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DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree of Doctor of Philosophy in the Graduate
School of The Ohio State University

By

James Howard Cook, B.E.E., M.S.

* * * * * * *

The Ohio State University
1970

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CHAPTER I
INTRODUCTION

1.1 The Importance of Interferometric Techniques in Radio Astronomy

In 1932 Karl G. Jansky, a research engineer for the Bell Telephone Laboratories, accidentally recorded radio noise originating from our galaxy while studying the direction of arrival of atmospheric radio noise. This marked the birth of radio astronomy. Since then this new astronomy has undergone considerable development as astronomers have exploited the techniques of radio engineering to measure cosmic electromagnetic radiation. Although these techniques have been in use only about 30 years, the results have revolutionized astronomy.

Radio astronomy deals with the radiation coming from celestial objects just as optical astronomy does. The difference between radio and optical astronomy lies in the wavelength range each uses. Radio astronomers observe radiation in the millimeter to dekameter wavelength range while optical astronomers observe light radiation which is in the $3 \times 10^{-5}$ cm. to $10^{-3}$ cm. range. It is this wavelength difference which makes the observational problems of radio astronomy so different from those of optical astronomy. Optical radiation
is disturbed by the atmosphere so that even under ideal conditions ground based observations can usually attain effective resolutions only slightly better than 1" arc. If the atmospheric disturbances could be eliminated the largest optical telescopes could theoretically provide much higher resolutions. Radio radiation is not affected by the atmosphere to the extent that optical radiation is and the resolution obtained in radio astronomy is limited only by the size of the radio telescope used. The size of the radio telescope is in turn mostly limited by the money available, since the cost of building it increases nearly exponentially with size. Typical large radio telescopes of the filled aperture type will have resolutions of at best a few minutes of arc. To obtain a 1" arc resolution at 10 cm. wavelength would require a telescope about 13 miles in diameter. Building a single telescope even a mile in diameter is clearly out of the question. Faced with this problem radio astronomers have used interferometric techniques to synthesize large radio telescopes with smaller less expensive telescopes.

1.2 Aperture Synthesis

The exact details involved in using interferometric techniques for the synthesis of large radio telescopes (aperture synthesis) will be given in Chapter 2. Here, only the general procedure will be given. The principles involved in the use of an interferometer for radio astronomy may also be
found in the literature.\textsuperscript{13,14,19,54,56,57,77,80}

Before proceeding further it should be pointed out that the radio telescopes considered here have filled circular apertures. The circular shape is not the only one used in practice, but it is used here for its simplicity. Also, in many radio telescopes one part of the aperture may be separated by more than one wavelength from the nearest part of the remaining aperture. One example of such an unfilled aperture is the Mills Cross telescope in Australia.\textsuperscript{53}

An example of a filled aperture would be a dish type antenna such as that commonly used in radar. As with the circular shape, filled apertures are considered here for their simplicity.

Let us now consider the synthesis of a large radio telescope, one with an aperture 13 miles in diameter. Imagine that the telescope is of the parabolic dish type. (See figure 1.) Under normal operation the radiation from the source at which the telescope is pointed is reflected from the surface of the dish and collected by the feed horn. To obtain a picture of the source it is necessary to scan the telescope over the source and build a picture similar to the manner of a television picture tube. The principle behind aperture synthesis is as follows: The time average of the intensity distribution over the source at which the telescope is pointed, i.e., the source picture built up TV style under
Figure 1. Parabolic reflector radio telescope.
normal operation, is the Fourier transform of the spacial autocorrelation of the electric field distribution over the aperture. Referring again to figure 1, let the aperture be marked off in squares. The spacial autocorrelation function of the field at the aperture may be obtained if one has \(<E_iE_j^*>\) for all i-j pairs. (The i and j subscripts refer to particular squares on the aperture, \(< ... >\) refers to a time average, and E refers to the electric field.) When using an interferometer to synthesize a large aperture all possible \(<E_iE_j^*>\) pairs are collected sequentially and combined in a computer to give the source picture. It may take several days to collect all \(<E_iE_j^*>\) pairs. It is assumed that the source does not change its radiation characteristics during that time.

Figure 2 contains an illustration of some of the measurements that are taken to synthesize a large aperture. As shown in figure 2, an interferometer may consist of two small radio telescopes connected together such that the output is the time average product of the fields at each telescope. The time average product is often referred to as the correlation of the signals, and this interferometer is called a correlation interferometer. Some interferometers add the signals from each telescope and are called addition interferometers. Only correlation interferometers are considered here since that type is most commonly used for the aperture synthesis work.
Figure 2. The aperture synthesis procedure. Each day an observation is taken at a different position on the aperture until all possible $<E_iE_i^*>$ measurements are obtained. These are then Fourier transformed in a computer to provide a source map. The resolution of the source map is determined by the size of the synthesized aperture.
1.3 Recent Work Involving Two-Element Interferometric Aperture Synthesis

Accounts of the early development of interferometry in radio astronomy may be found in the literature. Here only the more important work done in this area since 1960 will be considered.

At present there are six research groups which are engaged in extensive aperture synthesis work. Ryle's group at Cambridge, England has the most experience in this area. The main instrument there has a maximum separation of one mile and obtains resolutions of 80" arc and 23" at 408 and 1407 MHz respectively. Ryle pioneered in the development of both the phase switching interferometer and the supersynthesis technique. This technique makes use of the earth's rotation to facilitate the aperture synthesis procedure.

The interferometer at Owens Valley, California has been used by a group from the California Institute of Technology to do one-dimensional aperture synthesis of many radio sources. The instrument there consists of two 90 ft. parabolic reflectors which can give a resolution of 45" arc at 1425 MHz. Two baselines are available, one E-W and one N-S. The group at Owens Valley was the first to obtain the polarization brightness distribution of cosmic sources by interferometric techniques.

The 21 cm. interferometer at Nancay, France has been
used by Lequeux to observe continuum sources. Each antenna is a 7.5 m. paraboloid. The baseline is in the shape of a "T" and the antennas are moved on a railroad track. The resolving power is 18" arc in the E-W direction and 67" arc in the N-S direction.

Two interferometers have recently been used at Jodrell Bank, England. An extensive program to measure the angular diameter of radio sources is being carried out using the Mark 1 telescope as one element of a very long baseline interferometer. Conway and Kronberg have recently used the Mark 1 and Mark 2 telescopes as an interferometer to obtain one-dimensional polarization distributions of 1.2 minute arc resolution over the Crab Nebula and 3C315.

The interferometer of the National Radio Astronomy Observatory (henceforth referred to as NRAO) at Green Bank, West Virginia has also been used for studies of radio sources. The interferometer consists of three 85 ft. parabolic reflector antennas located on one baseline such that a maximum resolution of 8" arc can be obtained at 2695 MHz. Three independent, two-element interferometers are operated simultaneously between the three antennas. Recently, a portable 40 ft. antenna has been added to the system which should enable even higher resolutions to be obtained.
The Australians have recently built a two-element interferometer. Morris and Whiteoak used the 210 ft. telescope at Parks, Australia in conjunction with a smaller antenna to find the one-dimensional linear polarization over 13 extended sources at 21.2 cm. wavelength.72

1.4 Present Objective

This work represents the first part of a group effort, directed by Dr. H. C. Ko, to gain a working familiarity with polarization interferometry. Initially use is being made of the interferometer facility at the National Radio Astronomy Observatory (NRAO) in Green Bank, West Virginia. As a "trial run" on the NRAO interferometer, observations of about 50 radio sources were undertaken for two weeks during September and October 1968. The author assumed the responsibility for developing the necessary computer software at Ohio State to calibrate and reduce these observations so that further observations could be brought back from the NRAO facility in the "raw data" stage, to be reduced using the Ohio State computer facility. Some of the necessary software already was developed at NRAO. It was felt, however, that independently developing our own system of software would not only teach us much about the little detailed matters which might otherwise be overlooked, but also would decrease our dependence upon the NRAO facility. Also, the interferometer at NRAO had not yet been used to obtain polarization
information, and so the software to reduce this did not exist at NRAO.

The results obtained from two of the sources observed in 1968, the Crab Nebula and 3C33, are presented here along with a discussion of the procedures which were subsequently developed to calibrate the interferometer. Thus these observations fulfill a dual role. They are of interest observationally since the NRAO interferometer yields resolutions much higher than had previously been used to observe the Crab Nebula or 3C33, and they provided a good first test of the software system since both sources were expected to yield significant polarization.

The analysis of the interferometer output is carried out using the coherency matrix formalism as set forth by Ko.\textsuperscript{56,57} Since this formalism has not been widely used for antenna systems in the past, two chapters are devoted to a description of the coherency matrix and its application to the description of the interferometer antenna system.

At the time this work was started it was the highest resolution yet to be obtained for complete one-dimensional polarization and brightness distributions by the aperture synthesis technique. While this work was being finished Fomalont\textsuperscript{33} published the polarization and brightness distributions of 3C20 which he obtained using the NRAO interferometer at the same resolution. Thus this work cannot
claim the record for the highest resolution. It does represent a state-of-the-art measurement, however; and the methods used here to analyze the instrument are unique.

1.5 Definitions

It is now convenient to define a few of the terms which will be used in the remainder of this dissertation. The type of interferometer which will be considered here is a two-element, double side-band, correlation interferometer. The adjective "correlation" has been defined above. "Double side-band" refers to the type of heterodyning in which the RF passband extends far enough on each side of the local oscillator frequency that the image of the IF passband is also included in the IF signal. "Two-element" means that at each end of the interferometer only one antenna receives the signal. Some interferometers have several antennas at each end and may be called multi-element interferometers. Here the word "element" will be used synonymously with antenna. The line between the elements is referred to as the baseline of the interferometer. This will be designated by a vector \( \vec{B} \). The units of \( \vec{B} \) are RF signal wavelengths. The magnitude of \( \vec{B} \), in meters, will be designated \( D \). The power received from a radio source will be given in flux units. One flux unit is equal to \( 10^{-26} \) watts per square meter per Hertz (W/m\(^2\)Hz).
2.1 **Introduction**

In this chapter the theory involved in using an interferometer to make brightness distribution measurements is discussed so that certain concepts basic to interferometry may be presented. The term "scalar" is used here to indicate that only brightness distribution measurements are to be considered. Later the term "tensor" will be used to indicate that polarization distribution measurements are to be considered. The use of these terms to distinguish between polarization interferometry and brightness interferometry originated with Ko. It stems from the use of the electric coherence tensor (often called the coherency matrix) to characterize the polarized radiation.

The following sections contain a theoretical description of a two-element, double side-band, correlation interferometer. The interferometer elements are considered to have identical characteristics. Thus, for instance, both may receive only left circular polarization. The interpretation of the output expected from a completely unpolarized point source is given first, assuming isotropic elements.
The effects of the bandwidth and the size of the elements are then discussed with specific reference to the NRAO interferometer. The visibility function is presented and the Fourier transform relationship between the visibility function and the brightness distribution is discussed.

2.2 The Response to a Point Source

Read obtained the following expression for the power response of a correlation interferometer to a point source radiating in the frequency range between \( f \) and \( f + df \).

(See figure 3.)

\[
(2.2-1) \quad dS = \frac{I^2}{8} \cos \left[ \omega_{IF}(\tau + \frac{\Delta x}{c} + \frac{\Delta z}{c}) - \Delta \phi \right] \left( \pm w_L(\tau + \frac{\Delta x}{c} - \frac{\Delta y}{c}) \right) df
\]

In this equation

- \( I^2 \) is proportional to the average intensity of the point source in the frequency range.
- \( \tau \) is the time delay for the signal between antennas. (\( \tau \) is positive if the signal reaches antenna 1 first.)
- \( \Delta x \) is the difference in the path lengths between the antennas and the mixers.
- \( \Delta y \) is the difference in the path lengths between the local oscillator and the mixers.
- \( \Delta z \) is the difference in the path lengths between the mixers and the correlator.
- \( \Delta \phi \) is the difference between the phase shifts introduced by the IF amplifiers.
c is the velocity of light.

$w_{IF}$ is the intermediate frequency in radians per second.

$w_{LO}$ is the local oscillator frequency in radians per second.

The upper sign before the $w_{LO}$ term is to be used if the signal is in the upper side-band and the lower sign is to be used if it is in the lower side-band. The $\tau$ term is a slowly varying function of time as the source drifts across the sky. The $\Delta x$, $\Delta y$, $\Delta z$, and $\Delta \phi$ terms should remain constant in theory, but in practice they vary slightly.

The schematic diagram in figure 3 is that of a two-element, double side-band, correlation interferometer. The advantage of using the double side-band technique may be easily demonstrated using equation 2.2-1. In general radio sources radiate continuously over large ranges of frequency and may be considered to radiate a constant power flux density over the narrow frequency range covered by the IF passband. Let the IF frequency response be approximated by a rectangular function of width $b$ radians per second as shown in figure 3. Integrating the response as given by equation 2.2-1 over the IF passband yields the following:
Figure 3. A schematic diagram of a two-element, double side-band, correlation interferometer.
1) The single side-band case (The RF amplifier passes only the upper side-band.)

\[ S = b \left( \frac{1}{4} \right) \sin \left[ \frac{b}{2} (\gamma + \frac{\Delta X}{c} + \frac{\Delta Z}{c}) \right] X \]

\[ \cos[w_{IF}(\gamma + \frac{\Delta X}{c} + \frac{\Delta Z}{c}) - \Delta \phi + w_{LO}(\gamma + \frac{\Delta X}{c} - \frac{\Delta V}{c})] \]

2) The double side-band case (The RF amplifier passes both the upper and lower side-bands.)

\[ S = b \left( \frac{1}{4} \right) \sin \left[ \frac{b}{2} (\gamma + \frac{\Delta X}{c} + \frac{\Delta Z}{c}) \right] X \]

\[ \cos[w_{IF}(\gamma + \frac{\Delta X}{c} + \frac{\Delta Z}{c}) - \Delta \phi] X \]

\[ \cos[w_{LO}(\gamma + \frac{\Delta X}{c} + \frac{\Delta V}{c})] \]

As \( \gamma \) varies as a function of time the output \( S \) given for both the single side-band case and the double side-band case will behave in a sinusoidal manner. The phase of this sinusoidal variation will depend entirely upon the value for \( \gamma \) at which \( S \) crosses zero. Since \( w_{LO} \) is a great deal larger than \( w_{IF} \), \( w_{LO} \gamma \) will primarily determine these zero points. In the single side-band case the \( \Delta Z(w_{IF}/c) \) and \( \Delta \phi \) terms are added to the \( w_{LO} \gamma \) term and have equal effect on the position of the zero points. In the double side-band case the \( \Delta Z(w_{IF}/c) \) and \( \Delta \phi \) terms are in a separate cosine function.
which varies much more slowly with $\gamma$ than does the cosine function containing the $w_{L0}^{\gamma}$ term. The result of this is that a double side-band interferometer is much less sensitive to phase instabilities due to small IF amplifier phase shifts ($\Delta\phi$) and small baseline cable expansions or contractions ($\Delta z$) than a single side-band interferometer is.

Equation 2.2-3 gives the response of a two-element, double side-band, correlation interferometer to a polychromatic point source. The elements of the interferometer have been considered to be isotropic up to this point. The effect of introducing directive elements may be analyzed in two ways. The directivity of an element is a consequence of its size. Thus the inclusion of directive elements necessitates the inclusion of elements which cover a finite area, and the baseline between the interferometer elements will no longer be a discrete value but will include a range of values. One way to account for the directional elements is to integrate, over the range of baseline values, the interferometer response as given in equation 2.2-3 weighted by an appropriate function. This procedure involves much labor and is not recommended.

The effect of the directional elements may also be determined by obtaining the power patterns of the elements and multiplying the response of equation 2.2-3 by their
geometric mean. It may be shown that this procedure gives the same result as the procedure given in the last paragraph. Since the power pattern of each element is usually well known and no integration is involved, this procedure is the easier to apply.

Let us proceed to include the effect of directional elements in the response. Consider that each element is a parabolic dish antenna. The power pattern is approximately

\[ P = A e^{-\ln[2(\frac{a}{a_{HP}})^2]} \]

where \( a \) and \( a_{HP} \) are defined as in figure 4 and "A" is the effective aperture of the antenna. Assuming that both antennas are identical, the response as given by equation 2.2-3 may be multiplied by \( P \) of equation 2.2-4 to yield

\[ S = bA\left(\frac{\pi}{4}\right)\left\{ \frac{\sin\left[\frac{b}{2}(\gamma + \frac{\Delta x}{c} + \frac{\Delta y}{c})\right]}{\sin\left[\frac{b}{2}(\gamma + \frac{\Delta x}{c} + \frac{\Delta y}{c})\right]} \right\} \times \]

\[ \cos\left[w_{IP}(\gamma + \frac{\Delta x}{c} + \frac{\Delta y}{c}) - \Delta \theta\right] \times \]

\[ \left\{ e^{-\ln[2(\frac{a}{a_{HP}})^2]} \right\} \times \]

\[ \cos\left[w_{LO}(\gamma + \frac{\Delta x}{c} - \frac{\Delta y}{c})\right] \]

The parameter \( \gamma \) in equation 2.2-5 may be replaced by \( (D\sin \theta)/c \). The parameter "D" is the separation between
\( \alpha_{HP} \) represents the half power width of the antenna power pattern.

\( s \) is a unit vector pointing at the radio source which is being tracked by the interferometer.

\( \theta \) is the angle between the plane perpendicular to \( B \) and \( s \).

**Figure 4.** Parameters describing the orientation of a two-element interferometer.
interferometer elements in meters, and $\theta$ is the angle of the source position from a plane perpendicular to the interferometer baseline. (See figure 4.) Making this substitution for $r$ in equation 2.2-5 and rearranging the terms yields

$$S = S_0 \left\{ \frac{\sin \left[ \frac{b}{2} \left( \frac{D \sin \theta + \Delta x + \Delta z}{c} \right) \right]}{\frac{b}{2} \left( \frac{D \sin \theta + \Delta x + \Delta z}{c} \right)} \right\} \times \left\{ \cos \left[ \frac{w_{IF}}{2} \left( \frac{D \sin \theta + \Delta x + \Delta z}{c} - \Delta \phi \right) \right] \right\} \times \left\{ \exp \left[ -\ln \left( 2 \frac{\alpha}{\alpha_{HP}} \right)^2 \right] \right\} \times \left\{ \cos \left[ \frac{w_{LO}}{2} \left( \frac{D \sin \theta + \Delta x + \Delta z}{c} - \Delta \Gamma \right) \right] \right\}$$

where

$$S_0 = b A \left( \frac{\lambda^2}{4} \right)$$

Figure 5 contains a plot of both bracketed terms in equation 2.2-6 as functions of $|\theta|$ and $|\alpha|$, assuming $\Delta \phi$, $\Delta x$, and $\Delta z$ are zero. The values taken for $b/2\pi$ and $w_{IF}/2\pi$ are 10 MHz and 7 MHz respectively. The exponential term in figure 5 will be called the element pattern, and the other term will be called the bandwidth pattern.

The bandwidth and element patterns cause a source to be attenuated for large values of $|\theta|$ or $|\alpha|$. It is desirable to be able to measure the interferometer response to sources at any point in the sky. To do this the bandwidth and element patterns must be continuously moved so that
Figure 5. The bracketed terms in equation 2.2-6.
their maxima remain on the source as it drifts through the sky. This process is called tracking the source. The bandwidth pattern may be moved along with the source by varying the $\Delta z$ parameter such that $D \sin \theta + \Delta z + \Delta x$ remains nearly equal to zero. In practice this is what is done, and the only cases for which the bandwidth pattern has to be considered are if the position of the source is in error or if the source is very large. Similarly, the element pattern may be moved along with the source by continually turning both antennas so that they always point at the source.

Assuming now that the interferometer is tracking the point source, the bandwidth and element patterns may be ignored and the response is

\[(2.2-7) \quad S = S_0 \cos \left( w_0 \frac{D \sin \theta + \Delta x - \Delta y}{c} \right) \]

Further, if complex notation is used and $S_0$ is replaced by $S_0^*$ where

\[(2.2-8) \quad S_0^* = S_0 e^{j w_0 \frac{\Delta x - \Delta y}{c}} \]

the response may be written in the following concise manner.

\[(2.2-9) \quad S = S_0^* e^{j w_0 \frac{D \sin \theta}{c}} \]

Although $S_0^*$ is only proportional to the flux from the point source, in practice the interferometer is calibrated so that
the proportionality constant is unity. From this point \( S \) will be considered to be equal to the flux which is received by the interferometer.

2.3 The Visibility Function

Let us now examine the response of the interferometer to a point source whose true position differs from its assumed position. To relate this response to the position of the source on the celestial sphere and to the orientation of \( \hat{e} \), the hour angle (\( H \)) and declination (\( \delta \)) coordinates will be used in conjunction with the rectangular coordinates \((\hat{x}, \hat{y}, \hat{z})\) shown in figure 6.

\[
\begin{align*}
\hat{x} & \text{ points toward } H = 0^\circ, \delta = 0^\circ \text{ and is of unit length} \\
\hat{y} & \text{ points toward } H = 6^\circ, \delta = 0^\circ \text{ and is of unit length} \\
\hat{z} & \text{ points toward } \delta = 90^\circ \text{ and is of unit length}
\end{align*}
\]

Figure 6. Coordinate systems.
In what follows a position on the celestial sphere will be specified by a unit vector \( \hat{s} \). The true position of a source may be designated \( \hat{s}_t \) and its assumed position may be designated \( \hat{s}_a \). Using the above coordinates, \( \hat{s}_t, \hat{s}_a, \) and \( \hat{B} \) are written as follows:

\[
(2.3-1) \quad \hat{s}_t = \cos \delta_t \cos \lambda_t \hat{x} + \cos \delta_t \sin \lambda_t \hat{y} + \sin \delta_t \hat{z} \\
\hat{s}_a = \cos \delta_a \cos \lambda_a \hat{x} + \cos \delta_a \sin \lambda_a \hat{y} + \sin \delta_a \hat{z} \\
\hat{B} = B_x \hat{x} + B_y \hat{y} + B_z \hat{z}
\]

The response of the interferometer to a point source at \( \hat{s}_a \) is given by equation 2.2-9 as

\[
(2.3-2) \quad S = S_0 e^{j \frac{wL \sin \theta}{c}}
\]

This may be rewritten as

\[
(2.3-3) \quad S = S_0 e^{j 2\pi B_0 \sin \theta_a}
\]

since

\[
(2.3-4) \quad wL_0 \left( \frac{D}{c} \right) = 2\pi B_0
\]

where \( B_0 \) is the magnitude of \( \hat{B} \), the baseline vector.

Using the vector notation described above

\[
(2.3-5) \quad S = S_0 e^{j 2\pi \hat{B} \cdot \hat{s}_a}
\]
If the source is moved from \( \hat{s}_a \) to \( \hat{s}_t \), \( S \) may be written as

\[
(2.3-6) \quad S = V e^{j2\pi B^* \hat{s}_a}
\]

where \( V = S_0 e^{j2\pi B^*(\hat{s}_t - \hat{s}_a)} \). The parameter "\( V \)" is the visibility function of the source with respect to the assumed position.

Let us examine equation 2.3-6 in detail. The output from the correlator is actually equal to the real part of \( S \). Thus as \( B^* s_a \) changes, the correlator output will oscillate sinusoidally. These sinusoidal oscillations are called fringe. The visibility function, \( V \), is complex; and to extract \( V \) from the correlator output two numbers have to be obtained. One number, the amplitude of \( V \), is obtained from the amplitude of the fringe. The second number is determined by comparing the phase of the fringe (the sine wave) to the phase of a reference sine wave generated by an on-line digital computer. The phase of this reference sine wave is kept equal to \( 2\pi B^* s_a \) by the computer so that the difference between the phase of the fringe and the phase of the reference sine wave will be \( 2\pi B^*(\hat{s}_t - \hat{s}_a) \), the phase of \( V \).

In the NRAO interferometer the output from the correlator must be digitized so that the on-line computer can compare the fringe phase to that of the reference...
signal. Some interferometers do this comparison by analogue techniques. One such way is to construct two correlators instead of one. The input from one element of the interferometer is shifted by a quarter wavelength before being correlated in one of the correlators, producing a sine output. The other correlator is fed unshifted signals, producing a cosine output. A continuously increasing phase of $2\pi B \cdot \hat{s}_a$ is then introduced into both correlators so that their outputs will have a phase which depends only on $2\pi B \cdot (\hat{s}_t - \hat{s}_a)$. The amplitude of $V$ for an interferometer of this type is the square root of the sum of the squares of the sine and cosine outputs. The phase is given by the arctan of the sine/cosine quotient. In the analogue case just described the continuously increasing phase shift of $2\pi B \cdot \hat{s}_a$ takes the place of the reference signal generated by the computer at NRAO. The analogue system uses two correlators and an analogue device (to generate the $2\pi B \cdot \hat{s}_a$ phase shift), while the NRAO system uses one correlator and an on-line digital computer.

Continuing with the analysis, the expression $B \cdot (\hat{s}_t - \hat{s}_a)$ may be expanded as follows:
\[
(2.3-7) \quad \vec{B} \cdot (\hat{s}_t - \hat{s}_a) = \vec{B} \cdot \left( \frac{d\hat{S}}{d\delta} \right|_{\delta_aH_a} (\delta_t - \delta_a) + \frac{d\hat{S}}{dH} \right|_{\delta_aH_a} (H_t - H_a) \right] + \text{Higher Order Terms}
\]

\[
(2.3-8) \quad \frac{d\hat{S}}{d\delta} \bigg|_{\delta_aH_a} = -\sin\delta_a \cos H_a \hat{x} - \sin\delta_a \sin H_a \hat{y} + \cos\delta_a \hat{z}
\]

\[
(2.3-9) \quad \frac{d\hat{S}}{dH} \bigg|_{\delta_aH_a} = -\cos\delta_a \sin H_a \hat{x} + \cos\delta_a \cos H_a \hat{y}
\]

Define

\[
(2.3-10) \quad v = \vec{B} \cdot \frac{d\hat{S}}{d\delta} \bigg|_{\delta_aH_a}
\]

\[
(2.3-11) \quad u = (\vec{B} \cdot \frac{d\hat{S}}{dH} \bigg|_{\delta_aH_a}) \frac{-1}{\cos\delta_a}
\]

\[
(2.3-12) \quad \beta = \delta_t - \delta_a
\]

\[
(2.3-13) \quad \sigma = (H_a - H_t) \cos\delta_a
\]

Here \(v\) is the projection of \(\vec{B}\) in the direction of increasing declination at \(\delta_aH_a\). The parameter \(u\) is the projection of \(\vec{B}\) in the direction of increasing right ascension at \(\delta_aH_a\).

Using the parameters defined above and ignoring the higher order terms in equation 2.3-7 the expansion of \(\vec{B} \cdot (\hat{s}_t - \hat{s}_a)\) may be written as

\[
(2.3-14) \quad \vec{B} \cdot (\hat{s}_t - \hat{s}_a) = \sigma u - \beta v
\]
The visibility function may now be written as

\[ V = S \cdot e^{j2\pi(\sigma u + \beta v)} \]  \hspace{1cm} (2.3-15)

and \( S \) may be written as

\[ S = S \cdot e^{j2\pi(\sigma u + \beta v)} \cdot e^{j2\pi \delta a} \]  \hspace{1cm} (2.3-16)

It may be observed from equation 2.3-15 that the effect of a small deviation of a point source from its assumed position is a phase shift in the interferometer response. A westward displacement of the true source position \( H_t \delta_t \) corresponds to a negative \( \sigma \) and a southern displacement to a negative \( \beta \). The phase error introduced into \( S \) by neglecting the higher order terms of equation 2.3-7 is less than 5° for a source position error of less than 4° arc with a 25,000 wavelength baseline.

