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FROM 0.5 TO 20 MeV.

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A STUDY OF THE REACTION

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DISSEMINATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

Alan Morris Young, B.S., M.S.

* * * * *

The Ohio State University
1970

Approved by

Adviser
Department of Physics
To my parents

whose sacrifices made this endeavor possible.
ACKNOWLEDGEMENTS

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CHAPTER I
INTRODUCTION

Many investigations dealing with nuclear structure are concerned with the properties of nuclear energy levels and how various nuclei interact with other nuclei. The ultimate goal of such investigations is to arrive at a complete description of nuclear forces. Radiative capture reactions in which one nucleus is captured by another nucleus with subsequent emission of only gamma radiation are well suited for nuclear structure investigations. They permit not only the study of nuclear energy levels in the final nucleus but also the interaction mechanism of the two nuclei participating in the capture process.

The radiative capture reaction $^3\text{He}, \gamma)^6\text{Li}$ has been investigated below 3 MeV by Kohler and Austin, and between 2 and 18 MeV by Nusslin et al. The reaction $^6\text{Li}(\gamma, t)^3\text{He}$, inverse to the ground state capture reaction, has been reported by Bazhanov et al. and by Sherman et al., in an energy range equivalent to 6-16 MeV $^3\text{He}$ bombarding energy. In the regions of overlap, there is no agreement among these sets of cross section data, with some discrepancies.
up to a factor 20. These discrepancies make it difficult to compare the data with the general theoretical predictions of the dipole sum rule for $^6$Li. Other considerations also point up the necessity for obtaining additional data. Although some data on the relative capture rates to the first three states of $^6$Li are given by Kohler and Austin, further data on captures to these states, as well as to higher excited states, are necessary for an evaluation of the importance of $^3$He + T cluster structure in the low-lying states of $^6$Li. The single previously measured angular distribution, for the ground state transition only, needs to be supplemented by data at several energies and for the other transitions in order to obtain an understanding of the mechanism through which the capture reaction proceeds, and possibly of the structure of $^6$Li at high excitation energies.

In the present experiment, data necessary for clarification of the problems stated above have been obtained. Measurements have been made of the ground state capture cross section from 0.5 to 20.0 MeV (laboratory energy of the incident $^3$He particles). Cross sections for transitions to the first two excited states were studied up to 5.3 MeV. Possible transitions to higher excited states were investigated through several good-statistics runs, and a gamma ray which appears to go to the second T=1 state was discovered. Angular distributions were determined for the three strong transitions at
energies of 1.0, 2.7, and 5.0 MeV and for the ground state transition at 8.25 MeV.

The experimental results are discussed in the framework of the direct-capture model. The resonating-group method of Thompson and Tang was used to generate the wave functions employed in the direct-capture calculation. Additional calculations were performed to test the quality of the resonating-group wave functions as to how well they describe the ground state of the \(^6\)Li nucleus, assuming only \(T + \^3\)He parentage. The charge form factor and r.m.s. radius for \(^6\)Li was calculated and is compared to results from elastic electron scattering data. Also, the form factor for the quasi-elastic knockout reaction \(^6\)Li(p, p \(^3\)He)T was calculated and compared with experimental results.
CHAPTER II

EXPERIMENTAL PROCEDURE AND RESULTS

The beam of $^3$He ions, obtained from the 6-MeV Ohio State University Van de Graaff accelerator, was magnetically analyzed and directed onto targets of tritium in thin evaporated films of titanium on copper backings. Gamma rays were detected in a 4-in. diameter x 6-in. long NaI(Tl) spectrometer with a surrounding NE102 plastic scintillator used in anticoincidence. Use of this anticoincidence shield enhances the full-energy peak of the monoergic NaI(Tl) gamma ray response, and also serves as an efficient cosmic ray suppressor. Data above 6 MeV were obtained at the Stony Brook FN Tandem Accelerator Laboratory. The detection system used there is similar to the one mentioned above but employs a 10" x 10" NaI(Tl) crystal. Details of both detection systems are discussed in Appendix A.

Gamma-ray spectra were recorded for 23 bombarding energies between 1.0 and 20.0 MeV at a detector angle of 90° with respect to the beam. A typical spectrum, taken at $E(^3\text{He}) = 3.77$ MeV in the simple anticoincidence mode, is shown in Fig. 1, together with a $^6\text{Li}$
level diagram. Capture radiations were identified by the following criteria: a) correct energies as determined by the Q-value (15.793 MeV) and energy spacings in $^6$Li; b) dependence on bombarding energy $E_\gamma = Q + \frac{1}{2}E(^3\text{He});$ and c) angular dependence of gamma ray energy as given by the Doppler-shift formula. The capture radiation to the ground and first two excited states are clearly resolved in Fig. 1. Below 6 MeV, gamma ray intensities were obtained from such spectra with the aid of a least-squares fitting program$^{11}$ using monoergic detector lines shapes obtained from the reactions $^7\text{T}(p,\gamma)^{4}\text{He}$ and $^{11}\text{B}(p,\gamma)^{12}\text{C}$. Above 6 MeV, only the intensity of $\gamma_0$ was determined from the spectra. This was due to the fact that the background presented by high energy neutrons prevented an unambiguous determination of the intensities of $\gamma_1$ and $\gamma_2$. However, $\gamma_1$ does appear to become more intense relative to $\gamma_0$ above 15 MeV. A spectrum, taken at 18 MeV, is shown in Fig. 2a. An interactive spectrum-stripping program, STRIP, was used to extract $\gamma_0$ from these spectra in the following manner. The spectrum of interest is displayed on an interactive display oscilloscope and then an appropriate line shape is gain-shifted to the position of the peak of interest. Then, any fraction of that line shape's intensity can be subtracted from the spectrum. The result of subtraction is then displayed. Various subtractions are performed until an acceptable intensity is determined and a reasonable background remains. The
Fig. 1. Anticoincidence-shielded detector spectrum of gamma rays emitted at $90^\circ$ from $T(\text{^3He}, \gamma)^6\text{Li}$ at $E_{\text{Lab}}(\text{^3He}) = 3.77$ MeV. The least-squares fit for transitions to the first three states of $^6\text{Li}$ is shown as a solid line, the components making up this fit are indicated by broken lines. (The small shaded component is from $^{12}\text{C}$ contamination.)
Fig. 2a. Gamma-ray spectrum from $^3$He, $\gamma$ at 18 MeV bombarding energy and $\gamma_0$ line shape analysis. 1 refers to the position of $\gamma_0$. 
Fig. 2b. Gamma-ray spectrum from $T(\text{^3He, } \gamma)^6\text{Li}$ at 6.0 MeV bombarding energy and $\gamma_0$, $\gamma_1$, and $\gamma_2$ line shape analysis. 1, 2, 3, 4 refer to $\gamma_0$, $\gamma_1$, $\gamma_2$ and $\gamma_4$ respectively.
results of analyzing the spectrum recorded at 18 MeV in this manner is also shown in Fig. 2a. (For comparison, a spectrum taken at 6.0 MeV containing little background in the region of interest was also analyzed using STRIP and is shown in Fig. 2b.) The counts remaining in the region of $\gamma_0$, attributed to the neutron background, can be seen to vary smoothly with gamma-ray energy. The gamma-ray intensities were corrected for variations of detector efficiency with gamma-ray energy and for the fraction of pulses rejected by the pileup circuit. Random coincidences, which have the effect of rejecting valid events, were recorded in a separate coincidence unit during each run, and used as an additional correction factor. Small contributions at 15.11 MeV from the reaction $^{13}\text{C}(^3\text{He}, \alpha \gamma)^{12}\text{C}$ on built-up carbon were also taken into account.

The gamma ray intensities were calibrated by measuring the $90^\circ$ yield from the $T(p, \gamma)$ reaction on the same target employed in the $T(^3\text{He}, \gamma)$ measurement, with no changes in detector geometry or electronic processing. Using the value measured by Perry and Bame$^{12}$ at $E(p) = 1.0$ MeV of $\frac{d\sigma}{d\Omega}|_{90^\circ} = 3.65 \mu\text{b/sr}$ for the proton capture reaction, the $90^\circ$ differential cross sections for radiative capture to the first three states of $^6\text{Li}$ were obtained. Fig. 3 shows the cross section for ground-state capture. The points have been adjusted along the bombarding-energy axis to account for the finite
Fig. 3. Measured 90° differential cross section for ground-state capture in the reaction T(³He, γ)⁶Li.
³He-particle energy loss within the target. This effective energy thickness, arising primarily from the titanium, was measured by comparing the observed γ₀ resolution with the unbroadened 17.2 MeV detector response obtained from the reaction ¹¹B(⁰p,γ)¹²C using a thin boron target. Quoted probable errors include statistics; uncertainties in the least-squares fitting, charge collection and current integration, target thickness and nonuniformity, and count-rate correction factors; and the estimated error of the T(p,γ) comparison cross section. These errors were added quadratically.

In order to investigate the gamma-ray energy region corresponding to transitions to higher excited states in ⁶Li, the subtraction technique⁹ for improving the detector line shape, as mentioned above, was applied to several spectra recorded with good statistics. Such a spectrum, obtained at E(³He) = 4.5 MeV, is shown in Fig. 4. Evidence appears for a transition to the state¹³,¹⁴ at 5.36 MeV in ⁶Li which persists in most of the spectra up to 15.0 MeV, above which energy it becomes obscured by background radiation.

