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REFERENCE PHASE MODULATION EFFECTS IN
OPTICAL HOLOGRAPHY

DISSERTATION
Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By
The Ohio State University
1973

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However most outstanding throughout this period has been the unwavering cooperation and encouragement of Irene, my partner in life, and that of our children, Karen and Matthew. Many thanks to you all.
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LIST OF SYMBOLS

\(a(x), a\) - amplitude of time-variant phase change due to object vibration

\(a_x\) - object wave amplitude at coordinate \(x\)

\(a^*_x\) - complex conjugate of \(a_x\)

\(A_r\) - amplitude of reconstruction EM wave

\(A_x\) - total EM field amplitude at the holographic plate and at coordinate \(x\)

\(A^*_x\) - complex conjugate of \(A_x\)

\(b\) - amplitude of reference wave phase modulation (time variant)

\(B\) - constant, equal to the slope of the film characteristic transmission curve in the region of exposure

\(c\) - net phase modulation of a holographic system due to vibrations of the test object and/or the reference wave

\(c_o\) - reference wave phase modulation amplitude with compensation for misalignment of incident and reflected beams

\(C(c), C\) - reconstructed image contrast

\(d(t), d\) - object displacement vector

\(f(t), f\) - functional time dependence of object motion

\(G_C(c)\) - system gain at constant contrast

\(i\) - percent contrast or \(\sqrt{-1}\)

\(I(c)\) - light intensity, dependent upon the net phase modulation, \(c\)

\(I_f\) - light intensity at the film plane

\(I_m\) - mean value of the light intensity

\(I_{\text{max}}, I_{\text{min}}\) - maximum and minimum light intensities measured at interference plane of interest

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$I_t$ - light intensity transmitted through processed hologram

$I(x_n), I_x$ - light intensity at coordinate $x$

$J_0(c)$ - zero-order Bessel function of the 1st kind with argument, $c$

$J_1(c)$ - first-order Bessel function

$k$ - propagation constant $= 2\pi/\lambda$

$k_x$ - $x$ component of propagation constant

$K_1$ - propagation vector of light wave incident on test object

$K_2$ - propagation vector of any plane light wave scattered by test object

$K$ - difference vector of $(K_2 - K_1)$

$M_{x,y}$ - mode number with $x =$ number of vibration nulls in $x$ direction; $y =$ number of vibration nulls in $y$ direction

$M(\phi), M$ - modulation function equal to time-average phase variation during holographic exposure

$m$ - displacement multiplier which compensates for the noncollinear alignment of illuminating, viewing and displacement vectors

$m_r$ - reference beam modulator displacement amplitude

$n(x)$ - noise intensity distribution along $x$ direction of test bar

$O$ - complex expression for the EM light waves scattered from the test object

$O^*$ - complex conjugate of object wave $O$

$O_x$ - amplitude of object wave at coordinate $x$

$r_1$ - time-average argument per unit contrast at $i^{th}$ bias point

$R$ - complex expression for the Reference EM light wave

$R^*$ - complex conjugate of reference wave $R$

$R_x$ - amplitude of reference wave at coordinate $x$
\( T_a \) - hologram transmittance, ratio of light amplitude transmitted by the hologram to that incident on it

\( V(c), V \) - fringe visibility equal to \( \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} \)

\( w(x) \) - static-bar intensity distribution along \( x \) direction of test bar

\( x \) - direction coordinate

\( x_n \) - \( n^{th} \) location in the \( x \) direction

\( y \) - direction coordinate

\( z \) - direction coordinate, usually normal to the holographic plate and/or test object
\( \alpha \) - time-invariant phase of object wave

\( \beta \) - time-invariant phase of reference wave

\( \gamma(c), \gamma \) - slope of the \( J_0^2(c) \) Bessel function = \( dI(c)/dc \)

\( \delta \) - approximate spacing between hologram interference fringes

\( \delta_1 \) - angle between modulator illumination vector and mirror displacement vector

\( \delta_2 \) - angle between modulator reflected light vector and mirror displacement vector

\( \Delta c \) - incremental change in argument, \( c \)

\( \Delta I(c) \) - incremental change in intensity \( I(c) \)

\( \Theta \) - angle of intersection of the normals to the reference and object wave fronts

\( \Theta_1 \) - angle between object illumination vector and surface displacement vector

\( \Theta_2 \) - angle between observation vector and surface displacement vector

\( \lambda \) - wavelength of light = 6328 \( \hat{\mu} \) for HeNe CW laser

\( \mu(c), \mu \) - amplification function = \( -2J_1(c)/J_0(c) \)

\( \mu_{T.A.} \) - amplification factor for time-average holographic interferometry

\( \pi \) - constant of 3.1416

\( \phi_x \) - object phase function at coordinate \( x \)

\( \omega \) - angular frequency of vibration (both object and reference modulation)
CHAPTER I

INTRODUCTION

The field of optical holography became a potentially useful tool in materials evaluation once the techniques of holographic interferometry became established. Double-exposure, real-time and time-average holography have all been shown to respond to surface and near-surface anomalies in ways which have proven beneficial in establishing the integrity of the material. However a limitation in all of these approaches has been an inability to detect surface variances much smaller than a quarter of the wavelength of the light used to make the hologram. Several attempts have been made to extend this limitation.

The methods studied used holograms which were modified so that the object background field was made dark. These modification approaches form the basis of sub-fringe holography.\(^1\) Aleksoff discussed the general mathematical solution to the temporal modulation problem. (19) Mottier investigated the effects of triangular modulation functions. (13) Shajenko and Johnson did extensive work using stroboscopic holography. (14) Kersch has more recently used the techniques of sub-fringe holography to show its potential in the area of materials

\(^1\)Sub-fringe holography is the interferometric fringe formation due to object displacements which are less than a quarter wavelength of the illuminating laser light.
evaluation. In Kersch's studies it was noted that insufficient technical information was available to determine the performance characteristics of sub-fringe holography accurately. Questions of ultimate sensitivity, enhancement over time-average holography, improvement in contrast and fringe visibility remained to be answered.

The primary purpose of this dissertation is therefore to discuss the operating characteristics of sub-fringe holography. The analytical basis for predicting the variations in light intensities in the sub-fringe region is also discussed. The case of reference beam phase modulation is considered in detail.

Chapter II deals with the fundamental concepts of holography. Chapter III discusses temporal modulations of reference and object beams as added to a simple mathematical treatment of holography. It introduces the concepts of modulation functions. Sinusoidal object and/or reference beam modulations are shown to have \( J_0(c) \), the zero-order Bessel function, as a modulation function. The argument \( c \) is identified as the phasor sum of the phase modulation contributions from both the object and reference beams.

Changes in the amplitude of the reference-beam phase modulation and their effects on interferometric fringes are explored in detail in Chapter IV.\(^2\) It is established that the system's sub-fringe sensitivity, the resulting fringe contrast and fringe visibility are all described by the universal gain function, \( \mu(c) \), shown in Fig. 10.

\(^2\)The sub-fringe holographic condition requires that stationary regions of the test object lie in the first null of the \( J_0(c) \) Bessel function.
This function, derived from the Bessel function, \( J_0(c) \) and its first
derivative, are found to be fundamental in predicting system perform-
ance in sub-fringe time-average holography.

For example, this gain function, multiplied by the object's
surface displacement predicts the light intensity contrast to be
expected under sub-fringe biasing. The sub-fringe system's improvement
in detection of small vibration amplitudes is described by the constant-
contrast system gain, \( C_c \). This measure of system sensitivity is equal
to the product of \( \mu(c) \) and the time-average argument required to
achieve the minimum contrast detectable. It is found that a contrast
of 2% and a bias on the \( J_0^2(c) \) curve at \( c = 2 \) (about 1/4 of a wavelength)
yields a system gain of 50. Thus sub-fringe surface-vibrations holo-
graphy can detect displacements which are 1/50 as large as those
required to achieve the same contrast using time-average techniques.

The experimental apparatus is discussed in Chapter V. The
holographic configuration used, facts on exposures, modulator charac-
teristics, and the conjugate-image method of light recording are all
explained in detail. Of particular interest, however, is the discussion
of the method for viewing holographic multi-images. Figure 15 shows
an example of this technique. By exposing four or more adjacent
regions of the photographic plate to differing experimental conditions,
very subtle changes in fringe patterns can be readily detected and
accurately compared to expectations.

\[ C_c = \text{time-average displacement/sub-fringe displacement, all at the same fringe contrast.} \]
The experimental results of Chapter VI, both generally and specifically, support the $J_0(c)$ expected response. It was found that surface vibrations of differing phases could be easily detected by the variations in each region's peak intensity. Of utmost importance was the level of background noise present in the hologram. This noise level, in conjunction with the gain function, $\mu(c)$, establishes the improvement of sub-fringe holography over conventional time-average holography. Figure 28 shows a comparison of time-average and sub-fringe holography based on the noise levels measured experimentally, as discussed in Chapter VI.

It is concluded that the measured variations in light intensity correspond to those predicted by the phasor-modified sinusoidal modulation function, $J_0(c)$. The operable range for vibration studies can be extended to the sub-fringe region through temporal reference-beam modulation. However the extent to which this procedure improves system performance is not equal to that suggested by previous authors (about 100 to 1000X improvement in sensitivity had been suggested). Based on this study, an improvement of only about 20X over time-average holography can be expected.

The detectable contrast and the noise level of the holographic image are the limiting characteristics of sub-fringe holography. Visibility of sub-fringe patterns improves at higher vibrational excitation frequencies because the alternating light and dark regions are spaced more closely. However a limitation exists here as the spatial frequency of the pattern approaches that of the detector's
resolution. An upper excitation frequency for the system studied would be about 600 kHz, based on fringe spacings of about .01-inch.
CHAPTER II

THE HOLOGRAPHIC PROCESS

The holographic process is that sequence of steps involved in making a hologram and subsequently viewing the images stored therein. Fundamentally, holography is the recording of two interfering waves incident on a suitable recording medium, processing the latent interference pattern to full development, and then reilluminating the developed pattern with a suitable wave (usually this is one of the original waves). The result is a reconstruction of the other original waves. Since the entire wave system is recorded during the exposure, it is practically impossible to discern any difference between the original wave system and the reconstructed wave system.

Historical Background

This basic concept of holography was first recognized by Gabor in his attempts to improve the resolution capabilities of electron microscopes. (1, 2, 3) Following his initial discovery and his development of the mathematical theory of holography, little progress was made for more than a decade. Leith and Upatnieks later recognized that the highly-intense and coherent laser beam, in conjunction with an off-axis method of illumination, could yield bright, undisturbed holographic images. (4, 5) The field of holography then
began to expand again. These discoveries gave rise to an exceptional increase in the interest of the technical community in holography. Certain restrictions inherent in the Gabor holograms, such as the interference of "twin" images and the requirement for careful photographic material processing, were removed by off-axis holography. In addition, objects could be imaged both in reflection as well as transmission and objects of continuous tonal quality could be recorded. Detailed histories of the development of holography can be found in the early chapters of most of the books on Holography listed in the Bibliography.

**Characteristics of Holography**

In comparing holography and photography, one immediately notes that characteristics such as parallax and extreme depth of focus which are inherent in holography are not present in conventional photography. It is further noted that few restrictions exist for processing of holographic materials, thus giving the system a wide performance latitude. Because of the three-dimensional characteristic and since the buildup of an equivalent image using conventional photography would be almost an impossibility, it has been stated that one hologram is equivalent to a thousand pictures. (6)

The extended depth of focus arises from the hologram's ability to record both the amplitude and phase of the light wave front incident upon the holographic plate. This is accomplished despite the fact that photographic films can record only the intensity of incident light (proportional to the square of the light amplitude). This phase
information is stored by recording the interference pattern established between the light scattered from the subject and that from a reference beam. The light for both beams originates from the same illumination source. When the photographic film is placed within the region of wave interaction, lines, similar to grating lines, are recorded on the photographic film. Since each object point contributes partially to each of the lines formed at the plate, the optical information about the object, as seen from the position of the plate is stored in the plate's latent image.

Reconstruction of the image is achieved by reillumination of the processed film using the reference beam. Upon reilluminating, wave systems which are proportional to the original light waves are established and the object reappears as if by magic. The faithful reconstruction of the image occurs because the interference fringes on the film correlate the positions of each object point with respect to the reference beam.

**Mathematical Description**

The basic mathematical elements of the holographic process can be expressed rather simply. Light intensity, I, is proportional to the amplitude of the propagating electric wave, multiplied by its complex conjugate. The light intensity at a point within the
film plane is given by:

\[ I_x = (R + O)(R^* + O^*) = RR^* + OO^* + RO^* + OR^* \]

\[ = |R|^2 + |O|^2 + RO^* + OR^* \]  \[1\]

where:

- \( R \) - reference beam light (electric field) amplitude
- \( R^* \) - the complex conjugate of \( R \)
- \( O \) - object beam light amplitude
- \( O^* \) - the complex conjugate of \( O \)

It is assumed that the polarization of the light throughout the holographic system is the same (usually perpendicular to the plane of the holographic-components table). Thus the mathematical description of the interference patterns established by interacting electric fields can be handled algebraically and vector notation is not necessary. It is recognized that the intensities discussed above are the same as the magnitude of the Poynting vector and are expressible in units of watts/meter\(^2\). Equation \([1]\) shows that the incident intensity at the film plane consists of four terms. The first and second terms correspond to the individual light intensities due to the reference and object waves, respectively.

Reconstruction is observed following processing of the film and upon illuminating the hologram with the reference beam, \( R \).\(^4\) The

\(^4\)The film used to record the hologram is developed to within the linear portion of its H & D curve. This usually corresponds to a film density of 0.5 H & D—a relatively transparent condition.
light intensity transmitted through the hologram during reconstruction is proportional to:

\[ I_t = I_t R = RR^2 + R|O|^2 + R^2O^* + |R|^2O \]  

The last term in Eq. [2], \( |R|^2O \), is recognized as the reference wave intensity, modulated by the object amplitude. This is clearly an optical rendition of the object-wave amplitude, \( O \), modulated by the reference-wave intensity, \( R \). This last term, mathematically, describes the virtual image of the original object.

The third term in Eq. [2], \( R^2O^* \), is often called the "real" or more generally, the "conjugate" image. It comes into focus in front of the holographic plate and corresponds to the troublesome "twin" image of Gabor's in-line holograms. The presence of both a real and a virtual image is characteristic of all holographic processes.

In off-axis, pictorial holography, the recording geometry is arranged so that the real image does not interfere with the viewing of the virtual image. The virtual image appears to exist behind the plate through which it is viewed.

In order to establish satisfactory, stationary interference patterns at the film plate, the light scattered from the object and the light from the reference beam must originate from the same highly-coherent source, \( \text{i.e.} \), there must be a definite, continuing phase relationship between object and reference illumination. This requirement is achieved when all the light originates from a single source, is of the same wavelength and is generated synchronously. This is true
of the light generated by a laser. The recording by the photographic film of the interference (fringe) pattern, caused by the simultaneous presence of both object and reference beams, constitutes formation of the hologram.

**Hologram Formation**

In order to understand some of the limitations of optical holography it is necessary to understand how the object and reference beam interference fringes are formed. Limitations related to the required degree of system stability, film resolution, and light coherence are all dependent on fringe formation. Figure 1 shows a pictographic representation of a possible arrangement for exposing and subsequently viewing a hologram. The coherent light from the CW He-Ne gas laser is incident upon a beam-splitter. This element divides the light beam into components for subsequent use as object and reference beams. Approximately 95% of the incident light intensity is directed toward the object while 5% is used for the reference illumination of the holographic plate.

Both laser beams pass through a microscope-objective lens which changes the collimated beams into diverging cones of light. A pinhole the size of the central Airy disc of the lens is placed at the focal spot of the lens. This pinhole serves to filter out (optically stop) any nonaxial light rays. This combination of lens and pinhole is known as a spatial filter.
Fig. 1. Schematic diagram of a holographic arrangement for recording and/or viewing a hologram.
The diverging object beam is directed toward the subject to be recorded. The light scattered in the direction of the photographic plate from the subject is the object wave front which the hologram stores. The scattering from a diffuse body results in a very complicated wave front containing both amplitude and phase variations which change radically in distances of the order of the wavelength of light. This complex wave front contains all the visual information about the object. This scattered light would darken the film uniformly if recorded alone.

The 5% reference beam component, reflected from the front surface of the beam-splitter, travels a separate path to the holographic plate. In Figure 1 the reference beam is incident upon a second front-surface mirror which directs the light onto the photographic plate. The spatial filter removes nonparallel light ray irregularities caused by imperfections on the surfaces of the beam-splitter, mirrors and subsequent lenses. The reference beam's wave front is a spherical surface of approximately constant amplitude and phase over the plate area.

