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EDGES AS SPECIFIERS OF IMAGE QUALITY

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

by

Leon G. Thompson, B.Sc., M.Sc.

* * * * * *

The Ohio State University
1972

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Adviser
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ACKNOWLEDGEMENTS

A study of this scope would be impossible without the assistance of many people. I would like to express my sincere appreciation and gratitude to:

1. The U. S. Army Corps of Engineers for giving me the opportunity of engaging in the studies which led to the completion of this dissertation.

2. Dr. Desmond O'Connor who proposed the initial idea and method and continuously supplied guidance and counsel.

3. Dr. Sanjib Ghosh, who as my adviser, offered invaluable assistance throughout my tenure at the Ohio State University.

4. The Research Institute, Engineer Topographic Laboratories, from where I received technical assistance, the apparatus used in the experiment, literature and other support.

5. Mr. Stephan Eckstrand, a technician at Wright-Patterson Air Force Base, who operated sophisticated equipment in support of this project.

6. Mrs. Guytanna Swisher, who displayed great patience in the typing and proof-reading of this paper and assisted in many other ways.

7. My observers, Hank J. Hietkamp, Sebastian Ekenobi, Chellappah Kanagalingam, who gave many hours of their time in the collection of nearly 18,000 observations.

8. My family, Babs, Brian and Mathew, who graciously allowed me to devote an inordinate amount of time to my studies at their expense.
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PUBLICATIONS

"Study of the Effects of Pollutants on the Index of Refraction," *Applied

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Studies in Geodesy and Map Projections. Professors I. Mueller
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1. INTRODUCTION

1.1 Background and Purpose

In recent years the literature on the subject of image quality has undergone an enormous growth without producing very much of tangible value to photogrammetrists and others concerned with the measurement and detection of photographic images [103]. The only criterion in wide general usage is resolution. It has always been a comparatively simple matter to point out the deficiencies of that quantity as a specifier of image quality for the photogrammetrist concerned with measurements on a photograph. (See Appendix A) But it has not been so simple to produce a reasonable alternative. When resolution was discredited, a vacuum was created and a number of terms sprung up in an attempt to take its place. None has yet succeeded, although some of the new terms, such as spread function and modulation transfer function, appear to be relevant. Scientists have only recently realized that a single number does not, nor will not exist which will completely define the value of a photographic system. As Brock states, "The quality of an image is so very complex, and involves so many elusive factors, that it cannot be reduced to one number without sacrificing something vital and essential." [22, p361].

The many and varied users of photogrammetry have in the past been attempting to use the same terms in defining image quality. "In general, the concept of image quality does not emerge until we have defined
some task for the system to perform...." [22, p357]. The following statement by Brock ought to be considered a fundamental law for researchers in this area. "The more limited and precisely defined we can make the task, the more accurately can the performance of the system be evaluated...." [22, p357].

Within the broad acknowledgement that different criteria should be used for different tasks, it is proposed here that the edge should be the quantity most intimately related to photographic mensuration. When the dimensions of an object are determined from its photographic image, the measuring mark in the measuring instrument is brought into coincidence with the edge and the necessary dimensions deduced. The point at which the measuring mark is set on the edge is obviously of prime importance. Yet there is no available material in the literature to suggest just where the subjective judgement is made. The selection of the edge is also considered valuable because it is very elegantly related to the spread function whose Fourier transformation gives the corresponding modulation transfer function. In addition to the significance in measurement, the characteristics of the edge can be shown to be highly significant in influencing whether or not an object is detected against its background. The edge is consequently regarded as a very appropriate subject for fundamental research.

For the purposes of this research, to overcome the difficulties concerned with studying an actual photographic edge, the method of O'Connor will be used which consists of generating an edge through fusion of rapidly moving patterns mounted on a drum. Four trained and
experienced observers will repeatedly set a measuring mark tangent to where they subjectively determine the edge to be located. To tie this subjective edge determination to the edge and its measurable properties, each edge will be scanned with a photometer mounted behind a 200 mm lens which examines an area 1/4 mm in diameter. Armed with these subjective edge determinations and the objective luminance vs. distance measurements a detailed analysis will be made of each edge including the role that the maximum gradient, acutance, spread function, higher derivatives and modulation transfer function play in influencing the decision of the observers. In this way it is hoped that for the first time characteristics of an edge which enhance measuring accuracy and precision may be isolated and thereafter incorporated into photographic systems.

1.2 The Relevance of This Study to Photogrammetry

The objective of this study is primarily to search for a meaningful specifier of image quality. Image quality is defined as the measurability of an edge as determined by the degree of agreement between subjective edge locations from four experienced observers. A secondary objective is to investigate where the subjective edge is determined to be located in reference to some measurable quantity.

It is important to better understand where observers point and what seems to cause observers to agree more strongly as to where an edge is located. What causes one edge to be of higher quality than another and what easily measurable parameter would indicate which edge is of the higher quality? These factors are important to the researcher who is interested in designing a system which produces edges of high quality. The researcher obviously can not make hundreds of subjective observations
on a newly designed system's edge to determine its quality. He needs to be able to quickly measure some objective quantity which he knows correlates highly with image quality. A search for that quantity is the primary objective of this study.

It is obvious that if in the future the human observer is still a part of the photogrammetric measuring process as a desire for greater accuracy increases, the confusion over the subjective edge location will become increasingly important. Edges which minimize this confusion would thus become more and more desirable.

One might ask if the subjective edge location problem could be important in present day photogrammetry. Presently photogrammetrists speak of obtaining or hoping to obtain one micrometer accuracy at the photo scale. It is difficult to put this in terms of seconds of arc because most photogrammetric instruments differ in magnification, technique, etc. However, O'Connor [104, p8] determined that a comparator with a least count of 1μm at 12 X was comparable to a device with a least count of 10 seconds of arc. Let us assume that the dispersion in the means of four experienced observers for a particular edge was 30 seconds of arc. This would mean that on the photo an ambiguity of 3μm exists between the observers as to where the edge is located.
2. THEORY - THE EDGE & IMAGE QUALITY

2.1 The Edge

2.1.1 General Discussion

The edge is surely the most important and most basic element of an image. If the observer's eyes do not detect an edge, border or boundary, no object will be seen. The shape and location of an object is directly related to the detectable tone variance between the object and its background which is, of course, its edge or boundary. Fry states that "... a point is merely a small patch bounded by a border, and a line is a strip bounded on opposite sides by borders." [48, p63]. Nearly every descriptive adjective and noun in the photogrammetric vocabulary has been applied to the edge in an attempt to describe why, how and where one locates an image on a photograph. To study the edge the first and most obvious topic should be the edge trace. For the edge trace, besides being a graphical representation of the edge, is also the starting point for discussions of the spread function (See Section 2.2.2.3), higher derivatives and Fourier analysis (See Section 2.2.2.4). It seems only proper that the edge should be discussed in more detail.

2.1.2 Edge Trace

There are two kinds of edge traces, density and intensity. Both are a plot of density (or intensity) vs. distance. Both are derived through the use of some type of scanning apparatus, usually a microdensitometer. An edge trace reveals the distribution of light energy over a
particular region and when the distribution varies sufficiently that an observer can detect a change in tone, it is said that an edge or border exists. Another commonly used name is the "edge gradient."

Brainard and Ornstein, when studying image enhancement concluded that "The ability to extract information from an image 'depends in large measure on the ability to perceive borders' and the perception of borders depends significantly on the nature of the edge-gradients defining the border." [18, p5]. It is recognized that the image of a bar could be considered to be composed of an infinite number of infinitely thin lines, the spread function of each combining to give the over-all spread function. This type of analysis, made popular by Perrin [76], will not be directly considered in this paper. An edge trace here will be the actual distribution of light energy, over some distance, of a visible edge on a photograph. And, as was stated earlier, to circumvent the problems concerning the attainment of this edge trace from an actual photograph, an edge will be generated having the same characteristics as a photographic edge except that it will be wide enough to permit detailed study. Another advantage of the generated edge is that the edge scan gives the intensity trace directly. Both Scott and Barrow warn against the errors which can develop when a researcher attempts to get the intensity trace from the density trace [6, 115, 116]. This problem arises because nearly everyone starts with a microdensitometer trace (which is at best confusing) and must obtain an intensity trace in order to find the spread function or perform a Fourier analysis of the edge.

Colcord, et al. in an introductory study of edge gradients noted that apparently tangent pointing was related to the steepness of the
edge gradient [38]. They also found that "...the Mach Band location plays a significant role in the visual perception of the edge contours and, hence, affects the pointing accuracy and position." [38, piii]. Mach, Colcord, et al. maintain that when pointing to an edge we seem to point near to the inflection point if one occurs [110, 38]. However, Fry points out that it is not known at this time exactly where an observer points [47].

The issue of where an observer points will be discussed more completely in the following section.

2.1.3 Edge Location

Where the edge is located is one of the two most important facets of this study, (the other being the search for a specifier of image quality). Is the location of an edge a subjective decision? If it is subjective, is the subjective portion significant as far as measuring accuracy is concerned?

Hempenius, in an exhaustive study of the physiological and psychological aspects of visibility concludes that edge location is definitely a subjective decision [64]. O'Connor was one of the first to attempt to relate to photogrammetry the concept of pointing being subjective [104]. His efforts acted as a catalyst for other studies. Trinder has done considerable work in this area, but he has always been concerned with the task of centering a measuring mark on a target [129]. One cannot be certain how these results would correlate with the task of setting a measuring mark tangent to an edge, but it may be important to note that he found that "... background density is only a very marginally significant property of the target affecting pointing accuracies." [129,p196].
This may agree somewhat with Colcord's findings that standard deviations of pointing "...are essentially constant until the 15% contrast level when they increase." [38].

Colcord seems to agree with Hempenius that the edge location is a subjective decision and states that each observer pointed rather consistently to the subjective edge, but the "same" edge position was not seen by all the observers [38].

In general an edge may vary in two ways; in width and in contrast ratio; i.e. the maximum intensity divided by the minimum intensity. However, within these two general categories, the light distribution may assume any number of shapes, all of which are a smooth curve not necessarily symmetrical in nature. Hopefully this study will reveal information about every case.

Notwithstanding the earlier comments concerning the possibility that we point at or near the inflection point of the above mentioned smooth curves, more needs to be said about the apparent subjectiveness of the edge location process. A thorough discussion of this topic would lead one down the psychic road of Mach bands which is not one of the purposes of this paper. (For information on Mach bands, the reader is referred to Colcord and Ratliff [38, 110].) However, it is important to realize what actually happens when we view an edge.

It was Ernst Mach who first studied the topic of edge fusion at about the midpoint of the 19th Century. Using a simple apparatus he showed that if a white belt with black patterns (See Figure 2.1) were rapidly rotated, an observer would see a fused image as in the lower
part of Figure 2.1 [110]. If the edge were sufficiently wide, bright stria would appear at the junction of the two slopes. As the edge decreases in width, one merely sees two uniform bands of unequal brightness.

However, it is known from frequent edge scans that in photogrammetry edges are represented by smooth curves which may or may not be symmetrical. Mach further showed that if such an edge were fused by rotating a belt with a pattern, as in Figure 2.2, that the resulting fused edge could consist of two uniform bands, a darker band on top and a lighter band on the bottom [110]. Mach claimed that the line e'f' would be located at the "midpoint" of the edge which would possibly only be true for a perfectly symmetrical edge. The line e'f' is more correctly
located by saying that it would appear at the point of inflection of the curve although even this statement is often debated.

This amazing phenomenon was apparent in all the fused edges produced by the edge generating apparatus for this paper. A typical occurrence was to look at an edge and measure it with a linear scale arriving at an abrupt edge, possibly 1.5 cm in width; only to discover upon scanning the edge with a photomultiplier probe that the edge trace was a smooth curve and possibly 3.5 cm in width.
If the pattern on the left were wrapped around the drum and rapidly rotated, the observer would see a fused edge similar to the sketch. The line (or edge) $e'f'$ is near the inflection point (See analysis in Section 4.0). Yet if this fused edge were scanned with a photometer, an edge trace from $d''c''$ to $g''h''$ would be obtained. This figure is meant to illustrate the general situation only. The geometry, contrast, edge location, etc., are not necessarily valid for all cases.
This is a photograph of an actual fused edge generated by the device to be explained later. Note the "sharp" boundary or edge. An edge scan would reveal that the light is distributed in a smooth curve over a distance much beyond where the subjective edge appears to be located.

The point that is of extreme interest to the photogrammetrist is that "For the eye, the transition point from convex to concave forms an almost sharp border between bright and dark." [110, p311]. It is important to note that Mach maintains that barring small changes according to one's disposition, "... all persons see the described phenomena in almost the same way." [110, p284]. To conclusively separate this subjectively observed difference from normal pointing differences would be most difficult, although not impossible. An attempt will be made in a later section to separate the two phenomena.
If observers do in fact, point at the inflection point or some other objectively measurable location, and if the factors of an edge which affect the precision of pointing could be isolated, it is highly possible that the photogrammetrist would be well on his way to designing systems which would increase pointing precision and accuracy. It is hoped that this paper will provide some illumination in this area.

2.2 Image Quality

2.2.1 General Discussion

Webster defines quality as that property, characteristic or attribute which is inherent in a thing, thereby allowing you to describe and classify a species or type. That, in a somewhat general sense, describes one of our present goals, i.e. to be able to classify or rank images. In keeping with our discussion in Section 1 concerning the definition of our specific task, it can be stated that an image of good quality is any image which allows us to recognize and measure with high precision and accuracy an edge or boundary.

It is important to realize that the true location of an edge must be defined as that location where the reasonable and prudent, experienced observer, subjectively sets the measuring mark. As an edge's characteristics change, so will its true location as determined by this reasonable and prudent, experienced observer. And quite probably some edges will exhibit those characteristics which will make measurement to it more precise and accurate than to others. However, at this time the state of the art has not progressed to the point where we know for certain where this experienced observer will point nor do we know how a specific edge change will affect his pointing ability.
The lack of knowledge as to what constitutes an image of good quality for the photogrammetrist concerned with measuring to an edge has not kept authors from expounding at great length on the factors comprising image quality. The *Manual of Photographic Interpretation* is one of the few references which freely admits that they are unable to define good photography because the experts in the field are not in sufficient agreement in this area [95, p52]. Other authors that do discuss image quality criteria and factors do not explain for which tasks their parameters are valid. Most certainly all photogrammetrically and photographically related tasks do not have the same quality criteria.

An example of the diversity, and possibly confusion, which exists is seen in the following collection of comments on image quality.

A. The quality of an image formed by an optical system is determined by three factors:

   a. Aberration
   b. Wave nature of light
   c. Inaccuracies of manufacturing processes [1, p56].

B. The major factors governing the quality of photographic images are tone contrast and sharpness. Tone contrast is governed by:

   a. Spectral reflectivity of an object
   b. Spectral sensitivity of the film
   c. Spectral scattering by the atmosphere
   d. Spectral transmission by the photographic filter

and sharpness is governed by:

   a. Aberrations
   b. Focus
c. Image Motion

and d. Characteristics of photographic materials [95, p58].

C. The factors controlling image quality are:
   a. Angular field
   b. Definition
   c. Character of emulsion
   d. Altitude
   e. Ground speed
   f. Vibration
   g. Distortion

and h. Character of illumination [95, p50].

D. The amount of information in a photograph depends on:
   a. Graininess
   b. Sharpness
   c. Resolving power

and d. Tone reproduction [95, p52].

F. The factors affecting the sharpness of a boundary are:
   a. Refraction in the atmosphere
   b. Diffusion
   c. Diffraction
   d. Lens quality
   e. Sensitivity of film
   f. Graininess
   g. Irradiation

and h. Image motion [72].
From the above one might assume, (perhaps justly so) that someone's
definition of image quality will be affected by virtually every element
in the photographic system.

It would seem that any meaningful discussion of image quality would
firstly define the task and then list the criteria which will be used in
this discussion. For the purposes of this paper the task has been amply
defined, i.e. setting a measuring mark tangent to an edge. The quality
criteria has yet to be determined.

2.2.2 Possible Image Quality Criteria

2.2.2.1 Common Terms

There is a collection of terms which photogrammetrists find neces­
sary to use in their discussions of edges and images which nearly every­
one admits cannot be objectively defined. Examples are blur, definition,
graininess, tone, sharpness and in some cases contrast. Everyone has a
general "feel" for the meaning of these terms and no problem arises as
long as they are used in a general descriptive manner.

Frequently the terms are defined in the following manner:

a. Blur - lacking a definite outline

b. Definition - clarity of detail

c. Sharpness - subjective impression of the abruptness in the
change in density across an edge

d. Tone - Each distinguishable shade variation from black to
white [95]

e. Graininess - subjective impression of granularity [72]

and f. Contrast - the degree of differentiation between tones [95].
Just simply defining these terms does not reflect the thought and consideration that has been devoted to this subjective area. Fry has proposed that blur, magnification, focus and change in viewing distance are all interrelated [47]. Brock suggests that the blurring of an image corresponds to the stripping off of the harmonics [24]. Fry has related an index of blur to the Modulation transfer function [49]. O'Connor correlates blur with the edge gradient and suggests that it may be important in visual acuity tasks [104]. He hints at the possibility that the extent of blur may be reflected in the second or fourth derivatives of the edge trace. Trinder studied the effect of pointing to blurred signals and divided blur into density profile and annulus width [129]. His conclusion that pointing accuracies depend primarily on the grade of the density profile of the target and secondly on annulus width probably does not apply to this work in as much as he was concerned with centering a measuring mark on a circular or nearly circular target.

Higgins and Wolfe in 1955 maintained that definition was "... the composite effect of several subjective factors, such as sharpness, resolving power, graininess, and tone reproduction." [68]. They attempted to hold graininess and tone constant and tested the effects of sharpness and resolving power on definition. Twenty observers rated nine different photographs. They concluded that, "When graininess and tone reproduction are constant and resolving power is adequate to reproduce all the detail that can be observed under the conditions of viewing, acutance correlates well with definition." [68, p129]. They further stated that if the resolving power was low, then it too correlated well with definition [68]. They devised an equation defining definition in terms of
resolving power and acutance which, in the opinion of this author, has not received great acclaim.

In 1958 Higgins and Perrin spoke more harshly against resolving power and stated simply that it correlates very poorly with definition [67, p71].

Stulz and Zweig had 50 observers rate nine photographs in various categories. The terms picture quality and definition "... had definite meanings for the observers, but the term 'picture quality' led the observers to weight sharpness and graininess about equally while the term 'definition' led to a high correlation with sharpness and a low correlation with graininess." [126, p45]

Tone is important in the simple recognition of detail. If there is no tone contrast there will be no border and hence nothing will be seen.

The Manual of Photographic Interpretation suggests that tone may be substantially improved through the intelligent use of photographic films and filters which will emphasize our areas of concern by eliminating the effects of haze, unwanted wavelengths, etc., and capitalizing on the reflectance of our surface [95].

Numerous attempts have been made to find an objectively measurable quantity which would correlate well with the subjective decision of sharpness. Acutance is the most widely known and will be discussed under a special heading. The maximum edge gradient has also been suggested. Higgins and Jones investigated this index and found that the maximum gradient failed to distinguish an edge gradient having a sweeping toe and shoulder from one with an abrupt toe and shoulder [66]. They maintained
that the abrupt toe and shoulder produced the sharper image. The average gradient was also ruled out [66].

The composition of the edge-gradient may be informative and will also be discussed under a special heading.

