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THE EFFECT OF HEAT CURRENT MODULATION
ON THE VELOCITY FIELDS
AND THE CRITICAL REYNOLDS NUMBER IN HELIUM II

DISSERTATION

Presented in Partial Fulfillment of the Requirements
for the Degree Doctor of Philosophy in the Graduate
School of the Ohio State University

By
Charles Evan Oberly, B.Sc.

The Ohio State University
1971

Approved by

[Signature]
Adviser
Department of Physics
PLEASE NOTE:

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UNIVERSITY MICROFILMS
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The myriad of event prescribed and proscribed choices available along my world line led in their curious causal way to the study of low temperature physics. The most significant event was a thermodynamics course taught by Prof. J.G. Daunt, my first graduate adviser. Appreciation for thermodynamic concepts and, in particular, entropy developed while studying under Prof. Daunt was the prime motivation for eventual study of low temperature physics. The beauty of the progression of specific heat prominences that announced unsettling discovery after discovery at low temperatures was made quite clear by Prof. Daunt, whose personal researches were a part of the first great thrust in low temperature physics.

Following my connection with the origins of modern low temperature physics, Prof. D.O. Edwards became my adviser. The experience of observing the physical intuition of the current master of helium has been invaluable in setting personal standards of achievement.

Superfluidity was of great interest to me, because of previous work on vortex pinning in superconductors.
I am especially indebted to Prof. J.T. Tough for the suggestion and guidance of my research on hydrodynamic stabilization of heat flow in helium II combined with the study of superfluid vorticity in a new context. I am grateful for his jovial support and forbearance with my sometimes rather unorthodox working methods. The seemingly endless stream of physical models suggested by Prof. Tough were often of the proper timing to stimulate a hopelessly stymied student into renewed action.

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Finally, the usual thanks are due my wife who typed, endured with surprising good humor, and enchantingly embraced the attendant poverty.
VITA

September 2, 1939Born-Columbus, Ohio

1957Graduated, Westville High
School, Westville, Ohio

1961B.Sc., Ohio State
University, Columbus, Ohio

1961-1967Physicist, Aero Propulsion
Laboratory, Wright-Patter-
son AFB, Ohio

1967-1968Graduate Teaching Assis-
tant, Physics Department,
The Ohio State University,
Columbus, Ohio

1968-1971Graduate Research
Associate, Physics Depart-
ment, The Ohio State
University, Columbus, Ohio

PUBLICATIONS

"A Scaling Parameter for He II Thermal Counterflow and
Critical Heat Currents," J.T. Tough and C.E. Oberly,

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"An Apparatus for the Neutron Irradiation of Superconduc-
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CHAPTER I

INTRODUCTION

1.1 Classical Fluid Flow

Osborne Reynolds (1, 2, 3, 4) conducted flow experiments with classical incompressible fluids in tubes and found that a linear relation existed between the pressure gradient along the flow tube and the mean cross-sectional velocity $<v>$ of the fluid, so long as laminar flow conditions existed. Further increase of the mean velocity resulted in the development of eddies and a serious departure from the linear law of Poiseuille (5, 6, 7, 8). Reynolds discovered that this departure from linearity took place at a constant value of a characteristic dimensionless parameter, the Reynolds number $R$, for all classical fluids (1, 3, 4, 8)

$$ R = \frac{\rho <v>d}{\eta} \quad (1-1) $$

where $<v>$ is the value of the mean velocity in the tube, $d$ is the tube inside diameter, $\eta$ is the absolute viscosity, and $\rho$ is the fluid density. The flow transition from
linearity occurs at a critical Reynolds number \( R = R_0 \approx 2000 \). For a given fluid in a given tube, equation (1-1) defines a critical velocity \(<v_c>\). Reynolds also observed visually the production of turbulence throughout the tube at velocities greater than \(<v_c>\).

From dimensional aspects of classical fluid flow, the Reynolds number can be shown to be characteristic of similar flow conditions \((7,9)\). The Reynolds number is often considered to be the ratio of the inertial force term to the viscous frictional force term in the classical Navier-Stokes equation. From this viewpoint fluctuations in the flow will grow and the critical Reynolds number will define the transition point from laminar to turbulent flow, when the inertial driving force is sufficiently larger than the frictional damping force. For velocities less than that associated with the critical Reynolds number, the viscous damping in the flow is sufficient to quell the effects of any local disturbance and the flow remains well ordered. While it may be possible to exceed the critical Reynolds number for very smooth tubes with proper inlets that do not seriously perturb the flow, the flow will be metastable, and the introduction of local disturbances can result in a turbulent transition, an effect quite like supercooled liquid solidification upon introduction of nucleation sites. Below the critical Reynolds number the
introduction of local disturbances, no matter how severe, will only result in decaying oscillations and an eventual return to laminar flow. Some of these ideas are discussed further in Appendix A.

Turbulence is not easily defined in strict denotation, although several highly qualified definitions have been offered (10,11). The problem in defining turbulence is that the critical Reynolds number $R_c$ only defines the beginning of a flow region in which some disturbances are amplified as the flow proceeds downstream. As the disturbances become larger, the nonlinear effects eventually result in an irregular condition of flow such that the flow parameters appear to vary at random with time and space coordinates. The condition of complete randomness in the flow parameters at each point in the flow defines 'fully developed turbulence' from which statistically distinct average values may be discerned. These average values near the critical Reynolds number may be determined from the perturbation theory of Lin (12). In the following discussion 'turbulence' will apply to all flows for which $R>R_c$.

As Reynolds observed, the Poiseuille linear relation between the pressure gradient and the mean cross-sectional velocity no longer holds above the critical velocity. For large velocities such that fully developed turbulence is present in tube flow, a relationship between
the pressure gradient and the rate of flow through the tube has not been obtained theoretically as in laminar flow, and an empirical rule must be used.

This rule was expressed in a dimensionless form by Blasius and involved the relation between a coefficient of resistance and the Reynolds number (13). In terms of the pressure gradient this rule appears as

\[ \nabla p \propto \left( \frac{\eta \rho}{d} \right)^{\frac{3}{4}} <\hat{v}> \frac{7}{4}. \]  

(1-2)

The pressure gradient for large velocities therefore depends on the \( \frac{7}{4} \) power of the mean velocity.

The motion of a viscous fluid is described by the Navier-Stokes equation (14)

\[ \rho \left[ \frac{\partial \hat{v}}{\partial t} + (\hat{v} \cdot \nabla) \hat{v} \right] = -\nabla p + \eta \nabla^2 \hat{v} + (\zeta + \frac{\eta}{3}) \nabla (\hat{v} \cdot \hat{v}) \]  

(1-3)

where \( \rho \) is the fluid density, \( p \) is the pressure and \( \eta \) and \( \zeta \) are the first and second absolute viscosity coefficients, respectively. As mentioned in Appendix A we may use the hydrodynamic derivative and ignore the latter right hand term for incompressible flow so that equation (1-3) becomes

\[ \rho \frac{\partial \hat{v}}{\partial t} = -\nabla p + \eta \nabla^2 \hat{v}. \]  

(1-4)
Equation (1-4) can be used to describe laminar flow conditions and instantaneous situations for turbulent flow. Because of the complexity of turbulent flow, it is necessary to analyze mean values of the flow parameters, and the instantaneous values are of no utility.

Mean or average parameters in the Navier-Stokes equation (1-4) can be used to analyze fully developed turbulent flow with the substitution for velocities much greater than the critical velocity

\[ \tilde{v} = \tilde{v}_o + \tilde{v}'(t) \]  

(1-5)

where \( \tilde{v}_o \) is the time average of the velocity and \( \tilde{v}' \) is the fluctuating part, which is zero when time averaged. Still considering incompressible flow so that fluctuations in density may be ignored, and noting that both the mean and the fluctuating velocities satisfy continuity, the time average of equation (1-4) with substitution (1-5) is taken term by term to yield (15,16)

\[ \rho \frac{D\tilde{v}_o}{Dt} = - \tilde{\nabla}p + \eta \nabla^2 \tilde{v}_o - \tilde{\nabla} \cdot \left[ \rho(\tilde{v}' \tilde{v}') \right] \]  

(1-6)

where \( \frac{\partial \tilde{v}_o}{\partial t} = 0 \). This equation is just the Navier-Stokes equation applied to the mean velocity of the fluid with an additional apparent stress tensor term on the right-hand side involving the time averaged dyadic \( \rho \tilde{v}' \tilde{v}' \) of the
fluctuating velocity. These apparent stresses in turbulent flow have been termed Reynolds stress (16,17) because Reynolds originally discovered them. Reynolds stress must be added to the stresses caused by laminar flow, and the Reynolds stress due to turbulent flow will usually be much larger than that due to laminar flow, so the latter is neglected in turbulent flow. It is also possible to describe the stresses due to turbulent fluctuations by an effective or 'eddy' viscosity which merely adds to the fluid viscosity.

In this discussion of laminar and turbulent flow it is well to recall that only flow in tubes filled completely with incompressible fluid has been considered.

1.2 Stabilization of Classical Flow

Numerous physical systems are capable of being stabilized with the proper modulation of a suitable system parameter. The possibility of achieving a supercritical laminar flow has been discussed in section 1.1. In that case the flow metastability depended on the relative smoothness and inlet shape of the flow channel. There are, however, methods of stabilizing classical fluid flow beyond critical Reynolds number, as has been amply demonstrated by theory and experiment (18-29). This
stabilization is achieved by generating an oscillatory viscous shear wave in the fluid at an appropriate frequency and amplitude.

Landau and Lifshitz (30) discuss the characteristic flow established by oscillations of a solid body immersed in a viscous liquid. Assuming simple harmonic fluid motion at frequency \( \omega \), the velocity can be substituted into the Navier-Stokes equation (1-4) which then has a solution representing a transverse wave with velocity perpendicular to the direction of propagation. The amplitude of this transverse wave in the viscous fluid is damped with penetration into the fluid. At a distance \( \delta \) from the body surface the amplitude of the wave has dropped off by a factor of \( e \). This length is called the viscous penetration depth and is defined by

\[
\delta = \left( \frac{2\pi}{\alpha} \right)^{\frac{3}{2}}.
\]  

(1-7)

In the following discussion a useful dimensionless parameter for oscillating flow in tubes is \( a/\delta \) where \( a \) is the radius of the flow tube.

Gilbrech and Combs (18) experimentally determined a criterion for predicting laminar to turbulent transitions in an oscillating flow in circular cross section tubes. By application of suitable sinusoidal oscillations in
pressure, they were able to achieve a critical Reynolds number greater than $R_c = 2220$ which was obtained for steady flow conditions. (Their viscous wave parameter $k$ is related to $\delta$ by $\delta = (2)^{\frac{3}{2}}/k$.)

The experimental apparatus consisted of plastic tubing of nominal 1" or $\frac{3}{4}"$ inside diameter through which water containing yellow dye was forced by a variable speed steady flow pump in parallel with a variable stroke-variable frequency pressure oscillator. The onset of turbulence was determined by measuring the growth rate of turbulent 'plugs' introduced into the tube by a 'disturbance generator'. The 'plugs' were generated in laminar flow at different times throughout the oscillation cycle. The velocity of the leading and trailing edges of each 'plug' was then monitored by photocells at two positions along the flow tube. The critical Reynolds number of the flow was defined when the velocity of the leading edge of the 'plug' just exceeded that of the trailing edge, so that the 'plug' was indeed growing.

The results of these experiments have been plotted using the penetration depth as a parameter in Figure 1. It can be seen that for appropriate values of the velocity ratio $\tau = \langle v_p \rangle / \langle v_o \rangle$ and $a/\delta$ that the critical Reynolds number has increased by more than 100%, where $\langle v_o \rangle$ is the usual cross-sectional mean velocity of the steady component.
Stabilization by viscous wave modulation of the critical Reynolds number in water flow through tubes. The curves are interpolated from the data of Gilbrech and Combs (18) where the frequency parameter $k$ of the original data is related to the penetration depth by $\delta = (2)^{\frac{3}{2}}/k$. The tube radius is $a$, and the percent figures denote curves with a constant ratio $U$ of the cross-sectional mean amplitude of the periodic velocity $<v^*_p>$ to the cross-sectional mean steady velocity $<v_0>$. 

Figure 1
and \( <v_p'> \) is the amplitude of the periodic component of the cross-sectional mean velocity as determined by the oscillator piston stroke.

These results demonstrate a very significant enhancement of the stability of the flow using viscous wave modulation. The results were contrary to those anticipated by Gilbrech and Combs, and they were unable to explain their data.

The important experimental results of Donnelly and others (21,22) should not have been neglected, however. In these experiments the enhancement of Couette flow stability by modulation of the viscous flow between large diameter rotating cylinders was investigated. The mobility of ion species present in the carbon tetrachloride between the cylinders was measured with a sensitive electrometer to determine the onset of instability (32,33). Subcritical rotational frequencies produced no changes in the ion detection current, but a distinct change in the ion current was noted when the peak angular velocity was identical to the critical angular velocity for steady flow. The onset of instability resulted in enhanced ion mobility due to the development of radial flow.

The onset of instability could be strongly inhibited by modulating the rotational angular velocity of the inner cylinder. However, at relatively high or low frequencies, enhanced stability was not observed. At high
modulation frequencies the thickness of the annulus was much greater than the penetration depth characteristic of the rotation frequency, and the viscous shear wave did not significantly penetrate the fluid. At low frequencies there was no phase shift across the annulus so that an oscillatory shear wave does not appear in the fluid. At a given modulation frequency, the degree of stability enhancement increases with the amplitude of oscillation. The maximum degree of stability enhancement in these rotation experiments is not as pronounced as in the tube flow experiments of Gilbrech and Combs.

Donnelly et al. have suggested that the mechanism responsible for the stabilization in Couette flow is an interference between the viscous shear wave propagated across the annulus and the shear wave of the disturbance. Various widths of annuli were investigated which revealed that the shear wave stabilization was not as effective when larger annuli were used at the same modulation frequency. This demonstrates that the enhanced stabilization is quite dependent on the penetration depth.

Snyder (23) has investigated steady rotating flow between noncircular cylinders. Although these results were quite complex, it appeared that flow between a circular cylinder rotating inside a fixed square cylinder was slightly stabilized by the addition of an obstruction.
to the flow in the annulus. The obstruction apparently generates the proper time-dependent modulation of flow parameters for enhanced stability.

In an effort to understand these stabilization phenomena, Grosch and Salwen (19,20) theoretically investigated the hydrodynamic stability of both steady and modulated plane Poiseuille flow in a viscous fluid. A truncated expansion of orthogonal functions which satisfied the boundary conditions to the time-dependent Orr-Sommerfeld equation (31) was manipulated to yield the stability coefficient (the imaginary part of the eigenvalue of the least stable mode) for the flow. When this coefficient was compared with those of other workers for time-independent Poiseuille flow, it was found to agree within 1%. When the analysis was applied to the modulated flow, the results were that the modulated pressure gradient stabilized the flow, or at least decreased the growth rate of disturbances whenever complete stabilization did not occur.

The results of the calculation reveal that an initially unstable steady flow to which modulation is added will evolve through a stabilization peak as the modulation frequency is increased. The stability peak is observed regardless of whether the pressure amplitude or the velocity amplitude is held constant, i.e. the damping
coefficient in the Orr-Sommerfeld solutions goes from negative to positive, increases through a peak, and begins to decrease again as frequency increases. Viewed differently, the flow was considered with several different Reynolds numbers such that it was initially unstable. The modulation was then 'turned on' at a fixed frequency and the modulation pressure amplitude was increased from zero. Again, the damping stability coefficient goes from negative to positive in a smooth fashion. Although results are not extended far enough to discern a peak in stability, these results do indicate that even quite unstable flows can be stabilized by sufficiently large modulation amplitudes.

In his classic work on hydrodynamic stability, Lin (12) has shown that the unstable disturbance of frequency $\omega_d$ leading to the breakdown of laminar flow and the onset of turbulence appears as a shear wave in a viscous fluid very near the wall, and that this is the region in which energy is transferred from the base flow to the disturbance. The characteristic dimension of this region is the penetration depth given by equation (1-7) where $\omega_d$ is the frequency of the disturbance wave. Grosch and Salwen discovered that maximum stabilization of the flow occurs when the penetration of the modulation shear wave of frequency $\omega$ is near the penetration depth
of the disturbance wave, as had been observed by Donnelly et al. It was therefore suggested by Donnelly et al. and Grosch and Salwen that the viscous shear wave due to the modulation interferes with the disturbance shear wave in such a way that the process of energy transfer from the base flow to the disturbance is inhibited.

There has been a much more extensive theoretical effort expended in the area of viscous shear modulation of rotational flow than for tube flow. Conrad and Criminale (24, 25) investigated the stability of time-dependent Couette flow using the Reynolds-Orr perturbation energy equation, and found that a stabilization effect occurs. The experimental points of Donnelly et al. are found to fit the theoretical predictions for small amplitude modulation. Very large amplitude modulation causes a quite large destabilization. Shortly after the above report the problem was analyzed by a different method by Meister and Münzner (26), who obtained the same behavior for modulated rotational flow including agreement at small amplitudes with the data of Donnelly et al. Rosenblat (27) has also analyzed the problem for inviscid rotational flow and found that modulation could either stabilize or destabilize the flow.

There are other more qualitative investigations which provide further examples of stabilization in
classical flow when suitable parameters are modulated. Donnelly and Glaberson (29), motivated by the modulated rotational flow experiment, discovered experimentally that the stability of a gravity-driven water jet could be enhanced by vibrating the flow apparatus. These vibrations caused an oscillatory pressure gradient in the flow tube. Drazin (28) calculated the stability of parallel flow of a viscous conductive fluid in an oscillating magnetic field and found enhanced stabilization for suitably constrained flow.

A number of recent theoretical papers (34-36) have analyzed the effect of modulation on the onset of thermal convection in a horizontal layer of viscous fluid between two plates. Two of these efforts (34,35) deal with a steady temperature difference between the plates on which is superposed an oscillating temperature gradient of small amplitude. The third introduces the modulation by shaking the configuration vertically. These analyses are analogous to those required to explain the experiments of Donnelly et al., and, generally speaking, conditions for stabilization analogous to those observed by Donnelly et al. could be identified. A very valuable work by Betchov and Criminale (37) discusses the problem of analyzing stability in parallel flows although it does not specifically develop solutions to the problems of importance here.
In summary, there is a large body of experimental and theoretical evidence that stabilization of classical fluid flow is indeed possible with properly applied viscous wave modulation. These effects occur for Poiseuille flow in tubes and between infinite planes as well as in rotational flow between cylinders of various types. Experimental and theoretical conditions indicate that stabilization is likely in flows where the amplitude of modulation is not too large, and the frequency is such that $\delta$ is of the order of a characteristic dimension of the geometry. It has been suggested that the stabilization is caused by the interference between the modulation induced shear wave and the disturbance shear wave, both of which have a penetration depth of order $d$, the tube diameter. The parameterization of the pulsating flow is complete if the Reynolds number, a frequency parameter (usually the penetration depth), and the ratio of the modulation amplitude to the mean flow amplitude are known.

The following report will concentrate on modulated tube flow, so it is important to mention the work of Uchida (38) who solved the hydrodynamic equations of motion exactly for pulsating viscous flow superposed on a steady laminar flow of incompressible fluid in a circular cross section tube. Uchida does not investigate the question of stability, but he does develop expressions
for the time dependent velocity and pressure gradient parameterized by the radius, modulation frequency, and pressure amplitude ratio. These expressions are given in exact form as well as an asymptotic limit for low frequencies where \( a/\delta \ll 1 \). An abbreviated discussion of Uchida's results has been included in Appendix B. Uchida identified the total time averaged mass flow with that given by the Poiseuille law for the steady component of the pressure gradient.

1.3 Flow in Liquid Helium II

The preceding analysis of stability in classical fluid flow cannot be extended to flow in liquid helium without qualification. When the temperature of liquid helium drops below about 2.2°K (the so-called lambda point) at the saturated vapor pressure, the liquid goes through a remarkable transformation. Bulk thermal conductivity becomes so great that bubbles no longer form, even though this liquid has an extremely low latent heat of vaporization. Numerous other properties of the fluid develop in strange and dramatic ways if viewed from the standpoint of classical fluids. One of the more spectacular triumphs and mysteries in physics has been the discovery and description of the macroscopic quantum behavior of liquid helium below the lambda point (39-42). The dramatic difference in the behavior of liquid helium above and below
the lambda point has encouraged the labeling of the two
distinct fluid behavior regions as helium I and helium II,
respectively. The flow and stability properties of helium I
are classical and will not be of further interest here.
The flow and stability properties of helium II remain
enigmatic and constitute the major concern of this report.

The superfluid property of helium II was not
discovered until 30 years after the first liquefaction of
helium. All indications of this dramatic property must
have been observed at one time or another, but the rami-
fications of the observations were just too astounding to
be believed. Proper thermal conductivity measurements in
the mid-1930's finally provided convincing evidence of the
'super' heat conduction properties of helium II. Somewhat
later, Kapitza (43) experimentally observed the nearly
inviscid flow of helium II through narrow channels and
termed this behavior 'superfluidity'. The viscosity
measured in Kapitza's experiments was found to be quite
small and began to disappear altogether as the channel
dimensions were narrowed.

These strange and marvelous effects were inexpli-
cable until Landau (44), following a suggestion of Tisza
(45), in an intuitive burst of genius formulated the
highly successful phenomenological two fluid model.
According to this model helium II consists of two
components which coexist without interaction, one of which exhibits classical flow properties and is termed 'normal', and the other exhibits 'super' flow properties and is labeled accordingly. The normal component is associated with thermal excitations called phonons and rotons, the existence of which has been proved by neutron excitation experiments as well as numerous thermal phenomena. The normal component is considered to have a density $\rho_n$ and the remainder of the fluid is the super component of density $\rho_s$ such that the total density is $\rho = \rho_n + \rho_s$.

The entire entropy of the liquid is assumed to be contained in the normal component. The density of the components is strongly temperature dependent so that $\rho_n = \rho$ and $\rho_s = 0$ at the lambda point, and $\rho_n = 0$ and $\rho_s = \rho$ at absolute zero. The normal fluid excitations are then considered to exist in a noninteracting background of superfluid.

The strange flow effects in helium II can now be understood, at least in a heuristic sense, and the success of the two fluid model in explaining physical phenomena has been well established experimentally. The flow of helium II through small channels is explained by requiring all the viscous frictional losses to be associated with the normal component ($\gamma_n = \gamma$) while the curl-free super component flows without experiencing any frictional losses ($\gamma_s = 0$) with the normal component or physical
obstructions for flow paths larger than atomic dimensions. Then as the flow channel decreases from some large dimension, the normal component experiences more and more viscous friction losses until, at some critical size, it becomes effectively clamped by the friction forces and can no longer participate in the flow. For a channel size greater than atomic dimensions the super component can still flow in a friction free manner so that an infinitesimally small pressure difference will drive an enormous mass flow for small super component velocities. Critical velocity effects eventually limit the super flow and will be discussed in detail later.

The utility of the two fluid model is most apparent in describing heat flow. A heat source in helium II induces phonon and roton thermal excitations at the heater which travel away from the heater. If the total fluid density remains fixed, the continuity equation for the two fluids for zero source and sink terms will be

$$\rho_n \dot{v}_n + \rho_s \dot{v}_s = 0,$$  \hspace{1cm} (1-8)

where $\dot{v}_n$ and $\dot{v}_s$ are the respective velocities of the normal and super components. The efflux of normal component from the heat source must then be balanced by an influx of super component to satisfy the continuity equation. The entropy flow will be carried away from the
heater by the normal component while no entropy is assumed to be associated with the super component. Arp (46) has recently presented a complete review of heat transport through liquid helium II.

