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LIFE INSURANCE PORTFOLIOS:  
A STUDY IN NON-LINEAR ESTIMATION

by SHAKIL AHMAD FARUQI

A thesis submitted to  
The Graduate School  
of  
Rutgers University  
in partial fulfillment of the requirements  
for the degree of  
Doctor of Philosophy

Written under the direction of  
Professor Manoranjan Dutta  
of the Department of Economics  
and approved by  

New Brunswick, New Jersey  
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ABSTRACT OF THE THESIS

Life Insurance Portfolios:
A Study in Non-linear Estimation

by SHAKIL AHMAD FARUQI, Ph.D.

Thesis director: Professor Manoranjan Dutta

This study reports on the estimation of econometric models, involving non-linearities associated with the parameters of lag distributions and autoregressive processes. A recent and a more elaborate version of scanning technique has been used to obtain maximum likelihood estimates of a given structural relationship, specified within the frameworks of independent as well as interdependent systems. The transformations associated with the dual non-linearities and employed in the reduction processes, are of a special nature, regardless of the system under consideration. The solution algorithm and its replication routine, likewise, is of a special type, and has been used in this study to ascertain the optimality of structural parameters.

In the applied context, a model has been estimated on the quarterly data of U.S. life insurance companies' portfolio for the period 1957-70. The structural relationships of the model consist mainly of the asset demand functions, obtained from the extensions of the conventional theory of decision making under uncertainty, with a special focus on the institutionalized managerial behavior. Each asset in the portfolio has been specified as a function of its rate of return, risk surrogate, and substitutability indices. These functions have been grouped into (1) an idealized model, consisting of independent relations, and (2) a simultaneous model, involving interdependent relations.
Two basic sets of estimates of the portfolio model have been obtained; (1) using classical procedures, viz, OLS, 2SLS, LIML methods, and, (2) the double scanned maximum likelihood (DSML) procedure. The latter involves lagged approximations of the expected variables in the system as well as corrections for autoregressive processes. It also requires replications on the admissible range of non-linear parameters to ascertain optimal estimates. The DSML procedure has been followed with a considerable degree of success in the estimation of the portfolio model of life insurance companies.

The results of the study suggest that the scanned estimates, in each specification of the portfolio model, are superior to their counterpart classical estimates, on the a priori considerations of the behavioral relationships, as well as on purely technical grounds. It has been established that the classical procedures are inadequate if in a model we are faced with expectational elements and autoregressive error terms; where the latter may be due to the nature of the data set or may occur as an implication of the approximations of the lagged variables.
PREFACE

My aim in this study has been to determine the feasibility of certain newly developed procedures of non-linear estimation. It has involved aligning the a priori constructs of the portfolio behavior on one end, with an operational system of estimation on the other. As expected, the transition from an almost pedagogical framework to a restricted, but accessible and operational scheme, has not been a smooth one. The procedures of estimation used in the study, however, represent an improvement over the commonly used ones as demonstrated by the results.

I am greatly indebted to Professor Manoranjan Dutta for his help and encouragement during my graduate work at Rutgers University. His instruction and guidance has been invaluable to me in the completion of this study.

I would also like to thank Professor Gerald Childs for his comments and suggestions. Thanks are due in addition to Professor Roger Hinderliter for his help in clarifying various issues in the management of portfolios and for serving on my committee.

I am indebted to Mr. Koji Shinjo of the Department of Economics, University of Pennsylvania, for helping me with the computer routine "Double Search", which has been used extensively in this study. The computations were performed at the Computer Center of Rutgers University. I am thankful for the financial assistance made available to me for the use of these facilities. Mrs. Ruth Agans and Mrs. Mildred Evans provided invaluable instructions in computer programming.

A special thanks is due to my friend, Mr. William Carmichael, for his painstaking assistance in editing the manuscript. Needless to
say, all errors and omissions are my own.

Finally, I am very grateful to my wife, Rebecca, who cheerfully put up with me during the many evenings that I worked on this study.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>I. THE ISSUES</strong></td>
<td><strong>1</strong></td>
</tr>
<tr>
<td>1.1 Non-linearities and Estimation</td>
<td></td>
</tr>
<tr>
<td>1.2 The Economic Setting</td>
<td></td>
</tr>
<tr>
<td>1.3 Nature of Constructs</td>
<td></td>
</tr>
<tr>
<td>1.4 Contents, Procedures, and Presentation</td>
<td></td>
</tr>
<tr>
<td><strong>II. ANATOMY OF A FINANCIAL INVESTOR, THE LIFE INSURANCE COMPANIES</strong></td>
<td><strong>8</strong></td>
</tr>
<tr>
<td>2.1 The Sub-Group</td>
<td></td>
</tr>
<tr>
<td>2.2 Generalizations of Investment Behavior</td>
<td></td>
</tr>
<tr>
<td>2.2.1 The Capital Security Portfolio</td>
<td></td>
</tr>
<tr>
<td>2.2.2 The Income Security Portfolio</td>
<td></td>
</tr>
<tr>
<td>2.2.3 The Competitive or Yield Portfolio</td>
<td></td>
</tr>
<tr>
<td>2.3 Life Insurance Functions and Operations</td>
<td></td>
</tr>
<tr>
<td>2.4 Distribution of Assets</td>
<td></td>
</tr>
<tr>
<td>2.5 Forward Commitments</td>
<td></td>
</tr>
<tr>
<td>2.6 Cyclical Factors and Portfolios</td>
<td></td>
</tr>
<tr>
<td>2.7 Liquidity Considerations</td>
<td></td>
</tr>
<tr>
<td>2.8 Monetary Controls</td>
<td></td>
</tr>
<tr>
<td><strong>III. A MODEL OF PORTFOLIO BEHAVIOR, SOME GENERALIZATIONS FROM THE CONVENTIONAL THEORY</strong></td>
<td><strong>35</strong></td>
</tr>
<tr>
<td>3.1 Preliminaries</td>
<td></td>
</tr>
<tr>
<td>3.2 The Framework</td>
<td></td>
</tr>
<tr>
<td>3.3 The Model</td>
<td></td>
</tr>
<tr>
<td>3.4 Some Extensions</td>
<td></td>
</tr>
<tr>
<td>3.5 Issues in the Theory of Decision Making Under Conditions of Risk</td>
<td></td>
</tr>
<tr>
<td>3.5.1 The Implications of Risk Aversion</td>
<td></td>
</tr>
<tr>
<td>3.5.2 Static and Dynamic Models</td>
<td></td>
</tr>
<tr>
<td>3.5.3 Institutional Portfolio Behavior</td>
<td></td>
</tr>
<tr>
<td>3.5.4 Implications for Institutional Investors</td>
<td></td>
</tr>
<tr>
<td>3.6 Comments</td>
<td></td>
</tr>
</tbody>
</table>
# TABLE OF CONTENTS (Continued)

<table>
<thead>
<tr>
<th>Chapter</th>
<th>SURROGATE RISK MEASURES</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV.</td>
<td>4.1 Preliminaries</td>
<td>54</td>
</tr>
<tr>
<td></td>
<td>4.2 Risk Surrogates in Mean-Variance Models</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.3 Indices of Investment Performance</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.4 Systematic Risk as Risk Surrogate</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.5 Risk Surrogate for the Life Insurance Portfolio</td>
<td></td>
</tr>
<tr>
<td>V.</td>
<td>LIFE INSURANCE PORTFOLIO MODEL</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>5.1 Preliminaries</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.2 The System: Variables and Equations</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.3 The Idealized Model</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.4 The Simultaneous Model</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.5 Some Comparisons</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.6 Comments</td>
<td></td>
</tr>
<tr>
<td>VI.</td>
<td>DISTRIBUTED LAGS, SOME THEORETICAL CONSIDERATIONS</td>
<td>87</td>
</tr>
<tr>
<td></td>
<td>6.1 Preliminaries</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.2 Lag Distributions</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.3 The Geometrically Distributed Lags</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.3.1 The Direct Expectations Model-- I</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.3.2 The Direct Expectations Model-- II</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.4 The Geometrically Distributed Lags, Some Extensions</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.4.1 Stock Adjustments of Rate Approximations</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.4.2 The Truncation Procedure</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.5 Simultaneous Estimation</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.6 Comments</td>
<td></td>
</tr>
<tr>
<td>VII.</td>
<td>ESTIMATION AND RESULTS</td>
<td>119</td>
</tr>
<tr>
<td></td>
<td>7.1 Preliminaries</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.2 A Note on Interpretations</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.3 Comparisons of Classical (OLS) and DSML</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Estimates of Singel Equatio, Idealized System</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.4 Comparisons of Classical (2SLS, LIML) and DSML</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Estimates of Simultaneous System, Model II</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.5 Life Insurance Portfolio Model, Abbreviated</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.6 Comments</td>
<td></td>
</tr>
<tr>
<td>VIII.</td>
<td>CONCLUSIONS</td>
<td>157</td>
</tr>
<tr>
<td></td>
<td>BIBLIOGRAPHY</td>
<td>160</td>
</tr>
<tr>
<td></td>
<td>APPENDICES</td>
<td>165</td>
</tr>
<tr>
<td>Table</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>----------------------------------------------------------------------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>1. Distribution of Assets of U.S. Life Insurance Companies (Percentages)</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>2. Percentage Distribution of Bond Holdings</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>4. Single Equation System, Model I</td>
<td>124</td>
<td></td>
</tr>
<tr>
<td>Ordinary Least Square Estimates</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Single Equation System, Model I</td>
<td>127</td>
<td></td>
</tr>
<tr>
<td>Double Scanned, Maximum Likelihood Estimates of Dynamic Model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Single Equation System, Model I</td>
<td>129</td>
<td></td>
</tr>
<tr>
<td>Comparisons of the Classical (OLS) and DSML Estimates With a Special Focus on the Non-linearities of the Dynamic Model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. Simultaneous System, Model II</td>
<td>136</td>
<td></td>
</tr>
<tr>
<td>Two Stage Least Square Estimates</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. Simultaneous System, Model II</td>
<td>140</td>
<td></td>
</tr>
<tr>
<td>LIML Estimates</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9. Simultaneous System, Model II</td>
<td>144</td>
<td></td>
</tr>
<tr>
<td>Double Scanned Maximum Likelihood Estimates</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10. Simultaneous System, Model II</td>
<td>147</td>
<td></td>
</tr>
<tr>
<td>Comparisons of the Classical (2SLS, LIML) and DSML Estimates With a Special Focus on the Non-linearities of the Dynamic Model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11. Single Equation System, Abbreviated Model</td>
<td>152</td>
<td></td>
</tr>
<tr>
<td>Comparisons of the Classical and DSML Estimates</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER I

THE ISSUES

1.1 Non-Linearities and Estimation

Non-linearities in econometric models have been primarily associated with the specification of distributed lags or autoregressive schemes; but whereas the latter depends on the assumptions of the stochastic processes of the structural error terms, the distributed lag scheme needs to be derived as an implication of some behavioral hypothesis. Invariably, these a priori rationalizations are concerned with the theoretical validity of a particular type of economic relationship, hence most often we encounter non-linearities embodied in the uni-equation models. Further, in most of the empirical studies, attention has been focussed on the estimation of models involving an autoregressive or distributed lag scheme, but implications of a model containing both types of non-linearities in a single relationship have largely remained unexplored.

Our concern here will be precisely with these issues. We will like to specify a set of interdependent stochastic relationships in which some or all of the equations would be subject to non-linearities generated by geometrically distributed lags or first order autoregressive scheme or both. For estimation purposes, we will need to linearize each member of the system by suitably defined transformations whose structure will largely depend on the character of the lag scheme. We shall discuss the transformations at an appropriate stage, however, since these are
transitional entities between states of a given model, they have considerable implications for the distributional aspects of the stochastic processes and ultimately for estimation procedures.

As a first approximation, the model will be estimated by the ordinary least squares (OLS), the two stage least squares (2SLS), and limited information maximum likelihood (LIML) methods. Since these procedures are unsatisfactory considering the special characteristics of the model, we shall use scanning technique to obtain the estimators of structural parameters. It should be noted that this is not an iterative procedure, rather it is a more comprehensive scheme to ascertain the optimal values of the parameters involved—the optimal being defined in a global sense with reference to some operational criterion. It requires essentially a search over the admissible range of values non-linear parameters both for the single equation and the simultaneous systems.

Thus in the following we hope to establish the operational feasibility of certain relatively new estimation techniques which promise us better estimates of non-linear systems. Within the general class of rational distributed lag models, emphasis will be placed on the formulation and estimation of the geometrically distributed lags, its variants and applications. Though for the purposes of this dissertation we will concentrate on geometric lags, we shall explore the possibility of using other lag schemes as well. Some new results will be discussed embodying significant implications for the estimation procedures.

1.2 The Economic Setting

In the applied context we will consider investments in financial assets by U.S. life insurance companies. Our objective is to analyse the underlying factors which determine the composition and size of portfolio
balances of this financial group, within the general framework of existing micro-monetary models. The central proposition is that the decision makers seek to obtain a portfolio which is consistent with their expectations and which reflects in some manner their immediate experience with the prevailing economic conditions.

Following two considerations are pertinent in this regard. First, a glance at the portfolios of all financial institutions in the U.S. reveals that the combined assets of non-bank institutions are substantially large. For example, the total assets of life insurance companies rank second only to the holdings of commercial banks, and the total assets of savings and loans associations are claimed to be larger than the total assets of major industrial corporations in United States. In view of these facts it is naive to presume that the intra-institutional flows of funds proceed without regard to changing economic conditions and are neutral with respect to changes in a given portfolio balance within the sector. Specifically, during periods of economic stress a restructuring of portfolios is likely to occur among institutions which place a premium on liquidity. This necessitates a rigorous analysis of the present conditions and future directions not only of firms and their competitors, but also of the general economic conditions and of their likely impact on these institutions.

Secondly, the revived interests in monetary mechanisms has led to the proliferation of material both in theory and in application. On the theoretical level, some excellent work has been done in portfolio analysis as it relates to the general considerations of utility maximization under uncertainty, and also with specific reference to the structure of portfolios, given the postulated validity of axiomatic managerial
behavior. But on the empirical level, only a beginning has been made in analysing the behavior of financial intermediaries as reflected in their response to changing economic conditions in general and selective monetary controls in particular. Obviously, from the point of view of policy making, a knowledge of transmission mechanisms will be of considerable practical significance.

1.3 Nature of Constructs

The intricacies of the problem stated above require the construction of a model of portfolio behavior which will have solid foundations in the theory of risk behavior with explicit recognition of expectational relationships, and will be operationally significant. Ideally, the latter should not be a motivation in a given specification of the model, but the complexity of estimation problems requires a careful consideration of this aspect. A theoretically specified model will be operationally significant if only its important parameters can be estimated and some meaningful inferers can be obtained.

Accordingly, an attempt will be made to derive an asset-return relationship based on extensions of the conventional theory of decision making under uncertainty. This will satisfy the imperatives of obtaining the distributed lag scheme as an implication of some behavioral hypothesis. The stochastic version of the function will involve geometrically distributed lags since the functions contain expectational variables which can not be rationalized otherwise. We shall estimate these relationships on the portfolio data of U.S. life insurance companies over the sample period 1957-70. In the past, attempts were made to estimate financial models involving portfolios, but invariably the drawback has been lack of consistent a priori constructs and inadequate
or outmoded estimation procedures. Whereas no claim is being made here with respect to the theoretical validity of portfolio analysis beyond that it represents a consistent scheme, it is hoped that estimation procedure used here represents a substantial improvements over commonly used techniques.

1.4 Contents, Procedures, and Presentations

In Chapter II we shall offer some insights into the mechanisms of financial operations of U.S. life insurance companies. Specifically, a description of the processes of accumulation of assets and liabilities will be given with an emphasis on the various types of background considerations of life insurance fund managers with regard to 'desired' portfolio balance. We shall also examine the distribution of total investment funds into various asset categories over the sample period---the so-called allocative behavior of financial institutions---and its implications for the processes of intermediation. A brief discussion on forward commitments would be offered which are typical to life insurance companies. We will also have a look into the arrangements of cash inflows and outflows and will examine the possible effects of cyclical factors, liquidity considerations, and monetary controls on these flows.

In Chapter III we will be concerned with the theoretical aspects of the portfolio behavior, involving a rigorous analysis of decision making with regard to investment in risky assets. A choice-theoretic approach will be adopted to specify a micro-monetary model of portfolio management, mainly in terms of individual securities and their associated attributes. A quasi-demand function for securities will be derived from constrained maximization processes and rationalised for life insurance portfolio. A substantial part of the Chapter will be
devoted to the discussion of problems in the area of portfolio analysis, their bearing on our model, and the resulting implications. Finally, we shall consider alternative risk surrogates and examine their operational suitability for estimation purposes.

The models to be estimated will be discussed in Chapter V. But this discussion will be preceded by consideration of various measures of riskiness associated with investment in financial assets which will be the subject matter of Chapter IV. For the life insurance portfolio, we would like to obtain an operationally feasible measure of risk defined preferably for each of micro-relationships specified in the model. Of course, it is not possible for a single measure to characterize all the dimensions of risk hence we will have to select a relevant risk attribute of life insurance portfolio and proceed accordingly.

Essentially, we shall consider two kinds of model containing a set of asset-return relationships specified for each type of securities: (1) the idealised model consisting of independent relationships, and (2) the simultaneous model, involving interdependent systems. The variables and equations of each model will be defined with respect to their economic content and later the two models will be compared with some existing ones with respect to their specifications.

In Chapter VI a rigorous treatment of estimation problem will be offered in the framework of uni-equation and simultaneous models. In particular, we will like to establish the accessibility of the scanning method for both the systems, taking for granted the econometric validity of estimated coefficients. Main emphasis will be on geometrically distributed lag models, their variants, and their implications for the underlying hypothesis. The computer analogue of solution algorithm will
also be discussed in view of synthetic variable generated during estimation.

Finally, in Chapter VII we shall discuss the results of the estimated models. Basically, it will consist of two sets. The first one among them will correspond to the direct estimation of models by OLS, 2SLS, and LIML methods. The second will be based on the scanning procedure to take account of the non-linearities arising from the specification of the two types of models. In Chapter VIII some comments will be offered on the whole exercise.
Financial institutions differ widely in terms of their asset acquisition patterns which requires a careful consideration of the special characteristics of their investment behavior. This diversity, in part, is attributable to the fact that the liability contract between a depositor and the institution exerts an independent influence on the institution's demand for securities. For each financial institution the nature of the liability contract by which the funds are received is different, hence each of them has somewhat a different demand function for securities. The life insurance companies are a specialized type of financial institution not only as regards the nature of their liability contracts but also from the point of view of their asset holdings. In what follows we shall present a brief description of these aspects along with an outline of the modus operandi of the sub-group.

To begin with, it should be noted that the total assets of life insurance companies rank second only to the assets of commercial banks, hence their collective response to the changing economic conditions could have considerable implications for the stability of the remaining sectors in the economy. It has been generally assumed that their regular inflows and outflows of funds proceed normally without regard to such changes, but recent developments indicate a greater degree of
sensitivity on the part of the life insurance companies to economic fluctuations.

The interlinks underlying these relationships are to be found in the analysis of life insurance portfolio. We need to examine the factors which determine the composition of a given portfolio, the new acquisitions, and in general the overall investment policy of portfolio managers. Hence, our objective is the elucidation of a rather modicum of managerial experience in the context of real world phenomena involving uncertainties and other imperfections in the financial markets.

2.2 Generalizations of Investment Behavior

Whereas the life insurance fund managers confront an identical opportunity set in the sense that their choice is restricted to the available types of securities in the market, their individual portfolios may exhibit considerable variation. This is because (1) managers may visualize the needs of their companies differently, e.g., for some yield may be more important than risk, etc., or (2) preference orderings may not be the same, or (3) even if the preferences were the same, there are alternate ways of obtaining the same basket of attributes in different portfolios. These considerations have led to the several generalizations of investment behavior of life insurance companies.

2.2.1 The Capital Security Portfolio

The capital security viewpoint elevates default risk to the paramount position leading to an overall defensive posture in security acquisitions. If the management regards the life insurance function primarily as that of a trustee, the investment behavior of firms is
likely to follow the maxims of trustee investment, viz, first security; second, yield; and third, liquidity. Increased yield does not necessarily compensate for increased default risk. The risk of unexpected cash drain is relevant but is subsidiary to default risk. In short, this view treats the guarantee of future capital security as the essential guarantee of life insurance contract and hence is the primary objective of life insurance investment.

2.2.2 The Income Security Portfolio

In accordance with this viewpoint, emphasis is to be placed on the guarantee of long term yield. This implies that income risk is the most important aspect of investment in securities; hence, the firm should try to insulate the portfolio from any unfavorable effects of future movements of interest rates. One way to minimize the income risk would be to purchase long maturities which in some sense match the maturities of the liabilities of the policy contracts. Notice that the guarantee of the long term yield is not necessarily consistent with the objective of earning higher yield, specially, if the latter involves higher risks of insolvency. Avoidence of income risk is the appropriate objective for this portfolio.

2.2.3 The Competitive or Yield Portfolio

Proponents of this view emphasize the yield aspect of the portfolio on two grounds. Firstly, they observe that most of the risks associated with life insurance investments are avoidable, and that excessive efforts devoted to reducing these risks are not only unrewarding but dangerous in the sense that they divert attention from the possibility of increasing the yield. Secondly, for safeguarding the
the competitive position of the firm within the industry it is necessary to emphasize increased yield by maintaining a flexible approach towards investments.

Several methods of increasing the yield are available. Some of these are:

1. Following the maturity yield function, choose securities from among the entire range of maturities, and/or from maturities longer than 10 or 15 years.
2. Form expectations of the cyclical patterns of business activity and of interest rates, then invest accordingly. If this policy is pursued, the life insurance companies will go 'short' in low interest periods, i.e., sell currently high priced long maturities; and go 'long' in high interest periods, i.e., sell short maturities.
3. Form expectations of the average level of the long term interest rates over an extended future period and go 'short' when the current long rate is less than the expected long rate; and go 'long' when the current long rate is above expected long rate.

Besides purchases which increase yield, it is possible to make sales which increase yield. In any case, the competitive portfolio stresses the importance of increased yield while not ignoring the risks associated with this process.

2.3 Life Insurance Functions and Operations

The primary function of life insurance companies is to provide family financial protection and this, to a large degree, explains the general lack of volatility of inflows of funds. Their role as a financial investor is at best secondary, though quite significant considering
The life insurance function could be adequately achieved through the sale of 'term insurance' to individuals, which entails no savings on the part of the purchaser. Protection is purchased for a specified period of time and if the individual dies within that period, his beneficiary receives the specified payment. The charge for this primary service of life insurance companies is low relative to the cost of premiums for plans under which the policy holder acquires a cash reserve. The latter type of plan is termed as 'whole life' plan. The cash reserve mounts in proportion to the premium paid. Since the policy holder can at any time surrender his policy to obtain the cash value of the policy at a fixed rate of interest, life insurance companies may assume some characteristics of lending institutions.

Policy holders do not buy insurance with the intent of saving or borrowing from life insurance company; the options serve as additional inducement to purchase whole life rather than term insurance. Both parties to the transaction regard a term or whole life policy as being of a long term character guaranteeing a highly stable flow of funds to and from the company. The whole life policy, by providing alternatives to the policy holders otherwise unavailable, carries a sufficiently higher premium to more than offset the cost of additional services offered; and is favored by the insurance companies over the term insurance. Although the return to the policy holder on his savings under a whole life plan is less than he might obtain from most saving institutions, the interest rate charged him for borrowing against his policy is similarly less than would be levied by most lending institutions.

The policy premium and interest rate charged for borrowing
against the policy are set for the life of the policy. Factors beyond the control of the life insurance companies, such as changing economic conditions, changing propensities to save, varying market interest rates, or changes in disposable income, could encourage policy cancellations or policy borrowing and disrupt the companies' well planned flow of funds. Since the companies consider their liabilities to be long term obligations, they channel most of their funds into long term high yield loans and investment. If, however, economic conditions arise such that a substantial portion of their funds' inflow is disrupted, they might encounter serious destabilizing effects and face difficulties in meeting their long term commitments. To prepare for such contingencies, the managers must analyze the present conditions and future directions of not only their company and their competitors, but the general economy as well. If the companies set premiums too high, they may lose potential customers, but if they set rates too low in relation to future economic developments, they may face policy cancellations which could prove equally disrupting.

During periods of non-economic stress, liquidity may not be as important a consideration in structuring a portfolio as other times. Since knowledge of future economic conditions is not perfect, life insurance companies like all other financial institutions, must hedge against the time when circumstances elevate the importance of being liquid. Life insurance companies maintain liquidity by holding cash and federal government securities. Although these securities provide the companies some return on their investment, they are held less for yield purposes than liquidity. The segment of life insurance asset which is held primarily for yield consists of mortgages, business
bonds, and government bonds issued by almost all levels of governments including foreign.  

The life insurance companies are a dominant supplier of funds to corporate bond market in as much as they hold about one half of all corporate long term debts outstanding. They have also moved into private placements, an arrangement by which an institution or group of institutions make a loan in security form. Because terms agreed upon between borrowers and lenders are more flexible than if transactions were registered with the Securities and Exchange Commission, life insurance companies have found such arrangements increasingly profitable. This sort of financial technique is also well suited to the companies' future commitments' practices since they are able to match their stable inflows with investment outflows often on a staggered schedule of take-downs. By minimizing the length of the time their inflows remain idle or in low-yield, or highly liquid assets, the companies maximize return. The borrowers, correspondingly, appreciate the fact that they are guaranteed funds at a specified rate of interest regardless of changes in the capital market between the time the loan is closed and the period the funds are extended.

Life insurance companies compete primarily with the large private, state and local pension funds for such investment outlets, as well as competing with these and other financial intermediaries for funds' inflows. Although increased life expectancy has contributed to the demand for services provided by the life insurance companies, other financial institutions have organized to meet the rising demand and have flourished to the extent that they have slowed the once rapid growth of the life insurance companies.
2.4 Distribution of Assets

Financial investment by the life insurance companies over the years 1920-70 have exhibited some distinct characteristics of their own. In Table 1 we present an overall view of the distribution of investment funds in terms of proportions of assets held. Traditionally, the largest concentration of investment has been in corporate securities and mortgages, except for the years 1940-45. These two categories together account for nearly two thirds of entire assets of the life insurance companies, specially during the post-war years. On the other hand, government securities have shown a secularly declining trend which has continued during the recent period. The proportion of governments has plummeted down to 5.3% in 1970 from an all time high of 50.3% in 1945. This represents a substantial reduction in governments in the life insurance portfolio, specially in view of the fact that there has been a continued increase in the volume of public debt. Except for the World War II years, government securities have remained a small portion of the life insurance portfolio.

Equally significant is the substantial increase in the proportion of stocks held--it has nearly tripled during the post-war years. Frequently, this is attributed to the continually bullish market except for the brief recession of the late fifties. Policy loans have also increased over these years, but much of this increase occurred during the liquidity crunch of the late sixties. If we delete the last four years of our sample period, the portion of policy loans turn out to be stable around 5%. As regards miscellaneous assets, their share has remained constant, more or less, for the entire period.

A further breakdown of total bond holdings, both corporate and
TABLE 1

DISTRIBUTION OF ASSETS OF U.S.
LIFE INSURANCE COMPANIES (PERCENTAGES)

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<tr>
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Notes:
Source: Life Insurance Fact Book, 1971
Key to the column headings: t, t-th year; 1, government securities; 2, corporate securities; 3, mortgages; 4, real estate; 5, policy loans; 6, miscellaneous assets.
governments, shows a relative preponderance for industrials in this category.\textsuperscript{12} (Table 2) The proportion of industrials has been steadily rising, though the rate of increase has slowed down during the post-war years. Virtually all of the holdings are in North American corporations. For example, in 1969, 93\% of the corporate securities held by the life insurance companies were of U.S. corporations, 6\% of Canadian corporations, and the insignificant remainder of foreign corporations. Further an analysis of industrial bond holdings in 1965 revealed that over one half of these investments are in manufacturing industries, and the remainder in the bonds of non-manufacturing concerns.

From 1920 to 1950 the proportion of life insurance investments showed a generally rising trend. Since 1950, however, there has been a gradual decline; from a peak of 16.5\% in 1950 the proportion fell to 9.1\% in 1969. In spite of this decline, public utility bonds continue to represent an important part of the life insurance portfolio. The majority of these holdings consist of the bonds of U.S. utilities with the remainder in Canadian public utilities.

A long term shift in bond holdings has resulted in substantial changes in the overall portfolio of the life insurance companies, as is evident from Table 2. For example, during the World War II years, the life insurance companies had increased the portion of their funds invested in government bonds at the expense of all other types. These bonds represented nearly 46\% of total assets towards the end of 1945, but in 1969 the proportion had fallen to 2.1\%.

