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THE HYDRODYNAMIC APPROACH FOR DESCRIBING THE
MACROSCOPIC BEHAVIOR OF TRAFFIC FLOW ON A
SINGLE LANE OF A MULTI-LANE, ONE-WAY ROADWAY

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

Jeffrey Alan Myers, B.C.E., M.Sc.

*****

The Ohio State University
1973

Reading Committee:
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This study was conducted while the author was employed at the Transportation Research Center, Engineering Experiment Station, of The Ohio State University. The study was undertaken in conjunction with Research Project EES 278, "Investigation of Traffic Dynamics by Aerial Photogrammetry Techniques", sponsored by the Ohio Department of Transportation in cooperation with the Federal Highway Administration of the United States Department of Transportation. The author wishes to express his appreciation to these sponsors for the financial support which made this investigation possible.

The author would also like to thank his adviser, Dr. Joseph Treiterer, Professor of Civil Engineering, under whose direction this study was conducted. Acknowledgement is also given to Dr. Michael B. Godfrey and Mr. Ussamah Salaam for their consultation during the course of the study. In addition thanks is extended to the staff of the Transportation Research Center for their help in the data reduction phase of the investigation.

Finally, special mention is given to Mrs. Barbara Austin for her ceaseless and skillful efforts relative to the typing and editing of this text.
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"A Study of Planning Techniques for the Transportation Research Center", M.S. Thesis, Department of Civil Engineering, The Ohio State University, Columbus, Ohio, 1967


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Studies in Transportation Planning: Professors Zoltan Nemeth and Arthur Hawnn

Studies in Operations Research: Professors Walter C. Giffin and Thomas Rockwell
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<tr>
<td>( q )</td>
<td>traffic volume or flow rate, often referred to for the sake of brevity as traffic flow or simply flow; measured as vehicles per time</td>
<td></td>
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<tr>
<td>( k )</td>
<td>traffic density or concentration; measured as vehicles per distance</td>
<td></td>
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<tr>
<td>( v )</td>
<td>space mean speed of traffic, computed as the average speed of all vehicles occupying a given length of roadway at an instant of time; measured as distance per time</td>
<td></td>
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<tr>
<td>( x )</td>
<td>position on the roadway as measured from an arbitrary origin</td>
<td></td>
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<tr>
<td>( t )</td>
<td>time of observation as measured from an arbitrary origin</td>
<td></td>
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<tr>
<td>( c )</td>
<td>kinematic wave speed, defined either as ( \left( \frac{\Delta x}{\Delta t} \right) ) or ( \left( \frac{dx}{dt} \right) ); measured as distance per time</td>
<td></td>
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<td>( U )</td>
<td>shock wave speed, defined as ( \frac{\Delta x}{\Delta t} = \frac{q_s - q_{s}}{k_s - k_t} ); measured as distance per time</td>
<td></td>
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<tr>
<td>( c_r )</td>
<td>constant, used by Richards in theoretical derivation; measured as distance per time</td>
<td></td>
</tr>
<tr>
<td>( c_g )</td>
<td>constant, used by Greenberg in theoretical derivation; measured as distance per time</td>
<td></td>
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<tr>
<td>( s(x, t) )</td>
<td>source-sink term, used in modification of the one-dimensional equation of continuity to account for lane changing; measured as vehicles per distance-time</td>
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<tr>
<td>( n )</td>
<td>number of vehicles occupying a given domain either in space or time</td>
<td></td>
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<tr>
<td>( \tau )</td>
<td>interval of time of predetermined length</td>
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X  interval of space of predetermined length such as roadway section

$s_1$  intervehicular spacing, the distance from the front bumper of a given car to the front bumper of the car behind it

$a$  average acceleration, defined as $\frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$; measured as distance per time squared

$v_f$  free flow speed, the average speed of vehicles on a roadway section where density approaches zero; measured in distance per time

$k_j$  jam density, the maximum possible density on a roadway section at which average speed equals zero; measured as vehicles per distance

$k_m$  optimum density, the density for which volume is a maximum; measured in vehicles per distance

$A, B, a_1, a_2, a_3$  coefficients, used in various speed-density and volume-density equations

$R^2$  coefficient of determination, a measure of the amount of variation in a dependent variable explained by regressing it on one or more independent variables; dimensionless

$\alpha_3$  first shape factor, defined as the ratio of the third central moment of a distribution to the cube of the corresponding standard deviation; dimensionless, used as a measure of departure from symmetry

$CV$  coefficient of variation, defined as the ratio of the standard deviation of a distribution to the corresponding mean; dimensionless, used as a relative measure of variation
CHAPTER 1

INTRODUCTION

1.1 BACKGROUND

It is difficult to overemphasize the importance of transportation to twentieth century society. Transportation is essential to modern man in his business pursuits, in the operation of his government, in the quest for his education, in the provision of his recreational opportunities and in the preservation of his national liberties. The ability to travel to the destination of his choice at the time of his choice comfortably, economically, rapidly and safely has come to represent to many members of society a right rather than a privilege. Such an attitude is especially apparent in technologically developed countries such as the United States. At present approximately 20% of the Gross National Product of this country is being spent on transportation and transportation-related expenditures. Every day the factories of Detroit are turning out additional vehicles. In addition every year more and more people are acquiring the affluence necessary to purchase these vehicles. Each new owner expects to enjoy the increased mobility which his personal automobile offers him. He looks expectantly to those employed in the transportation related professions to provide him the accessibility he feels he deserves.

For many years the construction of new facilities served as the answer
of the transportation industry to this cry for accessibility. Highway expenditures were increased manyfold. Money was also poured into subsidies for railroads, airlines and even canals. Extensive systems of new facilities sprang up across the face of the nation. Each new addition was greeted by a cry for still increased construction expenditures. In 1956 the demand for transportation culminated in the funding of the National System of Interstate and Defense Highways, a project which represented the largest single public works expenditure in the history of the world. Nevertheless the call for new and improved transportation facilities continued unabated.

In recent years it has become apparent that new construction alone cannot satisfy the transportation needs of a complex and affluent society. Instead, construction must be supplemented by programs aimed at providing for more efficient utilization of existing facilities. The heart of these programs is the development of improved methods of traffic control. Before such control methodologies can be meaningfully developed, however, it is first necessary that the mechanisms involved in traffic flow be fully understood. In effect, a unified theory of traffic flow is required.

1.2 DESCRIPTION OF THE PROBLEM

The need for a comprehensive body of knowledge relevant to the important characteristics of traffic movement has long been recognized by professionals working in the transportation field. The first step toward such a theory of traffic was taken nearly 40 years ago in the early work of Greenshields. It has been only within the past 15 years, however, that the study of traffic has become
an accepted field of endeavor for researchers from the scientific disciplines. In this short time an extensive body of literature has sprung up which can be generally categorized under the heading of investigations into the theory of traffic flow. Contributors to this literature are found in all parts of the technically advanced world and include practitioners from the fields of physics, applied mathematics, statistics, operations research, economics, psychology and engineering to name but a few.

Topics considered in the literature vary widely in nature and reported studies range from the very primitive to the very sophisticated. In each case the underlying purpose is the same, however, that being to provide some contribution to the understanding of the complex phenomenon known as vehicular traffic. Much has been accomplished but much more remains to be done. One specific topic area in which study has been particularly lacking is that of traffic flow in a multiple-lane environment. The major factor inhibiting study in this area has been the lack of a suitable data base through which existing theories of traffic behavior could be compared with actual behavior. The aerial photogrammetric data collection technique developed by the Transportation Research Center of The Ohio State University offers the potential for collecting the required type of data on multiple-lane traffic flow. The availability of this data presents a unique opportunity for the researcher interested in providing a meaningful contribution to the traffic flow theory literature.

Insufficient data has been collected to date to provide the comprehensive data base needed to permit the type of exhaustive studies which will eventually
be required if a full understanding of the mechanisms of multi-lane flow is to be attained. Enough data is presently available, however, to make a beginning toward this end. The study documented in the following chapters is an attempt to provide that beginning.

1.3 OBJECTIVE OF THE STUDY

There are many approaches available for investigating multi-lane traffic flow characteristics. These approaches can be classified into four general categories as follows: car-following and control system approaches, statistical approaches, simulation approaches and analogical approaches. One interesting analogical approach treats traffic flow in a lane as being analogous to a one-dimensional fluid flowing in a long duct. This approach is referred to as the hydrodynamic approach and is the one that was chosen for investigation in this preliminary study. It was chosen because it represents an interesting approach to the analysis of single lane traffic flowing in a multi-lane environment and because it offers the potential of providing a valuable qualitative description of macroscopic traffic behavior under crowded roadway conditions. Such crowded conditions have become the rule rather than the exception on the modern urban freeway.

Accordingly the objective of this study is to investigate the applicability of the hydrodynamic approach for describing the macroscopic behavior of traffic flow on a single lane of a multi-lane, one-way roadway. In the course of the study both the qualitative and the quantitative descriptions of flow behavior provided by the fluid theory will be studied and evaluated.
1.4 SCOPE OF THE STUDY

There are two prevailing constraints that will act to limit the scope of the experimental investigation. The most significant restriction results from the scarcity of available data. As a result only a single real world flow situation can be analyzed during the experimental study. The second constraint involves the time available for the study. The experimental investigation reported herein constitutes only one part of a larger research project. Thus it must adhere to a time schedule dictated by the agreed-upon completion date of that project. This time constraint will not affect the fundamental nature of the study in any way except that it prohibits the collection of additional traffic flow data. Thus currently available data must be used in spite of its scarcity.

1.5 STRUCTURE OF THE REPORT

This report is organized into five chapters in addition to this one. Chapter 2 provides a brief review of the state of the art of traffic flow theory. No attempt is made to document all the important contributions made to date for such is an impossible task. The purpose is rather to describe the general approaches which have been used to study traffic behavior and to present studies representative of each approach. In this way a framework is provided into which the present study can be fit. This chapter can be readily omitted by the reader well acquainted with the annals of traffic flow theory.

Chapter 3 provides a detailed description of the major studies conducted to date using the hydrodynamic approach. Documentation is provided of the original theoretical work published by Lighthill and Whitham in 1955 and
Richards in 1956. In addition discussion is devoted to a number of other theoretical investigations that have attempted to build on these theoretical foundations as well as to a group of experimental studies that have been conducted for the purposes of testing the applicability of the theory. For the most part these experimental studies have been conducted on test tracks and in tunnels. Little experimental effort has been directed toward the treatment of a multi-lane case where lane changing is permitted.

Chapter 4 discusses the fundamental assumptions of the hydrodynamic approach as they apply to the specific flow situation being proposed for analysis. Specific issues treated in this discussion include the question of the continuity of the traffic fluid, the implications of scale to the experimental investigation and the problems attendant to the establishment of an appropriate mathematical form for the traffic equation of state. Each of these issues is an essential factor in determining the applicability of the hydrodynamic approach. The data available for use in the experimental investigation is also described.

Chapter 5 reports the results of the actual application of the approach to the study of flow behavior in the southbound median lane of Interstate 71 in northern Columbus, Ohio. The most notable attribute of the hydrodynamic theory is the existence of traffic waves (termed kinematic waves) along which changes in the fundamental parameters of flow are propagated through the traffic fluid. The existence of such waves is confirmed in the median lane flow and their characteristics are studied. The characteristics of the empirical waves are then compared with those predicted by the theory. An attempt is also
made to isolate the existence of a discontinuous wave (termed a shock wave) in the traffic fluid. The potential for the existence of such a wave is predicted by the theory but no such wave was located in the experimental data.

Chapter 6 provides a summary of the results of the study as well as the conclusions and recommendations for further study made as a result of this preliminary research. Briefly it is concluded that the hydrodynamic approach can be applied to certain carefully defined multi-lane flow problems. In return it will yield a reasonable qualitative description of macroscopic flow behavior on a per lane basis but no reliable quantitative results can be expected. A major reason for the failure of the theory to provide quantitative results is identified as the difficulty of properly defining the traffic equation of state in mathematical terms.
CHAPTER 2

THE THEORY OF TRAFFIC FLOW:

AN OVERVIEW

2.1 INTRODUCTION

From somewhat humble beginnings in the pioneering work of Greenshields in 1934 (1) the study of the theory underlying the behavior of traffic flow has become the arena for extensive research by professionals in the fields of physics, applied mathematics, statistics, psychology, systems analysis and engineering. The reasons for this burgeoning interest in identifying and defining the nature of the mechanism of traffic flow are many and varied. They embrace both the changing emphasis within the scientific community which encourages the scientist to leave his ivory tower and to come to grips with the pressing problems of everyday life and the increasing necessity of modifying the products of on-rushing technology to better serve mankind. One such product is the automobile which was created to benefit mankind but now threatens to enslave him. The endless proliferation of the automobile renders the study of traffic flow, in the words of Herman (2), "...a very meaningful discipline" which until quite recently has been virtually unexplored.

Over the past 15 years a rather extensive body of literature has grown up documenting preliminary as well as a few refined efforts at developing a
theory describing the flow of traffic. Contributors to this literature come from all of the aforementioned disciplines as well as others and include representatives from most of the technologically developed countries of the world. In addition to the literature this group of "traffic scientists" has met in a number of conventions including five international symposia to discuss the past, the present and the future of traffic science. Each article and each meeting has brought them closer to the attainment of a comprehensive understanding of the phenomenon of traffic flow. Much more, however, remains to be learned. It is hoped that the study documented in this paper will offer a small contribution to this quest for knowledge.

In order to provide a proper context for the experimental study reported herein a brief summary of the state of the art of traffic flow theory as it exists today is required. The purpose of this summary is not to provide a comprehensive review of all the significant work that has been done for such is impossible. It is designed rather to provide a framework into which the current study can be fit.

Basic areas of investigation into the mechanism of the traffic flow process pursued to date can be conveniently divided into four categories of approach. These are:

1. Car-following and control system approaches
2. Statistical approaches
3. Simulation approaches
4. Analogical approaches
An overview of some of the work completed in each of these respective categories is given in the following sections.

2.2 CAR-FOLLOWING AND CONTROL SYSTEM APPROACHES

The theory of car-following treats the traffic flow as a composite of a number of distinct vehicle pairs, one following the other along the roadway. It attempts to explain the behavior of an individual vehicle-driver unit in terms of its relation to the other vehicles in its immediate vicinity. Most models relate unit behavior to the behavior of only the vehicle immediately in front of it while a few more complicated models attempt to incorporate the relation between a given follower and two or more leading vehicles and perhaps even the trailing vehicle. In all cases the car-following or control system approach utilizes a microscopic view of the traffic stream. Due to the fundamental assumption of the theory that total traffic behavior is a composite of the behavior of individual units following one another, the car-following approach is strictly valid only for dense, single-lane traffic where passing is not permitted. It can be applied on a less rigorous basis, however, to any real world situation in which the above conditions are approximated.

The basic relation of car-following theory is a psychological rather than a physical one. Specifically it is stated that the action taken by a given driver-vehicle unit depends upon the stimulus applied to that unit and the sensitivity of that unit in responding to a given stimulus factor. In equation form this relation can be expressed as:

$$\text{Response} = \text{Sensitivity} \times \text{Stimulus}$$

(2.1)
Response is measured in terms of the acceleration of the following vehicle since this is the parameter of motion most directly under a given operator's control. Positive acceleration can be achieved immediately through application of the vehicle's accelerator while negative acceleration is available through a simple braking action. The stimulus factor could be represented by any of a number of microscopic traffic parameters which the driver evaluates in the process of decision-making. A number of experiments have indicated, however, that the rate of closure between a driver-vehicle unit and the vehicle ahead of it is well correlated with driver action. Thus relative speed between leader and follower is generally used as the stimulus measure.

Many different parameters have been proposed as quantitative representations of driver sensitivity. Among these are the spacing between leader and follower, the square of the spacing and the headway between the related units. A particularly simple model utilizes a constant sensitivity factor. In addition more complex models have been proposed which hypothesize a different sensitivity factor for accelerating as compared to decelerating behavior and which use a combination of sensitivity factors to further relate the follower's behavior to that of the next nearest neighbor and even the trailing vehicle.

Pioneering work employing the car-following type of approach was done by Reuschel and Pipes (3). These authors discussed a traffic flow model which utilizes a differential-difference equation to relate the motion of a vehicle to that of the vehicle in front of it. Extensions of the basic concept have been proposed by many researchers with extensive work in this area being conducted...
at the General Motors Research Laboratories by Herman, Montroll, Chandler, Gazis, Rothery and Potts. One of the important results of the early General Motors studies was the introduction of a lag factor or reaction time into the basic car-following relationship. This lag factor expresses the need of the following driver for some time to evaluate a given stimulus and decide upon his intended course of action before making his response. Another result of the General Motors studies was the formulation of a general car-following law incorporating the lag factor. The proposed law is given by:

\[ \dot{X}_{n+1}(t+T) = \frac{c \left[ \dot{X}_{n+1}(t+T) \right]^1}{\left[ \dot{X}_n(t) - \dot{X}_{n+1}(t) \right]^m} \left[ \ddot{X}_n(t) - \ddot{X}_{n+1}(t) \right] \]  

(2.2)

where:

- \( \ddot{X}_{n+1}(t+T) \) = acceleration of the follower at time \( t+T \)
- \( \dot{X}_{n+1}(t+T) \) = velocity of the follower at time \( t+T \)
- \( \dot{X}_{n+1}(t) \) = velocity of the follower at time \( t \)
- \( \dot{X}_n(t) \) = velocity of the leader at time \( t \)
- \( X_{n+1}(t) \) = position of the follower at time \( t \)
- \( X_n(t) \) = position of the leader at time \( t \)
- \( c \) = constant
- \( 1, m \) = integer exponents
- \( T \) = lag factor

A series of studies was conducted to determine the ability of the theoretical model, using different values for the exponents \( 1 \) and \( m \), to describe actual traffic behavior as represented by data collected on the General Motors test track and in the various tunnels under the control of the Port of New York Authority. Specific models tested included:
a) $l = 0, m = 0$: linear model or relative velocity follower

b) $l = 0, m = 1$: reciprocal spacing model

c) $l = 0, m = 2$: squared reciprocal spacing model

d) $l = 1, m = 2$: headway-reciprocal spacing model

These studies were documented in a lengthy series of publications (4, 5, 6, 7, 8, 9, 10).

As a result of these studies the researchers concluded that the concept of car-following yields valuable insights into the mechanisms governing the flow of vehicular traffic. These insights include the car-following behavior itself as well as certain aspects of the macroscopic behavior of the traffic stream. The two most important macroscopic aspects related to the car-following approach are the concept of the stability of traffic flow and the derivation of various macroscopic equations of state starting from the basic car-following law. With respect to the applicability of the various alternative models it was determined that non-linear forms such as those shown above as b, c, and d provide a better description of actual traffic behavior than does the linear model formulation. However, the available data was insufficient to establish the absolute superiority of any specific non-linear model (11).

Certain studies have been conducted employing extensions of the basic car-following concept. Newell proposed a model in which different sensitivity factors are used for accelerating as compared to decelerating traffic conditions (12). The rationale for such an approach is based on a group of studies in which it was shown that drivers react more slowly to accelerations than to decelerations (13, 8). This extension helps to explain many of the differences
noted between the behavior of traffic in a queue forming as compared to a queue releasing situation. Testing of this model to date, however, has not been extensive due to the lack of adequate empirical data.

