DIEWALD, Walter Joseph, 1944-
INVESTIGATION OF A COMBINED PHOTOGRAPHIC AND
COMPUTER SIMULATION TECHNIQUE FOR USE IN THE
STUDY OF ISOLATED INTERSECTIONS.

The Ohio State University, Ph.D., 1971
Engineering, civil

University Microfilms, A XEROX Company, Ann Arbor, Michigan
INVESTIGATION OF A COMBINED PHOTOGRAPHIC AND COMPUTER SIMULATION TECHNIQUE FOR USE IN THE STUDY OF ISOLATED INTERSECTIONS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

Walter Joseph Diewald, B.C.F., M.S.

The Ohio State University

Approved by

Adviser
Department of Civil Engineering
PLEASE NOTE:

Some Pages have indistinct print. Filmed as received.

UNIVERSITY MICROFILMS
ACKNOWLEDGMENTS

This study was made at the Transportation Engineering Center, Engineering Experiment Station, The Ohio State University, under project, EES 274, "Development of New Intersection Study Techniques". The project was sponsored by the Ohio Department of Highways in cooperation with the Federal Highway Administration, Department of Transportation. The author wishes to express his appreciation to the sponsors for their financial support which made this study possible.

The author wishes to express his appreciation to his adviser, Dr. Joseph Treiterer, Department of Civil Engineering, who also served as Research Supervisor for the project, for his guidance and assistance throughout the study. Acknowledgment is also made to Dr. Zoltan Nemeth, Department of Civil Engineering, for his advice during the course of the study.

The author would also like to acknowledge the assistance of the staff of the Transportation Engineering Center in the data collection and data reduction activities.
VITA

January 11, 1944

Born - Detroit, Michigan

1966

B. C. E. - Department of Civil Engineering, University of Dayton - Dayton, Ohio

1966-1967

Research Assistant, Transportation Engineering Center, The Ohio State University, Columbus, Ohio

1967

M. S. - Department of Civil Engineering, The Ohio State University, Columbus, Ohio

1967-1971

Research Associate, Transportation Engineering Center, The Ohio State University, Columbus, Ohio

PUBLICATIONS


FIELDS OF STUDY

Studies in Urban and Transportation Geography: Professors Reginald Golledge, Howard Gauthier

Studies in Operation Research: Professors Walter Giffin, Thomas Rockwell
TABLE OF CONTENTS

ACKNOWLEDGMENTS .............................................................. ii
VITA ............................................................................. iii
LIST OF TABLES .............................................................. vii
LIST OF ILLUSTRATIONS ............................................. viii

CHAPTER I INTRODUCTION ............................................... 1
   1.1 General
   1.2 Research Objectives
   1.3 Scope of Study

CHAPTER II DATA COLLECTION ............................................ 7
   2.1 The Photographic Technique
   2.2 ROBOT Camera System
   2.3 Camera Installation
   2.4 Intersection Site Selection
   2.5 Amount Collected

CHAPTER III RESULTS OF DATA COLLECTION ......................... 19
   3.1 Data Reduction Procedure
   3.2 Results of Data Collection

CHAPTER IV INTERSECTION SIMULATION MODEL .................... 35
   4.1 Past Studies
   4.2 The Purpose of the Simulation Model
   4.3 Description of the TEC Simulation Model
   4.4 Operational Characteristics of the TEC Model
   4.5 TEC Model Output
   4.6 TEC Model Validation
CHAPTER V INTERSECTION SIMULATION STUDY .......................... 57

5.1 Left-turn Lanes  
5.2 Intersection Study Design  
5.3 Selection of Independent Variables  
5.4 Preparation of the TEC Model; Single-lane Approach  
5.5 Preparation of the TEC Model; Two-lane Approach  
5.6 Results of the Simulation Study; Unchannelized Approach  
5.7 Results of the Simulation Study; Channelized Approach  
5.8 Regression Analysis  
5.9 Comparison with Existing Design Criteria

CHAPTER VI SUMMARY .............................................................. 78

6.1 Summary of Results  
6.2 Research Recommendations

APPENDIX A ............................................................................. 81
APPENDIX B ............................................................................. 82
APPENDIX C ............................................................................. 85
APPENDIX D ............................................................................. 105
REFERENCES ............................................................................. 120
## LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ROBOT Camera Mount System</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>Identification of Intersection Legs on Data Photographs</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>Typical Data Collection Photographs</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>Enlargement of a Pair of Data Photographs</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>Rear Projection Console</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>Plot of Composite Exponential C.D.F.</td>
<td>41</td>
</tr>
<tr>
<td>7</td>
<td>Gap Acceptance Distributions Obtained from the Literature</td>
<td>46</td>
</tr>
<tr>
<td>8</td>
<td>Program Statement Listing Utilizing Regression Equations</td>
<td>74</td>
</tr>
<tr>
<td>9</td>
<td>Sample Program Output</td>
<td>75</td>
</tr>
</tbody>
</table>
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Data Collection Locations</td>
<td>18</td>
</tr>
<tr>
<td>2</td>
<td>Summary of Data</td>
<td>23</td>
</tr>
<tr>
<td>3</td>
<td>Arrival Distribution Test Results</td>
<td>27</td>
</tr>
<tr>
<td>4</td>
<td>Start Delay Test Results</td>
<td>31</td>
</tr>
<tr>
<td>5</td>
<td>Gap Acceptance by Left-turn Vehicles on Four-lane, Two-way Streets</td>
<td>48</td>
</tr>
<tr>
<td>6</td>
<td>Gap Acceptance by Left-turn Vehicles on Two-lane, Two-way Streets</td>
<td>49</td>
</tr>
<tr>
<td>7</td>
<td>Delay Distribution Test Results</td>
<td>54</td>
</tr>
<tr>
<td>8</td>
<td>Traffic Data Comparisons</td>
<td>55</td>
</tr>
<tr>
<td>9</td>
<td>Single-lane Unchannelized Approach Fractional Factorial Results</td>
<td>67</td>
</tr>
<tr>
<td>10</td>
<td>Two-lane Unchannelized Approach Fractional Factorial Results</td>
<td>68</td>
</tr>
<tr>
<td>11</td>
<td>Single-lane Channelized Approach Fractional Factorial Results</td>
<td>69</td>
</tr>
<tr>
<td>12</td>
<td>Two-lane Channelized Approach Fractional Factorial Results</td>
<td>71</td>
</tr>
</tbody>
</table>
CHAPTER I

INTRODUCTION

1.1 General

In the development of a safe and efficient transportation system for a modern, urbanized society, many factors must be given attention. Moreover, since the most dominant mode of transportation in urban areas is the private automobile, the highway transportation system plays a significant role in the movement of people and goods. Highway and traffic researchers are currently engaged in numerous studies directed toward some improvement of the highway transportation system. Emphasis toward improving existing networks has recently been heightened by such programs as the TOPICS (Traffic Operation Program to Increase Capacity and Safety) program which specifically emphasizes improvement of existing networks.

The complex problem of improving existing roadway networks involves the two specific elements of such a network, the streets and the intersections. Intersections limit and often interrupt traffic flow on a roadway and, therefore, they control, to a large extent, the ability of major and secondary arterial streets to accommodate the flow of vehicles. Moreover, less is known about traffic flow through intersections and the factors affecting this flow than traffic flow on the street links.
The accident potential of intersections, where various traffic flows must intersect, merge, and diverge, adds emphasis to the desirability of a greater understanding of intersection traffic flows. More specifically, since two of the most common accident types at intersections, turning and rearend collisions, frequently involve left-turning vehicles, considerable attention to left-turn vehicles and their effect upon intersection traffic flow is not unwarranted. Left-turn movements also contribute substantially to a loss of intersection capacity and an increase of vehicular delay, particularly during periods of peak flow.

Operationally, high percentages of left-turns can be treated in a number of ways, one of which is the provision of a special turning lane through construction or marking. Existing warrants for the addition of a left-turn channel of a signalized intersection can be found in the geometric design handbooks published by The American Association of State Highway Officials (1,2). The total statement regarding left-turn channels and their length is that "the storage length should be based on 1.5 times the average number of vehicles that would store per cycle". This warrant appears to be empirically or intuitively based, although no origin is cited. Moreover, little research has been conducted to determine the interrelationships between intersecting traffic streams. Perhaps the reasons for this apparent lack are the complexity of traffic flow at an intersection and the difficulty involved in collecting the necessary data.
Past studies of intersection operation have, however, shown that photography is an adequate means of collecting intersection data, although its usefulness is somewhat limited with existing techniques. If these limitations could be eliminated, the kinds of data made available would be greatly expanded and the usefulness of the photographic technique greatly enhanced.

Simulation of traffic situations on high-speed computers can be used to provide a large amount of data under controlled laboratory conditions which would be difficult, if not impossible, to obtain through field studies. Simulation of a system is the operation of a model or simulator which involves certain types of mathematical or logical models that describe the behavior of a system over extended periods of real time. Naylor, et al., have pointed out a number of reasons for utilizing computer simulation techniques (3). These reasons include the following:

- the observed system may be so complex that it is impossible to describe it in terms of a set of mathematical equations for which it is possible to obtain analytic solutions which could be used for predictive purposes;
- even though a mathematical model can be formulated to describe some system of interest, it may be impossible to obtain a solution to the model by straightforward analytical techniques and in turn make predictions about the future behavior of the system; and
- It may be impossible or very costly to perform the necessary validating experiments on the mathematical models describing the system. Any of these reasons can be seen to apply to the study of intersection operation to some degree. Moreover, variables or controls can be changed and their effect analyzed in a simulation model.

The desirability of developing an intersection study technique which combines efficient photographic data collection with computer simulation of intersection operation is evident. The data collection system would provide data to be used as input for the simulation model as well as data necessary for the simulation model validation. The simulation model would provide an effective tool for generating the amounts of intersection data necessary for determining the interrelationships among the variables involved.

1.2 Research Objectives

The overall aim of this study was to investigate more efficient methods of studying intersection operation under varying traffic conditions. More specifically, four objectives were identified as the following:

(1) develop a photographic data collection system which is capable of recording continuous intersection traffic data, including information regarding stopped time delay of left-turn vehicles and suitable for the collection of data regarding left-turn gap acceptance characteristics.
(2) develop an intersection simulation model so that, with information provided by the data collection system, the intersection of two and four lane roadways, with or without left-turn channels, can be simulated.

(3) examine the adequacy of existing design warrants for the addition of separate left-turn lanes at the intersection of two and four lane roadways.

(4) develop guidelines for estimating the required length of additional turning lanes based upon delay criteria, for given traffic characteristics.

1.3 Scope of Study

The scope of this study can be described in terms of the three major phases of coordinated research which were pursued. The first phase consisted of a review of past intersection data collection systems to provide the basis for an efficient and compact data collection system utilizing a commercially made 35 mm camera. Following the development and testing of the system a data collection study was performed to obtain information on each of the four types of intersections under study.

The second phase of this study consisted of the development of a simulation model utilizing an efficient simulation language and capable of simulating the four types of intersections under study. The validation of this model
was performed utilizing the data made available through the data collection study.

The final phase consisted of a simulation study which provided a means for predicting intersection delay on the basis of certain intersection traffic characteristics. By providing a comparison of expected delay for both unchannelized and channelized approaches with various lengths, suitable channel lengths can be selected on the basis of decrease in delay.
2.1 The Photographic Technique

The use of photography for traffic engineering studies is desirable for a number of reasons. Photographic data collection provides a continuous and permanent record of all the events taking place within the view of the camera. As a result many kinds of data are available from the films. Photography also helps to minimize error that can result from continuous manual data collection. Furthermore, a properly developed photographic data collection procedure enables the researcher to collect data with a minimum of support personnel.

Since Greenshields' pioneering attempt at photographic data collection for traffic engineering purposes in 1933 (4), numerous traffic researchers have successfully utilized photography for data collection. Greenshields, Shapiro, and Erickson applied motion-picture photography to study of intersection characteristics (5). In a study designed to obtain traffic performance data for evaluating the capacity of signalized diamond interchanges, Capelle and Pinnell also made use of motion-picture photography (6). Dart's study of left-turn gap acceptance is an example of the many studies which have attempted
to apply photographic data collection to the study of specific traffic characteristics (7). Moreover, in the studies concerned with intersection operation, the importance of camera placement was readily recognized and the difficulty of obtaining satisfactory data on more than two approaches simultaneously was not overcome.

One solution to this problem was developed by Treiterer and Nemeth (8). In this approach a camera and mirror system permits the simultaneous photographing of all four approaches but because of the mirror configuration the center of the intersection is not photographed and, as a result, data on the waiting times of left-turn vehicles and left-turn gap acceptance information is not available.

2.2 Robot Camera System

The desirability of developing a data collection system for intersection studies which is small and compact, yet with a high capacity and utilizing a convenient film size such as 35 mm led to the selection of the Robot camera. This camera, manufactured in West Germany, is a compact 35 mm camera which is available with a 200 ft. film magazine. Utilizing an Estar-base Kodak Linagraph film, the camera has a maximum capacity for 30 minutes of continuous data at the rate of one frame per second. A unique camera and mirror system was developed so that with the use of two cameras four intersection approaches can be photographed simultaneously.

The system utilized two Robot cameras and mirror units. Each
Robot camera and mirror unit is housed within an aluminum frame which is attached to a tripod head. The tripod head is inserted into a pipe pole mount. This pole mount is then secured to a pole by two U-bolts and a firm grip is ensured by rubber sheeting positioned between the U-bolts and the pole and also between the steel plate and the pole. The entire camera mount system for one camera, shown in Figure 1, provides a maximum of adjustment movements as well as portability and ease of use.

The manner in which the two-camera system is able to record simultaneous data on all four approaches to an intersection is illustrated in Figure 2, which indicates the location of the pole-mounted cameras and the views recorded by the cameras. It should be remembered that the intersection legs labeled A and C are photographed through a mirror, resulting in a reverse image. Mounting of the cameras at the corners of the intersection avoids the major disruption to flow caused by the mounting of an overhead, center-mounted system such as Nemeth's (8), which necessitated parking a skyworker-type truck in the center of the intersection. It also minimized the accident potential that can exist during the mounting activity. Figure 3 shows two strips of photographs taken simultaneously, indicating how all four approaches to an intersection are recorded. An enlargement of a pair of photographs is shown in Figure 4 and it can be seen that vehicle position, turning movements, and arrival information can be recorded from the photographs. (It should be pointed out that in actual data reduction the photo negatives are projected onto
Figure 1. ROBOT Camera Mount System
Figure 2. Identification of Intersection Legs on Data Photographs
Figure 3. Typical Data Collection Photographs
Figure 4. Enlargement of a pair of data photographs.
a screen and appear much larger.)

Other components of the data collection system include the following:
an intervalometer which provides the pulsing mechanism necessary for the
actuation of the camera shutters every second; a 24-volt power supply for
the film-advance motors; a frame-counter which provides a means for record­
ing signal indication and frame count; and a specially designed control box
for the camera and power actuation.

The accuracy of the intervalometer was checked by the use of an
electronic counter. A sample of 100 readings resulted in a mean interval of
1.007 seconds; the standard deviation for the sample was 0.0058. As a result,
the 95 percent confidence interval consists of the range of values between
0.9956 and 1.0184 seconds, considered acceptable for this study.

2.3 Camera Installation

The mounting and positioning of the camera system for actual data
collection was carried out in two different ways. Initially, an International
Harvester Skyworker truck and crew were rented from the Service Department
of The Ohio State University. However, the crew as well as the truck had
other duties within the University community which were of higher priority
than data collection activities. As a result data collection was highly depend­
ent upon the availability of both the crew and the truck.

As a result, a platform ladder was purchased for the data collection
activities. The ladder provides a platform 18 feet above ground, a height which is most adequate for camera mounting. Furthermore, its use enables data collection to be carried out by a minimum of support personnel.

The power for the camera motor was obtained at times by connecting an AC/DC power converter to the power source within the traffic signal control box at the site and connecting the camera control box to the power converter. An alternative power supply system utilized two 12-volt wet cell batteries connected in series to provide 24-volt DC power directly to the camera control box.

During the data collection phase of the study it was observed that two people could adequately perform the necessary activities for data collection. The time necessary to prepare and mount both cameras took approximately forty-five minutes. Actual data collection and removal of the equipment each took approximately thirty minutes so that on-site activities necessary for data collection took approximately one hour and forty-five minutes.

2.4 Intersection Site Selection

Four different intersection approach configurations were being studied and, therefore, data collection on each of these approach types was necessary. It was also desirable to select sites which could be conveniently reached by the data collection crew. Furthermore, since isolated intersections were specifically under study, suburban and rural sites appeared to be
the most useful for the study.

A study of the signalized intersections within the Franklin County area resulted in the selection of four intersections. Each of the four intersections selected has four approaches of a particular type. Thus data collected at one intersection resulted in four examples of approach data for a particular approach configuration. The following is a list of the approach type and the corresponding intersection selected:

1. single-lane approach: intersection of 17th and Woodland Avenues (City of Columbus)

2. single-lane approach with left-turn channels: intersection of State Route 3 and State Route 161 (Franklin County)

3. two-lane approach: intersection of Northwest Boulevard and Zollinger Road (City of Upper Arlington)

4. two-lane approach with left-turn channel: intersection of McCoy Road and Reed Road (City of Upper Arlington).

2.5 Amount Collected

The total data collected during the study consisted of nine data sets. The term, set, is used to define the collection of data on four approaches at a single site location. The results of a data set for this study are two films with from seventeen to thirty minutes of continuous data for all four approaches.

Of the nine sets collected, two sets were rendered useless since the
power source used was not adequate for the film advance systems of both cameras, resulting in numerous double exposures on one of the two films in each case. Another data set, the first data collected, was not utilized because improper camera positioning resulted in less than adequate data on each of the four approaches. Two other data sets were not used because of accidents at the study site. In one film an accident within the center of the intersection caused total disruption to flow on all four approaches. In the second film a driver ran off the road into a ditch a few hundred feet from the intersection, totally disrupting flow on that approach. Fortunately, neither of these accidents was caused by the data collection procedures. A summary of the data collected and the films used in the study is shown in Table 1.
<table>
<thead>
<tr>
<th>Film No.</th>
<th>Location</th>
<th>Approach Type</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>S.R. 3 and S.R. 161</td>
<td>2</td>
<td>Not used</td>
</tr>
<tr>
<td>2.</td>
<td>Northwest Blvd. and Zollinger Rd</td>
<td>3</td>
<td>Power source failure</td>
</tr>
<tr>
<td>3.</td>
<td>Northwest Blvd. and Zollinger Rd</td>
<td>3</td>
<td>Power source failure</td>
</tr>
<tr>
<td>4.</td>
<td>S.R. 3 and S.R. 161</td>
<td>2</td>
<td>30 minutes data</td>
</tr>
<tr>
<td>5.</td>
<td>Northwest Blvd. and Zollinger Rd</td>
<td>3</td>
<td>20 minutes data</td>
</tr>
<tr>
<td>6.</td>
<td>McCoy and Reed Roads</td>
<td>4</td>
<td>Accident in approach</td>
</tr>
<tr>
<td>7.</td>
<td>McCoy and Reed Roads</td>
<td>4</td>
<td>25 minutes data</td>
</tr>
<tr>
<td>8.</td>
<td>17th and Woodland Avenues</td>
<td>1</td>
<td>Accident in approach</td>
</tr>
<tr>
<td>9.</td>
<td>17th and Woodland Avenues</td>
<td>1</td>
<td>17 minutes data</td>
</tr>
</tbody>
</table>

Approach Types:
(1) Single-lane
(2) Single-lane with left-turn channel
(3) Two-lane
(4) Two-lane with left-turn channel.
CHAPTER III
RESULTS OF DATA COLLECTION

3.1 Data Reduction Procedure

The reduction of the four data sets was accomplished through the use of the Rear Projection Console (RPC) and a modified Viewlex filmstrip projector. The RPC was used in a study by Treiterer and Nemeth (8), and provides for convenient viewing of the film (see Figure 5). The RPC is designed so that the film to be viewed is projected onto the first-surface mirror in the console. The image is reflected upward through a translucent screen. A sheet of tracing paper is placed over this screen and thus the image can be seen as projected upon the paper.

The projector is a model V-25R Viewlex filmstrip projector which has been slightly modified to accommodate the large reel of film used. Further modifications were also made to the film advance mechanism to allow for remote film advance and adjustment by the operator at the RPC.

Actual data reduction consisted of recording individual vehicle positions for each frame of film, followed by an analysis of this position information to determine vehicle arrival information, delay values, and other information to be discussed later.
Figure 5. Setup of the Rear Projection Console.