In view of the significance of the monetary action let us examine in some detail holdings of government securities. As we noted
### TABLE 2

PERCENTAGE DISTRIBUTION OF BOND HOLDINGS

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Notes:

Key to the column headings: t, t-th year; 1, U.S. government bonds; 2, foreign government bonds; 3, state and local government bonds; 4, railroad bonds; 5, public utility bonds; 6, industrial and miscellaneous bonds. Proportions refer to total assets held in t-th period.
earlier, these assets are primarily held for considerations of liquidity than yield or income—a hedge against unforeseen changes in the liquidity position. At the end of 1970, government securities were 5.3% of total life insurance assets. A glance at Table 3 will reveal that except for the years 1940-45, life insurance funds have ordinarily been invested in the private sector. The detailed breakdown of this category is as follows.

Securities of U.S. government include Treasury securities, state and local government securities, and Federal Agency securities. Treasury securities have been the largest component in this category, though in recent years the proportion of state and local governments has grown substantially large. For example, in 1970, Treasury securities constituted 36% of total government securities whereas the share of state and local government securities was 30%, and of federal agency securities was 5 per cent.¹³ A substantial part of the investment was in securities of Canadian government and international agencies. In 1970 these foreign investments amounted to nearly 29% of the total investment in governments.¹⁴

2.5 Forward Commitments

The life insurance companies maintain one of the most diversified portfolios among the financial institutions, given the legal and traditional constraints on their operations. In the past changes in the portfolio composition have been mainly due to long run factors; recently, however, the life insurance companies have exhibited increased vulnerability to cyclical factors. This could lead to serious repercussions considering the fact that life insurance companies are probably not as well equipped to cope with the large economic...
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**Notes:**

Key to the column headings: 1, treasury securities, U.S. government; 2, federal agency securities; 3, state and local government securities; 4, foreign government securities.

The total of these four columns has been used to arrive at the proportions of government securities in total assets reported in column #1 of Table 1.
fluctuations as some other financial institutions.

The reason for this is to be found in the nature of their forward commitments. Relying on normal stability of their flow of funds, life insurance companies make commitments to borrowers up to three years in advance of the actual loan extension. The duration of forward commitment generally varies directly with the size of borrowings. Since these commitments bind the life insurance companies to lend a certain amount of money at a set rate for a specified period, the company makes commitments decisions on the basis of interest rates and state of economic activity prevailing three years before funds actually flow from company's portfolio.

Another source of relative stability of aggregate life insurance portfolio is the adversity of opinions of life insurance fund managers about the future direction of the economy. Institutions which do not utilise forward commitments react more to changes in the present state of economic activity than future expected states. Although knowledge of the present directions of economic activity is not perfect, such knowledge should be more accurate than the knowledge of economic activity two or three years hence. Through different expectations of future developments the life insurance fund managers alter their portfolios in a rather non-concurring fashion; but the effects of such a response, based as it is on life insurance transactions in futures is to help smooth the fluctuations in total assets.

2.6 Cyclical Factors and Portfolios

The short run factors responsible for causing changes in portfolios can be generalised in terms of cyclical fluctuations in economic activity. From the point of view of financial institutions as a
whole and the life insurance companies in particular, the best indicators of these changes are certain key market rates. These rates may fluctuate as a result of discretionary monetary policy, or in normal circumstances, the fluctuations may be induced by underlying economic forces. In general, on the liability side, interest rate variation, specially changes in bank loan rates, create disturbances in the flow of premiums; on the asset side, variations in the rates of return of competing assets are the major source of re-adjustments.

Further, changes in the level of income, while a major indicator of economic activity, are not very relevant to portfolio decisions--at least not in a direct manner. With a general increase or decrease in income, a priori, it is expected that life insurance liabilities will not change as much as liabilities at a predominantly savings institutions, because at the present levels of income in U.S. the marginal propensity to save is higher than than the marginal propensity to secure insurance. Once a whole life policy is secured, premiums become fixed payments for the policy holders while funds placed in or taken from savings institutions generally remain more flexible. Savings and loans share, for example, may be postulated to be more income elastic than life insurance policy reserves. Similarly, consumer expenditures on durables are more income sensitive than payments of insurance premiums. Hence, income variations are only remotely effective on the portfolios of life insurance companies.

Within the legal, traditional and supply constraints, the response of most life insurance companies to yield differentials is not very different than the response of any economic entity. Assuming that the elasticity of supply of a given asset is infinite for a
single firm, the fund managers examine present and future expected yields, relative risks, marketability, and tax factors before making commitments. The differential in nominal interest rates has not changed over the years, but a secular shift among assets due to factors other than nominal rates has occurred.

For example, the previously noted decline in the holdings of railroad bonds may be attributed to risk and marketability factors while the increased attractiveness of the industrials relative to public utility bonds is somewhat more complex to analyse. Many industrials have convertible features not found with the public utility bonds as well as tax benefits accruing to holders of certain preferred industrials. Apparently, the tax benefits concomitant to state and local bonds have not changed much relative to their yields in recent years, and this may explain the relative consistency of its proportion or in other words, the neutrality of cyclical influences.

To meet expanding competition, the life insurance companies have found investments in stocks of private corporations to be one of the most promising. Since yields on certain stocks are potentially higher than alternative uses of funds, it is not surprising that the companies have recently decided to organize their own mutual funds for stock investment purposes. Although the legal constraints to stock holdings were relaxed by New York State law, most companies have been slow to maximize the use of stock investment of opportunities due to risks of stock trading. There is little indication, however, that life insurance stock investment follows cyclical patterns except in so far as the stock market itself responds to capital market movements. Neither total assets, mortgages, nor corporate bonds demonstrate any
significant cyclical movements in response to generally rising or falling interest rates. All three have exhibited steady secular increase over the past fifteen years.

2.7 Liquidity Considerations

The life insurance companies have recently been concerned with their liquidity position more deeply due to a re-apportionment of their investible funds between different assets, specially the treasury obligations and the policy loans. Treasury securities constitute one of the reserve sources of funds to the basic cash flow of the life insurance companies. Outflows, which occur when commitments are taken down, must be met through mortgages, normal security inflows, etc. If fund inflow falls short of expectations because mortgage repayments, for example slacken, or if fund outflow exceeds expectations due to increase in some factors such as policy loans, new funds must be generated by the sale of securities or bank borrowings. Declining amounts of treasury securities in the portfolio for the past 20 years reflect the reduced liquidity positions on which the life insurance companies have come to operate. In this respect they have followed a trend common to other financial institutions who have become increasingly vulnerable to liquidity crisis. Such institutions must cut back their commitments to the borrowers until their cash position improves. The borrowers in turn, seek alternative sources of funds or curtail their activities. A chain reaction working through financial intermediaries such as life insurance companies or savings and loans associations to such credit sensitive industries as commercial construction or home building occurs.
The impact of the liquidity crunch on life insurance companies can be seen in terms of their schedules of new commitments. In 1966 the new commitments were more than halved, falling from an annual rate of over 15 billion dollars to approximately 6 billion dollars—an unprecedented decline in the post-war era. A small but unanticipated change in policy loans from 4.8% of all assets in 1965 to 5.5% in 1966 was responsible for much of cash flow disruptions. During the years 1967-70 these changes became all the more pronounced leading to substantial re-evaluations of policies at life insurance companies.16

The rise of policy loans has been seen as one important manifestations of disintermediation.17 In its purest form, in case of policy loans, disintermediation phenomenon is the interest arbitrage differential that develops as market rates of interest rise to the level substantially above the contractual policy loan rates. The actual impact of enlarged policy loans on investment and forward commitment process has in fact become a major problem since late sixties. In 1969 this increase accounted for 17% of cash flow available for investment as against only 4% in 1965. In terms of magnitudes, the policy loans had doubled in 1969 over 1968.

Normally, the size of loans have not been of any substantial amount but there is some indication of a trend toward large loans to either individuals or others holding large policies. From the policy loan data of 1965-69 it appears that a number of 5% policy loans were requested by policy holders in order to repay other loans bearing higher interest rates.18 Even the 5% rate charged by the insurance companies is effectively diminished by the costs of administration of the loan. Further, the companies in extending the 5% loan
must forego opportunities to obtain 6% or more on investment, in addition to the loss incurred by liquidating 6% securities already owned to make the policy advance.

The close relation between policy loans and market interest rates can be observed by examining changes in policy loans against interest charged by banks on short term business loans between 1957 and 1970. It can be shown that a close positive relationship exists between the two variables although the bank loan rates for earlier years is less than 5%. It must be remembered, however, that the rates displayed are close to prime rates to good customers while rates to individuals would be at least 1 to 2 per cent higher. For life insurance companies, the level of market interest rates when near of above 5% is probably as significant a factor as changes in market rates. This explains why policy loans, as a percent of all life insurance assets, made sizable increases in the predominantly rising interest rate years and continued to climb steadily throughout the 1960's, culminating into exceptional increases during 1966-1970.

Although policy loans involve no risk, in the usual sense, for the insurance companies, since failure to repay a loan merely signifies cancellation of an equal liability, insurance officials discourage such loans because of their disruptive influences on basic cash flows. Insurance agents, however, often utilize the policy loans as a selling point to obtain new customers. Recent developments in the insurance field give the policy loan feature, perhaps unintentionally, the appearance of a loss leader.

In periods when the economy is booming there is generally greater demand for loans from all lending institutions. Rising demand
is met with a reduced supply of loanable funds as stabilization policies force most institutions to curtail their supply of loanable funds. The spillover of the excess demand falls heavily on life insurance companies, who like most credit unions, maintain moderate fixed rates of interests. Further, due to rising interest rates in such periods, life insurance companies have found themselves no longer insulated from changing economic conditions. Apparently, the relevant change of causality runs from high interest rates to policy loans to cash flow to curtailment of new commitments. Consequently, policy loans exercise influence far beyond their magnitude, on life insurance companies' commitments to credit seeking industries. New commitments were curtailed slightly in 1960, but more than halved in 1966, thus augmenting the credit crisis of 1966 and the subsequent recession. Some reforms of life insurance liquidity in general and policy loans in particular, appears necessary to avert detrimental effects on sectors which obtain credit from life insurance companies, policy holders and the companies themselves.

This brings us to the considerations of monetary controls on the non-bank financial intermediaries, particularly, the life insurance companies.

2.8 Monetary Controls

Three characteristics of life insurance investment are pertinent to the consideration of the influences exerted by monetary policy. First, since the life insurance companies invest their funds in a wide variety of outlets and maintain a highly diversified portfolio, there are several points at which Federal Reserve policy actions can and do exert their influences. Second, it is noteworthy that the bulk of life
insurance investment are placed in long term, fixed income obligations hence, wide swing in market price of these obligations are likely to occur due to changes in long terms interest rates. This feature of life insurance portfolio hampers the ability of the firms to convert existing assets into cash if such an action is needed in view of changing economic conditions. Third, as we pointed out before, the life insurance companies are quite sensitive to changing differentials in investment yields which makes them susceptible to discretionary monetary controls.

General credit controls, as distinguished from the selective credit controls, affect life insurance companies in various ways. These controls may:

(a) influence the direction of insurance investment, e.g., during the period of credit ease, mortgages are encouraged; but in periods of credit restraint corporate bond investment are encouraged;

(b) change the rate of interest, and thus affect the capital value of assets held by the life insurance companies, which in turn may affect the liquidity position of the companies;

(c) influence the total cash flow for investment through the effects of interest rate changes on policy loans and mortgage payments;

(d) affect the forward commitment policy of the companies through the influence exerted on investor expectations of the interest rate movements. For example, when the general monetary action contributes to the expectations of rising interest rates, life insurance companies tend to be less willing to expand their forward commitments; the reverse being true when policy actions lead to expectations of falling interest rates.
In similar fashion, federal debt management operations affect the life insurance investment, i.e., they cause shifts in direction of the flows of funds, affect the capital value of assets held by life insurance companies, lead to changes in liquidity position and affect the forward commitment policies.

Besides the general controls, there are a number of selective controls administered by federal agencies of states. For instance, the government insurance and guarantee of residential mortgages; the regulation by Securities and Exchange Commission of redemption features in bond issues of utilities, specially their insistence that bonds be immediately callable at call prices; and specific tax regulations.

Of all these measures, monetary controls, which affect directly or indirectly the market rates of interest, are more significant. The life insurance companies, being a vital part of supply of long term funds, are very responsive to a policy oriented monetary ease or restraint. For instance, if monetary authorities move to tighten the reserve position of the commercial banks and the general availability of bank credit, the life insurance companies become less willing to assume fully committed position with respect to the cash flow, since they expect interest rates to rise. Further, as the yield on corporate bonds rises, both on new offerings and direct placement, life insurance companies are induced to invest in corporate bonds and shift away from government insured mortgages where interest rates are rigid. During period of monetary ease, the life insurance companies are less reluctant to allow cash outflows as they expect rates to decline. Investible funds are then channeled into residential mortgages and away from corporate bonds where yields are declining.
The cash flow of life insurance companies from mortgage repayments is increased and refunding of debt at lower interest rates occurs.

The foregoing analysis is a direct repudiation of Gurley-Shaw thesis which suggests that monetary authorities have direct control over the process of credit creation by commercial banks and some control over the volume of time deposits through interest rate control mechanism, but they have little control over the non-monetary financial intermediaries, such as life insurance companies, mutual savings banks, and savings and loans associations, to sell their liquid claims for money and thus to add to the supply of close substitutes of money in the hands of public. Likewise, it is argued that monetary authorities have no control over the volume of loanable funds of these institutions. Concerning the life insurance companies, the authors of LIAA monograph observe:  

"Gurley and Shaw thesis has little applicability to life insurance companies for a number of reasons. One is that life insurance policy reserves are not regarded as liquid assets by the great majority of policy holders and are not close substitute for money. Secondly, the life insurance companies are not deposit institutions, a key assumption made in application of the thesis...Thirdly, life insurance companies have characteristics which differentiate them importantly from deposit institutions."

Besides, they argue that life insurance companies are affected by discretionary monetary policy through various money market mechanisms, and their participation in financial dealings can be regulated the same way as the operations of the banking institutions. These intermediaries do exert a stabilizing effect, if we mean by that primarily a prompt response to restrictive monetary activity. Some life insurance economists, however, do recognize the disintermediation potential—adverse to the stabilizing influences of monetary activity.
actions-- as exemplified by the policy loan experience of the life insurance companies during 1966 and again in 1969. These observations re-inforce the need to investigate the precise linkage mechanisms between monetary policy impulse and institutional response.
Footnotes

1. Wehrle has pointed out that these viewpoints are not new; in fact an interesting discussion on these issues is available in British actuarial journals. Leroy S. Wehrle, "Life Insurance Investment; The Experience of Four Companies," in Studies of Portfolio Behavior, ed. by Donald Hester and James Tobin (New York: John Wiley and Sons, Inc., 1967), p. 193.

2. For instance, the default risk is never zero unless only governments are purchased; or for that matter income risk is never zero because the portfolios of life insurance companies do contain 'shorts', although keeping 'long' is viewed as the general aim.


4. In recent years the life insurance companies have supplied nearly 10% of the financial capital flowing from all investment sources in United States. In fact these funds are a major source of capital to the investors in real assets. See 1971 Life Insurance Fact Book, published by Institute of Life Insurance, (New York: 1972), p. 67.

5. The risk associated with such a contract stems from the fact that the interest rate to be earned in the future on both current and future premium receipts is guaranteed at the time the policy is sold. This income risk has been considered as one of the paramount aspects of the investment undertakings of the life insurance companies.

6. In post-war years the proportion of government securities in total assets has been declining, (see Table 1) but this does not imply that the life insurance companies have come to regard the liquidity aspects of their portfolio as of being less significance. In the late 1960's liquidity position has been one of their major concerns. As a matter of fact, the restructured portfolio demonstrates a greater emphasis on yield and income placed by the life insurance fund managers.

7. Goldsmith has observed that the expansion of various retirement funds and pension funds have adversely affected the position of the life insurance companies as users of funds and hence also their importance as suppliers of funds. R.W. Goldsmith, Financial Institutions, (New York, Random House, 1968), pp. 102-104.

8. As far as possible we will like to confine ourselves to the description of proportions rather than magnitudes, i.e., the dollar value of investments; though the latter will also be considered in certain cases. Of course for estimation purposes we will need data in terms of dollar values of the respective assets held.
Footnotes (Continued)

9. The major part of corporate securities is comprised of corporate bonds including public utilities, railroads, and industrials. A detailed description of these appears in Table 2 in terms of proportions of total holdings.

10. Government securities consist of treasury securities, state and local government securities, foreign government and international agencies' securities, and securities issued by some of the federal agencies. A description of these in terms magnitudes appears in Table 3.

11. Miscellaneous assets are comprised of (a) cash holdings, (b) due and deferred payments of premiums, (c) due and accrued investment incomes, (d) and others. The proportion of these assets are stable and is larger than the fraction of total assets held in real estates for the entire sample period.

12. These proportions refer to the relative share of each type of bonds in total assets, rather than in 'total bond holdings'.


14. For instance, in 1970 investment in securities of Canadian central and local governments amounted to 2.6 billion dollars, or nearly 78% of total foreign investments.

15. Wehrle suggests that if insurance companies were to follow a cyclical policy, they will (a) go short in low interest periods and (b) will go long in high interest periods, possibly sell short maturities. We have already covered these issues in previous sections. Assuming that most of the Treasury securities held have shorter maturity than bonds or mortgages, life insurance activity in federal government bonds is consistent with the criteria stated above for the 1952-1967 period. Wherle found that there is no evidence to substantiate the existence of cyclical policy behavior on the part of the life insurance companies. His data, however, reflect the activities of only four life insurance companies for the 1947-48 period. Federal government bond holdings reflect some cyclical behavior on the part of life insurance companies in that period as well as in 1960's. Polci loans, in particular, have brought about an increased awareness of the effects of high interest rates on life insurance companies in the sixties. Wherle, op.cit., pp. 196,241.

16. The yearly net increase in policy loans in 1968 over 1967 was only 5.2%, but it jumped to nearly 17% in 1969 and in the subsequent year the net increase was nearly 14%. On a quarterly basis, the net increase may be larger for some observations of these years.
Footnotes (Continued)


18. The 5% rate is a statutory obligation in most states and is guaranteed to be fixed for the duration of the contract. If the insurance company did not advance policy loan, a loan could be secured using the cash value of the life policy as a collateral. Since an individual taking out a policy loan assumes no real obligation to re-pay, a large proportion of such loans lead to policy lapses.


20. For a description of federal and state laws see:
   a. Ibid., Chapter V


   Special attention needs to be given to the footnote #5 in this article. Note that we are not concerned here with the issues of intermediation, per se, rather we would like to concentrate on the efficacy of controls on the operations of the life insurance companies. The former has led to a lively controversy in monetary economics on some fundamental issues, e.g., what constitutes 'money', 'monetary institutions', and the relevance of control processes. For details see, W.L. Smith, "On Some Current Issues in Monetary Economics: An Interpretation," *Journal of Economic Literature*, 1970, pp. 767-782.

CHAPTER III

A MODEL OF PORTFOLIO BEHAVIOR, SOME GENERALIZATIONS FROM THE CONVENTIONAL THEORY

"... Monetary microeconomics concerns the balance sheet or portfolio choices of individuals units---households, businesses or financial institutions. The choices are constrained by its opportunities to buy and sell assets and to incur or retire debts. Within these constraints, the choices are affected by the objectives, expectations and uncertainties of the unit......" 1

3.1 Preliminaries

Investment in financial assets by micro decision making units -- firms and individuals-- is essentially a problem of choice between 'objects' involving uncertain outcomes which can be characterized in terms of probability distribution of risk and return. In conventional theory, these decision making units supposedly indulge in some sort of maximizing behavior, though there is no consensus regarding the objective function being maximized or its contents, thereby leading to pronounced differences in the resulting propositions. Our purpose in this chapter is not to present a catalogue of theoretical models of portfolio behavior, rather we will limit ourselves to developing propositions from a generalized expected-utility -maximization hypothesis. At a later stage we hope to extend these propositions to characterize behavior of life insurance companies.

Our pre-occupation with the considerations of a priori will supposedly furnish us consistent constructs regarding investment
behavior under conditions of risk. But the empirical investigation of
the resulting propositions for the study of life insurance companies
requires a careful interpretation of these generalizations. The infe­
rences obtained here are necessarily limited by the specific theore­
tical construct of the model, its assumptions, and rationalizaitons.
Briefly, we plan to examine the constrained investment behavior of a
micro unit--the specific financial intermediary under consideration--
involving indices of profitability, risk, and substitution. The
objective is to formulate these elements in a consistent manner so as
to reflect their influence on decisions to invest in risky ventures.

3.2 The Framework

Our analysis is couched in terms of a choice theoretic approach
in the sense that it incorporates:

1. preference orderings
2. conservation relations, and
3. market clearing conditions.

Assumptions of the model are:

1. Single period time horizon.
2. Expectations of future performance of assets can be expressed in
terms of a probability distribution of prices or returns.
3. Investors maximize expected utility of future wealth.
4. The utility function is unique up to a linear monotone transformation,
except for the unit of measure and origin.
5. It is well behaved, i.e., $u'(.)$ and $u''(.)$ exist.
7. The number of shares of security $j$, $q^*_j$, are completely divisible,
   but are held constant.
8. Investor is a price taker.

9. The equilibrium prices, \( p_j \), are such that market is cleared. Further, let us define:

1. \( W = \sum_j p_jq_j \), existing wealth; \( \overline{W} \) a composite risk; \( W^* \), the certainty equivalent of the composite risk.

2. \( \overline{W} = \sum_j p^* j q_j \), expected future wealth.

3. \( g = u(\overline{W}) = u \), risk measure in terms of utility.

4. \( h = \overline{W} - W^* \), risk measure in terms of dollar values.

5. \( c_j = \frac{p_jq_j}{v^0} \), relative investment in \( j \)-th security; \( 0 < c_j < 1 \)

6. \( r_j = \frac{p^* j}{p_j} - 1 \), expected rate of return on \( j \)-th security.

7. \( \rho = \sum_j r_j c_j \), expected return on portfolio.

8. \( \sum_j (p_j q_j) = \sum_j (p^*_j q^*_j) \), for all \( i, j = 1, \ldots, m \), the investors; and \( j = 1, \ldots, n \), the securities.

9. \( p_j q_j = y_j \), the dollar value of security \( j \); \( v_j = p_jq_j \), dollar value of \( j \)-th endowment; and \( \sum_j v_j = v^0 \), dollar value of total endowments.

Portfolio analysis, based as it is on the hypothesis of expected utility maximization a la Markowitz and Tobin, involves the generation of efficient frontier that maximizes investor's \( u(.) \); or what is the same, if \( u(.) \) is specified as \( u(\rho, \sigma) \), where \( \rho \) and \( \sigma \) are portfolio return and risk measures respectively, optimization of this function subject to the above constraint yields us the condition of optimum that we are looking for. In the following we present a generalized model of portfolio behavior where \( u(.) \) does not involve \( \rho \) and \( \sigma \) explicitly, though it can be specialized to incorporate the celebrated mean-variance criterion.
3.3 The Model

Suppose, operating with risk measure \( g \) as defined above, we write:

\[
(3.3.1) \quad u = u(W) - g
\]

We can now state our maximization problem as:

\[
(3.3.2) \quad \max L = u(W) - g - \lambda \left[ \sum_j p_j(q_j - \hat{q}_j) \right]
\]

The first order conditions are:

\[
(3.3.3) \quad \frac{\partial L}{\partial q_j} = 0 = u'(W)p^*_j - g_j - \lambda p_j, \quad g_j = \frac{\partial g}{\partial q_j}
\]

\[
(3.3.4) \quad \frac{\partial L}{\partial \lambda} = 0 = \sum_j p_j(q_j - \hat{q}_j)
\]

Eliminating by solving for the set of equations (3.3.3) and (3.3.4) on the assumption that the \( n \)-th security is a reference security and has a risk free return \( r_n \), we obtain:

\[
(3.3.5) \quad u'(W)p^*_j - g_j = u'(W)p^*_n s_j - g_n s_j
\]

\[
(3.3.6) \quad g_j - g_n s_j = u'(W)[p^*_j - p^*_n s_j]
\]

where \( s_j = \frac{p_j}{p_n} \), the \( j \)-th relative price. Since we have defined the expected rate of return for \( j \)-th security as \( r_j = \frac{p^*_j}{p_j} - 1 \), we can express:

\[
(3.3.7) \quad \frac{g_j}{p_j} - \frac{g_n}{p_n} = u'(W)[r_j - r_n]
\]

for the \( j \)-th security of portfolio. Writing an equivalent expression for the \( k \)-th security and suitably re-arranging the terms, we get:

\[
(3.3.8) \quad \frac{[g_j/p_j - g_n/p_n]}{[r_j - r_n]} = \frac{[g_k/p_k - g_n/p_n]}{[r_k - r_n]}
\]

This equation characterizes the quilibrium condition of a single
investor. The proposition states that utility maximizing investors choose securities such that all the assets have same relative marginal risk per dollar relative to risk premium -- a condition very similar in its essence to the one obtained under certainty models. The risk measure of equation (3.3.8), however, is hard to specify in operational terms. Hence let us proceed with an alternate risk parameter along the same lines as above. As we shall see, the equilibrium conditions obtained with risk measure $h$, can be extended to yield an asset-return function with less cumbersome operational problems.

### 3.4 Some Extensions

We have already established:

$$g = u(\bar{W}) - E[u(\bar{W})] = u(\bar{W}) - u$$

If $g > 0$, it implies risk aversion; $u(.)$ is concave:

$g = 0$, it implies risk neutrality; $u(.)$ is linear

$g < 0$, it implies risk loving; $u(.)$ is convex.

Since we have defined $h = \bar{W} - W^*$; therefore, $u(\bar{W} - h) = E[u(\bar{W})]$, by definition; hence $g \geq 0$ implies $h \leq 0$. Further, notice that the measure $g$ is not invariant under linear monotonic transformation but the alternate risk measure, $h$, is invariant, which in itself is a desirable property. The implications of this will be clear as we proceed. Working with the measure $h$ we obtain the following results.

We have defined $h$ as $h = \tilde{W} - W^*$, where $W^*$ is the certainty equivalent of the risk $\tilde{W}$, which is a composite random (risky) choice. Let $W^*$ be $u(W^*) = \tilde{u} = E[u(\bar{W})]$ holds, i.e., the utility of certainty equivalent $W^*$ is deemed to be the same as the expected utility of future wealth; then max $u$ implies max $u(W^*)$. Hence:
(3.4.1) \( u(W*) = u(W - h) \)

(3.4.2) \( \text{Max } L = u(W - h) - \left[ \sum J p_j (q_j - \hat{q}_j) \right] \)

(3.4.3) \( \partial L/\partial q_j = u'(.)(p^* - h_j) - \lambda p_j = 0, \quad h_j = \hat{\partial} h/\partial q_j \)

(3.4.4) \( \partial L/\partial \lambda = \sum J p_j [q_j - \hat{q}_j] = 0 \)

Solving for \( \lambda \) from nth equation of the above system:

(3.4.5) \( u'(.)(p^*_n - h_n) - \lambda p_n = 0 \)

(3.4.6) \( \text{hence, } u'(.)(p^*_j - h_j) = [u'(.)(p^*_n - h_n)/p_n] p_j \)

(3.4.7) \( u'(.)(p^*_j - h_j) l/p_l = u'(.)(p^*_n - h_n) l/p_n \)

Since \( r_j = p^*_j/p_j - 1 \), for all \( j \), simplifying (3.4.6) we obtain:

(3.4.8) \( r_j - r_n = h_j/p_j - h_n/p_n \)

Multiplying equation (3.4.8) by \( c_j, [c_j = p_j q_j/V_o] \), and summing it:

(3.4.9) \( \sum J r_j c_j - r_n \sum J c_j = \sum J [h_j/p_j] c_j - [h_n/p_n] \sum J c_j \)

If the nth security is risk less, \( h_n = 0 \); and we know that \( \sum J c_j = 1 \)

(3.4.10) hence we have, \( \sum J r_j c_j - r_n = \sum J [h_j/p_j] c_j \)

Operating on equation (3.4.10) we can develop an asset-return function as follows:

(3.4.11) \( [(r_1 p_1 q_1)/V_o + \sum J (r_j p_j q_j)/V_o] - r_n = [p_1 q_1/V_o] h'_1 + \sum J h'_j c_j, \quad \text{where } h'_j = h_j/p_j \)

Rearranging the terms, we can write:
(3.4.12) \[ h_1 q_1 = r_1 p_1 q_1 + \sum_{j \neq i} r_j p_j q_j - V^o r_n - \sum_{j \neq i} h_j p_j q_j \]

Defining the terms as:

(3.4.13) \[ r_i [ p_i q_i ] = r_i y_i = R_i \]

(3.4.14) \[ \sum_{j \neq i} R_j = [TR - R] = R^* i \]

(3.4.15) \[ V^o r_n = \sum_j (p_j q_j) r_n = V^* \]

(3.4.16) \[ \sum_{j \neq i} h_j p_j q_j = Z^* i \]

We can write equation (3.4.12) in new notations:

(3.4.17) \[ p_1 q_1 = f (r_1, h, V^o, r_n, p_j q_j, R_j) \]

In general terms, we have:

(3.4.18) \[ y_i = f(r_1; \Theta_1, V^*, R^*, Z^*) \]

This equation expresses the dollar value of i-th asset, \( y_i \), as a function of the variables defined in above equations. Interpretation of these elements is crucial to the asset-return function in (3.4.18).

