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TIME DEPENDENT DEFORMATION OF COHESIVE SOILS

D I S S E R T A T I O N

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

by


* * * * *

The Ohio State University
1973

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ACKNOWLEDGEMENTS

The author wishes to express his sincerest appreciation to his adviser, Dr. T.H. Wu, Professor of Civil Engineering, for his valuable advice, constant encouragement and constructive criticism during the course of this research and in the preparation of the thesis. Thanks are also due to Mr. T. Chang, Graduate Research Associate, for the help in the finite element computer program. The financial assistance made available to this research from The Ohio Department of Highways and the U.S. Bureau of Public Roads is gratefully acknowledged.

The author wishes to express his sincere appreciation to his wife for her patience and understanding throughout this study.
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INTRODUCTION

The stress-strain properties of cohesive soils have been the subject of considerable research in the recent years (6, 10, 24, 28, 30, 34, 47, 51, 52).* These properties are of fundamental importance in any attempt to predict long term deformations of earth structures and foundations.

The overall objective of this research is to investigate the creep behavior of cohesive soils. In a comprehensive analysis of creep behavior, both volumetric and shear strains should be considered. The stresses were resolved into isotropic and pure shear components. Creep behavior was first studied under these two simple conditions. Then, the creep behavior was investigated under more complicated stress conditions (undrained, general state of stress and zero lateral strain conditions). Triaxial tests and long-term one-dimensional consolidation tests were carried out.

Whenever possible, the behavior observed in laboratory tests should be compared with the performance of actual structures. In order to verify the developed techniques in predicting the long-term deformations of earth structures, the embankment and the

*Numbers in parentheses refer to references in Reference List.
cut in Project 33, Cuy. 77-8.69 and Cuy. 80-17.37, near Cleveland, Ohio, were chosen for this study. The deformations were computed using soil properties measured in the laboratory. Then the computed and measured deformations were compared.

Chapter I contains the analysis of the time dependent deformations of cohesive soils under different conditions of Loadings. Chapter II contains the viscoelastic characterization of the soil. The comparison between the measured and the computed deformations is presented in Chapter III. Details of the project are presented in the Appendix.
CHAPTER I

Analysis of the Stress-Strain Time Behavior of Cohesive Soils
I-1 Introduction

Time dependent deformation in soils may occur under a variety of conditions. These include hydrostatic or shear stresses or their combinations; drained or undrained loading; normally consolidated or over-consolidated soils; saturated or partially saturated soils. A considerable amount of research has been conducted on two types of stress-strain-time behavior: deformation with no volume change (also called creep) and secondary consolidation. In the common creep test a saturated soil specimen is subjected to a constant deviator stress with no drainage permitted. Secondary consolidation is commonly studied by the one-dimensional consolidation test. During secondary consolidation the excess-hydrostatic pore water pressure is nearly zero. Therefore, it also represents creep under approximately constant stress.

In a comprehensive analysis of time-dependent deformation both volumetric and shear strains should be considered. To develop stress-strain-time relationships in a form that can be readily applied in analytical work, the stress and strain tensors are separated into hydrostatic (isotropic) and deviatoric (shear) components. A stress field can be expressed by the stress tensor:
\[
\begin{bmatrix}
\sigma_{ij}
\end{bmatrix} = \begin{bmatrix}
\sigma_{11} & \sigma_{12} & \sigma_{13} \\
\sigma_{21} & \sigma_{22} & \sigma_{23} \\
\sigma_{31} & \sigma_{32} & \sigma_{33}
\end{bmatrix} \quad \text{(I. 1)}
\]

Where the normal stress components \((\sigma_{ij})\) are in terms of effective stresses. The stress tensor can be resolved into two components

\[
\begin{bmatrix}
\sigma_{ij}
\end{bmatrix} = \begin{bmatrix}
\sigma_{ij}\end{bmatrix}^\sigma + \begin{bmatrix}
\sigma_{ij}\end{bmatrix}^\tau \quad \text{(I. 2)}
\]

where

\[
\begin{bmatrix}
\sigma_{ij}\end{bmatrix}^\sigma = \begin{bmatrix}
\frac{(\sigma_{11} + \sigma_{22} + \sigma_{23})}{3} & 0 & 0 \\
0 & \frac{(\sigma_{11} + \sigma_{22} + \sigma_{33})}{3} & 0 \\
0 & 0 & \frac{(\sigma_{11} + \sigma_{22} + \sigma_{33})}{3}
\end{bmatrix} \quad \text{(I. 3)}
\]

and is called the hydrostatic (isotropic) stress tensor and

\[
\begin{bmatrix}
\sigma_{ij}\end{bmatrix}^\tau = \begin{bmatrix}
\frac{(2\sigma_{11} - \sigma_{22} - \sigma_{33})}{3} & \sigma_{12} & \sigma_{13} \\
\sigma_{21} & \frac{(2\sigma_{22} - \sigma_{11} - \sigma_{33})}{3} & \sigma_{23} \\
\sigma_{31} & \sigma_{32} & \frac{(2\sigma_{33} - \sigma_{11} - \sigma_{22})}{3}
\end{bmatrix} \quad \text{(I. 4)}
\]

and is called the deviatoric or shear stress tensor. In addition, the octahedral normal and octahedral shear stresses, defined as

\[
\sigma_0 = \frac{1}{3} \left( \sigma_{11} + \sigma_{22} + \sigma_{33} \right) \quad \text{(I. 5a)}
\]
\[ \tau_0 = \frac{1}{3} \left[ (\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6 (\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{13}^2) \right]^{\frac{1}{2}} \]  

(I. 5b)

are useful in data analysis.

For axial symmetry, \( \sigma_{11} = \sigma_z, \ \sigma_{22} = \sigma_{33} = \sigma_r \) and

\( \sigma_{12} = \sigma_{13} = \sigma_{23} = 0 \), Then Equations (I. 3) and (I. 4) can be simplified to:

\[
\begin{bmatrix}
\sigma_{11}
\end{bmatrix}
\begin{bmatrix}
\sigma
\end{bmatrix} =
\begin{bmatrix}
\frac{(\sigma_z + 2\sigma_r)}{3} & 0 & 0 \\
0 & \frac{(\sigma_z + 2\sigma_r)}{3} & 0 \\
0 & 0 & \frac{(\sigma_z + 2\sigma_r)}{3}
\end{bmatrix}
\]  

(I. 6)

and

\[
\begin{bmatrix}
\sigma_{11}
\end{bmatrix}
\begin{bmatrix}
\tau
\end{bmatrix} =
\begin{bmatrix}
\frac{2}{3} (\sigma_z - \sigma_r) & 0 & 0 \\
0 & \frac{2}{3} (\sigma_r - \sigma_z) & 0 \\
0 & 0 & \frac{2}{3} (\sigma_r - \sigma_z)
\end{bmatrix}
\]  

(I. 7)

and

\[
\sigma_0 = \frac{1}{3} (\sigma_z + 2\sigma_r) \]  

(I. 8)

\[
\tau_0 = \sqrt{\frac{2}{3}} (\sigma_z - \sigma_r) = \frac{\sqrt{6}}{3} \sigma_d \]  

(I. 9)
Similarly the strain tensor:

\[
[\varepsilon_{ij}] = \begin{bmatrix}
\varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\
\varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\
\varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33}
\end{bmatrix}
\] (I. 10)

can be resolved into two components:

\[
[\varepsilon_{ij}] = [\varepsilon_{ij}]^v + [\varepsilon_{ij}]^d
\] (I. 11)

where

\[
[\varepsilon_{ij}]^v = \begin{bmatrix}
\frac{(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33})}{3} & 0 & 0 \\
0 & \frac{(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33})}{3} & 0 \\
0 & 0 & \frac{(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33})}{3}
\end{bmatrix}
\] (I. 12)

and is called the spherical strain tensor and

\[
[\varepsilon_{ij}]^d = \begin{bmatrix}
\frac{(2\varepsilon_{11} - \varepsilon_{22} - \varepsilon_{33})}{3} & \varepsilon_{12} & \varepsilon_{13} \\
\varepsilon_{21} & \frac{(2\varepsilon_{22} - \varepsilon_{11} - \varepsilon_{33})}{3} & \varepsilon_{23} \\
\varepsilon_{31} & \varepsilon_{32} & \frac{(2\varepsilon_{33} - \varepsilon_{22} - \varepsilon_{11})}{3}
\end{bmatrix}
\] (I. 13)

and is called the deviatoric strain tensor.
In terms of the three principal strains $\varepsilon_1$, $\varepsilon_2$ and $\varepsilon_3$ the octahedral normal and octahedral shear strains are defined as:

$$\varepsilon_0 = \frac{1}{3} (\varepsilon_1 + \varepsilon_2 + \varepsilon_3) = \frac{\varepsilon_y}{3} \quad (I. 14)$$

$$\gamma_0 = \frac{2}{3} \left[ (\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_1 - \varepsilon_3)^2 + (\varepsilon_2 - \varepsilon_3)^2 \right]^{\frac{1}{2}} \quad (I. 15)$$

For axial symmetry $\varepsilon_1 = \varepsilon_2$ and $\varepsilon_2 = \varepsilon_3 = \varepsilon_r$, then Equation (I. 15) is simplified to:

$$\gamma_0 = \frac{2\sqrt{2}}{3} (\varepsilon_2 - \varepsilon_r) = \frac{2\sqrt{2}}{3} \varepsilon_d \quad (I. 16)$$

For convenience, the deviator stress $\sigma_d$ and the deviator strain $\varepsilon_d$ are used in this thesis.
I-2 Review of Creep Behavior

Rheological models composed of spring and dashpot combinations have been extensively used to characterize the time-dependent deformation of clay (3, 10, 32, 33, 34, 30). Models commonly used are shown in Figure 1.1. To simulate creep and secondary consolidation over a wide range of stress conditions, the viscosity of the flow element is generally treated as a nonlinear function of stress. Barden (3) used a nonlinear dashpot characterized by a power law to describe the settlement due to secondary compression.

