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THE DEVELOPMENT AND EVALUATION OF A
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FOR STUDENTS IN THE BEHAVIORAL SCIENCES

DISSertation
Presented in Partial Fulfillment of the
Requirements for the Degree Doctor of
Philosophy in the Graduate School of
The Ohio State University

By
James Jackson Barnette, B.S., M.A.

The Ohio State University
1972

Approved by
Robert P. Benzon
Advisor
Department of
Educational Development
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To my wife, Susan, I owe a majority of the thanks. Her encouragement and support have been a continued and substantial motivation in my educational and professional career. All of this would not have been possible without her.
VITA

July 22, 1943........ Born-Charleston, West Virginia

1961............... Graduated, Troy High School, Troy, Ohio

1966............... B. Sc., The Ohio State University, Columbus, Ohio

1966-1967......... Dean of Men's Staff, Capital University, Bexley, Ohio

1967-1969......... Director, Drackett Tower, The Ohio State University, Columbus, Ohio

1968............... M.A., The Ohio State University, Columbus, Ohio

1969-1970......... Director, Scholarships and Grants, Student Financial Aids, The Ohio State University, Columbus, Ohio

1970-1972......... Teaching Associate, Department of Educational Development, The Ohio State University, Columbus, Ohio

1970-1972......... Assistant to the Dean, College of Engineering, The Ohio State University, Columbus, Ohio
PUBLICATIONS

"The Objective Evaluation of Student Performance", Proceedings of the North Central Section of the American Society for Engineering Education, Cleveland, Ohio, April 1972.

FIELD OF STUDY

Major Field: Educational Development, Professor Robert Bargar

Studies in Higher Education, Professor Richard Frankie

Studies in Statistics, Professors George Briggs, Raymond Sletto, and James Gunnell

Studies in Educational Administration, Professor Carl Candoli
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Chapter I

Introduction

The purpose of this investigation is to develop and test a set of materials designed to prepare behavioral science students, including educators, for a successful experience in an introductory behavioral science statistics course.

Statement of the Problem

Will a programmed pre-statistics course manual which relates a review of mathematical concepts to actual uses in an introductory statistics course aid graduate and/or undergraduate students in their orientation to and performance in an introductory statistics course in education?

It is obvious that the mere mention of the word "statistics" strikes fear into the heart of many educators, especially if they have not had a course in introductory statistics in behavioral research.

To a great extent, the anxiety or fear of introductory statistics among educators stems from a high
anxiety level regarding the mathematics needed for the course. Many of the students, especially the graduate students, have been away from mathematics for several years. Many students have a hard time remembering the mathematics they have learned because they have not used it. Coupled with this is the introduction of new concepts and new symbolization which tends to increase anxiety toward the course.

Also, many students have been conditioned to fear statistics because of the attitudes of others. This has been a factor in the current attitude toward statistics.

In addition, introductory statistics courses require students to perform operations that are different than those found in most other behavioral science and education courses. Most students are not in a habit of thinking quantitatively. Also, the types of examinations are fairly rigorous and for many students are seen as a very threatening experience.

It has been demonstrated (11,63) that students tend to avoid learning situations which have high levels of anxiety associated with them. Many students facing an introductory statistics course have anxiety
levels of an increased degree. There is often a need to reduce student anxiety toward statistics and to develop confidence about the probability of success in the course.

One of the purposes of this study will be to examine the relationships between attitude, math proficiency level, and performance in an introductory statistics course.

This study deals with the development and evaluation of a programmed manual to: (1) reduce anxiety toward the course, and (2) provide a mathematics review specifically related to actual uses of mathematics in introductory behavioral research statistics.

Importance of the Study

The primary purpose of this study is to assess a method of assisting educational career oriented undergraduate and graduate students in their orientation to and performance in an introductory statistics course, through the use of a pre-statistics orientation programmed manual.

The evaluation of the effectiveness of this manual will be conducted on the basis of several attitudinal and performance variables.
There will be two major integrated components of the manual: (1) orientation to statistics in behavioral research, and (2) a review of selected mathematical concepts and how they are used in statistics.

Use of the manual should aid students as follows:
A. Increase understanding of the function of statistics in behavioral research.
B. Reduce anxiety concerning the course.
C. Prepare students for meeting the mathematical requirements for introductory statistics.
D. Aid in the understanding of initial statistical concepts.
E. Result in greater knowledge of statistical concepts.
F. Result in increased ability to apply statistical concepts.

Null Hypotheses to be Tested

1. The pre-course math proficiency level of the undergraduate students who have the manual will not be significantly higher than the pre-course math proficiency level of the undergraduate students who do not have the
manual.

2. The pre-course math proficiency level of the graduate students who have the manual will not be significantly higher than the pre-course math proficiency level of the graduate students who do not have the manual.

3. There will not be a significant decrease in variance in the math proficiency level of the undergraduate students who have the manual as compared with the undergraduate students who do not have the manual.

4. There will not be a significant decrease in variance in the math proficiency level of the graduate students who have the manual as compared with the graduate students who do not have the manual.

5. There will not be a significant change in pre-course attitude toward the statistics course between those undergraduate students who have the manual and those undergraduate students who do not have the manual.

6. There will not be a significant change in pre-course attitude toward taking the statistics course between those graduate students who have the manual and those graduate students who do not have the manual.

7. There will not be a significant correlation between
amount of time spent using the math oriented units of
the manual and increase in pre-course math proficiency
level for the undergraduate students.
8. There will not be a significant correlation between
amount of time spent using the math oriented units of
the manual and increase in pre-course math proficiency
level of the graduate students.
9. There will not be a significant correlation between
amount of time spent using the manual and change in
pre-course attitude toward taking the statistics course
for the undergraduate students.
10. There will not be a significant correlation between
amount of time spent using the manual and change in
pre-course attitude toward taking the statistics course
for the graduate students.
11. Prior to the course, the undergraduate students
who have the manual will not find it useful in their
preparation for taking the statistics course.
12. Prior to the course, the graduate students who have
the manual will not find it useful in their preparation
for taking the statistics course.
13. There will not be a significant correlation between
pre-course math proficiency level and final class score
for all students.

14. There will not be a significant correlation between pre-course attitude and final class score for all students.

15. Performance on the midterm exam will not be significantly higher for the undergraduate students who have the manual than for the undergraduate students who do not have the manual.

16. Performance on the midterm exam will not be significantly higher for the graduate students who have the manual than for the graduate students who do not have the manual.

17. Performance on the final exam will not be significantly higher for the undergraduate students who have the manual than for the undergraduate students who do not have the manual.

18. Performance on the final exam will not be significantly higher for the graduate students who have the manual than for the graduate students who do not have the manual.

19. Final course performance will not be significantly higher for the undergraduate students who have the manual than for the undergraduate students who do not
have the manual.

20. Final course performance will not be significantly higher for the graduate students who have the manual than for the graduate students who do not have the manual.

21. At the end of the course, undergraduate students who have used the manual will not find it useful in their preparation for taking the statistics course.

22. At the end of the course, graduate students who have used the manual will not find it useful in their preparation for taking the statistics course.

Definition of Terms

Pre-statistics Orientation Manual - A manual developed by the author designed to (1) orient the students to the basic topics of an introductory statistics course and (2) provide a mathematics review of concepts needed in an introductory statistics course. (A copy of this manual is included in the Appendix.)

Education 541 and 786 - An introductory statistics course offered by the Faculty of Educational Development at The Ohio State University. The course has two numbers, 541, for a group of selected undergraduate
students in education and 786, for graduate students in education and related fields. The course covers introductory descriptive and inferential statistics.

**Pre-course math proficiency level** - An evaluation of student math competency in areas relating to math uses in introductory statistics. The instrument includes: operations with signed numbers, operations with fractions, square-roots, elementary algebra, summation notation, and elementary linear graphing concepts. Pre-test and post-test forms were constructed. These are found in the Appendix.

A parallel-forms reliability coefficient was computed for the pre and post-test forms, based on subjects who do not receive the manual.

**Pre-course attitude** - An evaluation of student attitude toward mathematics and statistics. Students were requested to respond to a series of items constructed to give an indication of the degree of positive or negative attitude toward the course. Certain pre-selected items were used as indicators. Pre-test and post-test forms were constructed. These are found in the Appendix.

A test-retest reliability coefficient is not ap-
propriate since subject attitude could change as a function of time, since the pre-test was given three weeks prior to the course and the post-test was given during the first week of the quarter. A split-half reliability coefficient was calculated for the attitudinal instruments.

Pre-course student evaluation of the manual -
Student responses evaluating each of the manual units that they completed in terms of usefulness, clarity, and any suggestions for improvement. These forms are found in the Appendix along with the other instruments.

Post-course student evaluation of the manual -
Student responses at the end of the course evaluating the manual in terms of usefulness, format, areas of need, and suggestions for improvement. A copy is found in the Appendix.

Mid-term exam - An objective examination based on approximately the first half of the course. The exam was comprised of multiple-choice questions and problems on descriptive statistics.

Final exam - An objective examination based on the second half of the course. The exam was comprised of multiple-choice questions and problems on basic infer-
ential statistics.

**Final course score** - The final course score was the average standard score based on the midterm and final examinations.

**Organization of the Dissertation**

Chapter I presents the introduction, a discussion of the importance and objectives of the study, the hypotheses, and definition of terms.

An examination of the related literature is presented in Chapter II.

Chapter III presents the description of procedures, methodology, instrumentation, and data analysis.

Research findings are presented in Chapter IV.

The summary and conclusions are discussed in Chapter V.
Chapter II

Review of the Literature

Early in the 1920's W. W. Charters began to look at the process of learning in terms of a task analysis. His position was that curricula should be broken down into small behavioral segments to be sequenced into a pedagogically sound order. The ideal was to put the educational program in small enough increments that the student could learn without teacher assistance.

Perhaps the foremost pioneer of programmed learning theory was B. F. Skinner. Skinner's (21, 29) rationale is presented as, "the whole process of becoming competent in any field must be divided into a very large number of very small steps, and reinforcement must be contingent upon the accomplishments of each step." In order to acquire complex behaviors, students "pass through a carefully designed sequence of steps, often of considerable length. Each step must be so small that it can always be taken, yet in taking it, the student moves somewhat closer to fully competent behavior (20, 14)." More recently, Gagne (2) has pre-
presented a hierarchical diagram of subordinate skills in sequence based on his theory that identified competencies serve as mediators of positive transfer between lower level competencies to higher level competencies.

Most educators agree that instruction is more effective if it is individualized. Glaser (9,167-170) presents the benefits of individualized instruction as: (1) nurturing independent learning resulting in individuals who are resourceful and self-appraising learners, and (2) it is instruction based on notions of competence mastery, and the attainment of standards. Many studies have been conducted to attempt to determine learner variables related to effective programmed learning. These include research on general and specific ability, post specific achievement, personality variables, demographic characteristics, and attitudes toward programmed instruction. Research results have been inconclusive and somewhat contradictory. Also, a great deal of research has been conducted comparing a programmed mode of instruction with conventional modes. Most of this research has resulted in "no significant difference" in measures of achievement. However, achievement cannot be the only dependent var-
iable of interest. Programmed instruction has been shown to be efficient in terms of time factors for learning and use of teacher time (3,370). Branching programs have been shown to be usually more effective for groups with heterogeneous ability levels (3,370).

Robinson (18,133) cites as two primary uses of programmed materials in higher education: (1) a review of basic content necessary for pre-course preparation and (2) for supplementary course instruction. If pre-course review of certain needed skills is effective, this should result in more efficient course instruction. Brown et al. (4,114) cite several advantages of programmed instruction as: (1) permitting the individualization of learning, (2) reducing the amount of teaching time, (3) improving the level of performance and reducing the incidence of failure, and (4) permitting the assessment of reasons for successful and unsuccessful experiences. Pipe (17,p.3) states, "the real strength of programmed instruction is that it is relevant instruction".

The process of developing instructional programmed material is presented in the National Society for the Study of Education, Sixty-sixth Yearbook, Part II (16,
p.58). This process is presented, in general, in Figure 1. The development of the manuals used in this study was based on this framework.

In writing of an actual program, Cooper and Mertens (7, p.234) present several considerations which the author must make. First, the prospective author should acquaint himself with various techniques of programming. Secondly, a decision must be made as to what is to be taught and how it can be well illustrated and supported by meaningful factual material. Defining the behavioral objectives is the third consideration. These should be put in writing for careful consideration. Fourth, the program should be written in such a way that no additional instruction on the topic is required. Considering the age and type of student is the fifth consideration. The sixth consideration is the need for logical "breaks" in the program. Seventh, is a warning not to "over-program" which can bore the student.

Three types of program modes have been identified. First, is the linear mode where every student goes through each frame of the program in the same sequence
Branching, the second mode, guides the student through the program using differential sequencing based upon
his performance on previous frames. The third mode, which is very similar to branching, is the adaptive mode. The student enters the programmed material based on a pre-test of the content area to find areas which would be most beneficial to him.

Evaluation of programmed materials has usually been based on one of two methods. A very common method is the comparison of student achievement for students using programmed materials with students participating in a "conventional" class. Usually, in these evaluations, there has been a favorable evaluation of the programmed instruction, either in terms of significant differences in study time or relatively more effective teacher assistance in helping individual students (3, p.370). The other technique of evaluation has been in the form of analytical studies which attempt to identify factors that make programmed instruction effective and workable (3,p.354).

In a study, conducted by Collagan (6,p.364), the effectiveness of a programmed manual to prepare non-science students for the mathematics skills needed for a course in physical science was evaluated. Collagan concluded that "programmed mathematics instruction
for many remedial students is both more effective and more time-saving than conventional instruction".

In the area of pre-statistics course mathematics preparation, programmed materials have been developed by such authors as Baggaley (1) and Kearney (13). For the most part, these materials deal with the appropriate mathematical concepts, however, the instruction is somewhat abstract in terms of how the mathematics concepts are used in statistical computations. I feel programmed materials designed to review mathematics for introductory statistics should be based on the presentation of the essential mathematics concepts in terms of their actual use in statistical concepts and applications. Therefore, it will be necessary to provide some introduction to various statistical concepts in the body of the programmed text. By basing instruction on actual application rather than on abstract mathematical examples, the instruction should be more meaningful and more easily retained throughout the course.
Chapter III

Methodology

In order to assess the need for such a pre-statistics orientation manual, 89 opinionnaires were mailed on August 3, 1971, to undergraduate and graduate students who had completed Education 786 (introductory statistics) during the Winter, Spring, or Summer quarters of 1971. After six weeks, 58 or 65.2% had been returned. (A copy of this opinionnaire and the cover letter are in the Appendix.) When asked how much of a need they felt there was for a pre-statistics orientation manual, 31% said "some need", 38% said "much need", and 28% said "very much need". Thus, almost everyone responding felt that there was a need for such a manual.

Almost all of the content areas mentioned in the structure of the opinionnaire were included in the composition of the manual. No other suggestions were made for an other than programmed manual method of accomplishing the task. There were no other arithmetic or algebraic concepts which respondents felt were needed. All other concepts which respondents felt were needed
were considered to be actual course concepts which
did not lend themselves well to the pre-course orien-
tation manual.

Development of the Manual

Each unit of the manual was constructed on the
basis of the pilot opinionnaire. A rough draft was
prepared and reviewed by two undergraduate math stu-
dents. Revisions were made and each unit was examined
by at least four undergraduate students who had recent-
ly completed the introductory statistics course. The
evaluators were asked to: estimate the amount of
time needed to complete the unit, evaluate the clarity
and flow of the material, check problems for correct
answers, and make any suggestions for improving the man-
uals. Final revisions were made and the manual was
constructed. Figure 2 is a listing of the units in the
final manual.

Population and Sample Selection

It is assumed that the results of this study or
the determination of the usefulness of the manual, may
be generalized to the very broad population of graduate
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| XV.  | The Scattergram |
and undergraduate students who take introductory behavioral research statistics. However, for the purposes of this study, the population was those graduate and undergraduate students enrolled in Education 785 and Education 541, during Autumn quarter 1971, who intend to be enrolled in the introductory statistics course during Winter quarter 1972. Half of each group was assigned randomly to either the experimental or control group. The experimental groups were given the manual at the time they were pre-tested, which was during the first week of December 1971. The control groups were pre-tested at the same time, but did not receive the manual. Experimental groups students were encouraged to use the manual during the between quarter break. Control group students were encouraged to review math and were given the names of math review sources. In order to test the possible effect of the pre-test and the encouragement, a group of graduate students who enrolled in Education 786, but were not enrolled in Education 785 during Autumn quarter 1971, were given the attitudinal and math proficiency post-test at the beginning of Winter quarter when the other four groups were tested.
**Instrumentation**

I. Attitude Inventory, pre-test and post-test forms. This instrument was in the form of a check-list and rating-scale composed of items designed to determine why the student was taking the course, attitude toward math, and attitude toward taking the course. Reliability was assessed using a split-half method.

II. Math Proficiency Inventory, pre-test and post-test forms. This instrument, which was about 30 minutes in length, tested the student's ability to solve arithmetic problems which involved algebraic signs, sequencing of sub-operations, manipulation of equations, determining the location of decimal places, square roots, elementary graphing concepts, and the summation notation. Reliability was assessed by correlating the performance of the control group in the pre-test situation with the post-test situation.

III. Student Estimate of Time using manual and usefulness of manual in pre-course preparation. This instrument was in the form of specific questions regarding time spent and a rating-scale evaluating each unit of the instrument in terms of pre-course preparation.
IV. Midterm and Final Exams. These instruments were objective tests designed to measure student level of cognitive and application performance.

V. Final course performance. The average T score based on the T scores of the midterm and the final exams.

VI. Student post-course evaluation of the manual. A brief evaluation form completed by the student at the end of the course to determine usefulness of the manual, and suggestions for improvement.

Design

A modified combination of the pre-test, post-test control group design and the post-test only control group design was used to evaluate the possible relationship of the use of the manual and the various dependent variables previously specified.

The design is diagrammed in Figure 3.

Observations $O_1, O_2, O_3, O_4$ were in the form of a combination math proficiency test and attitude toward taking the statistics course pre-test.

Observations $O_5, O_6, O_7, O_8$ were the post-test
form of the math proficiency test and attitude toward taking the statistics course.

Observations $0_{9}$ and $0_{10}$ were estimates made by the student of the amount of time spent using the manual prior to the course and the student's evaluation of its usefulness.

Observations $0_{11}$, $0_{12}$, $0_{13}$, $0_{14}$ were the course mid-term exam grades.

Observations $0_{15}$, $0_{16}$, $0_{17}$, $0_{18}$, were the course final exam grades.

Observations $0_{19}$, $0_{20}$, $0_{21}$, $0_{22}$, were the course final course grades in the form of the student's average T score. Observations $0_{23}$ and $0_{24}$ were manual evaluation forms completed by the student at the end of the course.

Observation $0_{25}$, was a measure of pre-course math
proficiency level and attitudinal measure for a group of graduate students who did not take the pre-test. This was used to assess the possible effects of pre-test sensitization.

This design was strong in terms of internal validity. All of the sources of internal validity should be controlled to a satisfactory degree.

However, there are some possible problems in terms of external validity. There is a fair chance of problems in the area of interaction of testing and X. Observation 0_{21} was used to assess the possible effect of the pre-test in sensitizing subjects to the post-test measure of math proficiency. Multiple testing might be a weakness, although in this situation most of the observations would occur naturally as a part of the course, and thus should not be a major concern. It is assumed that the interactions of history, selection, or maturation with the treatment were not a serious concern, since the subjects had the treatment only for a period of three weeks.

Perhaps the greatest weakness is in terms of reactive arrangements. Some students in the same class will have used the manual while others will not. This could
have resulted in attitudinal differences. In an attempt to counteract this, the manuals were collected on the first day of the class so that no students would have them during the course.

Students who received the manual were asked to not let anyone else use them. However, the author had no control over this situation. This weakness could have effected various measures to some extent, hopefully in the direction of making any results biased on the conservative side. However, it should not have effected the student's evaluation of the usefulness of the manual.

**Analysis of the Data**

Hypotheses 1 and 2 were tested using analysis of covariance, the pre-test math proficiency test being the covariate. Alpha was set at 0.05.

Hypotheses 3 and 4 were tested using a t test of the variances. Alpha was set at 0.05.

Hypotheses 5 and 6 were tested using a chi-square test with alpha equal to 0.05.

Hypotheses 7 and 8 were tested using a Pearson correlation coefficient and testing it for significance.
Alpha was set at 0.05.

Hypotheses 9 and 10 were tested using Spearman correlation coefficients, corrected for ties, and testing them for significance at alpha equal to 0.05.

Hypotheses 11 and 12 were judged on the basis of percentage responses. They were rejected if more than 75% of the students found the manual at least somewhat useful.

Hypothesis 13 was tested using a Pearson r and testing for significance at the 0.05 level.

Hypothesis 14 was tested using a Spearman r, corrected for ties, and testing it for significance at the 0.05 level.

Hypotheses 15, 16, 17, 18, 19, and 20 were tested using analysis of covariance with pre-course math proficiency level as the covariate. Alpha equals 0.05.

Hypotheses 21 and 22 were judged on the basis of percentage responses. They were rejected if more than 75% of the students found the manual at least somewhat useful.
Prior to discussing the hypotheses, it is essential to discuss the validity and reliability of the primary instruments. The Math Proficiency Level Examination (pre and post-test forms) had face validity since the test items were based on the types of uses of mathematics in elementary statistics such as: operations with signed numbers, operations with fractions, square-roots, manipulation of equations, order of operations, use of the summation notation, and two-variable graphing concepts. Reliability was estimated by correlating the pre and post-test (parallel forms) performance of students who did not receive the manuals. Based on n=30, the correlation between pre and post-test performance was +0.912, which when compared with r=0 resulted in a t value of 11.8. This was significant at p<0.01.

The pre and post-test attitudinal measures were comprised of several items, eight of which on each instrument were used to measure attitude toward math...
and statistics. A parallel forms reliability estimate is not appropriate since attitude could change as a result of time proximity to the start of the course. Therefore, the estimate of reliability was based on a split-half reliability correlating the responses of four randomly selected items with the other four items of the instrument. For the pre-test, the split-half reliability, based on n=67 was +0.712. The estimate of total-test reliability is given by

\[
 r_{tt} = \frac{2r}{1+r} = +0.832,
\]

which was significant at p<0.01.

The post-test split-half reliability for the post-test instrument, based on n=67 was +0.705. The estimate of total-test reliability was +0.827, which is significant at p<0.01.

It is of interest to look at the relationships between the variables of attitude, math proficiency level, and performance in an introductory statistics course. The rex for attitude-final course performance was +0.324 and the rex for math proficiency-final course performance was +0.401. These values must be considered in terms of the rex between the attitude var-
iable and the math proficiency level, which was +0.578. In order to look at the relationships between these three variables, a partial-correlation coefficient was calculated for each pair while the third variable is partialed out.

The partial correlation between attitude and course performance, with math proficiency partialed out, was +0.123 (t=0.984, N. S.). Between math proficiency level and course performance, with attitude partialed out, was +0.277 (t=2.289, p<0.05).

Therefore, math proficiency level is a better predictor of course performance than is attitude. However there was a significant relationship between math level and attitude (r_s=+0.578, t=5.62, p<0.01). For these reasons, math proficiency level was used as the covariate in many of the comparisons.

Before discussing the hypotheses, another analysis should be considered. Did the math pre-test have an influence on post-test scores? Table 1 presents the math post-test data for the graduate students in the experimental, control, and post-test only groups.
A one-way analysis of variance was conducted between the three groups, comparing the math proficiency level post-test scores. Table 2 presents the results of this analysis.

**TABLE 2**

**PRE-COURSE MATH PROFICIENCY LEVEL**
**GRADUATE STUDENTS**
**ANALYSIS OF VARIANCE SUMMARY**

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between</td>
<td>1518.585</td>
<td>2</td>
<td>759.293</td>
<td>4.192*</td>
</tr>
<tr>
<td>Within</td>
<td>9056.397</td>
<td>50</td>
<td>181.128</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>10574.982</td>
<td>52</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*F* (2, 50) = 3.19, *p* < 0.05

On the basis of this analysis, at least one of the pairs of means is significantly different. Table 3 describes the mean differences as well as the comparative ratio (*t*/*s*) for the Scheffe' multiple comparison method.
TABLE 3
PRE-COURSE MATH PROFICIENCY LEVEL
GRADUATE STUDENTS
POST HOC COMPARISON SUMMARY

<table>
<thead>
<tr>
<th>Comparison</th>
<th>Mean Difference</th>
<th>$\Phi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental-</td>
<td>4.48</td>
<td>0.945</td>
</tr>
<tr>
<td>Control</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experimental-Post</td>
<td>12.27</td>
<td>2.880*</td>
</tr>
<tr>
<td>only</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Control-Post only</td>
<td>7.79</td>
<td>1.583</td>
</tr>
</tbody>
</table>

* Critical value $0.05 = \sqrt{(J-1)}_{\alpha} \ F_{J-1}, N-J = 2.53$

The comparison of primary consideration is the control group with the post-test only group. This comparison which had a mean difference of 7.79 was not significant at the less than 0.05 level. Thus, it seems as though the pre-test may have had some influence, but not a statistically significant one.

Hypothesis number one, stated in null form is:
The pre-course math proficiency level of the undergraduate students who have the manual will not be significantly higher than the pre-course math proficiency level of the undergraduate students who do not have the manual. Table 4 describes the summary statistics of the math proficiency level scores for the undergraduate students.

Analysis of covariance was used to test the differ-
TABLE 4
PRE-COURSE MATH PROFICIENCY LEVEL
UNDERGRADUATE STUDENTS
SUMMARY STATISTICS

<table>
<thead>
<tr>
<th></th>
<th>Experimental</th>
<th>Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-test Mean</td>
<td>53.63</td>
<td>56.71</td>
</tr>
<tr>
<td>Pre-test σ</td>
<td>12.02</td>
<td>11.18</td>
</tr>
<tr>
<td>Post-test Mean</td>
<td>60.19</td>
<td>56.24</td>
</tr>
<tr>
<td>Post-test σ</td>
<td>11.18</td>
<td>12.57</td>
</tr>
<tr>
<td>n</td>
<td>16</td>
<td>17</td>
</tr>
</tbody>
</table>

ences for significance. Analysis of covariance provides a statistical control of variance by accounting for pre-test variance in the post-test scores. Math pre-test scores were used as the covariate.

Table 5 presents the analysis of covariance of the math proficiency level scores for the undergraduates.

TABLE 5
PRE-COURSE MATH PROFICIENCY LEVEL
UNDERGRADUATE STUDENTS
ANALYSIS OF COVARIANCE SUMMARY

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between (adjusted)</td>
<td>357.41</td>
<td>1</td>
<td>357.41</td>
<td>8.19**</td>
</tr>
<tr>
<td>Within (adjusted)</td>
<td>1308.97</td>
<td>30</td>
<td>43.63</td>
<td></td>
</tr>
<tr>
<td>Total (adjusted)</td>
<td>1666.38</td>
<td>31</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**F (1,30) = 7.56, p<0.01
The observed $F$ ratio of 8.19 is significant at the less than 0.01 level. Therefore, hypothesis number one is rejected and it is concluded that undergraduates who received the manual did perform at a significantly higher level on the math proficiency post-test exam than did the undergraduates who did not receive the manual.

Hypothesis number two, stated in null form is:
The pre-course math proficiency level of the graduate students who have the manual will not be significantly higher than the pre-course math proficiency level of the graduate students who do not have the manual. Table 6 describes the summary statistics for this hypothesis.

<table>
<thead>
<tr>
<th>TABLE 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRE-COURSE MATH PROFICIENCY LEVEL</td>
</tr>
<tr>
<td>GRADUATE STUDENTS</td>
</tr>
<tr>
<td>SUMMARY STATISTICS</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Pre-test Mean</td>
</tr>
<tr>
<td>Pre-test $\sigma$</td>
</tr>
<tr>
<td>Post-test Mean</td>
</tr>
<tr>
<td>Post-test $\sigma$</td>
</tr>
<tr>
<td>$n$</td>
</tr>
</tbody>
</table>

These data were analyzed using analysis of covariance and the results are presented in Table 7.
### TABLE 7
PRE-COURSE MATH PROFICIENCY LEVEL
GRADUATE STUDENTS
ANALYSIS OF COVARIANCE SUMMARY

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between (adjusted)</td>
<td>291.81</td>
<td>1</td>
<td>291.81</td>
<td>5.09*</td>
</tr>
<tr>
<td>Within (adjusted)</td>
<td>1777.07</td>
<td>31</td>
<td>57.32</td>
<td></td>
</tr>
<tr>
<td>Total (adjusted)</td>
<td>2068.88</td>
<td>32</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*F (1,31) = 4.17, p<0.05

On the basis of this analysis, the hypothesis is rejected at the less than 0.05 significance level. Therefore, it is concluded that graduate students who received the manual did perform at a significantly higher level on the math proficiency post-test exam than did the graduates who did not receive the manual.

Hypothesis number three, in null form is: There will not be a significant decrease in variance in the math proficiency level of the undergraduate students who have the manual. This is a measure of the change toward a more homogeneous group in terms of math proficiency level. Table 8 presents the comparison of the pre-test and post-test variance using a t test for dependent groups.

This analysis indicates that there was not a significant decrease in variance for the undergraduate students who had the manual. Thus, hypothesis three is
TABLE 8
PRE-COURSE MATH PROFICIENCY LEVEL
UNDERGRADUATE STUDENTS
VARIANCE SUMMARY STATISTICS

<table>
<thead>
<tr>
<th></th>
<th>Pre-test</th>
<th>Post-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^2$</td>
<td>144.48</td>
<td>124.88</td>
</tr>
<tr>
<td>$n$</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>$r_{xy}$</td>
<td>+0.773</td>
<td></td>
</tr>
<tr>
<td>$t$</td>
<td>0.428</td>
<td>N.S.</td>
</tr>
</tbody>
</table>

not rejected.

Hypothesis number four, in null form is: There will not be a significant decrease in variance in the math proficiency level of the graduate students who have the manual. The comparison between pre-test and post-test variance is presented in Table 9.

TABLE 9
PRE-COURSE MATH PROFICIENCY LEVEL
GRADUATE STUDENTS
VARIANCE SUMMARY STATISTICS

<table>
<thead>
<tr>
<th></th>
<th>Pre-test</th>
<th>Post-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^2$</td>
<td>292.06</td>
<td>185.50</td>
</tr>
<tr>
<td>$n$</td>
<td>21</td>
<td>21</td>
</tr>
<tr>
<td>$r_{xy}$</td>
<td>+0.846</td>
<td></td>
</tr>
<tr>
<td>$t$</td>
<td>1.871*</td>
<td></td>
</tr>
</tbody>
</table>

*t (19) = 1.729, p<0.05

On the basis of this analysis, there is a significant decrease in variance between the pre-test and post-
test measures for the graduate students. Thus, hypothesis four is rejected.

Hypothesis number five, in null form is: There will not be a significant change in pre-course attitude toward taking the statistics course between those undergraduate students who have the manual and those undergraduate students who do not have the manual. Eight congruent items on each of the pre and post-test instruments were used to measure pre-course attitude. The post-test item response was compared with the appropriate pre-test response and any change of direction, either positive or negative, was noted. Table 10 presents the frequency of responses which changed in direction for the undergraduate experimental and control groups.

<table>
<thead>
<tr>
<th>TABLE 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHANGE IN PRE-COURSE ATTITUDE</td>
</tr>
<tr>
<td>UNDERGRADUATE STUDENTS</td>
</tr>
<tr>
<td>CHI-SQUARE SUMMARY</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Group</th>
<th>Negative</th>
<th>Positive</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental</td>
<td>22</td>
<td>33</td>
<td>55</td>
</tr>
<tr>
<td>Control</td>
<td>34</td>
<td>19</td>
<td>53</td>
</tr>
<tr>
<td>Total</td>
<td>56</td>
<td>52</td>
<td>108</td>
</tr>
</tbody>
</table>

$x^2 = 5.376, \ p<0.05$
The chi-square test, corrected for continuity, was used to analyze the data. The result was a chi-square value of 5.376, which was significant at less than the 0.05 level. Looking at Table 10, it is interesting to note that the group who received the manual (experimental) tended to move in a positive direction while the group that did not receive the manual (control) tended to move in a negative direction. Therefore, hypothesis number five is rejected.

Hypothesis number six, stated in null form is:
There will not be a significant change in pre-course attitude toward taking the statistics course between those graduate students who have the manual and those graduate students who do not have the manual. Table 11 presents the frequencies of directional response changes for the graduate students.

<table>
<thead>
<tr>
<th>TABLE 11</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHANGE IN PRE-COURSE ATTITUDE</td>
</tr>
<tr>
<td>GRADUATE STUDENTS</td>
</tr>
<tr>
<td>CHI-SQUARE SUMMARY</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Group</th>
<th>Negative</th>
<th>Positive</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental</td>
<td>13</td>
<td>55</td>
<td>68</td>
</tr>
<tr>
<td>Control</td>
<td>16</td>
<td>18</td>
<td>34</td>
</tr>
<tr>
<td>Total</td>
<td>29</td>
<td>73</td>
<td>102</td>
</tr>
</tbody>
</table>

\[ \chi^2 = 6.099, \ p < 0.02 \]
The chi-square test results in a chi-square of 6.099, which is significant at less than 0.02. Note that for both groups there were about the same number of changes in a negative direction, while the group which received the manual had a higher proportion of changes in a positive direction. Thus, hypothesis number six is rejected.

Hypothesis number seven, in null form is: There will not be a significant correlation between amount of time spent using the math oriented units of the manual and increase in pre-course math proficiency level for the undergraduates. A Pearson product-moment correlation coefficient of +0.013 was calculated, which is not significant. Thus, hypothesis number seven cannot be rejected.

Hypothesis number eight, in null form is: There will not be a significant correlation between amount of time spent using the math oriented units of the manual and increase in pre-course math proficiency level for the graduates. A Pearson product-moment correlation coefficient of +0.452 was calculated, which with 19 d.f. was significant at less than the 0.05 level. Thus, hypothesis number eight is rejected.
Hypothesis number nine, in null form is: There will not be a significant correlation between amount of time spent using the manual and change in pre-course attitude toward taking the statistics course for the undergraduates. A Spearman rank-order correlation coefficient of +0.359 was calculated, which was not significant. Therefore, hypothesis number nine is not rejected.

Hypothesis number ten, stated in null form is: There will not be a significant correlation between amount of time spent using the manual and change in pre-course attitude toward taking the course for the graduates. A Spearman rank-order correlation coefficient of +0.176 was calculated. This was not significant and thus, hypothesis number 10 cannot be rejected.

Hypothesis number eleven, in null form is: Prior to the course, undergraduates who have the manual will not find it useful in their preparation for taking the statistics course. Table 12 presents the distribution of responses on the manual evaluation forms for each of the primary units of the manual.
TABLE 12
STUDENT EVALUATION OF HELPFULNESS OF MANUAL
UNDERGRADUATE STUDENTS
RESPONSE SUMMARY

<table>
<thead>
<tr>
<th>Unit</th>
<th>Not Helpful</th>
<th>Little</th>
<th>Somewhat</th>
<th>Much</th>
<th>Very Much</th>
</tr>
</thead>
<tbody>
<tr>
<td>II</td>
<td>0</td>
<td>0</td>
<td>6</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>III</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>IV</td>
<td>1</td>
<td>3</td>
<td>8</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>V</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>VI</td>
<td>0</td>
<td>3</td>
<td>5</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>VII</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>VIII</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>IX,1</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>IX,2</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>X</td>
<td>0</td>
<td>0</td>
<td>6</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>XI</td>
<td>1</td>
<td>0</td>
<td>7</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>XII</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>XIII,1</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>XIII,2</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>XIII,3</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>XIII,4</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>XIV</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>XV,1</td>
<td>0</td>
<td>1</td>
<td>5</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>XV,2</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>XV,3</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>XV,4</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Totals: 3, 21, 73, 86, 89 \( \Sigma=272 \)
Percent: 1.1, 7.7, 26.3, 31.6, 32.7

Ninety-one percent of the responses were "somewhat" helpful or higher. In addition to this distribution, an item on the post-test attitudinal instrument asked students to rate the helpfulness of the manual. Thirty-eight percent rated it as "somewhat helpful" and fifty-six percent rated it as "very helpful". Therefore, on the basis of these responses, hypothesis number eleven is rejected.
Hypothesis number twelve, stated in null form is:
Prior to the course, graduate students who have the manual will not find it useful in their preparation for taking the statistics course. Table 13 presents the distribution of those responses to the evaluation forms.

**TABLE 13**

<table>
<thead>
<tr>
<th>STUDENT EVALUATION OF HELPFULNESS OF MANUAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>GRADUATE STUDENTS</td>
</tr>
<tr>
<td>RESPONSE SUMMARY</td>
</tr>
<tr>
<td>Unit</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>II</td>
</tr>
<tr>
<td>III</td>
</tr>
<tr>
<td>IV</td>
</tr>
<tr>
<td>V</td>
</tr>
<tr>
<td>VI</td>
</tr>
<tr>
<td>VII</td>
</tr>
<tr>
<td>VIII</td>
</tr>
<tr>
<td>IX,1</td>
</tr>
<tr>
<td>IX,2</td>
</tr>
<tr>
<td>X</td>
</tr>
<tr>
<td>XI</td>
</tr>
<tr>
<td>XII</td>
</tr>
<tr>
<td>XIII,1</td>
</tr>
<tr>
<td>XIII,2</td>
</tr>
<tr>
<td>XIII,3</td>
</tr>
<tr>
<td>XIII,4</td>
</tr>
<tr>
<td>XIV</td>
</tr>
<tr>
<td>XV,1</td>
</tr>
<tr>
<td>XV,2</td>
</tr>
<tr>
<td>XV,3</td>
</tr>
<tr>
<td>XV,4</td>
</tr>
<tr>
<td>Total</td>
</tr>
<tr>
<td>Percent</td>
</tr>
</tbody>
</table>

Ninety-three percent of the responses were "somewhat helpful" or higher. The responses for the item on the attitudinal post-test were twenty-eight percent
"somewhat helpful" and fifty-seven percent "very helpful". Thus, on the basis of these responses, hypothesis number twelve is rejected.

Hypothesis number thirteen, in null form is:
There will not be a significant correlation between pre-course math proficiency level and final class score for all students. A Pearson product-moment correlation coefficient of +0.442 was calculated for this comparison. This results in a t ratio of 3.91, which is significant at the less than 0.01 level. Therefore, hypothesis thirteen is rejected. Breaking this down into the undergraduate and graduate groups, the undergraduate correlation coefficient was +0.257 (t=1.44) which is not significant and the graduate correlation coefficient was +0.606 (t=4.31) which is significant at the 0.01 level.

Hypothesis number fourteen, in null form, is:
There will not be a significant correlation between pre-course attitude and final class score for all students. A Spearman rank-order correlation coefficient, corrected for ties, of +0.325 (t=2.73), was obtained which is significant at less than the 0.01 level. Therefore, hypothesis fourteen is rejected. The under-
graduate $r_s$ was $+0.354\ (t=2.04)$, which is significant at the 0.05 level. A $r_s$ of $+0.313\ (t=1.86)$ was calculated for the graduate group. This is significant at the 0.05 level.

