhat their background. The most interesting part of the data
shown here is the shift that occurs in relative standing of the three groups as the quarter progresses. The commitments to faster or slower paces would seem to have been made in a sensible way. The students who chose a faster pace have not fallen behind either of the other groups.
CHAPTER V
SUMMARY AND CONCLUSIONS

Restatement of the Problem

The primary concern of the study was to compare the effects of the traditional Math 101-150 sequence with the new CRIMEL course (Math 159) for those entering college students who were judged to be marginally prepared for college algebra on the basis of their entrance examinations. Its secondary purpose was to gain some insight into the effects of the various CRIMEL paces on the students who chose them. Thus, the study was designed to answer the following questions:

1. Of the students who entered Autumn Quarter and placed into Level III on the basis of their ACT and OSUM scores, did those who took Math 101 have a higher level of achievement in Math 150 than those who entered Math 159 directly?

2. Was the performance of students in the CRIMEL program who chose the accelerated pace adversely affected by that choice?

3. Was the performance of those who chose the reduced pace improved by that choice?

Since there is no way to let a student try each choice independently and then obtain a comparison, it was necessary to compare the students who took advantage of the new paces with students who did
not, either because they chose not to (Math 159 - regular pace) or because they were not given a choice (Math 150). So answers to questions 2 and 3 were attempted by looking for answers to the questions:

2'. How did the performance of students recommended for the accelerated CRIMEL course who subsequently finished in the accelerated pace compare with the performance of students a) recommended for the accelerated pace who chose regular pace, and b) recommended for accelerated pace who were in Math 150?

3'. How did the performance of students recommended for reduced pace who subsequently went at the reduced pace compare with the performance of a) students who were recommended for reduced pace but chose regular pace, b) students who were recommended for reduced pace but were in Math 150?

Since the CRIMEL course was arranged in such a way that students might choose a pace faster or slower than their recommended pace it would help answer questions 2 and 3 above if the questions below were considered.

4'. How did the performance of students recommended for the regular pace who chose the regular pace compare with the performance of a) students who were recommended for regular but chose the accelerated pace, b) students who were recommended for regular pace but chose reduced pace, and c) students who were recommended for regular pace but were in Math 150?
5'. How did students who went faster or slower than their recommended pace compare with those who followed the department recommendation?

In order to answer the questions above it was necessary to test the eight null hypotheses listed in Chapter Three. The hypotheses are repeated below in the summary of the study results.

Summary of Results

A study of the results displayed in Chapter 3 in conjunction with the hypotheses and their motivating questions shows the following.

$H^1_0$: Among students entering Autumn Quarter placing Level III, there were no significant differences in achievement on Test 2 or Test 3 between those entering the experimental course and those enrolling in Math 101 and then in Math 150, the control course.

$H^1_0$ was not rejected. The students who placed Level III and entered Math 159 directly were not shown to be significantly different in achievement from those who took the Math 101-150 sequence.

$H^2_0$: There were no significant differences in achievement on Test 2 or Test 3 between the accelerated group in the experimental course and the student group designated accelerated by Test 1 in the control course.
was deleted from the study because there were only 7 students recommended for accelerated pace in Math 150.

$H_0^2$: There was no significant differences in achievement on Test 2 or Test 3 between the regular group in the experimental course and the students designated regular by Test 1 in the control course.

$H_0^3$: was rejected for T2 and T3 for Level II students, but not rejected for T2 or T3 for Level IV and V students.

$H_0^4$: There were no significant differences in achievement on Test 2 or Test 3 between the reduced pace group of the experimental course and the student group designated reduced pace in the control course.

$H_0^4$: was not rejected for T2 or T3 for either Level II or Level IV and V students.

$H_0^5$: Among students in the experimental course who were recommended for the accelerated pace, there were no significant differences in achievement on Test 2 or Test 3 between those who went at the accelerated pace and those who went at the regular pace.

$H_0^5$: was rejected for both T2 and T3. The students who were recommended for accelerated pace and chose to go into the accelerated course were significantly better on Test 1, Test 2 and Test 3.
Among students in the experimental course who were recommended for the regular pace, there were no significant differences on Test 2 or Test 3 between those who went at the regular pace, those who went at the accelerated pace, and those who went at the reduced pace. $H_0^6$ was rejected for T2 and T3. Since the ANOVA procedure indicated significant differences for Tests 1, 2 and 3, t-tests were run to determine which of the new paces were different from the regular pace in achievement. The students who chose the accelerated pace were significantly better on Tests 1, 2 and 3. The students who chose the reduced pace were significantly worse than those who chose the regular pace on Tests 1 and 3, but were significantly better than those in the regular pace on T2.

Among students in the experimental course who were recommended for the reduced pace, there were no significant differences on Test 2 or Test 3 between those who went at the regular pace and those who went at the reduced pace. $H_0^7$ was rejected for T3 but not for T2. The students who chose reduced pace were significantly worse on Test 1 and Test 3, but not on Test 2. Although they had a lower average on Test 2 than those who chose the regular pace it was not significantly so and, in fact, the differences on Test 3 were not as pronounced as on Test 1.
Among students in the experimental course there were no significant differences in achievement as measured by Test 2 and Test 3 between those who chose a pace faster than their recommended pace, those who chose a pace slower than their recommended pace, and those who chose the recommended pace. $H_0^8$ was rejected for Test 2 but not for Test 3. There were significant differences on the Test 1 scores with the slower paces having a higher average than the 'same' paces which in turn had a higher average than the faster paces. On Test 2 the faster and slower paces each had significantly higher averages than the 'same' pace (i.e. all those who chose their recommended pace.). There were no significant differences on Test 3.

With these results in mind it is now possible to formulate answers to the questions posed at the beginning of the chapter.

**Interpretation of the Conclusions**

Question 1 asked whether students who placed Level III and took Math 101 had higher achievement scores in Math 150 than those who placed Level III and entered Math 159 directly. The answer is that the students who took the Math 101-150 sequence had no significantly higher averages than those in Math 159 on any of the three achievement tests. The CRIMEL program allowed 42 of the 93 Level III students assigned to Math 159 to finish the course by the end of their first
quarter in school with no sacrifice in average achievement level. This option was not open under the traditional program. Those who finished Math 159 in the reduced pace section also did as well as the Math 101-150 students in achievement. However, both the latter groups needed two quarters to complete the course, and, although the Math 159 reduced pace students received 5 hours credit over these two quarters those in the Math 101-150 sequence received 10 hours credit for the two courses.

Question 2 asked how the achievement of students who were recommended for and finished at the accelerated pace compared with that of others recommended for the accelerated pace. Since there were only 7 students recommended for accelerated pace within the Math 150 study population the only valid accelerated pace for comparison was the group of students who finished at the regular pace and had been recommended for accelerated pace. Among these students recommended for the accelerated pace those who chose the accelerated pace had significantly higher scores on Test 1 than those who chose regular pace and, although this was also true for both Test 2 and Test 3 as well, the level of significance fell from Test 1 to Test 3. The accelerated students did not maintain their original edge in the course. A study of these two groups of students and students who placed Level I and began in Math 151 Autumn Quarter, 1970 was done by Lawrence Coon (4). He compared their achievement in Math 254 (the fourth quarter of calculus) and the rate at which they dropped out of the calculus sequence. He concluded that the number of students
dropping out of the calculus sequence was lower for the accelerated group than for the other two groups and that, of those surviving through Math 254, their grade average was approximately 1/2 grade below the average of the other groups (2.54 for the accelerated group vs. 2.95 for the other groups out of a possible 4.00). The results were mixed on the achievement test given as part of their final exam, however the accelerated students again had a lower average score although it was not significantly lower than that of each of the other groups. Coon's study seems to confirm the results of this study that the accelerated students lose some competitive edge over students who choose the regular pace, although the loss does not appear to be large enough to be alarming.

Question 3 asked how the achievement of students who were recommended for and finished at the reduced pace compared to that of others recommended for reduced pace (in Math 159 or 150). Tables showed that there were no significant differences in achievement between students who were recommended for reduced pace in Math 159 and those who would have been recommended for reduced pace in Math 150. There was a big difference in the rate at which students dropped out of the two courses, however. Of the 132 students who dropped Math 150 76 had scores recorded for Test 1. Of these, 62 would have been recommended for reduced pace; 14 would have been recommended for the regular pace. Of the 97 students who dropped out of Math 159 51 had scores recorded for Test 1. Of these, 32 were recommended for reduced pace, 19 were recommended for regular pace. A large number of the students who dropped Math 150 who would have been
recommended for reduced pace may have been in effect electing the only reduced pace available to them - postponing the course to a later date. Those students who were recommended for the reduced pace and elected to finish Math 150 would then be comparable to the students in Math 159 who were recommended for reduced pace and chose regular pace.

The other comparison between groups of students recommended for reduced pace was between those within CRIME! who chose reduced pace and those who chose regular pace. Hypotheses $H_0^7$ indicates the results of this comparison. The students who chose reduced pace had significantly lower scores on Test 1 than those who chose regular pace. However, on Test 2 although the students in the reduced pace still had a lower average than those in the regular pace the difference was not significant. On Test 3 the scores of the reduced pace group were again significantly lower than those in the regular pace, but the difference was not as pronounced as on Test 1. It should be recalled that the treatment of the reduced pace students between Test 1 and Test 2 was not the same as the treatment they received between Test 2 and Test 3. Between Test 1 and Test 2 the students received additional instruction on Test 1 material and then received instruction on Test 2 material for the remainder of the quarter. They received more in-class instruction per topic than the students in the regular pace classes. Between Test 2 and Test 3 they received almost the same in-class time per topic but had more study time on their own between days of classroom instruction. The extra time for instruction which was provided in Math 159.01 reduced pace
seemed to be more effective than the extra study time available (without extra instruction) in Math 159.02 reduced pace. The opportunity to review the Test 1 material and retest was extremely valuable in allowing students to improve their grades enough to remain in the course. The grades (Test 1 + Test 2) of the regular pace students were significantly lower than the grades of the reduced pace students (Test 1 Retest + Test 2) in Math 159.01, but the grades of the regular students were higher in Math 159.02 (Test 3). This indicates that with additional instruction students with low initial scores in algebra, who might traditionally drop out of this course, can achieve at a reasonable level in college algebra without returning to an intermediate algebra course to review.

Question 4 asked how students who were recommended for the regular pace but chose another pace compared with those who were so recommended and remained in the regular pace. Comparing those who chose accelerated pace instead of the recommended regular pace it can be seen from Table 15 that they had significantly higher scores on all three tests than their fellows who chose the regular pace. However, the level of significance of the difference dropped as the students progressed through the quarter. The students who chose to reduce their pace had significantly lower scores on Test 1 than their fellow students who chose to remain in the regular pace. But, on Test 2 these reduced pace students had significantly higher scores than those who chose to remain at the regular pace. On Test 3 the reduced pace students again scored significantly lower than the
regular pace students, but the difference between Test 3 averages was not as great as the original difference in Test 1 averages. Again the difference between the two quarters of the reduced pace course with respect to instructional strategy indicates that the additional in-class instruction has more impact than the additional time provided between class meetings without additional classroom time.

Question 5' asked how students who chose paces faster or slower than the pace recommended to them compared to students who chose the recommended pace. Students who were recommended for regular pace but chose accelerated pace and those who were recommended for reduced pace but chose regular pace formed the group of students in the study population that chose a 'faster' pace. Students who were recommended for accelerated pace but chose regular pace and those who were recommended for regular pace but chose reduced pace formed the group of students within the study population who chose a 'slower' pace. The students who chose the pace recommended to them were of course in the 'same' pace. Table 16 shows the results of comparing these groups. On Test 1 the 'faster' group had a lower average than the 'same' group which in turn had a lower average than the 'slower' group. But, on Test 2 both the 'faster' and 'slower' groups had higher averages than the 'same' group, and on Test 3 there were no significant differences between the three averages. The students who chose to go at a faster pace than recommended did not as a group show lower achievement than the students who went at the recommended pace,
they seemed to be reasonable at assessing their own abilities in making this decision. The scores of those who chose to go at a slower pace than recommended seemed also to reflect reasonable choices since their scores were not so high that they appeared to have chosen their pace just to 'make a good grade' when they could have made adequate progress at a faster pace.

Implications and Recommendations for Action

At the time of the study certain changes in the mathematics placement procedure were under consideration. The trend of these changes was the raising of requirements for entering the college algebra courses i.e. allowing only Level II students to take Math 115.01, 120.01 and 159.01, and allowing Level III students to take these courses only after successfully completing Math 101. At the same time more students would be placed in Level III; those students taking the 'B' placement test would be placed as before, but those students taking the 'D' placement test who received scores of five or below would be placed in Level III instead of Level II as in the past. In the 1971-72 school year Math 115.01 and 115.02 were removed from the CRIMEL project. Since this was the only course in the project that had admitted Level III students directly into the course, it was decided to make Level II or completion of Math 101 prerequisite to the CRIMEL project. Since the data from this study had not been analysed the decision was made to go ahead with the strengthening
of the requirements to place into Level II as well. Thus some students who had been considered as Level II during this study would have been designated Level III had they entered during the 1971-72 school year.

On the basis of this study it seems feasible to allow students in Level III (as first defined) to choose either Math 159.01 or 101 as their first course at Ohio State. Those entering Math 101 would probably have Math 101, 159.01 and 159.02 (ten hours credit) by the end of their second quarter, while those entering Math 159.01 would probably have both Math 159.01 and 159.02 at the end of the second quarter but might have Math 151 (Calculus I) by then as well. A student's choice would depend on his own assessment of his abilities, and the pressures of his chosen program. For example, a biology major on a scholarship which required 15 hours per quarter minimum might be better off taking the Math 101 - 159 sequence because the reduced pace would be inconvenient under the circumstances, but an engineering student whose whole program is thrown into disorder unless he can take Math 151 his second quarter might choose to enter Math 159 directly since his only chance of following a full engineering curriculum his first year is to finish Math 159.01 and 159.02 the first quarter. This recommendation is made with the understanding that this kind of choice can be allowed only if the students involved are given adequate counseling during orientation and scheduling for their first quarter.
Since the 1970-71 school year was the last in which students with five points or less on the 'D' placement test were allowed to schedule 159.01 their first quarter, students in the study population which fell into this category, 4 finished at the accelerated pace, 132 finished at the regular pace and 64 finished at the reduced pace. While this indicates that a sizable percentage of these students were not prepared to complete a traditionally paced course in college algebra and trigonometry, the majority of these students did complete the course within one quarter and probably would not have gained enough from taking Math 101 to justify the extra time and money which would have been spent. However, the performance of these students was different from that of students who scored six to thirteen points on the 'D' placement test, those who would still have been Level II under the new placement rules. Of the 811 students in this category, 121 finished at the accelerated pace, 609 finished at the regular pace, and 81 finished at the reduced pace. From the above distributions it appears to be true that the department is justified in placing the former group in Level III and the latter group in Level II if the Level III students are given the option of trying Math 159 in the first quarter.

Thus it is the recommendation of this author that the placement level assignments continue to be made according to the 1971-72 rules, but that those students who place Level III and intend to enroll in Math 159 be given the option of either enrolling directly in Math 159...
or enrolling in Math 101 first. Again, this change would put a greater burden on the counseling staff during registration, but it could add the flexibility to allow for student attributes such as motivation which, although hard to measure, can be the deciding factor in determining achievement.

It is also the recommendation of the author that accurate information relating precourse data to in-course performance be made available to students contemplating enrollment in Math 159. Student advisors must be kept alert to the options available to their students and the implications of those options. In addition, the mathematics instructors in the CRIMEL program should be alert to their responsibilities in advising their students within the program. Information relating in-course measures to final achievement in Math 159.01 and 159.02 should be available for this purpose.

Some recommendations which follow from the results of the study had already been recognized and implemented before the data analysis was completed. They are listed below.

1. A more efficient administrative system should be developed to handle grades and pace changes and utilize the teaching staff to the fullest. This has been done over the last years by the invention of the 'cluster leader', an instructor who supervises all the sections of a course given at one hour. This instructor distributes students and instructors into sections at the three paces, supervises student movement between paces in that course and keeps central records of all the students' grades. This addition to the CRIMEL system has
provided continuity to the students' progress through the course while allowing freedom of movement between paces when necessary.

2. There should be more in-class instruction days for the Math 159.02 reduced pace group than were provided during the initial year. The reduced pace Math 159.02 is now scheduled for three days per week for the ten week quarter. Thus, these students have 30 days of instruction over material traditionally taught in 20 days. This compares well with the ratio of instructional days in reduced pace to instructional days in the traditional course that was used in the more successful Math 159.01 reduced pace sections during this study.

**Implications for Further Study**

In any developing system such as CRIMEL constant evaluation is necessary for successful growth. Only a few very specific recommendations for further study are listed here.

1. Follow-up studies of the type done by Coon (4) for the accelerated pace students should be done for the reduced pace students to see what impact the program has on courses taken after completing the CRIMEL courses.