TRANSLUCENT SCREEN (30" X 30")

FRONT SURFACE MIRROR

CONSOLE

35 mm PROJECTOR

PROJECTOR Remote Controlled From Reduction Console
The determination of vehicle position is based upon a grid system superimposed on the projected image of the roadway. The origin of this grid system is a set of white marks painted at 25 feet intervals along the approach lanes. The painted marks were approximately three inches wide and twelve inches long so that they would be inconspicuous to the drivers yet could be seen on the projected image of the roadway. The location of the 25 feet intervals are marked on the tracing paper, providing the basis for the reference scale. To obtain a grid system for 5 feet intervals the theory of cross ratio as discussed by Hallert is utilized (9). This fundamental principle of projectivity states that the cross ratio of distances between four points on a straight line is invariant under projection. Consequently, if the distance between four collinear points on the pavement are known and three of these points are identifiable on the photograph, then the location of the missing fourth point can be computed. The actual computations for this project were facilitated by the use of a computer program developed by Riopelle (10) and revised for this study. A statement listing of this program is included in Appendix A.

For the first phase of the data reduction activity, the actual frame by frame determination of vehicle position, it was found that from 40 to 50 man-hours were necessary at the RPC for each 250 feet of film. The subsequent analysis of the information taken from the film took approximately 60 man-hours to complete for each data set. Thus, for a complete data set, from 100 to 110 man-hours were required for complete data reduction.
3.2 Results of Data Reduction

As a result of the reduction of the four data sets a number of different kinds of information regarding the traffic characteristics of the intersections under study was made available. The following discussions detail the specific information made available.

Volume and Left-turn Data

For each intersection approach it was possible to determine the approach volumes and the number of left-turning vehicles. A summary of these results is shown in Table 2.

Vehicle Delay Values

The amount of time that each vehicle was delayed as it passed through the intersection is available from the frame by frame analysis of the data. As will be shown later, this information is essential in the simulation model validation process. The total delay that each vehicle accumulates as it passes through the intersection consists of stopped delay plus the delay due to queue or platoon movement. The first component of delay is stopped delay, defined as the period of time during which a vehicle is in a stopped position, waiting to proceed through the intersection. Such a vehicle can be stopped by a red signal indication, waiting for a queue of vehicles to dissipate once a green signal indication has been provided, or waiting for preceding vehicle to complete a left turn maneuver.

The second component of total delay is more complex than stopped delay.
<table>
<thead>
<tr>
<th>Approach</th>
<th>Total Vehicles</th>
<th>Left Turns</th>
<th>Stopped Vehicles</th>
<th>Total Delay (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northwest Blvd.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. B.</td>
<td>223</td>
<td>59</td>
<td>123</td>
<td>3011.94</td>
</tr>
<tr>
<td>S. B.</td>
<td>113</td>
<td>25</td>
<td>62</td>
<td>1195.10</td>
</tr>
<tr>
<td>Zollinger Road</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E. B.</td>
<td>114</td>
<td>10</td>
<td>75</td>
<td>1699.97</td>
</tr>
<tr>
<td>W. B.</td>
<td>176</td>
<td>13</td>
<td>100</td>
<td>2729.56</td>
</tr>
<tr>
<td>S. R. 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. B.</td>
<td>136</td>
<td>22</td>
<td>72</td>
<td>1696.81</td>
</tr>
<tr>
<td>S. B.</td>
<td>201</td>
<td>8</td>
<td>114</td>
<td>3071.02</td>
</tr>
<tr>
<td>S. R. 161</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E. B.</td>
<td>128</td>
<td>31</td>
<td>67</td>
<td>1993.82</td>
</tr>
<tr>
<td>W. B.</td>
<td>194</td>
<td>52</td>
<td>82</td>
<td>2301.38</td>
</tr>
<tr>
<td>Reed Road</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. B.</td>
<td>173</td>
<td>8</td>
<td>93</td>
<td>1699.99</td>
</tr>
<tr>
<td>S. B.</td>
<td>141</td>
<td>29</td>
<td>78</td>
<td>1222.75</td>
</tr>
<tr>
<td>McCoy Road</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E. B.</td>
<td>119</td>
<td>33</td>
<td>67</td>
<td>1342.99</td>
</tr>
<tr>
<td>W. B.</td>
<td>153</td>
<td>13</td>
<td>88</td>
<td>1458.60</td>
</tr>
<tr>
<td>Woodland Ave.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. B.</td>
<td>62</td>
<td>38</td>
<td>31</td>
<td>579.38</td>
</tr>
<tr>
<td>S. B.</td>
<td>85</td>
<td>4</td>
<td>32</td>
<td>642.51</td>
</tr>
<tr>
<td>17th Avenue</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E. B.</td>
<td>156</td>
<td>66</td>
<td>92</td>
<td>2060.24</td>
</tr>
<tr>
<td>W. B.</td>
<td>36</td>
<td>1</td>
<td>23</td>
<td>415.08</td>
</tr>
</tbody>
</table>
When a vehicle becomes a member of a queue or platoon, full freedom of movement is lost and the vehicle, even though it may be moving along the intersection approach, is not moving unimpeded. As a result its total delay can include the time during which it is a member of this queue and being delayed by it. It is recognized that only stopped delay is easily measured in field studies. Nevertheless, simulation model validation also requires estimates of the queue delay. The photographic technique utilized in this study provides a permanent record of all intersection vehicular movements and provides for an accurate determination of total delay.

In order to collect total delay data on all vehicles approaching the intersection, some measure must be used to determine that a vehicle is being delayed. Since previous studies offered no usable guides, it was decided that vehicles whose approach speed went below 15 feet per second were being delayed and this delay continued until they passed the intersection stop lines or turned left out of the approach.

For this study through and right-turn vehicles were treated similarly. Using this criterion, values for vehicular delay, also shown in Table 2, were determined. Information regarding the distribution of delays for each approach was also gathered from the films. These distributions were used in the simulation model validation studies and are discussed later.

Vehicle Arrivals

Information regarding the headway distributions for each of the inter-
section approaches under study is one of the results of the data collection effort. One of the input requirements for the simulation models is a headway distribution, used to simulate vehicular arrivals. Further, as will be discussed later, it has been shown that the headway distribution developed by Schuhl is an effective means of modeling vehicular headways in the traffic stream (11). This headway distribution, the composite exponential distribution, has been studied by Kell (12) in order to provide a means for its use in simulating headway distributions. Since it was not possible to develop an entire family of headway distributions for this study, it was decided that the method developed by Kell would be used to determine the cumulative distribution function for the arrival headway distributions. As a result, it was decided that a test of the arrival distributions found in the field studies against the composite exponential distributions would be appropriate.

The method used for this testing is the Kolmogorov-Smirnov one-sample test (13). The Kolmogorov-Smirnov, or K-S, test is concerned with the degree of agreement between the distribution of a set of sample values and some theoretical or generated distribution. Briefly, the test is concerned with the agreement between two cumulative distributions. The point of maximum divergence, D, between the two distributions is determined from

\[ D = \text{maximum} \left| F_o (X) - S_n (X) \right| \]

where \( F_o (X) \) is a completely specified cumulative distribution function, the
theoretical cumulative distribution under $H_0$; and, $S_n(X)$ is the observed cumulative frequency distribution of a random sample of $N$ observations.

The results of these tests are shown in Table 3. For the interval chosen, 5 seconds, the hypothesis, $H_0$, that the data come from the specified distribution, is accepted at the 0.05 level in 21 of the 24 cases tested. Of the three cases which failed the test, two represent the northbound traffic on Northwest Boulevard. Since this is the major direction of flow during the evening peak period of traffic, vehicles quite possibly could have been affected by traffic signals downstream of the study site and may have been traveling in platoons along the boulevard. Nevertheless, on the basis of these tests, it can be said that the composite exponential distribution is a reasonable choice for modeling arrival distributions.

Starting Delays

Vehicular starting delays are an important input for the simulation process since they can affect the cumulative delay time for each vehicle in a waiting queue. Furthermore, there are two specific sets of starting delay values which might be used to simulate vehicular starting delays. Thus before selecting either of the two sets for use in the simulation process some tests of the sample data with these two populations were completed.

The two sets of starting delay values are those of Greenshields, et al., and Drew. Greenshields, et al. reported that the first car of a waiting queue enters the intersection 3.8 seconds after the beginning of the green and that
<table>
<thead>
<tr>
<th>APPROACH</th>
<th>D</th>
<th>$D_{\text{critical}}$</th>
<th>RESULTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>S. R. 161 W. B.</td>
<td>0.109</td>
<td>0.120</td>
<td>accept</td>
</tr>
<tr>
<td>258 vph</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S. R. 161 E. B.</td>
<td>0.143</td>
<td>0.117</td>
<td>reject</td>
</tr>
<tr>
<td>270 vph</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S. R. 3 S. B.</td>
<td>0.034</td>
<td>0.095</td>
<td>accept</td>
</tr>
<tr>
<td>410 vph</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S. R. 3 N. B.</td>
<td>0.110</td>
<td>0.113</td>
<td>accept</td>
</tr>
<tr>
<td>292 vph</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ZOLLINGER RD.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W. B.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LANE 1: 186 vph</td>
<td>0.053</td>
<td>0.173</td>
<td>accept</td>
</tr>
<tr>
<td>LANE 2: 330 vph</td>
<td>0.026</td>
<td>0.129</td>
<td>accept</td>
</tr>
<tr>
<td>E. B.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LANE 1: 84 vph</td>
<td>0.168</td>
<td>0.257</td>
<td>accept</td>
</tr>
<tr>
<td>LANE 2: 255 vph</td>
<td>0.060</td>
<td>0.148</td>
<td>accept</td>
</tr>
<tr>
<td>NORTHWEST BLVD.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. B.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LANE 1: 321 vph</td>
<td>0.159</td>
<td>0.132</td>
<td>reject</td>
</tr>
<tr>
<td>LANE 2: 351 vph</td>
<td>0.198</td>
<td>0.126</td>
<td>reject</td>
</tr>
<tr>
<td>S. B.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LANE 1: 144 vph</td>
<td>0.050</td>
<td>0.196</td>
<td>accept</td>
</tr>
<tr>
<td>LANE 2: 195 vph</td>
<td>0.165</td>
<td>0.169</td>
<td>accept</td>
</tr>
<tr>
<td>APPROACH</td>
<td>D</td>
<td>D\text{critical}</td>
<td>RESULTS</td>
</tr>
<tr>
<td>----------</td>
<td>----</td>
<td>------------------</td>
<td>---------</td>
</tr>
<tr>
<td>WOODLAND AVE.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. B. 219 vph</td>
<td>0.158</td>
<td>0.192</td>
<td>accept</td>
</tr>
<tr>
<td>WOODLAND AVE.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S. B. 300 vph</td>
<td>0.060</td>
<td>0.156</td>
<td>accept</td>
</tr>
<tr>
<td>17th AVENUE</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E. B. 550 vph</td>
<td>0.042</td>
<td>0.111</td>
<td>accept</td>
</tr>
<tr>
<td>17th AVENUE</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W. B. 125 vph</td>
<td>0.149</td>
<td>0.194</td>
<td>accept</td>
</tr>
<tr>
<td>McCoy ROAD</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W. B.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LANE 1: 165 vph</td>
<td>0.135</td>
<td>0.164</td>
<td>accept</td>
</tr>
<tr>
<td>LANE 2: 203 vph</td>
<td>0.076</td>
<td>0.152</td>
<td>accept</td>
</tr>
<tr>
<td>E. B.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LANE 1: 178 vph</td>
<td>0.067</td>
<td>0.158</td>
<td>accept</td>
</tr>
<tr>
<td>LANE 2: 108 vph</td>
<td>0.100</td>
<td>0.178</td>
<td>accept</td>
</tr>
<tr>
<td>Reed ROAD</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S. B.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LANE 1: 187 vph</td>
<td>0.151</td>
<td>0.157</td>
<td>accept</td>
</tr>
<tr>
<td>LANE 2: 151 vph</td>
<td>0.086</td>
<td>0.177</td>
<td>accept</td>
</tr>
<tr>
<td>N. B.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LANE 1: 216 vph</td>
<td>0.031</td>
<td>0.145</td>
<td>accept</td>
</tr>
<tr>
<td>LANE 2: 199 vph</td>
<td>0.072</td>
<td>0.153</td>
<td>accept</td>
</tr>
</tbody>
</table>
successive cars enter at 3.1, 2.7, 2.4, and 2.2 second intervals until the sixth-in-line and all the following cars which enter at 2.1 second intervals. These values were determined for use in a technique for computing the capacity of a single traffic lane.

Drew, in his capacity-design procedure for intersection studies, makes use of a headway study reported by Capelle and Pinnell (6). The intervals which develop when a queue of vehicles is released at a stoplight were found to tend toward a constant of 2.0 seconds as the speeds between successive vehicles as they cross the stop line become constant and equal to the vehicles' desired speed for that facility. The first car was found to pass the stop line 2.8 seconds after the beginning of the green and the next three cars followed at 2.6, 2.1, and 2.1 second intervals; all following cars entered at 2.0 second intervals.

It was decided that each of the data sets provided a sample of starting delay values to be tested against the two sets of values provided by Greenshields, et al., and Drew. Because the intersection type for each data set differed and because past studies have shown that there is a significant difference in starting delay among approaches at different intersections (14), no attempt was made to group the data from all four data sets into a single sample to be tested against the two sets of population values.

For this test the statistic \( z = \frac{\bar{X} - \bar{X}'}{\frac{S}{\sqrt{N}}} \) where \( \bar{X} \) is the sample mean, \( \bar{X}' \) the population mean, \( S \) is the sample standard deviation, and \( N \) is the sample
size, was used. The values determined by Greenshields, et al. and Drew have been tested against the starting delays obtained from each of the data sets. For the 0.05 level of significance, the critical value of $Z$ is 1.96. The hypothesis tested, $H_0$, is that the sample starting delay values came from the populations of values proposed by Greenshields, et al., and by Drew. The results of these tests are shown in Table 4.

The results are somewhat inconclusive. It appears that either set of starting delay values is equally representative of the samples collected and thus either could be used to model the situation at the intersections under study. However, it should be pointed out that the study by Greenshields, et al. was performed in 1947. Since that time vehicular operating characteristics have changed somewhat. Drew's study was performed in the early 1960's and thus can be assumed to be more representative of present vehicle-driver characteristics. As a result it was decided to use the newer values in the simulation model.

Left-Turn Gap Acceptance Values

It was possible to determine the gap acceptance and rejection values for left-turning vehicles on each of the approaches studied. It was decided, however, that the sample of values obtained from the four primary films was not sufficient for a complete study of gap-acceptance characteristics at the study sites. However, the camera system was effective in collecting data for this type of study.
### Table 4

**Starting Delay Test Results**

**Northwest Blvd. - Zollinger Road Data**

<table>
<thead>
<tr>
<th>Vehicle in Queue</th>
<th>Mean Delay ($\bar{x}$)</th>
<th>Variance</th>
<th>Standard Deviation</th>
<th>Sample Size ($N$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.43</td>
<td>2.23</td>
<td>1.49</td>
<td>37</td>
</tr>
<tr>
<td>2</td>
<td>2.13</td>
<td>2.20</td>
<td>1.79</td>
<td>36</td>
</tr>
<tr>
<td>3</td>
<td>2.38</td>
<td>0.49</td>
<td>0.70</td>
<td>35</td>
</tr>
<tr>
<td>4</td>
<td>2.23</td>
<td>0.44</td>
<td>0.66</td>
<td>23</td>
</tr>
<tr>
<td>5</td>
<td>2.37</td>
<td>0.58</td>
<td>0.76</td>
<td>14</td>
</tr>
<tr>
<td>6</td>
<td>2.29</td>
<td>0.52</td>
<td>0.72</td>
<td>11</td>
</tr>
</tbody>
</table>

**Testing Greenshields' Values**

<table>
<thead>
<tr>
<th>Vehicle in Queue</th>
<th>$\bar{x}'$</th>
<th>$\bar{x}' - \bar{x}$</th>
<th>$z$</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.8</td>
<td>0.30</td>
<td>1.52</td>
<td>accept</td>
</tr>
<tr>
<td>2</td>
<td>2.1</td>
<td>0.31</td>
<td>0.10</td>
<td>accept</td>
</tr>
<tr>
<td>3</td>
<td>2.7</td>
<td>0.32</td>
<td>2.69</td>
<td>reject</td>
</tr>
<tr>
<td>4</td>
<td>2.4</td>
<td>0.37</td>
<td>1.23</td>
<td>accept</td>
</tr>
<tr>
<td>5</td>
<td>2.2</td>
<td>0.33</td>
<td>0.15</td>
<td>accept</td>
</tr>
<tr>
<td>6</td>
<td>2.1</td>
<td>0.39</td>
<td>0.87</td>
<td>accept</td>
</tr>
</tbody>
</table>

**Testing Drew's Values**

<table>
<thead>
<tr>
<th>Vehicle in Queue</th>
<th>$\bar{x}'$</th>
<th>$\bar{x}' - \bar{x}$</th>
<th>$z$</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.8</td>
<td>0.63</td>
<td>2.58</td>
<td>reject</td>
</tr>
<tr>
<td>2</td>
<td>2.6</td>
<td>0.53</td>
<td>1.78</td>
<td>accept</td>
</tr>
<tr>
<td>3</td>
<td>2.1</td>
<td>0.28</td>
<td>2.36</td>
<td>reject</td>
</tr>
<tr>
<td>4</td>
<td>2.0</td>
<td>0.23</td>
<td>1.67</td>
<td>accept</td>
</tr>
<tr>
<td>5</td>
<td>2.0</td>
<td>0.37</td>
<td>1.81</td>
<td>accept</td>
</tr>
<tr>
<td>6</td>
<td>2.0</td>
<td>0.29</td>
<td>1.33</td>
<td>accept</td>
</tr>
</tbody>
</table>
TABLE 4 - Continued

<table>
<thead>
<tr>
<th>VEHICLE IN QUEUE</th>
<th>MEAN DELAY(X)</th>
<th>VARIANCE</th>
<th>STANDARD DEVIATION</th>
<th>SAMPLE SIZE(N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.50</td>
<td>2.24</td>
<td>1.50</td>
<td>53</td>
</tr>
<tr>
<td>2</td>
<td>2.62</td>
<td>0.74</td>
<td>0.86</td>
<td>55</td>
</tr>
<tr>
<td>3</td>
<td>2.42</td>
<td>0.59</td>
<td>0.77</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>2.10</td>
<td>0.34</td>
<td>0.58</td>
<td>26</td>
</tr>
<tr>
<td>5</td>
<td>2.06</td>
<td>0.34</td>
<td>0.58</td>
<td>16</td>
</tr>
<tr>
<td>6</td>
<td>2.23</td>
<td>0.81</td>
<td>0.90</td>
<td>13</td>
</tr>
</tbody>
</table>

TESTING GREENSHIELDS' VALUES

<table>
<thead>
<tr>
<th>VEHICLE IN QUEUE</th>
<th>( \bar{X} )</th>
<th>( \bar{X} - \bar{X}' )</th>
<th>Z</th>
<th>RESULTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.8</td>
<td>0.30</td>
<td>1.46</td>
<td>accept</td>
</tr>
<tr>
<td>2</td>
<td>3.1</td>
<td>0.48</td>
<td>4.14</td>
<td>reject</td>
</tr>
<tr>
<td>3</td>
<td>2.7</td>
<td>0.28</td>
<td>2.30</td>
<td>reject</td>
</tr>
<tr>
<td>4</td>
<td>2.4</td>
<td>0.30</td>
<td>2.63</td>
<td>reject</td>
</tr>
<tr>
<td>5</td>
<td>2.2</td>
<td>0.14</td>
<td>0.96</td>
<td>accept</td>
</tr>
<tr>
<td>6</td>
<td>2.1</td>
<td>0.13</td>
<td>0.52</td>
<td>accept</td>
</tr>
</tbody>
</table>

TESTING DREW'S VALUES

<table>
<thead>
<tr>
<th>VEHICLE IN QUEUE</th>
<th>( \bar{X} )</th>
<th>( \bar{X} - \bar{X}' )</th>
<th>Z</th>
<th>RESULTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.8</td>
<td>0.70</td>
<td>3.40</td>
<td>reject</td>
</tr>
<tr>
<td>2</td>
<td>2.6</td>
<td>0.02</td>
<td>0.17</td>
<td>accept</td>
</tr>
<tr>
<td>3</td>
<td>2.1</td>
<td>0.32</td>
<td>2.62</td>
<td>reject</td>
</tr>
<tr>
<td>4</td>
<td>2.0</td>
<td>0.10</td>
<td>0.88</td>
<td>accept</td>
</tr>
<tr>
<td>5</td>
<td>2.0</td>
<td>0.06</td>
<td>0.41</td>
<td>accept</td>
</tr>
<tr>
<td>6</td>
<td>2.0</td>
<td>0.23</td>
<td>0.92</td>
<td>accept</td>
</tr>
</tbody>
</table>
### Table 4 - Continued

