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FOR EIGHTH GRADERS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

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The Ohio State University
1971

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The cooperation of the Mercer School District and the Lakeview School District in providing the facilities for this study is appreciated. My sincere gratitude goes to the teachers, Mary Wiese and David G. Cook, for the cooperative manner in which they performed their contributing tasks.

Finally, this manuscript is dedicated to my lovely wife and children who have sacrificed so much during the many years of my education. It is now my goal to repay them somehow for the postponed vacations, cancelled activities, and the many exhaustive hours of typing which are so very much a part of this endeavor.
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CHAPTER I
INTRODUCTION

What effects do organized laboratory experiments have upon the motivation and achievement of eighth grade mathematics students? The number of teachers and administrators seeking an answer to this question has substantially increased over the past few years. The recent interest in mathematics laboratories is indicated by the number of related articles appearing in contemporary mathematics and educational journals.

The teacher acceptance of the recent attempts to revitalize this almost forgotten teaching technique will greatly depend upon the results of their attempts to interest the students in laboratory experiments. These attempts, in turn, will be influenced to some extent by recent reports of educational research with laboratory methods. Mathematics laboratories are in a crucial stage of development.

Because of the many interpretations of the meaning of a mathematics laboratory, numerous questions arise in the minds of interested teachers. These questions need to be answered to a considerable degree of satisfaction before a critical, reflective teacher will seriously attempt to utilize mathematics laboratories in his classes.
As answers are sought for the obvious questions concerning general methods of approach, other questions, more pertinent to the local problems, become the focus of attention.

Listed below are some of the questions which this researcher considered during the time of the study.

1. Are mathematics laboratories effective in promoting achievement?

2. Are mathematics laboratories effective in motivating students to a greater interest in mathematics?

3. Will average and above average students be as interested as slower learners in the physical applications that arise in the laboratory?

4. Are mathematics laboratories effective in changing the attitudes of students toward mathematics?

5. Can experiments be enjoyable and educational simultaneously?

6. Is it relatively easy to devise experiments to teach most mathematical concepts?

7. Is the success of the laboratory experiment directly proportional to the elaborateness of its design? Is extensive, commercially produced equipment essential?

8. Are physical applications of mathematical concepts better presented before or after a logical, abstract discussion of the concept?

9. Within the limitations of the available school plants and student populations is it possible to design a study which will provide some answers to these questions?
10. How must the teachers prepare for this technique of presentation? Can every teacher be expected to use the laboratory approach effectively?

**Purpose and Design of the Study**

This study was designed to specifically answer two of the above questions related to mathematics laboratories. Does their use at the eighth grade level result in greater mathematical achievement? Do eighth grade students prefer mathematics laboratories to other methods of presentation?

Two experienced teachers of eighth grade mathematics in distinct school districts of Mercer County volunteered to participate in an organized attempt to study the merits of the laboratory methods of teaching. Six classes of eighth grade students, three from each school, were arbitrarily selected from those taking the usual eighth grade mathematics program. These classes constituted the student sample for the research. In both schools the administrators had class enrollments pre-established by a homogeneous grouping procedure based on overall achievement. Therefore, the use of intact groups was necessitated.

The first six weeks of the 1970-71 school year were used as an acclimation and refinement period. Even though the teachers and the author held weekly organizational meetings during the spring and summer months, more time was required by the teachers and their students before the research study could be satisfactorily refined. Following this initial, introductory period, each participating class was randomly assigned a laboratory type approach to be used for one unit of
mathematics over a six weeks time period. Within each school one class used a pre-lab approach calling for the laboratory experiments before the class discussed the related concepts in regular class recitation. A second class in each school used a post-lab approach in which concept related experiments followed class recitations. A third class in each school was assigned the standard approach commonly used in these schools in former years. This was named the no-lab approach.

The Hypotheses to be Tested

The null hypothesis to be statistically tested in six distinct cases is that there is no significant difference in achievement among eighth grade mathematics classes being taught either without laboratory techniques or with one of the two direct laboratory approaches.

The hypothesis to be tested by questionnaire is that the majority of eighth grade students, regardless of their ability levels, prefer the use of some laboratory experiments in their mathematics classroom; that is, experiments generate an improved climate for the motivation to learn mathematics.

The relative effects of the three approaches in each of the schools was compared through analysis of covariance on the post-test achievement scores. Covariates were employed in an attempt to equate differences in intelligence and prior knowledge of the mathematical concepts.

To compensate for the possible bias relative to the adaptability of experiments to the particular unit and to the novelty of the
laboratory method for students and teachers, research data were similar­ly collected for two additional six weeks units of study. Every class had the opportunity to experience all three laboratory approaches since the laboratory assignments passed through a complete rotation during the three six weeks periods of the program. That is, every student in the sample experienced one six weeks unit of mathematics with each type of laboratory.

To further compensate for the variable of teaching techniques, the data from each of the schools were analyzed separately. Thus, for all practical purposes, six comparisons of laboratory method were made by analysis of covariance.

Periodic attempts were made throughout the eighteen weeks to determine the students' reactions to the methods of presentation. Experiment rating forms were used but with only partial success; the major tool for gauging the students' preferences and interests was a questionnaire completed by each student after his class had experienced all three types of laboratory methods.

Need for the Study

The year a junior high school student spends in the eighth grade is a most crucial period in his search for mathematical maturity. This year is not only a time of learning and assimilating, but it is a time of decision. The eighth grade student is in the final steps of selecting the curriculum he will pursue for at least the next four years of school. His choice at this time will directly influence every aspect of his future.
The nature of the applied logic of a thirteen year old is at times rather shallow. That is, it is often the case that the decisions he makes are more influenced by recent experiences than by his totality of experience. Thus, the curricular choices upon which his future plans will stand or fall are largely based upon his experiences in the eighth grade. For many students it is a make or break year in the study of mathematics.

Psychologists agree that the life of an adolescent is very complex. This confusing complexity often negatively influences his academic progress. The vast majority of junior high school students underachieve relative to their mathematical potentialities. It is a task of tremendous magnitude for the eighth grade teacher to awaken these students to the reality that mathematics plays an important, continuing role in their lives.

Primary objectives of the junior high school mathematics program are to provide the students with the opportunities for relating and coordinating their mathematical knowledge learned in the elementary grades, to motivate the students through well conceived, challenging experiences to discover contemporary applications of mathematics, to prepare the students with a useable knowledge of the basic mathematical concepts necessary for future endeavors, and to enlighten the students on the magnitude of the contributory role assigned to mathematics in tomorrow's world.

Many students in the eighth year of school occasionally find it difficult to concentrate on mathematics. More often than teachers like to admit, the eighth grade syllabus closely resembles that of the
seventh grade. In addition, the sixth and seventh grade topics in mathematics often appear as carbon copies. This duplication, or possible triplication, of presented materials tends to diminish the motivational aspect required for measurable progress in mathematical comprehension.

A large percentage of teachers of eighth grade mathematics quickly become frustrated by the lack of desire their students display shortly after the school year has begun. As a consequence the classroom atmosphere degenerates to a pedagogical routine through which the students and their teacher must suffer, each hoping for long vacations punctuated with only brief periods of academia.

Fortunately, most of the students who succeed in enduring this painful process, while sufficiently meeting the sometimes meager standards of the bored teacher, are required to spend no more than one year in this situation before they are graduated to an often more interesting form of mathematics called algebra.

Unfortunately, each year the eighth grade teacher is allowed only three months to recuperate before being subjected to another group of unmotivated eighth graders. After a few years of being assigned to this comparatively unproductive role, the teacher looks for greener pastures in algebra or geometry and willingly steps aside for some unsuspecting neophyte who will eagerly attempt to create an atmosphere conducive to learning and appreciating mathematics. The cycle starts again. Is there any way to break this cycle?
Get the students involved, is a cry heard from all corners of the pedagogical world today. Let them see for themselves, let them use trial and error, let them make some educated guesses, let them refine their thinking through the experience of participation.

As Jo Phillips wrote in "Putting the Tic in Arithme,"

Gone forever are the days when the mathematics classrooms are a book, a pencil, and a ream of paper. Multisensory aids to learning, often things commonly found in the ordinary classroom environment, are essential to a successful contemporary program. The contemporary classroom is in a literal sense a laboratory. ... Until a child can make some simple classifications and see some simple relations, he is not ready to learn about numbers (28:216).

The Des Moines Public Schools use mathematics laboratories in the L.A.M.P. (Low Achievement Motivational Project). Some of their ideas are expressed as follows:

Primarily, a mathematics laboratory is a state of mind. It is characterized by a questioning atmosphere and a continuous involvement with problem solving situations. Emphasis is placed upon discovery resulting from student experimentation. A teacher acts as a catalyst in the activity between students and knowledge (21).

Unfortunately, ideal situations exist only in the minds of men, while reality provides the dented and battle-scarred settings in which educational research must take place.

In recent years the commercial market has been flooded with educational games and toys whose manipulation allegedly offers not only pleasure but a learning experience of one sort or another. Thousands of dollars and thousands of hours have been involved in testing these toys through customer participation.
The real educational value of these toys and games may not be highly correlated with the financial success of the marketed product. The inherent benefits of an educational nature are extremely difficult to measure. This is partially due to the fact that benefits from such an experience may not be individually recognized or appreciated.

Contemporary schools are becoming involved on a large scale in attempting to teach mathematical concepts via physical models or concrete examples. This is not to be confused with the applied mathematics which was emphasized so strongly for the first few decades of this century. Applications are important, but demonstrations of mathematical concepts through physical models may treat application of these concepts only as a possible by-product. The main emphasis is the concept itself, which may have a multitude of applications as the learner matures in his comprehension of the concept and of the problems which must be solved.

Setting of the Study

Knowing the importance of the entire junior high school mathematics program, many eager teachers are continually seeking for improved pedagogical techniques. During the spring of 1970 a seminar was held for the mathematics teachers of three Mercer County, Pennsylvania school districts. The author was invited to participate. The objectives of the seminar were to collectively pool the effective techniques used in the mathematics classrooms throughout the County and to determine which methods could best be used to improve the teaching of mathematics in the represented junior high schools.
Mathematics laboratories and their effectiveness was a pre-assigned topic proposed to stimulate discussion. Although every junior high school teacher at the seminar had used some models and games to demonstrate mathematical concepts, none had any experience with teaching in a mathematics laboratory setting.

The available literature on mathematics laboratories surveyed prior to the seminar dealt for the most part with experiments for slow learners in mathematics. Nearly all of the laboratories described in these materials were well equipped rooms, separate from the classroom, with laboratory tables and relatively extensive equipment for experimenting.

The average junior high school in this locality cannot afford the expense of separate laboratory facilities nor elaborate experimental apparatus. However, neither can they afford to jeopardize the academic welfare of their students by neglecting to take advantage of innovative ideas available through mathematics laboratory techniques.

Though the need for some type of program to rejuvenate junior high mathematics was unanimously agreed upon at this seminar, it was not practical at that time to make a wholesale adoption of an untested laboratory program. However, the interest in the laboratory experiments was sufficient to germinate the idea of a research study to determine the effectiveness of laboratory techniques on the motivational and achievement levels of eighth grade students.
Difficulties in Studying Mathematics Laboratories

The hodgepodge of ideas concerning mathematical laboratories which are verbalized either at seminars or in the literature generally serve to confuse rather than organize the thinking of the interested teacher. Thus, one facet of the problem confronting potential users of the laboratory method is to assimilate the numerous opinions, after they have been gathered, for the purpose of designing an organized study of laboratory methods.

To complicate the problem, the study must be realistic relative to existing curricular designs at the involved schools. Research in education, which is done with the cooperation of school administrators, most often involves a student sample in an actual school setting. This authentic school environment frequently dictates certain restrictions to the study due to the impracticality of manipulating students for experimental purposes.

Another hurdle on the path to a successful study is the presence of so many variables. Of course, every attempt should be made to minimize the effect of unmeasurable variables if the results of the study are to be generally meaningful. Two variables, closely related to each other, are the teacher and the adaptability of the school plant to laboratory teaching.

Every teacher has his own unique set of characteristics influencing the effectiveness of his teaching. His academic preparation in general areas as well as in mathematics and closely related subjects, his experiences in the classroom both as a student and as a teacher,
his personality traits which psychogenically control the degree of rapport with the students, and his confidence in the methods he is using to teach a lesson are a few of the many characteristics to which no guage or measuring device can be applied with accuracy.

Teacher variability is accentuated by the difference in physical facilities provided by the school districts. Continually expanding school districts have a very difficult time merely providing sufficient classrooms and teachers in their overcrowded conditions. Except in wealthy districts where keen foresight has produced sufficient building programs, rapid population growth practically eliminates the possibility of separately located and well equipped laboratory facilities. Confounding this problem is the size of the class enrollment. Conducting mathematics laboratories within the confines of the classroom with thirty to thirty-five eighth graders is a challenge many teachers are reluctant to face.

The effect of these existing deterrents cannot be overlooked. It appears obvious that the purpose of learning mathematics through the use of laboratory techniques can best be served under ideal conditions.

**Significance of the Study**

The junior high schools in Mercer County, Pennsylvania, have problems with achievement and motivation in their eighth grade mathematics classes which are not unique. It is possible that the results of a research study conducted in this locality will have contributory application for junior high schools in many other regions of the country. Even with all the inherent variables, within and between the
school districts across the nation, involving teacher preparation and pedagogical methods, urban and suburban desires and needs, and opportunities for advanced education in colleges or training schools, there exists a common problem, the underachievement of unmotivated junior high school students. Any and all efforts to solve this shared problem should be of unanimous interest.

If organized attempts are made to seek solutions to the problems in one school setting, whether they are satisfactory or not, there is merit in presenting the planned input, the resulting output, and the conclusions in a document available for all interested parties to examine. It does not logically follow that every workable plan in one school district will be successful for another. For that matter, it may not be advantageous for another teacher within the same school system to attempt it. However, even a cursory reading of the findings in one study may result in a gleaned idea contributing to an easing of the readers' related problems.

The significance of this study largely rests upon the validity of the above asserted premises.

**Overview of the Study**

Chapter II reviews the historical and contemporary literature related to mathematics laboratories and examines for relevancy the limited amount of related research dealing with laboratory methods of teaching mathematics.

Chapter III presents detailed descriptions of the environmental aspects of the school districts, the characteristics of the student
sample, the experiences and preparatory backgrounds of the teachers, 
the limitations of the physical facilities, the events during the pre-
paratory and trial periods, rules of procedure for the laboratory 
types, kinds of records maintained, content of the units covered, and 
the limitations of the study.

Chapter IV includes the numerical data collected and an analysis 
of these data by covariance. Also, analyzed are the students' opinions 
expressed through the student interest questionnaire.

Chapter V summarizes the preceding chapters, draws conclusions 
based upon the analysis of data, states the scope and limitations of 
the study, and presents related problems for further research.
CHAPTER II
RELATED LITERATURE AND RESEARCH

The primary purpose for searching the literature was to find material relevant to the practical problems confronting the Mercer County junior high school mathematics teachers. Hopefully, some reported experiences and some completed research studies involving mathematics laboratories would have conclusions which are applicable to the local situation and to others of a similar nature.

This primary purpose, then, delimits the scope of the literature review. The theoretical considerations that underlie the mathematics laboratory movement are minimized. Such large scale movements as the Nuffield Project and the School Mathematics Project in England, which have been widely discussed in recent years, are not in this chapter. Rather, the literature search was directed toward practical reports of projects in localities similar to the Mercer County school districts.

Descriptions of a Mathematics Laboratory

One often forgets what he has heard, ignores or disregards what he has seen, but somehow tends to remember that which he has touched. To handle, feel, bend, twist, stretch, cut, spin, throw, manipulate, guide, steer, control, or just simply touch seems to have a power which increases the retention spans of those involved.
No one realizes this more than a mother who watches her preschool child delight in the touch of a special toy or snuggle in the comforting warmth of his parents' arms. School teachers of K - 6 have always been cognizant of the greater interest their pupils have in their work when they can physically participate in a learning experience. Gimmicks and games, balls and string, cubes and spheres, scissors and paper, and models of paper mache can be found in the elementary classrooms of nearly every school in the country at one time or another.

A small boy of five came into the kindergarten one morning with radiant face and sparkling eyes, cry-out in joyful tones: "I have something for you! It's hard and long and has four edges and two ends!"

The precious object was held behind him, while he danced around in fond anticipation of the pleasure he was about to give his teacher, of whom he was very fond. "What can it be?" she answered, entering sympathetically into his pleasure. "Do show it to me." In proud triumph the hand which held the treasure was extended, and in the palm lay a burnt match. And the kindergarten teacher accepted it as a gift of value, for had it not helped to unlock the great world of form and its elements—faces, corners, and edges?

(From a nineteenth-century kindergarten teacher's report) (19:372).

A definition, or at least a general description, of mathematics laboratories may help with the decision whether or not to use them in our classes. Piaget has worked with children of all ages attempting to open the doors of their intricate minds and discover how children learn. The individualistic nature of a child's learning experiences, as emphasized by Piaget's reports, makes the laboratory methods even more important as it may provide the children with easy opportunity to choose experiments appropriate to their particular stage of development. Clarkson expresses a similar approach (5:495).
What is this classroom environment called the mathematics laboratory? There are many kinds of math labs, but one of the most common is simply the provision, by the classroom teacher, of materials and some time when children may choose to work on a mathematical problem that interests them, either alone or with a partner or small group of children sharing a similar interest. The distinguishing features of such lab periods are the independence of children from large group or teacher-directed lessons, and the possibilities for individualistic and active solutions to a wide variety of problems. Needless to say, independence and active participation produce an extremely high degree of motivation in the children. Anyone who has observed laboratory sessions can attest to this fact!

