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INTRODUCTION

The expected reply to the question "What are the competing theories of truth?" would be given by mentioning the correspondence theory, the coherence theory, the pragmatic theory and the semantic theory of truth. Yet, if one accepts some claims about these so-called theories, it is clear that there can be no competition. Tarski, who defends the semantic theory (conception) of truth, claims that this theory captures the essence of the correspondence theory.\(^1\) Thus, if we are to believe Tarski, we must narrow the above four candidates for competition to three. But Austin, who claims that he is defending the correspondence theory of truth, claims that one who defends this theory is defending "a series of truisms."\(^2\) If we are to believe Austin, then if the coherence theory and the pragmatic theory stand opposed to the correspondence theory, they must contain denials of truisms and thus cannot be served up as serious rival theories of truth.

If Aristotle has captured the correspondence theory, as Tarski suggests, by contending that "To say of what is that it is not, or of what is not that it is, is false, while to say of what is that it is,\


or of what is not that it is not, is true,"\(^3\) then it seems that we would have to agree with Austin that the correspondence theory is truistic. How could one possibly disagree with Aristotle's contention?

Contrary to what some philosophers have suggested, I think that there are competing "theories of truth." My intention in this dissertation is that of constructing some of the logically possible rival theories. Furthermore, I wish to indicate that at least some of these theories have not failed to gain adherents. Since the theories have had followers they, of course, have received some attention, but the theories have not been compared and contrasted nor have they been perspicuously presented by the device of employing artificial languages as models of the theories. Shortly, I will have more to say about uses of artificial languages.

The phrase "theories of truth" was put within double quotation marks in the above paragraph in order to indicate that this phrase will be used in a special fashion. According to my usage of the term one does not present a theory of truth unless he specifies what it is in virtue of which the bearers of truth-values are true. In other words, a theory of truth must state what it is that makes true "things" true and false "things" false.

Moore is a notable example of one who wrote about truth but did not present a theory of truth. According to his views as

expressed in his paper, "The Nature of Judgment," true "things" (propositions in his case) are just true. That is, there is nothing to which a true proposition is connected in virtue of which it is true. Truth is a simple, unanalyzable quality which is a constituent of true propositions.

It is interesting to note that Moore also did not possess a "theory of good" provided this phrase is defined in a manner parallel to that in which "theory of truth" was defined above. Moore's contention is that good is not an analyzable quality. Contrast Moore with those philosophers who claim that X is good in virtue of the connection X has to happiness or the connection X has to pleasure. The latter would be presenting "theories of good."

The following schema can be viewed as the starting point for the construction of the rival theories of truth which will be presented:

(T) X is true only if X is connected to Y.

Now X can be connected to Y but not be a constituent of Y, or X can be connected to Y and be a constituent of Y. Let us represent these alternatives by the following sub-schema:

(T') X is true only if X designates Y.

(T'') X is true only if X is a part of Y.

The labels "correspondence theory" and "coherence theory" could be used to distinguish between those theories which are constructed by

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embellishing on schema T' and those theories which are constructed by embellishing upon schema T'', respectively. Bradley has been said to have held a coherence theory and it will be made clear that Bradley's theory is a development of schema T''.

Although association of the correspondence theory with Frege, Bergmann, Wittgenstein, and Ockham is not as immediate as the association of the coherence theory with Bradley, there would be no offense to tradition if their theories which are developments of schema T' are called correspondence theories.

The terms "is a natural sign of," "means," "intends," "is intentionally tied to," "projects itself upon," "stands in a projective relationship to," "is about," and "denotes" would serve as replacements for the occurrence of "designates" in schema T'. It should be clear that as long as we are operating with schema T', sentences or articulated sounds are not the bearers of truth. Of course, it is not uncommon to talk about sentences designating facts or states of affairs. But this is a misleading manner of speaking if it is not understood that this is a shorthand way of saying that the sentence is made to designate the fact or state of affairs by someone.

One of the things that bothers me about Tarski's article, "The Semantic Conception of Truth," deals with precisely this point. Tarski chooses the sentence "The expression 'the father of his country' designates (denotes) George Washington," as an illustration of the

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manner in which he uses the term "designates."\textsuperscript{6} But Tarski never talks about this way of speaking as a shorthand device. Consequently the value of his remarks is diminished for me. Surely the linguistic expression \textit{per se} does not designate anything. It stands in certain spatiotemporal relationships to George Washington but it does not "get at" George Washington. When one talks about sentences as designatory, designation must be a three-termed relation involving a sentence, an object, and a language-user or group of language-users. In schema T' "designates" is a two-termed relation. Thus, any theory based on schema T' has it that the truth-bearers designate (get at)(denote)(are intentionally tied to) Y independently of the participation of a third party.

A schema which would serve as a basis for a revised-Tarski kind of theory of truth would be the following:

\[(\text{L-T'}) \text{ X is true-for-Z only if X is made to designate Y by Z.}\]

A very fundamental difference between those theories based on schema T' and those based on L-T' is evident. One cannot develop a theory of truth by developing schema L-T' without admitting that truth is relative to a language-user or group of language-users.

Put from an ontological point of view the theories based on L-T' are much less demanding. It is easier to grant the existence of sentences and articulated sounds which designate in virtue of the presence of language-users and sound-producers than it is to grant the

\textsuperscript{6}Ibid., p. 345.
existence of propositions which designate "on their own." But if truth can be established as an absolute rather than a relativistic notion by paying the price of a more demanding ontology, the price seems to be worth paying.

Ockham contrasts those entities which enter into a two-term and three-term relationship in the following passage:

According to Boethius, in the first book of the De interpretatione, language is threefold: written, spoken and conceptual. The last named exists only in the intellect. Correspondingly the term is threefold, viz., the written, the spoken and the conceptual term. A written term is part of a proposition written in some material and is or can be seen with the bodily eye. A spoken term is part of a proposition uttered with the mouth and able to be heard with the bodily ear. A conceptual term is a mental content or impression which naturally possesses signification or consignification, and which is suited to be part of a mental proposition and to stand for that which it signifies.  

Ockham's (and Boethius') conceptual terms signify naturally, i.e., unlike spoken and written terms, they are not made to signify. They signify on their own. Ockham states:

Certain differences are to be found among these [three] sorts of terms. One is the following: A concept or mental impression signifies naturally whatever it does signify; a spoken or written term, on the other hand, does not signify anything except by free convention. From this follows another difference. We can change the designation of the spoken or written term at will, but the designation of the conceptual term is not to be changed at anybody's will.

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8 Ibid., p. 48.
All of the theories of truth which are developments of schema T' require the existence of entities which are natural or primary signifiers (designators).

According to Ockham, the parts of propositions possess natural signification and it is in virtue of the natural signification of the parts that propositions taken as a whole designate entities. Thus, it is clear that Ockham's theory is not a straightforward development of schema T', viz., X is true only if X designates Y. The following schema more accurately suggests the type of theory that is Ockham's:

\[(P-T') \text{ } X \text{ is true only if a part of } X \text{ designates } Y.\]

The common feature shared by those theories which are developments of T' and P-T' is that either the truth-bearer or a part of the truth-bearer naturally signifies an entity. Those theories which are developments of schema L-T' (linguistic-correspondence theories) and the related schema:

\[(L-P-T') \text{ } X \text{ is true for } Z \text{ only if a part of } X \text{ is made to designate } Y \text{ by } Z\]

have it that neither the truth-bearers nor parts of the truth-bearers naturally signify entities.

My interest in the succeeding chapters will be with the theories which are elaborations of T' and P-T' (non-linguistic correspondence theories) rather than with their linguistic counterparts. But I would like to briefly suggest at this point that Church and Geach have defended linguistic counterparts to theories which are developments of T' and P-T', respectively.
Church writes:

... we postulate two abstract objects called truth-values, one of them being truth and the other one falsehood. And we declare all true sentences to denote the truth-value truth, and all false sentences to denote the truth-value falsehood.9

As has been argued earlier, the sentence per se does not denote anything. To make Church's claim intelligible we must alter in some fashion such as this:

A sentence, S, is true-for-Z only if Z uses S to designate truth.

As such, we have a theory of truth which fits the schema L-T'.

Geach states:

As used in this work, the terms "subject" and "predicate" will always be linguistic terms. I shall never call a man a logical subject, but only the name of a man -- the name "Peter," not the Apostle is the subject of "Peter was an Apostle," and not the property of being an Apostle, but its verbal expression is a predicate. I shall say, however, that what the predicate in "Peter was an Apostle" is predicated of is Peter, not his name; for it is Peter, not his name, that is being said to have been an Apostle. In saying that something is predicated of Peter, I do not mean that this predicate is true of or applies to Peter, but only that in some significant sentence, true or false, it is predicated of Peter. I shall say that a predicate is attached to a subject, is predicated of what the subject stands for, and applies to or is true of this if the statement formed is true.10

According to Geach, "Peter was an Apostle" is true only if "was an Apostle" applies to (designates) Peter. Geach, like Tarski, gives no indication that this is an abbreviated way of speaking. Surely the


words "was an Apostle" per se do not apply to anything. Geach denies that the predicate is other than a linguistic term thus we cannot view him as maintaining a theory based on schema P-T'. To make sense out of Geach's remarks we must revise the claim so that it reads something like this:

"Peter was an Apostle" is true only if "was an Apostle" is applied by a certain language-user or body of language-users to Peter.

But since there may be language-users who may not apply the predicate in this manner, the sentence may be false for them. Thus, a clearer statement of Geach's criterion would be:

"Peter was an Apostle" is true-for-Z only if Z applies "was an Apostle" to Peter.

As such, it appears that a theory which is based on L-P-T' is in the making.

The non-linguistic counterparts to Church's and Geach's theories can now be adumbrated. Corresponding to Church's theory we have a theory according to which a mental content (a proposition), which is not defined by its correspondence to a sentence but can be thought of as the meaning of a sentence, is true only if it designates (is the natural sign of) truth. The non-linguistic counterpart of Geach's theory would have it that a proposition which corresponds to a sentence such as "Peter was an Apostle" is true only if the predicate part of the proposition (not the sentence) designates Peter.

When we are dealing with non-linguistic correspondence theories, the designating relation is a two-term relation and furthermore it is asymmetrical. If X designates Y, it is not the case that Y
designates X. Though there is this agreement, there is disagreement concerning whether it is the case that:

1) Given any X which is a designator, there may or may not be a Y such that X designates Y, or

2) Given any X which is a designator, there must be a Y such that X designates Y and if Z designates Y then Z = X, or

3) Given any X which is a designator, there must be a Y such that X designates Y but it may be the case that Z designates Y and Z ≠ X.

Thus, given these alternative ways of viewing the designating relation, there are three distinct kinds of theories which are based on schema T'. According to the non-linguistic counterpart to Church's theory, a theory which I think has been improperly ascribed to Frege, all true propositions designate the same entity. Thus, the designating relationship is interpreted in the third manner listed.

In Chapter III a theory will be developed which treats the designating relation in the second manner. At least some form of this theory has been held by Bergmann. According to it, mental contents (propositions) designate facts and no two propositions designate the same fact. In Chapter III I also allude to a theory which I call "the deviant theory." According to it, a proposition is true if there is a fact which the proposition designates and is false if there is nothing which the proposition designates. To accept this theory, one must treat the designating relation in the first manner.

In the discussion of Frege below I will claim that Frege held a theory which is based on schema P-T' rather than on T'.
There is even a greater proliferation of kinds of theories which are based on schema P-T' given the three interpretations of the designating relation which were mentioned. You will recall that for those theories which are based on schema P-T' the parts of propositions are the natural signs or designators of entities. Thus, one may claim that one part designates \(_1\) (i.e., there may not be an entity which it designates), whereas another part may designate \(_2\) (i.e., there cannot be two such parts which designate the same entity but there is one entity which this part designates), and so on. If all propositions had just two parts, there would be exactly nine kinds of theories based on schema P-T' given the three interpretations of the designating relationship. But if there is a large number of constituents of propositions, as some philosophers have contended, it is clear that the number of alternative theories based on schema P-T' even exceeds this number. Of course, I have no intention of even alluding to all of the different varieties of theories in the succeeding chapters. But I have selected some representative theories for development. In Chapter II, in which I discuss Frege, I contend that the "saturated" parts of Frege's "atomic propositions" designate \(_1\) whereas the unsaturated parts of Frege's propositions designate \(_2\). In Chapter IV a theory which occurred to me after reading Wittgenstein's *Tractatus Logico-Philosophicus*\(^{12}\) will be developed according to

which all of the parts designate. According to the theory developed in Chapter V, Ockham's theory, none of the parts of propositions designate.

Let us consider some examples in order to illustrate the three types of designators. If we grant the existence of distinct mental contents corresponding to "The Morning Star" and the "Evening Star" and claim they must designate something and do get at the same thing, then we are treating the mental content corresponding to "The Morning Star" as a designator. Someone who contends that the mental content corresponding to "The Morning Star" must designate something and furthermore contends that the content corresponding to "The Evening Star" must be identical to the content corresponding to "The Morning Star," since they designate the same thing, would be treating this designator as being of type 2.

Suppose we grant that there is a mental content corresponding to "The last rapidly convergent series has a limit" and that there is a part of this mental content which corresponds to "The least rapidly convergent series." If we make the plausible assumption that there is nothing which this part designates, then we are treating this part as a designator of type 1.

Unlike the theories which are based on schemas T' and P-T', there is no designating relation which is involved in those theories which are based on schema T'''. Schema T''' is stated as follows:

(T''') X is true only if X is a part of Y.
Just as sentences and sounds per se cannot designate, sentences and sounds per se cannot be a part of anything in any manner that can be relevant to their truth. A sentence can be a part of a collection of sentences, but to say that if it is true then it is a part of a particular collection of sentences is a claim that is indefensible. But if we attend to mental contents rather than to sentences or sounds, a reasonable theory of truth can be developed which is based on schema T'''. In Chapter I, I will develop such a theory and ascribe it to Bradley.

Though Bradley's mental contents (propositions) do not designate, there are still grounds for comparing and contrasting his mental contents with those which do designate. In particular, Bradley's propositions, like Bergmann's, are simple, but are unlike Frege's propositions which are complex in structure.

Only a limited collection of propositions will be discussed in the opening chapter and in the succeeding chapters. I will not deal with particular and universal propositions--propositions "corresponding to" sentences of the form "Some X are . . ." or "All X are . . ." Furthermore, there will be no discussion of intensional propositions--propositions "corresponding to" intensional sentences such as "X believes that . . .," "X doubts that . . .," and "It is

13. The phrase "corresponding to" is put in quotation marks to indicate that the correspondence alluded to is a three-term relation and not a two-term relation. Nothing (or everything) corresponds to the sentence per se. In Ockham's terminology the sentence is a conventional, not a natural, sign. We associate the sentence with the proposition.
necessarily the case that . . ." I will restrict my attention to the more "manageable" propositions which "correspond to" sentences such as "This is red," "This is to the left of that," and to truth-functional sentences--sentences which are constructed by employing the standard connectives such as "if," "and" and "or." In other words, the theories of truth which will be disclosed will handle what have traditionally been called atomic propositions and those propositions which are such that if one knows the truth-value of them, there is a finite number of atomic propositions concerning which one can make non-trivial pronouncements about their truth-values. (An example of a trivial pronouncement about the truth-values of a finite number of atomic propositions would be the claim that each proposition is either true or false. An example of non-trivial pronouncement about the truth-values of a finite number of atomic propositions would be the claim that at least one of them is true.) From this description of the propositions which will be discussed it is evident that particular, universal and intensional propositions will not be discussed. (Of course, I do not wish to suggest that the developments of theories of truth for such propositions would not be of philosophical interest; quite the contrary.) To see this, consider the propositions "expressed by" "Something is red," "Everything is red," and "I believe that a is red." Suppose that it is true that something is red. Knowing this does not enable one to say anything of a non-trivial nature about the truth-value of a finite collection of atomic propositions. In particular, one cannot say that one of the atomic propositions expressed
by "a is red," "b is red," . . . , "m is red" is true, nor can one say that one of them is false. Of course, if one granted that there is only a finite number of entities in the world, then one could justifiably assert that at least one of a finite collection of atomic propositions is true given that it is true that something is red. But there are good reasons for not assuming that there is a finite number of entities in the world. Classical mathematics rests on the assumption that there is an infinite number of entities. Concerning the universal proposition expressed by "Everything is red," the falsity of this proposition would provide no basis for any non-trivial declarations concerning the truth-values of a finite number of atomic propositions. The likely prospects would be propositions expressed by sentences such as "a is red," "b is red," . . . , "m is red." But granting the reasonable assumption that there is an infinite number of entities, one has no basis for asserting that any of these propositions are false given that the universal proposition in question is false. In the case of the intensional proposition expressed by "I believe that a is red," it is evident that the truth of this proposition provides no basis for asserting that the atomic proposition expressed by "a is red" is true, nor is there any basis for asserting that the atomic proposition is false.

Lest the reader think that only atomic propositions will be discussed, consider the proposition expressed by "a is red and b is red." If this proposition is true, one has grounds for making the non-trivial assertion that the atomic proposition expressed by "a is red"
is true; if this proposition is false, one has grounds for making the non-trivial assertion that at least one of the atomic propositions expressed by "a is red" and "b is red" is false.

Why not delimit the propositions to be discussed by simply saying that only atomic propositions and molecular propositions which have atomic propositions as truth-functional constituents will be discussed? The reason is that I wish to examine accounts of the truth of certain non-atomic propositions which presuppose that these propositions are simple as well as those accounts of the truth of non-atomic propositions which operate on the presupposition that these propositions have atomic propositions as constituents.