17th AVENUE - WOODLAND AVENUE DATA

<table>
<thead>
<tr>
<th>VEHICLE IN QUEUE</th>
<th>MEAN DELAY($X$)</th>
<th>VARIANCE</th>
<th>STANDARD DEVIATION</th>
<th>SAMPLE SIZE($N$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.29</td>
<td>1.28</td>
<td>1.13</td>
<td>26</td>
</tr>
<tr>
<td>2</td>
<td>2.90</td>
<td>2.92</td>
<td>1.71</td>
<td>26</td>
</tr>
<tr>
<td>3</td>
<td>2.22</td>
<td>0.60</td>
<td>0.77</td>
<td>26</td>
</tr>
<tr>
<td>4</td>
<td>2.19</td>
<td>0.46</td>
<td>0.68</td>
<td>26</td>
</tr>
<tr>
<td>5</td>
<td>2.32</td>
<td>0.94</td>
<td>0.97</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>2.26</td>
<td>1.30</td>
<td>1.14</td>
<td>13</td>
</tr>
</tbody>
</table>

TESTING GREENSHIELDS' VALUES

<table>
<thead>
<tr>
<th>VEHICLE IN QUEUE</th>
<th>$\bar{X}'$</th>
<th>$\bar{X} - \bar{X}'$</th>
<th>$Z$</th>
<th>RESULTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.8</td>
<td>0.51</td>
<td>2.30</td>
<td>reject</td>
</tr>
<tr>
<td>2</td>
<td>3.1</td>
<td>0.20</td>
<td>0.60</td>
<td>accept</td>
</tr>
<tr>
<td>3</td>
<td>2.7</td>
<td>0.48</td>
<td>3.18</td>
<td>reject</td>
</tr>
<tr>
<td>4</td>
<td>2.4</td>
<td>0.21</td>
<td>1.58</td>
<td>accept</td>
</tr>
<tr>
<td>5</td>
<td>2.2</td>
<td>0.12</td>
<td>1.58</td>
<td>accept</td>
</tr>
<tr>
<td>6</td>
<td>2.1</td>
<td>0.16</td>
<td>0.72</td>
<td>accept</td>
</tr>
</tbody>
</table>

TESTING DREW'S VALUES

<table>
<thead>
<tr>
<th>VEHICLE IN QUEUE</th>
<th>$\bar{X}'$</th>
<th>$\bar{X} - \bar{X}'$</th>
<th>$Z$</th>
<th>RESULTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.8</td>
<td>0.49</td>
<td>2.21</td>
<td>reject</td>
</tr>
<tr>
<td>2</td>
<td>2.6</td>
<td>0.30</td>
<td>0.89</td>
<td>accept</td>
</tr>
<tr>
<td>3</td>
<td>2.1</td>
<td>0.12</td>
<td>0.79</td>
<td>accept</td>
</tr>
<tr>
<td>4</td>
<td>2.0</td>
<td>0.19</td>
<td>1.43</td>
<td>accept</td>
</tr>
<tr>
<td>5</td>
<td>2.0</td>
<td>0.32</td>
<td>1.68</td>
<td>accept</td>
</tr>
<tr>
<td>6</td>
<td>2.0</td>
<td>0.26</td>
<td>1.16</td>
<td>accept</td>
</tr>
</tbody>
</table>
### TABLE 4 - Continued

**REED ROAD - MCCOY ROAD DATA**

<table>
<thead>
<tr>
<th>VEHICLE IN QUEUE</th>
<th>MEAN DELAY((X))</th>
<th>VARIANCE</th>
<th>STANDARD DEVIATION</th>
<th>SAMPLE SIZE(N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.77</td>
<td>1.44</td>
<td>1.20</td>
<td>32</td>
</tr>
<tr>
<td>2</td>
<td>2.46</td>
<td>0.80</td>
<td>0.89</td>
<td>32</td>
</tr>
<tr>
<td>3</td>
<td>2.40*</td>
<td>0.45</td>
<td>0.67</td>
<td>32</td>
</tr>
<tr>
<td>4</td>
<td>2.07</td>
<td>0.35</td>
<td>0.59</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>SAMPLE TOO SMALL : NOT TESTED</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TESTING GREGSHELD'S VALUES**

<table>
<thead>
<tr>
<th>VEHICLE IN QUEUE</th>
<th>(\bar{X}')</th>
<th>(\bar{X} - \bar{X}')</th>
<th>Z</th>
<th>RESULTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.8</td>
<td>0.03</td>
<td>0.14</td>
<td>accept</td>
</tr>
<tr>
<td>2</td>
<td>3.1</td>
<td>0.64</td>
<td>4.08</td>
<td>reject</td>
</tr>
<tr>
<td>3</td>
<td>2.7</td>
<td>0.30</td>
<td>2.54</td>
<td>reject</td>
</tr>
<tr>
<td>4</td>
<td>2.4</td>
<td>0.33</td>
<td>2.50</td>
<td>reject</td>
</tr>
</tbody>
</table>

**TESTING DREW'S VALUES**

<table>
<thead>
<tr>
<th>VEHICLE IN QUEUE</th>
<th>(\bar{X}')</th>
<th>(\bar{X} - \bar{X}')</th>
<th>Z</th>
<th>RESULTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.8</td>
<td>0.97</td>
<td>4.58</td>
<td>reject</td>
</tr>
<tr>
<td>2</td>
<td>2.6</td>
<td>0.14</td>
<td>0.89</td>
<td>accept</td>
</tr>
<tr>
<td>3</td>
<td>2.1</td>
<td>0.30</td>
<td>2.54</td>
<td>reject</td>
</tr>
<tr>
<td>4</td>
<td>2.0</td>
<td>0.07</td>
<td>0.53</td>
<td>accept</td>
</tr>
</tbody>
</table>
4.1 Past Studies

A number of simulation models have been developed to investigate various aspects of intersection operation and vehicular movement within the intersection. Each model has been developed for a specific purpose and as a result direct comparison of the many models is, at best, difficult. Kell (15) subjected four specific models to a thorough analysis in order to provide a basis for future simulation model development; the four models are those of Bleyl (16), Gerlough (17), Kell (18), and Lewis (19). He concluded that field data indicated that some components of each model are valid and some are not.

More recent simulation models include those of Nemeth (8) and Dart (7). Nemeth's model considers an intersection with four similar approaches, each with a left-turn lane and a through-or-right-turn lane; it also permits the simulation of pretimed, semi-actuated, or fully actuated signals, since it was developed to assist in determining whether the best use is being made of traffic control devices.

Dart's simulation model represents an intersection with two lanes approaching from each of four directions; the left lane of each approach can
accommodate both left-turn and through vehicles and the right lane can accommodate through and right-turn vehicles; the signal operation is pretimed.

Each of the above studies made use of one of the many general purpose algebraic computer languages which have the inherent disadvantage that an extensive amount of programming is required for simulation. The system to be simulated must be modeled in detail with the language used and this usually requires a large amount of programming. As a result, it was decided to perform the desired simulation for this project with the specialized simulation language, GPSS/360.

The GPSS/360 language (General Purpose Systems Simulator) is designed for use on the IBM 360 computer (20). The specific language is built around a set of abstract elements called entities. These entities are divided into four classes: dynamic, equipment, statistical, and operational. Each of these classes can be further detailed, and each event within the system can be modeled by the use of one or more of about 50 "Blocks", each of which has a specific function. The system being simulated is structured in the form of a flow-chart-like block diagram.

Blum has shown how the GPSS language can be used for intersection and network simulation (21). Dare has performed a study of the signal funnel concept for a T-intersection with an advisory speed signal on the main approach using the GPSS language (22).
4.2 The Purpose of the Simulation Model

The purpose of the intersection simulation model is to provide a means by which the operation of a fixed-time signalized intersection can be simulated so that, when utilized with the information obtained from the newly developed Robot camera data collection system, the effects of the installation of left-turn channels of different lengths can be estimated. The effects which are studied here are expressed in terms of the delay to the vehicles passing through the intersection. This model, developed for use by the Transportation Engineering Center at The Ohio State University, is referred to as the TEC model.

4.3 Description of the TEC Simulation Model

The TEC simulation model simulates two opposing legs of an intersection and is capable of modeling any one of the following four geometric configurations:

1. single-lane approaches; no left-turn channels
2. single-lane approaches; with left-turn channels
3. two-lane approaches; no left-turn channels
4. two-lane approaches; with left-turn channels.

In order to simplify identification of the TEC model, the number of each of the above situations is used to refer to the simulation of that situation. For example, TEC #2 refers to the simulation of situation 2 above, single-lane approaches, with left-turn channels.
The TEC model simulates an intersection subject to the following basic assumptions regarding vehicular flow:

1. traffic consists solely of passenger cars of similar dimensions and operating characteristics,
2. no passing or lane changing (except into left-turn channels) is permitted in the approach lanes,
3. all vehicles enter the system with some preselected speed, such as the observed running speed.

4.4 Operational Characteristics of the Intersection Model

The significant traffic operation characteristics of a signalized intersection which are simulated in the TEC model and the manner in which they are simulated are presented in the following discussions.

Arrival Distributions

In the simulation of an intersection approach it is necessary to include the rate of arrival of vehicles, and the size and distribution of gaps between them. Actually this arrival rate is not the arrival rate for vehicles at the stop line of the intersection, but it is the arrival rate at a point some distance prior to the intersection where vehicles arrive randomly, without being influenced by the intersection traffic signal.

For the TEC model, traffic approaching an intersection is randomly generated from a headway distribution to achieve a preselected hourly volume. Numerous statistical distributions have been used for modeling vehicular head-
ways. The data collected in this study does not provide the vast amounts of headway data necessary to develop a family of distributions over the range of volumes considered in the simulation study. However, the composite exponential distribution has been successfully used in a number of previous studies and, as was shown in Chapter 2, it provides a reasonable model for the arrival data collected in this study. Moreover, Kell has provided an efficient method for calculating the cumulative distribution function for the composite exponential. As a result the composite exponential distribution was used for modeling vehicle arrivals in this study.

The composite exponential, as proposed for modeling a traffic stream by Schuhl, is based on the assumption that a traffic stream is divided into two groups. One group, referred to as the free-flowing or unrestrained vehicles, consists of those vehicles in the traffic stream that travel as their drivers wish and are not influenced by the vehicles or vehicles in front of them. The other group, known as the restrained vehicles, are influenced and delayed by the vehicle or vehicles in front of them. The theoretical distribution for the total stream is a composite or summation of two separate sub-distributions.

The composite exponential is defined by

\[ P(\text{headway} \geq t) = (1 - \alpha)(1 - \exp \left[-\frac{t - \lambda}{T_1 - \lambda}\right]) + \alpha \left(1 - \exp \left[-\frac{t - \gamma}{T_2 - \gamma}\right]\right) \]  

where

\[ P(\text{headway} \geq t) = \text{probability of headway greater than or equal to some time, } t; \]
\( \alpha = \) proportion of "restrained" traffic;
\( 1 - \alpha = \) proportion of "free-moving" traffic;
\( T_1 = \) average headway of "free-moving" traffic
\( T_2 = \) average headway of restrained group;
\( \lambda = \) minimum headway of unrestrained group;
\( \gamma = \) minimum headway of restrained group.

Kell (12) has investigated the suitability of this distribution for use in simulating headway distributions. Furthermore, he has provided a means for determining equation (1) by calculating three of the unknowns, \( \alpha \), \( T_1 \), and \( T_2 \), once the two other unknowns, \( \lambda \) and \( \gamma \), are estimated. In attempting to fit the composite exponential curve to a wide range of observed data, Kell found that \( \lambda \) and \( \gamma \) were relatively constant throughout the volume range and that best agreement occurred where \( 0.9 \leq \lambda \leq 1.0 \) and \( 1.20 \leq \gamma \leq 1.36 \).

In order to randomly generate vehicles from some probability distribution in the GPSS model it is necessary to provide as input information the cumulative distribution function (cdf) of the distribution used.

Thus it was necessary to develop a means for calculating the exponential cdf, based upon the method developed by Kell. A FORTRAN program was written which performed the calculations for determining the exponential cdf for any volume. (The statement listing for this program is included in Appendix C) Figure 6 displays an example of the resulting plot of the cdf for the composite exponential.
Figure 6. Plot of composite exponential cdf.

\[ p(h \geq t) = (1 - \alpha) \left( 1 - e^{\frac{t - \lambda}{t_1 - \lambda}} \right) + \alpha \left( 1 - e^{\frac{t - \tau}{T_2 - \tau}} \right) \]

\[ \lambda = 0.90 \]

\[ \tau = 1.22 \]
Car-following Behavior

Use of the stimulus-response equations of car-following theory for modeling car-following behavior within the intersection approach is not efficient in the GPSS/360 language. Therefore, in the TEC model vehicles are considered to arrive at some point outside the influence of the intersection, moving at some predetermined speed. The vehicles are then moved along the roadway by basing their movement on average speed and acceleration values until they become part of the queue waiting to pass through the intersection, or until they actually pass through the intersection undelayed, whichever occurs first.

This is presently accomplished by generating vehicular arrivals at a point 200 ft. from the approach stopline. The current length of the queue waiting behind the stopline is then tested and the vehicle is either passed through a section of the roadway or it joins the queue. Presently, this test is repeated for a total of three times, if necessary, until the vehicle has joined the queue or passed through the intersection. As explained previously, a vehicle is considered delayed if its approach speed decreases to 15 fps, thus when this occurs it joins the waiting queue. In the present form of the TEC model each vehicle in the queue is assumed to occupy 25 ft. of approach lane space. Furthermore, once a vehicle has become a member of the queue it accumulates delay time and simply waits until it becomes the first vehicle in the queue, whereupon it is processed by the model.
The general manner in which the TEC model provides for vehicular movement through the intersection for each of the four situations simulated is described below. Statement listings for each of the four situations are included in Appendix D.

TEC #1. Single-lane Roadway Without Left-turn Channel

In the case of a single-lane approach, the approach procedure described above is followed. Once a vehicle becomes the first in a waiting queue a probabilistic decision is made as to whether or not it turns left. If the vehicle does not turn left, a test is made to determine whether or not it has been delayed. If it has been delayed, either by the traffic signal or by preceding queued vehicles, it is passed through the intersection with an appropriate starting delay, provided the signal indication is green. If the vehicle has not been delayed up to that point, it passes through the intersection with no starting delay, provided the signal is green.

If the first vehicle in the queue is a left-turning vehicle, however, a different procedure is followed. The left-turn vehicle is assigned an initial turning delay, equivalent to the time it would take for the vehicle to pass through the opposing lane and then the opposing lane is tested for the presence of an acceptable gap. The left-turning vehicle leaves the queue only when an acceptable gap exists. In all cases, if a vehicle occupies the first position in the queue when the signal turns amber, it is passed through the intersection. The left-turn procedure is explained in more detail later in this report. In the TEC model right-turning vehicles are treated much the same as through vehicles.
in that they are assigned turning delays equal in length to the starting delays of through vehicles.

**TEC #2. Single-lane Roadway with Left-turn Channel**

In TEC #2 the left-turn vehicles are entered into a channel of a specific capacity and are then processed in a manner similar to the left-turn vehicles in TEC #1. The channel capacity can be changed for different computer runs in order to determine the effect of channel capacity on the vehicular delays. For determining channel capacity it is assumed that each vehicle occupies 25 ft. of roadway length, as suggested in the ASSHO Blue Book (1). Through vehicles are delayed behind the left-turn channel until the left-turn queue is less than or equal to the channel capacity, if more vehicles are waiting to turn left than the channel has capacity for. All other through and right-turn vehicles are handled similar to those in TEC #1.

**TEC #3. Two-lane Approach without Left-turn Channel**

For this variation of the simulation model vehicles are generated in the usual manner for each of the two lanes independently. The vehicles in the left-most lane (Lane 1) are processed similarly to those in TEC #1. The right lane (Lane 2) vehicles are processed as either through or right-turning vehicles.

**TEC #4. Two-lane Approach with Left-turn Channel**

In this variation vehicles are generated in the same manner as in TEC #3. Vehicles in the median lane (Lane 1) are processed in the same manner as those vehicles in TEC #2 except that there are no right-turning vehicles in this lane. The outside lane (Lane 2) vehicles are processed as either through or
right-turning vehicles.

Gap Acceptance Behavior

Gap acceptance behavior is handled quite effectively in all models by a unique decision process. Each left-turn vehicle, once it occupies the first position in its queue, is assigned a gap acceptance value from a distribution of gap acceptance values. The left-turning vehicle is delayed for a time interval equal in length to the time it would take the vehicle to turn left and clear the traffic lane.

The vehicles in the opposing traffic lane, once they have completed the starting delay times, are assigned to a storage (a specific GPSS/360 entity) for a period of time equal in length to the left-turn vehicle's gap acceptance time. If, after the period of time it takes for the left-turn maneuver, the storage is found to be empty, the left-turn vehicle is considered to have passed through the intersection without further delay. If, however, this storage is found to be occupied, the left-turn vehicle is delayed until the storage is empty. In the event that the signal turns red while a left-turn vehicle is waiting for an acceptable gap, the waiting vehicle is sent through the intersection without further delay.

Information regarding acceptable gap distributions is a necessary input for the TEC model. Numerous investigations of gap acceptance characteristics have been reported in the literature. Desrosiers has summarized the results of a number of these studies in the form of Figure 7 and has pointed out that intersection geometrics, sight obstructions, type of entering maneuver, volume,
Figure 7. Gap Acceptance Distributions Obtained from the Literature
and other traffic characteristics all affect gap acceptance distributions (23). Although some of these distributions have been used in computer simulation efforts to model left-turn gap acceptance, they originated in studies of vehicles leaving a cross street under stop sign control.

Gerlough and Wagner, however, have made use of a specific left-turn gap acceptance distribution which is used in the TEC model (17). This distribution is shown in Table 5. It is appropriate for TEC #3 and TEC #4 since it is based upon data taken at intersections of two-lane roadways.

The gap acceptance distribution used in TEC #1 and TEC #2 is one reported by Dart (7). This distribution is based upon data collected at intersections of single-lane approaches and is, therefore, appropriate for these two models. The distribution is shown in Table 6.

Traffic Signal Operation

The operation of the fixed-time signal is simulated by a routine which, in effect, runs simultaneous to the traffic routine described above. The routine performs somewhat like a clock capable of blocking intersection movement on the red signal indication. During the green signal indication the traffic routine simulates the vehicular movement of two opposing approaches. When the signal shows amber the first vehicle in each of the queues is allowed to pass through the intersection without any further delay. During the remainder of the amber indication and during the entire red indication approaching vehicles are allowed to join the queues waiting for a green signal indication but denied entry to the intersection.
<table>
<thead>
<tr>
<th>GAP SIZE (sec)</th>
<th>CUMULATIVE PERCENT ACCEPTING</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0 - 3.5</td>
<td>15.0</td>
</tr>
<tr>
<td>3.5 - 4.0</td>
<td>32.0</td>
</tr>
<tr>
<td>4.0 - 4.5</td>
<td>52.0</td>
</tr>
<tr>
<td>4.5 - 5.0</td>
<td>69.0</td>
</tr>
<tr>
<td>5.0 - 5.5</td>
<td>82.0</td>
</tr>
<tr>
<td>5.5 - 6.0</td>
<td>90.0</td>
</tr>
<tr>
<td>6.0 - 6.5</td>
<td>95.0</td>
</tr>
<tr>
<td>6.5 - 7.0</td>
<td>97.0</td>
</tr>
<tr>
<td>7.0 - 7.5</td>
<td>98.6</td>
</tr>
<tr>
<td>7.5 - 8.0</td>
<td>99.3</td>
</tr>
<tr>
<td>8.0 - 8.5</td>
<td>99.7</td>
</tr>
<tr>
<td>8.5 - 9.0</td>
<td>99.8</td>
</tr>
<tr>
<td>9.0 - 9.5</td>
<td>99.9</td>
</tr>
<tr>
<td>9.5 - 10.0</td>
<td>100.0</td>
</tr>
</tbody>
</table>
### TABLE 6

**GAP ACCEPTANCES BY LEFT TURNING VEHICLES ON TWO-LANE, TWO-WAY STREETS**

<table>
<thead>
<tr>
<th>GAP SIZE (sec)</th>
<th>CUMULATIVE PERCENT ACCEPTING</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0 - 1.0</td>
<td>0.0</td>
</tr>
<tr>
<td>1.0 - 1.5</td>
<td>1.4</td>
</tr>
<tr>
<td>1.5 - 2.0</td>
<td>10.2</td>
</tr>
<tr>
<td>2.0 - 2.5</td>
<td>18.3</td>
</tr>
<tr>
<td>2.5 - 3.0</td>
<td>31.3</td>
</tr>
<tr>
<td>3.0 - 3.5</td>
<td>50.0</td>
</tr>
<tr>
<td>3.5 - 4.0</td>
<td>64.6</td>
</tr>
<tr>
<td>4.0 - 4.5</td>
<td>85.3</td>
</tr>
<tr>
<td>4.5 - 5.0</td>
<td>94.7</td>
</tr>
<tr>
<td>5.0 - 5.5</td>
<td>96.4</td>
</tr>
<tr>
<td>5.5 - 6.0</td>
<td>97.9</td>
</tr>
<tr>
<td>6.0 - 6.5</td>
<td>98.2</td>
</tr>
<tr>
<td>6.5 - 7.0</td>
<td>98.5</td>
</tr>
<tr>
<td>7.0 - 7.5</td>
<td>99.3</td>
</tr>
<tr>
<td>7.5 - 8.0</td>
<td>99.4</td>
</tr>
<tr>
<td>8.0 - 8.5</td>
<td>100.0</td>
</tr>
</tbody>
</table>
Amber Decision

If the amber phase begins while a vehicle approaches the stop line, the driver must decide whether to pass through the intersection or to stop and wait for the green signal. This situation is simulated in the TEC model by a method which permits the vehicle in the first queue position behind the stop line to pass through the intersection as soon as the amber phase begins.

