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THE RELATIONSHIP OF VARIOUS HIGH SCHOOL MATHEMATICS PROGRAMS TO ACHIEVEMENT IN THE FIRST COURSE IN COLLEGE CALCULUS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

Howard William Paul, A.B., M.A.

* * * * * *

The Ohio State University

1970

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CHAPTER I

INTRODUCTION

The evolution of twelfth-grade mathematics courses with the general title of "Advanced Mathematics," has been an interesting phenomenon. Incorporating the essential concepts of solid geometry in the tenth-grade course was one of the initial steps toward expanded offerings for students in the final year of high school. Deleting tedious computations with logarithms from the traditional trigonometry was another. Advanced courses evolved that centered around the concept of "function" or "analysis."

Many high school seniors are exposed to these "new" courses. But certainly not all. A significant number (see Chapter IV) of students still enter the university with their highest level of preparation consisting of a semester of solid geometry and a semester of trigonometry. Algebra (one semester) and trigonometry (one semester) is another frequent combination, as is a semester of analytic geometry and one of trigonometry. A combined or integrated algebra-trigonometry is not uncommon. Semester courses in probability, linear algebra, and matrix algebra are also in evidence.
One of the most rapidly growing programs seems to be the introduction of a unit, a semester, or a full year of college-level calculus. Many criticisms of such a program have been heard \(^1\). Advocates of such plans have also voiced their opinions \(^3\). The effects of high school calculus on achievement in college calculus have been investigated by several persons. The work by Tillotson \(^43\), McKillip \(^32\), and Robinson \(^39\) will be discussed in detail in Chapter II.

A vital question at this point concerns whether or not differences in these preparation programs warrant the claim that any one of them should be preferred to another. There are many factors to consider when the "best" program is sought by a particular school. One point of view is that it doesn't really matter what course is taught as long as it is taught well. Closely related to this is the notion that the qualifications of the teachers should determine what is selected. If the teacher is capable and also comfortable with analytic geometry, then this may produce the best results for the pupils in that particular school.

Bedient \(^3\) showed that teachers in schools with more effective mathematics programs had a better knowledge of the mathematical content of the more modern materials.

\(^1\)The symbol \((x,y)\) will be used to refer to page "y" of entry "x" in the numbered bibliography. The symbol \((x)\) will refer to entry "x" in the bibliography.
He measured the effectiveness of programs by the performance in collegiate mathematics of students trained in these programs. The role of the high school teacher appears to be an important variable.

If, however, teacher competence can be assumed, then what curricular choices should be made? In a large school system where perhaps eight or ten teachers are adequately qualified to teach nearly any appropriate course, a decision must be made. One basis for determining a satisfactory program is to present the set of concepts prerequisite to university calculus or to a college mathematics program in general.

Little research work has been done to identify these prerequisite concepts explicitly. It is not unusual to read or hear the opinions of teachers concerning the nature of a satisfactory or desirable twelfth-grade mathematics program for college-bound students. Research findings by Buchanan (6,223) provide some interesting statistics regarding the opinions of college teachers of mathematics about what should be taught to high school seniors. There are limited research findings suggesting relationships between high school mathematics and success in college. Waggoner (47) has investigated this aspect.

One criterion for evaluating the high school preparation of students is their performance in a subsequent college calculus course. As mentioned earlier, McKillip
and Tillotson measured the effects of high school calculus on college calculus. Robinson (38) and Bedient (3) took a broader view of looking at different methods of meeting prerequisite mathematics, and at teacher characteristics affecting achievement. If it can be determined that students of comparable abilities but with different background preparations actually show significant difference in achievement, then it follows that at least some of the variation can be attributed to differences in their preparation.

Statement of the Problem

The purpose of this study was to determine if certain characteristics in high school background preparation actually relate to achievement of students in a beginning course in college calculus. These characteristics are identified with the concepts presented in a given mathematics course rather than with external factors or variables such as knowledge and training of the teacher or with student attitude.

The study was limited in the first place by the choice of students attending a single state university and in the second place by not considering the high school teacher as a variable in the preparation.

An ever-expanding group of high school seniors (nearly 14,000 in 1969 compared to 9,000 in 1965 and 2,908 in 1960) (12) are taking the Advanced Placement Examination
in Mathematics sponsored by the College Entrance Examination Board. This is an examination given to high school students who have studied calculus. It is assumed they have had the equivalent of a full year (two semesters) of college calculus during their high school years. Many (36) of these students receive credit and/or placement based upon their scores. While no specific recommendations are made by CEEB, a procedure is suggested which colleges may consider if they choose. Grades are reported on a 5-point college level scale:

5 - High honors  
4 - Honors  
3 - Good  
2 - Credit  
1 - No credit

Most colleges develop their own method of interpreting these scores. As an example of test results, in 1968 the scores on the Mathematics Examination were as found in Table 1.

TABLE 1
DISTRIBUTION OF SCORES ON THE 1968 ADVANCED PLACEMENT MATHEMATICS EXAMINATION

<table>
<thead>
<tr>
<th>Score</th>
<th>Number of Students</th>
<th>Percent</th>
</tr>
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<tbody>
<tr>
<td>5</td>
<td>965</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>1667</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>3259</td>
<td>28</td>
</tr>
<tr>
<td>2</td>
<td>2894</td>
<td>25</td>
</tr>
<tr>
<td>1</td>
<td>2838</td>
<td>24</td>
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</table>
In 1969 two different mathematics examinations were given; one for students with a basic minimum course and the other for a stronger or enriched course. The first examination was labeled Mathematics Calculus AB and the second Mathematics Calculus BC. Over 10,000 students took the AB test while approximately 3600 took the more difficult examination. These results are tabulated in Table 2.

### TABLE 2

**DISTRIBUTION OF SCORES ON THE 1969 ADVANCED PLACEMENT MATHEMATICS EXAMINATIONS**

<table>
<thead>
<tr>
<th>Score</th>
<th>AB Number</th>
<th>AB Percent</th>
<th>BC Number</th>
<th>BC Percent</th>
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<tbody>
<tr>
<td>5</td>
<td>931</td>
<td>9</td>
<td>554</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>1738</td>
<td>17</td>
<td>770</td>
<td>21</td>
</tr>
<tr>
<td>3</td>
<td>2726</td>
<td>27</td>
<td>943</td>
<td>26</td>
</tr>
<tr>
<td>2</td>
<td>2974</td>
<td>29</td>
<td>989</td>
<td>27</td>
</tr>
<tr>
<td>1</td>
<td>1911</td>
<td>19</td>
<td>418</td>
<td>11</td>
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</table>

Based on the recommendations of the College Entrance Examination Board for the granting of credit, and observation of Table 1, it is apparent that at least 24 percent of the examinees received no placement or credit for their work. There are strong indications that this is a conservative estimate, on the low side. Vance and Pieters (46,492) claimed that 44 percent was more nearly correct. This is substantiated by the fact that many
schools do not grant placement or credit for all of the scores higher than 1. For example, at The Ohio State University, the scores are examined and the examinations are read, and then each application is processed individually. Students with scores of 5 and 4 are readily granted placement into levels of calculus above the beginning course. Some of those with a score of 3 are given credit for one quarter of work. Other students (with scores of 3 and 2) are invited to take departmental examinations if they believe they qualify at a higher level. Some are granted advanced placement and some are not. Approximately 70 percent of the total number of Ohio State freshmen who applied for credit through the Advanced Placement Examination were denied such credit.

