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SEARCH FOR LOW-LYING LEVELS OF THE GIANT DIPOLE RESONANCE IN $^{40}$Ca BY THE $^{39}\text{K}(p,\gamma_0)$ REACTION

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of the Ohio State University

By

Donald Paul D'Amato, B.A., M.S.

* * * * * *

The Ohio State University
1972

Approved by

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PUBLICATIONS

"Analog Resonances in $^{64}$Mo(p,p')," J. J. Kent, S. L. Blatt, D. C.

"$^{64}$Zn(p,$\gamma$) Reaction over Analog Resonances," J. J. Kent, D. C.

"$^{39}$K(p,$\gamma_0$) Angular Distribution," D. P. D'Amato, S. L. Blatt, J. J.
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CHAPTER I

INTRODUCTION

The dominant characteristic in the photo-reaction cross-section for nuclei is the giant dipole resonance (GDR). The energy of the resonance varies from about 30 MeV for carbon to about 16 MeV for uranium and displays a slow and steady shift with mass number. The gross structure of the GDR varies smoothly through the magic numbers while exhibiting a splitting in deformed nuclei. The excitation would seem to involve the entire nucleus rather than only the outer-most shell. Indeed, the first attempts at its description interpret it as an oscillation of the protons with respect to the neutrons.¹ These collective model calculations achieved a fair measure of success in predicting the resonance energy and the gross structure variation with mass number. Despite the apparent collective nature of the GDR, it can be treated in the framework of the shell model provided a sufficient number of single particle excitations are taken into account. Recent work has been in this direction.

Improved energy resolution has shown intermediate and fine structure to exist throughout the GDR region. The resolution improvements
have come about by improved production and measurement techniques for gamma rays and by the use of the inverse reactions (i.e. radiative capture). The cross-section for the inverse reaction is related to the forward reaction by the principle of detailed balance. At the present time, charged particle capture reactions provide the highest resolution studies of the GDR. The observed gross structure and a fair amount of the intermediate structure have been reproduced by shell model calculations, while the fine structure is not well understood.

If examined with poor resolution, the peak in the $(\gamma,p)$ or $(\gamma,n)$ cross-section for the doubly-closed shell nucleus $^{40}\text{Ca}$ occurs at an excitation energy of about 20 MeV with a 3 to 4 MeV half-width.\(^3\) With its better resolution, the inverse reaction, proton capture by $^{39}\text{K}$ followed by gamma decay, has shown considerable structure to exist in the excitation function. A positive $Q$-value of 8.333 MeV for the capture reaction enables low-energy accelerators to achieve the necessary excitation energies for the study of the $^{40}\text{Ca}$ GDR region. The $^{40}\text{Ca}$ energy level diagram is displayed in figure 1. As may be seen, gamma decay to the ground state will be well separated in energy from decay to the first few excited states.

This work is concerned with the range of excitation energies 11 to 14 MeV, where several low-lying components of the GDR ($J^\pi = 1^-$) have been predicted to exist. Bartko and Thwaites\(^3\) (BT)
Figure 1. $^{40}$Ca Energy Level Diagram. \(^4\)
MANY LEVELS FROM
\(^{40}\text{Ca}(p,p')^{40}\text{Ca}\)
AND \(^{39}\text{K}(p,y)^{40}\text{Ca}\)
investigated this energy range by the $^{36}$K(p, γ) reaction with proton energy steps of 3 to 4 keV. Their $0^0$ excitation function for the ground state transition is shown in figure 2. Several attempts have been made to explain the observed structure in $^{40}$Ca using a shell model calculation, the common basis being the one particle-one hole approximation. Gillet and Sanderson\textsuperscript{5}) calculated the odd parity states in $^{40}$Ca by a conventional bound state computation with isospin mixing. They predict five $J^\pi = 1^-$ states between 11 and 14 MeV. Although there are many more than five resonances in this energy range, some correspondence between these predicted states and the resonances found by BT can be noted.\textsuperscript{3)}

In a more recent calculation, the coupled-channel method was used by Marangoni and Saruis\textsuperscript{6}) and they find a definite correlation between the resonant states in their model and the bound states of Gillet and Sanderson. In addition, the coupled-channel calculation predicts a significant dependence on energy of the $a_2$ coefficient in the Legendre polynomial expansion of the angular distribution of emitted gamma rays,

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_0}{4\pi} \left[ 1 + \sum_{m=-1}^{\infty} a_m P_m (\cos \theta) \right].$$

A sensitive test of this calculation would be a measurement of this coefficient.
Figure 2. Bartko and Thwaites$^3$ $^{38}K(p, \gamma_0)$ $0^0$ Excitation Function.
In a paper published in 1970, Heimlich and Mausberg \(^7\) (HM) describe work in which they simultaneously measured the \(^{39}\text{K}(p, \gamma \gamma)\) differential cross-sections at 0° and at 90° for excitation energies between 10.8 and 14.1 MeV. They make a comparison between their cross-section ratio, \(\frac{d\sigma(0^\circ)}{d\sigma(90^\circ)}\) and a computed ratio based on the \(a_2\) coefficient of Marangoni and Saruis. If in the actual distribution, coefficients other than \(a_2\) exist, the comparison cannot be valid. Their results show the cross-section ratio to be rapidly changing with energy. Little agreement with theory can be seen.

The present work was undertaken with the goal of obtaining complete angular distribution data in the stated energy range in order to permit better theoretical comparison. Preliminary results \(^8\) indicated discrepancies with the work of HM. The data were taken utilizing the Ohio State University Van de Graaff accelerator, a High Voltage Engineering Corp. model CN, 6 MV machine. An anti-coincidence shielded NaI(Tl) detector was used to obtain the gamma spectra. The gamma detector mechanical support and its electronics were designed and built in part by this author. Angular-distribution measurements were made at 10 of the stronger resonances noted by BT. The final results show some agreement and some discrepancy with those of HM and bear little resemblance to the theoretical predictions.
CHAPTER II
EXPERIMENTAL METHOD

In the energy range of this experiment, the differential cross section for $^{39}\text{K}(p, \gamma)$ is typically $10 \, \mu b/sr$ at energies corresponding to a large resonance and considerably less otherwise. This necessitated the use of a reasonably efficient detection system. On the other hand, if reasonable angular resolution and measurements at back angles were to be achieved, the detector would have to be placed at a distance from the target chamber. Another consideration that had to be taken into account in the choice of a detection system was the fact that natural potassium targets were to be used. The possible proton induced reactions on natural potassium, which contains 7% $^{40}\text{K}$, are shown in table 1. Since the $Q$-value for $^{40}\text{K}(p, n)$ is $-1.21 \, \text{MeV}$, the detector would have to tolerate a moderate neutron flux.

The gamma ray detector used represents a compromise between these requirements. It consists of a 10.2 cm diameter, 15.2 cm long NaI(Tl) crystal, surrounded by a 30.5 cm diameter ($\sim 8.9$ cm wall thickness) NE102 plastic scintillator. Six photo-
Table 1. Proton Induced Reactions on Natural Potassium

(93.1% $^{39}$K, 6.9% $^{41}$K) Below $E_p = 6$ MeV.
<table>
<thead>
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<th>Reaction</th>
<th>Q-Value (MeV)</th>
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<td>$^{39}\text{K}(p,\gamma)$</td>
<td>8.333</td>
</tr>
<tr>
<td>$^{39}\text{K}(p,\alpha)$</td>
<td>1.293</td>
</tr>
<tr>
<td>$^{41}\text{K}(p,\gamma)$</td>
<td>10.276</td>
</tr>
<tr>
<td>$^{41}\text{K}(p,n)$</td>
<td>-1.210</td>
</tr>
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<td>$^{41}\text{K}(p,\alpha)$</td>
<td>4.030</td>
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tubes are used on the plastic, the output of which is in anti-coincidence with the NaI(Tl) crystal for suppression of escape events and cosmic ray events. The plastic scintillator shield extends in both the forward and backward directions relative to the NaI(Tl) crystal for improved detection of escape events. The rear of the plastic, encasing the 58 DVP phototube, enables much of the escaping bremsstrahlung from fast electrons in the main crystal to be detected. This entire assembly is shielded by lead of thickness 7.6 cm on the front face, 10.2 cm on the sides and top and 5.1 cm on the bottom (figure 3).

The apparatus is mounted on a 2.5 cm thick aluminum cart supported by cam-followers (figure 4). Radial motion of this cart is possible from a position touching the target chamber to a position 1.2 meters back, while angular motion is possible from back angles of about 140° to 0°, depending on the type of target chamber used and the chamber to detector distance.

