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A FUNDAMENTAL MATHEMATICAL SIMULATION OF WATERSHEDS

DISSERTATION

Presented in Partial Fulfillment of the Requirement for the Degree of Doctor of Philosophy in the Graduate School of The Ohio State University

By

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* * * * *

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1971

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<td>( A )</td>
<td>Area of cross-section of an element of water ((\text{cm}^2))</td>
</tr>
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<td>( A )</td>
<td>A tridiagonal matrix</td>
</tr>
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<td>( A_{1} )</td>
<td>Factor in continuity and momentum equations</td>
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<tr>
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<td>Water capacity ( (\frac{\Delta \theta}{\Delta h} \text{; } \frac{1}{\text{cm}}))</td>
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<td>( C_{1}^{'}) \</td>
<td>Dimensionless water capacity</td>
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<td>( D )</td>
<td>A column vector</td>
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<td>( D )</td>
<td>Depth of an inclined soil slab measured perpendicular to the slope ((\text{cm}))</td>
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<td>( F )</td>
<td>External force acting on a water element ((\text{gm}))</td>
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<td>( F_{x} )</td>
<td>Force acting on a water element in the ( x )-direction ((\text{gm}))</td>
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<td>( F_{t} )</td>
<td>Total external frictional force resisting flow ((\text{gm}))</td>
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<td>( f_{2}^{'} )</td>
<td>Vertical infiltration rate ((\text{cm/sec}))</td>
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<td>( f_{2} )</td>
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<td>A column vector</td>
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\( \varepsilon \) Acceleration due to gravity (cm/sec^2)

\( \mathbf{H}^{4} \mathbf{M}' \) Maximum vertical depression and detention storage depth (cm)

\( \mathbf{H}^{4} \mathbf{M} \) Dimensionless storage depth (\( \mathbf{H}^{4} \mathbf{M}' / L \))

\( \mathbf{H}^{4} ' \) Vertical depression storage depth at any time \( t \) (cm)

\( \mathbf{H}^{4} \) Dimensionless vertical depression storage depth at any time \( t \) (\( \mathbf{H}^{4} ' / L \))

HR hour

h Capillary pressure (cm of H\(_2\)O)

\( I_x \) Inflow rate into a soil element in the x-direction (cm\(^3\)/sec)

\( I_z \) Inflow rate into a soil element in the z-direction (cm\(^3\)/sec)

i x-direction subscript for the finite difference mesh

I Maximum i

j z-direction subscript for the finite difference mesh

J Maximum j

K Soil hydraulic conductivity (cm/sec)

\( K_S \) Saturated hydraulic conductivity (cm/sec)

K3 Expression for a weighted conductivity around a given node

k Number of soil porosity classes

L Length of the sloping plane measured down the slope (cm)

L A lower triangular matrix

\( M_{P2} ' \) Change in momentum due to infiltration outflow from a water element (gm-cm/sec)

\( M_{RI} \) Change in momentum due to rainfall inflow into a water element (gm-cm/sec)

\( M_u \) Change of momentum of a flowing fluid in the x-direction (gm-cm/sec)

\( M_t \) Total change in momentum (gm-cm/sec)
m  Mass (gm)

m  Number representing the number of iterations during a computation

NDX  Number of divisions of the flow section along the x-axis

NDZ  Number of divisions of the flow section along the z-axis

n  Time parameter

n  Manning's roughness coefficient

Ox  Outflow rate out of a soil element in the x-direction (cm^3/sec)

Oz  Outflow rate out of a soil element in the z-direction (cm^3/sec)

P(x,y)  Any point inside the soil flow system

P1  Any point at the lower end of the soil flow system

P2  Any point at the upper end of the soil flow system

Q_o  Reference flow rate at the outlet (cm^3/sec)

Q'  Flow rate at the outlet (cm^3/sec)

Q  Dimensionless flow rate at the outlet (Q'/Q_o)

RI  Vertical rainfall rate (cm/sec)

RE  Dimensionless rainfall rate (RI/K_s)

R_T  Total rate of change of moisture in a soil element (cm^3/sec)

R  Ratio = Δx/Δz

r  Subscript used in general to represent i or j

r_1  Vertical distance from the datum plane to a point on the lower impermeable layer of the soil section (cm)

r_2  Vertical distance from the lower impermeable layer to any point in the soil section (cm)

STOR  Volume of water stored between sections 1 and 2 of the overland flow cross-section (cm^3)

S_f  Friction slope or slope of the energy grade line
S  The distance from the lower corner of the soil section to any point on the lower impermeable layer (cm)

\( t \)  Time (sec)

\( U \)  Upper triangular matrix

\( u \)  Overland flow velocity (cm/sec)

\( \text{UP} \)  Reference overland flow velocity (cm/sec)

\( u' \)  Dimensionless overland flow velocity (\( u/\text{UP} \))

\( V_x \)  Soil water flow velocity in the x-direction (cm/sec)

\( V_y \)  Soil water flow velocity in the y-direction (cm/sec)

\( V_z \)  Soil water flow velocity in the z-direction (cm/sec)

\( W \)  Weight of water in a control volume (gm)

\( XD \)  Dimensionless depth (D/L)

\( x \)  Distance along the slope of the watershed cross-section (cm)

\( y \)  Vertical overland flow depth (cm)

\( \bar{z} \)  Distance from the top of the water element to its centroid (cm)

\( z \)  Distance perpendicular to the watershed cross-section (cm)

\( \alpha \)  Slope angle (degrees)

\( \alpha \)  Forward-characteristic

\( \beta \)  Backward-characteristic

\( \gamma \)  Surface tension of water (dynes/cm = gm/sec^2)

\( \Delta t \)  Time increment (sec)

\( \Delta x \)  Distance increment along x-axis

\( \Delta y \)  Distance increment along y-axis

\( \Delta z \)  Distance increment along z-axis

\( \varepsilon \)  Soil porosity (cm^3/cm^3)
\( \eta \)  
Dynamic viscosity of water (poise = g m/cm sec)

\( \theta \)  
Dimensionless moisture content \((\theta / \theta_s)\)

\( \theta \)  
Volumetric moisture content \((cm^3/cm^3)\)

\( \theta_s \)  
Saturated volumetric moisture content \((cm^3/cm^3)\)

\( \kappa \)  
Dimensionless hydraulic conductivity \((K/K_s)\)

\( \lambda \)  
Element of triangular matrix \(L\)

\( \mu \)  
Unit weight of water \((gm/cm^3)\)

\( \nu \)  
Kinematic Viscosity \((cm^2/sec)\)

\( \xi \)  
Dimensionless distance along z-axis \((Z/L)\)

\( \pi \)  
\(\pi = 3.14159265\)

\( \rho \)  
Density of water \((gm/cm^3)\)

\( \sigma \)  
Element of the lower triangular matrix \(L\)

\( \tau \)  
Dimensionless time

\( \phi \)  
Total Potential \((cm)\)

\( \phi' \)  
Dimensionless capillary pressure \((h/L)\)

\( \phi'' \)  
Dimensionless total potential

\( \psi \)  
Dimensionless overland flow depth \((y/L)\)

\( \omega \)  
Dimensionless distance along the watershed slope \((x/L)\)
INTRODUCTION

The aim of hydrologic simulation is to recreate the past, observe the unusual, or anticipate the future without the limitations of time. The need for simulation in hydrology is emphasized by the increasing demands of expanding populations for more hydrologic and hydraulic structures; the design of most of these structures requires many years of records of precipitation, surface runoff, and stream flow. At the present time, there are very few data collecting stations which can boast of having periods of records that could be considered adequate. In any case, it is inconceivable that an attempt will ever be made to gauge every watershed within which structures will be built.

In the past, an engineering project could be designed without paying any attention to its effect on the environment or without worrying about the psychological effect on the population, provided the project was beneficial to that population. Those days are over. It is therefore obvious that the concepts of simulation and full synthesis must receive more attention in future studies of hydrology.

The major components of the hydrologic cycle include precipitation, interception, surface storage, evapotranspiration, infiltration, soil moisture, interflow, ground water flow, overland flow, and channel flow. These are all extremely complex processes. The raindrop size and shape, and the areal distribution of rainfall over a watershed are not easy to
characterize; the problem is further complicated when precipitation takes the form of snow. Interception and surface storage are processes that are difficult to analyze mathematically. Evapotranspiration and infiltration have been subjected to a great deal of analysis; most of the efforts have been spent in developing empirical relationships for these processes. Hydrologists and others interested in these fields are aware of the difficulties encountered in trying to analyze soil moisture, interflow, ground water, and channel flow systems. It is quite easy to look at the above processes, which are extremely complex, and conclude that a complete physically-based simulation of the hydrologic cycle will never be possible. But the rate of advance of mathematics and numerical analysis, computer technology, characterization and measurement of soil parameters, and remote sensing, is a hopeful sign that such a model, conceptually presented by Freeze and Harlan (1969), would be developed some day. Meanwhile, we should continue with the present approach: using sound theory and mathematics, when they are available, together with experience and good judgment to get the job done. The gaps will be filled when new knowledge becomes available.

The objective of this study is to solve the two-dimensional Darcian partial differential equations to obtain the infiltration component of the hydrologic cycle and to use this component to solve the equations of flow over an infiltrating watershed.
I. REVIEW OF LITERATURE

Some Watershed Models

There are two major approaches to watershed modeling, the deterministic approach and the parametric and stochastic approach. The deterministic approach seeks to study the components of the hydrologic cycle and their inter-relationships and attempts to manipulate these relationships to produce a watershed model. The stochastic and parametric approach depends on the stochastic properties of the input and output variables, and on statistical analysis of these variables to produce a model. A detailed treatment of the different approaches to the hydrologic system analysis is given by Amoroco and Hart (1964). No attempt will be made to review all the substantial number of available watershed models but some of the deterministic models will be mentioned here.

Dawdy and O'Donnell (1965) developed a catchment model based on four storage elements - surface storage, channel storage, soil moisture, and ground water. An empirical equation was used to calculate infiltration and nine parameters were used as controls to determine when moisture was ready to move from one storage element to the other. From their experience with the model, the authors concluded that the land phase simulation of the hydrologic cycle was feasible.

Using the analog computer Riley, Chadwick and Israelsen (1968)
and Amisial and Riley (1968) developed a model and applied it to a particular runoff event on the Walnut Gulch watershed in Arizona. The rainfall excess was obtained by subtracting the retention rate - the combined loss due to depression and vegetation storage - from the effective rainfall rate. The infiltration capacity rate was obtained from the Horton formula, and infiltration was assumed to occur at capacity rate rather than actual rate as long as there was water in retention storage. The equations of motion and continuity, the de Saint Venant equations, were solved to obtain the surface runoff component of the model. With additional tests, the authors expected the model to be useful for predicting runoff events on their particular watershed.

In the words of the authors, Holtan and Lopez (1970), the USDAHL-70 Model of Watershed Hydrology, "is currently a series of empiricisms selected to provide a mathematical continuum from ridge top to watershed outlet in terms of input information readily available to the analyst." Huggins and Monke (1966) also stressed the importance of developing a model for which the input data must be obtainable from maps and generalized tables. Kutchment and Koren (1968) started their model by estimating the antecedent soil moisture; the input functions for the model were precipitation, moisture deficit, and discharge up to the time that the design flood started. An optimization principle was used to improve the approximate initial parameters. The model produced some promising results.

Other models include those of Nemec (1968), Schultz (1968), Kozak (1968), and Amaftiesei and Ionescu (1968). But the most widely known hydrologic model is the Stanford Watershed Model developed by Crawford
and Linsley (1966) for a continuous modeling of soil moisture, stream flow and evapotranspiration for a watershed. Hourly precipitation, daily pan evaporation, and the physical and hydraulic watershed properties are the model input parameters. For a given watershed, a 5-year period of rainfall and runoff records is selected to optimize the watershed parameters for the model. Sound judgment and experience are required to select initial parameters. This model is well suited for large watersheds.

Almost all the models mentioned above use empiricism, and block building rules, to apportion the major input, precipitation, to the different subsystems of the watershed. This approach is used because the different equations governing the flow of water through the watershed subsystems are highly non-linear and very complex to solve. Recently, Smith and Woolhisier (1971) developed a model for which they solved the one-dimensional, vertical, unsaturated flow equation to obtain the infiltration component of the hydrologic cycle; the approximate unsteady overland flow equation was solved for the surface runoff system. The model was tested on a laboratory soil flume and a small experimental watershed and the results were considered satisfactory.

Infiltration and Flow Through Porous Materials

Most of hydrology is concerned with what happens to precipitation after it reaches the ground surface. Since the soil is a porous medium, the solution of hydrologic problems is closely tied up with the study of flow through porous materials. Klute (1952) advanced this subject by being the first to apply the finite-difference technique to the solution of the flow equation for unsaturated soils. Nelson (1962)
solved the equation for saturated and partially saturated heterogeneous medium and for partially saturated homogeneous media. Reisenauer (1963) described the extension of Nelson's solution to include two-, and three-dimensional problems.

Liakopoulos (1965) and Singh and Franzini (1967) used different techniques to solve the unsteady unsaturated flow problem, and Rubin (1967) took hysteresis into consideration in his solution. Klute, Scott, and Whisler (1965) treated the problem of steady state flow in a saturated inclined slab and developed the condition under which the two dimensional set up can be treated essentially as a one-dimensional problem. Rubin (1968) used an implicit scheme to analyze the unsaturated and partially unsaturated problem in two dimensions.

In solving the problem of flow through porous materials, a lot of researchers confine themselves to a medium that is either completely saturated or completely unsaturated. It is obvious in nature that both systems often exist in continuum and in many practical situations, we need to deal with them as such. Freeze (1968, 1969) gave excellent reports of his work on the mathematical and physical continuity between the saturated and unsaturated flow system. Taylor and Luthin (1969), without setting out to do so, demonstrated this continuity in their analysis of water-table aquifers. In a Ph. D. thesis, Amerman (1969) treated the flow of fluids through porous materials in two-dimensions.

Infiltration is one of the most significant components of the hydrologic cycle. It is the chief means by which water is supplied to the soil system. In a seven-part paper, Philip (1957) presented a thorough analysis of the infiltration process. He dealt mostly with
ponded cases and the soil system was semi-infinite. Hanks and Bowers (1962) treated the cases of infiltration into a semi-infinite homogeneous horizontal system, and a finite stratified soil column with reasonable results; and Whisler and Klute (1965) considered hysteresis in their treatment of infiltration into a vertical, initially equilibrated soil column of finite length. Rubin (1966) used a special transformation to study preponding and ponding stages of rainfall infiltration; and Staple (1966) accounted for hysteresis in his study of infiltration and moisture redistribution in vertical soil columns.

Compared to ponded-water infiltration, very little work has been done on unsteady state rainfall infiltration. Among the first to tackle this problem were Youngs (1960) who considered infiltration into a slate dust system at low infiltration rates, Rubin and Steinhardt (1963, 1964) and Rubin, Steinhardt and Reiniger (1964), who analyzed soil-water relations during rain infiltration into a vertical semi-infinite soil column with a constant initial moisture distribution. Whisler and Klute (1967) extended the analysis of Rubin and Steinhardt (1963) using a soil column with a non-uniform initial moisture content. In his analysis of recharging or discharging ground-water flow system, Freeze (1969) allowed for ponded as well as constant rate rainfall upper boundary conditions; Smith and Woolhisser (1971) also used the same type of upper boundary conditions in their infiltrating watershed model.

A few studies have dealt with the two-dimensional infiltration in groundwater recharge ditches or irrigation furrows, but to date, almost no analysis of the two-dimensional rainfall infiltration has been carried
out. This situation exists, not because there are no practical situations requiring two-dimensional analyses, but because of the complexities involved.

Methods of Solving the Equations of Flow through Porous Materials

Several methods have been used in attempting to solve the problems of flow through porous media. Klute (1952) used an approximate analytical method and Philip (1957) used a special transformation and a series approach. But these problems are usually not amenable to closed form solutions and the most common recourse is to finite differencing. There are two types of finite differencing, explicit and implicit. In the explicit approach one unknown quantity can be expressed in terms of other quantities that are all known, whereas an implicit approach involves solving simultaneous equations for the unknown quantities.

Hanks and Bowers (1962), Nelson (1962), Reisenauer (1963), Staple (1966), and Taylor and Luthin (1969) used the explicit method to analyze their problems. This approach is comparatively simple and straightforward but it requires very restrictive time and space mesh sizes to meet the convergence and stability conditions. The implicit scheme is not as restrictive as the explicit method but it is much more complicated. Rubin and Steinhardt (1963), Liakopoulos (1965), Rubin (1967), Whisler and Klute (1967), Freeze (1969), and Smith and Woolhiser (1971) all used the implicit method in their investigations.

In 1955, Peaceman and Rachford (1955) came up with the Alternating Direction Implicit (ADI) method of solving parabolic and elliptic partial differential equations. They used this method to solve the problems of unsteady-state heat flow in a square, and two-dimensional steady-state
heat flow in a square -- Laplace's equation. They found the ADI method to be stable and to be faster than the better known over-relaxation methods. The usual restrictions of the explicit methods did not apply. Henderson, Dempsey, and Nelson (1967) successfully used the ADI method for gas reservoir studies, and Rubin (1968) appears to be the first to use it for soil moisture analysis; Bishop, Green and Buzzelli (1969) adapted the ADI procedure for the hybrid computer solution of two-dimensional gas reservoir system. Amerman (1969) was able to use the ADI method successfully for soil moisture flow situations that were either completely saturated or completely unsaturated; but for his particular boundary conditions, he could not get it to work for the case of two-dimensional drainage from saturation and, therefore, resorted to the explicit approach.

Recently, Witherspoon, Havandel, and Newman (1968) reported a new numerical approach, the Finite Element method, for solving transient flow problems. This method is reported to have a number of advantages over the conventional finite difference methods particularly in being able to handle complicated boundary conditions in complex systems, but the method is still being studied. The ADI procedure is used for the two dimensional analysis of the soil moisture movement in this investigation.

**Determination of Hydraulic Conductivity**

Methods for measuring the hydraulic conductivity can be grouped into three general classes; namely, steady-state, transient outflow, and pore size distribution methods. The steady-state method applies to systems in which the moisture content, tension, and flux do not change with time; and the transient outflow method depends on the measurement of pressure and outflow using a pressure plate with, preferably, negligible
membrane impedance.

A theory in which permeability is related to pore size distribution was proposed by Childs and Collis-George (1950). The theory is based on the probability that pores of different radii are continuous in a porous medium. A matching factor given by the ratio of measured saturated conductivity to calculated hydraulic conductivity was necessary for the computation. Marshall (1958) facilitated the computation by using equal classes of porosity and Millington and Quirk (1960) eliminated the matching factor of Childs and Collis-George, and the reciprocal of suction, and replaced them with pore radii and a constant coefficient. Using the methods of Childs and Collis-George (1950), Marshall (1958), and Millington and Quirk (1960), Jackson, Reginato, and Van Bavel (1965), calculated conductivity values for graded sand and concluded that the method of Millington and Quirk with the matching factor of Childs and Collis-George gave the best results. Recently Skaggs, Monke and Huggins (1970) presented an approximate method for determining the hydraulic conductivity function of unsaturated soils. A thorough review of the methodology for determining hydraulic conductivities is given by Nwa (1967). The method of Millington and Quirk together with a matching factor is used in this analysis.

Overland Flow

After the soil surface has become saturated, and the depression storage satisfied, the amount of rain in excess of infiltration is free to runoff. This is another component of the hydrologic cycle that is difficult to handle because the equations governing the flow of water over the watershed surface are complicated and difficult to solve.
Consequently, attempts have been made to solve the overland flow equations in various simplified forms. The most common simplifying assumption is that a functional relationship exists between the flow velocity and depth. The resulting equations are often referred to as the kinematic wave equations as opposed to the dynamic wave equations which are derived from Newton's second law of motion. The kinematic wave approximations were used by Henderson and Wooding (1964), Wooding (1965), Brakensieck (1967), and Smith and Woolhiser (1971) in their analyses of overland flow. These approximations were generally considered to be adequate for their purposes by the authors.

