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DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

by

David G. Mader, B.S., M.S.

The Ohio State University
1971

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  Pre-Course Differences
  Achievement Differences
Development of Selection Strategies
Data Gathered for the Test Group
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CHAPTER I

INTRODUCTION

Effort to individualize instruction in mathematics has assumed many forms over the past few decades. Programmed texts, video tapes, and a range of computer-assisted instructional techniques have provided substantial material aid. Auto-tutorial systems and ability (or other homogeneous) groupings of students and creation of differently paced classes have been viewed as a means of narrowing the range of individual differences between students and, hence, of allowing more students to achieve at a level and pace appropriate to their abilities.

Statement of the Problem

The main purpose of this study was to develop and evaluate strategies for the selection of students for individually paced sections of a college pre-calculus mathematics course. Primary consideration was given to the generation of the selection strategies. Predictive criteria included pre-course as well as in-course measures. Evaluation of each strategy consisted of two components. The first was a determination of the effect on
achievement which accrued from the creation of the differently paced sections. At the conclusion of a specified amount of content, differences in achievement were measured. Comparisons were made between the achievement of students in the individually paced sections and students who were randomly grouped in a traditional treatment of the course during a different school term. The second component consisted of a measurement of the efficiency of each selection strategy; that is, the percentage of students who were correctly placed under a given strategy.

Existing reports on educational and psychological studies contain a wealth of information on placement and grouping of students. Most of these conclude that it is not the grouping itself which generates significant achievement patterns, but the treatment which accompanies or follows the grouping. A detailed review of this literature will be presented in the second chapter.

Setting of the Study

During the Autumn Quarter 1970, the Department of Mathematics of The Ohio State University initiated the first stage of a program designed to individualize freshman mathematics instruction. This phase consisted of dividing the content of existing courses into smaller "modules" and providing for
three levels of pacing for each module. The term "regular pace" is used in this study to designate the traditional pace utilized in covering a specified amount of content during one term of the school year. The term "accelerated pace" denotes treatment which allowed students to cover additional content during one term; "reduced pace," that which used the full term to cover the first 60 percent of the regular material. A term at The Ohio State University lasts ten weeks and is designated as Autumn, Winter, Spring or Summer Quarter of the proper year.

The traditional ten credit mathematics sequence for liberal arts students was restructured to include four modules with 3, 2, 3 and 2 credits, respectively. A detailed description of the content relationships between the traditional and restructured courses is reported in Appendix I.

During the Autumn Quarter 1970, the regular sections covered the first two modules (5 credits). The reduced pace section covered only the first 3-credit module while the accelerated section completed three modules (8 credits). Similar plans were in effect for other freshman mathematics sequences as a part of a department project known as CRIMEL (Curriculum Revision and Instruction in Mathematics at the Elementary
Level (23). *

All students in a given sequence started together under regular pacing. At the beginning of the third week, the group was tested, selection of students for the three levels was determined, and differential pacing begun. Students in the reduced pace section restudied the initial material for two weeks, were retested, and then proceeded through the remainder of the first module. This study was restricted to the modules which were offered during the Autumn Quarter. Pilot data was generated from the students enrolled in Math 116 during the Winter and Spring Quarters 1970.

Limitations

It should be noted that the subjects in this study do not represent the whole spectrum of pre-calculus students in mathematics at Ohio State. Students who major in natural sciences, physical sciences, engineering, mathematical sciences or administrative sciences enroll in different sequences of courses. The subjects in this study are typically liberal arts students who plan to major in social sciences, behavioral sciences, humanities or the arts. Before a student can enroll

* The symbol (m;n) will be used to refer to page "n" of entry "m" in the numbered bibliography. The symbol (m) will refer to entry "m" in the bibliography.
in Math 116, he must score above a minimal level on a placement test (OSU Math Test). This test is administered to students as part of an orientation program prior to their first quarter of enrollment at the University. Students whose scores are unsatisfactory must enroll in an algebra refresher course.

There were other limitations and problems related to this study which were beyond the control of this researcher. One of the basic goals of the CRIMEL project is to allow a student to proceed at a rate appropriate to his ability. It departs from the usual "lock-step" approach to grades and credit hours in that the amount of class time devoted to a given amount of content is not a determining factor. Students in this system are to be given considerable personal attention and guidance. However, due to the necessity to comply with University rules regarding deadlines for course registration and reporting of grades, there were obvious limitations on the extent of pacing ranges.

Lack of adequate financial resources resulted in operation of the program on an understaffed basis. Hence, some instructors were unable to give students as much individual help as was originally planned.

This study considered only those students for whom complete data was available. Subject loss occurred when students
withdrew from the course or did not take tests at the scheduled times. Specific data on the number of students who withdrew during the time of this study will be presented in a later chapter.

Hypotheses

Several hypotheses were tested in this study. The Winter 1970 students will be denoted as the pilot group and the Autumn 1970 students as the test group. The hypotheses, in null form, are as follows:

1. Students within a given expectancy range in the test group will achieve at the same level at the completion of the first module as students in the pilot group within the same expectancy range.

2. For test group students within a given expectancy range, there is no difference in achievement between the subsets of those students who complete the first module at different pacing levels.

3. At the completion of the first module, students in the test group will achieve at the same level as the students in the pilot group.

The first hypothesis deals with differences in achievement between students recommended for a given pacing level in the test group and the students in the pilot group who would have
been recommended for the same pacing level. Rejection of this hypothesis in favor of the test reduced pace group, for example, would indicate positive effects on achievement under the CRIMEL approach to instruction. A special t-test (described in Chapter V) was employed to determine significance of differences.

The second hypothesis tests, for example, among those students who were recommended for an accelerated section the difference in achievement between those students who finished at an accelerated pace and those who finished at a regular pace. Evaluation of these differences was made using analysis of variance.

Data related to the efficiency of the various criteria used to define expectancy levels will be provided in descriptive form for both the pilot and test group subjects.

The third hypothesis tests the differences in achievement between the entire test group and the entire pilot group. Test of this hypothesis was evaluated using a t-test.

Assumptions

A number of assumptions have been made which are basic to acceptance of the findings of this research. Specific attention will be given to each in later chapters. The five assumptions
are as follows:

1. There would be like variation within the test group and pilot group relative to pre-course criteria.

2. Within a given group of students, common lectures and syllabi together with central administration of the course would result in no significant differences in achievement between sections due to instructor differences.

3. Different instruments used in evaluating a common achievement measure would not result in significant differences if administered to equivalent populations.

4. Student responses to a questionnaire used to determine certain pre-course measures are valid.

5. Pre-course measures used to determine student placement at The Ohio State University are reliable.

Design of the Study

Pilot group data was collected for students enrolled in Math 116 during the Winter Quarter 1970. Achievement in the course was predicted using regression equations based on pre-course data as well as first examination scores. A study of the pilot data also suggested two alternative prediction strategies.
Each of the three strategies involved an attempt to identify candidates for the accelerated, regular and reduced pace sections to be created during the following Autumn Quarter. A detailed explanation of the instructional treatment for the pilot and test group subjects is presented in Chapter III.

In an attempt to identify a method of selecting candidates for differently paced sections during the Spring and Autumn Quarters, the following variables were investigated for the Winter group subjects:

1. standard score on mathematics portion of the ACT examination;
2. raw score on The Ohio State University Math Placement Test;
3. success in high school mathematics courses;
4. raw scores on in-course examinations.

Not all of the above data was available for every student. In order to generate regression equations, the Winter Math 116 students were divided into five subgroups identified according to available pre-course data. Linear (stepwise) regression equations were generated for each of the groups. Total points earned after the first test were predicted as a function of the score on the first test together with scores on pre-course
measures. These equations were used to section students in the Spring and constituted part of the selection criteria used in the Autumn. Summary data on the variables used for the various subgroups is included in a later chapter.

Results of research findings based upon the pilot data indicated that additional pre-course measures should be generated for students entering the course during the Autumn Quarter 1970. These variables were the time (in quarters) since the student last took a mathematics course and the sex of the student.

For some students, suggested pacing levels were determined effectively by considering only first midterm scores. Regression equations were generated for the remainder of the students.

The task of evaluating the Winter data in view of the content and testing plan to be used in the Autumn led to an increase in the number of students for whom identification of pacing level was not clear. This was due, in part, to the fact that the number of items on the first test was twenty (20) in the Autumn as opposed to twenty-five (25) in the Winter and Spring.

Pre-course data which was not used in selection criteria in the Autumn was considered in post-course evaluation.

Methods of Evaluation

Once the evaluation of the pilot data was completed, the
following items were determined:

1. For each of the pilot subgroups, statistical evidence related to the desirability of including or rejecting given measures as elements in the selection criteria.

2. Data relating to the instruments used in the courses involved in the pilot study.

3. Correlation matrix for variables employed and Multiple R between the actual and predicted achievement of subjects for whom regression equations were generated (pilot data).

4. Percentages of pilot group students correctly identified by the selection process to be employed in the Autumn.

5. Percentages of test group students correctly identified.

Since there were ranges of first midterm scores for which the technique of regression analysis did not produce highly significant results, several alternative selection procedures were tested during the Autumn Quarter 1970. These methods included (1) recommendations to every student based on the first midterm score only, and (2) suggesting that the student make his own choice of pacing level.

All students were free to choose a pacing level different
from that which was recommended. A student could also move to a slower paced section if his performance in an accelerated or regular section was unsatisfactory. Student records, then, included (1) the pacing level recommended to the student, (2) the pacing level chosen by the student, and (3) the pacing level at which the student completed a given module.

Overview

In this chapter the problem has been identified, the target population described, limitations of the study noted, hypotheses and assumptions outlined and a survey of the design and statistical methodology employed in the study presented. In Chapter II the relevance of other research studies will be discussed. Chapter III will include a detailed description of the design of the study. An evaluation of the criteria used to select students for the various pacing levels will be the basis for Chapter IV. Analyses of achievement data and test of hypotheses will be made in Chapter V. Chapter VI will include conclusions based on data generated throughout the study and some reflections relative to topics worthy of future study.
CHAPTER II

REVIEW OF RELATED LITERATURE

A search of available literature has revealed many studies which are related to the various aspects of this research. Evaluation of techniques of selecting students for individually paced sections of a college mathematics course after the subjects have had a short experience with the content and treatment of the course appears to be unique to this study. The description of literature related to this study is organized in this chapter according to the following categories:

1. Homogeneous grouping of students
2. Treatment of grouped students
3. Strategies for identification of ability groups.

Homogeneous Grouping

There is an abundance of literature reporting position statements of mathematics teachers and educators as well as results of research projects regarding the issue of homogeneous versus heterogeneous grouping of students. The majority of these articles pertain to elementary and secondary level students.
in a wide variety of subject matter areas. Charles Day (8) has provided a comprehensive review of the literature on homogeneous grouping. The bibliography of his report contains 245 references available prior to 1964. Evidence from research studies on the effects of homogeneous grouping is, at best, inconclusive. The conclusion reached by a number of authors is that grouping, in itself, does not contribute to increased levels of achievement, but what appears to count is what treatments are used after the students have been grouped.

Several authors, including Della-Dora and Ekstrom, have objected to the manner in which students have been grouped and the methods used to measure the effects of grouping.

Della-Dora (19), in 1960, commented that

"Grouping on the basis of academic ability, marks, or intelligence tests may inhibit opportunity for development of creativity on the part of the potentially creative pupils.

"How learning takes place is more affected by factors such as self-concept, social class background of the student, level of student aspiration, attitudes of teachers and students, and certain other social-psychological forces than it is by any method or combination of methods of grouping.

"Time expended in studying ways to improve learning could probably be more profitably devoted to examination of the crucial factors just cited rather than to the efficacy of various methods of grouping in isolation from these factors."
Ekstrom (11), as part of a review of experimental studies on homogeneous grouping, concluded that

"... controlled experimental studies comparing the effectiveness of homogeneous and heterogeneous grouping, as evaluated by student achievement, showed a great variety of experimental designs and no consistent patterns of results. Many experiments failed to control the type of teaching and to provide differentiation of teaching according to ability levels. Poor experimental design, such as the use of available data only and the use of matched pairs on unwarranted assumptions made many studies less effective.

"In experiments that specifically provided for differentiation of teaching methods and materials for groups at each ability level, and made an effort to push bright homogeneous classes, results tended to favor the homogeneous groups."

The study by French (12) supports some of the assertions of Della-Dora and Ekstrom. Based upon data collected at the high school level, French reported that not only do bright students do better in high ability classes, but slow children also do better in high ability classes. He concluded that ability grouping may be harmful to slow students.

Treatment of Grouped Students

At this time in the history of mathematics education, the issue at the college level appears to be how to group students effectively rather than whether to group. This is especially true of mathematics departments which serve large numbers of
students representing a broad spectrum of interests and abilities.

There is a growing trend at both the high school and college levels to "individualize" mathematics instruction, in many cases within the spirit suggested by Della-Dora. This does not mean that special grouping of mathematics students at the college level is novel to this decade.

In 1957, Steer (28) surveyed mathematics departments which included colleges and universities which had both small and large enrollments, open and selective admissions policies, and represented both public and non-public supported institutions. At that time grouping consisted of either special "honors" sections containing enrichment material, remedial courses for students not prepared for the regular courses, or creation of different content courses designed for students whose major academic interests were in areas other than mathematics or the physical sciences. It is in this last category that recent innovations have occurred and this is the particular focus of this study.

Two studies, although done at the high school level, have some relevance to this study in that they are concerned with the effects of content pacing. Smith (27) investigated students grouped by STEP scores. In considering the effects on
achievement resulting from (1) grouping with traditional instruction, (2) grouping with enrichment, and (3) grouping with content acceleration, results indicated that acceleration was more effective than either of the other two methods.

Frye (13) considered the effects of group versus individual pacing for homogeneous and heterogeneous groups of Algebra I high school students. Instruction for all students was via programmed materials. The time required to correctly complete a unit on "Completing the Square and the Quadratic Formula" was the dependent variable. Four treatment groups were identified as (1) Heterogeneous Group Paced, (2) Heterogeneous Individually Paced, (3) Homogeneous Group Paced, and (4) Homogeneous Individually Paced. Pairs of subjects were matched on the basis of the Primary Mental Abilities Test and the Orleans Algebra Prognosis. One member of the matched pair was assigned to a group paced treatment and the other to an individually paced group. The homogeneous groups consisted of students in the second and third quartiles of the distributed abilities; the heterogeneous groups, the first and fourth quartiles.

Evaluation of the data revealed the following results:

1. The time required by the Heterogeneous Group Paced group was significantly greater than that of the
Heterogeneous Individually Paced group.

2. The time required by the Homogeneous Group Paced group did not differ significantly from that required by the Homogeneous Individually Paced group.

3. The time required by the Heterogeneous Group Paced group was significantly greater than that required by the Homogeneous Group Paced group.

One of the main purposes of this study is to determine the effects on achievement which result when homogeneous subgroups of students are differentially paced; that is, time is an independent variable with achievement as the dependent variable. In Frye's study, the role of these variables was reversed.

Experimental and correlational studies carried out at the college level and which are concerned with grouping of students deal primarily with prediction of success and placement in a range of traditionally taught courses. There are, of course, many studies which attempt to compare various instructional techniques and treatments at the college level. There is a scarcity of studies dealing with attempts to individualize instruction within the framework of courses in pre-calculus mathematics. Schrank (26) attempted to determine whether ability
grouping was more effective in producing achievement in pre-calculus mathematics than random grouping. In that study, certain classes of bright students were accelerated. Data for the non-accelerated sections favored the ability grouped sections and was significant at the one per cent level. Data for the accelerated sections did not produce overall significant differences but did favor the ability grouped sections.

The basic goal of most programs of individualized mathematics instruction is to maximize the achievement potential of each student in the program. Evaluation of achievement data, however, is not, in all cases, the highest priority. The developers of an audio-tutorial program at Fullerton Junior College, for example, felt that measurement of student attitudes and evaluations should be their primary concern (18). A twenty-six question attitude survey of 124 students enrolled in the audio-tutorial program at Fullerton revealed, in part, the following findings:

1. At the beginning of the course approximately 50 per cent of the students tended to fear and dislike mathematics, had low self-confidence in their mathematics ability and preferred to avoid mathematics courses.
2. A large percentage of A-T students tended to increase in their mathematics self-confidence and their willingness to take another A-T mathematics course (83%) in contrast to their willingness to take a traditional mathematics course (34%).

3. A large percentage of students (74%) like the A-T method from the beginning of the semester and continued to like it as the semester progressed. Of the 25 per cent who did not like it at first, 19 per cent liked it by the end of the semester and 6 per cent continued to dislike it.

Bashford (2) attempted to determine the relationships between the organizational innovation of the CRIMEL Program, student attribution of causality, and student achievement in a study which was carried out concurrently with the second stage of the pilot study of the present research report. The extremes on a scale measuring attribution of causality consisted of external (causality beyond the control of the subject) and internal (causality within the control of the subject). Differences between pretest and post-test scale measures were noted. Although no differences were highly significant, there was a shift toward the internal end of the scale on the part of the accelerated and
reduced pace students. The disruption of planned procedures by student demonstrations during the time of the study imposed a serious limitation on the interpretation of results from Bashford's study. The attitudes of both students and teachers, as measured by Bashford through controlled interviews, were quite favorable with respect to both the placement of students into the three pacing levels and also the manner in which the different pacing levels were treated. Reaction was most favorable from students in the accelerated and reduced pace sections. Students in the regular sections felt that the homogeneous nature of their classes was the best aspect of the program.

Bashford's study reinforced the proposition asserted by Della-Dora regarding the inhibition of initiative in low ability homogeneous classes. Some students in the reduced pace sections of the pilot study felt that their teachers did not expect them to perform at a high level and, hence, that no student in a reduced pace section would be eligible for a high grade at the end of the course. These same students, then, felt obliged to make only the minimum effort required to satisfactorily complete the course.
Strategies for Identification of Ability Groups

There have been a number of studies at the college level which have employed various strategies to predict mathematical achievement and to place students in appropriate mathematics courses. The use of regression analysis was predominant with these studies. Predictor variables consisted of a variety of measures available to the researchers prior to the subjects' enrollment in given courses.

Anderson, Weaver and Wolf (1) investigated the difference in achievement between freshman elementary education majors who were instructed via large group lectures and those who were taught in small groups. These authors generated a regression equation whose variables were the College Board Mathematics Aptitude score, the number of years of academic mathematics studied in high school and the average grade earned in high school mathematics to predict a course grade for each student. The technique of matched pairs was then used to compare the effectiveness of the two methods of instruction.

Morgan (21) used a discriminant equation to determine placement for students in an engineering technology pre-calculus mathematics course at the University of Toledo. The predictor variables consisted of the Cooperative Mathematics Test score,
the number of years of academic high school mathematics, average grade in high school mathematics, and age in months beyond the seventeenth birthday. Significant losses in predictive ability occurred when an attempt was made to delete one or more of the above variables. Based on pilot data, mean scores were generated for prior successful and unsuccessful groups. The critical score was considered as midway between the above two scores. If a student's predicted score was above the critical score, he was identified as a probable successful student; otherwise, he was identified as a probable unsuccessful student. Predicted performance was then compared with actual performance. Predictions were found to be correct in 90 per cent of the cases. Morgan emphasized the usefulness of employing multiple variables in determining placement by noting that previous methods which used only the Cooperative Mathematics Test score were accurate in only 50 per cent of the cases.

A study of the factors associated with success in first-year college mathematics courses conducted by Wick (29) at six Minnesota and Wisconsin colleges and universities support the findings of Morgan. Wick concluded that

"The use of multiple regression techniques
for predicting success represented a considerable improvement over the use of single predictor variables. The resulting multiple correlation coefficients were high enough to warrant the use of predictions based on the multiple regression equations for selection and placement of students, and for other group-counseling procedures."

The independent variables in Wick's study consisted of mathematics grade average in grades 10 through 12, high school rank, scholastic aptitude test score, mathematics aptitude test score, and a mathematics placement test score.

Paul (22) investigated the relationships of high school mathematics courses to achievement in the standard calculus sequence at The Ohio State University. Paul specifically considered differences between students who had taken calculus in high school and those who had not. The portion of Paul's study which is of major interest to this writer is the role of predictor variables in selecting students for special honors sections of the calculus sequence. Paul found that the practice of not choosing honors students until these students had been exposed to the standard course for part of the school term was desirable. The best predictor was found to be the second of four mid-course examinations. The best pre-course predictors were quality points earned in high school mathematics courses (number of semesters times average grade) and the raw score on The Ohio
State Math D-Test placement examination. Paul's dissertation, as well as an earlier similar study by Crosswhite (7), contains many references to other studies which utilized the technique of regression analysis to predict achievement.

The technique recommended by Paul for selecting students for honors sections represents a method strongly recommended by Cronbach and Gleser (6). This involves the concept of sequential or multiple stage decision making as opposed to single stage strategies. Earlier references cited in this current report indicated the value of using multiple variables rather than single variables in predicting achievement or success in mathematics courses. The use of single regression equations, however, represents what Cronbach and Gleser classify as a single stage selection procedure. It is clear that such single stage processes help account for differences in subjects prior to exposure to the content and treatment of a given course. However, they are not very sensitive to subject reaction to the particular setting within which the course is conducted. Cronbach and Gleser drew upon data from business and industry to illustrate their assertions. Of particular interest is the process by which management can select employees to certain tasks and responsibilities. The
practice of not choosing employees for supervisory or management positions until these employees had experience within the setting of the company was found to be superior to strategies which exclusively used the pre-accept and pre-reject strategies.

In applying the findings of Cronbach and Gleser to the setting of a university mathematics course in which individually paced sections are to be created, it does not seem unreasonable to expect that selecting students for these sections after they have had some experience with the course would be more effective than selecting these students by means of pre-course measures only. Specific attention to these differences is given in later chapters of this paper.

Fujita and O'Reilly (14) applied a two-stage sequential selection model proposed by Cronbach and Gleser as a means of determining student eligibility for a pre-calculus course at the University of Hawaii. The model, as constructed by Fujita and O'Reilly, is reproduced in Figure 1.

The decisions made utilizing this model were based on the Scholastic Aptitude Test of Mathematical Ability, the Mathematics Test of the Cooperative Test Service and sex. The investigators found that of the number of students rejected for course admission at the first stage, approximately half could possibly perform
satisfactorily in the course. The predictor variables used in
the above study were found to be highly efficient for the high
performance groups, but not sensitive in identifying the low
performance groups. Rather than deny these students admission
to the course, it was decided to allow those who wished to take
a second test together with the intermediate group identified in
the first stage. A modified strategy was then developed and is
reproduced in Figure 2.

Figure 1. A Two-Stage Sequential Strategy.
Figure 2. A Modified Sequential Strategy or Pre-Accept Selection Strategy.

The decision of Fujita and O'Reilly to allow students for whom the selection strategy was not efficient exercise a personal judgment whether or not to continue to the second stage is of major interest to this current study. Unfortunately, Fujita and O'Reilly's planned strategy has not yet been evaluated. A question which arises in the study undertaken by this author is the extent to which students can correctly predict their own achievement levels.
Summary

A review of existing literature has revealed a number of findings that are relevant to the purpose of this study. There is some indication that ability grouping with subsequent differentiation of pacing levels may be able to provide for improvement in achievement, or at least that differentiation of pacing may be more effective than other treatment of ability grouped students. Efforts to individualize mathematics instruction may have positive effects on students' attitudes toward mathematics and self-concepts of their ability to do mathematics. The use of multiple variables is more efficient in predicting success than the use of single variables. Regression analysis is an efficient method for predicting success when multiple predictors are employed. Multiple-stage or sequential decision models may have advantages over single-stage models in selecting subjects for various treatments.
CHAPTER III

DESIGN OF THE STUDY

A sketch of the design of this study was given in Chapter I. The purpose of this chapter is to provide a more detailed description of (1) the treatment of students in the pilot and test groups, (2) the methods used in gathering the data, (3) the instruments used in the study, (4) the student subgroups identified, and (5) the selection strategies used in the study. A detailed analysis of the criteria used to determine the student subgroups and the rationale used to generate selection strategies is presented in Chapter IV.

Treatment of Students

During the pilot quarter, Winter 1970, the 800 students in Math 116 were taught via daily recitation (24 minutes) and television lecture (24 minutes). Assignment of students to teachers was determined by the order in which the students reported to a dispatcher on the first day of class. The first testing occurred at the end of the third week. All students in the course were given a common test at the same time (evening).
The same policy was used for examinations during the sixth and ninth weeks as well as the comprehensive final examination.

In the Spring Quarter 1970, Math 116 was taught on an individual instructor basis to 350 students. There was no television usage. Student placement into sections was the same as in the pilot quarter. All students took their first test at the same time. However, two forms of this test were constructed and administered, each equivalent to the first test of the pilot quarter on the basis of factor analysis of corresponding items.

After the first test the students were resectioned into accelerated, regular and reduced pace groups. The criteria used to select these subgroups will be discussed later. The regular group followed the same syllabus as the pilot Math 116 course. The accelerated group was paced at a rate that would insure coverage of the basic concepts of both Math 116 and 117. The reduced pace group was reinforced on material from the first three weeks and then proceeded at a pace that would cover about 60 per cent of the Math 116 content. The final examination for the reduced pace group was a subset of the examination used for the regular paced group.

Full implementation of this plan for the Spring Quarter
was interrupted during the fifth week by student demonstrations and a two-week shutdown of the University. Since data collected after the first test was subject to unknown effects due to the deviation from the planned treatment, no attempt was made to interpret it.

During the test quarter, Autumn 1970, all students started the course as in the pilot quarter with television used for the lectures. After the first test, students were resectioned. The reduced pace and regular sections utilized the television lecture/recitation method. All accelerated sections were taught by individual instructors. At a given pacing level all sections followed equivalent syllabi.

In order to create sensible breaks in content between modules during the test quarter, there was a slight change in the content covered in class prior to the first test. Test quarter students received a more detailed treatment of the axiomatic structure of the real number system than did students in the previous two groups. A section on exponents and radicals was also introduced prior to the first test. Unlike the pilot and Spring quarters, the basic concepts related to functions were treated after the first test in the test quarter. The relationship between the modules and the traditional sequence is as follows
(see Appendix A for detailed content comparison):

<table>
<thead>
<tr>
<th>Course Code</th>
<th>Credits</th>
<th>Course Title</th>
<th>Credits</th>
</tr>
</thead>
<tbody>
<tr>
<td>115.01</td>
<td>3</td>
<td>Algebra</td>
<td>116</td>
</tr>
<tr>
<td>115.02</td>
<td>2</td>
<td>Linear Algebra</td>
<td>117</td>
</tr>
<tr>
<td>115.03</td>
<td>3</td>
<td>Calculus with application to Economics</td>
<td></td>
</tr>
<tr>
<td>115.04</td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

To give able students an early opportunity to prepare for accelerated pacing during the test quarter, material on topics to be covered in the later part of Math 115.01 was made available via video tapes for student study outside of class time. The first test in this quarter consisted of two parts. The first part was used to determine achievement of content covered in class lectures; the second, the content the student studied on his own initiative. The score on the optional second part of the test was used in certain cases to determine whether or not a student was recommended for an accelerated section.

Several important aspects of the study require the reader to be familiar with the relationships with respect to time between the various pacing levels and the examinations given. These relationships are illustrated in Figure 3. Test I is the first examination given in the course, the one used as part of the selection criteria for the differently paced sections. Test I' was a retest over the material included on Test I as well as approximately 20 per cent of the material on Test II. Test II'
Figure 3. Profile of Test Group Pacing Levels and Tests with Respect to Time.
and the Final examination in the reduced pace sections were comprehensive examinations over the entire content of the three credit course, Math 115.01. The broken lines in Figure 3 indicate shifts of students from one pacing level to another.

All examinations used during the course of this study consisted of multiple-choice items. Students recorded their responses on IBM scanning sheets. The sheets were then scanned and a deck of computer cards was generated. The data on the cards indicated the student's name, section number, test form, raw score and responses to each item on the examination. From these cards, complete item analyses were generated. The examinations used in this study as well as a summary of the statistics available from the item analyses of these examinations is included in Appendices C and D, respectively.

**Data Selected for Evaluation of the Pilot Group**

As indicated in Chapter I, during the pilot quarter it was decided to develop a method for identification and selection of students for individually paced sections of Math 116. A small scale experiment was to be undertaken during the Spring Quarter 1970 for the purpose of clarifying logistic procedures and for testing and refining the method of selecting students for the large scale test program during the Autumn Quarter 1970. A review
of the literature (Chapter II) indicated that the use of regression equations based on multiple variables was a desirable method for predicting success of students. Based on previous experience, those members of the Mathematics Department at The Ohio State University who designed the CRIMEL program were convinced that prediction of success would be greatly enhanced if students were first exposed to the introductory material of the course. Thus, the examination over this material was to be included in the set of variables used in the regression analysis. An investigation of pre-course data available for Ohio State students yielded three potentially predictive variables.

The Ohio State University Testing Center maintains records on every student who enrolls in the University. Included in these records are the standard score on the mathematics portion of the ACT battery, a raw score on one of two Ohio State Mathematics Placement tests, and the quality points earned by the student in high school mathematics courses. The quality points are determined by multiplying the number of semesters or units of high school mathematics courses by the average grade in those courses.

Math ACT scores are required for all students who enter the University immediately after completing their high school
programs. These scores are not available for students who transfer to this University from other institutions of higher learning.

Placement in mathematics courses at The Ohio State University covers a broad range of mathematical expectations. Five placement levels are possible.

Level 1 - eligibility for enrollment in the standard course in differential calculus taken by students in the physical sciences and engineering.

Level 2 - eligibility for either a pre-calculus course in algebra and trigonometry or one in algebra, probability, and statistics.

Level 3 - eligibility for Math 116 (algebra and linear algebra) or the first course for elementary school teachers.

Level 4 - eligibility for a programmed course in algebra.

Level 5 - not eligible for mathematics courses offered for credit. Students in this level are encouraged to enroll in a remedial course offered by the local school system and to then retake the placement test.

Due to the large range in placement levels, two placement tests are given to students. The Math "D" Test is given to
students whose ACT Math scores are 25 or more. These students are placed in either Level 1 or Level 2. The study by Crosswhite (7) and the more recent investigation by Paul (22) concurred their recommendations relative to the use of the "D" Test score and the quality points earned by the students as the criteria for determining the difference between the two placement levels. The Math "B" Test is given to students whose ACT Math scores are less than 25 or for whom ACT Math scores are unavailable. Placement levels are determined solely on the basis of the "B" Test score as follows:

- Level 2 - score greater than 28.
- Level 3 - score greater than 18 and less than 29.
- Level 4 - score greater than 15 and less than 19.
- Level 5 - score less than 16.

