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THE MULTI-LINEAR SPEED-DENSITY RELATIONSHIP AND ITS IMMEDIATE APPLICATIONS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

by

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1971

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Studies in Operations Research, Professors Walter C. Giffin and Thomas H. Rockwell
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CHAPTER I

INTRODUCTION

One major area of traffic flow theories is the study of functional relationships between traffic density, volume and speed. The basic relationship of traffic flow assumes that traffic volume equals the product of speed and density. Therefore, only one pair of the three basic parameters is needed to study the relationships. Of the possible pairings of the three parameters, speed-density data seem to exhibit the most simple functional pattern when compared with volume-density data and speed-volume data. Therefore, it is used most often when the relationships among the three parameters are of interest.

Previous work in dealing with speed-density relationships has been summarized in the paper "A Statistical Analysis of Speed Density Hypotheses" by J. Drake, J. Schofer and A. May. The seven speed-density hypotheses involved in their analyses are presented in Figure 1 to provide an immediate reference on past attempts. The authors applied several statistical tests and intuitive discriminating criteria to these hypotheses in an attempt to establish the most desirable speed-density hypothesis. As a conclusion to this investigation, the authors stated that their results tend to support the three-regime-linear hypothesis and Edie's hypothesis above all other tested.
Figure 1 Proposed Velocity Versus Density Relationships
However, from a less rigid standpoint of application, all hypotheses (except the two linear regimes) performed well enough to warrant continued use.

From this study it seems that there are many acceptable speed-density hypothesis, yet none of them can be proven superior. It is felt that because the data used to develop these hypotheses and the data used to verify them were subjected to many undeterminable influences produced by the variance in driver behavior, different vehicles and possible different traffic performances prior to and after a kinematic disturbance, a true speed-density relationship has not been established.

Recently, an improved data collection process was developed at the Transportation Research Center at The Ohio State University by employing aerial photogrammetric techniques. This process enables one to observe a platoon of the same vehicles through various operating conditions for a period of time. In this manner the influences due to different drivers, different vehicles, and possible confusions before and after a kinematic disturbance can be recognized and compensated. It is felt that traffic data obtained by the aerial survey method provide an opportunity to have a fresh look at the speed-density problem.

It is intended in this study, by using data collected from aerial photogrammetric surveys, to establish and validate a speed-density hypothesis and to explore its immediate application on traffic operations.
CHAPTER II

DEVELOPMENT OF THE MULTI-LINEAR MODEL FROM OBSERVED TRAFFIC CHARACTERISTICS

2.1 Data Presentation

The data used for this study were selected from the vehicle trajectory plot shown in Figure 2. These vehicle trajectories were obtained by employing aerial photogrammetric techniques in a research project conducted at the Transportation Research Center of The Ohio State University\(^3\). The trajectories show that the vehicles initially moved in a relatively low density region, then were compressed through a kinematic wave, and finally were released to a uniform flow condition.

Two platoons, Platoon A and Platoon B, were chosen from these vehicle trajectories for detailed analyses. The two platoons are shown in Figures 3 and 4, respectively. The main reason for selecting these two platoons is trying to keep lane changing to a minimum. In this case, Platoon A had no vehicles entering or leaving, Platoon B had one vehicle leaving long before the platoon entered the kinematic wave and had two vehicles entering near the end of the study period. The selected sizes of the platoons were chosen primarily on the high observed maximum densities. It can be
seen from Figure 2 that the larger the size of platoons selected the smaller their observed maximum densities.

2.2 Observed Hysteresis Phenomena of Traffic Flows

When a freeway operates at or beyond maximum capacity, fluctuations in traffic movement occur. Such fluctuations may deteriorate to a stop-and-go type of operation with increasing demand. Temporary stop-and-go operations may also be caused by accidents of disabled vehicles. This occurrence is somewhat similar to the situation in which queues are formed and released by traffic signals. Intuitively, one would imagine that drivers have to be more alert in a queue-forming condition than in a queue-releasing condition to avoid possible collisions. Several studies have confirmed that drivers tend to react more quickly to traffic changes in a queue-generating condition than in a queue-releasing condition \(^4, 5, 6\). This tends to result in a less efficient movement of vehicles in a queue-releasing condition than in a queue-generating condition when the rate of flow is considered.

It seems that research in the direction of understanding the queue phenomenon might have very significant contributions to the field of traffic flow characteristics; however, there has been no direct investigation of the phenomenon. In this study, the use of aerial data provides data on a platoon of vehicles during the generation and dissipation stage after a complete traffic stoppage. The data provide an ideal case for studying the queue phenomenon.
Figure 2  Vehicle Trajectories
Figure 3  Identification of Platoon A Vehicles (Enclosed Area)
Figure 4  Identification of Platoon B Vehicles (Enclosed Area)
Observed Hysteresis Phenomenon

For the selected Platoon A and Platoon B, average velocities and densities were calculated for one second time intervals. The relationships between these two parameters are shown in Figures 5 and 6. Data points before and after the kinematic disturbance are differentiated in the following manner: small circles represent "before" conditions and crosses represent "after" conditions. Arrowheads are used to show the time sequence of the data points. By following the time sequence it can be observed that retarded traffic movement occurred immediately after the maximum density, i.e., the traffic moved slower in the queue-releasing condition than the queue-forming condition for the same density values. Unlike the plastic material which exhibits a hysteresis loop during an unloading and reloading cycle, the traffic platoons analyzed showed that their retarded performance was temporary and they recovered to their "before" performance levels at certain densities. For Platoon A, the traffic recovered at a density of 73 vpm. For Platoon B, the point of recovery is at 53 vpm.

The interesting phenomenon is that when traffic, after going through the hysteresis loop, reached its point of recovery, the density stopped decreasing but the average speed of the platoon kept increasing. Traffic for some time stayed at this density before a further reduction in density occurred. Eventually, Platoon A can be seen joining the "before" condition again at a much lower density of about 33 vpm. This is termed the Point
of Returning to Normalcy (P.O.R.N.). For Platoon B, this return to normal situation was not shown due to the lack of data observed below the density of 43 vpm on its recovery phase. However, the trend of the data indicates that Platoon B would return to the "before" condition at a traffic density of about 33 vpm.

This hysteresis phenomenon can be seen also on volume-density plots presented in Figures 7 and 8 and speed-volume plots presented in Figures 9 and 10.

From these graphs, two loops formed by data "before" and "after" can be observed going through a kinematic disturbance.

(a) The hysteresis loop.

Between the point of observed maximum density and the point of recovery, the normal behavior of traffic before maximum density and the retarded behavior of traffic immediately following maximum density form a loop. This is termed the hysteresis loop to emphasize the retarded performance of traffic flow on its recovery phase.

(b) The energy-gain loop.

Between the point of recovery and the point of returning to normalcy, the normal behavior of traffic before the maximum density and the higher than normal performance of traffic on its recovery path form a loop. Since hysteresis phenomena of plastic materials are caused by energy dissipation,
Figure 5. Observed Speed-Density Relationship for Platoon A.
Figure 6. Observed Speed-Density Relationship for Platoon B
Figure 7. Observed Volume-Density Relationship for Platoon A
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in an analogous manner, the performance of traffic on its recovery phase
is felt to be caused by gaining energy. In a later analysis on traffic energy
variations before and after a maximum density, this assumption is partially
verified. Therefore, this loop is termed the energy-gain loop to emphasize
the superior performance of traffic on its releasing path.

**Energy Discussions of the Hysteresis Phenomenon**

For plastic materials, the hysteresis loop which occurs on a stress-
strain diagram during an unloading and reloading cycle has been explained
by energy dissipation. It seems quite logical then that energy should be
considered for the discussion of hysteresis phenomena of traffic flows.

The kinetic energy of a traffic flow is defined as the product of density
and the square of the corresponding mean speed. In reference 13 the
coefficient of variation of speed (CVS) obtained by dividing the standard
deviation of speed by the average speed has been shown to be a good indica-
tion of "internal energy".

In Figure 11, the kinetic energy and the coefficient of variation of
speed for Platoon A are plotted against density. In Figure 12, the same
parameters for Platoon B are presented.

Following the time sequence of the data, it can be seen that when
traffic recovers from maximum density, a decrease of kinetic energy
results from an increase of the coefficient of variation of speed. At the
point of recovery the coefficient of variation of speed first crosses its
Figure 11. Observed Kinetic Energy and Coefficient of Variation of Platoon A Versus Density
Figure 12. Observed Kinetic Energy and Coefficient of Variation of Platoon B Versus Density
inbound path and keeps on decreasing. This is coupled with a significant build-up of kinetic energy. When traffic flow acquires further momentum from the point of recovery, the "after" condition shows higher kinetic energy and lower coefficient of variation of speed when compared with "before" condition.

The evidence presented seems to indicate that the hysteresis phenomenon of a traffic flow can be explained by the transfer from internal to kinetic energy.

**Supporting Analyses**

In order to obtain more insight into the hysteresis phenomenon of traffic so that proper interpretation of the phenomenon can be made, several traffic parameters were analyzed.