In spite of the difficulties several authors have solved the complete or partially complete dynamic wave equations with varying degrees of successes. Morgali and Linsley (1965) used an explicit finite difference technique to solve the overland flow equations. They used the method of characteristics to check and make sure that their solution points were within the unique solution region. Grace and Eagleson (1966) developed the similarity criteria necessary before the behavior of a model and prototype could be considered dynamically similar. If the similarity criteria are met, the prototype could be used to test the model and the results obtained from the test could be applied to other watersheds. The theory and characteristics of overland flow were presented by Chen and Hansen (1966). Using an implicit scheme, Brakensieck (1966) solved the overland flow equations in his investigation of watershed performance. Brakensieck concluded that the use of the hydrodynamic equations to simulate small upland watersheds appeared feasible.

In his Ph. D. dissertation, Liggett (1959) presented the theory of
characteristics and used it to study the unsteady flow in open channels with lateral inflow. The same treatment can also be applied to overland flow. Fletcher and Hamilton (1967) used the same technique to route flood through irregular channels and Chen and Chow (1968) applied it in their analysis of overland flow over an impervious surface. Quite often the success or failure in solving the hydrodynamic equations of overland flow depends on the method used. In an excellent paper, Liggett and Woolhiser (1967) presented analyses and comparison of the different techniques of solving the shallow-water equations. They found the explicit method unsatisfactory due to convergence and stability difficulties while the method of characteristics was the most accurate for the same initial point spacing. The method of characteristics was also found to be fast but the data at intermediate points in the x-t plane are difficult to obtain. The implicit scheme was also found to be satisfactory, but it is quite involved computationally.

In this study the explicit method explained by Morgali and Linsley (1965) is used; the method of characteristics is used to overcome the stability and convergence difficulties inherent in the explicit approach.
II. A BASIC MATHEMATICAL WATERSHED MODEL

A basic mathematical watershed model is made up of a complete description of the watershed - the size, the shape, and the boundary properties - together with the mathematical equations that describe the hydrologic processes. The sophistication and scope of the model depend on the availability of the equations that describe the hydrologic processes, how well these equations describe the processes, the ability to solve these equations, the availability of the required data, the size of the computer, and the availability of funds for such a model. Consequently, the usual practice is to make simplifying assumptions that will enable an adequate model to be developed. A conceptual watershed model is shown in Figure 1 and the components considered in this study are indicated with asterisks.

For the purpose of this investigation, it is assumed that evapotranspiration can be neglected during the period of precipitation. The rainfall rate used is assumed to be the effective rate, i.e., the actual rainfall rate minus the interception rate. Before surface runoff can take place, the retention, and the detention storages must be satisfied. Surface retention is the amount of water stored in surface depressions, and surface detention is the amount of water that must accumulate above the depression storage before surface runoff can begin. Surface detention becomes part of the overland flow towards the end of the runoff process.
Figure 1. The Basic Components of the Hydrologic Cycle

*These components are considered in this model.
The depth of the detention storage is included in the retention depth in this analysis. The other assumptions of this model will be specified as the rest of the components of the hydrologic cycle are dealt with.
III. INFILTRATION AND SOIL MOISTURE FLOW SYSTEM

Assumed Watershed Section

The assumed watershed section is shown in Figure 2. The x-axis is taken down the slope and the z-axis is perpendicular to the slope. The left hand end, and the bottom of the section are assumed to terminate at impermeable layers. The lower corner of the right hand end of the section is at the bottom of the stream channel; the stream channel is also assumed to lie on an impermeable base. For the analysis of moisture movement in the assumed section, these additional assumptions are made:

1. Darcy's law is valid.
2. The soil is homogeneous and isotropic.
3. During the infiltration process hysteresis is negligible.

Figure 2. Diagram of the flow geometry.
Derivation of the Flow Equations

Consider the soil element shown in Figure 3; the origin is at its center and the dimensions are as shown in the figure. The coordinate system is also shown in Figure 3. Any point in the coordinate system will be designated as \((i, k, j)\) which is equivalent to the designation \((x, y, z)\).

Let \(V_x\), \(V_y\), and \(V_z\) be the components of the flow velocity in the \(x\)-, \(y\)-, and \(z\)- directions, respectively. Assuming that the flow across the watershed slope is negligible, then:

Figure 3. A soil element and the coordinate system.
Since \( V_y \) is taken as zero, we have a two-dimensional situation; therefore a point in the flow system will be designated as \((i, j)\).

The inflow rate (cm\(^3\)/sec) in the \(x\)-direction is given by:

\[
I_x = V_{i-\frac{1}{2}, j} \Delta y \Delta z \quad \ldots \quad (2)
\]

and the outflow rate in the \(x\)-direction is:

\[
O_x = V_{i+\frac{1}{2}, j} \Delta y \Delta z \quad \ldots \quad (3)
\]

where \( i-\frac{1}{2} \) is used to designate the point between \( i-1 \) and \( i \) on the \(x\)-axis. The other half interval designations follow the above explanation.

From equations (2) and (3), we have:

\[
I_x - O_x = (V_{i-\frac{1}{2}, j} - V_{i+\frac{1}{2}, j}) \Delta y \Delta z \quad \ldots \quad (4)
\]

Similarly,

\[
I_z = V_{i, j-\frac{1}{2}} \Delta x \Delta y \quad \ldots \quad (5)
\]

\[
O_z = V_{i, j+\frac{1}{2}} \Delta x \Delta y \quad \ldots \quad (6)
\]

\[
I_z - O_z = (V_{i, j-\frac{1}{2}} - V_{i, j+\frac{1}{2}}) \Delta x \Delta y \quad \ldots \quad (7)
\]

Let the total rate of change of moisture be designated by \( R \), and
let \( \theta \) be the volumetric moisture content; it follows that:

\[
R_T = I_x - O_x + I_z - O_z
\]

or

\[
R_T = (V_{i-\frac{1}{2}, j} - V_{i+\frac{1}{2}, j}) \Delta y \Delta z + (V_i, j-\frac{1}{2} - V_i, j+\frac{1}{2}) \Delta x \Delta y
\]

But

\[
R_T = \frac{\partial \theta}{\partial t} \Delta x \Delta y \Delta z
\]

therefore

\[
\frac{\partial \theta}{\partial t} = -\left[ \frac{V_{i+\frac{1}{2}, j} - V_{i-\frac{1}{2}, j}}{\Delta x} \right] + \frac{V_i, j+\frac{1}{2} - V_i, j-\frac{1}{2}}{\Delta z}
\]

i.e.

\[
\frac{\partial \theta}{\partial t} = -\left[ \frac{\partial}{\partial x} (V_x) + \frac{\partial}{\partial z} (V_z) \right]
\]

From Darcy's law:

\[
V_x = -K(\theta) \frac{\partial \phi}{\partial x}
\]

\[
V_z = -K(\theta) \frac{\partial \phi}{\partial z}
\]

where \( \phi \) (cm) is the total potential to be defined shortly. Substituting equations (13) and (14) into (12), we have:

\[
\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} \left[ K(\theta) \frac{\partial \phi}{\partial x} \right] + \frac{\partial}{\partial z} \left[ K(\theta) \frac{\partial \phi}{\partial z} \right]
\]

which is the transient equation of moisture flow through porous materials.
**Definition of Total Potential**

Consider the point \( P(x, z) \) in Figure 4; the total potential or the hydraulic head, \( \phi \), at any point \( P \) is given by:

\[
\phi (x, z) = h(x, z) + z' \quad \cdots (16)
\]

where \( h(x, z) \) is the pressure head, and \( z' \) is the elevation of the point \( P \) from the datum plane. From Figure 4,

\[
z' = r_1 + r_2
\]

Define

\[
S = L - x - r
\]

it follows that

\[
S = L - x - z \tan \alpha
\]

But

\[
r_1 = S \sin \alpha = (L - x) \sin \alpha - \frac{z \sin^2 \alpha}{\cos \alpha}
\]

and

\[
r_2 = z \cos \alpha
\]

therefore

\[
z' = (L - x) \sin \alpha + z \cos \alpha
\]

The total potential, equation (16), is therefore given by:

\[
\phi(x, z) = h(x, z) + (L - x) \sin \alpha + z \cos \alpha \quad \cdots (17)
\]
Substituting the expression of equation (17) into (15), we have:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} \left( K(\theta) \left( \frac{\partial h}{\partial x} - \sin \alpha \right) \right) + \frac{\partial}{\partial z} \left( K(\theta) \left( \frac{\partial h}{\partial z} + \cos \alpha \right) \right).$$

(18)

Defining $C_l(h) = \frac{\partial \theta}{\partial h}$, the left hand side of equation (18) can be expressed in terms of the moisture capacity $C_l$, and the capillary pressure $h$; equation (18), therefore, becomes:

$$C_l(h) \frac{\partial h}{\partial t} = \frac{\partial}{\partial x} \left( K(h) \left( \frac{\partial h}{\partial x} - \sin \alpha \right) \right) + \frac{\partial}{\partial z} \left( K(h) \left( \frac{\partial h}{\partial z} + \cos \alpha \right) \right).$$

(19)

**Dimensionless Variables**

For convenience, the following dimensionless parameters are used,

\[ \phi' = \frac{h}{L} \quad \ldots \quad (20) \]

\[ \xi = \frac{z}{L} \quad \ldots \quad (21) \]

\[ \omega = \frac{x}{L} \quad \ldots \quad (22) \]

\[ \theta = \frac{\theta}{\theta_s} \quad \ldots \quad (23) \]

\[ \kappa (\phi') = \frac{K(h)}{K_s} \quad \ldots \quad (24) \]

\[ \tau = \frac{K_s}{\theta_s L} t \quad \ldots \quad (25) \]

where \( \theta_s \) is the saturated moisture content and \( K_s \) is the saturated hydraulic conductivity. It follows that the dimensionless form of \( Cl \) is given by \( Cl' = \frac{\partial \theta}{\partial \phi} \). Making appropriate substitutions, the total potential, equation (17), in dimensionless form, becomes:

\[ \phi''(\omega, \xi) = \phi'(\omega, \xi) + (1-\omega) \sin \alpha + \xi \cos \alpha \quad \ldots \quad (26) \]

Also substituting equations (20) - (26) into equation (19), the dimensionless form of (19) can be obtained. Equation (19) is reproduced below but should now be considered dimensionless:

\[ Cl(h) \frac{\partial h}{\partial t} = \frac{\partial}{\partial x} \left( K(h) \left( \frac{\partial h}{\partial x} - \sin \alpha \right) \right) + \frac{\partial}{\partial z} \left( K(h) \left( \frac{\partial h}{\partial z} + \cos \alpha \right) \right) \quad (27) \]

**Determination of Soil Parameters**

In order to solve equation (27), the relationships between moisture content and capillary pressure (\( \theta, h \)), moisture capacity and capillary pressure (\( Cl, h \)), and hydraulic conductivity and capillary
pressure \((K, h)\), must be determined. At least one of these relationships must be determined experimentally for the watershed in question because there is no simple functional relationship between moisture content and tension, moisture content and conductivity, or tension and conductivity. From an experimentally determined moisture-tension curve other soil parameters are derived as explained below. Experimental moisture-tension curves are available for the different soil types of the North Appalachian Experimental Watershed, Coshocton, Ohio; the moisture-tension data for the different soil types were averaged for use in this analysis. Figure 5 is a plot of the average moisture-tension relationship mentioned above. Figure 6 is the dimensionless form of Figure 5.

The hydraulic conductivity was derived from Figure 5 using the theories of Childs and Collis-George (1950), and Millington and Quirk (1960) as modified by Jackson, Reginato, and Van Bavel (1965). The equation used for this computation was reported by Nwa (1967) and is reproduced below:

\[
K = \frac{\gamma^2 \varepsilon^{4/3}}{2 \rho g \eta n k^2} \left( h_1^{-2} + 3h_2^{-2} + 5h_3^{-2} + \ldots + (2k-1) h_k^{-2} \right). \quad (28)
\]

where

\[
K = \text{hydraulic conductivity (cm/sec)}
\]

\[
\gamma = \text{surface tension of water (dynes/cm = gm/sec}^2\text{)}
\]

\[
\rho = \text{density of water (gm/cc)}
\]

\[
g = \text{gravity (cm/sec}^2\text{)}
\]

\[
\eta = \text{viscosity of water (poise = gm/cm sec)}
\]

\[
k = \text{number of pore classes}
\]
Figure 5. Average experimental moisture-tension curve from Coshocton, Ohio

Figure 6. Dimensionless moisture-tension curve derived from Figure 5
\[ \epsilon = \text{soil porosity up to the moisture content of interest (cc/cc)} \]

\[ h = \text{capillary pressure (cm of H}_2\text{O)} \]

The interval between the maximum and minimum moisture contents (see Fig. 5) is divided into \( k \) equal subintervals; each subinterval is called a pore class, and the average moisture content of each pore class is used as the porosity for that class. Several \( k \) values from 4 through 40 inclusive were tested and the value of 20 was considered adequate for this investigation. The surface tension, density, and viscosity of water were obtained at 15° C. The derived dimensionless conductivity-tension curve is shown in Figure 7.

By numerically differentiating the curve of Figure 6, the moisture capacity-tension function of Figure 8 was obtained. The portion of the curve in Figure 8, from the peak to zero, was arbitrarily drawn to comply with the theory, since \( C_l = \frac{\partial \theta}{\partial h} \), it means that \( C_l \) must be equal to zero at or near saturation. However, the actual field samples used did not display this property. The three curves of Figures 6, 7, and 8 were read into the computer and a non-linear interpolation routine was used to obtain any particular value required.
Figure 7. Dimensionless conductivity-tension curve calculated from Figure 5.

Figure 8. Dimensionless moisture capacity-tension curve derived from Figure 6.
**Initial Conditions**

For the purpose of this analysis, it was assumed initially that the system was at static equilibrium, that is, the total potential at every point of the flow system was zero; this means that the capillary pressure at each point was equal to the negative of the elevation of that point from the datum plane. In other words, the initial condition is given by:

\[ h = -z' \]  

where \( z' \) is the vertical distance of any point from the datum plane (see Figure 4). The above initial condition is not a restriction, any initial condition should be satisfactory.

**Boundary Conditions**

Up to the time that the surface becomes saturated, the upper boundary condition is:

\[ V_z i, J = -RE \cos \alpha \]  

where \( RE \) is equal to \( RI/K_s \), and \( RI \) is the rainfall rate. After the surface is saturated the upper boundary condition is given by:

\[ V_z i, J = -K \frac{\partial \phi}{\partial z} \bigg|_{i, J} \]  

Also, after the surface is saturated, and before the surface runoff begins, it is assumed that water will accumulate on the surface up to a maximum vertical depth, \( h'_{4M} \); this is the sum of the maximum surface retention and the maximum surface detention. \( h'_{4M} \) was assumed to be 1.27 cm. If \( h' \) is the depth of water on the surface at any time, before
$H_4'$ is satisfied, we have:

$$\frac{dH_4'}{dt} = RI - f_2', \quad 0 < H_4' \leq H_4M'$$  \hspace{1cm} (32)

where $f_2'$ is the vertical infiltration rate. Defining $H_4 = H_4'/L$, and making use of the dimensionless parameters of equations (20) through (25), equation (32), in dimensionless form, becomes:

$$\frac{dH_4}{dt} = \theta_s (RE - f_2), \quad 0 < H_4 \leq H_4M$$  \hspace{1cm} (33)

where $f_2$ is equal to $f_2/K_s$. Equation (33) is solved for $H_4$ which is taken as the capillary potential, $h$, on the surface after surface saturation and before surface runoff begins. The value of $H_4$ or $h$ is needed in equation (31) to define the upper boundary condition. When the surface runoff starts, the vertical depth of water on the surface, which is made up of $H_4M$ plus the runoff depth, is used in equation (31) to define the upper boundary condition. The right hand boundary (see figure 2) depends on the existing condition; if there is water in the stream channel, the depth of water, and the capillary fringe are taken into consideration. In this investigation, it was assumed that there was no water standing in the channel but the bottom was assumed to be saturated; the right boundary condition is therefore given by:

$$V_x I, j = -K \frac{\partial \phi}{\partial x} \bigg|_{I, j} \hspace{1cm} (34)$$

The other boundary conditions are as follows:

$$V_x I, j = 0 \hspace{1cm} (35)$$

$$V_z I, l = 0 \hspace{1cm} (36)$$
**Infiltration Rate**

Before the surface is saturated, the vertical infiltration rate, $f_2$, is equal to the rainfall rate, $R_E$. After the surface is saturated, the infiltration rate is determined from Darcy's law as follows:

$$V_z i, J = - K \frac{\partial \phi}{\partial z} \bigg|_{i, J} \quad \cdots \quad (37)$$

i.e.

$$V_z i, J = - K_i, J \frac{\partial \phi}{\partial z} \bigg|_{i, J}$$

$$= - K_i, J + K_i, J \frac{\partial \phi}{\partial z} \bigg|_{i, J}$$

$$= - K_i, J + K_i, J + K_i, J \frac{\partial \phi}{\partial z} \bigg|_{i, J}$$

$$= - K_i, J - 1 + 3 K_i, J \frac{\partial \phi}{\partial z} \bigg|_{i, J}$$

This means that:

$$V_z^n i, J = - K_{i, J} - 1 + 3 K_{i, J} \left( \frac{n-1}{\Delta z} - \frac{n-1}{\Delta z} \cos \alpha \right) \quad \cdots \quad (38)$$

But

$$V_z i, J = - f_2 \cos \alpha \text{ (see Figure 2)}$$

therefore

$$f_2^n i, J = K_{i, J} - 1 + 3 K_{i, J} \left( \frac{n-1}{\Delta z} \cos \alpha \right) \quad \cdots \quad (39)$$

where $n$ denotes the time dimension.
IV. THE ADI PROCEDURE

The ADI method of numerical analysis is an implicit scheme which breaks a system of differential equations into two subsystems. One subsystem is implicit in one direction and explicit in the other, and the second subsystem interchanges the order of the implicit-explicit directions. The two subsystems are solved alternately, the solution being complete for each time step after the second subsystem has been solved.

In this investigation, the soil moisture flow system is considered in three categories: the parabolic, in which the flow region (Figure 2) is completely unsaturated except possibly at the upper and right hand boundaries; the elliptic, in which the flow region is completely saturated; and the parabolic-elliptic category, in which mixed flow is allowed, that is, a portion of the flow region is saturated while the other portion is unsaturated.

The Parabolic System

As long as the flow region remains unsaturated the governing partial differential equation, equation (27), remains parabolic and therefore a solution is possible without iteration. For this case the ADI equivalent of equation (27) are given by equations (40) and (41) below:

30.
\[ \text{Cl}_{1,j}^{n+\frac{1}{2}} = \left( \frac{h_{i,j} - h_{i-1,j}}{\Delta t} \right)^{\frac{n+1}{2}} \]

where \( \text{Cl}_{1,j}^{n+1/2} \) denotes Cl value between the time \( n \) and \( n+1 \); \( \Delta t \) is the length of the time increment, and

\[ n = 0, 1, 2, \ldots, N, \]
\[ t = \frac{n \Delta t}{2} \]

and \( N \) is two times the total number of time increments of the study period.
Using the known parameters of the previous time step, equation (40) is first solved, new parameters are obtained, and then (41) is solved to obtain the required answer for the time step. The process is repeated until \( n \) is equal to \( N \). The details of the solution are described towards the end of this chapter.