The discussion will now be restricted even further to channel constrained heat flow. Figure 2 illustrates the behavior of the two fluid velocity fields when a heater which communicates with a large constant temperature reservoir of liquid helium through a flow channel is located at the thermally isolated end of the channel. The net flow of mass through the channel is zero, and the two fluids have antiparallel velocity fields so that heat is conducted from heater to reservoir by the normal component flow. The terms 'internal convection' or 'thermal counterflow' are applied to this heat transfer process in helium II.

An increase in normal component density caused by a temperature rise will result in flow of the super component to the higher temperature region to restore the equilibrium of the helium II. If this flow takes place through small pores which block the normal component, the pressure in the heated region will increase as the superfluid influx is attracted by the increased temperature. For an equilibrium temperature difference \( \Delta T \), the equilibrium pressure difference \( \Delta P \) is described by the
Figure 2

Velocity fields in a channel supporting helium II heat flow.
important relation discovered by London (47) with the assumption of thermodynamic reversibility

$$\Delta P = \rho S \Delta T$$ (1-9)

where S is the entropy per mass of the bulk liquid. Then, heat flow in helium II may be thought of as being driven by a thermomechanical pressure given by the London equation (1-9).

The two fluid equations of motion may be obtained analogous to the Navier-Stokes equation (1-4) for classical viscous fluids. The development of the two fluid equations has been briefly discussed in Appendix A. The appropriate equations (A-3) to (A-6) are combined to describe laminar flow in the two fluid model of helium II, with the assumption of incompressibility.

$$\rho_n \frac{D\mathbf{v}_n}{Dt} = \eta \nabla^2 \mathbf{v}_n - \frac{\rho_n}{\rho} \nabla P - \rho_s S \nabla T$$ (1-10)

$$\rho_s \frac{D\mathbf{v}_s}{Dt} = -\frac{\rho_s}{\rho} \nabla P + \rho_s S \nabla T$$ (1-11)

The viscosity of course, only enters into the normal fluid equation (1-10).
At small heat currents such that the two fluid counterflow is laminar and noninteracting, the velocities may be assumed to be time independent and small, and equation (1-10) becomes

\[ -\frac{\rho_n}{\rho} \hat{\nabla} p = \rho_s S \hat{\nabla} T - \eta \nabla^2 \hat{v}_n \tag{1-12} \]

Combining this equation with the London equation (1-9) yields

\[ \hat{\nabla} p = \eta \nabla^2 \hat{v}_n \tag{1-13} \]

which is equivalent to the Poiseuille equation (48) in classical hydrodynamics. In the case of flow through a circular cross section tube of radius \( a \) under a constant pressure gradient, equation (1-13) may be solved to yield

\[ <\hat{v}_n> = \frac{\int_0^a 2\pi \hat{v}_n(r) r dr}{\int_0^a 2\pi r dr} = -\frac{a^2}{\eta} \frac{\hat{\nabla} p}{\hat{\nabla} p} \tag{1-14} \]

Since the phonon and roton excitations comprising the normal fluid contain all the entropy in the helium II, the heat flow per cross-sectional area \( \hat{W} \) is given by

\[ \hat{W} = \frac{\hat{Q}}{\pi a^2} = \rho ST <\hat{v}_n> \tag{1-15} \]
where \( \langle \vec{v}_n \rangle \) is the usual mean cross-sectional velocity, \( \dot{Q} \) is the total heat current, and \( \dot{W} \) is mean cross-sectional heat current density. Inverting this equation yields

\[
\langle \vec{v}_n \rangle = \frac{\dot{W}}{\rho_{ST}} \tag{1-16}
\]

and using the equation of continuity (1-8), the super component mean velocity can be obtained.

\[
\langle \vec{v}_s \rangle = -\frac{\rho_n}{\rho_S} \langle \vec{v}_n \rangle = -\frac{\rho_n}{\rho_{ST}} \frac{\dot{W}}{\rho_{ST}} \tag{1-17}
\]

If equations (1-14) and (1-15) are combined to yield

\[
\nabla p = -\frac{8\eta}{a^2\rho_{ST}} \dot{W} \tag{1-18}
\]

and the London equation (1-9) is again called upon, the temperature gradient is given by

\[
\nabla T = -\frac{8\eta}{a^2(\rho_S)^2T} \dot{W} \tag{1-19}
\]

These last two equations have been used to compute the viscosity from measurements of the temperature gradient and the pressure gradient in thermal counterflow and are well substantiated experimentally, where the flow of the normal fluid has been assumed to be laminar and the
superfluid has been assumed to flow without friction losses. Both these gradients are proportional to the heat current density \( \tilde{W} \), and the thermal counterflow region for which equations (1-18) and (1-19) are applicable is termed the linear region (49,50). This linear region is analogous to the linear region found by Reynolds in classical laminar flow discussed in section 1-1.

The thermal resistance equivalent to electrical resistance in an electrical-thermal analogy may be written as follows using equation (1-19).

\[
\frac{\Delta T}{Q} = -\frac{8\gamma l}{\pi a^4 (\rho S)^2 T}
\] (1-20)

This thermal resistance is proportional to the viscosity, and \( \tilde{W} = \frac{Q}{\pi a^2} \). An expression has been obtained for the slip correction to the thermal resistance (51), but the temperatures and tube diameters used in the present experiments are large enough that the excitation mean free path is much smaller than the tube diameter, and no slip correction is required.

The results obtained for helium II heat flow to this point are valid only for small heat flow where laminar conditions prevail. For heat currents greater than some critical value, the pressure and temperature gradients rise rapidly above the values given by equations (1-18)
and (1-19). The question of the critical heat current and critical velocity will be deferred to section 1.4.

The excess thermal resistance that appears above the critical heat current may be approximately accounted for phenomenologically by postulating an interaction between the two fluids called the mutual friction force. Gorter and Mellink (50) suggested that the large relative velocity \( \vec{v} = \vec{v}_s - \vec{v}_n \) between the two fluids at high heat currents results in the development of friction losses. The equations of motion (1-10) and (1-11) must therefore be modified to

\[
\rho_n \frac{D\vec{v}_n}{Dt} = \eta \nabla \cdot \vec{v}_n - \frac{\rho_n}{\rho} \nabla p - \rho_s \nabla T + F_{sn}(\vec{v}) \quad (1-21)
\]

\[
\rho_s \frac{D\vec{v}_s}{Dt} = - \frac{\rho_s}{\rho} \nabla p + \rho_s \nabla T - F_{sn}(\vec{v}) \quad (1-22)
\]

where \( F_{sn}(\vec{v}) \) is the mutual friction force which effectively couples the motion of the two fluids.

If the steady state is considered so that equations (1-21) and (1-22) are identically zero and the resulting equations are added, equation (1-13)' is obtained which will again have solution (1-14) for circular tube flow. This result is the Allen and Reekie rule (52) which is remarkable for its independence of the mutual friction
force. Equation (1-18) still obtains allowing its substitution into equation (1-22) to yield

$$\nabla T = - \frac{8\eta}{a^2(\rho S)^2} \dot{W} + \bar{F}_{sn} \left[ \frac{-W}{(\rho S)^2_T} \right] .$$  \hfill (1-23)

The first term in this equation varies as \((1/a)^2\) and may be ignored for large tubes and heat currents. The temperature gradient has been shown to vary as \(W^3\), and Gorter and Mellink (50) therefore postulated that the mutual friction force should be of the form

$$F_{sn} = A\rho_s\rho_n(v_s-v_n)^3$$ \hfill (1-24)

so that (1-23) may be written

$$\nabla T = - \frac{8\eta}{a^2(\rho S)^2} \dot{W} - \frac{A\rho_n}{S(\rho S T)^3} W^3$$ \hfill (1-25)

where \(A\) is termed the Gorter-Mellink or mutual friction coefficient which varies slowly with temperature.

Substitution of equation (1-15) into (1-25) would yield a dependence of the temperature gradient on \(\langle v_n \rangle^3\), where we assume large heat currents and tube diameters in order to ignore the first term. This velocity dependence of the temperature gradient, for what is presumed to be some type of turbulent flow, is quite different from the
Blasius expression (1-2) for classical turbulent flow at large velocities. In addition, measurements of the pressure gradient in heat flow (53-55) not only disagree with the Allen and Reekie rule (1-14), but are in fair agreement with the Blasius 7/4 power dependence on velocity above the critical heat current. These results suggest a distinct type of turbulence associated with each component ('mutual friction type' for $v_s$ and 'classical type' for $v_n$), and each with its own critical heat current. Tough (56) has focused attention on this matter using the heat flow data of Brewer and Edwards (53) and Chase (57), and was able to discern three distinct regions in the temperature and pressure gradient data so that two critical heat currents could be defined. The nature of the three flow regions had already been verified to a certain extent by the velocity field measurements of Allen et al. (58). This experiment used a fine quartz fiber detector in a thermal counterflow to distinguish the three separate flow regions. In the first (subcritical) region the quartz fiber is undisturbed by the flow except for a steady force due to the viscous normal fluid, and any vorticity attached to the fiber is persistent. The second (intermediate) region is distinguished by a small but varying amounts of attached vorticity. The third region (supercritical) is characterized by random fluctuations of the fiber as would be anticipated from a turbulent normal component flow.
1.4 Critical Velocities

The critical heat current density associated with the onset of mutual friction and the transition from the subcritical to the intermediate state will be termed \( W_0 \) where \( W_0 = \frac{\dot{Q}_0}{\pi a^2} \). The superfluid velocity related to \( W_0 \) (equation (1-17)) will be called \( v_{sc} \) and will be assumed to represent a critical superfluid velocity in the sense that for \( v_s > v_{sc} \) the effects of vorticity are observed.

Feynman (59), following a suggestion by Onsager (60), theoretically determined a condition under which flowing helium II would develop quantized vorticity. In other words both fluids would flow in a laminar fashion in thermal counterflow until the velocity of the superfluid reached the critical value \( v_{sc} \), at which point a continuous quantized vortex production occurs in the super component. If the super component velocity were increased further, additional quantized vortex lines of single quantum circulations would be formed. Feynman developed the following expression for the velocity \( v_{sc} \) at which vortex production begins.

\[
v_{sc} \approx \frac{\hbar}{md} \ln \left( \frac{d}{2a_0} \right) \tag{1-26}
\]

where \( m \) is the helium-4 atomic mass and \( a_0 \) is the vortex core radius. Using equation (1-17), and equation (1-26),
the heat current required for the generation of the initial vortex in the flow could be calculated. Numerous experiments by Vinen and others (61-64) using different techniques have been conducted which substantiate the insight of Feynman and Onsager. Other derivations (65-67) of $v_{so}$ have been published which differ in detailed result, but most yield an expression in order of magnitude agreement with equation (1-26). These derivations as well as others are discussed in detail in Appendix C.

The presence of vorticity in thermal counterflow opens the question of possible scattering of the normal component excitations. Such a scatter interaction would be a natural explanation of the mutual friction force of Gorter and Mellink. The scattering of normal component excitations off vorticity in the super component would result in a coupling force between the two fluids like the mutual friction force in equations (1-21) and (1-22).

Although the present discussion has been limited to heat flow in helium II, the rotation experiments and theory of Hall and Vinen (68-71) provide great insight into the nature of vorticity and its connection with the establishment of the mutual friction force. The results of these experiments prove that there is vorticity in the rotating superfluid, that mutual friction depends on a relative velocity between the two fluids, and that the
model of roton scatter off vortex lines is a mechanism for mutual friction.

Returning to heat flow experiments, Vinen (72) in a very thorough series of experiments investigated the development of vorticity and the mutual friction. A model of mutual friction in heat flow was proposed by Vinen that differs from the rotation experiments mentioned above. The mutual friction force was first related to the length of vortex line per unit volume \( \mathcal{L}(v) \) present in the superfluid. A dynamical equation for \( \mathcal{L}(v) \) was then derived, and an expression for the equilibrium length of line \( \mathcal{L}_0(v) \) was given for the case of homogeneous, isotropic line distribution (\( \mathcal{L}_0^{\frac{3}{2}} d \geq 1 \)). The mutual friction due to this 'tangled mass of vortex line' was found to agree rather well with various experimental observations of Vinen, including steady state and transient behavior. The Vinen theory was tested by Brewer and Edwards (53,73,74) using their thermal resistance data and was found to be in good qualitative agreement. The difficulty with their analysis was the implicit assumption of only a single critical heat current. Chase (55) has reanalyzed their data and finds excellent agreement with the Vinen model. Since the Vinen model is only appropriate for a 'tangled mass' of vortex line, or what might be termed 'fully developed superfluid turbulence', its relation to \( v_{sc} \) and \( W_0 \) is somewhat vague.
The critical heat current density associated with the onset of turbulence in the normal component and the transition from the intermediate state to the supercritical flow region will be called $W_c$. A normal component velocity may be obtained from the Reynolds number $R_n$ given by equation (A-7) of Appendix A, but there is little evidence that it has been observed. The critical heat current or critical normal component velocity $v_{nc}$ given by $R_{nc} = 2000$ is much greater than any observed values.

If the superfluid turbulence discussed above develops prior to the normal component turbulence, an effective coupling between the two fluids occurs through interaction of the thermal excitations with vorticity. This interaction results in the mutual friction force discussed above, as well as an 'entrainment' of the superfluid by the normal component so that an effective normal fluid density $\rho^*_n$ develops. This effective density appears to saturate at $\rho^*_n = \rho$ for superfluid velocities somewhat greater than the critical superfluid velocity (75-77). The effective Reynolds number for the onset of normal component turbulence might therefore be taken to be $R$ (equation (A-13)), when superfluid turbulence is present.

As discussed in Appendix A, a constant critical value of Reynolds number $R$ has not really been effective in explaining the temperature dependence of $W_c$. A more
The characteristic dimensionless parameter $P$ has been developed (78), which comes much closer to characterizing the onset of normal component turbulence than does the Reynolds number. This parameter accounts for both the entrainment of the superfluid and the mutual friction force.

The parameter $P$ is given in terms of the mean cross-sectional velocity of the normal component by equation (A-20).

$$P = \frac{\rho \langle v_n \rangle d}{(\rho_s / \rho) \left( \frac{\eta_v}{\eta} \right)^{\frac{1}{2}}}$$

Critical values of this parameter $P_c$ for the data reported in the following chapters as well as all other relevant critical heat current data reported in the literature are shown in Figure 3. Apparently the flow of the normal fluid in the intermediate region becomes unstable when $P = P_c \approx 55$, and a flow closely resembling classical turbulent flow develops.

The basis for discussing critical velocities for either the superfluid or the normal fluid has been developed. It is quite evident that heat flow in helium II requires the simultaneous consideration of both critical velocities. If the superfluid critical velocity for heat flow above 1.00 K is calculated from Feynman's result
Critical values of the flow parameter $P$ (Equation (1-27)) for a variety of thermal counterflow data. All square symbols are for tubes of rectangular cross section, and $d$ is the hydraulic diameter ($4$ area/perimeter). The open symbols ($\bigcirc \bigtriangleup \nabla \diamond \square$) are from Chase (57) as follows: $\bigcirc$, $d=0.80\text{cm}$; $\bigtriangleup$, $d=0.159\text{cm}$; $\nabla$, $d=0.262\text{cm}$; $\diamond$, $d=0.404\text{cm}$; $\square$, $d=0.068\text{cm}$. The closed triangular symbols are from this work: $\blacktriangle$, $d=0.056\text{cm}$ (both tubes A and B at $1.2^0\text{K}$); $\blacktriangledown$, $d=0.010\text{cm}$. The other data are: $\bullet$, Reference (79), $d=0.106$; $\times$, References (53,73), $d=0.0108\text{cm}$; $+$, References (53,73,74), $d=0.0366\text{cm}$; $\blacklozenge$, Reference (54), $d=0.0255\text{cm}$; $\blacksquare$, Reference (72), $d=0.350\text{cm}$; $\blacksquare$, Reference (72), $d=0.530\text{cm}$.
(equation (1-26)) it is found to be much less than the critical superfluid velocity calculated from $R_n$ (equation (A-7)) and the continuity equation (1-8). This result demonstrates that the superfluid critical velocity will be exceeded before that of the normal fluid in thermal counterflow. The intermediate region with mutual friction coupling of the two fluids must therefore be anticipated prior to normal fluid turbulence. The term 'intermediate region', as well as numerous other suggested terms, was first suggested by Hung et al. (80) who experimentally investigated transport phenomena in narrow slits.

The heat flow data of Allen, Griffiths, and Osborne (58) have already been discussed in section 1.3. The dimensional character of the heat flow channel in this experiment was such that quantitative critical flow velocities could not be obtained, but the behavior of the quartz fiber vorticity detector left no doubt that three regions were present. This data clearly delinicates three flow regions: 1) the subcritical region in which both fluids flow in a laminar fashion, and $W < W_o = W(v_{sc})$, 2) the intermediate region in which quantized vorticity forms a turbulent superfluid flow, and the behavior of the normal fluid flow is not indicative of classical turbulence, i.e. $W(v_{sc}) < W < W(v_{nc})$, and 3) a supercritical region in which both fluids are involved in turbulent flow where
The mean heat current density \( W \) is related to the cross-sectional velocities according to equations (1-16) and (1-17).

Other heat flow measurements (81) have also uncovered the three flow regions. In these measurements the deflection of a negative ion beam was used to detect instability in the thermal counterflow. Three distinct regions of flow were noted, two of which involved vorticity. The lower velocity transition was attributed to a critical velocity in the super component and the upper transition was assumed to develop from classical normal component turbulence and its associated critical velocity.

In general, heat flow data for which temperature and pressure gradients are measured as a function of heat current density, have apparently not all been sufficiently sensitive to yield the detail of the three flow regions. It is possible, but not unambiguous, to discern the three flow regions in the data of Brewer and Edwards (53,73), as may be seen in Figure 4. The lower critical heat current is \( Q_0 \) which the authors call \( W_1 \). The upper critical heat current is taken at the point of the sharp increase in the thermal resistance and corresponds to \( Q_0 \).

1.5 **Purpose of This Research**

The major goal of the experiments to be described below is to explore the stability of the transition to
Figure 4

The thermal and mechanical resistance (defined as an effective viscosity $\gamma_{\text{eff}}$) measured in thermal counterflow by Brewer and Edwards (53,73). The circular cross-section tube diameter was $1.08 \times 10^{-2}$ cm, and the temperature $1.56^0\text{K}$. The dashed line is increasing heat current and the solid line is decreasing heat current.
turbulence in the normal component of liquid helium II by the modulation of suitable flow parameters. If modulation techniques similar to those which lead to the enhancement of stability in classical hydrodynamics are successful in stabilizing helium II flow, the case for the normal component hydrodynamic origin of the critical heat current $W_0$ would be very strong indeed. For these experiments involving helium II thermal counterflow, the modulated parameter will be the heat current, i.e. the temperature gradient which is related through the London equation to the pressure gradient in direct analogy to the experiments of Gilbreath and Combs (18).

If stability enhancement is achieved, i.e. if the critical heat current in a suitably modulated flow can be made significantly greater than the steady case, the variation of the effect with frequency and temperature can indicate whether $\rho$ or $\rho_n$ is involved in the penetration depth $\delta$. The considerations of entrainment leading to the parameter $P$ (equation (1-27)) suggest $\rho$ should be the appropriate density.

If stability enhancement is achieved by the modulation experiments, the intermediate state would be extended to higher heat currents. The extended intermediate region would then allow the determination of whether the transition $W_0$ is preceded by $W_0$. The superfluid
transition should become more distinct as the obscuring effects of the second transition are removed to higher heat currents.

Finally, the experiments will determine whether there are any stabilization effects on the superfluid turbulence transition $W_0$ due to the modulation. No effects are anticipated for this transition.
CHAPTER II

APPARATUS DESIGN AND FABRICATION

2.1 Introduction

An apparatus has been designed to conduct thermal counterflow experiments in helium II. A circular cross-section flow tube is arranged with one end in a temperature regulated bath of helium II and the other end in a small reservoir (the 'pot') which contains heaters and is thermally isolated from the bath. Figure 5 shows the apparatus in a highly schematic fashion, including the support post arrangement for the pot.

Two thermometers are located at either end of the flow tube for measuring the temperature difference along the tube. Two heaters are required for this apparatus to develop a sinusoidal modulation of the heat current and the resulting temperature gradient. The entire flow tube assembly is surrounded by a brass can which is sealed to the flange with an indium o-ring.

The flow tube thermometers are calibrated against the equilibrium vapor pressure of the bath according to the 1958 helium-4 scale of temperatures (82). The
Figure 5

Thermal Counterflow Apparatus Schematic
helium bath pressure is regulated by properly valving the mechanical vacuum pump used to reduce the bath pressure and by an Artronix\(^{(a)}\) Model 5301 electronic temperature controller.

Vapor pressures are measured with differential manometers using mercury and dibutyl phthalate as working fluids. The temperature corrected density conversion between the mercury and the dibutyl (equivalent of butyl) phthalate is provided by Johnson (83).

2.2 Flow Channel

The flow channels used in this apparatus are all stainless steel tubes of circular cross section. Table 1 includes the nominal dimensions of these tubes. The stainless steel tubing retains sufficient strength at low temperatures to allow very thin walls which conduct little heat compared with the thermal counterflow in the helium II which they contain. The ratio of the thermal resistance of helium II flow through the tube in the linear region to the thermal resistance of the stainless steel tube walls will be much less than \(10^{-5}\), even at 1.2\(^{0}\)K where the thermal resistance of the flow is greatest. Even in the fully turbulent flow region of interest where the flow thermal resistance increases markedly, the ratio will

\(^{(a)}\)Artronix Incorporated, 716 Hanley Industrial Court, St. Louis, Missouri.
TABLE 1. NOMINAL FLOW TUBE DIMENSIONS

<table>
<thead>
<tr>
<th>Tube Designation</th>
<th>Material</th>
<th>O.D. (cm)</th>
<th>d=I.D. (cm)</th>
<th>l=Length (cm)</th>
<th>l/d</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>SS 347</td>
<td>0.107</td>
<td>0.056</td>
<td>152.9</td>
<td>2730</td>
</tr>
<tr>
<td>B</td>
<td>SS 347</td>
<td>0.107</td>
<td>0.056</td>
<td>12.41</td>
<td>222</td>
</tr>
<tr>
<td>C</td>
<td>SS 304</td>
<td>0.020</td>
<td>0.010</td>
<td>10.15</td>
<td>1015</td>
</tr>
</tbody>
</table>
still be much less than $10^{-3}$. Therefore, the helium II effectively shorts out the tube wall in the thermal sense, and the tube wall thermal resistance may be neglected.

The flow tubes were prepared in a number of ways. The first experiments were conducted in flow tube C, the dimensions of which are given in Table 1. Working with tubes this small requires the use of a good microscope for cleaning the tube ends after cutting them to length. The tubes were initially cut to size with a powered diamond abrasive disc. This technique did not work too well because the tube ends were quite jagged, and the tiny holes were usually filled with chips, making them nearly impossible to clean.

Better cutting results were obtained with a spark erosion cutter. This tool allowed the tube to be cleanly cut with no chips or jagged edges. The cut is made by using the tube as an electrode to create an electrical breakdown in an oil bath between the tube and an adjustable wire on the machine. During the spark cutting operation, the tubes were held in place by a conductive silver epoxy called DAG415 Dispersion which was later removed with butanone.

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(b) Acheson Colliods Co., Port Huron, Michigan.
The inside diameter of the tube at the ends was tapered slightly using standard sizes of stainless steel root canal files and reamers which are more commonly utilized in dental work. The files and reamers are tapered slightly and consist of right-hand and left-hand spiral cutting edges, respectively. Twirling these instruments between one's fingers allowed minute chips and imperfections to be removed from the ends of the tubes as well as providing a slight entrance and exit taper which should reduce disturbance of the flow at the tube ends.

