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DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

Ting-yu Lo, B.S., M.S.

* * * * *

The Ohio State University

1972

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INTRODUCTION

The feasibility of utilizing sonic energy for drilling brittle materials, such as concrete or rock, has been known for some time. The basic process employs a sonic transducer mounted on a support structure and a slug of metal, called the "tool", as shown in Figure 1. The sonic transducer is an electromechanical device consisting of piezoelectric ceramic rings sandwiched in a resonant horn assembly. Transmission lines in multiples of half wave lengths can be attached to the tip of the resonant horn. The tools are of a general cylindrical shape with various tip and end geometries. For hard rocks, sonic energy is transmitted by means of impact coupling, or in other words, by repeated impact of the tool between the transducer and load at rates of hundreds to thousands of cycles per second. The tool impacts against the rock after impacting with and acquiring energy from the vibrating transmission line. Some energy of the tool is transmitted into the rock and results in some fracture of the rock surface. The tool then rebounds from the rock surface due to elastic recovery and again impacts against the transmission line tip. Due to the energy withdrawal from the transducer by the tool during tool-line impact, the internal energy of the transducer is at a lower level right after impact. It recovers before the next impact occurs.

Although the impact coupling process is rapid and somewhat random, it is a continual sequence of two clearly defined events: (1) impact and rebound of the tool against the vibrating line, and (2) impact
Figure 1 Sonic rock drilling system
and rebound of the tool from the load. Therefore, a rational analysis of impact coupling becomes possible and can be divided into three areas of study -- tool-line impact, tool-load impact and transducer analysis. Each area of these analyses involves several controlling parameters. The mass and tip geometry of the tool, the rock properties, and the energy level of each impact affect the incident-rebound velocity relationship as well as the characteristics of the impact zone. The end geometry and mass of the tool and the geometry and vibration amplitude of transmission line tip contribute to the characteristics of tool-line impact. The transducer is a rather complicated energy conversion system. Many parameters aside from the supplied terminal voltage affect its vibration characteristics. All of the previously mentioned parameters interact with each other; any one parameter can easily affect several others in the overall performance of the sonic system. The static force applied by the support structure is another factor in determining the energy transfer of the impact coupling process.

The problem of tool-rock impact has been studied extensively in the past two decades in the field of conventional percussive drilling. The main features of these studies are the relatively long drill steel and relatively high energy level per impact compared to the sonic rock drilling that we are now interested in. Mahban\textsuperscript{1} conducted extensive studies on the stress wave transmission along a transmission line into rock aimed at establishing theories which would predict the energy transfer ratio, reflected stress waves, and dynamic force-penetration relationship for various tip configuration of the rod. Later, he extended this work to studies of rock fracture under impact and determining the dynamic
force-penetration slopes with various fracture theories for brittle materials\(^2\). Graff, Feng and Kendall\(^3\) utilized Mahban's rock response model in the studies of energy transfer into rock and also used Mahban's experimental procedure for establishing the force-penetration relationship for concrete. In Chapter I of this study, the impact of a short tool on rock is analyzed based on Mahban's experimental results and it is established that a rigid mass-spring model of the tool is reasonable for such conditions of impact. An experiment in which sonic tools were propelled into single impact with the rock surface has been performed. The impact-rebound relationship and the energy loss characteristics were established. Studies are also extended to the impact zone characteristics and the discussion of specific energy.

Feng and Graff\(^4\) started the study of tool-line impact in 1968. A spherical ball and a rod five inches in length were used to impact a P-7 transducer having a thirty-inch transmission line. The stress waves were measured. The concept of "intercept" was first brought out here. Feng\(^5\) continued this study and established the impact-rebound velocity relationship for a spherical ball impacting a vibrating transmission line tip. Graff\(^6\) extended the concept of ball-line impact to the impact of more practical tools, i.e., more complicated tool geometries, while using the contact theory for the purpose of simplification. Shieh\(^7\) conducted experiments on the characteristics of energy transfer along a rod to tools for differing contact geometry between the tool and line. The results revealed that the simplification made by Graff was too severe to predict the contact time between tool and line accurately.
In Chapter II of this text, the study of tool impact on the transducer tip has been carried further. Using a different experimental method than Shieh, tools of various geometries were used to impact directly on sonic lines. Impact and rebound velocity and the transmitted stress wave were measured. The influence of the local contact geometry was assessed.

Study in the area of transducer characteristics has been extensive, although much is yet unknown on the performance of these devices. McMaster, Dettloff and Minchenko reviewed the basic principles of piezoelectric materials, while Dettloff and Minchenko studied the resonant horn. The overall transducer characteristics was first touched from the view point of classical circuit and transmission line theory by Hoffman and Swartz. Following this line of approach, Mahban and Graff developed the transfer function of a sonic power system. Graff, in 1969, reviewed the problems of vibrations of crystals and stepped horn resonators. The energy dissipation in a vibrating transmission line was analyzed by Fretwell and Graff. Ma, as part of an extensive study of the transducer, has measured the transient response of the transducer to impact. With the knowledge of energy dissipation and the observed transducer vibration characteristics, modeling of the recovery of transducer internal energy is made in Chapter III of the present study.

Feng studied the overall impact coupling process of a spherical tool between a vibrating and a static semi-infinite rod. The vibrating rod has a constant amplitude of vibration and is a simulation of a sonic transducer. The static line represents the load. The steady state
impact condition and energy transfer have been established and experimentally proved. This work was summarized by Feng and Graff\textsuperscript{17} and, in part, has also been presented in reference 18. The work of incorporating the characteristics of the sonic transducer and properties of rock into the impact coupling process should be accomplished to get a full understanding of the sonic drilling process. A qualitative explanation of the general interaction problem of a sonic system was first put forth by Graff\textsuperscript{19}. Here in Chapter IV, a detailed analysis on the effects of various parameters, such as drive voltage, transducer energy storage, static force, tool-load and tool-line parameters, has been conducted. Finally, all the analyses are united in the prediction of sonic drilling rate of rocks. A numerical example is given at the end of the chapter.

Sonic rock cutting is achieved in a series of percussive action of the tool. It differs with the usual percussive drilling in relatively lower energy level at each impact, relatively shorter drill steel and the high frequency of impact action. Even though the amount of rock removal per impact is small for a sonic system, the high frequency of impact may offset this. In any event, the effectiveness of the sonic rock drilling system in comparison with other existing commercial rock removal systems in the fields of rotary, percussive and rotary-percussive drilling remains to be established.
CHAPTER I TOOL-ROCK IMPACT

The phase of tool-rock interaction is a fundamental problem in conventional percussive drilling systems as well as sonic systems. A significant number of papers have been published in the past two decades. A large part of the efforts have been concentrated on two main areas: (1) Crater formation due to various forms of indentors, such as wedges, dies etc. is one area. A great deal of theoretical as well as experimental work has been done. It is generally agreed that penetration starts with the crushing of surface irregularities. As elastic deformation continues, subsurface cracks radiate out from lines of stress concentration along the cutting edges. The region directly under the tip is then crushed into fine fragments. Finally, large fragments chip out on both sides of the cutting tip along the maximum shear trajectories. (2) The contact force-penetration relationship is the second main area. This relationship varies considerably with various factors, such as bit mass and geometry, total energy in each impact, velocity of the bit as well as the characteristics of the rock sample. All of the results obtained by previous investigators show a common phenomenon. A bilinear spring representation is found in the loading curve with the unloading along a curve with much larger slope. In general, the curve can be represented approximately by two straight lines as shown in Figure 2. The bilinear representation is simple in appearance; but the slopes are difficult to find. Experiments are required to find these values.
The test methods used by previous investigators can be classified into static and dynamic testing. In the dynamic testing, a vertical drop tester is usually used. A cutting bit is attached to a lumped mass which is allowed to fall from a certain height upon rock sample. An air gun is also used to deliver bits at high velocity.

The energy level for all previous work is much higher than sonic impact, and the mass of the drill bit is also much larger than the sonic tool. Therefore, whether the results obtained by previous investigators can be used in our present analysis is in doubt. This leads to the analysis and experiments in this chapter.

Mahban suggested that when a rectangular stress wave is incident on rock along a semi-infinite rod, the contact force-penetration characteristics can also be represented by a bilinear model as shown earlier in Figure 2, where \( k_1 \) and \( k_2 \) are slopes of the loading and unloading path of the model. The energy is transmitted into rock via stress wave
propagation. In the sonic drilling process, a short cylindrical tool is used for transmitting sonic energy from the transducer to rock instead of a long rod. The difference between these two methods of energy transfer is apparent. Therefore, we do not know whether the bilinear model used by Mahban can be used for present problem of tool-rock impact. To tackle this problem, two modeling attempts are given based on Mahban's experimental result in Section 1-1. In Section 1-2, an experiment on single impact between tool and rock is conducted to study the actual tool-rock impact condition. This is followed by a comparison of actual and analytical results for a flat tool (Section 1-3).

1-1 ANALYSIS OF TOOL ON ROCK

Based on Mahban's experimental result, two different analytical approaches are possible. One is to assume that the tool is rigid and the response of the rock is the same as a spring of bilinear character with spring constants equal to the slopes of Mahban's bilinear model (Figure 3). Of course, the assumption just made is a bold one and needs justification. The obvious reason for this approach is the simplicity of the spring-mass system. It is simple in nature as well as in mathematical manipulation. The rebound velocity and maximum penetration, the rebound velocity and maximum penetration can be obtained readily without invoking complication.
Another way of solving this problem is by considering the stress wave characteristics in the tool during impact with bilinear rock property. In this approach, we are able to obtain more reasonable results at the price of the complicated numerical calculations involved. The differential equation governing the penetration of a tool against the rock was set up and solved. The results for the two different approaches are found to be close to each other. Therefore, in solving the tool-rock impact problem, the simpler approach can be used, i.e., one can assume a perfectly rigid tool impacting directly on a spring of bilinear character without invoking much error in the result.

1-1-1 Rigid Tool Impact on a Bilinear Spring

A rigid tool with initial velocity $V_0$ before impact possesses a total energy
where \( \rho \), \( A \) and \( l \) are the mass density, cross-sectional area and length of the cylindrical tool, respectively. The maximum contact force and penetration occurs when the velocity of the tool becomes zero and can be obtained by equating the tool energy, \( E \), to the maximum potential energy of the spring.

\[
E = \left( \frac{\rho A l V_0^2}{2} \right) = k_1 u_m^2 / 2
\]  

(2)

This gives

\[
u_m = V_0 \sqrt{\frac{\rho A l}{k_1}}
\]  

(3)

where \( u_m \) is the maximum penetration. For the maximum contact force, \( F_m \), we have

\[
F_m = k_1 u_m = V_0 \sqrt{\rho A l k_1}
\]  

(4)

or

\[
\frac{F_m}{A} = V_0 \sqrt{\rho \frac{k_1 l}{A}}
\]  

(5)

Part of the energy is generally lost due to the impact. Unloading is along \( k = k_2 \). The energy loss, \( E_L \), is the energy used to induce cracking in the rock and lost through stress waves, and is equal to the shaded area between lines of loading and unloading in Figure 3. This amount is given by

\[
E_L = k_1 u_m^2 \left(1 - k_1/k_2\right)/2
\]  

(6)
The rebound velocity is obtained without difficulty by considering the energy after impact and is given by:

\[ V_r = V_0 \sqrt{\frac{k_1}{k_2}} \]  

(7)

The time, \( t_m \), required to reach the peak (maximum penetration) and the time required from peak to complete separation of tool and load, \( t_r \), are, according to one-dimensional spring mass theory, respectively,

\[ t_m = \frac{2 \pi}{4} \sqrt{\frac{\rho A l}{k_1}} \]  

(8)

\[ t_r = \frac{2 \pi}{4} \sqrt{\frac{\rho A l}{k_2}} \]  

(9)

The total contact time is the sum of \( t_m \) and \( t_r \). It is interesting to note that \( t_m \) and \( t_r \) are independent of impact velocity of the tool, \( V_0 \), after the tool leaves the rock surface, there is a permanent deformation of the rock which is equal to \((1-k_1/k_2)u_m\).

From the previous discussion, it is easy to see that with the assumption of a rigid tool, the problem of tool-rock impact is much simplified and all the results can be obtained without complicated numerical calculations.

1-1-2 One Dimensional Elastic Tool Impact on a Bilinear Model

In the previous section, the discussion is based upon the assumption that the response of rock is the same as a bilinear spring with spring constants equal to the slopes of Mahban's bilinear model. An
alternate approach is given here. It is based on Mahban's results obtained from incident stress waves along a semi-infinite rod, and also based on the assumption that the stress wave characteristics in the tool is essentially one dimensional even though the length-to-radius ratio for a tool is rather small. In fact, with the above assumption, we are imposing the classical rod theory on tools of short length. Thus, the rigid-tool versus the elastic tool model is under examination.