A complementing description of laboratories is given by Spencer and Brydegaard (32:5).

The classroom for mathematics should be a learning laboratory. This does not imply that equipment or fancy gadgets in a room or things that children build make a "learning laboratory." Rather, a classroom becomes a learning-laboratory when it produces mental and physical activity that results in experimentation; this in turn should lead to formulation of procedures and to generalizations based upon reliable and sufficient information. The materials for the laboratory are within the reach of every teacher. The materials consist, for the most part, of things that children and teachers bring into the classroom for the lessons under consideration. Cups, glasses, bottles, cans, jars, boxes, labels, cartons, string, measuring sticks, and innumerable other things to use in experiments with measuring should be a part of every mathematics classroom.

The force behind the scenes for a laboratory for learning is the classroom teacher. ... He is the type of teacher who can teach without textbooks.

If through experiments in the classroom the teacher can succeed in getting the students interested, yet even excited, then the atmosphere for learning is more fertile. Perhaps essential interest in
mathematics can be best generated and maintained through experiments which simultaneously provide for entertainment, increased computational skills, and deeper insight into the underlying, unifying concepts. This is not a recently made conjecture. Marx reports on a book written two centuries ago (23:123).

Elementary education, mathematics and geometry—because they are the most abstract—are usually the dullest subjects from many children's viewpoint; yet these subjects could be made as exciting and interesting as arts and crafts is now to the brighter pupils without expensive equipment if we take some inspiration from a book published just about 200 years ago in Europe. Generously illustrated with wood-cuts the volume contains among more than 500 items of information (and mis-information) numerous practical experiments which were provided for the solution of what were then important everyday questions. These experiments are still as interesting to perform today as they were in 1764, and because they are based on the basic mathematical and geometrical principles they should make for some interesting class sessions in a school or be fun for a scientifically minded parent to try with the children. Because mathematics and geometry are here brought down from their abstract plane to a level where we deal with everyday objects, readily measurable quantities, and concepts easily understood, children who are normally curious will be stimulated to take a greater interest in these so important subjects. If similar experiments would have been a part of the curriculum in the writer's own school days, retention, interest and probably grades would have shown a substantial improvement!

To understand the abstract nature of mathematics it appears not only desirable but, for most of us, necessary to consider the ideas as they relate to material phenomenon. Use of a model to establish a pattern, to provide data, and to demonstrate a concept should not be underestimated.
In a recent issue of the Mathematics Teacher, Lick also described the dual nature of mathematics (20:86).

Mathematics is abstract; mathematics is not nature. However, the key to the study of nature and natural phenomena is the concept of the mathematical model. That is, a mathematical system may be so chosen that its terms and assumptions have some meaningful relation to the physical situation. This is another beautiful aspect of mathematics; in one instance it may be used as a 'tool with application to models representing physical phenomena, and in another it may be an abstract discipline in and of itself.

Luboratories in the Elementary Schools

Based on his study of the comparative merits of a manipulative approach with second graders, Nasca concludes:

A procedure which provides children with the opportunity to perform operations with concrete materials and encourages them to abandon such models in favor of mental or "abstract" manipulation can obviously provide superior gains in achievement (25:225).

Sensory materials have often been developed by good teachers of mathematics because they found them necessary. Students entering school for the first time may know how to count in a rote manner, but they have considerable difficulty understanding what this process accomplishes. The author recently observed a twenty two month old child correctly identify the letters of the alphabet and the digits when his attention was drawn to them on a calendar and in newsprint. Television programs of today provide numerous opportunities for children to observe and learn. However, correct identification and recitation of the symbols may not imply that the real meaning of these symbols is understood.
Understanding results in the combined uses of the concrete and the abstract. Simple use of the natural numbers to count a set of objects requires these combinations. The basic computational exercises require that students can freely think in an alternating manner between the concrete and the abstract.

One of the most popular topics for discussion and research in bridging the gap from the concrete to the abstract is the use of Cuisenaire rods.

During the process of abstract computation, we ignore the concrete things, and then when the computation is completed, we apply it to the concrete things which we have temporarily ignored. We begin with the concretion, then turn from it while we compute abstractly, and finally bring these abstractions back to the concretion (8:317).

Cuisenaire rods are a physical model for rational numbers and for a rational field. This makes them valuable for studying the commutative, associative and distributive laws. And by using a special convention, the properties of set algebra are made particularly evident. Other mathematical relationships explored with Cuisenaire rods include fractions, place value, different number bases and positive and negative integers. Because the colored rods help make mathematics real to students, they understand and enjoy discoveries made from manipulating these brightly-colored models (7:4).

Hollis (17:683) compared the effects of using the Cuisenaire-Gattegno method with a traditional method over a two year period encompassing first and second grade. At the end of the first year, pupils taught with the Cuisenaire-Gattegno method achieved as well as the control group on traditional tests and had acquired additional concepts and skills. By the end of the second year both achievement and concept knowledge were superior for the Cuisenaire-Gattegno classes.
Dealing with retarded children with intelligence quotient less than 80, Callahan and Jacobson report,

A distinct advantage of the rods is the ability to see the problem! It did not take the children long to visualize the number concepts which heretofore had been illusive abstractions. ... Gains were made in the area of transfer. This was evidenced in oral (mental) arithmetic. Mathematical stories and games, and also in their workbook examples. ... Throughout the experiment the teacher reported that the interest remained at a high level (3:12).

Nasca studied the merits of a manipulative approach to second grade arithmetic, choosing Cuisenaire rods as the structured concrete model. With due caution dictated by the design of his study, he states,

It is safe to conclude that primary students receiving instruction based on pupil exploration of concrete models can achieve traditionally established goals in mathematics (25:221).

Passy (27:440) reports a negative effect from use of Cuisenaire rods. Thus, it is evident that materials alone do not produce miraculous results. Many other devices are now available for teachers interested in using laboratory methods.

Laboratories in the Junior High School

Experiences which are good for elementary students may also be good for junior or senior high school students. If laboratory techniques increase the achievement levels of elementary classes, there is no real reason to doubt that the same thing will happen at the eighth grade level. At the present it appears that the elementary schools are leading the secondary schools in their use of laboratory
techniques. However, the ideas of the discovery approach can definitely be enhanced at all levels, kindergarten through graduate school, through effective use of the mathematics laboratories.

Glowing reports of successes with mathematics laboratories for below average pupils have been published in magazines and journals during the past few years. Ruth Irene Hoffman, University of Denver, wrote in The Bulletin of The National Association of Secondary-School Principals, April 1968, concerning "The Slow Learner - Changing His View of Math" (16:86-89). Hoffman cited projects and case studies to show the constructive effects of the laboratory approach upon low achievers in Grades K through 12.

Eighth graders in Bingham High School (Kansas City) spend from thirty minutes to two-and-a-half hours a week participating in an instructional dialogue on supplementary and enrichment topics in grade 8 science and mathematics. They use the IBM 1500 Instructional System.

To accommodate the varied backgrounds of their students, three Kansas City secondary schools have adopted modular scheduling. Under this plan students meet in large groups for presentation of new material by the teacher, for summaries of student reports, for speakers from industry, or for testing. Laboratory periods provide for study with teachers or other students and for learning by discovery through many relative devices or through games or puzzles.

In Bellport Middle School, Bellport, N. Y., students were encouraged to enter into a project of building such things as toboggans, kiddie rockers, tool boxes, and wagons from a special type of cardboard (33:209). These constructions were teaching aids for other subject areas such as mathematics. Freedom to work with the material, to make errors, to correct mistakes and discover new ideas were an important
part of the project. A summary of some of the advantages cited by Tinti includes 1) Satisfaction by handling and experimenting, 2) Creativity of expression, 3) Self-realization in constructing, and 4) Bridging the abstract to the concrete.

The National Education Association is also aware of recent attempts to incorporate laboratory methods in mathematics classes (26:50).

Basic ideas are approached intuitively through discovery and experiment. ... This includes operations on the abacus; use of cuisenaire rods for reinforcing the concepts of number relations; ratio, and fractions; practice in use of the hundred board, the number line, the geo-board; the use of meaningful logic puzzles; graphing games; and topological puzzles.

In Washington, D.C. junior high students meet for two laboratory periods a week in small groups. They work with electric calculators, tapes and earphones, filmstrips, overhead projectors, and mathematics frames.

Clearly, for some children, these programs have added a dimension and provided insight not readily available from the regular chalkboard and printed page (26:51).

In Winnetka Public Schools, Winnetka, Illinois, mathematics laboratories are held to help children become independent learners (24:501).

As the pupils work with concrete materials, they record their observations and then make generalizations from the data. ... The main idea is to keep students so motivated by their own work that they want to learn more. There is no failure, because all students are free to ask questions whenever they need help. The atmosphere of the laboratories is a carnival with a purpose.
The responses of the students quoted in this article indicate that Winnetka students are motivated not only to do more mathematics, but to like doing it. They look forward to the laboratory periods with anticipation.

**Research Studies**

As this chapter indicates, most of the literature on mathematics laboratories consists of general program descriptions, subjective evaluations, and considered opinions. In a recent research review by Kieren the indication is that serious researchers in education are giving more attention to manipulative activity in mathematical instruction (18:228). Among the questions which Kieren proposes for consideration are the following:

Does manipulative experience fit at a certain level into a particular hierarchy in the promotion of learning?
What are the best qualities of manipulative experience in promoting learning?

In addition to Kieren's questions the question of attitude changes has also been asked in two research studies.

Bruner suggests that learning mathematical ideas over a period of years involves enactive, iconic, and symbolic representations (2:10-11). Respectively these three representations are manipulation of an object using the sense of touch, organizing what one sees by observing patterns, and finally symbolically coding the activity for compacting and condensing the results for easy reference. The validity of the action-image-symbol sequence was considered in numerous studies. Armstrong, working with mentally retarded children, found activity to have
a pronounced effect on learning (1). Fennema (11), Trueblood (34), and Vance (35) found that normal school children in grades 2, 4, 7, and 8 can achieve as well at the symbolic level as at the manipulative level.

Whether or not a similar sequence of presentation should be followed in teaching a single concept over a short period of one or two lessons has not been investigated. The design of the present study involving different orderings of experiments and recitations may provide relevant information.

Cuisenaire rods, previously discussed in this chapter, provide an example of a manipulative device generally possessing the proper qualities to promote learning. Sources for other materials and devices which may have these desired qualities are mentioned later in this chapter.

There is little doubt that people of all ages perform tasks, simple or complicated, with much more success if they entertain the proper positive attitudes toward their assignment. Conversely, success very often generates contentment and pleasure. For some, hoeing the garden can provide as much recreational pleasure as playing a round of golf will for others. The difference surely is attitude. Pleasure is a relative status of satisfaction and joy, measured individually by personally established standards.

Higgins made a five week study involving 853 students of 29 eighth grade teachers in Santa Clara County, California at the beginning of which the students were assigned to attitude groups based on the results of a battery of tests (14). The materials on teaching of
mathematics through science prepared by SMSG were studied. It was found that differences in attitude patterns among groups is not reflected in significant differences in either ability or achievement. In addition, only 8 per cent of the students made a positive change in their attitudes toward mathematics while 6 per cent developed an unfavorable attitude toward the content of this unit.

A similar study was made by Wilkinson involving achievement and attitudes of sixth graders studying geometry under either verbally directed or discovery oriented techniques (36). The children with average and below average intelligence indicated a positive change in attitude toward mathematics while above average students seemed to appear passive. No significant differences in achievement were found.

The Nature of a Continuing Program

At the time of the revolution in mathematics during the 1960's, educators were highly critical of junior high school curricula emphasizing quantitative concepts and procedures which did not appear to serve their proposed objectives. Such topics as life insurance, discounts, commissions, banking, and taxation in the eighth grade were often condemned as having too little to offer in building solid foundations for either the terminal or the college bound students.

Many modern programs emphasize abstract structure and form to provide for proficiency in the use of mathematical concepts, principles, and skills and have been criticized for leaning too far in the opposite direction.
The author believes that an intelligent and logical approach for average students of eighth grade is one of compromise. True, it is the responsibility of the elementary school to lay the fundamental groundwork of basic concepts upon which arithmetical structure is built. However, the junior high school should not assume 100 per cent efficiency on the part of the elementary school. Apparently a continuing process of foundation building must be evidenced at all levels if mathematical growth toward maturity is to take place. Mathematical problem situations presented to provide for strengthening concepts and principles and at the same time lending themselves to a practical application, may be most efficiently made in a laboratory setting. The resulting solutions may be experienced through several of the senses as well as tabulated and recorded. Calculations may be meaningful beyond satisfying the teacher or agreeing with the answers in the back of the book. It is presumed that a program so designed will better serve contemporary society provided enough time, effort, and flexibility are employed in its construction.

The Laboratory Teacher

Every student should be fully challenged if possible, but prior to the challenge the teacher must necessarily have the attention and the interest of the student. Thus, the teachers of junior high school mathematics who search for improved techniques, will find increasing need to be well informed in mathematical subject matter, history of mathematics, psychology of individual differences, effective techniques of presentation of materials, and evaluation of results.
When teachers are provided with insufficient time for inservice preparation and adjustment to the laboratory methods of instruction, extraneous factors in the experience may greatly influence the decision to continue the endeavor. Following a one year program at David Jr. High School in Cleveland to establish a sound laboratory oriented program for seventh graders, Woodby recorded these statements summarizing the findings (37).

(1) Organization for small group instruction or individualized instruction is difficult. Discipline was a major concern.

(2) The goals of mathematics instruction often get lost in the mechanics of the laboratory activity.

(3) Belief of the teacher in a discovery approach is not sufficient to accomplish the appropriate behavior. Teachers will revert to telling, explaining, and showing students. There is a wide gap between teacher belief and teacher behavior.

(4) The teachers expected too much of the materials; it was assumed that the material would provide the motivation.

(5) Rewards for the teachers were different from their expectations. They had anticipated great increases in the achievement by the group as a whole. The rewards turned out to be unusual accomplishments by individuals and those were infrequent and unpredictable.

(6) The teachers became more concerned with how students learned than with the achievement; questions asked by students became more important to the teacher later in the semester than they were at the beginning.

(7) The teachers worked longer and more intensively than they did before the project. Even if they had much more time for preparation they stayed late and usually took work home to (a) organize for instruction, (b) devise activities and write instructions, and (c) evaluate results.

(8) Teachers in this learning situation need someone to talk to. Supervisors and consultants are important to the teacher in this situation.
From the teacher point of view, the learning was more nearly guided discovery than true discovery.

The teachers became better teachers because of what they learned about students learning. For example, they talked less and listened more at the end than they had at the beginning.

The two Cleveland teachers involved gleaned some valuable information from this experience with low achievers in the seventh grade. Through a summer training session conducted by the two participating teachers, fourteen other teachers worked with the materials in the laboratory and gained some insight into the proper uses of laboratory techniques.

Woodby lists ten desirable behaviors expected of a laboratory teacher (37).

1. The teacher asks questions that cause exploration and inquiry by the student.

2. The teacher devises and uses tasks that relate to fundamental mathematical concepts and techniques. A good example is Rosenbloom's simulated computer in which the student discovers the distributive principle.

3. The teacher uses materials other than the textbook.

4. The teacher provides individual and small group activities of an exploratory nature that results in the student trying something, gathering data, analyzing data, and testing conclusions.

5. The teacher uses cues that come from the student in making teacher decisions about questions asked or tasks assigned.

6. The teacher plans for and uses the basic strategy of student discovery.

7. The teacher employs the strategy of asking the students to make decisions on the basis of observation of events.
(8) The teacher provides situations for the student to play an active role in learning, rather than a passive one.

(9) The teacher creates a new problem or task that is easier or more familiar to the student when difficulty occurs, then allows the student to return to their original problem.

(10) The teacher provides the student with a means for determining whether an answer is right or wrong, independently of the teacher or the textbook.

A statement in the preface of *Laboratory Manual to Elementary Mathematics* by Fitzgerald, Bellamy, Boomstra, Oosse, and Jones, provides a summary of the role of the laboratory teacher (12).

The essence of the laboratory concept in learning mathematics is the fostering of inquiry and internal motivation to seek answers to questions. It is a fair generalization that teachers, at all levels, talk too much. A laboratory instructor must be wary of the temptations to provide excessive direction for the student, and thus rob the student of the experiences of finding answers for themselves. And some students will demand excessive direction, which may be evidence of the lack of the use of laboratory techniques in schools in the past.

**Sources for Materials and Devices**

An annotated bibliography of manipulative devices used in United States and England is presented by Davidson (7:509) under the general headings of blocks, calculators and computers, cards, construction, drawing tools, geo-boards, measuring devices, models, numerical games, puzzles, shapes and tile, strategy games, and student instructional materials. The suppliers' names and addresses are also listed.
A current listing of Mathematics Laboratory material has been compiled by Thomas P. Hillman through the distribution of a questionnaire to mathematics educators throughout the country. This list may be found in the June 1968 issue of *School Science and Mathematics* (15:488).