The tubes were cleaned with acetone in an ultrasonic cleaner. A tungsten wire of nominally 3/4 the inside diameter of the tube was then passed through the tube with some difficulty to ensure a clear passage. This wire was left in place until after the flow tube was soldered.

The larger flow tubes A and B were more easily cut and cleaned, and the same procedures as described above were followed. These tubes could also be cut with the abrasive disc, but more cleaning was required than with the spark erosion cutter.

The tube lengths of B and C were measured using a vernier calipers after examining the ends under a microscope to ensure that no burrs or chips were present.
The length measurement is quite accurate, but other difficulties to be discussed below erode this accuracy. The long tube A was measured with a common rule to within ±2mm. The length required for the tubes is determined by the inside diameter of the tube and the thermometer sensitivity required for accurate measurements of the temperature difference along the tube. In classical flow experiments the length to diameter ratio $l/d$ is usually taken to be of the order of 100 or greater so that end effects can be minimized. The thermometry problem will be discussed below, but a lack of sensitivity was observed in the short fat tube B due to its relatively small $l/d$.

Tube A was much too long to be contained in the vacuum can without bending it into a spiral. Since the long tube was absolutely required for the experiment, there was no other choice. The spiral tube is supported by tying each turn with cotton thread to the three copper support posts on the outside of the spiral. (Only a straight tube is shown in Figure 5.) The resulting spiral had 21 turns and each turn had a 2cm inside diameter. The turns were separated by about 3mm.

The spiral tube was carefully formed about grooved mandrels of progressively smaller diameters. This ensured that no kinks would result, and that deformation of the circular cross section tube to an elliptical cross section
would be minimized. The tube was not very ductile and could not be successfully bent until heat treated at 1875°F for 10 minutes in an inert gas atmosphere.

The selection of the diameter of the flow tube was a rather complicated process involving numerous design criteria. The tube diameter could not be extremely large or the \( l/d \) criterion mentioned above would require too long a tube for the space provided in the vacuum can. If the tube were short and fat, the thermometer sensitivity would not be great enough.

The properties of helium II flow must be considered for the smallest diameter tubes. The tube diameter must not be too small to allow clamping of the normal fluid. At the lowest temperature to be used in this experiment (1.1°K), the mean free path for phonon excitations becomes large (5x10^{-4} cm) (84). In order to maintain hydrodynamic flow conditions, the tube diameter will therefore have to be much larger than this mean free path. The tube should have a diameter greater than 10^{-2} cm.

It would not be desirable to generate second sound propagation in the tube due to the pulsating temperature. For typical velocities of second sound (2000 cm/sec) in the temperature range under consideration here) and experimental modulation frequencies, the wavelength of the second sound will be at least an order of magnitude longer than the tube, so second sound will not propagate.
Another factor entering into the tube diameter selection is the penetration depth for the viscous wave modulation. The oscillating normal component velocity wave associated with the oscillating heat current was initially assumed to have a penetration depth $\delta_n$ given by equation (1-7) with $\rho = \rho_n$. The classical fluid modulation experiments and theory discussed in section 1.2 indicate that the optimum ratio of the tube radius to the penetration depth for maximum stabilization will occur for

$$1 \leq \left( \frac{a}{\delta_n} \right) \leq 10 \quad (2-1)$$

The penetration depth is a function of the temperature and the modulation frequency of the viscous wave, so that the tube radius is fixed by the frequency range of the oscillator driving the heater and the temperature range of the cryostat.

The heater power supply to be discussed below contains integrated circuit modules which are limited to frequencies of about 10Hz or greater. The viscous wave in the fluid is twice the oscillator frequency, so the minimum frequency allowable in the fluid and equation (1-7) is 20Hz in order to use the heater power supply to its fullest capabilities. The initial tube radius ($C$) was chosen to conform with this minimum frequency condition.
Solving equation (1-7) for the tube radius in the range for maximum stabilization yields

\[ 6_n \leq a \leq 106_n \leq 10 \left( \frac{\eta_{\text{max}}}{\rho_{\text{min}}^n f_{\text{min}}} \right)^{\frac{1}{3}} \]  

(2-2)

where \( \eta_{\text{max}} \) and \( \rho_{\text{min}}^n \) are evaluated at 1.2\( ^0 \)K for their respective maximum and minimum values and \( f_{\text{min}} = 20 \)Hz which is the lower limit of the power supply. Calculation of the range of the tube radius from equation (2-1) yields the limits \((2.5 \times 10^{-3} \text{cm}) \leq a \leq (2.5 \times 10^{-2} \text{cm})\). This initial tube C was then chosen near the larger dimension for convenience of handling.

The experimental results to be discussed in the following chapter revealed that the penetration depth depending upon the total density and not the normal fluid density is the correct parameter. For this reason as well as other difficulties, flow modulation and stabilization were never achieved in flow tube C.

The other two flow tubes (A and B) were selected so that the radius would satisfy the conditions \(1 < a/\delta < 10\) and \(1 < a/\delta_n < 10\) over at least part of the frequency range. Figure 6 illustrates the constant \(a/\delta\) and \(a/\delta_n\) curves. For a given tube size the frequency range required to generate the required range of \(a/\delta\) or \(a/\delta_n\) may be determined. The thermal time constant \(\tau\) also enters into
Flow tube design criteria at 1.2 K plotted as the oscillator frequency vs. a dimensionless tube radius $X$, where $a_0 = 5.1 4 \times 10^{-3}$ cm is the measured radius of flow tube C, i.e. $X = 1$ for flow tube C. The value of $X$ for tubes B and C is about 5.5. The values of $\omega \tau$ (---) calculated using the length of flow tube B are close but not identical to those of tube C. The other values of $\omega \tau$ (------) are calculated for flow tube A. The constant values of $a/\delta$ are given by the solid lines, and the constant $a/\delta_\Pi$ are the dashed lines.
OSCILLATOR $f$ (Hz)

- $\omega T = 1.0$
- $\omega T = 0.1$
- $\omega T = 0.0$

$X = \alpha/a_c$

- $1/a_0 = 9/d$
- $1/a_1 = 6/d$
- $d = 2.86$
- $d = 5.66$
- $d = 2.86$
- $d = 5.66$
- $d = 5.66$

$1/a_0 = 4/d$

$1/a_1 = 2/6$

$1/a_1 = 1/6$

$1/a_1 = 1/14$

$1/a_1 = 1/1$

$1/a_1 = 1/14$

$1/a_1 = 1/1$

$1/a_1 = 1/14$

$1/a_1 = 1/1$

$1/a_1 = 1/14$

$1/a_1 = 1/1$
the choice of frequency range. This condition will be discussed more fully in the following section, and it will only be pointed out here that the angular frequencies \( \omega \) will be restricted to values of \( \omega \tau < 0.1 \). This condition on the frequency is quite restrictive and requires that the heater power supply be operated manually for the tube sizes finally selected for A and B in Table 1.

Having selected the tube radii and length using the above criteria, the tubes were installed on the brass flange by soldering them into appropriate diameter stainless steel sleeve tubes about 1cm long. The end of the flow tube was made flush with the top of the brass as shown in Figure 5. The sleeve tube and the brass serve as thermal paths which might disturb the temperature gradient at the tube end. Very rough calculations indicate that this disturbance may be neglected.

2.3 Helium II Pot

The helium II pot design is constrained by the thermal time constant \( \tau \) required to generate the viscous wave inside the flow tube. The steady state and time dependent behavior of the pot and flow tube assembly has been analyzed in detail in Appendix B. The result of this analysis is that the modulation heat current will be
averaged to a steady heat current in the tube unless the following condition holds

$$\omega \tau = (2\pi f) R_f C_l < 1$$  \hspace{1cm} (2-3)$$

where $$\omega = 2\pi f$$ is the angular frequency of the viscous wave in the liquid, $$R_f$$ is the thermal resistance of the flow tube, and $$C_l$$ is the heat capacity of the liquid helium II contained in the pot.

The condition (2-3) requires that all three variable quantities remain as small as possible. It has already been mentioned that the frequency $$f$$ should be greater than 20Hz for efficient operation. The thermal resistance of the flow tube has been defined in equation (1-20), and may be calculated by obtaining tube dimensions from Table 1. A maximum heat capacity for the pot can then be defined from condition (2-3).

There are two methods of constructing the apparatus:
1) the heaters can be directly immersed in the helium II inside the pot, or 2) the heaters can be wrapped on the outside of a thermally conductive pot wall. The second technique was attempted initially because there would be no requirement for wire feedthroughs into the pot from the vacuum can. A problem develops with this technique in that the Kapitza boundary resistance (85,86) between
the heater and the helium II must be considered. If the Kapitza boundary resistance is too large, the temperature difference between thermometer $T_2$ on the pot and the helium II will be too large. In order to circumvent this problem, the contact surface area between the helium II and the pot was maximized by using sintered copper sponge inside the pot. The temperature difference criterion between the external thermometer and the internal helium II was $10^{-4}$ K, which required a very large pot in order to achieve the necessary surface area.

The sintered copper pots were encased in a copper cylinder with a 1.3cm O.D., 2.2cm inside depth, and 0.1cm wall and end thickness. Copper powder which was 99.9% pure 325 mesh was pressed into the cylinder layer by layer at 2000psi using a hydraulic ram. After packing the powder, the sintering was done in an oven with a 1psig atmosphere of hydrogen gas at 575°C for four hours. The volume and packing fraction of the sintered material were determined to be $0.75\text{cm}^3$ and 45%, respectively. Goodstein et al. (87) report that there are about $4000\text{cm}^2$ of surface area in a $1\text{cm}^3$ copper sponge of this type.

The pot was capped with another copper piece which was soldered in place, and then the entire pot was soldered to flow tube C described above. Vacuum leaks developed in the copper walls due to displacements during
the sintering operation, and the entire pot had to be coated with solder to prevent these leaks.

The sintered copper pot combined with flow tube C allowed steady state measurements of heat flow with good agreement between the externally measured temperature and the temperature of the helium II. However, the heat capacity of the large amount of helium II present in the pot prevented the application of any modulation. This large heat capacity was acting as a very effective thermal ripple filter and only a steady state heat current and temperature gradient were present in the flow tube. Typical values of $\tau$ for this apparatus over the temperature range of interest were $10 < \tau < 50$ seconds. Then to satisfy condition (2-3), a very low frequency, e.g. 0.01 Hz, would be required to create modulation in the tube. Since this condition would occur only for $\alpha/\delta << 1$, this apparatus was of no further application to the modulation experiments.

In order to retain flow tube C and introduce modulation into the tube, the heat capacity of the pot had to be considerably reduced. The low Kapitza boundary resistance, while still an important consideration, would have to be compromised. The heat capacity in the helium II contained in the sintered copper pot was 3 to 4 orders of magnitude greater than that of the copper. Therefore, reduction of the helium space was mandatory. In order to
best reduce this space it was decided to place the heaters inside the pot as originally suggested above in 1) so that the heater wires would be in good thermal contact with the helium II.

Figure 7 illustrates the details of this second pot arrangement. A copper cylinder was machined, as suggested by Anderson (88) for electrical feedthroughs, with a feathered edge as shown in the figure. An Epibond 100A(a) insert was cast according to the manufacturer's instructions and machined to fill most of the volume inside the copper cylinder. The cylindrical helium II space remaining is typically 0.5mm in depth and 7.5mm in diameter. A small flange was left on the Epibond insert so that it had to be press fit into the copper. Two holes were drilled through the insert so that the two heaters could be passed through. The two wires in each hole were insulated from each other by a #30 fiberglass sleeve(b) which filled the hole and extended about 2mm outside the assembled pot. Four shallow holes were drilled into the insert to receive small brass pins for electrical terminals. The heater wires were coiled and soldered to these pins before further assembly.

(a) Furane Plastics Inc., 16 Spielman Rd., Fairfield, N.J.
(b) Bentley-Harris Manufacturing Co., Conshohocken, Pa.
Figure 7

Epibond Pot Schematic Prior to Baking.
Before assembling the two major pieces of the pot, a hole was drilled through the boss on the copper piece to admit a stainless steel sleeve tube about 1 cm long. This tube was silver soldered in place to provide some rigidity when the flow tube was inserted and soldered. After pressing the insert into the copper, the assembly was placed in a Teflon cylindrical form. Additional Epibond 100A powder was then added at a temperature of 150°C in an oven until the feathered edge of the copper was immersed about 1 mm in liquid Epibond 100A. The flange on the Epibond insert prevented the flow of the rather viscous molten Epibond into the helium II space.

After filling the Teflon form to the desired level with Epibond, the heat treatment was continued for 6 hours. Large bubbles have a tendency to form between the feathered copper and the Teflon mold for shorter heat treatments. A number of these pots were constructed, none of which leaked even when large bubbles were present in the Epibond. The fiberglass in the heater electrical feedthrough holes was effective in sealing these holes without wicking through and filling the helium space.

After cooling, the Teflon form was removed, and the pot was cleaned. The heater contact pins were scraped clean to allow external electrical connections to be made at a low soldering heat. The flow tube was
soft-soldered to the pot without damaging the Epibond vacuum seal or the soldered heater connections. The tungsten wire inside the flow tube was not removed until the soldering was completed to demonstrate that the flow tube was still open.

The heat capacity of this pot is still nearly all that of the helium II contained. The time constant associated with this type of pot and flow tube C is $0.2 < \tau < 1$ second. While this represents a significant improvement over the sintered copper pot, very low modulation frequencies are still required. The time constant condition discussed in Appendix B was not successfully met until this Epibond pot was combined with the much larger flow tube A with its small thermal resistance.

2.4 Thermometry

Carbon resistors serve as very sensitive thermometers at low temperatures. The temperature-resistance curve has a steep negative slope at lower temperatures, and sensitivity is approximately proportional to the inverse of temperature. The carbon is a granular composition material, and the contact between the granules can be greatly affected by thermal cycles. No adequate theory of the resistance of carbon is available, and the temperature dependence of the resistance must be
fitted with an empirical relation. The usual form of this empirical equation is a truncation of the semiconductor equation (89-91).

Unfortunately, the reproducibility of carbon thermometers is poor. The shape of the characteristic temperature-resistance curve for all carbon resistors of the Allen-Bradley type has been found to remain constant upon thermal cycling from room temperature to liquid helium temperature (92). If the carbon is protected from excessive thermal and mechanical shock, it is possible to retain the calibration of the carbon resistors to 0.1% (93). This requires keeping the resistors at liquid nitrogen temperature or below between experiments. Long term drift has also been observed over a period of several weeks for resistors immersed in liquid helium (94). This drift can be as large as several millidegrees. Edlow and Plumb (94) also noted that the resistance tended to increase at 4.2°K after cycling the thermometer below the lambda point and back to 4.2°K while the thermometer was immersed in the helium.

In this apparatus ordinary Allen-Bradley type carbon resistors are used to measure the temperature difference along the flow tube. Initially, 1/8 watt resistors were used to measure temperatures at three locations. These thermometers were calibrated against
the vapor pressure of the helium bath. The temperatures associated with the measured resistances were read directly from the calibration graph. Because the maximum temperature range of any one experiment is only 200 m°K, the graphical method is quite accurate. Since temperatures were determined graphically, knowledge of the day-to-day shift of the calibration curve was not required, and constants for the semiconductor relation did not have to be determined. Since the shape of the characteristic curve did not change noticeably, fewer calibration points were required after the curve shape was firmly established.

The three carbon thermometers were located as follows (see Figure 5): 1) $R_1$ was placed in the helium bath near the entrance to the flow tube, but not coaxial with the tube, 2) $R_2$ was attached to the copper surface of the helium II pot, and 3) $R_3$ was also attached to the pot at the same point with a constantan wire and was suspended by nylon threads so that it was thermally isolated (not shown in Figure 5). The resistors $R_2$ and $R_3$ were eventually wrapped with alternate layers of thin mylar and pure copper foil and packed in grease as described by Anderson et al. (95). This method of encapsulating the resistors ensured better thermal equilibrium in these thermometers which were isolated in the vacuum. The thermometer in the bath did not require this special
treatment because it was in intimate thermal contact with the superfluid. The factory applied insulation was not removed from the resistors.

Thermometer $R_2$ was glued in place on the copper surface of the pot using GE 7031 varnish. The constantan wire which connected thermometer $R_3$ with the pot was soldered to a copper lug on the sintered copper pot and is not shown in Figure 5. This #29 constantan wire was 16.5cm long and 0.286mm in diameter which resulted in a thermal resistance of about $10^7 \ (°K/watt)$ at $1.5°K$. The heat capacity of an 1/8 watt carbon resistor was interpolated from existing data (89,96) to yield a temperature dependent expression of the form

$$C_{1/8} \approx (1.01 \times 10^{-5}) \ T + (2.05 \times 10^{-6}) \ T^3$$

in units of (joule/°K) assuming only the carbon, resistor leads, and insulation to be present. At $1.5°K$ the heat capacity will be approximately $2.2 \times 10^{-5} \ (\text{joule/°K})$. The time constant for the response of thermometer $R_3$ at $1.5°K$ will therefore be $\tau' = 220 \ \text{seconds}$. This time constant was selected to be large enough that $R_3$ would not respond to any temperature oscillations so long as the frequency of the temperature oscillation obeys the condition $\omega \tau' >> 1$.

In order to measure the true average temperature, $R_3$ must be extremely well isolated thermally. The
electrical lead wires connecting \( R_3 \) and the terminal board at the bath temperature must therefore have thermal resistance much greater than the constantan wire. For this reason 150 cm of manganin #40 wire was used, and the resulting thermal resistance at 1.5°K was greater than 10^9 (°K/watt) for all the lead wires. This ensured that there would be very little temperature drop along these wires.

The difficulty in arranging the averaging thermometer \( R_3 \) was deemed worthwhile because electronic averaging is undesirable at the lowest modulation frequencies anticipated. The problem of reading a fluctuating temperature difference, which developed later, caused even greater appreciation for the fact that the average temperature of this thermometer could be much more quickly and accurately determined.

The average temperature could not be successfully measured, however. The thermal isolation of thermometer \( R_3 \) was so good that the measuring current heated the thermometer well above the equilibrium temperature even at the smallest current setting. All subsequent measurements on flow tube C were performed using only thermometers \( R_1 \) and \( R_2 \).

The effect of heating the isolated thermometer \( R_3 \) should have been anticipated, because the thermal conductivity of the carbon is very small at low temperature.
Berman (97) had measured the temperature increase of a much larger $\frac{1}{8}$ watt, 100 ohm Allen-Bradley resistor and found that there was a $13m^0K$ increase for a 1 microampere measuring current. He developed an empirical relation which described the temperature dependent behavior of the temperature increase. His measurements indicated that the heating effect would be negligible at $4^0K$.

Thermal equilibrium problems also became apparent in the $1/8$ watt resistor attached to the pot. The GE 7031 varnish apparently provided a large enough thermal resistance between $R_2$ and the pot that the measuring current in the thermometer produced a sizeable temperature drop across this thermal resistance. This problem was eliminated by soldering one of the thermometer leads directly to the pot with low melting point indium solder.

The sensitivity of any physical property which has a temperature dependence such as the electrical resistance $R(T)$ may be defined as $(1/R)(\Delta R/\Delta T)$ where $T$ is the temperature. The sensitivity and other characteristics of the carbon resistors used in this apparatus are provided in Table 2.

The $1/8$ watt thermometers $R_1$ and $R_2$ were eventually replaced with $\frac{1}{4}$ watt carbon resistors because greater sensitivity was required for the short flow tube B of Table 1. The larger carbon resistors have more sensitivity
TABLE 2

CHARACTERISTICS OF CARBON RESISTANCE THERMOMETERS

<table>
<thead>
<tr>
<th>Flow Tube</th>
<th>Power (watt)</th>
<th>$R_{270^°K}$ (ohm)</th>
<th>$R_{1.2^°K}$ (ohm)</th>
<th>$\Delta R/\Delta T$ (ohm/m^°K) at 1.2^°K</th>
<th>$(1/R)(\Delta R/\Delta T)$ (°K)^{-1} at 1.2^°K</th>
<th>Day-to-Day Drift at 1.2^°K</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1/4</td>
<td>148</td>
<td>R_1=160,400</td>
<td>808</td>
<td>5.04</td>
<td>±1m°K</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>R_2=146,700</td>
<td>675</td>
<td>5.47</td>
<td>±1m°K</td>
</tr>
<tr>
<td>B</td>
<td>1/4</td>
<td>148</td>
<td>R_1=160,400</td>
<td>808</td>
<td>5.04</td>
<td>±1m°K</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>R_2=146,700</td>
<td>675</td>
<td>5.47</td>
<td>±1m°K</td>
</tr>
<tr>
<td>C</td>
<td>1/8</td>
<td>47</td>
<td>R_1=9925</td>
<td>30</td>
<td>3.25</td>
<td>±1m°K</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>R_2=10,050</td>
<td>32.5</td>
<td>3.01</td>
<td>±1m°K</td>
</tr>
</tbody>
</table>
and are slightly more reproducible. These thermometers were installed in the same manner as those removed.

The additional sensitivity of the ¼ watt resistors was required because the temperature difference across the tube is proportional to the tube length and inversely proportional to the fourth power of the tube radius as may be seen from equation (1-20). While the tube lengths of B and C remain the same, the radius of tube B is over 5 times greater than tube C (see Table 1), and the required sensitivity for measuring the temperature difference increases by more than two orders of magnitude. Even the ¼ watt carbon resistors do not provide sufficient sensitivity for accurate measurements with tube B. These thermometers were accurate for the major experimental results obtained with tube A, however.

In order to eliminate any spurious heating in the carbon thermometers, 120pf capacitors were connected to shunt radio frequency pickup to ground from the plus and minus connections on the thermometers. The electrical lead wires connecting thermometer \( R_2 \) (and the heaters to be discussed below) with the terminal board inside the vacuum can are superconductive lead-coated manganin. The manganin is a nominal 0.05mm diameter wire coated with a thickness of about 0.01mm of lead. Below 7.2°K the lead becomes superconductive and the thermal resistance of the
wire increases markedly. Seven of these wires connect with the pot through the thermometer and the two heaters to give a combined thermal resistance of greater than $10^5(°K/watt)$ which is sufficient to eliminate any spurious thermal conduction effects in the pot temperature measurement. The negligible electrical resistance of the superconductive wires eliminates the need for exactly balancing the resistance of these tiny wires.

The effect of helium II penetration into the immersed thermometer $R_1$ will not be noticeable since no measurements will be made above the lambda point. $R_1$ is always calibrated while immersed in helium II, and $R_2$ is calibrated while in contact with helium exchange gas. The Kapitza boundary resistance of helium II will affect the temperature measured by $R_2$, however. The reduction of the pot size resulted in the very small contact area of about 0.5cm$^2$ between the helium II and the copper to which $R_2$ is soldered. If reasonable values of the Kapitza boundary resistance are chosen (85,86), the temperature difference between the thermometer $R_2$ and the helium II due to this boundary resistance will be of the order of 5 microKelvin.

The resistance of the thermometers was measured with an AC resistance bridge using integrated circuit amplification. The bridge was designed and constructed for this experiment to provide a resistance measurement
precision of 1:5000 at low power levels. The bridge was designed for a three-wire connection (98) to the thermometers to minimize electrical lead wire effects. Details of the bridge circuit and its operation have been included as Appendix D.