By definition, \( r_i \) is the i-th expected rate of return; \( \Theta_1 \) is an appropriate risk surrogate; \( V^* \) is an opportunity cost since it depends on the risk free return and initial endowments; \( R^* \) is the dollar value of returns of investment in remaining assets; and \( Z^* \) is the dollar value of investment in remaining assets, excluding the i-th one under consideration--- a constraint variable. Thus the value of i-th asset in a given time period is posited to be a function of risk variable, returns, rate of return of the associated asset, opportunity cost and total wealth.

The relationship in (3.4.18) based as it is on the hypothesis of expected utility maximization, will be the cornerstone of our estimation effort in the study of life insurance portfolios. It may
also be regarded as a demand function for the i-th asset in the portfolio, though we must point out that it is at best a quasi-demand function. The reason is that whereas for a normal good we expect the price coefficient to be negative, in the above case \( \frac{\partial y_i}{\partial r_i} \) is positive on the a priori basis. A negative value will be contrary to the basic postulates our model. We expect the demand for the i-th asset to vary directly in relation to the i-th rate of return. But note that only a step removed, we expect \( \frac{\partial y_i}{\partial p_i} \) to be negative since \( r_i \) and \( p_i \) are inversely related. Further, if risk aversion is assumed to be the appropriate kind of managerial behavior, we may expect \( \frac{\partial y_i}{\partial \Theta_i} \) to be negative, though the operational equivalents of \( \Theta_i \) may preclude such results. Substitution and complementarity may likewise be defined keeping in view the peculiarities of relationships derived here.

For estimation purposes, we will specify a set of demand functions for the life insurance companies in Chapter V, corresponding to the relation derived in (3.4.18). The specialized functions will pertain to each major type of the asset in the said portfolio and will later on be estimated over the sample period 1957-70.

We recognize, however, that the foregoing analysis leaves some of the outstanding issues unresolved; worse yet, it creates some of its own. There are serious gaps which may inhibit a clear cut interpretation of the obtained results even in the limited context of life insurance portfolio behavior. We shall devote the remainder of this chapter to a discussion of these problems.
3.5 **Issues in the Theory of Decision Making**

**Under Conditions of Risk**

At the outset it may be pointed out that in this section we will limit ourselves to the considerations of only some of the prominent issues both in the theoretical and applied field. In some respects it may be regarded as an attempt to provide an overview of these problems in relation to the model being considered here. We hope, it is obvious by now that we are not directly concerned with the classic problem in portfolio theory, viz, the nature of relationship between the portfolio risk and portfolio return, or for that matter security risk and security return, though the leading propositions stated above have been based on a model purporting to examine such relationships. We have instead focussed our attention on the propositions concerning the nature of the relationship between assets and other relevant variables. In an applied framework, the above implies that instead of fitting a relation between mean and variance of returns of the life insurance portfolio, we will try to estimate asset return function of the type discussed above.

### 3.5.1 The Implications of Risk Aversion

The assumption of risk aversion underlying the above analysis implies that an investor, starting from a position of certainty, is unwilling to take a risk which is actuarially unfair to him. Given two choices $A$ and $B$ where:

$A = [pR_1, (1-p)R_2]$, is a composit risky choice in two outcomes, $R_1$ and $R_2$; and $B$ is a choice with a certain outcome, $R_0$; the actuarial value of choice $A$ is its expected value, i.e.,

$\hat{A} = p R_1 + (1-p) R_2$. In this case a risk is said to be fair if
\[ \bar{A} = R_0 \text{ holds, since the participant gets same actuarial value over each alternate.} \]

Holdings of assets, though it connotes risk taking, is reconcilable with a general predominance of risk aversion, particularly when large amounts are involved, and has been shown to be compatible with expected utility maximization hypothesis.\(^7\) The crucial point at this stage is whether or not we need to specify certain restrictions on the \(u(.)\) to conform with risk aversion. In the general model, it seems that no explicit restriction need be placed other than the assumptions of the system, to arrive at the derived relationships. This implies that \(u(.)\) could either be rationalized in terms of the von Neumann-Morgenstern axioms of behavior, or straightforwardly can be postulated to possess the properties requisite to the analysis.

Suppose we do specify \(u(.)\) to be a quadratic function, retaining the assumptions of the investors being price takers and that the conservation relation holds, such that the vector of prices, \(P\), is market clearing. Let:

1. \(u(W) = aW - bW^2\), for \(b > 0\), hence
2. \(u'(W) = a - 2bW = 0\), and
3. \(u''(W) = -2b\), therefore for \(b > 0\), \(u''(.) < 0\), hence risk aversion. Also note that:
4. \(\bar{u} = E[u(W)] = a\bar{W} - bE(W^2)\); and since we have defined \(g\) as
5. \(g = u(W) - \bar{u}\), in above case we get,
6. \(g = a\bar{W} - b\bar{W}^2 - [a\bar{W} - bE(W^2)]\), whence
7. \(g = -b[\bar{W}^2 - E(W^2)] = -b\sigma_W^2\).

Since \(b\) and \(g\) are positive, the above implies risk aversion. This is the familiar characterization of the mean-variance models.\(^8\) In
this scheme, preference orderings can be structured in terms of the ratios \( \frac{dW}{d\xi^2} \); the indifference contours will have positive slope; and thus we could go on. But quadratic utility functions have found to be unsatisfactory on the following basis. \(^9\)

1. If \( b \) is positive and \( u''(.) \) is negative, then as \( W \) increases the quadratic \( u(.) \) implies that marginal utility of wealth decreases.

2. If for two portfolios \( \bar{W} \) and \( \bar{Q} \) are the same for all possible states of the world, a preference ordering, stating that two portfolios are equally valued, is not permissible, because the nature of the states may preclude such a valuation--in other words, the ultimate objects of choice are not some specific parameters of risk and return rather than the states of world.

3. Since the quadratic \( u(.) \) is not invariant under some types of transformation, it implies that the value of \( u''(.) \) by itself has no significance. The variance of returns is non-unique.

4. If the ratio \( u''(.) / u'(.) \) or \( [W \cdot u''(.) / u'(.)] \) is used as a measure of risk, quadratic \( u(.) \) is inconsistent with these measure.

In the general model some of these inadequacies have been taken care of. The object of choice is defined to be the utility of future wealth as measured in terms of capital value of the assets in a more or less static scheme, rather than the utility of returns and income, which seems to have led to these anamolies. Further, explicit incorporation of the conservation relations in the optimizing processes introduces a fundamental element of choice-theoretic structure, which together with the market clearing conditions, completes the specification of a general equilibrium model of portfolio behavior. In a purely technical sense, the model is at least as fully specified as
the state preference models. No doubt that the earlier versions of M-V models considered only the behavioral solution on the assumption of fixed endowments and static preferences, these have been extended to provide solutions for market equilibrium. To achieve this in our model, we need to add market clearing conditions to provide the general solution.

3.5.2 Static and Dynamic Models

It has been argued that the general model is static since it assumes that investors maximize $E[u(.)]$ at the end of a single period which spans the entire investment horizon. This does not permit portfolio revisions between initial investment and final assessment. The single period model is then extended to an arbitrary number of the periods where portfolio adjustments are constantly occurring on the basis of current asset holdings and current prices, given that the joint distribution of risk and return parameters in time period $t$ is independent of the one in $t'$ for all $t \neq t'$. Further, in the static context, the optimal risky asset ratios are shown to be independent of the investor's initial wealth.

The alternate to this scheme will be to formulate a dynamic model, combining the time and uncertainty dimensions of optimal asset theory. This will permit analysis of the effects of duration of time horizon and future of investment possibilities on current portfolio decisions. Hence if we consider preferences over time where objects of choice are dated entities, e.g., consumer claims, it may be possible to build a dynamic theory. But the existing literature in this area does not extend beyond the derivation of propositions of a very general nature which are operationally insignificant.
3.5.3 Institutional Portfolio Behavior

Thus far we have tacitly assumed that the general portfolio model deals with the individual investors who own the financial assets. Extensions of the model to the portfolio problems of financial intermediaries creates additional difficulties which deserve our attention. Some of the main issues in this regard have been discussed below.

As regards the nature of the constraints on the maximand, the model assumes the absolute levels of funds available for investment as fixed and concerns itself only with the allocation of these funds over candidate opportunities. For most of the financial intermediaries and specially for the life insurance companies, this restriction is undesirable. Accurately specified it turns out to be an inequality constraint on the system. Further, in order to bring under consideration the overall financial posture of the firm, we need to examine not only the structure of portfolio but also of the liabilities.\textsuperscript{11} If so, the inequality constraints on the total assets may be defined to be the sum of current and long term liabilities plus net worth. But this is altogether a different kind of exercise. It involves a different type of constraint and will change the nature of the objective function being maximized, because we need to incorporate additional terms in the maximand and define all over again, new parameters of risk and return. Above all, addition of a set of inequality constraints prohibits use of ordinary maximization techniques and requires a reformulation of the problem in terms of a non-linear programming model—an exercise which is beyond the scope of this study.

In the standard theory of the firm, given perfect knowledge and foresight, efficacy of the economic analysis is largely due to
the appropriate specification of the constraints on the opportunity set open to the firm, rather than due to accurate specification of the maximand. Since the standard constraints are production possibilities and an externally given cost of capital schedule, marginal quantities which offer criteria for decision making refer only to these two constraints. The financial analyst's preoccupation with the cost of capital as a criterion for investment cut-off is a single, direct and inescapable result of the assumption that this indeed is the proper model. To the extent that other more immediately binding constraints on the firm are held to be important, this optimum will be altered and the traditional capital budgeting axiom will no longer be appropriate.

Next, let us consider implications of alternate criteria to expected utility maximization. We may, if we wish, dwell upon profit maximization \[ F = R - C \], or market value maximization \[ V = sR_i \], where \[ sR_i = \sum R_i/(1+i) \]. In the conventional theory, given certainty, whether a firm maximizes \( F \) or \( V \), both procedures lead to equivalent propositions; viz, in case of \( F \)-maximization the proposition is that marginal value product of \( i \)-th input be equal to its marginal cost; and in case of \( V \)-maximization, we obtain the result that the additions to the capitalized value of owner's equity be at least as great as the cost of borrowing. Extensions of these analysis have found expression in recent, and rather sophisticated versions of net worth maximization models. Under general neo-classical conditions, maximization of net worth yields more or less a similar proposition as above, namely that marginal yield on capital be equal to user cost of capital, which in turn is a function of a host of...
availability and cost parameters. Specifically, in a dynamic version of the model, it has been shown that net worth maximization yields a stock adjustment type relationship which indeed is ideal from a purely operational viewpoint.\textsuperscript{13}

Under conditions of uncertainty, the above stated equivalence of the two propositions vanishes, more so if we take into account the peculiarities of investment in financial assets as opposed to investment in physical stock of capital.\textsuperscript{14} The firm no longer expects a unique profit outcome associated to each investment decision, instead it faces a host of mutually exclusive outcomes which can be characterized only in terms of subjective probability distributions. Profit becomes a random variable itself and its maximization has no operational significance. Further, in the article cited above, Modigliani and Miller contend that the problem can not be solved by proposing that we need to start off with the hypothesis of expected profit maximization instead of simply F-maximization, and in their view it is this difficulty which has led to the enunciation of subjective preferences hypothesis. To the authors expected utility maximization is equally unsatisfactory. As they view it, this criterion has some serious normative drawbacks which precludes meaningful generalization about micro investment behavior under uncertainty. Alternately, they propose a hypothesis of their own, based on the critical assumption that it is possible to group firms in homogenous classes of risk, such that the differences in returns on shares, issued by firms of different groups are identifiable by a scale factor. Further, they contend that the risk characteristics of assets are exogenously determined, which implies that individual investor's attitude towards risk can
have no direct influence on the risk aspects of portfolio decisions of large, publicly held financial intermediaries.

### 3.5.4 Implications for Institutional Investors

These issues help to focus our attention on a significant problem with the expected utility maximization hypothesis, viz, does the hypothesis permit generalizations about institutionalized behavior regarding investment in risky assets? Admittedly, in its present form, the conventional theory relates to investment in financial assets by individual investors, considers their preferences with regard to risk and return, and offers some propositions about behavior under uncertainty. Recent extensions of the analysis attempt to provide a general version of the M-V models, by introducing explicitly considerations of market equilibrium, and the conditions under which investors' equilibria are mutually consistent. But in the standard exposition, the implications of the dichotomy between individual and institutionalized investment behavior have not been explored.

In applied pursuits, the appropriateness of the u(.) hypothesis to institutional decisions has apparently never been questioned—perhaps due to a lack of a theory as respectable or as widely accepted as the one under consideration. It has been assumed that an asset demand function exists, or can be derived from the optimizing conditions of the u(.) hypothesis, and that it possesses requisite properties. Given this we need only to specify the process by which the expectations are generated. In absence of equivalence between net worth maximization and investment in non-risky assets, the stock adjustment principle is inoperative. This leaves only the adaptive expectations hypothesis which has been applied for corporate investment
behavior.

Assuming that the risk characteristics of assets are exogenously determined and that it is possible to classify financial intermediaries into homogenous risk classes, it is proposed that within a single type the portfolio managers possess a utility function common to all. As a behavioral hypothesis, it is also proposed that the managers maximize expected utility. This approach effectively reduces the problem of diversity in preferences. Financial firms can be classified in groups, specially in view of the differences in the procedures by which they raise funds in the money market. Risk and return aspects of both liabilities and assets provide additional bases for such grouping. This allows us to specify the so-called institutional preference function, which together with the conservation relations and market clearing conditions, provides the necessary ingredients of a general equilibrium model of portfolio behavior.

3.6 Comments

What have we accomplished thus far? We have derived in a rigorous fashion an asset demand function, based on expected utility maximization hypothesis. We have offered a tentative interpretation of the obtained functional form for the portfolio of life insurance companies. In view of the limitations of the model, we have offered some rationalizations for its applicability in applied pursuits. The next step will be to specify an econometric model for estimation purposes, based on the derived relationship. Before we plunge into the specifics of life insurance portfolio model, however, we must select appropriate risk variable(s) from amongst a host of risk surrogates, a task which will occupy us in the next chapter.
Footnotes

1. Donald Hester and James Tobin in Foreword of Cowles Foundation Monograph in Monetary Economics.

2. In the literature, the choice-theoretic approach to portfolio behavior usually refers to a particular class of models, dealing with intertemporal choices of dated consumption claims in a general framework.

3. For the analysis of the behavior of a single investor, a vector of market clearing security prices, \( P \), is assumed to be given.


5. Note that we have obtained (3.3.7) by dividing (3.3.6) with \( p_n \). In its final form it represents the security risk as a function of marginal utility of wealth, times the difference in returns of \( j \)-th security and \( n \)-th security. From these equations we can also define rate of substitution of risk for returns which can be shown to be equal to the marginal utility of wealth for risk.

6. For an excellent exposition of various issues in the theory of risk aversion see,


8. Markowitz, loc. cit.
   Tobin, loc. cit.

   Arrow, loc. cit.
Footnotes (Continued)

10. Sharpe, loc. cit., pp. 425-442


15. Stone, loc. cit.
   Sharpe, loc. cit.

CHAPTER IV

SURROGATE RISK MEASURES

4.1 Preliminaries

One of the most controversial issue in the analysis of investment involving risk has been the measure of riskiness associated with a proposed venture. In recent portfolio models a number of these measures have been proposed, based on the conventional theory, incorporating the well known mean-variance criterion. But the polarity of views even in this narrowed class of portfolio models is almost overwhelming; the most negative one being that the term: risk as applied to security investment has no clearly defined meaning, and that there is no theoretical justification for any generalized statement as to how risk affects security valuation and thus security investment. Extremes of opinion aside, it is rather encouraging to note that attempts have been made to incorporate various types of risk variables as an essential ingredient of models of portfolio behavior.

4.2 Risk Surrogates in Mean-Variance Models

In applied pursuits a number of risk measures have been proposed but in view of serious theoretical limitations, it is legitimate to regard these measures as surrogates of the risk characteristics of a given portfolio. In the class of mean-variance models, or its extensions a la Sharpe\(^2\), Lintner\(^3\), Mossin\(^4\) and others, the scheme is
to posit a relationship between portfolio return, \( r \), and portfolio risk, \( \Theta \), or security return \( r_i \), and security risk \( \Theta_i \). If these were our objectives, we need only to determine the parametric distribution of the return variable \((r, r_i)\) and subsequently test the relationship on a given data. The estimated relationship turns out to be:

\[
(4.2.1) \quad m_r = a + b \nu_r + \epsilon
\]

where \( m_r \) is the expected return and \( \nu_r \) is the associated risk variable; in fact mean and variance of returns, respectively. This relationship may be surmised in a purely ad hoc manner, or may be derived as an implication of some sort of maximizing behavior on the part of economic unit under consideration. For instance, we may work with a quadratic utility function to arrive at this kind of relationship. Be that as it may, invariably we end up with a line fitted in the risk-return quadrant, the intercept value being risk free rate.

Consider, for instance, Fama's characterization of equilibrium in the framework of mean-variance models presented in figure 1. Let \( \rho \) and \( \sigma \) be portfolio return and standard deviation of return respectively, where \( \frac{\partial \mu}{\partial \rho} = [1 - \sum c_j] r_n + r \). The equilibrium condition is given by the tangency solution:
\( \frac{\partial g}{\partial \theta} = (\text{slope } jt') = \frac{\sigma_r}{(r - r_n)} \)

which is quite similar to standard generalizations obtained from the conventional analysis. The only difference here is with respect to the slope of the indifference contour.

In empirical studies, the focus of attention has been a line fitted in the \((P, \sigma)\) quadrant. In a recent work, Pike and Gentry have reported on the empirical testing of the relationship in \( (4.2.1) \) on the common stock data of 34 U.S. life insurance companies, for the period 1956-67. The authors hypothesize a positive linear relationship between expected return of \( i \)-th asset and the risk associated with it; and what could be a better surrogate for \( (\mu_i, \sigma_i) \) than \( r_i \), the observed ex-post \( i \)-th rate of return, and \( \sigma_i \), the standard deviation of \( r_i \). Thus we end up with the estimated relationship given in the above diagram.

One of the basic premises of the portfolio models is that the decision making is an ex-ante activity, involving expectation variables; however, in actual applications the proposed surrogates have invariably been reduced to a measure designed to evaluate ex-post performance of investment behavior. In the study referenced above, Pike and Gentry have acknowledged this, nevertheless, they have fitted a relationship of the type considered in \( (4.2.1) \) on ex-post data for lack of a better alternative.

For our purposes, however, we would like to obtain some measure of riskiness of each security as well as an overall measure for the entire portfolio of life insurance companies. In the model discussed in Chapter III, it would be recalled that the risk parameter for \( i \)-th asset has been obtained as some function of risk premium per dollar
of marginal investment. Since it involves undefined values such as the expected future value of assets and the certainty equivalent of composite random risky choice, we need to obtain some surrogate for empirical investigation.

4.3 Indices of Investment Performance

As a first approximation, we may consider as possible surrogates, the indices of investment performance, based on some parameters of pure risk. Extensions of the conventional theory provide us with the criteria necessary for the evaluation of ex-post performance in terms of certain composite measures, \( c^* \), involving both the risk and return aspects of a portfolio. Three basic versions of these measure will be considered here. A common feature of all these variants is that \( c^* \) involves expectational values and is defined as a ratio of return differential to some measure of pure risk. For instance, in Sharpe-Lintner models it has been interpreted as excess portfolio return per unit of portfolio risk. Specially, in case of mutual funds, Sharpe\(^8\) presented a composite measure, \( c^*_s \), as \( c^*_s = (r - r_n)/s \), where \( r \) is the average yield over \( N \) periods, \( r_n \) is a certain return on risk free investments, and \( s \) is the standard deviation of the observed yields; all based on ex-post data. It is obvious that \( c^*_s \) can be regarded as the excess portfolio yield per unit of portfolio risk—a return to variability measure, but it is not clear whether it can be used meaningfully as a surrogate for risk.

This specification of the composite measure is almost identical to the one given by Lintner.\(^9\) It has been viewed by him as a parameter obtained from the optimal solution of the portfolio selection. In so far as Sharpe-Lintner models are considered as extensions of the
formulations involving two parameter criterion, \((\mu, \sigma)\), the similarity is not so surprising.

Notice that the measure of pure risk, \(\sigma\), which is commonly taken as a measure of risk, can be decomposed into two components:

1. the residual risk, which is unique to the particular security or the portfolio under consideration, and

2. the systematic risk, which refers to the riskiness of the portfolio that is inherent in the market itself.

This distinction has been used by Treynor to define a composite measure, \(c^*_T\), as follows:

\[
c^*_T = \frac{(r - \mu)}{\beta}
\]

where \(r\) and \(\mu\) are the same as defined for \(c^*_S\), and \(\beta\) is the slope of the following regression.

\[
r_t = a + b \cdot r^m_t + u_t
\]

The variable \(r_t\) is the \(t\)-th period yield of the portfolio and the explanatory variable is the corresponding yield achieved by the market as a whole. The coefficients \(a\) and \(b\) are the constant and slope of the fitted regressions, called the characteristic line of the fund or portfolio. The component of total variation in \(r_t\) explained by the characteristic line is the measure of systematic risk and the remainder is the residual risk. Treynor argues that since investors are assumed to be averse to risk as measured by variation in \(r_t\) and have little, if any, effect on this component of risk, therefore any attempt on the part of portfolio managers to reduce measured risk via diversification must be reflected in residual risk component. Thus, in the case of risk averse diversifiers, the coefficient \(b\) can be taken as a
measure of ex-post volatility of the fund and \( c^*_{T} \) can be inter-
reted as a composite measure of overall performance, given the level
of risk free return.\(^{11}\)

If diversification is perfect, it can be shown that the two
composite measures, \( c^*_S \) and \( c^*_{T} \), are consistent with each other.
Let \( r_t \) be a random variable, characterized by two-parameter distr-
ibution, namely, \((r, \sigma_r)\), where:
1. \( \bar{r} = a + b \bar{r}_m \), and
2. \( \sigma_r = \text{var}(r_t) = b \text{var}(r^m_t) + \text{var}(u_t) \)

hence systematic risk is equal to \((b \text{var}(r^m_t))\) and residual risk
is given by the last term in 2 above. Substituting these values for
\( r \) and \( s \) in \( c^*_S \) we obtain:
3. \( c^*_S = \frac{(r - r_n)}{s} = \frac{(a + b \bar{r}_m - r_n)}{b \sigma_{r_m}} = \frac{[\bar{r} - r_n]}{b} / \sigma_{r_m} \)
or,
4. \( c^*_S = c^*_{T} / \sigma_r \),

therefore the two measures are equivalent.

Finally, let us consider the measure proposed by Jensen, which
is similar to Treynor's in terms of the approach adopted.\(^{12}\) To
determine the systematic and residual risk, Jensen proposes the
following regression:
5. \( r - r_n = a_1 + b_1 (r^m - r_n) + e \)

Given the estimators for the two parameters, the composite measure
can be expressed as \( c^*_J = r_n - a_1/b_1 \), which has the property that
for a given portfolio, the higher the intercept value, the better is
the performance.\(^{13}\) Thus, in addition to the slope coefficient, the
intercept also plays a role in determining the performance.
4.4 Systematic Risk as Risk Surrogate

It is obvious from the foregoing analysis that the composite measures incorporate both risk and return aspects of a portfolio, and do no treat risk, per se, as a particularly useful parameter. This is entirely consistent with the fundamental objectives of the conventional theory, viz, the analysis of the relationship between risk and return. However, if we were to address ourselves to the appropriateness of a risk surrogate in the framework of the problem posed above, it appears, at least as an implication of the foregoing analysis, that considerations of only the systematic risk offers a promising line of approach. In other words, we need to assert that the measure of risk relevant to investors relates the tendency of return on securities to the returns on a collection of all securities in the market, in proportion to their total outstanding values. Given the assumptions of the extended portfolio model considered in Chapter III, systematic risk provides all necessary information about security risk for selecting an optimal portfolio, and in so far as portfolio risk is specified to be a combination of security risks, both the parameters are readily accessible. The rationale of the scheme will, of course, depend on the underlying model.

If we were to operate with the market model, we know that equilibrium conditions require security price adjustments until we arrive at an efficient portfolio, and that a linear relationship is to be posited between expected returns and variance of returns. If we retain the assumptions of the model it is possible to define a particularly simple measure of risk of i-th security, $\Theta_i$, as:

$$\Theta_i = \frac{\sigma_i(r_m)}{\sigma^2_{r_m}}$$

where the numerator is the covariance

1. $\sigma_i(r_m)$

where the numerator is the covariance
of the i-th and the market returns, and $\sigma_{ym}^2$ is the variance of market portfolio returns. If $r_i$ and $r_m$ are perfectly correlated we have $\Theta_i = (\sigma_{r_i}, \sigma_{r_m}) / \sigma_{ym}^2$. Once $\Theta_i$ are known, we can construct an index $\Theta$ as a linear combination of $\Theta_i$, and interpret $\Theta$ as the systematic risk of the portfolio. Hence,

$$\Theta = \sum_{i=1}^{n} c_i \Theta_i , \quad \sum_{i} c_i = 1 , \quad 0 < c_i < 1$$

where $c_i$ is the i-th proportion of security investment. Thus the volatility of a portfolio is presented as the surrogate for risk associated with it, and is defined as the surrogate for risk in terms of the weighted average of volatilities of the component securities, the weights being the proportions $c_i$ of wealth invested in i-th asset.

In another context, it has been demonstrated that the measure of systematic risk are equivalent to another class of risk measures arrived at under state preference models. The equivalence will hold only if the assumptions of the Sharpe-Lintner models are invoked with regard to the securities market.

If we further assume that investors evaluate the risk of a portfolio as a whole rather than the risk of each individual security, it is proposed that the risk inherent in the portfolio, howsoever measured, is the only relevant parameter. In an extreme, but not unlikely case, it is possible that the security risks may be compensating to such a degree that the level of portfolio risk is undisturbed as measured in terms of future aggregate returns. For the i-th security, let $[E(r_i) - r_n]$ be defined as the i-th risk premium, and $[E(r) - r_n]$ the risk premium for market portfolio, then the relation between the two can be characterized as: $[E(r_i) - r_n] = b [E(r) - r_n]$. Thus the i-th risk
premium is suggested to be proportional to the portfolio risk premium and hence the constant of proportionality may be interpreted as yet another measure of risk for i-th security.

4.5 Risk Surrogates for Life Insurance Portfolio

For purposes of estimation of the life insurance portfolio model, we shall operate with the risk measures discussed in section 4.4 of this Chapter. If we were to follow the derivations in Chapter III, the risk measures could be treated as a parametric value for i-th security function—a scale factor. But considering the complexities of scaling a host of independent variables by different indices as well as the relative meaninglessness of scale factor, it may be worthwhile to treat the surrogates as legitimately independent variables. This scheme will preserve the main features of the conventional portfolio theory and will provide us a mechanism to introduce a vitally significant element in considerations of investment in risky assets. We must point out, however, that the operational feasibility of the proposed measures remains a problem and will be given due consideration at the estimation stage of the model.
Footnotes


6. Operating on the equilibrium considerations Fama obtains the following as an extension of the above results:

\[ r_j - r_n = \left( \frac{r - r_n}{\sigma_r} \right) \sigma_{jw} \]

Similarly, if we let \( h_k = \frac{c_k}{\sum_j \sigma_{jk}^2} \), we can express an equivalent condition arrived at by Lintner by multiplying the equation by \( h_k \). In both the cases, if we let \( \Theta \) represent the risk surrogate for the expression on the left hand side of the equation, it is possible to arrive at results similar to the ones obtained in Chapter III.