Queue Discharge

The discharge of vehicles from a queue can best be handled by assigning starting delay values from tables of average values or from a distribution based upon its position within the queue. As was discussed in Chapter 3, values from the data collection phase of the study were compared with values from studies by Greenshields, et al., and from Drew. As was pointed out previously it was decided to utilize the values of Drew to model starting delays in the TEC model.

4.5 TEC Model Output

The TEC model makes extensive use of a specific GPSS/360 language block known as the QUEUE block. Each vehicle that approaches the intersection enters a specific QUEUE block. The QUEUE block provides the means for gathering statistics on vehicles which are delayed by a common cause or set of causes. For each simulation run a set of data concerning each QUEUE block indexed in the program is automatically provided in the output listing. The key QUEUE statistics include:

1. the average contents of the queue which the specific QUEUE block represents;
2. the total number of vehicles which entered the QUEUE;
3. the number of vehicles which spent zero time in the QUEUE;
4. the percentage of vehicles which spent zero time in the QUEUE;
5. the average time that all vehicles spent in the QUEUE;
6. the average time that delayed vehicles spend in the QUEUE;
7. the current QUEUE contents (at the end of the simulation run), and
the maximum contents observed during the simulation run.

From the output information of the QUEUE blocks the average delay per vehicle
and the average delay per stopped vehicle for each approach under consideration
is easily determined.

4.6 TEC Model Validation

In the desire to simulate intersection operation some characteristics of
both the vehicle and driver have been randomized and others averaged in order
to simplify the situation which is modeled. As a result of this simplification,
as well as the modeling process itself, certain tests are necessary to validate
the TEC simulation model. Naylor, et al., have suggested that the validation of
any type of model requires attention at three stages or levels of development
of the model (3). These stages include the actual model formulation, tests of
individual parts of the model, and prediction of the performance of the system
under study.

More specifically, in the formulation of the TEC simulation model, care
was taken in each consecutive stop to build an accurate and reasonable logic
system. This is important in model development but does not guarantee model validity in itself.

Individual sections of the simulation model which could be tested in a limited manner were also checked. Such testing provided information on individual vehicles moving through the intersection system, thereby checking the logic of that section of the model. Particular sections, such as the left-turn gap acceptance routine, were rewritten as individual programs and checked under varying conditions. This type of testing is also an essential part of model development but, it too does not provide a sufficient test of total model validity.

In order to determine the validity of the complete TEC model a number of tests were performed. Since comparison data for four different intersections was available from the data collection phase of study, the effectiveness of the TEC model for modeling each of these intersection types was tested. The basis for the validity tests is the Kolmogorov-Smirnov one-sample test for goodness of fit, which has been explained previously. The values which have been compared between the collected data and the simulation output are the delays for vehicles passing through the intersection.

The strategy of the validation tests was to develop the population distribution of delays for each simulated situation. This was accomplished by compiling the results of ten independent runs of the TEC model for each set of conditions specified by the collected data. For example, the test procedure for the data from film #4 concerning the State Route 3 traffic was the following:
The TEC model was prepared to simulate two opposing approaches of traffic, the northbound approach with a volume of 272 vehicles per hour and 16% left turns and the southbound approach with a volume of 204 vehicles per hour and 4% left turns. This situation was run on the computer a total of ten times, each run with an initialization period of five cycles, and a data collection period of thirty minutes. For each computer run the random number generators are randomly assigned a new "seed" number which randomizes the vehicle generation and turn selection routines. The population distribution of delays for both approaches was then estimated. Each of the sample distributions provided by the collected data was then tested against the appropriate population distribution. A summary of the results of these tests is shown in Table 7.

These results indicate that in all but two cases the hypothesis that the sample distribution can be reasonably thought to have come from the population distribution is accepted at the 0.95 level of significance for the five second increment chosen. The two cases in which the hypothesis was rejected both represent examples of low approach volumes. Both cases, however, differed considerably and it was difficult to attribute the failure of fit to any one specific characteristic. Nevertheless, these tests do indicate that the TEC model provides adequate simulation of the types of intersections under study.

The simulation results were further subjected to a comparative analysis similar to that of Gerlough and Wagner (17). Table 8 lists a number of traffic statistics compiled from the collected data and from the simulation runs. This comparison provides further evidence of the validity of the TEC model.
### TABLE 7

DELAY DISTRIBUTION TEST RESULTS

<table>
<thead>
<tr>
<th>APPROACH</th>
<th>D</th>
<th>$D_{critical}$</th>
<th>RESULTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>S. R. 3 - S. B.</td>
<td>0.088</td>
<td>0.095</td>
<td>accept</td>
</tr>
<tr>
<td>S. R. 3 - N. B.</td>
<td>0.112</td>
<td>0.113</td>
<td>accept</td>
</tr>
<tr>
<td>S. R. 161 - W. B.</td>
<td>0.043</td>
<td>0.117</td>
<td>accept</td>
</tr>
<tr>
<td>S. R. 161 - E. B.</td>
<td>0.051</td>
<td>0.120</td>
<td>accept</td>
</tr>
<tr>
<td>Zollinger Road - W. B.</td>
<td>0.065</td>
<td>0.102</td>
<td>accept</td>
</tr>
<tr>
<td>Zollinger Road - E. B.</td>
<td>0.045</td>
<td>0.127</td>
<td>accept</td>
</tr>
<tr>
<td>Northwest Blvd. S. B.</td>
<td>0.053</td>
<td>0.134</td>
<td>accept</td>
</tr>
<tr>
<td>Northwest Blvd. N. B.</td>
<td>0.091</td>
<td>0.095</td>
<td>accept</td>
</tr>
<tr>
<td>Reed Road - S. B.</td>
<td>0.150</td>
<td>0.115</td>
<td>Reject</td>
</tr>
<tr>
<td>Reed Road - N. B.</td>
<td>0.068</td>
<td>0.103</td>
<td>accept</td>
</tr>
<tr>
<td>McCoy Road - W. B.</td>
<td>0.077</td>
<td>0.110</td>
<td>accept</td>
</tr>
<tr>
<td>McCoy Road - E. B.</td>
<td>0.063</td>
<td>0.125</td>
<td>accept</td>
</tr>
<tr>
<td>17th Avenue - W. B.</td>
<td>0.100</td>
<td>0.228</td>
<td>accept</td>
</tr>
<tr>
<td>17th Avenue - E. B.</td>
<td>0.105</td>
<td>0.109</td>
<td>accept</td>
</tr>
<tr>
<td>Woodland Avenue - S. B.</td>
<td>0.143</td>
<td>0.147</td>
<td>accept</td>
</tr>
<tr>
<td>Woodland Avenue - N. B.</td>
<td>0.200</td>
<td>0.173</td>
<td>Reject</td>
</tr>
<tr>
<td>APPROACH</td>
<td>AVERAGE DELAY PER VEHICLE (min)</td>
<td>AVERAGE DELAY PER STOPPED VEHICLE (min)</td>
<td>PROPORTION STOPPED</td>
</tr>
<tr>
<td>--------------</td>
<td>---------------------------------</td>
<td>------------------------------------------</td>
<td>---------------------</td>
</tr>
<tr>
<td></td>
<td>Measured</td>
<td>Simulated</td>
<td>Measured</td>
</tr>
<tr>
<td>North West Blvd.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. B.</td>
<td>0.225</td>
<td>0.232</td>
<td>0.420</td>
</tr>
<tr>
<td>S. B.</td>
<td>0.176</td>
<td>0.181</td>
<td>0.326</td>
</tr>
<tr>
<td>Zollinger Road</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E. B.</td>
<td>0.249</td>
<td>0.229</td>
<td>0.378</td>
</tr>
<tr>
<td>W. B.</td>
<td>0.259</td>
<td>0.246</td>
<td>0.433</td>
</tr>
<tr>
<td>S. R. 161</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E. B.</td>
<td>0.260</td>
<td>0.249</td>
<td>0.496</td>
</tr>
<tr>
<td>W. B.</td>
<td>0.286</td>
<td>0.297</td>
<td>0.468</td>
</tr>
<tr>
<td>S. R. 3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. B.</td>
<td>0.208</td>
<td>0.242</td>
<td>0.392</td>
</tr>
<tr>
<td>S. B.</td>
<td>0.255</td>
<td>0.283</td>
<td>0.448</td>
</tr>
<tr>
<td>APPROACH</td>
<td>AVERAGE DELAY PER VEHICLE (min)</td>
<td>AVERAGE DELAY PER STOPPED VEHICLE (min)</td>
<td>PROPORTION STOPPED</td>
</tr>
<tr>
<td>------------------</td>
<td>---------------------------------</td>
<td>----------------------------------------</td>
<td>---------------------</td>
</tr>
<tr>
<td></td>
<td>Measured</td>
<td>Simulated</td>
<td>Measured</td>
</tr>
<tr>
<td>Reed Road</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. B.</td>
<td>0.164</td>
<td>0.186</td>
<td>0.305</td>
</tr>
<tr>
<td>S. B.</td>
<td>0.145</td>
<td>0.202</td>
<td>0.261</td>
</tr>
<tr>
<td>McCoy Road</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E. B.</td>
<td>0.188</td>
<td>0.211</td>
<td>0.334</td>
</tr>
<tr>
<td>W. B.</td>
<td>0.159</td>
<td>0.179</td>
<td>0.276</td>
</tr>
<tr>
<td>17th Avenue</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E. B.</td>
<td>0.220</td>
<td>0.364</td>
<td>0.373</td>
</tr>
<tr>
<td>W. B.</td>
<td>0.192</td>
<td>0.170</td>
<td>0.301</td>
</tr>
<tr>
<td>Woodland Avenue</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. B.</td>
<td>0.156</td>
<td>0.269</td>
<td>0.312</td>
</tr>
<tr>
<td>S. B.</td>
<td>0.126</td>
<td>0.193</td>
<td>0.333</td>
</tr>
</tbody>
</table>
CHAPTER V
INTERSECTION SIMULATION STUDY

The TEC model has been shown to provide a reasonable simulation of traffic operations at four different kinds of intersections. Thus the TEC model provides a convenient tool for studying the effect of change of various traffic characteristics upon the delay to vehicles passing through these intersections. A study was conducted to evaluate the effect of various traffic characteristics, particularly the addition of left-turn channels, upon vehicular delay.

5.1 Left-turn Lanes

The provision of a special lane for left-turning vehicles at a signalized intersection by construction or marking is an accepted technique for increasing safety and capacity. However, criteria for determining the optimum length for such lanes have not yet been generally accepted.

The geometric design handbooks for urban highways (2) and rural highways (1) published by the American Association of State Highway Officials provide only guidelines for the inclusion of the left-turn lanes at both signalized and unsignalized intersections. For unsignalized intersections in urban areas the suggested storage lengths for a left-turn channel is based upon the number of left-turning vehicles which are likely to accumulate in two minutes. The
The formula is

\[ L = \frac{Ns}{30} \]

where \( L \) is the storage length in feet, \( N \) is the design volume of left-turning movements in vehicles per hour, and \( s \) is the length occupied by each vehicle. For signalized intersections it is further suggested that the length be based on 1.5 times the average number of vehicles that would store per cycle. The formula would be

\[ L = \frac{1.5 C Ns}{3600} \]

where \( C \) is the signal cycle length and the other terms are as above.

### 5.2 Intersection Study Design

In order to evaluate the design criteria for signalized intersections, and to provide a basis for new criteria, a simulation study based upon experimental design considerations was initiated. An experimental design provides a convenient strategy for the systematic variation of certain independent variables under consideration. It is obvious that an attempt to investigate a wide range of a large number of variables at various levels would involve a large number of combinations of these variables. Since each possible combination would require a single simulation run in order to study the effect of changing the variables, the total number of simulation runs can be prohibitive. For example, consider an experiment which involves five factors. If three different values are considered for each factor, it would be necessary to initiate \( 3^5 \), or 243 computer runs, to consider all the possible combinations. Consideration of more factors and/or levels of factors obviously increases the total number of combinations.
The type of experiment in which a number of factors are examined in all possible combinations is known as complete factorial design, and is an accepted technique for this type of investigation provided the number of factors is not large. Moreover, when a complete replication of a factorial experiment is beyond the resources of the investigator or when higher order interactions are not expected to be of appreciable magnitude, the complete factorial design may not be necessary.

There does exist a technique that permits an experimenter to investigate only a fraction of the total factorial combinations. Studies involving this smaller number of combinations often contain enough information to fulfill the original experiment objectives. This specialized technique is known as a fractional factorial design and it provides the basis for a design utilizing only a fraction of the combinations of factors. In this manner a design requiring only a specified fraction of a complete replication can be used to determine the effects of the independent variables under study.

Thus the experimental design technique, the fractional factorial, provided the strategy for the systematic variation of the independent variables, as well as information regarding the effect of these independent variables upon the dependent variable, delay per vehicle. A regression analysis of the experimental results provided the predictive tool for design purposes.

The first phase of the simulation study consisted of identifying the factors affecting delay to vehicles passing through an intersection without a left-turn channel. The second phase was concerned with intersections with a
left-turn channel. Each phase consisted of two parts since both single-lane and two-lane approaches were considered. Each part of the study was based upon a fractional factorial design for a $1/3$ of a $3^5$ design. Thus, each part consists of 81 experiments of simulation runs. This design provides information for evaluating the significance of each of the five factors as well as all ten two-factor interactions. A more detailed description of the fractional factorial design for the $1/3 \cdot 3^5$ design which was used is given in Appendix D. The results of the experiments in each part also provide information necessary for the development of a regression model for use in predicting vehicular delay under conditions within the range of the variables chosen.

5.3 Selection of Independent Variables

As described previously, the TEC model simulates two opposing approaches to an intersection. In the TEC model these two approaches are referred to as the northbound (NB) and the southbound (SB) approaches for convenience. In order to consider as wide a range of intersection operating characteristics as possible, and, also to keep the size of the simulation study within reason, five independent variables were chosen to represent various conditions at an intersection for the first phase of the simulation study. These include the following:

1. approach volume (SB)
2. opposing volume (NB)
3. percentage of left-turns (SB)
4. signal cycle length

5. percent green time.

The result or effect under study is the average delay encountered by the vehicles on the southbound approach; this result is in terms of seconds per vehicle.

5.4 Preparation of the TEC Model; Single Lane Approach

Prior to any simulation run or group of runs a number of input parameters must be specified in the TEC model. The parameters and the levels chosen for use throughout the study include the following:

1. Percentage of left-turns-NB approach; a value of 10% left turns for the northbound approach was used throughout the simulation study. This value was used to express average conditions and to help limit the size of the experiment model since its effect upon SB delay was considered to be minimal.

2. Left-turn gap acceptance values; the left-turn gap acceptance distributions specified in the validation study for both TEC #1 and TEC #3 were also utilized for the simulation study.

3. Start delay values; individual values for vehicular starting delay used for the simulation study were the values suggested by Drew.

4. Approach speeds; each vehicle was assumed to be approaching the intersection approach at a speed of 30 mph.

5. Data collection period; in order to collect data on vehicles approaching the intersection for each combination of factors, or experiment, it was
necessary to select some period of time over which the conditions specified would be assumed to exist and to collect data for this period. Also it was necessary to assume some period of initialization during which the approaches would attain a steady state of operation. After some preliminary testing utilizing the TEC model, it was found that a length of time equal to five cycle lengths provided an adequate period of initialization.

The selection of the data collection period was a bit more complex. Drew and Pinnell have shown that peak periods exist within peak hours and have concluded that the use of the average hourly volume as a design basis may render the facility underdesigned for the entire peak period within the design hour (24). As a result it was desirable to assume that the simulation study represented the simulation of conditions existing during some peak period within the peak hour. Furthermore, since the length of the peak period was found to vary at different locations, the peak-hour factor presented in the Highway Capacity Manual, provided the basis for the data collection period used.

The peak-hour factor, which is a measure of the consistency of demand at an intersection approach, is defined as the ratio between the number of vehicles counted during the peak hour and four times the number of vehicles counted during the highest fifteen consecutive minutes. It was felt that a fifteen minute period provided an adequate representation of the peak period.

Prior to the simulation study a number of computer runs were made to determine the adequacy of the 200 foot approaches used successfully in the
validation studies. Based on the information provided by these runs it was decided that the approaches would be extended to 400 feet to accommodate higher volume levels, based upon the same strategy used for the shorter approaches. A number of simulation runs using the data from the validation studies showed that this change was accomplished successfully.

Levels of Variables; Single-Lane Approach

The independent variables which were altered to represent various conditions existing at an intersection and the levels used in TEC #1 for the first phase of the simulation study are the following:

1. traffic volume - NB; 200, 400 and 600 vehicles per hour
2. traffic volume - SB; 200, 400 and 600 vehicles per hour
3. percentage of left-turns - SB; 0, 10 and 20%
4. signal cycle length; 60, 75 and 90 seconds
5. percent green time; 40, 50 and 60%.

5.5 Preparation of the TEC Model; Two Lane Approach

The values chosen for the input parameters which required specification in the study of two-lane approaches are similar to those used in the study of single-lane approaches. These values can be summarized as follows:

(1) percentage of left-turns-NB approach; 10%
(2) left-turn gap acceptance distribution: the same as specified in the validation study.
(3) start delay values: those suggested by Drew.
(4) approach speed: 30 mph.

(5) data collection period: 5 cycle initialization period followed by 50 minutes data collection.

An additional consideration which had to be taken into account in the use of TEC #3 and TEC #4 was the split of approach volume between the two approach lanes. It has been pointed out that information regarding lane distributions for intersection approaches is quite scarce (17). Gerlough and Wagner did, however, attempt to develop a regression model for identifying lane distributions. Their model is stated as:

\[ y = 0.495 - 0.00015x \]

where \( x \) is the two-lane approach volume rate, in vehicles per hour, and \( y \) is the regression estimate of the proportion of traffic using the left lane. Use of this model, however, can give results which do not appear reasonable. For example, assuming an intersection approach volume of 1200 vehicles per hour, the number of vehicles expected in the left lane is 378 and the number expected in the right lane is 822. With an approach volume of this magnitude it would not seem likely that the split would be quite this extreme. For, although the percentage of traffic using the left lane can be expected to decrease with increases in total approach volume, eventually traffic volumes will reach a point at which left lane percentages will begin increasing again. Furthermore, in such a model, it can be expected that left-turn percentages should be a contributing factor.
Therefore, it was decided to select reasonable lane distributions to model the conditions expected to exist. The approach volumes considered in the two-lane approach studies were 400, 800, and 1200 vehicles per hour. The lane distributions were selected as follows:

<table>
<thead>
<tr>
<th>Total Volume</th>
<th>Lane 1 Volume</th>
<th>Lane 2 Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>150</td>
<td>250</td>
</tr>
<tr>
<td>800</td>
<td>300</td>
<td>500</td>
</tr>
<tr>
<td>1200</td>
<td>500</td>
<td>700</td>
</tr>
</tbody>
</table>

Levels of Variables; Two Lane Approach

The independent variables which were altered to represent various conditions existing at an intersection and the levels used in TEC #3 for the first phase of the simulation study include the following:

1. traffic volume - NB, 400, 800 and 1200 vehicles per hour
2. traffic volume - SB, 400, 800 and 1200 vehicles per hour
3. percentage of left-turns - SB; 0, 10 and 20%
4. signal cycle length; 60, 75 and 90 seconds
5. percent green time; 40, 50 and 60%.