Resolution is generally an unreliable predictor of sharpness. Brainard and Ornstein, in an article on image enhancement, quote from Stulz that "... the sharpness of an image has no fixed relation to the limit of resolution of the system..." [18, p2]. Shack also states that resolving power and sharpness are not that closely related. [121]

Perrin's discussion of the structure of the developed image in The Theory of the Developed Image is ultra complete [76]. In particular his outline of graininess and granularity is a thorough recapitulation of the state of the art. For the purposes of this paper it should only be emphasized that the presence of grains in the emulsion definitely have an effect on image quality. However, one should not state categorically that grains have an adverse effect on image quality.

It is apparent, in Appendix B, that the presence of clumps of grains complicates the analysis of the photographs and microdensitometer scans. Besides the physical harassment resulting in noise, another complication is the fact that the presence of grains impairs the eye's contrast discrimination mechanism [64]. However, without grains no image would appear because, of course, they are an important link in the photographic process. In fact, Perrin points out that attempts to reduce the presence of grains invariably adversely affect the speed and sharpness of the film [76]. The technique that is being used in this paper to generate an edge eliminates graininess as a parameter. In reality, of course,
graininess is a problem, but the conclusions drawn in this paper should most certainly apply to actual photographic edges. However, this should be verified through experimentation.

Contrast has been previously defined as the degree of tone differentiation. Geltmacher, Fry, Brock and others have listed many ways of objectively defining contrast by measuring the luminance of the object, background, etc. [51, 49, 72]. These definitions have acquired such names as contrast ratio, modulation, differential contrast, and others. Contrast is exceedingly important in a discussion of edges for an edge is a boundary or border between two areas of different contrast. The nature of this edge, i.e. the finite region over which the contrast difference takes place, is really a part of the edge-gradient and will be discussed under that section. It is true that one's ability to accurately measure to an object is not necessarily correlated with how readily detectable the object may be on the photograph. However, it is also true that unless one can detect an object one will be unable to measure it. Thus detection is directly correlated with contrast. Geltmacher claims that resolution is nearly constant for contrast ratios of 10:1, but falls off rapidly thereafter [51]. Bousky and Geltmacher [15, 51] in separate studies revealed that high contrast on the ground may become very low contrast on the photograph. For example, a newly painted target of diffuse black and flat white may have a contrast ratio of 20:1. But at 30,000 feet, the atmosphere reduces the contrast to only 5:1 [51]. Carmen and Carruthers claim that 90% of all fine detail (1-3 feet) exists at contrasts of 0.1 or less [51]. Bousky feels this to be slightly pessimistic, but admits it is feasible. He claims that "... the eye when
viewing even a high quality transparency under optimum conditions cannot see detail contrast below about .05 on most present photographic materials." [15, p6]. Hempenius is in general agreement, stating that normally an observer can distinguish two lines from one if the middle has at a minimum 3% less brightness [64]. The eye's performance when it is finally presented this contrast differential is discussed under the section on edge-gradients.

These few paragraphs concerning contrast were to emphasize that we perceive things because they have an edge or border and things have borders because our eyes perceive a sufficient contrast differential.

2.2.2.2 Acutance

Acutance was first announced by Higgins and Jones in 1952 [66]. A slight modification was made shortly thereafter [67] and today nearly every textbook on image quality carries a definition similar to the following:

Acutance (Ac) is the objective correlate of sharpness [76] and is defined mathematically as,

$$ Ac = \frac{G^2_x}{(D_B - D_A)} $$

where,

$$ G^2_x = \frac{\sum (\Delta D/\Delta X)^2}{N}, $$

and

$$ D_B, D_A \text{ are densities at B and A (see below)} $$

$$ \Delta D = \text{change in density over an interval } \Delta X $$

$$ N = \text{number of intervals.} $$

The points B and A are the locations where $\Delta D/\Delta X$ starts being greater than .005 when the distance is measured in micrometers and the negative is viewed at 4X.
Although acutance is now used by many people, there are still differences in opinion as to what it measures or what it indicates. Higgins and Perrin showed that resolving power and acutance are independent of each other, basing their experimental findings on a study of the spread function [67]. They emphasized that "... resolving power is sensitive to the shape of the neck of the spread function while acutance is sensitive to the width of the base or skirt." [67, p75].

Brainard, when using observers in photo interpretation type tasks (detection, recognition, etc.), found that image quality, based on the number of correct responses, correlated well with acutance [19]. Perrin states that acutance correlates well with sharpness "... when the edge has the sigmoid that is common to photographic materials." [76, p511]. He found it to be less good for other shapes. Both Lund and O'Connor thought that Greene's deduction that the response to a contour is a function of the second and fourth derivatives of the edge gradient, possibly implies that acutance, a function of the first derivative, is unimportant as a specifier of image quality in photogrammetry [90, 104, 54].

Support was supplied to the theory of the importance of the second derivative by an experiment done by Brainard [19]. He converted the light transmitted by a photograph through a linear relationship to a voltage. From this he obtained the second order differential and fed this back through the system. He noted a marked improvement in the correct responses of his observers when performing photo interpretation type tasks. Brock in 1966, stated that acutance seems to relate directly to images as the eye sees them [21]. Of course, this does not mean
that acutance would have to relate to image quality based on one's abil-
ity to measure to an edge.

Brock emphatically concludes that acutance has no place in lens
testing nor does it display quality vs. size that is important in aerial
photography. However, he further states, "Nevertheless, its emphasis on
edge quality is very appropriate, since edges are common in the highly
detailed scenes we examine. In some form it may yet find a place for
comparative evaluation of aerial negatives..." [22, p361]. The Manual
of Photogrammetry says acutance is "An objective measure of the ability
of a photographic system to show a sharp edge between contiguous areas
of low and high illuminance." [96, p1126].

From the above statements it would seem that considerable confusion
exists as to what can be done with acutance. No one has shown that it
relates to our ability to measure to an edge. Part of the problem may
be that, as with resolution, photogrammetrists have read too much into
it. At an ITEK symposium in 1963 Dr. Higgins stated that more has been
read into the term acutance than what was originally planned. It is a
"... psychophysical quantity which can only be applied to an actual
image after it is formed to tell how the edge looks in terms of sharp-
ness." [72, p7].

The question of whether acutance correlates with our ability to
measure to an edge should be somewhat clarified as a result of this
study.

2.2.2.3 Spread Function and Higher Derivatives

Once an intensity edge trace is obtained the spread function may be
derived by, (1) fitting a function to the observed discrete points, and
(2) finding and plotting the differential of the edge trace. This, of course, makes the spread function, as well as its higher derivatives, totally dependent upon the edge trace. Nonetheless, it is possible that a close study of the characteristics revealed by each derivative may disclose some parameter that correlates well with image quality as determined by the "closeness-together" of the edge locations of the four observers.

Four derivatives of each edge were found, plotted and studied. Figure 2.4 shows how the theoretical derivatives would appear for four different theoretical edge traces. In reality, of course, the edge traces could exhibit noise or more technically the presence of adjacency effects or border effects. The interested reader is referred to Perrin [76].

Because the edges used in this paper were manufactured, there were no adjacency effects and this subject was not considered. Despite this, the graphs of the various edge derivatives did not strictly assume the theoretical shapes because of discrepancies due to the imperfection in the fit of the function. This subject will be discussed in more detail in Section 5.7 concerned with the analysis of the higher derivatives.
Figure 2.4

Plot of Four Theoretical Edge Traces and Their First Four Derivatives
2.2.2.4 Modulation Transfer Function

2.2.2.4.1 Definition

The reader that is unfamiliar with Fourier transform principles and applications would find it advantageous to read Appendix E prior to continuing with this section. An understanding of these principles is relevant to an understanding of the often quoted photogrammetric phase; the modulation transfer function is the Fourier transform of the spread function. The modulation transfer function could be referred to in a number of different ways. Jones [77] lists twelve different basic names and then states that the prefix sine-wave or sinusoidal could be attached to any or all of the basic terms rendering thirty-six possibilities. At Itek's symposium [72] in 1963, an attempt was made to standardize these terms. Although no general agreement could be reached, it was emphasized that Modulation transfer function was the term recommended by the International Committee for Optics in 1961 for "... the modulus of the Fourier transform of a line spread function." [72, p12]. Brewer summarizes it very well by stating that the modulation transfer function (or MTF) is "The measure of the ability to transfer contrast from the object to the image..." [20, p1]. This is directly related to what Welander calls the fundamental purpose of a photographic system, that is, to receive varying brightness values of different objects on the ground and record them on the negative as density differences [136].

Modulation (M) is generally defined as

\[ M = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} \]

where I is the intensity of a sine wave target [104,48, 136]. If a
ratio of the modulations of the image and object is formed, the result is
called the modulation of the system, but only for one particular fre­
quency. [156]. Welander illustrates this through a figure similar to
Figure 2.5 [136, p16]. As the frequency (f) increases, the amplitude
of the imaged wave decreases, thereby lowering the system modulation at
that particular frequency. "A plot of modulation transmitted against
spatial frequency, yields the modulation transfer function of the element
concerned." [104, p48-49]. Brock is quick to point out that "Strictly
speaking, we cannot use the terms transfer function and modulation for
anything but sinusoidal targets." [25, p31]. This somewhat distressing
fact is circumvented by noting that the Fourier transform reveals the
spectrum or sinusoidal wave composition of an intensity spread function. This point will be discussed further in Sections 2.2.2.4.3 & 4 on advantages and disadvantages of the MTF principle.

Welander expresses the MTF principle in practical terms by noting that wide black and white bars of a pattern will be reproduced as black and white while more narrow bars become shades of gray. [136]. It is sometimes easier to grasp this concept if one considers, as does Perrin, the similarity to an audio system. He alleges that one could measure the frequency response of a phonograph by putting on pure notes of varying frequency. Then by measuring the sound energy coming out and plotting the ratio, power out/power in, versus frequency, one would arrive at the frequency response curve or MTF. [76]. Similarly, Welch, [137] feels that MTF's could be valuable in determining image quality once a correlation is proven. Welch [137] asserts that photogrammetrists have been unconcerned about MTF's, possibly because of their complexity and their undemonstrated value. Shaw [123] confirms that photogrammetrists have indeed been slow in adopting the MTF, pointing out that as early as 1946 Duffieux proposed that Fourier techniques from communication theory be applied to photography.

2.2.2.4.2 Characteristics

Birch and Welch [14,137] are only two of the many researchers who hint at the possibility of the modulation transfer function becoming a measure of image quality. It has been stated before that the imaging process does not alter the shape of a sinusoidal target. [122,22]. However, the targets encountered in every day photogrammetry are not
sinusoidal and the application of the MTF to these targets is not obvious if even possible.

Zimmerman [143] stresses that the MTF is independent of the nature of any real object while Brock [22] continuously emphasizes that the MTF is not a graph of image quality versus diminishing detail size. He maintains that "... the only direct physical application of the MTF is to show how the system reduces modulation on sinusoidal inputs as a function of the spatial frequency in the image plane." [22, p360]. Walker [133] points out the interesting fact that the modulation is unity for the limiting case of zero spatial frequency which corresponds to a target of uniform brightness. Welander admits that, although the MTF is not a graph of image quality versus diminishing detail, "... the smaller the object is on the ground, the more reduced is the object contrast on the aerial photograph." [135, p4]. That statement is one of the more practical illustrations of the MTF concept.

Brock, [30] who has studied this subject for years, introduces the somewhat confusing fact that, in terms of frequency spectrum, a small target is not a high frequency, but a wide-band input. A large target represents a narrow frequency band. Band width is usually expressed in cycles per second where, for example, 100 cycles/second would represent the range of frequencies from some value \( x \) to a value \( x + 100 \). For example, high notes require a large band width to be adequately represented; similarly a small object requires a large band width.

The fact that the MTF is a function of the object means that systems cannot be compared by merely comparing MTF's; what was imaged is also important [21].
Welch, while admitting that studies of the precision and accuracy of MTF analysis on edges were limited, stated in 1971 that "It is possible, however, to determine MTF's that indicate system performance by analysis of aerial photographs containing images of periodic targets or sharp edges." [137, p248]. Abbot, in the following quotation, explains specifically the procedure which will be used in this study, i.e., that of computing the MTF from an edge.

The optical transfer function can be computed indirectly by measuring the intensity distribution in the image of an edge. If the energy distribution in the image of an edge E(x) is measured as a function of x, the distance across the image, then the line spread function can be shown mathematically to be equivalent to the derivative of the edge function dE/dx. Since the optical transfer function is the inverse Fourier transform of the line spread function, then it is possible to calculate the O.T.F. from the measurement of the edge [3, p55].

But, as Birch [13] maintains, unless the user is capable of relating the results of this MTF analysis to "... past experience or previously used criteria.", it may not be of any value. This point leads into the next sections which represent the current opinions on the pros and cons of the MTF.

2.2.2.4.3 Advantages of MTF Analysis

It is probably correct to state that at this time photogrammetrists are undecided as to whether they wish to get involved with modulation transfer analysis. Nearly everyone will agree that lens designers or systems designers could advantageously use the MTF principle. (Which may in itself be a sufficient reason for the practicing photogrammetrist to get better acquainted with this technique.) Photogrammetrists must ask themselves if MTF's have any practical value or, more simply, are MTF's worth all the trouble?
Brock claims "... that good definition is associated with a good modulation transfer function, low granularity, and high resolving power," which probably is not too surprising [28, p42]. Welander states that an ... expression of results as modulation versus spatial frequency gives a useful general impression of system performance over a range of sizes, as distinct from the limiting size, which is the only information conveyed by resolving power [135, p2].

Brock [25] agrees by emphasizing that the transfer function reveals the system performance over its entire frequency range. Welander comments that "... the MTF offers objective values of the contrast reduction for small as well as large objects." [136, p17]. Rosenberg [112] when writing about resolution and its affect on detectability concluded that the MTF was more meaningful than lines-per-millimeter as a measure of true resolution and image quality.

Shack listed the advantages of the MTF in image evaluation as the following: the MTF is ...

a. A bounded function
b. Simple in form
c. Easy to measure
d. Easily manipulated
e. Allows convenient analysis of cascaded systems
and f. Allows communication theory to be applied to optics [120, p757]. Perrin's list of advantages includes the proposition that the MTF...

a. Exposes the fundamentals even though it removes what we actually see
b. Enables faults to be separated and is easy for people of other
disciplines to grasp

c. Is analytical and leads to improvements
and d. Is applicable when photogrammetry is associated with other mediums as communications, etc. [76].

Possibly one of the leading advantages of MTF analysis lies in the procedure for arriving at the total system performance. If a researcher wishes to combine the spread functions of two elements, he has two choices, (1) utilize the principle of convolution which is a rather complicated mathematical procedure, or (2) find the Fourier transform of both elements, simply multiply these and then find the original function from the inverse transform. The latter procedure is usually much easier [28,76,136,16].

It is easy, however, to find counter arguments for the above advantages which will be discussed in the next section.

2.2.2.4.4 Disadvantages of MTF Analysis

Shack's comment in the previous section that the MTF is easily measured is questioned by many people. Zimmerman, for example, states that "The direct measurement of the MTF requires sinusoidal test objects of great precision, as well as sophisticated electronic circuitry." [143, p7]. But this is a minor criticism compared to the doubtful value of the principle as a whole.

In 1964 Brock wrote,

By itself this [MTF] tells us nothing about the image quality we can obtain, since it is merely a statement of relative modulation transfer. We cannot even rank systems by their transfer functions alone: to say that one is better than another we must specify the purpose for which it is better, and this requires a knowledge of the input spectrum on which it is to operate [25, p352].
Brock [24,21] later stressed two important facts, (1) our minds cannot interpret the frequency spectra so to derive any specific information, one may have to transform back to the space domain, and (2) the MTF is really only applicable for sinusoidal targets. He states that "Any deductions made by the expert about image quality for shapes other than sinusoidal will be found, on reflection, to be based on intuitive or intellectual translation of the object into its spectrum." [21,22]. Kelson agrees with Brock stating that the MTF "... conveys no direct information about the developed photographic image." [124, p2].

Another consideration is the problem of linearity of a system and the subsequent applicability of Fourier analysis. At a NATO Symposium Beurle explained that while he thought the transfer function could be useful, he reminded everyone that "Fourier transform techniques were developed for use with hardware systems which were known to be linear. They are only valid when applied to a linear system." [11, p64]. Brock concurs and points out that the photographic emulsion "... is non-linear and for this reason alone cannot be said to have a transfer function unless we confine ourselves to very small modulations." [25, p32]. Welanders [136] suggests, however, that through special developing one may be able to consider the emulsion as nearly linear for practical purposes.

Other facts will be briefly presented in the following section, but first Perrin’s comment will be presented as an optimistic summary of the criticisms of the MTF.

Indeed, any assemblage of lines and edges (e.g., a scene recorded in a photograph) can be regarded as a pattern of superposed sinusoidal elements and treated by an extension of the methods described
here, but at present this possibility is of more theoretical interest than practical importance [76, p504].

2.2.2.4.5 Special Areas

This section is included to assist the interested reader who wishes to delve more deeply into the mysteries of the transfer function. These areas are considered to be beyond the region of interest to one who is concerned in only investigating the possibility of a correlation between the MTF and our ability to measure to an edge.

The various methods for arriving at the transfer function are discussed by Abbot [3]. Perrin [107, 76] discusses the mathematics involved but Bracewell's book [16] should be the starting point for the interested reader. Perrin [107] discusses the normalization of the MTF. Shaw [123] attempts to relate the MTF to the information capacity of a photograph. Both Brock and Schwidefsky [27,114] discuss the threshold curve and its applications. Zimmerman [143] and Bracewell [16] discuss the problem of the two dimensional transfer function with Bracewell's treatment being more mathematical and Zimmerman's more photogrammetric.
3. EXPERIMENTAL METHOD

3.1 General Technique

The basic approach is to use the method of O'Connor by which one manufactures an edge which closely resembles an actual photographic edge by rotating a white drum on which is placed a black geometric pattern. As the drum rotates rapidly the contrasts fuse and an edge of any desired width is generated. The characteristics of the edge can be varied by placing a different curve or pattern on the drum. From a distance of 6 meters four experienced observers bring a black measuring mark tangent to each edge many times.

An edge trace is formed by reading the luminance of a 1/4 mm diameter circle on the drum every 0.05 cm by scanning the edge with a translatable photomultiplier and lens assembly. The subjective and objective distance measurements are referenced to a small black fiducial mark painted on the drum. (See Photo 2). After this initial battery of data is collected the remaining work consists of analyzing the results and manipulating the data in an organized search for trends and measurable quantities which could serve as indicators of where one thinks the edge is located and values representative of image quality.
3.2 Description of the Apparatus

3.2.1 Edge Generator

Edges have not been the subject of very extensive nor intensive study in the past. This is probably a result of the difficulty in making measurements on or gathering data from a microscopic border, edge or boundary. Two techniques generally come to mind; unfortunately, neither is very fruitful. One technique is to scan an edge with a microdensitometer, thereby arriving at a plot of density (or transmittance if desired) vs. distance. However, assuming that the microdensitometer is
properly calibrated and that the slit is aligned with the edge, the
graphical output still contains so much noise that one trace is nearly
useless. To overcome this problem usually three traces are run, the
slit is moved slightly, and three more traces are plotted. This would
usually be done three times. Now the researcher is faced with nine
graphical portrayals of what he hopes is a plot of density vs. distance
for a particular edge. In his endeavors to arrive at a smooth curve
from which numerical data may be extracted, he could possible integrate
out unusual and distinguishing edge characteristics. (See Appendix B).
But more importantly the researcher still has no idea of how his data
relates to the subjective edge location.