In a recent study V.K. Smith and D. Tito report on the risk measures in the framework of ex-post portfolio performance. For details, see,


Footnotes (Continued)

11. It is possible to interpret the regression relation as:

\[ r - r_f = b \times \gamma_m \]

The slope of the regression line is the measure of volatility, and \( c^* \) is the corresponding measure of composite performance.


13. The risk free rate \( r_f \) has been assumed to be constant. Although this is reasonable within a given period, it may not be so over long periods. It has been suggested that it may be worthwhile to ignore \( r_f \) in certain cases and concentrate on the ratio of the estimates of parameters, \( a \) and \( b \).

14. In fact this measure is similar to the one considered by Jensen as far as systematic risk is concerned.


Another work by the same author needs to be referred with respect to the preceding footnote; since it deals with the problems of measurement of systematic risk.


CHAPTER V

LIFE INSURANCE PORTFOLIO MODEL

5.1 Preliminaries

In this chapter we will be concerned with the specification of a life insurance portfolio model based on the theoretical consideration of portfolio management discussed previously. For estimation purposes, we need to define each of the relationship corresponding to various types of assets held by life insurance companies. We then must treat them either independently of each other, as in uni-equation models, or interdependently, as in simultaneous systems. In fact we plan to do both; we will specify and estimate a set of asset-return relationships in isolation from each other on the supposition that each represents a self contained relation, and later on reformulate the whole set in a simultaneous system.

5.2 The System: Variables and Equations

We know from the previous discussion that the general functional form of the i-th relation is:

\[ y_i = f \left( r_i ; \theta_i, R^*i, Z^*i, V^* \right) \]

where,

\[ y_i = p_i q_i \], is the dollar value of i-th asset
\[ r_i \], is the i-th rate of return
\[ \theta_i \], is the i-th risk surrogate
\[ R^*i \], is the dollar value of remainder returns
\( Z_i \), is the dollar value of remainder investment and, \( V^* \), is the dollar value of risk free returns.

According to the above specification, the amount of i-th asset held in any given period depends upon its own rate of return, the riskiness associated with it, the alternative returns in total dollar values, total holdings less the i-th asset, and risk free returns. For the life insurance portfolio, we have interpreted (5.2.1) as the demand function for the i-th asset by the life insurance companies. We have, however, argued that the above relation is at best a quasi-demand function, in as much as \( \partial y_i / \partial r_i \) is positive. But this should not unduly disturb us because \( r_i \) is negatively related to \( p_i \), the price of i-th security, and if this correlation is perfect we may infer that the above function is indeed only a step removed from a traditional demand function.

We maintain, however, that in contrast to previous treatments, the relation in (5.2.1) is a quite rigorous specification of an asset return relationship. Among the class of portfolio considered in this study, we have most often encountered a conceptually unsatisfactory and empirically insignificant relation involving financial assets and its corresponding rate of return. Most of the models lack consistent a priori constructs, a situation which normally has the effect of suppressing some crucial aspects of the investment decisions. The espoused relationship between assets and their corresponding rates of return, remains at best ad hoc in nature. Even in the estimation context no explicit recognition is offered to substitution and constraint variables, though it is conceivable that adjustments may occur consequent to changes in opportunities for substitution and be limited by the size of overall holdings in a given period. These are significant exclusions.
and may well introduce distortions and inadequacies into an estimated model. The element of risk is likewise improperly treated or ignored.

An important source of the problem is the treatment of the rate of return variable. From the discussions in Chapter III it is obvious that the conditions of equilibrium of a micro-unit involve ex ante values which requires us to specify some valid approximations to arrive at their counterpart ex post variables. But this transition, while necessary for the estimation effort, need not obscure the fundamental characteristics of the rate of return variable. In a given relationship we can not substitute in a purely altruistic manner, the ex post values of the specified rate of return, rather we must develop a viable mechanism for the stated transition.

We hope to incorporate most of these considerations in our study. Our specification includes substitution and constraint variables, besides the appropriate surrogate for risk, which hopefully will have the effect of encompassing all the important dimensions of portfolio decision making by a given micro-economic unit. In addition we shall offer a rigorous treatment of the approximation mechanisms for pivotal variables, the expected rates of returns, and proceed for estimation accordingly.

Let us now define, as accurately as possible, the variables of equation (5.2.1) for the life insurance portfolio. First, let us consider the endogenous variables of the system:

1. U.S. Government securities, \( y_1 \)
2. State and Local Government securities, \( y_2 \)
3. Industrials and miscellaneous bonds, \( y_3 \)
4. Public Utilities' bonds, y_8
5. Railroad bonds, y_5
6. Common Stocks, y_6
7. Preferred stocks, y_7
8. Mortgages, y_8
9. Policy loans, y_9
10. Total business bonds, y_10
11. Total government securities, y_11

The above classification is based on monthly flow of funds data and represents considerable aggregation over its respective categories. Further, these categories do not necessarily contain perfectly homogeneous entities, though we shall assume so for our purposes. The problem of aggregation may be acute in some cases, e.g., the category on mortgages is an aggregate of various important but disparate types of mortgages extended by the life insurance companies. But a more serious problem seems to be the discrepancy between the flow of funds data, which essentially is a record of the end of the period of acquisitions, and the commitments data which correspond more closely to the timing of investment decisions. Conceptually, this may be regarded as a lag in the reporting of the data except for the fact that commitments are options in futures for the borrowers, and may or may not materialize. From the forthcoming discussions on the estimation techniques, it would be obvious that the lags, thus generated, can be taken care of substantially; but if the discrepancies are serious and persistent, some kind of adjustments will be needed to arrive at satisfactory results.

The above nine categories of assets together account for the largest proportion of total assets; in fact, for the year 1970 these
assets were nearly 92% of the total holdings of the life insurance companies. The distributional aspects of these assets have already been analysed in Chapter II and need not be repeated here. Briefly, we may describe the contents of the classification since it will be used to specify a set of stochastic relations.

The two categories on government securities, \( y_1 \) and \( y_2 \), are:

1. an aggregate of federal agencies' and Treasury securities, and
2. state and local securities, including municipal issues. The composite category of 'business bonds' has been broken down into its components, viz, \( y_3 \), \( y_4 \) and \( y_5 \), and excludes all foreign bonds. Holdings of common and preferred stocks, \( y_6 \) and \( y_7 \) likewise refer only to the stocks issued by U.S. owned corporations. We would like to operate primarily with the disaggregated data with respect to the government and business securities, but aggregate relations may also be specified for \( y_{10} \) and \( y_{11} \) and estimated on sectoral aggregates.

With regard to mortgages, \( y_8 \), we are using a highly aggregated category which includes,

1. non-farm residential mortgages-- FHA, VA, and conventional-- and
2. non-farm non-residential mortgages, held at the end of the period by the life insurance companies. Conceivably, it would be better to obtain a series on each member of the category as we have in the case of business securities. Though such data is not included explicitly, we will make suitable transformations on \( y_8 \) to distinguish between residential and non-residential mortgages and obtain regressions accordingly.

Finally, the policy loans have, in the late sixties, gained much significance due to their substantially increased magnitude in a period characterized by high rates of interest and a general liquidity crisis.
Traditionally, policy loans have not been prominent in portfolio considerations except for the above mentioned period. Again, these are the end of the period magnitudes in current dollar values.

The exogenous variables of the system are:

1. Yield on long term U.S. securities, \( r_1 \)
2. " state and local securities, \( r_2 \)
3. " industrials and miscellaneous bonds, \( r_3 \)
4. " public utilities' bonds, \( r_4 \)
5. " railroad bonds, \( r_5 \)
6. " common stocks, \( r_6 \)
7. " preferred stocks, \( r_7 \)
8. " mortgages, \( r_8 \)
9. Prime commercial rate, \( r_9 \)
10. Yield on business bonds, \( r_{10} \)
11. " short term government securities, \( r_{11} \)

Besides these rates some additional variables will also be considered. Most of them will be included in the explanatory set but whether we would regard them as exogenous or not will depend on the specification of the model.

1. Total investment income, total returns, \( R \)
2. 'Remainder returns', \( R^*i \), where \( R^*i = R - R^{\#i} \), and \( R^{\#i} \) is the weighted return of \( i \)-th asset.
3. 'Remainder investments', \( Z^*i \), where \( Z^*i = Z - \sum_j Z_j, \ j \neq i \) it is dollar value of all remaining assets— a sort of constraint variable, imbeded in the estimating equation.
4. Surrogate for portfolio risk, \( \Theta \)
5. Risk free returns, \( V^* \)
The first nine rates, \( r_1 \) to \( r_9 \) inclusive, correspond exactly to the types of assets described in the endogenous set, \( y_1 \) to \( y_9 \) inclusive. In addition we have an average yield variable for the composite category of business bonds and one short run rate for government securities. In all, only two types of yield on governments is being considered here, whereas in fact, we have a multiplicity of these rates corresponding to a variety of bonds, bills and certificates issued by the Treasury, federal agencies, state and local governments. For relatively disaggregated data on governments it would be necessary to include a more detailed data on yields; however, for our purposes the above will suffice.

Similar is the case of yield on mortgages. As noted above, we are using a highly aggregated data which includes a variety of mortgages. Ideally we would like to have disaggregated data both on yields and holdings. The rate of returns on mortgages refers only to yields on residential mortgages. For estimation we would use \( r_8 \) as the appropriate return variable for 'residential mortgages' and only as a proxy variable for the remainder term.

We have rather disaggregated data on return variables other than the two categories described above. For business securities we have five yield variables corresponding to the three types of business bonds and two types of business stocks. This is a fairly disaggregated category and hopefully will be adequate for our purposes. Finally, in light of the nature of these borrowings, no direct estimates of the yield on policy loans is available. It is a common practice in applied pursuits to use the prime commercial rate as a proxy variable for returns on policy loans.
The exogenous set will consist of the rates of return, $r_i$, $i = 1, \ldots, n$; the risk surrogate, $\theta$, and risk free returns, $V^*$. These are truly exogenous in the sense that none of these variables is predetermined. For the OLS estimation of $i$-th relationship, the explanatory set will consist of the exogenous variables so defined as well as the remainder returns, $R^*$, and remainder investments, $Z^*$. Obviously the last two will be differednt for each relationship and have to be suitably modified. Since both are functions of endogenous variables, we will have to make appropriate adjustments for simultaneous estimation. But $\theta$ and $V^*$ will remain the same for each function regardless of the estimation procedure. Ideally we would like the risk surrogate of $i$-th security, $\theta_i$, to characterize the appropriate risk attributes of its corresponding assets; or for that matter, the portfolio risk surrogate, $\theta$, should sum up the uncertainty dimensions of a given portfolio. Among the various surrogate for $\theta$ and $\theta_i$ suggested in Chapter IV, it seems that most of them would not be available in an operational sense. For the $i$-th relation we would like $\theta_i$ to be a ratio of $\sigma_{\text{im}}^2 / \sigma_{\text{m}}^2$, where $\sigma_{\text{im}}$ is the covariance between the $r_i$ and $r_m$ rates, $r_m$ being the portfolio return and $\sigma_{\text{m}}^2$ the variance of this rate. But notice that $\sigma_{\text{im}}$ is defined only for all $t$ observations, $t = 1, \ldots, T$ of $r_i$ and $r_m$; hence, we have only a single valued vector, i.e., only a scale factor. The alternative is to obtain the $t$-th period variance of return and use it as a surrogate for the portfolio risk.

Another exogenous variable common to all relations will be the proxy variable for risk free returns on endowments. The endowments have been defined as the sum of dollar value of all assets in the
In the initial period, \( \sum_t (p_i q_t) = V^o \); and the risk free returns as \( r_n V^o = V^* \), where \( r_n \) is the rate of return on non-risky asset and is considered to be generally lower than the rate of return on differentiated, risky asset. In view of this it is not unrealistic to surmise that for any period \( V^* \) would be lower than the total returns on all investments reported by the life insurance companies. Further, note that both the variables, \( r_n \) and \( V^o \), preclude any strict specification, leaving to us a less than desirable alternative of using instrumental variables. Description of the instrument variables will be postponed to the estimation stage of the model.

The remainder returns variable \( R^*_i \) will be obtained from the data on total investment income of all life insurance companies. Since information is available only on the aggregate variable, \( R \) - the total returns - we have to construct \( R_i \), returns attributable to \( i \)-th asset. Theoretically, \( R_i = (p_i q_i)^r_i \) for \( i = 1, \ldots, 9 \) hence a direct approach will be to obtain \( R_i \) simply by multiplying \( y_i \) by \( r_i \). But we will follow a different rather indirect approach to ensure the identity \( \sum_i R_i = R \). First, we will construct a column of proportions, \( c_i \), where \( c_i = y_i / Y \), for all \( i \), given the restrictions that \( 0 < c_i < 1 \), and that \( \sum_i c_i = 1 \). Next, we obtain the weighted \( i \)-th return, \( R^w_i \) as follows:

\[
R^w_i = \left[ \frac{c_i r_i}{\sum_t c_t r_t} \right] \quad \text{for } i = 1, \ldots, 9
\]

For the \( i \)-th relation we must deduct \( R^w_i \) thus obtained from the total returns, \( R \), to arrive at remainder returns attributable to the remaining investments in the portfolio, i.e., \( R^*_i = R - R^w_i \); otherwise we end up with an inadmissible functional form. Obviously, the identity \( R^*_i + R^w_i = R \) holds for any \( i \)-th relation.
A similar approach is needed to arrive at the remainder investment variable, $Z^*_i$, for the $i$-th function, i.e., we need to deduct the dollar value of $i$-th asset from the aggregate investment, $Z^*_i = Z - Z_1^*$, or what is the same, $Z^*_i = Y - y_1^*$. Thus, unlike variables $\Theta$ and $V^*$, the two variables, $R^*$ and $Z^*$ would be different for each of the asset demand functions. Presently, the last two variables are being considered in their aggregate form but later modifications will be made according to the dictates of the estimation model. Whereas it would be legitimate to regard $\Theta$, $V^*$, $R^*$, and $Z^*$ as independent variables for the idealized model, it is obvious that for the simultaneous model, $Z^*$ will have to be excluded from the set of exogenous variables.

5.3 The Idealized Model

As a first approximation we would like to estimate the following equations independently of each other. We will presume, though only for the time being, that the $i$-th asset equation of the system represents a self-contained relationship and is adequate to characterize investment behavior with respect to the $i$-th asset. Treating each of the equation in isolation, we could obtain OLS and maximum likelihood estimators of the coefficients and the desired test values.

For the life insurance portfolio the preceding classification of assets is exhaustive enough in the sense that it takes care of almost the entire spectrum of investment activity of the firms involved. Corresponding to each of these assets, the following equations can be specified:
These nine equations form a set of asset return relationships which may be called as Model I of the life insurance portfolio. Following strictly the notational scheme laid down earlier, each member of the set can be interpreted accordingly. For instance, in the first equation, U.S. government securities, $y_1$, are expressed as a function of the corresponding rate of return, $r_1$, and other independent variables, viz, the risk surrogate, $\theta$, remainder returns, remainder investments, and risk free returns; thus we could go on for all the members of the system in Model I. The streamlined notational scheme can be used to rewrite the entire set as:

$$y_i = \phi \left( r_i, \theta, R^*_i, Z^*_i, V^* \right)$$

where $y_i$ is the $i$-th asset, $r_i$ is the corresponding rate of return, and

$$R^*_i = R - \sum_j R^*_j, \quad \text{for } i, j = 1, \ldots, 9$$

$$Z^*_i = Z - Z_i = \sum_j Z^*_j,$$

$$R = \sum_j R^*_j, \quad Z = \sum_j Z^*_j$$

for $R^*_i \neq R^*_j$, and $Z^*_i \neq Z^*_j$.

The last two relations follow from the definitions of these two variables. We may also add the following aggregated relationships to the basic set of nine equations:

$$y_{10} = \phi \left( r_{10}, \theta, R^{*10}, Z^{*10}, V^* \right)$$

$$y_{11} = \phi \left( r_{11}, \theta, R^{*11}, Z^{*11}, V^* \right)$$
where \[ Z^{*10} = Z - \sum_{j=3}^{5} Z_j; \quad Z^{*11} = Z - \sum_{j=3}^{9} Z_j, \quad \text{and} \]
\[ R^{*10} = R - \sum_{j=3}^{5} R_j; \quad R^{*11} = R - [R^{*1} + R^{*2}].\]

These two equations are for the aggregated category of business bonds and government securities, respectively. The corresponding \( R^* \) and \( Z^* \) variables have been defined accordingly. These relations may be regarded complementary to the basic system in Model I in the sense that the estimation of these two equations is not likely to add to the information already gleaned from the basic set because these are sectoral aggregates. By the same token, we may not expect any significant relation between the specified asset and return variables, given the state of flow of funds data. These will be retained as a part of the single equation system and will be dropped from consideration in the simultaneous set of relationships.

The stochastic version of the functional forms may be expressed in familiar terms as follows:

\[
(5.3.14) \quad X_i = [x_{tk}]_i \quad t = 1, \ldots, T \\
\quad i = 1, \ldots, 9 \text{ or } 11 \\
\quad k = 1, \ldots, K
\]

For the \( i \)-th relation, let vector \( x_k \) be defined as:
\[
\begin{align*}
  x_1 &= r_1, \\
  x_2 &= \theta, \\
  x_3 &= v^*, \\
  x_4 &= R^*i, \\
  x_5 &= Z^*i, \\
\end{align*}
\]

i.e., \( K = 5 \) in a typical relationship. This enables us to write the \( i \)-th equation compactly as:

\[
(5.3.15) \quad y_i = X_i B_i + u_i
\]

If we now specify a suitable distribution for the stochastic error term \( u_i \) and define appropriate transformations to mitigate the non-linearities associated with the expectational elements of \( X_i \), the parameters in (5.3.15) would be accessible. Problems of estimation
will be discussed adequately later, suffice to say here that estimation of the single equation system by OLS procedure would be unsatisfactory in view of the peculiarities of the model. It is proposed that it may be worthwhile to treat the set of relationships in Model I as being interdependent, viz, simultaneity may be regarded as the appropriate feature of the system as a whole. In the following we shall outline a simultaneous model based essentially on the member relations of equations in (5.3.1 to 5.3.9) after some of the variables have been redefined to suit the purpose.

5.4 The Simultaneous Model

The economic rationale of considering the asset return relationships as being interdependent lies in the view that the investment decisions of the life insurance fund managers bear more heavily on the collective features of a given portfolio rather than on the risk and return aspects of a given security considered in isolation. Whereas the individual rate of return still remains the crucial explanatory variable in a given relation, some provisions have to be made for the possible interlinks among various kinds of assets, either in terms of their absolute quantities or in terms of some indices of portfolio characteristics. Two possible indices are the weighted average rate of return and a weighted risk surrogate which can be constructed from the predefined values of individual components, but it is not clear in what manner the inclusion of these two variables in the exogenous set would improve upon the existing specification. Further, we know that the equations of the Model I do incorporate some element of simultaneity through the inclusion of the remainder term with respect to asset investment, the Z* variables. In a truly interdependent system, however,
we would like to incorporate the members of $Z^*$ explicitly among the set of explanatory variables. Accordingly, let us specify the following model:\textsuperscript{10}

\begin{equation}
(5.4.1) \quad y_1 = f( r_1, \theta, R^{*1}, Z^{*1}, V^*, Y_1 )
\end{equation}

\begin{equation}
(5.4.9) \quad y_9 = f( r_9, \theta, R^{*9}, Z^{*9}, V^*, Y_9 )
\end{equation}

or, for the $i$-th relation, we have:

\begin{equation}
(5.4.10) \quad y_i = f( r_i, \theta, R^{*i}, Z^{*i}, V^*, Y_1 )
\end{equation}

Thus, we have nine interdependent equations in the simultaneous system, which may be called as Model II of the life insurance portfolio. The set has been defined over practically the same type of explanatory variables except for the $Z^{*i}$ of Model I. In each of the equations of the Model II, it has been replaced by the remaining endogenous variables in the set.\textsuperscript{11} To write the stochastic version of the Model II let:

- $Y$, ($y_i$, $y_j$ $i,j = 1, \ldots, m$), the endogenous set
- $X$, ($x_s$ $s = 1, \ldots, k$), the exogenous set
- $A$, ($a_s$ $s = 1, \ldots, k$), the coefficients of $x_s$
- $B$, ($b_j$ $j = 1, \ldots, k$), the coefficients of $y^*_1$

$A_i$, $B_i$ are the $i$-th vector of coefficients for subsets, $X_i$, $Y_i$.

For the $t$-th observation and $i$-th relationship the following holds:

\begin{equation}
(5.4.11) \quad y_{it} = \sum_j y_{tj} b_{ji} + \sum_s x_{ts} a_{si} + u_{ti}
\end{equation}

In matrix notations, for all $T$ observations, the entire set could be written as:

\begin{equation}
(5.4.12) \quad Y = YB + XA + U
\end{equation}
We know that if the system of simultaneous relation in (5.4.12) deter-
mines uniquely the values of the current endogenous variables in terms
of the predetermined and random variables of the system, the matrix
\((I - B)\) must be non-singular, whence the system may be solved to yield
the reduced form:

\[(5.4.13) \quad Y = X [I - B]^{-1} + U [I - B]^{-1}, \quad \text{or} \]
\[Y = X \Pi + U\]

But as we shall observe, the uniqueness does not hold because the system
is over-identified. However, after we have subjected the system to the
so-called zero restrictions we could write the i-th relationship for
all T observations as follows:

\[(5.4.14) \quad y_i = Y_i B_i + X_i A_i + U_i \quad i = 1, \ldots, 9\]

which corresponds to the set of relations in (5.4.1, 5.4.9) and
specifically to (5.4.10). Note that:

\[y_i, (y_j \quad j \neq i, \quad i, j = 1, \ldots, 9)\]
\[x_i , (x_{i1} = r_i , \quad x_{12} = \Theta , \quad x_{31} = R^* i , \quad x_{41} = V^* )\]

and \(B_i\), \(A_i\) are the respective vectors of coefficients for the two sets.
Investment in i-th asset is thus regarded as a function of expected
rate of return and other exogenous variables included in the set \(X_i\),
as well as the remaining endogenous variables of the system. We know
that the two variables of the exogenous set, \(r_i\) and \(R^* i\) will be different
for each relation whereas \(\Theta\) and \(V^*\) will remain the same, hence
the non-uniqueness of the reduced form in (5.4.13). Further, for the
system in (5.4.14), the following identities hold:

\[(5.4.15) \quad \sum_i R^* i \equiv R, \quad \text{and} \quad \sum_i y_i \equiv Y\]
For any given period, the sum of weighted returns is equal to total investment income, and the sum of all i-th investments is equal to the total outlay for the entire life insurance group.

In the present form, the equations of the system in (5.4,14) can be estimated by standard procedures of single equation estimation, viz, the 2SLS or LIML; or we could opt for simultaneous estimation procedures, i.e., 3SLS or FIML methods. Whatever technique is adopted we have to provide some reasonable approximation for the crucial variable of the explanatory set—the ex ante rate of return—if any significant results are to be expected. We propose that the best way to arrive at this approximation for the relations of both the idealized and simultaneous models is through a geometrically distributed lag scheme, operating on the observed, ex post rate. This in effect replaces the ex ante rate by its ascertained 'permanent level' of the observed rate, commensurate to the hypothesis that the investment decisions are in fact based on some permanent rate of return as an approximation to the expected rate of return. Admittedly, in a technical sense, the use of the geometrically distributed lags is only one of the available approximations, perhaps the simplest one among them. The possibility of employing other kinds of approximations will be discussed, but a large part of our estimation effort will hinge upon the indicated procedure. The theoretical underpinnings of this approach have already been discussed rigorously in Chapter III.

5.5 Some Comparisons

The models outline above have been specified exclusively for the life insurance portfolio. Conceivably, the generalities of the theoretical construct could permit a wider application than the one
attempted here. We could possibly extend the model to consider investment activity of other kinds of financial intermediaries. For the present purposes, however, we shall limit ourselves to the estimation of life insurance portfolio model only. Next, we would like to consider briefly some of the models that have already been estimated for the portfolios of different types of financial intermediaries. But in the narrow class of models being considered here, only a few attempts have been made so far, and perhaps none with an exclusive focus on the life insurance investment. Generally, these models lack a cohesive theoretical treatment of the proposed hypothesis, i.e., the estimated relation is most often stipulated in an ad hoc manner. The posited relationship, while supposedly relevant in an empirical sense, does not draw upon the established generalizations of investment behavior of a micro-unit. In addition, the postulated equivalence of ex ante variables to their counterpart ex post variables further distorts the obtained results. We shall demonstrate that such an equivalence does not hold, and that a plain substitution of ex post series for ex ante rates is indeed an oversimplification which may be seriously misleading.

In a recent study, Silber has estimated a monetary econometric model of the portfolio behavior of major U.S. financial intermediaries. The model contains forty-two equations, thirty-four of which deal with financial markets for high powered money, government and corporate bonds, mortgages, loans, and various types of deposits; the remaining equations form a rudimentary real sector. The objective is to investigate the portfolio behavior—the demand for various kinds of assets—of financial institutions, their interest rate sensitivity and the response of the real economy to financial impulses. There are
eighteen asset demand equations arranged and specified according to the nature of the activities of financial intermediaries, including three such equations for the life insurance companies. These are:

1. Govt: Bonds = f (i_g, dP/P, dA, A, S_i)

2. Corp: Bonds = f (i_c, i_g, i_m, CB_{t-1}, dP/P, A, S_i)

3. Mortgages = f ([i_c - i_m], [i_g - i_m], M_{t-1}, M_{t-2}, L/V)

where:

i are the respective rates of interest

dp/p, a rate of change in GNP deflator

A, total assets of life insurance companies

S_i, seasonal dummy variables

M, mortgages (acquisitions)

M*, mortgages (commitments)

Asset demand equations of the above type have been estimated for almost all the major groups of financial intermediaries. A brief reference to the stock adjustment principle notwithstanding, Silber has made no attempt to provide any satisfactory theoretical basis for his specifications. Why investment decision makers behave as they do remains largely-explored. The ad hoc nature of the equations becomes all the more clear if we consider the arbitrariness in the inclusion of different kinds of explanatory variables in different equations. No explanation is offered as to why the own rate of interest is not uniformly appropriate for the set of asset demand functions.

From a technical viewpoint, there are considerable problems with the above model. While Silber recognizes the need to approximate the expected rate of returns with some appropriate proxy, he refrains
from doing so in his own estimation effort. We shall demonstrate in Chapter VI that such an approximation is necessary, and if affected, it involves important implications for estimation procedures which can not be easily ignored. The estimates of the three equations listed above good fit and a lack of serial correlation. This result, however, is superficial, because the interest rate variable is not significant in any of the equations listed above, and the high $R^2$ value is due perhaps largely to the inclusion of lagged endogenous variables. For the simultaneous estimation of the system, we must adopt more sophisticated procedures than a straightforward 2SLS scheme because of the needed approximations for the ex ante variables.

Some further points may be noted in this regard.

1. Whereas Silber claims access to commitments data for the life insurance companies, he has, nevertheless, elected to use flow of funds series.

2. All three types of assets are aggregates, involving considerable oversimplification, specially with respect to the rate of interest. The categories can be disaggregated.

3. Considerable improvement can be made by using techniques which provide the facility for needed approximations.

5.6 Comments

The foregoing completes the considerations of the a priori with respect to the life insurance portfolio model which we began in Chapter II. We have discussed operations of the life insurance companies in financial markets, have offered a rigorous treatment of the a priori considerations pertinent to portfolio behavior, and have specified model of estimation of our own. Our next task will be to discuss the
various procedures of estimation keeping in view the peculiarities of the models outlined here. Specifically, we shall demonstrate the procedures of approximation in the framework of single equation estimation of the idealized model as well as the simultaneous model.
Footnotes

1. For details, see section 3.4 of Chapter III.

2. We may extend the analysis a little further. We know that in a normal demand function, quantity \( q_j \) depends on the set of relative prices and the level of income, and that properly \( \frac{dq_j}{dp_j} \) is negative. Further, if \( \frac{dq_j}{dp_j} \) is positive then \( q_j \) and \( q_4 \) are substitutes, if \( \frac{dq_j}{dp_4} \) is negative, they are compliments, otherwise independent. Since we know that, in case of life insurance portfolio model, \( \frac{dy_j}{dr_1} \) is positive on an a priori basis, and that \( r_1 \) is negatively related to \( p_1 \), hence we can state:
   i. if \( \frac{dy_j}{dp_j} \) is negative then \( i \)-th \( j \)-th securities are substitutes.
   ii. if the derivative in i. is positive, the respective securities are compliments
   not compensated for the 'constraint effect' associated with a change in the rate of return, \( r_1 \). In our case the constraint is the dollar value of endowments at the beginning of the period—the initial wealth.
   Also note that all the derivatives listed above are partial derivatives rather than total derivatives.