Measurements were made to obtain information on the angular distributions of the capture gamma rays at incident energies of 1.0, 2.7, 5.0, and 8.25 MeV. Spectra at 2.7 MeV are shown in Fig. 5. The gamma rays are seen to shift to higher energies at forward angles, in agreement with calculated Doppler shifts for this reaction. The
Fig. 4. Good-statistics gamma-ray spectrum at 4.5 MeV. The subtraction technique of Ref. 9 has been used to improve the energy resolution.
Fig. 5. Spectra taken, using the subtraction technique, to determine angular distributions at 2.70 MeV. The shaded area indicates the residual spectrum after $\gamma_0$, $\gamma_1$, and $\gamma_2$ have been stripped out with intensities given by direct capture angular distribution calculations.
angular dependence for $\gamma_0$ is found to be $W(\theta) \sim 1 - (1 \pm 0.08) P_3(\cos \theta)$; thus within the small experimental error, $W(\theta) = \sin^2 \theta$ for $\gamma_0$.

Due to the background at forward angles arising from high-energy neutrons, the angular distributions of $\gamma_1$ and $\gamma_2$ cannot be extracted unambiguously from the data; theoretical predictions may nevertheless be compared directly with the data for qualitative agreement or disagreement. The angular distributions obtained at the other three energies appear to be identical. A small departure from a $\sin^2 \theta$ dependence for $\gamma_0$ at 9 MeV, was reported by Nusslin et al. A small departure from the $\sin^2 \theta$ dependence would not be inconsistent with the 5.0 and 8.25 MeV angular distributions of the present experiment.

Again, the uncertainty here is the result of the neutron background present at forward angles and higher bombarding energies.

Since the $\sin^2 \theta$ distribution for ground state capture appears to hold at all energies, the total cross section is obtained simply by multiplying the measured 90° differential cross section by $8\pi / 3$.

Fig. 6 shows this total cross section, plus the data of references 1-4. The two photo-disintegration measurements were converted to the radiative capture scale by use of the reciprocity relation$^{15}$

$$
\frac{(2J_{\text{He}} + 1)}{T} \frac{(2J_{\text{He}} + 1)p^2}{\text{He cap}} \sigma_{\text{He}} = \frac{2(2J_{\gamma} + 1)}{T} \frac{(E/c)^2}{\gamma_{\text{photo}}} \sigma_{\text{Li}}^{\gamma_{\text{photo}}} \quad (1)
$$
Fig. 6. Total cross section for ground-state capture in the reaction $T(^3\text{He}, \gamma)^6\text{Li}$ as reported in the literature, in comparison with the present experiment.
where $p^\text{\text{He}}_x$ is the c.m. momentum of $^{3}\text{He}$ particle, $J_x$ is the ground state spin of nucleus $x$, $\sigma_{\text{cap}}$ is the $T(^{3}\text{He}, \gamma_0)^6\text{Li}$ cross section, and $\sigma_{\text{photo}}$ is the $^6\text{Li}(\gamma, t)^3\text{He}$ cross section.

The present data agree very well with the values calculated from the $90^\circ$ measurements of Kohler and Austin,\textsuperscript{1} using the $\sin^2\theta$ distribution of the present experiment. The results of Nüsslin et al.,\textsuperscript{2} appear to be in reasonable agreement, but those data include capture to the first and second excited states of $^6\text{Li}$ as well as the ground state.\textsuperscript{16} Subtracting the contributions of $\gamma_1$ and $\gamma_2$, as measured in the present experiment and discussed below, the data of Nüsslin et al. should be reduced by a factor of about 0.7. Agreement among the three capture measurements, however, is much better than any comparisons with the inverse reaction.\textsuperscript{3,4} The $^6\text{Li}(\gamma, t)^3\text{He}$ reaction has recently been measured by Makhnovskii, and the integrated cross section has been reported.\textsuperscript{17,18} This new work indicates that the values reported by Bazhanov et al. are in fact too large.

Experimental results obtained for $\gamma_1$ and $\gamma_2$ are discussed in the next chapter.
CHAPTER III
DISCUSSION

A. Capture Mechanism

The ground-state cross section results of the present reaction exhibits a broad structure centered around an excitation energy of about 20.0 MeV, with a center-of-mass width of about 5.5 MeV. This structure may be a broad compound nucleus resonance, although the analysis given below makes this interpretation doubtful. Consideration will first be given to an estimate of a lower limit for the radiative width of a "resonance" at this energy; then the angular distributions will be discussed.

The peak cross section for a single \(^{(9}\text{He}, \gamma)\) resonance can be written as

\[
\sigma_{\text{res}} = 4\pi R^2 \frac{(2J_{\text{res}} + 1)}{(2J_{\text{He}} + 1)(2J + 1)} \left( \frac{\Gamma_{\text{He}} \Gamma_{\gamma}}{\Gamma_{\gamma}} \right)
\]

By using the values \(\sigma_{\text{res}} = 90 \mu\text{b}\) and \(\Gamma = 5.5 \text{ MeV}\), and noting that \((\Gamma_{\text{He}} / \Gamma) \leq 1\), a lower limit on \(\Gamma_{\gamma}\) may be found:
\[(2J_{res} + 1)\Gamma_\gamma \geq 4.6 \text{ keV.} \quad (3)\]

The Weisskopf estimate for a 20-MeV E1 transition in \(^6\text{Li}\) is
\[\Gamma_\gamma W = 1.8 \text{ keV;}\] if the radiating "single particle" is taken to be a
3-nucleon cluster, the center-of-mass correction\(^{19}\) gives a theoretical estimate of \(\Gamma'_\gamma W = 0.45 \text{ keV.}\) For any \(J_{res}\) compatible with an
E1 ground state transition, the experimental lower limit for \(\Gamma_\gamma\)
exceeds this single particle estimate. In comparison, the large
number of E1 transitions in light nuclei whose measured radiative
widths have been catalogued by Skorka et al.\(^{31}\) have a typical \(\Gamma_\gamma\)
about 0.5% of the Weisskopf estimate. Since few high-energy
electric dipole transitions are catalogued by Skorka et al., one
cannot be sure what is really "typical" in the present case. Never­
theless, the present transition appears to be enormously enhanced.

Additional difficulties arise in trying to explain the observed
angular distributions with a compound nucleus model. The observed
\(\sin^2\theta\) ground-state distribution cannot be duplicated for any single
value of \(J_{res}\), if incoming partial waves up to \(l=2\) and outgoing
radiations up to \(L=2\) are considered. Theoretical calculations\(^{21}\)
indicate the possibility of a group of states in \(^6\text{Li}\) in this energy
region, with \(L=1\), \(S=1\), and \(T=1\). If all the levels in this group
(with \(J^m = 0^-\), \(1^-\), and \(2^-\)) contribute coherently in the ground-state
capture reaction, an angular distribution $\sin^2 \theta$ is produced for the unique combination in which the amplitudes from all three states are equal. That the same distribution persists over a wide range of energies then implies a complete degeneracy of the three compound levels, an assumption which does not appear theoretically justified.

Although a broad peak may be present in the excitation curve, the energy variation is slow enough to allow consideration of the "direct" capture mechanism.\textsuperscript{22,23,24} In this model, radiative capture is assumed to be a one-step process. The direct capture and compound-nucleus capture models are two extreme simplifications of the exact capture mechanism. In the present case, these two extremes may not be too far apart; a 5.5 MeV wide compound state has a lifetime of about $7 \times 10^{-22}$ sec., which is the same order of magnitude as the time for an incident $^3$He particle to transit the nucleus in a direct reaction.

The direct capture cross section can be calculated in first order perturbation theory by considering matrix elements of the electromagnetic multipole operators between initial scattering states and final bound states of the incident and target nuclei. In the notation of ref. 24, the differential cross section for direct capture is given by
where $K$ and $P$ are the wave number and polarization state, respectively, of the gamma ray, $v_i$ is the relative speed of the projectile-target system, and $i$ and $f$ refer to the incoming and final state, respectively.

For $E1$ radiation, the interaction Hamiltonian can be written as

\[
H_{\text{int}}^P = -i \left( \frac{4\pi}{3} \right)^{\frac{1}{2}} K e \mu \left( \frac{Z_{\text{He}}}{M_{\text{He}}} - \frac{Z_T}{M_T} \right).
\]

where $M$ is the magnetic quantum number of the radiation and

\[
\mu = \frac{M_{\text{He}} M_T}{(M_{\text{He}} + M_T)}\]

is the reduced mass. The rotation matrices are functions of the angles describing the direction of gamma-ray emission with respect to the beam axis, and the spherical harmonics depend on the orientation of the target-projectile system. $\mathcal{O}_{E1}(r)$ is the radial part of the electric dipole operator. A discussion of how $H_{\text{int}}^P$ and $\mathcal{O}_{E1}$ are obtained can be found in Appendix C.
The initial and final state wave functions can be written as

\[ |i, m_i\rangle = \sum_{l=0}^{\infty} \left[ \frac{4\pi(2l+1)}{\xi} \right] \frac{i(\sigma_l - \sigma_{l+1})}{l\varepsilon} R_{lS_i}(k, r) \frac{1}{kr} Y_{l}^{\alpha} \chi_{S_i} m_i \]  

(6)

and

\[ |f, m_f\rangle = \theta_f \frac{U_{L_f(r)}}{r} \sum_{\beta} C(L_f S_f J_f; m_f - \beta, \beta) \cdot Y_{L_f}^{m_f - \beta} \chi_{S_f}^{\beta}. \]  

(7)

In these expressions, \( k \) is the wave number of the incident particle, \( \chi_{S}^{m} \) is the spin function for channel spin \( S \), and \( \theta_f \) is the coefficient of fractional parentage of the \( T + ^3\text{He} \) cluster in the total wave function for state \( f \).