2. An evaluation should be done of the Math 120.01 and 120.02 courses and data relating pre-course and in-course measures to final achievement in the course should be made available for purposes of advising students.
3. Further consideration should be given to the placement process for students in Levels II and III since this was a first attempt at evaluating these levels with respect to success in a first course in college algebra.
APPENDIX A

MATHEMATICS PLACEMENT AT THE OHIO STATE UNIVERSITY

INTRODUCTION

Each student entering The Ohio State University is assigned to one of five Mathematics Placement Levels on the basis of his performance on a Mathematics Placement test, his ACT Mathematics score, and his high school mathematics background. This determines the point at which a sequence can be entered and whether remedial work is necessary before it can be entered.

The necessary data are obtained by the University Testing Office during freshman orientation. Students who have an ACT Mathematics score of twenty-five or above (the sixty-sixth percentile) are given the D form of the Mathematics Placement test (which has twenty-five questions); those whose score is twenty-four or below are given the B form (which has thirty questions).\(^1\) A "multiple score" is also needed for those who take the D form. This is equal to twice the D form score plus the total quality points; where quality points are awarded on the basis of four points for each semester of A, three points for each semester of B, etc., for all high school mathematics courses beginning with the ninth grade (Algebra of General Mathematics).

\(^1\)Sample questions from the B and D forms of the Mathematics Placement test cannot be given, as they are kept confidential. Interested persons should contact the Mathematics Department.
The exact procedure used in assigning students to these various Placement Levels is described in the next section, which consists of a description of the materials sent by the Mathematics Department to the University Counselors in 1970.

**Mathematics Counseling Office Materials**

If the student's ACT Mathematics score is 24 or below, the student is given the Mathematics Placement test form B, and is placed as follows:

- **Level II** if B score is 30 or above.
- **Level III** if B score is 19 to 20 inclusive.
- **Level IV** if B score is 16, 17, or 18.

*OR*

- ACT Composite Standard score is 20 or above.

- **Level V** if B score is 15 or below

*AND*

- ACT Composite Standard score is 19 or below.

If the student's ACT Mathematics score is 25 or above, the student is given the Mathematics Placement test form D, and is placed as follows:

- **Level I** if D score is 12 or above

*AND*

- Multiple score* is 55 or above.
**Level II** If D score is 11 or below.

**OR**

D score is 12 or above and Multiple score is 54 or below.

*Multiple score is computed by the following:

\[ M.S. = 2 \times D \text{ score} + \text{Total quality points} \]

i.e. 4 points for each semester of A,

3 points for each semester of B, etc.

using every semester of high school

math beginning with ninth grade

(Algebra of General Math).
On the answer sheet provided, choose the one best response to each of the following questions by marking with PENCIL ONLY in the appropriate space after the question number. None means "None of the preceding".

1. \([-3, 6] \cap [6, 8]\) \cup [2, 6] =
   (a) \([-3, 6]\),  (b) \([2, 6]\),  (c) \([2, 8]\),  (d) \([-3, 8]\),
   (e) None.

2. Which axiom is used to factor \(x^2y + x^2z = x^2(y + z)\)?
   (a) Associative law,  (b) Commutative law,
   (c) Inverse law,  (d) Distributive law,  (e) None.

3. The sum of the solutions to \(|x - 5| = 8\) is:
   (a) 10,  (b) 16,  (c) 5,  (d) -10,  (e) None.

In Questions 4, 5, and 6, \(N\) is the set of natural numbers, \(Z\) is the set of integers, \(A = \{x : x \in N \text{ and } x < 4\}\) and \(B = \{x : x \in N \text{ and } x \leq 4\}\).

4. The set \(B\) is equal to:
   (a) \([0, 4]\),  (b) \([0, 4]\),  (c) \([0, 1, 2, 3, 4]\),
   (d) \([1, 2, 3]\),  (e) None.

5. Which of the following is true?
   (a) \(B \subseteq A\),  (b) \(A = B\),  (c) \(A \subseteq B\),  (d) \(A \cap B = \emptyset\),
   (e) None.
6. \( A \cup (Z \cap [-1, 3]) = \)
   (a) \{1, 2, 3\}, (b) \{-1, 0, 1, 2, 3\}, (c) \{0, 3\},
   (d) \{-1, 3\}; (e) None.

7. \( \frac{a^2 \cdot a^{3/2}}{a^5} = \)
   (a) \( a^{-2} \), (b) \( a^{3/2} \), (c) \( a^{-3/2} \), (d) \( a^{-15} \),
   (e) None.

8. \( \sqrt[3]{-4} = \)
   (a) \(-2\), (b) \(4^{1/3}\), (c) 2, (d) Undefined,
   (e) None.

9. \( 1 - \sqrt{x} + \frac{\sqrt{x}}{1 + \frac{1}{\sqrt{x}}} = \)
   (a) \(-\sqrt{x}\), (b) \(\frac{1}{1 + \sqrt{x}}\), (c) \(1 + \sqrt{x}\), (d) \(1 - \sqrt{x}\),
   (e) None.

10. The solution set for \( 3 - 2(x - 1) \leq 4x - 1 \) is:
    (a) \( \{x : x \geq 1\} \), (b) \( \{x : x \leq 1\} \), (c) \( \{x : x \leq 1/2\} \),
     (d) \( \{x : x \geq 1/2\} \); (e) None.

11. \( \{x : |x^2 - x| = x^2 - x\} \) is equal to:
    (a) \([\infty, \infty]\), (b) \([\infty, 0]\), (c) \([1, \infty]\), (d) \([0, 1]\),
     (e) \([\infty, 0] \cup [1, \infty] \).

12. \( (x^{1/m^2} \frac{m}{x^{1/n^2} \frac{n}{m+n}} \frac{m}{x^{-1/n}} \) equals:
    (a) \( x^{1/m} \), (b) \( x^{1/n} \), (c) \( x^{m-n} \), (d) \( x^{m+n} \),
     (e) None.
13. The solution set for $|3x - 6| \leq 15$ is:
   (a) $[0, 15]$, (b) $[-5, 5]$, (c) $[-3, 3]$, (d) $[-3, 7]$, (e) None.

14. The solution set for $x^2 + 1 > 0$ is:
   (a) $[-1, 1]$, (b) $(-\infty, \infty)$, (c) $(-1, 1)$, (d) $\emptyset$, (e) None.

15. \text{\{x : } \sqrt{1 - \frac{x^2}{x^2}} \text{ is a real number\}} \text{ is:}
   (a) \{x : x \neq 0\}, (b) \{x : -1 \leq x \leq 1\},
   (c) \{x : -1 \leq x \leq 1 \text{ and } x \neq 0\}, (d) \{x : x \leq 1\},
   (e) None.

16. The solution set for $(x - 1)(x + 3) \leq 0$ is:
   (a) $[-1, 3]$, (b) $[-3, 1]$, (c) $(-\infty, -1) \cup (3, \infty)$,
   (d) $(-\infty, -3) \cup (1, \infty)$, (e) None.

17. The solution set for $x^2 \leq 6x$ is:
   (a) $[-\infty, 6]$, (b) $[0, \infty]$, (c) $[0, 6]$, (d) $[6, \infty]$, (e) None.

18. The solution set for $\frac{x}{|x+3|} \leq 0$ is:
   (a) $[-\infty, 0]$, (b) $(-\infty, \infty)$, (c) $(-\infty, -3)$,
   (d) $(-\infty, -3) \cup (-3, 0]$, (e) None.

19. The solution set for $|x| + |x - 3| < 5$ is:
   (a) $(-1, 4)$, (b) $[0, 4]$, (c) $(-1, 1)$, (d) $(-4, 4)$,
   (e) None.

20. The interval $(-2, 10) = \{x : |x - a| < p\}$ where:
   (a) $a = -2, p = 12$; (b) $a = 4, p = 10$;
   (c) $a = 5, p = 6$; (d) $a = 4, p = 6$; (e) None.
The items on Form B are the same as the items on Form A except for the order of appearance. The correspondence of items between the two forms is presented in the following table.

**CORRESPONDENCE OF ITEMS BETWEEN FORMS A AND B OF TEST GROUP TEST ONE**

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On the answer sheet provided, choose the one best response to each of the following questions by marking with PENCIL ONLY in the appropriate space after the question number. None means "None of the preceding".

1. If \( f(x) = 3 - x^2 \), then \( f(-2) = \)
   (a) 7  (b) 1  (c) -1  (d) 12  (e) None

2. If \( f(t) = 1 - t \) and \( g(x) = x^2 - 1 \), then \( g(f(2)) = \)
   (a) -3  (b) -2  (c) 0  (d) 2  (e) 3

3. A composition of mappings under which the line \( y = -2 + 3x \) is the image of the line \( y = x \) is:
   (a) a \( Y \) -distortion by 2 followed by a \( Y \) -translation by -3
   (b) a \( Y \) -distortion by -2 followed by a \( Y \) -translation by 3
   (c) a \( Y \) -distortion by 3 followed by a \( Y \) -translation by -2
   (d) a \( Y \) -distortion by -3 followed by a \( Y \) -translation by 2
   (e) None

4. The image of the point \((-6,3)\) under a \( Y \) -distortion by 3 followed by an \( X \) -reflection is:
   (a) \((6,9)\)  (b) \((-6,9)\)  (c) \((6,-9)\)  (d) \((-6,-9)\)
   (e) None

5. Which of the following subsets of \( R^2 \) ((a), (b), (c) or (d)) is symmetric about the \( X \) -axis
   (a) \( \{(x,y) : y = |x|\} \)  (b) \( \{(x,y) : y = x^2\} \)
   (c) \( \{(x,y) : y = x^3\} \)  (d) \( \{(x,y) : x = y^2\} \)
   (e) None
6. Which of the following ((a), (b), (c) or (d)) is not a function:
   (a) \((x, y): y = |x|\)    (b) \((x, y): y - 3 = \frac{x + 1}{2}\)
   (c) a \(Y\)-translation by 3    (d) \((x, y): x^2 + y^2 = 1 \text{ and } x \geq 0\)
   (e) All of the above are functions.

7. Which of the following ((a), (b), (c) or (d)) is not a one-to-one function:
   (a) \(f(x) = \sqrt{x}\)    (b) \(f(x) = \frac{1}{x}\)    (c) \(f(x) = \frac{3x - 1}{2}\)
   (d) \(f(x) = 2^x\)    (e) a \(Y\)-translation by 3.

8. Which of the following graphs best represent \((x, y): y = (x - 3)^2 - 4\)?

9. Which of the following graphs best represent \((x, y): |y - 1| = x\):
10. The zeros of \( f(x) = 6x^2 - x - 15 \) are:
   (a) -\(2/3\), 3/5  (b) 3/2, -5/3  (c) 2/3, -3/5  
   (d) -3/2, 5/3  (e) None

11. The equation of the line parallel to the line \( x - 3 = y + 2 \) containing the point \((2, -3)\) is:
   (a) \(y - x + 5 = 0\)  (b) \(y + x + 1 = 0\)  (c) \(2x - y = 7\)  
   (d) \(x - 2 = y - 3\)  (e) None

12. If \( q(x) = \frac{x^2 - 1}{3} \), then the domain of \( g \) is:
   (a) \(\emptyset\)  (b) \(\{x \in \mathbb{R}: x \neq 1 \text{ or } x \neq -1\}\)  (c) \((-\infty, 3) \cup (3, \infty)\)  
   (d) \([1, \infty]\)  (e) None

13. If \( h(u) = u^3 \), then the range of \( h \) is:
   (a) \((-\infty, \infty)\)  (b) \(\emptyset\)  (c) \([0, \infty]\)  (d) \([0, \infty]\)  (e) None

14. If \( P = (x, 3) \), \( Q = (-1, 2) \), and \( \frac{\overrightarrow{PQ}}{2} = \sqrt{5} \), then the sum of all such \( x \) is:
   (a) 3  (b) -2  (c) 1  (d) -1  (e) None

15. If \( f(x) = \sqrt{x - 3} \), then
   (a) \(f^{-1}(x) = (x - 3)^2\)  (b) \(f^{-1}(x) = \frac{1}{x + 3}\)  
   (c) \(f^{-1}(x) = (x + 3)^2\)  (d) \(f^{-1}(x) = 3 + x^2\)  (e) \(f\) has no inverse.

16. If \( f \) is the inverse function of \( g \), then:
   (a) \(f(x)g(x) = x\)  (b) \(f(g(x)) = 1\)  (c) \(f(g(x)) = g(f(x))\)  
   (d) \(f(x) + g(x) = x\)  (e) None

17. If \( h(t) = \frac{\sqrt{1 - t}}{1 + t} \), then the domain of \( h \) is:
   (a) \([-\infty, 1]\)  (b) \((1, \infty) \cup (-1,1)\)  (c) \((-\infty, -1) \cup (-1,1)\)  
   (d) \(\{x \in \mathbb{R}: x \neq -1\}\)  (e) None

18. If \( \log_{25} \frac{1}{5} = x \), then \( x = \)
   (a) 2  (b) 1/2  (c) -2  (d) \(\sqrt{5}\)  (e) None
19. If \( \exp 3^x = \exp 9^{x-4} \), then \( x = \)

(a) 2 or -2  (b) 1/2  (c) 8  (d) 4  (e) None

20. The sum of the solutions for \( \log_2(x+2) + \log_2(x-1) = 2 \) is:

(a) -1  (b) -3  (c) 2  (d) 1  (e) None
The items on Form B are the same as the items on Form A except for the order of appearance. The correspondence of items between the two forms is presented in the following table.

### CORRESPONDENCE OF ITEMS BETWEEN FORMS A AND B OF THE OPTIONAL TEST GROUP TEST TWO

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On the answer sheet provided, choose the one best response to each of the following questions by marking with PENCIL ONLY in the appropriate space after the question number. **None** means "None of the preceding".

1. If \( g(s) = \sqrt{2 - s} \), then \( g(-3) = \)
   (a) 1,  (b) \( \sqrt{5} \),  (c) undefined,  (d) 2,  
   (e) none.

2. If \( g(x) = x + 3 \) and \( f(t) = 1 - t^2 \), then \( f(g(-1)) = \)
   (a) -3,  (b) 2,  (c) 3,  (d) -2,  (e) none.

3. A composition of mappings under which the line \( y = 2x - 3 \) is the image of the line \( y = x \) is:
   (a) A \( Y \)-distortion by 2 followed by a \( Y \)-translation by -3,  
   (b) A \( Y \)-distortion by -2 followed by a \( Y \)-translation by 3,  
   (c) A \( Y \)-distortion by 3 followed by a \( Y \)-translation by -2,  
   (d) A \( Y \)-distortion by -3 followed by a \( Y \)-translation by 2,  
   (e) none.

4. Which of the following subsets of \( \mathbb{R}^2 \) (a), (b), (c), or (d) is symmetric about the \( Y \)-axis?
   (a) \( \{(x,y): y = \frac{1}{x}\} \),  (b) \( \{(x,y): x = y^2\} \),  
   (c) \( \{(x,y): y = \sqrt{x}\} \),  (d) \( \{(x,y): x^2 + y^2 = 1\} \),  
   (e) none.
5. Which of the following ((a), (b), (c), or (d)) is not a function?
   (a) \((x,y): y = \sqrt{x}\),  (b) \((x,y): y = 2^x\),  
   (c) A Y-distortion by 2,  (d) \((x,y): |y| = x\),  
   (e) All of the above are functions.

6. Which of the following ((a), (b), (c), or (d)) is not a one-to-one function?
   (a) \(f(x) = |2^x|\),  (b) \(g(x) = |\log_2 x|\),  
   (c) \(h(x) = \frac{\sqrt{x-1}}{2}\),  (d) \(F(x) = x^3\),  
   (e) none.

7. Which of the following graphs best represent \((x,y): y = 4 - (x + 3)^2\)?

   (a)   
   (b)   
   (c)   
   (d)   
   (e)   

8. Which of the following graphs best represent \((x,y): x - 2y \geq 2\)?
9. Which of the following graphs best represent \((x, y): y - 1 = |x|\):

(a) (b) (c) (d) (e)

10. The zeros of \(f(x) = 6x^2 + x - 15\) are:

(a) \(-2/3, 3/5\);  (b) \(3/2, -5/3\)  (c) \(2/3, -3/5\);
11. The equation of the line containing the points \((-1, 0)\) and \((3, -2)\) is:
(a) \(2y + x + 1 = 0\),  \(\text{(b)} \ x - y = 5\),  \(\text{(c)} \ x + 2y - 1 = 0\),
(d) \(x + y = 1\),  \(\text{(e)} \) none.

12. If \(f(x) = \sqrt{x - 2}\), then the understood domain of \(f\) is:
(a) \([-\infty, 0]\),  \(\text{(b)} [0, \infty]\),  \(\text{(c)} [2, \infty]\),  \(\text{(d)} [-\infty, 2]\),
\(\text{(e)} \) none.