3. Towards the end of this Chapter we shall discuss some of the empirical work done in this area using alternative specifications of asset demand functions for different types of financial intermediaries. It should be noted, however, that our concern is only with studies involving asset return relationships in contrast to the studies of risk return relationship. The two are obviously concerned with different types of issues.

4. An exception to this statement would be Silber's model of portfolio behavior which seeks to establish the hypothesis that different categories of securities are good substitutes in the lender portfolio. We will examine the model in some detail in section 5.5 of this chapter.

5. If we were to examine the nature of relationship between security return and security risk or portfolio return and portfolio risk, as is the case in the usual types of portfolio models, all we need to do is determine the parametric distribution of the rate of return assuming it to be a random variable. In this case, the second moment is usually taken as the measure of riskiness. However, for a quasi-demand function of the type being stipulated here, we need to define a risk surrogate which would be accessible as well as operationally significant.

6. Since, in the guaranteed rate of interest on policy loans has been generally lower than rates elsewhere, nearly 74% of the families who borrowed on their insurance policy in 1969 chose a policy loan over other types of loans.
Footnotes— (Continued)

7. For similar reasons, a linear combination of \( \Theta_1 \) cannot be used as a surrogate for risk, i.e., it is not possible to construct an index \( \Theta \) such that:
\[
\Theta = \sum_i c_i \Theta_i , \quad \text{for} \quad 0 < c_i < 1 , \quad \sum_i c_i = 1
\]
holds, because, again, \( \Theta_1 \) is not defined as a \( \Theta \)-valued vector.

8. The reason for having dual notational scheme will be clear as we proceed.

9. For the detailed layout of each equation in Model I, please see Appendix I.

10. For the detailed layout of each equation in Model II, please see Appendix I.

11. Notice the equivalence of the \( Z^*i \) and \( Y_1 \) variables. We know that \( Z^*i \) is the total investment in dollar terms minus the dollar value of the \( i \)-th investment appearing in the \( i \)-th equation of the idealized model, section 5.3. On the other hand, variable \( Y_1 \) represents the same magnitudes but in its component form; it consists of the set of remainder endogenous variables. For instance, if \( i = 1 \), \( Y_1 \) would consist of \( (y_2, \ldots, y_9) \), thus excluding the first variable. Hence, typically we will have eight endogenous variables as explanatory variables in the \( i \)-th relation, i.e., \( Y_1 \) consists of \( y_j \) \( j \neq i \) for \( i, j = 1, \ldots, 9 \).

12. Thus, in addition to using the simplified or sophisticated geometric lag scheme for both types of models, we shall also investigate the possibility of using the finite polynomial lag distribution. Much would depend on the nature of rationalizations permissible within the theoretical constructs of lag models and their compatibility with the a priori considerations elaborated thus far. Hopefully these have been embodied in the specification of idealized model as well as the simultaneous model, outlined above.

CHAPTER VI

DISTRIBUTED LAGS:

SOME THEORETICAL CONSIDERATIONS

6.1 Preliminaries

For the estimation of the life insurance portfolio model outlined previously, it will be necessary to provide some reasonable approximation to the crucial explanatory variable of the system— the expected rate of return. A simple substitution of the ex-post values of the observed rates will not be appropriate and very likely will lead to insignificant results. We suggest that a valid approximation may be obtained through the use of a suitable distributed lag scheme, perhaps with the simplest one among them, viz, the geometrically distributed lags. As we shall explain in the following, the use of this particular approximation has the effects of replacing the ex-ante variables with the geometric weighted average of its observed values, which has often been interpreted as the permanent level of the variable in question. We shall offer comments on these aspects as we proceed.

6.2 Lag Distributions

It has been customary to regard distributed lag schemes as being purely ad hoc in nature, or at best derived from some primitive assumptions concerning the behavior of the economic units. From the preceding discussions, specially in Chapter III, it is obvious
that these views are unfounded in the present case. We have offered a rigorous treatment of the portfolio behavior, based on the existing micro-monetary models, and have ventured some generalizations leading to the asset return functions—the demand functions—which are the subject of estimation here. The derived relationship does involve expectational variables which need to be treated properly.

At the outset it may be stated that among the class of distributed lag models, the rational distributed lag scheme (RDL) is a more comprehensive one, because in addition to its general form, it can be approximated to Pascal Lags (PAL) or the geometrically distributed lags (GDL) by placing appropriate restrictions on the lag coefficients. In a general model:

\[(6.2.1) \quad y_t = \sum_{i=0}^{T} w_i y_{t-i} + u_t \quad t = 1, \ldots, T\]

If we normalize the lag sequence \(w_i\), we get:

\[(6.2.2) \quad w_i = 1; \text{ and } 0 < w_i < 1\]

It can be shown that \(w_i\) has a rational generating function \(W(s)\) such that it can be expressed as a ratio of two polynomials, \(A(s)\) and \(B(s)\), i.e., \(W(s) = A(s)/B(s)\). If so, the model in (6.2.1) is said to be characterized by a RDL function and has the convenient property that any arbitrary distributed lag can be approximated to a desired degree of accuracy by a member of its class, e.g., the PAL or the GDL. In the following we shall limit ourselves to the consideration of the GDL scheme, its variants and their implications for estimation procedures.\(^4\)
6.3. The Geometrically Distributed Lags

Among the various types of lags, perhaps the GDL have been used most frequently. The popularity of these models may be attributed to the ease with which behavioral hypothesis such as habit persistence, permanent 'something', or flexible accelerator, etc., can be rationalized in terms of GDL models. The introduction of the well known transformation by L.M. Koyck in 1954 and its subsequent use by Cagan, Nerlov and Friedman in different types of settings, clearly demonstrated the superiority of the model over the classical Gauss-Markov processes. As stated earlier, in case of the life insurance companies' portfolio model the motivation to use GDL scheme stems from the required approximation for expected rates of return.

We will consider essentially three types of geometric lags with an emphasis on the econometric implications of the model, though we will also point out the underlying implications of the hypothesis.

6.3.1 The Direct Expectations Model -- I

The simplest form of GDL will be to consider the following model where variable $x_m$ is specified to be subject to infinite lag structure of the type:

$$\begin{equation}
E \sum_{j=0}^{\infty} \lambda^j x_{t-j}, \ m
\end{equation}$$

Incorporating it in a general scheme:

$$\begin{equation}
y_t = \sum_{L=1}^{m-1} a_L x_{it} + a_m \sum_{j=0}^{\infty} \lambda^j x_{t-j}, m + u_t, \text{ where}
\end{equation}$$

$$\begin{equation}
E(u_t) = 0, \ E(u_t u_{t'}) = \sigma^2 I, \text{ for all } t, t'
\end{equation}$$

We are retaining here the assumption of absence of serial correlation
in the structural error term \( u_t \); this has some interesting implications which will be evident as we proceed. Transforming (6.3.2), we get:

\[
(6.3.4) \quad y_t = \sum_{i=1}^{m-1} a_i x_{it} + a_m I[I - \lambda L]^{-1} x_{mt} + u_t
\]

\[
[I - \lambda L] y_t = [I - \lambda L] \sum_{i=1}^{m-1} a_i x_{it} + a_m x_{mt} + [I - \lambda L] u_t
\]

\[
(6.3.5) \quad y_t - \lambda y_{t-1} = \sum_{i=1}^{m-1} a_i [x_{it} - \lambda x_{it-1}] + a_m x_{mt} + [u_t - \lambda u_{t-1}]
\]

Let, \( y^*_t = y_t - \lambda y_{t-1} \) and \( x^*_it = x_{it} - \lambda x_{it-1} \)

\( x^*_mt = x_{mt} \); \( v_t = u_t - \lambda u_{t-1} \)

Rewriting (6.3.5) in matrix notations:

\[
(6.3.6) \quad Y^* = X^* A + V
\]

Estimation of the parameters \( A \) depends on the distribution of the transformed error term \( V \). Note that the distribution of \( V \) is:

\[
(6.3.7) \quad E(v_t) = 0 , \quad E(v_t v_t') = \sigma^2 \Omega^{9}
\]

It is obvious that whereas the structural model is assumed to be free of autocorrelation, the transformed model does involve autoregressive processes. Accordingly, the variance covariance matrix of the error term \( V \) is quite different from its counterpart in structural relationship. However, if we are interested in OLS estimation of \( A \) in (6.3.6), it requires minimizing:

\[
(6.3.8) \quad V'V = [Y^* - X^* A]' [Y^* - X^*A]
\]

Hence,

\[
\hat{A} = [X^* X^*]^{-1} X^* Y^* \quad (6.3.9)
\]

But \( \hat{A} \) has some undesirable properties; particularly, it is biased
and inconsistent, because:

\[(6.3.10) \quad E(\hat{A}) = E[A + (X^*X^*)^{-1} X^*V] , \quad \text{since} \]

\[(6.3.11) \quad E[X^*V] \neq 0 , \quad \text{therefore} \quad E[\hat{A}] \neq A . \quad \text{Note that} \]

\[(6.3.12) \quad E[y_{t-1} v_t] = -\lambda s^2 \quad \text{hence inconsistency.} \]

A far more serious problem is that \( A \) in (6.3.9) is solely a function of \( X^* \) and \( Y^* \) without involving the peculiar character of \( V \).

In view of the properties of matrix \( \Omega \) --- positive definite, symmetric --- if we suppose that there exists a permissible transformation, \( R' \), on \( y_t \) and \( x_{it} \) such that equation (6.3.6) is obtained, it may be possible to estimate \( A \) through the generalized Aitkin's method. It can be shown that these estimators possess similar properties as the maximum likelihood estimators. But the problem is that the suggested transformation is infeasible.\(^{11}\) Let the transformation matrix be defined as:

\[
R = \begin{bmatrix}
-\lambda & 0 & \ldots & 0 \\
0 & -\lambda & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & \ldots & 0 & -\lambda
\end{bmatrix}
\]

Since \( \Omega = RR' \), it appears that if we transform equation (6.3.2) by the matrix \( R \) we would obtain:

\[(6.3.13) \quad RY = RXA + RU , \quad \text{or in star notations we have} \]

\[Y^* = X^* A + V\]

which is exactly the same as equation (6.3.6) in appearance, but
in fact is different from it with respect to the last vector of the matrix $X^*$ which involves variable $x_m$, the geometric lag variable. The transformation $R$, therefore, does not lead to the desired results.

Next, suppose we wish to operate on equation (6.3.6) to obtain estimates of $A$ from the maximum likelihood procedure. To this effect, let us assume, in addition to the assumptions in (6.3.3), that the structural error term, $u_t$, is normally distributed, i.e., $u_t \sim N(0, \Sigma)$ hence, $v_t \sim N(0, \sigma^2 I)$. This enables us to write the likelihood function of joint normal distribution of $v$ as:

\[
L(v, \Sigma) = (2\pi)^{-T/2} |\Sigma|^{-T/2} \exp\left(-\frac{1}{2} v' \Sigma^{-1} v\right)
\]

In log form:

\[
L(v, \Sigma) = -\frac{T}{2} \ln (2\pi) - \frac{T}{2} \ln \sigma^2 + \frac{1}{2} \ln |\Sigma| - \frac{1}{2} \sigma^2 [Y^* - X^* A]' \Sigma^{-1} [Y^* - X^* A]
\]

since the above distribution is conditional on the value of $\lambda$, therefore, for ascertain optimal $\hat{\lambda}$, we can obtain $\hat{A}$ by solving the following normal equations:

\[
\frac{\partial L}{\partial A} = 0 = -\frac{1}{\sigma^2} [Y^* - X^* A]' \Sigma^{-1} [Y^* - X^* A]
\]

\[
\frac{\partial L}{\partial \sigma^2} = 0 = -\frac{T}{2\sigma^2} + \frac{1}{2} \sigma^2 [Y^* - X^A]' \Sigma^{-1} [Y^* - X^A]
\]

The estimates of parameters are:

\[
\hat{A}(\lambda) = [X^* X^*]^{-1} X^* \Sigma^{-1} Y^*
\]

\[
\hat{\sigma}^2(\lambda) = \frac{1}{T} [Y^* - X^A]' \Sigma^{-1} [Y^* - X^A]
\]

It is obvious that these maximum likelihood estimators would crucially
depend on the value of \( \lambda \). If they yield global maximum value for (6.3.14) we may substitute \( \hat{\lambda} \) and \( \hat{\sigma}^2 \) in (6.3.15) to obtain the concentrated likelihood function:

\[
(6.3.20) \quad L(y, x; \lambda) = - \frac{T}{2} \ln \left( 2\pi \right) - \frac{T}{2} \ln \frac{\sigma^2}{(\lambda)}
\]

The above function is maximized if \( \sigma^2 \) is minimized with respect to some value of \( \lambda \). Unfortunately, if we attempt to obtain an explicit expression for \( \lambda \) by solving \( \partial L/\partial \lambda = 0 \), we shall be led to a highly non-linear equation, from which it will not be possible to get \( \hat{\lambda} \) and \( \sigma^2 \). On the other hand, we observe that for a \( \lambda \), \( \sigma^2 \) is simply the square of standard error of the estimates in the ordinary regression model. If the permissible range of \( \lambda \) is divided in \( n \) equi-distant points, where \( n \) is as large as we wish it to be, then for each \( \lambda_j \quad j = 1, \ldots, n \quad 0 < \lambda < 1 \), we can obtain estimators in (6.3.18) where \( \hat{\lambda}_j \) is such that it minimizes \( \sigma^2 \). Observe that estimators \( \hat{\lambda} \) and \( \hat{\sigma}^2 \) possess the usual properties of maximum likelihood estimators such as asymptotic unbiasedness, efficiency and consistency. \(^{13} \)

Finally, comparing the OLS and maximum likelihood estimators in (6.3.9) and (6.3.18), respectively, we find that they differ in terms of their values, besides the obvious differences in their properties. Both of these involve transformed variables but the maximum likelihood estimators explicitly incorporate the inverse of the variance-covariance matrix of error term \( V \).

This in essence is the maximum likelihood scheme which we shall closely follow in all subsequent expositions of GDL models. For the purposes of this dissertation, eventually we will employ these procedures to obtain empirical results.
6.3.2 The Direct Expectations Model— II

We shall now consider a model essentially similar to the one discussed in the previous section, except that we specify the structural error term, $u_t$, is autocorrelated and that it obeys the first order Markov scheme. Thus in the model:

(6.3.21) \[ y_t = \sum_{i=1}^{m-1} a_i x_{it} + a_m \sum_{j=0}^{\infty} \lambda^j x_{t-j}, \ m + u_t, \ \text{let} \]

(6.3.22) \[ E(u_t) = 0, \ E(u_t u_t') = \Sigma, \ \text{where} \]

\[ u_t = \rho u_{t-1} + e_t, \ E(e_t) = 0, \ E(e_t e_t') = \sigma^2 I \]

Rewriting (6.3.21) in operator notations:

(6.3.23) \[ y_t = \sum_{i=1}^{m-1} a_i x_{it} + I [I - \lambda L]^{-1} a_m x_{tm} + I [I - \rho L]^{-1} e_t \]

Multiplying it by suitable lag operator expressions and rearranging the terms, we obtain:

(6.3.24) \[ y_t = [\lambda + \rho] y_{t-1} + [\lambda \rho] y_{t-2} = \]

\[ \sum_{i=1}^{m-1} a_i [x_{it} - (\lambda + \rho) x_{it-1} + (\lambda \rho) x_{it-2}] + \]

\[ a_m [x_{mt} - \rho x_{mt-1}] + [e_t - \lambda e_{t-1}] \]

Equation (6.3.24) represents a double transformation on the above model; one for the lag parameter, $\lambda$, and the other for autocorrelation parameter, $\rho$. Thus the estimation of (6.3.24) involves ascertaining the optimal values of $\lambda$ and $\rho$ and then estimating $a_i$ for $i = 1, \ldots, m$, such that the associated standard error is minimal.

If we also assume that $u_t$ is normally distributed, we could apply maximum likelihood procedure, outlined previously, to estimate the desired parameters. To write the maximum likelihood function of the
equation (6.3.24), involving the two step transformation, let us proceed as follows.

We know that the transformation for the geometric lag on the model in (6.3.21) leads to equation (6.3.6). Since matrix $\Sigma$ is positive definite, symmetric matrix, there exists a matrix $M$ such that $M'M = \Sigma^{-1}$ holds. Transforming the equation (6.3.6) by $M$ we obtain:

(6.3.25) $MY^* = MX^* + MV$, or, in double star notations:

(6.3.26) $Y^{**} = X^{**} A + e^*$

Notice that (6.3.26) is nothing else but (6.3.24) written in matrix notations. Also, note that the elements of $e^*$ in (6.3.26) are the same as elements of $v$ in (6.3.6), except for the notational differences. Hence the variance-covariance matrix of $e^*$ is also the same:

(6.3.27) $E(e^*e^*_t) = 0$, $E(e^*_t e^*_t) = \sigma^2 \Omega$

This establishes the equivalence of the distribution of the error terms. Writing the log likelihood function of (6.4.26), we have:

(6.3.28) $L^*(...,.) = - \frac{T}{2} \ln[2\pi] - \frac{T}{2} \ln \sigma^2 + \frac{1}{2} \ln |\Omega(G)|$

\[ - \frac{1}{2} \sigma^2 [Y^{**} - X^{**} A]' \Omega^{-1} [Y^{**} - X^{**} A], \]

and the maximum likelihood estimators, conditional on the value of $\lambda$ and $\Phi$ are:

(6.3.29) $\hat{A}(\lambda, \Phi) = [X^{**}' \Omega^{-1} X^{**}]^{-1} X^{**}' \Omega^{-1} Y^{**}$

$\hat{\sigma}^2 (\lambda, \Phi) = \frac{1}{T}. [Y^{**} - X^{**} A]' \Omega^{-1} [Y^{**} - X^{**} A]$
Estimators of $\hat{A}$ and $\hat{\theta}^2$ involve a double search on the pre-specified values of $\lambda$ and $\theta$. We may divide the admissible range of the two parameters in equal or unequal intervals:

For each combination in the $(\lambda, \theta)$ space we may estimate parameters in (6.3.29) and retain the one with the lowest standard error. But this procedure may not lead to the global maximum of the function in (6.3.28), since the number of possible combinations is very large. Ideally, we would like to obtain a unique maximum of the function which may involve a very large number of replications and may very well depend upon how finely the intervals are defined. The exercise would also require obtaining the inverse of matrix which could prove cumbersome for large $T$. In any case the maximum likelihood estimators of (6.3.29) possess the optimal properties.

It may be pointed out at this stage that in order to estimate the models of life insurance portfolio, we shall rely heavily on the procedure discussed in the preceding sections. We know that for the $i$-th asset a typical equation of Model I, Chapter V, is:

$$y_{it} = \sum_j x_{tj} a_{ji} + u_{it}$$

where elements of $x_j$ are defined on page , Chapter V. Performing the double transformation on (6.3.30) for the geometric lags associated with the approximation of the expected rate of return, $r_1$, and
serial correlation, we obtain:

\[
(6.3.31) \quad y_{it} - (\lambda + \rho) y_{it-1} + (\lambda \cdot \rho) y_{it-2} = \\
\sum_{j} a_{ij} [x_{jt} - (\lambda + \rho) x_{jt-1} + (\lambda \cdot \rho) x_{jt-2}] + \\
\cdot a_m [x_{mt} - \xi x_{mt-1}] + [e_{it} - \lambda e_{it-1}]
\]

or, in matrix notations,

\[
(6.3.32) \quad Y** = X** A + V
\]

Notice the similarity of expressions in (6.3.32) and (6.3.26). Hence, it can be asserted that the coefficients of Model I (Chapter V) can be obtained as:

\[
(6.3.33) \quad \hat{A} = [X**' \Omega^{-1} X**]^{-1} X**' \Omega^{-1} Y**
\]

for the i-th equation of the system. The estimators in (6.3.33) are obviously for the equations of the idealized model, ascertained after scanning over the permissible range of the two parameters, \( \lambda \) and \( \rho \).

The estimators of the simultaneous model will be discussed in a later section of this chapter.\(^{17}\)

6.4 The Geometrically Distributed Lags

Some Extensions

A model similar to the above is obtained from the hypothesis of adaptive expectations, which posits a relationship of the following type:

\[
(6.4.1) \quad y = f (x_i, x_m; u)
\]

Despite the apparent similarity between the direct expectations model...
and the above characterization, we must state that adaptive expectations differs from direct expectations model in the manner the expectations are generated. The hypothesis proposes that the decision makers revise their expectations according to their recent most experience with the accuracy of their predictions; and the extent of the revision depends on some fraction of the error they made in the previous period. Thus we end up with a relationship of the type:

\[(6.4.2) \quad x^*_t - x^*_{t-1} = \lambda (x^*_{t-1} - x^*_{t-2})\]

Solving for \(x_t\) we get:

\[x^*_t = \sum_{j=0}^{\infty} (1 - \lambda)^j \lambda x_{t-j-1}\]

Substituting it in the general model, we obtain the following structural equation:

\[(6.4.3) \quad y_t = \sum_{i=1}^{m-1} a_i x_{it} + a_m (1 - \theta) \sum_{j=0}^{\infty} \theta^j x_{t-j-1} + \mu_t\]

Several interpretations of the model in (6.4.3) are possible depending on the nature of the problem at hand. In his pioneering work, Koyck suggested that capital stock, \(K\), may be considered to be proportional to some weighted average of previous output, \(Q\), extending over many periods. That is:

\[(6.4.4) \quad K_t = a (1 - \lambda) \sum_{j=0}^{\infty} \lambda^j Q_{t-j}\]

Since the above can not be estimated in its present form due to the non-linearity of parameter \(\lambda\), hence let us write an equation for \(K_{t-1}\), multiply it by \(\lambda\), and subtract from (6.4.4) to arrive at:

\[(6.4.5) \quad K_t = a (1 - \lambda) Q_t + \lambda K_{t-1}\]

This equation represents the well known Koyck transformation which has
been used extensively in econometric investigations. For instance, in the studies on consumption function, in one of the earlier attempts, T.M. Brown explored the nature of lagged effects of income changes on consumption levels due to habit persistence. But a more illustrative example would be Friedman's 'permanent income' hypothesis, which states that consumption levels depend on permanent income, approximated by a weighted average of previous income levels. The technique has already spilled over into other areas. Thus, in the case of agricultural supply models, if $x^*_t$ of (6.4.2) is interpreted as the expected price level, we can state, using the expression in (6.4.3), that the supply in year $t$ depends on a string of previously experienced price levels, the recent ones having a greater degree of influence, i.e., supply depends on some 'permanent level' of prices. Eventually, we end up with a hypothesis which is based on permanent 'something' type of a priori reasoning.

The foregoing raises some important issues for the life insurance portfolio model proposed here. The foremost being that if the direct expectations or adaptive expectations generate equivalent transformed relations, though in a technical sense only, it may be worthwhile to seek rationalizations in terms of the latter. The processes are feasible indeed, but it is doubtful whether the imperatives of the a priori reasoning are satisfied by ad hoc formulations. Besides, a model based on the generalizations of behavior under conditions of risk and derived from the established processes of maximization has a more sound footing than the one obtained from ad hoc hypothesis of adaptive expectations. The former provides us a more reliable basis of formulating model of investment behavior involving risky ventures.
Applying appropriate transformations on (6.4.3) for parameters \( \theta \) and \( \phi \) to mitigate the dual non-linearities, we obtain:

\[(6.4.6) \quad Y** = X** A + e^* \]

For the \( t \)-th observation, the elements in (6.4.6) are:

\[ y**_t = y_t - (\theta + \phi) y_{t-1} + (\phi \theta) y_{t-2} ; \]
\[ x**_{it} = x_{it} - (\theta + \phi) x_{it-1} + (\phi \theta) x_{it-2} ; \]
\[ x**_{mt} = x_{mt} - \phi x_{mt-1} ; \text{ and } e^*_t = e_t - \theta e_{t-1} \]

Equation (6.4.6) is similar to (6.3.26), except for the \( t \)-th index of the last column of \( X** \), and parameter \( \theta \); hence all the considerations of the section 6.3.2 are pertinent here with respect to the estimation of parameters. There is a complete equivalence between the two models. In view of this we can write the estimators as:

\[(6.4.7) \quad A(\theta, \phi) = [X** \Omega^{-1} X**]^{-1} X** \Omega^{-1} Y** \]

In spite of the similarity, however, the use of direct expectations or adaptive expectations hypothesis would depend on the postulates of underlying behavior.

6.4.1 Stock Adjustments or Rate Approximations?

Within the framework of life insurance portfolio model, we have so far concentrated on the processes of generating expectations on the appropriate rates of return. We have also examined the implications of linearizing the parameter associated with expectational variable for the idealized model. In some cases, stock adjustment hypothesis has been proposed as the appropriate behavioral postulate,
concerning investment in risky assets. In general terms, the hypothesis suggests that in the model \( y = f(x) \), adjustments occur on variable \( y \) in the following manner:

\[
(6.4.8) \quad y_t - y_{t-1} = h(y^*_t - y_{t-1})
\]

the model is characterized by stock adjustment process. Further, specify,

\[
(6.4.9) \quad y^*_t = f(\cdot), \quad \text{or} \quad y^*_t = a_1 + a_2 x_t + u_t
\]

which describes the functional form of the adjustment variable. Interpreting the above two equations would require a detail of the problem under consideration. In aggregate investment analysis, the flexible accelerator hypothesis furnishes us a good example of the application of stock adjustment principle. It is stipulated that the desired capital stock, \( K^*_t \), is some function of the level of output, \( Q_t \), and that the net investment, in turn, is some function of \( K^*_t \) and the actual capital stock, lagged one period. In terms of the variables of equations (6.4.8) and (6.4.9) we can write the transformed relation as:

\[
(6.4.10) \quad y_t = h [a_1 + a_2 x_t + u_t] + (1 - h) y_{t-1}
\]

where \( y_t \) is to be interpreted as the capital stock and \( x_t \) the level of output.

The practice has been carried over to the financial investment models with some measure of success. As noted earlier, attempts have been made to rationalize asset demand functions, or the like of it, in terms of adjustment principle. The important point to be noted here is that adjustments occur on the dependent variable, \( y_t \), rather than on the independent variable. In the case of life insurance portfolio
models, the behavioral hypothesis of stock adjustment seems to imply that expectations are operative on the asset variable, \( y_i \), rather than on the rate of return, \( r_i \). This represents a considerable departure from the stated propositions of Chapter III. If we were to operate with this scheme, we need no longer worry about linearization of the parameter associated with \( r_i \), though in the process we have acquired a similar parameter associated with \( y_i \). The technocracy of the models is the same, but the underlying behavioral propositions are quite different. Some other peculiarities of the model will be noted as we proceed along. For the general case, let us consider:

\[
(6.4.11) \quad y^*_t = \sum_{i=1}^{m} a_i x_{it} + u_t
\]

Substituting for \( y^*_t \) in (6.4.9) and rearranging terms:

\[
(6.4.12) \quad y_t - (1-h) y_{t-1} = h \sum_{i=1}^{m} a_i x_{it} + h u_t
\]

In matrix notations:

\[
(6.4.13) \quad Y^* = X^* A + u^*
\]

Given the assumptions regarding the error term \( u_t \):

\[
E(u_t) = 0, \quad E(u_t u_{t'}) = \sigma^2 I, \quad \text{notice that}
\]

\[
E(u^*_t) = h E(u_t) = 0, \quad E(u^*_t u^*_{t'}) = h^2 \sigma^2 I
\]

Note that the transformed error term, \( u^*_t \), is free of autocorrelation, if to begin with, the structural error terms are assumed to be so. We would recall that one of the major problem with the previous two models has been the induced autocorrelation in the transformed model. Further, since \( E[y_{t-1} u_t] = 0 \), and \( E[x_{it} u_t] = 0 \) holds by our assumptions of
the structural model, therefore same is true of the transformed model. The implication is that the OLS estimators of \( A \) will not be beset with the problems that we considered before in the context of expectations models.

In view of the above it would appear that the stock adjustment model has a built-in superiority over the previous ones, besides the considerations of the underlying hypothesis. This is not necessarily true because if we were to solve the difference equation in (6.4.12), we obtain:

\[
(6.4.14) \quad y_t = I(I - cL)^{-1} h \sum_{l=1}^{m} a_l y_{lt} + h I(I - cL)^{-1} u_t
\]

This suggest that the partial adjustment hypothesis produces a GDL model, but one in which the error term is also subject to same type of lag structure as the explanatory variable, because if we specify:

\[
(6.4.15) \quad \eta_t = h I(I - cL)^{-1} u_t
\]

we get:

\[
(6.4.16) \quad v_t = c v_{t-1} + h u_t
\]

which indeed is the first order Markov scheme with parameter \( c \).