5.6 Results of Simulation Study: Unchannelized Approach

The fractional factorial design, besides providing the basis for the regression model for predicting delay, yields information on the significance of the five factors and ten two-factor interactions with respect to the response being studied. As pointed out in Appendix D an analysis of variance can be
performed on the simulation results. The analysis of variance for the single-
lane, unchannelized approach study is shown in Table 9. The significant fac-
tors include approach volume (A), opposing volume (B), percent left-turns (C),
and percent green time (E); the interaction between approach volume and per-
cent left-turns (AC) is also significant.

The analysis of variance for the two-lane, unchannelized approach study
is shown in Table 10. The significant factors in this case are the same as in
the single-lane study; approach volume (A), opposing volume (B), percent left-
turns (C), and percent green time (E). However, three interactions are signif-
icient: the interaction between approach volume and percent left-turns (AC);
the interaction between approach volume and percent green time (AE); and the
interaction between opposing volume and percent left-turns (BC).

From these results it can be seen that cycle length was not a significant
factor and none of the interactions involving cycle length were significant for
either the single-lane case or the two-lane case. As a result it was decided
that signal cycle length would be dropped from consideration in the second phase
of the simulation study. In its place, the variable, channel storage capacity,
was used. This simple replacement permitted the use of the same $1/3 \cdot 3^5$
design in the second phase. A cycle length of 75 seconds was used throughout
the second phase.

5.7 Results of Simulation Study: Channelized Approach

The analysis of variance for the single lane, channelized approach study
is shown in Table 11. For the results of this phase, factor D, which was con-
TABLE 9

SINGLE-LANE UNCHANNELIZED APPROACH

FRACTIONAL FACTORIAL RESULTS

<table>
<thead>
<tr>
<th>FACTOR</th>
<th>SUM OF SQUARES</th>
<th>MEAN SQUARES</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>8266.69</td>
<td>4133.34</td>
<td>9.25*</td>
</tr>
<tr>
<td>B</td>
<td>3113.21</td>
<td>1556.60</td>
<td>3.48*</td>
</tr>
<tr>
<td>C</td>
<td>8570.06</td>
<td>4285.03</td>
<td>9.59*</td>
</tr>
<tr>
<td>D</td>
<td>893.90</td>
<td>446.95</td>
<td>1.00</td>
</tr>
<tr>
<td>E</td>
<td>8248.67</td>
<td>4124.33</td>
<td>9.23*</td>
</tr>
<tr>
<td>AB</td>
<td>2924.22</td>
<td>731.06</td>
<td>1.64</td>
</tr>
<tr>
<td>AC</td>
<td>5420.01</td>
<td>1355.00</td>
<td>3.03*</td>
</tr>
<tr>
<td>AD</td>
<td>368.56</td>
<td>92.14</td>
<td>0.21</td>
</tr>
<tr>
<td>AE</td>
<td>3175.53</td>
<td>793.88</td>
<td>1.78</td>
</tr>
<tr>
<td>BC</td>
<td>2069.71</td>
<td>517.43</td>
<td>1.16</td>
</tr>
<tr>
<td>BD</td>
<td>421.23</td>
<td>105.31</td>
<td>0.24</td>
</tr>
<tr>
<td>BE</td>
<td>829.37</td>
<td>207.34</td>
<td>0.46</td>
</tr>
<tr>
<td>CD</td>
<td>212.45</td>
<td>53.11</td>
<td>0.12</td>
</tr>
<tr>
<td>CE</td>
<td>1778.52</td>
<td>444.63</td>
<td>0.99</td>
</tr>
<tr>
<td>DE</td>
<td>1245.29</td>
<td>311.32</td>
<td>0.70</td>
</tr>
</tbody>
</table>

\[ \bar{y} = 27.43 \]

\[ \text{S. S. E.} = 13,406.98 \quad \text{M. S. E.} = 446.90 \]

FACTORS:
- A = approach volume
- B = opposing volume
- C = percent left turns
- D = cycle length
- E = percent green time

(* indicates significance at the 0.95 level)
### TABLE 10

**TWO-LANE UNCHANNELIZED APPROACH**

**FRACTIONAL FACTORIAL RESULTS**

<table>
<thead>
<tr>
<th>FACTOR</th>
<th>SUM OF SQUARES</th>
<th>MEAN SQUARES</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>26859.67</td>
<td>13429.83</td>
<td>34.81*</td>
</tr>
<tr>
<td>B</td>
<td>5647.27</td>
<td>2823.63</td>
<td>7.32*</td>
</tr>
<tr>
<td>C</td>
<td>16473.56</td>
<td>8236.78</td>
<td>21.35*</td>
</tr>
<tr>
<td>D</td>
<td>573.79</td>
<td>286.89</td>
<td>0.74</td>
</tr>
<tr>
<td>E</td>
<td>13108.20</td>
<td>6554.10</td>
<td>16.99*</td>
</tr>
<tr>
<td>AB</td>
<td>2860.36</td>
<td>715.09</td>
<td>1.85</td>
</tr>
<tr>
<td>AC</td>
<td>7736.93</td>
<td>1934.23</td>
<td>5.01*</td>
</tr>
<tr>
<td>AD</td>
<td>56.62</td>
<td>14.15</td>
<td>0.04</td>
</tr>
<tr>
<td>AE</td>
<td>5766.87</td>
<td>1441.72</td>
<td>3.74*</td>
</tr>
<tr>
<td>BC</td>
<td>5022.12</td>
<td>1255.53</td>
<td>3.25*</td>
</tr>
<tr>
<td>BD</td>
<td>204.83</td>
<td>51.21</td>
<td>0.13</td>
</tr>
<tr>
<td>BE</td>
<td>961.25</td>
<td>240.31</td>
<td>0.62</td>
</tr>
<tr>
<td>CD</td>
<td>360.36</td>
<td>90.09</td>
<td>0.23</td>
</tr>
<tr>
<td>CE</td>
<td>1669.76</td>
<td>417.44</td>
<td>1.08</td>
</tr>
<tr>
<td>DE</td>
<td>917.53</td>
<td>229.38</td>
<td>0.59</td>
</tr>
<tr>
<td></td>
<td><strong>88219.12</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \bar{y} = 35.10 \]

**S. S. E. = 11573.69**  **M. S. E. = 385.79**

**FACTORS:**
- **A** = approach volume
- **B** = opposing volume
- **C** = percent left turns
- **D** = cycle length
- **E** = percent green time

(* indicates significance at the 0.95 level)
### TABLE 11

**SINGLE-LANE-CHANNELIZED APPROACH**

**FRACTIONAL FACTORIAL RESULTS**

<table>
<thead>
<tr>
<th>FACTOR</th>
<th>SUM OF SQUARES</th>
<th>MEAN SQUARES</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3897.32</td>
<td>1948.66</td>
<td>3.36*</td>
</tr>
<tr>
<td>B</td>
<td>588.96</td>
<td>294.48</td>
<td>0.51</td>
</tr>
<tr>
<td>C</td>
<td>2114.23</td>
<td>1057.11</td>
<td>1.62</td>
</tr>
<tr>
<td>D</td>
<td>6004.53</td>
<td>3002.26</td>
<td>5.18*</td>
</tr>
<tr>
<td>E</td>
<td>3269.11</td>
<td>1634.55</td>
<td>2.82*</td>
</tr>
<tr>
<td>AB</td>
<td>251.02</td>
<td>62.76</td>
<td>0.11</td>
</tr>
<tr>
<td>AC</td>
<td>1167.10</td>
<td>291.77</td>
<td>0.50</td>
</tr>
<tr>
<td>AD</td>
<td>2421.76</td>
<td>605.44</td>
<td>1.04</td>
</tr>
<tr>
<td>AE</td>
<td>548.95</td>
<td>137.24</td>
<td>0.24</td>
</tr>
<tr>
<td>BC</td>
<td>395.80</td>
<td>98.95</td>
<td>0.17</td>
</tr>
<tr>
<td>BD</td>
<td>640.62</td>
<td>160.16</td>
<td>0.28</td>
</tr>
<tr>
<td>BE</td>
<td>1401.26</td>
<td>350.31</td>
<td>0.60</td>
</tr>
<tr>
<td>CD</td>
<td>3226.55</td>
<td>806.64</td>
<td>1.39</td>
</tr>
<tr>
<td>CE</td>
<td>381.76</td>
<td>95.44</td>
<td>0.16</td>
</tr>
<tr>
<td>DE</td>
<td>1327.44</td>
<td>331.86</td>
<td>0.57</td>
</tr>
</tbody>
</table>

\[
\sum \text{SUM OF SQUARES} = 27636.41
\]

\[
\bar{y} = 23.58
\]

\[
\text{S. S. E.} = 17401.21 \quad \text{M. S. E.} = 580.04
\]

**FACTORS:**
- A = approach volume
- B = opposing volume
- C = percent left turns
- D = storage capacity
- E = percent green time

(* indicates significance at the 0.95 level)
sidered at levels of 1, 3 and 5, represents left-turn channel storage capacity for 1, 3 or 5 vehicles based upon a required 25 feet storage per vehicle. The significant factors include approach volume (A), storage capacity (D), and percent green time (E), no interactions were significant.

The analysis of variance for the two-lane, channelized approach study are shown in Table 12. Again no interactions were significant, but the significant factors include approach volume (A), percent left-turns (C), storage capacity (D), and percent green time (E).

5.8 Regression Analysis

From the results of these two phases of the simulation regression in the form of

\[ Y = \beta_0 + \sum \beta_i X_i + \sum \beta_{ii} X_i^2 + \sum \beta_{ij} X_i X_j \]

was performed on both the single-lane approach data and the two-lane approach data to provide quadratic relationships suitable for predicting vehicle delay from traffic operational characteristics. The regression was performed through the use of a stepwise multiple regression program which is a part of the Biomedical Computer Programs series (26). The program computes a sequence of multiple linear regression equations in a stepwise manner in which one variable is added to the regression equation at each step. The variable added is the one which makes the greatest reduction in the error sum of squares and, equivalently, it is the variable which, if added, would have the highest F value. A minimum value of \( F = 4.0 \) was specified for inclusion into the equation.
### TABLE 12

**TWO-LANE CHANNELIZED APPROACH**

**FRACTIONAL FACTORIAL RESULTS**

<table>
<thead>
<tr>
<th>FACTOR</th>
<th>SUM OF SQUARES</th>
<th>MEAN SQUARES</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>13352.57</td>
<td>6176.29</td>
<td>5.24*</td>
</tr>
<tr>
<td>B</td>
<td>3600.87</td>
<td>1800.44</td>
<td>1.53</td>
</tr>
<tr>
<td>C</td>
<td>9607.73</td>
<td>4803.86</td>
<td>4.08*</td>
</tr>
<tr>
<td>D</td>
<td>8024.70</td>
<td>4012.35</td>
<td>3.40*</td>
</tr>
<tr>
<td>E</td>
<td>7011.27</td>
<td>3505.63</td>
<td>2.97*</td>
</tr>
<tr>
<td>AB</td>
<td>473.34</td>
<td>118.34</td>
<td>0.10</td>
</tr>
<tr>
<td>AC</td>
<td>2671.37</td>
<td>667.84</td>
<td>0.57</td>
</tr>
<tr>
<td>AD</td>
<td>650.74</td>
<td>162.69</td>
<td>0.14</td>
</tr>
<tr>
<td>AE</td>
<td>349.19</td>
<td>87.30</td>
<td>0.07</td>
</tr>
<tr>
<td>BC</td>
<td>4371.10</td>
<td>1092.78</td>
<td>0.93</td>
</tr>
<tr>
<td>BD</td>
<td>1169.31</td>
<td>292.33</td>
<td>0.25</td>
</tr>
<tr>
<td>BE</td>
<td>2314.03</td>
<td>578.51</td>
<td>0.49</td>
</tr>
<tr>
<td>CD</td>
<td>4403.84</td>
<td>1100.96</td>
<td>0.93</td>
</tr>
<tr>
<td>CE</td>
<td>2290.55</td>
<td>572.64</td>
<td>0.49</td>
</tr>
<tr>
<td>DE</td>
<td>1214.93</td>
<td>303.73</td>
<td>0.26</td>
</tr>
<tr>
<td><strong>T</strong></td>
<td><strong>61705.54</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \bar{y} = 34.57 \]

\[ \text{S. S. E.} = 35352.64 \quad \text{M. S. E.} = 1178.42 \]

**FACTORS:**
- A = approach volume
- B = opposing volume
- C = percent left turns
- D = storage capacity
- E = percent green time

(* indicates significance at the 0.95 level)
The resultant equation for the single-lane approach is

\[ Y = 66.01265 - 1.24792 X_5 + 0.00010 X_1 X_2 + 0.00311 X_1 X_3 
- 0.01167 X_1 X_4 - 0.00828 X_2 X_4 - 0.26231 X_3 X_4 
+ 0.15052 X_4 X_5 \]

where \( Y \) = average delay (seconds per vehicle)

\( X_1 \) = approach volume (vehicles per hour)

\( X_2 \) = opposing volume (vehicles per hour)

\( X_3 \) = percent left turns (X 100)

\( X_4 \) = channel storage capacity (vehicles)

\( X_5 \) = percent green time (X 100)

The multiple correlation coefficient, \( r \), is 0.81. Therefore, \( r^2 \), is 0.65, or 65% of the variation is explained by the regression.

The equation for the two-lane approach is

\[ Y = 60.64511 - 0.86324 X_5 + 0.00003 X_1 X_2 + 0.00286 X_1 X_3 
+ 0.00330 X_2 X_3 - 0.24953 X_3 X_4 - 0.05974 X_3 X_5 
- 0.00002 X_4^2 \]

where the variables are as defined above. The multiple correlation coefficient for this equation is 0.863 and \( r^2 = 0.74 \). Thus, 74% of the variation is explained by the regression equation.

This pair of equations provides an adequate means for the prediction of average vehicular delay under specific intersection traffic conditions and permits the designer to estimate expected delay when a left-turn storage lane is added to the approach. Use of a desk calculator can supply the designer with
the results in a reasonably short time. However, to further simplify this approach, a computer program was written in FORTRAN to handle the calculations necessary for either of the two equations. Figure 8 is a statement listing for this program and, by following the instructions included in the program, the output shown in Figure 9 can be obtained.

5.9 Comparison with Existing Design Criteria

In order to provide a comparison of the regression technique results with the existing criteria for design as well as to illustrate how the two can be used to complement each other as design tools, the following analysis was performed. The left-turn channel storage requirements for a volume of 600 vehicles per hour on a single-lane approach and for left-turn percentages of 10 and 20 based upon the AASHO geometric design criteria are 46.9 feet and 93.8 feet, respectively. A storage requirement of 25 feet per vehicle and a 75 second cycle length is assumed. If the designer selects storage capacity based upon the next highest 25 feet increment, the storage capacities for these two examples would be 2 vehicles and 4 vehicles.

The regression technique provides information on the average delay per vehicles for these same conditions if a number of other factors are specified. If, for example, opposing volume is 400 vehicles per hour, green time is 50%, and left-turns are 10%, during a peak period, the expected delay per vehicle for various channel capacities is as follows:
C VARIABLE DEFINITIONS FOLLOW
C A=APPROACH VOLUME (VEHICLES PER HOUR)
C B=OPPOSING VOLUME (VEHICLES PER HOUR)
C C=PERCENT LEFT TURNS X 100
C E=PERCENT GREEN TIME X 100
C J=INDICATOR  1= SINGLE LANE APPROACH
C 2= TWO LANE APPROACH
C INPUT IS FORMAT FREE - VALUES MUST BE
C SEPARATED BY BLANKS OR BY A COMMA
C DATA DECK MUST INCLUDE A LAST CARD WITH
C J=3, OR CAN BE PUNCHED AS FOLLOWS
C J=3

21 READ,A,B,C,E,J
   IF(J.GE.3) GO TO 101
   IF(J.GE.2) GO TO 102
   D=0.0
   I=0
   DELY=66.01265-1.24792*E+.00010*A*B+.00311*A*C-.01167*A*D-.00828*B*
   1       D-.26231*C*D+.15052*D*E
   PRINT2
   2 FORMAT(/,5X,'APPROACH VOLUME',5X,'OPPOSING VOLUME',5X,'% LEFT TURNS',5X,'% GREEN TIME',5X,'STORAGE CAPACITY',5X,'DELAY / VEHICLE')
   PRINT3,A,B,C,E,DELAY
   DO 10 I=1,5,1
   D=D+1.0
   DCHA=66.01265-1.24792*E+.00010*A*B+.00311*A*C-.01167*A*D-.00828*B*
   1       D-.26231*C*D+.15052*D*E
   PRINT4,A,B,C,E,D,DCHA
10 CONTINUE
   GO TO 21
102 D=0.0
   I=0
   DELY=60.64511-.86324*E+.00003*A*B+.00286*A*C+.00330*B*C-.24953*C*D
   1       -.05974*C*E-.00002*D*D
   PRINT2
   PRINT3,A,B,C,E,DELAY
   DO 11 I=1,5,1
   D=D+1.0
   DCHA=60.64511-.86324*E+.00003*A*B+.00286*A*C+.00330*B*C-.24953*C*D
   1       -.05974*C*E-.00002*D*D
   PRINT4,A,B,C,E,D,DCHA
11 CONTINUE
   GO TO 21
   1 2)
101 STOP
END

Figure 8

Vehicle Delay Calculation Program Statement Listing
<table>
<thead>
<tr>
<th>APPROACH VOLUME</th>
<th>OPPOSING VOLUME</th>
<th>% LEFT TURNS</th>
<th>% GREEN TIME</th>
<th>STORAGE CAPACITY</th>
<th>DELAY / VEHICLE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>NO CHANNEL</td>
<td></td>
</tr>
<tr>
<td>600.00</td>
<td>400.00</td>
<td>10.0</td>
<td>50.00</td>
<td>46.28</td>
<td>46.28</td>
</tr>
<tr>
<td>600.00</td>
<td>400.00</td>
<td>10.0</td>
<td>50.00</td>
<td>40.87</td>
<td>40.87</td>
</tr>
<tr>
<td>600.00</td>
<td>400.00</td>
<td>10.0</td>
<td>50.00</td>
<td>35.04</td>
<td>35.04</td>
</tr>
<tr>
<td>600.00</td>
<td>400.00</td>
<td>10.0</td>
<td>50.00</td>
<td>30.04</td>
<td>30.04</td>
</tr>
<tr>
<td>600.00</td>
<td>400.00</td>
<td>10.0</td>
<td>50.00</td>
<td>24.63</td>
<td>24.63</td>
</tr>
<tr>
<td>600.00</td>
<td>400.00</td>
<td>10.0</td>
<td>50.00</td>
<td>19.22</td>
<td>19.22</td>
</tr>
</tbody>
</table>

**Figure 9**

Sample Program Output
The addition of a left-turn channel of storage equal to 2 vehicles decreases delay under the specified conditions by 10.83 seconds per vehicle. A channel with a storage capacity of 4 vehicles decreases the delay an additional 10.82 seconds per vehicle for a total decrease in delay of 21.65 seconds. Furthermore, if left-turns are assumed to be 20% and all other factors are the same as above, the regression technique yields the following expected delay values:

<table>
<thead>
<tr>
<th>Storage capacity</th>
<th>Delay per vehicle</th>
</tr>
</thead>
<tbody>
<tr>
<td>no channel</td>
<td>64.94</td>
</tr>
<tr>
<td>1</td>
<td>56.90</td>
</tr>
<tr>
<td>2</td>
<td>48.87</td>
</tr>
<tr>
<td>3</td>
<td>40.83</td>
</tr>
<tr>
<td>4</td>
<td>32.80</td>
</tr>
<tr>
<td>5</td>
<td>24.77</td>
</tr>
</tbody>
</table>

The addition of a channel of storage for 4 vehicles decreases the expected delay 32.14 seconds. This information should provide the designer with an additional decision-making tool. At the least the regression technique provides delay information over a range of possible storage capacities, so that if delay is of prime consideration during the peak period the storage capacities suggested by the AASHO technique can be analyzed relative to a number of alternative capacities. Furthermore, where peak conditions vary greatly
such that it is possible to encounter a wide range for the various factors involved, each of these combinations can be analyzed by the regression technique to provide a more substantial comparison for the decision-making process.
CHAPTER VI
SUMMARY

The overall aim of this research was to investigate more efficient methods of studying intersection operation under varying traffic conditions. The results of this research are applicable to the types of intersections studied and which have specific traffic characteristics that fall within the range of values studied.

6.1 Summary of Results

The specific results of this research are related to the stated objectives of the study. The accomplishments of this research can best be discussed relative to these objectives.