Another technique is to magnify an edge many times and photograph
it, hopefully capturing the inner secrets of this mysterious edge. Ap­
pendix B contains examples of the results of these endeavors. The emul­
sion grains become overwhelming and as the edge increases in magnifica­
tion, it loses its identity and hence its usefulness.

Thus the interested researcher is forced to generate his own edge.
This can be done in a number of ways, rotating prisms, mal-focusing, use
of aperatures, etc., but the method of O'Connor, i.e. that of rapidly
rotating a drum on which is placed a pattern was used by this author.

One could justifiably inquire as to whether the conclusions drawn
from generated edges would be applicable to actual edges on a photo­
graph. It is believed that they are for the following reasons.
The drum, on which the pattern is placed, is rotated at 60 cps by a small electric motor.

It is a fact that the optical system of a human when presented with light distributed as a smooth curve will form an edge in a manner compatible with the Mach phenomena. From frequent edge scans it is well known that light is distributed across an edge on a photograph in a smooth curve, the same type of light distribution that was generated in this study. The eye is not concerned about where the light distribution comes from, but is only concerned with the actual light distribution.
There can be no doubt that the eye performs the same type of fusion process when it is looking at an edge on a photograph. It is realized that at the photo scale the discrepancies in edge location that are discussed in this paper can be significantly large so as to be of concern in present day photogrammetric practice. However, for the researcher and for the future generations of photogrammetry, the problems in subjective edge location will be even more important.

In order that the edge should be uniformly fused it was believed that the drum had to rotate at a rate greater than the flicker frequency of the human eye. The flicker frequency of the eye is greatly dependent upon distance and luminance, becoming asymptotic after approximately one meter and increasing with increasing luminance [63, p59-61]. Harvey states that he would expect a flicker frequency of approximately 29 cycles per second [63], but a literature search revealed a source [61] which claimed that the visual flicker is paralleled by a cortical activity of up to 53 cps. Consequently it was decided that if the drum rotated at 60 cps, the flicker frequency problem would be eliminated.

Throughout these experiments the drum was allowed to rotate at 60 cps. However, the author is of the opinion that this great a rotation rate is unnecessary. Although this was not a part of the overall study two simple tests demonstrated that the rotation rate was of minor importance. With the drum rotating at 60 cps the measuring mark was made precisely tangent to the edge with the assistance of a Wild T-3 theodolite at 40 X magnification. The motor turning the drum was then turned off. The location of the edge did not noticeably change as the
drum slowed down. It was not until the drum was rotating at a ridiculously slow rate did any measurement confusion result.

The same test was run with the scanning photometer aimed at a section of the drum. The luminance did not change until the drum was rotating very slowly, approximately 10-15 cps.

The author found these points interesting but possibly not scientifically conclusive, so the drum was continuously rotated at 60 cps.

The pattern which was placed on the drum was primarily cut from black "Scotch" brand pressure sensitive tape manufactured by the Industrial Tape Division of 3-M Company. The tape (Y-9244) provided a smooth even textured surface with very low reflectance (light absorption of 93%). When placed on the drum, which had been painted with a satin white paint from a spray can, a high contrast edge was developed.

Later the drum was painted gray and different shades of tape were used to create edges of varying contrast.

The patterns had to be systematically cut. Initially a paper template, whose width equaled that of the black tape (3 inches), was cut out of graph paper. Using French curves a series of smooth curves were drawn on the template. Each curve was different than the others; either the inflection point was moved or the width of the edge was altered.

A strip of tape was placed beneath the template and very small pinpricks were made through the template into the tape. This marked the desired curve or pattern on the tape. Using a French curve and a sharp knife, the desired pattern was cut and carefully wrapped on the drum. Mechanical limitations kept the maximum width of the fused edge to 3.5 cm. The edge could be made as narrow as desired except that it became
difficult to construct a smooth curve with only one inflection that was 48 cm high and any narrower than 2.5 cm. This range of edge width was deemed adequate for this study. One can appreciate the ease in studying this edge vs. an actual photographic edge whose nominal width may be only about 60 μm.

3.2.2 Light Source

The key to proper lighting for an experiment of this sort is consistency and diffusion. The drum was indirectly illuminated by 54, 12 volt, direct current, automobile headlight bulbs. Each bulb was attached to its own rheostat with the power being supplied by a Lambda Model LK-360 FM regulated power supply which converts AC to DC with a ripple of only 0.5 millivolts \[73\]. By using a uniformly diffusing placard and a scanner (to be described later) it was shown that the light source was constant and uniform over the entire horizontal width of the drum. At this point, because the light source supplied constant illumination on the drum and because the drum was painted a diffuse white, it was suspected that the surface would be perfectly diffusing. This was verified experimentally by showing that its luminance remained constant no matter what the direction of view.\(^1\) (See [61], p814 and [65], p21).

After a one-half hour warmup period the light source remained exactly constant as determined from continuous photometer readings.

\(^{1}\)To this must be added the phrase "within reason." The scanner was sensitive to mal-focus and when rotated through too extensive an angle, the scanner would go out of focus as well as the circular target area becoming enlarged and naturally the luminance reading would change.
General view of edge generating device at 6 meters. The black box houses the 54, 12 volt, automobile headlight bulbs. The observer views the edge through the large circular tube. The rectangular bar (arrow) supports the flexible shaft which is connected to a smooth wheel by the observer and runs to the gear linkage which eventually drives the measuring mark. The power supply for the lights can be seen in the lower right corner of the photograph.
Photo 5

Side view of black box containing the 54 bulbs which supply the indirect, diffuse illumination.

Photo 6

Array of rheostats which allows the intensity of the drum to be altered. Each bulb has its own rheostat.
3.2.3 Scanning Photometer

All of the initial objective data was obtained by traversing a photomultiplier across the edge and measuring the luminance every 0.05 cm. A scanning system (See Photo 7), generally designed by the author and built by CINTRA, was used to read the luminance of the drum. The system was composed of the following elements:

1) Model 150 Digital Photometer,
2) Model 5686 Photomultiplier Probe,
3) Model 100 Photomultiplier Power Supply,
4) Model 5303-A Transverse Drive and Reflex Adapter,
5) Model 5015 Reflex Light,

and 6) F2.5, 200 mm focal length objective lens assembly.

Because of the variable sensitivity of the eye to radiation of different wavelengths, a standard function has been established by the Commission Internationale de l'Eclairage (CIE), formerly called in English translations, International Commission on Illumination (ICI), for converting radiant energy into luminous (i.e. visible) energy [131, p1045].

The photomultiplier detector was corrected by CINTRA to the CIE response to within 2%, with the calibration being performed against a 2870°K source using standard NBS techniques [36]. The optics system connected to the photomultiplier was designed to allow the probe to measure the luminance of a 1/4 mm diameter spot 20 inches from the input lens.

Luminance is defined as that "... quantitative attribute of light that correlates with the sensation of brightness and is the evaluation of radiance by means of the standard luminosity function." [131, p1044].
Scanning photometer with components. The digital photometer is resting on the power supply both of which are on the shelf of the scanning table. Both the objective lens assembly and the photomultiplier probe are traversed during scanning.

The foot-lambert is a unit of photometric brightness (luminance) equal to $1/\pi$ candles per square foot, or to the uniform photometric brightness of a perfectly diffusing surface emitting or reflecting light at the rate of one lumen per square foot, or to the average photometric brightness of any surface emitting or reflecting light at that rate. The foot-lambert is the same as the "apparent foot-candle." [7, p31].

Both the foot-lambert and the foot-candle are widely used illumination terms and are really expressions for similar quantities, namely,
lumens per square foot [122, p3]. "When the lumens are coming from the surface, the term foot-lambert is used." [122, p3]. A foot-lambert is a unit of luminance equivalent to 1.076 millilamberts [61, p62]. The amount of illumination to which the drum was exposed is compatible to what Lund and Colcord determined as optimum for measuring accuracy [90].

So the scanner measures the luminance of the drum in foot-lamberts. The quantitative measure was displayed by the digital photometer.

The model ISO Digital Photometer is a digital autoranging nanometer that is designed to monitor the output current of optical detectors. The current from the detector (photomultiplier tube in this case) is amplified by a low noise, low drift amplifier. It is then fed back into the detector packager where it is referenced to correct for detector sensitivity, area, temperature, and other calibration variables. The corrected signal is then presented to the second amplifier which increases the signal level and provides zero to -10 volts as an analog signal through a 100 ohm isolation resistor. This same signal is directly applied to the input of an analog-to-digital converter. The signal is eventually counted by a three-decade counter and the resultant reading is presented on three numerical indicator (nixie) tubes with a power-of-ten exponent. With a voltage setting of -1054.3 volts on the Cintra model 100 Power Supply, the Cintra model 150 Digital Photometer system reads directly in foot-lamberts [36].

So in summary, a photomultiplier is traversed across the edge. The photomultiplier converts light energy into electrical energy. The photometer receives this electrical energy and displays digits which correspond to the luminance in foot-lamberts.
3.2.4 Subjective Edge Measurement

3.2.4.1 Basic Principle

The basic principle involved consists of four trained and experienced observers setting a black measuring mark tangent to where each observer decides the edge to be located. A numerical distance reading is obtained for each setting from a shaft encoder which is connected to devices which both display the linear distance and punch the digits on paper tape. The tie between these subjective measurements and the objective measurements from the scanner is obtained by setting the measuring mark (MM) tangent to a black fiducial mark (small rectangle) with the drum stationary. Exactness of the setting is insured by using a Wild T-3 with 40 X magnification. All measurements are referenced to this point.

The measuring mark is of such a diameter that it subtends an angle of 3.41 min. which was considered an optimum size in a study done by Lund [90]. The MM was black simply because this is what one finds in photogrammetric practice.

3.2.4.2 Number of Observers

When using the method of repeated measurements or, as Guilford says, the method of average error [56, p86], one must be concerned with the number of observers and the number of observations to be executed by each observer. The number of observations can be reached by considering confidence intervals, similar experiments, a desire to have a normal distribution, etc. The problem of how many observers to use can not be solved so easily. It would be advantageous to have a large number, but in a task of this sort, under the conditions that this report is being
written, one has to be practical. O'Connor, Lund, Colcord, and Trinder [104, 90, 38, 129] all using a similar apparatus under somewhat the same conditions justified the use of four or less observers. Trinder states, "In any investigations on visual tasks, it is reasonable to assume that the pattern of results obtained by one or two experienced observers is indicative of the pattern which would occur for all experienced observers." [129, p192]. Lund verified that this was in fact true by bringing in a group of observers which he labeled a control group. He found that the results obtained by the major observers fell within the range exhibited by the control group [38, p61].

It seems logical and reasonable to this author that the above statements should be valid and, as such, four observers were trained on the apparatus. The training procedure consisted of hundreds of observations taken over the period of approximately three weeks. All of the observers were already familiar with basic photogrammetric pointing procedures and were either students in or employees of the Geodetic Science Department.

All observers were given extensive eye examinations to insure that they had no eye irregularities which would preclude their being effective. A summary of the eye examinations may be found in Appendix C.

3.2.4.3 Mechanical Assembly

The observer is always sitting 6 meters from the rotating drum. One views the edge by looking through a tube 6 inches in diameter. The value of 6 meters was chosen firstly, because at this distance even the most conservative of experts would agree that the human eye will have focused at infinity, and secondly, the greater distance means that the
angle subtended by small movements of the measuring mark will indeed be very small. For example, a distance of 1 mm at the drum subtends an angle of only 34.3775 seconds of arc. (See Appendix D).

At the observer's immediate right is located a smooth wheel which when turned causes the MM to be translated either to the left or right. This is accomplished through a series of gear-shaft connections ending with a drum micrometer.

The lateral position of the MM could be read directly from the drum micrometer. This procedure would be both tedious and time consuming so an automatic system was assembled and the drum micrometer was used merely to transform rotatory motion into translatory motion. (See photographs).

A Perkin-Elmer, 100/1000, shaft, rotary encoder is linked directly with the shaft which turns the drum micrometer. This is a rugged, positive and extremely reliable device which sends electrical signals which correlate with a lateral position of the MM. Instructions from the shaft encoder are sent to a Model 5RLS Perkin-Elmer Encoder Readout which converts the electrical signals into digits and displays five such digits on nixie tubes [44]. Repeated tests proved that one millimeter at the drum corresponded exactly to 1575 encoder readout units. Similarly one encoder readout digit corresponds to 0.635 μm. It was simple and convenient to perform many calculations using the digits displayed on the encoder readout. As such these digits were arbitrarily referred to as being in "ER" units. Appendix D lists the simple conversions.

Large quantities of data were obtained which needed to be processed (find mean, standard deviation, etc.) before it was useful. To
Photo 8

The shaft encoder (s.e.). When the observer turns the smooth wheel located by his chair (which is six meters from the drum) a shaft and gear linkage eventually turns a micrometer (mic.). The micrometer converts rotary motion into linear motion and pushes the measuring mark. The shaft encoder converts rotary motion into electrical impulses which are eventually displayed as digits by the encoder readout. The location of the measuring mark is therefore precisely located by the displayed digits. (1575 encoder readout digits corresponds exactly to a 1 mm movement of the measuring mark.)
Encoder Readout, Digital Data Recorder and Paper tape punch. The small keyboard device on the Digital Data Recorder is a means of manually punching symbols and numbers on paper tape.

facilitate putting this data on computer cards a Perkin-Elmer DDR2C Digital Data Recorder was attached to the encoder readout. The Digital Data Recorder converted the displayed digits to computer-compatible paper tape by sending instructions to a paper tape punch. (See photographs). Commands to record data could be generated either by pushing the RECORD button on the DDR or activating the foot-pedal located on the floor in front of the observer. Even though the backlash in the micrometer was unmeasurably small, good photogrammetric procedure was followed and the edge was always approached from the same side, i.e. from the lighter region towards the darker.
3.2.5. **Tieing the Subjective to the Objective**

The scanner supplies an objective measurement of luminance against distance. By setting the measuring mark to the edge and recording the "x-axis" reading, the subjective location of the edge could be determined. However, it was imperative for comparative purposes to reference both distance measurements from the same point. Consequently a black rectangle (see Figure 3.1 or Photo 2) was painted on the drum. By sighting through the reflex sight the circular target was centered on

![Image](image.png)

**Figure 3.1**

*Reflex sight of scanner is centered on the edge of the fiducial mark. The measuring mark is made tangent to the fiducial mark.*

the edge of the fiducial mark. The standard deviation of the mean of 10 pointings was of the order of ±0.004 mm at the drum.
Using a Wild T-3 theodolite with 40 X magnification the measuring mark (MM) was made tangent to the same fiducial mark. The standard deviation of the mean of 30 pointings was of the order of ±0.002 mm at the drum. This now established a common reference point which tied the objective luminance readings to the subjective edge location readings. But to make the numbers compatible with our concept of the x-axis increasing to the right one additional manipulation was performed. (Refer to Figure 3.2). The numbers in Figure 3.2 are representative values; 13.500 cm and 60000 ER are the measured values of the scanner sight and MM when set on the fiducial mark.

![Figure 3.2](image)

The value of b is computed in ER units which corresponds to the measured value of 9.5 cm. Then by shifting the scales the initial values are set to zero and all other values proportionately changed. Now the edge trace starts at zero and increases to the right which is aesthetically pleasing.
The number 9.5 cm represents the x-axis value corresponding to the first luminance reading accepted as part of the edge. Let, \( b \), be the number in ER units which corresponds to 9.5 cm. This can be computed from the following equation,

\[
b = 60000 - 15750 (13.500 - 9.500),
\]

where 15750 is the number of ER units in one centimeter. Once \( b \) is determined, the number 9.500 and \( b \) are both assumed to be zero and all values corrected accordingly. This allows both the subjective and objective readings to be referenced to a common zero point and increase in magnitude to the right. The only purpose in this last exercise was to make graphs and traces of the edges aesthetically pleasing.

3.3 Observational Procedure

A study of confidence intervals revealed that 90 observations per observer on each edge would yield a mean with a standard deviation of \( \sim 1 \) sec. of arc in visual acuity. This was considered to be totally adequate. Initially each observer performed three sets of 30 observations each with approximately a 10 minute rest period between sets. The following statistical tests were performed for each observer;

1. Three sigma rejection of observation criteria,
2. Run test to examine the randomness of each set of 30 observations [41, 57],
3. Bartlett's criteria as a test for the homogeneity of variances of the three sets [59, p115-116],
and
4. An analysis of variance technique as expressed by Hamilton [59, p100-102] to determine if the theoretical means of the three sets of each observer were significantly different (\( \alpha = 5\% \)).
It was found after observing 5 different edges that tests 1, 2 and 3 were consistently and easily satisfied, but that the observers were failing test 4 with regularity. This meant that the means of the three sets of each observer were significantly different, i.e., the observer was not measuring to the same location each time. It must be emphasized that at this time all observers were considered to be thoroughly trained and experienced and that the precision of the three sets for each observer was generally the same. (Naturally each observer did not have the same precision). The possibility of a mechanical irregularity was investigated and ruled out. A hint as to why the means of the three sets of each observer were often significantly different was supplied when a pattern to the degree of failure seemed to present itself.

Figure 3.3 shows the edges that were initially studied. Each edge was approximately 3.5 cm in width, had a contrast ratio of 18 to 1 and was observed in the same manner. The only difference in the edges was

![Figure 3.3](image)

Figure 3.3
Graphical representation of the first five edges studied. Only the location of the inflection point was changed.
the location of the inflection point. Edge 3 was symmetrical, edges 2 & 4 had the inflection point moved approximately 0.6 cm to the left and right respectively and similarly with edges 1 and 5.

During the course of performing the four statistical operations on each observer's data, it became apparent that everyone was failing the test, for the possibility of the theoretical means being different, more drastically for edges 3, 4 and 5 than for edges 1 and 2. This seemed to imply that the observers were arriving at a significantly different subjective decision as to where the edge was located for each set. Normally when an observer's means are significantly different, the procedure is to re-observe. This was initially done and when three compatible sets appeared the observer was released. However, if he was in fact pointing to a different location because he had unconsciously arrived at a different subjective location, then the re-observation procedure was biasing the results and furthermore, 3 sets of 30 observations really only gave the observer three opportunities to subjectively locate the edge.

A simple test was run. It was hypothesized that the observers experienced more difficulty in arriving at the same subjective decision as to where the edge was located for edge 3 than for edge 1. Each observer was asked to point to edges 1 and 3 ten times per set and for nine sets. That is, 9 sets of 10 instead of 3 sets of 30. The standard deviation of the 9 means of each observer was calculated. Table 3.1 lists the results and a definite improvement can be witnessed for all observers when viewing edge 1 versus edge 3.
Table 3.1

The dispersion of the 9 means per each observer was significantly less for Edge 1 than for Edge 3.