Actually, there are two radial tracks with similar carts and at the present time, a large Ge(Li) with or without an associated NaI(Tl) annulus can be mounted on the other cart. A large center column serves both as the target chamber support and as the pivot point for the radial tracks. The tracks are constructed of 15.2 cm steel channels with hardened steel plates welded on.
Figure 3. Anti-coincidence Shielded NaI(Tl) Detector--Vertical Cross-Section View. The Mallory 2000 is a high density tungsten alloy collimator.
Figure 4. Heavily Shielded NaI Detector and Mounting. Except for beam pipe requirements, the detector is completely rotatable about the target position. In addition, approximately 1.2 meters of radial travel is possible.
TARGET CHAMBER MOUNTS HERE

SECOND TRACK PIVOTS ON THIS SHOULDER
The plates were afterwards ground flat and parallel. It was calculated that the maximum vertical load of 2270 kg weight applied to the center of these tracks would produce a deflection of less than .25 mm. The tracks are supported at the outer ends by steel wheels running on a circular steel plate with leveling screws every 5°.

The large radial travel of the carts was originally designed for neutron-gamma discrimination by time of flight techniques. The necessary pulsed beam system, although partially constructed, has not yet been installed. However, the motion has proved to be valuable since the detector can be moved quite close for an excitation curve measurement while it can be backed off far enough to achieve the angular resolution and reach back angles necessary in an angular distribution measurement when the counting rate is sufficient.

Two target chambers have been built and used with this apparatus (figure 5). One target chamber is of a T-type configuration, usually used for targets with a solid backing. The other chamber, used with targets transparent to the beam, has a faraday cup attached in which the beam is stopped 2.4 meters behind the chamber. The faraday cup is lead-lined and shielded from the detector by 10.2 cm of lead. Of course, the latter chamber limits angular motion of the carts to about 40° in the forward direction, once again depending on detector to target distance.
Figure 5. Target and Evaporation Chambers. This target chamber (with the Faraday cup) is for use with targets transparent to the beam. A similar target chamber without the attached beam pipe leading to the Faraday cup is used for targets opaque to the beam or when it is desirable to have a detector at 0°.
Both target chambers have provision for an evaporation chamber to be attached to the top of them. Thus, targets may be prepared by evaporation onto a suitable backing and then moved into position for data taking without losing vacuum. Most of the metallic potassium targets used in this experiment were prepared in such a manner.

This experiment, in common with other particle capture-radiative emission reactions can be characterized by a high total count rate combined with a low rate for pulses of interest. The gamma rays of interest are typically of the highest energy and the counting rate is subject to rapid changes due to accelerator beam fluctuations. Such changes usually result in gain shifting in photomultiplier tubes caused by the changing dynode resistor bypass current. To minimize this fast gain shift, the last six dynodes of the 58 DVP phototube were voltage stabilized by an external zener diode resistor string.

Slow gain shifts were compensated for by a variable gain amplifier coupled to a discriminator. The reference pulse for this system was derived from 4 light-emitting diodes† illuminating the photocathode of the 58 DVP. The diodes were imbedded in a short light pipe between the NaI and the photocathode and were

†Electro-Nuclear Labs., type 484; Gallium Phosphide Visible Light Source.
pulsed with a fast rise-slow decay pulse at a repetition rate of 1 kHz. They proved to be a very stable reference for the stabilization system. (Appendix B).

The low count rate for pulses of interest combined with the high total rate made the use of a certain amount of pulse pile-up suppression necessary. To minimize the time during which a pile-up event could occur, phototube anode signals were delay-line clipped and gated by a fast linear gate before being shaped and processed. Nano-second logic (E.G. & G. M100 series) was extensively used for anti-coincidence protection and pile-up detection circuitry. Based upon some arbitrary fast discriminator level, pile-up events may be classified as of a low-low, low-high, or a high-high type with the low-low type causing significantly more events.

Since the total count rate was usually well below 50 kHz, in order to simplify the electronics, pile-up protection only for the low-low type was used.

The data consisted of 512 channel spectra, digitized by a Northern Scientific analogue to digital converter and stored by the laboratory IBM 1800 on-line computer, first in magnetic core storage and then on magnetic disk. The first 256 channels of each spectrum constituted the coincidence or rejected part; that is, the pulses from the NaI were in coincidence with a pulse from the
plastic scintillator. The 2nd 256 channels were the anti-coincidence or accepted part; in other words, in this case no plastic pulse occurred (within a specified resolving time) during a NaI pulse. Thus, in channels 1-256, escape events and cosmic rays were recorded while in 257 to 512, mainly full energy gamma-ray peaks were recorded. The reasons for storing both coincidence and anti-coincidence portions of a spectrum may be summarized as follows:

1. Possible changes in anti-coincidence shield efficiency due to changes in NE102 phototube gain may later be compensated for if both accepted and rejected portions are known.

2. Random coincidence rate changes due to counting rate changes are likewise provided for.

3. Cosmic ray background may be clearly seen and subtracted.

4. Secondary subtraction in which a fraction of the rejected spectrum is subtracted from the accepted may be used to further improve the resolution for separation of closely lying gamma peaks.
The geometry of the present detection system is similar to that developed earlier by our group, but several significant changes were instituted prior to the beginning of this experiment. These changes included the use of the new bi-alkali photocathode 58 DVP phototube, the repositioning and replacement of the six phototubes for the NE102 plastic and the use of the new light-emitting diode gain stabilization. We felt that because of these changes, a new resolution check should be made. The 20.5 MeV gamma ray from the reaction \( T(p, \gamma)^4\text{He} \), \( E_p = 0.95 \) MeV, was used for this check. Figure 6 shows the results. As can be seen, the resolution is approximately 7% full-width at the half maximum point. The cosmic ray rejection ratio, those appearing in the coincidence portion over those in the anti-coincidence portion, is (depending on energy) on the order of 300 to 1. While these numbers do not represent an improvement over the previous values, the reconfigured detector is more versatile, allowing measurements to be made over a greater angular range.

The extraction of areas for the \( ^8K(p, \gamma_0) \) peak proved to be somewhat of a problem. Maximum accuracy was desired because it was soon discovered that most of the resonances within the energy range that we were studying had relatively small non-isotropic components. We desired to know whether the gamma ray in question was truly isotropic or did indeed have an angular
Figure 6. \( T(p, \gamma)^4\text{He} \) Spectrum. The first 256 channels is the coincidence spectrum while the second 256 is the anti-coincidence. The counts seen above the peak in the coincident portion come from cosmic rays. Virtually none are seen in the anti-coincidence spectrum.
$T(p, \gamma)$

COUNTS

CHANNEL NUMBER
distribution with other than an \( a_0 \) component in the Legendre polynomial expansion. A reasonably simple integration technique based on a discriminator "window" or a channel by channel summation usually did not provide the consistent results desired because of gain shifts, contaminant gamma rays and inability to pick the peak position accurately by eye. Also, the low energy tail of the peak (figure 7) contained a significant fraction of the peak area and did not have any "minimum" on which to base a summation.

It was felt that a computer peak fitting routine based on a "standard" line-shape might provide consistent areas or at least it might provide the peak channel information on which an area integration could be based. The line-shape itself, would be derived from a "very good statistics" spectrum of a well separated gamma ray with an energy close to the gamma rays of interest. There of course has been considerable work done along similar lines and our techniques were patterned after those of King et al.\(^{12}\) They found that an anti-coincidence shielded NaI(Tl) detector gave a line-shape that might be approximated by a constant term and a first order exponential on the low side of the peak and a gaussian on the high side. We found that significantly better fits might be obtained by fitting both sides of the peak to a function
Figure 7. $^{38}$K(p,γ) - Typical Spectrum. Here the NaI(Tl) discriminator threshold has been lowered to show the complete spectrum. During data collection, it was usually set higher to prevent possible pulse pile-up from the intense lower energy (∼7 MeV) gamma rays resulting mostly from thermal neutron capture in the NaI(Tl) crystal. It might be noted that although the channel number is displayed as 0 through 256, this and all the following are anti-coincidence spectra.
$^{39}\text{K} (p, \gamma)$

$E_p = 5.80 \text{ MeV}$

$\theta_{\text{LAB}} = 90^\circ$
of the form:

\[ f(x) = \exp \left[ b_0 + b_1 x + b_2 x^2 + b_3 x^3 \right] \]

where \( x = \) channel number.

Also, this type function has the advantage that by first taking the logarithm of each point to be fit, a linear least squares fitting technique may be used to calculate the coefficients. However, it was still necessary to divide the line-shape into two halves and fit each half separately. The transition point from one set of coefficients to the other was determined empirically on the basis of a minimum overall chi-squared per degree of freedom.

\[
\frac{\chi^2}{M} = \frac{1}{m-m} \sum_{i=1}^{n} \left[ \frac{f(x_i) - y_i}{\sigma_i} \right]^2
\]

- \( n = \) number of data points,
- \( m = \) number of parameters in fit,
- \( M = \) number of degrees of freedom.