Hypothesis number fifteen, stated in null form is: Performance on the midterm exam will not be significantly higher for the undergraduate students who have the manual than for undergraduate students who do not have the manual. Raw scores on the midterm were converted to T scores. The pre-test math proficiency scores were used as the covariate in the analysis of covariance. Table 14 describes the midterm exam scores for the undergraduates.

| TABLE 14 |
|---|---|---|---|
| MIDTERM EXAMINATION PERFORMANCE |
| UNDERGRADUATE STUDENTS |
| SUMMARY STATISTICS |

<table>
<thead>
<tr>
<th></th>
<th>Experimental</th>
<th>Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math Pre-test Mean</td>
<td>55.80</td>
<td>56.71</td>
</tr>
<tr>
<td>Math Pre-test $\sigma$</td>
<td>8.86</td>
<td>11.18</td>
</tr>
<tr>
<td>Midterm Mean T Score</td>
<td>49.03</td>
<td>50.81</td>
</tr>
<tr>
<td>Midterm T Score $\sigma$</td>
<td>8.75</td>
<td>10.91</td>
</tr>
<tr>
<td>n</td>
<td>15</td>
<td>17</td>
</tr>
</tbody>
</table>

The analysis of covariance results are presented in Table 15.
TABLE 15
MIDTERM EXAMINATION PERFORMANCE
UNDERGRADUATE STUDENTS
ANALYSIS OF COVARIANCE SUMMARY

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between (adjusted)</td>
<td>14.88</td>
<td>1</td>
<td>14.88</td>
<td>0.172 N.S.</td>
</tr>
<tr>
<td>Within (adjusted)</td>
<td>2501.74</td>
<td>29</td>
<td>86.27</td>
<td></td>
</tr>
<tr>
<td>Total (adjusted)</td>
<td>2516.62</td>
<td>30</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The F ratio of 0.172 is not significant. Therefore, hypothesis fifteen cannot be rejected.

Hypothesis number sixteen, in null form is:
Performance on the midterm exam will not be significantly higher for the graduate students who have the manual than for the graduate students who do not have the manual. Again, analysis of covariance was used to compare the groups described in Table 16.

TABLE 16
MIDTERM EXAMINATION PERFORMANCE
GRADUATE STUDENTS
SUMMARY STATISTICS

<table>
<thead>
<tr>
<th></th>
<th>Experimental</th>
<th>Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math Pre-test Mean</td>
<td>47.76</td>
<td>49.85</td>
</tr>
<tr>
<td>Math Pre-test σ</td>
<td>17.09</td>
<td>15.16</td>
</tr>
<tr>
<td>Midterm Mean T Score</td>
<td>50.29</td>
<td>50.45</td>
</tr>
<tr>
<td>Midterm T Score σ</td>
<td>9.10</td>
<td>11.28</td>
</tr>
<tr>
<td>n</td>
<td>21</td>
<td>13</td>
</tr>
</tbody>
</table>

The analysis of covariance results are presented in Table 17.
TABLE 17
MIDTERM EXAMINATION PERFORMANCE
GRADUATE STUDENTS
ANALYSIS OF COVARIANCE SUMMARY

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between (adjusted)</td>
<td>4.57</td>
<td>1</td>
<td>4.57</td>
<td>0.091 N.S.</td>
</tr>
<tr>
<td>Within (adjusted)</td>
<td>1552.43</td>
<td>31</td>
<td>50.08</td>
<td></td>
</tr>
<tr>
<td>Total (adjusted)</td>
<td>1557.00</td>
<td>32</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

An F ratio of 0.091 was obtained, which is not significant. Thus, hypothesis number sixteen cannot be rejected.

Hypothesis number seventeen, in null form is: Performance on the final exam will not be significantly higher for the undergraduates who have the manual than for undergraduates who do not have the manual. Table 18 presents the data used in this comparison.

TABLE 18
FINAL EXAMINATION PERFORMANCE
UNDERGRADUATE STUDENTS
SUMMARY STATISTICS

<table>
<thead>
<tr>
<th></th>
<th>Experimental</th>
<th>Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math Pre-test Mean</td>
<td>55.80</td>
<td>56.50</td>
</tr>
<tr>
<td>Math Pre-test σ</td>
<td>8.86</td>
<td>11.49</td>
</tr>
<tr>
<td>Final Mean T Score</td>
<td>50.50</td>
<td>49.50</td>
</tr>
<tr>
<td>Final T Score σ</td>
<td>10.14</td>
<td>9.87</td>
</tr>
<tr>
<td>n</td>
<td>15</td>
<td>16</td>
</tr>
</tbody>
</table>

The analysis of covariance summary is presented in Table 19.
TABLE 19
FINAL EXAMINATION PERFORMANCE
UNDERGRADUATE STUDENTS
ANALYSIS OF COVARIANCE SUMMARY

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between (adjusted)</td>
<td>10.66</td>
<td>1</td>
<td>10.66</td>
<td>0.100 N.S.</td>
</tr>
<tr>
<td>Within (adjusted)</td>
<td>2985.15</td>
<td>28</td>
<td>106.61</td>
<td></td>
</tr>
<tr>
<td>Total (adjusted)</td>
<td>2995.80</td>
<td>29</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The F ratio of 0.100 is not significant. Therefore, hypothesis seventeen cannot be rejected.

Hypothesis eighteen, stated in null form is: Performance on the final exam will not be significantly higher for the graduates who have the manual than for the graduates who do not have the manual. The data for this comparison are presented in Table 20.

TABLE 20
FINAL EXAMINATION PERFORMANCE
GRADUATE STUDENTS
SUMMARY STATISTICS

<table>
<thead>
<tr>
<th></th>
<th>Experimental</th>
<th>Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math Pre-test Mean</td>
<td>47.76</td>
<td>49.83</td>
</tr>
<tr>
<td>Math Pre-test σ</td>
<td>17.09</td>
<td>15.16</td>
</tr>
<tr>
<td>Final Mean T Score</td>
<td>50.92</td>
<td>52.55</td>
</tr>
<tr>
<td>Final T Score σ</td>
<td>8.19</td>
<td>10.94</td>
</tr>
<tr>
<td>n</td>
<td>21</td>
<td>13</td>
</tr>
</tbody>
</table>

Table 21 presents the results of the analysis of covariance.
TABLE 21
FINAL EXAMINATION PERFORMANCE
GRADUATE STUDENTS
ANALYSIS OF COVARIANCE SUMMARY

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between (adjusted)</td>
<td>9.78</td>
<td>1</td>
<td>9.78</td>
<td>0.128</td>
</tr>
<tr>
<td>Within (adjusted)</td>
<td>2370.72</td>
<td>31</td>
<td>76.47</td>
<td></td>
</tr>
<tr>
<td>Total (adjusted)</td>
<td>2380.50</td>
<td>32</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

An F ratio of 0.128 is not significant. Thus, hypothesis eighteen cannot be rejected.

Hypothesis number nineteen stated in null form is: Final course performance will not be significantly higher for the undergraduate students who have the manual than for the undergraduate students who do not have the manual. The summary statistics are presented in Table 22.

TABLE 22
FINAL COURSE SCORE
UNDERGRADUATE STUDENTS
SUMMARY STATISTICS

<table>
<thead>
<tr>
<th></th>
<th>Experimental</th>
<th>Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math Pre-test Mean</td>
<td>55.80</td>
<td>56.50</td>
</tr>
<tr>
<td>Math Pre-test σ</td>
<td>8.86</td>
<td>11.49</td>
</tr>
<tr>
<td>Final Course T Score Mean</td>
<td>49.79</td>
<td>50.21</td>
</tr>
<tr>
<td>Final Course T Score σ</td>
<td>7.82</td>
<td>9.36</td>
</tr>
<tr>
<td>n</td>
<td>15</td>
<td>16</td>
</tr>
</tbody>
</table>

Analysis of covariance was conducted and the results are presented in Table 23.
TABLE 23
FINAL COURSE SCORE
UNDERGRADUATE STUDENTS
ANALYSIS OF COVARIANCE SUMMARY

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between (adjusted)</td>
<td>0.042</td>
<td>1</td>
<td>0.042</td>
<td>0.001N.S.</td>
</tr>
<tr>
<td>Within (adjusted)</td>
<td>1981.048</td>
<td>28</td>
<td>70.75</td>
<td></td>
</tr>
<tr>
<td>Total (adjusted)</td>
<td>1981.90</td>
<td>29</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The F ratio of 0.001 is not significant. Therefore, hypothesis nineteen cannot be rejected.

Hypothesis number twenty, in null form is: Final course performance will not be significantly higher for the graduate students who have the manual than for the graduate students who do not have the manual. Table 24 presents the summary statistics for this comparison.

TABLE 24
FINAL COURSE SCORE
GRADUATE STUDENTS
SUMMARY STATISTICS

<table>
<thead>
<tr>
<th></th>
<th>Experimental</th>
<th>Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math Pre-test Mean</td>
<td>47.76</td>
<td>49.85</td>
</tr>
<tr>
<td>Math Pre-test $\sigma$</td>
<td>17.09</td>
<td>15.16</td>
</tr>
<tr>
<td>Final Course T Score Mean</td>
<td>50.61</td>
<td>51.50</td>
</tr>
<tr>
<td>Final Course T Score $\sigma$</td>
<td>8.19</td>
<td>10.39</td>
</tr>
<tr>
<td>n</td>
<td>21</td>
<td>13</td>
</tr>
</tbody>
</table>

The analysis of covariance is presented in Table 25.
TABLE 25
FINAL COURSE SCORE
GRADUATE STUDENTS
ANALYSIS OF COVARIANCE SUMMARY

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between (adjusted)</td>
<td>0.232</td>
<td>1</td>
<td>0.232</td>
<td>0.005 N.S.</td>
</tr>
<tr>
<td>Within (adjusted)</td>
<td>1521.713</td>
<td>31</td>
<td>49.08</td>
<td></td>
</tr>
<tr>
<td>Total (adjusted)</td>
<td>1521.945</td>
<td>32</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The F ratio of 0.005 is not significant, so hypothesis twenty cannot be rejected.

Hypothesis number twenty-one, null form is: At the end of the course, undergraduate students who have used the manual will not find it useful in their preparation for taking the statistics course. A questionnaire was given to the undergraduates who used the manual at the end of the course. A copy of this instrument is provided in the Appendix. In response to "Rate the overall helpfulness of the manuals in helping you prepare for this course", 47% answered "very helpful" and 53% answered "somewhat helpful". When asked "How helpful were the manuals as a math review?", 53% answered "very helpful" and 40% answered "somewhat helpful". Responding to the question "How helpful were the manuals as an introduction to statistics?", 60% responded "very helpful" and 40% responded "somewhat helpful". For each of these three items, the responses of "some-
what helpful" or higher are greater than 75%.

On the basis of the responses to these first three items, hypothesis twenty-one is rejected and it is assumed that undergraduates found the manual useful in their preparation for the statistics course.

When asked "To what extent did the use of the manuals influence your anxiety level?", 7% of the undergraduates answered "decreased it greatly", 67% answered "decreased it somewhat", and 27% answered "didn't influence it at all". Sixty-seven percent of the undergraduates felt the manuals would have probably been more useful had they been able to use them more than they did before the beginning of the course. Sixty-seven percent of the undergraduates felt the manuals would have been more useful had they been able to use them during the course in addition to prior to the beginning of the course. When asked "To what extent did the manuals make you look forward to the course?", 27% of the undergraduates answered "very much" and 53% answered "somewhat". Eighty percent of the undergraduates found it useful to have a separate Reference and Answer Manual rather than incorporated together with the primary manual.
Hypothesis number twenty-two, stated in null form is: At the end of the course, graduate students who have used the manual will not find it useful in their preparation for taking the statistics course. A copy of the questionnaire is presented in the Appendix. In response to "Rate the overall helpfulness of the manuals in helping you prepare for this course", 48% of the graduates answered "very helpful" and 48% answered "somewhat helpful". When asked "How helpful were the manuals as a-math review?", 76% answered "very helpful" and 24% answered "somewhat helpful". Responding to the question "How helpful were the manuals as an introduction to statistics?", 33% responded "very helpful" and 52% responded "somewhat helpful". For each of these three items, the responses of "somewhat helpful" or higher are greater than 75%. On the basis of the responses to the first three items, hypothesis twenty-two is rejected. When asked "To what extent did the use of the manuals influence your anxiety level?", 33% of the graduates answered "decreased it greatly", 52% answered "decreased it somewhat", and 14% answered "didn't influence it at all". Sixty-seven percent of the graduates felt the manuals would have probably been
more useful had they been able to use them more than they did before the beginning of the course. Fifty-two percent of the graduate students felt the manuals would have probably been more useful had they been able to use them during the course in addition to prior to the course. When asked "To what extent did the manuals make you look forward to the course?", 19% responded "very much" and 62% responded "somewhat". Sixty-two percent of the graduate students found it useful to have a separate Reference and Answer Manual rather than incorporated together with the primary manual.
Chapter V

Summary and Conclusions

Both the attitudinal and math proficiency measures demonstrated a very high level of reliability. The pre-tests did not seem to have a significant influence on post-test performance.

For the undergraduates, the students who received the manuals demonstrated a significant increase in pre-course math proficiency level as compared with the undergraduate students who did not receive the manuals. The group which received the manuals increased from a mean 73% correct to a mean 78% correct while the group of undergraduates which did not receive the manuals decreased from a mean 82% correct to a mean 77% correct. This indicates that there was some lack of control through randomization. However, the analysis of co-variance provided this control.

Although there was a decrease in the math proficiency post-test variance as compared with the pre-test variance for the undergraduates who received the manual, it was not a significant decrease. This tends to indi-
cate that the undergraduate group which used the manual was more homogeneous in terms of mathematics ability.

There was not a significant correlation between amount of time spent using the math oriented units of the manual and increase in math proficiency level for the undergraduates. Since the range of the increase for the undergraduate was only 5%, this result could be expected.

Undergraduates who received the manual demonstrated a positive change in attitude while undergraduates who did not receive the manual demonstrated a negative change in attitude. This difference was a significant one. However, there was not a significant correlation between the amount of time spent using the manual and positive change in pre-course attitude.

Prior to the course, undergraduates who received and used the manual felt it was useful in their preparation for the introductory statistics course. At the end of the course, they still indicated that they felt it was useful in their preparation. Undergraduates tended to perceive the manual as more helpful as an introduction to statistics than as a mathematics review.
Undergraduates who received the manuals did not perform significantly higher on the midterm, on the final, or in terms of the final course grade than did the undergraduates who did not receive the manuals. One reason which could partially explain this result is that the post-test performance on the math proficiency exam was almost identical for the two groups. Also, students who did not receive the manual would have reviewed math concepts as they needed them during the course. In addition, undergraduate students typically have not been away from taking a mathematics course for more than a year prior to taking this course.

For the graduates, the students who received the manuals demonstrated a significant increase in pre-course math proficiency level. This was coupled with a significant decrease in the math proficiency post-test variance as compared with the pre-test variance for the graduate students who received the manuals. The graduate students who received the manuals increased from a mean 65% correct to a mean 77% correct while the graduate students who did not receive the manual increased from a mean 68% correct to a mean 71% correct. The significant decrease in variance tends to indicate
that the group which used the manual was a more homogeneous group in terms of mathematics proficiency level. With a more homogeneous group, an instructor should be able to spend less time helping students review math and thus be able to spend more time discussing statistics concepts.

Graduate students who received the manuals demonstrated a significant positive change in attitude. The number of positive and negative changes for the group which did not receive the manual was almost equivalent, which would be expected. However, for the graduate student group which received the manual there were more than four times the number of positive changes compared with negative changes.

There was a significant correlation between the amount of time spent using the math oriented units of the manual and increase in math proficiency level for the graduates. However, there was not a significant correlation between the amount of time spent using the manual and positive change in attitude for the graduates. It seems as though the amount of time using the manual did not effect attitude but the fact that the students used the manual tended to result in
a more positive attitude toward taking the course.

Prior to the course, graduate students who received and used the manual felt it was useful in their preparation for the introductory statistics course. At the end of the course, they still indicated that they felt it was useful in their preparation. Graduate students using the manuals tended to find them more helpful as a mathematics review than as an introduction to statistics.

Graduates who received the manuals did not perform at a significantly higher level on the midterm, or on the final, or in terms of the final course grade than did the graduates who did not receive the manuals. One possible reason for this could be that the graduate students who did not receive the manuals did some review on their own prior to the course. These students were provided information about what mathematics concepts should be reviewed as well as the names of previously prepared review sources. This could be reflected in the increase on the mathematics proficiency tests from a mean 68% correct to a mean 71% correct. Also, students in the control groups were provided with a brief mathematics review, as needed, during the course.
Most of the graduate students felt the manuals would have been more useful had they been able to use them prior to and during the course.

**Conclusions**

In general, the conclusions of this investigation may be stated as follows:

1. There is a significant relationship between mathematics skill and performance in an introductory behavioral science statistics course for both undergraduate and graduate education students.

2. There is a significant relationship between mathematics skill and attitude toward introductory behavioral statistics for both undergraduate and graduate education students.

3. The use of the Pre-Statistics Orientation Manual tends to result in improved mathematics skills for both undergraduate and graduate education students.

4. The use of the Pre-Statistics Orientation Manual tends to result in an increase in positive attitude toward taking an introductory behavioral statistics course for both undergraduate and graduate education students.
5. The use of the Pre-Statistics Orientation Manual did not tend to result in improved course performance for undergraduate or graduate education students.

6. Both undergraduate and graduate education students felt the Pre-Statistics Orientation Manual was useful in their preparation for the introductory behavioral statistics course.

In general, the use of the manual tends to result in a higher mathematics proficiency level and a more positive attitude toward taking the introductory course. However, there was no difference in terms of actual course performance between groups which received the manual and groups which did not receive the manual. This was probably due to the fact that the course was taught in a way that did not assume mastery of the material in the manual. Thus, the group which did not receive the manual did receive a mathematics review during the course. This review may not have been necessary had all of the students received the manual a time prior to the beginning of the course and had they been able to use the manual during the course. If the manual provided this review then it would seem likely
that more material could have been discussed during the course rather than spending a great deal of time reviewing and introducing very elementary statistical concepts. Thus, instruction in introductory behavioral research statistics could be more efficient.

In addition, if students have a more positive attitude toward taking introductory statistics as a result of using the manual, then the manual must be seen as worthwhile whether or not achievement in the course is increased.

Limitations of the Study

Several limitations must be considered in assessing the value of this investigation. In terms of instrumentation, the validity of the instruments could be a concern. The only type of validity which would be considered was face validity, which was a somewhat subjective evaluation made by the investigator. In addition, the pre-course instruments could have had a sensitization effect on the students, although the analysis indicated that there was not a significant effect of the pre-tests.

Another limitation which tended to reduce the
possible helpfulness of the manuals was a time constraint imposed by the investigator. Students were given the manuals during the third week of December 1971, and were required to return them during the first week of January 1972. Thus, the students had the manual for only about three weeks, which was during the Christmas-New Years period. This was probably not an ideal time for many students to use them.

The possibility of a "halo effect" is another limitation. Students who received the manuals could have changed their attitude based on the attention given them. The investigator tried to minimize this by giving the group that did not receive the manual other information that they could use to review mathematics concepts prior to the course. A copy of this information is presented in the Appendix. Another limitation closely associated with this is the possibility of students responding positively to please the investigator. It was necessary to have students place their names on the instruments in order to make the comparisons required in the analysis.
Implications for the Future

The Pre-Statistics Orientation Manual will be revised to clarify certain areas and correct erroneous problem answers which were pointed out by students in the unit evaluations. These changes will be of a minor nature.

Another evaluation, using a more diagnostic approach, rather than an experimental approach, should then be conducted. This could take the form of a pre-test used to identify specific problem areas for a student. The student would then be directed to spend time working in specific sections of the manual to remedy these weaknesses prior to the course.

Yet another possible investigation has been pointed out. It would be valuable to compare a class which received the manual with a class that did not receive the manual in terms of the possibility of covering more of the statistics content area in the same amount of time, with essentially the same level of competency achieved in the congruent topics.

It is hoped that this Pre-Statistics Orientation Manual will be suitable for publication. Another possibility is the adaptation of the manual, or parts
of the manual, to a computer-assisted instruction format.
BIBLIOGRAPHY


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22. Tobias, Sigmund, "Effect of Attitudes to Programmed Instruction and Other Media on Achievement from Programmed Materials", AV Communications Review, Fall 1969, pp. 299-306.
Appendix A

Instruments and Student Instructions
August 31, 1971

I am attempting to determine the need for an orientation and review manual for students to use prior to taking an introductory statistics course. Since you have recently completed an introductory course, I would like your opinions of the need for such a manual and also the specific needs which you personally had prior to taking the course.

Please complete the enclosed opinionnaire as soon as possible and return it to me in the enclosed envelope.

Your response is greatly appreciated.

Sincerely,

Jack Barnette
Educational Development

JJB/sjn

Enclosures
The purpose of this opinionnaire is to attempt to ascertain the need for and perhaps the components of a pre-statistics programmed manual for an orientation and review of some of the basic mathematical concepts and techniques frequently used in introductory statistics.

Please respond to the instrument thinking of needs which would have been best met prior to the discussion of statistical concepts in your course. Please rank your opinion of each of the following statements, using this scale:

1 - No need
2 - Little need
3 - Some need
4 - Much need
5 - Very much need

1. A pre-statistics orientation manual as described above. 1 2 3 4 5

2. Do you see a better way of accomplishing the same orientation purpose? If yes, please elaborate.

3. Is there a need for providing examples of how statistics is used in educational research? 1 2 3 4 5

4. How much of a need is there for review of the following arithmetic and algebraic techniques?
   a. Addition (positive and negative numbers) 1 2 3 4 5
   b. Subtraction (" " " " ) 1 2 3 4 5
   c. Multiplication(" " " " ) 1 2 3 4 5
   d. Division (" " " " ) 1 2 3 4 5
   e. Multiplication with fractions or decimal numbers 1 2 3 4 5
   f. Division with fractions or decimal numbers 1 2 3 4 5
   g. Finding the number of non-decimal places after multiplication, division, square, square root 1 2 3 4 5
   h. Finding squares using a table 1 2 3 4 5
   i. Finding square roots using a table 1 2 3 4 5
   j. Arithmetic operations which include parentheses, such as 20 x (6-6+9) + (16+30+2) = and (3+4)² = and \sqrt{580+10-9} = 1 2 3 4 5
   k. Inequality notation; > , < , ≠ , etc. 1 2 3 4 5
5. Are there any other arithmetic or algebraic concepts which you feel should be included? ______________________

6. Is there a need for a graphical description of the measurement scales? 1 2 3 4 5

7. How much of a need is there for introduction of the summation notation? 1 2 3 4 5

8. Relating to frequency distributions, is there a need to discuss:
   a. relative areas under different parts of the distribution? 1 2 3 4 5
   b. various properties of a distribution such as skewness and kurtosis? 1 2 3 4 5
   c. interval size, interval limits, and interval midpoints? 1 2 3 4 5

9. Is there a need to include elementary probability concepts? 1 2 3 4 5

10. Relating to a scatter diagram such as plotting scores on X with scores on Y, is there a need to discuss:
    a. the line of best fit (least squares) 1 2 3 4 5
    b. the slope of the line of best fit 1 2 3 4 5
    c. the Y intercept of the line of best fit 1 2 3 4 5
    d. deviations from the line of best fit 1 2 3 4 5

11. Are there any other introductory concepts that you feel should be included in such an orientation program?

   ______________________

   ______________________

   ______________________

   ______________________
12. Please cite the three introductory or review concepts which would have been of most use to you in understanding introductory statistics.

1. 

2. 

3. 

13. Do you have any additional comments you would like to make about this project?
TO: Future 786 students  
FROM: Jack Barnette  
Please do not discuss the contents of this handout with other members of the class.

Since you are planning to take Education 786 during Winter quarter, you may wish to review certain mathematics concepts during the quarter break.

We will be providing various kinds of review during the course, but you may want to do this for yourself.

If so, here are a few concepts you might consider:
1. Arithmetic symbols
2. Arithmetic operations with signed numbers
3. Arithmetic operations with fractions
4. How to locate a decimal point after performing an arithmetic operation
5. Calculating or using a table to find square-roots
6. The order of arithmetic operations when several different operations are found in an equation
7. The elementary manipulation of equations, how to move parts of an equation around to get all the knowns on one side of the equation and the unknown on the other side of the equation
8. Plotting points on a graph
9. The use of a straight-line equation
It is not necessary that you have complete mastery of these concepts at the beginning of the course. We will review them in class as we go and this may be sufficient for your purposes.

If you are interested in using a programmed review here are suggested aids:


To: Future 786 students
From: Jack Barnette
Please do not discuss the contents of this handout with other members of the class.

Since you are taking introductory statistics next quarter, I would like to provide you with some materials that may help you in the course.

You may pick these materials up in my office:
N-122 Hitchcock Hall
College of Engineering
2070 Neil Ave.
during the following times:

Dec. 13 Monday 3-4:30
Dec. 14 Tuesday 8-12:30 or 1:30-3:30
Dec. 15 Wednesday 8-11:30 or
            Wednesday 5:15-6:30 (in Arps 142)
Dec. 16 Thursday 8-11:30 or 1:30-3:30
Dec. 17 Friday 8-11:30

If you cannot pick up the materials during one of these times, please call me at
422-2651 or 262-8919
Instructions: Please complete this questionnaire and answer every question.

1. Have you taken an introductory statistics course before Education 786 or 541-544? (____ yes) (____ no)
   If so, what were your feelings about the course?
   (____ very negative) (____ negative) (____ neutral) (____ somewhat positive) (____ very positive)

2. Why are you taking introductory statistics? Check all that apply.
   (____ requirement)
   (____ not a requirement)
   (____ advised by a faculty member)
   (____ advised by a friend)
   (____ felt it would be a useful course)
   (____ felt having the course would look good on my credentials)
   (____ other, please specify)

3. Are you looking forward to taking the course? (____ not at all) (____ somewhat reluctantly) (____ neutral) (____ somewhat) (____ very much)

4. In general, would you rank the attitude of faculty members, which you have been in contact with, toward behavioral research statistics? (____ very negative) (____ somewhat negative) (____ neutral) (____ somewhat positive) (____ very positive)

5. In general, how would you rank the attitude of other students toward behavioral research statistics? (____ very negative) (____ somewhat negative) (____ neutral) (____ somewhat positive) (____ very positive)

6. Have you put off taking an introductory statistics course because you felt your mathematical skills were inadequate? (____ yes) (____ no)

7. How do you rank your present level of math skills in relation to taking introductory statistics? (____ very poor) (____ somewhat inadequate) (____ sufficient) (____ more than sufficient) (____ very good)

8. Would you rather not take an introductory statistics course? (____ yes) (____ no)

9. How much do you enjoy working with numbers? (____ not at all) (____ not much) (____ somewhat) (____ very much)
10. What is your attitude toward mathematics? (____ very negative) (____ somewhat negative) (____ neutral) (____ somewhat positive) (____ very positive)

11. How difficult do you think the course will be for you? (____ very difficult) (____ somewhat difficult) (____ neutral) (____ somewhat easy) (____ very easy)

12. Do you think you will be able to use statistics after taking this course? (____ yes) (____ no) (____ don't know)

13. Do you think you will be better prepared to read and evaluate the literature in your field after taking the course? (____ yes) (____ no) (____ don't know)

14. When did you have your last math course?
   Year _______
   Was it: ______ high school algebra
            ______ college algebra
            ______ calculus
            ______ other (specify) ______

15. In general, what has been your average grade in the math courses you have taken?
   ______ D, ______ C, ______ B, ______ A

16. In general, how would you rank your anxiety level toward this course as compared with other courses? (____ much lower) (____ somewhat lower) (____ about the same) (____ somewhat higher) (____ much higher)

17. If you were to predict the grade level which you feel you would finish the course with, what would it be?
   ______ C-, ______ C, ______ C+, ______ B-, ______ B, ______ B+, ______ A-, ______ A

18. Do you think a pre-statistics course math review would be beneficial for you? (____ yes) (____ no) (____ don't know)

19. Would you be willing to spend your own time, prior to the course, for such a review? (____ yes) (____ no)
Name

Pre-statistics Math Proficiency Exam

1. -4. Add the following numbers:
   1. 3.5 + 1.6 = ______
   2. 6.8 + (-3.2) = ______
   3. -18 + (-12) = ______
   4. -3.5 + (-1.5) + 0.5 + (-4.0) = ______

5. -8. Subtract the following:
   5. 7.2 - 4.1 = ______
   6. 19 - (-4) = ______
   7. -45 - (-8) = ______
   8. -7.8 - (-13.4) = ______

9. -11. Multiply the following:
   9. 6 \times 1.6 = ______
   10. 0.008 \times (-8) = ______
   11. (-0.02) \times (-96) = ______

12. -14. Divide the following:
   12. \frac{-15}{3} = ______
   13. \frac{1.69}{-13} = ______
   14. \frac{(-24)}{-0.02} = ______

15. What is the reciprocal of 25? ______

STOP DO NOT GO TO THE NEXT PAGE
16.-17. Add the following fractions:

16. \( \frac{1}{4} + \frac{2}{3} = \) 

17. \( \frac{1}{6} + \left(-\frac{2}{3}\right) = \)

18.-19. Subtract the following fractions:

18. \( \frac{7}{9} - \frac{1}{3} = \)

19. \( \frac{3}{5} - \left(-\frac{7}{8}\right) = \)

20.-21. Multiply the following:

20. \( \frac{1}{4} \times \frac{1}{3} = \)

21. \( \frac{3}{8} \times \left(-\frac{4}{5}\right) = \)

22.-23. Divide the following:

22. \( \frac{1}{3} \div \frac{1}{4} = \)

23. \( \frac{5}{3} \div \left(-\frac{7}{8}\right) = \)

STOP DO NOT GO TO THE NEXT PAGE
24. Calculate the square-root of 162.8 to the nearest one-decimal place.

\[ \sqrt{162.8} = \underline{\hspace{2cm}} \]

25. If \( \sqrt{4} = 2 \) and \( \sqrt{40} = 6.32 \)
25. What is the \( \sqrt{400} \)? \underline{\hspace{2cm}}

26. What is \( \sqrt{0.4} \)? \underline{\hspace{2cm}}

27. What is \( \sqrt{400000} \)? \underline{\hspace{2cm}}

STOP DO NOT GO TO THE NEXT PAGE
28. If $a - x = b$, what is the value of $x$?

$x = \underline{\phantom{0000}}$

29. If $ax^2 = 3b + c$, what is the value of $x$?

$x = \underline{\phantom{0000}}$

30. If $b = (3+1)^2 + 2^2$

$b = \underline{\phantom{0000}}$

31. If $y = \sqrt{\left(\frac{30}{2} - 1\right)} / 4$

$y = \underline{\phantom{0000}}$

STOP DO NOT GO TO THE NEXT PAGE
32.-35. If $X_1 = 3$, $X_2 = 4$, $X_3 = 1$, and $X_4 = 2$

What are the values of:

32. $\sum X = \underline{\phantom{000}}$
33. $\sum X^2 = \underline{\phantom{000}}$
34. $(\sum X)^2 = \underline{\phantom{000}}$
35. $n = \underline{\phantom{00}}$

36.-40. If you have a straight-line equation of

$Y = aX + b$

36. What is the $Y$-intercept? $\underline{\phantom{000}}$
37. What is the slope? $\underline{\phantom{000}}$
38. Which of the following is true?

_____ The slope is negative.
_____ The slope is positive.
_____ The slope can be positive or negative.

39. If $a = 4$, $X = 6$, and $b = -5.2$, what is the value of $Y$?

$Y = \underline{\phantom{000}}$

40. If $a = 3$, $Y = -6$, and $b = 3$, what is the value of $X$?

$X = \underline{\phantom{000}}$
Name ________________________________
Columbus address ___________________________ phone ____________
Office address _______________________________ phone ____________
Degree pursued ____________________________
Major area ________________________________
Please list any other statistics course taken at this or any other university __________________________

Please answer the following questions.

1. How much are you looking forward to taking this introductory statistics course? 
   (very much) (somewhat) (neutral) 
   (somewhat reluctantly) (not at all)

2. Have you put-off taking an introductory statistics course because you felt your mathematical skills were inadequate? 
   (yes) (no)

3. If you had a choice would you rather not take this course? 
   (yes) (no)

4. How would you rank your present level of math skills in relation to taking this course? (very poor) (somewhat inadequate) (sufficient) (more than sufficient) (very good)

5. How much do you enjoy working with numbers? (very much) 
   (somewhat) (not much) (not at all)

6. How difficult do you think this course will be for you? 
   (very difficult) (somewhat difficult) (neutral) 
   (somewhat easy) (very easy)

7. What is your attitude toward mathematics? (very positive) 
   (somewhat positive) (neutral) (somewhat negative) 
   (very negative)
8. During the past three weeks, how has your anxiety level about this course changed? (___increased greatly) (___increased somewhat) (___no change) (___decreased somewhat) (___decreased greatly)

9. If you were to predict the grade level which you feel you will finish the course with, what would it be? 
___C-, ___C, ___C+, ___B-, ___B, ___B+, ___A-, ___A

10. About how much time did you spend during the past three weeks preparing to take this course? (___none) (___less than 1 hr.) (___1-3 hrs.) (___3-5 hrs.) (___5-10 hrs.) (___10-15 hrs.) (___more than 15 hrs.)

11. What, if any, source(s) of assistance did you use in this preparation?
1. _____________________________________________________________
2. _____________________________________________________________
3. _____________________________________________________________

12. Rate the helpfulness of each of these source(s) in helping you prepare for this course?
1. (___very helpful) (___somewhat helpful) (___don't know) (___little help) (___very little help)
2. (___very helpful) (___somewhat helpful) (___don't know) (___little help) (___very little help)
3. (___very helpful) (___somewhat helpful) (___don't know) (___little help) (___very little help)
Pre-statistics Math Proficiency Exam

1.-4. Add the following numbers:

1. \( 6.5 + 14.8 = \) ______
2. \( 13.6 + (-3.6) = \) ______
3. \( -35 + (-18) = \) ______
4. \( -3.5 + (-2.5) + 0.5 + (-6.0) = \) ______

5.-8. Subtract the following:

5. \( 6.3 - 5.6 = \) ______
6. \( 35 - (-12) = \) ______
7. \( -61 - (-32) = \) ______
8. \( -4.2 - (-8.1) = \) ______

9.-11. Multiply the following:

9. \( 7 \times 1.7 = \) ______
10. \( 0.05 \times (-13) = \) ______
11. \( (-0.06) \times (-42) = \) ______

12.-14. Divide the following:

12. \( \frac{-35}{7} = \) ______
13. \( \frac{2.56}{-16} = \) ______
14. \( \frac{-48}{-0.004} = \) ______

15. What is the reciprocal of 10? ______

STOP. DO NOT GO TO NEXT PAGE UNTIL TOLD TO DO SO.
16.-17. Add the following fractions:
16. \( \frac{1}{8} + \frac{2}{3} = \) 
17. \( \frac{1}{16} + \left( -\frac{3}{4} \right) = \)

18.-19. Subtract the following fractions:
18. \( \frac{5}{9} - \frac{3}{5} = \)
19. \( \frac{1}{5} - \left( -\frac{3}{8} \right) = \)

20.-21. Multiply the following:
20. \( \frac{1}{3} \times \frac{1}{3} = \)
21. \( \frac{5}{16} \times \left( -\frac{1}{5} \right) = \)

22.-23. Divide the following:
22. \( \frac{1}{2} \div \frac{1}{8} = \)
23. \( \frac{2}{9} \div \left( -\frac{3}{8} \right) = \)

STOP. DO NOT GO TO THE NEXT PAGE UNTIL TOLD TO DO SO. YOU MAY GO BACK TO PAGE 1 IF YOU COMPLETE THIS PAGE BEFORE YOUR TIME IS UP.
24. Calculate the square-root of 184.0 to the nearest one-decimal place.

\[ \sqrt{184.0} = \quad \]

25. - 27. If \( \sqrt{6} \approx 2.45 \) and \( \sqrt{60} = 7.75 \),
25. What is \( \sqrt{600} \)?
26. What is \( \sqrt{0.6} \)?
27. What is \( \sqrt{600000} \)?

STOP. DO NOT GO TO THE NEXT PAGE UNTIL TOLD TO DO SO.
YOU MAY GO BACK TO THE PREVIOUS PAGE.
28. If \( c - a = b + d \), what is the value of \( a \), in terms of \( b \), \( c \), and \( d \)?

\[ a = \]

29. If \( 6x^2 = \frac{y - z}{2} \), what is the value of \( x \), in terms of \( y \) and \( z \)?

\[ x = \]

30. If \( a = \left( \frac{(6 + 3)^2 - 7}{2} \right)^2 \), what is the value of \( a \)?

\[ a = \]

31. If \( y = \sqrt{(11 + 5 - 4)^2/4} \)

\[ y = \]

STOP. DO NOT GO TO THE NEXT PAGE UNTIL TOLD TO DO SO, YOU MAY GO BACK TO PREVIOUS PAGES.
32.-35. If $X_1 = 4, X_2 = 5, X_3 = 1, X_4 = 4$, and $X_5 = 6$

What are the values of:

32. $\bar{X} = ______$
33. $\bar{X}^2 = ______$
34. $(\bar{X})^2 = ______$
35. $n = ______$

36.-40. If you have a straight-line equation of

$Y = mX + b$

36. What is the $Y$-intercept? ______
37. What is the slope? ______
38. Which of the following is true?

- The slope is positive.
- The slope is negative.
- The slope can be positive or negative.

39. If $m = 4, X = 9$, and $b = -32$,

What is the value of $Y$?

$Y = ______$

40. If $m = -3, Y = -5$, and $b = -7$,

What is the value of $X$?

$X = ______$
Since you were one of the students who received one of the Pre-Statistics Orientation Manuals, I would like for you to answer the following few questions for me. Your participation in this evaluation is deeply appreciated.

Jack Barnette

1. Rate the overall helpfulness of the Manuals in helping you prepare for this course.
   
   (_____ very helpful)  (_____ somewhat helpful)  (_____ little help)  (_____ not helpful)

2. How helpful were the Manuals as a math review?

   (_____ very helpful)  (_____ somewhat helpful)  (_____ little help)  (_____ not helpful)

3. How helpful were the Manuals as an introduction to statistics?

   (_____ very helpful)  (_____ somewhat helpful)  (_____ little help)  (_____ not helpful)

4. Rate your anxiety level prior to using the Manuals.

   (_____ very high)  (_____ high)  (_____ average)  (_____ low)  (_____ very low)

5. To what extent did the use of the Manuals influence your anxiety level?

   (_____ decreased it greatly)  (_____ decreased it somewhat)
   (_____ didn't influence it at all)  (_____ increased it somewhat)
   (_____ increased it greatly)

6. Would the Manuals have been more useful had you been able to use them more before the beginning of the course?

   (_____ yes)  (_____ probably)  (_____ don't know)  (_____ probably not)  (_____ no)

7. Would the Manuals have been more useful had you been able to use them during the course in addition to prior to the beginning of the course?

   (_____ yes)  (_____ probably)  (_____ don't know)  (_____ probably not)  (_____ no)

8. Did you find it useful to have a separate Reference and Answer Manual rather than incorporated together?

   (_____ yes)  (_____ don't know)  (_____ it would have made no difference)

9. To what extent did the Manuals make you look forward to the course?

   (_____ very much)  (_____ somewhat)  (_____ little)  (_____ very little)  (_____ not at all)

10. Are there any topics which you feel should be included in the Manuals which were not?
11. Are there any sections of the Manuals which you feel should be emphasized more?