The reference beam mirror is arranged so that the two wave fronts pass through one another at an angle, \( \theta \). The intensity of the reference beam is adjusted so that it is about three times as strong as that of the object beam when measured at the photographic plate.\(^5\) Everywhere in the region of common overlap an interference pattern is

\(^5\)Conceptually the reference beam is considered a signal carrier while the light contribution from the object is considered the modulation on the carrier. To avoid "over modulation," the reference beam intensity is always greater than that of the object beam.
created. Therefore there are nodes of destructive light interference and antinodes of constructive light interference. Of prime significance, however, is the fact that the interference pattern remains stationary with respect to both time and space.

In the interference region the neighboring nodes are separated by a distance

$$\delta \approx (\lambda/2) \sin(\theta/2)$$

where: \( \lambda \) - wavelength of light (6328 Å for He-Ne lasers)
\( \delta \) - approximate spacing between fringes on hologram

Equation [3] shows the fringe separation as being only approximate because of the complex nature of the wave fronts reflected from the object. Had these been emanating from an optically-flat surface, the expression for the node separation would be exact.

**Holographic Restrictions**

A hologram is a photographic recording of any portion of the complex and stationary interference pattern generated by the interaction of reference and object beams. There are no general restrictions on which part of the pattern is chosen or on plate orientation, if one assumes an infinite film resolution and unlimited laser coherence. However practical materials require that the recording medium have sufficient resolution to distinguish between the most-closely-spaced nodes and antinodes. For angles shown in Fig. 1, \( \delta \approx \lambda \), which means a resolution of about 2,000 line pairs per millimeter is required for a He-Ne laser of 6328 Å wavelength. This
resolution requirement can be met in practice by using high-resolution photographic emulsions presently available. High-resolution photographic emulsions normally possess low sensitivity. Thus holographic photographic plates typically have film speeds of less than 0.1 ASA. Exposures taken using a 50 milliwatt, 0.6328 μ laser, are typically one second in duration when recording on Agfa 10E75 photographic plates.

If any random motion of the system occurs during the exposure, and has an amplitude of the order of a half-wavelength of light, the latent image of a node would be partially superimposed on the interference pattern of an antinode. The resulting interference pattern would average to a generally uniform intensity and the optical-grating effect would be lost. It should be evident that any significant movement giving rise to shifts in phase of either beam along any portion of its path beyond the beam-splitter will also contribute to a loss in the stability of the interference pattern.6

After exposure the film plate is developed by standard darkroom techniques. Thirty-second development in fine-grain high-resolution developer, ten seconds in a stop bath, and 1-3 minutes in the hardener is sufficient processing to produce a good holographic grating on the plate. The plate is then washed and dried. If the film was presoaked and exposed while in a water tank, it can be viewed while still wet.

6The problem of stability is critical when using continuous-wave lasers since the slightest movements in objects and components can destroy the recording of a stationary pattern. Consequently, much of this work is done on specially stabilized tables of granite or pneumatically isolated benches. The problem has been eased through the use of pulsed lasers with extremely short pulse durations.
Reconstruction of the Image

After the plate is dried it can be viewed by using a reference beam similar to that used in the recording process. It need not be from the same light source but it must be coherent. Upon looking through the hologram, with reference beam illumination, one sees a reconstruction of the wave fronts which originally were emanating from the object. It is impossible for the observer to discriminate between this virtual image and the actual object. Changes in the observer's point of view show the characteristic three-dimensional nature of holography. Parallax effects are particularly dramatic in those objects which are of a three-dimensional nature and are positioned relatively close to the holographic plate.

Figure 2 shows several views of a sample three-dimensional object depicting three focal plane conditions of the camera used to photograph the reconstructed images. The photographs were taken with a conventional lens camera through the hologram plate with the original object (OSU Welding) removed. The reference beam illumination reconstructed the light waves established originally by the object. The camera recorded the depth characteristics by use of a very low aperture setting (f-1.4) and adjusting the range-finder to three distinct focal planes. Note that the camera images are identical to those seen by direct visual observation of the holographic image.
Fig. 2. Three photographs of the reconstructed virtual image of a hologram which show the results of focusing at three different distances within the object-image field.
Holographic Interferometry

A unique feature of optical holography is its ability to reconstruct an almost-exact copy of the original object. The image rendition is so precise that the reconstructed image, when compared to the original object, establishes interference fringes which appear on the image surface. This requires that both object and image be viewed simultaneously. The processed hologram must be replaced exactly in the same position it held during exposure.

In holographic interferometry the observer, looking through an illuminated hologram, sees the reconstructed image beyond the plate. The original object (also beyond the hologram) is left in the place it occupied during the exposure. The object is also illuminated. The observer can see the object directly (through the low density hologram) as well as its reconstructed image. If the hologram was accurately replaced into its holder and no massive movement had occurred at the object, the observer would see interference fringes across the surface of the object. Upon the slightest mechanical stressing of the part, the fringes will move accordingly. This type of interferometric viewing is called real-time holography.

Real-time holography is particularly useful in observing how test objects deform under numerous types of stimuli including (1) direct mechanical displacements, (2) thermal stressing, (3) vacuum stressing, and (4) sonic stressing (excitation by mechanical vibrations). The resulting fringe contrast is rather low but it is more than sufficient to see how a part responds to stimuli which give rise to surface displacements.
A second form of holographic interferometry uses the interference which may exist between sets of images, stored in the same hologram. This condition occurs when two (or more) successive exposures are made of the same test object. If the object is subjected to two (or more) states of stress and the exposures are made under the stressed conditions, two (or more) slightly-displaced replicas of the object are stored in the hologram. If the object's surface displacements between the exposures were between $\frac{1}{2}$ and about 100 wavelengths of the illuminating light, the two images will exhibit a set of interference fringes with a very high fringe contrast. This multi-image form of holographic interferometry is called double-exposure holography. It is extremely effective in permanently storing the two (or more) states of surface displacement which existed during each of the exposures.

Figure 3 shows an example of a doubly-exposed hologram. A cantilever beam, supported at the right end, is the object. The first exposure was made with the beam in an initially unstressed position. Following the first exposure and with the laser light off, the beam was mechanically displaced by several wavelengths of light. The second exposure was then made. The resulting effect is the clear interference of the first and second exposures. The displacement of the beam is largest at the left end and hence the finer fringe pattern is observed there.
Fig. 3. Photograph of a double-exposure hologram showing interference fringes on a cantilever beam. The meter stick is an indication of the coherence length of the laser. The lower letters are used to show the parallax effects of holography.
A third form of holographic interferometry is used in exposures of vibrating test objects. In this case, a large collection of individual exposures (i.e., an ensemble of exposures) is recorded in the hologram for the duration of the exposure. If, on the average, the vibrating surface assumes two (or more) relatively-stationary positions, interference fringes (similar to those of the double exposure case) will be seen upon reconstruction of the image. Hence the technique is called time-average holography. The fringe contrast is reduced for surface displacements in excess of a few light wavelengths but the technique is excellent for permanently storing vibrations mode patterns. Most of the photographs throughout this dissertation were made from time-average holograms.

Time-average holography is a more general case of holographic interferometry, double-exposure holography being a very special case of time-average holography. Both of these techniques fall into the class of holograms which have been temporally-modulated by shifts in object position. The concepts of temporally-modulated holograms and how they are analyzed are discussed in the next chapter.
CHAPTER III

TEMPORALLY-MODULATED HOLOGRAPHY

Holographic interferometry, in general, has become a useful tool for precise measurement of both static and dynamic surface displacements. (7,8) Time-average holography in particular has made possible the analysis of surface vibrations. (9,10) This holographic technique takes advantage of the temporal phase modulation of the object beam as follows. The object beam's phase modulation is proportional to the amplitude of the vibrational surface displacement. For periodic surface displacements, interferences between the images formed during sequential segments of the exposure give rise to fringe formation on the resultant image. The fringes of the reconstruction correspond to loci of constant modulation depth and, hence, of constant vibration amplitude. The analysis for this type of exposure is based on the superposition of the ensemble of exposures.

However, temporal modulation can be applied to the reference beam as well as to the object beam. Such temporally-modulated reference and/or object beams have been used to control the interferometric-fringe characteristics of contrast, location and shape. As shown in the literature, various forms of pulse-train modulation can be more useful in holographic interferometry than simple time-average holography. (11,12,13) Strobe holography is a particularly good example. (14)
In this case the modulation form is a series of pulses, synchronized with the vibrating surface of the object and at twice its frequency. The resulting image yields high contrast fringe patterns which are similar to those found in double-exposure holography.

A generalized analysis of the formation of interference fringes due to object modulation has been developed by Stetson. (15,16) It is based on the formation of a unique characteristic modulation function for each type of object motion. A review of these characteristic modulation functions is outlined later, following discussion of a simple theory of holography. A detailed discussion of the effects of adding phase modulation to the plate-exposing reference wave is given in Chapter IV.

A Theory of Holography

The mathematical basis for image formation by holography is stated using the electromagnetic field vectors of the coherent laser beam's light. (17) Figure 4 shows a cross section of a photographic plate and the light waves incident upon (and/or diffracted by) it.

Exposing the Hologram

The mathematical form of the complex electric-field wave from the test object, \( O_x \), and incident onto the plate from the left is

\[
O_x = a_x e^{-i\phi(x)}
\]

[4]

where:

- \( a_x \) - electric field vector amplitude at coordinate \( x \)
- \( \phi(x) \) - object-wave phase at coordinate \( x \)
Fig. 4. Schematic diagram of the holographic process. Shown are the incident reference and object waves as well as the three waves established upon reconstruction. These are: (1) the straight-through illumination; (2) the reconstructed (virtual image) object wave; and (3) the reconstructed (conjugate image) object wave.

$O_x$ is called the object wave. Equation [4] emphasizes the location ($x$) dependence of the object wave's electric field amplitude and phase. No attempt is usually made to explicitly define the form of either $a_x$ or $\phi(x)$ since they would be extremely complex functions for a real test object.\textsuperscript{7}

\textsuperscript{7}The electric field component of a traveling EM wave is normally expressed as a vector to show its polarization effects. However, most CW lasers emit only a linearly polarized beam and thus a first order approximation to the beam's behavior is the simple scalar form shown in Eq. [4].
A monochromatic plane wave, \( R_x \), of unit amplitude and incident at an angle \( \theta \) is designated as the reference wave. The reference electric field is expressed as

\[
R_x = e^{-ik_x x}
\]

where: \( k_x = (2\pi/\lambda) \sin \theta \) - propagation constant in the \( x \) direction

The total electric field component of the electromagnetic light waves incident on the plate at coordinate, \( x \), is

\[
A_x = O_x + R_x = a_x e^{-i\phi(x)} + e^{-ik_x x}
\]

The electric field intensity, \( I_x \), is equal to \( A_x A_x^* \) where \( A_x^* \) is the complex conjugate of \( A_x \). Thus the light intensity at any position \( x \) is

\[
I_x = (1 + a_x^2) + e^{ik_x x} a_x e^{-i\phi(x)} + a_x^* e^{-i\phi(x)} = 2k_x x
\]

Developing the Hologram

The transmittance, \( T_a \), of the exposed and developed hologram (the ratio of light amplitude transmitted by the hologram to that incident on it) contains a term proportional to the exposure (hence proportional to the intensity). Thus the transmittance can be written as

\[
T_a = BI_x
\]

where: \( T_a \) - amplitude transmittance

\( B \) - constant, equal to the slope of the \( T_a - I_x \) characteristic curve
This linear relationship exists when the photosensitive plate is exposed to the interference pattern formed by \( R_x \) and \( O_x \), provided the ratio of \( R_x \) to \( O_x \) is carefully controlled at about 3:1 to assure linear recording. The plate must be properly developed. The hologram is assumed to be of the absorption type.

**Reconstructing the Image**

The image is reconstructed by illuminating the processed hologram with the monochromatic reference beam, \( R_x = e^{-ikx} \). The amplitude of the transmitted wave, \( A_r \), is therefore the product of the amplitude transmittance, \( T_a \), and the illuminating reference beam's E-field amplitude. Thus

\[
A_r = T_a e^{-ikx} = B |e^{-ikx}|
\]

or

\[
A_r = B \left[ (1 + a_x^2) e^{-ikx} + a_x e^{-i\phi(x)} + a_x^* i[\phi(x) - 2k_x x] \right]
\]

In Eq. [10] the first term within the brackets describes the attenuated light that passes directly through the hologram. It is shown as ray (1) in Fig. 4. The second term in Eq. [10] is the light diffracted to form the reconstructed wave of the object. It is shown

---

8 The linearity of the \( T_a - I_x \) transfer characteristic depends on the exposure and development of the photo plate. However this is still a first-order approximation to the transfer characteristic and \( O_x \) is kept smaller than \( R_x \) to avoid "overmodulation" and operation in the nonlinear portion of the transfer characteristic.
as ray (2) in Fig. 4. The third term in Eq. [10] is the first-order diffracted wave on the other side of the reference beam which forms the conjugate wave. This corresponds to ray (3) in Fig. 4.

These three terms are recognizable through their general mathematical forms. The original reference wave, \( R_x = e^{ikx} \), is modified to become \( B(1 + a^2_x) e^{ikx} \). This is a wave traveling in the same direction as the original wave but reduced in intensity. The second term, \( B_a e^{-i\phi(x)} \), is an amplitude-reduced reconstruction of the original object wave, \( a_x e^{-i\phi(x)} \). The conjugate term is recognized by the 180° reversal in the phase, \( \phi(x) \), and the new propagation direction.

In essence, Eqs. [4]-[10] constitute a simple mathematical theory of holography. These equations (1) start with definitions of the mathematical forms of the object and reference waves, (2) express the intensity variations stored in the hologram, and (3) outline the diffracted and undiffracted components of the reconstruction reference beam.

**Introduction to Phase Modulation Effects**

Aleksoff has shown that the form of the modulation function does not change when either the object or reference waves are phase modulated. (19) The modulation function is the function describing the variations in fringe patterns and intensity. However, the complexity of the argument of the modulation function does change with variations in phase modulation. When both reference and object beams are modulated at the same frequency, Aleksoff shows that E-field
phasor addition can be used to determine the resultant magnitude of the argument.

The effects of phase modulation are introduced into the previous analysis of the holographic process by rewriting Eqs. [4] and [5] for the object and reference beams with terms showing the vibrational sinusoidal time dependence.\(^9\)

\[
O_x(t) = a_x e^{-i\phi(x)} e^{ia \sin(\omega t + \alpha)} \quad [11]
\]

\[
R_x(t) = e^{-ik_x x} e^{ib \sin(\omega t + \beta)} \quad [12]
\]

where:

- \(a\) - amplitude of time-variant phase of object wave
- \(b\) - amplitude of time-variant phase of reference wave
- \(\alpha\) - time-invariant phase of object wave
- \(\beta\) - time-invariant phase of reference wave

The total E-field, \(A_x\), at the holographic film plane is equal to the sum of the two component fields. Thus:

\[
A_x(t) = a_x e^{-i[\phi(x) - a \sin(\omega t + \alpha)]} + e^{-i[k_x x - b \sin(\omega t + \beta)]} \quad [13]
\]

\(^9\)Note that all quantities have now become time dependent and this fact is reflected in their notation (i.e., \(O_x\) becomes \(O_x(t)\), \(R_x\) becomes \(R_x(t)\)).
The intensity of this field (given by the total field multiplied by its complex conjugate) becomes

\[ I_x(t) = A_x^* A_x = (1 + a_x^2) + a_x e^{-i(\phi - k_x x)} e^{i[a \sin(\omega t + \alpha) - b \sin(\omega t + \beta)]} + a_x^* e^{i(\phi - k_x x)} e^{-i[a \sin(\omega t + \alpha) - b \sin(\omega t + \beta)]} \]

[14]

The phase terms due to the temporal modulation can be re-written as

\[ c \sin(\omega t + \gamma) = a \sin(\omega t + \alpha) - b \sin(\omega t + \beta) \]

[15]

where: \[ c^2 = a^2 + b^2 - 2ab \cos(\alpha - \beta) \]
\[ \gamma = \alpha - \beta \]

Therefore the second term in Eq. [14], representing the object wave system, can be written as

\[ a_x e^{-i(\phi - k_x x)} e^{i c \sin(\omega t + \gamma)} \]

[16]

The processed photosensitive plate is assumed to have an amplitude transmittance, \( T_a \), proportional to the intensity, \( I_x \), \( T_a = B I_x \) [Eq. 8]). Thus the wave corresponding to the image of the reconstructed object becomes

\[ A_x(t) = T_a e^{-i k_x x} = B a_x e^{-i(\phi - k_x x)} e^{i c \sin(\omega t + \gamma)} e^{-i k_x x} \]

\[ = B a_x e^{-i(\phi - k_x x)} e^{i c \sin(\omega t + \gamma)} \]

[17]
$A_x(t)$ is of the same form as that found for the nonmotion case of Eq. [10], except for the temporal phase-modulation term. During the time of plate exposure, the time-average, $\langle A_x(t) \rangle$, of this field is recorded. Taking the time-average of Eq. [17]

$$
\langle A_x(t) \rangle = \frac{1}{T} \int_{-T/2}^{T/2} A_x \, dt = \frac{B a}{T} \int_{-T/2}^{T/2} e^{i \gamma} \sin(\omega t + \gamma) \, dt
$$

[18]

Using the identity $J_0(c) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{i c \sin x} \, dx$

and substituting $x = \omega t + \gamma$; $dx = \omega dt$

$$
\langle A_x(t) \rangle = B a e^{-i\phi} J_0(c)
$$

[19]

where: $J_0(c)$ - zero order Bessel function of the 1st kind

c - net phase modulation

$B$ - film characteristic proportionality constant

$a_x$ - amplitude of original object field

$\phi$ - stationary-object phase function

It is evident that $J_0(c)$ and $B$ (multipliers of the original object wave) modify the reconstruction's amplitude. However, $J_0(c)$ has numerous zeros which would force the reconstructed $A_x$ to zero. It is these zeros that appear as dark fringes on the image. The multiplier $J_0(c)$ is referred to as the modulation function for $O_x$. Note that
the argument of the Bessel function is given by \( c \) which is the phasor sum of the object and reference modulation amplitudes.

\[ \langle A_x(t) \rangle \] is the time-average E-field amplitude established by the processed holographic plate upon reconstruction of the holographic image by the original reference wave. This wave is detected by photographic means or by viewing with the human eye. Both of these methods are responsive to light intensity and thus their responses are proportional to \( A_r A_r^* \). The reconstructed image intensity is therefore given by

\[ I_r = B^2 a_x^2 J_0^2(c) \]  \[ \text{[20]} \]

Thus the observed image of a time-average hologram will possess an intensity modulation across its surface proportional to \( J_0^2(c) \). The argument (c) of the Bessel function is the phasor sum of the object and reference wave phase modulation amplitudes. Figure 5 shows the phasor relationship of the object-reference wave components, \( a, \alpha \) and \( b, \beta \).