At The Ohio State University the performance of freshmen granted advanced placement has been quite good. But, Advanced Placement Examination scores must reflect considerable knowledge of calculus before placement is granted. A check of the Performance for Autumn Quarter of 1969 indicated that 70 percent of the students granted advanced standing in calculus received a grade of "A" in the higher level course. The other 30 percent received a grade of "B". All but two of these students had scored a 4 or 5 on the Advanced Placement Examination.

There is apparently little cause for concern at most colleges and universities about the achievement of students
granted advanced standing through the Advanced Placement Program. There have been several research studies on this topic and most have indicated that almost 100 percent of these students realize a high degree of success in university mathematics when granted advanced placement. For example, Bergeson (4) at Western Michigan University checked academic performance of students granted placement. This study showed that acceleration did lead to above average performances.

The problem of identifying students for advanced standing is made even more complex by two other factors. One is that large numbers of students enrolled in high school advanced placement courses do not take the Advanced Placement Examination. The other is that if the Examination is taken, the results and the Essay part of the Examination itself are sent to only one school; the student's first choice. Many students are not accepted at the college they identify as their first preference. Many of these students receive no credit even though they may have qualified through their Advanced Placement Examination score. For this reason, one important category of students observed in this research had been exposed to the fundamental concepts of a basic first year calculus course but were enrolled in a beginning calculus course in college.

A second category for the study included students with a one-semester background of high school calculus.
These students generally do not seek advanced standing. For the most part they had an eighth grade algebra course so that a five year program was possible.

The third category included students from specific "advanced mathematics" sequences as noted earlier. The majority of these students were introduced to the fundamental idea of limit and to the definition of derivative. Their exposure to calculus ranged from several days to six weeks.

The remaining freshmen were classified in two ways. One group studied a formal course in analytic geometry, usually combined with a semester of trigonometry. The other group basically had a background in algebra and trigonometry. More precisely, this final group consisted of students with specific semester courses which contained no calculus or analytic geometry.

To obtain additional information and data which appeared to have utility, some of the upper classmen who were randomly mixed in the calculus classes with freshmen were also classified. This was done in two ways. One group had taken the course before and achieved poorly. The other had received pre-calculus training at The Ohio State University and not in the high school. These categories or condition groups will be discussed in detail in Chapter IV.
Hypotheses

Two primary hypotheses were tested in this study. The first involved achievement in a course and the other was related to the length of time that a given preparation maintained an effect on achievement. Stated as null hypotheses, they are:

(1) Students with relatively similar expectancy of success, but who have had different background preparation will achieve equally well in a first course in college calculus. A sub-hypothesis would claim that students with high school calculus as background will achieve no better than those without such preparation.

(2) If a variation in achievement is found among students with different backgrounds, it will not be maintained as progress in the calculus course, or in a subsequent course, is made.

The second hypothesis deals with observations throughout a quarter of work measured by five examinations. An attempt was made to determine if variation on the first examination would be maintained on the fifth examination. Also considered, was whether there might be any indication that initial variation would still prevail during the second course in calculus.
Research Design

The population for this study was the first-quarter freshmen (some upper classmen) enrolled in the beginning calculus course at The Ohio State University during the Autumn Quarter of 1969. Initially 567 freshmen and 351 upper classmen were classified into condition groups. There were 526 freshmen for whom complete background data were available so they might qualify as subjects for purposes of this study. In addition, 240 upper classmen served to form additional categories for investigation. One hundred and eleven non-freshmen were dismissed from consideration for a variety of reasons. Many were transfer students and others had a long lapse of time since their previous mathematics course. Further details are found in Chapter IV.

A preliminary aspect of the study was the categorization of the available subjects into condition groups. The second phase involved the recording of achievement scores from the calculus course itself. Predictive equations were calculated which served to classify groups as having the same relative expectancy of success. Finally, variation among the mean scores of the condition groups defined earlier, was analyzed.

These calculus classes were taught by thirty-one different instructors, in thirty-seven sections. All were coordinated carefully by one member of the mathematics
department. This writer was one of those thirty-one instructors. The procedure followed in assigning students to these classes was to have all those enrolled in the calculus course report to a common dispatching location on the first day of the quarter. At this location each student was handed a card marked with a room number to which he was to report. This was done at the 9:00 A.M. class hour and also at 3:00 P.M.

Assumptions

Several assumptions have been made regarding the relevancy of the study and the application of the desired tests for the statistical inferences. The consideration given to each of these will be found in later chapters.

(1) A reasonable criterion for teaching a high school mathematics course is that there will be beneficial effects on achievement in collegiate mathematics.

(2) There would be equal variation within the condition groups to be investigated.

(3) Coordinating efforts would result in individual instructor differences not contributing significantly to condition group differences.

(4) Examinations administered in the morning would not result in significant differences from the examinations administered in the afternoon.

(5) The A.M. and P.M. examinations were equivalent as evaluative criteria.
(6) Student responses to the questionnaire used to determine the categories, were valid.

Overview

In this chapter, the study has been introduced and the problem defined. In Chapter II, a discussion of pertinent literature and research will be presented. Chapter III provides a detailed account of the design and organization of the study. The means by which the final condition groups were determined and how students were selected as subjects are presented in Chapter IV. Chapter V gives a detailed description of the statistical inferences made as a result of an analysis of the data, and Chapter VI extends this analysis to the second course in calculus. The final Chapter serves as a summary and lists recommendations and conclusions made on the basis of the statistical analysis.
CHAPTER II

REVIEW OF RELATED LITERATURE

A review of the literature pertaining to research on achievement in college mathematics revealed many studies with considerable variation in purpose and design. Several researchers considered the effect of some single background characteristic on college level achievement in mathematics. Three types of research projects seemed most relevant to the purposes of this study.

The first category included three principal studies which were investigations of the effect of high school calculus on achievement in beginning college calculus. These studies, by Tillotson (43), McKillip (32), and Robinson (39) will be described in this chapter. They are particularly significant in view of the literature relating to the high school calculus program. Numerous position papers have been written from both positive and negative points of view. Although this still appears to be a controversial issue there is little doubt that high school calculus programs are here to stay.

A second category consists of studies that attempt to delineate factors from the high school background
preparation that affect achievement in college mathematics. The college courses observed were varied and the background factors were associated with many different items other than mathematical content. The study by Bedient (3) is pertinent to the present work, as is that of Blanton (5). To a lesser degree, Robinson (38) and Waggoner (47) have contributed related information as have Wick (50), Wampler (48), Smith (42), and Knight (26).

The final category included several studies related to placement procedures at colleges and universities. These refer specifically to the placement of freshmen into appropriate mathematics courses. The literature pertaining to research on placement procedures is voluminous and a thorough review cannot be considered. A study by Kurtz (27) surveyed placement procedures at thirty-three institutions of higher learning and resulted in the discovery of placement criteria suggesting a need for this current study. Of considerable importance is the study by Crosswhite (13), since the environment was the same as for this study. He investigated placement procedures at The Ohio State University and formulated the method by which students are currently being placed.