The first objective was to develop a photographic data collection system capable of recording continuous intersection traffic data, including information regarding stopped time delay of left-turn vehicles and suitable for the collection of data regarding left-turn gap acceptance characteristics. This objective was accomplished by the development of a new camera and mirror system which utilizes 35 mm ROBOT cameras and magazines capable of recording data for thirty minutes at a rate of one frame per second.

The unique data collection system was tested in the field and was shown to be an adequate means for collecting various traffic data. Coupled with
previously developed data reduction equipment, this system provided the data necessary for this study.

The second objective of the study was to develop a computer simulation model which was capable of simulating the intersection of two and four lane roadways, with and without left-turn channels. A simulation model, the TEC Model, was written which can simulate these four different types of intersections. The TEC Model was written in the simulation language, GPSS/360. The data which was collected was used to provide information some of the inputs into the TEC Model. Delay data collected in the field was used to validate the simulation model.

Two further objectives of this study were also specified. These objectives consisted of an examination of the adequacy of existing design warrants for the addition of separate left-turn lanes at the intersection of two and four lane roadways and the development of guidelines for estimating the required length of additional turning lanes based upon delay criteria. A simulation study was conducted in which various traffic characteristics were systematically varied based upon experimental design considerations. This study resulted in a pair of predictive equations which relate average vehicle delay to various traffic characteristics, including left-turn storage channel capacity. One equation is used for single-lane approaches, the other for two-lane approaches.

It is recognized that these equations provide an estimate for vehicular delay and that, when coupled with existing design criteria, they provide the highway designer with a significant decision-assisting tool.
6.2 Future Research Recommendations

A significant research tool has been developed which combines a photographic data collection technique with a computer simulation model for use in the study of intersection traffic characteristics. As a result of the development of this study a number of specific recommendations for future research can be stated. These include the following:

1. Expansion of the study to include development of design criteria based upon construction costs and design safety, as well as vehicular delay is desirable. It appears that all three factors (and, possibly, more) should be considered so that a more meaningful decision can be made by the highway designer in the design process.

2. The newly developed data collection system can be used to collect the amounts of data necessary for an extensive study of left-turn gap acceptance characteristics. This particular traffic characteristic has not received much attention in past studies, possibly because of the difficulty involved in collecting the necessary data for an adequate analysis.

3. It is also quite possible that a study of this type can be made for many other different kinds of intersection configurations and that such information as might be generated by these studies can be extremely useful to highway designers.

4. Simulation models such as the TEC Model can also be developed to provide information for the design of intersections of other than fixed-time signalization and also for the use in experimentation of various signal timing schemes.
APPENDIX A

Program Statement Listing for Cross-Ratio Calculations
INTEGER P, PA
DIMENSION SAA1(120)
READ(5, 51) NSITES
51 FORMAT(11)
NS=1
1 WRITE(6, 101)
101 FORMAT(12H1GRID SYSTEM//)
READ(5, 52) JREC, STREET, NBLKS
52 FORMAT(2A4, I2)
WRITE(6, 102) JREC, STREET
102 FORMAT(1H02A4)
NB=1
AA1=0.0
2 READ(5, 52) BLOCK1, BLOCK2
READ(5, 53) STEP, AB, BC, SAB, SBC
53 FORMAT(8F10.2)
WRITE(6, 103) BLOCK1, BLOCK2, AB, SAB
103 FORMAT(1H0A6, 4H TO, A6, 4H, 6H FT. (, F5.2, 5H IN.)/)
P=1
IF(NB.LT.NBLKS) A1C+(AB=BC)-AA1
IF(NB.EQ.NBLKS) A1C=(BC-AB)+AA1
X=SAB/SBC/AB*BC
3 Y=X*AA1/A1C
IF(NB.LT.NBLKS) SAA1(P)=(SAB+SBC)*Y/(Y+1.)
IF(NB.EQ.NBLKS) SAA1(P)=(SBC-SAB)*Y/(1.-Y)
AA1=AA1+STEP
IF(AA1.GT.AB) GO TO 4
P=P+1
IF(NB.LT.NBLKS) A1C=A1C-STEP
IF(NB.EQ.NBLKS) A1C=A1C+STEP
GO TO 3
4 WRITE(6, 104)(SAA1(PA), PA=1, P)
104 FORMAT(10F8.2)
IF(NB.GE.NBLKS) GO TO 5
NB=NB+1
AA1=AA1-AB
GO TO 2
5 IF(NS.GE.NSITES) GO TO 6
NS=NS+1
GO TO 1
6 NAME=1
STOP
END
APPENDIX B

Program Statement Listing for Composite Exponential Calculations
DIMENSION PROB(300),PPP(300)

C COMPOSITE EXPONENTIAL CALCULATION KELL

M=0

1000 READ,VOL,BLAM,TAU
ACON=-0.046-0.0448*(VOL/100.0)
CON=4827.9/(VOL**1.024)
CCON=2.655-0.12*(VOL/100.0)
BPA=-10.503+2.829*(ALOG(VOL))-0.173*(ALOG(VOL)**2)
CCC=EXP(BPA)-2.0
SUM=ACON-(BLAM/TAU)
ALPH=EXP(SUM)
SUMS=CCC-(TAU/CCON)
BALPH=EXP(SUMS)
PRINT27,VOL

27 FORMAT('I',10X,'VOL=',F5.1,4X,'I',6X,'PROB(I)',5X,'PPP(I)')

T=0.0
DO 10 I=2,110,2
T=T+2.0
BBB=ACON-(T/CON)
DDD=CCC-(T/CCON)
PROB(I)=1-(EXP(BBB)+EXP(DDD))
PRINT9,I,PROB(I),PPP(I)

9 FORMAT(22X,I3,6X,F5.3,7X,F5.3)

10 CONTINUE
M=M+1
IF(M.LE.5) GO TO 1000
STOP
END
APPENDIX C

Program Statement Listings for TEC Model
SIMULATE

* GPSS/360 INTERSECTION SIMULATION *
* INTERSECTION MODEL TEC 1 *
* SINGLE LANE ROADWAY***NO LEFT TURN CHANNEL *

RMULT 69,87,99;83,47,99;83,81
1 VARIABLE (5-QSBQ1)*5+14
2 VARIABLE (X5*23)+25
3 VARIABLE (X5*10)+30
4 VARIABLE (5-QSBQ2)*5+14
5 VARIABLE (5-QNBQ1)*5+14
6 VARIABLE (5-QSBQ2)*5+14

* FUNCTION 1 IS SOUTHBOUND ARRIVAL DISTRIBUTION *
1 FUNCTION RN2,C27
0,0/.149,20/.379,40/.520,60/.616,80/.688,100/.745,120/.790,140/.827,160
.999,660/1.0,780

* FUNCTION 3 IS NORTHBOUND ARRIVAL DISTRIBUTION *
3 FUNCTION RN4,C41
0,0/.087,20/.225,40/.319,60/.391,80/.450,100/.502,120/.547,140/.588,160

* FUNCTION 2 IS SOUTHBOUND LEFT-TURN GAP ACCEPTANCE VALUES *
2 FUNCTION RN3,C15
0,0/.014,15/.102,20/.183,25/.313,30/.500,35/.646,40/.853,45
.947,50/.964,55/.979,60/.982,65/.985,70/.993,75/.994,80/1.0,85

* FUNCTION 4 IS NORTHBOUND LEFT-TURN GAP ACCEPTANCE VALUES *
4 FUNCTION RN5,C15
0,0/.014,15/.102,20/.183,25/.313,30/.500,35/.646,40/.853,45
.947,50/.964,55/.979,60/.982,65/.985,70/.993,75/.994,80/1.0,85

* FUNCTION 7 CALCULATES STARTING DELAY VALUES *
7 FUNCTION *1,L4
1,28/2,26/3,21/4,20
GENERATE , ,1
SAVEVALUE 1,K1
SAVEVALUE 2,K1
SAVEVALUE 50,FN4
SAVEVALUE 51,FN2
STORAGE S5-S6,10
TERMINATE

GENERATE , ,1
BACK
ADVANCE 260
ADVANCE 20
ADVANCE 10
LOGIC S 4
LEAVE 5,S5
LEAVE 6,S6
ADVANCE 1
PREEMPT SB11,,SBL9,,RE
PREEMPT NB11,,NBL9,,RE
SAVEVALUE 1,K1
SAVEVALUE 2,K1
ADVANCE 19
ADVANCE 300
RETURN SB1
RETURN NB1
LOGIC R 4

SIGNAL OPERATION

26 SECONDS GREEN TIME

SIGNAL IS RED

30 SECONDS RED TIME

SIGNAL IS GREEN
TRANSFER BACK
GENERATE 1,FN1
ADVANCE 20
TEST GE O$SBQ1,14,LAB1
TRANSFER ,LAB2
LAB1- TEST E O$SBQ1,8,LAB3
ADVANCE V2
TRANSFER ,LAB2
LAB3- TEST E O$SBQ1,9,LAB4
ADVANCE V3
TRANSFER ,LAB2
LAB4- TEST LE O$SBQ1,5,LAB10
ADVANCE V1
TRANSFER ,LAB2
LAB10- ADVANCE 14
LAB2- ADVANCE 20
TEST GE O$SBQ1,6,STL1
TRANSFER ,STL2
STL1- TEST E O$SBQ1,0,STL3
ADVANCE V2
TRANSFER ,STL2
STL3- TEST E O$SBQ1,1,STL4
ADVANCE V3
TRANSFER ,STL2
STL4- ADVANCE V1
STL2- QUEUE SBQ1
SEIZE SB11
TEST E X15,5,SB11
TRANSFER .100,SB11,SB12
STL1- TRANSFER SIM,SB11,SB14
SBL4- ASSIGN 1,X1
.TEST GE P1,K4,SB15
ASSIGN 1,K4
SBL5- ADVANCE 1,FN7
SAVEVALUE 1+,K1
TRANSFER ,SB15
SBL2- QUEUE SB02
ADVANCE 29
GATE SE 6
RELEASE SB11
SAVEVALUE 51,FN2
SAVEVALUE 1,K4
SBL7- DEPART SB02
SBL8- DEPART SBQ1
SBL3- ENTER 5,1
RELEASE SB11
DEPART SBQ1
ADVANCE X50
TEST NE S5,0,SB11
LEAVE 5,1
SBL11- TERMINATE
SBL9- DEPART SBQ1
TEST NE O$SBQ2,K0,SB10
SAVEVALUE 51,FN2
DEPART SBQ2
SBL10- TERMINATE
GENERATE 1,FN3
NORTHBOUND TRAFFIC ROUTINE
ADVANCE 20
TEST GE O$NBQ1,14,NAB1
TRANSFER ,NAB2
NAB1- TEST E O$NBQ1,8,NAB3
ADVANCE V2
TRANSFER, NAB2
NAB3 TEST E Q$NB01, 9, NAB4
ADVANCE V3
TRANSFER, NAB2
NAB4 TEST LE Q$NB01, 5, NAB10
ADVANCE V5
TRANSFER, NAB2
NAB10 ADVANCE 14
NAB2 ADVANCE 20
TEST GE Q$NB01, 6, NTL1
TRANSFER, NTL2
NTL1 TEST E Q$NB01, 6, NTL1
ADVANCE V2
TRANSFER, NTL2
NTL3 TEST E Q$NB01, 0, NTL3
ADVANCE V2
TRANSFER, NTL2
NTL4 ADVANCE V6
NTL2 QUEUE NBQ1
SEIZE NB11
TEST E X15, 5, NBL1
TRANSFER, 100, NBL1, NBL2
NBL1 TRANSFER SIM, NBL3, NBL4
NBL4 ASSIGN 1, X2
TEST GE P1, K4, NBL5
ASSIGN 1, K4
NBL5 ADVANCE 1, FN7
SAVEVALUE 2+, K1
TRANSFER, NBL3
NBL2 QUEUE NBQ2
ADVANCE 29
GATE SE 5
RELEASE NB11
SAVEVALUE 50, FN4
SAVEVALUE 20, K4
NBL7 DEPART NBQ2
NBL8 DEPART NBQ1
TERMINATE
NBL3 ENTER 6, 1
RELEASE NB11
DEPART NBQ1
ADVANCE X51
TEST NE S6, 0, NBL11
LEAVE 6, 1
NBL11 TERMINATE
NBL9 DEPART NBQ1
TEST NE Q$NB02, K0, NBL10
SAVEVALUE 50, FN4
DEPART NBQ2
NBL10 TERMINATE
GENERATE ..., 1
CLOCK COUNTER
GEN1 SAVEVALUE 25+, K1
TEST LE X25, K5, AVD2
ADVANCE 600
SPLIT 1, TERM
TRANSFER, GEN1
AVD2 SAVEVALUE 15, 5K
ADVANCE 9000
SPLIT 1, TERM
TRANSFER, GEN1
TERM TERMINATE 1
10 PERCENT LEFT TURNS
60 SECOND CYCLE
15 MINUTES OF DATA
START
RESET
START
END

5, NP

5 CYCLE INITIALIZATION PERIOD
SIMULATE
*
* GPSS/360 INTERSECTION SIMULATION
* *
* INTERSECTION MODEL TEC 2
* *
* SINGLE LANE ROADWAY***WITH LEFT TURN CHANNEL
* *
1 VARIABLE (5-QSBSQ1)*5/14
2 VARIABLE (X5*23)+25
3 VARIABLE (X5+10)+30
4 VARIABLE (5-QSBSQ2)*5/14
5 VARIABLE (5-QSBSQ1)*5/14
6 VARIABLE (5-QSBSQ2)*5/14
* FUNCTION 12 IS S B ARRIVAL DIST 600 VPH
12 FUNCTION RN2,C22
0,0/.197,20/.500,40/.661,60/.759,80/.824,100/.903,140/.928,160
*996,360/.997,380/.998,400/.999,440/1.0,520
* FUNCTION 20 IS N B ARRIVAL DIST 200 VPH
20 FUNCTION RN4,C41
0,0/.087,20/.225,40/.319,60/.391,80/.450,100/.502,120/.547,140/.588,160
* FUNCTION 7 CALCULATES STARTING DELAY VALUES
7 FUNCTION =1,1
1,28/2,26/3,21/4,20
GENERATE ,,,1
SAVEVALUE 12,K1
SAVEVALUE 13,K1
SAVEVALUE 14,K80
SAVEVALUE 50,FN4
SAVEVALUE 51,FN2
STORAGE S5=S6,10
STORAGE S2,5
SAVEVALUE 25,K0
TERMINATE
GENERATE ,,,1
SIGNAL OPERATION
BACK ADVANCE 340
ADVANCE 20
LOGIC 4
LEAVE S5
LEAVE S6
LEAVE S1
ADVANCE 1
NEW1 PREEMPT SB12,SBL9,RE
GATE LS 1,ALT1
PREEMPT SB11
TRANSFER ,ALT2
ALT1 PREEMPT SB11,SBL10,RE
ALT2 PREEMPT NB12,NBL9,RE
GATE LS 2,ALT3
PREEMPT NB11
TRANSFER ,ALT4
SIGNAL IS RED
GATE SE 6
RELEASE SBL12
SBL9 DEPART SBL02
SAVEVALUE 51, FN2
LEAVE 1,1
SBL10 DEPART SBL01
TERMINATE
SBL4 ENTER 5, 1
SBL7 RELEASE SBL11
DEPART SBL01
ADVANCE X50
TEST NE S5, 0, SBL11
LEAVE 5, 1
SBL11 TERMINATE
GENERATE 1, FN20
ADVANCE 20
TEST GE 0 S SBL01, 14, NAB1
TRANSFER S NAB2
NAB1 TEST E 0 S SBL01, 8, NAB3
ADVANCE V2
TRANSFER S NAB2
NAB3 TEST E 0 S SBL01, 9, NAB4
ADVANCE V3
TRANSFER S NAB2
NAB4 TEST LE 0 S SBL01, 5, NAB10
ADVANCE V5
TRANSFER S NAB2
NAB10 ADVANCE 14
NAB2 ADVANCE 20
TEST GE 0 S SBL01, 6, NTL1
TRANSFER S NTL2
NTL1 TEST E 0 S SBL01, 0, NTL3
ADVANCE V2
TRANSFER S NTL2
NTL3 TEST E 0 S SBL01, 1, NTL4
ADVANCE V3
TRANSFER S NTL2
NTL4 ADVANCE V6
NTL2 QUEUE NBL01
SEIZE NBL11
TEST E X25, 5, NBL1
TRANSFER S 10, NBL1, NBL2
NBL1 GATE SNF 2, NBL3
TRANSFER S 1M, NBL4, NBL5
NBL12 GATE LR 4
NBL5 ASSIGN 1, X13
LOGIC R 2
TEST GE P1, K4, NBL6
ASSIGN 1, K4
NBL6 ADVANCE 1, FN7
ENTER 6, 1
SAVEVALUE 13+, K1
TRANSFER S NBL7
NBL3 LOGIC S 2
GATE SNF 2
GATE LR 4, NBL12
ADVANCE 60
SAVEVALUE 13, K4
TRANSFER S NBL4
NBL2 QUEUE NBL02
ENTER 2, 1
RELEASE NBL11

NORTHBOUND TRAFFIC ROUTINE

10 PERCENT LEFT TURNS
GATE LR 4
SEIZE NB12
ADVANCE 29
GATE SE 5
RELEASE NB12
DEPART NB02
SAVEVALUE 50, FN4
LEAVE 2,1
NBL9 DEPART NB01
TERM
NBL10 ENTER 6,1
NBL4 RELEASE NB11
DEPART NB01
ADVANCE X51
TEST NE S6,0,NBL11
LEAVE 6,1
NBL7 ENTER 6,1
NBL11 RELEASE NB01
DEPART NB01
ADVANCE S6,0,NBL11
LEAVE 6,1
NBL11 TERMINATE
GENERATE,,,,1
GEN1 SAVEVALUE 25+, K1
TEST LE X25+, K5, AVD2
ADVANCE 750
SPLIT 1, TERM
TRANSFER , GEN1
AVD2 SAVEVALUE 25, K5
ADVANCE 9000
SPLIT 1, TERM
TRANSFER , GEN1
TERM
TERMINATE 1
START 5, NP
RESET
START 1
END

CLOCK COUNTER

15 MINUTES DATA

5 CYCLE - INITIALIZATION PERIOD
SIMULATE

* GPSS/360 INTERSECTION SIMULATION

* INTERSECTION MODEL TEC 3

* TWO LANE ROADWAY***WITHOUT LEFT TURN CHANNEL

* RMULT 69,87,99,83,47,99,83,81
1 VARIABLE (5-0$SBQ1) x 5 + 14
2 VARIABLE (X5*23)+25
3 VARIABLE (X5*10)+30
4 VARIABLE (5-0$SBQ2) x 5 + 14
5 VARIABLE (5-0$NBQ1) x 5 + 14  #NEW
6 VARIABLE (5-0$NBQ2) x 5 + 14  #NEW

* FUNCTION 1 IS SOUTHBOUND LANE 2 ARRIVAL DISTRIBUTION

1 FUNCTION RN2,C17
0,0/.221,20/.552,40/.717,60/.810,80/.869,100/.908,120/.935,140/.954,160

* FUNCTION 5 IS SOUTHBOUND LANE 1 ARRIVAL DISTRIBUTION

5 FUNCTION RN6,C22
0,0/.173,20/.442,40/.596,60/.696,80/.766,100/.818,120/.858,140/.888,160
*990,360/.994,400/.998,480/.999,520/1.0,620

* FUNCTION 2 IS S.B. LEFT-TURN GAP ACCEPTANCE VALUES

2 FUNCTION RN3,C15
0,0/.150,32/.320,37/.520,42/.600,47/.82,52/.90,57/.95,62/.97,67/.986,72
*993,77/.997,82/.998,87/.999,92/1.0,97

* FUNCTION 3 IS NORTHBOUND LANE 2 ARRIVAL DISTRIBUTION

3 FUNCTION RN4,C27
0,0/.134,20/.341,40/.471,60/.564,80/.636,100/.693,120/.74,140/.78,160
*992,560/1.0,900

* FUNCTION 6 IS NORTHBOUND LANE 1 ARRIVAL DISTRIBUTION

6 FUNCTION RN7,C33
0,0/.127,20/.322,40/.446,60/.538,80/.609,100/.666,120/.714,140/.755,160

* FUNCTION 4 IS N.B. LEFT-TURN GAP ACCEPTANCE VALUES

4 FUNCTION RN5,C15
0,0/.150,32/.320,37/.520,42/.600,47/.82,52/.90,57/.95,62/.97,67/.986,72
*993,77/.997,82/.998,87/.999,92/1.0,97