If the first three states of \( ^6\text{Li} \) are described in \( L-S \) coupling, they are given by \( ^{13}\text{S}, ^{13}\text{D}, ^{31}\text{S} \), for the ground state, first and second excited states, respectively. (The first superscript is the isospin multiplicity, the second is the spin multiplicity.) Two incoming partial waves are considered, \( ^3\text{P} \) and \( ^1\text{P} \). E1 transitions from the former will go only to the \( ^{13}\text{S} \) and \( ^{13}\text{D} \) states, while the transition to the \( ^{31}\text{S} \) state comes from the latter only. The calculated angular distributions are shown in Table 1. These angular distributions should be unchanged.
Table 1. Direct-capture electric dipole angular distribution calculations. The assumed configurations are given in the notation $(2T+1)(2S+1)_{L}$.

<table>
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<tr>
<th>final state configuration</th>
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<th>$\gamma_2$</th>
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<tr>
<td>$^{13}S(1^+)$</td>
<td>$^{13}D(3^+)$</td>
<td>$^{31}S(0^+)$</td>
<td></td>
</tr>
<tr>
<td>$^{33}P$</td>
<td>$^{33}P$</td>
<td>$^{11}P$</td>
<td></td>
</tr>
<tr>
<td>$W(\theta)$</td>
<td>$\sin^2\theta$</td>
<td>$1 - 0.1 P_2(\cos \theta)$</td>
<td>$\sin^2 \theta$</td>
</tr>
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over the energy range where p-wave capture is dominant. When these calculated distributions are subtracted from the 3-MeV data, a reasonably smooth spectrum, presumably due to the high-energy neutron background, plus a small contribution around 15 MeV due to the reaction $^{13}\text{C}(^{3}\text{He}, \alpha \gamma)^{12}\text{C}^*$ (15.11), remains (see Fig. 5). The data is thus in agreement with these direct-capture angular distribution calculations.

By assuming explicit forms for the radial wave functions of eqs. 6 and 7, the complete energy dependence of the capture cross sections can be calculated and will be discussed in Section C. Qualitative conclusions can be drawn from a consideration of ratios of cross sections. Let a "reduced dipole capture cross section ratio" $\left[\frac{\gamma_n}{\gamma_m}\right]$ be defined as follows:

$$\left[\frac{\gamma_n}{\gamma_m}\right] = \frac{\sigma_{\gamma_n}/E_{\gamma_n}^3}{\sigma_{\gamma_m}/E_{\gamma_m}^3}$$  \hspace{1cm} (8)

where $n$ and $m$ refer to the various final states. From eqs. 4-7, ratios for $\gamma_0$, $\gamma_1$, $\gamma_2$ have been calculated to be

$$\left[\frac{\gamma_1}{\gamma_0}\right] = \frac{14}{15} \left(\frac{\theta}{\theta_0}\right)^2 \left(\frac{\int_0^{\infty} U_2^{(n)} O_{e1} R_{\|} \, dr}{\int_0^{\infty} U_0^{(n)} O_{e1} R_{\|} \, dr}\right)^2$$  \hspace{1cm} (9)
\[
\left[ \frac{\gamma_1}{\gamma_0} \right] = \frac{1}{3} \left( \frac{\theta_2}{\theta_0} \right)^2 \left( \frac{\sum_0^\infty U_0^{(\nu)} \delta E_1 R_{10} \, dn}{\sum_0^\infty U_0^{(\nu)} \delta E_1 R_{11} \, dn} \right)^2
\]

where the superscript on the final state radial function labels the particular final state involved. Because of identical energy behavior of numerator and denominator, \[\gamma_1/\gamma_0\] should be approximately constant over a wide range of energies. On the other hand, the ratio \[\gamma_2/\gamma_0\] is not expected to be constant, since the transitions proceed through different initial states. The two reduced cross section ratios \[\gamma_1/\gamma_0\] and \[\gamma_2/\gamma_0\] are shown in Fig. 7, and can be seen to exhibit the qualitative behavior indicated here. In calculating the ratios in Fig. 7, the measured 90° differential cross sections for \(\gamma_1\) and \(\gamma_2\) transitions were multiplied by factors of \(8\pi/2.1\) and \(8\pi/3\), respectively, under the assumption that the angular distributions are well described by the direct capture calculations. These qualitative remarks about the intensity ratios for the various transitions are verified by the calculations of Section III-C.
Fig. 7. Reduced cross section ratios for $\gamma_0$, $\gamma_1$, and $\gamma_2$. The ratio is defined as
\[ \left[ \frac{\gamma_n}{\gamma_m} \right] = \frac{(\sigma_n/\gamma_n)(\sigma_m/\gamma_m)}{(\sigma_n/\gamma_n)(\sigma_m/\gamma_m)}. \]
B. $\alpha + d$ and $T + ^3\text{He}$ Cluster States in $^6\text{Li}$

In 1960, Phillips and Tombrello proposed a simple cluster model for light nuclei. If only two body cluster configurations are important in describing a nucleus we may write the wave function of the nucleus $\psi$ as

$$\psi = \sum_{\lambda} \theta_{2,\lambda} \psi_{2,\lambda}$$

A measurement of the expansion parameter $\theta_{2,\lambda}$ allows a determination of the spatial ordering of nucleons in the nucleus. Each wave function $\psi_{2,\lambda}$ represents a particular two-cluster or parentage and is a measure of the probability that the nucleus will be found with that particular parentage assuming the $\psi_{2,\lambda}$ to be orthogonal. For well defined energy eigenstates, $|\theta_{2,\lambda}|^2$ can be determined approximately by

$$|\theta_{2,\lambda}|^2 \approx \gamma_{\lambda}^2/(3\pi^2/2\mu a_{\lambda}^2)$$

where $\gamma_{\lambda}^2 = \Gamma_{\lambda}/(2P_{\lambda})$, $P_{\lambda}$ = penetrability for channel $\lambda$, $\Gamma_{\lambda}$ is the partial width for decay into the two-cluster channel $\lambda$, and $a_{\lambda}$ is the channel radius. Now $|\theta_{2,\lambda}|^2 = 1$ so that if, at a certain eigenenergy $E_\lambda$, $|\theta_{2,\lambda}|^2 \approx 1$ then $|\eta_{2,\mu} \gamma_{\lambda}|^2 << 1$. This means that the wave function for the nucleus in that particular state can be described almost entirely by $\psi_{2,\lambda}$. If a particular $\theta_{2,\lambda}$ is near unity it is inferred
that the state is described primarily by the corresponding two body cluster.

In the paper of Phillips and Tombrello, the low lying states of $^6$Li were described as being solely of a $\alpha + d$ parentage. A qualitative reason for favoring $\alpha + d$ clustering in low lying states of the $^6$Li nucleus is that the two clusters are bound by only 1.47 MeV which is smaller than the internal energies of either of the two clusters (28.3 MeV for $\alpha$ and 2.22 MeV for d). However, a triton and $^3$He cluster are bound by 15.8 MeV which exceeds the internal binding energies of 8.48 MeV and 7.72 MeV respectively for the T and $^3$He clusters. The considerable attention devoted to the description of the low lying states of the $^6$Li nucleus has overwhelmingly favored $\alpha + d$ clustering with T + $^3$He clustering assigned a non-existent or at best, a minor role. The neglect of T + $^3$He clustering is in part due to a lack of experiments sensitive to the T + $^3$He channel from which quantitative estimates could be made regarding the relative importance of T + $^3$He and $\alpha + d$ clustering in the $^6$Li nucleus.

Shell model calculations by Eramzhyan indicate that, in the vicinity of 17 MeV excitation in $^6$Li, there should exist a group of three levels of $^{33}$P character which would have large T + $^3$He widths.
These levels, with $J^\pi = 0^-, 1^-, \text{and} 2^-$, should be separated by the spin-orbit coupling. Bazhanov et al.\textsuperscript{3} suggested that the peak apparent in their photodisintegration experiment at about 22 MeV be ascribed to this group of levels. However, Kurdyumov,\textsuperscript{30} in a later calculation, claims that the only $^6$Li levels that can disintegrate into $T + ^3\text{He}$ have energies between 16 and 18 MeV.

Resonating-group calculations of the $^3\text{He} + T$ system by Thompson and Tang\textsuperscript{31} indicate that elastic scattering data are consistent with the presence of a $^{33}\text{P}$ resonance at 22.3 MeV and a $^{11}\text{P}$ resonance at 21.3 MeV. The singlet and triplet phase shifts are sufficiently different to allow the observed behavior of the ratio $\left[\gamma_2/\gamma_0\right]$ (Fig. 7).