**Summary**

In summary, the reviewed literature which seemed pertinent to the particular problem of organizing a study regarding effective laboratory experiences, led to the following conclusions:

1. Every mathematics classroom is a potential laboratory.
2. The teacher is probably the most important element in the laboratory setting.
3. Essential physical tools may vary from expensive, commercially produced items to homemade devices of paper and string.
4. One objective is to interlace the abstract ideas of mathematics and the concrete materials at hand to weave a useable fabric of concepts understandable by the students.
5. Laboratory experiences with physical models, for both elementary and junior high school students, have very often motivated students to be more inquisitive toward mathematics.
6. The implementation of laboratory methods in the mathematics classrooms should be preceded by a carefully planned, preparatory period including organizational features and instructional methods.
CHAPTER III
FEATURES AND ORGANIZATION OF THE STUDY

The School Districts

The two neighboring school districts, Mercer and Lakeview, from which the population samples were selected are located in northwestern Pennsylvania. They are consolidated districts with a large majority of their school population bussed in from surrounding townships. The school communities are rural in nature with few industries located within their boundaries. A large percentage of the adult population living in these districts commute fifteen or twenty miles to work in the Shenango Valley steel mills or the Westinghouse Corporation transformer plant. Another considerable percentage of the population own and operate farms within the districts. Mercer is a county seat which means that many professional people live nearby and send their children to one of these schools. Both districts have junior-senior high schools with the total population of children in grades 7 - 12 between 1,000 and 1,500 students. They both have the problems of being overcrowded due to abnormal population growth. This is partially due to the recent completion of two Interstate highways intersecting within the County boundaries. There has been an influx of trucking industries within the County lines and many employees have moved into these school districts.
Additional State law enforcement officers were required because of the highways, and the State Police barracks is located in Mercer. The major problem of providing classroom space for the offspring of these new residents has arisen in the elementary schools. However, the junior high school was affected also. A building program at Mercer further complicated the space problem since some former junior high school classrooms were absorbed in an expanded library before the new classrooms were available for occupancy.

This information is relevant to the study only in that separate laboratory rooms were not available and the classes were generally overcrowded in both school districts.

The administrators of these districts were extremely cooperative and are open-minded toward innovative ideas in teaching. Yet this receptiveness to new ideas is balanced with sufficient cautions to avoid the exploitation of the students for the sake of experimentation. The teaching responsibilities were strictly assigned to the regular teachers during this study, although a student teacher was assigned to each cooperating teacher during parts of the second semester. Little fanfare was involved and the author was strictly a behind the scenes director and observer.

**Physical Provisions**

The school plants at Mercer and Lakeview were not constructed with mathematics laboratories in mind. Classrooms are of standard sizes according to State building codes; there are no work tables nor is there room for any to be moved in; storage cupboards are
conspicuously missing and window sills are often used for book shelves; expensive laboratory equipment is perhaps available in small quantity, but most devices are of paper or wood which can be conveniently tucked in the corner of a desk drawer or placed on the small shelf of the teacher's locker. This is not a description of an atypical eighth grade classroom. Neither is it a description of a classroom in which mathematics laboratories can be conveniently conducted.

During the brief period of preparatory time for this study little could be done to alleviate the problems with the school plants. Perhaps this is fortunate for if ideal conditions existed for an experiment of this nature, it would be so out of the ordinary that its results would not be generalizable.

The Teachers

In each of the school districts, Mercer and Lakeview, one teacher has the responsibility of teaching all or a major part of the eighth grade mathematics. Both teachers have five or more years of teaching experience and thoroughly understand the problems facing eighth grade mathematics teachers. Because of their experience and knowledge they willingly volunteered to participate in organizing and administering a laboratory type program. Games, models, and gadgets had been used in their classrooms many times before. Particular attention to individualized programs within the classes resulted in the Lakeview teacher's having prepared batteries of achievement tests which must be successively passed before completion of the eighth grade. The Mercer teacher attempted to keep all students of a given class together, but individual
classes were permitted and encouraged to learn at their own rates of comprehension. Both teachers established similar minimum standards for satisfactory achievement based on their professional experiences.

Preparatory and Trial Periods

A short time after the County seminar in the spring of 1970, the two cooperating teachers met with the author to establish some ground rules for the preparatory period. A great deal of planning was necessary before a study of any consequence could be initiated and completed during the next school year. Weekly meetings during the summer months were scheduled when vacations and military obligations did not interfere. During these meetings the content of the first semester of eighth grade mathematics was reviewed with its potentiality for adaptable experiments in mind.

Catalogues of devices and pertinent literature from methods books and mathematics journals were searched for ideas related to the course content. By the beginning of school in September some ideas for experiments relevant to sets, the decimal system, and other number base systems had been formulated. However, the methods for presenting these experiments and the type of records to keep for analyzing the results were not at a sufficiently refined stage to begin the research. It was realized that the methods of mathematics laboratories were new to both teachers and to all students. A time of adjustment and refinement was required. The first six weeks of the school year were set aside for a continuation of the planning period.
One class in each school was arbitrarily assigned to a type of laboratory approach in which the experiments precede the discussion of the related concept in class recitations. Another class was arbitrarily chosen to have a laboratory type in which the experiments would follow the class recitations. The third class in each school was to have the traditional approach with no organized laboratory experiments. These three approaches were respectively named pre-lab, post-lab, and no-lab.

During the first six weeks trial period the decision was made with mutual agreement that regularly scheduled laboratory experiments were not convenient. Rather, experiments would be conducted when the respective classes were ready for them. The teachers also experimented with and refined the differing teaching and questioning techniques used to distinguish the pre-lab and post-lab approaches. At the conclusion of the trial period the plans and procedures for an organized study had been formulated.

**Student Samples**

Of the eighth grade classes taking the regular mathematics program, three were arbitrarily selected by each teacher to make up the student sample. The homogeneously grouped classes were ranked by the teachers as high, medium, and low in expected achievement levels. However, no special effort was made to select the highest or the lowest of the school-ranked classes.

The combined total of eighth grade students in both schools is approximately 350. Of these students, about 180 were involved in at least a part of the laboratory methods program while 80 Mercer students
and 66 Lakeview students actually completed all phases of the work and are therefore included in the data. For reasons such as families moving from or into the districts or students having prolonged absences due to illness during the periods when material was presented and tests were administered, it was impossible to include all 180 students in the analysis. However, all students who actually completed the entire study program are included since no other process of elimination was employed after the initial selection of the six classes.

The students were not informed that any kind of study was in progress. They, of course, realized that some classes were using approaches to the subject areas different from theirs, but they were not made aware of any particular assignment design. This information was intentionally withheld and the author's visitations to the classrooms were minimized to avoid any possible Hawthorne Effect.

Class Grouping

The 146 students included in this study were members of six eighth grade classes, three each from Mercer High School and Lakeview High School, Mercer County, Pennsylvania. On the one hand, the true class enrollments are slightly higher than the sample indicates, but only those students who were physically present to complete the entire study are included in the data. On the other hand, all of those students completing the program were included in the statistical data. The range of involved students per class was 18 to 27.

One experienced teacher from each school taught the three classes involved and conducted the laboratory sessions. In both schools the
students were homogeneously grouped according to overall achievement ratings. The three arbitrarily selected classes did not include the class of highest achievers in the eighth grades since their course work is enriched to include more algebra than is generally studied in the eighth grade at these schools.

As is often true when a study involves students from a real school situation, the format must be designed to utilize the existing intact groups. This is necessary to avoid unwanted interference with the schools' programs which are already established. The impracticality of having the school administration and teachers cater to the researchers' wishes makes it necessary for the researcher to design his study according to the existing classes and their curricula.

In each school the three homogeneous classes included one of relatively high achievers, one of average achievers, and one of below average achievers as rated by the teachers. The reader is reminded that the first six weeks of the 1970-71 school year served as a trial period for some of the ideas which were developed by the author and the two teachers during the summer months of 1970.

Beginning with the second six weeks and continuing through the fourth six weeks, the study was organized to provide criteria which when analyzed would determine the effects of laboratory techniques on achievement in mathematics at the eighth grade level. The null hypothesis to be tested is that there is no significant difference in achievement between eighth grade mathematics classes being taught either without laboratory techniques or with one of the two direct laboratory approaches.
Because it was highly impractical to select the students of participating classes randomly from the entire eighth grade population, there are definite class differences in intelligence quotient means and in average pre-knowledge of the concepts to be taught.

The schools' records show that the Lakeview students were administered intelligence tests (Otis MAT AS) in 1970 and the Mercer students were also tested this year for intelligence quotients (SRA High School Placement Test, 1970 edition). Thus, reasonably accurate mean intelligence quotients were readily available (see Table 1).

<table>
<thead>
<tr>
<th>Classes by Teacher Rankings*</th>
<th>Intelligence Quotient Means</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mercer</td>
</tr>
<tr>
<td>Class 1 — High</td>
<td>111.27</td>
</tr>
<tr>
<td>Class 2 — Medium</td>
<td>107.56</td>
</tr>
<tr>
<td>Class 3 — Low</td>
<td>96.63</td>
</tr>
</tbody>
</table>

*Note that the teachers ranked the learning potentials of the classes in the order of the numerical classifications in Table 1 based on their experiences with them during the trial period. This ranking corresponds with the intelligence score rankings in Mercer, but with the closeness of the mean scores for classes 2 and 3 at Lakeview, it happens that the teacher rankings are numerically reversed. In this paper the rankings of high, medium and low will represent the teacher rankings corresponding respectively to classes 1, 2, and 3.
Instructional Procedures

One class of the three in each school was assigned a pre-lab approach for the first unit of work on fractions, ratios, and proportions. This type of laboratory was designed to see what the students could do with predesigned experiments involving a particular concept and to allow them freedom to make conjectures concerning generalizations from the specific experiments performed. All of this took place before the concept was discussed in the usual manner during class recitation. Naturally there was some carryover from related ideas explored in previous years. It was hoped that this pre-lab approach would serve to bridge any existing, retention gaps between seventh and eighth grade when this continuity of concepts was involved.

The students in a pre-lab were given the physical materials for the experiment and the essential instructions for performing the first steps of the experiment. The nature of the pre-lab experiments was such that the teacher explained relatively simple procedures to initiate the experiment and then the students were encouraged to apply their own ideas in expanding and extending these procedures.

For example, consider Experiment D (see Appendix A) involving the rolling of a ball down an inclined plane. In this case, the students may work in committees of five or six students, each committee having its own set of materials. If a trough is not available down which to roll the ball, any smooth board will suffice, though it may take more trials to keep the ball on the board for the entire length.
The students were instructed to set up the inclined plane and time the descent of the rolling ball for a predetermined distance.

The teacher may promote student participation in the formulation of the succeeding steps of this experiment by asking questions like those stated below.

Will the time of the ball's descent be any different if the ball is released above the first mark so that it has a running start?

Now that we have one recorded time for the descent of the ball, is there any method we can use to get a better estimate of the actual time it takes to roll from one mark to another? How do we find the average of 5 readings of the stopwatch?

Is the speed of the ball constant for the entire distance down the inclined plane? How fast was it going the instant you released it?

If the distance is known and the time of descent is known, can we find the average speed of the ball? What is a formula involving time, speed or rate, and distance?

What units of measure for the time and the distance should we use? What unit of measure applies to the average rate of descent?

If we know the speed of the ball in feet per second can we convert this measure to miles per hour? If a moving object goes 88 feet per second, it is traveling at a speed of 60 miles per hour. Can we use this relationship between units of rate measure to convert feet per second to miles per hour in our experiment? How?

What is a ratio? What is a proportion statement?

Other leading questions are suggested in the written experiment in the appendix.
Through this pre-lab the stage has been set for a class recitation on ratios and proportions as they apply to rate problems of various kinds.

For the same unit of work a second class of the three was assigned a post-lab approach. This laboratory type was designed to follow the discussion of the concept in class recitation. Here the materials for the equipment were provided but specific directions for their utilization were minimized. Exploration and discovery in an applicatory setting was the objective. Improvisation with other readily available materials was encouraged. Thus, specific applications from the generalized concepts already discussed as well as extension of these generalizations resulted from student participation in the post-lab.

For a comparison of the pre-lab and post-lab approaches, again consider Experiment D in a post-lab setting. These students have already discussed ratios and proportions and have solved problems using proportions and rate pairs in regular periods of class recitation.

Each committee is supplied with an inclined plane, a ball, and a stop watch. The objective of the post-lab is to promote innovation and exploration through the application of the previously discussed concepts. Teacher guidance is ever present, but specific instructions for implementing the basic concepts are minimized to allow as much freedom as possible.

What can we do with these items, the board, the ball, and the watch, to get an application of a time-rate-distance problem? Not all committees need do the same type problem. Is the board or inclined
plane really necessary? With these relatively simple leading questions, the committees are in an atmosphere of exploration, searching for some application of what they already know in theory. The teacher should move among committees to determine what ideas they have and to ask other leading questions to promote new ideas. If one group cannot think of a good application, the teacher can often explain the methods of a working committee with slight alterations suggested. For example, if Committee A is rolling the ball from a dead start on one mark to a second mark on the inclined plane, suggest to Committee B that three marks be placed on the inclined plane at X, Y, Z, that the ball be released at X, but that the clock be used to time the descent of the ball from point Y to point Z. Committee C might simply drop the ball from a fixed height to the floor. Committee D might use two balls, different in size and weight, and compare the times of descent. However, suggestions by the teacher should be kept at a minimum to allow for freedom of thought and action on the part of the students.

Thus, for all fifteen experiments which were used in the eighteen weeks of this research study the pre-lab and post-lab differed by the approach to the experiment. The mathematical concepts being emphasized were the same for each laboratory type.

The experiments as they are presented in the appendix are not written specifically for either laboratory approach. The required materials are listed and a set of suggested questions are stated which the teachers may or may not use according to the type of laboratory approach desired. The teachers of the eighth grade students in this study soon found that general plans for an experiment are essential,
but flexibility is a most important property if the students are to willingly participate in the experiments. No ideas are ignored. Every suggestion is considered for its value before it is either used, adapted for use, or discarded. The role of the teacher is to get the experiment started by planting some general ideas and then let the students nurture, cultivate, weed, seed, and hopefully harvest a worthwhile crop of mathematical concepts from their experimental endeavors.

The remaining class from each school was assigned a no-lab approach during this six weeks on fractions, ratios, and proportions. The traditional class recitations were employed. Some models and devices were involved, as they normally are when concepts are explained by the teacher, but direct student participation in experiments in a laboratory setting was not permitted.

**Nature of the Units Covered**

During the first six weeks of the school year, the trial and refinement period, both teachers agreed to include the study of sets, and set notation even though one text did not begin with this information. The knowledge of sets provided a unifying concept used throughout the year in the study of mathematics. Also included in this trial period were the following topics:

One-to-one correspondences, cardinal number, number lines, names for numbers, place value, factors, exponents, decimal system, polynomial expressions, base 7 and base 12 systems, and the commutative, associative, and distributive properties.
Unit I, the first unit of work involved in the statistical study, included the following material: fractions as quotients, equivalent fractions, common denominators, comparing fractions, computational manipulation with fractions, reciprocals, complex fractions, decimals, repeating and terminating decimals, ratios, equivalent ratios, rates and proportions, scale drawings, ratio and per cent, and interest problems.

Unit II included: points, lines and planes, angles, congruency, constructions, perpendiculars, angle bisectors, parallel lines, simple closed figures, triangles, quadrilaterals, polygons, perimeters, circles, circumference, and the number Pi.

Unit III included: diagonals of polygons, sums of interior and exterior angles in polygons, areas of special polygons and circles, inscribed and circumscribed polygons, space figures, and surface areas and volumes of prisms, cylinders and spheres.

For the most part, the teachers covered the pertinent material contained in the 1961 (30) and 1964 (31) editions of eighth grade mathematics (Book Two) published by Silver Burdett Company. They re-organized and supplemented the related chapters where they deemed it necessary to produce essentially the same mathematics courses in the two schools.

Treatment Variations with Ability Levels and Units

The instructional and testing procedures were repeated for two more units of eighth grade mathematics, called Geometry I and Geometry II. The study of each of these units began with a diagnostic test and
concluded with an analogous achievement test. The exact number of days or weeks in which the units were to be taught was not specified. Some classes easily completed the units in six weeks, while the lower achievers required slightly more time. Enrichment work took up the slack and tests were administered at the same time for all classes.

To assure that all students from the sample had the same opportunities to study mathematics during the school year, a rotation of laboratory assignments was made at the end of each of the three units.

The laboratory type assignments in each school for the three units included in this study appear in Table 2 and Table 3. Other convenient views of those same assignments are presented in Tables 4 and 5 where ability grouped classes in the schools are mapped onto units of work.