Effects of High School Calculus

All of the studies reviewed in this section considered the relationship of high school calculus to
achievement in a beginning college calculus course. In Chapter I works had been cited that considered achievement in higher level calculus courses when advanced standing was given.

Tillotson (1962)

The study by Tillotson (43) was an evaluation of the effect of a short introductory unit of calculus in high school on achievement in the first course in calculus and analytic geometry at the University of Kansas. He identified 133 students as the group from which to select the "unit of calculus" sample. A "no calculus" group of 130 students was similarly identified. Two different courses were being offered at the University of Kansas with one being more theoretical than the other. By use of a table of random numbers, a sample was drawn which contained forty-eight cases in each of four categories formed by the two levels of calculus and by the two types of preparation.

Tillotson's criteria of achievement were the score on the final examination and the letter grade awarded for the semester. These were adjusted to equalize factors of expectancy of success and mathematical preparation. This was done by computing a regression equation using normalized high school rank and the score on a placement test as variables. Analysis of covariance was then used to test the basic hypothesis that there would be no significant variation in achievement between the four
selected groups. None of the F-values obtained were significant at the .05 level. Thus the null hypothesis could not be rejected. Tillotson concluded that a brief unit on calculus did not have significant relationship to achievement in university calculus. Justification of presenting such content would have to be made on some other basis.

Tillotson reviewed thoroughly the works of Pingry (37), Robinson (38), and Hassinger (22) as the primary background studies for his research. The reader is referred to these dissertations or their review by Tillotson as containing further relevant information relating to achievement in calculus.

McKillip (1965)

The study conducted by McKillip (32) at the University of Virginia was similar to that by Tillotson. He investigated the effect of a semester or more of high school calculus on students' grades in the first semester of calculus at the University of Virginia. Subjects were from among 973 entering first-year men in the College of Arts and Sciences and the School of Engineering and Applied Science. Three groups were identified. Students from the College of Arts and Sciences taking calculus during their first semester; students from Arts and Sciences who took their pre-calculus work at the University of Virginia during the first semester and calculus the
second semester; and students from the School of Engineering and Applied Science taking pre-calculus the first semester and calculus the second semester, constituted the three groups. Subgroups were then formed, based upon the following conditions:

1. Less than one semester of high school calculus
2. One or more semesters of high school calculus
3. At least one but less than two semesters of calculus
4. Two or more semesters of high school calculus

These groups were further subdivided according to their public or private high school backgrounds.

Regression formulas were calculated for the three primary groups using subjects with no previous record of having calculus. This was to predict grades in the course based on expectancy when no previous calculus had been in the background preparation. The independent variables were (1) average grades in high school mathematics, (2) rank in the high school graduating class, (3) SAT mathematics score, (4) SAT verbal score, and (5) CEEB mathematics achievement examination score. In the case of the two groups with prior mathematical experience at the university, the pre-calculus course grade was additionally considered as an independent variable. The course grade in the first semester of calculus was the dependent variable.
The regression formulas were used to predict grades of the subjects in each group. These were then compared, by subtraction, with the actual grade received to determine signed differences. The comparison instrument was a Wilcoxon Matched-Pairs Signed-Ranks Test. A t-test for the significance of the mean differences was also used.

When less than two semesters of calculus were considered, no significant difference in achievement was found. Only when two or more semesters was the identifying characteristic of a group, was a significant difference noted. Public and private institutions seemed equally effective in this respect. McKillip suggested a need to observe more closely the effects of a full two semesters of high school calculus as well as to investigate the effect of such courses as analytic geometry and elementary functions on achievement in calculus at the university level.

Robinson (1968)

Robinson (39) investigated the effects of two semesters of secondary school calculus on students' first and second quarter calculus grades at the University of Utah. This study was quite similar to that performed by McKillip. From five Utah high schools subjects for the research were chosen. Regression equations were calculated to predict grades in the first and second quarters of calculus. The variables used were the high school
analytic geometry grade, the average of high school mathematics grades, rank in the high school graduation class, ACT English score, and the ACT mathematics score. Predicted grades were compared with actual grades by means of a Wilcoxon Matched-Pairs Signed-Ranks Test and a t-test for determining the significance of differences. Robinson rejected the hypothesis that two semesters of high school calculus would not affect the achievement in a beginning college calculus course, compared with achievement of students with no calculus as background preparation.

The three studies reviewed here do not completely answer the questions which arise when the high school calculus program is viewed as a controversial issue. However, support seems to be given to advocates of a full two-semester course. Anything less has not generally been recommended and these studies seem to substantiate this position. As a critical negative point of view, the classic "The Case Against Calculus" by Allendoerfer (1,482) certainly stands out. But, for each critic there is a staunch supporter. Admittedly, supporters usually give qualified endorsements. Lynch (31), Grossman (21,560), and Ferguson (15,451) have written professional viewpoints on the subject.

But, there is more to be considered than high school calculus when achievement in college mathematics is being considered. Other types of studies will now be described.
Factors in the High School Background

The affect of other factors in high school background preparation on success in college calculus has not been clearly defined as a research topic. Many diverse studies can be found relating factors in a preparatory program to achievement at the college level. The factors selected vary greatly and the observed achievement has been in several different areas. Chaney (8,699), for instance, studied the effect of a formal study of "limit" in high school on achievement in college calculus. Waggoner (47) investigated the relationship of high school mathematics to success in a general college curriculum. Smith (42) and Knight (26) developed instruments for predicting success in pre-calculus college courses. Smith developed an instructional unit in the area of modular arithmetic but statistical results indicated no significant correlations of this work with the letter grade in a subsequent college mathematics course. Wick (50) has also contributed to the research literature in this area. He obtained correlations between achievement in first-year college mathematics and high school achievement in mathematics, high school class rank, number of semesters of high school mathematics studied, and several other factors. Most of these correlation values were between .35 and .45. Content of the mathematical background was not included in his study.
Of greater significance to this study, is the work done by Blanton (5) at the University of Georgia. Blanton investigated certain programs in high school mathematics and measured their effect on the achievement in college mathematics. He sought an answer to the question "Does the amount and type of high school mathematics affect achievement of college freshmen?" Three groups of subjects were considered. Two of these were made up of students enrolled in college-algebra courses at two different schools. The third group was made up of students enrolled in a business mathematics course at one of the same colleges. Results indicated greater success when the amount of background mathematics was greater. This was true for both the algebra classes but not for the business mathematics course. Blanton concluded that more advanced mathematics programs were needed to improve achievement in college programs.

Bedient (1966)

The study performed by Bedient (3) at the University of Colorado provides interesting statistics relating to the mathematics teachers of the high schools whose programs were being evaluated. The observed characteristic of high school preparation was, in this instance, the teacher. Achievement at all levels of freshmen mathematics was the criterion for deciding that
certain Colorado high schools had programs that were superior to others. Seventeen high schools were found to have ten or more students enrolled at the University of Colorado. They were ranked with respect to the performance of their former students in freshmen mathematics courses. A weighting system based on the relative difficulty of the course was considered in the ranking process. Four schools were identified as having students who performed at a high level in college mathematics. Four others in the lower half of the ranking were also identified. A matching was made between the two groups. This was done by pairing schools that had subjects with approximately the same mean high school class rank.