(i) Initial Condition of a Tool with Constant Velocity

It can be shown that the motion of a cylindrical tool with velocity \( V_0 \) can be represented by two rectangular stress waves of opposite sign propagating in two different directions as shown in Figure 4. The magnitude of the stress waves are equal to \( V_0 \frac{c_0}{2} \), where \( c_0 \) is the longitudinal wave velocity in rods. Since both ends of the tool are free, the wave is reflected back without distortion in shape, but with different sign.

\[
\sigma_0 = \frac{V_0 c_0 \rho}{2}
\]

Figure 4 Impact of one-dimensional tool against rock
(ii) Bilinear Model and Governing Differential Equation

During impact of the tool with the load, the tip of the tool is compressed against the load and is no longer a free end. The reflected wave travels with velocity \( c_o \) away from the point of contact. When it reaches the other end of the tool, it is reflected back as an incident wave. There may be several incident-reflection wave cycles before the tool bounces back from the rock surface. A large force occurs between the load and tool. As a result, some fracture and penetration occurs. The velocity of the tip, or the penetration velocity, \( \dot{u} \), according to one-dimensional theory of wave propagation in a rod is given by

\[
\dot{u} = -\frac{c_0}{Y}(\sigma_i - \sigma_x)
\]

where \( Y \) is Young's modulus, \( \sigma_i \) is the incident wave, and \( \sigma_x \) is the corresponding reflected stress wave. During the first wave cycle of contact, \( \sigma_i \) is equal to \( -\frac{V_0 c_o \rho}{2} \).

The contact force between tool and load is

\[
F = A (\sigma_i + \sigma_x)
\]

Eliminating \( \sigma_x \) in equation (10) and equation (11), we obtain the following differential equation

\[
\dot{u} = \frac{c_0 F}{Y A} = -\frac{2c_0 \rho}{Y}
\]

To solve the above differential equation, we can use the contact force-
penetration characteristics pointed out by Mahban as already shown in Figure 3. This model can be expressed in the following form:

\[
F(t) = \begin{cases} 
  k_1 u(t) & \text{if } \dot{u} > 0 \\
  (k_1 - k_2) u_m + k_2 u(t) & \text{if } \dot{u} < 0 
\end{cases}
\]  \tag{13}

The values of \( k_1 \) and \( k_2 \) are obtained from experiments of rectangular stress waves propagating into rock along a semi-infinite rod as conducted by Mahban. The stress cycles after the first are generally not rectangular, but we assume the same values of \( k_1 \) and \( k_2 \) can still be used. Then, the problem can be solved with equation (12) and equation (13). The solution follows:

(a) For \( 0 \leq t \leq t_1 \)

\[
\begin{align*}
  u &= A_1 + B_1 e^{-p_1 \overline{t}} \\
  t_1 &= \frac{2l}{c_0} \\
  \overline{t} &= t \\
  p_1 &= \frac{c_0 k_1}{EA} \\
  A_1 &= \frac{2 \sigma_0 A}{k_1} \\
  B_1 &= -A_1 = -\frac{2 \sigma_0 A}{k_1} \\
  u_1 &= u(t_1) 
\end{align*}
\]  \tag{14}

(b) For \( t_1 \leq t \leq 2t_1 \)

\[
\begin{align*}
  u &= A_2 + (B_2 + C_2 \overline{t}) e^{-p_1 \overline{t}} \\
  \overline{t} &= t - t_1 
\end{align*}
\]  \tag{15}
\( A_2 = -2 \sigma_0 A / k_1 \)
\( B_2 = u_1 + 2 \sigma_0 A / k_1 \)
\( C_2 = 4 \sigma_0 \sqrt{E} \)
\( u_2 = u(t_1) \)

(c) For \( 2t_1 \leq t \leq t_m \leq 3t_1 \).

\[ t_m = \text{time required to reach maximum penetration} \]

\[ u = A_3 + (B_3 + C_3 \tilde{t} + D_3 \tilde{t}^2)e^{-p_1 \tilde{t}} \]

\[ \tilde{t} = t - 2t_1 \]

\[ A_3 = 2 \sigma_0 A / k_1 \]
\[ B_3 = u_2 - 2 \sigma_0 A / k_1 \]
\[ C_3 = -2p_1 u_1 \]
\[ D_3 = -4p_1 \sigma_0 \sqrt{E} \]

\[ 2t_1 \leq t_m = \frac{1}{2}(-a \pm \sqrt{a^2 - 4b}) \leq 3t_1 \]

\[ a = \frac{(u_1 k_1 - 2)/p_1}{2 \sigma_0 A} \]
\[ b = \frac{B}{2p_1 \sigma_0 \sigma_0} \left( u_1 + \frac{u_2}{2} \right) + \frac{1}{2p_1^2} \]
\[ u_m = u(t_m) \]

(d) For \( t_m \leq t \leq 3t_1 \).

\[ u = A_4 + (B_4 + C_4 \tilde{t})e^{-p_1 \tilde{t}} + E_4 e^{-p_2 \tilde{t}} \]

\[ A_4 = \alpha + \frac{2 \sigma_0 A}{k_2}, \quad \alpha = (k_2 - k_1)u_m / k_2, \quad \tilde{t} = t - t_m \]

\[ B_4 = \frac{2 \sigma_0}{E(p_2 - p_1)} \left[ - \frac{k_1 u_1}{A} + \frac{4p_1 \sigma_0}{p_2 - p_1} \right] \]
\[ \eta_d z_d z_d = g_d \]
\[ \frac{\eta d - z_d}{(\eta d + z_d) o_o z_d o_o} = g_d \]
\[ \frac{\eta d - z_d}{(\eta d + z_d) o_o z_d o_o = g_d} \]
\[ (\eta d + z_d) o_o z_d o_o = g_d \]
\[ \eta_d - \eta_d = g_d \]
\[ \frac{v}{(\eta d + z_d) o_o z_d o_o = g_d} \]
\[ \frac{v}{(\eta d + z_d) o_o z_d o_o = g_d} \]
\[ \eta_d - \eta_d = g_d \]
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\[ \frac{v}{(\eta d + z_d) o_o z_d o_o = g_d} \]
\[ \frac{v}{(\eta d + z_d) o_o z_d o_o = g_d} \]
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\[ \frac{v}{(\eta d + z_d) o_o z_d o_o = g_d} \]
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\[ \frac{v}{(\eta d + z_d) o_o z_d o_o = g_d} \]
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\[ \frac{v}{(\eta d + z_d) o_o z_d o_o = g_d} \]
\[ \frac{v}{(\eta d + z_d) o_o z_d o_o = g_g} \]
\[ \frac{v}{(\eta d + z_d) o_o z_d o_o = g_g} \]
\[ \frac{v}{(\eta d + z_d) o_o z_d o_o = g_g} \]
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\[ \frac{v}{(\eta d + z_d) o_o z_d o_o = g_g} \]
\[ \frac{v}{(\eta d + z_d) o_o z_d o_o = g_g} \]
The solution is carried on in this fashion until the contact force $F$ becomes zero. Further, once the value of $u(t)$ is known, the reflected stress, $\sigma_r$, can be obtained from equations (11) and (13). Also, it can easily be observed by considering equation (14) that the maximum penetration never occurs within the first stress cycle of the incident wave. Usually, it happens during the third cycle of the incident wave; at the same time, the penetration velocity becomes zero.

The approximate value of the rebound velocity can be obtained by taking the mean value of the elastic waves in the tool when the tool is no longer in contact with the rock surface. A more detailed explanation is given in the following section.

1-1-3 Numerical Examples and Illustration

Two example problems are given here to compare and illustrate the results discussed in Section 1-1-1 and Section 1-1-2. The values of $k_1$ and $k_2$ of the bilinear model represent mainly the effect of tip geometry, initial velocity $V_o$ (or equivalently, stress level $\sigma_o$) and the rock properties. Only the flat tool with high velocity ($V_o = 520$ in./sec.) and low velocity ($V_o = 78.67$ in./sec.), which corresponds to the high stress level ($\sigma_o = V_o c_o^2/2 = 39.0$ ksi) and low stress level ($\sigma_o = 5.9$ ksi) respectively, in Mahban's experiments, will be discussed here. The same discussion can be applied to other cases.
EXAMPLE 1: Flat Tip Tool at High Velocity \( (V_o = 520 \text{ in./sec.}) \)

Impact Velocity \( V_o = 520 \text{ in./sec.} \)

Corresponding equivalent stress level \( \sigma_o = 39.0 \text{ ksi} \)

Length of tool \( l = 1 \text{ in.} \)

From Mahban's experimental result for Indiana limestone,

\[
k_1/A = 2.74 \times 10^6 \text{ lb./in.}^3, \quad k_2/A = 9.10 \times 10^6 \text{ lb./in.}^3
\]

Table 1 Comparison of Impact of Elastic and Rigid Tools

<table>
<thead>
<tr>
<th>Tool</th>
<th>Maximum penetration ( u_m ) ( \times 10^{-3} \text{ in.} )</th>
<th>Max. contact force per unit area, ( F/A ) (ksi)</th>
<th>Rebound velocity ( \text{in./sec} )</th>
<th>Time req. to reach peak (( \mu \text{sec} ))</th>
<th>Time from peak to separation (( \mu \text{sec} ))</th>
<th>Total contact time (( \mu \text{sec} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic</td>
<td>8.459</td>
<td>23.18</td>
<td>287</td>
<td>25.67</td>
<td>16.1</td>
<td>41.8</td>
</tr>
</tbody>
</table>

EXAMPLE 2: Flat Tip Tool at Low Velocity \( (V_o = 78.67 \text{ in./sec.}) \)

Impact Velocity \( V_o = 78.67 \text{ in./sec.} \)

Corresponding equivalent stress level \( \sigma_o = 5.9 \text{ ksi} \)

Length of tool \( l = 1 \text{ in.} \)

From Mahban's experimental result for Indiana limestone

\[
k_1/A = 3.17 \times 10^6 \text{ lb./in.}^3, \quad k_2/A = 6.67 \times 10^6 \text{ lb./in.}^3
\]

Table 2 Comparison of Impact of Elastic and Rigid Tools

<table>
<thead>
<tr>
<th>Tool</th>
<th>Maximum penetration ( u_m ) ( \times 10^{-3} \text{ in.} )</th>
<th>Max. contact force per unit area, ( F/A ) (ksi)</th>
<th>Rebound velocity ( \text{in./sec} )</th>
<th>Time req. to reach peak (( \mu \text{sec} ))</th>
<th>Time from peak to separation (( \mu \text{sec} ))</th>
<th>Total contact time (( \mu \text{sec} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic</td>
<td>1.189</td>
<td>3.77</td>
<td>54</td>
<td>24.31</td>
<td>17.1</td>
<td>41.4</td>
</tr>
<tr>
<td>Rigid</td>
<td>1.210</td>
<td>3.836</td>
<td>54.2</td>
<td>24.16</td>
<td>16.66</td>
<td>40.82</td>
</tr>
</tbody>
</table>
Figure 5 and Figure 8 give the penetration of the tool against the rock surface for both cases. It is easy to see that the results obtained by considering the tool to be completed rigid is very close to the results obtained by stress wave approach. With the closeness of $u(t)$ for the two approaches, we could imagine that there should not be much deviation for values of the contact forces. The results are shown in Figure 6 and Figure 9 for high stress level and low stress level, i.e., the elastic tool and the rigid tool. In short, these two approaches give very close results as we can plainly see from Table 1 and Table 2. The deviation of maximum penetration and maximum contact force for the rigid tool from the elastic tool is less than 2% for both problems.

To understand the wave propagation in the elastic tool, the incident and reflected stress waves at the tip for all time are plotted as shown in Figure 7 and Figure 10. The solid line gives the magnitude of the incident stress wave at the tip of the tool at any given time while the dashed line represents the corresponding value of the reflected stress wave. It is interesting to note that the maximum contact force or maximum penetration occurs at a time when incident and reflected waves have the same value; i.e., at a time when these two curves cross each other. The solid lines in the lower figure represent two waves propagating in opposite directions. The total stress in the tool is the sum of these two stress waves and is shown as the dashed line, which is maximum at the contact point and is zero at the free end. The stress distribution is approximately linear. The above results are not surprising, since the particle velocity is equal to zero at the tip when the tool reaches
Figure 5 Time variation of penetration for flat tip tool with $V_o = 520$ in./sec.