Thus, if the two parameters are postulated to have a mechanistic relationship, OLS procedure is applicable and gives us \( \hat{A} \) with desirable properties.

Apart from this, the adjustment model is beset with a more serious problem than is usually recognized, i.e., if we compare the transformed equation (6.4.12) with the one obtained from the transformation of an explicitly autocorrelated model, the two would be statistically indistinguishable in the sense that from the empirical results alone it would not be possible to determine whether we are
dealing with a partial adjustment model or with a simple regression in which the error term exhibits a first order autoregressive structure. For purposes of this exposition, let us suppose, however, that there is autocorrelation, i.e., the structural error term obeys:

\[ E(u_t) = 0, \quad E(u_t u_t') = \Sigma, \quad \text{where:} \]

\[ u_t = \rho u_{t-1} + e_t, \quad \text{and} \quad e \sim (0, \sigma^2 I) \]

Transforming (6.4.12) for autocorrelation parameter , we have:

\[
(6.4.16) \quad y_t - c y_{t-1} = h \sum_{i=1}^{m} a_i x_{it} + h \left[ I - \rho I \right]^{-1} e_t \\
(6.4.17) \quad y_t - [c + \rho] y_{t-1} + [c.\rho] y_{t-2} = h \sum_{i=1}^{m} a_i \left[ x_{it} - \rho x_{it-1} \right] + h e_t
\]

In matrix notations:

\[
(6.4.18) \quad Y^{**} = X^{**} A + e^{*}
\]

The generalized Aitken's estimators of \( A \) and \( \rho \) are:

\[
(6.4.19) \quad \hat{A}(h, \rho) = [X^{**} X^{**}]^{-1} X^{**} Y^{**} \quad \quad \text{or:} \quad \hat{A}(h, \rho) = [X^{*} \Sigma^{-1} X^{*}]^{-1} X^{*} \Sigma^{-1} Y^{*}
\]

\[
(6.4.20) \quad \hat{\sigma}^2(h, \rho) = 1/T [Y^{**} - X^{**} A]'[Y^{**} - X^{**} A]
\]

If instead, we follow the maximum likelihood procedure it can be shown that we obtain similar estimators as in (6.4.16). Again, the technique involves search over the permissible range of values of \( h \) and \( \rho \) such that the standard error of estimates is minimized. 25
6.4.2 The Truncation Procedure

Ideally, we would like to estimate the parameters of life insurance portfolio model as given in equation (6.3.33). But these are operationally inaccessible, as normally is the case with the ideal estimators. Obtaining (6.3.33) involves a very costly computational procedure. In the following we will consider a scheme which reduces this problem considerably. As a matter of fact, it has been used in developing the Fortran routine for scanning method.

Consider a simple geometric lag model involving autocorrelated structural error terms:

\[(6.4.21) \quad y_t = \sum_{i=1}^{m-1} a_i x_{it} + a_m \sum_{j=0}^{\infty} \lambda^j x_{t-j,m} + u_t\]

\[E(u_t) = 0, \quad E(u_t u_t') = \Sigma\]

Let the infinite sum associated with the geometric lag variable, \(x_m\), be truncated at an appropriate interval so as to emphasize the effects of operator conditions, vis-a-vis some initial conditions according to the dictates of a priori reasoning. Thus if we truncate the above sum at \(j = t-1\) and rewrite the above as:

\[(6.4.22) \quad y_t = \sum_{i=1}^{m-1} a_i x_{it} + a_m \sum_{j=0}^{t-1} \lambda^j x_{t-j,m} + a_m \sum_{j=t}^{\infty} \lambda^j x_{t-j,m} + u_t\]

We can also express the last sum as:

\[a_m \sum_{j=t}^{\infty} \lambda^j x_{t-j,m} = a_m \lambda^t \sum_{s=0}^{\infty} \lambda^s x_m, o-s = a_0^* \lambda^t\]

The term \(a_0^*\) is the truncation remainder; a consistent estimator of the summation part in \(a_0^*\) will be, in two variable case, \(a_0/a_1\).

Redefining the terms:

\[(6.4.23) \quad y_t^* = y_t, \quad x_0^* t = \lambda^t, \quad x_m^* t = \sum_{j=0}^{t-1} \lambda^j x_{t-j,m}\]
\[
\begin{align*}
a^*_i = a_i, \text{ for } i = 1, \ldots, m; \quad a^*_0 = a_m \sum_{j=0}^{t-1} \lambda^j x_{m-j}, 0-s
\end{align*}
\]

and finally, \( x^*_{it} = x_{it}, \) for \( i = 1, \ldots, m-1. \) This enables us to write (6.4.22) in matrix notations as:

\[
(6.4.24) \quad y^* = x^* A^* + U
\]

Transforming (6.4.24) for autocorrelation by matrix \( M \) defined earlier, we get:

\[
(6.4.25) \quad M y^* = M x^* A^* + M U, \quad \text{whence:}
\]

\[
(6.4.26) \quad y^{**} = x^{**} A^* + U^*
\]

The generalized Aitkin's estimators of parameters in (6.4.26) are:

\[
(6.4.27) \quad \hat{A}^*(\lambda, \rho) = [x^{**} x^{**}]^{-1} x^{**} y^{**}, \quad \text{or;}
\]

\[
(6.4.28) \quad A^*(\lambda, \rho) = [x^* \Sigma^{-1} x^*[^{-1}] x^* \Sigma^{-1} y^*
\]

It is obvious that we could also obtain an estimator for \( \hat{\sigma}^2 \) from the equation (6.4.26); and that these estimators possess the maximal properties. 27

Notice that the truncation scheme generates a synthetic variable in the process of transformation for the geometric lag parameter and is associated with the expectational variable of the equation under consideration. It may very well exhibit certain statistical properties similar to the ones obtained for expectational variable.

6.5 Simultaneous Estimation

The estimation of simultaneous model of life insurance portfolio (Model II, Chapter V) poses some additional problems which
need our attention. To begin with, the specification of the model requires some corrections for the problem of inconsistency arising from the simultaneity of the model. In addition, we have to follow the standard procedures for linearizing lag and autocorrelation parameters. It will be recalled that the \( i \)-th equation of the model is:

\[
y_{it} = \sum_j y_{tj} b_{ji} + \sum_s x_{ts} a_{si} + u_{ti}
\]

\( i,j = 1, \ldots, m; \quad s = 1, \ldots, n; \quad t = 1, \ldots, T \)

In matrix notations:

\[
y_{i} = Y_i B_i + X_i A_i + U_i
\]

The \( y_j \) are the endogenous variables of the system— the assets in the life insurance portfolio— and \( x_s \) are the exogenous variables, the most important one among them being the \( i \)-th expected rate of return. In specific terms, our problem is to eliminate the inconsistency before any transformations are affected for the non-linear parameters.

Our only recourse seems to be the use of instrumental variables, specially since the lagged values of both endogenous and exogenous variables are involved in the transformations. In an earlier work, Liviatan proposed application of standard instrument variable technique to estimate sophisticated GDL model.\(^{29}\) He uses lagged values of exogenous variables appearing in the equation as instruments which has drawn criticism to the effect that such instruments are unsatisfactory in the case of time series estimation.\(^{30}\) In fact Hannan has shown that Liviatan's estimators are inefficient, and has proposed an estimator based on spectral technique. Specifically, he has used a
method which is asymptotically equivalent to generalized Aitkin's estimators, given that consistent estimates of elements in covariance matrix $\Sigma$ are available. It has been argued that this method is relatively more useful when the knowledge of stochastic processes of structural error terms is minimum.\(^{31}\)

Recently, some schemes have been proposed dealing with the use of instrumental variables in the estimation of simultaneous systems where lags are generated by the autoregressive processes only.\(^{32}\) It should be noted that whereas autoregressive processes do generate a transformed relation involving lagged explanatory variables, the implications of resulting lag structure for the estimated parameters are not the same as the ones emanating from a GDL model. The reason for this lies in the distributional aspects of reduced form error terms. For the moment, however, we are concerned only with simultaneous autoregressive scheme. Consider the model:

\[(6.5.3) \quad BY + AX = U\]

The dimensions will be defined as we proceed. For all the equations in (6.5.3) let us assume that the model obeys first order autoregressive scheme. Hence:

\[(6.5.4) \quad U = R U_{-1} + E, \quad \text{where:} \quad U_{-1} = BY_{-1} + AX_{-1}\]

Substituting and rearranging, we can write the reduced form as:

\[(6.5.5) \quad Y = B^{-1} R B Y_{-1} + B^{-1} R A X_{-1} - B^{-1} A X + B^{-1} E\]

We can write the first equation of the system as:

\[(6.5.6) \quad y_1 = \sum_j b_{1j} y_{jt} + \sum_s a_{1s} x_{st} + u_{1t}\]
where, \( j = 1, \ldots, m \); \( s = 1, \ldots, n \). In matrix terms:

\[
(6.5.7) \quad y_1 = -B_1 Y_1 - A_1 X_1 + U_1 , \quad \text{where}
\]

\[
U_1 = r_{11} U_{1-1} + e_1
\]

Writing lagged equation for (6.5.7) we have:

\[
(6.5.8) \quad y_{1-1} = -B_1 Y_{1-1} - A_1 X_{1-1} + U_{1-1}
\]

The index (\( -1 \)) should be read as (\( t - 1 \)) for each observation.

Multiplying (6.5.8) by \( r_{11} \) and subtracting from (6.5.7):

\[
(6.5.9) \quad y_1 - r_{11} y_{1-1} = -B_1 [Y_1 - r_{11} Y_{1-1}] - A_1 [X_1 - r_{11} X_{1-1}]
\]

\[
+ [r_{11} U_{1-1} + e_1]
\]

To estimate (6.5.9) a two step procedure is followed as in the estimation of two stage least squares. In the first step, \( Y_1 \) is to be regressed on a set of instrumental variables which include exogenous variables as well as all the lagged exogenous variables, and lagged endogenous variables, appearing in the equation (6.5.9) only. Of course, if \( r_{11} \) is equal to zero the whole system in (6.5.9) collapses to the simple two-stage least square scheme. In the second step, \( Y_1 \) thus obtained is substituted for \( Y_1 \) in (6.5.9) and estimators for \( A_1 \) and \( B_1 \) are obtained in the usual manner. This procedure is repeated until for some value of \( r \), \( A_1 \) and \( B_1 \) are optimal. This is essentially an iterative two-stage least square procedure. It can be shown that the resulting estimators are consistent.33

We are now in a position to offer some comments on Sargan’s iterative instrumental variable approach for the estimation of
autoregressive simultaneous systems. In the first stage of estimation of i-th equation in the system, Sargan proposes using a set of instrumental variables which include (1) all the lagged endogenous variables, \( Y \), (2) all the lagged exogenous variables, \( X \), and (3) all the pre-determined variables, \( X \), appearing in the equation as well as (4) all the lagged exogenous variables contained in the entire system. This is a long list of instrument variables. The procedure may not be feasible if \( T \) is not very large relative to the number of variables being used and is superfluous in so far as consistent estimators can be obtained for the same system by using a small number of instruments. Also, it should be remembered that Sargan's scheme considered here deals with a reduced model of autocorrelated structures only. Distributed lags would involve further complications in so far as we would have to consider behavior of the transformed error terms.

A further simplification has been proposed over the iterative 2SLS procedure. Notice that the iterative 2SLS, though computationally less expensive than Sargan's 2SLS, it still requires knowledge of the reduced form and saves instruments only to the extent that a given exogenous variable appears in only one equation of the model. The number of instruments will be substantially reduced if we assume that the \( r_{ij} \) are the same for all equations, say \( R \) such that \( R = r_{0}I \). Hence:

\[
(6.5.10) \quad Y = r_{0} Y_{-1} + B^{-1} A [X - r_{0} X_{-1}] + [B^{-1} (R - r_{0}I) U_{-1} + B^{-1} E]
\]

Apparently, the above assumption is unrealistic, but if we could so assume, it is obvious that first part of the error term in (6.5.10)
is zero; hence in the estimation of the equation (6.5.9), in the first stage, \( Y_1 \) should be regressed on \( Y_1 \) and \([X_2 - r_0 X_2^{-1}]\), where \( X_2 \) denotes all variables not appearing in the first equation. Since the number of variables in \( X_1 \) is likely to be small relative to the number included in \( X_2 \), the number of instruments saved by using the difference \([X_2 - r_0 X_2^{-1}]\) rather \( X_2 \) and \( X_2^{-1} \) separately, is likely to be substantial. Thus we reach closer to Sarga's 2SLS without apparently incurring the disadvantages associated with it.

Returning to our model in (6.5.2), it is obvious that in the first stage of estimation we have to eliminate the stochastic elements in \( Y_1 \) by using a feasible number of instruments. We shall list these variables for each member of simultaneous system in the forthcoming chapter. In the next stage of estimation, we will use scanning method to estimate coefficients \( B_i \) and \( A_1 \) corresponding to some optimal values of \( \rho \) and \( \lambda \). The final estimating equation for the \( i \)-th asset is, after linearization and correction for inconsistency:

\[
(6.5.11) \quad y_{**i} = y_{**i} B_i + X_{**i} A_1 + V_i 
\]

If we let, \( w = y_{**i} ; \ Z = [Y_{**i}, X_{**i}] \); \( d = [B_i] \), we can write:

\[
(6.5.12) \quad w = Z d + v 
\]

The double-scanned, maximum likelihood estimators of \( d \) conditional on the globally optimum values of \( \hat{\lambda} \) and \( \hat{\rho} \) are:

\[
(6.5.13) \quad d = [Z' \Omega^{-1} Z]^{-1} Z' \Omega^{1} w 
\]

We know from the discussions in section 6.4.2 that the scanning
method is operational only for the truncated version of the model. Hence, the results on scanned estimators would be available only after suitable modifications have been affected.

6.6 Comments

The material discussed in the preceding section covers the basic issues in the specification and estimation of geometrically distributed lags in the framework of the models of estimation of life insurance portfolio. We have excluded the general polynomial and rational distributed lags from our study. We have also refrained from providing esoteric proofs of the results obtained; instead we have concentrated on the analytical and technocratic aspects of the systems considered here.
Footnotes

1. Recall that the expected rate of return for the $i$-th asset, $r_i$, has been defined as:

$$r_i = \left[ \frac{p_{i+1}}{p_i} \right] - 1$$

where $p_{i+1}$ is the expected price and $p_i$ is the current price of the $i$-th security. Unquestionably, the variable $r_i$ involves expectations and must be treated accordingly while estimating the demand functions of the Chapter V.

2. We shall also examine, rather briefly, the possibility of using other lag distributions, e.g., the general polynomial or the rational distributed lags, of which the GDL are a special case. In fact a purely technical approach may be adopted, i.e., let the data determine itself the appropriate kind of lag distribution. In view of our limited objectives, however, we shall concentrate on the geometrically distributed lags only.


4. Briefly, the Pascal Lags and its approximation to geometrically distributed lags is as follows. In view of the restriction (6.2.2) on $w_i$, if we assume that the $w_i$ are characterized by a negative binomial distribution such that:

$$i. \quad w_i = (1-\lambda) \left[ r + i - 1 \right]^i \lambda^{i}; \quad 0 < r, \lambda < 1$$

then in the model

$$ii. \quad y_t = W(L) x_t + u_t = A(L)/B(L) x_t + u_t$$

we can have:

$$iii. \quad y_t = [1 - \lambda]^r \frac{I}{[I - \lambda L]^{-r}} x_t + u_t$$

whose lag structure depends on two parameters, $\lambda$ and $r$. By an appropriate choice of the two parameters we can construct lag coefficients which first increase and then decline, i.e., the resulting lag structure has a Pascal distribution. It is evident that if $r=1$ we end up with a monotonically declining coefficient, $\lambda$, the parameter of the geometrically distributed lags scheme.


9. To obtain $\Omega$ let us proceed as follows:

$$E(v_t v_{t'}) = E(u_t - \lambda u_{t-1}) [u_t - \lambda u_{t-1}]'$$

$$= E(u_t u_{t'}) - \lambda E(u_t u_{t-1}) - \lambda E(u_{t-1} u_{t'}) + E(u_{t-1} u_{t-1})'$$

If $t = t'$, 
$$E(v_t v_{t'}) = \sigma^2 (1 + \lambda^2)$$

If $t = t'-1$, 
$$= -\sigma^2 \lambda$$

If $t = t'+1$, 
$$= -\sigma^2 \lambda$$

otherwise, 
$$= 0$$

Hence matrix $\Omega$ has only three non-zero diagonals; is symmetric and has dimensions of $(T-1)(T-1)$. 

$$
\begin{bmatrix}
\lambda^2 & -\lambda & 0 & \cdots & 0 \\
-\lambda & 1 + \lambda & -\lambda & \cdots & 0 \\
0 & -\lambda & 1 + \lambda & \cdots & -\lambda \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1 + \lambda \\
\end{bmatrix}
$$

Notice that matrix $\Omega$ is similar to the inverse of variance-covariance matrix of the structural error term of an autocorrelated model. More on this will follow in a later section.

10. For a detailed discussion of the underlying procedures and proofs of properties, see:


Footnotes—(Continued)


12. The maximum likelihood estimator of $\sigma^2$ in (6.3.19) is different from other estimator, $\hat{\sigma}^2$, by the proportionality factor $T-m/T$, where for large $T$, $T-m/T$ is unity.

$$\hat{\sigma}^2 = [V'\Omega^{-1}V]/[T-m-2], \quad \hat{\sigma}^2 = [(T-m)/T] \sigma^2$$

The two estimators, however, are equivalent for large $T$.


14. The matrix $M$ is:

$$M = \begin{bmatrix}
-p & 0 & \cdots & 0 \\
0 & -p & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & -p \\
\end{bmatrix}$$

Notice the similarity of this matrix with matrix $R$, section 6.3.1.

15. For large $T$, the inverse of $\Omega$ is given by:

$$\hat{\Omega}^{-1} = \frac{1}{1-\lambda} \begin{bmatrix} \lambda & \cdots & \lambda \\
\vdots & \ddots & \vdots \\
\lambda & \cdots & \lambda \\
\end{bmatrix}$$

Notice the similarity of this matrix with matrix $\Lambda$.

Footnotes— (Continued)

17. In view of the transformations for the non-linearities associated with $\lambda$ and $\rho$, and the peculiarities of the scanning method, ordinary regression programs would clearly be inadequate. To compute the coefficients of the model in (6.3.33), a new Fortran routine was developed at the University of Pennsylvania. The program is now operational at CCIS, Rutgers University.

18. In a study of inflationary trends, Cagan (footnote #6) suggested a model:

i. $y_t = a p^* t + u_t$, where $p^*_t = b \sum_{j=0}^{\infty} c^j p_{t-j}$
and is obtained from:

$c(t) = (1-b) t' = t+1$

ii. $p^*_{t-1} - p^*_t = b (p_t - p^*_t)$

In view of the above we can write:

iii. $y_t = a [ b \sum_{j=0}^{\infty} (1-b)^j p_{t-j} + u_t$

Re-indexing the model in iii we find that the resulting equation is very similar to the one given in (6.3.6).


21. Formally, consumption level, $C_t$, is suggested to be a function of the permanent level of income, $Y_p$:

i. $C_t = k [ Y_p ]_t$, where $Y_{pt} = \sum_{j=0}^{\infty} Y_{t-j}$

Applying Koyck's transformation, we obtain:

ii. $C_t = k Y_t + \lambda C_{t-1}$, or, in stochastic version,

iii. $C_t = k Y_t + \lambda C_{t-1} + [u_t - \lambda u_{t-1}]$


22. For instance, while considering Silber's model in Chapter V we observed that the asset demand function for different types of financial intermediaries have been rationalized in terms of stock adjustment principle. Similar is the case with Jeffer's model.
Footnotes—(Continued)

23. In case of agricultural supply models, we can rewrite (6.4.8) and (6.4.9) to interpret $y_t$ as the agricultural supply, and $x_t$ the corresponding market price; implying that the desired supply is a function of prevailing price, and that in a given period, adjustments on $y_t$ are constrained by exogenous factors, resulting only in partial adjustment. This type of model has been widely used with a considerable degree of success. Like its counterpart in expectational models, the adjustment parameter is a positive fraction. For details, see:

Nerlov, *loc. cit.*, (footnote #7)


24. This formulation is quite close to Koyck's transformation introduced earlier. It has led to the belief that the proponents of stock adjustment principle combined the adaptive expectations model with Koyck's reduction procedure to provide an acceptable rationale as well as a feasible estimation technique, applicable to a wide variety of problems.

25. It is possible that the assumption of the constancy of adjustment parameter is invalid, specially in the case of optimal adjustment paths being unknown due to continuously changing levels of equilibrium magnitudes. Worse yet, as Waud has pointed out, a misspecification of the kind where estimation from the reduced form or its counterpart is not permissible due to the possibility that the standard model involves expectational elements on both sides of the equation, may preclude meaningful estimates. For example, consider:

1. $y^* = f(x^*, u)$, where

2. $y^*_t = b x_t + u_t$, and is relevant to (6.4.8); and

3. $x^*_t = \sum_{j=0}^{\infty} c(1-c)^j x_{t-j-1}$, and pertains to (6.4.2)

If the resulting reduced form is the valid formulation on some a priori basis, estimation of (6.4.6) or (6.4.19) is unwarranted.
Footnotes-- (Continued)

See:

26. For $j = t-1$, appears in the truncation remainder, then after that appears, involving initial observations on $x_m$.

27. For proofs, see;
   Amemiya and Fuller, loc. cit., (footnote #10)

28. Dhrymes, loc. cit., (footnote #10)


30. Dhrymes, loc. cit., (footnote #16)


32. Amemiya and Fuller, loc. cit., (footnote #10)


34. Sargan, loc. cit., (footnote #32)

35. Sargan, loc. cit., (footnote #32)
CHAPTER VII

ESTIMATION AND RESULTS

7.1 Preliminaries

We are now in a position to consider the results of empirical investigation of the life insurance portfolio model, specified in Chapter V. We would like to examine the behavior of the so-called asset demand functions, both in the static and dynamic frameworks, with an emphasis on the role of rate of return as the pivotal variable. For reasons set forth earlier, the appropriateness of this variable for each of the specified relationships will be of special interest to us. Furthermore, to determine the validity of the central propositions of this study, we would like to compare results obtained from the two broad categories of estimation procedures considered in Chapter VI, viz, (1) OLS, 2SLS, and LIML methods, and (2) the double scanned, maximum likelihood method (DSML), involving non-linearities. These two sets of estimates will be examined closely in terms of appropriately defined test statistics.

The scheme is as follows. We will offer four basic sets of estimates corresponding to the Models I and II, discussed in Chapter V. In addition, we shall offer two sets of estimates on an abbreviated model which would be specified in this chapter at a later stage.

A. For the single equation, idealized system, Model I, we will have:

1. OLS estimates of nine core equations of the system as well as the two aggregate relationships. In each of the demand functions
we will utilise the ex post value of the rate of return, \( r_1 \), for a given equation. These will be the non-scanned estimates.

2. Double Scanned Maximum Likelihood estimates (DSML) of the single equation system, where we will provide for scanning on the non-linear parameters associated with the geometric lag variable and the autocorrelated errors. Obviously, in this case we will treat the rate of return as the expected variable, \( r^*i \), instead of using the observed values.

B. For the simultaneous system, Model II, we will have:

1. 2SLS estimates, non-scanned, treating the rate of return variable as in the case of A.1, and

2. Limited Information Maximum Likelihood (LIML) estimates;

3. Double Scanned Maximum Likelihood estimates (DSML) of the simultaneous system, treating the rate of return variable as in A.2, the \( r^*i \), involving lags, and autocorrelation.

C. We will specify an abbreviated life insurance portfolio model, consisting of the rates of return and remainder investment as the only explanatory variables. We will offer estimates for this model (Model I-a) obtained from OLS and DSML procedures to provide a comparison of the two techniques.

Complete results obtained from a particular procedure for a given specification of the life insurance portfolio model will be listed in a single table, though at times this may become rather unwieldy. For purposes of comparisons, however, two summary tables will be included focusing on the main features of DSML estimation—the non-linearities associated with lag and autocorrelation parameters.
7.2 A Note on Interpretations

The OLS, 2SLS, and LIML estimates are fairly straightforward. Within these frameworks we have proceeded to obtain estimates of coefficients and their test values, for a given relationship, in a purely classical manner. The special features of the DSML procedure, however, need to be reiterated briefly, both for the idealized and simultaneous systems. In case of the idealized system, DSML method requires double transformation on a given structural relationship to eliminate non-linearities associated with autocorrelation and geometric lag parameters. The maximum likelihood estimates of the transformed relationships are then obtained with reference to some optimal values of these parameters as determined by the scanning procedure.

Essentially, a similar approach has been adopted with respect to the DSML estimates of simultaneous systems, except that in the initial stage we have tried to eliminate the stochastic element of the explanatory endogenous variables in a structural relationship—a standard method adopted for the correction of the inconsistency problem in simultaneous estimation.

We have maintained that if in a relationship expectational variables are involved, a feasible and effective method of approximation to their counterpart observed values would be via geometric lags. Transformation for the non-linearities associated with the lag parameter \(\lambda\), however, induces autoregressive processes of its own which need to be neutralized with yet another transformation for the autocorrelation parameter \(\rho\). We expect the DSML estimates to perform better than the OLS estimates in terms of the appropriate test statistics.
For these reasons, we need to take care of the rate of return variable in both straightforward OLS and dynamized DSML versions of the Model I for its significance and appropriateness of the sign. Also we need to examine the Durbin-Watson statistic for indications of the degree of autocorrelation. A priori, we expect $\partial y_i / \partial r_i$ to be positive for i-th equation regardless whether we are considering the static or the dynamic version of the relationship. A negative sign and an insignificant t-value would be indicative of supurious relationship because we have asserted that the i-th rate of return and demand for i-th asset are positively related. This holds irrespective of the goodness of fit statistic, which traditionally turns out to be very high for estimates on monetary data.\(^2\)

As regard the $R^{*i}$ variable, we expect $\partial y_i / \partial R^{*i}$ to be negative since by definition $R^{*i}$ represents alternative profitability, but not in the strict sense of a perfect substitute. Ideally, if we were to regress $y_i$ on $r_i$ and $r_j$ along with other explanatory variables, we could then define $\partial y_i / \partial r_j$ as indicative of substitution or complimentarity between i-th and j-th assets, depending on the sign of the coefficient.\(^3\) In the present specification, since $R^{*i}$ is a function of $r_j$ as well as $y_j$, $j \neq i$, = 1, ......., 9, the strict definition of complimentarity or substitution does not hold. Similarly, we expect $\partial y_i / \partial Z^{*i}$ to be negative in so far as $Z^{*i}$ represents alternative investment possibilities in terms of the remaining assets in the portfolio. If we were to redefine $Z^{*i}$ as sum of all ex post dollar investments, it may be possible to treat the aggregate variable as a constraint on the opportunity set open to an investor.

Two of the explanatory variables, $\Theta$ and $V^*$, are the same for all of the asset demand relationships. Ideally, we would like to have
individual risk surrogates, \( \theta_i \), for each security in the portfolio, but in view of the considerations stated earlier we must, for the moment, use the portfolio risk surrogate, \( \theta \). If risk aversion is assumed we may expect \( \frac{\partial y_i}{\partial \theta} \) to be negative appropriately, and in so far as \( \theta \) is being used as the instrument variable, similar sign of the coefficient needs to be obtained. Further, since \( V^* \) is also unobservable, we are using \( V^*_t = r_{gt} y_{it} \) as the instrument for the \( t \)-th observation of \( V^* \), where \( r_{gt} \) is the weighted rate of return on default-risk-free securities. We may expect \( \frac{\partial y_i}{\partial V^*} \) to be positive in so far as we are stipulating a positive relation between assets and their rate of return.

7.3 Comparisons of Classical (OLS) and DSML Estimates of Single Equation, Idealized System

The OLS and DSML estimates of the single equation, idealized system-- Model I-- have been presented in tables 4 and 5 respectively. Both of these tables include results on 11 equations of the life insurance portfolio model; nine of these are asset demand relationships for basic categories-- \( y_1 \) through \( y_9 \)-- and the remaining two are sectoral aggregates, as defined in Chapter V. A summary of the results has been given in Table 6.