The discussion of the theory based on schema T'' which is found in the opening chapter will be followed by a chapter in which I develop a theory based on schema P-T'--a theory which I attribute to Frege. In Chapter III, a theory, some version of which has been held by Bergmann, will be developed. This theory, even though it is based on schema T', is in some respects strikingly close to Frege's. In the first three chapters complex entities play a role in the development of the theories. In Chapter IV, I develop a theory--a theory which I think the Wittgenstein of the Tractatus held at least some version--which attempts to avoid complex entities entirely. The substitute for such entities will be called complexes of entities. But it will be argued that this theory is ultimately unsuccessful. In Chapter V a theory will be outlined which is an interesting alternative to the theories of the preceding three chapters. According to
this theory, the designata of propositions are simple entities rather than complex entities, i.e., facts. But, this theory provides no adequate manner of handling relational propositions and thus must be rejected.

The theories of the first three chapters, unlike those of the last two, are satisfactory in the sense that given the presuppositions of the respective theories, one can lay down the conditions under which what I have called the more manageable propositions have one truth-value as opposed to another. Furthermore, these theories do justice to our intuitions concerning the truth-value relationships which propositions have to one another. For example, one of our intuitions is that if the proposition "expressed by" "a is red" is true it is also the case that the proposition "expressed by" "A is red or b is red" is true. Each of the theories accords with this intuition and similar such intuitions.

I have no intention of determining which of the three satisfactory theories is the correct theory. To argue for the correctness of one of the three theories one would have to give evidence for his affirmative or negative answers to questions such as the following:

"Are there propositions?"

"If there are propositions, are they simple?"

"If there are propositions, is there an intentional nexus which connects propositions and non-propositions?"

I think that the arguments for one's answers to these questions would be of great philosophical interest. But my philosophical interests
lie elsewhere in what follows. I intend to show what kind of theories, if any, can be constructed given the various answers to questions such as the above.

Artificial languages will be constructed for each of the theories which is presented. I think that without using such a device it would be extremely difficult, if not impossible, to give clear portraits of the various theories. The inspiration for using such a device stems to a large extent from an article by Wilfrid Sellars. It was only after reading this article that I was able to get a grasp on Aristotle's puzzling contention that a primary substance consists of matter and form which are "somehow one." Sellars provides a linguistic model of an entity which has parts but does not really have parts. Sellars' symbols which are written in various styles of type are suitable for representing such entities. The fact that a boldface cannot be written without writing some symbol represents the fact that the form of a primary substance is not a separate entity. The fact that a symbol cannot be written without being written in some style of type represents the fact that the matter is not a separate component of the primary substance.

The term "models" has been subjected to a variety of uses. As I am using the term, a model is a heuristic device. Models such as

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the one to which I have briefly alluded provide a means of representing or showing what it is very difficult to say.

Though artificial languages will be used to represent propositions, it does not follow that the theories of truth which involve propositions which are so represented are linguistic. If one claimed that the truth of a proposition is dependent upon the truth of an expression in an artificial language, then his account of the truth of propositions would be linguistic. Although I will be using artificial languages to represent the structure of propositions, I am not contending that the truth of expressions in artificial languages helps explicate the truth of propositions. It is the truth of propositions, not the truth of linguistic expressions which is my concern.
As was mentioned in the introduction, the theory of truth of this chapter is based on schema T' -- it is a non-linguistic coherence theory of truth. The truth-bearer will be called thoughts or propositions. According to the theory, a proposition is true only if it is the content of and thus a part of the world. Since there is only one world, there is only one true thought. But there are, of course, other thoughts. Some of these thoughts are partially true. The major problem the theory must deal with is that of explaining how thoughts can be partially true given that the true thought has no parts. If the one true thought had parts, then one could easily account for the partial truth of thoughts. A partially true thought would be a part of the one true thought. But according to the theory at hand all thoughts, even those expressed by compound sentences involving the connectives "and" and "or," are simple.

Given the grammatical complexity of sentences such as "Jones is tall," it is perhaps most natural to think that the thought which "corresponds to" "Jones is tall" is complex. One may, for example, think of the thought as composed of a part which corresponds to "Jones," and a part which corresponds to "is tall." Or the thought
may be construed as divided into three parts which correspond to "Jones," "is," and "tall." But in order to understand the theory at hand, we must construe all thoughts as simple. There are other linguistic correlates of thoughts than the standard ones which are useful in construing thoughts as simple. For example, consider the expression "a Jones being tall." We can use this expression to allude to the same thought as we use the expression "Jones is tall" to allude to. Yet we are less likely to think of the thought-correlate of "a Jones being tall" as complex. It may be that the non-naturalness of associating "a Jones being tall" with a complex thought stems merely from the fact this phrase is not ordinarily used. But in any case, by using this phrase instead of the standard ones, we may be more likely to think of the corresponding thought as simple. Unless we construe thoughts as simple, it will be impossible to appreciate the theory under consideration.

But what linguistic phrase can aid us in construing the sentence, "The wolf eats the lamb," as having a simple thought-correlate? I would recommend associating the thought with phrases such as "a

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1 There is a good reason for using the phrase "a Jones being tall" instead of "Jones' being tall." The former phrase, unlike the latter, suggests a tenet of the theory, viz., the uniqueness of the application of a thought cannot be built into the thought itself. According to the theory, there is nothing about a thought per se which guarantees that it is a content of (is true of) exactly one complex entity. I will say more about this point in the next section of this chapter.

2 F. H. Bradley discusses the simplicity of the thought corresponding to this sentence on pp. 11-12 of his *The Principles of Logic*, Vol. I (2nd ed; London: Oxford University Press, 1922).
wolf's eating of a lamb" or "a lamb's being eaten by a wolf." Neither of these phrases would be considered as having the standard grammatical parts such as subjects and predicates and thus they hopefully will not lead us to construe the thought which corresponds to them as simple.

Another thought whose simplicity Bradley makes relevant comments concerning is the thought corresponding to "The thing is a man, woman or child." It is helpful to think of this thought as being correlated with "A thing's humanity." The grammar of this phrase should not lead one to partition the corresponding thought.

Of course, there are sentences which do not suggest ready "translations." For example, what translations can be offered for "The thing is red or green," or "The thing is red unless I'm mistaken"? For such sentences, it appears that we must invent "translations," if we are to avoid having grammatical considerations lead us to construe the thoughts envisioned by the theory as being complex in structure. Hyphens are useful for this purpose. Thus, we might give the following "translations":

"A thing's redness-or-greennes."  
"A thing's redness-or-a person's mistakenness."

Though it is not my purpose to determine whether propositions are simple, but rather to develop theories which are based on the presupposition that thoughts are complex, I would like to allude in passing to an argument which Bradley gives for the simplicity of

\[3\text{Ibid.}, \text{p. 130.}\]
thoughts in order to prevent the reader from thinking that a theory of truth which is based on the presupposition that thoughts are simple must be absurd. Bradley writes:

... We have the idea of a wolf and we call that one idea. We imagine the wolf eating the lamb, and we say, There are two ideas, or three or perhaps even more. But is this because the scene is not given as a whole? Most certainly not so. It is because in the whole there exist distinctions, and those groupings of attributes we are accustomed to make. But, if we once start on this line and deny the singleness of every idea which embraces others, we shall find the wolf himself is anything but one. He is the synthesis of a number of attributes, and, in the end, we shall find that no idea will be one which admits any sort of distinction in itself. Choose then which you will say, There are no single ideas, save the ideas of those qualities which are too simple to have any distinguishable aspects, and that means there are no ideas at all -- or, Any content whatever which the mind takes as a whole, however large or however small, however simple or however complex, is one idea, and its manifold relations are embraced in an unity.\textsuperscript{4}

Bradley's argument can be paraphrased as follows: If one construes thoughts as being divisible into parts, then there is no good reason for not also construing the parts of thoughts, the parts of parts of thoughts, the parts of parts of parts of thoughts, ..., as divisible into parts. But then there would be no ultimate or basic constituents of thoughts. A thought without a basic constituent would not be a thought. The basic constituent of a thought is the thought itself. I.e., all thoughts are simple.

Those who reject Bradley's argument would claim that there are proper constituents of thoughts which have no proper parts. So the issue is whether there are such proper constituents of thoughts. I

\textsuperscript{4}Ibid., pp. 11-12.
think the reader will grant that neither answer to the question is absurd, but my purpose is not to settle which view is "correct."\(^5\)

Given that all thoughts are simple, how does one account for the partial truth of thoughts? In order to hint at the solution, let us make the fanciful assumption that the thought corresponding to "a Jones being tall" is the content of the world and is thus, given the theory being developed, the true thought. Though the thoughts corresponding to "a Jones" and "being tall" are not parts of the thought corresponding to "a Jones being tall," they would still be partially true. The thought corresponding to "a Smith," given that Smith is different from Jones, and the thought corresponding to "being short" would not be partially true. The thoughts corresponding to "a Jones" and "being tall" would be partially true since (1) these thoughts are tied to all things to which the thought corresponding to "a Jones being tall" is tied, and (2) these thoughts are tied to things other than those to which the thought corresponding to "a Jones being tall" is tied. I.e., the thoughts corresponding to "a Jones" and "being tall" are "true of" all of those things which the thought corresponding to "a Jones being tall" is true of and other things as well. Given (1) the thought corresponding to "a Jones" is at least partially true; given (2) the thought corresponding to "a Jones" is at most partially true. Now if the world had as its content the thought

\(^5\)I put the term "correct" in quotes since the notion of being philosophically correct is a very technical notion. It would be a mistake, for example, to confuse philosophical correctness with scientific correctness.
corresponding to "a Jones being tall," then this content would be
tied to things to which the content corresponding to "a Smith" and
the content corresponding to "being short" would not be tied. These
two thoughts would be "discrepant" with the world and would be neither
true nor partially true.

What is the thing to which the thought corresponding to "a
Jones being tall" would be tied if this thought were the content of
the world? It would be that which made the world with its content
this world and not some other world with the same content. It is what
I call an individuator. The world thus consists of a content and an
individuator. Partially true thoughts are not contents of the world
but they are contents of worlds other than the world. A thought is
partially true provided it is constituent of a world which has the
individuator of the world as a constituent. Think of an entity cor-
responding to "This" as being the individuator of the world. Now the
claim is that the thought corresponding to "a Jones" is only partially
true since it is tied to at least one individuator in addition to
This to which the thought corresponding to "a Jones being tall" is not
tied.

To account for the partial truth of the thought corresponding
to "a Jones" given that the thought corresponding to "a Jones being
tall" is the content of the world, we need at least two complex enti-
ties in addition to the complex entity which is the world. (The
world is a complex entity since it consists of a content and an in-
dividuator.) These two complex entities and other such complex
entities which have contents and individuators as constituents will be called worlds or facts. Before saying more about partial truth, I would like to say more about contents (thoughts) and individuators. Some symbolism will be introduced to aid the development of the theory.

**Whats and Thats**

Bradley's description of what he calls "real things" fits what I call worlds. Bradley writes, "... anything real has two aspects, existence and character, ..."\(^6\) Something cannot be a real thing unless it is a particular thing, but it must be more than just a particular thing. Pertaining to the world, Bradley states, "The subject [the real thing] ... is never mere reality, or bare existence without character."\(^7\) The world is not a bare particular. But also the world is not to be identified with a mere content or a mere character.

Bradley says, "... the subject [the real thing] ... is neither the mere 'what' of the predicate, nor is it any other mere 'what.'"\(^8\) The real thing, Reality, could be schematized in the following fashion:

\[ R = T + p. \]

'T' denotes the constituent of Reality which makes it what it is. 'p' designates the constituent which makes it a 'that' instead of a mere 'what.' In other words, if p were not a constituent of Reality, 

\(^7\)Ibid., p. 149.
\(^8\)Ibid., p. 148.
Reality would not be individual. The plus-sign signifies that Reality is a union of a 'what' and a 'that.'

Even though T and p are not the real thing, Reality, they have to be given an ontological status. Neither of them can be constituents of Reality unless they are entities.

In order to get a better insight into the roles which T and p play as constituents of Reality, it would be helpful to find something (even though it is not the real thing) in our everyday experience which has constituents which behave like them.

The following remarks of Bradley provide us with such a heuristic device:

In all that is we can distinguish two sides, (i) existence and (ii) content. In other words we perceive that it is and what it is. . . . For unless it has a character which is different or distinguishable from that of other facts. And thus, which makes it what it is, we call its content. We may take as an instance any common perception. The complex of qualities and relations it contains, makes up its content, or that which it is; and, while recognizing this, we recognize also, and in addition, that it is. Every kind of fact must possess these two sides of existence and content, . . ."\(^9\)

Even though there is only one Reality, viz., T + p, we can think of ordinary objects such as particular flowers as being models of this fact. A particular flower is not something which is barren of all qualities. For example, it is yellow and has a green stem. These qualities help make it what it is. They give it a content. But it is also an individual. It is this yellow flower with a green stem rather than that yellow flower with a green stem. Think of T as making

Reality what it is in the way in which you think of the qualities of a particular flower as making it what it is. Think of p as individuating Reality in the manner in which you think of some constituent of the yellow flower as making it this flower.

The fact that Jones is tall also provides a model of Reality. This fact has a content. I.e., there is a quality of being a tall Jones which is a content of this fact. There is nothing about this simple quality which prohibits it from being attached to another entity. So there is an entity in addition to the content of the fact which is a constituent of the fact. This constituent "particularizes" the content. Without such an entity there would be no way of distinguishing between the fact that this Jones is tall and the fact that that Jones is tall provided these are facts. (If one took spatial and temporal attributes as being somehow built into the quality of being a tall Jones, then he could give good reasons for claiming that there could not be two such facts.)

To accept the theory of truth which is being outlined one must claim that there are entities which correspond to the following three expressions: "T," "p," and "T + p." These are not the only entities which there are, according to the theory. But there is no restriction to the effect that there is one number rather than some other number of contents (whats). Likewise, the theory does not rule out a given number from being the number of individuators or the number of worlds which there are.
Let us suppose that there are no more than a denumerable number of contents (thoughts) and let us use the following labels for them: "T," "T_1," "T_2," .... Because of the use of this labeling device, one should not conclude that any serial ordering of all thoughts is possible. We will note in the subsequent discussion that some thoughts are "parts of" other thoughts, but we will also note that it is impossible to give a sequential ordering of all thoughts.

Let us also suppose that there are at most a denumerable number of individuators and label them as follows: "p," "p_1," "p_2," .... Individuators are also not ordered.

The number of worlds which there are is not a function of the number of thoughts and individuators which there are. But there cannot be more worlds than the product of the number of thoughts and the number of individuators. This feature of the theory should not be surprising. It amounts to the claim that there is no way of determining \textit{a priori} how many facts (worlds) there are.

Expressions of the form "X + y" where "X" is replaced by the name of a thought and "y" by the name of an individuator will be used to name worlds. Every expression of this form does not designate an entity. If this were the case, there would be only one true thought.

Those expressions of the form "X + y" which do not designate entities do not designate possible entities. In the theory of truth under discussion, there is no dichotomy between actual and possible entities. Such a dichotomy is not needed. Entities are just
entities. The presuppositions of the theory do not require us to talk
about different "modes of existence" or entities which might be called
the Actual and the Possible. In contrast, the theory developed in
Chapter III will require the existence of such entities. The theory
developed in Chapter II requires two entities which function like
the two special entities required for the theory of Chapter III.

**Partially True Thoughts**

Bradley writes about thoughts as follows:

... an idea, if we use idea of the meaning, is neither
given nor presented but is taken. It cannot as such exist.
It cannot ever be an event, with a place in the series of
time or space. It can be a fact no more inside our heads
than it can outside them. And, if you take this mere
idea by itself, it is an adjective divorced, a parasite cut
loose, a spirit without a body seeking rest in another, an
abstraction from the concrete, a mere possibility which by
itself is nothing.¹⁰

I construe Bradley to be saying that thoughts are not real things,
i.e., they are not entities which can be designated by expressions of
the form "X + y," but not that they are not entities. If Bradley held
the view that thoughts have no ontological status, he certainly would
not have tried so hard to describe them.

Concerning the partial truth of thoughts Bradley writes:

... to be more or less true, and to be more or less
real, is to be separated by an interval, smaller or
greater, from all-inclusiveness of self-consistency. Of
two given appearances the one more wide, or more harmoni-
ous, is more real. It approaches nearer to a single, all-
containing, individuality. To remedy its imperfections,
in other words, we should have to make a smaller altera-
tion. The truth and the fact, which, to be converted into

¹⁰Ibid., pp. 7-8.
the Absolute, would require less rearrangement and addition, is more real and truer. To possess more the character of reality, and to contain within oneself a greater amount of the real, are two expressions for the same thing.11

Bradley's poetic language is suggestive of what it is for a thought to have a degree of truth. In order to attempt a more rigorous formulation of the notion of the partial truth of thoughts, given that thoughts are simple, let us invoke the symbolism which has been introduced.

Let "T + p" represent Reality. T would then be the true thought since it would be the content of Reality. Suppose the thought $T_1$, which is different from T, is a constituent of two worlds which are represented by "$T_1 + p" and "$T_1 + p_1." Then the thought $T_1$ is partially true. In general, if T is the content of Reality and the set whose members are the individuators of the worlds of which T is a constituent is included in the set containing the individuators of the worlds of which a thought $T_1$ is a constituent, then $T_1$ is partially true.

To illustrate this notion suppose that there is a blue book on my table and another blue object on my table. There would be three facts (worlds): the fact that this is a blue object on my table $(T + p)$, the fact that this is a blue object on my table $(T_1 + p)$, and the fact that that is a blue object on my table $(T_1 + p_1)$. Given that the thought T expressed by "being a blue book on a person's table" is true, then the thought $T_1$ expressed by "being a blue object on a person's table" would be partially true.