Bedient's problem was to discover similarities and differences in the traits of the mathematics teachers of these eight schools. He collected data through a series of interviews with forty teachers in the two groups of schools. The purpose was to determine the degree of association between student performance in freshmen mathematics and the teachers': (1) knowledge of modern curriculum content, (2) recency of training, (3) familiarity with modern instructional materials, and (4) professional activity. He also looked for association with the amount of modern materials being used in the school system and the estimated morale of the teachers in the school. A socio-economic index was computed for each school and a
ranking of the eight schools established. The Spearman Rank Correlation Coefficient of the two rankings was .31. Since this was not significant at the .05 level, association between socio-economic rank and performance rank could not be established.

Some pronounced differences were established between the teachers of the two groups of schools. The teachers of the high performance schools were younger and more recently trained. They were more familiar with modern mathematical content and materials. There did not appear to be a difference with respect to professional activity, although it was low in both groups. High performance schools practiced extensive ability grouping. Low performance schools did not. Adequate administrative staffs were observed in both groups of schools. Morale contrasts were noted but were not sufficient to claim measurable differences. The better performing schools were characterized as having administrators who were interested in promoting an improved mathematics curriculum.

The implication of this research is that there is a significant association between knowledge of modern mathematical content possessed by the teachers and the performance of their students at the university level in mathematics.
Placement Procedures

Some of the studies previously reviewed could be considered relevant to college placement policies. If background factors can be used to predict success in college mathematics courses, persons in charge of the placement of incoming freshmen should consider such factors. There are, however, many studies designed specifically to determine such placement procedures at some given institution of higher learning. One by Crosswhite (13) at The Ohio State University will be discussed in this section. His is significant to the current study since all subjects chosen by this writer were placed in beginning calculus as first quarter freshmen according to a formula developed in that study. First, another study, by Kurtz (27) will be described.

Kurtz (1967)

The research study by Kurtz at the University of Nebraska was a critical analysis of the placement procedures in mathematics for freshmen of thirty-three selected institutions of higher learning in six different states. His primary purpose was to analyze provisions of institutions of higher learning to take into account the nature of high school courses and the level of achievement of students prior to their admission. To do this, Kurtz held personal interviews with department chairmen or their
designated representatives to establish the status of the mathematics courses available for freshmen at each school. He then analyzed the placement examinations and how the scores were interpreted. A study of the catalogs as well as the texts of the freshmen courses was made.

The analysis of the collected information revealed some interesting statements of position by department chairmen as well as information pertaining to placement criteria. For example, some representatives were quoted as saying that entering freshmen do not possess the necessary manipulative skills required for college level mathematics and that they would prefer that high schools concentrate on developing these skills and mechanical processes. This has been a frequent criticism of "modern" mathematics programs. However, it seems to be in complete disagreement with the current philosophy of emphasizing "understanding" rather than repetitive drill in the secondary school curriculum.

This last fact is not in complete accord with another frequent comment cited by Kurtz. He found that department chairmen claimed that so few of their freshmen have been exposed to modern mathematics programs that they are not justified in modifying the existing freshmen courses.

There seemed to be consistent agreement at the thirty-three institutions involved that a repetition of
content in college courses of material studied in high school was not undesirable. This was especially true if the college work was more sophisticated and presented in greater depth.

The analysis of statements involving the placement criteria used in the thirty-three schools revealed the following order of reliance, with the most frequent listed first.

1. High school mathematics courses and grades received.
2. National standardized examinations
3. Locally constructed examinations
4. Overall high school grades
5. Quality of high school mathematics

Also included, although not explicitly, was a consideration of the desires and interests of the students.

A problem associated with the first and fifth mentioned criteria was that high school transcripts made it difficult to judge what concepts had actually been taught. Also noted was that locally constructed placement examinations contained little, if any, modern terminology and symbolism.

Kurtz later wrote in The Mathematics Teacher (28,557) that it was his belief that advisement for placement of freshmen students in mathematics should be done by the students' adviser in mathematics, and if possible the advisers should be the ones who will be teaching the courses.
The dissertation by Kurtz contains an extensive review of studies pertaining to placement procedures at some given institution. Many are similar to that by Howlett (24,651) as reported in *The Mathematics Teacher*. He was seeking a simple formula whereby all incoming freshmen might be properly placed at Michigan Technological University. A multiple regression procedure was applied to determine a prediction equation. Class rank in high school was the variable that independently predicted achievement with the highest correlation. Overall results were not completely satisfactory. Thus, Howlett recommended that placement not be on a command basis but on a recommendation basis. When a new student's variable scores are entered in the prediction formula, the value computed should result in a recommendation of several courses of action that could be taken by the student. But, ultimately the student makes the final choice.

Perhaps the time saved in pre-enrollment testing would make such a proposal worth considering for some colleges and universities.

Crosswhite (1964)

The significance of the study by Crosswhite (13) has previously been mentioned. By systematically reviewing the policy at The Ohio State University for placing freshmen in appropriate mathematics courses and investigating many additional factors (a total of fifteen independent
variables), he hoped to develop an accurate, reliable, and practical placement procedure. The method employed to do this was multiple regression analysis. The definition of a simple, yet optimum, equation which was a linear combination of two of the original fifteen independent variables was made.

One of the limitations inherent in Crosswhite's study was caused in part by placement procedures at that time. This pertained to the special honors sections of beginning calculus. In the Fall of 1963 the sections of calculus were formed in two ways. All first-quarter freshmen scoring sufficiently high on the locally constructed mathematics placement test to qualify for calculus were placed in honors sections with regular staff members instructing. All students who had taken the normal pre-calculus sequence as freshmen were placed in regular sections with graduate students serving as instructors. Differences in grading procedures were in evidence. Also significant was the fact that class size for honors sections averaged twenty-three and for regular sections thirty-six. To provide a common measure of achievement, two one-hour examinations were scheduled for all sections. The sum of these scores and the course grade were considered as dependent variables. Achievement was associated with the examination score and success associated with the course grade. Multiple correlations using a derived
variable were significantly higher for achievement than for success.

The first quarter freshmen who received advanced standing in calculus did so by scoring fifteen or more on a test of twenty-five questions. This was known as the OSU Math D Test. The optimum regression equation mentioned earlier was in terms of the score on the "D" test and also in terms of the total quality points for semesters of high school mathematics. With 4 points for A, 3 points for B, etc., a student could compile 32 points if he received "A" for each of eight semesters of work. Students in a five-year program could exceed this total. The derived equation was: 

\[ y = 2x_1 + x_2 \]

where \( x_1 \) was the placement test score and \( x_2 \) the quality points.

A \( y \)-value of 55 or more is sufficient today for placement into beginning calculus at The Ohio State University. An additional factor in current placement procedure is that the "D" test score must exceed 11 regardless of the \( y \)-value. If a student scores below the cut-off and yet believes in his ability to achieve at a higher level, he may take a proficiency test for the course or courses to be by-passed.