In Table 4 we have non-scanned OLS estimates of the Model I. It includes values of the estimated coefficients and t-test for each of the explanatory variables in the equation; the \( R^2 \), adjusted for the degrees of freedom; the Durbin-Watson (DW) and F-statistics. In all, we have 11 subsets of these results corresponding to each of the equations in the system. As noted earlier the variables \( \theta \) and \( V^* \) -- surrogate for risk and risk free returns, respectively-- are the
|| \( r_i \) | \( R*i \) | \( Z*i \) | \( \theta \) | \( V* \) | \( C^2 \) | \( R^2 \) | \( DW \) | \( F \) |
|---|---|---|---|---|---|---|---|---|
| \( y_1 \) | 1.79 | 0.32 | -0.05 | -2.95 | 0.003 | 111.2 | 0.97 | 1.01 | 538.1 |
| | 1.85 | 1.82 | -5.58 | -2.75 | 0.43 | 23.4 |  |  |  |
| \( y_2 \) | -12.01 | -0.55 | 0.04 | -28.64 | -0.02 | 124.13 | 0.16 | 0.11 | * |
| | -0.74 | -0.87 | 1.18 | -0.98 | -1.4 | 0.6 |  |  |  |
| \( y_3 \) | 56.26 | 23.68 | -1.15 | -86.13 | -0.36 | 261.32 | 0.99 | 0.92 | 9202.5 |
| | 3.31 | 11.62 | -41.02 | -3.29 | -2.93 | 5.6 |  |  |  |
| \( y_4 \) | 12.07 | 19.73 | -1.09 | -37.97 | -0.19 | 532.77 | 0.91 | 0.34 | 124.75 |
| | 15.85 | -14.36 | -7.64 | -1.9 | -2.10 | 22.67 |  |  |  |
| \( y_5 \) | 12.63 | -0.07 | -0.09 | 15.21 | -0.003 | 387.95 | 0.96 | 1.32 | 313.09 |
| | 7.38 | -0.18 | -0.25 | 4.64 | -0.23 | 56.34 |  |  |  |
| \( y_6 \) | -29.39 | 16.24 | -0.92 | 3.72 | -0.04 | 568.66 | 0.99 | 0.79 | 4810.8 |
| | -6.03 | 37.16 | -16.72 | 0.38 | -0.79 | 13.2 |  |  |  |
| \( y_7 \) | -4.12 | 3.70 | -0.02 | 5.16 | -0.02 | 88.29 | 0.99 | 0.49 | 1473.01 |
| | -1.38 | 6.67 | -0.54 | -1.19 | -1.45 | 4.06 |  |  |  |
| \( y_8 \) | 11.8 | 6.57 | 0.04 | -45.01 | -0.07 | 66.81 | 0.99 | 0.36 | 1131.74 |
| | 1.47 | 5.42 | 0.38 | -6.15 | -1.76 | -1.63 |  |  |  |
The format of this table should be noted carefully. It has been used with slight variations in almost all the tables included in this chapter.

The equations of the Model I have been described in Chapter V, and individually detailed in Appendix I. Briefly, in this table the value of the coefficient for each explanatory variable is given in the appropriate column for i-th equation of the system, immediately followed by the t-value; and in the last three columns we have R-square, adjusted for the degrees of freedom, the Durbin-Watson statistic, and the F-value, respectively.

The \( y_i \) (i = 1, \ldots, 11) represent the dependent variable of the i-th equation. The system, as reported in this table, includes nine core equations of the Model I and two aggregate relationships. For each equation we should interpret the explanatory variables by their i-th index. The key to the variables and the equations has been given in Chapter V. However, for OLS estimates note that \( r_i \) is the observed, ex-post value of i-th rate of return, and differs from \( r*_{i1} \), which is the expected rate of return. The subscripts of each explanatory variable are to be matched with \( y_i \) to locate the desired coefficient and its t-value.
same for each equation; however, the rate of return variable—\( r_1 \) or \( r^*_1 \), the remainder return variable, \( R^*_i \), and remainder investment variable, \( Z^*_i \), are different for each relationship and need to be interpreted accordingly. This holds for the single equation, idealized system in general, regardless of whether we are considering OLS estimates or DSML estimates.

We have adopted a similar format for Table 5, which contains the results of the DSML estimates of the dynamized life insurance portfolio model. This will facilitate comparisons of the two sets of results for each equation of the system. A special feature of the DSML estimates are the optimal values of the non-linear parameters \( \rho \) and \( \lambda \) which are used in the transformation of a given equation before the coefficients are estimated. It is noteworthy that the solution algorithms of the autocorrelated GDL model—the dynamized version of the Model I—generates a synthetic variable, associated with the geometrically distributed lag variable of the equation. Its coefficient and t-test have also been included here. The OLS estimates of the system does not involve any synthetic or dummy variable.

The estimates of the first equation of the idealized system, \( y_1 \), involving regression of U.S. government securities as dependent variable on its realized rate of interest, \( r_1 \), and other explanatory variables of the set (\( R^*_1, Z^*_1, \theta, V^*, \text{Con} \)) shows a good fit, though autocorrelated, (Table 4, item \( y_1 \)). The coefficient of the pivotal variable has the appropriate sign and may be argued to be nearly significant. For this equation the constant coefficient has a large t-value, whereas variables \( \theta \) and \( Z^*_1 \) seem to be moderately significant and possess appropriate signs. However, the coefficient of \( V^* \) has a low t-value, and the coefficient of \( R^*_1 \) has an inappropriate sign.
TABLE 5

SINGLE EQUATION SYSTEM, MODEL I

DOUBLE SCANNED, MAXIMUM LIKELIHOOD ESTIMATES OF DYNAMIC MODEL

<table>
<thead>
<tr>
<th>r*i</th>
<th>R*i</th>
<th>Z*i</th>
<th>φ</th>
<th>V*</th>
<th>C</th>
<th>SYN</th>
<th>R²/DW</th>
<th>̂μ/λ</th>
</tr>
</thead>
<tbody>
<tr>
<td>y_1</td>
<td>6.02</td>
<td>-0.37</td>
<td>-0.03</td>
<td>-0.12</td>
<td>-0.002</td>
<td>-837.4</td>
<td>1995.5</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>4.11</td>
<td>-1.24</td>
<td>-1.53</td>
<td>-0.01</td>
<td>-0.36</td>
<td>-1.61</td>
<td>4.1</td>
<td>2.11</td>
</tr>
<tr>
<td>y_2</td>
<td>5.61</td>
<td>-0.73</td>
<td>-0.05</td>
<td>5.64</td>
<td>0.004</td>
<td>1390.4</td>
<td>-488.2</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>4.64</td>
<td>-2.24</td>
<td>-2.2</td>
<td>0.55</td>
<td>0.28</td>
<td>2.8</td>
<td>-1.1</td>
<td>1.95</td>
</tr>
<tr>
<td>y_3</td>
<td>22.28</td>
<td>3.57</td>
<td>-0.94</td>
<td>-62.26</td>
<td>3.08</td>
<td>1483.6</td>
<td>-6792.4</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>2.56</td>
<td>1.97</td>
<td>-23.26</td>
<td>-1.31</td>
<td>1.4</td>
<td>4.3</td>
<td>-1.9</td>
<td>1.54</td>
</tr>
<tr>
<td>y_4</td>
<td>8.96</td>
<td>-0.13</td>
<td>-0.02</td>
<td>-2.66</td>
<td>-0.0001</td>
<td>1375.34</td>
<td>264.26</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>9.20</td>
<td>-0.73</td>
<td>-2.03</td>
<td>-0.94</td>
<td>-0.47</td>
<td>15.8</td>
<td>3.92</td>
<td>1.52</td>
</tr>
<tr>
<td>y_5</td>
<td>3.56</td>
<td>-0.10</td>
<td>-0.001</td>
<td>1.70</td>
<td>1.52</td>
<td>325.86</td>
<td>106.03</td>
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<tr>
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<td>9.10</td>
<td>-2.65</td>
<td>-0.61</td>
<td>1.56</td>
<td>0.14</td>
<td>15.29</td>
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<td>1.52</td>
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<td>-0.01</td>
<td>-4.47</td>
<td>0.005</td>
<td>-64.9</td>
<td>-82.57</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>-1.9</td>
<td>2.11</td>
<td>-0.25</td>
<td>-0.20</td>
<td>0.55</td>
<td>-0.12</td>
<td>-0.62</td>
<td>2.00</td>
</tr>
<tr>
<td>y_7</td>
<td>-1.6</td>
<td>0.09</td>
<td>0.01</td>
<td>-2.10</td>
<td>-0.0001</td>
<td>0.68</td>
<td>-5.6</td>
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<td>-1.8</td>
<td>1.01</td>
<td>2.69</td>
<td>-0.80</td>
<td>-0.9</td>
<td>0.01</td>
<td>-0.3</td>
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</tr>
<tr>
<td>y_8</td>
<td>13.60</td>
<td>-0.013</td>
<td>-0.07</td>
<td>-9.20</td>
<td>-0.0001</td>
<td>5625.8</td>
<td>-272.9</td>
<td>0.99</td>
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<tr>
<td></td>
<td>2.61</td>
<td>-0.024</td>
<td>-0.74</td>
<td>-0.34</td>
<td>-0.717</td>
<td>2.6</td>
<td>-0.9</td>
<td>0.81</td>
</tr>
</tbody>
</table>
The format of Table 5 is quite similar to Table 4 with the following exceptions/additions. Hence, previous considerations are applicable here.

i. The R-square and DW statistics have been reported in the same column, the upper value is R-square and the lower value is DW statistic.

ii. The explanatory variable, \( r^*i \), is to be interpreted differently from its counterpart \( r^i \) in the OLS system. It is now the expected variable and needs to be treated so.

iii. An extra column has been added, reporting the coefficient and t-values of synthetic variable associated with variable \( r^*i \). For details on the nature of this variable see Chapter VI.

iv. The last two columns report the 'optimal' value of scanned non-linear parameters.

---

### TABLE 5—Continued

<table>
<thead>
<tr>
<th>( y_9 )</th>
<th>( r^*i )</th>
<th>( R^*i )</th>
<th>( Z^*i )</th>
<th>( \phi )</th>
<th>( V^* )</th>
<th>( C )</th>
<th>SYN</th>
<th>( R^2/DW )</th>
<th>( \hat{\phi}/\hat{\lambda} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20.46</td>
<td>0.92</td>
<td>-0.86</td>
<td>-173.3</td>
<td>0.002</td>
<td>1799.6</td>
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<td>0.99</td>
<td></td>
</tr>
<tr>
<td>3.61</td>
<td>0.62</td>
<td>-9.97</td>
<td>-3.6</td>
<td>0.07</td>
<td>6.4</td>
<td>-3.7</td>
<td>1.46</td>
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</tr>
<tr>
<td>( y_{10} )</td>
<td>33.94</td>
<td>1.25</td>
<td>-0.77</td>
<td>-39.41</td>
<td>-0.001</td>
<td>1051.4</td>
<td>-2121.7</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>6.12</td>
<td>1.40</td>
<td>-17.81</td>
<td>-0.79</td>
<td>-0.6</td>
<td>3.2</td>
<td>-0.09</td>
<td>1.98</td>
<td>0.99</td>
<td></td>
</tr>
<tr>
<td>( y_{11} )</td>
<td>9.5</td>
<td>-0.82</td>
<td>-0.1</td>
<td>12.66</td>
<td>-0.001</td>
<td>1695.8</td>
<td>469.9</td>
<td>0.93</td>
<td>0.79</td>
</tr>
<tr>
<td>3.22</td>
<td>-1.56</td>
<td>-2.7</td>
<td>0.82</td>
<td>-0.29</td>
<td>1.9</td>
<td>0.5</td>
<td>-</td>
<td>0.99</td>
<td></td>
</tr>
</tbody>
</table>

---

It is worthwhile to note that the scanning procedure adopted here is quite comprehensive. It involves a search for the optimal values of \( \lambda \) and \( \phi \) over their respective range of \((-0.99, 0.99)\) and \((0.05, 0.99)\) in steps of 0.098 for initial pass and subsequently over the narrowed range in the neighborhood of already located global optimum in same number of steps. The total number of computations involved is a staggering 840, far more than any of the iterative procedures would promise. In a sense it is equivalent to computing 840 equations each transformed for the same number of pair of values of the non-linear parameters for each structural relationship.
<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
<th>DSML</th>
<th>OLS</th>
<th>DSML</th>
<th>DSML</th>
</tr>
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<tr>
<td></td>
<td>r₁</td>
<td>rᵣ₁</td>
<td>Rᵣ/DW</td>
<td>Rᵣ/DW</td>
<td>Rᵣ/ᵣ</td>
</tr>
<tr>
<td>y₁</td>
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<td>0.97</td>
<td>0.98</td>
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</tr>
<tr>
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</tr>
<tr>
<td>y₂</td>
<td>-12.01</td>
<td>5.61</td>
<td>0.16</td>
<td>0.95</td>
<td>0.69</td>
</tr>
<tr>
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<td>-0.74</td>
<td>4.64</td>
<td>0.11</td>
<td>1.95</td>
<td>0.99</td>
</tr>
<tr>
<td>y₃</td>
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<td>22.28</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
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<td>0.92</td>
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<tr>
<td>y₄</td>
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<td>0.89</td>
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<td>1.52</td>
<td>0.99</td>
</tr>
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<td>y₅</td>
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<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>7.38</td>
<td>9.19</td>
<td>1.32</td>
<td>1.08</td>
<td>0.85</td>
</tr>
<tr>
<td>y₆</td>
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<td>0.99</td>
<td>0.93</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
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<td>-1.9</td>
<td>0.79</td>
<td>2.00</td>
<td>0.37</td>
</tr>
<tr>
<td>y₇</td>
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<td>0.99</td>
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</tr>
<tr>
<td></td>
<td>-1.38</td>
<td>-1.88</td>
<td>0.49</td>
<td>1.83</td>
<td>0.74</td>
</tr>
<tr>
<td>y₈</td>
<td>11.8</td>
<td>13.60</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
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<td>2.61</td>
<td>0.36</td>
<td>0.81</td>
<td>0.99</td>
</tr>
<tr>
<td>y₉</td>
<td>21.74</td>
<td>20.46</td>
<td>0.99</td>
<td>0.97</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>3.69</td>
<td>3.61</td>
<td>0.51</td>
<td>1.46</td>
<td>0.99</td>
</tr>
<tr>
<td>y₁₀</td>
<td>20.38</td>
<td>33.94</td>
<td>0.98</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
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<td>6.12</td>
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<td>1.98</td>
<td>0.99</td>
</tr>
<tr>
<td>y₁₁</td>
<td>7.2₄</td>
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<td>0.36</td>
<td>0.93</td>
<td>0.79</td>
</tr>
<tr>
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<td>2.0₈</td>
<td>3.2₂</td>
<td>0.4₈</td>
<td>0.5₀</td>
<td>0.99</td>
</tr>
</tbody>
</table>

This table is based on the results of OLS and DSML estimates reported in full in Tables 4 and 5. The purpose in isolating the coefficients of $r_1$ and $r*_1$ is to focus our attention on the behavior of pivotal variable whose lagged approximations have been used in obtaining DSML estimates. In a similar fashion, DW supposedly reveals the effects of scanning for autocorrelation.
For the same equation the DSML estimates (Table 5, item $y_1$) have performed better in terms of criteria set forth earlier. The coefficient of $r*1$ has the proper sign and is consistently significant. This seems to suggest that the lagged approximations of the expectational variable, $r*$, is the appropriate specification for the asset demand function. The DW statistic also demonstrates improvement over its counterpart in the OLS estimates, for the optimal value of the autocorrelation parameter $\hat{\rho} = 0.49$. The remaining variables in the equation have appropriate signs, except $R*1$, though they are not as significant as before. There is marginal improvement in the value of $R^2$ as well.

The second equation of the idealized system, $y_2$, involving local and state government securities, affords a valuable contrast between OLS and DSML estimates. As regards the OLS results, (Table 4, item $y_2$), the crucial variable $r_2$ is not only insignificant but carries a negative coefficient. Besides, the DW statistic shows a highly autocorrelated regression. The remaining variables are also insignificant, and $V*$ and $Z*2$ have inappropriate signs. The OLS estimates of the $y_2$ equation suggest that there is inaccuracy in the specification, but if we look into the DSML estimates of the same equation (Table 5, item $y_2$), we find that the lagged approximation of $r*2$ is not only significant but changes sign, i.e., it has a positive coefficient for $\hat{\lambda} = 0.99$. In addition, the correction for autocorrelation seems to have improved the estimates considerably. The DW statistic shows no autocorrelation for $\hat{\rho} = 0.69$. Thus, on both the counts we feel that the DSML procedure has performed better than the OLS procedure. Finally, the $R^2$ also shows marked improvement
for DSML estimates, indicating a better fit than the one obtained in the OLS estimates.

The OLS estimates of the third equation of the system, $y_3$, involving industrial bonds as the dependent variable suggest that the explanatory variables ($r_3$, $R*3$, $Z*3$, $\Theta$, $V*$), are mostly significant, but the DW statistic reveals autocorrelated error terms. By far it is the most successful equation in the system if evaluated in terms of criteria that we have established. The DSML estimates of the same equation retain most of the features described above, i.e., $r*3$ is significant and has the appropriate sign. There is slight improvement in the DW statistic as well. The corresponding optimal values of the nonlinear parameters have been given in the last column of Table 5.

More or less similar observations can be made for the fourth equation of the system, $y_4$, involving public utilities' bonds as the dependent variable. Again the OLS estimates of the equation (Table 4, item $y_4$) are largely acceptable as regards the significance of the rate of return variable, $r_4$, and the other variables in the explanatory set, except for the autocorrelated error terms. The DSML estimates of the same equation (Table 5, item $y_4$) exhibit significant $r*4$ variable as well as the remaining explanatory variables. It seems that for equations $y_3$ and $y_4$ most of the features of OLS and DSML estimates compare favorably, though there is a gain involved in scanning for autocorrelation parameter.

The only equation of the idealized model which seems to have a close similarity of estimates obtained from both the procedures is the one for railroad bonds, $y_5$. The rate of return variable is significant in both the cases and has appropriate sign, but the equation seems to be autocorrelated in both the cases; scanning does not seem
to have improved much in case of DSML estimates.

The next two equations, $y_6$ and $y_7$, involve common and preferred stocks respectively. These two equations have negative signs of the coefficients of their pivotal variables ($r_6, r_7; r^*_6, r^*_7$) for both OLS and DSML estimates, (Tables 4 and 5; items $y_6$ and $y_7$). Though this is contrary to the a priori specifications of the asset demand function, it nevertheless seems to be appropriate if we consider the underlying factors, which govern the choice of stocks in the life insurance portfolio. Apparently, stocks are primarily held for the purpose of capital gains, and the extent of the gains supposedly neutralizes the loss differentials on yields. Further, in so far the yields on stocks are calculated as a ratio of dividends to prices, then as prices rise, $r_1$ declines and is consistent with an increase in $y_1$, and hence we have a negative coefficient. This would specifically be true if the sample period encompasses mostly a bullish market and a buoyant economy. It is no surprise, therefore, that we have obtained negative coefficients consistently for both the stocks in all the versions of the life insurance portfolio model.

Keeping the above in view, if we compare the OLS and DSML estimates of the two equations, we find that DSML estimates have performed better than OLS estimates; specifically as regards the problem of autocorrelation. The coefficients of $r^*_6$ and $r^*_7$ are significant, but the rate approximation does not seem to be vital for the estimation of these two equations.

The OLS estimates of equation $y_8$, involving non-farm mortgages as the dependent variable show an insignificant though appropriate coefficient for its rate of return, $r_8$. (Table 4, item $y_8$). This may
be partly due to the fact that we are using flow of funds data. For the same equation we have inappropriate signs of coefficients for $R*8$ and $V*$, and insignificant coefficient for $Z*8$. The DSML estimates, on the other hand, exhibit marked difference for the same specification and same data. In this case the pivotal variable, $r*8$, turns out to be significant enough to be acceptable, however, it seems legitimate to speculate that if commitments data were used the picture would not change relative to DSML estimates, except that we may end up with a considerably more significant $r*$ variable. The equation has also improved, though marginally, with respect to autocorrelation for the optimal values of autocorrelation parameter given in the table.

The last equation of the core relationships, $y_9$, deals with policy loans as the dependent variable. The OLS and DSML estimates of this equation seem to possess common features with respect to the rate of return variable— it is significant and has the appropriate sign in both the cases, (Tables 4 and 5, item $y_9$). Further, variables $Z*9$, $\theta$, and $V*$ have similar types of coefficients in both the cases though the coefficient of $R*9$ is insignificant in case of DSML estimates and continues to have perverse sign. The goodness of fit statistic is identical in both the cases, but again, the OLS equation turns out to be autocorrelated. For an optimal value of autocorrelation parameter (0.99) the DSML estimates improve substantially.

The foregoing equations constitute the core of life insurance portfolio model. Both the idealized and simultaneous systems consist basically of these nine behavioral relations, however, we have also estimated the aforementioned two aggregate relationships, pertaining to the total of corporate and government securities respectively.
These have been regressed on the averages of $r_i$, $R^*_i$, and $Z^*_i$ variables of the respective categories, with $\Theta$ and $\psi^*$ remaining the same. Thus $r_{10}$ and $r_{11}$ are the averages of the rates of return on corporate and government securities respectively, corresponding to which we have the expectational counterparts, $r^*_{10}$ and $r^*_{11}$, used in the DSML estimates. For both the equations we find that the pivotal variables are significant and possess appropriate signs in the OLS as well as DSML procedures. The OLS estimates are autocorrelated, however, DSML estimates are largely free of this bias, (Tables 4 and 5; items $y_{10}$ and $y_{11}$). In addition, there is significant improvement in the $R^2$ value for equation $y_{11}$ in case of DSML estimates.

The above analysis clearly and understandably demonstrates the inherent superiority of the DSML procedure in almost all the cases. Lacunae of model specifications notwithstanding, the theoretical optimality considerations of the dual scanning procedure over the entire spectrum of non-linear parameters, virtually ensures a better fit on the data for a given equation. The DSML procedure provides an improved scheme even in relation to iterative procedures, not to mention the simplified OLS scheme.

7.4 Comparisons of Classical (2SLS, LIML) and DSML

Simultaneous System, Model II

We have presented detailed results on the estimates of the simultaneous system (Model II, Chapter V) in Tables 7, 8, and 9. A summary of these results appears in Table 10. In all, we have three basic sets of estimates on the nine behavioral equations of the model. Two of these have been obtained from the classical procedure, viz, the 2SLS and LIML methods, and have been catalogued in Tables 7 and 8.
respectively. The third set listed in Table 9 pertains to the application of DSML procedure in simultaneous estimation.

The format of these tables is similar so as to facilitate detailed comparisons of these results. On the title page of each of these tables we have a matrix of coefficients of explanatory endogenous variables of the system and their t-values. In the last two columns we have $R^2$, adjusted for the degrees of freedom, the DW statistic and the optimal values of non-linear parameters, $\lambda$ and $\rho$. These last two parameters are, of course, assumed to be zero for classical estimates. On the following page of each of these tables, we have a matrix of coefficients of endogenous variables of the system and their t-values.

As before, variables $y_1$ through $y_9$ represent the i-th equation of the simultaneous system and should be interpreted accordingly. In each equation there are 13 explanatory variables including the constant; eight of these are explanatory variables of the endogenous set and the remaining are the explanatory exogenous variables. For 2SLS and LIML estimation, the exogenous variables are the same and comprise of $(r_1, R*1, Q, \text{ and } V*)$. The first two of these variables need to be indexed appropriately to match the listing of equations, $y_i$, in a manner similar to the one described in the preceding section. In fact these are same explanatory variables as used in the idealized system, with the exclusion of the $Z*i$ variables, which have now been replaced by the set of explanatory endogenous variables.

For the DSML estimates, we have the same explanatory exogenous and endogenous variables, except that we interpret the pivotal variable differently. Instead of using the ex post values of $r_1$ we have obtained lagged approximations of $r_1$, treating it as an expectational
### TABLE 7

**SIMULTANEOUS SYSTEM, MODEL II**

**TWO STAGE LEAST SQUARE ESTIMATES**

<table>
<thead>
<tr>
<th>EXOGENOUS VARIABLES</th>
<th>( r_i )</th>
<th>( R^* )</th>
<th>( \theta )</th>
<th>( V^* )</th>
<th>( C )</th>
<th>( R^2 )</th>
<th>( DW )</th>
</tr>
</thead>
<tbody>
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**Notes:**

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Notes:

a In this table we have listed 2SLS estimates of the simultaneous system detailed in Chapter V. The key to the variables is given in the same chapter and the equations have been listed in Appendix I. The results on nine structural equations of the system have been reported in two sets:

i. In the first set, reported on the title page, coefficients and t-values of explanatory exogenous variables have been listed in addition to the R-square and DW statistic. As before, the subscripts of the exogenous variables, specially $r_1$, $R^x_1$, and $Z^x_1$ need to be matched with $y_1$ to locate the desired coefficient of a given variable.

ii. In the second set, we have reported the coefficients and t-values of explanatory endogenous variables, the $y$-hats, for each structural equation of the system.

This format will be broadly followed in reporting the results on simultaneous systems obtained from different estimation procedures.

b No estimates of the third equation of the system are available possible because of the determinant value being zero. The equation refers to 'industrial and miscellaneous bonds' category in the life insurance portfolio.
variable. Again, we have to match the subscripts of $r_i$ and $R_i$ with $y_i$, $i = 1, \ldots, 9$ to read accurately the desired results.

The classical and DSML procedures of estimation of the simultaneous system retain a similarity as regards the treatment of the inconsistency problem associated with explanatory endogenous variables. In the initial stage of estimation, we have regressed the endogenous variables on all the exogenous variables of the system to eliminate the stochastic element of $y_i$, and in the final stage we have obtained estimates as prescribed by the particular procedure.

As regards the comparisons of 2SLS, LIML, and DSML estimates, we shall concentrate on the appropriateness and significance of the rate of return variable, and the DW statistic. This is not to suggest that we will overlook entirely the results on the remaining explanatory variables of the equation, however, in order to ascertain the efficacy of the scanning technique, we have to focus our attention on the salient aspects of the DSML estimates. We have, therefore, offered a summary of results in Table 10 which gives the coefficients and t-values of the pivotal variable in the 2SLS, LIML, and DSML estimates, as well as $R^2$, DW, and optimal values of non-linear parameters.

The 2SLS estimates of the first equation of the simultaneous system, $y_1$, yield a negative and insignificant coefficient for the rate of return variable, $r_1$; and the equation is autocorrelated, (Tables, 7, 8, and 9; item $y_1$). Similar is the case of LIML estimates of the equation. But on the other hand, the DSML estimates of $y_1$ exhibit a positive as well as significant coefficient for $r^*_1$ variable. Scanning for the autocorrelation parameter also shows marked improvement as demonstrated by the DW statistic. As a whole, we
### TABLE 8

**SIMULTANEOUS SYSTEM, MODEL II**

**LIML ESTIMATES**

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**Notes:**

This table has been continued on the following two pages with a substantial change in the format made necessary by the large size of the so-called B-matrix---the matrix of coefficients of explanatory endogenous variables. It is followed by some clarifying notes and comments.
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</table>
Notes:

a In this table we have listed limited information maximum likelihood (LIML) estimates of the simultaneous system. These are also known as limited information single equation (LISE) estimates in the literature. The set of equations and variables is the same as catalogued in Table 7. The format is also the same, i.e., on the title page we have coefficients and t-values of explanatory exogenous variables, R-square and DW statistics, and on the following page we have given estimates for the explanatory endogenous variables of the system.

b The constant coefficient is too large for equations \( y_1 \) and \( y_9 \). Estimates of \( y_1 \) equation seem to be dubious enough, because in addition to an extraordinary large constant coefficient it has inappropriate value for R-square--- it is a negative. The remaining equations seem to have proper estimates.

c For two equations of the LIML system no estimates are available. These are \( y_3 \) and \( y_5 \). For the third equations no estimates are available in any simultaneous system, but the 5th equation is peculiar to LIML system only.
find that DSML estimates of the first equation are superior to the classical estimates.

Among the remaining exogenous variables of the same equation, the coefficient of $R^*1$ changes sign in the proper direction and becomes significant for the DSML estimates, however, the coefficients of $\Theta$ and $V^*$ have perverse signs and/or remain insignificant. In 2SLS estimates, the coefficient of $V^*$ has a proper sign and is also significant but this is not true of $R^*1$ variable. Similarly, we seem to have obtained mixed results for the classical and DSML procedures when we compare the coefficients of the explanatory endogenous variables of the first row of the so-called $B$ matrix. A definite picture is obtained if we confine ourselves to the salient aspects of the classical and DSML procedures.