**The Physical Significance**

If a portion of a test object's surface vibrations are in phase with the sinusoidal modulation of the reference beam and their respective displacement amplitudes are equal, the field incident on the photographic plate (from that object location) will be stationary. Thus although the object's surface is physically moving, the in-phase reference modulation phasors vectorially sums to yield an apparent stable field. Thus the reconstructed time-average fields which
Fig. 5. Phasor diagram showing object wave phasor, $\mathbf{a}$, and reference wave phasor, $\mathbf{b}$, yielding resultant phasor, $\mathbf{c}$.

emanate from the same object location are all stable and establish a diffraction pattern exhibiting a high visibility (i.e., the fields reconstruct with a high image intensity).

Conversely, those regions where an "apparent null" does not exist will establish weak diffraction patterns in the hologram. Upon reconstruction those regions (on the image) appear low in intensity, (i.e., the region on the image of the object appears significantly darker than the "apparent null" positions which are as bright as the nonvibratory reconstructed images). If the relative phase between reference and object displacement is $180^\circ$ during exposure, the
reference beam enhances the apparent displacement of the surface (by
doubling it). The reconstruction in this case shows a decided drop
in image intensity as the effective phase displacement, \( c \), increases.
This follows since the \( J_0(c) \) response decreases as \( c \) increases
(up to the first null).

Figure 6 shows a vibratory pattern recorded using conventional
time-average techniques (above) and the shift in "apparent null"
position due to reference beam phase modulation (below). In the conven­
tional case the brightest image is located at the clamped region
(\text{i.e.,} around the periphery of the vibrating plate). Reference modula­
tion succeeded in establishing an "apparent null" within the vibratory
pattern which is well removed from the clamped regions. This effect
has been discussed by Neumann, \textit{et al.} (13) Note that the static regions
of the image (\text{i.e.,} the clamped periphery) are dark in the reference
modulation case. With real-time visual observations the shift in null
positions could be observed during both amplitude and phase variations
of both reference and object modulators.

\textbf{Modulation Functions}

The concepts of holographic modulation functions have been
discussed in detail by Stetson. (15,16) Starting from the diffraction
integral representing the waves generated by the hologram, he concludes
that the systems of fringes to be expected in time-average holography
are readily predictable. His analysis shows that the modulation
Fig. 6. Sample of apparent null shift due to synchronous reference wave modulation.
function for a time-average hologram is given by

\[ M(\phi) = \frac{1}{T} \int_{0}^{T} e^{i f(t) \phi} dt \]  

[21]

where: \( \phi \) - phase change due to variations in geometrical orientation of object, observer or illumination
\n\n\( f(t) \) - function describing the time-dependent form of the object's modulation\(^{10}\)
\n\( T \) - time duration of the exposure,

For the experimental geometry of Figure 7,

\[ \phi = (K_2 - K_1) \cdot d = K \cdot d \]  

[22]

where: \( K_1 \) - object illumination vector
\( K_2 \) - diffracted plane wave vector (observation direction)
\( K \) - difference of \( K_2 \) and \( K_1 \)
\( d \) - surface displacement vector

Of utmost importance is Stetson's conclusion that the light distribution of the static object case, when multiplied by the modulation function, yields the time-average light distribution. Thus:

\[ \langle A_r(t) \rangle = M(\phi) A_r \]  

[23]

where: \( \langle A_r(t) \rangle \) - time-average reconstructed wave system
\( A_r \) - static reconstructed wave system
\( M(\phi) \) - vibrational modulation function

\(^{10}\)The assumption is made that the form of the modulation can be written as the product of \( f(t) \) and \( \phi \).
Thus $M(\phi)$ contains the hologram fringe information. The argument, $\phi$, is the functional relationship which relates the magnitude of the modulation and the geometry of the system to fringe formation.

**Some Typical Examples**

The term $M(\phi)$ is merely the time average of the phase displacement functions (Eq. [21]). Results for several simple time-dependent cases are listed in Table 1.
# TABLE 1

## SUMMARY OF COMMON MODULATION FUNCTIONS

<table>
<thead>
<tr>
<th>Object Motion</th>
<th>Modulation Function</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type</strong></td>
<td><strong>d(t)</strong>(^a)</td>
</tr>
<tr>
<td>None</td>
<td>0</td>
</tr>
<tr>
<td>Random</td>
<td>&gt; (\lambda/4)</td>
</tr>
<tr>
<td>Jump</td>
<td>(d_0 \text{ sgn}(t))^b</td>
</tr>
<tr>
<td>Sinusoidal</td>
<td>(d_0 \sin(\omega t))</td>
</tr>
<tr>
<td>Linear Velocity</td>
<td>(v_0 t)</td>
</tr>
</tbody>
</table>

\(^a\) \(d(t) = d_0 f(t)\) with \(d_0\), the object's peak displacement amplitude

\(^b\) \(\text{sgn}(t) = t/|t| = +1\) for \(t > 0\); \(-1\) for \(t < 0\)

\(^c\) \(\text{sinc}(t) = \sin\pi t/\pi t\)

---

**Case 1, \(d(t) = 0\)**

The case where \(d(t) = 0\) corresponds to the reconstruction of a stationary object. Thus, \(M(\(\phi\)) = M(0) = \frac{1}{T} \int_0^T dt = 1\). Thus the average value \(<A_{\text{r}}(t)\>\) given by Eq. [23] is identical to that of the stable test object, \(A_{\text{r}}\). Therefore no fringes appear on the image of the object upon reconstruction.
Case 2, $d(t) > \lambda/4$ and Random

The case where $d(t) > \lambda/4$ and random corresponds to the presence of unwanted disturbances during exposure. Random vibrations of system components prevent the formation of stable interference patterns on the light-sensitive holographic plate. Physically this results in an unsatisfactory reconstruction (usually characterized by a very low intensity). If the object surface displacements are greater in magnitude than $\lambda/4$, a resultant system of fringes may occur. For fringes to occur, the probability density of the displacement must tend to establish two or more quasi-stable regions. Upon reconstruction these regions will interfere with each other. In this case the absence of any truly-stable regions will also result in a low intensity reconstructed image.

Case 3, The Jump Function, $d_0 \operatorname{sgn}(t)$

The case where $d(t) = d_0 \operatorname{sgn}(t)$ corresponds to two separate stable positions of the test object. This is the case in double-exposure holography. The magnitude of the physical separation is $2d_0$ and the change occurs at $t = 0$. The exposure is considered to extend from $t = -T/2$ to $t = +T/2$, where $T$ is the exposure time. Thus both positions receive the same duration of exposure.

Using Eq.[21],

$$M(\phi) = \int_{-T/2}^{0} e^{-ikd_0} dt + \int_{0}^{T/2} e^{ikd_0} dt = e^{-ikd_0} \left( T/2 \right) + e^{ikd_0} \left( T/2 \right)$$

$$= T \cos(Kd_0)$$

For unity exposure time, this becomes simply $\cos(Kd_0)$. 
Case 4, The Sinusoidal Case, \( d_0 \sin \omega t \)

The sinusoidal case where \( d(t) = d_0 \sin \omega t \) corresponds to the typical situation of time-average holography. The sinusoidal phase variation yields the modulation function \( J_0(K \cdot d_0) \), a Bessel function. This function has a maximum at zero argument and decreases in peak values as the argument increases. It goes through a series of alternating nulls and peaks and therefore a characteristic fringe pattern is seen across the surface of the test object. However since the peaks of the modulation function rapidly decrease in magnitude with increasing argument, the appearance of the reconstructed image shows a decided decrease in fringe intensity for regions of maximum deflection.

Summary

The purpose of this section has been to discuss the concept of a characteristic modulation function and to indicate how this function describes the fringe formation on test object images due to subject surface displacements during exposure. Now that the case for sinusoidal object displacements has been developed, (Eq. [19]), it will next be rigorously discussed by considering the effects introduced by phase modulating the reference waves synchronously during the time of exposure.

Of particular significance is the use of reference-wave phase modulation for the purpose of extending the useful range of time-average holographic interferometry. The lower limit of time-average holography is based on the wavelength of light used to make the
hologram. (The first dark fringe occurs at object surface-displacement amplitudes yielding phase changes of slightly more than a quarter wavelength of the illuminating light.) Smaller surface displacements can be detected if reference-wave modulation is used to establish a totally-dark object (i.e., bias the hologram to the first null of $J_0(c)$).

This approach is the dynamic (time-average) version of sub-fringe holographic interferometry. The basic premise is that reconstructed images with totally-dark backgrounds show very small surface displacements better than those observed when bright backgrounds are present. The fundamental principles of sub-fringe holographic interferometry are discussed in the succeeding chapters.
CHAPTER IV

SUB-FRinge HOLOGRAPHIC INTERFEROMETRY

The techniques of reference-wave phase modulation extend the useful range of time-average interferometric holography. Test object surface vibration amplitudes yielding object beam phase changes of less than a quarter-wavelength of the light used to make the hologram, have been easily detected. Since the detected vibration amplitudes correspond to conditions yielding no time-average fringes at all, the method has been called sub-fringe holography. Sub-fringe holography shifts the static operating point of the hologram from a modulation function's maximum to a minimum (null). This method of biasing the operating point to the modulation function's null results in a low intensity visibility of static reconstructed images. Thus locations on the object's surface which (on the average) are stationary during exposure reconstruct with little or no image intensity. However, locations on the object's surface which do move during exposure are readily seen. These regions appear as increases in light intensity and are adjacent to the stationary locations which remain dark (i.e., they have a low light intensity).

The Zero-Order Bessel Function, $J_0^2(c)$

Chapter III, Temporally-Modulated Holography, established the theoretical basis which showed that sinusoidally-modulated object...
light waves, upon reconstruction, have a zero-order Bessel function, $J_0(c)$, amplitude modulation function. The argument, $c$, of the Bessel function is proportional to the amplitude of the object's surface displacement during exposure. When reference wave phase modulation is introduced in addition to object beam modulation during the exposure, the form of the reconstructed image's amplitude modulation function is still $J_0(c)$. However, the argument, $c$, becomes the phasor sum of both the object- and reference-modulation amplitudes. The intensity of the image as viewed by the human eye or by photographic techniques is proportional to the square of the image $E$-field. Thus the intensity modulation observed on test surfaces subject to sinusoidal vibrations during exposure follows a $J_0^2(c)$ response.

Figure 8 shows the general shape of the $J_0^2(c)$ Bessel function. The maximum value of $J_0^2(c)$ occurs at the origin. Here $c = 0$ and thus neither object nor reference beams are being phase modulated. Introduction of reference wave modulation displaces the static-surface operating point from the central peak to elsewhere on the $J_0^2(c)$ function. Thus the location on the $J_0^2(c)$ function corresponding to zero object modulation is set by the magnitude of the reference modulation. Assignment of the reference modulation's amplitude is referred to as setting the operating (bias) point. By adjusting the amplitude of the reference wave modulation, an operating point can be selected at any desired radial distance from the origin, 0. The reference modulation is measured on a radial line from the center of the $J_0^2(c)$ function and in the $z = 0$ plane.
Fig. 8. General shape of the $J^2_0(c)$ function. The argument, $c$, corresponds to the radial distance from the origin, 0, in the plane $Z = 0$. 
Sub-fringe holographic interferometry uses as a bias point the first null encountered in moving radially outward from the central peak (See Fig. 8). Recalling that the modulation function, \( M(\phi) = J_0(c) \) multiplies the static object amplitude function, \( A_x \), this bias null point becomes the multiplying factor for the static portions of the reconstructed image (See Eq.[23]). Thus the light intensity of the static regions of the virtual (or conjugate) image of a sub-fringe holographic interferogram is zero (dark image). If some locations on the object surface are vibrating synchronously with the reference wave phase modulation during exposure, these locations may show some degree of increased light intensity upon reconstruction. The degree of intensity increase depends critically upon the relative phase angle of the object surface displacements with respect to the reference wave phase modulation.

Figure 8 and Plate 1 show that changes in the magnitude of the argument \( c \) display a maximum rate of change in modulation function when \( \Delta c \) is measured along a radial line from the origin. Displacements which occur along tangent lines (i.e., \( \Delta c \) perpendicular to \( c \)) give a minimum rate of change. The phase effects of reference wave biasing are shown diagramatically in Plate 1.

If \( a \), the object displacement modulation, is in phase (or \( 180^\circ \) out-of-phase) with the reference beam modulation, \( b \), maximum rate of change of the modulation function occurs (See Plate 1(a). If \( a \) is orthogonal to \( b \), a minimum rate change is obtained. The \( J_0^2(c) \) response for \( b \) biased at the null and \( a \) oriented collinearly (Plate 1(b)) and
Plate 1. Phasing effects of reference-wave biasing \( \beta \).
(a) Typical resultant argument \( c = a + b = a/\alpha + 2.4 \beta = c/\gamma \); (b) Modulation function response for radial changes in \( c \) (SECTION AA'); (c) response for orthogonal changes in \( c \) (SECTION BB').
orthogonally (Plate 1(c)) shows this significant relative-phase dependence. Note that the magnitude of \( a \) is equal to \( b \) in the cases shown in Plate 1(b) and (c).

**Theoretical Performance**

The preceding observations show that relative phase and amplitude of reference- and object-wave modulations dictate the appearance of intensity variations in sub-fringe holography. The theoretical variations in intensity expected from reference beam phase modulations of a sinusoidally-vibrating bar, based upon the \( J_0^2(c) \) solution to the problem, are outlined in the following paragraphs.

**Geometrical Arrangement**

The geometrical arrangement shown in Fig. 9 represents the basic method used to introduce the effects of sub-fringe holography. It defines the location of the test object, as well as the directions of light illumination, observation and surface displacement. The phase modulation of the reference beam is accomplished by movement of a mirror with displacement magnitude, \( m_r \). The angles between illumination, observation and displacement for the reference modulator are also shown.

The light from the laser is divided at the beam-splitter, BS, into two components. One is eventually used to illuminate the test object (\( I_t \)). The other is reflected by the reference modulator, expanded by a lens and finally falls on the film plate as the reference
Fig. 9. Geometrical arrangement of test object within the holographic system showing the angle relationships between the surface displacement vector, $a_x$, and the illumination vector, $I_1$, and the observation direction vector, $I_2$.

Beam. The observer views the test object through the hologram. The observed intensity is in the $I_2$ direction.