Calculations\textsuperscript{6, 31} in the resonating-group model have also been made of bound states with a $T + ^3\text{He}$ cluster configuration; states with L-S coupling descriptions similar to the ground and first two excited states are predicted between 3.47 and 10.39 MeV. These levels should have $\theta_f$ values near unity (in contrast to the lower states which are expected to be primarily $\alpha + d$ cluster states\textsuperscript{27}). They should therefore have substantial direct-capture transition strengths. No evidence for these states exists in the data of the present experiment. The lowest energy level (which should be correct to within 1 MeV) should show up as a very strong peak somewhere between $\gamma_2$ and the
high-energy neutron background edge. In this region, only the transition $\gamma_4$, discussed below, is seen. Transitions to the higher states would be hidden, in the present experiment, by the neutron background. The strong transitions to the lowest three states of $^6$Li suggest that there is significant configuration mixing of the higher energy $T + ^3$He cluster states in these lower states.

Berman et al.\textsuperscript{33} have suggested that there is present in the ground state of $^6$Li an appreciable mixing of a bound $T + ^3$He configuration. Evidence from other reactions\textsuperscript{5,33,34} indicates sizable $T + ^3$He clustering in the ground state and somewhat less in the first two excited states of $^6$Li. Attempts to estimate the values of the $\theta_1$ parameter for these levels from the present experiment are discussed in Section C. Preliminary estimates of ratios of these fractional parentage coefficients can be obtained by comparing the calculated ratios of Eq. 9 with the experimental values of Fig. 7. If all the radial integrals are assumed to have similar values, $\theta_1^2 \approx 0.6 \theta_0^2$ and $\theta_2^2 \approx \theta_0^2$. The coefficients $\theta_1$ and $\theta_2$ both appear, in the extreme approximation, to be larger (in comparison with $\theta_0$) than suggested by the results of other reactions.\textsuperscript{6}

The one substantial transition to states above the first $T=1$ state appears (Fig. 4) to go to a state at about 5.4 MeV which has been proposed as the analog of the first excited states of $^6$Be and
\(^6\text{He}\). If the angular distribution is assumed to be nearly isotropic, 
\[ \sigma_{\gamma_4} \approx 10^{\mu\text{b}} \text{ at } 4.5 \text{ MeV}. \]
The reduced cross section ratio for the two \( T=1 \) states, \( \left[ \gamma_4/\gamma_2 \right] \), is of order 1. The second \( T=1 \) state would be \(^{31}\text{D} \) in L-S coupling, and an E1 transition from the \(^{11}\text{P} \) incoming wave would be expected to have a strength, compared to the transition to the first \( T=1 \) state, given by

\[ \left[ \gamma_4 \right] = 2 \left( \frac{\theta_4}{\theta_2} \right)^2 \left( \frac{\int_{0}^{\infty} U_2^{(1)} \theta_{E1} R_{10} \, d\tau}{\int_{0}^{\infty} U_2^{(3)} \theta_{E2} R_{10} \, d\tau} \right)^2 \]  \hspace{1cm} (11)

assuming p-wave capture. If, again, the assumption of nearly identical radial integrals is made, \( \theta_4^2 \approx \frac{1}{2} \theta_2^2 \).

Since no isospin singlets should exist in \( T_Z = \pm 1 \) nuclei, analogous dipole transitions in \(^6\text{Be} \) or \(^6\text{He} \) are not possible. However, radiative capture of \(^3\text{He} \) by \(^3\text{He} \) has been observed recently by Harrison et al.\(^{35} \) No ground state transition was seen, but capture to the first excited state of \(^6\text{Be} \) was seen. The reaction was interpreted as direct capture from the \(^{31}\text{S} \) partial wave, which would not radiate to the \( 0^+ \) ground state but could have an E2 transition to the (2+) first excited state. The analog of this transition should contribute to \( \gamma_4 \) in \(^6\text{Li} \).
The cross section for this contribution at a given gamma-ray energy can be estimated by correcting the $^3\text{He}(^3\text{He}, \gamma)^6\text{Be}$ cross section at the same gamma-ray energy for the different penetrabilities and quadrupole effective charge factors for $^3\text{He} + ^3\text{He}$ and $^3\text{He} + \text{T}$.

(These two effects go in opposite directions.) Using the cross section found by Harrison et al. at $E_\gamma = 12.7 \text{ MeV}$, the E2 s-wave capture would be expected to contribute at least half of the observed $T(^3\text{He}, \gamma_4)^6\text{Li}$ cross section at 4.5 MeV incident energy.
C. Direct Capture Calculations

From the discussion given in Section A of this chapter, the most likely mechanism by which the reaction $^{3}\text{He}, \gamma^{6}\text{Li}$ proceeds is that of direct capture. The reasons for favoring this mechanism were that the angular distributions appear to be energy independent and the capture cross sections vary smoothly with energy. This apparent lack of resonances implies that no intermediate state is formed, i.e. the capture process is dominated mostly by electromagnetic forces and not nuclear forces. The direct process thus lends itself quite readily to theoretical treatment and interpretation since electromagnetic forces are well understood.

The nuclear model which is chosen to generate the wave functions used in a direct capture calculation (Equations 6 and 7) must adequately describe the scattering of the two nuclei under consideration. This is because the scattering phase shifts are needed to completely specify the initial state prior to capture. Using resonating group theory, D. R. Thompson and Y. C. Tang have been able to describe satisfactorily the scattering of tritons from $^{3}\text{He}$ over a wide energy range. The resonating group formalism was thus chosen to generate the wave functions to be employed in the capture calculation. A brief discussion of this method is given in Appendix B.
The wave functions for both the initial scattering states $R_{ls}$ of Equation 6 and final bound states $U_{Lf}$ of Equation 7 were generated using a resonating group computer code obtained from Dr. D. R. Thompson. The calculations were performed on the OSU IBM 360/75 computer. In order to check whether the code was working properly, elastic scattering phase shifts, cross sections, and bound state energy eigenvalues were determined for the $T + ^3\text{He}$ case and found to be in agreement with those obtained by Thompson and Tang. While the more salient features of the calculations performed are discussed below, a more detailed description of the calculation and the numerical techniques employed are discussed in Appendix D.

For each energy, the initial state wave function $R_{ls}$ of Equation 6 was normalized to behave asymptotically as the Coulomb wave function $F_l \cos \delta_{ls} + G_l \sin \delta_{ls}$.

As discussed in Section A of this chapter, the first three states of $^9\text{Li}$ can be described as $^1\text{S}$, $^1\text{D}$ and $^2\text{S}$ for the ground state, first and second excited states, and the resonating group method predicts these levels to have excitation energies (relative to the experimentally measured ground state) at 3.47, 8.18 and 7.37 MeV respectively. For each of these bound states the corresponding final state wave functions $U_{Lf}$ appearing in Equation 7 were obtained by requiring that $U_{Lf}$ asymptotically behave as the
Whittaker function (see Appendix D) and were normalized by the requirement $\int_0^{\infty} U_{Lf}^2 \, dr = 1$. These wave functions were considered to be appropriate for the final states actually observed experimentally.

The radial wave functions $R_{ls}$ and $U_{Lf}$ so obtained were used to compute, as a function of bombarding energy, the capture cross section given by Equation 4 for the ground state transition. Also computed as a function of energy were the reduced capture cross section ratios $[\gamma_1/\gamma_0]$ and $[\gamma_2/\gamma_0]$ given by Equations 9 and 10 respectively.

The calculated total cross section for $\gamma_0$ is compared with the results of the present experiment (assuming $\sin^2 \Theta$ angular dependence at all energies) in Fig. 8. As can be seen in this figure, the calculated cross section agrees quite well with experiment. It should be emphasized that the energy dependence of the matrix elements of $H^{p}_{\text{int}}$ (see Equation 4) is governed solely by the triplet p-wave scattering phase shifts. The only adjustable parameter in the entire calculation is $\Theta_0^2$ which determines the normalization of the theoretical cross section to that measured experimentally.

Physically, $\Theta_0^2$ is the probability that the ground state of $^6\text{Li}$ has the assumed cluster configuration (or parentage) namely $T + ^3\text{He}$. 
Fig. 8. Calculated cross section for $\gamma_0$ compared to results of experiment.
$T({}^3\text{He},\gamma_0)^6\text{Li}$

$\sigma_{\text{TOTAL}} (\mu b)$

$^3\text{He}$ Lab Energy (MeV)
That is, the closer $\theta_0^3$ is to unity, the better one can describe the $^6$Li ground state as being comprised solely of a triton and $^3$He. However, caution must be exercised with the interpretation of the magnitude of $\theta_0^3$ since other cluster substructures such as $\alpha + d$ are also present in the $^6$Li ground state, and their wave functions are not necessarily orthogonal to that of the $T + ^3$He component. Since we have not taken into account these other channels in the calculation, the magnitude we obtain for $\theta_0^2$ should be considered as a rough upper limit. However, a large value of $\theta_0^2$ obtained in a single channel calculation would still indicate appreciable clustering for that component.

The actual magnitude of $\theta_0^2$ was obtained in two ways. A least squares fit of the theoretical cross section to the experimental points yielded a value of 0.69 for $\theta_0^2$. Secondly, the area under the calculated curve was equated to the experimental area which gave a value of 0.64. For purposes of discussion a value of 0.6 will be used for $\theta_0^2$.