Thus, students who appear in the Math 116 course do not have a consistent or uniform pre-course data base. The student population would have to be partitioned if regression prediction equations were to be utilized. Preliminary consideration of a method of classifying the student population yielded five subgroups. They are listed below together with identifying labels which will be used as subgroup names in the remainder of this paper.
DTES - Those students who took the Math "D" Test.

Data for these students consists of placement
test score, quality points and Math ACT score.

BAYES - Those students who took the Math "B" Test,
who have Math ACT scores available and who
placed Level 3 or higher.

BOYES - Those students who took the Math "B" Test,
who do not have Math ACT scores available and who
placed Level 3 or higher.

BANOS - Those students who took the Math "B" Test,
who have Math ACT scores available and who
placed Level 4 or lower.

BONOS - Those students who took the Math "B" Test,
who do not have Math ACT scores available
and who placed Level 4 or lower.

Students who transfer from other institutions and whose
transcripts indicate a transfer of mathematics credit are not
required to take the Mathematics Placement Test. The number
of such students who enroll in Math 116 is small and these
students were eliminated as subjects of this study.

The student enrollment at the beginning of Math 116 during
the pilot quarter was 903. Of those students, 161 dropped the
course during the quarter. One hundred forty-one students were eliminated from the statistical analysis due to missing data. Missing data resulted from inability to locate pre-course measures for certain students or from students who either did not take one or more of the mid-course examinations or did not take an examination at the scheduled time. The analysis of the pilot data was based on the 601 students for whom complete information was available.

During the pilot quarter 3 one-hour examinations were administered as well as a two-hour final examination. Each examination was given to all students at the same time (early evening). Each mid-course test was scored on the basis of 100 points; the final examination, on the basis of 200 points. For the purpose of identifying expectancy levels for the test group, the following achievement ranges were defined:

0 - 299 total points - Deficiency Range. Students with scores in this range either failed the course or received a grade of D.

300 - 424 total points - Average Range. Students with scores in this range received grades between a high D and a B.

425 - 500 total points - Superior Range. Students in this
Development of Selection Strategies

If an accurate prediction method could be developed, students in the deficiency range would be recommended for reduced pace sections; those in the superior range for the accelerated pace; the remaining students for regular paced sections.

Two linear regression equations predicting total points were generated for each of the five subgroups identified earlier in this chapter. The first equation used pre-course data only, while the second was based on the first examination score as well as the pre-course data. As expected, addition of the first test score as a predictor variable produced an improved prediction equation. A comparison of the correlations of the predicted total points with actual total points for the two prediction equations for each subgroup is presented in Table 1.

For each of the five subgroups, the large difference between the correlation coefficients is due to a high correlation between the first test score and total points earned in the course. There is an equally high correlation between the first test score and the remainder of points earned in the course. Detailed data supporting the use of the regression equations which include the first test score to predict total points is included in Chapter IV.
### TABLE 1

**COMPARISON OF MULTIPLE CORRELATION OF A REGRESSION EQUATION BASED ON PRE-COURSE DATA ONLY AND ONE INCLUDING THE FIRST IN-COURSE EXAMINATION**

<table>
<thead>
<tr>
<th>GROUP</th>
<th>N</th>
<th>RR* (pre-course only)</th>
<th>RR* (including Test I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DTEST</td>
<td>119</td>
<td>0.388</td>
<td>0.730</td>
</tr>
<tr>
<td>BAYES</td>
<td>167</td>
<td>0.374</td>
<td>0.723</td>
</tr>
<tr>
<td>BOYES</td>
<td>85</td>
<td>0.335</td>
<td>0.803</td>
</tr>
<tr>
<td>BANOS</td>
<td>189</td>
<td>0.198</td>
<td>0.688</td>
</tr>
<tr>
<td>BONOS</td>
<td>41</td>
<td>0.055</td>
<td>0.728</td>
</tr>
</tbody>
</table>

* multiple correlation

A question then arose as to whether accurate prediction could be accomplished by considering only the first test score. An investigation revealed that the overall predictive ability of the first test was due mainly to the fact that extremely high and low scores on the first test were accompanied by corresponding extremes when total course performance was considered. For the remainder of the students the initial set of prediction equations was much less sensitive. After studying plots of first test scores versus total points, it was noted that distinction could be made between non-deficient and non-superior groups. (See Table 12, Chapter IV)

A summary of the above findings is included in Table 2.
TABLE 2

DECISION TABLE BASED ON TEST I

<table>
<thead>
<tr>
<th>TEST I RANGE</th>
<th>PREDICTED PERFORMANCE</th>
<th>% CORRECT PREDICTIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>22 - 25</td>
<td>SUPERIOR</td>
<td>89</td>
</tr>
<tr>
<td>17 - 21</td>
<td>NOT DEFICIENT</td>
<td>94</td>
</tr>
<tr>
<td>13 - 16</td>
<td>NOT SUPERIOR</td>
<td>98</td>
</tr>
<tr>
<td>0 - 12</td>
<td>DEFICIENT</td>
<td>85</td>
</tr>
</tbody>
</table>

The efficiency of the extreme ranges of Test I scores in predicting total performance was determined to be acceptable so that regression equations would not be needed for these subgroups of students. The remaining two ranges, although highly efficient in eliminating one of the performance levels, do not distinguish between the remaining two levels. The decision was then made to investigate the possibility of determining separate regression equations for the "not deficient" and "not superior" groups indicated above. The correlations of the predictions based on these new equations with actual performance is indicated in Table 3.
TABLE 3
CORRELATION OF PREDICTED TO ACTUAL POINTS FOR MIDRANGE SUBGROUPS

<table>
<thead>
<tr>
<th>GROUP</th>
<th>RR (Test I, 17 - 21)</th>
<th>RR (Test I, 13 - 16)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DTEST</td>
<td>0.404</td>
<td>0.241</td>
</tr>
<tr>
<td>BAYES</td>
<td>0.434</td>
<td>0.369</td>
</tr>
<tr>
<td>BOYES</td>
<td>0.462</td>
<td>0.165</td>
</tr>
<tr>
<td>BANOS</td>
<td>0.531</td>
<td>0.277</td>
</tr>
<tr>
<td>BONOS</td>
<td>0.533</td>
<td>0.180</td>
</tr>
</tbody>
</table>

A two-stage sequential strategy evolved as a result of the above findings. The first stage consisted of identifying students for accelerated or reduced pace sections if their scores on Test I were above 21 or below 13, respectively. The second stage involved the use of the regression equations generated for the midrange subgroups. This strategy correctly identified levels of achievement for 79 per cent of the students in the pilot group. The above strategy will be labeled as Strategy I in the remainder of this paper. This strategy was used as the basis for making recommendations to students during the Spring Quarter 1970. As mentioned earlier, evaluation of the effectiveness of
this strategy was not attempted due to the campus disruptions.

A second strategy consisting solely of recommendations based on Test I scores appeared as a reasonable alternative. This was especially suggested by the high correlation between Test I scores and total performance in the pilot group data. This single stage strategy is outlined in Table 4. This strategy will be referred to as Strategy II in the remainder of this paper. Strategy II efficiency for the pilot data was 70 per cent.

Two-thirds of the incorrect recommendations in the Test I range 13-21 of Strategy II occur in the ranges 13-15 and 19-21. Recommending the regular pace for students with Test I scores in the range 16-18 would be correct for 76 per cent of the cases. For the other two ranges, accurate prediction of achievement was, at best, difficult. Since one of the behavioral objectives of the CRIMEL program was to give students a larger share of responsibility in determining their own performance levels, this problem provided an opportunity to measure the ability of the student to identify for himself the pacing level appropriate to his abilities. Thus, a third strategy evolved. Strategy III is outlined in Table 5.

The effectiveness of each of the above three strategies was measured during the test quarter. A detailed report of
### TABLE 4

SINGLE STAGE STRATEGY BASED ON
TEST I SCORES ONLY

<table>
<thead>
<tr>
<th>TEST I RANGE</th>
<th>RECOMMENDATION</th>
<th>% CORRECT</th>
</tr>
</thead>
<tbody>
<tr>
<td>22 - 25</td>
<td>ACCELERATED</td>
<td>89</td>
</tr>
<tr>
<td>13 - 21</td>
<td>REGULAR</td>
<td>65</td>
</tr>
<tr>
<td>0 - 12</td>
<td>REDUCED PACE</td>
<td>85</td>
</tr>
</tbody>
</table>

### TABLE 5

SELECTION STRATEGY UTILIZING THE FACTOR
OF STUDENT CHOICE

<table>
<thead>
<tr>
<th>TEST I RANGE</th>
<th>RECOMMENDATION</th>
<th>% CORRECT</th>
</tr>
</thead>
<tbody>
<tr>
<td>22 - 25</td>
<td>ACCELERATED</td>
<td>89</td>
</tr>
<tr>
<td>19 - 21</td>
<td>STUDENT TO CHOOSE BETWEEN ACCELERATED AND REGULAR</td>
<td>95*</td>
</tr>
<tr>
<td>16 - 18</td>
<td>REGULAR</td>
<td>76</td>
</tr>
<tr>
<td>13 - 15</td>
<td>STUDENT TO CHOOSE BETWEEN REGULAR AND REDUCED PACE</td>
<td>99*</td>
</tr>
<tr>
<td>0 - 12</td>
<td>REDUCED PACE</td>
<td>85</td>
</tr>
</tbody>
</table>

* under the assumption that students will select the correct pace.
the data supporting each of the selection strategies, as determined by the pilot data, is given in Chapter IV.

Data Gathered for the Test Group

Since Math ACT scores and Ohio State Math Placement scores were found to be predictive for certain subsets of students in the pilot group, these pre-course measures were also generated for the test group students. The quality points collected for the DTEST subgroup of the pilot students were found not to be efficient predictors of course achievement. This may have been due to the fact that the Testing Center at this University makes no distinction (except for grades earned) between the quality points earned by a student who took two semesters of "general mathematics" at the high school level and one who took two semesters of a standard course in algebra. A reasonable conjecture would be that grades in high school courses at or above the level of Algebra I would be better indicators of potential achievement in a pre-calculus college mathematics course than high school courses consisting of pre-algebra content. Determining quality points based on courses starting with Algebra I is consistent with the methods used by Anderson, Weaver and Wolf (1). Morgan (21) found that the age of a
student was a significant predictor of achievement. An alternative choice would be to determine the time lapse since a student last studied mathematics in a classroom setting. Fujita and O'Reilly (14) determined that significant differences in achievement resulted when their subjects were partitioned according to sex. The above studies were discussed in Chapter II.

During the first class meeting of the test quarter, students were given a questionnaire which determined three additional pre-course measures: (1) quality points earned in high school mathematics courses starting with Algebra I, (2) time lapse, in quarters of a year, since the student's last mathematics course, and (3) the sex of the student. These measures were considered in post-course evaluation. A copy of the questionnaire used to determine this data is included in Appendix B.

The course during the test quarter was offered at 10:00 a.m. and 4:00 p.m. on the central campus of the University and consisted of 15 and 12 sections of students, respectively. The day after Test I was administered, each student was given a recommended pacing level according to one of the three selection strategies discussed earlier. All students in a given section received pacing recommendations under the same strategy. The distribution of strategies among the various sections of the
course is given in Table 6.

The decision to use the 4:00 p.m. sections for Strategy I recommendations was based on the time needed to match background data with Test I scores for use in the regression equations. In order to provide efficient distribution of instructions to teachers coordination of discussions with students regarding the rationale for suggested pacing levels, it was decided to retain Strategy III recommendations within the 10:00 a.m. sections and to use the remaining sections at each hour for Strategy II recommendations. The instructions given to the recitation instructors regarding the pacing level recommendations to be given to their students is included in Appendix B.

Not all students were willing to accept the pacing level for which they were recommended. Approximately 92 per cent of the students, however, did elect to accept the recommended pacing level. Not all students were able to achieve at an acceptable level within the pace which they chose. Records were maintained for each student in the test group indicating the selection strategy utilized, the pacing level recommended, the pacing level chosen and the pacing level at which the student completed the course.

Examinations during the test quarter were administered to
TABLE 6  
DISTRIBUTION OF SELECTION STRATEGIES  
AMONG TEST GROUP SECTIONS

<table>
<thead>
<tr>
<th>STRATEGY</th>
<th>HOUR</th>
<th># SECTIONS</th>
<th># STUDENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>4:00</td>
<td>8</td>
<td>253</td>
</tr>
<tr>
<td>II</td>
<td>10:00</td>
<td>8</td>
<td>282</td>
</tr>
<tr>
<td>III</td>
<td>10:00</td>
<td>7</td>
<td>247</td>
</tr>
<tr>
<td>III</td>
<td>4:00</td>
<td>4</td>
<td>128</td>
</tr>
</tbody>
</table>

all students in the early evening. Due to the large number of students taking each test, it was necessary to conduct the testing at consecutive hours. Hence, two equivalent forms of each test were prepared. Test records for each student, then, consisted of the test number (Test I, Test I', etc.), the test form, the test score and the student's response to each multiple-choice item.

Summary

This chapter has considered the instructional treatment for pilot and test group students. Pre-course and in-course measures were identified. Three selection strategies, based on pilot group data, were identified for the test group. A detailed
analysis of the criteria basic to these selection strategies will be presented in the next chapter.
CHAPTER IV

SELECTION STRATEGIES

In the preceding chapter a brief description of each of the selection strategies used in this study was presented. The purpose of this chapter is to provide a detailed statistical analysis of the pilot data as it relates to the various selection strategies.

The rationale for partitioning the pilot group into five subgroups was based on differences in pre-course measures available for use as predictor variables. Considerable variation was found among the five subgroups with respect to both pre-course and in-course measures. In Table 7 the means and standard deviations of the pre-course variables (Ohio State University Math Test score, Math ACT score and quality points earned in high school mathematics) as well as the Test I score and total points earned in Math 116 are presented for each of the pilot subgroups.

As a preliminary to determining regression prediction equations for each of the population subgroups, two sets of
correlations were obtained. The correlation of potential predictor variables with the total points earned in the course is given in Table 8. The correlation of pre-course measures with the first examination appears in Table 9.

Multiple (stepwise) regression analysis was then applied to the data for each of the subgroups. The regression coefficients for the equations which resulted are presented in Table 10.

The value of including Test I as a predictor variable was determined by considering the loss in multiple correlation which resulted when Test I was deleted from the regression analysis. A comparison of the last columns of Tables 10 and 11 defines the effect of deleting Test I.

Since Test I had a high correlation with total points for all subgroups, a study was made of the plot of Test I versus total points. The objective of this effort was to determine if efficient prediction of achievement in the course could be made on the basis of Test I alone. The joint distribution of these two data sets is presented in Table 12.

Subjects with Test I scores in the ranges of 22-25 and 0-12 could be recommended for accelerated and reduced pace sections, respectively, with high efficiency. High efficiency is also obtained by not recommending the reduced pace treatment
TABLE 7  
MEANS AND STANDARD DEVIATIONS  
FOR PILOT VARIABLES

<table>
<thead>
<tr>
<th></th>
<th>MID</th>
<th>TP</th>
<th>OSUM</th>
<th>ACT</th>
<th>QP</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAYES</td>
<td>M</td>
<td>14.9</td>
<td>324.2</td>
<td>23.3</td>
<td>20.4</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>3.3</td>
<td>56.7</td>
<td>3.6</td>
<td>3.4</td>
</tr>
<tr>
<td>BOYES</td>
<td>M</td>
<td>16.0</td>
<td>360.9</td>
<td>25.1</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>4.1</td>
<td>62.8</td>
<td>4.3</td>
<td>--</td>
</tr>
<tr>
<td>DTEST</td>
<td>M</td>
<td>16.8</td>
<td>361.7</td>
<td>7.5</td>
<td>27.5</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>3.3</td>
<td>56.0</td>
<td>2.8</td>
<td>2.1</td>
</tr>
<tr>
<td>BANOS</td>
<td>M</td>
<td>13.65</td>
<td>299.4</td>
<td>14.7</td>
<td>18.0</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>3.9</td>
<td>56.9</td>
<td>2.8</td>
<td>3.7</td>
</tr>
<tr>
<td>BONOS</td>
<td>M</td>
<td>14.1</td>
<td>309.6</td>
<td>15.6</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>3.5</td>
<td>55.1</td>
<td>2.3</td>
<td>--</td>
</tr>
</tbody>
</table>
### TABLE 8
CORRELATION OF PREDICTOR VARIABLES WITH TOTAL POINTS

<table>
<thead>
<tr>
<th>GROUP</th>
<th>N</th>
<th>TEST 1</th>
<th>ACT</th>
<th>OSUM</th>
<th>QP</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAYES</td>
<td>167</td>
<td>.703</td>
<td>.198</td>
<td>.366</td>
<td>--</td>
</tr>
<tr>
<td>BOYES</td>
<td>85</td>
<td>.800</td>
<td>--</td>
<td>.335</td>
<td>--</td>
</tr>
<tr>
<td>DT TEST</td>
<td>119</td>
<td>.728</td>
<td>.277</td>
<td>.331</td>
<td>.209</td>
</tr>
<tr>
<td>BANOS</td>
<td>189</td>
<td>.687</td>
<td>.188*</td>
<td>.091*</td>
<td>--</td>
</tr>
<tr>
<td>BONOS</td>
<td>41</td>
<td>.721</td>
<td>--</td>
<td>.055*</td>
<td>--</td>
</tr>
</tbody>
</table>

* Not different from zero at .05 level of significance.

### TABLE 9
CORRELATION OF PRE-COURSE VARIABLES WITH TEST I

<table>
<thead>
<tr>
<th>GROUP</th>
<th>N</th>
<th>ACT</th>
<th>OSUM</th>
<th>QP</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAYES</td>
<td>167</td>
<td>.239</td>
<td>.293</td>
<td>--</td>
</tr>
<tr>
<td>BOYES</td>
<td>85</td>
<td>--</td>
<td>.335</td>
<td>--</td>
</tr>
<tr>
<td>DT TEST</td>
<td>119</td>
<td>.321</td>
<td>.411</td>
<td>.254</td>
</tr>
<tr>
<td>BANOS</td>
<td>189</td>
<td>.222</td>
<td>.137*</td>
<td>--</td>
</tr>
<tr>
<td>BONOS</td>
<td>41</td>
<td>--</td>
<td>.221</td>
<td>--</td>
</tr>
</tbody>
</table>

* Not different from zero at .05 level of significance.
### TABLE 10

**COEFFICIENTS IN MULTIPLE REGRESSION EQUATIONS**
**PREDICTING TOTAL POINTS USING TEST I TOGETHER**
**WITH PRE-COURSE DATA**

<table>
<thead>
<tr>
<th>GROUP</th>
<th>TEST I</th>
<th>OSUM</th>
<th>ACT</th>
<th>QP</th>
<th>CONSTANT</th>
<th>RR</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAYES</td>
<td>11.18</td>
<td>2.89</td>
<td>-0.34</td>
<td>--</td>
<td>96.89</td>
<td>.723</td>
</tr>
<tr>
<td>BOYES</td>
<td>11.75</td>
<td>1.09</td>
<td>--</td>
<td>--</td>
<td>145.08</td>
<td>.803</td>
</tr>
<tr>
<td>DTEST</td>
<td>11.84</td>
<td>0.49</td>
<td>1.18</td>
<td>0.11</td>
<td>125.89</td>
<td>.730</td>
</tr>
<tr>
<td>BANOS</td>
<td>9.96</td>
<td>-0.16</td>
<td>0.59</td>
<td>--</td>
<td>155.35</td>
<td>.688</td>
</tr>
<tr>
<td>BONOS</td>
<td>11.70</td>
<td>-2.57</td>
<td>--</td>
<td>--</td>
<td>184.66</td>
<td>.728</td>
</tr>
</tbody>
</table>

### TABLE 11

**COEFFICIENTS IN MULTIPLE REGRESSION EQUATIONS**
**PREDICTING TOTAL POINTS USING ONLY PRE-COURSE DATA**

<table>
<thead>
<tr>
<th>GROUP</th>
<th>OSUM</th>
<th>ACT</th>
<th>QP</th>
<th>CONSTANT</th>
<th>RR</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAYES</td>
<td>5.38</td>
<td>1.40</td>
<td>--</td>
<td>170.08</td>
<td>.374</td>
</tr>
<tr>
<td>BOYES</td>
<td>4.88</td>
<td>--</td>
<td>--</td>
<td>238.67</td>
<td>.335</td>
</tr>
<tr>
<td>DTEST</td>
<td>4.83</td>
<td>5.05</td>
<td>0.63</td>
<td>172.62</td>
<td>.388</td>
</tr>
<tr>
<td>BANOS</td>
<td>1.25</td>
<td>2.74</td>
<td>--</td>
<td>231.75</td>
<td>.198</td>
</tr>
<tr>
<td>BONOS</td>
<td>1.30</td>
<td>--</td>
<td>--</td>
<td>289.39</td>
<td>.055</td>
</tr>
</tbody>
</table>
for students in the Test I range 17-21 and not recommending the accelerated pace for students in the Test I range 13-16. For each set of midrange students, new regression equations were generated. The data related to the multiple regression analysis is presented in Tables 13 through 18.

The two-stage strategy defined above was titled Strategy I. The overall efficiency of Strategy I was found to be 80 per cent. From the following tables, it is evident that the application of regression analysis did not result in a highly predictive strategy for the students in Test I range 13-16. The inability of predictor
<table>
<thead>
<tr>
<th>GROUP</th>
<th>TEST I</th>
<th>T. PTS.</th>
<th>OSUM</th>
<th>ACT</th>
<th>QP</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAYES</td>
<td>M 18.39</td>
<td>369.37</td>
<td>25.26</td>
<td>21.37</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>SD 1.27</td>
<td>42.10</td>
<td>3.89</td>
<td>3.52</td>
<td>--</td>
</tr>
<tr>
<td>BOYES</td>
<td>M 18.48</td>
<td>391.94</td>
<td>25.14</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>SD 1.27</td>
<td>40.87</td>
<td>4.13</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>DTEST</td>
<td>M 18.76</td>
<td>386.54</td>
<td>8.05</td>
<td>27.88</td>
<td>23.75</td>
</tr>
<tr>
<td></td>
<td>SD 1.34</td>
<td>40.33</td>
<td>2.91</td>
<td>2.00</td>
<td>7.55</td>
</tr>
<tr>
<td>BANOS</td>
<td>M 18.40</td>
<td>350.11</td>
<td>14.84</td>
<td>19.40</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>SD 1.14</td>
<td>38.72</td>
<td>3.20</td>
<td>3.52</td>
<td>--</td>
</tr>
<tr>
<td>BONOS</td>
<td>M 19.00</td>
<td>372.86</td>
<td>16.00</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>SD 1.73</td>
<td>47.45</td>
<td>1.91</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>
**TABLE 14**

**CORRELATION OF PREDICTOR VARIABLES FROM PILOT SUBGROUPS (TEST I BETWEEN 17 AND 21) WITH TOTAL POINTS**

<table>
<thead>
<tr>
<th>GROUP</th>
<th>N</th>
<th>TEST I</th>
<th>OSUM</th>
<th>ACT</th>
<th>QP</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAYES</td>
<td>49</td>
<td>.304</td>
<td>.324</td>
<td>.103*</td>
<td>--</td>
</tr>
<tr>
<td>BOYES</td>
<td>35</td>
<td>.452</td>
<td>.022*</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>DTEST</td>
<td>64</td>
<td>.389</td>
<td>.039*</td>
<td>.122*</td>
<td>.120*</td>
</tr>
<tr>
<td>BANOS</td>
<td>45</td>
<td>.493</td>
<td>-.076*</td>
<td>-.026*</td>
<td>--</td>
</tr>
<tr>
<td>BONOS</td>
<td>7</td>
<td>.448</td>
<td>-.336</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

* Not significantly different from zero at the .05 level.
<table>
<thead>
<tr>
<th>GROUP</th>
<th>TEST I</th>
<th>OSUM</th>
<th>ACT</th>
<th>QP</th>
<th>CONSTANT</th>
<th>RR</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAYES</td>
<td>9.81</td>
<td>3.44</td>
<td>-0.56</td>
<td>--</td>
<td>113.96</td>
<td>.434</td>
</tr>
<tr>
<td>BOYES</td>
<td>15.03</td>
<td>0.93</td>
<td>--</td>
<td>--</td>
<td>90.78</td>
<td>.462</td>
</tr>
<tr>
<td>DTEST</td>
<td>11.77</td>
<td>-0.85</td>
<td>0.06</td>
<td>0.58</td>
<td>157.00</td>
<td>.404</td>
</tr>
<tr>
<td>BANOS</td>
<td>18.42</td>
<td>-1.98</td>
<td>-1.28</td>
<td>--</td>
<td>65.28</td>
<td>.531</td>
</tr>
<tr>
<td>BONOS</td>
<td>11.80</td>
<td>10.67</td>
<td>--</td>
<td>--</td>
<td>269.88</td>
<td>.533</td>
</tr>
</tbody>
</table>
TABLE 16

MEANS AND STANDARD DEVIATIONS FOR PILOT SUBGROUPS WITH TEST 1 SCORES BETWEEN 13 AND 16

<table>
<thead>
<tr>
<th>GROUP</th>
<th>TEST I</th>
<th>T.PTS.</th>
<th>OSUM</th>
<th>ACT</th>
<th>QP</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAYES</td>
<td>M 14.56</td>
<td>314.60</td>
<td>22.37</td>
<td>20.11</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>SD 1.14</td>
<td>42.46</td>
<td>2.94</td>
<td>3.54</td>
<td>--</td>
</tr>
<tr>
<td>BOYES</td>
<td>M 14.63</td>
<td>343.59</td>
<td>25.07</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>SD 0.97</td>
<td>35.07</td>
<td>3.82</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>DTEST</td>
<td>M 14.92</td>
<td>339.59</td>
<td>7.05</td>
<td>27.03</td>
<td>21.51</td>
</tr>
<tr>
<td></td>
<td>SD 1.21</td>
<td>37.96</td>
<td>2.05</td>
<td>1.98</td>
<td>10.17</td>
</tr>
<tr>
<td>BANOS</td>
<td>M 14.47</td>
<td>304.11</td>
<td>15.21</td>
<td>18.16</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>SD 1.08</td>
<td>42.28</td>
<td>2.43</td>
<td>3.41</td>
<td>--</td>
</tr>
<tr>
<td>BONOS</td>
<td>M 14.95</td>
<td>316.00</td>
<td>16.09</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>SD 1.09</td>
<td>35.25</td>
<td>2.11</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>
### TABLE 17

**CORRELATION OF PREDICTOR VARIABLES FROM PILOT SUBGROUPS (TEST I BETWEEN 13 AND 16) WITH TOTAL POINTS**

<table>
<thead>
<tr>
<th>GROUP</th>
<th>N</th>
<th>TEST I</th>
<th>OSUM</th>
<th>ACT</th>
<th>QP</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAYES</td>
<td>75</td>
<td>.306</td>
<td>.181*</td>
<td>.170*</td>
<td>--</td>
</tr>
<tr>
<td>BOYES</td>
<td>27</td>
<td>.160</td>
<td>.054*</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>DTEST</td>
<td>37</td>
<td>.132*</td>
<td>.119*</td>
<td>.162*</td>
<td>-.031*</td>
</tr>
<tr>
<td>BANOS</td>
<td>70</td>
<td>.251</td>
<td>.144*</td>
<td>.048*</td>
<td>--</td>
</tr>
<tr>
<td>BONOS</td>
<td>22</td>
<td>.094*</td>
<td>-.165*</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

* Not significantly different from zero at the .05 level.
<table>
<thead>
<tr>
<th>GROUP</th>
<th>TEST I</th>
<th>OSUM</th>
<th>ACT</th>
<th>QP</th>
<th>CONSTANT</th>
<th>RR</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAYES</td>
<td>11.26</td>
<td>2.59</td>
<td>0.77</td>
<td>--</td>
<td>77.33</td>
<td>.369</td>
</tr>
<tr>
<td>BOYES</td>
<td>5.68</td>
<td>0.37</td>
<td>--</td>
<td>--</td>
<td>251.06</td>
<td>.165</td>
</tr>
<tr>
<td>DTEST</td>
<td>3.98</td>
<td>2.18</td>
<td>2.80</td>
<td>-0.42</td>
<td>198.31</td>
<td>.241</td>
</tr>
<tr>
<td>BANOS</td>
<td>9.26</td>
<td>1.63</td>
<td>0.81</td>
<td>--</td>
<td>130.62</td>
<td>.277</td>
</tr>
<tr>
<td>BONOS</td>
<td>2.33</td>
<td>-2.58</td>
<td>--</td>
<td>--</td>
<td>322.62</td>
<td>.180</td>
</tr>
</tbody>
</table>

TABLE 18

COEFFICIENTS IN REGRESSION EQUATIONS PREDICTING TOTAL POINTS (PILOT GROUPS, TEST I BETWEEN 13 AND 16)
variables to account for individual differences in achievement for students at the lower ranges of performance when utilizing sequential strategies was a phenomenon which also occurred in the study by Fujita and O'Reilly (14).

At this point some mention should be made of the use of Test I in regression equations whose dependent variable is total points earned in the course. A potential problem lies in the fact that the Test I score represents approximately 20 per cent of the total points earned by a student. Although this is a small percentage, it could introduce enough bias into the regression coefficients to warrant further investigation. A method which eliminates this bias (if any) would be to first predict the points earned in the course after Test I as a function of Test I and the pre-course measures. Predicted total points would then be determined by summing the Test I points with the predicted "other" points. The Test I points are desirable to include in the selection procedure in some form since, except for students who enroll in the reduced pace sections, the Test I score is considered in determining the course grade.

If the regression equation predicting "other" points for the group BAYES, for example, is

\[
\text{Other Points} = (a) \ (\text{Test I}) + (b) \ (\text{ACT}) + (c) \ (\text{OSUM}),
\]
then the equation predicting total points is

\[ \text{Total Points} = (a + 4) \times \text{(Test I)} + (b) \times \text{(ACT)} + (c) \times \text{(OSUM)}. \]

The latter equation is due to the fact that Test I consisted of twenty-five questions with a perfect score yielding 100 points for the student.