2.4 Response to an Extended Source

Consider an extended source centered at \( H_a \delta_a \) with a brightness distribution \( R(\sigma, \beta) \). (See figure 7.)

\[ \beta \text{ (North)} \]
\[ \sigma \text{ (East)} \]
\[ H_a \delta_a \]

**Figure 7.** The extended source.
The response of an interferometer to this extended source may be written as the superposition of many point sources, each with a flux $R(\sigma,\beta)d\sigma d\beta$. Integrating equation 2.3-16 over the source yields

\[ S = \int \int R(\sigma,\beta) e^{j2\pi(\sigma u + \beta v)} d\sigma d\beta \]

This may be rewritten as

\[ S = V(u,v)e^{j2\pi(\vec{B} \cdot \vec{S}_a)} \]

where

\[ V(u,v) = \int \int R(\sigma,\beta) e^{j2\pi(\sigma u + \beta v)} d\sigma d\beta \]

The visibility function in terms of $u$ and $v$ is seen to be the two dimensional Fourier transform of the brightness distribution of the extended source in terms of $\sigma$ and $\beta$.

An extended source may be represented by its brightness distribution or its visibility function. The visibility function may be measured directly by an interferometer. Once the visibility function is completely known (for all possible values of $u$ and $v$) the brightness distribution may be found by Fourier inversion. The visibility function is the representation of the source in the spacial frequency domain (the $u$-$v$ plane), and the brightness distribution is the representation of the source in the space domain (the $\sigma$-$\beta$ plane).
A word of caution is perhaps appropriate here. Remember that the phase of the visibility function is linear with respect to $\beta$ and $\sigma$ only for small values of $|\beta|$ and $|\sigma|$. (See equation 2.3-7.) Further, the error in the visibility phase which is produced by making this approximation is multiplied by $\overline{\varepsilon}$. Thus for a given baseline length ($|\overline{B}|$) there is a size limit above which a source will no longer produce a visibility function which obeys the Fourier transform relationship of equation 2.4-3. Another way of looking at the same thing is to realize that for a given source there is an upper limit to the baseline length at which an interferometer may be operated in a tracking mode and still produce a visibility function which obeys the Fourier transform relationship of equation 2.4-3.

2.5 **The u-v Plane Locus**

The values of $u$ and $v$ at which the visibility function of a source may be sampled by an interferometer are limited by the declination of the source and the number of baseline spacings available to the interferometer. Since $u$ and $v$ are the parametric equations of an ellipse when written in terms of $\delta$ and $H$, an elliptical track will be followed on the u-v plane as the interferometer tracks the source. Each new separation of the interferometer results
in a different elliptical track on the u-v plane. By tracking a source at many different spacings the visibility function may be sampled over most of the u-v plane. This technique, called supersynthesis, was pioneered by Martin Ryle at Cambridge, England.\(^{80,81,82}\) The elliptical tracks which are sampled by the NRAO interferometer are shown in figures 8 through 14 for several source declinations. Here a source is considered observable only if it is at least 15° above the horizon.
Figure 8. Tracks sampled on the u-v plane by the NRAO interferometer when tracking a radio source at declination $-40^\circ$. ($\lambda = 11.1$ cm.)
Figure 9. Tracks sampled on the u-v plane by the NRAO interferometer when tracking a radio source at declination -20°. 
(λ = 11.1 cm.)
Figure 10. Tracks sampled on the u-v plane by the NRAO interferometer when tracking a radio source at declination 0°.

(\(\lambda = 11.1 \text{ cm.}\))
Figure 11. Tracks sampled on the u-v plane by the NRAO interferometer when tracking a radio source at declination 20°.  
(\(\lambda = 11.1\text{ cm}\).)
Figure 12. Tracks sampled on the $u$-$v$ plane by the NRAO interferometer when tracking a radio source at declination $40^\circ$. ($\lambda = 11\,\mu\text{m}$.)
Figure 13. Tracks sampled on the u-v plane by the NRAO interferometer when tracking a radio source at declination $60^\circ$.
($\lambda = 11.1$ cm.)
Figure 14. Tracks sampled on the u-v plane by the NRAO interferometer when tracking a radio source at declination 80°. ($\lambda = 11.1$ cm.)
2.6 **Sampling Theorem Considerations**

The following two generalizations may be made by applying the principles of the sampling theorem to the visibility-brightness function Fourier pair.

1) The greater the extent of the u-v plane sampled by the interferometer, the higher will be the resolution of the brightness distribution obtained on the σ-β plane.

2) The higher the density of sampled points on the u-v plane, the larger will be the area on the σ-β plane which is transformed. A point of diminishing returns occurs here when the area on the σ-β plane gets so large that the bandwidth and element patterns significantly limit the field of view or when the area becomes so large that the higher order terms of equation 2.3-7 may no longer be ignored.

These generalizations are made clearer in Appendix A where the fast Fourier transformation is discussed.
CHAPTER III
THE COHERENCY MATRIX

3.1 Introduction

The coherency matrix is one way to completely describe the polarization properties of electromagnetic radiation. It was first introduced by Wiener\textsuperscript{108} as an application of his generalized harmonic analysis to the description of coherent and incoherent sources of light. It was later used in a slightly different form by Wolf\textsuperscript{109} to suggest a formulation of the theory of optics in terms of observable quantities. This formulation of optics has undergone considerable development in the last ten years.\textsuperscript{11,74,110} Ko, realizing the generality of this formulation, used the coherency matrix in his analysis of the interaction of a radio antenna with statistical radiation.\textsuperscript{51} Since the coherency matrix of the incoming radiation may be easily related to the output of an interferometer it is used in this dissertation to describe the polarization properties of the radiation emitted by a radio source. This chapter contains a description of the coherency matrix and its relationship to the polarization state of the radiation.
3.2 The Composition of a Coherency Matrix

The coherency matrix for an arbitrarily directed, polychromatic, partially polarized plane wave may be written as follows: 52, 58

\[
J = P^T \begin{bmatrix} \rho'_{xx} & \rho'_{xy} & \rho'_{xz} \\
\rho'_{yx} & \rho'_{yy} & \rho'_{yz} \\
\rho'_{zx} & \rho'_{zy} & \rho'_{zz} \end{bmatrix}
\]

\[(3.2-1) \quad J = P^T \begin{bmatrix} \rho'_{xx} & \rho'_{xy} & \rho'_{xz} \\
\rho'_{yx} & \rho'_{yy} & \rho'_{yz} \\
\rho'_{zx} & \rho'_{zy} & \rho'_{zz} \end{bmatrix}\]

\(J\) is the symbol for the coherency matrix, and \(P^T\) is a function of frequency representing the total power in the wave per unit frequency. The \(\rho\) terms are dimensionless functions of frequency.

\[
P^T(f) \rho'_{xx}(f) \longleftrightarrow \frac{<E_x(t)E_x^*(t+\tau)>}{2Z_0} = \frac{T_{xx}(\tau)}{2Z_0}
\]

\[
P^T(f) \rho'_{xy}(f) \longleftrightarrow \frac{<E_x(t)E_y^*(t+\tau)>}{2Z_0} = \frac{T_{xy}(\tau)}{2Z_0}
\]

\[
P^T(f) \rho'_{xz}(f) \longleftrightarrow \frac{<E_x(t)E_z^*(t+\tau)>}{2Z_0} = \frac{T_{xz}(\tau)}{2Z_0}
\]

etc.

In the above expressions \(Z_0\) is the impedance of free space, and \(<...>\) represents a time average. \(\longleftrightarrow\) represents the Fourier transform relationship. \(E_x(t), E_y(t), \ldots\) are the analytic signal representations of the \(x, y, \ldots\) components of the time varying \(E\) field. 11, 52, 98 Each element of the coherency matrix is proportional to the Fourier transform
of a cross-correlation or auto-correlation function. If the wave is considered to be quasi-monochromatic at frequency $f_1$ the Fourier transforms may be eliminated. In this case $P^T(f) \rho'_{xx}(f)$ becomes approximately $P^T \rho_{xx} \delta(f - f_1)$ where $P^T$ and $\rho_{xx}$ are constants, $P^T$ has dimensions of power and $\rho_{xx}$ is dimensionless. Fourier transforming $P^T \rho_{xx} \delta(f - f_1)$ and equating the result with $T_{xx}(\tau)/2Z_0$ will yield

$$\rho_{xx} = \frac{T_{xx}(0)}{2Z_0 P^T}$$

Similarly

$$\rho_{xy} = \frac{T_{xy}(0)}{2Z_0 P^T}$$
$$\rho_{yx} = \frac{T_{yx}(0)}{2Z_0 P^T}$$
$$\rho_{yy} = \frac{T_{yy}(0)}{2Z_0 P^T}$$

The $P^T(f)$, $\rho'_{xx}(f)$, $\rho'_{xy}(f)$, etc. functions may be replaced by the $P^T$, $\rho_{xx}$, $\rho_{xy}$, etc. constants in the coherency matrix for the quasi-monochromatic case. This represents a slight redefinition of the coherency matrix in the sense that $P^T$ now has been replaced by $P^T$ which represents the total power in the wave integrated over all frequencies instead of per unit frequency interval. Also, the $\rho'$s are no longer functions of frequency. From now on the wave will always be considered quasi-monochromatic and the quasi-monochromatic definition will be used.
If the plane wave is directed in the positive $\hat{z}$ direction, $E_z = 0$. In this case

$$\rho_{xz} = \rho_{yz} = \rho_{zz} = \rho_{zx} = \rho_{zy} = 0$$

and the coherency matrix is given as a 2 by 2 matrix.

\[(3.2-2) \quad J = P^T \begin{bmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{bmatrix} \]

In what follows the coordinate systems will always be oriented with the $\hat{z}$ coordinate pointing in the direction of wave propagation.

If the $E$ field vector were represented as the vector sum of two orthogonal circularly polarized waves ($E_L$, $E_R$) instead of two orthogonal linearly polarized waves ($E_x$, $E_y$), the coherency matrix would be written

\[(3.2-3) \quad J = P^T \begin{bmatrix} \rho_{LL} & \rho_{LR} \\ \rho_{RL} & \rho_{RR} \end{bmatrix} \]

where

$$\rho_{LL} = \frac{<E_L^*E_L>}{2\sigma_0}P^T$$

$$\rho_{LR} = \frac{<E_L^*E_R>}{2\sigma_0}P^T$$

etc.
The IRE convention is used here for right and left circular polarization. Thus a right-handed wave rotates counterclockwise when approaching. Also, $e^{-j\omega t}$ time dependence is assumed. Using this convention a monochromatic wave, $\vec{E}$, may be represented as follows:

$$\vec{E}(z,t) = [E_L \hat{\xi}_L + E_R \hat{\xi}_R] e^{j(z - wt)}$$

(3.2-4)

$$\vec{E}(z,t) = [E_x \hat{x} + E_y \hat{y}] e^{j(z - wt)}$$

where

$$\begin{bmatrix} \hat{\xi}_L \\ \hat{\xi}_R \end{bmatrix} = \begin{bmatrix} 1 \\ \sqrt{2} \end{bmatrix} \begin{bmatrix} 1 \\ \sqrt{2} \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix}$$

(3.2-5)

and

$$\begin{bmatrix} E_L \\ E_R \end{bmatrix} = (E_x \ E_y) \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ +i/\sqrt{2} & -i/\sqrt{2} \end{bmatrix}$$

(3.2-6)

To distinguish between the two different representations the coherency matrix will be prefaced circular or linear. The circular coherency matrix is

$$J_c = P^T \begin{bmatrix} \epsilon_{LL} & \epsilon_{LR} \\ \epsilon_{RL} & \epsilon_{RR} \end{bmatrix}$$

(3.2-7)

and the linear coherency matrix is

$$J_l = P^T \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} \\ \epsilon_{yx} & \epsilon_{yy} \end{bmatrix}$$

(3.2-8)
Using equations 3.2-6 it may be shown that

\[
\begin{bmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\
-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{bmatrix}
\]

Notice that the transformation between \( J_1 \) and \( J_c \) is a similarity transformation. This yields two important properties of \( J_1 \) and \( J_c \): First, the determinants of \( J_1 \) and \( J_c \) are equal:

\[
\begin{vmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\
-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{vmatrix} = \begin{vmatrix}
\frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}}
\end{vmatrix}
\]

Second, the trace of \( J_1 \) is equal to the trace of \( J_c \):

\[
\rho_{LL} + \rho_{RR} = \rho_{xx} + \rho_{yy}
\]

3.3 Description of Polarization by Coherency Matrices

The manner by which the polarization configuration of a partially polarized wave can be described by its coherency matrix will now be discussed. Consider an elliptically polarized, quasi-monochromatic plane wave traveling in the \( \hat{z} \) direction. (See figure 15.)
Let $\alpha$ be the absolute value of the axial ratio of the ellipse described by the end point of the $\vec{E}$ field vector at $z = 0$. Also, let $\phi$ be the angle, measured positive in the counter-clockwise direction, between the $\hat{x}$ axis and the major axis of the ellipse. In the $\hat{x}'$, $\hat{y}'$, $\hat{z}'$ coordinates the linear coherency matrix of the wave may be written

$$
J_{1e}^e = P_e \begin{bmatrix}
\frac{1}{1+\alpha^2} & \frac{\pm j\alpha}{1+\alpha^2} \\
\frac{\pm -j\alpha}{1+\alpha^2} & \frac{\alpha^2}{1+\alpha^2}
\end{bmatrix}
$$

where $P_e$ is the power of the elliptically polarized wave and the upper sign is for left hand circular polarization, the lower for right hand circular polarization.

To convert from $\hat{x}'$, $\hat{y}'$, $\hat{z}'$ coordinates to $\hat{x}$, $\hat{y}$, $\hat{z}$ coordinates observe that $J_1^e$, the coherency matrix of the
wave in the \( \hat{x}, \hat{y}, \hat{z} \) coordinates, may be written

\[
(3.3-2) \quad J^e_1 = \frac{1}{2\pi c} \left[ \begin{array}{c} E_x \\ E_y \end{array} \right] \times \left[ \begin{array}{cc} E_x^* & E_y^* \end{array} \right]
\]

\[
= \frac{1}{2\pi c} \left[ \begin{array}{cc} E_x E_x^* & E_x E_y^* \\ E_y E_x^* & E_y E_y^* \end{array} \right]
\]

where \( \times \) represents a Kronecker product. The transformation from primed to unprimed coordinates is as follows:

\[
(3.3-3) \quad \left[ \begin{array}{c} E_x \\ E_y \end{array} \right] = \left[ \begin{array}{cc} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{array} \right] \cdot \left[ \begin{array}{c} E_x^* \\ E_y^* \end{array} \right]
\]

The "\cdot" represents ordinary matrix multiplication. Substituting \( \left[ \begin{array}{c} E_x \\ E_y \end{array} \right] \) as given by equation 3.3-3 into equation 3.3-2 yields

\[
(3.3-4) \quad J^e_1 = \frac{1}{2\pi c} \left[ \begin{array}{cc} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{array} \right] \cdot \left[ \begin{array}{c} E_x^* \\ E_y^* \end{array} \right] \times \left[ \begin{array}{cc} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{array} \right]
\]

\[
= \left[ \begin{array}{cc} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{array} \right] \cdot J^e_1 \cdot \left[ \begin{array}{cc} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{array} \right]
\]
The associative property assumed in equation 3.3-4 is not valid in general but may be shown to be valid here. (See reference 74, page 376.) Substituting \( J^e_1 \) from equation 3.3-1 into equation 3.3-4 gives

\[
J^e_1 = p^e \begin{pmatrix}
\left( \frac{\alpha^2 \sin^2 \phi + \cos^2 \phi}{\alpha^2 + 1} \right) & \left( \frac{1 - \alpha^2 \sin \phi \cos \phi}{1 + \alpha^2} \right)
\end{pmatrix}
\]

Using the transformation given in equation 3.2-9, the circular coherency matrix may be written as follows:

\[
J^e_c = p^e \begin{pmatrix}
\frac{1}{2} \pm \frac{\alpha}{\alpha^2 + 1} & \frac{1 - \alpha^2 (\cos 2\phi + j \sin 2\phi)}{2} \\
\frac{1 - \alpha^2 (\cos 2\phi - j \sin 2\phi)}{2} & \frac{1}{2} \pm \frac{-\alpha}{\alpha^2 + 1}
\end{pmatrix}
\]

Now consider a completely unpolarized quasimono-chromatic wave. The linear and circular coherency matrices of the wave may be written as follows:

\[
J^u_l = J^u_c = p^u \begin{pmatrix}
1/2 & 0 \\
0 & 1/2
\end{pmatrix}
\]

where \( p^u \) is the total power in the unpolarized wave.
Every partially polarized wave may be represented as a vector sum of an unpolarized wave of power $P^u$ and an elliptically polarized wave of power $P^e$. Since the unpolarized wave and the elliptically polarized wave are incoherent, their individual coherency matrices may be added to obtain the coherency matrix of their vector sum. (See reference 110, page 1177.) Using this procedure the coherency matrices of a partially polarized wave may be constructed by adding $J^e_1$ and $J^u_1$ or $J^e_c$ and $J^u_c$ with the following results.

\((3.3-8)\) \[ J_1 = J^u_1 + J^e_1 \]

\[
\begin{bmatrix}
\frac{1}{2} + \rho_n^e \left( \frac{1 - \alpha^2}{2} \right) & \left( \frac{1 - \alpha^2}{2} \right) \\
\frac{1}{2} \rho_n^e \left( \frac{1 - \alpha^2}{2} \right) & \left( \frac{1 - \alpha^2}{2} \right)
\end{bmatrix}
\]

\[(3.3-9)\] \[ J_c = J^u_c + J^e_c \]

\[
\begin{bmatrix}
\frac{1}{2} \rho_n^e \alpha \left( \frac{1 - \alpha^2}{2} \right) & \rho_n^e \left( \frac{1 - \alpha^2}{2} \right) \\
\rho_n^e \left( \frac{1 - \alpha^2}{2} \right) & \frac{1}{2} \rho_n^e \alpha \left( \frac{1 - \alpha^2}{2} \right)
\end{bmatrix}
\]

The upper sign stands for left hand polarization and the lower sign stands for right hand polarization. The total
power in the partially polarized wave is $P^T = P^u + P^e$, and $p^e_n = P^e / P^T$ is the ratio of the elliptically polarized component power to the total power. This ratio, $p^e_n$, has been called the degree of polarization by many authors, but here it will be referred to as the degree of elliptical polarization.

One may also define a quantity $p^l_n$ as the degree of linear polarization,

$$(3.3-10) \quad p^l_n = \frac{P^l_n}{P^T}$$

where $P^l$ is the linearly polarized power $= \text{Max}[P^T|\rho_{xx}-\rho_{yy}|]$. Similarly a quantity $p^c_n$ may be defined which is the degree of circular polarization,

$$(3.3-11) \quad p^c_n = \frac{P^c_n}{P^T}$$

where $P^c$ is the circularly polarized power $= P^T|\rho_{LL}-\rho_{RR}|$. The quantities $p^l_n$ and $p^c_n$ may also be expressed in terms of $p^e_n$ and $\alpha$ if the expressions for $\rho_{xx}$, $\rho_{yy}$, $\rho_{LL}$, and $\rho_{RR}$ in equations 3.3-8 and 3.3-9 are substituted into equations 3.3-10 and 3.3-11. One then obtains the following:

$$(3.3-12) \quad p^l_n = p^e_n \frac{1-\alpha^2}{1+\alpha^2}$$

$$(3.3-13) \quad p^c_n = p^e_n \frac{2\alpha}{1+\alpha^2}$$
Equations 3.3-8 and 3.3-9 may now be rewritten using $p_n^1$ and $p_n^c$.

\[(3.3-14) \quad J_1 = P_T \begin{bmatrix} \frac{1}{2}(1 + p_n^1 \cos 2\phi) & \frac{1}{2}(p_n^1 \sin 2\phi \pm j p_n^c) \\ \frac{1}{2}(p_n^1 \sin 2\phi \mp j p_n^c) & \frac{1}{2}(1 - p_n^1 \cos 2\phi) \end{bmatrix}\]

\[(3.3-15) \quad J_c = P_T \begin{bmatrix} \frac{1}{2}(1 \pm p_n^c) & \frac{1}{2}(p_n^1 \cos 2\phi + j \sin 2\phi) \\ \frac{1}{2}(p_n^1 \cos 2\phi - j \sin 2\phi) & \frac{1}{2}(1 \mp p_n^c) \end{bmatrix}\]

As in equations 3.3-8 and 3.3-9, the upper sign is for left circular polarization and the lower sign is for right circular polarization.

Equations 3.3-14 and 3.3-15 give the relationships between the elements of both circular and linear coherency matrices and certain parameters $\phi$, $\alpha$, $p_n^c$, $p_n^1$, and $P_T$ which completely describe the polarization configuration of a partially polarized wave. Formulas for finding these parameters when one has the coherency matrix of a wave are given in table 1.

It may be shown that

\[(3.3-16) \quad (p_n^1)^2 + (p_n^c)^2 = (p_n^e)^2\]

The physical significance of $p_n^1$ and $p_n^c$ is straightforward. Every elliptically polarized wave may be decomposed into
two circularly polarized waves which are coherent and oppositely polarized. One wave may have more power in it than the other, and the difference between these two powers accounts for a net circular polarization. The remaining power accounts for a net linear polarization. This decomposition is unique as far as the parameters $p_n^c$ and $p_n^l$ are concerned.
Table 1

Polarization Parameters in Terms of Coherency Matrix Elements

<table>
<thead>
<tr>
<th>Circular $J_c = \begin{bmatrix} J_{LL} &amp; J_{LR} \ J_{RL} &amp; J_{RR} \end{bmatrix}$</th>
<th>Linear $J_1 = \begin{bmatrix} J_{xx} &amp; J_{xy} \ J_{yx} &amp; J_{yy} \end{bmatrix}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P^T = \text{Trace } J_c$</td>
<td>$P^T = \text{Trace } J_1$</td>
</tr>
<tr>
<td>$p_n^e = \sqrt{1 - 4 \frac{J_{c}}{(P^T)^2}}$</td>
<td>$p_n^e = \sqrt{1 - 4 \frac{J_{1}}{(P^T)^2}}$</td>
</tr>
<tr>
<td>left hand polarization if $J_{LL} &gt; J_{RR}$</td>
<td>left hand polarization if $\text{imaginary part } J_{xy} &gt; 0$</td>
</tr>
<tr>
<td>right hand polarization if $J_{RR} &gt; J_{LL}$</td>
<td>right hand polarization if $\text{imaginary part } J_{xy} &lt; 0$</td>
</tr>
<tr>
<td>$p_n^c = \frac{</td>
<td>J_{LL} - J_{RR}</td>
</tr>
<tr>
<td>$l_n = \sqrt{\frac{4J_{LR}J_{RL}}{J_{LL} + J_{RR}}}$</td>
<td>$l_n = \sqrt{\frac{(J_{xx} - J_{yy})^2 + (J_{xy} + J_{yx})^2}{J_{xx} + J_{yy}}}$</td>
</tr>
<tr>
<td>$\frac{\alpha}{1 + \alpha^2} = \frac{p_n^c}{2p_n^e}$</td>
<td>$\frac{\alpha}{1 + \alpha^2} = \frac{p_n^c}{2p_n^e}$</td>
</tr>
<tr>
<td>$\phi = \frac{1}{2} (\text{Phase of } J_{LR} )$</td>
<td>$\phi = \frac{1}{2} \sin^{-1} \left[ \frac{J_{xy} + J_{yx}}{p_n^e(J_{xx} + J_{yy})} \right]$</td>
</tr>
</tbody>
</table>
CHAPTER IV

PRACTICAL ASPECTS OF TENSOR INTERFEROMETRY

4.1 Introduction

Tensor interferometry has been discussed by Ko.\textsuperscript{56,57} Essentially it is the use of an interferometer to measure the visibilities of each component of the coherency matrix associated with a source. Inversion of these visibilities then gives the coherency matrix distribution across the source. (See figure 16.) In theory this is all there is to it. In practice, however, the outputs from the interferometer are contaminated by instrumental effects; and a certain amount of instrumental calibration is necessary. These instrumental effects are discussed in this chapter. As in the previous chapters reference will be made to the NRAO interferometer. Thus the theory will be presented for an interferometer with circularly polarized feeds.

Before progressing further it may be useful to stress the difference between the coherency matrix as discussed in Chapter 3 and the coherency matrices which will be used in this chapter. In Chapter 3 the coherency matrix was used to describe the radiation coming from a point source. In this chapter the coherency matrix will be used in two ways.
Scalar Interferometry (As discussed in Chapter 2)

\[ T_{ii}(u, v) \leftrightarrow B_{ii}(\sigma, \beta) \]

\[ i = L, R, x, \text{ or } y \]

Tensor Interferometry (Using circularly polarized feeds)

\[ T_{LL}(u, v) \]
\[ T_{LR}(u, v) \]
\[ T_{RL}(u, v) \]
\[ T_{RR}(u, v) \]

\[ \begin{bmatrix} T_{LL}(u, v) & T_{LR}(u, v) \\ T_{RL}(u, v) & T_{RR}(u, v) \end{bmatrix} \]

\[ \begin{align*}
T_{LL}(u, v) & \leftrightarrow B_{LL}(\sigma, \beta) \\
T_{LR}(u, v) & \leftrightarrow B_{LR}(\sigma, \beta) \\
T_{RL}(u, v) & \leftrightarrow B_{RL}(\sigma, \beta) \\
T_{RR}(u, v) & \leftrightarrow B_{RR}(\sigma, \beta) 
\end{align*} \]

Figure 16. The measurements of tensor interferometry.
First it will be used to describe the radiation which would be emitted from an antenna as seen in the far field, thereby characterizing the receiving properties of that antenna. Second, it will be used to describe the radiation coming from an extended source.

Using a coherency matrix to describe the radiation emitted from an antenna in the far field is straightforward. The theory as developed in Chapter 3 may be used with the emitting antenna fulfilling the role of the point source. To use a coherency matrix to describe the radiation from an extended source is perhaps not as straightforward. The 2 by 2 coherency matrix is used but the elements of the coherency matrix are now functions of direction. In the case of a point source in Chapter 3 this functional relationship was implied when the coherency matrix was given for a source lying on the z axis. Here the source lies on and near the z axis. The direction specifying the particular part of the source that the coherency matrix terms are referred to will be given using the β and σ direction cosines defined by equations 2.3-12 and 2.3-13 in Chapter 2. In this manner the coherency matrix may be used to describe the distribution of the polarization over the extended source.
From this point on the coherency matrix of a source will be given as

\[ J_c = \begin{bmatrix}
  B_{LL}(\sigma, \beta) & B_{LR}(\sigma, \beta) \\
  B_{RL}(\sigma, \beta) & B_{RR}(\sigma, \beta)
\end{bmatrix} \]

or

\[ J_1 = \begin{bmatrix}
  B_{xx}(\sigma, \beta) & B_{xy}(\sigma, \beta) \\
  B_{yx}(\sigma, \beta) & B_{yy}(\sigma, \beta)
\end{bmatrix} \]

The coherency matrix elements \( B_{LL}, B_{LR}, B_{xx}, B_{xy}, \) etc. have the dimensions of \( \text{W/m}^2 \text{sr} \). (\( \text{sr} \) represents steradians.) These elements replace the \( P_{LL}^T, P_{LR}^T, P_{xx}^T, P_{xy}^T \), etc. elements used in Chapter 3. This involves a slight redefinition of the coherency matrix as given in Chapter 3 to accommodate spatial distributions. The \( \sigma \) and \( \beta \) direction cosines are written in parenthesis after each matrix element to stress the fact that the matrix elements are functions of \( \sigma \) and \( \beta \). The Fourier transform or visibility of each coherency matrix element, which is the quantity that the interferometer measures, will be represented by "\( T \)".

4.2 The LR Coherency Matrix of a Correlation Interferometer

It has been shown that the coherency matrix formalism which was used in the last chapter to describe electromagnetic radiation may also be used to describe the
receiving properties of a single antenna. Briefly, a coherency matrix, \((a_{ij})\), which describes the radiation transmitted by the antenna is assigned to the antenna. Let the coherency matrix of the incoming radiation which the antenna is receiving be \((B_{ij})\). Then the output power available from the antenna terminals may be expressed as the trace of the matrix product.

\[
(4.2-1) \quad S = \text{Trace} \left[ (a_{ij}) \cdot (B_{ij})^T \right]
\]

where \((B_{ij})^T = (B_{ji})\)

This formalism will now be applied to obtain the complex output power from a correlation interferometer.