Murayama and Shibata (33) are the first to use the rate process theory (Glasstone, Laidler and Eyring, 19) for modeling the viscous flow. The flow rate may be expressed as (Christensen and Wu, 10 and Wu et al., 52):

$$\dot{\varepsilon}_f = \beta \sinh (\alpha \sigma)$$  \hspace{1cm} (I.17)

where $\dot{\varepsilon}_f$ if the flow rate of the viscous element under a stress $\sigma$ and $\alpha$ and $\beta$ are model parameters that represent the flow characteristics. Under a constant stress the creep of the model shown in Figure 1.1 may be expressed as:

$$\varepsilon = F(t)$$  \hspace{1cm} (I.18a)

where $\dot{\varepsilon}$ is a dimensionless strain and $t$ is a dimensionless time
defined by

\[ \varepsilon^* = \frac{\varepsilon - \varepsilon_0}{\varepsilon_{\infty} - \varepsilon_0} \]  
(I. 18b)

\[ \varepsilon^* = \frac{1}{2} \alpha \beta \frac{k_1}{k_1 + k_2} t \]  
(I. 18c)

For different values of \( \alpha, \beta, K_1, \) and \( K_2, \) Equation (I. 17) may be plotted as shown in Figure 1.2.

Other studies of creep as a rate process have been published by Mitchell (29), Mitchell and Campanella (30) and Mitchell et al. (31). However, the practical utility of the expressions remains doubtful, since there is not direct and simple way of accurately evaluating the various parameters involved.

A phenomenological approach was adopted by Singh and Mitchell (46) who found that the logarithm of the strain rate in the undrained creep test varies linearly with the logarithm of time according to the Equation:

\[ \dot{\varepsilon} = At^{-m} \]  
(I. 19)

in which \( \dot{\varepsilon} \) is the strain rate at any time \( t \) and \( A \) and \( m \) are constants. Most of \( \dot{\varepsilon} \) versus time plots examined in Singh and Mitchell paper extends only over a limited time interval of about 1000 minutes. The question remains whether the value of \( m \) remains constant over long periods of time.
Equation (I. 19) indicates that the strain increases as time increases and approaches an asymptote at \( t = \infty \). Integration of Equation (I. 19) yields for the case, \( m = 1 \)

\[
\varepsilon = \frac{A}{1-m} t^{1-m} + c
\]

(I. 20)

where \( c \) is the constant of integration.

For \( m < 1 \), let \((1-m) = n\), then \( n > 0 \). The constant of integration \( c \) may be obtained from the condition: at \( t = 0 \)

Then Equation (I. 20) becomes

\[
\varepsilon_1 = \varepsilon_0 + \frac{A}{n} t^n
\]

(I. 21a)

For \( m < 1 \), let \((1-m) = -n\), then \( n > 0 \). The constant \( c \) can be obtained from the condition: at \( t = \infty \) \( \varepsilon = \varepsilon_\infty \). Then Equation (I. 19) becomes

\[
\varepsilon_1 = \varepsilon_\infty - \frac{A}{n} t^n
\]

(I. 21b)

For the case \( m = 1 \), integration of Equation (I. 18) gives

\[
\varepsilon = \varepsilon_1 + A \ln t
\]

(I. 21c)

where \( \varepsilon_1 \) is the strain at one minute.

Examination of Equations (I. 21) shows that only for the case \( m = 1 \), does the strain vary linearly with the logarithm of time. The strain versus log time curves for the three cases \( m < 1, m = 1, \) and
m > 1 are shown as curves A, B and C in Figure (1.3).

Perloff (37) expressed the shear strain versus time relation as,

$$\epsilon_1 - \epsilon_3 = c t^n$$  \hspace{1cm} (I. 22)

where

\(\epsilon_1 = \text{axial strain}\)

\(\epsilon_3 = \text{radial strain}\)

\(c, n = \text{constants}\)

He also suggested that \(n\) is a constant, irrespective of the stress level, confining pressure and moisture content and \(c\) is a function of the ratio of the stress level to the failure stress. This is essentially the same Equation (I. 21a). If \(m\) is smaller than 1.0 the strain-time relationship is linear on the log-log plot.
I. 3 Experimental Work

I. 3.1 Introduction

In the present investigation, the stress-strain-time behavior was studied under the conditions of increments of hydrostatic stress, drained; increments of pure shear stress, drained; increments of axial stress, undrained; and zero lateral strain.

The first three conditions were studied by means of triaxial tests. The behavior under the fourth condition was studied by performing long term on-dimensional consolidation tests.

I. 3.2 The Soils Used

To provide a wide range in clay types, four types of soils were used in the tests. The soils and their index properties are listed in Table 1.1. The use of remoulded Grundite offers the advantage that the variations between individual specimens would be small and a large number of variables could be studied. The other three clays are included to provide a comparison between the behaviors of remoulded and undisturbed specimens. The soils are described in the following paragraphs.

Grundite Clay: This is an illite clay known by the trade name Grundite. It is mined at Goose Lake of Grundy County, Illinois, and is marketed by the Illinois Clay Products Company.
A batch of clay was mixed with double distilled water to achieve a water content of about 49%. The soil was then put into plastic bags and stored in a humid room for at least one month before testing. Hand moulded specimens were used for the tests.

**Maine Clay:** Undisturbed samples of a silty clay were obtained from the site of the reconstruction of route I-77 near Cape Elizabeth, Maine.

The natural water content of this clay is equal to or higher than its liquid limit. The clay is a normally consolidated, sensitive clay and its vane shear strength is of the order of 0.1 to 0.15 tons per square foot.

**Nevada Clay:** Undisturbed samples of this clay were obtained from the Gelack area in Nevada near the Oregon border. It is normally consolidated and the sensitivity of the clay is greater than 4.0. According to the Chemical Engineering Branch, U.S.B.R., Denver, Colorado, the clay material consists mainly of poorly crystallized montmorillonite type clay (probably nontronite) with moderate amounts of illite and some kaolinite.

**Cleveland Clay:** The undisturbed samples were obtained from the site of the I77-I80 interchange at Independence, Ohio. The subsoil consists of over-consolidated glacial lake deposits which include a thick layer of "varved clay". The average preconsolidation
pressure ranges from 2.5 to 3.5 kg/cm². The varved clay is a gray silty clay with silt and fine sand laminations. The distance between the silt and fine sand laminations varied from a small fraction of an inch to three inches so that some triaxial specimens do not contain any silt and fine sand laminations. The samples described in this section are all taken from the varved clay layer.
### Table (1.1)

**Index Properties of the Soils Used**

<table>
<thead>
<tr>
<th>Clay</th>
<th>L. L. (%)</th>
<th>P. L. (%)</th>
<th>W%</th>
<th>Pc kg/cm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grundite</td>
<td>54</td>
<td>26</td>
<td>46-49</td>
<td>Remoulded</td>
</tr>
<tr>
<td>Maine</td>
<td>35-40</td>
<td>18-20</td>
<td>35-40</td>
<td>Normally</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Consolidated</td>
</tr>
<tr>
<td>Nevada</td>
<td>110-140</td>
<td>50-60</td>
<td>95-100</td>
<td>Normally</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Consolidated</td>
</tr>
<tr>
<td>Cleveland</td>
<td>30-45</td>
<td>10-25</td>
<td>24-32</td>
<td>Preconsolidated</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.5-3.5</td>
</tr>
</tbody>
</table>
I.3.3 Triaxial Tests

The triaxial tests were carried out in the standard Geonor triaxial cells with rotating bushings. The stress increments were applied by means of dead loads. The specimens were 1.4 inches in diameter and 2.8 inches long.

The initial consolidation and loading conditions used in the triaxial tests are shown diagramatically in Figure 1.4. In case of the Grundite specimens, the hand moulded specimens had very low strength. To simulate in situ conditions, three types of initial consolidation were used: isotropic consolidation to \( \sigma_1 = \sigma_3 = \sigma_0 \), anisotropic consolidation to \( \sigma_1 + 2\sigma_3 = 3\sigma_0 \) and isotropic preconsolidation to \( \sigma_1 = \sigma_3 = 2\sigma_0 \) followed by isotropic unloading to \( \sigma_1 = \sigma_3 = \sigma_0 \). The three series of tests are denoted by the letter symbols I, K and P, respectively. These three types of initial consolidation are shown diagramatically in Figure 1.4a. The other soils used were undisturbed and isotropic stresses were used for initial consolidation.

After initial consolidation increments of \( \sigma_0 \) were applied by increasing the hydrostatic pressure in the triaxial cell as for isotropic consolidation. Increments of \( \tau_0 \) were applied by changing the axial and radial stresses such that the octahedral normal stress is kept constant during the test. This condition may be
obtained by increasing the axial stresses and decreasing the radial stresses (compression) or by increasing the radial stresses and decreasing the axial stresses (extension). These loading conditions are shown in Figure 1.4b. The three cases are denoted by the letter symbols I, OC and OE, respectively. Thus, an anisotropically consolidated specimen subjected to an extension shear would have the designation K-OE. All I, OC and OE tests were carried out in the drained condition. In undrained tests, designated as CU, the axial stress was increased while the radial stress was kept constant. For the series EU the axial stress was kept constant while the radial stress was increased. These two cases are shown in Figure 1.4c.