12. Do you have any suggestions for improving the Manuals?
Evaluation of Unit II: Overview
1. Approximate amount of time spent in this Unit: ____________
2. How helpful was this Unit in presenting an overview of introductory statistics?
   (____Not helpful) (____Little) (____Some)
   (____Much) (____Very Much)
   Comments: ____________________________

3. Was the Unit easy to read and understand?
   (____No) (____Not completely) (____Yes)
   Please cite any parts that you did not understand.
   Page   Paragraph No.   Comments
   ______  ____________   ______________________________________
   ______  ____________   ______________________________________
   ______  ____________   ______________________________________
   ______  ____________   ______________________________________

4. Do you have any suggestions for improving this Unit and/or its' questions? Please elaborate.
Evaluation of Unit 4: Measurement Scales

1. Approximate amount of time spent in this Unit: _____

2. How helpful was this Unit in presenting the four basic measurement scales?
   (______ Not Helpful) (______ Little) (______ Some)
   (______ Much) (______ Very Much)

   Comments: _____

3. Do any of the scales need further explanation? If so, which one (s)?
   _____ Nominal
   _____ Ordinal
   _____ Interval
   _____ Ratio

4. Was the Unit easy to read and understand?
   (______ No) (______ Not completely) (______ Yes)
   Please cite any parts which were not.

   Page   Paragraph No.   Comments
   _____   ________      ____________________________
   _____   ________      ____________________________
   _____   ________      ____________________________
   _____   ________      ____________________________

5. Do you have any suggestions for improving this Unit and/or its' questions? Please elaborate.
Evaluation of Unit IV Arithmetic Definitions

1. Approximate amount of time spent in this Unit: ____

2. Was this Unit a helpful review for you?
   (____ Not helpful) (____ Little) (____ Some)
   (____ Much) (____ Very Much)
   Comments:

3. Was the Unit easy to read and understand?
   (____ No) (____ Not completely) (____ Yes)
   Please cite any parts that you did not understand.
   Page Paragraph No. Comments
   ____ _______ ______________________
   ____ _______ ______________________
   ____ _______ ______________________

4. Do you have any suggestions for improving this Unit and/or its' questions? Please elaborate.
Evaluation of Unit V, Summation Notation

1. Approximate amount of time spent in this Unit: ___

2. How helpful was this Unit in introducing the summation notation to you?
   (____ Not helpful) (____ Little) (____ Some)
   (____ Much) (____ Very Much)

3. Was the Unit easy to read and understand?
   (____ No) (____ Not completely) (____ Yes)
   Please cite any parts that were not.
   Page  Paragraph No.  Comments

   ___  ______________________________

   ___  ______________________________

   ___  ______________________________

4. Do you have any suggestions for improving this Unit and/or its' questions? Please elaborate.
Evaluation of Unit VI, Operations with Signs

1. Approximate amount of time spent in this Unit: 

2. How helpful was this Unit in providing this review? 
   (_____Not Helpful) (_____Little) (_____Some) 
   (_____Much) (_____Very Much) 
   Comments: 

3. Was the Unit easy to read and understand? 
   (_____No) (_____Not Completely) (_____Yes) 
   Please cite any parts that were not. 
   Page Paragraph No. Comments
   
   
   

4. Do you have any suggestions for improving this Unit and/or its' questions? Please elaborate.
Evaluation of Unit VII: Operations with Fractions

1. Approximate amount of time spent in this Unit: ____________

2. How helpful was this Unit in providing this review?
   (____ Not Helpful) (_____ Little) (_____ Some)
   (_____ Much) (_____ Very Much)
   Comments: ____________________________________________________________________

3. Was the Unit easy to read and understand?
   (_____ No) (_____ Not completely) (_____ Yes)
   Please cite any parts that were not.
   Page   Paragraph No.   Comments
   ___________   ____________________________
   ___________   ____________________________
   ___________   ____________________________

4. Do you have any suggestions for improving this Unit and/or its' questions? Please elaborate.
   ____________________________________________________________________
Evaluation of Unit VIII, Location of Decimal Point

1. Approximate amount of time spent in this Unit:_____

2. How helpful was this Unit?
   (_____Not Helpful) (_____Little) (_____Some)
   (_____Much) (_____Very Much)

Comments:

3. Did this Unit help you develop a skill you did not have before? (_____No) (_____Yes)

4. Was the Unit easy to read and understand?
   (_____No) (_____Not completely) (_____Yes)

Please cite any parts that were not.

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5. Do you have any suggestions for improving this Unit and/or its' questions? Please elaborate.
Evaluation of Unit IX, Square-roots.
1. Approximate amount of time spent in this Unit: ___

2. How helpful was this Unit in reviewing the direct calculation of a square-root?
   (___Not helpful) (___Little) (___Some)
   (___Much) (___Very Much)
   Comments: ___

3. How helpful was this Unit in helping you learn how to find square-roots using a table?
   (___Not helpful) (___Little) (___Some)
   (___Much) (___Very much)
   Comments: ___

4. Was the Unit easy to read and understand?
   (___No) (___Not completely) (___Yes)
   Please cite any parts that were not.
   Page     Paragraph No.     Comments
   ___       ___              ___
   ___       ___              ___
   ___       ___              ___

5. Do you have any suggestions for improving this Unit and/or its' questions? Please elaborate.

Evaluation of Unit X, Order of Operations.

1. Approximate amount of time spent in this Unit: __________

2. How helpful was this Unit in reviewing the order of arithmetic operations?
   (___ Not helpful) (___ Little) (___ Some) 
   (___ Much) (___ Very much)

3. Was the Unit easy to read and understand?
   (___ No) (___ Not completely) (___ Yes)

   Please cite parts that were not.

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4. Do you have any suggestions for improving this Unit and/or its' questions? Please elaborate.
Evaluation of Unit XI, Manipulation of Equations

1. Approximate amount of time spent in this Unit:

2. How helpful was this Unit in providing a review?
   (____Not helpful) (____Little) (____Some)
   (____Much) (_____Very much)

   Comments:

3. Was the Unit easy to read and understand?
   (____No) (____Not completely) (_____Yes)

   Please cite points that were not.

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4. Do you have any suggestions for improving this Unit and/or its questions? Please elaborate
Evaluation of Unit XII, Frequency Distributions

1. Approximate amount of time spent in this Unit: __________

2. How helpful was this Unit in introducing the frequency distribution, the grouped frequency distribution and polygons to you?
   (____ Not helpful) (____ Little) (____ Some)
   (____ Much) (____ Very Much)
   Comments:

3. Was the Unit easy to read and understand?
   (____ No) (____ Not completely) (____ Yes)
   Please cite any parts that are not.

   Page  Paragraph No.  Comments
   _____  ___________  _______________________
   _____  ___________  _______________________
   _____  ___________  _______________________

4. Do you have any suggestions for improving this Unit and/or its' questions? Please elaborate.
Evaluation of Unit XIII, Interval %, Cf, Cef, -p

1. Approximate amount of time spent in the Unit: ____

2. How helpful was this Unit in introducing:
   a. Interval percentages
      (___ Not helpful) (___ Little) (___ Some) (___ Much) (___ Very much)
   b. Cumulative percentages
      (___ Not helpful) (___ Little) (___ Some) (___ Much) (___ Very much)
   c. Cumulative frequencies
      (___ Not helpful) (___ Little) (___ Some) (___ Much) (___ Very much)
   d. Probability of a score occurring
      (___ Not helpful) (___ Little) (___ Some) (___ Much) (___ Very much)

Comments:

3. Was the Unit easy to read and understand?
   (___ No) (___ Not completely) (___ Yes)
   Please cite any parts that were not.

   Page | Paragraph No. | Comments
   ---- | ------------- | ------------
   ____ | ____________ | ___________
   ____ | ____________ | ___________
   ____ | ____________ | ___________

4. Do you have any suggestions for improving this Unit and/or its questions? Please elaborate.
Evaluation of Unit XIV, Unit Normal Distribution

1. Approximate amount of time spent in this Unit: ____

2. How helpful was this Unit in introducing the Unit Normal Distribution?
   (____ Not helpful) (____ Little) (____ Some)
   (____ Much) (____ Very Much)
   Comments:

3. Was the Unit easy to read and understand?
   (____ No) (____ Not completely) (____ Yes)
   Please cite any parts that were not.

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4. Do you have any suggestions for improving this Unit and/or its' questions? Please elaborate.
Evaluation of Unit XV, Scattergram

1. Approximate time spent in this Unit: _____

2. How helpful was this Unit in:
   a. reviewing graphing
      (_____ Not helpful) (_____ Little) (_____ Some)
      (_____ Much) (_____ Very Much)
   b. introducing the scattergram
      (_____ Not helpful) (_____ Little) (_____ Some)
      (_____ Much) (_____ Very much)
   c. introducing the line of least fit
      (_____ Not helpful) (_____ Little) (_____ Some)
      (_____ Much) (_____ Very much)
   d. reviewing a straight-line equation
      (_____ Not helpful) (_____ Little) (_____ Some)
      (_____ Much) (_____ Very much)

   Comments:

3. Was the Unit easy to read and understand?
   (_____ No) (_____ Not completely) (_____ Yes)

   Please cite points that were not.

   Page   Paragraph No.   Comments
   _______  ___________  __________________________
   _______  ___________  __________________________
   _______  ___________  __________________________

4. Do you have any suggestions for improving this Unit and/or its' questions? Please elaborate.
Appendix B

Pre-Statistics Orientation Manual
The Use of These Manuals

This set of manuals is being developed and evaluated as a dissertation project. Your participation in this project will hopefully be as much or more benefit to you as it is to the author.

These manuals are designed to provide you with a math review as well as an introduction to certain widely used concepts and techniques in introductory statistics. If you are not confident in your math skills, you will wish to work through the complete manual which should take you about seven to ten hours. On the other hand, if you are reasonably confident in your math skills, you may wish to work only in certain Units of the manual. Some of the units have pre-tests which should help you decide whether or not working in that Unit would be beneficial to you.

The pages in the manual are color-coded for your convenience. In the primary manual, yellow pages separate the Units and pre-tests have been printed on blue paper. In the Reference and Answer Manual, reference pages are in pink, answers to pre-tests are in blue, and Unit evaluation forms are in green.

It is hoped that having two manuals will be a further convenience for you. It will be possible for you to look at various pages simultaneously to compare problems and their answers and to be more convenient for using reference pages and tables as related to the primary manual discussions.
Due to the nature of this evaluation, it will be necessary to permit you the use of the manual only during the time before the course begins. You will be required to return the manual on the first day of the course. Therefore, please use it during the quarter break. Also, please do not share or discuss this manual with other students taking the course. They have been provided with other sources of assistance if they so desire to review math concepts prior to the course.

The key to the successful evaluation of this manual is in your evaluation of each Unit that you complete. Please complete the evaluation forms found at the end of the Reference and Answer Manual as soon as you complete the Unit.

Your cooperation is sincerely appreciated. We wish to do everything possible to make your instruction in introductory statistics as successful and enjoyable as possible.

Jack Barnette
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Unit I
Introduction
Unit I. Introduction

(Estimated time-5 minutes)

Man has always sought to explain the world through looking for relationships among phenomena and describing these relationships in terms of symbols and sets of symbols, called equations, which simplify the complex nature of the world. He has developed a language for qualification and quantification which follows specified rules and is communicable to all mankind. This language is mathematics. Mathematical symbolization is used to represent as parsimoniously (simply) as possible real events that occur in the world. Evidence of this is quite common in the areas of physical science. However, the use of mathematics in the behavioral sciences is relatively new. This use of mathematics in the behavioral sciences is primarily the use of an area of applied mathematics called statistics.

You are currently planning on taking a course in introductory statistics as they are used in behavioral research. You are probably somewhat apprehensive about this course. Perhaps you have heard people talk about how hard such a course is and you perhaps have some doubts about your mathematics skills, especially if you have not used your math recently, except when trying to balance your checkbook or file your income-tax form. So you may feel you are going into the course somewhat ill-prepared. However, you do have the basic mathematics skills needed for such a
course, though they may be a little rusty from non-use, you should be able to attain a sufficient understanding to handle all the mathematics used in an introductory course. You do not have to know calculus to understand and use any of the elementary statistical techniques or most of the more advanced statistical techniques. However, you will need to have skill in basic arithmetic, in some elementary algebra, and in some graphing. There are probably no math skills that you have not already used. You will have to learn some new symbolization and a few new operations, but the math is the same as you were taught in high school. It may look a little different because of the different symbols, but it is not.

You will not be expected to memorize a lot of formulas that you may never use after the course. There are a few very basic formulas which you will learn primarily because you use them often. But we're not interested in your ability to memorize formulas. We are interested in helping you understand what formulas mean and how they are used.

The purpose of this manual is to assist you in your review of mathematics skills in relation to how they are used specifically in introductory statistics. It is hoped that after using this manual you will not only have the skills needed, but also a certain high level of confidence about your preparation in using these skills.
Basic mathematics concepts will be reviewed and examples of how they are related to statistics concepts will be presented. You will have the opportunity to check on your own progress by doing the problems included in each section and at the end of the sections.

Provided with this manual are several evaluation forms. Please keep track of the approximate amount of time you spend working on each section and also provide your evaluative comments relative to each section.

There is every reason to believe that you will have a successful and even perhaps a very enjoyable experience taking introductory statistics.

"Yes Virginia, statistics can be fun as well as a very valuable asset."
Unit II
An Overview of Introductory
Behavioral Research Statistics
Reference Manual pages 1 to 5
A. The function of statistics in behavioral research is to provide a system for qualifying and/or quantifying behavioral research variables. Assume you are interested in the variable of intelligence. There are a great many theories regarding this construct of intelligence; it is hard to define in an absolute sense. However, researchers have identified various observable behaviors that seem to be related to the relative degree of possession of this attribute. On the basis of these observable behaviors, test developers have provided a method of measuring relative intelligence with a fairly high degree of accuracy. Assume that you gave an intelligence test to 100 eighth-grade students. How would you communicate the results of this test to anyone else? You could list the IQ scores in no particular order and provide the list as the results. However, this alone would not be a very good description of the results. In what ways could you describe the results in a manner that would be accurate and concise? One of the primary uses of statistics is in describing a set of data. We could describe it graphically or numerically or both ways. Describing the results graphically would probably be done using a frequency distribution or a frequency polygon (a polygon is a many sided figure). You could arrange
the scores in order of low to high and count the number of times (frequency) each score occurred to get a frequency distribution. The frequency polygon provides a picture of the 100 IQ scores. There are many numerical ways to describe the 100 IQ scores. What if you were interested in using one score to represent all 100 IQ scores? It would be possible to use as the representative score the most frequently occurring score (this is called the mode). It would also be possible to use the score which represents the point where 50% of the 100 scores or 50 scores fall below that point and 50% fall above that point (this is called the median). Another way to find a representative score is to add up all the scores and divide by the total number of scores to get what is commonly called an "average" (this is called the arithmetic mean). These three (the mode, median, and the mean) are referred to as measures of central tendency.

B. A measure of central tendency is a very useful descriptive statistic. However, it only tells part of the story. Along with a measure of central tendency, we are usually interested in the spread or variability of the scores in terms of a range or interval which contains a specified theoretical or actual percentage or proportion of the total number of scores. One such measure of variability is the range, which is an interval which contains all of the scores, essentially the (high score minus the low score) + 1. Another common
measure of variability is the standard deviation which theoretically is the interval which contains about 68% of all of the scores. Measures of variability are also descriptive statistics.

C. Another type of descriptive statistics is used to look at the relationship between two variables. Graphically we could plot the variables on a two-axis graph, called a scattergram, to describe the relationship or we could look at the relationship numerically in terms of a correlation coefficient. Correlation coefficients are also descriptive statistics.

D. Descriptive statistics "tell it how it is". However, "telling it how it is" is not the only purpose of statistics. We are also interested in making generalizations or inferences beyond the actual data in hand. In other words, we want to infer characteristics of a population, which has not been measured, from the measures of a representative sample of that population. Also, we want to compare the statistics of groups to make a decision regarding the probability that the groups are significantly different from each other in their possession of some attribute. These uses of statistical techniques are called inferential statistics. Examples of inferential statistical techniques which are discussed in an introductory statistics course are the t test, the chi-square test, and the F test. (You may have heard of these tests before.) They are sometimes called significance tests. Inferential
statistics might best be considered a technique for comparing descriptive statistics.

E. To summarize, there are two primary types of statistical techniques. The first is descriptive statistics which "tell it how it is". Measures of central tendency, variability, and correlation are some of the descriptive statistics. The second type is inferential statistics which "tell it how it probably is". The various significance tests are inferential statistical techniques.

F. Let's now define a few terms we will use many times in the course. The first set of terms is "a population" as compared with "a sample". A population is a usually large group of subjects which possess certain characteristics which you have identified. For example, a population which you might be interested in is "all freshman students at The Ohio State University". You could be more specific about your population and identify it as "all freshman students who live in residence halls at The Ohio State University". You could be even more specific and define your population as "all male, freshman students who live in Drackett Tower at The Ohio State University". You as the researcher must identify your population.

G. From an identified population you might be interested in drawing a sample, using one of the random sampling techniques. You want this sample to be representative of the larger population.
Using the sample of subjects, we measure some behavioral science variable and we describe the data in terms of descriptive statistics. A statistic is based on the observation of a sample. The mean, mode, median, standard deviation, etc. of a set of sample data are statistics.

I. Theoretically, it is possible to measure the entire population. However, this is usually not practical because of its large size. If we could find the mean, mode, median, standard deviation, etc. of the population we would have a parameter. A parameter is a descriptor based on the total population. Thus, remember that a parameter is to a population as a statistic is to a sample.

J. Since we usually don't know the value of a parameter, we often estimate the value of the parameter from the value of the sample statistic. This is an important use of statistical techniques.

K. All of the statistical techniques which you will encounter are easy to use even though you do not completely understand the mathematical concepts behind
them. You will not be expected to derive any formulas and you will seldom be expected to understand precisely why certain statistical techniques are more appropriate in different situations.

1. We want you to be able to calculate statistics using pre-determined formulas, to be able to make some judgements regarding the statistical results, and make some decisions about when, but not necessarily why, to use certain statistics instead of others. We want you to become a user not a theoretician, unless, of course, you want to become a theoretician, and if so, maybe we can help you along your way.

Unit II. Post-test Questions. Check your answers on page 3 of the Reference and Answer Manual.

1. The mode, median, and mean are called
   ___a. measures of description.
   ___b. measures of central tendency.
   ___c. measures of congruency.

2. A descriptor based on population data is called
   ___a. an inferential statistic.
   ___b. a descriptive statistic.
   ___c. a parameter.

3. A graphical description of the relationship between two variables is called
   ___a. a scatter-gram.
   ___b. a frequency polygon.
   ___c. a correlation coefficient.

4. The standard deviation is a descriptive statistic
which describes
___a. a population.
___b. a sample.
___c. variability of a set of data.

5. We often estimate _____ from _____ in statistics.
___a. a population from a sample.
___b. a parameter from a statistic.
___c. an inferential statistic from a descriptive statistic.

6. The kind of statistic which we use to compare two groups to see if they are significantly different is called
___a. an inferential statistic.
___b. a sample statistic.
___c. a descriptive statistic.

7. When a researcher chooses a sample from a population, he wants the sample to be
___a. as large as possible.
___b. as random as possible.
___c. as representative as possible.

Please evaluate Unit II. on page 77 of the Reference and Answer Manual.
Unit III
A Description of the Four
Basic Measurement Scales
Reference Manual pages 6 to 7
Unit III. A Description of the Four Basic Measurement Scales

(Expected time-15 to 20 minutes)

A. In statistics, as in all mathematical operations, we work with numbers. We use numbers as symbols to mean something in terms of the measurements we make. All too often we do not look at these numbers in any other way than that way which permits our using them in arithmetic operations. The important criterion of the relationship between a number and a measure is the concept of isomorphism. An isomorphic measure is a measure true to reality. However, numbers which we use can take on a variety of levels of measurement precision. The researcher must be aware of these levels of precision and make decisions regarding possible limitations inherent in the use of these numbers in arithmetic operations and, subsequently, in statistical calculations.

B. Basically, there are four so-called measurement scales; one qualitative and three quantitative in nature. These scales represent precision of measurement. The qualitative scale is called the nominal scale. The three quantitative scales are the ordinal scale, the interval scale, and the ratio scale, in increasing order of measurement precision.

In this unit, each of the scales will be discussed giving characteristics and examples of each.

C. 1. The Nominal Scale
The nominal scale is not a measurement scale in the strict sense of the definition. However, it does provide for discrimination of classes of individuals, objects, or processes. It is a naming or classificatory scale. We use numbers only as an identification. Arithmetic operations using these numbers are meaningless. For example, we may assign all females a number of 1 and all males a number of 2. The numbers reflect no quantitative difference, they are merely labels. If we had six classes of seventh-graders, we could assign each a number of 1 through 6, but we could not say that class number 6 possessed a greater amount of any attribute than class number 1, we could only say that class 6 is a different group than class 1. Examples of nominal classifications are: racial origin, sex, social security numbers, eye color, religious belief, etc.

D. 2. The Ordinal Scale

The ordinal scale is a continuum relating to the relative degree of possession of some attribute or property. On the basis of low to high possession of an attribute we can order or rank individuals, objects, or processes relative to each other. We can assign a number of 1 to the lowest entity and a number of 10 to the highest, if there are 10 in the set. Entity number 10 possesses the highest degree of this attribute for the 10 we have, but it does not necessarily possess twice as much as does number 5. Also, there
is not necessarily the same degree of possession of
the attribute between entities number 3 and number 4,
as compared with entities number 6 and number 7. Per­
haps a diagram will make this a little more obvious.

We can make statements such as: "10 is higher
than 9", "7 is lower than 8", etc. But, we don't know
the true distance between the points.

One of the best examples of this scale of measure­
ment is the hardness of minerals. If mineral A scrat­
ches mineral B, then mineral A has a higher degree of
the possession of the hardness property. We can, on
the basis of several observations, rank a set of min­
erals according to this property. Examples of ordinal
measurements are: military rank, achievement test
scores, attitude measures, personality trait rankings,
etc.

Arithmetic operations with the numbers of the
ordinal scale are usually questionable and are usually
uninterpretable. However, we do have some techniques
specifically designed for ordinal type data.

E. 3. The Interval Scale

This scale has not only the properties of the
ordinal scale, but it also has the property of equal
units of measurement. An interval scale might look
like this:
We can interpret differences in a meaningful way. The magnitude of difference between entities 1 and 2 is equal to the magnitude of difference between entities 4 and 5. There is no naturally occurring zero point on an interval scale. Any zero point is arbitrary and does not indicate the absolute absence of the attribute.

A good example of an interval scale is temperature on a Fahrenheit or centigrade scale. The units are equal, but the 0 is arbitrary. You can have a negative temperature.

Measures of IQ are considered to be close to interval measures. They are more precise than ordinal measures, but not quite precise enough to be considered completely interval.

Most arithmetic operations are possible with interval scale measurements. Almost, all statistical calculations are appropriate using interval data.

F. 4. The Ratio Scale

The ratio scale has all the properties of the interval scale as well as a naturally occurring zero point. The zero point representing the absence of the attribute.
With this scale we are able to say that point 8 possesses twice as much of the attribute as does point 4.

Examples of ratio measures are: height, weight, drug dosage, time, etc.

All arithmetic operations are possible with a ratio scale, however, few statistical techniques require data of such precision.

G. 5. Points to Consider in Classifying a Measurement as a Specific Scale

The focus in this consideration must be on what is being measured and not any frequency of a certain measure. Is it possible to put what is being measured on a continuum? For example, you can say you have 5 males and 10 females. You would be right in saying you have twice as many females as males. But, this is a frequency count and not a measure of sex. The attribute of sex cannot be placed on a continuum. It is a nominal measure. You do not have twice the "sexness".

Consider the attribute being measured not the frequency of its occurrence.

Unit III. Post-test Questions. Check your answers on page 7 of the Reference and Answer Manual.

Classify the following measures in terms of nominal, ordinal, interval, or ratio.

a. Zip-code numbers

b. Number of spelling words correct on a test

(see next page)
as a measure of spelling ability

c. Time, measured using a stop-watch, starting at zero time

d. Calendar time

Please evaluate Unit III. on page 78 of the Reference and Answer Manual.
Unit IV
Arithmetic Definitions
Reference Manual pages 8 to 10
Unit IV. Arithmetic Definitions

(Estimated time-10 minutes)

Before going into arithmetic operations, let's review the terminology and symbols which we must know. You may already know all of these but let's make sure.

We will consider the six basic arithmetic operations of addition, subtraction, multiplication, division, squaring, and finding square-roots.

A. Addition (operational symbol: +)
   Addends - the numbers being added together.
   Sum - the final result of the addition operation

B. Subtraction (operational symbol: -)
   Subtrahend - the number being subtracted from another number
   Difference - the final result of the subtraction operation

C. Multiplication (operational symbols: \times, \cdot, \text{or many times understood, as in } a \times b = ab)
   Multiplicand and Multiplier - the numbers being multiplied; also called factors
   Product - the result of the multiplication operation

D. Division (operational symbols: \div, \text{or } \frac{a}{b})
   Dividend - the number being divided into; the numerator (top value) in \( \frac{a}{b} \)
   Divisor - the number divided by; the denominator (bottom value) in \( \frac{a}{b} \)
   Quotient - the final result of the division operation
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ation or the result of \( b \) divided into \( a \) in \( a/b \)

E. Squaring (Operational symbol: \( 2 \) in \( x^2 \))

Square - the result of a number multiplied by itself

Square-root (Operational symbol: \( \sqrt{} \), radical)

Square-root - the result of the determination of the value of the number, which if multiplied by itself, would give the value of the number under the square-root operational symbol

F. Other symbols:

( ) - Parentheses - the quantity enclosed in the parentheses, the result of the completion of arithmetic operations indicated within the parentheses

[ ] - Brackets - the quantity within the brackets, usually used to include quantities that contain parentheses

\( \Sigma \) - Summation notation, the addition of a series of values, this will be discussed in Unit V. of this manual

\( x^a \) - "a" is called an exponent or a power

G. Inequality Notation:

\( a<b \) - "a" is less than "b"

\( a\leq b \) - "a" is less than or equal to "b"

\( a>b \) - "a" is greater than "b"

\( a\geq b \) - "a" is greater than or equal to "b"

\( a\neq b \) - "a" is not equal to "b"

Note: the open end of the symbol is the larger
value

H.  "A Non-Decimal Place" - a place to the left of the decimal point
"A Decimal Place" - a place to the right of the decimal point

I. | |, is the symbol for an absolute value. An absolute value is a value without regard to sign. Absolute value are treated as though they were positive.


1. \( a \times b = \)
   ___a. a quotient
   ___b. a product
   ___c. a square

2. In the division \( a/b \), "a" is called the
   ___a. numerator
   ___b. divisor
   ___c. denominator

3. Put this set of symbols in words, \( c > d \)
   ___a. c is less than d
   ___b. c is greater than or equal to d
   ___c. c is greater than d

4. The expression \( 10^4 \) is
   ___a. ten to the fourth power
   ___b. ten times four
   ___c. the fourth-root of ten
5. Fill in the names of the parts of this equation:
\[ \frac{e}{f} = g \]

_a. e = denominator, f = numerator, g = quotient
_b. e = dividend, f = divisor, g = product
_c. e = numerator, f = denominator, g = quotient

6. \( \Sigma \) is the
_a. exponent symbol
_b. summation notation
_c. square-root symbol

7. In the number 63,125, the numbers 63 are called
_a. non-decimal place numbers
_b. decimal place numbers
_c. unreal numbers

8. Find the absolute values of +25, -25, and -8
_a. +25, -25, -8
_b. +25, +25, +8
_c. +25, +25, -8

Please evaluate Unit IV, on page 79 of the Reference and Answer Manual.
Unit V
Introduction to the
Summation Notation
Reference Manual pages 11 to 14
Unit V. Introduction to the Summation Notation

(Estimated time-20 minutes)

A. If you have ever looked at a statistics book you have probably seen a rather foreboding symbol that sort of looks like E, but not quite. What it is, is the Greek letter sigma (Σ) and it is used to symbolize the result of an addition of several scores. It is the summation notation. Often you will see the symbol as simply:

ΣX, where X represents scores

B. However, this is not the complete symbol. The complete symbol for the result of an addition operation of 10 scores would look like this,

\[ \sum_{i=1}^{10} X_i \]

C. Let's break this down and look at each part independently and in relation to each other. We assign each of the ten scores a number from 1 to 10 to identify it, this is the symbol (i) which labels the score with a number. (This is an example of the use of the nominal scale which was discussed in Unit III.) It does not effect in any way the value of the score. It allows us to distinguish between any specific scores.

Each of the 10 scores is represented by the symbol \( X_i \), where i can be 1, 2, ..., 10. In our case, the ten scores are symbolized as: \( X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10} \).

D. Below the summation notation symbol Σ there is
an indication of the first value of X or the score to be added. For

\[ \sum_{i=1}^{\ast} X_i \]

the designation \( i=1 \) below the summation notation tells us that the first score to be added is the score with the designation \( X_1 \) (where \( i=1 \)). We automatically then add the next score (\( X_2 \)) and so on until we have some indication to stop adding scores. This indication is given by a symbol above the summation notation. In our example the last score to be added is the one identified as number 10.

E. We can summarize the operation as

\[ \sum_{i=1}^{10} X_i \]

"Add together all scores X which are identified as scores labeled 1 through 10." "\( i \)" takes on the values or labels beginning with 1 and ending with 10. If the total number of scores in the data set is 10, then 10 is the value of \( n \), the number of scores.

Assume we have the following 10 scores:

<table>
<thead>
<tr>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
</tbody>
</table>
First let's give each score a numerical label,

\[ X_1 = 5 \quad X_6 = 7 \]
\[ X_2 = 8 \quad X_7 = 3 \]
\[ X_3 = 3 \quad X_8 = 5 \]
\[ X_4 = 6 \quad X_9 = 6 \]
\[ X_5 = 2 \quad X_{10} = 7 \]

F. The complete data set has 10 scores, so \( n = 10 \).

If we want, as we usually do, to add up or find the sum of all 10 scores. Using the summation notation, the result of the addition process is symbolized:

\[
\sum_{i=1}^{10} X_i \quad \text{or} \quad \sum_{i=1}^{n} X_i \quad \text{where} \quad n = 10
\]

This is a shorthand symbol for the result of:

\[ X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 + X_{10} \]

In terms of our scores,

\[ 5 + 8 + 3 + 6 + 2 + 7 + 3 + 5 + 6 + 7 = \sum_{i=1}^{n} X_i = 52 \]

G. More often than not we are interested in finding the sum of all the scores. So we start with \( i = 1 \) and add up all the \( n \) scores (the total number) to find the sum. When this is the case we sometimes drop some of the symbolization since it is understood. For example, if we had 23 scores and we wanted to find the sum of all 23 scores, the summation notation would look like this,

\[
\sum_{i=1}^{23} X_i = \sum_{i=1}^{n} X_i
\]

H. We know that we want to start with score number
and end with the score which has the label of the value of \( n \). So, for further simplicity, we might write the summation notation as

\[ \sum EX \]

with the below notation understood as \( i=1 \) and the above notation understood as \( n \).

I. There are times when we want to find the sum of certain scores, but not all the scores. If, using our scores

\[
\begin{align*}
X_1 &= 5 \\
X_2 &= 8 \\
X_3 &= 3 \\
X_4 &= 6 \\
X_5 &= 2
\end{align*}
\]

\[
\begin{align*}
X_6 &= 7 \\
X_7 &= 3 \\
X_8 &= 5 \\
X_9 &= 6 \\
X_{10} &= 7
\end{align*}
\]

we wanted to find a sum of only the first five scores. Our symbol and answer would be,

\[
\sum_{i=1}^{5} X_i = 5+8+3+6+2 = 24
\]

If we were interested in finding the sum of scores \( X_4 \) through \( X_8 \), our symbol and answer would be

\[
\sum_{i=4}^{8} X_i = 6+2+7+3+5 = 23
\]

(Start with score number 4 and end with score number 8).

J. You will see the summation notation used in various ways in relation to other arithmetic operations. The most common ways will be in the following four forms:
1. $\Sigma X$
2. $(\Sigma X)^2$
3. $\Sigma X^2$
4. $\Sigma XY$

K. We've already considered number 1. $\Sigma X$ is the sum of all the scores in the data set. Number 2 is the sum of the scores, the quantity squared. To find this value, we first find the sum of the scores and then we square the value of the sum. Assume we have the following five scores: $X_1 = 6$, $X_2 = 3$, $X_3 = 8$, $X_4 = 5$, $X_5 = 2$. The value of $\Sigma X = 6 + 3 + 8 + 5 + 2 = 24$. The value of $(\Sigma X)^2$ is equal to the sum of scores, the quantity squared or $(24)^2 = 576$.

L. It is easy to confuse $(\Sigma X)^2$ with $\Sigma X^2$. $\Sigma X^2$ is the sum of the individual scores squared. Each score is squared first and then the summation operation is carried out. Consider the scores $X_1 = 6$, $X_2 = 3$, $X_3 = 8$, $X_4 = 5$, $X_5 = 2$. To find $\Sigma X^2$ we first square each score ($X$). $X_1^2 = 36$, $X_2^2 = 9$, $X_3^2 = 64$, $X_4^2 = 25$, $X_5^2 = 4$.

We then find the sum,

$\Sigma X^2 = 36 + 9 + 64 + 25 + 4 = 138$

Note that 138 is not equal to 576, and $\Sigma X^2$ is not equal to $(\Sigma X)^2$.

M. Form number four $\Sigma XY$, is another common use of the summation notation, especially when we consider the concept of correlation between two variables $X$ and $Y$. Assume we have five students and we measure them on $X$ which is a test for intelligence and we
measure them on Y which is a test of achievement.

<table>
<thead>
<tr>
<th>Student number</th>
<th>Score on X</th>
<th>Score on Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>6</td>
</tr>
</tbody>
</table>

N. The symbol $\Sigma XY$ represents the sum of products of the score X and Y for each student. The score on X for student number 1 is multiplied with the score on Y for student number 1. You can only multiply scores for the same students, not between students. Once these products are found, they are added together to find $\Sigma XY$. Consider an example,

<table>
<thead>
<tr>
<th>Student Number</th>
<th>Score on X</th>
<th>Score on Y</th>
<th>XY</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>7</td>
<td>42</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>6</td>
<td>48</td>
</tr>
</tbody>
</table>

Therefore,

$\Sigma XY = 15 + 6 + 42 + 48 = 113$

Unit V. Post-test Questions

Using the following set of 12 scores, answer the first set of questions. Check your answers on page 12 of the Reference and Answer Manual.

$X_1 = 2$          $X_7 = 1$

$X_2 = 4$          $X_8 = 6$
Set I. Questions

1. If you wanted to find the sum of all 12 scores what would the symbol be for the summation operation?

   a. \( \sum_{i=1}^{n} \)
   b. \( \sum_{i=12}^{1} \)
   c. \( \sum_{1}^{12} \)

2. Find the value indicated by the summation notation:

   \( \sum_{i=1}^{4} \)

   a. 43
   b. 12
   c. 17

3. Find the value of \( \Sigma X \).

   a. 2
   b. 43
   c. 1849

4. Find the value of \( (\Sigma X)^2 \).

   a. 185
   b. 1849
   c. 43
5. Find the value of $\sum X^2$.
   \[a. \quad 1849\]
   \[b. \quad 43\]
   \[c. \quad 185\]

6. What is the value of $\sum_{i=1}^{9} X_{i}^2$?
   \[a. \quad 121\]
   \[b. \quad 11\]
   \[c. \quad 53\]

Using the following data for $X$ and $Y$ measured on the same subjects, answer the questions in set two. Check your answers on page 14 of the Reference and Answer Manual.

<table>
<thead>
<tr>
<th>Subject no.</th>
<th>$X$</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>

Set II Questions

7. What is the value of $(\sum X)^n$?
   \[a. \quad 441\]
   \[b. \quad 21\]
   \[c. \quad 125\]

8. Find the value of $\sum XY$.
   \[a. \quad 101\]
   \[b. \quad 44\]
   \[c. \quad 483\]
9. Find the value of $(\Sigma X)(\Sigma Y)$.

   a. 44
   b. 483
   c. 101

Please evaluate Unit V. on page 80 of the Reference and Answer Manual.
Unit VI
Arithmetic Operations
with Signs
Reference Manual pages 15 to 19
Unit VI. Arithmetic Operations with Signs: Pre-test

Instructions: Complete this test as much as possible placing your answers in the "answer" blank. Then check your answers on page 17 of the Reference and Answer Manual placing the correct answer from the manual in the "correct answer" blank. If you're incorrect in your answer, see the pre-test question for directions concerning the starting place in the Unit which discusses that type of problem. After working through the unit take the sub-unit test items to check your proficiency in solving those types of problems.

1. a. Add the following numbers:

<table>
<thead>
<tr>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
</tr>
<tr>
<td>25</td>
</tr>
</tbody>
</table>

answer, correct answer

If you got the wrong answer, check your addition.

b. Add:

<table>
<thead>
<tr>
<th>-6</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
</tr>
<tr>
<td>-12</td>
</tr>
</tbody>
</table>

answer, correct answer

See paragraph A, page 40.

c. Add:

<table>
<thead>
<tr>
<th>-6</th>
</tr>
</thead>
<tbody>
<tr>
<td>+5</td>
</tr>
<tr>
<td>-12</td>
</tr>
</tbody>
</table>

answer, correct answer
d. Add:

\[-24\]
\[+36\]
\[-18\]

\[\text{answer, correct answer}\]

See paragraph A, page 40.

e. Add:

\[1.32\]
\[-0.85\]
\[-0.81\]
\[+0.04\]

\[\text{answer, correct answer}\]

See paragraph A, page 40.

2. a. Subtract:

\[12-7=\text{answer, correct answer}\]

Check your subtraction.

b. Subtract:

\[10-(-5)=\text{answer, correct answer}\]

See paragraph B, page 41.

c. Subtract:

\[0-25=\text{answer, correct answer}\]

See paragraph B, page 41.

d. Subtract:

\[-35-(-35)=\text{answer, correct answer}\]

See paragraph B, page 41.
3. Find the products of the following.
   a. \(3 \times 14 = \) ____, correct answer ____.  
      Check your multiplication.
   b. \(-6 \times 13 = \) ____, correct answer ____
      See paragraph C, page 43.
   c. \(-25 \times 7 = \) ____, correct answer ____.  
      See paragraph C, page 43.
   d. \((-6) \times (-3) \times (-5) = \) ____, correct answer ____.  
      See paragraph C, page 43.
   e. \((-3) \times (-4) \times (-2) \times (-1) = \) ____, correct answer ____.  
      See paragraph C, page 43.
   f. \((5) \times (-1.06) = \) ____, correct answer ____
      See paragraph C, page 43.

4. Find the quotients of the following.
   a. \(12 \div 5 = \) ____, correct answer ____
      Check your answer.
   b. \(-35 \div 7 = \) ____, correct answer ____
      See paragraph D, page 44.
   c. \(45 \div (-5) = \) ____, correct answer ____
      See paragraph D, page 44.
   d. \(-7.50 \div (-1.25) = \) ____, correct answer ____
      See paragraph D, page 44.
   e. \((-6/2) \div (-1) = \) ____, correct answer ____
      See paragraph D, page 44.
5. Find the squares of the following.
   a. \(7^2=\), correct answer ____.  
      See paragraph E, page 44.
   b. \((-13)^2=\), correct answer ____.  
      See paragraph E, page 44.
   c. \((-1.5)^2=\), correct answer ____.  
      See paragraph E, page 44.
   d. \(0.60^2=\), correct answer ____.  
      See paragraph E, page 44.
   e. \((-0.90)^2=\), correct answer ____.  
      See paragraph E, page 44.

6. Find the positive square-roots of the following
   a. \(\sqrt{25}=\), correct answer ____
      Check your answer.
   b. \(\sqrt{144}=\), correct answer ____
      See paragraph F, page 45.
   c. \(\sqrt{0.64}=\), correct answer ____
      See paragraph F, page 45.