Given the correlation interferometer shown in figure 17, let \(S\) equal the average complex output.

\[
(4.2-2) \quad S = G <V_1 V_2^*> \]

where

\[
(4.2-3) \quad V_1 = \vec{E} \cdot \vec{h}_1(\theta) \ e^{j\omega \tau \sin \theta} \\
V_2 = \vec{E} \cdot \vec{h}_2(\theta) \ e^{-j\omega \tau \sin \theta}
\]

The parameters \(\vec{h}_1(\theta)\) and \(\vec{h}_2(\theta)\) are the vector complex antenna heights of Sinclaire for antennas 1 and 2 respectively. \(\vec{E}\) is the incident electric field and \(G\) is a proportionality constant. The parameters \(V_1\) and \(V_2\)
Figure 17. A correlation interferometer. $B_o$ is the length of the interferometer baseline in wavelengths.
represent the voltages at antennas 1 and 2. Expressing the parameters in circular basis

\[ \hat{h}_1(\theta) = h_{1L} \hat{e}_L + h_{1R} \hat{e}_R \]
\[ \hat{h}_2(\theta) = h_{2L} \hat{e}_L + h_{2R} \hat{e}_R \]

\[ \hat{e}^* = E_L \hat{e}_L^* + E_R \hat{e}_R^* \]

The conjugates are needed in equation 4.2-5 since the \( \hat{e}_L \) and \( \hat{e}_R \) unit vectors will be defined for a wave receding from the antennas and \( \hat{e} \) is approaching. The \( \hat{e}_L \) and \( \hat{e}_R \) unit vectors are related in the following way to the unit vectors \( \hat{x} \) and \( \hat{y} \): (See Chapter 3 and reference 48.)

\[ \hat{e}_L = \frac{\hat{x}}{\sqrt{2}} - \frac{i \hat{y}}{\sqrt{2}} \]
\[ \hat{e}_R = \frac{\hat{x}}{\sqrt{2}} + \frac{i \hat{y}}{\sqrt{2}} \]

Note that
\[ \hat{e}_L \cdot \hat{e}_L^* = \hat{e}_R \cdot \hat{e}_R^* = 0 \]
\[ \hat{e}_L \cdot \hat{e}_L^* = \hat{e}_R \cdot \hat{e}_R^* = 1 \]

and \( e^{-j\omega t} \) time dependence is used.

Substituting 4.2-3, 4.2-4, and 4.2-5 into 4.2-2 yields

\[ S = G < (E_L h_{1L} + E_R h_{1R})(E_L^* h_{2L} + E_R^* h_{2R}) > e^{j2\pi B_0 \sin \theta} \]
Expanding equation 4.2-7

\[
S = G \left[ (h_{1L}^* h_{2L} e^{j2\pi B_0 \sin \theta}) <E_L E_L^*> + (h_{1L}^* h_{2R} e^{j2\pi B_0 \sin \theta}) <E_L E_R^*> + (h_{1R}^* h_{2L} e^{j2\pi B_0 \sin \theta}) <E_R E_L^*> + (h_{1R}^* h_{2R} e^{j2\pi B_0 \sin \theta}) <E_R E_R^*> \right]
\]

Equation 4.2-8 may be written in the same form as equation 4.2-1 if one defines the coherency matrix of the interferometer to be

\[
\begin{bmatrix}
\rho_{LL} & \rho_{LR} \\
\rho_{RL} & \rho_{RR}
\end{bmatrix}
\]
as in equations 4.2-9 and the spacial Fourier transform of the coherency matrix of the incoming radiation to be

\[
\begin{bmatrix}
T_{LL} & T_{LR} \\
T_{RL} & T_{RR}
\end{bmatrix}
\]
as in equations 4.2-10.

\[
\begin{align*}
\rho_{LL} &= h_{1L}^* h_{2L} e^{j2\pi B_0 \sin \theta} \\
\rho_{LR} &= h_{1L}^* h_{2R} e^{j2\pi B_0 \sin \theta} \\
\rho_{RL} &= h_{1R}^* h_{2L} e^{j2\pi B_0 \sin \theta} \\
\rho_{RR} &= h_{1R}^* h_{2R} e^{j2\pi B_0 \sin \theta}
\end{align*}
\]

\[
\begin{align*}
T_{LL} &= <E_L E_L^*>/2\zeta_0 \\
T_{LR} &= <E_L E_R^*>/2\zeta_0 \\
T_{RL} &= <E_R E_L^*>/2\zeta_0 \\
T_{RR} &= <E_R E_R^*>/2\zeta_0
\end{align*}
\]
In equation 4.2-8 assume that the $1/2Z_0$ factor has been absorbed by the $G$ proportionality constant. The complex output of the interferometer as given by equation 4.2-8 may now be written as

$$S = G \left[ \text{Trace} \left( \Phi \cdot (T)^T \right) \right]$$

in analogy with the single antenna case given in equation 4.2-1. A difference between the formalism developed here for an interferometer and that developed for a single antenna is that the output signal one wishes to express is complex in the case of an interferometer (fringe amplitude and phase), while it is real for a single antenna. Also, the definition of the coherency matrix of the incoming radiation is different for the two cases. In the single antenna case both field quantities in the pointed brackets are measured at the same point in space. In the interferometer case they are measured at different points in space.

The coherency matrix of the interferometer may be expressed in a form which is more useful than that of equation 4.2-9. If $(a)$ is the coherency matrix of antenna 1 and $(b)$ is the coherency matrix of antenna 2, then

$$\begin{pmatrix} h_{1L} & h_{1L}^* \\ h_{1R} & h_{1R}^* \end{pmatrix}$$
\[(4.2-13) \quad (b) = \begin{bmatrix} h_{2L}h_{2L}^* & h_{2L}h_{2R}^* \\ h_{2R}h_{2L}^* & h_{2R}h_{2R}^* \end{bmatrix} \]

Define \( f_{1L} \) by \( h_{1L} = |h_{1L}| e^{j f_{1L}} \)

(4.2-14) \( f_{1R} \) by \( h_{1R} = |h_{1R}| e^{j f_{1R}} \)

	etc.

Now from equations 4.2-9, 4.2-12, 4.2-13, and 4.2-14 one may write the LR coherency matrix of the correlation interferometer in terms of the coherency matrices of the interferometer elements.

\[
\begin{align*}
\rho_{LL} &= \sqrt{a_{LL}b_{LL}} e^{j(f_{1L} - f_{2L})} j2\pi B_0 \sin \theta \\
\rho_{LR} &= \sqrt{a_{LL}b_{RR}} e^{j(f_{1L} - f_{2R})} j2\pi B_0 \sin \theta \\
(4.2-15) \quad \rho_{RL} &= \sqrt{a_{RR}b_{LL}} e^{j(f_{1R} - f_{2L})} j2\pi B_0 \sin \theta \\
\rho_{RR} &= \sqrt{a_{RR}b_{RR}} e^{j(f_{1R} - f_{2R})} j2\pi B_0 \sin \theta
\end{align*}
\]

Thus equation 4.2-8 may now be written as follows:

\[(4.2-16) \quad S = G e^{j2\pi B_0 \sin \theta} \left[ \sqrt{a_{LL}b_{LL}} e^{j(f_{1L} - f_{2L})} T_{LL} \\
+ \sqrt{a_{LL}b_{RR}} e^{j(f_{1L} - f_{2R})} T_{LR} \\
+ \sqrt{a_{RR}b_{LL}} e^{j(f_{1R} - f_{2L})} T_{RL} \\
+ \sqrt{a_{RR}b_{RR}} e^{j(f_{1R} - f_{2R})} T_{RR} \right] \]
4.3 The Complex Output of a Correlation Interferometer Operating in the Tensor Mode

It was shown in the previous section that the average complex power output from a correlation interferometer may be written as

\[
S = \text{G} e^{j2\pi B_0 \sin \theta} \left[ \sqrt{a_{LL}^* b_{LL}} e^{j(f_{1L} - f_{2L})} T_{LL} + \sqrt{a_{LL}^* b_{RR}} e^{j(f_{1L} - f_{2R})} T_{LR} + \sqrt{a_{RR}^* b_{LL}} e^{j(f_{1R} - f_{2L})} T_{RL} + \sqrt{a_{RR}^* b_{RR}} e^{j(f_{1R} - f_{2R})} T_{RR} \right]
\]

where

- \( (a_{i,j}) \) is the coherency matrix of antenna 1,
- \( (b_{i,j}) \) is the coherency matrix of antenna 2,
- \( (T_{i,j}) \) is the visibility of the coherency matrix of the incoming radiation,
- \( f_{1L} \) is the phase of \( h_{1L} \),
- \( f_{2L} \) is the phase of \( h_{2L} \),
- \( f_{1R} \) is the phase of \( h_{1R} \),
- \( f_{2R} \) is the phase of \( h_{2R} \).

The interferometer at NRAO is designed so that each element may receive either left or right hand circular polarization. Thus by throwing the proper switches one can have the interferometer operate in four different polarization modes. Table 2 describes the modes in detail.
The coherency matrix for an antenna will change depending upon which polarization mode the antenna is in. To designate which mode the antenna is in a superscript L or R will be used with each element of the coherency matrix. Thus when antenna 1 is in the left circular polarization mode (LCP)

\[
(a_{ij}) = \begin{bmatrix}
    a_{LL}^L & a_{LR}^L \\
    a_{LR}^L & a_{RR}^R \\
    a_{RL}^L & a_{RR}^R \\
\end{bmatrix}.
\]

If antenna 1 were actually capable of receiving only left circular polarization when it is in the LCP mode the elements of \((a_{ij})\) would be as follows:

- \(a_{LL}^L = A\)
- \(a_{LR}^L = a_{RL}^L = a_{RR}^R = 0\)
("A" is the effective aperture of antenna 1 when in the LCP mode.) However, such a condition is never completely attainable, and in practice all elements of \((a_{ij})\) will be non-zero. The \(a_{LL}^L\) term will be much larger than the others, however.

Similar notation can be devised for the phase terms. Thus let \(f_{IL}^L\) be the phase of \(h_{1L}\) when antenna 1 is in the LCP mode. Table 3 summarizes this notation.

<table>
<thead>
<tr>
<th>Table 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coherency Matrix Notation</td>
</tr>
<tr>
<td>( (a_{ij}) = \begin{bmatrix} a_{LL}^L &amp; a_{LR}^L \ a_{RL}^L &amp; a_{RR}^L \end{bmatrix} )</td>
</tr>
<tr>
<td>( (b_{ij}) = \begin{bmatrix} b_{LL}^L &amp; b_{LR}^L \ b_{RL}^L &amp; b_{RR}^L \end{bmatrix} )</td>
</tr>
<tr>
<td>( f_{1L}^L )</td>
</tr>
<tr>
<td>( f_{2L}^L )</td>
</tr>
<tr>
<td>( f_{1R}^L )</td>
</tr>
<tr>
<td>( f_{2R}^L )</td>
</tr>
<tr>
<td>( a_{LL}^L \gg a_{LR}^L, a_{RL}^L, a_{RR}^L )</td>
</tr>
<tr>
<td>( b_{LL}^L \gg b_{LR}^L, b_{RL}^L, b_{RR}^L )</td>
</tr>
</tbody>
</table>
Now suppose the interferometer is in the LL mode.

Equation 4.3-1 may be written as follows:

\[
S = \left[ C_{1LL} T_{LL} + C_{2LL} T_{LR} + C_{3LL} T_{RL} + C_{4LL} T_{RR} \right] e^{j2\pi B_0 \sin \theta}
\]

where

\[
C_{1LL} = G \sqrt{a_{LL} b_{LL}} e^{j(f_{1L} - f_{2L})}
\]
\[
C_{2LL} = G \sqrt{a_{LL} b_{RR}} e^{j(f_{1L} - f_{2R})}
\]
\[
C_{3LL} = G \sqrt{a_{RR} b_{LL}} e^{j(f_{1R} - f_{2L})}
\]
\[
C_{4LL} = G \sqrt{a_{RR} b_{RR}} e^{j(f_{1R} - f_{2R})}
\]

Similarly if the interferometer is in the LR mode

\[
S = \left[ C_{1LR} T_{LL} + C_{2LR} T_{LR} + C_{3LR} T_{RL} + C_{4LR} T_{RR} \right] e^{j2\pi B_0 \sin \theta}
\]

Table 4 summarizes this notation for all polarization modes.

It is now possible to make several observations concerning the relative effects of the various terms which contribute to \( S \). The consensus among radio astronomers is that most radio sources contain very little (less than 1%) circular polarization. Thus the \( T_{LL} \) and \( T_{RR} \) terms should be approximately equal. Also, few sources show linear polarization greater than 10%. This enables one to assume that on the average the \( T_{LR} \) and \( T_{RL} \) terms should be at
<table>
<thead>
<tr>
<th>Polarization Mode</th>
<th>Interferometer Output</th>
<th>Constants</th>
</tr>
</thead>
<tbody>
<tr>
<td>LL</td>
<td>(C1 LL T&lt;sub&gt;LL&lt;/sub&gt; + C2 LL T&lt;sub&gt;LR&lt;/sub&gt; + C3 LL T&lt;sub&gt;RL&lt;/sub&gt; + C4 LL T&lt;sub&gt;RR&lt;/sub&gt;) e&lt;sup&gt;j2πB&lt;sub&gt;0&lt;/sub&gt;sinθ&lt;/sup&gt;</td>
<td>C1 LL = G/\sqrt{a_{LL}} / b_{LL} e&lt;sup&gt;j(f&lt;sub&gt;1L&lt;/sub&gt; - f&lt;sub&gt;2L&lt;/sub&gt;)&lt;/sup&gt;</td>
</tr>
<tr>
<td>LR</td>
<td>(C1 LR T&lt;sub&gt;LL&lt;/sub&gt; + C2 LR T&lt;sub&gt;LR&lt;/sub&gt; + C3 LR T&lt;sub&gt;RL&lt;/sub&gt; + C4 LR T&lt;sub&gt;RR&lt;/sub&gt;) e&lt;sup&gt;j2πB&lt;sub&gt;0&lt;/sub&gt;sinθ&lt;/sup&gt;</td>
<td>C1 LR = G/\sqrt{a_{LL}} / b_{LL} e&lt;sup&gt;j(f&lt;sub&gt;1L&lt;/sub&gt; - f&lt;sub&gt;2L&lt;/sub&gt;)&lt;/sup&gt;</td>
</tr>
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</table>
### Table 4 cont.

<table>
<thead>
<tr>
<th>Polarization Mode</th>
<th>Interferometer Output</th>
<th>Constants</th>
</tr>
</thead>
<tbody>
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<td><strong>RL</strong></td>
<td></td>
<td>( C_{1RL} = G/\sqrt{a_{LL}^R b_{LL}^L} e^{j(f_{1L}^R - f_{2L}^R)} )</td>
</tr>
<tr>
<td></td>
<td>( (C_{1RL} T_{LL} + C_{2RL} T_{LR} + C_{3RL} T_{RL} + C_{4RL} T_{RR}) e^{j2\pi B_0 \sin \theta} )</td>
<td>( C_{2RL} = G/\sqrt{a_{LL}^R b_{RR}^R} e^{j(f_{1R}^R - f_{2R}^R)} )</td>
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<tr>
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<td></td>
<td>( C_{3RL} = G/\sqrt{a_{RR}^R b_{LL}^L} e^{j(f_{1R}^R - f_{2L}^R)} )</td>
</tr>
<tr>
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<td></td>
<td>( C_{4RL} = G/\sqrt{a_{RR}^R b_{RR}^R} e^{j(f_{1R}^R - f_{2R}^R)} )</td>
</tr>
<tr>
<td><strong>RR</strong></td>
<td></td>
<td>( C_{1RR} = G/\sqrt{a_{LL}^R b_{LL}^R} e^{j(f_{1L}^R - f_{2L}^R)} )</td>
</tr>
<tr>
<td></td>
<td>( (C_{1RR} T_{LL} + C_{2RR} T_{LR} + C_{3RR} T_{RL} + C_{4RR} T_{RR}) e^{j2\pi B_0 \sin \theta} )</td>
<td>( C_{2RR} = G/\sqrt{a_{LL}^R b_{RR}^R} e^{j(f_{1L}^R - f_{2R}^R)} )</td>
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<td>( C_{3RR} = G/\sqrt{a_{RR}^R b_{LL}^L} e^{j(f_{1R}^R - f_{2L}^R)} )</td>
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<td>( C_{4RR} = G/\sqrt{a_{RR}^R b_{RR}^R} e^{j(f_{1R}^R - f_{2R}^R)} )</td>
</tr>
</tbody>
</table>
least 10 times smaller than the $T_{LL}$ and $T_{RR}$ terms. Further, the $(a_{ij})$ and $(b_{ij})$ coherency matrix terms are related as follows:

$$a_{LL} \approx b_{LL} \approx a_{RR} \approx b_{RR} > a_{LL}, a_{RR}, b_{LL}, \text{ and } b_{RR}$$

This, in turn, enables one to predict that

$$\sqrt{a_{LL}b_{LL}} \gg \sqrt{a_{LL}b_{RR}} \sim \sqrt{a_{RR}b_{LL}} \gg \sqrt{a_{RR}b_{RR}}$$

$$\sqrt{a_{LL}b_{RR}} \gg \sqrt{a_{LL}b_{LL}} \sim \sqrt{a_{RR}b_{RR}} \gg \sqrt{a_{RR}b_{LL}}$$

etc.

From the above considerations one can now eliminate some of the terms in the expressions for $S$ according to the mode of polarization. Table 5 contains the revised expressions.

<table>
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<tr>
<th>Polarization</th>
<th>Output</th>
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<tr>
<td>LL</td>
<td>$S = C1LL \ T_{LL}$</td>
</tr>
<tr>
<td>LR</td>
<td>$S = (C1LR + C4LR) \ T_{LL} + C2LR \ T_{LR}$</td>
</tr>
<tr>
<td>RL</td>
<td>$S = (C1RL + C4RL) \ T_{RR} + C3RL \ T_{RL}$</td>
</tr>
<tr>
<td>RR</td>
<td>$S = C4RR \ T_{RR}$</td>
</tr>
</tbody>
</table>

Examination of Table 5 reveals the consequence of the instrumental effects mentioned at the beginning of this chapter. Measurements of the LL or RR polarizations will
only be proportional to the $T_{LL}$ or $T_{RR}$ visibilities of the coherency matrix. However the crossed polarization measurements will be proportional to both the $T_{LR}$, $T_{RL}$ terms and the $T_{LL}$, $T_{RR}$ terms.

One point which is important but hasn't been mentioned yet is that it is unnecessary to measure the components of the $(T)$ matrix over the entire $u$-$v$ plane. Half of the $u$-$v$ plane will be sufficient since

$$T_{LL}^*(u,v) = T_{LL}(-u,-v)$$

$$T_{LR}^*(u,v) = T_{RL}(-u,-v)$$

$$T_{RL}^*(u,v) = T_{LR}(-u,-v)$$

$$T_{RR}^*(u,v) = T_{RR}(-u,-v)$$

These relationships may be easily proven by writing out the Fourier transformations of the $B_{LL}(\sigma, \beta)$, $B_{LR}(\sigma, \beta)$, $B_{RL}(\sigma, \beta)$, and $B_{RR}(\sigma, \beta)$ terms and changing the signs of $u$ and $v$. 
CHAPTER V

THE DATA REDUCTION PROCEDURE

5.1 Introduction

The data reduction procedure refers to everything that must be done to the interferometer output in order to obtain the brightness or polarization distributions of the sources being observed. In theory the output from the interferometer should be the visibility function of the source. However in practice there are several instrumental effects which have to be accounted for.

Figure 18 contains a flow chart of the various steps in the data reduction procedure. The remaining sections of this chapter will elaborate on the individual steps and the manner in which they were carried out for the September and October 1968 observations. Often each step refers to a separate computer program.

5.2 Observations Taken at Green Bank

This step is a data organizing procedure developed and carried out by the staff at NRAO. Referring to figure 19, the data are stored on a magnetic disk at the time of the observation by the on-line DDP-116 computer at Green Bank. Once a day the contents of this disk are dumped onto
Figure 18. The data reduction procedure. The KINGKONG program output is used by all the steps below the AVERAGING step. The CALL program incorporates all corrections to produce the visibility function.
Figure 19. Observations taken at Green Bank.
a 7 track magnetic tape and delivered to Charlottesville. This data is transferred from the 7 track tape to a 9 track tape, called tape number one, at Charlottesville by a program called INTREAD on the NRAO IBM 360-50. The INTREAD program also produces a paper copy of all the data plus instrumental parameters which were on the 7 track tape. Another program called IBALL is available upon the observer's request which reads the 9 track tape and produces a graphical display of the data.

5.3 The Editing and Averaging Steps

IEDIT is a program developed by the staff of NRAO. Most of the input for it consists of scans or parts of scans which are to be edited from the data due to interference or instrumental malfunction. (See figure 20.) IEDIT also makes several corrections to the data which at one time were automatically done by the DDP-116 computer. One of these corrections is for atmospheric refraction, another is a clock correction. The only input required for the corrections is a specification of the clock correction at the time of observation. This is provided by the staff at NRAO. Since the interferometer is capable of finding positions of celestial objects to within a few seconds of arc it is necessary to know the position of the local meridian to
Figure 20. The IEDIT program.
even greater accuracy. If the local clock were off by one second of time the meridian would be in error by as much as fifteen seconds of arc at the celestial equator. In practice the clock correction is given in milliseconds of time, implying a correction of an error of less than a few seconds of arc at the most. Judging from the average performance of the integrated interferometer phase over several days (less than ±20° phase drift) one may surmise that the clock error after correction must be much less than thirty milliseconds of time.

The averaging step is carried out by the KINGKONG program developed by the staff at NRAO. The name of the program is probably due to the fact that it may be asked to punch the vector averaged data onto cards, resulting in an enormous amount of output. In the work reported here the data were averaged over five minute intervals (time) and each datum point punched on a separate card. This gave about 44,000 cards for the two weeks of observations. These data were later transferred to a magnetic tape at Ohio State.

The organization of the output data from the KINGKONG program on the magnetic tape is in card image form. Each card contains the real and imaginary part of a five minute average of the interferometer output for a specific polarization (LL, LR, RL, or RR) and a specific correlator (1, 2,
or 3). (The correlator number specifies which of the three possible independent interferometers the data was obtained from. As was stated in Chapter 1 the NRAO interferometer contains three antennas enabling three independent interferometers to be operating at once between them.) Twelve cards are needed to give all the information obtained during one five minute interval. Other information is also on each card and is explained in section 5.9. The CALL program listing given in section 5.9 contains statements designed to read the KINGKONG output tape. (See statement 100 and format 930 of the CALL program.)

5.4 Extinction and AGC Calibration

The extinction and AGC calibration is designed to account for two errors which are caused by the increased length of the path through the atmosphere which the signal must traverse when a source is near the horizon. Traversing this increased path length causes a source to be attenuated as it nears the horizon. This effect is called extinction. At the same time the automatic gain control (AGC) responds to the increased atmospheric radiation and ground pickup in the sidelobes by reducing the IF gain. This further reduces the signal from the source.

Corrections for these errors have been determined for both the September and October data. Each correction is
designated as a function $F(za)$ of the zenith angle. A separate $F(za)$ is found for each correlator. $F(za) \times 100$ is the percentage decrease in amplitude of a point source from its amplitude at $za = 0$. To correct the data once $F(za)$ is known, one must multiply all amplitudes by $1/(1 - F(za))$.

The corrections were obtained by averaging the normalized amplitude of many point sources versus zenith angle $"za"$.

$$za = \cos^{-1}(\hat{s}_z \cdot \hat{s})$$

where $\hat{s} = \text{a unit vector in the direction } \delta, \lambda$

$$= \cos \delta \cos \lambda \hat{x} + \cos \delta \sin \lambda \hat{y} + \sin \delta \hat{z}$$

and $\hat{s}_z = \text{a unit vector in the zenith direction}$

$$= \cos(38.5^\circ) \hat{x} + \sin(38.5^\circ) \hat{y}$$

The sources and fluxes used are listed in table 6. Figure 21 summarizes the results for all cases. The correction used by the staff at NRAO is

$$F(za) = 0.35(\sec(za) - 1).$$

The curve corresponding to this correction is also shown in figure 21. Table 7 gives the equations for the curves shown in figure 21.
### Table 6
**Extinction and AGC Calibrators**

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<th>Sept. Data</th>
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<td><strong>Source</strong></td>
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<td>3C345</td>
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<tr>
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</table>

* The flux is given in flux units.

One flux unit = $10^{-26}$ W/m²Hz.
Figure 21. Extinction and AGC corrections.
Table 7
Equations* for Extinction and AGC Corrections

<table>
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<tr>
<th>Correlator No.</th>
<th>Date</th>
<th>1-F(za)</th>
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<td>1-2.54 x 10^{-7} za^3</td>
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<td>Oct.</td>
<td>1-2.265 x 10^{-5} za^2</td>
</tr>
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<td>Sept.</td>
<td>1-3.400 x 10^{-5} za^2</td>
</tr>
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<td>Oct.</td>
<td>1-3.400 x 10^{-5} za^2</td>
</tr>
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<td>Sept.</td>
<td>1-3.400 x 10^{-5} za^2</td>
</tr>
<tr>
<td>1</td>
<td>Oct.</td>
<td>1-4.450 x 10^{-5} za^2</td>
</tr>
<tr>
<td>(NRAO Correction)**</td>
<td></td>
<td>1-0.35 ( sec(za)-1 )</td>
</tr>
</tbody>
</table>

* The zenith angle, represented by "za", is in degrees.

** The NRAO correction is applied to all correlators and is independent of the date of observation.
5.5 Delay Correction

It was mentioned in Chapter 2 that in order to track a source the signal from one antenna must be delayed. As the source drifts through the sky this delay must be constantly changed by switching different lengths of cable into the signal path between the antenna which is closer to the source and the correlator. Since the attenuation of the cables is not uniform an amplitude error is thus introduced into the interferometer output. Table 8 contains the average normalized amplitudes of several point sources as a function of the delay at which they were observed. The delay may include any number of wavelengths between plus and minus the maximum separation of the antennas. The delay range in table 8 has been divided into 128 parts and is represented as an integer between 2 and 129. Equation 5.5-1 gives the correspondence between the delay (DLA) and the position of the source which the interferometer is observing. A later computer program uses the data in table 8 to correct the interferometer output for this delay amplitude error.

\[
(5.5-1) \quad DLA = \frac{64}{24200} \left[ B_x \cos(\delta) \cos(H) + B_y \cos(\delta) \sin(H) + B_z \sin(\delta) + 65 \right]
\]

where \( B_x, B_y, \) and \( B_z \) are the baseline parameters as defined in equation 2.3-1.
\( \delta \) is the source declination.
\( H \) is the source hour angle.
### Table 8
The Delay Table

<table>
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<tr>
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<td>1.0000</td>
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<td>1.0000</td>
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<td>1.0000</td>
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<td>1.0000</td>
<td>129</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
</tbody>
</table>
5.6 Instrumental Linear Polarization Corrections

The output of an interferometer, if one assumes little circular polarization, has been shown in Chapter 4 to be as follows:

\[
\begin{align*}
S_{LL} &= C_{1LL} T_{LL} \\
S_{LR} &= (C_{1LR} + C_{4LR}) T_{LL} + C_{2LR} T_{LR} \\
S_{RL} &= (C_{1RL} + C_{4RL}) T_{RR} + C_{3RL} T_{RL} \\
S_{RR} &= C_{4RR} T_{RR}
\end{align*}
\]

(5.6-1)

The \( S_{ij} \) terms in equations 5.6-1 represent the uncorrected or "raw" data for the \( ij \)th polarization mode. The \( T \) terms are the corrected or true data. The \( C \) terms represent the imperfections in the measurement system which distort the true data as it is being measured. If the \( C \) terms are known one is able to obtain the true data from the raw data by solving equations 5.6-1 for the \( T \) terms. In this section the procedures followed to obtain the ratios of certain \( C \) terms which determine the instrumental linear polarization will be discussed. Later, in the section covering the description of the CALL program, the use of these ratios to help obtain the true data will be described.