The stress state in the undrained test is analyzed in Figure 1.5. In order to have the effective stresses of the undrained test equal to those in a constant octahedral normal stress test, it is necessary that:

\[ \Delta \sigma_z + 2 \Delta \sigma_r = 0 \]  \hspace{1cm} (1.23)

\[ \Delta u_0 = \frac{1}{2} (\Delta \sigma_z - \Delta \sigma_r) - \Delta u_a \]

or

\[ \Delta u_0 = \frac{1}{3} (\Delta \sigma_z - \Delta \sigma_r) \]  \hspace{1cm} (1.24)

This implies that the pore water pressure parameter \( A \) must equal to \( 1/3 \).
Stress path tests were also carried out. In the stress path test, the soil sample was consolidated anisotropically under the condition of no lateral deformation ($K_0$ consolidation). After initial consolidation the sample was loaded in increments to failure according to the particular stress paths chosen.
The one-dimensional consolidation test is a simple and popular laboratory test for the study of the long term deformation of clays. The stresses during the consolidation test are shown in Figure 1.6. It is commonly assumed that the ratio $k_0 = \sigma_r / \sigma_z$ is constant during primary consolidation. This is the period where large volumetric strains are taking place and creep (or secondary consolidation) is relatively unimportant. However, after the end of primary consolidation the axial strain continues to increase (secondary consolidation). Long term consolidation tests were carried out on different types of soils in order to study the strain-time behavior during secondary consolidation.
I. 4 Results

I. 4. 1 Volumetric Deformation versus Time Relationship

Upon the application of a hydrostatic stress to a soil sample in a drained condition, deformation occurs in the forms of volumetric strain \( (\varepsilon^\nu) \) and shear strain \( (\varepsilon^\sigma) \). For an isotropic material the shear deformation is theoretically equal to zero. Any deviatoric strain that occurs in a specimen subjected to an isotropic stress field may be attributed to the anisotropy in either the material or the stress condition in the soil specimen (DeJong and Verwijst, 11).

Grundite Clay: Volumetric strain versus time curves plotted on the semi-log scales are presented in Figures 1.7, 1.8 and 1.9 for isotropically consolidated (I-I), anisotropically consolidated (K-I) and over consolidated (P-I) specimens, respectively. The stress increment ratio \( R_I \) indicated on each curve is defined by:

\[
R_I = \frac{\Delta \sigma_0}{\sigma_{0i}} \tag{I. 25}
\]

where \( \Delta \sigma_0 \) denotes the stress increment and \( \sigma_{0i} \) denotes the initial consolidation stress.

The data shown in Figures 1.7 and 1.8 indicate that the volumetric strain versus time relationship is strongly affected by the stress increment ratio \( R_I \) for isotropically consolidated and anisotropically consolidated specimens.
It is rather difficult to determine the end of primary consolidation by the graphic construction for tests with very small values of \( R_j \), even though primary consolidation should be complete before 24 hours. The end of primary consolidation was checked by measurement of the pore pressure. Figure 1.10 shows the pore water pressure measured at the middle of isotropically consolidated specimen. Very little pore pressure remained after 800 minutes. The end of primary consolidation is marked on the curves.

Following the phenomenological approach of Singh and Mitchell, (46) the strain rate \( \dot{\varepsilon} \) obtained from isotropic consolidation tests is plotted against time on the logarithmic scale in Figure 1.11. The curves represent the general behavior of normally consolidated soils. The end of primary consolidation is also marked on the curves. This figure indicates that for values of \( R_j \) less than 0.2, the slope \( m \) during creep is smaller than 1.0. It is also noticed that \( m \) does not change very much with time and it may be considered constant within the duration of the tests. It also indicates that the slope \( m \) is greater than 1.0 if \( R_j \) is 1.0 or larger. In between is a transition zone where \( m \) is approximately 1.

On the other hand the volumetric strain-time and the volumetric strain rate-time curves for preconsolidated samples (Figures 1.9 and 1.12), indicate that the slope \( m \) may be considered constant and
its value does not differ very much from 1.0 for all values of $R_I$. The steep slope at around 1000 minutes corresponds to the curvature in the time consolidation curves, near 90% consolidation (Figure 1.9).

The semi-log plot has been widely used to present time-consolidation curves and such plots facilitate comparison with published data. To study the time deformation behavior, the log $\varepsilon$ - log $t$ plot offers some advantages. The data in Figure 1.7 are plotted on the log $\varepsilon$ - log $t$ scale in Figure 1.13. The end of primary consolidation is also marked on the curves. For times beyond the completion of primary consolidation, the data may be satisfactorily represented by a linear relationship although $m$ is not always less than 1.0. Thus the log $\varepsilon$ - log $t$ plot may serve as a reasonable approximation for all the tests of this series.

**Maine Clay:** The data obtained from (I-I) tests are plotted on the semi-log scale in Figures 1.14. The data are in agreement with these for normally consolidated Grundite in that the curves show $m > 1$ for large values of $R_I$ and $m < 1$ for small values of $R_I$.

**Nevada Clay:** The data obtained from (I-I) tests on Nevada Clay are plotted on the semi-log scale in Figure 1.15. The data are in agreement with those for the Main Clay and the normally consolidated Grundite.
Cleveland Soil: Volumetric strain-time and volumetric strain-rate-time curves obtained from (I-I) tests are shown in Figures 1.16 and 1.17. The data indicate that during creep the slope $m$ does not differ very much from 1.0. It may also be noted that the strain rate during creep is nearly the same for all values of $R_I$ shown in Figure 1.17. The curves are similar to those of (P-I) tests on Grundite.

The log$\sigma_y$ - log $t$ plots for Maine, Nevada and Cleveland clays are similar to those of Grundite and they are not shown here.

Under isotropic stresses, shear strain is observed in all the tests described above. However, its magnitude is much smaller than the other component and is not shown.

The above results indicate that the magnitude of $m$ is not the same for all types of soils tested. For normally consolidated clays, the value of $m$ is strongly affected by the stress ratio $R_I$. For small values of $R_I$, the slope $m$ is less than 1.0 and $m$ increases slightly with time. For values of $R_I > 0.4$, the slope $m$ is approximately constant and for very large stress increment ratios, $R_I > 1.0$, $m$ exceeds 1.0. Values of $m$ for different values of $R_I$ for normally consolidated Grundite are listed in Table 1.2. For over-consolidated clays, the value of $m$ may be taken equal to 1.0 for all values of $R_I$. However, in spite of the fact that $m$ ranges from 0.75 to 2.10, the data deviates very little from a linear relationship between log $t$ and log $\sigma_y$. Hence, this comparatively simple relation is considered satisfactory.
Table 1.2

Values of $m$ for Normally Consolidated Soils

<table>
<thead>
<tr>
<th>Stress Ratio $R_I$</th>
<th>0.2</th>
<th>0.4</th>
<th>1.0</th>
<th>3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>0.75</td>
<td>1.0</td>
<td>1.0</td>
<td>2.10</td>
</tr>
</tbody>
</table>
I. 4. 2 Shear Deformation Time Characteristics

Typical shear strain versus time curves obtained from IOC tests on Grundite, Nevada and Cleveland clays are plotted on the semi-log scale in Figures 1.18, 1.19, 1.20 and 1.21. The stress ratio $R_T$ is defined by:

$$R_T = \frac{\sigma_d}{\sigma_{df}}$$  \hspace{1cm} (I. 26)

where $\sigma_d$ is the deviator stress and $\sigma_{df}$ is the deviator stress at failure.

The data show a noticeable departure from the linear relationship during the first 500 minutes. The data indicate that for values of $R_T$ larger than 0.2 to 0.3 and for large values of time, the shear strain-time relationship may be approximated by a linear relationship on the semi-log scale. In other words, $m$ approaches 1.0 for large values of time. The data from (IOC) tests on Grundite and Cleveland clays are plotted on the log $\varepsilon_d$ - log $t$ scale in Figure 1.22. These represent the general behavior of the (IOC) tests. The data indicate that $m$ changes considerably during the interval $1 < t < 500$ min.

This indicates that the method of Singh and Mitchell (1968) in which the test duration is 100 minutes or less, may lead to appreciable error if extrapolation is attempted.
This strain-time relationship may be expressed as:

\[ \epsilon_d = A_1 + B_1 \log(t-t_0) \]  

(I.27)

in which

- \( \epsilon_d \) is the deviator strain at any time \( t \), \( t_0 \).
- \( t_0 \) is the time after which the strain-log time curve approaches a straight line,
- \( A_1 \) is the deviator strain for \( t - t_0 = 1.0 \),
- and \( B_1 \) is a constant.

Previous studies on Grundite, Wu (53), showed that \( B \) is strongly affected by the stress ratio \( R_T \). Their data indicate that \( B \) increases as the stress ratio increases.

To explore alternative methods of representation, the data in Figures 1.18 through 1.21 are plotted as \( \log \epsilon_d - \log t \) in Figures 1.23 through 1.25. Except for stress levels approaching failure stresses the linear representation appears satisfactory.

The undrained triaxial tests (IUC) have been widely used for measurement of shear creep. As shown in Section 3.3, if \( A = 1/3 \), then the undrained test is equivalent to the IOC test. In the present study the values of \( A \) for Grundite ranges from 0.57 to 1.06.

Nevertheless, it is of practical interest to compare the results of the
two series of tests. The creep curves for IUC tests on Grundite and Nevada clays are shown in Figures 1.26, 1.27 and 1.28.

It can be seen from these figures that the shear strain-time relationship may be approximated by straight lines on the logε - log t plots. The slope of the lines does not change too much with the shear stress level. However, it is noticed that the slope for the first load increment is sometimes considerably larger than the slope for subsequent load increments. The physical implications of this are not clear. However, Prerloff (37) observed the same phenomena in his tests on compacted clay and he found that cycling the first increment of loading may eliminate this. This may indicate that probably there exist some seating effects during the first load increment.