13. If \(g(x) = \log_3(x^2 - 1)\), then the understood domain of \(g\) is:
(a) \((-\infty, -1) \cup (1, \infty)\),  \(\text{(b)} (0, \infty)\),  \(\text{(c)} [0, \infty]\),
(d) \((-\infty, \infty)\),  \(\text{(e)} \) none.

14. If \(f(x) = 3^x\), the range of \(f\) is:
(a) \([3, \infty]\),  \(\text{(b)} (-\infty, \infty)\),  \(\text{(c)} [0, 3]\),  \(\text{(d)} (0, \infty)\),
\(\text{(e)} \) none.

15. Solve: \(x^2 - x + 1 > 0\).
(a) \((1/2 - \sqrt{5}/2, 1/2 + \sqrt{5}/2)\);  \(\text{(b)} (0, 1)\),
(c) \((-\infty, 1) \cup (1, \infty)\);  \(\text{(d)} (-\infty, \infty)\);  \(\text{(e)} \) none.

16. If \(P = (-1, 2)\) and \(Q = (3, -5)\), then the distance \(PQ\) is:
(a) \(\sqrt{50}\),  \(\text{(b)} \sqrt{35}\),  \(\text{(c)} \sqrt{13}\),  \(\text{(d)} \sqrt{65}\),  \(\text{(e)} \) none.

17. If \(f = ((x, (x - 1)^2): x \geq 1)\), then
(a) \(f^{-1}(x) = \sqrt{x - 1}\),  \(\text{(b)} f^{-1}(x) = \sqrt{x} - 1\),
(c) \(f^{-1}(x) = 1 + \sqrt{x}\),  \(\text{(d)} f^{-1}(x) = \sqrt{x + 1}\),
\(\text{(e)} f^{-1} \) does not exist.
18. If \( \log_x 25 = \frac{2}{3} \), then \( x = \) 
(a) 5, (b) 125, (c) -5, (d) 1/5, (e) none.

19. The solution set for \( \log_2(x + 4) - \log_2(x - 1) = 1 \) is:
(a) \{2,4\}, (b) \{6\}, (c) \{4,8\}, (d) \{3\}, (e) \emptyset

20. If \( \exp 2^x = \exp 4^x \), then \( x = \)
(a) 0 or 2, (b) 1, (c) -1 or 1, (d) -2 or 2, (e) none.
On the answer sheet provided, choose the one best response to each of the following questions by marking with PENCIL ONLY in the appropriate space after the question number. None means "none of the preceding".

1. If \( f(x) = |2^{-x}| \), then \( f(-1) = \)
   
   (a) 1/2  (b) 2  (c) -1/2  (d) -2  (e) None

2. If \( g(t) = \sqrt{t+3} \) and \( f(x) = 1 + x^2 \), then \( f(g(-1)) = \)
   
   (a) -3  (b) 2  (c) 3  (d) -2  (e) None

3. A composition of mappings under which the curve \( y = -\frac{3}{2} + \frac{1}{2}x^2 \) is the image of \( y = x^2 \) is:
   
   (a) a Y-distortion by 3/2 followed by a Y-translation by -1/2
   (b) a Y-distortion by 1/2 followed by a Y-translation by -3/2
   (c) a Y-distortion by 1/2 followed by a Y-translation by 2/3
   (d) a Y-distortion by 1/2 followed by a Y-translation by 3/2
   (e) None

4. Which of the following subsets of \( \mathbb{R}^2 \) ((a), (b), (c) or (d)) is symmetric about the Y-axis?
   
   (a) \( \{(x,y): \ y = |1/x|\} \)  (b) \( \{(x,y): \ x = y^2\} \)
   (c) \( \{(x,y): \ y = \sqrt{x}\} \)  (d) \( \{(x,y): \ |y| = x\} \)  (e) None

5. Which of the following ((a), (b), (c) or (d)) is not a function?
   
   (a) \( \{(x,y): \ y = |x^2|\} \)  (b) \( \{(x,y): \ y = |2^{-x}|\} \)
   (c) A Y-distortion by 3 followed by a Y-translation by -1.
   (d) \( \{(x,y): \ |y| = |x|\} \)  (e) All of the preceding are functions.
6. Which of the following ((a), (b), (c) or (d)) is not a one-to-one function?
   (a) \( f(x) = |2^x| \)  
   (b) \( g(x) = \log_2 x \)  
   (c) \( h(x) = (x+1)^3 \)  
   (d) \( P(x) = \frac{2}{x} \)  
   (e) All of the preceding are one-to-one.

7. Which of the following best represent \((x,y): y = 2 - 3^x\)

8. Which of the following best represent \((x,y): 2x - y \leq 2\)
9. Which of the following graphs best represent \( \{(x, y): |y| + 1 = x\} \)

(a) ![Graph A]  
(b) ![Graph B]  
(c) ![Graph C]  
(d) ![Graph D]  
(e) ![Graph E]

10. The sum of the zeros of \( f(x) = x^2 + 4x + 51 \) is:
   (a) \(-14\)  
   (b) \(14\)  
   (c) \(19\)  
   (d) \(-19\)  
   (e) None

11. The equation of the line containing the points \((-1, 0)\) and \((-1, 3)\) is:
   (a) \(x + 1 = 0\)  
   (b) \(y + 1 = 0\)  
   (c) \(y = 3\)  
   (d) \(x + y = 1\)  
   (e) None

12. If \( f(x) = \frac{\sqrt{x}}{x-1} \), then the understood domain of \( f \) is:
   (a) \([0, \infty)\)  
   (b) \((-\infty, -1) \cup [0, \infty)\)  
   (c) \([0, 1) \cup (1, \infty)\)  
   (d) \((-\infty, 1) \cup (1, \infty)\)  
   (e) None

13. If \( f(x) = \log_3(x+1) \), then the range of \( f \) is:
   (a) \([3, \infty]\)  
   (b) \((-\infty, \infty)\)  
   (c) \([0, 3]\)  
   (d) \((0, \infty)\)  
   (e) None

14. For \( f \) and \( g \) linear functions, which of the following \((a), (b), (c) \text{ or } (d)\) is not a linear function?
   (a) \( h(x) = f(x) + g(x) \)  
   (b) \( k(x) = f(x) - g(x) \)  
   (c) \( F(x) = f(g(x)) \)  
   (d) \( H(x) = 3f(x) - 2g(x) \)  
   (e) All of the preceding are linear functions.
15. Solve: \( x^2 + 2x - 5 < 0 \)
   (a) \((-1 - \sqrt{12}), -1 + \sqrt{12}\)  \( (b) \((-1 - \sqrt{5}, -1 + \sqrt{5}) \)
   (c) \((-\sqrt{5}, \sqrt{5}) \)  \( (d) (-\infty, \infty) \)  \( (e) \text{None} \)

16. If \( P = (1, 2) \) and \( Q = (-3, 4) \), then the distance \( PQ \) is:
   (a) \( \sqrt{13} \)  \( (b) 2\sqrt{5} \)  \( (c) 2\sqrt{5} \)  \( (d) \sqrt{10} \)  \( (e) \text{None} \)

17. If \( f = \{(x, (x+1)^2) : x \geq -1\} \), then
   (a) \( f^{-1}(x) = \sqrt{x-1} \)  \( (b) f^{-1}(x) = \sqrt{x} - 1 \)  \( (c) f^{-1}(x) = 1 + \sqrt{x} \)
   (d) \( f^{-1}(x) = \sqrt{x} + 1 \)  \( (e) \text{None} \)

18. If \( \log_{27} x = -1/3 \), then \( x = : \)
   (a) 3  \( (b) -3 \)  \( (c) -1/3 \)  \( (d) 1/3 \)  \( (e) \text{None} \)

19. The solution set for \( \log_{5}(x+12) = 2 + \log_{5} x \) is:
   (a) \( \{12\} \)  \( (b) \{1/2\} \)  \( (c) \{25\} \)  \( (d) \{2\} \)  \( (e) \text{None} \)

20. If \( (\exp_{3} x)^2 = \exp_{3} 5x \), then \( x = : \)
   (a) 1 or 5  \( (b) 0 \)  \( (c) 3 \)  \( (d) 15 \)  \( (e) \text{None} \)
The items on Form B are the same as the items on Form A except for the order of appearance. The correspondence of items between the two forms is presented in the following table.

**CORRESPONDENCE OF ITEMS BETWEEN FORMS A AND B OF TEST GROUP TEST TWO**

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On the answer sheet provided, choose the one best response to each of the following questions by marking with PENCIL ONLY in the appropriate space after the question number. None means "None of the preceding".

1. \([-1,4) \cup (0,5) =\]
   (a) \([-1,5]\), (b) \((0,4]\), (c) \((4,5]\), (d) \([-1,0]\),
   (e) none.

2. \((-3,8] \cap [8,10) \cup (8,9] =\)
   (a) \((-3,10]\), (b) \((8,9]\), (c) \([8,9]\), (d) \(\emptyset\)
   (e) none.

3. If \(A = (-2,2)\), \(B = \{x: x \in \mathbb{N} \text{ and } x < 1\}\), and \(C = \{x: x \in \mathbb{Z} \text{ and } x \leq 3\}\), then \((A \cap C) \cup B\) equals
   (a) \((-\infty,2]\), (b) \([0,1]\), (c) \([-1,0,1]\), (d) \([0,1]\),
   (e) none.

4. The reason that \(\left(\frac{1}{a} \cdot a\right) \cdot b = b \cdot \left(\frac{1}{a} \cdot a\right)\) is:
   (a) The commutative law, (b) The associative law,
   (c) The inverse law, (d) The distributive law,
   (e) none.

5. \((-\frac{27}{8})^{-2/3} =\)
   (a) \(9/4\), (b) \(-9/4\) (c) \(-4/9\) (d) \(4/9\),
   (e) none.
6. \( \left( \frac{a^2 - b^2}{a^2 + b^2} \right)^{-2} = \) (a) \( \frac{a}{b} \), (b) \( \left( \frac{b}{a} \right)^{10} \), (c) \( \frac{a^2}{b^2} \), (d) \( \frac{b^{12}}{a^{12}} \), (e) none.

7. \( \sqrt[3]{2\sqrt{16} - (1/2)^{-1} + \sqrt{12}} = \) (a) 0, (b) \( 2\sqrt{3} \), (c) \( \frac{3\sqrt{12}}{2} - 2 \), (d) \( 5\sqrt{2} \), (e) can not be determined.

Find the solution sets in questions 8 - 13.

8. \( 3(x - 4) \leq 2(4x + 9) \)
   (a) \( \{x: x \leq -6\} \), (b) \( \{x: x \geq -6\} \), (c) \( \{x: x \geq 6\} \), (d) \( \{x: x < 6\} \), (e) none.

9. \( (x + 2)(x - 8) > 0 \)
   (a) \((-2, 8)\), (b) \((-8, 2)\), (c) \((-\infty, -8) \cup (2, \infty)\), (d) \((-\infty, -2) \cup (8, \infty)\), (e) none.

10. \( |x - 4| < 2 \):
    (a) \((4, 6)\), (b) \((-\infty, 6)\), (c) \((4, \infty)\), (d) \((-6, 6)\), (e) none.

11. \( |x^2 + x - 4| \geq -2 \):
    (a) \([-\infty, -2] \cup [1, \infty]\) (b) \([1, \infty]\), (c) \(\mathbb{R}\), (d) \(\emptyset\), (e) none.

12. \( \frac{|x - 4|}{x + 1} < 0 \):
    (a) \((-\infty, -1)\), (b) \((-\infty, 4)\), (c) \((-1, 4)\), (d) \((4, \infty)\), (e) none.

13. \( \frac{4}{1 - x} > 2 \):
    (a) \((-\infty, -1)\), (b) \((-1, \infty)\), (c) \((0, 1)\), (d) \((-1, 1)\), (e) none.
14. The sum of the solutions to $|2x + 5| = 7$ is:
   (a) -1,  (b) 0,  (c) 1,  (d) 2,  (e) none.

15. Which of the following best represents $\{(x,y): x = 1 - 2y\}$?
   (a)  
   (b)  
   (c)  
   (d)  
   (e) none.

16. If $f(x) = (x + 2)(x^2 + 4x + 1)$, then the sum of the zeros of $f$ is:
   (a) 1,  (b) -2,  (c) -4,  (d) -6,  (e) none.

17. Which of the following ((a), (b), (c), or (d)} is not a function?
   (a) $\{(x,y): x = y^2 \text{ and } y \geq 0\}$,  (b) $\{(x,y): y = |x|^3\}$,
   (c) $\{(x,y): x = 2 \text{ and } y \in \mathbb{R}\}$,  (d) $\{(x,y): y = -3 \text{ and } x \in \mathbb{R}\}$,
   (e) All of the preceding are functions.

18. Which of the following best represents $\{(x,y): y = 1 + 2|x|\}$?
   (a)  
   (b)  
   (c)  
   (d)  
   (e)  

19. If \( g(x) = \frac{x^2 - 9}{x + 1} \), then \( g \) has how many real zeros?
(a) 0, (b) 1, (c) 2, (d) 3, (e) none.

20. Which of the following \((a), (b), (c), \) or \((d)\) is not a one-to-one function?
\[(a) \{(x,y): x + y = 4\}, \quad (b) \{(x,x^2): x \geq 0\}, \]
\[(c) \{(x,y): y = |x - 2|\}, \quad (d) \{(x,y): |y| = x \text{ and } y \leq 0\}, \]
\[(e) \text{ All of the preceding are one-to-one.} \]

21. If \( f(x) = \frac{x + 2}{2x - 1} \), then the understood domain of \( f \) is:
(a) \( \{x \neq -2, x \neq 1/2\} \), (b) \( \{x \neq 1/2\} \), (c) \( \{x \neq -2\} \),
(d) \( \mathbb{R} \), (e) none.

22. If \( g(x) = \sqrt{x^2 - 7x - 8} \), then the understood domain of \( g \) is:
(a) \( [0,\infty] \), (b) \([1,\infty]\), (c) \([8,\infty]\), (d) \( \mathbb{R} \), (e) none.

23. If \( h(x) = 1 - x^2 \), then \( h(\sqrt{2}) \) equals:
(a) 3, (b) -3, (c) 1, (d) -1, (e) none.

24. If \( f(x) = \sqrt{1 - 2x} \), then \( f(1 - 2x) \) equals:
(a) \( \sqrt{4x - 1} \), (b) \( 1 - 2x \), (c) \( \sqrt{1 - 2x} \), (d) \( 1 - 4x^2 \),
(e) none.
25. If \( g(x) = 1 + x \) and \( f(x) = x^2 \), then \( g(f(x)) \) equals:

(a) \( 1 + x^2 \),  (b) \( (1 + x)^2 \),  (c) \( x^2 + x^3 \),

(d) \( 1 + x \),  (e) none.
Reduced Pace Test I
November 12, 1970

The items on Form B are the same as the items on Form A except for the order of appearance. The correspondence of items between the two forms is presented in the following table.

**CORRESPONDENCE OF ITEMS BETWEEN FORMS A AND B OF REDUCED PACE TEST ONE**

<table>
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</table>
1. \([-1, 2) \cup (0, 3) = \) (a) \([-1, 0, 1, 2]\)  (b) \([-1, 0, 1, 2, 3]\)  (c) \([0, 2]\)  (d) \([-1, 3]\)  (e) None

2. Let \(\mathbb{N}\) denote the set of natural numbers, \(\mathbb{Z}\) the set of integers and \(\mathbb{Q}\) the set of rational numbers. Which of the following is not true?  (a) \(\mathbb{N} \subseteq \mathbb{Q}\)  (b) \(\mathbb{Q} \cup \mathbb{Z} = \mathbb{Q}\)  (c) \(\mathbb{Z} \cup \mathbb{N} \subseteq \mathbb{N}\)  (d) \(\mathbb{N} \subseteq \mathbb{Z}\)  (e) None

3. \((1+3\sqrt{2})^2 = \) (a) \(19+6\sqrt{2}\)  (b) \(37\)  (c) \(19\)  (d) \(13+3\sqrt{2}\)  (e) \(7+3\sqrt{2}\)

4. The sum of the integers in the set \((-8, 3) \cap [-2, 3]\) is:  (a) \(0\)  (b) \(1\)  (c) \(2\)  (d) \(3\)  (e) None

5. \((\sqrt{x} \cdot \frac{3}{\sqrt{x}})^{5/6} = \) (a) \(\frac{5}{\sqrt{x}}\)  (b) \(\sqrt{x}\)  (c) \(\sqrt{\frac{5}{x}}\)  (d) \(\sqrt{x}\)  (e) None

6. \(\left(\frac{4x-3y}{xy-2}\right)^{-1/2} = \) equals:  (a) \(\frac{y}{2x}\)  (b) \(\frac{x}{2y}\)  (c) \(\frac{x^2}{2y}\)  (d) \(\frac{x}{2y}\)  (e) None

Let \(P(-1, 1), Q(5, 9), R(1, 4)\) and \(S(x, 8)\) be points in \(\mathbb{R}^2\).