Dividing the second and third of equations 5.6-1 by the first and fourth yields equations 5.6-2.
The quotients \( \frac{C1LR + C4LR}{C1LL} \), \( \frac{C2LR}{C1LL} \), \( \frac{C3RL}{C4RR} \), and \( \frac{C1RL + C4RL}{C4RR} \) are the ratios referred to above. The \( \frac{T_{LR}}{T_{LL}} \) and \( \frac{T_{RL}}{T_{RR}} \) terms are equal to \( p^1/2 \phi \) and \( p^1/2 \phi \) respectively for a point source with little (<1%) circular polarization. The terms \( p^1 \) and \( \phi \) were described in Chapter 3. In Chapter 3, \( \phi \) was defined as being measured counterclockwise from the \( \hat{x} \) axis. The convention used in the literature is to put the \( \hat{x} \) axis to the north. This convention will be followed here, and so \( \phi \) will be measured positive east from north.

To obtain the C term ratios the raw data of several point sources was fitted in a least squares sense to the known polarizations for those sources using equations 5.6-2. The known polarizations were obtained from the literature. (References 32, 37, and 38) Where two or more different values of the polarization were found in the literature an average value was used if the difference was small. If the difference was large the source was not used. Table 9 contains the point sources which were used and their polarizations.
Table 9

Linear Polarization Calibrators

<table>
<thead>
<tr>
<th>Source</th>
<th>Linear Polarization*</th>
</tr>
</thead>
<tbody>
<tr>
<td>3C286</td>
<td>8.95(\pm)60.2</td>
</tr>
<tr>
<td>3C309.1</td>
<td>1.66(\pm)166.8</td>
</tr>
<tr>
<td>3C345</td>
<td>3.42(\pm)115.0</td>
</tr>
<tr>
<td>3C48</td>
<td>1.96(\pm)128.4</td>
</tr>
<tr>
<td>CTA21</td>
<td>0.58(\pm)128.0</td>
</tr>
<tr>
<td>3C147</td>
<td>0.41(\pm)122.2</td>
</tr>
<tr>
<td>CTA102</td>
<td>4.98(\pm)40.0</td>
</tr>
<tr>
<td>3C454.3</td>
<td>2.96(\pm)43.8</td>
</tr>
<tr>
<td>3C119</td>
<td>0.57(\pm)107.6</td>
</tr>
<tr>
<td>3C138</td>
<td>8.32(\pm)26.0</td>
</tr>
<tr>
<td>3C287</td>
<td>3.35(\pm)146.0</td>
</tr>
<tr>
<td>NRA0530</td>
<td>4.50(\pm)78.2</td>
</tr>
<tr>
<td>3C418</td>
<td>1.38(\pm)146.2</td>
</tr>
</tbody>
</table>

* The amplitude is given in units of per cent. The phase is given in degrees.
It was discovered that the instrumental parameters tend to be a function of the delay length.\textsuperscript{32} To correct for this effect as much as possible, four sets of the C term ratios were found for the September and October data. Each set refers to a fourth of the signal delay range. The first set refers to that quarter of the delay range when the source is in the far west. The fourth set refers to the quarter when the source is in the far east.

Since it is desired to obtain the true values of the source polarization from the raw data, once the C term ratios were found for equations 5.6-2 these equations were solved for the T ratio terms. The new equations are listed below.

\[
\frac{T_{LR}}{T_{LL}} = G_{LR} \frac{S_{LR}}{S_{LL}} + GD_{LR}
\]

(5.6-3)

\[
\frac{T_{RL}}{T_{RR}} = G_{RL} \frac{S_{RL}}{S_{RR}} + GD_{RL}
\]

The G and GD terms are functions of the C term ratios of equations 5.6-2. There are 48 different sets of G and GD terms since different ones exist for each correlator (1, 2, or 3), delay range (1, 2, 3, or 4), polarization (LR or RL), and observation period (September or October). Tables 10 and 11 contain the final G and GD parameters for the September and October observation periods.
Table 10

Linear Polarization Parameters for September

<table>
<thead>
<tr>
<th>Correlator</th>
<th>Polarization</th>
<th>Delay</th>
<th>( G^* )</th>
<th>( 100 \times GD^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>LR</td>
<td>1</td>
<td>0.81/−18.6</td>
<td>1.4/−99.4</td>
</tr>
<tr>
<td>1</td>
<td>LR</td>
<td>2</td>
<td>0.827/−12.8</td>
<td>1.9/−84.4</td>
</tr>
<tr>
<td>1</td>
<td>LR</td>
<td>3</td>
<td>0.777/−13.8</td>
<td>0.1/−88.4</td>
</tr>
<tr>
<td>1</td>
<td>LR</td>
<td>4</td>
<td>0.817/−11.3</td>
<td>0.3/−120.2</td>
</tr>
<tr>
<td>1</td>
<td>RL</td>
<td>1</td>
<td>0.90/17.4</td>
<td>1.9/−161.4</td>
</tr>
<tr>
<td>1</td>
<td>RL</td>
<td>2</td>
<td>0.927/8.9</td>
<td>1.4/−177.7</td>
</tr>
<tr>
<td>1</td>
<td>RL</td>
<td>3</td>
<td>0.867/10.3</td>
<td>1.9/−137.6</td>
</tr>
<tr>
<td>1</td>
<td>RL</td>
<td>4</td>
<td>0.887/11.3</td>
<td>2.0/−133.0</td>
</tr>
<tr>
<td>2</td>
<td>LR</td>
<td>1</td>
<td>0.867/−62.2</td>
<td>2.4/−60.6</td>
</tr>
<tr>
<td>2</td>
<td>LR</td>
<td>2</td>
<td>0.867/−56.7</td>
<td>2.0/−31.5</td>
</tr>
<tr>
<td>2</td>
<td>LR</td>
<td>3</td>
<td>0.857/−65.4</td>
<td>1.5/−33.9</td>
</tr>
<tr>
<td>2</td>
<td>LR</td>
<td>4</td>
<td>0.957/−59.8</td>
<td>2.2/−28.0</td>
</tr>
<tr>
<td>2</td>
<td>RL</td>
<td>1</td>
<td>0.827/69.6</td>
<td>4.0/177.4</td>
</tr>
<tr>
<td>2</td>
<td>RL</td>
<td>2</td>
<td>0.807/55.8</td>
<td>3.2/167.3</td>
</tr>
<tr>
<td>2</td>
<td>RL</td>
<td>3</td>
<td>0.837/61.9</td>
<td>3.3/177.3</td>
</tr>
<tr>
<td>2</td>
<td>RL</td>
<td>4</td>
<td>0.797/56.7</td>
<td>3.1/177.0</td>
</tr>
<tr>
<td>3</td>
<td>LR</td>
<td>1</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>3</td>
<td>LR</td>
<td>2</td>
<td>0.837/−14.5</td>
<td>4.3/−149.2</td>
</tr>
<tr>
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<td>LR</td>
<td>3</td>
<td>0.767/−14.9</td>
<td>4.2/−151.2</td>
</tr>
<tr>
<td>3</td>
<td>LR</td>
<td>4</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
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<td>RL</td>
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<td>0.0</td>
<td>0.0</td>
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<tr>
<td>3</td>
<td>RL</td>
<td>2</td>
<td>0.95/12.0</td>
<td>1.3/108.8</td>
</tr>
<tr>
<td>3</td>
<td>RL</td>
<td>3</td>
<td>0.90/14.8</td>
<td>1.3/127.0</td>
</tr>
<tr>
<td>3</td>
<td>RL</td>
<td>4</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

* The phases of these complex numbers are given in degrees.
<table>
<thead>
<tr>
<th>Correlator</th>
<th>Polarization</th>
<th>Delay</th>
<th>G*</th>
<th>100 x GD*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>LR</td>
<td>1</td>
<td>0.90/14.4</td>
<td>1.98/99.0</td>
</tr>
<tr>
<td>1</td>
<td>LR</td>
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<td>0.89/3.5</td>
<td>2.60/61.4</td>
</tr>
<tr>
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<td>0.99/20.8</td>
<td>1.06/118.3</td>
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<td>LR</td>
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<td>0.92/9.3</td>
<td>0.68/134.8</td>
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<td>RL</td>
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<td>0.93/19.4</td>
<td>2.78/171.4</td>
</tr>
<tr>
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<td>RL</td>
<td>2</td>
<td>0.98/11.7</td>
<td>2.56/172.8</td>
</tr>
<tr>
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<td>RL</td>
<td>3</td>
<td>0.94/9.4</td>
<td>2.47/150.1</td>
</tr>
<tr>
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<td>RL</td>
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<td>0.98/8.4</td>
<td>2.66/144.6</td>
</tr>
<tr>
<td>2</td>
<td>LR</td>
<td>1</td>
<td>1.37/74.4</td>
<td>3.83/88.2</td>
</tr>
<tr>
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<td>LR</td>
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<td>1.00/74.0</td>
<td>2.29/71.2</td>
</tr>
<tr>
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<td>LR</td>
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<td>0.97/76.2</td>
<td>1.70/51.6</td>
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<tr>
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<td>4</td>
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<td>2.19/62.1</td>
</tr>
<tr>
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<td>0.84/62.1</td>
<td>3.98/168.7</td>
</tr>
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<td>2</td>
<td>0.80/78.0</td>
<td>3.56/162.5</td>
</tr>
<tr>
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<td>RL</td>
<td>3</td>
<td>0.86/73.1</td>
<td>3.49/169.3</td>
</tr>
<tr>
<td>2</td>
<td>RL</td>
<td>4</td>
<td>0.86/73.7</td>
<td>3.67/167.9</td>
</tr>
<tr>
<td>3</td>
<td>LR</td>
<td>1</td>
<td>0.94/13.2</td>
<td>4.93/156.0</td>
</tr>
<tr>
<td>3</td>
<td>LR</td>
<td>2</td>
<td>0.90/14.6</td>
<td>4.57/156.7</td>
</tr>
<tr>
<td>3</td>
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<td>3</td>
<td>0.93/14.7</td>
<td>5.28/160.3</td>
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<tr>
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<td>LR</td>
<td>4</td>
<td>0.94/13.2</td>
<td>4.93/156.0</td>
</tr>
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<td>0.84/14.4</td>
<td>1.59/125.7</td>
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<td>1.44/128.6</td>
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<td>1.86/130.6</td>
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<tr>
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<td>RL</td>
<td>4</td>
<td>0.84/14.4</td>
<td>1.59/125.7</td>
</tr>
</tbody>
</table>

* The phases of these complex numbers are given in degrees.
It is difficult to estimate the accuracy of the G and GD parameters. A rough estimate of the uncertainties introduced into any polarization measurement due to the G and GD parameters was obtained by computing the linear polarizations of several point sources and comparing the values obtained with previously published values. Typical vector residuals of about 0.5% polarization were found. This would lead to large phase errors (>30%) for polarizations of less than 1%. However, for polarizations greater than 5% the phase error should be less than 5°.
5.7 The Phyphit Prep Step

The Phyphit Prep step consists of running the PHYPHIT PREP computer program. The PHYPHIT PREP program is designed to correct the data from certain point sources called baseline calibrators and provide information derived from these data. Since the baseline calibrators are point sources the visibility functions associated with them should all have constant amplitudes and vary in phase only as the true position of the source varies from its assumed position. The baseline calibrators are used primarily for their phase information, and so only sources whose positions are well known are chosen for this function. Part of the information PHYPHIT PREP provides is in the form of punched cards which are in turn used as input for the PHYPHIT program. The PHYPHIT program will be discussed in the next section.

A clear understanding of just what the function of the PHYPHIT PREP program is can be obtained by discussing what it does in the order in which it does it. The first section of the program reads the baseline calibrator LL and RR data and stores it for further reference. As it reads the data it corrects and modifies it as follows:

1) Each datum has a scan number associated with it. This scan number is checked against a list of scans containing bad data. If it is on the list the datum is discarded.
2) The Tuesday during the September 1968 observation period something happened to telescope number three causing the phase in correlators two and three to jump by +125° and -125° respectively. This is corrected by subtracting and adding 125° to the phases of these two correlators for all the data obtained in September after the phase jump.

3) The interferometer has three gain settings. The highest gain setting is 1250 and is to be used while observing any source having a total flux of less than 8 flux units. The second gain setting is 125 and is used for sources of less than 80 flux units. The third gain setting is 25 and is used for sources of less than 800 flux units. The third gain is set higher by a factor of two to counteract the effect of the automatic gain control (AGC). The PHYPHIT PREP program adjusts all the source data to an effective gain of 1250. Thus the numbers representing the amplitudes of point sources will be proportional to their fluxes regardless of what gain they were originally observed in. Sources observed in the 25 gain or sources having fluxes greater than 80 flux units form an exception to this due to the effect of the AGC.

A slight digression here is necessary to explain the effect of the AGC mentioned above. Since the correlators are linear only for a certain range of input voltage the IF signal level from each antenna must be reduced if the
source which the antenna is pointed at is too strong. This reduction is done by the AGC. As a result of the AGC the output of each correlator is

\[ S = K \frac{GT_c}{T_u + T_c} \]

where \( K \) is a proportionality constant, \( G \) is the gain, \( T_c \) is the correlated input noise temperature, and \( T_u \) is the uncorrelated input noise temperature plus the system noise temperature. Notice that for a given source, \( T_u + T_c \) is independent of the length of the baseline. For sources less than 80 flux units the deviation of \( V \) from a linear response to \( T_c \) is less than 10%. However, for sources greater than 80 flux units the deviation is considerable. The actual effect of the AGC may be easily computed. For the NRAO interferometer the system noise temperature is about 110°K and 1°K corresponds to about 8 flux units. Consider two point sources, one 4 flux units observed with gain 1250 and the other 400 flux units observed on gain 25. The calculations for the ratio of the outputs expected are as follows:

\[
\frac{S_{4}}{S_{400}} = \frac{1250 \left( \frac{4/8}{110 + 4/8} \right)}{25 \left( \frac{400/8}{110 + 400/8} \right)} = 0.72
\]
The PHYPHIT PREP program would have multiplied the data from the 400 flux unit source by 50 to bring it to an effective 1250 gain. Now if the 4 flux unit source had been used as a calibrator it would have appeared that the 400 flux unit source actually had \((4/0.72) \times 50 = 278\) flux units, down from the true value of 400 flux units by a factor of 0.69.

4) As the observations are obtained they are grouped in scans, each scan identified by a scan number. The scan numbers increase in the order in which the scans were obtained. Sometimes, for unknown reasons, the phase response of the interferometer drifts slightly over the course of a few hours. Thus a point source with an accurate assumed position will yield a phase which varies slightly from scan to scan. If the amount of this phase drift is known as a function of the scan numbers it is possible to feed the information to the PHYPHIT PREP program in the form of a table. The drift is then removed by PHYPHIT PREP. After this step in the program is completed any point source with an accurate assumed position should have a visibility phase of 0°.

5) It was shown in Chapter 2 that accurate knowledge of the length and orientation of the baseline is necessary to properly interpret the output from an interferometer. This will be discussed in detail in the next section. Here it is
only necessary to point out that the baseline which was assumed at the time the observations were taken will sometimes be slightly in error. The PHYPHIT program is designed to obtain baseline corrections from the observations of the baseline calibrators. Once these baseline corrections are known PHYPHIT PREP can modify all the data so that it appears as if the correct baseline was assumed at the time of observation. The baseline corrections are optional, however; and the usual procedure is to run PHYPHIT PREP to try to find if baseline corrections are needed before using PHYPHIT to find them.

6) Sometimes after a source has been observed a more accurate position than the one which was assumed at the time of the observation becomes available. Changing the data to correspond to the new position merely involves a change in the phase of the visibility function. PHYPHIT PREP will make this change if it is instructed to.

7) In addition to the baseline corrections mentioned above there is a parameter "k" which is also discussed in the next section. This parameter has to do with the separation between the two perpendicular axes about which each antenna of the interferometer rotates as it tracks a source. Ideally "k" should be zero, but in actuality it is finite. PHYPHIT PREP will modify the data so as to remove the effect of the "k" term.
8) The extinction and AGC calibration has already been discussed. PHYPHIT PREP has the facility to use the corrections in Table 7 to correct the "raw" data by multiplying it by \( \frac{1}{1-F(z)} \).

9) The delay correction has also been discussed. PHYPHIT PREP also has the facility to read the corrections in Table 8 and use them to correct the "raw" data by dividing it by the proper correction.

After reading, correcting, and storing all the LL and RR data from the baseline calibrators the PHYPHIT PREP program lists the amplitude and phase of this LL and RR data for each calibrator separately as a function of hour angle. The phase is also plotted as a function of hour angle after each listing. This enables two things to be checked very easily. First, if the assumed baseline constants are in error the phase of all calibrators will vary sinusoidally with a period of 24 hours. Second, if the assumed position of one source is badly in error the phase of that source will vary over a much wider range of angles than the phases of the other sources. These effects will be discussed in greater detail in the next section.

The PHYPHIT PREP program finishes by listing and plotting the vector average amplitude and phase for each scan. The amplitude is given in computer counts per flux unit, the amplitude for each source having been divided by
the magnitude of the flux from that source in flux units. These plots provide an indication of any long term instrumental phase drift, and the average counts per flux unit information gives the C1LL and C4RR parameters discussed in Chapter 4.

The PHYPHIT PREP program was run for the baseline calibrators of the September and October 1968 observation periods. The average values for the instrumental phase, C1LL, and C4RR are given in table 12.
Table 12

Average Instrumental Phase, C1LL, and C4RR for Sept. and Oct. 1968

<table>
<thead>
<tr>
<th></th>
<th>Av. Inst. Phase LL</th>
<th>Av. Inst. Phase RR</th>
<th>C1LL**</th>
<th>C4RR**</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sept.* Cor. 1</td>
<td>-45°±15°</td>
<td>35°±15°</td>
<td>288±30</td>
<td>304±30</td>
</tr>
<tr>
<td>Sept.* Cor. 2</td>
<td>-75°±15°</td>
<td>55°±15°</td>
<td>281±30</td>
<td>269±30</td>
</tr>
<tr>
<td>Sept.* Cor. 3</td>
<td>-155°±8°</td>
<td>165°±10°</td>
<td>248±30</td>
<td>263±30</td>
</tr>
<tr>
<td>Oct. Cor. 1</td>
<td>-105°±15°</td>
<td>-28°±15°</td>
<td>305±15</td>
<td>314±15</td>
</tr>
<tr>
<td>Oct. Cor. 2</td>
<td>-65°±10°</td>
<td>75°±15°</td>
<td>285±10</td>
<td>283±15</td>
</tr>
<tr>
<td>Oct. Cor. 3</td>
<td>138°±7°</td>
<td>74°±10°</td>
<td>275±11</td>
<td>264±12</td>
</tr>
</tbody>
</table>

* The average phases for September do not apply to the data taken on the first day of the observation period (scans 12201 to 12263). There was some phase drift during this period.

**The dimensions of C1LL and C4RR are computer counts per flux unit.
5.8 The PHYPHIT Program

In Chapter 2 the response of an interferometer which is observing a point source was given as follows:

\[ S = V e^{j2\pi B \cdot \hat{s}_a} \]  
\[ (5.8-1) \]

The parameter V is the visibility function of the point source with respect to the assumed source position represented by \( \hat{s}_a \).

\[ V = S_0 e^{j2\pi B \cdot (\hat{s}_t - \hat{s}_a)} \]  
\[ (5.8-2) \]

Before one can recover V from the output S one must know \( \vec{B} \) very accurately. The PHYPHIT program is designed to find \( \vec{B} \) from the phase behavior of certain point sources called baseline calibrators and an approximate value of \( \vec{B} \) which will be designated \( \vec{B}_a \).

The NRAO interferometer contains an on-line digital computer which automatically performs the first step in the data reduction procedure to recover V from S. This step consists of dividing the output from the correlator, S, by \( e^{j2\pi \vec{B} \cdot \hat{s}_a} \). Unfortunately at the time of observation \( \vec{B} \) is usually not known with sufficient accuracy and the division is actually by \( e^{j2\pi \vec{B}_a \cdot \hat{s}_a} \). The result of this is that the computer output \( S_c \) is
differing from the true visibility function by a phase term. Once the true value of the baseline is found $V$ may easily be recovered from $S_c$.

The analysis given in Chapter 2 was for an ideal interferometer. In reality there is some contribution to the interferometer output which is a result of non-ideal characteristics. Two of these characteristics must be accounted for by the PHYPHIT program. First, the PHYPHIT PREP program has often not been instructed properly to remove the instrumental phase contribution, and the output $S_c$ examined by PHYPHIT may contain a residual instrumental phase $\phi$. Second, the true baseline vector may vary slightly with hour angle due to a slight difference in the distance between the polar axis and declination axis for each antenna. Introducing these two effects into the equation for $S_c$ yields

$$(5.8-3) \quad S_c = V e^{j2\pi(\vec{B} - \vec{B}_a) \cdot \hat{S}_a}$$

where $\phi$ is the residual instrumental phase, $k$ represents the polar axis-declination axis distance error, and $\hat{x}$ and $\hat{y}$ are unit vectors as described in Chapter 2.
In order to find $\vec{B}$ the PHYPIT program must also find $k$ and $\phi_0$. It must be pointed out that the input to the PHYPIT program is generated by the PHYPIT PREP program and may have had the effects of $k$ and $\phi_0$ removed.

Expanding the scalar products in the phase of $S_c$ as given by equations $5.8-4$ yields

$$(5.8-5) \quad \phi = 2\pi \phi_0$$

$$+ 2\pi \Delta \delta \left[ B_x \sin \delta \cos \alpha + B_y \sin \delta \sin \alpha - B_z \cos \delta \right]$$

$$+ 2\pi \Delta \alpha \left[ B_y \cos \delta \cos \alpha - B_x \cos \delta \sin \alpha \right]$$

$$- 2\pi \Delta B_x \left[ \cos \delta \cos \alpha \right]$$

$$- 2\pi \Delta B_y \left[ \cos \delta \sin \alpha \right]$$

$$- 2\pi \Delta B_z \left[ \sin \delta \right]$$

$$+ 2\pi k \left[ \cos \delta \right]$$

+ Second Order Terms

where $\phi$ is the phase of $S_c$ ($S_c = S_\circ e^{i\phi}$).

$\Delta \alpha = \alpha_0 - \alpha_t$, the assumed right ascension of the source minus the true right ascension.

$\Delta \delta = \delta_0 - \delta_t$, the assumed declination of the source minus the true declination.

$\Delta B_x$, $\Delta B_y$, and $\Delta B_z$ are the $\hat{x}$, $\hat{y}$, and $\hat{z}$ components of the vector $\vec{B}_0 - \vec{B}$.

The second order terms in equation $5.8-5$ may be ignored if $\Delta \alpha$ and $\Delta \delta$ are less than a few minutes of arc and $\Delta B_x$, $\Delta B_y$, and $\Delta B_z$, are less than a few wavelengths.
The terms in equation 5.8-5 may be rearranged in the following manner to show the hour angle dependence more clearly.

\[ (5.8-6) \quad \phi = A + D\cos H_a + E\sin H_a \]

where

\[ (5.8-7) \quad A = 2\pi [\phi_0 + (k - \Delta \delta)\cos \delta_a - \Delta B_z \sin \delta_a] \]

\[ (5.8-8) \quad D = 2\pi [\Delta B_x \sin \delta_a + \Delta B_y \cos \delta_a - \Delta B_x \cos \delta_a] \]

\[ (5.8-9) \quad E = 2\pi [\Delta B_y \sin \delta_a - \Delta B_x \cos \delta_a - \Delta B_y \cos \delta_a] \]

The PHYPHIT program determines a set of A, D, and E parameters for each baseline calibrator by a least squares fit of equation 5.8-6 to the phase data of that calibrator. Since the \( \Delta \delta \) and \( \Delta \alpha \) parameters of each calibrator are independent, the average of all the D parameters so found will yield \( \Delta B_x \cos \delta_a \) and the average of all the E parameters will yield \( \Delta B_y \cos \delta_a \). The \( \Delta B_z \) and \( k \) parameters are found by a least squares fit of the A parameters to equation 5.8-7.

The methods used by the PHYPHIT program will yield approximate results only. It is difficult to calculate the standard errors to be expected from the methods used. PHYPHIT has been programmed to give standard errors along with the \( \Delta B_x, \Delta B_y, \Delta B_z, k, \Delta \alpha \), and \( \Delta \delta \) parameters it finds. These standard errors aren't too good, however; and in practice a better idea of how accurate the results are may
be obtained by running PHYPHIT PREP again with the new parameters.

On the day that we observed baseline calibrators during the September 1968 observation period there was quite a bit of phase drift in the data, and we were unable to obtain good baselines for correlators 1 and 2. Fortunately Dr. C. Wade had determined accurate baselines for correlator 1, and we used his values for that correlator. For correlator 2 we used the difference between Dr. Wade's correlator 1 baselines and our correlator 3 baselines. We did not observe many baseline calibrators during the October 1968 observation period since Dr. Wade had already determined all the baselines for that configuration. Table 13 summarizes the final baseline parameters for the September and October 1968 observation periods.

PHYPHIT PREP was run with the baseline parameters for both September and October listed in table 13. On the average the phase drifts for correlator 1 were less than ±20° for September and ±15° for October. The phase drifts for correlators 2 and 3 were even less. From this one may estimate that the baselines are good to at least ±0.06 wavelengths. Table 14 contains the names of the point sources which were used as baseline calibrators along with the positions which were assumed for them.
<table>
<thead>
<tr>
<th></th>
<th>B_x</th>
<th>B_y</th>
<th>B_z</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sept.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cor. 1</td>
<td>-6812.303</td>
<td>-21439.951</td>
<td>9115.349</td>
<td>-0.062</td>
</tr>
<tr>
<td>Cor. 2</td>
<td>-6048.552</td>
<td>-19056.570</td>
<td>8107.244</td>
<td>0.033</td>
</tr>
<tr>
<td>Cor. 3</td>
<td>-763.751</td>
<td>-2383.381</td>
<td>1008.105</td>
<td>-0.095</td>
</tr>
<tr>
<td>Oct.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cor. 1</td>
<td>-6812.303</td>
<td>-21439.951</td>
<td>9115.349</td>
<td>-0.062</td>
</tr>
<tr>
<td>Cor. 2</td>
<td>-4533.398</td>
<td>-14292.661</td>
<td>6083.324</td>
<td>-0.093</td>
</tr>
<tr>
<td>Cor. 3</td>
<td>-2278.904</td>
<td>-7147.292</td>
<td>3032.028</td>
<td>0.037</td>
</tr>
</tbody>
</table>
### The Baseline Calibrators and Their Assumed Positions

<table>
<thead>
<tr>
<th>Name</th>
<th>Right Ascension</th>
<th>Declination</th>
</tr>
</thead>
<tbody>
<tr>
<td>3C286</td>
<td>13^h 28^m 49.66^S ± 0.04^S</td>
<td>30° 45' 58.3&quot; ± 0.7&quot;</td>
</tr>
<tr>
<td>3C309.1</td>
<td>14^h 58^m 56.50^S ± 0.07^S</td>
<td>71° 52' 10.8&quot; ± 0.6&quot;</td>
</tr>
<tr>
<td>3C345</td>
<td>16^h 41^m 17.56^S ± 0.04^S</td>
<td>39° 54' 10.6&quot; ± 0.6&quot;</td>
</tr>
<tr>
<td>3C48</td>
<td>1^h 34^m 49.82^S ± 0.04^S</td>
<td>32° 54' 20.4&quot; ± 0.6&quot;</td>
</tr>
<tr>
<td>CTA21</td>
<td>3^h 16^m 9.14^S ± 0.04^S</td>
<td>16° 17' 40.3&quot; ± 0.7&quot;</td>
</tr>
<tr>
<td>3C147</td>
<td>5^h 38^m 43.49^S ± 0.04^S</td>
<td>49° 49' 42.4&quot; ± 0.6&quot;</td>
</tr>
<tr>
<td>CTA102</td>
<td>22^h 30^m 7.80^S ± 0.04^S</td>
<td>11° 28' 22.8&quot; ± 0.7&quot;</td>
</tr>
<tr>
<td>3C454.3</td>
<td>22^h 51^m 29.52^S ± 0.04^S</td>
<td>15° 52' 53.7&quot; ± 0.7&quot;</td>
</tr>
<tr>
<td>1055+01</td>
<td>10^h 55^m 55.34^S ± 0.07^S</td>
<td>1° 50' 2.8&quot; ± 1.6&quot;</td>
</tr>
<tr>
<td>NRA0530</td>
<td>17^h 30^m 13.55^S ± 0.07^S</td>
<td>-13° 2' 45.6&quot; ± 1.6&quot;</td>
</tr>
</tbody>
</table>
5.9 The CALL Program

The CALL program brings together all the corrections which have been determined and applies them to the KINGKONG output data. The output from the CALL program consists of the amplitude and phase of the source visibility function for every point on the u-v plane at which the source was observed. Most of the corrections applied by the CALL program are identical to those applied by the PHYPHIT PREP program. The remaining corrections involve the parameters discussed in Chapter 4 and will be discussed here.