A. Speed dispersion analysis.

In Figures 13 and 14, the standard deviations of vehicular speeds of Platoon A and Platoon B were plotted against density. By following the time sequence for Platoon A it can be seen that initially the standard deviation of speeds started at low values, proceeded to a maximum at 55 vpm and dropped to an absolute minimum at 92 vpm. It rose again to a local maximum of 125 vpm and then decreased as the density approached its maximum value. After the platoon recovered from the disturbance, the standard deviation assumed values lower than the "before" values, but it soon increased and surpassed the "before" values, reaching a local maximum at
Figure 13. Observed Standard Deviation of Speeds Versus Density of Platoon A
Figure 14. Observed Standard Deviation of Speeds Versus Density for Platoon B
about 100 vpm. It then decreased to values lower than "before" values at about 78 vpm. After the standard deviation had passed the "before" values, it kept decreasing but without substantial changes in density. The data remained at around 73 vpm for quite a while and reached a local minimum before further recovery of traffic flow. When traffic regained momentum again, the standard deviations of speeds increased slowly with detectable oscillations.

For Platoon B the variations of the standard deviation of speeds exhibit a similar pattern, even though the density values corresponding to the local minimums and local maximums of Platoon A are not the same.

B. Spacing dispersion analysis.

In Figures 15 and 16, standard deviations of vehicle spacings of Platoon A and Platoon B are plotted against density. Different variational patterns of the standard deviations of spacings before and after the maximum density can be clearly identified and can be described in a similar manner as was done for the speed dispersions.

C. Average headway analysis.

In Figures 17 and 18, variational patterns of average headway versus density for Platoon A and Platoon B are presented. By following the time sequence of Platoon A data, it can be seen that the traffic maintained an average headway of two seconds in the density region between 33 vpm to about 110 vpm. The average time headway then increased, first slowly and
Figure 15. Observed Standard Deviation of Spacings Versus Density for Platoon A
Figure 16. Observed Standard Deviation of Spacings Versus Density for Platoon B
Figure 17. Observed Average Headway - Density Relationship for Platoon A
Figure 18. Observed Average Headway - Density Relationship for Platoon B
then sharply, as traffic flow approached its maximum density. When the platoon was released from the maximum density, the average headway decreased but maintained higher values when compared to "before" conditions. The average headway finally reached the "before" value of two seconds at 73 vpm and kept decreasing without pronounced variations in density. The average headway then increased again with a reduction in density after it reached a local minimum of about one second, and again assumed the "before" condition at 33 vpm.

For Platoon B, the density region in which a nearly constant average headway was maintained is from 33 vpm to 90 vpm. The point where the recovery path intersected the "before" path is around 59 vpm. Other than those values, the average headway of Platoon B exhibits a similar pattern when compared with Platoon A.

**General Interpretation of the experimental analyses.**

Relying on the information presented in the foregoing sections, the observed hysteresis phenomenon of a traffic stream can be interpreted as follows:

Consider a platoon of vehicles starting from low densities such that drivers are little affected by other vehicles and intervehicular spacings are large. When density increases, headways must be reduced due to the shortened spacings between vehicles. Since a headway represents the time available for a driver to respond to changes by the lead car, it seems
logical to assume that a driver would choose to maintain a headway which is greater than, or at least equal to, his response time. The result is that faster moving vehicles tend to slow down, causing the entire platoon to decrease its speed and spacing dispersions, while the average headway remains at a constant value of about two seconds. As the process continues, the dispersion of speeds and spacings eventually reaches a minimum value where vehicles in the platoon are equally spaced and are moving at about the same speed. Driving at a constant speed, however, requires continuous attention in dense traffic and places a considerable strain on the drivers. Hence the dispersion of speeds and spacings increase again. As the density continues to increase, a further reduction of mean speed and an increase in the average headway results. The traffic stream finally slows down to a stop and a maximum density is obtained. Drivers obviously take care in the process to avoid possible conflicts.

When the queue begins to release itself from a stopped position, drivers tend to respond to changing traffic conditions more slowly than in the queue forming condition. This is because there is no immediate danger of accidents. The slow response by the drivers results in larger headways than in the queue generating condition. This results in a less efficient performance of traffic flow than that exhibited when entering the kinematic disturbance. In other words, the average speed of the traffic stream is lower than the speed of the queue generating condition at the same density level. As density decreases,
with relatively large average headways available, trailing vehicles tend to catch up; the result is reduced speed and spacing dispersions, increased mean speeds, and decreased average headways. When this process continues, the traffic eventually recovers from its retarded performance and reaches its "before" performance level when average headway reduces to the two second value. Since the dispersion of speed is relatively low at the point of recovery, the first vehicle of the platoon moves at about the same speed as the last vehicle; therefore, the length of the platoon remains unchanged. However, since the momentum of vehicles adjusting their speeds and spacings still prevails, vehicles are able to increase their speeds due to reduced speed and spacing dispersions instead of physically expanding the platoon length. The result is that the average speed of the platoon keeps increasing at the point of recovery without a change in density. As this process continues and the average headway reaches a low value where drivers feel their provided headways are no longer sufficient for them to respond to possible traffic changes, trailing vehicles tend to slow their acceleration. This results in a decrease in density, increases in average headways, spacing dispersion and speed dispersion. Finally, traffic flow assumes the "before" condition again.

Conclusions of the observed hysteresis phenomenon.

The above interpretation of the hysteresis phenomenon is only an attempt to describe and explain observed traffic behavior. Further work is
necessary to validate some of the assumptions made in the interpretation. However, there are two major conclusions which can be drawn at this stage of the investigation.

1. The movement patterns exhibited by traffic streams before entering a kinematic disturbance are different from the movement patterns exhibited after departing from a kinematic disturbance. Traffic performance is retarded when released from the congestion and recovers at a much lower density value. A hysteresis loop occurs between maximum density and the point of recovery when a traffic stream passes through jam conditions. When traffic is further released from the point of recovery, its performance is superior to the condition prior to the congestion. The traffic later returns to the normal condition at a density of about 33 vpm. An energy gain loop exists between the point of recovery and the point of returning to normalcy.

2. At respective density levels, the average speed of the vehicles in a traffic stream is affected by the dispersion of vehicle speeds. A low coefficient of variation of speeds is associated with a high average speed.

Besides the affirmative statements made, it is felt that the whole traffic density domain from zero to jam density is divided into three different operating regions defined by the point of recovery and the point of returning to normalcy. It is felt that the behavior of traffic is analogous to the behavior of a plastic material in the region between the point of recovery and jam density. The evidence is the observed hysteresis loop. In the
region between the point of returning to normalcy and the point of recovery, the observation is that traffic always operates at or below an average headway of two seconds. In the region between zero density and the point of returning to normalcy, no data are available to present the actual traffic performance, but this region should provide a different operating environment to drivers than the other two regions due to its low density nature.

**Effects of the hysteresis analysis on speed-density relationships.**

As far as the speed-density relationship is concerned, the discussed hysteresis phenomenon provides two major recommendations.

1. Since speed-density exhibits different relationships between a queue generating and a queue releasing condition, it seems appropriate that the speed-density relationship of queue generating conditions be treated separately from the speed-density relationship of queue releasing conditions.

2. Since the hysteresis phenomenon tends to suggest that traffic can be divided into three different operating regions such that density changes may have different effects on speed changes in each of the regions, it seems that speed-density relationship can be analyzed on a region by region basis.

From the above discussion, it was decided to evaluate the speed-density relationship only for the queue generating condition. The speed-density relationship of queue releasing conditions is left for future investigation.
2.3 Detailed Analyses of Traffic Parameters for Queue Generating Conditions

It has been stated in the previous section that the purpose of the study is to establish a speed-density relationship for queue generating conditions. A possible approach to this problem was suggested by dividing the domain into several operating regions, each of which characterizes a different operating condition. Then, by analyzing speed-density relationships in each of the regions, one may be led to an acceptable speed-density hypothesis for the whole traffic domain.

Although the hysteresis analysis tended to suggest that traffic can be divided into three different operating regions, it is felt that detailed analyses of several traffic parameters on queue generating conditions are needed to confirm these regions and possibly to identify sub-regions.

Absolute and marginal safety considerations

Absolute and marginal safety concepts in car-following conditions have been introduced in previous research work conducted at the Transportation Research Center at The Ohio State University. The absolute safety concept is defined as follows: "the leading vehicle is brought to a sudden stop by some object in the roadway (running into a suddenly appearing obstacle, heavy truck or container). The driver of the trailing vehicle reacts on the incidence of the collision and is able to stop his car without hitting the leading car in a rear-end collision. No space is left between the vehicles after the stopping maneuver."
The marginal safety concept is defined as follows: "the driver of the leading car is forced to bring his car to a standstill in an emergency and tries to stop his vehicle in the shortest possible distance. After some delay caused by reacting on the maneuver of the leading car, the driver of the trailing vehicle duplicates the braking maneuver of the leading car, and both vehicles come to a safe stop. No rear-end collision will occur although no space remains between the vehicles after stopping."

The absolute safety concept requires a spacing, $S_a$, between vehicles equal to the safe stopping distance and is given by

$$S_a = 1.47 VT + \frac{V^2}{30f}$$

where $V =$ vehicle speed in mph

$T =$ the reaction time of the driver in seconds, and

$f =$ the coefficient of friction.

The marginal safety concept requires less distance between successive vehicles and is mainly dependent on the reaction time of the driver. The spacing, $S_m$, required can be expressed as:

$$S_m = 1.47 VT$$

Since speed is a function of traffic density, these safe spacings can be related to density.

It is felt that a driver, based on his driving experience, is able to judge the risk involved in his driving in accordance with his speed and the spacing available between his vehicle and the leading vehicle, and he will
Figure 19. Comparison of Average Spacing to Marginally Safe Spacing for Platoon A (Prior-Queue Conditions)
Figure 20. Comparison of Average Spacing to Marginally Safe Spacing for Platoon B (Prior-Queue Conditions)
Figure 21. Comparison of Average Spacing to Absolutely Safe Spacings for Platoon A (Prior-Queue Conditions)
Figure 22. Comparison of Average Spacing to Absolutely Safe Spacing for Platoon B (Prior-Queue Conditions)
respond to the traffic accordingly. Therefore, the comparison between required safe spacings and the actual spacings should provide some insight into different traffic operating conditions.