Drew has proposed a concept which he terms car maneuvering which involves a further extension of the car-following approach (14). In his model Drew hypothesizes that a driver is stimulated to action not only by the car immediately ahead but also by the motions of several proceeding vehicles and the car immediately behind. He then formulates an equation of motion with the following form:

\[ \ddot{x}_i = -C_i^2 K_i^n K'_i(x) \]  

(2.3)

where \( i \) refers to a portion of the traffic stream, \( C_i^2 \) is the sensitivity coefficient and \( K \) is traffic density as a function of position. Using this format the behavior of any number of vehicles can be analyzed. Although this approach has considerable intuitive and logical appeal its ability to describe actual driver behavior is still unestablished.

2.3 STATISTICAL APPROACHES

Traffic flow by its very nature is a stochastic phenomenon. It is a process made up of varying elements with varying characteristics and capabilities intent on the pursuit of varying goals. As a result of its innate variability the traffic stream is extremely susceptible to problems of interaction between elements manifested in congestion. This congestion can greatly diminish the efficiency with which traffic moves. The investigation of the causes and extent
of traffic congestion, therefore, becomes an important part of the overall study of the mechanism of traffic flow.

Due to the stochastic nature of traffic a substantial number of researchers have turned to the methods of probability and mathematical statistics in an effort to find suitable descriptions of traffic characteristics. Specific analytical techniques employed for this purpose include distribution modeling, queuing theory, Markov processes and a number of optimization procedures. Models employed vary widely both in terms of degree of mathematical complexity and in terms of the realism with which the traffic situation in question is treated. Many of the models are quite primitive but each is an attempt to provide further insight into the complex entity known as vehicular traffic. Two valuable summaries of some of the modeling efforts performed to date using probabilistic and statistical techniques are provided by Highway Research Board Special Report 79 (15) and a book on mathematical approaches to traffic flow by Haight (16).

Two of the most popular areas of investigation for traffic scientists armed with an arsenal of statistical methods have been the study of the distribution of various parameters of the traffic stream in both the time and space domain and the analysis of delay characteristics at roadway intersections. These areas both offer a number of specific problems amenable to mathematical attack.

Volume, density and average speed are the parameters generally used by the practicing traffic engineer to describe the state of a traffic flow. In this usage each parameter is defined as an average value over some slice of the
space-time plane and the statistical variability of these averages is only of interest from a large scale point of view. Arrivals, spacing, headway and spot speed, however, are parameters which can be used to characterize the microscopic nature of traffic flow. The distributions of these parameters in both time and space provide a valuable indication of the local nature of the traffic stream. The distribution of gap sizes acceptable to drivers desiring to cross or merge into a moving traffic stream is another valuable descriptor of local flow conditions. Consequently much research effort has been devoted to finding suitable theoretical probability distributions for characterizing the variability displayed by these parameters in actual traffic flows. Representative studies of this type have been made and reported by Gerlough (17), Haight (18), Schuhl (19), Oliver (20), Raff (21) and Brieman (22) to name but a very few. Distributions which have been tested and found useful for certain applications include Poisson, Generalized Poisson and Uniform for arrivals; Translated Negative Exponential and Pearson Type III for spacings; Negative Exponential, Translated Negative Exponential, Composite Exponential, Pearson Type III and Log-normal for headways; Normal and Pearson Types I and III for spot speeds; and the Log-normal and Pearson Type III for gap acceptance.

Congestion at roadway intersections and the resulting delay to motorists is one of the most serious problems facing the practicing traffic engineer. It is not surprising, therefore, that the analysis of congestion and delay has become an important topic of concern to the traffic scientist. Since the parameters which enter into the intersection delay problem are of a stochastic nature
probabalistic and statistical approaches are well suited for predicting the
resulting delay characteristics. A number of models have been developed to
describe intersection delays. Analysis has included delays to non-priority
traffic at stop sign controlled intersections, delays to vehicles arriving at a
signalized intersection using fixed time control, delays caused to traffic by
vehicle actuated signals and delays to pedestrians attempting to cross a stream
of vehicular traffic. A brief review of a representative selection of this work is
presented below.

Jewell studied the distribution of waiting times of non-priority vehicles
stopped at a stop sign (15). He assumed an arbitrary main street headway
distribution and analyzed the behavior of the mean, variance and distribution of
waiting time for a number of main street conditions at the time of arrival of a
side street vehicle. A primary result of the study was the computation of the
expected side street discharge during a given interval of time assuming that
multiple vehicle discharges into a single gap are impossible. In addition he
showed that expected delay to a side street vehicle increases as the second or
third power of the critical gap and linearly with increasing volume. The variance
of delay was shown to be related to a third or higher power of the critical gap.

Tanner has modeled a T-intersection controlled by a stop sign (23). He
assumed random arrivals on both the major and minor roads and specified
minimum allowable time intervals between both consecutive major and minor
road vehicles passing through the intersection. There was also a restriction
imposed on how closely a minor road vehicle could follow a major road vehicle
into the intersection crossing area but no restriction on major road vehicles following minor road vehicles. Expressions were derived for computing the expected delay to both a major and a minor road vehicle. An expression was also derived for the maximum possible flow on the minor road. This flow combined with the major road flow establishes the expected capacity of the intersection for a given major road flow condition.

Little analyzed the delays occurring at a fixed time signal controlled intersection under a set of idealized conditions (24). Arrivals are assumed to be random and are restricted to a single lane. Stopping and starting at the intersection are assumed to be instantaneous maneuvers and vehicles are discharged into the intersection with constant headways. No turning is allowed at the intersection. Based on these assumptions an expression was developed for average queue length at the signal. Since discharge headways are taken to be constant the expected time required for a queue to be dissolved could also be computed. Little then considered two possible extensions of his model for multi-lane traffic. The first extension assumes that arriving vehicles join the shortest existing queue at the traffic signal while the second model assumes that arrivals form separate and independent streams of traffic. Of the two models the second one seems to come closer to predicting a realistic average queue length. Newell used the method of Markov analysis to study the flow through a signal controlled intersection (25). He defined three possible states of the intersection system based on the desires of motorists arriving from opposing directions and developed a set of transition probabilities for moving from state to state. Using this
approach a mathematical expression was derived for the average number of vehicles expected to clear the intersection in any one signal cycle. The expression is too complex to be computationally practical but can be used to analyze a number of special cases.

Delay to traffic at intersections controlled by vehicle-actuated signals has been studied by Haight (26) and Garwood(27). Haight analyzed the case of a semi-actuated signal using the techniques of statistical analysis. He assumed random arrivals on both main and side street approaches and derived expressions for the distribution of main street red phases and the number of vehicles which can pass through the intersection during a given main street green phase. He then describes how these expressions can be used to derive the probability of overflows on the main street. His expressions, like Newell's, are too complicated for general use. Garwood considered the case of a fully actuated signal with arrivals on all approaches assumed to be random. He then developed expressions for the cumulative waiting time distribution and for the expected waiting time of a vehicle arriving at the intersection in the absence of an existing queue. Analytical solution of both expressions is possible but requires much tedious effort. Limited graphical results are available for the solution of cases for a selected group of traffic flow and signal timing parameters.

Preliminary results describing the delay to pedestrians attempting to cross a traffic stream were reported by Adams (28). He assumed that both vehicles and pedestrians arrive according to a Poisson distribution and developed an expression relating probability of delay to the vehicular volume and the
length of the critical gap acceptable to allow safe crossing. Expressions were also developed for the expected delay per pedestrian for all pedestrians and for those pedestrians delayed. A more comprehensive study of delay to pedestrians was conducted by Tanner (29). This researcher analyzed a variety of situations involving conflict between vehicle and pedestrian streams. In addition to the traditional approach which considers pedestrians crossing the street as individual entities, Tanner analyzed the case of pedestrians serviced in bulk and also compared the delay to pedestrians crossing an entire roadway at a single time with that experienced by those stopping midway at a refuge island. Expressions were developed for computing the average size of a pedestrian bunch waiting to cross and the expected delay suffered by both the full-street and refuge island crossing pedestrians. Notable work on delay to pedestrians has also been done by Weiss (30) and Moskowitz (31).

2.4 SIMULATION APPROACHES

Many problems that need to be studied with regard to the mechanisms of traffic flow are too large, too complex and involve too many variables to be modeled effectively using analytical techniques. Such problems can often be treated, however, using the method of computer simulation. With the phenomenal rise in the sophistication of computer technology that has occurred during the last ten years many researchers have turned to simulation techniques to solve problems that have been previously intractable. Among the traffic situations that have been successfully approached to date using simulation models are flow behavior through complex intersections, flow through a system of
adjacent intersections, flow in an underwater vehicular tunnel, merging behavior at a freeway entrance ramp and weaving behavior on a multi-lane roadway section.

The underlying concept of the simulation approach is the preparation of a model of a real world system which extracts the fundamental essence of the system without including unnecessary components which would cause the model to become bulky, overly sensitive to the influences of unimportant variables or prohibitively expensive to use. An ideal simulation model is simple and logical in construction, contains the essential elements needed to describe the behavior of the modeled system with the desired degree of accuracy, excludes all unnecessary and redundant elements, allows the important parameters of the system to be varied within the desired range of study and possesses a low simulation time–real time ratio. The validity of the model output must always be checked against field data collected in the real world system environment.

A typical computer simulation study requires the accomplishment of the following steps:

1. Definition of the Problem.—The traffic system to be modeled must be defined in specific detail. Items to be considered at this preliminary stage include the identification of parameters and variables of importance, the delineation of rules governing system operation, the determination of relevant system inputs and outputs and the specification of alternative measures of effectiveness which can be used to evaluate model validity.

2. Formulation of the Mathematical Model.—The most important step in
computer simulation study is the construction of a mathematical model which adequately characterizes the operation of the system to be studied. The usual approach to model formulation divides the total system into a number of distinct parts or sub-systems. A separate mathematical model is then prepared for each sub-system with careful consideration given to the design of the interface between sub-systems. During the model formulation stage a decision must be made regarding which of the parameters, variables, operating rules, inputs, outputs and measures of effectiveness identified in Step 1 are to be actually incorporated in the simulation model.

3. Preparation of the Computer Program.—Once the individual sub-system models have been formulated in mathematical terms and the proper relationship between sub-models has been established, the total model must be coded for computer analysis. There are several languages available which can be used to communicate the details of the model from its mathematical form to a form suitable for the computer. These include both general programming languages such as FORTRAN and a host of special simulation languages such as GPSS, DYNAMO and SIMSCRIPT. Although choice of language is optional, the special simulation languages generally yield a more flexible modeling capability. This flexibility is very desirable since all possible system alternatives to be evaluated using the model can often not be foreseen in the model formulation stage. In addition, the simulation languages usually require less programming time thus expediting the completion of the modeling task.

4. Validation of the Computer Model.—Before a computer simulation
model can be used as a predictor of system behavior in the context of an experimental study, its reliability for producing accurate output for a given set of input conditions must be established. This model testing procedure is commonly referred to as validating the model. Model validity can be checked either using existing historical data on system behavior or employing fresh data collected especially for validation purposes. If historical data has been used to formulate the model, however, it is generally not a good procedure to validate the model using the same data. The validity of the model should be checked as completely as possible over the full range of parameters and variables expected to be used during the experimental study. If the model behavior agrees with actual system behavior with the desired degree of accuracy the experimental study can be begun. If substantial disagreement occurs a minor or major reformulation of the model may be required.

5. Experimentation.—The simulation technique is really no more than a method of studying a system by conducting a series of sampling experiments on it. Since it is usually impossible or at least very difficult and quite expensive to work with the actual system, a model is constructed and the experiments are conducted on the model. The behavior of the model under a given set of conditions is then used to predict the behavior of the real world system under the corresponding conditions. The experimental part of a simulation study thus consists of a number of runs of the model for each of a variety of pertinent operating conditions. The input parameters and variables are systematically changed and the resulting output for each set of input conditions is recorded.
Each input condition is run often enough to provide a suitable degree of statistical accuracy in the evaluation of the corresponding output. Since each run of the model may require a substantial outlay of both time and money the experimental study must be very carefully planned such that acceptable results are achieved with a minimum number of model runs.

6. Evaluation of Experimental Results.—Once the experimental study has been completed the results are evaluated with regard to their implications relative to system behavior. Based on the experimental results a judgment can be made as to which set of inputs should be used to provide a desired output or which decision-rule produces the most acceptable system behavior. This information can then be used to make suitable adjustments to the real world system.

One of the first applications of the technique of simulation to the study of roadway traffic was reported by Mathewson, Trautman and Gerlough in 1954 (32). The traffic situation considered was a standard four approach signal controlled intersection with traffic confined to one lane on each approach. Pedestrians were permitted and drivers on each approach had the choice of proceeding straight ahead or turning left or right. Various simulation approaches were considered including the use of a discrete-variable simulator, a continuous-variable simulator and a specially programmed general purpose computer. A diagrammatic representation of a typical program for use on a general purpose computer is presented but no actual application of the proposed simulation model is made. A completed simulation of the four approach intersection
situ atio n  was rep o rted  the next y e a r, however, by Goode, Pollmar and Wright (33). T his model assumed random arrivals on all four approaches and simu­ lated intersection operation under a number of volume, turn fraction and signal timing conditions. Average delay was used as the measure of effectiveness for evaluating different input parameter combinations. No attempt was made to validate the completed model.

Webster carried out an extensive simulation study of traffic behavior at a signal controlled intersection for the purpose of developing a logical method for determining signal settings (34). The basic premise of his modeling tech­ nique was that each approach of a fixed time signalized intersection operates independently of the other approaches and thus can be studied separately. He assumed that vehicles arrive at random and are discharged from an approach queue at a constant rate called the saturation flow. Discharge continues through­ out an interval of time referred to as the effective green time and no flow is allowed during the red time. Parameters considered in the construction of the model include approach width, parking conditions, traffic composition, per­ centage of turning maneuvers, demand volume and the signal timing. Total delay to all approach vehicles is used as the measure of effectiveness for evalu­ ating different signal settings. As a result of the study Webster derived an analytical expression which can be used to compute an optimum cycle length which will minimize total delay. A procedure is also suggested for allocating green time among competing approaches. Webster's study was performed for conditions commonly found in England. Wentzel has done a similar study for
conditions common to the United States (35). Computer models for simulating a series of adjacent intersections have been developed by Goode and True (15), Francis and Lott (36) and Vecellio (37).

Drew describes a model developed to simulate vehicle merging behavior in the vicinity of a freeway entrance ramp (14). Variables considered by the model include the number of freeway lanes, the length of the freeway section, the number of on and off ramps in the section, the ramp locations, the length of the on ramp acceleration lane, the overall ramp length, the grade of the freeway section and the location of the grade relative to the beginning of the section. Model operation is begun with vehicles placed at random on the freeway section with each vehicle placed a safe distance from its neighbors. Each vehicle is assigned a desired speed, a current speed and a position relative to a zero reference point. The vehicles are then processed along the freeway until they depart onto an exit ramp or exit the study section. A given vehicle can proceed straight ahead or weave either left or right whenever possible. It accelerates or decelerates depending upon its current speed, its desired speed and its position relative to the vehicle directly in front of it. A weaving vehicle increases its speed to a new speed called the weaving speed. New vehicles can enter the system via an entrance ramp or at the input end of the simulated section. Entrance ramp vehicles merge into the shoulder lane of the freeway according to a merging logic and are then processed as freeway vehicles. The simulation continues until a preselected number of vehicles have been processed. The output of the model includes a table of average values of a number of flow
parameters. A space-time diagram representing the behavior of all processed vehicles can also be plotted.

Helly developed a simulation model for analyzing the bottleneck behavior of automobile traffic traveling in a single lane with no passing or lane changing permitted (38). This model uses a general car-following type of control equation to describe the interaction mechanism existing between a given vehicle and the vehicle ahead of it. The basic control methodology is developed around the hypothesis that each driver tries to minimize both the difference between his actual headway and some desired headway and the relative velocity between himself and the car ahead. The parameters of the control equation are determined from experimental data collected from a variety of sources. Helly uses his model to study the effect of bottlenecks on the flow of vehicles through vehicular tunnels. Two distinct types of bottlenecks are defined; a type one bottleneck in which a driver's behavior is independent of his position in the traffic stream and a type two bottleneck in which it is not. The consequences of each type of bottleneck to the flow of traffic are explored and certain recommendations are offered for alleviating the effect of type two bottlenecks.

Other simulation studies of interest have been reported by Kell for intersections (39), Perchonok and Levy for freeway on-ramps (40) and Glickstein, Findley and Levy for interchanges (41).

2.5 ANALOGICAL APPROACHES

The three foregoing approaches all attempt to treat the traffic stream directly through the development of models specifically designed to describe
different traffic situations. Another method of attack treats the traffic flow as being analogous to some selected physical system for which rules of system behavior have been thoroughly investigated and are well established. These behavior rules are then used to make inferences about the behavior of traffic under a similar set of external conditions. The accuracy with which these inferences actually describe real world traffic behavior depends upon the extent to which the traffic system obeys the postulates that underlie the operation of the analogous physical system. The analogical approach, generally speaking, takes a macroscopic view of traffic and provides results which are indicative of flow behavior as a whole but ignore the individuality of the single driver-vehicle combination. Thus models derived in this manner often provide qualitative rather than strictly quantitative flow descriptions.

The best known of the analogical traffic models is the hydrodynamic approach due to Lighthill and Whitham (42) and Richards (43). In this approach traffic is treated as a compressible fluid moving through a duct in a one dimensional flow. The density of the fluid varies as it moves and the flow rate is assumed to follow an approximate functional relationship with density at each point along the length of the duct. The only other basic assumption required to complete the analogy is that matter (vehicles) is conserved throughout the flow. Using this approach it can be shown that small inhomogeneities in the fundamental traffic parameters (speed, volume or density) will be propagated back through the traffic stream as waves similar to those found in flood movements in long rivers. Along the length of such a wave volume, speed and density
remain constant. The wave will travel in a straight line through the space-time plane if a single flow-density relation prevails throughout the length of the duct (roadway) and will follow a curved path if the flow-density relation varies with position. It is also shown that two or more such waves can run together to form a discontinuous wave or "shock wave" such as is commonly found in gaseous flows. This shock wave will also propagate back through the traffic stream and vehicles encountering it will be forced to change speeds abruptly as they pass through the shock.

Although the hydrodynamic analogy pertains strictly only to single lane flows on long, crowded roads it can also be used to provide approximate descriptions for small scale problems such as flows through bottlenecks and the starting and stopping waves created by traffic lights. It may also be possible to generalize the theory for application to high density, multi-lane flows where lane changing is difficult and thus uncommon. Such is the application described in the succeeding chapters of this paper. Hence, no more will be said about the approach at this time as it will be described in great detail in the next chapter.