<table>
<thead>
<tr>
<th>Observer</th>
<th>Edge 1</th>
<th>Edge 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>±6.7 Secs. of Arc</td>
<td>±10.8 Secs. of Arc</td>
</tr>
<tr>
<td>2</td>
<td>±7.1 &quot;</td>
<td>±16.9 &quot;</td>
</tr>
<tr>
<td>3</td>
<td>±8.9 &quot;</td>
<td>±18.4 &quot;</td>
</tr>
<tr>
<td>4</td>
<td>±9.9 &quot;</td>
<td>±14.8 &quot;</td>
</tr>
</tbody>
</table>

This simple test convinces the author that a better estimate of where an observer subjectively locates an edge, in an experiment of this type, is obtained from 9 sets of 10 observations rather than 3 sets of 30 observations. Consequently the final observational procedure for this paper was chosen as having each observer set the measuring mark tangent to where he sees the edge 10 times, rest for a few minutes and repeat the process until 9 sets were obtained. The standard deviation of the grand mean of the 9 sets was still approximately ±1 second of arc and a better estimate of each observer's subjective edge location was obtained.
4. DATA ANALYSIS

4.1 General Discussion

The purpose of this section is to discuss what has been learned about the location of the subjective edge and how it relates to other points which are objectively measurable.

After a discussion of the characteristics of each edge the problem of fitting a function to the observed points is discussed. Naturally the function alters the observed points somewhat but not excessively. Furthermore, it was realized that where so many readings of high quality were taken across the edge it was possible to merely use the observed data in most computations.

Because the subjective edge location was discovered to be related to the location of the inflection point of the intensity edge trace a discussion of various techniques for finding the inflection point follows. It is discussed how the location of the maximum gradient is synonymous to the location of the inflection point. Thus the inflection point was located by finding the location of the maximum observed edge gradient. The effect of a systematic personal bias is discussed in this section also.

The next section illustrates how a change in luminance affects the subjective edge.

And finally the last section is a discussion, with derivations, relating the intensity and density edge trace inflection points. This
last section was included because most observed data from actual photographs are density values and this section will facilitate the location of the intensity edge trace inflection point which, as was stated, correlates highly with the location of the subjective edge.

4.2 Characteristics of the Generated Edge

The purpose of this section is to illustrate to the reader the characteristics of each edge and to explain the identification procedure which will allow the reader to conjure in his mind a particular edge when reference is made to its identifying symbol.

Not including observations on non rotating patterns and the many practice edges, it can be said that 18 edges were observed and studied in detail. The edges may be divided into three general categories based on the contrast of the pattern and of the drum. Case I consisted of black patterns wrapped on a white drum producing edges with a contrast ratio of approximately 18 to 1 or a density range of 1.3 to 0. It seems reasonable to refer to this category as the "black-white edges" or simply "B-W".

Case II consisted of gray patterns wrapped on a white drum producing edges with a contrast ratio of approximately 2.5 to 1 and density range of 0.4 to 0. It should be sufficiently clear if they are referred to as the "G-W edges".

Lastly, Case III, the black pattern on a gray drum, (B-G edges) produced a contrast ratio of about 3.2 to 1 and a density range of 1.4 to 0.9. Figure 4.1 should help establish these edge categories in the reader's mind.
The above patterns with contrast ratios of 18:1, 2.5:1 and 3.2:1 were individually placed on the drum. The density range for these edges varied from 1.3-0, 0.4-0 and 1.4-0.9. Different edge traces could be obtained by varying the individual patterns.

Additionally, within each color category just mentioned, the edges were allowed to have two general widths, 3.5 cm and 2.5 cm. Because there are only two widths the edges may be called either "wide" or "narrow" with no resulting confusion.

Lastly, by allowing the inflection of the point of the pattern to move to the left and right of its symmetrical position three edges within each of the following categories were obtained: B-W wide, B-W narrow, G-W wide, etc. Figure 4.2 shows the approximate three edges for the B-W narrow case.
Inflection Point
Left

Symmetrical

Inflection Point
Right

Approximate Height of Patterns was 48 cm

Figure 4.2

Within the B-W (G-W, B-G) narrow category, the inflection point was allowed to assume three approximate positions. The same technique was applied to the wide edges, thereby producing the 18 edges which were studied in detail.

It would be clumsy and time consuming to refer to specific edges by their numerical inflection point location so a simple numbering system was adopted. Table 4.1 is self explanatory and could serve as an identification summary of the edges which were studied. For example, edge B-W 6, would obviously refer to the narrow edge, black pattern on a white drum with the inflection point located to the left of the symmetrical edge's inflection point location as seen by the observer.
Table 4.1

<table>
<thead>
<tr>
<th>Edge Identification</th>
<th>Edge Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Edge Number</td>
<td>Edge Color</td>
</tr>
<tr>
<td>1</td>
<td>B-W, G-W, or B-G</td>
</tr>
<tr>
<td>3</td>
<td>&quot;</td>
</tr>
<tr>
<td>5</td>
<td>&quot;</td>
</tr>
<tr>
<td>6</td>
<td>&quot;</td>
</tr>
<tr>
<td>7</td>
<td>&quot;</td>
</tr>
<tr>
<td>8</td>
<td>&quot;</td>
</tr>
</tbody>
</table>

4.3 Edge Trace & Curve Fitting

This section discusses the problems encountered when one is attempting to fit a function to a set of discrete observed points. It must be emphasized at the start that the problems mentioned herein were generally avoided by using the observed data which was considered of high quality. Appendix F contains a computer plot of each observed point connected by a straight line for each edge. An inspection of these plots makes it obvious that the discrete points display a remarkable continuity and that light is distributed across an edge in the form of a smooth curve.

However, in some cases, it may be advantageous to fit a function to the observed points for the purpose of finding the higher derivatives or simply for ease in handling the data, hence the curve-fitting problem which has often been the nemesis of researchers in the study of edges. A search of the literature reveals that authors are apparently reluctant to divulge the type of function they use in situations such as this. The phrase "... fitting a function to the edge ..." continuously appears
but no function is suggested. Maune [146] was one of the few who admitted being plagued by this problem and finally chose a hyperbolic tangent function. However, he was working with symmetrical edges only which would make the hyperbolic tangent more applicable. Through trial and error it was found that a simple cubic equation would handle a symmetrical edge quite well, but was totally inadequate for the unsymmetrical edge. Mandel states that "By choosing a polynomial of sufficiently high degree, it is generally possible to achieve a good empirical fit for any reasonably smooth set of data." [145, p245]. Trial and error, a desire for simplicity and Mandel's statement led to a seven degree polynomial as the function which was fit to all edge traces in this paper. The author will admit that other possibilities do exist but feels that this polynomial is totally adequate and that whatever shortcomings it may possess are no more severe than one would find in any adequate substitute.

Figure 4.3 is an attempt to graphically describe the problems that arise when a fitted function is used. Discrete points form a plot of the observed luminance readings against distance for edge B-W3. The smooth curve is a plot of the edge trace as derived from the fitted seven degree polynomial. Whether one considers this a good fit would possibly depend on the purpose for finding the function in the first place. The fit in Figure 4.3 is the second worse fit of the 18 edges. The greater the luminance range the poorer was the polynomial fit. This means for the G-W and B-G edges, the fit was excellent with a standard deviation of the fit as low as 0.02 ft-lams. It was found that
even increasing the degree of the polynomial from 7 to 11 only lowered
the standard deviation 25%. However for some researchers it would be
advantageous to use an eleven degree polynomial. For the purposes of
this study the seven degree polynomial fit was adequate. Much of the
work could be done by working with the discrete observed points, the
differences between these, etc. and for the comparative studies in the
image quality section the 7 degree polynomial's small effect on the
shape of the edge trace would make virtually no difference in the qual-
ity ranking of the edges.

The effect of the imperfect fit is graphically apparent although
one would have to say that the function fits the observed points rather
well.

![Graph showing the effect of the imperfect fit](figure43.png)

**Figure 4.3**

Edge trace of edge B-W 3 showing the agreement between the observed
discrete points and a plot of the fitted function.
The small imperfections in the fit of the function mushroom as the higher derivatives are found. Figure 4.4 is an attempt to depict the difference in the first derivative of edge B-W 3 as obtained by differentiating the fitted seven degree polynomial versus a free-hand sketch of how the derivative should actually appear.

![Figure 4.4](image)

First derivative (spread function) of edge B-W 3 as determined from differentiation of fitted function versus a sketch of the expected shape. (Thick line represents sketch)

The same type of discrepancy existed for all the edges and for each of the higher derivatives. The question arose as to what range along the abscissa should one consider when comparing one spread function to another. The answer lay in the decision to use the discrete observed points as much as possible. If $Y_i$ is an observed luminance value, then
a plot of \((Y_{i+1} - Y_i)/\Delta x\) is a plot of the spread function. Because of the large number of luminance readings taken across each edge (71 readings or every 0.05 cm) a plot of these discretely computed points approximated a smooth curve. If it was possible to use the discrete observed points directly as in a discussion of acutance, maximum gradient etc., this was done. In other cases, as in the study of the higher derivatives, other approaches were tried. In every section it is explicitly stated as to what was tried and finally accepted.

4.4 Edge Location and the Inflection Point of the Intensity Edge Trace

4.4.1 General Discussion

From earlier work it was suspected that the subjective edge would lie in the near vicinity of the inflection point of the intensity edge trace. As such the inflection point location for each edge was computed. For a smooth curve such as was studied here the inflection point can be defined as:

1) The point where the radius of curvature changes from negative (positive) to positive (negative),
2) The point where the slope of the curve is a maximum (minimum),
3) The point which locates the maximum gradient (if the distance increment is sufficiently small),

or

4) The value on the abscissa where the second derivative of the edge trace is zero.

If a function has been fit to the observed data, the location of the inflection point as computed from all of the above methods will be identical.
Now if the discretely observed values were accurate enough and in large enough quantity and if the fit of the function was good enough there would be no difference in the location of the inflection as computed from the observed data versus the fitted function.

Column 2 of Table 4.2 lists the location of the inflection point as determined from the fitted function. Column 3 lists the inflection point as determined from the location of the observed maximum gradient. The columns are nearly identical with the exception of the 1 & 6 edges. The question arose as to which location was correct and why.

The inflection point location was adopted as the location of the observed maximum gradient for the following two reasons both of which will be elaborated on in the ensuing paragraphs:

1) The fit of the function in every case was poorer for the 1 & 6 edges than for the other edges,

and 2) The observed values were great in quantity and of high precision.

The scanning apparatus was capable of translating with an accuracy of one micrometer. However, there was no need to read to this accuracy and in fact a reading was taken every 0.05 cm. As such the abscissa value could be considered exact with the only observed error coming from the luminance reading of the photometer. The detector was pre-calibrated by Cintra with the calibration being "... traceable to NBS with an absolute accuracy of ±5%." [36 p4]. However, the absolute accuracy is not as important in this case as is the precision or repeatability. In an experiment designed to test the repeatability of the
detector, it was found that the average per cent variation in the luminance reading was only 0.2%.

By comparing computer plots of the functional values and the observed values it was visually apparent that the function moved the inflection point to the left or towards the direction of the decreasing abscissa.

Thus, Column 3, the inflection point location as determined from the maximum gradient of the observed luminance, was taken as being most correct and was compared to the subjective edge location.

The subjective edge location was determined from a grand mean of the individual means resulting from each observer setting the MM tangent to the edge for 9 sets of 10 observations each. This is commensurate to simply finding the mean of the 360 observations made on each edge. The possibility of attempting to weight the observations was considered but rejected. All observers were generally of equal pointing ability with particular observers excelling on particular edges or possibly simply because of his personal well-being on that day. Column 4 of Table 4.2 lists the subjective location of each edge. To this is applied a systematic bias correction which is discussed in the next section.

Column 5 shows the difference between the subjective edge location and the inflection point location. To Column 5 a small "correction" (to be explained in the next section) is applied to allow for the systematic bias of each observer. Column 6 shows, in centimeters at six meters, the amount that the four observers set the measuring mark to the
right or into the less dense region for each edge. The average of Column 6 expressed in seconds of arc is 33 seconds. This number could be considered as a representative nominal value for the amount that the fused edge forms into the less dense region or, in this case, to the right of the inflection point. In addition to this, most observers possess a systematic bias, causing them to set the MM even further into the less dense region. That point will be discussed in the next section. What is important here is the fact that the edge fuses slightly to the right of the inflection point or slightly into the less dense region. Unfortunately, as can be easily seen from Table 4.2, the amount that the edge fuses into the less dense region is apparently dependent upon the density in some manner. For example, the averages of Column 6 for the G-W, B-W, and B-G edges are 0.13, 0.11 and 0.05 centimeters while the average densities are 0.20, 0.62, and 1.17 respectively. Although there is not enough evidence here to arrive at any definite conclusions, a tendency has been indicated and a possible direction for future research is revealed.

It can be concluded, however, that an observer will subjectively determine an edge to be located slightly into the less dense region from the intensity edge trace inflection point. A representative value for this amount would be approximately 30 seconds of arc. However, the amount apparently varies according to the density situation across the edge. As the average density increases the observers seemed to determine the subjective edge to be nearer to the intensity edge trace inflection point, but there is not enough evidence here to suggest that
some other relation, possibly the density difference across the edge is not equally important.

### Table 4.2

<table>
<thead>
<tr>
<th>(1) Edge #</th>
<th>(2) Location of the Inflection point from function (cm)</th>
<th>(3) Location of the Inflection point from observed max grad (cm)</th>
<th>(4) Subjective edge location (mean of 360 obs) (cm)</th>
<th>(5) Column (4) minus Column (3) (cm)</th>
<th>(6) Column (5) after applying correction for system bias (cm at 6m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-W 1</td>
<td>0.58</td>
<td>0.68</td>
<td>0.82</td>
<td>0.14</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>1.59</td>
<td>1.58</td>
<td>1.77</td>
<td>0.19</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>2.48</td>
<td>2.48</td>
<td>2.60</td>
<td>0.12</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>0.53</td>
<td>0.58</td>
<td>0.72</td>
<td>0.19</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>1.12</td>
<td>1.02</td>
<td>1.23</td>
<td>0.21</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>2.12</td>
<td>2.10</td>
<td>2.20</td>
<td>0.10</td>
<td>0.05</td>
</tr>
<tr>
<td>G-W 1</td>
<td>0.42</td>
<td>0.52</td>
<td>0.72</td>
<td>0.20</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>1.78</td>
<td>1.78</td>
<td>1.99</td>
<td>0.21</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>2.88</td>
<td>2.82</td>
<td>2.99</td>
<td>0.17</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>0.48</td>
<td>0.52</td>
<td>0.73</td>
<td>0.21</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>1.22</td>
<td>1.22</td>
<td>1.45</td>
<td>0.23</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>1.78</td>
<td>1.78</td>
<td>1.92</td>
<td>0.14</td>
<td>0.08</td>
</tr>
<tr>
<td>B-G 1</td>
<td>0.62</td>
<td>0.72</td>
<td>0.90</td>
<td>0.18</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>1.72</td>
<td>1.68</td>
<td>1.83</td>
<td>0.15</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>2.76</td>
<td>2.78</td>
<td>2.82</td>
<td>0.04</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>0.35</td>
<td>0.38</td>
<td>0.52</td>
<td>0.14</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>1.20</td>
<td>1.20</td>
<td>1.29</td>
<td>0.09</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>1.82</td>
<td>1.85</td>
<td>1.94</td>
<td>0.09</td>
<td>0.02</td>
</tr>
</tbody>
</table>

#### 4.4.2 Personal Observer Bias

Lund found that "... each observer points consistently but at different positions compared to the true location." [90, p40]. He attempted to partially explain these differences based on the peculiarities of each observer's vision but admitted that one of the unknowns in this type of task is the observer himself [90, p41]. The tendency for each
observer to point to one side or the other (usually the brighter side or into the area of lighter density of the true edge was observed in this study. It is recognized that the position to which an observer points will oscillate by as much as 10-20 seconds of arc. However, this author is of the opinion that an observer still maintains a rather constant bias. The above statement is based on the following facts.

With all conditions remaining the same the observers were asked to set the measuring mark tangent to an edge of the three contrasts studied with the drum stationary. Table 4.3 shows the results. The "true edge location" was determined in this case by using a Wild T-3 theodolite and determining the "correct" or expected reading. This was possible because by using the T-3 at 40 X the standard deviation of the mean was approximately two orders of magnitude smaller than with the unaided eye. A positive value in Table 4.3 means that the observer set the MM too far to the right, i.e. too far into the less dense region. The table clearly reflects the same tendency as noted by Lund.

Table 4.3

<table>
<thead>
<tr>
<th>Edge</th>
<th>Obs 1</th>
<th>Obs 2</th>
<th>Obs 3</th>
<th>Obs 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-W</td>
<td>+19.6 Secs</td>
<td>+9.0 Secs</td>
<td>+20.9 Secs</td>
<td>+15.3 Secs</td>
</tr>
<tr>
<td>G-W</td>
<td>+28.4</td>
<td>+9.2</td>
<td>+18.5</td>
<td>+26.7</td>
</tr>
<tr>
<td>B-G</td>
<td>+31.3</td>
<td>+33.4</td>
<td>+2.8</td>
<td>+23.3</td>
</tr>
</tbody>
</table>

Further evidence of the presence of a personal bias in each observer can be found by looking at the individual means of the 9 sets of 10 observations. Table 4.4 is extremely interesting because it reflects a
remarkable consistency on the part of the observers to always point to the same relative location. If an observer's edge location was greater than the mean for that edge a "+" was placed in that column. Similarly, if smaller, a "-" was placed in the column. The numbers were purposefully not included so that the purpose of the table would not be lost. It can be seen from the table, for example, that Observer Two always had a mean less than that of the other observers except for the B-G edges where his personal systematic bias was larger.

In all fairness it must be admitted that this table could be a result of the subjective edge for a particular observer simply "forming" in a constant relative position. However, because the relative location seems to reflect the measured systematic personal bias it is felt that this consistency is not predominantly a result of the subjective edge location. For example, Observer Three had a high positive bias for the B-W and G-W edges and the row of "+"s" reflects this fact. Then for the B-G edges his personal bias dropped to only +2.8 seconds. This bias was much less than that of the other observers and this change is reflected in the "-"s" appearing in Observer Three's column across from the B-G edges. A similar tendency can be noted for Observer Two.

In conclusion it seems that all observers point generally into the less dense region, but that the amount is related to the contrast or density of the edge and is unique for each observer.

The question as to whether the measured values for each observer as obtained from the fused edges could be corrected by an amount as determined from setting on a stationary known edge must be considered.
Table 4.4

<table>
<thead>
<tr>
<th>Edge</th>
<th>Observers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>B-W 1</td>
<td>+</td>
</tr>
<tr>
<td>3</td>
<td>+</td>
</tr>
<tr>
<td>5</td>
<td>+</td>
</tr>
<tr>
<td>6</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>+</td>
</tr>
<tr>
<td>G-W 1</td>
<td>+</td>
</tr>
<tr>
<td>3</td>
<td>+</td>
</tr>
<tr>
<td>5</td>
<td>+</td>
</tr>
<tr>
<td>6</td>
<td>+</td>
</tr>
<tr>
<td>7</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>+</td>
</tr>
<tr>
<td>B-G 1</td>
<td>+</td>
</tr>
<tr>
<td>3</td>
<td>+</td>
</tr>
<tr>
<td>5</td>
<td>+</td>
</tr>
<tr>
<td>6</td>
<td>+</td>
</tr>
<tr>
<td>7</td>
<td>+</td>
</tr>
<tr>
<td>8</td>
<td>+</td>
</tr>
</tbody>
</table>

O'Connor [103] believes it is not possible to apply the stationary edge results. He reasons that the stationary edge is physically present while the fused edge is formed within the mind of the observer and to compare the two would be like comparing "apples and oranges." The author, after generating, scanning and measuring many different edges is of the opposite opinion. Even though the fused edge is formed in the observer's mind, it is nonetheless real. It is important to realize that when one is measuring to a fused edge, one is not measuring to a misty, ghostly image which floats through the mind but to a sharp well-defined edge that is so distinct as to make casual observers question...
the validity of the actual luminance distribution. It may be true that each observer sees the fused edge in a slightly different location but to one observer when he is attempting to measure to the fused edge, it is as real and distinct as the tape placed on the stationary drum. As such it seems reasonable that if an observer consistently points to one side of the true stationary edge he may also point to the same side and with the same relative amount when viewing a fused edge.