$k_1/A = 2.74 \times 10^6$ lb./in.$^3$

$k_2/A = 9.10 \times 10^6$ lb./in.$^3$

- Elastic tool
- Rigid tool
$k_1/A = 2.74 \times 10^6 \text{ lb./in.}^3$

$k_2/A = 9.10 \times 10^6 \text{ lb./in.}^3$

---

**Figure 6** Time variation of contact force for flat tip tool with $V_o = 520 \text{ in./sec.}$
Figure 7 Incident and reflected stress wave at flat tip of tool with $V_0 = 520$ in./sec.

$k_1/A = 2.74 \times 10^6$ lb./in.$^3$

$k_2/A = 9.10 \times 10^6$ lb./in.$^3$

--- Incident Wave

--- Reflected Wave

Stress waves in tool during max. penetration
Figure 8 Time variation of penetration for flat tip tool with $V_o = 78.67$ in./sec.

$\frac{k_1}{A} = 3.17 \times 10^6$ lb./in.$^3$

$\frac{k_2}{A} = 6.67 \times 10^6$ lb./in.$^3$

--- Elastic tool

--- Rigid tool
$k_1/A = 3.17 \times 10^6 \text{ lb./in.}^3$

$k_2/A = 6.67 \times 10^6 \text{ lb./in.}^3$

--- Elastic tool

--- Rigid tool

Figure 9 Time variation of contact force for flat tip tool with $V_o = 78.67 \text{ in./sec.}$
Figure 10 Incident and reflected stress waves at flat tip of tool with $V_0 = 78.67$ in./sec.

\[ k_1/A = 3.17 \times 10^6 \text{ lb./in.}^3 \]

\[ k_2/A = 6.67 \times 10^6 \text{ lb./in.}^3 \]
maximum penetration. At this stage, maximum stress occurs at the contact end while zero stress happens at the free end.

At the end of contact between tool and rock, the stress waves in the tool are approximately equal to two rectangular waves moving in opposite directions. The rebound velocity can be obtained by using the average value of the stress waves, $\sigma_{av}$, and the argument cited in Section 1-1-2. Thus,

$$V_r = \frac{2 \sigma_{av}}{c_0 f} \quad (20)$$

The values of rebound velocity for both elastic and rigid tool are given in Table 1 and Table 2. The results are still in good agreement with each other.

1-1-4 Conclusion

In the above two approaches, we make assumption in both cases; but it is easy to see that the elastic tool method has stronger theoretical background than the rigid tool method since the stress waves consideration in the former approach resembles the elastic waves in the long rod of Mahban’s experiment. By looking at the results obtained in Section 1-1-3, it shows that the rigid tool method yields results very close to the elastic tool method. This much simplifies the mathematical manipulation that the latter approach requires and, therefore, is more acceptable from a practical point of view. The closeness of these two approaches can also be understood by considering the energy and the bilinear contact force-penetration
relation. During maximum penetration, all the energy of the rigid tool has been used in compressing the bilinear spring while some energy is still retained in the elastic tool. Therefore, the maximum penetration and maximum contact force should be larger in the rigid tool case than in elastic tool case. Since the elastic energy that the elastic tool possesses during maximum penetration is rather small compared to the original total energy, the closeness of maximum penetration and maximum contact force is also understandable. All of the above discussion have been confirmed in Figures (5), (6), (8) and (9).

1-2 EXPERIMENTS OF TOOL IMPACT ON ROCK

A previous study of a rectangular stress wave propagating along a semi-infinite rod and incident on a rock surface by Mahban shows that the dynamic contact force-penetration characteristics can be approximately represented by a bilinear model as shown in Figure 2, where \( k_1 \) and \( k_2 \) are slopes of the loading and unloading paths of the bilinear model. Based on Mahban's model for rock, the problem of tool-rock impact has been theoretically analyzed in Section 1-1. In the rigid tool approach, we assumed that the tool is completely rigid and the response of rock is the same as a bilinear spring; in the elastic tool approach, we assumed that the wave phenomenon in the tool is essentially one-dimensional and the values of \( k_1 \) and \( k_2 \) remain unchanged during the contact period. To study the actual tool-rock impact problem, a series of experiments were conducted. Attempts were made toward finding the impact-rebound velocity relationship; then the actual results were compared with the
analytical results obtained from the two approaches cited previously in Section 1-3. Special emphases was also given to the impact zone characteristics, energy loss, specific energy, and the effect of various tool geometries and number of impacts.

Specific energy in rock cutting \( (e_s) \) is defined as the energy required to remove a unit volume of rock. It is a very important parameter in discussing a rock drilling process. Two factors are involved in the economy of a rock drilling system, the amount of time and the amount of energy required to cut a certain volume of rock. The less the time and energy the higher the economy. The time factor of a system is usually controlled by the power supplied and the ability of the system to transform energy into mechanical work. This is where the high power sonic transducer system comes in when we talk about sonic rock cutting. The specific energy is governed mainly by the tip geometry and impact velocity of a sonic tool.

1-2-1 Experimental Apparatus and Procedure

A photograph and schematic diagram of the experimental set-up are shown in Figure 11. A spring gun, which can deliver tools at a specific velocity by adjusting the spring deformation, is placed directly in front of the surface of a rock specimen. The movement of tools of various tip geometry is guided by the gun barrel, with the direction of incidence and rebound of the tool being approximately perpendicular to the rock surface. The incident and rebound velocities of the tools are measured by the use of two photo cells which are
Figure 11 Experimental set-up.
attached to the end of the spring gun and spaced one inch apart. Two oscilloscopes are used to record the time required for the tool to pass the one-inch distance both in incident and rebound movement. The velocity of the tool is simply equal to the reciprocal of the time measured. The incident velocity of the tools ranged from 200 in./sec, to 600 in./sec.

Figure 12 shows the dimensions and letter designations of the tools used. All of the tools are one inch in diameter and length. The design of the tip is more an art than science. Starting from the hand held chisel cutting tool, people have used experimental methods to find the most efficient cutting bit. Two general types of tip geometry are used here - four wing bits and graved wing bits - aside from the flat tip which is the most simple and basic geometry. Cross wing bits resemble a popular type of bit geometry used in percussive drilling. The graved wing bits are contrary to the cross wing bits. They are graved in instead of bulged out. Variation of contact area has been given in each case.

1-2-2 Incident and Rebound Velocity Relationship

Figure 13 to Figure 16 are plots of incident versus rebound velocity relationships for various tools impacting against Indiana limestone and granite. All the figures show a fairly wide scatter in the rebound velocity data. This may be due in part to the irregularity of both the rock surface and the impact conditions. An approximate best fit straight line has been drawn for each case. Type B tool has the
Figure 12 - Dimensions of tools used in the experiments.
Figure 13 Incident-rebound velocity of Type A tool
Figure 14 Incident-rebound velocity of Type B tool
Figure 15 Incident-rebound velocity of Type D2 tool
Figure 16 Incident-rebound velocity of Type E1 tool
highest rebound velocity among all the tools tested, and it happens to have the least contact area with the rock surface. Generally, the slope for granite is larger than for limestone.

The straight line approximation represents a constant ratio of $k_1/k_2$ for a specific rock and tool geometry if a bilinear response model is assumed.

1-2-3 Effect of Number of Impacts on The Rebound Velocity

Figure 17 and Figure 18 show the variation of rebound velocity as the number of impacts at a specific location increases for Type B and Type D2 tools, respectively. Figure 19 shows the impact zone condition after twenty-five impacts. The incident velocity of the tools was kept constant for all impacts and was equal to 550 in./sec. The figures show that the rebound velocity decreases as the number of impacts increases. The effect of chips caused by continuous impact may be the possible explanation for this trend. For each impact, fracture occurs in the impact zone and part of the chips remain in that region, while part of them leave the impact zone and rebound with the tool. When the tool again impacts against the rock, part of the energy of the tool is lost in further crushing of the chips between the tool and rock. In sonic drilling, we usually call this the pulverizing effect. The more chips between tool and rock the more energy is absorbed, and the less the rebound velocity. Therefore, solving the problem of chip removal is one of the important tasks in improving the rock cutting process. Micro-fractures caused by previous impacts might be another reason since
Figure 17 Effect of the number of impacts on the rebound velocity for Type B tool.
Figure 18 Effect of number of impacts on the rebound velocity for Type D2 tool
Figure 19  Impact zone in Indiana limestone after twenty-five impacts ($V_0 = 550$ in./sec.)
micro-fractures change the properties of the rock. In future discussions of the impact-rebound relationship, we will by-pass the effect of chips and micro-fractures and assume that every impact of the tool is on a fresh surface, or in other words, the number of impacts has no effect on rebound velocity.

1-2-4 Characteristics of the Impact Zone

As stated early in this chapter, the sequence of rock crushing due to wedges, dies etc, is as follows; crushing of surface irregularities, elastic deformation and propagation of microfractures, crushing of the region under the tip into fine fragments, and chipping along maximum shear trajectories. This rock crushing sequence happens only when the tip of drill steel is sharp and the energy level in each impact is high. But, in sonic drilling, the energy level is low. It is, therefore, interesting to see the characteristics of the impact zone.

Figure 20 shows the impact zone condition of Indiana limestone at different number of impacts for various tip geometries. The first few impacts generally do not show any damage to the rock at all except traces of the tip contact area. This suggests that only slight crushing of the surface irregularities has occurred. As the number of impacts increases, chipping starts at the boundary of the contact area. Elastic deformation is also present since the tool usually rebounds from the rock surface. Micro-fracture is not detectable but is believed to be present in some degree. For tools with large contact area, such as the flat tool, very little crushing in the contact region occurs due to the
Figure 20  Impact zone condition in Indiana limestone at different number of impacts.
large area of contact and small contact force. Chipping after a number of impacts is believed to be due to the accumulation of microfracture and stress concentration effects at the edge of contact. Studies of the punch problem always indicate stress concentration at the boundary of contact. For example, I. N. Sneddon has solved for the stresses produced by a circular flat punch on a half-space - the static analogy of flat tool impact on a rock surface. He found that maximum shearing stress goes to infinity at the boundary, and decreases either inward or outward. The result for flat tool-rock impact in Figure 20 shows what is to be expected from such stress distribution.

The crushed region can be observed for the Type B tool only after a large number of impacts. This can be explained as the effect of accumulated microfractures which weaken the region under impact. The Type B tool has the least area of contact. Therefore, the impact stress is higher than the rest of the tools used if the tool incident energy is kept the same for all tools. It is, therefore, not surprising to see that Type B tool causes more damage to rock than another tool while the flat tip tool has the lowest efficiency in rock drilling.

1-2-5 Specific Energy

To study the specific energy of rock removal for granite, the Type F tool which has a sharp tip made of carbide inserts was used (Figure 12). The tool was propelled into a series of impacts with the rock surface at a specific location. The total energy used was the sum of all the individual impact energies; and the volume of rock removal
was obtained by dividing the total weight of the chips and dust collected by the average density of the rock.

Figure 21 shows the relationship between incident velocity $V_o$ and specific energy $e_s$ and volume $v$ for the Type F tool impacting on granite. A linear relationship between $v$ and $V_o$ is observed. This result strengthens the belief that the higher the energy level per impact the more fracture occurs for an equal amount of work. Rock as well as concrete is a brittle material. It is weak in both tension and shear. Fracture occurs when the tensile and shear stress reach a certain level. High energy level impact would cause not only more fractures but also cause them to propagate a greater distance. Therefore, it would cause larger fragments than would a low energy impact and save energy by not breaking rock into small fragments than necessary.

1-3 COMPARISON OF ANALYTICAL AND EXPERIMENTAL RESULTS OF TOOL-ROCK IMPACT (TYPE A FLAT TOOL)

The tool-rock impact problem was analyzed and was based on Mahban's experimental results and certain assumptions of tool behavior. It was also found that the tool-rock impact analysis can be replaced approximately by a rigid mass-spring system. We will now compare the experimental and analytical results.

Mahban obtained the values of $k_1$ and $k_2$ in his experiment on limestone for four different stress levels ($\sigma_o$) of incident rectangular stress waves along a 40-inch rod. The incident velocities ($V_o$) of a
Figure 21 Variations of specific energy and volume removal ($m_e=1.08^*$)
tool corresponding to his stress levels can be obtained according to the equation for one-dimensional elastic wave in the tool (Figure 4).

\[ V_o = \frac{2\sigma_o}{c_o^2} \tag{21} \]

where \( c_o \) and \( f \) are one-dimensional wave velocity and the mass density of the tool, respectively.

The values of stress level (\( \sigma_o \)) and bilinear slopes obtained by Mahban, and the corresponding incident velocities of a tool are shown in Table 3. The rebound velocity \( V_r \) of the tool can be calculated approximately by considering the energy of the rigid mass-bilinear spring system, and is given in equation (7). The values of rebound velocity, \( V_r \), of the tool corresponding to Mahban's stress level are also listed in the following table.