Prigogine and a number of associates have presented a series of papers documenting another interesting analogical approach to analyzing traffic flows (44, 45, 46, 47). This approach utilizes the principles of statistical physics to derive a "kinetic equation" that describes the space-time behavior of the velocity distribution of cars in a manner similar to that by which the standard kinetic equation describes the behavior of gas molecules. The theory begins by
postulating that the velocities of individual cars follow a probability density function which depends upon their position in both space and time. This function is represented by \( f(x, v, t) \). Certain important quantities can then be defined in terms of the velocity distribution function as follows:

**Local concentration:** \( C(x, t) \)

\[
C(x, t) = \int_{v} f(x, v, t) \, dv \quad (2.4)
\]

**Local flow:** \( q = C \bar{v} \) \( (x, t) \)

\[
q = \int v f(x, v, t) \, dv \quad (2.5)
\]

**Velocity dispersion:** \( C(\bar{v} - \bar{v})^2 \)

\[
C(\bar{v} - \bar{v})^2 = \int (v - \bar{v})^2 f(x, v, t) \, dv \quad (2.6)
\]

The theory then postulates that the distribution function satisfies a kinetic type equation with the following form:

\[
\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} = \left( \frac{\partial f}{\partial t} \right)_{\text{relaxation}} + \left( \frac{\partial f}{\partial t} \right)_{\text{interaction}} 
\]

where the first term on the right hand side is a result of the discrepancy between the actual distribution function, \( f \), and the distribution function desired by the participating drivers, \( f_0 \), and the second term results from the interaction between drivers. A specific functional form is then proposed for each of these two terms. The first term depends upon the quantity \( (f - f_0) \) while the interaction term is represented as a function of the probability of passing, \( p \).

Solution of the kinetic traffic equation reveals the existence of two distinct traffic flow regimes. The first regime prevails at low densities and is dominated by the desires of individual drivers. This flow region is called the
individual flow regime. The second regime prevails at medium and high density and is referred to as the collective flow regime. In collective flow the effect of vehicular interactions is dominant and the average speed of traffic is independent of the desires of drivers. Some effort is also devoted to the investigation of the question of traffic stability using the kinetic equation approach. Here it is shown that waves of the type described by Lighthill and Whitham are only one of a number of types of waves explained by the kinetic equation.

Analogical approaches have also been proposed by Pipes (48) and Harr and Leonards (49). Pipes developed a series of differential equations describing the dynamics of a line of traffic proceeding under the guidance of a postulated safe spacing rule. He then described a procedure for solving these equations using an electric circuit analogy. Harr and Leonards discussed the possibility of analyzing traffic behavior based on an equation of diffusion such as is used to describe the flow of a compressible, viscous fluid reacting to a motivating pressure potential. Both approaches are interesting but offer limited potential as realistic models of actual traffic flow.

2.6 SUMMARY

The material presented in the preceding sections is indicative of the types of studies that have been pursued to date in the quest for a theory of traffic flow. Although it by no means represents a comprehensive documentation of all the important contributions, it is sufficient to illustrate the scope which the field has come to embrace. In spite of the vast amount of work already completed, however, much remains to be done. New models must be developed
and the applicability of the existing models must be tested under a variety of real world conditions. Extensive experimental programs must be carried out to provide the necessary data to, in the words of Weiss (50), "...put more flesh onto the theoretical skeleton".

One area in which little has been done toward evaluating the applicability of existing theory is that of multiple-lane, one-way traffic flow. The major factor inhibiting study in this area has been the lack of a sufficient data base describing the movement of traffic on a multi-lane roadway in both time and space (51). The aerial data collection technique developed at the Transportation Research Center of The Ohio State University makes the collection of the required kind of data possible. Although insufficient data has been collected thus far to constitute the extensive data base desired, adequate data is available to make a beginning.

The traffic theory selected as the subject of the preliminary investigation is the hydrodynamic analogy model of Lighthill and Whitham. It was chosen because it represents an interesting approach to the analysis of multi-lane traffic flows and because it offers the potential of providing a valuable qualitative description of macroscopic traffic behavior under crowded roadway conditions. Such crowded conditions have become the rule rather than the exception on the modern urban freeway. It is imperative, therefore, that the mechanism of traffic flow under these conditions be fully understood.
CHAPTER 3

TRAFFIC CONSIDERED AS A FLUID:

THE HYDRODYNAMIC APPROACH

3.1 INTRODUCTION

The hydrodynamic approach to the analysis of traffic flow was first advanced in a classic paper prepared in 1954 by M. J. Lighthill and G. B. Whitham of the Department of Mathematics of the University of Manchester and published the following year in the Proceedings of the Royal Society of London (42). The approach assumes that traffic can be treated as a continuum and that traffic flow on a long, crowded roadway is analogous to the flow of water in a long river. At about the same time and without knowledge of their work, P. I. Richards of Technical Operations, Incorporated, of Arlington, Massachusetts, advanced a nearly identical theory in which traffic is treated as a continuous fluid (43). The only difference between this theory and that of Lighthill and Whitham is that Richards assumes a specific functional form for the relation between speed and density while Lighthill and Whitham treat the general case. Thus the Lighthill-Whitham Theory is somewhat less restrictive in terms of the assumptions required for application.

Based on the theoretical foundation of Lighthill and Whitham further theoretical work using the fluid analogy was done by Greenberg (52), Bick and
Newell (53), Newell (54), Foster (55) and Pipes (56, 57, 58). In addition, a limited number of experimental studies have been performed in an attempt to test the applicability of the fluid flow approach to certain real world traffic flow situations. Edie and Foote (59, 60), and Edie and Baverez (61) studied the propagation of disturbances through single lane tunnel traffic, Franklin (62) and Herman and Rothery (63) analyzed the behavior of a platoon of vehicles traveling along a test track and Leutzbach (64) investigated the continuity of traffic flow in a single lane of the Karlsruhe-Mannheim Autobahn with the remaining lanes closed due to construction. Little experimental effort has been devoted to date, however, to the investigation of the applicability of the fluid analogy to multi-lane flow situations where lane changing is permitted. This is the application which this paper attempts to treat.

The remainder of this chapter is devoted to a detailed description of the fluid flow approach. Documentation is provided of the original theoretical work, the additional theoretical studies noted above and the existing experimental investigations.

3.2 THE FLUID ANALOGY: LIGHTHILL AND WHITHAM

The concept of describing traffic using a fluid analogy was originally suggested by the study of flood movements in rivers and the flow of fluids about supersonic projectiles. In both cases it was noted that inhomogeneities in the flow parameters are propagated through the fluid medium in the form of waves. These waves are continuous in form and travel through the fluid at a velocity dependent upon the prevailing state of the fluid. Since a number of different
velocities are possible two or more such continuous waves may run together. When this happens the continuous waves coalesce to form a discontinuous wave or "shock wave". This shock wave then moves through the fluid carrying with it abrupt changes in the fundamental parameters of flow. This phenomenon was observed to be quite similar to the accordion-like movement of traffic flow commonly found on long, crowded roads in which the parameters of flow undergo rapid changes.

The fluid analogy to traffic flow is founded upon two basic relations. First, it is assumed that matter (vehicles) is conserved in a traffic flow. This requirement is expressed in terms of a one dimensional equation of continuity.

\[ \frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = 0 \]  

(3.1)

Second, it is assumed that a functional relationship exists between flow and density which may remain constant with position along the roadway or may vary as a function of \( x \). Thus

\[ q = q(k, x) \]  

(3.2)

The existence of traffic waves can then be shown directly through the use of a new variable, \( c \), which represents the wave velocity and is defined as the partial derivative of flow with respect to density at a given position on the roadway.

Hence,

\[ c = \left( \frac{\partial q}{\partial k} \right)_x = \text{constant} = c(k, x) \]  

(3.3)

Multiplying (3.1) by (3.3) results in

\[ \frac{\partial q}{\partial t} + c \frac{\partial q}{\partial x} = 0 \]  

(3.4)
This equation states that \( q \) is constant along the curves \( c \) given by equation (3.3) traveling past the point \( x \). If the roadway is assumed to be uniform such that \( q \) becomes a function of \( k \) alone then these same curves represent lines of constant density and, hence, also constant average speed. Each curve represents a continuous wave moving with a velocity \( c \). Assuming once again a uniform roadway the wave velocity will remain constant along the roadway length and the wave paths will take the form of straight lines in the space-time plane. The appropriate value of \( c \) for a given \( q \) is given by the slope of the flow-density curve at the corresponding point \( (q^0, k^0) \).

Since equation (3.1) is a first order equation the flow waves described by (3.4) possess only one wave velocity at each point in space. This is to say that equation (3.4) has only one system of characteristics given by \( dx = c \cdot dt \). This is in contrast to classical dynamic waves which possess at least two velocities (forward and backward) at each point. In order to distinguish these waves from dynamic waves Lighthill and Whitham termed them "kinematic waves". The existence of these kinematic waves depends only on the equation of continuity and the existence of a functional relation between \( q \) and \( k \) and is independent of more complicated physical considerations.

A fundamental relation of traffic flow defines flow as the product of space mean speed and density. Assuming a uniform roadway

\[
q = k v
\]  

(3.5)

Using this expression the wave velocity, \( c \), of a given flow condition can be related to the space mean speed of the traffic for the same flow condition.
\[
c = \left( \frac{\Delta q}{\Delta k} \right) = v + k\frac{\Delta v}{\Delta k}
\]  

(3.6)

Since \( \frac{\Delta v}{\Delta k} \) is generally accepted to be a negative quantity for a traffic flow, equation (3.6) means that \( c \) is always less than the space mean speed of the traffic and, hence, kinematic waves always travel backward through the traffic stream. They may travel either forward or backward relative to the roadway, however, depending upon the relative magnitudes of \( v \) and \( k\frac{\Delta v}{\Delta k} \).

Each small change in flow is propagated as a kinematic wave traveling with a velocity \( c \) through the traffic fluid. A phenomenon of interest with respect to these waves is now illustrated through reference to Figure 1. This figure shows the general relation assumed to exist between \( q \) and \( k \) for a traffic stream. The shape of this curve is a direct result of the fact that \( q = kv \) and the condition that \( \frac{\Delta v}{\Delta k} < 0 \). No specific functional relationship between \( q \) and \( k \) is implied. Since the curve is concave upwards it is apparent from consideration of equation (3.3) that flow waves will travel at increasing speeds as density decreases. Conversely, if density is increasing along the roadway, newly generated waves will travel at a slower pace than those waves representing lower density values which were generated earlier. In this situation the newly formed waves will eventually be overtaken by earlier waves leading to an impossible situation in which different values of \( q \), \( k \) and \( v \) exist at the same point in space and time.

This conflict is resolved through the formation of a discontinuous wave termed a shock wave since its process of formation is identical to that of a shock wave in a gas flow. Flow conditions within the shock front are assumed to adjust instantaneously such that the different values of \( q \), \( k \) and \( v \) represented
Figure 1 Schematic Representation of the General Relation Between $q$ and $k$
by the intersecting kinematic waves may exist simultaneously on either side of
the shock.

The motion of the shock front can be investigated using conservation
considerations. Obviously since vehicles are neither created nor destroyed
(at least not entirely) on the roadway the quantity of vehicles entering the shock
from one side must equal the quantity exiting on the other side. Thus:

\[ q_1 - U k_1 = q_2 - U k_2 \quad (3.7) \]

where \( U \) is the speed of the shock wave. Therefore

\[ U = \frac{q_2 - q_1}{k_2 - k_1} = \frac{\Delta q}{\Delta k} \quad (3.8) \]

Hence the speed of the shock wave separating two distinct flow conditions is
given by the slope of the chord connecting the corresponding points on the flow-
density curve.

A second condition of interest can also be noted by reference to Figure 1.
Due to the shape of the generalized \( q-k \) curve a point exists at which \( c = \frac{\Delta q}{\Delta k} \) is
equal to zero. This point is of special interest since it corresponds to the
maximum possible flow of the roadway in question. Hence, it is seen that the
flow wave carrying the maximum possible flow through the traffic fluid remains
stationary relative to the roadway. This observation suggests a potential pro-
cedure for measuring roadway capacity. Consider a long line of vehicles
stopped on the roadway due to some mechanism such as a traffic light. As the
line is started again a complete system of kinematic flow waves is emitted.
Each particular wave travels at its own particular speed, \( c \), depending upon the
flow rate carried. The wave carrying the maximum flow rate, however, has zero velocity relative to the road and stands still. By measuring the flow rate in the immediate vicinity of the starting point, therefore, a good estimate of the roadway capacity can be obtained.

The manner in which the Lighthill-Whitham theory of kinematic waves and shock waves can be used to provide a description of macroscopic traffic behavior will now be illustrated. Consider a stream of vehicles moving along a uniform roadway. Assume that the flow-density relation for this vehicle flow is shown in the upper part of Figure 2. Also assume that the input flow to the roadway section is constant and corresponds to the point \((q^0, k^0)\) on the flow-density curve. Thus the flow characteristics passing the input point all carry the same flow and are parallel in the space-time plane with a slope given by \(c_0 = \left( \frac{\partial q}{\partial k} \right)_{k = k_0} \). These flow waves are shown in space-time representation in the lower part of Figure 2.

Now assume that this constant flow is suddenly brought to a stop at some point downstream such as point \(x_1\) shown in Figure 2, remains stopped for the time period \(t_1\) to \(t_2\) and is then restarted. Accordingly at \(x_1\) a shock front will be created which will travel back along the roadway at a speed \(U_a\) as defined by the appropriate chord on the \(q-k\) curve of Figure 2. Traffic arriving at this shock front will be abruptly brought to a stop causing a queue of vehicles to form behind point \(x_1\). The shock front will continue to travel at speed \(U_a\) as long as the stoppage at \(x_1\) persists.

As traffic is restarted at time \(t_2\) a whole series of flow characteristics
Figure 2 Illustration of the Utilization of the Lighthill - Whitham Theory
are emitted at point \( x_1 \) carrying different rates of flow. One such characteristic corresponds to zero density flow and progresses rapidly down the roadway at speed \( c_{00} \). Others correspond to flow rates at densities \( k_a, k_b, k_c \) and \( k_d \) and travel as shown in Figure 2. As each of these characteristics intersects the shock front the speed of the front is modified and becomes that described by the chord connecting the appropriate flow rate with the original input flow rate on the curve of Figure 2. As a result the shock front slows to a stop, changes direction and proceeds downstream with ever increasing velocity. Eventually it passes point \( x_2 \) restoring the original flow conditions to the roadway section.

While the shock conditions persist many vehicles will either be brought to rest or have their speeds substantially reduced from the desired speed value. In addition the flow disturbance caused by the shock front can last much longer than the original stoppage which generated the shock. This is graphically illustrated in the lower part of Figure 2. The flow interruption at \( x_1 \) lasts only from \( t_1 \) to \( t_2 \) but some vehicles remain at rest until \( t_3 \) and normal flow is not re-achieved until \( t_4 \). The construction in Figure 2 shows that a vehicle moving along the roadway will encounter dense traffic conditions suddenly but will be able to leave them behind only slowly. This description coincides exactly with conditions experienced in real world traffic flows.

### 3.3 THE FLUID ANALOGY: RICHARDS

The analysis of Richards proceeds exactly like that of Lighthill and Whitham except that Richards treats density as the perturbable quantity rather
than volume. Also Richards assumes a specific functional form for the relationship between speed and density which thus implies a specific volume-density relation. As with the theory of Lighthill and Whitham the two basic assumptions of the Richards approach are the equation of continuity and the equation relating speed (or volume) to density. Using these equations a general equation for wave motion is derived. Except for the use of density as the dependent variable and a difference in notation this equation is identical to equation (3.4).

The movement of the resulting density waves is then investigated using a graphical technique. Specific cases described include the behavior of a line of vehicles pulling away from a high density flow on a clear road and the overtaking of a slow-moving line of vehicles by a faster moving, lower density line. In this latter case the graphical construction results in a density pattern in which two distinct values of density must exist at the same point in space and time. Richards, like Lighthill and Whitham, uses the concept of a discontinuous shock wave to explain this phenomenon. An expression identical to equation (3.8) is derived to describe the motion of the shock front along the roadway and an extension of the graphical technique is described for characterizing the shock wave. Using this technique it is shown that shock waves will always develop on roadways where \( \frac{\partial k}{\partial x} > 0 \); that is where the density downstream exceeds the upstream density.

A result derived by Richards that is not contained in the work of Lighthill and Whitham is an expression describing the acceleration of a vehicle at a given
place and time. Specifically it is shown that

\[ \frac{dv}{dt} = -c_r^2 \frac{k}{\partial x} \frac{\partial k}{\partial x} \quad (3.9) \]

which indicates that a driver changes his speed in accordance with the density conditions he perceives around him on the roadway as described by the density gradient \( \frac{\partial k}{\partial x} \). Although the specific form of equation (3.9) is dependent upon Richards' choice of speed-density relationship, the dependence of acceleration on the density gradient is a general result that has been shown by other authors (47, 57).

3.4 FURTHER THEORETICAL INVESTIGATIONS

Whereas Lighthill and Whitham provided only a qualitative description of the flow-density curve and Richards assumed a functional form relating the parameters, Greenberg (52) used the fluid analogy to derive a specific volume-density expression. His approach begins with an equation of motion for a one-dimensional fluid similar to that of Richards.

\[ \frac{dv}{dt} = -\frac{c_g^2}{k} \frac{\partial k}{\partial x} \quad (3.10) \]

Then since velocity is a function of both distance and time equation (3.10) becomes

\[ \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} + \frac{c_g^2}{k} \frac{\partial k}{\partial x} = 0 \quad (3.11) \]

This equation is then solved simultaneously with a modified version of the equation of continuity shown below

\[ \frac{\partial k}{\partial t} + v \frac{\partial k}{\partial x} + k \frac{\partial v}{\partial x} = 0 \quad (3.12) \]

with the following result for the non-trivial case
\[ \frac{dv}{dk} = -\left( \frac{c_g}{k} \right) \]  

(3.13)

This equation is then solved using the technique of separation of variables to yield an expression relating speed and density. Thus

\[ v = c_g \ln\left( \frac{k_j}{k} \right) \]  

(3.14)

where \( c_g \) is a constant and \( k_j \) is the density for jam conditions (i.e. when \( v = 0 \)).

Greenberg then offers a limited experimental verification of his \( v-k \) relationship using data collected in the Lincoln Tunnel and on the Merritt Parkway. He shows that equation (3.14) provides an excellent least-squares fit to the experimental data.

Bick and Newell (53) proposed a generalization of the basic fluid theory in which traffic flow in opposing lanes of a two-lane, two-way undivided highway is considered. The highway is assumed to be uniform in both time and space and the two traffic lanes are taken to be identical. Traffic behavior is described by two one-dimensional equations of continuity and the average velocity in a given lane is assumed to be a function of the densities existing in both lanes. An empirical expression is formulated to describe this velocity-density relation. Wave behavior is then investigated for a number of different initial traffic conditions. Results are presented in graphical form.

Newell (54) proposed a second generalization of the basic theory in which he hypothesized that traffic behavior is described by not one but two distinct equations of state. One equation expresses the relationship existing between speed and density while the traffic stream is undergoing a period of deceleration while the second equation describes conditions corresponding to a period of
acceleration. The curves representing the two types of traffic behavior are then joined by a series of connecting curves which allow transition from one to the other. Newell based his idea on the results of a pair of previous studies (8, 13) which indicated that driver behavior during acceleration and deceleration maneuvers is indeed different.