Tables 19 and 20 contain the coefficients in the alternate prediction equations for use in Strategy I recommendations. A comparison of Table 19 with Table 15 and Table 20 with Table 18 with respect to pacing levels recommended for individual students revealed only a small number of changes in recommendations. The efficiency of the Strategy I selection procedure remained unchanged. Hence, the original equations were considered appropriate.

A second strategy to be applied during the test quarter was the sole use of Test I scores as the basis for selecting students for the differently paced sections. This strategy would correctly identify 76 per cent of the pilot group students, and was titled Strategy II.

An important criterion to be considered in the evaluation of effectiveness of a given selection strategy is the nature of the incorrect recommendations. Placement of a student at too high a level is a false positive recommendation. If the recommendation
### TABLE 19

**COEFFICIENTS IN ALTERNATE REGRESSION EQUATIONS PREDICTING TOTAL POINTS (PILOT GROUP, TEST I BETWEEN 17 AND 21)**

<table>
<thead>
<tr>
<th>GROUP</th>
<th>TEST I</th>
<th>OSUM</th>
<th>ACT</th>
<th>QP</th>
<th>CONSTANT</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAYES</td>
<td>9.92</td>
<td>3.47</td>
<td>-0.52</td>
<td>--</td>
<td>110.36</td>
</tr>
<tr>
<td>BOYES</td>
<td>15.21</td>
<td>1.06</td>
<td>--</td>
<td>--</td>
<td>83.89</td>
</tr>
<tr>
<td>DTTEST</td>
<td>11.49</td>
<td>-0.20</td>
<td>0.35</td>
<td>0.46</td>
<td>163.14</td>
</tr>
<tr>
<td>BANOS</td>
<td>15.39</td>
<td>-1.23</td>
<td>-1.11</td>
<td>--</td>
<td>109.16</td>
</tr>
<tr>
<td>BONOS</td>
<td>3.77</td>
<td>-7.80</td>
<td>--</td>
<td>--</td>
<td>416.84</td>
</tr>
</tbody>
</table>
TABLE 20

COEFFICIENTS IN ALTERNATE REGRESSION EQUATIONS PREDICTING TOTAL POINTS (PILOT GROUP, TEST I BETWEEN 13 and 16)

<table>
<thead>
<tr>
<th>GROUP</th>
<th>TEST I</th>
<th>OSUM</th>
<th>ACT</th>
<th>QP</th>
<th>CONSTANT</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAYES</td>
<td>11.23</td>
<td>2.39</td>
<td>1.01</td>
<td>--</td>
<td>77.60</td>
</tr>
<tr>
<td>BOYES</td>
<td>5.68</td>
<td>0.38</td>
<td>--</td>
<td>--</td>
<td>251.06</td>
</tr>
<tr>
<td>DTEST</td>
<td>1.60</td>
<td>-1.85</td>
<td>1.97</td>
<td>0.28</td>
<td>277.19</td>
</tr>
<tr>
<td>BANOS</td>
<td>10.80</td>
<td>2.14</td>
<td>0.93</td>
<td>--</td>
<td>97.73</td>
</tr>
<tr>
<td>BONOS</td>
<td>2.33</td>
<td>-2.58</td>
<td>--</td>
<td>--</td>
<td>322.61</td>
</tr>
</tbody>
</table>
is too low, then it is known as a false negative recommendation.
The distribution of correct, false positive, and false negative recommendations for each of the five pilot subgroups under the strategies discussed to this point is presented in Table 21.
Summary statistics are given in Table 22. Recommendation levels are 1, 3 and 5 for accelerated, regular and reduced pace, respectively. Level 1 false negative and Level 5 false positive recommendations are considered undefined.

In all three of the preceding strategies, approximately one-half of the incorrect recommendations appear as false positive placements into the regular paced group. Since students would have the opportunity of moving to the reduced pace group from the regular group if they remained in the regular group after the initial resectioning of the test group, such false positive recommendations were judged to be acceptable. However, since students could not move into a higher pacing level later in the test quarter, the number of false negative recommendations should be small. Nine per cent of the recommendations under Strategy I and 10 per cent under Strategy II fall into this latter category. As indicated earlier in this paper, students could request to enroll in a pacing level different from that which was recommended.
### TABLE 21

**COMPARISON OF SINGLE AND MULTIPLE STAGE PREDICTION STRATEGIES ACCORDING TO PILOT GROUP SUBGROUPS**

<table>
<thead>
<tr>
<th>GROUP</th>
<th>REC</th>
<th>Single Stage Regression</th>
<th>Multiple Stage Strategy I</th>
<th>Single Stage Strategy II</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>BAYES</td>
<td>c</td>
<td>4</td>
<td>78</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>f.p.</td>
<td>2</td>
<td>24</td>
<td>--</td>
</tr>
<tr>
<td>n = 167</td>
<td>f.n.</td>
<td>-</td>
<td>6</td>
<td>20</td>
</tr>
<tr>
<td>BOYES</td>
<td>c</td>
<td>10</td>
<td>46</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>f.p.</td>
<td>2</td>
<td>10</td>
<td>-</td>
</tr>
<tr>
<td>n = 85</td>
<td>f.n.</td>
<td>-</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>DTEST</td>
<td>c</td>
<td>9</td>
<td>74</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>f.p.</td>
<td>5</td>
<td>10</td>
<td>-</td>
</tr>
<tr>
<td>n = 119</td>
<td>f.n.</td>
<td>-</td>
<td>9</td>
<td>2</td>
</tr>
<tr>
<td>BANOS</td>
<td>c</td>
<td>0</td>
<td>76</td>
<td>63</td>
</tr>
<tr>
<td></td>
<td>f.p.</td>
<td>0</td>
<td>31</td>
<td>-</td>
</tr>
<tr>
<td>n = 189</td>
<td>f.n.</td>
<td>-</td>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>BONOS</td>
<td>c</td>
<td>0</td>
<td>20</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>f.p.</td>
<td>0</td>
<td>4</td>
<td>-</td>
</tr>
<tr>
<td>n = 41</td>
<td>f.n.</td>
<td>-</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>TOTALS</td>
<td>c</td>
<td>23</td>
<td>294</td>
<td>126</td>
</tr>
<tr>
<td></td>
<td>f.p.</td>
<td>9</td>
<td>79</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>f.n.</td>
<td>-</td>
<td>24</td>
<td>46</td>
</tr>
</tbody>
</table>
### Table 22

**Efficiency Distribution for Three Selection Strategies**

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Correct</th>
<th>False Positive</th>
<th>False Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Single Stage</td>
<td>443</td>
<td>9</td>
<td>79</td>
</tr>
<tr>
<td>Regression</td>
<td>74%</td>
<td>1</td>
<td>13</td>
</tr>
<tr>
<td>Multiple Stage</td>
<td>N 479</td>
<td>10</td>
<td>56</td>
</tr>
<tr>
<td>Strategy I</td>
<td>80%</td>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>Single Stage</td>
<td>N 460</td>
<td>2</td>
<td>81</td>
</tr>
<tr>
<td>Strategy II</td>
<td>% 76</td>
<td>1</td>
<td>13</td>
</tr>
</tbody>
</table>

Strategy III, which incorporates the factor of student choice, has been explained in the previous chapter. Evaluation of the effectiveness of all three strategies with respect to the test group is presented in the next chapter.
CHAPTER V

ANALYSIS OF THE DATA

The analysis of the data generated during the course of this study consisted of several stages. The evaluation of the pilot data relative to the selection strategies which were used to identify suggested pacing levels for students in the test quarter was the focus of Chapter IV. The analysis of the remaining data is organized in this chapter as follows:

1. Comparison of the pilot and test groups with respect to pre-course measures.
2. Determination of the equivalence of the different forms of examinations administered during the test quarter.
3. Tests of the three hypotheses stated in Chapter I.
4. Comparison of the effectiveness of the three selection strategies for the test group.

The interpretation of the tests of hypotheses comparing the pilot and test groups is strongly affected by the extent to which
the student populations of the two groups are similar. A comparison of pre-course measures for each of the five population subgroups revealed no significant differences at the 10 per cent level for six of the eight data sets considered. Two comparisons were significant; one result favored the test group, the other, the pilot group. Test of significance was made by using a t-test (see Appendix F). The results of these comparisons appears in Table 23.

Table 7 (see Chapter IV) revealed a wide variation in achievement levels between the five student subgroups. The achievement of the entire pilot group can be obtained by making use of the distribution of students among the five subgroups. Table 23 indicates the difference in distribution of the five subgroups between the pilot and test groups. Under the assumptions that the pilot and test group subgroups are equivalent relative to pre-course measures, and that the testing during the pilot and test quarters was equivalent, a comparison of expected achievement levels can be generated. In Table 24 mean achievement for the test group is given for the entire group as well as for the subgroups of students which were treated under the various selection strategies. In order to provide control for the differences which did exist between the
### TABLE 23

**COMPARISON OF PILOT AND TEST GROUP PRE-COURSE MEASURES**

<table>
<thead>
<tr>
<th>Group</th>
<th>Measure</th>
<th>PILOT GROUP</th>
<th></th>
<th>TEST GROUP</th>
<th></th>
<th>t</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>N</td>
<td>M</td>
<td>SD</td>
<td>N</td>
<td>M</td>
<td>SD</td>
</tr>
<tr>
<td>BAYES</td>
<td>ACT</td>
<td>167</td>
<td>20.4</td>
<td>3.4</td>
<td>198</td>
<td>21.0</td>
<td>2.8</td>
</tr>
<tr>
<td>BAYES</td>
<td>OSUM</td>
<td>167</td>
<td>23.3</td>
<td>3.6</td>
<td>198</td>
<td>23.4</td>
<td>3.5</td>
</tr>
<tr>
<td>BOYES</td>
<td>OSUM</td>
<td>85</td>
<td>25.1</td>
<td>4.3</td>
<td>75</td>
<td>24.6</td>
<td>4.5</td>
</tr>
<tr>
<td>DTEST</td>
<td>ACT</td>
<td>119</td>
<td>27.5</td>
<td>2.1</td>
<td>298</td>
<td>27.3</td>
<td>2.5</td>
</tr>
<tr>
<td>DTEST</td>
<td>OSUM</td>
<td>119</td>
<td>7.5</td>
<td>2.8</td>
<td>298</td>
<td>6.8</td>
<td>3.5</td>
</tr>
<tr>
<td>BANOS</td>
<td>ACT</td>
<td>189</td>
<td>18.0</td>
<td>3.7</td>
<td>113</td>
<td>17.5</td>
<td>3.8</td>
</tr>
<tr>
<td>BANOS</td>
<td>OSUM</td>
<td>189</td>
<td>14.7</td>
<td>2.8</td>
<td>113</td>
<td>14.1</td>
<td>3.5</td>
</tr>
<tr>
<td>BONOS</td>
<td>OSUM</td>
<td>41</td>
<td>15.6</td>
<td>2.3</td>
<td>15</td>
<td>14.2</td>
<td>3.8</td>
</tr>
<tr>
<td>GROUP</td>
<td>DT/TEST</td>
<td>BAYES</td>
<td>BOYES</td>
<td>BANOS</td>
<td>BONOS</td>
<td>EXPECTED ACHIEVEMENT</td>
<td></td>
</tr>
<tr>
<td>------------</td>
<td>---------</td>
<td>-------</td>
<td>-------</td>
<td>-------</td>
<td>-------</td>
<td>---------------------</td>
<td></td>
</tr>
<tr>
<td>PILOT</td>
<td>119</td>
<td>167</td>
<td>85</td>
<td>189</td>
<td>41</td>
<td>328.0</td>
<td></td>
</tr>
<tr>
<td>TEST</td>
<td>298</td>
<td>198</td>
<td>75</td>
<td>113</td>
<td>15</td>
<td>339.8</td>
<td></td>
</tr>
<tr>
<td>STRATEGY I</td>
<td>76</td>
<td>38</td>
<td>15</td>
<td>18</td>
<td>1</td>
<td>346.4</td>
<td></td>
</tr>
<tr>
<td>STRATEGY II</td>
<td>73</td>
<td>48</td>
<td>25</td>
<td>22</td>
<td>4</td>
<td>341.9</td>
<td></td>
</tr>
<tr>
<td>STRATEGY III</td>
<td>94</td>
<td>62</td>
<td>20</td>
<td>69</td>
<td>11</td>
<td>333.5</td>
<td></td>
</tr>
</tbody>
</table>
test and pilot groups relative to background, treatment and testing, standardized measures were used in the tests of the hypotheses.

A second test which was performed prior to evaluation of the hypotheses concerned the equivalence of the two forms of Test I which were administered to the test group students. Comparison of the means using a t-test revealed that larger differences would occur more than 31 per cent of the time by chance. It was concluded that the differences between the means of the two test forms would not adversely affect the analysis of the data.

The next stage in the data analysis involved the test of the first two hypotheses. The first hypothesis, stated in null form, was

Students within a given expectancy range in the test group will achieve at the same level at the completion of the first module as students in the pilot group within the same expectancy range.

Test of the first hypothesis determines differences in achievement between students recommended for a given pacing level in the test group and the students in the pilot group who would have been recommended for the same pacing level. The test of the first hypothesis was performed for each of the three
recommendation strategies employed during the test quarter.
The evaluation of the first hypothesis represents a partial
determination of the difference in achievement between a
heterogeneous group paced group and homogeneous individually
paced groups. Hence, the pilot group which would have been
recommended for the regular pace and the test group which was
recommended for the regular pace were considered as pivotal
points in the tests of the first hypothesis.

The second hypothesis, stated in null form, was

For test group students within a given expectancy
range, there is no difference in achievement
between the subsets of those students who complete
the first module at different pacing levels.

The second hypothesis tests the differences in achievement
between those students who finished Math 115.01 at different
pacing levels but who were given the same recommendations
under the same selection strategy. Analysis of variance was
employed as the statistical technique to test the difference in
means.

Achievement was defined as the examination score on the
second test for the pilot and test groups. Test II in the test
group marked the end of Math 115.01. For the accelerated
and regular pace groups in the test quarter, and the entire
pilot group, Test II was an examination on the content covered in the course after Test I. However, for the reduced pace groups in the test quarter, Test II' and the Final were comprehensive examinations over the entire content of the course. The subset of questions on Test II' and the Final which most closely paralleled items on Test II (administered to the accelerated and regular pace sections) were used to determine achievement for the reduced pace students. The subset of questions from the Final were used unless the student completed Math 115.01 with Test II'. The equivalence relation between the items on Test II and the items on Test II' and the Final is presented in Appendix E.

Test of hypotheses 1 and 2 under the above assumptions are reported in Tables 25, 26 and 27 (pp. 83-5). Pacing levels recommended to students under the various selection strategies are coded as follows:

1. Accelerated pace.
2. Student to choose between accelerated and regular pace.
3. Regular pace.
4. Student to choose between regular and reduced pace.
5. Reduced pace.
The pacing level at which students finished the course are coded 1, 3 or 5 as above. In each table the measures used to evaluate Hypothesis 1 are to the left of the \( t \) values; the measures used to evaluate Hypothesis 2 are to the right of the \( F \) values.

An explanation of the format used in Tables 25, 26 and 27 as well as a guide to the interpretation of the \( t \) and \( F \) values which are reported is given below using data in Table 25 as an example.

Relative to Hypothesis 1, a positive \( t \)-score indicates a difference favoring the pilot group; a negative \( t \)-score, the test group. The \( t \)-scores are reported only for the non-regular pace since the scores of students recommended for the regular pace in the pilot and test groups are used as reference measures. The \( t \)-value 0.018 reported for recommended Level 1 (accelerated pace) under Strategy I is computed utilizing the following steps:

1. Determine the mean and standard deviation of achievement scores for the pilot students who would have been recommended for the accelerated pace.

2. Determine the same measures for the pilot students
who would have been recommended for the regular pace.

3. Represent the difference of the means in (1) and (2) as a standardized score, \( Z_p \).

4. Determine the mean and standard deviation of the achievement for the test group students who were recommended for the accelerated pace.

5. Determine the same measures for the test group students recommended for the regular pace.

6. Represent the difference of the means in (4) and (5) as a standardized score, \( Z_t \).

7. Use a one-tailed \( t \)-test to test the hypothesis \( Z_p = Z_t \) against the alternative \( Z_p > Z_t \).

The mathematical formula used to compute the value of \( t \) in (7) above is given in Appendix F.

The procedure used to test Hypothesis 1 relative to other pacing levels is analogous to steps (1) through (7) above with "accelerated" replaced by the pacing level being tested.

A pictorial representation of the relationships of \( Z_p \) and \( Z_t \) to the pilot and test group achievement distributions is given in Figure 4. \( R_m \) and \( \bar{x}_m \) indicate the distribution of achievement scores and mean of the distribution, respectively, for
Figure 4. Pictorial Representation of $Z_p$ and $Z_t$ Measures.
students recommended for pacing level \( m \); \( Z_p \) and \( Z_t \),

the \( Z_p \) and \( Z_t \) measures, respectively, for pacing level \( n \).

Interpretation of the \( t \)-values can be made, relative to

the various pacing levels, as outlined in the steps below. Since

all \( t \)-values were found to be positive, (that is, differences

favoring the pilot group) the following discussion is restricted to

positive values of \( t \).

1. Pacing Levels 1 and 2

   (a) \( t > 1.645 \) --- reject the hypothesis that no

       significant differences exist at the five per cent

       level. Conclude that the students in the test

       group did not achieve at as high a level as

       expected. The scores of these students were

       significantly closer to the scores of the students

       recommended for the regular pace in the test

       group than was the case in the pilot group.

   (b) \( t \leq 1.645 \) --- no conclusion can be made with a

       high degree of confidence.

2. Pacing Levels 4 and 5

   (a) \( t > 1.645 \) --- reject the hypothesis as in 1 (a).

       Students in test group achieved at a higher level

       (relative to those recommended for the regular
pace) than those students in the pilot group. In cases where most of these students finished the course at the reduced pace, rejection of Hypothesis 1 indicates a positive effect on achievement related to these test group students being able to study certain mathematical concepts at a slower pace than was possible for similar pilot group students.

(b) $t \leq 1.645$ --- no conclusion can be made with a high degree of confidence.

Thus, for Strategy I recommendations, the following conclusions can be drawn. The $t$-value of 0.018 for Level 1 recommendations indicates that the performance of students in the test group identified as potential high achievers (relative to those recommended for the regular pace) was not significantly different from the achievement of a similar set of students in the pilot group. The $t$-value of 2.660 for the Level 5 recommendations allows rejection of Hypothesis 1. Thus, the test group students recommended for the reduced pace under Strategy I achieved at a significantly higher level (relative to those recommended for the regular pace) than similar students in the pilot group.


<table>
<thead>
<tr>
<th>REC</th>
<th>N</th>
<th>MEAN</th>
<th>VAR</th>
<th>Zp</th>
<th>N</th>
<th>MEAN</th>
<th>VAR</th>
<th>Zt</th>
<th>t</th>
<th>F</th>
<th>N</th>
<th>MEAN</th>
<th>FINISH</th>
</tr>
</thead>
<tbody>
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<td>43</td>
<td>17.02</td>
<td>7.93</td>
<td>5.51</td>
<td>14</td>
<td>15.14</td>
<td>3.82</td>
<td>5.48</td>
<td>0.018</td>
<td>0.21</td>
<td>4</td>
<td>14.75</td>
<td>1</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td>15.30</td>
<td>3</td>
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<tr>
<td>3</td>
<td>359</td>
<td>14.51</td>
<td>8.41</td>
<td>120</td>
<td>11.88</td>
<td>9.72</td>
<td></td>
<td></td>
<td></td>
<td>4.23*</td>
<td>91</td>
<td>12.17</td>
<td>3</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>29</td>
<td>10.62</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>186</td>
<td>11.62</td>
<td>9.45</td>
<td>10.61</td>
<td>36</td>
<td>7.76</td>
<td>9.31</td>
<td>6.84</td>
<td>2.660*</td>
<td>0.52</td>
<td>4</td>
<td>9.25</td>
<td>3</td>
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<td></td>
<td></td>
<td>32</td>
<td>7.57</td>
<td>5</td>
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</tbody>
</table>

* Significant at .05 level.
TABLE 26

COMPARISON OF ACHIEVEMENT ON TEST TWO BETWEEN PILOT AND TEST GROUPS
UNDER STRATEGY II RECOMMENDATIONS

<table>
<thead>
<tr>
<th>REC</th>
<th>N</th>
<th>MEAN</th>
<th>VAR</th>
<th>Z_p</th>
<th>N</th>
<th>MEAN</th>
<th>VAR</th>
<th>Z_t</th>
<th>t</th>
<th>F</th>
<th>N</th>
<th>MEAN</th>
<th>FINISH</th>
</tr>
</thead>
<tbody>
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<tr>
<td>1</td>
<td>19</td>
<td>18.36</td>
<td>2.24</td>
<td>10.37</td>
<td>10</td>
<td>15.30</td>
<td>14.68</td>
<td>3.32</td>
<td>4.985*</td>
<td>1.78</td>
<td>3</td>
<td>17.66</td>
<td>1</td>
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</tr>
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<td>400</td>
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<td>8.88</td>
<td>215</td>
<td>11.20</td>
<td>12.69</td>
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<td>144</td>
<td>11.65</td>
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<td></td>
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<td></td>
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<td>9.65</td>
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<td>5</td>
<td>146</td>
<td>11.59</td>
<td>9.64</td>
<td>9.73</td>
<td>55</td>
<td>8.66</td>
<td>13.58</td>
<td>4.41</td>
<td>3.760*</td>
<td>4.34*</td>
<td>2</td>
<td>14.00</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Significant at .05 level.
### Table 27
COMPARISON OF ACHIEVEMENT ON TEST TWO BETWEEN PILOT AND TEST GROUPS
UNDER STRATEGY III RECOMMENDATIONS

<table>
<thead>
<tr>
<th>TEST OF HYPOTHESIS 1</th>
<th>TEST OF HYPOTHESIS 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PILOT GROUP</strong></td>
<td><strong>TEST GROUP</strong></td>
</tr>
<tr>
<td><strong>REC</strong></td>
<td><strong>N</strong></td>
</tr>
<tr>
<td>1</td>
<td>19</td>
</tr>
<tr>
<td>2</td>
<td>95</td>
</tr>
<tr>
<td>3</td>
<td>226</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>146</td>
</tr>
</tbody>
</table>

* Significant at .05 level.
The F-value reported for each recommended pacing level was obtained using a one-way analysis of variance and is a measure of the significance of the difference between the means of the subgroups which finished the course at different pacing levels. For $F \geq 1$, the right tail of the F-distribution was used; for $F < 1$, the left tail. Interpretation of F-values appearing in the left tail of the F-distribution is subject to the violation of the assumption of normal distributions of the subgroups. This violation is sometimes characteristic of cases when the mean within-group sum of squares exceeds the mean between-group sum of squares; that is, when the F-value is less than one. In the test of Hypothesis 2 reported in these tables, F-values less than one occurred when (1) one of the subgroups contained a very small number of subjects, or (2) the difference in the two means was quite small. The assumption of non-significant differences in the means for either of the above two cases was considered acceptable.

The means and frequencies of the subgroups which were recommended for a given pacing level, but who completed the course at different pacing levels, can be used to further refine the interpretations related to the t-values reported under Hypothesis 1. For example, in Table 25 the condition which
may have contributed most to the significant increase in achievement on the part of the students who were recommended for the reduced pace in the test group was the fact that 89 percent of those students finished the course in the reduced pace. If the majority of these students had finished the course in the regular pace, the reported t-value would have been spuriously high due to the ineffectiveness of the recommendation strategy.

Two questions of validity impose limitations upon the interpretation of the data presented in Tables 25, 26 and 27. The first relates to the equivalence of Test II in the test quarter to Test II in the pilot quarter. As indicated in the Introduction, the order of presentation of content differed slightly between the two quarters. The second limitation is the choice of items from the comprehensive examinations used to generate Test II scores for the reduced pace students. The rationale for using this method was based on a desire to eliminate Test I scores (used to determine pacing level recommendations) from subsequent achievement measures.

Defining achievement as the sum of scores on Test I and Test II for the pilot group and the accelerated and regular pace students in the test group, and the score on the last comprehensive examination taken by students in the reduced pace test
group greatly reduced the validity problem relative to achievement comparisons. However, the double use of the Test I score is then a limitation. The amount of Test I bias is consistent with both the pilot and test groups since the selection criteria under a given recommendation strategy was identically applied to both groups.

Hypotheses 1 and 2 were tested under the alternate criteria of achievement. These results are presented in Tables 28, 29 and 30 using the same format as Tables 25, 26 and 27.

There are three patterns which are consistent for all six of the preceding tables. The first indicates that there is typically no significant difference in achievement between subsets of students in the test group who finished at different pacing levels but who were recommended for the same pacing level. Secondly, the students in the test group who were recommended for the accelerated pace did not achieve as high as similar students in the pilot group. Finally, students in the test group recommended for the reduced pace achieved at a level significantly closer to the average group than did similar students in the pilot group.

Earlier in this chapter, data was presented which indicated that, by subgroups, the pilot and test groups were equivalent relative to pre-course measures, but that by considering the
<table>
<thead>
<tr>
<th>REC</th>
<th>N</th>
<th>MEAN</th>
<th>VAR</th>
<th>Zp</th>
<th>N</th>
<th>MEAN</th>
<th>VAR</th>
<th>Zt</th>
<th>t</th>
<th>F</th>
<th>N</th>
<th>MEAN</th>
<th>FINISH</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>43</td>
<td>33.90</td>
<td>12.77</td>
<td>10.57</td>
<td>14</td>
<td>31.69</td>
<td>7.06</td>
<td>7.01</td>
<td>2.512*</td>
<td>0.30</td>
<td>10</td>
<td>31.40</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>359</td>
<td>27.77</td>
<td>14.22</td>
<td></td>
<td>120</td>
<td>25.55</td>
<td>23.42</td>
<td></td>
<td>0.16</td>
<td></td>
<td>91</td>
<td>25.66</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>186</td>
<td>20.13</td>
<td>13.05</td>
<td>23.06</td>
<td>36</td>
<td>20.56</td>
<td>35.34</td>
<td>4.71</td>
<td>12.972*</td>
<td>0.18</td>
<td>32</td>
<td>20.85</td>
<td>5</td>
</tr>
</tbody>
</table>

* Significant at .05 level.
### TABLE 29

**COMPARISON OF ACHIEVEMENT ON TEST ONE AND TEST TWO BETWEEN PILOT AND TEST GROUPS UNDER STRATEGY II RECOMMENDATIONS**

<table>
<thead>
<tr>
<th>Test of Hypothesis 1</th>
<th>Test of Hypothesis 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pilot Group</strong></td>
<td><strong>Test Group</strong></td>
</tr>
<tr>
<td>REC</td>
<td>N</td>
</tr>
<tr>
<td>-----</td>
<td>---</td>
</tr>
<tr>
<td>1</td>
<td>19</td>
</tr>
<tr>
<td>2</td>
<td>53</td>
</tr>
<tr>
<td>3</td>
<td>400</td>
</tr>
<tr>
<td>5</td>
<td>146</td>
</tr>
</tbody>
</table>

* Significant at .05 level.
## TABLE 30

COMPARISON OF ACHIEVEMENT ON TEST ONE AND TEST TWO BETWEEN PILOT AND
TEST GROUPS UNDER STRATEGY III RECOMMENDATIONS

<table>
<thead>
<tr>
<th>TEST OF HYPOTHESIS 1</th>
<th>TEST OF HYPOTHESIS 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>PILOT GROUP</td>
<td>TEST GROUP</td>
</tr>
<tr>
<td>REC</td>
<td>N</td>
</tr>
<tr>
<td>1</td>
<td>19</td>
</tr>
<tr>
<td>2</td>
<td>95</td>
</tr>
<tr>
<td>3</td>
<td>226</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>146</td>
</tr>
</tbody>
</table>

* Significant at .05 level.
distribution of subgroups within each of the two quarters, the
test group could be expected to achieve at a slightly higher
level than the pilot group. This last expectation was based
upon the assumption that the treatment of content and the level
of difficulty of the examinations would be equivalent for the
pilot and test groups.

The test of Hypothesis 3 measures the difference in
achievement between the pilot and test groups. Achievement
here is defined to be the sum of the first two examination scores
for the pilot group students and the test group students who
completed the course in the accelerated or regular pace, and
the score on the last comprehensive examination taken by students
in the test group reduced pace. The data relevant to the test of
Hypothesis 3 is presented in Table 31. In addition to the data
on the entire pilot and test groups, the achievement of the
students in the group is also presented according to the pacing
levels at which the students finished Math 115.01.

The next stage in the analysis of the data consisted of the
evaluation of the effectiveness of each of the three selection
strategies used during the test quarter. In Table 32 the
distribution of students under each selection strategy is generated
according to three criteria: (1) the pacing level recommended to
## TABLE 31

**COMPARISON OF ACHIEVEMENT OF THE PILOT AND TEST GROUPS**

<table>
<thead>
<tr>
<th>GROUP</th>
<th>N</th>
<th>MEAN</th>
<th>VAR</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>PILOT</td>
<td>601</td>
<td>25.8</td>
<td>31.9</td>
<td>2.135*</td>
</tr>
<tr>
<td>TEST</td>
<td>812</td>
<td>24.9</td>
<td>44.2</td>
<td></td>
</tr>
<tr>
<td>TEST (ACC)</td>
<td>20</td>
<td>33.0</td>
<td>6.8</td>
<td></td>
</tr>
<tr>
<td>TEST (REG)</td>
<td>463</td>
<td>25.4</td>
<td>18.1</td>
<td></td>
</tr>
<tr>
<td>TEST (RED)</td>
<td>329</td>
<td>23.7</td>
<td>42.4</td>
<td></td>
</tr>
</tbody>
</table>

* Significant at .01 level.

the student, (2) the pacing level chosen by the student, and (3) the pacing level at which the student finished Math 115.01. The data is reported both in terms of the number of students in each subcategory as well as the percentage distribution of students recommended for a given pacing level under a given selection strategy.