The linear relationship on the log ε - log t plot implies that the strain-time relationship may be expressed by:

$$ε_1 - ε_3 = c t^n$$  \hspace{1cm} (I.28)

where

ε₁ is the axial strain,

ε₃ is the radial strain.

The constant n is the slope of the lines and the constant c is the shear strain at one minute.
The data obtained from IOC and IUC tests on Grundite and from IOC tests on Cleveland clay indicate that the value of \( n \) does not change too much with shear stress level and may be taken as a constant. On the other hand \( c \) is strongly affected by the shear stress level. Tests on Grundite (see Figures 1.26 and 1.27) also show that \( n \) is not affected by the sequence of loading. It remains about the same whether the shear stress is applied in one or two increments. Thus, Equation 1.28 is valid for the drained, as well as, undrained creep, and for single, as well as, multi-increment loading.

Figure 1.29 shows the relationship between the creep parameters \( c \) and \( n \) and the stress ratio \( R_T \) for IUC and IUE tests on Grundite clay and Figure 1.30 shows the same plot for IOC tests on Grundite clay. These figures indicate that the effect of confining pressure is accounted for if \( c \) is expressed as a function of \( R_T \). The results of the tests using single increment loading are in good agreement with those using two increment loading. However, the \( c \) versus \( R_T \) curve for IOC tests differs considerably from that for IUC tests.

The creep parameters \( c \) and \( n \) obtained from IOC tests on Cleveland soil are plotted in Figure 1.31 as a function of the stress ratio \( R_T \). It can be seen that for different values of confining pressure \( \sigma_0 \) and different initial water contents, the data can be
described approximately by a single curve. The scatter in the data can be traced to variations in the initial water content. The variations in the water content reflect primarily the variation in the clay content. Hence the scatter may be considered as reflection of natural variations in the subsoil. The data on Grundite (Figure 1.30) do not show this scatter because the initial water content is almost the same in all samples.

The stress paths used in the stress path tests on Cleveland clay are shown in Figures 1.32 and 1.33. These approximate the changes in \( \sigma_1 \) and \( \sigma_3 \) in the slope adjacent to the cut. The rotation in the principal axes is not considered. The measured strains after 24 hours of loading are indicated by the numerals.

The deviator strain versus time curves obtained from the stress path tests are plotted on the logarithmic scales in Figure 1.34. The values of the principal strain difference at one minute obtained from these tests are shown in Figure 1.31. The data are in general agreement with the data obtained from IOC tests.
I. 4. 3 One Dimensional Consolidation Tests

One dimensional consolidation tests are commonly used to measure compressibility. Hence a large amount of published information is available. Furthermore, the simplicity of this test makes it possible to perform tests of long durations. Since volumetric strain is predominant in these tests, it is reasonable to compare the results with those of the I-I tests. In the data interpretation we use a load increment ratio defined by:

\[ R_c = \frac{\Delta p}{p} \]  

(I. 29)
in which \( p \) is the vertical stress on the soil sample before applying the stress increment \( \Delta p \).

The vertical deformation versus log time curves obtained from one dimensional consolidation tests are presented in Figures 1.35 through 1.37 for Maine clay and in Figures 1.38 through 1.40 for Nevada clay and in Figures 1.41 through 1.43 for Cleveland clay.

In general the pattern is similar to that of the (I-I) tests. The shape of the time consolidation curves is strongly affected by the stress increment ratio \( R_c \) and the magnitude of the vertical stress relative to the preconsolidation pressure \( p_c \). For stresses below \( p_c \) and load increment ratios greater than 0.4, the slope \( m \) during secondary consolidation is equal or greater than 1.0.

For values of \( R_c \), greater than 0.4, and vertical stresses above \( p_c \),
$m$ is equal to 1.0. For all values of $R_c$ below 0.3, $m$ is smaller than 1.0 irrespective of the stress level.

However, the data obtained from a long term test, Figure 1.41, show that beyond $10^4$ minutes the shape of the curve indicates that $m < 1$. At the end of the test it was found that the cadmium coating on the inside wall of the consolidation ring was stuck to the soil sample. In two other long-term tests in which stainless steel rings were used, brown spots on the rings were noticed at the end of the tests. In a fourth long-term test, a change in the shape of the time consolidation curve was noticed after a period of time of 12,000 minutes. An increase in room temperature was recorded at about the same time due to a malfunction in the temperature control system. The above incidents indicate that long-term tests may suffer from factors such as sample disturbance, temperature changes and electrochemical reactions. The extent to which these factors influence our results has not been evaluated.

Comparing the strain-time curves from one dimensional consolidation tests with those obtained from (I-I) tests and IOC or IOE tests, (for example Figure 1.44), we can see that the slope $n$ of the curve for one dimensional consolidation tests is larger than those for (IOC) and (I-I) tests on the same material. The stresses during one dimensional consolidation tests are shown in Figure 1.6.
It is noted in Section 3.4 that during primary consolidation the ratio \( K_0 = \frac{\bar{\sigma}_r}{\bar{\sigma}_z} \) is assumed constant. However, after the end of primary consolidation, the axial strain continues to increase.

Consider the results of an (I-K) test on Grundite shown in Figure 1.45. In the (I-K) test, the soil sample was consolidated isotropically to some pressure \( \sigma_0 \), then the cell pressure was increased and the axial stress was adjusted continuously to maintain a constant cross-sectional area of the specimen. In Figure 1.45 the strain-time curve and the \((\sigma_z - \sigma_r)\)-time curve for the same load increments are shown. It can be seen that to maintain the condition of \( \varepsilon_r = 0 \) it was necessary to increase \((\sigma_z - \sigma_r)\) after the end of primary consolidation. This indicates that \( K_0 \) decreases as time increases. This holds for normally consolidated clays. However, it was reported by T. K. Tan (47) that \( K_0 \) would increase with time for over-consolidated clays.

The preceding analysis shows that the stresses in one-dimensional consolidation tests are not constant. Thus, the measured strain-rates may not represent those for the condition of constant stress.

To calculate the time dependent deformation of earth structures, the creep data should be expressed by a mathematical formulation commonly called constitutive equations. This is the subject of the following chapter.
Figure 1.1 Rheological Model for Clays.

\[ D = \text{Principal Stress Difference} = \sigma_0 \]

\[ \epsilon^* = 1 + \frac{1}{A} \ln \tanh [t^* + \tanh^{-1}(-A)] \]

\[ A = \frac{\sqrt{2}}{3} \alpha D \frac{k_1}{k_1 + k_2} \]

D = Principal Stress Difference = \(\sigma_d\)

Figure 1.2 Theoretical Creep Curves

(Christensen and Wu, 1964)
Figure 1.3 Strain-Log Time Curves
Figure 1.4a Consolidation

Isotropic

Anisotropic

Preconsolidation

Figure 1.4b Drained Loading

Drained Isotropic

Drained Shear Compression

Drained Shear Extension

Figure 1.4c Undrained Loading

Undrained Creep Compression

Undrained Creep Extension

Figure 1.4 Loading Condition in the Triaxial Test
\[ \Delta u = \Delta u_a + \Delta u_b = \Delta u_a + \Delta \sigma_r \]

\[ \Delta \sigma_z^2 = \Delta \sigma_z - \Delta u = (\Delta \sigma_z - \Delta \sigma_f) - \Delta u_0 \]

\[ \Delta \sigma_f = \Delta \sigma_f - \Delta u = \Delta \sigma_f - \Delta \sigma_f - \Delta u_0 \]

\[ \Delta \sigma_r = -\Delta u_0 \]

\( \Delta u \) = Total Pore Pressure
\( \Delta u_a \) = Pore Pressure Due to Deviatoric Stresses
\( \Delta u_b \) = Pore Pressure Due to Isotropic Stresses
\( \Delta \sigma_r \) = Pore Pressure Parameters

A and B are Skempton's Pore Pressure Parameters

Figure 1.5 Stress State in the Undrained Test
\[ \Delta \sigma_2 = \Delta \sigma_r, \Delta \sigma_2^0 = 0 \]

\[ \Delta \sigma_2^r = \Delta \sigma_2 - u \]

\[ \Delta \sigma_2 \rightarrow \Delta \sigma_2 \]

(a) \( t = 0 \)

(b) \( t < t_{90} \)

(c) \( \rightarrow t_{90} \)

\[ \Delta \sigma_2 = \text{Axial Stress Increment} \]
\[ \Delta \sigma_r = \text{Radial Stress Increment} \]
\[ u = \text{Pore Water Pressure} \]
\[ u_1 = \text{Initial Pore Water Pressure at } t = 0 \]
\[ t = \text{Any Time} \]
\[ t_{90} = \text{Time of 90\% Consolidation} \]

Figure 1.6 Stresses in the One-Dimensional Consolidation Test
Grundite Clay

I - I Tests

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Stress Increment</th>
<th>$R_1$</th>
<th>$\omega_1$ %</th>
</tr>
</thead>
<tbody>
<tr>
<td>327</td>
<td>1.66-5.0 ksc</td>
<td>3.0</td>
<td>46.16</td>
</tr>
<tr>
<td>501</td>
<td>2.5 - 5.0</td>
<td>1.0</td>
<td>49.28</td>
</tr>
<tr>
<td>318-1</td>
<td>2.5 - 3.50</td>
<td>0.4</td>
<td>48.44</td>
</tr>
<tr>
<td>319-5</td>
<td>4.5 - 5.0</td>
<td>0.1</td>
<td>48.08</td>
</tr>
<tr>
<td>319-1</td>
<td>2.5 - 3.0</td>
<td>0.2</td>
<td>48.08</td>
</tr>
</tbody>
</table>