The flat object (a test bar) is assumed to be vibrating in a sinusoidal mode, excited possibly by acoustic waves from a loudspeaker or by direct piezoelectric stimulation. The peak amplitude of the surface vibration at any position $x$ along the length of the bar is denoted $a(x)$. The net phase change in the object beam due to surface displacements is therefore dependent on the angles $\theta_1$ and $\theta_2$. 
When (1) the light propagation vector incident on the test object, \( \mathbf{I}_1 \), (2) the surface displacement vector, \( \mathbf{a}(x) \), and (3) the vector corresponding to the viewing direction, \( \mathbf{I}_2 \), are all collinear, the argument of the Bessel function is given by \( c = (2\pi/\lambda)\mathbf{a}(x) \) (i.e., as \( \mathbf{a}(x) \) varies from zero (corresponding to a vibration null) to \( \mathbf{a}_{\text{max}}(x) \), the argument \( c \) varies proportionately). Usually these three vectors are not collinear and only the components of the incident and viewing direction vectors which are collinear with the displacement vector contribute to the argument \( c \). Thus misalignment of these vectors is compensated through use of the sum of the vector products, 
\[
\mathbf{I}_1 \cdot \mathbf{a}(x) + \mathbf{I}_2 \cdot \mathbf{a}(x) = \cos \theta_1 + \cos \theta_2.
\]
The same relationship holds for the reference beam with \( \delta_1 \) and \( \delta_2 \) substituted for \( \theta_1 \) and \( \theta_2 \). The total expressions for the phase variations due to object and reference mirror vibrations, respectively, are

\[
(2\pi/\lambda)(\cos \theta_1 + \cos \theta_2)\mathbf{a}(x) \tag{24}
\]
and

\[
(2\pi/\lambda)(\cos \delta_1 + \cos \delta_2)\mathbf{m}_r \tag{25}
\]

where:

- \( \theta_1 \) - angle between the object illumination beam and the object displacement vector
- \( \theta_2 \) - angle between the observation vector and the object displacement vector
- \( \delta_1 \) - angle between the light beam incident on the reference wave modulation mirror and the modulator displacement vector
- \( \delta_2 \) - angle between the reflected light beam and the modulator displacement vector
Appearance of Intensity Variations: Simple Time-average Case

A simple time-average hologram of a vibrating surface appears to have a series of fringes on it, the dark regions corresponding to the zeros of the $J_0^2(c)$ function. The fringe patterns of standing-wave modes in planar test objects are symmetrical about the nodal regions of object surface displacement because $J_0^2(c)$ is an even function about the $c = 0$ operating point.

Schematically, the surface displacement along a vibrating bar as well as the predicted intensity variations in one direction can be drawn as in Plate 2. Plate 2(a) shows a typical sinusoidal displacement pattern, $a(x)$, along one dimension of a planar test object. The mode of vibration has a null in the center of the bar with the ends clamped.

Compensation for noncollinear illumination, observation and displacement vectors as well as reference modulation biasing is given in Plate 2(b). The expression for the magnitude of the net

---

The case cited represents the bar rigidly clamped at both ends. Excitation could be by any number of sources including loudspeakers, piezoelectric elements, magnetostrictive elements or electrostatic modulators.
Plate II. Form of transverse bar vibrations and resultant intensity distribution for time average exposure without reference wave bias.
phase modulation argument function, \( c \), is given by:\textsuperscript{12}

\[
c = ma(x) \pm c_o
\]  \[26\]

and

\[
m = (2\pi/\lambda)(\cos \theta_1 + \cos \theta_2)
\]  \[27\]

The displacement multiplier, \( m \), compensates for any misalignment of illuminating, viewing and displacement vectors. \( c_o \), given by

\[
c_o = (2\pi/\lambda)(\cos \delta_1 + \cos \delta_2)m_r
\]  \[28\]

is the angle-compensated reference modulation multiplier.

The argument, \( c \), applied to the \( J^2_0(c) \) function in Plate 2(c) yields the resultant intensity distribution in Plate 2(d). The procedure for using Plate 2 is as follows:

1. For each coordinate location, \( x_n \), in Plate 2(a), the amplitude of the displacement, \( a(x_n) \), is found and transferred laterally to the geometry-compensation curve Plate 2(b).

2. Depending on the slope and lateral displacement of the compensation curve (i.e., the magnitudes of \( m \) and \( c_o \)), a value of \( \pm c \) is found and transferred down to the squared Bessel function response in Plate 2(c).

\textsuperscript{12}Eq.[26] applies for the special case of reference and object modulations which are in-phase (+) or 180° out-of-phase (-). Eq. [26] shows that both the geometry-compensated object modulation, \( ma(x) \), and reference modulation, \( c_o \), contribute to the net phase modulation, \( c \), which affects the ultimate image intensity distribution upon holographic reconstruction.
(3) The magnitude of the squared Bessel function is transferred laterally to the plot of the intensity distribution along the x coordinate in Plate 2(d).

(4) At the coordinate, \( x_n \), the value \( I(x_n) \) is plotted, based upon the original coordinate point and the transferred squared Bessel function. The resultant intensity is normalized at a value of unity.

It is evident from Eq. [27] that if the directions of object illumination or viewing are changed, or if the wavelength of the light is changed, a corresponding change in scaling of the \( J^2_o(c) \) function will occur. However through the scheme of Plate 2, these changes can be accommodated easily. For simple time-average holography, the reference beam modulation mirror displacement, \( m_r = 0 \) and thus \( c_o = 0 \). Therefore the compensation curve of Plate 2(c) is centered about \( c = 0 \).

**Bias to Null Case**

As the reference beam modulation increases from zero, it shifts the compensation curve away from \( x = 0 \) and thus establishes a biased operating point at a location other than the central peak. At the first null of \( J^2_o(c) \), \( c_o = 2.405 \). Using a collinear reference modulation system (\( \delta_1 = \delta_2 = 0 \))

\[
c_o = 2.405 = (2\pi/\lambda)(1 + 1)m_r \tag{29}
\]

or

\[
m_r = (2.405/4\pi)\lambda = 0.192\lambda \tag{30}
\]
This condition is shown in Plate 3(b) by the shift to the right in the compensation curve by an amount $c_0$. This shift results in an asymmetry occurring in the $I(x)$ distribution in Plate 3(d). This deviation from symmetry has been used to discriminate regions of unequal phase in vibrations studies. (13)

An "apparent null" exists on the surface of the object provided the object's displacement magnitude exceeds that of the reference mirror. The location of this null depends on the shift in the compensation curve (i.e., the "depth" of the reference modulation). Note that this linear compensation can only be used for the in-phase or $180^\circ$ out-of-phase case. To accommodate other relative phase conditions, either the intensity modulation function, $J_0^2(c)$, would have to be modified or a nonlinear geometry-compensation curve would have to be developed.

**Small-Amplitude Intensity Variations: The Sub-fringe Case**

The cases shown in Plates 2 and 3 correspond to object-vibration amplitudes which yield Bessel function arguments larger than $c = 2.4$ (the first null of $J_0^2(c)$). For object vibration displacements less than the first fringe condition, very small changes in the intensity of the reconstructed image can be expected. No formation of actual fringes will occur. Plate 4 shows this case with the scale of the $J_0^2(c)$ function expanded to include only the region around the central and the immediately adjoining peaks. The vibration amplitude, $a(x)$, is adjusted so that $a_{\text{max}}(x)$, the peak vibration amplitude, yields a 75% intensity drop upon reconstruction.
Plate III. Development of reconstructed light intensity for reference beam modulated case. $c_o = 0.19\lambda$ = first null of $J_0^2(c)$ function.
Plate IV. Development of reconstructed light intensity for reference bias set for null, 2/3 drop in peak intensity, 1/3 drop and no bias conditions.
Plate 4(d) shows the results expected for four levels of reference beam modulation: (1) none, (2) image intensity, I(x), down by 1/3 in the zero-displacement regions, (3) image intensity down by 2/3 in the displacement nulls, and (4) image intensity equal to zero in the null regions. The lateral displacement of the reference compensation curve, Plate 4(b), is determined by drawing a vertical line from the peak, 1/3 down, 2/3 down, and at the sub-fringe null region of the \( J_0^2(c) \) curve. The intensity variation, I(x), expected along the reconstruction of the test object is constructed point-by-point as described earlier. Note that the zero reference modulation case is identical to that shown in Plate 2(d).

**Changes in Number of Apparent Fringes**

At I(x) equal to about 50% of the static case intensity, the number of intensity peaks per unit length along the test piece drops by a factor of two. This implies that the transfer function for the bar vibrations has changed from a double-valued function (near the \( J_0^2(c) \) peak at zero bias) to a single-valued function (along the peak's side slope at a bias of 2.1). This effect is analogous to the d.c. biasing of magnetostrictive transducers, used to obtain quasi-linear operation and to avoid frequency-doubling.

Plate 5 is a series of photographs showing the sub-fringe intensity variations observed on a planar test piece. The vibrations of the test bar correspond to the modulation levels of Plate 4. Shown above the photographs is a sinusoidal waveform representing the
Plate V. Photographs of reconstructed images showing intensity variations due to changes in reference beam modulation amplitudes. a) time average (no bias); b) bias at 2/3 peak intensity; c) bias at 1/3 peak intensity; and d) bias at null intensity. (Zero vibration locations change from light to dark.)
surface displacement during exposure. Note that the null displacement zones are bright in the time-average case and dark in the sub-fringe case.

With object vibrations as large as those shown in Plate 4, the resulting intensity patterns along a test object will always exhibit uneven peak intensities. This intensity variation arises because of the large discrepancy between the zero-bias peak, \( J^2_0(0) = 1 \) and the next lower peak of the \( J^2_0(x) \) function (\( J^2_0(3.83) = 0.16 \)). For a more uniform peak intensity distribution along \( x \), the object displacements should not exceed 25% of the reference modulation. These are the object-displacement amplitudes which respond most dramatically to sub-fringe holographic interferometry.

Based upon the definitions of contrast and visibility stated in the next section, the theoretical improvement in observation of small surface displacements derived from the use of sub-fringe holography, can be assessed quantitatively.

**Contrast Enhancement**

The ease with which a person can detect changes in light intensity depends upon the magnitude of the change, relative to the average light level. Contrast, \( C \), can be defined as the ratio of the change in light intensity to the average light intensity level.
Mathematically it is expressed as:

\[ C = \frac{\Delta I(c)}{I(c)} \]  

where:  
- \( C \) - reconstructed image contrast  
- \( \Delta I(c) \) - incremental change in light intensity due to an incremental change in the intensity function argument, \( \Delta c \).  
- \( I(c) \) - light intensity level associated with argument, \( c \) (\( c \) is the net phase shift)

In a sub-fringe mode of operation, the reconstructed image itself represents the average light level. Thus \( I(x) = I(c) \) in the sub-fringe case. The light intensity, \( I(c) \), is modified by deviations introduced by surface and/or reference beam vibrations. When sinusoidal vibrations are used, \( I(c) \) varies as \( J_0^2(c) \). The derivative of \( I(c) \), taken with respect to the argument, \( c \), is

\[ \frac{dI(c)}{dc} = 2J_0(c) J'_o(c) = -2J_0(c) J_1(c) = \gamma(c) \]  

where:  
- \( J'_o(c) \) - 1st derivative of \( J_0(c) \) with respect to argument, \( c \)  
- \( J_1(c) \) - 1st order Bessel function  
- \( \gamma(c) \) - slope (gradient) of the \( I(c) \) function
It follows that the contrast, \( C \), for the sinusoidal vibration case is given by

\[
C = \frac{\gamma(c)/I(c)}{I(c)} \Delta \phi = -\frac{2J_1(c)/J_0(c)}{J_0(c)} \Delta \phi = n(c) \Delta \phi \tag{33}
\]

where: \( n(c) \) - amplification function = \( -2J_1(c)/J_0(c) \)
\( \Delta \phi \) - incremental change in argument, \( c \)

Plots of \( J^2_0(c) \), the intensity modulation function, as well as \( J_0(c) \), \( J_1(c) \) and \( n(c) = -2J_1(c)/J_0(c) \), the amplification function, are shown in Figure 10 for the region \( 0 \leq c \leq 4.8 \). Numerical values are shown in Appendix A.

The Universal Gain Function

It will be seen in this section as well as the next that the amplification function, \( n(c) \), is rather universal. It is useful in most of the expressions for system enhancement as a gain multiplier. It varies dramatically with the argument, \( c \) and accounts for the visibility of low level vibration patterns.

As seen in Eq. [33] the numerical value for the contrast observed on the image of the test object due to vibrations and sub-fringe biasing is dependent on both the argument deviation, \( \Delta \phi \), and the amplification function, \( n(c) \). At a bias point of \( c = 2.0 \) for example, \( n(2.0) = 5.0 \). A change in light intensity of 10% would require an argument deviation of \( \Delta \phi = 0.02 \) since

\[
0.1 = 5\Delta \phi
\]

from Eq. [33], and thus

\[
\Delta \phi = 0.02
\]
Fig. 10. Plots of the modulation function $J^2(c)$ and the amplification function $\mu(c) = -2J_1(c)/J_0(c)$. Also shown are $J_0(c)$ and $J_1(c)$ for reference.
Noting that for conventional (zero bias) time-average holography a 10% change in image intensity requires $\Delta c = 0.45$, an effective gain in sensitivity to surface displacements of 22.5X is realized by use of sub-fringe techniques.

Consider conventional time-average holography where the zero-vibration position intensity is normalized to unity. The light intensity at the first fringe is zero, and thus the contrast, $C = \Delta I/I = 1/1 = 1$. Since $I(c)$ goes to zero at $c = 2.4$, a crude approximation to an equivalent $\mu_{T.A.}$ for time-average holography would be

$$\mu_{T.A.} = C/\Delta c = 1/2.4 = 0.416$$

$\mu(c)$ equals 0.416 at $c = 0.38$. Thus any biasing of $c$ greater than about 0.4 results in a system contrast superior to that found with time-average holography for a comparable displacement, $\Delta c$.

The function $\mu(c)$ decreases monotonically toward minus infinity as $c$ approaches the null condition, $J^2_o(2.4) = 0$ (See Figure 10). As $c$ approaches 2.4 from the left, $\mu(c)$ goes to minus infinity. As $c$ approaches 2.4 from the right, $\mu(c)$ goes to plus infinity. Thus a unique value for $\mu(2.4)$ does not exist. This in itself is not of any great concern since in practical work, the effects of system noise and instabilities are likely to predominate prior to $|\mu|$ reaching 100 or more.
Improved Vibration's Sensitivity

A major benefit attained by using sub-fringe holographic techniques is the extension of the holographic system response to small surface displacements (phase change less than the normal holographic limit of a quarter-wavelength of light). A measure of the improved sensitivity to small surface displacements is the system gain at constant contrast. The constant-contrast gain of the system, \( G_C \), is defined as the ratio of the time-average argument, \( c_i \), to the sub-fringe change in argument, \( \Delta c_i \), required to obtain the same contrast (1% contrast).

\[
G_C = \frac{c_i}{\Delta c_i}
\]

where: \( G_C \) - constant contrast system gain
\( c_i \) - time-average argument for a contrast of 1%
\( \Delta c_i \) - change in argument to achieve a contrast of 1%

Figure 11 (inset) shows the relationship between \( c_i \) and \( \Delta c_i \).

Substituting from Eq. [33] into Eq. [34], the expression for system gain becomes

\[
G_C = \mu(c)\left(\frac{c_i}{C}\right) = \mu(c)r_i(c)
\]

where: \( r_i(c) \) - time-average argument per unit contrast
\( \mu(c) \) - amplification function

A plot of \( r_i(c) \) as a function of percent contrast is shown in Figure 11. A table of numerical values of \( r_i(c) \) is given in Appendix B.
CONTRAST – Percent

Fig. 11. Plot of the time-average argument per unit contrast, $r_1$, as a function of percent contrast, $i$. 
A theoretical calculation of system gain requires an estimate of the system's detectable contrast. If, for example, a contrast of 2% is just detectable, the time-average argument per unit contrast is $r_{2\%} = 10$ (See Appendix B). The system gain, $G_{2\%}$ is therefore $10\mu(c)$.

If an operating point of $c = 2.0$ is chosen, $\mu(2.0) = 5$ (from Appendix A). It follows that the system gain for 2% contrast and an operating point of $c = 2.0$ is 50. This means that a vibration amplitude $1/50$ of that required by time-average techniques yields equivalent intensity variations (contrast of 2%) upon biasing at $c = 2.0$.

**Fringe Visibility Enhancement**

In interferometric work, overall system performance is often described in terms of the fringe visibility based on light intensity variation. Visibility, $V$, has been defined as the difference of the maximum and minimum observed intensities divided by their sum. Thus

$$V = \frac{(I_{\text{max}} - I_{\text{min}})}{(I_{\text{max}} + I_{\text{min}})} \quad [36]$$

where: $V$ - visibility of the interference pattern

$I_{\text{max}}$ - maximum observed intensity.

$I_{\text{min}}$ - minimum observed intensity.

It is noted from Eq. [36] that $(I_{\text{max}} - I_{\text{min}})$ approaches zero for very small variations in intensity. Thus the visibility, $V$, approaches zero. In addition, maximum visibility of unity occurs when $I_{\text{min}}$ is equal to zero (i.e., the dark fringes are entirely black). This visibility maximum is independent of the value of $I_{\text{max}}$. 
Thus in the sub-fringe mode with its dark fringes the visibility is expected to be high. The changes which occur in observed visibility due to sub-fringe biasing of the reference wave are discussed in the following paragraphs.