Tables 33, 34 and 35 report, for a given selection strategy, the distribution of correct, false positive and false negative
<table>
<thead>
<tr>
<th>REC</th>
<th>CHO</th>
<th>FIN</th>
<th>STRATEGY I</th>
<th>STRATEGY II</th>
<th>STRATEGY III</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>N</td>
<td>%</td>
<td>N</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>29</td>
<td>3</td>
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<td>3</td>
<td>2</td>
<td>14</td>
<td>4</td>
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<tr>
<td>1</td>
<td>3</td>
<td>3</td>
<td>8</td>
<td>57</td>
<td>3</td>
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<td>2</td>
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<td>1</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3</td>
<td>--</td>
<td>--</td>
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<tr>
<td>3</td>
<td>3</td>
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<td>91</td>
<td>75</td>
<td>142</td>
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<td>3</td>
<td>3</td>
<td>5</td>
<td>28</td>
<td>23</td>
<td>57</td>
</tr>
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<td>11</td>
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</tr>
<tr>
<td>5</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>19</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>5</td>
<td>26</td>
<td>70</td>
<td>50</td>
</tr>
</tbody>
</table>
### TABLE 33

DISTRIBUTION OF CLASSES OF STRATEGY I
RECOMMENDATIONS FOR THE TEST GROUP

<table>
<thead>
<tr>
<th>REC</th>
<th>CORRECT</th>
<th>F. POS.</th>
<th>F. NEG.</th>
<th>EFF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>10</td>
<td>--</td>
<td>.29</td>
</tr>
<tr>
<td>3</td>
<td>91</td>
<td>31</td>
<td>0</td>
<td>.75</td>
</tr>
<tr>
<td>5</td>
<td>33</td>
<td>--</td>
<td>4</td>
<td>.89</td>
</tr>
<tr>
<td>TOTAL</td>
<td>128</td>
<td>41</td>
<td>4</td>
<td>.74</td>
</tr>
</tbody>
</table>

### TABLE 34

DISTRIBUTION OF CLASSES OF STRATEGY II
RECOMMENDATIONS FOR THE TEST GROUP

<table>
<thead>
<tr>
<th>REC</th>
<th>CORRECT</th>
<th>F. POS.</th>
<th>F. NEG.</th>
<th>EFF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>7</td>
<td>--</td>
<td>.30</td>
</tr>
<tr>
<td>3</td>
<td>144</td>
<td>69</td>
<td>2</td>
<td>.67</td>
</tr>
<tr>
<td>5</td>
<td>53</td>
<td>--</td>
<td>2</td>
<td>.96</td>
</tr>
<tr>
<td>TOTAL</td>
<td>200</td>
<td>76</td>
<td>4</td>
<td>.71</td>
</tr>
</tbody>
</table>
### TABLE 35

**DISTRIBUTION OF CLASSES OF STRATEGY III RECOMMENDATIONS FOR THE TEST GROUP**

<table>
<thead>
<tr>
<th>REC</th>
<th>CORRECT</th>
<th>F. POS.</th>
<th>F. NEG.</th>
<th>EFF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>2</td>
<td>--</td>
<td>.66</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
<td>0</td>
<td>--</td>
<td>.83*</td>
</tr>
<tr>
<td>3</td>
<td>59</td>
<td>11</td>
<td>0</td>
<td>.84</td>
</tr>
<tr>
<td>4</td>
<td>87</td>
<td>--</td>
<td>0</td>
<td>.52*</td>
</tr>
<tr>
<td>5</td>
<td>42</td>
<td>--</td>
<td>3</td>
<td>.93</td>
</tr>
<tr>
<td>TOTAL</td>
<td>190</td>
<td>13</td>
<td>3</td>
<td>.77</td>
</tr>
</tbody>
</table>

* per cent of these students who chose correctly

recommendations for each of the pacing levels as well as the overall efficiency of the selection strategy.

Tables 36, 37 and 38 indicate the number of correct and false positive choices of pacing levels by the students under a given selection strategy. Since students were not able to move to a higher pacing level than that which they chose, no false negative choices are reported. For each selection strategy, the overall efficiency of student choice is presented.
### TABLE 36
**DISTRIBUTION OF CLASSES OF STUDENT CHOICE UNDER STRATEGY I RECOMMENDATIONS**

<table>
<thead>
<tr>
<th>REC</th>
<th>CORRECT CHOICE</th>
<th>F, POS. CHOICE</th>
<th>EFF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>2</td>
<td>.86</td>
</tr>
<tr>
<td>3</td>
<td>94</td>
<td>28</td>
<td>.77</td>
</tr>
<tr>
<td>5</td>
<td>30</td>
<td>7</td>
<td>.81</td>
</tr>
<tr>
<td>TOTAL</td>
<td>136</td>
<td>37</td>
<td>.78</td>
</tr>
</tbody>
</table>

### TABLE 37
**DISTRIBUTION OF CLASSES OF STUDENT CHOICE UNDER STRATEGY II RECOMMENDATIONS**

<table>
<thead>
<tr>
<th>REC</th>
<th>CORRECT CHOICE</th>
<th>F, POS. CHOICE</th>
<th>EFF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>4</td>
<td>.60</td>
</tr>
<tr>
<td>3</td>
<td>156</td>
<td>59</td>
<td>.72</td>
</tr>
<tr>
<td>5</td>
<td>52</td>
<td>3</td>
<td>.95</td>
</tr>
<tr>
<td>TOTAL</td>
<td>214</td>
<td>66</td>
<td>.76</td>
</tr>
</tbody>
</table>
TABLE 38
DISTRIBUTION OF CLASSES OF STUDENT CHOICE
UNDER STRATEGY III RECOMMENDATIONS

<table>
<thead>
<tr>
<th>REC</th>
<th>CORRECT CHOICE</th>
<th>F. POS. CHOICE</th>
<th>EFF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>1</td>
<td>.83</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>3</td>
<td>.83</td>
</tr>
<tr>
<td>3</td>
<td>60</td>
<td>10</td>
<td>.86</td>
</tr>
<tr>
<td>4</td>
<td>35</td>
<td>32</td>
<td>.52</td>
</tr>
<tr>
<td>5</td>
<td>43</td>
<td>2</td>
<td>.96</td>
</tr>
<tr>
<td>TOTAL</td>
<td>158</td>
<td>48</td>
<td>.76</td>
</tr>
</tbody>
</table>

Summary

In this chapter the equivalence of the pilot and test groups relative to pre-course measures was determined. Data related to the equivalence of the two forms of Test I administered to the test group students was presented. Tests of the three hypotheses basic to this study were performed and the effectiveness of the three recommendation strategies together with the effectiveness of student acceptance or rejection of recommended pacing levels were described. Interpretation of the results presented in this chapter will be discussed in the next chapter.
CONCLUSIONS, INTERPRETATIONS, REFLECTIONS AND RECOMMENDATIONS

The preceding chapters contain a detailed account of the design of this study, the selection strategies used to recommend students for differently paced sections of a freshman college mathematics course, a statistical analysis of data related to the achievement of the students recommended for given pacing levels under the various strategies and a description of the relationships between recommended pacing levels, pacing levels chosen by students and the levels at which students finished the course.

This final chapter will be organized as follows:

1. A summary of the findings of the study based on data collected during the pilot and test quarters of the study.

2. Interpretations of the findings.

3. Reflections and observations relative to factors which became apparent during the course of the study, but which were not part of the original focus of the investigation.
4. Recommendations for action based on findings of this study.

5. Recommendations for additional research in the area of individualized mathematics instruction.

Findings

Based on data collected during the pilot quarter, Winter 1970, three selection strategies were chosen as the basis for suggesting pacing levels to students during the test quarter, Autumn 1970.

Strategy I utilized multiple regression equations and predicted course achievement as a function of the first examination score in the course as well as Math ACT scores and Ohio State University Math Placement Test scores. The efficiency of Strategy I (per cent of correct predictions) for the pilot subjects was 80 per cent. Strategy II recommendations were based on the first examination scores only. It's efficiency for the pilot group was 78 per cent. Strategy III recommendations were the same as for Strategy II with the exception that students in the upper-middle range on Test I were asked to choose between the accelerated and regular pace; the students in the lower-middle range, between the regular and reduced pace. The projected efficiency of Strategy III was 86 per cent.
For students recommended under a given strategy, two hypotheses were tested. Hypothesis 1 asserted that test group students recommended for a given pacing level would achieve at the same level as those pilot group students identified as candidates for the same pacing level. Hypothesis 2 asserted that for students recommended for the same pacing level there would be no difference in achievement between those subgroups of students who completed the course at different pacing levels. Achievement was measured in two ways: (1) the examination score on content treated after the first test, and (2) the sum of examination scores on the entire content treated during the course. Hypotheses 1 and 2 were tested under both definitions of achievement.

Analysis of the data relative to Hypothesis 1 revealed the following findings with achievement defined as the score on Test II.

1. Strategy I

(a) There was no significant difference between the pilot and test group students identified as accelerated pace candidates.

(b) The test group students recommended for reduced pace achieved at a significantly higher level than similar students in the pilot group.
2. Strategy II

(a) The test group students recommended for the accelerated pace achieved at a significantly lower level than similar students in the pilot group.

(b) The test group students recommended for the reduced pace achieved at a significantly higher level than similar students in the pilot group.

3. Strategy III

(a) The test group students recommended for the accelerated pace appeared to achieve at a lower level than similar students in the pilot group, but the difference was not significant.

(b) The test group students who were asked to choose between the accelerated and regular pace achieved at a significantly lower level than similar students in the pilot group.

(c) The test group students who were asked to choose between the regular and reduced pace achieved at a significantly higher level than similar students in the pilot group.

(d) The test group students recommended for the reduced pace achieved at a significantly higher
level than similar students in the pilot group.

When achievement was taken to be total performance in the course, Hypothesis 1 was rejected in every case. In summary form, the conclusions are as follows:

1. All three strategies
   (a) The test group students recommended for the accelerated pace achieved at a significantly lower level than similar students in the pilot group.
   (b) The test group students recommended for the reduced pace achieved at a significantly higher level than similar students in the pilot group.

2. Strategy III
   (a) The test group students who were asked to choose between the accelerated and regular pace achieved at a significantly lower level than similar students in the pilot group.
   (b) The test group students who were asked to choose between the regular and reduced pace achieved at a significantly higher level than similar students in the pilot group.

Analysis of the data relative to Hypothesis 2 revealed the following findings with achievement defined as the score on Test II:
1. Strategy I

(a) Of the test group students recommended for the accelerated pace, there was no significant difference in achievement between the students who completed the course at the accelerated pace and those who finished at the regular pace.

(b) Of the test group students recommended for the regular pace, students who finished at the regular pace achieved at a significantly higher level than those who finished at the reduced pace.

(c) Of the test group students who were recommended for the reduced pace, there was no significant difference in achievement between the groups who finished the course at the regular and reduced paces.

2. Strategy II

(a) Of the test group students who were recommended for either the accelerated or regular pace, there was no significant difference between the respective groups who finished at the accelerated pace and those who finished at the regular pace.
(b) Of the test group students who were recommended for the regular pace, the students who finished at the regular pace achieved at a significantly higher level than those who finished at the reduced pace.

(c) Of the test group students who were recommended for the reduced pace, the students who finished at the regular pace achieved at a significantly higher level than those who finished at the reduced pace.

3. Strategy III

(a) Of the test group students recommended for the reduced pace, the students who finished at the regular pace achieved at a significantly higher level than those who finished at the reduced pace.

(b) Of the test group students who were recommended for pacing levels other than the reduced pace, there was no significant difference in achievement between the respective subgroups who finished at different pacing levels.

'When achievement was considered as the sum of scores earned on examinations covering the entire content of the course,
evaluation of Hypothesis 2 revealed the following findings:

1. **Strategy I**
   
   For every recommended pacing level there was no significant difference in achievement, respectively, between the subgroups who finished the course at different pacing levels.

2. **Strategy II**
   
   For every recommended pacing level there was no significant difference in achievement, respectively, between the subgroups who finished the course at different pacing levels.

3. **Strategy III**
   
   (a) Of the test group students who were asked to choose between the regular and reduced pace, the students who finished at the reduced pace achieved at a significantly higher level than those who finished at the regular pace.

   (b) Of the remaining test group students, there was no significant difference between the subgroups who finished at different pacing levels.

Hypothesis 3 asserted that there would be no difference in achievement between the test group and pilot group students.
Achievement was defined to be performance over the entire content of the first module as described in the second tests of Hypotheses 1 and 2 above. Hypothesis 3 was rejected and the following conclusion was made:

Test of Hypothesis 3

The test group students achieved at a significantly lower level than the students in the pilot group.

In addition to the tests of the above three hypotheses, a major purpose of this research study was to determine the efficiency of the various selection strategies and also the efficiency of choice of pacing levels by the students. The pilot data indicated that the efficiency of the three selection strategies were as follows: Strategy I --- 80 per cent; Strategy II --- 78 per cent; Strategy III --- 86 per cent. When applied to the test group, the three selection strategies maintained the same rank-ordering as in the pilot group. However, consistent decreases in efficiency occurred as follows: Strategy I --- 74 per cent; Strategy II --- 71 per cent; Strategy III --- 77 per cent.

The overall efficiency of student choice of pacing levels was consistent among the three strategies (78, 76 and 76 per cent, respectively). Since 92 per cent of the students in the test
group accepted the recommended pacing level, many false recommendations also appear in the data as false choices on the parts of the students. Of the eight per cent of the students who chose a pacing level different from that which was recommended, three-fourths made the correct decision. Of the 12 per cent of the students who were asked to choose their own pacing level, only three-fifths chose correctly. This latter statistic is heavily weighted by the fact that only one-half of the lower-middle subgroup of these students chose correctly.

As a result of the data related to the efficiency of recommendations and student choice, it was concluded that

1. The small gain in predictive efficiency resulting from using both in-course and pre-course data rather than in-course data only does not warrant the cost of obtaining the pre-course measures.

2. Due to its consistently high ranking efficiency, a strategy which combines recommendations based on first examination scores together with ranges of free choice by the students may be preferred to a strategy which prescribes pacing levels for all students.

3. Students in the upper half of the achievement range on the first examination tend to make choices of pacing
levels with much higher efficiency than the students
in the lower half of the achievement range.

**Interpretation of the Findings**

Evaluation of Hypothesis I indicated a drop in achievement
of test group students recommended for the accelerated pace.
Although the number of students in this category is small, two
factors are suggested as reasons for the achievement differences
between the pilot and test group students. The first is related
to the effect of content acceleration. It is very possible that
many of the students who achieve at a high level in an introduc-
tory liberal arts course (such as the one within which the frame-
work of this study was undertaken) are already learning at a rate
close to their maximum potential. For those students, acceler-
ation may be accompanied by a decrease in the level of achieve-
ment. A supporting bit of evidence for this assertion is the fact
that in the only instance in which the test group students
recommended for the accelerated pace achieved at the same
level as the pilot students (Strategy I, achievement based on
Test II) 70 per cent of the test group students finished the course
at the regular pace and achieved at a slightly higher level than
those who finished at the accelerated pace. The second factor
relates to the relative difficulty of the test group content
compared to the content for the pilot group. Although the statistical testing was designed to account for such differences, the fact that the test group students achieved at a level significantly lower than was expected (Hypothesis 3) prevents eliminating this factor as an important consideration.

In view of the discussion of the latter of the above two factors, the significant increase in achievement on the part of the students recommended for the reduced pace may be related to a positive effect of the reduced pacing treatment rather than other factors. This is especially the case since 95 per cent of the students recommended for the reduced pace finished the course at the reduced pace. It should also be noted that under every selection strategy those students who were recommended for the regular pace but who finished at the reduced pace achieved at a higher level than those students who were recommended for the reduced pace and finished there. Included in the treatment of the reduced pace was the opportunity for these students to re-study and re-test over material on which prior achievement was not satisfactory.

An interesting pattern which resulted from the analysis of Hypothesis 2 is the fact that the only cases where significant differences occur are between students who finished at the regular
pace and those who finished at the reduced pace. In four of the five such cases, the differences favor the students who finished the course in the regular pace.

The data related to the efficiency of student choice of pacing levels revealed a factor that, on the one hand, contributed toward decreased efficiency, but on the other may be desirable. A great majority of incorrect choices were among students who chose the regular pace but who finished at the reduced pace. The untested factor of the student's perceptual base relative to his self-expectancies is strongly suggested. The hesitancy of some students to accept the reduced pace as a means of maximizing their achievement potential was revealed during the Spring Quarter 1970. A number of students in the reduced pace during that quarter reported negative feelings based on the association of the reduced pace with "inferiority." It is quite possible that for students who can choose between regular and reduced pace treatment, affective factors play a role that dominates the available in-course and pre-course cognitive measures.

Some Further Reflections and Observations

A number of informative items, although not part of the major focus of this investigation, became available during and
after the time of this study. These factors are presented for the purpose of relating the findings of this study to the context of broader aspects of approaches to individualized instruction in mathematics.

Consideration will be given to the following:

1. Additional data related to achievement of the test group during the test and subsequent quarters.
2. Student reaction to the CRIMEL approach to instruction.
3. Some anecdotes related to student rationale for choice of pacing levels.
4. The role of re-testing --- the second stage in the CRIMEL program.

One clear indication of benefit derived from the introduction of differently paced sections as a means of providing improved individual treatment for students with a wide range of interests and abilities is a marked decrease in the number of students who drop out of the course before its completion. During the pilot quarter, 161 of 903 students dropped the course. Examination of records for these students revealed that almost all of them were either failing the course at the time they dropped it or were achieving at a minimum passing level. It was students
like these that the reduced pace was designed to help. During the test quarter, only 83 of 1,099 students dropped the course.

Tests of hypotheses in this study were carried out with respect to achievement scores rather than the success measures of grades earned. Using the scale $A = 4.0$, $B = 3.0$, $E = 0.0$, the grade point average for the students in Math 115.01 during the test quarter was 2.04. In the second module, Math 115.02, the grade distributions by pacing levels were --- accelerated - 3.67, regular - 2.58, reduced - 2.24. The grades earned by the reduced pace students in the second module very closely approximate the distribution of grades earned by students in Math 116 in previous quarters. This indicates a marked improvement since the reduced pace students were the kind who, in previous courses, typically dropped the course or received grades of $D$ or $E$.

A profile of the pre-course measures of quality points earned in high school mathematics courses (starting with Algebra I), time since last enrollment in a mathematics course, and sex was generated for each pacing level population at the end of Math 115.01. The average quality points for the students in each pacing level were --- accelerated - 9.53, regular - 6.36, reduced - 4.68. The mean time lapse (in years) distribution
was --- accelerated - 0.78, regular - 1.45, reduced - 1.51. This data supports the findings of prediction studies reported in Chapter II. Had the experimental study conducted during the Spring Quarter 1970 not been interrupted by student disruptions, it is possible that the above two variables could have significantly improved the efficiency of the Strategy I recommendations.

Investigation of the variable, sex, revealed that about one-half of the regular pace students were female, while only one-third of the accelerated and reduced pace students were female. In all three pacing levels, the females achieved at a higher level than the males.

The success of any program which attempts to individualize instruction cannot be judged solely on the basis of achievement records. This is especially true when one of the main objectives is to allow students to learn at a rate appropriate to each student's interest and ability. Success of the program as viewed by the student is also an important consideration. As part of the evaluation of the CRIMEL program, students were asked to indicate whether or not they thought the program should be continued. The percentages of students in the accelerated, regular and reduced pace sections who thought that the program should be continued were 65, 48 and 64, respectively; those who
thought it should not be continued were 7, 27 and 16 respectively.
The remainder of the students either had no opinion or did not respond to this item on the questionnaire. Since the treatment in the regular sections was not very different from traditional instructional techniques, the apparent neutral reaction from these students was not unpredictable.

That achievement potential cannot be predicted solely on the basis of achievement measures is an established principle. The existence of numerous instruments designed to measure attitude toward a variety of criteria attests to this principle. Assignment of students to pacing levels in the form of recommendations, with provisions for student choice, is important. At the time during the test quarter when students were given recommended pacing levels, this writer interviewed many students who wished to choose a pacing level other than that which was recommended. The need to account for differences in attitude and motivation not determined by the selection strategy can be illustrated by citing some examples of the rationale which students offered as bases for their choices.

The first case is a student who scored very close to the class mean on the first examination. Her academic record indicated that she was an above average student, but had never
had the opportunity to participate in educational experiences other
than standard courses with traditional treatments. She asked to
be allowed to enroll in the accelerated pace because, in her
words, "I have never had the chance to do anything other than
what other people would consider 'ordinary.'" She was convinced
that she could succeed in the accelerated pace if given the
opportunity. Her grades in the four accelerated pace modules
were B, A, A and B.

Another student who was recommended for the accelerated
pace decided to return to the regular pace after one week due
to pressure from parents who were concerned that the student
might not earn an A if she remained in the accelerated pace.

A number of students who requested permission to enter
the reduced pace in spite of satisfactory performance in the
regular section felt that the rest of their courses were being
slighted because an inordinate amount of time was spent studying
mathematics. Other students wished to reduce the pace of their
study because of the employment loads required to finance their
educations. Some students expressed great apprehension about
remaining in the regular pace either because of prior bad
experiences with mathematics courses or due to the fact they
had not been enrolled in a mathematics course for several years.
A number of students who had failed the first test did not want to go to the reduced pace. Some felt their academic programs would be endangered if they did not complete five credit hours of mathematics during that quarter. Others did not believe they were "inferior" students and preferred to stay in the regular pace. Neither of these groups was concerned with the implication of possible failure in the course because of their decisions. Some of these students did go to the reduced pace after the second test in the regular pace in order to avoid receiving a failing grade.

One of the advantages available to reduced pace students was the opportunity to re-test over material on which prior testing resulted in unsatisfactory performance. During the second stage in the development of the CRIMEL program this opportunity will be extended to all students in the course. During the Spring Quarter 1971, a small scale experiment with about 150 Math 116 students was conducted in an attempt to identify logistical and instructional implications of a program which would allow a student to take an examination over specified content at the time the student felt he was prepared. If the student was not satisfied with his performance, he could take another examination at a later date. During the course of this experiment,
the students averaged over two attempts at each test. Preliminary evaluation indicates that, as a group, the students did not make significant gains in achievement by re-testing. One problem in such an evaluation is the attempt to distinguish between those students who casually re-took an examination with the hope of getting a higher score and those students who made an effort to first re-study the material before taking a second examination.

One serious drawback was that most of the students in the course spent so much time re-testing that they completed only the first three-credit module of the course. The role of the teacher in making recommendations to students regarding possible re-testing may be the key to the efficiency of the revised program.

As was done in the test quarter of the main study, students in the second stage of the CRIMEL program were asked to evaluate their experiences. Returns from the questionnaires indicated that 65 per cent of the students thought that the flexible testing program should be continued; 17 per cent indicated that the program should not be continued. Interviews with students revealed a very strong positive attitude toward the re-testing aspects of the program. Students' attitudes seemed to be based on the fact that they had a chance to improve the level of achievement on a given examination. The fact that actual
improvement did not occur as a result of re-testing did not seem to be a factor in many students' verbal praise of the program. Giving students a larger share of the responsibility for their academic performance was considered to be an important factor relative to student attitude toward mathematics instruction.

Recommendations for Action Based on Findings of the Study

A number of recommendations can be made to the Department of Mathematics of The Ohio State University as a result of this study.

The first recommendation is that the Department continue its efforts to provide for variable pacing of content in its introductory mathematics courses. The practice of allowing students with unsatisfactory examination scores to have the opportunity to re-study the material and to then take another examination should be continued and expanded to groups other than those students in the reduced pace sections. Students in the reduced pace sections of the test group of this study achieved at a level significantly closer to that of the regular group than similar students in the pilot group. It is quite possible that students who wish to complete the five credit hour course during one school term could increase their achievement potential if they also had the opportunity to re-test over certain portions of the material.
The benefits for students participating in accelerated pacing will consist of either earning additional credit during the term or finishing their mathematics coursework before the end of the term. Accelerated pacing should not be abandoned because of a drop from the high achievement level maintained by superior students in the regular sections of traditionally paced courses.

The focus of a second recommendation is the method by which students should be selected for the differently paced sections. The Mathematics Department should continue to allow students to have a short exposure to the setting and content of the course and to take an in-course examination before recommendations to students are given. It is important to allow students to share in the decision-making regarding pacing levels to be followed. It is recommended that Math ACT and OSU Math Placement Test scores not be used as selection criteria. Those persons in the CRIMEL program who have responsibility for discussing potential pacing levels with a given student should be aware of the relationships between (1) the amount of mathematics background (starting with high school algebra) of the student, (2) the time since the student last took a mathematics course and (3) success in the various pacing levels.
Identification of Areas for Further Research

The results of this study cannot be immediately generalized as expectancies with regard to other mathematics courses at this University nor mathematics courses at other colleges and universities. Currently, certain aspects of this study are being replicated with respect to a pre-calculus course in algebra and trigonometry. This course serves as an entry to the standard course sequence in calculus taken by science and engineering students at this University.

There has been a major effort on the part of both school and university educators to develop programs of individualized instruction. It has been the experience of this writer that many of these programs do not contain any formal methods of evaluation. Studies which have been reported with regard to selecting students for a range of academic experiences do not account for factors other than those related to measures of achievement. The factor of student choice has been shown to be effective for some groups of students. In this study it was found that 92 per cent of the students followed the recommendations provided by the department.

If the criteria for generating selection recommendations are to be more efficient, additional research needs to be done relative to psychological factors related to both achievement of students
in the setting of an individualized approach to instruction and to types of decisions made by students regarding their choices of instructional experiences. Factors to be investigated should include students' (1) attitudes toward their abilities to do mathematics, (2) motivations to learn mathematics, and (3) self-expectancies with respect to achievement levels in mathematics as well as other academic subjects.

Programs of individualized instruction require behavioral changes on the part of both teachers and students. The teacher whose prior experience has been with traditional heterogeneous groups of students who study under his leadership over an extended period of time may have difficulty in adjusting to a different teaching style and to different expectancies on the part of the students. The student who is suddenly faced with the responsibility of sharing to a great extent in decisions regarding his academic progress may also experience a number of adjustment problems.

The effect of an individualized instructional approach utilized in one area of a student's coursework on other areas of his academic program should provide additional areas of interesting research. For example, are individualized instructional programs in mathematics and the physical and life sciences
applicable to programs in humanities and social sciences? How do students view the value of individualized instructional techniques used in a certain mathematics course compared to traditional treatments in (1) other mathematics courses, and (2) courses other than mathematics? What behavioral changes occur in both teachers and students during their first exposure to a system of individualized instruction? What is the later profile of behavioral characteristics after (1) prolonged experience with an individualized system, and (2) change to a traditional system of instruction?

Evaluation of the psychological factors, together with achievement measures related to the teaching-learning experience within the context of individualized instruction, may help bring educators closer to the goal of providing each student with experiences which will maximize his learning potential.
APPENDICES

A. Content Comparison between the Pilot and Test Group.

B. Instrument Used to Collect Background Information from the Test Group Students.
   Instruction to Recitation Instructors Regarding Pacing Level Recommendations.

C. Examinations Administered to the Pilot and Test Groups.

D. Summaries of Item Analysis of the Examinations.

E. Comparison of Means between Forms A and B of Examinations.

F. Formula Used to Generate t-scores.
APPENDIX A

CONTENT COMPARISON BETWEEN PILOT AND TEST GROUPS

In Table 39 the content of the courses involved in this study is categorized by the following code:

1. Pilot Group (Math 116-117)
   (a) $P_1$ --- content for Test I, Math 116
   (b) $P_2$ --- content for Test II, Math 116
   (c) $P_3$ --- content for Test III, Math 116
   (d) $P_4$ --- content for Math 117

2. Test Group (Math 115.01 thru 115.04)
   (a) $T_1$ --- content for Test I, Math 115.01
   (b) $T_2$ --- content for Test II, Math 115.01
   (c) $T_3$ --- content for Math 115.02
   (d) $T_4$ --- content for Math 115.03
   (e) $T_5$ --- content for Math 115.04
TABLE 39

COMPARISON OF CONTENT BETWEEN THE PILOT AND TEST QUARTERS

<table>
<thead>
<tr>
<th>CONTENT</th>
<th>PILOT</th>
<th>TEST</th>
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<tbody>
<tr>
<td>Sets, Operations on Sets, Notation</td>
<td>(P_1)</td>
<td>(T_1)</td>
</tr>
<tr>
<td>The Number System and Number Line</td>
<td>(P_1)</td>
<td>(T_1)</td>
</tr>
<tr>
<td>Subsets of Reals, Intervals</td>
<td>(P_1)</td>
<td>(T_1)</td>
</tr>
<tr>
<td>Axioms for Addition and Multiplication</td>
<td>(P_1)</td>
<td>(T_1)</td>
</tr>
<tr>
<td>Order Axioms, Inequalities</td>
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<td>(T_1)</td>
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<tr>
<td>Absolute Value</td>
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<td>(T_1)</td>
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<tr>
<td>Solution Sets for Equations, Inequalities, and Absolute Value Problems</td>
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<td>(T_1)</td>
</tr>
<tr>
<td>Exponents and Radicals</td>
<td>(P_2)</td>
<td>(T_1)</td>
</tr>
<tr>
<td>Functions, Domain and Range</td>
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<td>(T_2)</td>
</tr>
<tr>
<td>Examples of Functions: Polynomials, (\sqrt{x}), ([x]), (</td>
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<td>)</td>
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<tr>
<td>Composition of Functions</td>
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<td>(T_2)</td>
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<td>Graphing Subsets of (R^2)</td>
<td>(P_2)</td>
<td>(T_2)</td>
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<td>Logarithmic and Exponential Functions</td>
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<td>Equations of Lines</td>
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<tr>
<td>Vectors in (R^2) and (R^3)</td>
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<td>(T_3)</td>
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<td>CONTENT</td>
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<td>TEST</td>
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<tr>
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</tr>
<tr>
<td>Equations of Planes and Spheres</td>
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<td>$T_3$</td>
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<td>Linear Programming</td>
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<tr>
<td>Matrices and Determinants</td>
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<td>Linear Transformations</td>
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<td>Introduction to Basic Concepts and Properties of Differentiation and Integration</td>
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</tr>
<tr>
<td>Applications of Calculus to Economics</td>
<td>$P_4$</td>
<td>$T_5$</td>
</tr>
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</table>
APPENDIX B

DEPARTMENT OF MATHEMATICS

STUDENT INFORMATION SHEET

I. Identification Data

A. Print your name in the space provided on the response sheet and darken the matching grid below your name.

B. Write the numeric portion (five numbers) of your student number (found on your schedule or fee cards) in the first five columns of the Student Number section. Enter a zero in column six. Leave columns seven and eight blank. Fill in the grid.