**TABLE 2**

ASSIGNMENT OF ABILITY LEVELS TO LABORATORY TYPES PER UNIT AT MERGER

<table>
<thead>
<tr>
<th>Lab - Type</th>
<th>Unit I</th>
<th>Unit II</th>
<th>Unit III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-lab</td>
<td>High</td>
<td>Medium</td>
<td>Low</td>
</tr>
<tr>
<td>Post-lab</td>
<td>Medium</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>No-lab</td>
<td>Low</td>
<td>High</td>
<td>Medium</td>
</tr>
</tbody>
</table>
### TABLE 3
**ASSIGNMENT OF ABILITY LEVELS TO LABORATORY TYPES PER UNIT AT LAKEVIEW**

<table>
<thead>
<tr>
<th>Lab - Type</th>
<th>Unit I</th>
<th>Unit II</th>
<th>Unit III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-lab</td>
<td>Low</td>
<td>High</td>
<td>Medium</td>
</tr>
<tr>
<td>Post-lab</td>
<td>High</td>
<td>Medium</td>
<td>Low</td>
</tr>
<tr>
<td>No-lab</td>
<td>Medium</td>
<td>Low</td>
<td>High</td>
</tr>
</tbody>
</table>

### TABLE 4
**ASSIGNMENT OF LABORATORY TYPES TO ABILITY LEVELS PER UNIT AT MERCER**

<table>
<thead>
<tr>
<th>Ability Levels</th>
<th>Unit I</th>
<th>Unit II</th>
<th>Unit III</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>Pre-lab</td>
<td>No-lab</td>
<td>Post-lab</td>
</tr>
<tr>
<td>Medium</td>
<td>Post-lab</td>
<td>Pre-lab</td>
<td>No-lab</td>
</tr>
<tr>
<td>Low</td>
<td>No-lab</td>
<td>Post-lab</td>
<td>Pre-lab</td>
</tr>
</tbody>
</table>
### TABLE 5

**ASSIGNMENT OF LABORATORY TYPES TO ABILITY LEVELS PER UNIT AT LAKEVIEW**

<table>
<thead>
<tr>
<th>Ability Levels</th>
<th>Unit I</th>
<th>Unit II</th>
<th>Unit III</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>Post-lab</td>
<td>Pre-lab</td>
<td>No-lab</td>
</tr>
<tr>
<td>Medium</td>
<td>No-lab</td>
<td>Post-lab</td>
<td>Pre-lab</td>
</tr>
<tr>
<td>Low</td>
<td>Pre-lab</td>
<td>No-lab</td>
<td>Post-lab</td>
</tr>
</tbody>
</table>

**Testing**

Before any classes or laboratories were held for a particular unit of work, a teacher-made diagnostic test was administered to determine the pre-knowledge of the included mathematical concepts (see Appendix B). These tests were constructed jointly by the teachers and the author and the same test was given at each school at the appropriate times. Tables of these test scores appear in Chapter IV.

At the conclusion of this unit a teacher-made achievement test was administered (see Appendix C). The items on this test were analogous to those of the diagnostic test previously given. These sets of test scores along with the students' intelligence quotients comprised the essential data for the statistical analysis of this unit.
Questionnaire

An attempt was made to determine the immediate reactions of the students and the teachers to each of the fifteen experiments through evaluation check sheets (see Appendix D). Unfortunately, these forms were not always returned due to a lack of sufficient time during the laboratory period to satisfactorily check them. However, an evaluation of those responses received by the author will be given in Chapter IV.

When junior high school students are asked to react to statements regarding their opinions of a school activity, there are many influencing factors. This is especially true when they are requested to recall events which occurred over a long period of time and to make comparisons concerning these events.

Students in this age group are whimsical to the extent that the most recent experiences often become the chief criteria by which all other experiences are evaluated. These experiences may be good or bad as seen from the students' viewpoint. In either case, the emotional state of the students and the degree of teacher-student rapport are often reflected in the attitudes of the students toward their assigned work.

There is no way of knowing the real reasons for the instantaneous opinions of junior high school students. In spite of this known shortcoming in evaluating students' opinions, another Student Interest Questionnaire was constructed and distributed one week after the close of the third unit (see Appendix E). It was hoped that an interest trend would be evident when the entire set of responses was viewed. The results of this endeavor are also summarized in Chapter IV.
Features Limiting the Scope of the Study

Although the statistical analysis of data in Chapter IV treats each school separately, the fact that the two school districts from which the samples were selected possess many similar characteristics has a limiting effect on the application of the results of this study for the teacher in another type district. The rural setting, the homogeneity of the living standards and cultural experiences, and the rather conservative approach toward untested ideas are characteristics which many school districts do not have. Consequently, the students have similar environmental backgrounds which influence their adaptability to a different type of learning experience. This influence is difficult, if not impossible, to measure.

Who is to say what makes a good class or a bad class? Yet, in one of the schools the teachers of all eighth grade subjects agreed that this group of students on the whole was one of the poorest in recent years; poor in the sense of their lackadaisical attitude toward school in general. Perhaps the national crises and the uncertainty of the times contribute to this listlessness in many geographical areas.

Only two teachers were involved in this study. Both of them were raised and educated in northwestern Pennsylvania; though they both have many years of teaching experience, their teaching loads are not light. There was no released time from usual school responsibilities for these teachers. One is assistant coach of a major sport whose team became involved in tournament play for the State Championships. There was no hard evidence that this caused any distraction from, nor dereliction in
fulfilling the classroom responsibilities, but undoubtedly the con-
fusion made the task more difficult. The students, too, must have been
affected by the pressures of tournament involvement at this level.

Ill health necessitated that either a substitute teacher or the
assigned student teacher replace one of the regular teachers for part
of the time during this study. Again, there is no evidence that this
influenced the results of the study, but students may have reacted dif-
ferently under these conditions.

The large class enrollments relative to the size of the class-
rooms diminished perceptively the amount of individualized attention
given the students during laboratory periods. Both teachers were as-
signed student teachers during part of the project, and their assist-
ance probably helped to alleviate this situation.

All diagnostic tests and achievement tests were constructed
through the combined efforts of the two teachers and the author. The
reliability coefficients for these tests were calculated by a Kuder-
Richardson formula discussed by Harry A. Greene et al., in Measurement
and Evaluation in the Secondary School (13:74). The coefficients
calculated were all sufficiently high to show reasonable reliability
(see Table 6). The formula used provides a "footnote coefficient"
which may underestimate, but never overestimates, the reliability coef-
ficient.

Because of its simplicity and because it furnishes
a result of sufficient accuracy for many purposes,
this method is recommended for use by teachers in
estimating the reliability of their informal ob-
jective examinations.
TABLE 6
RELIABILITY COEFFICIENTS FOR DIAGNOSTIC
AND ACHIEVEMENT TESTS

<table>
<thead>
<tr>
<th></th>
<th>Diagnostic Test</th>
<th>Achievement Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit I</td>
<td>.97</td>
<td>.95</td>
</tr>
<tr>
<td>Unit II</td>
<td>.94</td>
<td>.90</td>
</tr>
<tr>
<td>Unit III</td>
<td>.89</td>
<td>.92</td>
</tr>
</tbody>
</table>

This completes the chapter on the organization of the study. Chapter IV presents the methods of collecting and analyzing the data and the results of this analysis.
CHAPTER IV

DATA AND ANALYSIS OF DATA

Separation of Schools

The common factors of similar textbooks, units, and evaluative devices used in the two schools and discussed in Chapter V were not considered sufficient grounds for an analysis of combined data. Two teachers were involved. Their knowledge of subject matter, their teaching methods, and their laboratory facilities and equipment were in all probability not equivalent. The data from the three classes in each school district were treated separately in the analysis. That is, the data are reported, recorded, and treated as results of two separate experiments with mathematics laboratories. No attempt is made to draw statistical comparisons across schools. Mercer data are reported and analyzed first.

Control Variables—Mercer

There were two sets of control variables, the set of intelligence quotients and the set of diagnostic test scores for each class. Table 7 reports the means of these variables for each class for each unit of work at Mercer.
TABLE 7
CONTROL VARIABLE MEANS BY CLASS AND UNIT
AT MERCER

<table>
<thead>
<tr>
<th>Class</th>
<th>IQ</th>
<th>Unit I Diagnostic Test</th>
<th>Unit II Diagnostic Test</th>
<th>Unit III Diagnostic Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>111.27</td>
<td>66.73</td>
<td>86.23</td>
<td>66.42</td>
</tr>
<tr>
<td>2</td>
<td>107.56</td>
<td>65.04</td>
<td>81.30</td>
<td>73.48</td>
</tr>
<tr>
<td>3</td>
<td>96.63</td>
<td>36.67</td>
<td>38.37</td>
<td>52.44</td>
</tr>
</tbody>
</table>

It is interesting to note that the test means are ranked in the same order as the IQ means except for Unit III where class 2 has the highest mean score. Further discussion of this fact appears in the analysis of data.

Dependent Variable—Mercer

The variable being tested for effect upon achievement and motivation is the type of laboratory treatment. Table 4 in Chapter III presented the laboratory assignments for the several classes at Mercer for the three units of study. Recall that a rotation of laboratory assignments at the conclusion of each unit made it possible for each class to experience each laboratory type once during the study. A comparison of the laboratory types may be found in Chapter III.
Independent Variable—Mercer

The percentage scores of the achievement tests for each unit were vital to the analysis. Table 8 reports the class means for these scores, although more important criteria for the analysis of data are the adjusted mean scores of the achievement tests in Table 9. Table 10 records the same set of adjusted mean scores as Table 9, but maps units into laboratory type assignments. The reasons for and the methods of making these adjustments in the achievement scores are now discussed.

TABLE 8
MEAN ACHIEVEMENT TEST SCORES FOR EACH CLASS BY UNITS AT MERCER

<table>
<thead>
<tr>
<th>Class</th>
<th>Unit I</th>
<th>Unit II</th>
<th>Unit III</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>73.38</td>
<td>70.69</td>
<td>78.77</td>
</tr>
<tr>
<td>2</td>
<td>73.63</td>
<td>68.22</td>
<td>69.41</td>
</tr>
<tr>
<td>3</td>
<td>52.59</td>
<td>60.00</td>
<td>63.00</td>
</tr>
</tbody>
</table>

TABLE 9
ADJUSTED MEAN ACHIEVEMENT TEST SCORES FOR EACH CLASS BY UNITS AT MERCER

<table>
<thead>
<tr>
<th>Class</th>
<th>Unit I</th>
<th>Unit II</th>
<th>Unit III</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>68.0</td>
<td>65.6</td>
<td>77.1</td>
</tr>
<tr>
<td>2</td>
<td>70.3</td>
<td>65.0</td>
<td>64.3</td>
</tr>
<tr>
<td>3</td>
<td>61.2</td>
<td>68.1</td>
<td>69.7</td>
</tr>
</tbody>
</table>
TABLE 10

ADJUSTED MEAN ACHIEVEMENT TEST SCORES FOR EACH LABORATORY TYPE AT MERCER

<table>
<thead>
<tr>
<th>Lab Type</th>
<th>Unit I</th>
<th>Unit II</th>
<th>Unit III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-lab</td>
<td>68.0</td>
<td>65.0</td>
<td>69.7</td>
</tr>
<tr>
<td>Post-lab</td>
<td>70.3</td>
<td>68.1</td>
<td>77.1</td>
</tr>
<tr>
<td>No-lab</td>
<td>61.2</td>
<td>65.6</td>
<td>74.3</td>
</tr>
</tbody>
</table>

Analysis of Covariance--Mercer

Three F statistics, one for each of the three sets of variables in a unit, were obtained by running the scores for each of the eighty students through two consecutive computer programs on the IBM 360/40 computer at Slippery Rock State College, Pennsylvania (6). The first program which acts upon the control variables and the dependent variable is a multivariate analysis of variance called MANOVA. The output from this program becomes the input for a MANOVA with covariates called COVAR. The output of COVAR is the F statistic which relates the possibly significant effect of the laboratory treatments on the achievement scores for each unit after an attempt is made to partial out the effect of the two control variables. Tables 11, 12, and 13 report the results of COVAR for the Mercer classes.
### TABLE 11

**ANALYSIS OF COVARIANCE FOR ACHIEVEMENT DIFFERENCES AMONG THREE EXPERIMENTAL CLASSES WITH ASSIGNED LABORATORY TYPES CONTROLLING ON PRIOR MATHEMATICS ACHIEVEMENT AND INTELLIGENCE ON MERCER UNIT I**

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>df</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between</td>
<td>2</td>
<td>594.04</td>
<td>297.02</td>
<td></td>
</tr>
<tr>
<td>Within</td>
<td>75</td>
<td>15493.50</td>
<td>206.58</td>
<td>1.44</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>77</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### TABLE 12

**ANALYSIS OF COVARIANCE FOR ACHIEVEMENT DIFFERENCES AMONG THREE EXPERIMENTAL CLASSES WITH ASSIGNED LABORATORY TYPES CONTROLLING ON PRIOR MATHEMATICS ACHIEVEMENT AND INTELLIGENCE ON MERCER UNIT II**

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>df</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between</td>
<td>2</td>
<td>38.84</td>
<td>19.42</td>
<td>0.24</td>
</tr>
<tr>
<td>Within</td>
<td>75</td>
<td>5993.25</td>
<td>79.91</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>77</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TABLE 13

ANALYSIS OF COVARIANCE FOR ACHIEVEMENT DIFFERENCES AMONG THREE EXPERIMENTAL CLASSES WITH ASSIGNED LABORATORY TYPES CONTROLLING ON PRIOR MATHEMATICS ACHIEVEMENT AND INTELLIGENCE ON MERCER UNIT III

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>df</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between</td>
<td>2</td>
<td>1940.88</td>
<td>970.44</td>
<td>10.19</td>
</tr>
<tr>
<td>Within</td>
<td>75</td>
<td>7145.00</td>
<td>95.26</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>77</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For the degrees of freedom indicated, based on the number of treatments minus one and the number of students in the sample minus five, an F-ratio of at least 3.12 is required before significant differences in achievement due to laboratory treatment is indicated beyond the 0.05 level. An F-ratio of at least 4.90 indicates significance beyond the 0.01 level (29:402).

In addition to the F-scores, COVAR prints out the adjusted mean achievement scores for each class in every unit. These were recorded in two ways in Tables 9 and 10.

The F-ratios of 1.44 and 0.24 for Unit I and Unit II respectively at Mercer indicate that in neither case is there justification for rejecting the null hypothesis. However, the F-score of 10.19 for Unit III shows a significant effect upon the achievement scores due to the types of laboratory assignments. This significance is beyond the 0.01 level.
With the assumption that COVAR, as it was applied to the available data, was able to equalize the control variables, a comparison of the adjusted mean achievement scores will similarly compare the effects of the laboratory treatments. No significant differences are reported for Unit I or Unit II. For Unit III in Mercer the post-lab ranks highest, followed by pre-lab and no-lab in that order. It is interesting to note that though the differences of adjusted means were not significant for Unit I or Unit II they did share the same direction as those for Unit III. In all cases, the class with the post-lab assignment consistently had the highest mean score. Because of the rotation of laboratory assignments every class had its highest achievement when assigned the post-lab approach. The temptation is to arrive hastily at the conclusion that the post-lab approach yields superior achievement for students of all ability levels. However, only the Unit III F-score is significantly large. Since only high ability students received the post-lab treatment for Unit III, we can conclude no more than that post-lab was superior for high ability students for the topics included in Unit III.

To further test this conclusion and to possibly determine other significant differences in achievement due to treatments, a t-test was run on the adjusted mean scores to make a pair-wise comparison of the treatments for Unit III. The results of these tests are shown in Table 14.
<table>
<thead>
<tr>
<th>Pairs</th>
<th>df</th>
<th>Required t-score*</th>
<th>t-score</th>
<th>Superior Treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-Post</td>
<td>51</td>
<td>2.68</td>
<td>2.80</td>
<td>Post</td>
</tr>
<tr>
<td>Pre-No</td>
<td>51</td>
<td>2.68</td>
<td>2.07</td>
<td>—</td>
</tr>
<tr>
<td>Post-No</td>
<td>52</td>
<td>2.68</td>
<td>15.35</td>
<td>Post</td>
</tr>
</tbody>
</table>

*Required for significance at the 0.01 level; 2.01 is required at the 0.05 level.

This table clearly shows that there are significant differences in achievement for the post-lab, pre-lab comparison and especially for the post-lab, no-lab comparison. Post-lab students achieved significantly higher on the average than either of the other two classes. The t-score of 2.07 for the pre-lab, no-lab pairing was not sufficiently high to warrant any decision about superior laboratory treatments at the 0.01 level, but it did indicate significance beyond the 0.05 level in favor of the pre-lab.

**Another Analysis—Mercer**

From another point of view, utilizing standard deviations and standard scores, the total effect of the laboratory types across all three units may be observed. Within each unit the standard deviation for the distribution of adjusted achievement scores for the pooled sample is part of the COVAR print out. The differences between each
class adjusted mean and the adjusted mean for the entire sample is easily calculated. Dividing these differences by the standard deviation for the sample yields the variations of the class adjusted means from the sample adjusted means in terms of standard scores. Table 15 tabulates this information for the Mercer sample.