For sinusoidal object vibrations and a 0 or π phase angle between reference and object beams, it has been shown that the image intensity variations follow a \( J^2_0(c) \) response upon reconstruction. For the sub-fringe case, the Bessel function argument, \( c \), falls between 0 and 2.4. Figure 12 (inset) shows the \( J^2_0(c) \) function with the variations in argument, \( \Delta c \), and the corresponding variations in image intensity, \( \Delta I = (I_{\text{max}} - I_{\text{min}}) \).

For conventional time-average vibration studies; bias, \( c = 0 \), \( I_{\text{max}} = 1 \) and \( I_{\text{min}} = J^2_0(\Delta c/2) \). The argument is given by \( \Delta c/2 \) since the reference beam bias level equals zero and the object vibrations have a peak-to-peak magnitude excursion of \( \Delta c \). However due to the double-valued nature of \( J^2_0(c) \) about the point \( c = 0 \), only a change in argument of \( \Delta c/2 \) contributes to the fringe visibility. It follows that the time-average visibility is given by

\[
V_{\text{T.A.}} = \frac{1 - J^2_0(\Delta c/2)}{1 + J^2_0(\Delta c/2)}
\]

[37]
Fig. 12. Plot of changes in visibility due to bias of time-average hologram in the sub-fringe region.
Table 2 shows the variation in fringe visibility due to changes in object displacement amplitudes.

### Table 2

**FRINGE VISIBILITY OF TIME-AVERAGE INTERFEROGRAMS**

<table>
<thead>
<tr>
<th>Δc</th>
<th>$I_{min}^a = J_0^2(Δc/2)$</th>
<th>$V(Δc)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.000</td>
<td>0.0000</td>
</tr>
<tr>
<td>0.2</td>
<td>0.995</td>
<td>0.0025</td>
</tr>
<tr>
<td>0.4</td>
<td>0.980</td>
<td>0.0096</td>
</tr>
<tr>
<td>0.6</td>
<td>0.956</td>
<td>0.0227</td>
</tr>
<tr>
<td>0.8</td>
<td>0.922</td>
<td>0.0404</td>
</tr>
</tbody>
</table>

$\frac{a}{I_{max}} = 1$

It is evident from Table 2 that the visibility of small object vibrations is very low, representing a lower limit for time-average holography. Not until $Δc = 4.8$ will the visibility reach a peak value of unity.

In sinusoidal sub-fringe interferometry (where a biased reference beam is used to shift the operating point further out on the $J_0^2(c)$ curve), the visibility, $V$, follows a response proportional to $μ(c)$. This function was described earlier as the amplification function and is equal to $-2 \frac{J_1(c)}{J_0(c)}$. 
Using the definition of Eq. [36], the fringe visibility for 
$0 \leq c \leq 2.4$ is derived as follows:

\[
V = \Delta I / 2I_m = [\Delta J^2_0(c)/2J^2_0(c)]
\]
\[
= -2J_0(c)J_1(c) \Delta c / 2J^2_0(c)
\]
\[
= -[J_1(c)/J_0(c)] \Delta c
\]

where:  
- $V$ - fringe visibility
- $\Delta I$ - observed change in intensity = $I_{\text{max}} - I_{\text{min}}$
- $I_m$ - mean value of intensity = $(1/2)(I_{\text{max}} + I_{\text{min}})$
- $\Delta c$ - incremental change in displacement-dependent argument, $c$

Using the definition for the amplification function, $\mu(c)$, Eq. [38] becomes

\[
V = V(c) = \mu(c) (\Delta c / 2)
\]

Figure 12 shows a plot of $V$ as a function of bias argument, $c$, for $\Delta c = 0.1, 0.2, 0.3,$ and $0.4$. Numerical values for $V$ are listed in Appendix C. As the bias on the reference wave increases from $c = 0$ (conventional time-average case) to $c = 2.4$ (sub-fringe case), the visibility increases monotonically to its maximum value of unity. The effect of biasing is very dramatic, especially as $c$ approaches 2.4. For $\Delta c = 0.1$, $V_{T,A.} = 0.001$. At $c = 2.0$, $V(2) = 0.25$ giving a 250X increase in visibility due to sub-fringe biasing.

The tops of the visibility curves in Figure 12 reach unity at smaller and smaller values of bias argument $c$ with increasing $\Delta c$. 

\[\text{[39]}\]
This apparent saturation of $V$ is introduced because as soon as $I_{\text{min}}$ reaches zero ($c = 2.4$), the visibility is always at its maximum, regardless of $I_{\text{max}}$. Thus whenever the sum of the bias argument, $c$, and $\Delta c/2$ equals 2.4, $V(c)$ will be a maximum. The bias, $c$, needed to just reach maximum visibility is given by

$$c_{\text{max}} = 2.4 - (\Delta c/2)$$  \hspace{1cm} [40]

A discrepancy exists between the predicted values of $V$, based upon $V = \mu(c)(\Delta c/2)$, and $V = 1$ at $c = 2.4 - (\Delta c/2)$. This discrepancy arises from the use of the first-order approximation for $\Delta c$. Near $c = 2.4$ the $J_0^2(c)$ function behaves very nearly like a quadratic function. Thus when a linear relationship like $\mu(c)(\Delta c/2)$ is used to represent the visibility, $V$, the extrapolated $I_{\text{min}} = \text{zero}$ point is in error by a factor of 2. This effect is most noticeable near $c = 2.4$, the $J_0^2(c)$ null point, for larger values of $\Delta c$. The plots of fringe visibility in Figure 12 have been drawn with straight lines for $0.7 < V < 1$. The intercept points at $V = 1$ are according to Eq. [40]. An analysis of the discrepancy between $\mu(c)(\Delta c/2)$ and Eq. [40] is given in Appendix D.

**Summary**

From this and the previous chapter it is evident that an improvement in reconstructed image contrast (visibility) can be expected through the use of reference beam modulation in synchronism with test object surface vibrations. The predicted interference patterns follow a $J_0^2(c)$ response (for sinusoidal modulations). The
argument of the Bessel function, c, has been shown to depend on the magnitudes as well as the relative phases of the object and reference beam modulations during plate exposure. These concepts have been explored experimentally and the laboratory results obtained are discussed in the following chapters.
CHAPTER V

EXPERIMENTAL TECHNIQUES

Several laboratory systems were needed in order to experimentally evaluate the effects of reference beam phase modulation. This chapter describes these systems and discusses their performance. They include the apparatus used in obtaining phase modulated holograms and the equipment used to analytically analyze the image reconstructions. Specific system elements included 1) the holographic optical components and their geometrical arrangements; 2) the electronic control systems and their influence on optical element behavior during exposures; 3) the optical reconstruction setups and image-scanning apparatus; and, 4) the test object and reference beam modulator.

A conventional off-axis holographic system was used throughout the study. The only necessary addition was the light beam phase modulator introduced into the reference beam. This modulator was electronically excited in synchronism with test object vibrations. The effects of changing object displacement amplitudes and phase angles (relative to the reference wave phase) were examined experimentally. These changes were facilitated by the use of independent controls for the electrical sources driving the test object and the reference modulator. Variations in light intensity, measured at the
reconstructed image of the object, were recorded by scanning through the conjugate image of the reconstruction with an apertured photometer. The details of all of these systems are discussed in the immediately succeeding sections.

**Holographic Configuration**

The holographic configuration of optical components and test object used in studying the effects of sub-fringe holography is shown in Figures 13 and 14. The light beam emitted by the laser was split into two paths by the variable beam-splitter. The light reflected by the beam-splitter was used to illuminate the test object. The scattered light from the test object became the object beam. The light transmitted through the variable beam-splitter was phase modulated, expanded and eventually it illuminated the photographic plate. This light was called the reference beam. The plate, upon exposure, recorded some of the light scattered by the test object as well as the light of the reference beam. When a coherent light source like a laser is used, the plate stores the resulting interference pattern of the two light beams (object and reference).

The light source used was a 50 mw, Spectra Physics Model 125, He-Ne continuous-wave laser. This model laser emitted a continuous light in the red part of the visual spectrum at a wavelength of 6328 Å. The exit beam of the laser was only a few millimeters wide. In order to use such a small beam for large-area holographic work it had to be expanded, optically.
Fig. 13. Schematic diagram of the general holographic configuration of optical components and test object used in the study of sub-fringe characteristics.
Fig. 14. Photograph of general holographic configuration used in the study of sub-fringe characteristics.
The expansion was done either through the use of a single lens (which yielded a diverging light field), or with a multi-lens beam expander (which yielded a wider but collimated light field). Both techniques were used in the system shown in Fig. 13. A single lens, in conjunction with a pinhole (GCO 400 spatial filter), was used to expand the beam illuminating the test object. A beam expander and laser collimator (Tropel Model 1557-C5) was used to expand the reference wave.

Redirecting and distribution of light beams was done using front-surface mirrors and high-efficiency beam-splitters. The mirrors used in redirecting unexpanded laser beams were GCO 210, 1.5-inch-square mirrors. The illumination to the object was directed by a GCO 200, 4-inch-diameter mirror. The main beam-splitter of the system had a variable-density reflective coating (Jodon Model VBA 200). This element allowed for simple redistribution of relative light intensities into the reference and object beams. It also made viewing in real time easier since the reconstructed image intensity could be set equal to the object illumination by rotating the beam-splitter. This equal setting yielded maximum real-time fringe visibility (i.e., maximum fringe contrast). The 50-50, transmission/reflection ratio, beam-splitter used as a part of the reference beam phase modulation system was a GCO 100, 1.5-inch-square unit.

Other components used in the holographic system included:

1. Electronic Timer/Shutter (GCO 700)
2. 4 x 5 Dual Plateholder (GCO 300)
3. Power Meter (GCO 760) (Silicon photovoltaic detector)
4. Vibration Isolation System (GCO 1309X)(Table)
Agfa-Gevaert Scientia 10E75 high-resolution film plates were used throughout the program. They were obtained through GCO, Inc. under their designation of HR-1, 4 x 5 plates. The emulsion has a flat spectral sensitivity from 6000-7000 Å and a resolution of 2000 line-pairs/mm. The emulsion is 7 microns thick and the glass plates are 0.05 inches thick.

The plates were usually exposed for 1-2 sec. and then were processed at room temperature as follows:

(1) Develop 1-2 min. with agitation in Kodak HRP developer,
(2) Rinse 10-20 sec. in acetic acid stop bath,
(3) Fix 3-10 min. in Kodak rapid fixer,
(4) Wash 4 min. in water.

Method of Viewing Multi-images

The changes in test part vibration patterns caused by reference beam phase modulation were often very subtle. If examination of these changes was made by first looking through one plate and subsequently through another, the subtle effects of temporal modulation were easily missed and many tests were deemed worthless for this reason. However, a scheme was devised that permitted simultaneous viewing of several vibration patterns. In addition, this procedure permitted rapid recording of holograms in quick succession. Thus the danger of parameter variation due to thermal or similar drift effects between exposures was reduced.
The procedure entailed use of a mask which covered all but one quarter of the film plate during exposures. Following a first exposure, appropriate test conditions were altered and the mask was rotated 90°. Thus another quarter of the plate was exposed to the revised test condition. This procedure was repeated twice more, allowing four exposures to be made on each plate. Each exposure corresponded to a different test condition. After development, the images stored in each quadrant could be viewed independently (with mask in place) or simultaneously (with mask removed). This was done by looking through the hologram as if it were a window. The four quadrants then are similar to four discrete panes of the window. The focus of the observer's eye is beyond the "window" (the plate) at the image of the object, not at the film plane.

Simultaneous viewing of two or more images had the advantage of seeing the discrete change from one vibration pattern to the next as the observer moved his head through the transition from one quadrant to the next. High light intensity regions often become dark (and vice versa) when simple time-average patterns are compared to sub-fringe patterns. These changes are readily seen using the technique of multiple-image viewing.

Figure 15 shows a dramatic example of the technique. Four vibrational states of a square plate are shown. They correspond to four distinct resonance frequencies, one of them being zero. The photograph of Figure 15 was made by positioning the camera lens at the center of the hologram so that light from each of the four quadrants passed on to the film plane. Thus portions of each of the four images were recorded simultaneously.
Fig. 15. Multi-image viewing allows three frequencies of vibration to be observed simultaneously along with the non-vibratory case.
The multi-image technique proved extremely helpful throughout the entire program. It not only allowed for multi-view observation, but it also expedited the scanning of reconstructed conjugate images. This follows since only a slight shift of the 3mm-diameter illuminating laser beam was needed to completely change the image being recorded from one quadrant to the next. Thus many test conditions could be viewed and recorded with a minimum of plate repositioning. Although Fig. 15 shows the general concept, it does not show the true capabilities evident in real-time, double-exposure, time-average and phase-modulated dynamic viewing.

Multi-image storage was extended even further through rotating the photographic plate 180° about its normal. The plate was masked again in this new orientation and another series of four exposures made. Upon development, the eight stored images are found to be totally independent of each other. Thus a total of eight test conditions can be quickly stored in the plate. A minimum of plate positioning and removal is required between the exposures. All exposures were obviously subjected to identical photo processing and this made comparative measurements of light intensity more uniform and reliable.

Four or eight exposures on a single plate permitted sequences of exposures to be made rapidly. Therefore undesirable transient effects such as temperature variations in electronic or test part characteristics and general environment vibrations were minimized. This was particularly important since a uniform level of vibration amplitude was difficult to maintain. A definite down-shift in test part resonant frequency existed with increasing temperatures.
The frequency-temperature dependence of the bar test samples was -4 cyc/°F.

**Test Object for Sub-fringe Studies**

The test piece used to examine the light intensity distributions in sub-fringe holography was a sheet of 2024-T6 aluminum with dimensions of 6" x 1" x 0.037". It was mounted in the fixture shown in Fig. 16. Both ends of the sheet were rigidly clamped. A loudspeaker placed directly behind the test bar excited the vibrational modes of the bar. The loudspeaker was air-coupled to the bar. The speaker was acoustically isolated from the holographic table by several layers of acoustical insulation. This minimized mechanical cross talk via direct mechanical conduction. The massive base of the steel clamping fixture proved sufficiently stable for the sub-fringe tests. Numerous time-average and real-time experiments showed no appreciable movement of the base nor clamps.

The test bar was sprayed with a diffusely-reflecting white coating. The substance was a developing agent used in dye-penetrant nondestructive testing. The bar was subsequently scribed at one-inch intervals for the purpose of fringe calibration.

**Test Object Vibratory Modes**

The vibratory frequency response of the test object was obtained by real-time holography. This technique uses the interference of the holographic image with the object itself. The
Fig. 16. Photograph of a planar test object and its mounting used in sub-fringe holographic evaluations.
interference occurs when the processed hologram is placed precisely where it was during exposure of the test object. The object image is reconstructed via the reference beam passing through the hologram and being diffracted into the original object wave system. If while viewing the reconstructed image the precisely-positioned object is also illuminated, a system of interference fringes is found to appear on the object. Subsequent movement of the object exhibits a change in the fringe positions and spacings. If the object is vibrated during real-time viewing, the antinodal displacement regions show no presence of fringes. However, the nodal regions continue to show the static fringes.

Modes of surface displacement can be identified using this real-time technique. Each mode has a unique pattern of nodal regions which can be seen in real time. The test bar was vibrated by air-coupling from the loudspeaker over the frequency range from 200 Hz to 10.3 kHz. The frequencies of the first eighteen distinct displacement modes which were observed are shown in Fig. 17.

Mode patterns are identified by the symbol $M_{x,y}$. The $x$ represents the number of nulls in the $x$ direction while the $y$ corresponds to the number of nulls in the $y$ direction. $M_{1,0}$ therefore, represents the single vertical null condition. $M_{1,1}$ corresponds to a bar divided into four equal quadrants by a null region. More modes were seen at higher excitation frequencies but are not shown in Fig. 17. The test frequency for speaker excitation was limited to below 10 kHz. The speaker and the audio power amplifier driving it became excessively hot at higher frequencies. Higher order modes
Fig. 17. Plot of the resonances found in the planar test object as a function of excitation frequency. $M_{x,y}$ is the mode number with $x$ nulls in the horizontal direction and $y$ nulls in the vertical direction.
were observed using direct coupling to the test piece via piezoelectric crystals driven by ultrasonic-frequency power generators.

Several modes of vibration were available for work in the sub-fringe studies. The $M_{3,0}$ mode at 1.9 kHz was specifically chosen for several reasons. These were:

1. it gave a null in the geometric center of the bar which made a convenient reference location for fringe analyses,
2. an additional node existed on either side of the central node, reducing the immediate effects of the clamped boundaries which alter the spatial purity of the sinusoidal bar displacements,
3. the fringe patterns existed over a sufficiently large area for ease in viewing and resolution by the scanning system,
4. all of the $M_{x,0}$ modes yield vibration patterns with light intensity variations in only one dimension (horizontally).

The $M_{3,0}$ mode yielded two sets of data for each test condition (to the left and the right of the central null). The only difference between the two was their relative spatial phase.