11 Bradley, Appearance and Reality, pp. 322-23.
Consider another example. Suppose that it is a fact that there are two wolves, one of which is eating a lamb. There would be three facts: the fact that this wolf is eating a lamb \((T_2 + p_3)\), the fact that this is a wolf \((T_3 + p_4)\), and the fact that that is a wolf \((T_3 + p_5)\). Suppose that the world is limited in content to the thought expressed by "a lamb being eaten by a wolf." Then this thought \(T_2\) would be true and the thought \(T_3\) would be partially true.

To further illustrate the criterion for partial truth, suppose that all of the worlds are given by the following formulae:

\[
W^T = T + p
\]

\[
W^T_1 = T_1 + p
\]

\[
W^T_3 = T_1 + p_3
\]

\[
W^T_2 = T_2 + p_2
\]

\[
W^T_4 = T_4 + p
\]

\[
W^T_5 = T_4 + p_5
\]

\[
W^T_6 = T_4 + p_6
\]

Suppose that \(W^T\) is Reality. Then the content \(T\) would be the true thought. All of the worlds are distinct from each other, since no two worlds have all of their constituents in common. The thought \(T_1\) is a constituent of two worlds. Since the set whose member is \(p\) is included in the set whose members are \(p\) and \(p_3\), the thought \(T_1\) is partially true. The thought \(T_2\) is a constituent of one world. Since
the set whose member is \( p \) is not included in the set whose member is \( p_2 \), the thought \( T_2 \) is not partially true. One can easily determine that the thought \( T_4 \) must be partially true given the criterion of partial truth.

Notice that this account of partial truth in no way requires that thoughts be complex. Rather than appealing to the notion that thoughts have parts in order to account for partial truth, we appeal to the notion that there is a variety of worlds (facts).

If two thoughts are constituents of worlds which have identical individuators, it does not follow that the thoughts are identical. What does follow is that the thoughts, if partially true, have the same degree of partial truth. Suppose it were the case that all green things were squares and all squares were green things. Let us suppose that the thought \( T_1 \) is expressed by "being a green thing" and the thought \( T_2 \) is expressed by "being a square." Suppose, for the sake of simplicity, that there are only five green things. These five green things could be represented by "\( T_1 + p \)," "\( T_1 + p_2 \)," "\( T_1 + p_4 \)," "\( T_1 + p_6 \)," and "\( T_1 + p_8 \)." Given the hypotheses, the five square things would be represented by "\( T_2 + p \)," "\( T_2 + p_2 \)," "\( T_2 + p_4 \)," "\( T_2 + p_6 \)," and "\( T_2 + p_8 \)." Given the existence of these worlds we are in no way committed to maintaining that \( T_1 \) and \( T_2 \) are identical. They are not. But we are committed to the claim that \( T_1 \) and \( T_2 \) have the same degree of partial truth, given that they are partially true.
Neither True nor Partially True Thoughts

The thoughts which are neither true nor partially true are either "partially rejected" or "completely rejected" by the true thought T. In order to get a rough analogue of the notion of partial and complete rejection, let us consider the thoughts corresponding to the following phrases: "being male," "being a bachelor," and "being female." The second thought would be partially rejected by the first whereas the third thought would be completely rejected by the first.

In order to give a rigorous formulation of the notion of being neither true nor partially true, let us again refer to the symbolism. Suppose there are three men, two bachelors, and one female. Suppose that A, B, and C are men, A and C are bachelors, and D is a female. There would be the following facts (worlds): the fact that A is a man, the fact that B is a man, the fact that C is a man, the fact that A is a bachelor, the fact that C is a bachelor, and the fact that D is a female. These worlds could be represented, respectively, by:

"$T_2 + p_2$," "$T_2 + p_3$," "$T_2 + p_5$," "$T_4 + p_2$," "$T_4 + p_5$," and "$T_5 + p_7$." $T_2$ is the thought expressed by "being a man"; $T_4$ is the thought expressed by "being a bachelor"; $T_5$ is the thought expressed by "being female." $p_2$, $p_3$, and $p_5$ individuate the three men; $p_2$ and $p_5$ individuate the two bachelors; $p_7$ individuates the female. Since the set whose members are $p_2$, $p_3$, and $p_5$ is neither included in the set whose members are $p_2$ and $p_5$ nor the set whose member is $p_7$, neither the thought expressed by "being a bachelor" nor the thought expressed by "being female" is partially true given that the thought expressed
by "being male" is true. But the thought, being a bachelor, is not in the same category as the thought, being female, relative to the thought, being male. The intersection of the set whose members are \( p_2, p_3, \) and \( p_5 \) and the set whose members are \( p_2 \) and \( p_5 \) is non-empty whereas the intersection of the former set with the set whose member is \( p_7 \) is empty. For this reason we will say that the thought, being a bachelor, is partially rejected and the thought, being a female, is completely rejected by the thought, being a male.

To further illustrate the conditions under which thoughts are neither true nor partially true, let us imagine that all of the worlds, including Reality as the first one represented on the following list, are represented by this list of expressions:

\[
\begin{align*}
\text{"WT} &= T + p \\
\text{"W}_1^T &= T_1 + p_2 \\
\text{"W}_3^T &= T_1 + p_3 \\
\text{"W}_2^T &= T_2 + p_3 \\
\text{"W}_3^T &= T_3 + p_2 \\
\text{"W}_4^T &= T_4 + p_5
\end{align*}
\]

Given that \( T \) is the content of Reality and is thus the true thought, we can assert that \( T_1 \) is neither true nor partially true (the set whose member is \( p \) is not included in the set whose members are \( p_2 \) and \( p_3 \)), that \( T_2 \) is neither true nor partially true (the set whose member is \( p \) is not included in the set whose member is \( p_3 \)), and that neither \( T_3 \)
nor $T_4$ is true or partially true. Suppose that $W^T_1$ were Reality instead of $W^T$. Then $T_1$ would be the true thought. $T$, $T_2$, $T_3$, and $T_4$ would be neither true nor partially true, but $T$ and $T_4$ would be completely rejected by $T_1$ whereas $T_2$ and $T_3$ would be partially rejected by $T_1$. If $W^T_2$ were Reality, then $T_1$ would be partially true, and $T$, $T_2$, and $T_4$ would be neither true nor partially true. $T$, $T_2$, and $T_4$ would be completely rejected by $T_3$.

Consider another example. Compare the thoughts expressed by "A wolf's eating of a lamb," "A timber wolf's eating of a lamb," and "A wolf's sleeping." The second thought is partially rejected by the first, but the third thought, in the order listed, is completely rejected by the first. If the first thought is true, neither the second nor the third is partially true. The idea governing the criterion is simply that if there are more instances of the true thought than of another thought, the latter thought cannot be partially true, and if no instances of the true thought are instances of another thought, then the latter thought cannot be partially true. We can easily imagine that there are more instances of a lamb being eaten by a wolf than there are instances of a lamb being eaten by a timber wolf. We can also easily imagine that there are no instances of a lamb being eaten by a wolf which are also instances of the same wolf being asleep.

Although Bradley does not make the distinction between partial and complete rejection we have just made, his comments about rejection in general may be helpful. Bradley writes:
Every negation must have a ground, and this ground is positive. It is that quality \( x \) in the subject which is incompatible with the suggested idea. \( A \) is not \( B \) because \( A \) is such that, if it were \( B \), it would cease to be itself. Its quality would be altered if it accepted \( B \); and it is by virtue of this quality, which \( B \) would destroy, that \( A \) maintains itself and rejects the suggestion. In other words its quality \( x \) and \( B \) are discrepant. And we cannot deny \( B \) without affirming in \( A \) the pre-existence of this discrepant quality.\(^2\)

The quality \( x \) to which Bradley refers is the thought \( T \). The subject \( A \) is the world whose constituents are \( T \) and \( p \), viz., Reality. "\( B \)" might be interpreted in two ways. It may be construed as being the name of a particular thought \( T_2 \) (say) or the particular world \( W_2^T \) (say). If it is interpreted in the former way, then since "\( A \)" stands for a world (a fact) one wonders why Bradley chose another capital letter to stand for a thought. But if \( B \) is identical with the world \( W_2^T \), then Bradley is saying that a thought and a world are discrepant rather than saying two thoughts are discrepant. There would be no real problem in speaking in this fashion. We can say that the quality \( T \) rejects the world \( W_2^T \) provided that the quality \( T \) rejects the quality \( T_2 \). If the quality \( T \) rejects the quality \( T_2 \), then the quality \( T \) and the world \( W_2^T \) are "discrepant."

Thoughts Corresponding to Compound Sentences

The simplicity of all thoughts may be as surprising a feature of the theory as any feature of the theory which has been mentioned to this point, since it may be very natural to think that the expressions, "the thought that it is not the case that \( A \)" and "the thought

that either A or B," for example, must refer to entities which are composed of other thoughts. Of course, if there were one thought which had another thought as a constituent, then one could not maintain the thesis that thoughts are simple. Though there are many complex expressions which could be dealt with, I will only make the effort to show that the following expressions do not stand for complex thoughts: "the thought that it is not the case that T," "the thought that either A or B," and "the thought that A and B."

The treatment which is given by the theory to the expression "the thought that it is not the case that T" can be focused on by considering the following passage:

It is impossible for anything to be only Not-A. It is impossible to realize Not-A in thought. . . .

Not-A must be more than a bare negation. It must also be positive. It is a general name for any quality which, when you make it a predicate of A, or joint predicate with A, removes A from existence. The contradictory idea is the universal idea of the discrepant or contrary. In this form it must keep its place in logic. It is a general name for any hypothetical discrepant; but we must never for a moment allow ourselves to think of it as the collection of discrepants.13

The expression "the thought that it is not the case that T" does not stand for any particular thought. It is a variable which ranges over those thoughts which are neither true nor partially true given that the thought that T is true. That is, it ranges over those thoughts which are either partially or completely rejected by the thought T. Although the range of the expression "the thought that it is not the

13 Ibid., p. 123.
case that T" is relative to T, the thoughts which are within the range of the expression do not have T as a constituent.

If the expression "the thought that it is not the case that T" stood for a collection of thoughts, then one could not insist that thoughts are simple. For there would be a thought which had thoughts as constituent entities, viz., the members of the collection. But as Bradley states in the last sentence of the above quotation, the expression does not have a collection of discrepants as a referent.

To illustrate the role which the expression "the thought that it is not the case that T" plays, consider the following example:
The only entities are T, T_1, T_2, p, p_1, p_2, T + p, T + p_1, T + p_2, T_1 + p, T_1 + p_1, T_2 + p, and T_2 + p_2. Both T_1 and T_2 are rejected (partially) by T. The expression "the thought that it is not the case that T" is a variable which ranges over T_1 and T_2. (T_1 and T_2 are neither true nor partially true.)

According to the theory, when one is thinking that it is not the case that a wolf is eating a lamb, he is not entertaining a single thought. Rather he is entertaining several thoughts such as those expressed by phrases such as "a wolf's being asleep" and "a lamb being eaten by a timber wolf." Though there is no single simple thought which corresponds to the expression "a lamb is not being eaten by a wolf," there are several simple thoughts which correspond to the expression. According to the theory, there are no thoughts which have "negative constituents." By rejecting the notion that thoughts are complex, one rejects the notion that thoughts have
negative constituents. In the next chapter, a theory will be developed according to which thoughts have negative constituents.

The following remarks will be helpful in explaining the role which the expression "the thought that either A or B" plays:

The assertion in "A is b or c" is not that A is b or c. What then do we affirm? We say in the first place that A exists. In the next place we certainly give it some quality. What quality do we give it? If it cannot be either b or c, can it possibly be something that falls between them? No, for that would be neither. For instance, grey is not white or black, and it excludes both colours. The predicate of A, while neither b nor c, must not be a quality exclusive of either. It must then be a quality common to both, which is not yet either, but is further determinable as one or the other.

If we like to call this basis x, then "A is x" is categorically true. We may in some cases have distinguished x and given it a name, but in other cases it is unnamed and implicit. "Man, woman, and child," have a common basis in "human being." In "white or black" the quality "coloured, and coloured so as to exclude other hues," is the attribute asserted. . . . So, in every disjunction and as the ground of it, there must be the assertion of a common quality, the sphere within which the disjunction is affirmed.

But x is not any universal whatever which happens to be common to b and c. It is particularized further. It excludes the opposite of each of these qualities, and cannot be the negative of "b or c." It is affirmed as fully determined not outside the region which is covered by bc. But since b and c, as predicates of A, are incompatible, it cannot be both of them. The conclusion remains that it must be one. "One single element of the region enclosed by bc" is the predicate common to b and c. And this predicate it is which, in disjunction, we categorically assert of A.14

To illustrate the manner in which the truth of those simple thoughts which correspond to expressions such as "being a man, woman, or child" is handled by the theory, let us suppose that there are

14Ibid., pp. 130-31.
two men, two women, and a child. There would be the following ten facts: the fact that A is a man, the fact that B is a man, the fact that C is a woman, the fact that D is a woman, the fact that E is a child, and the five facts that A through E are human beings. Let us represent these ten facts by: \( T_1 + P_1 \), \( T_1 + P_2 \), \( T_2 + P_3 \), \( T_2 + P_4 \), \( T_3 + P_5 \), \( T_4 + P_1 \), \( T_4 + P_2 \), \( T_4 + P_3 \), \( T_4 + P_4 \), and \( T_4 + P_5 \) respectively. If we suppose that \( T_1 \) is true, then \( T_4 \) is partially true since the set whose members are \( P_1 \) and \( P_2 \) is included in the set whose members are \( P_1 \) through \( P_5 \). If we suppose that \( T_2 \) is true, as it would be if \( T_2 + P_3 \) were reality, then \( T_4 \) would be partially true since the set whose members are \( P_3 \) and \( P_4 \) is included in the set whose members are \( P_1 \) through \( P_5 \). If \( T_3 \) were true, then again \( T_4 \) would be partially true. Of course, if \( T_4 \) were true, it would not follow that either \( T_1 \), \( T_2 \), or \( T_3 \) would be partially true. This is as it should be. We should not be able to infer that someone is a man given that he is a human being. Likewise, the inferences to being a child and being a man from being a human being should not be and are not justified. The converse inferences are and should be justified.

According to the theory at hand, all thoughts are simple. Those who claim that thoughts are constituted by other thoughts must have different "levels" of thoughts. In Chapter II we will examine a theory which requires thoughts to exist on various levels. Bradley, for one, tries to avoid this inelegance. According to him, all thoughts occur on the same level. But it seems to be a universal rule
that one elegance requires another inelegance. Bradley provides no exception to this apparent law.

In our discussion of worlds, only worlds which just are have been recognized. In addition to the worlds which just are, there are other worlds which must be. These worlds are dependent upon the worlds which just are. It is by means of this inelegance that the theory can account for the fact that the thought that X or Y must be partially true if either the thought that X or the thought that Y is true or partially true.

To show how this works, suppose that only the following worlds just are:

\[ W^T = T + p \]
\[ W^T_1 = T_1 + p \]
\[ W^T_2 = T_2 + p \]
\[ W^T_2 = T_2 + p_1 \]
\[ W^T_2 = T_2 + p_2 \]

Call the first world a T-world, Reality, the next two worlds T\(_1\)-worlds, and the last three worlds T\(_2\)-worlds. The worlds which are required in order to account for the partial truth of the simple thought referred to by expression of "the thought that T\(_1\) or T\(_2\)" are dependent upon these worlds. The worlds which must be since there are T\(_1\)-worlds and T\(_2\)-worlds will be called T\(_1\)/T\(_2\) worlds. (The worlds dependent upon the
existence of $T_2$-worlds and $T/T_1$-worlds will be called $T_2/T/T_1$-worlds
and so on.)

Let us consider the structure of some of these worlds which
must be. The $T_1/T_2$-worlds are representable by means of the following
formulae:

\[ W_{T_1/T_2} = T_1/T_2 + p \]
\[ W^1_{T_1/T_2} = T_1/T_2 + p_1 \]
\[ W^2_{T_1/T_2} = T_1/T_2 + p_2 \]

The following are $T/T_1/T_2$-worlds:

\[ W^{T/T_1/T_2} = T/T_1/T_2 + p \]
\[ W^1_{T/T_1/T_2} = T/T_1/T_2 + p_1 \]
\[ W^2_{T/T_1/T_2} = T/T_1/T_2 + p_2 \]

There are as many $T_1/T_2$-worlds as there are distinct individuators of
the $T_1$-worlds and of the $T_2$-worlds. There are as many $T/T_1/T_2$-worlds
as there are distinct individuators of the $T$-worlds and the $T_1/T_2$-
worlds.

It is a principle of the theory that thoughts which are linked
to exactly the same individuators have the same degree of truth.
(I.e., they are both partially true to the same degree or are both
neither true nor partially true.) Since $T_1/T_2$ is a content of worlds
with the same individuators as the individuators of the worlds whose
contents are $T_2/T_1$, it follows that these contents have the same degree
of truth. It is also the case, for example, that $T/T_1/T_2$ and $T_1/T_2$ are identical in truth-value.

Notice that the thought that $T_1$ or $T_2$ is partially true given that $T_1$ is partially true or $T_2$ is partially true. This is the case since the set whose members are $p$ and $p_1$ is included in the set whose members are $p$, $p_1$, and $p_2$, and the set whose members are $p$, $p_1$, and $p_2$ is included in the set whose members are $p$, $p_1$, and $p_2$.