To evaluate the reduced capture cross section ratios $[\nu_1/\nu_0]$ and $[\nu_2/\nu_0]$ given by Equations 9 and 10 respectively, the radial integrals appearing in those expressions were computed. The calculated reduced capture cross section ratios were adjusted to those
determined by experiment by a least squares fit, thus permitting values for $\theta_1^2$ and $\theta_2^2$ to be obtained. Figure 9 shows the calculated ratios compared to the present experimental results. The magnitudes of $\theta_1^2$ and $\theta_2^2$ were found to be 0.48 and 0.42 respectively, assuming $\theta_0^2$ to be 0.6. Thus in addition to the ground state the first two excited states also exhibit substantial $T + ^3He$ components. The computed ratios, besides agreeing well with experiment, also display the qualitative behavior with energy as discussed in Section A of this chapter. An estimate of $[\gamma_4/\gamma_2]$ given by Equation 11 can also be made. If the radial integrals appearing in this expression are about the same for the second and fourth excited state $[\gamma_4/\gamma_2] = 2 (\theta_4/\theta_2)^2$. As discussed in Section A of this chapter $[\gamma_4/\gamma_2]$, is of order unity. Thus, $\theta_4^2 \sim \frac{1}{2} \theta_2^2$ and since $\theta_2^2 = 0.5 \theta_0^2$, $\theta_4^2$ is the order of 0.15.

Additional calculations were performed to test the quality of the resonating-group wave functions as to how well they described the ground state of the $^6Li$ nucleus, assuming only $T + ^3He$ parentage. The r.m.s. radius for $^6Li$ was calculated to be 2.55 fm which is precisely the currently accepted experimental value. The charge form factor for elastic electron scattering from $^6Li$ was computed and found to be in good agreement with experiment.
Fig. 9. Calculated cross section ratios compared to experimental results.
up to momentum transfers of 2 fm$^{-2}$. However, at higher momentum transfers the calculated form factor disagrees with experiment by several orders of magnitude. The present calculation does predict a diffraction maximum at a momentum transfer of 6.5 fm$^{-2}$, compared to the experimental finding of a maximum at 8 fm$^{-2}$; many other theories are not capable of reproducing such a maximum. Since electron scattering is sensitive to more than just the $T + {}^3$He component in the $^6$Li ground state, it is clear that other components also are present. This agrees qualitatively with the present experimental result, since $\theta_0^2 < 1$.

The reaction $^6$Li(p, p $^3$He)$^T$, is sensitive to $T + {}^3$He cluster structure in the $^6$Li ground state. The differential cross section $d^3\sigma/dE dp d\Omega_{^3\text{He}}$ for this reaction can be expressed as the product of a kinematical factor, the free $T + {}^3$He differential cross section, and a form factor. This form factor is just the square of the Fourier transform of the $^6$Li ground state radial wave function. This form factor was computed and is shown in comparison to the data of reference 39 in Fig. 10, and appears to agree satisfactorily with the results of that experiment. Also, the present calculation agrees as good with the data as the results of calculations given in reference 40.
Fig. 10. Form factor for the reaction $^6\text{Li}(p, p^3\text{He})T$ compared to experiment of Bachlier et al. Normalization of the calculated curve to the data points is arbitrary.
$|\phi(q)|^2$ (ARBITRARY UNITS) vs. MOMENTUM TRANSFER $q$
D. The Dipole Sum Rule in $^6$Li

The classical dipole sum rule, calculated from a consideration of the combined dipole strengths of each nucleon within a nucleus, is\textsuperscript{15}

$$\int \sigma(E_\gamma) dE_\gamma = \frac{2\pi^2 e^2 h}{Mc} \frac{NZ}{A} \tag{12}$$

For $^6$Li, this rule says that the total E1 photoabsorption cross section, integrated over all energies, is $= 90 \text{ MeV-mb}$. Integrated photoabsorption cross sections in light nuclei have, in general, been found to be lower than the sum rule value,\textsuperscript{16} but only photonucleon emission has been taken into account in most of these experiments. The $^6$Li($\gamma$,n) and $^6$Li($\gamma$,p) reactions account for only about 1/2 of this total below $E_\gamma = 32 \text{ MeV}$,\textsuperscript{30} and about 2/3 when energies up to 60 MeV are included.\textsuperscript{3} The phototriton cross section, as measured earlier\textsuperscript{2,3} appeared to be sufficient to account for the remaining 1/3, but those measurements (or the reciprocity calculations contained in ref. 2) do not seem to be correct in the light of the present work.

The $^6$Li($\gamma$,t)$^3$He cross section, integrated up to $E_\gamma = 25.8 \text{ MeV}$, is $10.1 \pm 1.6 \text{ MeV-mb}$. This number may be compared to other quoted values in Table 2. It should be noted that angular distributions of the $^6$Li phototriton emission reaction have not been reported; thus the reciprocal nature of the reaction considered here is, to some extent, still an assumption.
Table 2. Reported values for the integrated photo-triton cross section in $^6\text{Li}$ up to $E_\gamma = 25$ MeV.

<table>
<thead>
<tr>
<th>References</th>
<th>$\int \sigma(E_\gamma) , dE_\gamma$ MeV·mb</th>
<th>Reaction Studied</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nusslin, et al. (2)</td>
<td>$26 + 9^a$</td>
<td>capture</td>
</tr>
<tr>
<td>Sherman, et al. (4)</td>
<td>$&lt; 5$</td>
<td>photo</td>
</tr>
<tr>
<td>Bazhanov, et al. (3)</td>
<td>$30 + 10 - 15$</td>
<td>photo</td>
</tr>
<tr>
<td>Makhnovskii (17, 18)</td>
<td>$10 - 15$</td>
<td>photo</td>
</tr>
<tr>
<td>Present work</td>
<td>$10.1 \pm 1.6$</td>
<td>capture</td>
</tr>
</tbody>
</table>

Notes: a. This value disagrees with the data from ref. 2 converted to the inverse reaction using eq. 1.
The integrated cross section for dipole radiation in the T + \(^3\)He cluster system can be calculated in the manner used\(^{15}\) to derive (10). The mass M of the radiating particle is set equal to the 3-nucleon cluster mass, and the effective charge \(\mu \left( \frac{Z_{\text{He}}}{A_{\text{He}}} - \frac{Z_T}{A_T} \right) e\) is used instead of the proton and neutron effective charges. If the radiative cross section is summed over the two clusters, one obtains

\[
\frac{\int \sigma, T(E_\gamma) dE_\gamma}{\int \sigma, \text{all}(E_\gamma) dE_\gamma} = 1/9
\]

so the integrated cross section for the phototriton component alone should be about 10 MeV·mb. The fact that the value measured in the present experiment comes so close to completely satisfying this modified sum rule emphasizes the importance of T + \(^3\)He clustering in the \(^6\)Li ground state.
CHAPTER IV
CONCLUSIONS

The agreement noted among the three sets of ground-state capture measurements\(^1,2\) and the fact that the integrated phototriton cross section of 10.1 MeV-mb calculated from the present data comes so close to satisfying the phototriton dipole sum-rule prediction of 10 MeV-mb indicates that the present data may be more reliable than those obtained from \(^6\text{Li}(\gamma, t)^3\text{He}\) measurements\(^3,4\) which disagree both with one another and the capture reaction data. The direct capture model calculation employing resonating-group wave functions accounts very well for the energy dependence of the capture cross sections and angular distributions measured in the present work. The agreement of the present data with the general predictions of the dipole-sum rule and the success of the direct-capture model in accounting for what was observed experimentally should help to remove much of the controversy (see Fig. 6) regarding the strength of the reaction as well as to explain the nature of the mechanism by which it proceeds.

The value of 0.6 obtained for \(\theta_0^3\) in conjunction with the above noted agreements between theory and data, indicates the im-
portance of $T + ^3\text{He}$ clustering in the ground state of $^6\text{Li}$. The amount of $T + ^3\text{He}$ clustering in the ground state as well as the first, second, and fourth excited states is more substantial than has previously been appreciated.

The precise magnitude of the $\delta^2$ determined for the various $^6\text{Li}$ states in this work are to be considered as rough upper limits (refer to Section C, Chapter III) for the actual $\delta^2$. This is because we have neglected in the calculation transitions between the $T + ^3\text{He}$ channel and the $\alpha + d$ channel. The substantial $T + ^3\text{He}$ clustering in various $^6\text{Li}$ states exhibited in the present work and the significant evidence for $\alpha + d$ clustering in these same states from other reactions indicates the desirability of a coupled-channel calculation to generate more realistic wave functions to be employed in the direct capture calculation. It is by performing this type of calculation that more reliable quantitative estimates could be made to determine the relative importance of $T + ^3\text{He}$ and $\alpha + d$ clustering in $^6\text{Li}$.

Table 3 summarizes the results of the values obtained for $\delta^2_{T + ^3\text{He}}$ in the present work for the four final states studied.

Finally, additional information could be obtained on the behavior of $\gamma_1$, $\gamma_2$, and $\gamma_4$ (and possibly $\gamma_0$) if the present measurements would be repeated with a pulsed $^3\text{He}$ beam. This would allow time-of-flight rejection of the unwanted neutron background which
obscured some of these transitions at the higher bombarding energies and forward angles. By performing the measurements in this manner the gamma ray intensities could unambiguously be extracted from the spectra at all energies and angles. This would allow a further comparison of the experimental results with the theoretical capture calculations presented in this work.
Table 3. $^3\text{He}$ Fractional Parentage Coefficients for Low Lying States of $^6\text{Li}$.