Newell's concept has two interesting and important consequences. First, the system of curves proposed to describe the volume-density relation for the two types of maneuvers and for transitions from one to the other are not everywhere concave upwards. Thus acceleration shock waves are permitted to form while such shock waves are impossible under the basic theory. Second, if a vehicle traveling at a given speed accelerates briefly and then decelerates back to his original speed, two sets of flow waves are emitted. In a Lighthill-Whitham approach these two sets of waves would be a shock wave (deceleration) and an expanding wave (acceleration) but the expanding wave would eventually collide with the shock destroying it and returning stability to the flow. Using Newell's approach the two wave sets would not necessarily ever meet since they are generated by different equations of state. Thus a potential for persisting instability is introduced into the theory which could result in a series of rapid decelerations and accelerations. Such stop and go traffic flow is often observed on crowded roadways.

Foster (55) investigated the effect of utilizing different theoretical forms relating speed and density on the applicability of the hydrodynamic traffic model. Specific relationships considered were a linear function such as the one
previously studied by Richards, Greenberg's logarithmic function and a reciprocal function resulting from integration of the microscopic car-following model for the case of constant sensitivity. The wave properties appropriate to each relationship were derived and then applied to find the speed of a shock wave caused by a short, sudden interruption to flow in a crowded lane of traffic. An important result of this theoretical study is the demonstration of the relationship existing between wave properties and the various parameters of the alternative speed-density equations. Specifically it is shown that the constant of both the linear and logarithmic speed-density expressions can be estimated by measuring the jam density and the speed of the characteristic which carries this density through the traffic fluid.

Foster also reports the results of a limited experimental investigation carried out for the purpose of verifying his theoretical findings. Wave behavior was studied in a crowded traffic stream resulting from flow interruptions caused by a pedestrian actuated traffic signal. Vehicle crossing times were measured at a series of six vehicle detectors spaced at 50 foot intervals back from the stop line in the median lane of a divided highway. The resulting space-time data was used to construct the path of each vehicle on a space-time diagram. Speed-density data was constructed from this diagram by associating average speeds at 2.5 ft./sec. intervals with the mean spacing existing at that speed.

Using this data the applicability of the alternative speed-density relationships is investigated. All curves are found to fit the data very well but the logarithmic form is chosen as being most reasonable based on certain intuitive
discriminating criteria. The existence of characteristics which carry constant
values of space mean speed and follow straight line paths in the space–time
plane is then demonstrated. Along these characteristics values of volume and
density are shown to remain relatively constant. The velocity at which each
characteristic travels is shown to be approximately linearly related to the space
mean speed of the traffic. No definite contradictions of the hydrodynamic
theory are uncovered.

Pipes has conducted a series of theoretical studies using the fluid
analogy. In a study reported in 1965 (56) traffic wave phenomena are investigated
using both a car following approach and the macroscopic hydrodynamic approach.
A unique feature of the hydrodynamic part of the study is the use of a trigono­
metric functional relating flow and density. Expressions are derived to describe
wave behavior under the action of such a functional. A second study reported in
1968 (57) analyzes wave behavior for a variety of different speed–density rela­
tionships. In addition an extension of the basic fluid theory is discussed which
incorporates the effects of diffusion and inertia into the standard equation for
wave motion. The diffusion effect is considered to be a function of the density
gradient with position while the inertia effect depends upon the density gradient
with time. A similar extension was proposed but not discussed by Lighthill and
Whitham. Implications of the extended theory are illustrated assuming that
density varies harmonically over space and time. A third study reported in
1969 (58) is concerned with the consideration of different forms for the expres­
sion relating the acceleration of a vehicle in the traffic stream to the density
gradient with position. It is shown that a vehicle moving in the presence of a positive density gradient will decelerate while acceleration will result from a negative gradient. It is also shown that different forms for the traffic equation of state can be derived beginning with specific forms for the acceleration-density gradient expression.

3.5 EXPERIMENTAL STUDIES

The majority of the experimental investigations conducted to date relevant to the hydrodynamic approach to describing traffic flow have been performed in vehicular tunnels or on special test tracks. This dependence upon idealized traffic situations has been necessitated by the extreme difficulty involved in collecting the required type of data for open road traffic flow. One notable study has been reported, however, in which flow in a single freeway lane was studied using standard ground-based data collection techniques (64).

Extensive studies of the characteristics of traffic flow in vehicular tunnels have been conducted by the staff of the Port of New York Authority. Three representative studies are described below.

Edie and Foote (59) utilized the principles of the fluid analogy to study the effect of a geometric bottleneck on flow through the Lincoln Tunnel. Volume-density diagrams were prepared using empirical data for locations upstream, within and downstream from a previously identified geometric bottleneck. These diagrams were then compared to theoretical results predicted from the hydrodynamic approach. Although the actual traffic behavior did not correspond exactly to that predicted by fluid considerations enough similarity did exist to
substantiate the value of the theory for providing general qualitative descriptions of macroscopic behavior. Traffic wave behavior was also investigated using a sequential volume counting technique. Analysis of the data revealed that flow waves do exist and are propagated through the traffic fluid. The speed of these waves was found to be reasonably consistent with that predicted by kinematic wave theory.

A second study reported by Edie and Foote (60) concentrated on the effect of shock waves on tunnel traffic. Three distinct types of flow were found to exist in the Holland Tunnel: relatively constant flow in a bottleneck section, severely fluctuating flow upstream of the bottleneck and less severely fluctuating flow downstream. The severe flow conditions upstream were the direct result of the passage of shock waves emanating from the bottleneck back through the traffic stream. Once a low speed queue was formed due to the passage of such a wave front the downstream end of the queue became a secondary bottleneck which acted to inhibit flow to a greater extent than the original geometric bottleneck. The flow deterioration caused by the shock front is accentuated because drivers display significantly faster reaction times to decelerating maneuvers than to accelerating maneuvers of the same magnitude and because the decelerating capabilities of vehicles are greater than their accelerating capabilities. Consequently as the shock wave moves backward it increases in thickness thereby involving increasing numbers of vehicles.

Edie and Baverez (61) also studied the generation and propagation of traffic waves in the Holland Tunnel. They found for the specific data available
that a stoppage wave was generated by the arrival of a pulse of high level flow into a portion of the tunnel in which traffic densities were relatively high. Accordingly drivers found themselves too close together in time and began reducing their speeds to lower and lower levels with some drivers eventually reaching a level below 5 mph. The concentration rose until a state of relative incompressibility was reached at which point the wave began to propagate upstream. Once again since deceleration and acceleration response was different the wave grew with respect to the number of vehicles involved as it moved. The speed of the wave was measured as approximately 10 mph. Based on their observations the authors presented a rationale by which the occurrence of consecutive shock waves could be explained. Such a phenomenon cannot be explained directly using the fluid analogy.

Franklin (62) studied the behavior of single-lane traffic flow on both circular and straight test tracks. The experimental work for the study was conducted at the Road Research Laboratory in England. Specific areas investigated included the nature of the flow-density relationship, the mechanism of platoon flow and the propagation of disturbances. As a result of his investigation Franklin suggested that the flow-density relationship for steady flow is not a single continuous curve as was postulated by Lighthill and Whitham but rather consists of three or four separate flow regimes. In one of these regimes flow remains relatively constant independent of changing density conditions. In the study of disturbance propagation Franklin found indications that both deceleration and acceleration maneuvers can be propagated as shock waves. This finding is
contrary to the original Lighthill-Whitham theory but is consistent with the ex-
tension proposed by Newell (54).

Herman and Rothery (63) conducted an extensive study of platoon behav-
ior on a test track at the General Motors Proving Grounds for the purpose of
identifying the characteristics of disturbance propagation. The experimental
situation consisted of a platoon of eleven identical vehicles traveling on a
straight, level roadway with no entrances, exits or cross roads for a length of
2.5 miles. Thus, to some degree, the platoon represents a homogeneous traffic
stream traveling on a limited access highway. Three instrumented vehicles were
positioned in the platoon as the first, sixth and eleventh members allowing the
behavior of the platoon to be measured with respect to both space and time.

The primary purpose of the study was to provide information on the
speed of propagation of disturbances relative to the moving platoon. Toward
this end a series of platoon runs was made with the platoon traveling at different
average speeds and with the lead vehicle executing a variety of different acceler-
ation and deceleration maneuvers. The speed of propagation of the resulting
disturbances was computed from the space-time records of vehicle behavior
provided by the instrumented vehicles.

Results of the study revealed that the speed of propagation with respect
to the traffic stream increases in a direct fashion with increases in the average
speed of the traffic stream. It was also determined that acceleration and decel-
eration disturbances propagate with different speeds. Finally, it was shown
that the time of propagation of both acceleration and deceleration disturbances
can be described with a good degree of accuracy as a linear function of the logarithm of platoon length. Different linear functions are required, however, to represent the two types of maneuvers. These results lend further credence to the idea that acceleration and deceleration behavior are governed by two distinctly different equations of state. No definitive suggestion is offered, however, as to what the functional form of these equations might be.

Leutzbach (64) studied the flow of traffic in a single lane of the Karlsruhe-Mannheim Autobahn to determine the usefulness of the theory of continuity for describing actual traffic behavior. The study site used was a four lane section of two-way highway in which one lane in each direction was closed for repair. Four observation points were selected each separated from the next by 300 meters. At each point data was collected using a pair of road tubes spaced 10 meters apart. Thus speed and volume data could be collected at each observation point and the corresponding density value derived. Using this procedure the pattern displayed by density over time could be established at each observation point.

If the theory of continuity applies then the density pattern existing at a given observation point should be predictable using the pattern at an upstream observation point and the density characteristics determined by the Lighthill-Whitham Theory. Leutzbach attempted to make such a prediction for each of the three downstream observation points using density characteristics determined from an empirically derived flow-density curve incorporating data from all observation points. In general he found that although the predicted and
observed density patterns were similar there were very definite discrepancies.
It was not possible to establish, however, whether these discrepancies indicated
invalidity of the theory of continuity or whether they resulted from difficulties
in determining a functional description for the flow-density relationship. In
fact it could not be stated with confidence as to whether a single q-k curve
described the roadway in question or if two or more curves were appropriate.

Although Leutzbach's study was inconclusive relative to establishing the
applicability of the theory of continuity, it was valuable in that it points out
some of the difficulties involved in applying traffic theory to real world situations.
These difficulties become greater as more complex flow situations are investi­
gated. The problems inherent in the application of such theory to multi-lane
situations thus become apparent.

3.6 SUMMARY

The hydrodynamic analogy of Lighthill and Whitham and Richards repre­
sents a unique and interesting approach to the analysis of traffic flow behavior.
It has limitations, however, that must be recognized by any researcher attempting its application. First, vehicles obviously are not fluid molecules. They
are not identical, they have finite dimensions and they possess physical limita­
tions in their accelerating and decelerating capabilities. Second, vehicles are
controlled by human beings and thus do not respond directly to physical laws.
Drivers differ in their capabilities and possess minds of their own full of desires
and goals which affect their behavior within the traffic stream. Finally, the
description of traffic as a fluid represents the limiting behavior of a stochastic
process involving large numbers of individual units. Such an approach can only apply strictly, therefore, to large scale problems such as the movement of a dense stream of traffic on a long, crowded roadway.

The application of the fluid analogy relies on the validity of the equation of continuity and depends upon the establishment of a functional relationship between flow and density. The determination of such a relationship is a difficult problem that has been attacked with varying degrees of success by many researchers. A number of possible forms have been proposed but none can be shown to be definitely superior. In fact it can be shown that different q-k functionals may result depending upon the extent of the time and space domains over which the data is collected. In addition, there are definite indications that two rather than one equation of state exists for the description of traffic behavior. One equation describes traffic flow under conditions of acceleration while the other depicts deceleration behavior. The question of whether the equation of state varies with position on the roadway also arises to further complicate the issue. Thus, the analysis of even simple idealized flow conditions becomes a difficult problem.

The introduction of multi-lane traffic flow into the analysis increases the difficulties involved. First, an additional variable is brought into the establishment of the flow-density relationship since vehicles can now change lanes. Second, the equation of continuity no longer strictly applies since vehicles can be created and destroyed from a given lane by changing lanes. Although this can be accounted for mathematically by adding a source and sink term to the continuity relationship, there is little hope of determining a usable expression
for quantifying the space-time behavior of this term. There is also difficulty involved in designing a data collection methodology capable of supplying the required type of data on vehicle movements along a multi-lane roadway. A technique must be developed which provides records of vehicle movement in both space and time without disturbing the traffic flow to the point of completely biasing vehicle movements.

In spite of these difficulties or perhaps because of them the study reported in the next two chapters attempts to investigate the applicability of the hydrodynamic analogy for describing macroscopic traffic behavior in a single lane of a multi-lane roadway. It is emphasized that the study described represents only a preliminary effort due to the scarcity of data available at this time. It is hoped, however, that it will provide sufficient results of interest to stimulate further research in this area.
CHAPTER 4

DESCRIPTION OF THE STUDY DATA, DISCUSSION OF
FUNDAMENTAL ASSUMPTIONS AND ESTABLISHMENT
OF THE EQUATION OF STATE

4.1 INTRODUCTION

The fluid analogy for the description of the macroscopic behavior of
traffic is founded on two basic relations: the one-dimensional equation of con­
tinuity and an expression describing traffic flow as a function of traffic density.
Its treatment of traffic as a continuous fluid theoretically restricts its use to
large scale problems involving large populations of vehicles. Any experimen­
tal study designed to evaluate the applicability of the fluid analogy, therefore,
must operate within this framework and consider very carefully any departure
from the fundamental assumptions. The discussion contained in this chapter is
devoted to that purpose.

The equation of continuity states in mathematical terms the condition
that vehicles are conserved in a traffic flow. In a multi-lane situation where
flow in a single lane is to be studied the continuity requirement is not strictly
met. This is due to the possibility which exists for vehicles to change lanes
thus being either created or destroyed with reference to the lane in question.
The effect which this departure from continuity will have on the applicability of the fluid theory is discussed in a succeeding section.

The establishment of a mathematical functional relating flow and density is a difficult problem which, as was pointed out in Chapter 3, has been studied in much detail by many different researchers. Experience to date indicates that substantially different $q$-$k$ relationships may be obtained by:

1. Varying the size of the sampling domain with respect to both space and time over which average values of $q$ and $k$ are measured
2. Varying the position on the roadway at which measurements are made
3. Aggregating $q$ and $k$ measurements made during conditions of traffic acceleration and deceleration or treating the two types of behavior separately.

Accordingly much care must be used in deciding which methodology for measuring $q$ and $k$ is most appropriate to the given flow situation being studied. This question is also treated later on in this chapter.

Finally, the question of what constitutes a large scale problem must be answered. Does a large scale flow condition require thousands of vehicles to be studied or hundreds or how many? In fact, what are the implications of scale to the problem being investigated? A section of this chapter deals with this consideration in the context of the specific flow situation of concern.
Before these questions are treated, however, we will begin by describing the study section for which data on the space-time behavior of vehicles is available and the unique data collection technique used to provide this information.

4.2 DESCRIPTION OF THE STUDY DATA

4.2.1 Definition of the Study Site

The study site used for the experimental investigation is a portion of Interstate 71 located in the north side of Columbus, Ohio. At the time of data collection this section of I-71 was comprised of two northbound and two southbound lanes and represented the principle link between the heavily populated northern suburbs and the CBD. The study section is 3.5 miles in length and is located on a high type alignment with a maximum grade of 2% and a maximum curvature of 1°15'. Three interchanges are located in the study area each with a full complement of on and off ramps resulting in a total of three entrance and three exit points for each direction of flow.

Specific attention was devoted to traffic flowing in the southbound median lane during the morning peak period. This lane was selected for several reasons. First, it carries the heaviest flow of traffic thus assuring crowded roadway conditions. Second, it serves a very low proportion of trucks (less than 1%) and, hence, provides a relatively homogeneous flow of vehicles. Third, it is much less affected by vehicles entering and exiting the freeway than is the shoulder lane. Accordingly, it is subject to less lane changing
especially during times of heavy congestion. Therefore violation of the continuity requirement is less pronounced.

The morning peak hour on I-71 in the area of interest extends from 7:00 to 8:00 AM. The data to be used in this study was collected on Tuesday, July 25, 1967, between 7:45 and 7:50 AM. The weather was clear and no unusual incidents that might affect the normal flow of traffic were noted. The prolonged deceleration and acceleration maneuvers that can be observed in the data are the result of natural traffic interaction and were not artificially introduced as part of the data collection methodology.

4.2.2 Collection and Reduction of the Data

Data was collected using a unique aerial data collection technique developed at the Transportation Research Center of The Ohio State University. The data collection device is a KA 62A aerial reconnaissance camera manufactured by Chicago Aerial Industries. This camera has a 3 inch focal length and produces photographs having a 4.5 inch by 4.5 inch format. This format provides approximately 1500 feet of longitudinal ground coverage for each 1000 feet of flight altitude as measured from ground level. The camera is mounted in a specially designed, vibration attenuating mount and is carried by a Bell Ranger helicopter. The helicopter with the camera in place is shown in Figure 3. The mount is so designed as to allow rotation of the camera through a horizontal angle of approximately 120 degrees. Thus photography can be taken while the helicopter is banking and the camera can be adjusted to compensate for helicopter drift.
Figure 3 Bell Helicopter with Data Collection Camera
The camera is equipped with a 250 foot film magazine allowing for a film capacity of about 600 frames. The film used to collect the data used in this study was Kodak Plus-X Panchromatic having an exposure index of 80. Photographs were taken at one second intervals and provide a total time coverage of 3 minutes and 58 seconds. A typical data collection photograph is shown as Figure 4. Flight altitude during data collection was maintained at approximately 4000 feet above ground. The data collection crew attempted to maintain a flight speed corresponding to the speed of the traffic being photographed.

Reduction of the photography was accomplished using the Nistri AP/C Analytical Plotter of the Ohio Department of Transportation. The x, y photo coordinates of each vehicle were determined for each of the 238 photographs. The corresponding ground coordinates were then established using a battery of computer programs and previously defined ground control points. These ground coordinates were then manipulated by the computer to provide information on spacing, headway, longitudinal position on the roadway and velocity for each vehicle at each one second interval. In addition a graphical summary of vehicle behavior in time and space was prepared for each lane of travel in the form of vehicle trajectories. The trajectories for the median lane are shown as Figure 5. The reader interested in a more detailed description of data collection and reduction procedures is referred to Interim Report EES 278-2, "Investigation of Traffic Dynamics by Aerial Photogrammetry Techniques", published by the Engineering Experiment Station of The Ohio State University (1969).
Figure 4  Typical Data Collection Photograph
Figure 5 Vehicle Trajectories for the Median Lane of I-71
4.2.3 Characteristics of the Data

The data represented by the trajectories of Figure 5 provides a description of traffic behavior that could not be provided using standard ground-based data collection techniques. It provides a complete record of the movement of some 115 individual vehicles in both time and space over a domain containing approximately 12,000 separate data points. In addition each data point is established with a high degree of accuracy. It is estimated that longitudinal positions on the roadway are accurate within ± 0.50 foot and velocities within ± 1.00 mph. Data of this quality allows a unique opportunity for investigating traffic flow in a multi-lane environment.