Usually, where this minimum was found, the two calculated functions joined with reasonable smoothness. Also, this point was found to coincide with the channel where the small remaining single escape peak joined the full energy peak.
Two different gamma-ray producing reactions were used to provide the line-shape spectra. The reaction

\[ Q = 19.81 \text{ MeV} \]

\[ _3^7\text{He}(p, \gamma)^4\text{He} \]

\[ E_p = 0.95 \text{ MeV} \]

was used for the line shape and the fit shown in figure 8. \( \chi^2/M \) for this fit is 1.09, which shows that the fit is probably close to the parent distribution. Although this is a good fit over a good proportion of the line-shape, our gamma rays of interest were in the 12-14 MeV range and hence the line shape could not be expected to be nearly as close to their parent distributions.

The other reaction utilized was

\[ Q = 15.63 \text{ MeV} \]

\[ ^{12}\text{C}(^3\text{He}, \alpha \gamma)^{12}\text{C} \]

\[ E_{^3\text{He}} = 3.75 \text{ MeV} \]

\[ E_\gamma = 15.11 \text{ MeV} \]

This reaction had the advantage of a high cross section with consequent prolific gamma production. However, the gamma ray produced will be doppler broadened (\( \pm 0.190 \text{ MeV} \)) by the alpha emission. Also, a gamma ray of lower intensity is produced at 12.71 MeV, in addition to the \(^3\text{He} \) capture gamma rays above the 15.11 MeV peak. The line-shape and the fit from this reaction are also shown in figure 8.
Figure 8. $T(p, \gamma)^4$He and $^{13}$C($^3$He, $\alpha\gamma$) Spectra and Associated Fits.

Only the portions of the spectrums that were actually used to produce the line shapes are displayed. A lower energy gamma ray in the $^{13}$C($^3$He, $\alpha\gamma$) spectrum prevents fitting the data in the lower channels.
$T(p, \gamma)$

$E_{\gamma} = 20.5 \text{ MeV}$
\[\theta_{\text{LAB}} = 90^\circ\]

$^{13}\text{C}(^3\text{He}, \alpha \gamma)$

$E_{\gamma} = 15.11 \text{ MeV}$
\[\theta_{\text{LAB}} = 90^\circ\]
In fitting to the actual data, the coefficients obtained from both of the above reactions gave relatively comparable fits with perhaps the $^{12}$C($^3$He, $\alpha\gamma$) coefficients giving a somewhat lower $\chi^2/M$.

The fitting of the functional form containing the eight specified coefficients to the actual data was accomplished by a three parameter grid search technique. The three parameters were the function position along the energy axis, its width about its maximum position, and its intensity. These parameters were changed without changing the eight input coefficients by the use of a function of the form:

$$F(x) = \mathcal{R} \left( Z(x) \right)$$

$$G(z) = \begin{cases} \exp \left[ a_1 + a_2 z + a_3 z^2 + a_4 z^3 \right] & \text{for } z \leq z_T \\ \exp \left[ b_1 + b_2 z + b_3 z^2 + b_4 z^3 \right] & \text{for } z > z_T \end{cases}$$

$$Z(x) = \frac{1}{\omega} \left[ x - x_{\text{max}} (1 - w) - n \right]$$

and \( h = \) height ratio

\( w = \) width ratio

\( n = \) shift in channels from original position

\( a_1, a_2, a_3, a_4 \) are the input coefficients

\( b_1, b_2, b_3, b_4 \)

\( x_{\text{max}} = \) original position of the maximum

\( Z_t = \) transition point from use of "low" coefficients to "high" set.
The grid search consisted of separately minimizing $\chi^2$ for each parameter (n, h, and w) in turn. By successive iterations of locating the minimum, an absolute minimum was found. At this point, the integrated area $\left[ h \cdot w \cdot (\text{input area}) \right]$ is computed and outputed along with the best values for the function height, width and channel of the peak. A typical $39K(p, \gamma_0)$ spectrum showing the $\gamma_0$ peak is displayed in figure 9 along with the associated fit.

The peak areas returned from the peak fitting program were corrected for random events and NE102 phototube gain changes. Since random coincidence rate changes and phototube changes only serve to change the ratio of accepted to rejected, a simple multiplicative factor, namely

$$\frac{\text{anti-coincidence} + \text{coincidence}}{\text{anti-coincidence}}$$

will provide the necessary correction (providing cosmic ray background is subtracted). The pulse height analyzer live timer was used to provide for possible dead time changes.

Following the above corrections, the peak areas for each angle were used in a linear least squares Legendre polynomial fitting routine. For each angular distribution, fits were done
Figure 9. High Energy End of $^{38}\text{K}(p,\gamma)$ Spectrum and Fitted Shape.

The shape obtained from the $T(p,\gamma)$ reaction has been used for the fit displayed in the upper spectrum. The data indicated by crosses in the upper spectrum are the same as that shown by the histogram in the lower spectrum.
$^{39}\text{K}(p,\gamma)$

$E_{\gamma_0} = 13.20 \text{ MeV}$

$\theta_{\text{LAB}} = 62^\circ$

$^{39}\text{K}(p,\gamma)$

$E_{\gamma_0} = 13.20 \text{ MeV}$

$\theta_{\text{LAB}} = 62^\circ$
for a varying number of Legendre polynomials. Also, the fitting program was run with both even and odd polynomials and with even only. If a table is constructed of $\chi^2/M$ versus an increasing number of parameters in the fit (and hence, decreasing number of degrees of freedom), there will generally be found a point at which $\chi^2/M$ will become minimum. As the number of degrees of freedom is further decreased, $\chi^2/M$ will remain the same or perhaps increase somewhat. This minimum value point then represents the most probable number of coefficients needed to completely fit the data. If sufficient statistics are obtained, $\chi^2/M$ will, depending on the number of degrees of freedom, be somewhat less than one. Too great a deviation from this value is suspect. This is the criterion we applied to determine whether to use even or even and odd polynomials and at what point to terminate the fitting. In table 2 are shown the various fits obtained for the distribution measured at 5.72 MeV. As may be seen, the $\chi^2/M$ found with $P_0$ and $P_2$ terms included is good at 0.592 (this might be somewhat low but standard errors had to be estimated). Certainly, there does not seem to be
Table 2. Measured Angular Distribution Coefficients for the Resonance at 5.72 MeV. The coefficients that resulted from fitting the distribution with even and odd Legendre polynomials and with even polynomials only are displayed along with the associated chi-squared and chi-squared per degree of freedom. The subscripts on the \( w \)'s refer to the Legendre polynomials used for the fit (e.g. \( w_{0,2,4}(\theta) = A_0 + A_2 P_2(\cos \theta) + A_4 P_4(\cos \theta) \)).
<table>
<thead>
<tr>
<th></th>
<th>Even and Odd Legendre Polynomials</th>
<th></th>
<th>Even Legendre Polynomials Only</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( w_0(\theta) )</td>
<td>( w_{0,1,2}(\theta) )</td>
<td>( w_{0,1,2,3,4}(\theta) )</td>
</tr>
<tr>
<td>( A_0 )</td>
<td>396.5 ( \pm ) 6.3</td>
<td>373.3 ( \pm ) 7.2</td>
<td>363.8 ( \pm ) 11.8</td>
</tr>
<tr>
<td>( A_1 )</td>
<td>10.9 ( \pm ) 14.8</td>
<td>33.5 ( \pm ) 26.8</td>
<td></td>
</tr>
<tr>
<td>( A_2 )</td>
<td>98.3 ( \pm ) 17.3</td>
<td>63.0 ( \pm ) 34.6</td>
<td></td>
</tr>
<tr>
<td>( A_3 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( A_4 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \chi^2 )</td>
<td>81.5</td>
<td>5.38</td>
<td>3.91</td>
</tr>
<tr>
<td>( \chi^2/M )</td>
<td>7.41</td>
<td>.598</td>
<td>.559</td>
</tr>
</tbody>
</table>
an $A_1^+$ term needed. A slight improvement is noted when $P_4$ through $P_4$ terms are included but it is probably not significant.

\[ \frac{d\sigma}{d\Omega} \propto W(\theta) = A_0 + \sum_{m=1}^{\infty} A_m P_m(\cos \theta) \]

\[ = A_0 \left[ 1 + \sum_{m=1}^{\infty} a_m P_m(\cos \theta) \right] \]

\[ \text{In the remainder of this work, the following convention will be used to distinguish the normalized and unnormalized Legendre polynomial coefficients:} \]
CHAPTER III
RESULTS AND ANALYSIS

A. Angular Distribution Results and Comparison with Other Work

An excitation curve measurement for $\gamma_\circ$ at $90^\circ$ was taken in 10 keV steps from 4.9 to 6.0 MeV proton energy prior to the start of this experiment as a rough check for anistropy and reproducibility by comparison with BT's $0^\circ$ measurement. The result (figure 10), which was completed prior to the publication of the work of HM, used only a very simple discriminator "window" type integration of the peak to measure the yield. The target thickness was apparently much greater than BT's and the accuracy is probably not great enough to come to any conclusions regarding the relative resonance strengths or widths. It can be said that no large resonances were found that BT had not measured with the 3 to 4 keV steps they used to cover the 2.9 to 6.0 MeV range. HM, who used 20 keV steps except near the stronger resonances, differ in the energy and strength assignments. In table 3, a comparison of the two works is shown. HM found consistently lower energy assignments for the resonances, differing by 10-20 keV for the lower
Figure 10. $S(p, \gamma) \text{ Excitation Function Measured at } \theta_{cm} = 90^\circ$.