C. Your classroom teacher will provide you with the section number. Enter this number in the space provided and fill in the grid.

D. Identify your sex. Omit the item labeled Test Form.

E. Use the section to identify your first quarter of enrollment at OSU. In filling out the grid use the following key for the Quarter you entered:
   1) Summer, 2) Autumn, 3) Winter, 4) Spring

F. Write your course number and instructor's name in the space provided.

II. Background Information

In responding to questions in this section, use items 1-6 on the response sheet.

1. Did you transfer to Ohio State from some other college or university? a) yes, b) no
2. How many years of high school mathematics (starting with Algebra I) have you taken?
   a) 1,  b) 2,  c) 3,  d) 4,  e) 5 or more

3. What was your average grade in high school mathematics courses?
   a) A,  b) B,  c) C,  d) D,  e) F

4-5. When was the last time you took a math course (high school or college)?
   4. a) 1970,  b) 1969,  c) 1968,  d) 1967,  e) 1966 or before
   5. a) Autumn,  b) Summer,  c) Spring,  d) Winter

6. Have you taken any previous math courses in college?
   a) yes,  b) no
INSTRUCTIONS TO RECITATION INSTRUCTORS
REGARDING PACING LEVEL RECOMMENDATIONS

Strategy I

If a student's score on the optional Test II is greater than or equal to 15, then the student should be recommended for an accelerated section.

The following recommendations are made based on Test I scores as well as information on students available prior to Test I.

If a student does not agree with the recommendation, he (she) should contact Dave Mader in Room 3, Alumni House on Friday or prior to the class on Monday.

The following students are recommended for an accelerated section. They should report to Room RL 2143 on Monday morning.

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Strategic I (continued)

The following students are recommended for the regular section.

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The following students are recommended for the reduced pace section. They should report to Room RL 2007 on Monday morning.

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Strategy II

If a student's score on the optional Test II is greater than or equal to 15, then the student should be recommended for an accelerated section.

The following recommendations are made based on Test I scores.

If a student does not agree with the recommendation, he (she) should contact Dave Mader in Room 3, Alumni House on Friday or prior to the class on Monday.
Strategy II (continued)

The following students are recommended for an accelerated section. They should report to Room RL 2143 on Monday morning.

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---

---

The following students are recommended for the regular section.

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---

---

The following students are recommended for the reduced pace section. They should report to Room RL 2007 on Monday morning.

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Strategy III

If a student's score on the optional Test II is greater than or equal to 15, then the student should be recommended for an accelerated section.
Strategy III (continued)

The following recommendations are made based on Test scores.

If a student does not agree with our recommendation, he (she) should contact Dave Mader in Room 3, Alumni House on Friday or prior to the class on Monday.

The following students are recommended for an accelerated section. They should report to Room RL 2143 on Monday morning.

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We are unable to make a firm recommendation to the following students. They are asked to choose either the accelerated section (Room CA 214) or remain in the regular section.

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Strategy III (continued)

We are unable to make a firm recommendation to the following students. They are asked to choose either a reduced pace section (Room RL 2009) or remain in the regular section.

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The following students are recommended for the reduced pace section. They should report to Room RL 2009 on Monday morning.

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On the answer sheet provided, choose the one best response to each of the following questions by marking with PENCIL ONLY in the appropriate space after the question number.

1. \((-3, 8) \cap [8, 9] = (\text{a}) (-3, 17], (\text{b}) [8], (\text{c}) \emptyset, (\text{d})(-3,9], (\text{e}) \text{None of these}

2. \((0, 3] \cup (-1, 2) = (\text{a}) (-1, 3], (\text{b}) (0, 2), (\text{c}) \emptyset, (\text{d}) (-1, 5), (\text{e}) \text{None of these}

In questions 3, 4, and 5, \(N\) denotes the set of Natural numbers and \(Z\) denotes the set of Integers.

3. If \(A = \{x \mid x \in N \text{ and } x < 2\}\) and \(B = \{x \mid x \in N \text{ and } x > -2\}\), then
   (a) \(A \subseteq B\), (b) \(B \subseteq A\), (c) \(A = B\), (d) \(A \cap B = \emptyset\), (e) None of these

4. \(N \cap [-1, 3] = (\text{a}) [-1, 0, 1, 2], (\text{b}) [-1, 3], (\text{c}) [-1, 3], (\text{d}) \{1, 2\}, (\text{e}) \text{None of these}

5. If \(C = \{x \mid x \in N \text{ and } x \leq 4\}, D = \{x \mid x \in Z \text{ and } -5/2 < x < 5\}\) and, \(E = \{x \mid x \in Z \text{ and } |x| < 3\}\) then \(C \cup (D \cap E) = (\text{a}) \{1,2,3,4\}, (\text{b}) \{1\}, (\text{c}) \{0, 1, 2, 3, 4\}, (\text{d}) \{-2, -1, 0, 1, 2, 3, 4\}, (\text{e}) \text{None of these}

6. The solution set for \(2x + 3 \leq -5\) is: (a) \(x > 0\), (b) \(x \leq 4\), (c) \(x \geq -4\), (d) \(x > 4\), (e) None of these

7. The solution set for \(3(1 + x) + 1 \geq 2x - 1\) is: (a) \(x \geq -5\), (b) \(x \leq -5\), (c) \(x \geq 5\), (d) \(x \leq 5\), (e) None of these

8. The solution set for \((x + 6)(x - 2) \leq 0\) is: (a) \(x \geq 2\), (b) \(x \leq -6\), (c) \(x \geq 1 \text{ or } x \leq -6\), (d) \(-6 \leq x \leq 1\)
9. The solution set for $x^2 \leq 25$ is: (a) $\{-5 \leq x \leq 5\}$, (b) $[x \geq -5]$, (c) $[x \leq 5]$, (d) $[x \leq -5 \text{ or } x \geq 5]$, (e) None of these.

10. The sum of the solutions to $|x + 3| = 5$ is: (a) $\{2\}$, (b) $\{8\}$, (c) $\{5\}$, (d) There is no solution, (e) None of these.

11. The solution set for $|2x - 3| \leq 13$ is: (a) $\{x \geq 8\}$, (b) $[-5 \leq x \leq 8]$, (c) $[x \geq -5]$, (d) $[x \leq 8]$, (e) None of these.

12. The sum of the solutions to $|x + 2| + |x + 1| = 5$ is: (a) $-6$, (b) $-3$, (c) $0$, (d) $3$, (e) None of these.

13. The solution set for $|x^2 - 5x - 3| < 3$ is: (a) $\emptyset$, (b) $[5 < x < 6]$, (c) $[-1 < x < 0]$, (d) $[5 < x < 6] \cup [-1 < x < 0]$, (e) None of these.

14. The solution set for $\frac{|x + 1|}{(x - 1)(x^2 + 4x + 4)} < 0$ is: (a) $\{x > -1\}$, (b) $\{x < -1\}$, (c) $\{x < 1\}$, (d) $(-1, 1)$, (e) None of these.

15. The solution set for $|x - 1| \geq 2$ is: (a) $\{x \geq -1\}$, (b) $\{x \geq -3\}$, (c) $\emptyset$, (d) all reals, (e) None of these.

16. If $f(x) = \sqrt{x^2 + 1}$, then $f([\pi])$ is: (a) $\sqrt{10}$, (b) $\sqrt{\pi^2 + 1}$, (c) $\sqrt{5}$, (d) $\pi$, (e) None of these.

17. The domain of $f(x) = \sqrt{\frac{1-x}{x}}$ is: (a) $\{x \geq 1\}$, (b) $\{x \leq 1\}$, (c) $\{x \neq 0\}$, (d) all reals, (e) None of these.

18. The range of $f(x) = |x - 2|$ is: (a) $\{y \geq 2\}$, (b) $\{y \leq 2\}$, (c) $\{y \geq 0\}$, (d) $\emptyset$, (e) None of these.

19. Which of the following functions is not one-to-one? (a) $f(x) = 2x + 1$, (b) $f(x) = \frac{2 - x}{3}$, (c) $f(x) = x$, (d) $f(x) = x^2$, (e) None of these.
(d) \( f(x) = x^2 \),  (e) \( f(x) = x^3 \)

20. If \( f(x) = x^2 + x \), then \( \frac{f(x + h) - f(x)}{h} \) = (a) \( 2xh + h^2 + h \), (b) \( 2xh + h^2 \), (c) \( 2x + 1 + h \), (d) \( 2x + 1 \), (e) None of these

21. If \( f(x) = x + 3 \) and \( g(x) = x^2 + 2x + 1 \), then \( g(f(x)) = \) (a) \( x^2 + 2x + 4 \), (b) \( x^2 + 11x + 16 \), (c) \( x^2 + 2x + 3 \), (d) \( x^2 + 8x + 16 \), (e) None of these

22. The Range of \( h(x) = \frac{3 + 2x}{x} \) is: (a) all reals, (b) \{ \( y \neq 2 \} \), (c) \{ \( y \neq 3 \} \), (d) \{ \( y \neq 0 \} \), (e) None of these

23. The property that \( f(x_1 + x_2) = f(x_1) + f(x_2) \) holds for which kind of linear functions \( f(x) = mx + b \)?
   (a) no linear functions.
   (b) only those linear functions with \( m = 1 \).
   (c) only those linear functions with \( m = 0 \).
   (d) only those linear functions with \( b = 0 \).
   (e) all linear functions.

24. If \( x, a, b \in \mathbb{R} ; b > 0 \) and \( |x - a| < b \), then which of the following is true?
   (a) \( x \in (b - a, b + a) \).
   (b) \( x \neq a \) and \( x > a + b \) or \( x < a - b \).
   (c) \( x^2 + ax + b = 0 \).
   (d) the distance on the number line between \( x \) and \( a \) is less than \( b \). (e) none of these.

25. If \( A \) is a subset of the domain of a function \( f \), then the set \( f(A) \) is defined by \( f(A) = \{ f(x) \mid x \in A \} \). If \( A = [-2, 3] \) and \( f(x) = 3 - x^2 \) then \( f(A) \) is: (a) \([-1, 3]\), (b) \([-6, -1]\), (c) \([-6, 3]\), (d) \( \emptyset \), (e) none of these
Directions: Place your name, section code number, and test form (green—form A, yellow—form B) in the proper areas on the answer sheet provided. Select the one best response—a, b, c, d, or e—to each of the following questions and indicate your choice on the answer sheet (mark with a soft lead pencil).

1. A segment of the graph of \( y - 2x = 10 \) in \( \mathbb{R}^2 \) is best represented by:

   (a) \[
   \begin{array}{c}
   y \\
   \hline
   x
   \end{array}
   \]

   (b) \[
   \begin{array}{c}
   y \\
   \hline
   x
   \end{array}
   \]

   (c) \[
   \begin{array}{c}
   y \\
   \hline
   x
   \end{array}
   \]

   (d) \[
   \begin{array}{c}
   y \\
   \hline
   x
   \end{array}
   \]

   (e) \[
   \begin{array}{c}
   y \\
   \hline
   x
   \end{array}
   \]

2. The graph of \( x > 2 - y \) in \( \mathbb{R}^2 \) is best represented by:

   (a) \[
   \begin{array}{c}
   y \\
   \hline
   x
   \end{array}
   \]

   (b) \[
   \begin{array}{c}
   y \\
   \hline
   x
   \end{array}
   \]

   (c) \[
   \begin{array}{c}
   y \\
   \hline
   x
   \end{array}
   \]
3. The graph of \( x + |y - 2| < 4 \) in \( \mathbb{R}^2 \) is best represented by:
(a) the x-axis (b) the origin and the positive x-axis and the y-axis (c) the x-axis and the y-axis (d) the y-axis (e) none of these

4. The graph of \( y|x| = 0 \) in \( \mathbb{R}^2 \) is best represented by:
(a) the x-axis (b) the origin and the positive x-axis and the y-axis (c) the x-axis and the y-axis (d) the y-axis (e) none of these

5. The graph of \( y + z = 2 \) in the first octant of \( \mathbb{R}^3 \) is best represented by:
(a) (b) (c)
6. The graph of \( x = y = z \) in the first octant of \( \mathbb{R}^3 \) is best represented by:

(a) \( \gamma \)

(b) \( \gamma \)

(c) \( \gamma \)

(d) \( \gamma \)

(e) \( \gamma \)

In problems 7 through 16, let \( P = (-1, 0), Q = (2, -1), R = (-2, 4), S = (3, -2), X = (x, y) \) and \( O = (0, 0) \).

7. The equation of the line through the points \( R \) and \( S \) is:

(a) \( 2x + 5y = 6 \)  
(b) \( 6x + 5y = 8 \)  
(c) \( x + 2y = 6 \)  
(d) \( 2x + y = 0 \)  
(e) None of these are correct

8. The equation of the line through point \( Q \) and parallel to the \( y \)-axis is:

(a) \( y = 2x \)  
(b) \( y = 2 \)  
(c) \( x = y \)  
(d) \( x = 2 \)  
(e) none

9. The slope of the line perpendicular to the line through the points \( P \) and \( Q \) is:
10. The equation of the line through point S and parallel to the line through points Q and R is:
(a) \(x + y = 1\) (b) \(5x + 4y = 7\) (c) \(3x + 2y = 5\)
(d) \(x - y = 5\) (e) none of these

11. The midpoint of the segment PS is:
(a) \((1, -1)\) (b) \((2, 1)\) (c) \((-2, -1)\) (d) \((1, 2)\)
(e) none of these

12. The point one-fourth of the way from P to S is:
(a) \((0, -1)\) (b) \((0, -1/2)\) (c) \((2, 3/4)\) (d) \((-1/2, -3/5)\)
(e) none

13. Find X so that \(\overline{OX} = \overline{SP}\). X = :
(a) \((-4, 2)\) (b) \((-1, 4)\) (c) \((-1, 6)\) (d) \((0, 3)\)
(e) none of these

14. Find X so that \(\overline{FX} + 2\overline{XP} = -3\overline{SQ}\). X = :
(a) \((1, 2)\) (b) \((4, 5)\) (c) \((3, 1)\) (d) \((2, 3)\)
(e) none of these

15. \(d(P, Q) = :\)
(a) \(\sqrt{23}\) (b) \(3\) (c) \(\sqrt{10}\) (d) \(2\sqrt{2}\) (e) none of these

16. The length of \(3\overline{PQ} = 2\overline{SO}\) is :
(a) \(\sqrt{274}\) (b) \(\sqrt{307}\) (c) \(35\) (d) \(14\) (e) none of these

17. The slope of the line with equation \(2y - 4x - 6 = 0\) is :
(a) \(1/2\) (b) \(2\) (c) \(-2\) (d) \(-1/2\) (e) none of these

18. If a and b are in \(R\) such that \(a(-1,3) + b(1,-2) = (0,1)\), then the sum of a and b is :
(a) \(3\) (b) \(2\) (c) \(0\) (d) \(-1\) (e) none of these
19. Which of the following is not a convex subset of $\mathbb{R}^2$?

(a) ![Diagram A](image1)
(b) ![Diagram B](image2)
(c) ![Diagram C](image3)
(d) All are convex subsets
(e) None are convex subsets

In problems 20 through 22, let $P = (0, 1, 0)$, $Q = (-2, 1, 1)$, and $R = (4, 0, 3)$.

20. The equation of the line through points $P$ and $Q$ is:

(a) $y = 1$
(b) $x = -2z, y = 1$
(c) $-x = 2z$
(d) $x = y = z = 1$
(e) None of these

21. The equation of the set of points equidistant from the points $Q$ and $R$ is:

(a) $x + 2y - 3z = -1$
(b) $3x + 2y + z = 6$
(c) $12x - 2y + 4z = 19$
(d) $5x + 4y + 6z = 18$
(e) None of these

22. $d(Q, R) =$

(a) $\sqrt{37}$
(b) $3$
(c) $6$
(d) $\sqrt{39}$
(e) None of these

23. The best sketch of $R$ is given by:

(a) ![Sketch A](image4)
(b) ![Sketch B](image5)
(c) ![Sketch C](image6)
24. The sum in $\mathbb{R}^2$ of the vertices of the region R is:
   (a) (16, 4) (b) (16, 2) (c) (10, 6) (d) (6, 16) (e) none

25. The minimum value of $F(x, y) = 3y - 4x + 10$ defined in the region R is:
   (a) -30 (b) -40 (c) 0 (d) -126 (e) none of these
On the answer sheet provided, choose the one best response to each of the following questions by marking with PENCIL ONLY in the appropriate space after the question number. None means "None of the preceding".

1. \([-3, 6] \cap [6, 8]) \cup [2, 6] =
   (a) \([-3, 6])
   (b) \([2, 6])
   (c) \([2, 8])
   (d) \([-3, 8])
   (e) None.

2. Which axiom is used to factor \( x^2 y + x^2 z = x^2(y + z) \)?
   (a) Associative law,
   (b) Commutative law,
   (c) Inverse law,
   (d) Distributive law,
   (e) None.

3. The sum of the solutions to \(|x - 5| = 8\) is:
   (a) 10,
   (b) 16,
   (c) 5,
   (d) -10,
   (e) None.

In Questions 4, 5, and 6, \(N\) is the set of natural numbers, \(Z\) is the set of integers, \(A = \{x : x \in N \text{ and } x < 4\}\) and \(B = \{x : x \in N \text{ and } x \leq 4\}\).

4. The set \(B\) is equal to:
   (a) \([0, 4])
   (b) \((0, 4])
   (c) \([0, 1, 2, 3, 4])
   (d) \([1, 2, 3])
   (e) None.

5. Which of the following is true?
   (a) \(B \subseteq A\)
   (b) \(A = B\)
   (c) \(A \subseteq B\)
   (d) \(A \cap B = \emptyset\)
   (e) None.
6. \( A \cup (Z \cap [-1, 3]) = \)
   
   (a) \{1, 2, 3\},  (b) \{-1, 0, 1, 2, 3\}, (c) \{0, 3\},
   (d) \{-1, 3\},  (e) None.

7. \( \frac{a^2 \cdot a^{3/2}}{a^5} = \)
   
   (a) \(a^{-2}\),  (b) \(a^{3/2}\), (c) \(a^{-3/2}\), (d) \(a^{-15}\),
   (e) None.

8. \( \sqrt[3]{-4 \cdot 4} = \)
   
   (a) \(-2\),  (b) \(4^{1/3}\), (c) \(2\),  (d) Undefined,
   (e) None.

9. \( 1 - \sqrt{x} + \frac{\sqrt{x}}{1 + \frac{1}{\sqrt{x}}} = \)
   
   (a) \(-\sqrt{x}\),  (b) \(\frac{1}{1 + \sqrt{x}}\), (c) \(1 + \sqrt{x}\), (d) \(1 - \sqrt{x}\),
   (e) None.

10. The solution set for \(3 - 2(x - 1) \leq \frac{4x - 1}{2}\) is:
    
    (a) \(\{x : x \geq 1\}\),  (b) \(\{x : x \leq 1\}\), (c) \(\{x : x \leq 1/2\}\),
    (d) \(\{x : x \geq 1/2\}\),  (e) None.

11. \(\{x : |x^2 - x| = x^2 - x\}\) is equal to:
    
    (a) \([-\infty, \infty]\),  (b) \([-\infty, 0]\), (c) \([1, \infty]\),  (d) \([0, 1]\),
    (e) \([-\infty, 0]\) \(\cup [1, \infty]\).

12. \(x^{1/m^2} \cdot x^{1/n^2} = \frac{m}{n} \cdot x^{-1/n}\) equals:
    
    (a) \(x^{1/m}\),  (b) \(x^{1/n}\), (c) \(x^{m-n}\), (d) \(x^{m+n}\),
    (e) None.
13. The solution set for \(|3x - 6| \leq 15\) is:
   (a) \([0, 15]\), (b) \([-5, 5]\), (c) \([-3, 3]\),
   (d) \([-3, 7]\), (e) None.

14. The solution set for \(x^2 + 1 > 0\) is:
   (a) \([-1, 1]\), (b) \((-\infty, \infty)\), (c) \((-1, 1)\), (d) \(\emptyset\),
   (e) None.

15. \(\{x : \sqrt{1 - \frac{x^2}{x^2}} \text{ is a real number}\}\) is:
   (a) \(\{x : x \neq 0\}\), (b) \(\{x : -1 \leq x \leq 1\}\),
   (c) \(\{x : -1 \leq x \leq 1 \text{ and } x \neq 0\}\), (d) \(\{x : x \leq 1\}\),
   (e) None.

16. The solution set for \((x - 1)(x + 3) \leq 0\) is:
   (a) \([-1, 3]\), (b) \([-3, 1]\), (c) \((-\infty, -1) \cup (3, \infty)\),
   (d) \((-\infty, -3) \cup (1, \infty)\), (e) None.

17. The solution set for \(x^2 \leq 6x\) is:
   (a) \((-\infty, 0]\), (b) \([0, \infty)\), (c) \([0, 6]\), (d) \([6, \infty)\),
   (e) None.

18. The solution set for \(\frac{x}{|x+3|} \leq 0\) is:
   (a) \((-\infty, 0]\), (b) \((-\infty, \infty)\), (c) \((-\infty, -3)\),
   (d) \((-\infty, -3) \cup (-3, 0]\), (e) None.

19. The solution set for \(|x| + |x - 3| < 5\) is:
   (a) \((-1, 4)\), (b) \([0, 4]\), (c) \((-1, 1)\), (d) \((-4, 4)\),
   (e) None.

20. The interval \((-2, 10) = \{x : |x - a| < p\}\) where:
   (a) \(a = -2, p = 12\); (b) \(a = 4, p = 10\);
   (c) \(a = 5, p = 6\); (d) \(a = 4, p = 6\); (e) None.
The items on Form B are the same as the items on Form A except for the order of appearance. The correspondence of items between the two forms is presented in the following table.

**CORRESPONDENCE OF ITEMS BETWEEN FORMS A AND B OF TEST GROUP TEST ONE**

<table>
<thead>
<tr>
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On the answer sheet provided, choose the one best response to each of the following questions by marking with PENCIL ONLY in the appropriate space after the question number. None means "none of the preceding".

1. If \( f(x) = |2^{-x}| \), then \( f(-1) = \)
   (a) 1/2  (b) 2  (c) -1/2  (d) -2  (e) None

2. If \( g(t) = \sqrt{t+3} \) and \( f(x) = 1 + x^2 \), then \( f(g(-1)) = \)
   (a) -3  (b) 2  (c) 3  (d) -2  (e) None

3. A composition of mappings under which the curve \( y = -\frac{3}{2} + \frac{1}{2}x^2 \)
   is the image of \( y = x^2 \) is:
   (a) a Y-distortion by \( 3/2 \) followed by a Y-translation by \(-1/2\)
   (b) a Y-distortion by \( 1/2 \) followed by a Y-translation by \(-3/2\)
   (c) a Y-distortion by \( 1/2 \) followed by a Y-translation by \( 2/3\)
   (d) a Y-distortion by \( 1/2 \) followed by a Y-translation by \( 3/2\)
   (e) None

4. Which of the following subsets of \( \mathbb{R}^2 \) ((a), (b), (c) or (d)) is symmetric about the Y-axis?
   (a) \( \{(x,y): y = |1/x|\} \)  (b) \( \{(x,y): x = y^2\} \)
   (c) \( \{(x,y): y = \sqrt{x}\} \)  (d) \( \{(x,y): |y| = x\} \)  (e) None

5. Which of the following ((a), (b), (c) or (d)) is not a function?
   (a) \( \{(x,y): y = |x^2|\} \)  (b) \( \{(x,y): y = |2^{-x}|\} \)
   (c) A Y-distortion by 3 followed by a Y-translation by -1
   (d) \( \{(x,y): |y| = |x|\} \)  (e) All of the preceding are functions.
6. Which of the following (a), (b), (c) or (d) is not a one-to-one function?
(a) \( f(x) = |2^x| \)  
(b) \( g(x) = \log_2 x \)  
(c) \( h(x) = (x+1)^3 \)  
(d) \( F(x) = \frac{2}{x} \)  
(e) All of the preceding are one-to-one.

7. Which of the following best represent \((x, y): y = 2 - 3^x\)
(a)  
(b)  
(c)  
(d)  
(e)  

8. Which of the following best represent \((x, y): 2x - y \leq 2\)
(a)  
(b)  
(c)  
(d)  
(e)
9. Which of the following graphs best represent \( (x, y): |y| = 1 = x \)

(a) ![Graph A]
(b) ![Graph B]
(c) ![Graph C]
(d) ![Graph D]
(e) ![Graph E]

10. The sum of the zeros of \( f(x) = x^2 + 14x + 51 \) is:
   (a) -14  (b) 14  (c) 19  (d) -19  (e) None

11. The equation of the line containing the points \((-1, 0)\) and \((-1, 3)\) is:
   (a) \( x + 1 = 0 \)  (b) \( y + 1 = 0 \)  (c) \( y = 3 \)  (d) \( x + y = 1 \)
   (e) None

12. If \( f(x) = \frac{\sqrt{x}}{x - 1} \), then the understood domain of \( f \) is:
   (a) \([0, \infty)\)  (b) \((-\infty, -1) \cup [0, \infty)\)  (c) \([0, 1) \cup (1, \infty)\)
   (d) \((-\infty, 1) \cup (1, \infty)\)  (e) None

13. If \( f(x) = \log_3(x + 1) \), then the range of \( f \) is:
   (a) \([3, \infty)\)  (b) \((-\infty, \infty)\)  (c) \([0, 3]\)  (d) \((0, \infty)\)
   (e) None

14. For \( f \) and \( g \) linear functions, which of the following ((a), (b), (c) or (d)) is not a linear function?
   (a) \( h(x) = f(x) + g(x) \)  (b) \( k(x) = f(x) - g(x) \)
   (c) \( F(x) = f(g(x)) \)  (d) \( H(x) = 3f(x) - 2g(x) \)
   (e) All of the preceding are linear functions.
15. Solve: \( x^2 + 2x - 5 < 0 \)
   
   (a) \((-1 - \sqrt{12}, -1 + \sqrt{12}) \)  
   (b) \((-1 - \sqrt{6}, -1 + \sqrt{6}) \)  
   (c) \((-\sqrt{6}, \sqrt{6}) \)  
   (d) \((-\infty, \infty) \)  
   (e) None

16. If \( P = (1, 2) \) and \( Q = (-3, 4) \), then the distance \( \overline{PQ} \) is:
   
   (a) \(\sqrt{18} \)  
   (b) \(2\sqrt{3} \)  
   (c) \(2\sqrt{5} \)  
   (d) \(\sqrt{10} \)  
   (e) None

17. If \( f = (x, (x+1)^2): x \geq -1 \), then
   
   (a) \(f^{-1}(x) = \sqrt{x} - 1 \)  
   (b) \(f^{-1}(x) = \sqrt{x} + 1 \)  
   (c) \(f^{-1}(x) = 1 + \sqrt{x} \)  
   (d) \(f^{-1}(x) = \sqrt{x + 1} \)  
   (e) None

18. If \( \log_{-1/3} x = -1/3 \), then \( x = \):
   
   (a) 3  
   (b) -3  
   (c) -1/3  
   (d) 1/3  
   (e) None

19. The solution set for \( \log_{2}(x+12) = 2 + \log_{2}x \) is:
   
   (a) \([12]\)  
   (b) \([1/2]\)  
   (c) \([25]\)  
   (d) \([2]\)  
   (e) None

20. If \( (\exp_3 x)^2 = \exp_3 5x \), then \( x = \):
   
   (a) 1 or 5  
   (b) 0  
   (c) 3  
   (d) 15  
   (e) None
The items on Form B are the same as the items on Form A except for the order of appearance. The correspondence of items between the two forms is presented in the following table.

**CORRESPONDENCE OF ITEMS BETWEEN FORMS A AND B OF TEST GROUP TEST TWO**

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On the answer sheet provided, choose the one best response to each of the following questions by marking with PENCIL ONLY in the appropriate space after the question number. None means "None of the preceding".

1. If \( f(x) = 3 - x^2 \), then \( f(-2) = \)
   (a) 7  (b) 1  (c) -1  (d) 12  (e) None

2. If \( f(t) = 1 - t \) and \( g(x) = x^2 - 1 \), then \( g(f(2)) = \)
   (a) -3  (b) -2  (c) 0  (d) 2  (e) 3

3. A composition of mappings under which the line \( y = -2 + 3x \) is the image of the line \( y = x \) is:
   (a) a Y-distortion by 2 followed by a Y-translation by -3
   (b) a Y-distortion by -2 followed by a Y-translation by 3
   (c) a Y-distortion by 3 followed by a Y-translation by -2
   (d) a Y-distortion by -3 followed by a Y-translation by 2
   (e) None

4. The image of the point \((-6, 3)\) under a Y-distortion by 3 followed by an X-reflection is:
   (a) \((6, 9)\)  (b) \((-6, 9)\)  (c) \((6, -9)\)  (d) \((-6, -9)\)
   (e) None

5. Which of the following subsets of \( \mathbb{R}^2 \) (((a), (b), (c) or (d)) is symmetric about the X-axis
   (a) \(\{(x, y): y = |x|\}\)  (b) \(\{(x, y): y = x^2\}\)
   (c) \(\{(x, y): y = x^3\}\)  (d) \(\{(x, y): x = y^2\}\)
   (e) None
6. Which of the following ((a), (b), (c) or (d)) is not a function:
(a) \( ((x, y): y = |x|) \)  (b) \( ((x, y): y - 3 = \frac{x + 1}{2}) \)
(c) a Y-translation by 3  (d) \( ((x, y): x^2 + y^2 = 1 \quad \text{and} \quad x \geq 0) \)
(e) All of the above are functions.

7. Which of the following ((a), (b), (c) or (d)) is not a one-to-one function:
(a) \( f(x) = \sqrt{x} \)  (b) \( f(x) = \left| \frac{1}{x} \right| \)  (c) \( f(x) = \frac{3x - 1}{2} \)
(d) \( f(x) = 2^x \)  (e) a Y-translation by 3.