TABLE 15

VARIATION OF THE ADJUSTED MEAN ACHIEVEMENT SCORES FROM THE ADJUSTED MEAN OF THE POOLED SAMPLE IN TERMS OF STANDARD SCORES AT MERCER

<table>
<thead>
<tr>
<th>Lab-Type</th>
<th>Unit I Variation</th>
<th>Class</th>
<th>Unit II Variation</th>
<th>Class</th>
<th>Unit III Variation</th>
<th>Class</th>
<th>Sum of Variations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-Lab</td>
<td>0.11</td>
<td>1</td>
<td>-0.14</td>
<td>2</td>
<td>-0.06</td>
<td>3</td>
<td>-0.09</td>
</tr>
<tr>
<td>Post-Lab</td>
<td>0.27</td>
<td>2</td>
<td>0.22</td>
<td>3</td>
<td>0.71</td>
<td>1</td>
<td>1.20</td>
</tr>
<tr>
<td>No-Lab</td>
<td>-0.30</td>
<td>3</td>
<td>-0.07</td>
<td>1</td>
<td>-0.63</td>
<td>2</td>
<td>-1.00</td>
</tr>
<tr>
<td>Totals</td>
<td>0.08</td>
<td>0.01</td>
<td>0.02</td>
<td>0.11</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In all cases, the relative effects of the three laboratory treatments across all three units may be discerned by comparing the sums of the respective variations.

For the Mercer sample the expected occurs. The post-lab treatment has a larger variation sum across the three units than either the pre-lab or no-lab treatments. The pre-lab sum ranks second and the no-lab sum is smallest. Regardless of the unit studied or the general
ability level of the classes taking the post-lab experiments, the adjusted mean scores for the post-lab treatment are greater than those for the pooled sample mean in every case. For Unit III, where the COVAR F-ratio indicated significant differences in achievement, the mean variation for the post-lab is considerably larger than the sample mean.

Table 16 reports the facts and figures of Table 15, but maps classes onto laboratory treatments at Mercer. In this table one can view the sums of variations by classes across the three treatments. For the entire study, the highest ability class ranks first, the lowest ability class ranks second, and the middle ability class ranks third. The comparatively poor showing for Class 2 during the no-lab treatment seems to account for the drop to third place in total affect. The reader is reminded that Class 2 had the highest mean diagnostic score for Unit III, but their achievement was far below expectation.

<table>
<thead>
<tr>
<th>Class</th>
<th>Pre-Lab</th>
<th>Post-Lab</th>
<th>No-Lab</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.11</td>
<td>0.71</td>
<td>-0.07</td>
<td>0.75</td>
</tr>
<tr>
<td>2</td>
<td>-0.14</td>
<td>0.27</td>
<td>-0.63</td>
<td>-0.50</td>
</tr>
<tr>
<td>3</td>
<td>-0.06</td>
<td>0.22</td>
<td>-0.30</td>
<td>-0.14</td>
</tr>
<tr>
<td>Totals</td>
<td>-0.09</td>
<td>1.20</td>
<td>-1.00</td>
<td>0.11</td>
</tr>
</tbody>
</table>
Mercer Questionnaire Analysis

The questions and the percentages of students in each class who checked corresponding responses are listed in Table 17. The first student opinion expressed on the questionnaire indicated their preferences for pre-lab, post-lab, and no-lab in that order. For all three classes at Mercer there were no significant differences in the percentages of students selecting pre-lab or post-lab treatments. In each of the two upper ability classes whose IQ means are more nearly the same, only one student chose no-lab as the preferred treatment. Such was not the case for the lower ability class. Approximately one-third of the students in this class, with mean IQ less than 100, chose no-lab as their favorite type.

Responses for item two indicate that more than two-thirds of the students in the upper ability classes found mathematics laboratories made this school year more interesting. The same student in each class, who selected no-lab in item one, found the year more boring, and a few students could sense little difference due to the changed approach. The responses to this item for the lower ability class are nearly evenly divided, with a slight margin favoring the "more interesting" choice.

The medium class responded to item three as one might predict in light of their responses to items one and two. However, at least two students from the high class preferred some lab to no-lab and were not bored by the approaches, but suggest no-labs at all for next year's eighth graders. Sixty per cent of the low class suggest no-labs at
**TABLE 17**

RESPONSES TO THE STUDENT INTEREST QUESTIONNAIRE
BY PERCENTAGES FOR EACH CLASS AT MERCER

<table>
<thead>
<tr>
<th>Classes</th>
<th>High</th>
<th>Medium</th>
<th>Low</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>111.3*</td>
<td>107.6</td>
<td>96.6</td>
</tr>
</tbody>
</table>

1. The kind of lab I like best was
   a) before class discussion of the topic 52 46 30
   b) after class discussion of the topic 44 50 37
   c) no lab at all 4 4 33

2. Mathematics labs have made this year compared with last year
   a) more interesting 69 82 41
   b) more boring 4 4 26
   c) no different 27 14 33

3. I suggest that next year’s eighth graders
   a) continue having at least the same 44 30 16
      number of labs as we had
   b) have more labs than we had 44 66 24
   c) have no labs at all 12 4 60

4. These lab experiments on the whole
   a) helped me to understand mathematics 52 82 60
      better
   b) confused me more than they helped me 8 4 21
   c) made no difference in my ability to 40 14 19
      understand mathematics

5. When my teacher announced that we were
   going to have another laboratory
   experiment
   a) I looked forward to it 69 72 45
   b) I said to myself, "So what."
   31 21 33
   c) I said to myself, "No, no again."
   0 7 22

*Mean IQ to nearest tenth.
all. This change also indicates an hypothetical attitude of "Well, it was all right, but don't do it again."

Better than one-half of the students in each class at Mercer felt an increase in understanding concepts resulted from laboratory experiments. Sixty per cent for the low class is uncommonly high in light of their responses to the first three items. The fact that about one-fifth of the students in this class were confused is consistent with the previously recorded attitudes in items one and two, but inconsistent with their responses to item three. Looking ahead to item five, one can see that once again about one-fifth of the lower class registered disgust with the entire procedure. Nearly half of this class looked forward to experiments possibly because they felt it aided their understanding of mathematics. This is essentially the same group who found mathematics laboratories more interesting (item 2). Clearly, the percentages of responses to items two and five are consistent for all three classes at this school.

In general, the responses to all items reflecting positive interest and a sense of increased understanding are encouraging. However, the personal reasons for these responses are not totally known. Two contrary statements written by students who favored laboratories emphasize the complexity of analyzing the responses to this questionnaire; "I'd like to have more of them because they made me understand math a lot better," and "They got us out of part of the class--they were o.k."
The student evaluation forms (see Appendix D), which were sup-
posed to be checked by students and collected by the teachers following
each experiment, were not used with enough regularity to warrant much
discussion. Any attempt to draw conclusions from the small number of
evaluations and comments received could not be justified.

Before any conclusions based on this study at Mercer are pre-
maturely stated, an examination of the data from the Lakeview school
is in order. Data, completely analogous to that from Mercer, were col-
lected and recorded. The content of each unit for the two schools was
basically the same throughout the study.

Control Variables--Lakeview

Identical diagnostic tests were administered to the Mercer and
the Lakeview students during the study. Table 18 reports the means for
the set of intelligence quotients and the set of diagnostic test scores
for each class per unit at Lakeview.

| TABLE 18 |
| Control Variable Means by Class and Unit at Lakeview |

<table>
<thead>
<tr>
<th>Class</th>
<th>IQ</th>
<th>Unit I Diagnostic Test</th>
<th>Unit II Diagnostic Test</th>
<th>Unit III Diagnostic Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>111.89</td>
<td>68.22</td>
<td>72.93</td>
<td>72.19</td>
</tr>
<tr>
<td>2</td>
<td>102.44</td>
<td>54.39</td>
<td>45.44</td>
<td>51.61</td>
</tr>
<tr>
<td>3</td>
<td>102.95</td>
<td>54.38</td>
<td>48.14</td>
<td>63.71</td>
</tr>
</tbody>
</table>
Class 2 and Class 3 were ranked by the Lakeview teachers as medium and low ability classes respectively. Throughout this report these rankings will be used even though their mean IQ scores are essentially equal. Except for a slight variation in Unit III, the ranges of the means of the diagnostic scores for these two classes are relatively small. Class 1 consistently scored higher on the diagnostic tests.

**Dependent Variable—Lakeview**

Table 6 in Chapter III shows the order of laboratory assignments and their rotations during the study. The correspondence between types of laboratory approaches initially assigned for Unit I and the ability levels of the classes differs from that of Mercer by one rotation. This was not a result of design since assignments were randomly made by the respective teachers. The point is, no attempt was made to duplicate the treatment assignments at the two schools.

**Independent Variable—Lakeview**

Table 19 tabulates the class means for achievement test scores for each unit at Lakeview. The more revealing adjusted mean achievement scores appear in Tables 20 and 21. These tables contain the same figures, but Table 20 maps classes into units while Table 21 maps laboratory types into units.
### TABLE 19
MEAN ACHIEVEMENT TEST SCORES FOR EACH CLASS
BY UNITS AT LAKEVIEW

<table>
<thead>
<tr>
<th>Class</th>
<th>Unit I</th>
<th>Unit II</th>
<th>Unit III</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>70.56</td>
<td>90.22</td>
<td>70.19</td>
</tr>
<tr>
<td>2</td>
<td>41.39</td>
<td>45.67</td>
<td>48.06</td>
</tr>
<tr>
<td>3</td>
<td>39.29</td>
<td>64.43</td>
<td>56.67</td>
</tr>
</tbody>
</table>

### TABLE 20
ADJUSTED MEAN ACHIEVEMENT TEST SCORES FOR EACH
CLASS BY UNITS AT LAKEVIEW

<table>
<thead>
<tr>
<th>Class</th>
<th>Unit I</th>
<th>Unit II</th>
<th>Unit III</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>61.5</td>
<td>87.1</td>
<td>62.9</td>
</tr>
<tr>
<td>2</td>
<td>47.8</td>
<td>48.1</td>
<td>55.8</td>
</tr>
<tr>
<td>3</td>
<td>45.5</td>
<td>66.4</td>
<td>59.3</td>
</tr>
</tbody>
</table>
### TABLE 21

**ADJUSTED MEAN ACHIEVEMENT TEST SCORES FOR EACH LABORATORY TYPE AT LAKEVIEW**

<table>
<thead>
<tr>
<th>Lab-Type</th>
<th>Unit I</th>
<th>Unit II</th>
<th>Unit III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-Lab</td>
<td>45.5</td>
<td>87.1</td>
<td>55.8</td>
</tr>
<tr>
<td>Post-Lab</td>
<td>61.5</td>
<td>48.1</td>
<td>59.3</td>
</tr>
<tr>
<td>No-Lab</td>
<td>47.8</td>
<td>66.4</td>
<td>62.9</td>
</tr>
</tbody>
</table>

**Analysis of Covariance—Lakeview**

MANOVA and COVAR were consecutively run on the Lakeview data. The number of students in this sample was 66. Thus, the degrees of freedom for within variation was 61. An F-ratio of at least 3.14 indicates significance of treatment beyond the 0.05 level while an F-ratio of 4.95 or greater means significant differences in achievement are attributable to laboratory treatments at the 0.01 level of probability (24:402). COVAR results for Lakeview were printed in Tables 22, 23, and 24.
### TABLE 22

**Analysis of Covariance for Achievement Differences Among Three Experimental Classes With Assigned Laboratory Types Controlling on Prior Mathematics Achievement and Intelligence at Lakeview Unit I**

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>df</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>( f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between</td>
<td>2</td>
<td>2711.54</td>
<td>1355.77</td>
<td>6.89</td>
</tr>
<tr>
<td>Within</td>
<td>61</td>
<td>12007.85</td>
<td>196.85</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>63</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### TABLE 23

**Analysis of Covariance for Achievement Differences Among Three Experimental Classes With Assigned Laboratory Types Controlling on Prior Mathematics Achievement and Intelligence at Lakeview Unit II**

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>df</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>( f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between</td>
<td>2</td>
<td>10972.32</td>
<td>5486.16</td>
<td>26.57</td>
</tr>
<tr>
<td>Within</td>
<td>61</td>
<td>12594.06</td>
<td>206.46</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>63</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### TABLE 24

**Analysis of Covariance for Achievement Differences Among Three Experimental Classes With Assigned Laboratory Types Controlling on Prior Mathematics Achievement and Intelligence at Lakeview Unit III**

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>df</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>( f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between</td>
<td>2</td>
<td>362.66</td>
<td>181.33</td>
<td>0.88</td>
</tr>
<tr>
<td>Within</td>
<td>61</td>
<td>12520.86</td>
<td>205.26</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>63</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
For Units I and II at Lakeview, the F-ratios of 6.89 and 26.59 were both sufficiently large to register significance beyond the 0.01 level. Referring back to Table 21, and comparing adjusted mean achievement scores, it was found that for Unit I the post-lab treatment was superior. Since only high ability students received the post-lab treatment for Unit I, we can only conclude that post-lab was superior for high ability students for the topics included in Unit I.

An inspection of the adjusted means for Unit II at Lakeview shows a different pattern. As a matter of fact, the post-lab has the smallest adjusted mean among the three types. The pre-lab class had an extremely high adjusted mean of 87.1 and the no-lab students ranked second with an adjusted mean of 66.4. Only high ability students received the pre-lab treatment for Unit II, and so we can only conclude that pre-lab was superior for high ability students for the topics included in Unit II.

The F-score for Unit III is so low that no comparison of the equal adjusted means is warranted.

A pair-wise comparison of the adjusted mean achievement scores for each treatment for Unit I and Unit II presents a clear view of the treatment relationships. The t-scores resulting from these comparisons are presented in Tables 25 and 26.
TABLE 25

t-SCORES FROM PAIR-WISE COMPARISONS OF TREATMENTS
FOR UNIT I AT LAKEVIEW

<table>
<thead>
<tr>
<th>Pairs</th>
<th>df</th>
<th>Required t-score*</th>
<th>t-score</th>
<th>Superior Treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-Post</td>
<td>46</td>
<td>2.40</td>
<td>3.74</td>
<td>Post</td>
</tr>
<tr>
<td>Pre-No</td>
<td>37</td>
<td>2.43</td>
<td>0.05</td>
<td>--</td>
</tr>
<tr>
<td>Post-No</td>
<td>43</td>
<td>2.42</td>
<td>3.26</td>
<td>Post</td>
</tr>
</tbody>
</table>

*Required t-score for significance at the 0.01 level.

TABLE 26

t-SCORES FROM PAIR-WISE COMPARISONS OF TREATMENTS
FOR UNIT II AT LAKEVIEW

<table>
<thead>
<tr>
<th>Pairs</th>
<th>df</th>
<th>Required t-score*</th>
<th>t-score</th>
<th>Superior Treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-Post</td>
<td>43</td>
<td>2.42</td>
<td>9.07</td>
<td>Pre</td>
</tr>
<tr>
<td>Pre-No</td>
<td>46</td>
<td>2.40</td>
<td>5.01</td>
<td>Pre</td>
</tr>
<tr>
<td>Post-No</td>
<td>37</td>
<td>2.43</td>
<td>4.04</td>
<td>No</td>
</tr>
</tbody>
</table>

*Required t-score for significance at the 0.01 level.

The superiority of the post-lab treatment over both pre-lab and no-lab for Unit I is quite evident from the respective t-scores of 3.74 and 3.26. Class 3 with a mean IQ of 102.95 and a diagnostic test mean of 54.38 was assigned pre-lab. Class 2 with a mean IQ of 102.44 and a diagnostic test mean of 54.39 was assigned no-lab. The equivalent
control variables and the nearly equivalent adjusted mean achievement scores of 45.5 and 47.8 indicate no significant difference between pre-lab and no-lab treatments for Unit I. A t-score of 0.05 verifies this conclusion.

For Unit II both the pre-lab class and the no-lab class achieved more satisfactorily than the post-lab students. The respective t-scores for these comparisons of 9.07 and 4.04 are sufficiently high to indicate significance beyond the 0.01 level. It is worthwhile to note that pre-lab students achieved significantly better than no-lab students as indicated by a t-score of 5.01. Thus, even though post-lab treatment appeared least effective for Unit II, pre-lab was more beneficial than no-lab for this unit. Again, the high ability class was assigned the laboratory treatment which proved most effective for Unit II.

Another Analysis—Lakeview

Tables 27 and 28 for Lakeview are analogous to Tables 15 and 16 for Mercer. The sums of the variations of adjusted means for each treatment from the adjusted mean for the pooled sample are found across all three units in Table 27.
TABLE 27

VARIATION OF THE ADJUSTED MEAN ACHIEVEMENT SCORES FROM THE ADJUSTED MEAN OF THE POOLED SAMPLE IN TERMS OF STANDARD SCORES AT LAKEVIEW

<table>
<thead>
<tr>
<th>Lab-Type</th>
<th>Unit I</th>
<th>Unit II</th>
<th>Unit III</th>
<th>Sum of Variations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Variation</td>
<td>Class</td>
<td>Variation</td>
<td>Class</td>
</tr>
<tr>
<td>Pre-Lab</td>
<td>-0.52</td>
<td>3</td>
<td>1.20</td>
<td>1</td>
</tr>
<tr>
<td>Post-Lab</td>
<td>0.64</td>
<td>1</td>
<td>-1.54</td>
<td>2</td>
</tr>
<tr>
<td>No-Lab</td>
<td>-0.36</td>
<td>2</td>
<td>-0.24</td>
<td>3</td>
</tr>
<tr>
<td>Totals</td>
<td>-0.24</td>
<td>-0.58</td>
<td>-0.10</td>
<td>-0.92</td>
</tr>
</tbody>
</table>

According to the sums of variations in this table, the ranking of the laboratory types in descending order of contributing effect on achievement was pre-lab, no-lab, and post-lab. Even though the post-lab rated highest in Unit I and nearly equal to the average of the others in Unit III, this treatment suffered a particularly hard setback during Unit II. One can only speculate on the reasons for this unexpected decline in performance (see Chapter V).