For Platoon A and Platoon B, absolutely safe spacings and marginally safe spacings are calculated based on a response time of two seconds and the appropriate velocity dependent coefficient of friction of a dry pavement. The calculated marginally safe spacings versus density are shown in Figures 19 and 20. Absolutely safe spacings versus density can be seen in Figures 21 and 22. Solid curves shown in the graphs represent the average spacing. Dotted lines are estimated safe spacings in the low density region. They are obtained by extending existing data points and by assuming a free flow speed of 70 mph. This approximation is felt acceptable because it can be observed from the graphs that a small change of speed in the low density range does not affect the results very much.

From the graphs, the observed general pattern is that the required safe spacing data intersect the average spacing curve at two places, thus separating the whole traffic domain into three regions. In the graphs which describe density and marginally safe spacing relationships, the three regions identified can be seen to correspond to the three operating regions suggested in the hysteresis analysis, i.e., the two intersecting points of marginally safe spacing and average spacing are coincident with the Point of Recovery and the Point of Returning to Normalcy identified in the hysteresis analysis.
For absolutely safe spacings, Figure 15 shows that Platoon A data intersect the average spacing at 14 vpm and 112 vpm. Figure 16 shows that the two intersecting points are at 12 vpm and 99 vpm.

To summarize, the combined considerations of marginally safe spacings and absolutely safe spacings divide the traffic domain into five different operating regions. Take Platoon A as an example. The first region is from zero density to 14 vpm; in this region average spacing is greater than both the required absolutely safe spacing and the marginally safe spacing. The second identified region is between 14 vpm and 33 vpm. In this region average spacing is less than the required absolutely safe spacing but greater than the required marginally safe spacing. In the third region, between 33 vpm and 73 vpm, the average spacing is less than both the absolutely safe spacing and the marginally safe spacing. This region seems to offer the lowest degree of safety as far as safe spacings are concerned. The fourth region, from 73 vpm to 112 vpm, has a similar safe spacing relationship as the second region; however, the operating speed and actual spacings are considerably lower than in the second region. The last region, from 112 vpm to jam density, has average spacings greater than both the required absolutely safe spacing and marginally safe spacing. This condition is similar to the first region; however, actual spacings in this region are very low and vehicles are moving at very low speeds.
In addition to the five operating regions that can be identified by the safe spacing analysis, several findings can be extracted from the graphs.

1. The result of the marginally safe spacing analysis tends to confirm the assumption that the Point of Recovery and the Point of Returning to Normalcy defined from the hysteresis analysis do separate different traffic operating conditions.

2. It appears that traffic is more concerned about marginally safe spacings than absolutely safe spacings. Marginal safety was maintained nearly all of the time.

3. From the absolutely safe spacing graphs, it can be seen that only at very low densities (less than 10 vpm) can drivers drive at speeds exceeding 70 mph and be considered safe since the absolutely safe spacing requirement is satisfied. This seems to be the region with no interaction between vehicles, that is, the region of free flow.

**Average headway analysis**

It is assumed that when a driver drives on a crowded highway he tries to maintain a headway which allows him enough time to respond to any possible traffic changes to assure safe driving. Therefore, the headway values obtained from a traffic stream can be expected to provide information on the traffic condition and it should be possible to use the headway values to establish different operating regions.
Since there was no information collected at low densities, a theoretical analysis of headways at low densities was conducted.

According to its definition, the average headway (in seconds) of a traffic stream can be expressed as

$$ h = \frac{3600}{Q} = \frac{3600}{KV} $$

where Q is the volume of the traffic stream in vph, K is the density of the traffic stream in vpm and V is the average speed of the stream in mph. For different values of speeds, the theoretical relationship between h and K is plotted in Figure 23. It can be seen from the graph that for any speed, the average headway drops very quickly at first and then tapers off. In reality, since the average speed of the vehicles will be 60-70 mph in the low density region, it can be expected that the actual headway-density curve would be well confined in the narrow band bounded by the 50 mph and 80 mph curves. With the help of this theoretical analysis, headway-density relationships were estimated in the low density region for the two groups of vehicles. This information was added to the actual data collected and is shown in Figures 24 and 25.

On the graphs, five division lines that separate different operating regions are also shown. The positions of the first four division lines are determined by the previously discussed hysteresis phenomenon and the safe spacing analyses. The estimate of the last division line is based on the average headway analysis.
Figure 23. Theoretical Headway Variations at Low Density Region
Figure 24. Average Headway Versus Density for Platoon A (Prior-Queue Conditions)
Figure 25. Average Headway Versus Density for Platoon B (Prior-Queue Conditions)
Considering Platoon A, the variance of its average headway with density can be described on a region-by-region basis in the following manner. In the region between zero density and 14 vpm, average headway drops sharply from infinity to about 5 seconds. In the region between 14 vpm and 33 vpm, average headway drops from 5 seconds to 2 seconds at a moderate rate. After the average headway reaches the 2 second value, a further increase of density no longer causes a reduction in average headway. In the region between 33 vpm and 73 vpm, average headway remains constant at 2 seconds. In the region between 73 vpm and 112 vpm, average headway assumes a value of 2.2 seconds most of the time, but increases slowly when density approaches 112 vpm. When density further increases, average headway increases with increasing density, first slowly and then sharply, as the density approaches its maximum value. The important point of the above description is that each of the operating regions defined previously characterizes a different average headway pattern.

Since a larger headway means less urgency in responding to traffic changes than a smaller headway, changes of density should have less effect on speed changes in a region with longer headways than in a region with smaller headways. In other words, traffic operating under different average headway conditions should perform differently. The observation presented in this analysis seems not only to confirm that the operating regions defined
in previous analyses are reasonable, it also indicates that average headway itself can be used to define different operation regions.

By further examining the average headway data of Platoon A, it seems that an additional density break-point that separates different operating regions can be positioned at 156 vpm, based on the average headway argument. It can be seen that the region between 112 and 156 vpm average headway exhibits a similar variational pattern as in the region between 73 vpm and 112 vpm; i.e., average headway remains constant at first but increases as traffic approaches another operating region. It can also be seen that average headway values in the two neighboring regions which are separated by \( k = 156 \) vpm fall into two different brackets.

For Platoon B, the additional density break-point defined based on average headway considerations is estimated to be at 128 vpm.

Besides the above discussions, the observed average headway seems to have a minimum value of 2 seconds when the hysteresis effect is not considered.

**Speed dispersion analysis**

In Figure 26 the standard deviation of speeds of vehicles in Platoon A is plotted against density. The corresponding information for Platoon B is shown in Figure 27. The previously defined operating regions are separated by dividing lines shown on the graphs. Since no data was observed in the low density regions, the dividing line in this region is not shown.
Figure 26. Standard Deviation of Speeds Versus Density for Platoon A (Prior-Queue Conditions)
Figure 27. Standard Deviation of Speeds Versus Density for Platoon B  
(Prior-Queue Conditions)
By examining the graphs, speed dispersions are evidently different in each of the operating regions. Since speed dispersion contains information on vehicle interactions which affects the performance of traffic, the different speed dispersion patterns observed in each region tend to confirm that the previously defined operating regions are indeed reasonable.

2.4 Summary of Observed Traffic Characteristics and Their Influence on Speed-Density Relationships

From previously observed traffic characteristics, the traffic domain seems to contain six different traffic operating regions. In order to provide an all-in-one reference to this claim, previously discussed parameters and their defined operating regions are shown collectively in Figures 28 and 29 for Platoon A and Platoon B, respectively. The presented parameters are average speed ($v$), absolutely safe spacing ($S_a$), average headway (H), and the standard deviation of speeds ($\sigma_v$). Marginally safe spacing is not included due to the limited graph size. The variation of average speed before and after a kinematic disturbance is presented to illustrate the hysteresis effect on different operating regions. All other parameters represent only the conditions prior to a kinematic disturbance.

From the graphs, the six regions separated by the dividing lines are characterized by a specific operating condition according to the parameters presented.
Figure 28. Summarization of Studied Parameters and Their Defined Traffic Operating Region for Platoon A
Figure 29. Summarization of Studied Parameters and Their Defined Traffic Operating Regions for Platoon B
As far as the speed-density relationships with respect to the operating regions are concerned, a discussion based on driving responsibility can be made. This discussion is mainly a collective interpretation of traffic behavior as exhibited by the analyzed traffic parameters.

Considering Platoon A, when vehicles travel in the density region from zero to 14 vpm, drivers enjoy an average headway of more than 5 seconds and "absolutely safe spacings" are provided. There is no urgency for any drivers to respond to the change of the traffic environment occurring in their journey. Therefore, changes in density will have no or little effect on drivers' speeds.

When the vehicles travel in the density region from 14 vpm to around 33 vpm, the average headway is reduced from 5 seconds to 2 seconds and a transition from absolutely safe car-following conditions to marginally safe car-following conditions occurs. Considering the observed minimum average headway of 2 seconds under normal operations, this region seems to provide drivers with some freedom in their maneuverability. Therefore, changes of density will have some effect on speeds, but not very much.

When the vehicles travel in the density region between 33 vpm and 73 vpm, drivers are driving under a condition where average headways are at two seconds and the average spacing is less than the spacing required for maintaining marginal safety. Changes of density should have the greatest
effect on speeds in this region. Since the average headway value remains the same and the speed distribution does not change very much throughout this region, the unit change of average speed due to a unit change of density remains constant.