The ultimate conclusion from this is that although the readings from the fused edge may be corrected, they can only be corrected if the personal bias or systematic error for an observer at that particular contrast and density is known.

If the above statement is true and if reasonably accurate observations were taken it should be possible to correct the edges with nearly the same contrast as that for which the observers were calibrated. This is what was eventually done and Column 6 of Table 4.2 reflects the final result.

4.5 Edge Location and a Change in Luminance

Five edges at the high contrast (B-W) were observed by Observer One at three different luminances. The objective was to determine the effect on edge location from a change in luminance. It is interesting to consider what results when the luminance is reduced in an experiment of this type and what parallel may be drawn between this result and an actual photograph. Density (D) is given by this well known equation,

\[ D = \log \frac{I_i}{I_t} \]

where \( I_i \) = incident light, \( I_t \) = transmitted light.
The photometer in this experiment can be considered to be measuring $I_t$. $I_t$ would be the maximum photometer reading, thereby making the density at one point zero which would correspond to a completely transparent piece of film. The maximum and minimum measured intensity values ($I_t$) across the edge at the three luminances were ...

<table>
<thead>
<tr>
<th>Luminance</th>
<th>$I_t$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>45 ft-lam</td>
<td>2.5 ft-lam</td>
</tr>
<tr>
<td>Medium</td>
<td>18.5</td>
<td>1.04</td>
</tr>
<tr>
<td>Low</td>
<td>10.5</td>
<td>0.58</td>
</tr>
</tbody>
</table>

The contrast ratios remained approximately 18:1. However, using the equation for density ($D$),

$$D = \log \frac{I_t}{I} = \log \frac{45}{I}$$

the maximum and minimum density at the three luminances would be ...

<table>
<thead>
<tr>
<th>Luminance</th>
<th>$D$</th>
<th>$D_{min}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>1.24</td>
<td>0.0</td>
</tr>
<tr>
<td>Medium</td>
<td>1.63</td>
<td>0.4</td>
</tr>
<tr>
<td>Low</td>
<td>1.88</td>
<td>0.6</td>
</tr>
</tbody>
</table>

If the assumption is made that all measurements were taken from the same photograph and if an actual microdensitometer were being used, then $I_t$ would not change and the change in density could only be attributed to the scanning of a more dense region on the photograph.

As can be seen from the maximum and minimum density values at the three luminances the change in density across the edge remained nearly the same while the average density at the three luminances rose sharply as the luminance was lowered. This change in average density may also be introduced by changing the contrast ratio of the pattern. It was learned in Section 4.3 that the observers determined the subjective edge
to be nearer to the intensity edge trace (IET) inflection point for the B-G edges than for the B-W edges which suggested some dependence upon density. The results in Table 4.5 show that as the luminance decreased the observer pointed more into the less dense region or farther away from the IET inflection point. This movement is believed to be a result of a change in the personal systematic bias of the Observer. In an attempt to verify that the personal systematic bias of Observer One changed as the luminance was lowered the following experiment was conducted. With the luminance at the high setting Observer One set the measuring mark tangent to a B-W edge with the drum stationary. The mean of 9 sets of 10 observations was 37768 ER units. The luminance was then lowered to correspond to what was previously defined as the "low" setting. Now the mean of the 90 observations was 38060 ER units. This meant that at the lower luminance, Observer One determined the subjective edge to be slightly more than 6 seconds of arc farther into the less dense region or away from the IET inflection point.

Although Lund and Colcord found a similar type of movement it must be emphasized that each observer may react differently. It is not known at this time how to predict where an observer will point or what his personal edge preferences will be. This is another point in support of the unpopular concept of "calibrating an observer", that is, through repeated pointings on stationary edges of varying contrasts to determine the amount that an observer points to one side or the other of the true edge. These calibration values could then be applied to actual edge measurements.
Table 4.5 presents the results of Observer One pointing on the five B-W edges with the drum rotating. For all five edges as the luminance decreased the observer pointed more into the less dense region. These results compare favorably to Lund and Colcord’s findings [90]. The trend is not so obvious at the medium luminance, but becomes more apparent at the lower luminance. A positive movement means a movement to the right or into the less dense region.

<table>
<thead>
<tr>
<th>Edge Lum</th>
<th>Edge Location (cm)</th>
<th>Subjective Edge Movement (secs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>0.7209</td>
<td>+1</td>
</tr>
<tr>
<td>Med</td>
<td>0.7235</td>
<td>+9</td>
</tr>
<tr>
<td>Low</td>
<td>0.7481</td>
<td></td>
</tr>
<tr>
<td>High</td>
<td>1.2271</td>
<td>-3</td>
</tr>
<tr>
<td>Med</td>
<td>1.2174</td>
<td>+15</td>
</tr>
<tr>
<td>Low</td>
<td>1.2721</td>
<td></td>
</tr>
<tr>
<td>High</td>
<td>1.7386</td>
<td>+3</td>
</tr>
<tr>
<td>Med</td>
<td>1.7480</td>
<td>+28</td>
</tr>
<tr>
<td>Low</td>
<td>1.8199</td>
<td></td>
</tr>
<tr>
<td>High</td>
<td>1.9216</td>
<td>0</td>
</tr>
<tr>
<td>Med</td>
<td>1.9216</td>
<td>+7</td>
</tr>
<tr>
<td>Low</td>
<td>1.9433</td>
<td></td>
</tr>
<tr>
<td>High</td>
<td>2.6544</td>
<td>-2</td>
</tr>
<tr>
<td>Med</td>
<td>2.6494</td>
<td>+4</td>
</tr>
<tr>
<td>Low</td>
<td>2.6658</td>
<td></td>
</tr>
</tbody>
</table>

4.6 Relation of the Density and Intensity Edge Trace Inflection Points

It has thus far been established that an observer apparently locates an edge some 30 seconds into the less dense region as measured from the
inflection point of the intensity edge trace (IET). However, the relation of this point to the density edge trace (DET) inflection point is both interesting and relevant. When working from a photograph what is usually obtained is the DET possibly from a direct scan with a microdensitometer. To arrive at the IET from the DET is sometimes a tedious and exacting procedure involving the D log E curve of the film. It would seem advantageous to locate the IET inflection point directly from the observed density values.

It will be shown later that the abscissa value of the inflection point from the intensity and density edge traces can not be equal. Table 4.6 contains a summary of the inflection point location as determined from the DET and IET. Columns 2 and 3 reflect the location of the inflection point as determined from the maximum gradient of the respective observed values. The fact that these two columns are nearly identical only reflects that the amount of the difference is less than the accuracy of the table. These two columns are considered correct to ±0.02 cm or approximately ±7 seconds of arc. If further accuracy were possible a definite difference would be noticed. However, these two columns are important because it shows that one would generally not introduce an error greater than ±7 seconds of arc if one measured the edge from the maximum observed density gradients versus the maximum observed intensity gradient.

Columns 4 and 5 reflect the location of the inflection point as a result of fitting a seven degree polynomial to first the observed density values and then the observed intensity values. The differences in
these two columns are a result of the fact that the density and intensity edge traces are simply not located at the same place, but predominantly as a result of the imperfect fit of the functions causing the inflection point to be shifted slightly. The differences are greater for the B-W edges because the fit of the curves was not as good. If one had a function which fit both observed data perfectly the only difference would be the slight difference (less than ±7 seconds of arc) in the locations of the two inflection points.

Lastly a study was undertaken to determine the effect of fitting a function to either the observed intensity or density values and then computing the difference in the inflection point as a result of the mathematics involved.

Let us start with the very common expression for density (D),

\[ D = \log \frac{I_i}{I_t} \]

where \( I_i \) is the incident light and \( I_t \) is the transmitted light. There are two cases which may be examined. In Case I let us assume that we are measuring light intensity, such as with the generated edge. The incident light (\( I_i \)) is a constant which we shall represent by the letter "a". Let \( f(x) \) represent a function which describes the light intensity across the edge. It has previously been discussed that the intensity edge trace inflection point is located at the point where the slope of the curve is a maximum or minimum. This means that the first derivative, \( f'(x) \), of the IET is a maximum or minimum. It is commonly known in mathematics that the first derivative of a function will be a maximum (minimum) at the inflection point and that the second derivative will be
zero at this same location. This is reasonable when one considers that the second derivative is merely a plot of the slope of the first derivative and if the first derivative has peaked, then its slope must equal zero at that point. This fact is significant and can be expressed similarly by saying that the value of \( x \) which makes the second derivative, \( f''(x) \), of the intensity edge trace equal to zero is the abscissa value of the location of the inflection point which for edges is our only concern.

Symbolically,

\[ f''(x) = 0 \]

when \( x \) = location of the inflection point.

Now let \( D(x) \) represent a function describing the density distribution across an edge. By the same reasoning the value of \( x \) which will make the second derivative, \( D''(x) \), of the density trace equal to zero will be the location of the inflection point of the DET.

We have then,

\[ D = \log \frac{I_l}{I_t} = \log \frac{a}{I_t} \]

or in terms of functions,

\[ D(x) = \log \frac{a}{f(x)} \]

Finding the first derivative of \( D(x) \) ...

\[ D'(x) = -(\log_{10} e) \frac{f'(x)}{f(x)} \]

Let \( k = \log_{10} e \) (\( \approx .434 \)), then

\[ D''(x) = -k \frac{f''(x) f(x) + k [f'(x)]^2}{f(x)^2} \]
Now we know that the abscissa of the density trace inflection point will be equal to the x value which makes $D''(x) = 0$, or the value of x which will satisfy this equation,

$$f''(x) = \frac{[f'(x)]^2}{f(x)}.$$  \hspace{1cm} (1)

If we compare this x value to that obtained when we set $f''(x) = 0$ we will have the difference in the locations of the density and intensity edge trace inflection points.

Case II would occur when one measures the density values directly. Let $D(x)$ be the function which is fit to the observed density values. Again let $f(x)$ be a function representing the intensity distribution across an edge. Then using a similar argument as in Case I we get,

$$D = \log \frac{I_i}{I_t} = \log \frac{a}{I_t},$$

or in terms of a function,

$$D(x) = \log \frac{a}{f(x)}.$$  

But we know $D(x)$ and want $f(x)$. So solving for $f(x)$ ...

$$D(x) = \log a - \log f(x)$$

$$10^{D(x)} = a - f(x)$$

$$f(x) = a - 10^{D(x)}$$

The first derivative of $f(x)$ is ...

$$f'(x) = -10^{D(x)} (\log_{10} 10) \cdot D'(x)$$

Let $p = \log_{10} 10$ (= 2.30), then

$$f'(x) = -p \cdot 10^{D(x)} \cdot D'(x)$$

The simplified second derivative becomes,

$$f''(x) = -p \cdot 10^{D(x)} \cdot D''(x) - p^2 (D'(x))^2 \cdot 10^{D(x)}.$$
Once again, the intensity trace inflection point will be located where \( x \) makes \( f''(x) = 0 \) which is the same \( x \) value that solves this relationship:

\[
D''(x) = -p [D'(x)]^2.
\]  

(2)

This equation is significant because it means that the inflection point of the intensity edge trace may be located by fitting a function to the observed density values and solving Eq (2) for \( x \). This \( x \) value is the abscissa of the IET inflection point and the point to which the subjective edge location is most nearly related. Both equations were numerically verified.

The final column of Table 4.6 contains the often used edge location estimate of average density defined as

\[
D_{ave} = \frac{D_{max} + D_{min}}{2}.
\]

As can be seen this edge determination can be in very great error. For example, the difference between the location of the average density point and the IET inflection point for edge B-W 5 was 0.63 cm at the drum minus the amount that an observer would locate the subjective edge to the right of the inflection point (0.14 cm from Table 4.2) cm or nearly 3 minutes of arc.
Table 4.6

The Inflection Point (IP) as a Function of the Density and Intensity Edge Traces. Column 3 Most Nearly Corresponds to the Subjective Edge Location.

<table>
<thead>
<tr>
<th>(1)</th>
<th>(2) IP Location from Obs Max Density Grad</th>
<th>(3) IP Location from Obs Max Intensity Grad</th>
<th>(4) IP Location from Density Function</th>
<th>(5) IP Location from Intensity Function</th>
<th>(6) Location of Dave</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-W</td>
<td>0.68</td>
<td>0.68</td>
<td>0.43</td>
<td>0.58</td>
<td>0.61</td>
</tr>
<tr>
<td>3</td>
<td>1.58</td>
<td>1.58</td>
<td>1.36</td>
<td>1.59</td>
<td>1.20</td>
</tr>
<tr>
<td>5</td>
<td>2.48</td>
<td>2.48</td>
<td>2.38</td>
<td>2.48</td>
<td>1.85</td>
</tr>
<tr>
<td>6</td>
<td>0.48</td>
<td>0.58</td>
<td>0.32</td>
<td>0.52</td>
<td>0.45</td>
</tr>
<tr>
<td>7</td>
<td>1.02</td>
<td>1.02</td>
<td>1.00</td>
<td>1.02</td>
<td>0.67</td>
</tr>
<tr>
<td>8</td>
<td>2.08</td>
<td>2.10</td>
<td>2.05</td>
<td>2.12</td>
<td>1.52</td>
</tr>
<tr>
<td>G-W</td>
<td>0.52</td>
<td>0.52</td>
<td>0.40</td>
<td>0.42</td>
<td>0.52</td>
</tr>
<tr>
<td>3</td>
<td>1.78</td>
<td>1.78</td>
<td>1.73</td>
<td>1.78</td>
<td>1.72</td>
</tr>
<tr>
<td>5</td>
<td>2.82</td>
<td>2.82</td>
<td>2.80</td>
<td>2.88</td>
<td>2.60</td>
</tr>
<tr>
<td>6</td>
<td>0.52</td>
<td>0.52</td>
<td>0.42</td>
<td>0.48</td>
<td>0.52</td>
</tr>
<tr>
<td>7</td>
<td>1.22</td>
<td>1.22</td>
<td>1.20</td>
<td>1.22</td>
<td>1.20</td>
</tr>
<tr>
<td>8</td>
<td>1.78</td>
<td>1.78</td>
<td>1.78</td>
<td>1.78</td>
<td>1.72</td>
</tr>
<tr>
<td>B-G</td>
<td>0.72</td>
<td>0.72</td>
<td>0.56</td>
<td>0.62</td>
<td>0.70</td>
</tr>
<tr>
<td>3</td>
<td>1.68</td>
<td>1.68</td>
<td>1.63</td>
<td>1.72</td>
<td>1.60</td>
</tr>
<tr>
<td>5</td>
<td>2.78</td>
<td>2.78</td>
<td>2.68</td>
<td>2.76</td>
<td>2.52</td>
</tr>
<tr>
<td>6</td>
<td>0.58</td>
<td>0.58</td>
<td>0.32</td>
<td>0.35</td>
<td>0.40</td>
</tr>
<tr>
<td>7</td>
<td>1.20</td>
<td>1.20</td>
<td>1.15</td>
<td>1.20</td>
<td>1.16</td>
</tr>
<tr>
<td>8</td>
<td>1.82</td>
<td>1.85</td>
<td>1.78</td>
<td>1.82</td>
<td>1.70</td>
</tr>
</tbody>
</table>
5. DATA ANALYSIS - IMAGE QUALITY

5.1 General Discussion

The purpose of this section is to illustrate the results of a search to find some objective and easily measurable quantity which would correlate highly with image quality.

The image-quality criterion used in this paper was the ability of the four observers to arrive at the same subjective edge location. A number which was representative of this ability was the standard deviation of the four observers' means for each edge. For example, for edge B-W 1 the four observers' means were, 0.7662, 0.7624, 0.7878 and 0.7839 centimeters as measured from the zero point. The grand mean was 0.7751 cm. The standard deviation (the dispersion) of the four observers' means was 0.0115 cm at the drum or 4.0 seconds of arc. The number 4.0 was then used to rank this edge as far as image quality is concerned.

The image quality rank of each edge was determined both with and without the inclusion of the individual observer pointing bias. There was very little difference in the order of the edges and no major changes in position. The sample correlation coefficient as defined by Mandel [145, p55] was computed to be +0.94 for the two rankings. Because it is felt that the inclusion of the systematic bias correction is more correct, that ranking will be used in the remainder of the comparisons made in this paper.
Table 5.1 lists the edge, its rank and its image quality criteria value. This table will be referred to often and comparisons made between the image quality of the edges and other measurable quantities.

The sample correlation coefficient ($\gamma$) will be computed for any two quantities being discussed. It is generally recognized that

$$-1 < \gamma < +1$$

and to help settle the question of whether a particular value of $\gamma$ is significantly large or small, the technique expressed by Hamilton will be used [49, p186]. In essence we are testing the hypothesis that the two quantities in question are not correlated.

Let $\gamma_{xy}$ be the sample correlation coefficient. Then

$$t_{xy} \text{ (computed)} = \left[ \right]^{1/2} \text{(Deg of freedom)} \frac{\gamma_{xy}^2}{1-\gamma_{xy}^2}$$

By referring to a tabulation of the Student's t distribution, a value is extracted corresponding to $t_{DF,\alpha}$. The computed value of t would exceed $t_{DF,\alpha}$ only $\alpha(100)$ times out of 100 opportunities. If the computed t is greater than the tabulated t, then the two quantities in question are significantly correlated at the $\alpha$ significance level. For the purposes of this paper an $\alpha$ of 0.05 will be considered significant, $\alpha = 0.01$ highly significant and $\alpha = 0.005$ very highly significant. Another way of stating this is that the probability of the computed t exceeding, for example, the tabulated $t_{DF, 0.005}$ by chance is 0.005% or, in other words, extremely rare.

Because the degrees of freedom always remain 17, it is possible to compute the minimum value of $\gamma_{xy}$ which would be significant, highly
significant and very highly significant. Performing these computations reveals that the correlation between two quantities will be:

<table>
<thead>
<tr>
<th>Type of Correlation</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Significant</td>
<td>$</td>
</tr>
<tr>
<td>Highly Significant</td>
<td>$</td>
</tr>
<tr>
<td>Very Highly Significant</td>
<td>$</td>
</tr>
</tbody>
</table>

This type of approach is relevant if one is to insure that a particular quantity is indeed a meaningful specifier of image quality.

Lastly one must consider the accuracy of Table 5.1, that is, what is the chance that through observational error the order of rank of the edges is incorrect. One could proceed in the following manner. Let $\sigma$ be the dispersion of the four observer means which, of course, is the number used to rank the edges by quality. Then the error in $\sigma$, $\varepsilon_{\sigma}$, is a measure of the reliability of the values in the table.