In the following let $S_{LL}$ mean the raw data value of the interferometer output when in the LL mode, and let $T_{LL}$ mean the true or corrected output for the LL mode. Make similar definitions for $S_{LR}$, $T_{LR}$, $S_{RL}$, $T_{RL}$, $S_{RR}$, and $T_{RR}$. Using the above parameters the expressions for the output of a correlation interferometer given in table 5 may be written as follows:

\[
egin{align*}
S_{LL} & = C1_{LL}(T_{LL}) \\
S_{LR} & = (C1_{LR} + C4_{LR})(T_{LL}) + C2_{LR}(T_{LR}) \\
S_{RL} & = (C1_{RL} + C4_{RL})(T_{RR}) + C3_{RL}(T_{RL}) \\
S_{RR} & = C4_{RR}(T_{RR})
\end{align*}
\]
This system of equations may be solved for the T parameters yielding

\[
T_{LL} = \frac{1}{C_{1LL}} (S_{LL})
\]

\[
T_{LR} = -\frac{C_{1LR} + C_{4LR}}{(C_{2LR})(C_{1LL})} (S_{LL}) + \frac{1}{C_{2LR}} (S_{LR})
\]  
\[
(5.9-2)
\]

\[
T_{RL} = -\frac{C_{1RL} + C_{4RL}}{(C_{3RL})(C_{4RR})} (S_{RR}) + \frac{1}{C_{3RL}} (S_{RL})
\]

\[
T_{RR} = \frac{1}{C_{4RR}} (S_{RR})
\]

The PHYHPHIT PREP program has provided the \(1/C_{1LL}\) and \(1/C_{4RR}\) parameters. The linear instrumental polarization parameters have provided

\[
\frac{T_{LR}}{T_{LL}} = G_{LR} \frac{S_{LR}}{S_{LL}} + G_{DLR}
\]

\[
(5.9-3)
\]

\[
\frac{T_{RL}}{T_{RR}} = G_{RL} \frac{S_{RL}}{S_{RR}} + G_{DRL}
\]

Multiplying by \(T_{LL}\) and \(T_{RR}\) in the first and second of equations 5.9-3 respectively yields

\[
T_{LR} = \frac{G_{LR}}{C_{1LL}} (S_{LR}) + G_{DLR} (\frac{S_{LL}}{C_{1LL}})
\]

\[
(5.9-4)
\]

\[
T_{RL} = \frac{G_{RL}}{C_{4RR}} (S_{RL}) + G_{DRL} (\frac{S_{RR}}{C_{4RR}})
\]

Using equations 5.9-4, equations 5.9-2 may now be written as follows:
The CALL program generates the T data from the S data according to equations 5.9-5 for one observation period. As each datum is read from the KINGKONG output it is corrected for various errors as described in the section on PHYPHIT PREP. The datum is then placed in one of twelve arrays, corresponding to its correlator (three possibilities) and polarization (four possibilities) combination. The position in the array is a function of the hour angle at which the observation was taken. As each datum is entered in an array it is vector averaged with any data which may have been previously entered at the same position in the array. After all data has been entered in the proper arrays the elements of each array are divided by the C1LL or C4RR parameters corresponding to the proper correlator. Then the arrays of each correlator are combined element by element (combining data taken at identical hour angles) according to the second or third equation of equations 5.9-5.
Remember that a different set of G and GD parameters exists for each correlator and delay set. The data is finally written out along with the position on the u-v plane at which it applies.

Since the CALL program contains all the corrections, its source listing has been included in this section to make the documentation as complete as possible. Table 15 contains an explanation of each of the important terms used in the CALL program.
### Table 15
Terms Used in the CALL Program

<table>
<thead>
<tr>
<th>Terms</th>
<th>Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSC1</td>
<td>Only data with scan numbers greater than LSC1 and less than LSC2 are examined by CALL.</td>
</tr>
<tr>
<td>LSC2</td>
<td>This term is used as an array subscript. It signifies the Sept. observation period when it is equal to 1 and the Oct. observation period when it is equal to 2.</td>
</tr>
<tr>
<td>NS</td>
<td>This term is used as an array subscript. It signifies the correlator number.</td>
</tr>
<tr>
<td>ICORR</td>
<td>This term is used as an array subscript. It signifies the delay set. It attains the values 1, 2, 3, or 4.</td>
</tr>
<tr>
<td>KT</td>
<td>This term represents the amplitude for a G term. The G term is used to specify the instrumental linear polarization. See Chapter 5, section 6.</td>
</tr>
<tr>
<td>MT</td>
<td>This term represents the phase of a G term. See GA for further details.</td>
</tr>
<tr>
<td>GA</td>
<td>This term represents the amplitude of a GD term. The GD term is used to specify the instrumental linear polarization. See Chapter 5, section 6.</td>
</tr>
<tr>
<td>Terms</td>
<td>Descriptions</td>
</tr>
<tr>
<td>-----------</td>
<td>-----------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>GDP</td>
<td>This term represents the phase of a GD term. See GD for further details.</td>
</tr>
<tr>
<td>BLNX(I)</td>
<td>These terms represent the X, Y, Z components of the baseline vector of the I\textsuperscript{th} correlator which were used at the time of the observations.</td>
</tr>
<tr>
<td>BLNY(I)</td>
<td></td>
</tr>
<tr>
<td>BLNZ(I)</td>
<td></td>
</tr>
<tr>
<td>CBX(I)</td>
<td>These terms represent the X, Y, Z components of the correct baseline vector of the I\textsuperscript{th} correlator.</td>
</tr>
<tr>
<td>CBY(I)</td>
<td></td>
</tr>
<tr>
<td>CBZ(I)</td>
<td></td>
</tr>
<tr>
<td>DBX(I)</td>
<td>DBX(I) = BLNX(I) - CBX(I) etc</td>
</tr>
<tr>
<td>DBY(I)</td>
<td></td>
</tr>
<tr>
<td>DBZ(I)</td>
<td></td>
</tr>
<tr>
<td>ISCDFT(J,M)</td>
<td>This term represents the scan number of the M\textsuperscript{th} member of a list concerning the J\textsuperscript{th} correlator.</td>
</tr>
<tr>
<td>PHDL(J,M)</td>
<td>These terms represent the instrumental phase of data from the J\textsuperscript{th} correlator. The M subscript represents their position in a list. The L refers to LL or LR data, the R to RR or RL data.</td>
</tr>
<tr>
<td>PHDR(J,M)</td>
<td></td>
</tr>
<tr>
<td>FUPCL(J,M)</td>
<td>These terms represent the flux units per count (1/C1LL or 1/C4RR of Chapter 4) of data from the J\textsuperscript{th} correlator. The M subscript represents their position in a list. The L refers to LL or LR data, the R to RR or RL data.</td>
</tr>
<tr>
<td>FUPCR(J,M)</td>
<td></td>
</tr>
<tr>
<td>AXIS(M)</td>
<td>This term represents the k parameter for the M\textsuperscript{th} correlator. See Chapter 5, section 8.</td>
</tr>
<tr>
<td>Terms</td>
<td>Descriptions</td>
</tr>
<tr>
<td>------------</td>
<td>----------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>DELAY(J,I)</td>
<td>This term represents the instrumental amplitude delay correction for the J&lt;sup&gt;th&lt;/sup&gt; correlator. See Chapter 5, section 5.</td>
</tr>
<tr>
<td>NFRST</td>
<td>To find the data associated with a particular source the CALL program reads the KINGKONG output tape until it comes to NFRST, the first scan number for that source which appears on the tape.</td>
</tr>
<tr>
<td>ISCAN</td>
<td>This term is used to store the scan number associated with the datum currently being processed by the CALL program.</td>
</tr>
<tr>
<td>NA ME IS</td>
<td>These terms are used to store the name of the source currently being processed by the CALL program.</td>
</tr>
<tr>
<td>MINHA</td>
<td>The MINHA term contains the minimum hour angle encountered by the program at certain stages of the data search.</td>
</tr>
<tr>
<td>MAXHA</td>
<td>The MAXHA term contains the maximum hour angle encountered by the program at certain stages of the data search.</td>
</tr>
<tr>
<td>ANGLE(M)</td>
<td>The position on the u-v plane is specified in polar coordinates by the CALL program. The ANGLE(M) array contains the angles that the radial vectors make with the v axis. Positive angles are measured toward the u axis.</td>
</tr>
<tr>
<td>WUV(M)</td>
<td>This array contains the magnitude of the radial vectors in the polar coordinate system used by CALL to specify positions on the u-v plane. See ANGLE(M) for further information.</td>
</tr>
</tbody>
</table>
Table 15 cont.

<table>
<thead>
<tr>
<th>Terms</th>
<th>Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{NOPT}(J,K,M)$</td>
<td>The $\text{NOPT}(J,K,M)$ array contains, in each array element, the number of datum which have been averaged to obtain the value of the data in the corresponding elements of the $\text{RE}(J,K,M)$, $\text{FIM}(J,K,M)$, $\text{FM}(J,K,M)$, and $\text{PHA}(J,K,M)$ arrays.</td>
</tr>
<tr>
<td>$\text{RE}(J,K,M)$, $\text{FIM}(J,K,M)$</td>
<td>These arrays are used to store the real and imaginary parts of the interferometer output data at various stages of its reduction. The $J$ subscript stands for the correlator number. The $K$ subscript stands for the polarization, $K = 1$, $2$, $3$, or $4$ for $\text{LL}$, $\text{LR}$, $\text{RL}$, or $\text{RR}$ respectively. The $M$ subscript refers to the hour angle, $M = 0$ at $-6$ hrs. and runs to $144$ at $+6$ hrs. One unit of $M$ corresponds to $5$ minutes of time.</td>
</tr>
<tr>
<td>$\text{FM}(J,K,M)$, $\text{PHA}(J,K,M)$</td>
<td>These arrays contain the same information that the $\text{RE}(J,K,M)$ and $\text{FIM}(J,K,M)$ arrays contain but in polar form instead of rectangular.</td>
</tr>
<tr>
<td>$\text{IPOLL}$</td>
<td>The $\text{IPOLL}$ term is set equal to $1$, $2$, $3$, or $4$ depending upon whether the polarization of the datum being considered by the CALL program is $\text{LL}$, $\text{LR}$, $\text{RL}$, or $\text{RR}$ respectively.</td>
</tr>
<tr>
<td>$\text{ISG}$</td>
<td>The $\text{ISG}$ term contains the minus or plus sign for the hour angle term in BCD form.</td>
</tr>
<tr>
<td>$\text{IHAH}$, $\text{FHAH}$, $\text{IHAM}$, $\text{FHAM}$, $\text{FHAS}$</td>
<td>These terms contain the hours, minutes, and seconds of the hour angle.</td>
</tr>
<tr>
<td>Terms</td>
<td>Descriptions</td>
</tr>
<tr>
<td>-------</td>
<td>--------------</td>
</tr>
<tr>
<td>FHAR</td>
<td>This term is used to store the hour angle in radian measure.</td>
</tr>
<tr>
<td>IRAH</td>
<td>These terms contain the hours, minutes, and seconds of the right ascension of the source being observed.</td>
</tr>
<tr>
<td>IRAM</td>
<td></td>
</tr>
<tr>
<td>FRAS</td>
<td></td>
</tr>
<tr>
<td>FDECR</td>
<td>This term is used to store the declination in radian measure.</td>
</tr>
<tr>
<td>ISGD</td>
<td>The ISGD term contains the minus or plus sign for the declination term in BCD form.</td>
</tr>
<tr>
<td>IDEG</td>
<td>These terms contain the degrees, minutes, and seconds of the declination of the source being observed.</td>
</tr>
<tr>
<td>FDEG</td>
<td></td>
</tr>
<tr>
<td>IMIND</td>
<td></td>
</tr>
<tr>
<td>FMIND</td>
<td></td>
</tr>
<tr>
<td>FSECD</td>
<td></td>
</tr>
<tr>
<td>IAVS</td>
<td>These terms appear in the CALL program but are not used and may be neglected.</td>
</tr>
<tr>
<td>IAVD</td>
<td></td>
</tr>
<tr>
<td>IRMS</td>
<td></td>
</tr>
<tr>
<td>NPTEM</td>
<td>As a datum is read by the CALL program the number of independent measurements which were made by the interferometer to obtain the datum is read and stored in NPTEM. Each independent measurement consists of 15 seconds of observation.</td>
</tr>
<tr>
<td>DATAR</td>
<td>As a datum is read it is temporarily stored in DATAR AND DATAI in rectangular form.</td>
</tr>
<tr>
<td>DATAI</td>
<td></td>
</tr>
<tr>
<td>IGAIN</td>
<td>The IGAIN term represents the gain which the interferometer was in when a datum was recorded. This parameter proved unreliable and was neglected.</td>
</tr>
</tbody>
</table>
### Table 15 cont.

<table>
<thead>
<tr>
<th>Terms</th>
<th>Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>FASR</td>
<td>This is the name given a subroutine which changes complex numbers from rectangular to polar form.</td>
</tr>
<tr>
<td>AMP</td>
<td>These terms are often used to store the amplitude of a complex number.</td>
</tr>
<tr>
<td>W</td>
<td>These terms are often used to store the phase of a complex number.</td>
</tr>
<tr>
<td>TEMA</td>
<td>These terms are sometimes used for temporary storage.</td>
</tr>
<tr>
<td>P</td>
<td></td>
</tr>
<tr>
<td>PP</td>
<td></td>
</tr>
<tr>
<td>TEMP</td>
<td></td>
</tr>
<tr>
<td>TSV1, IZ</td>
<td></td>
</tr>
<tr>
<td>TSV2, FA</td>
<td></td>
</tr>
<tr>
<td>TSV3, BX, BY, BZ</td>
<td></td>
</tr>
<tr>
<td>TSV4, L, FL, FNP</td>
<td></td>
</tr>
<tr>
<td>TSV5, FNPL, KT, KK</td>
<td></td>
</tr>
<tr>
<td>TSV6, SHA, CHA, CD, SD</td>
<td></td>
</tr>
<tr>
<td>TEMP, MT</td>
<td></td>
</tr>
<tr>
<td>Z</td>
<td>The Z term is used sometimes for the zenith angle and sometimes for the signal delay in wavelengths.</td>
</tr>
<tr>
<td>U</td>
<td>The U, V terms are used to specify the position on the u, v plane in rectangular coordinates.</td>
</tr>
<tr>
<td>V</td>
<td></td>
</tr>
<tr>
<td>LARK(M)</td>
<td>The LARK(M) array is used to store the number of datum to be averaged. The M subscript takes the values 1, 2, or 3 corresponding to correlators 1, 2, or 3.</td>
</tr>
<tr>
<td>SCA(M)</td>
<td>These arrays are used to obtain the scalar averaged amplitude and phase.</td>
</tr>
<tr>
<td>SCP(M)</td>
<td></td>
</tr>
<tr>
<td>VTR(M)</td>
<td>These arrays are used to obtain the vector averaged amplitude and phase.</td>
</tr>
<tr>
<td>VTI(M)</td>
<td></td>
</tr>
<tr>
<td>Terms</td>
<td>Descriptions</td>
</tr>
<tr>
<td>---------------</td>
<td>----------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>SCAS(M)</td>
<td>These arrays are used to obtain the scalar and vector averages of the square amplitude, phase, real, and imaginary parts as part of the procedure for finding the standard deviations.</td>
</tr>
<tr>
<td>SCPS(M)</td>
<td></td>
</tr>
<tr>
<td>VTRS(M)</td>
<td></td>
</tr>
<tr>
<td>VTIS(M)</td>
<td></td>
</tr>
<tr>
<td>IPLT(I)</td>
<td>This array is used for temporary storage by the plot routine.</td>
</tr>
<tr>
<td>LAP(I)</td>
<td>The LAP(I) array contains BCD information for use in the output formats. See the data statements immediately after the format statements.</td>
</tr>
<tr>
<td>MINUS</td>
<td>The MINUS term contains a minus sign in BCD format.</td>
</tr>
<tr>
<td>IBLANK</td>
<td>The IBLANK term contains BCD blanks for use in the plot routine.</td>
</tr>
<tr>
<td>ISTAR</td>
<td>The ISTAR term contains a BCD &quot;*&quot; for use in the plot routine.</td>
</tr>
<tr>
<td>IMRK</td>
<td>The IMRK term contains a BCD &quot;I&quot; for use in the plot routine.</td>
</tr>
<tr>
<td>LBP(I)</td>
<td>The LBP(I) array contains BCD information for use in the output formats. See the data statements immediately after the format statements.</td>
</tr>
</tbody>
</table>
THIS PROGRAM PROVIDES THE CORRECTED, UNNORMALIZED
VISIBILITY VS. HA FOR ALL ELEMENTS OF THE COHERENCY
MATRIX FOR ANY SOURCE FROM DATA OF THE K002 TAPE
USE NO MORE THAN 30 SOURCES AT ONE TIME OR THE
PROGRAM WILL BECOME QUITE SICK

DATA
K002 TAPE
LSC1, LSC2 BEGINNING AND ENDING SCAN NUMBERS
THE G'S AND GD'S FOR ALL THREE CORRELATORS
THE G'S ARE RATIOS    GD'S ARE IN PERCENT UNITS
OLD BASELINE PARAMETERS
CURRENT BASELINE PARAMETERS (FORMAT 920)
THREE SETS OF THE FOLLOWING TWO STATEMENTS
NUMBER OF CARDS TO FOLLOW
ISCDPT(I), PHDL(I), PHDR(I), FUPCL(I), FUPCR(I) IN ORDER
OF SCAN NUMBER.
THE ABOVE THREE ARRAYS ARE FOR INSTRUMENTAL PHASE
DRIFT.
THE THREE AXIS SEPARATION PARAMETERS AXIS(1), AXIS(2)
, AND AXIS(3) FOR CORRELATORS 1, 2, and 3
AS MANY OF THE FOLLOWING AS NEEDED
NFRST,NGAIN (FORMAT NO. 965)
NFRST ENDS THE PROGRAM IF .EQ. ZERO
NFRST RECYCLES TO LSC1,LSC2 INPUT IF .LT. ZERO
NGAIN IS 1250, 125, OR 25

DOUBLE PRECISION BLNX(3), BLNY(3), BLNZ(3), CBX(3),
1CBY(3), CBZ(3)
DIMENSION IPLT(120), NOPT(3,4,150), RE(3,4,150),
1FIM(3,4,150), LAP(4), LARK(3), DBX(3), DBY(3), DBZ(3),
1LBX(3), DELAY(3,150), MN(3,4,150), PHA(3,4,150),
1SCA(3), SCP(3), VTR(3), VTI(3), SCAS(3), SCPS(3),
1VTRS(3), VTIS(3), FK(3), ISCDPT(3,400), PHDL(3,400),
1PHDR(3,400), AXIS(3), FUPCL(3,400), FUPCR(3,400),
1ANGLE(150), GA(2,3,2,4), CP(2,3,2,4), GDA(2,3,2,4),
1GDP(2,3,2,4), WUV(3,150)
Call Source Listing cont.

900 FORMAT (1H1, 36X, 'PLOTS OF VECTOR AVERAGE PHASE VS. 1HOUR ANGLE')
901 FORMAT(1H1,36X,'PLOTS OF VECTOR AVERAGE PHASE VS. 1SCAN NUMBER')
902 FORMAT(1H1, 36X, 'PLOTS OF VECTOR AVERAGE COUNTS/FLUX UNIT VS. SCAN NUMBER')
903 FORMAT(1H ,F6.2,3(3X,I3,1X,2F9.1),5X,A4)
904 FORMAT(1H ,F6.2,3(1X, I3, 1X, F9.3, F8.1, 3X),1X,A4,
    1 F6.1,2X,3(F5.2,2X))
905 FORMAT(1H , F6.2, 109A1, A4)
906 FORMAT(1H , 17X,'CORRELATOR 1',24X,'CORRELATOR 2',24X,
    1 'CORRELATOR 13')
907 FORMAT(//,9X,'---CORRELATOR(1)---',12X,' SQRT(U*U+V*V)/1000.'
908 FORMAT(' --HA-- -PTS- -AMP-- --PH-- -PTS- -AMP
    1-- --PH-- -PTS- -AMP-- --PH-- POL PA COR 1
    COR 2 COR 3')
910 FORMAT(1H ,F6.0,3(5X,I3,1X,F9.4,F7.1),5X,A4)
911 FORMAT(1H , F6.0, 109A1, A4)
915 FORMAT(4F10.5)
916 FORMAT(' GA=",F10.5, ' GP=", F10.5, ' GDA=", F10.5,
    1GDP=" , F10.5, 5X, 'NS=' , I1, ' I10RR=' , I1, ' KT=" ,
    111, ' HT=' , I1)
920 FORMAT(3F10.3/3F10.3/3F10.3)
921 FORMAT(1H,1H ,6X,3(2F9.1,F7.4),3X,7HVERAGE)
922 FORMAT(1H ,6X,3(2F9.1,F7.4),3X,18HSTANDARD DEVIATION)
926 FORMAT(1H ,F6.2,3(F9.1,F7.4))
930 FORMAT( I5, 2I1, A1, I2, I3, F5.1, 2I3, F5.1, A1, 2I3,
    1 F5.1, I4, 1X, I4, 1X, I4, I2, 2F7.1, I1 )
940 FORMAT (1X,2A4,2X,A4,15)
941 FORMAT(6F10.5, 2X, 2A4, / 6F10.5)
950 FORMAT(1H1,3X,3HRA="2I5,F5.1,3X,4HDEC="2I5,F5.1,13X,9H
    1SOURCE IS ,3A4, 'EPOCH IS THE TIME OF OBSERVATION',3X,
    1 ' ABOVE POSITION WAS USED AT THE TIME OF OBSERVATION
    1--DATA FROM SCANS',I6, ' TO',I6)
951 FORMAT(4F10.5,15)
955 FORMAT(1H1,27HOLD BASELINE PARAMETERS ARE ,3(F10.3,5X)
    1/28X,3( F10.3,5X)/28X,3(F10.3,5X))
956 FORMAT (1X ,27HOLD BASELINE PARAMETERS ARE ,3(F10.3,5
    1X)/28X,3( F10.3,5X)/28X,3(F10.3,5X))
957 FORMAT(1X ,27THE OLD-NEW DIFFERENCES ARE ,3(F10.3,5X)
    1/28X,3 F10.3,5X)/28X,3(F10.3,5X))
Call Source Listing cont.

960 FORMAT (I6,2I1,A1,I2,I3,F5.1,I4,I3,F5.1,I4,1H,,I4,  
11H,,I4, I2,2F7.1,I1,1X,4HAMP=,F7.1,2X,2HP=,F7.1,2X,  
12Hw=,F8.1,2X,3HPA=,F6.1)
965 FORMAT(215)
967 FORMAT (15, 4F10.5)
968 FORMAT(1H1, 10X, 'PHASE CORRECTIONS IN DEGREES AND  
1FLUX UNITS/COUNT T FOR CORRELATOR NUMBER',12,/,  
1'SCAN',5X, 'DPLL',5X, 'DPRR',5X, 'FUPCL',5X, 'FUPCR')
969 FORMAT(5X,I5,2X,F8.2,2X,F8.2,2X,F8.6,2X,F8.6)
970 FORMAT(I5,3A4)
975 FORMAT(/,14X,3(F8.3,1X,F6.1,10X), 'SCALOR AVERAGE')
976 FORMAT(/,14X,3(F8.3,1X,F6.1,10X), 'STANDARD DEVIATION')
977 FORMAT(/,14X,3(F8.3,1X,F6.1,10X), 'VECTOR AVERAGES')
978 FORMAT(/,14X,3(F8.3,1X,F6.1,10X), 'STANDARD DEVIATION')
980 FORMAT (6F10.5,4X,A1)
982 FORMAT ( 'AXIS SEPARATION PARAMETERS ARE', 3F10.3,  
1' FOR CORR 1,2, AND 3 RESPECTIVELY')
983 FORMAT( 'THE DELAY TABLE FOLLOWS')
984 FORMAT(5X,3F10.5,15)
985 FORMAT(3A4,8X,I5,5X,15)
986 FORMAT(F5.2,3(I3,F6.3,F6.1),A4,F6.1,3F6.2)
989 FORMAT(7H ERROR )

C***********************************************************************
C
DATA LAP(1), LAP(2), LAP(3), LAP(4)/ 'LL ', ' LR ', ' RL  
1 ', ' RR '/  
DATA MINUS / 1H-/  
DATA IBLANK, ISTAR, IMRK / 4H , 1H*, 1HI /  
DATA LBP(1), LBP(2), LBP(3) / 1H1, 1H2, 1H3 /  
C***********************************************************************
10 CONTINUE  
LCL=ISTAR
C
C***********************************************************************
C
C NKK IS THE UNIT WITH THE KINGKONG TAPE  
C NIN IS THE UNIT WITH THE DATA CARDS ON IT  
C NOUT IS THE UNIT ONTO WHICH THE OUTPUT MUST BE WRITTEN  
NKK=3
NIN=5
NOUT=6
NPUN=7
C***********************************************************************
CALL SOURCE LISTING cont.

C INITIALIZE THINGS
PMIN=-180.0
PMAX= 180.0
DO 80 M=1,400
DO 80 J=1,3
ISCDFT(J,M)=0
PHDL(J,M)=0.0
PHDR(J,M)=0.0
FUPCL(J,M)=0.0
FUPCR(J,M)=0.0
80 CONTINUE
DO 81 M=1,150
ANGLE(M)=0.0
DO 81 J=1,3
81 DELAY(J,M)=0.0

C**READ BEGINNING AND ENDING SCAN NUMBERS
READ (NIN,965) LSC1,LSC2
C READ THE G'S AND GD'S
DO 83 NS=1,2
DO 83 ICORR=1,3
DO 83 KT=1,2
DO 83 MT=1,4
READ (NIN,915) GA(NS,ICORR,KT,MT),GP(NS,ICORR,KT,MT),
1GDA(NS,ICORR,KT,MT), GDP(NS,ICORR,KT,MT)
WRITE(NOUT,916)GA(NS,ICORR,KT,MT),GP(NS,ICORR,KT,MT),
1GDA(NS,ICORR,KT,MT), GDP(NS,ICORR,KT,MT) ,NS,ICORR,KT,
1MT
83 CONTINUE
C READ BASELINE PARAMETERS
READ (NIN,920) (BLNX(I), BLNY(I), BLNZ(I), I=1,3)
WRITE(NOUT,955)(BLNX(I),BLNY(I), BLNZ(I), I=1,3)
READ (NIN,920) (CBX(I),CBY(I),CBZ(I),I=1,3)
DO 88 I=1,3
DBX(I)=BLNX(I)-CBX(I)
DBY(I)=BLNY(I)-CBY(I)
DBZ(I)=BLNZ(I)-CBZ(I)
88 CONTINUE
WRITE (NOUT,956) (CBX(I),CBY(I),CBZ(I),I=1,3)
WRITE (NOUT,957) (DBX(I),DBY(I),DBZ(I),I=1,3)
C READ INSTRUMENTAL PHASE CORRECTIONS VS. SCAN NUMBER
C AND FLUX/COUNT VS. SCAN NUMBER
DO 89 J=1,3
READ (NIN,965) I
Call Source Listing cont.