Arrows indicate points at which target or integration window width were changed.
$^{39}K (p, \gamma_0) ^{40}Ca$

$\theta_{c.m.} = 90^\circ$

$\gamma_0$ COUNTS / 60 $\mu$C

$E_p$ (LAB)

0 4.80 5.00 5.20 5.40 5.60 5.80 6.00
Table 3. Resonance Energies and Strengths as Measured by Bartko and Thwaites\textsuperscript{3)} and Heimlich and Mausberg.\textsuperscript{7)}
<table>
<thead>
<tr>
<th><strong>Bartko and Thwaites (BT)</strong></th>
<th><strong>Heimlich and Mausberg (HM)</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_p^{\text{lab}}$ (MeV)</td>
<td>$E_p^{\text{lab}}$ (MeV)</td>
</tr>
<tr>
<td></td>
<td>$E_x$ (MeV)</td>
</tr>
<tr>
<td>3.863</td>
<td>12.099</td>
</tr>
<tr>
<td>3.875</td>
<td>12.111</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>3.970</td>
<td>12.204</td>
</tr>
<tr>
<td>4.104</td>
<td>12.334</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>* 4.195</td>
<td>12.423</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>* 4.380</td>
<td>12.604</td>
</tr>
<tr>
<td>* 4.446</td>
<td>12.668</td>
</tr>
<tr>
<td>* 4.467</td>
<td>12.688</td>
</tr>
<tr>
<td>* 4.658</td>
<td>12.875</td>
</tr>
<tr>
<td>* 4.766</td>
<td>12.980</td>
</tr>
<tr>
<td>4.875</td>
<td>13.086</td>
</tr>
<tr>
<td>* 4.903</td>
<td>13.113</td>
</tr>
<tr>
<td>* 4.986</td>
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<tr>
<td>4.995</td>
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<td>5.083</td>
<td>13.289</td>
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<td></td>
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<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>* 5.723</td>
<td>13.913</td>
</tr>
<tr>
<td>* 5.805</td>
<td>13.993</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Indicates resonances studied in this work.
energies and 30-40 keV for the higher ones. BT's resonance strengths, uncorrected from the 0° measurement, vary over a considerably wider range than do those of HM who give the values measured at 90° obtained from the 5 keV and 20 keV energy steps. The reasons for this are not clear but they are probably associated with the different target thicknesses and window widths used.

Due to the strongly resonant characteristics of the excitation function with a relatively small non-resonant contribution, we decided to concentrate our effort on obtaining angular distribution data for the prominent resonances between 3 and 6 MeV proton energy. There are ten such structures that we have studied. In each case, an excitation curve for the region of interest was first taken. The targets were generally thick for the resonances studied and the distributions were measured slightly above the step in the yield function. Two of the levels measured had half-widths on the order of thickness of the targets; namely the resonances at 5.72 and 5.80 MeV.

It can be seen from figure 2 that there exist small structures near four (3.86, 4.45, 4.90 and 4.99 MeV) of the resonances for which angular distributions have been measured in this experiment. Attempts were made to minimize the effect that these might have on our measurements but it is clear that the contributions from these overlapping resonances cannot be eliminated.
the 3.86 MeV resonance appears to be the most difficult to separate. The strengths\(^+\) for the 10 levels that we have examined vary from 112 eV to 4.2 eV (BT values). Several resonances of strength intermediate to these extremes were not examined because we felt that they probably could not be separated with our thicker targets.

The measured distributions along with the most probable fits are shown in figures 11 to 15. The probable coefficients along with the associated standard errors and the chi-squared per degree of freedom for the fits are listed in table 4. It can be seen that most of the distributions are fit with a \(P_0\) and a \(P_2\) term only although some seem to need an odd term, with the \(P_1\) term generally small. Two of the distributions, those at 4.66 and 4.99 MeV, have been found to be fit best with only an isotropic component. The latter distribution has the smallest error bars associated with it, having been done a number of times and the results summed. A chi-squared per degree of freedom of 0.801 shows the fit to be quite good. An improvement in \(\chi^2/M\) to 0.494 could be achieved when very small \(P_1\) through \(P_4\) terms were included, but this probably isn't physically meaningful. Table 4 shows the \(a_2\) coefficient that was obtained when the distribution was fit with \(P_0\) and \(P_2\) terms and it is seen that this is quite small.

BT mention that this resonance was given a \(J^\pi = 2^+\) assignment by

\(^+\)Resonance strength = \((2J + 1) \frac{\Gamma P^\gamma}{\Gamma} \).
Figure 11. Angular Distributions Measured at $E_p = 3.86$ and $4.38$ MeV.
$E_p = 3.86 \text{ MeV}$

$E_p = 4.38 \text{ MeV}$
Figure 12. Angular Distributions Measured at $E_p = 4.45$ and $4.66$ MeV.
$E_p = 4.45 \text{ MeV}$

$E_p = 4.66 \text{ MeV}$

LABORATORY ANGLE

ARBITRARY UNITS
Figure 13. Angular Distributions Measured at $E_p = 4.77$ and $4.90 \text{ MeV}$.
\[ \frac{d\sigma}{d\Omega} \]

LABORATORY ANGLE

\[ E_p = 4.77 \text{ MeV} \]

\[ E_p = 4.90 \text{ MeV} \]
Figure 14. Angular Distributions Measured at $E_p = 4.99$ and 5.08 MeV.
$E_p = 4.99\text{ MeV}$

$E_p = 5.08\text{ MeV}$

[Graph showing the angular distribution of scattered particles for two different proton energies, with arbitrary units on the y-axis and laboratory angle on the x-axis.]
Figure 15. Angular Distributions Measured at $E_p = 5.72$ and $5.80$ MeV.
$E_p = 5.72 \text{ MeV}$

$E_p = 5.80 \text{ MeV}$
Table 4. Legendre Polynomial Coefficients for the Fits to the Angular Distributions at the Ten Resonances Studied. Chi-squared per degree of freedom (\(\chi^2/M\)) has been included to estimate the quality of fit. An asterisk (*) indicates that the best fit was obtained without this coefficient, but it has been included for error bar comparison. The \(a_2\) coefficient attributed to Heimlich and Mausberg\(^7\) was obtained from their cross-section ratio by assuming only \(P_0\) and \(P_2\) terms in the angular distribution.
<table>
<thead>
<tr>
<th>Resonance Energy $E_p$ (MeV)</th>
<th>$\chi^2/M$ for best fit</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$\frac{d\sigma(0^\circ)}{d\sigma(90^\circ)}$</th>
<th>$\frac{d\sigma(0^\circ)}{d\sigma(90^\circ)}$</th>
<th>$a_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.86</td>
<td>.894</td>
<td>-.069+0.025</td>
<td>.147+0.031</td>
<td>1.18+0.06</td>
<td>1.18</td>
<td>.11</td>
</tr>
<tr>
<td>4.38</td>
<td>1.15</td>
<td>-.063+0.042</td>
<td>.98+0.07</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.45</td>
<td>3.13</td>
<td>.080+0.028</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>4.13</td>
<td>.073+0.035</td>
<td>.014+0.043</td>
<td>1.20+0.06</td>
<td>1.11</td>
<td>.07</td>
</tr>
<tr>
<td>4.66</td>
<td>.943</td>
<td>(.008+0.02)*</td>
<td>1.05+0.04</td>
<td></td>
<td>1.11</td>
<td>.07</td>
</tr>
<tr>
<td>4.77</td>
<td>.445</td>
<td>-.099+0.028</td>
<td>.84+0.04</td>
<td></td>
<td>.96</td>
<td>-.03</td>
</tr>
<tr>
<td>4.90</td>
<td>.749</td>
<td>-.086+0.023</td>
<td>-.063+0.029</td>
<td>.84+0.04</td>
<td>.96</td>
<td>-.03</td>
</tr>
<tr>
<td>4.99</td>
<td>.801</td>
<td>(-.001+0.014)*</td>
<td>1.00+0.03</td>
<td></td>
<td>1.28</td>
<td>.17</td>
</tr>
<tr>
<td>5.08</td>
<td>2.57</td>
<td>-.028+0.014</td>
<td>.166+0.018</td>
<td>1.25+0.05</td>
<td>1.36</td>
<td>.21</td>
</tr>
<tr>
<td>5.72</td>
<td>.592</td>
<td>.286+0.033</td>
<td>1.38+0.06</td>
<td></td>
<td>1.41</td>
<td>.24</td>
</tr>
<tr>
<td>5.80</td>
<td>1.34</td>
<td>.043+0.023</td>
<td>1.12+0.05</td>
<td></td>
<td>1.18+0.08</td>
<td>.11+0.05</td>
</tr>
</tbody>
</table>
means of an angular distribution measurement although they do not substantiate this further. Presumably, BT observed a $P_4$ term in the angular distribution, indicating d-wave capture to a $2^+$ level. This is the only way a $2^+$ assignment might be made and as mentioned, we find the distribution to be isotropic. An isotropic distribution would most likely mean s-wave formation of a $1^+$ or a $2^+$ level.