Let the individual means of the four observers for each edge be $a$, $b$, $c$ and $d$. The standard error of these means was computed numerous times and always was less than ±1 second of arc. Let us assume that the error in $a$, $b$, $c$ & $d$ is

$$\varepsilon_a = \varepsilon_b = \varepsilon_c = \varepsilon_d = \pm 1 \text{ sec.}$$

Now let the grand mean, $m$, of the four individual means be

$$m = \frac{a + b + c + d}{4}.$$

From simple error propagation

$$\varepsilon_m^2 = \frac{1}{16}(\varepsilon_a^2 + \varepsilon_b^2 + \varepsilon_c^2 + \varepsilon_d^2)$$

$$\varepsilon_m^2 = \frac{1}{16}(\pm 4) = \pm \frac{1}{4}$$

and

$$\varepsilon_m = \pm 0.5 \text{ seconds of arc.}$$
From the common equation for the dispersion or standard deviation we get ...

\[ \sigma^2 = \frac{\sum y^2}{n-1} - \frac{(m-a)^2 + (m-b)^2 + (m-c)^2 + (m-d)^2}{3} \]

Differentiating this expression and letting \( \sigma = \varepsilon_\sigma \), etc., we arrive at ...

\[ 2 \sigma \varepsilon_\sigma = \frac{2}{3} [(m-a)(\varepsilon_m - \varepsilon_a) + (m-b)(\varepsilon_m - \varepsilon_b) + (m-c)(\varepsilon_m - \varepsilon_c) + (m-d)(\varepsilon_m - \varepsilon_d)] \]

Solving for \( \varepsilon_\sigma \) and substituting the numerical values for \( \varepsilon_m, \varepsilon_a, \varepsilon_b, \) etc.,...

\[ \varepsilon_\sigma = \frac{1}{6\sigma} [(m-a) + (m-b) + (m-c) + (m-d)]. \]

This expression gives the uncertainty (or error) in \( \sigma \) as a result of the measurement errors in \( a, b, c \xi d \). It can be seen to depend upon two quantities, the size of the residuals and the size of \( \sigma \). It is possible to simplify this further to remove the \( \sigma \) but there is no need to do so. Quickly computing the quantity, \( \varepsilon_\sigma \), for each edge, one finds that it is always less than 0.6 seconds of arc. This value will then be used as a measure of the reliability of Table 5.1.
Table 5.1

<table>
<thead>
<tr>
<th>Rank</th>
<th>Edge</th>
<th>Image Quality Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>B-W 8</td>
<td>3.2 Secs</td>
</tr>
<tr>
<td>2</td>
<td>B-W 1</td>
<td>4.0</td>
</tr>
<tr>
<td>3</td>
<td>B-W 6</td>
<td>6.5</td>
</tr>
<tr>
<td>4</td>
<td>B-G 6</td>
<td>8.3</td>
</tr>
<tr>
<td>5</td>
<td>B-W 7</td>
<td>14.9</td>
</tr>
<tr>
<td>6</td>
<td>G-W 7</td>
<td>15.1</td>
</tr>
<tr>
<td>7</td>
<td>G-W 6</td>
<td>15.9</td>
</tr>
<tr>
<td>8</td>
<td>B-G 8</td>
<td>17.0</td>
</tr>
<tr>
<td>9</td>
<td>B-W 3</td>
<td>17.8</td>
</tr>
<tr>
<td>10</td>
<td>B-W 5</td>
<td>18.5</td>
</tr>
<tr>
<td>11</td>
<td>B-G 7</td>
<td>21.4</td>
</tr>
<tr>
<td>12</td>
<td>G-W 8</td>
<td>21.6</td>
</tr>
<tr>
<td>13</td>
<td>G-W 5</td>
<td>24.3</td>
</tr>
<tr>
<td>14</td>
<td>G-W 1</td>
<td>25.9</td>
</tr>
<tr>
<td>15</td>
<td>B-G 5</td>
<td>26.0</td>
</tr>
<tr>
<td>16</td>
<td>G-W 3</td>
<td>27.7</td>
</tr>
<tr>
<td>17</td>
<td>B-G 3</td>
<td>31.5</td>
</tr>
<tr>
<td>18</td>
<td>B-G 1</td>
<td>33.2</td>
</tr>
</tbody>
</table>

5.2 Image Quality and the Slope of the Density and Intensity Edge Trace

Table 5.2 lists the edge and its intensity and density edge trace maximum slopes as computed by fitting the seven degree polynomial to the observed points and finding the maximum (minimum) value of the first derivative. The correlation coefficients between image quality as expressed in Table 5.1 and the intensity and density slopes are -0.63 and +0.78 respectively. Both of these are very highly significant. It is interesting to note that the correlation of the density trace slope is approximately of equal significance as is acutance. (Section 5.4). This means that the researcher can determine the relative quality of an edge simply by tracing it with a microdensitometer.
and calculating the slope of the trace.

Table 5.2

<table>
<thead>
<tr>
<th>Edge</th>
<th>Maximum Intensity Slope</th>
<th>Maximum Density Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-W 1</td>
<td>44.81</td>
<td>-1.575</td>
</tr>
<tr>
<td></td>
<td>33.21</td>
<td>-0.779</td>
</tr>
<tr>
<td></td>
<td>40.70</td>
<td>-0.782</td>
</tr>
<tr>
<td></td>
<td>55.86</td>
<td>-1.976</td>
</tr>
<tr>
<td></td>
<td>46.54</td>
<td>-1.095</td>
</tr>
<tr>
<td></td>
<td>64.94</td>
<td>-1.019</td>
</tr>
<tr>
<td>G-W 1</td>
<td>34.55</td>
<td>-0.565</td>
</tr>
<tr>
<td></td>
<td>24.24</td>
<td>-0.351</td>
</tr>
<tr>
<td></td>
<td>26.43</td>
<td>-0.348</td>
</tr>
<tr>
<td></td>
<td>38.62</td>
<td>-0.640</td>
</tr>
<tr>
<td></td>
<td>36.94</td>
<td>-0.538</td>
</tr>
<tr>
<td></td>
<td>38.00</td>
<td>-0.527</td>
</tr>
<tr>
<td>B-G 1</td>
<td>3.51</td>
<td>-0.547</td>
</tr>
<tr>
<td></td>
<td>3.02</td>
<td>-0.398</td>
</tr>
<tr>
<td></td>
<td>3.64</td>
<td>-0.440</td>
</tr>
<tr>
<td></td>
<td>5.91</td>
<td>-0.953</td>
</tr>
<tr>
<td></td>
<td>4.76</td>
<td>-0.621</td>
</tr>
<tr>
<td></td>
<td>4.95</td>
<td>-0.622</td>
</tr>
</tbody>
</table>

5.3 Image Quality and The Maximum Gradient

The maximum gradient may be studied in four different ways:

1) The measured maximum intensity gradient,
2) The measured maximum density gradient,
3) The maximum intensity gradient as determined from a function fit to the observed intensity values,
4) The maximum density gradient as determined from a function fit to the observed density values.
It is interesting to note that a change in intensity of, for example, 8 ft-lams could result in a smaller change in density than an intensity difference of, for example, only 6 ft-lams. The numerical size of the density gradient is also a factor of where the change occurs, i.e. in the high or low density regions. The interested reader can establish this for himself by quickly sketching a few representative edge traces. Even though intensity was actually measured, the measured density values could be simulated by solving this equation for density, \(D_i\).

\[ D_i = \log \frac{I_{\text{max}}}{I_i} \]

where \(0 \leq i \leq n\) and \(n\) is the number of times an intensity reading was taken.

Fitting a function to a set of observed points has the effect of dampening out abrupt changes or smoothing the data which helps explain why the observed and computed maximum gradients differ.

It was found experimentally, and later mathematically verified, that the edge with the greatest slope also possessed the greatest computed maximum gradient. (This of course assumes that the incremental abscissa distance, \(\Delta x\), remains constant.) This is only logical when one considers that the maximum slope means the point where

\[ \frac{dy}{dx} = \max \]

which is the same as saying

\[ \frac{y_{i+1} - y_i}{\Delta x} = \max \]

which is of course the maximum computed gradient.
Therefore the maximum computed gradient had the same correlation coefficient as the maximum slope of the curve. Consequently Table 5.3 contains only a listing of the edges and the maximum observed intensity and density gradients. The correlation coefficients for these two quantities were -0.36 and -0.62. This means that the maximum gradient of the observed intensity value tells us nothing about image quality but that the correlation between the observed maximum density gradient and image quality is highly significant. This point is of importance for a researcher who is attempting to design a system which produces edges of high measuring quality. He could ascertain the relative value of an edge simply by measuring the density at discrete points near the density trace inflection point and determining the value of the maximum density gradient. The higher the maximum density gradient, the more measurable will be the edge.
Table 5.3

<table>
<thead>
<tr>
<th>Edge</th>
<th>Intensity</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-W 1</td>
<td>8.2</td>
<td>0.205</td>
</tr>
<tr>
<td>3</td>
<td>4.0</td>
<td>0.077</td>
</tr>
<tr>
<td>5</td>
<td>7.8</td>
<td>0.128</td>
</tr>
<tr>
<td>6</td>
<td>4.2</td>
<td>0.123</td>
</tr>
<tr>
<td>7</td>
<td>6.0</td>
<td>0.162</td>
</tr>
<tr>
<td>8</td>
<td>5.3</td>
<td>0.076</td>
</tr>
<tr>
<td>G-W 1</td>
<td>5.7</td>
<td>0.088</td>
</tr>
<tr>
<td>3</td>
<td>5.1</td>
<td>0.071</td>
</tr>
<tr>
<td>5</td>
<td>4.7</td>
<td>0.059</td>
</tr>
<tr>
<td>6</td>
<td>3.8</td>
<td>0.060</td>
</tr>
<tr>
<td>7</td>
<td>8.0</td>
<td>0.111</td>
</tr>
<tr>
<td>8</td>
<td>5.3</td>
<td>0.070</td>
</tr>
<tr>
<td>B-G 1</td>
<td>0.35</td>
<td>0.050</td>
</tr>
<tr>
<td>3</td>
<td>0.36</td>
<td>0.046</td>
</tr>
<tr>
<td>5</td>
<td>0.42</td>
<td>0.043</td>
</tr>
<tr>
<td>6</td>
<td>0.77</td>
<td>0.130</td>
</tr>
<tr>
<td>7</td>
<td>0.70</td>
<td>0.102</td>
</tr>
<tr>
<td>8</td>
<td>0.50</td>
<td>0.052</td>
</tr>
</tbody>
</table>

5.4 Image Quality and Acutance

The acutance of each edge was determined from the equation of Higgins discussed in Section 2.2.2.2. The distance interval was taken as 0.05 cm which for these generated edges is not excessively large. The sample correlation coefficient, $r_{xy}$, was computed to be -0.80 which is very highly significant. This reveals the very interesting fact that an excellent specifier of image quality is acutance; the higher the acutance the better is the image quality as defined in this paper. The individual values are separately tabulated in Table 5.4 because of their particular importance.
One might expect acutance to be highly correlated because the slope of the density edge trace was highly correlated.

Table 5.4

<table>
<thead>
<tr>
<th>Edge</th>
<th>Image Quality</th>
<th>Acutance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Secs of Arc</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B-W 8</td>
<td>3.2</td>
<td>.2760</td>
</tr>
<tr>
<td>B-W 1</td>
<td>4.0</td>
<td>.4376</td>
</tr>
<tr>
<td>B-W 6</td>
<td>6.5</td>
<td>.5929</td>
</tr>
<tr>
<td>B-G 6</td>
<td>8.3</td>
<td>.4037</td>
</tr>
<tr>
<td>B-W 7</td>
<td>14.9</td>
<td>.4113</td>
</tr>
<tr>
<td>G-W 7</td>
<td>15.1</td>
<td>.3684</td>
</tr>
<tr>
<td>G-W 6</td>
<td>15.9</td>
<td>.2384</td>
</tr>
<tr>
<td>B-G 8</td>
<td>17.0</td>
<td>.2100</td>
</tr>
<tr>
<td>B-W 3</td>
<td>17.8</td>
<td>.1938</td>
</tr>
<tr>
<td>B-W 5</td>
<td>18.5</td>
<td>.2023</td>
</tr>
<tr>
<td>B-G 7</td>
<td>21.4</td>
<td>.3167</td>
</tr>
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<td>G-W 8</td>
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<td>.2340</td>
</tr>
<tr>
<td>G-W 5</td>
<td>24.3</td>
<td>.0953</td>
</tr>
<tr>
<td>G-W 1</td>
<td>25.9</td>
<td>.1926</td>
</tr>
<tr>
<td>B-G 5</td>
<td>26.0</td>
<td>.1024</td>
</tr>
<tr>
<td>G-W 3</td>
<td>27.7</td>
<td>.1190</td>
</tr>
<tr>
<td>B-G 3</td>
<td>31.5</td>
<td>.0970</td>
</tr>
<tr>
<td>B-G 1</td>
<td>33.2</td>
<td>.1327</td>
</tr>
</tbody>
</table>

5.5 Image Quality and Precision of Pointing

As was previously mentioned the closeness together of the means of the four observers was used as a measure of image quality. This could be considered a test of the measurability of an edge. If an edge was so sharply formed, that is, so unambiguous that all the observers saw and measured to the same location, this edge was considered of high quality.

However, this type of quality test does not examine the precision within the measurements of each observer. Instead of computing the standard deviation of 90 pointings for each observer for each edge, the
dispersion of the nine means from each observer setting the MM tangent to an edge for 9 sets of 10 observations was computed. This procedure was used in an attempt to determine or measure the varying degrees of difficulty in arriving at the same subjective decision as to where an edge is located. For example, if to an observer a particular contrast, illumination, density, etc., across an edge was especially pleasing to him one would expect the dispersion of his 9 means for that edge to be rather small or at least smaller than the other edges.

One might think that an edge of high quality or measurability as defined in this paper would also result in a small dispersion of the means of the four observers. However, this was not found to be the case. Table 5.5 lists the edge, its quality rank and the dispersion of the means of each observer for each edge in seconds of arc. This dispersion of the means should not be confused with the standard error of the mean. (That value was generally ±1 second of arc). The dispersion of the mean is merely a measure of the amount that the mean of each of the 9 sets differed for a particular edge and observer. It is probably the best measure available for the degree of difficulty that each observer experienced in arriving at the same subjective decision as to where an edge was located. The last column depicts the mean dispersion of the four observer dispersions for that edge.

The sample correlation coefficient between image quality and the mean dispersion was computed to be -0.28 which displays virtually no correlation. This fact is surprising and can only be explained by saying that apparently different observers can measure more precisely
Table 5.5
Dispersion of the 9 Means of Each Observer Per Edge

<table>
<thead>
<tr>
<th>Edge</th>
<th>Rank</th>
<th>Dispersion of the 9 Means Per Observer (Secs of arc)</th>
<th>Mean Dispersion (Secs)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Obs 1</td>
<td>Obs 2</td>
</tr>
<tr>
<td>B-W 1</td>
<td>2</td>
<td>6.7</td>
<td>7.1</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>10.1</td>
<td>16.9</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>11.1</td>
<td>8.0</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>8.0</td>
<td>4.6</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>8.8</td>
<td>11.6</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>13.1</td>
<td>12.8</td>
</tr>
<tr>
<td>G-W 1</td>
<td>14</td>
<td>11.8</td>
<td>10.9</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8.7</td>
<td>6.7</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>6.8</td>
<td>7.7</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>15.1</td>
<td>16.1</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>6.3</td>
<td>13.1</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>6.1</td>
<td>8.6</td>
</tr>
<tr>
<td>B-G 1</td>
<td>18</td>
<td>4.2</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>7.4</td>
<td>5.2</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>5.7</td>
<td>5.0</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>5.5</td>
<td>4.6</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>8.8</td>
<td>3.8</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>7.9</td>
<td>8.9</td>
</tr>
</tbody>
</table>

on different edges. This would explain the absence of any correlation between quality and precision. For example, Observer One measured with more precision on edge B-G 1, however, for Observer Four, B-G 1 would be ranked 17th out of the 18 edges. It would seem that precision of pointing is a personal thing with each observer having his own edge preferences. The reader must be careful not to infer that an image of high quality would yield poor measuring precision. What is implied here is that observers point with generally the same precision on all reasonable edges with particular individuals excelling on particular edges.
5.6 Image Quality and the Density Spread Function

The term spread function is normally used to represent the first derivative of the intensity edge trace. Upon studying this element only marginal importance could be placed upon the shape of the intensity spread function. However, because of the high correlation of the slope of the density edge trace and image quality, it was hypothesized that possibly the shape of the density spread function would be more relevant. The density spread function will then be defined as the first derivative of the density edge trace. It was discussed in Section 5.2 that generally as the slope of the density edge trace increased image quality also increased. However, the maximum slope is merely the maximum value of \((y_{i+1} - y_i)/\Delta x\), and nothing is said of what happens on either side of this point. It is possible to draw two edges of varying width which have the same maximum slope but which can not, by all orders of reason, be of the same quality. Figure 5.1 depicts two such edges. If the \(\Delta x\) increment were small enough, both edges would have the same maximum slope. However, it is unreasonable to expect them to be of the same quality. In addition to the slope it would seem that what is happening on either side of the maximum slope must also be of prime importance.

Consequently the density spread function was considered. An approximate density spread function (DSF) is plotted in Figure 5.2 for the two edges shown in Figure 5.1. It would seem that the narrower DSF would represent the edge of higher measuring quality. Subsequently, two terms, amplitude \((A)\) and dispersion \((\sigma)\) were defined
Both edges would have the same maximum slope if $\Delta x$ were small enough. However one would not expect them to be of the same quality.

Approximate density spread functions for edges in Fig. 5.1. The more narrow DSF would seem to represent the edge of higher quality.

According to the sketch in Figure 5.3, the amplitude ($A$) must be carefully defined. It is the total change in density from where the DSF peaks to where it reaches its lowest point on the side of the DSF which faces the region of lighter density. If a function is fit to
observed density values the lowest density value may actually be negative in which case it would be added to the value of the DSF at its peak.

The term, dispersion \((\sigma)\), is defined somewhat as in statistics. It is the distance from a vertical line, perpendicular to the abscissa and passing through the maximum point of the DSF, to the inflection point of the portion of the DSF facing the region of lighter density. The purpose of always considering the portion of the curve facing the region of lighter density is because it is in this region that we measure. This was established in Section 4. Figure 5.3 shows a typical DSF with the terms A and \(\sigma\) defined.

Figure 5.3

Depicts quantities defined as amplitude \((A)\) and Dispersion \((\sigma)\) which are measured on the density spread function.
A new term called, DECUTANCE, (De) is defined and is equal to

$$De = \frac{A}{o}.$$  

The name is derived from the similarity of this quantity to acutance. However, this term is much simpler than acutance, is more easily computed and has a higher correlation with image quality than acutance. The units would be density/distance; or, for this paper \((cm^{-1})\). (Considering density as being unitless.) For an actual photograph the units would most probably be \((\mu m^{-1})\).