End of Primary Consolidation

Figure 1.7 $\epsilon_\sigma$ versus Log t Curves: (I-I) Tests on Grundite
### Grundite Clay K-I Tests

<table>
<thead>
<tr>
<th>Test #</th>
<th>Stress Increment</th>
<th>$R_I$</th>
<th>$\omega_I$ %</th>
</tr>
</thead>
<tbody>
<tr>
<td>#425</td>
<td>2.5 - 5.0 ksc</td>
<td>1.0</td>
<td>45.27</td>
</tr>
<tr>
<td>#424-2</td>
<td>3.5 - 5.0</td>
<td>0.42</td>
<td>45.56</td>
</tr>
<tr>
<td>#417-1</td>
<td>2.5 - 3.05</td>
<td>0.22</td>
<td>46.92</td>
</tr>
<tr>
<td>#417-2</td>
<td>3.05 - 3.55</td>
<td>0.165</td>
<td>46.92</td>
</tr>
<tr>
<td>#417-3</td>
<td>3.55 - 4.05</td>
<td>0.14</td>
<td>46.92</td>
</tr>
<tr>
<td>#417-4</td>
<td>4.05 - 4.56</td>
<td>0.126</td>
<td>46.92</td>
</tr>
<tr>
<td>#417-5</td>
<td>4.56 - 5.06</td>
<td>0.11</td>
<td>46.92</td>
</tr>
</tbody>
</table>

End of Primary Consolidation

### Figure 1.8

$\varepsilon^\sigma_v$ versus Log $t$ Curves: (K-I) Tests on Grundite
Grundite Clay
P-I Tests.

<table>
<thead>
<tr>
<th>Test #</th>
<th>Stress Increment</th>
<th>( R_t )</th>
<th>( \omega_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>516-1</td>
<td>2.5-3.0 ksc</td>
<td>0.2</td>
<td>41.44</td>
</tr>
<tr>
<td>502-2</td>
<td>3.50-5.0</td>
<td>0.43</td>
<td>47.27</td>
</tr>
<tr>
<td>514</td>
<td>2.50-5.0</td>
<td>1.0</td>
<td>47.18</td>
</tr>
</tbody>
</table>

End of Primary Consolidation

Figure 1.9 \( \varepsilon_{\sigma} \) versus Log t Curves: (P-I) Tests on Grundite
Figure 1.10 Pore-Pressure Dissipation During Consolidation
Figure 1.11 Log $\dot{\varepsilon}_v^p$ versus Log $t$ Curves: (I-I) Tests on Grundite Clay

Grundite Clay
Test #  $R_x$
- 327  3.0
- 425  1.0
- 316-1  0.4
- 319-1  0.2

End of Primary Consolidation
Figure 1.12 Log $\varepsilon^\sigma$ versus Log t Curves: (P-I) Tests on Grundite Clay

Test # | $R_\lambda$
---|---
514 | 1.0
502-2 | 0.43
516-1 | 0.2

End of Primary Consolidation
Figure 1.13 Log $\varepsilon_\sigma$ versus Log $t$ Curves: (I-I) Tests on Grundite Clay
1.08 — O
Maine Clay
Test # 706 (I-I)
\( \omega_i = 34\% \)
Stress Increment
\( R_z \)
- 0.3 - 0.625 ksc 1.08
- 0.625 - 1.25 1.0
- 1.25 - 1.567 0.25
- End of Primary Consolidation

Figure 1.14 \( \varepsilon_V^\sigma \) versus Log t Curves: (I-I) Test on Maine Clay
Figure 1.15 $\varepsilon_v$ versus Log $t$ Curves: (I-I) Test on Nevada Clay
Figure 1.16 $\epsilon^\sigma$ versus Log $t$ Curves: (I-I) Test on Cleveland Clay

Cleveland Clay
Test No. 2 II I (I-I)
$\omega_1 = 26.16$

Stress Increment : $R_I$

- $\Delta$ 2.0 - 2.25 ksc 0.125
- $\bullet$ 2.25 - 2.75 0.22
- $\bigcirc$ 2.75 - 3.50 0.272
- $\triangle$ 3.50 - 5.0 0.43
- $\square$ 5.0 - 10 1.0

End of Primary Consolidation
Figure 1.17 Log $\varepsilon'_{\sigma}$ - Log t Curves: (I-I) Test on Cleveland Clay
Figure 1.18 $\varepsilon_d$ versus Log $t$ Curves: (IOC) Test on Grundite Clay
Figure 1.19 $e_d$ versus Log t Curves: (IOC) Test on Nevada Clay

Nevada Clay
Test # 703 IOC
$\sigma_0 = 2.50$ ksc
$\omega_i = 93\%$
Figure 1.20 $\epsilon_d$ versus Log $t$ Curves: (IOC) Test on Cleveland Clay

Cleveland Clay
Test # 2-8-4 (IOC)
\[\sigma_0 = 4.0 \text{ kg/cm}^2, \quad \omega_i = 32\%\]

\[\mathcal{R}_T = 0.16 \quad \sigma_d = 0.52 \text{ ksc}\]

\[\mathcal{R}_T = 0.32 \quad \sigma_d = 1.0 \text{ ksc}\]
Figure 1.21 $\varepsilon_d$ versus Log $t$ Curves: (IOC) Test on Cleveland Clay

Cleveland Clay
Test # 2-8-4 (IOC)
$\sigma_0 = 4.0$ kg/cm$^2$
$\omega_t = 32\%$
Figure 1.22 Log $\dot{\varepsilon}_d$ versus Log $t$ Curves: (IOC) Tests on Grundite and Cleveland Clays
Figure 1.23 Log $\varepsilon_d$ versus Log $t$ Curves: (IOC) Test on Grundite Clay
Figure 1.24 Log $\varepsilon_d$ versus Log $t$ Curves: (IOC) Test on Nevada Clay

Nevada Clay
Test #: 703 (IOC)
$\sigma = 25$ ksc
$\omega_i = 93\%$

$\sigma_d = 0.80$ ksc
$\sigma_d = 0.26$ ksc
Figure 1.25 Log $\varepsilon_d$ versus Log $t$ Curves: (IOC) Test on Cleveland Clay
Grundite Clay

Test # 203 (IUE)

\( R_T = 0.77 \)  \( \sigma_d = 0.38 \text{ ksc} \)

\( R_T = 0.38 \)  \( \sigma_d = 0.77 \text{ ksc} \)

\( \sigma_c = 1.25 \text{ ksc} \)

\( \omega_i = 48.55 \% \)

Figure 1.26 Log \( \varepsilon_d \) versus Log \( t \) Curves: (IUC) Test on Grundite Clay
Grundite Clay

Test #1 (IUC)

\[ \sigma_0 = 2.5 \text{ ksc} \]

\[ \omega_i = 47.7\% \]

One Increment

\[ R_T = 0.79 \]

\[ \sigma_d = 1.57 \text{ ksc} \]

Grundite Clay

Test #11 (IUC)

\[ \sigma_0 = 2.50 \text{ ksc} \]

\[ \omega_i = 47.44\% \]

Two Increments

\[ R_T = 0.835 \]

\[ \sigma_d = 1.67 \text{ ksc} \]

\[ R_T = 0.41 \]

\[ \sigma_d = 0.82 \text{ ksc} \]

Figure 1.27 Log \( \epsilon_d \) versus Log \( t \) Curves: (IUC) Test on Grundite Clay
Figure 1.28 Log $\varepsilon_d$ versus Log $t$ Curves: (IUC) Test on Nevada Clay
Figure 1.29 Creep Parameters c and n versus $R_T$: (IUC) and (IUE) Tests on Grundite Clay
Figure 1.30 Creep Parameters c and n versus $R_T$: (IOC) Tests on Grundite Clay
Figure 1.31 Creep Parameters $c$ and $n$: (IOC) and Stress Path Tests on Cleveland Clay

\[ R_T = \left( \frac{\sigma_d}{\sigma_{df}} \right) \]
<table>
<thead>
<tr>
<th>Sample #</th>
<th>Orientation</th>
<th>W-Ci %</th>
<th>Type of Test</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-23-1</td>
<td>VL</td>
<td>27.0</td>
<td>IOC</td>
<td></td>
</tr>
<tr>
<td>2-20-1</td>
<td>VL</td>
<td>30.0</td>
<td>IOC</td>
<td></td>
</tr>
<tr>
<td>2-15-3</td>
<td>VL</td>
<td>28.0</td>
<td>Stress Path</td>
<td>△ Loading, △ Unloading</td>
</tr>
<tr>
<td>2-13-1</td>
<td>VL</td>
<td>29.0</td>
<td>Stress Path</td>
<td>□ Loading, ■ Unloading</td>
</tr>
</tbody>
</table>

Numerals Indicate $\varepsilon_d$

Figures 1.32 Stress Path Tests (Cleveland Clay)
Figure 1.33 Stress Path Tests (Cleveland Clay)
Figure 1.34 Log $\varepsilon_d$ versus Log $t$ Curves: Stress Path Tests on Cleveland Clay
Figure 1.35 Time Consolidation Curve: Maine Clay (One-Dimensional Consolidation)
Figure 1.36 Time Consolidation Curve: Maine Clay (One-Dimensional Consolidation)
Figure 1.37 Time Consolidation Curve: Maine Clay (One-Dimensional Consolidation)
Figure 1.38 Time Consolidation Curve: Nevada Clay (One-Dimensional Consolidation)
Figure 1.39 Time Consolidation Curve: Nevada Clay
(One-Dimensional Consolidation)
Figure 1.40 Time Consolidation Curve: Nevada Clay
(One-Dimensional Consolidation)
Figure 1.41 Time Consolidation Curve: Cleveland Clay
(One-Dimensional Consolidation)
Dial Change (inches)