The Parabolic-Elliptic System

Four sets of equations and an iteration parameter are required for solving the parabolic-elliptic system. The first two sets, for the portion of the flow medium that is unsaturated, are given by equations (42) and (43) below:

\[
\frac{C_{l_{i,j}}^{n+\frac{1}{2},2m+1}}{2\Delta t} \left( \frac{n+1,2m+1 - n}{\frac{1}{2} \Delta t} \right) + \phi_m \left( \frac{n+1,2m+1 - n}{h_{i,j}} \right) = \frac{n+\frac{1}{2}}{2\Delta x} \left( \frac{n+1,2m+1 - n+1,2m+1}{\Delta x} - \sin \alpha \right) + \frac{n+\frac{1}{2}}{K_{i-1,j} + K_{i,j}} \left( \frac{n+1,2m+1 - n+1,2m+1}{\Delta x} - \sin \alpha \right) + \frac{n}{2\Delta z} \left( \frac{n}{\Delta z} + \cos \alpha \right) - \frac{n}{K_{i,j-1} + K_{i,j}} \left( \frac{n}{\Delta z} + \cos \alpha \right). \tag{42}
\]
In the above two equations, \( m \) is an iteration number and \( \rho \) is an iteration parameter; \( \rho^m \) simply means that we are dealing with \( \rho \) of \( m \)th iteration. The iteration parameter used here was determined by trial and error and is a combination of those used by Rubin (1968) and Amerman (1969) and is given by:

\[
\rho = \frac{k_3}{(\Delta z)^2} \sin^2\left(\frac{I_g \pi}{4 \times XD}\right), \quad I_g = 0, 1, 2, \ldots, m \quad \ldots \quad (44)
\]

where

\[
k_3 = \frac{1}{2} \left( \frac{n+\frac{1}{2}}{K_{i-1,j}} + \frac{n+\frac{1}{2}}{K_{i+1,j}} + \frac{n+\frac{1}{2}}{K_{i,j}} + \frac{n+\frac{1}{2}}{K_{i,j-1}} + \frac{n+\frac{1}{2}}{K_{i,j+1}} \right)
\]

and \( XD \) is the dimensionless form of \( D \) shown in figure 2. The iteration parameter varies cyclically, the same value being used in both equations (42) and (43) for each iteration. Starting with known soil parameters of the previous iteration equation (42), and then (43), is solved; new
soil parameters are obtained after solving equation (43). An iteration is complete for a time step after both equations have been solved. Note that the soil parameters of the 5th and 6th terms of equation (42) remain at the values of the previous time step.

The next two sets of equations for the parabolic-elliptic system are for the portion of the flow medium that is completely saturated. These equations are given below:

\( \rho_m \left( \frac{h_{i,j}^{n+1,2m+1} - h_{i,j}^n}{\Delta t} \right) = \frac{n + \frac{1}{2}}{K_{i,j}} + \frac{n + \frac{1}{2}}{K_{i-1,j}} \left( \frac{h_{i+1,j}^{n+1,2m+1} - h_{i,j}^{n+1,2m+1}}{2\Delta x} - \sin \alpha \right) \)

\( \frac{n + \frac{1}{2}}{2\Delta x} \left( \frac{h_{i,j}^{n+1,2m+1} - h_{i-1,j}^{n+1,2m+1}}{\Delta x} - \sin \alpha \right) \)

\( + \frac{n + \frac{1}{2}}{2\Delta x} \left( \frac{h_{i,j}^{n} - h_{i+1,j}^{n}}{\Delta x} + \cos \alpha \right) \)

\( - \frac{n + \frac{1}{2}}{2\Delta x} \left( \frac{h_{i,j}^{n} - h_{i,j-1}^{n}}{\Delta x} + \cos \alpha \right) \) ...

... (45)
\[\rho^m \left( \frac{n+1, 2m+2 - n+1, 2m+1}{h_{1,j}} \right) = \frac{n+\frac{1}{2}}{k_{i,j} + k_{i,1,j}} \left( \frac{n+1, 2m+1 - n+1, 2m+1}{h_{1,j}} \right) - \sin \alpha \]

\[+ \frac{n+\frac{1}{2}}{k_{i-1,j} + k_{i,j}} \left( \frac{n+1, 2m+2 - n+1, 2m+2}{h_{1,j+1}} \right) \sin \alpha \]

\[+ \frac{n+\frac{1}{2}}{k_{i,j}} \left( \frac{n+1, 2m+2 - n+1, 2m+2}{h_{i,j}} \right) \cos \alpha \]

\[+ \frac{n+\frac{1}{2}}{k_{i,j-1} + k_{i,j}} \left( \frac{n+1, 2m+1 - n+1, 2m+1}{h_{i,j-1}} \right) \cos \alpha \] \text{(46)}

Equations (45) and (46) are solved just like equations (42) and (43).

The iteration parameter is also given by equation (44).

If any point in the flow region is in the unsaturated zone, equations (42) and (43) are used, but if the point is in a saturated zone, equations (45) and (46) are used. If a point is at or near the saturated-unsaturated boundary, the coefficients of the neighboring nodes needed depend on the zone in which the nodes are located.

The Elliptic System

When the flow region of figure 2 becomes completely saturated, equation (27) becomes Laplace's equation:

\[\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \] \text{... (47)}

Equations (48) and (49) below represent the ADI system for solving equation (47):
The iteration parameter, \( \rho \), in the above two equations is the same as the one given in equation (44) except that it does not contain the \( K_3 \) term. Equations (48) and (49) are solved alternately as described for the other systems.

**The Method of Solving the Flow Equations**

When all the terms are collected and combined appropriately, equations (40) - (43), (45), (46), (48), and (49) can be expressed as:

\[
\rho^n \left( \begin{array}{c} h_{i,j}^{n+1,2m+1} - h_{i,j}^{n+1,2m} \\ h_{i,j}^{n+1,2m+1} - h_{i,j}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+1} - h_{i,j}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+1} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+1} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+1} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+1} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+1} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+1} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+1} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+1} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+1} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+1} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+1} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+1} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+1} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+1} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+1} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+1} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+1} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+1} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \end{array} \right) = \frac{h_{i+1,j}^{n+1,2m+1} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1}}{\Delta x^2}
\]

\[
+ \frac{h_{i,j-1}^{n+1,2m} - 2h_{i,j}^{n+1,2m} + h_{i,j+1}^{n+1,2m}}{\Delta z^2}
\]

\[
\rho^n \left( \begin{array}{c} h_{i,j}^{n+1,2m+2} - h_{i,j}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+2} - h_{i,j}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+2} - h_{i,j}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+2} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+2} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+2} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+2} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+2} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+2} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+2} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+2} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+2} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+2} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+2} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+2} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+2} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+2} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+2} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+2} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+2} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+2} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+2} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \\ h_{i,j}^{n+1,2m+2} - 2h_{i,j}^{n+1,2m+1} + h_{i,j+1}^{n+1,2m+1} \end{array} \right) = \frac{h_{i+1,j}^{n+1,2m+2} - 2h_{i,j}^{n+1,2m+2} + h_{i,j+1}^{n+1,2m+2}}{\Delta x^2}
\]

\[
+ \frac{h_{i,j-1}^{n+1,2m+2} - 2h_{i,j}^{n+1,2m+2} + h_{i,j+1}^{n+1,2m+2}}{\Delta z^2}
\]

Where \( A_r, B_r, \) and \( C_r \) are the coefficients of the unknown \( h \) terms, and \( D_r \) is made up of all the known terms of the equations; \( r \) represents \( i \) when the solution in the \( x \)-direction is desired, and \( j \) when the \( z \)-direction solution is sought. Equation (50) can be expressed in a matrix form:

\[
A \mathbf{h} = \mathbf{D}
\]
The matrix $A$ is tridiagonal. For clarity, the solution of equation (51) in one direction, the $x$-direction, will be explained below.

With the help of equation (50), equation (51) can be written in the form:

$$
\begin{bmatrix}
B_1 & C_1 & 0 & 0 \\
A_2 & B_2 & C_2 & 0 \\
0 & A_3 & B_3 & C_3 \\
\vdots & \vdots & \vdots & \vdots \\
0 & A_{I-2} & B_{I-2} & C_{I-2} \\
0 & A_{I-1} & B_{I-1} & C_{I-1}
\end{bmatrix}
\begin{bmatrix}
h_1 \\
h_2 \\
h_3 \\
\vdots \\
h_{I-2} \\
h_{I-1}
\end{bmatrix}
= 
\begin{bmatrix}
D_1 \\
D_2 \\
D_3 \\
\vdots \\
D_{I-2} \\
D_{I-1}
\end{bmatrix}
\quad (52)
$$

The matrix $A$ in equation (52) can be decomposed into the form $LU$, where $L$ and $U$ are lower and upper triangular matrices, respectively.

i.e.

$$
A = LU
\quad \cdot \cdot \cdot (53)
$$
Let
\[ L = \begin{bmatrix}
\lambda_1 & 0 & 0 & \cdots & 0 \\
0 & \lambda_2 & 0 & \cdots & 0 \\
0 & 0 & \lambda_3 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & \sigma_{I-2} & \lambda_{I-2} & 0 \\
0 & 0 & 0 & \cdots & \sigma_{I-1} & \lambda_{I-1} & \cdots & \cdots & \cdots & \cdots \end{bmatrix} \tag{54} \]

and
\[ U = \begin{bmatrix}
1 & B_{11} & 0 & \cdots & 0 \\
0 & 1 & B_{12} & \cdots & 0 \\
0 & 0 & 1 & B_{13} & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \cdots & \cdots & \cdots \\
0 & 0 & 0 & \cdots & 1 & B_{I-1} \\
0 & 0 & 0 & \cdots & 0 & 1 \\
\end{bmatrix} \tag{55} \]

It follows that
\[ LU = \begin{bmatrix}
\lambda_1 & \lambda_1 B_{11} & 0 \\
0 & \sigma_2 B_{11} + \lambda_2 B_{12} & \lambda_2 B_{12} \\
0 & \sigma_3 B_{12} + \lambda_3 B_{13} & \lambda_3 B_{13} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & \sigma_{I-1} B_{I-1} + \lambda_{I-1} B_{I-2} & \cdots & \cdots & \cdots & \cdots \\
\end{bmatrix} \tag{56} \]
Comparing the elements of $A$ and $LU$, we have:

\[
\lambda_1 = B_1
\]

\[
B_{l1} = \frac{C_1}{\lambda_1}
\]

i.e. $B_{l1} = \frac{C_1}{B_1}$ \hspace{1cm} \ldots \ldots (57)

\[
\sigma_1 = A_1
\]

\[
\lambda_1 = B_1 - \sigma_1 B_{l1-1}
\]

i.e. $\lambda_1 = B_1 - A_1 B_{l1-1}$ \hspace{1cm} \ldots \ldots (58)

\[
B_{l1} = \frac{C_1}{B_1 - A_1 B_{l1-1}}
\]

\hspace{1cm} \ldots \ldots (59)

From equations (51) and (53),

$$A h = L U h = D$$

Let

$$U h = G l$$ \hspace{1cm} \ldots \ldots (60)

then

$$L G l = D$$

Making use of the matrix $L$ and the fact that $\sigma_1 = A_1$, we obtain

\[
G l_1 = \frac{D_1}{B_1}
\]

\hspace{1cm} \ldots \ldots (61)

and

\[
G l_1 = \frac{D_1 - A_1 G l_{i-1}}{B_1 - A_1 B_{l1-1}}
\]

\hspace{1cm} \ldots \ldots (62)

By back substitution equation (60) is solved for $h$ beginning with $h_{i-1}$ which is given by:
The rest of the \( h \) values are obtained from:

\[
h_i = G_{li} - B_{li} h_{i+1}
\]  

(64)

Therefore, to solve equation (50) \( B_{lr} \) and \( G_{lr} \) are first computed. For the \( x \)-direction, the computation starts from the left boundary of figure 2. The boundary coefficients are first computed from:

\[
B_{l1} = \frac{C_1}{B_1}
\]  

(57)

\[
G_{l1} = \frac{D_1}{B_1}
\]  

(61)

The remaining of the \( B_l \) and \( G_l \) terms are computed from the following recursive relationships:

\[
B_{li} = \frac{C_i}{B_i - A_i B_{l-1}} , \quad l < i \leq I-2
\]  

(59)

\[
G_{li} = \frac{D_i - A_i G_{l-1}}{B_i - A_i B_{l-1}} , \quad 1 < i \leq I-1
\]  

(62)

The first \( h \) term to be computed is given by:

\[
h_{I-1} = G_{lI-1}
\]  

(63)

The rest of the \( h \) values are computed from:

\[
h_i = G_{li} - B_{li} h_{i+1}
\]  

(64)

It is assumed here that the right hand boundary value \( h_{I-1} \) is known or can be computed by other means. The way the upper and the right boundary values were determined was described previously under boundary...
conditions.

After all the rows have been solved in the above manner, the column values are solved in exactly the same way. More information concerning the solution of the tridiagonal matrix systems can be found in publications by Peaceman and Rachford (1955), Richtmyer and Morton (1967), and Conte (1965).
V. OVERLAND FLOW

The Law of Conservation of Mass

Consider an element of fluid shown in Figure 9; during one time increment, $\Delta t$,

\[
\text{Inflow} = \left( (A - \frac{\partial A}{\partial x} \frac{\Delta x}{2})(u - \frac{\partial u}{\partial x} \frac{\Delta x}{2}) + RI \Delta x \cos \alpha \right) \Delta t
\]

\[
\text{Outflow} = \left( (A + \frac{\partial A}{\partial x} \frac{\Delta x}{2})(u + \frac{\partial u}{\partial x} \frac{\Delta x}{2}) + f_2' \Delta x \cos \alpha \right) \Delta t
\]

Change in storage = $\frac{\partial A}{\partial t} \Delta x \Delta t$

From the law of conservation of mass:

Figure 9. Water element for deriving the continuity equation
Inflow-Outflow = change in storage

It follows that:

\[
\frac{\partial A}{\partial t} = (RI - f_2') \cos \alpha - A \frac{\partial u}{\partial x} - u \frac{\partial A}{\partial x}
\]  \hspace{1cm} (65)

where A is the average area of flow cross-section, RI is the vertical rainfall rate, \( f_2' \) is the vertical infiltration rate, and u is the flow velocity. In Figure 9, y is the vertical depth of water and z is the depth measured perpendicular to the slope.

Taking a unit width,

\[ A = z \]

and equation (65) becomes:

\[
\frac{\partial z}{\partial t} = (RI - f_2') \cos \alpha - z \frac{\partial u}{\partial x} - u \frac{\partial z}{\partial x}
\]  \hspace{1cm} (66)

But \( z \approx y \cos \alpha \)

and so equation (66) becomes:

\[
\frac{\partial y}{\partial t} + u \frac{\partial y}{\partial x} + y \frac{\partial u}{\partial x} = RI - f_2'
\]  \hspace{1cm} (67)

which is the equation of continuity for overland flow.

The Law of Conservation of Momentum

The law of conservation of momentum or Newton's second law of motion states that the resultant of all the body and surface forces acting on the water element in Figure 10, in a given direction, is equal to the change in momentum in that direction. The surface forces are created by the shear on the bottom, the hydrostatic pressure on the bottom and ends, and the surcharge pressure or overpressure on the bottom.
and ends of the element. The surcharge pressure is the pressure in excess of the hydrostatic value and is caused by the difference in vertical momentum flux between the incoming rainfall and the outgoing infiltration. The body forces are caused by the effect of gravity on the water element.

Several distribution models of the surcharge pressure have been suggested or adopted; one assumption is that the surcharge pressure is constant with depth, a second assumption is that it varies directly with depth, and a third suggested model is that the excess pressure starts at zero at the surface, varies non-linearly for a short distance below the surface, and then remains constant for the rest of the depth. These assumptions are discussed by Lyons (1967) and by Grace and Eagleson (1965). As shown in Figure 10, the assumption that the excess pressure is constant with depth is adopted in this study.

Referring to a unit width of the element in Figure 10, the components of the forces acting on the four faces in the x-direction are as follows:

\[
F_x(1) = F - \frac{\partial F}{\partial x} \frac{\Delta x}{2} + (P_s - \frac{\partial P_s}{\partial x} \frac{\Delta x}{2})(z - \frac{\partial z}{\partial x} \frac{\Delta x}{2})
\]

\[
F_x(2) = 0
\]

\[
F_x(3) = -F - \frac{\partial F}{\partial x} \frac{\Delta x}{2} - (P_s + \frac{\partial P_s}{\partial x} \frac{\Delta x}{2})(z + \frac{\partial z}{\partial x} \frac{\Delta x}{2})
\]

\[
F_x(4) = -F_f + P_s \Delta x \tan \alpha + W \sin \alpha
\]

By summing the 4 forces we have:

\[
\sum F_x = -\frac{\partial F}{\partial x} \Delta x - z \frac{\partial P_s}{\partial x} \Delta x - \Delta x P_s(\frac{\partial z}{\partial x} - \tan \alpha) - F_f + W \sin \alpha . (68)
\]
where

$$\Sigma F_x = \text{resultant of all the forces acting on the element in the x-direction}$$

$$F = \text{force acting on the element}$$

$$F_f = \text{total external frictional force resisting flow}$$

$$P_s = \text{surcharge pressure; the excess pressure distribution is assumed constant with depth}$$

$$W = \text{weight of water in the control volume}$$

The total change in momentum in the x-direction is determined as follows:

Figure 10. Water element for deriving the momentum equation
The change of momentum of the flowing fluid is given by:

$$ M_u = m \frac{du}{dt} \quad \ldots \quad (69) $$

where $m$ is the mass of the flowing fluid in the control volume and is equal to $\frac{W}{g}$, $g$ is the acceleration due to gravity, and $u$ is the flow velocity; the momentum coefficient is assumed to be unity. The acceleration term, $\frac{du}{dt}$, can be expressed in the form:

$$ \frac{du}{dt} = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} \frac{dx}{dt} $$

or

$$ \frac{du}{dt} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \quad \ldots \quad (70) $$

Substituting equation (70), into (69), we have:

$$ M_u = \frac{W}{g} \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) \quad \ldots \quad (71) $$

The change in momentum due to the rainfall inflow is expressed as:

$$ M_{RI} = \frac{\mu \Delta x}{g} \frac{RI \cos \alpha}{g} (u - RI \sin \alpha) $$

and the change in momentum due to the infiltration outflow is given by:

$$ M_{f_2} = -\frac{\mu \Delta x}{g} \frac{f_2' \cos \alpha}{g} (u - f_2' \sin \alpha) $$

where $RI$ is the vertical rainfall rate, $f_2'$ is the vertical infiltration rate, and $\mu$ is the unit weight of water. The total change in momentum therefore is given by:
From the above law of conservation of momentum

\[ \Sigma F_x = M_T \]

it follows that:

\[
- \frac{\partial F}{\partial x} \Delta x - z \frac{\partial P_s}{\partial x} \Delta x - \Delta x P_s (\frac{\partial z}{\partial x} - \tan \alpha) - F_f + W \sin \alpha
\]

\[
= \frac{W}{g} (\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x}) + \frac{\mu \Delta x u \cos \alpha}{g} (RI - f_2') - \frac{\mu \Delta x \cos \alpha \sin \alpha}{g} (RI^2 - f_2'^2)
\]

Most of the authors who have worked on the overland flow problems (see References) have ignored the overpressure terms because of the difficulties involved in evaluating them; several of the authors have also ignored a number of the other terms and have been satisfied with the results. In this analysis the surcharge pressure terms will be ignored as well as the 2nd and 3rd terms on the right hand side of equation (72). This equation therefore becomes:

\[
- \frac{\partial F}{\partial x} \Delta x = F_f + W \sin \alpha = \frac{W}{g} (\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x})
\]

Referring to Figure 10:

\[ F = \frac{\mu}{2} z^2 \]

\[ \frac{\partial F}{\partial x} = \mu z \frac{\partial z}{\partial x} \]

\[ W = \mu z \Delta x \]
Since
\[ z = y \cos \alpha \]
\[ \frac{\partial z}{\partial x} = \cos \alpha \frac{\partial y}{\partial x} \]

we have:
\[ \frac{\partial F}{\partial x} = \mu y \cos^2 \alpha \frac{\partial y}{\partial x} \]
\[ W' = \mu y \Delta x \cos \alpha \]

Assuming that the friction slope of a uniform steady flow can be used for a non-uniform unsteady flow of the same depth and average velocity,
\[ F_f = \mu z \Delta x S_f \]
or
\[ F_f = \mu y \Delta x \cos \alpha S_f \]  \hspace{1cm} \ldots (74) \]

where \( S_f \) is the friction slope or the slope of the energy grade line.

Substituting the above relationships into equation (73) and rearranging we have:
\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \cos \alpha \frac{\partial y}{\partial x} = g (\sin \alpha - S_f) \]  \hspace{1cm} \ldots (75) \]

which is the momentum equation used in this analysis. Other derivations of the momentum and continuity equations can be found in the publications by Grace and Eagleson (1965), and Lyons (1967).

From Manning's formula:
\[ S_f = \frac{n^2}{2.2082} \frac{u^2}{y^{4/3}} \]  \hspace{1cm} \ldots (76) \]
where \( n \) is Manning’s roughness coefficient and the hydraulic radius is taken to be equal to the depth of flow, \( y \).

If the flow is assumed to be laminar, Albertson, Barton and Simons (1960) define the friction slope, \( S_f \), as:

\[
S_f = \frac{3n u}{\mu y^2}
\]  

(77)

where \( n \) is the dynamic viscosity and \( \mu \) is the unit weight of water.