Table 5.6 displays the 18 edges in their proper rank, the edge identification and the corresponding decutance value. The computed correlation coefficient was an amazing \(-0.84\). This means that the greater the decutance the higher in quality is the edge. Considering that an \(\gamma_{xy}\) of 0.63 is very highly significant an \(\gamma_{xy}\) of 0.84 is phenomenal.
<table>
<thead>
<tr>
<th>Edge Rank</th>
<th>Edge Identification</th>
<th>Decutance (cm⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>B-W 8</td>
<td>7.23</td>
</tr>
<tr>
<td>2</td>
<td>B-W 1</td>
<td>3.96</td>
</tr>
<tr>
<td>3</td>
<td>B-W 6</td>
<td>5.63</td>
</tr>
<tr>
<td>4</td>
<td>B-G 6</td>
<td>3.38</td>
</tr>
<tr>
<td>5</td>
<td>B-W 7</td>
<td>2.33</td>
</tr>
<tr>
<td>6</td>
<td>G-W 7</td>
<td>1.67</td>
</tr>
<tr>
<td>7</td>
<td>G-W 6</td>
<td>2.08</td>
</tr>
<tr>
<td>8</td>
<td>B-G 8</td>
<td>2.02</td>
</tr>
<tr>
<td>9</td>
<td>B-W 3</td>
<td>1.28</td>
</tr>
<tr>
<td>10</td>
<td>B-W 5</td>
<td>1.61</td>
</tr>
<tr>
<td>11</td>
<td>B-G 7</td>
<td>1.78</td>
</tr>
<tr>
<td>12</td>
<td>G-W 8</td>
<td>1.69</td>
</tr>
<tr>
<td>13</td>
<td>G-W 5</td>
<td>0.92</td>
</tr>
<tr>
<td>14</td>
<td>G-W 1</td>
<td>1.55</td>
</tr>
<tr>
<td>15</td>
<td>B-G 5</td>
<td>1.00</td>
</tr>
<tr>
<td>16</td>
<td>G-W 3</td>
<td>0.73</td>
</tr>
<tr>
<td>17</td>
<td>B-G 3</td>
<td>0.80</td>
</tr>
<tr>
<td>18</td>
<td>B-G 1</td>
<td>1.28</td>
</tr>
</tbody>
</table>

### 5.7 Image Quality and the Higher Derivatives

Considerable study and labor was devoted to computing and analyzing the 2nd, 3rd and 4th derivatives of each edge trace. It was concluded that no information is contained in these higher derivatives that is not already contained in the original edge trace or its 1st derivative. These derivatives are purely mathematical and can only reflect the characteristics of the spread function. For example, refer to the edge trace and spread function in Figure 5.4. Just as the 1st derivative (spread function) is a plot of the slope of the edge trace, the 2nd derivative is a plot of the slope of the 1st derivative. If one breaks the 1st derivative up into segments a-b and b-c then it can be seen that the 2nd derivative over these segments acts exactly
as the 1st derivative did on the original edge trace. This type of reasoning can be carried through to the 4th derivative. It is believed for this reason that nothing astonishing was found in the higher derivatives.

Figure 5.4

Just as the 1st Derivative is a plot of the slope of the edge trace, the 2nd Derivative is a plot of the slope of the 1st Derivative. The same reasoning can be carried through to the 4th Derivative and higher.
5.8 Image Quality and the Modulation Transfer Function

Considerable effort was devoted to the problem of determining if anything of value could be learned about image quality by studying the modulation transfer function (MTF) of the edge. Image quality is again defined as the measurability of an edge. And, as was stated earlier, the modulation transfer function is defined as a plot of the normalized modulus of the Fourier transform of the intensity spread function. The principle of and current thinking on the MTF as applied to photogrammetry and other disciplines is presented in Section 2.2.2.4 and Appendix E. In essence, if \( g(x) \) is the intensity edge trace then \( f(x) = g'(x) \) is the intensity spread function. The Fourier transform, \( F(S) \), of \( f(x) \) is generally given by this expression

\[
F(S) = \int_{a}^{b} f(x) e^{-ixs} dx.
\]

\( F(S) \) will be a complex number,

\[
F(S) = X + iY.
\]

The modulus (\( M \)) of \( F(S) \) is given by this expression,

\[
M = (x^2 + y^2)^{1/2}.
\]

Finally the values of \( M \) are normalized by dividing all the values by the largest value of \( M \). (This will normally be the value of \( M \) when the spatial frequency is zero.) A plot of the normalized \( M \) values against the spatial frequency is the MTF of an edge.

The simplicity in stating the principle erroneously conceals the quantity of thought and effort which can, in fact, be applied to this concept. A brief example follows.
Earlier a discussion was held concerning the applicability of fitting a seven degree polynomial to the observed intensity values. After fitting anddifferentiating this polynomial, it was observed that the Fourier transform, \( F(S) \), approaches infinity as \( S \) approaches zero. This disturbing fact could not be circumvented by applying mathematical techniques [142]. This meant that the modulus was infinite and that the values could not be normalized. A number of approaches led to the conclusion that the spread function could not be represented by a polynomial if one intended to calculate the MTF.

The next avenue of approach was to choose a function to represent the spread function whose Fourier transform was known to be bounded, for example, \( e^{-ax^2} \). Bracewell [16, p357] shows that the Fourier transform of \( e^{-\pi x^2} \) is \( e^{-\pi s^2} \) which is most certainly bounded. It was then realized that a similarly bounded function is the "error function", \( f(x) \), given by

\[
f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x^2)}{2\sigma^2}}.
\]

With difficulty it can be shown that the Fourier transform of \( f(x) \) is

\[
F(S) = \sqrt{\pi} e^{-\frac{(s^2\sigma^2)}{2}},
\]

which is of course bounded. However, more thought revealed that the error function would not represent the unsymmetrical nature of some of the spread functions and in fact emphasized the dispersion of the spread function at the expense of the amplitude. This approach was subsequently abandoned.

At this point the Fast Fourier Transform algorithm as devised by Cooley and Tukey [148] was applied. This is defined as "... a method
for efficiently computing the discrete Fourier transform of a series of data samples." [138, p45]. Although to the non-mathematician it may seem that this technique is based on magic, it has received wide acclaim and is a popular and accepted computational tool.

An advantage in using the Fast Fourier Transform (FFT) algorithm is that the discrete observed values can be used directly. If $y_i$ is the observed luminance, then an array composed of $(y_{i+1} - y_i) / 0.05$ is formed when 0.05 represents the distance between the observations. This array, representing the intensity spread function, is transformed point by point into the Fourier transform through a subroutine contained in the Ohio State University Computer library.

The reader who is interested in the details of how the FFT algorithm works is urged to see Cooley and Tukey or Cochran [148, 149].

Now that the MTF of the edges was available the question arose as to what facet of the MTF could possibly be a specifier of image quality. Two quantities were investigated; the maximum value of the modulus and the cut-off frequency. The cut-off frequency for the wide edges was arbitrarily chosen as the frequency where the MTF was less than or equal to 0.149. For the narrow edges it was more difficult to determine the cut-off frequency because of the high noise level for which the author was pre-warned by Trinder [147]. It was obvious that the cut-off frequency was much lower for the narrow edges and its value could only be determined by noting where the MTF seemed to dip and then start to rise again. For the narrow edges the maximum value of the modulus did not occur exactly (although very near) at the point
where the spatial frequency equalled zero. The author can offer no explanation for this except to say that Cochran [149] displays the same possibility and it is apparently a peculiar but accepted quirk of the transforming process.

Table 5.7 displays the maximum modulus and cut-off frequency for each edge. The sample correlation coefficient was computed and no significant correlation was revealed between image quality as defined in this paper and these two elements of the MTF as computed by Cooley's FFT algorithm.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Edge</th>
<th>Maximum Modulus</th>
<th>Cut-off Frequency (CPS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>B-W 8</td>
<td>1220</td>
<td>2.2</td>
</tr>
<tr>
<td>2</td>
<td>B-W 1</td>
<td>800</td>
<td>4.7</td>
</tr>
<tr>
<td>3</td>
<td>B-W 6</td>
<td>1378</td>
<td>0.9</td>
</tr>
<tr>
<td>4</td>
<td>B-G 6</td>
<td>145</td>
<td>1.2</td>
</tr>
<tr>
<td>5</td>
<td>B-W 7</td>
<td>1410</td>
<td>0.6</td>
</tr>
<tr>
<td>6</td>
<td>G-W 7</td>
<td>1230</td>
<td>0.6</td>
</tr>
<tr>
<td>7</td>
<td>G-W 6</td>
<td>1211</td>
<td>0.9</td>
</tr>
<tr>
<td>8</td>
<td>B-G 8</td>
<td>139</td>
<td>1.2</td>
</tr>
<tr>
<td>9</td>
<td>B-W 3</td>
<td>809</td>
<td>1.9</td>
</tr>
<tr>
<td>10</td>
<td>B-W 5</td>
<td>813</td>
<td>4.4</td>
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<td>11</td>
<td>B-G 7</td>
<td>153</td>
<td>0.9</td>
</tr>
<tr>
<td>12</td>
<td>G-W 8</td>
<td>1162</td>
<td>1.2</td>
</tr>
<tr>
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</tr>
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<td>16</td>
<td>G-W 3</td>
<td>528</td>
<td>4.1</td>
</tr>
<tr>
<td>17</td>
<td>B-G 3</td>
<td>72</td>
<td>1.9</td>
</tr>
<tr>
<td>18</td>
<td>B-G 1</td>
<td>69</td>
<td>2.5</td>
</tr>
</tbody>
</table>
6. CONCLUSIONS

6.1 General Discussion

The stated primary objective of this study was to search for an easily measurable objective quantity which correlated highly with image quality. An image of high quality was defined to be an image whose edge was located at the same location by four different experienced observers. As the dispersion in the subjective edge location of the four observers increased the edges were said to be of decreasing quality.

Through repeated edge points 18 edges were located and ranked as to quality. The edge location process enabled the secondary objective to be studied, i.e. that of relating the subjective edge to some objective quantity.

6.2 Edge Location

It was found that observers locate the subjective edge slightly into the less dense region as measured from the inflection point of the intensity edge trace. A nominal value for this quantity is 30 seconds of arc; however it appears that for the edges studied as the average density across the edge increased the amount that the observers pointed into the less dense region decreased. In other words, although the value of 30 seconds of arc is approximately correct, the subjective edge location appears to be a function of some quantity, possibly density.

A tendency for observers to set the measuring mark too far into the lighter region was detected by having them point on edges whose location
was known. This tendency, which was called a systematic personal bias, adds an additional parameter to the measurement process. The amount that an observer sets the measuring mark too far into the lighter region is a function of the observer and of the characteristics of the edge being observed. It is obvious that this systematic bias should be removed, which means that an observer would have to be calibrated. That is, the amount of bias under many different edge situations would have to be measured. Then the measured bias could be subtracted from the subjective edge location.

By varying the incident light in the relation Density = \log \left( \frac{I_i}{I_t} \right) the personal bias of one observer was shown to change. The tendency, that has been noted by others, for the observer to set the measuring mark further into the less dense region was detected here. Although it is believed all observers would demonstrate the same tendency, there is no assurance that the amount would be the same.

The dispersion in subjective edge location of the four observers varied from ±3.2 to ±33.2 seconds of arc. Using the relationship of 10 seconds of arc corresponding to 1 \( \mu \text{m} \) at the photograph on a comparator it can be seen that the error introduced just by the ambiguity in the subjective edge location could be ±3 \( \mu \text{m} \). It has been shown by O'Connor [104, p9] that the error connected with centering a measuring mark on a target is much less. As such, in any photogrammetric task where it is possible, greater accuracy will be achieved if edge measurement is avoided.
The difficulty in calibrating all observers under all situations combined with the yet remaining subjective edge location error seems to indicate that as the desire for greater accuracy in photogrammetry is increased the role of the human observer will have to be decreased. A type of edge scanning device which could swiftly locate the same objective point (e.g. the density or intensity edge trace inflection point) for each edge would render more uniform and exacting results.

Lastly a relationship was derived (Section 4.5) between the inflection point location for the intensity and density edge traces which could be useful in edge location studies.

6.3 Image Quality Conclusions

It was found that any quantity related to density correlated with image quality much more than its corresponding intensity quantity. The correlations of the slopes of the intensity and density edge traces are very highly significant. Acutance is obviously an excellent specifier of image quality.

The new quantity, decutance, has the highest correlation with image quality of all the possible specifiers studied. Additionally it is easily computed from the observed density values.

The precision of pointing did not correlate with image quality which suggests an individual personal edge preference by each of the observers. That is, observers point more precisely on some edges than on others, but the particular edge producing high pointing precision may vary according to the observer.
The higher derivatives (2nd, 3rd & 4th) of the intensity edge trace revealed no information that was not already discernible from the edge trace or its first derivative.

Neither the modulus of the Fourier transform of the intensity spread function nor the cut-off frequency as computed by Cooley's Fast Fourier Transform algorithm displayed any correlation with image quality.

Table 6.1 lists the quantities which had a significant correlation with image quality. A negative correlation coefficient means that as the quantity increases in value the image quality increases (dispersion of means decreases).

For the various reasons stated herein the following quantities appear not to be specifiers of image quality: precision of pointing, observed maximum intensity gradient, the higher derivatives, and the MTF of the edge.

Table 6.1

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Sample Correlation Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decutance</td>
<td>-0.84</td>
</tr>
<tr>
<td>Acutance</td>
<td>-0.80</td>
</tr>
<tr>
<td>Max. Slope Density Edge Trace (funct.)</td>
<td>+0.78</td>
</tr>
<tr>
<td>Max. Observed Density Grad.</td>
<td>-0.62</td>
</tr>
<tr>
<td>Max. Slope Intensity Edge Trace (funct.)</td>
<td>-0.63</td>
</tr>
</tbody>
</table>
6.4 Brief Summary of the Results

Based on the results of the experiments described herein, the following brief statements can be made:

1. Observers locate the subjective edge approximately 30 seconds of arc into the region of lighter density as measured from the inflection point of the intensity edge trace. The exact amount may be a function of density.

2. Observers possess a systematic pointing bias which should be removed when pointing of high accuracy is desired. The amount of this bias may be a function of density.

3. The ambiguity in subjective edge location can cause an error as great as ±3μm in edge location at the photo scale.

4. The location of the inflection point of the intensity edge trace may be located by solving a derived equation based on observed density values.

5. The subjective variance and personal bias of observers indicates that the role of the human observer in photogrammetric edge location must be decreased if extremely high accuracy is desired.

6. A new objective quantity called decutance was derived which is easily measured and correlates very highly with image quality.

7. Acutance and the slope of the density edge trace correlate highly with image quality.

8. The higher derivatives of the intensity edge trace revealed nothing about image quality that was not obtainable from the original edge trace and its spread function.
9. Quantities based on intensity generally show very little correlation with image quality.

10. The precision of pointing did not correlate with image quality.

11. Neither the maximum modulus of the MTF nor the cut-off frequency as computed by Cooley's Fast Fourier transform algorithm correlated with image quality.

12. The individual who desires an objective quantity which correlates with image quality should choose decutance, acutance, or the slope of the density edge trace and in that order.
7. RECOMMENDATIONS

An apparent need is present to lessen the role of the human observer in the photogrammetric process where edge measurement is concerned. As such it is recommended that new impetus be given to studies related to automatic mapping and measuring devices. This type of device would require film of very low granularity and some type of magnification and scanning mechanism. Investigations along these lines should be encouraged.

Should someone have a need for more accurate human measurement a technique for calibrating an observer should be devised. A function could possibly be derived which would describe his personal pointing bias in terms of density and possibly even relate his subjective edge location to some objectively measurable point as the inflection point.

It would be meaningful if the image quality conclusions reached in this paper could be verified from measurements on actual photographic edges. If various observers measured well magnified edges which were later scanned with a microdensitometer, the correlation between decussance (acutance, etc.) with image quality could be verified.
APPENDIX

A. The Argument Against Resolution as a Specifier of Image Quality

Resolution may be defined as "The minimum distance between two adjacent features, or the minimum size of a feature, which can be detected by a photographic system...." [96, p1153]. This distance is usually expressed in lines per millimeter. Resolving power is a measure of the ability of a photo-optical system (or component thereof) to produce an image of recognizable configuration at minimum size [72, p15]. In practice it is common to find these terms used interchangeably.

It must be emphasized that resolution is a useful tool. It has been the fault of men, and not a shortcoming in resolution, that it has been placed on a throne where it simply does not belong. Brock states that "Perhaps the greatest fallacy in resolving power testing and reporting procedure is the specification of a single value to characterize the resolving power of a film at a given target contrast, exposure, and processing." [28, p63-64]. In other words resolution would be more adequate if, before specifying a number, the author would list the multitude of conditions to which this value pertains.

The following is simply a listing of the disadvantages of resolution all of which can easily be found in the existing literature:

1. There is no precise criteria for resolution, that is, no accepted set of ground rules which must be obeyed when attempting to determine the resolving power of a component or system [52, p22].
2. "For some lenses, the image plane for maximum resolution does not correspond to the plane for maximum sharpness; focusing is normally done to the latter plane." [42, p4].

3. Resolving power measurements are made under a specific set of conditions and are valid only for that set of conditions [28, p63].

4. Resolving power is a statistical quantity and if measured many times for one system would reveal a gaussian distribution [28, p63].

5. It is difficult to pinpoint the cause of degradation in a system or to determine resolution capabilities of each component to arrive at an overall prediction of system capability [28, p65].

6. Resolving power does not take into account the ability of the system to image frequencies lower than the cut-off frequency [12, p4].

7. It is difficult to standardize because it depends heavily on human judgement [23, p33-34].

8. Resolving power is a threshold phenomenon and as such tells us nothing about how larger objects are imaged.

For the photogrammetrist whose task is to measure to an edge the shortcoming of resolution is made obvious by the following diagrams which are based on Figure 23.50, page 537 of The Theory of the Photographic Process (Third Edition edited by T. H. James) [76].

Let Figure 8.1 represent two hypothetical spread functions. For a large object, one to which a photogrammetrist may wish to measure to the edge, it is apparent that the image from which spread function "a" was derived would display the most clear and sharp edge. However, if two
Figure 8.1

Two Hypothetical Spread Functions

Figure 8.2

Two objects with Spread Functions "a" would be difficult to differentiate but their edge easily measured
Two objects with Spread Functions "b" would be difficult to measure but readily discernible smaller objects were placed side by side, each with the spread functions of "a" and then "b", it is easily seen in Figures 8.2 and 8.3 that that spread function "b" is derived from the system with the higher resolution.

The conclusion is obvious; resolution is not a specifier of image quality for the photogrammetrist who is concerned with measuring to an edge.
B. Microdensitometer Edge Traces and Edge Magnification

To better understand the problems concerned with studying an edge through the two techniques of microdensitometer scans and magnification the author traveled to Wright Patterson Air Force Base where the equipment and expertise of the Aerial Photograph Group, Photographic Branch, Reconnaissance Division of the Avionics Laboratory was graciously placed at his disposal for a few days. The mission was to study a few "typical" edges. The "typical" edge was generally defined to be an edge which would result from a good aerial photograph, using Kodak film, using a popular precision frame camera, flying between 2-30 thousand feet and with an expected resolution of 30-40 lines/mm.

Frame number 4 of a roll of film to which the following data applies was chosen:

- **Taking unit**: ACGS (WPABFB)
- **Date**: March 9, 1971
- **Time**: 1637 - 1947 hrs.
- **Focal Length**: 151.410 mm
- **Altitude**: 25,000 ft. (above grad. level)
- **Project number**: AF 71-37
- **Camera**: KCIB (Serial #63-151)
- **Lens**: Planigon
- **Filter**: B
- **Film**: Kodak Plus X Aerocon
A number of edges which appeared to the unaided eye to be extremely sharp and of high contrast were chosen to be examined. The first step was to scan these edges with a microdensitometer. An experienced technician, Mr. Stephan A. Eckstrand, actually operated the equipment. Various slit sizes were tried with the final slit being 1 \( \mu m \) wide and 171 \( \mu m \) long. The graph paper moved 0.2 inch/sec and the scanner moved 250/\( \mu m/min \). This meant that after one second the scanner had traveled 4,167 \( \mu m \) and plotted this information over 0.2 inches of graph paper. Therefore each division on the graph paper (in the following figures), which is 0.1 inch in width, represents approximately 2.09 \( \mu m \).