7. Find the reciprocal of the following numbers.
   a. \(2, \text{ reciprocal}=\), correct answer ____
      See paragraph G, page 46.
   b. \(-4, \text{ reciprocal}=\), correct answer ____
      See paragraph G, page 46.
   c. \(-25, \text{ reciprocal}=\), correct answer ____
      See paragraph G, page 46.
Unit VI. Basic Arithmetic Operations with Signed Numbers

(Estimated time-15 to 50 minutes)

In this unit we will review the procedures of the basic arithmetic operations of addition, subtraction, multiplication, division, squares, square-roots, and reciprocals with emphasis on rules regarding the use of positive and negative signs.

A. Addition

The numbers added together to get a sum are called the addends. If all of the addends have the same sign, then the sum has the same sign.

Examples:

\[
\begin{array}{c|c}
+6 & -3 \\
+4 & -6 \\
\hline
+3 & -6 \\
\hline
+13 & -14
\end{array}
\]

However, many times the addends do not have the same sign. Often we have addends which are positive and addends which are negative in the same problem. Add the positive addends and then add the negative addends. Subtract the sum with the smallest absolute value (an absolute value is a magnitude without a sign) from the sum with the greatest absolute value. The difference is the sum of the two numbers and is given the initial sign of the largest absolute sum.

Examples:

\[
\begin{array}{c|c}
+6 & -4 \\
-3 & +3 \\
\hline
40
\end{array}
\]
Solve the following problems.

Addition Problems
1. $3 + (-4) =$
2. $-6 + 1 =$
3. $6 + (-4) + (-2) + 7 =$
4. $-8 + 13 + (-14) + 2 =$

Check your answers on page 18 of the Reference and Answer Manual. Check them before going on to the next section.

B. Subtraction

The number to be subtracted is called the subtrahend. To subtract, change the sign of the subtrahend and add the two numbers, paying attention to the sign. It is sometimes useful to picture the number scale to make sure you move in the right direction.

Subtracting a positive is a move to the left, while subtracting a negative is a move to the right. Subtracting 8 from 6, move 8 places to the left from +6, to get -2. Or, \( \frac{6}{-8} = \frac{6}{-2} \)

Consider, subtracting -4 from 6, \( \frac{6}{-4} + (+\frac{6}{10}) \) (moving right)
Consider, subtracting 6 from -10.

One of the common uses of this procedure in statistics is in finding what is called a mean deviation (x), symbolized with a small x, which is the directional amount a score (X) is different than the mean (average) ($\bar{X}$) of the set of scores, of which X is one of the scores.

The mean deviation is given by the formula,

$$x = X - \bar{X}$$


For example, if the mean ($\bar{X}$) of a set of 4 scores is equal to 5.2, what are the mean deviations of the 4 scores of 4.1, 6.3, 2.8, and 7.6?

$$x = X - \bar{X}$$

for 4.1

$$x = 4.1 - 5.2 = -1.1$$

for 6.3

$$x = 6.3 - 5.2 = +1.1$$

for 2.8

$$x = 2.8 - 5.2 = -2.4$$

for 7.6

$$x = 7.6 - 5.2 = +2.4$$

One way to check to see if you have calculated the mean deviations properly, is to see if the sum of all of the mean deviations for a set of scores is equal to zero. If the sum of deviations is not zero, then there is a mistake someplace, perhaps an incorrect sign.

Solve the following subtraction problems,

1. $-3 - (+2) =$
2. $13 - (-3) =$
3. $-24 - (-12) =$
4. What are the mean deviations of scores of 6.3, 8.6, and 4.9? The mean of the three scores is 6.6. Is the sum of the deviations equal to 0?

Check your answers on page 18 of the Reference and Answer Manual. Check before continuing.

C. Multiplication

In finding the product of two numbers, the sign is positive if the signs of the numbers are the same.

\[
\begin{align*}
10 & \quad -10 \\
6 & \quad -6 \\
+60 & \quad +60
\end{align*}
\]

If one of the signs is positive and the other sign is negative, then the product is negative.

\[
\begin{align*}
-10 & \quad 10 \\
6 & \quad (-6) \\
-60 & \quad -60
\end{align*}
\]

Many times we multiply a series of numbers such as: \((-3)\times2\times6\times(-4)\) =

To find the sign of the product of a series of multiplication operations, follow these rules.

Add the number of negative signs, then: If the number of negative signs is even, the product is positive. If the number of negative signs is odd, the product is negative. Consider the example,

\((-3)\times2\times6\times(-4) = (?) \ 144 \ \text{No. of - signs = 2, product is positive, +144.}\)

Consider another example,

\((-4)\times3\times2\times(-6)\times(-1) = (?) \ 144 \ \text{No. of - signs = 3, product is negative, -144.}\)
Find the answers to these multiplication problems.

1. \((-6) \times (-4) = \) 
2. \((-5) \times 6 = \) 
3. \((-3) \times 4 \times (-2) = \) 
4. \(6 \times (-5) \times 2 = \) 
5. \((-4) \times (-5) \times (-3) = \)

Check your answers on page 18 of the Reference and Answer Manual. Check before continuing.

D. Division

Division rules are similar to the multiplication rules. If both the dividend and the divisor have the same sign, the quotient is positive.

\[
\frac{+10}{+6} = +1.67 \quad \frac{-10}{-6} = +1.67
\]

If one of the signs of either the dividend or divisor is positive and the other is negative, then the quotient is negative.

\[
\frac{+10}{-6} = -1.67 \quad \frac{-10}{+6} = -1.67
\]

Solve the following division problems.

1. \(12 \div (-2) = \) 
2. \(-35 \div (-7) = \) 
3. \((-6 \times 4) \div 3 = \) 
4. \(-16 \div (-5) = \)

Check your answers on page 18 of the Reference and Answer Manual. Check before going on.

E. Squaring

The square of a number is always positive, since both signs are the same.
Examples:
\[ 7^2 = 7 \times 7 = 49 \]
\[ (-6)^2 = (-6) \times (-6) = 36 \]
\[ (-0.5)^2 = (-0.5) \times (-0.5) = 0.25 \]

Quite often in statistics we square numbers so we can get away from using negative values. We almost always square mean deviations,

\[ x^2 = (X - \bar{X})^2 \]

this can be negative or positive, squaring we get,

\[ x^2 = (X - \bar{X})^2 \]

this is always positive.

We then use the sum of the mean deviations squared in many other operations.

Find the squares of the following numbers,

1. \(-9^2 =\)
2. \(0.4^2 =\)
3. \(-0.8^2 =\)
4. \(1.2^2 =\)

Check your answers on page 19 of the Reference and Answer Manual. Check before continuing.

F. Square-roots

Please refer to Unit IX. for methods of finding square-roots either by direct calculation or by using a table.

A square-root can take on either the positive or negative sign.

\[ \sqrt{49} = +7 \quad \text{and} \quad \sqrt{49} = -7 \]

However, in our work we are always only interested in the positive square-root value.
Find the positive square-roots of the following.

1. \( \sqrt{100} = \)
2. \( \sqrt{.25} = \)
3. \( \sqrt{2500} = \)
4. \( \sqrt{400} = \)

Check your answer on page 19 of the Reference and Answer Manual. Check before continuing.

G. Reciprocals

The reciprocal of a number "a" is \( \frac{1}{a} \)

For example, the reciprocal of 5 is \( \frac{1}{5} \) or 0.20.

The reciprocal of .5 is \( \frac{1}{.5} \) or 2.0. The reciprocal of -8 is \( \frac{1}{8} = -0.125 \) and the reciprocal of -.8 is \( \frac{1}{-8} = -1.25 \).

Find the reciprocals of the following numbers.

1. \( 3 = \)
2. \( .2 = \)
3. \( -10 = \)
4. \( -\frac{1}{4} \) or -0.25 =

Unit VII
Arithmetic Operations with Fractions
Reference Manual pages 20 to 30
Unit VII. Operations with Fractions, Pre-test

Complete the test as much as possible. Check your answers on page 21 of the Reference and Answer Manual and write the correct answers in the "correct answer" blank. Enter the unit where indicated.

1. Add the following fractions, simplify the result where possible.
   a. $\frac{3}{5} + \frac{1}{2} = ___\text{answer, correct answer}$. See paragraph A, page 51.
   b. $\frac{1}{5} + \frac{1}{20} = ___\text{answer, correct answer}$. See paragraph A, page 51.
   c. $\frac{7}{8} + \frac{3}{5} = ___\text{answer, correct answer}$. See paragraph A, page 51. If you have the sign wrong, see paragraph A, page 40.

2. Subtract the following fractions; simplify the result where possible.
   a. $\frac{7}{8} - \frac{1}{8} = ___\text{answer, correct answer}$. See paragraph B, page 52.
   b. $\frac{4}{7} - \frac{1}{2} = ___\text{answer, correct answer}$. See paragraph B, page 52.
   c. $\frac{3}{5} - \frac{7}{8} = ___\text{answer, correct answer}$. See paragraph B, page 52. If you have the wrong sign see paragraph B, page 41.

3. Multiply the following fractions; simplify result
where possible...

4. Divide the following fractions; simplify result where possible.

a. \( \frac{6}{7} \div \frac{3}{7} = \) ___answer, correct answer____. See paragraph D, page 55.

b. \( \frac{1}{8} \div \frac{3}{5} = \) ___answer, correct answer____. See paragraph D, page 55.

c. \( \frac{8}{9} \div \left( \frac{-1}{6} \right) = \) ___answer, correct answer____.

5. Square the following fractions.

a. \( \left( \frac{1}{12} \right)^2 = \) ___answer, correct answer____. See paragraph E, page 56.

b. \( \left( \frac{4}{5} \right)^2 = \) ___answer, correct answer____. See paragraph E, page 56.

c. \( \left( \frac{-6}{7} \right)^2 = \) ___answer, correct answer____. See
paragraph E, page 55. If you have the wrong sign see paragraph E, page 44.

6. Find the square-roots of the following fractions.
   a. \( \sqrt{\frac{1}{4}} = \) \hspace{1em} answer; correct answer \hspace{1em} \ . See paragraph F, page 57.
   b. \( \sqrt{\frac{9}{49}} = \) \hspace{1em} answer, correct answer \hspace{1em} . See paragraph F, page 57.
Unit VII. Arithmetic Operations with Fractions

(Estimated time 10 to 25 minutes)

Occasionally, but not frequently, we must perform arithmetic operations involving fractional numbers. In this unit we will discuss operations involving fractions of the form \( \frac{a}{b} \), (or also written as \( a/b \)). We will not discuss fractions that are in the form of decimal-place values such as (one-half = 0.5).

A. Addition of Fractions

In order to add fractions the denominator (bottom part) of the fraction must be the same value. We can add \( \frac{1}{10} \) and \( \frac{3}{10} \), because the denominators are the same. We add the numerators and then place the sum over the common denominator to get \( \frac{1}{10} + \frac{3}{10} = \frac{4}{10} \).

When the denominators are not the same for the fractions to be added, we must perform a pre-addition operation to make them the same. Consider the addition of two fractions,

\[
\frac{a}{c} + \frac{b}{d} = ?
\]

In order to get the same denominator we take the numerator of the first fraction (a) times the denominator of the second fraction (d), to get \( a \times d \). To this add the denominator of the second fraction (c) times the numerator of the first fraction (b), to get \( c \times b \). Next divide this sum by the product of the two denominators \( b \times d \). Thus,
Let's put in some values and add the two fractions.

\[ \frac{1}{10} + \frac{4}{9} = (1 \times 9) + (10 \times 4) = \frac{90}{90} + \frac{40}{90} = \frac{130}{90} \]

Add the following fractions. Check the answers on page 22 of the Reference and Answer Manual.

1. \( \frac{7}{8} + \frac{5}{8} = \)
   - a. \( \frac{35}{64} \)
   - b. \( \frac{12}{16} = 1 \frac{1}{2} \)
   - c. \( \frac{12}{16} = \frac{3}{4} \)

2. \( \frac{2}{4} + \frac{1}{3} = \)
   - a. \( \frac{4}{7} \)
   - b. \( \frac{4}{12} = \frac{1}{3} \)
   - c. \( \frac{13}{12} = 1 \frac{1}{12} \)

3. \( \frac{7}{9} + \frac{3}{8} = \)
   - a. \( \frac{83}{72} = 1 \frac{11}{72} \)
   - b. \( \frac{21}{72} \)
   - c. \( \frac{10}{17} \)

B. Subtraction of Fractions

We do the same thing in the subtraction of fractions that we did in the addition of fractions except the operation sign becomes - where it was a plus. If the denominators are the same, such as in the case, \( \frac{3}{5} - \frac{2}{5} = ? \), we simply subtract the numerators and divide by the common denominator to get,

\[ \frac{a}{b} - \frac{c}{d} = \frac{3}{5} - \frac{2}{5} = \frac{ad-bc}{bd} = \frac{15-10}{25} = \frac{5}{25} = \frac{1}{5} \]
If the denominators are different, we follow the same procedure as before to make them the same. Take the numerator of the first fraction \(a\) times the denominator of the second fraction \(d\), to get \((a \times d)\), minus the numerator of the second fraction \(c\) times the denominator of the first fraction \(b\) and divide this difference by the product of the two denominators.

\[
\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd} = ?
\]

Notice that you drew the arrow lines in a way that they do not intersect after the initial cross multiplication. For example, the difference between,

\[
\frac{1}{2} - \frac{2}{9}
\]

is given by,

\[
\frac{(1 \times 9) - (2 \times 2)}{(2 \times 9)} - \frac{9 - 4}{18} = \frac{5}{18}
\]

Subtract the following fractions. Check the answers on page 23 of the Reference and Answer Manual.

1. \[
\frac{7}{8} - \frac{4}{8} =
\]
   \[\_\_\_a. \frac{3}{8}\]
   \[\_\_\_b. \frac{7}{12}\]
   \[\_\_\_c. -\frac{3}{8}\]

2. \[
\frac{3}{4} - \frac{1}{11} =
\]
   \[\_\_\_a. \frac{29}{44}\]
   \[\_\_\_b. \frac{1}{22}\]
   \[\_\_\_c. -\frac{29}{44}\]
C. Multiplication of Fractions

Multiplication of fractions is quite straightforward. We don't have to put the fractions in a form with a common denominator. We simply multiply the numerators and divide by the product of the denominators. For example,

\[
\frac{a}{b} \times \frac{e}{f} = \frac{a \times e}{b \times f}
\]

If we multiply \(\frac{4}{7}\) times \(\frac{6}{5}\) we get,

\[
\frac{4}{7} \times \frac{6}{5} = \frac{24}{35}
\]

Multiply the following fractions. Check your answers on page 24 of the Reference and Answer Manual.

1. \(\frac{2}{9} \times \frac{5}{9} = \)
   
   ____a. \(\frac{7}{9}\)  
   ____b. \(\frac{11}{9}\)  
   ____c. \(\frac{11}{36}\)

2. \(\frac{23}{90} \times \frac{3}{4} = \)
   
   ____a. \(\frac{92}{270}\)  
   ____b. \(\frac{23}{120}\)  
   ____c. \(\frac{11}{23}\)
Division of Fractions

The division of fractions is not quite as simple as the multiplication of fractions. We have two steps. The first step is done with the divisor fraction. We invert this fraction (make the numerator the denominator and the denominator the numerator). If we inverted the fraction \( \frac{7}{8} \) we would have \( \frac{8}{7} \), the inverted fraction \( \frac{1}{2} \) would be \( \frac{2}{1} \). After we invert the divisor fraction, we multiply the dividend fraction times the inverted fraction as we did in multiplication. For example,

\[
\frac{a}{b} \div \frac{g}{h} = \frac{a}{b} \times \frac{h}{g} = \frac{a \times h}{b \times g}
\]

Using a numerical example,

\[
\frac{2}{3} \div \frac{7}{6} = \frac{2}{3} \times \frac{6}{7} = \frac{12}{21}
\]

Divide the following fractions. Check the answers on page 26 of the Reference and Answer Manual.
1. \( \frac{4}{5} \times 3 = \)
   
   \[ a. \quad 2 \frac{2}{5} \]
   \[ b. \quad 3 \frac{3}{4} \]
   \[ c. \quad 4 \frac{1}{15} \]

2. \( \frac{1}{2} + \frac{1}{4} = \)
   
   \[ a. \quad 2 \]
   \[ b. \quad 1/2 \]
   \[ c. \quad 1/8 \]

3. \( \left( -\frac{5}{16} \right) \times \frac{3}{5} = \)
   
   \[ a. \quad 15/80 \]
   \[ b. \quad -15/80 \]
   \[ c. \quad -3/8 \]

4. \( \left( -\frac{1}{12} \right) \times \left( -\frac{1}{8} \right) = \)
   
   \[ a. \quad -1/96 \]
   \[ b. \quad 2/3 \]
   \[ c. \quad -2/3 \]

**E. The Square of a Fraction**

A fraction squared is the fraction multiplied by itself.

\[ \frac{a}{b} \times \frac{a}{b} = \frac{a^2}{b^2} \]

Thus, the square of \( \frac{1}{4} \) is,

\[ \left( \frac{1}{4} \right)^2 = \frac{1}{4} \times \frac{1}{4} = \frac{1^2}{4^2} = \frac{1}{16} \]

Find the squares of the following fractions. Check your answers on page 28 of the Reference and Answer Manual.
The Square-root of a Fraction

The square-root of a fraction \( \frac{a}{b} \) is

\[
\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}
\]

Thus, the square-root of \( \frac{9}{25} \) is

\[
\sqrt{\frac{9}{25}} = \frac{\sqrt{9}}{\sqrt{25}} = \frac{3}{5}
\]

Find the square-roots of the following fractions. Check the answers on page 29.

1. \( \sqrt{\frac{4}{9}} = \)
   
   \[\begin{array}{l}
a. \frac{2}{3} \\
b. \frac{2}{9} \\
c. 1 \frac{1}{3}
\end{array}\]
2. \( \sqrt{\frac{121}{64}} = \)
   - a. \( \frac{11}{64} \)
   - b. \( 1 \frac{1}{8} \)
   - c. \( 1 \frac{3}{8} \)

3. \( \sqrt{\frac{1}{81}} = \)
   - a. \( 0 \)
   - b. \( \frac{1}{9} \)
   - c. \( \frac{1}{81} \)

Please evaluate Unit VII. on page 82 of the Reference and Answer Manual.
Unit VIII
Location of Decimal Point
Reference Manual pages 31 to 34
Unit VIII. Location of Decimal Point Pre-test

(Estimate time-35 minutes)

Find the decimal point for the following problems. Try to find the decimal without actually doing the arithmetic. After completing the test, check your answers on page 32 of the Reference and Answer Manual. For the problems you missed, you will find instructions in the manual. You will be directed to specific points in the manual.

1. Addition
   a. \(4.327 + 0.026 = 4.353\), correct answer.
      See paragraph A, page 62.
   b. \(135.42 + 6.1 = 141.52\), correct answer.
      See paragraph A, page 62.
   c. \(6.25 + 3.1 + (-1.59) = 7.830\), correct answer.
      See paragraph A, page 62.

2. Subtraction
   a. \(40.35 - 5.0 = 35.35\), correct answer.
      See paragraph A, page 62.
   b. \(0.063 - (-1.35) = 1.413\), correct answer.
      See paragraph A, page 62.

3. Multiplication
   a. \(6.2 \times 0.3 = 1.86\), correct answer.
      See paragraph B, page 63.
   b. \(4001 \times 2.41 = 9606.4\) (rounded-off), correct answer.
      See paragraphs B, C, D, and E, page 63, and/or F, and G, page 65.
c. \(0.1352 \times 0.1450 = 0.00196\) (rounded off), correct answer. See paragraphs B, C, D, E, page 63 and/or F, and G, page 65.

4. Division
   a. \(75.0 \div 1.5 = 50.00\), correct answer. See paragraph F, page 65 and then paragraph I, page 68.
   b. \(0.140 \div 35 = 0.0004\), correct answer. See paragraph F, page 65 and then paragraph I, page 68.
   c. \(9000.0 \div 4.5 = 2000.00\), correct answer. See paragraph F, page 65 and then paragraph I, page 68.

5. Squares
   a. \(500^2 = 250000\), correct answer. See paragraph F, page 65 and then paragraph H, page 68.
   b. \(0.06^2 = 0.0036\), correct answer. See paragraph F, page 65 and then paragraph H, page 68.

6. Square-roots
   a. \(\sqrt{4.41} = 2.10\), correct answer. See paragraph F, page 65 and then paragraph J, page 69.
   b. \(\sqrt{0.0064} = 0.08\), correct answer. See paragraph F, page 65 and then paragraph J, page 69.
Location of the Decimal Point After Performing an Arithmetic Operation

Many students who have been away from performing arithmetic operations for a long period of time tend to have some problems locating the decimal place of an answer. The purpose of this unit is to provide you with some general rules which will help you locate the decimal location of an answer.

A. Addition and Subtraction

In addition and subtraction, the decimal place is fixed and will not vary with the operation. If you add a series of numbers, the numbers must be arranged in accordance with a fixed decimal location. For example, if you add the numbers 25.000, 1.635, and 0.013, they must be added in the following arrangement:

\[
\begin{array}{c}
\text{Addition} \\
25.000 \\
1.635 \\
0.013 \\
\hline
26.648
\end{array}
\]

decimal in fixed location

The same is true of subtraction, the decimal location is fixed. Consider subtracting 6.343 from 25.000, the arrangement is:

\[
\begin{array}{c}
\text{Subtraction} \\
25.000 \\
0.343 \\
\hline
24.657
\end{array}
\]

decimal in fixed location

Consider subtracting 14.187 from 6.235. The arrangement is:

\[
\begin{array}{c}
6.235 \\
-14.187 \\
\hline
62
\end{array}
\]
Remember we change the sign of the number being subtracted and we add it to the number being subtracted from. So, it looks like this:

\[
\begin{array}{c}
6.235 \\
\frac{+(-14.187)}{ } \quad \frac{-7.952}{ }
\end{array}
\]

B. Multiplication

You may remember the rule of finding the decimal location by adding up the number of decimal places in the multiplicand and the multiplier and placing the decimal that number of places to the left of the right-most number of the complete answer.

\[
\begin{array}{c}
2.4 \\
\times 3.6 \\
\hline
8.64 \\
+ 1 \text{ place} \quad + 1 \text{ place} \quad 2 \text{ places to the left}
\end{array}
\]

This rule is fine for relatively small numbers where you have the complete product.

But what if you have large numbers and it is not necessary to find the complete product out to so many decimal places. In this case you may want to find the decimal by moving from the left of the answer rather than the right. For example, multiplying

\[
\begin{array}{c}
2.43786 \\
\times 6.34589 \\
\hline
\end{array}
\]

You could round off both numbers and use 2.438 and 6.346 and find the decimal location using the sum of the number of places method.

Let me show you two other methods that might be used either to find the location or to check the location after using another method. These methods are
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particularly useful when using a square-root table.

C. The first method is a method where you find the location of the decimal moving from the left rather than the right. Look at the following set of products:

\[
\begin{align*}
1.3 \times 1 &= 1.3 \\
1.3 \times 2 &= 2.6 \\
1.3 \times 5 &= 6.5 \\
1.3 \times 7 &= 9.1 \\
1.3 \times 8 &= 10.4
\end{align*}
\]

D. Notice that as the multiplier increases the number of places to the left of the decimal stays the same until the product goes past 10.0, then there are two places to the left rather than one. We can use this relationship to locate the decimal moving from the left rather than the right. If the multiplicand and the multiplier left values would result in a product less than 10, then the number of decimal places is one less than the total number of non-decimal places in the two numbers.

Example: \(113.28563 \times 52.7358\)

Think of 1.1 times 5.2, this result would be less than 10, so the number of places to the left of the decimal would be \((3 \text{ places} + 2 \text{ places}) - 1 = 4 \text{ places}\) left of the decimal. The answer would be,

\[
\begin{align*}
\text{5974.21} \\
\text{1234}
\end{align*}
\]
E. If the left values result in a product greater than 10, then the result would have the total number of non-decimal places of the multiplicand and the multiplier.

Example: $634.8256 \times 3563.32$

Think of $6.3 \times 3.5$, this result would be greater than 10, so the number of places to the left of the decimal would be $(3 \text{ places} + 4 \text{ places}) = 7$, and the answer would be,

$$2262089.93$$

$7 \text{ places}$

This is often a good way to check the location of your decimal place.

F. The second method is to use powers of 10. We arrange the numbers in a form where we can get an initial location of the decimal and we then make adjustments based on the exponent of 10. In order to use this method, we must first be able to arrange numbers in terms of powers of 10. Any value may be expressed in terms of a power of 10. Consider the value of 1, it may be written as,

$$
\begin{align*}
1.00 & = 10^0 \\
1.00 \times 10^0 & = 10^0 \\
0.100 \times 10^1 & = 10^1 \\
0.010 \times 10^2 & = 10^2 \\
0.001 \times 10^3 & = 10^3 \\
10.0 \times 10^{-1} & = 10^{-1}
\end{align*}
$$
Notice that as the decimal moves from left to right, for each place it moves, the exponent is increased by -1. For each place the decimal moves from right to left, the exponent increases by +1. Memorize this rule, after you use it a few times, you will be able to use it with some confidence.

Practice putting these values in the power of 10 form so that there is one non-decimal place to the left of the decimal.

Example: \(362.5 = 3.625 \times 10^2\)

1. \(13.6 =\)
2. \(635.8 =\)
3. \(0.5 =\)
4. \(0.0035 =\)
5. \(645.3 \times 10^{-3} =\)
6. \(0.432 \times 10^2 =\)

Check the answers on page 33 of the Reference and Answer Manual.

G. Finding the product of two numbers using this system is as follows. If one number is \(a \times 10^x\) and the other is \(b \times 10^y\), then the product is: \((a \times b) \times 10^{x+y}\). The product of the non-exponent values is found first, then it is multiplied by 10 with an exponent of the sum of the two original exponents.
Example: $634.8256 \times 3563.325$  
\[
\frac{22621}{22621} \quad (\text {? decimal point})
\]

$634.8256 = 6.348256 \times 10^2$

$3563.325 = 3.563325 \times 10^3$

You know that 6.3 times 3.5 will be about 20.0; so the initial product is (after rounding) 22.621 and the power of ten is $10^{2+3} = 10^5$, so the complete answer is $22.621 \times 10^5$ or 2262100. (count over five places to the right and place the decimal). Consider another example,

\[
\frac{25.1354}{0.0785} \quad (\text {? decimal point})
\]

$25.1354 = 2.51354 \times 10^1$

$0.0785 = 7.85 \times 10^{-2}$

You know that 2.5 times 7.8 is about 20, so the initial product (after rounding) is 19.731 and the power of 10 is $10^{1+(-2)} = 10^{-1}$, so the complete answer is $19.731 \times 10^{-1}$ or 1.9731 (count over one place to the left and place the decimal).

Find the following products using powers of 10.

1. $6.00 \times 250.0 = $
2. $0.382 \times 0.0053 = $
3. $-0.02 \times 763.0 = $
4. $351.0 \times 1643.0 = $
5. $(-3.56 \times 10^{-3}) \times (6 \times 10^{-4}) =$

Check your answer on page 33 of the Reference and Answer Manual.
H. Squaring a Number

This is the same as multiplying a number times itself, so the previous procedures hold. If \( a \times 10^x \), is the number, its square is \( (a^2) \times 10^{2x} \).

Consider \( 0.4^2 \),
\[
0.4 = 4 \times 10^{-1}
\]
\[
0.4^2 = 4^2 \times 10^{-1+(-1)} = 16 \times 10^{-2} = 0.16
\]

Consider \( 48^2 \)
\[
48 = 4.8 \times 10^1
\]
\[
48^2 = 4.8^2 \times 10^{1+1} = 23.04 \times 10^2 = 2304
\]

I. Division

The power of 10 method is particularly useful for finding the decimal location after a division operation. Consider \( a \times 10^x \) as the dividend and \( b \times 10^y \) as the divisor,

\[
\frac{a \times 10^x}{b \times 10^y} =
\]

The quotient is equal to \( \frac{a}{b} \times 10^{x-y} \). Care must be used to make sure \( y \) is subtracted from \( x \) and not added. Consider the example of \( 8/2 \) in exponent form.
\[
8 = 8.0 \times 10^0
\]
\[
2 = 2.0 \times 10^0
\]
\[
8/2 = (8.0 \times 10^0)/(2.0 \times 10^0) = (8.0/2.0) \times 10^{0-0}
\]
\[
= 4.0 \times 10^0 = 4.0 \times 1 = 4.0
\]

That’s a rather simple example, but it shows how it works. Let’s consider a little more complex example,
\[
4.40 \div 0.08 = 550 \text{ (?) decimal point)
\]

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4.40 = 4.40 \times 10^0
0.08 = 8.0 \times 10^{-2}

Think of dividing 4.4 by 8.0, this is about 0.5, so the initial part of the answer is 0.550. Take this times the power of 10 which is \(10^{0-(-2)} = 10^2\) (minus a minus is a plus). Put them together,

\[0.550 \times 10^2 = 55.0\]

Consider this example,

\[-435.82 = \frac{-3352}{-435.82} = 4.3582 \times 10^2
\]
\[0.0013 = 1.3 \times 10^{-3}\]

-4.35 divided by 1.3 is about -3, so the initial part of the answer is \(-3.352\) (after rounding) and this is taken times the power of 10, which is \(10^{2-(-3)} = 10^5\). This is equal to \(-3.352 \times 10^5 = -335200\) (move five places to the right). Division problems using powers of 10.

1. \(775 \div 2.5 =\)
2. \(-169 \div 0.013 =\)
3. \(0.140 \div 35 =\)
4. \((4.82 \times 10^{-6}) \div (-1.205 \times 10^{-5}) =\)
5. \((6 \times 10^4) \times (-3 \times 10^5) =\)

Check your answers on page 33 of the Reference and Answer Manual.

J. Square-roots

The powers of 10 method is very useful in finding
the decimal location of the answer and it also aids in using the table to find the proper value of the square-root. Refer to Unit IX, regarding the use of square-root tables. Here we will primarily be concerned with locating the decimal place. The square-root of the value $\sqrt{a \times 10^x}$ is equal to $\sqrt{a} \times \sqrt{10^x}$.

The $\sqrt{a}$ may be found in a table or by using a slide rule. The $\sqrt{10^x}$ is equal to $10^{x/2}$. The exponent ($x$) is divided by 2. Therefore, the exponent ($x$) must be an even number, so that $x/2$ is a whole number, and not a fraction. So in arranging the number from which we extract the square-root, the exponent must be divisible by 2, giving a whole number.

Consider the $\sqrt{9}$,

$$\sqrt{9} = \sqrt{9} \times 10^0 = \sqrt{9} \times 1$$

$$\sqrt{9} = 3, \quad \sqrt{10^0} = 10^{0/2} = 10^0 \quad (10^0 = 1)$$

Put them together, $3 \times 10^0 = 3 \times 1 = 3$

Consider the $\sqrt{490}$,

$$\sqrt{490} = \sqrt{49} \times 10^1 \text{ or } \sqrt{4.9} \times 10^2$$

What about $\sqrt{49} \times 10^1$? No, because the exponent (1) is not divisible by 2, giving a whole number.

What about $\sqrt{4.9} \times 10^2$? Yes.

Thus, $\sqrt{490} = \sqrt{4.9} \times 10^2 = \sqrt{4.9} \times \sqrt{10^2}$

$\sqrt{4.9}$ is between 2.0 and 3.0, so looking in the table we find the value as 2.21.

$$\sqrt{10^2} = 10^{2/2} = 10^1$$

So, putting them together we have

$$2.21 \times 10^1 = 22.1$$
Consider another example,

\[ \sqrt{0.0016} = \sqrt{1.6 \times 10^{-3}} \text{ or } \sqrt{16 \times 10^{-4}} \]

\[ \sqrt{16 \times 10^{-4}} \text{ is the one we want} \]

\[ \sqrt{16 \times 10^{-4}} = \sqrt{16} \times \sqrt{10^{-4}} = 4 \times 10^{-4/2} = 4 \times 10^{-2} = 0.04 \]

What about \( \sqrt{36900} \) ?

\[ \sqrt{36900} = \sqrt{3.69 \times 10^4} \text{ or } \sqrt{36.9 \times 10^3} \]

\[ \sqrt{3.69 \times 10^4} \text{ is the one we want} \]

\[ \sqrt{3.69 \times 10^4} = \sqrt{3.69} \times \sqrt{10^4} = 1.92 \times 10^{4/2} = 1.92 \times 10^2 = 192 \]

Find the following square-roots using powers of 10.

1. \( \sqrt{4900} = \)
2. \( \sqrt{360} = \)
3. \( \sqrt{0.16} = \)
4. \( \sqrt{0.0025} = \)
5. \( \sqrt{1.44} = \)

Check your answer on page 34 of the Reference and Answer Manual. Please evaluate Unit VIII, on page 83 of the Reference and Answer Manual before continuing to Unit IX.
Unit IX
Finding the Square-root
Reference Manual pages 35 to 42
Unit IX. Finding the Square-root.

(Estimated time: 45 to 60 minutes)

A. Many times in doing statistical calculations we must find the square-root of a number. You probably knew at one time how to calculate a square-root, but you may have forgotten. A square-root may be found in a few ways. One way is to calculate it directly, but this is a time consuming task. Another way is through the use of a Square-root Table. This way is fast, but not as accurate as a direct calculation beyond three or four digits.

In this section we will review the direct calculation method and also present the method using a table.

Remember that the square-root of a number can be either positive or negative. A square is always positive.

\[ 7 \times 7 = 49 \]
\[ (-7) \times (-7) = 49 \] (remember a minus times a minus is a plus)

In statistics we are only interested in the positive square-root, or +7 in our example. You might wonder why we use so many square-roots in statistics. We often square a series of numbers and then end up taking a square-root. We square to remove negative values; you know that the square of a negative number is positive. For example, one formula we use in statistics to calculate a standard deviation is

\[ \sigma = \sqrt{\frac{\sum (X-x)^2}{n}} \]
Notice we square the values of \((X-\bar{X})\), which is the difference between a score and the mean of the set of scores and can be positive or negative in sign. After we square each value of \((X-\bar{X})\), (we'll have one for each score), we add these squares, which will all be positive. We then divide this sum by \(n\) (the sample size) and then take the square-root of the quotient for the value of the standard deviation. We do this kind of computation often in statistics, so we must calculate square-roots quite frequently.

B. In the direct calculation of a square-root you must know the squares of the integers 1 through 9 and 10. Here they are for review if you have forgotten some of them.

\[
\begin{align*}
1 \times 1 & = 1 \\
2 \times 2 & = 4 \\
3 \times 3 & = 9 \\
4 \times 4 & = 16 \\
5 \times 5 & = 25 \\
6 \times 6 & = 36 \\
7 \times 7 & = 49 \\
8 \times 8 & = 64 \\
9 \times 9 & = 81 \\
10 \times 10 & = 100
\end{align*}
\]

One of the most important steps in calculating a square-root is in arranging the number in terms of places from the decimal point. Look closely at these examples of various square-root and square values using the digit 6.

\[
\begin{align*}
(600.00)^2 &= 360000.00 \quad (60.00)^2 = 3600.00 \quad (6.00)^2 = 36.00 \quad (0.60)^2 = .36 \quad (0.06)^2 = .0036
\end{align*}
\]

The square-root of 3600 is equal to 60. 60x60=3600.
Notice that for each move of the decimal point in the number to be squared \((60.00)^2\), the decimal point in the square of that number moves two places in the same direction.

C. Let's group some numbers in the proper way to prepare to extract a square-root. We group a number in terms of pairs of digits from the decimal point. For example, if we want to find the square-root of \(743.0\), we group each pair of numbers to the left of the decimal point,

\[
\begin{array}{c}
743.0 \\
2 \quad 43.0 \\
\end{array}
\]

Grouping the number \(786431.0276\) would be,

\[
\begin{array}{c}
78 \quad 64 \quad 31.02 \quad 76 \\
\end{array}
\]

If the grouping is not done correctly, then the answer will be wrong. Now let's go through the procedure for a direct calculation of a square-root. Find the square-root of \(625.0\). First, group

\[
\begin{array}{c}
625.0 \\
6 \quad 25.0 \\
\end{array}
\]

D. Set up the number in the following form,

\[
\begin{array}{c}
6 \quad 25.0 \\
\end{array}
\]

We start with the far left pair of digits which in this case is 6, or 06 if you must see two to have a pair. What one place digit squared is less than 6, (while the digit plus 1, squared, is greater than 6)?

\[
2^2 = 4, \quad 3^2 = 9
\]

The answer is 2. Place the 2 above the 6 in the basic form.
E. Now square the 2 and put the answer below the 6.

\[
\begin{array}{c}
2 \\
6 25.0 \\
\hline \\
4 \\
\end{array}
\]

F. Subtract 4 from 6 and then bring down the next pair to the right of 6.

\[
\begin{array}{c}
2 \\
6 25.0 \\
\hline \\
4 \\
\end{array}
\]

remainder \( \frac{2}{25} \)

G. Now double the value on top of the line, and place the result below the last remainder and to the left. This is the "trial divisor".

\[
\begin{array}{c}
2 \\
6 25.0 \\
\hline \\
4 \\
\end{array}
\]

(4)

H. Now how many times does 4 go into 22, and still be equal to or less than 22? The answer is 5. We now place the 5 in three places:

(1) above the next pair in the number, top line, 

\[
\begin{array}{c}
2 \\
6 5.0 \\
\hline \\
4 \\
\end{array}
\]

(4)

(2) to the left of the parenthesis in the trial divisor 

\[
\begin{array}{c}
2 \\
6 5.0 \\
\hline \\
4 \\
\end{array}
\]

\( 5(4) \)

(3) and directly after the trial divisor.
I. Now multiply the trial divisor.

\[
\begin{array}{c|c}
2 & 5. \\
6 & 25.0 \\
4 & \hline
\end{array}
\]

\[
5(45) = \frac{2}{5(25)}
\]

J. Subtract the trial divisor product from the remainder.

\[
\begin{array}{c|c}
2 & 5. \\
6 & 25.0 \\
4 & \hline
2 & 25 \\
\end{array}
\]

\[
5(45) = \frac{2}{25} \quad \frac{2}{25} \\
0
\]

And since there are no more non-zero values, the answer is complete. Thus, the square-root of 625.0 is 25.0.

Now to check, take 25 times 25 and you should get 625.0.