Table 3 Stress Levels of Mahban's Experiment and Corresponding Incident and Rebound Velocity of a Tool

<table>
<thead>
<tr>
<th>Stress Level</th>
<th>( \sigma_o ) (ksi)</th>
<th>High</th>
<th>Med-High</th>
<th>medium</th>
<th>Low</th>
</tr>
</thead>
<tbody>
<tr>
<td>k_1/A (10^6 lb./in.(^3))</td>
<td>2.74</td>
<td>2.89</td>
<td>3.69</td>
<td>3.17</td>
<td></td>
</tr>
<tr>
<td>k_2/A (10^6 lb./in.(^3))</td>
<td>9.10</td>
<td>7.21</td>
<td>6.82</td>
<td>6.67</td>
<td></td>
</tr>
<tr>
<td>Incident Velocity</td>
<td>( V_o ) (in./sec.)</td>
<td>520.0</td>
<td>326.5</td>
<td>104.0</td>
<td>78.7</td>
</tr>
<tr>
<td>Rebound Velocity</td>
<td>( V_r ) (in./sec.)</td>
<td>283</td>
<td>207</td>
<td>76.4</td>
<td>54</td>
</tr>
</tbody>
</table>
Figure 22 is a plot of incident velocity versus the rebound velocity for a flat tool (Type A) according to the analytical results presented in Table 3 and the results obtained in this experiment. The figure shows that the actual rebound velocity is always lower than calculated, i.e., the analytical value of $V_r$ is the upper bound of the actual case. The reasons for a lower actual value may be due to the following factors:

1. The actual rebound velocity can not be measured accurately due to the friction between the tool and the spring gun barrel in the experiment. This effect is especially important when the rebound velocity is small.

2. Misalignment of the axis of tool and the normal line of the rock surface will cause the impact to initiate at a point on the periphery of the tool before the surfaces come into complete contact with each other; i.e., two impacts actually occur in one single incident-rebound process. This would increase the energy loss. The direct result would be a lower rebound velocity than a perfect surface-to-surface single impact as we assumed in evaluating the value of $V_r$ by analytical means. Work is also done in compressing the air between the tool and rock surface.

3. Assumptions were made in both cases in the theoretical analysis. Therefore, the models of analysis may not be able to represent the actual condition.

4. The value of $k_1$ and $k_2$ are chosen according to the best fit straight lines. Therefore, the calculated $V_r$ is only an approximation according to equation (7).
Figure 22  Incident -rebound velocity of a flat tip tool (Type A)
CHAPTER II TOOL IMPACT ON A TRANSMISSION LINE

In the sonic drilling process, a short bouncing cylindrical tool is used between the transmission line of a transducer and the load. Therefore, the problem of tool-line impact is of importance in analyzing the overall drilling process. The problem involves vibration and wave phenomena in both short (tool) and long (transmission line) cylindrical rods in the analysis of tool-line impact. The problem of impact on long cylindrical rods has been studied extensively by researchers since Hopkinson in 1914. Almost all the researchers were interested in the wave phenomena in long rods. Little attention has been given to rods of small length-to-radius ratio; the reason being that the behavior of short cylindrical rods is very complicated according to the three-dimensional theory of elasticity. With the help of the computer, McNiven and Percy found the approximate natural frequencies of axial vibration of short cylindrical rods. Bertholf considered the wave propagation in a finite rod due to a step stress applied at one end of the rod while the other end is free. The above two studies on the short rod still give very little direct help in solving the present problem in which the contact force between tool and line is very short in duration (measured time is less than 50 microseconds). Because of the difficulties involved, previous analysis on the tool-line impact are based on two simplifying assumptions. One is to assume that the one-dimensional theory of elastic wave propagation for long rods is still applicable to tools of short length during impact. The other is to treat the tool as a rigid body.
The stress waves produced in the transmission line, according to the above assumptions, are shown in Figure 23 and Figure 24. Here we substitute for the transmission line a semi-infinite rod of the same diameter as the tool. A typical stress wave measured in laboratory experiments of tool-line impact is shown in Figure 25. This shows a considerably different wave shape from those shown in Figures 23 and 24. Therefore, the above two theoretical approaches do not describe the problem of tool-line impact. Feng conducted an experimental investigation of the longitudinal impact of steel balls on the tip of a vibrating sonic transmission line. In his work a theoretical study on the ball-line impact was found to be in very good agreement with the experimental results. It is very interesting to note that the shape of a stress wave produced in his ball-line impact is very similar to the result obtained in the tool-line impact experiment mentioned previously (Figure 25). The theoretical background for Feng's analysis is that the tip of the tool must be a curved surface.

In the discussion of specific energy of rock removal, it was found that the higher the energy level of tool per impact the more fracture results for an equal amount of work; in other words, the less the specific energy required for rock removal. Therefore, in discussing tool-line impact, the increase of rebound velocity from the line is an important criteria in the search for an effective end geometry of the tool. Another thing that should be noted here is that the rotation of the tool about one of its diameters should be avoided during drilling because it is only a waste of energy.
Figure 23 Stress wave in the transmission line due to impact of one-dimensional elastic tool

\[ \sigma_o = \frac{1}{2} V_o \sqrt{\rho Y} \]

Figure 24 Stress wave in the transmission line due to impact of rigid tool

\[ \sigma_o = V_o \sqrt{\rho Y} \]

Figure 25 Measured form of the stress wave in the transmission line due to impact of a tool

\[ F(t) = \sigma A \]
An experimental study of tool-line impact was performed. The study was restricted to impact on a static transmission line. However, as shown by Feng, this impact behavior is of use in understanding impact on the vibrating transmission line. Special attention in this experiment was given to the effect of end geometry on the rebound velocity of the tool, the energy loss during impact, the relationship between the incident velocity and rebound velocity, the stress waves produced, and the effect of the tip geometry of the transmission line.

2-1 EXPERIMENTAL PROCEDURE

A photograph and schematic diagram of the experimental set-up is shown in Figure 26. This set-up is essentially the same as the one used in the tool-rock impact experiment (Figure 11). The only difference is that one of the two oscilloscopes is substituted by a counter and is used to measure the strain waves in the transmission line due to impacts of the tool. The counter is used to measure the incident velocity while the other oscilloscope is used to measure the rebound velocity.

Figure 27 shows the various tools used in this study; all of them are one inch in diameter and one inch long. Four different end geometries of the tool were used in this experiment. All tools shown were made of cold-rolled steel. Since only a few impacts per tool were produced, it was not necessary to use special materials or hardened steel for the tools.
Figure 26 photograph and schematic diagram of experimental set-up.
2-2 RESULTS

2-2-1 End Geometry of the Tool:

Four different end geometries were investigated for approximately the same impact velocity of 550 in./sec. Strain waves and rebound velocity were obtained and studied. Some of the results are summarized below.

1. Secondary impact: When a tool approaches the transmission line, the longitudinal axes of the tool and line usually are not perfectly parallel to each other. For tool types 2, 3, and 4, the contact between tool and line generally starts at a point on the periphery of the tool and a point on the surface of the transmission line tip. A very short time later, the surface of the tool rotates about the contact point and comes into complete contact with the surface of the transmission line tip. This phenomenon is called secondary impact. The stress wave that goes into the transmission line has two peaks (Figure 28). If the axis of the tool is perfectly parallel to the line, only single impact will occur, and there is only one peak in the transmitted wave. The Type 1 tool has a slightly curved surface and for this, there is only a single impact in every circumstance. This illustrates why the curves are more consistent for the Type 1 tool for several impacts at the same incident velocity.

2. Percentage of rebound energy from the line (Re) and the energy loss due to impact: We now define the percentage of rebound energy Re as the ratio of $E_r$ and $E_o$. 
Figure 28  Stress waves in transmission line due to impacts of various tools  
(Time base: 5 μsec./division, Ordinate (stress): 5 ksi/division)
\[ \text{Re} = \frac{E_r}{E_o} = \left(\frac{V_r}{V_o}\right)^2 \times 100 \]  

(22)

The value of Re for tools of different end geometry are shown in Table 4 for \( V_o = 550 \text{ in./sec.} \).

Table 4  Percentage of Rebound Energy from The Line  
\((V_o = 550 \text{ in./sec.})\)

<table>
<thead>
<tr>
<th>Type of Tool</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re</td>
<td>7.16</td>
<td>2.39</td>
<td>3.00</td>
<td>4.21</td>
</tr>
</tbody>
</table>

The main difference between the Type 1 tool and the rest of the tools is that Type 1 tool has a slightly curved surface with a large of curvature while the other tools have essentially flat surfaces. The contact between the Type 1 tool and the transmission line starts at a point close to the center of the tip surface while it starts at the periphery of the other tools. With this difference, the Type 1 tool is expected to yield higher rebound velocity because; (1) the moment arm of the couple caused by the inertia force of the tool and the reactive force from the transmission line is smaller for the Type 1 tool, or in other words, a smaller couple is applied to the Type 1 tool. Therefore, it possesses less rotational energy after impact, and; (2) even though the contact starts at a point for the Type 1 tool, with the large radius of curvature, the contact area can be larger during impact than with other tools where contact starts at the periphery; this is because the curved surface is rather flat and little elastic deformation at the tip will bring a large area into contact with the transmission line. Hence, less plastic deformation occurs for the Type 1 tool and, therefore, less
energy loss from this cause. With less energy loss and less rotational energy, it is not surprising to see, in Table 4, that the Re value is larger for the Type 1 tool. Therefore, this tool should be more favorable for the sonic drilling system.

3. Rebound velocity and stress waves in the transmission line:

During impact between a cylindrical tool and a transmission line, a large force $F(t)$ occurs at the contact surface. The force is equal to the area times the stress level of the wave that is transmitted into the line, as shown in Figure 25, where $V_o$ is the incident velocity, $V_r$ is the rebound velocity, $c$ is the wave velocity, $\sigma(t)$ is the stress wave in the line, and $m$ is the mass of the tool. According to the principle of conservation of linear momentum, we have

$$A \int_0^T \sigma(t) \, dt = m (V_o - (-V_r))$$  \hspace{1cm} (23)

where $T$ is the duration of contact. From the experimental result, $\sigma(t)$ is a positive function of time (i.e., it is always compressive). If we have $\sigma(t)$, then $V_r$ can be calculated from the above equation as

$$V_r = \frac{A}{m} \int_0^T \sigma(t) \, dt - V_o$$  \hspace{1cm} (24)

where the integral gives the area enclosed by the stress wave. Therefore, the rebound velocity $V_r$ is directly proportional to the enclosed area according to equation (24). Figure 28 shows that the area enclosed by the stress wave for the Type 1 tool is larger than the rest of the tools,
and, therefore, it is expected that the Type i tool has a larger rebound velocity.

2-2-2 Tip Geometry of the Transmission Line:

To investigate the effect of tip geometry of a transmission line on the rebound velocity and stress produced, a tip was attached directly on the end of a long rod in two different ways. Figure 29 shows the dimension of the tip and the means of connecting it to the transmission line. The stress waves produced under the impact of Type 1 and Type 2 tools are shown in Figure 30 and Figure 31, respectively. The incident velocity of the tools was approximately 550 in./sec. There are two important features shown in these figures. The maximum stress is reduced by using an enlarged tip and the duration of contact is increased due to a secondary pulse shown. The reason for the above two phenomena is because the transmission line with an enlarged tip is no longer a prismatic cylindrical rod. Wave reflection occurs at sections where the cross-sectional area changes. This result suggests the possible desirability of having a lumped mass at the tip of the transmission line to reduce the peak of the stress wave into the body of the transducer, therefore reduce the possibility of ceramic damage.

In the following table (Table 5), the percentage of rebound energy \( Re \) for \( V_0 = 550 \) in./sec. is shown. These results show that the rebound velocity is not affected by changing the area of the transmission line tip. More tests must be given to get more conclusive results. Comparing the value of \( Re \) in Table 4 (7.16%) and Table 5 (11.4%) for the Type 1 tool without an enlarged transmission line tip, there is a significant
Figure 29 Transmission line tip dimension and tip-transmission line assemblies.
Figure 30 Stress waves from tool impact. (Type 1 tool)

Figure 31 Stress waves from tool impact. (Type 2 tool)
Table 5 Values of Re for Three Different Transmission Line Tip Geometries

<table>
<thead>
<tr>
<th>Tip Geometry</th>
<th>(A) Without Enlarged Tip</th>
<th>(B) With Enlarged Tip</th>
<th>(C) With Inverted Tip</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type 1 Tool</td>
<td>11.4%</td>
<td>9.5%</td>
<td>9.4%</td>
</tr>
<tr>
<td>Type 2 Tool</td>
<td>2.7%</td>
<td>3.1%</td>
<td>2.7%</td>
</tr>
</tbody>
</table>

difference noted in these values. The reason for this difference is the following: the value in Table 4 was obtained with a new tool made of cold-rolled steel while the value in Table 5 was obtained when the tip of the tool was flattened out due to plastic deformation in persistent impacts which increased the area of contact between the tool and transmission line. Therefore, less plastic deformation loss, or higher rebound energy, is detected in the latter case (Table 5).