* FUNCTION 7 CALCULATES STARTING DELAY VALUES

7 FUNCTION #1,L4
1,28/2,26/3,21/4,20

GENERATE ,1
SAVEVALUE 1,K1
SAVEVALUE 2,K1
SAVEVALUE 3,K1
SAVEVALUE 4,K1
SAVEVALUE 50,FN4
SAVEVALUE 51,FN2
STORAGE S1-S2,3/S5-S6,10
TERMINATE

GENERATE ,1
BACK ADVANCE 210
ADVANCE 20
LOGIC S 4
SAVEVALUE 5,K1
LEAVE 5,S5

SIGNAL OPERATION

SIGNAL IS RED
SIGNAL IS GREEN

REPEAT CYCLE

SOUTHBOUND TRAFFIC ROUTINE

LANE 2

LEAVE 6,56
ADVANCE 1
PREEMPT NB11,,NBL13,,RE
PREEMPT SB11,,SBL13,,RE
PREEMPT NB12,,NBL10,,RE
PREEMPT SB12,,SBL10,,RE
SAVEVALUE 1,K1
SAVEVALUE 2,K1
SAVEVALUE 3,K1
SAVEVALUE 4,K1
ADVANCE 19
ADVANCE 350
RETURN NB11
RETURN NB12
RETURN SB11
RETURN SB12
LOGIC R 4
SAVEVALUE 5,K0
TRANSFER $ BACK
GENERATE 1,FN1
ADVANCE 20
TEST GE Q$SB01,14,LAB1
TRANSFER $LAB2
LAB1 TEST E Q$SB01,8,LAB3
ADVANCE V2
TRANSFER $LAB2
LAB3 TEST E Q$SB01,9,LAB4
ADVANCE V3
TRANSFER $LAB2
LAB4 TEST LE Q$SB01,5,LAB10
ADVANCE V1
TRANSFER $LAB2
LAB10 ADVANCE 14
LAB2 ADVANCE 20
TEST GE Q$SB01,6,STL1
TRANSFER $STL2
STL1 TEST E Q$SB01,0,STL3
ADVANCE V2
TRANSFER $STL2
STL3 TEST E Q$SB01,1,STL4
ADVANCE V3
TRANSFER $STL2
STL4 ADVANCE V1
STL2 QUEUE SB01
QUEUE SUM1
SEIZE SB11
TRANSFER $SIM,SBL1,SBL2
SBL2 ASSIGN 1,K1
TEST GE P1,K4,SBL3
ASSIGN 1,K4
SBL3 ADVANCE 1,FN7
SAVEVALUE 14,K1
SBL1 ENTER 5,1
SBL4 RELEASE SB01
DEPART SB01
DEPART SUM1
ADVANCE X50
TEST NE S5,K0,SBL14
LEAVE 5,1
SBL14 TERMINATE
SBL13 DEPART SB01
DEPART SUM1
TERMINATE 1, FN5
ADVANCE 20
TEST GE QB02, 14, LAB5
TRANSFER, LAB6

LAB5 TEST E QB02, 8, LAB7
ADVANCE V2
TRANSFER, LAB6

LAB7 TEST E QB02, 9, LAB8
ADVANCE V3
TRANSFER, LAB6

LAB8 TEST LE QB02, 5, LAB9
ADVANCE V4
TRANSFER, LAB6

LAB9 ADVANCE 14
LAB6 ADVANCE 20
TEST GE QB02, 6, STL5
TRANSFER, STL6

STL5 TEST E QB02, 0, STL7
ADVANCE V2
TRANSFER, STL6

STL7 TEST E QB02, 1, STL8
ADVANCE V3
TRANSFER, STL6

STL8 ADVANCE V4
STL6 QUEUE SBQ2
QUEUE SUM1
SEIZE SB12
TEST E X15, 5, SBL5
TRANSFER, 240, SBL5, SBL6

SBL5 TRANSFER SIM, SBL7, SBL8
SBL8 ASSIGN 1, X2
TEST GE P1, K4, SBL9
ASSIGN 1, K4

SBL9 ADVANCE 1, FN7
SAVEVALUE 2+, K1
TRANSFER, SBL7

SBL6 QUEUE SBQ3
ADVANCE 29
GATE SE 6
RELEASE SB12
SAVEVALUE 51, FN2
SAVEVALUE 2, K4
DEPART SB03
DEPART SBQ2
DEPART SUM1
TERMINATE

SBL7 ENTER 5, 1
RELEASE SB12
DEPART SBQ2
DEPART SUM1
ADVANCE X50
TEST NE S5, K0, SBL12
LEAVE 5, 1

SBL12 TERMINATE
SBL10 DEPART SBQ2
DEPART SUM1
SAVEVALUE 51, FN2
TEST NE QB02, K0, SBL11
DEPART SBQ3

SBL11 TERMINATE
GENERATE 1, FN3

LANE 1

24 PERCENT LEFT TURNS

LEFT TURN ROUTINE

NORTHBOUND TRAFFIC ROUTINE
TRANSFER, NTL6

NTL7 TEST E 0S,NB02,1,NTL8
ADVANCE V3
TRANSFER, NTL6

NTL8 ADVANCE V6

NTL6 QUEUE NB02
QUEUE NUM1
SEIZE NB12
TEST E X15,5,NBL5
TRANSFER *300,NBL5,NBL6

NBL5 TEST EQ ENB Q2
TRANSFER SIM,NBL7,NBL8

NBL8 ASSIGN 1,X4
TEST GE P1,K4,NBL9
ASSIGN 1,K4

NBL9 ADVANCE 1,FN7
SAVEVALUE 4,K1
TRANSFER, NBL7

NBL6 QUEUE NB03
ADVANCE 29
GATE SE 5
RELEASE NB12
SAVEVALUE 50,FN4
SAVEVALUE 4,K4
DEPART NB03
DEPART NB02
DEPART NUM1
TERMINATE

NBL7 ENTER 6,1
RELEASE NB12
DEPART NB02
DEPART NUM1
ADVANCE X51
TEST NE S6,K0,NBL12
LEAVE 6,1

NBL12 TERMINATE

NBL10 DEPART NB02
DEPART NUM1
SAVEVALUE 50,FN4
TEST NE O$NB03,K0,NBL11
DEPART NB03

NBL11 TERMINATE
GENERATE ",1

GEN1 SAVEVALUE 29,K1
TEST LE X25,K5,AVD2
ADVANCE 750
SPLIT 1,TERM
TRANSFER ,GEN1

AVD2 SAVEVALUE 15,K5
ADVANCE 9000
SPLIT 1,TERM
TRANSFER ,GEN1

TERM TERMINATE 1
START 5,NP
RESET
START 1

END

/*/ CLOCK COUNTER

75 SECOND CYCLE

15 MINUTES OF DATA

5 CYCLE INITIALIZATION PERIOD

15 MINUTES OF DATA
SIMULATE

* GPSS/360 INTERSECTION SIMULATION
* INTERSECTION MODEL TEC 4
* TWO LANE ROADWAY WITH LEFT TURN CHANNEL
* 
* RMULT 69,87,59,83,47,99,83,81
1 VARIABLE (5-0$SBQ1)\times5+14
2 VARIABLE (X5=23)+25
3 VARIABLE (X5=10)+30
4 VARIABLE (5-0$SBQ2)\times5+14
5 VARIABLE (5-0$NBQ1)\times5+14
6 VARIABLE (5-0$NBQ2)\times5+14
* FUNCTION 1 IS SOUTHBOUND LANE 2 ARRIVAL DISTRIBUTION
1 FUNCTION RN2,C18
0,0/.21,20/.529,40/.649,60/.789,80/.85,100/.892,120/.922,140/.943,160
1,0,460
* FUNCTION 5 IS SOUTHBOUND LANE 1 ARRIVAL DISTRIBUTION
5 FUNCTION RN6,C31
0,0/.134,20/.341,40/.471,60/.564,80/.636,100/.693,120/.74,140/.78,160
*992,560/.996,640/.997,680/.998,720/.999,780/1,0,920
* FUNCTION 2 IS EASTBOUND LEFT-TURN GAP ACCEPTANCE VALUES
2 FUNCTION RN3,C15
0,0/.150,32/.320,42/.690,47/.82,52/.90,57/.95,62/.97,67/.986,72
*993,77/.997,82/.998,87/.999,92/1,0,97
* FUNCTION 3 IS NORTHBOUND LANE 2 ARRIVAL DISTRIBUTION
3 FUNCTION RN4,C18
0,0/.21,20/.529,40/.649,60/.789,80/.85,100/.892,120/.922,140/.943,160
1,0,460
* FUNCTION 6 IS NORTHBOUND LANE 1 ARRIVAL DISTRIBUTION
6 FUNCTION RN7,C31
0,0/.134,20/.341,40/.471,60/.564,80/.636,100/.693,120/.74,140/.78,160
*992,560/.996,640/.997,680/.998,720/.999,780/1,0,920
* FUNCTION 4 IS WESTBOUND LEFT-TURN GAP ACCEPTANCE VALUES
4 FUNCTION RN5,C15
0,0/.150,32/.320,42/.690,47/.82,52/.90,57/.95,62/.97,67/.986,72
*993,77/.997,82/.998,87/.999,92/1,0,97
* FUNCTION 7 CALCULATES STARTING DELAY VALUES
7 FUNCTION *1,L4
128/2,26/3,21/4,20
GENERATE ...,1
SAVEVALUE 1,K1
SAVEVALUE 2,K1
SAVEVALUE 3,K1
SAVEVALUE 4,K1
SAVEVALUE 5,FN4
SAVEVALUE 51,FN2
STORAGE S1-S2,3/S5-S6,10
TERMINATE
GENERATE ...,1
BACK ADVANCE 270 27 SECONDS GREEN TIME
ADVANCE 20
LOGIC S 4
SAVEVALUE 5,K1
LEAVE 5,S5
SIGNAL OPERATION
BACK ADVANCE 270 27 SECONDS GREEN TIME
ADVANCE 20
LOGIC S 4
SAVEVALUE 5,K1
LEAVE 5,S5
SIGNAL IS RED
LEAVE 6,S6
ADVANCE 1
PREEMPT SB11,,SBL5,,RE
PREEMPT SB13,,SBL12,,RE
GATE LS 1,ALT1
PREEMPT SB12
TRANSFER ,ALT2
ALT1 PREEMPT SB12,,SBL13,,RE
ALT2 PREEMPT NB11,,NBL5,,RE
PREEMPT NB13,,NBL12,,RE
GATE LS 2,ALT3
PREEMPT NB12
TRANSFER ,ALT4
ALT3 PREEMPT NB12,,NBL13,,RE
ALT4 SAVEVALUE 1,K1
SAVEVALUE 2,K1
SAVEVALUE 3,K1
SAVEVALUE 4,K1
ADVANCE 19
ADVANCE 440
RETURN SB11
RETURN SB12
RETURN SB13
RETURN NB11
RETURN NB12
RETURN NB13
LOGIC R 4
SAVEVALUE 5,K0
TRANSFER ,BACK
GENERATE 1,FN1
ADVANCE 20
TEST GE QSB01,14,LAB1
TRANSFER ,LAB2
LAB1 TEST E QSB01,8,LAB3
ADVANCE V2
TRANSFER ,LAB2
LAB3 TEST E QSB01,9,LAB4
ADVANCE V3
TRANSFER ,LAB2
LAB4 TEST LE QSB01,5,LAB10
ADVANCE V1
TRANSFER ,LAB2
LAB10 ADVANCE 14
LAB2 ADVANCE 20
TEST GE QSB01,6,STL1
TRANSFER ,STL2
STL1 TEST E QSB01,0,STL3
ADVANCE V2
TRANSFER ,STL2
STL3 TEST E QSB01,1,STL4
ADVANCE V3
TRANSFER ,STL2
STL4 ADVANCE V1
STL2 QUEUE SB01
QUEUE SUM1
SEIZE SB11
TRANSFER SIM,SBL1,SBL2
SBL2 ASSIGN 1,X1
TEST GE P1,K4,SBL3.
ASSIGN 1,K4
SBL3 ADVANCE 1,FN7
SAVEVALUE 1+,K1
SBL1  ENTER  5,1
SBL4  RELEASE  SB11
       DEPART  SB01
       DEPART  SUM1
       ADVANCE  X50
       TEST NE  S5,K0,SBL20
       LEAVE  5,1
SBL20  TERMINATE
SBL13  DEPART  SB01
       DEPART  SUM1
       TERMINATE
       GENERATE  1,FN5
       ADVANCE  20
       TEST GE  0$SB02,14,LAB5
       TRANSFER  0,LAB6
LAB5  TEST E  0$SB02,8,LAB7
       ADVANCE  V2
       TRANSFER  0,LAB6
LAB7  TEST E  0$SB02,9,LAB8
       ADVANCE  V3
       TRANSFER  0,LAB6
LAB8  TEST LE  0$SB02,5,LAB9
       ADVANCE  V4
       TRANSFER  0,LAB6
LAB9  ADVANCE  14
LAB6  ADVANCE  20
       TEST GE  0$SB02,6,STL5
       TRANSFER  0,STL6
STL5  TEST E  0$SB02,0,STL7
       ADVANCE  V2
       TRANSFER  0,STL6
STL7  TEST E  0$SB02,1,STL8
       ADVANCE  V3
       TRANSFER  0,STL6
STL8  ADVANCE  V4
STL6  QUEUE  SB02
       QUEUE  SUM1
       SEIZE  SB12
       TRANSFER  0,290,SB66,SB77
SBL6  GATE SNF  1,SB87
       TRANSFER  1,SB89,SB10
SBL14  GATE LR  4
SBL10  ASSIGN  1,X2
       LOGIC R  1
       TEST GE  P1,K4,SBL11
       ASSIGN  1,K4
SBL11  ADVANCE  1,FN7
       SAVEVALUE  2,K1
       TRANSFER  0,SBL9
SBL8  LOGIC S  1
       GATE SNF  1
       GATE LR  4,SBL14
       ADVANCE  60
       SAVEVALUE  2,K4
       TRANSFER  0,SBL9
SBL7  QUEUE  SB03
       ENTER  1,1
       RELEASE  SB12
       GATE LR  4

LANE 1
NORTHBOUND TRAFFIC ROUTINE

SEIZE SB13
ADVANCE 29
GATE SE 6
RELEASE SB13
SBL12 DEPART SBQ3
SAVEVALUE 51,FN2
LEAVE 1,1
DEPART SBQ2
DEPART SUM1
TERMINATE

SBL9 ENTER 5,1
RELEASE SB12
DEPART SBQ2
DEPART SUM1
ADVANCE X50
TEST NE SB5,K0,SBL15
LEAVE 5,1

SBL15 TERMINATE
GENERATE 1,FN3
ADVANCE 20
TEST GE O$NBQ1,14,NAB1
TRANSFER ,NAB2
NAB1 TEST E O$NBQ1,8,NAB3
ADVANCE V2
TRANSFER ,NAB2
NAB3 TEST E O$NBQ1,9,NAB4
ADVANCE V3
TRANSFER ,NAB2
NAB4 TEST LE O$NBQ1,5,NAB10
ADVANCE V5
TRANSFER ,NAB2
NAB10 ADVANCE 14
NAB2 ADVANCE 20
TEST GE O$NBQ1,6,NTL1
TRANSFER ,NTL2
NTL1 TEST E O$NBQ1,0,NTL3
ADVANCE V2
TRANSFER ,NTL2
NTL3 TEST E O$NBQ1,1,NTL4
ADVANCE V3
TRANSFER ,NTL2
NTL4 ADVANCE V5
NTL2 QUEUE NBQ1
QUEUE NUM1
SEIZE NB11
TRANSFER S1H,NBL1,NBL2
NBL2 ASSIGN 1,X3
TEST GE P1,K4,NBL3
ASSIGN 1,K4
NBL3 ADVANCE 1,FN7
SAVEVALUE 3+,K1
NBL1 ENTER 6,1
NBL4 RELEASE NB11
DEPART NBQ1
DEPART NUM1
ADVANCE X51
.TEST NE SB6,K0,NBL20
LEAVE 6,1
NBL20 TERMINATE
NBL13 DEPART NBQ1
DEPART NUM1
NBL5 DEPART NBQ1
RELEASE NBL15
DEPART NBQ2
DEPART NUM1
ADVANCE X51
TEST NE S6,K0,NBL15
LEAVE 6,1
NBL15 TERMINATE
GENERATE ...,1
GEN1 SAVEVALUE 25+,K1
TEST LE X25,K5,AVD2
ADVANCE 750
SPLIT 1,TERM
TRANSFER ;GEN1
AVD2 ADVANCE 9000
SPLIT 1,TERM
TRANSFER ;GEN1
TERM TERMINATE 1
START 5,NP
RESET
START 1
END

CLOCK COUNTER
75 SECOND CYCLE
15 MINUTES OF DATA
5 CYCLE INITIALIZATION PERIOD
APPENDIX D

Fractional Factorial Discussion
Description of Experiment Plans (for factors at 3 levels)

INTRODUCTION

A full factorial experiment with $n$ factors (variables), each at three levels, results in $3^n$ different factorial combinations. The $n$ factors are denoted by capital letters. (For $n = 5$, the factors are $A$, $B$, $C$, $D$, and $E$.) The actual treatment combinations are designated by the term $(x_1 x_2 x_3 \ldots x_n)$, where $x_1 = 0$, $1$, or $2$; thus $x_1$ is the level of factor $A$, $x_2$ is the level of factor $B$, $\ldots$, $x_n$ is the level of factor $n$ for the combination being considered. (For $n = 5$, the term 10121 indicates factor $A$ at level 1, factor $B$ at level 0, factor $C$ at level 1, factor $D$ at level 2, and factor $E$ at level 1.)

In addition to using capital letters to represent the various factors, the capital letters are also used to represent the various main effects and interactions associated with the respective factors. Main effects have two degrees of freedom (df) and are designated by capital letters alone. Two factor interactions have four df and can be split into two parts, each with two df. For two factors, $A$ and $B$, the two parts of the two factor interaction are $AB$ and $AB^2$; there is no physical significance attached to either of these two parts, they are simply two orthogonal components of interactions. They are used to reduce the complexity of calculations.
LOSS OF INFORMATION

One result of using an $r^{th}$ fractional replicate of a full set of factorial combinations is to lose information on one or more of the higher order interactions and to have all main effects and interactions inextricably mixed or aliased with other main effects or interactions. However, in most cases second or higher order interactions are negligible or physically impossible to interpret. Hence an objective of a fractional factorial design is that the design (1) has no main effects aliased with other main effects or aliased with two factor interactions; and, (2) has as few two factor interactions as possible aliased with other two factor interactions. Two factor interactions which are only aliased with higher order interactions are termed measurable.

For every $1/3^P$ fractional design there is a fundamental identity represented by the symbol I and $(3^P - 1)/2$ groups of letters connected by equal signs, i.e.,

$$I = A^{a_1}B^{b_1}C^{c_1} = A^{a_2}B^{b_2}C^{c_2} = \cdots = A^{a_t}B^{b_t}C^{c_t}.$$ 

where $t = (3^P - 1)/2$ and the $a_i, b_i, c_i, \ldots$ $(i=1, \ldots, t)$ take on values 0, 1, 2. A group of such letters is called a "word". The words in the fundamental identity are such that the first letter of every word always has unity as an exponent, and a letter having a zero exponent is omitted from the word, or can be regarded as being a unity element, i.e., $A^0 = B^0 = C^0 = \ldots = 1$. 
The fundamental identity serves the following two purposes:

1. it is used to select the appropriate subset of treatments from the full factorial; and

2. it determines the manner in which the various main effects and interactions are aliased with one another as a result of considering only a fraction of measurements from the full factorial.