**Summary**

Review of the literature has shown that there is considerable variation in the manner in which educators and educator-researchers have considered resolving the
problem of identifying characteristics related to achievement in college mathematics. In some studies a background factor was selected and corresponding achievement observed. In other cases, achievement was noted and characteristics of those considered successful were identified. Many studies were predictive in nature. None has successfully identified a twelfth-year mathematics program most likely to guarantee success in college calculus. The fact that "success" in a college course of calculus varies among institutions of higher learning, must be considered. It seems to be a function of "how" it is taught and "where" it is taught. More studies are needed to determine if achievement in "problem solving" courses and "theoretical" courses can be predicted in terms of the same variables.
CHAPTER III

DESIGN OF THE STUDY

The first chapter included a brief description of the design of the study so no attempt will be made to repeat what has already been said. However, there are important details of the research which require additional explanation. The procedure followed in collecting the necessary data will be discussed in this chapter as will be the manner in which the data was analyzed. The subjects used in the study will be the central topic of Chapter IV.

Planning and Procedures

At The Ohio State University the beginning calculus sequence is numbered in the following manner.

First Quarter Calculus  Math 151
Second Quarter Calculus  Math 152
Third Quarter Calculus  Math 153
Fourth Quarter Calculus  Math 254
Advanced Calculus  Math 550
(Highest Freshmen Honors)  Math 190

Some calculus sections become designated as "honors" sections and are labeled, for example, Math 151H. The
highest honors section for freshmen, Math 190, begins a sequence that in three quarters covers the content of the 151, 152, 153, and 550 courses. This section will be mentioned only briefly in this work. The regular classes are a heterogeneous grouping of freshmen and upper classmen. However, a majority of those enrolled in the Autumn Quarter are first-quarter freshmen.

During the summer of 1969, plans were made to utilize the thirty-seven sections of Math 151 scheduled for Autumn Quarter. This involved approximately 1100 students when the Math 190 section and a special class of Math 151 for engineering students were initially considered. This brought the total to thirty-nine sections. The pre-planning consisted of two primary phases. One was to determine the most efficient method of collecting information so that background mathematics programs might be studied. The second was the construction of a testing instrument to be used as a reliability check and as a possible predictor variable in the regression analysis.

It was decided that questionnaires would be used to collect the desired data relating to the high school background. A questionnaire was developed to be used by the instructors on the first day of classes at the beginning of Autumn Quarter (see Appendix A). The information collected included the name of the high school attended, mathematics courses taken, grades for these mathematics
courses, and comments concerning the high school calculus that they might have had.

After these questionnaires had been examined, a second questionnaire was written and sent to the high schools of a subset of the subjects. This was designed to obtain additional information in some cases and to verify student responses in others. This served as the first reliability check (see Appendix B).

The testing instrument was a simple multiple choice test consisting of ten questions. These questions were pertinent to the most fundamental concepts of beginning calculus but were ones not likely to be answered without having studied calculus (see Appendix C). This test will be referred to as the pretest in all subsequent instances. The pretest and questionnaire were both distributed to and collected from the students on the first day of class. In the case of a few late entries, information was gathered on the second day. Only two later entries were considered in the study. Students were unaware of the reasons for the request of information or for the examination. Scores were not revealed except to interested instructors. Primary emphasis of this study was placed on the observation of achievement of freshmen but the questionnaire and pretest were administered to all students enrolled in Math 151.
One consideration in administering the test described here was to observe whether or not there was an obvious relationship between results and the students' claims of having studied high school calculus. A second consideration was associated with the possibility the pretest score might be predictive of midterm examination scores.

The second major phase of planning the study was the determination of how "achievement" or "success" in the course was to be measured. Actually, the two works used here seem to imply different meanings. For purposes of this study, it was assumed that "achievement" pertained to examination results and "success" pertained to the course grade, as suggested in earlier research (13).

Achievement was measured by a series of four midterm examinations and a final examination. The examinations were administered at the end of the second, fourth, sixth, and eighth weeks and the final examination was scheduled for the eleventh week. The course grade was the indicator of "success," where the numerical equivalents were the usual: A - 4, B - 3, C - 2, D - 1, and E - 0. These were used to determine group means.

Each of the first four examinations (see Appendix C) was graded on the basis of 20 points. Each was a multiple choice type examination with five possible answers for each question. The Math 151 course was
offered to students at 9:00 A.M. and 3:00 P.M. daily. Students were about equally divided with slightly more than half attending at 3:00. Different examinations, intended to be equivalent, were administered at the two hours, on the same day. Conscientious efforts were made to construct these examinations as nearly similar as possible without permitting students to aid one another by revealing test items. A comparison of the means of the 9:00 and 3:00 sections for examinations one, two, and three revealed no significant differences. The F-values for the three variables were respectively 2.105, 0.005, and 1.389 which were significant at levels of .147, .942, and .239. This indicates that the differences could occur approximately 15, 94, and 24 percent of the time, by chance.

It was concluded from the calculated F-values that the examinations were sufficiently similar to not have adverse effects on the data analysis.

An additional factor taken into consideration was the distribution of the members of the condition groups within the two time periods. They were evenly divided. The greatest difference occurred with a group for which approximately 53 percent of the subjects had been randomly assigned at the 3:00 hour.

The fourth examination and the final examination (see Appendix C) were administered in the evening with all students taking the same test simultaneously. The final
examination was similar in structure to the midterms but consisted of forty questions instead of twenty.

All examinations were machine-scored, so a punched deck of computer cards was available for each test. Cards indicated not only the score but the individual response to questions. This procedure also eliminated any differences in scores resulting from instructor differences in grading procedures. For the student's benefit, letter equivalences were uniformly established within all sections for these test scores. These letter equivalences were of no significance except that they ultimately determined the letter course grade. They are noted here:

- 16 to 20  - A
- 14 to 15  - B
- 10 to 13  - C
- 8 to 9    - D

Any score below 8 was considered as failing. For the final examination, the numbers were just doubled.

The text used in all Math 151 classes at The Ohio State University is *Calculus and Analytic Geometry* by Fisher and Ziebur (18). Chapters One, Two, Three and Four were covered during the Autumn Quarter. (Chapters Five, Six, and Seven are included in the second quarter, Math 152.) The first examination covered material in Section 1 through Section 7. The second examination tested material in Sections 8 through 15. The third
examination covered Sections 16 to 20 and the fourth considered the content in Sections 21 to 25. The final examination was comprehensive and included the content of all of the first thirty-one sections.

Statistical Analysis

Regression equations predicting examination scores in Math 151 were calculated. The course grade was similarly predicted. The purpose of the regression analysis was two-fold. First, there was a desire to place students into categories according to expectancy of achievement and success in the course. The grouping according to "expectancy" was done in several different ways and is described in detail in Chapter IV. The second purpose was to determine the significance of instructor differences. A predicted mean score for each instructor's class was computed. Then, the actual mean for each class was computed so a comparison could be made. A t-test was used to determine the significance of the mean differences.

One set of six condition groups with the same relative "expectancy" was determined. Mean scores for each of the condition groups were calculated for each of the common examinations and the significance of the differences of means was investigated. Another set, with seven condition groups, was treated similarly. Consideration of the variation among the means as well as within each classification was necessary. The F-test was used
to demonstrate differences among the means which could be used to provide a comprehensive test of the significance of differences among the means, it does not tell us which ones differ significantly. Thus, Duncan's Multiple Comparison Test was used to evaluate these differences. Bartlett's test for homogeneity could be used to test the assumption of equal variance within a group but is considered unnecessary when groups are large.