Of the four resonances that seem to have an $a_1$ term in the Legendre polynomial expansion (3.86, 4.45, 4.90, and 5.08 MeV), the lowest three seem most definite. However, the fit at 4.45 MeV has a rather high $\chi^2/M$ and indeed the fit does not appear "correct", although no improvement was found when polynomials higher than $P_1$ were included. It might be noted that these lowest three resonances apparently needing an odd term are three of the four which have close-lying smaller resonances. It must be said however, that the distributions for some of the forward and backward angles for two of these resonances (3.86 and 5.08 MeV) were measured separately and then normalized to each other by means of several common angles. Due allowance was made in the error bars for the propagation of error that would occur in such a normalization, but if the energies or target thicknesses were significantly different and the real distribution depended sensitively
on the energy, odd polynomial terms could certainly result. Since the same kind of normalization to common angles was used for the distribution at 4.99 MeV without any odd terms occurring, the procedure is not unreasonable.

Also shown in table 4 is a comparison of the data with the ratio of the $0^\circ$ to $90^\circ$ cross sections as measured by HM. In addition, we have extracted from HM's cross section ratio an $a_2$ coefficient by assuming only $P_0$ and $P_2$ terms in the angular distribution. These cross section ratios were obtained from the cross section graphs of figure 3 in reference 7 and therefore due to graphing and interpolation errors, they cannot be expected to be very accurate. Since HM give error bars at only a few positions on the graphs, none were included in the table except for the 5.80 MeV resonance at which they are only approximate. Included in this table are the values of $\frac{d\sigma(0^\circ)}{d\sigma(90^\circ)}$ which we measured based only on the $0^\circ$ and $90^\circ$ data points and hence they are considerably less accurate than the quoted values for the Legendre polynomial coefficients which are based usually on 6 or more data points.

Despite the aforementioned difficulties in assigning experimental uncertainties, a comparison of the two cross section ratios is possible and proves interesting. It can be seen that there is generally good agreement within error bars that might be assigned,
with HM's values being almost consistently greater. The one exception to this is the 4.45 MeV resonance and these are the data that we feel least sure about. The greatest difference between the two ratios occurs at 4.99 MeV and is probably well outside error bars that might be included. As was previously mentioned, this is the energy at which we are most confident of our data.

HM have mentioned that they have found variation in the \( \frac{dc(0^\circ)}{d\sigma(90^\circ)} \) ratio over an apparently individual resonance. If this is the case, it is surprising that the peak ratios are as close as they seem to be. Certainly, target thickness and beam energy differences would change the measured distributions considerably. The reasons for the consistent variation in one direction and the great difference at 4.99 MeV are unknown.

B. Possible Spins and Parities

The process of particle capture with consequent radiative decay may be schematically diagrammed (following the notation of Carr and Baglin\textsuperscript{13}):
where:
\[
p = \begin{cases} 
0 & \text{for magnetic multipole radiation} \\
1 & \text{for electric multipole radiation} 
\end{cases}
\]

\( L \) = multipolarity of radiation

\( j_a \) = spin of incident particle with orbital angular momentum \( \ell \)

\( s = \text{channel spin} = j_a + j_{A-a} \).

The differential cross section may be written:
\[
\frac{d\sigma}{d\Omega} = \propto^2 \left[ \frac{(2j_{R,a} + 1)(2j_{R,a} + 1)}{2} \right]^{-1} \leq B_{t_1, t_2}
\]
\[
\propto \Re \left\{ \langle p_1, p, \alpha | R_{t_1}^J | l_1, s \alpha \rangle^* \right\}
\]
\[
\times \langle p_2, p_2, \alpha' | R_{t_2}^J | l_2, s \alpha \rangle
\]

where the summation extends over \( s, p_1, p_2, l_1, l_2, t_1, t_2, J_1 \) and \( J_2 \).

\( \alpha \) and \( \alpha' \) specify the internal structure of the initial and final states.

\( t_1 \) and \( t_2 \) each denote a combination of values \( \{ p, L, J, \ell, s \} \).

\( \langle | R_{t}^J | \rangle \) denotes the reaction matrix element for some reaction channel.

\( 2\pi \chi \) is the center-of-mass wave-length of the proton.
The geometrical part may be separated out and $B_{t_1 t_2}$ may be written

$$B_{t_1 t_2} = \sum_j A_{t_1 t_2 j} P_0 (\cos \Theta)$$

with the allowed values of $\nu$ restricted by

$$\max (|l_1 - l_2|, |l_1 - \not{l}_2|, |j_1 - j_2|) \leq \nu$$

$$\nu \leq \min (l_1 + l_2, \not{l}_1 + \not{l}_2, j_1 + j_2)$$

and $l_1 + l_2 + \nu$ must be even. The $A_{t_1 t_2 \nu}$ may be calculated and are tabulated in several sources.

If at a particular energy, the reaction proceeds through a single channel, the angular distribution is entirely predictable. Of course, this is rarely the case, there generally being at least the two channel spins contributing. If the reaction proceeds through more than one channel, the different intensity terms along with possible interference terms enter in, weighted by the unknown reaction matrix elements. (It might be noted however, that different channel spins do not interfere.) While an exact solution of the data usually cannot be done, reasonable assumptions can be made concerning the reaction channels and spins involved.
In tables 5 and 6, these theoretical Legendre polynomial coefficients obtained from reference 13 are shown. The $J^\pi$ values have been limited to $1^-$, $1^+$, and $2^+$ and since the final state $j^\pi_A$ value is $0^+$, the type of radiative transition is uniquely specified. It seems likely that these are the only spins and parities that need be considered. Enhancement factors of 65 and 1842 above the Weisskopf estimates for $M2$ and $E3$ transitions respectively, would be needed for the weakest resonance that was studied in this work ($4.775$ MeV, $2(J + 1) \frac{\Gamma_P \Gamma_y}{\Gamma} = 4.2$ eV; BT's measurement). These transition strengths are considerably above the distribution of the known transition strengths for $M2$ and $E3$ transitions in the compilation of Skorka et al.\textsuperscript{15})

Table 5 shows the distribution coefficients assuming non-interference. It is seen that the values of the angular distribution coefficients that we have measured are all consistent with the limits that might be set for p-wave formation of a $J^\pi = 1^-$ level or d-wave formation of $1^+$ or $2^+$ levels. It seems likely d-wave formation of a $2^+$ level can be neglected based on the apparent absence of a $P_4$ term in the measured distributions. The $P_4$ term as shown in the table would be sizeable compared to the $P_2$ term unless there occurred accidental cancellation of the contributions from
Table 5. Theoretical Legendre Polynomial Coefficients\textsuperscript{13) } --

Non-Interference Terms (The expansion is of the form

\[ W(\Theta) = \sum_{a=0}^{\infty} c_a P_a(\cos \Theta) \]
<table>
<thead>
<tr>
<th>( \ell )</th>
<th>s</th>
<th>( J^\pi )</th>
<th>Transition Type</th>
<th>Legendre Polynomial Coefficients</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1^+</td>
<td>M1</td>
<td>( c_0 )</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2+</td>
<td>E2</td>
<td>3.0</td>
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<tr>
<td>1</td>
<td>1</td>
<td>1^-</td>
<td>E1</td>
<td>3.0</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1^-</td>
<td>E1</td>
<td>3.0</td>
</tr>
<tr>
<td>2</td>
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<td>1^+</td>
<td>M1</td>
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<tr>
<td></td>
<td>2</td>
<td>1^+</td>
<td>M1</td>
<td>3.0</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2^+</td>
<td>E2</td>
<td>5.0</td>
</tr>
<tr>
<td>3</td>
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<td>1^-</td>
<td>E1</td>
<td>3.0</td>
</tr>
<tr>
<td></td>
<td></td>
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</table>
Table 6. Theoretical Legendre Polynomial Coefficients \(^{13}\) --