**Electronic Control Systems**

The amplitude and phase of vibration of both the test bar and the reference beam modulator were controlled by a system of several electronic elements. Figure 18 shows a photograph of the elements. Figure 19 shows the system's circuit diagram. The primary
Fig. 18. Photograph of the elements of the Electronic Control System.
Fig. 19. Schematic wiring diagram of the electronic control system.
electrical source which established the vibration frequency was a Hewlett-Packard, HP 650A or HP 4204 A, test oscillator. The frequency was continuously monitored by a Hickock Frequency Counter, DP150.

The test oscillator's output was the foundation for two separate drive signals. (1) The oscillator output went directly to the first channel of the McIntosh, MC275, dual-output power amplifier. The corresponding output signal drove the loudspeaker. (2) The oscillator output also went to the Hewlett-Packard, HP 3300/3302A, phase-lock oscillator which controlled the phase shift of the reference beam modulator signal. The frequency of the HP 3300 phase-lock oscillator's output signal was identical to its input signal. However the relative phase between the input and output signals was continuously adjustable throughout a range of 360° (± 10°). The variable-amplitude, phase-shifted output signal from the phase lock oscillator was then applied to the second channel of the MC275 power amplifier. The corresponding output was used to drive the reference beam phase modulator. These amplifier inputs were made adjustable via calibrated attenuators both for convenience and to assure accurate setting of signal levels in either channel.

The twin outputs of the MC275 power amplifier were monitored by two Hewlett-Packard, HP 400E, RMS Voltmeters. These were equipped with analog outputs which were monitored by a dual channel Tektronix 535A oscilloscope.
The overall functioning of the electronic control system was as follows. The primary operating frequency was established by varying the test oscillator. This signal was amplified sufficiently to observe vibrational mode patterns being formed on the test object when viewed in real-time. Reference beam modulation effects were observed by amplifying the output signal of the phase lock oscillator to a level sufficient to "bias" the operating point at the first null of the $J_0^2(c)$ function. The phasing effects were introduced by varying the relative phase of the phase lock oscillator's input and output signals.

Reference Beam Phase Modulator

The aspects of sub-fringe holography examined in this study were based on phase modulation of the reference beam. The holographic system required a device which could change the path length of the reference beam in a controlled manner and by at least a half wavelength of the laser light. A piezoelectric tube, rigidly mounted at one end and with a mirror at the other end, was used for this purpose. A small, front-surfaced mirror was bonded to the free end of the tube. Figure 20 shows the general construction of the modulator and how it was incorporated into the holographic system.

The piezoelectric tube was supplied by the Vernitron Division of the Clevite Corporation. The PZT-4 tube's dimensions were 0.5-inch long, 0.255-inch O.D. and 0.025-inch wall thickness. The length to wall-thickness ratio was about 20:1.
Fig. 20. Photograph of the phase modulator used in the reference beam. Light incident on the 50-50 beam splitter is reflected onto the modulator mirror. The light is then reflected back through the beam splitter to the beam expander.
For thickness-polarized tubes, the change in length due to an applied voltage, \( V \), is:

\[
\Delta L = d_{31} \times V \times (L/T) \tag{41}
\]

where:
- \( \Delta L \) - overall change in length, \( \text{\AA} \)
- \( d_{31} \) - piezoelectric strain constant, \( \text{\AA}/\text{volt} \)
- \( V \) - applied voltage between electrodes, volts
- \( L/T \) - length to wall-thickness ratio

for PZT-4, \( d_{31} = 2.8 \text{\AA}/\text{volt} \), therefore the end displacement per unit applied voltage is:

\[
\frac{\Delta L}{V} = (2.8)(20) = 56, \text{\AA}/\text{volt} \tag{42}
\]

Using the folded system of Fig. 20, the mirror displacement needed to just reach the null of the \( J^2(c) \) curve (\( c = 2.4 \)) is just \((1/2) 2400\text{\AA} = 1200\text{\AA}\). It follows that

\[
\Delta L = 1200\text{\AA} = 56(\text{\AA}/V) \times V
\]

Therefore:

\[
V = 1200/56 = 21.43 \text{ Volts, d.c.}
\]

or

\[
V = 30.3 \text{ Volts}_{\text{rms}}
\]

It was found experimentally that \( 45 \text{ Volts}_{\text{rms}} \) was needed to drive the vibrator to the 1st null throughout the nonresonant region of the modulator's response.
The frequency response of the vibrator is shown in Fig. 21. Plotted in Fig. 21 is the a.c. voltage applied to the modulator which just rendered the static fringes seen in real-time viewing invisible due to the vibrations of the modulator. At this voltage the reference wave phase changes correspond to the null of the \( J_0^2(c) \) function. The drop in the voltage requirements above 20 kHz represents the first major resonance of the piezoelectric tube. Satisfactory performance was found at frequencies as high as 400 kHz using this particular modulator.

**Method of Optical Reconstruction**

Holograms recorded during this study were reconstructed in a manner which permitted direct recording of the observed variations in interferometric fringe intensities. The conjugate image of the test object was used in every case. The conjugate (or real) image was established by passing a narrow laser beam through the hologram, antiparallel to the original alignment of the exposing reference beam. By doing so, an undistorted real image of the test object was formed. This image was projected on a ground glass screen and subsequently scanned with a light level detector. The detector was a photometric microscope having unity magnification. Detection of random room light and laser scatter from the hologram were avoided by placing a ground glass sheet at the image plane of the hologram. Figure 22 shows a schematic representation of the physical arrangement. Figure 23 is a photograph of the actual apparatus used during the study.
Fig. 21. Frequency response of the piezoelectric modulator used in the reference beam.
Fig. 22. System schematic for scanning the reconstructed conjugate images for intensity variations.

The scanning system was based on a Gamma-Scientific microphotometer. This system was composed of a Photometric Microscope (Model 700-10A) with a 1X objective lens, NA = 0.06 and a 10X eyepiece. A 0.050-inch diameter portion of the 1X image within the eyepiece was directed via a beam-splitter and fibre-optic link to the S-11 Photomultiplier Detector (Model 2020-1). The detector converted the scanned light levels to d.c. voltages which were indicated by the Digital Display Photometer (Model 2400).

The image to objective lens distance was 6-inches. The microscope was automatically scanned in the X-direction by a motor-driven lead-screw mechanism. The X-position data was obtained by
Fig. 23. Photographs of system used for scanning the reconstructed conjugate images for intensity variations.
either (1) a proportional linear potentiometer and power supply combination or (2) by a synchronized time-base drive. Analog voltages, proportional to the photometer d.c. output voltage were sent as Y-data to the X-Y Plotter (Mosely 2d).

The spatial resolution of the detection system was checked by traversing the detector across a single-edge target. The measured light level changed from maximum to minimum within 1.5 mm. The detector diameter was 1.27 mm (0.05 inch). A typical single-edge response curve is shown in Appendix E.

Random (dark current) noise generated within the Photomultiplier tube (PMT) of the detector was 1 mv$_{\text{rms}}$ at maximum sensitivity ($10^3$) and an operating accelerating potential (700 V$_{\text{d.c.}}$). At lower sensitivities, ($10^2$,10), the dark current noise dropped to less than 0.1 mv$_{\text{rms}}$. These measurements were made with the smoothing filter in the "slow" position. The noise level increased to about 2 mv and 3.5 mv with the filter set at "medium" and fast," respectively.

The d.c. dark current of the PMT increased with accelerating potential. This increase was compensated in the Photomultiplier Detector by a counter-balancing current generated in the Photometer. The dark current of the system could be compensated (by nulling) for PMT acceleration potentials of up to 1.4 kv. The presence of random noise makes the use of the system above 1 kv impractical, however.

Appendix E shows the dark current response of the photometer as well as the increase of noise with increasing accelerating potential. Gain of the photometer was $10^4$ to 1 for a change in accelerating potential of 5 to 1.
The holographic, electronic and optical instrumentation characteristics and responses have been shown in this chapter as well as in the referenced appendixes. The next chapter summarizes the experimental results obtained in the study of sub-fringe holography using the instrumentation and equipment described in this chapter.
CHAPTER VI

EXPERIMENTAL RESULTS

The experimental investigation of the characteristics of reference beam temporal modulation fell into the following three segments:

1) general observation of phase modulation effects on a one-dimensional vibrating bar,
2) careful photo-multiplier scanning of the reconstructed images of a one-dimensional vibrating bar,
3) observation of modulation effects on typical test parts.

Thus the experimental work proceeded from the general recognition of phase and amplitude dependences of object vibration to the specific verification of intensity variations according to the \( J_0^2(c) \) modulation function. The results of these two phases permitted intelligible observation of reference modulation effects on the complex geometries of typical test parts.

General Observations of Phase Modulation

Reference phase modulation effects were observed both using time-average holography and real-time holography. The real-time holographic observations were used to establish operating parameters such as amplitudes of excitation, frequencies of modulation and relative
dynamic phase. Then the time-average technique was used to permanently record the results. Thus the observed behavior of reference beam phase modulation was made available for subsequent photographing.

**Time-average Observations of Phase Modulation**

The phase detection capabilities of reference wave modulation systems are easily seen by using relatively low amplitude object vibrations ($a_{\text{max}} = \lambda/4$). A conventional time-average reconstruction for such low amplitudes would appear as a series of light and dark bands, each light band representing a nodal position. Plate 6(b) shows a photograph of such a reconstruction. Bright regions exist at both ends of the bar corresponding to the clamped end-conditions. The intervening bright regions correspond to nodal positions along the bar. The surface vibration pattern for this case is shown in Plate 6(a) above the top photograph corresponding to the time-average case (Plate 6(b)).

Introducing sufficient phase modulation to shift the operating point partially down the $J_0^2(c)$ slope, a shift in location of apparent nulls occurs and the number of light and dark regions is reduced by one half. This decrease in the number of intensity reversals occurs because the $J_0^2(c)$ function is double-valued when the operating point is at an extremum (peak or null) but is single-valued on the side slope of the central peak.

The regions in which the pattern gets lighter in Plate 6(c) represents the case where the reference phase modulation tends to
Plate VI. General appearance of phase modulated vibrations. 
(a) surface displacement pattern; (b) time-average fringe pattern due to (a); (c) phase modulated fringe pattern of (a) and (b); (d) sub-fringe intensity pattern of (a).
cancel the effects of object vibrations. The darkened regions, on the other hand, represent the additive effects of both object and reference modulations driving the intensity modulation function into a deeper portion of the first $J_0^2(c)$ null.

Eventual operation right at the $J_0^2(c)$ null results in a complete reversal of the regions of black and white from that of the time-average case. Plate 6 also shows the comparison of the time-average case (b) and the sub-fringe case (d) where the operating point is at the $J_0^2(c)$ null. Note how the relatively broad light regions of the time-average case have become rather narrow dark regions in the sub-fringe case. This change in width is caused by the profile of the $J_0^2(c)$ function around $c = 0$ and $c = 2.4$. The width of the $J_0^2(c)$ central peak, $J_0^2(0)$, (taken at half of the central peak amplitude) is 80% larger than the width of the first valley, $J_0^2(2.4)$, (taken at half of the second peak amplitude). Thus the reduction in nodal-region width is to be expected in going from the time-average case to the sub-fringe case.

Adjacent regions on opposite sides of the dark, null regions have unequal levels of intensity in Plate 6(d). This arises because one side represents light from the $J_0^2(c)$ zero-argument peak while the opposite side represents the light from the next lower peak. The next peak has a magnitude equal to only 16% of that of the central peak. Thus a characteristic of reference beam phase modulation holography is the appearance of alternate regions having two different light levels about the central dark regions.
For relatively-uniform intensities to exist, the vibration amplitude, $\Delta c$, must not exceed $\pm 0.4$ around the operating point of $c = 2.4$. Therefore surface displacements must satisfy the relationship

$$(2\pi/\lambda)(\cos \theta_1 + \cos \theta_2)a(x) < 0.4$$

For normal incidence $a(x)$, is simply

$$a(x) < (0.1/\pi)\lambda = 0.0319\lambda = (1/30)\lambda$$

**Real-Time Observations of Phase Modulation**

The time-average holograms of Plate 6 were made after real-time holograms of the same object had been viewed in detail. Real-time holography permitted detailed visualization of the vibratory modes of vibration as well as vibration amplitude and phase effects. This was done by viewing the laser illuminated object while simultaneously viewing the reconstructed image through the reference beam illuminated hologram. Interference bands appeared on the object when the hologram had been replaced precisely in the location in which it was originally exposed. For all vibratory studies of surface displacements over a quarter wavelength of laser light, the regular array of real time interference bands seemed to disappear in the vibrating regions. The nonvibrating regions (nodal regions) maintained their regular array of interference bands. It was through this real time technique that normal modes of test object vibration were identified holographically.
Similarly the effects of the variation of relative phase between reference and object waves could be observed in real time. For the in-phase (and 180°) condition of reference beam and test object modulations, the regions of apparent nulls were very clear. As the phase control of the phase lock oscillator was rotated through 180°, an angle existed at which the apparent null region could be clearly identified. Using sinusoidally-vibrating test panels, a 180° reversal in phase was visually characterized by a shift in apparent nulls to alternate vibrating regions.

Using this real time technique it was possible to verify that

1. effective null regions could be established anywhere throughout the vibrating test plates by varying the amplitudes of either the test object vibrations or the reference wave modulation.

2. maximum effective null areas could be found at two relative phase points, 180° apart.

3. phase adjustments at 90° to these maximum null angles did not yield any discernible effective null regions.

4. low amplitude test object vibrations, undetectable without reference wave modulation, could be made observable by the proper selection of relative phase angles.

These results all supported the premise that the effective null regions were established by simultaneous application of both the object vibrations and the reference beam vibrations. They also supported the premise that the location of the effective null regions
are dependent on the phase of the reference beam mirror, relative to the phase of the object vibrations. They further supported the notion that object vibrations with associated object beam phase angle amplitudes less than a quarter of the laser light wavelength could be made visible by introduction of a properly phased reference beam modulation. Thus the technique does in fact make sub-fringe data available to the holographer. This can easily represent an increase of 5X in holographic test sensitivity (as noted in Table 5 on p.130).

Although these general results are encouraging, they do not give any information regarding the mathematical form of the modulation function. The manner chosen for demonstrating the correspondence of intensity variations and the \( J^2_o(c) \) function relied upon the point-by-point plotting of image intensity along a vibrating test bar. The observed results are outlined below using the scanning technique described earlier in Chapter V.

**Photomultiplier Scanning of Reconstructed Images**

The reconstructed conjugate images were scanned using the Digital Photometer System described in Chapter V. The photomultiplier tube (PMT) technique was chosen after nonlinear film response characteristics proved film exposing techniques to be unsatisfactory.

Initial scan results using the photomultiplier tube showed two unexpected trends. The first trend was a nonuniform "background-
noise" level throughout the length of the test piece.\textsuperscript{13} Both of these effects were investigated and the results shown below.

Sources of Background Noise

An unexpectedly-high background noise level prompted an investigation into its causes. A hologram using the test bar as an object was exposed under three operating conditions. These were:

1. no exposure at all (i.e., the holographic plate was not subjected to any direct laser light)
2. reference beam illumination without object beam (i.e., the test object was not illuminated)
3. normal object beam plus reference beam.

Upon reconstruction and scanning by the PMT, three distinct levels of noise could be observed. These are shown as the lower three curves in Fig. 24. The base line at 0 mv was established with the aperture to the PMT closed. Curve (a) (the lowest curve) shows the background light level of the room in which the scans were made. It was caused by general white-light leakage within the darkened laboratory. The level is uniform in amplitude across the scan region.

Curve (b), the second-lowest curve, decreases from a maximum of about 12 mv near the right end of the plot to about 5 mv at the left end of

\textsuperscript{13}"Noise" in this case refers to a light level greater than that recorded when the photo-multiplier tube is shuttered close. The photometer system records it as an increase in the d.c. average level of the PMT output voltage. This noise is not to be confused with the rapidly changing (a.c.) output voltages representing the effects of thermal noise of the PMT and speckle effects observed as the detector is scanned along the image.
Fig. 24. Scanned intensity levels of a static test bar and contributions to noise due to variations in the film emulsion condition.
the plot. This noise signal is caused by the light scattered by the processed but unexposed photographic film emulsion. Curve (c), the third noise curve, corresponds to the intensities measured due to the exposed and developed hologram. It also decreases from right to left.

Judging from the above observations, the nonuniform component of the background noise signal is due to the scattering of the collimated reconstruction laser beam by the holographic emulsion. Note in Fig. 22 of Chapter V that the undiffracted portion of the reconstruction laser beam is located near one end of the ground glass scan region. This end corresponds to the high noise end of Fig. 24. It follows that the noise trend is coupled to the scatter (flare) light given off by the laser beam transmitted through the hologram.

The relative, average noise contribution each source made individually was estimated near the center of the test bar. The noise contribution due to the stray light in the scanning room was a uniform 4 mv. The additional contribution by the unexposed (but processed) emulsion brought the level up to 10 mv. The total noise level for an exposed and processed hologram was 13 mv. Table 3 shows the relative contributions of each source.