8. Which of the following graphs best represent \( ((x, y): y = (x - 3)^2 - 4) \)?

9. Which of the following graphs best represent \( ((x, y): |y - 1| = x) \):
10. The zeros of \( f(x) = 6x^2 - x - 15 \) are:
   (a) \(-2/3, 3/5\)  
   (b) \(3/2, -5/3\)  
   (c) \(2/3, -3/5\)  
   (d) \(-3/2, 5/3\)  
   (e) None

11. The equation of the line parallel to the line \( x - 3 = y + 2 \) containing the point \((2, -3)\) is:
   (a) \(y - x + 5 = 0\)  
   (b) \(y + x + 1 = 0\)  
   (c) \(2x - y = 7\)  
   (d) \(x - 2 = y - 3\)  
   (e) None

12. If \( q(x) = \frac{x^2 - 1}{3} \), then the domain of \( g \) is:
   (a) \(\emptyset\)  
   (b) \((x \in \mathbb{R}: x \neq 1 \text{ or } x \neq -1)\)  
   (c) \((-\infty, 3) \cup (3, \infty)\)  
   (d) \([1, \infty)\)  
   (e) None

13. If \( h(u) = u^3 \), then the range of \( h \) is:
   (a) \((-\infty, \infty)\)  
   (b) \(\emptyset\)  
   (c) \([0, \infty]\)  
   (d) \([-\infty, 0]\)  
   (e) None

14. If \( P = (x, 3) \), \( Q = (-1, 2) \), and \( \overline{PQ} = \sqrt{5} \), then the sum of all such \( x \) is:
   (a) \(3\)  
   (b) \(-2\)  
   (c) \(1\)  
   (d) \(-1\)  
   (e) None

15. If \( f(x) = \sqrt{x - 3} \), then:
   (a) \(f^{-1}(x) = (x-3)^2\)  
   (b) \(f^{-1}(x) = \frac{1}{x+3}\)  
   (c) \(f^{-1}(x) = (x+3)^2\)  
   (d) \(f^{-1}(x) = 3+x^2\)  
   (e) \(f\) has no inverse.

16. If \( f \) is the inverse function of \( g \), then:
   (a) \(f(x)g(x) = x\)  
   (b) \(f(g(x)) = 1\)  
   (c) \(f(g(x)) = g(f(x))\)  
   (d) \(f(x) + g(x) = x\)  
   (e) None

17. If \( h(t) = \frac{\sqrt{1 - t}}{1 + t} \), then the domain of \( h \) is:
   (a) \([\infty, 1]\)  
   (b) \((1, \infty) \cup (-1, 1)\)  
   (c) \((-\infty, -1) \cup (-1, 1)\)  
   (d) \((x \in \mathbb{R}: x \neq -1)\)  
   (e) None

18. If \( \log_{25} \frac{1}{x} = x \), then \( x = \)
   (a) \(2\)  
   (b) \(1/2\)  
   (c) \(-2\)  
   (d) \(\sqrt{5}\)  
   (e) None
19. If \( e^{3x} = e^{9x-4} \), then \( x = \)
(a) 2 or -2 (b) 1/2 (c) 8 (d) 4 (e) None

20. The sum of the solutions for \( \log_2(x+2) + \log_2(x-1) = 2 \) is:
(a) -1 (b) -3 (c) 2 (d) 1 (e) None
Chapter 2
Autumn 1970

The items on Form B are the same as the items on Form A except for the order of appearance. The correspondence of items between the two forms is presented in the following table.

### CORRESPONDENCE OF ITEMS BETWEEN FORMS

**A AND B OF THE OPTIONAL TEST GROUP TEST TWO**

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1. If \( g(s) = \sqrt{2 - s} \), then \( g(-3) = \)
   (a) 1,  (b) \( \sqrt{5} \),  (c) undefined,  (d) 2,  (e) none.

2. If \( g(x) = x + 3 \) and \( f(t) = 1 - t^2 \), then \( f(g(-1)) = \)
   (a) -3,  (b) 2,  (c) 3,  (d) -2,  (e) none.

3. A composition of mappings under which the line \( y = 2x - 3 \) is
   the image of the line \( y = x \) is:
   (a) A Y-distortion by 2 followed by a Y-translation by -3,
   (b) A Y-distortion by -2 followed by a Y-translation by 3,
   (c) A Y-distortion by 3 followed by a Y-translation by -2,
   (d) A Y-distortion by -3 followed by a Y-translation by 2,
   (e) none.

4. Which of the following subsets of \( \mathbb{R}^2 \) (a), (b), (c), or (d)
   is symmetric about the Y-axis?
   (a) \( \{(x,y): y = \frac{1}{x}\} \),  (b) \( \{(x,y): x = y^2\} \),
   (c) \( \{(x,y): y = \sqrt{x}\} \),  (d) \( \{(x,y): x^2 + y^2 = 1\} \),
   (e) none.
5. Which of the following \((a), (b), (c),\) or \((d)\) is not a function?
\[(a) \{(x,y): y = \sqrt{x}\}, \quad (b) \{(x,y): y = 2^x\}, \]
\[(c) \text{ A } Y\text{-distortion by 2}, \quad (d) \{(x,y): |y| = x\}, \]
\[(e) \text{ All of the above are functions}.\]

6. Which of the following \((a), (b), (c),\) or \((d)\) is not a one-to-one function?
\[(a) f(x) = |2^x|, \quad (b) g(x) = |\log_2 x|, \quad (c) h(x) = \frac{\sqrt{x-1}}{2}, \]
\[(d) F(x) = x^3, \quad (e) \text{ none}.\]

7. Which of the following graphs best represent \(\{(x,y): y = 4 - (x + 3)^2\}\)?

8. Which of the following graphs best represent \(\{(x,y): x - 2y \geq 2\}\)
9. Which of the following graphs best represent \[(x,y): y - 1 = |x|\]:

(a) \[\text{Graph A}\]
(b) \[\text{Graph B}\]
(c) \[\text{Graph C}\]
(d) \[\text{Graph D}\]
(e) \[\text{Graph E}\]

10. The zeros of \[f(x) = 6x^2 + x - 15\] are:

(a) \[-2/3, 3/5\];
(b) \[3/2, -5/3\];
(c) \[2/3, -3/5\];
11. The equation of the line containing the points (-1,0) and (3,-2) is:
(a) 2y + x + 1 = 0,  (b) x - y = 5,  (c) x + 2y - 1 = 0,
(d) x + y = 1,  (e) none.

12. If f(x) = \sqrt{x - 2}, then the understood domain of f is:
(a) [-\infty,0],  (b) [0,\infty],  (c) [2,\infty],  (d) [-\infty,2],
(e) none.

13. If g(x) = \log_2(x^2 - 1), then the understood domain of g is:
(a) (-\infty,-1) \cup (1,\infty),  (b) (0,\infty),  (c) [0,\infty],
(d) (-\infty),  (e) none.

14. If f(x) = 3^x, the range of f is:
(a) [3,\infty],  (b) (-\infty),  (c) [0,3],  (d) (0,\infty),
(e) none.

15. Solve: x^2 - x + 1 > 0.
(a) (1/2 - \sqrt{5}/2, 1/2 + \sqrt{5}/2);  (b) (0,1),
(c) (-\infty,1) \cup (1,\infty);  (d) (-\infty);  (e) none.

16. If P = (-1,2) and Q = (3,-5), then the distance PQ is:
(a) \sqrt{50},  (b) \sqrt{33},  (c) \sqrt{13},  (d) \sqrt{65},  (e) none.

17. If f = \{(x, (x - 1)^2): x \geq 1\}, then
(a) f^{-1}(x) = \sqrt{x - 1},  (b) f^{-1}(x) = \sqrt{x - 1},
(c) f^{-1}(x) = 1 + \sqrt{x},  (d) f^{-1}(x) = \sqrt{x + 1},
(e) f^{-1} does not exist.
18. If \( \log x^{25} = \frac{2}{3} \), then \( x = \)
   (a) 5, (b) 125, (c) -5, (d) \( \frac{1}{5} \), (e) none.

19. The solution set for \( \log_2(x + 4) - \log_2(x - 1) = 1 \) is:
   (a) \{2, 4\}, (b) \{6\}, (c) \{4, 8\}, (d) \{3\}, (e) \emptyset

20. If \( \exp 2^{x^2} = \exp 4^x \), then \( x = \)
   (a) 0 or 2, (b) 1, (c) -1 or 1, (d) -2 or 2,
   (e) none.
On the answer sheet provided, choose the one best response to each of the following questions by marking with **PENCIL ONLY** in the appropriate space after the question number. **None** means "None of the preceding".

1. \([-1,4) \cup (0,5] =\
   \(a) [-1,5]\\n   (b) (0,4)\\n   (c) (4,5]\\n   (d) [-1,0)\\n   (e) none.\)

2. \(([-3,8] \cap [8,10]) \cup (8,9] =\
   \(a) (-3,10)\\n   (b) (8,9)\\n   (c) [8,9]\\n   (d) \emptyset\\n   (e) none.\)

3. If \(A = (-2,2)\\n   B = \{x: x \in \mathbb{N} \text{ and } x < 1\}\\n   C = \{x: x \in \mathbb{Z} \text{ and } x < 3\},\) then \((A \cap C) \cup B\) equals
   \(a) (-\infty,2)\\n   (b) [0,1]\\n   (c) [-1,0,1]\\n   (d) [0,1]\\n   (e) none.\)

4. The reason that \(\left(\frac{a}{a} \cdot a\right) \cdot b = b \cdot \left(\frac{a}{a} \cdot a\right)\) is:
   \(a) The\ commutative\ law,\hspace{1cm} (b) The\ associative\ law,\\n   (c) The\ inverse\ law,\hspace{1cm} (d) The\ distributive\ law,\\n   (e) none.\)

5. \((-\frac{27}{8})^{-2/3} = (a) 9/4, (b) -9/4\hspace{1cm} (c) -4/9 (d) 4/9,\\n   (e) none.\)
6. \( \left( \frac{a^2 - 3}{a - 3} \right)^{-2} \) = (a) \( \frac{a}{b} \), (b) \( b^{10} \), (c) \( \frac{a^2}{b^2} \),
(d) \( \frac{b^{12}}{a^{12}} \), (e) none.

7. \( 3 \sqrt[3]{2 \sqrt{16} - (1/2)^{-1} + \sqrt{12}} = \) (a) 0, (b) \( 2 \sqrt{3} \),
(c) \( 3 \sqrt{42} - 2 \), (d) \( 5 \sqrt{2} \), (e) can not be determined.

Find the solution sets in questions 8 - 13.

8. \( 3(x - 4) \leq 2(4x + 9) \)
   (a) \( \{x: x \leq -6\} \), (b) \( \{x: x \geq -6\} \), (c) \( \{x: x \geq 6\} \),
   (d) \( \{x: x < 6\} \), (e) none.

9. \( (x + 2)(x - 8) > 0 \)
   (a) (-2,8), (b) (-8,2), (c) (-\( \infty \),-8) U (2,\( \infty \)),
   (d) (-\( \infty \),-2) U (8,\( \infty \)), (e) none.

10. \( |x - 4| < 2 \) : (a) (4,6), (b) (-\( \infty \),6), (c) (4,\( \infty \)),
    (d) (-6,6), (e) none.

11. \( |x^2 + x - 4| \geq -2 \) : (a) [-\( \infty \),-2] U [1,\( \infty \)] (b) [1,\( \infty \]),
    (c) \( \mathbb{R} \) (d) \( \emptyset \), (e) none.

12. \( \frac{|x - 4|}{x + 1} < 0 \) : (a) (-\( \infty \),-1), (b) (-\( \infty \),4), (c) (-1,4),
    (d) (4,\( \infty \)), (e) none.

13. \( \frac{4}{1 - x} > 2 \) : (a) (-\( \infty \),-1), (b) (-1,\( \infty \)), (c) (0,1),
    (d) (-1,1), (e) none.
14. The sum of the solutions to $|2x + 5| = 7$ is:
   (a) -1, (b) 0, (c) 1, (d) 2, (e) none.

15. Which of the following best represents $\{(x,y): x = 1 - 2y\}$?
   (a) \begin{tikzpicture}
       \draw[->] (0,0) -- (3,0);
       \draw[->] (0,-1) -- (0,3);
   \end{tikzpicture}
   (b) \begin{tikzpicture}
       \draw[->] (0,0) -- (3,0);
       \draw[->] (0,-1) -- (0,3);
   \end{tikzpicture}
   (c) \begin{tikzpicture}
       \draw[->] (0,0) -- (3,0);
       \draw[->] (0,-1) -- (0,3);
   \end{tikzpicture}
   (d) \begin{tikzpicture}
       \draw[->] (0,0) -- (3,0);
       \draw[->] (0,-1) -- (0,3);
   \end{tikzpicture}
   (e) none.

16. If $f(x) = (x + 2)(x^2 + 4x + 1)$, then the sum of the zeros of $f$ is:
   (a) 1, (b) -2, (c) -4, (d) -6, (e) none.

17. Which of the following ((a), (b), (c), or (d)) is not a function?
   (a) $\{(x,y): x = y^2$ and $y \geq 0\}$,
   (b) $\{(x,y): y = |x|^3\}$,
   (c) $\{(x,y): x = 2$ and $y \in \mathbb{R}\}$,
   (d) $\{(x,y): y = -3$ and $x \in \mathbb{R}\}$,
   (e) All of the preceding are functions.

18. Which of the following best represents $\{(x,y): y = 1 + 2|x|\}$?
   (a) \begin{tikzpicture}
       \draw[->] (0,0) -- (3,0);
       \draw[->] (0,-1) -- (0,3);
   \end{tikzpicture}
   (b) \begin{tikzpicture}
       \draw[->] (0,0) -- (3,0);
       \draw[->] (0,-1) -- (0,3);
   \end{tikzpicture}
19. If \( g(x) = \frac{x^2 - 9}{x + 1} \), then \( g \) has how many real zeros?
   (a) 0, (b) 1, (c) 2, (d) 3, (e) none.

20. Which of the following \((a), (b), (c), \) or \( (d) \) is not a one-to-one function?
   (a) \((x,y): x + y = 4)\), (b) \((x,x^2): x \geq 0)\),
   (c) \((x,y): y = |x - 2|)\), (d) \((x,y): |y| = x \) and \( y \leq 0)\),
   (e) All of the preceding are one-to-one.

21. If \( f(x) = \frac{x + 2}{2x - 1} \), then the understood domain of \( f \) is:
   (a) \( x \neq -2, x \neq \frac{1}{2} \), (b) \( x \neq \frac{1}{2} \), (c) \( x \neq -2 \),
   (d) \( \mathbb{R} \), (e) none.

22. If \( g(x) = \sqrt{x^2 - 7x - 8} \), then the understood domain of \( g \) is:
   (a) \( [0,\infty] \), (b) \( [1,\infty] \), (c) \( [8,\infty] \), (d) \( \mathbb{R} \), (e) none.

23. If \( h(x) = 1 - x^2 \), then \( h(\sqrt{2}) \) equals:
   (a) 3, (b) -3, (c) 1, (d) -1, (e) none.

24. If \( f(x) = \sqrt{1 - 2x} \), then \( f(1 - 2x) \) equals:
   (a) \( \sqrt{4x - 1} \), (b) \( 1 - 2x \), (c) \( \sqrt{1 - 2x} \), (d) \( 1 - 4x^2 \),
   (e) none.
25. If \( g(x) = 1 + x \) and \( f(x) = x^2 \), then \( g(f(x)) \) equals:

(a) \( 1 + x^2 \),  
(b) \( (1 + x)^2 \),  
(c) \( x^2 + x^3 \),

(d) \( 1 + x \),  
(e) none.
Reduced Pace Test I
November 12, 1970

The items on Form B are the same as the items on Form A except for the order of appearance. The correspondence of items between the two forms is presented in the following table.

CORRESPONDENCE OF ITEMS BETWEEN FORMS
A AND B OF REDUCED PACE TEST ONE

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1. \([-1, 2) \cup (0, 3) = \) (a) \([-1, 0, 1, 2]\) (b) \([-1, 0, 1, 2, 3]\) (c) \((0, 2]\) (d) \([-1, 3]\) (e) None

2. Let \(N\) denote the set of natural numbers, \(Z\) the set of integers and \(Q\) the set of rational numbers. Which of the following is not true? (a) \(N \subseteq Q\) (b) \(Q \cup Z = Q\) (c) \(Z \cup N \subseteq N\) (d) \(N \subseteq Z\) (e) None

3. \((1 + 3\sqrt{2})^2 = \) (a) \(19 + 6\sqrt{2}\) (b) \(37\) (c) \(19\) (d) \(13 + 3\sqrt{2}\) (e) \(7 + 3\sqrt{2}\)

4. The sum of the integers in the set \((-8, 3) \cap [-2, 3]\) is: (a) \(0\) (b) \(1\) (c) \(2\) (d) \(3\) (e) None

5. \((\sqrt{x} + 3, x^2)^{5/6} = \) (a) \(5\sqrt{x}\) (b) \(x^5\) (c) \(x^{5/3}\) (d) \(x^{5/18}\) (e) None

6. \(\left(\frac{4x - 3y}{x^2 + y^2}\right)^{-1/2}\) equals: (a) \(\frac{v}{2x}\) (b) \(\frac{x}{2v}\) (c) \(\frac{x^2}{2y}\) (d) \(\frac{y}{2}\) (e) None

Let \(P(-1, 1), Q(5, 9), R(1, 4)\) and \(S(x, 8)\) be points in \(\mathbb{R}^2\).

Questions 7 - 9 refer to these points.

7. \(\overline{PQ} = \) (a) \(10\) (b) \(9\) (c) \(8\) (d) \(7\) (e) None

8. The equation of line \(PQ\) is: (a) \(4x - y = -5\) (b) \(3x - 4y = 1\) (c) \(x - 4y = -5\) (d) \(3x + 4y = 1\) (e) None
9. If the line through P and Q is parallel to the line through R and S, then x equals:
   (a) 3    (b) 3 1/2  (c) 4    (d) 4 1/2  (e) None
10. The slope of the line 3x = 2y - 1 is: (a) 3   (b) 2
     (c) 2/3   (d) 3/2   (e) None
11. The equation of the line through (1, -3) parallel to 5x = y - 1 is:
    (a) 5x - y = 2   (b) y = 5x + 8   (c) 5x + y = 2
    (d) 10x - 2y = 16   (e) None
12. Which of the following best represents \((x, y): y \leq 1 - 4x\)?
    (a) 
    (b) 
    (c) 
    (d) 
    (e) None
13. The sum of the solutions to \(|x + 10| = 12\) is:
    (a) 0   (b) 1   (c) 2   (d) 3   (e) None
14. The graph of \(y = (x + 1)^2 + 2\) is:
    (a) 
    (b) 
    (c) 
    (d) 
    (e) None
15. If \( f(x) = 8^x \), then \( f(-2/3) = (a) -16/3 \) (b) \( \frac{1}{4} \) (c) 1/4 (d) \( \frac{1}{4} \) (e) None

16. Which of the following sets is not a function:
   (a) \( \{(x, y) : y = |x+3| \} \) (b) \( \{(x, y) : |x-2y| \leq 0 \} \)
   (c) \( \{(x, y) : y = x^2 - 1 \} \) (d) \( \{(x, y) : y^2 = x - 1 \} \)
   (e) More than one of the above are not functions.

17. Which of the following best represents \( \{(x, y) : y = |x - 3| \} ? \)
   (a)  
   (b)  
   (c)  
   (d)  
   (e) None

Let \( F \) represent a \( Y \)-distortion by 2, \( G \) an \( X \)-translation by -5. Questions 18-20 refer to these mappings.

18. \( F(-6,8) = (a) (-6,16) \) (b) (-12,8) (c) (-6,4) (d) (-12,16) (e) None

19. The image of \( y = x^2 \) under \( G \) followed by \( F \) is:
   (a) \( y = 2(x-5)^2 \) (b) \( y = 2(x+5)^2 \) (c) \( y = 2x^2 - 5 \)
   (d) \( y = 2x^2 + 5 \) (e) None

20. \( G^{-1}(-10,15) = (a) (-5,15) \) (b) (-1/10,15) (c) (10,-15) (d) (-10,3) (e) None

Find the solution sets in Problems 21-26.

21. \( 2(x-3) \geq 3x-5 \). (a) \([-1, \infty]\) (b) \([-\infty, -1]\) (c) \([1, \infty]\)
   (d) \([-\infty, 1]\) (e) None
22. \((x + 2)(x - 7) \leq 0\).  
(a) \([7, \infty) \cup [-\infty, -2]\)  
(b) \([7, \infty) \cap [-\infty, -2]\)  
(c) \([-7, 2]\)  
(d) \([-2, 7]\)  
(e) None

23. \(|2x - 1| \leq 9\).  
(a) \([-4, 5]\)  
(b) \([-\infty, 5]\)  
(c) \([-9, \infty]\)  
(d) \([5, \infty]\)  
(e) None

24. \(x^2 + 3x \geq 10\).  
(a) \([2, \infty]\)  
(b) \([-5, 2]\)  
(c) \([-\infty, -5]\)  
(d) \([2, \infty]\)  
(e) None

25. \(|x + 3| > 5\).  
(a) \([2, \infty]\)  
(b) \([-\infty, -2]\)  
(c) \([8, \infty]\)  
(d) \(R\)  
(e) None

26. \(\frac{x - 1}{x + 3} > 2\).  
(a) \((-7, -3)\)  
(b) \((-\infty, -7)\)  
(c) \((-7, \infty)\)  
(d) \((-\infty, -3)\)  
(e) None

27. Let \(f(x) = 1 + x^2\) and \(g(t) = \sqrt{1 - t}\), then \(f(g(-8)) = \)  
(a) 7  
(b) 8  
(c) 9  
(d) 10  
(e) None

28. If \(f(x) = x^2 - 16\), then the sum of the zeros of \(f\) is:  
(a) 0  
(b) 2  
(c) 4  
(d) 6  
(e) None

29. Which of the following best represents \(\{(x, y): y = 3 + 2x\}\)?  
(a)  
(b)  
(c)  
(d)  
(e) None

30. If \(f(x) = (x + 1)^{-1/2}\), then the understood domain of \(f\) is:  
(a) \(R\)  
(b) \((0, \infty)\)  
(c) \(\{x: x \neq -1\}\)  
(d) \((-1, \infty)\)  
(e) None
31. If \( g(x) = \frac{1}{4-x} \), then \( g^{-1}(x) \) equals:
(a) 4 - x  (b) \( 4 - \frac{1}{x} \)  (c) 4 + x  (d) \( \frac{4x}{1-x} \)  (e) None

32. If \( h(x) = x^3 - x^2 - 3x \), then the sum of the zeros of \( h \) equals:
(a) 0  (b) 1  (c) 2  (d) 3  (e) None

33. Which of the following best represents \( \{(x, y): |x| = y + 3\} \)?

34. The understood domain of the function \( g(x) = \sqrt{x^2 - 2x} \) is:
(a) \([-\infty, 0] \cup [2, \infty]\)  (b) \([0, \infty]\)  (c) \([2, \infty]\)  (d) \(\mathbb{R}\)
(e) None

35. If \( f(x) = \frac{x+1}{x} \), then \( f(x-1) \) equals:
(a) \(\frac{x}{x-1}\)  (b) \(\frac{1}{x}\)  (c) \(\frac{x-1}{x}\)  (d) \(\frac{x}{x+1}\)  (e) None

36. If \( h(x) = 3x^3 - 2x - 1 \), then \( h^{-1}(h(x)) \) equals:
(a) \(3x^6 - 2x^2 - 1\)  (b) \(x\)  (c) \(x^2\)  (d) \(3x^3 - 2x - 1\)  (e) None

37. The understood domain of the function \( f(x) = \sqrt{\frac{x - 2}{|x + 1|}} \) is:
(a) \((-\infty, -1) \cup [2, \infty]\)  (b) \([x: x \neq -1]\)  (c) \([0, \infty]\)
(d) \([2, \infty]\)  (e) None

38. The sum of the coordinates of the minimum point on the graph of \( y = x^2 - 10x \) is:
(a) -25  (b) -20  (c) -15  (d) -10
(e) None
The items on Form B are the same as the items on Form A except for the order of appearance. The correspondence of items between the two forms is presented in the following table.

### CORRESPONDENCE OF ITEMS BETWEEN FORMS A AND B OF REDUCED PACE TEST TWO

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CRIMEL (Reduced Pace) Final Exam

December 15, 1970

FORM A

On the answer sheet provided, choose the one best response to each of the following questions by marking with PENCIL ONLY in the appropriate space after the question number. None means "None of the preceding".

1. \(([-2,3) \cap (3,6]) \cup (3,5] = \) (a) \(\emptyset\) (b) \((3,5]\) (c) \([3,5]\) (d) \([-2,6]\) (e) None

2. If \(f(z) = 4 - 2^{-z}\), then \(f(-2) = \) (a) 0 (b) 4 (c) 7 (d) 8 (e) None

3. The equation of the line containing \((-1, 2)\) and \((2, -1)\) is: (a) \(2x + y = 0\) (b) \(x + 2y = 0\) (c) \(y = x + 3\) (d) \(y + 3 = x\) (e) None

4. The sum of the solutions to the equation \(\log_3 |x| = 2\) is: (a) 0 (b) 8 (c) 9 (d) 3/2 (e) None

5. If \(P = (3, -1)\) and \(Q = (1, 5)\), then the distance \(PQ\) is: (a) \(4\sqrt{2}\) (b) \(2\sqrt{10}\) (c) \(2\sqrt{5}\) (d) \(2\sqrt{2}\) (e) None

6. If \(h(x) = 1 - x - x^2\), then the sum of the zeros of \(h\) is: (a) 1 (b) \(-1\) (c) 2 (d) \(-2\) (e) None

7. If \(\log_{14} \frac{1}{2} = \frac{1}{2}\), then \(x = \) (a) 2 (b) 4 (c) 8 (d) 16 (e) None

8. The slope of the line \(x = \frac{5}{4}y + 3\) is: (a) \(-\frac{5}{4}\) (b) \(\frac{5}{4}\) (c) \(\frac{4}{5}\) (d) \(-\frac{4}{5}\) (e) 3

9. If \(f(x) = x^2 + 1\) and \(g(x) = \sqrt{1-x}\), then \(f(g(-1)) = \) (a) 0 (b) 1 (c) 3 (d) Undefined (e) None
10. A composition of mappings under which \( y = 4(x+3)^2 \) is the image of \( y = x^2 \) is:

(a) An X-translation by 3 followed by an X-distortion of 4
(b) An X-translation by -3 followed by an X-distortion by 4
(c) An X-translation by 3 followed by a Y-distortion by \( k \)
(d) An X-translation by -3 followed by a Y-distortion by \( k \)
(e) None

11. Which of the following subsets of \( \mathbb{R}^2 \) is not a function?

(a) \( \{(x, y): x^2 + 1 = y\} \)
(b) \( \{(x, y): y = |x|\} \)
(c) \( \{(x, y): y = 3 - 6x\} \)
(d) \( \{(x, y): y = 2^{-x}\} \)
(e) All of the above are functions.

12. If \( f(x) = \frac{\sqrt{x+3}}{x+2} \), then the understood domain of \( f \) is:

(a) \([-3, \infty)\]
(b) \([-\infty, -3] \cup (-2, \infty)\]
(c) \([-3, -2] \cup (-2, \infty)\]
(d) \((-2, \infty)\)
(e) None

13. Which of the following best represents \( \{(x, y): 2x = 1 - y\} \)?

(a) \( \) \( \) \( \) \( \)
(b) \( \) \( \) \( \) \( \)
(c) \( \) \( \) \( \) \( \)
(d) \( \) \( \) \( \) \( \)
(e) None

14. \( (\sqrt{10} - \sqrt{2})^2 = \)

(a) 8
(b) 12
(c) \( 12 - 4\sqrt{5} \)
(d) \( 12 - \sqrt{20} \)
(e) None

15. The sum of all values for \( x \) for which the expression \( \frac{x - 1}{x^2 - 1} \) is undefined is:

(a) -1
(b) 0
(c) 1
(d) 2
(e) None
16. \( \left( \frac{(x+y)^{-1}}{(x+y)^2} \right)^{-2/3} = \) (a) \( x^2 + 2xy + y^2 \)  (b) \( (x+y)^{1/3} \)  
(c) \( (x+y)^{3/2} \)  (d) \( x^2 + y^2 \)  
(e) None

17. Which of the following best represents \( \{(x, y): y - 1 = |x - 1|\} \)?

(a) \[\begin{array}{c}
1 \\
| |
\end{array}\]
(b) \[\begin{array}{c}
1 \\
/\n\end{array}\]
(c) \[\begin{array}{c}
1 \\
\setminus
\end{array}\]
(d) \[\begin{array}{c}
1 \\
/\n\end{array}\]
(e) None

18. Which of the following is not a one-to-one function?

(a) \( f(x) = \sqrt{x} \)  (b) \( g(x) = \log_3(x^2 + 1) \)  (c) \( h(x) = 2^{-3x} \)  
(d) \( r(x) = 2x + |x| \)  (e) None of the above are one-to-one functions.

19. If \( f(x) = \sqrt{1-x} \), then \( f(1-x) = \) (a) \( \sqrt{1-x} \)  (b) \( (1-x)^2 \)  
(c) \( 1-x \)  (d) \( \sqrt{x} \)  (e) None

Find the solution sets in Questions 20 - 24.

20. \( (x+1)(x-3) \geq x(x-1) \):  
(a) \([-\infty, -3]\)  (b) \([3, \infty]\)  
(c) \([-1, 1]\) \( U \) \([3, \infty]\)  (d) \([-\infty, -1]\) \( U \) \([1, 3]\)  (e) None

21. \( x^2 + x > 12 \):  
(a) \( \mathbb{R} \)  (b) \((-\infty, -3]\) \( U \) \([4, \infty]\)  (c) \((-4, 3]\)  
(d) \((-\infty, -4]\) \( U \) \([3, \infty]\)  (e) None

22. \( |x-4| < 10 \):  
(a) \((-\infty, 6]\)  (b) \((-\infty, 14) \)  (c) \((-6, 14) \)  
(d) \((-14, 6) \)  (e) None

23. \( (x-1)|x+4| \leq 0 \):  
(a) \([-4, 1]\)  (b) \([-\infty, 1]\)  
(c) \([-\infty, -4]\) \( U \) \([1, \infty]\)  (d) \([-\infty, -4]\)  (e) None
24. \( \log_2 \sqrt{x+5} - \log_2 \sqrt{x-1} = \frac{1}{2} \) :  
(a) \([-5, 1]\)  (b) \([-1, 3]\)  (c) \([3, 7]\)  (d) \([7, \infty)\)  (e) None

Let \( F \) represent an X-distortion by \(-2\) \( G \) a Y-reflection and \( H \) a Y-translation by \(-1\). Questions 25-27 refer to these mappings.