Cleveland Clay
Sample # 2-2-2
\( \frac{D}{H} = 8.0 \)
\( \sigma_z = 3.69 \text{ kg/cm}^2 \)
\( R_C = 0.50 \)

Figure 1.42 Time Consolidation Curve: Cleveland Clay (One-Dimensional Consolidation)
Figure 1.43 Time Consolidation Curve: Cleveland Clay (One-Dimensional Consolidation)
Figure 1.44 Log $\varepsilon$ versus Log $t$: Tests on Cleveland Clay
Figure 1.45 Stress and Strain versus Log $t$ Curves: (I-K) Test on Grundite

Test #: 704

$\sigma_{ri} = 2.5 \text{ kg/cm}^2$  ○ ($\sigma_z - \sigma_r$)

$\Delta \sigma_r = 1.0 \text{ kg/cm}^2$  ● ($\varepsilon_z$)
CHAPTER II

Viscoelastic Characterization of

Time Dependent Deformation of Clays
II.1 Introduction

The data presented in the previous Chapter indicate clearly that the time dependent deformation of clays is of a nonlinear viscoelastic nature. Various methods used in material characterization have been summarized by Sandhu (40). Shapery (42, 43, 45) introduced a method of characterizing nonlinear viscoelastic solids based on the principal of irreversible thermodynamics. He developed constitutive equations and used experimental data to evaluate the material property functions in these equations. The material characterization described in Chapter I may be fitted to the general formulation by Shapery. For this reason, an outline of the method is presented in the following section.
II.2 Shapery's Non Linear Viscoelastic Material Characterization

II.2.1 Linear Equations:

When a constant stress $\sigma$ is applied at $t = 0$, the ratio of the strain response to stress input is a function only of time; viz.,

$$\frac{\varepsilon}{\sigma} = D(t) \quad (\text{II.1})$$

where $D(t)$ is the creep compliance and is a function of $t$. The stress-strain relations for this hypothetical creep test can be put in the equivalent form

$$\varepsilon = D_0 \sigma + \Delta D(t) \sigma \quad (\text{II.2})$$

where

$D_0 = D(0) = $ initial value of the creep compliance, and

$\Delta D(t) = D(t) - D_0 = $ transient creep compliance

when $D(t)$ is known, we can calculate the strain response to an arbitrary stress input by means of the Boltzmann superposition integral,

$$\varepsilon = D_0 \sigma + \int_0^t \Delta D(t - \tau) \frac{d\sigma}{d\tau} \, d\tau \quad (\text{II.3})$$
II.2.2 Nonlinear Equations:

For nonlinear constitutive equations, we observe that the thermodynamic theory developed by Shapery (42, 43) permits us to express material properties in terms of either stress or strain. When the stress is treated as an independent state variable, Shapery's theory (45) yields

\[ \epsilon = g_0 D_0 \sigma + g_1 \int_0^t \Delta \dot{D} (\psi - \dot{\psi}) \frac{dg_2 \sigma}{d\tau} \, d\tau \quad \text{(II.4)} \]

where \( D_0 \) and \( \Delta \dot{D} (\psi) \) are the components of the linear viscoelastic creep compliance defined above, and \( \psi \) is the "reduced time" defined by

\[ \psi = \int_0^t \frac{dt}{\sigma \sigma [\sigma(t)]} \quad \text{(II.5a)} \]

and

\[ \psi' = \psi_\tau = \int_0^\tau \frac{dt}{\sigma \sigma [\sigma(t)]} \quad \text{(II.5b)} \]

The material properties \( g_0, g_1, g_2 \) and \( a \sigma \) are functions of the stress. Comparing Equations (II.3) and (II.4) we see that \( g_0 = g_1 = g_2 = a \sigma = 1 \) for linear viscoelastic material.

Substituting a constant stress \( \sigma \) into the last term of Equation (II.4), and recognizing that \( \frac{dg_2 \sigma}{d\tau} \) is zero except at \( \tau = 0 \), yields
Thus, the nonlinear creep compliance, $D_n$, is

$$D_n = \frac{\varepsilon}{\sigma} = g_0D_0 + g_1 g_2 \Delta D(t/a\sigma)$$

(II. 7)

For a two step uniaxial loading test,

$$\sigma = \begin{cases} \sigma_a & 0 < t < t_a \\ \sigma_b & t_a < t < t_b \end{cases}$$

(II. 8)

where $\sigma = 0$ when $t<0$, and $\sigma_a$ and $\sigma_b$ are constants. Substitution of the stress history Equation (II. 8) into Equation (II. 4) yields for $0 < t < t_a$

$$\varepsilon = \left[g_0^aD_0 + g_1^a g_2^a \Delta D(t/a^a\sigma)\right] \sigma_a = D_n^a \sigma_a$$

(II. 9)

For $t_a < t < t_b$

$$\varepsilon = g_0^b D_0 \sigma_b + g_1^b \left[\int_0^{t_a} \Delta D(\psi-\psi') \frac{dg_2^b\sigma}{d\tau} \, d\tau + \int_{t_a}^{t} \Delta D(\psi-\psi') \frac{dg_2^b\sigma}{d\tau} \, d\tau\right]$$

(II. 10)

where $t_a$ is the time immediately before the second stress $\sigma_b$ is applied, and the superscript on a material property indicates the particular stress at which it is to be evaluated. The integrations in Equation (II. 10) are carried out by recognizing that $\frac{dg_2^b\sigma}{d\tau}$ is zero except at $\tau = 0$ and $\tau = T_a$; hence
In the following we will show the applications of Equations (II. 9) and (II. 11) for treating creep and recovery data. For these cases

$a^b_\sigma = 1, g_1^b = 1, g_2^b = 0$ and the superscripts $a$ and $b$ on the material properties will be dropped.

\[
\varepsilon = \phi_0^b D_0 \sigma_b + g_1^b g_2^a \sigma_a \Delta D(\psi) + (g_2^b \sigma_b - g_2^a \sigma_a) \Delta D(\frac{t-t_a}{\sigma_b})
\]

where \( \psi = \frac{t_0}{\phi_0^a} + \frac{t-t_a}{\phi_0^b} \)
II.2.3 Use of Creep and Recovery Data

The nonlinear creep compliance, \( D_n \), given in Equation (II.9) may be expressed as

\[
\log (D_n - g_0 D_0) = \log (g_1 g_2) + \log \Delta D(t/\sigma) \quad (\text{II.12})
\]

where \( D_n - g_0 D_0 = \Delta D_n \) is the transient component of the nonlinear creep compliance. This has the same form as the experimental behavior expressed by Equation (I.28).

Instead of the usual form

\[
D(n) = D_0 + D_1 \psi^n \quad (\text{II.13})
\]

the creep compliance may be expressed as:

\[
\Delta D(n) = D_1 \psi^n \quad (\text{II.14})
\]

In such a case we will not be interested in obtaining the value of \( g_0 \) and the stress functions to be determined are \( g_1, g_2 \) and \( \sigma \).

However, if the stress \( \sigma \), is removed at \( t = t_a \), strain measured during the "recovery" period, \( t > t_a \), provides the additional information necessary for the evaluation of all the properties.

Strain during the recovery period, \( t > t_a \), is denoted as \( \varepsilon_r \); it is found from Equation (II.11) by setting \( \sigma = 0 \).
\[ \varepsilon_r = \left[ \Delta D \left( \frac{t_0}{a_\sigma} + t - t_0 \right) - \Delta D \left( t - t_0 \right) \right] g_2 \sigma \]  

(II.15)

The creep strain at \( t = t_a \), \( \varepsilon(t_a) \), obtained from Equation (II.9) is:

\[ \varepsilon(t_a) = g_1 g_2 \Delta D \left( \frac{t_a}{a_\sigma} \right) \sigma \]  

(II.16)

Substitution of the power law \( \Delta D_n = D_1 \psi^n \) into Equations (II.16) and (II.15) we get:

\[ \varepsilon(t_a) = g_1 g_2 D_1 \sigma \frac{t^n}{a_\sigma^n} \]  

(II.17)

\[ \varepsilon_r = g_2 \sigma \left[ D_1 \left( \frac{t_0}{a_\sigma} + t - t_0 \right)^n - D_1 \left( t - t_0 \right)^n \right] \]  

(II.18)

From Equations (II.17) and (II.18) we get:

\[ \frac{\varepsilon_r}{\varepsilon(t_a)} = g_1^{-1} \left[ (1 + a_\sigma \lambda)^n - (a_\sigma \lambda)^n \right] \]  

(II.19a)

where

\[ \lambda = \left( \frac{t - t_0}{t_0} \right) \]  

(II.19b)

At relatively long times, Equation (II.19a) may be written as:

\[ \frac{\varepsilon_r}{\varepsilon(t_a)} = g_1^{-1} a_\sigma^{n-1} \lambda^{n-1} \]  

(II.19c)

Thus, the creep and recovery data enable us to obtain the functions \( g_1 \), \( g_2 \) and \( a_\sigma \).
II.3 Determination of the Creep Functions for Cleveland Clay

It may be noted here that the characterization process described in this section may be used to describe shear creep as well as volumetric creep. The formulation described in this Chapter are applied to estimate the deformations of the embankment and the cut of the interchange. The details of the project are given in the Appendix.

However, because of the nature of the problem (see Chapter III) we are only interested in the characterization of shear creep of the soil. The following section describes the process used to determine the material property functions required for the characterization of the shear creep behavior for Cleveland Clay.