Table 28, mapping the variations by classes onto treatments, indicated that the highest ability class performed best regardless of the laboratory type assigned. Class 3 rated second in overall achievement, and Class 2 was last in the ranking. Class 2 had a low variation score of -1.54 for post-lab treatment which largely accounts for the low
overall showing of these students and for the negative sum of variations for post-lab across units.

**TABLE 28**

**VARIATION OF THE ADJUSTED MEAN ACHIEVEMENT SCORES FROM THE ADJUSTED MEAN OF THE POOLED SAMPLE MAPPING LAKEVIEW CLASSES ONTO TREATMENTS**

<table>
<thead>
<tr>
<th>Class</th>
<th>Pre-Lab</th>
<th>Post-Lab</th>
<th>No-Lab</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1.20</td>
<td>0.64</td>
<td>0.22</td>
<td>2.06</td>
</tr>
<tr>
<td>2</td>
<td>-0.28</td>
<td>-1.54</td>
<td>-0.36</td>
<td>-2.18</td>
</tr>
<tr>
<td>3</td>
<td>-0.52</td>
<td>-0.04</td>
<td>-0.24</td>
<td>-0.80</td>
</tr>
<tr>
<td>Totals</td>
<td>0.40</td>
<td>-0.94</td>
<td>-0.38</td>
<td>-0.92</td>
</tr>
</tbody>
</table>

**Lakeview Questionnaire Analysis**

Table 29 presents the results of the interest questionnaire for the Lakeview students. The responses for Classes 2 and 3 with comparable IQ scores are similar for all questions with Class 3 registering slightly higher on an attitude scale. A considerable number of students from these classes were bored by the laboratory experiments (item 2), and perhaps this is why they preferred no-lab (item 1). These same students recommended no-labs for next year's classes (item 3) perhaps due to their general feeling of disgust for laboratory experiments (item 5). Confusion seemed to play a major role for the one-fifth of Class 2 with this relatively negative reaction to the program (item 4).
TABLE 29
RESPONSES TO THE STUDENT INTEREST QUESTIONNAIRE BY PERCENTAGES FOR EACH CLASS AT LAKEVIEW

<table>
<thead>
<tr>
<th>Classes</th>
<th>High</th>
<th>Medium</th>
<th>Low</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>111.9*</td>
<td>102.4</td>
<td>103.0</td>
</tr>
</tbody>
</table>

1. The kind of lab I like best was
   a) before class discussion of the topic 74 19 40
   b) after class discussion of the topic 23 38 44
   c) no lab at all 3 43 16

2. Mathematics labs have made this year compared with last year,
   a) more interesting 80 46 36
   b) more boring 3 16 28
   c) no different 17 38 36

3. I suggest that next year's eighth graders
   a) continue having at least the same number of labs as we had 51 34 52
   b) have more labs than we had 46 45 28
   c) have no labs at all 3 21 20

4. These lab experiments on the whole
   a) helped me to understand mathematics better 71 65 64
   b) confused me more than they helped me 0 19 8
   c) made no difference in my ability to understand mathematics 29 16 28

5. When my teacher announced that we were going to have another laboratory experiment
   a) I looked forward to it 51 50 28
   b) I said to myself, "So what." 40 34 52
   c) I said to myself, "No, not again." 9 16 20

*Mean IQ to nearest tenth.
Perhaps a more positive approach to the percentages in Table 29 will be more revealing and encouraging. For Class 1, little needs to be said; the record speaks for itself. Seventy-four per cent of Class 1 chose pre-lab first, and this class had their highest mean achievement scores with pre-lab, Unit II. Even though Class 2 performed so poorly with post-lab treatment, 38 per cent of them preferred this type of approach. From selections (a) and (b) of item three we see that nearly 80 per cent of this class recommends at least as many experiments for next year's classes. Sixty-five per cent were of the opinion that their understanding of mathematics was improved by the laboratory experiments (item 4), and half of this group looked forward to the next laboratory experience during the study (item 5).

A summary of the study and further interpretations of the data are included in Chapter V.
CHAPTER V

SUMMARY AND INTERPRETATIONS

The Problem

The school year which the academically maturing, junior high school students spend in the eighth grade is a very important year. Their experiences during this year directly influence the decisions for future endeavors which they are called upon to make before entering the ninth grade. Students in this age group are subject to split-second decisions and spontaneous actions largely based on recent experiences. If their decisions for the future are to lead satisfactorily to mental and social productivity, these students must be effectively challenged, motivated, and taught by their eighth grade teachers. The teachers must be properly prepared for this task, and their search for improved pedagogical techniques must never cease.

In their search for better methods, the mathematics teachers of three junior-senior high schools in Mercer County, Pennsylvania decided during a seminar to weigh the merits of mathematics laboratories. Two eighth grade teachers in particular from distinct school districts were interested enough in the further study of the effectiveness of mathematics laboratories to volunteer their services. The author, who was invited to preside at the seminar, and the two volunteers proceeded to establish a program of inquiry and experimentation.
Related Literature

Before any attempts were made to include the use of mathematics laboratories in their classrooms, some pertinent literature was reviewed. The perusal of available literature provided useful guidelines for the implementation of laboratory experiments in mathematics classrooms.

It was discovered that most of the published material pertaining to the subject was directed toward teachers and students in the elementary grades. However, comprehension of the abstract nature of mathematics is often difficult for older students also. Mathematical models of all kinds may provide an avenue of return from the concrete to the abstract.

Some equipment for mathematics laboratories has been commercially prepared and tested. Examples of this type are desk-top computers, logic puzzles, games, geoboards, and Cuisenaire rods. Generally the bridges from the concrete to the abstract have been constructed and traversed with some degree of success using these and similar models. The equipment which other teachers have successfully used to motivate students to actively participate in the process of learning mathematical concepts consists simply of those physical items which are either already in or are brought by the students into the ordinary classroom.

Those teachers who have been able to ask the proper questions of their students leading them to discover basic concepts, regardless of the type of equipment employed, teachers who have allowed the students to use their ingenuity to expand and extend these initial ideas, have
indicated that laboratory techniques do enhance the learning process at the elementary school level.

A few teachers of junior high school children have attempted to test the effectiveness of mathematics laboratories. The results of these informal evaluations are similar to those involving elementary students. Thus, the conclusion that properly used laboratory techniques will probably produce positive results is supported by nearly all of the reviewed, pertinent literature. But this literature primarily reports informal studies.

It was also discovered that the idea of using laboratory methods in a mathematics class is neither restricted to schools in the United States nor is it a recently developed idea. European textbooks, some written over 200 years ago, include adaptable ideas for contemporary experiments.

**Organization of the Study**

After approximately five months of searching, planning, attempting, and refining, an organized plan to test the effectiveness of laboratory techniques was put into operation by the two eighth grade teachers at the two Mercer County school districts.

Three arbitrarily chosen classes from the two eighth grades were randomly assigned distinct laboratory approaches for a six weeks unit of work on fractions, ratios, and proportions. These classes were homogeneously grouped according to overall academic achievement. This grouping is standard procedure in both schools and necessitated that
intact classes with differing intelligence averages be involved in the study.

Two more units of work on geometry, each approximately six weeks in length, were completed using laboratory techniques. The class assignments to distinct laboratory approaches were rotated for each unit of work. Thus, every class had the opportunity to study mathematics for six weeks under each laboratory approach.

The three laboratory approaches were called pre-lab, post-lab, and no-lab. For pre-lab and post-lab several similar experiments per unit were incorporated into the class sessions. The laboratory was the classroom, and the equipment for the experiments consisted of environmental gadgets. The differences between pre-lab and post-lab were accentuated by the types of leading questions asked by the teachers. In pre-lab, predesigned experiments were used in the hope of guiding the students to conjecture about the concepts to be discussed in following recitation sessions. In post-lab, less teacher guidance was involved while materials for student innovated applications of previously discussed concepts were sought. The teachers played an extremely important part in making the laboratory approaches distinctive as they initiated the use of the tools and motivated the students through their questioning to discover more ideas and/or applications. The no-lab approach was the standard classroom procedure of five class recitations per week with no experiments involving student participation included.

The null hypothesis that was tested statistically was that there is no significant difference in achievement between eighth grade
classes taught either without laboratory techniques or with one of the two direct laboratory approaches.

**Statistical Analysis of Data**

The data collected to test this null hypothesis, for the 146 students in the sample, included recently obtained intelligence quotients, diagnostic test scores for each unit recorded from teacher-made tests administered before each unit began, and achievement test scores for each unit derived from teacher-made tests administered at the conclusion of each unit. The class averages from each of these sets of scores provided the input for a computer calculated analysis of covariance called COVAR.

The two control variables, which the COVAR attempted to equalize for each class per unit, were the IQ scores and the scores on the unit diagnostic tests. The independent variable was the laboratory approach and the dependent (criterion) variable was the unit achievement test scores. An F-ratio was printed out for each unit of work at each school district. Of the six resulting F-scores, three were of sufficient magnitude to warrant a rejection of the null hypothesis beyond the 0.01 level.

For the Lakeview students significant differences in achievement due to laboratory treatment were indicated for Unit I and Unit II, while for the Mercer students the significant F-score occurred for Unit III.

Pair-wise comparisons of the adjusted mean achievement scores were made for the classes in the three cases with large F-ratios. The
post-lab treatment was most effective for Unit III at Mercer and for
Unit I at Lakeview. The pre-lab treatment was most effective for Unit
II at Lakeview. It is interesting that the class with the post-lab ap-
proach for this Unit II had the lowest adjusted mean achievement score
of all three classes. Naturally, this generates a question regarding
the reasons for this complete reversal of the achievement rankings.

The reversal of the effectiveness of laboratory patterns for Unit
II at Lakeview should caution us that changes in other variables can
change the laboratory treatment effect. These other variables include
the degree of difficulty of the unit being studied and the teacher-
student rapport which directly influences attitudes of the students.

Possibly due to the relative ease of relating the basic geometric
concepts in Unit II to the previously studied and intuitively under-
stood geometry, and perhaps, due to more enthusiastic teacher involve-
ment because a student teacher began assisting during this unit, the
achievement average for the pre-lab class was considerably higher than
for either of the other two classes. Enthusiasm for a different sub-
ject area, which eighth grade students usually exhibit, may have been
unsatisfactorily enhanced because of the delayed physical applications
for the post-lab class. By the time experiments were scheduled a pas-
sive attitude leading to poor achievement may have resulted. When this
relatively low ability class was asked to select the laboratory type
they most preferred, 19 per cent of them chose pre-lab, 38 per cent
selected post-lab, and 43 per cent of them preferred no-lab. This gen-
eral disenchantment with experiments in mathematics is consistent with
the relatively poor showing class 2 made throughout the study (refer to Tables 20 and 28).

*Limitations of COVAR for this Study*

Elashoff (10:383) calls analysis of covariance a delicate instrument. Several assumptions are listed in this revealing article which must be met before the results of COVAR can be accepted without reservations. The first of these assumptions deals with random selections. Covariance assumes a random selection of students for the classes and random assignments of classes to treatments. However, since this situation is rarely possible in educational research, Elashoff states that this analysis may be used with caution if intact classes are used, but treatments are randomly assigned. This is the situation for this study as necessitated by the actual schools' environments.

There is a possibility that COVAR could not remove all of the bias caused by differing ability levels among the three classes at each school. Of course, the range of ability levels from the lowest possible to the highest possible in each school could have caused unalterable bias. This was not the case. The two eighth grade classes with highest ability at Mercer were not considered nor was the highest ranked class at Lakeview. Whether or not the resulting ranges of ability are too great to be overcome is a matter of judgment.

As can be easily discerned from Table 20, the highest ability class at Lakeview had the highest adjusted mean achievement regardless of the laboratory treatment employed. It was because of this
occurrence that a second view of the effects of laboratory treatments was made using the amounts of variation of the adjusted mean achievement scores from the adjusted mean for the total sample in terms of standard scores. After these variations were calculated for each class per unit, they were summed across all three units. These sums were then ranked to compare effects of laboratory treatments. As might be expected, these rankings did not completely clarify the analysis. The Mercer scores (see Table 15) show a decided advantage for those students taking the post-lab treatment. This confirms the analysis by COVAR. Lakeview sums of variations (see Table 27) ranked treatments in descending order of effect as pre-lab, no-lab, post-lab.

Apparently this resulted from a combination of two factors. One, the so-called medium class made an extremely poor showing for Unit II under the post-lab treatment, and two, the bias of intelligence could not be completely overcome by COVAR resulting in a relatively good score for the highest ability class in Unit III under a no-lab treatment.

Another assumption which, according to Elashoff, must be met is that within each treatment, criterion scores have a linear regression on the control variable scores. Also, the slope of the regression lines should be the same for each treatment. To test for these requirements, scattergrams plotting criterion scores against control variable scores were made for all three treatments. In the judgment of the author, there appears to be no serious violation of these
assumptions regarding linearity. The basis for assuming at least some reliability on the results of COVAR is Elashoff's statement,

> Generally, violation of the assumptions of linearity, homogeneity of regressions, normality, or homogeneity of variances will be less serious if individuals have been assigned to the treatments at random and the x variable has a normal distribution (10:398).

The laboratory treatments were initially assigned at random and then assignments were rotated for each new unit. The two control variables, IQ scores and diagnostic test scores, were plotted on distribution curves to check for normality. The only seriously skewed distribution is for the Mercer Class 2, Unit III diagnostic test which was decidedly skewed negatively. In other words, this class performed exceptionally well on this test. This class was assigned the no-lab approach for Unit III, and their raw achievement test mean score was ranked second as one might expect for this medium ability group (see Table 8). However, when COVAR adjusted the achievement means, this class fell to the lowest ranked class. Apparently the good performance on the diagnostic test followed by average or below average achievement influenced the adjustment. Expectations were not met by this class. Even the low ability class relatively overachieved the medium ability class for this unit. This tends to substantiate that including some laboratory experiments, regardless of their type, motivates the students to higher achievement than does totally excluding experiments. The question of COVAR's ability to make the proper adjustment in spite of the skewed distribution is still an open one.
Final Interpretations

1. Laboratory methods of teaching mathematical concepts to eighth grade students, where they are encouraged to actively participate in the manipulation of devices for discovery purposes, can produce higher achievement test scores than does a traditional, no laboratory method of teaching.

2. Of the two laboratory approaches, pre-lab before class recitations and post-lab after class discussions, the post-lab approach more often than not produces higher achievement test scores. This conclusion is based on the COVAR analysis and also on the summation of effects using standard score evaluations.

3. Due to hard to measure variables such as students' personal problems, excessive extra-curricular activities, student-teacher rapport, and the degree of challenge in the laboratory experiments, the reactions of eighth graders to any laboratory approach are somewhat unpredictable. The skill of the teacher to guide, but not over-guide, and the amount of contagious enthusiasm exhibited by the instructor seem to be very important elements in the classroom procedures.

4. Students with average or above average ability almost unanimously prefer some type of mathematics laboratory activity. Nearly one-third of the lower ability students, with IQ scores less than 100, prefer no laboratory experiments of the discovery type. Perhaps this is because they become confused by the absence of specific instructions. The hypothesis that the majority of eighth grade students, regardless of their ability levels, prefer some experiments in the classroom is accepted as valid.
5. Approximately two-thirds of the students in the sample at each school were of the opinion that laboratory experiments were helpful in explaining mathematical concepts. Nearly one-fourth of the higher ability students were not aware of any appreciable improvement in their abilities to comprehend the concepts through experiments, but they prefer laboratory methods to traditional class recitations.

6. The majority of the students in the sample, regardless of their ability levels, stated that they looked forward to the next experiment. Thus, the pre-lab or post-lab approach to discovering mathematical concepts through discovery experiments acted, at least, as motivating factors for most eighth grade students in this study.

7. Generalizing from a specific set of experiments is risky. When judging the effects of mathematics laboratories on students at Mercer and Lakeview schools, one must judge the internal validity of this study. When one attempts to decide to what other populations these results can be applied in general, external validity is involved. Campbell and Stanley warn that this type of guessing at laws involving situations similar to those studied must be done with caution.

The experiments we do today, if successful, will need replication and cross validation, at other times under other conditions before they can become an established part of science, before they can be theoretically interpreted with confidence (4:3).
Recommendations for Further Study

With the cautionary words of Campbell and Stanley uppermost in the author's mind, the following recommendations are made for related studies.

1. A research study, similar to the one described in this paper, could be conducted with junior high school classes whose students are selected completely at random from a large student population and whose laboratory treatments are also randomly assigned. The results of an analysis of covariance on such a study would be more reliable, and could be used to either confirm or deny the conclusions of this study.

2. Special effort could be made to design effective experiments for a mathematics laboratory leading to a discovery of basic concepts for classes with above average ability. Apparently these students can be highly motivated to eagerly explore mathematics if the experiments are challenging enough and effectively designed to make them worth the time and effort.

3. A study could be made to improve the standard experiments for low achievers to encourage them to initiate trials of their own making for the sake of conjecturing about generalizations. The effect of these refined experiments may be compared through statistical analysis with the results of the cookbook variety commonly used with lower ability students.