4.3 THE QUESTION OF CONTINUITY

The basic equation of one-dimensional continuity was given in Chapter 3 as equation (3.1) and is repeated below.

\[ \frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = 0 \]  (4.1)

The interpretation is that for a given section of highway the difference between the vehicle inflow and the vehicle outflow is reflected as a change in the number of vehicles in the section. This equation will always hold in single-lane, no passing flow situations. It does not hold in general for multi-lane situations.

Two possible approaches are available for treating the problem of loss of continuity in a multi-lane flow. The first approach, based on the work of Pipes (56), involves the modification of the basic equation (4.1) by the addition of a space and time dependent source-sink term. The resulting equation would then be:
Thus continuity in a given lane is maintained by systematically adding and subtracting traffic from it according to a defined space-time function. Although this approach could account mathematically for lane changing effects it is objectionable for two important reasons. First, there is no obvious way to determine the nature of the source term, \( s(x, t) \), such that it could be expressed as a quantitative space-time function. In fact the idealization required to establish a form for \( s(x, t) \) would undoubtedly obviate its intended purpose. Second, the addition of \( s(x, t) \) to the equation of continuity obscures the resulting wave properties. Although characteristics would still exist along which \( q, k \) and \( v \) are constant they would no longer follow straight line paths in the physical plane. Indeed, identifying the nature of the characteristics would become a very difficult problem in itself. For these reasons the use of an equation of continuity such as (4.2) does not seem to be a reasonable alternative.

The second approach is to select a portion of the single-lane flow for study in such a manner that the amount of lane changing contained in the data is minimal. This is the approach taken in the study reported herein. By minimizing lane changing the basic continuity relationship can be used as an approximate description of traffic behavior. The accuracy of the approximation depends upon the amount of lane changing occurring and the extent of the space-time region over which it occurs. Reference to Figure 5 reveals that substantial lane changing takes place in the median lane of I-71 during the time of study. However, the vast majority of this lane changing takes place either
well before or well after the region of medium to high density in which the fluid analogy can be expected to provide a reasonable description of flow behavior. Almost no lane changing occurs in the immediate vicinity of the point of stoppage separating the deceleration and acceleration phases of the traffic flow. Thus, although it is not strictly valid, it is thought that the standard continuity relation will adequately serve the purposes of this study. The departure from reality caused by this approximation will be reconsidered once the experimental results have been presented.

4.4 DEFINITION OF THE CONTINUUM VARIABLES

4.4.1 General Considerations

A difficult but also very important question to be dealt with in attempting to apply the fluid analogy to real world traffic situations revolves around the determination of the appropriate definition of the continuum variables q, k and v. Lighthill and Whitham provided the following specific definitions for measuring flow, density and average speed using ground based data collection techniques (42). If dx represents a short slice of roadway with the point of interest x as the midpoint, τ represents a moderate length of time, long enough for many vehicles to pass, with t as the midpoint, then:

\[ q = \frac{n}{\tau} \]  \hspace{1cm} (4.3)

where q is the flow and n is the number of vehicles crossing the slice dx in time τ. Also the density, k, can be expressed as:

\[ k = \frac{\sum dt}{\tau dx} \]  \hspace{1cm} (4.4)
with $\Sigma dt$ representing the sum of the times taken by the individual $n$ vehicles to cross the slice and $\tau$ and $dx$ as previously defined.

The average velocity is then computed as a derived quantity as follows:

$$v = \frac{q}{k} = \frac{dx}{\frac{1}{n} \Sigma dt} \quad (4.5)$$

This is the "space mean speed" as defined by Wardrop (65) and represents an average of vehicle speeds weighted according to the time they remain on the slice of road $dx$. According to these definitions $q$, $k$ and $v$ are to be measured over a "short" section of roadway with averages taken over a "moderate" length of time. Further specification is not given.

Wardrop provides a second set of definitions suitable for computing $q$, $k$ and $v$ from aerial photography. Using a notation similar to that of Lighthill and Whitham the following expressions result:

$$k = \frac{n}{X} \quad (4.6)$$

$$v = \frac{\Sigma dx_i}{ndt} \quad (4.7)$$

$$q = kv = \frac{\Sigma dx_i}{Xdt} \quad (4.8)$$

In this equation system flow is the derived quantity and is expressed as the product of density and space mean speed. $X$ represents the length of a long section of roadway, and $dt$ is the short time interval between successive photographs. The distance traveled by the $i$th vehicle during a given time interval $dt$ is represented by $dx_i$. It is summed over the $n$ vehicles occupying roadway length $X$ during time $dt$. Once again the decision of what constitutes a "long"
section of road and a "short" interval of time is left to the judgment of the researcher.

Wright (66) has performed an interesting study that pertains to the question at hand. The basic purpose of his investigation was to determine the effect of different space-time sampling domains on the fundamental parameters of traffic flow. He also explored the effect of serial correlation on the resulting parameter relationships. As a result of his study he determined that sampling domain size has a definite effect on the nature of the average parameter values obtained. Specifically he found that serial correlation effects can affect flow relations for freely flowing traffic conditions and that these effects could be minimized by increasing the time domain over which averages are obtained. For congested traffic, however, he observed that the use of large time intervals for calculating parameter values tends to mask the instantaneous relationships existing between the parameters. Thus for these flow conditions very small time intervals are most appropriate. He recommends, therefore, that the sampling domain appropriate for a given study be determined based upon the prevailing flow conditions and the purpose for which the q, k and v relationships are desired.

The fluid analogy to traffic flow is an instantaneous theory which attempts to predict how traffic conditions change with changing demand conditions. Thus it seems that a very small time interval is appropriate for determining the q and k values required to construct the equation of state. Accordingly it was decided that traffic conditions would be sampled at one second intervals.
The choice of an appropriate space domain remains. For the reasons specified above it is logical that a local domain should be more appropriate than a domain consisting of a long section of roadway. The question then becomes what should be the size of the local domain and where along the study section should it be located. The size portion of the question has no definite answer but certain guidelines are available. First, the section length selected should be sufficiently long to include a large enough number of vehicles to damp out erratic individual driver behavior. Second, it should not be so long as to include portions of the traffic stream having widely different characteristics such that the average parameter values obtained have no meaning.

Assuming an appropriate section length is selected, where should it be located along the study section? Theoretically, if the roadway is uniform, the location of the sampling section should make no difference. From a practical standpoint, however, since data is available for only about four minutes of time random variations in flow will render even a uniform roadway non-homogeneous. In addition, the assumption of a uniform roadway is an idealization for even a high type real world roadway such as Interstate 71. One attractive solution to this dilemma employs a sliding sampling section which moves along the roadway with the traffic stream. Using this methodology average values are obtained which represent the aggregated characteristics of the entire study section while maintaining the advantages of a local sampling domain. This procedure has previously been used successfully for making measurements on multi-lane roadways by Clear (67) and Lee (68).
The problem of sampling domain in space has thus been simplified to the following considerations: how long should the sliding section be and in which portion of the traffic stream should it be located. These considerations are approached experimentally in the following sections.

4.4.2 The Effect of Sampling Section Length

A variety of different section lengths have been employed by different researchers. The section lengths reported in the literature range from a single car length up to about 600 feet. No real justification is ever given, however, for the length used. To investigate the effect of section length on the resulting \( q \), \( k \) and \( v \) values two sections were studied having significantly different lengths. The locations of these sections are shown superimposed on the vehicle trajectories in Figure 6. In order to isolate the effect of section length from other possible effects that could influence parameter measurements fixed rather than moving sections were used. The time duration of study was identical in both cases.

The alternative sampling sections are referred to as Section X and Section Y. Section X is 600 feet in length while Section Y is 300 feet long. To eliminate effects due to roadway nonuniformity to the greatest possible extent Section Y is included within Section X. The values of \( q \), \( k \) and \( v \) were computed for the two sections at one second intervals using the parameter definitions of Wardrop which for the sampling domain in question are more appropriate than those of Lighthill and Whitham. The results of these computations are summarized in the form of speed versus density plots for the two sections. These
Vehicle Trajectories
Median Lane
Interstate 71
Southbound
Film 4
July 25, 1967
7:45 AM

Figure 6 Identification of Section X and Section Y
plots are presented as Figures 7 and 8. The plots were obtained by averaging all the instantaneous average speed values existing at a given density. The speed value thus obtained represents a grand average over time.

Observation of Figures 7 and 8 leads to the following conclusions. The data points calculated for Section X seem to give a slightly better behaved relationship between speed and density in the medium to high density region (above 60 vehicles per mile) than those computed for Section Y. The Section X data points are more erratic, however, at low densities. The shorter section length provides a large coverage of both the speed and density domain but, in doing so, provides a less reasonable estimate of the free flow speed for traffic traveling on a highway with a 60 mph speed limit. Both groups of data provide an estimate of jam density that is somewhat lower than that expected for freeway traffic. In short, the results of the study are inconclusive.

Rather than belabor the issue further, it was decided that a section length of approximately 600 feet would be used as the space sampling domain for the experimental study. This decision was based on two premises. First, that the longer length would more effectively damp out random variations caused by erratic drivers and second, that it would provide a suitably stable mathematical relationship between speed and density (and thus volume and density) at medium to high density values. This is the portion of the density domain of particular interest with respect to the fluid analogy.

4.4.3 The Effect of Sampling Section Location

The effect of varying the location of the sampling section within the
Figure 7 Plot of Speed vs. Density for Section X
Figure 8 Plot of Speed vs. Density for Section Y
traffic stream was investigated by dividing the total stream into vehicle groups each with an average length of about 600 feet as measured along the length of the 3.5 mile study section. Six different vehicle groups were identified in all. The identity of the individual groups is illustrated in Figure 9. The breakpoints dividing the vehicle groups were so chosen as to attempt to isolate groups which represent relatively cohesive flow units and such that a given group would maintain the same leading and trailing vehicle during the course of its travel. The selection of breakpoints also had to satisfy the choice of a nominal 600 foot sampling section.

Flow, volume and average speed values were then computed for each vehicle group at one second intervals using the parameter definitions of Wardrop. A minor modification was required, however, in the density definition. Since the sampling section is at any point defined by the group length rather than a constant length, it was considered more realistic to define density as the ratio of the number of individual spacings making up the platoon to the sum of these spacings. Thus

\[ k = \frac{n-1}{\sum s_i} \]  

(4.9)

The average speed is still computed as given by equation (4.7) and volume is equal to average speed times density.

The resulting \( q, k \) and \( v \) values were then plotted versus one another resulting in three separate plots for each vehicle group. In each case the plot relating speed and density provided the most regular pattern for the entire group of data points considered. Figures 10, 11 and 12 represent a typical set
Figure 9 Identification of Alternative Vehicle Groups
Figure 10 Plot of Speed vs. Density for Vehicle Group 4
Figure 11 Plot of Volume vs. Density for Vehicle Group 4
Figure 12 Plot of Speed vs. Volume for Vehicle Group 4
of parameter relationship plots. These figures represent the relationships existing for vehicle group 4.

Since speed versus density provided the least scattered data, this data was used to determine the effect of sampling space location on the parameter values obtained. By studying the speed-density plots for the six vehicle groups it was determined that a sampling section moving with groups 1, 2 and 6 did not provide sufficient coverage of the density domain to allow a reliable speed-density pattern to be identified which would remain usable over the entire range of densities desired. Thus these groups were dropped from consideration. Vehicle group 3 data provided sufficient density coverage but, like Section Y, provided an estimated value for free flow speed that was lower than is to be expected for the given flow conditions. Thus this group was also discarded. The two remaining groups, groups 4 and 5, provided wide density coverage, a reasonable estimate of free flow speed and a reasonable estimate of jam density. The speed-density pattern provided by the data of these two groups was quite similar but not identical. No logical criteria presented itself for choosing between the two groups of data. Therefore, it was decided that the equation of state corresponding to both groups would be formulated and studied.

The sample data to be used in the experimental study of wave behavior consists therefore of parameter values calculated by moving a sliding sampling section nominally 600 feet long through two portions of the total traffic stream and averaging the results over one second intervals in the time domain. The
data for group 4 has already been presented. Group 5 data is shown in Figures 13, 14 and 15.

4.4.4 The Effect of the Acceleration-Deceleration Dissymmetry

A number of researchers have suggested that rather than one equation of state, two are required to accurately depict traffic behavior. One equation of state would describe behavior during accelerating conditions while the other would correspond to decelerating conditions. The implications of this dissymmetry of acceleration and deceleration with regard to application of the fluid analogy have been discussed by Newell (54).

In order to determine the relevance of two distinct equations of state for describing flow conditions in the median lane of Interstate 71 during the period of study, an investigation of the acceleration and deceleration behavior of vehicles in groups 4 and 5 was undertaken. Average acceleration and deceleration values were computed for individual vehicles at one second intervals by dividing the speed change occurring in that interval by the interval length. Thus:

\[
\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}
\]  

(4.10)

The resulting average values were then plotted versus the total speed change in which the subject vehicle was involved. A separate plot was prepared for acceleration and deceleration conditions. These plots are shown as Figures 16 and 17.

Observation of Figures 16 and 17 reveals three interesting and important points. First, it is noted that a wide variation occurs among individual vehicles with regard to both the average acceleration and deceleration values used to
Figure 13 Plot of Speed vs. Density for Vehicle Group 5
Figure 14 Plot of Volume vs. Density for Vehicle Group 5
Figure 15 Plot of Speed vs. Volume for Vehicle Group 5
Figure 16 Plot of Average Acceleration Rate vs. Speed Change
Figure 17 Plot of Average Deceleration Rate vs. Speed Change
negotiate a given speed change. This is further proof that vehicles do not behave like fluid molecules. Second, deceleration rates at a given speed change value are in general substantially greater than acceleration rates corresponding to the same amount of speed change. The difference is especially pronounced at higher speed change values. Third, although the data points are quite scattered, it is apparent that deceleration rates employed increased with increasing speed change while acceleration rates remain basically the same over the speed change domain investigated.

Thus it seems that vehicle behavior during acceleration is indeed different than that displayed during deceleration. This is a characteristic that must be considered in formulating the equation of state. Accordingly the values for the continuum variables q, k and v computed for vehicle groups 4 and 5 were separated into two classes corresponding to whether they represented behavior before or after the stoppage zone separating the deceleration and acceleration regions. The speed-density data resulting from this disaggregation is shown in Figures 18 and 19 for group 4 and Figures 20 and 21 for group 5. The data shown in Figures 18 and 20 are for decelerating conditions while that of Figures 19 and 21 are for accelerating conditions. It can be seen by studying these figures that the speed-density patterns for the two conditions are different.

4.5 ESTABLISHMENT OF THE EQUATIONS OF STATE

4.5.1 Investigation of Alternative Mathematical Forms

The speed-density data presented in Figures 18 through 21 represents
Figure 18  Plot of Speed vs. Density for Vehicle Group 4  
Deceleration Phase
Figure 19  Plot of Speed vs. Density for Vehicle Group 4
Acceleration Phase
Figure 20 Plot of Speed vs. Density for Vehicle Group 5 Deceleration Phase
Figure 21 Plot of Speed vs. Density for Vehicle Group 5
Acceleration Phase
in empirical form the relationship existing between the fundamental flow parameters for vehicle groups 4 and 5. Before proceeding to an investigation of the behavior of traffic waves, however, it is necessary to formulate a mathematical expression describing the four distinct equations of state. This section is devoted to that purpose.

Many alternative mathematical forms are possible. Forms previously investigated for use with the fluid analogy include the following:

\[ a. \quad v = v_f(1 - \frac{k}{k_j}) \]  \hspace{1cm} (4.11)

\[ b. \quad v = c_g \ln \left(\frac{k_j}{k}\right) \]  \hspace{1cm} (4.12)

\[ c. \quad v = \frac{A}{k} - B \]  \hspace{1cm} (4.13)

\[ d. \quad v = v_f e^{-\frac{k}{k_m}} \]  \hspace{1cm} (4.14)

Equation (4.11) is a simple linear function first proposed by Greenshields (1) and adapted by Richards in his original paper on shock waves (43). Equation (4.12) is the logarithmic form derived by Greenberg from basic fluid considerations. It can also be derived starting with car-following considerations and thus represents a bridge between microscopic and macroscopic theory.

Equation (4.13) is a reciprocal form derived from car-following theory and investigated by Foster (55). Equation (4.14) was proposed by Underwood (69) to correct the deficiency of Greenberg's relation which is asymptotic to the speed axis. Each proposed speed-density relation results in a continuous, well behaved flow-density curve which is an essential requirement of the fluid analogy approach. The ability of each of these forms to describe the data of vehicle groups 4 and 5 will be investigated. In addition a fifth mathematical
form is proposed for study. This fifth form considers average speed to be a quadratic function of density. Thus:

\[
e. \quad v = a_1 + a_2 k + a_3 k^2
\]  

(4.15)

The procedure used to determine the degree of fit displayed by the five proposed mathematical forms with respect to the data of vehicle groups 4 and 5 involves the regression of speed on density. Since by an appropriate transformation of variables each proposed equation can be expressed in linear form, the regression technique employed is least squares linear regression. The regression was performed on the IBM 370/165 digital computer of The Ohio State University Instruction and Research Computer Center using the regression subroutine of the OMNITAB programming language.

Each of the five linear forms was fit to each of the four separate groups of speed-density data. The resulting regression equations were then transformed to retrieve the desired equation form. The results of the regressions are summarized in Tables 1 and 2. In the equations shown in these tables density is measured in vehicles per mile and speed is given as mph. Since the coefficient of the second degree term of the quadratic form equation was quite small in each case, a test was performed to determine if this coefficient was significantly different from zero. The null hypothesis that \( a_3 = 0 \) was rejected in each case at the 5% level of significance. Thus the quadratic form equation
TABLE 1

Results of the Speed-Density Linear Regression

Group 4 Data

<table>
<thead>
<tr>
<th>Mathematical Form</th>
<th>Equation</th>
<th>$R^2$</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deceleration Phase</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linear</td>
<td>$v = 68.43 - 0.41k$</td>
<td>0.833</td>
<td>9.22</td>
</tr>
<tr>
<td>Greenberg</td>
<td>$v = 33.23 \ln(180/k)$</td>
<td>0.977</td>
<td>3.42</td>
</tr>
<tr>
<td>Reciprocal</td>
<td>$v = \left( \frac{1727.31}{k} \right) + 0.931$</td>
<td>0.949</td>
<td>5.08</td>
</tr>
<tr>
<td>Underwood</td>
<td>$v = 122.0e^{-k/47.14}$</td>
<td>0.957</td>
<td>3.51</td>
</tr>
<tr>
<td>Quadratic</td>
<td>$v = 87.30 - 0.98k + 0.003k^2$</td>
<td>0.983</td>
<td>2.95</td>
</tr>
<tr>
<td>Acceleration Phase</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linear</td>
<td>$v = 55.31 - 0.31k$</td>
<td>0.621</td>
<td>6.37</td>
</tr>
<tr>
<td>Greenberg</td>
<td>$v = 33.32 \ln(185/k)$</td>
<td>0.697</td>
<td>5.70</td>
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<tr>
<td>Reciprocal</td>
<td>$v = \left( \frac{2761.94}{k} \right) - 8.52$</td>
<td>0.690</td>
<td>5.77</td>
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<tr>
<td>Underwood</td>
<td>$v = 176.0e^{-k/39.82}$</td>
<td>0.909</td>
<td>3.12</td>
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<tr>
<td>Quadratic</td>
<td>$v = 81.96 - 0.88k + 0.002k^2$</td>
<td>0.705</td>
<td>5.64</td>
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TABLE 2

Results of the Speed-Density Linear Regression

Group 5 Data.