From the fact that

\[
\mu = \rho g
\]

and

\[
\nu = \frac{\eta}{\rho}
\]

equation (77) becomes:

\[
S_f = \frac{3\nu u}{g y^2}
\]  

(78)

where \( \rho \) is the density, and \( \nu \) is the kinematic viscosity of water. If a non-laminar flow is assumed, equation (76) is used for the friction slope, if a laminar flow is assumed (78) is used.

**Overland Flow Assumptions**

The assumptions made in deriving the overland flow equations are summarized as follows:

1. The flow cross-section has a unit width.
2. The pressure distribution in the flow section is hydrostatic, i.e., the pressure distribution coefficient = 1.
3. The momentum correction coefficient = 1.
4. The surcharge pressure can be ignored.
5. The change in momentum in the direction of flow due to the incoming rainfall and the outgoing infiltration is negligible.

6. The average velocity is uniform.

7. The hydraulic radius, the ratio of the water cross-sectional area to its wetted perimeter, is equal to the depth of flow.

8. The friction slope of a uniform steady flow can be used for a non-uniform unsteady flow of the same depth and average velocity.

**Overland Flow Equations in Dimensionless Form**

In addition to the following dimensionless variables of Chapter 3,

\[ \omega = \frac{x}{L} \]

\[ \tau = \frac{K_s}{\theta_s L} t \]

\[ \text{RE} = \frac{RI}{K_s} \]

\[ f_2 = \frac{f_2'}{K_s} \]

define

\[ \psi = \frac{y}{L} \]

\[ u' = \frac{u}{u_p} \]

If follows that:

\[ \frac{\partial \psi}{\partial t} = \frac{K_s}{\theta_s} \frac{\partial \psi}{\partial \tau} \]

\[ \frac{\partial u}{\partial t} = \frac{u_p}{L} \frac{K_s}{\theta_s} \frac{\partial u'}{\partial \tau} \]

\[ \frac{\partial u}{\partial x} = \frac{u_p}{L} \frac{\partial u'}{\partial \omega} \]
Substituting the above equations into (67) and (75), we have:

\[ \frac{1}{\sigma_s} \frac{\partial \psi}{\partial t} + \frac{u}{K_s} \frac{\partial u}{\partial \xi} + \frac{u}{K_s} \frac{\partial \psi}{\partial \omega} = RE - f_2 \]  \tag{79}

\[ \frac{1}{\sigma_s} \frac{\partial u}{\partial t} + \frac{u}{K_s} \frac{\partial u}{\partial \xi} + \frac{g L \cos \alpha}{up K_s} \frac{\partial \psi}{\partial \omega} = \frac{g L \sin \alpha}{up K_s} \frac{u^2}{2.2062 K_s L^{1/3}} \frac{1}{\psi^{4/3}} \]  \tag{80}

i.e.

\[ A_1 \frac{\partial \psi}{\partial t} + A_2 u \frac{\partial \psi}{\partial \xi} + A_2 y \frac{\partial u}{\partial \xi} = RE - f_2 \]  \tag{81}

\[ A_1 \frac{\partial u}{\partial t} + A_2 u \frac{\partial u}{\partial \xi} + A_3 \frac{\partial \psi}{\partial \xi} = A_4 - A_5 \frac{u^2}{y^{4/3}} \]  \tag{82}

The factors \( A_1, A_2, A_3, A_4, \) and \( A_5 \) are the coefficients shown in equations (79) and (80). Equations (81) and (82) are still considered dimensionless, and they are the equivalent of the overland flow equations given by (67) and (75).

**Methods of Solving Overland Flow Equations**

As indicated previously, the methods described by Morgali and Linsley (1965) are used to solve the overland flow equations in this study. The parts of these methods used here are explained in the following sections.

**The Method of Characteristics**

Since \( y \) and \( u \) are functions of both time and distance, the total derivatives, \( \partial y/\partial t \) and \( \partial u/\partial t \), can be written as follows:
The momentum and continuity equations, equations (82) and (81), together with the above expressions for the total derivatives, form a system of equations that are used to route water over a surface by the method of characteristics. The system of equations are reproduced below:

\[
A_1 \frac{\partial y}{\partial t} + A_2 u \frac{\partial y}{\partial x} + A_2 y \frac{\partial u}{\partial x} = RE - f_2 \tag{81}
\]

\[
A_1 \frac{\partial u}{\partial t} + A_2 u \frac{\partial u}{\partial x} + A_3 y \frac{\partial y}{\partial x} = A_4 - A_5 \frac{u^2}{y^{4/3}} \tag{82}
\]

\[
\frac{\partial y}{\partial t} dt + \frac{\partial y}{\partial x} dx = dy \tag{83}
\]

\[
\frac{\partial u}{\partial t} dt + \frac{\partial u}{\partial x} dx = du \tag{84}
\]

Equations (81) - (84) above can be written in the following matrix form:

\[
\begin{pmatrix}
A_1 & A_2 u & A_2 y & 0 \\
0 & A_3 & A_2 u & A_1 \\
dt & dx & 0 & 0 \\
0 & 0 & dx & dt
\end{pmatrix}
\begin{pmatrix}
\frac{\partial y}{\partial t} \\
\frac{\partial u}{\partial x} \\
\frac{\partial y}{\partial x} \\
\frac{\partial u}{\partial t}
\end{pmatrix}
= \begin{pmatrix}
RE - f_2 \\
A_4 - A_5 \frac{u^2}{y^{4/3}} \\
dy \\
du
\end{pmatrix} \tag{85}
\]

If the determinant of the elements of the square matrix of equation (85) does not vanish, then equations (81) - (84) have a unique solution. If the determinant vanishes then the equations have an infinite number of
solutions, and the equations are classified as elliptic, parabolic, or hyperbolic, depending on the criteria explained below.

Setting the determinant of the square matrix equal to zero, expanding, and collecting the terms, we have:

\[
\left[ \frac{dx}{dt} \right]^2 - \frac{2}{A_1} \frac{A_2 u}{dx} \left[ \frac{dx}{dt} \right] + \left[ \frac{A_2 u}{A_1} \right]^2 \frac{A^2 A^2 y}{(A_1)^2} = 0 \quad \ldots \quad (86)
\]

Equation (86) is a quadratic in \( \frac{dx}{dt} \). If the discriminant of (86) is negative, zero, or positive, we will have two complex roots, one real root, or two real roots accordingly. These are the criteria that classify equations (81) - (84) as elliptic, parabolic, or hyperbolic, respectively.

The discriminant of equation (86) is given by:

\[
\left[ \frac{2}{A_1} \frac{A_2 u}{dx} \right]^2 - \left[ \frac{2}{A_1} \frac{A_2 u}{dx} \right]^2 \frac{A_2 A_3 y}{(A_1)^2} = A_2 A_3 y \quad (A_1)^2
\]

which is a positive quantity. Equations (81) - (84) are therefore hyperbolic. Solving (86) for \( \frac{dx}{dt} \), we have:

\[
\frac{dx}{dt} = \frac{A_2}{A_1} u \pm \sqrt{\frac{A_2 A_3}{(A_1)^2} y} \quad \ldots \quad (87)
\]

i.e.

\[
\frac{dx}{dt} \bigg|_x = \frac{A_2}{A_1} u + \sqrt{\frac{A_2 A_3}{(A_1)^2} y} \quad \ldots \quad (88)
\]

\[
\frac{dx}{dt} \bigg|_y = \frac{A_2}{A_1} u - \sqrt{\frac{A_2 A_3}{(A_1)^2} y} \quad \ldots \quad (89)
\]

This means that at every point in the solution domain there are two slopes. The direction \( \frac{dx}{dt} \) is known as the characteristic direction. A curve which
at every point has the slope given by equation (88) is called an α characteristic, and a curve whose slope at every point is given by equation (89) is known as a β characteristic.

Consider the solution domain shown in figure 11; assuming that the values of $u$ and $y$ at the time $n$ had been determined, the time step $\Delta t$ must be such that the values of $u$ and $y$ at the time $n+1$ lie within the unique solution domain. There will be no convergence if this is not the case.

Figure 11. Solution domain for overland flow
If equations (88) and (89) are satisfied, the solutions obtained for \(u\) and \(y\) will lie within the unique solution domain. In this study, the two equations were used to check whether convergent or divergent solutions were being obtained.

**An Explicit Method**

The following finite difference expressions are written with reference to the point \(i\) on the x-axis in figure 12.

\[
\frac{\partial y}{\partial t} = \frac{y_{i+1}^n - y_i^n}{\Delta t}
\]

\[
\frac{\partial y}{\partial x} = \frac{y_{i+1}^n - y_{i-1}^n}{2 \Delta x}
\]

\[
\frac{\partial u}{\partial x} = \frac{u_{i+1}^n - u_{i-1}^n}{2 \Delta x}
\]

\[
\frac{\partial u}{\partial t} = \frac{u_{i+1}^n - u_i^n}{\Delta t}
\]

Substituting the above equations into (81), we have:

\[
y_{i+1}^n = y_i^n + \frac{\Delta t}{A_1} (RE - f_2) - \frac{A_2 \Delta t u_i^n}{2 A_1 \Delta x} \left( \frac{y_{i+1}^n - y_{i-1}^n}{2} \right) - \frac{A_2 \Delta t y_i^n}{2 A_1 \Delta x} \left( \frac{u_{i+1}^n - u_{i-1}^n}{2} \right)
\]

Also substituting the above equations into (82), we obtain:
Figure 12. Overland flow cross-section

\[
\begin{align*}
\left[ \frac{u_{i+1}}{u_1} \right]^2 + \frac{A_1 \left( \frac{y_1}{A_5} \right)^{4/3}}{\Delta t} & \quad u_{i+1} \\
+ \frac{n+1}{\left( \frac{y_1}{A_5} \right)^{4/3}} & \quad \left[ \frac{A_2 u_i}{2 \Delta x} \left( \frac{u_{i+1} - u_{i-1}}{u_{i+1} - u_{i-1}} \right) \right] \\
+ \frac{A_3}{2 \Delta x} \left( \frac{u_{i+1} - u_{i-1}}{2 \Delta x} \right) = 0 & \quad \ldots \quad (91)
\end{align*}
\]

Equation (91) is a quadratic in \( u_{i} \). Solving it for \( u_{i} \) and disregarding the negative root, which would give negative velocities, we have:
\[ u_{i}^{n+1} = - \frac{Al \left( \frac{n+1}{y_{i}} \right)^{4/3} A5 \Delta t}{2 A5 A5} \]
\[ + \left[ \left( \frac{Al \left( \frac{n+1}{y_{i}} \right)^{4/3} A5 \Delta t}{2 A5} \right)^{2} - \left( \frac{n+1}{y_{i}} \right)^{4/3} \right] \left( \frac{A2 u_{i}^{n}}{2 \Delta x} \right) \left[ \frac{u_{i+1}^{n} - u_{i-1}^{n}}{A_{i}} \right] \]
\[ + \frac{A3}{2 \Delta x} \left[ \frac{n}{y_{i+1} - y_{i-1}} - \frac{Al^{n}}{\Delta t u_{i} - A_{i}} \right] \]
\[ \quad \frac{1}{2} \quad . . . \quad (92) \]

In deriving equation (92), it is assumed that the friction slope, equation (76), for the time increment \( \Delta t \) is valid at the end of the increment.

Therefore, equation (90) is first solved for \( y_{i}^{n+1} \) and this value of \( y \) is used in (92) to obtain \( u_{i}^{n+1} \).

**Initial Conditions**

After the depression and detention storages have been satisfied, surface runoff is ready to start; this time is considered the zero time to start solving equations (90) and (92). At that time \( n = 1 \), and

\[ y_{1}^{1} = 0 \]
\[ u_{1}^{1} = 0 \]

**Boundary Conditions**

**The Upstream Boundary**

At the upstream boundary (see figure 12) the velocity is assumed to be zero

i.e.

\[ u_{1}^{n+1} = u_{1}^{n} = 0 \]

Consider the section between the points \( i = 1 \) and \( i = 2 \) in figure 12; let STOR be the volume of storage in this section at any time. From the law
of conservation of mass:

\[ \text{STOR}^{n+1} = \text{Inflow}^{n+1} - \text{Outflow}^{n+1} + \text{STOR}^n \]

i.e.

\[ \text{STOR}^{n+1} = (\text{RE} - f_2)^{n+1} \Delta x \Delta t - y_2^{n+1} u_2^{n+1} \Delta t + \text{STOR}^n \quad \ldots \quad (93) \]

But for the unit width section,

\[ \text{STOR}^{n+1} = \frac{y_1^{n+1} + y_2^{n+1}}{2} \Delta x \quad \ldots \quad (94) \]

Therefore

\[ y_1^{n+1} = 2 (\text{RE} - f_2)^{n+1} \Delta t - y_2^{n+1} \left[ 1 + \frac{2 \Delta t}{\Delta x} u_2^{n+1} \right] + \frac{2}{\Delta x} \text{STOR}^n \quad (95) \]

Since \( y_2^{n+1} \) had been obtained from (90), \( y_1^{n+1} \) is the only unknown in equation (95). If the value of \( y_1^{n+1} \) computed from (95) is less than zero, it is set equal to zero and \( y_2^{n+1} \), and then \( u_2^{n+1} \), are recalculated from equations (90) and (92), respectively.

**The Downstream Boundary**

For the downstream boundary, the following finite difference approximations were used:

\[ \frac{\partial y}{\partial t} = \frac{y_1^{n+1} - y_1^n}{\Delta t} \]

\[ \frac{\partial y}{\partial x} = \frac{y_1^n - y_1^{n-1}}{\Delta x} \]

\[ \frac{\partial u}{\partial x} = \frac{u_1^n - u_1^{n-1}}{\Delta x} \]
\[
\frac{\partial u}{\partial t} = \frac{u_n^{n+1} - u_n^n}{\Delta t}
\]

Substituting the above equations into (81) and (82), we obtain equations (96) and (97) below which are similar to equations (90) and (92).

\[
y_I^{n+1} = y_I^n + \frac{\Delta t}{A_l} (R_E - r_2) - \frac{A_2 \Delta t}{A_l \Delta x} \left[ u_I^n - u_I^{n+1} \right] - \frac{A_2 \Delta t y_I^n}{A_l \Delta x} \left[ u_I^n - u_{I-1}^n \right] \quad \ldots \ldots \quad (96)
\]

\[
u_I^{n+1} = - A_l \left( \frac{n+1}{y_I^n} \right)^{4/3}
+ \left( \frac{A_l y_I^n}{2 A_5 \Delta t} \right)^2 - \left( \frac{y_I^n}{A_5} \right)^{4/3} \left( \frac{A_2 u_I^n}{A \Delta x} \left( u_I^n - u_{I-1}^n \right) \right)
+ \frac{A_2}{\Delta x} \left[ u_I^n - u_{I-1}^n \right] \left( \frac{A_1 u_I^n}{A_l \Delta x} - A_1 \right) \left( y_I^n - y_{I-1}^n \right) \quad \ldots \ldots \quad (97)
\]

The outflow at the downstream end is given by:

\[
Q_I^{n+1} = y_I^{n+1} u_I^{n+1}
\quad \ldots \ldots \quad (98)
\]

Since \( y \) and \( u \) in (98) are dimensionless, \( Q \) is also dimensionless and can be expressed as:

\[
Q = \frac{Q'}{Q_o}
\]

where \( Q' \) is the dimensional flow rate and \( Q_o = U P L \). The parameters \( U P \) and \( L \) are the same factors used for the dimensionless analysis in this chapter.

**Solution Procedure**

The general procedure for solving the overland flow equations is
as follows:

1. Initial values are first determined.
2. Equations (90) and (92) are solved for \( i = 2 \) through \( i = I-1 \).
3. Equation (95) is solved for \( i = 1 \).
4. Equations (96), (97), and (98) are solved for \( i = I \).
5. If the convergence conditions of equations (88) and (89) are met, the solution moves on to the next time step, otherwise \( \Delta t \) or \( \Delta x \) is adjusted and the above procedure is repeated until the end of the solution period is reached.
VI. THE STRATEGY FOR LINKING THE COMPONENTS OF THE MODEL

The equations governing the soils phase of this study were analyzed under three major categories -- the parabolic, the elliptic, and the parabolic-elliptic category; and the overland flow equations were said to be hyperbolic. Invariably, the solution of one system is needed to solve the next system. It is the solution of the equations in each system, and the logical linking of all the solutions in the systems, that make up the model. The logical flow chart of Figure 13 shows the decision points at which the solutions of the different systems are linked to produce the model. The symbols in the figure are as previously defined; the terms IOUT2 and IOUT5 are used for recalling decisions that were made during previous time steps. The detailed methods of solving the equations of the different systems were given in the previous chapters; this chapter is only concerned with the general procedure and the decision points mentioned above.

The general procedure is described below; this is not a step by step description. The points mentioned here will refer to the points of the flow geometry of Figure 2.

1. Equations (40) and (41) are solved if no point, other than (I,1), is saturated (the point (I,1) was initially assumed to be saturated). Once any other point is saturated, equations
Figure 13. The Solution Strategy.
(40) and (41) are no longer used.

2. If any point other than (1,1) is saturated, we will have to deal with the parabolic-elliptic system. In this system, if a point is not saturated, equations (42) and (43) are solved but if the point is saturated, then equations (45) and (46) are solved.

3. If all the points are saturated, equations (48) and (49) of the elliptic system are solved.

4. If the surface soil is saturated, equation (39) is used to find the infiltration rate and equation (33) is then solved to find the depth of the depression storage. Equation (33) is used until the depth at the point (1,J) is equal to H^M, the maximum depression storage. If other points on the surface attain the depth of H^M before the point (1,J), their depths are set at H^M until the point (1,J) also attains this depth.

5. After the maximum depression storage has been satisfied at (1,J), the surface runoff equations, equations (81) and (82) are solved, and the flow at the outlet computed. At this time the depth of water at any point on the surface is equal to H^M plus the overland flow depth at that point.
VII. KIRKHAM'S ANALYTICAL SOLUTION OF A DITCH DRAINAGE PROBLEM FOR COMPARISON WITH THE COMPUTER SOLUTION

The Ditch Drainage Problem

The theory of seepage into auger holes is the basis of the ditch drainage problem to be described in this chapter. Kirkham and van Bavel (1948) solved analytically the problem of seepage into an auger hole that just reaches or penetrates the impervious layer. The solution started immediately after the hole had been completely or partially emptied; previous to this the water in the hole was at the level of the water table. In solving the problem the authors assumed that the soil was homogeneous down to the impermeable layer; that Darcy's law is valid; and that Laplace's equation is applicable. They obtained formulae for the potential and the stream functions and for the rate of rise of water in the auger hole.

Noting that the auger hole solution is exact for auger holes of all possible radii, including an infinite radius, Kirkham (1950) used this solution to solve the problem of seepage into parallel, equally spaced ditches. The ditch problem is illustrated in Figure 14. The ditches are equally spaced, 2s being the distance between them from wall to wall; d is the depth to the impermeable layer from the surface; and h is the height of water in the ditch above the impermeable layer. The soil
surface is maintained at complete saturation by rainfall or by some other means. Water seeps through the soil into the ditch and any excess amount flows into the ditch as surface runoff; there is no accumulation on the surface. Since \( h \) is constant it means that water runs out of the ditch at a rate equal to the inflow rate, i.e. steady state prevails. The solution is for a unit length of ditch. The same notations used by Kirkham are used in this chapter even though they may conflict with the notations used in the rest of the dissertation; this makes comparison easier.