More or less the same is the case with the second equation of the simultaneous system, $y_2$, which involves state and local government securities, (Tables, 7, 8, and 9; item $y_2$). The coefficient of rate of return variable, $r_2$, is insignificant and worse yet has a negative sign in both the 2SLS and LIML estimates. If, however, we treat it as expected variable, $r^*2$, and use lagged approximations instead of its ex post observed value, it turns out to be a significant variable with a changed sign for the DSML estimates. The error terms, however, remain autocorrelated; there is no improvement in this regard. Among the remaining explanatory exogenous variables, $R^*2$ and $V^*$ are significant and possess appropriate sign in both classical and DSML estimates. As regards the explanatory endogenous variables, for the 2SLS and LIML estimates variables $y_3$ to $y_9$ inclusive, are significant; but for the DSML estimates only $y_1$, $y_3$, $y_4$, and $y_5$ are so. Nevertheless, on the
### TABLE 9
SIMULTANEOUS SYSTEM, MODEL II
DOUBLE SCANNED MAXIMUM LIKELIHOOD ESTIMATES

<table>
<thead>
<tr>
<th>EXOGENOUS VARIABLES</th>
<th>r*1</th>
<th>R*1</th>
<th>θ</th>
<th>V*</th>
<th>C</th>
<th>R²/DW</th>
<th>( \hat{\theta}/\hat{\lambda} )</th>
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</thead>
<tbody>
<tr>
<td>y₁</td>
<td>7.38</td>
<td>-3.12</td>
<td>18.5</td>
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<td>-543.4</td>
<td>0.98</td>
<td>0.42</td>
</tr>
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<td>2.32</td>
<td>-2.18</td>
<td>0.29</td>
<td>-1.00</td>
<td>-0.7</td>
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<tr>
<td>y₂</td>
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<td>-1.43</td>
<td>87.0</td>
<td>0.02</td>
<td>710.5</td>
<td>0.95</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>1.8</td>
<td>-1.48</td>
<td>1.55</td>
<td>1.58</td>
<td>1.6</td>
<td>0.88</td>
<td>0.99</td>
</tr>
<tr>
<td>y₃</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
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<tr>
<td>y₄</td>
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<td>1336.6</td>
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<td>0.89</td>
</tr>
<tr>
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</tr>
<tr>
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</tr>
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<td>0.005</td>
<td>2.4</td>
<td>1.16</td>
<td>0.88</td>
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<tr>
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<td>397.88</td>
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</tr>
<tr>
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</tr>
<tr>
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<td>0.06</td>
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<td>1.87</td>
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</table>

**Notes:**
This table has been continued on the following two pages with a substantial change in the format made necessary by the large size of the so-called B-matrix—the matrix of coefficients of explanatory endogenous variables. It is followed by some clarifying notes and comments.
<table>
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<th>$y_1$</th>
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<th>$y_3$</th>
<th>$y_4$</th>
<th>$y_5$</th>
<th>$y_6$</th>
<th>$y_7$</th>
<th>$y_8$</th>
<th>$y_9$</th>
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</thead>
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<td>-0.2</td>
<td>-0.54</td>
<td>0.02</td>
<td>3.4</td>
<td>1.89</td>
<td>-0.01</td>
</tr>
<tr>
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<td>-0.02</td>
<td>-1.4</td>
<td>-1.09</td>
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<td>-0.01</td>
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<td>0.03</td>
<td>-0.03</td>
<td>0.005</td>
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<td>$y_8$</td>
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<td>-12.38</td>
<td>0.01</td>
<td>-13.9</td>
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<td>-0.16</td>
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<td>-0.65</td>
<td>7.05</td>
<td>0.16</td>
<td>17.74</td>
<td>1.89</td>
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<td>0.76</td>
<td>4.0</td>
<td>8.94</td>
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</table>

**TABLE 9 -- Continued**
TABLE 9—Continued

Notes:

a In this table we have listed the double scanned maximum likelihood estimates (DSML) of the simultaneous system which differ from the previous two sets of estimates (Tables, 7 and 8) in many respects, specially as regards the treatment of the rate of return variable. The format of the table is the same as in the preceding two tables, except that we have reported R-square and DW statistic in the same column of the title page. The coefficients and t-values of the explanatory endogenous set is given on the following page.

b The scanning procedure for simultaneous system differs slightly from the one used in case of the single equation system. (Table 4) Here we have scanned for each equation on only 420 pairs of values of the non-linear parameters, instead of 840 pairs used earlier, since the enlarged set of explanatory variables is very expensive to replicate in terms of CPU time usage. Nevertheless, it remains a comprehensive scheme as compared to any of the iterative procedures, and we have compromised only the second decimal point accuracy of the optimal values of non-linear parameters.

c As before, no estimates are available for the third equation of the system.
### TABLE 10
SIMULTANEOUS SYSTEM, MODEL II
COMPARISONS OF THE CLASSICAL (2SLS, LIML) AND DSML ESTIMATES WITH
A SPECIAL FOCUS ON THE NON-LINEARITIES OF DYNAMIC MODEL

<table>
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<tr>
<th></th>
<th>2SLS</th>
<th>LIML</th>
<th>DSML</th>
<th>2SLS</th>
<th>LIML</th>
<th>DSML</th>
<th>R²/DW</th>
<th>R²/DW</th>
<th>R²/DW</th>
<th>ε/λ</th>
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<tr>
<td>r_i</td>
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<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>r*_i</td>
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</table>

This table is similar to Table 6. It presents the results of estimates simultaneous system in a condensed form. We have isolated here the coefficients of the pivotal variable of the portfolio model to offer a direct comparison between simultaneous DSML estimates with their counterpart classical estimates reported in full in Tables 7, 8, and 9. For purposes of comparisons we need to take note of the coefficient of rate of return variable, its sign, and t-value, as well as DW statistic.

b No estimates are available.
criteria set forth earlier, we find that the DSML estimates have shown better results than their counterpart classical estimates.

In the corporate bond category we have three equations, \( y_3 \), \( y_4 \), and \( y_5 \), for industrial bonds, public utilities' bonds, and railroad bonds respectively. No estimates are available for \( y_3 \) because of a zero determinant in all the procedures. As regards \( y_4 \), however, we have a positive as well as significant coefficient for its rate of return in 2SLS and LIML estimates, but the coefficients of \( R*4, V* \) and \( \theta \) have inappropriate signs, (Tables 7, 8, and 9; item \( y_4 \)). In the DSML estimates the coefficients of \( R*4 \) has an appropriate sign and is significant; in fact its t-value has increased appreciably. There is no visible improvement in the degree of autocorrelation, perhaps because, to begin with, the equation is largely free of this problem. Variables \( R*4 \) and \( \theta \) change sign for the better, but \( V* \) continues to have perverse sign in DSML estimates. Among the explanatory endogenous variables of equation \( y_4 \), the coefficients of \( y_2 \) and \( y_5 \) through \( y_9 \) are significant both in case of 2SLS as well as LIML estimates, but for DSML estimates \( y_5 \) and \( y_7 \) drop out from this set. In view of the above we can state that if no visible improvement is offered by DSML estimates in an already well behaved stochastic relationship, DSML, at least, causes no deterioration of the results.

For equation \( y_5 \) we do not have results from the LIML procedure, but 2SLS estimates are available and can be compared with those of DSML estimates. The equation behaves relatively well in both the cases as judged by the estimates of the coefficients for rates of return in the two specifications, and the DW statistic, (Tables 7, and 9; item \( y_5 \)). For both \( r_5 \) and \( R*5 \) we have positive as well as significant
coefficient. The remaining exogenous variables, \( R^s \), \( \theta \), and \( V^* \) have appropriate signs in DSML estimates but these are largely insignificant. With the exception of \( y_1 \), the same is true of explanatory endogenous variables. In case of the 2SLS estimates, however, \( R^s \) has an appropriate and significant coefficient. This is also the case with the explanatory endogenous variables. Thus, again we have an equation where classical and DSML estimates compare favorably.

The next two equations, \( y_6 \) and \( y_7 \), involve common and preferred stocks respectively. Their estimates are similar to those obtained in case of the idealized model discussed at some lengths in the preceding section. The common stock equation, \( y_6 \), has consistently yielded a negative and significant coefficient for the rate of return variable in all the three variants of its estimates, (Tables 7, 8, and 9; item \( y_6 \)); and is largely free of autocorrelation in case of DSML estimates. If the portfolio behavior with respect to acquisitions of stocks is primarily motivated by the prospects of capital gains, as we argued previously, the signs of the coefficients seem to be appropriate. As regards the preferred stock equation, \( y_7 \), the coefficient of the rate of return, \( r^s \), is positive in case of 2SLS and LIML estimates, but it is properly negative in case of DSML estimates when lagged approximations are used, (Tables 7, 8, and 9; item \( y_7 \)). A switch in the sign of the coefficient of above variety is indicative of the appropriateness of DSML estimation procedure vis-a-vis the classical procedures.

The estimates of the 8th equation of the system, involving non-farm mortgages, offer a better comparison between classical and DSML estimates, (Tables 7, 8, and 9; item \( y_8 \)). The coefficients of \( r^g \)
and $r^*8$ have positive signs and are significant for 2SLS and DSML estimates but, surprisingly enough, we have a negative coefficient in the LIML estimates. If for the moment we concentrate only on the 2SLS and DSML estimates, we find that the two sets of results compare favorably in so far as the pivotal variable is concerned. The DW statistic, however, reveals autocorrelated error terms for the 2SLS estimates. In this case, scanning is successful, and there is considerable improvement in the DSML estimates.

Among the remaining exogenous variables, $R^*8$ and $V^*$ continue to have inappropriate and insignificant coefficients in both the cases, whereas $\Theta$ has the appropriate sign but remains insignificant. As regards the explanatory endogenous variables, for 2SLS, $y_1$ and $y_5$ are significant. For the equation as a whole we find that DSML estimates have performed better than their counterpart classical estimates.

The last of the system, $y_9$, involving policy loans, has perverse signs for coefficients of its rate of return variable, $r_9$ and $r^*9$, in all the case. Scanning for the autocorrelation parameter seems to have helped in eliminating the problem of serially correlated error terms, but the inappropriateness of the coefficients of the pivotal variable precludes meaningful interpretation of the results. If we disregard this aspect of the estimates, and instead adopt a purely technocratic approach, we may state that DSML procedure has been very successful in dealing with at least one of the problems in the estimation of this relationship.

7.5 Life Insurance Portfolio Model, Abbreviated

In order to highlight the differences between the classical and DSML procedures it may be worthwhile to consider an abbreviated single
equation system, holding the a priori specification of the functional forms in abeyance, (Models I and II, Chapter V). Towards this end, let us recast the dependent variables of the Model I solely in terms of their respective rates of returns, \( r_i \) or \( r_i^* \), as the case may be— and the remainder investment variable, \( Z_i^* \), for each relationship. Thus we have:

\[
(7.5.1) \quad y_1 = \phi (r_1, Z_1^*) \\
\ldots \\
(7.5.9) \quad y_9 = \phi (r_9, Z_9^*)
\]

In general:

\[
(7.5.10) \quad y_i = \phi (r_i, Z_i^*)
\]

The nine equations in (7.5.10) constitute what may be called as the 'abbreviated model' (Model I-a) of the life insurance portfolio, specified only on two explanatory variables. The definition of these variables remains the same, so does the role of rate of return variable, viz, these rates will continue to be treated as the pivotal variable of the system. The OLS estimates of the equations in Model I-a are based on the assumption that we can regard the ex post values of rates of return, \( r_i \), as the pivotal variable and that the structural error terms are free of autocorrelation. The DSML estimates, on the other hand, require lagged approximations of the variables \( r_i \), similar to the scheme followed earlier. In addition we will also provide for the transformation associated with an autocorrelated structure.

The results of the OLS and DSML estimates of the above model have been listed in Table 11. As regards the coefficients of the rates
TABLE 11

SINGLE EQUATION SYSTEM, ABBREVIATED MODEL
COMPARISONS OF THE CLASSICAL (OLS) AND DSML ESTIMATES

<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
<th>DSML</th>
<th>OLS</th>
<th>DSML</th>
<th>OLS</th>
<th>DSML</th>
<th>DSML</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>r_1</td>
<td>r*_i</td>
<td>Z*_i</td>
<td>Z*_i</td>
<td>R^2/DW</td>
<td>R^2/DW</td>
<td>( \hat{\epsilon}/\lambda )</td>
</tr>
<tr>
<td>( y_1 )</td>
<td>37.35</td>
<td>5.20</td>
<td>-0.04</td>
<td>-0.05</td>
<td>0.92</td>
<td>0.98</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>6.45</td>
<td>4.24</td>
<td>-24.97</td>
<td>-4.79</td>
<td>0.97</td>
<td>2.13</td>
<td>0.99</td>
</tr>
<tr>
<td>( y_2 )</td>
<td>-35.45</td>
<td>4.34</td>
<td>0.01</td>
<td>-0.04</td>
<td>0.17</td>
<td>0.95</td>
<td>0.79</td>
</tr>
<tr>
<td></td>
<td>-3.49</td>
<td>2.97</td>
<td>3.3</td>
<td>-6.06</td>
<td>0.13</td>
<td>2.05</td>
<td>0.96</td>
</tr>
<tr>
<td>( y_3 )</td>
<td>-19.03</td>
<td>39.64</td>
<td>0.38</td>
<td>-0.91</td>
<td>0.95</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>-0.44</td>
<td>5.96</td>
<td>16.90</td>
<td>-22.10</td>
<td>2.02</td>
<td>1.32</td>
<td>0.99</td>
</tr>
<tr>
<td>( y_4 )</td>
<td>2.72</td>
<td>8.72</td>
<td>0.01</td>
<td>-0.03</td>
<td>0.58</td>
<td>0.99</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>0.27</td>
<td>9.76</td>
<td>4.26</td>
<td>-6.13</td>
<td>0.04</td>
<td>1.45</td>
<td>0.94</td>
</tr>
<tr>
<td>( y_5 )</td>
<td>20.95</td>
<td>3.42</td>
<td>20.95</td>
<td>-0.008</td>
<td>0.93</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>23.01</td>
<td>8.75</td>
<td>23.09</td>
<td>-6.28</td>
<td>0.92</td>
<td>1.08</td>
<td>0.83</td>
</tr>
<tr>
<td>( y_6 )</td>
<td>36.36</td>
<td>-14.82</td>
<td>0.09</td>
<td>0.09</td>
<td>0.95</td>
<td>0.99</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>1.96</td>
<td>-1.23</td>
<td>35.26</td>
<td>12.34</td>
<td>0.27</td>
<td>1.94</td>
<td>0.58</td>
</tr>
<tr>
<td>( y_7 )</td>
<td>-5.73</td>
<td>-1.96</td>
<td>0.02</td>
<td>0.19</td>
<td>0.99</td>
<td>0.99</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>-4.40</td>
<td>-2.61</td>
<td>56.50</td>
<td>29.13</td>
<td>0.52</td>
<td>1.87</td>
<td>0.71</td>
</tr>
<tr>
<td>( y_8 )</td>
<td>61.99</td>
<td>27.76</td>
<td>0.07</td>
<td>-0.88</td>
<td>0.78</td>
<td>0.96</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>2.69</td>
<td>5.21</td>
<td>5.73</td>
<td>-18.77</td>
<td>1.43</td>
<td>1.41</td>
<td>0.99</td>
</tr>
<tr>
<td>( y_9 )</td>
<td>-166.14</td>
<td>13.90</td>
<td>0.68</td>
<td>-0.08</td>
<td>0.97</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>-2.31</td>
<td>3.05</td>
<td>15.02</td>
<td>-0.99</td>
<td>0.10</td>
<td>0.87</td>
<td>0.99</td>
</tr>
</tbody>
</table>

The equations of this system have been outlined in Appendix II. The model consists of the basic set of nine relationships, \( y_1 \) through \( y_9 \), and each of them has been regressed on their respective rates of return, \( r_1 \) or \( r*_i \), and the remainder investment variable, \( Z*_i \).

The scanning procedure is the same as followed in the case of Model I, Table 5; viz, there are 840 replications for each equation of the system over the range \(-0.99 < \epsilon < 0.99\), and \(0.05 < \lambda < 0.99\) to obtain the optimal values of these parameters. As before, these are reported in the last column of this table.
of return variable, the OLS estimates (column 1) are highly unsatisfactory for equations $y_2$, $y_3$, $y_4$, $y_6$, and $y_9$. Thus, in majority of cases, we find that these estimates are untenable in view of the a priori considerations elaborated previously. In all these equations the coefficients of $r_4$ are either insignificant or worse yet have negative signs. Specifically, $r_2$, $r_3$, and $r_9$ have a negative sign, whereas these should be positive; on the other hand, the coefficient of $r_6$ should be negative but the estimated coefficient is positive. Further, in case of $r_4$, its coefficient is insignificant, though it has an appropriate sign. In view of the above we may conclude that for the abbreviated model as whole, OLS estimates have been unsuccessful, specially because of the undesirable coefficients of the pivotal variables of the system.

On the other hand, the DSML estimates (column 2) of the coefficients of $r^*i$ variable have offered consistently good results. The approximation mechanism for the expected variables of the model seems to have performed better than a simple substitution of the observed values as explanatory variables. In all of the nine equations, we find that the coefficients of $r^*i$ are significant and have appropriate signs. In fact the performance of DSML estimates in this respect has been much better than witnessed thus far. In no less than four cases there is a reversal of sign of the coefficients in the desired direction. Exclusion of the insignificant variables from the explanatory set seems to have improved the estimates of the model, though we are not in a position to offer theoretically valid reasons for such an exclusion except in terms of ad hoc hypothesis.

Similar is the case as regards the remainder investment variables, $Z^*i$, (columns 3 and 4). We expect, a priori, that these
variables would have negative coefficients in so far as Z*i represent alternative investment possibilities, except for the stock equations of the model. The OLS estimates, again, are largely unsatisfactory as compared to their counterpart DSML estimates. Mostly, these coefficients have inappropriate signs. In case of DSML estimates a reversal of sign is involved usually in the appropriate direction. In addition, the estimates have a high level of significance as exhibited by their t-values.

Correction for the autocorrelation problem seems to have further contributed to the differences in the two sets of estimates. The DW statistic has invariably shown an improvement in case of the DSML estimates or at least has preserved its level. The only exception to this seems to be the equation yg which also has bad results with respect to the coefficient of its rate of return. The optimal value of autocorrelation parameters as well as the lag parameter have been given in the last column of Table 11. The DSML estimates are, of course, conditional on these values.

7.6 Comments

The foregoing discussion seems to be instructive in regards to the feasibility and efficacy of the scanning procedure. The results, on the whole, seem to be quite encouraging. We have rearranged the estimates from the idealized and simultaneous systems in Tables 6 and 10 to highlight the difference between classical (OLS, 2SLS, LIML) and DSML procedures. In addition, we have also offered estimates on the abbreviated model which further emphasizes the difference between the two procedures. A glance at these tables would confirm that in terms of the criteria established earlier, DSML estimates perform consistently
better than their counterpart classical estimates, or at least maintain the same level of accuracy. Further, if in a given economic relationship expectational variables are involved, scanning method can be used to provide us with sophisticated information regarding the behavior of crucial variables in the functional forms.

For the life insurance portfolio model, in general, we have found that the expected rate of return, \( r^*i \), and the indices of substitutibility—variables \( R^*i \) and \( Z^*i \)—are significant, though the latter have inappropriate signs in some cases. The performance of the risk surrogate was disappointing. This seems to emphasize the need of a more accurate specification of this variable than the one offered in this study. These limitations in the applied framework stem directly from the existing state of arts in the realm of theoretical models of portfolio behavior.
Footnotes

1. For details of the procedure, see sections 6.3, 6.4, and 6.5 of Chapter VI.

2. There is an important exception to this rule. It is not likely that stock equations of the life insurance portfolio will have positive coefficients for the rate of return variable. The reason for this will be discussed as we proceed.

3. We have argued previously that \( y_1 / r_1 \) is positive, and \( y_1 / p_1 \) is negative, because of the negative relation between \( r_1 \) and \( p_1 \); similarly, if \( y_1 / r_j \) is negative, \( i \)-th \( j \)-th securities are substitutes, if it is positive, the securities are complements, otherwise independent, see footnote #2, Chapter V.

4. In fact we have to index these variables suitably for each equation of the model. In the tabulated form, however, the respective for the corresponding coefficients and t-test have been put together in the appropriate column, titled for each variable; so are the statistics \( R^2 \) and DW.

5. It may be argued that the DW statistic in case of DSML estimates is inoperable in so far as the autoregressive processes involve lagged values of variables as regressors. But a glance at the transformations of DSML procedure and the accompanying solution algorithm would confirm that this does not hold in our case. The set of regressors does not include lagged variables in an explicit manner.

6. The DW values have been computed for the 95% confidence level.

7. The yield rates on stocks have been obtained from the flow of funds data as reported in the monthly Federal Reserve Bulletins. These have been calculated as the ratio of dividend to prices for the \( t \)-th observation.
CHAPTER VIII

CONCLUSIONS

It is clear from the empirical evidence presented here that if in a given functional form the explanatory variable(s) contains expectational elements, it would be inappropriate to estimate the stochastic version of the model by classical procedures. Further, if autoregressive processes are involved, either as an implication of a lag model or autocorrelation or both, it is imperative to provide the needed correctives to mitigate the problem, otherwise we could be faced with a serious bias in the estimates.

In the specific framework of the life insurance portfolio model, we have found that DSML estimates have performed better in the case of single equation idealized system. The lagged approximations of the expectational variables of the system have invariably given us better estimates than their counterpart ex post variables. In addition, scanning for the autocorrelation parameter has also improved the results substantially, even in cases where the pivotal variable happens to have an inappropriate sign. The double transformation associated with the two types of non-linearities and the particular solution algorithm adopted here does indeed affect, in a significant manner, the estimates of a given relationship.

Similarly, in the case of a simultaneous system, we have found that DSML estimates have performed better than the classical estimates. In fact, in some cases the difference between the two sets of results
is quite prominent. As regards the problem of autocorrelation, however, the improvement in results is not as significant as in the case of single equation system, but in most instances we find that scanning technique has helped to improve, or at least maintain, the accuracy of classical results.

It should be noted here that the DSML procedure used in the case of simultaneous estimation is rather more comprehensive than any attempted so far. At the most, we have found in the existing literature simultaneous estimation of an explicitly autocorrelated model using iterative methods of convergence to an optimal solution. In the present study, we have demonstrated that, to begin with, the iterative scheme is inadequate in locating the global minimum of the likelihood function using the same criterion as employed in case of DSML procedure. Secondly, we have established the feasibility of estimation of a simultaneous system, involving prespecified autoregressive processes, generated by the lag and serial correlation in a structural relationship. These aspects of DSML procedure embody significant implications for existing methods of estimation.

In view of the above it would be safe to generalize that if we are confronted with the estimation of non-linear model, where the non-linearities are of the type as specified here, it is indeed preferable to employ the DSML procedure, which takes account of the peculiarities of the system. The classical approach in this case would clearly be inadequate.

As regards the monetary aspects of the estimated life insurance portfolio model, we have found that the rate of return, if properly specified, is the crucial variable in managerial decision making with
regard to investment in risky assets. Unfortunately, the surrogate for risk, $\Theta$, and risk free returns, $V^*$, did not turn out to be as significant as we expected, however, the substitutability indices—$R^*i$ and $Z^*i$—behaved well enough to warrant their inclusion. We have strictly adhered to the dictates of the a priori considerations as outlined in Chapter III, and have included all the variables thus specified in the set of explanatory variables for a given asset demand function. Presumably, if we were to follow a more pragmatic approach, we would eliminate the insignificant variables, as we did in the case of the abbreviated model.

In most cases we have been able to obtain a good fit to the data as well as significant relationships. For purposes of forecasting, this aspect of estimates may prove quite helpful and would undoubtedly facilitate simulation of the portfolio model. In addition, the estimates may also be of significance in formulating policies of portfolio management, though, for a greater degree of realism it would be necessary to effect some adjustments in the specification of the equations of the model.
Bibliography


APPENDIX I

LIFE INSURANCE PORTFOLIO MODEL:

DETAILED LIST

The idealized system—Model I—consists of 11 stochastic relationships corresponding to each of the nine types of assets in the life insurance portfolio and the two aggregate categories. Retaining the notational scheme developed in Chapter V, we could write the following equations belonging to the set in Model I:

Governments:

(1.1) U.S. Government, \( y_1 = f (r_1, \theta, V^*, R^*_1, Z^*_1) \)
(1.2) State and Local, \( y_2 = f (r_2, \theta, V^*, R^*_2, Z^*_2) \)

Corporate Bonds:

(1.3) Industrials, \( y_3 = f (r_3, \theta, V^*, R^*_3, Z^*_3) \)
(1.4) Public Util: \( y_4 = f (r_4, \theta, V^*, R^*_4, Z^*_4) \)
(1.5) Railroads, \( y_5 = f (r_5, \theta, V^*, R^*_5, Z^*_5) \)

Corporate Stocks:

(1.6) Common Stocks, \( y_6 = f (r_6, \theta, V^*, R^*_6, Z^*_6) \)
(1.7) Preferred Stocks, \( y_7 = f (r_7, \theta, V^*, R^*_7, Z^*_7) \)

Others:

(1.8) Mortgages, \( y_8 = f (r_8, \theta, V^*, R^*_8, Z^*_8) \)
(1.9) Policy Loans, \( y_9 = f (r_9, \theta, V^*, R^*_9, Z^*_9) \)

These nine equations are the core of the life insurance portfolio
portfolio model, which in the above specification consists of independent asset demand functions. In addition, there are two aggregate relationships, defined for total business bonds \((y_3 + y_4 + y_5)\), and total governments \((y_1 + y_2)\). Thus we have:

\[(1.10) \quad \text{Total Governments, } y_{10} = f(r_{10}, \Theta, V^*, R^{10}, Z^{10})\]
\[(1.11) \quad \text{Total Corporate Bonds, } y_{11} = f(r_{11}, \Theta, V^*, R^{11}, Z^{11})\]

Corresponding to these 11 equations we will have eleven sets of estimated coefficients and appropriate test values obtained from the OLS and DSML procedures. The key to the variables has been given in the main text, Chapter V, and need not be repeated here.

The simultaneous system (Model II) consists of the first nine equations of the model outlined above on slightly redefined variables. The last two equations have been replaced by the identities

1. defined for the total dollar value of investments \(y_1 = Y\), and
2. total returns, \(R = R^*1\). Besides, the endogenous set, \(y_1\), would now be explicitly incorporated in the set of explanatory variables, instead of the aggregate form used above as \(Z^*1\) variables.

The equations of the simultaneous system are:

\[(1.12) \quad y_1 = f(r_1, \Theta, V^*, R^*1, Y_1\ [y_2, \ldots, y_9])\]
\[(1.13) \quad y_2 = f(r_2, \Theta, V^*, R^*2, Y_2\ [y_1, y_3, \ldots, y_9])\]
\[(1.14) \quad y_3 = f(r_3, \Theta, V^*, R^*3, Y_3\ [y_1, y_2, y_3, \ldots, y_9])\]
\[(1.15) \quad y_4 = f(r_4, \Theta, V^*, R^*4, Y_4\ [y_1, y_2, y_3, y_5, \ldots, y_9])\]
\[(1.16) \quad y_5 = f(r_5, \Theta, V^*, R^*5, Y_5\ [y_1, \ldots, y_4, y_6, \ldots, y_9])\]
\[(1.17) \quad y_6 = f(r_6, \Theta, V^*, R^*6, Y_6\ [y_1, \ldots, y_5, y_7, y_8, y_9])\]
\[(1.18) \quad y_7 = f(r_7, \Theta, V^*, R^*7, Y_7\ [y_1, \ldots, y_6, y_7, y_8, y_9])\]
\[(1.19) \quad y_8 = f(r_8, \Theta, V^*, R^*8, Y_8\ [y_1, \ldots, y_7, y_8])\]
\[(1.20) \quad y_9 = f(r_9, \Theta, V^*, R^*9, Y_9\ [y_1, \ldots, y_8])\]
Thus in a typical relationship of the simultaneous model, there will be at least 12 explanatory variables, 8 of them will be the explanatory endogenous variables, and four exogenous variables, besides the constant. The exogenous variables will include rate of return, risk surrogate, risk free returns, and remainder returns.

Corresponding to these nine equations we will have nine sets of estimates of the coefficients of explanatory variables, and the test statistics obtained from the application of (1) 2SLS, and LIML methods, and (2) DSML procedure. The simultaneous estimation procedure is a hybrid of two methods, viz, in the first stage of estimation we would like to eliminate the stochastic elements in the set of endogenous variables, $Y_1$, by regressing them on all exogenous variables of the model; and in the second stage, we will use $Y_1$ for scanned maximum likelihood estimates. Details of the estimation techniques have been discussed in Chapter VI, and need not be repeated here.
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