READ (NIN,967) (ISCDFT(J,M), PHDL(J,M), PHDR(J,M),
1 FUPCL(J,M), FUPCR(J,M), M=1,1)
WRITE (NOUT,968) J
WRITE(NOUT,969)(ISCDFT(J,M), PHDL(J,M), PHDR(J,M),
1 FUPCL(J,M), FUPCR(J,M), M=1,1)
89 CONTINUE
READ (NIN,980) (AXIS(M), M=1,3)
WRITE (NOUT,982) (AXIS(M), M=1,3)
C READ DELAY VALUES
DO 85 I=2,135
READ (NIN,980) FA,(DELAY(J,I),J=1,3)
IF(FA.EQ.0.0) I=135
85 CONTINUE
WRITE (NOUT,983)
WRITE (NOUT,984)(DELAY(1,J), DELAY(2,J), DELAY(3,J), J,
1 J=2,135)
C*****************************************************************************
90 CONTINUE
C
SEARCH FOR SOURCE WHOSE FIRST SCAN IS NFRST
C NFRST.LE.ZERO WILL STOP THE PROGRAM
READ (NIN,965) NFRST, NGAIN
IF (NFRST) 106,106,92
92 READ (NKK,965) ISCAN
IF (ISCAN) 105,9^,92
9^ READ (NKK,970) ISCAN, NA, ME, IS
IF (ISCAN-NFRST) 92,96,92
96 CONTINUE
C*****************************************************************************
C INITIALIZE THINGS------------------------------------
MINHA=150
MAXHA=-150
DO 95 M=1,150
ANGLE(M)=0.0
DO 95 J=1,3
WUV(J,M)=0.0
DO 95 K=1,4,1
NOPT(J,K,M)=0
RE(J,K,M)=0.0
FIM(J,K,M)=0.0
FM(J,K,M)=0.0
PHA(J,K,M)=0.0
95 CONTINUE
C*****************************************************************************
READ ONE KINGKONG CARD AND TEST ISCAN

ISCAN.LET.0 ERROR-STOP
ISCAN=0 GO TO CIRPOL COMPUTATION

J=0
100 READ (NKK,930) ISCAN,1CORR,IPOLI,ISG,IHAH,IANH,FRAS,
    IRAH,IRAM,FRAS,ISGD,IDEG,IMIND,FSECD,IAVS,IAVD,
    IRMS,NPTEM,DATAR,DATAI,IGAIN
    J=J+1
    IF(ISCAN) 105,110,115
105 WRITE (NOUT,999)
    WRITE (NOUT,950) IRAH,IRAM,FRAS,IDEG,IMIND,FSECD,NA,
    1ME,IS
106 STOP
110 GO TO 500
115 CONTINUE

TEST TO SEE IF CARD IS NEEDED
IF(ISCAN.LT.LSC1.OR.ISCAN.GT.LSC2) GO TO 117

ELIMINATION OF BAD SCANS
IF (ISCAN.EQ.12207) GO TO 117
IF (ISCAN.GT.1^-175.AND.ISCAN.LT.14179) GO TO 117
IF (ISCAN.GT.14190) GO TO 117
GO TO 118
117 J=J-1
GO TO 100
118 CONTINUE

CORRECTION FOR PHASE JUMPS AFTER MAINTENANCE
IF (ISCAN.GT.14000) GO TO 130
IF (ISCAN.LT.12339) GO TO 130
IF(1CORR-2) 130,127,128
127 CALL FASR (DATAR,DATAI,AMP,P)
P=(P+125.0)*0.01745
    DATAR=AMP*COS(P)
    DATAI=AMP*SIN(P)
    GO TO 130
128 CALL FASR (DATAR,DATAI,AMP,P)
P=(P-125.0)*0.01745
    DATAR=AMP*COS(P)
    DATAI=AMP*SIN(P)
130 CONTINUE
C GAIN ADJUSTMENT
IF (NGAIN-125) 134, 132, 138
132 IF (IPOLL.GT.1.AND.IPOLL.LT.4) GO TO 138
133 FTEMP=10.0
GO TO 135
134 IF (IPOLL.GT.1.AND.IPOLL.LT.4) GO TO 133
FTEMP=100.0/2.0
135 CONTINUE
DATAR=DATAR*FTEMP
DATAI=DATAI*FTEMP
138 CONTINUE
C TEST ISG FOR A MINUS SIGN
IF (ISG.NE.MINUS) GO TO 140
IHAH=-1*IHAH
IHAM=-1*IHAM
FHAS=-1.0*FHAS
140 FHAH=IHAH
FHAM=IHAM
PHAR=(FHAH+(FHAM*0.01667)+(FHAS-0.000277b))*0.2618
C TEST ISGD FOR A MINUS SIGN
IF (ISGD.NE.MINUS) GO TO 153
IDEG=-1*IDEG
IMIND=IMIND*(-1)
FSECD=FSECD*(-1.0)
153 CONTINUE
C COMPUTE HA IN FLOATING RADIANS
FDEG=IDEG
FMIND=IMIND
FDECR=(FDEG+(FMIND*0.01667)+(FSECD*0.000277b))*0.01745
WRITE(NOUT, 950) IRAH, IRAK, FRAS, IDEG, IMIND, FSECD, 
NA, ME, IS, LSC1, LSC2
WRITE(NPUN, 955) NA, ME, IS, LSC1, LSC2
TSV1=IRAH
TSV2=IRAM
TSV3=FRAS
TSV4=IDEG
TSV5=IMIND
TSV6=FSECD
160 CONTINUE
C
Call Source Listing cont.

C CORRECT FOR PHASE DRIFT AND INST. PHASE
C CORRECT AMP. TO FLUX UNITS
CALL FASR (DATAR,DATAI,TEMA,TEMP)
  I=0
200  I=I+1
   IF(ISCDFT(ICORR,I).EQ.0) GO TO 204
   IF(ISCDFT(ICORR,I).LT.ISCAN ) GO TO 200
204 CONTINUE
   IF(IPOLL.GT.2) GO TO 205
   TEMP=TEMP-PHDL(ICORR,I)
   TEMA=TEMA*FUPCL(ICORR,I)
   GO TO 206
205 CONTINUE
   TEMP=TEMP-PHDR(ICORR,I)
   TEMA=TEMA*FUPCR(ICORR,I)
206 CONTINUE
C CARRY TEMA AND TEMP TO THE NEXT SECTION
C ******************************************************************************************
C CORRECT FOR BASELINE ERRORS
   TEMP=TEMP+((DBX(ICORR)*COS(FHAR)+DBY(ICORR)*SIN(FHAR))
              *COS(FDECR)+DBZ(ICORR)*SIN(FDECR))*360.0
C CARRY TEMA AND TEMP TO THE NEXT SECTION
C ******************************************************************************************
C CORRECT FOR AXIS SEPARATION
   TEMP= (TEMP-AXIS(ICORR)*360.0*COS(FDECR))*0.01745
   DATAR=TEMA*COS(TEMP)
   DATAI=TEMA*SIN(TEMP)
C ******************************************************************************************
C EXTINCTION CORRECTION
   Z=ARCOS((0.754)*COS(FDECR)*COS(FHAR)+(0.623)*SIN(FHAR))
   IF(ISCAN.GT.13000) GO TO 207
   IF(ICORR-2) 210,210,220
207 IF(ICORR-2) 225,210,215
210 FTEMP=(3.4DE-5)*Z**2
   GO TO 230
215 FTEMP=(2.265E-5)*Z**2
   GO TO 230
220 FTEMP=(2.54E-7)*Z**2
   GO TO 230
225 FTEMP=(4.45E-5)*Z**2
230 FTEMP=1.0/(1.0-FTEMP)
   DATAR=DATAR*FTEMP
   DATAI=DATAI*FTEMP
C******************************************************************************************
CALL SOURCE LISTING cont.

C DELAY CORRECTION
BX = BLNX(ICORR)
BY = BLNY(ICORR)
BZ = BLNZ(ICORR)
Z = BX * COS(FDECR) * COS(FHAR) + BY * COS(FDECR) * SIN(FHAR) +
    BZ * SIN(FDECR)
IZ = 65.0 + 64.0 * Z / 24200.0
IF(DELAY(ICORR,IZ), EQ, 0.0) GO TO 240
FA = 1.0 / DELAY(ICORR,IZ)
DATAR = DATAR * FA
DATAI = DATAI * FA
240 CONTINUE
C CARRY BX, BY, BZ, AND Z ON TO THE NEXT THREE SECTIONS
C***************************************************************************
C COMPUTE HA INDEX AND STORE IN IHAH
C HA INDEX STARTS AT ZERO FOR -6 HRS AND RUNS TO 144
C FOR +6 HRS
C ONE UNIT OF HA INDEX CORRESPONDS TO FIVE MINUTES OF
C TIME
FHAM = FHAR * 45.837 + 72.5
IHAH = FHAM
C***************************************************************************
C FILL WUV ARRAY WITH THE PROJECTED BASELINE IN WAVELENGTH UNITS
IF(WUV(ICORR, IHAH), NE, 0.0) GO TO 250
Z = SQRT(BX * BX + BY * BY + BZ * BZ - Z * Z)
WUV(ICORR, IHAH) = Z / 1000.0
250 CONTINUE
C***************************************************************************
C FILL THE ANGLE ARRAY WITH PA (MEASURED POSITIVE EAST FROM N)
IF (ANGLE(IHAH), NE, 0.0) GO TO 260
U = BX * SIN(FHAR) - BY * COS(FHAR)
V = SQRT(Z * Z - U * U)
CALL FASR(U, V, Z, P)
ANGLE(IHAH) = 90.0 - P
260 CONTINUE
C***************************************************************************
C CHECK IHAH TO FIND MINHA AND MAXHA
IF(IHAH - MINHA), 400, 405, 405
400 MINHA = IHAH
405 IF (MAXHA - IHAH), 410, 415, 415
410 MAXHA = IHAH
415 CONTINUE
C***************************************************************************
CALL Source Listing cont.

C CHECK NOPT(ICORR, IPOLL, IHAH) TO FIND NO. OF PTS.
C AVERAGED THIS FAR AND STORE DATAR AND DATAI IN RE
C AND FIM ARRAYS BY HOUR ANGLE
C USE THE (,,1,), (,,2,), (,,3,), (,,4,) ARRAYS
I= NOPT(ICORR, IPOLL, IHAH)
FL=L
FNP=NPTEM
FNPL=FNP+FL
RE(ICORR, IPOLL, IHAH)=(FL* RE(ICORR, IPOLL, IHAH)+
1DATAR*FNP)/FNPL
FIM(ICORR, IPOLL, IHAH)=(FL*FIM(ICORR, IPOLL, IHAH)+
1DATAI*FNP)/FNPL

C UPDATE THE NOPT ARRAY
NOPT(ICORR, IPOLL, IHAH)=NPTEM+L

GO TO 100

500 CONTINUE

C CONVERT TO AMP AND PHASE
DO 510 KPOL = 1,4,1
DO 510 I = MINHA,MAXHA
DO 505 M=1,3
CALL FASR(RE(M,KPOL,I),FIM(M,KPOL,I), FM(M,KPOL,I),
1PHA(M,KPOL,I))
505 CONTINUE

510 CONTINUE

C CORRECT LR AND RL ARRAYS
DO 650 K=2,3
KT=K-1
KK=K/3
KK=KK*3+1
DO 650 M=1,3
BX=CBX(M)
BY=CBY(M)
BZ=CBZ(M)
DO 650 I=1,144
AMP=FM(M,KK,I)*FM(M,K,I)
IF (AMP.EQ.0.0) GO TO 650
FHAR=I
FHAR=(FHAR/144.0)*3.14159
FHAR=FHAR-1.57079
SHA=SIN(FHAR)
CHA=COS(FHAR)
Call Source Listing cont.

CD=COS(FDECR)
SD=SIN(FDECR)
FHAM=(BX*CHA+BY*SHA)*CD+BZ*SD
FHAM=((FHAM/24273.0)+1.0)*2.0
MT=FHAM+1.0
IF(MT.LT.1) MT=1
IF(MT.GT.4) MT=4
NS=1
IF(NFRST.GT.13000) NS=2
AMP=FM(M,KK,I)*GDA(NS,M,KT,MT)*0.01
P=(PHA(M,KK,I)*GDP(NS,M,KT,MT))*0.01745
W=FM(M,K,I)*GA(NS,M,KT,MT)
PP= (PHA(M,K,I)+GP(NS,M,KT,MT))*0.01745
RE(M,K,I)= AMP*COS(P)+ W*COS(PP)
FIM(M,K,I)= AMP*SIN(P)+ W*SIN(PP)
CALL FASR (RE(K,K,I),FIM(M,K,I), FM(M,K,I),
650 CONTINUE

WRITE OUT SORTED MATERIAL IN AMP AND PHASE FORM
AND COMPUTE THE AVERAGE AND STANDARD DEVIATION
FOR AMP AND PHASE

C ******************************
C WRITE OUT SORTED MATERIAL IN AMP AND PHASE FORM
C AND COMPUTE THE AVERAGE AND STANDARD DEVIATION
C (VECTOR AND SCALAR) FOR AMP AND PHASE
C
J=1
I=1
K=1
DO 750 K=1,4,1
WRITE (NOUT,907)
WRITE (NOUT,908)
DO 700 M=1,3
LARK(M)=0
SCA(M)=0.0
SCP(M)=0.0
VTR(M)=0.0
VTI(M)=0.0
SCAS(M)=0.0
SCP(M)=0.0
VTR(M)=0.0
VTI(M)=0.0
700 CONTINUE
DO 715 I=MINHA,MAXHA
DO 710 M=1,3
AMP=FM(M,K,I)
IF (AMP) 702,702,701
701 CONTINUE
Gall Source Listing cont.

P = PHA(M,K,I)
P = P * 0.01745
W = AMP * COS(PR)
PP = AMP * SIN(PR)
LARK(M) = LARK(M) + 1
SCA(M) = SCA(M) + AMP
SCAS(M) = SCAS(M) + AMP * AMP
SCP(M) = SCP(M) + P
SCPS(M) = SCPS(M) + P * P
VTR(M) = VTR(M) + W
VTRS(M) = VTRS(M) + W * W
VTI(M) = VTI(M) + PP
VTIS(M) = VTIS(M) + PP * PP
GO TO 710

702 PHA(M,K,I) = 0.0
FM(M,K,I) = 0.0

710 CONTINUE
FL = I
FL = FL * 0.0833 - 6.0
M = NOPT(1,K,I) + NOPT(2,K,I) + NOPT(3,K,I)

REMOVE THE GO TO 715 IF YOU WANT PUNCH OUT
GO TO 715
IF (M.EQ.0) GO TO 715
WRITE (NPUN, 986) FL, (NOPT(M,K,I), FM(M,K,I), PHA(M,K,I),
1M = 1, 3), LAP(K), ANGLE(I), (WUV(M,I), M = 1, 3)

715 WRITE (NOUT, 904) FL, (NOPT(M,K,I), FM(M,K,I), PHA(M,K,I),
1M = 1, 3), LAP(K), ANGLE(I), (WUV(M,I), M = 1, 3)
DO 720 M = 1, 3
FL = LARK(M)
SCA(M) = SCA(M) / FL
SCP(M) = SCP(M) / FL
VTR(M) = VTR(M) / FL
VTI(M) = VTI(M) / FL
SCAS(M) = SCAS(M) / FL
SCPS(M) = SCPS(M) / FL
VTRS(M) = VTRS(M) / FL
VTIS(M) = VTIS(M) / FL
U = SQRT(VTRS(M) - VTR(M) * VTR(M))
V = SQRT(VTIS(M) - VTI(M) * VTI(M))
SCAS(M) = SQRT(SCAS(M) - SCA(M) * SCA(M))
SCPS(M) = SQRT(SCPS(M) - SCP(M) * SCP(M))
W = ABS(VTR(M))
PP = ABS(VTI(M))
VTRS(M) = SQRT((W+U)*(W+U) + (PP+V)*(PP+V)) - SQRT(W*W + PP*PP)
CALL FASR(W,PP,TEMA,TEMP)
W = W - U
PP = PP + V
CALL FASR (W,PP,U,V)
VTIS(M)=V-TEMP
CALL FASR (VTR(M),VTI(M),W,PP)
VTR(M)=PP
VTI(M)=VTR(M)-W
VTIS(M)=VTI(JYI)=PP
WRITE(NOUT,975) (SCA(M),SCP(M),M=1,3)
WRITE(NOUT,976) (SCAS(M),SCPS(M),M=1,3)
WRITE(NOUT,977) (VTR(M),VTI(M),M=1,3)
WRITE(NOUT,978) (VTRS(M),VTIS(M),M=1,3)
750 CONTINUE

C
C CONSTRUCT THE PLOTS OF PHASE VS. HOUR ANGLE FOR THE
C SOURCE
DO 753 I=1,120
IPLT(I)=IBLANK
753 CONTINUE
K=1
DO 755 K=1,4,1
WRITE (NOUT,900)
WRITE (NOUT,906)
DO 755 I=MINTA,MAXHA
IPLT(I)=IMRK
IPLT(I)=IMRK
IPLT(I)=IMRK
DO 754 M=1,3
J=PHA(M,K,I)*0.1-18.0+FL*36.0+1.0
IF (PHA(M,K,I).NE.0.0) IPLT(J)=LBP(M)
754 CONTINUE
FL=FL
FL=FL*0.0833-6.0
WRITE (NOUT,905) FL, (IPLT(M),M=1,109), LAP(K)
DO 755 M=1,109
IPLT(M)=IBLANK
755 CONTINUE

C
C
. 821 CONTINUE
READ (NIN,965) MFRST,NGAIN
IF (MFRST) 824,825,9^
SUBROUTINE FASR (RE, FI, AMP, P)
AMP = SQRT(RE*RE + FI*FI)
P = 0.0
IF (AMP) 90, 90, 10
10 IF (RE) 20, 30, 40
20 P = ATAN(FI/RE) * 57.3 - 180.0
   IF (FI .GT. 0.0) P = P + 360.0
   GO TO 90
30 P = -90.0
   IF (FI .GT. 0.0) P = 90.0
   GO TO 90
40 P = ATAN(FI/RE) * 57.2958
90 CONTINUE
RETURN
END
5.10 FFT Program

The FFT program performs a one-dimensional Fourier inversion of the visibility function using a fast Fourier transform algorithm. Details of the fast Fourier transform algorithm are given in Appendix A. Certain aspects of the FFT program such as how one uses it and what it is capable of doing are discussed here.

Input to the FFT program consists of an array of complex numbers. Each element of the array represents the visibility function at one point on the u-v plane. Since the Fourier inversion is one-dimensional the points must lie along a straight line which passes through the origin of the u-v plane. Let the distance along this line in wavelengths be represented by \( w \). The distance between points must be uniform and the number of points must be even. Let \( \Delta w \) represent the distance in wavelengths between adjacent points and \( N \) represent the number of points. Also let \( \phi \) be the angle that the line makes with the v axis, measured positive towards the positive u axis.

The output of the FFT program consists of an array of complex numbers. Each element of the array represents the brightness observed by the synthesized fan beam as it is swept across the source oriented perpendicular to a line making an angle \( \phi \) with the line of constant right ascension at the source. The angle \( \phi \) is measured positive towards the
east. Each brightness point is situated $\Delta \theta$ radians from the adjacent ones where $\Delta \theta = 1/(N \Delta w)$. The brightness is in units of $W/m^2(\Delta \theta)Hz$.

Figure 22 contains a flow chart of the FFT program. One comment should be made concerning the taper function. The sidelobes of the synthesized beam may be reduced by artificially attenuating the visibility function at high values of $w$. Unfortunately this also widens the main beam response. A feature was incorporated in the FFT program which allows the first array accepted by the program to be a taper function which specifies the attenuation to be applied to the following visibility functions. This taper function is then applied to all the succeeding visibility functions which the FFT program reads until another taper function is specified. Immediately after each taper function is read in it is Fourier transformed and read out, giving a plot of the synthesized beam shape. While this feature exists, it has not been used in the work reported here.
Read $W$ and $N$. 

Read the visibility or taper function.

If the above is not real and imaginary convert it to real and imaginary.

Reorder the array to prepare for the FFT algorithm.

Is this a taper function?  

Yes

No

Apply the taper function.

FFT Main Routine

FFT Sort Routine

Write output.

Plot output.

If this was a taper function store it for later use.

Read a Key.  Key = Blank

Key = Stop

Stop

Figure 22. The FFT program.
Read and count V's
NC=NC1= # of V's

If NC isn't a power of two
set NC and NC1 to next highest
power of two.

Compute \( W, W^2, W^3, W^4, \ldots W^{\frac{NC-1}{2}} \)

\( I_{\text{EXP}} = \frac{NC1}{NC} \)

\[ \text{DO 155} \]
\[ I = 1, NC1-NC+2, NC \]

\[ \text{DO 150} \]
\[ K = 0, \frac{NC}{2} -1, 1 \]

\( J = K + I \)
\( JT = K \times I_{\text{EXP}} \)
\( TV = V(J) \)

\( V(J) = V(J) + V\left(\frac{NC}{2} + J\right) \)

\( V\left(\frac{NC}{2} + J\right) = \left[ TV - V\left(\frac{NC}{2} + J\right) \right] W^{JT} \)

\( NC = NC/2 \)

\( NC \leq 1 \)

End

Figure 22 cont. FFT main routine.
Figure 22 cont. FFT sort routine.
FFT Source Listing

C ********************************************************************
C THIS DECK BELONGS TO JIM COOK 293-6310
C FFT FFT FFT FFT FFT FFT FFT FFT FFT FFT FFT FFT FFT FFT
C FAST FOURIER TRANSFORM PROGRAM
C
C INPUT
C DELW,NC1,LSAVE
  DELW IS DELTA W
  LSAVE IS NO. OF PTS IN THE VISIBILITY FUNCTION
C NC1 CARDS CONTAINING THE FOLLOWING
VR AND VI 1-4 (FORMAT 900)
  (AS MANY OF THE ABOVE CARDS AS NECESSARY TO HOLD
  THE VISIBILITY FUNCTION. THE NUMBER OF POINTS
  (N) MUST BE EVEN. POINT #((N/2 + 1) IS THE ZERO
  BASELINE POINT. POINTS LESS THAN (N/2 + 1) ARE
  NEGATIVE, THOSE GREATER ARE POSITIVE.)
C ITEST (FORMAT 930)
  ITEST = (BLANK CARD), 'POLR', OR 'TAPR'
  BLANK CARD -- VR AND VI ARE REAL AND IMAGINARY
  POLR -- VR AND VI ARE IN PASOR POLAR FORM
  TAPR -- VR AND VI ARE A TAPER FUNCTION
C IKEY (FORMAT 930)
  IKEY = (BLANK CARD) ------ A VISIBILITY FUNCTION
  follows
  IKEY = 'STOP' --- STOP
C THE FIRST SET OF VR AND VI MUST BE A TAPER FUNCTION
C (NC1=NC1 OF ALL VISIBILITIES TO WHICH THE TAPER IS
C TO BE APPLIED) THIS TAPER FUNCTION WILL THEN BE
C APPLIED TO ALL THE FOLLOWING VISIBILITIES UNTIL
C ANOTHER TAPER FUNCTION IS INTRODUCED
C ********************************************************************
C
C THE BRIGHTNESS IS GIVEN IN UNITS OF INTEGRATED FLUX
C PER ARC-SEC DIVISION INSTEAD OF FLUX PER RADIANT.
C STRAIGHT APPLICATION OF THE FFT ON THE VISIBILITY
C FUNCTION YIELDS T*X(T). THIS IS MULTIPLIED BY
C EVASL=1/(N)=DELTA(T)/T TO GIVE DELTA(T)*X(T).
C
C ********************************************************************
C
INTEGER PLV(55), PLB(55), TAPR
DIMENSION VR(1025), VI(1025), VRP(1025), VIP(1025),
  WR(513), WI(513), VRPT(1025)
DATA IB, TAPR, IPOL / 4H , 4HTAPR, 4HPOLR /
DATA IIA, IIP / 1HA, 1HP /
FFT Source Listing cont.

900 FORMAT(8F10.5)
910 FORMAT(1X,F9.2,3X,F10.1,3X,F6.1,12X,F9.2,3X,F10.1,
13X,F6.1)
915 FORMAT( 1H1,11X, 'VISIBILITY FUNCTION',23X
  1'BRIGHTNESS DISTRIBUTION', /, 2X, 'W(KWLS)', 5X,
  1'AMPLITUDE PHASE', 13X, 'ARC(SEC)', 4X
  1'AMPLITUDE PHASE', 110X, 'T=', F15.2)
916 FORMAT( 1H1, 11X, TAPER ',23X,
  1' BEAM ', /, 2X,'W(KWLS)', 5X,
  1'AMPLITUDE PHASE', 13X, 'ARC(SEC)', 4X,
  1'AMPLITUDE PHASE', 10X, 'T ', F15.2)
920 FORMAT( 1H1, 19X, 'VISIBILITY', 45X, 'BRIGHTNESS')
921 FORMAT( 1H1, 19X, ' TAPER ', 45X, ' BEAM ')
925 FORMAT( 1X, F7*2, 51A1, 'IS ', F8.2, 51A1, '1')
930 FORMAT(A4)
935 FORMAT(F10.5,215)

C*********************************************************************************************
C
C NC1=NO. OF CARDS TO FOLLOW FOR ONE CASE
80 READ (5,935) DELW,NC1, LSAVE
  EVASL=LSAVE
  EVAS=1.0/EVASL
  DO 105 I=1,NC1
    II=4*(I-1)
105 READ (5,900) (VRP(II+M), VIP(II+M), M=1,4)
    NC=II+4
    READ (5,930) ITEST
    IF (ITEST .EQ. IB) GO TO 115
    IF (ITEST .EQ. TAPR) GO TO 115
    DO 110 I=1,NC
      P=VIP(I)* 3.1415927/180.0
      A=VRP(I)
      VRP(I)=A*COS(P)
      VIP(I)=A*SIN(P)
110 CONTINUE
115 CONTINUE
C NC1=1
120 NC1=NC1*2
    IF (NC.GT.NC1) GO TO 120
    NC12=NC1/2
    NC2=NC/2
    FN=NC1
    T=FN*DELW
    DELA =(1.0/T) *57.3*3600.0
T=FN/T
DO 125 I=1,NC1
  VR(I)=0.0
125 CONTINUE
DO 130 I=1,NC2
  VR(I)=VRP(NC2 I)
  VI(I)=VIP(NC2 I)
  VR(NC1+1-I)=VRP(NC2+1-I)
  VI(NC1+1-I)=VIP(NC2+1-I)
130 CONTINUE
DO 135 I=1,NC1
  VRP(I)=VR(I)
  VIP(I)=VI(I)
135 CONTINUE
C******************************************************************************
C
C SKIP THE APPLICATION OF THE TAPER FUNCTION IF THIS IS
C A TAPER FN.
IF (ITEST .EQ. TAPR ) GO TO 138
C******************************************************************************
C
C APPLY THE TAPER FUNCTION
DO 137 I=1,NC1
  FLAG=VRPT(I)
  VR(I)=VR(I)*FLAG
  VI(I)=VI(I)*FLAG.
137 CONTINUE
138 CONTINUE
C******************************************************************************
C
FN=-6.283/FN
WRT=COS(FN)
WIT=SIN(FN)
WR(1)=1.0
WI(1)=0.0
DO 140 J=2,NC12
  I=J-1
  WR(J)=WR(I)*WRT-WI(I)*WIT
  WI(J)=WI(I)*WRT+WR(I)*WIT
140 CONTINUE
C******************************************************************************
C
C PERFORM THE FFT
C
C
FFT Source Listing cont.