K. Now let's find the square-root of another number.

Find the square-root of 883161.25. First, group the number

\[
\begin{array}{c|c}
88 & 31 \ 61.25 \\
\end{array}
\]

Set up,

\[
\begin{array}{c|c}
88 & 31 \ 61.25 \\
\end{array}
\]

Find the digit which squared is less than 88, while the digit plus 1, squared, is greater than 88.

\[
g^2 = 81 \quad 10^2 = 100
\]

Thus, 9 is our first entry,

\[
\begin{array}{c|c}
9 & \frac{88}{31} \ 61.25 \\
\end{array}
\]
Square 9 and subtract from first pair, bring down next pair,

\[
\begin{array}{c}
\underline{9} \\
\underline{88} \\
\underline{31} \\
\underline{61.25}
\end{array}
\]

\[
\underline{81} \\
7 \underline{31}
\]

remainder \( \frac{7}{31} \)

Double 9 and place as trial divisor, left and underneath remainder,

\[
\begin{array}{c}
\underline{9} \\
\underline{88} \\
\underline{31} \\
\underline{61.25}
\end{array}
\]

\[
\underline{81} \\
7 \underline{31}
\]

(18)

L. How many times does 18 go into 73? About 4.

Place 4 in three places as below,

\[
\begin{array}{c}
\underline{9} \\
\underline{88} \\
\underline{31} \\
\underline{61.25}
\end{array}
\]

\[
\underline{81} \\
7 \underline{31} \\
2 \underline{36}
\]

No, can't subtract to a negative value. The number 4 is too high, so reduce it to three and you now have,

\[
\begin{array}{c}
\underline{9} \\
\underline{88} \\
\underline{31} \\
\underline{61.25}
\end{array}
\]

\[
\underline{81} \\
7 \underline{31} \\
2 \underline{36}
\]

Remainder \( \frac{1}{82} \)

Now bring down the next pair,

\[
\begin{array}{c}
\underline{9} \\
\underline{88} \\
\underline{31} \\
\underline{61.25}
\end{array}
\]

\[
\underline{81} \\
7 \underline{31} \\
2 \underline{36}
\]

3(183) = \( \frac{549}{18261} \)

Double the top value (93) and place it below and to the left of the remainder,
186 goes into 182 about 10 times, but not quite, so let's try 9 as our next answer value. Put 9 in the three proper places. Multiply and find the remainder.

\[
\begin{array}{c}
939 \\
883161.25 \\
81 \\
731 \\
3(183)=
\end{array}
\]

\[
\begin{array}{c}
549 \\
9(186)= 168261 \\
144025 \\
(186)=
\end{array}
\]

Bring down the next pair,

\[
\begin{array}{c}
939 \\
883161.25 \\
81 \\
731 \\
3(183)=
\end{array}
\]

\[
\begin{array}{c}
549 \\
9(186)= 168261 \\
144025 \\
(186)=
\end{array}
\]

Double the top value (939) and place left and below the remainder,

\[
\begin{array}{c}
939 \\
883161.25 \\
81 \\
731 \\
3(183)=
\end{array}
\]

\[
\begin{array}{c}
549 \\
9(186)= 168261 \\
144025 \\
(186)=
\end{array}
\]

19 goes into 144 about 7 times, so let's try 7 as our next answer value. Put 7 in the three places, multiply and find the remainder,
M. We now have the answer except for rounding off the last value of .7. Is the answer closer to .7 or .8? To do this, we add a pair of zeros to the right end and go through the process one more time.

\[
\begin{array}{c}
\sqrt{883161.25} \approx 939.8 \\
883161.2500 \\
81 \\
731 \\
3(183) = 549 \\
18261 \\
9(1869) = 16821 \\
144025 \\
7(18787) = 131509 \\
12516 \\
6(187946) = 1127676 \\
125 = about 6 times 19
\end{array}
\]

Since .76 is closer to .8 than .7, we take .8 as the final value. Thus,

\[\sqrt{883161.25} = 939.8\]

Now to check, we square 939.8.

\[939.8^2 = 883224.04\]

This value is relatively close to 883161.25. It is closer than 939.7^2 which is equal to 883036.09.

Sometimes it is helpful to make up a set of symbols to memorize to aid you in remembering the procedures for a given calculation. You may be able to make one to use in calculating a square-root. Here is one that the author has composed which might help...
you calculate a square-root.

\[
\begin{align*}
G2, S &, D2, 2T, 3X \\
\hline
G2, & \text{group by pairs from the decimal point} \\
S, & \text{square of integer for first pair} \\
, & \text{subtract from previous value for partial remainder} \\
D2, & \text{drop next pair for complete remainder} \\
2T, & \text{double top of line to get trial divisor} \\
3X, & \text{place trial multiplier in three places and multiply} \\
\hline
\end{align*}
\]

go back and repeat from subtraction

Calculate the following square-roots using the direct calculation approach. Check each one before continuing.

1. \( \sqrt{45} \) = (to one decimal place)
   - a. 2.1
   - b. 6.7
   - c. 6.5

2. \( \sqrt{182.631} \) = (to one decimal place)
   - a. 13.5
   - b. 42.7
   - c. 13.6

3. \( \sqrt{0.183} \) = (to two decimal places)
   - a. 1.40
   - b. .14
   - c. 0.43

Check your answers on page 38 of the Reference and Answer Manual.
other ways of finding a square-root

N. Calculating a square-root directly is very time consuming and it is easy to make an error in the procedure. We have other ways of finding square-roots. The fastest, most accurate way is to use a calculator which extracts square-roots. However, most students don't have a calculator handy when they want to find a square-root. Another way to find a square-root is by using a slide-rule. However, the slide-rule is not very accurate for more than two or three places, so it is not quite accurate enough for most work using square-roots.

C. Another way to find a square-root, which is fast and usually accurate enough for our purposes is to use a square-root table. Most introductory statistics texts have a square-root table. You will find excerpts of the table from Guilford's text on page 37 of the Reference and Answer Manual. We will use this table in our following examples and problems.

P. This table has seven columns. The first is a number, in this table going from 1 to 1,000. The number squared is found in column two. Column three is the value of the square-root of that number. Column four is the square-root of 10 times the number, or the number with the decimal place moved one place to the right. Columns five, six, and seven, are not frequently
used in the course. Column five is the reciprocal which is 1 divided by the number. Column six is the reciprocal of the square-root of the number and column seven is the reciprocal of the square-root of 10 times the number.

Q. You can find the squares of a number using this table. Notice that the square value is grouped by pairs of digits which aid in finding the location of the decimal place. For example, if the decimal is at the extreme right of the value then the decimal in the answer is at the extreme right of the square value.

The square of 26.0 is 676.0. If the decimal is one place to the left of the extreme right digit then the decimal is moved in one pair of digits to the left in the answer. For example, 1.9 squared is equal to 3.61 and 61.1 squared is equal to 3733.21. Using the table, find the squares of

1. 292 =
2. 1.4² =
3. 6.19² =
4. 0.601² =

Check your answers on page 39 of the Reference and Answer Manual.

R. Using the table to find a square-root is a little more complicated, but you'll pick it up with a little practice and common sense. The square-root of a number in the table, if the decimal point is to the extreme right of the number, is found in column three. The
square-root of 12.0 is 3.4641 and the square-root of 616.0 is 24.8193. Now, moving the decimal point causes us to make a new consideration. If the decimal point moves an odd number of times, either right or left, then we must use column four, the $\sqrt{10n}$. If the decimal point moves an even number of times, either right or left, then we must use column three. This relates to the pairing of two digits as we discussed before.

S. Let's take the number 17 and move the decimal point around and find the square-roots. The square-root of 17.0 is 4.1231. Now move one place (odd number) to the right. Square-root of 170.0 is found in column four, which is 13.0384. Move two places (even number) right from the table value, which is 1700. Now we must use column three, but we move the decimal one place to the right. No. of places to move in answer = No. of places moved in number to be squared /2. 

$(2/2 = 1)$ Only valid for a column three value.

$\sqrt{1700} = 41.231$

Now consider the square-root of 17000, decimal point moved three places (odd number) to the right. Since it is an odd number we use column four and move the decimal point one place to the right of the table value. The number of decimal places the point moves, if moving to the right, is given by (number of places moved from table value -1)/2, or $(3-1)/2 = 1$ place. Do this for a column four value.

$\sqrt{17000} = 130.384$

84
195

1. Now, let's go the other direction and find the square-root of 1.7. This is a move of one decimal place, so we use column four, and move the decimal point one place to the left of the table value. The number of decimal point moves, if moving to the left, is given by \((\text{number of moves} + 1)/2\), or \((1+1)/2 = 1\).

\[\sqrt{1.7} = 1.3038\]

2. Now, what is the square-root of 0.17? Decimal point is moved two places from table value, so we use column three and move the decimal one place to the left.

\[\sqrt{0.17} = 0.4123\]

What is the value of the square-root of 0.017? We've moved the decimal three (odd number) places to the left so we use column four and we move the decimal place \((3+1)/2 = 2\) places to the left.

\[\sqrt{0.017} = 0.130384\]

Now, find the square-root of 0.0017. We've moved four places to the left, so we'll use column three, and move the decimal place \(4/2 = 2\) places left in the table value.

\[\sqrt{0.0017} = 0.130384\]

3. Let's move the decimals in the value 616 to find some square-root values. What is the square-root of 61600.0? The decimal was moved two places right. Use column three and move the decimal point \(2/2 = 1\) places right of the table value.

\[\sqrt{61600.0} = 248.193\]

Find the square-root of 0.00616. We've moved five
places to the left. Use column four and move the decimal \((5+1)/2 = 3\) places to the left of the table value.

\[ \sqrt{0.00616} = 0.0784857 \]

W. Now let's summarize the procedure.

1. Find the numerical value in column 1 of the table.
2. Count the number of decimal places the number to be extracted has different than the table value. Note the direction of the decimal point move.
3. If the number of places is even, use column 3, and move the decimal of the table value (number of places moved, either left or right/2) left or right.
4. If the number of places is odd, use column 4 and
   a. If the move is to the right, move the decimal in the table value (number of places -1)/2 to the right.
   b. If the move is to the left, move the decimal in the table value (number of places +1)/2 to the left.

After you have found several of these square-roots, you should be able to find the decimal point without a great deal of difficulty. Always check your answer keeping in mind these guidelines:

X. 1. The square-root of a number less than one is always less than one.
2. Any number whose numerical part (not considering the decimal point) is less than 316 when squared will have a number of non-decimal places equal to (twice the number of non-decimal places in the number to be squared) minus 1 place. For example, 3.16\(^2\) is equal to 9.9856. The number of non-decimal places is equal to \((2\times1)-1 = 1\). Consider 213.0\(^2\), which is 45369.0. There are \((2\times3)-1 = 5\) non-decimal places in the answer.
3. Any number whose numerical part (not considering the decimal point) is equal to or greater than 317 when squared will have a number of non-decimal places equal to twice the number of non-decimal places in the number to be squared. For example 38.0\(^2\) will be equal
to 1444.0. The number of non-decimal places equals \((2 \times 2) = 4\) places. Consider the square of 6140.0, there will be \((2 \times 4) = 8\) non-decimal places in the answer, so it is (look at your table) 379456.00.0.

These guidelines can be used as check procedures which can help you make sure you have the right answer.

Using the square-root table, find the square-roots.
Check your answers on page 40 of the Reference and Answer Manual.

1. \(\sqrt{28.0} = \)
   _a. 7.84
   _b. 529
   _c. 1.67

2. \(\sqrt{6040} = \)
   _a. 77.7
   _b. 246
   _c. 36.48

3. \(\sqrt{6.20} = \)
   _a. 3.844
   _b. 2.49
   _c. 7.87

4. \(\sqrt{0.28} = \)
   _a. 0.53
   _b. 0.167
   _c. 0.053

5. \(\sqrt{614000} = \)
   _a. 783.6
   _b. 247.8
   _c. 78.36
6. \( \sqrt{0.00806} = \)
   - a. 0.0246
   - b. 0.778
   - c. 0.0778

7. \( \sqrt{260000} = \)
   - a. 676
   - b. 509.9
   - c. 1612.5

Please evaluate Unit IX. on page 84 of the Reference and Answer Manual before continuing to Unit X.
Unit X
Order of Arithmetic Operations
Reference Manual pages 43 to 47
Unit X. Order of Arithmetic Operations  
(Estimated time-30 minutes)  
A. In many statistics equations there are several arithmetic operations which must be performed in proper sequence for the answer to be correct. This unit should aid you in determining the order of preference for arithmetic operations within an equation. The general order of sequencing arithmetic operations is, from first priority to last priority:
  1. Parentheses ( )  
  2. Brackets [ ]  
  3. Exponents and Square-roots  
  4. Multiplication and Division  
  5. Addition and Subtraction  
B. The key to performing operations in the proper sequence is in the adherence to the brackets and the parentheses. We do not use brackets unless they have parentheses enclosed within them. We work within parentheses and then once we've found the value of the parentheses then we continue to other operations outside the parentheses. For example, if we have the expression \((1+6)^2\), the first thing we do is find the value of the parentheses which is 7 in this case. Then we perform the operation on the value of the parentheses which is \((7)^2\) or 49. Let's consider the expression without the parentheses, \(1+6^2\). The first operation is finding the square of 6 and then adding 1 to the result to get \(1+36 = 37\).
Other examples of this procedure are:

\[
(2x4)^2 = (8)^2 = 64 \\
[1+(3x5)]^2 = (1+15)^2 = 16^2 = 256 \\
[(2+1)^2x2]^2 = [(3)^2x2]^2 = [9x2]^2 = 18^2 = 324 \\
(3^2)^2 = (9)^2 = 81
\]

C. When multiplication and/or division is combined with addition and/or subtraction on the same expression then the multiplication/division takes priority over the addition/subtraction operation.

For example, here are some problems worked the correct way and the incorrect way.

<table>
<thead>
<tr>
<th>Correct</th>
<th>Incorrect</th>
</tr>
</thead>
<tbody>
<tr>
<td>4+6+2+1 =</td>
<td>4+6+2+1 =</td>
</tr>
<tr>
<td>4+3+1 = 8</td>
<td>10÷3 = 3 1/3</td>
</tr>
<tr>
<td>Correct</td>
<td>Incorrect</td>
</tr>
<tr>
<td>4x3+6x5 =</td>
<td>4x3+6x5 =</td>
</tr>
<tr>
<td>12+30 = 42</td>
<td>4x9x5 = 180</td>
</tr>
</tbody>
</table>

Solve the following problems. Check your answers on page 44 of the Reference and Answer Manual.

1. \[4x6+3+2 = \]
   - a. 29
   - b. 44
   - c. 38

2. \[10÷2+6x3 = \]
   - a. 33
   - b. 23
   - c. 3.75
D. When two operations which have the same priority level such as multiplication and division occur together, the rule is to do the operations in order from left to right. Here are some correct and incorrect solutions.

### Correct
- $10 \times 2 + 4 \div 2 = 20 + 4 \div 2 = 20 + 2 = 22$
- $20 \div 4 \div 2 = 20 \div 4 = 5$
- $5 \div 2 = 2.5$

### Incorrect
- $10 \times 2 + 4 + 2 = 10 \times 2 + 2$
- $10 \times 1 = 10$

### Correct
- $4 \div 2 \times 6 \div 4 = 2 \times 6 \div 4 = 3$
- $12 \div 4 = 3$

### Incorrect
- $4 \div 2 \times 6 \div 4 = 4 \div 12 \div 4$
- $1 \div 3 \div 4 = 1/12$

Solve the following problems. Check your answers on page 44 of the Reference and Answer Manual.

1. $6 \times 4 + 2 \times 3 =$
   - a. 4
   - b. 8
   - c. 36

2. $16 \div 8 \times 14 \div 7 =$
   - a. 0.020
   - b. 4
   - c. 2

Solve the following. Answers are on page 44. Check them before continuing.

1. $(3 \times 5) + (6 \times 2) =$
   - a. 42
   - b. 27
   - c. 180
2. \((2^4)^3 = \)
   ___a. 16
   ___b. 4
   ___c. 64

3. \([3+(2+1)^2]^2 = \)
   ___a. 12
   ___b. 144
   ___c. 49

4. \(\sqrt{(1x3)^2+(2x2)^2} = \)
   ___a. 25
   ___b. 7
   ___c. 5

E. In many statistical problems, we find the use of the symbol Σ which is the summation (addition) notation. This is discussed in greater detail in Unit V. However, let's show how it works in relation to the other arithmetic operations. The symbol itself (Σ) does not stand for a numerical value. It must be coupled with a symbol which indicates what is added or summed. For example, ΣX is the value of the sum of all scores labeled X. If \(X_1 = 4, X_2 = 2, X_3 = 2,\) and \(X_4 = 3,\) then \(\Sigma X = 4+2+2+3 = 11 = \) the sum of scores of X. We could also have ΣY which would symbolize the addition of all scores labeled Y.

You will undoubtedly see this symbol used: ΣX^2 which, remember the order of operations, is the result of the summation of each score labeled X after it is squared. For example, if \(X_1 = 4, X_2 = 2, X_3 = 2,\) and
\(x_4 = 3\), then \(\Sigma X^2 = 16 + 4 + 4 + 9 = 33\) = the sum of the squared scores labeled \(X\).

You will also come in contact with the symbol \((\Sigma X)^2\) which looks much like \(\Sigma X^2\), but it is not the same. In finding \((\Sigma X)^2\), the first thing we do is find the value of the parentheses. We found in one of our previous examples that \(\Sigma X = 11\), thus the value of \((\Sigma X)^2 = (11)^2 = 121\). You'll note that this is not the same value as we got when we found \(\Sigma X^2\) which was equal to 33.

P. Several times during the course, you will have to use formulas to calculate various statistics. Let's take a few of these and determine the order of operations.

Here is a formula we use to calculate the median. It has been abbreviated for simplicity.

\[
\text{Mdn.} = L + W\left(\frac{n/2 - C}{f}\right)
\]

First, we find the value within the parentheses. Find \(n/2\) and subtract \(C\) from the quotient. Next, divide this value by \(f\). Now we have the value of the parentheses. We take this value times \(W\) and then add the product to \(L\) to find the value of the median.

If \(L = 14.5\), \(W = 3\), \(n = 20\), \(C = 8\), and \(f = 4\), find the value of the median. Show your work.
The median is equal to ____. Check your answer on page 46 of the Reference and Answer Manual.

G. Another formula that you will use is for the Pearson product-moment correlation coefficient. This formula is:

\[ r = \frac{n \Sigma XY - (\Sigma X)(\Sigma Y)}{\sqrt{(n \Sigma X^2 - (\Sigma X)^2)(n \Sigma Y^2 - (\Sigma Y)^2)}} \]

This may look like a very complex formula, but it is no problem if you follow the proper steps. First, find the value of the top of the equation. Multiply \( n \) times the value of \( \Sigma XY \) (\( \Sigma XY \) already represents the result of a previous summation operation). Next multiply \( \Sigma X \) times \( \Sigma Y \) and then subtract this product from the value you calculate for \( n \Sigma XY \). Now you have the numerator of the equation. Next, we work on finding the value under the square-root radical. First, find the value of \((\Sigma X)^2\). Next, find \( n \Sigma X^2 \) which is \( n \) times the value of \( \Sigma X^2 \). Then subtract \((\Sigma X)^2\) from \( n \Sigma X^2 \) which you just found. Follow the same procedure for the \( Y \) bracket. Now, multiply the value of the \( X \) bracket times the value of the \( Y \) bracket. Take the square-root of the product. Now, you have the value of the denominator for the value of \( r \).

If \( n = 10 \)

\[ \Sigma XY = 4.0 \]
\[ \Sigma X = 5.0 \]
\[ \Sigma Y = 2.0 \]
\[ \Sigma X^2 = 10.5 \]
\[ \Sigma Y^2 = 2.4 \]

(These numbers are fictitious; They are only used in this example for simplicity.)
Find the value of r

\[ r = \frac{nEXY - (EX)(EY)}{\sqrt{nEX^2 - (EX)^2} \sqrt{nEY^2 - (EY)^2}} \]

The answer is on page 46 of the Reference and Answer Manual.

H. Another formula which you will use is the formula for the chi-square statistic \( \chi^2 \). It is:

\[ \chi^2 = \sum \frac{(0-E)^2}{E} \]

The steps in using this formula are easy to get mixed up. First, we find the value of the parentheses, \((0-E)\), and then square the result, \((0-E)^2\). We then divide this square by the value of \(E\). We will have a series of these operations. When we find each of the \((0-E)^2/E\) values, we add them up (\(\Sigma\)) to find the value of \(\chi^2\).

Assume you have a series with three 0 and E values.
(These values are fictitious).

Number in Series 1 2 3

Value of 0 8 10 5

Value of E 4 2 2

We find the value of $x^2$ by

$$x^2 = \sum \frac{(0-E)^2}{E} = \frac{(8-4)^2}{4} + \frac{(10-2)^2}{2} + \frac{(5-2)^2}{2} = \frac{4^2}{4} + \frac{8^2}{2} + \frac{3^2}{2} = 16 + 64 + 9 = 4.5 + 32 + 4.5 = 40.5$$

With the following data, calculate $x^2$. (These values are fictitious). Number in Series of two

1 2

Value of 0 12 7

Value of E 2 2

$$x^2 = \sum \frac{(0-E)^2}{E} =$$

Check your answer on page 46 of the Reference and Answer Manual. Please evaluate Unit X, on page 85 of the Reference and Answer Manual before continuing to Unit XI.
Unit XI
Elementary Manipulation
of Equations
Reference Manual pages 48 to 57
Unit XI. Elementary Manipulation of Equations Pre-test.

Check your answers on page 49 of the Reference and Answer Manual. Write the manual answers in the "correct answer" blank. Solve the following equations for the value indicated.

1. \( m + n = k \)
   \( n = \underline{\hspace{2cm}} \) answer, correct answer \underline{\hspace{2cm}}. See paragraphs A. and B., page 101.

2. \( x + 3y = 13a \)
   \( x = \underline{\hspace{2cm}} \) answer, correct answer \underline{\hspace{2cm}}. See paragraphs A. and B., page 101.

3. \( z = 4a + 7b - c \)
   \( c = \underline{\hspace{2cm}} \) answer, correct answer \underline{\hspace{2cm}}. See paragraphs A. and B., page 101.

4. \( 4a - a = 6f - 3g \)
   \( a = \underline{\hspace{2cm}} \) answer, correct answer \underline{\hspace{2cm}}. See paragraphs A. and B., page 101.

5. \( 3b = xy \)
   \( b = \underline{\hspace{2cm}} \) answer, correct answer \underline{\hspace{2cm}}. See paragraph F, page 106.

6. \( h = k + m \)
   \( \theta \) \( h = \underline{\hspace{2cm}} \) answer, correct answer \underline{\hspace{2cm}}. See paragraph E, page 105.

7. \( 7m + 3b = r \)
   \( b = \underline{\hspace{2cm}} \) answer, correct answer \underline{\hspace{2cm}}. See paragraph G, page 106.

8. \( x + 2 = z + 1 \)

99
9. \( \sqrt{b} = ac+2 \)
\( b = \) _______answer, correct answer_______  See paragraph J, K, and L, page 110.

10. \( x^2 - y^2 = 3h \)
\( y = \) _______answer, correct answer_______  See paragraphs J, K, and L, page 110.
A. In an introductory statistics course, we work with equations. Usually, these equations are set up in terms of the value you wish to find on the left of the equal sign and the values which you know on the right of the equal sign. Consider: \( b=ax \), \( b \) equals \( a \) times \( x \). We usually know the values of \( "a" \) and \( "x" \) and then we find the value of \( b \). But what if you know the values of \( b \) and \( x \), but not \( a \)? Is there a way to get \( "a" \) alone on one side of the equal sign with the two known values on the other side? You might already see this revised equation as:

\[ a=b/x \]

The purpose of this unit is to help you learn to make some of these manipulations. We will consider only relatively elementary manipulations, since this will be all, you should need for a course in introductory statistics.

Consider an equation as a simple balance with the equal sign as the fulcrum.

\[ b = ax \]

If the balance is balanced, then the equal sign is true. If something is added to one side of the balance, it will tip and the equal sign will no longer be true (\( \neq \)).
B. Now if you added the same weight to the other side of the balance, the equal sign would again be true.

\[ ax + b = ax + m \quad \text{if} \quad b = ax \]

Therefore, if equals are added to both sides of an equation, the equation is still true. Of course, the same is true of subtraction. Subtraction equals from an equation results in a true equation.

Let's now look at a few examples of the use of this principle in relation to manipulating an equation. If we had an equation such as:

\[ a - b = cd \]

What would we do if we wanted to find \( a \)? We must move "b" to the other side. If we add "b" to both sides, what would we get?

\[ (-b+b) = 0 \quad a-b+b = cd+b \]

Plus b minus b = 0, therefore "b" drops out of the left side of the equation and we now get

\[ a = cd+b \]

Let's use a numerical example. If \( cd = 10 \) and \( b = 4 \), what is the value of \( a \)?

\[ (-4+4) = 0 \quad a-4 = 10 \]

\[ \overset{\rightarrow}\text{a-4+4 = 10+4} \]

\[ a = 10+4 \]

\[ a = 14 \]

Does this make sense in the first equation?

What if we have,

\[ g+h = i-j \]
and we know "g", "i", and "j", what is the value of "h"? Let's subtract g from both sides to remove it from the left side.

\[(g-g) = 0 \quad g+h = i-j\]
\[g-g+h = i-j-g\]
\[h = i-j-g\]

With a numerical example, \(g=4\), \(i=8\), \(j=3\). What is the value of h?

\[(4-4) = 0 \quad 4+h = 8-3\]
\[4+h-4 = 8-3-4\]
\[h = 8-3-4\]
\[h = 1\]

C. There are times when this addition or subtraction method will work to simplify an equation, but there are times when it will not. This method will only work if the value to be manipulated is not a part of a product, quotient, square, or square-root. For example, in the equation,

\[ab = c+d\]

It is not legitimate to get "a" alone on the left side of the equation by subtracting or adding "b". "b" must be alone and not a part of another arithmetic operation other than addition or subtraction. By the same token, it is not possible to simplify the equation,

\[a(-b) = c+d\]

by adding "b" to both sides.

However, say you had this equation,
and you wanted to solve for "a", you could subtract "(bc)" to get

\[ a = d - (bc) \]

D. Remember any value which you manipulate by addition or subtraction cannot be a part of a product, quotient, square, or square-root. It must be all or none.

Solve the following problems. Check your answers on page 50 of the Reference and Answer Manual, before going on.

1. \( x + y = rs - t \)
   \[ x = ? \]
   \[
   \begin{align*}
   &a. \quad x = rs - t - x \\
   &b. \quad x = rs - t + y \\
   &c. \quad x = rs - t - y
   \end{align*}
   \]

2. \( a - x + y = (q/h) - r \)
   \[ a = ? \]
   \[
   \begin{align*}
   &a. \quad a = (q/h) - r - x - y \\
   &b. \quad a = (q/h) - r + x - y \\
   &c. \quad r = (q/h) + r + x - y
   \end{align*}
   \]

3. \( m + n = h - k \)
   \[ h = ? \]
   \[
   \begin{align*}
   &a. \quad h = m + n + k \\
   &b. \quad h = k - m - n \\
   &c. \quad h = (m + n) / k
   \end{align*}
   \]

4. \( a + 2b = cd \)
   \[ a = ? \]
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a. \( a = cd - 2b \)
b. \( a = \frac{(cd-b)}{2} \)
c. \( a = cd - b \)

E. We found that sometimes it is possible to add or subtract equal values from both sides of an equation to simplify it. Similarly, it is sometimes possible to simplify an equation by multiplying both sides of an equation by the same value. Think again of your balance.

\[
b = ax
\]

If we place 3 b's on the left, 3 ax's on the right will balance the scale if \( b = ax \).

\[
3b = 3ax
\]

if \( b = ax \)

Let's show how we might use this concept in simplifying an equation. Assume

\[
a/3 = b
\]

what is the value of "a"? We must remove the 3 from the left side of the equation. If we multiply both sides by 3, we have

\[
3a/3 = 3b
\]

Now, note that \( 3a/3 = a/1 = a \). The 3's cancel out and the final result is

\[
a = 3b
\]

We multiply to remove a value from the denominator of one of the sides. For example, if \( b = 9 \), what is the value of "a"?
\[ \frac{a}{3} = b \]

\[ a = 3b \]

\[ a = 27 \]

Now check and make sure that our value works in both equations. Does,

\[ \frac{27}{3} = 9? \quad \text{Yes!} \]

F. We can also sometimes use division to manipulate an equation. We divide both sides of an equation by the same value. We still have balance.

\[ 5a = (bc) \]

What is the value of \( a \)? Divide both sides by 5 to remove 5 from the left side of the equation.

\[ \frac{5a}{5} = \frac{(bc)}{5} \]

The 5's on the left cancel out, \( \frac{5a}{5} = \frac{a}{1} = a \).

Therefore, if \( 5a = bc \), then

\[ a = \frac{bc}{5} \]

Let's check this with numbers. If \( bc = 15 \), what is the value of "a"?

\[ 5a = (bc) \]

\[ 5a = 15 \]

\[ a = \frac{15}{5} = 3 \]

Does this fit in the original equation?

\[ 5(3) = 15 \quad \text{Yes!} \]

G. As with addition and subtraction, there are requirements regarding when we can and when we cannot use multiplication or division to simplify an equation. This
will only work when the value to be manipulated is multiplying the complete side of the equation or dividing the complete side of the equation. For example, if we have,

\[ \frac{a}{b} + c = d \]

we can solve for "a" by multiplying both sides by "b" and then subtracting bc. To do this we would have,

\[ \frac{a}{b} + c = d \]

(multiply by b) \[ \frac{ba}{b} + bc = bd \]

(subtract bc) \[ a = bd - bc \]

(factor b) \[ a = b(d - c) \]

Or we could first subtract c and then multiply by b. This is what we would have.

\[ \frac{a}{b} + c = d \]

(subtract c) \[ \frac{a}{b} = d - c \]

(multiply by b) \[ \frac{ba}{b} = b(d - c) \]

\[ a = b(d - c) \]

Notice that you multiply the complete right side of the equation by b.

We can also divide to simplify some equations.

For example, if

\[ a(b - c) = de + f \]

What if we want to solve for "a"? We divide both sides of the equation by (b - c).

\[ \frac{a(b - c)}{b - c} = \frac{de + f}{b - c} \]

\[ a = \frac{de + f}{b - c} \]
Notice that the complete right side is divided by the value of \((b-c)\).

H. You may simplify an equation using this logic or you can simply learn a few rules which will help you.

1. If you change a value which is added or subtracted on one side of an equation to the other side of the equation simply change its sign.
   
   \[ a - b = c \]
   
   \[ a = c + b \]

   Changing \(b\) from one side to the other, changes the sign. Another example,
   
   \[ a + b = cd \]
   
   \[ a = cd - b \]

2. When changing a value which is a multiplier or a divisor on one side of the equation to the other side of the equation make a multiplier a divisor and a divisor a multiplier.

   For example,
   
   \[ ab = c + d \]

   Want to solve for \(b\). "a" is a multiplier. To change it to the other side makes it a divisor.
   
   \[ b = \frac{c + d}{a} \]

   Another example,
   
   \[ r = \frac{t - u}{s} \]

   Solve for \(r\). "s" is a divisor so changing it to the other side makes it a multiplier.
   
   \[ r = s(t - u) \]
I. Consider solving for $k$ in this equation

$$m + \frac{1}{k} = n-p$$

We've got to get "$k" alone on one side of the equation as a numerator not a denominator. Move "m" by subtraction, to the right side so the "$k" is a divisor on the left side.

$$\frac{1}{k} = n-p-m$$

Now move "$k" to the right side as a multiplier rather than a divisor,

$$1 = k(n-p-m)$$

Now to get "$k" alone on the right side, move the multiplier ($n-p-m$) to the other side as a divisor. Therefore,

$$\frac{1}{n-p-m} = k$$

Solve the following equations for the specified value. Check your answers on page 52 of the Reference and Answer Manual.

1. Solve for "$a" in

$$abc = \frac{e}{f}$$

   a. $a = \frac{e-bc}{f}$
   b. $a = \frac{e}{bcf}$
   c. $a = \frac{e}{bc+f}$
2. Solve for "t" in
\[ p - s = q - r \]
\[ t = \frac{s}{p - q + r} \]

b. \[ t = \frac{p \cdot s}{q - r} \]

c. \[ t = \frac{p - s - q + r}{9} \]

3. Solve for "g" in
\[ g = \frac{h(i + j - 10)}{2} \]

a. \[ g = \frac{h(i + j - 10)}{3} \]

b. \[ g = \frac{h(i + j - 8)}{3} \]

c. \[ g = \frac{h(i + j - 10 - 2)}{3} \]

J. Sometimes we need to solve an equation which has a square or square-root involved. If we have an equation, we can square both sides and still maintain our equality, if we want to remove a square-root radical, we can square both sides. However, we must square the complete side with only a radical, which may have a multiplier or divisor but not an addition or subtraction operation. Perhaps the examples below will help you understand this.

We can remove these radicals by squaring.

Side of equation
Square
\( \sqrt{n} \) \[ (\sqrt{n})^2 = n \]
\( \sqrt{a} \) \[ (\sqrt{a})^2 = a \]
\( \sqrt{\frac{3}{a}} \) \[ (\sqrt{\frac{3}{a}})^2 = \frac{3}{a} \]
\( \sqrt{5a} \) \[ (\sqrt{5a})^2 = 5a \]

2. Solve for "t" in
\[ p - s = q - r \]

b. \[ t = \frac{p \cdot s}{q - r} \]

c. \[ t = \frac{p - s - q + r}{9} \]
K. We cannot remove the radicals of the examples below, until the addition or subtraction value is moved to the other side of the equation.

<table>
<thead>
<tr>
<th>Side of equation</th>
<th>Remove, then Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( \sqrt{n} + 5 )</td>
<td>+5 ( (\sqrt{n})^2 = n )</td>
</tr>
<tr>
<td>b. ( 3\sqrt{a} - 6 )</td>
<td>-6 ( (3\sqrt{a})^2 = 9a )</td>
</tr>
<tr>
<td>c. ( \sqrt{5/a} - 7 )</td>
<td>-7 ( (\sqrt{5/a})^2 = 5/a )</td>
</tr>
</tbody>
</table>

Let's look at a few examples of the use of squaring to simplify an equation. Consider the equation,

\[ \sqrt{a} = b+c \]

Solve for "a". To remove the radical, square both sides

\[ (\sqrt{a})^2 = (b+c)^2 \]
\[ a = (b+c)^2 \]

What is we have this equation and we want to solve for "h"?

\[ \frac{\sqrt{h}}{k} = m-n \]
\[ \sqrt{h} = k(m-n) \]
\[ (\sqrt{h})^2 = [k(m-n)]^2 \]
\[ h = [k(m-n)]^2 \]

L. If we have a value that is squared on one side we can remove the square by taking the square-root of both sides of the equation. For example,

\[ a^2 = bc+d \]
\[ \sqrt{a^2} = \sqrt{bc+d} \]
\[ a = \sqrt{bc+d} \]

Also, if we want to solve for "m", in
First multiply both sides by $n$

$$\frac{m^2}{n} = n(p-q)$$

Take the square-root of both sides.

$$\sqrt{\frac{m^2}{n}} = \sqrt{n(p-q)}$$

$$m = \sqrt{n(p-q)}$$

Solve the following problems involving squares and square-roots. Check your answers on page 54 of the Reference and Answer Manual.

1. Solve for "b" in

$$\frac{3\sqrt{b}}{a} = \frac{c+d}{3}$$

   a. $b = \frac{a(c+d)}{3}$

   b. $b = \left[\frac{a(c+d)}{3}\right]^2$

   c. $b = \left[\frac{a(c+d)}{3}\right]^3$

2. Solve for "s" in

$$t + r = \frac{3u - v}{s^3}$$

   a. $s = \sqrt[3]{\frac{t}{3u-v-r}}$

   b. $s = \frac{t}{3u-v-r}$

   c. $s = \left(\frac{t}{3u-v-r}\right)^2$


1. Solve for "a" in

$$\frac{b^2}{c} = \frac{a}{3} + 6$$
Please evaluate
Unit XI. on page
86 of the Reference
and Answer Manual.

2. Solve for \( \sigma^2 \)
\[
\sigma = \frac{\Sigma x^2}{n-1}
\]
___ a. \( \Sigma x^2 = \sigma/\sqrt{n-1} \)
___ b. \( \Sigma x^2 = \sigma^2/(n-1) \)
___ c. \( \Sigma x^2 = \sigma^2(n-1) \)

3. Solve for "C" in,
\[
AB = C^2 + D
\]
___ a. \( C = \sqrt{AB - D} \)
___ b. \( C = \sqrt{AB - D} \)
___ c. \( C = (AB-D)^2 \)

4. Solve for "N" in,
\[
\bar{x} = \frac{\Sigma x}{N}
\]
___ a. \( N = \frac{\Sigma x}{\bar{x}} \)
___ b. \( N = \frac{\bar{x}}{\bar{x}} \)
___ c. \( N = \Sigma X \bar{x} \)

5. Solve for "r" in
\[
s_e = \frac{s_y \sqrt{1-r^2}}{1-r^2}
\]
___ a. \( r = \frac{1-s_e^2}{s_y^2} \)
___ b. \( r = \frac{1-s_e^2}{s_y^2} \)
___ c. \( r = 1-\left( \frac{s_e}{s_y} \right) \)
Unit XII
Graphing

Frequency Distribution and Polygon

Grouped Frequency Distribution and Polygon

Reference Manual pages 58 to 64
Describing data is one of the most important functions of statistics. We describe data in basically two ways: numerically and/or graphically. Visually describing data can be very useful in providing a quick and easy look at the total distribution of scores. There are many examples of the use of graphing in statistics. However, we are primarily interested in two types of graphs. The first type is called a frequency polygon. A frequency polygon is a graph for only one variable, where the X axis is in the form of occurring score intervals and the Y axis is in the form of frequencies or the number of times each score or score interval occurs. The second type of graph is called a scattergram. A scattergram is a two variable graph where each point on the graph represents measures on both variables for one subject. One variable is plotted along the X axis and the other is plotted on the Y axis.

The combination of numerical and graphical descriptions of data provides a particularly powerful tool for making statistical conclusions.

All of the graphing in introductory statistics is done in terms of the Cartesian coordinate system. The Cartesian coordinate system should be familiar to you from your previous algebra or geometry experience. You will find this graphic system on page 75 of the Reference and Answer Manual.
Unit XII. A. Frequency Distribution and Polygon

A. The frequency polygon uses both the X and Y axis, but there is only one variable. The variable scores are plotted on the X axis and the frequency of occurrence for each score is plotted on the Y axis. This permits us to condense a large amount of data into a picture which describes the overall distribution of the data.

Let's take a set of data which has ten scores which are: 16, 15, 14, 15, 16, 15, 13, 14, 15, 18. The first step is to arrange the score categories from low to high keeping in mind any scores that could have occurred between the high and the low scores. The lowest score is 13 and the highest score is 18. Therefore, the score categories would be in order (usually the low score is on the bottom):

18
17
16
15
14
13

Next, we count the number of times each score category occurs or its frequency.

B. The score 13 occurs once, 14 occurs twice, 15 occurs four times, 16 occurs twice, 17 does not occur, and 18 occurs once. Thus our frequency distribution looks like this: (X represents score category, f represents frequency)
In constructing our frequency polygon, we use the X axis for the score category, based on the possible score categories. (Usually the low score is to the left). X axis 13 14 15 16 17 18

Only the top of the Y axis (the positive portion) is used since there cannot be a negative frequency. The Y axis is moved to the left or right until it is close to the low score category.

To do it as below would not be practical, especially if the low score was very high.

C. The Y axis is used to plot the frequencies from
zero occurrences to the largest number of frequencies of a score category.

Now we put the X and Y scales together in the following form.

\[ Y \]
\[ f \]
\[ 6 \]
\[ 5 \]
\[ 4 \]
\[ 3 \]
\[ 2 \]
\[ 1 \]
\[ 0 \]
\[ 12 \]
\[ 13 \]
\[ 14 \]
\[ 15 \]
\[ 16 \]
\[ 17 \]
\[ 18 \]
\[ 19 \]

Score categories for variable X.

D. We now plot the number of frequencies on the Y axis that we determined in the frequency distribution.
Score 13 had 1, 14 had 2, 15 had 4, 16 had 2, 17 had 0, and 18 had 1.

\[ f \]
\[ 4 \]
\[ 3 \]
\[ 2 \]
\[ 1 \]
\[ 0 \]
\[ 12 \]
\[ 13 \]
\[ 14 \]
\[ 15 \]
\[ 16 \]
\[ 17 \]
\[ 18 \]
\[ 19 \]

E. To finish our frequency polygon, we draw lines between the points including the next score lower than the lowest score and the next score higher than the highest score (12 and 19).

So our completed frequency polygon looks like this:
With the following 20 scores, construct a frequency distribution and a frequency polygon. Check your answers and polygon on page 60 of the Reference and Answer manuals. Label the axes of the polygon.