Additional regression equations were calculated to predict some of the test scores and the course grade. These equations utilized variables not considered in the initial analysis and their formulation was motivated by the procedure being used to form honors sections of Math 151. At first, one additional variable was added. It was the score on the first examination. Then the scores of the first two examinations were considered as independent variables in determining predictors of final examination achievement and of success in the course. (Students were being promoted to honors classes when they achieved well on the first two or three examinations.)

Since "additional" variables have just been mentioned, some word should be said about the original variables. The data collected included several measures required as entrance information by the University. Considered initially as independent variables were the following:
\[ x_1 \quad \text{Pretest score} \]
\[ x_2 \quad \text{ACT Mathematics score (percentile)} \]
\[ x_3 \quad \text{Normalized high school class rank} \]
\[ x_4 \quad \text{Placement test scores; OSU Math "D" test} \]
\[ x_5 \quad \text{Quality points} \]

A sixth variable, \( x_6 \), was \( 2x_4 + x_5 \) and was considered in a separate analysis replacing \( x_4 \) and \( x_5 \). Each of the following was then individually considered as a dependent variable:

\[ x_7 \quad \text{Examination \# 1} \]
\[ x_8 \quad \text{Examination \# 2} \]
\[ x_9 \quad \text{Examination \# 3} \]
\[ x_{10} \quad \text{Examination \# 4} \]
\[ x_{11} \quad \text{Final Examination} \]
\[ x_{12} \quad \text{Sum of} \ x_7 \ \text{through} \ x_{11} \]
\[ x_0 \quad \text{Course grade} \]

Finally, \( x_7 \) and \( x_8 \) were combined with the five initial variables serving as independent variables, with \( x_{11} \) and \( x_0 \) as dependent variables. The equation calculated to indicate similar expectancy groups was determined with \( x_{12} \) as the dependent variable.

The selection of these variables resulted from a review of predictive studies performed earlier at The Ohio State University and at other schools. These were discussed in Chapter II.
A word of explanation is perhaps necessary in several instances. Variables $x_4$ and $x_5$ were described earlier in the review of the study by Crosswhite (13). Variable $x_5$ represents the quality points for all semesters of high school mathematics for a given student. Variable $x_4$ refers to the score on the mathematics placement examination administered to all students as an entrance requirement. Variable $x_6$ was considered because of its importance to the mathematics program at Ohio State.

Summary

The two principal phases in the development of this study were the data-collecting and the statistical analysis of the data. Each was comprised of two major sub-phases. Data were collected via questionnaires directed at the study-population and to the school officials of the high schools from which the subjects had graduated. A description of the manner in which the questionnaire data were utilized is presented in detail in Chapter IV. This information was used to determine condition groups consisting of students with similar mathematical background experiences. The other aspect of data-collecting involved the scores on five common examinations.

Regression analysis and analysis of variance were the two major statistical procedures utilized in the study. The regression analysis technique was employed to determine
condition groups with nearly similar "expectancy of success." The analysis of variance was employed to determine if differences of achievement among condition groups was significant. This was accomplished by calculating mean scores for each condition group on each of the common examinations. These mean scores were compared using an F-test and the Duncan's Multiple Comparison Test was used to determine which differences were actually significant.
CHAPTER IV

SELECTION OF CONDITION GROUPS

At The Ohio State University students are placed into the first quarter of calculus by scoring 55 or higher when the formula \( y = 2x_4 + x_5 \) is applied; \( x_4 \) being the placement test score and \( x_5 \) the total quality points received for high school mathematics. Approximately 8 percent of the total Autumn Quarter freshmen enrollment qualified for beginning calculus. This totaled nearly 800 students. This is considerably lower than the 25 percent which Kurtz (27) found when he studied thirty-three college programs, but it still represents a sizable group of students that is prepared to begin calculus. This also represents a significant reduction in percent from four years ago when 12 percent of the Autumn enrollment qualified at this level at Ohio State. Many of these 800 students never intend to enroll in a mathematics course. Others postpone their required mathematics courses until a later date. Still others enrolled in optional courses intended as terminal sequences when mathematics was not to be pursued further. The total number of first-quarter freshmen who finally enrolled in Math 151 was 567.
Nearly 500 non-freshmen were also enrolled in Math 151 for Autumn Quarter and classroom sections were assigned with no attempts to keep freshmen separate.

The Student Questionnaire

One of the major tasks of this study was to organize more than 1000 students, but primarily 567 freshmen, into condition groups. In order to identify a set of students who had studied from the same text, or who had studied similar units of high school mathematics, a questionnaire was prepared. All students enrolled in Math 151 were asked to complete this form on the first day of class on October 1, 1969 (see Appendix A).

The information provided by these questionnaires gave some indication of the different types of backgrounds to be found, and the groupings that might be considered. The information requested related primarily to high school mathematics courses and the grades received in them. A description of the highest level mathematics course was also requested.

Five reliability checks were made to assure the dependability of background information. One check was essentially a clarification procedure and several others served as verification as well as clarification. Still another served as a check against the claim of having studied calculus in high school. These checks will now be described.
Personal Contact

Each student whose questionnaire was incomplete was personally approached by this investigator. Many clarifications were made. Textbooks were discussed and the topics covered in high school were often revealed. This aspect of considering the questionnaire provided a greater confidence in using the information.

The purpose of the questionnaire was to obtain classifications of backgrounds and at this point these began to take shape. However, some students were uncertain about the mathematical content of their twelfth-grade course and the reliability of their questionnaires as well as those which appeared clearly identifiable, had not been checked. Thus the writer sought additional clarification and verification.

The High School Questionnaire

During the month of November in 1969 a questionnaire was mailed to ninety-two high schools, the majority of which were in Ohio (see Appendix B). This was mailed to the head of the mathematics department in each school at which a student had reported studying content he could not positively identify. This included approximately twenty schools. The other schools were selected at random and the replies were intended to serve as a check of the earlier responses on the student questionnaire. A letter
(see Appendix B) was included with each questionnaire describing the research project briefly. It also contained the names of the students from the school to which it was addressed. In every case but one, the student had graduated from the contacted school during the preceding June. In the one case, the student had moved during the early Spring of his senior year and graduated from a school in a different state.

This high school questionnaire requested information about the upper levels of the mathematics program, especially the twelfth-year course. Names and authors of textbooks and information concerning the duration of the calculus unit were requested. The most important item was intended to determine if the student(s) named actually participated in the most advanced program. If not, further details were requested.

Replies were received from 95 percent (87 schools) of the ninety-two schools to which the questionnaire was sent. This included 92 percent (150 students) of 163 students about whom information was requested. Of the thirteen students not accounted for, ten were from two high schools whose mathematics program was known by this writer. Full year advanced placement programs in calculus existed at these schools.

Students' claims made on the first questionnaire were substantiated by the high school questionnaire in
all cases but one and a full explanation of this single case was made available. All clarifications sought were obtained. The details provided by the mathematics department chairmen enabled a further refinement in the formation of the condition groups.