The bridge was driven by the output of a PAR\(^{(a)}\) Model 120 lock-in amplifier which also served to display the null output of the bridge using phase sensitive detection. The internal frequency of the PAR 120 was 17Hz. The combined bridge and lock-in amplifier exceeded the resistance measurement design criterion of 1:5000. The practical limit of temperature measurement was actually ±10 microKelvin. The day-to-day drift in the bridge was 1:10,000 which is mainly caused by a very sensitive common mode adjustment. The bridge was calibrated against an internal precision resistor. If the bridge does not provide an identical calibration resistance reading when the bridge arms are reversed, the common mode voltage must be adjusted. This adjustment was only rarely required.

A switching arrangement was provided so that any one of the original three thermometers and/or the temperature difference between any two thermometers could be measured quickly. In practice the thermometers were

\(^{(a)}\)Princeton Applied Research, Princeton, N.J.
closely matched and the bath was well regulated so that a significant temperature drift did not occur between the measurements of resistance of two thermometers. The switching arrangement was normally used only to select either of two thermometers.

2.5 Heaters

The introduction of a sinusoidal heat current modulation in the pot cannot be accomplished with only one heater. The heat current is proportional to the square of the electrical current and will result in a double frequency term if an electrical sinusoidal current is superposed on a steady current. In order to develop a pure sinusoidal heat current modulation, and thus velocity modulation, the steady and oscillating electrical currents must drive two separate heaters (see Appendix E for additional discussion).

The heaters were fabricated from Evanohm wire and were nominally 1000 ohms in order to attain a maximum power of 0.1 watt at a maximum power supply voltage of 10 volts. All heaters were coiled noninductively to limit spurious heating due to high frequency pickup. The space available for the heaters in the Epibond pot was very small. These heaters were made from 40cm of 0.025mm diameter wire which was tightly coiled. Since nearly 1cm of the heater wire was encased by the Epibond 100A, an error of several
per cent is encountered in measuring the effective heater resistance after it has been installed.

The Evanohm heater wire experiences a very small change in electrical resistivity with temperature (99), and is within 1% of its room temperature value at 4.2°K. There is also a small heater resistance correction necessary due to the 10 ohm lead wires connecting the heaters to the cryostat header.

A heater power supply was designed to provide a variable, steady state, electrical current and a sinusoidal electrical current for the separate heaters. The ripple in the DC output was designed to be less than 1% so that false modulation would not be present at line frequency which is comparable to anticipated modulation frequencies. The power supply provides a 10 volt maximum rms output for both the DC and the AC heaters.

In addition to providing either DC or AC outputs, the power supply contains an analog circuit which maintains the total power from both outputs at a preset constant level. In other words, the power supply can be set at a prescribed steady state power level, and then the AC power can be turned on and increased with a corresponding decrease in the DC power such that the total power output remains fixed at the initial value. This allows the ratio of AC power to total power to be continuously adjusted at a preset total power level. The details of operation and
limiting factors of the power supply are provided in Appendix E.

A Wavetek Model 116(a) oscillator is used to drive the power supply. This oscillator can supply a variety of waveforms with a low frequency limit of 0.005Hz. The response of the integrated circuits in the heater power supply is limited to 10Hz, however. The modulation frequency in the liquid will be that of the heat current or twice the oscillator frequency (see Appendix E). The heater power supply was therefore limited to experiments with fluid modulation frequencies of greater than 20Hz. This power supply was still used to provide separate DC and AC current for the heaters at lower frequencies, but the analog circuits were defeated, and the proper heat current ratio had to be set manually.

The reproducibility and drift of the power supply is well within 1% for the higher voltages. Steady state voltages below 100mv tend to drift considerably, but this does not seriously affect the measurements, since the total heat currents involved are less than 10 microwatts. The device is still usable down to 10 millivolt outputs. A small DC offset of the order of several millivolts was present, but the heat current created by this offset is always negligible.

(a) Wavetek Inc., San Diego, California
Voltages across the two heaters were measured at the cryostat header using a number of different digital display and pointer movement meters. Measurements of rms AC voltages were obtained above 10Hz, and peak-to-peak voltages were measured below 10Hz. No systematic deviations were noted in all the voltage measurements. The heater power could then be determined from the voltages and the heater resistances measured at the pot at room temperature.

2.6 Temperature Regulation

Each experimental data set required that the heat current be increased from zero to a value corresponding to the region of fully developed turbulence in helium II. These experimental runs required from one to two hours to obtain the temperature difference along the flow tube at each heat current setting. During this time the bath temperature must not drift significantly because the thermal resistance of the flow tube given by equation (1-20) is strongly temperature dependent. A 10 m°K change in temperature results in a 10% change in the thermal resistance of any size flow tube.

The temperature regulation of the bath is provided by the previously mentioned Artronix Model 5301 electronic temperature controller. The vapor pressure of the bath was controlled mechanically near the point at which
temperature was to be regulated. The vacuum pump was throttled so that the bath temperature was still dropping slowly, and then the Artronix controller was turned on. A low sensitivity Speer carbon resistance thermometer (100) was used to monitor the helium bath temperature near the top of the vacuum can and is shown in Figure 5.

The Artronix device contained a resistance bridge which could be set to balance at the desired resistance and temperature of the thermometer. Further cooling of the bath below this temperature by mechanical pumping was countered by the controller in the following way. The regulator bridge imbalance which measured the bath temperature change was fed back into a heater power amplifier. This amplifier produced the required voltage across a 1000 ohm Evanohm heater in the bath (shown in Figure 5) to bring the bath back to the required temperature. If the bath became too hot, the bridge imbalance completely switched off the regulator heater. Continuous heater power was usually applied to the bath to maintain efficient regulation.

When the regulator was first turned on, a large drift toward lower temperature was always noted. In order to avoid these large initial drifts, measurements were not begun until about 30 minutes after regulation had been accomplished. Typically, the temperature drift
would decrease as the experiment progressed. At the beginning of the data taking, after the 30 minute wait, the drift would be as great as 200 microKelvin per minute. Toward the end of the experiment, the long term drift was as low as 10 microKelvin per minute, with extended periods in which no drift occurred. The regulator was always adjusted when the drift became several milliKelvin.

The drift of the regulator probably could have been reduced by using a heater with a larger surface area to reduce Kapitza boundary effects and a more sensitive thermometer of the Allen-Bradley type. The most likely cause of the drift was a servo motor used as a phase sensitive detector in the Artronic controller. This servo motor required a large starting torque and was found to be sticking on numerous occasions.

The bath regulation was not of any value unless the pot at the end of the flow tube was also cold. Helium II film flow down the tube was not sufficient to cool the pot. Only the short fat tube B (see Table 1) could be cooled by this technique, and this required more than an hour. The other two tubes would have required a factor of ten longer to cool by film flow. Therefore, the pot had to be cooled by admitting a small amount of helium gas into the vacuum can. This exchange gas was evacuated after the apparatus had cooled below the lambda point, and
helium II flow maintained the pot at equilibrium with the bath. The exchange gas was pumped for 30 to 60 minutes to minimize desorption cooling effects when the pot was heated.

Exchange gas was readmitted to the vacuum can at the end of the experiment to obtain calibration data for resistor $R_2$. The gas pressure in the can during calibration was usually 500 milliTorr. Bath regulation remained good until the level of the helium was actually below the regulator heater and resistor.

2.7 Vibration

Brewer and Edwards (73) reported that the steady state thermal resistance in tubes similar to the ones used in this experiment was quite sensitive to vibration. Initial measurements with tube C (see Table 1) revealed that vibrational heating would be a problem. This tube is very thin and additional support was required to suspend the heavy pot. The combined flow tube and support assembly vibrated fiercely with little damping when the cryostat header was struck. Vibrations could be felt by fingertip several seconds after a light tap. It therefore was no surprise to find that tapping on the cryostat while the apparatus was cold greatly changed the temperature difference along the tube. In fact there seemed to be so
much vibration that the hysteresis loops reported by Brewer and Edwards were not observed.

The thermometers inside the vacuum can were also heating when the exchange gas was removed. This spurious heating was blamed on vibration due to the intricate suspension required for thermometer $R_3$ as discussed in section 2.4. The heating of the thermometers was later found to be mainly caused by bridge currents and poor thermal contact although vibration was still a likely contributor to the problem.

Wheatley et al. (101) has reported that externally forced vibrations caused a heat deposition of as great as $\frac{1}{2}$ microwatt in a pyrex tube comparable to this flow tube, but much thicker and stiffer. These authors also found that the heat leak continued to increase at a constant vibration amplitude up to the 13Hz frequency limit of their apparatus.

In order to eliminate vibrations from the apparatus, a number of measures were taken. Initial attempts were made to 'tune' the elements of the apparatus inside the vacuum can. Various types of $\frac{1}{4}$ inch diameter support posts were used including thin wall stainless steel and solid copper. The pot end of the tube was tied down with different types of string at various tensions with and without a nylon bumper between the pot and the support
The range of tension on the flow tube allowed it to hang slack or produce a credible middle C when plucked. The flow tube could be pulled so tight that the flow tube solder joint was actually pulled loose once.

When the tuning adjustments inside the vacuum can were checked with helium II flow, no significant improvements were made over the original arrangement. The worst case developed for a very tightly strung flow tube where three nylon strings were used to tie the flow tube laterally to the support post. The temperature of the pot continued to rise after 3.5 hours of monitoring and the temperature difference along the flow tube was nearly 1°K. In the second worst case, no nylon bumper was used to support the pot. The pot was only tied down to the support post in line with the flow tube which allowed transverse vibrations of the assembly. In this case the temperature difference became quite large although not as large as the previous case, and thermal equilibrium was achieved after about seven hours. The helium II in either case was conducting heat well into the supercritical region.

These results indicated that the pot and flow tube must be tied down tightly, but as is well known, nylon string should not be used because vibrational heating seems to develop quite strongly in nylon. The stiffer
support posts also seemed to cause less vibrational heating. A symmetrical 3-post support assembly was selected as the most stable platform for supporting the pot both laterally and horizontally. The flow tube tension could be tightened at will using ordinary cotton thread so nylon support bumpers between pot and post were eliminated.

Attempts were also made to brace the support post against the vacuum can and the vacuum can against the helium dewar wall to no avail. The liquid nitrogen radiation shield was supercooled by admitting exchange gas into the helium dewar wall to determine the effects of nitrogen bubbling on the vibrational heating. Finally, the three tubular stainless steel downlegs from the cryostat header to the vacuum can were braced by forcing Teflon cylinders between them. The downlegs were then tightly tied together with many wraps of heavy string. None of these adjustments significantly affected the vibrational heating level.

With the cryostat as finely tuned as possible, attention was directed toward eliminating vibration sources outside the cryostat. The mechanical pump for evacuating the bath, which was only a few feet from the cryostat, was set on fiberglass shock mounts. The 4-inch diameter pumping line was isolated from the pump by a flexible hose and from the cryostat by a double bellows
arrangement. All other mechanical pumps were placed on rubber mats and pumping lines were isolated from the cryostat by rubber hoses. These rubber hoses were immersed in boxes of sand where practical in order to damp out any residual vibration in the hose. While the vibration of the cryostat was markedly reduced by these measures, the residual vibration in the room was still quite intense. The room in which the experiment was located was directly above a machine shop and several floors below a giant blower which provided ventilation for the entire building. The resonant frequency of the walls and floor as well as the cryostat was measured and found to be about 100Hz. Two different types of transducers were used for these measurements. A quartz piezoelectric transducer was used for most measurements. This transducer is not particularly sensitive to very low frequency vibrations. The other transducer was a strain gauge accelerometer.

Before the large mechanical pump was mounted on shock pads, it contributed two-thirds of the room vibration even though the pump shaft turns at only 8Hz. The entire room and cryostat seemed to resonate at 100Hz independent of the driving frequency. For example, dragging a chair across the floor still caused a 100Hz signal to appear on either transducer.
It is relatively easy to damp out the vibration entering the cryostat from the floor. Four Model FBS-3-500 spring mounts were obtained from Consolidated Kinetics(a), each of which had a rated deflection of 3 inches at a loading of 500 pounds. These springs were capable of damping vibration entering through the floor down to very low frequencies. High frequency vibrations along the spring coil itself were damped by a fiberglass pad between the springs and the floor.

A heavy steel frame was constructed, mounted on the springs, and loaded with nearly a ton of sand. The cryostat was mounted in this frame and the resulting resonant frequency of the vertical frame vibration was measured to be 2.5Hz using a capacitive distance gauge.

This same distance gauge was mounted such that vibrations of the pot support posts could be measured while the apparatus was mounted in the floating steel frame. These results indicate that the resonant frequency of the three ¼ inch tubular stainless steel support posts is 17Hz. Maximum vibrational amplitudes were obtained by tapping on these ¼ inch posts, tapping laterally on the vacuum can flange, or tapping on the cryostat downlegs. When the large mechanical pump was turned on, only a

(a) Consolidated Kinetics, 249 Fornof Lane, Columbus, Ohio.
barely discernible increase in the vibrational amplitude over the background was observed.

Installation of nylon bumpers between the pot support posts decreased the resonant frequency several per cent. These were subsequently removed because of their ineffectuality. Replacement of the ⅛ inch stainless steel support posts with ¼ inch solid copper posts reduced the resonant frequency an insignificant amount.

The final configuration for heat flow measurements used the three copper support posts, to which the pot was firmly tied with cotton string. The vibration level was reduced sufficiently to observe the hysteresis loops reported by Brewer and Edwards. The copper posts provided no measurable heat leak away from the pot and also served as a thermal anchor for the lead wires going to the pot.
CHAPTER III

EXPERIMENTAL PROCEDURE

3.1 Introduction

Much of the experimental procedure has been delineated in the previous chapter. The difficulties encountered in eliminating spurious heating due to vibration and heat deposition in the thermometers by the bridge currents have been discussed in detail. All measurements have been obtained with properly mounted thermometers and with all effective antivibration mechanisms in operation. This chapter will concern itself with the details of the several techniques used to obtain data points in order to graphically determine the critical heat currents.

The basic data required from the experiment are the two critical heat currents \( W_0 = W(v_{sc}) \) associated with the superfluid critical velocity \( v_{sc} \) and \( W_c = W(v_{nc}) \) associated with the normal component velocity \( v_{nc} \). The heat current density \( W \) is related to the heater power by \( W = Q/\pi a^2 \).
The difficulty in selecting the two critical points is illustrated by the data of Brewer and Edwards in Figure 4. The thermal resistance curve for decreasing heat current is relatively smooth throughout and a certain amount of guesswork is required to select the critical points and three distinct flow regions. However, if the effective viscosity data of Brewer and Edwards are examined in Figure 4, the selection of the critical points is much easier. The curve changes slope at rather sharply defined points so that both critical heat currents can be accurately defined.

In the experiments under consideration here, the critical heat currents are obtained by plotting the temperature difference along the tube vs. the steady component of the heater power \( \dot{Q} \) (see equation (B-6)), or alternately, the thermal resistance vs. \( \dot{Q} \). Each data run consists of 30-40 points \((\Delta T, \dot{Q})\) as shown in abbreviated form in Figures 8 through 10 for each of the three types of tubes used in steady flow experiments. The power at which a deviation from the linear relation is first noted is termed \( \dot{Q}_o=\dot{Q}(v_{sc}) \), and the subsequent strong deviation from the intermediate region behavior is termed \( \dot{Q}_o=\dot{Q}(v_{nc}) \). The critical velocities \( v_{nc} \) and \( v_{sc} \) may be calculated from the heat currents using equations (1-16) and (1-17).
Figure 8

Steady state temperature difference along flow tube C at 1.211°K.
Figure 9

Steady state temperature difference along flow tube B at 1.200°K.
Figure 10

Steady state temperature difference along flow tube A at 1.206°K.
The critical Reynolds number or the critical parameter P suggested in Appendix A may then be evaluated from equation (1-27).

No measurements were undertaken until more than 30 minutes had passed after the regulation of the bath temperature had been achieved. This procedure allowed the drift in the bath temperature to be minimized as already discussed in section 2.6. After each experimental run had been completed and the helium II flow was supercritical, the apparatus was allowed to set idle with no heater power applied for a period of 10 minutes or longer. This presumably allowed the remanent vorticity in the flow tube to decay to some more or less uniform background level before subsequent power was applied. As a result the effects of remanent vorticity on the onset of superfluid turbulence and the intermediate region are hopefully eliminated.

3.2 Heater Control

Before steady state measurements are performed, the AC heater is shorted at the cryostat to prevent spurious heating due to electrical noise. The DC heater voltage is then switched on and increased in small increments. The temperature difference is measured after each DC voltage increase. As the onset of the intermediate region is approached, the increment in heater voltage is
decreased to even smaller steps to ensure the best resolution of the first critical transition point \( W_0 \). The minimum consistent voltage increment possible was several millivolts.

The steady state temperature difference was achieved almost simultaneously with the adjustment of the heat current in the subcritical region. No apparent time constant was noted in the intermediate region, but a time constant of up to several minutes could be noted upon entering the supercritical region, as has been reported by Vinen (72). As the heat current continued to increase in the supercritical region, the time constant for the temperature difference progressively decreased until it again became negligible.

The fluctuating heat current could also be used alone by shorting the DC heater. The AC power was increased gradually as in the case of the DC heater. For frequencies greater than 10Hz, the oscillating voltage was read directly from a voltmeter with an rms output. For frequencies less than 10Hz the peak-to-peak values of the voltage could be accurately determined.

When the rather small oscillating heater voltages were applied, the temperature difference also began to fluctuate. This problem did not occur in measurements with tube C because \( \omega \tau \gg 1 \) at all frequencies attempted,
and modulation of the velocity fields was not successfully accomplished in the tube. In the two larger tubes modulation frequencies were sufficiently low that the condition \( \omega \tau \ll 1 \) was obeyed. In these tubes the temperature of the pot \( T_2 \) began to fluctuate for a pure AC power of less than 10 microwatts.

Because very low frequencies were required to satisfy the \( \omega \tau \) condition, the time constant on the PAR 120 could not be adjusted so that the resistance bridge null output could be averaged. This averaging process was therefore accomplished by an 'eyeball' average based on the peak-to-peak swing of the PAR 120 meter. As the heat current continued to increase, the meter fluctuations required that the sensitivity of the PAR 120 be reduced, and the accuracy in reading \( T_2 \) was degraded.

Eventually, the fluctuations of the meter became full scale on the lowest sensitivity scale. As a result sensible measurements of \( R_2 \) could no longer be made. Fortunately, at this point the heat currents in all cases were well into the supercritical region. At the very high oscillating heat currents, even \( T_1 \) in the bath began to fluctuate, e.g. tube B at \( 1.2^\circ K \) and 1Hz resulted in \( T_1 \) fluctuations at an average power of 160 microwatts. This thermometer was not coaxial with the flow tube and was located at least 0.3mm from the end of the tube. These results certainly demonstrate that modulation of the
temperature gradient could be accomplished in the larger tubes.

When both heaters were connected to the heater power supply described in Appendix E, the total heat current could be adjusted in two ways. The DC voltage could be increased by increments to a predetermined value, and then the AC voltage could be increased by small increments. The resultant total power would rise with the increasing DC voltage, but would remain constant during the AC voltage increase. The ratio of the amplitude of the modulated power $\delta Q$ to the steady power $Q$ is

$$\sigma = \frac{\delta Q}{Q}$$

(3-1)

and would increase steadily from zero with the increasing AC voltage. The ratio $\sigma$ may be related to the heater resistances and voltages by inspection of equations (E-1) and (E-3) of Appendix E. The second method for adjusting the total heat current required simultaneous adjustment of both AC and DC heater controls. The DC voltage could be increased incrementally, while the AC voltage was increased by a pretabulated amount to maintain a constant ratio $\sigma$. A steady incremental rise in the total power resulted while the ratio was maintained nearly constant by the analog circuit of the heater power supply. The latter technique
was almost universally applied to experiments in which an oscillator frequency of greater than 10Hz was required.

When both AC and DC heater voltages were used, the fluctuations in $T_2$ became apparent at a greater total heat current than in the case of purely oscillatory heat current.

At oscillator frequencies less than 10Hz, the heater powers had to be adjusted manually. In order to eliminate the numerous calculations required to maintain the ratio between the powers constant, a method similar to the first one described for the analog circuit was used. In these experiments the analog circuit was defeated, and both heater voltages could be adjusted independently. If a given ratio $\sigma$ was desired at a power corresponding to the critical power $\dot{Q}_c$ for the steady state, the following method was used. The DC voltage was increased incrementally to a level which corresponded to the desired ratio at the steady state critical point and then was held constant. Then the AC voltage was increased incrementally providing the desired ratio at the critical point. Only the AC voltage was increased beyond this point. See Figure 11 for a highly schematic picture of a heat current adjustment with constant power increments. In other words the combined power produced by the two heaters at a prescribed ratio $\sigma = \dot{\delta}Q/\dot{Q}$ would be equal to the steady
Figure 11

Manual heat current adjustment in constant steps. The value of $t_o$ is the duration of time that the oscillating heat current component exceeds the steady state critical heat current $Q_c$ for the transition to supercritical flow.
STEADY STATE $\dot{Q}_c$

CONSTANT DC

TIME
state critical power $Q_c$ in the same tube at the same temperature. Unfortunately, the ratio $\sigma$ changes continuously while the AC voltage is increased as may be seen in Figure 11.

3.3 **Hysteresis**

Figure 4 provides an excellent illustration of hysteretic behavior from the data of Brewer and Edwards. When the steady state heat current was increased in a system relatively free from vibration, the linear subcritical flow region extended well beyond the usual critical heat current. However, when the heat current is subsequently decreased after the transition to turbulence, the flow is characterized by the upper curve in Figure 4. The three regions of flow appear and the critical heat currents can be defined.

Brewer and Edwards noted that externally induced vibrations could cause transition between the two types of behavior illustrated in Figure 4. The data analysis is therefore complicated by the presence and absence of vibrational perturbations.

Before the antivibration measures discussed in section 2.7 were taken, no hysteresis was observed in the heat flow in tube C. After most of the external vibration was eliminated, the hysteresis loops were quite pronounced particularly in the smallest flow tube C. If externally
applied vibrational perturbations were applied near the transition \( W_0 \), the characteristic curve of tube C for an increasing DC heat current was identical with that for a decreasing heat current as shown in Figure 12. The larger tubes in this experiment did not exhibit large hysteresis loops. Hysteresis was present in the larger tubes to the extent that the critical heat current density \( W_0 \) would be in error by 20%, if steady state measurements were obtained with an increasing heat current.

In order to standardize the effects of vibration on the stability at the normal fluid critical transition, a standard amplitude vibrational perturbation was applied as the critical transition was approached. The perturbation consisted of a series of three or four solid raps on the antivibration frame with a standard model VACO A 316-8 screwdriver handle after each DC heat current was established. The resulting curves more nearly followed the steady state decreasing heat current curves, but the transition was still not as accurately determined, especially for the small tube C. All steady state critical heat currents for these experiments have been determined with decreasing heat currents to eliminate these hysteretic effects.

When modulated steady state heat currents were used, at low ratios \( \sigma \), there was essentially no
Figure 12

Hysteresis in flow tube C at 1.213°K.
\( \Delta T \) (mK)

\( Q \) (\( \mu \) watts)

- ○ INCREASE NO TAPPING
- × DECREASE NO TAPPING
- + INCREASE TAPPING

\( Q_0 \)
\( Q_C \)
hysteretic behavior observed. Even though the critical transitions were reasonably repeatable for these measurements using increasing heat currents, the 'standard amplitude perturbation' was applied at each heat current setting as the transition for the steady state was approached and until the transition was actually observed.