A series of creep and recovery tests was performed on samples of Cleveland clay to obtain the functions required for the material characterization. The soil samples were initially consolidated isotropically to some confining pressure. An increment of deviator stress $\sigma_d$ was applied to the soil sample and then removed. The deviator strain-time curves obtained for the varved clay are shown in Figures 2.1 through 2.3. The failure envelope is shown in Figure 2.4. The values of the principal strain difference at one minute, $c$, obtained from single incremental loading are plotted against the stress ratio $R_T$ in Figure 2.5.
The values of $g_1$ at different stress levels were obtained from Equation (II.19a) by setting $t = t_a$. Values of $\sigma$ were obtained from Equation (II.19c) by setting $t = 2t_a$. The results show that even at the lowest stress level, the values of $g_1$ and $\sigma$ are not equal to 1.0. Hence, the range in which linear viscoelasticity may hold is lower than the lowest stress level used.

Nonlinear viscoelastic behaviors have been summarized by Thorkildsen (49). Of particular interest is the expression reported by Findley (15) on many monolithic and composite polymeric materials. Setting the initial strain $\varepsilon_0 = 0$, the form of $D_n$ suggested by Findley may be written as

$$D_n = \left[ \frac{\sinh (\sigma/\sigma_m)}{(\sigma/\sigma_m)} \right] A_t^n \quad (II.20)$$

Substituting the creep law, Equation (II.13), into Equation (II.7) and setting $g_0 = 0$, we obtain

$$D_n = \frac{g_1 g_2}{A_o \sigma} D_1 t^n \quad (II.21)$$

Using Equations (II.20) and (II.21), the creep compliance $D_n$ is expressed as

$$D_n = \left[ \frac{\sinh (\sigma/\sigma_m)}{(\sigma/\sigma_m)} \right] D_1 t^n \quad (II.22)$$

where $\sigma_m$, $D_1$, and $n$ have values that depend on the particular material.
This equation has the same form as Equation (1.28) if we let
\( c = D_1 \sinh(\sigma/\sigma_m) \), \( \sigma = R_T \) and \( \sigma_m = R_0 \) is a constant. In
fact, the term \( D_1 \sinh(\sigma/\sigma_m) \) has the same shape as \( c \) and the
values of \( \sigma_m \) and \( D_1 \) were obtained by fitting Equation (II.22) to
the plot of the shear strain at one minute versus \( R_T \).

Once \( \sigma_m \) and \( D_1 \) are known, the value of \( g_2 \) can be obtained
from Equation (II.21)

\[
g_2 = \left[ \frac{\sinh \frac{R_T}{R_0}}{\frac{R_T}{R_0}} \right] \frac{\sigma^n}{g_1}
\]

Values of \( c, n, a_\sigma \), \( g_1, g_2 \) and \( D_1 \) as functions of the stress
ratio \( R_T \) for Cleveland varved clay, silty clay, glacial till and
compacted clay are shown in Figures 2.6 through 2.16.
Figure 2.1 Log $\varepsilon_d$ versus Log $t$: (IOCR) Test on Cleveland Varved Clay, $R_T = 0.34$
Figure 2.2 Logεd versus Log t: (IOCR) Test on Cleveland Varved Clay, R_τ = 0.50
Figure 2.3  Log $\varepsilon_d$ versus Log $t$: (IOCR) Test on Cleveland Varved Clay, $R_T = 0.796$
Figure 2.4 Shear Strength Envelope for Cleveland Varved Clay
Figure 2.5 $c$ versus $R_T$ for Cleveland Varved Clay
Figure 2.6 Values of $g_1$ and $a_{\sigma}$ for Cleveland Varved Clay
Figure 2.7 Values of $g_2$ and $D_1$ for Cleveland Varved Clay
Cleveland
Gray Silty Clay

$R_T = \left(\frac{\sigma_d}{\sigma_{df}}\right)$

Figure 2.8 $c$ versus $R_T$ for Cleveland Silty Clay
Figure 2.9 Values of $g_1$ and $\sigma$ for Cleveland Silty Clay
Figure 2.10 Values of $g_2$ and $D_1$ for Cleveland Silty Clay
Figure 2.11  c versus $R_T$ for Cleveland Glacial Till
Figure 2.12 Values of $g_1$ and $a_\sigma$ for Cleveland Glacial Till
Figure 2.13 Values of $g_2$ and $D_1$ for Cleveland Glacial Till
Figure 2.14  $c$ versus $R_{\tau}$ for Cleveland Compacted Fill
Figure 2.15 Values of $g_1$ and $a_\sigma$ for Cleveland Compacted Fill
Figure 2.16 Values of $g_2$ and $D_1$ for Cleveland Compacted Fill
CHAPTER III

Calculations of the Time Dependent Earth Movements
III.1 Introduction

In this Chapter, the calculation of the time dependent movements of the cut and the embankment of the I-77 and I-80 interchange is described. The computed movements are then compared with the measured movements.

The finite element method was used in the calculations. The incremental linear elastic finite element computer program developed by Sandhu (41) was modified in order to account for the nonlinear viscoelastic material properties. The steps in the finite element analysis are similar to those developed by Ramaswamy (39). The total applied loads are divided into steps of loadings. The induced stresses and strains due to instantaneously applied loads are determined at some initial time $t_1$. The time is then increased to $t_2$ and the material properties are assumed to change abruptly. Having obtained the new material properties at $t_2$, the stresses and strains are obtained at $t_2$ under the same applied loads. In the same way the solution is obtained at the desired values of time. The procedure is repeated for subsequent load increments.
III.2 Material Properties Used in the Calculations

The material properties required for calculations are the Young's modulus $E$ and Possion's ratio $\mu$ or the shear and bulk moduli $G$ and $K$. In some cases $G$ and $K$ are preferred because they may be evaluated independently.

To obtain the shear modulus Equation (II.23) is written as

$$\gamma_{\text{oct}} = \frac{g_1 g_2}{a_{\sigma}^n} D_1 t^n \tau_{\text{oct}}$$  \hspace{1cm} (III.1)

where $\gamma_{\text{oct}}$ and $\tau_{\text{oct}}$ are the octahedral shear strain and stress, respectively. The shear modulus $G(t)$ is given by

$$G(t) = \frac{\tau_{\text{oct}}}{\gamma_{\text{oct}}}$$  \hspace{1cm} (III.2)

Using Equations (III.1) and (III.2) the shear modulus may then be written as

$$G(t) = \frac{a_{\sigma}^n}{g_1 g_2 D_1 t^n}$$  \hspace{1cm} (III.3)

The method of obtaining $a_{\sigma}$, $g_1$, $g_2$, and $D_1$ as functions of $R_T$ for different types of soils is given in detail in Chapter II.

For the embankment, the analysis of the pore water pressure showed that there was little pore water pressure dissipation and a value for $\mu$ equal to 0.49 was used in the calculations. The un-
drained shear strength was used to obtain the stress ratio $R_{T}$ and total stresses were used in the calculations.

The ground water table in the area of the cut was lowered gradually due to excavation and the case of the cut was considered as drained. Effective stresses were used in the deformation analysis and the average value of $K = 285 \text{ kg/cm}^2$ obtained from isotropic rebound tests was used in the calculations.
III.3 Computed and Measured Deformations

The computed and measured deformations for the embankment (Sec C-C, Figure A.2) and the cut (Sec. D-D, Figure A.5) are shown in Figures 3.1 through 3.4. The step loads used in the calculations and the field construction rates are also. The dashed lines in Figures 3.1 and 3.3 represent the displacements at the end of the time interval of the construction or excavation steps.

The computed vertical deformation for the embankment exceeds the measured value (Figure 3.1). Since the computations are made for the undrained condition the difference between the computed and measured vertical deformations is not surprising. The measured movements certainly included some volumetric compression of the sandy materials near the ground surface. The computed and measured horizontal movements are in good agreement. We note that slope indicator S-2-70 was installed in August 1970, after embankment construction had begun. Hence the computed movement after August 1970 should be used in making the comparison.

The computed horizontal deformations for the cut are somewhat larger than the measured deformations. In making the comparison we note that the tips of slope indicator tubes 3 and 4 were not located in the stiff bottom which turned out to be 20 feet deeper than as indicated in the preliminary borings. Thus, the measured movements
are in error and the actual movements should be somewhat greater. If we compare the horizontal deformations above Elev. 540; (Figure 3.3) we find good agreement between the computed and measured values. The computed and measured vertical movements are in good agreement.

The computed and measured deformation rates for the embankment are also reasonably close to each other. For the cut the computed rate of horizontal displacement differs from the measured rate. We should note the irregular geometry of the cut section near the slope indicator tubes (Figure A.1). The condition is quite different from that of plane strain. Also, excavation went on intermittently in the main cut and along the east side of I-77. These are impossible to duplicate in the calculations. In spite of these complications it seems reasonable to conclude that the computed and measured displacements are of the same order of magnitude. It must be realized that simpler loading and subsoil conditions that may be more suitable for comparison of computed and measured deformations are not likely to be encountered in practice.
Figure 3.1 Displacements of the Embankment
Figure 3.2 Measured and Computed Settlements of the Embankments
Figure 3.3 Displacements of the Cut
Figure 3.4 Measured and Computed Horizontal and Vertical Displacements of the Cut

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CHAPTER IV

Conclusions
The analysis of the volumetric deformation time properties of the soil tested indicates that the shape of the time consolidation curves during creep is strongly affected by the load increment ratio for normally consolidated clays. For over-consolidated clays the curves are of the same shape for all load increment ratios.

For times beyond the completion of the primary consolidation the data may be satisfactorily represented by a linear relationship on the log $\epsilon_v$ - log $t$ plots.