<table>
<thead>
<tr>
<th>$J^\pi(E_{ex})$</th>
<th>$1^+(\text{G.S.})$</th>
<th>$3^+(2.18 \text{ MeV})$</th>
<th>$0^+(3.56 \text{ MeV})$</th>
<th>$2^+(5.36 \text{ MeV})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Theta^2$</td>
<td>0.60</td>
<td>0.48</td>
<td>0.42</td>
<td>0.15</td>
</tr>
</tbody>
</table>
APPENDIX A

GAMMA-RAY DETECTION SYSTEM

In attempting to study \(^3\text{He}, \gamma\) reactions, certain experimental difficulties arise which are not ordinarily encountered in the study of \((p, \gamma)\) or \((\alpha, \gamma)\) processes. The large mass excess of \(^3\text{He}\) nuclei causes the Q value for \(^3\text{He}\) capture to be quite high, typically 15 MeV, thereby allowing many reaction channels to be open and causing \(^3\text{He}\) radiative capture cross sections to be quite small (nominally, several \(\mu\text{b/sr}\)). There are usually many intense low-energy gamma rays produced in competing reactions in which particles are emitted while leaving the final nuclei in excited states. Neutrons from the \((^3\text{He}, n)\) reactions provide another source of background. Gamma-ray detectors for these studies, therefore, must have a good response to high-energy radiation, and an efficient method for removing any pileup resulting from high-counting-rate low-energy pulses. Cosmic-ray rejection is another important feature, since the total counting rate limitation of many appropriate detectors forces the experiment to be run under conditions in which the counting rate due to the capture reactions of interest is of the same magnitude as that due to cosmic radiation.
The main detection system used here* is the detector reported in ref. 8. A 10.2-cm diameter by 15.2 cm long NaI(Tl) crystal is surrounded by a 10.2-cm thick shield of NE 102 plastic scintillator used in anticoincidence. This provides cosmic ray rejection, as well as a substantial improvement in the gamma-ray line shape. Since most spectrometers which employ the anticoincidence technique to improve the line shape are generally made with a NaI(Tl) annulus, it is appropriate to point out here some of the advantages gained through the use of a plastic scintillating shield. At the high energies under consideration (15-25 MeV), pair production is the primary form of interaction in the NaI(Tl) crystal. Most of the events in which radiation escapes from the NaI(Tl) detector arise from one, or both, of the following mechanisms: a) a single .511-MeV annihilation quantum may escape, most likely from the front face of the crystal if any collimation is used; b) bremsstrahlung accompanying the stopping of the electron pair may give rise to multiple-quantum escape, primarily from the back face of the crystal. The ease of fabrication of plastic scintillator ingots to the exact geometry required allows better coverage of the front and back faces than can be made with a simple NaI(Tl) annulus. Due to the multiple-quantum character of many of

* The system was designed and built by Professor S. L. Blatt with the assistance of Dr. S. C. Ling and the author.
the events in which energy is lost in the crystal, the less-efficient plastic scintillator (which has, however, a reasonably high probability of detecting at least one of the escaping gamma-rays) gains in effective efficiency, and becomes competitive with NaI(Tl) for this application. The short decay-constant characteristic of plastic scintillators provides very high anticoincidence efficiency in the electronic circuitry, while allowing dead-time to be kept to a minimum.

The geometry of the detection system is shown in Fig. 11

To allow the use of fast electronics in a pileup-rejection configuration, the NaI(Tl) crystal is mounted on a 58 AVP fast photomultiplier. The plastic scintillator is viewed by 6 medium-speed 10-stage photomultipliers. Both CBS 7818 and Amperex XP 1031 tubes have been employed. Use of scintillation pulses down to about 30 keV equivalent gamma-ray energy in the plastic shield is possible, but no substantial line-shape improvement was noted when pulses below about 60 keV are included. Fig. 12 shows a block diagram of the electronics. An anticoincidence resolving time of 20 ns was used, with the accompanying dead-time about 50% of that figure.

To enhance the cosmic-ray rejection characteristics of the system, the entire detector is surrounded by at least 10.2 cm of lead. The combination of lead and anticoincidence shield reduces cosmic-ray counts in the 15-25 MeV region of the gamma ray spectrum.
Fig. 11. Schematic diagram of the NaI(Tl) gamma ray detection system. The acceptance angle is determined by a Mallory 2000 high-density tungsten alloy collimator. The fibre-optics light guide transmits external light pulses, used in conjunction with the gain-stabilization system, to the photomultiplier.
Fig. 12. Electronics block diagram for NaI(Tl) detector system.
by a factor of $10^3$. Shielding of the NE 102 scintillator from direct radiation produced at the target is accomplished with a Mallory 2000 tungsten-alloy collimator.

Rejection of spectrum-distorting pileup pulses is accomplished in fast logic, using primarily commercially available modules in the E. G. & G. M100 series. Pulse pileup distortion was reduced by a factor of over 200 using the logic that is described in detail elsewhere.\textsuperscript{10} With this system, the approximately 4 $\mu$s long time interval surrounding each linear amplifier pulse within which pileup can occur was reduced to only 5 or 20 ns within which pileup could still occur and remain undetected. The 5 ns residual pileup resolving time is characteristic of pileup events in which two low-amplitude pulses sum to simulate a pulse in the high-energy region of interest. Pileup events in which one of the pulses involved is itself a pulse of interest have a residual pileup resolving time of 20 ns. Although the system performs properly at total counting rates as high as $7 \times 10^6$ pulses per second, the present experiment was conducted at counting rates in the neighborhood of $5 \times 10^4$ pps.

Gain in the linear electronics was held constant through the use of two separate stabilizing systems. Rapid gain fluctuations due to dynode loading from sudden counting-rate changes or high-current pulses were reduced by a factor of about 500 by employing
an external string of 1N3000B-series zener diodes as low-impedance voltage sources for the last six dynodes of the 58 AVP. Longer-term stability was controlled to about 1% by a circuit adapted from a design by Marlow, used in conjunction with a stable external light pulser fed into the light-pipe between the NaI(Tl) crystal and the photomultiplier through a fiber-optics bundle.

The response of this system to 20.5 MeV gamma rays produced in the T(p, γ)⁴He reaction is shown in Fig. 13. The upper curve shows the response of the NaI(Tl) crystal alone which gives a resolution of 16%. Curve (b) is the anticoincidence spectrum with 6.8% resolution at the expense of 53% loss of peak height compared to curve (a). The line shape can be further improved by subtracting about 20% of the coincident spectrum from the anticoincident spectrum, as seen in curve (c).

Data above 6 MeV were obtained at the S.U.N.Y.--Stony Brook FN tandem accelerator laboratory. The detection system used there is similar to the one described in this work but employs a 10'' x 10'' NaI(Tl) crystal. Details of this system have been reported elsewhere.

The response of this detection system to 22.9 MeV gamma radiation is shown in Fig. 14. Curves (a) and (b) are the anticoincident and coincident spectra, respectively. Note the improved resolution of 4.4% obtained in the accepted spectrum, curve (a), as compared to
Fig. 13. Response of NaI(Tl) detector system to 20.4 MeV gamma rays produced in the reaction $T(p, \gamma)^4\text{He}$ at 0.95 MeV bombarding energy. (a) and (b) show the response of the detector system without and with the anticoincidence shield in use. (c) represents the improvement by using the subtraction technique discussed in the text.
T(p,γ) Line Shapes

E_γ = 20.5 MeV
Fig. 14. Response of the Stony Brook detector system to 22.9 MeV gamma rays produced in the reaction $T(p, \gamma)^{4}\text{He}$ at 4.0 MeV bombarding energy. (a) and (b) show the response of the detector system with and without the anticoincident shield in use. (c) is the singles response of the NaI(Tl) crystal. (d) represents the improvement by using the subtraction technique discussed in the text.
6.8% for the smaller Ohio State system. Curve (c) characterizes the "bare crystal" response and is the sum of curves (a) and (b).

The subtraction technique discussed earlier was employed in obtaining curve (d) for which the resolution is 3.5%. 
APPENDIX B

METHOD OF RESONATING-GROUP
NUCLEAR STRUCTURE CALCULATIONS

The theory of resonating-group nuclear structure calculations has been applied extensively by many authors for the treatment of scattering problems where both the incident and target nuclei are comprised of only a few nucleons. In particular, Thompson and Tang have used this method, in the single-channel approximation, to successfully describe the scattering of tritons from $^3$He. The discussion which follows closely follows that of Thompson and Tang in reference 31.