The theory of this chapter can be summarized as follows:

1) There is one truth thought which is true in virtue of being a constituent of the one fact which is Reality; 2) Thoughts are partially true or false depending upon the facts (worlds) of which they are constituents. According to the theories of the succeeding chapters, thoughts are not parts of entities but designate entities. Furthermore, thoughts will not have truth-values other than truth and falsity. No notion of partial truth will be involved in the remaining theories.
Frege's account of truth can be roughly summarized as follows:

1) A proposition\(^1\) is true if and only if there is a fact designated by this proposition which has the object the True as a constituent, and

2) A proposition is false if and only if there is a fact designated by this proposition which has the object the False as a constituent.

In this chapter I will enlarge upon and make more explicit the above two claims.\(^2\)

**Propositions**

Unlike Bradley, Frege claims that thoughts are composed of entities. According to Frege, no thought is simple. But all of the thoughts discussed by Frege do not have the same degree of complexity.

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\(^1\)In what follows I will use the terms "proposition" and "thought" interchangeably.

\(^2\)I am more interested in the account of facts and propositions which will be given than in proving that Frege actually gave this account. Yet the references to Frege will provide some evidence that he gave the account which is ascribed to him.
The grammatical complexity of sentences makes Frege's view that propositions are complex more natural than Bradley's view that propositions are simple. You will recall that we claimed that it is very natural to think of the thought expressed by "Jones is tall" as complex given that the correlated linguistic expression is divided into parts.

The alternative ways of dividing the sentence into parts suggest alternative ways of dividing the corresponding thought into parts. The following two ways are most frequently suggested:

1. "Jones" is a part, "is" is a part, and "tall" is a part, and
2. "Jones" is a part and "is tall" is a part. If one assumes that the notation of the standard predicate calculus depicts the correct grammatical structure of sentences, then he is granting that the first way of viewing the grammatical complexity of sentences is the correct one. This is so since "Jones is tall" is represented by an expression such as "F(a)" which has the three parts "F," "a," and "( )" which correspond to "tall," "Jones," and "is," respectively. One who thinks that the predicate calculus reflects the correct structure of sentences and thinks that the structure of sentences reflects the structure of thoughts would posit three proper propositional constituents of the thought which corresponds to "Jones is tall."

In Chapter V, I will examine a theory of truth which is based on the claim that the structure of propositions is revealed by the structure of sentences in the predicate calculus. In this chapter
a theory will be developed according to which the structure of the thought corresponding to "Jones is tall" is reflected by the second "natural" way mentioned for dividing sentences into parts. Thus there are two propositional constituents of the thought corresponding to "Jones is tall" instead of three, according to this theory. The part corresponding to "Jones" will be called a saturated constituent; the part corresponding to "is tall" will be called an unsaturated constituent.

It is thus clear that we cannot use the predicate calculus to adequately reflect the structure of thoughts envisioned by the theory of this chapter. In particular the language which will be used to represent the structure of the thoughts of the theory of this chapter will not employ parentheses. The notation will bear similarities to what is called the Polish notation which avoids the use of parentheses.

Because it is natural to think that sentences have parts and to think that the structure of sentences reflects the structure of propositions, it does not follow that propositions are complex. A Bradley would be the first to point this out. Also because it is a matter of "common sense" that a single word such as "Jones" is not true and it is natural to think that there is a thought corresponding to this word, it does not follow that there is a thought
which is not true. One who states that there is no true thought corresponding to this term would entertain the view that only complex thoughts have truth-values; one like Bradley would claim that any thought has a truth-value.

Though no air-tight arguments have been advanced for the claim that propositions are complex, I think that the reader will grant that the assumption that thoughts are complex is both common and plausible. My purpose in this chapter will be that of constructing a theory of the truth of propositions given that it is granted that propositions are complex.

Atomic propositions

If a thought T has no thought distinct from T as a constituent, then T will be called an atomic thought (proposition). If there is a thought distinct from T which is a constituent of T, then T is not an atomic thought. According to Frege there are propositions of both types.

Though atomic propositions do not have thoughts as constituents, they have constituents nonetheless. These constituents are of two types. To use Frege's terminology, a constituent of an atomic proposition is either "saturated" or "unsaturated." Though there are many saturated constituents of atomic propositions, none of these differ in type from one another. (Two entities differ in type if and

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3There are cases where it is natural to say that a single word is true. Imagine the following dialogue: A. "Who is guilty?" B. "Jones." It would be natural to say that what Jones said was true even though Jones uttered a single word.
only if the result of replacing the occurrence of one constituent of a proposition by another entity would not yield a proposition.) Only the following expressions will be used to label at least some of these constituents: "a_1^p," "a_2^p," "a_3^p," . . . . These constituents are at least similar to what Carnap calls individual concepts. The only difference between a propositional constituent such as a_1^p and one of Carnap's individual concepts is that there must be exactly one thing which an individual concept is "about." (An individual concept is a concept such as the concept of Sir Walter Scott.) But there is no guarantee that there is exactly one thing which a_1^p is "about." Given this difference, Carnap's propositions are always either true or false whereas there are Fregean propositions which are neither true nor false. More will be said about this point after the notion of being about is discussed in some detail.

Unlike saturated components, the unsaturated components of atomic propositions do fall into different categories. One unsaturated constituent differs in type from another if one is a constituent of an atomic proposition having m saturated constituents and the other is a constituent of a proposition having more or less than m constituents. If an unsaturated element is a constituent of a proposition which has m saturated constituents, it cannot be a constituent

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4Throughout the discussion the superscript "p" will be used to form symbols which designate propositional constituents. It is a mnemonic device. One should not assume that the propositional constituents are complex since complex labels are being employed.

of a proposition which has other than m constituents. An unsaturated constituent of an atomic proposition will be called: unsaturated_1 if there is exactly one saturated constituent of the atomic proposition, unsaturated_2 if there are exactly two saturated constituents of the atomic proposition, unsaturated_3 if there are exactly three constituents of the proposition, and so on. There must be exactly one unsaturated constituent of an atomic proposition, but no upper bound is placed on the number of saturated constituents which an atomic proposition can have. But there is an upper bound placed on the number of saturated constituents a proposition can have given the unsaturated constituent of the proposition.

I shall use the following expressions to label unsaturated constituents of atomic propositions:

\[ _1^U ' _1, _1^U ' _2, _1^U ' _3, \ldots \]
\[ _2^U ' _1, _2^U ' _2, _2^U ' _3, \ldots \]
\[ _3^U ' _1, _3^U ' _2, _3^U ' _3, \ldots \]

The expressions in the second row stand for the unsaturated_2 constituents of atomic propositions. "\[ _{10}^U ' _5 \]" stands for a specific unsaturated_10 constituent of an atomic proposition. The left-hand subscript indicates the degree of unsaturatedness of the unsaturated constituent. The propositional constituent corresponding to "is red"
would be unsaturated, and would be labeled by an expression from the first row. The propositional constituent corresponding to "is between" would be labeled by an expression from the third row.

Why not simply represent the propositional constituent corresponding to "is red" by a standard symbol from the first order calculus such as "F_1" instead of using the above more cumbersome notation? There are two good reasons for not doing so. In the first place, the notation "F_1" gives us no clue concerning whether it stands for a propositional constituent or the constituent of a fact. The superscript "p" clears up this ambiguity. In the second place, the standard translation of "a is red" into the predicate calculus as "F_1(a)" indicates that "F_1" does not represent a propositional constituent corresponding to "is red" but it represents a propositional constituent corresponding to "red" or perhaps "redness." The parentheses evidently correspond to "is." Of course, one could claim that though "F_1" does not stand for "is red," "F_1( )" does. My only complaint about using this notation would be that it does not adequately represent the simplicity of the unsaturated constituents which are represented by the above more cumbersome notation. For example, if we used "F_1( )" and "G_1( )" to stand for two unsaturated constituents, the notation could easily mislead one to think that the two unsaturated constituents shared a constituent represented by the parentheses since the parentheses occur in both of the representatives. As long as one keeps in mind that the unsaturated constituents are simple and
that symbols such as "F_3( , , )" stand for propositional constituents, there is no harm in thinking of this symbol as doing the same job as "\( 3 \mathcal{U}_1 \)," for example.

Atomic propositions will be labeled by expressions of the form "\( \mathcal{U}^{p}_{i \ j \ k_1 \ k_2} a^p_{a \ a \ a \ a} \)" provided that \( i, j, k_1, k_2, \ldots \) are equal to 1, 2; 3, . . . . Thus, "\( \mathcal{U}^{p}_{1 \ 6 \ a \ a} \)" and "\( \mathcal{U}^{p}_{2 \ 2 \ a \ a} \)" are names of atomic propositions.

Frege remarks:

... not all the parts of a thought can be complete; at least one must be 'unsaturated,' or predicative; otherwise they would not hold together. For example, the sense of the phrase 'the number 2' does not hold together with that of the expression 'the concept prime number' without a link. We apply such a link in the sentence 'the number 2 falls under the concept prime number'; it is contained in the words 'falls under,' which need to be completed in two ways--by a subject and an accusative; and only because their sense is thus 'unsaturated' are they capable of serving as a link. Only when they have been supplemented in this twofold respect do we get a complete sense, a thought.⁶

Frege's discussion centers around an atomic proposition which has three constituents: two saturated constituents which can be represented in our notation by "\( a^p_1 \)" and "\( a^p_2 \)" and an unsaturated constituent which can be represented in our notation by "\( \mathcal{U}^{p}_{2 \ 1} \)." The entire proposition would then be represented by "\( \mathcal{U}^{p}_{2 \ 1} a^p_1 a^p_2 \)." The order in which the expressions occur makes a difference. "\( a^p_1 \)" stands for the sense of the phrase "the number 2."

The proposition which is expressed by the sentence "Vallachi is a philosopher" has two constituents. The saturated constituent and the unsaturated constituent can be represented in our notation by \( a_4^P \) and \( U_2^P \) respectively. The entire atomic proposition would be represented by \( U_2^P a_4^P \).

The sentence "Vallachi is a philosopher" is not identical with the referent of the expression \( U_2^P a_4^P \). Moreover, the atomic proposition exists independently of physical entities such as linguistic marks. But to write and talk about atomic propositions we must employ physical objects. Yet the things talked about do not depend upon physical objects for their existence. (There are or at least can be thoughts which are not named.)

Frege uses expressions of the form "the sense of \( x \)," where \( x \) is a proper name of an object, to refer to the saturated constituents of atomic propositions. Two features of this device are misleading. Firstly, it may suggest that Frege wishes to limit the number of unsaturated constituents of atomic propositions to the number of proper names of objects which there are. But Frege does not mean to impose any such limit. Secondly, the labeling device suggests that the number of saturated constituents of propositions is limited to the number of objects which can receive proper names. Frege does not intend this either. (In the terminology of the introduction, the saturated constituents of Frege's atomic propositions designate.) Frege writes:

The words 'the celestial body most distant from the Earth' have a sense, but it is very doubtful if they also have a reference. The expression 'the least
rapidly convergent series' has a sense but demonstrably has no reference, since for every given convergent series, another convergent, but less rapidly convergent, series can be found. In grasping a sense, one is not certainly assured of a reference. 7

Frege is not saying that only expressions of the form "the sense of x," where x is a proper name of something, refer to the saturated constituents of atomic propositions. By employing the symbols "a^P_1," "a^P_2," . . . to refer to these constituents of atomic propositions, there seems to be no danger of being mislead in either of these ways.

Molecular Propositions

What I shall call molecular propositions are among those propositions which are not atomic propositions. There are propositions which are not atomic and also not molecular. For example, the proposition which is the sense of the sentence "I believe that the least rapidly convergent series has a limit" is neither atomic nor molecular. My discussion of propositions will be restricted to atomic and molecular propositions.

Molecular propositions

A molecular proposition has an atomic proposition which is not identical to the molecular proposition as a constituent. In other words, a molecular proposition has an atomic proposition as a proper constituent. No atomic proposition has an atomic proposition

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as a proper constituent. The only saturated constituents which are proper constituents of molecular propositions are atomic propositions and the saturated constituents of atomic propositions.

Molecular propositions have unsaturated constituents which are not the constituents of any atomic propositions. Such an unsaturated constituent will be said to be unsaturated if it is a constituent of molecular propositions which have exactly one atomic proposition as a constituent. Where there are exactly two atomic propositions which are the constituents of a molecular proposition, the unsaturated constituent which is a constituent of neither of the atomic propositions will be said to be unsaturated.

There are four unsaturated constituents which are not the constituents of any atomic proposition but are constituents of molecular propositions. I label them as follows:

"1_1^P," "2_1^P," "3_1^P," and "4_1^P."

These four constituents are the only "logical constituents" of propositions which are unary in character.

The sixteen unsaturated entities which are constituents of molecular propositions but are not constituents of any atomic propositions will be referred to by the following symbols:

"1_2^P," "2_2^P," "3_2^P," . . . "16_2^P."

These are all of the "logical constituents" of propositions which are binary in character. All "logical constituents" are either unary or binary.
Why not use standard symbols such as "¬" and "∨" to represent the entities represented by "1^p" and "2^p," respectively? The same two reasons for not using "F_1" instead of "1^P" can be given for not using "¬" and "∨" together with a third. There is nothing about the symbols "¬" and "∨" which suggest that they stand for propositional constituents. They may equally well stand for constituents of thoughts. This ambiguity is cleared up by using the superscript "p."
Moreover, the symbols such as "¬" and "∨" occur in contexts such as "¬(p ∨ (q ∨ r))." The symbol "¬(" instead of the symbol "¬" would have to be used to play the same role as the symbol "1^P"; the symbol "(" would have to be used to play the same role as the symbol "2^P." But the symbols "¬(" and "(" may suggest that the entities which they designate are complex since the symbols have parts—one part being the parentheses. By using symbols such as "1^P" and "2^P" one should not be mislead and think that the unsaturated constituents of propositions which these symbols designate have parts. There is a third reason for not using the standard symbols. There are not enough to go around. Though the symbol can be used to indicate a proposition which necessarily has a truth-value which opposes that of another proposition, there is no standard notation for a proposition which is distinct from another proposition which necessarily share the same truth-value. According to the theory under discussion, the distinct propositions expressed by "It is true that A" and "A" necessarily have the same truth-values, but there is no standard notation for representing the constituent of the former
proposition which is not a constituent of the latter. Copi suggests a means of representing this constituent and the others which are not covered by the standard notational devices. He uses the following twenty symbols: "f₁( )," "f₂( )," "f₃( )," "f₄( )," "f₁( , )," "f₂( , )" . . . , "f₁₆( , )." The first four symbols in this list may be used to refer to the ₁UP, ₂UP, ₃UP and ₄UP; the last sixteen expressions in the above list may be used to represent the entities ₂UP, ₂UP, . . . , ₁₆UP. But Copi's notation has the following two drawbacks: (1) it is not clear that the twenty symbols represent propositional constituents instead of the constituents of facts, and (2) it is not clear that the twenty constituents stand for constituents of propositions which are simple. For these reasons, I recommend the notation which I am employing rather than those notations which have had more common use.

Expressions of either of the following two forms refer to molecular propositions:

1) ₀UPX

2) ₀UPXY

provided that: i = 1, 2, 3, or 4; j = 1, 2, 3, . . . , or 16; X is the name of an atomic proposition; and Y is the name of an atomic proposition.

Consider the proposition expressed by the sentence "It is not the case that the least rapidly convergent series has a limit." This

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is a molecular proposition and can be represented by the expression 
"1^P 1^P 1^P." The English sentence can be broken up into the following 
three parts: "It is not the case that . . .," " . . . has a limit," 
and "the least rapidly convergent series." It is useful to think of 
these three parts as corresponding to 1^P, 1^P, and 1^P, respectively.

The molecular proposition expressed by the sentence "It is 
true that the least rapidly convergent series has a limit" has two 
constituents which are also constituents of the proposition just con­sidered and one which is not a constituent of the proposition just 
considered. This proposition can be denoted by "2^P 1^P 1^P." It is 
helpful to think of "It is true that . . ." as being a part of the 
sentence in the way in which 1^P is a part of the proposition.

There are molecular propositions which have 1^P and 1^P as 
constituents, but since there are no parts of English sentences, at
least none that I know of, that correspond to these parts of propo­sitions, I will wait for further developments before making more com­ments about these entities.

The sentence "The least rapidly convergent series has a 
natural number as a limit or the least rapidly convergent series has 
a transcendental number as a limit" expresses a molecular proposition. 
In our notation the symbol "1^P 2^P 3^P" can be used to denote this 
proposition. It is useful to think of " . . . or . . ." as being a 
part of the sentence which expresses this molecular proposition in 
the way 2^P is a part of the proposition. Some other parts of
molecular propositions can be usefully thought of as correspondents to "... and ..." and "... if ..."

Molecular propositions

A proposition is a molecular proposition if and only if there is at least one molecular proposition which is a proper constituent of it and the only other constituents are those to which we can refer by using the symbols which have been concocted.

Rather than providing formation rules, I shall exhibit by example how names of molecular propositions are constructed.

"It is not the case that it is not the case that three is greater than five" expresses a molecular proposition. This proposition can be represented by the expression "\(^{1}_{1}p_{1}^{p_{1}}p_{2}^{p_{2}}p_{a1}^{a_{1}}a_{2}^{a_{2}}\)."

If we used this expression to stand for this particular proposition, we would use "\(^{2}_{2}p_{1}^{p_{1}}p_{a1}^{a_{1}}a_{2}^{a_{2}}\)" to stand for the proposition expressed by the sentence "Three is greater than five." Notice that \(^{2}_{2}p_{1}^{p_{1}}p_{a1}^{a_{1}}a_{2}^{a_{2}}\) is an atomic proposition, not a molecular proposition. The following phrase indicates that Frege considers these thoughts to be distinct:

"... of the two thoughts: A, and the negation of the negation of A: ... ."

"It is true that it is true that both three is greater than five and five is greater than three" expresses a molecular proposition.