The results obtained from triaxial pure shear tests (IOC) and undrained creep tests indicate that the shear strain-time relationship may be approximated by straight lines on the log $\epsilon_d$ - log $t$ plots. This relationship is given by

$$\epsilon_t - \epsilon_3 = c_t^n$$

The data indicate that $n$ is approximately constant irrespective of the stress level and the initial water content. When the applied stresses are expressed as a fraction of the failure stress, $c$ may be considered as independent of the confining pressure and the initial water contents.

The results also indicate that the values of $c$ and $n$ remain approximately the same irrespective of whether the stresses are applied in one or more than one increment. It is also noticed that the values of $c$ for (IOC) tests differ considerably from that for IUC
tests.

The analysis shows that the stresses in the one-dimensional consolidation test are not constant. For normally consolidated clays, $K_0$ decreases as time increases. Thus, the measured strain-rates may not represent these for the condition of constant stress.

The computational method developed in this research appears to give satisfactory results. The deformations computed by this method for the embankment and the cut of the Independence interchange are in reasonable agreement with the observed deformations.
APPENDIX

Description of the Project
A.1 History of Construction

The interchange for I-77 and I-80 is located near Independence, Ohio. Figure A.1 shows the general plan of the project. It contains a deep cut from Station 480+00 to Station 489+35 and a high embankment north of Station 489-35. Cross-sections of the cut and the embankment are shown in Figures A.2 through A.5.

The construction of the project began in May of 1969. There were no major problems in the construction of the cut. The highest point of the embankment is 75 feet above original ground and the height decreases toward the north and the south as shown in Figure A.1. It was originally intended to construct the embankment with material obtained from the cut. This was done for the first 45 feet of the embankment beginning from original ground surface at elevation 600+ feet. It remained at elevation 645 feet from September, 1970 to January, 1971. The necessity of drying this material before compaction delayed the project to such an extent that it was decided to mix broken shale with the material obtained from the cut to facilitate compaction. In March of 1971, the fill was completed to the final elevation of 685 feet from Station 489+00 to Station 493+00. A short time later, a slide occurred on the west slope of the embankment. The slide area is shown by the dashed line in Figure A.1. At the
same time large shear displacements were recorded by a slope indicator readings at Station 489+10, 55 feet left of I-77. Construction of the embankment was stopped in April, 1971 while investigation of the stability was being made. The analyses in this study pertain to the embankment as it was in April of 1971.
A.2 Field Instrumentation

An extensive system of field instrumentation was installed to monitor the earthwork performance. The instrumentation consisted of slope indicator tubes, piezometers and settlement platforms. The locations of all instruments are shown in Figure A.6.

The slope indicator tubes were installed to monitor the horizontal and vertical earth movements. Slope indicators 1 through 6 were installed before construction. S-1-70 through S-3-70 were installed during construction and S-1-71 through S-5-71 were installed after the slide. Inclination readings were taken at regular intervals of depth and were subsequently used to determine the horizontal ground displacements. They were also used to determine the vertical earth movements. The elevation of the joints of the slope indicator tubes were obtained at different time intervals. From this, vertical displacement was computed over the entire length of the slope indicator.

Piezometers were installed to observe the pore water pressure. Before construction, Cassagrange-type piezometers with double leads were installed. These were all designated by numerals (Figure A.6) and had tip elevations between 570 and 575 feet. In 1970 nine pneumatic piezometers were installed in groups of three.
The piezometers of each group had tip elevations of 598, 590 and 570 feet. The piezometers were designated by numerals with prefix "P" and followed by "70". Then in the late 1971 additional Cassagrande-type piezometers were installed near the failure area. These were designated by numerals with a prefix "P" and followed by "71".

The settlement platforms were installed on the original ground surface near elevation 600 feet to measure the settlement of the embankment. The elevation of the top of the rods were determined by leveling.
A.3 Geological History of the Area

The project site is located in Cuyahoga River Valley. The Cuyahoga River follows an old drainage channel of the Teays period (Newberry, 1873). During some major geological events this river valley was cut deep into bedrock due to a lower lake level in the Erie basin. This level was as much as 200 feet below the level of the present Cuyahoga River near Independence, Ohio.

During the later stages of glaciation (both Illinoian and Wisconsin), this area was repeatedly covered by the continental ice sheet. During intervals of glacial retreat, a series of seven lakes of various levels occupied this area. Both glaciers and the lakes combined to produce a very complex subsoil profile. Borings show that the subsoil consists of sandy surface deposits over-laying thick deposits of varved clays, clay tills and some sand.

The lower sands are probably remnants of ancient beaches that developed along the glacial lakes. The varved clays are lacustrine deposits that consist of stratified silt and clay. The silts were deposited during summers when the water was turbulent. When the lake was frozen in winter, the finer clay sediments settled out. Thus, alternating layers of silt and clay were deposited. The till was deposited by glaciers and consisted of silty clay with sand and gravel.
The ground surface (Figure A.1) varies from elevation 590 feet at the bottom of the valley to elevation 680 feet at the top of the lake plane. In the stream valleys, the material has been eroded away to approximately elevation 590 feet. Some alluvium has been found in these locations.
A.4 Subsoil Conditions

The subsoil conditions were investigated by 18 borings and 3 test trenches. The locations of the borings are shown in Figure A.6 and the dates and the top and bottom elevations of each boring are given in Tables A.1 and A.2. The subsoil at the site (Figure A.7) consists of a layer of silty sand near the ground surface underlain by layers of varved clay, silty clay and glacial till. A layer of gray silty clay 10 to 15 feet thick is found above elevation 660. In some areas (borings T-1-71 through T-6-71 and boring 2) this material is weathered and has a brownish color. Below this a layer of varved clay which extends down to elevation 630 feet. The varved clay is a gray silty clay with silt and fine sand laminations. The natural water content of the varved clay varies between 24% and 28% depending on the amount of the silt (smaller water content is associated with higher silt content).

Shear displacements in the varves (Figure A.8) have been observed in several samples taken from this zone. The material between elevation 630 and 610 feet appears to be similar to the varved clay; however, the thickness of the alternating layers of clay and silt becomes quite variable and layers up to 24 inches thick have been encountered. For example, in borings T-1-71 through T-6-71
a layer of soft gray silty clay with small pockets of silt and fine sand was traced between elevations 630 and 625 feet. The natural water content of the gray silty clay layers varies between 29% and 34% depending on the amount of clay. The strength of the clay is also variable and some samples appear somewhat softer than the varved clay. Slickensides have been found in several samples taken from this zone (Figure A.9). Below approximately elevation 610 feet, there are 30 to 35 feet of glacial till. This material consists of stiff gray silty clay with sand and gravel and stone fragments. Its water content ranges from 17% to 20%. Below elevation 580 and down to elevation 540 there exists a thick layer of varved clay similar to that above elevation 610 but its strength is somewhat higher. Within this varved clay, a layer of gray silty clay up to 5 feet thick was encountered in borings 2a, A, T-2-71, 5-3-71 and in a boring at Station 57+10, 90 feet right, in the fill area. This material is similar to the gray silty clay between elevations 630 and 625. Below elevation 540 there is a layer of stiff glacial till.

It should be noted that the ground surface at the site of the embankment is near elevation 600. Hence, the upper varved clay is not present in the subsoil profile beneath the embankment.
<table>
<thead>
<tr>
<th>Boring #</th>
<th>Station</th>
<th>Elevation (ft.)</th>
<th>Date Started</th>
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<tr>
<td></td>
<td></td>
<td>Top</td>
<td>Bottom</td>
</tr>
<tr>
<td>2</td>
<td>489+10, 55 Lt.</td>
<td>669.00</td>
<td>588.00</td>
</tr>
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<td>3</td>
<td>483+18, 5 Rt.</td>
<td>670.40</td>
<td>600.40</td>
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<tr>
<td>A</td>
<td>493+70, 12 Lt.</td>
<td>599.00</td>
<td>538.00</td>
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<tr>
<td>2a</td>
<td>59+87, 27 Lt.</td>
<td>597.80</td>
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<td>582.00</td>
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<td>632.90</td>
<td>590.90</td>
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<td>630.00</td>
<td>613.00</td>
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<td>663.50</td>
<td>542.50</td>
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<tr>
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<td>661.40</td>
<td>540.40</td>
</tr>
<tr>
<td></td>
<td>57+10, 90 Rt.</td>
<td>604.60</td>
<td>543.60</td>
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</table>
## Table A.2

### Test Trenches

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<th>Elevation (ft)</th>
<th>Date Started</th>
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<tbody>
<tr>
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<td>489+30 ±</td>
<td>628 ±</td>
<td>4-15-71</td>
</tr>
<tr>
<td>Trench 2</td>
<td>491+ to 489+50  ±</td>
<td>610 to 620 ±</td>
<td>9-3-72</td>
</tr>
<tr>
<td>Trench 3</td>
<td>491+00 ±</td>
<td>613 to 623 ±</td>
<td>4-4-72</td>
</tr>
</tbody>
</table>
Figure A.1 Plan of the Project
Figure A.2 Cross-Section A-A (The Embankment)
Figure A.3 Cross-Section B-B (The Embankment)
Figure A.4 Cross-Section C-C (The Embankment)
Figure A.5 Cross-Section D-D (The Cut)
Figure A.6 Instrumentation Plan

**LEGEND**
- ■ Slope Indicator with Slot Orientation
- △ Pneumatic Piezometer
- ▲ Porous Tube Piezometer
- □ Settlement Platform

**INDEPENDENCE INTERSECTION**
(I-77, I-80)
**INSTRUMENT LOCATION PLAN**
Figure A.7 Subsoil Profile
Figure A. 8  Shear Zone in Varved Clay
BIBLIOGRAPHY