TABLE 3

<table>
<thead>
<tr>
<th>Noise Source</th>
<th>PMT Output (mv)</th>
<th>% Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scan-room light</td>
<td>4</td>
<td>31</td>
</tr>
<tr>
<td>Unexposed emulsion</td>
<td>6</td>
<td>46</td>
</tr>
<tr>
<td>Exposed emulsion</td>
<td>3</td>
<td>23</td>
</tr>
</tbody>
</table>

*aBoth cases represent processed film.
It is evident from Table 3 that the emulsion on the holographic plate contributed the most to the noise signal detected by the PMT. The introduction of silver platelets (due to development of sensitized silver halide gains) contributed another portion, equal to one half of the unexposed emulsion contribution. The contribution of 31% due to the stray light of the room in which the holograms were scanned is relatively high. However a limit of 1 mw existed on the laser used to reconstruct the holograms. Thus although the room was very dark, the weakly illuminated plates required a very high sensitivity of the PMT. Although this background noise is undesirable, its uniform nature makes it less objectionable.

Since these noise sources are consistent from plate to plate and predictable, it seems safe to consider that this background light which is now the level of minimum intensity was always added to the light measured from the test object. Thus for the case of Fig. 24 the level of the static test object light intensity always is high by an amount equal to the noise level at each location. The light intensities measured from the static test object are also shown in Fig. 24. Put another way, the expected variations of intensity due to phase modulation are expected to fall between the noise level and the static bar light intensity level.

Causes for Nonuniform Bar Illumination

The uppermost curve (d) shown in Fig. 24 corresponds to the light intensity distribution measured from the reconstruction along the length of the static test bar shown in Figs. 13 and 16. The test
bar d.c. light level is about 5X higher than that of the background noise discussed earlier. However the a.c. (rms) variation in noise level is only about 1 mV. The difference in the static bar d.c. level and the dark region d.c. noise level is about 40 mV. Thus the peak signal to noise ratio is about 40:1. Measurements taken of modulation fringe function effects never exceeded this figure.

In measurements of the illuminating light scattered from the test object itself (taken in the direction of the hologram), it was found that they followed the static object intensity curve of Fig. 24(d) to within ±5% throughout the test bar region of interest. The shape of the curve recording the light distribution along the bar is a composite result of (1) the object-illuminating beam profile, (2) the angle between test object and hologram plane, and (3) the scattering directivity of the diffusive coating deposited on the test object.

Mathematical Representation of Noise and Bar Illumination Distribution

The experimental variations in light-intensity for vibrating test bars had to be compensated (for the noise and static bar intensity distributions discussed above) before correlations with theory could be made. This compensation was made using straight-line approximations for both the noise and the static bar distributions. Figure 25 shows the noise and static bar distributions for the test case which was analyzed in detail. Using a slope-intercept form for the distributions

\[ n(x) = 0.9x + 5.6 \]
Fig. 25. Static bar and background noise intensities used to establish the noise spatial dependence, \( n(x) \), and weighting function spatial dependence, \( w(x) \).
\[
w(x) = \begin{cases} 
16.75x + 34.5 & 0 \leq x \leq 1.8 \\
-14x + 80.0 & 1.8 \leq x \leq 5 
\end{cases}
\]

where: \( n(x) = \text{noise distribution} \)
\( w(x) = \text{static bar light intensity distribution} \)

It follows that for theory and experiment to correlate, the \( J_0^2(c) \) function must be weighted by the static bar distribution, \( w(x) \), and then added to the noise function, \( n(x) \). Schematically and mathematically this can be represented by

\[
J_0^2(c) \overset{w(x)}{\rightarrow} I(x)
\]

and

\[
I(x) = w(x) J_0^2(c) + n(x)
\]

where: \( I(x) = \text{measured light intensity} \)
\( n(x) = \text{noise intensity distribution} \)
\( w(x) = \text{static bar intensity distribution} \)
\( J_0^2(c) = \text{modulation function (Bessel function)} \)
\( c(x) = \text{net phase modulation of system} \)
Correlation of Measured Light Intensity and the \( J_0^2(c) \) Modulation Function

Using \( a(x) \) as a sinusoidally varying function (See Chapter IV) and Eqs. [43] and [44], \( I(x) \) was computed for 0.1-inch increments along the test bar. The region used was in the center of the test bar, corresponding to 0.8 and 2.0 inches from the recorded end of the bar. This region corresponds to the node at the geometrical center of the bar and the adjacent areas on either side of it. The adjacent areas extend to the next closest vibrational nodes. Table 4 lists the numerical results of these computations.

Precise correlation of computed values of \( I(x) \) (Table 4) and scanned intensity patterns (Fig. 26) were established at specific locations on the bar. For example, the locations of vibratory nulls were established through the observed scan patterns recorded for the time-average and sub-fringe modes. These two cases give definitive patterns for null locations. Positioning for the other two curves was determined by setting the 2/3- and 1/3- peak intensities at their respective vibrational null positions.

Figure 26 shows the actual data curves recorded by the PMT system for the central region of the vibrating bar. The surface displacement, measured by time-average techniques, gave a phase change of \( \lambda/4 \). The theoretical values, tabulated in Table 4, are plotted as triangles. Figure 27 shows the extent of the \( J_0^2(c) \) curve used for each case of Fig. 26.
**TABLE 4**

**TABULATION OF CALCULATED INTENSITIES**

<table>
<thead>
<tr>
<th>Position x (inch)</th>
<th>Noise n(x)</th>
<th>Weight w(x)</th>
<th>$I_0(x)^b$</th>
<th>$I_1(x)^c$</th>
<th>$I_2(x)^d$</th>
<th>$I_3(x)^e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>4.3</td>
<td>46.8</td>
<td>51.1</td>
<td>37.1</td>
<td>11.3</td>
<td>4.3</td>
</tr>
<tr>
<td>0.9</td>
<td>4.5</td>
<td>50.0</td>
<td>39.5</td>
<td>54.4</td>
<td>33.3</td>
<td>15.1</td>
</tr>
<tr>
<td>1.0</td>
<td>4.7</td>
<td>53.4</td>
<td>26.1</td>
<td>53.4</td>
<td>50.7</td>
<td>30.1</td>
</tr>
<tr>
<td>1.1</td>
<td>4.8</td>
<td>52.3</td>
<td>21.1</td>
<td>48.5</td>
<td>53.8</td>
<td>36.2</td>
</tr>
<tr>
<td>1.2</td>
<td>5.1</td>
<td>50.9</td>
<td>26.1</td>
<td>51.5</td>
<td>49.0</td>
<td>29.3</td>
</tr>
<tr>
<td>1.3</td>
<td>5.2</td>
<td>49.6</td>
<td>40.0</td>
<td>54.8</td>
<td>33.7</td>
<td>15.7</td>
</tr>
<tr>
<td>1.4</td>
<td>5.3</td>
<td>48.2</td>
<td>53.5</td>
<td>39.0</td>
<td>12.5</td>
<td>5.3</td>
</tr>
<tr>
<td>1.5</td>
<td>5.5</td>
<td>46.9</td>
<td>38.4</td>
<td>14.3</td>
<td>5.5</td>
<td>10.2</td>
</tr>
<tr>
<td>1.6</td>
<td>5.7</td>
<td>45.5</td>
<td>23.9</td>
<td>7.0</td>
<td>6.0</td>
<td>12.9</td>
</tr>
<tr>
<td>1.7</td>
<td>5.8</td>
<td>44.2</td>
<td>19.5</td>
<td>5.7</td>
<td>10.7</td>
<td>12.9</td>
</tr>
<tr>
<td>1.8</td>
<td>6.0</td>
<td>42.8</td>
<td>23.2</td>
<td>7.2</td>
<td>9.2</td>
<td>12.7</td>
</tr>
<tr>
<td>1.9</td>
<td>6.2</td>
<td>41.5</td>
<td>35.3</td>
<td>14.0</td>
<td>6.2</td>
<td>10.4</td>
</tr>
<tr>
<td>2.0</td>
<td>6.3</td>
<td>40.2</td>
<td>46.5</td>
<td>34.4</td>
<td>12.3</td>
<td>6.3</td>
</tr>
</tbody>
</table>

*aAll "intensity" values measured in PMT output voltage, expressed in mv.*

$bI_0(x)$ - time-average case (no reference beam modulation).

c$I_1(x)$ - intensity at null location set for 2/3 of peak value by reference beam modulation adjustment.

d$I_2(x)$ - intensity at null location set for 1/3 of peak intensity value.

$eI_3(x)$ - sub-fringe case (intensity at null location set for minimum).
Fig. 26. Comparison of experimentally derived data (direct plots) and the predicted intensity variations (▲) for a surface displacement peak amplitude of \(\lambda/4\).
Fig. 27. Extent of $J_0^2(c)$ function used for $\Delta c$ of $\lambda/4$, reference adjusted from 0, 1/3, 2/3, and 3/3 of first null.
As is evident from Fig. 26, there is good agreement between the measured curves and the predicted test points. The general agreement of shapes between Figs. 26 and 27 is also evident. It follows that the prediction of a $J^2_0(c)$ functional response is valid.

**Useful Range of Time-average / Sub-fringe Holography**

The useful range of time-average holography has an upper limit dictated by the surface displacements which yield $J^2_0(c)$ variations which are comparable to the rms noise level of the system. Typically this occurs at about fifteen fringes corresponding to a phase shift of approximately 8 wavelengths and a surface displacement of about 4 wavelengths of the illuminating light. The lower limit for time-average holographic interferometry is set by the visibility of a single fringe. This corresponds to test object surface displacements near $\lambda/4$.\(^\text{14}\)

Sub-fringe holography becomes useful at the single fringe phase displacement of $\lambda/4$ and extends the useful range to still smaller phase changes. The practical limits to which sub-fringe holography can be extended have been of major concern during this study.

\(^{14}\)At the first $J^2_0(c)$ null, $c = (2\pi/\lambda)(\cos \theta_1 + \cos \theta_2)a(x) = 2.4$. Assuming normal incidence, $\theta_1 = \theta_2$ and $2.4 = a(x)\pi(4/\lambda)$. Thus $a(x)$ at the first time-average null is equal to 0.76 ($\lambda/4$).
Using the data of Fig. 26, it is evident that the PMT output voltage representing the rms noise level is approximately 1 mv.

This noise level was measured near the low intensity conditions. The peak values of Fig. 26 are 50 mv (in the center of the bar) when compensated for background noise. Thus the peak of the $J_o^2(c)$ function is related to 50 mv as the minimum intensity deviation is to 1 mv. Therefore the smallest detectable change in the $J_o^2(c)$ function is 0.02.

Near the first null of $J_o^2(c)$ this corresponds to a change in argument, $\Delta c$, of 0.3 (See Appendix D). Using Eq. [24] of Chapter IV,

$$\Delta c = 0.3 = \left(\frac{2\pi}{\lambda}\right)(\cos \theta_1 + \cos \theta_2)a(x)$$  \[46\]

For normal incidence of the illuminating beam and viewing direction on the test object, $\theta_1 = \theta_2 = 0$. Thus the minimum surface displacement detectable by sub-fringe holographic techniques (limited by rms system noise) is therefore equal to about 1/40 of the signal level, i.e.,

$$a(x)\bigg|_{\text{min}} = 0.3\lambda/4\pi = \lambda/42$$  \[47\]

A change in argument, $\Delta c$, of 2.4 yields the first fringe in time-average holography. A $\Delta c$ of 0.3 (minimum detectable by sub-fringe) is therefore equivalent to (1/8) of a time-average displacement fringe. Thus sub-fringe holography is eight times more sensitive to small surface displacements than is 1st fringe time-average holography with a signal to noise ratio of 50:1. The range of displacements detectable using time-average and sub-fringe holography is outlined in Table 5.
### TABLE 5

USEFUL RANGES OF TIME-AVERAGE AND SUB-FRINGE HOLOGRAPHY

<table>
<thead>
<tr>
<th>Units</th>
<th>Sub-Fringe</th>
<th>Time-Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelengths</td>
<td>$(\lambda/40 - \lambda/4)$</td>
<td>$(\lambda/4 - 8\lambda)$</td>
</tr>
<tr>
<td>Microinches*</td>
<td>$(0.623 - 6.25)$</td>
<td>$(6.25 - 200)$</td>
</tr>
<tr>
<td>Å*</td>
<td>$(158.2 - 1582)$</td>
<td>$(1582 - 50,624)$</td>
</tr>
<tr>
<td>Microns, mm(10)$^{-3}$*</td>
<td>$(0.0158 - 0.158)$</td>
<td>$(0.158 - 5.06)$</td>
</tr>
</tbody>
</table>

* For HeNe Lasers, $\lambda = 6328Å$

**Comparison of Time-average and Sub-fringe Holography**

A comparison between time-average and sub-fringe holography was made in the previous section of this chapter. An analysis, based on the measured noise level, showed an improved sensitivity to small surface displacements of about 8 to 1. This was a rough approximation intended for general comment.

Another comparison can be made, based on the fringe visibility of both time-average and sub-fringe holography. Recall that Eq. [36] defined the fringe visibility as

$$ V = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} \quad [36] $$

where: $I_{\text{max}}$ = maximum fringe intensity

$I_{\text{min}}$ = minimum fringe intensity
When $I_{\text{min}} = 0$, the visibility, $V$, is forced to unity. But if the minimum is other than zero, than $V < 1$.

Using the noise level measured experimentally (corresponding to $I_{\text{min}} = 0.02$), a plot was made of the fringe visibility as a function of the net phase shift. This plot is shown as a solid line in Fig. 28 and labeled $V_{T,A}$. Note that the net phase shift corresponds to twice the surface displacement using normal directions of incident illumination and observation.

In the region where the net phase shift was greater than $\lambda/2$, the numerical values for $V$ were calculated using $I_{\text{min}} = 0.02$. Throughout this region $I_{\text{max}}$ was chosen as the peak value nearest the maximum $\Delta c$ excursion but at a lesser argument. Since the peaks of $J^2_0(c)$ decrease with increasing argument, $V$ decreases with increasing phase shift (argument). Since the values of $V$ for $c > 2.4$ are based on the "previous" intensity peak value, $V_{T,A}$ is step-like throughout this region.

In the region $\phi < \lambda/2$, $I_{\text{max}} = 1$ while $I_{\text{min}} = J^2_0(c)$. Thus the actual visibility for the time-average case does not go to zero abruptly at $\phi = \lambda/2$ but decreases smoothly. For example, the visibility of a time-average hologram of a test part vibrating with a displacement amplitude equal to $0.1\lambda$ (which yields a phase shift of $0.2\lambda$) has a visibility equal to 0.4.

The fringe visibility calculated for the sub-fringe case also uses a minimum intensity of 0.02 (representing a signal to noise ratio of 50:1). In this case the maximum intensity is given by $J^2_0[2.4 - (\Delta c/2)]$. The results for both the in-phase ($0^\circ$) and out-of-phase ($180^\circ$) cases are shown in Fig. 28 by dashed and broken lines, respectively. The
Fig. 28. Fringe visibility for time-average and sub-fringe holography using the measured noise level of 1 mv = 0.02/1.0 of the peak value.
0° case represents intensity increases up the central peak of the
J_0^2(c) function and thus can get as high as the time-average case for
sufficiently large phase displacements (0.4λ). The 180° case can only
reach the height of the next lower peak, namely, 0.16. Thus the sub-
fringe and time-average results are equal in the transition region of
φ ≈ 0.5λ. The numerical results for the above cases are listed in
Appendix F.

It is evident from Fig. 28 that the fringe visibility is
higher in the sub-fringe case than in the time-average case for φ < 0.3λ.
It is this region that is considered extended due to the sub-fringe
mode of operation. The shape of the V_{T,A} curve in this region does
not change appreciably with different S/N ratios since the reference
for V_{T,A} is the peak intensity value. However the sub-fringe
visibility improves with better signal to noise (S/N) ratios.
Table 6 shows the variation of sub-fringe visibility at a phase
change of 0.05λ and 0° relative phase as a function of S/N ratio.
The signal to noise (S/N) ratios are varied from 50:1 (observed
experimentally) to 1000:1. These data are also shown in Fig. 28 as
+x's on dotted-line estimates of the sub-fringe visibilities expected
at the improved S/N ratios.
TABLE 6

SUB-FRINGE VISIBILITY AT 0.05\(\lambda\)^a

<table>
<thead>
<tr>
<th>S/N Ratio</th>
<th>(I_{\min})</th>
<th>(I_{\max} - I_{\min})</th>
<th>(I_{\max} + I_{\min})</th>
<th>(V_{S.F.})</th>
</tr>
</thead>
<tbody>
<tr>
<td>50:1</td>
<td>0.02</td>
<td>0.01</td>
<td>0.05</td>
<td>0.200</td>
</tr>
<tr>
<td>100:1</td>
<td>0.01</td>
<td>0.02</td>
<td>0.04</td>
<td>0.500</td>
</tr>
<tr>
<td>200:1</td>
<td>0.005</td>
<td>0.025</td>
<td>0.035</td>
<td>0.714</td>
</tr>
<tr>
<td>500:1</td>
<td>0.002</td>
<td>0.028</td>
<td>0.032</td>
<td>0.876</td>
</tr>
<tr>
<td>1000:1</td>
<td>0.001</td>
<td>0.029</td>
<td>0.031</td>
<td>0.935</td>
</tr>
</tbody>
</table>

^a_{I_{\max}} = 0.03 for in-phase (0°) case.