25. \( F(8, 10) : \)  
(a) \((8, 20)\)  (b) \((16, 20)\)  (c) \((16, 10)\)  (d) \((4, 5)\)  (e) None

26. The image of \( y = x \) under \( G \) followed by \( H \) is:
(a) \( x+y = -1 \)  (b) \( x-y = 1 \)  (c) \( y-x = 1 \)  (d) \( x+y = 1 \)  (e) None

27. If \( H^{-1} \) denotes the inverse of the mapping \( H \), then \( H^{-1}(-2, 3) \) =  
(a) \((3, -2)\)  (b) \((2, -3)\)  (c) \((-2, 2)\)  (d) \((-2, 4)\)  (e) None

28. Which of the following best represents \( \{(x, y) : 3x - y \geq -2\} \)?
(a) \( \)  (b) \( \)  (c) \( \)  (d) \( \)  (e) None

29. If \( f(x) = 3 + 2^x \), then the range of \( f \) is:  
(a) \((3, \infty)\)  (b) \([0, \infty)\)  (c) \(\mathbb{R}\)  (d) \((1, \infty)\)  (e) None

30. Which of the following greetings is most appropriate to the season?  
(a) Happy Birthday  (b) WOW  (c) *$!----?/$*  (d) Hi  (e) Merry X-mas
31. Which of the following best represents \( \{ (x, y) : y = (x+2)^2 - 3 \} \)?

(a) \( \)
(b) \( \)
(c) \( \)
(d) \( \)
(e) None

32. If \( g(x) = \log_2(1-x) \), which of the following is defined?

- (a) \( g(1) \)
- (b) \( g(3) \)
- (c) \( g(0) \)
- (d) \([g(2)]^2 \)
- (e) All are defined.

33. If \( f(x) = x^2 - 1 \) with \( x \geq 0 \), then \( f^{-1}(x) = \)

- (a) \( \sqrt{x} + 1 \)
- (b) \( 1-x^2 \)
- (c) \( \sqrt{x+1} \)
- (d) \( \frac{1}{x^2 - 1} \)
- (e) None

34. The understood domain of the function \( h(x) = \log |x-1| \) is:

- (a) \( (0, \infty) \)
- (b) \( (-1, \infty) \)
- (c) \( \mathbb{R} \)
- (d) \( (-1, \infty) \)
- (e) None

35. Which of the following best represents \( \{(x, y) : y = \left(\frac{1}{2}\right)^x \} \)?

(a) \( \)
(b) \( \)
(c) \( \)
(d) \( \)
(e) None
36. The understood domain of the function \( f(x) = \sqrt[3]{\frac{x-1}{x}} \) is:
   (a) \([1, \infty]\)  (b) \((0, \infty)\)  (c) \((-\infty, 0) \cup [1, \infty]\)  (d) \((0, 1]\)
   (e) None

37. Which of the following best represents \( \{(x, y) : y = \log_2(x+3)\} \)?
   (a)  
   (b)  
   (c)  
   (d)  
   (e) None

38. The understood domain of the function \( g(x) = \sqrt{\log x} \) is:
   (Hint: Look at the graph of \( y = \log x \).)  (a) \([1, \infty]\)
   (b) \((0, \infty)\)  (c) \(\mathbb{R}\)  (d) \((0, 1]\)  (e) None
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Final Exam

Math 115.02 FORM A
Winter 1971

On the answer sheet provided, choose the one best response to each of the following questions by marking with PENCIL ONLY in the appropriate space after the question number. None means "None of the preceding".

1. The distance between \( P(-1, -2, -3) \) and \( Q(1, 2, -3) \) is:
   (a) 0  (b) 4  (c) \( 2\sqrt{14} \)  (d) \( 2\sqrt{7} \)  (e) None

2. The components of \( \overrightarrow{PQ}, \) \( P(-1, -2, 3) \) and \( Q(1, 2, -3) \) are:
   (a) \((-6, 6, 6)\)  (b) \((6, 6, 6)\)  (c) \((-6, -6, 6)\)
   (d) \((6, 6, -6)\)  (e) None

3. If \( \overrightarrow{ST} \) has components \((-3, 1)\) with \( T(1, 2) \) then \( S \) has coordinates:  (a) \((-2, 3)\)  (b) \((-4, -1)\)  (c) \((4, 1)\)
   (d) \((2, -3)\)  (e) None

4. The equation of the line in \( \mathbb{R}^3 \) which passes through \( P(1, 0, 1) \) and \( Q(3, -2, 1) \) is:  (a) \( \frac{x-1}{2} = \frac{y}{-2} = \frac{z-1}{0} \)  (b) \( \frac{z-1}{2} = \frac{y}{-2} \) and \( z = 1 \)
   (c) \( \frac{x-1}{3} = \frac{y}{-2} = \frac{z-1}{1} \)  (d) \( \frac{x-3}{1} = \frac{y+2}{2} = \frac{z-1}{1} \)
   (e) None

5. The equation of the plane equidistant from the points \( P(1, 0, 1) \) and \( Q(3, -2, 1) \) is:  (a) \( x+y = 3 \)  (b) \( x-y = 3 \)
   (c) \( x+y = -3 \)  (d) \( x-y = -3 \)  (e) None

6. The center and radius of the sphere \( x^2 + (y-1)^2 + (z+3)^2 = 4 \) are:  (a) \((0, -1, 3), 4\)  (b) \((0, -1, 3), 2\)
   (c) \((0, 1, -3), 16\)  (d) \((0, 1, -3), 2\)  (e) None
7. The matrix \( A = (a_{ij})_{3 \times 3} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \).

The set \( \{a_{ij}\}_{i=1, j=1}^{3, 2} \) is:

(a) \( \{1, 2, 3, 4, 5, 6\} \)
(b) \( \{1, 4, 7, 2, 5, 8\} \)
(c) \( \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \)
(d) cannot be determined
(e) None

8. \( \sum_{i=1}^{3} \sum_{j=1}^{4} i \cdot j = \begin{cases} (a) 20 & (b) 42 & (c) 48 \end{cases} \) (d) 56

(e) None

9. If \( A = (a_{ij})_{3 \times 4} \) and \( a_{ij} = i^2 \cdot j \) then \( A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 8 & 12 & 16 \\ 9 & 18 & 27 & 36 \\ 16 & 32 & 48 & 64 \end{pmatrix} \)

(a) \( \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 8 & 12 & 16 \\ 9 & 18 & 27 & 36 \end{pmatrix} \)
(b) \( \begin{pmatrix} 1 & 4 & 9 & 16 \\ 2 & 8 & 18 & 32 \\ 3 & 12 & 27 & 48 \end{pmatrix} \)
(c) \( \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 8 & 12 & 16 \end{pmatrix} \)
(d) \( \begin{pmatrix} 1 & 4 & 9 & 16 \\ 2 & 8 & 18 & 32 \end{pmatrix} \)

(e) None

10. The graph in \( \mathbb{R}^3 \) of \( x+y = 3 \) is:

(a) \( \begin{pmatrix} \end{pmatrix} \)
(b) \( \begin{pmatrix} \end{pmatrix} \)
(c) \( \begin{pmatrix} \end{pmatrix} \)
(d) \( \begin{pmatrix} \end{pmatrix} \)

(e) \( \begin{pmatrix} \end{pmatrix} \)
11. In \[
\begin{pmatrix}
1 & 4 & 1 & 2 \\
0 & 3 & 0 & 1 \\
2 & 2 & 2 & 1 \\
5 & 1 & 4 & 3 \\
\end{pmatrix}
\]
the minor \( A_{31} \) is: 
(a) \( \begin{pmatrix} 0 & 3 & 1 \\ 2 & 2 & 1 \\ 5 & 1 & 3 \end{pmatrix} \)

(b) \( \begin{pmatrix} 0 & -3 & 1 \\ -2 & 2 & -1 \\ 5 & -1 & 3 \end{pmatrix} \)

(c) \( \begin{pmatrix} 4 & 1 & 2 \\ 3 & 0 & 1 \\ 1 & 4 & 3 \end{pmatrix} \)

(d) \( \begin{pmatrix} 4 & -1 & 2 \\ -3 & 0 & -1 \\ 1 & 4 & 3 \end{pmatrix} \)

(e) None

12. In the set of 2x3 matrices, the additive inverse of 
\[
\begin{pmatrix}
2 & 0 & 3 \\
-1 & 2 & 2 \\
\end{pmatrix}
\]
is: 
(a) \( \begin{pmatrix} -2 & 0 & -3 \\ 1 & -2 & -2 \end{pmatrix} \)

(b) \( \begin{pmatrix} -1 & 2 & 2 \\ 2 & 0 & 3 \end{pmatrix} \)

(c) \( \begin{pmatrix} -2 & 1 \\ 0 & 2 \\ -3 & -2 \end{pmatrix} \)

(d) \( \begin{pmatrix} 2 & -1 \\ 0 & 2 \\ 3 & 2 \end{pmatrix} \)

(e) None

13. The multiplicative inverse of \( \begin{pmatrix} 2 & 1 & 3 \end{pmatrix} \) is: 
(a) \( \begin{pmatrix} \frac{1}{2} & -1 \\ -1 & 2 \\ \frac{1}{2} \end{pmatrix} \)

(b) \( \begin{pmatrix} -2 & 1 & -3 \\ 1 & -2 & -2 \end{pmatrix} \)

(c) \( \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \)

(d) does not exist

(e) None

14. \( \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \end{pmatrix} \) = 
(a) cannot be multiplied

(b) \( \begin{pmatrix} -3 & 2 & 7 \\ -5 & 4 & 13 \end{pmatrix} \)

(c) \( \begin{pmatrix} 4 & 2 & 8 \\ -5 & 3 & 11 \end{pmatrix} \)

(d) \( \begin{pmatrix} 0 & 3 & 2 \\ 4 & 0 & 1 \end{pmatrix} \)

(e) None
15. In the product \[
\begin{pmatrix}
1 & 0 & -1 & 2 \\
3 & -1 & 4 & 7 \\
1 & 1 & 1 & 1 \\
-1 & 2 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
3 & 4 & 1 & 2 \\
-5 & 1 & 1 & 0 \\
0 & 2 & 2 & 1 \\
3 & 6 & -6 & 0
\end{pmatrix}
\]
, the entry in the 3rd row, 4th column will be: (a) 3 (b) 1 (c) -3 (d) -1 (e) None

16. If
\[A = \begin{pmatrix}
1 & 2 \\
3 & 4 \\
-7 & -10 \\
-15 & -22
\end{pmatrix}
\]
then \(A \cdot A^{-1}\) = (a) \(\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}\) (b) \(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\) (c) (d) A has no inverse (e) None

17. The determinant of
\[
\begin{pmatrix}
1 & 2 \\
5 & 6
\end{pmatrix}
\]
is: (a) 0 (b) -1 (c) -4 (d) -3 (e) None

18. The determinant of
\[
\begin{pmatrix}
1 & 2 & 3 \\
1 & 2 & 3 \\
4 & 1 & -2
\end{pmatrix}
\]
is: (a) 0 (b) -1 (c) -4 (d) -3 (e) None

19. The determinant of
\[
\begin{pmatrix}
5 & 4 & 0 & 2 \\
1 & 0 & 0 & 0 \\
7 & 9 & 1 & 12 \\
3 & 4 & 0 & -2
\end{pmatrix}
\]
is: (a) 0 (b) 16 (c) -16 (d) -12 (e) 12

20. In the adjoint matrix of
\[
\begin{pmatrix}
1 & 2 & -1 \\
3 & 0 & 4 \\
1 & 2 & 3
\end{pmatrix}
\]
, the entry in the 2nd row, 1st column is: (a) 3 (b) 8 (c) -8 (d) 5 (e) None
21. The inverse of
\[
\begin{pmatrix}
2 & 0 & 0 \\
0 & 1 & 0 \\
0 & 1 & 3
\end{pmatrix}
\]
is: (a) \[
\begin{pmatrix}
\frac{1}{2} & 0 & 0 \\
0 & 1 & \frac{1}{3} \\
0 & 0 & 1
\end{pmatrix}
\]
(b) \[
\begin{pmatrix}
\frac{1}{2} & 0 & 0 \\
0 & 1 & \frac{1}{3} \\
0 & 0 & 1
\end{pmatrix}
\] (c) \[
\begin{pmatrix}
\frac{1}{2} & 0 & 0 \\
0 & 1 & \frac{1}{3} \\
0 & 0 & 1
\end{pmatrix}
\]
does not have an inverse (e) None

22. The inverse of
\[
\begin{pmatrix}
1 & 0 & -1 \\
2 & 0 & 3 \\
3 & 0 & 7
\end{pmatrix}
\]
is: (a) \[
\begin{pmatrix}
\frac{1}{10} & 0 & \frac{1}{15} \\
\frac{1}{10} & 0 & \frac{1}{15} \\
\frac{1}{10} & 0 & \frac{1}{15}
\end{pmatrix}
\]
(b) \[
\begin{pmatrix}
0 & 0 & 0 \\
-5 & 10 & -5 \\
0 & 0 & 0
\end{pmatrix}
\] (c) \[
\begin{pmatrix}
0 & -5 & 0 \\
0 & 10 & 0 \\
0 & -5 & 0
\end{pmatrix}
\]
does not have an inverse (e) None

23. The matrix of direct paths is:
(a) \[
\begin{pmatrix}
0 & 1 & 1 \\
1 & 0 & 2 \\
1 & 2 & 0
\end{pmatrix}
\]
(b) \[
\begin{pmatrix}
0 & 1 & 1 \\
1 & 0 & 2 \\
1 & 1 & 0
\end{pmatrix}
\]
(c) \[
\begin{pmatrix}
0 & 1 & 1 \\
1 & 0 & 1 \\
1 & 2 & 0
\end{pmatrix}
\]
does not have an inverse (e) None

24. If
\[
\begin{pmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
1 & 0 & 0
\end{pmatrix}
\]
is a matrix of direct paths, then the matrix of
3-stage paths is: (a) \[
\begin{pmatrix}
0 & 3 & 0 \\
0 & 0 & 3 \\
3 & 0 & 0
\end{pmatrix}
\]
(b) \[
\begin{pmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1
\end{pmatrix}
\]
24. (c) \[
\begin{pmatrix}
0 & 0 & 1 \\
1 & 0 & 0 \\
0 & 1 & 0 \\
\end{pmatrix}
\] (d) \[
\begin{pmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
1 & 0 & 0 \\
\end{pmatrix}
\] (e) None

25. If \[
x + y + z = 6
\]
and if the adjoint of \[
\begin{pmatrix}
1 & 1 & 1 \\
1 & -1 & 2 \\
-3 & 1 & -1 \\
\end{pmatrix}
\]
is \[
\begin{pmatrix}
-1 & 2 & 3 \\
7 & 2 & -1 \\
-2 & \frac{1}{2} & -2 \\
\end{pmatrix}
\] then \(x, y\) and \(z\) are: (a) 2, 2, 2 (b) -2, -2, -2 (c) 2, -2, 2 (d) -2, 2, -2 (e) None

26. The set of equations \(y + z = 6\) could be written as the matrix equation
\[
\begin{pmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
0 & 1 & 1 \\
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
z \\
\end{pmatrix}
= \begin{pmatrix}
3 \\
-2 \\
1 \\
\end{pmatrix}
\]
(a) \[
\begin{pmatrix}
1 & 1 & 0 \\
0 & 1 & 1 \\
0 & 0 & 1 \\
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
z \\
\end{pmatrix}
= \begin{pmatrix}
3 \\
-2 \\
1 \\
\end{pmatrix}
\]
(b) \[
\begin{pmatrix}
1 & -1 & 0 \\
2 & 1 & 1 \\
0 & 0 & 1 \\
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
z \\
\end{pmatrix}
= \begin{pmatrix}
3 \\
0 \\
1 \\
\end{pmatrix}
\]
(c) \[
\begin{pmatrix}
1 & -1 & 0 \\
0 & 1 & 1 \\
0 & 0 & 1 \\
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
z \\
\end{pmatrix}
= \begin{pmatrix}
3 \\
-2x \\
1 \\
\end{pmatrix}
\]
(d) \[
\begin{pmatrix}
1 & 1 & 3 \\
-2 & 1 & 1 \\
0 & 0 & 1 \\
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
z \\
\end{pmatrix}
= \begin{pmatrix}
0 \\
0 \\
1 \\
\end{pmatrix}
\]
(e) None

27. The function \(a\) induced by matrix \(A = \begin{pmatrix}
-1 & 2 & 3 \\
1 & 1 & 1 \\
\end{pmatrix}\) is:
(a) \(a(x, y) = (-x + y, 2x + y)\) (b) \(a(x, y, z) = (-x + 2y + 3z, x + y + z)\)
(c) \(a(x, y) = (-x + 2y, x + y, 3x + y)\)
(d) \(a(x, y) = (-x + y, 2x + y, 3x + y)\) (e) None
28. The geometric transformation induced by matrix $A = \begin{pmatrix} \frac{1}{2} & 1 \\ 1 & \frac{1}{2} \end{pmatrix}$
is:  
(a) A projection onto the line $y = \frac{1}{2}x$  
(b) A rotation of 180 degrees  
(c) A reflection through the line $y = x$  
(d) A projection onto the line $y = x$  
(e) None

29. The matrix that induces $a(x, y) = (3x - y, x + y, x + 2y)$ is:
(a) $\begin{pmatrix} 3 & 1 & 1 \\ -1 & 1 & 2 \end{pmatrix}$  
(b) $\begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix}$  
(c) $\begin{pmatrix} 3 & 1 & 1 \\ -1 & 1 & 2 \\ 1 & 2 \end{pmatrix}$  
(d) does not exist.  
(e) None

30. The matrix that induces $a(x, y) = (x + 2, y)$ is:
(a) $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$  
(b) $\begin{pmatrix} 1 & 0 \\ \frac{2}{y} & 1 \end{pmatrix}$  
(c) $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$  
(d) does not exist.  
(e) None
The items on Form B of this test are the same as the items on Form A, including the order of appearance of the items. Differences occur only in the order of appearance of the multiple choice responses.
1. The fifth term in the set given by \( \left\{ \frac{1}{k^2 - 1} \right\}_{k=3}^{10} \) is:

(a) \( \frac{1}{63} \), (b) \( \frac{1}{48} \), (c) \( \frac{1}{35} \), (d) \( \frac{1}{24} \),

(e) None of these.

2. \( \sum_{i=1}^{4} (2i + 1) = \)

(a) 28, (b) 24, (c) 20, (d) 18, (e) None of these.

3. The matrix \( (c_{ij})_{2x3} \) where \( c_{ij} = j - i \) is:

(a) \( \begin{pmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \end{pmatrix} \), (b) \( \begin{pmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \end{pmatrix} \),

(c) \( \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \end{pmatrix} \),

(d) \( \begin{pmatrix} 0 & -1 & -2 \\ -1 & 0 & -1 \end{pmatrix} \), (e) None of these.

4. If \( \begin{pmatrix} 1 & 2 & -1 \\ 3 & 1 & 0 \end{pmatrix} + A = \begin{pmatrix} 2 & 1 & 5 \\ -1 & 4 & 3 \end{pmatrix} \), then \( A \) is:

(a) \( \begin{pmatrix} 1 & -1 & 6 \\ -4 & 3 & 3 \end{pmatrix} \), (b) \( \begin{pmatrix} 3 & 3 & 4 \\ 2 & 5 & 3 \end{pmatrix} \),

(c) \( \begin{pmatrix} 0 & 2 & 1 \\ 0 & 1 & 0 \end{pmatrix} \)

(d) Not defined, (e) None of these.

5. Let \( A = \begin{pmatrix} 1 & 3 & -2 \\ 0 & 1 & 4 \end{pmatrix} \) and \( B = \begin{pmatrix} -1 & 0 \\ -3 & 1 \\ 2 & 1 \end{pmatrix} \), then

(a) \( A + B \) and \( A \cdot B \) both exist.

(b) \( A + B \) exists but \( A \cdot B \) is not defined.

(c) \( A + B \) is not defined but \( A \cdot B \) exists.

(d) Neither \( A + B \) nor \( A \cdot B \) is defined.

(e) None of these.
6. The additive inverse of \( \begin{pmatrix} 2 & -1 & 3 \\ -1 & 4 & 0 \end{pmatrix} \) is:

(a) \( \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \),
(b) \( \begin{pmatrix} 2 & -1 & 3 \\ 1 & 4 & 0 \end{pmatrix} \),
(c) \( \begin{pmatrix} -2 & 1 & -3 \\ -1 & 4 & 0 \end{pmatrix} \),
(d) \( \begin{pmatrix} -2 & 1 & 3 \\ -1 & 4 & 0 \end{pmatrix} \),
(e) None of these.

7. If \( A = \begin{pmatrix} 1 & -1 & 3 \\ 0 & 1 & 2 \\ 1 & 4 & 1 \end{pmatrix} \) then \( |A_{23}| \) is:

(a) -5, (b) -1, (c) -13, (d) 5 ,
(e) None of these.

8. The matrix equation representing the system \( \frac{2x - y + z = 4}{y + 2x - 3z = 3} \) is:

\( \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix} \)

(a) \( \begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 1 & -1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix} \),
(b) \( \begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 1 & -1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix} \),
(c) \( \begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 1 & -1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix} \),
(d) \( \begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 1 & -1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix} \),
(e) None of these.

9. The projection of points onto the line \( y = 0 \) can be given by which of the following matrices.

(a) \( \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \),
(b) \( \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \),
(c) \( \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \),
(d) \( \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \),
(e) None of these.

10. If \( A = (a_{ij})_{3x3} \), which is the following is not a real number.

(a) \( |A| \),
(b) \( |A_{11}| \),
(c) \( |\text{adj } A| \),
(d) \( |a_{11}| \),
(e) All of the above are real numbers.
11. \[
\begin{pmatrix}
2 & 1 \\
-1 & 2
\end{pmatrix}
\begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix} = 
\]
(a) \[
\begin{pmatrix}
1 & 0 \\
2 & 1
\end{pmatrix}
\]
(b) \[
\begin{pmatrix}
2 & 1 \\
-1 & 2
\end{pmatrix}
\]
(c) \[
\begin{pmatrix}
-1 & 2 \\
2 & -1
\end{pmatrix}
\]

12. \[
\begin{pmatrix}
3 & -1 \\
1 & 0
\end{pmatrix}^{-1} = 
\]
(a) \[
\begin{pmatrix}
0 & 1 \\
-1 & 3
\end{pmatrix}
\]
(b) \[
\begin{pmatrix}
-3 & 1 \\
-1 & 0
\end{pmatrix}
\]
(c) \[
\begin{pmatrix}
1/3 & -1 \\
1 & 0
\end{pmatrix}
\]
(d) \[
\begin{pmatrix}
1/3 & 1/3 \\
0 & 1
\end{pmatrix}
\]
(e) None of these.

13. \[
\begin{vmatrix}
1 & 0 & 2 \\
0 & 2 & 1 \\
-1 & 1 & 0
\end{vmatrix} = 
\]
(a) 5, (b) 1, (c) -6, (d) 3,
(e) None of these.

In questions 14 and 15 let the matrices \( A_{2\times3} \), \( B_{3\times3} \), \( C_{3\times2} \), \( D_{2\times2} \), \( E_{2\times2} \) be given.

14. Which of the following exist?
(a) \( |A| \) (b) \( D \cdot C \) (c) \( A^2 \) (d) \( B + C \)
(e) None of these.

15. Which is the following statement is true?
(a) \( D \cdot E = E \cdot D \) (b) \( A(B \cdot C) = (A \cdot B)C \) (c) \( A \cdot B \in M_{3\times2} \)
(d) \( C^{-1} \in M_{2\times3} \) (e) All are true.

16. The matrix which counts the two-stage or three stage paths in the network \( A \) \( \rightarrow \) \( B \) is:
(a) \[
\begin{pmatrix}
0 & 3 \\
3 & 10
\end{pmatrix}
\]
(b) \[
\begin{pmatrix}
0 & 3 \\
3 & 1
\end{pmatrix}
\]
(c) \[
\begin{pmatrix}
9 & 30 \\
30 & 19
\end{pmatrix}
\]
(d) \[
\begin{pmatrix}
18 & 33 \\
33 & 29
\end{pmatrix}
\]
(e) None of these.
17. If $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}$, $C = \begin{pmatrix} -1 & 1 \\ 2 & -2 \end{pmatrix}$, then $|B| A^{-1} C =$

(a) $2 \begin{pmatrix} 2 & -2 \\ -1 & 1 \end{pmatrix}$  
(b) $-2 \begin{pmatrix} -1 & 1 \\ 2 & 2 \end{pmatrix}$  
(c) $2 \begin{pmatrix} 1 & -1 \\ -2 & 2 \end{pmatrix}$  
(d) $\begin{pmatrix} 4 & -4 \\ 2 & -2 \end{pmatrix}$  
(e) None of these.

18. The sum of the numbers $x$ and $y$ which satisfy

$\begin{pmatrix} 2 & 1 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 2 \end{pmatrix}$ is:

(a) 4,  
(b) -4,  
(c) -6,  
(d) 6,  
(e) None of these

19. Which of the following is not a linear transformation?

(a) A reflection through the line $y = 2x$,  
(b) $f(x, y) = (x-1, y+2)$,  
(c) $f(x, y) = (x, y)\begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$,  
(d) Y-distortion by 2,  
(e) All are linear transformations.

20. Let $g$ be the function induced by the matrix $\begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix}$ and $f(x, y) = (x+y, x-y)$. Find $f(g(1, 2))$.

(a) $(7, -3)$,  
(b) $(1, 1)$,  
(c) $(-2, 5)$,  
(d) $(3, -7)$,  
(e) None of these.

21. The sum of the values of $s$ and $t$ which satisfy the equation $(5, 4) - 2(t, s) = (7, t+2)$ is:

(a) -1,  
(b) 1,  
(c) 0,  
(d) -2,  
(e) None of these.

In Questions 22-24, let $P : (-2, 3)$; $Q : (1, -1)$; $S : (1, 2)$; $X : (x, y)$; and $O : (0, 0)$.

22. If $X$ is 1/3 of the distance from $P$ to $Q$ then the coordinates of $X$ are:

(a) $(-1, 5/3)$,  
(b) $(1, 4/3)$,  
(c) $(-1, 2)$,
23. The coordinates of $X$ so that $\overline{PX} = \overline{QS} + 2\overline{PS}$ are:
(a) $(4, 4)$,  (b) $(1, 3)$,  (c) $(5, 5)$,  (d) $(1, 5)$,
(e) None of these.

24. The length of $\overline{OX} = \overline{PQ} + \overline{QS}$ is:
(a) $3 + \sqrt{5}$,  (b) $8$,  (c) $\sqrt{34}$,  (d) $10$,
(e) None of these.

In Questions 25-27 let $P : (1, 2, 3)$ and $Q : (3, 0, -1)$.

25. The midpoint of $PQ$ is:
(a) $(4, 2, 2)$,  (b) $(-2, 2, 4)$,  (c) $(2, 1, 1)$,
(d) $(2, -2, -4)$,  (e) None of these.

26. The distance between $P$ and $Q$ is:
(a) $\sqrt{24}$,  (b) $\sqrt{36}$,  (c) $\sqrt{8}$,  (d) $\sqrt{10}$,
(e) None of these.

27. The equation of the plane equidistant from $P$ and $Q$ is:
(a) $(x - 1)^2 + (y - 2)^2 + (z - 3)^2 = 6$ ,
(b) $4x - 4y - 8z + 4 = 0$ ,
(c) $x^2 + y^2 + z^2 = 36$ ,
(d) $2x + 4y + 8z = 10$ ,
(e) None of these.

28. The best sketch of the graph of $2x + 3y + z = 6$ in the first octant is:
(a) \[ \text{Graph (a)} \]  (b) \[ \text{Graph (b)} \]
29. \( \sum_{i=1}^{3} \sum_{j=3}^{5} (2i - j) = \)

(a) 4, (b) -14, (c) 0, (d) -7, (e) None of these.

30. Let \( A_{3 \times 3} \) be given. If \( B = A^2 \), then \( b_{22} = \)

(a) \( \sum_{i=1}^{3} (a_{2i})^2 \), (b) \( \frac{3}{3} (a_{i2})^2 \), (c) \( \left( \sum_{i=1}^{3} a_{2i} \right)^2 \),

(d) \( \sum_{i=1}^{3} a_{2i} \cdot a_{i2} \), (e) None of these.
Mathematics 115.02 (Regular)  Final Exam  Autumn, 1970

Directions: Select the one best response - a, b, c, d or e, - to the following questions.

1. The fifth term in the set given by \( \left\{ \frac{1}{k^2 - 1} \right\}_{k=3}^{10} \) is

   (a) \( \frac{1}{63} \),  (b) \( \frac{1}{48} \),  (c) \( \frac{1}{35} \),  (d) \( \frac{1}{24} \)

   (e) None of these.

2. \( \sum_{i=1}^{4} (2i + 1) = \)

   (a) 28  (b) 24  (c) 20  (d) 18  (e) None of these.

3. The matrix \( (c_{ij})_{2\times3} \) where \( c_{ij} = |j-i| \) is:

   (a) \( \begin{pmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \end{pmatrix} \)  (b) \( \begin{pmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \end{pmatrix} \)  (c) \( \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \end{pmatrix} \)

   (d) \( \begin{pmatrix} 0 & -1 & -2 \\ -1 & 0 & -1 \end{pmatrix} \)  (e) None of these.

4. If \( \begin{pmatrix} 1 & 2 & -1 \\ 3 & 1 & 0 \end{pmatrix} + A = \begin{pmatrix} 2 & 1 & 5 \\ -1 & 4 & 3 \end{pmatrix} \), then \( A \) is:

   (a) \( \begin{pmatrix} 1 & -1 & 6 \\ -4 & 3 & 3 \end{pmatrix} \)  (b) \( \begin{pmatrix} 3 & 3 & 4 \\ 2 & 5 & 3 \end{pmatrix} \)  (c) \( \begin{pmatrix} 0 & 2 & 1 \\ 1 & 0 & 2 \end{pmatrix} \)

   (d) Not defined  (e) None of these.