Large ratio modulated flow exhibited hysteresis of a different kind. Following the critical transition to the supercritical flow region for an increasing modulated heat current, the heat current was decreased and the flow returned to the intermediate region at a larger heat current than expected. When the surprising results of this type of experiment, discussed in the following chapter, were observed, the following experiment was performed to complete the work described here. A DC heat current was increased beyond its critical value $W_c$ and then was decreased slightly, but not below the critical value. The total heat current in this supercritical region was then maintained constant while the modulation was turned on and incrementally increased. When the ratio $\sigma$ increased to 70%, while the total heat current remained constant, the flow returned to the intermediate flow region. This unexpected stabilization of a flow which was already supercritical will be discussed in the following chapter.
CHAPTER IV

EXPERIMENTAL RESULTS

4.1 Tube C

Table 1 provides the dimensions of tube C which was soldered to an Epibond pot of the type shown in Figure 7. Because the tube has such a small diameter, its thermal resistance is large, and condition (2-3) \( \omega \tau << 1 \) is not obeyed. The heat current modulation is averaged by the heat capacity of the pot, and a viscous wave cannot be generated in the flow at a reasonable frequency. The calculations presented in Figure 6 indicate that the frequency would have to be lower than the oscillator limit (0.005Hz), in order to have \( \omega \tau < 0.1 \) where significant modulation of the flow velocities in the tube might be expected. As a result, enhanced stabilization of the flow was not found in this tube.

The oscillation amplitude of the pot temperature \( T_2 \) was measured to determine that the absence of stabilization in this tube was due to the condition \( \omega \tau > 1 \). The arrangement of the electronics for detection of the temperature fluctuation in carbon resistor \( R_2 \) is shown
in Figure 13. The oscillator frequency is established at exactly \( \frac{1}{2} \) the reference frequency output of the 17Hz PAR 120 by displaying the appropriate Lissajous figure on the oscilloscope. The oscillator is then connected to drive the AC heater with an 8.5Hz frequency.

The heat flow \( Q(t) \) between the heater and thermometer \( R_2 \) is proportional to the square of the heater voltage and results in an oscillation frequency of 17Hz in the temperature of the helium II. The resistance of thermometer \( R_2 \) will follow the temperature oscillations, so that if the switch in the battery circuit is closed, the constant measuring current through \( R_2 \) will generate a 17Hz voltage. This signal is amplified by a PAR 112 preamplifier before entering the signal input of the PAR 120. The oscillator is connected to the frequency doubling reference input of the PAR 120 so that the thermometer output signal may be phase-sensitive detected. The only low frequency available in the PAR 120 was 17Hz, so frequency dependence of the oscillation amplitude of \( R_2 \) was not measured.

At 1.2°C using the carbon resistor described in Table 2 for tube C, the maximum voltage amplitude across the carbon resistor was found to be one microvolt. At the thermometer sensitivity listed in the table, this amplitude corresponds to a maximum temperature fluctuation
Figure 13

Block Diagram for Measurement of $R_2$ Fluctuations.
OSCILLOSCOPE

PAR 120
(f = 17 Hz)
INPUT f REF

2 TRIGGER

PAR 112

oscillator (f₀ = 8.5 Hz)
OUTPUT

POT

22 K

250 K

1.5 V

R₂

Q(t)

(17 Hz)

R_AC
of less than one microdegree. This maximum temperature fluctuation corresponds to a heat current fluctuation amplitude of less than $10^{-3}$ microwatt in the pot. The actual heat current amplitude entering the pot from the oscillator is approximately 10 microwatts, so that it is quite apparent that 17 Hz oscillations in the helium II velocity fields are nonexistent. The $\omega T > 1$ condition has therefore prevented stabilization of the flow in tube C by viscous wave modulation.

The absence of stabilization or destabilization of the critical heat current in this tube at high frequencies is apparent from Figure 14. Even at high amplitudes of modulation corresponding to a $\sigma$ near one, the critical heat current remains constant at about 16.5 microwatts. Most of the scatter of the data points may be attributed to slight differences in temperature. For example, practically all the data points below 16.5 microwatts are at temperatures which are as much as 7 m°K below 1.210°K, and most of the data points above this baseline are at temperatures several m°K higher than 1.210°K.

Since tube C could not be successfully modulated, only DC data were obtained at 1.2°K. The critical flow parameter $P_c$ has been calculated from equation (1-27) and has been plotted in Figure 3. The characteristic
Figure 14

Critical heat current in tube C at $1.21^\circ$K for various heat current ratios $\sigma$ and oscillator frequencies.
$Q_c (\mu \text{watts})$

$\sigma = \delta Q/\dot{Q}$

- $\circ$ 50 Hz
- $\triangle$ 25 Hz
- $\triangledown$ DC

Oscillator Frequency
heat flow curves have already been provided in Figures 8 and 12. The intermediate region can be clearly delineated on these curves as shown in the figures. The data obtained from flow tube C is tabulated in Table 4 of Appendix F.

Calculation of the cross-sectional mean velocity, Reynolds number, and the parameter P is dependent on knowing the tube radius. For tube C which is very small the radius cannot be measured mechanically. The heat flow characteristic of the tube may be used to determine its radius with reasonable accuracy. If the thermal resistance $\Delta T/Q$ is plotted as a function of the heat current, and the subcritical region is extrapolated to its intercept with the vertical axis, the thermal resistance at zero heat current will be determined. This thermal resistance at zero heat current is given by equation (1-20) and is a function of temperature dependent quantities and the tube radius $a$ to the inverse four power. Although the thermal resistance can only be empirically determined to within several per cent, the error in estimating the tube radius will be quite small. The radius of tube C has been calculated in this manner and has been evaluated to be $a=5.145 \times 10^{-3}$ cm.
4.2 **Tube B**

To ensure that modulation could be introduced into the tube flow, tube C was replaced by the larger diameter tube B. The pot soldered to tube B was rebuilt but was a replica of that used with tube C. Since these two tubes are also of the same approximate length, the \( \omega \tau \) condition will remain the same, as shown in Figure 6. The larger radius of tube B allows larger modulation frequencies to be generated in the tube flow while the condition \( \omega \tau < 1 \) is retained.

The much lower thermal resistance of this larger diameter tube necessitated the installation of more sensitive carbon thermometers (see Table 2). Even these thermometers were not really sensitive enough for flow experiments conducted with this tube, as may be seen by comparing the steady state data scatter in Figure 9 with the scatter for the other tubes in Figures 8 and 10. The scatter in the data does not allow the intermediate region to be defined.

It can be noted from Figure 6 that stabilization enhancement cannot be anticipated for this tube which is about 5.5 times the diameter of C unless the oscillator frequency is less than 5Hz. Critical heat current measurements were obtained for this tube over an oscillator frequency range of 0.1-10Hz. In no case was any
consistent repeatable stabilization of the critical heat current observed.

In fact the heat flow in tube B was found to be destabilized by the modulated heat current. Figure 15 illustrates this behavior rather well, considering the difficulty in determining the critical heat current from this relatively insensitive data. Instead of enhancing stability as the heat current ratio $\sigma$ increased or the oscillator frequency was reduced, the flow became more and more destabilized. It can also be noted in Figure 15 that the destabilization did not occur near 10Hz where $\omega \tau > 1$ as shown in Figure 6.

The destabilization effect of the modulated heat current immediately led to the suspicion that a short tube effect was appearing at low frequencies. The critical velocity of the normal component in tube B at 1.2⁰K is approximately 5cm/sec. The tube itself is only 12.4cm long so that the wavelength of the modulation is comparable to or greater than the tube length at fluid frequencies less than 0.4Hz. These fluid frequencies correspond to an oscillator frequency of 0.2Hz as discussed in Appendix E.

Effects of short tube lengths may therefore be anticipated for oscillator frequencies of less than about 0.5Hz. The oscillatory heat current amplitude
Figure 15

Destabilization in tube B shown as the critical heat current $Q_c$ for the transition to supercritical flow vs. the heat current ratio $\sigma$ at various oscillator frequencies.
The diagram shows the relationship between $Q_c$ (in µwatts) and $\sigma$ for different frequencies. The curves represent data for 10 Hz, 0.5 Hz, and 0.3 Hz, along with DC. The frequency range is indicated by markers on the x-axis.
which is greater than the steady state critical heat current is shown as the cross-hatched area in Figure 11. As the oscillation amplitude is increased or the frequency is decreased the time \( t_0 \) during which the heat current is supercritical increases. When the time \( t_0 \) becomes comparable to the time required for normal fluid to travel the length of the tube, the flow can be considered to be essentially steady state. Turbulence in the normal fluid will be generated during the time \( t_0 \) and will probably decay slowly. As a result the critical transition appears at lower heat currents in the modulated flow.

When the wavelength of the flow modulation is longer than the tube, the stabilization due to the modulation viscous wave should no longer be present. The stabilization effect is nonexistent because the viscous wave is a slow continuous variation throughout the tube and may be considered to be steady state.

Since the modulated wave will produce normal fluid turbulence in discontinuous bursts, the behavior of the modulated turbulent flow at low frequency should differ somewhat from the steady state characteristic flow. The steady state curve of Figure 9 has been compared with the modulated flow data in Figure 16. The critical heat current is reduced significantly for the modulated flow, even when tapping is not applied, although the general shape of the \( \Delta T \) vs. \( Q \) curves remain quite similar.
Destabilization of modulated heat flow at $f_{osc} = 0.5\text{Hz}$, $\sigma = 70\%$, and $T = 1.204^\circ\text{K}$. 
\[ \Delta T (\text{mK}) \]

AC_1 \quad f_{\text{osc}} = 0.5 \text{ Hz}, \sigma = 70\%, \text{ NO TAPPING}

AC_2 \quad f_{\text{osc}} = 0.5 \text{ Hz}, \sigma = 70\%, \text{ TAPPING}

DC \quad \text{STEADY STATE}

\[ \dot{Q}(\text{\textmu watts}) \]

\[ \dot{Q}(\text{\textmu watts}) \]

\[ \dot{Q}_C(\text{\textmu watts}) \]
The modulated flow data of Figure 16 were obtained for a heat current ratio $\sigma=70\%$. The destabilization due to the short tube is presented at this ratio because destabilization effects can also result from viscous wave modulation at small penetration depths and large ratios as may be seen in Figure 1 from the dashed line for $T=90\%$. The frequency and amplitude for the data of Figure 16 should not be in the viscous wave destabilization region.

Viscous wave destabilization may have been present in this tube for oscillator frequencies greater than 1Hz. The presence of such destabilization below 1Hz would be obscured by potential short tube effects. The viscous wave destabilization effect is quite small, and probably would not be much greater than the experimental error for this flow tube. In any event, very little data was obtained in the region where the maximum destabilization might be anticipated, i.e. $1<f_{osc}<10$Hz.

The tube radius calculated from the thermal resistance of tube B could not be accurately determined because of the large experimental error. The combination of poor thermometer sensitivity and the short tube effect made further data taking undesirable. The data obtained is presented in Table 5 in Appendix F, but is not very trustworthy.
4.3 Tube A

In order to increase the temperature difference sensitivity and to eliminate the short tube effect at low frequency, a longer flow tube was required. The vacuum can shown in Figure 5 limited flow tube length in straight sections to about 12cm. The longer flow tube A (see Table 1), which was cut from the same length of tubing as B, was coiled into a spiral in order to fit into the vacuum can. The same carbon resistors were used to measure temperatures for both tubes A and B.

The steady state flow characteristics of tube A has already been depicted in Figure 10. The three regions of steady flow in this tube were easily discernible at 1.2°K, but the superfluid transition $Q_0$ was not at all clearly defined at 1.5°K in most cases.

Modulation experiments for this tube were begun at an oscillator frequency of 1.0Hz. No stabilization effects were noted until the second experimental run when the oscillator frequency had been reduced to 0.1Hz. A significant enhancement of the transition stability was noted for heat current ratios of 70% and 80% at 1.2°K. These results were repeatable in the subsequent experimental run, and the behavior of the stability region was mapped out below 0.1Hz at this temperature and at 1.5°K.

Examination of Figure 6 reveals that stability enhancement could not be expected above 1.0Hz due to the
condition which enters at lower frequencies than tubes B and C. In fact, significant velocity field modulation in the helium II should not have been expected until oscillator frequencies of less than 0.5Hz were attempted. Several experimental runs were attempted at 0.3 and 0.5Hz with no apparent stabilization effects. The modulation therefore seems to become appreciable between 0.3 and 0.1Hz which is in reasonable agreement with the calculations displayed in Figure 6.

Typical plots of $\Delta T$ vs $Q$ are provided in Figures 17 and 18 for the two different temperatures at a frequency and amplitude ratio near the maximum stabilization effect. The data has been labeled with the ratio $a/\delta$ rather than $a/\delta_n$ for reasons discussed below. The critical heat current $Q_c$ has been increased by more than 50% due to the viscous wave stabilization.

It can be noted from these figures that the stability of the superfluid transition has also been affected by the modulation. This shift in $Q_c$ is temperature dependent and completely unexpected. At 1.5°K the superfluid transition is destabilized whereas the transition stability at 1.2°K is enhanced.

The stabilization of the supercritical transition provides a more pronounced intermediate region for analyzing the approximate $W^3$ contribution of mutual friction to the temperature gradient. The stabilization
Figure 17

$\Delta T$ vs. $\dot{Q}$ for tube A at 1.2$^0$K. Both steady and modulated flow characteristics are shown including their respective critical heat currents.
$\Delta T$ vs. $Q$ for tube A at $1.5^oK$. Both steady and modulated flow characteristics are shown including their respective critical heat currents.
\( a/\delta = 1.14, \sigma = 90.5\% \)
\( \Delta a/\delta = 0.0, \sigma = 0 \)

\( T = 1.5 \text{ K} \)
of the normal fluid transition resulted in such an extended intermediate region that the value of the mutual friction contribution to the temperature difference was quite large. When the normal fluid critical transition finally occurred the change in $\Delta T$ was so small that the 'jump' was sometimes not noticeable while the data was being taken (see Figure 18).

The extension of the intermediate region by viscous wave stabilization provided enough data to evaluate the mutual friction coefficient $A$ of Gorter and Mellink (50). The definition of the mutual friction force in (1-24) allows the temperature gradient in tube flow to be written as equation (1-25). A better expression for the mutual friction term in (1-25) was suggested by Vinen (72) and has recently been amply vindicated (102).

$$\nabla T' = -A \frac{\rho n}{S(\rho_S\nu T)^3} (W-W_0)^2 W \quad (4-1)$$

If the temperature gradient of the linear subcritical region is subtracted from the measured gradient, the remaining temperature gradient will be $\nabla T'$. If this is divided by $W$ and the resulting quantity plotted against $(W-W_0)^2$ on log-log graph paper, the slope of the resulting straight line will yield the mutual friction coefficient in product with known temperature dependent terms.
The value of $A$ calculated by this method was found to be 56 (cm sec/gm) at 1.5°K and 52 (cm sec/gm) at 1.2°K. The temperature dependence of these values is not as strong as determined by some observers, but the values of $A$ are well within the range reported by others (72,74). Because sufficient data for the calculation of $A$ could only be obtained in modulated flow, the temperature differences can involve large errors. The error in this calculation of $A$ is about ±20%.

There was initial concern that resorting to a spiral tube might disturb the flow in such a way that the mutual friction coefficient might be affected. The flow would no longer be curl-free when constrained by the spiral tube walls. Atkins (103) and others (104) have reported flow measurements in spiral tubes. Although Atkins reports reduced fluid velocities in his spiral tube, he attributes this to deformation of the circular cross section of the tube. Great care was taken in this experiment in order that the tube would not be deformed, as reported in section 2.2. Peshkov and Tkachenko (104) report a mutual friction coefficient in their very long spiral tube that is within the range of values reported by others.

Figures 19 and 20 show the families of curves for various heat current ratios $\sigma$ (equation (3-1)) at fixed
Figure 19

$\Delta T$ vs. $\dot{Q}$ for various $\sigma$ at $1.2^0K$ in flow tube A. The origin for the temperature difference is shifted upward $5^0K$ for each successive data set to improve the data point separation.
$T = 1.2 \, \text{K}, \frac{a}{b} = 1.14$

- $\sigma = 61\%$
- $\sigma = 100\%$
- $\sigma = 81\%$
- $\sigma = 90\%$

Graph:

- $\dot{Q}_c (\Delta)$
- $\dot{Q}_c (\nabla) = 213$
- $\dot{Q}_c (\bigcirc)$
Figure 20

$\Delta T$ vs. $Q$ for various $\sigma$ at $1.5^\circ K$ in flow tube A. The origin for the temperature difference is shifted upward $2m^\circ K$ for each successive data set to improve the data point separation.
$T = 1.5 \text{ K, } a/b = 1.47$

$\Delta \sigma = 41.5\%$

$\sigma = 70\%$

$\sigma = 80\%$

$\sigma = 90.5\%$
temperature and frequency. The origins of each curve have been displaced along the vertical axis to improve the data point separation. The effect of increasing the amplitude of modulation on both critical heat currents is quite pronounced.

The data obtained from experiments with this tube have been included as Table 6 in Appendix F. The tube radius has been calculated from equation (1-20) where the thermal resistance at 1.20°C has been taken to be 15(°K/watt), so that \( a = 0.0285 \text{cm} \). The thermal resistance values at 1.2°C varied by ±10%, but the thermal resistances evaluated at 1.5°C were not nearly as accurate. Since tube B was cut from the same length of material, it may also be assumed to have this radius.

The main result of the experiments is presented in Figures 21 and 22 for the two quite different fluids provided by helium II at 1.2 and 1.5°C, respectively. These plots of the critical flow parameter \( P_0 \) against the heat current ratio \( \sigma \) for various values of \( a/\delta \) reveal the similarity of the stabilization effect for the two different fluids. This similarity would not be apparent if the curves were labeled with values of \( a/\delta_n \). The maximum stabilization appears for \( \sigma \) between 80 and 90% for all frequencies at which stabilization can be identified.
Figure 21

Critical parameter $P_c$ vs heat current ratio $\sigma$ at $1.2^0K$. 
\[ \circ \quad a/\delta = 1.14 \]
\[ \triangle \quad a/\delta = 1.47 \]
\[ \diamond \quad a/\delta = 4.59 \]
\[ \times \quad a/\delta = 0.0 \]

\[ T = 1.2K \]
Figure 22

Critical parameter $P_c$ vs heat current ratio $\sigma$ at $1.5^\circ K$. 

140
\[ \frac{a}{\delta} = 0.72 \]
\[ \frac{a}{\delta} = 1.14 \]
\[ \frac{a}{\delta} = 1.47 \]
\[ \frac{a}{\delta} = 0.00 \]

\[ T = 1.5 \, \text{K} \]
The results for helium II flow may be compared directly with those of Gilbrech and Combs (18) in water as shown in Figure 23. The water data shows a peak in the stabilization of the critical Reynolds number near a velocity amplitude ratio of 80% and $a/6 \leq 2.83$. The parameter $a/6$ for maximum stabilization in helium appears to be slightly greater than one, where the total density is used to calculate the penetration depth. Gilbrech and Combs do not provide low enough $a/6$ data to conclude that the stabilization effect has been maximized as frequency is varied. The strong qualitative agreement between the data in Figures 21-23 suggests a common origin for the results.

4.4 Restabilization of Turbulent Flow

The restabilization of turbulent flow in helium II already alluded to in Chapter 3 has been observed in three ways: 1) by inducing vibrations in the intermediate region, 2) by reducing the modulated heat current in the supercritical region, and 3) by holding the total heat current constant while the ratio $\sigma$ was increased in the supercritical region. These striking results were not reported in the water data of Gilbrech and Combs.

Figure 24 illustrates an observation of the first type. Data is obtained in the usual manner for a modulated flow ($\sigma=90\%$, $a/6=1.14$), i.e. the heat current $Q$
Figure 23

$R_c$ vs. velocity amplitude ratio in water as measured by Gilbrech and Combs (18). The velocity amplitude ratio is the cross-sectional mean oscillating velocity amplitude $\langle v_p \rangle$ divided by the cross-sectional mean steady velocity $\langle v_o \rangle$. 
Restabilization with changing heat current in flow tube A. Vibrational perturbations induce momentary excursions from the open to the closed circles for increasing heat current. After the transition to the supercritical state has occurred, the heat current is decreased. A restabilization transition occurs at a heat current less than the critical value for the increasing modulated flow, but much greater than the steady flow critical value.
$T = 1.2 \text{ K}, \alpha / \delta = 1.14, \sigma = 90\%$

- **O** INCREASING HEAT
- **●** VIBRATION INDUCED
- **△** DECREASING HEAT
- --- DC CURVE
is increased to some value and the apparatus is tapped. For \( Q \) less than the steady flow critical power (125 microwatts), the tapping produces a momentary increase in \( \Delta T \) with an immediate return to the subcritical value. For \( Q \geq 125 \) microwatts however, the tapping produces a larger \( \Delta T \) (compare open and filled circles in Figure 2). The maximum value of these temperature differences is quite difficult to measure because the modulated flow period is of the same order as the vibration induced excursion. With some experience the maximum temperature difference can be obtained, and it was discovered that these maximum points approximately follow the curve for steady state supercritical flow in the same tube at the same temperature. One interpretation of these observations is that the tapping drives the flow into the supercritical state, but the viscous wave modulation restabilizes the flow to the intermediate state.

The second type of observation is shown by the triangles in Figure 2. In this case the modulated heat current is reduced in steps from a value well into the supercritical region. A transition into the intermediate region occurs at \( Q \approx 180 \) microwatts which is less than the modulated \( Q_c \) but greater than the steady \( Q_c \). If the modulation only acts to inhibit the transition from the intermediate to the supercritical state, one might expect
the helium to remain in the supercritical state down to the steady \( \dot{Q}_0 \) (125 microwatts). It appears, however, that the modulation can restabilize the supercritical state.

The third type of observation is similar to the previous one although more dramatic. Steady flow is initially established in the supercritical region. The modulation heat current \( \delta \dot{Q} \) is then increased keeping the heat current \( \dot{Q} \) constant. As the heat current ratio \( \sigma \) is increased, restabilization to the intermediate region occurs as shown in Figure 25. The slight increase in the temperature difference just prior to the restabilization occurs at both 1.2 and 1.5°K at various frequencies and is probably due to a slight difference in character between steady and modulated supercritical flow. In any event the flow is quite evidently restabilized as a result of the viscous wave modulation. Flow data from other experiments equivalent to the restabilized region are also plotted.

4.5 Error Estimates

The validity of equations (1-10) and (1-11) is based on the assumption of incompressibility so that second viscosity effects may be ignored. The subsequent analysis of the data depends on the reasonableness of this assumption in a flow tube where sizeable temperature gradients occur. If tube C supports fully developed
Restabilization with increasing $\sigma$ at a constant heat current. The total time-averaged heat current $Q$ is held constant while the modulation heat current amplitude $\delta Q$ is increased in small steps. The restabilized flow occurs at a ratio $\sigma = \delta Q/Q$ of about 75%. The value of $\Delta T$ for $\sigma > 75\%$ is equivalent to the other stabilized runs at identical heat currents and ratios in the intermediate region.
A T(m K)

\[ T = 1.2 \text{ K}, \theta/\delta = 1.14 \]

- INCREASING \( \sigma \)
- \( \Delta T(Q, \sigma) \) FROM OTHER DATA
turbulent flow, the temperature difference can be greater than 100\,mK. The density of the normal component can vary by as much as a factor of two at 1.2\,K. In the subcritical and intermediate regions, the normal density can still vary by 10\%. The normal fluid and superfluid are certainly not incompressible in tube C, although corrections have not been made. In tube A where nearly all the significant measurements were obtained, the maximum temperature difference in the intermediate region is about 1.5\,mK which results in a normal density correction of less than 1\%. Even in the supercritical region the temperature difference is usually less than 10\,mK and the density correction is still less than 5\%, which is acceptable within the other error limits of these experiments. Tube B involves an order of magnitude less temperature difference than in tube A.