In the operating region between 73 vpm and 112 vpm, drivers face similar driving conditions as in the previous region but with a slightly increased average headway. The most important difference is that vehicles are driving at more uniform speeds than in the previous region. Therefore, the effect of a change in density on speed should be less pronounced than in the previous region.

For the two remaining operating regions which are defined by 112 vpm to 156 vpm and 156 vpm to jam density, the previously stated arguments on average headway, safe spacings and speed dispersions can still be applied and the result is that changes of density do have little effect on speeds.

2.5 Proposed Multi-Linear Speed-Density Hypothesis

In general, the relationships between speed V and density K can be represented by a differential equation

\[ \frac{dV}{dK} = -C \quad (0 \leq K \leq k_j) \]

where the minus sign indicates an increase in density causes a decrease in speed and "C" indicates the particular effect of density changes on changes of speed. By assuming different "C" functions, different speed-density
relationships can be presented. For example, a constant "C" value indicates speed and density have a linear relationship.

From previous discussion, if only prior queue conditions of traffic are to be considered, it seems that changes in density have different effects on changes in speed in the six previously defined operating regions. In other words, the speed-density differential equation can be rewritten as:

\[
\frac{dV}{dK} = \begin{bmatrix}
-b_1 & \text{(for the first region)} \\
-b_2 & \text{(for the second region)} \\
-b_3 & \text{(for the third region)} \\
-b_4 & \text{(for the fourth region)} \\
-b_5 & \text{(for the fifth region)} \\
-b_6 & \text{(for the sixth region)}
\end{bmatrix}
\]

\((0 \leq K \leq k_j)\)

In the above expression, \(b_i\) represents the specific effect of density changes on changes of speed in the \(i\)-th operating region. From the material presented in Section 2.4, it seems that two assumptions with regard to the properties of the \(b_i\)'s can be made. The first one is that the \(b_i\)'s are constants. The second one is that the \(b_i\)'s of neighboring operating regions are different. It also can be claimed that \(b_1 < b_2 < b_3 > b_4 > b_5 > b_6\).

Based on the first assumption, a speed-density hypothesis can be obtained by integration. The resulting relationships are:
\[ V = \begin{cases} 
  a_1 - b_1 & \text{(for the first region)} \\
  a_2 - b_2 & \text{(for the second region)} \\
  a_3 - b_3 & \text{(for the third region)} \\
  a_4 - b_4 & \text{(for the fourth region)} \\
  a_5 - b_5 & \text{(for the fifth region)} \\
  a_6 - b_6 & \text{(for the sixth region)} 
\end{cases} 
(0 \leq K \leq k_j) \]

where \( a_i \)'s (i = 1, \ldots, 6) are integration constants.

Physically, this proposed speed-density relationship presents a multi-linear function which consists of six linear segments. It can also be claimed that the slope of the linear line in the third region exhibits the steepest slope. Judging from the nature of traffic that speed and density are varied in a continuous manner with respect to time, the six linear segments present a continuous function with no discontinuities when traffic moves from one operating region to another.

Before the proposed speed-density hypothesis is to be verified, it is intended to stress again that this proposed hypothesis is for prior queue conditions only.
CHAPTER III

VERIFICATION OF THE PROPOSED MULTI-LINEAR SPEED-DENSITY HYPOTHESIS

3.1 Method of Investigation

Since the proposed multi-linear speed-density model hypothesizes that speed and density have a linear relationship in the previously defined operating regions, a statistical verification of the model must rest mainly upon the proof of the linearity of the speed-density data in each of the operating regions. This process can be accomplished by applying linear regression analyses to speed-density data in each of the regions to obtain their linear correlation coefficients. However, fluctuations of data most likely will cause discontinuity of least-square lines obtained at the junction of two neighboring operating regions. Since continuity is essential for the model to be a valid representation of the speed-density relationship, one must prove that gaps created between neighboring least-square lines are due to errors and not due to any weakness of the model. Besides the above mentioned linearity and continuity of the model, it is also desirable to prove that any two neighboring least-square lines have different slopes. The proof of this matter would confirm the statement that density changes have different effects on speeds in different operating regions.
From the above discussion, the verification of the multi-linear speed-density model is to include the following phases.

1. Apply linear regression analyses to prior-queue speed-density data in every operating region previously defined. For every operating region, the output would involve a linear correlation coefficient of speed-density data, a least-square line to describe the speed-density relationship, and a 90% confidence interval for the least-square line.

2. From the previous phase, constructed least-square lines will most likely create gaps at density break-points that separate adjoining operating regions. In order to prove that these gaps are due to chance, so that continuity of the model can be maintained, this phase is designed to test the null hypothesis that gaps created by two neighboring least-square lines at density break-points have an expected value of zero. A t-test is employed for this purpose.

3. This phase is designed to test whether the slopes of two adjacent least-square lines are significantly different. A t-test is used.

3.2 Statistical Analyses

Data

In order to utilize information obtained in the previous chapter, speed-density data of Platoon A and Platoon B prior to a traffic jam are used in verifying the proposed multi-linear speed-density hypothesis.
Linear regression analysis of the data

Theoretically, for a random sample of $n$ pairs of values of $x$ and $y$ represented by $(x_i, y_i), \ (i = 1, 2, \ldots n)$, the linear correlation coefficient $r$ can be estimated by

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\left[\sum (x_i - \bar{x})^2\right] \left[\sum (y_i - \bar{y})^2\right]}}$$

where

$$\bar{x} = \frac{\sum x_i}{n}, \quad \bar{y} = \frac{\sum y_i}{n}$$

The least-square line $y = a + bx$ for these $n$ pairs of $x$ and $y$ values can be constructed by calculating its slope "$b$" according to the equation

$$b = \frac{n \sum x_i y_i - \sum x_i \sum y_i}{n \sum x_i^2 - (\sum x_i)^2}$$

and its $y$-intercept "$a$" can be calculated by

$$a = \bar{y} - b\bar{x}$$

According to statistical theory, the "$a$" and "$b$" have normal distributions, which provide the ability to calculate confidence intervals. For "$b$", the 100 $(1 - \alpha)$ percent confidence interval is between

$$b + t_{n-2, 1-\alpha/2} \cdot \hat{\sigma}_b \quad \text{and} \quad b - t_{n-2, 1-\alpha/2} \cdot \hat{\sigma}_b$$

where $t_{n-2, 1-\alpha/2}$ is the $t$ value obtained for $(n-2)$ degrees of freedom and a cumulative probability of $1 - \alpha/2$. The $\hat{\sigma}_b$ is the estimated standard
deviation of "b" and it can be calculated from

\[ \hat{\sigma}_b = \left( \frac{\sum(y_i - a - bx_i)^2}{(n-2)\sum(x_i - \bar{x})^2} \right)^{1/2} \]

For the y-intercept "a", the 100 (1 - \alpha) percent confidence interval is between

\[ a \pm t_{n-2,1-\alpha/2} \hat{\sigma}_a \] and \[ a \pm t_{n-2,1-\alpha/2} \hat{\sigma}_a \]

where \( \hat{\sigma}_a \) is the estimated standard deviation of "a" and it can be calculated from

\[ \hat{\sigma}_a = \left( \frac{\sum(y_i - a - bx_i)^2}{n-2} \right)^{1/2} \left( \frac{1}{n} + \frac{\bar{x}^2}{\sum(x_i - \bar{x})^2} \right) \]

For Platoon A and Platoon B data, linear regression analyses were applied. Due to the lack of observations in the low density regions, only four out of the six previously defined operating regions can be analyzed. Figures 30 and 31 show the speed-density data and the fitted least-square lines for each region. The obtained correlation coefficients, the equations of the least-square lines, and the 90% confidence intervals for slopes and y-intercepts are given in Table 1.

From the graphs, it can be seen that Platoon A has an estimated jam density of 222 vpm and Platoon B has an estimated jam density of 214 vpm.

Test for continuity of the multi-linear model

From Figures 30 and 31 gaps can be seen between the two neighboring least-square lines at every break-point that separates two adjacent operating regions. Since speed and density of a platoon of vehicles should vary continuously, the discontinuation of neighboring least-square lines must be due to
Figure 30. Multi-linear Speed-density Function for Platoon A (Prior Queue Conditions)
Figure 31. Multi-linear Speed-density Function for Platoon B (Prior-Queue Conditions)
TABLE 1

RESULTS OF REGRESSION ANALYSES FOR SPEED-DENSITY DATA IN EACH OPERATING REGION

<table>
<thead>
<tr>
<th>Platoon</th>
<th>Operating Regions</th>
<th>Least Square Line $V = a + bk$</th>
<th>$r$ (%)</th>
<th>$\hat{a}$</th>
<th>$\hat{b}$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>33 - 73</td>
<td>$V = 71.24 - 0.81k$</td>
<td>99.56</td>
<td>0.77</td>
<td>0.015</td>
<td>69.90</td>
<td>72.58</td>
</tr>
<tr>
<td></td>
<td>73 - 112</td>
<td>$V = 46.10 - 0.29k$</td>
<td>94.27</td>
<td>2.16</td>
<td>0.024</td>
<td>42.36</td>
<td>49.84</td>
</tr>
<tr>
<td></td>
<td>112 - 156</td>
<td>$V = 23.62 - 0.13k$</td>
<td>93.30</td>
<td>3.03</td>
<td>0.023</td>
<td>19.77</td>
<td>37.47</td>
</tr>
<tr>
<td></td>
<td>156 - 222</td>
<td>$V = 24.09 - 0.10k$</td>
<td>99.24</td>
<td>1.38</td>
<td>0.008</td>
<td>26.84</td>
<td>27.34</td>
</tr>
<tr>
<td>B</td>
<td>33 - 59</td>
<td>$V = 93.33 - 1.10k$</td>
<td>98.34</td>
<td>2.15</td>
<td>0.051</td>
<td>89.60</td>
<td>97.06</td>
</tr>
<tr>
<td></td>
<td>59 - 160</td>
<td>$V = 54.33 - 0.15k$</td>
<td>99.48</td>
<td>0.99</td>
<td>0.012</td>
<td>52.57</td>
<td>56.09</td>
</tr>
<tr>
<td></td>
<td>100 - 128</td>
<td>$V = 24.45 - 0.12k$</td>
<td>98.96</td>
<td>0.98</td>
<td>0.009</td>
<td>22.36</td>
<td>26.54</td>
</tr>
<tr>
<td></td>
<td>128 - 214</td>
<td>$V = 20.26 - 0.09k$</td>
<td>97.60</td>
<td>1.64</td>
<td>0.010</td>
<td>16.77</td>
<td>23.75</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
errors of observation. Otherwise the multi-linear speed-density model cannot be accepted as a valid representation of speed-density relationships. To prove that the gaps are due to chance is essential to the verification of the model.