Interference Terms. (The expansion is of the form

\[ W_2(\theta) = \sum_{i=0}^{\infty} c_i P_i(\cos \theta) \]
<table>
<thead>
<tr>
<th>$l$</th>
<th>$l'$</th>
<th>Radiation</th>
<th>$l$</th>
<th>$l'$</th>
<th>Radiation</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
</tr>
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<tr>
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<td>E1</td>
<td>0</td>
<td>M1</td>
<td>-7.348</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>M1</td>
<td>-5.196</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>E2</td>
<td>9.000</td>
<td></td>
<td></td>
<td>6.000</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>E1</td>
<td>2</td>
<td>M1</td>
<td>-6.971</td>
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<td></td>
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<td></td>
<td></td>
<td>0</td>
<td>E2</td>
<td>-7.348</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>E2</td>
<td>6.148</td>
<td></td>
<td></td>
<td>-1.757</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>E1</td>
<td>4.409</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>E1</td>
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<td>M1</td>
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<td></td>
<td></td>
<td>0</td>
<td>E2</td>
<td>-2.151</td>
<td></td>
<td></td>
<td>-6.000</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>E2</td>
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<td></td>
<td></td>
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<tr>
<td>1</td>
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<td>E2</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>0</td>
<td>E2</td>
<td>-5.976</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>E2</td>
<td>-5.669</td>
<td></td>
<td></td>
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</tbody>
</table>
the two different channel spins. As mentioned, the two distributions that are apparently isotropic most likely result from s-wave formation of $1^+$ or $2^+$ levels. It should be noted that the measured $a_2$ coefficient when examined as a function of energy apparently changes sign in the vicinity of the two isotropic distributions, which means that accidental cancellation of the contributions from the two channel spins for a $1^-$ level does not appear as unlikely as may first be thought. It also seems likely that $f$-wave formation represents at best a small contribution to any $1^-$ level that exists because in none of the measured distributions did the $P_2$ coefficient become more negative than could be explained by using only $p$-waves.

The $P_1$ terms that apparently exist in the angular distributions imply an interference between states of opposite parity. Only even coefficients can be obtained from interference between states of like parity. Presumably, the interference would be between a $1^-$ level and a $1^+$ or $2^+$ level. To better visualize the relationship between the non-interference and the interference terms, the total distribution may be written in the form,

$$\frac{d\sigma}{d\Omega} \propto W(\theta) = W_1(\theta) + x^2 W_2(\theta) + 2x W_{12}(\theta)$$
where \( W_1(\theta) \) = the angular distribution due to
resonance number 1 alone,
\( W_2(\theta) \) = the angular distribution due to
resonance number 2 alone,
\( W_{12}(\theta) = W_{21}(\theta) \) = the interference distribution,
and \( x = \frac{\langle |R_2^J \pi| \rangle}{\langle |R_1^J \pi| \rangle} \) is the ratio of the reaction matrix
elements.

\( x \) is, of course, unknown and may assume either sign.

Table 6 is a tabulation of the theoretical coefficients obtained
from interference between various combinations of \( 1^- \), \( 1^+ \), and \( 2^+ \)
levels. Values of the proton orbital angular momentum between
0 and 3 have been considered. Interference occurs between the
primed and the unprimed terms only for the same channel spin s.
Examining only those combinations resulting in a coefficient of \( P_1 \),
it may be seen that there are many ways to produce the observed
odd terms. It seems that no particular type of interference may
be singled out as being the likely cause of the \( P_1 \) terms, although
perhaps one or two might be neglected as they produce a reasonably
large \( P_3 \) term in addition to the \( P_1 \) term.
As mentioned in the introduction, several theoretical works have been published in which predictions of the $1^{-}$ states in $^{40}\text{Ca}$ have appeared$^{16,5,6}$ with the common basis of these works being the one particle-one hole picture. The general procedure is to start with the undisturbed configurations and introduce a residual interaction which mixes the particle-hole basis states. As first demonstrated by Brown and Bolsterli,$^{17}$ one or two of the resulting states in the simple picture have much higher energies and greatly enhanced dipole transition matrix elements connecting them to the ground state. This then appears as the main mechanism for the production of the GDR.

A major theoretical difficulty is encountered in most of these works.$^{18}$ Generally, an infinite harmonic oscillator potential is used for the generation of the basis states despite the fact that the strongly collective dipole levels in question are well above the particle emission thresholds. Thus, the basis states cannot be very suitable for the description of these particle
unstable states since they have the wrong asymptotic character. However, it is difficult to apply conventional shell-model diagonalization techniques to find the shifted energies when the basis states are generated with a finite potential well. In this case, the spectrum obtained from the unperturbed Hamiltonian contains both discrete bound states and a continuum of positive energy scattering states. Thus the wave functions of the interacting system must be expanded as a sum over the bound states and as an integral over the continuum. The problem of diagonalizing an infinite matrix results.

In a 1964 publication, Gillet and Sanderson describe how they carried out particle-hole calculations for the odd parity states in \(^{40}\text{Ca}\) utilizing an approximation known as the (Symmetric) Random Phase Approximation (RPA). They later refined these calculations in the work mentioned in the introduction when better experimental single particle energies became available. In addition they were able to use a better value for the oscillator length parameter and diagonalize the complete RPA problem instead of its symmetric approximation. Five \(J^\pi = 1^-\) states are predicted between 11 and 14 MeV. The energies and the main configurations of these states are:
As BT pointed out, the 3 highest of these $1^-$ states fall close to 3 large resonances they observed. The other two fall below the majority of the resonances in this region. Since most of the resonances we measured had distributions consistent with a p-wave $1^-$ level, probably no one-to-one correspondence with the predicted energy levels can be made and there are probably more levels than can be accounted for in this theory. Also, since the theory concerns only the negative parity states, nothing can be said about the positions of $1^+$ or $2^+$ levels.

The difficulties that are inherent in and the approximations that are necessary for the aforementioned type of calculations can be overcome by using the coupled-channel approach and formulating the theory as a scattering theory. In this manner, discrete state methods are avoided. Cross sections as functions of energy are calculated directly without the need to introduce $\gamma$-widths or partial widths for nucleon decay.
In reference 6, Marangoni and Saruis have computed the dipole photo-reaction cross-section for $^{40}$Ca with a 1p-1h continuum approximation. Their purpose in doing the work was to see how much of the structure found experimentally might be reproduced by a theoretical calculation. Also, they wished to see how the inclusion of scattering states modified previous bound state calculations. Their development of the coupled equations is outlined in Appendix C.

HM show the results of a comparison of their yield curve with the calculated total cross section from the Marangoni and Saruis coupled-channel calculations with isospin mixing in figure 6 of reference 7. (Actually, in this case, their graph comes from a private communication with Saruis.) There seems to be little agreement between the experimental results and the theoretical predictions. There is a considerably greater amount of structure in the experimental yield curve and it does not appear that any energy averaging process could give much better correlation. An indirect comparison of the $a_2$ coefficient in the angular distribution was also made as mentioned earlier. Once again, there can be seen little in common with the predictions.

The results of this work in which an $a_2$ coefficient is obtained directly allow a somewhat better check on the theory. Nevertheless,
since these results are not greatly different from those of HM, little better agreement can be expected. In figure 16, the results of the coupled-channel calculation carried through with and without mixing between the proton and neutron channels is compared with the results for the ten resonances studied. It is seen that the experimental $a_2$ coefficient is positive for most of the energies studied while the prediction is mainly negative. Neither the calculations with isospin mixing (dashed curve) nor without (solid curve) match the data points.
Figure 16. Marangoni and Saruis$^6$ Theoretical $a_2$ Coefficient Compared with the Measured Values. The solid curve is the value calculated without isospin mixing and the dashed curve is that calculated with isospin mixing.
CHAPTER V
SUMMARY

The original goal of the present work was to identify which of the many resonances found in the $^3K(p, \gamma)$ reaction between $E_p = 3$ and 6 MeV might be low-lying GDR components. Unfortunately, we have not been able to specify the spins and parities of the levels as we had hoped to. Nevertheless, we have been able to eliminate some possible assignments and describe the angular distributions better than any previous work has done. In particular, BT gave a $J^\pi = 2^+$ assignment to a resonance for which we find that either a $1^+$ or a $2^+$ assignment seems likely, without any possibility of distinguishing between the two. Even a $1^-$ assignment cannot be ruled out. The measurements of the angular distributions have shown several resonances to have $P_1$ interference components. Because of the latter, a better comparison of the $a_2$ coefficient with the Marangoni and Saruis theoretical prediction is possible with this work than with the HM work. It might be mentioned however, that except for a few resonances, there is generally good agreement between the HM data and the results in this work.
The $a_2$ coefficient that we have measured varies by a good amount over the energy range studied, yet it does not exhibit the large swings seen in the theoretical results. Also, there doesn't seem to be any obvious correlation between swings in the experimental and the theoretical work, although there is a region around $E_x = 13$ MeV where there might be some correspondence. At higher energies, again there seem to be theoretical-experimental discrepancies. Diener et al.\textsuperscript{20} have measured angular distributions for the $^6K(p,\gamma_0)$ reaction at some twelve energies throughout the GDR region. Their results indicate a more or less constant $a_2$ coefficient of approximately $-0.4 \pm 0.1$ and an $a_1$ coefficient of about 0.1. While the Marangoni and Saruis prediction for $a_2$ is generally negative in the region measured, the average value is about $-0.1$. 
APPENDIX A