The data scatter in Figures 21 and 22 shows how really large the experimental error is for these types of measurement. The critical heat current \( \dot{Q}_c \) as well as the other parameters determining \( P_c \) are strongly temperature dependent. The data must be carefully selected so that the temperatures at which the heat current is determined are all identical. In addition the classical flow experiments are equally messy and critical Reynolds numbers are strongly affected by flow channel entrance conditions, vibration, etc.
The heat current and temperature measurements are straightforward as described in Chapter 3. The steady state voltage and heater resistance are determined to \( \pm 2\% \). When the AC voltage is turned on, the error in the heat current measurement will remain about the same. However, at very low frequencies, the error in the AC voltage measurement may be slightly more uncertain because the peak-to-peak voltage must be read.

Steady state temperatures can be read to \( \pm 10 \) microKelvin. In tube C this error is negligible, even at very low heat currents, as may be seen in Figures 8 and 12. For tube B the error is still nearly \( \pm 5\% \) at \( Q_0 \) (see Figure 9). The data obtained with this tube is very difficult to interpret at low heat currents. The error in temperature measurement for tube A is again small. At heat currents of 30 microwatts, the temperature error is only several per cent and is negligible at the transition heat currents.

For a modulated flow the error in estimating the temperature difference at low frequencies can become quite large. Comparison of the DC and AC data in Figures 17 and 18 reveal this problem. The fluctuating output of the pot thermometer must be 'eyeball' averaged to determine the mean temperature. As the oscillations in temperature become quite large, the sensitivity of the PAR 120 used to
monitor the resistance bridge must be reduced. The reduction in sensitivity results in an increase in the error estimate for the temperature difference. As the oscillatory heat current increases, the temperature amplitude increases and the sensitivity decreases until the peak temperature swings can no longer be read on the PAR 120. The sensitivity has usually decreased by an order of magnitude in the region of fully developed turbulence in flow tube A. This corresponds to an error of ±100 microKelvin in the temperature difference measurement. The smaller amplitude oscillations, will, of course, have smaller errors caused by the fluctuating temperature. The error in estimating the temperature difference in the intermediate region is probably ±30-40 microKelvin. This would correspond to an error of as high as ±10% in locating ΔT for a given ˙q, although the error will normally be much lower than this.

Error in determining the heat current ratio σ for low frequency measurements is a result of the continuous change of this ratio when the steady heat current is fixed, and only the modulation heat current is increased. The ratio is interpolated between successive heat current settings in order to evaluate it at the steady flow value of the critical heat current. The error in this ratio is about ±1%.
The superfluid critical heat current $Q_0$ was very difficult to estimate. In the small tube C it could be estimated to within several per cent. The poorer temperature resolution in the other tubes resulted in an interpolation error of $\pm 10\%$ or greater. In numerous cases the transition could not even be located. These values of $Q_0$ must be viewed with great suspicion, although the general behavior of this transition can be deciphered.

The large errors for fluctuating heat currents are not really sufficient to explain the scatter in the $P_0$ values of Figures 21 and 22. In fact, the steady state $P_0$ values are nearly as scattered as in the modulated flow. The problem of error analysis in this kind of an experiment where transition phenomena are involved is complicated by the possibility of metastable states and hysteresis. The best estimate of error must be obtained from the data plots themselves.

Much of the data obtained has not been included in Appendix F because of behavior not typical of the preponderance of data. For example, the temperature difference was observed to jump from one linear curve to another, and perhaps, would jump back to the original behavior at a somewhat higher heat current. This behavior was observed in the laminar flow region when no vibrational perturbations were induced. In the intermediate flow region at a given heat current, the average
temperature difference would sometimes oscillate very slowly, with periods of as long as several minutes. The amplitude of this slow oscillation could be quite large, sometimes approaching 1m°K. The flow characteristics well above \( Q_0 \) were also not reproducible as may be seen in Figures 19 and 20. Data exhibiting odd effects have been purposely excluded for clarity. Over 300 individual data runs were obtained in these experiments which provided a great deal of experience in determining which data was relevant.

Reproducibility of data was not good. The problems of metastable flow states and drifting temperature resulted in an extremely large amount of data being obtained. The stability of the flow was then analyzed on the basis of the average behavior supported by the preponderance of data. The chances of repeating any given data in a successive run were perhaps as low as 60-70%.

4.6 Steady Flow Comparison with Other Experiments

The original tube C was selected, among other reasons, to be comparable to tube number 2 of Brewer and Edwards (73). The steady flow measurements obtained from tube C are compared with the results of Brewer and Edwards in Table 3 at 1.212°K. These results are quite comparable, especially in consideration of the fact that the \( Q_0 \) values
of Brewer and Edwards are generally acknowledged to be low in comparison with other experiments.

The restrictions required in the selection of the diameter and length of tubes B and C for successful flow modulation precluded matching the steady state flow tubes of other workers.
TABLE 3

STEADY FLOW COMPARISON OF TUBES

| Tube Designation | Measured I.D. (cm) | Length (cm) | $\dot{Q}_0$ (microwatt) | $\dot{Q}_s$ (microwatt) | $\frac{\Delta T}{Q} | \dot{Q}_0$ (°K) watt |
|------------------|-------------------|-------------|------------------------|------------------------|-----------------------------|
| C                | 0.0103\(^{(a)}\)  | 10.15       | 12.5                   | 16.5                   | 0.87                        |
| B&E 2            | 0.0108\(^{(b)}\)  | 10.2        | 9                      | 14                     | 0.625                       |

(a) From heat flow in helium II.

(b) From Poiseuille law in helium gas flow.
CHAPTER V

CONCLUSIONS AND DISCUSSION

The assertion that the critical heat current density $W_c$ is associated with the onset of turbulent flow in the normal component has been continually supported in this research. Previous uncertainty about this identification, based on the unsatisfactory behavior of the critical Reynolds number, has been partially alleviated by the introduction of the parameter $P$. The development of this parameter grew out of attempts to better understand the data in the research reported here and previously reported work. The success of $P_c$ in describing the critical heat current density $W_c$ in steady flow recommends its use in these modulation experiments.

The primary objective of these experiments, to determine whether the critical heat current can be increased by suitable modulation, has been successfully achieved (see Figures 21 and 22). What remains for discussion and analysis are the conditions under which enhanced stability occur, i.e. how does $P_c$ depend upon the frequency and amplitude of the modulation and the properties of the liquid helium II.
Evidence has been presented that the small tube C could not be stabilized because the thermal time constant was too large. The oscillating component was essentially filtered from the helium II flow so that only negligible viscous wave modulation was present in the tube flow.

In the larger diameter tube B oscillating flow conditions were possible at low frequencies, but the tube was so short that the wavelength of the modulation was of the same order as the length of the flow tube. In this case essentially steady heat flow characteristics prevailed in the sense of a very slowly varying heat flow. The critical transition was therefore destabilized when the modulation heat current was applied because the normal fluid flow became turbulent when the instantaneous value of the heat current exceeded the steady state critical value. The thermal resistance of this tube was so small that adequate temperature difference measurements could not be obtained.

After a considerable period of trial and error, the proper flow modulation conditions were established in tube A. The stabilization of heat flow in helium II was achieved at extremely low frequencies \( f_{\text{osc}} \leq 0.1 \text{Hz} \), much lower than had been originally anticipated. All the data to be discussed below comes from tube A unless otherwise specified.
The theory and experiments on stabilization in classical fluids discussed in section 1.2 suggest that the critical parameter, $P_c$, in this case, will depend upon the amplitude of modulation $\sigma$ and the frequency and liquid properties through the dimensionless ratio $a/\delta$. For $\sigma$ such that stabilization occurs (typically 50-100%), the function $P_c(a/\delta, \sigma)$ has a maximum for $a/\delta \geq 1$. Examination of the data for helium II at $1.5^0K$ given in Figure 22 is in excellent agreement with this behavior, i.e. $P_c$ is maximized at $\sigma \approx 80\%$ and $a/\delta \approx 1$. The important feature of this result is that it is $a/\delta$ not $a/\delta_n$ which is used to label the curves. For this data, $a/\delta = 1.14$ corresponds to $a/\delta_n = 0.34$ which is somewhat less than the expected value for maximizing $P_c$.

It is certainly quite possible, however, that the function $P_c(a/\delta, \sigma)$ is not precisely the same as the classical function $R_c(a/\delta, \gamma)$ for the water data. Indeed, the maximum of $P_c$ occurs between 80-90% while the maximum of $R_c$ occurs between 70-80%. (This difference most likely is the result of the respective definitions of the ratio, and will be discussed below.) The choice between $a/\delta$ and $a/\delta_n$ can be made unambiguously by consideration of the $1.2^0K$ data in Figure 21. At $1.2^0K$, the kinematic viscosity of helium II $\eta/\rho$ is increased by about 20% while $\eta/\rho_n$ is increased by about a factor of 5 when compared to
1.5°K values. Thus, while the maximum in $P_c$ seen in Figure 21 occurs at $a/\delta \approx 1.14$ in agreement with the data at 1.2°K, the corresponding value of $a/\delta_n$ is about 0.19 compared with 0.34 at 1.2°K. In other words the stabilization of helium II at both 1.2°K and 1.5°K can be described by the single function $P_c(a/\delta, \sigma)$, but not by $P_c(a/\delta_n, \sigma)$. (The results in Figure 3, by the way, could be regarded as agreement of $P_c(\sigma=0)$ over the entire temperature range.) Further, the comparison of the helium II results (Figures 21 and 22) and the results for water (Figure 23) suggests rather strongly that similar phenomena are being observed, i.e. the onset of turbulence in the normal component.

The appearance of $\delta$ rather than $\delta_n$ ($\rho$ rather than $\rho_n$) in the description of helium II stability is not surprising in view of the model for thermal counterflow employed in Appendix A. Indeed, the result is quite in harmony with this model which naturally defines the parameter $P$. The presence of super component vorticity in the intermediate region ($W_0 < W < W_c$) produces both mutual friction and an effective normal fluid density $\rho_n^*$, and $\rho_n^* \approx \rho$ has been assumed at $W_c$. It is precisely this density $\rho_n^*$ (the coefficient of $D\vec{\rho}_n/Dt$) which appears in $\delta$.

The amplitude of the modulation appears in $P_c$ as the ratio $\sigma = \delta\dot{Q}/\dot{Q}$. The ratio $\tau$ in the work of Gilbrech and
Combs is the amplitude of the cross-sectional mean velocity divided by the cross-sectional mean steady velocity. An attempt was made in Appendix B to relate the ratio \( \sigma \) used here to the equivalent velocity ratio of the normal component. Equation (B-26), which is obtained from thermal analysis, reveals that the velocity amplitude ratio is not just proportional to \( \sigma \), but is proportional to the strongly temperature dependent term \( (\rho_s / \rho) \sigma \).

If the analysis of Uchida (38) discussed in Appendix B is followed, a rather strong frequency dependence of the velocity amplitude ratio is also present (equation (B-31)). It must be noted that the analysis of Uchida is only valid for laminar flow of both \( v_s \) and \( v_n \) so that the analysis can no longer be applied in the intermediate region. In the final analysis the actual velocity amplitude ratio remains indeterminate in the intermediate region. The velocity amplitude ratio will remain proportional to \( \sigma \), but other temperature and frequency dependent factors are probably required. One interesting experiment suggested by this result would be a direct measurement of the velocity ratio (equation B-31) both for \( v_s < v_{sc} \) and \( v_s > v_{sc} \).

The enhanced stability resulting from the viscous wave stabilization uncovered an extended intermediate region with its characteristic nonlinear dependence of the temperature gradient on \( W \). The intermediate region
under the same flow conditions (a/δ, σ) was more extended at 1.5⁰K than at 1.2⁰K. The mutual friction coefficient could be determined from the extended intermediate region data and was found to be comparable to that found in other experiments. The intermediate region in steady flow was not extensive enough to determine mutual friction coefficients with any precision, although the results did not disagree with those in modulated flow.

The behavior of the data in the intermediate region is quite consistent with the Vinen model of 'a tangled mass of vortex line'. The 'dirty' case in which vorticity is initially present and mutual friction losses appear at W₀ always occurred for modulated flow. In steady flow, especially at the beginning of an experiment when the apparatus remains idle for 30 minutes or longer, the flow would sometimes remain in the linear region for heat currents several times their critical value, e.g. see Figure 12. This 'clean' case was never observed in any of the modulated flow measurements, although many were at the beginning of an experiment. One could speculate that the dirty case is more easily established with modulated flow, although these results are hardly definitive.

A surprising feature of this experiment is that the superfluid transition W₀ is affected by the modulation. The behavior of the superfluid transition at 1.5⁰K could
be adequately described in a qualitative fashion by applying the vortex mill model of Glaberson and Donnelly (105). In this model a residual vortex pinned transversely in the flow tube would be acted on by the Magnus force and the vortex image force in the wall as heat flow is initiated. The vortex would remain in a fixed position without removing flow energy for steady flow below the critical velocity \( v_{sc} \). When the steady velocity exceeds \( v_{sc} \), the vortex line will grow and extend down the tube at the expense of the flow.

For the modulated flow the vortex model at an average flow velocity less than \( v_{sc} \) would require an oscillating pinned vortex line. When the peak velocity exceeds the critical velocity, the vortex mill generates additional line which removes energy from the flow. This results in the appearance of a mutual friction at a depressed heat current as may be seen in the data of Figures 18 and 20. As the velocity continues its oscillation, the vortex line will be restored to its sub-critical value and no additional vortex line will be generated, until the cycle repeats. The larger the amplitude of the modulation, the smaller the average heat current required to create the vortex mill. The longer the period of the modulation, the longer the period over which the vortex mill will generate additional line.
As the heat current becomes larger, the additional generation of vorticity due to the peak amplitude of the velocity modulation will no longer be effective and the modulated intermediate region should join that of the steady state.

The vortex mill model does not appear applicable to the enhanced stabilization of the superfluid transition which occurs at 1.2°K, however. It could be possible that the resonant frequency of the vortex is such that the vortex cannot respond to the higher frequency velocity fluctuations at lower temperatures. Accordingly, the resonant frequencies of the normal modes of the vibrations of a columnar vortex were calculated after Thomson (106). The lowest resonant frequencies possible in the given tube dimensions were of the order of one kilohertz, which means that the vortex line should follow the velocity fluctuations at any frequency used in this experiment. Since the dynamics of quantized vorticity in helium II are not adequately understood, nor is the relation of \( v_{so} \) and \( W_0 \) to the Vinen model and mutual friction entirely clear, the variation of \( W_0 \) in modulated flow will have to remain a puzzle. Numerous other models have been attempted to explain this behavior with no success.

Another surprising result was that fully developed turbulent flow could actually be restabilized by the
application of the appropriate modulation frequency and amplitude. The most dramatic demonstration of restabilization is observed when modulation applied to a steady flow in the supercritical region produces a flow in the intermediate region. There is no evidence in the literature that such a restabilization has been attempted in classical flow.
APPENDIX A

SELECTION OF A CRITICAL FLOW SCALING PARAMETER FOR HELIUM II

Using the notation of the Introduction, the two-fluid model hydrodynamic equations generally used (107,108) are

\[
\rho_n \frac{\partial \vec{v}_n}{\partial t} + \rho_n (\vec{v}_n \cdot \nabla) \vec{v}_n = -\frac{\rho_n}{\rho} \nabla p - \rho_s S \nabla T + \eta \nabla^2 \vec{v}_n \tag{A-1}
\]

\[
\rho_s \frac{\partial \vec{v}_s}{\partial t} + \rho_s (\vec{v}_s \cdot \nabla) \vec{v}_s = -\frac{\rho_s}{\rho} \nabla p + \rho_s S \nabla T \tag{A-2}
\]

for \( v_s < v_{s0} \), where both the super and normal fluids are assumed to be incompressible. The incompressibility assumptions result in the approximations \( \nabla \cdot \vec{v}_n \approx 0 \approx \nabla \cdot \vec{v}_s \) which are usually quite satisfactory for thermal counter-flow (42). These approximations allow the irreversible dissipations described by the coefficients of second viscosity to be ignored. Ignoring higher order terms and using the hydrodynamic derivative \( \frac{D\vec{v}}{Dt} = \frac{\partial\vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} \),
equations (A-1) and (A-2) may be written

\[ \rho_n \frac{D \nabla \nabla \nabla n}{D t} = \gamma \nabla ^2 \nabla n - \nabla p_n \]  \quad (A-3)

\[ \rho_s \frac{D \nabla \nabla s}{D t} = - \nabla p_s \]  \quad (A-4)

where

\[ \nabla p_n = \frac{\rho_n}{\rho} \nabla p + \rho_s \nabla T \]  \quad (A-5)

\[ \nabla p_s = \frac{\rho_s}{\rho} \nabla p - \rho_s \nabla T. \]  \quad (A-6)

The Reynolds number is often defined as the ratio of the inertial force term to the viscous frictional term in the equation of motion. Equation (A-3) then gives the Reynolds number \( R_n \) (9)

\[ \frac{\rho_n \frac{D \nabla \nabla n}{D t}}{\gamma \nabla ^2 \nabla n} \rightarrow \frac{\rho_n \langle v_n \rangle d}{\gamma} = R_n \]  \quad (A-7)

where the symbol \( \rightarrow \) indicates the result of dimensional analysis, and where \( \langle v_n \rangle \) is the cross-sectional mean velocity of the normal component in a flow channel of characteristic dimension \( d \), and the Reynolds number \( R_n \) is a scaling parameter of the flow.
When $v_s$ exceeds $v_{sc}$, a mutual friction force $F_{sn}$ is established, as well as other interaction forces $F_s$ and $F_n$ in the super and normal components respectively. The complete hydrodynamic equations for $v_s > v_{sc}$ are then

$$\rho_n \frac{\partial \vec{v}_n}{\partial t} + \rho_n (\vec{v}_n \cdot \nabla) \vec{v}_n = -\frac{\rho_n}{\rho} \vec{\nabla} p - \rho_s \vec{\nabla} T + \gamma \vec{\nabla}^2 \vec{v}_n + \vec{F}_n - \vec{F}_{sn} \tag{A-8}$$

$$\rho_s \frac{\partial \vec{v}_s}{\partial t} + \rho_s (\vec{v}_s \cdot \nabla) \vec{v}_s = -\frac{\rho_s}{\rho} \vec{\nabla} p + \rho_s \vec{\nabla} T - \vec{F}_{sn} - \vec{F}_n \tag{A-9}$$

Ignoring the interaction forces $F_s$ and $F_n$, and using definitions (A-5) and (A-6) equations (A-8) and (A-9) become:

$$\rho_n \frac{D \vec{v}_n}{Dt} = \gamma \vec{\nabla}^2 \vec{v}_n - \vec{\nabla} p_n + \vec{F}_{sn} \tag{A-10}$$

$$\rho_s \frac{D \vec{v}_s}{Dt} = -\vec{\nabla} p_s - \vec{F}_{sn} \tag{A-11}$$

When mutual friction is present, values of $R_{nc}$ in helium II flow are found not only to be much smaller than the expected value $R_{nc} \approx 2000$, but the critical Reynolds number also has a strong temperature dependence. Although
$R_n$ does approximately follow the linear dependence on $d$, it can be concluded that another dimensionless parameter is required to characterize helium II heat flow in the intermediate region.

Experiments which are sensitive to the inertial term in equation (A-3), 'acceleration experiments' (109) and 'periodic boundary layer experiments' (75), can be interpreted to mean that vorticity production in the intermediate region increases the normal fluid inertial mass. Then equation (A-3) may be replaced by

$$
\rho \frac{D \nabla \cdot \rho}{Dt} = \eta \nabla^2 \nabla - \nabla p_n
$$

(A-12)

for $v_s > v_{sc}$. The effective inertial mass of the normal component $\rho_{n}^{*}$ will be $\rho_{n}^{*} = \rho_{n}$ for $v_s < v_{sc}$ and appears to saturate at $\rho_{n}^{*} = \rho_{n} + \rho_{s} = \rho$ for $v_s$ sufficiently greater than $v_{sc}$. This description of the coupling between normal and super component flow motivates the modification of the flow parameter $R_n$ (Equation (A-7)) to the following description.

Guided by the success of Stass, et al. (54) in describing the apparent onset of normal fluid turbulence in some results not involving thermal counterflow, Chase (57) considered the total Reynolds number $R$

$$
R = \frac{\rho <v_n> d}{\eta}
$$

(A-13)
as a scaling parameter for his thermal counterflow data. The replacement of the normal component density \( \rho_n \) by the total fluid density \( \rho \) is physically appealing when the transition to turbulence in the super component proceeds that in the normal component, as is assumed here. The mutual friction force present in the intermediate region presumably entrains the super component and the flow would be better characterized by the total density.

Chase found that \( R_c \) determined from \( \langle v_{no}(T,d) \rangle \) was approximately constant for \( T<1.4^\circ K \), but decreased rapidly as the temperature approached the lambda point. The inability to correlate similar flows with this total Reynolds number was discussed in a later report by Chase (55) in which he proposed another scaling parameter on purely empirical grounds. Chase recognized the importance of the precedent intermediate region in establishing normal component turbulence, and that the correct flow parameter would contain some measure of the super component vorticity. He was thus led to construct dimensionless parameters employing an effective superfluid viscosity \( \gamma_e \), and found that the quantity

\[
R_{ns} = \rho d \left( \frac{\langle v_n \rangle}{\gamma} + \frac{\langle v_s \rangle}{\gamma_e} \right)
\] (A-14)
could be made constant at $W_o$ by choosing $\eta_e$ between 3 and 6 micropoise. The effective superfluid viscosity or 'eddy' viscosity was found to be close to that measured by Brewer and Edwards (53,73,74) and is presumed to be associated with the radial transfer of momentum due to the interaction between vortex lines. The effective viscosity also agrees with that calculated from the Vinen vortex line model (55,72).

Tough (56) has attempted to analyze the data of Chase (57,110) and others (54,74) using a different phenomenological theory. This theory treats the total critical Reynolds number $R$ as being dependent on another dimensionless parameter $g$ which is defined such that it depends on the mutual friction coupling between the two fluids. For a small coupling force the value of $R_c$ assumes its classical value and will decrease with increasing $g$. The theory was unable to account for the behavior of the large relative velocity data of Staas et al. Except for this anomaly the dependence of $R_c(g)$ on $g$ appeared qualitatively the same for all available data. This approach, although successful in obtaining a picture of the intermediate flow region, remains highly phenomenological and does not explain what is physically causing the Reynolds number depression for large coupling.

The more effective models described above which approach the goal of yielding a dimensionless parameter
descriptive of dynamic similarity in helium II flow require an intermediate region wherein vorticity is present in the super component but dissipative turbulence is not present in the normal component. None of the critical Reynolds number formulations described appear characteristic of the onset of turbulence in the normal component and a better indicator of the transition out of the intermediate state must still be found.