5. Let \( A = \begin{pmatrix} 1 & 3 & -2 \\ 0 & 1 & 4 \end{pmatrix} \) and \( B = \begin{pmatrix} -3 & 1 \\ 2 & 1 \end{pmatrix} \), then

   (a) \( A + B \) and \( A \cdot B \) both exist.

   (b) \( A + B \) exists but \( A \cdot B \) is not defined.

   (c) \( A + B \) is not defined but \( A \cdot B \) exists.

   (d) Neither \( A + B \) nor \( A \cdot B \) is defined.

   (e) None of these.
6. The additive inverse of \( \begin{pmatrix} 2 & -1 & -3 \\ 1 & 4 & 0 \end{pmatrix} \) is:

(a) \( \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \)
(b) \( \begin{pmatrix} 2 & -1 & 3 \\ 1 & 4 & 0 \end{pmatrix} \)
(c) \( \begin{pmatrix} -2 & 1 & -3 \\ 1 & 4 & 0 \end{pmatrix} \)
(d) \( \begin{pmatrix} -2 & 1 & 3 \\ -1 & 4 & 0 \end{pmatrix} \)
(e) None of these

7. If \( A = \begin{pmatrix} 1 & -1 & 3 \\ 0 & 1 & 2 \\ 1 & 4 & 1 \end{pmatrix} \) then \( |A_{13}| \) is:

(a) -5  
(b) -1  
(c) -13  
(d) 5  
(e) None of these

8. The matrix equation representing the system
\[
\begin{align*}
2x - y + z &= 4 \\
y + 2x - 3z &= 3 \\
x - z + 2y &= 5
\end{align*}
\]
is:

(a) \( \begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 1 & -1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix} \)
(b) \( \begin{pmatrix} 2 & 1 & -3 \\ 1 & 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} \)
(c) \( \begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 1 & -1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix} \)
(d) \( \begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \)
(e) None of these.

9. The projection of points onto the line \( y = 0 \) can be given by which of the following matrices.

(a) \( \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \)
(b) \( \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \)
(c) \( \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \)
(d) \( \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \)
(e) None of these.

10. If \( A = (a_{ij})_{3x3} \), which of the following is not a real number.

(a) \( |A| \)
(b) \( |A_{11}| \)
(c) \( \text{adj } A \)
(d) \( a_{11} \)
(e) All of the above are real numbers.
11. \[
\begin{pmatrix}
2 & 1 & 1 & 0 & 2 \\
1 & 2 & 0 & -1 & 1
\end{pmatrix}
\]

(a) \[
\begin{pmatrix}
-1 & -2 & 0 \\
2 & 1 & 5
\end{pmatrix}
\]
(b) \[
\begin{pmatrix}
2 & -1 & 5 \\
-1 & -2 & 0
\end{pmatrix}
\]
(c) \[
\begin{pmatrix}
-1 & 2 \\
-2 & -1 \\
0 & 5
\end{pmatrix}
\]

(d) Does not exist  
(e) None of these.

12. \[
\begin{pmatrix}
3 & -1 & -1 \\
1 & 0 & 0
\end{pmatrix}
\]

(a) \[
\begin{pmatrix}
0 & 1 \\
-1 & 3
\end{pmatrix}
\]
(b) \[
\begin{pmatrix}
-3 & 1 \\
-1 & 0
\end{pmatrix}
\]
(c) \[
\begin{pmatrix}
1/3 & -1 \\
1 & 0
\end{pmatrix}
\]

(d) \[
\begin{pmatrix}
1/3 & 1/3 \\
0 & 1
\end{pmatrix}
\]
(e) None of these.

13. \[
\begin{pmatrix}
1 & 0 & 2 \\
0 & 2 & 1 \\
-1 & 1 & 0
\end{pmatrix}
\]

(a) 5  
(b) 1  
(c) -6  
(d) 3  
(e) None of these.

In questions 14 and 15 let the matrices \(A_{2x3}, B_{3x3}, C_{3x2}, D_{2x2}, E_{2x2}\) be given.

14. Which of the following exist?

(a) \(|A|\)  
(b) \(D\cdot C\)  
(c) \(A^2\)  
(d) \(B + C\)  
(e) None of these.

15. Which of the following statements is true?

(a) \(D\cdot E = E\cdot D\)  
(b) \(A(B\cdot C) = (A\cdot B)C\)  
(c) \(A\cdot B \in M_{3x2}\)  
(d) \(C^{-1} \in M_{2x3}\)  
(e) All are true.

16. The matrix which counts the two-stage or three stage paths in the network \(A\) is:

(a) \[
\begin{pmatrix}
0 & 3 \\
3 & 10
\end{pmatrix}
\]
(b) \[
\begin{pmatrix}
0 & 3 \\
3 & 1
\end{pmatrix}
\]
(c) \[
\begin{pmatrix}
9 & 30 \\
30 & 19
\end{pmatrix}
\]
(d) \[
\begin{pmatrix}
18 & 33 \\
33 & 29
\end{pmatrix}
\]
(e) None of these.
17. If \( A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \), \( B = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix} \), \( C = \begin{pmatrix} -1 & 1 \\ 2 & -2 \end{pmatrix} \), then \(|B|A^{-1}C = \)

(a) \( \begin{pmatrix} 2 & -2 \\ -1 & 1 \end{pmatrix} \)  
(b) \( \begin{pmatrix} -2 & -1 \\ 2 & 2 \end{pmatrix} \)  
(c) \( \begin{pmatrix} 1 & -1 \\ -2 & 2 \end{pmatrix} \)  
(d) \( \begin{pmatrix} 4 & 1 \\ 2 & -2 \end{pmatrix} \)  
(e) None of these.

18. The sum of the numbers \( x \) and \( y \) which satisfy \( \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 2 \end{pmatrix} \) is:

(a) 4  
(b) -4  
(c) -2  
(d) 2  
(e) None of these.

19. Which of the following is not a linear transformation?

(a) A reflection through the line \( y = 2x \)
(b) \( f(x, y) = (x-1, y+2) \)
(c) \( f(x, y) = (x, y) \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix} \)
(d) Y-distortion by 2
(e) All are linear transformations.

20. Let \( g \) be the function induced by the matrix \( \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix} \) and \( f(x, y) = (x+y, x-y) \). Find \( f(g(1, 2)) \).

(a) \( (7, -3) \)  
(b) \( (1, 1) \)  
(c) \( (-2, 5) \)
(d) \( (3, -7) \)  
(e) None of these.

21. The sum of the values of \( s \) and \( t \) which satisfy the equation \((s, 2) - 2(t, s) = (7, t-2)\) is:

(a) \(-1\)  
(b) \(1\)  
(c) \(0\)  
(d) \(-2\)  
(e) None of these.

In questions 22-24 let \( P: (-2, 3) \), \( Q: (1, -1) \), 
\( S: (1, 2) \), \( X: (x, y) \), \( 0: (0, 0) \)
22. If $X$ is $1/3$ of the distance from $P$ to $Q$, then the coordinates of $X$ are:
(a) $(-1, 5/3)$  (b) $(1, 4/3)$  (c) $(-1, 2)$
(d) $(1, 1/3)$  (e) None of these.

23. The coordinates of $X$ so that $FX = QS + 2PS$ are:
(a) $(4, 4)$  (b) $(1, 3)$  (c) $(5, 5)$  (d) $(1, 5)$
(e) None of these.

24. The length of $OX = PQ + QS$ is:
(a) $3 + \sqrt{5}$  (b) 8  (c) $\sqrt{34}$  (d) 10
(e) None of these.

In questions 25-27 let $P: (1, 2, 3)$ and $Q: (3, 0, -1)$.

25. The midpoint of $PQ$ is:
(a) $(4, 2, 2)$  (b) $(-2, 2, 4)$  (c) $(2, 1, 1)$
(d) $(2, -2, -4)$  (e) None of these.

26. The distance between $P$ and $Q$ is:
(a) $\sqrt{24}$  (b) $\sqrt{36}$  (c) $\sqrt{8}$  (d) $\sqrt{10}$
(e) None of these.

27. The equation of the plane equidistant from $P$ and $Q$ is:
(a) $(x - 1)^2 + (y - 2)^2 + (z - 3)^2 = 6$
(b) $4x - 4y - 8z + 4 = 0$
(c) $x^2 + y^2 + z^2 = 36$
(d) $2x + 4y + 8z = 10$
(e) None of these.
28. The best sketch of the graph of $2x + 3y + z = 6$ in the first octant is:

(a) [Sketch A] 
(b) [Sketch B] 
(c) [Sketch C] 
(d) [Sketch D] 
(e) [Sketch E]

29. $\sum_{i=1}^{3} \sum_{j=3}^{5} (2i - j) =$

(a) 4  (b) -34  (c) 0  (d) -7  (e) None of these.

30. Let $A_{3\times3}$ be given. If $B = A^2$, then $b_{22} =$

(a) $\sum_{i=1}^{3} (a_{21})^2$  
(b) $\sum_{i=1}^{3} (a_{12})^2$  
(c) $(\sum_{i=1}^{3} a_{2i})^2$  
(d) $\sum_{i=1}^{3} a_{2i} \cdot a_{12}$  
(e) None of these
Directions: Select the one best response to each of the following questions. Record your answer in the blank to the left of each question as well as on the response sheet.

1. The sum of the values of $s$ and $t$ which satisfy the equation $(s,2) - 2(t,s) = (7, t - 2)$ is:
   a) $-1$   b) $1$   c) $0$   d) $-2$   e) none of these.

2. If $X$ is $1/3$ of the distance from $P$ to $Q$, then the coordinates of $X$ are:
   a) $(-1, 5/3)$   b) $(1, 4/3)$   c) $(-1, 2)$   d) $(1, 1/3)$
   e) none of these.

3. The coordinates of $X$ so that $\overline{PX} = \overline{QS} + 2\overline{PS}$ are:
   a) $(4, 4)$   b) $(1, 3)$   c) $(5, 5)$   d) $(1, 5)$
   e) none of these.

4. The length of $\overline{OX} = \overline{PQ} + \overline{QS}$ is:
   a) $3 + \sqrt{5}$   b) $8$   c) $\sqrt{34}$   d) $10$
   e) none of these.

5. The midpoint of $\overline{PQ}$ is:
   a) $(4, 2, 2)$   b) $(-2, 2, 4)$   c) $(2, 1, 1)$
   d) $(2, -2, -4)$   e) none of these.

6. The distance between $P$ and $Q$ is:
   a) $\sqrt{24}$   b) $\sqrt{36}$   c) $\sqrt{8}$   d) $\sqrt{10}$
   e) none of these.
7. The equation of the plane equidistant from $P$ and $Q$ is:
   (a) $(x - 1)^2 + (y - 2)^2 + (z - 3)^2 = 6$,
   (b) $4x - 4y - 8z + 4 = 0$,
   (c) $x^2 + y^2 + z^2 = 36$,
   (d) $2x + 4y + 8z = 10$,
   (e) none of these.

8. The best sketch of the graph of $2x + 3y + z = 6$ in the first octant is:
   (a) \[ \text{Sketch A} \]
   (b) \[ \text{Sketch B} \]
   (c) \[ \text{Sketch C} \]
   (d) \[ \text{Sketch D} \]
   (e) none of these.

9. \[ \sum_{i=1}^{3} \sum_{j=3}^{5} (21 - j) = \]
   (a) 4, (b) -34, (c) 0, (d) -17, (e) none of these.

10. Let $A_{3 \times 3}$ be given. If $B = A^2$, then $b_{22} =$
    (a) $\sum_{i=1}^{3} (a_{2i})^2$,
    (b) $\sum_{i=1}^{3} (a_{1i})^2$,
    (c) $\left( \sum_{i=1}^{3} a_{2i} \right)^2$, 
    (d) \[ \text{Expression} \] 
    (e) none of these.
(d) $\sum_{i=1}^{3} a_{2i} \cdot a_{12'}$, (e) none of these.

11. $(2 \ 1) \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \end{pmatrix} = (a) \begin{pmatrix} -1 & -2 & 0 \\ 2 & -1 & 5 \end{pmatrix}$, (b) \begin{pmatrix} -1 & -2 & 0 \end{pmatrix},
(c) \begin{pmatrix} -1 & 2 \\ -2 & -1 \end{pmatrix}, (d) does not exist, (e) none of these.

12. $(3 \ 0)^{-1} = (a) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, (b) \begin{pmatrix} -3 & 1 \\ -1 & 0 \end{pmatrix}, (c) \begin{pmatrix} 1/3 & -1 \\ 1 & 0 \end{pmatrix},
(d) \begin{pmatrix} 1/3 & 1/3 \\ 0 & 1 \end{pmatrix}, (e) none of these.

13. $\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ -1 & 1 & 0 \end{vmatrix} = (a) 5$, (b) 1, (c) -6, (d) 3,
(e) none of these.

In questions 14 and 15 let the matrices $A_{2x3}$, $B_{3x3}$, $C_{3x2}$, $D_{2x2}$, $E_{2x2}$ be given.

14. Which of the following exist?
(a) $|A|$, (b) $D \cdot C$, (c) $A^2$ (d) $B + C$
(e) none of these.

15. Which of the following statements are true?
(a) $D \cdot E = E \cdot D$ (b) $A(B \cdot C) = (A(B)C)$
(c) $A \cdot B \in M_{3x2}$ (d) $C^{-1} \in M_{2x3}$ (e) all are true.

16. The matrix which counts the two-stage or three stage paths in the network is:
(a) \begin{pmatrix} 9 & 3 \\ 5 & 10 \end{pmatrix}, (b) \begin{pmatrix} 0 & 3 \\ 5 & 1 \end{pmatrix}, (c) \begin{pmatrix} 9 & 30 \\ 30 & 19 \end{pmatrix},
(d) \begin{pmatrix} 18 & 33 \\ 33 & 29 \end{pmatrix}, (e) none of these.
17. If \( A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ B = \begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}, \ C = \begin{pmatrix} -1 & 1 \\ 2 & -2 \end{pmatrix}, \) then \(|B| \cdot A^{-1} \cdot C =
(a) \begin{pmatrix} 2 & -2 \\ -1 & 1 \end{pmatrix}, \ (b) \begin{pmatrix} -2 & 1 \\ 2 & 2 \end{pmatrix}, \ (c) \begin{pmatrix} 1 & -1 \\ -2 & 2 \end{pmatrix},
(d) \begin{pmatrix} 4 & -4 \\ 2 & -2 \end{pmatrix}, \ (e) \text{none of these.}

18. The sum of the numbers \( x \) and \( y \) which satisfy
\[
\begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}
\]
is:
(a) 4, \ (b) -4 \ (c) -2, \ (d) 2, \ (e) none of these.

19. Which of the following is not a linear transformation?
(a) A reflection through the line \( y = 2x, \)
(b) \( f(x,y) = (x - 1, y + 2), \)
(c) \( f(x,y) = (x,y) \begin{pmatrix} 1 & 2 \\ 3 & -4 \end{pmatrix}, \)
(d) Y-distortion by 2, \ (e) all are linear transformations.

20. Let \( g \) be the function induced by the matrix \( \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix} \)
and \( f(x,y) = (x + y, x - y) \). Find \( f(g(1,2)). \)
(a) \((7,-3), \) (b) \((1,1), \) (c) \((-2,5), \)
(d) \((3,-7), \) (e) none of these.
APPENDIX D

1) Test One (Pilot Group)

Number of students taking test: 828
Number of items in test: 25

Mean test score: 14.26  Standard deviation: 4.15
Median : 15.00    Skewness : -0.23
Mode : 16.00    Kurtosis : -0.50

Maximum: 25
Minimum: 2

Reliability estimates:

Kuder-Richardson 20: 0.758
Kuder-Richardson 21: 0.672

Mean item difficulty: 0.430  Mean item discrimination: 0.427

2) Test Two (Pilot Group)

Number of students taking test: 773
Number of items in test: 25

Mean test score: 16.79  Standard deviation: 3.94
Median : 17.00    Skewness : -0.17
Mode : 17.00    Kurtosis : -0.62

Maximum: 25
Minimum: 4

Reliability estimates:

Kuder-Richardson 20: 0.694
Kuder-Richardson 21: 0.637

Mean item difficulty: 0.413  Mean item discrimination: 0.405
3) Test One (10:00 a.m. Test Group)  Form A - 115.01

Number of students taking test: 248
Number of items in test: 20

Mean test score: 11.73  Standard deviation: 3.34
Median : 12.00  Skewness : -0.49
Mode : 12.00  Kurtosis : -0.01

Maximum: 19
Minimum : 2

Reliability estimates:

Kuder-Richardson 20: 0.678
Kuder-Richardson 21: 0.594

Mean item difficulty: 0.413  Mean item discrimination: 0.414

4) Test One (10:00 a.m. Test Group)  Form B - 115.01

Number of students taking test: 233
Number of items in test: 20

Mean test score: 11.44  Standard deviation: 3.06
Median : 11.00  Skewness : -0.16
Mode : 11.00  Kurtosis : -0.10

Maximum: 19
Minimum : 3

Reliability estimates:

Kuder-Richardson 20: 0.607
Kuder-Richardson 21: 0.501

Mean item difficulty: 0.428  Mean item discrimination: 0.372
5) Test One (11:30 a.m. Test Group) Form A - 115.01

Number of students taking test: 95
Number of items in test: 20

Mean test score: 12.68
Median: 13.00
Mode: 13.00

Standard deviation: 3.07
Skewness: -0.65
Kurtosis: 1.12

Maximum: 19
Minimum: 1

Reliability estimates:

Kuder-Richardson 20: 0.638
Kuder-Richardson 21: 0.533

Mean item difficulty: 0.366
Mean item discrimination: 0.361

6) Test One (11:30 a.m. Test Group) Form B - 115.01

Number of students taking test: 91
Number of items in test: 20

Mean test score: 12.25
Median: 12.00
Mode: 11.00

Standard deviation: 2.85
Skewness: 0.06
Kurtosis: -0.09

Maximum: 19
Minimum: 6

Reliability estimates:

Kuder-Richardson 20: 0.569
Kuder-Richardson 21: 0.438

Mean item difficulty: 0.387
Mean item discrimination: 0.355
7) Test One (4:00 p.m. Test Group) Form A - 115.01

Number of students taking test: 176
Number of items in test: 20

Mean test score: 11.57
Median: 11.00
Mode: 11.00

Standard deviation: 3.28
Skewness: -0.20
Kurtosis: -0.10

Maximum: 19
Minimum: 2

Reliability estimates:
Kuder-Richardson 20: 0.660
Kuder-Richardson 21: 0.575

Mean item difficulty: 0.422
Mean item discrimination: 0.403

8) Test One (4:00 p.m. Test Group) Form B - 115.01

Number of students taking test: 180
Number of items in test: 20

Mean test score: 11.91
Median: 12.00
Mode: 12.00

Standard deviation: 3.02
Skewness: -0.08
Kurtosis: -0.49

Maximum: 20
Minimum: 4

Reliability estimates:
Kuder-Richardson 20: 0.597
Kuder-Richardson 21: 0.495

Mean item difficulty: 0.405
Mean item discrimination: 0.402
9) Test Two (Regular Sections) Form A

Number of students taking test: 770
Number of items in test: 20

Mean test score: 11.74  Standard deviation: 3.68
Median: 12.00  Skewness: -0.07
Mode: 12.00  Kurtosis: -0.56

Maximum: 20
Minimum: 1

Reliability estimates:

Kuder-Richardson 20: 0.715
Kuder-Richardson 21: 0.676

Mean item difficulty: 0.413  Mean item discrimination: 0.464

10) Test Two (Regular Sections) Form B

Number of students taking test: 720
Number of items in test: 20

Mean test score: 11.23  Standard deviation: 3.56
Median: 11.00  Skewness: -0.07
Mode: 11.00  Kurtosis: -0.55

Maximum: 20
Minimum: 1

Reliability estimates:

Kuder-Richardson 20: 0.690
Kuder-Richardson 21: 0.643

Mean item difficulty: 0.438  Mean item discrimination: 0.445
11) Test Two (Accelerated Pace Hopefuls) Form A

Number of students taking test: 551
Number of items in test: 20

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<th>Standard deviation:</th>
<th>3.42</th>
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<td>Mode</td>
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</tbody>
</table>

Maximum: 19
Minimum: 0

Reliability estimates:

- Kuder-Richardson 20: 0.677
- Kuder-Richardson 21: 0.602

Mean item difficulty: 0.493
Mean item discrimination: 0.448

12) Test Two (Accelerated Pace Hopefuls) Form B

Number of students taking test: 496
Number of items in test: 20

<table>
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<th>Mean test score:</th>
<th>10.55</th>
<th>Standard deviation:</th>
<th>3.47</th>
</tr>
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<tbody>
<tr>
<td>Median</td>
<td>11.00</td>
<td>Skewness</td>
<td>-0.17</td>
</tr>
<tr>
<td>Mode</td>
<td>10.00</td>
<td>Kurtosis</td>
<td>-0.41</td>
</tr>
</tbody>
</table>

Maximum: 19
Minimum: 1

Reliability estimates:

- Kuder-Richardson 20: 0.679
- Kuder-Richardson 21: 0.617

Mean item difficulty: 0.472
Mean item discrimination: 0.420
13) Test Two  (Accelerated Sections)

Number of students taking test: 327
Number of items in test: 20

<table>
<thead>
<tr>
<th>Mean test score: 15.88</th>
<th>Standard deviation: 2.29</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median: 16.00</td>
<td>Skewness: -0.52</td>
</tr>
<tr>
<td>Mode: 17.00</td>
<td>Kurtosis: -0.25</td>
</tr>
</tbody>
</table>

Maximum: 20
Minimum: 9

Reliability estimates:

- Kuder-Richardson 20: 0.541
- Kuder-Richardson 21: 0.399

Mean item difficulty: 0.206  Mean item discrimination: 0.289
14) Test One (Retake, Reduced Pace) Form A

Number of students taking test: 702
Number of items in test: 25

Mean test score: 15.08  Standard deviation: 3.93
Median: 15.00  Skewness: -0.36
Mode: 16.00  Kurtosis: -0.21

Maximum: 24
Minimum: 3

Reliability estimates:

Kuder-Richardson 20: 0.711
Kuder-Richardson 21: 0.673

Mean item difficulty: 0.397  Mean item discrimination: 0.388

15) Test One (Retake, Reduced Pace) Form B

Number of students taking test: 658
Number of items in test: 25

Mean test score: 15.03  Standard deviation: 4.12
Median: 16.00  Skewness: -0.41
Mode: 17.00  Kurtosis: -0.25

Maximum: 24
Minimum: 1

Reliability estimates:

Kuder-Richardson 20: 0.733
Kuder-Richardson 21: 0.674

Mean item difficulty: 0.399  Mean item discrimination: 0.399
16) Test Two (Reduced Pace) Form A

Number of students taking test: 620
Number of items in test: 38

Mean test score: 22.87  Standard deviation: 6.53
Median : 24.00       Skewness : -0.61
Mode : 23.00         Kurtosis : 0.18

Maximum: 37
Minimum: 2

Reliability estimates:

Kuder-Richardson 20: 0.844
Kuder-Richardson 21: 0.808

Mean item difficulty: 0.398  Mean item discrimination: 0.412

17) Test Two (Reduced Pace) Form B

Number of students taking test: 649
Number of items in test: 38

Mean test score: 23.27  Standard deviation: 6.15
Median : 24.00       Skewness : -0.43
Mode : 23.00         Kurtosis : -0.20

Maximum: 37
Minimum: 3

Reliability estimates:

Kuder-Richardson 20: 0.822
Kuder-Richardson 21: 0.782

Mean item difficulty: 0.388  Mean item discrimination: 0.399
18) Final (Reduced Pace) Form A

Number of students taking test: 393
Number of items in test: 38

Mean test score: 20.87
Median: 21.00
Mode: 26.00

Standard deviation: 5.77
Skewness: -0.03
Kurtosis: -0.43

Maximum: 35
Minimum: 5

Reliability estimates:

Kuder-Richardson 20: 0.787
Kuder-Richardson 21: 0.737

Mean item difficulty: 0.451
Mean item discrimination: 0.369

19) Final (Reduced Pace) Form B

Number of students taking test: 338
Number of items in test: 38

Mean test score: 19.29
Median: 20.00
Mode: 19.00

Standard deviation: 5.77
Skewness: -0.08
Kurtosis: -0.40

Maximum: 33
Minimum: 5

Reliability estimates:

Kuder-Richardson 20: 0.787
Kuder-Richardson 21: 0.734

Mean item difficulty: 0.492
Mean item discrimination: 0.382
20) 115.02 Final (Reduced Pace Sections) Form A

Number of students taking test: 122
Number of items in test: 30

Mean test score: 17.95
Median: 19.00
Mode: 20.00

Standard deviation: 4.52
Skewness: -0.68
Kurtosis: 0.33

Maximum: 27
Minimum: 5

Reliability estimates:

Kuder-Richardson 20: 0.733
Kuder-Richardson 21: 0.670

Mean item difficulty: 0.402
Mean item discrimination: 0.371

21) 115.02 Final (Reduced Pace Sections) Form B

Number of students taking test: 136
Number of items in test: 30

Mean test score: 19.79
Median: 20.00
Mode: 23.00

Standard deviation: 4.30
Skewness: -0.60
Kurtosis: 0.09

Maximum: 29
Minimum: 7

Reliability estimates:

Kuder-Richardson 20: 0.703
Kuder-Richardson 21: 0.658

Mean item difficulty: 0.340
Mean item discrimination: 0.327
22) 115.02 Final (Reduced Pace Sections) Retake

Number of students taking test: 94  
Number of items in test: 30

Mean test score: 16.97  
Median: 17.00  
Mode: 17.00  
Standard deviation: 4.29  
Skewness: 0.16  
Kurtosis: -0.43  
Maximum: 27  
Minimum: 7

Reliability estimates:

Kuder-Richardson 20: 0.716  
Kuder-Richardson 21: 0.621

Mean item difficulty: 0.434  
Mean item discrimination: 0.343

23) 115.02 Final (Regular Sections)

Number of students taking test: 432  
Number of items in test: 30

Mean test score: 22.33  
Median: 23.00  
Mode: 23.00  
Standard deviation: 4.30  
Skewness: -0.73  
Kurtosis: 0.38  
Maximum: 30  
Minimum: 7

Reliability estimates:

Kuder-Richardson 20: 0.766  
Kuder-Richardson 21: 0.715

Mean item difficulty: 0.256  
Mean item discrimination: 0.346
Number of students taking test: 24
Number of items in test: 20

Mean test score: 17.13
Median: 17.00
Mode: 18.00

Standard deviation: 2.09
Skewness: -0.31
Kurtosis: -0.43

Maximum: 20
Minimum: 12

Reliability estimates:

Kuder-Richardson 20: 0.643
Kuder-Richardson 21: 0.594

Mean item difficulty: 0.241
Mean item discrimination: 0.388
## APPENDIX E

### TABLE 40

**COMPARISON OF MEANS BETWEEN FORMS A AND B OF EXAMINATIONS USED IN THE STUDY**

<table>
<thead>
<tr>
<th>GROUP</th>
<th>TEST</th>
<th>FORM A MEAN</th>
<th>FORM B MEAN</th>
<th>t</th>
<th>p &gt; t</th>
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</thead>
<tbody>
<tr>
<td>Regular</td>
<td>I (10:00)</td>
<td>11.73</td>
<td>11.44</td>
<td>0.993</td>
<td>.32</td>
</tr>
<tr>
<td>Regular</td>
<td>I (11:30)</td>
<td>12.68</td>
<td>12.25</td>
<td>0.991</td>
<td>.32</td>
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<tr>
<td>Regular</td>
<td>I (4:00)</td>
<td>11.57</td>
<td>11.91</td>
<td>1.015</td>
<td>.31</td>
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<tr>
<td>Regular</td>
<td>II (Screen)</td>
<td>10.14</td>
<td>10.55</td>
<td>1.925</td>
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<td>Regular</td>
<td>II</td>
<td>11.74</td>
<td>11.23</td>
<td>2.713</td>
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<tr>
<td>Reduced</td>
<td>I</td>
<td>15.08</td>
<td>15.03</td>
<td>0.228</td>
<td>.82</td>
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<td>Reduced</td>
<td>II</td>
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<td>23.27</td>
<td>1.685</td>
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<td>Final</td>
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<td>19.29</td>
<td>3.674</td>
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</tr>
<tr>
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<td>17.95</td>
<td>19.79</td>
<td>3.345</td>
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</table>
TABLE 41

EQUIVALENCE PAIRINGS FOR ITEMS ON TEST TWO (REGULAR) WITH TEST TWO AND FINAL (REDUCED PACE)

<table>
<thead>
<tr>
<th>TEST II* (Regular)</th>
<th>TEST II* (Reduced)</th>
<th>FINAL* (Reduced)</th>
<th>TEST II* (Regular)</th>
<th>TEST II* (Reduced)</th>
<th>FINAL (Reduced)</th>
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</table>

* Form A
APPENDIX F

Formulas Used to Generate t-scores

Given two normal distributions X and Y with sample measures $\bar{x}, s_x^2, n_x$ and $\bar{y}, s_y^2, n_y$, the statistic

$$t = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}}$$

with the associated degrees of freedom

$$v = \frac{\left(\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}\right)^2}{\frac{(s_x^2/n_x)^2}{n_x+1} + \frac{(s_y^2/n_y)^2}{n_y+1}} = 2$$

has an approximate $t$ distribution. This statistic tests the hypothesis $\mu_x = \mu_y$ when $\sigma_x$ and $\sigma_y$ are unknown and not necessarily equal.
Test of Hypothesis 1 in this study involved a two-step use of the above t-test. Using Strategy I recommendations reported in Table 25, the t-score for the accelerated pace recommendations was generated as follows:

1. Let \( \bar{x} , s_x^2 , n_x \) and \( \bar{y} , s_y^2 , n_y \) be the pilot measures for the groups recommended for the accelerated and regular pace levels, respectively.

2. Compute \( t_1 \) using (1) above.

3. In the same way, compute \( t_2 \) for the test group.

4. \( t_1 \) and \( t_2 \) are approximate standard scores with population mean zero and standard deviation one.

5. \( H: t_1 = t_2 \) is tested using (1) to obtain

\[
  t = \frac{t_1 - t_2}{\sqrt{2}}
\]

the value reported in Table 25 as 0.018.

The t-values for other strategies and pacing levels were computed in a manner analogous to (1) through (5) above, with \( t_1 \) and \( t_2 \) generated as positive numbers.
BIBLIOGRAPHY