2-2-3 Rebound Velocity of the Tool and Stress Waves in the Static Transmission Line:

The relationships between maximum stress and incident velocity of the tool are shown in Figures 32 and 33 for the Type 1 tool and Type 2 tool, respectively. The relationship between rebound velocity and incident velocity of the tool is shown in Figure 34 and 35 for the Type 1 (curved tool) and Type 2 (Flat tool), respectively.

The linear relationship between rebound and incident velocity shown in Figure 34 is in agreement with the results found by Feng in ball-line impact. The ratio $\frac{V_r}{V_o}$ is smaller in this experiment because the tip
\[ \sigma_{\text{max}} (\text{psi}) = C v_o (\text{in./sec.}) \]

\[ C = 56.5 \text{ lb.-sec./in}^3 \]

**TYPE 1 TOOL**

2.5" radius

27" Flattened area due to persistent impact

**Figure 32** Incident velocity-maximum stress for Type 1 tool.
Figure 33 Incident velocity-maximum stress for Type 2 tool
Figure 34 Incident-rebound velocity relationship for Type 1 tool.

\[ V_r = D V_o \quad D = 0.315 \text{ (linear region)} \]
Figure 35 Incident-rebound velocity relationship for Type 2 tool
of the tool has less curvature and has been flattened out due to persistent impact. Also, plastic deformation of the tool, which is made of relatively soft cold-rolled steel, is believed to have a large effect. Figure 35 does not have as good a linear relationship between rebound and incident velocity as in Figure 34. This is mainly due to the fact that impact of a flat tool against the transmission line is not consistent for all impacts. The rebound velocity varies considerably even for the same incident velocity.

2-3 REBOUND VELOCITY OF TOOL FROM A VIBRATING TRANSMISSION LINE

It was found experimentally, as described in Section 2-2-1, that the tool with a slightly curved surface at the end is favorable for use in sonic impact. Feng has solved the impact problem of a spherical ball against a vibrating transmission line by theoretical means. The same approach is used here for the case of a cylindrical tool with a curved contact surface.

The tip displacement of a resonant transducer (or transmission line) can be expressed as:

\[ u_0(t) = c_1 \sin(\omega t + \phi) \quad (25) \]

where

- \( \phi \) = phase angle
- \( c_1 \) = amplitude of tip displacement
- \( \omega \) = angular frequency of resonance
- \( t \) = time measured from the start of tool-line impact
The displacement $\beta_1$ of the center of the transmission line tip during impact is a combination of: (1) the displacement due to transducer tip vibration $(u_0(t) - u_0(0))$, (2) the displacement due to the stress wave traveling down the transmission line, $u_1$, and (3) the local deformation, $w_1$. The positive directions of $u_1$, $u_0$ and $w_1$ are shown at the upper left hand corner of Figure 36. Therefore, we have

$$\beta_1 = u_1 + w_1 + u_0(t) - u_0(0) \quad (26)$$

Similarly, the displacement at the tip of the tool, $\beta_2$, is

$$\beta_2 = u_2 + w_2 \quad (27)$$

where $u_2$ is the rigid body displacement of the tool, and $w_2$ is the local deformation of the tool tip. The positive direction is assumed to be the same as $\beta_1$.

During the course of impact,

$$\beta_1 = \beta_2$$

or

$$w_2 - w_1 = u_1 - u_2 + u_0(t) - u_0(0) \quad (28)$$

The impact period is short and the transmission line is usually long enough that the wave front of the reflected stress wave will not reach the tip during this period. We also assume that the stress wave due to previous impact has been damped out. Then, the displacement $u_0$ can be expressed as
\[ u(x,t) = f(x-c_0 t) \]  \hspace{1cm} (29)

From equation (29), we can obtain

\[ \dot{u} = -c_0 f'(x-c_0 t) \]  \hspace{1cm} (30)

and

\[ F = EA \frac{\partial u}{\partial x} = E A f'(x-c_0 t) \]  \hspace{1cm} (31)

Eliminating \( f'(x-c_0 t) \) in equation (30) and equation (31), we obtain

\[ \ddot{u} = -\frac{F}{P A c_0} \]  \hspace{1cm} (32)

and

\[ \dot{u}_1 = \dot{u}_o(0,t) = -\frac{F}{P A c_0} \bigg|_{x=0} \]  \hspace{1cm} (33)

By considering the conservation of linear momentum of the tool, one gets the following equation

\[ \frac{d^2 u}{dt^2} = V_o + \frac{1}{m} \int_T^T F \, dt \]  \hspace{1cm} (34)

Substituting equations (33) and (34) into equation (28), and defining the "approach", \( \alpha \), as \( \left( w_1 - w_2 \right) \), we have

\[ \dot{\alpha} = V_o + \frac{1}{m} \int_T^T F \, dt + \frac{F}{P A c_0} \bigg|_{x=0} - c_1 \omega \cos(\omega t + \phi) \]  \hspace{1cm} (35)

Hertz contact theory states that
\[ F = -K \alpha^{3/2} \]  
(36)

where \( K \) is a constant depending on elastic properties and geometry of the contact surface, and is given as:

\[ K = 2ER_2^{1/2}/(1-v^2) \]  
(37)

for a flat transmission line tip where it is assumed that tool and line are made of the same material. Also, \( R_2 \) is the radius of curvature of the tool end, and \( v \) is Poisson's ratio. With equations (35), (36) and (37), a differential equation for \( \alpha \) is given by:

\[ \frac{d^2\alpha}{dt^2} + \frac{K}{\rho A_0} \frac{d\alpha^{3/2}}{dt} + \frac{K}{m} \alpha^{3/2} = c_1 \omega^2 \sin(\omega t + \phi) \]  
(38)

A computer program similar to Feng's was written to solve the incident-rebound relationship for tools of different mass and various curvature at the impact end. Figure 36 is a typical example of the impact-rebound relationship for various tip vibration amplitudes. All the tools are one inch in diameter but with different tip and end geometries. The mass of the tools is designated as \( m_e \). In Figure 36, a tool with curved surface of ten inches radius is used. The tool has a \( \frac{1}{2}'' \) equivalent length \( (m_e = \frac{1}{2}'' \) ), i.e., the tool has a mass which is equivalent to a flat ended rod of one inch in diameter and \( \frac{1}{2}'' \) in length. One thing that should be pointed out here is that the rebound velocity is calculated in the sense of average as stated by Feng, because the actual rebound velocity varies as to where the tool comes into contact with the
Figure 36 Incident-rebound velocity relationship
($R=10^{	ext{in.}}$, and $m_e=\frac{1}{2}^\text{in.}$)
transmission line tip.

Figure 37 is a plot similar to Figure 35 but with four different values of $m_e$. The main purpose of such a plot is to examine the effect of tool mass on the rebound velocity. It is easy to see that the rebound velocity drops as the mass increases. The value of rebound velocity not only depends upon the mass of the tool but also depends upon the curvature of tool tip. Figure 38 shows the effect of curvature of the tool end on the rebound velocity for $m_e = \frac{1}{4}$. It is interesting to note that the rebound velocity increases at the left-hand side of this figure while it decreases at the right-hand side for all $c$ larger than zero if we increase the radius of tool tip, $R$. The transition between these two regions along a straight line $V_r = 2.27V_o$, which is also shown in the figure.
Figure 37 Variation of incident-rebound velocity relationship with respect to equivalent length $m_e$ ($R = 5''$)
Figure 38 Effect of curvature on the rebound velocity ($m_e = 4''$)
CHAPTER III TRANSUDER VIBRATION AND RESPONSE TO IMPACT

In the previous two chapters, we discussed the tool-load and tool-line phases of the impact coupling process of sonic rock drilling. In this chapter, we will study certain aspects of the transducer operating characteristics. The capability of a sonic tool to do work depends upon the energy it possesses, i.e., the mass and velocity before it hits the rock. As we learned in the study of tool-line impact, this velocity increases as the tip vibration amplitude of the transducer increases. That is, the rebound velocity of the tool from the vibrating transmission line increases as the internal energy increases because the internal energy is proportional to the second power of vibration amplitude. Therefore, the focusing point of this study is also extended to the study of transducer internal energy as well as the tip vibration amplitude.

As we pointed out previously, study in the area of transducer characteristics has been extensive, yet, much is still unknown of transducer performance. This holds particularly for the response of the transducer during and immediately after impact from the sonic tool. Thorough theoretical as well as experimental study is needed to get a complete understanding on this matter. However, it is not the purpose at this stage to go into detailed research in that area. Instead, it is simply to study the internal energy and tip vibration amplitude of a sonic transducer for the purpose of analyzing the sonic interaction process and finding the power transmission of the transducer system. The transducer
response to impact will be modeled based on observed transducer behavior and basic vibrational principles governing solid materials.

Ma 16 conducted an experiment in the study of energy removal and transducer recovery during tool impact. From this experiment, an interesting phenomenon can be observed. The recovery of the tip vibration amplitude (velocity as well as displacement) is along an exponential curve as shown within the dotted region of Figure 39. Two experimental results of such removal and recovery curves, after data reduction, are shown in Figure 40. The ordinate is non-dimensionalized with respect to the total amount of amplitude drop immediately after impact. We will give some theoretical background in addition to the previous experimental observations to justify the resulting modeling of the recovery of a sonic transducer after impact.

For convenience of analysis, we will start with the discussion of the most simple and basic piezoelectric resonator, a cylindrical sandwich type, as shown in Figure 41. Then, the results of this analysis will be expanded to the study of the vibration and damping characteristics of a more complicated sonic resonator.

3-1 FORCED VIBRATION OF A CYLINDRICAL SANDWICH TYPE RESONATOR

The most simple and basic piezoelectric resonator is the cylindrical sandwich type as shown in Figure 41. Because of its simple geometry, the analysis is straightforward compared with resonators of more
Figure 40 Recovery of tip vibration amplitude

Figure 39 Transducer tip vibration

Figure 38 Tool line impact
complicated shape. It is the purpose of this section to discuss this resonator in hope of expanding the results to finding the general vibration and damping characteristics of a more complicated sonic resonator.

![Piezoelectric element](image)

End mass  End mass

Figure 41 Cylindrical sandwich type resonator

Usually two kinds of damping exist in a vibration system, viscous damping and Coulomb damping. In a sonic transducer, viscous damping is present within the metal and ceramic materials while the Coulomb damping exists as interfacial friction between the metal and ceramic parts. The viscous damping force is along the direction of particle velocity. In the sonic transducer, radial vibration is always coupled with longitudinal even though the first radial resonant mode is usually a higher order mode than the first longitudinal mode. In the discussion of longitudinal resonance of a transducer, the effect of radial motion is usually neglected and the problem treated as one dimensional. The direction of the Coulomb friction force in the sonic resonator is radial. The effect of interfacial friction on longitudinal vibration is, therefore, indirect. With the above arguments, the Coulomb friction
and viscous damping force in the radial direction can be neglected if the approximation is made to discuss longitudinal vibration only.

In a steady state resonance condition, the amplitude of longitudinal vibration is a constant. The force that the ceramic applies to the metal part also reaches a steady state constant amplitude. In the following discussion, we will analyze the vibrational characteristics of a simplified model of a cylindrical sandwich type resonator as shown in Figure 42. Two equal and opposite forces $P_0 \sin \omega t$ are applied on a cylinder of length $l$ and area $A$ at each side of the midpoint.