Questions 7 - 9 refer to these points.

7. \(\overline{PQ} = \) (a) \(10\)  (b) \(9\)  (c) \(8\)  (d) \(7\)  (e) None

8. The equation of line \(PQ\) is:  (a) \(4x-y = -5\)  (b) \(3x-4y = 1\)  (c) \(x-4y = -5\)  (d) \(3x+4y = 1\)  (e) None
9. If the line through P and Q is parallel to the line through R and S, then x equals:
   (a) 3  (b) $3\frac{1}{2}$  (c) 4  (d) $4\frac{1}{2}$  (e) None

10. The slope of the line $3x = 2y - 1$ is:
    (a) 3  (b) 2
    (c) $\frac{2}{3}$  (d) $\frac{3}{2}$  (e) None

11. The equation of the line through (1, -3) parallel to $5x = y - 1$ is:
    (a) $5x - y = 2$  (b) $y = 5x + 8$  (c) $5x + y = 2$
    (d) $10x - 2y = 16$  (e) None

12. Which of the following best represents $\{(x, y) : y \leq 1 - 4x\}$?
    (a) $\text{Diagram A}$  (b) $\text{Diagram B}$  (c) $\text{Diagram C}$
    (d) $\text{Diagram D}$  (e) None

13. The sum of the solutions to $|x + 10| = 12$ is:
    (a) 0  (b) 1  (c) 2  (d) 3  (e) None

14. The graph of $y = (x + 1)^2 + 2$ is:
    (a) $\text{Graph A}$  (b) $\text{Graph B}$  (c) $\text{Graph C}$
    (d) $\text{Graph D}$  (e) None
15. If \( f(x) = 8^x \), then \( f(-2/3) = \) (a) -16/3 (b) 4 (c) 1/4 (d) 4 (e) None

16. Which of the following sets is not a function:

(a) \( \{(x, y): y = |x+3|\} \)  (b) \( \{(x, y): |x-2y| \leq 0\} \)
(c) \( \{(x, y): y = x^2 - 1\} \)  (d) \( \{(x, y): y^2 = x - 1\} \)
(e) More than one of the above are not functions.

17. Which of the following best represents \( \{(x, y): y = |x-3|\} \):

(a) \( V \)  (b) \( X \)  (c) \( H \)
(d) \( A \)  (e) None

Let \( F \) represent a \( Y \)-distortion by 2, \( G \) an \( X \)-translation by -5. Questions 18 - 20 refer to these mappings.

18. \( F(-6,8) = \) (a) \((-6,16)\)  (b) \((-12,8)\)  (c) \((-6,4)\)
   (d) \((-12,16)\)  (e) None

19. The image of \( y = x^2 \) under \( G \) followed by \( F \) is:

(a) \( y = 2(x-5)^2 \)  (b) \( y = 2(x+5)^2 \)  (c) \( y = 2x^2 - 5 \)
   (d) \( y = 2x^2 + 5 \)  (e) None

20. \( G^{-1}(-10,15) = \) (a) \((-5,15)\)  (b) \((-1/10,15)\)  (c) \((10,-15)\)
   (d) \((-10,3)\)  (e) None

Find the solution sets in Problems 21 - 26.

21. \( 2(x-3) \geq 3x - 5 \)  (a) \([-1, \infty]\)  (b) \([-\infty, -1]\)  (c) \([1, \infty]\)
   (d) \([-\infty, 1]\)  (e) None
22. \((x+2)(x-7) \leq 0\). (a) \([7, \infty) \cup (-\infty, -2]\)  
(b) \([7, \infty) \cap (-\infty, -2]\)  
(c) \([-7, 2]\)  
(d) \([-2, 7]\)  
(e) None

23. \(|2x-1| \leq 9\). (a) \([-4, 5]\)  
(b) \([-5, 5]\)  
(c) \([5, \infty)\)  
(d) \([5, \infty)\)  
(e) None

24. \(x^2 + 3x \geq 10\). (a) \([2, \infty)\)  
(b) \([-5, 2]\)  
(c) \([\infty, -5]\)  
(d) \([2, \infty)\)  
(e) None

25. \(|x+3| > 5\). (a) \([2, \infty)\)  
(b) \([-\infty, -2]\)  
(c) \([-\infty, -3]\)  
(d) \([2, \infty)\)  
(e) None

26. \(\frac{x-1}{x+3} > 2\). (a) \((-7, -3]\)  
(b) \((-\infty, -7]\)  
(c) \((-7, \infty)\)  
(d) \((-\infty, -3]\)  
(e) None

27. Let \(f(x) = 1 + x^2\) and \(g(t) = \sqrt{1 - t}\), then \(f(g(-8)) = \)
(a) 7  
(b) 8  
(c) 9  
(d) 10  
(e) None

28. If \(f(x) = x^2 - 16\), then the sum of the zeros of \(f\) is:
(a) 0  
(b) 2  
(c) 4  
(d) 6  
(e) None

29. Which of the following best represents \((x, y): y = 3 + 2^x\)?

(a) 
(b) 
(c) 
(d) 
(e) None

30. If \(f(x) = (x+1)^{-1/2}\), then the understood domain of \(f\) is:
(a) \(\mathbb{R}\)  
(b) \((0, \infty)\)  
(c) \(\{x: x \neq -1\}\)  
(d) \((-1, \infty)\)  
(e) None
31. If \( g(x) = \frac{1}{4 - x} \), then \( g^{-1}(x) \) equals:
   (a) \( 4 - x \)  (b) \( 4 - \frac{1}{x} \)  (c) \( 4 + x \)  (d) \( \frac{4x}{1 - x} \)  (e) None

32. If \( h(x) = x^3 - x^2 - 3x \), then the sum of the zeros of \( h \) equals:
   (a) 0  (b) 1  (c) 2  (d) 3  (e) None

33. Which of the following best represents \( \{(x, y): |x| = y + 3\} \)?
   (a) ![Graph A]  (b) ![Graph B]  (c) ![Graph C]  (d) ![Graph D]  (e) None

34. The understood domain of the function \( g(x) = \sqrt{x^2 - 2x} \) is:
   (a) \([0, \infty) \cup [2, \infty)\)  (b) \([0, \infty]\)  (c) \([2, \infty]\)  (d) \(\mathbb{R}\)
   (e) None

35. If \( f(x) = \frac{x + 1}{x} \), then \( f(x - 1) \) equals:
   (a) \( \frac{x}{x - 1} \)  (b) \( \frac{1}{x} \)  (c) \( \frac{x - 1}{x} \)  (d) \( \frac{x}{x + 1} \)  (e) None

36. If \( h(x) = 3x^3 - 2x - 1 \), then \( h^{-1}(h(x^2)) \) equals:
   (a) \( 3x^6 - 2x^2 - 1 \)  (b) \( x \)  (c) \( x^2 \)  (d) \( 3x^3 - 2x - 1 \)  (e) None

37. The understood domain of the function \( f(x) = \sqrt{\frac{x - 2}{|x + 1|}} \) is:
   (a) \((-\infty, -1) \cup [2, \infty]\)  (b) \([x: x \neq -1]\)  (c) \([0, \infty]\)
   (d) \([2, \infty]\)  (e) None

38. The sum of the coordinates of the minimum point on the graph of \( y = x^2 - 10x \) is:
   (a) -25  (b) -20  (c) -15  (d) -10  (e) None
The items on Form B are the same as the items on Form A except for the order of appearance. The correspondence of items between the two forms is presented in the following table.

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On the answer sheet provided, choose the one best response to each of the following questions by marking with PENCIL ONLY in the appropriate space after the question number. None means "None of the preceding".

1. \([-2, 3) \cap (3, 6) \cup (3, 5] = (a) \emptyset \quad (b) (3, 5]
   (c) [3, 5] \quad (d) [-2, 6] \quad (e) None

2. If \( f(z) = 4 - 2^{-z} \), then \( f(-2) = (a) 0 \quad (b) 4 \quad (c) 7 \)
   (d) 8 \quad (e) None

3. The equation of the line containing \((-1, 2)\) and \((2, -1)\) is:
   (a) \(2x + y = 0\) \quad (b) \(x + 2y = 0\) \quad (c) \(y = x + 3\)
   (d) \(y + 3 = x\) \quad (e) None

4. The sum of the solutions to the equation \(\log_3 |x| = 2\) is:
   (a) 0 \quad (b) 8 \quad (c) 9 \quad (d) 3/2 \quad (e) None

5. If \( P = (3, -1) \) and \( Q = (1, 5) \), then the distance \(PQ\) is:
   (a) \(4\sqrt{2}\) \quad (b) \(2\sqrt{10}\) \quad (c) \(2\sqrt{5}\) \quad (d) \(2\sqrt{2}\) \quad (e) None

6. If \( h(x) = 1 - x - x^2 \), then the sum of the zeros of \( h \) is:
   (a) 1 \quad (b) -1 \quad (c) 2 \quad (d) -2 \quad (e) None

7. If \(\log_x 4 = \frac{1}{2}\), then \( x = (a) 2 \quad (b) 4 \quad (c) 8 \quad (d) 16 \)
   (e) None

8. The slope of the line \( x = \frac{5}{4} y + 3 \) is: \(a) -\frac{5}{4} \quad (b) \frac{5}{4} \quad (c) \frac{4}{5} \quad (d) -\frac{4}{5} \quad (e) 3 \)

9. If \( f(x) = x^2 + 1 \) and \( g(x) = \sqrt{1 - x} \), then \( f(g(-1)) = (a) 0 \quad (b) 1 \quad (c) 3 \quad (d) Undefined \quad (e) None \)
10. A composition of mappings under which \( y = 4(x+3)^2 \) is the image of \( y = x^2 \) is:
   
   (a) An \( X \)-translation by 3 followed by an \( X \)-distortion of \( k \)
   
   (b) An \( X \)-translation by -3 followed by an \( X \)-distortion by \( 4 \)
   
   (c) An \( X \)-translation by 3 followed by a \( Y \)-distortion by \( 4 \)
   
   (d) An \( X \)-translation by -3 followed by a \( Y \)-distortion by \( 4 \)

11. Which of the following subsets of \( \mathbb{R}^2 \) is not a function?

   (a) \( \{(x, y): x^2 + 1 = y\} \)  
   (b) \( \{(x, y): y = |x|\} \)
   
   (c) \( \{(x, y): y = 3 - 6x\} \)  
   (d) \( \{(x, y): y = 2^{-x}\} \)
   
   (e) All of the above are functions.

12. If \( f(x) = \frac{\sqrt{x+3}}{x+2} \), then the understood domain of \( f \) is:

   (a) \([-3, \infty)\)  
   (b) \([-\infty, -3] \cup (-2, \infty)\)  
   (c) \([-3, -2) \cup (-2, \infty)\)
   
   (d) \((-2, \infty)\)  
   (e) None

13. Which of the following best represents \((x, y): 2x = 1 - y\)?

   (a) 
   
   (b) 
   
   (c) 
   
   (d) 
   
   (e) None

14. \( (\sqrt{10} - \sqrt{2})^2 = \)  

   (a) 8  
   (b) 12  
   (c) 12 - \( \frac{4\sqrt{5}}{} \)  
   (d) 12 - \( \sqrt{20} \)

   (e) None

15. The sum of all values for \( x \) for which the expression \( \frac{x-1}{x^2-1} \) is undefined is:

   (a) \(-1\)  
   (b) 0  
   (c) 1  
   (d) 2  
   (e) None
16. \left(\frac{x+y}{x+y^2}\right)^{-2/3} = (a) x^2 + 2xy + y^2 \quad (b) (x+y)^{1/3} \\
(c) (x+y)^{3/2} \quad (d) x^2 + y^2 \quad (e) None

17. Which of the following best represents \{(x,y): y-1 = |x-1|\}? 
(a) \hspace{1cm} (b) \hspace{1cm} (c) \hspace{1cm} (d) \hspace{1cm} (e) None

18. Which of the following is not a one-to-one function? 
(a) \( f(x) = \sqrt{x} \) \hspace{1cm} (b) \( g(x) = \log_3(x^2 + 1) \) \hspace{1cm} (c) \( h(x) = 2^{-3x} \)
(d) \( r(x) = 2x + |x| \) \hspace{1cm} (e) None of the above are one-to-one functions.

19. If \( f(x) = \sqrt{1-x} \), then \( f(1-x) = \) (a) \( \sqrt{1-x} \) \hspace{1cm} (b) \( (1-x)^2 \)
(c) \( 1-x \) \hspace{1cm} (d) \( \sqrt{x} \) \hspace{1cm} (e) None

Find the solution sets in Questions 20-23.

20. \((x+1)(x-3) \geq x(x-1)\) : \hspace{1cm} (a) \([\infty, -3]\) \hspace{1cm} (b) \([3, \infty]\)
   (c) \([-1, 1]\) \(\cup\) \([3, \infty]\) \hspace{1cm} (d) \([\infty, -1]\) \(\cup\) \([1, 2]\) \hspace{1cm} (e) None

21. \(x^2 + x > 12\) : \hspace{1cm} (a) \(\mathbb{R}\) \hspace{1cm} (b) \((-\infty, -3) \cup (4, \infty)\) \hspace{1cm} (c) \((-4, 3)\)
   (d) \((-\infty, -4) \cup (3, \infty)\) \hspace{1cm} (e) None

22. \(|x-4| < 10\) : \hspace{1cm} (a) \((-6, 14)\) \hspace{1cm} (b) \((-\infty, 14)\) \hspace{1cm} (c) \((-6, 14)\)
   (d) \((-14, 6)\) \hspace{1cm} (e) None

23. \((x-1)(x+4) \leq 0\) : \hspace{1cm} (a) \([-4, 1]\) \hspace{1cm} (b) \([-\infty, 1]\)
   (c) \([-\infty, -4] \cup [1, \infty]\) \hspace{1cm} (d) \([-\infty, -4]\) \hspace{1cm} (e) None
24. $\log_2 \sqrt{x+5} - \log_2 \sqrt{x-1} = \frac{1}{2}$:  
(a) $(-5,1)$  
(b) $(-1)$  
(c) $(3)$  
(d) $(7)$  
(e) None

Let $F$ represent an $X$-distortion by $2$, $G$ a $Y$-reflection and $H$ a $Y$-translation by $-1$. Questions 25-27 refer to these mappings.

25. $F(8,10)$:  
(a) $(8,20)$  
(b) $(16,20)$  
(c) $(16,10)$  
(d) $(4,5)$  
(e) None

26. The image of $y = x$ under $G$ followed by $H$ is:
(a) $x+y = -1$  
(b) $x-y = 1$  
(c) $y-x = 1$  
(d) $x+y = 1$  
(e) None

27. If $H^{-1}$ denotes the inverse of the mapping $H$, then $H^{-1}(-2,3)$:
(a) $(3,-2)$  
(b) $(2,-3)$  
(c) $(-2,2)$  
(d) $(-2,4)$  
(e) None

28. Which of the following best represents $\{(x,y) : 3x-y \geq -2\}$?
(a)  
(b)  
(c)  
(d)  
(e) None

29. If $f(x) = 3x, x^2$, then the range of $f$ is:  
(a) $(3, \infty)$  
(b) $[0, \infty]$  
(c) $R$  
(d) $(1, \infty)$  
(e) None

30. Which of the following greetings is most appropriate to the season?  
(a) Happy Birthday  
(b) WOW  
(c) *$!----?/$*  
(d) Hi  
(e) Merry X-mas
31. Which of the following best represents \( (x, y): y = (x+2)^2 - 3 \)?

(a) ![Graph A](image1)

(b) ![Graph B](image2)

(c) ![Graph C](image3)

(d) ![Graph D](image4)

(e) None

32. If \( g(x) = \log_2(1-x) \), which of the following is defined?

(a) \( g(1) \) 
(b) \( g(3) \) 
(c) \( g(0) \) 
(d) \( [g(2)]^2 \) 
(e) All are defined.

33. If \( f(x) = x^2 - 1 \) with \( x \geq 0 \), then \( f^{-1}(x) = \)

(a) \( \sqrt{x} + 1 \) 
(b) \( 1 - x^2 \) 
(c) \( \sqrt{x+1} \) 
(d) \( 1/(x^2 - 1) \) 
(e) None

34. The understood domain of the function \( h(x) = \log |x-1| \) is:

(a) \( (0, \infty) \) 
(b) \( (1, \infty) \) 
(c) \( \mathbb{R} \) 
(d) \( (-1, \infty) \) 
(e) None

35. Which of the following best represents \( (x, y): y = \left(\frac{3}{2}\right)^x \)?

(a) ![Graph A](image1)

(b) ![Graph B](image2)

(c) ![Graph C](image3)

(d) ![Graph D](image4)

(e) None
36. The understood domain of the function $f(x) = \sqrt{\frac{x-1}{x}}$ is:
(a) $[1, \infty]$  (b) $(0, \infty)$  (c) $(-\infty, 0) \cup [1, \infty]$  (d) $(0, 1)$  (e) None

37. Which of the following best represents $\{(x, y): y = \log_2(x+3)\}$?
(a) $\text{Graph A}$  (b) $\text{Graph B}$  (c) $\text{Graph C}$  (d) $\text{Graph D}$  (e) None

38. The understood domain of the function $g(x) = \sqrt{\log x}$ is:
(Hint: Look at the graph of $y = \log x$.)  (a) $[1, \infty]$  (b) $(0, \infty)$  (c) $\mathbb{R}$  (d) $(0, 1)$  (e) None
The items on Form B are the same as the items on Form A except for the order of appearance. The correspondence of items between the two forms is presented in the following table.