Equations (A-3) to (A-6) correctly describe experiments if the flow is assumed to be both steady and parallel. In the intermediate region \( v_s > v_{sc} \), the scattering of normal fluid excitations off quantized vorticity in the superfluid results in a mutual friction force \( F_{sn} \) (62, 68, 69) and an effective normal fluid density \( \rho_n^* \) (75, 109), as discussed above. If it is assumed that \( F_{sn} \) and \( \rho_n^* \) are in some way measures of any unsteady nonparallel components of the velocity fields in the intermediate region, then the numerator in equation (A-7) becomes \( \left( \rho_n^*/\rho_n \right) F_{sn} \). The flow similarity parameter replacing the Reynolds number \( R_n \) of equation (A-7) is then

\[
p^2 = \frac{\left( \rho_n^*/\rho_n \right) F_{sn}}{\eta \nabla^2 v_n}.
\]  
(A-15)
The usual simplified form of $F_{sn}$ (50)

$$F_{sn} = A \rho_s \rho_n (\langle v_s \rangle - \langle v_n \rangle)^3$$  \hspace{1cm} (A-16)

is inserted into equation (A-15) to yield

$$p^2 = \frac{\rho_n^* A \rho_s (\langle v_s \rangle - \langle v_n \rangle)^3}{\eta (\langle v_n \rangle/d^2)}$$ \hspace{1cm} (A-17)

where dimensional analysis (9) similar to that leading to the Reynolds number has been employed. Assuming no fluid sources or sinks to be present, the continuity equation is

$$\rho_n \dot{v}_n + \rho_s \dot{v}_s = 0$$  \hspace{1cm} (A-18)

which may be rewritten

$$\langle v_s \rangle - \langle v_n \rangle = - (\frac{\rho_n}{\rho_s} + 1) \langle v_n \rangle = - \frac{\rho}{\rho_s} \langle v_n \rangle.$$  \hspace{1cm} (A-19)

The entrainment of the superfluid occurs sufficiently quickly that we may assume $\rho_n^* \approx \rho$ near $W=W_c$. With this assumption the substitution of equation (A-19) into (A-17) yields

$$P = \frac{\rho \langle v_n \rangle d}{(\rho_s/\rho)} \left( \frac{A}{\eta} \right)^{\frac{2}{3}}$$ \hspace{1cm} (A-20)
where $P^2$ was defined so that $P$ would be linear in $\langle v_n \rangle$.

The units of $(A/\gamma)^{1/2}$ are just $\gamma^{-1}$ units, and apart from a dimensionless ratio ($\rho_s/\rho$) the equations (A-7) and (A-20) are remarkably similar.

The above assumptions suggest that the transition from the intermediate region to the supercritical region at $W_0$ marks the limit of stability of the normal fluid flow in the intermediate region and may be characterized by the scaling parameter $P$ given by equation (A-20). That is for $W > W(v_{nc})$, fluctuations in $v_n$ are no longer damped by the normal fluid viscosity, but grow and consume energy from the base flow.

The steady flow hydrodynamic equations equivalent to (A-8) and (A-9) are

$$0 = \gamma \nabla^2 \langle v_n \rangle - \nabla P_n + \bar{F}_{sn} - \bar{F}_n$$  \hspace{1cm} (A-21)

$$0 = - \nabla P_s - \bar{F}_{sn} - \bar{F}_s$$ \hspace{1cm} (A-22)

Using equations (A-5) and (A-6) and the London equation these equations may be written

$$\nabla P = \gamma \nabla^2 \langle v_n \rangle - \bar{F}_s - \bar{F}_n$$ \hspace{1cm} (A-23)

$$\rho_s \nabla T = \frac{\rho_s}{\rho} \gamma \nabla^2 \langle v_n \rangle + \bar{F}_{sn} + \frac{\rho_n}{\rho} \bar{F}_s - \frac{\rho_s}{\rho} \bar{F}_n$$ \hspace{1cm} (A-24)
Equations (A-23) and (A-24) have been shown to agree reasonably well with experimental data in the intermediate region (56).

Note that the $F_s$ term in equation (A-23) for the pressure gradient can be interpreted in terms of the effective super component viscosity observed by Brewer and Edwards (53,73,74), and utilized by Chase (55) in equation (A-14). This second dissipative mechanism is characterized by the force $F_s$ acting on $v_s$ only.

Rather convincing experimental evidence for the existence of $F_s$ has recently been given by Rosenshein, et al. (111).

The use of equation (A-20) as the scaling parameter for determining $P_c$ at the critical transition from intermediate flow to supercritical flow has been evaluated using data from this experiment as well as all other relevant data, (53,54,57,72-74,79). The results of plotting $P_c$ vs. temperature are shown in Figure 3 which indicates $P_c \approx 55$ independent of temperature and tube geometry. The viscosity data used in these calculations is that of Tough, et al. (112) and the mutual friction constant values are those of Vinen (72). Below 1.2°K the data for $A$ and $\eta$ were taken from Cornelissen and Kramers (79). The characteristic dimension for the flow channels was taken to be the diameter for circular cross-sections and the hydraulic diameter which is four
times the cross-sectional area divided by the perimeter of the cross section for the rectangular channels of Chase (57) and Vinen (72).

The data in Figure 3 still show a decrease in $P_c$ above 1.7°K as is the case for critical Reynolds numbers. However, the drop is not nearly as large and the general agreement for all these experiments in different tubes is much better than for the various critical Reynolds numbers. Since $P_c$ depends directly on the mutual friction constant, the deviations in behavior of each type of tube may result from a geometry dependence of the mutual friction constant. For example, if the values of $A$ measured by Brewer and Edwards (53, 73, 74) are used to evaluate $P_c$ for their data, $P_c \approx 50$ which is in far greater agreement with the other data in Figure 3. It must also be noted that the self interaction dissipation terms have not been included explicitly in this scaling parameter. The scaling parameter $P_c$ represents a significant improvement over $R_c$ for the analysis of helium II thermal counterflow at the transition to normal component turbulence.

Griffiths (113) has provided an estimate of the mutual friction constant $A$ based on a particular form of coupling which is quite compatible with the above ideas. In the limit of low temperature Griffiths finds

$$A \approx 2.5 \times 10^{-3} \left(\frac{\rho_s}{\rho}\right)^2 / \eta$$

which can be used in equation (A-20).
to yield

\[ P \approx 5 \times 10^{-2} R \quad (A-25) \]

where \( R \) is given by equation (A-13). It follows from Griffiths' model that \( R_c \) will be constant at low temperature in agreement with the observations of Chase \((55,57)\).

The nonthermal counterflow of Staas et al. \((54)\) was arranged to provide \( v_s - v_n \approx 0 \). In this case \( F_{sn} \approx 0 \) even though \( v_s > v_{sc} \). The intermediate flow region is described for this case by the parameter \( R \) (equation (A-13)). The limit of stability of the normal component flow in this intermediate region will then occur at a critical value of \( \langle v_n \rangle \) corresponding to a constant critical value \( R_c \), which was discovered to be true by Staas et al. Because of the nature of the apparatus, they were able to perform a thermal counterflow measurement only at 1.7°K. The value \( v_{nc} \) in this case also agreed with \( R_c \approx 1.2 \times 10^3 \) and prompted Staas et al. to suggest \( R \) as a universal scaling parameter. This is not correct, however, and the single thermal counterflow datum of Staas et al. is plotted in Figure 3 and can be seen to agree with other counterflow results for the scaling parameter \( P \). The equality of \( R_c \) for the nonthermal counterflow results and the one thermal counterflow measurement must be regarded as coincidental.
In view of the range of variables involved and the difficulty in scaling classical fluids, the results of Figure 3 are regarded as indicative that P is a reasonable scaling parameter for the intermediate flow of helium II. The distinction between thermal counterflow and nonthermal counterflow results is also clarified.
APPENDIX B

VELOCITY AMPLITUDE ANALYSIS

Consider the flow tube arrangement shown in Figure 3 with a bath temperature $T_1$ and a pot temperature $T_2$. If an amount of heat $Q_1$ is put into the pot and $Q_0$ escapes the pot through the flow tube, the heat content of the helium in the pot will increase in amount

$$C_1 dT = Q_1 - Q_0$$

where $C_1$ is the heat capacity of the liquid helium II in the pot and $dT$ is the increase in the pot equilibrium temperature $T_2$. Differentiating this equation with respect to time yields

$$C_1 \frac{dT}{dt} = \dot{Q}_1 - \dot{Q}_0$$

(B-1)

where $\dot{Q}_0$ may be written in terms of the flow tube thermal resistance $R_f$

$$\dot{Q}_0 = \frac{T_2 - T_1}{R_f} = \frac{dT}{R_f}$$
Combining the two previous equations

\[ \dot{q}_1 = c_1 \frac{dT}{dt} + \frac{dT}{R_f} \]

and differentiating again with respect to time yields

\[ \frac{\ddot{q}_1}{c_1} = \ddot{T} + \frac{1}{c} \dot{T} \]  
(B-2)

where

\[ c = R_f c_1 \]  
(B-3)

has the units of time.

In a steady state analysis, \( \dot{q}_1 = \text{constant} \) so that \( \ddot{q}_1 = 0 \), and equation (B-2) becomes

\[ q = \dot{T} + \frac{1}{c} \dot{T} \]

which has the solution

\[ T_2(t) = c'e^{-\frac{t}{c}} + c'' \]  
(B-4)

where \( c' \) and \( c'' \) are constants to be evaluated at \( t=0 \)

where \( T_2 = T_1 \) and for \( t \gg 2, \) i.e. \( t \to \infty \). Application of these conditions to equation (B-4) yields the following expressions.
\[ T_2(0) = T_1 = C' + C'' \]

\[ T_2(\infty) = C''. \]

But, as time approaches infinity, the temperature will just approach its equilibrium value \( T_2 \) which may be written as

\[ T_2(\infty) = T_1 + \Delta T \]

where \( \Delta T = T_2 - T_1 \). Combining these last three relations with equation (B-4) provides the steady state relationship with determined constants

\[ T_2(t) = T_1 + \Delta T \left(1 - e^{-t/\tau}\right). \quad (B-5) \]

The time dependent analysis of equation (B-2) begins with the assumption of an oscillating heat current of the form

\[ \dot{Q}_1(t) = \dot{Q} + \delta\dot{Q} \sin \omega t \quad (B-6) \]

where \( \omega \) is the oscillatory frequency, \( \dot{Q} \) is the steady state value of \( \dot{Q}_1 \) in the preceding analysis as \( t \to \infty \), and \( \delta\dot{Q} \) is the oscillating heat current amplitude.
Differentiating equation (B-6)

\[
\frac{\dddot{Q}_1(t)}{C_1} = \frac{w \dot{Q}_1 \cos \omega t}{C_1}
\]

which may be identified with equation (B-2)

\[
\dddot{T} + \frac{1}{\tau} \dot{T} = \frac{w \dot{Q}_1 \cos \omega t}{C_1} \tag{B-7}
\]

which has a complete solution of the form

\[
T_2(t) = C'e^{-t/\tau} + C'' + T_p(t) \tag{B-8}
\]

where \(T_p(t)\) is the particular solution to the inhomogeneous differential equation (B-7). Assuming the solution to the homogeneous equation to be the real part of

\[
T_p(t) = \delta T e^{i\omega t} = \delta T'(t) \tag{B-9}
\]

which may be differentiated with respect to time and substituted into equation (B-7) to yield

\[
\delta T = \frac{\delta Q}{C_1} \frac{\tau}{(1 - \omega^2 \tau)} . \tag{B-10}
\]

For \(\omega \tau >> 1\), the real part of \(T_p\) becomes

\[
T_p(t) \approx -\frac{\delta Q}{\omega C_1} \cos \omega t \tag{B-11}
\]
which may be combined with equations (B-5) and (B-8) to yield the complete time dependent solution for $\omega \tau >> 1$

$$T_2(t) \approx T_1 + \Delta T(1-e^{-t/\tau}) - \frac{\delta Q}{\omega c_1} \cos \omega t$$

For $t >> \tau$, transient effects may be ignored and this equation becomes

$$T_2(t) \approx T_1 + \Delta T - \frac{\delta Q}{\omega c_1} \cos \omega t \quad (B-12)$$

which may be written

$$\Delta T(t) \approx T_2(t) - T_1 = \Delta T - \frac{\delta Q}{\omega c_1} \cos \omega t. \quad (B-13)$$

Substituting equation (B-6) into equation (B-1) yields

$$\dot{Q}_o = -c_1 \dot{T} + (\dot{Q} + \delta Q \sin \omega t) \quad (B-14)$$

and differentiating equation (B-12) with respect to time and substituting for $\dot{T}$ in the above equation provides the result

$$\dot{Q}_o = \dot{Q}. \quad (B-15)$$
This result proves that there is no heat current modulation in the tube \( \dot{Q}_0 \) for \( t \gg t' \), because \( \dot{Q} \) has been defined to be the time independent value of the heat current entering the pot.

The Navier-Stokes equation of motion (A-3) for the normal component in the two fluid model is provided in Appendix A, using the following definition (A-5) of the effective normal component pressure gradient.

\[
\nabla P_n = \frac{\rho_n}{\rho} \nabla P + \rho_s S \nabla T
\]

Assuming the gradients in this equation to be linear and using equation (B-13), the above equation may be written

\[
\Delta P_n(t) \approx \frac{\rho_n}{\rho} \Delta P + \rho_s S (\Delta T - \frac{\delta \dot{Q}}{\omega c_1} \cos \omega t)
= \Delta P_{no} + \delta P_n'(t) \tag{B-16}
\]

where the following definitions have been used

\[
\Delta P_{no} = \frac{\rho_n}{\rho} \Delta P + \rho_s S \Delta T \tag{B-17}
\]

\[
\delta P_n'(t) = -\rho_s S \frac{\delta \dot{Q}}{\omega c_1} \cos \omega t \tag{B-18}
\]

The explicit time dependence of the densities of the two fluids and the entropy has been ignored.
Consideration of equation (B-16) under reversible conditions reveals that a time-dependent normal fluid velocity may be driven by the time-dependent effective normal component pressure $P_n$ which includes the thermomechanical force. The normal component velocity will then have the form

$$v_n(t) = v_{no} + \delta v_n^t(t) \quad \text{(B-19)}$$

where $v_{no} = \dot{Q}_0 / \rho S \Delta T A$ and $A$ is the tube cross-sectional area.

Then for $\omega T \gg 1$, the ratios of the root-mean-square (rms) value of the time dependent term to the steady term of equations (B-16) and (B-19) are proportional

$$\frac{\delta v_n^t(t)}{v_{no}} \propto \frac{\delta P_n^t(t)}{P_{no}} = \frac{-\rho S \frac{\delta Q}{\omega C_1} \cos \omega t}{\rho_n \Delta P + \rho_s S \Delta T}$$

where definitions (B-17) and (B-18) have been used.

Appealing to the London equation $\Delta P = \rho S \Delta T$ and $\rho = \rho_n + \rho_s$ and equations (B-9) and (B-11), the above proportionality may be written

$$\frac{\delta v_n^t(t)}{v_{no}} \propto \frac{\delta P_n^t(t)}{P_{no}} = \frac{-\rho S \frac{\delta Q}{\omega C_1} \cos \omega t}{\rho \Delta T} \propto \frac{\delta T^t(t)}{\Delta T}$$
Using equation (B-3), the definition of the thermal resistance for the steady state $R_f = \frac{\Delta T}{Q_o}$, and taking the rms value of $\cos \omega t$, the above equation may be written

$$\frac{\delta V_n(t)}{V_{no}} \propto \frac{\delta Q}{Q_o} = \frac{\rho_s}{\rho} \frac{1}{\omega \tau} \frac{\delta Q}{Q_o}. \quad (B-20)$$

Then for $\omega \tau >> 1$, $\delta Q/Q_o$ must become very large before any modulation can appear in the flow tube velocity fields. If $\delta Q/Q_o$ is of order unity or smaller, then effectively no velocity field modulation will appear in the tube.

For $\omega \tau << 1$, equations (B-9) and (B-10) provide the real solution

$$T_p(t) \approx \tau \frac{\delta Q}{C_1} \sin \omega t \approx \delta T'(t). \quad (B-21)$$

Combining equations (B-5) and (B-8) with the above equation for $t >> \tau$, the equivalent of equation (B-12) may be written for $\omega \tau << 1$.

$$T_2(t) \approx T_1 + \Delta T + \tau \frac{\delta Q}{C_1} \sin \omega t \quad (B-22)$$

This equation may be written

$$\Delta T(t) = \Delta T + \tau \frac{\delta Q}{C_1} \sin \omega t \quad (B-23)$$
To evaluate the heat current \( \dot{Q}_o \) in the flow tube, equation (B-14) may again be used with the time differentiation of equation (B-22) to yield

\[
\dot{Q}_o = \dot{Q} + \dot{\delta Q}(\sin \omega t - \omega \tau \cos \omega t).
\]

This equation reveals that the oscillating heat current entering the pot, which is given by equation (B-6), appears in the flow tube with a very slight phase shift for \( \omega \tau \ll 1 \). This time dependent behavior may be compared with the steady heat current in the flow tube in equation (B-15) for \( \omega \tau \gg 1 \).

Again assuming gradients to be linear in the flow tube and using equation (B-23), equation (A-5) of Appendix A is modified to yield

\[
\Delta P_n(t) = \frac{\rho_n}{\rho} \Delta P + \rho S \left( \Delta T + \tau \frac{\dot{Q}}{C_1} \sin \omega t \right)
\]

\[
= \Delta P_{no} + \delta P_n(t) \tag{B-24}
\]

where the definition (B-17) holds, but

\[
\delta P_n(t) = \rho S \tau \frac{\dot{Q}}{C_1} \sin \omega t \tag{B-25}
\]
Again assuming the normal component velocity of the form (B-19), the velocity ratio equivalent to (B-20) can be established using equations (B-17) and (B-25) and the steady state thermal resistance

\[ \frac{\delta v_n(t)}{v_{no}} \propto \frac{\rho_s}{\rho} \frac{C}{R \tau C_l} \frac{\delta Q}{Q_o} = \frac{\rho_s}{\rho} \frac{\delta Q}{Q_o}. \]  

(B-26)

Then the velocity ratio will be just proportional to the heat current ratio in the flow tube for \( \omega \tau \ll 1 \), and appreciable modulation of the velocity fields should be possible.

For \( \omega \tau \ll 1 \), equation (B-26) provides the dependence of the velocity amplitude ratio on the temperature and the heat current ratio in the flow tube. In order to determine the frequency dependence of this ratio, the discussion is deferred to Uchida (38). Uchida has solved exactly the classical Navier-Stokes equation (A-3) in cylindrical coordinates for circular cross-section tubes in the laminar flow region. The relationship between the pressure gradient ratio of Uchida \( x_{sn}/x_o \) and the notation used here is

\[ \frac{x_{sn} \sin nt}{x_o} = \frac{\delta p_n(t)}{\Delta P_{no}} = \frac{\rho_s}{\rho} \frac{\delta Q}{Q_o} \sin \omega t \]  

(B-27)

where \( n = \omega \tau \).
The Navier-Stokes equation is solved by assuming a Fourier expansion solution. This assumed solution is substituted into the Navier-Stokes equation and the unknown coefficients are determined. This solution involving Bessel functions is integrated over the radius to provide the following dimensionless mean cross-sectional velocity

\[
\frac{\langle v_n(t) \rangle}{<v_{no}>} = 1 + \sum_{w=1}^{\infty} \frac{x_{sw}}{x_o} \frac{8}{(k_n a)^2} \left[ \frac{2D}{k_n a} \cos \omega t + (1 - \frac{2C}{k_n a}) \sin \omega t \right] + \sum_{w=1}^{\infty} \frac{x_{sw}}{x_o} \frac{8}{(k_n a)^2} \left[ \frac{2D}{k_n a} \sin \omega t - (1 - \frac{2C}{k_n a}) \cos \omega t \right]
\]

(B-28)

with the following definitions.

\[
C = \frac{\text{ber}(k_n a) \text{bei}'(k_n a) - \text{bei}(k_n a) \text{ber}'(k_n a)}{\text{ber}^2(k_n a) + \text{bei}^2(k_n a)}
\]

\[
D = \frac{\text{ber}(k_n a) \text{ber}'(k_n a) + \text{bei}(k_n a) \text{bei}'(k_n a)}{\text{ber}^2(k_n a) + \text{bei}^2(k_n a)}
\]

\[
k_n a = (2)^{\frac{1}{2}} \frac{a}{b_n} = (\rho_n \omega / \eta)^{\frac{1}{2}} a
\]

(B-29)
Reviewing equation (B-27), it may be noted that only the time dependent odd part of the solution (B-28) is of interest, therefore

\[ \frac{\langle \delta v_n'(t) \rangle}{\langle v_{no} \rangle} = \frac{\rho_s}{\rho} \frac{\delta Q}{Q_0} \frac{8}{(k_n a)^2} \left[ \frac{2D}{k_n a} \sin \omega t - (1 - \frac{2C}{k_n a}) \cos \omega t \right]. \]  

(B-30)

Uchida tabulates the coefficients of the trigonometric functions in equation (B-30) as a function of \( k_n a \). This tabulation reveals that the very complex frequency dependence of equation (B-30) may be approximately written for \( k_n a \approx 0.5 \)

\[ \frac{\langle \delta v_n'(t) \rangle}{\langle v_{no} \rangle} \approx \frac{\rho_s}{\rho} \frac{\delta Q}{Q_0} \left[ \frac{8}{(k_n a)^2} \left( \frac{2D}{k_n a} \sin \omega t \right) \right]. \]  

(B-31)

For these values of \( k_n a \) and \( \omega \ll 1 \), the velocity field modulation will be approximately in phase with the thermodynamic driving functions. Again using Uchida's tabulated results, the amplitude correction for the frequency (equation (B-31)) will be only several per cent.

It must be carefully noted that the conditions for use of the Uchida equations is based on the restrictions \( \omega \ll 1 \) and subcritical conditions for both \( v_s \) and \( v_n \).
Because these flow conditions are assumed, the frequency parameter $k_n$ in equation (B-29) is assumed to be correct. The condition $k_n \leq 0.5$ is satisfied for all the data presented in this report.
APPENDIX C

SUPERFLUID CRITICAL VELOCITIES
AND THE LANDAU CRITERION

The Feynman critical velocity is neither the first nor only formulation of critical velocity limit in heat flow. As early as 1941, Landau (44) proposed a critical velocity limit for a pure superfluid at T=0°K. He assumed the superfluid to flow through a tube and that the only possibility for the fluid to lose momentum was through the creation of a phonon or roton excitation in the liquid.

When the flow is considered in the comoving reference frame, the excitation has energy ε and momentum \( \vec{p} \). The energy of the excitation in the stationary reference frame is ε' = ε + pv for parallel flow in the tube. Assuming all the energy and momentum of the superfluid flow is converted into the excitation, the total energy of the flow will be

\[
E = (\epsilon + pv) + \frac{1}{2}mv^2
\]  

(C-1)

where \( \frac{1}{2}mv^2 \) is just the initial kinetic energy of the flowing superfluid. Then (ε+pv) is just the change in
energy due to the elementary excitation which must be negative because the energy of the flowing liquid must decrease.

Therefore, \( v > -\varepsilon/p \) must hold to produce an excitation. The flow will remain superfluid and cannot decay as long as the velocity remains below the critical value

\[
v_c = \left| \frac{\varepsilon}{p} \right|.
\]  

(C-2)

Calculations have been made (114) for the Landau critical velocity, and for rotons the critical velocity is \( 6 \times 10^3 \) (cm/sec), while phonon excitation requires a critical velocity of \( 2.4 \times 10^4 \) (cm/sec). These velocities are extremely high for heat flow experiments, and, as Landau speculates, lower critical velocities due to other mechanisms may dominate the flow. For example the Feynman velocity \( v_{sc} \) is several orders of magnitude less.

In an extension of the Landau critical velocity theory, Ginsburg (115) suggested that other types of excitations could exist. One example of such an excitation behaves like an ideal gas particle with associated kinetic energy, and could have a value of \( \varepsilon/p \) less than phonons and rotons. The critical velocity associated with this sort of excitation was of the appropriate order of magnitude to explain the experimental results, but
differs in its dependence on the channel dimension d. The dependence discovered by Ginsburg is that the product $(v_{so}d)$ is a constant, which differs from the later empirical rule discussed below. In any event the excitations required for Ginsburg's theory have never been discovered.