At one density break-point, a least-square estimate of speed, $V_1$, can be obtained from the data in the operating region to its left.

According to statistical theory, if a least-square line $y = a + bx$ is constructed from $n$ pairs of $x$ and $y$ values, the $y_0$ of a point $(x_0, y_0)$ on this line has an estimated variance, $\sigma^2_{y_0}$, which is given by

$$\frac{\hat{\sigma}^2_{y_0}}{\sigma^2_{y_0}} = \sum_i \left[ (x_i - \bar{x}) + (\bar{x} - x_0) \right]^2$$

Where $\bar{x} = \frac{1}{n} \sum x_i$ and $\sigma^2_{y_0}$ has degrees of freedom equal to ($n-2$). From this formula, the variance $\sigma^2_{V_1}$ of $V_1$ can be obtained.

Considering the operating region to the right of the break-point, similar calculations can be made and a speed value $V_2$ and its variance $\hat{\sigma}^2_{V_2}$ can be obtained.

In this case, it is obvious that $(V_1 - V_2)$ represents the gap between the two adjacent least-square lines intersecting at the density break-point. Since continuity of the multi-linear model requires that no gap can exist between any two least-square lines at their density break-point, the null hypothesis that the gap $(V_1 - V_2)$ equals zero can be made.
From the nature of observation errors, \( V_1 \) and \( V_r \) can be assumed to have normal distributions. Therefore, \( (V_1 - V_r) \) is also normally distributed with a variance which equals the sum of \( \sigma_1^2 \) and \( \sigma_r^2 \). The aforementioned null hypothesis can be tested by using a t-test. In this case, the t value is obtained from

\[
t = \frac{V_1 - V_r}{\sqrt{\sigma_1^2 + \sigma_r^2}}
\]

(degree of freedom = \( n_1 + n_r - 4 \))

where \( n_1 \) is the number of data points to the left of the break-point and \( n_r \) is the number of data points to the right of the break-point.

The t-test has been applied to Platoon A and Platoon B data, and the results are presented in Table 2.

**Test of differences of slopes**

Since the slope of a speed-density least-square line of one operating region represents the effect of changing density or changes in speed, failure to prove that slopes are significantly different for any two adjacent least-square lines would mean that speed and density follow a smooth curve. It is therefore essential to show that least-square lines in two adjacent regions have significantly different slopes. From linear regression analyses of the data, slopes and their variances of least-square lines in every operating region have been calculated. For two slopes, \( b \) and \( b' \), and their variances, \( \sigma_b^2 \) and \( \sigma_{b'}^2 \), a t-statistic can be obtained by using the following formula:

\[
t = \frac{b - b'}{\sqrt{\sigma_b^2 + \sigma_{b'}^2}}
\]

(degree of freedom = \( n + n' - 4 \))
## Table 2

**T-Test Results of Continuity of the Multi-Linear Speed Density Model**

<table>
<thead>
<tr>
<th>Platoon</th>
<th>Density Break Points (vpm)</th>
<th>$V_i$ (mph)</th>
<th>$\hat{\sigma}_i^2$ (mph)</th>
<th>$V_r$ (mph)</th>
<th>$\hat{\sigma}_r^2$ (mph)</th>
<th>$t = \frac{V_i - V_r}{\sqrt{\hat{\sigma}_i^2 + \hat{\sigma}_r^2}}$</th>
<th>Degree of Freedom</th>
<th>5% Critical $t$ Value</th>
<th>$V_i - V_r = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>73</td>
<td>23.72</td>
<td>.1586</td>
<td>24.59</td>
<td>.1903</td>
<td>1.471</td>
<td>27</td>
<td>2.052</td>
<td>Accepted</td>
</tr>
<tr>
<td></td>
<td>112</td>
<td>12.95</td>
<td>.3233</td>
<td>13.15</td>
<td>.2265</td>
<td>0.279</td>
<td>21</td>
<td>2.080</td>
<td>Accepted</td>
</tr>
<tr>
<td></td>
<td>156</td>
<td>6.94</td>
<td>.4532</td>
<td>7.24</td>
<td>.0403</td>
<td>0.427</td>
<td>5</td>
<td>2.571</td>
<td>Accepted</td>
</tr>
<tr>
<td>B</td>
<td>59</td>
<td>27.90</td>
<td>.9410</td>
<td>29.84</td>
<td>.0766</td>
<td>1.840</td>
<td>28</td>
<td>2.048</td>
<td>Accepted</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>12.83</td>
<td>.0751</td>
<td>12.25</td>
<td>.0122</td>
<td>1.970</td>
<td>16</td>
<td>2.120</td>
<td>Accepted</td>
</tr>
<tr>
<td></td>
<td>128</td>
<td>8.83</td>
<td>.0327</td>
<td>8.52</td>
<td>.1452</td>
<td>0.736</td>
<td>8</td>
<td>2.306</td>
<td>Accepted</td>
</tr>
</tbody>
</table>
where \( n \) is the number of data points used to derive \( b \) and \( \frac{1}{\sigma_b^2} \), and \( n \) is the number of data points used to derive \( b' \) and \( \frac{1}{\sigma_b'^2} \).

From the calculated \( t \) value, the null hypothesis that \( b \) is not significantly different from \( b' \) can be tested. This test is applied to Platoon A and Platoon B data; results are given in Table 3.

3.3 Discussion and Conclusions of Statistical Testing

Shortcomings of the data

The major shortcoming of data used in the verification phase is that insufficient data points have been observed. Even though the material presented in Chapter II indicated that the whole traffic domain is capable of being divided into six different operating regions, no data was observed in the first two regions. Consequently, only four regions can be verified. It can be seen also that data points in the fifth and the sixth region are rather scarce.

Discussion of the experimental results

In the linear regression analysis, high linear correlation coefficients were found for speed-density data in every operating region analyzed. The lowest value found is 94.27% and others exceed 97%. These high linear correlation coefficients seem to indicate that speed and density have a strong tendency of being linearly related in each of the operating regions tested.

In the test of continuity of the multi-linear model, all gaps which appeared at the density break-points were found to merely due to chance.
<table>
<thead>
<tr>
<th>Platoon</th>
<th>Regions Compared (vpm)</th>
<th>( b )</th>
<th>( \hat{\sigma}_b^2 )</th>
<th>( b' )</th>
<th>( \hat{\sigma}_b'^2 )</th>
<th>( t = \frac{b - b'}{\sqrt{\frac{\hat{\sigma}_b^2 + \hat{\sigma}_b'^2}{n}}} )</th>
<th>Degree of Freedom</th>
<th>5% Critical t value</th>
<th>Null Hypothesis</th>
<th>( b = b' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>33 - 73 to 73 - 112</td>
<td>0.651</td>
<td>0.000225</td>
<td>0.296</td>
<td>0.000576</td>
<td>12.54</td>
<td>27</td>
<td>2.052</td>
<td>Rejected</td>
<td></td>
</tr>
<tr>
<td></td>
<td>73 - 112 to 112 - 156</td>
<td>0.296</td>
<td>0.000576</td>
<td>0.139</td>
<td>0.000529</td>
<td>4.76</td>
<td>21</td>
<td>2.080</td>
<td>Rejected</td>
<td></td>
</tr>
<tr>
<td></td>
<td>112 - 156 to 156 - 222</td>
<td>0.139</td>
<td>0.000529</td>
<td>0.108</td>
<td>0.00064</td>
<td>1.28</td>
<td>5</td>
<td>2.571</td>
<td>Accepted</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>33 - 59 to 59 - 100</td>
<td>1.109</td>
<td>0.002600</td>
<td>0.415</td>
<td>0.00144</td>
<td>13.33</td>
<td>28</td>
<td>2.048</td>
<td>Rejected</td>
<td></td>
</tr>
<tr>
<td></td>
<td>59 - 100 to 100 - 128</td>
<td>0.415</td>
<td>0.000144</td>
<td>0.122</td>
<td>0.00081</td>
<td>13.96</td>
<td>16</td>
<td>2.120</td>
<td>Rejected</td>
<td></td>
</tr>
<tr>
<td></td>
<td>100 - 128 to 128 - 214</td>
<td>0.122</td>
<td>0.000081</td>
<td>0.091</td>
<td>0.000100</td>
<td>2.38</td>
<td>8</td>
<td>2.306</td>
<td>Rejected</td>
<td></td>
</tr>
</tbody>
</table>
The result seems to indicate that the proposed multi-linear model satisfies the continuity requirement.