The equation solved by Kirkham (1950), Laplace's equation, is given by:

\[
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \quad \cdots (99)
\]

where \( \phi \) is the total potential (hydraulic or piezometric head) at any point \((x,y)\) in the flow region. Using the dimensionless variables of Chapter 3, equation (99) remains the same but should now be considered

![Diagram of Kirkham's ditch drainage problem](image)

**Figure 14.** Kirkham's ditch drainage problem.

The ditches are equally spaced and they penetrate into an impermeable layer; the soil surface is maintained at saturation.
dimensionless. In Figure 14, the impervious layer is the datum plane; the origin for solving equation (99) was taken at A'; x increases to the right and y increases upwards. Kirkham's solution was for the section ABA'B'. In order to satisfy the boundary conditions of Figure 1 for comparison purposes, it was necessary to solve equation (99) for the section CDC'D' of Figure 14. Fortunately the solution field for this section is simply a mirror image of the solution field for section ABA'B'; the solution and its image are discussed later in this chapter. Figure 1 and the section CDC'D' become one and the same if:

\[ y = z \]
\[ \alpha = 0 \]
\[ s = L \]

**Boundary Conditions**

**Kirkham's Boundary Conditions**

Kirkham solved equation (99) analytically for the section ABA'B' of Figure 14. The origin is at A'; x increases to the right and y increases upwards. His boundary conditions are as follows:

\[ \phi = h, \quad 0 \leq y \leq h \quad x = 0 \]
\[ \phi = y, \quad h \leq y \leq d \quad x = 0 \]
\[ \phi = d, \quad y = d \quad 0 \leq x \leq s \]
\[ \frac{\partial \phi}{\partial x} = 0, \quad 0 \leq y \leq d \quad x = s \]
\[ \frac{\partial \phi}{\partial y} = 0, \quad y = 0 \quad 0 \leq x \leq s \]

**Boundary Conditions for the Computer Solution**

The computer solution of equation (99) was for the section CDC'D' of Figure 14. The origin is at C'; x increases to the right and y
increases upwards. The boundary conditions for the computer solution are as follows:

\[ \begin{align*}
\phi &= h , \quad 0 \leq y \leq h \quad x = s \\
\phi &= y , \quad h \leq y \leq d \quad x = s \\
\phi &= d , \quad y = d \quad 0 \leq x \leq s \\
\frac{\partial \phi}{\partial x} &= 0 , \quad 0 \leq y \leq d \quad x = 0 \\
\frac{\partial \phi}{\partial y} &= 0 \quad y = 0 \quad 0 \leq x \leq s
\end{align*} \]

Kirkham's analytical solution field for equation (99) for the section ABA'B' of Figure 14 is simply a mirror image of the solution field for the same equation for the section CDC'D' by the ADI method of this study subject to the above boundary conditions. The results of the analytical and computer solutions are discussed in the next chapter.

**The Analytical Solution**

Subject to the boundary conditions given on page 72 the analytical solution of equation (99) for the equipotentials, \( \phi \), was reported by Kirkham (1950, 1957) and is given by:

\[
\begin{align*}
\phi &= d - \frac{8d}{\pi^2} \left( \cos \frac{\pi h}{2d} \cos \frac{\pi y}{2d} \cosh \frac{\pi}{2d} \cosh \frac{\pi s}{2d} \cosh \frac{3\pi (s - x)}{2d} \right) \\
&\quad + \frac{1}{3^2} \cos \frac{3\pi h}{2d} \cos \frac{3\pi y}{2d} \cosh \frac{3\pi s}{2d} \cosh \frac{2d}{2d} \\
&\quad + \frac{1}{5^2} \cos \frac{5\pi h}{2d} \cos \frac{5\pi y}{2d} \cosh \frac{5\pi s}{2d} \cosh \frac{2d}{2d} + \ldots \right) \quad (100)
\end{align*}
\]
where the variables are as previously defined and as shown in Figure 14. He constructed a flow net for ditches 20 arbitrary units deep, 80 units apart and half filled with water. In other words:

\[ d = 20 \]
\[ h = 10 \]
\[ s = 40 \]

The equipotentials were arbitrarily labelled from 0 to 100 in steps of 10. Figure 15 shows some of Kirkham's and the reflected equipotentials for the above case.
Figure 15. Equipotentials for the ditch drainage problem when $d = 20$, $h = 10$, and $s = 40$ arbitrary units.
VIII. RESULTS AND DISCUSSION

Theoretical Results

Soil Moisture Movement

An arbitrary length, L, of 304.80 cm (10 ft.) and depth, D, of 121.92 cm (4 ft.) were chosen for the dimensions of the flow geometry of Figure 2; the dimensionless length and depth were 1.0 and 0.40, respectively. The saturated volumetric moisture content, $\theta_s$, was 46.58%, and the saturated hydraulic conductivity, $K_s$, was 0.0006676 cm/sec. These and the other soil properties used in this study were obtained from the North Appalachian Experimental Watershed, Coshocton, Ohio.

A great deal of the study effort went into the investigation of the moisture movement in porous materials. Specifically, an attempt was made to determine whether the Alternating Direction Implicit (ADI) method of numerical analysis could be used to solve the problem of moisture movement through a watershed section. This method was reported to be stable and to be faster than the better known over-relaxation methods. The methods had been used with success by others to solve problems in flow regions that were either completely saturated or completely unsaturated, but at least one report indicated the failure of the method in handling as one unit, a flow region that was partly saturated and
partly unsaturated. In a lot of flow situations, it would be extremely
cumbersome to attempt to break the problem into two sub-regions, one
saturated and the other unsaturated, and then to handle each sub-region
separately. One would have to decide how to handle the moving bound-
aries that are usually involved. The main interest in the ADI method,
therefore, was to determine whether, in addition to its reported
advantages, it can handle as a unit, the mixed saturated-unsaturated
flow problems mentioned above.

In this part of the analysis overland flow was not considered;
water was permitted to accumulate on the surface up to the maximum depth
allowed for the depression and detention storages. This depth, $H_{\text{d}}$, was assumed to be 1.27 cm. Any amount of water above this depth was
allowed to run off freely.

As expected, the completely saturated and the completely unsatura-
ated cases presented the least problems. It was possible to set an
error limit of $10^{-6}$ for h and still have convergence in 2 to 4 iterations
in those cases. The solution of the mixed flow system was more difficult;
in this case the most common error limit was between $10^{-2}$ and $10^{-4}$. Part
of the difficulty was probably due to the use of the same iteration
parameter for the saturated and unsaturated regions, but the most
important reason was the large differences between the hydraulic
conductivities at the saturated-unsaturated boundary nodes. The largest
errors always occurred at these mixed boundary nodes and the deviation
from the usual exponential (Horton-type) infiltration rate-time curve
occurred during the period of mixed flow. This can be seen in Figures
16, 17, 18, 19, and 20.
The points on the curve were the same for the following NDX tested:

<table>
<thead>
<tr>
<th>NDX</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5.00</td>
</tr>
<tr>
<td>4</td>
<td>2.50</td>
</tr>
<tr>
<td>6</td>
<td>1.67</td>
</tr>
<tr>
<td>8</td>
<td>1.25</td>
</tr>
</tbody>
</table>

Figure 16. Infiltration rate vs time for four $\Delta x$ and R values. $\Delta t = 5$ min, rainfall rate = 0.00282 cm/sec, $R = \Delta x/\Delta z$, NDX = 4. The dashed-lines are drawn in to show the deviation of the solution points from the usual exponential infiltration rate-time curve.

Figure 17. Infiltration rate vs time for different plane slopes. $\Delta t = 5$ min, rainfall rate = 0.00282 cm/sec, NDX = NDZ = 4. The dashed-lines are drawn in to show the deviation of the solution points from the usual exponential infiltration rate-time curve.
**Figure 18.** Infiltration rate vs time for five \( \Delta z \) and \( R \) values

\[ \Delta t = 5 \text{ min, rainfall rate} = 0.00282 \text{ cm/sec, NDX} = \frac{1}{4}. \] The dashed-lines are drawn in to show the deviation of the solution points from the usual exponential infiltration rate-time curve.

NDX = no. of intervals along x-axis
NDZ = no. of intervals along z-axis
R = \( \Delta x/\Delta z \)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>NDZ</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \triangle )</td>
<td>2</td>
<td>1.25</td>
</tr>
<tr>
<td>( \square )</td>
<td>4</td>
<td>2.50</td>
</tr>
<tr>
<td>( \circ )</td>
<td>6</td>
<td>3.75</td>
</tr>
<tr>
<td>( \times )</td>
<td>8</td>
<td>5.00</td>
</tr>
<tr>
<td>( + )</td>
<td>9</td>
<td>5.63</td>
</tr>
</tbody>
</table>
Rainfall Rate

<table>
<thead>
<tr>
<th>cm/sec</th>
<th>in/hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0028</td>
<td>4</td>
</tr>
<tr>
<td>0.0021</td>
<td>3</td>
</tr>
<tr>
<td>0.0014</td>
<td>2</td>
</tr>
<tr>
<td>0.0007</td>
<td>1</td>
</tr>
</tbody>
</table>

Figure 19. Infiltration rate vs time for 4 rainfall rates, Δt = 5 min, NDX = NDZ = 4. The dashed-lines are drawn in to show the deviation of the solution points from the usual exponential infiltration rate-time curve.

Figure 20. Infiltration rate vs time for different values of Δt. Rainfall rate = 0.00282 cm/sec, NDX = NDZ = 4. The dashed-lines are drawn in to show the deviation of the solution points from the usual exponential infiltration rate-time curve.
Figure 16 shows the influence of the size of \( \Delta x \) on the infiltration rate; for the 4 ratios of \( \Delta x/\Delta z \) shown in the figure, the infiltration rate was not affected. This could be expected since \( \Delta x \) does not enter the infiltration rate equation, equation (39), directly. A similar comparison was made by changing the sizes of \( \Delta z \), and keeping \( \Delta x \) at one value; there was no effect during the first four hours of infiltration and during the period of steady state. But after an extended period of mixed flow large values of \( \Delta z \) tended to reduce the infiltration rate. The infiltration rate-time curve for this case is shown in Figure 18. There was little or no effect of the plane slope on infiltration rate as shown in Figure 17.

Before the surface is saturated, the infiltration rate is assumed to be equal to the rainfall rate; this means that before surface saturation the infiltration rate increases as the rainfall rate increases. Figure 19 indicates that after the surface is saturated the rainfall rate has no influence on the infiltration rate. The usual reduction of infiltration by high rainfall rates is due to the destruction of the surface soil structure by the impact of the rain drops.

If the rainfall rate is greater than or equal to the saturated conductivity (RI \( \geq K_s \)) the soil surface first becomes saturated after a limited length of time and then saturation moves down the profile to the impermeable layer; the movement of the "surface of saturation" is simultaneous, starting from the lower right hand end (Figure 2) to the impermeable boundary at the upper left hand end. Figure 21 shows this movement, oscillations in the capillary pressure values having been ignored. The rainfall rate for Figure 21 was 0.0028 cm/sec and the time increment was
The above situation is reversed if the rainfall rate is less than the saturated conductivity \( (R_i < K_s) \). For Figure 22, the rainfall rate was 0.0003338 cm/sec, and was less than the saturated conductivity \( (K_s = 0.0006676 \text{ cm/sec}) \), and the time increment was also 5 min. In this case, saturation started at the lower impermeable boundary and moved towards the top; it does not matter whether the initial moisture content at the surface layer is greater than that of the lower impermeable boundary or not. According to Rubin and Steinhardt (1963), rain infiltration can continue indefinitely without ponding if \( R_i \leq K_s \); it is assumed here that they meant semi-infinite systems with which they worked. The present result does not contradict the above statement; in fact, it confirms it because if saturation begins at the lower impermeable layer even when the initial moisture content of the upper layer is greater than that of the lower layer, it means that the surface layer could not become saturated if we had a semi-infinite system.

In the two cases illustrated in Figures 21 and 22, the maximum depression storage, \( H_{4M} \), was satisfied starting from the lower end of the plane and moving to the left end; surface saturation was also satisfied in a similar manner. It is therefore important to note that surface runoff does not start along the whole length of a sloping plane at the same time.

Figure 20 shows the effect of different values of \( \Delta t \) on the infiltration rate. The smaller the time increment, the closer the curve gets to the usual theoretical infiltration rate-time curve. This figure demonstrates the latitude we have with the ADI method as far as the time
Figure 21. Position of zero capillary pressure with time for rainfall rate greater than saturated conductivity.

Figure 22. Position of zero capillary pressure with time for rainfall rate less than saturated conductivity.
step is concerned, but it also indicates that we should expect errors to be introduced in the solution if the time step is too large.

**Overland Flow**

Figure 20 shows the result of varying the time steps in the analysis of the soil moisture movement. If one is prepared to accept the degree of error involved, the time increment could be made quite large without introducing stability and convergence problems. But this was not the case in the overland flow analysis. The \( \alpha \)- and \( \beta \)-characteristics, equations (88) and (89), place limits on the sizes of \( \Delta t \) and \( \Delta x \); if the time or distance increment was such that the solution point was outside the unique solution domain (see Figure 11) the computer actually stopped processing the program. This is illustrated in Table 1. When a time increment of 60 seconds (equivalent to the dimensionless time increment of \( 300.0 \times 10^{-6} \)) was used, the dimensionless time increment calculated from equation (88) was \( 20.58 \times 10^{-6} \). The calculation was done at the outlet after 34 minutes from the beginning of rainfall. The calculated time increment was much smaller than the actual time step used. This caused error messages to be printed out during the next time step. The actual time increment used cannot be greater than the increment calculated from the characteristic equations, otherwise the values of \( y \) and \( u \) obtained will give a divergent solution. For all time increments greater than or equal to 0.4 seconds, solutions were obtained up to the time the characteristic equations were violated; at this time the program always failed to run. The solutions were therefore restricted to time increments less than or equal to 0.2 second.

Since the solution of the soil moisture phase of the problem did
not have serious time step limitations as illustrated above, larger time steps were used for this part of the solution. But the movement of the soil phase solution and the overland flow solution from one time period to the next had to be synchronized. For example, if a time step of one minute was used for the soil phase, and 0.2 second, used for the overland flow, the soils phase solution would move from one minute to the next in one step while the overland flow solution would move in 300 steps.

As long as the characteristic equations were satisfied, there was no significant effect of the size of the time step on the solutions obtained. This is illustrated in Figures 23 and 24, and also in Table 1.

In the characteristic equations, one parameter, \( \Delta x \) or \( \Delta t \) could be fixed, and the other chosen to satisfy the equations. If \( \Delta x \) is fixed, as was done in Figures 23 and 24, and in Table 1, it means that the value of \( \Delta t \) used cannot be greater than the value of \( \Delta t \) calculated from the equations if convergence is required. If \( \Delta t \) is fixed, the value of \( \Delta x \) used cannot be less than the value computed from the equations. Figure 25 did not show any significant effect of \( \Delta x \) on the infiltration rate; but Figure 26 and Table 2 indicate that after a while the runoff decreases as \( \Delta x \) decreases.

When overland flow was not considered, it was indicated that the rainfall rate had no effect on the infiltration rate after the soil surface became saturated. This is still the case even when overland flow is considered, as is shown in Figure 27. But as could be expected, the runoff rate increases as the rainfall rate increases; this result is shown in Figure 28 for three rainfall rates.

Manning's roughness coefficient has a large influence on the runoff
Table 1. The Control of the Solution of the Overland Flow Equations by the Characteristic Lines */

<table>
<thead>
<tr>
<th>Dimensionless Flow Depth</th>
<th>Dimensionless Flow Velocity</th>
<th>Calculation from α - Characteristic Equation (Equation 88)</th>
<th>Calculation from α - Characteristic Equation</th>
<th>Actual Dimensionless Time Increment Used</th>
<th>Actual Dimensional Time Increment Used</th>
<th>Time at which computation was done. From beginning of rainfall.</th>
<th>Time at which computer solution diverged. From beginning of rainfall.</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>u</td>
<td>$\frac{dx}{dt} \times 10^{-4}$</td>
<td>$dt \times 10^6$</td>
<td>$\Delta t \times 10^6$</td>
<td>$\Delta t$ seconds</td>
<td>Min</td>
<td>Min</td>
</tr>
<tr>
<td>0.0006555</td>
<td>0.231</td>
<td>1.214</td>
<td>20.58</td>
<td>300.0</td>
<td>60.0</td>
<td>34</td>
<td>35</td>
</tr>
<tr>
<td>0.0008157</td>
<td>0.268</td>
<td>1.366</td>
<td>18.81</td>
<td>5.0</td>
<td>1.0</td>
<td>34</td>
<td>79</td>
</tr>
<tr>
<td>0.010455</td>
<td>1.457</td>
<td>5.530</td>
<td>4.52</td>
<td></td>
<td></td>
<td>132</td>
<td>133</td>
</tr>
<tr>
<td>0.0008183</td>
<td>0.269</td>
<td>1.368</td>
<td>8.27</td>
<td>3.0</td>
<td>0.6</td>
<td>34</td>
<td>78</td>
</tr>
<tr>
<td>0.0230559</td>
<td>2.486</td>
<td>8.386</td>
<td>2.98</td>
<td></td>
<td></td>
<td>132</td>
<td>224</td>
</tr>
<tr>
<td>0.0364976</td>
<td>3.421</td>
<td>10.864</td>
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<td></td>
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<td>223</td>
</tr>
<tr>
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<td>0.269</td>
<td>1.369</td>
<td>18.26</td>
<td></td>
<td></td>
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<td>78</td>
</tr>
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<td>0.0231150</td>
<td>2.490</td>
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<td>2.0</td>
<td>0.4</td>
<td>78</td>
<td>132</td>
</tr>
<tr>
<td>0.0479406</td>
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<td>12.585</td>
<td>1.99</td>
<td></td>
<td></td>
<td>132</td>
<td>223</td>
</tr>
<tr>
<td>0.0431272</td>
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<td>12.017</td>
<td>2.08</td>
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<td>223</td>
</tr>
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<td>1.370</td>
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<td></td>
<td>34</td>
<td>78</td>
</tr>
<tr>
<td>0.0231408</td>
<td>2.492</td>
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<td>1.0</td>
<td>0.2</td>
<td>78</td>
<td>—</td>
</tr>
<tr>
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<td>12.588</td>
<td>1.99</td>
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<td></td>
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<td>223</td>
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<td>0.0788460</td>
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<td>16.577</td>
<td>1.51</td>
<td></td>
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<td>223</td>
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<tr>
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<td>0.269</td>
<td>1.370</td>
<td>18.25</td>
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<td>0.1</td>
<td>78</td>
<td>—</td>
</tr>
<tr>
<td>0.0231247</td>
<td>2.491</td>
<td>8.401</td>
<td>2.98</td>
<td>0.5</td>
<td>0.1</td>
<td>78</td>
<td>—</td>
</tr>
<tr>
<td>0.0478366</td>
<td>4.041</td>
<td>12.570</td>
<td>1.99</td>
<td></td>
<td></td>
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<td>—</td>
</tr>
<tr>
<td>0.0784083</td>
<td>5.603</td>
<td>16.549</td>
<td>1.51</td>
<td></td>
<td></td>
<td>132</td>
<td>—</td>
</tr>
</tbody>
</table>

* This computation was done at the outlet point. RI = 0.0028 cm/sec, n = 0.06, soils phase time increment = 1 min. NDX = NDZ = 4.
Table 2. Comparison of the Actual Interval Along the X-Axis and the Interval Computed from the $a$-Characteristic Equation *

<table>
<thead>
<tr>
<th>Dimensionless Flow Depth $y$</th>
<th>Dimensionless Flow Velocity $u$</th>
<th>Computed from $a$-Characteristic $\frac{dx}{dt} \times 10^{-4}$</th>
<th>Computed from $a$-Characteristic $dx$</th>
<th>Actual Dimensionless Intervals Used $\Delta x$</th>
<th>No. of Intervals Along X-Axis NDX</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0252</td>
<td>2.628</td>
<td>21.78</td>
<td>0.218</td>
<td>0.500</td>
<td>2</td>
</tr>
<tr>
<td>0.0258</td>
<td>2.542</td>
<td>6.55</td>
<td>0.065</td>
<td>0.250</td>
<td>4</td>
</tr>
<tr>
<td>0.0228</td>
<td>2.466</td>
<td>8.33</td>
<td>0.083</td>
<td>0.167</td>
<td>6</td>
</tr>
<tr>
<td>0.0219</td>
<td>2.405</td>
<td>8.16</td>
<td>0.082</td>
<td>0.125</td>
<td>8</td>
</tr>
</tbody>
</table>

* This computation was done at the outlet point 4 hours after the rainfall started. $RI = 0.0014$ cm/sec $n = 0.06$, soils phase time increment, $\Delta t = 1$ min. Overland flow time increment $\Delta t_1 = 0.2$ sec. No. of intervals along $z$-axis = 4.
Table 2. Comparison of the Actual Interval Along the X-Axis and the Interval Computed from the 
\( \alpha \) - Characteristic Equation */