The Landau critical velocity is far too great to explain experimental observations, in terms of phonon or roton excitations, however, the creation of vortices in the superfluid could provide the proper sort of excitation energy in equation (C-2). Feynman (59) has suggested that the only way to slow down the superfluid flow is for small parts of the fluid to accept energy from the flow to form irrotational flow. The excitation energy for the formation of these vortices would be much less than that required to slow down the entire flow, so more reasonable critical velocities would be obtained from equation (C-2).

The Feynman calculation of the critical superfluid velocity for the initial production of a quantized vortex used the classical vortex energy spectrum in equation (C-2) to yield equation (1-26). Feynman speculated that very close to this critical velocity, the resistance of the flow will be irregular and fluctuations will appear. However, he was unable to quantify the flow resistance as a function of vortex production. Equation (1-26) does not include dependence on the structure of the vortex core,
on interactions between vortices, or flow interactions between vortices and flow boundaries.

Atkins (114) has also attempted the Feynman approach to the Landau criterion using the model of a closed ring of vortex line in a flow tube and ignoring boundary impulse effects on the ring. Again calculating the energy spectrum of this vortex, Atkins obtained an expression for the critical velocity from the Landau criterion which was twice the value obtained by Feynman in equation (1-26). Wilks (116) corrects Atkin's expression for the vortex ring expression.

To account for boundary effects, Fineman and Chase (67) have calculated the energy spectrum for a bounded classical vortex ring with an empty core. Substitution of the results into equation (C-2) yields a critical velocity which approaches zero as the vortex diameter approaches the diameter of the flow channel. In other words a vortex could be created at the walls for infinitesimal superfluid flow velocities, so they concluded that the vortex formation responsible for the critical superfluid velocity could not take place at the channel walls.

Other workers have analyzed the problem of the formation of a bounded vortex near the wall and found that the energy spectrum is sensitive to the structure assumed for the vortex core (117) or that the assumption of
boundary independent impulse in the above analyses may not hold near solid boundaries (118). The lack of a unique impulse definition for equation (C-2) requires that it be applied with considerable care to vortex ring excitations. In the case of a core structure dependent energy, it was found that the ring energy does decrease but could remain finite at the wall so that equation (1-26) still applied approximately except very near the boundary.

Vinen (119,120) has discussed the difficulty in establishing a mechanism for the creation of vortex line in channel flow. While it is possible to devise numerous configurations of vortex lines that develop in a flow channel and satisfy the Landau criterion, the formation of the vortices in wide channels is quite difficult to rationalize. The wide channels will have low critical velocities for the superfluid, and the necessary vortex configuration that must be created would have to extend over a large volume, e.g. a vortex ring would have a diameter of nearly a millimeter for the lowest experimental velocities. The quantum mechanical transition required to generate this large vortex ring becomes increasingly improbable with volume. Then there may exist a kind of potential barrier to the formation of the initial vortex in superfluid flow. Since the critical velocities predicted for the very large
channels are often larger than experimental values, Vinen proposed that small sections of vortex line are always present in the flow channel. These lines are perhaps attached to protuberances from the nonideal channel walls and are metastable. These remanent lengths of line may therefore act as nuclei for the growth of additional line when the superfluid flow exceeds its critical velocity and may explain very low critical velocities. Vinen's (61) measurements of apparent vorticity adherence to a fiber for long periods of time, even when the fiber was shaken, support his views. The conclusion from Vinen's work is that additional superfluid turbulence develops from the expansion of existing vortex line in the channel instead of the creation of the initial vortex at a higher energy and critical velocity.

In an extension of Vinen's conjectures Glaberson and Donnelly (105) have analyzed the effect of tube flow on a metastable vortex pinned to rough spots on the tube wall. The superfluid flow results in a Magnus force on the pinned vortex line which causes the initially straight line to curve out into the center of the tube until the total force on any element of the line at the prescribed flow velocity is zero. The restoring force acting on the line is due to the attractive image of the vortex in the tube wall.
As the flow velocity continues to increase, the vortex line becomes a semicircle with endpoints at the same pinning sites on the tube wall. At this flow condition the critical velocity for a vortex ring is attained which is equivalent to equation (1-26). The critical velocity in this case is the local velocity induced in one element of the vortex line by all the other elements of the line and is equal to the flow velocity for balanced forces.

Once this critical velocity is exceeded, the Magnus force will be greater than the restoring force on the line. The line will then grow steadily in length from its semicircular shape and presumably will extend down the tube with the superfluid flow, eventually developing into a Vinen vortex tangle.

When the flow velocity is subcritical, no flow energy is required to maintain the bowing vortex line, but when the velocity exceeds the critical value, the tangling vortex line will grow at the expense of the flow energy causing pressure and temperature differences between the channel ends. This model for the onset of superfluid turbulence is most satisfying in comparison with the development of turbulence in classical flow. Glaberson and Donnelly also propose that a vortex ring 'mill' is established by this model in that the tangled vortex line can join together to form rings as the line threads down the tube.
Fetter (121) and Amit and Gross (122) have derived the energy of vortex pairs in a tube, and obtain an inverse dependence of the critical velocity on the flow tube diameters without the logarithemic term in $d$ of Feynman's equation (1-26). Numerous additional approaches have been taken, but all develop the same form of the dependence of the critical velocity on the inverse of the flow tube diameter. This behavior fits the data approximately for $d>10^{-3}\text{cm}$. Other models have been advanced which yield a $d^{-\frac{1}{3}}$ dependence for the critical velocity, but none describe the experimental data over the entire range of measurements.

The results of numerous experimental superfluid critical velocities have been compiled (123-127) including many of the results already described, as well as film flow, rotation, and oscillation experiments. These data all seem to adhere to the empirical law ($v_{so} d^{-\frac{1}{3}}=\text{constant}$) over a remarkably wide range of flow dimensions. This empirical law has been successfully used from channel widths of $10^{-4}\text{cm}$ to $1\text{cm}$ at intermediate hydrodynamic region temperatures. None of the theories available yet describe this empirical dependence of the critical velocity on the channel dimension. Craig (128) has developed an expression which fits the data very well, but his expression requires a highly arbitrary choice of constants and is somewhat unphysical.
Another approach to the critical velocity in the superfluid near the lambda point has been taken by Langer and others (129,130). Langer and Fisher have proposed a thermal fluctuation model of the intrinsic critical-velocities found in superfluid gyroscope experiments (131). These experiments revealed that an intrinsic critical velocity was present which was independent of the size and shape of the container in which pure superfluid flow was constrained. A characteristic temperature dependence of the critical velocity near the lambda point was also discovered experimentally.

In their theory Langer and Fisher assert that a nonzero superfluid flow must be regarded as a metastable state with properties analogous to those of a super-saturated vapor. For a large sample of the metastable fluid phase there will be a finite probability for nucleation of the stable or zero velocity phase. For low velocities the transition probability is so small that transitions are not observable, and the system will appear stable. The critical velocity condition will apply when the transition probability becomes appreciable compared with experimental times. The nucleation sites in this model are vortex rings which have the topological properties required by analysis of the superfluid order parameter to nucleate a transition from uniform superflow to a state of lower velocity. The rate of fluctuation
varies as a Boltzmann exponential factor involving the critical vortex ring energy for growth. Above this energy the ring lowers its free energy by expanding to the tube walls where it annihilates at the wall with a resultant decrease of the flow energy in the same amount as annihilated.

The Langer-Fisher theory successfully yields the temperature dependence and lack of size dependence of the critical velocity for the Clow and Reppy experiments. In addition, Notarys (132) and Liebenberg (133) have modified the theory to apply to pressure-driven superfluid flow and thermally driven superfluid film flow, respectively. The experimental results of these two authors are in excellent agreement with the Langer-Fisher thermal fluctuation description using vortex rings for the thermal nucleations, even at temperatures quite far from the lambda point. It must be noted, however, that the Langer-Fisher critical velocities are nearly an order of magnitude greater than the Feynman critical velocity.

The subject of the critical flow velocity $v_{sc}$ of the super component has not been exhausted by any means. The preceding discussion, though lengthy, leads us to the salient fact that there is no mechanism to which the critical superfluid velocity may be universally attributed. The empirical dependence of the critical velocity on the
flow channel dimension has never been obtained from the myriad of suggested physical models. The requirements for the development of the first quantized vortex from the ideal superfluid state are still in question for channels of practical dimensions.

The best model for the generation of the superfluid critical velocity seems to be that of Glaberson and Donnelly based on the Landau criterion. The model depends on remanent pinned vorticity in the flow tube, which is reasonable for practical experiments. The development of superfluid turbulence occurs in a natural way, and the extension of the tangled vortex line from the pinning sites on the tube wall as the line absorbs energy from the flow is quite analogous with classical turbulence. The generation of the tangled vortex line fits into the Vinen picture of mutual friction. Beyond the superfluid critical velocity the mutual friction increases rapidly with relative velocity due to the scatter of thermal excitations off the extending vortex line. If no vorticity is present in the initial flow, then the Langer-Fisher intrinsic critical velocity theory should become appropriate for very large velocities.
APPENDIX D

AC RESISTANCE BRIDGE

The resistance of the carbon thermometers must be precisely measured with very low internal heat dissipation. The AC resistance bridge is uniquely suited to the restrictions of low temperature resistance measurements. The rather large thermal emf that must be accounted for in DC measurements is eliminated by the AC bridge with equal or better precision than DC methods. If a low frequency sampling current is used for the AC measurement, there will be little electrical interference with other experimental measurements. Reports of AC methods for low temperature thermometry have appeared in the recent literature (134,135). Several books provide details of bridge design and operation (136-138).

The AC bridge circuit used for resistance measurements in these experiments is shown in Figure 26. The integrated circuits used in the bridge are all Fairchild Model μA709C operational amplifiers which exhibit good temperature stability. An oscillator input of 10 volts rms or less is applied at the attenuator A1 which has a
Figure 26

AC Resistance Bridge Circuit Diagram
maximum gain of one. The attenuated signal is then impressed on the bridge circuit. The signals at the other two junctions of the bridge are fed separately into operational amplifiers A2 and A3. These two operational amplifiers act as infinite input impedance buffers of unity gain. The output of A3 is connected to the inverting input of A4. The common mode voltage (139) resulting from the bridge ground connection is nulled by adjusting the common mode control, as discussed below. The output of A2 and the inverted, corrected output of A3 are summed and amplified by A5 to provide the bridge null output signal. The gain of A5 is controlled from the front panel of the bridge and is normally full on at the null balance. All these operational amplifiers require an external power source of \( \pm 15 \) volts DC.

The bridge is designed to provide a 3-wire connection (98) to the unknown to minimize electrical lead wire effects. The third wire effectively cancels any thermally induced changes in lead wires of the same type and length.

All fixed resistors used in the bridge are Corning No. NA60 metal-oxide film resistors which have a temperature coefficient of 100 (ppm/°C). The fixed 1:1 ratio bridge arms are matched to within 0.0025%. The
calibration resistance $R_c$ and capacitance $C_c$ are nominally 47.5 kohm and 470 pf, respectively.

The variable arm of the bridge consists of a gang of decade resistors $R_v$ which are Electro Scientific Industries Series DS6 and a variable capacitor for quadrature balancing. The decade resistors allow unknown resistances $R_x$ of 0.1-10 megohm to be balanced. The variable capacitor in this arm can be switched in 100 pf steps while a second control provides a continuous 0-100 pf variation of $C_v$ in order to balance $C_x$ in the unknown.

The common mode adjustment turned out to be quite sensitive. Initially, a Spectrol Model 510 precision potentiometer was used to control the common mode operational amplifier $A_4$. The sensitivity of this 0-1 kohm potentiometer was not sufficient to control the common mode voltage, so a Spectrol Model 860 potentiometer was added in series with the Model 510 to provide better fine adjustment control. The Model 860 has resistance range of 0-25 ohm and a temperature coefficient of 1000 (ppm/°C) while the temperature coefficient of the Model 510 is 20 (ppm/°C).

The common mode control is adjusted by balancing the bridge in the normal-calibrate positions of the arms switches. The reverse-calibrate positions are then selected and the bridge is again balanced. If the two
measurements do not agree, the common mode control must be adjusted until the bridge balance displays the average of the two readings. The procedure should then be repeated until the two readings agree.

The bridge is capable of providing 1:5000 precision for resistances in the range of carbon resistors at low temperature. The power dissipated in the carbon resistor due to the bridge current is typically $10^{-7}$ watt in attenuator position number 4 at an oscillator voltage of 1 volt rms. Lower bridge power attenuator settings did not result in reliable resistance readings, probably because of excessive noise. The day-to-day drift of the bridge was 1:10,000 although a hard tap on the instrument rack could cause the very sensitive common mode adjustment to shift considerably. The common mode adjustment should not be attempted until the bridge has warmed for about 30 minutes.
APPENDIX E

HEATER POWER SUPPLY

A power supply was required to provide both a steady voltage $V_{DC}$ and an alternating voltage $V_{AC}(t)$ to a heater. If a single heater of resistance $R_H$ were used and the form of $V_{AC}(t)$ is

$$V_{AC}(t) = V_0 \sin(\omega t/2) \quad (E-1)$$

then the instantaneous power at the heater would be

$$\dot{Q}_1(t) = \frac{(V_{DC} + V_{AC}(t))^2}{R_H}$$

$$= \frac{V_{DC}^2}{R_H} + 2 \frac{V_{DC}V_0}{R_H} \sin \left(\frac{1}{2} \omega t\right) + \frac{V_0^2}{R_H} \sin^2 \left(\frac{1}{2} \omega t\right)$$

$$= \frac{V_{DC}^2}{R_H} + \frac{1}{2} \frac{V_0^2}{R_H} + 2 \frac{V_{DC}V_0}{R_H} \sin \left(\frac{1}{2} \omega t\right) - \frac{1}{2} \frac{V_0^2}{R_H} \cos \omega t \quad (E-2)$$

where a trigonometric identity has been used. This result was also reported for a thermophone by Arnold and Crandall (140). The superposition of the two frequency terms in equation (E-2) results in a complex waveform.
In order to develop a pure sinusoidal variation of the power or heat current, two separate heaters must be used. The total instantaneous power generated by $V_{DC}$ applied across a resistor $R_{DC}$ and $V_{AC}(t)$ applied across resistor $R_{AC}$ is

$$
\dot{Q}_1(t) = \frac{V_{DC}^2}{R_{DC}} + \frac{V_{0}^2}{R_{AC}} \sin^2(\frac{1}{2}\omega t)
$$

$$
= \frac{V_{DC}^2}{R_{DC}} + \frac{1}{2} \frac{V_{0}^2}{R_{AC}} + \frac{1}{2} \frac{V_{0}^2}{R_{AC}} \sin(\omega t - \frac{1}{2}\pi)
$$

$$
= \dot{Q} + \delta\dot{Q} \sin(\omega t - \frac{1}{2}\pi)
$$

(E-3)

where $\dot{Q} = (\frac{V_{DC}^2}{R_{DC}} + \frac{1}{2}(\frac{V_{0}^2}{R_{AC}}))$ and $\delta\dot{Q} = \frac{1}{2}V_{0}^2/R_{AC}$.

This equation is identical to the assumed form of the oscillating heat current in equation (B-6) of Appendix B apart from a phase factor. The double frequency term has been removed from equation (E-3) and it should be noted that $\omega$ is the frequency of the heat current oscillation in the fluid which is twice the oscillator frequency that provides the voltage.

The root mean squared (rms) value of the power in equation (E-3) is
\[
\frac{Q_1}{Q_1} = \frac{V_{DC}^2}{R_{DC}} + \frac{V_O^2}{R_{AC}} (1 + (2)^{-\frac{1}{3}})
\]

\[
= \frac{V_{DC}^2}{R_{DC}} + 0.854 \frac{V_O^2}{R_{AC}}
\]

\[
= \frac{V_{DC}^2}{R_{DC}} + 1.707 \frac{V_{AC}^2}{R_{AC}}
\]

(E-4)

where \(V_{AC}\) is the rms value of \(V_{AC}(t)\), i.e. \(V_O = 2^{\frac{3}{4}}V_{AC}\).

If the initial rms power is \(Q_1 = (V'_{DC})^2/R_{DC}\), where \(V_{DC}\) is the initial steady voltage and the alternating voltage is zero, then equation (E-4) may be written

\[
\frac{(V'_{DC})^2}{R_{DC}} = \frac{V_{DC}^2}{R_{DC}} + 1.707 \frac{V_{AC}^2}{R_{AC}}
\]

(E-5)

which can be solved for \(V'_{DC}\) to yield

\[
V'_{DC} = (V_{DC}^2 + 1.707 \frac{R_{DC}}{R_{AC}} V_{AC}^2)^{\frac{1}{2}}
\]

(E-6)

In these experiments the resistance ratio in equation (E-6) is usually one and may be ignored. The heater resistances should also include the lead wire resistance which can be several per cent of the heater resistance. Since the lead wire resistances for both heaters are practically identical, these effects may also be ignored.
The objective of the power supply design is to hold expression (E-6) constant for a preselected $V'_{DC}$ while $V_{AC}$ is varied from zero to the maximum value consistent with the equality. A feasible method of maintaining $V'_{DC}$ at the value of the square root is to use integrated circuit function modules combined with a feedback loop.

An analog circuit using integrated circuit modules was designed to provide constant total power to the two heaters independent of the setting of the AC heater power. The circuit depicted in Figure 27 depends on the use of two Philbrick/Nexus Model 4352 Average-rms-Vector Modules. (a) These modules may be connected to yield the average value or the root mean squared value of one input signal or the square root of the sum of the squares of two input signals.

The circuit must provide constant total power as the AC level is adjusted. This is accomplished by maintaining an expression of the form

$$ (X^2 + B^2Y^2)^{\frac{1}{2}} = \text{Constant} \quad \text{(E-7)} $$

where the constant is the initial DC voltage setting before any AC voltage is turned on.

---

(a) Philbrick/Nexus Research, Dedham, Mass.
Figure 27

Heater Power Supply Circuit Diagram
One of the 4352 modules A2 is used to convert the AC signal to its equivalent rms value Y. This rms signal is then fed into A1 the other 4352 module after passing through an appropriate multiplier amplifier K4. The multiplier is a Philbrick/Nexus Model 1017 operational amplifier which provides the product (BY) at its output. The multiplier factor \( B^2 = 1.707 \left( \frac{R_{\text{DC}}}{R_{\text{AC}}} \right) \) is adjustable to allow for different heater resistance ratios.

Module A1 provides the square root of the sum of the squares at its output where the product (BY) is one of the two inputs. The output of this module is then fed back into the X input of the module effecting a feedback control loop. The resultant DC output which drives the DC heater is then modified by the level of AC input so that the square root of the sum of the squares (equations (E-6) and (E-7)) is maintained at the constant initial setting as the AC voltages are varied.

There is a problem, however, in that the AC voltage may be increased such that its product with the preset multiplier B alone causes the square root of the sum of the squares to exceed the initial DC level. The AC voltage at which this occurs could be determined by a time-consuming and inconvenient calculation for various initial DC voltage settings, but a transistorized warning light detector has been included in the circuit to signal this eventuality.
The warning light circuit works as follows. The output of the 4352 module A1 is positive and is connected to the inverting input of operational amplifier K3 shown in the figure. The non-inverting input is connected to the +15 volt supply through the DC voltage control. A positive DC level is preset by this control to correspond to the desired total or DC heater power. When \( (x^2 + y^2)^{\frac{1}{2}} \) becomes greater than this input, as it does when the AC heater power exceeds the preset initial DC heater power, the operational amplifier inputs become unequal. The large inverting input results in a large negative output. This negative DC voltage now turns transistor Q2 off because the diode prevents current flow, and transistor Q1 turns on which causes the warning light to operate.

Limitations of the device are mainly the limitations of the 4352 modules. The modules may not exceed 10 volts rms at input or output terminals. This, of course, requires that the root square sum function must have both inputs restricted to 10 volts or less because the output is the square root of the sum of the square voltages. The rms function is limited to frequencies greater than 10Hz, and the output accuracy of the 4352 modules is \( \pm 3\% \).

The circuit works quite well at relatively large voltages, i.e. greater than 100mv. Fluctuations begin
to appear in the output when the DC voltage is less than 100mv, but the device is still usable down to about 10mv. The multiplier retains its setting within several per cent down to 10mv where the multiplier may be in error by as much as 10%. One disturbing feature of the circuit is that the warning light will turn on at very low DC voltages (<5mv). This in turn locks the DC output at a slightly negative voltage due to the enormous power drain of the warning light. The situation cannot be corrected unless the power is switched off, or the DC voltage is set at a high level (>100mv). Extremely low DC output voltages are to be avoided.

The power supply is provided with accurate metering circuits for both $V_{DC}$ and $V_{AC}$ at the output to the respective heaters. Six full scale ranges from 0.01 to 10 volts are provided. The heater output voltages are each controlled by separate 10-turn potentiometers with calibrated dials in two voltage ranges. Other controls include another 10-turn potentiometer with calibrated dial for setting the multiplier between zero and ten. A bypass switch provides for setting a calibrated multiplier of 1.00 so that the output power can be easily checked against the preset DC power without complicated mathematics. The multiplier may also be checked by measuring the multiplier amplifier (K4) output and the rms module (A2) output.
Division of the multiplier output by the rms output yields the multiplier B. A switch and output terminal are provided at the rear of the power supply for these measurements.

This so-called power supply is really only a power control device. A separate oscillator input is required to provide the voltage that powers both heaters. The oscillator must have a variable frequency and should never be set at an output of greater than 10 volts rms. The operational amplifiers and function modules also require an external power source capable of delivering ±15 volts DC.
APPENDIX F

DATA TABULATION

The heat flow data collected from flow tubes C, B, and A are listed in Tables 4, 5, and 6, respectively. The Run Number is a chronological listing of the data collection at a given nominal temperature and refers to an identical listing in the original data books.

The temperature $T$ is the bath temperature. If a temperature drift was present, $T$ was taken to be the interpolated temperature at $Q_0$. The ratio $\sigma$ is the amplitude of the oscillating heat current $\delta Q$ divided by the average total heat current defined at $Q=Q'_0$ where $Q'_0$ is the steady state transition heat current. The oscillator frequency is $f_{osc}$ which drives the viscous wave modulation of penetration depth $\delta$ where the total fluid density has been used. The radii $a$ of the tubes are $5.145 \times 10^{-3}$ cm for tube C and $0.0285$ cm for tubes A and B, as determined from the thermal resistance to heat flow at zero heat flow $\Delta T/Q | Q=0$. The smallest heat current at which the intermediate region can be detected is $Q_0$, and

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the onset of turbulence in the normal fluid is defined by $Q_0$. The critical flow parameter $P_c$ has been justified and defined in Appendix A and may be calculated from equation (1-27) when the temperature and $Q_0$ are known.

Nonexistent data in the table for the thermal resistance at zero heat current simply means the thermal resistance was not calculated and plotted. Lack of data for $Q_0$ usually indicates that insufficient data were available or that the data scatter was too great to evaluate $Q_0$. 
### TABLE 4

**TUBE C DATA**

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<th>Run No.</th>
<th>( T(°K) )</th>
<th>( \sigma(°/o) )</th>
<th>( f_{osc}(Hz) )</th>
<th>( \frac{\Delta T}{Q} )</th>
<th>( \dot{Q} ) (( °K )/W)</th>
<th>( \dot{Q}_o(\mu W) )</th>
<th>( \dot{Q}_o(\mu W) )</th>
<th>( P_c )</th>
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<td>•</td>
<td>14</td>
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<td>$f_{osc.}(HZ)$</td>
<td>$\Delta T$</td>
<td>$\dot{\sigma}$</td>
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<td>$q_o (\mu W)$</td>
<td>$\dot{q}_c (\mu W)$</td>
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REFERENCES

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