<table>
<thead>
<tr>
<th>FORM A</th>
<th>FORM B</th>
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</table>
Directions: On the answer sheet provided choose the one best response to each of the following questions. "None" means "None of the preceding is true."

1. \( \sec(-\pi) \) is a) 0, b) 1, c) -1, d) undefined, e) none.

2. \( \cos(2^\circ \theta) = \) a) \( \frac{1}{2} \), b) \( -\frac{1}{2} \), c) \( \frac{\pi}{2} \), d) \( \frac{\pi}{2} \), e) none.

3. \( 7 - 2i - (3 - i)(1 + i) = \) a) \( 3 - 4i \), b) \( 5 - 2i \), c) \( 3 \), d) \( 4 \), e) none.

4. \( \sin \frac{\pi}{4} = \) a) \( \cos \frac{\pi}{4} \), b) \( \cos(\pi - 4) \), c) \( \sqrt{1 - \cos^2 \frac{\pi}{4}} \), d) \( \sin(\pi - \pi) \), e) none.

5. \( \sin 250^\circ = \) a) \( \sin 70^\circ \), b) \( \sin \left(\frac{25\pi}{18}\right) \), c) \( \sin 4.5 \), d) \( \cos 290^\circ \), e) none.

6. \( \frac{5}{3} - 4i = \) a) \( 3 - 4i \), b) \( 3 + 4i \), c) \( .6 + .8i \), d) \( \frac{5}{3} + \frac{5}{4}i \), e) none.

7. Which of the following statements is true?
   a) \( \sin 2 < \sin \frac{\pi}{4} \), b) \( \cos 2 < \cos 3 \), c) \( \tan 1 < \sec 1 \), d) \( \sin 2 < \cos 3 \), e) none.

8. A sector is generated in a circle of radius 3 by a central angle of \( 120^\circ \). The area of this sector is
   a) \( 540\pi \), b) \( 360\pi \), c) \( 9\pi \), d) \( 3\pi \), e) None.
9. If \( \tan^{-1}(\arccos x) = 0 \), then \( x \) is
   a) 0 ,  b) 1 ,  c) \( \pi/2 \) ,  d) indeterminate,  e) None.
10. If \( \sin x = 2/3 \), then \( |\sin 2x| = \)
   a) 4/3 ,  b) 2/3 ,  c) \( \sqrt{5}/9 \) ,  d) 4\( \sqrt{5}/9 \) ,  e) None.
11. \( \cos \frac{2\pi}{3} = \) a) \( \cos \frac{5\pi}{3} \) ,  b) \( \sin \frac{4\pi}{3} \)  c) \( \sin(-\frac{\pi}{3}) \),
    d) \( -\sin^2 \frac{\pi}{4} \)  e) None.
12. In the right triangle at the right, \( x \) equals
   a) \( r \sec \alpha \) ,  b) \( r \cot \alpha \) ,  c) \( r \cos \alpha \),
    d) \( r \tan \alpha \) ,  e) None.
13. \( \frac{1 + \sin x - \cos^2 x}{1 + \sin x} = \)
    a) \( \sin x \) ,  b) 0 ,
    c) \( \sin x \cos^2 x \) ,  d) \( \sin^2 x \) ,  e) None.
14. \( (\tan \frac{\pi}{4} - i \cot \frac{\pi}{4})^6 = \)
    a) \( \tan \frac{3\pi}{2} - i \cot \frac{3\pi}{2} \) ,
    b) 6 - 6i ,  c) -i ,  d) 8i ,  e) None.
15. In the triangle at the right, \( y \) equals
   a) \( r \sin \beta \csc \alpha \) ,  b) \( r \sec \beta \),
    c) \( r \tan \beta \) ,  d) \( r \sin \alpha \),
    e) None.
16. The solution set for the equation \( \sin^2 x = \sin x \) is
    a) \( \{0\} \) ,  b) \( \{k\pi : k \in \mathbb{Z}\} \) ,  c) \( \{0, 1\} \),
    d) \( \{\frac{\pi}{2} + 2k\pi : k \in \mathbb{Z}\} \) ,  e) None.
17. \( \cos(\sin^{-1} 1/3) = \) a) \( 2\sqrt{2}/3 \) ,  b) \( 1/3 \) ,  c) \( -1/3 \),
    d) \( \sqrt{2}/3 \) ,  e) None.
18. The graph of \( y = -3 \sin \pi x \) is

a)
\[
\begin{array}{c}
3 \\
\pi \\
2\pi \\
-3
\end{array}
\]

b)
\[
\begin{array}{c}
3 \\
1 \\
2 \\
3
\end{array}
\]

c)
\[
\begin{array}{c}
1 \\
-1
\end{array}
\]

d)
\[
\begin{array}{c}
3 \\
\pi \\
2\pi \\
-3
\end{array}
\]

e)
\[
\begin{array}{c}
3 \\
1 \\
2 \\
3
\end{array}
\]

19. The graph of \( y = \frac{\pi}{2} - \sin^{-1} x \) is

a)
\[
\begin{array}{c}
\text{Graph 1}
\end{array}
\]

b)
\[
\begin{array}{c}
\text{Graph 2}
\end{array}
\]

c)
\[
\begin{array}{c}
\text{Graph 3}
\end{array}
\]

d)
\[
\begin{array}{c}
\text{Graph 4}
\end{array}
\]

e)
\[
\begin{array}{c}
\text{Graph 5}
\end{array}
\]

20. The graph of \( \{(x,y) : x = \sin t, y = \cos t, 0 \leq t \leq \pi\} \) is

a)
\[
\begin{array}{c}
1 \\
\text{Graph 6}
\end{array}
\]

b)
\[
\begin{array}{c}
1 \\
\text{Graph 7}
\end{array}
\]

c)
\[
\begin{array}{c}
1 \\
\text{Graph 8}
\end{array}
\]
Directions: On the answer sheet provided choose the one best response to each of the following questions. "None" means "None of the preceding is true."

1. \( \csc(-\pi/2) \) is  
   a) 0,  b) 1,  c) -1,  d) undefined,  
   e) none.

2. \( \cos 210^\circ \) =  
   a) 1/2,  b) -1/2,  c) \( \sqrt{3}/2 \),  d) -\( \sqrt{3}/2 \),  
   e) none.

3. \( \sin 5 = \)  
   a) \( \cos 5 \),  b) \( \cos(2\pi-5) \),  c) \( \sqrt{1 - \cos^2 5} \),  
   d) \( \sin(2\pi - 5) \),  e) none.

4. Which of the following statements is true?  
   a) \( \sin 2 < \sin 3 \),  b) \( \cos 2 < \cos 3 \),  c) \( \sin 2^\circ < \sin 2 \),  
   d) \( \csc 1 < \cot 1 \),  e) none.

5. What is the solution set of \( \cos(-t) = -1 \)?  
   a) \( \{t : t = 2k\pi, k \in \mathbb{Z}\} \),  b) \( \{t : t = (2k + 1)\pi, k \in \mathbb{Z}\} \),  
   c) \( \emptyset \),  d) \( \{t : t = -\pi/4 + k\pi, k \in \mathbb{Z}\} \),  e) none.

6. If \( \sin 40^\circ = .64 \) and \( \cos 40^\circ = .77 \), then \( \sin(-230^\circ) = \)  
   a) .64,  b) -.64,  c) .77,  d) -.77,  e) none.

7. A sector of a circle of radius 4 is generated by an angle of \( \pi/5 \) radians. The area of this sector is  
   a) \( 8\pi/5 \),  b) \( 4\pi/5 \),  c) \( 2\pi/5 \),  d) \( 16\pi/5 \),  e) none.

8. \( \sin(v - \frac{\pi}{2}) = \)  
   a) \( \sin v \),  b) \( \cos v \),  c) -\( \sin v \),  
   d) -\( \cos v \),  e) none.
9. If a circle of radius 2 contains a central angle \( \theta \) that intercepts an arc 5 units long, then \( \theta \) is
   a) \( \left( \frac{50}{\pi} \right)^{\circ} \),  
   b) \( \frac{5\pi}{2} \),  
   c) \( \frac{2\pi}{5} \),  
   d) \( \frac{3611}{\pi} \),  
   e) none.

10. In the right triangle at the right, \( x \) equals
   a) \( d \sin \alpha \),  
   b) \( d \csc \alpha \),  
   c) \( d \cos \alpha \),  
   d) \( d \sec \alpha \),  
   e) none.

11. Which of the following is true?
   a) \( \sin(\pi+1) = \sin \pi + \sin 1 \),  
   b) \( \cos(\pi+1) = \cos \pi + \cos 1 \),  
   c) \( \tan(\pi+1) = \tan \pi + \tan 1 \),  
   d) \( \sec(\pi+1) = \sec \pi + \sec 1 \),  
   e) none.

12. \( \sin(2u + 2v) = \)
   a) \( 2 \sin(u+v) \),  
   b) \( \sin 2u + \sin 2v \),  
   c) \( 2 \sin u + 2 \sin v \),  
   d) \( 2 \sin(u+v) \cos (u+v) \),  
   e) none.

13. \( \sin(-t) \cos(-t) - \tan(-t) \) equals
   a) \( (1 + \cos^2 t) \tan t \),  
   b) \( -(1 + \cos^2 t) \tan t \),  
   c) \( 0 \),  
   d) \( \sin^2 t \tan t \),  
   e) none.

14. In the triangle at the right, \( x = \)
   a) \( r \sin \beta \csc \alpha \),  
   b) \( r \sin \alpha \csc \beta \),  
   c) \( r \cot \alpha \),  
   d) \( r \sin \alpha \),  
   e) none.

15. \( \{ t : \sin 2t > 0 \} \cap [0, 2\pi] = \)
   a) \( (0, \pi) \),  
   b) \( (0, \pi/2) \cup (3\pi/2, 2\pi) \),  
   c) \( (0, \pi/2) \cup (\pi, 3\pi/2) \),  
   d) \( (0, \pi) \cup (3\pi/2, 2\pi) \),  
   e) none.

16. A X-translation by 2 followed by a Y-translation by 3 transforms the graph of \( y = \sin x \) into the graph of \( y = \)
   a) \( 3 + \sin(x-2) \),  
   b) \( 3 \sin(x+2) \),  
   c) \( 3 \sin(x-2) \).
d) $3 + \sin(x+2)$, e) none.

17. Which number is not in the domain of $f(x) = \tan(\sin x)$?
   a) 1, b) $\pi/2$, c) 2, d) 0, e) All are in the domain.

18. The graph of $y = \tan 2x$ is
   a) 
   b) 
   c) 
   d) 
   e) 

19. The graph of $\{(x, 2 \sin(\pi x + \pi))\}$ is
   a) 
   b) 
   c) 
   d) 
   e)
20. The graph of \((x, y) : x = \cos t, y = \sin t, t \in [-\pi/2, \pi/2]\) is

a)

\[ \text{Diagram a) here.} \]

b)

c)

\[ \text{Diagram c) here.} \]

d)

\[ \text{Diagram d) here.} \]

e)

\[ \text{Diagram e) here.} \]
On your answer sheet, indicate the one best response to each of the following 50 questions. Answer "None" means "None of the above is true".

1. If \((1 + i)z = 4i - 6\), then \(z =\)
   \(\begin{align*}
   & (a) \quad -1 + 5i \\
   & (b) \quad -6 + 4i \\
   & (c) \quad -6 - 4i \\
   & (d) \quad 3 - 2i \\
   & (e) \quad \text{None}
   \end{align*}\)

2. If \(6 + 4i - 2^x + y^2i = 3i - 2\), then \(x + y\) equals
   \(\begin{align*}
   & (a) \quad 1 \\
   & (b) \quad 2 \\
   & (c) \quad 3 \\
   & (d) \quad 4 \\
   & (e) \quad \text{None}
   \end{align*}\)

3. Which of the following is true?
   \(\begin{align*}
   & (a) \quad \cos 2^o < \cos 2 \\
   & (b) \quad \sin 3 < \cos 3 \\
   & (c) \quad \tan 4 < \tan 5 \\
   & (d) \quad \cos 3 < \cos 4 \\
   & (e) \quad \text{None}
   \end{align*}\)

4. If \(g(x) = \cos(Sin^{-1}x)\), the domain of \(g\) is:
   \(\begin{align*}
   & (a) \quad [-1,1] \\
   & (b) \quad [-\frac{\pi}{2}, \frac{\pi}{2}] \\
   & (c) \quad [0,\pi] \\
   & (d) \quad (-\infty, \infty) \\
   & (e) \quad \text{None}
   \end{align*}\)

5. If \(\cos 20^o = x\) and \(\sin 20^o = y\), then \(\sin 250^o =\)
   \(\begin{align*}
   & (a) \quad -x \\
   & (b) \quad x - \pi \\
   & (c) \quad -y \\
   & (d) \quad -y - \pi \\
   & (e) \quad \text{None}
   \end{align*}\)

6. If \(y = \tan(sin x)\), \(x \in (0,\frac{\pi}{2})\), \(y \in (0,\frac{\pi}{2})\) then \(x\) is:
   \(\begin{align*}
   & (a) \quad \tan^{-1}(\sin^{-1}y) \\
   & (b) \quad \tan^{-1}(\sin y) \\
   & (c) \quad \sin^{-1}(\tan y) \\
   & (d) \quad \sin^{-1}(\tan^{-1}y) \\
   & (e) \quad \text{undefined.}
   \end{align*}\)

7. \((\tan \frac{\pi}{4} + i \cot \frac{\pi}{4})^4 = \)
   \(\begin{align*}
   & (a) \quad 4 + 4i \\
   & (b) \quad \tan \pi + i \cot \pi \\
   & (c) \quad 2\sqrt{2}i \\
   & (d) \quad -4 \\
   & (e) \quad \text{None}
   \end{align*}\)
8. Which of the following statements (a), (b), (c), or (d) is not true for every complex number \( z \)?

(a) \(|z|^2 = |z|^2\)  
(b) \(|z| = |\bar{z}|\)  
(c) \(|z\bar{z}| = z\bar{z}\)  
(d) \(z^2 = (\bar{z})^2\)  
(e) All of the preceding are true.

9. \(\frac{5}{4} + 3i\) = (a) \(.8 -.6i\)  
(b) \(\frac{5}{4} + \frac{5}{3}i\)  
(c) \(\frac{5}{4} - \frac{5}{3}i\)  
(d) \(4 - 3i\)  
(e) None

10. \(\cos\left(\frac{3\pi}{2} - t\right)\) = (a) \(\sin t\)  
(b) \(-\sin t\)  
(c) \(\cos t\)  
(d) \(-\cos t\)  
(e) None

11. The function \( I \) is defined such that \( I(z) \) is the imaginary part of \( z \). Which of the following statements is true for every complex number \( z \)?

(a) \( I(z) = I(\bar{z}) \)  
(b) \( I(|z|) = |I(z)| \)  
(c) \( I(z_1 + z_2) = I(z_1) + I(z_2) \)  
(d) \( I(z_1z_2) = I(z_1)I(z_2) \)  
(e) All the above are true.