Because of these changes in \(V_{S.F.}\) with improved S/N ratio, the effective range of sub-fringe holography can be expected to be improved with increasing S/N ratios. (This is also true of the time-average range for large displacements.) Table 7 shows the useful range of sub-fringe holography based on a criterion of \(V = 0.4\) (\(I_{\max}/I_{\min} = 2.33\)). At this visibility, \(V_{T.A.} = 0.4\) occurs at \(\phi = 0.2\lambda\) (i.e., \(a(x) = 0.1\lambda\) and \(c_x = 1.26\)).
TABLE 7

RANGE OF SUB-FRINGE HOLOGRAPHY AT V = 0.4

<table>
<thead>
<tr>
<th>S/N Ratio</th>
<th>$I_{\text{min}}$</th>
<th>$I_{\text{max}}$</th>
<th>$\Delta c$</th>
<th>$c_{1}/\Delta c^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50:1</td>
<td>0.02</td>
<td>0.0466</td>
<td>0.38</td>
<td>3.32</td>
</tr>
<tr>
<td>100:1</td>
<td>0.01</td>
<td>0.0233</td>
<td>0.28</td>
<td>4.50</td>
</tr>
<tr>
<td>200:1</td>
<td>0.005</td>
<td>0.0117</td>
<td>0.20</td>
<td>6.30</td>
</tr>
<tr>
<td>500:1</td>
<td>0.002</td>
<td>0.0047</td>
<td>0.12</td>
<td>10.50</td>
</tr>
<tr>
<td>1000:1</td>
<td>0.001</td>
<td>0.0023</td>
<td>0.08</td>
<td>15.75</td>
</tr>
</tbody>
</table>

$^a c_{1}/\Delta c$ is the improvement in system sensitivity over that of the
time-average case at a constant visibility of 0.4 corresponding to
$I_{\text{max}}/I_{\text{min}} = 2.33$. Time-average argument at $V = 0.4$ is $c_1 = 1.26$.

Modulation Effects on Typical Test Parts

The illustrations used to discuss sub-fringe holography so
far have been for a specially-designed aluminum test bar. This bar
was excited in a vibrational mode ($m_{0x}$), which assured a variation
in light intensity in only one direction (i.e., along the length of
the bar). The excitation frequency ($\approx 2$ kHz) was chosen to have a
transverse wave acoustic wavelength of about three inches. All of
these conditions were used to facilitate photomultiplier scanning (as
discussed earlier).

Most test objects subjected to vibrational excitation,
however, do not display such a simple mode pattern. Especially-
complex vibrational patterns appear on irregularly-shaped objects.
In spite of this fact it is possible to determine local regions of inhomogeneities by looking at the overall mode shapes on a test object. Thus the methods of time-average and sub-fringe holography are useful for materials evaluation. Used in this context, the technique is sometimes called holographic nondestructive testing (HNDT).

General Characteristics of Sub-fringe Holography

Certain characteristics have been found inherent in transferring from time-average to sub-fringe holograms while viewing industrial test parts. These include the following:

1. light regions on the test object become dark.

2. all vibrating regions do not exhibit the same light intensity.

3. if the reference beam modulation is not sufficient to transfer the pattern entirely into a sub-fringe mode, the number of vibratory regions drops to half of those seen in time-average holography.

All of these effects are seen in Plates 7 and 8 and are accounted for according to the theories described earlier.

Reversal of Light to Dark

Plate 7 shows the reversal of dark regions for light regions arising from the operating point being moved from \( c = 0 \) (top) to the \( J_0^2(c) \) null (bottom) (i.e., from time-average to sub-fringe). The effect is very dramatic when low level vibrations are present. Plate 7 shows the test part under time-average exposure (top) being basically
Plate VII. Dimming effect of sub-fringe holography, showing the phase detection capabilities due to intensity differences.
Plate VIII. Three photographs of a typical test part examined using time-average holography (top), sub-fringe holography (bottom) and partial reference beam modulation (center).
bright with a slight amount of darkening in the peak vibratory areas. Upon reference beam modulation (bottom) the entire part recon structs dark with light regions at the vibration sites. At first viewing it is often difficult to see any image at all due to the reversal from light regions to dark regions. However, careful examination with a high intensity reference illumination (and a very dark room) usually allows the vibration patterns to be seen.

The darkening effect is less noticeable if the time-average holograms display a first or second fringe, corresponding to displace ments of a wavelength or so. In this case, a shift in null positions of half a fringe occurs but the overall brightness is not greatly affected. This is seen in Plate 8 in the center photograph. The top of Plate 8 is the time-average case while the bottom is the sub-fringe case.

Phase Dependence of Light Intensity

Regions adjoining the vibratory nulls in time-average holography usually darken equally under uniform sinusoidal excitation. All the regions look alike regardless of phase relationships. How ever in sub-fringe holography this does not hold true. Of the adjacent regions in the sub-fringe case, one lies on the central peak while the other \( (180^\circ \text{ out-of-phase}) \) lies on the second peak of the \( J^2_0(c) \) function. The out-of-phase region can therefore only increase to 16\% of the central peak's intensity. Thus the complex sub-fringe mode patterns of typical test parts show their relative phase relationships directly. This is evident from several of the more regular mode
patterns shown in Plates 7 and 8. The differences in intensity correspond to the in-phase and out-of-phase regions. Therefore

sub-fringe holography yields phase information directly which is not available using time-average techniques.

Reduction in Fringe Spatial Frequency

The reduction in the number of vibrating regions by a factor of two at reference beam modulations less than 0.19λ arises from biasing on the side of the central $J^2_0(c)$ peak. At the extremes of peaks and nulls of the $J^2_0(c)$ function, the intensity variations are double-valued (i.e., an increase, + or -, in test object vibration about the null ($m_r = 0.19\lambda$) results in an increase in intensity). If $m_r$, the reference modulator's vibrating mirror amplitude, is about half the value required for sub-fringe excitation, the system response is single valued (i.e., for each vibration amplitude, a unique intensity exists for object amplitudes less than 0.1λ). This effect is very pronounced in the center photograph of Plate 8.

Therefore a reference beam modulation of about 0.1λ changes half of the dark vibration regions of a time-average hologram into light regions. The other half of the dark vibration regions are made still darker since they are closer to the $J^2_0(c)$ null.
Summary of Experimental Results

The general observations of both real-time and sub-fringe holograms supported the theoretical predictions of earlier chapters regarding test object mode patterns and their dependence on vibrational amplitudes, frequencies and relative phase. Apparent nulls were established and translated as desired by adjustment of vibrational amplitudes and/or relative phase between object and reference modulators. Partial reference modulation dropped the number of fringes by a factor of two and sub-fringe excitation reversed the reconstructed light regions by dark regions.

Subsequent scanning with a PMT established static image intensity distributions as well as noise components attributable to hologram emulsions and laboratory conditions. Based on the measured signal-to-noise ratio of 50:1, the improvement of sub-fringe holographic sensitivity over that of time-average holography was set at 8:1. A minimum detectable surface displacement of \(\lambda/40\) was established for the measured noise limitation.

In comparing time-average and sub-fringe holography it is evident that both their ranges of performance are limited by the system's noise. Tables 6 and 7 show how the useful range of both techniques can be extended using high S/N ratios. Table 8 summarizes their respective image characteristics.
<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Time-average</th>
<th>Sub-fringe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed light-intensity level</td>
<td>Bright at vibration null, decreasing rapidly to noise level at about 15 fringes.</td>
<td>Dark at vibration null, adjacent regions brighter but usually not of equal intensity. Overall appearance quite dark.</td>
</tr>
<tr>
<td>Relative object's vibrations phase dependence</td>
<td>None</td>
<td>Dictates visibility of fringes. Can be used for adjusting system sensitivity independent of object excitation amplitude</td>
</tr>
<tr>
<td>Fringe visibility</td>
<td>Varies according to Fig. 28 and Tables 6 and 7</td>
<td>Varieties with frequency amplitude and frequency of excitation. Amplitude independent within operating range.</td>
</tr>
<tr>
<td>Fringe spatial frequency</td>
<td>Varies with vibration amplitude and frequency of excitation. Amplitude independent within operating range.</td>
<td></td>
</tr>
</tbody>
</table>

Prepared by the study of a one-dimensionally vibrating test bar's responses and followed by the viewing of mode patterns by time-average and sub-fringe holography in typical test parts, the experimental work of this dissertation was completed.
The following conclusions are based on the theoretical developments of Chapters III and IV and the experimental results of Chapter VI.

1. The measured light intensity variations of a time-average hologram of a sinusoidally-vibrating bar exposed under controlled conditions confirm that the modulation function for time-average holography follows a $J_0^2(c)$ response as predicted by Eq. [20].

2. Addition of a synchronized reference wave modulation extends the usefulness of time-average holography by rendering measurable the relative phase displacement variations on the test object. (See Chapter VI.)

3. Control of the relative vibrational phase angle between the object- and reference-modulator electrical drive signals permits adjustment of the holographic system's sensitivity. This flexibility allows observation of small object surface displacement which would go unnoticed otherwise. (See Plate 1)

4. The overall operating range of a reference beam phase modulated system extends from an upper surface displacement of about $8\lambda$ (time-average, noise limited intensity variations) down to about $(1/40)\lambda$ (sub-fringe, noise limited intensity variations). (See Table 5)
5. Holographic nondestructive testing is enhanced through the use of reference beam phase modulation since it:
   a) permits the placement of apparent nodal regions anywhere throughout the test piece.
   b) permits detection of low level surface displacements not observed using conventional time-average holography. (See Chapter VI)

6. The relative performance characteristics of dynamic sub-fringe holography have been shown analytically to depend upon a universal gain function, $\mu(c)$, which predicts the improvement in contrast, visibility and system gain due to reference beam phase modulation. (See Eq.[33]) The validity of this function follows from the agreement of measured reconstructed light intensities with the predicted intensities based on the $J_0^2(c)$ modulation function.

7. A straight-forward method is now available for translating test object surface vibrations into expected intensity variations for a one-dimensionally vibrating structure. (See Chapter IV) The method accounts for (1) reference beam phase biasing, (2) illumination-observation direction misalignments and (3) general scaling variation based on object displacement amplitude and wavelength of the illuminating radiation.

8. Deviations in expected results derived from scans of conjugate images are accounted for by (1) the nonuniform illumination of the test object, (2) the directivity of the diffusely-scattering object surface, (3) the flare light from the reconstructing laser,
(4) the scattering of light due to the emulsion and silver-grains of the hologram and (5) the general background illumination of the test laboratory. (See Eq. [45] and Table 3).

9. A laboratory technique of multi-image storage within single holograms has (1) made detection of subtle changes in fringe patterns possible, (2) assured equal development conditions for all of the exposures stored on the plates, and (3) made possible a cost reduction of 8 to 1 in the use of holographic plates. (See Fig. 15)

10. This dissertation has examined the fundamental concepts of temporal phase modulation in time-average holography. It has outlined the mathematical basis for (1) the variations in light intensity of the sub-fringe holographic image along a one-dimensionally vibrating bar, (2) the enhancement of contrast due to reference modulation, and (3) the improvement in system sensitivity to small surface displacements. It has been shown experimentally that these mathematically-predicted results, based on a correlation with the $J_0^2(c)$ function, are observed in the laboratory to within an experimental error of less than 20%.
APPENDIX A

NUMERICAL VALUES OF $J_0(c)$, $J_1(c)$, $J_0^2(c)$ AND $\mu(c)$, 

$0 \leq c \leq 5.2$

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<tr>
<th>$c$</th>
<th>$J_0(c)$</th>
<th>$J_1(c)$</th>
<th>$J_0^2(c)$</th>
<th>$\mu(c)$</th>
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<td>0</td>
<td>1.000</td>
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APPENDIX B

NUMERICAL VALUES OF $r_i$

\[ r_i = \frac{c_i}{C} \]

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APPENDIX C

NUMERICAL VALUES FOR VISIBILITY V(c)

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APPENDIX D

VISIBILITY PREDICTION NEAR NULL OF $J_o^2(c)$

The expression given for visibility in Eq. [39], uses an exact amplification point function, $\mu(c)$ (i.e., mathematically correct at each bias point $c$). This function, however, is multiplied by an incremental change in argument, $\Delta c$, which is taken as linear. Near $c = 2.4$, $\mu(c)$ is changing rapidly and this leads to a discrepancy between the visibility predicted by Eq. [39] and that of Eq. [40], based on the known bias point and the $\Delta c$ in question. Experimentally this difference is found to be a factor of two in the value of $c$ at which $V$ just becomes equal to unity. This factor of two arises from the following.

$J_o^2(c)$, in the vicinity of $c = 2.4$, very nearly follows a quadratic functional response, i.e., $J_o^2(c) = a(c - 2.4)^2$. Figure 29 shows this equivalence with normalization of the quadratic function having been made at $c = 2.0$. For a quadratic function, $y(x)$, a linearized equivalent of the function based upon the first two terms of a Taylor's series expansion will always yield an error of two in the prediction of the linear $y = 0$ intercept. This is shown in Figure 30.

For the quadratic form, $y = x^2$. $\frac{dy}{dx} = 2x$. At $x = x_o$, $y = y_o = x_o^2$, and $\frac{dy}{dx}\bigg|_{x=x_o} = 2x_o$. The form of the linear approximation is

$$y = (\frac{dy}{dx})x + y(o)$$

[48]
Fig. 29. Equivalence of $J_0^2(c)$ and $a(c-2.4)^2$ near $c = 2.4$. 

$$0.3125(c-2.4)^2$$
Fig. 30. Effect of Taylor Series approximation of $y = x^2$
by $y = (dy/dx)x + y_o$. 
At a specific point,

\[ y_o = (2x_o)x_o + y(o) \]  \[ [49] \]

and

\[ y_o = x_o^2 \]  \[ [50] \]

Solving both equations simultaneously for \( x_o^2 \) one finds

\[ x_o^2 = y_o = (y_o - y(o))/2 \]

or

\[ y(o) = -y_o \]  \[ [51] \]

Therefore the linear approximation to \( y = x^2 \) becomes

\[ y = (2x_o)x - y_o \]  \[ [52] \]

The x intercept occurs at \( y = 0 \). Using Eq. [50] it follows that

\[ x_{\text{intercept}} = y_o / 2x_o = x_o / 2 \]  \[ [53] \]

Thus all attempts to use linear approximations near the \( c = 2.4 \) point of a \( J_o^2(c) \) light intensity response result in discrepancies which are in error by a factor of two.
APPENDIX E

PERFORMANCE CHARACTERISTICS OF PHOTOMETER SYSTEM

The Gamma Scientific Digital Photometer described in Chapter IV was tested for general operating characteristics. These included the variation of dark current and rms noise level as a function of the accelerating potential on the photomultiplier tube (PMT). In addition, the spatial resolution of the scanning optical system was examined. Figure 31 shows the dark current response of the PMT for changes in the accelerating potential throughout its operating range. The uncompensated dark current increases rapidly with applied kilovoltage. This current can be compensated internally to remain, on the average, near zero for any chosen kilovoltage. However the rms noise of the dark current still increases with higher kilovoltage. The increase in rms noise is shown as the compensated curve in Fig. 31. It is evident from Fig. 31 that the presence of random noise does not become troublesome until the accelerating potential approaches 700 volts.

The resolution of the PMT scanning system was gaged by scanning through a single edge test pattern. The resulting PMT output is shown in Fig. 32. The bright part of the pattern (left) at an output of 70 mv drops to the dark region (10 mv) within 1 mm. The accelerating potential was set at 410 volts.
Fig. 31. Dark current response of the S-11 photomultiplier detector Model 2020-1 as a function of accelerating potential.
Fig. 32. Photomultiplier Scanning System single-edge resolution and noise effects at accelerating potential of 700 volts and maximum sensitivity. Single-edge trace at $V_{\text{PMT}} = 410$ volts, filter = slow.
The filtering action of the digital photometer was recorded for a period of 10 sec. in the three operating positions of slow, medium and fast. The results are shown in Fig. 32 as the inset. All curves are plotted to the same scale factors. The accelerating potential duplicated the scanning test conditions at 700 volts.
## APPENDIX F

**TIME-AVERAGE VISIBILITY**

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$^a_{I_{\text{min}}} = 0.020$

$^b_{I_{\text{max}}} = 1.000$
### SUB-FRINGE VISIBILITY

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\(^a_{I_{min}} = 0.020\)
BIBLIOGRAPHY


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BOOKS