![Simplified model of cylindrical sandwich type resonator](image)

**Figure 42** Simplified model of cylindrical sandwich type resonator

Viscous damping in metal is usually of the Voigt type. The stress-strain relation can be expressed as

$$\sigma_x = Y \epsilon_x + Y' \frac{\partial \epsilon_x}{\partial x}$$

(39)
where \( x \) is the direction of stress and strain, \( Y \) is the Young's modulus, and \( Y' \) is a damping factor. The one dimensional equation governing the cylinder is

\[
\frac{Y}{2\pi} \frac{\partial}{\partial x} \left( A \frac{\partial u}{\partial x} \right) + \frac{Y'}{\pi} \frac{\partial}{\partial t} \left( A \frac{\partial^2 u}{\partial x^2} \right) - \rho A \frac{\partial^2 u}{\partial t^2} = 0.
\]

\( A(x) \) = cross-sectional area at point \( x \)

\( u(x,t) \) = longitudinal displacement

For the simplified model, we can express \( u(x,t) \) as an infinite series of orthogonal vibrational modes \( u_n(x) \) as follows

\[
u(x,t) = \sum_{n=0}^{\infty} \phi_n(t) u_n(x) = \sum_{n=0}^{\infty} \phi_n(t) \cos(p_n x)
\]

with

\[
u_n(x,t) = \cos(p_n x)
\]

\[
p_n = \frac{n\pi}{l} \quad (n = 1, 2, 3, \ldots)
\]

Using the principle of virtual work, a set of differential equations for \( \phi_m(t) \) can be obtained as

\[
M \ddot{\phi}_m + \gamma_m \dot{\phi}_m + E M \phi_m = -2P_0 \sin\left(\frac{mn}{l}\right) \sin\left(\frac{m\pi}{2}\right) \sin(\omega t)
\]

where

\[
M = Ap_1/2, \quad \gamma_m = AY^m \pi^2/2l, \quad E_m = AYm^2 \pi^2/2l, \quad P_m = -2P_0 \sin\left(\frac{mn}{l}\right) \sin\left(\frac{m\pi}{2}\right)
\]

The transient solution of equation (42) is
\[
\phi_m = e^{-\frac{\gamma_m t}{2M_0}} \left\{ A_m \sin(q_m t) + B_m \cos(q_m t) \right\}
\]  \hspace{1cm} (43)

where

\[
q_m = \left\{ \frac{E_m}{M_0} - \frac{\gamma_m^2}{4M_0^2} \right\}^{\frac{1}{2}}
\]

The steady state solution is

\[
\phi_m = \frac{p_m}{\omega Z_m^*} \sin(\omega t - \psi_m)
\]  \hspace{1cm} (44)

where

\[
Z_m = \left( \frac{E_m}{\omega} - M_0 \omega \right)^2 + \gamma_m^2
\]

\[
\psi_m = \frac{\omega \gamma_m}{E_m - M_0 \omega^2}
\]

and, the steady state displacement \( u_s(x,t) \) is

\[
u_s(x,t) = \sum_{n=1}^{\infty} \frac{p_m}{n \omega Z_m} \sin(\omega t - \psi_m) \cos(p_m x)
\]  \hspace{1cm} (45)

When the exciting frequency \( \omega \) reaches the value \( (E_1/M_0 - \gamma_1^2/2M_0^2)^{\frac{1}{2}} \), the coefficient \( P_1/\omega Z_1 \) in equation (44) becomes a maximum, and

\[
\frac{P_1}{\omega Z_1} = \frac{p_1}{\gamma_1^2} = \frac{-2p_0 \sin(\pi a)}{\gamma_1^2}
\]  \hspace{1cm} (46)

In this condition, the system resonates at its lowest mode because the terms higher than one in equation (45) can be neglected in comparison
with the first term. Therefore,

\[ u_s(x,t) = \frac{-2p_0}{\eta_1^2} \cos\left(\frac{\pi x}{1}\right) \sin(\omega t - \frac{\pi}{1}) \sin(\frac{\pi a}{1}) \]  

(47)

Since \( Y' \) (or \( \eta_1 \)) is usually very small for metals, we have

\[ \omega \approx \eta_1 = \sqrt{\frac{E_1}{M_0}} = \frac{\pi}{1} \frac{Y}{P} \]

\[ \eta_1^2 = \frac{A\pi^2 c_\theta Y'}{212} \sqrt{1 - \left(\frac{\omega Y'}{2Y}\right)^2} \]  

(48)

It is known that \( Q \) (factor of merit) = \( \frac{F}{\Delta F} = \frac{2\pi I}{\Delta I} = \frac{Y}{\omega Y'} \) 

(49)

where \( F \) : resonant frequency

\( \Delta F \) : bandwidth (between half-power points)

\( I \) : internal energy

\( I' \) : energy dissipated in a cycle

Equation (48) can be rewritten as

\[ \eta_1^2 = \frac{A\pi^2 c_\theta Y'}{212} \sqrt{1 - \left(\frac{1}{2Q}\right)^2} \]  

(50)

\( Q \) is usually very large (on the order of 50 to \( 10^4 \)), so we can neglect the second term in the radical sign and get

\[ \eta_1^2 \approx \frac{A\pi^2 c_\theta Y'}{212} \]  

(51)

With equations (43), (47), and (51), we have the general solution for the vibration of the resonator.
\[ u(x,t) = \sum_{n=1}^{\infty} e^{-2M^o \frac{n^2}{\eta_n^2}} \left\{ A_n \sin(q_n t) + B_n \cos(q_n t) \right\} \cos(\frac{n\pi x}{L}) \]

\[ -\frac{4P_0^2 \sin\left(\frac{n\pi}{L}\right)}{A_n^3 \eta_0 Y'} \cos\left(\frac{n\pi x}{L}\right) \sin(\omega t - \psi_1) \]

(52)

In actual operation, the value of \( P_0 \) depends on the terminal voltage applied to the ceramics and prestress in the transducer. These two variables are assumed to be fixed during vibration in this study. Another variable that affects the value of \( P_0 \) is the previously mentioned interfacial friction. Since the interfacial friction force depends upon the dynamic behavior of the transducer, the value of \( P_0 \) is a function of time. In the previous analysis, the effect of interfacial friction is neglected and the value of \( P_0 \) is assumed to be constant to simplify the problem.

The time factor \( S_n \), which is equal to \( 2M^o / \eta_n \) in Equation (52), is a function of \( Y' \); the exponential coefficient \( e^{-t/S_n} \) represents the effect of damping in the system. We know that

\[ S_n = \frac{2M^o}{\eta_n} \]  

(53)

Since

\[ \omega^2 \approx \frac{E_1}{M^o} \]  

(54)

we have
In the previous section, we discussed the forced vibration of a simplified cylindrical sandwich type resonator. It was found that the resonator will vibrate in a natural mode if the exciting frequency is close to the natural mode. On the transient part of the solution, the time factor $S_n$ depends on the damping characteristics of the materials used. The amplitude of steady state vibration is controlled by damping as well as the voltage applied to the ceramics.

Now we will expand the results obtained for a simple cylindrical sandwich type resonator to a more complicated resonator such as the P-11 transducer with body, resonant horn and transmission line. It is believed that the same type of solution will exist, except with different mode shapes, time factor and steady state amplitude. That is to say, we have a steady state solution with a lowest natural mode if the frequency of excitation is at that mode, and an amplitude which increases as the terminal voltage increases and damping decreases. The transient solution is an infinite series combination of all the natural modes with exponential time functions. The time factors depends on all the damping characteristics of transducer. In written form, we

\[ S_n = \frac{2q_1}{\omega n^2} = \frac{S_1}{n^2} \]  

Where

\[ S_1 = \frac{2Q_0}{\omega} \]  

3-2 TRANSDUCER VIBRATION AND RECOVERY OF TRANSDUCER TIP AMPLITUDE
will have the following:

\[ u(x,t) = A_1(x) \sin(\omega t - \phi_1) - \sum_{n=1}^{\infty} e^{-t/S_n} \left\{ A_n \sin(q_n t) + B_n \cos(q_n t) \right\} f_n(x) \]

(57)

where \( f_n(x) \) is the nth mode shape. The lowest mode of vibration is \( f_1(x) \).

In general, since the mass of the tool is rather small, the energy loss of the transducer for each impact is small compared to the total stored energy. If, for example, a tool of mass 0.003439 slug \( (m_e = \frac{1}{3} \text{ slug}) \) impacts against a transducer, which has a tip vibration amplitude of 1.15 mils, and the incident and rebound velocity of the tool are equal to 66.6 in./sec. and 111.1 in./sec., respectively; the energy loss of the transducer is equal to the energy gain of the tool and is about 1.124 in.-lb. while the internal energy is approximately equal to 17.25 in.-lb. (we will talk more about the internal energy in Section 3-3). The percentage of energy loss is about 7.54%. Since the energy loss of the transducer is small per impact, the mode of vibration after impact is essentially \( f_1(x) \), i.e., the value of \( A_n \) and \( B_n \) are small compared to \( A \).

According to previous analysis,

\[ S_n = S_1/n^2 \]

(55)

This tells us that the higher order terms in the transient solution damp out much faster than the lowest mode. Therefore, we make another assumption here to neglect the effect of higher order terms in equation (57), giving
The above equation represents approximately a harmonic vibration with gradually varying amplitude. In another words, the recovery of the transducer from energy loss due to tool-line impact is exponential. The above analysis has been confirmed by the experimental results stated early in this chapter. Therefore, we may model the recovery of transducer tip vibration amplitude $c(t)$ as follows

$$c(t) = C - A e^{-t/S}$$

Where $C$ represents the maximum amplitude at the no load condition, $A$ is the amplitude drop of the transducer after impact and is dependent on the energy drain by the tool. Right after impact, $t=0$, the tip amplitude is

$$c(0) = C - A$$

As time goes on, $c(t)$ increases in an exponential order with a time factor $S$ which depends on the total damping characteristics of a sonic transducer, i.e., the damping in ceramic as well as in metal is also considered.

3-3 INTERNAL ENERGY AND TIP VIBRATION AMPLITUDE

The ability of a sonic transducer to do work is dependent on the amount of energy in the transducer. It is known that internal stored energy in a longitudinally vibrating resonator is in the form of strain
and kinetic energy. The amount of energy is proportional to the square of maximum displacement (or amplitude) at any point. In Chapter II, we used the tip vibration amplitude in discussing the tool impact-rebound relationship on a vibrating transmission line. We again use the amount of tip vibration amplitude as the measurement of the internal energy of a transducer.

It is not difficult to find the stored energy in terms of tip amplitude in the resonant horn and transmission line once the mode of vibration is known. It is more difficult to find the energy which is stored in the body of transducer. This difficulty arises due to the complicated structure of this part. An approximate value is obtained by assuming the approximate mode shape in the body and neglecting the energy stored in the ceramics. The proportional constant, $K$, for the P-11 transducer with a 10 inch transmission line in the equation

$$I = K c^2$$

is found approximately to be equal to 11.2 in.-lb./(mil)$^2$. In equation (60), $I$ is the internal energy, and $c$ is the tip vibration amplitude. The energy stored in the body of a stepped transducer has been calculated and is much smaller than in the resonant horn and transmission line. Hence the approximate value of $K$ thus obtained is expected to be not very far from the true value. The reason is that the stored energy in the resonant horn and transmission line can be calculated exactly while approximation is made only in calculating the stored energy in the body. If the energy in the body is small, the approximation just made would not affect the total stored energy drastically. The value of $K$ can be
considered as a measurement of the energy storage capacity of a sonic transducer. The larger the value of $K$ the more the energy can be stored.

The value of the tip displacement is important in the impact coupling process and difficult to obtain from purely theoretical analysis. The value is also dependent on the terminal voltage of the electric supply and temperature of the transducer system. An experiment was conducted to find the effect of temperature and voltage on the tip amplitude of an unloaded P-11 transducer with a ten-inch transmission line. It is found that the amplitude of vibration increases as the applied voltage increases as shown in Figure 43. Figure 44 shows the peak-to-peak amplitude and power supply as a function of frequency of vibration. The applied voltage is approximately 1,200 volts.

To find the effect of temperature change on the vibration characteristics of the P-11 transducer with a 10 inch transmission line, a set of curves similar to Figure 44 but at different temperatures is shown in Figure 45. It is interesting to note that the resonant frequency, amplitude and input power varies as the temperature changes. At room temperature the amplitude is largest while the power required to maintain steady state vibration is least. As time goes on, the heat accumulates in the transducer and causes the temperature to rise. The amplitude drops drastically while more energy is required to maintain resonance. This means that more energy is consumed as heat and acoustic noise at high temperature. There is no doubt that the temperature has a large effect on the properties of the piezoelectric material since the properties of metal
Figure 43 Amplitude-voltage relationship
(Temperature of transducer = 75°F)

Figure 44 Variations of amplitude and input power as function of driving frequency
Figure 45 The effect of transducer temperature on amplitude, resonant frequency and power.
are fairly stable for such small ranges of temperature change. Another important fact is the drop in resonant frequency of the system as the temperature rises. This is not particularly desirable if a single frequency power supply is to be used.
CHAPTER IV DRILLING RATE OF A SONIC TRANSDUCER IN HARD ROCK

As stated previously in the Introduction, the sonic rock cutting process is achieved by using an intermittent tool between the transmission line tip and the work surface. The tool acquires energy from impacting with the vibrating transmission line tip; it then impacts against the rock and rebounds from the surface. During impact between tool and rock, the tool loses part of its energy and causes some fracture of the rock. In previous chapters, we discussed details of each impact process separately and the recovery of the tip vibration amplitude after impact. In this chapter, we will combine the separate studies and see how work is performed by the system. One thing should be mentioned here is that we will talk about drilling of hard rocks only. The reason for this will be explained later in more detail.