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proposition which can be represented by

\[ 2_{U1}^2 u_1^2 v_2^2 u_2^1 p_1^1 p_2^1 p_2^2 p_2^2 p_2^1 p_1^2 p_1^1 p_2^1. \]

Note that this proposition has proper constituents which are molecular propositions.

Molecular propositions are either molecular \(_1^1\) propositions or molecular \(_2^2\) propositions.

**Facts**

Facts, like propositions, possess saturated and unsaturated constituents. I will call the saturated constituents of facts "objects" and the unsaturated constituents of facts "functions" in conformity with the manner in which Frege uses the German equivalents of these terms.

**Objects**

I shall represent objects by the following expressions:

\[ b_1, b_2, b_3, \ldots \]

This notation is not meant to suggest any restriction on the number of objects.

The saturated constituents of propositions and the saturated constituents of facts are not unrelated though they are not dependent upon one another for their existence. Corresponding to each object there is at least one saturated constituent of an atomic proposition. In some cases there may be several. Frege refers to an object which corresponds to at least two saturated constituents of atomic propositions.
Frege writes:

If we say 'the Evening Star is a planet with a shorter period of revolution than the Earth,' the thought we express is other than in the sentence 'the Morning Star is a planet with a shorter period of revolution than the Earth'; for somebody who does know that the Morning Star is the Evening Star might regard one as true and the other as false. And yet both sentences must have the same reference; . . . 10

Let $1_{1}^{p}a_{1}^{p}$ and $1_{1}^{p}a_{2}^{p}$ stand for the thoughts expressed by the two sentences which Frege mentions in the above quotation. Let $b_{1}$ stand for the object which is the Morning Star and the Evening Star. Frege's view is that both $a_{1}^{p}$ and $a_{2}^{p}$ correspond to $b_{1}$.

But there may be no saturated object which corresponds to a saturated constituent of an atomic proposition. Such a saturated constituent designates. Frege writes:

The sentence 'Odysseus was set ashore at Ithaca while sound asleep' obviously has a sense. . . . The thought [expressed by this sentence] remains the same whether 'Odysseus' has reference or not. 11

According to Frege there is no object which corresponds to $a_{54}^{p}$ if the thought is represented by "$1_{10}^{p}a_{54}^{p}.$"

Physical objects such as stars are not the only objects which there are. Frege says, "Places, instants, stretches of time, are,


logically considered, objects; ... Numbers are objects. In addition the True and the False are objects. These two objects play an extremely crucial role in Frege's account of truth. After I talk about functions, I will have more to say about these two objects.

**Functions**

Functions are the unsaturated constituents of facts. But a function does not depend upon the existence of a fact for its existence.

Corresponding to each unsaturated constituent of an atomic proposition or a molecular proposition is exactly one function. (The unsaturated constituents of propositions designate.) Functions will be labeled in such a way that this correspondence is indicated. "\( k_i^U \)" stands for the function which corresponds to \( k_i^P \). Thus the function \( 2^U \) corresponds to \( 1^P \), and the function \( 4^U_15 \) corresponds to \( 3^P_15 \), for example.

I shall use expressions of the following forms to denote facts:

\[
2^{U_1} b_{1_2} b_{1_3}, \quad 3^{U_1} b_{1_2} b_{1_3}, \quad 4^{U_1} b_{1_2} b_{1_3} b_{1_4}, \quad \ldots
\]

\[
1^{U_1} b_{1_2}, \quad 2^{U_1} b_{1_2}, \quad 3^{U_1} b_{1_2}, \quad 4^{U_1} b_{1_2}, \quad 5^{U_1} b_{1_2}.
\]

\[
1^{U_1} b_{1_2} b_{1_3}, \quad 2^{U_1} b_{1_2} b_{1_3}, \quad \ldots, \quad 16^{U_1} b_{1_2} b_{1_3}.
\]

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\[12\text{Ibid., p. 71.}\]
If \( i_1, i_2, \ldots \) are replaced by "0," "1," \ldots \ the resulting expression may stand for a fact. The expressions which have been constructed to denote facts do not necessarily denote facts. That is, there may be an expression for a fact and no corresponding fact. The expressions used to denote facts are in this respect like the expressions used in Chapter I to denote worlds.

It is helpful in thinking about Frege's facts to have a way of "reading" the expressions which designate facts. We can "read" the expressions \( 2U_{i_1}\ b_{i_2}\ b_{i_3},\ "\2Ub_{i_1}\ b_{i_2},\ "\3Ub_{i_1}\ b_{i_2}\ b_{i_3},\ " \) respectively, as follows:

The function \( 2U_{i_1} \) maps the object \( b_{i_2} \) into the object \( b_{i_3} \).

The function \( 2U \) maps the object \( b_{i_1} \) into the object \( b_{i_2} \).

The function \( 3U \) maps the objects \( b_{i_1} \) and \( b_{i_2} \) into the object \( b_{i_3} \).

**Truth**

Not all propositions are true or false.\(^{13}\) Frege writes:

The sentence 'Odysseus was set ashore at Ithaca while sound asleep' obviously has a sense. But since it is doubtful whether the name 'Odysseus,' occurring therein, has reference, it is also doubtful whether the whole sentence has one. Yet it is certain, nevertheless, that anyone who seriously took the sentence to be true or false would ascribe to the name 'Odysseus' a reference, not merely a

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\(^{13}\) As I read Frege, he would assign no truth-value to the famous sentence "The present king of France is bald." I see Frege as neither a Russell nor a Meinong, but as a Strawson who is primarily interested in thoughts rather than sentences or statements. Cf. Bertrand Russell, "On Denoting," Mind, XIV (October, 1905), pp. 479-93; cf. P. F. Strawson, "On Referring," Mind, LIX (July, 1950), pp. 320-44.
sense; for it is of the reference of the name that the predicate is affirmed or denied. Whoever does not admit the name has reference can neither apply nor withhold the predicate.\footnote{Ibid., p. 62.}

The condition which a proposition must meet in order for it to be either true or false is:

For each saturated constituent of any atomic proposition which is a constituent of the proposition there is a corresponding object.

An atomic proposition would have only one atomic proposition as a constituent, \textit{viz.}, itself. If "$_1U^P_1a^P_1$" denotes the proposition expressed by the sentence "Odysseus was set ashore at Ithaca while sound asleep," then this proposition is true or false only if there is an object $b_2$ (say) which corresponds to the entity $a^P_1$.

There is at least one fact which corresponds to any atomic or molecular proposition which has a truth-value. Exactly one fact corresponds to an atomic proposition which has a truth-value. Consider the atomic proposition denoted by "$_1U^P_3a^P_4$." Suppose this proposition has a truth-value. Corresponding to this proposition is a fact which has as constituents the object which corresponds to $a^P_4$, the function $2U_3$ (the function which corresponds to $1U^P_3$), and either the object the True or the object the False. If $b_5$ is the object which corresponds to $a^P_4$, $b_1$ is the True, and $b_3$ is the False, then there is a fact denoted by "$_2U_3b_5b_1$" or a fact denoted by "$_2U_3b_5b_3$" which corresponds to the proposition. Exactly one of these expressions denotes a fact.
If the former expression denotes a fact, the proposition to which it corresponds is true. If the latter denotes a fact, the proposition to which it corresponds is false. Both of the expressions \(2U_3b_5b_1\) and \(2U_3b_5b_3\) do not designate facts. If they did, then the proposition \(U_5a^P\) would be both true and false.

There is more than one fact which corresponds to any molecular proposition. To illustrate this point, consider the proposition expressed by the sentence "It is not the case that it is not the case that the Morning Star is a body illuminated by the Sun." This proposition can be denoted by \(^1U_1P^1U_2P^aP\). If this proposition has a truth-value, then there is an object which corresponds to \(a^P\). Suppose there is such an object. Label it \(b_1\). Suppose \(b_2\) is the True and \(b_3\) is the False. Either \(2U_2b_1b_2\) or \(2U_2b_1b_3\) (but not both) denotes a fact which corresponds to the proposition. Suppose the former expression denotes a fact which corresponds to the proposition. In addition to this fact there is a fact denoted by \(^1U_3b_3\) which corresponds to the proposition. The constituents of this fact are dependent upon the constituents of the former fact. If \(2U_2b_1b_3\) denoted a fact, then \(^1U_3b_2b_2\) would have to have been the label of the second fact which would have corresponded to the proposition. There is also a third fact which corresponds to the proposition. It is the fact denoted by the expression \(^1U_3b_3b_2\). The presence of \(b_3\) in the third fact is dependent upon the presence of \(b_2\) in the second fact. Since \(b_3\), the True, is a constituent of the third fact and
"last fact" which corresponds to the proposition, the proposition is true.

You will note that it was merely supposed that \(2U_2^1b_1b_2\) denoted a fact, whereas it was asserted that \(1U_2^1b_2b_3\) denoted a fact. What accounts for the difference? \(2U_2\) is a function which may have either the True or the False (but not both) for a given argument, but \(1U\) is a function which only maps the True into the False and the False into the True. Since the True was the argument of the function \(1U\) in the above illustration, the False had to be the value of the function. That is, \(1U_2b_2b_3\) had to denote a corresponding fact.

The four functions designated by \(1U_2\), \(2U_2\), \(3U_2\), and \(4U_2\) differ in the following respects:

1) One maps the True into the False and the False into the True.
2) One maps the True into the True and the False into the False.
3) One maps the True into the False and the False into the False.
4) One maps the True into the True and the False into the True.

The senses of "It is not the case that . . . ." and "It is true that . . . ." correspond to the first two functions, respectively. These senses would be labeled \(1U^P\) and \(2U^P\) in our notation. Standard English seems to have no way of referring to the unsaturated
constituents of propositions which correspond to the last two functions. But \( \text{"}_1^3p \) and \( \text{"}_1^4p \) can serve as labels of these constituents.

The sixteen functions designated by \( \text{"}_1^1u, \text{"}_1^2u, \ldots, \text{"}_1^{16}u \) differ as follows:

1) One maps the True and the True into the False, the True and the False into the False, the False and the True into the False, and the False and the False into the False.

2) One maps the True and the True into the True, the True and the False into the False, the False and the True into the False, and the False and the False into the False.

16) One maps the True and the True into the True, the True and the False into the True, the False and the True into the False, and the False and the False into the True.

The sixteenth function which is listed would correspond to the unsaturated entity referred to by "the sense of '... if ...'" The second function listed would correspond to the entity referred to by "the sense of '... and ...'" English seems to provide no natural way of referring to the parts of propositions which correspond to some of the functions listed. For example, there seems to be
no way of denoting the unsaturated portion of a proposition which corresponds to the first function which was listed. But \( u_1 \) denotes this entity.

Let us consider some propositions which have corresponding facts which are such that some of them have the second function and the sixteenth function as constituents.

Consider, for example, the proposition expressed by the sentence "That Vallachi is a bachelor if Vallachi is married is not the case" will serve as one among many possible examples. The proposition can be designated by the expression:

\[
\begin{array}{c}
\text{"} 1 u_1 p 16 u_2 & u_2 p & u_2 p \text{"} \\
1 & 2 & 1 & 1 & 1 & 1 & 2 & 1 \end{array}
\]

Suppose that \( b_1 \) corresponds to \( a_1 \), \( b_3 \) is the True, and \( b_4 \) is the False. Either \( \text{"} 2 1 1 1 1 2 1 \text{"} \) or \( \text{"} 2 1 1 1 1 4 \text{"} \) (but not both) designates a fact which corresponds to the proposition, and either \( \text{"} 2 1 1 b_1 b_3 \text{"} \) or \( \text{"} 2 1 1 b_1 b_4 \text{"} \) (but not both) designates a fact which corresponds to the proposition. Thus there are four cases to be considered:

1) \( 2 1 1 b_1 b_3 \) and \( 2 1 1 b_1 b_3 \) are corresponding facts. (I.e., suppose that it is a fact that \( 2 1 \) maps \( b_1 \) into \( b_3 \) and it is a fact that \( 2 2 \) maps \( b_1 \) into \( b_3 \).) Since \( b_3 \) is the value of \( 2 1 \) and \( 2 2 \) for the argument \( b_1 \), \( \text{"} 16 u_2 b_3 b_4 \text{"} \) must be a corresponding fact. But since \( 16 \) maps \( b_3 \) and \( b_3 \) into \( b_4 \), \( b_4 \) must be the argument of the last corresponding fact, viz., \( 1 u_2 b_3 \). Since \( b_3 \) is the value of the function which is the constituent of the last corresponding
fact and since $b_3$ is the False, the proposition to which these facts correspond is false.

2) $2U_1b_1b_3$ and $2U_2b_1b_4$ are corresponding facts. The other correspondings facts are $\frac{16}{3}Ub_3b_4$ and $1Ub_3b_4$. The proposition is true.

3) $2U_1b_1b_4$ and $2U_2b_1b_3$ are corresponding facts. $\frac{16}{3}Ub_4b_3b_4$ and $1Ub_4b_3$ are corresponding facts. The proposition is false.

4) $2U_1b_4b_4$ and $2U_2b_1b_4$ are corresponding facts. The other corresponding facts are $\frac{16}{3}Ub_4b_4b_4$ and $1Ub_4b_3$. The proposition is false.

A final example will illustrate the dependence of the truth-values of atomic and molecular propositions upon facts. Consider the proposition expressed by the sentence "Vallachi is not married if Vallachi is a bachelor." This proposition can be represented by the expression

$$\text{"}16U_1b_1b_4\text{"}$$

Suppose that the object which corresponds to $a_1$ is again represented by "$b_1$." Let "$b_3$" and "$b_4$" stand for the True and the False, respectively. Again there are four mutually exclusive cases to consider:

1) $2U_1b_1b_3$ and $2U_2b_1b_4$ are corresponding facts. Since $2U_1$ maps $b_1$ into $b_3$, $1Ub_3b_4$ must be a corresponding fact. Since $1U$ maps $b_3$ into $b_4$ and $2U$ maps $b_1$ into $b_3$,
\[16^3_{\text{Ub}_4 b_4} b_4\] must be a corresponding fact. Since \(b_4\) is
value of the function which is a constituent of the
last fact given the arguments, \(b_3\) and \(b_4\), and since
\(b_4\) is the True, the proposition to which these facts
correspond is true.

2) \(2^1_{\text{Ub}_1 b_3}\) and \(2^2_{\text{Ub}_2 b_4}\) are corresponding facts. \(2^1_{\text{Ub}_2 b_4}\)
is a corresponding fact. The last corresponding fact is
\[16^3_{\text{Ub}_4 b_4} b_4\]. The proposition is true.

3) \(2^1_{\text{Ub}_1 b_4}, 2^2_{\text{Ub}_2 b_3}, 2^3_{\text{Ub}_4 b_3}\), and \(16^3_{\text{Ub}_3 b_3} b_4\) are corresponding
facts. The proposition to which these facts correspond
is true.

4) \(2^1_{\text{Ub}_1 b_4}, 2^2_{\text{Ub}_2 b_4}, 2^3_{\text{Ub}_4 b_3},\) and \(16^3_{\text{Ub}_3 b_3} b_4\) (the last) are
 corresponding facts. The proposition is false.

Corresponding to any atomic or molecular proposition which
has a truth-value, there is a collection of facts. The number of
facts in this collection depends upon the proposition to which they
correspond. But no matter how many there are, there is always a "last
fact" in the collection. This fact always has an unsaturated element,
a function which corresponds to the unsaturated element which is named
by the left-most expression which occurs in the name of the proposi­tion to which the fact corresponds. If this function has the value,
the True, for the argument(s) which is (are) determined by the
proposition, the proposition is true. Otherwise, it is false. If the
proposition is represented by \("^3_{2}\)...", then \(^3_{3}\) will be the func­tion
which is the constituent of "the last" corresponding fact.
The following statements capture the essential features of the theory which has been outlined:

1) Atomic and molecular propositions have saturated and unsaturated constituents.

2) If an atomic or a molecular proposition has a truth-value, then for each saturated constituent of an atomic proposition which is a constituent (not necessarily a proper constituent) of the proposition there is a corresponding object. Some atomic and molecular propositions do not have truth values.

3) If an atomic or a molecular proposition has a truth-value, then for each unsaturated constituent of this proposition there is a function which is the constituent of a fact. Each function is a constituent of exactly one fact. The order of the unsaturated constituents of the proposition suggest a means of ordering these facts. The "last fact" has either the True or the False "in a favored position." The truth or falsity of the proposition depends upon which of these two constituents is in the favored position.
CHAPTER III

BERGMANN

The theory of truth which will be developed in this chapter (Bergmann's Theory) bears similarities to both Bradley's and Frege's theories. If one were pressed for a short characterization of Frege's theory the following would do as well as any:

A proposition is true if and only if it corresponds to the True; a proposition is false if and only if it corresponds to the False.

An equally short and no less accurate characterization of Bergmann's theory is:

A proposition is true if and only if it corresponds to the Actual; a proposition is false if and only if it corresponds to the Possible.

These characterizations suggest some similarities but ignore the major differences. One of these differences marks a similarity between Bergmann and Bradley. Frege's propositions are complex; Bergmann's propositions are simple.

Frege's thoughts have constituents; Bergmann's do not. To mark this difference an artificial language which models Bergmann's theory must be unlike an artificial language which models Frege's
theory. Bergmann provides us with a symbolism for his thoughts. Any symbol which has the form \( \varphi X \varphi \) will be the name of a thought provided that \( X \) is replaced by the name of a fact. Later in the chapter more will be said about the names of facts, but \( A_1 \) is one example of an expression which will be used to name a fact. Thus, the expression \( \varphi A_1 \varphi \) will be used to name a proposition.