There are several advantages to using the resonating-group method. The principal one being that a two-body potential consistent with low energy nucleon-nucleon scattering is employed. Thus, the use of the optical model, which has a number of variable parameters, is avoided. Also, the indistinguishability of the nucleons is properly taken into account by fully antisymmetrizing the wave functions used in the calculation of matrix elements.
Assuming only the T-$^3$He channel, one takes the trial wave function of the six nucleon scattering problem to be

$$\Psi = A \left\{ \varphi_1 \varphi_2 F(\vec{R}_1 - \vec{R}_2) \xi(\sigma, \tau) \right\}$$

(1)

where the operator $A$ denotes complete antisymmetrization of the wave function. $\xi(\sigma, \tau)$ is the appropriate spin-isospin function for the system. The two functions $\varphi_1$ and $\varphi_2$ are the internal spatial wave functions for the triton and $^3$He clusters and have the form

$$\varphi_1 = \exp \left\{ - \frac{\alpha}{2} \sum_{i=1}^{3} (\vec{r}_i - \vec{R}_1)^2 \right\}$$

(2)

$$\varphi_2 = \exp \left\{ - \frac{\alpha}{2} \sum_{i=4}^{6} (\vec{r}_i - \vec{R}_2)^2 \right\}$$

(3)

where $\vec{R}_1$ and $\vec{R}_2$ denote the positions of the center of mass of each cluster. The widths of the two clusters $\alpha$ is chosen to be consistent with their rms radii as determined from electron scattering experiments. The saturating nature of nuclear forces is crudely accounted for in the calculation by allowing this width parameter to remain fixed. The function $F(\vec{R}_1 - \vec{R}_2)$ describes the relative motion of the two clusters and is determined by applying the variational principle:

$$\delta \int \Psi^* (H - E_{tot}) \Psi \, dt = 0$$

(4)
where $E_{\text{TOT}}$ is the total energy of the system. $H$ is a six-body Hamiltonian of the form

$$
H = -\frac{\hbar^2}{2m} \sum_{i=1}^{6} \nabla_i^2 + \sum_{i>j=1}^{6} V_{ij}
$$

where the two-body interaction is given by

$$
V_{ij} = -V_0 \exp(-\beta r_{ij}^2) \left( w + m P_{ij}^r + b P_{ij}^\sigma - h P_{ij}^c \right) + \frac{e^2}{V_{ij}} \left(1 + \tau_i^z\right) \left(1 + \tau_j^z\right)
$$

The quantities $P_{ij}^r$, $P_{ij}^\sigma$, and $P_{ij}^c$ are the space, spin and isospin exchange operators and $\tau_i^z$ is the third component of isospin (charge) of the $i$th nucleon. The constants multiplying the exchange operators satisfy the following relations

$$
w + m + b + h = 1
$$

$$
w + m - b - h = \chi
$$

where $\chi$ is the ratio of the strengths of the n-p singlet to triplet interaction. The constants $V_0$ and $\beta$ are determined by fitting low energy nucleon-nucleon scattering data. In this calculation $V_{ij}$ is rewritten as follows

$$
V_{ij} = y V_{\text{serber}} + (1-y) V_{\text{symmetric}}
$$
where $V_{\text{Serber}}$ is given by (6), (7) and (8) for $w = m$ and $b = h$.

Similarly, for $V_{\text{symmetric}}$ but with $h = 2w$ and $m = 2b$. If the one-channel approximation is indeed a valid assumption, then we expect a value of $y$ close to unity. The value of $y$ is the only adjustable parameter in the entire calculation for scattering cross sections.

If one sums over all spin and isospin coordinates and integrates over all internal spatial coordinates of the clusters, one obtains an integrodifferential equation for $F$ given by

$$
\left\{ \frac{K^2}{3m} V^2 + E - V_D(\vec{r}) - V_C(\vec{r}) \right\} F(\vec{r}) = \int K(\vec{r}, \vec{r'}) F(\vec{r'}) d\vec{r'}
$$

(10)

The potentials $V_D$ and $V_C$ are the direct nuclear and Coulomb potentials respectively, acting between the clusters and are obtained through using only the direct or unsymmetrized part of the wave function. They are expressed as

$$
V_D = -V_0 \left( \frac{3\alpha}{3\alpha + 4\beta} \right)^{3/2} \omega_L^2 \exp \left( \frac{-3\alpha\beta}{3\alpha + 4\beta} r^2 \right)
$$

(11)

$$
V_C = \frac{Z_1 Z_2 e^2}{r} \frac{2}{\sqrt{\pi}} \int_0^{(3\alpha/\beta)^{1/2} r} \exp (-t^2) dt
$$

(12)
The constant $w_{ls}$ in $V_D$ is a linear combination of the exchange constants and can be found in reference 31. The kernel $K(r, r')$ represents a nonlocal interaction containing all dynamical effects that arise from the exchange character of nuclear forces and effects of exchange from antisymmetrization. If (10) is expanded in partial waves one obtains the equation

$$\left\{ \frac{\hbar^2}{3m} \left[ \frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} \right] + E - V_0 - V_\Sigma \right\} f_{ls}(n) = \int_0^\infty K_{ls}(n, n') f_{ls}(n') \, dn'$$

This equation can be solved for $f_{ls}$ by a numerical technique developed by Robertson. A detailed expression for $k(r, r')$ can be found in reference 31.

In treating elastic scattering one requires that the function $f_{ls}$ match on to a linear combination of regular and irregular Coulomb wave functions thereby determining the scattering phase shifts for a given partial wave. When generating wave functions for bound states, $f_{ls}$ must match on to a Whittaker function thus determining a bound state energy eigenvalue.

Various values of $y$ ranging from 1.1 to 1.3 were found to yield fits of similar quality to the $T + ^3\text{He}$ scattering data. In calculating the phase shifts and differential cross sections, a value of 1.25 was chosen for $y$. 
Since the value of \( y \) is determined from the scattering data, there is no adjustable parameter in the bound-state problem. The results of the bound-state calculations made by Thompson and Tang for \( ^6\text{Li} \) are found in reference 6, and the \( ^6\text{Li} \) levels found to have a \( T + ^3\text{He} \) configuration are listed in Table I of that reference.
In this section an interaction Hamiltonian for EL radiation is developed for the purpose of obtaining more accurate expressions for electric multipole operators. The notation used in the following discussion is that of Rose and Brink. For simplicity, operators for absorption of radiation will be considered first, then, under Hermitian conjugation, be applied to the emission process.

The Hamiltonian for a single particle of charge $e$ and mass $m$ interacting with an electromagnetic field $\vec{A}$ is

$$H = H_0 - \frac{e\vec{k}}{2mc}(2\vec{P} \cdot \vec{A}) = H_0 + H_{\text{int}}$$  \hspace{1cm} (1)

The momentum operator $\vec{P}$ is measured in units of $\hbar$, i.e., $\vec{P} = -i\nabla$. Now for polarization $\vec{e}$ and wave number $\vec{k}$, the expansion of the corresponding vector potential $\vec{A}_e$, assuming EL radiation only, is

$$\vec{A}_e = \vec{e}_k \exp(i\vec{k} \cdot \vec{r}) = \frac{1}{\sqrt{2}} \sum_{LM} \vec{A}_e^{LM} D_{LM}^L$$ \hspace{1cm} (2)
where \( q = +1, -1 \) denotes right- and left-handed states of circular polarization respectively. The superscript \( e \) denotes electric radiation.

Upon combining equations (1) and (2) we obtain the following for the interaction Hamiltonian.

\[
H_{\text{INT}} = -\frac{e\hbar}{2mc} 2 \mathbf{P} \cdot \mathbf{E} \exp(i \mathbf{K} \cdot \mathbf{r}) \\
= \frac{e\hbar}{2mc} \frac{1}{\sqrt{2}} \sum_{LM} \mathbf{A}^e_{LM} \cdot \mathbf{P} \mathcal{D}^L_{M0}
\]

The field \( \mathbf{A}^e_{LM} \) is the electric multipole component of the transverse field \( \mathbf{A}^q \) and is given by

\[
\mathbf{A}^e_{LM} = \frac{1}{K\sqrt{L(L+1)}} \nabla \times \mathbf{L} \phi_{LM}
\]

where \( \phi_{LM} = i^L (2L+1) j_L(kr) \sqrt{\frac{4\pi}{2L+1}} Y_{LM}(\theta, \phi) \).

Equation (4) can be rewritten as

\[
\mathbf{A}^e_{LM} = \frac{1}{K\sqrt{L(L+1)}} i^L (2L+1) i \nabla \left\{ \left[ (L+1) j_L(kr) j_{L+1} \right] * \right\}
\]

\[
\sqrt{\frac{4\pi}{2L+1}} Y_{LM} \right\} + \frac{i K \phi_{LM}}{K\sqrt{L(L+1)}}
\]

\[(5)\]
The last term in (5) will be neglected since we are interested in obtaining only a more exact multipole operator.

Then $\tilde{A}_{LM}^e$ takes the form

$$\tilde{A}_{LM}^e = \frac{1}{R \sqrt{L(L+1)}} \hat{P} \mathbf{w}_{LM}$$

(6)

where $\mathbf{w}_{LM} = i^L (2L+1) \left[ (L+1) \mathbf{j}_L - L \mathbf{j}_{L+1} \right] \sqrt{\frac{4\pi}{2L+1}} Y_{LM}$.