4. Elementary and junior high school students who have been repeatedly exposed to mathematics laboratories over a period of three or more years may become so accustomed to this method that they passively
participate. Is an approach alternating laboratories and class recitations within each year's study better than alternating methods annually? The results of a carefully planned study to answer this questions could prove very valuable to the coordinators of mathematics curricula.
BIBLIOGRAPHY

1. Armstrong, J. R. Representation modes as they interact with cognitive and mental development of the retarded to promote mathematical learning. A research report presented at the annual meeting of the AERA, Los Angeles, 1969.


21. Low Achievement Motivation Project, 1164 26th Street, Des Moines, Iowa 50311.


UNIT I

EXPERIMENT A.

The only material needed for this simple experiment is a quantity of paper, preferably colored construction paper.

Each student selects one sheet which represents one whole part. The paper is torn in half, using a folding process to find a dividing line. The phrase "dividing line" is emphasized to equate the ideas of halving the paper and dividing it into two equal parts.

One of the halves is again folded and halved. What part of the whole is represented by one of the smallest pieces of paper? How do you know that it is one-fourth? Are you dividing one by four or are you taking one-half times one-half? Is there any difference in these two operations?

Now, divide one-fourth by two or halve a fourth. What fractional part of the whole do you have now in the smallest portion?

If we were to continue taking one-half of the smallest pieces of paper on our desks, would the pieces ever become larger? What is $\frac{1}{2} \times \frac{1}{2}$? If we multiply any number by one-half, will the product ever be a number larger than one multiplicand?

What would happen if we had started by tearing our paper into three equal parts? How would we accomplish this? How can we obtain a piece of paper representing $\frac{1}{9}$ of the whole? $\frac{1}{27}$?

Can you figure out how to take two-thirds of a whole by folding and tearing? What about $\frac{4}{9}$? Compare the products with the multiplicands in each case. Try another fractional value such as three-eighths.
UNIT I

EXPERIMENT B.

Using strips of construction paper and the methods of Experiment A, fasten pieces of paper together to form two strips measuring 1 1/4 and 2 3/8 units. Now, by folding these strips, divide them into eighths of a unit. How many eighths are there in 1 1/4? In 2 3/8?

If we combine all of these eighths together by placing the strips end to end, how many eighths do we have? How many units do we have? What is the sum of 1 1/4 + 2 3/8? Can you tell me why we chose to divide the units into eighths?

Start again with new strips representing 1 1/3 and 2 5/6 units. What should we use as a common fractional part? Why use sixths? Could we use twelfths? Why don’t we?

What would you use if you wished to find the sum 1 1/3 + 1 1/4? 1 1/3 + 2 3/8? 2 5/6 + 2 3/8?
Fractional parts of a unit measure can be related to fractional parts of one hundred units. The required apparatus consists of any stick divided into equal parts such as a one foot ruler, a meter stick, and several pieces of string three or four feet in length.

On a flat surface, the floor if necessary, form a triangle using the meter stick as one side and two pieces of string as the other two sides. The strings may or may not be the same length. Call the point where the strings meet, point P. The meter stick and point P are to remain fixed throughout this experiment.

Select a denominator for your fractions such as eight. Place the ruler parallel to the meter stick with its left end on the one string side of the triangle and move it up or down so that the number eight falls on the other string. See the picture above.
From $P$ stretch strings cutting the ruler at points 1, 2, 3, 4, 5, 6, 7. Each of these strings will cut the meter stick at a distinct numeral. $1/3 = ?/100, 3/8 = ?/100$, etc.

Select other denominators such as five, six, seven or eleven. If a measuring stick longer than a ruler is available, try 16 or 20 as a denominator.

Check your results by long division.
UNIT I

EXPERIMENT D.

A spherical object, such as a tennis ball, is to be rolled down an inclined plane. If possible use a trough made by folding material such as aluminum flashing. Measure a definite length, perhaps six feet, along the trough clearly marking both endpoints. With a stop watch, time the roll of the ball from one mark to the next. Do not push the ball, release it from a standstill at the top mark. Perform this part of the experiment four or five times without changing the angle of the inclined plane. Average the times recorded and use this average in the following calculations.

If the ball rolls $s$ feet in $t$ seconds, how many feet does it average per second? How many inches per second, yards per second, feet per minutes, feet per hour, miles per hour? Use the resulting formulas to calculate the speed of your ball.

Increase the slope of the inclined trough and see what happens to the average speed of the ball. If we raise the one end twice as high as it was originally, do we double the average speed of the ball? Try it three times higher than the first position. Can you determine the average speed of the ball if it is dropped from a specific height to the floor? Why do we use the phrase, average speed? Is the actual speed of the ball ever less than the average speed? Is it ever more than the average speed? When?
Everyone knows how to play tic-tac-toe using 0's and X's. This is a similar game using fractions. The idea is to take turns placing fractions whose values are less than one in the boxes of a three by three square attempting to get a sum of one either vertically, horizontally, or diagonally. Your opponent will attempt to block your efforts by carefully selecting his fractions and at the same time try to get a sum of one for himself.

Other restrictions to make the game more interesting are to use only fractions less than one-half, or to change the sought after sum to $5/8$, or $5/4$, or $3/2$.

Decimal fractions may also be used effectively in this game. It is interesting to mix fractions, decimals, and percents in the same game.

Computational skills decidedly improve as various versions of this experiment are repeatedly employed.

Can you think of other games like these?
UNIT II

EXPERIMENT P.

Protractors and straight edges are commonly used tools for mathematical exercises in an experimental setting. Some useful experiences to promote discovery are,

1. Measure the size of a given angle.

2. Construct an angle of specific size.

3. Construct an angle whose measure is the sum of two given angles.

4. Construct two intersecting lines and measure the four resulting angles.

5. Construct a triangle and measure the three angles. Find the sum.

6. Measure an exterior angle of the same triangle. Compare its size with the sum of the two non-adjacent interior angles.

7. Sketch two lines which appear to be parallel. Cut them with a transversal. Measure the resulting angles. Find equal measures.

8. Construct parallel lines using equal corresponding angles.

9. Construct a right angle. Construct a right triangle. Measure the two smaller angles. Find the sum of these two measures.

10. Use your protractor to draw a circle. Draw two radii forming an angle of 80°. Draw two chords from a fixed point on the circle to the endpoints of these radii. Measure the angle formed by these two chords. Try it again with other chords.
UNIT II

EXPERIMENT 6.

Gather numerous circular objects such as jar lids, plastic covers, paper plates, paper or plastic cups, small or large wheels, etc.

Using a flexible measuring tape, such as one used in sewing, to estimate the circumferences of the circular objects collected. Carefully measure the largest chord possible (the diameter) of each object. Record these in a table with columns for Circumference and Diameter. Using long division, find the ratio of the circumference to the diameter for each of the objects.

Should your answers be the same for all cases?

Should they all be 22/7, 3.14, or something else?


Why do we use the Greek letter π to represent the ratio of the circumference to the diameter?

What ratio was used in Biblical times? See I Kings 7:23.
EXPERIMENT II.

The construction of parallel rulers using strips of light cardboard connected with brass fasteners to allow for flexible joints, can be interesting and educational.

On two strips eight inches long and one inch wide, center two points six inches apart. Cut two other strips five inches in length and center two points each three inches apart. Join these strips with the fasteners at the points marked with the two equal length strips opposite each other.

What figure is formed? If the angle sizes are changed by pushing sideways on the figure, are the opposite sides still parallel? Measure the angles of a figure formed after marking on a sheet of paper along the edges of your parallel ruler. Are the corresponding angles equal in measure?

Given a line L and a point P not on L, can you use your ruler to construct a line M through P which is parallel to line L? How many such lines do you suppose can be thus drawn?

Sketch an angle of any convenient size on a sheet of paper. On the same paper draw two lines with your parallel ruler each of which is parallel to one of the rays of the given angle. Extend these lines until they intersect to form four angles. Measure these angles. How do they relate to the original angle you sketched? Which pairs of rays could you select to make the newly constructed angle congruent to the original? How do their directions compare with those of the original? Is there just one answer to the preceding question?
UNIT II

EXPERIMENT I.

The special materials needed for the following exercises are scissors and sheets of paper. Heavy wax paper is most suitable since the creases become white lines and its transparency makes it easier to superimpose points and lines.

Construct, by folding the wax paper, each of the following geometric figures.

1. straight lines
2. perpendicular lines
3. perpendicular bisector of a line segment
4. parallel lines
5. angles
6. angle bisectors
7. triangles with each angle bisected
8. triangles with three altitudes
9. triangles with three perpendicular bisectors of the respective sides
10. triangles with three medians
11. rectangles and squares
12. equilateral triangle.

These paper folded constructions are quite simple except for the equilateral triangle. Follow the steps outlined below to assure success.
1. Construct a pair of lines \( K \) and \( L \) perpendicular at point \( P \).

2. Construct two lines \( M, N \) parallel to \( L \) cutting \( K \) at \( Q \) and \( R \) so that \( PQ = QR \).

3. Fold the paper so that point \( R \) falls on line \( M \) at \( R' \) and the crease goes through point \( P \). The crease cuts line \( N \) at \( T \).

4. Now, \( PT \) is one side of the equilateral triangle and the other vertex \( S \) on line \( L \) can be easily located by folding \( T \) down to line \( L \) on a crease along \( PR' \). Triangle \( STP \) is equilateral. Can you explain why this construction works?
UNIT II

EXPERIMENT J.

The materials for this laboratory experiment are pick-up sticks and bits of plastic clay. The objective is to construct polygons of various sizes and shapes. Tinkertoy sticks and wheels can also be used effectively, but their flexibility is more restricted.

1. Construct an equilateral triangle. Is it also isosceles? Construct a second equilateral triangle using one of the sides of the first triangle as a base. Construct another with the same vertex shared by the first two. Do you see anything special about the positions of the sides? Are any of them collinear? Continue to construct triangles four and five. Can you make a sixth? How many sticks are needed to complete the sixth triangle? What is a hexagon? What is a regular hexagon?

2. Construct a parallelogram. How do you make it into a rectangle? Are the diagonals of a rectangle equal? Check those of a general rhombus. Are the diagonals of a rhombus perpendicular?

3. Construct a triangle by putting together in a line an even number of sticks on each side. Use a different number for each side such as 2, 4, and 6. Using as many sticks or parts of sticks as needed, join the midpoints of the sides forming a smaller triangle. Do you need any fractional parts of the sticks? How do the perimeters of the two triangles compare? Are there any parallel lines in the figure? How do the angles of the two triangles compare?
UNIT III

EXPERIMENT K.

On sheets of cardboard cut from sides of cardboard cartons, fasten a sheet or sheets of graph paper. By using thumb tacks and elastic strings, form various polygons such as triangles, squares, rectangles, parallelograms, rhombuses, regular hexagons, etc. Estimate the area of each figure by counting the squares enclosed by its sides.

Diagonals of polygons with more than three sides may be stretched to divide the figure into triangles. Estimate the sums of all interior angles. Can you guess what the sum of all the exterior angles might be?

Using a piece of regular string as a compass, construct a circle with a radius of ten units. Estimate the area by counting the squares enclosed.

Inscribe a regular polygon in the circle. Is its area more or less than the circle? Double the number of sides. How do the areas of polygon and circle compare now?

Circumscribe a regular polygon about the circle. Is its area more or less than the circle? Will doubling the number of sides make its area larger or smaller?

If we continued to increase the number of sides of both the inscribed and circumscribed polygons, what happens to their enclosed area as they relate to the area of the circle?

Can the area of a circle and the area of an inscribed or circumscribed polygon ever be exactly the same?
UNIT III

EXPERIMENT I.

The area of a curved surface is more difficult to measure than that of a flat surface. Therefore, the purpose of this experiment is to relate the lateral areas of cylinders to areas of rectangles and to relate the area of a hemisphere to the area of a circle by intuitive thinking.

Let's first review the formulas for the circumference and the area of a circle since these will be required. \( C = \pi d \) and \( A = \pi r^2 \).

If we take a rectangular piece of paper and bend its one edge in a circular manner, a cylinder is formed. Thus, the lateral area of a cylinder is exactly the same as the area of this rectangle. What is the formula for the area of a rectangle?

How can we determine the dimensions required to calculate the lateral area of a cylinder? The height is easily measured and the circumference can be calculated (estimated) after the diameter is determined. How can we use these values to get the lateral surface area of the cylinder?

The materials required for the next part of our experiment are half a sphere, made by cutting either a rubber ball or a wooden croquet ball in half, and a length of relatively heavy chord string.

On the circular cross section of the hemisphere tightly coil the heavy chord, starting at the center, until the entire surface is covered by one and only one layer of string. Carefully mark the end of the chord where it runs off the edge of the circle. Now, unwind the
chord and measure its length. How can the area of the circle be calculated now that we know the length of the coiled chord? Does the uncoiled chord in any way resemble a rectangle?

Using chord from the same ball, coil the chord over the curved surface of the hemisphere. Try to wind the chord with the same compactness as you used on the circular cross section. Completely cover the surface with one and only one layer of twine.

What can we do now to compare these two areas? Can you "guestimate" the formula for the area of a sphere when the radius is known?
EXPERIMENT M.

The following experiment requires a sufficient number of wooden cubes one inch on an edge to construct solid figures.

What units of measure are used to measure volumes? Of course, quarts and gallons are used, but what units can we use which are related to linear and area measure?

Build the following and "guestimate" their volumes.

1. Right rectangular prism.
2. Oblique rectangular prism.
3. Right triangular prism.
4. Cylindrical tower (estimate for blank areas).
5. Pyramid.

Check your estimated answers against those calculated through the formulas in our text.
UNIT III

EXPERIMENT N.

Rather special equipment is necessary for the experiment to determine or substantiate formulas for the volumes of pyramids, cones, and spheres.

Our desire is to relate the volumes of these objects to volumes of prisms and cylinders, the formulas for which we already know.

\[ V_p = lwh \quad V_c = \pi r^2 h. \]

Thus, we need a cone and a cylinder with the same base size and the same height, and a pyramid and a prism with the same base size and height.

Can you guess what we intend to do with these? Will you venture a guess as to how many cones will fill the cylinder? How many pyramids will fill the prism?

Sand, or some other granular substance, is better than a liquid since it cleans up more easily.

Check your "guestimates" by filling the larger volumes with the smaller containers.

Take a cylinder whose height is the same as its radius and completely fill it with water. Place the container in a large, empty pan. Make sure the pan is empty. Now, take a ball with the same diameter as the cylinder and completely immerse it in the water allowing the excess water to overflow into the pan. Remove the cylinder from the pan and completely empty it. Now, pour the water from the pan into the empty cylinder. This represents the volume of the sphere. What fractional part of the cylinder is filled?
Since \( V = \frac{2}{3} \pi r^2 h \) for the sphere, and since \( h = 2r \),

\[ V = \frac{2}{3} \pi r^2 \cdot 2r = \frac{4}{3} \pi r^3. \]

Do you agree?
UNIT III

EXPERIMENT 0.

Three dimensional figures are often difficult to sketch on a two dimensional plane. To assist the minds eye in these interpretations, the pick-up sticks and plastic clay can be very helpful.

Construct a cube, a rectangular prism, a rectangular pyramid, a triangular prism, and a triangular pyramid using the sticks and clay. Place them carefully on the table in front of you.

On a sheet of paper sketch exactly what you see. Try not to move your head as you sketch so that your perspective will be the same.

Another method which may be used involves the use of a light behind the object and a translucent paper secured by placing it in a frame of cardboard on wood. The shadow of the object can be traced on the paper producing a two dimensional look at a three dimensional object.

Can you sketch a sphere? What special markings are needed to give the illusion of three dimensions?

Try sketching a cone and show cross sections formed when the cone is sliced at various angles.
APPENDIX B

DIAGNOSTIC TESTS
EIGHTH GRADE DIAGNOSTIC TEST ON UNIT I.

Name______________________________

Part I

1. Is 12 a common denominator of the fractions 1/2 and 5/6?

2. What is the least common denominator of 2/3 and 1/9?

3. Express 1/3, 2/5, and 5/6 with the same denominator.

4. 3 3/8 is equal to 2 and how many eighths?

5. What is the reciprocal of 7, 1/2, 4/3, 2 1/4, .80?

6. Write out how you would read the following:
   a. .3985
   b. 36.384

7. Write in polynomial form: 35.793.

8. Write 1/3 as a decimal.
   Perform the indicated operations and express your answers in simplest form.
   9. \( \frac{1}{3} + \frac{2}{5} \div \frac{5}{6} = \)

10. \( \frac{7}{8} - \frac{3}{4} = \)

11. \( \frac{5}{6} - \frac{3}{10} = \)

12. \( 3 \frac{3}{8} - \frac{7}{8} = \)

13. \( \frac{21}{25} \times \frac{5}{7} = \)

14. \( 12.5 \times .06 = \)

15. \( \frac{1}{6} \div \frac{7}{8} = \)

16. \( .0048 \div .16 = \)
Part II

Simplify the following ratios.

1. $96 : 27$
2. $6 : 26$
3. $5 : 8$
4. $1.1$ to $.01$
5. In the proportion $3:4 = 30 : 40$ name the means.

Solve the following proportions for the missing numbers.

6. $1 : 2 = ? : 12$
7. $7 : 3 = 4 : ?$

Write the following ratios as per cents.