If the p generators of a 1/3^P fractional replicate are

\[ G_i = A^{q_i} B^{b_i} C^{c_i} \ldots \quad i = 1, 2, \ldots, p \]

then the levels in the fractional combinations \( x_1 x_2 x_3 \ldots \) selected for the 1/3^P fraction satisfy simultaneously the p equations:

\[ a_i x_1 + b_i x_2 + c_i x_3 + \ldots = 0 \quad (\text{modulo } 3) \quad i = 1, 2, \ldots, p \]

similar equations in which 1 or 2 is used are equally valid; e.g.

\[ a_i x_1 + b_i x_2 + c_i x_3 + \ldots = 1 \quad \text{(modulo } 3) \]

or \[ a_i x_1 + b_i x_2 + c_i x_3 + \ldots = 2 \quad \text{(modulo } 3) \]

If X represents a main effect then the aliases of X are obtained by multiplying the fundamental identity and the square of the fundamental identity by X. All the exponents should be reduced modulo 3 and the leading letter of every word must have an exponent of unity. (Modulo 3 means that when multiplying two effects the cube of any effect is equated to unity.) Thus, if ABCDE is the fundamental identity, the aliases of A are

\[ A(ABCDEFG) = A^2 BCDE \]

and \[ A(A^2 B^2 C^2 D^2 E^2) = B^2 C^2 D^2 E \]
but since the leading letter of every word must have unity as an exponent, the terms are squared to obtain

\[(A^2BCDE)^2 = A^4B^2C^2D^2E^2 = AB^2C^2D^2E^2\]

and

\[(B^2C^2D^2E^2)^2 = B^4C^4D^4E^4 = BCDE\]

Analysis of Fractional Factorial Plans

The model can be written as

\[\gamma(x_1 x_2 x_3 \ldots) = \eta(x_1 x_2 x_3 \ldots) = \epsilon(x_1 x_2 x_3 \ldots)\]

where

\[\gamma(x_1 x_2 x_3 \ldots) = \text{actual observations}\]

\[\eta(x_1 x_2 x_3 \ldots) = \text{true value of the effects}\]

\[\epsilon(x x x \ldots) = \text{error, a set of uncorrelated random variables with mean zero and variance.}\]

Further definitions for the full factorial design are:

\[\mu = (1/3^n) \sum \eta(x_1 x_2 x_3 \ldots)\]

\[(A)_{i} = (1/3^n) \sum_{x_1 = 1} \eta(x_1 x_2 x_3 \ldots) - \mu \quad i = 0,1,2\]

\[(B)_{i} = (1/3^n) \sum_{x_2 = 1} \eta(x_1 x_2 x_3 \ldots) - \mu \quad i = 0,1,2\]

\[\vdots\]

\[(AB)_{i} = (1/3^n) \sum_{x_1 + x_2 = i \text{ (mod 3)}} \eta(x_1 x_2 x_3 \ldots) - \mu \quad i = 0,1,2\]

\[(AB^2)_{i} = (1/3^n) \sum_{x_1 + 2x_2 = i \text{ (mod 3)}} \eta(x_1 x_2 x_3 \ldots) - \mu \quad i = 0,1,2\]

\[\vdots\]

\[(AB^2C \ldots)_{i} = (1/3^n) \sum_{x_1 + 2x_2 + 2x_3 \ldots = i \text{ (mod 3)}} \eta(x_1 x_2 x_3 \ldots) - \mu \quad i = 0,1,2\]

where \(\sum_{x_1 = 1}^{n-1}\) sums over all 3\(^n\) factorial combinations holding \(x_1 = i\) fixed,
\[ \sum_{x_2 = i}^{3^{n-1}} \text{sums over all } 3^{n-1} \text{factorial combinations holding } x_2 = i \text{ fixed}, \ldots, \]
\[ \sum_{x_1 + 2x_2 + 2x_3 = i}^{3^{n-1}} \text{sums over all } 3^{n-1} \text{factorial combinations such that } x_1 + 2x_2 + 2x_3 = i \text{ is fixed. From these definitions the sums } \left[ (A)_0 + (A)_1 + (A)_2 \right], \ldots, \]
\[ \left[ (AB)_0 + (AB)_1 + (AB)_2 \right], \ldots \text{ are all equal to zero. The quantities } (A)_i, (B)_i, (C)_i, \ldots \text{ are parameters associated with the main effects of factors } A, B, C, \ldots, \text{ respectively. The quantities } (AB)_i, (AB^2)_i, (AC)_i, (AC^2)_i, (BC)_i, (BC^2)_i, \ldots \text{ are parameters associated with the two factor interactions } AB, AB^2, AC, AC^2, BC, BC^2, \ldots, \text{ etc. With the above definitions the true value of a treatment combination, } \eta(x_1 x_2 x_3 \ldots) \text{ can be written as a linear function of these parameters, i.e.,} \]
\[ \eta(x_1 x_2 x_3 \ldots) = \mu + (A)x_1 + (B)x_2 + (C)x_3 + \ldots + (AB)x_1 + x_2 + (AB^2)x_1 + 2x_2 + \ldots + (BC)x_2 + x_3 + (BC^2)x_2 + 2x_3 + \ldots + (ABC)x_1 + x_2 + x_3 + \ldots + (ABC^2 \ldots)x_1 + 2x_2 + 2x_3 \ldots \]
where all indices are reduced modulo 3. For example, if } n = 4, \text{ then}
\[ \eta(x_1 x_2 x_3 x_4) = \mu + p_1 + p_2 + p_3 + p_4 \]
where
\[ p_1 = (A)x_1 + (B)x_2 + (C)x_3 + (D)x_4 \]
\[ p_2 = (AB)x_1 + x_2 + (AB^2)x_1 + 2x_2 + (AC)x_1 + x_3 + (AC^2)x_1 + 2x_3 + (AD)x_1 + x_4 + + (AD^2)x_1 + 2x_4 + (BC)x_2 + x_3 + (BC^2)x_2 + 2x_3 + (BD)x_2 + x_4 + (BD^2)x_2 + 2x_4 + (CD)x_3 + x_4 + (CD^2)x_3 + 2x_4 \]
\[ p_3 = (ABC)x_1 + x_2 + x_3 + (ABC^2)x_1 + 2x_2 + x_3 + (ABC^2)x_1 + x_2 + 2x_3 + + (AB^2 C)x_1 + 2x_2 + 2x_3 + (ACD)x_1 + x_3 + x_4 + (ACD^2)x_1 + x_3 + 2x_4 + (ACD^2)x_1 + x_3 + 2x_4 + (ACD^2)x_1 + x_3 + 2x_4 + (ABD)x_1 + x_2 + x_4 + + (ABD^2)x_1 + x_2 + 2x_4 + (ABD^2)x_1 + x_2 + 2x_4 + + (BCD)x_2 + x_3 + x_4 + (BC^2 D)x_2 + 2x_3 + x_4 + (BCD^2)x_2 + x_3 + 2x_4 + (BC^2 D^2)x_2 + 2x_3 + 2x_4 \]
\[ p_4 = (ABCD)x_1x_2x_3x_4 + (AB^2CD)x_1+2x_2+x_3+x_4 \\
+ (ABC^2D)x_1+x_2+2x_3+x_4 + (ABCD^2)x_1+x_2+x_3+2x_4 \\
+ (AB^2C^2D)x_1+2x_2+2x_3+x_4 + (ABC^2D^2)x_1+x_2+2x_3+2x_4 \\
+ (AB^2C^2D^2)x_1+2x_2+2x_3+2x_4 \\
+ (AB^2CD^2)x_1+2x_2+x_3+2x_4 \]

Suppose \( x_1 = 0, x_2 = 1, x_3 = 2, x_4 = 1 \), then the indices are replaced by their values, reduced modulo 3.

Fractional factorial design: if the design is a \( 1/3^P \) replicate of the full \( 3^n \) factorial, the estimates of the parameters defined above would then be:

\[ \hat{\mu} = \frac{1}{3^{n-p}} \sum y(x_1 x_2 x_3 \ldots) \]

(sum over all \( 3^{n-p} \) combinations)

\[ \hat{(A)}_i = \frac{1}{3^{n-p-1}} \sum_{x_1=i} y(x_1 x_2 x_3 \ldots) - \hat{\mu} \quad i = 0, 1, 2 \]

\[ \hat{(B)}_i = \frac{1}{3^{n-p-1}} \sum_{x_2=i} y(x_1 x_2 x_3 \ldots) - \hat{\mu} \quad i = 0, 1, 2 \]

\[ \hat{(AB)}_i = \frac{1}{3^{n-p-1}} \sum_{x_1+x_2=i} y(x_1 x_2 x_3 \ldots) - \hat{\mu} \quad i = 0, 1, 2 \]

\[ \hat{(AB^2)}_i = \frac{1}{3^{n-p-1}} \sum_{x_1+2x_2=i} y(x_1 x_2 x_3 \ldots) - \hat{\mu} \quad i = 0, 1, 2 \]

\[ \hat{(AB^2 C^2 \ldots)}_i = \frac{1}{3^{n-p-1}} \sum_{x_1+2x_2+2x_3+\ldots=i} y(x_1 x_2 x_3 \ldots) - \hat{\mu} \quad i = 0, 1, 2 \]
Since only a fraction of the full factorial is used, all the above estimates are biased. The way in which each estimate is biased depends upon the aliases with which it is entangled.

For a \(1/3^P\) fractional replicate, there are \((3^P - 1)/2\) words in the fundamental identity, denoted by

\[ u_1, u_2, u_3, \ldots, u_t \]

where \( u_j = A^{a_j} B^{b_j} C^{c_j} \ldots \) \( j = 1, 2, \ldots, t; \quad t = (3^P - 1)/2 \)

Let \( X = A^\alpha B^\beta C^\gamma \ldots \) be a particular main effect or interaction and let its \((3^P - 1)\) aliases be denoted by \(x_{u_1}, x_{u_2}, \ldots, x_{u_t}, x_{u_2}, \ldots, x_{u_t}^2\)

where the first non-zero exponent is always unity. Then the expected value of the estimate of \((X)_i\) is

\[ E(X)_i = (X)_i + \sum_{j=1}^{t} (x_{u_j}) \lambda_j + \sum_{j=1}^{t} (x_{u_j}^2) \rho_j \]

where \(\lambda_j\) and \(\rho_j\) take on the values 0, 1, and 2 are determined in the following manner:

\[ x_{u_j} = A^{a'_j} B^{b'_j} C^{c'_j} \ldots \]
\[ x_{u_j}^2 = A^{a''_j} B^{b''_j} C^{c''_j} \ldots \]

it is always possible to write

\[ a'_j x_1 + b'_j x_2 + c'_j x_3 + \ldots = \theta'_j (a_j x_1 + b_j x_2 + c_j x_3 + \ldots) \]
\[ + \phi'_j (a x_1 + \beta x_2 + \gamma x_3 + \ldots) \]

where \(\theta'_j, \phi'_j\) are equal to 1 or 2 and all coefficients are reduced modulo 3.
For particular values of \( (x_1, x_2, \ldots) \), \( i = \alpha x_1 + \beta x_2 + \ldots \) (mod 3), \( \lambda_j \) is given by \( i \phi_j^l \) (mod 3). Similarly, for \( xu_j^2 \) it is always possible to write

\[
a_j''x_1 + b_j''x_2 + c_j''x_3 + \ldots = \theta_j''(a_jx_1 + b_jx_2 + c_jx_3 + \ldots) + \phi_j''(a x_1 + \beta x_2 + \gamma x_3 + \ldots)
\]

and \( p_j \) is given by \( i \phi_j'' \) where \( i = (ax_1 + bx_2 + cx_3 + \ldots) \) mod 3 for specific values of \( x_1, x_2, x_3 \ldots \).

**Analysis of Variance**

Once the various main effects and interactions are obtained, the analysis of variance follows in the following manner:

If \( (X)_i \) is the estimate of the parameter \( (\alpha A, \beta B, \gamma C, \ldots)_i \), associated with the measurable interaction \( \alpha A, \beta B, \gamma C, \ldots \), the appropriate sum of squares (S.S.) having 2 df is given by

\[
3^{n-p-1} \sum_{i=0}^{2} (X)_i^2
\]

Hence the S.S. associated with the main effect A is

\[
3^{n-p-1} \sum_{i=0}^{2} (A)_i^2
\]

the S.S. associated with the main effect B is

\[
3^{n-p-1} \sum_{i=0}^{2} (B)_i^2
\]

the S.S. associated with \( AB \) and \( (AB)^2 \), each having 2 df is

\[
3^{n-p-1} \sum_{i=0}^{2} (AB)_i^2
\]

and

\[
3^{n-p-1} \sum_{i=0}^{2} (AB^2)_i^2
\]
Therefore the analysis of variance (ANOVA) table is built in the following manner:

<table>
<thead>
<tr>
<th>Source</th>
<th>S.S.</th>
<th>d.f.</th>
<th>M.S.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>$\sum [y(x_1 x_2 x_3 \ldots)^2]$</td>
<td>$3^{n-p}$</td>
<td></td>
</tr>
<tr>
<td>$R(\hat{\mu})$</td>
<td>$\sum [y(x_1 x_2 x_3 \ldots)^2 - \bar{y}^2]$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$R(\hat{A})$</td>
<td>$3^{n-p-1} \sum_{i=0}^2 (\hat{A})_i^2$</td>
<td>2</td>
<td>S.S./2</td>
</tr>
<tr>
<td>$R(\hat{B})$</td>
<td>$3^{n-p-1} \sum_{i=0}^2 (\hat{B})_i^2$</td>
<td>2</td>
<td>S.S./2</td>
</tr>
<tr>
<td>$R(\hat{AB})$</td>
<td>$3^{n-p-1} \sum_{i=0}^2 (\hat{AB})_i^2$</td>
<td>2</td>
<td>S.S./2</td>
</tr>
</tbody>
</table>

S.S.E. by subtraction = $a$ by sub. $= \beta$ $a/\beta$

The F-ratio for each factor is obtained by dividing the M.S. for each factor by the M.S.E. This F-ratio is compared to the F-value with 2 and $\beta$ degrees of freedom in an F-table (for some level of confidence). If the F-ratio obtained in the ANOVA table is larger than the corresponding value in the F-tables it can be concluded that the factor is significant, i.e., it makes some significant contribution to the response. If the F-value in the ANOVA table is less than the corresponding F-table value it can be concluded that that particular factor does not in fact make a significant contribution to the response being investigated.
As seen in the table, the sum of the squares due to error (S.S.E.) is obtained by subtraction after calculating the S.S. for the various factors. In order to be able to calculate the M.S.E. some df must be available. It can be seen that this would be impossible if all main effects and interactions were to be estimated and only one replication is made. However, this difficulty is overcome by assuming that the second and higher order interactions are negligible or physically impossible, then only the main effects and first order interactions are estimated and remaining df are used for error estimation.

For the following discussion the notation is:

- $A_L$ = linear effect of $A$
- $A_Q$ = quadratic effect of $A$
- $B_L$ = linear effect of $B$
- $B_Q$ = quadratic effect of $B$
- etc.

It can be seen from the ANOVA table that each main effect has 2 df associated with its S.S. Each main effect can be broken down into two parts, a linear effect and a quadratic effect in the following manner:

Assume that the three levels to be studied are equi-spaced from one another (equi-spacing quantitative levels make the analysis much simpler).
If $T_1$, $T_2$, and $T_3$ represent response totals in an experiment where the factor being considered is set at three equi-spaced levels, the results could be shown as follows:

![Diagram showing response totals $T_1$, $T_2$, and $T_3$]  

If factor $F_j$ produces a linear response the linear effect from the low level (0) to the intermediate level (1) is equal to $(T_2 - T_1)$ and the linear effect from the intermediate level (1) to the high level (2) is equal to $T_3 - T_2$.

The total linear effect is then $F_L = (T_2 - T_1) + (T_3 - T_2) = T_3 - T_1$ (note that $F_L$ is a contrast; if a linear function of $t_1$, $t_2$, ... , $t_n$, $l = a_1 t_1 + a_2 t_2 + ... + a_n t_n$, is called a linear contrast of $t_1$, $t_2$, ... $t_n$.

If the sum of the coefficients is zero, $\sum_{i=1}^{n} a_i = 0$.

If factor $F$ produces a quadratic effect on the response, the slope between level (0) and level (1) will be different from the slope between level (1) and level (2). The difference in slopes is $F_Q = (T_3 - T_2) - (T_2 - T_1) = T_3 - 2T_2 + T_1$.

$F_Q$ is also a contrast.

Note that the linear contrast $F_L$ is orthogonal to the quadratic contrast $F_Q$ so that the S.S. due to both linear and quadratic contrasts can be determined.
Suppose, then, it is desired to determine the linear effect of factor A, $A_L$. This is accomplished by multiplying all the response values at the low level of A by $-1$, all the response values at the intermediate level of A by 0, and all the response values at the high level of A by $+1$, and then summing all the results. The S.S. due to $A_L$ is then $\frac{(A_L)^2}{\sum_{i=1}^{3^n-p} c_i^2}$, where the $c_i$'s are the coefficients ($-1, 0, +1$) used in the contrast. The procedure can be further extended to break down the S.S. for the first degree interaction into four components each having 1 df.

**An Example: 1/3 Replicate of a $3^5$ Factorial Design**

The following plan for a 1/3 replicate of a $3^5$ factorial design was taken from (1):

<table>
<thead>
<tr>
<th></th>
<th>00000</th>
<th>01221</th>
<th>02112</th>
<th>10212</th>
<th>11100</th>
<th>1202</th>
</tr>
</thead>
<tbody>
<tr>
<td>00000</td>
<td>21222</td>
<td>22110</td>
<td>02210</td>
<td>01101</td>
<td>02022</td>
<td>10122</td>
</tr>
<tr>
<td>01221</td>
<td>21111</td>
<td>21000</td>
<td>21102</td>
<td>22020</td>
<td>00120</td>
<td>01011</td>
</tr>
<tr>
<td>02112</td>
<td>11001</td>
<td>12222</td>
<td>12021</td>
<td>21210</td>
<td>21212</td>
<td>12200</td>
</tr>
<tr>
<td>10212</td>
<td>12220</td>
<td>00111</td>
<td>11211</td>
<td>12102</td>
<td>12201</td>
<td>12120</td>
</tr>
<tr>
<td>11100</td>
<td>20112</td>
<td>01002</td>
<td>02100</td>
<td>00021</td>
<td>11212</td>
<td>12012</td>
</tr>
<tr>
<td>1202</td>
<td>22002</td>
<td>22221</td>
<td>20022</td>
<td>01212</td>
<td>02012</td>
<td>00201</td>
</tr>
<tr>
<td>00111</td>
<td>01002</td>
<td>10221</td>
<td>22222</td>
<td>22122</td>
<td>02022</td>
<td>01122</td>
</tr>
<tr>
<td>10101</td>
<td>12000</td>
<td>10101</td>
<td>10020</td>
<td>22122</td>
<td>22012</td>
<td>12201</td>
</tr>
<tr>
<td>00022</td>
<td>00222</td>
<td>01020</td>
<td>12210</td>
<td>10020</td>
<td>01200</td>
<td>12120</td>
</tr>
<tr>
<td>01200</td>
<td>20220</td>
<td>21111</td>
<td>02211</td>
<td>00102</td>
<td>02100</td>
<td>01012</td>
</tr>
<tr>
<td>11122</td>
<td>20001</td>
<td>20100</td>
<td>21021</td>
<td>21021</td>
<td>02121</td>
<td>00012</td>
</tr>
<tr>
<td>10002</td>
<td>11220</td>
<td>11022</td>
<td>20211</td>
<td>20010</td>
<td>21201</td>
<td>0202</td>
</tr>
<tr>
<td>20121</td>
<td>11020</td>
<td>0202</td>
<td>0202</td>
<td>0202</td>
<td>0202</td>
<td>0202</td>
</tr>
</tbody>
</table>

where each cell denotes the response under that particular combination of the factors; for example, 01221 denotes the value of the response being investigated at level 0 of factor A, level 1 of factor B, level 2 of factor C, level 2 of factor D, and level 1 of factor E.
The fundamental identity used here is \( I = ABCDE \); there are no main effects aliased with other main effects or with two factor interactions and all the two factor interactions are measurable. Using the definitions discussed above the values of the effects can be calculated. For example

\[
\hat{\mu} = \frac{1}{3^{5-1}} \sum y(x_1 x_2 x_3 x_4 x_5) \quad \text{average of all the observations}
\]

and

\[
\hat{\sigma}^2 = \frac{1}{3^{5-1-1}} \sum_{x_1=0} y(x_1 x_2 x_3 x_4 x_5) - \hat{\mu}
\]

\[
= \frac{1}{27} \left[ 00000 + 02220 + 01110 + 02001 + 01221 + 00111 \\
+ 01002 + 00222 + 02112 + 00210 + 02100 + 01020 + 02211 \\
+ 01101 + 00021 + 01212 + 00102 + 02022 + 00120 + 02010 \\
+ 01200 + 02121 + 01011 + 00201 + 01122 + 00012 + 02202 \right] - \hat{\mu}
\]

The remaining effects can be determined similarly.

The S. S. due to each of the above terms can be obtained as outlined previously. The error S.S. can be determined by subtracting the reduction due to the mean and the reduction due to each of the above terms from the total S.S. The number of df is calculated as follows:

- mean: 1 df
- Main effects: 5, each with 2 df = 10 df
- two factor interactions: \( \binom{5}{2} = 10 \), each with 4 df = 40 df

The total number of df consumed in estimating the parameters equals 51. The number of df remaining to estimate the error equals 81 - 51 = 30 df.
Tests for significance can be carried out and conclusions can be made as to which factors have a significant effect upon the response being investigated. The significant factors can then be used to develop a regression model for the response.
REFERENCES


22. Dare, Charles, "Development of an Advisory Speed Signal System for High-Speed Intersections under Traffic-Actuated Control", Highway Research Board Record #286, pp. 1-17.