This notational device gives the unfortunate impression that thoughts are complex entities since the designators of thoughts are complex in character. A left-hand "corner" quotation mark and a right-hand "corner" quotation mark are constituents of every expression which stands for a thought. The symbolism thus has the misleading feature of suggesting that these quotation marks designate "parts" of thoughts. But these constituents of the names of thoughts do not name constituents of thoughts. There are no proper constituents of thoughts.

This misleading feature of the symbolism is outweighed by a distinct advantage which the symbolism has. The symbolism indicates that to each fact there is a corresponding proposition. But the notation should not be construed as an indicator that the existence of a proposition is a function of the existence of a fact. Both propositions and facts are independently existing entities; both propositions and functions have ontological status.

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1For example, see Gustav Bergmann, Logic and Reality (Madison: The University of Wisconsin Press, 1964), p. 134.
We noted in the preceding chapter that Frege contends that there is a correspondence between propositions and facts. A notation was developed which would indicate this correspondence. The device was: Expressions of the form $^k_{i+j}$ stand for the unsaturated constituents of propositions which are related to facts whose unsaturated constituents are represented by $^k_{i+1+j}$. We need another notational device to indicate the correspondence which Bergmann envisions as holding between propositions and facts, since according to Bergmann's theory, the correspondence is between parts of propositions and parts of facts. The "corner" quotation mark device will accomplish this objective.

Though Bergmann's propositions are simple whereas Frege's propositions have differing degrees of complexity, the distinction which Frege makes between atomic and molecular propositions has its parallel in Bergmann's theory. Some of Frege's propositions correspond to exactly one fact and others correspond to more than one fact; some of Bergmann's propositions correspond to "simple facts" and others correspond to "complex facts." Both Frege and Bergmann would put the propositions expressed by "Vallachi is a philosopher" and "Vallachi is a philosopher or Vallachi is a psychiatrist" in different categories. Though Bergmann claims that both propositions are simple, the former corresponds to a simple fact whereas the latter corresponds to a complex fact. Frege's view is that both propositions are complex, but the former corresponds to exactly one fact, if it corresponds to a fact at all, and the latter corresponds
Facts

According to what I am calling Bergmann's theory, a simple fact has no facts as proper constituents. Complex facts have facts as proper constituents.

Let us let the symbols "A_1," "A_2," ... serve as representatives of simple facts. The additional constituents of complex facts will be represented by the following symbols: "^1_jU" (j = 1, 2, 3, 4) and "^2_jU" (j = 1, 2, ... 16). The expressions which will be used to denote facts can be recursively defined as follows:

a) An expression of the form "A_1" (i = 1, 2, ...) denotes a fact.

b) If an expression X denotes a fact, then the expression "^1_jUX" (j = 1, 2, 3, 4) denotes a fact.

c) If the expressions X and Y denote facts, then the expressions of the form "^2_jUXY" (j = 1, 2, ... , 16) denote facts.

d) The expressions of the above forms are the only expressions which denote facts.

Thus, the symbols "^2_1^3_1^4_1A_2" and "^1_2^4_1^11_1^4_1A_4" represent facts.
One of the expressions \( \frac{3}{2} \) corresponds to the expression "or." One of the expressions \( \frac{1}{1} \) corresponds to a negative fact; one of the expressions \( \frac{4}{1} \) corresponds to the expression "not." The artificial symbolism which I am using to represent Bergmann's facts has the advantage of indicating that there are sixteen binary operators and four unary operators on facts. Although there is a standard terminology for some of the types of facts, not all of the types have traditional names. Moreover, the symbolism is useful in getting clear about the similarities and dissimilarities between Frege's theory and Bergmann's theory.

Let us compare the four Bergmannian entities denoted by \( \frac{3}{1} \) with the four Fregean entities denoted by \( \frac{4}{1} \). The difference in left-hand subscripts marks an important difference in the manner in which they view facts. Bergmann's entities which are so-named require saturation by one entity (a fact) in order for an entity (a fact) to be formed, whereas Frege's entities require saturation by two entities (two objects) in order for an entity (a fact) to be formed.

Suppose that Bergmann's \( \frac{3}{1} \) and Frege's \( \frac{4}{2} \) correspond to the word "not." If \( \frac{3}{1} \) is completed by the fact \( A_1 \), the result is the fact \( \frac{3}{1} A_1 \). If \( \frac{4}{2} \) is completed by the objects, the True and the False, the result is a fact.

These remarks of the last two paragraphs apply mutatis mutandis to the entities named by \( \frac{1}{2} \) and \( \frac{1}{3} \) \( (j = 1, 2, \ldots, 16) \).
Truth

Just as Frege's account of the truth of propositions is dependent upon the connection between propositions and facts, so also is Bergmann's. But Bergmann does not and cannot posit the same kind of connection. You will recall that Frege's propositions are complex. The distinct parts of Frege's propositions correspond to distinct parts of facts. Since Bergmann's propositions are unitary, only the propositions in toto correspond to (are tied to) facts.

In Bergmann's world every proposition is tied to a fact. This marks another difference between Bergmann's world and Frege's world. According to Frege the proposition expressed by "The least rapidly convergent series has a limit" is not tied to a fact. There are some "dangling propositions" such as this, propositions which are not "about" anything. Bergmann parts ways at this point. Every proposition is about something (a fact).

Though Bergmann thinks that all propositions are intentionally tied to the world (facts) and Frege claims that some are not, Bergmann and Frege are closer to one another than either is to Bradley. You will recall that none of Bradley's propositions are "about" facts. For Bradley, propositions are constituents of facts. Propositions are not "above the world"; they are a "part of the world." Several traditional labels occur to me as ways of marking this most fundamental distinction. Bergmann and Frege are "realists"; Bradley is an "idealist." Bergmann and Frege have "correspondence theories of truth"; Bradley does not. The difference is that Bradley's
propositions are not intentionally tied to the world whereas all of Bergmann's and some of Frege's are.

Bergmann uses the symbol "M" to baptize the intentional tie between propositions and facts. If we were to baptize the ties between Frege's propositions and Frege's facts, two labels would be required, one for the tie between the saturated parts of propositions and the saturated parts of facts (a tie which doesn't always tie) and one for the tie between the unsaturated parts of propositions and the unsaturated parts of facts (a tie which always ties).

Since all of Bergmann's propositions stand in an "aboutness" relation to facts, the truth of a proposition is not simply a matter of its being about a fact. According to Bergmann a proposition is true if it is about a fact that has one "mode of existence" and false if it is about a fact that has another "mode of existence."

One can easily concoct a variant of Bergmann's theory. True propositions are M-tied (intentionally tied) to facts; false propositions are not M-tied to facts. According to this deviant theory, all false propositions are "dangling propositions." In terms of the existence of "dangling propositions" the theory is closer to Frege's theory. But there is still a major difference. For Frege, only the propositions which are neither true nor false are "wandering adjectives" whereas a proponent of the deviant theory claims that all false propositions are "wandering adjectives." Bergmann's world is

\[2\text{See the indices of Meaning and Existence and Logic and Reality under "Meaning."} \]
more "highly structured" than the world of Frege which is in turn more highly structured than the world of the proponent of the deviant theory.

Bergmann states clearly in the following passage that he will have nothing to do with what I have called the "deviant theory":

If all intentions are to exist, how about those of (the thoughts in) false beliefs, which, as one says, do not "exist"? How about perceptual error? The double quotes around 'exist' mark the philosophical use that limits existence to the actual. In my world, we know, the intention of, say, a false belief is a possibility (p-fact) and as such has ontological status (exists), even though it is neither actual, nor, as we shall see, real . . . . the mode of possibility provides the way out of the difficulties the realists encountered in assigning, as I believe they must, some ontological status to the intentions of all acts.\(^3\)

Bergmann's intentions are facts. True propositions and false propositions are M-tied to intentions. The intentions of true propositions are facts which exist in the mode of actuality; the intentions of false propositions are facts which exist in the mode of possibility.

Bergmann is not saying that the facts which are intended by true propositions and the facts which are intended by false propositions have different kinds of existence, viz., actual existence and possible existence. Bergmann states:

'Exist' is univocal. When I read in a philosophical book that there are several kinds of existing, I am tempted to put it aside. If I don't, it is because I hope that reading on I shall discover that what is so unfortunately expressed is the proposition that there are several kinds of existents. If this hope is disappointed then I know

that I could not possibly understand the book. So I put it aside. The philosopher's concern is only with the most general kinds of existents. These are the ontological modes, categories, sub-categories.4

Thus, an actual fact is one mode (kind) of existent and a possible fact is another mode (kind) of existent.

Any artificial language which is an accurate model of Bergmann's ontology must mark this difference in kinds of facts by using different kinds of expressions to stand for them. Bergmann shares this bias concerning artificial languages. He writes:

In some artificial languages, for instance, individuals and only individuals are represented by lower-case letters. The letter itself represents the thing, the letter's shape, i.e., its being lower-case, the thing's being an individual, or, as one says, individuality, which is a subsistent. Nor is this subsistent represented in any other way. This illustrates a characteristic feature of such languages. The several members of one mode or kind are all represented in the same way.5

It is interesting that Bergmann does not concoct an artificial language which marks the difference in the kinds of entities which are intended by true propositions and false propositions. Bergmann makes extensive use of artificial languages and employs different kinds of symbols to signify that his universals, individuals, ties, and facts are different kinds of existents. I am not sure why he does not employ symbols to distinguish between actual facts and possible facts. It is this lacuna which I will attempt to fill.

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4 Ibid., p. 306.
5 Ibid., p. 89. (The underlining is mine.)
It is not clear whether Bergmann contends that possible and actual facts are different in kind in virtue of possessing different kinds of constituents. At some points Bergmann talks about different kinds of ties which are the constituents of facts. These different kinds of ties could account for the difference in the kind of facts which they constitute. Bergmann writes:

The nexus connecting two simples, actually or P-wise, is represented by a relational thing connecting the two individuals which represent the simples.⁶

At another point he states:

The represented fact or P-fact consists of an individual exemplifying a character, actually or P-wise; .⁷

Yet at other points Bergmann apparently denies that there is a dichotomy between actual and possible nexus. He states:

All possible entities are complex. More precisely, they would be complexes if they were not mere possibilities.⁸

Let us suppose that the difference between actual and possible facts can be accounted for in terms of the difference between actual and P-wise ties. The symbols "A₁," "A₂," . . . will denote actual facts; the symbols "A'₁," "A'₂," . . . will denote possible facts. The left-hand subscript in the latter set of symbols indicates that the constituents of the facts denoted by these symbols are held

⁶Ibid., p. 248. (The underlining is mine.)
⁷Ibid., p. 260. (The underlining is mine.)
⁸Ibid., p. 92.
together by a different kind of tie than that which connects the constituents of actual facts.

According to Bergmann a proposition is true if it is intentionally tied to an actual fact and false if it is intentionally tied to a possible fact. If we preserve the same symbolism for propositions, then it is evident that none of the propositions named by "\( \neg A_1 \)," "\( \neg A_2 \)," ... can be false for they will be intentionally tied to actual facts. (\( \neg X \) is intentionally tied to \( X \).) Suppose we expand our symbolism for thoughts to include "\( \neg_p A_1 \)," "\( \neg_p A_2 \)," .... Then this awkward consequence follows: All true propositions have to be true and all false propositions have to be false. Consider the thoughts \( \neg_p A_2 \) and \( \neg_p A_3 \). \( \neg_p A_2 \) would have to be tied to \( A_2 \); thus \( \neg_p A_2 \) would have to be false. \( \neg_p A_3 \) would have to be tied to \( A_3 \); thus \( A_3 \) would have to be true.

One way of avoiding this embarrassment would be by maintaining that \( \neg X \) may be intentionally tied to either \( X \) or \( \neg X \). Then there could be false propositions and a true proposition could have been false and a false proposition could have been true. But the price is dear and Bergmann is unwilling to pay it. One has to abandon the thesis that a given thought is "about" one and only one fact—the thesis that thoughts designate.

We reached this apparent impasse by assuming that actual facts are different in kind from possible facts in virtue of possessing distinct kinds of nexūs. Let us abandon the view that there are distinct kinds of ties. Rather, let us separate facts in the following manner:
There are those which are tied to the Actual and those which are tied
to the Possible. (A fact which is tied to the Actual exists in the
mode of actuality; a fact which is tied to the Possible exists in the
mode of possibility.) A proposition is true if it is intentionally tied
to a fact which is tied to the Actual; a proposition is false if it
is intentionally tied to a fact which is tied to the Possible. The
symbols "A" and "P" will be used to denote the Actual and the Possible,
respectively. The symbol "0U" will be used to denote the entity
which ties facts to either the Actual or the Possible. The criterion
for the truth of a proposition can now be given a more formal state­
ment. "X" is true if and only if "0UXA" denotes an entity; "X" is
false if and only if "0UXP" denotes an entity. If "0UXA" denotes an
entity, then it denotes the fact that 0U maps the fact X into the
Actual. If "0UXP" denotes an entity, then it denotes the fact that
0U maps the fact X into the Possible. Both "0UXA" and "0UXP" cannot
denote entities. If they did, the thought "X" would have to both
true and false.

The theory as now presented is strikingly close to Frege's
theory. The Actual and the Possible play roles similar to those played
by the True and the False. To see how close the similarity is let us
consider the proposition expressed by "Vallachi is a philosopher."
According to Frege, this proposition is true if and only if the fact­
components corresponding to "Vallachi" and "is a philosopher" are
connected to the True. According to Bergmann the proposition is true
if and only if the fact-components corresponding to "Vallachi" and
"is a philosopher" are tied to the Actual. The difference is that in Frege's theory the fact-components corresponding to "Vallachi" and "is a philosopher" do not form a fact independently of their tie with the True, but in Bergmann's theory these components do form a fact independently of their connection with the Actual.

In their analyses of the truth of propositions expressed by "simple sentences" such as the one just considered, Bergmann invokes the Actual or the Possible just once and Frege invokes the True or the False just once. As we noted in Chapter II Frege's analysis of the truth of propositions expressed by "complex sentences" such as "Vallachi is a philosopher or Vallachi is a psychiatrist" makes more than one appeal to the True or the False. As you will recall, for Frege (at least my reading of Frege) there is an ordered sequence of facts which corresponds to a proposition expressed by a "complex sentence." Either the True or the False occurs in each fact which appears in such an ordered sequence. Corresponding to the proposition expressed by the sentence at hand there are three facts in the ordered sequence. The first has a constituent corresponding to "Vallachi," "is a philosopher," and either "the True" or "the False"; the second has a constituent corresponding to "Vallachi," "is a psychiatrist," and either "the True" or "the False"; the third also has three constituents and each of these three constituents corresponds to either "the True" or "the False." Bergmann, on the other hand, does not make more than one appeal to the Actual or the Possible in order to analyze the truth of propositions expressed by complex
sentences such as the one just considered. According to him, there is exactly one fact which corresponds to a proposition no matter how complex the sentence is which is necessary to express it. This fact is either tied to the Actual or the Possible.

But Bergmann pays a price for this elegance. Bergmann has hierarchies of facts--facts which are constructed out of other facts. Frege avoids hierarchies of facts by sequentially ordering facts. Thus, from the standpoint of the simplicity of their accounts of the truth of propositions expressed by complex sentences, there is not much reason for adopting the one theory over the other.

Logical Truths

It is in terms of the sequential ordering of facts that Frege accounts for the sameness of truth-values of the propositions expressed by sentence pairs such as "..." and "It is not the case that it is not the case that ..." Bergmann, of course, cannot use the same device since there is only one fact which corresponds to a proposition. Bradley does not have the problem that Frege and Bergmann have since according to him the propositions expressed by such distinct sentences are not distinct.

Frege's account is neat. Corresponding to each proposition there is an ordered sequence of facts. The favored constituent of the last fact in each sequence determines the truth-value of the proposition. There are n facts in the first sequence. The second sequence has the same facts plus an additional two facts. The additional two facts are what might be called negative facts. The next
to the last fact in the second sequence carries the True into the
False in which case the last fact carries the False into the True, or
the next to the last fact carries the False into the True in which
case the last fact carries the True into the False. If the favored
c constituent of the last fact of the first sequence is the True, then
the first alternative will hold. The favored constituent of the last
fact of the second sequence will be the True. If the favored constitu­
ent of the last fact of the second sequence is the False, then the
second alternative will hold and the favored constituent of the last
fact of the second sequence will be the False. It follows that if
the first proposition is true the second proposition must be true, and
if the first is false the second must be false.

What sort of an account can Bergmann give to explain why both
propositions must have the same truth-value? He would say that the
facts intended by the propositions are both tied to the Actual. But
why must they be? I do not think that an "answer" can be given by
one who adopts the theory as outlined. One can say that they both
just must be connected to the Actual or it is part of "the logical
form of the world" that they both are connected to the Actual. But
are these answers?

It appears as though Frege's theory has accounted for some­
thing which Bergmann's theory does not account for. But has he?
What reason does Frege give for saying that the two additional facts
which belong to the second sequence must be such and such? Frege has
no answer to this question just as Bergmann has no answer to his cor­relative question.

The pair of propositions which has just been considered is representative. Take any other pair of propositions whose members must have the same truth-value. Frege "accounts" for the sameness of truth-values in terms of the manner in which the corresponding sequences of facts are related. Bergmann "accounts" for the sameness of truth-values by saying that it is part of the logical form of the world that the related facts are both connected to the Actual or to the Possible.

To give an adequate grounding for the necessary truth of thoughts such as "T if and only if not not T" given the presupposi­tions of the theory ascribed to Bergmann, one would have to add entities of distinct kinds to the ontology which is already rich. Two functions would be required in place of the one function \( ^0_2 \). One of these functions would necessarily map facts into the Actual. The other function would contingently map facts into the Actual or the Possible. Thus the thought "T if and only if not not T" would be necessarily true since the fact to which this thought is intentionally tied is necessarily mapped into the Actual. The non-necessary truth of "T", if true, would be accounted for given that the fact to which "T" is tied is contingently mapped into the Actual.