The microdensitometer was programmed to measure transmittance. It was not calibrated which means each transmittance reading is not correct in the absolute sense. However, relative readings are correct which means that the shape and width of the edges have been correctly determined.

Figures 8.4, 8.5, and 8.6 on the following pages depict these actual edge traces of the same edge. The slit was slightly moved in a direction parallel to the edge after each trace. Smooth curve (A) in Figure 8.7 is the final product from the three scans. To arrive at this curve a separate smooth curve was drawn through each scan and these three curves combined through the use of a light table.

The remaining smooth curves are presented merely to demonstrate the edge differences which are apparent when using this technique.

Table 8.1 is a summary of the edges scanned. The column "A transmittance" is merely a listing of numbers which demonstrates the relative differences in transmittance. No units can be correctly applied to
Figure 8.4
First Microdensitometer
Edge Trace of Edge 1
Figure 8.5
Second Microdensitometer
Edge Trace of Edge 1
Figure 8.6
Third Microdensitometer
Edge Trace of Edge 1
Smooth curve A was Derived From the Three Previous Edge Traces. It Appears to be Nearly Symmetrical With a Width of Approximately 63 μm. Curves B, C, and D Were Obtained in the Same Manner as A. Note That Curves B and D are Obviously Unsymmetrical.
these numbers because the microdensitometer was not calibrated before
the scans were made. The information in Table 8.1 is meant for demon­
stration purposes only, but it would seem that the value 50-60 μm could
be quoted as a nominal width of a typical edge and it is also apparent
that not all edges are symmetrical.

Table 8.1

<table>
<thead>
<tr>
<th>Edge</th>
<th>Transmittance</th>
<th>Width (μm)</th>
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</thead>
<tbody>
<tr>
<td>1 (A)</td>
<td>4.6</td>
<td>63</td>
</tr>
<tr>
<td>2 (D)</td>
<td>2.3</td>
<td>48</td>
</tr>
<tr>
<td>3</td>
<td>5.1</td>
<td>54</td>
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<tr>
<td>4</td>
<td>2.6</td>
<td>63</td>
</tr>
<tr>
<td>5</td>
<td>3.8</td>
<td>56</td>
</tr>
<tr>
<td>6 (C)</td>
<td>2.6</td>
<td>54</td>
</tr>
<tr>
<td>7</td>
<td>3.6</td>
<td>54</td>
</tr>
<tr>
<td>8 (B)</td>
<td>2.6</td>
<td>48</td>
</tr>
</tbody>
</table>

Magnification - A Zeiss Ultrophot II (camera microscope) was used to
magnify the edges and to subsequently photograph this magnified image.
The camera used Polaroid 3000 film with a time exposure of approximately
5 seconds in duration. The format is 4 x 5 inches.

Photo 10 is the same edge that was traced with the microdensitome­
ter in Figures 8.4, 8.5, & 8.6. Photo 11 corresponds to edge trace D
in Figure 8.7. About all that can be learned from these photographs is
that grains and clumps of grains become very prevalent under high magni­
ification and that even sharp edges seem to lose their identity when
highly magnified.
Photo 10
Edge 1 Magnified 700 Times

Photo 11
Edge 2 Magnified 700 Times
C. Medical Reports on the Observer's Eyes

(The reports are listed in the order that the observers were numbered).
### Optometry Examination

**OPTOMETRY EXAMINATION**

**DATE:** 19 May 70

**HISTORY:**

**OPHTHALMOSCOPY-EXTERNAL:**

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<th>GLD RX</th>
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<td>-0.25</td>
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<tr>
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<td>-0.50</td>
<td>SPH</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**OPHTHALMOSCOPY-EXTERNAL:**

- NO LEAVERS

**REMARKS:**

**OTHER FINDINGS:**

- C.T. normal
  - Amy A.D.
Dr. S. K. Ghosh  
The Ohio State University  
1958 Neil Avenue  
Room 327 Cockins Hall  
Columbus, Ohio 43210

Re: Hank Hietkamp

Dr. Ghosh:

On January 20, 1972 Hank Hietkamp was examined at The Ohio State University College of Optometry Clinic. He was found to be a high myope, and was correctable to better than 20/20 in each eye. There was no evidence of eye disease or any restrictions upon the field of vision at the time of the examination.

This patient showed orthophoria at six meters. Normal single binocular vision was demonstrated both at six meters and forty centimeters.

It was recommended that Mr. Hietkamp wear the following prescription at all times:

R. E. - 8.25 - 1.25 x 15
L. E. - 8.50 - 0.75 x 170

A re-examination of Mr. Hietkamp's vision system is recommended within two years.

Sincerely,

Arb. R. Augsburger, O.D.  
Instructor O.S.U.  
College of Optometry

ARA/bjh
Dr. S. K. Ghosh
The Ohio State University
1958 Neil Avenue
Room 327 Cockins Hall
Columbus, Ohio 43210

Re: Sebastian Ekenobi

Dr. Ghosh:

On February 2, 1972 Sebastian Ekenobi was examined at The Ohio State University College of Optometry Clinic. Our examination procedures found this patient to be essentially emmetropic. Visual acuity in each eye was better than 20/20. There was no evidence of eye disease or any restrictions upon the field of vision at the time of the examination.

This patient showed orthophoria at six meters. Normal single binocular vision was demonstrated both at six meters and forty centimeters.

No spectacle prescription was recommended at this time. A re-examination is recommended within two years.

Sincerely,

Arol R. Augsburger, O.D.
Instructor, O.S.U.
College of Optometry

ARA/bjh
Dr. S.K. Ghosh
The Ohio State University
1958 Neil Avenue
Room 327 Cockins Hall
Columbus, Ohio 43210

Re: Chellapah Kanagalingham

Dr. Ghosh:

On February 2, 1972 Chellapah Kanagalingham was examined at The Ohio State University College of Optometry Clinic. The patient was found to be a very low hyperope. The visual acuity without spectacles, however, was found to be better than 20/20. There was no evidence of eye disease or any restrictions upon the field of vision at the time of the examination.

This patient showed 2 of exophoria at six meters. Normal single binocular vision was demonstrated both at six meters and forty centimeters.

No prescription was recommended for this patient at this time. A re-examination is recommended within two years.

Sincerely,

Arol R. Augsburger, O.D.
Instructor, O.S.U.
College of Optometry
APPENDIX

D. Relevant Data

At 6 meters (1" = 25.4 mm)

1 ER = .000025 inches
      = .000635 mm = .635 μm
      = .0218 seconds of arc

1 mm  = 34.3775 seconds of arc
      = 1575 ER

1 sec of arc = .029089 mm
      = .001145 inches
      = 45.8 ER

0.001 inch = 40 ER
      = .8752 seconds of arc

An "ER" is one digit as displayed by the Encoder Readout.
E. The Fourier Transform

1. Introduction

Webster says to "transform" means to change the outward form or appearance. Mathematically the word transform means the same thing, that is, the outward appearance of a mathematical expression is changed. The most general reason for changing the form of a mathematical expression is to enable the user to more readily solve a problem or to amplify a particular characteristic which may normally be somewhat obscure.

A very common transform is the logarithm. Given the problem of multiplying 359 by 582 we transform the outer appearance of the mathematical expressions into 2.55509 and 2.76492. This, of course, is done through the use of tables. Now following the rules for our transform, we add 2.5509 to 2.76492, arriving at 5.32001. This is the solution to the problem except it is still in the transformed mode so it is not as readily recognized. Finding the inverse transform (antilogarithm) we arrive at 208938, which is what one would find if a calculator or long multiplication were used.

The same basic principle is utilized when using, for example, the Laplace transform. The Laplace transform is found by following a set of rules (or using a table) just as is the logarithm. For example, given the problem of solving this differential equation,

\[ Y = Y' \quad (Y(0) = 1). \]
We first look up in tables the Laplace transform of both terms:

\[ Y(S) = SY(S) - Y(0) \]

or

\[ Y(S) = SY(S) - 1. \]

This is the simplified problem in our transformed system. Solving for \( Y(S) \) we get,

\[ Y(S) = \frac{-1}{1-S} = \frac{1}{S+1}, \]

which is the solution in our simplified system. Finding the inverse Laplace transform (from tables) we arrive at \( Y = e^Y \) which is the solution to the original differential equation.

This simple analogy between the frequently used logarithm and one of the many existing transforms was presented to underscore the fact that the Laplace, Fourier, etc., transforms are tools to be used to simplify a problem allowing the user to either more easily solve the problem or to present graphically some characteristic.

It is interesting to note that anyone could devise his own transform by establishing the rules for finding it. However, the key is that one must be able to find the inverse transform and arrive at the original function and more importantly, the inventor must demonstrate the usefulness of his transform.

2. History of the Fourier Transform

In the 19th Century many scholars were involved with the problems of heat conduction and the vibrating string. Mathematicians seemed to realize that if an arbitrary function could be represented by a series they would be able to solve these problems [32]. Euler, d'Alambert,
Bernoulli, and Lagrange were unsuccessful in their attempts although they were very close to the solution. [69]. It was Fourier in his *Théorie Analytique de la Chaleur*, published in 1822, who first demonstrated this fact. The amount of credit given him varies with the reference. The *Encyclopedia Britannica* states that Fourier discovered the fact that an arbitrary function could be represented by a series in 1807 [45]. Carslaw gives Lagrange credit with expressing a function as a series and limits Fourier's contribution to his expressing the coefficients of his series as integrals [32]. Fourier did demonstrate how his series could be applied to the heat conduction problem. His ideas were soon extended by Poisson, Riemann and Cauchy to such an extent and with such rapidity that it was forgotten from whence they come. Fourier, as his prestige increased, reminded the world from his seat in the French Academy of Mathematics [32].

The concept of the Fourier series was in existence for some time when, according to Titchmarsh, Cauchy made the substitution, 
$$e^{ix} = \cos x + i \sin x,$$
while working with the series and devised what is now called the Fourier sine and cosine transform [125]. This was indeed an important discovery. Tolstov stated that ....

This idea of reducing complicated mathematical operations on the original function to simple algebraic operations on its transform (and then taking the inverse transform of the final result) is the basis for the operational calculus, a very important branch of applied mathematics [128, p192].

Sneddon warns however, that "The use of finite Fourier transforms does not, of course, solve problems which are incapable of solution by
the direct application of the theory of Fourier series, but it does fa­cilitate the resolution of boundary value problems" [125, p72].

Having taken a brief look at the origin of the Fourier transform, it is now necessary to present mathematical definitions and become more specific. It is interesting to note that Laplace in 1812 published his Théorie Analytique des Probabilités in which he introduced the theory of well known Laplace transform [46]. Apparently he and Fourier worked independently which is intriguing because of the very close relationship between the Fourier and Laplace transforms.

3. The Fourier Series

Every function, f(x), which is quasi-differentiable and periodic may be represented by a Fourier series.

A function, f(x), is said to be quasi-differentiable in an interval [a, b] if we can divide [a, b] into a finite number of subintervals such that f(x) is differentiable in each open subinterval and the discontinuities at the edges of these subintervals are jumps [142]. If a function, f(x), is periodic with period T, then f(x+T) = f(x). Graphically, it is a function which continuously repeats itself. Oftentimes functions are simply defined to be periodic which does not subtract from the correctness of the series representation over the interval in which the function is defined [99].

If a function, f(x) is periodic, of period 2L, and quasi-differentiable it may be represented by a Fourier series which has the form

\[ f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L}) \]

where
One may wish to represent a function by a series simply to make it easier to handle. A function as simple as, \( \sin x \), cannot be evaluated exactly. By expressing it as a series this function may be calculated to any desired accuracy. The Fourier series is the first step towards the Fourier integral and transform.

4. The Fourier Integral and Transform

"The Fourier integral may be regarded as the formal limit of the Fourier series as the period tends to infinity" [87, p8]. Let the period of a function, \( f(x) \), be \( 2\pi \). Then

\[
f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)
\]

where

\[
a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx \quad \quad \quad (n = 1, 2, \ldots)
\]

\[
b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx.
\]

If \( f(x) \) is not periodic we can always look at this function as a function having the period \( \infty \). This function can not be represented by a Fourier series but if we impose the additional condition that,

\[
\int_{-\infty}^{\infty} |f(x)| \, dx < \infty,
\]

the function can be represented in a form similar to the Fourier series, i.e.,

\[
f(x) = \int_{0}^{\infty} \left[ a_n \cos nx + b_n \sin nx \right] \, dx
\]

where
\[ a_n = \frac{1}{\pi} \int_{-\infty}^{\infty} f(x) \cos nx \, dx \quad (n = 1, 2, \ldots) \]

\[ b_n = \frac{1}{\pi} \int_{-\infty}^{\infty} f(x) \sin nx \, dx. \]

This "... is one form of the so called Fourier Integral" [99, p147].

By using the well known relationships between \( \sin x \), \( \cos x \) and \( e^{ix} \) and following an exercise illustrated in various references [131, 99, 177, 125, etc.] the function, \( f(x) \) is made to exist in another form. Under the conditions that \( f(x) \) is quasi-differentiable and that,

\[ \int_{-\infty}^{\infty} |f(x)| \, dx < \infty \]

then the function,

\[ F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \, e^{-ixs} \, dx, \]

is called the Fourier transform of \( f(x) \) [141].

The Fourier transform is alternately defined as ....

\[ F(s) = \int_{-\infty}^{\infty} f(x) \, e^{-2\pi ixs} \, dx \]

or

\[ F(s) = \int_{-\infty}^{\infty} f(x) \, e^{-ixs} \, dx \]

or

\[ F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \, e^{-ixs} \, dx \] [16].

All three versions are in common use [16] and the only difference is in how the original function or inverse transform is defined. Respective to the above definitions of the transform we have the following definitions of the inverse transform:

\[ f(x) = \int_{-\infty}^{\infty} F(s) \, e^{i2\pi xs} \, ds, \]

\[ f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(s) \, e^{ixs} \, ds, \]

and

\[ f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) \, e^{ixs} \, ds \] [16].
Lighthill states that "... no general agreement has been reached on where the 2π's in the definition of Fourier transforms should be put" [87,p8]. He adds however, that all of the notations have advantages in some situations.

If the limit of the integral is not -∞ and ∞, that is, if the function, f(x), is defined over a finite interval, then the Fourier transform exists if f(x) is quasi-differentiable and if the integral,

$$F(s) = \int_{-L}^{L} f(x) e^{-ixs} \, dx,$$

exists.

4.1 Example of Fourier Transform

The following is an example of how the Fourier transform of a function, f(x), is obtained.

Let

$$f(x) = \begin{cases} x & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$F(s) = \int_{-1}^{1} xe^{-isx} \, dx = \left[ -\frac{e^{-isx}}{s^2}(-isx-1) \right]_{-1}^{1}$$

$$F(s) = \frac{ie^{-is}}{s} + \frac{ie^{is}}{s} + \frac{e^{-is}}{s^2} - \frac{e^{is}}{s^2}$$

But

$$e^{is} = \cos s + i \sin s.$$ Therefore ...

$$F(s) = \frac{2i}{s} \cos s - \frac{2i}{s^2} \sin s.$$

4.2 Cosine and Sine Transforms

The purpose of the following section is to illustrate what is meant by the cosine transform, $F_c(s)$, and sine transform, $F_s(s)$ which are frequently found in the literature.
Let
\[ F(s) = \int_{-\infty}^{\infty} f(x) \, e^{-isx} \, dx. \]

Making the substitution,
\[ e^{-isx} = \cos sx - i \sin sx, \]
we get ...
\[ F(s) = \int_{-\infty}^{\infty} f(x) \cos sx \, dx - i \int_{-\infty}^{\infty} f(x) \sin sx \, dx. \]

In other words \( F(s) \) has a real and imaginary part.
\[ F(s) = R(s) + iX(s) \quad \text{(if } f(x) \text{ is real)} \]

Now if \( f(x) \) is even (\& real) then \( F(s) \) will be only real because
\[ \int_{-\infty}^{\infty} \text{(odd function)} = 0 \]
and \( \sin x \) is odd and \( f(x) \) is even which makes the integral over an odd function and the imaginary part drops out.

Therefore,
\[ F(s) = \int_{0}^{\infty} f(x) \cos sx \, dx = 2 \int_{0}^{\infty} f(x) \cos sx \, dx. \]

The integral from 0 to \( \infty \) of \( f(x) \cos sx \, dx \) is called the cosine transform of \( f(x) \), i.e.,
\[ F_c(s) = \int_{0}^{\infty} f(x) \cos sx \, dx. \]

Similarly, the sine transform is defined as,
\[ F_s(s) = \int_{0}^{\infty} f(x) \sin sx \, dx. \]

In general terms, if \( f(x) \) is defined from \(-L\) to \( L\) and if the conditions for a Fourier transform exist and if \( f(x) \) is either odd or even then:
\[ F(s) = 2F_c(s) - 2iF_s(s). \]

4.2.1 Example of Cosine and Sine Transforms

Let
\[ f(x) = \begin{cases} x & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases} \]

Because \( f(x) \) is real and odd \( F(s) \) must be purely imaginary.
Bracewell lists the following transforms:

a) Fourier
b) Abel
c) Hankel
d) Hillert
e) Laplace
f) Mellin [16],

Each transform has a slightly different set of ground rules and was probably devised to fulfill a particular need.

Transforms are used to assist in problem solving in many areas, the following of which is an incomplete list:

a) Electrical oscillations in simple circuits
b) Equation of motion in a string
c) Heat conduction in solids
d) Motion of slow neutrons
e) Hydrodynamics
f) Radioactive transformations
g) Two dimensional stress [125]
h) Etc.

Webster defines a spectrum as an array of the components of an emission or wave separated and arranged in the order of some varying characteristic as wavelength, mass, or energy. The Fourier transform is a means of arriving at a spectral function, hence gives us the opportunity of examining the components making up some emission or wave [128].

For example, in electricity a pulse is broken down into its wave components.
Presently in photogrammetry, it is becoming popular to speak of breaking down the light distribution of an image to its wavelength components in an attempt to better understand what are the effects of various components on image quality. Fourier techniques have been used in optics for some time, but presently no one has conclusively demonstrated that they have any meaningful applications in photogrammetry. Research is being conducted in this area at the present time.
F. Computer Plots of the Observed Luminance Values

These computer plots are presented as an aid to the interested reader in determining the physical characteristics of the observed edges. The roughness of the graphs is a result of having the computer connect the individually plotted observed discrete points with straight lines.
FIGURE 8.8

EDGEO TRACERO OF EDGES
B-W 1, 3, 5

DISTANCE IN CM
LUMINANCE IN FT-LAM
FIGURE 8.30

LUMINANCE IN FT-LAM

0.00 10.00 20.00 30.00 40.00 50.00

0.00 1.00 2.00 3.00 4.00

DISTANCE IN CM

EDGE TRACE OF EDGES  
G-W 1, 3, 5
FIGURE 8.12
FIGURE 8.13
BIBLIOGRAPHY


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Since sending this work to the typist, the following relevant publications have come to the attention of the author.