12. Let \((x,y)\) be the coordinates of a point \( P \) on a circle of radius 2. If \( P \) is the terminal side of an angle with measure 240°, then

(a) \( x = -\sqrt{3}, y = -1 \)  
(b) \( x = -\sqrt{3}/2, y = -\frac{1}{2} \)  
(c) \( x = -\frac{1}{2}, y = \sqrt{3}/2 \)  
(d) \( x = -1, y = -\sqrt{3} \)  
(e) None

13. \([\sin(-t) + \cos t + \sin t + \cos(-t)]\tan t = \)

(a) 0  
(b) 2 \(\sin t\)  
(c) 2 \(\sin t\) \(\cos t\)  
(d) 2 \(\sin t\) \(\tan t\)  
(e) None

14. \(\cos \frac{\pi}{3}\) equals (a) \(\sin \frac{\pi}{3}\)  
(b) \(\cos \frac{2\pi}{3}\)  
(c) \(\cos \frac{\pi}{3}\)  
(d) \(\cos^2 \frac{\pi}{3}\)  
(e) None
15. \( \frac{2 + 24}{\sqrt{3} + 1} \) equals (a) \( \sqrt{2} (\cos 15^\circ + i \sin 15^\circ) \)
(b) \( \sqrt{2} (\cos 15^\circ - i \sin 15^\circ) \)
(c) \( \sqrt{2} (\cos 75^\circ + i \sin 75^\circ) \)
(d) \( \sqrt{2} (\cos 75^\circ - i \sin 75^\circ) \)
(e) None

16. If \( \cos t = \frac{1}{4} \), then \( \sin 2t = \)
(a) \( \frac{\sqrt{15}}{2} \)
(b) \( \frac{\sqrt{15}}{4} \)
(c) \( \frac{\sqrt{15}}{8} \)
(d) \( \frac{\sqrt{3}}{2} \)
(e) None

17. \( \frac{1 - \cos x}{\sin^2 x + \cos x - 1} \) equals (a) \( \csc^2 x \)
(b) \( \sec x \)
(c) \( \tan^2 x \)
(d) \( -\sin^2 x \cos x \)
(e) None

18. The area of a sector of a circle of radius 2 cut by an angle of 10\(^\circ\) is: (a) \( \frac{\pi}{9} \)
(b) \( \frac{\pi}{18} \)
(c) 20
(d) \( \frac{20}{\pi} \)
(e) None

19. \( \tan(\sin^{-1} \frac{3}{4}) = \)
(a) \( \frac{3}{5} \)
(b) \( \frac{4}{5} \)
(c) \( \frac{3}{\sqrt{7}} \)
(d) \( \frac{\sqrt{7}}{5} \)
(e) None

20. Let \( A = \{ x : (\sin x - 1) = 0 \} \) and \( B = \{ x : (\sin x - 1)(\sin x - 2) = 0 \} \). Which of the following is true?
(a) \( A \subseteq B \) but \( A \nsubseteq B \)
(b) \( B \subseteq A \) but \( B \nsubseteq A \)
(c) \( A = B \)
(d) \( A \nsubseteq B \) and \( B \nsubseteq A \)
(e) None

21. \( \{ t : \cos \frac{t}{2} \geq 0 \} \cap [0, 2\pi] = \)
(a) \( [0, \frac{\pi}{2}] \)
(b) \( [\pi, 2\pi] \)
(c) \( [0, \frac{\pi}{2}] \cup [\pi, \frac{3\pi}{2}] \)
(d) \( [0, \frac{\pi}{2}] \cup [\frac{3\pi}{2}, 2\pi] \)
(e) None

22. If \( \sin^{-1}(\cos^{-1} x) = 0 \), then \( x \) is:
(a) 0
(b) 1
(c) \( \frac{\pi}{2} \)
(d) undefined
(e) None
23. How many distinct solutions exist for the equation 

\[ 4 \sin \pi x = x \] 

(a) 4  (b) 5  (c) 6  (d) 7  (e) 8

24. \[ \cos^{-1} 0 \cdot \sin \frac{\pi}{2} - \sin^{-1} (\cos 0) \] equals

(a) -1  (b) 0  (c) 1  (d) \( \frac{-\pi}{2} \)  (e) None

25. Which of the following (a), (b), (c), or (d) is not true?

(a) \( -1 \leq 2 \sin x \cos x \leq 1 \)  (b) \( \sec^2 x - \tan^2 x = 1 \)

(c) \( -2 \leq 1 + \cos 2x \leq 2 \)  (d) \( \frac{-\pi}{2} \leq \sin^{-1} \frac{x}{2} \leq \frac{\pi}{2} \)

(e) All the above are true.

26. Which of the following functions is not an even function? Note: For any even function, \( f(-x) = f(x) \) for all numbers in the domain of \( f \).

(a) \( f(x) = \sin x^2 \)  (b) \( f(x) = \sin^2 x \)  (c) \( f(x) = \cos x \)

(d) \( f(x) = \cos^{-1} x \)  (e) All the above are even functions.

27. In the triangle at the right, 
\[ \alpha = 30^\circ, \beta = 105^\circ, \text{ and } y = 2. \]

Thus \( x = \)

(a) \( 2\sqrt{2} \)  (b) \( \sqrt{2} \)  (c) \( \sqrt{6} \)  (d) \( \sqrt{3} + 1 \)

(e) \( \frac{4}{\sqrt{3}} \)

28. The graph of \( y = 2 \cos(\pi x - \frac{\pi}{2}) \) is:

(a)

(b)
29. The graph of \( y = \cos(\cos^{-1}x) \) is:
   (a) 
   (b) 
   (c) 
   (d) 
   (e) 

30. The graph of \( y = -\sin^{-1}x \) is:
Math 159.02
Midterm
Winter 1971

On the answer sheet provided, choose the one best response to each of the following questions by marking with PENCIL ONLY in the appropriate space after the question number. None means "None of the preceding".

1. \( \cos(v - \frac{\pi}{2}) = \) a) \( \sin v \) b) \( \cos v \) c) \( -\sin v \) d) \( -\cos v \) e) none

2. \( \sin 210° = \) a) \( -\frac{\sqrt{3}}{2} \) b) \( \frac{\sqrt{3}}{2} \) c) \( -\frac{1}{2} \) d) \( \frac{1}{2} \) e) none

3. \( \sec\left(-\frac{\pi}{2}\right) = \) a) 1 b) -1 c) 0 d) undefined e) none

4. If \( \sin 40° = .64 \) and \( \cos 40° = .77 \), then \( \sin(-230°) = \) a) .77 b) - .77 c) .64 d) - .64 e) none

5. \( \sin 5 = \) a) \( \cos 5 \) b) \( \sqrt{1 - \cos^2 5} \) c) \( \cos(2\pi - 5) \) d) \( \sin(2\pi - 5) \) e) none

6. Which number is not in the domain of \( f \) if \( f(x) = \tan(\sin x) \) a) 0 b) 1 c) 2 d) \( \frac{\pi}{2} \) e) All these are in the domain

7. The solution set of \( \cos(-t) = -1 \) is a) \( \{ t : t = 2k\pi, k \in \mathbb{Z} \} \) b) \( \emptyset \) c) \( \{ t : t = - \frac{\pi}{4} + k\pi, k \in \mathbb{Z} \} \) d) \( \{ t : t = (2k + 1)\pi, k \in \mathbb{Z} \} \) e) none

8. Which of the following statements is true ? a) \( \sin 2 < \sin 2° \) b) \( \sin 2 < \sin 3 \) c) \( \cot 1 < \csc 1 \) d) \( \cos 2 < \cos 3 \) e) none
9. \( \sin (2u + 2v) = \) a) \( 2 \sin(u + v) \) b) \( 2 \sin u + 2 \sin v \) c) \( 2 \sin(u + v) \cos(u + v) \) d) \( \sin 2u + \sin 2v \) e) none

10. \( \sin(-t) \cos(-t) - \tan(-t) = \) a) \( \sin t \cos t - \tan t \) b) \( -\sin t \cos t - \tan t \) c) \( \sin t \cos t + \tan t \) d) \( -\sin t \cos t + \tan t \) e) none

11. \( \cos \left( \frac{7\pi}{3} \right) = \) a) \( \frac{\sqrt{3}}{2} \) b) \( -\frac{\sqrt{3}}{2} \) c) \( \frac{1}{2} \) d) \( -\frac{1}{2} \) e) none

12. If a circle of radius 1 has area \( \pi \) square units then the sector of that circle generated by an angle of \( \frac{\pi}{3} \) radians has area a) \( \frac{\pi}{10} \) b) \( \frac{\pi}{5} \) c) \( \frac{\pi}{20} \) d) \( \frac{\pi}{5} \) e) none

13. \( \cos|t| = \) a) \( \cos t \) b) \( \cos t \) c) \( \cos t^2 \) d) All of these e) none

14. \( \cos(1 - \pi) = \) a) \( \cos 1 \) b) \( \sin 1 \) c) \( -\cos 1 \) d) \( -\sin 1 \) e) none

15. Find a member of the solution set of \( \cos \frac{\pi}{6} = \sin x \) a) \( \frac{\pi}{6} \) b) \( \frac{\pi}{4} \) c) \( \frac{\pi}{3} \) d) \( \frac{\pi}{2} \) e) none

16. The range of \( \sin(\cos x) \) does not contain a) 1 b) 0 c) \(-\frac{1}{2}\) d) \( \frac{1}{2} \) e) none

17. \( \tan - \frac{\pi}{4} = \) a) 1 b) \( \frac{1}{2} \) c) \(-\frac{1}{2}\) d) \(-1\) e) none

18. \( \cos \frac{\pi}{12} = \cos \left( \frac{\pi}{4} - \frac{\pi}{6} \right) = \) a) \( \frac{\sqrt{6} + \sqrt{2}}{4} \) b) \( \frac{\sqrt{6} - \sqrt{2}}{4} \) c) \( \frac{\sqrt{6}}{2} \) d) \( \frac{\sqrt{6} - \sqrt{2}}{2} \) e) none

19. \( \cot \frac{\pi}{6} = \) a) \( \frac{1}{\sqrt{3}} \) b) \( \frac{1}{2} \) c) 2 d) \( \sqrt{3} \) e) none
20. \( \frac{\sin 2t}{1 + \cos 2t} \)  
   a) \( \cot t \)  
   b) \( \tan t \)  
   c) \( 1 + \tan t \)  
   d) \( \sin 2t + \tan 2t \)  
   e) none
Directions: On the answer sheet provided choose the one best response to each of the following questions. "None" means "None of the preceding is true."

1. \( \sin \frac{7\pi}{6} = \) a) \( \frac{\sqrt{3}}{2} \) b) \( \frac{1}{2} \) c) \( -\frac{\sqrt{3}}{2} \) d) \( -\frac{1}{2} \) e) none

2. \( \sec \frac{3\pi}{4} = \) a) \( -\frac{\sqrt{2}}{2} \) b) 0 c) \( \frac{1}{2} \) d) \( \frac{\sqrt{2}}{2} \) e) none

3. \( \tan 300^\circ = \) a) \( -\sqrt{3} \) b) \( -\frac{1}{\sqrt{3}} \) c) -1 d) \( \frac{1}{\sqrt{3}} \) e) none

4. \( \cos(\pi - t) = \) a) \( \sin t \) b) - \( \cos t \) c) \( \cos t \) d) - \( \sin t \) e) none

5. \( \sin(u-v) \cos v + \cos(u-v) \sin v = \) a) 2 \( \sin u \cos v \) b) \( \sin(u+v) \) c) \( \sin u \) d) \( \cos v \) e) none

6. \( \csc t - \cot t \cos t = \) a) \( \tan t \) b) \( \cot t \) c) \( \cos t \) d) \( \sin t \) e) none

7. \( \cot t + \tan t = \) a) 1 b) \( \sin t + \cos t \) c) \( \frac{2}{\sin 2t} \) d) \( 2 \) e) none

8. \( -2 \sin(2nx + 1) = \) a) \( 2 \sin(2nx + 1) \) b) \( 2 \cos(2nx + 1) \) c) \( 2 \sin(2nx + 1 + \frac{\pi}{2}) \) d) \( 2 \sin(2nx + 1 + \pi) \) e) none

9. A circle with a radius of 2 contains a central angle \( \theta \) that intercepts an arc 5 units long. \( \theta = \) a) \( \frac{\pi}{2} \) b) \( \frac{2}{5} \)
10. If \( \sin 50^\circ = .766 \) and \( \cos 50^\circ = .643 \) then \( \sin 130^\circ = \)
   a) \( -.766 \) b) \( .766 \) c) \( .643 \) d) \( .643 \) e) none

11. \( \{ \sin t = \frac{1}{4} \} \cap \{ \cos t = \frac{3}{5} \} = \)
   a) \( \frac{\pi}{4} \) b) \( \{ t \mid \tan t = \frac{1}{3} \} \)
   c) \( \emptyset \) d) \( \{(2n+1)\frac{\pi}{6} \mid n \text{ is an integer} \} \) e) none

12. \( \cos(\cos x) = 0 \) if \( x = \)
   a) \( \frac{\pi}{2} \) b) \( 1 \) c) \( 0 \) d) \( \pi \)
   e) none

13. If \( \tan t = \frac{3}{4} \) then \( |\sin 2t| = \)
   a) \( \frac{24}{25} \) b) \( \frac{\sqrt{25}}{2} \) c) \( \frac{1}{2} \)
   d) \( \sqrt{3} \) e) none

14. If the right triangle at right, \( x = \)
   a) \( 2 \) b) \( 2\sqrt{3} \)
   c) \( 3 \) d) \( 1 \) e) none

15. \( \arcsin \frac{\sqrt{3}}{2} = \)
   a) \( \frac{\sqrt{3}}{2} \) b) \( \frac{1}{2} \) c) \( \frac{\pi}{6} \) d) \( \frac{\pi}{3} \) e) none

16. \( \arccos(\sin^{-1}0) = \)
   a) \( 0 \) b) \( 1 \) c) \( \frac{\pi}{2} \) d) \( \pi \) e) none

17. \( \cos(\sin^{-1}.8) = \)
   a) \( .2 \) b) \( .6 \) c) \( .8 \) d) \( .36 \)
   e) none

18. \( \cot(\tan^{-1}\frac{3}{4}) = \)
   a) \( \frac{1}{4} \) b) \( \frac{\pi}{4} \) c) \( \frac{4}{3} \) d) \( \frac{\pi}{3} \) e) none

19. The graph of \( y = -2 \sin(x - \frac{\pi}{2}) \) is:
   a) \( \) b) \( \) c) \( \)
20. \( \{(x, y) \mid x = \sin t, y = \cos(t + \pi) \text{ and } 0 \leq t \leq \pi \} \) represented by:

a) \hspace{1cm} b) \hspace{1cm} c) \hspace{1cm} d) \hspace{1cm} e) 

\[ \begin{align*}
\cos 220^\circ &= a) \cos 40^\circ \hspace{1cm} b) \cos \frac{22}{18} \pi \hspace{1cm} c) \cos \frac{11}{18} \pi \\
&\hspace{1cm} d) \cos \frac{11}{36} \pi \hspace{1cm} e) \text{ none}
\end{align*} \]

22. Which of the following is true? 

a) \( \sin 3 > \sin 4 \)

b) \( \cos -1 < \cos 1 \)

c) \( \sin 2^\circ < \sin 200^\circ \)

d) \( |\sec t| < 1 \)

e) all of these are true

23. \( \tan^{-1}(\cos x) = 0 \) if \( x = \) 

a) 0 \hspace{1cm} b) 1 \hspace{1cm} c) \( \pi \) \hspace{1cm} d) \( \frac{\pi}{2} \) 

e) none

24. If \( \cos x = \frac{3}{5} \), then \( |\cos 2x| = \) 

a) \( \frac{24}{25} \) \hspace{1cm} b) \( \frac{7}{25} \) \hspace{1cm} c) \( \frac{6}{5} \) 

\hspace{1cm} d) \( \frac{9}{25} \) \hspace{1cm} e) none

25. In the right triangle at the right, \( x \) equals 

a) \( a \cos \alpha \) \hspace{1cm} b) \( a \sin \alpha \) \hspace{1cm} c) \( a \cot \alpha \)
26. In the triangle at the right, $b$ equals
   a) $\csc \alpha \sin \beta$  
   b) $a \sin \alpha \sin \beta$  
   c) $a^2 - a \cos(\alpha - \beta)$  
   d) $a \sin (\beta - \alpha)$  
   e) none

27. $\sin(\cos^{-1} \frac{2}{3}) =$
   a) $\frac{1}{3}$  
   b) $\frac{\sqrt{3}}{3}$  
   c) $\sin(\frac{2}{3})$  
   d) $\frac{\sqrt{3}}{3}$  
   e) none

28. $\frac{\sin 2t - \cos 2t}{\sin t - \cos t} =$
   a) $\sin t - \cos t$  
   b) 0  
   c) $\sec t$  
   d) $\cos t - \sec t$  
   e) none

29. The graph of $y = -\frac{1}{2} \sin(2x - \frac{\pi}{2})$
   a)  
   b)  
   c)  
   d)  
   e)  

30. The graph of $y = \sin^{-1}(x) - \frac{\pi}{2}$
   a)  
   b)  
   c)  
   d)  
   e)  

31. \((3 + 4i) - (6 - 2i)(1 - i) = \) a) \(7 - 4i\) b) \(-1\) c) \(-1 + 12i\) d) \(-5 - 4i\) e) none

32. \((3 - i)Z = 1 - i\) so \(Z = \) a) \(\frac{2 - i}{5}\) b) \(\frac{1 - 2i}{3}\) c) \(\frac{1}{3}\) d) \(\frac{3 - i}{1 - i}\) e) none

33. \(\frac{i}{1 - i} = \) a) \(1 + i\) b) \(-1 + i\) c) \(1 - i\) d) \(-i\) e) none

34. \(i^2 = \) a) \(-1\) b) \(1\) c) \(0\) d) All of these e) none

35. \(\sqrt{3} + i = \) a) \(\sqrt{2} \cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\) b) \(2 \cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\) c) \(\sqrt{2} \cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\) d) \(2 \cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\) e) none

36. \(2(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}) = \) a) \(-1 + i\sqrt{3}\) b) \(-\sqrt{3} + i\) c) \(1 - i\sqrt{3}\) d) \(\sqrt{3} - i\) e) none

37. \((\tan \frac{\pi}{4} - i \cot \frac{\pi}{4})^3 = \) a) \(-1 + i\) b) \(1 - i\) c) \(-1 - i\) d) \(-2(1 + i)\) e) none

38. \([3(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})]^4 = \) a) \(81(-\frac{1}{2} - i\frac{\sqrt{3}}{2})\) b) \(81\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)\) c) \(12\left(-\frac{\sqrt{3}}{2} - i\frac{1}{2}\right)\) d) \(81(-\frac{\sqrt{3}}{2} - i\frac{1}{2})\) e) none

39. \((1 + i)^8 = \) a) \(1\) b) \(8 + 8i\) c) \(16\) d) \(1 + i\) e) none

40. \(8^{1/3} = \) a) \(2\) b) \((2, -1 + i\sqrt{3}, -1 - i\sqrt{3})\) c) \((2, 1 + i\sqrt{3}, -1 - i\sqrt{3})\) d) \((2, -1 + i\sqrt{3})\) e) none
In questions 1, 2, and 3, \( P \) is the set of positive integers, \( Z \) is the set of integers, \( A = \{ x \mid x \in P \text{ and } x \leq 3 \} \) and \( B = \{ x \mid x \in Z \text{ and } x < 3 \} \).