Target Preparation

Targets were prepared by evaporation of natural metallic potassium in the target chamber onto a thin nickel (1/2 mg/cm²) or carbon (~30 μg/cm²) backing. Targets evaporated on both seemed to last about the same length of time and the nickel was usually used due to its ease of fabrication. Metallic potassium was used rather than the more easily handled salts of potassium (such as KI) because the latter gave evidence of decomposition under beam bombardment. In addition, there is greater yield possible from the pure potassium. As reported by R. J. De Meijer et al., the iodine is driven off of KI and metallic potassium is left. We felt this process could lead to uncertainties in effective target thickness and composition. Changes in composition which might occur even after several days of use by virtue of changes in the beam spot location or shape were judged unacceptable for angular distribution measurements.

A thin backing was chosen because it was found impossible to maintain the potassium on a thick backing without evaporation loss.
Direct target cooling did not seem to alleviate the problem which was probably due to localized heating. The small energy loss suffered in the thin backing seemed to make these targets last longer. Even so, loss of the potassium as a function of time could be noted in most experimental runs and attempts were made to minimize the effect on the data by designing data runs so that measurements were made at angles chosen randomly and by repeating measurements whenever possible.

For some of the data taken towards the end of the experiment, a potassium target which had been overcoated with a thin layer of gold was used and although this seemed better, deterioration still appeared to be present. The advantage with this target is that it can be stored in air and used on separate occasions. Uncoated targets always had to be made fresh and it was troublesome to try to obtain reasonably similar target thicknesses.
APPENDIX B

Electronics

A block diagram of the electronics is shown in figure 17. The NaI(Tl) anti-coincidence shielded detector utilizes a plastic (NE 102) scintillator shield seen by six 56 DVP phototubes whose outputs are summed in a 50 ohm matched impedance mixer. Two of these phototubes are positioned atop the plastic, near its front face, while the remaining four are symmetrically positioned on the rear face. The use of a plastic scintillator instead of NaI decreases the probability of detecting an escape event in the main crystal but greatly increases the count rate capability. The summed signals from the plastic scintillator are clipped and amplified and fed to a zero dead-time (updating) discriminator which is operated in an input DC mode. In this mode, the discriminator output pulse width is either a specified width or the pulse input width, whichever is longer. Thus there is an output pulse during the full time of very large saturated signals as might occur from a cosmic ray event.

The NaI(Tl) dynode signals are slightly integrated, clipped and
Figure 17. Electronics Block Diagram.
amplified, then fed to a fast discriminator operating in a lower level timing mode. In this mode, the timing information is obtained from the pulse leading edge at a level set slightly above the noise level, while an output pulse is obtained only if a preselected higher threshold is achieved. One output of this discriminator is used to open a linear gate for the NaI(Tl) linear signals obtained from the anode output of the phototube and is adjusted to have a dead-time equal to the shaping amplifier output pulse length. Another output of the discriminator is fed to a fast coincidence unit with veto pulses coming from the updating discriminator. If no veto pulse occurs in coincidence with a NaI(Tl) pulse, a routing signal is generated. The anti-coincidence signals are thus sent to the second 256 channels in the pulse height analyzer.

As suggested by Amsel\textsuperscript{22} and tried successfully by others,\textsuperscript{23} the anode signals of the 58 DVP phototube are clipped with an 80 nsec length cable terminated with a 12 ohm resistor. This serves to shorten the pulses and bring the tail of the pulse more quickly back to the baseline. The pulse width after this procedure is approximately 200 nsec. The system resolution has been found not to be significantly changed by this clipping procedure. These linear pulses are then gated with a gate width of slightly greater than 200 nsec by the fast discriminator pulses from the dynode signals.
Thus, pile-up of two low level signals (below the discriminator threshold) is prevented unless the two pulses occur within the resolving time of the discriminator (about 20 nsec). The linear signal is further shaped and amplified by the stretcher and shaping amplifiers. The limiter between the linear gate and stretcher amplifier serves to provide a high impedance output such that the RC time constant of the stretcher amplifier is not effectively lowered. The gain stabilizer is a linear amplifier with a variable gain which is controlled by an internal system consisting of three discriminators with coincidence logic. This system serves to keep the pulses fed back to its discriminator input at a constant amplitude, thereby gain stabilizing the linear electronics and phototube.

Reference pulses for the gain stabilization system are derived from the four light emitting diodes illuminating the photocathode that were mentioned in the main text. The two linear gates that exist after the shaping amplifier serve to sort out the light pulser signals from the scintillator signals by means of the logic pulse derived from the light pulser driver. The linear pulse associated with a true scintillation is passed to the pulse height analyzer while the linear pulse associated with the light emitting diodes is fed to the discriminator input of the gain stabilizer.
The light emitting diodes seem well suited for use as a reference light source for our detector. They emit in the visible light range with an emission peak at approximately 560 millimicrons. Although usual light emitting diodes have an output amplitude that is sharply temperature dependent, these have a quite flat output versus operating temperature characteristic when at about 20° C. This, of course, eliminates the need for precise temperature control when operated at low average power levels. At the present time, they seem to work well with a peak input power of 15 watts each and a duty factor of $2 \times 10^{-4}$. This gives an average power dissipation of about 3 milliwatts. The current driving the diodes is derived from a set of silver-mica capacitors charged to a constant voltage by a precision power supply. The entire charge on the capacitors is allowed to flow through the diodes at a 1 kHz repetition rate.

Four diodes are used in the present configuration because it was found it would be difficult to achieve the light output equivalent to about a 25 MeV scintillator pulse with a single diode. The diodes are extremely inefficient in terms of light output to power input. While it might have been possible to achieve the necessary light from two diodes and would certainly have been possible from three, four provided a more symmetric arrangement and allowed a certain amount of redundancy. The diodes each have their own
final driving stage and can be operated singly or in any combination. They are mounted in clear epoxy around the perimeter of the light pipe between the scintillator and the phototube. An attempt was made to shield the photocathode from direct illumination by the diodes but nevertheless it was been found that there is a significant difference between the focussing of the photoelectrons from a true scintillation and from the light diodes.
APPENDIX C

Giant Dipole Coupled-Channel Calculations: A Summary

Conventional bound state particle-hole calculations give the resonance positions and at best, the reduced width parameters. Coupled-channel (CC) calculations can yield both the resonance and the background contributions (with their relative phases) to the cross section.¹⁹) The CC calculations with a 1 particle-1 hole approximation (which limits the states of the target nucleus (A-1) to pure hole configurations) have been done for the four closed shell nuclei: ¹²C, ¹⁶O, ²⁸Si and ⁴⁰Ca. The calculations for ¹⁶O have been done by several authors¹⁸,²⁴,²⁵) who have found their results generally consistent with each other.²⁵) All of the calculations have yielded cross-sections which are considerably less structured than the experimental ones. The existence of the finer structure has been attributed by Gillet et al.²⁵) to the coupling of np-nh (n ≥ 2) quasi-bound states coupled to the lp-lh states. The following CC development closely parallels that of Marangoni and Saruis in reference 6, who have used the lp-lh continuum approximation to calculate the photo-reaction cross sections for
$^{12}\text{C}$ and $^{40}\text{Ca}$. 

The time independent Schrödinger equation for $A$ particles with a given angular momentum $J$ and parity $\pi$ may be written

$$H \left| \psi \right> = E \left| \psi \right>$$

Assuming the Hamiltonian can be separated into an independent particle Hamiltonian and a residual particle-hole interaction, we write,

$$H = H_0 + V$$

where the independent particle Hamiltonian is the sum of the contributions from each of the $A$ particles;

$$H_0 = \sum_{i=1}^{A} H_0 (i)$$

and

$$H_0 (i) = t_0 (i) + \nu_0 (i)$$

$\nu_0 (i)$ is a spherical potential and $t_0 (i)$ is the kinetic energy operator.