### Deceleration Phase

<table>
<thead>
<tr>
<th>Mathematical Form</th>
<th>Equation</th>
<th>$R^2$</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>$v = 68.24 - 0.49k$</td>
<td>0.892</td>
<td>6.70</td>
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<tr>
<td>Greenberg</td>
<td>$v = 36.04 \ln(149/k)$</td>
<td>0.981</td>
<td>2.82</td>
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<tr>
<td>Reciprocal</td>
<td>$v = \left(\frac{2046.95}{k}\right) - 6.71$</td>
<td>0.986</td>
<td>2.41</td>
</tr>
<tr>
<td>Underwood</td>
<td>$v = 116.0e^{-k/45.24}$</td>
<td>0.993</td>
<td>1.88</td>
</tr>
<tr>
<td>Quadratic</td>
<td>$v = 91.12 - 1.245k - 0.004k^2$</td>
<td>0.984</td>
<td>2.64</td>
</tr>
</tbody>
</table>

### Acceleration Phase

<table>
<thead>
<tr>
<th>Mathematical Form</th>
<th>Equation</th>
<th>$R^2$</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>$v = 52.91 - 0.37k$</td>
<td>0.716</td>
<td>5.49</td>
</tr>
<tr>
<td>Greenberg</td>
<td>$v = 32.61 \ln(146/k)$</td>
<td>0.790</td>
<td>4.72</td>
</tr>
<tr>
<td>Reciprocal</td>
<td>$v = \left(\frac{2483.01}{k}\right) - 14.52$</td>
<td>0.831</td>
<td>4.23</td>
</tr>
<tr>
<td>Underwood</td>
<td>$v = 120.0e^{-k/40.51}$</td>
<td>0.948</td>
<td>2.35</td>
</tr>
<tr>
<td>Quadratic</td>
<td>$v = 87.77 - 1.298k + 0.005k^2$</td>
<td>0.812</td>
<td>4.48</td>
</tr>
</tbody>
</table>
provides an estimate of average speed that is significantly different than that provided by the corresponding linear form. The reader interested in a more detailed description of the linear least squares regression procedure is referred to any of a number of good texts on the subject (70, 71).

4.5.2 Selection of Appropriate Mathematical Forms

Referring to Tables 1 and 2 it can be seen that any of four of the proposed mathematical forms provides a relatively good fit to the experimental data for the deceleration phase for both groups 4 and 5 while only one form, Underwood's, provides a decent description of the data for the acceleration phase for each group.

Considering first the deceleration phase, the equations proposed by Greenberg and Underwood, the reciprocal form and the quadratic form each explain 95% or more of the variation in the average speed. There is also relatively little difference in the size of the standard error of the estimate for these alternative equations for group 5 data and for the equations of Greenberg, Underwood and the quadratic form for group 4. The standard error of the reciprocal form is somewhat higher for group 4 data but not so large as to cause its rejection. From the standpoint of a volume-density relationship, however, the reciprocal speed-density form is objectionable because it implies a linear q-k curve. Such a straight line relation between q and k is in conflict with masses
of experimental work conducted in the traffic engineering field which implies that \( q \) versus \( k \) results in a concave upwards curvilinear relationship. Also the reciprocal speed-density form indicates that maximum flow occurs at zero density which is clearly untrue. For these reasons the reciprocal form will be dropped from consideration.

Thus three possible mathematical forms remain for describing the equation of state for the deceleration phase. The wave behavior predicted by each of these forms will be compared to that actually measured from the experimental data in the next chapter. The equation of state for the acceleration phase will be described based on Underwood's equation. The Underwood form explains about 90% of the variation in average speed for group 4 data and almost 95% of the variation for group 5. The volume-density relationships derived from the selected speed-density equations are summarized in Table 3.

4.6 THE QUESTION OF SCALE

The continuous flow approach represents the limiting behavior of a stochastic process for a large population of vehicles and, therefore, is only strictly applicable to large scale problems involving the movement of vehicles along long, crowded roadways. The question at hand becomes, does the problem proposed for study in this paper constitute such a large scale problem. To answer this question it is first necessary to consider what the implications of scale are to the hydrodynamic analogy.

The fluid approach results in the description of changes in traffic state in terms of the propagation of traffic waves. For these waves to form and
### TABLE 3

Summary of the Derived Equations of State

**Group 4 Data**

<table>
<thead>
<tr>
<th>Deceleration Phase</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Greenberg</strong></td>
<td>$q = 33.23k \ln(180/k)$</td>
</tr>
<tr>
<td><strong>Underwood:</strong></td>
<td>$q = 122.0k e^{-k/47.14}$</td>
</tr>
<tr>
<td><strong>Quadratic:</strong></td>
<td>$q = 87.30k - 0.98k^2 + 0.003k^3$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Acceleration Phase</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Underwood:</strong></td>
<td>$q = 176.0k e^{-k/39.82}$</td>
</tr>
</tbody>
</table>

**Group 5 Data**

<table>
<thead>
<tr>
<th>Deceleration Phase</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Greenberg:</strong></td>
<td>$q = 36.04k \ln(149/k)$</td>
</tr>
<tr>
<td><strong>Underwood:</strong></td>
<td>$q = 116.0k e^{-k/45.24}$</td>
</tr>
<tr>
<td><strong>Quadratic:</strong></td>
<td>$q = 91.12k - 1.24k^2 + 0.004k^3$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Acceleration Phase</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Underwood:</strong></td>
<td>$q = 120.0k e^{-k/40.51}$</td>
</tr>
</tbody>
</table>
propagate according to the theory the scale of the observation area in both
time and space must be large with respect to the scale of the disturbances
themselves. The observation space of 3.5 miles by 4 minutes adequately ful-
fills this requirement. Secondly, the parameter values calculated for the
traffic stream represent instantaneous values existing in a fluid. Thus the
scale of the sampling domain must be small with respect to the scale of the
observation area. A sampling space of 600 feet by 1 second is obviously
small compared to 3.5 miles by 4 minutes. The hydrodynamic analogy also
requires that the roadway be crowded. This requirement stems from the
fundamental assumption of the theory that volume varies as a function of
density. Since at very low densities a change in density does not directly imply
a change in flow, the roadway in question must be "crowded" enough so that the
two are in fact related. The median lane of Interstate 71 during the morning
peak hour is certainly crowded in this sense of the word. Finally, the theory
depends upon the establishment of a stable representation for the traffic equa-
tion of state. Thus, the mathematical form chosen to represent the equation
must be selected from a data base consisting of a large number of data points.
The equations selected for use in this study were based on a minimum of 57
data points for the deceleration phase of group 5 and a maximum of 146 points
for the acceleration phase of group 4. No definite guidelines are available
regarding a recommended minimum size for the data base. Although a sample
size of 57 seems somewhat small the available data points do seem to form a
stable pattern for the variation of speed with density. In addition the
mathematical forms selected provide an explanation for a quite high proportion of the existing variation. Thus it is thought that the available data points constitute an adequate sample size for the purposes of this study.

Accordingly, it is contended that the flow situation selected for study does constitute a large scale problem in the sense of Lighthill and Whitham. An extension of the study in which larger numbers of vehicles are sampled in the construction of the equations of state is, however, recommended for future research.

4.7 SUMMARY

This chapter was devoted to an investigation of the applicability of the fluid analogy to the study of the behavior of traffic traveling in one lane of a multi-lane, one-way roadway. Specific attention was paid to the question of continuity, to the establishment of the fundamental traffic equation of state and to the implications of scale on the experimental situation in question. Each of these considerations is an essential issue in determining the suitability of the hydrodynamic approach for attacking the traffic flow problem proposed.

It was decided that although the one-dimensional equation of continuity upon which the fluid analogy is based does not strictly apply to a multi-lane situation, the departure from reality can be minimized by selecting a study environment in which only a small number of lane changes takes place. Keeping lane changes to a minimum is especially important in regions of moderate and high density. Detailed consideration was given to the determination of the appropriate methodology for measuring the continuum variables q, k and v and
to the selection of a suitable mathematical form for the traffic equation of state. It was decided that measurements should be taken over a local sampling domain with short duration in both time and space. A time width of one second and a space width of 600 feet were selected for use. A sliding domain was employed moving in a selected portion of the traffic stream. Acceleration and deceleration characteristics were found to be substantially different and accordingly measured parameter values were separated into two classes. Data was collected for six different vehicle groups and two groups from the six were selected for analysis.

Three different mathematical speed-density forms were found to provide an adequate fit to the deceleration phase data while only one form provided a reasonable fit for acceleration data. The selected deceleration forms were those proposed by Greenberg and Underwood and a quadratic form of the type $v = a_1 + a_2 k + a_3 k^2$. Underwood's form was selected for the acceleration data. The volume-density equations corresponding to each selected speed-density form were then derived.

A discussion was then presented evaluating the implications of scale to the experimental situation to be studied. It was concluded that the specific flow problem does constitute a large scale problem in the sense of Lighthill and Whitham. Thus it is treatable using the hydrodynamic approach. It was recommended, however, that an extension of the present study be undertaken in which larger sample sizes are available for establishing the traffic equations of state.
Attention is now turned to an analysis of the wave behavior predicted by the hydrodynamic approach as compared to the behavior actually observed in the median lane of Interstate 71. This analysis is presented in the next chapter.
CHAPTER 5

INVESTIGATION OF TRAFFIC

WAVE BEHAVIOR

5.1 INTRODUCTION

If the hydrodynamic analogy is applicable to the study of traffic flow in the median lane of Interstate 71, then flow disturbances occurring in this lane should be propagated back through the stream as kinematic waves. These waves should travel at a speed equal to the slope of the tangent to the appropriate traffic equation of state at the point representing the corresponding flow condition. In addition these kinematic waves should have the potential for coalescing to form discontinuous traffic shock waves. The speed of these shock waves would then be equal to the slope of the chord connecting the flow conditions existing on either side of the shock. Further a definable relation should exist between the speed of propagation of a kinematic wave relative to the roadway and the space mean speed of the traffic corresponding to the flow value being transmitted. The form of this relation should be determined by the form of the mathematical function used to characterize the applicable traffic equation of state. Each of these predicted consequences of the fluid theory will be explored in this chapter.
5.2 DISCUSSION OF THEORETICAL WAVE BEHAVIOR

The basic equation governing the motion of kinematic waves is given in Chapter 3 as equation (3.4) and is repeated for ease of reference below.

\[
\frac{\partial q}{\partial t} + c \frac{\partial q}{\partial x} = 0
\]  
(5.1)

For a uniform roadway this equation requires that variations in all three continuum variables be propagated through the traffic fluid as waves along which flow, density and space mean speed remain constant. The speed of a given wave is computed as \( c = \left( \frac{\partial q}{\partial k} \right) \).

Using this condition expressions can be derived relating the speed of propagation of waves through a stream governed by a specific equation of state to traffic density. Such expressions have been formulated for the alternative equations of state selected in Section 4.5.2 to describe the accelerating and decelerating behavior of vehicle groups 4 and 5. The resulting wave speed equations are summarized in Table 4. In these equations density is measured in vehicles per mile and wave velocity is given in mph.

The speed of propagation of a given kinematic wave relative to the roadway can also be defined as a function of the space mean speed transmitted by the wave by equation (3.6) repeated below as (5.2).

\[
c = \left( \frac{\partial q}{\partial k} \right) = v + k \frac{\partial v}{\partial k}
\]  
(5.2)

Thus depending upon the specific form of the function \( \partial v/\partial k \) different functions relating \( c \) and \( v \) can be obtained. For example, assuming the applicable equation
TABLE 4

Summary of Wave Speed Equations

VEHICLE GROUP 4

<table>
<thead>
<tr>
<th>Deceleration Phase</th>
<th>Equation of State</th>
<th>Wave Speed Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Greenberg</td>
<td>$c = 33.23 \left[ \ln \left( \frac{180}{k} \right) - 1 \right]$</td>
</tr>
<tr>
<td></td>
<td>Underwood</td>
<td>$c = 122.0 \ e^{-k/47.14} (1 - 0.021k)$</td>
</tr>
<tr>
<td></td>
<td>Quadratic</td>
<td>$c = 87.30 - 1.98k + 0.009k^2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Acceleration Phase</th>
<th>Equation of State</th>
<th>Wave Speed Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Underwood</td>
<td>$c = 176.0 \ e^{-k/39.82} (1 - 0.025k)$</td>
</tr>
</tbody>
</table>

VEHICLE GROUP 5

<table>
<thead>
<tr>
<th>Deceleration Phase</th>
<th>Equations of State</th>
<th>Wave Speed Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Greenberg</td>
<td>$c = 36.04 \left[ \ln \left( \frac{149}{k} \right) - 1 \right]$</td>
</tr>
<tr>
<td></td>
<td>Underwood</td>
<td>$c = 116.0 \ e^{-k/45.24} (1 - 0.022k)$</td>
</tr>
<tr>
<td></td>
<td>Quadratic</td>
<td>$c = 91.12 - 2.49k + 0.012k^2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Acceleration Phase</th>
<th>Equation of State</th>
<th>Wave Speed Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Underwood</td>
<td>$c = 120.0 \ e^{-k/40.51} (1 - 0.025k)$</td>
</tr>
</tbody>
</table>
of state describing flow on a particular roadway conforms to Greenberg's expression, the appropriate wave speed expression is obtained as follows.

\[ v = c_g \ln \left( \frac{k_j}{k} \right) \]

and

\[ \frac{\delta v}{\delta k} = \frac{c_g k}{k_j} \left( -\frac{k_j}{k^2} \right) = -\frac{c_g}{k} \]

Therefore:

\[ c = v + k(-\frac{c_g}{k}) = v - c_g \quad (5.3) \]

Hence, for this case, the wave speed \( c \) is a linear function of the space mean speed being transmitted having unit slope and an intercept defined by the negative value of Greenberg's constant \( c_g \). Since the constant \( c_g \) can be shown to represent the traffic speed at which maximum flow occurs then if Greenberg's equation of state applies, equation (5.3) can be used to predict the optimum traffic speed from wave characteristic considerations.

Similar wave speed–traffic speed functionals can be derived for each of the mathematical speed-density forms discussed in Section 4.4.1. In general the following relations are obtained:

<table>
<thead>
<tr>
<th>Speed–Density Form</th>
<th>Wave Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>( c = 2v - v_f )</td>
</tr>
<tr>
<td>Greenberg</td>
<td>( c = v - c_g )</td>
</tr>
<tr>
<td>Reciprocal</td>
<td>( c = -B )</td>
</tr>
<tr>
<td>Underwood</td>
<td>( c = v \left[ 1 - \frac{k}{k_m} \right] )</td>
</tr>
<tr>
<td>Quadratic</td>
<td>( c = 2v - a_1 + a_3 k^2 )</td>
</tr>
</tbody>
</table>
The first three equations for wave speed so attained are linear functions of traffic speed while the equations derived for the expression of Underwood and the quadratic expression depend upon both speed and density and, since \( v = f(k) \), are nonlinear in speed. Each linear function, however, exhibits a different slope. Thus a procedure for investigating the suitability of various forms for the equation of state beginning with an empirical analysis of traffic wave behavior is suggested. The traffic waves are identified and their speeds determined. A plot is then prepared treating wave speed as a function of traffic speed. The shape of the \( c-v \) functional implied by this plot is identified and used to select the most appropriate equation of state.

It is reemphasized at this point, before the experimental studies of wave behavior are discussed, that although the speed of a traffic wave may be given in terms of a complicated expression of speed and density, if the roadway is uniform, the wave itself will propagate along a simple straight-line path in the space-time plane.

5.3 EXPERIMENTAL STUDIES OF WAVE BEHAVIOR

5.3.1 Identification of the Wave Characteristics

Since on a uniform roadway a given traffic wave carries constant values of flow, speed and density, any of these three parameters can be used for the purpose of identifying wave movements. The form of the data available for the present study is such that it is most convenient to identify lines in the space-time plane corresponding to conditions of constant space mean speed. Thus this parameter was chosen for use.
The procedure begins by determining the space-time coordinates of vehicles traveling at the speed value appropriate to the wave being isolated. Once the coordinates for a number of vehicles sufficient to define the general shape of the wave in the space-time plane have been collected, a regression of position on the roadway against time of observation is performed. Since waves are assumed to follow straight-line paths in the x, t plane, the technique of least-squares linear regression is employed for this purpose. A separate regression is performed for each value of space mean speed being considered. In addition, since acceleration and deceleration characteristics were shown to differ in the analysis of Chapter 4, a separate regression is performed at each traffic speed value for each of the two types of traffic maneuvers.

If the hydrodynamic analogy applies for the flow situation being considered, the characteristics isolated should be straight-lines. The tendency toward linearity can be investigated by examining the individual wave path plots in combination with the coefficient of determination and the standard error of the estimate values provided by the regression. In addition, since the speed of propagation of a kinematic wave can be defined as $c = \frac{dx}{dt}$, the slope of the regression line fit to the data representing a given space mean speed provides an estimate of the speed of the wave carrying that mean speed value through the traffic fluid. The wave speed determined in this manner is measured relative to the roadway.

5.3.2 Results of the Space–Time Regression

A separate regression was performed for speed values of 0 fps, 5 fps,
10 fps, 15 fps, 20 fps, 25 fps, 30 fps, 40 fps, 50 fps, 60 fps, 70 fps, and 80 fps for deceleration conditions and for speeds of 0 fps, 5 fps, 10 fps, 15 fps, 20 fps, 25 fps, 30 fps, 40 fps, and 50 fps for acceleration conditions. Regressions for higher speed values for both types of conditions would have been desirable but were impossible due to the limited data available. As before, the required regressions were carried out on the IBM 370/165 computer using the OMNITAB language.

Two typical sets of data showing the pattern displayed by points of constant speed in the space-time plane are shown as Figures 22 and 23. These data sets represent the two extreme speed values studied for the deceleration phase. Figure 22 corresponds to a mean speed value of 0 fps. The resulting wave path can be seen to be quite uniform with a remarkable tendency toward linearity. Figure 23 illustrates the path of the 80 fps wave. Points defining this wave are few in number and somewhat scattered. In spite of the scatter, however, an underlying linear trend seems evident. The degree of definition of the paths of the waves carrying each of the other speed values studied, lies somewhere in between the extremes of Figures 22 and 23. A complete set of wave path data is presented in the Appendix.