In the test on differences of slopes, the slopes of any two neighboring least-square lines, with the exception of one pair, were found to be significantly different from one another. This result then favors the assumption that changing density has different effects on speed changes in different operating regions.

Judging from the results of all of the statistical tests made, it appears that the proposed multi-linear speed-density model, which is continuous and consists of several linear segments, is statistically acceptable as a representation of speed-density relationships for the density range analyzed.

In Figures 32 and 33, volume-density plots of Platoon A and Platoon B are presented. In the low density regions three lines are drawn to represent volume-density relationships for constant speeds of 50, 60 and 70 mph. Since speeds in the low density region would most likely vary within the range from 50 mph to 70 mph, the actual volume-density curve is expected to fall within the narrow band bounded by the 50 mph and the 70 mph lines. Guided by these two lines and based on the assumption that vehicles travel at 70 mph when densities are below 10 vpm, the volume density curves in the low density region were estimated and are shown by the dotted lines. For the remaining portion of the graphs, actual density-volume data and theoretical curves derived from fitted multi-linear curves are presented. From the illustrated
Figure 32. Derived Volume-Density Function for Platoon A (Prior-Queue Conditions)
Figure 33. Derived Volume-Density Function for Platoon B (Prior-Queue Conditions)
graphs, it can be seen that the analyzed density range not only cover the high density, low volume regions where most traffic problems occur, but also the region where traffic reaches its maximum volume.

These are the regions which are most important in terms of freeway traffic control procedures. Therefore, the multi-linear speed-density model will lend itself well to the application of freeway control strategies.
CHAPTER IV
PROPOSED TRAFFIC OPERATING REGIONS
LEVELS OF SERVICE, AND CONTROL STANDARDS

4.1 Introduction

To increase the capacity of any roadway, physical expansion of the facility and operational control are the two major means to accomplish it. However, physical expansion of facilities is subject to many limitations and often is not feasible. Therefore, traffic control is frequently the only tool that a traffic engineer possesses to improve traffic conditions. Effective control requires traffic operating conditions to be distinguished and proper labels of level of service to be attached so that proper control standards can be proposed. However, this cannot be done before different traffic conditions are identified and their operational characteristics understood. This is why the analysis of traffic characteristics under different traffic conditions continues to be one of the most basic topics for traffic researchers. Various methods have been proposed and terms such as free flow, stable flow, normal flow, unstable flow, forced flow, etc., were used and loosely defined in an attempt to describe the different traffic conditions. Examples of research in this area can be found in References 9, 10, 11 and 12. A widely used parameter in such attempts has been the operating speed or the average speed of a traffic stream.
Although most of the previous work has considered the average speed as the most promising parameter, it is felt that density is an improved measure in distinguishing traffic operating regions. In general, speeds range from zero to 70 mph, the speed limit on most freeways; densities range from zero to jam density of 220 vpm. Consequently, using densities as the boundaries of the different traffic regions would be more accurate than using the average speed. Density can be used in all cases while the upper limit of speed is restricted by geometric and control conditions of roadway facilities. In addition, a density value can more correctly present a picture of traffic conditions than speed. For instance, let us consider a traffic stream moving at an average speed of 30 mph and at a density of 60 vpm. If only one of the two figures is to be used to describe the traffic stream, the density value of 60 vpm will suggest that the average vehicle spacing is at 88 feet (which is about 4 to 5 vehicle lengths). This traffic condition can be visualized in an imaginary manner and the associated traffic speed can often be estimated. On the other hand, merely mentioning the fact that traffic moves at 30 mph will not directly lead to such an imaginary picture of the actual condition.

4.2. Delineating Traffic Operating Regions

From previous analyses, it appears that the whole traffic domain can be divided into six operating regions; however, two problems have to be solved before the obtained information can be applied for control purposes.
The first problem is the determination of acceptable density-break-point values that can be used in every day traffic conditions. For Platoon A, the five density break points that separate the six operating regions were found to be 14 vpm, 33 vpm, 73 vpm, 112 vpm, and 156 vpm. For Platoon B, the values are 12 vpm, 33 vpm, 59 vpm, 100 vpm, and 128 vpm. In a study conducted at the Transportation Research Center of The Ohio State University, the behavior of eight platoons from aerial data were studied. From the analysis of the break points of these platoons it was felt that 30 vpm, 60 vpm, 105 vpm, and 145 vpm can be adopted as border lines for the second, third, fourth, and fifth density-break-points, respectively. The first density break point is determined to be 10 vpm for common usage, reasons for this decision are explained later.

The second problem is the determination of the effect of the hysteresis phenomenon on the characteristics of operating regions. From previous analyses, it is known that the hysteresis phenomenon can affect the maximum volume value of traffic, the position of density where the maximum volume occurs, and traffic behavior at a specified density level; however, the hysteresis phenomenon does not affect the positions of density-break-points that separate operating regions. It is also true that the hysteresis phenomenon does not change the relative performances of operating regions. For example, the third region is always the region where capacity, maximum volume, of a traffic stream occurs.
From the above discussions, the operating characteristics of each region can be generally described in the manner presented below; however, those traffic characteristics which are under the influence of hysteresis phenomena are not included so as to avoid increasing the complexity of the descriptions.

1. Free flow region. This region has a density span from zero to 10 vpm. From the absolutely safe spacing analyses of Figure 22, it can be seen that for a speed of 70 mph and a reaction time of 2 secs, the provided average spacing between vehicles at 12 vehicles per mile met the absolute safety requirements. It seems that driving at 70 mph and being able to maintain an absolute safe spacing are good criteria for free flow conditions. Therefore, the rounded figure of 10 vehicles per mile is thought to be a reasonable choice as the upper limit of the free flow region. From another viewpoint, since a small increase of density in this low density region would cause a tremendous decrease of average spacing between vehicles, the choice of 10 vehicles per mile is on the safe side.

2. Semi-free flow region. This region covers the density span from 10 vehicles per mile to 30 vehicles per mile. In this region the safe spacing analysis applies. Absolutely safe spacing is not maintained all the time, but marginally safe spacing is provided. The average speed of the vehicles in this range vary from 50 mph to 70 mph. The high operating speed indicates vehicles in this region still have a great deal of freedom in their movement. This is why this region is termed a semi-free flow region.
3. Capacity flow region. This region covers the density span from 30 vpm to 60 vpm. In this region the average headway is lower than 2 secs and varies in a narrow range of 1.8 sec to 2 secs. The 1.8 sec average headway is thought to be the minimum average headway value that is acceptable to most drivers in normal operating conditions. Marginally safe spacing is maintained in this region only when response times are less than 1.8 sec requiring considerable alertness on the part of the drivers. The nearly constant but low value of the average headway in this region indicates the tremendous influence of changing density on average speed. The average speed has a wide range of variation from 60 mph to 25 mph. The relatively wide range of speed variations (individual vehicles) as shown by the vehicle interaction study indicates that the choice of individual speed is not severely restricted which tends to suggest that this region is tolerable to most drivers. Since this is the region where the capacity, maximum volume, of traffic is obtained, it is termed the capacity flow region.

4. Restricted flow regions. This region is bounded by 60 vpm and 105 vpm. In this region, the average headway value is somewhat higher than 2 seconds but remains in a narrow range. Marginally safe spacing (for a response time of 2 seconds) is barely satisfied in this region. This indicates driver alertness is still required in this region. The average speed does not have a wide range as in the previous region which indicates the influence of density changes is not as great as in the previous region. This is because
vehicles are moving with approximately uniform speeds and spacings, thus the shortening of total platoon length due to increased density is absorbed by every vehicle instead of one or two vehicles. Since the vehicles operating in this region tend to float with the rest of the vehicles having little choice of their own speed, this region is called the restricted flow region. The volume in this region is quite high and according to some other speed-density hypotheses this is the region where the volume is at a maximum.

5. Disturbed flow region. This region is bounded by densities of 105 vpm and 145 vpm. In this region the average headway varies little in the range from 2.5 seconds to 3.5 seconds. The absolute and marginal safety spacings are provided in this region because of the low average speed. Despite the low average speed, the standard deviation of speeds is relatively high (much higher than in the previous region) which indicates the vehicles in this region no longer move in a uniform manner. It is termed the disturbed flow region to describe its non-uniform flow condition.

6. Forced flow region. This is the region where density exceeds the value of 145 vpm. The average headway value begins around 3.4 seconds when traffic enters this region and increases sharply as the density approaches kj. Both absolutely safe and marginally safe spacings are provided because of the low average speed. The term forced flow is self-explanatory of the condition in this region.