Local High School Contact

The third reliability check was an extension of the type just described. It concerned high schools in the greater Columbus area and was oral in nature rather than written. Telephone calls were made to representatives of sixteen schools involving approximately sixty freshmen. A high degree of clarification was made possible by this effort and no contradictions were discovered. Not only were no students eliminated from the study as a result of the high school contacts, but some were added for whom data had been questionable.

Checking Quality Points

A random selection of student questionnaires was checked against a previously determined listing of the total quality points for semesters of high school mathematics. This had been compiled by the Office of Evaluation at The Ohio State University during the Summer months of 1969, also by means of a student questionnaire. Very few discrepancies were noted and previous research suggest a high degree of accuracy in student reporting of
high school grades. No further details were sought regarding the grades of freshmen in prior courses.

The Pretest

The final reliability item served several purposes. It had been anticipated that many students would indicate that a unit, or more, of calculus had been studied in high school. The pretest, a ten-item calculus "quiz," was given to provide a check on the calculus background and an opportunity to investigate its predictive quality when used as an independent variable in a multiple regression analysis. Both aspects proved to have some merit. The pretest was designed to check such basic concepts of the calculus as the definition of derivative, limits, maximum and minimum, acceleration-velocity relationships, and concavity of graphs of functions. A copy of the test is found in Appendix C.

Results of the pretest in terms of scores were not good. This was not unexpected. However, an analysis of the results implied a degree of reliability in the questionnaire responses. In several instances, high scores could not be explained when no calculus was indicated in the background. It was easier to rationalize low scores when better results might have been expected. All of the pretest results are summarized in Table 8 presented later in this chapter. An item analysis of the pretest is summarized in Appendix D.
After carefully studying the questionnaires and the checks described above, the final condition groups were identified. The largest single group was classified according to the type of textbook which had been used in high school. Five texts were identified as being "modern" and "advanced mathematics" in nature and providing a "survey-type" background. These were high school texts which most frequently used by the freshmen enrolled in Math 151. All included at least a unit of work in calculus but none presented much more than a short introductory unit. Eventually all students who studied from this type of text were combined into a single classification considered as providing a "foundation" background with a short unit in calculus. But, initially there were two texts which were observed to have been used by large numbers of students. These were *Modern Introductory Analysis* (14) and *Principles of Mathematics* (2) and will be identified as Text A and Text B in later discussions. They were used to designate two initial condition groups. Students who studied from *Foundations of Advanced Mathematics* (25), *Advanced High School Mathematics* (45), and *Principles of Advanced Mathematics* (34) were combined with subjects whose high school mathematics was from an assortment of other texts which could be described with the similar title of "foundations" with "a unit of calculus," to form a third condition group for the study.
Forty-two percent (237) of the 567 freshmen in the study had a mathematics background consisting of modern content suggested by the above discussion. The 237 subjects which were initially divided into three groups were combined into one group for the initial statistical analysis.

The fourth condition group included those freshmen who had studied a semester of analytic geometry during their senior year in high school. In all cases, a textbook specifically for analytic geometry was used. The other semester of work was usually devoted to trigonometry but in a few instances it was advanced algebra. Seventy-five students (13 percent of the total) made up this group.

The next two condition groups were designated for students who had studied high school calculus more extensively. One group had a background of a full year of calculus and the other a full semester. Ninety-three (15 percent) and 37 (7 percent) were the totals, respectively, for these two condition groups.

There was one additional group identified which was relatively large. Sixty-four (11 percent) students had a high school preparation consisting of algebra and trigonometry as the most advanced course work. Approximately half of this total studied the algebra and trigonometry as separate semester courses and the other half used a text with an integrated or unified algebra-trigonometry format.
Five other condition groups were initially identified. However, the number of subjects classified in each was less than desirable. For the primary data analysis, students from all five categories were considered as a single condition group. These categories were formed from backgrounds of linear algebra, college algebra, probability trigonometry, and solid geometry. The numbers in each were, respectively, 8, 15, 13, 17, and 18. The solid geometry and probability were semester courses usually combined with a semester of trigonometry. In none of these cases was their evidence of calculus in the preparatory program.

Table 3 shows the distribution of freshmen into the twelve condition groups as initially identified.

A glance at Table 3 reveals that a large percent of the first-quarter freshmen had a high school preparation which could be considered "modern." Groups 1, 2, 3, 5, 6, and 8 are in this category and some mathematicians would include Group 10 as well. If Group 4 is not included, 64 percent of the freshmen could be considered as having a "modern" background. When analytic geometry is included, the total percent is increased to seventy-eight.

The total in Table 3 represents all of the first-quarter freshmen enrolled in Math 151 during Autumn quarter. Of this number, 526 were still enrolled at the end of the quarter and could be considered in the statistical analysis. Forty-one students dropped the
### TABLE 3
DISTRIBUTION OF FRESHMEN INTO 12 CONDITION GROUPS

<table>
<thead>
<tr>
<th>Number</th>
<th>Group</th>
<th>Number Classified</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Studied Text &quot;A&quot;</td>
<td>83</td>
</tr>
<tr>
<td>2</td>
<td>Studied Text &quot;B&quot;</td>
<td>38</td>
</tr>
<tr>
<td>3</td>
<td>All Other Adv. Math. Texts</td>
<td>116</td>
</tr>
<tr>
<td>4</td>
<td>Semester of Analytic Geometry</td>
<td>75</td>
</tr>
<tr>
<td>5</td>
<td>Year of Calculus</td>
<td>83</td>
</tr>
<tr>
<td>6</td>
<td>Semester of Calculus</td>
<td>37</td>
</tr>
<tr>
<td>7</td>
<td>Algebra and Trigonometry</td>
<td>64</td>
</tr>
<tr>
<td>8</td>
<td>Linear Algebra</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>College Algebra</td>
<td>15</td>
</tr>
<tr>
<td>10</td>
<td>Probability</td>
<td>13</td>
</tr>
<tr>
<td>11</td>
<td>Trigonometry</td>
<td>17</td>
</tr>
<tr>
<td>12</td>
<td>Solid Geometry</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>567</td>
</tr>
</tbody>
</table>
course from their schedule at some point during the term. This represents approximately 7.2 percent of the total. Table 4 shows the dispersion of "drops" among the condition groups.

In addition to the first-quarter freshmen who were categorized, 351 upperclassmen were classified according to their college-level preparatory program. The remaining non-freshmen could not be placed for various reasons. Some students had studied no mathematics during the previous two or three years. Many transfer students received credit for course work not exactly comparable to courses offered at Ohio State. Some were foreign students whose preparatory program could not be specifically described in terms of the selected condition groups.

The 351 non-freshmen were categorized into five subgroups. One classification identified students who had already had a course in calculus, either at Ohio State or another university. They had either failed the course the first time or had not achieved to their satisfaction. The sixty-eight students in this group comprised nearly 20 percent of the total number of upperclassmen originally taken into consideration.