Scores on X variable

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>3</td>
</tr>
<tr>
<td>26</td>
<td>4</td>
</tr>
<tr>
<td>27</td>
<td>2</td>
</tr>
<tr>
<td>28</td>
<td>2</td>
</tr>
<tr>
<td>29</td>
<td>2</td>
</tr>
<tr>
<td>30</td>
<td>1</td>
</tr>
</tbody>
</table>

Frequency Distribution
Unit XII. B. The Grouped Frequency Distribution

F. Many times the number of scores in our data set is quite high and there is the possibility that the range between the lowest score and the highest score may be relatively large. It is often not feasible to have each score as a score category on the X axis. We can further condense the data by combining a small range of scores into a small score interval which includes several individual scores. We then have a series of intervals rather than individual scores. From this, we develop a grouped frequency distribution.
and a grouped frequency polygon.

Assume that we have the following set of data.

These are scores on a reading achievement test for sixth graders.

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>1</td>
</tr>
<tr>
<td>33</td>
<td>1</td>
</tr>
<tr>
<td>42</td>
<td>1</td>
</tr>
<tr>
<td>48</td>
<td>1</td>
</tr>
<tr>
<td>25</td>
<td>1</td>
</tr>
<tr>
<td>23</td>
<td>1</td>
</tr>
<tr>
<td>35</td>
<td>1</td>
</tr>
<tr>
<td>41</td>
<td>1</td>
</tr>
<tr>
<td>43</td>
<td>1</td>
</tr>
<tr>
<td>32</td>
<td>1</td>
</tr>
<tr>
<td>16</td>
<td>1</td>
</tr>
<tr>
<td>41</td>
<td>1</td>
</tr>
<tr>
<td>27</td>
<td>1</td>
</tr>
<tr>
<td>35</td>
<td>1</td>
</tr>
<tr>
<td>43</td>
<td>1</td>
</tr>
<tr>
<td>25</td>
<td>1</td>
</tr>
<tr>
<td>38</td>
<td>1</td>
</tr>
<tr>
<td>32</td>
<td>1</td>
</tr>
<tr>
<td>28</td>
<td>1</td>
</tr>
<tr>
<td>29</td>
<td>1</td>
</tr>
<tr>
<td>36</td>
<td>1</td>
</tr>
<tr>
<td>32</td>
<td>1</td>
</tr>
<tr>
<td>36</td>
<td>1</td>
</tr>
<tr>
<td>35</td>
<td>1</td>
</tr>
<tr>
<td>43</td>
<td>1</td>
</tr>
<tr>
<td>37</td>
<td>1</td>
</tr>
<tr>
<td>36</td>
<td>1</td>
</tr>
<tr>
<td>41</td>
<td>1</td>
</tr>
<tr>
<td>40</td>
<td>1</td>
</tr>
<tr>
<td>40</td>
<td>1</td>
</tr>
<tr>
<td>33</td>
<td>1</td>
</tr>
<tr>
<td>30</td>
<td>1</td>
</tr>
</tbody>
</table>

G. To use each score as a score category on the X axis is possible, but not practical. We follow certain procedures to determine the number of equal size intervals and the size of the intervals. There are four primary steps to follow. First, we determine the inclusive range, which is 1 plus the difference between the highest and lowest scores. Our highest score is 53 and our lowest score is 16. The inclusive range is 1+(53-16) = 38

H. Secondly, we determine the size of the score interval. We base this on a convention that the ideal number of intervals should be 12 to 15 intervals. To find the size of the intervals, we divide the inclusive range by 12 and then by 15, and based on the results, we make a decision regarding the size of the intervals. In our example,

\[
\frac{38}{12} = 3.2
\]

\[
\frac{38}{15} = 2.5
\]

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On the basis of these results, an interval of size 3.0 would provide between 12 and 15 intervals. Also, for reasons which will be explained later, it is advisable to have an odd numbered interval size such as 3, 5, 7, etc.

I. The third step is to set up the intervals. The lowest interval must include the lowest score. The lowest score in our case is 16. We find the lowest multiple of the interval size below or equal to the lowest score. Five times 3 is 15, which is below 16, so we start our intervals at 15. Starting at 4 times 3 or 12 would be too low, and 6 times 3 or 18, would be too high.

J. Each score must be placed in only one interval. This presents a problem if the upper-limit of one interval is the same value of the lower-limit of the next highest interval and there is the possibility of a score falling on that part. Which interval would the score go into? The point that the intervals meet cannot be a possible score value. Therefore, we make the interval limit point a fractional score value that cannot occur. For example, if interval size were 3 and all scores were in whole numbers (no fractional part) the lower limit of an interval could be 14.5 and the higher-limit of the same interval would be 17.5, which would also be the lower limit of the next highest interval. No score can fall on the limit point. It can be placed in one and only one interval.
We have an exclusive interval.

K. Now, let's go back to our example with 50 scores. We determine that the interval size should be 3 and the lowest interval should start at 15. Actually, it should start at 14.5, so that a score of 15 cannot fall on the limit point. But in setting up the intervals, we can use the score of 15 and add 3 to find the lowest possible score to be included in the next highest interval. Continue to add 3 to each successive value until the highest value (54) is above the highest occurring score (53). Drop the highest value (54). This is the result (work up from the bottom).

84
51
48
45
42
39
36
33
30
27
24
21
18
15

Now add the interval size [(3 minus 1)/2] to each value for the upper-whole number for that interval. We now have,
Notice that each lower whole-number limit increases by 3 and each upper whole-number limit increases by 3 and each interval contains 3 possible scores. (i.e., 15-17 contains 15, 16, and 17).

In actuality, each limit point is midway between the upper limit of the lower interval and the lower limit of the higher interval. The limit point between interval 30-32 and 33-35 is equal to 32.5. This is called the exact limit. This is what we want; no score can fall on 32.5, because all scores are in whole-numbers. How many intervals did we end up with? Remember we want 12 to 15. We have 13 intervals.

The next step is to tally the number of scores within each interval. Our first grouped frequency distribution will look like this:

See page 125
Grouped Frequency Distribution

<table>
<thead>
<tr>
<th>Whole-number limits</th>
<th>Midpoint</th>
<th>Tally</th>
<th>Frequency (f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>51-53</td>
<td>52</td>
<td>I</td>
<td>1</td>
</tr>
<tr>
<td>48-50</td>
<td>49</td>
<td>I</td>
<td>1</td>
</tr>
<tr>
<td>45-47</td>
<td>46</td>
<td>III</td>
<td>2</td>
</tr>
<tr>
<td>42-44</td>
<td>43</td>
<td>IIII</td>
<td>4</td>
</tr>
<tr>
<td>39-41</td>
<td>40</td>
<td>NIII</td>
<td>8</td>
</tr>
<tr>
<td>36-38</td>
<td>37</td>
<td>NIII</td>
<td>13</td>
</tr>
<tr>
<td>33-35</td>
<td>34</td>
<td>NIII</td>
<td>6</td>
</tr>
<tr>
<td>30-32</td>
<td>31</td>
<td>NIII</td>
<td>5</td>
</tr>
<tr>
<td>27-29</td>
<td>28</td>
<td>NIII</td>
<td>4</td>
</tr>
<tr>
<td>24-26</td>
<td>25</td>
<td>NIII</td>
<td>2</td>
</tr>
<tr>
<td>21-23</td>
<td>22</td>
<td>NIII</td>
<td>2</td>
</tr>
<tr>
<td>18-20</td>
<td>19</td>
<td>I</td>
<td>1</td>
</tr>
<tr>
<td>15-17</td>
<td>16</td>
<td>I</td>
<td>1</td>
</tr>
</tbody>
</table>

M. To check to see that you included all scores find the sum of the frequencies. This sum must be equal to n, the total number of scores.

N. As we did before, this grouped frequency distribution may be graphed with the score categories on the X axis and the frequencies on the Y axis. The grouped frequency polygon is easily put in the form of a bar graph or histogram. The X axis score categories are put in terms of the midpoint of each interval. This is why it's good to have intervals which have an odd numbered size such as 3, 5, 7, etc. The midpoint of the interval 15-17 is 16.0. The midpoint splits the interval into two equal halves. Had an interval size of 4 been chosen such as 15-18, the midpoint would be the point that splits the interval into the equal halves. The midpoint would be \((15+18)/2 = 33/2 = 16.5\). It is much easier to work with an interval which has a non-fractional midpoint for purposes of calculations.
and graphing.

Let's set up the X axis for our data. First determine the midpoints for each interval. The midpoint can usually be determined by visual inspection, but it can also be calculated by adding together the lower limit and upper limit and dividing by two. For example, the midpoint of the interval 33-35 is \((33+35)/2 = 68/2 = 34.0\), but you probably know that anyway. Complete the midpoint column below. Check your answers on page 61 of the Reference and Answer Manual.

<table>
<thead>
<tr>
<th>Whole-number limits</th>
<th>Midpoint</th>
</tr>
</thead>
<tbody>
<tr>
<td>51-53</td>
<td></td>
</tr>
<tr>
<td>48-50</td>
<td></td>
</tr>
<tr>
<td>45-47</td>
<td></td>
</tr>
<tr>
<td>42-44</td>
<td>46</td>
</tr>
<tr>
<td>39-41</td>
<td>43</td>
</tr>
<tr>
<td>36-38</td>
<td></td>
</tr>
<tr>
<td>33-35</td>
<td>34</td>
</tr>
<tr>
<td>30-32</td>
<td>31</td>
</tr>
<tr>
<td>27-29</td>
<td></td>
</tr>
<tr>
<td>24-26</td>
<td></td>
</tr>
<tr>
<td>21-23</td>
<td>22</td>
</tr>
<tr>
<td>18-20</td>
<td>19</td>
</tr>
<tr>
<td>15-17</td>
<td></td>
</tr>
</tbody>
</table>

Once the midpoints are found, they can be plotted along the X axis as below. Fill in your values.

```
0   19  22     31  34     43  46   X
```

0. The midpoint is taken as the score representative of all of the scores within an interval. Now we can plot the frequencies in relation to the midpoints of the score intervals. Our points for plotting are
P. Now we connect the adjacent points with a straight line. We took the midpoint of the next lower interval from the lowest interval as the point (13) where the line intersects with the X axis at the left end of the graph and we take the midpoint of the next higher interval from the highest interval as the point (55) where the line intersects with the X axis at the right end of the graph. Putting it all together does not spell MOTHER, but it does look like this:
Q. This graph could be put in the form of a bar graph or histogram. The division point between the bars is the exact limit point, which is the point which exactly separates each interval from the next interval with the midpoint in the center of the interval. Our histogram would look like this:
Note that the exact interval limit separates the lines that make up the bars. Such as the point on the X axis of 23.5. This is the exact limit separating the 21-23 interval and the 24-26 interval. Now we have a frequency polygon and a histogram for the same set of data. Both are useful graphical descriptions of the data. However, the frequency polygon is usually preferred to the histogram because it gives a better indication of the contour of the distribution. Determine a grouped frequency distribution and grouped frequency polygon for the following 60 arithmetic test scores.

92 84 82 72 76 83 75 81 95 77
68 79 82 79 80 83 58 75 63 70
76 78 75 88 76 74 69 71 83 80
70 69 86 75 61 67 72 84 74 72
68 71 79 75 85 76 90 87 73 71
82 76 80 78 75 73 81 86 77 68

1. Determine the interval size. Remember you would like 12-15 intervals.

Inclusive range = \(1 + (\text{Hi-Lo})\) = _____

Check your answer on page 61 of the Reference and Answer Manual.

\[
\text{Inclusive range} = \frac{12}{12} = __
\]

\[
\text{Inclusive range} = \frac{15}{15} = __
\]

What is a good interval size? _____ Check your answer on page 61 of the Reference and Answer Manual.

130
2. What whole-number should start the lowest interval?
   Check your answer on page 61 of the Reference and Answer Manual.

3. Set up your whole-number score intervals using the lowest whole-number which is indicated as the "correct" answer in question number 2. Also, find the midpoints of each score interval. Check your answers on page 62 of the Reference and Answer Manual.

<table>
<thead>
<tr>
<th>Whole-number Interval</th>
<th>Midpoint</th>
<th>Tallies</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. Tally the scores in the proper intervals. Place your tallies in column (3) of the table presented in question number 3. Find the frequencies for column (4). Page 63 of Reference and Answer Manual.
5. Using the midpoints on the X axis, plot the grouped frequency polygon.

Compare your frequency polygon with the one on page 64 of the Reference and Answer Manual.

Please evaluate Unit XII. on page 87 of the Reference and Answer Manual.
Unit XIII
Interval Percent, Cum. Frequency, Cum. Percent
Probability of Score Occurrence
Reference Manual Pages 65 to 68
Unit XIII. A. Interval Percent, Cumulative Frequency, and Cumulative Percent.

(Estimated time-50 minutes)

Now that you know how to set up a frequency distribution, let us look at an additional use of this distribution other than providing us with a visual description of the data.

Consider our previous frequency distribution of the 50 reading achievement scores:

<table>
<thead>
<tr>
<th>Exact Score Interval</th>
<th>Score Interval</th>
<th>Midpoint</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.5-53.5</td>
<td>51-53</td>
<td>52</td>
<td>1</td>
</tr>
<tr>
<td>47.5-50.5</td>
<td>48-50</td>
<td>49</td>
<td>1</td>
</tr>
<tr>
<td>44.5-47.5</td>
<td>45-47</td>
<td>46</td>
<td>2</td>
</tr>
<tr>
<td>41.5-44.5</td>
<td>42-44</td>
<td>43</td>
<td>4</td>
</tr>
<tr>
<td>38.5-41.5</td>
<td>39-41</td>
<td>40</td>
<td>8</td>
</tr>
<tr>
<td>35.5-38.5</td>
<td>36-38</td>
<td>37</td>
<td>13</td>
</tr>
<tr>
<td>32.5-35.5</td>
<td>33-35</td>
<td>34</td>
<td>6</td>
</tr>
<tr>
<td>29.5-32.5</td>
<td>30-32</td>
<td>31</td>
<td>5</td>
</tr>
<tr>
<td>26.5-29.5</td>
<td>27-29</td>
<td>28</td>
<td>4</td>
</tr>
<tr>
<td>23.5-26.5</td>
<td>24-26</td>
<td>25</td>
<td>2</td>
</tr>
<tr>
<td>20.5-23.5</td>
<td>21-23</td>
<td>22</td>
<td>2</td>
</tr>
<tr>
<td>17.5-20.5</td>
<td>18-20</td>
<td>19</td>
<td>1</td>
</tr>
<tr>
<td>14.5-17.5</td>
<td>15-17</td>
<td>16</td>
<td>1</td>
</tr>
</tbody>
</table>

\[ n=50 \]

A. Often, we wish to take a closer look at this distribution. One way to do this is to compare the relative percentages of each interval. To find the percent for each interval we use this formula:

\[
\% \text{ in interval} = \frac{f_\text{in interval}}{n} \times 100\%
\]

For example, the percentage for interval 42-44 would be

\[
\% \text{ in 42-44} = \frac{f_\text{in 42-44}}{n} \times 100\% = \frac{4}{50} \times 100\% = 0.08 \times 100\% = 8.0\%
\]

1. What is the percentage for the 33-35 interval?_____
2. What is the percentage for the 36-38 interval?  
   Check your answer on page 66.

Fill in your values in the following table.

<table>
<thead>
<tr>
<th>Exact Interval</th>
<th>Score Interval</th>
<th>Midpoint</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.5-53.5</td>
<td>51-53</td>
<td>52</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>47.5-50.5</td>
<td>48-50</td>
<td>49</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>44.5-47.5</td>
<td>45-47</td>
<td>46</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>41.5-44.5</td>
<td>42-44</td>
<td>43</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>38.5-41.5</td>
<td>39-41</td>
<td>40</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>35.5-38.5</td>
<td>36-38</td>
<td>37</td>
<td>13</td>
<td>--</td>
</tr>
<tr>
<td>32.5-35.5</td>
<td>33-35</td>
<td>34</td>
<td>6</td>
<td>--</td>
</tr>
<tr>
<td>29.5-32.5</td>
<td>30-32</td>
<td>31</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>26.5-29.5</td>
<td>27-29</td>
<td>28</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>23.5-26.5</td>
<td>24-26</td>
<td>25</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>20.5-23.5</td>
<td>21-23</td>
<td>22</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>17.5-20.5</td>
<td>18-20</td>
<td>19</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>14.5-17.5</td>
<td>15-17</td>
<td>16</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

\[ n = 50 \quad \% = 100\% \]

B. Of course, the sum of the percent entries should be equal to 100%. If, using your values for the two categories the sum of the percentages is not 100%, check your addition.

C. In addition to being interested in the relative frequencies and percentages within each score interval, we often are interested in the change in frequencies and percentages as we go from the lowest to the highest interval. To do this, we make addition to the frequency distribution. This is in the form of the cumulative frequency and cumulative percentage. A cumulative frequency is that frequency for an interval which is the sum of all frequencies below and including that interval. This can be found by adding the interval...
frequency to the cumulative frequency of the previous interval.

Consider our previous example:

<table>
<thead>
<tr>
<th>Exact Interval</th>
<th>Score Interval</th>
<th>Midpoint</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.5-53.5</td>
<td>51-53</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>47.5-50.5</td>
<td>48-50</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>44.5-47.5</td>
<td>45-47</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>41.5-44.5</td>
<td>42-44</td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>38.5-41.5</td>
<td>39-41</td>
<td></td>
<td>8</td>
</tr>
<tr>
<td>35.5-38.5</td>
<td>36-38</td>
<td></td>
<td>13</td>
</tr>
<tr>
<td>32.5-35.5</td>
<td>33-35</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>29.5-32.5</td>
<td>30-32</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>26.5-29.5</td>
<td>27-29</td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>23.5-26.5</td>
<td>24-26</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>20.5-23.5</td>
<td>21-23</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>17.5-20.5</td>
<td>18-20</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>14.5-17.5</td>
<td>15-17</td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

D. The cumulative frequency of the lowest interval (15-17) is equal to 1. For the next interval (18-20) the cumulative frequency is 1+1=2. For the next interval (21-23) the cumulative frequency is equal to 2 (cumulative frequency in 18-20)+2 (interval frequency) +4. The next three cumulative frequencies are: 4+2=6, 6+4=10, and 10+5=15. Calculate the cumulative frequencies for the following intervals. Check your answers on page 66 and fill them in on the following table.

1. 33-35 interval? Cf =

2. 36-38 interval? Cf =

3. 39-41 interval? Cf =

n=50
Fill in your calculated values:

<table>
<thead>
<tr>
<th>Exact Interval</th>
<th>Score Interval</th>
<th>Midpoint Frequency</th>
<th>Cumulative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.5-53.5</td>
<td>51-53</td>
<td>52</td>
<td>50</td>
</tr>
<tr>
<td>47.5-50.5</td>
<td>48-50</td>
<td>49</td>
<td>49</td>
</tr>
<tr>
<td>44.5-47.5</td>
<td>45-47</td>
<td>46</td>
<td>48</td>
</tr>
<tr>
<td>41.5-44.5</td>
<td>42-44</td>
<td>43</td>
<td>46</td>
</tr>
<tr>
<td>38.5-41.5</td>
<td>39-41</td>
<td>40</td>
<td>--</td>
</tr>
<tr>
<td>35.5-38.5</td>
<td>36-38</td>
<td>37</td>
<td>--</td>
</tr>
<tr>
<td>32.5-35.5</td>
<td>33-35</td>
<td>34</td>
<td>--</td>
</tr>
<tr>
<td>29.5-32.5</td>
<td>30-32</td>
<td>31</td>
<td>15</td>
</tr>
<tr>
<td>26.5-29.5</td>
<td>27-29</td>
<td>28</td>
<td>10</td>
</tr>
<tr>
<td>23.5-26.5</td>
<td>24-26</td>
<td>25</td>
<td>6</td>
</tr>
<tr>
<td>20.5-23.5</td>
<td>21-23</td>
<td>22</td>
<td>4</td>
</tr>
<tr>
<td>17.5-20.5</td>
<td>18-20</td>
<td>19</td>
<td>2</td>
</tr>
<tr>
<td>14.5-17.5</td>
<td>15-17</td>
<td>16</td>
<td>1</td>
</tr>
</tbody>
</table>

E. Notice that the cumulative frequency in the highest interval is equal to the total number of scores (n).

Now we find the cumulative percentages in the same way as we found the interval percentages.

Cumulative \% = \frac{\text{Cumulative frequency} \times 100\%}{n}

For example, the cumulative percentages up through and including the 18-20 interval would be:

Cumulative \% (18-20) = \frac{2 \times 100\%}{50} = 0.04 \times 100\% = 4\%

Thus, the percent of all the scores included in the 18-20 interval and all intervals below that is equal to 4\%.

The cumulative percentage up through and including the 42-44 interval is:

Cumulative \% (42-44) = \frac{46 \times 100\%}{50} = 92\%

Find the cumulative percentages for the three intervals in the previous three questions and put your an-
Fill in all the blank spaces. Check C% answers on page 66.

<table>
<thead>
<tr>
<th>Exact Interval</th>
<th>Score Interval</th>
<th>Mid.</th>
<th>f</th>
<th>%</th>
<th>Cum. f</th>
<th>Cum. %</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.5-53.5</td>
<td>51-53</td>
<td>52</td>
<td>1</td>
<td>2</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>47.5-50.5</td>
<td>48-50</td>
<td>49</td>
<td>1</td>
<td>2</td>
<td>49</td>
<td>98</td>
</tr>
<tr>
<td>44.5-47.5</td>
<td>45-47</td>
<td>46</td>
<td>2</td>
<td>4</td>
<td>48</td>
<td>96</td>
</tr>
<tr>
<td>41.5-44.5</td>
<td>42-44</td>
<td>43</td>
<td>4</td>
<td>8</td>
<td>46</td>
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<tr>
<td>38.5-41.5</td>
<td>39-41</td>
<td>40</td>
<td>8</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>35.5-38.5</td>
<td>36-38</td>
<td>37</td>
<td>13</td>
<td>24</td>
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<tr>
<td>32.5-35.5</td>
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<td>6</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>29.5-32.5</td>
<td>30-32</td>
<td>31</td>
<td>5</td>
<td>10</td>
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<tr>
<td>26.5-29.5</td>
<td>27-29</td>
<td>28</td>
<td>4</td>
<td>8</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>23.5-26.5</td>
<td>24-26</td>
<td>25</td>
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<td>4</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>20.5-23.5</td>
<td>21-23</td>
<td>22</td>
<td>2</td>
<td>4</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>17.5-20.5</td>
<td>18-20</td>
<td>19</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>14.5-17.5</td>
<td>15-17</td>
<td>16</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Using your frequency distribution of 60 arithmetic scores, here is a table which contains some of the calculations for the interval percentages, cumulative frequencies, and cumulative percentages. Find the needed values by answering questions 1, 2, and 3.

<table>
<thead>
<tr>
<th>Whole-number Interval</th>
<th>Midpoint</th>
<th>f</th>
<th>%</th>
<th>Cf</th>
<th>C%</th>
</tr>
</thead>
<tbody>
<tr>
<td>93-95</td>
<td>94</td>
<td>1</td>
<td>—</td>
<td>60</td>
<td>100.0</td>
</tr>
<tr>
<td>90-92</td>
<td>91</td>
<td>2</td>
<td>—</td>
<td></td>
<td></td>
</tr>
<tr>
<td>87-89</td>
<td>88</td>
<td>2</td>
<td>—</td>
<td>57</td>
<td>95.0</td>
</tr>
<tr>
<td>84-86</td>
<td>85</td>
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</tr>
<tr>
<td>81-83</td>
<td>82</td>
<td>8</td>
<td>13.3</td>
<td>50</td>
<td>83.3</td>
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<td>42</td>
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<td>13</td>
<td>—</td>
<td>34</td>
<td>56.7</td>
</tr>
<tr>
<td>72-74</td>
<td>73</td>
<td>7</td>
<td>11.7</td>
<td>21</td>
<td>35.0</td>
</tr>
<tr>
<td>69-71</td>
<td>70</td>
<td>7</td>
<td>11.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>66-68</td>
<td>67</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>63-65</td>
<td>64</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60-62</td>
<td>61</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>57-59</td>
<td>58</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Check your values on page 67 after completing the table.

1. Find the needed values in the \( \% \) category.

\[
\% = \frac{f}{n} \times 100\%
\]

For \( f = 1 \), \( \% = \) ____________

For \( f = 4 \), \( \% = \) ____________

For \( f = 13 \), \( \% = \) ____________

For \( f = 5 \), \( \% = \) ____________

For \( f = 2 \), \( \% = \) ____________

2. Find the needed values in the cumulative frequency category.

\[ \text{Cf} = \text{Cf up to previous interval} + f \text{ within interval} \]

\[
\text{Cf}(57-59) = \___
\]

\[
\text{Cf}(60-62) = \___
\]

\[
\text{Cf}(63-65) = \___
\]

\[
\text{Cf}(66-68) = \___
\]

\[
\text{Cf}(69-71) = \___
\]

\[
\text{Cf}(84-86) = \___
\]

\[
\text{Cf}(90-92) = \___
\]

3. Find the cumulative percentages to fill in the blanks of the table.

\[
\% = \frac{\text{Cf}}{n} \times 100\%
\]

\[
\%\text{(57-59)} = \___ \quad \%\text{(69-71)} = \___
\]

\[
\%\text{(60-62)} = \___ \quad \%\text{(84-86)} = \___
\]

\[
\%\text{(63-65)} = \___ \quad \%\text{(90-92)} = \___
\]

\[
\%\text{(66-68)} = \___
\]
Unit XIII. B. The Probability of the Occurrence of
Selected Scores.

F. Look at the grouped frequency polygon on page 128
of this Manual. Notice that 100% of the scores fall between the scores of 15 and 53.
Now consider any one of the 50 scores that went into
the frequency distribution. How sure are you that this score is included in the frequency distribution? You
should be 100% sure that it has been included. Here we get into the notion of probability.
G. A probability is in terms of a proportion (0 to
1.00) or a ratio which has a value of from 0 to 1.00.

\[ p = \frac{\text{no. of ways a favored event can occur}}{\text{total no. of ways an event can occur}} \]

This cannot be greater than 1.00. What is the probability that any of the 50 scores will be in the distribution?

\[ p = \frac{50 \text{ scores possible}}{50 \text{ scores total}} = 1.00 \]

H. We relate a percentage to a proportion to a probability.

\[ \frac{\text{percentage}}{100} = \text{proportion} = \text{probability} \]

For example, back at your frequency distribution on page 135 of this Manual.

Notice that the interval 39-41 contains 16% of the scores. This is a proportion equal to 0.16, which is the probability that anyone of the 50 scores will fall in the 39-41 interval.
probability = p = \frac{8 \text{ scores possible}}{50 \text{ scores total}} = 0.16

I. So if you randomly chose one score out of 50, the likelihood or probability that it would fall in the 39-41 interval would be 0.16 or 16 times out of 100.

What would be the probability of a score falling in the series of intervals 15-17 through 27-29? How many scores out of 50 fall in this range? Look at your cumulative frequency column and you will see that 10 out of 50 scores fall in this range. So the probability would be

p = \frac{10 \text{ scores possible}}{50 \text{ scores total}} = 0.20

So, 20 times out of 100, you would, in picking a score at random from the original 50, choose a score that would fall in the 15-17 through 27-29 range. Notice that this is the cumulative \% divided by 100.

J. What if you were interested in the probability of a score falling in the range of intervals 30-32 through 42-44? How many scores out of 50 fall in this range? You could add the individual frequencies (5+6+13+8+4=36) and find the proportion or probability as, \frac{36 \text{ scores possible}}{50 \text{ scores total}} = 0.72.

K. Or you could find the percentage of scores in that range and divide by 100. Percent up to and including interval 42-44 is 92\%. Percent up to, but not including interval 30-32 is 20\%. Subtract 20\%
from 92% and you get 72%, which is equal to a probability of 0.72. This latter way is the way we usually find a probability of a score occurring because many times the frequencies are too large to use in the first calculation.

L. Now keep in mind that the maximum probability is 1.00. In addition to considering the probability that scores will fall in a specific interval, we are also interested in the probability that they will not.

For example, we found, using our distribution of 50 scores, that in choosing a score at random from the 50, the probability that the score would fall in the 39-40 interval was

\[ p = \frac{8 \text{ scores possible}}{50 \text{ scores total}} = 0.16 \]

What then would be the probability that a randomly selected score would not be in the 39-41 interval? We label this probability with a "q" so we don't confuse it with "p". We could set it up this way,

\[ q = \frac{42 \text{ scores possible}}{50 \text{ scores total}} = 0.84 \]

M. But a much easier way to find q would be using this relationship,

\[ p + q = 1.00 \] (move p to the right side and change its sign from + to −)

\[ q = 1.00 - p \] knowing that \( p = 0.16 \), therefore \( q = 1.00 - 0.16 = 0.84 \)

This is the probability that a randomly selected
score would not fall in the 39-41 interval.

N. The probability of an event occurring plus the probability of that event not occurring is equal to 1.00. Either the event can occur or it cannot.

Let's look at another one. We found that the probability that a score in the group of 50 would fall in the range of intervals 30-32 through 42-44 was equal to 0.72. What is the probability that it would not fall in this range?

\[ q = 1.00 - p = 1.00 - 0.72 = 0.28 \]

This means 28 times out of 100 choices of one of the 50 scores, the score would not fall in the range of intervals 30-32 through 42-44.

We will use this concept again in the next section relating to the unit normal distribution.

Answer the following questions. Check your answers on page __68__ of the Reference and Answer Manual.

1. a. Looking at your 60 arithmetic score grouped frequency distribution, (see page __63__ of the Reference and Answer Manual), what is the probability of a random score, falling in the 78-80 interval?

\[ p = \frac{\text{no. scores possible}}{\text{no. scores total}} = ____ \]

b. Therefore, in randomly choosing a score from the group 60, 100 times, how many times would the score probably fall in the 78-80 inter-
2. a. What is the probability that a randomly selected score of the 60 would fall in the range 57-59 interval through 72-74 interval? \( p = \) ____

b. How many times out of 100? ____

c. What is the probability that a randomly selected score would not fall in the range 57-59 interval through 72-74 interval? \( q = \) ____

3. a. What is the probability that a randomly selected score will fall in the range 60-62 interval through 90-92 interval? \( p = \) ____

b. What is the probability that a randomly selected score will not fall in this range? \( q = \) ____

Please evaluate Unit XIII on page 88 of the Reference and Answer Manual.
Unit XIV

Unit Normal Distribution

Reference Manual pages 69 to 72
Unit XIV. Introduction to the Unit Normal Distribution
(Estimated time-45 minutes)
A. Look back at the frequency polygon of the reading achievement scores on page 128 and also the frequency polygon which you constructed on page 132. You probably noticed that these polygons, if you smoothed out the lines into curves rather than straight lines, look like a distribution that you have probably called "Bell-shaped" or "normal". The so-called normal distribution is a widely used concept in statistics. But, just what is this thing we call the normal distribution? The normal distribution is not a phenomenon of nature. The normal distribution is a mathematical model which closely approximates the distributions of many behavioral science variables and the statistics of those variables. We use it because it is useful as a model.
B. The standard normal distribution model looks like this graphically:

![Graph of the normal distribution for μ = 0 and σ = 1.](image-url)
C. Small letter z is used as the symbol of the variable which is normally distributed. The original scores are transformed mathematically into standard scores called z scores. z scores have a mean (arithmetic average) of zero (note it on the graph). This is a standard mean value. A z value is determined using the following formula, for each value of X:

\[ z = \frac{X - \bar{X}}{\sigma} \]

X is one of many scores in the data set. \( \bar{X} \) is the mean of the distribution. The quantity \( (X - \bar{X}) \) is the mean deviation, note that some of the values will be positive if \( X \) (the score) is greater than \( \bar{X} \) (the mean) and some of the values will be negative if \( X \) (the score) is less than \( \bar{X} \) (the mean). \( \sigma \) is the standard deviation, of the scores which is always positive.

D. The z score is in terms of the number of standard deviations, in either direction from the mean, that a score may fall. For example, if we had a score of 25 in a set of normally distributed scores with a mean of 20 and a standard deviation of 5, in terms of a z score this would be:

\[ z = \frac{X - \bar{X}}{\sigma} = \frac{25 - 20}{5} = \frac{5}{5} = +1.0 \]

+1 on the z scale. In other words a score of 25 is one standard deviation above the mean.

What if we had a score of 15 in a set of normally distributed scores which had a mean of 25 and a standard deviation of 5, what would be the z score?
-2.0 corresponds to the -2 on the z scale, and is said to be two standard deviations below the mean. Z scores are useful because using them permits us to compare normal distributions by transforming them all into normal distributions that have a mean of zero and a standard deviation of one.

E. Look back at the normal distribution graph and you will notice that on the Y axis we do not have frequencies in terms of whole numbers. We'll tell you why in just a moment, but first let's describe what the Y axis is in this diagram. The Y axis describes the relative height of the curve at various points along its path. The highest point of the curve is at z = 0. The value of the Y axis is 0.3989. The height of a point on the curve is called the ordinate, which you might remember is another name given to the Y axis. It probably seems strange to have the Y axis in such an abstract fashion rather than in terms of whole number frequencies. The reason for this is based on another standard which we have in relation to the normal distribution. In statistics we are quite often interested in relative areas under the normal distribution. As a convention, we set the total area under the curve of the distribution at 1.00. Thus, below the curve and above the z score axis, the area is 1.00. Therefore, the ordinate or height must be in

\[ z = \frac{X - \bar{X}}{s} = \frac{15 - 25}{5} = \frac{-10}{-2.0} \]

* page 71 of the Reference and Answer Manual
those fractional terms in order for the area under
the curve to work out to be mathematically equal to 1.00.

F. We have a name for this normal distribution, which has a mean of zero, a standard deviation of one, and an area of one. We call it the "unit normal distribution".

G. The area of one is considered a proportion or probability. If you remember that all of the scores are included in the distribution, then it follows that the probability of a score being within the distribution is 1.00.

As we did in the previous section, we are interested in relative probabilities of scores being at certain points along the X axis, which in this case, the z scale.

H. We can divide the unit normal distribution into relative areas, the sum of which must equal 100% if we divide in terms of percentages, or the sum must be equal to 1.00 if we divide in terms of probabilities.

The relative percentage values for the areas under the unit normal distribution are shown below.
I. Now these areas can be converted into probabilities by dividing by 100. Thus, 34.1% = 0.341. Notice that between -1 standard deviation and +1 standard deviation there are 68.3% of the scores. Thus, there is a probability that in a normal distribution there is about a 0.68 probability that a score will fall between -1 standard deviation and +1 standard deviation. Also, notice that between -2 standard deviations and +2 standard deviations, there are about 68% + 27% = 95% of the scores. Thus the probability is about 0.95 or about 95 times out of 100 that a score should fall between -2 standard deviations and +2 standard deviations.

Answer the following questions. Check your answers on page 72.

1. a. In relation to the areas under the normal distribution what is the percent of the area below the mean? _____

   b. What is the probability associated with this area? p = _____

2. a. What percent of the area falls between -1 and -2 z units? _____%

   b. What is the probability? p = _____

   c. This probability represents how many times out of 100 would a score probably fall in the region -1 and -2 standard deviations? _____

J. The appendix of most statistics texts has a table
of areas and ordinates of the unit normal curve, in terms of the X axis in the form of z scores (standard scores).

Here are parts of this table found in the Guilford text.* Let's look at the parts of this table in relation to the unit normal curve reference on page 71 of the Reference and Answer Manual. Notice that there are five columns in the table. We'll consider each one independently. Column (1) is the z score, the standard score or the number of standard deviations from the mean. Look at the z score of 1.00 in the table. This entry has two meanings. It can be -1.00 or +1.00. Look at these two points on your diagram. Column (2) is the value of the proportion of the total area of 1.00 under the normal curve between the mean (0) and, going either direction, one (1) standard deviation. Notice this value is 0.3413 which corresponds to 34.13% of the area under the curve. This value is the same, for both directions you go from the mean. It does not take on a negative value (can't have a negative area).

K. What if you wanted to find the value of the areas above or below the standard score point of +1.00?

When we say above a certain point, we mean to the right of it and when we say below a certain point we mean to the left of it. Look at +1 on your diagram. Notice that the larger portion of the curve falls below

*See page 71 of Reference and Answer Manual.
or to the left of the point. Therefore, the area below +1.00 is the larger portion and you look in column 3 to find the value of the area which is 0.8413.

Check this by adding 0.3413, which is above the mean to the +1.00 point, and 0.5000 which is the area to the left of the mean (0). The area above the +1.00 is the smaller portion of the curve so you look in column 4 for the area above +1.00 which is 0.1587. You could have also gotten this result by subtracting 0.8413 from 1.0000 which would be 0.1587. Remember the sum of the two areas must equal 1.00.

What if you wanted to find the areas above and below the point -1.00? Look at your diagram. The portion to the right is the largest, so looking in column 3, we conclude that 0.3413 is the area above the point, and by column 4, 0.1587 is the area in the smaller portion, or below the point -1.00. It's a good idea to draw yourself a diagram to help you make sure you get the right values from the table.

Column 5 in the table gives the value of the ordinate at a given point. At either -1.00 or +1.00 standard scores the ordinate is 0.2420. This is the point on the curve at -1.00 and +1.00 in terms of the Y axis. Look at your diagram to check this out.

Answer the following questions. Check your answers on page 72 of the Reference and Answer Manual.

Using the Unit Normal Curve Reference Sheet on page 73.
1. a. What is the proportion of the area between the mean (0) and +1.03z? _____
   b. What is the proportion of the area between the mean (0) and -1.03z? _____
   c. What proportion of the area is below +1.03z? _____
   d. What proportion of the area is below -1.03z? _____
   e. What proportion of the area is above -1.03z? _____
   f. What proportion of the area is above +1.03z? _____
   g. What is the value of the ordinate at -1.03z? _____ at +1.03z? _____

2. a. What proportion of the area is above +1.96z? _____
   b. What proportion of the area is below -1.96z? _____
   c. What proportion of the area is between -1.96z and +1.96z? _____
   d. What proportion of the area is not between -1.96z and +1.96z? _____
   e. What probability is associated with between -1.96z and +1.96z? _____
   f. How many times out of 100? _____
   g. What probability is associated with the area not between -1.96z and +1.96z? _____
M. Now let's look into how this concept is actually used in statistics. Assume we have a population of 1000 subjects who are eighth grade students in a large city school system. We give them a test on vocabulary and find a score for each of the 1000 students. We calculate a population mean. Now we randomly choose a sample of 10 students and calculate a mean (arithmetic average) for this set of 10 scores. Put these 10 subjects back in the population (this is called "sampling with replacement"). Now take another random sample of 10 subjects and again calculate the mean for these scores. Now replace these back in the population. Suppose you continually went through this process and calculated several means (all based on sample sizes of 10), and then plotted these means into a frequency distribution of sample means. As the number of sample means increases the frequency distribution of sample means (not scores) approaches a normal distribution. If we could take all of the possible samples and calculate the means and plot these means we would have what we call a "theoretical sampling distribution" of sample means. The theoretical sampling distribution is not actually calculated but we estimate what it would be if we were able to take all the possible samples of a given size.

N. Assume that this is the theoretical sampling
distribution of sample means of samples of size 10.
Assume we have converted the sample means into z scores.