NC=NC1
IEXP=NC/NC
NC12=NC1-NC+2
NC2=NC/2
DO 155 I=1,NC12,NC
M1=NC2-1
DO 150 K=0,M1
J=K+I
JT=K*IEXP+1
TVR=VR(J)
TVI=VI(J)
VR(J)=VR(J)+VR(NC2+J)
VI(J)=VI(J)+VI(NC2+J)
FN=VR(NC2+J)
VR(NC2+J)=(TVR-VR(NC2+J))*WR(JT)-(TVI-VI(NC2+J))*WI(JT)
VI(NC2+J)=(TVI-VI(NC2+J))*WR(JT)+(TVR-FN)*WI(JT)
150 CONTINUE
155 CONTINUE
NC NC2
IF(NC.GT.1) GO TO 145
C ************************************************************
C SORT THE RESULT
C
K=1
NC12=NC1/2
200 CONTINUE
M1=K+1
DO 205 MM=M1,NC12
WR(MM)=VR(MM)
WI(MM)=VI(MM)
205 CONTINUE
J=0
I=K
L=K
II=NC12
215 LL=II+1
I=I+1
VR(I)= VR(LL)
VI(I)= VI(LL)
ICH=K*(2*I+2)
IF (I.GE. ICH ) GO TO 220
GO TO 215
J=J+1
FFT Source Listing cont.

225 CONTINUE
I=I+1
L=L+1
VR(I)=WR(L)
VI(I)=WI(L)
IF (I.LT. (K*(2*I+1))) GO TO 225
IF (I.LT. (NC1-K)) GO TO 215
K=2*K
IF (K.LT. (NC1)) GO TO 200

C******************************************************************************************************************
C
C
FN=NC1
DO 320 I=1,NC1
TVR=VR(I)
TVI=VI(I)
CALL FASR (TVR,TVI,AMP,P)
VR(I)=AMP
VI(I)=P
TVR=VRP(I)
TVI=VIP(I)
CALL FASR (TVR,TVI,AMP,P)
VRP(I)=AMP
VIP(I)=P
320 CONTINUE
IF (ITEST.EQ. TAPR) GO TO 322
WRITE (6,915) T
GO TO 323
322 WRITE (6,916) T
323 CONTINUE
DO 325 I=1,NC1
NC12=NC1/2
FN=I-1
IF (I.GT.NC12) FN=I-NC1-1
W=FN*DELD*0.001
FN=FN*DELA
VR(I)=VR(I)*EVSAL
WRITE(6,910) W, VRP(I), VIP(I), FN, VR(I), VI(I)
325 CONTINUE

C******************************************************************************************************************
C
C
C FIND VRP MAX AND MIN, VR MAX AND MIN
P.MAX=0.0
P.MIN=-900000.0
V.MAX=-900000.0
FFT Source Listing cont.

VMIN=900000.0
DO 330 I=1,NC1
IF (VRP(I).GT. PMAX) PMAX=VRP(I)
IF (VRP(I).LT. PMIN) PMIN=VRP(I)
IF (VR(I).GT. VMAX) VMAX=VR(I)
IF (VR(I).LT. VMIN) VMIN=VR(I)
330 CONTINUE
IF (PMAX.EQ.PMIN) GO TO 335
IF (VKAX.EQ.VKIN) GO TO 335
NORMIZE PVR AND VR, PVI AND VI TO A 0 TO 50 RANGE
DO 335 I=1,NC1
VIP(I)=VIP(I)/180.0)*25.0 26.0
VRP(I)=((VRP(I)-PMIN)/(PMAX-PMIN))*50.0+1.0
VR(I)=((VR(I)-VMIN)/(VMAX-VMIN))*50.0+1.0
VI(I)=(VI(I)/180.0)*25.0+26.0
335 CONTINUE
NC12=NC12+1
FLAG=0.0
IF (ITEST.EQ.TAPR) GO TO 336
WRITE(6,920)
GO TO 337
336 WRITE (6,921)
337 DO 340 I=NC12,NC1
T=NC1
FN=I-1-NC1
FN=FN+T*FLAG
W=FN*DELW*0.001
FN=FN*DELA
DO 338 J=1,55
PLV(J)=IB
338 PLB(J)=IB
J=VRP(I)
PLV(J)=I1A
J=VIP(I)
PLV(J)=I1P
J=VR(I)
PLB(J)=I1A
J=VI(I)
PLB(J)=I1P
FFT Source Listing cont.

WRITE (6, 925) W, (PLV(J), J=1, 51), FN, PLB(J), J=1, 51)

340 CONTINUE
  IF (FLAG.EQ.1.0) GO TO 342
  FLAG=1.0
  NC1=NC12-1
  NC12=1
  GO TO 337

342 CONTINUE
  IF (ITEST.NE.TAPR) GO TO 345
  NC1=NC1*2
  DO 343 I=1,NC1
  343 VRPT(I)=(VRP(I)-1.0)*0.02
  345 READ (5, 930) IKEY
  IF (IKEY.EQ.IB) GO TO 80
  STOP
END

SUBROUTINE FASR (RE, FI, AMP, P)
  AMP=SQRT(RE*RE, FI*FI)
  P=0.0
  IF (AMP) 90, 90, 10
  IF (RE) 20, 20, 40
  20 P=ATAN(FI/RE)*57.3-180.0
  IF (FI.CT.0.0) P=P+360.0
  GO TO 90
  40 P=ATAN(FI/RE)*57.2958
  90 CONTINUE
RETURN
END
CHAPTER VI

THE CRAB NEBULA

6.1 Introduction

It has been established that the Crab Nebula, also known as M1, 3C144, or Taurus A, is a remnant of a supernova which occurred in 1054 A.D. The event was recorded by Chinese astronomers who reported the appearance of a "guest star" in the constellation Taurus which was so brilliant that it cast shadows at night and was visible during the day. It gradually weakened and disappeared from view after a few years. In the middle of the eighteenth century astronomers, having the use of telescopes, observed a faint nebula in Taurus which came to be known as the Crab Nebula due to its crab-like shape. In 1928 Hubble linked the presence of the Crab Nebula to the "guest star" by proving that the rate of expansion of the nebula was consistent with a beginning in 1054 A.D.

In 1949 Bolton and Stanley obtained improved coordinates for a radio source which they discovered in Taurus the year before. These coordinates matched the position of the Crab Nebula and thus produced the first optical identification of a discrete cosmic radio source. The fact that it radiated at radio frequencies as well as at optical frequencies
was only the first of many surprises that the Crab Nebula held for astronomers.

In 1953 Shklovsky suggested that the radiation from the Crab Nebula could be generated by the synchrotron mechanism. Dombrovsky detected polarized optical emission from the Crab Nebula in 1954, thereby confirming Shklovsky's proposal. In 1957 Mayer, McCullough, and Sloanaker detected linear polarization in the 3.15 cm. radio radiation from the Crab Nebula indicating that the radio radiation was also due to the synchrotron process. This was the first time either optical or radio radiation had been found to originate in a synchrotron process.

Still another first was recorded by the Crab Nebula when in 1964 Bower, Byram, Chubb, and Friedman discovered that it was also radiating X-rays. Astronomers aren't yet sure what mechanism in the nebula causes the X-ray radiation. Initially it was speculated that the supernova event may have left a neutron star buried inside the nebula and that this star may be radiating X-rays. Then the possibility of a dilute very hot plasma radiating by the thermal bremsstrahlung process was considered. However, with the advent of pulsars attention has been again shifted to the neutron star theory.

Pulsars were first observed as radio sources which gave off pulsed radiation. In 1968 a pulsar was discovered in the Crab Nebula. In 1969 Cocke, Disney, and Taylor
discovered that there were optical pulses coming from the center of the nebula. Lynds, Maran, and Trumbo quickly identified a particular star in the center of the nebula as being the source of the optically pulsed radiation. Three months later Fritz, Henry, Meekins, Chubb, and Friedman reported the results of their X-ray observations which showed that the Crab Nebula was also giving off pulsed X-rays. Recently Vasseur et al. reported possible gamma ray emission of a pulsed nature from a region near the Crab Nebula. Since the pulses of the gamma ray, X-ray, optical, and radio radiation are all similar they are currently believed to originate from the same source, and current theories support the possibility that this source may be a neutron star.

It should be emphasized that while some radio, optical, and X-ray radiation has been observed to be of a pulsed nature this does not account for all of the radiation from the Crab Nebula. In 1964 Hewish and Okoye detected a strong low frequency radio source (38 MHz) in the Crab Nebula. This source, less than 1" arc in diameter, has spectral characteristics unlike any other known radio source and has attracted considerable attention. The latest position found for this low frequency radio source corresponds to the position of the pulsar and there has been speculation that it may indeed be the pulsar.

Thus some radiation from the Crab Nebula is polarized
and may be attributed to a synchrotron radiation process, some is pulsed and associated with the pulsar, and some forms a source of unusual spectral characteristics at low radio frequencies. The current problem in astrophysics is to determine the process or processes which are causing this complex radiation from within the nebula. The next step in solving this problem is to gain more precise information about this radiation. In the following sections the results of some high resolution observations of the 11.1 cm. polarized radiation will be presented and compared with previously published results in an attempt to gain an improved picture of the distribution of the radio polarization in the nebula.

6.2 Previous Radio Polarization Measurements

Before presenting the 11.1 cm. polarization results which we obtained let us examine some previous radio polarization measurements of the Crab Nebula. The best radio map of the total radiation from the Crab Nebula at this time is the one obtained by Hogg et al. with the NRAO interferometer at 11.1 cm. Figure 23 contains a tracing of Hogg's map on a grid of the right ascension and declination coordinates. In what follows we will use Hogg's map as a reference to show the position at which several workers have found significant amounts of radio polarization.

Table 16 contains the results of nine linear polarization measurements of the Crab Nebula over a wavelength range
of 1.55 cm. to 21.1 cm. The reference for each measurement is given in the bibliography. Except for a few, these are pencil beam measurements of rather low resolution. This is one of the reasons that we are investigating the Crab Nebula with the high resolution techniques of interferometry.

Figures 24, 25, and 26 are to be used with table 16. Those measurements listed in table 16 which were of such poor resolution that only the center of polarization could be given are plotted in figure 24. Letter codes are used to identify the respective positions for the polarization centers. The measurements of Hobbs and Hollinger at 2.07 cm. were more extensive and are plotted in figure 25. Those of Seielstad and Weiler at 21 cm. were obtained with a fan beam and are plotted in figure 26. Conway and Kronberg have measured the polarization at 21 cm. and their results are also shown in figure 26.

If one looks closely at the percentage polarization in table 16 it is seen that there is a slight tendency toward a decrease with increasing wavelength. This could indicate depolarization by any one of several mechanisms such as differential Faraday rotation within the source, spectral effects, etc. Since the percentage of polarization is also quite dependent upon the resolution of the observing instrument if that resolution is greater than the size of the polarized region, it is difficult to interpret the data given in table 16.
Table 16
Crab Nebula Linear Polarization Measurements

<table>
<thead>
<tr>
<th>Reference</th>
<th>Letter</th>
<th>Wavelength</th>
<th>Percentage Polarization</th>
<th>Polarization Angle</th>
<th>Resolution</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>MH</td>
<td>1.55 cm.</td>
<td>16%</td>
<td>154°</td>
<td>1.7' arc</td>
<td>3' arc gaussian</td>
</tr>
<tr>
<td>2</td>
<td>AB</td>
<td>1.97 cm.</td>
<td>11%</td>
<td>159°</td>
<td>2.8' arc</td>
<td>2' arc gaussian</td>
</tr>
<tr>
<td>43</td>
<td>HH</td>
<td>2.07 cm.</td>
<td>11.6±0.5%</td>
<td>154±2°</td>
<td>2' arc</td>
<td>central northern (see figure 6.2-3)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4.1±2.2%</td>
<td>117±4°</td>
<td></td>
<td>eastern</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10.9±1.6%</td>
<td>153±4°</td>
<td></td>
<td>southern</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>9.8±2.5%</td>
<td>140±7°</td>
<td></td>
<td>western</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6.7±1.7%</td>
<td>150±7°</td>
<td></td>
<td>southwestern</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10.5±1.6%</td>
<td>159±4°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>G</td>
<td>6.0 cm.</td>
<td>20%</td>
<td>141°</td>
<td>4.1' arc</td>
<td>2' arc gaussian</td>
</tr>
<tr>
<td>29</td>
<td>DT</td>
<td>9.8 cm.</td>
<td>6.1%</td>
<td>135°</td>
<td>1.55' by</td>
<td>2.3' arc gaussian</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.9' arc</td>
<td></td>
</tr>
<tr>
<td>71</td>
<td>MR</td>
<td>10.6 cm.</td>
<td>9%</td>
<td>141°</td>
<td>4' arc</td>
<td>1.9' arc gaussian</td>
</tr>
<tr>
<td>27</td>
<td>DGHM</td>
<td>21.1 cm.</td>
<td>2.5%</td>
<td>86°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>91</td>
<td>SW</td>
<td>21.1 cm.</td>
<td>2.0±0.5%</td>
<td>78°</td>
<td>&lt;1' arc</td>
<td>eastern (see figure 6.2-4)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.9%</td>
<td>132°</td>
<td></td>
<td>western</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2%</td>
<td>84°</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>≥25%</td>
<td>155°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>CK</td>
<td>21 cm.</td>
<td>&gt;10%</td>
<td>complex</td>
<td>1.2' arc</td>
<td>southeastern (see central figure 6.2-4)</td>
</tr>
</tbody>
</table>

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Figure 23. The total radiation from the Crab Nebula at 11.1 cm., Hogg et al.46
Figure 24. Positions for maximum polarization. (Refer to table 16.)
Figure 25. Positions associated with the observations of Hobbs and Hollinger. (Refer to table 16.)
Figure 26. Positions associated with the observations of Conway and Kronberg (CK) and Seielstad and Weiler (SW). (Refer to table 16.) The observations of Seielstad and Weiler were taken with a synthesized fan beam. The curve represents the fan beam response to the polarized radiation. The circles associated with the positions of Conway and Kronberg represent the approximate upper limit to the size of each source.
with respect to depolarization. Observations at higher resolutions will be very helpful here.

Looking closer at the data in table 16 the following rough picture concerning the polarization of the Crab Nebula is obtained. In the high and middle frequency range of the radio spectrum (wavelengths of a few centimeters to about ten centimeters) the polarization is reported as consisting of a single broad area centered near the center of the total radiation with possible extensions to the east and southwest. At lower frequencies (above about twenty centimeters wavelength) the polarization is reported as originating from two or three separate regions. It is possible that this separation into regions would also be observed in the higher frequency data if the observations had been obtained at higher resolutions. Since our observations are made at a much higher resolution than those reported in table 16 we should be able to resolve this question for the 11 cm. radiation.

6.3 Total Power and Polarization Measurements

We observed the Crab Nebula in September and October of 1968 with the NRAO radio interferometer at 11.1 cm. wavelength. Two different baseline configurations were used giving resolutions down to 8" arc. The tracks followed on the u-v plane are plotted in figure 27. The position assumed for the source during the observations is (epoch 1950)

\[ \alpha = 5^h 31^m 29.99^s \]
\[ \delta = 21^\circ 59' 0.0'' \]
The calibration of the baseline, instrumental polarization, etc. has been described in Chapter 5.

As shown in figure 27 the coverage of the u-v plane is along a line allowing the synthesis of a fan beam. The fan beam is oriented perpendicular to a line passing through the coordinates given above in a direction 71° to 72° east of north. The visibility function obtained for the total power is plotted in figure 28. It is only given out to 8,000 wavelengths since the source is essentially resolved at higher spatial frequencies. The Fourier transform of the total power visibility is plotted in figure 29. Figure 30 contains the visibility function for the polarization (twice the LR component of the coherency matrix). The Fourier transform of the polarization visibility function is plotted in figure 31.

Those spatial frequencies near zero were not obtained for either the polarization or total power measurements, and the visibility functions had to be approximated there. The approximation for the total power visibility was made so as to make the fan beam response match Hogg's map. The approximation for the polarization visibility was made to fit the 11 cm. integrated polarization results published by Gardner and Davies.\textsuperscript{37}

The polarization response is plotted to the scale of Hogg's total power map in figure 32. Recalling the rough
picture obtained for the polarization of the Crab Nebula by examining table 16 it appears that the 11.1 cm. radiation does originate from several smaller regions instead of one broad region. Four peaks in the polarization response have been labeled A through D to facilitate the following discussion. Since the fan beam is able to locate the position of a source only in one direction the position in the perpendicular direction must be obtained by reference to other observations. In general, examination of the previous polarization measurements reveals four separate areas from which polarization has been detected. Three of these areas were detected by Conway and Kronberg\textsuperscript{24} at 21 cm. and are plotted in figure 26. The fourth area corresponds to the position of Hobbs and Hollinger's southwestern measurement.\textsuperscript{43} The peaks in the 11.1 cm. polarization response reported here seem to correspond to these four areas. The letters A, B, C, and D of each peak have also been plotted on the total power map in figure 32 at the position of each corresponding area.

6.4 Discussion of Results

It has been shown that the linear polarization angle will vary linearly with the square of the wavelength due to Faraday rotation in the media between the source and the observer.\textsuperscript{36} The measured polarization angles listed in table 16 and the polarization angles reported here for each of the three areas B, C, and D (no data is available for A)
have been plotted against wavelength squared in figure 33. Examination of figure 33 reveals that the data is approximately linear. The derivative of the polarization angle versus wavelength squared curve in radians per square meter is called the rotation measure. The rotation measures computed for areas B and D correspond to those reported by Conway and Kronberg. The rotation measure for area C is quite different from what Conway and Kronberg reported (−90 radians/m² versus −35 radians/m²). This may be due to our data being confused by the B area of polarization.

It is difficult to compute accurate polarization percentages from strip scan data such as we have here. The true size of an area of polarization is unknown, and two areas may actually overlap as is probably the case with the B and C areas. Rough estimates, assuming circular symmetry and using the positions for the areas of polarization given in figure 32, yield the following:

- Area A is about 23±5% polarized.
- Area B is about 19±5% polarized.
- Area C is greater than 24% polarized.
- Area D is about 30±7% polarized.

Conway and Kronberg list area A as being greater than 10% polarized, area B as being about 2% polarized, and area C as being greater than 25% polarized. These all agree with the above estimates since one would expect a higher percentage
polarization at higher frequencies and higher resolutions. Gardner finds a 20\% polarization in the 6 cm. radiation from the D area, but his resolution was 4.1' arc and the size of the source at D which he reported was about 2' arc in diameter as opposed to our size of about 0.5' arc.

The strip scan polarization response obtained here (11.1 cm.) is compared in figure 34 with a strip scan response computed from the optical polarization measurements of Walraven. Figure 35 contains a comparison of the percentage polarizations along the strip scan for the radio and optical results shown in figure 34. It is apparent that the depolarization between optical and radio frequencies is much greater in the central and western part of the source than in the eastern part.

The following generalizations may be made from the 11.1 cm. polarization measurements reported here: (1) The 11.1 cm. polarization originates from several distinct regions within the boundaries of the total power radiation. (2) No significant polarization structure is seen at 11.1 cm. having dimensions less than about 30' arc. (3) At 11.1 cm. larger amounts of polarization are found in the eastern part of the source than in the western part. (4) Most of the polarization at 11.1 cm. comes from the central part of the total power radiation. (5) A rather abrupt change in the polarization angle is detected in the western part of the
source at 11.1 cm. (6) The depolarization between optical and radio frequencies is much greater in the central and western part of the source than in the eastern part.

Note that the above conclusions must apply only to the continuous radiation (non-pulsed) since our time constant was too large to resolve any pulsation such as that which has been observed from the pulsar.
Figure 27. Coverage of the u-v plane while observing the Crab Nebula.
The length of the baseline corresponding to each track is given in meters. In September 1968 the interferometer was positioned to give the 300m., 2400m., and 2700m. baselines. In October 1968 the position was changed so that the 900m. and 1800m. baselines could be obtained. ($\lambda = 11.1$ cm.)
Figure 28. Total power visibility for the Crab Nebula. The parameter "w" represents the distance from the origin of the u-v plane along a line about 71.5° east from north. ($\lambda = 11.1 \text{ cm}$.)}
Figure 29. Total power response to the Crab Nebula along a line 71.5° east of north passing through the assumed position

\[ \alpha = 5^h \ 31^m \ 29.99^s \ \text{and} \ \delta = 21^\circ \ 59' \ 0'' \text{ at zero.} \]
Figure 30. Linear polarization visibility for the Crab Nebula.
The parameter "w" represents the distance from the origin of
the u-v plane along a line about 71.5° east from north.
(\lambda = 11.1 \text{ cm.})
Figure 31. The linear polarization response to the Crab Nebula (twice the LR component of the coherency matrix) along a line $71.5^\circ$ east of north passing through the assumed position of the source, 

$\alpha = 5^h 31^m 29.99^s$ and $\delta = 21^\circ 59' 0''$, at zero.
Figure 32. The amplitude of the linear polarization response to the Crab Nebula overlaid on Hogg's map of the total power response.
Figure 33. Linear polarization angles for areas B, C, and D versus wavelength squared. This data was obtained from table 16 and figure 31.
Figure 34. Optical and radio linearly polarized strip scan response to the Crab Nebula. The amplitude scale is different for the optical and radio curves. Each is proportional to the linearly polarized power.
Figure 35. Optical and radio linear polarization percentages along the strip scans of figure 34.
CHAPTER VII

3C33

7.1 **Introduction**

While the Crab Nebula is located within our galaxy and is associated with a star that suffered an explosion, the radio source 3C33 is located about $1.7 \times 10^8$ psc from our galaxy (that's about 6,000 times the diameter of our galaxy) and is associated with a galaxy that is believed to have suffered an explosion. Optically 3C33 appears to have a circular nucleus about 2.5" arc in diameter imbedded in an elongated structure of dimensions 3.3" arc by 8.4" arc. These structures are in turn imbedded in a faint outer envelope measuring about 9.4" arc by 22" arc. To a radio telescope 3C33 appears to be two sources separated by about 250" arc. The optical image of 3C33 lies on a line between the two radio sources, and it is believed that the radio sources are a result of material expelled from the galaxy in the explosion. Due chiefly to its greater distance we receive only about 0.01 times as much flux from 3C33 at 11.1 cm. as we do from the Crab Nebula. The events occurring in 3C33 are, however, much more extensive than those in the Crab Nebula. For example, in linear dimensions
alone 3C33 is roughly 100,000 times larger than the Crab Nebula. If 3C33 were brought as close as the Crab Nebula we would receive about $2.5 \times 10^8$ times more flux from it at 11.1 cm. as we now do from the Crab Nebula.

The mechanisms responsible for the radiation from 3C33 may, as in the case of the Crab Nebula, be quite varied. Since the radio radiation is partially polarized it is felt that the synchrotron mechanism must play a significant role. It is possible that the material expelled in the explosion is highly ionized and producing radiation by the synchrotron process as it interacts with the local magnetic fields.

It was mentioned above that the radio picture of 3C33 consists of two sources. Some workers also detect a weaker third source somewhere between the two previously mentioned sources. The nature of this third source is still somewhat in doubt. It may be associated with material ejected from the galaxy in another explosion, or it could be associated with the optical object. This was one of the reasons which prompted us to observe 3C33.

7.2 Previous Radio Polarization Measurements

In order to compare the results obtained by various workers at several different frequencies certain parameters are defined in figure 36 which, taken together, describe
the 3C33 radio source. Table 17 contains the values for the parameters given in figure 36 which correspond to the results obtained by several workers at different frequencies. The parameters consistent with our results are also given in table 17. Our results and their relationship to the previous measurements will be discussed in the next sections.

- $D_E$ represents the dimensions of the east radio source.
- $D_W$ represents the dimensions of the west radio source.
- $S_E$ represents the radio flux of the east source.
- $S_C$ represents the radio flux of the central source.
- $S_W$ represents the radio flux of the west source.

**Figure 36.** Parameters describing 3C33.
<table>
<thead>
<tr>
<th>Reference</th>
<th>Wavelength</th>
<th>B</th>
<th>$D_E$</th>
<th>$D_W$</th>
<th>P.A.</th>
</tr>
</thead>
<tbody>
<tr>
<td>This Work</td>
<td>11.1 cm.</td>
<td>_____</td>
<td>20&quot; arc</td>
<td>±5&quot; arc</td>
<td>±2.5&quot; arc</td>
</tr>
<tr>
<td>Moffet$^69$</td>
<td>10.6 cm.</td>
<td>215&quot; arc to 250&quot; arc</td>
<td>16&quot; arc by 8&quot; arc$^*$</td>
<td>16&quot; arc by 8&quot; arc$^*$</td>
<td>19°</td>
</tr>
<tr>
<td>Seielstad$^89$</td>
<td>10.6 cm.</td>
<td>240±30&quot; arc</td>
<td>13±2&quot; arc</td>
<td>13±2&quot; arc</td>
<td>20°</td>
</tr>
<tr>
<td>Fomalont$^{31}$</td>
<td>21 cm.</td>
<td>_____</td>
<td>&lt; 0.5' arc</td>
<td>&lt; 0.5' arc</td>
<td>_____</td>
</tr>
<tr>
<td>Seielstad$^92$ &amp; Weiler</td>
<td>21.2 cm.</td>
<td>_____</td>
<td>_____</td>
<td>_____</td>
<td>_____</td>
</tr>
<tr>
<td>Macdonald et al.$^{61}$</td>
<td>21.3 cm.</td>
<td>244&quot; arc</td>
<td>20&quot; arc by &lt; 25&quot; arc by</td>
<td>20°</td>
<td></td>
</tr>
<tr>
<td>Maltby &amp; Moffet$^{62}$</td>
<td>31.3 cm.</td>
<td>228±36&quot; arc</td>
<td>&lt; 20&quot; arc</td>
<td>&lt; 20&quot; arc</td>
<td>20°</td>
</tr>
<tr>
<td>Macdonald et al.$^{61}$</td>
<td>73.5 cm.</td>
<td>_____</td>
<td>_____</td>
<td>_____</td>
<td>_____</td>
</tr>
<tr>
<td>Allen et al.$^1$</td>
<td>190 cm.</td>
<td>_____</td>
<td>4&quot; arc</td>
<td>4&quot; arc</td>
<td>_____</td>
</tr>
</tbody>
</table>

** Major axis is 20° east of north.
Table 17 cont.

Radio Measurements of 3C33

<table>
<thead>
<tr>
<th>Reference</th>
<th>$S_E$</th>
<th>$S_C$</th>
<th>$S_W$</th>
<th>$P_E$</th>
<th>$\phi_E$</th>
<th>$P_W$</th>
<th>$\phi_W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>This Work</td>
<td>1.3</td>
<td>—</td>
<td>5.1</td>
<td>$2.9%$</td>
<td>$95^\circ$</td>
<td>9.6$%$</td>
<td>$81^\circ$</td>
</tr>
<tr>
<td></td>
<td>$\pm0.4$</td>
<td>—</td>
<td>$\pm0.5$</td>
<td>$\pm1.5%$</td>
<td>$\pm30^\circ$</td>
<td>$\pm0.8%$</td>
<td>$\pm6^\circ$</td>
</tr>
<tr>
<td>Moffet$^{69}$</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Seielstad$^{89}$</td>
<td>2.7</td>
<td>—</td>
<td>4.6</td>
<td>$5.5%$</td>
<td>$88^\circ$</td>
<td>8.1$%$</td>
<td>$88^\circ$</td>
</tr>
<tr>
<td>Fomalont$^{31}$</td>
<td>2.5</td>
<td>3.3</td>
<td>7.0</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Seielstad &amp; Weiler$^{92}$</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>4.5$%$</td>
<td>$62^\circ$</td>
<td>11.5$%$</td>
<td>$62^\circ$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\pm0.5%$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Macdonald et al.$^{61}$</td>
<td>3.3</td>
<td>—</td>
<td>9.9</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Maltby &amp; Moffet$^{62}$</td>
<td>$S_E$</td>
<td>—</td>
<td>—</td>
<td>2.5$xS_E$</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>of &lt; 20% of total flux</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Macdonald et al.$^{61}$</td>
<td>7.4</td>
<td>—</td>
<td>16.1</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Allen et al.$^1$</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

* Given in flux units.
7.3 Total Power and Polarization Measurements

Figure 37 contains the tracks followed on the u-v plane while observing 3C33 at NRAO in September and October 1968. These tracks enable a fan beam to be synthesized which is oriented perpendicular to a line between 71° and 72° east of north passing through the assumed position of 3C33. This assumed position is (epoch 1950)

\[ \alpha = 1 \hbar 6 \m 13.2 \s \]
\[ \delta = 13^\circ \ 3' \ 33.0'' \]

The visibility functions which were obtained for the total and polarized flux are given in figures 38 and 39 respectively. Since no observations were taken at less than about 1000 wavelengths the visibility functions in this area were interpolated, using Kellerman's value for the total flux. The strip scans derived from the Fourier inversion of the total and polarized flux visibility functions are given in figures 40 and 41. The effective beam width in both cases is about 8'' arc. In figure 40 the peaks of the two sources are labeled "east" and "west" to facilitate the following discussion.