8. $40 : 100$
9. $1 : 1$
10. $283 : 1000$
11. $25\%$ of $484 = ?$
12. Write $5/8$ as a per cent.
13. Write $172.4\%$ as a decimal.
14. Write $100\%$ as a decimal.
15. $128$ is $2\%$ of ?
1. A straight line has
   a. no beginning and no end point
   b. a definite beginning and a definite end point
   c. a beginning point and no end point
   d. no beginning point but an end point

2. A line segment is
   a. a definite number of points
   b. sometimes curved
   c. a subset of a straight line
   d. the same as a straight line

3. Two points on a line and all the points between is
   a. an inch
   b. a line segment
   c. a ray
   d. an angle

4. If we have two lines in the same plane that don't meet, we call them
   a. parallel lines
   b. perpendicular lines
   c. oblique lines
   d. skew lines

5. Perpendicular lines meet to form a
   a. straight angle
   b. 45° angle
   c. 90° angle
   d. none of these because they don't meet

6. A simple closed figure divides a plane into how many sets?
   a. 1
   b. 2
   c. 3
   d. 4
7. A corner of a polygon is called
   a. an angle
   b. a vertex
   c. a diagonal
   d. two rays

8. A diagonal is a line segment that joins two non-adjacent vertices of a polygon.
   a. True
   b. False
   c. Sometimes, hard to tell

9. A square is a (an)
   a. octagon
   b. pentagon
   c. quadrilateral
   d. hexagon

10. An angle is formed by two intersecting
   a. rays
   b. lines
   c. segments
   d. diagonals

11. A ray has
   a. no beginning and no end point
   b. a definite beginning point and a definite end point
   c. a beginning point and no end point
   d. no beginning point but an end point

12. The instrument used to measure angles is called a
   a. compass
   b. ruler
   c. meter stick
   d. protractor
Approximate the measure of the following angles using your protractor.

13.

14.

15.
EIGHTH GRADE DIAGNOSTIC TEST ON UNIT III.

PART I.

1. In the figures above, which are closed figures?
   (A) B only (B) B and E (C) A, B, and C (D) A, C, and D (E) all of them

2. Figure C is called a (A) pentagon (B) hexagon (C) heptagon (D) octagon (E) none of these

3. A polygon with three sides is called a (A) square (B) triangle (C) quadrilateral (D) decagon (E) none of these

4. The sum of the measures of the angles of a quadrilateral is (A) 90° (B) 180° (C) 270° (D) 360° (E) 400°

5. The sum of the lengths of the sides of a polygon is called the (A) boundary (B) area (C) perimeter (D) circumference (E) size

6. A circle separates the plane into ___ sets of points. (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

7. The line segment connecting the center of a circle with a point on the circle is called the (A) arc (B) diameter (C) radius (D) semicircle (E) all of these.
8. A triangle with no two sides equal is called a(n) ______ triangle.
   (A) Acute  (B) Obtuse  (C) Scalene  (D) Isosceles
   (E) Equilateral

9. A triangle with two sides equal is called a(n) ______ triangle.
   (A) Acute  (B) Obtuse  (C) Scalene  (D) Isosceles
   (E) Equilateral

10. A triangle with three sides equal is called a(n) ______ triangle.
    (A) Acute  (B) Obtuse  (C) Scalene  (D) Isosceles
    (E) Equilateral

11. The sum of the measures of the angles of a triangle is
    (A) 90°  (B) 180°  (C) 270°  (D) 360°  (E) 400°

12. In the space at the right, draw a quadrilateral.

13. Shade its interior.

14. Mark a point M in the exterior.

15. Mark a point N on the boundary.

Part II.

Select the answer that best completes the following:

1. Area measures the
   (a) Exterior of a closed figure
   (b) Interior of a closed figure
   (c) Distance around a closed figure
   (d) None of the above

2. Units used to measure area must be
   (a) Cubic units
   (b) Linear units
   (c) Square units
   (d) None of these

3. Volume measures
   (a) Capacity
   (b) Surface
   (c) Distance around
   (d) None of these

4. Units used to measure volume must be
   (a) Square units
   (b) Linear units
   (c) Cubic units
   (d) None of these
5. Match the areas of the following figures with the corresponding formulas:

   _______ rectangle  a. \( \frac{1}{2} \, h \, (b_1 + b_2) \)
   _______ triangle       b. \( s^2 \)
   _______ trapezoid     c. \( 2 \, \pi \, rh \)
   _______ parallelogram d. \( lw \)
   _______ circle        e. \( l + w \)
   _______ cylinder      f. \( c + rd \)
   _______ parallelogram g. \( 2 \, \pi \, r \)
   _______ cylinder      h. \( \frac{1}{2} \, bh \)
   _______ cylinder      i. \( \frac{w_1}{w_2} \)
   _______ cylinder      j. \( \pi \, r^2 \)
   _______ cylinder      k. \( bh \)

6. Match the volumes of the following figures with the corresponding formulas:

   _______ cube            a. \( \frac{1}{3} \, \pi \, r^2 \, h \)
   _______ rectangular prism b. \( 2l + 2w \)
   _______ triangular prism  c. \( lwh \)
   _______ cylinder        d. \( 2 \, \pi \, h \)
   _______ square pyramid   e. \( \pi \, r^2 \, h \)
   _______ cone            f. \( \frac{4}{3} \, r^2 - \frac{4}{3} \, r^3 \)
   _______ sphere          g. \( s^3 \)
   _______ sphere          h. \( \frac{4}{3} \, \pi \, r^3 \)
   _______ sphere          i. \( Bh \)
   _______ sphere          j. \( l + w + h \)
   _______ sphere          k. \( \frac{1}{3} \, Bh \)
7. Find the area of a rectangle 3 ft. long and 2 ft. wide.

8. If the area of the base of a rectangular prism is 24 square feet and the volume is 96 cubic feet, what is its height?
APPENDIX C

ACHIEVEMENT TESTS
1. Name 3 common denominators of \( \frac{5}{3} \), \( \frac{7}{8} \) and \( \frac{1}{6} \) which are less than 200. 

2. What is the least common denominator for the fractions in number 1? 

3. Express \( \frac{1}{3} \), \( \frac{3}{5} \), and \( \frac{5}{12} \) with the same denominator. 

4. \( \frac{9}{7} = 10 + \frac{2}{7} + \frac{2}{7} = 10 + \frac{4}{7} \) 

5. What is the reciprocal of each of the following? 
   a. \( 2 + 5 \) 
   b. \( 1 \) 
   c. \( \frac{4}{3} \) 
   d. \( 3 \frac{1}{3} \) 
   e. \( 0.031 \) 

6. Write out how you would read the following. 
   a. \( 0.3985 \) 
   b. \( 36.384 \) 

7. Write in polynomial form: \( 35.703 \). 

8. Write \( \frac{2}{7} \) as a decimal. 

   Perform the indicated operations and express your answers in simplest form.

9. \( \frac{1}{3} + \frac{3}{8} + \frac{5}{12} = \) 

10. \( \frac{5}{6} - \frac{5}{12} = \)
11. \( \frac{5}{6} - \frac{3}{10} = \frac{11}{30} \)

12. \( 11 \frac{9}{7} - \frac{11}{7} \)

13. \( \frac{24}{9} \times \frac{3}{4} = \frac{2}{1} \)

14. \( .23 \times .09 = \frac{1}{50} \)

15. \( \frac{1}{10} \div \frac{4}{5} = \frac{1}{8} \)

16. \( .0048 \div .016 = \frac{1}{4} \)

17. Simplify the following ratios.
   a. 144 : 18 ________
   b. 2 in. : 2 ft. ________
   c. \( \frac{8}{5} : \frac{5}{6} \) ________
   d. 1.1 : .01 ________
EIGHTH GRADE ACHIEVEMENT TEST ON UNIT II.

Choose the best answer. Put the letter corresponding to the question number on the answer sheet. DO NOT write on the test copy. Mark only one answer per question. Erase or blot out any answer you wish to change.

1. When two lines intersect, their intersection is (a) a point (b) a line (c) a plane (d) a ray (e) the null set.

2. When two planes intersect their intersection is (a) a point (b) a line (c) a plane (d) a ray (e) the null set.

3. When two parallel lines intersect, their intersection is (a) a point (b) a line (c) a plane (d) a ray (e) the null set.

4. The intersection of two skew lines is (a) a point (b) a line (c) a half-line (d) a plane (e) the null set.

5. The angle formed by one ray and by another ray perpendicular to it with a common endpoint is called (a) acute (b) obtuse (c) right (d) skewed (e) none of the above.

FOR QUESTIONS 6-10, REFER TO DIAGRAM I.

6. Angle AOB divides the plane of the paper into how many regions? (a) 1 (b) 2 (c) 3 (d) 4 (e) none of these.

7. The rays OA and OB of angle AOB form the _____ of the angle. (a) rays (b) half-lines (c) interior (d) exterior (e) sides.

8. The shaded region is called the _____ of angle AOB. (a) vertex (b) boundary (c) interior (d) sides (e) exterior.

9. Point O is called the (a) vertex (b) boundary (c) interior (d) sides (e) exterior.

10. The unshaded portion is called the (a) side (b) vertex (c) exterior (d) interior (e) half-plane.
REFER TO DIAGRAM 2 FOR QUESTIONS 11-19. Choose the closest estimate.

11. The measure of angle AOE is
   (a) 60° (b) 90° (c) 120° (d) 135° (e) 180°

12. The measure of angle AOC is
   (a) 60° (b) 90° (c) 120° (d) 135° (e) 180°

13. The measure of angle DOE is
   (a) 45° (b) 90° (c) 120° (d) 135° (e) 180°

14. The measure of angle AOD is
   (a) 45° (b) 90° (c) 120° (d) 135° (e) 180°

15. The measure of angle AOB is
   (a) 45° (b) 60° (c) 90° (d) 120° (e) 135°

16. The measure of angle BOE is
   (a) 45° (b) 60° (c) 90° (d) 120° (e) 135°

17. Angle AOB is
   (a) acute (b) right (c) obtuse (d) all of these (e) none of these

18. Angle AOD is
   (a) acute (b) right (c) obtuse (d) all of these (e) none of these

19. Angle DOE is
   (a) acute (b) right (c) obtuse (d) all of these (e) none of these

FOR QUESTIONS 20-22 REFER TO DIAGRAM 3.

20. Angles APC and HPD are _____ angles.
    (a) complementary (b) supplementary (c) vertical (d) adjacent
    (e) none of these

21. Angles APB and HPD are _____ angles.
    (a) complementary (b) supplementary (c) vertical (d) all of these
    (e) none of these

22. Angles CPA and APB are _____ angles.
    (a) adjacent (b) vertical (c) complementary (d) all of these
    (e) none of these
23. The measure of the angle that is the complement of 35° is (a) 35° (b) 45° (c) 55° (d) 145° (e) 155°.

24. The measure of the angle that is the supplement of 80° is (a) 10° (b) 30° (c) 50° (d) 80° (e) 100°.

25. In geometry, two figures that are the same in size and shape are called (a) acute figures (b) adjacent figures (c) supplementary angles (d) congruent figures (e) complementary angles.

MATCHING: Choose the definition on the right that best fits the word on the left. Record the letter of the definition on the answer sheet beside the number of the word. The lines separate each group of words and definitions.

27. Right angle b. The study of angles.
28. Acute angle c. An angle with a degree measure less than 90.
29. Obtuse angle d. An angle with a degree measure greater than 90.

30. Perpendicular lines a. Lines NOT in the same plane that do NOT intersect.
31. Oblique lines b. Lines in the same plane that do NOT intersect.
32. Parallel lines c. Two lines that intersect to determine right angles.
33. Skew lines e. Lines that intersect in more than one point.

34. Adjacent angles a. Two angles with degree measure which when added together equal 180 degrees.
35. Complementary angles b. A set of points consisting of two different rays with a common endpoint.
36. Supplementary angles c. Two angles with (1) a common vertex, (2) a common ray, and (3) disjoint interiors.
37. Vertical angles d. Two non-adjacent angles determined by two intersecting lines.
            e. Two angles whose sum of their measures equal 90 degrees.
38. Midpoint
39. Angle bisector
40. Congruent

a. A ray that divides an angle into two congruent angles.
b. Two lines that intersect to determine right angles.
c. A point that bisects a line segment.
d. Two figures that are the same in size and shape.
e. Two angles whose sum of their measures equals 90 degrees.
EIGHTH GRADE ACHIEVEMENT TEST ON UNIT III.

Part I. Definitions:

1. __A simple closed figure that is the union of line segments.__
   a. four
b. trapezoid
c. square
d. parallelogram
e. rectangle
f. polygon
g. rhombus
h. three
i. trapezoid
j. five
k. equilateral
l. scalene
m. right

2. __A quadrilateral with exactly two pairs of opposite sides parallel.__

3. __A quadrilateral has how many sides?__

4. __A parallelogram with all of its angles right angles.__

5. __A rectangle with all of its sides equal in length.__

6. __A parallelogram with all of its sides equal in length.__

7. __A triangle with all of its sides equal in length.__

8. __A triangle with one angle equal to 90 degrees.__

Part II. Formulas:

___ 1. Rectangle

A. __V = 1/3 Bh__

___ 2. Triangle

B. __A = lw__

___ 3. Circle

C. __V = lwh__

___ 4. Prism

D. __V = π r^2 h__

___ 5. Pyramid

E. __A = π r^2__

___ 6. Cylinder

F. __A = 1/2 bh__

___ 7. Cone

G. __V = 1/3 π r^2 h__

Part III. Completion:

1. The sum of the measures of the angles of a triangle is ____ degrees.

2. A triangle may have no more than one _____ degree angle.

3. A triangle may have three _____ angles.

4. The _____ of any polygon is the sum of the lengths of its sides.

5. The circumference of any circle may be found by using the formula __C = _____.
Part III. Completion continued:

6. The perpendicular bisector of a segment passes through its __________.

7. A line segment whose endpoints are non-adjacent vertices of a polygon is called a _______ of the polygon.

Part IV. True or False:

_____ 1. All triangles are right triangles.

_____ 2. All squares are rectangles.

_____ 3. Lines that neither intersect nor are parallel are skew.

_____ 4. A triangle contains no adjacent vertices.

_____ 5. A septagon has 7 sides and 7 vertices.

_____ 6. \( \triangle ABC \) means the same as \( \triangle A \).

_____ 7. Two angles are vertical if they have a common vertex and a common ray between them.

_____ 8. There is exactly one line through two different points.

_____ 9. A closed solid figure divides space into three sets of points.

_____ 10. The faces of all polyhedra are polygons.

_____ 11. A cube is a rectangular solid.

_____ 12. A square pyramid has one base.

_____ 13. A hemisphere is half a circle.

_____ 14. Another name for the vertex of a cone is its apex.

_____ 15. An oblique prism contains no right angles.
Part V. Find the areas of the following:

1. Triangle: \( b = 34'' \), \( h = 17'' \)
2. Rectangle: \( l = 40'' \), \( w = 7'' \)
3. Circle: \( r = 7'' \)
4. Square: \( s = 8'' \)

Part VI. Find the volumes of the following:

1. Prism: \( l = 17, w = 2, h = 10 \)
2. Pyramid: \( B = 60 \text{ sq. ft.}, h = 9' \)
3. Cylinder: \( r = 7', h = 10' \)
4. Cone: \( r = 14'', h = 4'' \)
5. 1 cu. ft. = ______ cu. in.
6. 1 cu. yd. = ______ cu. ft.
APPENDIX D

TEACHER AND STUDENT EVALUATION FORMS
TEACHER'S EVALUATION

1. All or nearly all of the students willingly participated.  

2. The students were interested to the extent of offering suggestions for procedures.  

3. The LAB was more than "fun and games" in that mathematical concepts seemed clarified.  

4. The students' extemporaneous and/or unsolicited comments were favorable.  

5. Overall, the experience was satisfactory.  

Comments, particularly on the "No" checks:

STUDENT'S EVALUATION OF TODAY'S LAB EXPERIENCE

1. It was interesting.  

2. It helped me discover some different ideas about mathematics.  

3. It gave me a better understanding of the mathematics involved.  

4. I still have some questions about what we were trying to do.  

COMMENTS AND SUGGESTIONS:
APPENDIX E

STUDENT INTEREST QUESTIONNAIRE
STUDENT INTEREST QUESTIONNAIRE

Class__________________________

During this school year your class has had a number of mathematics laboratories. Sometimes these were given before you talked about the mathematical ideas in your class and sometimes the laboratories came after your class discussions.

There is a real interest in your ideas about these laboratory experiments. Please answer the following multiple choice questions by carefully selecting the best answer provided.

1. The kind of lab I liked best was
   a) before class discussion of the topic
   b) after class discussion of the topic
   c) no lab at all.

2. Mathematics labs have made this year compared with last year
   a) more interesting
   b) more boring
   c) no different.

3. I suggest that next year's eighth graders
   a) continue having at least the same number of labs as we had
   b) have more labs than we had
   c) have no labs at all.

4. These lab experiments on the whole
   a) helped me to better understand mathematics
   b) confused me more than they helped me
   c) made no difference in my ability to understand mathematics.

5. When my teacher announced that we were going to have another laboratory experiment,
   a) I looked forward to it
   b) I said to myself, "So What,"
   c) I said to myself, "No, not again."

You are requested to write more comments in the space below about the laboratory experiences. Be sincere, but feel free to write both favorable and unfavorable comments.

__________________________________________________________________________
__________________________________________________________________________
__________________________________________________________________________