<table>
<thead>
<tr>
<th>Dimensionless Flow Depth ( y )</th>
<th>Dimensionless Flow Velocity ( u )</th>
<th>Computed from ( \alpha )-Characteristic ( \frac{dx}{dt} \times 10^{-4} )</th>
<th>Computed from ( \alpha )-Characteristic ( dx )</th>
<th>Actual Dimensionless Intervals Used ( \Delta x )</th>
<th>No. of Intervals Along 'X-Axis' ( NDX )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0252</td>
<td>2.628</td>
<td>21.78</td>
<td>0.218</td>
<td>0.500</td>
<td>2</td>
</tr>
<tr>
<td>0.0238</td>
<td>2.542</td>
<td>6.55</td>
<td>0.065</td>
<td>0.250</td>
<td>4</td>
</tr>
<tr>
<td>0.0228</td>
<td>2.466</td>
<td>8.33</td>
<td>0.083</td>
<td>0.167</td>
<td>6</td>
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<tr>
<td>0.0219</td>
<td>2.405</td>
<td>8.16</td>
<td>0.082</td>
<td>0.125</td>
<td>8</td>
</tr>
</tbody>
</table>

* This computation was done at the outlet point 4 hours after the rainfall started. \( RI = 0.0014 \) cm/sec. \( n = 0.06 \), soils phase time increment, \( \Delta t = 1 \) min. Overland flow time increment \( \Delta t_1 = 0.2 \) sec. No. of intervals along z-axis = 4.
Figure 23. The effect of overland flow time increment on the infiltration rate.
RI = 0.0028 cm/sec; Manning's n = 0.06
NDX = NDZ = 4; $\Delta t = 1$ min
$\Delta t =$ Time increment for soils phase

Figure 24. The effect of overland flow time increment on the runoff rate.
RI = 0.0028 cm/sec; Manning's n = 0.06
NDX = NDZ = 4, $\Delta t = 1$ min
$\Delta t =$ Time increment for soils phase.
$q'/Q_o$ = Runoff Rate

$\text{NDX} = \text{No. of intervals along } x\text{-axis}$
$\text{NDZ} = \text{No. of intervals along } z\text{-axis}$
$\Delta t = \text{Time increment for soils phase}$
$\Delta t 1 = \text{Time increment for overland flow}$

$\Delta \text{NDX} = 2$
$\Delta \text{NDX} = 4$
$\Delta \text{NDX} = 6$
$\Delta \text{NDX} = 8$

**Figure 25.** The effect of $\Delta x$ on the infiltration rate.
$R_I = 0.0014 \text{ cm/sec; } \Delta t = 1 \text{ min},$
$\Delta t 1 = 0.2 \text{ sec, } n = 0.06, \text{ NDZ} = 4$

**Figure 26.** The effect of $\Delta x$ on runoff rate.
$R_I = 0.0014 \text{ cm/sec, } \Delta t = 1 \text{ min, } \Delta t 1 = 0.2 \text{ sec}$
$n = 0.06$
Figure 27. The effect of rainfall rate on the infiltration rate, allowing for surface runoff.
\( \Delta t_1 = 0.2 \text{ sec}, \Delta t = 1 \text{ min}, NDX = NDZ = 4, n = 0.06 \)

Figure 28. Runoff rate Vs time for three rainfall rates.
\( \Delta t_1 = 0.2 \text{ sec}. \ NDX = NDZ = 4, n = 0.06. \)
\( \Delta t = 1 \text{ min} \)
rate; small n values produce large runoff rates while large n values produce small runoff rates. Figure 30 shows the effect of n on the runoff rate and Figures 31 and 32 show a magnified effect by concentrating on a short time interval. As indicated in Figure 29, the effect of n on the infiltration rate is not significant.

As previously indicated in the soils analysis, the effect of the slope angle on the infiltration rate was not significant. Figure 33 also does not show a significant effect of the slope angle on the infiltration rate when overland flow is considered. However, the runoff rate should increase as the slope angle increases. Figure 34 indicates that the runoff rate does increase with the slope angle, but this effect occurs up to the first seven hours from the beginning of rainfall. The points for 10 and 15 degrees coincided up to the 7th hour. The point in Figure 34, where most of the curves cross, and where the contradiction begins, is close to the point in Figure 33, where the soil system becomes completely saturated. This discrepancy is most likely due to the error in the infiltration part of the solution.

In Figures 21 and 22, overland flow was ignored and the maximum depth over the surface was fixed but in Figures 35 and 36 overland flow was taken into account. In the later cases the positions of zero capillary pressure with time were smoother than in the former cases probably due to the higher depths of water over the surface.

Comparison of the Computer Solution of the Ditch Drainage Problem with Kirkham's Analytical Solution

Figure 37 shows equipotentials obtained for the ditch drainage problem of Figure 14 when the soil is 20 units deep and the spacing
Runoff Rate \( q' / Q_0 \)

Infiltration Rate \( f^A g \)

\[ \Delta t = \text{Time increment for soils phase} \]
\[ \Delta t_l = \text{Overland flow time increment} \]

- \( \triangle n = 0.01 \)
- \( \square n = 0.04 \)
- \( \circ n = 0.06 \)
- \( \times n = 0.20 \)
- \( + n = 0.60 \)

**Figure 29.** The Effect of Manning's \( n \) on the Infiltration Rate.

RI = 0.0014 cm/sec, \( \Delta t = 0.2 \) sec
\( \Delta t = 1 \) min, NDX = NDZ = 4

**Figure 30.** The Effect of Manning's \( n \) on Runoff Rate.

\( \Delta t_l = 0.2 \) sec, RI = 0.0014 cm/sec
\( \Delta t = 1 \) min, NDX = NDZ = 4
87.

Figure 31. Runoff Rate Vs Time for Manning's n of 0.06

Figure 32. Runoff Rate Vs Time for Manning's n of 0.60
**Figure 33.** The Effect of the Slope Angle on the Infiltration Rate

Infiltration Rate ($q'_{f}/A_g$)

- RI = 0.0014 cm/sec
- $\Delta t_1$ = 0.2 sec
- $\Delta t$ = 1 min
- $n$ = 0.06

- $\alpha$ = 5°
- $\alpha$ = 10°
- $\alpha$ = 15°
- $\alpha$ = 20°

**Figure 34.** The Effect of the Slope Angle on the Runoff Rate

Runoff Rate ($Q'_{a_0}/A_g$)

- RI = 0.0014 cm/sec
- $\Delta t_1$ = 0.2 sec
- $\Delta t$ = 1 min
- $n$ = 0.06

- $\alpha$ = 5°
- $\alpha$ = 10°
- $\alpha$ = 15°
- $\alpha$ = 20°
Figure 35. Position of zero capillary pressure with time for rainfall
rate = 0.0028 cm/sec.
$\Delta t = 1$ min, $\Delta t_1 = 0.2$ sec, $n = 0.06$

Figure 36. Position of zero capillary pressure with time for rainfall
rate = 0.0014 cm/sec.
$\Delta t = 1$ min, $\Delta t_1 = 0.2$ sec, $n = 0.06$
between the ditches is 80 units from ditch wall to ditch wall, the ditch is half full of water measured from the impermeable layer. The solid lines in Figure 37 represent Kirkham's analytical solution and the dashed lines represent the computer solution. The agreement is exact for equipotentials 0 and 100 and for 80, 90, and 95 up to a distance of 16 units from the ditch; it is almost exact for equipotentials 20, 40, and 60. The only equipotential which show disagreements between the analytical and the computer solutions that warrant any discussion are 90 and 95.

The last vertical line in Figure 37, 40 units from the ditch is a streamline and the equipotential 100 is perpendicular to it as it should be. Looking at the distribution of the equipotentials 80, 90, 95, and 100 it does not look likely that 95 would deviate as drastically from 100 as the analytical solution suggests. Besides it would be difficult, if not impossible, to draw a streamline from a point on the surface between 36 and 40 units from the ditch that would intersect the equipotential 95 at right angles as it should whereas streamlines can be drawn from any point on the surface to intersect the equipotentials 90 and 95 of the computer solution at right angles. I therefore believe that the equipotentials 90 and 95 should follow the computer solution values.
Figure 37. Comparison of computer solution with Kirkham's Analytical Solution for the ditch drainage problem of Figure 14 when d = 20, h = 10, and s = 40 arbitrary units.
IX. SUMMARY AND CONCLUSIONS

In the belief that understanding the complex problems of watershed hydrology lies in diligent studies of the component parts of the hydrologic cycle, a watershed model that simulates the soil moisture movement, and the overland flow, was developed. The Alternating Direction Implicit (ADI) method of numerical analysis was used to analyze the soil moisture aspect of the model and an explicit method was used for the overland flow analysis. The overland flow solution was controlled by the characteristic lines that define a region in which we have convergent solutions.

The watershed section is a sloping plane; the x-axis is taken down the slope and the z-axis perpendicular to the slope. The effects of several parameters on the infiltration rate and on the runoff rate were studied. The slope angle and the finite difference mesh size in the x-direction had little or no effect on infiltration; also, the length of the time step had a negligible effect on the infiltration rate. After the surface became saturated, the rainfall rate had no effect on the infiltration rate.

As long as the convergence and stability requirements as expressed by the characteristic equations for overland flow were met, the time increment had an insignificant effect on the runoff rate, but if the characteristic equations were not satisfied, the effect of the time step was
serious enough to cause the computer to stop processing the program. Also the mesh size in the x-direction had no effect on the runoff rate unless the convergence requirements were violated.

Small values of Manning's roughness coefficient produced large surface runoff rates and large values of n produced small rates of runoff. Also the runoff rates increased or decreased as the rainfall rates increased or decreased. Large slope angles were expected to increase the rate of runoff, but this only happened to a limited extent.

In the soil moisture analysis, it was found that if the rainfall rate is greater than or equal to the saturated hydraulic conductivity, saturation started from the surface of the plane to the lower impermeable layer. The situation is reversed if the rainfall rate is less than the saturated hydraulic conductivity.

As a test of its validity, the model was used to solve a ditch drainage problem for which an analytical solution is available. The problem involved parallel, equally spaced ditches penetrating an impermeable layer and half filled with water. The soil surface was maintained at complete saturation. Equipotentials were obtained by the model for comparison with Kirkham's analytical solution of the same problem. The agreement was excellent except at points midway between the parallel ditches. At these points the computer solution was taken to be more accurate because it fulfilled the requirement that it should be possible to draw streamlines from any points of the flow region to intersect the equipotentials at right angles whereas the analytical solution did not.

Based on the results of this study, the following conclusions are drawn:
The ADI procedure is a very powerful method of solving two-dimensional, completely saturated, and completely unsaturated flow problems. The method is also satisfactory for solving, as a unit, problems in which part of the flow region is saturated and the other part is unsaturated. The freedom to choose the space and time mesh sizes to fit particular problems is a significant advantage. The ADI method can, therefore, be used to model rainfall infiltration into a sloping plane with satisfactory results.

A "physically-based" mathematical model of the hydrologic system is feasible; the next steps are to decide the degree of sophistication of such a model, and the size of watershed it can adequately handle.
X. SUGGESTIONS FOR FURTHER RESEARCH

Besides the obvious use of a model of this nature to predict run-off hydrographs and to study the soil moisture movement, it can also be used for the following purposes:

The movement and distribution of chemicals in soils and the possible flow of these chemicals into the streams could be studied with the intention of determining their pollution potential or their effect on water quality.

The points on a hillside at which the interflow component of the infiltrated water could re-emerge, for a given set of conditions, could be determined for the purpose of designing hillside tile lines.

The spacing of open ditches and tiles could be determined in drainage situations.

The effect of induced changes in a drainage basin can be studied more easily with the use of this type of model. It can also be used for the classroom teaching of hydrology.

In view of the above purposes, the following recommendations are made:

The overland flow solution should be replaced by an implicit scheme so that the same time increment could be used for both the soils phase and the overland flow phase. This will make it possible to choose the time and space mesh sizes to suit the existing conditions and could
save the computer time.

At least the interflow and the channel flow components should be added to the model. This will increase its usefulness.

Experiments should be designed to test the accuracy and the applicability of the model developed in this study. The limiting size of the watershed for which this type of model could be used should be determined.
APPENDIX A

THE COMPUTER PROGRAM VARIABLES
THE COMPUTER PROGRAM VARIABLES

The computer program symbols are defined below. Some of the symbols that do not appear here have been defined in the List of Symbols and the meanings of others are obvious in the computer program itself.

\[\begin{align*}
\text{ALPHA} & \quad = \text{Plane angle} \\
A_1, A_2, \ldots, A_9 & \quad = \text{Constants} \\
A_{T1}, A_{T2}, \ldots, A_{T4} & \quad = \text{Constants} \\
B_{T1}, B_{T2}, \ldots, B_{T4} & \quad = \text{Constants} \\
B_l(I) & \quad = \text{Element of the upper diagonal matrix } U \text{ at any point during the ADI solution in the } x \text{- direction} \\
B_l(J) & \quad = \text{Element of the upper diagonal matrix } U \text{ at any point during the ADI solution in the } z \text{- direction} \\
C_{l}(I, J, N) & \quad = \text{Water capacity at any point and time (dimensionless)} \\
C_2(I) & \quad = \text{Dimensionless water capacity derived from dimension-less moisture-tension curve} \\
\text{CA} & \quad = \text{Cosine} \\
\text{CT} & \quad = \text{Maximum error in } H \text{ at any point during a time step} \\
\text{DH}^4 & \quad = \text{Change in the capillary pressure of the surface soil} \\
\text{DTHETA} & \quad = \text{Dimensionless } \Delta \theta \\
\text{DT} & \quad = \text{Dimensionless time increment for soils phase analysis} \\
\text{DTL} & \quad = \text{Dimensionless time increment for overland flow analysis} \\
\text{DT2} & \quad = \text{DT (sec)}
\end{align*}\]
DT3 = DT1 (sec)

DX = Dimensionless Δx

DX2 = Square of DX

DZ = Dimensionless Δz

DZ2 = Square of DZ

ETA = Dynamic viscosity of water

FLLT1(I, J) = Percolarion rate at any point (dimensionless)

FLLT2(I, J) = Infiltration rate at any point (dimensionless)

FM = Matching factor = \frac{\text{measured saturated conductivity}}{\text{calculated saturated conductivity}}

GAMMA = Surface tension of water

G = Acceleration due to gravity

G1(I) = Element of the column vector
        G1 at any point during the ADI solution in the x-direction

G1(J) = Element of the column vector
        G1 at any point during the ADI solution in the z-direction

HET(I) = Capillary pressure at any point.

HET(MZ) = Capillary pressure at saturated moisture content

HET1(I) = Dimensionless HET(I)

HET1(MZ) = Dimensionless HET(MZ)

H(I, J, N) = Capillary pressure at any point and time (dimensionless)

HI(I) = Input capillary pressure (cm of water)

H2(I) = Dimensionless capillary pressure derived from the moisture-tension curve

H4M = Dimensionless surface storage depth

II = Maximum number of columns in the grid system

IJ = Product of II and JJ
IN = Constant
IOUT1 = Number used to check whether any point other than the point (I, 1) is saturated
IOUT2 = Number used to check whether the depth of water at the point (I, J) is equal to H,M
IOUT5 = Number used to check whether all points in the flow system are saturated
IOUT7 = Number used to check whether overland flow has occurred for the first time or not
IS = A variable factor of the iteration parameter
IX = II - 1
IY = Variable used for counting saturated points in the soil system
IZ = Variable used for counting saturated points in the soil system
JJ = Maximum number of rows in the grid system
JX = JJ - 1
K (I, J, N) = Hydraulic conductivity at any point and time (dimensionless)
KS = Saturated hydraulic conductivity (cm/sec)
M = Number of pore classes used in the hydraulic conductivity computation
MZ = M + 1. This number is used to define the point of saturation on the 0-axis of the moisture-tension curve
MZ1 = Number of the capillary pressure values and the corresponding water capacity values read off the Cl-h curve for interpolation purposes
N = Time dimension
NLT = Total number of intervals along the time-axis
NDX = Number of divisions of the flow section along the x-axis
NDZ = Number of divisions of the flow section along the z-axis
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>NDZ</td>
<td>Number of divisions of the flow section along the z-axis</td>
</tr>
<tr>
<td>NN</td>
<td>Twice NDT</td>
</tr>
<tr>
<td>NPT</td>
<td>The present number of time increments from the start of the computation</td>
</tr>
<tr>
<td>NS</td>
<td>Constant in the overland flow subroutine</td>
</tr>
<tr>
<td>NZ</td>
<td>Number of values of the moisture content and the corresponding capillary pressure available for plotting the moisture-tension curve</td>
</tr>
<tr>
<td>PI</td>
<td>(3.14159265)</td>
</tr>
<tr>
<td>Q</td>
<td>Dimensionless flow rate at the outlet of overland flow</td>
</tr>
<tr>
<td>RE</td>
<td>Dimensionless rainfall rate</td>
</tr>
<tr>
<td>RI</td>
<td>Vertical rainfall rate (cm/sec)</td>
</tr>
<tr>
<td>RHO</td>
<td>Density of water (gm/cm)</td>
</tr>
<tr>
<td>SA</td>
<td>Sine (\alpha)</td>
</tr>
<tr>
<td>T</td>
<td>Total dimensionless time of precipitation</td>
</tr>
<tr>
<td>T1</td>
<td>Total time of precipitation (sec)</td>
</tr>
<tr>
<td>THEATA(I, J, N)</td>
<td>The dimensionless moisture content at any point and time</td>
</tr>
<tr>
<td>THETAS</td>
<td>Saturated moisture content</td>
</tr>
<tr>
<td>TETS</td>
<td>Saturated moisture content</td>
</tr>
<tr>
<td>TET(I)</td>
<td>Moisture content at any point</td>
</tr>
<tr>
<td>TET(MZ)</td>
<td>Moisture content at (I = MZ)</td>
</tr>
<tr>
<td>THET(NZ)</td>
<td>Saturated moisture content at (I = NZ)</td>
</tr>
<tr>
<td>TETL(I)</td>
<td>Dimensionless TET(I)</td>
</tr>
<tr>
<td>TETL(MZ)</td>
<td>Dimensionless TET(MZ)</td>
</tr>
<tr>
<td>TETL</td>
<td>Smallest moisture content value available on the moisture-tension curve</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td>$u(I, N)$</td>
<td>Overland flow velocity at any point and time (dimensionless)</td>
</tr>
<tr>
<td>$U_P$</td>
<td>Reference overland flow velocity (cm/sec)</td>
</tr>
<tr>
<td>$XD$</td>
<td>Dimensionless depth to the impermeable layer measured along the z-axis</td>
</tr>
<tr>
<td>$XD_1$</td>
<td>Dimensional XD (cm)</td>
</tr>
<tr>
<td>$XI$</td>
<td>Iteration parameter for solving the parabolic-elliptic system</td>
</tr>
<tr>
<td>$X3$</td>
<td>Iteration parameter for solving the elliptic system</td>
</tr>
<tr>
<td>$XX(I)$</td>
<td>Hydraulic conductivity at any point (cm/sec)</td>
</tr>
<tr>
<td>$XX(MZ)$</td>
<td>Hydraulic conductivity at $I = MZ$</td>
</tr>
<tr>
<td>$XX_1(I)$</td>
<td>Dimensionless $XX(I)$</td>
</tr>
<tr>
<td>$XX_1(MZ)$</td>
<td>Dimensionless $XX(MZ)$</td>
</tr>
<tr>
<td>$XXS$</td>
<td>Saturated hydraulic conductivity (cm/sec)</td>
</tr>
<tr>
<td>$XL$</td>
<td>Length of the watershed section (cm)</td>
</tr>
<tr>
<td>$XN$</td>
<td>Manning's roughness coefficient</td>
</tr>
<tr>
<td>$XPP$</td>
<td>Overland flow time increment calculated from the $\alpha$-characteristic</td>
</tr>
<tr>
<td>$Y(I, N)$</td>
<td>Dimensionless overland flow depth at any point and time</td>
</tr>
<tr>
<td>$ZP$</td>
<td>Vertical height of any point in the soil system from the datum plane (cm)</td>
</tr>
</tbody>
</table>
APPENDIX B

PROGRAM FLOW CHARTS
NOTE: When any point of the flow system is saturated, IOUT1 = 1, otherwise IOUT1 = 0. The point (II,1) is not included since it was initially assumed to be saturated.