Then,

$$\tilde{A}_{LM}^e \cdot \hat{P} = \frac{1}{R \sqrt{L(L+1)}} (\hat{P} \mathbf{w}_{LM}) \cdot \hat{P}$$

$$\tilde{A}_{LM}^e \cdot \hat{P} = -\frac{i}{R \sqrt{L(L+1)}} (\hat{n} \mathbf{w}_{LM}) \cdot \hat{P}$$

(7)

and we note that

$$[H, \mathbf{w}_{LM}] = -\frac{i \hbar^2}{m} (\hat{n} \mathbf{w}_{LM}) \cdot \hat{P}.$$  

Thus $\tilde{A}_{LM}^e \cdot \hat{P}$ can be written as

$$\tilde{A}_{LM}^e \cdot \hat{P} = \frac{m}{\hbar^2 R \sqrt{L(L+1)}} [H, \mathbf{w}_{LM}]$$

(8)

Upon substituting (8) into (3) one obtains

$$H_{int} = -\frac{e \hbar}{2 m c \sqrt{2}} \sum_{LM} \frac{1}{\sqrt{L(L+1)}} [H, \mathbf{w}_{LM}] D_{LM}^2$$

(9)
Now \( \langle \lambda | [H, \omega_{\ell m}] | \mu \rangle = (E_{\lambda} - E_{\mu}) \langle \lambda | \omega_{\ell m} | \mu \rangle = \hbar c k \langle \lambda | \omega_{\ell m} | \mu \rangle \)

so that we can write an effective interaction Hamiltonian as

\[
H_{\text{INT}} = -\frac{e^2}{2m c} \frac{2}{\sqrt{2}} \frac{\mu}{k^2 r} \sum_{\ell m} \frac{\omega_{\ell m}}{\sqrt{L(L+1)}} D_{m q}^L
\]

\[
H_{\text{INT}} = -\frac{e^2}{2} \sum_{\ell m} \frac{i^L(2L+1)}{\sqrt{2L+1}} \sqrt{\frac{4\pi}{2L+1}} Y_{\ell m} D_{m q}^L \left[(L+1) \hat{j}_L - kr \hat{j}_{L+1}\right]
\]

(10)

For computational purposes, it becomes helpful to identify more explicitly the radial and angular parts of (10). Previous direct capture calculations have assumed the wavelength of emitted radiation to be large compared to the dimensions of the system radiating. This assumption, known as the long-wavelength approximation, is valid so long as \( kr < 1 \). The radial operators for \( E_1 \) and \( E_2 \) radiation, in this approximation are \( r \) and \( r^3 \) respectively. At this point, a more exact radial operator \( \theta_{EL} \) can be defined which approaches that of the long-wavelength approximation when \( kr < 1 \), namely,
With this definition (10) can be re-expressed as

\[ H_{\text{int}} = -e \sum_{LM} \frac{(iR)^L}{(2L-1)!!} \sqrt{\frac{L+1}{2L}} O_{EL} \sqrt{\frac{4\pi}{2L+1}} Y_{LM} D_{MQ}^L \] (12)

Now, to obtain the proper Hamiltonian for emission, one must take the negative Hermitian conjugate of \( H_{\text{int}} \). (Negative because \( \langle \lambda | [H, \psi_{LM}] | \mu \rangle \) changes sign under Hermitian conjugation.)

Thus, the Hamiltonian for emission of EL radiation of polarization \( q \) is

\[ H_{\text{int}}^T = e \sum_{LM} \frac{(-iR)^L}{(2L-1)!!} \sqrt{\frac{L+1}{2L}} O_{EL}^* \sqrt{\frac{4\pi}{2L+1}} Y_{LM}^* D_{MQ}^L \] (13)

Then, in the case for \( L=1 \), the Hamiltonian for emission and the El operator can be written as

\[ H_{\text{int}}^T = -\sum_M i e R^T D_{MQ}^1 Y_{LM}^* \sqrt{\frac{4\pi}{3}} \] (14)

with

\[ O_{E1} = \frac{3}{2e^3} [e \cos \phi - (1-e^2) \sin \phi] \]
Finally, when two charged particles are radiating about their center-of-mass, one must consider the corrections to charge and mass (the "effective charge" and "reduced mass", respectively). With these corrections, $H_{\text{int}}$ becomes

$$H_{\text{int}}^b = -ie\kappa \mu \left( \frac{2e_1}{M_1} - \frac{2e_2}{M_2} \right) \sum_M D_M^{1*} \Omega_{e_1} Y_{e_1}^{*} \sqrt{\frac{4\pi}{3}}$$

(15)
APPENDIX D

Numerical Techniques and Calculational Details

In this section a discussion of numerical techniques employed in the direct capture calculation is presented.

The Schrödinger equation \( H\psi = E\psi \) for a two-body central force problem can be cast into the following form with the substitution \( \psi = f/r \):

\[
\frac{d^2 f}{dr^2} + U(r)f = 0
\]  

(1)

where \( U(r) = \frac{2U}{\hbar^2} (E - V(r)) - \ell(\ell + 1)/r^2 \). Fox and Goodwin have developed a simple and very accurate technique for numerically solving differential equations that have the form of Equation 1, where the function \( f(r) \) is evaluated at a succession of discrete points \( r_n = nh \) (\( n \) is a non-negative integer and \( h \) is the desired step size). The following recurrence relation holds among the \( f_n = f(r_n) \):

\[
(1 + \frac{1}{12} \hbar^2 u_{m+1}) f_{m+1} - (2 - \frac{5}{6} \hbar^2 u_m) f_m + (1 + \frac{1}{12} \hbar^2 u_{m-1}) f_{m-1} = 0
\]  

(2)
where \( U_n = U(r_n) \). One of the boundary conditions imposed on \( (1) \) is \( f(0) = f_0 = 0 \). By choosing \( f_1 = f(h) = \text{constant} \) (which affects only normalization of \( f \)) along with \( f_0 = 0 \) one can obtain other values of \( f_n \) by application of the Fox-Goodwin recurrence formula given in Equation 2.

For two particles bound by only a Coulomb field the appropriate solution of \( (1) \) is the Whittaker function which satisfies the differential equation

\[
\frac{d^2W}{d\epsilon^2} - \left[ 1 + \frac{2\eta}{\epsilon} + \frac{l(l+1)}{\epsilon^2} \right] W = 0. \tag{3}
\]

where \( \epsilon = kr \) and \( \eta = Z_1 Z_2 e^2/\hbar v \). The asymptotic expansion for \( \epsilon \gg 1 \) is

\[
W_{\eta, l} \sim \frac{e^{-\epsilon}}{(2\epsilon)^{\eta}} \left\{ 1 + \sum_{m=1}^{\infty} \frac{[(l+\frac{1}{2})-(\eta+\frac{1}{2})] \cdots [(l+\frac{1}{2})-(\eta+m+1/2)]}{m! (2\epsilon)^m} \right\} \tag{4}
\]

and the logarithmic derivative of \( W \) is given by

\[
\frac{1}{W} \frac{dW}{d\epsilon} = -\frac{1}{\epsilon} \left\{ \epsilon + \eta + (l-\eta)(l+\eta+1) \frac{W_{\eta+1, l}}{W_{\eta, l}} \right\} \tag{5}
\]
The calculation of the Whittaker functions involved computing its asymptotic value at two successive points followed by a downward recurrence on the $f_n$ using the Fox-Goodwin relation given by (2).

Values of $W_n, \zeta$ and its logarithmic derivative computed in this manner were found to agree precisely with the tabulated values of Hebbard and Robson\textsuperscript{44} for those $\nu$ and $\rho$ values in the region of interest.

In principle, to generate the bound state wave functions $U_{L_f}$ (in Equation 7 of Chapter III) the bound state energy eigenvalues must be known precisely. In practice however, this can never be achieved when solving for the wave functions numerically. The eigenvalues were determined for each of the bound states by observing between what two energies the tail of the wave function $U_{L_f}$ changes sign. This indicates that an eigenvalue has been passed through. These "eigenvalues" were determined to within one milli-electron volt where the $U_{L_f}$ would still change sign asymptotically but the wave function out to the vicinity of 11 fermis would remain unchanged to 4 decimal places. At this point (10.8 fm) the logarithmic derivatives of the $U_{L_f}$ were computed by the central difference formula\textsuperscript{45}

$$\frac{f_n'}{f_n} = \frac{-(f_{n+2} - 8f_{n+1} + 8f_{n-1} - f_{n-2})}{12 \ h \ f_n}$$
and compared to that of the Whittaker function at the same point. The ground state and second excited state's logarithmic derivatives could be matched to that of $W$ to within .1% and the first excited state wave function could be matched to better than 3%. Confident of the matching at 10.8 fermis the $U_{L_f}$ were continued out to 81.6 fermis and were normalized by requiring that they have unit norm:

$$\int_0^\infty U_{L_f}^2 \, dr = 1.$$  

All of the numerical integrations were performed through the use of the extended Simpson's rule given by

$$\int_a^b f(x) \, dx = \frac{h}{3} \left[ f_0 + 4(f_1 + f_3 + \cdots + f_{2m-1}) + 2(f_2 + f_4 + \cdots + f_{2m-2}) + f_{2m} \right]$$  

(8)

All integrations were performed up to 81.6 fermis with a step size of 0.24 fermis as was used by Thompson and Tang for the $T + \alpha$ He scattering problem.

The $E1$ Hamiltonian (Eq. 5 of Chapter III) and the exact $E1$ operator $\hat{O}_{E1}$ (Eq. 11 of Appendix C) both are functions of the gamma-ray energy $E_\gamma$ or wave number $k = E_\gamma / hc$. The exact gamma ray energies for the three transitions studied were used to determine $k$ rather than using the theoretical gamma ray energies.
REFERENCES