A summary of the important quantities from the regression analysis is presented in Table 5. These quantities include the sample size used to define the wave path in the x, t plane, the wave speed, the coefficient of determination and the standard error of the estimate for each of the twenty-one constant mean speed waves identified.
<table>
<thead>
<tr>
<th>TIME, SECONDS</th>
<th>0.0</th>
<th>1.0000E 01</th>
<th>2.0000E 01</th>
<th>3.0000E 01</th>
<th>4.0000E 01</th>
<th>5.0000E 01</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure 22 Points of Constant Space Mean Speed in the Space-Time Plane (0 fps Decel.)**
Figure 23 Points of Constant Space Mean Speed in the Space-Time Plane (80 fps Decel.)
### TABLE 5

Summary of Wave Characteristics

<table>
<thead>
<tr>
<th>Deceleration Phase</th>
<th>Wave Identity</th>
<th>Sample Size</th>
<th>Wave Speed c(fps)</th>
<th>Coefficient of Determination</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 fps</td>
<td>30</td>
<td>-18.14</td>
<td>0.992</td>
<td>20.68</td>
<td></td>
</tr>
<tr>
<td>5 fps</td>
<td>33</td>
<td>-18.89</td>
<td>0.997</td>
<td>12.39</td>
<td></td>
</tr>
<tr>
<td>10 fps</td>
<td>32</td>
<td>-18.73</td>
<td>0.995</td>
<td>16.95</td>
<td></td>
</tr>
<tr>
<td>15 fps</td>
<td>30</td>
<td>-19.66</td>
<td>0.997</td>
<td>13.51</td>
<td></td>
</tr>
<tr>
<td>20 fps</td>
<td>30</td>
<td>-19.62</td>
<td>0.997</td>
<td>14.00</td>
<td></td>
</tr>
<tr>
<td>25 fps</td>
<td>30</td>
<td>-17.00</td>
<td>0.954</td>
<td>27.48</td>
<td></td>
</tr>
<tr>
<td>30 fps</td>
<td>22</td>
<td>-12.95</td>
<td>0.811</td>
<td>42.96</td>
<td></td>
</tr>
<tr>
<td>40 fps</td>
<td>18</td>
<td>-13.35</td>
<td>0.825</td>
<td>40.98</td>
<td></td>
</tr>
<tr>
<td>50 fps</td>
<td>18</td>
<td>-5.96</td>
<td>0.646</td>
<td>44.55</td>
<td></td>
</tr>
<tr>
<td>60 fps</td>
<td>15</td>
<td>1.60</td>
<td>0.073</td>
<td>55.37</td>
<td></td>
</tr>
<tr>
<td>70 fps</td>
<td>18</td>
<td>5.67</td>
<td>0.735</td>
<td>40.57</td>
<td></td>
</tr>
<tr>
<td>80 fps</td>
<td>10</td>
<td>9.12</td>
<td>0.712</td>
<td>49.65</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Acceleration Phase</th>
<th>Wave Identity</th>
<th>Sample Size</th>
<th>Wave Speed c(fps)</th>
<th>Coefficient of Determination</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 fps</td>
<td>25</td>
<td>-15.02</td>
<td>0.991</td>
<td>17.67</td>
<td></td>
</tr>
<tr>
<td>5 fps</td>
<td>28</td>
<td>-14.80</td>
<td>0.997</td>
<td>10.65</td>
<td></td>
</tr>
<tr>
<td>10 fps</td>
<td>30</td>
<td>-14.66</td>
<td>0.998</td>
<td>9.75</td>
<td></td>
</tr>
<tr>
<td>15 fps</td>
<td>30</td>
<td>-14.21</td>
<td>0.996</td>
<td>12.70</td>
<td></td>
</tr>
<tr>
<td>20 fps</td>
<td>30</td>
<td>-13.74</td>
<td>0.990</td>
<td>20.50</td>
<td></td>
</tr>
<tr>
<td>25 fps</td>
<td>25</td>
<td>-13.51</td>
<td>0.975</td>
<td>32.27</td>
<td></td>
</tr>
<tr>
<td>30 fps</td>
<td>25</td>
<td>-15.10</td>
<td>0.961</td>
<td>37.06</td>
<td></td>
</tr>
<tr>
<td>40 fps</td>
<td>15</td>
<td>-4.29</td>
<td>0.799</td>
<td>53.08</td>
<td></td>
</tr>
<tr>
<td>50 fps</td>
<td>20</td>
<td>11.05</td>
<td>0.791</td>
<td>83.51</td>
<td></td>
</tr>
</tbody>
</table>
A number of interesting observations can be made by considering the results shown in Table 5 in conjunction with the wave path plots presented in the Appendix. First, and probably most important, kinematic waves do exist in the I-71 median lane traffic flow. Second, the great majority of these waves follow paths in the $x, t$ plane that can be well represented by straight lines. In fact, the tendency toward linearity is remarkably high for waves carrying speeds up to and including 25 fps for deceleration and up to and including 30 fps for acceleration. This seems to be the region, characterized by low space mean speed and high densities, in which the traffic fluid is fully cohesive. Third, the speed of propagation of the waves carrying these low speed values is basically constant. The wave speed for the deceleration waves carrying constant speeds between 0 and 25 fps varies only in the small range from -17.00 fps to -19.66 fps. Similarly the wave speed of the waves carrying low acceleration speed values varies only from -13.51 fps to -15.10 fps. This type of wave speed behavior coincides with that predicted by the reciprocal speed-density relationship. Fourth, the wave speed of waves carrying a given space mean speed value is definitely different for accelerating conditions as compared to decelerating conditions. In the region of low speeds referred to previously deceleration waves propagate backwards on the average approximately 23% faster than the corresponding acceleration waves. This result provides support for the concept of two distinct equations of state and confirms the results of a number of other researchers who have observed that deceleration waves travel faster than acceleration waves.

Fifth, as waves carrying higher speed values are reached, the speed of propagation begins to increase in a positive sense relative to the roadway.
Eventually for some space mean speed the wave speed becomes zero. The waves for higher speeds then change directions and move forward relative to the road. The speed of propagation continues to increase with increasing space mean speed. This type of wave behavior is consistent with the theory of Lighthill and Whitham. Sixth, as wave speed approaches zero, the wave paths begin to depart somewhat from linearity. Once \( c = 0 \) is passed, however, and waves change direction of propagation, the tendency toward linearity once more begins to increase. This wave behavior is also consistent with theory since in the vicinity of \( c = 0 \) the slope of the \( q-k \) curve defining wave speed is changing very rapidly with changing density. Thus it is logical that waves formed in this region will be less well-defined than those carrying flow rates less than maximum. This is also consistent with the well known traffic engineering maxim that flow near capacity is unstable. Seventh, the space mean speed value at which flow is a maximum (i.e., where \( c = 0 \)) is different for accelerating and decelerating conditions. Maximum flow occurs at higher space mean speed for decelerating conditions than for accelerating conditions. Finally, the sample sizes used to identify the waves were necessarily small since waves do not appear to persist for long intervals of time in the flow studied. The maximum length wave identified consisted of 33 constant speed vehicles and lasted for only 49 seconds. The short duration of wave existence is thought to be related to departures of the experimental situation from the continuity condition.

5.3.3 Analysis of the Wave Speed-Mean Speed Pattern

Additional results of interest can be obtained by plotting the wave speed
versus the space mean speed carried by the corresponding wave. This was done and the plots are presented as Figures 24 and 25. Figure 24 represents deceleration behavior while acceleration is described by Figure 25.

Observation of these plots reveals an extremely interesting phenomenon. The plateaus of nearly constant wave speed corresponding to the low space mean speed values discussed in the previous section are clearly visible on the left of the plots. Once wave speeds leave this plateau, however, they increase in a very direct fashion with increasing space mean speed for both flow conditions. In fact the relation between c and v beyond the plateaus appears quite linear. This wave speed–traffic speed pattern is very similar to the pattern found by Herman, Lam and Rothery in an investigation of the starting behavior of vehicle platoons (72). The traffic flow situation in this case was an idealized one formed on a test track. The occurrence of such similar patterns in two completely different flow situations is a remarkable result. Such a linear relation between wave speed and space mean speed conforms to the behavior predicted by either a linear form or a Greenberg form equation of state (see Section 5.2). Unfortunately, the available data is insufficient to determine the complete nature of the c–v pattern since the highest space mean speed wave observed was only 80 fps for deceleration and 50 fps for acceleration.

The form of Figures 24 and 25 is such that they can readily be used to provide an estimate of the space mean speed corresponding to maximum flow. This speed is determined by locating the point at which wave speed is zero.
Figure 24  Plot of Wave Speed vs. Space Mean Speed
Deceleration Phase
Figure 25 Plot of Wave Speed vs. Space Mean Speed
Acceleration Phase
If this is done the resulting optimum traffic speed values are 57.9 fps (39.4 mph) for deceleration conditions and 42.8 fps (29.1 mph) for acceleration conditions. The optimum speed value corresponding to deceleration coincides closely with the generally accepted optimum speed range for freeways (35 - 45 mph) while the speed given for acceleration is lower than expected.

5.3.4 Analysis of Spacings and Headways

A given kinematic traffic wave is supposed to carry constant values of flow and density as well as space mean speed. In order to determine if this characteristic pertains to the waves identified in this study, an analysis of volume and density along the speed waves was conducted. The parameters used to represent volume and density were headway and spacing. If the theory of kinematic waves applies the distributions of headways and spacings along a given wave should be symmetric and display a low variance.

Instantaneous spacing and headway values were computed from the experimental data corresponding to the constant speed values defining each of the acceleration and deceleration waves. The distributions of both parameters were then subjected to a statistical analysis to determine the shape and the amount of variation in the two distributions representing each speed wave. The first shape factor, $\alpha_3$, is used as a measure of departure from symmetry while the coefficient of variation of the appropriate parameter gives a relative measure of variability. For a symmetric distribution with low variance both these parameters should exhibit values approaching zero. The reader unfamiliar with these statistical measures is referred to any good introductory
The results of the statistical analyses are given in Table 6 for spacings and Table 7 for headways. In each table the mean value, the first shape factor and the coefficient of variation of the parameter in question are given for the distributions corresponding to each speed wave.

Before any attempt is made to interpret the results presented in these two tables, it must be emphasized that the sample sizes available for characterizing the spacing and headway distributions existing along a given wave are much too small to allow the nature of these distributions to be well defined. Thus no absolute conclusions regarding the constancy of volume and density along a given wave are possible. Certain observations are possible, however, that shed further light on the nature of the identified traffic waves. Regarding the question of symmetry it is observed that the shape factors for the various waves jump about rather randomly. This irregular behavior is exhibited for both acceleration and deceleration waves and for both spacing and headway distributions. The existence of some very small shape factors lends some credence to the hypothesis that the distributions do have a tendency toward symmetry but that the small sample size prohibits this tendency from being clearly illustrated. However, an alternative hypothesis that the existence of the small shape factors is a random occurrence can not be disproved. In fact, analysis of the third moment about the mean for the individual distributions indicates that, in most cases, the distributions tend towards a slight positive skew. Irrespective of the magnitudes of the shape factors it can be seen that,
TABLE 6

Summary of Results of Statistical Spacing Analysis

<table>
<thead>
<tr>
<th>Wave Identity</th>
<th>Mean Spacing (ft)</th>
<th>Coefficient of Variation of Spacing</th>
<th>First Shape Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 fps</td>
<td>26.22</td>
<td>0.21</td>
<td>1.58</td>
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<tr>
<td>5 fps</td>
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<td>0.15</td>
<td>0.25</td>
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<tr>
<td>10 fps</td>
<td>29.90</td>
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<td>1.23</td>
</tr>
<tr>
<td>15 fps</td>
<td>34.86</td>
<td>0.21</td>
<td>0.72</td>
</tr>
<tr>
<td>20 fps</td>
<td>39.94</td>
<td>0.18</td>
<td>0.32</td>
</tr>
<tr>
<td>25 fps</td>
<td>48.35</td>
<td>0.19</td>
<td>0.00</td>
</tr>
<tr>
<td>30 fps</td>
<td>57.84</td>
<td>0.25</td>
<td>0.10</td>
</tr>
<tr>
<td>40 fps</td>
<td>68.74</td>
<td>0.36</td>
<td>0.14</td>
</tr>
<tr>
<td>50 fps</td>
<td>90.27</td>
<td>0.40</td>
<td>0.30</td>
</tr>
<tr>
<td>60 fps</td>
<td>103.31</td>
<td>0.47</td>
<td>0.52</td>
</tr>
<tr>
<td>70 fps</td>
<td>116.35</td>
<td>0.51</td>
<td>0.88</td>
</tr>
<tr>
<td>80 fps</td>
<td>156.44</td>
<td>0.54</td>
<td>0.52</td>
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</table>

Deceleration Phase

<table>
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<th>Mean Spacing (ft)</th>
<th>Coefficient of Variation of Spacing</th>
<th>First Shape Factor</th>
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</thead>
<tbody>
<tr>
<td>0 fps</td>
<td>29.01</td>
<td>0.19</td>
<td>0.41</td>
</tr>
<tr>
<td>5 fps</td>
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<td>0.24</td>
<td>0.93</td>
</tr>
<tr>
<td>10 fps</td>
<td>51.94</td>
<td>0.24</td>
<td>0.58</td>
</tr>
<tr>
<td>15 fps</td>
<td>59.49</td>
<td>0.26</td>
<td>0.85</td>
</tr>
<tr>
<td>20 fps</td>
<td>67.18</td>
<td>0.28</td>
<td>1.06</td>
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<td>25 fps</td>
<td>74.77</td>
<td>0.32</td>
<td>1.10</td>
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<tr>
<td>30 fps</td>
<td>80.45</td>
<td>0.32</td>
<td>1.38</td>
</tr>
<tr>
<td>40 fps</td>
<td>95.54</td>
<td>0.27</td>
<td>0.72</td>
</tr>
<tr>
<td>50 fps</td>
<td>99.35</td>
<td>0.40</td>
<td>1.22</td>
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</table>

Acceleration Phase
TABLE 7

Summary of Results of Statistical Headway Analysis

<table>
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<tr>
<th>Deceleration Phase</th>
<th>Wave Identity</th>
<th>Mean Headway (sec)</th>
<th>Coefficient of Variation of Headway</th>
<th>First Shape Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 fps</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
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<td>6.88</td>
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<td>2.47</td>
</tr>
<tr>
<td></td>
<td>10 fps</td>
<td>3.34</td>
<td>0.19</td>
<td>1.03</td>
</tr>
<tr>
<td></td>
<td>15 fps</td>
<td>2.36</td>
<td>0.20</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>20 fps</td>
<td>2.03</td>
<td>0.18</td>
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</tr>
<tr>
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<td>30 fps</td>
<td>1.94</td>
<td>0.25</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>40 fps</td>
<td>1.72</td>
<td>0.36</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>50 fps</td>
<td>1.81</td>
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</tr>
<tr>
<td></td>
<td>60 fps</td>
<td>1.72</td>
<td>0.47</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>70 fps</td>
<td>1.66</td>
<td>0.51</td>
<td>0.88</td>
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<td></td>
<td>80 fps</td>
<td>1.95</td>
<td>0.54</td>
<td>0.53</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Acceleration Phase</th>
<th>Wave Identity</th>
<th>Mean Headway (sec)</th>
<th>Coefficient of Variation of Headway</th>
<th>First Shape Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 fps</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>5 fps</td>
<td>11.42</td>
<td>0.38</td>
<td>2.38</td>
</tr>
<tr>
<td></td>
<td>10 fps</td>
<td>5.71</td>
<td>0.29</td>
<td>1.16</td>
</tr>
<tr>
<td></td>
<td>15 fps</td>
<td>4.02</td>
<td>0.26</td>
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<td></td>
<td>20 fps</td>
<td>3.39</td>
<td>0.28</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td>25 fps</td>
<td>3.00</td>
<td>0.32</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td>30 fps</td>
<td>2.69</td>
<td>0.32</td>
<td>1.41</td>
</tr>
<tr>
<td></td>
<td>40 fps</td>
<td>2.39</td>
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<td>0.71</td>
</tr>
<tr>
<td></td>
<td>50 fps</td>
<td>1.99</td>
<td>0.40</td>
<td>1.22</td>
</tr>
</tbody>
</table>
in general, the distributions of the flow parameters existing along deceleration waves tend more toward symmetry than those along corresponding acceleration waves.

With respect to the variation exhibited by the spacing and headway distributions, it is observed that the coefficients of variation of both parameters corresponding to waves carrying low values of space mean speed are relatively low and remain fairly constant with increasing traffic speed. The coefficients of variation begin to increase steadily, however, as higher space mean speed waves are considered. This variation pattern is consistent with and lends further support to the observation made in Section 5.3.3 that the traffic fluid is cohesive for speeds below 25 fps during deceleration conditions and below 30 fps for acceleration conditions. When speeds rise above these levels the fluid begins to break up as illustrated by the steadily increasing variation in the spacing and headway distributions. While the condition of cohesiveness exists no significant difference is discernable between the variation of volume and density along deceleration waves as compared to that along the corresponding acceleration waves.
5.4 COMPARISON OF THEORETICAL AND EXPERIMENTALLY OBSERVED WAVE BEHAVIOR

5.4.1 Qualitative Behavior

The material contained in Sections 5.2 and 5.3 demonstrates that the behavior of kinematic waves as predicted using the Lighthill-Whitham fluid analogy is quite similar from a qualitative point of view to the wave behavior observed in the experimental study. Kinematic waves do exist and are propagated along straight line paths for an extensive portion of the average speed (and hence density) domain. Waves carrying low speed values travel backward relative to the roadway while waves carrying high speed values travel forward. In between there is a wave corresponding to maximum flow that stands still. Spacings and headways are relatively constant along a given speed wave with the amount of variation present increasing with the magnitude of the speed value carried. Different average spacing and headway values are exhibited by different speed waves. Definite differences exist between waves carrying deceleration speed values as compared to acceleration speed values. The question now arises: how good is the quantitative prediction of kinematic wave behavior offered by the theory.

5.4.2 Quantitative Behavior

The first attempt at evaluating the ability of the theory for predicting the quantitative behavior of traffic waves was predicated upon a comparison of the observed wave speeds with those predicted using the theoretical equations of Table 4. The results of such a comparison for the deceleration phase of
Vehicle Group 4 are shown in Table 8.

As can be seen from Table 8 although the predicted wave speeds are of the right order of magnitude and, for most waves, display the proper direction of travel, there is not good agreement between any of the three sets of theoretical results and the observed wave speeds in terms of actual speed value. If a selection from the three alternative equations of state has to be made, however, Underwood's form seems to yield the most comparable wave speed values.

In general actual waves travel at slower absolute speeds and exhibit less difference in wave speed between individual waves than is predicted by the theory. If the three sets of theoretical wave speeds are compared without reference to the observed speeds, it can be seen that substantially different wave speed predictions result depending upon the mathematical form chosen for the equation of state. This is true even though each mathematical form represented in Table 8 provides a good fit ($R^2 \geq 0.950$) to the experimental speed-density data. The basic conclusion is that the attainment of accurate quantitative results using the fluid analogy is very difficult due to the sensitivity of such results to the mathematical form representing the equation of state. Results very similar to those shown in Table 8 were obtained for the acceleration phase for Vehicle Group 4 and for both acceleration and deceleration phases for Vehicle Group 5. A complete set of results is provided in the Appendix.