Figure 42 contains a comparison of the two dimensional positions obtained for the east and west source maxima by Macdonald with the strip scans reported here. Examination of figure 42 reveals a good agreement. Thus at 11.1 cm wavelength we obtain essentially the same component separation (244'' arc) observed by Macdonald at 21.3 cm. Our component
size in one dimension also agrees with the results of MacDonald. The peaks of the sources as seen in figure 40 seem to be skewed away from the position of the optical object. This effect has also been reported by Moffet at 10.6 cm.

Figure 43 contains the total integrated flux from the east and west sources as given in table 17 plotted versus the wavelength on a log-log scale. The resulting spectra yield spectral indices for the east and west sources of about 0.8 and 0.6 respectively. All points fit the straight lines corresponding to these spectra fairly well with the exception of those representing the east source at 10.6 and 11.1 cm. This points out a rather large disagreement between our measurement and Seielstad's for the east source. This will be discussed in more detail later.

Figure 44 contains a plot versus frequency squared of the polarization angles listed in table 17. The data of figure 44 fits a straight line yielding a rotation measure of about -13.4 radians/m² for both sources. The percentage polarization of the integrated flux for the west source seems to agree with the measurements at other frequencies. The corresponding measurement for our east source is a bit low, however, when compared with Seielstad's measurement. (See table 17.)

The central source is just visible in our data. Figure 45 contains a plot of the total flux strip scan over 3σ33
convolved with a square pulse 20" arc wide to remove any noise and sidelobe response. The central source now shows up easily along with two new sources on the west side of the west source. Since it is difficult to accurately measure a weak source which is located near a strong one with an interferometer the size of the central and two new western sources must be viewed with caution. We detect the total integrated flux from the central source as being $1.5 \pm 0.5$ flux units. It is located, in one dimension, near or at the position of the optical object. It appears to be about 10" to 30" arc wide along our line of resolution. Our results for all the sources of 3C33 are summarized in table 18.

7.4 Discussion of Results

In general our results for the west source agree with the results other workers have obtained at other frequencies. Our results for the east source, however, do not agree very well with what Seielstad obtained. Let us compare the procedures used by Seielstad with those used here to see if an explanation of this discrepancy can be found. At 10.6 cm, Seielstad used an interferometer to obtain an east-west strip scan, the same type of an observation we have made except that his is along a line 90° east of north. This difference should not contribute to the discrepancy. However, Seielstad was only able to determine the visibility function at about 10 points between 0 and 5000 wavelengths on the u-v plane. Our
## Table 18
The Radio Source 3C33 at 11.1 cm.

<table>
<thead>
<tr>
<th>Source</th>
<th>Integrated Total Flux*</th>
<th>Integrated Percentage Pol.</th>
<th>Polarization Angle</th>
<th>Angular Extent in P.A. 71.5°</th>
</tr>
</thead>
<tbody>
<tr>
<td>East</td>
<td>1.3±0.4</td>
<td>2.9±1.5%</td>
<td>95±30°</td>
<td>20±5&quot; arc</td>
</tr>
<tr>
<td>Central</td>
<td>1.5±0.5</td>
<td>6±3%</td>
<td>80±30°</td>
<td>15±5&quot; arc</td>
</tr>
<tr>
<td>Original West</td>
<td>5.1±0.5</td>
<td>9.6±0.8%</td>
<td>81±6°</td>
<td>12.5±2.5&quot; arc</td>
</tr>
<tr>
<td>First New West</td>
<td>&lt; 1.7</td>
<td>0</td>
<td></td>
<td>40±15&quot; arc</td>
</tr>
<tr>
<td>(−110° arc in Figure 45)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Second New Western</td>
<td>&lt; 1.7</td>
<td>0</td>
<td></td>
<td>30±12&quot; arc</td>
</tr>
<tr>
<td>(−180° arc in Figure 45)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* In flux units—one flux unit equals 10⁻²⁶ Watts/m²Hz
Figure 37. Tracks followed on the u-v plane while observing 3633 in October and September 1968. ($\lambda = 11.1$ cm.)
Figure 38. Total power visibility for 3C33. $w$ represents the distance from the origin of the u-v plane along a line 71.5° east of north. $w$ is positive in the positive u direction. ($\lambda = 11.1$ cm.)
Figure 39. Linear polarization visibility for 3C33. \( w \) represents the distance from the origin of the u-v plane along a line 71.5° east of north. \( w \) is positive in the positive u direction. \( (\lambda = 11.1 \text{ cm}) \)
Figure 40. Total power response to 3C33 along a line 71.5° east of north passing through the assumed position 
\[ \alpha = 1^h 6^m 13.2^s \] and \( \delta = 13^\circ 3' 33.0'' \) at zero.
Figure 41. The linear polarization response to 3C33 (twice the LR component of the coherency matrix) along a line 71.5° east of north passing through the assumed position of the source, \( \alpha = 1^h 6^m 13.2^s \) and \( \delta = 13^\circ 3^\prime 33.0^\prime\prime \), at zero.
Figure 42. Source maxima obtained by Macdonald compared with the strip scan total power response of figure 45.
Figure 43. Total integrated flux from the east and west sources as a function of wavelength.
Figure 44. Polarization angle versus wavelength (in meters) squared.
Figure 45. Strip scan of the total power radiation from 3033 as shown in figure 40 convolved with a square pulse 20" arc wide.
data is essentially continuous out to 24,000 wavelengths. The result of this difference is that while we could obtain our strip scan by Fourier inverting our visibility function Seielstad had to use a technique called model-fitting. In doing this he first picked the simplest model for the strip scan which the visibility function seemed to suggest (a double source). He then manipulated the model parameters (source size, separation, etc.) until the Fourier transform of the model made a good fit to the observed visibility function.

Here lies the root of the matter. Examination of figure 45 reveals that we obtained more than just two sources, and the farther west of the two new western sources (between -160" and -200" arc from zero) lies almost exactly as far from the original west source as the east source does. Due to this the first few nulls of the visibility function amplitude will be approximately equal to the total flux of the original west source minus the total flux of the sum of the east source and the farther west of the two new western sources. If you add the flux we obtain for these two sources you will get about 2.5 flux units, nearly the flux Seielstad obtained for the east source. Thus it seems possible that Seielstad's east source flux may be in error, an unavoidable error caused by his method of analysis. Seielstad recognized this possibility and remarked that his results were "plausible, but not necessarily unique."
Now let us examine the two new western sources which, as we have remarked, have to be viewed with caution. It has been stated that our sampling of the visibility function was essentially continuous out to 24,000 wavelengths on the u-v plane. There were two exceptions to this. No observations could be made at spacings below 1000 wavelengths and in an area a few hundred wavelengths wide near 3000 wavelengths. The resulting interpolation at 3000 wavelengths is felt to have a negligible effect on the final result due to its relatively small size when compared to the smallest structure present in the rest of the visibility function. The interpolation below 1000 wavelengths is, however, likely to have an effect. The result of any error in that interpolation will give the strip scan (the Fourier inversion of the visibility function) false sources with sizes in excess of about 1' arc. Sources less than 1' arc in extent, such as the two new western sources, are primarily determined by that part of the visibility function above 1000 wavelengths and thus should be unaffected by the interpolation. The net effect of the interpolation below 1000 wavelengths is thus felt to be a slow baseline drift. The places in figure 45 where the flux goes negative are felt to be caused by this baseline drift. Of course this drift makes interpretation of our results a bit difficult, and until more measurements are taken to fill in the interpolated areas of our visibility
functions we are forced to view all our results, including the two new western sources, with caution. This caution is reflected in the standard errors quoted for our results.

We gave the results which our observations provided for the central source in the previous section. The discussion given above concerning the two new western sources also applies to the central source, and our standard errors reflect our caution in interpreting the results for the central source also. We can have a bit more confidence in the central source since several other workers have also reported its presence. Macdonald is the only other worker to provide a position for it, however. Fortunately our position agrees with his. (See figure 40.) His central source is somewhat broader than ours. This is probably due to the fact that our resolution is nearly three times higher than his.

In conclusion the radio source 3C33 appears to be composed of at least three and possibly five components. The central component lies nearly at the position of the optical object and appears to be slightly polarized. The stronger components appear to be skewed away from the optical object and also appear to be polarized. The polarization angles are nearly constant for all the components which appear to be polarized.
APPENDIX A

FAST FOURIER TRANSFORM

A.1 Introduction

Many problems in science require the computation of a continuous Fourier transformation. Since this computation is usually to be done on a digital computer the continuous Fourier transformation is usually approximated by a discrete Fourier transformation. The fast Fourier transformation (FFT) is a clever procedure for computing the discrete Fourier transform (DFT) of a finite sequence.\textsuperscript{22,26} The FFT reduces both the time and round-off error involved in the conventional computation by a factor of $2(\log_2 N)/N$, where $N$ is the number of terms in the sequence. The following table illustrates the usefulness of the FFT procedure.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
$N$ & $N/2(\log_2 N)$ \\
\hline
4 & 1.0 \\
16 & 2.0 \\
64 & 5.3 \\
256 & 16.0 \\
1024 & 51.0 \\
4096 & 170.0 \\
\hline
\end{tabular}
\caption{Improvement Factor for the FFT}
\end{table}

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This appendix describes the relationship between the discrete and continuous Fourier transforms and a form of the FFT called "decimation in frequency." The FFT may be used for any case in which \( N \) is not a prime number. Here \( N \) is always considered to be a power of two for simplicity.

### A.2 Definitions

Let us define \( x(t) \) and \( a(f) \) to be a continuous Fourier transform pair if they are related by the continuous Fourier transform relationship.

\[
\begin{align*}
x(t) &= \int_{-\infty}^{\infty} a(f)e^{-j2\pi ft} \, df \quad -\infty < t < \infty \\
a(f) &= \int_{-\infty}^{\infty} x(t)e^{+j2\pi ft} \, dt \quad -\infty < f < \infty
\end{align*}
\]

The following shorthand notation will be used to signify the Fourier transform relationship.

\[x(t) \leftrightarrow a(f)\]

The same notation will be used for both the discrete and continuous cases.

Let us also define \( x_n \) and \( a_n \) to be a discrete Fourier transform pair if they are related by the discrete Fourier transform relationship.
\[ x_i = \sum_{n=0}^{N-1} a_n e^{-j2\pi in/N}, \quad i = 0, 1, \ldots, N-1 \]

\[ a_n = \frac{1}{N} \sum_{i=0}^{N-1} x_i e^{j2\pi in/N}, \quad n = 0, 1, \ldots, N-1 \]

### A.3 Relationship Between Discrete and Continuous Transforms

If \( x(t), -\infty < t < \infty \), and \( a(f), -\infty < f < \infty \), are a Fourier transform pair, we may define two new functions

\[ x_p(i\Delta t) = \sum_{l=-\infty}^{\infty} x(i\Delta t + lT) \]

and

\[ a_p(n\Delta f) = \sum_{k=-\infty}^{\infty} a(n\Delta f + kF) \]

where

\[ \Delta f = \frac{1}{N\Delta t} = \frac{1}{T} = F/N \]

such that \( x_p(i\Delta t) \) and \( a_p(n\Delta f) \) are a discrete Fourier transform pair. This defines the relationship between discrete and continuous Fourier transform pairs. A proof of the above statements has been given by Cochran.\(^{22}\)

When one wishes to use a digital computer to compute \( x(t) \) given \( a(f) \) where \( x(t) \) and \( a(f) \) are a continuous Fourier transform pair one must first form \( a_p(f) \). From this function the sequence \( a_p(n\Delta f) \) is formed. The function \( x_p(i\Delta t) \) may then be found from \( a_p(n\Delta f) \) by a discrete Fourier transformation. It is in this step that the fast Fourier transformation procedure may be used. Once \( x_p(i\Delta t) \) is obtained it is
a simple procedure to obtain $x(t)$ by truncation and shifting. The above steps are illustrated in figure 46.

It should be pointed out that it is necessary to pick $T$ large enough to avoid any aliasing errors in the formation of $x_p(i\Delta t)$. Also $F$ must be picked large enough to obtain the desired resolution in $x(t)$. Together $T$ and $F$ then determine $N$, $\Delta t$, and $\Delta f$.

A.4 Decimation in Frequency

Now let us consider the fast Fourier transformation procedure called "Decimation in Frequency." Given the inverse of the discrete Fourier transformation

$$T x_1 = \sum_{r=0}^{N-1} A_r W^{-rl} + j2\pi/N$$

where $W = e^{-j2\pi/N}$, split the quantity being summed into two parts.

$$T x_1 = \sum_{r=0}^{(N/2)-1} (A_r W^{-rl} + A_{r+N/2} W^{-(r+N/2)l})$$

Now consider the even and odd subscripted terms in the summation separately.
Thus the inverse DFT of a sequence of terms has been replaced by the inverse DFT of two smaller sequences. This could not have been done if the original sequence had not contained an even number of terms. If the two smaller sequences also have an even number of terms, each of them may also be broken into two sequences. If $N$, the number of terms in the original sequence, is a power of two this process may be continued until the original sequence is broken into $N/2$ smaller sequences, each with only two terms. Figure 47 contains a flow diagram describing the FFT for a sequence of four terms using "Decimation in Frequency."
Given Form

Perform the FFT

Scaling and Reconstruction

Figure 46. Performing a continuous Fourier transformation using the FFT procedure.
\[ T_{x_1} = \sum_{r=0}^{N-1} A_r W^r \quad l = 0,1,2,3 \quad W = e^{j2\pi/4} = j \]

Figure 47. Flow diagram of the FFT using "Decimation in Frequency."
APPENDIX B

THE PHASE GRADIENT METHOD

B.1 Introduction

A procedure is described in this appendix which may be used to obtain the position of a point source with an interferometer. Since the procedure involves finding the gradient of the source visibility phase it shall be called the "Phase Gradient Method." The accuracy of a position found by this procedure depends upon the following five factors:

1) The position of the source
2) The length and orientation of the interferometer baseline
3) The accuracy with which the interferometer baseline is known
4) The length of time during which the source is observed
5) The phase stability of the interferometer

The relationships between these factors and their relative influence on the accuracy of the position measurements are also discussed in this appendix. In addition, some sample results from a small computer program which has been written to perform the calculations for the phase gradient method are described.
B.2 The Phase Gradient Method

The visibility phase obtained when tracking a point source with an interferometer of fixed baseline $\hat{B}$ may be written as follows:

\[(B.2-1) \phi \approx -360 \, u \cos \delta_a \Delta \alpha^R
-360 \, v \Delta \delta^R
-360 \, \cos H_a \cos \delta_a \Delta B_x
-360 \, \sin H_a \cos \delta_a \Delta B_y
-360 \, \sin \delta_a \Delta B_z
-360 \, \cos \delta_a (-k)
- \Delta \phi + \phi_o\]

$\phi$ is the phase in degrees measured by the interferometer.

$\phi_o$ is the instrumental phase in degrees.

$\Delta \phi$ is the phase error introduced by unknown effects.
(At NRAO $\phi \approx \pm 15^\circ$ for one five minute observation at $\hat{B} = 24,000 \lambda$)

$\delta_a$ is the assumed declination of the source.

$H_a$ is the assumed hour angle of the source.

$\Delta B_x$, $\Delta B_y$, and $\Delta B_z$ represent the errors in the assumed baselines. ($\Delta B + \hat{B}_{true} = \hat{B}_{assumed}$)

$\Delta \alpha^R$ is the error in the assumed right ascension in radians. ($\alpha_{true} + \Delta \alpha = \alpha_{assumed}$)

$\Delta \delta^R$ is the error in the assumed declination in radians. ($\delta_{true} + \Delta \delta = \delta_{assumed}$)

$k$ represents the misalignment of the polar-declination axis distance of the two interferometer elements.
Equation B.2-1 is a valid approximation when $\Delta \alpha$ and $\Delta \delta$ are smaller than a few minutes of arc and the parameters $\Delta B_x$, $\Delta B_y$, and $\Delta B_z$ are within a few wavelengths. The error in the approximation of $\phi$ amounts to about $3^\circ$ for a 2' arc position difference between the true position $(\alpha_t, \delta_t)$ and the assumed position $(\alpha_a, \delta_a)$.

The $H$ variable may be eliminated from equation B.2-1 if consideration is given only to sources not on the equator. Thus $\cos(H)$ and $\sin(H)$ may be found in terms of $u$, $v$, and $\delta$ for $\delta \neq 0$ as follows:

\begin{align*}
(B.2-2) \quad u &= B_x \sin H - B_y \cos H \\
(B.2-3) \quad v &= -B_x \sin \delta \cos H - B_y \sin \delta \sin H + B_z \cos \delta
\end{align*}

Equations B.2-2 and B.2-3 are derived in Chapter 2. From equation B.2-2 one may obtain

\begin{align*}
(B.2-4) \quad \sin H &= \frac{u}{B_x} + \frac{B_y}{B_x} \cos H
\end{align*}

Substituting equation B.2-4 into equation B.2-3 yields

\begin{align*}
(B.2-5) \quad v &= -\left[B_x \sin \delta + B_y \sin \delta \left(\frac{B_y}{B_x}\right)\right] \cos H \\
&\quad - B_y \left(\frac{u}{B_x}\right) \sin \delta + B_z \cos \delta
\end{align*}
Solving equation B.2-5 for \( \cos(H) \)

\[
(B.2-6) \quad \cos H = -u\left(\frac{B_y}{B_x^2 + B_y^2}\right) - v\left(\frac{B_x}{B_x^2 + B_y^2}\right) \frac{1}{\sin \delta} + \frac{B_x B_z}{B_x^2 + B_y^2} \frac{1}{\tan \delta}
\]

From equations B.2-4 and B.2-6 one obtains

\[
(B.2-7) \quad \sin H = u\left(\frac{B_x}{B_x^2 + B_y^2}\right) - v\left(\frac{B_y}{B_x^2 + B_y^2}\right) \frac{1}{\sin \delta} + \frac{B_y B_z}{B_x^2 + B_y^2} \frac{1}{\tan \delta}
\]

From equations B.2-1, B.2-6, and B.2-7 the following may be obtained.

\[
(B.2-8) \quad \phi \approx Au + Cv + E
\]

where

\[
A = -360 \cos \delta \left[ \Delta \alpha^R - \Delta B_x \left(\frac{B_y}{B_x^2 + B_y^2}\right) + \Delta B \left(\frac{B_x}{B_x^2 + B_y^2}\right) \right]
\]

\[
C = -360 \left[ \Delta \delta^R - \Delta B_x \left(\frac{B_x}{B_x^2 + B_y^2}\right) \frac{1}{\tan \delta} - \Delta B \left(\frac{B_y}{B_x^2 + B_y^2}\right) \frac{1}{\tan \delta} \right]
\]

\[
E = -360 \left[ \Delta B_x \left(\frac{B_x B_z}{B_x^2 + B_y^2} \cos \delta \right) + \Delta B_y \left(\frac{B_y B_z}{B_x^2 + B_y^2} \cos \delta \right)
+ \Delta B_z \sin \delta - k \cos \delta \right] - \Delta \phi + \phi.
\]

From equation B.2-8

\[
\frac{d\phi}{du} \approx A + \frac{dE}{du} = A + \frac{d\Delta \phi}{du}
\]
Substituting for $A$ and solving for $\Delta \alpha^R$ yields

\[ (B.2-9) \quad \Delta \alpha^R \approx \frac{-1}{360 \cos \delta} \left( \frac{\partial \phi}{\partial u} - \frac{\partial \phi}{\partial v} \right) + \Delta B_x \left( \frac{B_y}{B_x^2 + B_y^2} \right) - \Delta B_y \left( \frac{B_x}{B_x^2 + B_y^2} \right). \]

Similarly

\[ \frac{\partial \phi}{\partial v} \approx C + \frac{\partial E}{\partial v} = C + \frac{\partial \phi}{\partial v} \]

and by substituting for $C$ and solving for $\Delta \delta^R$ one obtains

\[ (B.2-10) \quad \Delta \delta^R \approx \frac{-1}{360} \left( \frac{\partial \phi}{\partial v} - \frac{\partial \phi}{\partial v} \right) + \Delta B_x \left( \frac{B_y}{B_x^2 + B_y^2} \right) \frac{1}{\tan \delta} \]

\[ + \Delta B_y \left( \frac{B_y}{B_x^2 + B_y^2} \right) \frac{1}{\tan \delta}. \]

The error terms $\Delta \phi$, $\Delta B_x$, and $\Delta B_y$ in equations $B.2-9$ and $B.2-10$ are not known exactly. The maximum magnitude for each may be approximated, however. Thus equations $B.2-9$ and $B.2-10$ may be rewritten as follows:

\[ (B.2-11) \quad \Delta \alpha^s \approx \frac{-240}{2\pi \cos \delta} \frac{\partial \phi}{\partial u} \pm \left[ E_{\Delta \alpha^s} + E_{\Delta B_x} + E_{\Delta B_y} \right] \]

\[ (B.2-12) \quad \Delta \delta^s \approx \frac{-3600}{2\pi} \frac{\partial \phi}{\partial v} \pm \left[ E_{\Delta \delta^s} + E_{\Delta B_x} + E_{\Delta B_y} \right] \]
In equations B.2-11 and B.2-12 $\Delta \alpha^S$ and $\Delta \delta''$ are given in seconds of time and arc respectively. The error terms are as follows:

\[
\frac{E^\phi}{\Delta \alpha^S} = \frac{240}{\pi} \frac{1}{\cos \delta} \frac{|\Delta \phi|_{\text{max}}}{\Delta u}
\]

\[
E^\phi_{\Delta \delta''} = \frac{3600}{\pi} \frac{|\Delta \phi|_{\text{max}}}{\Delta v}
\]

\[
\frac{E^{ABx}}{\Delta \alpha^S} = \frac{360(120)}{\pi} \frac{|B_y|}{B_x^2 + B_y^2} \cdot |\Delta B_x|_{\text{max}}
\]

\[
\frac{E^{ABx}}{\Delta \alpha^S} = \frac{360(120)}{\pi} \frac{|B_y|}{B_x^2 + B_y^2} \cdot \frac{1}{\tan \delta} \cdot |\Delta B_x|_{\text{max}}
\]

\[
\frac{E^{ABy}}{\Delta \alpha^S} = \frac{360(120)}{\pi} \frac{|B_x|}{B_x^2 + B_y^2} \cdot \frac{1}{\tan \delta} \cdot |\Delta B_y|_{\text{max}}
\]

where $|\Delta \phi|_{\text{max}}$ is the maximum uncertainty in $\phi$,

$\Delta u$ is the range of $u$ sampled,

and $\Delta v$ is the range of $v$ sampled.

Also, $|\frac{\partial \Delta \phi}{\partial u}|_{\text{max}}$ has been set equal to $\frac{2|\Delta \phi|_{\text{max}}}{\Delta u}$

and $|\frac{\partial \Delta \phi}{\partial v}|_{\text{max}}$ has been set equal to $\frac{2|\Delta \phi|_{\text{max}}}{\Delta v}$.

Equations B.2-11 and B.2-12 give the relationship between the phase gradient and the source position errors. The
The phase gradient method simply consists of finding $\frac{d\phi}{du}$ and $\frac{d\phi}{dv}$ and using equations B.2-11 and B.2-12 to find $\Delta \alpha^S$ and $\Delta \delta^"$. The following conclusions may be obtained if one studies the error terms of equations B.2-11 and B.2-12.

1) The higher the declination of the source, the larger will be the effect of the $|\Delta \phi|_{\text{max}}$ parameter on $\Delta \alpha^S$. (This is primarily due to the convergence of the right ascension system near the poles.)

2) At a given declination the errors due to $|\Delta \phi|_{\text{max}}$ are inversely proportional to $\Delta u$ or $\Delta v$.

3) The errors due to $\Delta B_x$ and $\Delta B_y$ are largest for small values of $|B|$.

4) The declination errors due to $\Delta B_x$ and $\Delta B_y$ blow up for sources near the equator.

One word of caution must be given concerning the error terms. One may be tempted to reduce $|\Delta \phi|_{\text{max}}$ by $\frac{1}{\sqrt{N}}$ if $N$ independent observations have been averaged. The correctness of such a maneuver is contingent upon the errors affecting $\Delta \phi$ having a normal distribution. While this is a reasonable assumption it probably is not strictly true. Also notice that while the $|\Delta \phi|_{\text{max}}$ term might be reduced by averaging, the $|\Delta B_x|_{\text{max}}$ and $|\Delta B_y|_{\text{max}}$ terms may not be since the $\Delta B_x$ and $\Delta B_y$ errors are constant from one scan to the next.
B.3 An Example

A small computer program has been written to perform the steps of the phase gradient method automatically. As a first test of this program, henceforth called PGP, the positions of 3C147, 3C286, V42.22.01, and CTA102 were found using data obtained at NRAO in October 1968. Only data from correlator 1 was used. The baselines assumed were those provided for us by Wade. They are listed below.

\[ B_x = -6812.301 \pm 0.01 \]
\[ B_y = -21439.949 \pm 0.01 \]
\[ B_z = -9115.348 \pm 0.01 \]

Table 20 lists the positions originally assumed for the sources \((\alpha_a, \delta_a)\) and the new positions which PGP found \((\alpha_t, \delta_t)\). The positions of three of these sources have been given to us by Wade and these are also listed for comparison. Figure 48 is a graphical comparison of our positions with Wade's.
Table 20

Source Positions

<table>
<thead>
<tr>
<th>Source</th>
<th>Assumed and true positions</th>
<th>Wade's result</th>
</tr>
</thead>
<tbody>
<tr>
<td>3C147</td>
<td>$\alpha_a = 5^h 38^m 43.52^s$</td>
<td>$43.49^s \pm 0.04^S$</td>
</tr>
<tr>
<td></td>
<td>$\delta_a = 49^\circ 49^\prime 42.90^\prime$</td>
<td>$42.4^\prime \pm 0.6^\prime$</td>
</tr>
<tr>
<td></td>
<td>$\alpha_t = 5^h 38^m 43.44^S \pm 0.08^S$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\delta_t = 49^\circ 49^\prime 42.30^\prime \pm 0.8^\prime$</td>
<td></td>
</tr>
<tr>
<td>3C286</td>
<td>$\alpha_a = 13^h 28^m 49.66^S$</td>
<td>$49.66^S \pm 0.04^S$</td>
</tr>
<tr>
<td></td>
<td>$\delta_a = 30^\circ 45^\prime 59.00^\prime$</td>
<td>$58.3^\prime \pm 0.7^\prime$</td>
</tr>
<tr>
<td></td>
<td>$\alpha_t = 13^h 28^m 49.64^S \pm 0.06^S$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\delta_t = 30^\circ 45^\prime 58.4^\prime \pm 1.1^\prime$</td>
<td></td>
</tr>
<tr>
<td>V42.22.01</td>
<td>$\alpha_a = 22^h 0^m 38.90^S$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\delta_a = 42^\circ 2^\prime 9.0^\prime$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\alpha_t = 22^h 0^m 39.34^S \pm 0.08^S$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\delta_t = 42^\circ 2^\prime 8.3^\prime \pm 0.8^\prime$</td>
<td></td>
</tr>
<tr>
<td>CTA102</td>
<td>$\alpha_a = 22^h 30^m 7.76^S$</td>
<td>$7.80^S \pm 0.04^S$</td>
</tr>
<tr>
<td></td>
<td>$\delta_a = 11^\circ 28^\prime 19.50^\prime$</td>
<td>$22.8^\prime \pm 0.7^\prime$</td>
</tr>
<tr>
<td></td>
<td>$\alpha_t = 22^h 30^m 7.79^S \pm 0.07^S$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\delta_t = 11^\circ 28^\prime 21.9^\prime \pm 3.0^\prime$</td>
<td></td>
</tr>
</tbody>
</table>
Figure 48. A graphical comparison of our positions with Wade's.
REFERENCES


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