The above traffic operating regions and their operating characteristics are summarized in Table 4.
### TABLE 4

TRAFFIC OPERATING REGIONS AND THEIR OPERATING CHARACTERISTICS OR UNINTERRUPTED FLOWS

<table>
<thead>
<tr>
<th>TRAFFIC FLOW CONDITIONS</th>
<th>DENSITY RANGE</th>
<th>OPERATING SPEED RANGE</th>
<th>AVERAGE HEADWAY</th>
<th>STANDARD DEVIATION OF VEHICLE SPEEDS</th>
<th>SAFETY SPACINGS CONSIDERATION</th>
<th>GENERAL DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free Flow</td>
<td>0 to 10</td>
<td>≥70</td>
<td>&gt; 5 secs, drops quickly when density increases</td>
<td>VARY</td>
<td>Absolute safety spacing provided at all times</td>
<td>Complete freedom of movement of individual vehicle</td>
</tr>
<tr>
<td>Semi-Free Flow</td>
<td>10 to 30</td>
<td>50-70</td>
<td>drops slowly from 5 secs to 2 secs as density increases</td>
<td>VARY</td>
<td>Marginal safety spacing provided at all times</td>
<td>Freedom of movement of individual vehicle is partially restricted</td>
</tr>
<tr>
<td>Capacity Flow</td>
<td>30 to 60</td>
<td>25-60</td>
<td>≤2.0 secs, most values stay around 1.9 secs.</td>
<td>Relatively high safety spacing only with driver alertness</td>
<td>Average speed is restricted, but freedom of choosing individual speed is not seriously restricted. (Mass, kinetic energy and capacity attained)</td>
<td></td>
</tr>
<tr>
<td>Restricted Flow</td>
<td>60 to 105</td>
<td>13-30</td>
<td>≥2.0 secs, most values stay around 2.1 secs</td>
<td>Relatively low</td>
<td>Marginal safety spacing barely provided</td>
<td>Average speed as well as individual speed are highly restricted; vehicles &quot;flow&quot; in the traffic stream.</td>
</tr>
<tr>
<td>Disturbed Flow</td>
<td>105 to 145</td>
<td>8-15</td>
<td>most values remain constant in the range between 2.5 secs to 3.0 secs</td>
<td>Relatively high</td>
<td>Absolute safety spacing provided at all times</td>
<td>Speed variation of individual speed associated with low average speed indicates the traffic flow is disturbed.</td>
</tr>
<tr>
<td>Forced Flow</td>
<td>145 to Kj</td>
<td>≤8</td>
<td>&gt; 3.5 secs, sharply as density approaches Kj.</td>
<td>Relatively low</td>
<td>Absolute safety spacing provided at all times</td>
<td>Vehicle maneuverability is severely restricted.</td>
</tr>
</tbody>
</table>
4.3. Level of Service for Uninterrupted Flow

The late Dr. Schwar, when he was the chairman of the Highway Research Board Committee on Quality of Traffic Service in 1966, wrote an article entitled 'Quality of Traffic Service.' The basic qualitative considerations involved in the problem of level of service were well summarized in the article. He quoted the tentative definition of level of service suggested by the committee as "The quality of level of traffic service is a measure of the adequacy of a road, street, highway or system when compared to desirable and practical standards . . . . A meaningful and easily measured parameter is average over-all speed. This measure of traffic service is influenced by and affects other parameters." The last sentence in the above quoted statement simplified the problem by recognizing that participating parameters are not independent of each other. Thus, one parameter should be representative of all parameters considered.

Apparently this philosophy was adapted by the authors of the 1965 Highway Capacity Manual and level of service standards for uninterrupted flow were accordingly suggested. The resultant tabular form of the HCM recommendation is duplicated and shown in Table 5.

From the information obtained in Section 4.2., one should be able to establish a system for level of service based on density considerations. However, before such a system is introduced, studying of the HCM system should be very helpful. In order that a common ground for comparison can be gained,
TABLE 5

LEVELS OF SERVICE AND MAXIMUM SERVICE VOLUMES FOR FREeways AND EXPRESSWAYS UNDER UNINTERRUPTED FLOW CONDITIONS (HCM 1965)

<table>
<thead>
<tr>
<th>LEVEL OF SERVICE</th>
<th>TRAFFIC FLOW CONDITIONS</th>
<th>SERVICE VOLUME/CAPACITY (s/c) RATE</th>
<th>MAXIMUM SERVICE VOLUME UNDER IDEAL CONDITIONS, INCLUDING 70-MPH AVERAGE HIGHWAY SPEED (TOTAL PASSENGER CARS PER HOUR, ONE DIRECTION)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>BASIC LIMITING VALUE FOR AVERAGE HIGHWAY SPEED (MPH) OF 70 MPH, FOR:</td>
<td>4-LANE FREEWAY (2 LANES/ ONE DIRECTION)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4-LANE FREEWAY (2 LANES/ DIRECTION)</td>
<td>6-LANE FREEWAY (3 LANES/ DIRECTION)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>OPERATING SPEED (MPH)</td>
<td>60 MPH</td>
</tr>
<tr>
<td>A Free flow</td>
<td></td>
<td>560</td>
<td>20.35</td>
</tr>
<tr>
<td>B Stable flow</td>
<td></td>
<td>555</td>
<td>20.30</td>
</tr>
<tr>
<td></td>
<td>Stable flow (upper speed range)</td>
<td>550</td>
<td>20.35</td>
</tr>
</tbody>
</table>

| C Stable flow    |                          | 50 | 20.75XPHF | 20.80XPHF | 20.83XPHF | 20.45XPHF | 1000 |
| D Approaching unstable flow |                          | 540 | 20.50XPHF | 20.80XPHF | 20.45XPHF | 1000 |
| E Unstable flow  | 30-35° | 21.00 | 4000 |
| F Forced flow    | <30° | Not meaningful | 2000 |

* Operating speed and head; s/c ratio are independent measures of level of service; both limits must be satisfied in any determination of level.
* Operating speeds required for level I is not attainable even at low volumes.
* Stable flow rate for freeways in the ratio of the whole-hour volume to the highest rate of flow occurring during a 3-min interval within the peak hour.
* A peak-hour factor of 1.00 is assumed; the volume listed here should be considered as maximum average flow rates likely to be achieved during the peak 3-min interval within the peak hour.
* Capacity.
the speed system suggested in Table 5 has to be transferred to a density-oriented system. This transformation can be achieved by employing the deterministic relationship between speed, density and volume. In Column 3 of Table 5, the minimum operating speeds for each service level are listed. Their corresponding upper density limits can be calculated by using the maximum volume values appearing in Column 9 of the same table divided by the speed values in Column 3. Such calculations have been made and the results are summarized in Table 6.

Comparing Table 5 and Table 4 with the help of values in Table 6, we can see that:

1. The region of Level of Service A suggested by the Highway Capacity Manual is the free flow region defined by this study.

2. The combined region of Levels of Service B and C specified in the HCM as the stable flow region covers the same range suggested as the semi-free flow region by this study.

3. The combined region of Levels of Service D and E specified in the HCM as the unstable flow region covers the same range suggested as the capacity flow region by this study.

4. The region of Level of Service F suggested by the HCM is further divided into a restricted flow region, a disturbed flow region, and a forced flow region in this study.
<table>
<thead>
<tr>
<th>Level of Service</th>
<th>Maximum Service Volume (Column 9 of Table 5) VPH/2 Lanes</th>
<th>Min. Operating Speed (Column 3 of Table 5) MPH</th>
<th>Upper Density Limit VPM/Lane</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1400</td>
<td>60</td>
<td>11.7</td>
</tr>
<tr>
<td>B</td>
<td>2000</td>
<td>55</td>
<td>18.5</td>
</tr>
<tr>
<td>C</td>
<td>3000</td>
<td>50</td>
<td>30.0</td>
</tr>
<tr>
<td>D</td>
<td>3600</td>
<td>40</td>
<td>45.0</td>
</tr>
<tr>
<td>E</td>
<td>4000</td>
<td>30</td>
<td>60.0</td>
</tr>
<tr>
<td>F</td>
<td>&lt;30</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The above described close association between the traffic regions suggested by the Highway Capacity Manual and the traffic regions defined by this study is not the only common ground found from the comparison. The Highway Capacity Manual suggested that the capacity under an ideal condition is 2000 vph per lane, which occurs when the average speed of the traffic is around 30 mph and within the density range of 45 vpm – 60 vpm (Level of Service E). The aerial photographic data collected in this study agree with these figures.

For a schematic presentation, a comparison between the HCM system and the system established at this study is summarized in Figure 34.

From the findings obtained in Section 2.3 and in the previous section, a new way of establishing the level of service for uninterrupted flow is suggested. This system is illustrated in Table 7.

The operating conditions for these designated levels of service can be described as follows:

Level of service A describes a free flow condition. The speeds of vehicles are mainly controlled by drivers' desires and/or the imposed speed limits and physical roadway conditions. A spacious average headway of greater than 5 seconds is provided so that even sudden speed changes of lead vehicles would not result in serious problems for the following driver. Freedom of driving in terms of maneuverability and comfort of driving in terms of responding to traffic changes are maintained.
Figure 34. Comparison of Traffic Regions Suggested by this Study and Traffic Conditions Suggested by the Highway Capacity Manual (1965)

<table>
<thead>
<tr>
<th>Traffic Conditions suggested at this study</th>
<th>Traffic Conditions suggested by HCM</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Free Flow</strong></td>
<td><strong>Free Flow</strong></td>
</tr>
<tr>
<td><strong>Semi-Free Flow</strong></td>
<td><strong>Stable Flow (upper speed range)</strong></td>
</tr>
<tr>
<td><strong>Stable Flow</strong></td>
<td><strong>Approaching Unstable Flow</strong></td>
</tr>
<tr>
<td><strong>Approaching Unstable Flow</strong></td>
<td><strong>Unstable Flow</strong></td>
</tr>
<tr>
<td><strong>Capacity Flow</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Restricted Flow</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Disturbed Flow</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Forced Flow</strong></td>
<td></td>
</tr>
</tbody>
</table>

**Level of Service suggested by HCM**

- A: Free Flow
- B: Stable Flow (upper speed range)
- C: Stable Flow
- D: Approaching Unstable Flow
- E: Unstable Flow
- F: Forced Flow

Density (v.p.m.)