When the depth of water at the point (l,JJ) is equal to or greater than the maximum surface depression depth H^M, IOUT2 = 1, otherwise IOUT2 = 0. Surface runoff computation starts when IOUT2 = 1.

When all points of the flow system are saturated IOUT5 = 1, otherwise IOUT5 = 0.

IOUT7 = 0 until the surface runoff subroutine START is called for the first time. From this time onwards IOUT7 = 1.
READ IN OR INCORPORATE IN PROGRAM THE CONSTANTS

ALPHA, ETA, GAMMA, G, MZ1, NZ, PI, RHO

NPT = 0
N = 1

CALCULATE DIMENSIONLESS QUANTITIES

DX, XD, DZ, DT, DT1, RE, T

CALCULATE CONSTANTS

A1, A2, A3, A4, A5, A6, A7, A8, A9, AT1, AT2, AT3, AT4, BT1, BT2, BT3, BT4, CA, DT2, DT3, DX2, DZ2, H^M, I, IX, IJ, JJ, JX, NN, SA

CALCULATE K 

BY THE METHOD OF MILLINGTON AND QUIRK

SET INITIAL CONDITIONS
1

MPT = NPT + 1

IY = 0
IZ = 0

IOUT1 = 1

CALL UNSADI

CHECK STATUS OF IOUT1

4

CALL START

IOUT7 = 1

3

IOUT5 = 1

CALL SATADI

4

CALL PASADI

CHECK STATUS OF IOUT5

WRITE

NPT, IOUT1, IOUT2, IOUT5, IOUT7, FILT2, THETA, H, K, C1, XPP, Y, U, Q

NPT = NDT

Yes

STOP

No

RESET PARAMETERS

2
SUBROUTINE UNSDI

DIMENSION COMMON REAL

COMPUTE TOP BOUNDARY CONDITIONS

COMPUTE RIGHT HAND SIDE BOUNDARY CONDITIONS

ADI SOLUTION IN X-DIRECTION (Sweeping the Rows)

J = I

I = 1

CALCULATE BL(I), GL(I)

CALCULATE
BL(I), 1<I<II-2
GL(I), 1<I<II-1

CALCULATE H(II-1,J,N)
USE INTERPOLATION ROUTINE TO DETERMINE CORRESPONDING K, C1
CALCULATE \( H(I, J, N) \)
Determine corresponding \( k, c_1 \) from interpolation routine

If \( I = 1 \) then go to 7

If \( J = J + 1 \) then go to 5

ADI solution in z-direction (sweeping the columns)

Calculate \( B_1(1), G_1(1) \)

Calculate
\[
B_1(J), \ 1 < J \leq J_1 - 2 \\
G_1(J), \ 1 < J \leq J_1 - 1
\]

Calculate \( H(I, J_1 - 1, N) \)
Determine corresponding \( \theta, k, c_1 \) from interpolation routine

If \( J = J_1 - 2 \) then go to 9

Else if \( I = I - 1 \) then go to 7

Else \( I = I + 1 \) then go to 7

Else \( J = J + 1 \) then go to 5

Else \( I = 1 \) then go to 8

Else \( J = J - 1 \) then go to 5

Else \( I = I + 1 \) then go to 7

Else \( J = J + 1 \) then go to 5

Else \( I = I - 1 \) then go to 7

Else \( J = J - 1 \) then go to 5

Else \( I = I + 1 \) then go to 7

Else \( J = J + 1 \) then go to 5

Else \( I = I - 1 \) then go to 7

Else \( J = J - 1 \) then go to 5

Else \( I = I + 1 \) then go to 7

Else \( J = J + 1 \) then go to 5
CALCULATE $H(I,J,N)$
Determine corresponding $\theta, k, c_1$ from interpolation routine

If $J = 1$, no; if $I = II - 1$, no; return end.

SUBROUTINE PASADI

DIMENSION COMMON REAL

Compute top boundary conditions

Compute right hand side boundary conditions

$IS = 0$
$IN = 0$

10

Compute iteration parameter

ADI solution in x-direction (sweeping the rows)

11
SATURATION
Yes
COMPUTE PARAMETERS USING SATURATED CONDITIONS

No
COMPUTE PARAMETERS USING UNSATURATED CONDITIONS

CALCULATE $H_1(1), G_1(1)$

CALCULATE $H_1(I), 1 < I < I + 1$

$G_1(I), 1 < I < I - 1$

CALCULATE $H(I-1, J, N)$

$I = I - 2$

CALCULATE $H(I, J, N)$

$I = 1$

Yes

$J = J - 1$

No

$J = J + 1$

$14$

$13$

$12$

$11$
ADI SOLUTION IN Z-DIRECTION (SWEEPING THE COLUMNS)

I = 1

15

J = 1

SATURATION

Yes

COMPUTE PARAMETERS USING SATURATED CONDITIONS

No

COMPUTE PARAMETERS USING UNSATURATED CONDITIONS

CALCULATE \( B_1(1), G_1(1) \)

CALCULATE \( B_1(j), 1 < j < JJ - 2 \)

\( G_1(j), 1 < j < JJ - 1 \)

CALCULATE \( H(i, JJ - 1, N + 1) \)

DETERMINE CORRESPONDING \( K, C_1 \) FROM INTERPOLATION ROUTINE

J = JJ - 2

16
CALCULATE $H(I, J, N+1)$
DETERMINE CORRESPONDING $K, C_1$ FROM INTERPOLATION ROUTINE

J=1

No  J=J-1  16

Yes

I=II-1

No  I=I+1  15

Yes

IN=0

Yes  17

No

FIND $C_T$, MAXIMUM DIFFERENCE IN ABSOLUTE VALUE BETWEEN $H$ OF PREVIOUS AND PRESENT ITERATION FOR $J=1,JX; I=1,IX$

I$S$=0  18

No

17

I$S$=I$S$+1

IN=1

$H(I,J,N+2)=H(I,J,N+1)$ FOR $J=1,JX; I=1,IX$
$H(I,J,N) = H(I,J,N+1)$ for $J = 1, JX; I = 1, IX$

Determine corresponding $\theta, k, c1$ using interpolation routine

RETURN

END

SUBROUTINE SATADI

DIMENSION COMMON REAL

Compute top boundary conditions

Compute right hand side boundary conditions

IS = 0

19

Compute iteration parameter

ADI solution in X-direction (sweeping the rows)
CALCULATE $H(l, J, N)$

CALCULATE $E_l(l)$, $G_l(l)$

CALCULATE $H(l-1, J, N)$

CALCULATE $H_{l}(l)$, $l \leq l \leq l-2$

CALCULATE $G_{l}(l)$, $1 \leq l \leq l-1$

CALCULATE $H(l-1, J, N)$

$I = l - 2$

CALCULATE $H(l, J, N)$
ADI SOLUTION IN Z-DIRECTION (SWEEPING THE COLUMNS)

1. \( I = 1 \)
   - \( J = J_{\text{FINAL}} - 1 \)
     - Yes: \( J = J_{\text{FINAL}} - 1 \)
     - No: \( I = I - 1 \)

2. \( I = 1 \)
   - \( J = 1 \)
     - \( \text{CALCULATE } B_l(1), G_l(1) \)
     - \( \text{CALCULATE } B_l(J), 1 < J < J_{\text{FINAL}} - 2 \)
     - \( \text{CALCULATE } G_l(J), 1 < J < J_{\text{FINAL}} - 1 \)
     - \( \text{CALCULATE } H(I, J_{\text{FINAL}} - 1, N+1) \)
     - \( J = J_{\text{FINAL}} - 2 \)
Calculate $H(I, J, N+1)$

If $J = 1$

If $I = I + 1$

If $IS = 0$

Find $CT2$, maximum difference in absolute value between $H$ of previous and present iteration for $J = 1, JX; I = 1, IX$

If $CT2 < 0.00001$

$IS = IS + 1$

$H(I, J, N+2) = H(I, J, N+1)$

$H(I, J, N-1) = H(I, J, N+1)$
\[ H(I,J,N) = H(I,J,N+1) \text{ for } J=1,JX; I=1,IX \]

RETURN
END

---

SUBROUTINE SURF

DIMENSION COMMON REAL

CALCULATE PB

CALCULATE DH4

\[ H(I,J,N) = DH4 \times H(I,J,N-1) \]

DETERMINE CORRESPONDING Theta, K, C1 FROM INTERPOLATION ROUTINE

RETURN
END
SUBROUTINE START

DIMENSION COMMON REAL

COMPUTE FILT2 FOR I=1,II

CALCULATE Y(II,N),U(II,N),Q(II,N)

CALCULATE Y(I,N),U(I,N) FOR I=2,II-1

I=1

COMPUTE STOR(1,N),Y(1,N),U(1,N)

Y(1,N)=0 Yes

U(1,N)=0

Y(1,N)<0.000 No

RECALCULATE Y(2,N),U(2,N)

RETURN

END
APPENDIX C

THE COMPUTER PROGRAM
THE COMPUTER PROGRAM

The computer program presented here was written in FORTRAN IV, version GL, originally for the IBM Computer 360/75 and then was later run on IBM 370/165 without any changes. The program was broken up into several sub-routines and the main program acted as a control center for linking the sub-routines in an appropriate manner.

The sub-routine UNSADI was used to solve the parabolic system, PASADI solved the parabolic-elliptic system, SATADI solved the elliptic system, and START solved the hyperbolic or overland flow system. The sub-routine SURF was used to calculate the surface infiltration rate until the depth of water at the point (l, JJ) was equal to $H^4M$. At this time overland flow solution started and SURF was no longer needed. The function TERP1 was used to obtain the soil parameters as they were needed. Any of the sub-routines or the function could be replaced by new or improved versions when they became available without affecting the logic of the overall model.

The length of time it takes to run the program on any computer depends very much on the physical condition of the watershed section; it takes longer if the section remains partly saturated and partly unsaturated during most of the solution period than it does if the section is mostly saturated or mostly unsaturated during the same period. On the
average the 360/75 computer took 1/50th of the physical time to complete the simulation while the 370/165 model took 1/160th of the physical time to complete the simulation. This underscores the belief that the speed of the computer will not be a hindrance in the future against the development of a physically-based mathematical model of the hydrologic system. The program listing is given in the following pages.
PROGRAM LISTING

```
// ADA:10,'MWA',E.U.
// (12000),CLASS=F
// $) EXEC PG=(I,FRT,T,M)=O(20)
// SYSPLNT DD SYSUT=A
// SYSTEM DO (CF=LLAESF1,UNIT=SYSDA,DISP=MOD,PASS),SPACE={CYL,(1,1)}
// OCH=(RECF=FB,RECL=RO,HLKSIZE=400)
// SYSIN DD *
C * * * * * * * * * * * * * * * * C
C MAIN PROGRAM FOR THE FUNDAMENTAL MATHEMATICAL SIMULATION OF
C WATERSHEDS - VERSION OF SEPTEMBER 28, 1971
C THE PRIMARY FUNCTION OF THE MAIN PROGRAM IS TO CONTROL THE USE OF
C ALL THE SUBROUTINES. IT ALSO COMPUTES THE HYDRAULIC CONDUCTIVITY
C * * * * * * * * * * * * * * * * C
DIMENSION THETA(100,10,3),X(100,10,3),K(100,10,3),M(110)
DIMENSION Y(21,1),X(21,1),ETAS,KS,R1,UP,XN
READ(5,2) HI
READ(5,500) H1
READ(5,502) P2
READ(5,502) H2
500 FORMAT(16F5.2)
502 FORMAT(16F5.2)
N=1
N1=15
DX=1.0/NX
XDX=NX/XL
DZ=NDX/N2
T1=K1/(ETAS*X)
D1=1/T1
D1=1/200.00
DT1=1/T1
DT2=1/T1
K=K1/KS
K1=1.00
J=2
JX=J-1
IX=1-1
N=2
MX=23
B1=1.0/DX
B2=1.0/DZ
B3=1.0/DT3
D2=DX
D2=D2
A1=B1+D1
A2=BT2PD1
A3=DT1+B1+B12
A4=MOX/DZ
ALPHA=0.17**33
SA=SIN(ALPHA)
N=1
```
GO TO 507

509  XX(1)=FM*XX*CS*(TET(1))**((4/3)
XX(1)=XX(1)/XS

514 CONTINUE

TET(M2)=0.438
TET(1)=1.00
XX(1)=0.40634
XX(1)=1.00
HET(M2)=0.00
HET(1)=0.00

C = END OF HYDRAULIC CONDUCTIVITY COMPUTATION

* * * * * *

C = INITIAL CONDITIONS

* * * * * *

DO 118 I=1,11
V(I,1)=0.00
U(I,1)=0.00
F(I,T)=0.00

118 CONTINUE

Q(0,0)

STOR(I,1)=0.00
DO 510 J=1,11
DO 510 J=1,11
2(J-J)=D2
2P(I)-1=I*M^/SA+.2*CA
5(I-1)+J)*PE

510 CONTINUE

WRITE(6,523) X1*XL,AGO,TETAS,UP,NOT,DX1,DX2,DT3,DT4,T

FORMT(60H) TOTAL SIMULATION TIME(TI(SEC)
1 = F0.3/60H LENGTH OF SLOPE X(LCM)
2 = F0.3/60H DEPTH OF SOIL X(LCM)
3 = F0.3/60H SLOPE ANGLE ALPMA(RADIANS)
4 = F0.3/60H RAINFALL RATE R/MM(SEC)
5 = F0.3/60H SATURATED HYDRAULIC CONDUCTION
6ACTIVITY KSICP/SCF
7CONTING TETAS/SCF
8TONF TETAS/SCF
9ACTIV TETAS/SCF

BND FLOW VELOCITY U(FM/SEC)

9TIME INCREMENTS FOR SOIL FLOW SYSTEM NOT = 15/60H NUMBER OF 0

DISTANCE INCREMENTS ALONG X-AXIS NDX = 15/60H NUMBER 0

2F INCREMENTS ALONG Z-AXIS NDZ = 15/60H TIME I

INCREDM FOR SOIL FLOW SYSTEM SECT DT2 = F0.4/60H T

TIME INCREMENTS FOR OVERLAND FLOW SECT DT3 = F0.4/60H T

TOTAL DIMENSIONLESS SIMULATION TIME T = F0.4/60H T

DISTANCE INCREMENTS ALONG X-AXIS DX = F0.4/60H T

TIME INCREMENTS ALONG Z-AXIS DZ = F0.4/60H T

DIMENSIONLESS TIME INCREMENT FOR SOIL FLOW SYSTEM T

DIMENSIONLESS TIME INCREMENT FOR OVERLAND T

FORDER 60H MANING'S ROUGHNESS COEFFICIENT

1XM = F0.4/76
WRITE(6,525) PRINT MOISTURE TENSION (cm of water), and correspond
10ING /22H MOISTURE CONTENT/45H MOISTURE TENSION HOI
23URE CONTENT)
WRITE(6,527) (M1(1),THE(1)),1=1,N2)
532 FORMAT(1X,F6.1,12X,F6.4)
WRITE(6,526)
526 FORMAT(60H INPUT DIMENSIONLESS MOISTURE TENSION AND CORRESPOND
10ING/34H DIMENSIONLESS WATER CAPACITY/49H MOIST
2URE TENSION/50F CAPACITY) WRITE(6,525) (M1(2),CZ1(1),1=1,N2)
533 FORMAT(20X,F6.3,13X,F5.3)
WRITE(6,515) NPT
515 FORMAT(1H,27H NPT /21X,46/8H)
1 INICIAL CONDITION/45S/72H STARTING FROM TOP AND
ZENING DOWNWARDS THE FOLLOWING FOUR GROUPS OF/66H
35 NUMBERS ARE MOISTURE CONTENT, TENSION, HYDRAULIC/63H
4WITY, AND WATER CAPACITY, IN EACH GROUP THE TOP ROW/63H
5NTS THE SOIL SURFACE, THE ACTUAL ROW REPRESENTS THE/67H
6EABLE LAYER. THE WATERSHED DIVIDE IS TO THE LEFT AND THE/28H
7ULET IS TO THE RIGHT)
J=J+1
16 J=J-1
IF(J.EQ.0) GO TO 19
WRITE(6,17) (TF(1,J,J),J=1,III)
17 FORMAT(1H,10X,10F12.5)
GO TO 16
18 WRITE(6,19)
19 FORMAT(1H,10X)
J=J+1
20 J=J-1
IF(J.EQ.0) GO TO 22
WRITE(6,21) (TF(3,J,J),J=1,III)
21 FORMAT(1H,10X,10F12.5)
GO TO 20
22 WRITE(6,23)
23 FORMAT(1H,10X)
J=J+1
24 J=J-1
IF(J.EQ.0) GO TO 26
WRITE(6,25) (TF(5,J,J),J=1,III)
25 FORMAT(1H,10X,10F12.5)
GO TO 24
26 WRITE(6,27)
27 FORMAT(1H,10X)
J=J+1
503 J=J-1
IF(J.EQ.0) GO TO 505
WRITE(6,504) (C1(1,J,J),J=1,III)
504 FORMAT(1H,10X,10F12.5)
GO TO 503
505 WRITE(6,511)
511 FORMAT(1H,10X)
512 FORMAT(6,527)
527 FORMAT(64H THE FOLLOWING NUMBERS IN THE TOP ROW ARE THE DIMEN
15LESS/64H OVERLAND FLOW DEPTHS, THE NUMBERS IN THE SECOND R
20W ARE THE/47H CORRESPONDING FLOW VELOCITIES, THE WATERSHED DI
3IDE IS TO THE LEFT/71H AND THE OUTLET IS TO THE RIGHT. THE SI
4ngle number in the third row/50H IS THE DIMENSIONLESS FLOW RA
5TE AT THE OUTLET)
WRITE(6,512) (TV(1,J),J=1,III),U(1,J=1,11),0
512 FORMAT(1H,14X,5F11.7//1M ,14X,5F11.7//1M ,14X,1F11.7///)
512 FORMAT(6,524)
104 IF (I=J,K,L) GO TO 113
FORMAT(1H, 10X, F6.2, 15, 17)
WRITE(*,534)
FORMAT(2H)
WRITE(*,530)
FORMAT(2H)
STARTING FROM TOP AND GOING DOWNWARDS THE FOLLOWING 1G FOUR GROUPS OF 64H DIMENSIONLESS NUMBERS ARE MOISTURE CONTENT 2ND TENSION, HYDRAULIC CONDUCTIVITY, AND WATER CAPACITY, IN EACH GROUP THE 1ND ROW REPRESENTS THE SOIL SURFACE, THE 2ND IMPERMEABLE LAYER, THE WATERSHED DIVIDE IS TO THE LEFT THE 28TH OUTLET IS TO THE RIGHT///
J=N+1
J=J-1
IF(1J.EQ.0) GO TO 83
WRITE(*,E6) (THETA(I, J, N), I=1, II)
FORMAT(1H, 10X, 10F12.5)
GO TO 81
WRITE(*,E4)
FORMAT(2H)
J=N+1
J=J-1
IF(1J.EQ.0) GO TO 87
WRITE(*,E6) (K(I, J, N), I=1, II)
FORMAT(1H, 10X, 10F12.7)
GO TO 89
WRITE(*,E2)
FORMAT(2H)
J=N+1
J=J-1
IF(1J.EQ.0) GO TO 90
WRITE(*,E5) (C(I, J, N), I=1, II)
FORMAT(1H, 10X, 10F12.7)
GO TO 92
WRITE(*,E3)
FORMAT(2H)
J=N+1
J=J-1
IF(1J.EQ.0) GO TO 94
WRITE(*,E6)
FORMAT(65H)
THE FOLLOWING NUMBERS IN THE TOP ROW ARE DIMENSIONLESS TIME/72H INCREMENTS FOR OVERLAND FLOW CALCULATED FROM THE ALPHA-CHARACTERISTIC EQUATION, THE ACTUAL TIME TACKER
3EAT USED MUST BE LESS THAN /72H THE SMALLEST OF THESE NUMBERS TH/2M THE DIMENSIONLESS OVERLAND FLOW OFF-THE-WATERSHED DIVIDE IS TO THE LEFT AND THE OUTLET IS TO THE RIGHT
9) WATERSHED DIVIDE IS TO THE LEFT THE OUTLET IS TO THE RIGHT///
WRITE(*,E2)
FORMAT(65H)
THE FOLLOWING NUMBERS IN THE TOP ROW ARE DIMENSIONLESS TIME/72H INCREMENTS FOR OVERLAND FLOW CALCULATED FROM THE ALPHA-CHARACTERISTIC EQUATION, THE ACTUAL TIME TACKER
3EAT USED MUST BE LESS THAN /72H THE SMALLEST OF THESE NUMBERS TH/2M THE DIMENSIONLESS OVERLAND FLOW OFF-THE-WATERSHED DIVIDE IS TO THE LEFT AND THE OUTLET IS TO THE RIGHT
9)
WRITE(*,E35)
WRITE(6,125)
WRITE(6,117) (Y(I, J, N), I=1, II)
WRITE(6,111) (U(I, J, N), I=1, II)
WRITE(6,120)
WRITE(6,100)
WRITE(6,119)
DO 97 J=1, JJ
DO 98 J=1, JJ
DO 99 J=1, JJ
DO 100 J=1, JJ
CONTINUE
IF(IOUT7.EQ.0) GO TO 119
GO TO 31
DO 119 J=1, JJ
GO TO 31
STOP
END
<table>
<thead>
<tr>
<th>Page</th>
<th>Line</th>
<th>Text</th>
</tr>
</thead>
</table>
| 23690 | 11 | P
| 19400 | 6 | I
| 22490 | 5 | G
| 16490 | 4 | L
| 11490 | 3 | C
| 6490 | 2 | D
| 1 | 1 | A