The second attempt at comparing theoretical and experimental wave
TABLE 8

Comparison of Theoretical and Observed Wave Speeds

Vehicle Group 4 - Deceleration Phase

<table>
<thead>
<tr>
<th>Wave Identity</th>
<th>Theoretical Wave Speed (fps)</th>
<th></th>
<th></th>
<th></th>
<th>Observed Wave Speed (fps)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Greenberg</td>
<td>Underwood</td>
<td>Quadratic</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 fps</td>
<td>-48.85</td>
<td>-</td>
<td>-</td>
<td>-18.14</td>
<td></td>
</tr>
<tr>
<td>5 fps</td>
<td>-43.96</td>
<td>-12.76</td>
<td>-</td>
<td>-18.89</td>
<td></td>
</tr>
<tr>
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<td></td>
</tr>
<tr>
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<td>-19.66</td>
<td></td>
</tr>
<tr>
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<td>-24.27</td>
<td>-30.83</td>
<td>-17.00</td>
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</tr>
<tr>
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<td>-23.67</td>
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<td>-12.95</td>
<td></td>
</tr>
<tr>
<td>50 fps</td>
<td>1.17</td>
<td>-13.95</td>
<td>-7.95</td>
<td>-5.96</td>
<td></td>
</tr>
<tr>
<td>60 fps</td>
<td>11.14</td>
<td>-4.03</td>
<td>7.39</td>
<td>1.60</td>
<td></td>
</tr>
<tr>
<td>70 fps</td>
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<td>23.80</td>
<td>5.67</td>
<td></td>
</tr>
<tr>
<td>80 fps</td>
<td>31.17</td>
<td>10.48</td>
<td>41.88</td>
<td>9.12</td>
<td></td>
</tr>
</tbody>
</table>
behavior quantitatively involved the prediction of the space mean speed value carried by the stationary wave (i.e., $c = 0$). Observed wave behavior demonstrates that the relevant traffic speed values are 57.9 fps for the deceleration phase and 42.8 fps for the acceleration phase. The theoretical speed values can be obtained for each alternative equation of state by solving the wave equations of Section 5.2 for the case where $\dot{c} = 0$. If this is done the results shown in Table 9 are obtained.

Table 9 reveals that each of the three equations of state selected for use in describing deceleration behavior provides an estimate of optimum traffic speed that is within 15% of the observed value. The quadratic form equation provides the best estimate for both vehicle groups. The estimate provided for acceleration behavior by Underwood's equation is much higher in both cases than the observed value. Thus, from the point of view of predicting the optimum speed of the traffic stream, the fluid analogy provides more reliable results for conditions of deceleration.

The final approach taken was to compare the theoretical functions relating wave speed and space mean speed derived in Section 5.2 to the c-v pattern displayed by the waves identified in the experimental study as shown in Figures 24 and 25. The hope was that the experimental pattern would match the pattern predicted by theory employing one of the alternative equations of state. If matching patterns could be identified, the next step would be to make quantitative estimates of the pertinent traffic stream parameters based on observed wave behavior and compare these estimates to measured parameter
TABLE 9

Speed Values Carried by the Stationary Wave

<table>
<thead>
<tr>
<th>Vehicle Group 4</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Deceleration Phase</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observed Value:</td>
<td>57.9 fps</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Greenberg:</td>
<td>48.8 fps</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Underwood:</td>
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<tr>
<td>Quadratic:</td>
<td>55.2 fps</td>
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<tr>
<td><strong>Acceleration Phase</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Observed Value:</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Underwood:</td>
<td>95.1 fps</td>
<td></td>
<td></td>
</tr>
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</table>

<table>
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<th>Vehicle Group 5</th>
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</thead>
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<td><strong>Deceleration Phase</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Observed Value:</td>
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<td></td>
</tr>
<tr>
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<td>53.0 fps</td>
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<td></td>
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<tr>
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<td>62.7 fps</td>
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<td></td>
</tr>
<tr>
<td>Quadratic:</td>
<td>60.1 fps</td>
<td></td>
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</tr>
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<td><strong>Acceleration Phase</strong></td>
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</tr>
<tr>
<td>Observed Value:</td>
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<td></td>
</tr>
<tr>
<td>Underwood:</td>
<td>64.9 fps</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
values. Once again no conclusive results were obtained. Whereas the theoretical functions depict the c-v relationship as being either linear or curvilinear throughout the entire traffic speed domain depending upon the equation of state chosen, the observed pattern consists of a basically constant wave speed plateau at low space mean speeds connected to a linear segment of positive slope which describes the relationship at higher space mean speeds. Although such a pattern could be explained in terms of a complex equation of state such as a combined linear-reciprocal or Greenberg-reciprocal form, it is more likely further evidence of the inability of the basic theory to provide accurate quantitative results. As pointed out in Chapter 2 such quantitative discrepancies are often the case with theory derived using an analogical approach.

5.5 SHOCK WAVES

No experimental studies relative to shock wave behavior could be conducted for the simple reason that no shock waves were present in the available traffic flow data. Although median lane volumes, densities and speeds exhibited wide variation within a relatively small portion of the space-time plane providing an environment that would be theoretically conducive to shock formation, no such discontinuous wave formed. In fact, from a practical point of view, it is doubtful that a fully discontinuous wave will ever be observed on the open roadway. The ability of a given driver to observe conditions several car lengths in front of him, his finite reaction time and the finite accelerating and decelerating capabilities of vehicles mitigate against
the occurrence of a completely abrupt change in the fundamental traffic flow parameters. Whereas shock waves in gases are formed as a result of the collision of fluid masses having different flow characteristics, drivers have an inherent tendency to avoid collision whenever possible. Waves of stopping and starting do exist in freeway traffic flows. The flow parameter values displayed on opposite sides of these waves are widely different. However, these waves possess substantial thickness and thus do not travel as true shock waves. Although the basic fluid theory provides some indication of the qualitative behavior of such waves, it does not offer a viable quantitative description.

A wave that approaches a true shock might be formed in a freeway flow as a result of a geometric bottleneck or a traffic incident. A geometric bottleneck is a section of roadway which due to its geometric configuration possesses a capacity significantly less than the sections upstream and downstream from it. Fluid theory predicts that a shock wave will be formed if a pulse of high level flow greater than bottleneck capacity arrives at such a geometric constriction. The resulting wave will then propagate upstream at a speed dependent upon the magnitude of the arriving flow pulse and the capacity of the bottleneck. The wave will continue to propagate until the upstream demand falls below bottleneck capacity. It will then change direction and eventually pass through the bottleneck returning stable flow to the upstream section. A traffic incident induced wave might be present as the result of a stalled vehicle or an accident which abruptly interrupts the freeway flow.
Such an abrupt change in flow conditions would theoretically also cause a shock front to be formed and to move upstream at a speed defined by the chord connecting the original flow conditions to \( k_j \) on the fundamental diagram. This wave would persist until the incident was cleared from the freeway.

Once again, since neither drivers or vehicles can react instantaneously, the actual wave formed in response to a bottleneck or an incident in a real world traffic flow would not be truly discontinuous but would rather possess a finite thickness. Such a wave might be conveniently termed a pseudo-shock. The ability of the basic theory to describe such pseudo-shocks in a multi-lane flow could not be evaluated since no geometric bottlenecks or traffic incidents were represented in the available data.

5.6 SUMMARY

This chapter has been devoted to an investigation of traffic wave behavior. A series of kinematic waves carrying different values of volume, speed and density were identified within the traffic flow in the median lane of southbound Interstate 71. The behavior of these waves was studied and compared to the behavior predicted by application of the hydrodynamic analogy. It was found that although the theory derived from the analogy provides a remarkably good qualitative description of actual wave behavior it does not provide very good quantitative results. One problem in this regard was identified as the extreme sensitivity of the predicted quantitative values to changes in the mathematical form of the equation of state. No studies of shock
wave behavior could be conducted since no shock waves were present in the experimental data. A rationale was presented to explain the absence of shock waves in a crowded traffic flow such as the one studied.
CHAPTER 6

SUMMARY OF RESULTS, CONCLUSIONS
AND RECOMMENDATIONS

6.1 REVIEW OF STUDY OBJECTIVE AND SCOPE

Before presenting a summary of the results achieved during the course of this study it is appropriate to review the objective with which the study was undertaken and to redefine the prevailing limitations that served to restrict the scope of the study. In this manner a framework can be established from which the value of the investigation can be judged.

The overall aim of the study was to investigate the applicability of the hydrodynamic approach for describing the macroscopic behavior of traffic flow on a single lane of a multiple lane, unidirectional roadway. The importance of such a study lies in the fact that existing attempts to model traffic behavior in a multi-lane environment have been both few in number and limited in success. Yet there is no question that the multi-lane traffic flow situation is an extremely important part of the present day transportation system. Many cities depend exclusively on their freeway system to provide the accessibility required for the successful functioning of society. As traffic demand increases problems of congestion and delay threaten to choke off this accessibility. In order to
eliminate or at least alleviate these problems it is essential that the fundamental nature of multi-lane traffic flow be fully understood. Hence, the development of approaches for modeling multi-lane traffic behavior becomes an important research priority.

The principal factor restricting the scope of the study reported herein was the scarcity of data of the type required on flow in a multi-lane environment. Comprehensive data on the space-time behavior of vehicles was available only for a section of roadway 3.5 miles in length and for a time period of about four minutes. A total of 115 vehicles were represented by the data. Although data existed for two separate lanes of travel only that describing flow in the median lane was satisfactory for use. Shoulder lane flow was subject to frequent interruption due to vehicles entering and exiting from the roadway. A second but less significant limitation placed on the investigation was the time frame in which it was to be conducted. Since the subject study was a part of a larger research project it was necessary that it be completed in compliance with the agreed termination date of that project. This time limitation brought no real hardship on the study except that it prohibited the collection and reduction of further traffic flow data.

6.2 SUMMARY OF RESULTS

The results achieved during the course of the study which are pertinent to the behavior of traffic flowing in a multi-lane environment
are summarized below. The specific results refer to the macroscopic description of single-lane flow as provided by the hydrodynamic approach.

1. Constant values of space mean speed are propagated back through the traffic stream in the form of kinematic waves. In general, these waves follow straight line paths in the space-time plane.

2. Volume and density measured in terms of headways and spacings remain relatively constant along a given wave. Different space mean speed waves exhibit different average headway and average spacing values.

3. The waves travel along the roadway at velocities which are in fact related to the traffic equation of state. Waves carrying low space mean speed values travel backward along the roadway while high space mean speed waves travel forward. In between there is a moderate speed wave which remains stationary. The space mean speed value carried by this latter wave provides a ballpark estimate of the optimum speed of the traffic flow.

4. The wave carrying a given space mean speed value for conditions of deceleration is distinctly different from the corresponding wave representing accelerating conditions. This difference is primarily manifested in the speed of propagation of the wave. In general acceleration waves travel at a slower absolute speed than the corresponding deceleration waves. Actual acceleration rates were also found to be smaller in magnitude and to exhibit less variation with respect to the size of the speed change being negotiated than deceleration rates.
5. The traffic waves carrying low space mean speed values travel at a nearly constant speed which is independent of the traffic speed. This wave speed plateau is characteristic of both acceleration and deceleration conditions although the magnitude and extent of the region of constant wave speed varies between the two conditions. Once the plateau is escaped wave speed increases directly (in fact almost linearly) with increasing traffic speed. The rate of increase is different for acceleration as compared to deceleration waves.

6. The traffic fluid seems to be most cohesive in the region between space mean speeds of 25 fps and 0 fps for deceleration conditions and between 0 fps and 30 fps for acceleration conditions. At these low traffic speeds the kinematic waves are very well defined and display a very marked tendency toward linearity in the space–time plane. Distributions of both spacings and headways are well behaved and have relatively low coefficients of variation. Certain of the distributions seem to be quite symmetrical while others do not. No reliable conclusion regarding symmetry can be drawn, however, due to the small sample sizes defining the distributions in question. Above 25 fps for deceleration and 30 fps for acceleration the waves begin to lose definition. At the same time the variation displayed by the spacing and headway distributions becomes more pronounced. Poorest definition occurs in the vicinity of zero wave speed. This loss of definition is understandable since this is the region on the fundamental diagram corresponding to capacity traffic flow and
such high volume flow is known to be unstable. Once traffic speeds above optimum are obtained the waves become better defined but do not attain the same degree of definition which characterizes the cohesive flow region.

7. For the flow situation studied the kinematic waves exist only for relatively short periods of time and involve only a relatively small number of vehicles. The longest duration wave identified involved only 33 vehicles and persisted for 49 seconds. The loss of fluid continuity due to the possibility for vehicles to change lanes contributes to the transient nature of the observed waves.

8. No discontinuous waves (shock waves) were identified in the study data even though the flow environment was suitable for the formation of such a wave. The absence of shock waves in the given flow situation is thought to be the result of the finite capabilities of drivers and vehicles and the inherent tendency of drivers to avoid abrupt and hazardous driving maneuvers. In fact it is doubtful that a truly discontinuous wave will ever be observed on the open roadway unless one considers the case of a multi-vehicle rear end collision.

9. Attempts to predict the quantitative behavior of the observed traffic waves from theoretical considerations did not produce reliable results. The predicted results relative to wave speed and the optimum speed of traffic flow were found to be very sensitive to the mathematical form chosen to represent the equation of state. The selection of an
appropriate mathematical form is complicated by a number of factors:

a. The problem of determining an appropriate space-time domain over which values of the continuum variables are to be sampled

b. The choice of the most appropriate definition for use in computing q, k and v values

c. The question of roadway uniformity and whether one or more fundamental diagrams are required to describe flow along a roadway section

d. The acceleration-deceleration dissymmetry

Even though a mathematical form is found that provides a good fit to the data chosen for use, there is no guarantee that the resulting wave predictions are reliable. In fact it was shown for deceleration conditions that three mathematical forms each of which exhibited a good fit to the available data provided widely varying predictions for wave speed. None of the three sets of predictions compared favorably to the observed values of wave speed.

6.3 CONCLUSIONS

It is concluded that the hydrodynamic approach does possess a significant potential for describing qualitatively the macroscopic behavior of traffic traveling in a single lane within a multi-lane environment. For this potential to be realized, however, careful attention must be given to
defining the flow situation to be studied such that flagrant violations of the fundamental assumptions of the approach are prevented. Specifically, consideration must be devoted to questions of continuity and scale and to the construction of an appropriate representation of the traffic equation of state. Each of these is an essential factor that must be carefully dealt with in the light of the specific flow situation being analyzed.

Certain quantitative results can also be obtained using the hydrodynamic approach. These results, however, are very sensitive to the mathematical form chosen to represent the traffic equation of state. Since the attainment of an appropriate equation of state itself constitutes an extremely difficult problem, the implications of which are not completely understood, the resulting quantitative predictions were found to be very unreliable. This does not necessarily represent a breach in the validity of the theory but rather emphasizes the difficulties involved in framing a real world traffic flow situation in theoretical terms.

6.4 RECOMMENDATIONS

The analysis of multi-lane flow situations constitutes a fertile ground for future research. It is both a fundamentally important problem area and one that is virtually unexplored. The following represent only a few of many possible avenues of attack.

1. The subject study was restricted by a paucity of data. It is recommended therefore that further study be conducted along identical
lines with a greatly increased data base. A desirable data base should include increased coverage of traffic behavior in both space and time. In addition it should include a variety of different demand conditions and different roadway cross-sections.

2. The establishment of an accurate representation of the traffic equation of state is essential to the analysis of multi-lane flow situations. A comprehensive study of the factors affecting the form of the equation of state (or equations of state) is needed. Such a study must be carried far beyond the end of finding a mathematical form (or forms) that provides an adequate fit to a given set of experimental data.

3. A question that also must be treated is the amount of dependency existing between adjacent lanes in a multi-lane, one-way traffic flow. Can valid results be obtained by analyzing lane flow independently or is such a simple approach too idealized to provide useful results? If adjacent flows are dependent can this dependency be characterized?

4. The statistical approach of Prigogine and his associates provides another interesting avenue for attacking multi-lane flows. Investigation of real world flow situations from this point of view could yield enlightening results. Once again the attainment of a comprehensive data base is required. In fact the provision of such a data base represents a worthy research goal in itself.
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29. Tanner, J. C., "The Delay to Pedestrians Crossing a Road", Biometrika 38, pp. 383-392, (1951)


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APPENDIX

A.1 INTRODUCTION

In Chapter 5 of this report it was stated that twenty-one distinct kinematic traffic waves carrying constant values of space mean speed were identified in the southbound median lane flow of Interstate 71. Plots depicting the path in the space-time plane of only two of these waves (0 fps and 80 fps - deceleration phase) were presented at that time. These plots were thought to sufficiently illustrate the method used to identify the waves and the remaining plots were omitted to maintain the cohesiveness of the text. A complete set of plots for all waves is included herein for the reader earnestly interested in the details of the study.

In addition in Chapter 5 a discussion was presented regarding the ability of the fluid theory to provide accurate quantitative results. As part of that discussion a comparison was made of theoretical wave speeds and those experimentally observed in the median lane flow. Illustrative results were presented for the deceleration phase of Vehicle Group 4. A complete set of results for the four groups of data analyzed are included below.
A. 2 PRESENTATION OF RESULTS

The wave path plots are shown as Figures A1 through A21. The first twelve plots depict deceleration waves while the remaining nine are for acceleration waves. Plots are arranged in order by increasing space mean speed. The comparisons of theoretical and observed wave speed are summarized in Tables A1 through A4. Tables A1 and A2 present results for Vehicle Group 4 while Tables A3 and A4 are for Vehicle Group 5.
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Figure A2 Kinematic Wave Path - 5 fps Decel.
Figure A3 Kinematic Wave Path - 10 fps Decel.
Figure A4 Kinematic Wave Path - 15 fps Decel.
Figure A5 Kinematic Wave Path - 20 fps Decel.
Figure A6 Kinematic Wave Path - 25 fps Decel.
Figure A7 Kinematic Wave Path - 30 fps Decel.
Figure A8  Kinematic Wave Path - 40 fps Decel.
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Figure A9 Kinematic Wave Path - 50 fps Decel.
Figure A10 Kinematic Wave Path - 60 fps Decel.
**OMNITAR SPEED CHARACTERISTICS STUDY**

**70 FPS CONTOUR - DECEL**

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**Figure A11 Kinematic Wave Path - 70 fps Decel.**
Figure A12 Kinematic Wave Path - 80 fps Decel.
Figure A13 Kinematic Wave Path - 0 fps Accel.
**Figure A14 Kinematic Wave Path - 5 fps Accel.**
Figure A15 Kinematic Wave Path - 10 fps Accel.
Figure A16 Kinematic Wave Path - 15 fps Accel.
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Figure A18 Kinematic Wave Path - 25 fps Accel.
Figure A19 Kinematic Wave Path - 30 fps Accel.
Figure A20 Kinematic Wave Path - 40 fps Accel.
Figure A21 Kinematic Wave Path - 50 fps Accel.
TABLE A-1

Comparison of Theoretical and Observed Wave Speeds

Vehicle Group 4 - Deceleration Phase

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Comparison of Theoretical and Observed Wave Speeds

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TABLE A-3

Comparison of Theoretical and Observed Wave Speeds

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### TABLE A-4

Comparison of Theoretical and Observed Wave Behavior

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