Theoretical Sampling Distribution of Sample Means

\[
\begin{align*}
&\mu (\text{Greek letter } \mu) \text{ is the mean of all sample means which is the true population mean.} \\
&\text{Now remember the unit normal distribution. The total area or probability under the curve is 1.00.} \\
&0. \text{ If you now have a new sample of 10 eighth grade subject's vocabulary scores. What is the probability that its mean will fall between } -1z \text{ and } +1z \text{ from the population mean? Remember that the area between } -1z \text{ and } +1z \text{ is 0.6813, so the probability is about .68, or 68 times out of 100 that the new sample mean is in this range. What is the probability that the new sample mean will be either below } -1z \text{ or above } +1z? \text{ This probability is } 1.00 - 0.68 = 0.32, \text{ thirty-two times out of 100, the new score will fall outside of the } -1z \text{ to } +1z. \\
&\text{P, The points on the z scale of } \pm 1.96z \text{ and } \pm 2.58z
\end{align*}
\]
are of particular importance in statistical work. Look on your diagram and notice the points marked -1.96 and +1.96. Between these two points fall 95.0% of the possible sample mean value or the area 0.950. The probability that a sample mean will fall in the area below -1.96z is 0.025 and the probability that a sample will fall above +1.96z is 0.025 (see column 4 in the table at 1.96) so the probability that a sample mean will be as far as 1.96 z units in either direction of the true mean is 0.025 + 0.025 = 0.050 or 5 times out of 100. Thus, is it likely that the new sample mean would be 1.96 z units below the mean or 1.96 z units above the mean? The chances of this occurrence are 5 times out of 100. This 0.05 probability has a name that you will hear more about in the course, it is called the "0.05 level of significance".

If our new sample mean did fall in this 0.05 region, we would tend to conclude that it is different than the population in terms of vocabulary because the chance of this happening is equal to or less than 5 times in 100.

We will use these concepts in this course when we discuss hypothesis testing and the significance tests. Answer the following questions. Check your answers on page 76.

1. What proportion of the area falls between -2.58 and +2.58z units? _____

2. What proportion of the area does not fall between
-2.58 and +2.58z units? _______

3. If your new sample mean fell on the z unit axis either below the -2.58z unit or above the +2.58z unit, how many times out of 100 would like this have occurred by chance? _______

Please evaluate Unit XIV. on page 89 of the Reference and Answer Manual.
Unit: XV
Scattergram
Line of Best Fit
Reference Manual pages 73 to 76
Unit XV. Scattergram

(Estimated time-30 minutes)

A. We concerned ourselves with how we graphically describe a one variable distribution in the previous section. The other most widely used graphing technique in introductory statistics is the graphical description of two variables, either both measured on the same subject or each measured on one of two subjects that are matched in some way. Both measures are taken on many subjects, not just one. We have two scores or measures based on some common reference point, usually the same person. Often we are interested in the statistical relationship of the two measures which is called the correlation. Let's call the first variable X and the other variable Y. Assume X is a measure of intelligence and Y is a measure of college grades measured on the same subject, for several subjects. We are interested in looking at the relationship of these two variables. In other words, if there is a high intelligence test score, does there tend to be a high grade and if there is a low intelligence test score, does there tend to be a low grade? If so, this is a positive correlation. When X is high Y tends to be high, when X is low Y tends to be low. This is a direct relationship.

Consider another set of two variables. Assume X is a measure of anxiety level and Y is a measure of
manual dexterity. We obtain this pair of measure for several subjects and we find that a high score on the anxiety scale tends to be related to a low manual dexterity score, and a low anxiety level score tends to be related to a high manual dexterity score. This is an inverse relationship and we would say this is a negative correlation.

B. You must keep in mind that correlation is a mathematical relationship and not necessarily a causal relationship. X does not have to cause Y, in order for these to be a significant correlation.

We have many ways of calculating a correlation coefficient which gives us a mathematical measure of the relationship between X and Y. However, many times it is useful to graphically look at the distribution of both X and Y, both taken for a group of subjects. We do this in the form of a scatter-gram and we use the Cartesian coordinate system to plot each pair of X and Y scores for several subjects.

C. For your review, here is the Cartesian coordinate system as we used before. (See page 75 of the Reference and Answer Manual.) In this situation, the Y scale becomes a score measure rather than a frequency count as it was in the frequency polygon discussed before. The frequencies are now in terms of the number of points plotted on the scatter-gram.

You may remember how to plot pairs of scores, but
let's briefly review this.

Here are four pairs of scores:

<table>
<thead>
<tr>
<th>Pair No.</th>
<th>Score on X</th>
<th>Score on Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>+3</td>
<td>+4</td>
</tr>
<tr>
<td>2.</td>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>3.</td>
<td>-1</td>
<td>+3</td>
</tr>
<tr>
<td>4.</td>
<td>+5</td>
<td>-2.5</td>
</tr>
</tbody>
</table>

Let's take each pair and plot it on the graph.

Consider pair number 1, \( X = +3, \ Y = +4 \). Move on the \( X \) axis to +3 and then up the \( Y \) axis to +4. The point which meets both of these criteria is the point to be plotted.

See below:
Pair number 2 is \(X=0\) and \(Y=-2\). The point corresponds to the \(X\) point equal to 0 which is the \(Y\) axis itself and the \(Y\) value is -2 which is below the \(X\) axis. See below:

Pair number 3 is \(X=-1\) and \(Y=+3\). This is plotted as below.
Pair number 4 is $X=5$, $Y=-2.5$, which would be plotted as below:

Let's now take a small set of data and plot the points:

<table>
<thead>
<tr>
<th>Pair number</th>
<th>$X$</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>2.5</td>
</tr>
<tr>
<td>5</td>
<td>-7</td>
<td>-2.5</td>
</tr>
<tr>
<td>6</td>
<td>-4</td>
<td>-1</td>
</tr>
</tbody>
</table>
Plot the points on the following graph. Draw a line to connect the points. Compare your graph with the one on page 76 of the Reference and Answer Manual.

D. The Equation for a Straight Line

When you place a ruler on the points you noticed that they all fell on a straight (linear) line and you drew a line to connect all of the points.

E. This line we have drawn is called "the line of best fit". It is the straight line which best represents the points relative to each other. We can describe this line using a mathematical equation. A
straight line equation has the formula of \( Y = aX + b \).

P. Now let's look at the parts of this equation. "Y" is the score on the Y scale corresponding to a given value of X which is the value of X in the part "aX".

G. "a" is what we call the slope of the line of best fit. The slope is a value that describes the angle of the line respective to the X and Y axes, and is given in terms of the ratio of the number of units of use or decrease in Y for each unit increase in X. The reference you must use is the increase in one unit of X, such as the increase from X=0 to X=+1. How many units does Y increase or decrease? The value of Y goes from +1 at X=0 to +1.5 at X=+1, which is an increase of 0.5, so the slope or the value of "a" is +0.5. Had there been a decrease in the value of Y as X increased one unit, the slope would have been a negative value. "b" is called the Y intercept, which is the point on Y that the line of best fit crosses. "b" in our example is equal to +1.0.

So, our equation for the line of best fit in our example is:

\[
Y = aX + b
\]

\[
Y = (+.5)(X) + 1.0
\]

Now using this equation, we can predict a value of Y, if we know a value of X. What is the value of Y is \( X = +2.0 \)?

\[
Y = (+.5)(2) + 1.0 = 1.0 + 1.0 = 2.0
\]
Look on your diagram and see if this is correct.

When $X=2$, does $Y=2$?

Using the equation $Y = (+.5) \times X + 1.0$,

1. What is the predicted value of $Y$ if $X = -5$
   
   $Y$ predicted =

2. What is the predicted value of $Y$ if $X = 15$
   
   $Y$ predicted =

Check your answers on page 76.

H. In the previous examples, we were able to identify a straight line that connected all the points on a scatter-gram. This is an idealized situation. In a scatter-gram using behavioral data, this will probably not happen. We'll not be able to connect all the points with one linear line. However, we look for a line that best represents the pattern of points on a graph and we call this line "the line of best fit". We sometimes call this line the "least squares line" or the "regression line". All three mean the same thing.

I. The equation which represents this line will be $Y = aX + b$, where $a$ is the slope of the line and $b$ is the $Y$ intercept of the line. The primary reason we are not able to connect all the points are usually because of errors of measurement due to lack of precision of a measuring instrument and random errors associated with subjects who are measured.

Let's take a brief example and follow it through the process of determining correlation and then prediction.
Here are scores for 10 college students. X scores are scores on the ACT test which students take prior to college enrollment and Y scores are the student's grade-point average at the end of the freshman year.

<table>
<thead>
<tr>
<th>Subject Number</th>
<th>ACT Score</th>
<th>G.P.A. for Freshman Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>1.50</td>
</tr>
<tr>
<td>2</td>
<td>27</td>
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<td>10</td>
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Plotting the points on the graph will look like the graph below. Check a few of the points to make sure you understand how they are plotted. If you have trouble refer to the discussion labeled paragraph C, page 160.
J. Looking at these points, you should see that there seems to be some relationship or correlation between the measures of these two variables. A high ACT score tends to be related to a high G.P.A. and a low ACT score tends to be related to a low G.P.A. This is a direct, thus positive relationship. We calculate a correlation coefficient to describe this relationship. At this point, we won't go into the mathematics of the calculation, but assume you did and found that you got a correlation coefficient of 0.60. The possible range of this correlation coefficient must be between -1 and +1.

K. If the value of the correlation coefficient, which we symbolize as "r" is greater than 0 or positive, we have a direct relationship (high X-high Y), if "r" is
less than 0, we have an inverse relationship (high $X$, low $Y$). If there is no relationship, then $r$ will be zero or very close to zero. "$r$" provides an index of the relative departure of the scores from the theoretical line of best fit.

L. You could look at the scores on the graph and imagine a line of best fit, but we have ways of calculating the values of "a" (slope) and "b" (Y-intercept) which will give us an equation ($Y = aX + b$) which we use to draw the line. Assume we calculate the value of "a" and find it is equal to +0.081. We also calculate the value of b and find it is equal to 0.861. Now substitute the calculated values in our basic equation.

Basic equation $Y = aX + b$

Our line of best fit equation $Y = 0.081X + 0.861$

M. Using this equation, we now determine two new points to plot that allow us to draw our line of best fit (remember from geometry that two points define a straight line). A value of $Y$ will be given if we enter an arbitrary value of $X$ into the equation. The value of $X$ we choose should be one that appears on our graph for ease of plotting.

Say we choose our first value of $X$ equal to 8.

What is the value of $Y$ when $X=8$?

$Y = 0.081X + 0.861$

$Y = 0.081(8) + 0.861$

$Y = 0.648 + 0.861 = 1.509$
Now we choose a second value of X to determine a second point on our line. It's best to choose a value on the other extreme of the X scale. Let's take X=32. What is the value of Y when X=32?

\[ Y = 0.081X + 0.861 \]

\[ Y = 0.081 (32) + 0.861 \]

\[ Y = 2.592 + 0.861 = 3.453 \]

Now we have two points to plot that determine the line of best fit. They are (8, 1.51) and (32, 3.45). Plotting these two points and drawing a line to connect them and extending this line beyond the points will look like this:
This is our line of best fit or our regression line. This line has a slope of +0.081 and a Y intercept of 0.861. Notice that the line of best fit intersects the Y axis at the value of the Y intercept. That should not seem strange to you.

Now let's put the line of best fit on the original scatter-gram to see if it looks like a good line.

In summary, we have plotted two variables on a scatter-gram. We have determined their relationship by calculating a correlation coefficient. We have determined an equation for a line of best fit which
is of the form $Y=aX+b$.

Q. Now let's consider a very important use of this information. One of the functions of statistics is to make predictions. The equation which we determine does permit us to make predictions. When we make predictions based on such an equation we call the process regression analysis. The line of best fit is also called the regression line. This line has certain properties that will be discussed in greater depth in the course.

R. Assume you are a college counselor and you are directing a program to help entering college students who are academically ill-prepared for college. One thing you would have to do is identify students who fall in this category. How can you predict beforehand what grade-point average a student will probably have at the end of his freshman year? First, you need to determine a measure that you can take prior to college enrollment that is correlated highly (either positive or negative) with freshman grade-point average. You determine that the ACT test does correlate highly, so you select a group of last year's freshman students and find out how their ACT scores were correlated with their grade-point averages. Next you determine the line of best fit or the regression line equation. This equation is the key to prediction. You assume that this equation will be valid for this year's group of students, even though it is based on last year's fresh-
man group. In our previous example, we determined our regression equation to be:

\[ Y = 0.081 \times +0.861 \]

(Usually this would be based on many more students than 10, but for ease of explanation, we will use only 10 pairs of score.) As the counselor, you decide that all students who have predicted grade-point averages of 2.10 or less should be offered assistance through your program. Therefore, at what ACT score, and below that score, would you predict a student would have a 2.10 or less at the end of his freshman year?

Predicted G.P.A. = ACT score times slope plus Y intercept

\[ Y = \text{ACT score times } 0.081 \text{ plus } 0.861 \]

The Y cut-off value is 2.10, the unknown is ACT score. We manipulate this equation to have ACT score alone on one side of the equation.

\[ Y = aX + b \quad \text{subtract } b \text{ from} \]
\[ Y - b = aX + b - b \quad \text{both sides; divide} \]
\[ \frac{Y - b}{a} = X \]

Therefore, \( X = \frac{Y - b}{a} \) = G.P.A. -Y intercept

\[ \frac{X}{a} \text{ slope} \]

\[ X = \frac{2.10 - 0.861}{0.081} = 1.239 = 15.3 \text{ ACT} \]

Conclusion, any entering freshman student who scores at 15 or less composite ACT score would be considered for assistance in the program. Actual ACT scores are not functional.)
We often use the previous regression analysis for another purpose. Assume you are working in financial aids and you have scholarships for students who demonstrate the most academic promise. You could use this same equation and find the ACT score that predicts that a student will have a 3.00 or above grade-point average at the end of the freshman year. What ACT score would that be? Calculate this value and check your answer on page _____.

ACT score which predicts G.P.A. of 3.00+ = __________

T. You must keep in mind that these are predictions. There is bound to be some error or inaccuracy. Sometimes the prediction will be too high, sometimes it will be too low. However, it will be the best or most probably estimate in terms of the data that you have.

Please evaluate Unit XV, on page 90 of the Reference and Answer Manual.
Appendix C

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Unit II. Overview of Introductory Behavioral Research Statistics

Reference Information

Vocabulary:

1. **Central tendency measures** - one score which best describes the set of scores; could be the mode and/or median and/or mean.

2. **Correlation coefficient** - a numerical index of the relationship between two variables.

3. **Descriptive statistic** - a statistic which describes the characteristics of a set of data; includes measures of central tendency and variability.

4. **Frequency** - the number of times a score occurs in a set of data.

5. **Frequency distribution** - a table which indicates the frequencies of occurrence of scores on one variable in a set of data.

6. **Frequency polygon** - a graph which describes the frequency distribution of a set of scores. Scores plotted on the X axis and frequencies plotted on the Y axis.

7. **Inferential statistic** - a statistic which is used to infer characteristics of a population based on a set of sample data; use of significance tests such as t test, chi-square, F test.

8. **Mean** - the arithmetic average; the sum of scores in a set of data divided by the number of scores
in the set of data.

9. **Median** - the point which separates a set of scores into two groups; one group contains the low 50% of the scores and the other group contains the high 50% of the scores.

10. **Mode** - the most frequently occurring score in a set of data.

11. **Parameter** - a numerical descriptor of the population; usually estimated by a sample statistic.

12. **Population** - a group of subjects having the same characteristics identified by the researcher.

13. **Range** - (inclusive) a measure of variability; the interval which contains all of the scores in a set of data; equal to (high score - low score) + 1.

14. **Scattergram** - a graph which plots two variables simultaneously in terms of frequencies.

15. **Standard deviation** - a measure of variability of a set of scores from the mean of the set of scores.

16. **Statistic** - a numerical descriptor of a set of data from a sample.

17. **Variability** - a measure of the spread of scores; includes range and standard deviation.
Unit II. Post-test Answers

1. a. The answer is b. They are what we call descriptive statistics, measures of central tendency, not measures of description. If you need additional clarification, re-read paragraph A., page 6.

   b. This is correct, measures of central tendency.

   c. The answer is b. They are what we call measures of central tendency. If you need additional clarification, re-read paragraph A., page 6.

2. a. The answer is c. An inferential statistic is a significance test. For further clarification see paragraph I., page 10.

   b. A statistic is based on sample data, not population data, see paragraph I., page 10.

   c. This is the correct answer, a parameter.

3. a. This is the correct answer, a scatter-gram.

   b. The correct answer is a. A frequency polygon is used for a one variable description. See paragraph A., page 6, then paragraph C., page 8.

   c. The answer is a. A correlation coefficient is a numerical description of the relationship between two variables. See paragraph C., page 8.

4. a. The answer is c. The standard deviation may describe the variability of data.
but not the population itself. See paragraph B., page 7.

b. The answer is c. The standard deviation may describe the variability of sample data, but not the sample itself. See paragraph B., page 7.

c. This is the correct answer. The standard deviation is a descriptive statistic which describes either sample or population data variability.

5. a. The answer is b. We often estimate a parameter from a statistic. See paragraphs I. and J., page 10.

b. This is the correct answer, a population parameter from a sample statistic.

c. The answer is b. See paragraphs I. and J., page 10.

6. a. This is the correct answer, inferential statistics.

b. The answer is a. See paragraph D., page 8.

c. The answer is a. See paragraph D., page 8.

7. a. The answer is c. We choose a sample many times for the convenience of not having to measure the entire population. See paragraph G., page 9.

b. The answer is c. It is possible for a random sample to be non-representative of the population. See paragraph G., page 9.
c. This is the correct answer. We want a representative sample and the greatest probability of getting a representative sample is in choosing a random sample.

Please evaluate Unit II. on page 77 before continuing to Unit III.
Unit III. Four Basic Measurement Scales

Reference Information

Measurement Scales:

1. **Nominal** - a label, identification, no quantitative value.
2. **Ordinal** - ability to rank scores, but no equal scale of measurement.
3. **Interval** - ability to rank scores plus equal interval scale of measurement, an arbitrary zero point.
4. **Ratio** - all of interval properties plus an absolute zero point.
Unit III. Post-test Answers

1. **Nominal** is the answer. Zip-code numbers are labels and do not measure anything. See paragraph C., page 14.

2. **Ordinal** is the answer. We are measuring spelling ability. Getting none of the words correct is not an indication of "no spelling ability". We are able to rank them from low to high. Different words would carry different weights in terms of actual spelling ability. Thus, we cannot say that we have equal intervals between measures. See paragraph D., page 15.

3. **Ratio** is the answer. We are measuring time starting from a natural zero point, the start of the stop-watch. We can say that 20 seconds is twice as much time as 10 seconds. See paragraph F., page 17.

4. **Interval** is the answer. We are measuring time, but not from a natural zero point. We can't define the year 0 in terms of absence of time. The year 0 is an arbitrary zero-point. There was time before calendar year 0. However, we do have equal intervals. We can say that there is the same amount of calendar time between 1700 and 1800, as there is between 1800 and 1900. See paragraph E., page 16.

Please evaluate Unit III, on page 78 before continuing to Unit IV.
Unit IV. Arithmetic Definitions

Reference Information

( ) - Parentheses

[ ] - Brackets

\[ \sum \] - Summation notation

\( x^a \) - "a" an exponent or power

\( a<b \) - a less than b

\( a\leq b \) - a less than or equal to b

\( a>b \) - a greater than b

\( a\geq b \) - a greater than or equal to b

\( a\neq b \) - a not equal to b

Non-decimal place - left of the decimal point values

Decimal place - right of the decimal point values
Unit IV. Post-test Answers

1. a. The answer is b. A quotient is the result of a division operation. See paragraph C., page 21.
   b. This is the correct answer. The result of a multiplication operation is a product.
   c. The answer is b. A square is the result of taking a number times itself. See paragraph C., page 21.

2. a. This is the correct answer. The top value is the numerator. It is also called the dividend.
   b. The answer is a. The divisor is another name for the denominator which is the bottom value. See paragraph D., page 21.
   c. The answer is a. The denominator is the bottom value. See paragraph D., page 21.

3. a. The answer is c. Remember that the greatest value is at the open end of the symbol. See paragraph G., page 22.
   b. The answer is c. If this were the case then the symbol would be >. See paragraph G., page 22.
   c. This is the correct answer. "c is greater than d."

4. a. This is the correct answer. \(10^4 = 10 \times 10 \times 10 \times 10\), ten to the fourth power. The exponent is 4.
b. The answer is a. See paragraph F., page 22.
c. The answer is a. See paragraph F., page 22.

5. a. The answer is c. See paragraph D., page 21.
b. The answer is c. See paragraph D., page 21.
c. This is the correct answer.

\[
\frac{\text{numerator}}{\text{denominator}} = \frac{\text{quotient}}{\text{divisor}}
\]

6. a. The answer is b. See paragraph F., page 22.
b. This is the correct answer. \(\Sigma\) is the summation notation.
c. The answer is b. The square-root symbol is \(\sqrt{\cdot}\). See paragraph F., page 22.

7. a. This is the correct answer. 63 in 63.125 are non-decimal place numbers.
b. The correct answer is a. The decimal place numbers are 125 in 63.125. See paragraph H., page 23.
c. The correct answer is a. See paragraph H., page 23.

8. a. The answer is b. Absolute values are treated as though they were positive. See paragraph I., page 23.
b. This is the correct answer. All are positive.
c. The answer is b. Absolute values are treated as though they were positive. See paragraph I., page 23.

Please evaluate Unit IV. on page 79 before continuing to Unit V.
Unit V. Introduction to Summation Notation

Reference Information

\[ \sum_{i=1}^{n} x_i \]

- starting point for summation operation of X scores; in this case \( x_1 \)

- finishing point for summation operation; in this case all of the scores

\[ \Sigma \] - Summation operation symbol

\[ \Sigma X \] - Sum of the scores on the X variable.

\[ (\Sigma X)^2 \] - Sum of the scores, the quantity squared.

\[ \Sigma X^2 \] - Sum of the scores after each is squared.

\[ \Sigma XY \] - Sum of the products of the X variable times the Y variable for each subject.
Unit V. Post-test Answers

Answers Set I

1. a. This is the answer $\sum_{i=1}^{n} X_i$, add all scores $X_i$ from $X_1$ through $X_n$, which is $X_{12}$.

b. The answer is a. In this choice, the 12 and the 1 have been reversed. See paragraphs C, D, and E., page 26.

c. The answer is a. In this choice a., 1 has been placed where the "i" should be in the subscript of $X$. $\sum_{i=1}^{n} X_i$ See paragraphs C, D, and E., page 26.

2. a. The answer is C. If you got 43, you probably added all the 12 scores rather than only $X_{12}$ through $X_4$. See paragraph I., page 29.

b. The answer is c. If you got 12, you probably added only $X_1$ through $X_7$. You must include the last $X$ value indicated by the index on top of the summation sign. See paragraph I., page 29.

c. This is correct, $\sum_{i=1}^{n} X_i = 4 + 4 + 6 + 5 = 17$.

3. a. The answer is b. If you got 2, you probably took only $X_1$. Remember if the symbol is not complete you assume we start with $X_1$ and end with $X_n$ which is $X_{12}$. See paragraph K., page 28.

b. This is the answer, $\sum_{i=1}^{n} X_i = X_1 + X_2 + \ldots + X_{12} = 2 + 4 + \ldots + 5 = 43$. 

12
c. The answer is b. If you got 1849, you probably squared 43. See paragraph H., page 28.

4. a. The answer is b. If you got 185, you found \( \Sigma X^2 \) not \((\Sigma X)^2\). Find the value inside parentheses before you do an outside operation. See paragraphs K. and L., page 30.

b. This is the correct answer, \((\Sigma X)^2 = (43)^2 = 1849\).

c. The answer is b. If you get 43, you found the \(\Sigma X\) and forgot to square it. See paragraph K., page 30.

The answer is c. You found the value of \((\Sigma X)^2\) not \(\Sigma X^2\). See paragraphs K. and L., page 30.

c. This is the correct answer, \(\Sigma X^2 = X_1^2 + X_2^2 + \cdots + X_{12}^2 = 2^2 + 4^2 + \cdots + 5^2 = 4 + 16 + \cdots + 25 = 185\). Square first and then add.

6. a. This is the answer. First find \(\sum_{i=7}^{9} X_i\) and then square the result, \(\sum_{i=7}^{9} X_i = 11, \ (\sum_{i=7}^{9} X_i)^2 = (11)^2 = 121\).

b. The answer is a. You found \(\sum_{i=7}^{9} X_i\), but you forgot to square it. \((11)^2 = 121\). See paragraph K. page 30.

c. The answer is a. You have found \(\sum_{i=7}^{9} X_i^2\) not \(\sum_{i=7}^{9} (X_i)^2\). See paragraphs K. and L., page 30.
Answers set II.

7. a. This is the correct answer. \((EX)^2 = (21)^2 = 441\).
   
b. The answer is a. You found \(EX\), but forgot to square it. See paragraph K., page 30.
   
c. The answer is a. You found \(EX^2\) rather than \((EX)^2\). See paragraphs K. and L. page 30.

8. a. This is the correct answer. \(EXY\) is the sum of the cross-products of each subjects \(X\) and \(Y\) scores. \(EXY = (6)(5)+(8)(2)+(4)(7)+(3)(9) = 101\).
   
b. The answer is a. If you got 44, you have probably found \(EX+EXY\) which is not the same as \(EXY\). See paragraph N., page 31.
   
c. The answer is a. If you got 483, you have probably found \((EX)(SY)\) which is not the same as \(EXY\). Remember the order of operations rules. Multiplication has precedence over addition in \(EXY\). See paragraph N., page 31.

9. a. The correct answer is b. You have found \(EX+SY\) not \((EX)(SY)\). Work within parentheses first.
   
b. This is the correct answer. \(EX = 21\) and \(SY = 23\), so \((EX)(SY) = (21)(23) = 483\).
   
c. The correct answer is b. You have found \(EXY\) rather than \((EX)(SY)\). Work within parentheses first.

Please evaluate Unit V. on page 80 before continuing to Unit VI.
Unit VI. Arithmetic Operations with Signs

Reference Information

**Addition** - add + scores, add - scores, give the difference the sign of the highest value.

**Subtraction** - change the sign of the number being subtracted and add, giving sign of the highest value.

**Multiplication** - if odd-number of negative signs, product is negative, if even-number of negative signs, product is positive.

**Division** - if both numerator and denominator are the same sign, quotient is positive; if one is positive and the other negative, quotient is negative.

**Square** - the square of a number is positive.

**Square-root** - can be positive or negative - in statistics we always use the positive square-root.

**Reciprocal** - a reciprocal is a number divided into 1 - can be positive or negative.
Unit VI. Definitions used in this unit relating to statistics:

\[ x = X - \bar{X} \]  

the mean deviation of a score

Where:

\( X \) is a given or individual score

\( \bar{X} \) is the mean (average) of all the scores

Each score in a set of data has a mean deviation, which is its difference from the mean of all the scores. A mean deviation can be positive or negative. The sum of all mean deviations (\( \Sigma x \)) is always equal to zero. The square of a mean deviation is always positive. The sum of the squares of each mean deviation is not equal to zero. \( \Sigma x^2 \neq 0 \)
Unit VI. Pro-test Answers

1. a. +35  
b. -24  
c. -14  
d. -6  
e. -0.30

2. a. +5  
b. -15  
c. -25  
d. 0  
e. -2.59

3. a. +42  
b. -78  
c. +175  
d. -90  
e. +24  
f. -5.30

4. a. 2.4 or 2 2/5  
b. -5  
c. -9  
d. +6.0  
e. +3

5. a. +49  
b. +169  
c. +2.25  
d. +0.36  
e. +0.81

6. a. +5  
b. +12  
c. +0.80

7. a. ½ or 0.5  
b. -½ or -0.25  
c. -1/25 or -0.04
Unit VI. Answers to addition problems.

1. \((-\cdot)\ (+)\)
\[ 4 - 3 = -1 \]

2. \((-\cdot)\ (+)\)
\[ 6 - 1 = -5 \]

3. \((+\cdot)\ (-\cdot)\)
\[ 13 - 6 = +7 \]

4. \((-\cdot)\ (+)\)
\[ 22 - 15 = -7 \]

Unit VI. Answers to subtraction problems.

1. \(-3-\cdot(+2) = -3 + (-2) = -5\)

2. \(13-(-3) = 13 + 3 = 16\)

3. \(-24-(-12) = -24 + 12 = -12\)

4. \(x = 6.3 - 6.6 = -0.3\)
   \(x = 8.6 - 6.6 = +2.0\)
   \(x = 4.9 - 6.6 = -1.7\)

Sum of the three deviations is equal to 0.

Unit VI. Answers to multiplication problems.

1. \((-\cdot)\ x\ (-\cdot)\ = +24\)

2. \((-\cdot)\ x\ 6\ = -30\)

3. \((-\cdot)\ x\ 4\ x\ (-\cdot)\ = +24\)

4. \(6\ x\ (-\cdot)\ x\ 2\ = -60\)

5. \((-\cdot)\ x\ (-\cdot)\ x\ (-\cdot)\ = -60\)

Unit VI. Answers to division problems.

1. \(12\div\ (-\cdot)\ = -6\)

2. \(-35\div\ (-\cdot)\ = +5\)

3. \((-\cdot)\ x\ 4\ /\ 3\ = -24/3 = -8\)

4. \(-16/0.5 = +32\)
Unit VI. Answers for squaring problems.

1. 81
2. 0.16
3. 0.64
4. 1.44

Unit VI. Answers for square-root problems.

1. 10
2. 0.5
3. 50
4. 20

Unit VI. Answers to reciprocal problems.

1. 1/3 or 0.333
2. 1/.2 = 5
3. -1/10
4. -1/.25 = -4

Please complete the evaluation for Unit VI. on page 81 before continuing to Unit VII.
Unit VII. Arithmetic Operations with Fractions

Reference Information

Addition of fractions
\[ \frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd} \]

Subtraction of fractions
\[ \frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd} \]

Multiplication of fractions
\[ \frac{a}{b} \times \frac{e}{f} = \frac{ae}{bf} \]

Division of fractions
\[ \frac{a}{b} \div \frac{e}{f} = \frac{a}{b} \times \frac{f}{e} = \frac{af}{be} \]

Squaring of fractions
\[ \left( \frac{a}{b} \right)^2 = \frac{a^2}{b^2} \]

Finding Square-root of fractions
\[ \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \]
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. a. $\frac{11}{10} = 1 \frac{1}{10}$</td>
<td>4. a. $\frac{42}{21} = 2$</td>
<td></td>
</tr>
<tr>
<td>b. $\frac{25}{100} = \frac{1}{4}$</td>
<td>b. $\frac{5}{24}$</td>
<td></td>
</tr>
<tr>
<td>c. $\frac{59}{40} = 1 \frac{19}{40}$</td>
<td>c. $-\frac{32}{9} = -3 \frac{5}{9}$</td>
<td></td>
</tr>
<tr>
<td>2. a. $\frac{6}{8} = \frac{3}{4}$</td>
<td>5. a. $\frac{1}{144}$</td>
<td></td>
</tr>
<tr>
<td>b. $\frac{1}{14}$</td>
<td>b. $\frac{16}{25}$</td>
<td></td>
</tr>
<tr>
<td>c. $-\frac{11}{40}$</td>
<td>c. $\frac{36}{49}$</td>
<td></td>
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<tr>
<td>3. a. $\frac{1}{40}$</td>
<td>6. a. $\frac{1}{2}$</td>
<td></td>
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<tr>
<td>b. $\frac{6}{28} = \frac{3}{14}$</td>
<td>b. $\frac{3}{7}$</td>
<td></td>
</tr>
<tr>
<td>c. $-\frac{15}{45} = -\frac{1}{3}$</td>
<td></td>
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</tbody>
</table>
Unit VII. Answers to addition of fractions problems.

1. a. The answer is b. If you got $35/64$, you multiplied the numerators and divided the product by the product of the denominators. See b.

   b. This is the correct answer. Since both have the same denominator,
   \[
   \frac{7}{8} + \frac{5}{8} = \frac{7+5}{8} = \frac{12}{8} = 1 \frac{1}{2}
   \]

   c. The answer is b. If you got $12/16 = 3/4$, you added the numerators and divided by the sum of the denominators. See b, and paragraph A, page 51.

2. a. The answer is c. If you got $4/7$, you added the numerators and divided by the sum of the denominators. See c, and paragraph A, page 51.

   b. The answer is c. If you got $4/12 = 1/3$, you probably correctly found the common denominator but added the numerators before adjusting them for the change in denominator. See c, and paragraph A, page 51.

   c. This is the correct answer. We must find the common denominator and then adjust the numerators.
   \[
   \frac{3}{4} + \frac{1}{3} = \frac{9+4}{12} = \frac{13}{12} = 1 \frac{1}{2}
   \]

3. a. This is the answer. Find the common denominator and adjust the numerators.
   \[
   \frac{7}{9} + \frac{3}{8} = \frac{56+27}{72} = \frac{83}{72} = 1 \frac{11}{72}
   \]
b. The answer is a. If you got 21/72, you found the common denominator, but did not adjust the numerators. See a, and paragraph A, page 51.

c. The answer is a. If you got 10/17, you did not find the common denominator and then adjust the numerators. See a, and paragraph A, page 51.

Unit VII. Answers to subtraction of fractions problems.

1. a. This is the correct answer. The denominators are already common.

\[
\frac{7}{8} - \frac{4}{8} = \frac{7-4}{8} = \frac{3}{8}
\]

b. The answer is a. If you chose 7/12, you probably found 28/64 by multiplying the numerators and dividing by the product of the denominators. See a, and paragraph B, page 52.

c. The answer is a. If you got \(-\frac{3}{8}\), you probably did this:

\[
\frac{7}{8} - \frac{4}{8} = \frac{32-56}{64} = \frac{-24}{64} = -\frac{3}{8} \text{ (wrong)}
\]

You have the numerators in the wrong orders. See a, and paragraph B, page 52.

2. a. This is the correct answer. Find the common denominator and subtract, in proper order, the adjusted numerators.

\[
\frac{3}{4} - \frac{1}{11} = \frac{33-4}{44} = \frac{29}{44}
\]

b. The answer is a. If you chose 3/44, you found the correct common denominator, but did not
adjust the numerators. See a, and paragraph B, page 52.

c. The answer is a. If you chose -29/44, you found the correct common denominator but had the adjusted numerator in reverse order. You did this:

\[
\frac{3}{4} - \frac{1}{11} = \frac{4-33}{44} = \frac{-29}{44} \quad \text{(wrong)}
\]

3. a. This is the correct answer. Find the common denominator and subtract in the proper order.

\[
\frac{3}{8} - \left(\frac{-11}{16}\right) = \frac{48-(-88)}{128} = \frac{48+88}{128} = \frac{136}{128} = \frac{1}{16}
\]

b. The answer is a. If you got -5/8, you did everything correct except change the sign of the final operation. You found

\[
\frac{3}{8} - \left(\frac{-11}{16}\right) = \frac{48-(-88)}{128} = \frac{-40}{128} = \frac{-5}{16} \quad \text{(wrong)}
\]

See a, and paragraph B, page 52.

c. The answer is a. If you got -1 1/16, you did the operations correctly but gave your result the wrong sign. See a, and paragraph B, page 52.

Unit VII. Answers to multiplication of fractions problems.

1. a. The answer is c. If you got 7/9, you added the fractions rather than multiplied. See c, and paragraph C, page 54.

b. The answer is c. If you got 1 1/6, you multi-
plied the numerators but not the denominator.
   See c, and paragraph C, page 54.

c. This is the correct answer. Multiply the numerators and divide by the product of the denominators.
   \[
   \frac{2}{9} \times \frac{5}{9} = \frac{10}{81}
   \]

2. a. The answer is b. If you chose \( \frac{92}{270} \), you probably did this:
   \[
   \frac{23}{90} \times \frac{3}{4} = \frac{92}{270} \quad \text{(wrong)}
   \]
   See b, and paragraph C, page 54.

b. This is the correct answer. Multiply numerators and divide by the product of the denominators:
   \[
   \frac{23}{90} \times \frac{3}{4} = \frac{69}{360} = \frac{23}{120}
   \]

c. The answer is b. If you got \( \frac{11}{23} \), you probably did this:
   \[
   \frac{23}{90} \times \frac{3}{4} = \frac{270}{80} = \frac{23}{23} = 3 \frac{11}{23} \quad \text{(wrong)}
   \]
   See b, and paragraph C, page 54.

3. a. The answer is c. If you got \( \frac{6}{65} \), you did everything correctly except remember that a positive times a negative is a negative. See c.

b. The answer is c. If you got \( -\frac{13}{30} \), you cross-multiplied when you should not have.
   \[
   \frac{6}{13} \times \left( \frac{-1}{5} \right) = -\frac{13}{50} \quad \text{(wrong)}
   \]
   See c, and paragraph C, page 54.
a. This is the correct answer.

\[ \frac{6}{13} \times \left( -\frac{1}{5} \right) = -\frac{6}{65} \]

4. a. The answer is b. If you got -5/51, you forgot that a negative times a negative is a positive. See b.

b. This is the correct answer.

\[ \left( -\frac{1}{3} \right) \times \left( -\frac{5}{17} \right) = +\frac{5}{51} \]

c. The answer is b. If you got 15/17, you cross-multiplied when you shouldn't have. See b, and paragraph C, page 54.

Unit VII. Answers to division of fractions problems.

1. a. The answer is c. If you chose 2 2/5, you probably did this:

\[ \frac{4}{5} + 3 = \frac{12}{5} = 2 \frac{2}{5} \quad \text{(wrong)} \]

You did not invert the divisor 3/1 to make it 1/3 and then multiply. See c, and paragraph D, page 55.

b. The answer is c. If you got 3 3/4, you probably inverted 4/5 rather than 3/1. See c, and paragraph D, page 55.

c. This is the correct answer. Invert the divisor, then multiply.

\[ \frac{4}{5} \div \frac{3}{1} = \frac{4}{5} \times \frac{1}{3} = \frac{4}{15} \]

2. a. This is the answer. Invert the divisor, then multiply.

\[ \frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1} = \frac{4}{2} = 2 \]

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b. The correct answer is a. If you got 1/2, you probably inverted 1/2 rather than 1/4. See a, and paragraph D, page 55.

c. The answer is a. If you chose 1/8, you probably multiplied before inverting the divisor. See a, and paragraph D, page 55.

3. a. The answer is c. If you got 15/80, you probably multiplied before inverting and then gave the result a positive rather than negative sign. See c, and paragraph D, page 55.

b. The answer is c. If you got -15/80, you probably multiplied before inverting. See c, and paragraph D, page 55.

c. This is the answer. Invert the divisor, multiply, and give the result a negative sign.

\[
\left( \frac{-5}{16} \right) \times \frac{3}{5} = \frac{-5 \times 3}{16 \times 5} = \frac{-15}{80} = -\frac{3}{16}
\]

4. a. The answer is b. If you chose -1/96, you multiplied before inverting and gave the result the wrong sign. See b, and paragraph D, page 55.

b. This is the answer. Invert, multiply, and give the result a positive sign.

\[
\left( \frac{-1}{12} \right) \times \left( \frac{-1}{3} \right) = \frac{-1 \times -1}{12 \times 3} = \frac{1}{36} = \frac{1}{36}
\]

c. The answer is b. If you chose -2/3, you did the operations correctly, but gave the result the wrong sign. See b.
Unit VII. Answers to the squaring of fractions problems

1. a. The answer is c. If you chose $\frac{1}{32}$, you probably did this:

$\left(\frac{1}{6}\right)^2 = \frac{2}{36} = \frac{1}{18}$ (wrong)

1$^2$ is not 2. See c. and paragraph E, page 56.

b. The answer is c. If you got $\frac{1}{8}$, you probably squared 1, but not 8. See c. and paragraph E, page 56.

c. This is the answer. Square the numerator and divide by the square of the denominator.

$\left(\frac{1}{8}\right)^2 = \frac{1}{8} \times \frac{1}{8} = \frac{1}{64}$

2. a. The answer is c. If you chose $\frac{9}{10}$, you probably squared 3 but not 10. See c. and paragraph E, page 56.

b. The answer is c. If you got $-\frac{9}{100}$, you did the operation correctly, but gave the result the wrong sign. Remember a negative times a negative equals a positive. See c.

c. This is the answer. Square the numerator and divide by the square of the denominator.