Frege's ontology could also be expanded to account for the necessary truth of the thought that T is true if and only if T. You will recall that Frege's account of the truth of thoughts rests on
the notion that thoughts "generate" sequences of facts. To account for the necessary truth of some thoughts, two "generating ties" would have to replace the one "generating tie." Those thoughts which are true and which necessarily generate the sequence of facts in virtue of which they are true would be necessarily true thoughts. Those thoughts which are true in virtue of sequences of facts which are contingently generated are not necessarily true.

To explicate adequately the notions of being necessarily mapped, contingently mapped, necessarily generated, and contingently generated would require an expansion of our notational systems which are already complex. I will not make the required adjustments in this work.

The Representation Thesis

I think that Bergmann accepts the major points of the theory which has been ascribed to him such as: propositions are unitary, propositions are true or false, propositions are intentionally tied to facts--true propositions to actual facts and false propositions to possible facts. Still I am reluctant to say that it is his theory since I believe that he subscribes to the following thesis:

A language suitable for representing the entities of the world can be constructed such that corresponding to any well-formed expression of the language there is an entity.

According to the theory of truth under discussion, no such language can be constructed. Either $\frac{0}{2}UXA$ or $\frac{0}{2}UXP$ does not stand for an
entity. If both of these expressions did stand for entities, then, since \( ^rX \) is intentionally tied to \( X \), \( ^rX \) would be both true and false.

Bergmann writes:

The representation theory must meet . . . major criticisms. . . . If the theory were true, every expression would represent an entity. But for some expressions there is no entity to represent. Or, what amounts to the same, the alleged entity does not exist. Hence I think the theory is false.\(^9\)

Since Bergmann attempts to meet this objection to what he calls the representation theory, it is clear that he cannot fully accept the theory of truth which has been presented in this chapter.

But the point is not merely that Bergmann does not accept the theory of this chapter. Given that one claims that propositions are simple, propositions are true or false, a proposition is intentionally tied to exactly one fact, he is logically compelled to reject the representation thesis. Thus, my contention is that Bergmann's remarks are inconsistent. The consistent theory of truth of this chapter is the result of making minor deletions and emendations in Bergmann's inconsistent remarks about truth.

\(^9\)Ibid., p. 86.
The three theories of truth of the preceding chapter can be summarized as follows:

1) A proposition is true if it is intentionally tied to a fact and false if there is nothing to which it is intentionally tied. (The "deviant theory." )

2) A proposition is true if it is intentionally tied to a fact possessing one kind of constituent and false if it is intentionally tied to a fact possessing another kind of constituent.

3) A proposition is true if it is intentionally tied to a fact which is tied to the Actual and false if it is intentionally tied to a fact which is tied to the Possible. (Bergmann's theory.)

One cannot accept the first two theories unless he abandons the thesis that those entities which intend entities intend exactly one entity. Yet one can accept the third theory and embrace this thesis.

In this chapter I will develop a theory which is compatible with the above thesis, but which unlike Bergmann's theory (and Frege's theory) does not require the positing of two special entities such
as the Actual and the Possible. The theory will be called Wittgenstein's theory.

**Propositions**

According to Wittgenstein's theory, a proposition is true if its constituents are intentionally tied to the constituent of a fact which has the same structure as that of the proposition and false if it has a different structure. The theory is obviously distinct from any of the theories which have been considered to this point.

Neither Bradley's nor Bergmann's propositions have any structure at all; their propositions are simple. Frege's propositions are complex, but the facts which are correlated with Frege's propositions do not even have the same mathematical multiplicity and thus cannot have the same structure. You will recall that the proposition \( \text{UP}a_1 \) is intentionally tied to a fact which has three ingredients, not two.

Yet even if Frege's propositions and correlated facts had the same mathematical multiplicity Frege's propositions do not have the sort of structure which Wittgenstein's have and thus the theories are distinct. Frege's propositions are complex entities; Wittgenstein's propositions are complexes of entities. Frege's propositions are entities but Wittgenstein's are not.

Firstly, I will attempt to make out the distinction between a complex entity and a complex of entities by giving an analogue. (Frege's treatment of propositions is to Wittgenstein's as Locke's treatment of minds is to Hume's.) Secondly, I will consider the
contrasting kind of artificial language which one must construct in
order to model complexes of entities.

A mind, according to Hume, is a bundle of perceptions. Locke
disagrees. According to Locke there is something (a mental sub­
stratum) in which the perceptions inhere. The mental substratum knits
the perceptions into one unit, a complex unit. For Hume there is no
thing which connects the perceptions. The perceptions are related but
there is no relator. Thus, I would say that Hume's minds are complexes
of entities (perceptions) whereas Locke's minds are complex entities.

There is a sense in which a language which provided a model
for Hume's complexes of entities would be a much simpler language
than one which provided a model for Locke's complex entities. It
would require fewer kinds of expressions. Only names for perceptions
would be necessary. In contrast, a language which reflected Locke's
complex entities would require names for perceptions, names for men­
tal substrata and a name for the entity which connects perceptions
to a mental substratum. An expression such as "m₄(p₁, p₂, p₅, p₁₀)"
might be used to stand for a Lockean complex entity where "m₄" stands
for a mental substratum, the "p₁'s" stand for perceptions and the
parentheses represent the connection between the two types of entities.

What Locke calls a complex entity Hume would call a complex of
entities and vice versa. Suppose that "m₂(p₁, p₂, p₅)" and
"m₃(p₁, p₂, p₅)" designate two complex entities. Hume would argue
that they are complexes of entities. Now how would a Humean differ­
entiate between the two complexes of entities by simply using the
names "p_1," "p_2," and "p_5"? Wouldn't the expression "p_1p_2p_5" (say) be ambiguous? An apparent way out of the difficulty is by arranging the symbols in determinate ways. The three symbols which are the names of perceptions could be arranged in two ways such that one of them would be paired with "m_2(p_1, p_2, p_5)" and the other would be paired with "m_3(p_1, p_2, p_5)." Thus the symbols for complexes of entities would not suffer from ambiguity just as the symbols for complex entities do not. The symbols "p_1," "p_2," and "p_5" when arranged vertically may stand for one determinate complex of entities and when arranged horizontally may stand for a distinct determinate complex, for example.

Since Wittgenstein's propositions are complexes of entities and Frege's propositions are complex entities, symbols such as "a_1^P_1a_2^P_2" and "a_2^P_1a_1^P_2" should not be used to stand for Wittgenstein's propositions. But distinct arrangements of "a_1^P_1" and "a_2^P_2" can be used to represent what Wittgenstein would say are distinct propositions.

If the arrangements are not determinate, they will be ambiguous. Wittgenstein emphasizes the need for determinacy. He writes:

A proposition is not a medley of words. --(Just as a theme in music is not a medley of notes.) (3.141)

It should be noted that Wittgenstein is using the term "proposition" in the above quoted passage in a manner in which I am not using the

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term. By a proposition I mean a thought. I use "proposition" in such a way that collections of linguistic entities and collections of noises are not propositions. But I think that Wittgenstein would also not hesitate to say that a thought is not a medley of entities but has a determinate structure.

One can see that there is an endless number of Wittgensteinean thoughts (propositions) that can be represented by the two symbols "a₁^" and "a₂^" since there is an endless number of ways of arranging these symbols on a page. The arrangements in the boxes below are distinct:

In order to give a formal treatment of the theory of truth under consideration it will be necessary to refer to all of the distinct arrangements which represent propositions. The symbols "A_n a₁^ a₂^ p_n" (n = 1, 2, . . .) will be used to refer to those arrangements of the symbols "a₁^" and "a₂^" which represent propositions which have exactly two constituents named by "a₁^" and "a₂^." In general, expressions of the form "A_n (a₁^ a₂^ a₃^ . . . a_m^)" (m, n, i₁, i₂, . . . = 1, 2, 3, . . .) will refer to the distinct arrangements of m symbols which represent propositions.

It is important to notice that the expression "A_1^" does not denote an entity. Its use is that of referring to a complex of
linguistic entities and this complex represents a complex (a proposition). Thus, though "2\(a_1^p a_2^p\)" may look like it plays the same role as "2\(a_1^p a_2^p\)" it does not. "2\(a_1^p\)" stands for an entity—the unsaturated part of a particular proposition. "2\(a_1^p\)" refers to a particular arrangement of linguistic entities, but the arrangement is not an entity.

**Truth**

As was mentioned earlier in the chapter, the truth of a proposition depends upon its connection with facts. Now facts, as well as propositions, are complexes of entities. We used the symbols "a\(i^p\)" (\(i = 1, 2, \ldots\)) to stand for the constituents of propositions. Let us use the symbols "a\(i\)" (\(i = 1, 2, \ldots\)) to stand for the constituents of facts. This notational device is especially useful since it correctly suggests that there is a connection between the constituents of propositions and the constituents of facts. Each constituent of a proposition intends (is intentionally tied to) exactly one constituent of a fact. (The constituents of propositions designate.) In general, a\(j^p\) is intentionally tied to a\(j\) (\(j = 1, 2, \ldots\)). (a\(j\) is the intention of a\(j^p\)).

To refer to the arrangements of expressions which represent facts we will use the same device as we used to refer to the

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I am using the term "facts" in a more restricted way than Wittgenstein does. According to him, all complexes of entities are facts. Thus we find Wittgenstein saying "A picture is a fact" on p. 15 of the *Tractatus* (2.141). On my usage of the term "fact" a complex of entities is not a fact unless its constituents are the intentions of propositional constituents.
arrangements of expressions which represent propositions. Thus, the expressions \\
\[ A_{m \times n} (a_{i_1}^P, a_{i_2}^P, \ldots, a_{i_m}^P) \] (m, n, i_1, i_2, \ldots = 1, 2, 3, \ldots) refer to the arrangements of expressions which represent facts.

The criterion for the truth of a proposition can now be stated as follows:

The proposition represented by the complex of linguistic expressions referred to by \\
\[ A_{m \times n} (a_{i_1}^P, a_{i_2}^P, \ldots, a_{i_m}^P) \] is true if and only if \\
\[ A_{m \times n} (a_{i_1}, a_{i_2}, \ldots, a_{i_m}) \] refers to a complex of linguistic expressions which represents a fact.

If the proposition represented by the complex of linguistic expressions referred to by \\
\[ A_{m \times n} (a_{i_1}^P, a_{i_2}^P, \ldots, a_{i_m}^P) \] is false, then \\
\[ A_{m \times n} (a_{i_1}, a_{i_2}, \ldots, a_{i_m}) \] refers to a complex of entities which does not represent a fact. 3 Thus, the theory appears to be not dissimilar to the so-called deviant theory. You will recall that according to the deviant theory a proposition is true if it is tied to a fact and false if it is tied to nothing. But it would be a mistake to assimilate the two theories. Each of the a_{i_1}^P's is tied to an a_{i_1} and the a_{i_1}'s must have some structure. Thus, every proposition whether it be true or false is tied to a fact. If the proposition

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3 The expression \[ A_{m \times n} (a_{i_1}, a_{i_2}, \ldots, a_{i_m}) \] is a complex entity not a complex of entities, but it can be used to refer to a complex of entities.
and the fact have the same structure, then the proposition is true; 
if the structures of the proposition and the fact differ, then the
proposition is false.

One Proposition-One Fact

But there is a fatal flaw in the theory which has been pre­
sented. According to the theory, (1) there are distinct propositions
and (2) propositions are complexes of entities. It is impossible to
defend both of these claims.

If these claims were correct, then there would be no diffi­
culty in maintaining that there are distinct facts and facts are
complexes of entities. Facts would be discrete in virtue or being
tied to discrete propositions. In other words propositions would
divide the world into facts. There would be a sense in which each
person would create his own world.

I think that Wittgenstein accepts the claims (which I think
are incompatible) and draws the above inference. He writes:

. . . what the solipsist means is quite correct; . . .
The world is my world: this is manifest in the fact that
the limits of language (of that language which alone I
understand) mean the limits of my world. (5.62)4

None of the accounts of truth that were examined in the pre­
ceding chapters led us to solipsism. What accounts for the difference?
The answer is simple. The facts entertained by the theories of the
foregoing chapters are complex entities; the facts entertained by the
theory developed in this chapter are complexes of entities. The facts

4Ibid., p. 115.
of the preceding chapters are single entities; they possess "unity in complexity." One of Bradley's facts consists of a thought (an ideal content) which is tied to an individuator (a real substantive). The singleness of Frege's facts is the result of the unsaturated component which unites certain saturated components to form one entity. Bergmann's facts also are composed of entities whose parts are held together. Bergmann's complex facts are held together by one or more of the twenty "special" constituents which were discussed in the preceding chapter. Even Bergmann's "simple facts" have a unity in complexity. His universals and particulars which comprise his simple facts (atomic facts) are tied together by what he calls the exemplification tie.  

In contrast to these accounts of facts, according to Wittgenstein's account, there is nothing which binds the constituents of his facts into one entity. In other words there is no constituent of a fact which individuates it. If there were nothing which were an external individuator of facts, it would be nonsense to talk about many facts or to talk about this fact as opposed to that fact. But there are external individuators of facts according to Wittgenstein. The propositions which a person entertains slice the world into facts. Thus "The world is my world."

5In both Logic and Reality and Meaning and Existence Bergmann discusses the nature of atomic facts at length. Since Bergmann's account of truth could be presented without discussing the structure of atomic facts, the omission of such a discussion from Chapter III is accounted for.
Since there is a one-to-one correspondence between the constituents of a particular proposition and certain objects in the world, the particularity of facts will have been established provided there is some means of particularizing propositions. There must be some method of showing that this proposition is separate from that proposition. But propositions are complexes of entities just as facts are. Just as there is no internal individuator of facts, there is no internal individuator of propositions.

We cannot solve the problem by invoking higher order complexes of entities to individuate propositions for they would require separation by even higher order complexes and so on. One apparent way out of the dilemma would be by supplying propositions with an internal individuator. For example, propositions may be construed in a Fregean fashion. Frege's $\sum_{1}^{P} a_{1} P_{1} a_{2} P_{2}$ has $a_{1} P_{1}$ and $a_{2} P_{2}$ bound together into one proposition from which $a_{3} P_{3}$, for example, is excluded. Thus, it is this proposition and not that proposition. But, as was pointed out earlier, a Fregean proposition is a complex entity and complex entities cannot have the same sort of structure as complexes of entities. It follows that if Wittgenstein's propositions were construed in a Fregean fashion and if one accounted for truth in terms of similarity of structure, then no proposition could be true.

But why not use expressions of the form \[ m n (a_{1} P_{1}, a_{2} P_{2}, \ldots, a_{m} P_{m}) \], which were introduced earlier, to individuate propositions? Such expressions were introduced in order to have a means of referring to propositions which were assumed to be distinct, but why couldn't
we use such expressions to play the role of separating propositions from one another? The absurd consequence of doing so would be that we would be maintaining that the distinctness of thoughts is grounded in the distinctness of linguistic expressions. The thought that snow is white would not be distinct from the thought that snow is black (say) until two linguistic expressions were coined which would make two distinct thoughts where there was only one.

Of course, it is possible to adopt the position that there is only one proposition (a complex of entities) and one fact (a complex of entities). The one proposition would be true if each constituent of this proposition were intentionally tied to exactly one constituent of the world and if the structure of the intentions were identical with the structure of the entities forming the one proposition. Otherwise, the proposition would be false.

According to this position, the world is one fact. Bradley also claimed that the world is one fact. But the differences are enormous. The content of Bradley's world (Reality) is a proposition; the proposition is not "above" the world. Bradley's world is one complex entity, not a complex of entities.

I find the claim that "the world is one" not nearly as bizarre as the claim that there is only one proposition. The latter claim has the consequence that I am not having two distinct thoughts when I think that it is snowing and when I think that it is not snowing. I take this to be a reductio ad absurdum of the theory outlined in this chapter.
CHAPTER V

OCKHAM

With the exception of Bradley's theory, according to all of the theories which we have considered, the bearers of truth-values are intentionally tied to facts. Bradley's propositions, unlike those of Frege, Bergmann, and Wittgenstein (for whom there can be only one) are constituents of facts. Thus the theory of the present chapter is radically different from any of the other theories considered. According to it, propositions are intentionally tied to "objects" (not facts). These objects will not be called facts since they are simple whereas facts are complex in character.

Singular Propositions

A major part of the theory of this chapter (Ockham's theory) can be roughly summarized as follows:

A subject-predicate singular proposition is true if and only if the predicate and the subject stand for the same object.

The following passage indicates that Ockham accepts this characterization:

... Let us first speak of singular propositions of inheritance in the present tense [and not determined by a modality], which have both the predicate and the subject in the
nominative case, and are not equivalent to a hypothetical proposition. For the truth of such a singular proposition, which is not equivalent to many propositions, it is not required that the subject and the predicate be really the same, nor that the predicate be really in the subject, or really inhere in the subject, nor that it be really united with the subject outside the mind. For instance, for the truth of the proposition 'This is an angel' it is not required that this common term 'angel' be really the same with that which has the position of subject in this proposition, or that it be really in it, or anything of the sort; but it is sufficient and necessary that subject and predicate should stand for the same thing.*

The use of "proposition" is consistent with the use which has been given to the term throughout the preceding chapters. A proposition is neither a series of words nor a series of noises but is a thought. Ockham's propositions (thoughts) are true in a primary sense, i.e., one cannot explicate the truth of written sentences or spoken sentences without analyzing the truth of thoughts, but one can explicate the truth of propositions without first explaining what it is for a sentence to be true. There is thus an important sense in which the theory of truth to be developed in this chapter is similar to the theories of the preceding chapters. Ockham's theory is also a non-linguistic theory of truth.