* * *

**Note:** The text appears to be a mix of numbers and symbols, possibly related to a program or algorithm.
DN 27 J=J+1, JX
C = .25*DT+H17*(K[I,J,N-1]+K[I,J+1,N-1])
IF(J.EQ.1) GO TO 24
IF(J.EQ.2) GO TO 75
K1=K[I-1,J,N-1]*Z+O*K[I,J,N-1]+K[I,J,N-1]
A = .25*DT+H17*(K[I,J,N-1]+K[I,J,N-1])
B = -(C[I,J,J-1]+.25*DT)*T2*K2
D=+25*DT*(K[I,J,N-1]+K[I,J,N-1])+CA/DZ
D3=.25*DT+H17*(K[I,J,N-1]+K[I,J,N-1]+K[I,J,N-1])
IF(J.EQ.1) GO TO 23
D2=K[I,J,N-1]-.25*DT*B*T1*K1+K[I,J,N-1]
D4=.25*DT*(K[I+1,J,N-1]+K[I-1,J,N-1]+SA/DX)
D = -.125*(S+D4)-05
IF(J.EQ.X) GO TO 22
B1(J) = C[I+1,A+1,J-1])
G1(J) = 10+G1(J-1))/8-A+81(J-1))
GO TO 27
32
D1=CMH1(J-1)
G1(J) = (.A+G1(J-1))/B+81(J-1))
M(I,J,N)=G1(J)
IF(IJ,J+1,FE,0,C) GO TO 207
THE1(I,J,N)=TEPL1(H[I,J,N]+HET1,TET1,M2+0)
K(I,J,N)=EFFM1(H[I,J,N]+HET1,TXL1,PZ,0)
C(I,J,N)=TEPL1(H[I,J,N]+H2,C2,M2,0)
GO TO 27
207
K(I,J,N)=1.00
THE1(I,J,N)=1.00
C(I,J,N)=0.00
GO TO 27
24
D4=.25*DT*(K[I+1,J,N-1]+K[I-1,J,N-1]+SA/DX)
D = -.125*(S+D4)-05
B1(J)=A+C/J/F
G1(J)=(.A+CMH02+CA)/B
27 CONTINUE
J=J+2
28
M(I,J,N)=G1(J)-E1(J)=H[I,J,N]
IF(IJ+,F),GE,0,C) GO TO 208
THE1(I,J,N)=TEPL1(H[I,J,N]+HET1,TET1,M2,0)
K(I,J,N)=TEPL1(H[I,J,N]+HET1,TXL1,PZ,0)
C(I,J,N)=TEPL1(H[I,J,N]+H2,C2,M2,M2,0)
GO TO 204
208
THE1(I,J,N)=1.00
K(I,J,N)=0.00
C(I,J,N)=0.00
205 CONTINUE
IF(J.EQ.1) GO TO 25
J=J-1
GO TO 28
29 CONTINUE
30 RETURN
END
SUBROUTINE PASAD(FILT2,THERA,C1,K,H)

THE SUBROUTINE PASAD IS USED WHEN SATURATED AND UNSATURATED
SUBREGIONS FIRST TOGETHER IN THE FLOW REGION

THE PARABOLIC-ELLIPTIC SYSTEM

DIMENSION THETA(10,10),M(10,10),K(10,10),TET1(21),XX(21)
DIMENSION CI(10,10),BI(10,10),GI(10,10),FILT1(10,10),M2(23),C2(23)
DIMENSION HFI(21),FILT2(10,10)
COMMON/BLCK2/J,J1,J2,J3,J4,DX,DZ,DT,CA,SA,RENPT,NOT,IK,JK,SJ,SZ,TZ
COMMON/BLCK3/R15,R14,R13,R12,R11,R10,R9,R8,R7,R6,R5,R4,R3,R2,R1
REAL KS,K1,K2,K3

ID = 200

C TOP BOUNDARY CONDITION COMPUTATION

J = JJ
J = 1
1 IF(META(J-1,J1),GE,TETS1) GO TO 11
F3=2.0*RE+CA/DZ
F4=(K(J-1,J1),K(J-1,1))+(M(J-1,J1)-M(I-1,J1))/DX+CA/DZ
IF(I,J-1) GO TO 1
IF(J1,1) GO TO 3
F1=0.5*(K(J,J1),K(J+1,1))+(M(J,J1)-M(I,J1))/DX-SA1/DX
F2=0.5*(K(J,J1),K(J-1,1))+(M(J,J1)-M(I-1,J1))/DX-SA1/DX
GO TO 4
2 F1=(K(J,J1),K(J+1,1))+(M(J+1,J1)-M(I,J1))/DX-SA1/DX
F2=0.0
GO TO 4
3 F1=0.5*(K(J,J1),K(J+1,1))+M(I,J1))/DX-SA1/DX
F2=0.0
4 F1=F1-F2+F3-F4
DTHETA=F4-F1
THETA(J1,J2)=META(J1,J2)+DTHETA
IF(META(J1,J2),GE,1.0) GO TO 101
M(J,J2)=TERP1(THETA(J1,J2),TET1,1)
GO TO 5
5 IF(I,J1,1) GO TO 6
J = J1
GO TO 1
C RIGHT HAND SIDE BOUNDARY COMPUTATION

1 I = 11
J = J1-1
7 IF(META(J,J1),GE,TETS1) GO TO 11
F3 = 0.5*(K(J,J1),K(J-1,1))+(M(J,J1)-M(I,J1))/DX-SA1/DX
F2 = 0.0
IF(J,J1,1) GO TO 8
F3 = 0.5*(K(J,J1),K(J-1,1))+(M(J,J1)-M(I,J1))/DX+CA1/DZ
F4 = 0.5*(K(J,J1),K(J+1,1))+(M(J,J1)-M(I,J1))/DX+CA1/DZ
GO TO 9

PA 0622
PA 0623
PA 0624
PA 0625
PA 0626
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PA 0678
PA 0679
PA 0680
19 X1=IS*PI/(2.0*XD)
20 X2=COS(X1)
21 DO 30 J=1,X
22 DO 50 I=1,X
23 IF (I.EQ.11) GO TO 25
24 C=0.25*UT*DT*(K(I,J,2)+K(I,J-1,2))+2.0*K(I,J,2)+K(I,J-1,2)
25 IF (I.EQ.1) GO TO 19
26 A=25*UT*DT*(K(I,J-1,2)+K(I,J,2))
27 K1,K2=K1,2*K(I,J-1,2)+Z.0*K(I,J,2)*K(I,J-1,2)
28 KI=K1,2+Z.0*K(I,J-1,2)*K(I,J-1,2)
29 K2=K1,2+Z.0*K(I,J-1,2)*K(I,J-1,2)
30 IF (I.EQ.1) GO TO 17
31 IF (I.EQ.11) GO TO 23
32 A=25*DT*BT2*K(I,J,1)+K(I,J-1,1)+K(I,J,1)
33 X3=0.5*12.0*UT*(K(I,J-1,2)+Z.0*K(I,J-1,2)+K(I,J-1,2))
34 IF (I.EQ.11) GO TO 24
35 D3=0.25*DT*BT2*K(I,J-1,2)+K(I,J,2)+K(I,J-1,2)
36 GO TO 10
K(1,J,2)=1.00
C(1,J,2)=0.00
CONTINUE
62 IF(K1,J,2) GO TO 63
J = J-1
GO TO 61
CONTINUE
63 IF(IN.EQ.0) GO TO 100
C11=0.0
C12=0.0
GO 65 J=1,JX
GO 65 I=1,JY
CT1=ABS(M1(J,J,N+1)-M1(J,J,N+2))
C12=C11+C13
IF(C11.GT.CT2) GO TO 64
GO TO 65
64 CT2=CT1
N1 = 1
NJ = J
CONTINUE
65 IF(IS.EQ.6) GO TO 68
100 IS=15+1
IN= 1
DO 67 J=1,JX
DO 67 I=1,JY
M1(J,J+2)=M1(J,J+1)
110 IF(M1(J,J+2).GE.0.00) GO TO 210
THETA(1,J,2) = TEP1(M1(J,J,2)+H1(J,J,2)+M2,J,0.0)
K(1,J,2) = TEP1(M1(J,J,2),J1,E1,K1,2,M2,J,0.0)
C11(1,J,2) = TEP1(M1(J,J,2)+H2,J2,C2,M2,J,0.1)
GO TO 211
210 THETA(1,J,2)=1.00
K(1,J,2)=1.00
C11(1,J,2)=0.00
211 CONTINUE
IF(J.EQ.1) GO TO 65
FILL1(J,J)=K1(J,J,2)*((M1(J,J+1,2)-M1(J,J-1,2))/(2.0*DZ*CA)+1.0)
GO TO 70
69 FILL1(J,J)=G.25*(M(DK1(J,J,2)-K1(J,J,2))*((M1(J,J+1,2)-M1(J,J-1,2))/(2.0*DZ*CA)+1.0)
4DZ*CA=1.00)
70 CONTINUE
RETURN
END

SUBROUTINE SATADI(FILT2,THETA,C1,K1,M)
C
* THE SUBROUTINE SATADI IS USED WHEN THE FLOW REGION IS COMPLETELY
C* SATURATED (THE ELLIPTIC SYSTEM)
C
* DIMENSION K(10,10),C1(10,10,2),F1(10,10),H(10,10,2),FIL1(10,10,2,10),M(10,10,2)
* COMMON/BLCK/1,J,J1,J2,J,JX,DX,DZ,DT,CA,SA,RE,NPT,NOT,IP,JX,BT2
* COMMON/BLCK/1,J,J1,J2,J,JX,DX,DZ,DT,CA,SA,RE,NPT,NOT,IP,JX,BT2
* JD = 300
C
* TOP BOUNDARY CONDITION COMPUTATION
C
*
J = JJ
DO 34 I=1,11
34 CALL SURF(FJLT2,1,ETA,C1,K,H)
CALL RHANSIDE HUNDARY CONDITION COMPUTATION
C
D 11
J = JJ-1
1 IF(J.EQ.1) GO TO 2
H(I,J,N)=D*H(I,J-1,N-1)+H(I,J+1,N-1)
J = J-1
GO TO 1
2 H(I,J,N)=D*CA+H(I,J+1,N-1)
C
C THE ADI SOLUTION IN THE X-DIRECTION (Sweeping the Rows)

IS = 0
3 X1=15*PI/(2.*XN)
X2=CSU(X1)
X3=2.*CSU(X2)
31 DO 14 J=1,JY
DO 12 I=1,JX
B = (-2.,0)*X2*(I-1)
D2 = ((X3*X3-2.*X2*I))**M(I,J)
D2 = (0.75)**M(I,J+1)
IF(J.EQ.1) GO TO 7
5 IF(I.EQ.1) GO TO 8
IF(J,EQ.1) GO TO 9
IF(I.EQ.1) GO TO 10

A D = 01-LJ-D+12
IF(I.EQ.1) GO TO 12
B(I,J-1)=0.-01(I-1)-1)
G(I,J)=0-G(I-1-I)/(8-B(I-I-1))
GO TO 12
32 DO=H(I-1,J,N)
G(I,J)=(0-C(I-1-I))/(8-B(I-I-1))
W(I,J,N)=G(I,J)
GO TO 12
7 DO = 12.0*03
GO TO 5
8 D1 =(-2.0*03*SA-BT3*2.0*02*CA)
GO TO 11
9 D1 = BT3*2.0*02*CA
GO TO 6
10 D1 =(-2.0*03*SA-BT3*H(I,J-1,1))
11 D = 01-D2-C2
B(I,J)=0.0/8
G(I,J)=D/8
12 CONTINUE
I=1-2
IF(J,EQ.1) GO TO 14
I = I-1
GO TO 13
14 CONTINUE
C
C THE ADI SOLUTION IN THE 2-DIRECTION (Sweeping the Columns)

DO 25 I=1,JX
DO 23 J=1,JX
B = (-2.0*X3*023)
25 CONTINUE
D2 = \( (X3 + D1^2 - 2.0 \times D1 + 1) \times H(I, J, N) \)
D3 = \( M4 + H(I, J, N) \)
IF I = EO = 11, GO TO 18
16
IF (I < EO + 1), AND J > EO, 1) GO TO 19
IF I = EO + 1), GO TO 20
IF J = EO + 1), GO TO 21
D1 = B4 + H(I - 1, J, N)

17
D = -D1 - D2 - D3
IF I < EO + 1) GO TO 33
B11 = \( 1.0 / (H(I, J, N) - 1)) \)
G111 = \( I - C1(I - 1) / 16 - B11 (I - 1) \)
GO TO 23

23
D = D - H(I, J, N) + 11
G111 = \( B11 (I - 1) / 16 - B11 (I - 1) \)
M1 = I, J, M1 = C1, J = 1
GO TO 23

18
D = 2.0 + D3
GO TO 16

19
D1 = \(- (2.0 \times D1 + DX \times FA - 2.0 \times D2 + CA) \)
GO TO 22

20
D = -B4 + 2.0 \times X \times SA
GO TO 17

21
D1 = 2.0 \times D2 + CA + BY + (I - 1, J, N)

22
D = -D1 - D2 - D3
B111 = \( 2.0 / 8 \)
G1111 = D / 8
CONTINUE
J = J - 2

24
M111 = J + 1, J + 1, N + 1
IF I = EO + 1, GO TO 25
J = J - 1
GO TO 24

25
CONTINUE
IF I = EO + 0, GO TO 160
C11 = 0.0
C12 = 0.0
C13 = 0.0
DG 27 J = 1, JX
DG 27 J = 1, IX
C1 = a5\( (H(I, J, N + 1) - M(I, J, N + 2)) \)
C12 = C1 + C13
IF I = G.E., C12 = 1.0
GO TO 26

26
C12 = C1
NJ = 1
NJ = J
CONTINUE
IF I \( \times C12 \times (L.T.0.0001) \) GO TO 30

100
IS = 15 + 1
DG 29 J = 1, JX
DG 29 J = 1, IX
M(I, J, N + 2) = M(I, J, N + 1)
29
M(I, J, N + 1) = M(I, J, N + 1)
GO TO 3

30
DG 35 J = 1, JX
DG 35 J = 1, IX
M(I, J, N + 1) = M(I, J, N + 1)
CONTINUE
RETURN
END
INPUT DATA

The first set of data required are those necessary for the computation of the hydraulic conductivity for the watershed. The constants are the soil temperature, the density, viscosity and surface tension of water at this temperature, the saturated hydraulic conductivity and the saturated moisture content for the soil, the number of pore classes, M, the acceleration due to gravity and the length of the watershed section. The moisture-tension curve must also be obtained preferably experimentally since there are no simple functions relating the soil parameters. At least five points must be obtained for the moisture-tension curve; the more the better; the number of points thus obtained was designated by NZ. The dimensionless form of the moisture-tension curve was plotted and the water capacity-tension relationship was obtained from this plot. The number of points obtained for the above C1-h curve was designed by MZ1.

The NZ values of \( \theta \) were read into the computer in ascending order and the corresponding values of \( h \) were also read into the computer. It does not matter which parameter is read in first, \( \theta \) or \( h \), the only restriction is that whichever is read in first has to be in ascending order of magnitude. This restriction is due to the way the interpolation function TERPl was written. The MZ1 values of C1 and \( h \) have to be read into the computer in a similar manner. The constants NZ and MZ1 must be supplied as input data. The other parameters required in the program are
as indicated in the READ statement of the MAIN program.

The initial values of the moisture content, tension, hydraulic conductivity and water capacity must be supplied for each point of the grid system. These could be read in or a rule could be used to calculate them as was explained in Chapter 3 of this report. The simulation begins after the hydraulic conductivity has been calculated and the initial conditions specified. The input data are printed out as part of the output for verification purposes.

**OUTPUT DATA**

The first set of data printed out are some of the input information and some of the calculated constants that have physical significance and the second set are the moisture-tension data used for plotting the moisture-tension curve and for deriving the other soil parameters needed in the program; the third set are the dimensionless moisture tension-water capacity data also derived from the moisture-tension curve. These three sets of data are shown on the next two pages.

In order to ascertain whether the watershed section is unsaturated, saturated or mixed and whether overland flow has started, IOUT1, IOUT2, IOUT5 and IOUT7 are printed out; their meaning were reported in Appendices A and B and in the body of this study. The number of time increments NPT is also printed out. The other printouts include the infiltration rate, FILT2, the moisture content, the capillary pressure, the hydraulic conductivity, the water capacity, the time increment for overland flow calculated from the \( \alpha \)-characteristic equation, the overland flow velocity and depth and the outflow rate. These printouts and further explanation are given in the following pages.
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<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Depth of Soil (m)</td>
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<tr>
<td>Slope Angle (Radians)</td>
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<tr>
<td>Rainfall Rate (mm/sec)</td>
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<tr>
<td>Saturated Hydraulic Conductivity (m/sec)</td>
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<tr>
<td>Saturated Moisture Content (cc/g)</td>
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<tr>
<td>Reference Overland Flow Velocity (m/sec)</td>
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<tr>
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<td>Number of Distance Increments Along Z-Axis</td>
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<td>Time Increment for Overland Flow (sec)</td>
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<td>Manning's Roughness Coefficient</td>
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**INPUT MOISTURE TENSION (CM OF WATER) AND CORRESPONDING MOISTURE CONTENT**

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**INPUT DIMENSIONLESS MOISTURE TENSION AND CORRESPONDING WATER CAPACITY**

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Starting from top and going downwards, the following four groups of dimensionless numbers are moisture content, tension, hydraulic conductivity, and water capacity. In each group, the top row represents the soil surface, the bottom row represents the impermeable layer, the watershed divide is to the left, and the cutlet is to the right.

<table>
<thead>
<tr>
<th>Number</th>
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</table>

The following numbers in the top row are dimensionless time increments for overland flow calculated from the alpha-characteristic equation. The actual time increment used must be less than the smallest of these numbers. The numbers in the second row are the dimensionless overland flow depths. The numbers in the third row are the corresponding flow velocities. The single number in the fourth row is the dimensionless flow rate at the cutlet. The watershed divide is to the left and the cutlet is to the right.
APPENDIX E

COMPUTER OUTPUT FOR KIRKHAM'S DITCH DRAINAGE PROBLEM
The output shown on the next page are all dimensionless in accordance with the dimensionless variables of Chapter 3. The elevation, capillary pressure, and total potential should be multiplied by $L = 40$ to make them dimensional. In each of the 6 groups of numbers the top row represents the soil surface and the bottom row represents the impermeable layer; the midway point between the ditches is to the left and the ditch is to the right.
### Moisture Content

<table>
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<th>Value 3</th>
<th>Value 4</th>
<th>Value 5</th>
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### Elevation

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</tbody>
</table>

### Water Capacity

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<th>Value 3</th>
<th>Value 4</th>
<th>Value 5</th>
<th>Value 6</th>
<th>Value 7</th>
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<tbody>
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</tbody>
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MOISTURE CONTENT

CAPILLARY PRESSURE

TOTAL POTENTIAL

HYDRAULIC CONDUCTIVITY

WATER CAPACITY