The bound eigenstates of $H_0$ will be denoted by $|c \rangle$ and the scattering eigenstates by $|c, \epsilon \rangle$, where $\epsilon$ denotes the total energy in channel $c$. Now $|\psi_E \rangle$ can be expanded as a sum over the bound eigenstates of $H_0$ and an integral over the scattering eigenstates.
\begin{equation}
|\gamma_E\rangle = \sum_c \alpha_c^E |c\rangle + \sum_c \beta_c^E(e) |c\rangle, e \geq \epsilon_c
\end{equation}

\(\epsilon_c\) is the threshold energy of channel \(c\) and in the lp-lh model is equal to the hole energy.

In the particle-hole development, the following notation will be used:

\[ B \equiv (t_B, j_B, \tau_B) \] refers to the quantum numbers of a particle, and

\[ b \equiv (n_b, j_b, l_b, \tau_b) \] refers to the hole.

\(\tau_B\) is the \(z\)-component of the isospin of the particle (and \(\tau_B = -\tau_b\)).

Each channel \(c\) of the particle-hole type can then be specified by the quantum numbers \((B, b, J, M)\), where \(J = j_b + j_B\), and \(M = m_b + m_B\). Then the lp-lh ket for \(A\) nucleons is:

\begin{equation}
|B, b, J\rangle = \sum_{m_b, m_a} \langle j_b, m_b j_a, m_a | J\rangle |B, m_a, \epsilon\rangle |b_j, m_a, J^J\rangle
\end{equation}

where \(\langle j_b, m_b j_a, m_a | J\rangle\) is the Clebsch-Gordon coefficient. The hole ket is defined by the wave function:

\[ \Phi_b(\vec{r}) = \langle \vec{r} | b_j, m_b \rangle = \frac{R_b(n_e, \epsilon)}{\lambda} \sum_{\tau_b} \int \chi_{j_b}^\tau(\vec{r}) \right]

The particle ket is defined by
if the particle is in the continuum or

\[ \Phi_{\text{B}}(\vec{r}, \epsilon) = \langle \vec{r} | B, m_B, \epsilon \rangle = \frac{u_B(n, \epsilon)}{\nu} t_o^{m_a} y_{j_e}^{m_e}(\hat{r}) \]

if the particle is bound.

\[ \Phi_{\text{B}}(\vec{r}, \epsilon) = \langle \vec{r} | B, m_B, \epsilon \rangle = \frac{R_B(n, \epsilon, \epsilon_0)}{\nu} t_o^{m_a} y_{j_e}^{m_e}(\hat{r}) \]

\[ \Phi_{\text{B}}(\vec{r}) \text{ and } \Phi_{\text{B}}(\vec{r}, \epsilon) \text{ are eigenfunctions of the single particle} \]

Hamiltonian \( h_0 \). The radial wave functions use the following normalization:

\[ \int R_i(n, \epsilon_i) R_j(n, \epsilon_j) \, dn = \delta_{ij} \]

\[ \int u_B(n, \epsilon) u_B'(n, \epsilon') \, dn = \int (\epsilon - \epsilon') \delta_{BB'} \]

The angular wave functions are given by:

\[ \sigma_{j_e}^{m_e}(\hat{r}) = \sum_{m_s} \langle l m_s s m_s | j m \rangle i^l \chi^{m_e}_l(\hat{r}) \chi^m_s \]

and \( t_t^r \) is the isospin wave function.

The radial part of the wave function of the particle interacting

with the hole may likewise be expanded and can be written:

\[ \Phi_{\text{B}}^{J^P}(n, \epsilon) = \sum_{\epsilon_0} \frac{\epsilon}{\epsilon_0} \beta_{B^I^P}(\epsilon) u_B(n, \epsilon) \, d\epsilon \]

\[ + \int_{\epsilon_0}^{\infty} \beta_{B^I^P}(\epsilon) u_B(n, \epsilon) \, d\epsilon \]
Marangoni and Saruis choose the residual particle-hole interaction to be a zero-range force of the form:

\[ \omega = - u_0 \sum (\tilde{\alpha}_a - \tilde{\alpha}_b) \left( \pi_t + \rho \pi_s \right) \]

\( \pi_t \) and \( \pi_s \) are the triplet and singlet projection operators in the particle-hole spin space.

Substitution of the appropriate previous equations into the Schrodinger equation leads to the following coupled integro-differential equations and the functions \( r^{J\Pi}_{\beta\beta'}(r, \varepsilon) \) are the solutions to these equations.

\[
\left[ -\frac{\hbar^2}{2\mu_{sb}} \frac{d^2}{dn^2} + \frac{\hbar^2 (l_b + 1)}{\lambda^2} + \lambda_b^0(n) + (\varepsilon_a - E) \right] f_{\beta\beta'}^{J\Pi}(n, \varepsilon) \\
- u_0 \sum_{\beta'\beta} G_{\rho} (\beta\beta', \beta'\beta'; J\Pi) R_{\beta'}(n, \varepsilon_{\beta'}) R_{\beta}(n, \varepsilon_{\beta}) \rho_{\beta'\beta'}^{J\Pi}(n, \varepsilon) \\
+ u_0 \sum_{\varepsilon_{\beta'}} R(\varepsilon_{\beta'}, n) \sum_{\beta\beta'} G_{\rho} (\beta\beta', \beta'\beta'; J\Pi) c_{\varepsilon_{\beta'}}^{\beta'\beta} = 0
\]
Here $\psi_B^0(r)$ is the optical single-particle potential for particle B.

$G_p(\ )$ is a geometrical factor depending on the spin term parameter $p$ of the residual interaction.

$$C_{\ell B}^{l_B^{e_B}} = \int_0^\infty \frac{R_B(\lambda', e') R_b(\lambda'_e, e_b)}{\rho^2} \int_{B_B}^{T_T} (\lambda'_e, e) d\lambda'_e$$

$\epsilon_B^0$ denotes the energies of the target with quantum numbers $l_B$, $j_B$.

The last term on the left hand side of the coupled integro-differential equations is neglected as it was shown by Raynal et al.\textsuperscript{34)\textsuperscript{34)} that it does not significantly affect the computed cross sections.

The definition of the radial wave function is completed when appropriate boundary conditions are specified. From the independent solutions of the coupled equations, a wave function $\psi_{Aa}^{J\pi}(r, \epsilon)$ can be constructed which has an incoming wave of unit amplitude only in the open channel ($Aa$, $J\pi$). The dipole transition probability is then proportional to the matrix elements

$$\left| \langle 0 \mid D \mid \psi_{Aa}^{J\pi} (n, E) \rangle \right|^2$$

where $D$ is the dipole operator and $|0\rangle$ is the vacuum state. It is then straightforward to calculate the dipole photo-nuclear cross-sections.

For the single-particle potentials, Marangoni and Saruis have used a Woods-Saxon potential with spin-orbit coupling and a Coulomb
term of the form:

$$\nu_{2d}^i(n) = -V_{2d}^i \left[ 1 + e^{(n-R_0)/2} \right]^{-1} + V_{2d}^i \left( \frac{d}{d\eta} \right)^2$$

$$\times \left[ \left( \frac{\nu}{\nu+1} \right) - \frac{\nu+1}{2} \right] \frac{d^n}{d\eta^n} \left[ 1 + e^{(n-R_0)/2} \right]^{-1}$$

$$+ V_{\text{coul}}$$

where

$$V_{\text{coul}} = (\varepsilon-1) \frac{e^2}{\hbar} \zeta(n)$$

with

$$\zeta(n) = \begin{cases} 1 & \text{for } \nu > R_0 \\ \frac{\nu}{2} \frac{n}{R_0} - \frac{1}{2} \left( \frac{n}{R_0} \right)^3 & \text{for } \nu \leq R_0 \end{cases}$$

and

$$R_0 = \rho_0 (A-1)^{1/2}$$

Rather than calculating the Woods-Saxon parameters in the single particle potentials, Marangoni and Saruis have chosen the parameters to reproduce the experimental single particle and hole energies. However, the latter are surprisingly not well known due to the fact that there exist practically no pure single particle states in nuclei adjacent in $A$ to $^{40}$Ca. The single particle energies that have been used are those of Gillet and Sanderson.5)

The radius parameter $r_0$ and the diffuseness parameter $d$ occurring in the single particle potentials have been fixed at 1.25 fm
and 0.53 fm respectively. An imaginary potential of the form

\[-i W \chi e^{(t-a^*R_0)} \left[ 1 + e^{(t-a^*R_0)} \right]^2.\]

is sometimes added by the authors to the single particle potentials to take into account configurations which are not considered in this lp-lh model. The general effect of this absorptive potential is to lower the computed cross-sections and indeed the parameter W was adjusted to obtain the correct magnitude for the cross-sections.

Marangoni and Saruis have also done the calculations with and without the assumption that the total isospin of the system is a good quantum number. A comparison of the two calculations of the $A_2$ coefficient with and without isospin mixing was shown in figure 16.
REFERENCES


16. V. Gillet and E. A. Sanderson, Nucl. Phys. 54 (1964) 472.