It is clear that Ockham's singular propositions must be complex entities since his account of the truth of singular propositions requires the existence of a subject-part and a predicate-part. Frege's theory is the only theory we have examined which construes propositions as complex entities. (Wittgenstein's proposition is a

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1Ockham, Philosophical Writings, trans. by Philotheus Boehner (Edinburgh: Thomas Nelson and Sons Ltd., 1957), p. 76.
complex of entities.) But there are alternatives to Frege's structuring of propositions. Rather than making the subject saturated and the predicate unsaturated one could: (1) make the subject unsaturated and the predicate saturated, or (2) make both the subject and the predicate unsaturated, or (3) make both the subject and the predicate saturated and posit a third entity which is unsaturated. I doubt that there is enough textual evidence to decide which of the four alternative structurings is Ockham's. The theory which I will develop will embrace the fourth alternative.

An artificial language which models singular propositions which are structured in the fourth manner must have names for the saturated subjects, names for the saturated predicates, and a name for the unsaturated tie (the copula). "s₁," "s₂," . . . will name the subjects; "p₁," "p₂," . . . will name the predicates; "C" will name the unsaturated tie. Complex linguistic expressions of the form "xCy" will name singular propositions provided that "x" is replaced by the name of a subject and "y" is replaced by the name of a predicate.

Now it is not difficult to imagine that the proposition expressed by the sentence "This is an angel" can be modeled by "s₄Cp₅," for example. "s₄" corresponds to "this"; "C" corresponds to "is"; and "p₅" corresponds to "an angel." But what about propositions expressed by sentences such as "This is larger than that"? It seems possible to treat this as a singular proposition, i.e., one can claim

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²An example used by Ockham on p. 76 of Philosophical Writings.
that it is modeled by an expression such as \( s_1 C s_2 \) where \( s_1 \) corresponds to "this," "C" corresponds to "is," and \( s_2 \) corresponds to "larger than that." By taking this gambit it is clear that this account of propositions is radically different from Frege's. According to Frege's theory an expression such as \( 1 \cup_1 p_1 \) models the proposition expressed by the sentence "This is an angel" where \( p_1 \) corresponds to "this" and \( 1 \cup_1 \) corresponds to "is an angel." For Frege an expression such as \( 2 \cup_1 a_2 a_3 \) models the proposition expressed by the sentence "This is larger than that" where \( a_2 \) corresponds to "this," \( a_3 \) to "that," and \( 2 \cup_1 \) to "is larger than." Thus according to the theory attributed to Ockham the two propositions have the same structure (they are modeled by expressions of the form "xCy") whereas for Frege the two propositions have different structure (one is modeled by an expression of the form "\( 1 Xx \)" and the other by an expression of the form "\( 2 Xxy \)"

But Ockham pays a price for this elegance. He must have propositional constituents corresponding to "larger than A," "larger than B," "larger than C," . . . in addition to constituents corresponding to "A," "B," "C," . . . . Frege requires only propositional constituents corresponding to "is larger than" in addition to those corresponding to "A," "B," "C," . . . .

Consider the singular proposition represented by \( s_1 C p_1 \)." According to Ockham this proposition is true if and only if \( s_1 \) and \( p_1 \) stand for the same thing. Let us call the things for which the subjects and predicates stand objects. \( O_1, O_2, \ldots \) will be used to
label objects. Thus, if both \( s_1 \) and \( p_1 \) stand for (are intentionally tied to) \( O_5 \), for example, then the proposition \( s_1 p_1 \) is true.

A major difference between Frege's theory and Ockham's theory is apparent. The subject of a Fregean proposition can never stand for the same thing as the predicate. A subject of a Fregean proposition stands for the saturated constituents of a fact, if it stands for anything, whereas the predicate of a Fregean proposition stands for an unsaturated constituent of a fact.

Frege and Ockham also differ concerning the number of entities to which the predicate of a proposition may be intentionally tied. According to Frege, there is exactly one entity which the predicate of a proposition intends, but Ockham would claim that there may not be exactly one. To illustrate this divergence in views consider the singular proposition expressed by "This is red." Frege would claim that there is exactly one intention of the predicate corresponding to "is red" which unites the intention of the subject, if there is such an intention, to either the True or the False. But it is Ockham's view that the predicate of the proposition intends more than one entity (object). Frege would claim that \( \langle U^p a^p \rangle \) represents the proposition expressed by "This is red." For Frege there is exactly one entity to which \( U^p \) is intentionally tied, viz., \( U^b \). Ockham would claim that \( s_1 p_1 \) represents the proposition expressed by "This is red." According to Ockham, \( p_1 \) may be intentionally tied to several objects, perhaps \( O_3, O_7, \) and \( O_{10} \). It is not that both Frege and Ockham cannot account for the commonsensical belief that there are
several things which are red. To see this, consider the proposition
expressed by "That is red." Suppose it is a matter of commonsense
that both "This is red" and "That is red" are true. Suppose \( \uparrow p \downarrow a \)
is Frege's representation of the latter sentence. Frege accounts for
the truth of the two propositions by claiming that \( \cup a \) connects both
\( a_1 \) and \( a_2 \) to the True. In contrast, Ockham would claim that the
common predicate \( p_1 \) of the two propositions represented by "\( s_1 p_1 \)"
and "\( s_2 p_1 \)" is intentionally tied to the intention of \( s_1 \) and also to
the intention of \( s_2 \).

Not only may a predicate of one of Ockham's propositions be
about more than one object but there may be no object to which it is
intentionally tied. Ockham would account for the truth that no thing
is red, if it were a truth, by claiming that the predicate \( p_1 \) which
-corresponds to "red" is about no thing. On the other hand, Frege
would account for it by claiming that \( \cup a \) unites all of the intentions
of the subjects of singular propositions to the False.

Though Frege's predicates of singular propositions stand in a
one-to-one relationship with their intentions and Ockham's may not,
is there any agreement between Ockham and Frege concerning the inten-
tions of subjects? There is at least this much: There can be at most
one entity which is the intention of the subject of a singular proposi-
tion. Frege would say that there need not be at least one. You will
recall that according to Frege the subject of the proposition ex-
pressed by "The least rapidly convergent series has a limit" is not
about anything. I doubt that there is enough textual evidence to
determine whether Ockham claims that the subject of every singular proposition must be about at least one thing. If we treat Ockham as saying that there are subjects which are not tied to at least one thing, then Ockham, like Frege, could claim that there are propositions which are neither true nor false. If we treat Ockham as saying that each subject of a singular proposition is tied to at least one thing, then it follows that each singular proposition would be true or false.

If one claims that there are propositions which are neither true nor false since there are subject constituents of propositions which do not refer, then he would also have to deny the law of excluded middle. That is, if the subject of the proposition P which is denied is non-refferring, then if the truth or falsity of the proposition which is the denial of P (Not P) is dependent upon whether the subject of P refers, then it would follow that if P is neither true nor false, Not P is neither true nor false. Thus it would follow that there are propositions P such that the proposition, P or not P, is neither true nor false. One could preserve the law of excluded middle and still maintain that there are propositions which are neither true nor false by giving an assymetrical treatment to the pair of propositions P and Not P. By claiming that the truth or falsity of Not P does not depend upon the existence of a referent for the subject constituents of P in Not P whereas the truth or falsity of P depends upon the existence of a referent for P, one could still preserve the law of excluded middle. Frege's theory of truth gives a
symmetrical treatment to the propositions $P$ and Not $P$ and thus by denying that all subject constituents of propositions have referents the theory must also deny the law of excluded middle. According to Bergmann's theory, there are no subject constituents of propositions but all propositions have a truth-value and the principle of excluded middle is preserved. Bradley's theory also preserves the law of excluded middle. The theory of truth ascribed to Ockham will remain neutral on the issue concerning whether the subject constituents of all propositions have referents. Thus, the theory will also be neutral to the issue of whether the law of excluded middle holds. But it should be clear how one would vary Ockham's theme in order to reject the law of excluded middle. I will not write the variations.

Ockham's theory seems to be very natural for propositions expressed by sentences such as "This is an angel" and "This is red." But how can the theory handle propositions which are expressed by sentences which employ "logical terms" such as "and" and "or"? Consider the proposition expressed by the sentence "This is red or that is blue." Does the subject of the proposition correspond to "this" or to "that"? If the subject corresponds to "this," is there a single predicate term corresponding to "is-red-or-that-is-blue"? If the subject corresponds to "that," is there a single predicate corresponding to "This-is-red-or-... is-blue"?

If there is no reason for saying that a constituent corresponding to "this" as opposed to a constituent corresponding to "that" is the subject, it is nonsense to talk about the subject of the
propagation. If either one can be the subject, then neither the one nor the other is the subject. This same argument can be used against the prior treatment which was given to propositions which are expressed by sentences such as "This is larger than that." We took a constituent corresponding to "this" to be the subject and a constituent corresponding to "is larger than that" to be the predicate. But why could not a constituent corresponding to "that" be the subject and a constituent corresponding to "This is larger than" be the predicate? If no good reason can be given for choosing between the two subject-candidates, then I do not see how one can say that the proposition has the form of a subject-predicate proposition.

This problem does not arise with any of the theories which have been previously considered. Bergmann's propositions are simple and thus there are none which are claimed to have a subject-predicate structure. The components of Wittgenstein's propositions have an equal status and there is no division in terms of types of constituents. Frege does recognize subject-predicate propositions (those of the form \(\mathbf{U}_i a_j\)) but refuses to call propositions expressed by sentences such as "This is larger than that" subject-predicate propositions. For Frege there is a constituent corresponding to "this" and a constituent corresponding to "that," but neither of the constituents can be identified as the subject.

If there were a means of answering this objection and of thus treating propositions expressed by "This is taller than that" and "This is red or this is blue" as subject-predicate propositions, it
is interesting to note that there would be neither relational constituents of propositions nor logical constituents of propositions, i.e., there would be no constituents corresponding to terms such as "is larger than," "is smaller than," etc., and also no constituents corresponding to terms such as "or," "and," etc.

A Revision of Ockham

Rather than dropping the theory at this point, I will attempt to revise the theory to overcome the objection which has been raised and still try to keep some of the spirit of the theory.

Rather than shoving relational propositions which are expressed by sentences such as "This is larger than that" into a subject-predicate mold, they will be treated as sui generis propositions. In other words, the treatment given to relational propositions is close to that given by Frege. But, there is still a major difference. Frege's predicates and relations are unsaturated constituents of propositions, whereas the predicates and the relations of the revision of Ockham's theory are saturated constituents of propositions. For Frege, no copula was required; for the present theory all propositions possess a copula. Parentheses will be used to mark the existence of the copula. I will use the symbols $\overline{p_1}$, $\overline{p_2}$, ..., $\overline{p_1}$, $\overline{p_2}$, ..., $\overline{p_1}$, $\overline{p_2}$, ..., and so on to stand for the predicates and relations which are saturated constituents of propositions. If a predicate or relation is labeled by a symbol which has the left-hand subscript n, then there are n other constituents which will be called subject constituents. I will use the symbols $\overline{s_1}$, $\overline{s_2}$, ...
to label these constituents. In general, the symbols of the following form will denote propositions:

\[ mP_j(s_{11}, s_{12}, \ldots, s_{1m}) \]

provided that \( m, j, s_{11}, s_{12}, \ldots, s_{1m} \) = 1, 2, 3, \ldots.

To illustrate the symbolism, consider the proposition expressed by the sentence "This is larger than that." Such a proposition could be represented by \( aP_1(s_3, s_4) \) where \( aP_1 \) corresponds to "larger than," \( s_3 \) corresponds to "this," \( s_4 \) corresponds to "that," and the parentheses correspond to "is." (The parentheses represent the entity which binds the other entities into a single proposition. It is this kind of entity which Wittgenstein wished to avoid. We have noticed the difficulties which befell him as a consequence of this bit of frugality.)

Let us also consider the proposition expressed by "A is between B and A." We can use the symbol \( 3P_2(s_1, s_2, s_1) \) to represent this proposition. \( 3P_2 \) corresponds to "between," \( s_1 \) corresponds to "A," \( s_2 \) corresponds to "B," and the parentheses denote the copula.

The symbolism developed up to this point is not equipped to deal with propositions expressed by sentences which employ the logical terms such as "not," "and," and "if." The symbolism will be expanded shortly. The propositions which the symbolism is suitable for representing will be called atomic propositions.

We are now in a position to state the criterion for the truth of atomic propositions:
An atomic proposition is true if and only if the predicate part of the proposition designates an ordered n-tuple whose constituents are denoted by the subject constituents of the proposition.

To illustrate this criterion, let us consider the proposition expressed by the sentence "This is larger than that" which has been represented by "$P_1(s_3, s_4)". $P_1$ designates ordered 2-tuples (pairs). Suppose that it designates only the following ordered pairs: $(0, 0)$, $(0, 0)$ and $(0, 0)$. Now if it is the case that $s_3$ denotes $0$ and $s_4$ denotes $0$, then the proposition $P_1(s_3, s_4)$ is true. But if $s_3$ denotes $0$ and $s_4$ denotes $0$, then the proposition is false.

Though the predicate of an atomic proposition may denote several entities (ordered n-tuples), the subjects of such propositions denote at most one entity. I will not restrict the theory in such a way that would require the subjects to denote exactly one entity. It was noted earlier in the chapter if such a restriction is made then every atomic proposition is either true or false. Without such a restriction, one is in a position to adopt the Fregean position that there are atomic propositions which are neither true nor false.

The truth of the proposition $P_1(s_1, s_2, s_3)$ depends upon whether there is an ordered 3-tuple designated by $P_1$ whose constituents are denoted by $s_1$, $s_2$, and $s_3$, respectively.

This non-linguistic account of the truth of atomic propositions is somewhat similar to the linguistic account of truth of
sentences which is found in Tarski's "The Concept of Truth in Formalized Languages." Tarski's account of truth is linguistic since linguistic expressions, not propositions, are the bearers of truth. According to Tarski infinite ordered sequences "satisfy" or fail to "satisfy" certain linguistic expressions and the truth of these expressions is dependent upon whether or not these expressions are satisfied. For Tarski, the connection between ordered sequences and linguistic expressions must be "set up." According to the non-linguistic theory at hand, the connection between ordered n-tuples and propositions (not: linguistic expressions) is "already there." Propositions designate ordered n-tuples independently of our conventions. (Designation can be considered the converse of satisfaction.) In Ockham's terminology, propositions are natural and not conventional signs.

The remaining problem with which I will deal is that of giving an account of the truth of propositions expressed by sentences involving logical terms, such as "and" and "or," which is in line with the above account of the truth of atomic propositions. There is a "natural" account which I will give which differs from the former accounts in a very significant respect.

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The account is essentially Ockham's. Ockham writes:

For the truth of a copulative proposition, it is required that each part be true.\(^4\)

Ockham also states:

For the truth of a disjunctive proposition it is necessary that one or the other part be true.\(^5\)

A negative proposition is true only if the negated part of the proposition is false. A Sheffer-stroke proposition is true only if neither of the two major parts is true.

The distinctive feature of this account is that compound propositions are true or false in a vicarious fashion. Unlike atomic propositions, compound propositions are not true or false in virtue of what they designate but are true or false in virtue of the truth or falsity of the parts. Consider the compound propositions denoted by "\(2P_1(s_1, s_2) \text{ And } 2P_2(s_3, s_4)\)" and "\(2P_1(s_1, s_2) \text{ Or } 2P_2(s_3, s_4)\)." The only parts of these two propositions which designate are \(2P_1, 2P_2, s_1, s_2, s_3,\) and \(s_4\). Thus, in so far as we can talk about the propositions per se as designating entities, they both designate the same things. But the latter proposition may be true and the former proposition may be false.

Frege's and Bergmann's compound propositions are not true in a vicarious fashion. No two of their compound propositions designate the same things and still have differing truth-values. (In fact, for Bergmann and Frege, distinct propositions always designate distinct

\(^4\)Ockham, *Philosophical Writings*, p. 80.

\(^5\)Ibid., p. 81.
things. For Frege, they designate distinct sequences of facts. For 
Bergmann, they designate distinct facts.) Given the inadequacy of 
Wittgenstein's account of propositions as complex entities the question 
of the truth of compound propositions could not even arise. Of course, 
Bradley's propositions do not designate at all and thus Bradley's 
propositions which are expressed by compound sentences cannot be 
vicariously true in the manner in which Ockham's are.

Though Ockham's propositions are true in a vicarious fashion, 
Ockham's view concerning the truth of thoughts such as the thought 
that A and B and the thought that A or B conforms with the truth 
tables which are found in contemporary logic texts. But so also do 
the views of Bradley, Frege, and Bergmann. The difference is that the 
accounts of truth given by the latter three philosophers ground the 
truth of thoughts such as the thought that A or B in the world, not 
in simple propositions. Ockham grounds the truth of thoughts such as 
the thought that A or B in the truth of the thought that A and the 
thought that B. Ockham's theory is flawed for this reason. The 
thought that A or B seems to me to be no more removed from the world 
than the thought that A. The truth of the thought that A or B is 
just as closely related to the state of the world as is the truth of 
the thought that A. Bradley's, Frege's, and Bergmann's theories do 
justice to these intuitions; Ockham's does not. By making the truth 
of thoughts like the thought that A or B dependent upon (not merely
related to) the truth of the thought that A, Ockham is giving some thoughts a parasitical status. The phenomenological evidence counts against Ockham. On this score, it is clear that the theories of Bradley, Frege, and Bergmann are victorious.
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