- 0
- 10
- 30
- 60
- 105
- 145

Traffic Conditions suggested at this study
### TABLE 7

**RECOMMENDED LEVELS OF SERVICE FOR UNINTERRUPTED FLOW CONDITIONS**

<table>
<thead>
<tr>
<th>Level of Service</th>
<th>Traffic Flow Conditions</th>
<th>Density Range (vpm)</th>
<th>Speed Range (mph)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Free flow</td>
<td>0 - 10</td>
<td>&gt; 70</td>
</tr>
<tr>
<td>B</td>
<td>Semi-free flow</td>
<td>10 - 30</td>
<td>50 - 70</td>
</tr>
<tr>
<td>C</td>
<td>Capacity flow</td>
<td>30 - 60</td>
<td>25 - 60</td>
</tr>
<tr>
<td>D</td>
<td>Restricted flow</td>
<td>60 - 105</td>
<td>13 - 30</td>
</tr>
<tr>
<td></td>
<td>Disturbed flow</td>
<td>105 - 145</td>
<td>8 - 15</td>
</tr>
<tr>
<td></td>
<td>Forced flow</td>
<td>&gt; 145</td>
<td>&lt; 8</td>
</tr>
</tbody>
</table>
Level of service B describes a semi-free flow condition. An average headway of more than 2 seconds is provided so that marginal safety is maintained (see Section 2.3 for marginal safety concept). Even though the average speed of the traffic is somewhat restricted, the freedom of selecting one's individual speed is, however, reasonable maintained. It is realized that driving is less restricted in the lower density portion than in the higher density portion of this region; however, no clear-cut dividing point can be found by which this region could be further sub-divided.

Level of Service C describes the capacity flow condition. The average headway maintains a constant value of about 2 seconds. An increase in density causes reduction of spacings between vehicles which in turn would immediately cause a reduction of average speed. Marginal safety is provided with alertness of driving (reaction times of drivers must be less than 1.8 sec). Vehicles are tied together as a continuous flow but individual movement is not really seriously restricted (evidenced from a large value of standard deviation of speeds). It has been found at the low density end of this region that the average speed can be as high as 60 mph and that the average speed at the high density end of the region can be as low as 25 mph. Since the average headway is the same (2 seconds) at both these ends, the alertness needed in driving and the maneuverability (freedom of choosing lanes and speed) enjoyed in the driving should not be very much different. Sub-division of this region into more than one service level according to operating speed is possible but
merely picking any speed value to serve as a boundary of one service level hardly seems logical. Therefore, this Level of Service C suggested here is not further divided. It can be noted that the maximum volume, or capacity, of the traffic stream occur in the range of this service level.

Level of Service D includes all the undesirable traffic conditions defined in this study. It includes the restricted flow region, the disturbed flow region and the forced flow region. Vehicle speeds are low and drivers' maneuverability is severely restricted. When the traffic releases from this stage, it will lead to a potentially dangerous state where chain collisions could happen. Even though several studies conducted showed that three different traffic conditions as described above can be identified, it is felt that the disturbed flow condition and the forced flow condition are the consequences of the restricted flow state instead of being independent of it. Since all of them are undesirable, it does not appear to be worthwhile to further separate them and create some new service levels.

4.4. Proposed Control Standards for Uninterrupted Flow

Traffic changes rapidly in the field. In order for any proposed control system to be effective, the parameter employed must:

1. possess a sound theoretical basis to assure its workability,
2. lend itself for easy and accurate field measurements,
3. be a sensitive indicator for traffic changes.
From all the information gathered so far, a density oriented control system seems promising. Nevertheless, direct use of density as the control parameter imposes some difficulties since the only way to determine density is by counting the number of vehicles in a section of roadway at a time instant. Photogrammetry is a relatively simple way to obtain density values but the time required for processing cannot cope with the change of traffic. Thus, it cannot be used for control purposes.

Fortunately, there is a parameter, lane occupancy, which can be related to density and fulfills all the requirements described above. For any kind of detector, the lane occupancy is defined as the ratio in percent of time that a sensor is occupied by the moving vehicles to the time period considered.

According to Athol, the relationship between the lane occupancy and the density can be expressed as:

\[ \text{Lane occupancy} = \text{average vehicle length} \times \text{density}. \]

Accept 17.6 ft. as the average vehicle length. This relationship can be reduced to an even simpler form:

\[ \text{Lane occupancy} \% = \frac{1}{3} \text{density (vpm)}. \]

From this relationship, standards for controlling traffic at different levels of service can be established by using lane occupancy as the control parameter. The appropriate numerical values are listed in Table 8.
### TABLE 8
RECOMMENDED CONTROL STANDARDS FOR DESIRED LEVELS OF SERVICE

<table>
<thead>
<tr>
<th>Level of service desired</th>
<th>Upper density limit (vpm)</th>
<th>Lane occupancy cannot exceed %</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10</td>
<td>3.3</td>
</tr>
<tr>
<td>B</td>
<td>30</td>
<td>10.0</td>
</tr>
<tr>
<td>C</td>
<td>60</td>
<td>20.0</td>
</tr>
</tbody>
</table>

In order that undesirable traffic conditions be avoided, the traffic should not be allowed to exhibit a lane occupancy value of more than 20%.
CHAPTER V

ACHIEVEMENTS AND RECOMMENDATIONS

5.1 Achievements

Based on the observed fact that traffic operating regions can be determined according to their operating characteristics, the objectives of the study were set (1) to define a functional relationship between the density and the average speed, (2) to define traffic operating regions, (3) to establish levels of service, and (4) to establish control standards. Through direct data manipulation and experimental analyses, the achievements toward fulfilling these objectives can be summarized as follows:

1. A multi-linear speed-density relationship is hypothesized and verified. Based on observed traffic characteristics, the whole traffic domain can be divided into six operating regions. By proposing that speed and density have a linear relationship in each of the six operating regions, a continuous multi-linear speed-density function consisting of six linear segments was hypothesized. The verification of the hypothesis consists of (1) a test of linearity of speed-density data in each of the regions, (2) a test of continuity of the model, and (3) to determine whether slopes of adjacent linear segments are significantly different. Even though no data was available in the first two
regions, test results obtained from data in the four remaining regions which cover the density range from 33 vpm to jam density indicate that the multi-linear speed-density model is statistically acceptable.

2. From analyses on some traffic operational parameters such as average headway, interaction between vehicles, and safety spacings, specific operational characteristics of the six defined regions were identified. According to their operational characteristics, these regions are termed as free flow (0-10 vpm), semi-free flow (10 vpm - 30 vpm), capacity flow (30-60 vpm), restricted flow (60 vpm - 105 vpm), disturbed flow (105 vpm - 145 vpm), and forced flow (145 vpm - kj), respectively.

3. From operational characteristics of the six defined traffic operating regions and the information on level of service documented in the 1965 Highway Capacity Manual, a system for level of service was established. There were four service levels designated; level of service A describes the free flow condition, level of service B describes the semi-free flow condition, level of service C describes the capacity flow condition, and the level of service D represents the combined conditions of restricted flow, disturbed flow and forced flow.

4. Standards for controlling traffic at different levels of service was established by using lane occupancy as the control parameter. Appropriate numerical values were suggested also.
5. While striving to fulfill the objectives of this study, an interesting phenomenon of traffic flow has been observed. The movement patterns exhibited by traffic streams before entering a kinematic disturbance were found to be different from the movement patterns exhibited by traffic after departing from a kinematic disturbance. This phenomenon is termed the hysteresis phenomenon of traffic flow to describe the retarded performance of traffic flow when it recovers from a kinematic disturbance. The detailed illustration and analysis of this phenomenon is presented in Chapter II.

5.2. Recommendations

In the future, work should be done on the following topics:

1. Since this study dealt with data collected from freeway traffic with its uninterrupted flow and its ideal physical environment, the findings, especially the quantitative items, of these studies will not be applicable to traffic under other geometric conditions. It seems research work similar to this study should be conducted for uninterrupted flows under not-ideal geometric environments and for traffic flows on signalized city streets.

   It is felt when the density of traffic exceeds a certain value, the vehicle's influence on each other would be more prominent than the influence of other physical factors (except the signals). Thus traffic would behave similarly no matter what the geometric and other conditions were. Certainly
this assumption needs to be verified. Nevertheless, the approach employed in this study, if applied to non-freeway traffic, is believed to be capable of bringing solutions to many of our present day traffic problems.

2. It has been shown that the whole domain of traffic can be divided into several operating conditions where traffic behaves differently. It is thus possible to evaluate the applicability of existing mathematical traffic models on a regional basis. For instance, the car-following theories certainly have limited application in the free-flow region. As for other regions, using different values of the sensitivity index of a car-following model seems more acceptable than using only one value for all conditions.

Another interesting topic would be to determine the regions where the fluid-analogy models can bring reasonable results. By doing this, the advantages of these models can be saved and their disadvantages can be eliminated.

3. In this study, realizing that traffic behaves differently after it recovers from a kinematic disturbance when compared with the conditions prior to the kinematic disturbance, the major effort was in analyzing the prior-queue conditions. Nevertheless, study of the hysteresis phenomenon of traffic revealed that recovering traffic from a kinematic disturbance resulted in high volume and low average headway conditions. High volume certainly is desirable, however, low average headway conditions may be the main reason for the chain-reaction type of rear-end collisions. A study of
this matter may contribute to increase the capacity and safety of our transportation system.
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