The resonant frequency of a sonic transducer can be affected by many factors, such as voltage, temperature, and static force. The direct result of these detuning effects is to change the tip vibration amplitude. Since the sonic transducer is a high Q system, any small amount of frequency change can affect the tip vibration amplitude drastically. This, in turn, reduces the capability of the transducer to do work. Future efforts should be given to making the transducer resonant at all times during operation. In another words, we will discuss the system interaction only on the assumed conditions that the transducer is always in a resonant state, and that the tip vibration amplitude is a constant during steady state, no load operation. One thing should be mentioned here is
that we will discuss fixed voltage operation only, otherwise, the tip vibration amplitude would vary from time to time as the voltage changes.

4-1 INTERNAL ENERGY DURING STEADY STATE OPERATION

We have discussed the recovery of the transducer after tool-line impact. It was found that the tip vibration amplitude will recover in an exponential manner. If no further impacts occur, the tip vibration amplitude will eventually reach the steady state no load amplitude. In actual operation, the tool will bounce back and forth in the gap between the transmission line tip and load. The impacts of the tool against the tip can be at hundreds and even thousands of times per second. The tip amplitude has no chance of fully recovering back to its steady state no load condition because the second impact follows the first one in a short period. Hence, the internal energy during the steady state, no load vibration represents a maximum value.

As discussed in Section 3-2, the tip vibration amplitude is assumed to be

\[ c(t) = C - A e^{\frac{-t}{S}} \]  \hspace{1cm} (59)

The corresponding internal energy, according to equation (60) is

\[ I(t) = K \left( C - A e^{\frac{-t}{S}} \right)^2 \]  \hspace{1cm} (61)

Figures 46 and 47 represent the variation of tip vibration amplitude
Figure 46 Variation of tip vibration amplitude

\[ c = C - A e^{-t/S} \]

Figure 47 Variation of internal energy

\[ I = K(C - A e^{-t/S})^2 \]
and internal energy during operation. The tip amplitude and internal energy at the steady state unloaded condition are \( C \) and \( I_0 \), respectively. When a load is applied to the system, the tool acquires energy from impacting with the transmission line tip. As time goes on, the amplitude and internal energy recovers due to the power supply input. Before the transducer recovers to its full energy, the tool bounces back from the rock surface, and again drains energy from the transducer. This process goes on until a steady state condition is reached. The dashed lines represent the recovery curves of the transducer if no further drain of energy from the tool occurs.

If we expand equation (61), we have

\[
I(t) = K(C-A+A(1-e^{-t/S}))^2 \\
= K(C-A)\cdot (C-A) + 2K(C-A)A(1-e^{-t/S}) + KA^2\cdot (1-e^{-t/S})^2 
\]

(62)

The first term represents the internal energy right after impact. The second and third terms represent the recovery. It is easy to see that, in the second term, the recovery is also dependent on \( C \) and \( K \) as well as the time factor \( S \). The larger the value of \( C \) and \( K \) the faster the energy recovery.

In the steady state operating condition, the energy recovered equals the energy drained from the transducer, assuming \( T_i \) is the impact interval. Therefore, we have the following

\[
I_1 = I(T_i) = K(C-Ae^{-t/S})^2 = K(C-A)^2 + \Delta 
\]

(63)
where $\Delta$ is the energy that the tool acquired from the transducer, i.e.,

$$\Delta = \frac{1}{2} m(v_2^2 - v_1^2) + E_L$$

(64)

with $E_L$ being the energy loss due to impact. $V_1$ and $V_2$ are the velocities of the tool as shown in the lower right hand corner of Figure 48. If the transmission line tip and tool are made of hard material, not much plastic deformation should occur. Therefore, $E_L$ is considered to be small compared to $m(v_2^2 - v_1^2)/2$ and can be neglected. Thus, we have

$$\Delta = m(v_2^2 - v_1^2)/2$$

(65)

During this steady state condition, the internal energy is at a specific value before impact. This implies that the tip vibration amplitude is also at a specific value. Now, there are several questions to be answered: What is the value of the internal energy and tip vibration amplitude? What are the values of impact and rebound velocity of the tool against the transmission line tip and rock? These problems will be answered as the discussion proceeds.

In actual operation, the impact and rebound velocity of tool are somewhat random and so is the internal energy and tip vibration amplitude; but, in the sense of averages, they are assumed to be deterministic in the above analysis.
In section 4-1, we discussed the general idea of internal energy during the steady state operating condition. Now we will study the tool behavior during this condition. The tool-rock and tool-line phases of impact coupling process have been studied extensively in Chapter I and Chapter II separately. The present task is to combine them for the study of this coupling process. Feng studied the impact coupling of a spherical ball between a static and a vibrating transmission line. The present analysis follows his method of approach.

In Chapter I, we represented the incident-rebound velocity relationship of a tool impacting against rock by a best fit straight line while in Chapter II, we obtained the incident-rebound velocity relationship of a tool impacting against a vibrating transmission line. If we plot these two relationships on the same figure with the notation of incident and rebound velocity as shown in the lower-right hand corner of Figure 48, we obtain an example figure as shown. Here, in this case, a tool with half-inch equivalent length \( \frac{m_e}{m_a} = \frac{1}{2} \) and ten-inch radius of curvature of the tool-tip contact surface is used. The rebound-incident velocity ratio and tip amplitude \( c_t \) are assumed to be equal to 0.6 and 1.0, respectively. The steady state velocities happen to be at the point of intersection of these two relationships, i.e., the point \( W (V_1 = 57.5 \text{ in./sec, and } V_2 = 96.0 \text{ in./sec.}) \). To illustrate this, let the tool start at a velocity \( V_{01} \) and impact against the vibrating line (point \( M_1 \)). The rebound velocity \( V_2 \) from the line becomes the incident velocity \( V_2 \) for tool-rock impact (point \( M_2 \)). With this rebound velocity \( V_1 \) from rock,
Figure 48 Steady state velocities for $c_1 = 1.0$ mil, $m_e = 1/2$" and $R = 10"$
the tool impacts again against the transmission line tip (point $M_3$).

This process continues until the point of intersection $W$ is reached.

The same result will be obtained if we start at a lower velocity $V_{01}'$.

The above is true so long as the tip vibration amplitude remains unchanged.

If the amplitude $c_1$ changes so is the point $W$ representing the steady

state velocities. Therefore, we can say that steady state velocities

$V_1$ and $V_2$ are functions of $c_1$ since the tool-rock impact relation and

the geometry of the tool are assumed to remain unchanged during operation.

It is interesting to note that the situation may arise if the slope

of the rebound-incident velocity relationship for tool-rock impact

(slope $= V_1/V_2$) is so small that no point of intersection $W$ can be obtained.

This means that no steady state velocities can be reached. The problem

then becomes more complicated. It is believed that, in some instances,

the tool tip is in close contact with the rock while the transmission

line hits the tool at the end, i.e., the tool-rock and tool-line impacts

happen at more or less the same time. The slope of the rebound-incident

velocity relationship for tool-rock impact $(V_1/V_2)$ depends on the tip

geometry of the tool and the properties of the rock. Usually harder rock

yields higher values of $V_1/V_2$. In the remainder of this analysis, we

will consider only the case where steady state impact coupling occurs.

Therefore, we restrict ourselves to hard rock drilling in which inter­

section point $W$ exists for a specific tool geometry.

Another interesting thing in previous analysis of impact coupling

is that the steady state velocities do not change so long as the tip

vibration amplitude remains unchanged; the static force $F_s$ applied by
the support structure of the transducer plays no direct role. It seems to have no effect on the impact process. Actually the static force affects the steady state velocities via affecting the value of steady state tip vibration amplitude $c_1$. This will be clear as the discussion of impact coupling continues.

Now let's start the discussion of static force by looking at the dynamics of a sonic transducer considered by Graff\(^{19}\) (Figure 49). The rigid body motion is governed by

$$M \ddot{x} = F_s - f(t)$$  \hspace{1cm} (66)

where $M$ is the mass of the transducer, $f(t)$ is the time varying force at the tip of the transducer due to the impact of a sonic tool, and $F_s$ is the static force applied by the support structure. Force $f(t)$ is a combination of a sequence of impact forces, each impact force is high in magnitude but short in duration. It usually is less than 50 $\mu$sec. In actuality, the impact forces would be of various amplitudes and intervals, but thinking in terms of averages it is justifiable to consider equal amplitudes and spacings. The interval of impacts is quite long compared with each impact duration in sonic process. For example, if the frequency of impact is 1,000 impacts per second, the time interval is in the order of 1,000 micro-seconds.

Integrating equation (66) gives

$$M(x - x_0) = F_s t - \int_0^t f(t) \, dt$$  \hspace{1cm} (67)
Figure 49 Transducer acted upon by the static force $F_s$ and the impact force $f(t)$

Figure 50 Rigid body motion (velocity) of a sonic transducer acted upon by static force and impact
Since we assumed that each impact force is the same and the duration is short, it is justifiable to represent the second term as follows

\[ \int_0^t f(t) \, dt = I_m \sum_{n=1}^{N} H(t - t_n) \]  

(68)

where \( N \) is the number of impacts between the time interval 0 and \( t \), \( H \) is a Heavyside function. Now we rewrite equation (67) as follows

\[ N(\dot{x} - \dot{x}_0) = F_s \cdot t - I_m \sum_{n=1}^{N} H(t - t_n) \]  

(69)

This equation shows that the velocity of a sonic transducer has a sawtooth form as shown in Figure 50. Thus, the constant force \( F_s \) causes a linear increase in velocity, but this periodically offset by sudden velocity decreases due to the impact pulses. If the proper balance holds between the static force and the impulse \( I_m \) and impulse interval \( T_1 \), the increase in velocity is completely offset by the drop due to impact, so that

\[ F_s \cdot T_1 = I_m \]  

(70)

This is a statement of impulse balance, so that the average net change in momentum of the transducer is zero.

According to the definition of impulse,

\[ I_m = m(V_1 + V_2) \]  

(71)

Therefore, from equation (70), we have
The value of $T_1$ consists of impact times and the travelling times of the tool between the transmission line tip and the rock surface. The frequency of impact coupling is just equal to the reciprocal of $T_1$; or,

$$f = 1/T_1 = \frac{F_s}{m(v_1 + v_2)}$$ (73)

As stated previously, steady state velocities, $V_1$ and $V_2$, are function of tip vibration amplitude $c_1$. Thus, the relationship between impact frequency and the tip vibration amplitude $c_1$ are related to each other according to equation (73). Generally, the increase or decrease of $V_1$ and $V_2$ follows the trend of the value of $c_1$. When we increase the impact frequency, the amplitude drops, therefore, reducing the internal stored energy of the transducer. If the static force is kept constant during operation, it is easy to see from equation (72) that $T_1$ is a function of $c_1$.

4-3 POWER TRANSMISSION

Now let's go back to the discussion of internal energy of the transducer, and rewrite equations (63), (65), (59), and (72) as follows:

Internal energy just before impact is

$$I_1 = k(c - a)^2 + \Delta = k(c - Ae^{-T_i/S})^2$$ (63)

The energy drain in each impact is

$$T_1 = \frac{m(v_1 + v_2)}{F_s}$$ (72)
\[ \Delta = \frac{m(V_2^2 - V_1^2)}{2} \quad (65) \]

Tip vibration amplitude was given previously by \( c(t) = C - A \exp(-t/T) \), while the interval of impulse is

\[ T_1 = \frac{m(V_1 + V_2)}{F_s} \quad (72) \]

Eliminating \( \Delta \) in equations (63) and (65), and solving for \( A \), we obtain

\[ A = \frac{C \exp(T_1/S) + \sqrt{C^2 - (m/2K)(V_2 - V_1)^2 (1 - \exp(-T_1/S))}}{1 - \exp(-T_1/S)} \quad (73) \]

Substituting (73) in equation (59) and letting \( t = T_1 \), we get

\[ c(T_1) = \frac{C \exp(T_1/S) + \sqrt{C^2 - (m/2K)(V_2 - V_1)^2 (1 + \exp(T_1/S))/(1 - \exp(T_1/S))}}{1 + \exp(T_1/S)} \quad (74) \]

To decide whether the positive or negative sign in the above equation should be chosen, let's consider a transducer with large \( K \), i.e., a transducer whose energy storage capacity is very large. Then, by neglecting the second term in the radical sign, we have

\[ c_1 = c(T_1) \approx C (e^{T_1/S} + 1)/(e^{T_1/S} + 1) \quad (75) \]

If we chose the positive sign, we have

\[ c_1 = c(T_1) = C \]
This means that, for a transducer with large energy storage capacity, the steady state vibration amplitude before impact is close to the unloaded vibration amplitude $C_t$. This is consistent with the statement made in Section 4-1 that the larger the value of $K$ the faster the energy recovery. If we choose the negative sign, we have

$$c(T_1) = C(e^{T_1/S} - 1)/(e^{T_1/S} + 1)$$

This has no special physical meaning. Therefore, we choose the positive sign in equation (73) and (74), that is

$$c_i = c(T_1) = \frac{C e^{T_1/S} + \sqrt{C^2 - (m/2K)(V_2^2 - V_1^2)(1 + e^{T_1/S})/(1 - e^{T_1/S})}}{1 + e^{T_1/S}}$$

As discussed at the end of Section 4-2, the static force $F_s$ is kept constant, and $V_1$, $V_2$, and $T_1$ are functions of $c_i$ (or $c(T_1)$). Then, the right handside of equation (76) is a function of $c_i$ only. Therefore, the value of $c_i$ can be solved numerically if not analytically, since no mathematical relationship between $V_1$, $V_2$ and $c_i$ exists.