1. The set \( A \) is equal to:
   (a) \((0,3]\)
   (b) \([0,3]\)
   (c) \(\{1,2,3\}\)
   (d) \([0,2]\)
   (e) none

2. Which of the following is an upper bound of \( B \)?
   (a) \(0\)
   (b) \(1\)
   (c) \(2\)
   (d) \(-1\)
   (e) none

3. \( A \cup (Z \cap [-1,3]) = \)
   (a) \([1,2]\)
   (b) \([1,2,3]\)
   (c) \([-1,3]\)
   (d) \([-1,3]\)
   (e) none

4. \(([-2,5] \cap [5,6]) \cup [-1,5] = \)
   (a) \([-2,5]\)
   (b) \([-1,5]\)
   (c) \([-1,5]\)
   (d) \([-2,5]\)
   (e) none

5. The least upper bound of the solutions to \(|x - 2| = 4\) is:
   (a) -2
   (b) 2
   (c) 4
   (d) 6
   (e) none
6. \( \sqrt{-1} \cdot \sqrt{64} = \)  
(a) 2  (b) \((64)^{2/3}\)  (c) -2  (d) undefined  (e) none

7. If \( a < 0, a \in \mathbb{R} \) then \( \frac{a^{2}}{a^{3}} = \)  
(a) -1  (b) 1  (c) a  (d) \( \frac{1}{a}\)  (e) none

8. Which axiom is used to factor \( yx^2 + yz^2 = y(x^2 + z^2) \)  
(a) Associative Axiom  (b) Distributive Axiom  (c) Inverse Axiom  (d) Commutative Axiom  (e) none

9. If \( x > 0 \) then \( 1 + \sqrt{x} = \frac{\sqrt{x}}{1 - \frac{\sqrt{x}}{1 + \sqrt{x}}} \)  
(a) \( -\sqrt{x}\)  (b) \( 1 + \sqrt{x}\)  (c) \( \frac{1}{1 + \sqrt{x}}\)  (d) \( 1 - x\)  (e) none

10. \( \{x \mid |x^2 - 1| = 1 - x^2\} \) is equal to:  
(a) \( (-\infty, \infty)\)  (b) \( (-1,1)\)  (c) \( (-\infty, -1) \cup (1, \infty)\)  (d) \( \emptyset\)  (e) none

11. The solution set for \( 1 - 3x \leq 4x + 15 \) is:  
(a) \( \{x \mid x \geq -2\}\)  (b) \( \{x \mid x \leq -\frac{16}{7}\}\)  (c) \( \{x \mid x \geq -\frac{16}{7}\}\)  (d) \( \{x \mid x \leq \frac{16}{7}\}\)  (e) none

12. \( \{x \mid \frac{\sqrt{2} - x}{x} \) is a real number\} is:  
(a) \( \{x \mid x \neq 0\}\)  (b) \( \{x \mid x \leq 2\}\)  (c) \( \{x \mid x \leq 2, x \neq 0\}\)  (d) \( \{x \mid -2 \leq x \leq 2, x \neq 0\}\)  (e) none
13. \( \{ x \mid (x - 2)(x + 1) \leq 0 \} \) is
   (a) \([-2,1]\)  (b) \([-1,2]\)  (c) \((\infty,-1) \cup (2,\infty)\)
   (d) \((\infty,-2) \cup (1,\infty)\)  (e) none

14. The solution set for \( |3 - 2x| \leq 1 \) is:
   (a) \([0,1]\)  (b) \([-\frac{1}{2},\frac{1}{2}]\)  (c) \([-1,2]\)  (d) \([1,2]\)
   (e) none

15. The solution set for \( x^2 \geq 5x \) is:
   (a) \([0,5]\)  (b) \((\infty,-5) \cup (5,\infty)\)  (c) \([5,\infty]\)
   (d) \((\infty,0) \cup (5,\infty)\)  (e) none

16. The solution set for \( |x + 1| < -1 \) is:
   (a) \((-2,0)\)  (b) \((\infty,-2) \cup (0,\infty)\)  (c) \((\infty,\infty)\)  (d) \(\emptyset\)
   (e) none

17. The solution set for \( \frac{x + 1}{|x - 1|} > 0 \) is:
   (a) \((-1,1) \cup (1,\infty)\)  (b) \((-1,\infty)\)  (c) \((1,\infty)\)
   (d) \((\infty,-1) \cup (1,\infty)\)  (e) none

18. \( \{ x \mid -3 < x < 5 \} = \{ x \mid |x - a| < p \} \) where
   (a) \( a = -3, p = 8 \)  (b) \( a = 3, p = 8 \)  (c) \( a = -1, p = 3 \)
   (d) \( a = 1, p = 4 \)  (e) none

19. For \( a > 0 \), \( \sqrt[3]{a^2} = \frac{\sqrt[4]{a^3}}{a} \)
   (a) \( a^{3/2} \)  (b) \( a^{3/4} \)  (c) \( \frac{1}{\sqrt[3]{a^2}} \)  (d) \( \sqrt[3]{a^2} \)  (e) none

20. The solution set for \( |x| + |x + 3| \leq 3 \) is:
   (a) \([0,6]\)  (b) \([0]\)  (c) \([0,3]\)  (d) \([-3,0]\)  (e) none
Form A

Math 150

Exam 2

February 10, 1971

On the answer sheet provided, choose the one best response to each of the following questions by marking with PENCIL ONLY in the appropriate space after the question number. None means "none of the preceding".

1. If \( f(x) = \lfloor x \rfloor \), then \( f(1/2) = \)
   a) -1 b) 0 c) 1/2 d) 1 e) none

2. If \( g(t) = \sqrt{1-t} \) and \( f(x) = 1 + x^2 \), then \( f(g(-1)) = \)
   a) -1 b) 0 c) 1 d) 2 e) none

3. A composition of mappings under which the curve with equation \( y = \frac{1}{2}x^2 - \frac{1}{3} \) is the image of \( y = x^2 \) is:
   a) a Y-distortion by 1/3 followed by a Y-translation by \(-\frac{1}{2}\)
   b) a Y-distortion by 1/2 followed by a Y-translation by \(-\frac{1}{3}\)
   c) a Y-distortion by 1/2 followed by a Y-translation by 3
   d) a Y-distortion by 1/2 followed by a Y-translation by 1/3
   e) none

4. Which of the following subsets of \( \mathbb{R}^2 \) (a), (b), (c) or (d)) is symmetric about the Y-axis?
   a) \( \{(x, y) \mid y = x^2\} \)
   b) \( \{(x, y) \mid x = |y|\} \)
   c) \( \{(x, y) \mid y = \sqrt{x}\} \)
   d) \( \{(x, y) \mid x = \frac{1}{y}\} \)
   e) none

5. Which of the following ((a), (b), (c) or (d)) is not a function?
   a) \( \{(x, y) \mid y = |3^x|\} \)
   b) \( \{(x, y) \mid y = |x^2 - 1|\} \)
   c) \( \{(x, y) \mid y = |x|\} \)
   d) \( \{(x, y) \mid y = \log_2 |x|, x \neq 0\} \)
   e) All of the preceding are functions.
6. Which of the following \((a), (b), (c)\) or \((d)\) is \textbf{not}\ a one-to-one function?
\[
\begin{align*}
\text{a) } f(x) &= |3^x| \\
\text{b) } g(x) &= \log_2 x \\
\text{c) } h(x) &= |x| \\
\text{d) } F(x) &= \frac{1}{x} \\
\text{e) } &\text{All of the preceding are one-to-one.}
\end{align*}
\]

7. Which of the following best represents \(\{(x, y) \mid y = 3 - x^2\}\)
\[
\begin{align*}
\text{a) } &
\text{b) } \\
\text{c) } &
\end{align*}
\]

8. Which of the following best represents \(\{(x, y) \mid y \geq |x|\}\)
\[
\begin{align*}
\text{a) } &
\text{b) } \\
\text{c) } &
\end{align*}
\]

9. Which of the following graphs best represents \(\{(x, y) \mid y = |x + 1|\}\)?
\[
\begin{align*}
\text{a) } &
\text{b) } \\
\text{c) } &
\end{align*}
\]
10. The sum of the zeros of \( f(x) = x^2 + 2x - 99 \) is:
   a) -2  b) -1  c) 20  d) 2  e) none

11. The equation of the line containing the points \((-1, 0)\) and \((1, 4)\) is:
   a) \( y = 4(x - 1) \)  b) \( y = 1 \)  c) \( y - 1 = \frac{1}{2}x \)
   d) \( y = 2(x + 1) \)  e) none

12. If \( f(x) = \sqrt{3 - x} \), then the understood domain of \( f \) is:
   a) \([3, \infty)\)  b) \((-3, 3)\)  c) \((\infty, 3]\)  d) \((\infty, -3]\) \(\cup\) \([3, \infty)\)
   e) none

13. If \( f(x) = 3^{x-1} \), then the range of \( f \) is:
   a) \([3, \infty)\)  b) \((0, \infty)\)  c) \([1, \infty)\)  d) \((-\infty, \infty)\)  e) none

14. If \( f(x) = \log_2 x^2 \), then the understood domain of \( f \) is:
   a) \((0, \infty)\)  b) \([2, \infty)\)  c) \((-2, 2)\)  d) \((\infty, 0) \cup (0, \infty)\)
   e) none

15. Solve: \( x^2 - x + 1 < 0 \).
   a) \( \left(\frac{1 - \sqrt{5}}{2}, \frac{1 + \sqrt{5}}{2}\right) \)  b) \((0, 1)\)  c) \(\emptyset\)  d) \((\infty, \infty)\)
   e) none

16. If \( P = (2, -1) \) and \( Q = (-1, 3) \) then the distance \( \overline{PQ} \) is:
   a) \(\sqrt{29}\)  b) 1  c) \(\sqrt{5}\)  d) 5  e) none

17. If \( f(x) = x^2 + 1 \) over domain \([0, \infty)\), then
   a) \( f^{-1}(x) = \sqrt{x - 1} \)  b) \( f^{-1}(x) = \sqrt{x + 1} \)  c) \( f^{-1}(x) = \sqrt{x} - 1 \)
   d) \( f^{-1}(x) = 1 + \sqrt{x} \)  e) \( f^{-1} \) does not exist
18. \( \log_8 4 = \)
   a) 2  b) 2/3  c) \(-\frac{1}{2}\)  d) 3/2  e) none

19. The solution set for \( \log_3 (x + 17) = 2 + \log_3 (x + 1) \) is :
   a) \{-17\}  b) 9  c) 1  d) \{-1\}  e) none

20. If \( b^{x^3} = b^{(2x)^2} \), then \( x = \)
   a) 0 or 4  b) 0 or 2  c) -1 or 1  d) 2  e) none
Math 150

Exam III - Form A

Name __________________________

Instructor _____________________

Mar. 3, 1971

Directions: On the answer sheet provided choose the one best response to each of the following question. "None" means "None of the preceding is true."

1. \( \cos\left(\frac{\pi}{3}\right) = \quad\begin{align*}
& a) -\frac{\sqrt{3}}{2} \\
& b) -\frac{1}{2} \\
& c) \frac{1}{2} \\
& d) \frac{\sqrt{3}}{2} \\
& e) \text{ none}
\end{align*}\)

2. \( \csc(-90^\circ) = \quad\begin{align*}
& a) 0 \\
& b) 1 \\
& c) -1 \\
& d) \text{ undefined} \\
& e) \text{ none}
\end{align*}\)

3. \( \cos 3 = \quad\begin{align*}
& a) \sin 3 \\
& b) \sin(\pi-3) \\
& c) \sqrt{1-\sin^2 3} \\
& d) \cos(-3) \\
& e) \text{ none}
\end{align*}\)

4. \( \cos 200^\circ = \quad\begin{align*}
& a) \cos\frac{5\pi}{9} \\
& b) \cos 20^\circ \\
& c) \cos\frac{10\pi}{9} \\
& d) \sin 70^\circ \\
& e) \text{ none}
\end{align*}\)

5. Which of the following statements is true? \( a) \cos 2 > \cos 2^\circ \\
\quad b) |\sin 2| \geq |2| \\
\quad c) \sin 2 < \sin 4 \\
\quad d) \sin\frac{3\pi}{4} < \sin\frac{3\pi}{5} \\
\quad e) \text{ none}
\)

6. A sector is generated in a circle of radius 3 by a central angle of 15°. The arc length of this sector is \( \quad\begin{align*}
& a) \pi/4 \\
& b) \pi/6 \\
& c) \pi/3 \\
& d) \pi/5 \\
& e) \text{ none}
\end{align*}\)

7. If \( \cos x = 2/3, \) then \( |\cos(x - \frac{\pi}{2})| = \quad\begin{align*}
& a) 1/3 \\
& b) 2/3 \\
& c) \sqrt{5}/3 \\
& d) \sqrt{5}/9 \\
& e) \text{ none}
\end{align*}\)

8. \( \cos\frac{2\pi}{3} = \quad\begin{align*}
& a) \sin(4\pi/3) \\
& b) \sin^2 \frac{\pi}{4} \\
& c) \cos(4\pi/3) \\
& d) \sin(-\pi/3) \\
& e) \text{ none}
\end{align*}\)
9. In the right triangle shown, x equals
   (a) \( \sin \alpha \)  (b) \( \csc \alpha \)  (c) \( \tan \alpha \)
   (d) \( \sec \alpha \)  (e) none

10. \((\sin x - \cos x)^2 + \sin 2x =\)
    (a) \( \sin x \)  (b) 0  (c) \( \cos 2x \)  (d) 1  (e) none

11. In the triangle shown at the right, \( \sin \alpha = \)
    (a) \( \frac{x}{y} \sin \beta \)  (b) \( \frac{x}{y} \cos \beta \)  (c) \( xy \sin \beta \)
    (d) \( \frac{y}{x} \sin \beta \)  (e) none

12. \( \sin(\cos^{-1} \frac{1}{2}) = \)
    (a) \( \sqrt{2}/2 \)  (b) \( \sqrt{3}/2 \)  (c) \( \tan \frac{1}{2} \)  (d) \( 1/2 \)  (e) none

13. The solution set of \( \cos t = -1 \) is
    (a) \{\pi\}  (b) \{t | t = k\pi, \ k \text{ an integer}\}  (c) \{\pi, 0, -\pi\}
    (d) \{(2k + 1)\pi | k \text{ an integer}\}  (e) none

14. \( \{x | \cos^2 x = \cos x\} \cap [0, 2\pi] = \)
    (a) \( [0, 2\pi] \)  (b) \( [0, \pi/2, \pi, \frac{3\pi}{2}, 2\pi] \)  (c) \( [0, \pi/2, \frac{3\pi}{2}, 2\pi] \)
    (d) \{0\}  (e) none

15. If \( \tan^{-1} 3x = \frac{\pi}{4} \), then \( x = \)
    (a) \( 1/3 \)  (b) \( \pi/12 \)  (c) \( 3 \)  (d) 0  (e) none

16. The domain of the function \( \sin(\cos^{-1} x) \) is
    (a) \( [-\frac{\pi}{2}, \frac{\pi}{2}] \)  (b) \( [0, \pi] \)  (c) \( [-1, 1] \)  (d) \( \mathbb{R} \)
    (e) none

17. \( \sin(-t)\sec(-t) - \tan(-t) = \)
    (a) \( \sin^2 t \tan t \)  (b) \( 1 + \tan t \)  (c) \( 2 \tan t \)  (d) \( 1 \)
    (e) none
18. The graph of $y = -2 \sin 3x$ is

(a) \[ \ldots \]

(b) \[ \ldots \]

(c) \[ \ldots \]

(d) \[ \ldots \]

(e) \[ \ldots \]

19. The graph of $y = \frac{x}{2} - \cos^{-1}x$ is

(a) \[ \ldots \]

(b) \[ \ldots \]

(c) \[ \ldots \]

(d) \[ \ldots \]

(e) \[ \ldots \]
20. The graph of \( \{(x,y) \mid x = \cos(t - \frac{\pi}{2}), y = \sin t, 0 \leq t \leq \frac{\pi}{2}\} \) is

(a) 
(b) 

(c) 
(d) 

(e)
Math 150

Final - Form A

Mar. 16, 1971

Directions: On the answer sheet provided choose the one best response to each of the following questions. "None" means "None of the preceding is true."