$\left(-\frac{3}{10}\right)^2 = \frac{-3}{10} \times \frac{-3}{10} = \frac{9}{100}$

3. a. This is the answer. Square the numerator and divide by the square of the denominator.

$\left(\frac{4}{13}\right)^2 = \frac{4}{13} \times \frac{4}{13} = \frac{16}{169}$

b. The answer is a. If you got $1 \frac{3}{13}$, you pro-
bably squared 4, but not 13. See a, and paragraph E, page 56.

c. The answer is a. If you chose 4/169, you probably squared 13, but not 4. See a, and paragraph E, page 56.

Unit VII. Answers to square-root of fractions problems.

1. a. This is the answer.
   \[ \sqrt{4/9} = \sqrt{4} / \sqrt{9} = 2/3 \]
   b. a is the answer. If you chose 2/9, you found the square-root of 4, but not of 9. See a, and paragraph F, page 57.
   c. a is the answer. If you got 1 1/3, you probably found the square-root of 9, but not of 4. See a, and paragraph F, page 57.

2. a. The answer is c. If you got 11/64, you found the square-root of 121, but not of 64. See c, and paragraph F, page 57.
   b. The answer is c. If you chose 1 1/8, you probably found the square-root of 64, but not of 121. See c, and paragraph F, page 57.
   c. This is the answer.
   \[ \sqrt{121/64} = \sqrt{121}/\sqrt{64} = 11/8 = 1 \ 3/8 \]

3. a. The answer is b. If you chose 0, you probably took the square-root of 1 as being 0, which is incorrect. See b.
   b. This is the correct answer.
   \[ \sqrt{1/81} = \sqrt{1}/\sqrt{81} = 1/9 \]
c. The answer is b. If you chose $1/81$, you probably found the square-root of 1, but not of 81. See b, paragraph F, page 57.

Please evaluate Unit VII. on page 82 of the Reference and Answer Manual.
Unit VIII. Location of Decimal Point

Reference Information

6 = 6.00
6 = 6.00 x 10^0
6 = .600 x 10^1
6 = .0600 x 10^2
6 = .00600 x 10^3
6 = 60.0 x 10^-1
6 = 600. x 10^-2
6 = 6000. x 10^-3

As the decimal point moves from left to right, the exponent increases by -1 for each place moved.

As the decimal point moves from right to left, the exponent increases by +1 for each place moved.

Multiplication \((a x 10^x) \times (b x 10^y) = (ab) \times 10^{x+y}\)

Division \((a x 10^x) \div (b x 10^y) = (a/b) \times 10^{x-y}\)

Square \((a x 10^x)^2 = a^2 \times 10^{2x}\)

Square-root \(\sqrt{a x 10^x} = \sqrt{a} \times 10^{x/2}\) (x must be divisible by 2 into a whole number)
Unit VIII. Answers to Pro-test

1. a. 4.353  
    b. 141.52  
    c. 7.830

2. a. 35.35  
    b. 1.413

3. a. 1.86  
    b. 9606.4  
    c. 00.0196

4 a. 50.00  
    b. 0.004  
    c. 2000.00

5 a. 250000.00  
    b. 000.0036

6 a. 2.10  
    b. 000.08
Unit VIII. Answers to multiplication problems.

1. \(6.00 \times 250.0 = (6.0 \times 10^0) \times (2.5 \times 10^2) = 15.0 \times 10^2 = 1500\)

2. \(0.382 \times 0.0053 = (3.82 \times 10^{-1}) \times (5.3 \times 10^{-3}) = 20.246 \times 10^{-4} = 0.0020246\)

3. \(-0.02 \times 763.0 = (-2.0 \times 10^{-2}) \times (7.63 \times 10^2) = -15.26 \times 10^0 = -15.26\)

4. \(351.0 \times 1643.0 = (3.51 \times 10^2) \times 1643 \times 10^3 = 5.76693 \times 10^5 = 576693.0\)

5. \((-3.56 \times 10^{-3}) \times (6 \times 10^{-4}) = 21.36 \times 10^{-7} = 0.000002136\)

Unit VIII. Answers for powers of 10 problems.

1. \(13.6 = 1.36 \times 10^1 \) (move 1 place right to left);
   \(10^{0+1} = 10^1\)

2. \(635.8 = 6.358 \times 10^2 \) (move 2 places right to left);
   \(10^{0+2} = 10^2\)

3. \(0.5 = 5.0 \times 10^{-1} \) (move 1 place left to right);
   \(10^{0+(-1)} = 10^{-1}\)

4. \(0.0035 = 3.5 \times 10^{-3} \) (move 3 places left to right);
   \(10^{0+(-3)} = 10^{-3}\)

5. \(645.3 \times 10^{-3} = 6.453 \times 10^{-1} \) (move 2 places right to left);
   \(10^{-3+2} = 10^{-1}\)

6. \(0.432 \times 10^2 = 4.32 \times 10^1 \) (move 1 place left to right);
   \(10^{2+(-1)} = 10^1\)

Unit VIII. Answers to Division Problems.

1. \(775 \div 2.5 = (7.75 \times 10^2)/(2.5 \times 10^0) = 3.1 \times 10^2 = 310\).
2. \(-169 \div 0.013 = \frac{-1.69 \times 10^2}{1.3 \times 10^{-2}} = -1.3 \times 10^4 = -13000\)

3. \(0.140 \div 35 = \frac{1.40 \times 10^{-1}}{3.5 \times 10^1} = 0.4 \times 10^{-2} = 0.004\)

4. \((4.82 \times 10^{-6})/(1.205 \times 10^{-5}) = 4.0 \times 10^{-1} = 0.40\)

5. \((6 \times 10^4)/(-3 \times 10^5) = \frac{-18 \times 10^2}{-2 \times 10^6} = 9 \times 10^{-1} = 0.9\)

Unit VIII. Answers to Square-root Problems.

1. \(\sqrt{4900} = \sqrt{49 \times 10^2} = \sqrt{49} \times \sqrt{10^2} = 7 \times 10^{2/2} = 7 \times 10^1 = 70\)

2. \(\sqrt{360} = \sqrt{3.60 \times 10^2} = \sqrt{3.6} \times \sqrt{10^2} = 1.90 \times 10^{2/2} = 1.90 \times 10^1 = 19.0\)

3. \(\sqrt{0.16} = \sqrt{16 \times 10^{-2}} = \sqrt{16} \times \sqrt{10^{-2}} = 4 \times 10^{-2/2} = 4 \times 10^{-1} = 0.4\)

4. \(\sqrt{0.0025} = \sqrt{25 \times 10^{-4}} = \sqrt{25} \times \sqrt{10^{-4}} = 5 \times 10^{-4/2} = 5 \times 10^{-2} = 0.05\)

5. \(\sqrt{1.44} = \sqrt{144 \times 10^{-2}} = \sqrt{144} \times \sqrt{10^{-2}} = 12 \times 10^{-2/2} = 12 \times 10^{-1} = 1.20\)

Please evaluate Unit VIII. on page 83 of the Reference and Answer Manual before continuing to Unit IX.
Unit IX. Finding Square-Roots

Reference Information

Direct calculation of square-roots

1. Group by pairs from the decimal point
2. Find the square which is lower than the left-most pair, place the value above the pair on the answer line.
3. Subtract the value squared, found in step 2 from the pair value. Bring down the next pair. This is the complete remainder.
4. Double the answer line and place to the left and below the remainder found in step 3.
5. Find the multiple of the doubled value which is as close as possible to the remainder, but still less than the remainder. Place this multiple in three places: (1) above the next pair on the answer line, (2) as the last value in the multiplicand, (3) and as the multiplier.
6. Multiply the multiplier times the multiplicand and place product under the remainder.
7. Subtract the product from the remainder.
8. Go back to step 4 and continue.

Using a table of square-roots

1. Find the value for which you are extracting the square-root in column 1 of the table.
2. Count the number of decimal places the number
to be expected has different from the table value. Note the direction of the decimal point move.

3. If the number of places moved is even, use column 3, and move the decimal given in the table (number of places moved/2) places in the direction of the original move.

4. If the number of places moved is odd, use column 4 and:
   a. If the move is to the right, move the decimal in the table value [(number of places moved right -1)/2] places to the right.
   b. If the move is to the left, move the decimal in the table value [(number of places moved left +1)/2] places to the left.
Table A  Squares, square roots, and reciprocals of numbers 1 to 1,000

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<th>√N</th>
<th>√10N</th>
<th>1/N</th>
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Unit IX. **Square-root problem answers.**

1. **a.** The answer is **b.** If you got 2.1, you probably found the square-root of 4.5. See paragraph D, page 75. Also, see below.

   **b. This is the correct answer.** \( \sqrt{45} = 6.7 \).

   Check your answer \( 6.7^2 = 44.89 \).

   **c. The answer is **b.** If you got 6.5, you probably doubled the initial top value rather than squared it and then subtracted from 45. See paragraph E, page 76. Also, see below.

   \[
   \begin{align*}
   \sqrt{45} &= 6.71 \\
   6^2 &= 36 \\
   7(127) &= 889 \\
   2(6) &= 1100 \\
   1(341) &= 1341 \\
   2(67) &= \text{STOP}
   \end{align*}
   \]

   The second decimal place is close to 1, round off to 6.7.

2. **a. This is the correct answer.** \( \sqrt{182.631} = 13.5 \).

   **b. The answer is a.** If you got 42.7, you probably grouped from the extreme right rather than with reference to the decimal place. Proper grouping 1 82.63 10 See paragraph C, page 75.

   **c. The answer is a.** If you got 13.6, you probably rounded off improperly. See paragraph M, page 80.

3. **a. The answer is c.** If you got 1.4, you have grouped improperly and have misplaced your decimal point. See paragraph C, page 75.
b. The answer is c. If you got .14, you have grouped improperly and have found the square-root of 0.0183 rather than 0.183.

c. This is the correct answer. \( \sqrt{0.183} = 0.43 \).
Square 0.43 to check your answer. \( 0.43^2 = 0.1849 \).

Unit IX. Answers to squaring problems.

1. 841
2. 1.96
3. 38,3161
4. 0.361201

If you have difficulty with these, see paragraph Q, page 82.
Unit IX. Square-root using table, problem answers.

1. a. The answer is b. If you chose 7.84, you looked at the square column rather than the square-root column. See b.

   b. This is the answer. Use column three and since the number is 28.0, there is no need to move the decimal point. $\sqrt{28.0} = 5.29$.

   c. The answer is b. If you chose 1.67, you looked in the wrong column. See paragraphs R. and S, page 83. See b.

2. a. This is the answer. Use column four since it is 10N which in our case is $604 \times 10 = 6040$. No decimal move is required. $\sqrt{6040} = 77.7$

   b. The answer is a. If you chose 24.6, you looked in column three rather than column four. See paragraph S, page 84.

   c. The answer is a. If you chose 36.48, you looked in the square column rather than a square-root column. See a.

3. a. The answer is b. If you chose 3.844, you looked in the square column rather than a square-root column. See b.

   b. This is the correct answer. Use column three since the decimal moved left an even (2) number of places. Move the decimal $\frac{2}{2} = 1$ place to the left of the table value. $\sqrt{6.20} = 2.49$
c. The answer is b. If you chose 7.87, you looked in the wrong column. Since the decimal place moved an even number of places, use column three. See paragraph W. 3, page 86. See b.

4. a. This is the answer. You moved two decimal places to the left, so use column three and move the table value decimal \( \frac{2}{2} = 1 \) place to the left. \( \sqrt{0.28} = 0.53 \).

b. The answer is a. If you chose 0.167, you looked in column four rather than column three. See a.

c. The answer is a. If you chose 0.053, you looked in the right column (three) but you moved the decimal one too many places to the left in the table value. See paragraph W. 3, page 86.

5. a. This is the correct answer. The decimal has been moved an odd number of places (3) to the right, so use column four. The answer decimal place is moved \( \frac{3-1}{2} = 1 \) place to the right of the table value. \( \sqrt{614000} = 783.6 \).

b. The answer is a. If you got 247.6, you used the wrong square-root column. You should use column four rather than column three. See paragraph W. 4. a, page 86. See a.

c. The answer is a. If you got 78.36, you used the correct column (four), but you moved the decimal \( \frac{3+1}{2} = 2 \) places rather than \( \frac{3-1}{2} = 1 \) place. See paragraph W. 4. a, page 86.
6. a. The answer is c. If you got 0.0246, you looked in column three rather than four. The decimal was moved an odd number (5) places to the left. Since it is an odd number, use column four. See paragraph W.4. b, page 86. See c.

b. The answer is c. If you chose 0.778, you used the right column but moved the decimal in the table value \((5-1)/2 = 2\) decimal places to the left rather than \((5+1)/2 = 3\) places. See paragraph W. 4. b, page 86. See c.

c. This is the correct answer. The decimal was moved 5 places to the left. So the column four value decimal place is moved \((5+1)/2 = 3\) places to the left. \(\sqrt{0.00606} = 0.0778\).

7. a. The answer is b. If you chose 676, you looked in the square column rather than a square-root column. See b.

b. This is the answer. The decimal was moved an even number of places (4) to the right. Use column three and move the decimal \(4/2 = 2\) places to the right. \(\sqrt{260000} = 509.9\).

c. The answer is b. If you chose 1612.5, you used the wrong column. See paragraph W. 3, page 86. See b.

Please evaluate Unit IX. on page 94 of the Reference and Answer Manual before continuing to Unit X.
Unit X. Order of Arithmetic Operations

Reference Information

General order of sequencing arithmetic operations.

First to last priority

1. Parentheses
2. Brackets
3. Squares and Square-roots
4. Multiplication and Division
5. Addition and Subtraction
Unit X. Answers to problems from page 91.

1. a. This is the correct answer. \(4 \times 6 + 3 + 2 = 24 + 3 + 2 = 29\).
   b. The answer is a. If you got 44, you probably found \(4 \times (6 + 3 + 2) = 4 \times 11 = 44\). See a.
   c. The answer is a. If you got 38, you probably found \(4 \times (6 + 3) + 2 = 4 \times 9 + 2 = 36 + 2 = 38\). See a.

2. a. The answer is b. If you got 33, you probably found \([10 \div 2] + 6\) \(\times 3 = 33\). See b.
   b. This is the answer. \(10 \div 2 + 6 \times 3 = 5 + 18 = 23\).
   c. The answer is b. If you got 3.75, you probably found \([10 \div (2 + 6)] \times 3 = (10 + 8) \times 3 = 1.25 \times 3 = 3.75\). See b.

Unit X. Answers to problems from page 92.

1. a. The answer is c. You probably found 24 + 6 = 4. See c.
   b. The answer is c. See c.
   c. This is the answer. \(6 \times 4 \div 2 \times 3 = 24 \div 2 \times 3 = 12 \times 3 = 36\).

2. a. The answer is b. If you got 0.020, you probably found \([16 \div (8 \times 14)] \div 7 = 0.020\). See b.
   b. This is the answer. \(16 \div 8 \times 14 \div 7 = 28 \div 7 = 4\).
   c. The answer is b. See b.

Unit X. Answers to problems from page 92.

1. a. The answer is b. If you chose 42 as the answer, you probably took 3 \(\times 5 + 6\) and then multiplied your answer by 2. Remember to work within the parentheses first before working outside.
of it. See b.

b. This is the answer. \((3x5)+(6x2) = 15 + 12 = 27\).

c. The answer is b. If you got 180, you probably did this \((3x5)x(6x2) = 180\). See b.

2. a. This is the correct answer \((2^2)^2 = (4)^2 = 16\).

b. The answer is a. If you got 4, you probably found the value of the parentheses and then forgot to square it. See a.

c. The answer is a. If you chose 64, you probably squared 8 rather than 4. See a.

3. a. The answer is b. If you got 12, you found the bracket value and forgot to square it. See b.

b. This is the correct answer \([3+(2+1)^2]^2 = [3+3^2]^2 = (3+9)^3 = 12^3 = 144\).

c. The answer is b. If you got 49, you probably found the value of \((2x1)\) in the parentheses. See b.

4. a. The answer is c. If you got 25, you have found the value under the radical sign but forgot to take the square-root of it. See c.

b. The answer is c. If you got 7, you probably decided that \(\sqrt{(1x3)^2} = 1x3\) and \(\sqrt{(2x2)^2} = 2x2\) and therefore \((1x3)+(2x2) = 7\). This is not correct. When addition or subtraction is involved, you must complete the operations under the radical first before taking the square-root. See c.

c. This is the correct answer. \(\sqrt{(1x3)^2+(2x2)^2} = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5\)
Unit X. Answer to median problem.

The answer is given by the formula: \( \text{Mdn} = L + \frac{\frac{n}{2} - C}{f} \)

\[
\text{Mdn.} = 14.5 + 3 \left( \frac{20/2-8}{4} \right) = 14.5 + 3 \left( \frac{10-8}{4} \right) = 14.5 + 3 \left( \frac{2}{4} \right) = 14.5 + 1.5 = 16.0
\]

Unit X. Answer to correlation coefficient problem.

\[
r = \frac{n \Sigma XY - (\Sigma X)(\Sigma Y)}{\sqrt{n \Sigma X^2 - (\Sigma X)^2} \sqrt{n \Sigma Y^2 - (\Sigma Y)^2}} = \frac{10(4.0)-(5.0)(2.0)}{\sqrt{[10(10.5)-(5.0)^2][10(2.4)-(2.0)^2]}} = \frac{40 - 10}{\sqrt{[10(10.5)-(5.0)^2][10(2.4)-(2.0)^2]}} \]

\[
= \frac{30}{\sqrt{[10(10.5)-25.0][10(2.4)-4.0]}} = \frac{30}{\sqrt{[105-25][24-4]}} = \frac{30}{\sqrt{80}(20)} = \frac{30}{40} = 0.75 = r
\]

Unit X. Answer for \( \chi^2 \) problem.

\[
\chi^2 = \frac{\Sigma (O-E)^2}{E} = \frac{(12-2)^2}{2} + \frac{(7-2)^2}{2} = \frac{(10)^2}{2} + \frac{(5)^2}{2} = \frac{1600}{46}
\]
\[
\frac{100}{2} + \frac{25}{2} = 50 + 12.5 = 62.5
\]

Please evaluate Unit X. on page 85 of the Reference and Answer Manual before continuing to Unit XI.
Reference Information

If you have an equation, it is permissible to:

1. Add the same value to both sides,
2. Subtract the same value from both sides,
3. Multiply both sides by the same value,
4. Divide both sides by the same value,
5. Square both sides of the equation,
6. Find the square-root of both sides of the equation,

and still maintain the validity of the equal sign.
Unit XI. Elementary Manipulation of Equations, Pre-test
answers.

1. \( n = k - m \)
2. \( x = 13a - 3y \)
3. \( c = 4a + 7b - z \)
4. \( a = 4! - 6f + 3g \)
5. \( b = \frac{xy}{3} \)
6. \( h = 6(k + m) \)
7. \( b = \frac{x - 7m}{3} \)
8. \( y = \frac{x}{z - 1} \)
9. \( b = (ac + 2)^2 \)
10. \( y = \sqrt{x^2 - 3h} \) or \( \sqrt{-3h + x^2} \)
Unit XI. Problem Answers from page 104.

1. a. The answer is c. If you chose \( x = rs-t-x \), you solved for \( y \) rather than \( x: \ y = rs-t-x \).
   See c. and paragraph B, page 102.

   b. The answer is c. If you chose \( x = rs-t+y \), you subtracted \( y \) from the left side, but added \( y \) to the right side. You must subtract from both sides. See c. and paragraph B, page 102.

   c. This is the answer. Here's how we get the answer:
   \[
   \begin{align*}
   x + y &= rs-t \\
   x + y - y &= rs-t - y \\
   x &= rs-t - y
   \end{align*}
   \]

2. a. The answer is b. If you chose \( a = (q/h)-r-x-y \), you added \( x \) to the left side and subtracted \( x \) from the right side. You must add \( x \) to both side. Also, you subtracted \( y \) from the left side and added \( y \) to the right side. You must subtract \( y \) from both sides. See b. and paragraph B, page 102.

   b. This is the correct answer. Here's how it's done:
   \[
   \begin{align*}
   a - x + y &= (q/h) - r \\
   a - x + x + y &= (q/h) - r + x \\
   a + y - y &= (q/h) - r + x - y \\
   a &= (q/h) - r + x - y
   \end{align*}
   \]

   c. The answer is b. If you chose \( a = (q/h)+r+x-y \), you correctly added \( x \) and subtracted \( y \) from both sides but you changed the sign of \( r \). See
b. and paragraph B, page 102.

3. a. This is the answer. Here's the solution:

\[ m+n = h-k \]

(Add \( k \)) \[ m+n+k = h-k+k \]

Thus, \[ h = m+n+k \]

b. The answer is a. If you got \( h = -k-m-n \), you moved \( h \) to the left side, but did not change its sign. You did correctly move \( m \) and \( n \). If you had done it this way correctly, you would have had,

\[ -h = -m-n-k = h = m+n+k \]

See a. and paragraph B, page 102.

c. The answer is a. If you chose \((m+n)/k\), you didn't add \( k \) to both sides properly. See a. and paragraph B, and 102.

4. a. This is the answer. Here's what we do,

\[ a+2b = cd \]

Subtract \( 2b \) \[ a+2b-2b = cd-2b \]

\[ a = cd-2b \]

b. The answer is a. If you chose \( a = (cd-b)/2 \), you improperly subtracted \( 2b \). See a. and paragraph C, page 103.

c. The answer is a. If you chose \( a = cd-b \), you subtracted \( 1b \), but not \( 2b \). See a. and paragraph C, page 103.
Unit XI. Problem Answers, from page 109.

1. a. The answer is b. If you chose \( a = \frac{e}{bc} \), you subtracted bc instead of dividing by bc. See b. and paragraph F, page 106.

   b. This is the answer. Divide both sides by bc, bc becomes a denominator.

\[
abc = \frac{e}{f}
\]

\[
\frac{abc}{bc} = \frac{e}{bcf}
\]

\[
a = \frac{e}{bcf}
\]

   c. The answer is b. If you chose \( a = \frac{e}{bc+f} \), you added bc to the denominator rather than multiplying it times the denominator. See b. and paragraph F, page 106.

2. a. This is the answer. Here's what we do,

\[
p - \frac{s}{t} = q - r
\]

add \( \frac{s}{t} \)

\[
p - \frac{s}{t} + \frac{s}{t} = q - r + \frac{s}{t}
\]

subtract q

\[
p - q = \frac{a + a - r + s}{t}
\]

add r

\[
p - q + r = \frac{a + a + s}{t}
\]

multiply by t

\[
t(p - q + r) = \frac{s\ell}{s}
\]

divide by \( (p - q + r) \)

\[
t = \frac{s}{p - q + r}
\]

   b. The answer is a. See a. and paragraphs E, on

52
c. The answer is a. See a. and paragraphs E, on page 105 and F, on page 106.

3. a. This is the answer. Here's the solution,

\[
\frac{3g-2}{h} = ij-10
\]

multiply \[
\frac{h(3g-2)}{h} = h(ij-10)
\]

by \( h \)

add 2 \[
3g-2+2 = h(ij-10)+2
\]

divide by \( \frac{3g}{3} = h(ij-10)+2 \)

Thus, \[
g = \frac{h(ij-10)+2}{3}
\]

b. The answer is a. See a. and paragraphs E, page 105 and F, page 106.

c. The answer is a. See a. and paragraphs E, page 105, and F, page 106.
Unit XI. Problems Answers from page 112.

1. a. The answer is b. If you chose $b = \sqrt{\frac{a(c+d)}{3}}$, you did everything correct except you took the square-root of the right side rather than square it. See b.

b. This is the answer. Here's the solution.

\[
\frac{3\sqrt{b}}{a} = c+d
\]

multiply by $a$ \[
\frac{3\sqrt{b}a}{a} = a(c+d)
\]

divide by 3 \[
\frac{3\sqrt{b}}{3} = a\left(\frac{c+d}{3}\right)
\]

find square \[
(\sqrt{b})^{2} = \left[\frac{a(c+d)}{3}\right]^{2}
\]

Thus, \[
b = \left[\frac{a(c+d)}{3}\right]^{2}
\]

c. The answer is b. If you got $b = \left[\frac{a(c+d)}{3}\right]^{2}$, you squared the numerator, but not the denominator. You must square the complete side. See b. and paragraph K, page 111.

2. a. This is the answer. Here's what we do,

\[
t + r = 3u - v
\]

\[
t^2 - r^2 = 3u - v - r
\]

\[
\frac{s^2}{a^2} = s^2(3u - v - r)
\]

\[
\frac{t}{3u - v - r} = s^2
\]

\[
\sqrt{\frac{t}{3u - v - r}} = s^2
\]

\[
s = \sqrt{\frac{t}{3u - v - r}}
\]
b. The answer is a. See a. and paragraph L, page 111.

c. The answer is a. If you chose $s = \left(\frac{t}{u-v-r}\right)^2$, you did everything right except you squared the right side of the equation rather than taking the square-root. See a. and paragraph L, page 111.

Manipulations of Equations, Post-test answers.

1. a. This is the answer.

\[
\frac{b^2}{c} = \frac{a+6}{3}
\]

- subtract 6 \[
\frac{b^2-6}{c} = \frac{a+6-6}{3}
\]
- multiply by 3 \[
\frac{2b^2-18}{c} = \frac{3a}{3}
\]

Thus, \[
a = \frac{2b^2-18}{c}
\]

b. The answer is a. If you chose $a = \frac{3b^2-6}{c}$, you didn't multiply the complete side of the equation by 3. See a. and paragraph E, page 105.

c. The answer is a. If you chose $a = \frac{3\sqrt{b^2}-18}{c}$, you changed $b^2$ to $b$. See a. and paragraph E, page 105.

2. a. The answer is c. If you chose $\Sigma x^n = a\sqrt{n-1}$, you forgot to square the right side of the equation. See c. and paragraph K, page 111.
b. The answer is c. If you chose $\Sigma x^2 = \sigma^2/(n-1)$, you divided rather than multiplied $(n-1)$. See c, and paragraph E, page 105.

c. This is the answer. Here's how we find it,

$$\sigma = \sqrt{\frac{\Sigma x^2}{n-1}}$$

square $\sigma^2 = \frac{\Sigma x^2}{n-1}$

multiply by $(n-1)$ $\sigma^2 (n-1) = \frac{\Sigma x^2 \sqrt{n/x}}{(x/x^2)}$

Thus, $\Sigma x^2 = \sigma^2 (n-1)$

3. a. The answer is b. If you chose $C = AB-D$, you improperly found the square-root. See b. and paragraph K, page 111.

b. Correct answer. $AB = C^2 + D$

subtracted $C$ $AB - D = C^2 + D - D$

find square-root $\sqrt{AB - D} = \sqrt{C^2}$

Thus, $C = \sqrt{AB - D}$

c. The answer is b. If you chose $C = (AB-D)^2$, you squared $(AB-D)$ rather than found it's square-root. See b. and paragraph L, page 111.

4. a. This is the answer.

$$\bar{X} = \frac{\Sigma x}{N}$$

multiply by $N$ $N\bar{X} = \frac{\Sigma x}{N}$

divide by $\bar{X}$ $\frac{N\bar{X}}{\bar{X}} = \frac{\Sigma x}{\bar{X}}$

Thus, $N = \frac{\Sigma x}{\bar{X}}$

b. The answer is a. See a. and paragraph E, page 56.
5. a. This is the answer.

\[ s_e = s_y \sqrt{1 - r^2} \]

divide by \( s_y \)

\[ s_e = \frac{s_y}{s_y} \sqrt{1 - r^2} \]

square

\[ \left( \frac{s_e}{s_y} \right)^2 = 1 - r^2 \]

add \( r^2 \)

\[ \left( \frac{s_e}{s_y} \right)^2 + r^2 = 1 \]

subtract \( \left( \frac{s_e}{s_y} \right)^2 \)

\[ r^2 = 1 - \left( \frac{s_e}{s_y} \right)^2 \]

find square-root

\[ r = \sqrt{1 - \left( \frac{s_e}{s_y} \right)^2} \]

b. The answer is a. See a.

c. The answer is a. See a.

Please evaluate Unit XI, on page 86 of the Reference and Answer Manual.
Unit XII. The Frequency Distribution and Frequency Polygon

Reference Information

Vocabulary:

1. **Frequency** - the number of times a score occurs in a set of data.

2. **Frequency distribution** - a table which indicates the frequencies of occurrence of scores on one variable in a set of data.

3. **Frequency polygon** - a graph which describes the frequency distribution of a set of scores. Scores plotted on X axis; frequencies plotted on Y axis.

4. **Grouped frequency distribution** - a frequency distribution which utilizes a small range of scores as the score category rather than individual scores. Used when there are a large number of scores in the set of data.

5. **Grouped frequency polygon** - a graph which describes a grouped frequency distribution.

6. **Inclusive range** - (High score - Low score) + 1

7. **Interval size** - the number of individual score units which are grouped to make a score interval. Usually ideal to have an odd number. intervals size (ie. 3, 5, 7).

8. **Exact limit** - a point which separates two adjacent intervals, no score can fall exactly on this value.
9. **Wholenumber limit** - the highest or lowest possible score which falls within an interval.

10. **Interval midpoint** - the point which exactly separates an interval into two equal parts; it is midway between the lower and upper limits. It is the best estimate of the scores in that interval. Would prefer to have the midpoint as a whole-number and not a fraction.

11. **Histogram** - a bar-graph which describes a frequency distribution.
Unit XII. Answers to (Frequency Distribution) problems

Score Category | f
---|---
31 | 1
30 | 1
29 | 2
28 | 3
27 | 4
26 | 5
25 | 3
24 | 1

If you have a problem, see paragraphs A and B, page 116.

If you have a problem, see paragraphs D and E, page 118.
### Unit XII. Answers to Midpoint Column Problem

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</tbody>
</table>

If you have a problem, see paragraph N, page 125.

Unit XII. Grouped Frequency Distribution and Polygon Problem.

1. Inclusive range = 1+ (Hi-Lo) = 1+(95-58) = 38
   
   \[
   \frac{38}{12} = 3.2 \\
   \frac{38}{15} = 2.5
   \]

   Interval size = 3. Remember, you would like an odd-number interval size. See paragraph H, page 121.

2. Start with the multiple of three which is less than the low score.

Low score = 58
$3 \times 19 = 57$

$3 \times 20 = 60$

Thus, start the interval at whole-number of 57.0. You could start at 56 or even 55 and your distribution would be about the same as if you had started at 57. Starting at a multiple of the interval size is a convention. See paragraph I, page 122.

3. Whole-number Interval Midpoint

| 93-95 | 94  |
| 90-92 | 91  |
| 87-89 | 88  |
| 84-86 | 85  |
| 81-83 | 82  |
| 78-80 | 79  |
| 75-77 | 76  |
| 72-74 | 73  |
| 69-71 | 70  |
| 66-68 | 67  |
| 63-65 | 64  |
| 60-62 | 61  |
| 57-59 | 58  |

See paragraph N, page 125.
<table>
<thead>
<tr>
<th>Whole-number interval</th>
<th>Midpoint</th>
<th>( f )</th>
</tr>
</thead>
<tbody>
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<td>93-95</td>
<td>94</td>
<td>1</td>
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<tr>
<td>90-92</td>
<td>91</td>
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<td>87-89</td>
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<td>72-74</td>
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<td>69-71</td>
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<td>60-62</td>
<td>61</td>
<td>1</td>
</tr>
<tr>
<td>57-59</td>
<td>58</td>
<td>1</td>
</tr>
</tbody>
</table>
5. Grouped Frequency Polygon.

See paragraph 0, page 126.
Unit XIII. Interval Percent, Cumulative Frequency, Cumulative Percent and the Probability of Occurrence of Selected Scores

Reference Information

Vocabulary:

1. **Interval percentage** - the percent of the scores in the data set that fall in a given interval.
   \[ \% \text{ in interval} = \frac{f \text{ in interval}}{n} \times 100\% \]

2. **Cumulative frequency** - that frequency which is the sum of the frequencies of all intervals below the given interval and the frequency within the given interval.

3. **Cumulative percentage** - the percentage of the scores below and including the given interval.
   \[ C\% = \frac{\text{Cumulative } f}{n} \times 100\% \]

4. **A Probability** - ratio of number of favored outcomes to the number of possible extremes.
   \[ p = \text{a proportion} = \frac{\text{percentage}}{100\%} \]
Unit XIII. A. Answers for interval percentage.

1. 33-35 interval
   \[ \% \text{ in 33-35 interval} = \frac{f \text{ in 33-35 interval}}{n} \times 100\% \]
   \[ \% = \frac{6}{50} \times 100\% = 0.12 \times 100\% = 12\% \]

2. 36-38 interval
   \[ \% \text{ in 36-38 interval} = \frac{f \text{ in 36-38 interval}}{n} \times 100\% \]
   \[ \% = \frac{13}{50} \times 100\% = 0.26 \times 100\% = 26\% \]

Unit XIII. A. Cumulative frequencies problem answers.

1. 33-35 interval
   \[ \text{Cf} = \text{Cf up to 33-35 interval} + f \text{ in 33-35 interval} \]
   \[ \text{Cf (33-35)} = 15 + 6 = 21 \]

2. 36-38 interval
   \[ \text{Cf} = \text{Cf up to 36-38 interval} + f \text{ in 36-38 interval} \]
   \[ \text{Cf (36-38)} = 21 + 13 = 34 \]

3. 39-41 interval
   \[ \text{Cf} = \text{Cf up to 39-41 interval} + f \text{ in 39-41 interval} \]
   \[ \text{Cf (39-41)} = 34 + 8 = 42 \]

Unit XIII. A. Cumulative percent answers.

1. 33-35 interval
   \[ \% = \frac{\text{Cf}}{n} \times 100\% \]
   \[ \% \text{ (33-35)} = \frac{21}{50} \times 100\% = 42\% \]

2. 36-38 interval
   \[ \% \text{ (36-38)} = \frac{34}{50} \times 100\% = 68\% \]
3. 39-41 interval

\[ \text{C\% (39-41)} = \frac{42 \times 100\%}{50} = 84\% \]

Unit XIII. A. Answers to frequency distribution.

<table>
<thead>
<tr>
<th>Whole-number interval Midpoint</th>
<th>f</th>
<th>%</th>
<th>Cf</th>
<th>C%</th>
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<td>57-59</td>
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<td>1</td>
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</tbody>
</table>
Unit XIII. B. Answers to questions.

1. a. \( p = \frac{\text{no. score possible}}{60} = \frac{8}{60} = 0.133 \)
   \( \text{no. scores total} = 60 \)

   See paragraph H, page 140.

   b. 13.3 times out of 100.

   See paragraph I, page 141.

2. a. \( p = \frac{\text{no. scores possible}}{60} = \frac{21}{60} = 0.350 \)
   \( \text{no. scores total} = 60 \)


   b. 35 times out of 100.

   See paragraph I, page 141.

   c. \( q = 1.000 - p = 1.000 - 0.350 = 0.650 \)

   See paragraph M, page 142.

3. a. \( p = \frac{\text{no. of scores possible}}{60} = \frac{58}{60} = 0.967 \)
   \( \text{no. of scores total} = 60 \)

   See paragraph J and K, page 141.

   b. \( q = 1.000 - p = 1.000 - 0.967 = 0.033 \)

   See paragraph M, page 142.
Unit XIV. Introduction to the Unit Normal Distribution

Reference Information

Vocabulary:

z score, standard score - a z score is a converted raw score; a ratio of the difference a score is from the mean of the scores and the standard deviation ($\sigma$) of the set of scores:

$$z = \frac{X - \overline{X}}{\sigma}$$

It is the number of standard deviation units a score is from the mean.

Unit Normal Distribution - a model which we use frequently in statistics. Raw scores are transformed into z scores. z scores have a mean of zero and a standard deviation of one. The area under the unit normal distribution is set at 1.000.

Ordinate - the height of the curve, which describes a distribution, at a given point on the X axis or z score axis.

Theoretical Sampling Distribution - a theoretical, not actually determined, distribution of a sample statistic, usually sample means. We plot sample means, not raw individual subject scores, on the X axis and the frequencies of the occurrence of these sample means on the Y axis. Based on the theoretical ability of the researcher to select an infinite number of samples from a popula-
tion and calculate and plot each different sample mean.

**0.05 level of significance** - an obtained result which could have occurred by chance 5 times, or less, out of 100 times.

**0.01 level of significance** - an obtained result which could have occurred by chance 1 time, or less, out of 100 times.
Univ Normal Distribution Reference Sheet

![Graph of the normal distribution for μ = 0 and σ = 1.]
Unit XIV. Answers to questions.

1. a. 50\%\textsuperscript{a}, The mean (0) separates the curve into two equal halves.
   b. \( p = \frac{50\%}{100\%} = 0.50 \)

2. a. 13.6\%\textsuperscript{a}
   b. \( p = \frac{13.6\%}{100\%} = 0.136 \)
   See paragraph J, page 150.
   c. 13.6 times out of 100.
   See paragraph I, page 150.

Unit XIV. Answers to questions.

1. a. .3485 See paragraph K, page 151.
   b. .3485 See paragraph K, page 151.
   c. .8485 See paragraph K, page 151.
   d. .1515 See paragraph K, page 151.
   e. .8485 See paragraph K, page 151.
   f. .1515 See paragraph K, page 151.
   g. .2347 (for both) See paragraph L, page 152.

2. a. .0250 See paragraph K, page 151.
   b. .0250 See paragraph K, page 151.
   c. .4750 + .4750 = .9500
   d. .0250 + .0250 = .0500
   e. .95
   f. 95 out of 100 times
   g. .95
   h. 5 out of 100 times

Unit XIV. Answers to questions.

1. 0.99 See paragraph K, page 151.
2. 0.01 See paragraph K, page 151.
3. 1 time out of 100 times See paragraph O, page 155.
Unit XV. The Scatterogram and Line of Best Fit

Reference Information

Vocabulary:

1. **Scattergram** - a graph which is used to plot two variable measures based on some common reference, usually the same person.

2. **Positive correlation** - a mathematical, direct relationship between two variables; a high score on X tends to be related to a high score on Y, low X tends to be related to low Y.

3. **Negative correlation** - a mathematical, inverse relationship between two variables; a high score on X tends to be related to a low score on Y, low X tends to be related to high Y.

4. **Line of best fit, Regression line (linear)** - the line on a scattergram which best represents the points relative to each other. Has the general equation of \( Y = aX + b \), where "a" is the slope and "b" is the Y-intercept.

5. **Slope** - the directional magnitude of change in the value of Y for the change in 1 unit of X. Can be either positive or negative.

\[
\text{Y} \quad \text{slope is positive} \quad \text{X}
\]
\[
\text{Y} \quad \text{slope is negative} \quad \text{X}
\]
6. **Y-intercept** - the point on the Y axis where the line of best fit intersects the Y axis. The value of Y when X=0. Can be positive or negative.

7. **Regression analysis** - the use of a regression line to predict the value of a variable on the basis of a known value of another variable.
Cartesian Coordinate System

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Unit XV. Answers

Graph from page 164.

Unit XV. Predicted values of Y.

1. \( X = -5 \)
   \[ Y = (+0.5)(-5)+1.0 = -2.5+1.0 = -1.5 \]

2. \( X = 15 \)
   \[ Y = (+0.5)(15)+1.0 = 7.5+1.0 = 8.5 \]

See paragraphs E, F, G, page 164.

Unit XV. ACT Score

\[ X = \frac{2.000-0.861}{0.081} = \frac{2.139}{0.081} = 26.4 = \text{ACT} \]

Therefore, students with above about a 26 ACT score

would be primary scholarship candidates. See paragraph R, page 172.