Now the questions stated at the end of Section 4-1 becomes clear once the values $c_i$, or $c(T_1)$, is obtained. The values of $V_1$ and $V_2$ can be found according to Section 4-2. The value of $T_1$ can be obtained by using equation (72) with the values of $V_1$ and $V_2$ just found. The internal energy just before impact can also be evaluated according to equation (60).

With all the information obtained previously, the power transmission into rock can be calculated by considering the energy loss of tool to
rock in each impact and the frequency of the sonic process. The energy absorbed per impact, $\Delta$, is

$$\Delta = m(v_2^2 - v_1^2)/2$$  \hspace{1cm} (77)

Therefore, the power transmission $P$ is the product of $\Delta$ and $f$, i.e.,

$$P = \Delta x f$$  \hspace{1cm} (78)

Substituting $f$ of equation (73) in the above equation, we have

$$P = F_s (v_2 - v_1)/2$$  \hspace{1cm} (79)

With the value of $c_1$ known, the value of $v_2 - v_1$ (or $P$) can be obtained. Figure 51 shows the values of $v_2 - v_1$ for $R = 10^4$ and various values of equivalent length of the tool, $m_e$. The ratios of the rebound-incident velocity relationship in tool-rock impact used here are equal to 0.7 and 0.6.

4-4 DRILLING RATE OF A SONIC TRANSDUCER

As we mentioned previously in discussing the specific energy ($e_s$) of rock removal, the time required to remove a certain volume of rock is important in the discussion of economy of a drilling system. Therefore, drilling rate is the focusing point for any kind of rock drilling system. With the power transmission ($P$) and specific energy known for a sonic transducer system and rock, the drilling rate ($R$) can be found.
Figure 51 Velocity difference for various values of equivalent length of the tool
very easily as follows

\[ R = \frac{P}{e_s} \]  

(80)

With all the studies on the sonic rock drilling being completed, the next best thing to do before closing up the discussion is to study an example problem by using the information presented. This would not only help understand the previous studies but also get a general idea of a practical situation in a sonic rock drilling process.

4-5 NUMERICAL EXAMPLE

The information with regard to the transducer, static force applied, the tool and the rock are given as follows:

TRANSUDER: P-11 transducer with one inch diameter and ten inch long transmission line

- Resonant frequency = 9,750 Hz
- Applied terminal voltage = 1,200 volts

STATIC FORCE: 100 lb.

TOOL: Dimensions - Type F tool as shown in Figure 12,

- Radius of curvature of the tool end = 10"
- Equivalent length \( m_0 = \frac{3}{4} \"

ROCK: Granite

\[ v_1/v_2 = 0.6 \]

(Laboratory experiment shows that the value of rebound-incident ratio is between 0.54 to 0.60 for Type F tool impacting against granite, 0.6 is used here for analysis.)

Specific energy \( e_s \) - Figure 21

The internal energy \( I \) according to equation (60) is

\[ I = K c^2 \]  

(1)
where $K$ is equal to $11.2 \text{ in.-lb.} / (\text{mil})^2$ for P-11 transducer with a 10" transmission line (page 88).

Angular frequency $\omega = 2\pi f = 61,260 \text{ rad./sec}$.

No load amplitude $G = 3.4/2 = 1.7 \text{ mils}$ according to Figure 43.

Time factor $S$ is dependent upon the total damping characteristics of the transducer. Here, we assume that equation (56) still holds after the modeling of transducer tip vibration amplitude recovery is made, i.e.

$$ S = \frac{2\omega}{\omega} \quad (56) $$

where $\omega$ can be obtained by using equation (49), or

$$ \omega = \frac{F}{\Delta F} \quad (49) $$

and Figure 44. The value of $\omega$ is found to be equal to 76.5 for the terminal voltage 1200 volts and temperature 75°F. Therefore,

$$ S = 2 \times 76.5/61,260 = 2.498 \text{ m sec} $$

$$ T_1 = m(V_1 + V_2)/F_s = 0.0055027 V_2 $$

$$ T_1/S = 0.001147536 V_2 $$

and

$$ (m/2K)(V_2^2 - V_1^2)(1+e^{T_1/S})/(-1+e^{T_1/S}) = 8.1885 \times 10^{-6} V_2^2 \frac{1+e^{T_1/S}}{2-1+e^{T_1/S}} \text{ (mil)}^2 $$

With the above information, the curves for left-hand side and right-hand side of equation (76) are

$$ \phi = c(T_1) $$
\[ \phi = \frac{Ce^{T_1/S} + \frac{e^{T_1/S}}{1 + e^{T_1/S}} \frac{1}{2} \frac{2}{2} \frac{1}{2} (v_1^2 - v_2^2)(1 + e^{T_1/S})}{(2 + 1)} \]

and are plotted in Figure 52. The solution of equation (76) is the intersection of the above two curves. That is

\[ c_1 = c(T_1) = 1.15 \text{ mils} \]

From Figure 51,

\[ V_1 - V_2 = 44.4 \text{ in./sec.} \]
\[ V_1 = 66.6 \text{ in./sec.} \]
\[ V_2 = 111.0 \text{ in./sec.} \]
\[ T_1 = 0.504 \text{ m sec.} \]
\[ I_1 = 11.2 \times (1.15)^2 = 17.25 \text{ in.-lb.} \]
\[ \Delta = \frac{1}{2} m(v_2^2 - v_1^2) = 1.124 \text{ in.-lb.} \]
\[ \Delta / I_1 = 1.124/17.25 = 0.07 \%

Power transmission \( P \) is

\[ P = \frac{1}{2} F_s(v_2 - v_1) = 2,220 \text{ in.-lb./sec.} = 11,100 \text{ ft-lb./min.} \]
\[ = 250.8 \text{ watts} \]

We do not have the specific energy chart for \( m_e = \frac{1}{2} \) but do have it for \( m_e = 1.08 \) (Figure 21). To find the specific energy for \( m_e = 0.5 \) and \( V_2 = 111.0 \text{ in./sec.} \), let's assume that equal incident energy of the tool would cause equal amount of damage to the rock if tip geometries are the same for tools of different mass (or \( m_e \)); then the corresponding incident velocity \( V'_2 \) of the tool with \( m'_e \) can be found as follows

\[ \frac{1}{2} m_e V'_2^2 = \frac{1}{2} m'_e V_2^2 \]
TIP VIBRATION AMPLITUDE, $c_1$

Figure 52 Solution of $c_1$ (graphical method)
For \( m_e = \frac{3}{8} \)" and \( m_e' = 1.08" \)

\[ V_2' = 75.5 \text{ in.}/\text{sec.} \]

The value of \( e_s \) corresponding to \( V_2' = 75.5 \text{ in.}/\text{sec.} \) and \( m_e' = 1.08" \) is 11,000 ft-lb./in.\(^3\) according to Figure 21. Therefore, the drilling rate in volume per minute is

\[ R_v = \frac{P}{e_s} = \frac{11,100}{11,000} = 1.01 \text{ in.}^3/\text{min}. \]

and the drilling rate in in./min. is

\[ R_L = R_v/A = 1.01/\pi(\frac{1}{2})^2 = 1.29 \text{ in.}/\text{min}. \]

Drilling rates and power transfer are also calculated for the case when the terminal voltage and static force applied to the transducer are equal to 2,000 volts and 200 lb., respectively. An estimate value of \( Q \) is used (\( Q = 60 \)) in this calculation since no experimental result is available for a transducer driven at 2,000 volts. The results are listed in the following table along with the previous example.

<table>
<thead>
<tr>
<th>( V = 1,200 \text{ volts} )</th>
<th>( V = 2,000 \text{ volts} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_s = 100 \text{ lb.} )</td>
<td>( F_s = 200 \text{ lb.} )</td>
</tr>
<tr>
<td>( R_L ) in./min.</td>
<td>1.29</td>
</tr>
<tr>
<td>( R_v ) in.(^3)/min.</td>
<td>1.01</td>
</tr>
<tr>
<td>Power transfer</td>
<td></td>
</tr>
<tr>
<td>watts</td>
<td>250.8</td>
</tr>
<tr>
<td>ft-lb./min.</td>
<td>11,100</td>
</tr>
<tr>
<td>( T_1 )</td>
<td>0.504</td>
</tr>
<tr>
<td>( \Delta /I_1 )</td>
<td>7.54</td>
</tr>
</tbody>
</table>
The drilling rates of 1.29 and 8.58 in./min. obtained here in this section are lower than conventional percussive drilling. Bruce in a study found the drilling rate with two special bits (four-wing) and percussive drilling apparatus is within the range of 5 to 40 in./min, while the maximum piston energy consumed is in the range of 46,000 ft-lb./min. to 162,000 ft-lb./min. Considering the power consumed and the drilling rate of a sonic transducer system, it is easy to see that the system is in the low range of percussive drilling system.
Sonic rock cutting is a rather practical problem, with many factors influencing the performance of the system. Even though we finally are able to calculate the drilling rate of a sonic transducer system in this study, much work, such as the improvement of drilling rate, and study of the sonic transducer, still needs to be done.

The properties of the rock are not so easy to monitor as other kinds of materials, such as metals. It differs from one piece to another due to its difference in composition and geographical location. Even in the same piece of rock, the inconsistency of rock properties is also shown in the experimental data. The lack of consistency makes it not only difficult to classify rock material for engineering purpose but also difficult to analyze problems related to rocks. Theoretical method usually yields even greater range of scattering. Therefore, experimental methods is heavily relied upon. It is understandable that the experimental results presented in Chapter I are scattered in wide ranges. Best efforts were always made in treating experimental data in this study.

The geometries of a tool, either the tip or the end geometry are important in sonic rock cutting. Much attention has been given to them in the present study. It is interesting to see that the sharp tip four wing bit still finds its merits in sonic drilling. Even though the sonic energy per impact is small compared to conventional percussive drill, method of transfer is still via percussive action, i.e., the impact of drill
steel tip with the rock surface. In the study of end geometry, it was found that the tool with a slightly curved surface is more favorable. It happens that the approach derived by Feng in ball-line impact is readily applicable for our need in finding the incident-rebound velocity relationship.

The sonic transducer is simple in structure but its behavior is very complicated. At the present time, there are not enough research results that can predict easily its performance under all circumstances. The complication arises due to the complicated properties of the ceramics, and the inter-coupling of the ceramics, metal and electric power supply system. Modeling and assumptions are necessary and were made to simplify the complicated problems yet still get results which would give us some insight in the rock drilling problem.

With the analysis of the subprocesses of impact coupling and the study of sonic response of a transducer to impact, we were able to find the drilling rate of a sonic system. No experiments have been conducted to find the actual drilling rate of the system in rock as yet. The tip vibration amplitude of a sonic transducer is not easily controllable if an experiment is conducted. It may not be possible to obtain consistent experimental data due to this fact. As stated in Chapter III, the amplitude drops as the temperature builds up in the transducer. Aside from this, detuning of the transducer resonant frequency away from the driving frequency of the power source would also reduce the vibration drastically.

In this analysis, we considered only the case for which impact
coupling exists, i.e., where the rebound-incident velocity ratio of rock is high enough to have an intersection point with the impact rebound relationship of tool with transmission line. For soft rocks, limestone for example, further research is required to get a real understanding in the behavior of tool in the drilling process.

It is too early to talk about whether the sonic transducer is an economic rock drilling system at this moment even if we can find the power required and drilling rate. Many other problems need to be solved and improved before we can find the maximum drilling rate of a sonic transducer, such as the effect of voltage change during operation, the temperature effect on the tip vibration amplitude, the most effective mass and geometry of the tool, the optimum value of static force, etc. But this work hopefully lays the foundation for further optimizing of the parameters to obtain a maximum drilling capacity of the system.
REFERENCES


7. Shieh, L. C., "Energy Transfer Into Sonic Impact Tools," Chapter 4, EES 380 Final Report (see ref. 6).


28. Thompson, J. H. C., Phil. Mag., 41, 1933.