1. $3\sqrt{-2^2 + \sqrt{16}} = \ a) \ 2 \ b) \ -2 \ c) \ undefined \ d) \ 0 \ e) \ none$

2. The solution set of $\frac{x + 1}{|x - 1|} > 0$ is \ a) \ (-1, 1) \cup (1, \infty) \ b) \ (-1, \infty) \ c) \ (1, \infty) \ d) \ (-\infty, -1) \cup (1, \infty) \ e) \ none$

3. In the simplification of \( c(a + ((-a) + b)) \) to be a real axiom whose use is unnecessary is
   \ a) \ Associative Axiom for Addition
   \ b) \ Commutative Axiom for Multiplication
   \ c) \ Identity Axiom for Addition
   \ d) \ Inverse Axiom for Addition
   \ e) \ All of the preceding are necessary

4. $\frac{xy - 1 - 4x - 1y}{2x - 1 + y - 1} = \ a) \ 2y - x \ b) \ x - 2y \ c) \ x + 2y \ d) \ \frac{xy}{x + 2y} \ e) \ none$

5. For $a < 0$, $\sqrt{a} \sqrt{a} = \ a) \ a^{3/2} \ b) \ a^{3/4} \ c) \ \frac{4a^3}{a^3} \ d) \ \sqrt[3]{3} \ e) \ none$

6. The solution set of $(x+2)(x-1) \leq 0$ is \ a) \ $[-2, 1]$ \ b) \ $(-\infty, 1]$ \ c) \ $(-\infty, -2] \cup [1, \infty)$ \ d) \ $[-2, 1]$ \ e) \ none
7. $\{x|a < x < b\} = \{x|x-2 < 1\}$
   a) $a = -\frac{1}{2}$, $b = 1/2$
   b) $a = 3$, $b = 3$  c) $a = 1$, $b = 3$  d) $a = -1$, $b = 3$
   e) none

8. $\frac{\sqrt{5} + 1}{\sqrt{5} - 1} = a) 1 + \frac{1}{2}\sqrt{5}$  b) $\frac{3}{2} + \frac{1}{6}\sqrt{5}$  c) $1 + \frac{1}{3}\sqrt{5}$
   d) $\frac{3}{2} + \frac{1}{2}\sqrt{5}$  e) none

In the following 2 problems $J$ is the set of integers, $Q$ is the
set of rational numbers, $A = \{1, 2, 3\}$, $B = \{x| x \leq 1\}$,
$C = \{x|x \in J \text{ and } x < \sqrt{2}\}$

9. The least upper bound of $(Q \cup A) \cap C$ is
   a) does not exist  b) 2  c) $\sqrt{2}$  d) 1  e) none

10. If $D = \{x|2x - 3 < 0\}$, then
   a) $D \subseteq C$  b) $D \subseteq A$
       c) $B \subseteq D$  d) $A \subseteq D$  e) none

11. The domain of $f(x) = \sqrt{2 - \frac{x}{x}}$ is
   a) $(-\infty, 2]$  b) $[2, \infty)$
       c) $(-\infty, 0) \cup (0, 2]$  d) $(-\infty, \infty)$  e) none

12. If $f(x) = \lceil x \rceil$, $g(x) = x^2$ then $g(f(3/2)) = a) 3/2$
       b) 1  c) $9/4$  d) 2  e) none

13. Which of the following ((a), (b), (c) or (d)) is not a one-to-one
    function?  a) $\{(x,y)|y = (x-2)^2\}$  b) $\{(x,y)|y = 3 \log_2 x + 1\}$
       c) $\{(x,y)|y = \sqrt{1-x^2}, x \in [0,1]\}$  d) $\{(x,y)|y = |2^x| + 1\}$
       e) All of the preceding are one-to-one

14. $\log_2 3 = a) \log_4 2$  b) 2  c) $1/3$  d) $\log_4 9^3$
       e) none

15. $\{x|b^{x^2} = b^{-2x-1}, x \text{ is a real number}\} =$  a) $\{1\}$  b) $\{-1\}$
       c) $\emptyset$  d) $\{1 + \sqrt{2}, 1 - \sqrt{2}\}$  e) none
16. The equation of the line through the points (-1, 1) and (3, 0) is
   a) $y = 12 - 4x$  
   b) $4y = 3 - x$  
   c) $2y = x - 3$  
   d) $y = 4x - 12$  
   e) none

17. The distance between the points (-1, 1) and (3, 0) is
   a) $\sqrt{5}$  
   b) 17  
   c) $\sqrt{10}$  
   d) $\sqrt{17}$  
   e) none

18. If $f(x) = \sqrt{1 - x^2}$ over the domain [-1, 0] then
   a) $f^{-1}(x) = \sqrt{1 - x^2}$  
   b) $f^{-1}(x) = 1 - x^2$  
   c) $f^{-1}(x) = \sqrt{x^2 - 1}$
   d) $f^{-1}(x)$ is undefined  
   e) none

19. Which of the following graphs best represents
   $\{(x, y)|y = \log_2(x - 1)\}$
   a) 
   b) 
   c) 
   d) 
   e) 

20. Which of the following graphs best represents
   $\{(x, y)|y = 2 - \left| x \right|\}$
   a) 
   b) 
   c) 
   d) 
   e)
21. \( \cos 220^\circ = \) a) \( \cos 40^\circ \) b) \( \cos \frac{22\pi}{18} \) c) \( \cos \frac{11\pi}{18} \)
d) \( \cos \frac{11\pi}{36} \) e) none

22. Which of the following is true?
a) \( \sin 3 \) > \( \sin 4 \) b) \( \cos -1 < \cos 1 \) c) \( \sin 2^\circ < \sin 200^\circ \)
d) \( |\sec t| < 1 \) d) all of these are true

23. \( \tan^{-1}(\cos x) = 0 \) if \( x = \) a) 0 b) 1 c) \( \pi \) d) \( \frac{\pi}{2} \)
e) none

24. If \( \cos x = \frac{3}{5} \), then \( |\cos 2x| = \) a) \( \frac{24}{25} \) b) \( \frac{7}{25} \)
c) \( \frac{6}{5} \) d) \( \frac{9}{25} \) e) none

25. In the right triangle at the right, \( x \) equals
a) \( a \cos \alpha \) b) \( a \sin \alpha \)
c) \( a \cot \alpha \) d) \( a \tan \alpha \) e) none

26. In the triangle at the right, \( b \) equals
a) \( a \csc \alpha \sin \beta \) b) \( a \sin \alpha \sin \beta \)
c) \( a^2 - a \cos(\alpha - \beta) \) d) \( a \sin(\beta - \alpha) \)
e) none

27. \( \sin(\cos^{-1} \frac{2}{3}) = \) a) \( \frac{1}{3} \) b) \( \frac{\sqrt{5}}{3} \)
c) \( \sin(\frac{2}{3}) \) d) \( \frac{\sqrt{5}}{3} \)
e) none

28. \( \frac{\sin 2t}{\sin t} - \frac{\cos 2t}{\cos t} = \) a) \( \sin t - \cos t \) b) 0 c) \( \sec t \)
d) \( \cos t - \sec t \) e) none

29. The graph of \( y = -\frac{1}{2} \sin(2x - \frac{\pi}{2}) \) a)

b) c) d) e)
30. The graph of \( y = \sin^{-1}(x) - \frac{\pi}{2} \)
   a) 
   b) 
   c) 
   d) 
   e) 

31. \((3 + 4i) - (6 - 2i)(1 - i) = \)
   a) \(7 - 4i\)
   b) \(-1\)
   c) \(-1 + 12i\)
   d) \(-5 - 4i\)
   e) none

32. \((3 - i)z = 1 - i\) so \(z = \)
   a) \(\frac{2 - i}{5}\)
   b) \(\frac{1 - 2i}{3}\)
   c) \(\frac{1}{3}\)
   d) \(\frac{3 - i}{1 - i}\)
   e) none

33. \(\frac{i}{1 - i} = \)
   a) \(1 + i\)
   b) \(-1 + i\)
   c) \(1 - i\)
   d) \(-i\)
   e) none

34. \(i^2 = \)
   a) \(-1\)
   b) \(1\)
   c) \(0\)
   d) All of these
   e) none

35. \(\sqrt{3} + i = \)
   a) \(\sqrt{2}(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6})\)
   b) \(2(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6})\)
   c) \(\sqrt{2}(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})\)
   d) \(2(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})\)
   e) none

36. \(2(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}) = \)
   a) \(-1 + i \sqrt{3}\)
   b) \(-\sqrt{3} + i\)
   c) \(1 - i \sqrt{3}\)
   d) \(\sqrt{3} - i\)
   e) none

37. \((\tan \frac{\pi}{4} - i \cot \frac{\pi}{4})^3 = \)
   a) \(-1 + i\)
   b) \(1 - i\)
   c) \(-1 - i\)
   d) \(-2(1 + i)\)
   e) none

38. \([3(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})^4 = \)
   a) \((-\frac{1}{2} - i \frac{\sqrt{3}}{2})\)
   b) \(8i(\frac{1}{2} - i \frac{\sqrt{3}}{2})\)

39. \((1 + i)^8 = \)
   a) \(1\)
   b) \(8 + 8i\)
   c) \(16\)
   d) \(1 + i\)
   e) none
40. \(8^{1/3} = \)
a) 2  
b) \(\{2, -1 + i\sqrt{3}\}\)  
c) \(\{2, 1 + i\sqrt{3}, -1 - i\sqrt{3}\}\)  
d) \(\{2, -1 + i\sqrt{3}\}\)  
e) none
1) Test One - Math 159.01
Number of students taking test: 242
Number of items in test: 20

Mean test score: 13.36
Median: 14
Mode: 16
Standard deviation: 3.47
Skewness: -0.42
Kurtosis: -0.07
Maximum: 20
Minimum: 2

Reliability estimates:
Kuder-Richardson 20: 0.741
Kuder-Richardson 21: 0.665

Mean item difficulty: .332
Mean item discrimination: .402

2) Test One - Math 159.01
Number of students taking test: 387
Number of items in test: 20

Mean test score: 12.70
Median: 12
Mode: 12
Standard deviation: 3.17
Skewness: -0.05
Kurtosis: -0.29
Maximum: 19
Minimum: 4

Reliability estimates:
Kuder-Richardson 20: 0.666
Kuder-Richardson 21: 0.568

Mean item difficulty: .365
Mean item discrimination: .386
9) Test Two (Regular Sections) Form A

Number of students taking test: 770
Number of items in test: 20

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Maximum: 20
Minimum: 1

Reliability estimates:

- Kuder-Richardson 20: 0.715
- Kuder-Richardson 21: 0.676

Mean item difficulty: 0.413
Mean item discrimination: 0.464

10) Test Two (Regular Sections) Form B

Number of students taking test: 720
Number of items in test: 20

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Maximum: 20
Minimum: 1

Reliability estimates:

- Kuder-Richardson 20: 0.690
- Kuder-Richardson 21: 0.643

Mean item difficulty: 0.438
Mean item discrimination: 0.445
11) Test Two (Accelerated Pace Hopefuls) Form A

Number of students taking test: 551
Number of items in test: 20

Mean test score: 10.14  Standard deviation: 3.42
Median : 10.00  Skewness : 0.03
Mode : 10.00  Kurtosis : -0.38

Maximum: 19
Minimum: 0

Reliability estimates:

Kuder-Richardson 20: 0.677
Kuder-Richardson 21: 0.602

Mean item difficulty: 0.493  Mean item discrimination: 0.448

12) Test Two (Accelerated Pace Hopefuls) Form B

Number of students taking test: 496
Number of items in test: 20

Mean test score: 10.55  Standard deviation: 3.47
Median : 11.00  Skewness : -0.17
Mode : 10.00  Kurtosis : -0.41

Maximum: 19
Minimum: 1

Reliability estimates:

Kuder-Richardson 20: 0.679
Kuder-Richardson 21: 0.617

Mean item difficulty: 0.472  Mean item discrimination: 0.420
13) Test Two (Accelerated Sections)

Number of students taking test: 327
Number of items in test: 20

Mean test score: 15.88  Standard deviation: 2.29
Median: 16.00  Skewness: -0.52
Mode: 17.00  Kurtosis: -0.25

Maximum: 20
Minimum: 9

Reliability estimates:
Kuder-Richardson 20: 0.541
Kuder-Richardson 21: 0.399

Mean item difficulty: 0.206  Mean item discrimination: 0.289
### 14) Test One (Retake, Reduced Pace) Form A

- Number of students taking test: 702
- Number of items in test: 25

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<td>-0.36</td>
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<tr>
<td>Mode</td>
<td>16.00</td>
<td>Kurtosis</td>
<td>-0.21</td>
</tr>
</tbody>
</table>

- Maximum: 24
- Minimum: 3

Reliability estimates:

- Kuder-Richardson 20: 0.711
- Kuder-Richardson 21: 0.673

- Mean item difficulty: 0.397
- Mean item discrimination: 0.388

### 15) Test One (Retake, Reduced Pace) Form B

- Number of students taking test: 658
- Number of items in test: 25

<table>
<thead>
<tr>
<th>Mean test score</th>
<th>15.03</th>
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<th>4.12</th>
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<td>Kurtosis</td>
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- Maximum: 24
- Minimum: 1

Reliability estimates:

- Kuder-Richardson 20: 0.733
- Kuder-Richardson 21: 0.674

- Mean item difficulty: 0.399
- Mean item discrimination: 0.399
16) Test Two (Reduced Pace) Form A

Number of students taking test: 620
Number of items in test: 38

Mean test score: 22.87  Standard deviation: 6.53
Median : 24.00       Skewness : -0.61
Mode : 23.00         Kurtosis : 0.18

Maximum: 37
Minimum: 2

Reliability estimates:

Kuder-Richardson 20: 0.844
Kuder-Richardson 21: 0.808

Mean item difficulty: 0.398  Mean item discrimination: 0.412

17) Test Two (Reduced Pace) Form B

Number of students taking test: 649
Number of items in test: 38

Mean test score: 23.27  Standard deviation: 6.15
Median : 24.00       Skewness : -0.43
Mode : 23.00         Kurtosis : -0.20

Maximum: 37
Minimum: 3

Reliability estimates:

Kuder-Richardson 20: 0.822
Kuder-Richardson 21: 0.782

Mean item difficulty: 0.388  Mean item discrimination: 0.399
18) Final (Reduced Pace) Form A

Number of students taking test: 393
Number of items in test: 38

Mean test score: 20.87  Standard deviation: 5.77
Median: 21.00  Skewness: -0.03
Mode: 26.00  Kurtosis: -0.43

Maximum: 35
Minimum: 5

Reliability estimates:

Kuder-Richardson 20: 0.787
Kuder-Richardson 21: 0.737

Mean item difficulty: 0.451  Mean item discrimination: 0.369

19) Final (Reduced Pace) Form B

Number of students taking test: 338
Number of items in test: 38

Mean test score: 19.29  Standard deviation: 5.77
Median: 20.00  Skewness: -0.08
Mode: 19.00  Kurtosis: -0.40

Maximum: 33
Minimum: 5

Reliability estimates:

Kuder-Richardson 20: 0.787
Kuder-Richardson 21: 0.734

Mean item difficulty: 0.492  Mean item discrimination: 0.382
APPENDIX D
EQUIVALENCE PAIRINGS FOR ITEMS ON TEST TWO (REGULAR)
WITH TEST TWO AND FINAL (REDUCED PACE)

<table>
<thead>
<tr>
<th>TEST II* (Regular)</th>
<th>TEST II* (Reduced)</th>
<th>FINAL* (Reduced)</th>
<th>TEST II* (Regular)</th>
<th>TEST II* (Reduced)</th>
<th>FINAL (Reduced)</th>
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</thead>
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* Form A
## EQUIVALENCE PAIRINGS FOR ITEMS ON TEST THREE

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<th>MATH 150 T3</th>
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a. Form A
b. Mid. refers to the regular pace 3rd midterm, all other numbers refer to the Math 159.02 regular pace final examination.
c. Mid. refers to the Math 150 3rd midterm, all other numbers refer to questions on the Math 150 final examination.
BIBLIOGRAPHY