At The Ohio State University the pre-calculus course, Math 150, is offered to those freshmen who do not score sufficiently high on the placement examination to be enrolled in Math 151. Approximately 38 percent of the
TABLE 4

DISPERSION OF DROPS IN CONDITION GROUPS

<table>
<thead>
<tr>
<th>Group</th>
<th>Original #</th>
<th>Drops</th>
<th>Final #</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>83</td>
<td>6</td>
<td>77</td>
</tr>
<tr>
<td>2</td>
<td>38</td>
<td>0</td>
<td>38</td>
</tr>
<tr>
<td>3</td>
<td>116</td>
<td>10</td>
<td>106</td>
</tr>
<tr>
<td>4</td>
<td>75</td>
<td>6</td>
<td>69</td>
</tr>
<tr>
<td>5</td>
<td>83</td>
<td>3</td>
<td>80</td>
</tr>
<tr>
<td>6</td>
<td>37</td>
<td>3</td>
<td>34</td>
</tr>
<tr>
<td>7</td>
<td>64</td>
<td>5</td>
<td>59</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>15</td>
<td>1</td>
<td>14</td>
</tr>
<tr>
<td>10</td>
<td>13</td>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>11</td>
<td>17</td>
<td>2</td>
<td>15</td>
</tr>
<tr>
<td>12</td>
<td>18</td>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>Total</td>
<td>567</td>
<td>41</td>
<td>526</td>
</tr>
</tbody>
</table>
freshmen class placed at this level in the Fall of 1969. Three separate condition groups were formed whose members had previously taken the Math 150 course. They were classified according to the grades received in the course. The purpose in doing this will be explained shortly. Of the non-freshmen, 252 were identified positively as having taken Math 150. For many, it was a repetition of high school content since they had received credit for four years of high school mathematics. Thirty-eight students indicated they had received a grade of "D." A grade of "C" was received by 125 students and eighty-nine achieved at the "A" or "B" level. A sample of these grades, as recorded on the student questionnaire, were checked with the records kept in the mathematics department. No discrepancies were found in any of the cases checked.

The final condition group consisted of thirty-one non-freshmen who had studied college level mathematics but not the usual Math 150 course. At Ohio State there are several sequences considered to be terminal in nature which are designed to satisfy the mathematics requirements of the various colleges. Occasionally these students decide they want more mathematics or they change their major, necessitating additional course work. Some of the content is overlapping, so credit is not given for both the terminal course work and Math 150. These students may then enroll in Math 151. This indicates that the
pre-calculus mathematics may differ even when taken at The Ohio State University.

At the end of the quarter only 240 of the original 351 non-freshmen were still enrolled. This represents a 32 percent "drop" rate. Many of these students recognized that they were having difficulties and dropped the course rather than take a failing grade. Table 5 indicates the distribution in the five non-freshmen condition groups and the number of "drops" within each group.

<table>
<thead>
<tr>
<th>Group</th>
<th>Original #</th>
<th>Drops</th>
<th>Final #</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math 150 &quot;D&quot;</td>
<td>38</td>
<td>21</td>
<td>17</td>
</tr>
<tr>
<td>Math 150 &quot;C&quot;</td>
<td>125</td>
<td>41</td>
<td>84</td>
</tr>
<tr>
<td>Math 150 &quot;A&quot; or &quot;B&quot;</td>
<td>89</td>
<td>20</td>
<td>69</td>
</tr>
<tr>
<td>Terminal Course</td>
<td>31</td>
<td>14</td>
<td>17</td>
</tr>
<tr>
<td>Previous Math 151</td>
<td>68</td>
<td>15</td>
<td>53</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>351</strong></td>
<td><strong>111</strong></td>
<td><strong>240</strong></td>
</tr>
</tbody>
</table>

Due to the less than desirable numbers found in several of the condition groups, it was decided that some consolidations should be made. This was accomplished by reducing the number of freshmen categories from twelve to five and the number of non-freshmen categories from five to three. Freshmen with a year of high school calculus
were retained as a condition group. It also seemed desirable to keep a condition group whose members had a background of one semester of high school calculus. The only other group retained intact, consisted of students who had studied a semester course of analytic geometry. Groups 1, 2, and 3 as listed in Table 3 were combined into one category, and considered as having a brief unit of calculus. Groups 7, 8, 9, 10, 11, and 12 were combined as having had no high school calculus and no specific unit in analytic geometry. Table 6 shows the distribution by numbers in each of the five freshmen groups.

The five non-freshmen classification were combined into three groups as indicated in Table 7. Students who received a grade of "A" or "B" in Math 150 formed one group and those who received "C" or "D" were combined with those who had been in a terminal sequence, to form a second group. Those with prior college-level calculus experience formed the third category.

It has been mentioned earlier in this chapter that a pretest was given to all students enrolled in Math 151. The questions on this test were all related to calculus. The examination papers were arranged according to the original seventeen condition groups. They were machine-scored and a mean for each of the twelve freshmen groups was computed. Also calculated was a mean for the non-freshmen who had previously had calculus at the college
### TABLE 6
DISTRIBUTION AMONG THE FIVE FRESHMEN GROUPS

<table>
<thead>
<tr>
<th>Group</th>
<th>Original #</th>
<th>Drops</th>
<th>Final #</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (Year of Calculus)</td>
<td>83</td>
<td>3</td>
<td>80</td>
</tr>
<tr>
<td>B (Sem. of Calculus)</td>
<td>37</td>
<td>3</td>
<td>34</td>
</tr>
<tr>
<td>C (Unit of Calculus)</td>
<td>237</td>
<td>16</td>
<td>221</td>
</tr>
<tr>
<td>D (No calculus or A.G.)</td>
<td>135</td>
<td>13</td>
<td>122</td>
</tr>
<tr>
<td>E (No calculus, Analyt)</td>
<td>75</td>
<td>6</td>
<td>69</td>
</tr>
<tr>
<td>Total</td>
<td>567</td>
<td>41</td>
<td>526</td>
</tr>
</tbody>
</table>

### TABLE 7
DISTRIBUTION AMONG NON-FRESHMEN CONDITION GROUPS

<table>
<thead>
<tr>
<th>Group</th>
<th>Original #</th>
<th>Drops</th>
<th>Final #</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1 Math 150 &quot;A&quot; or &quot;B&quot;</td>
<td>89</td>
<td>20</td>
<td>69</td>
</tr>
<tr>
<td>F2 Math 150 &quot;C&quot; or &quot;D&quot;</td>
<td>194</td>
<td>76</td>
<td>118</td>
</tr>
<tr>
<td>G Math 151 previously</td>
<td>68</td>
<td>15</td>
<td>53</td>
</tr>
<tr>
<td>Total</td>
<td>351</td>
<td>111</td>
<td>240</td>
</tr>
</tbody>
</table>
level. To provide an additional comparison, a mean for the "A" and "B" Math 150 students was included. The results were satisfying in that support was given to the fact that students' claims to have studied calculus seemed to be reflected in these means. The two groups with the highest means on the pretest were those with a background of a year of calculus and a semester of calculus. Next in order was one of the groups determined by a "modern" foundation type of textbook which included a unit of calculus. The lowest mean was for Group F1 whose pre-calculus course was Math 150. A summary of these means and standard deviations is presented in Table 8.

One feature of Table 8 that is difficult to explain is the relatively high mean for the group with strictly an algebra background. While no formal calculus was taught in these preparatory programs, perhaps more concepts relating to calculus were presented than was indicated by the questionnaires. The single problem on the test requiring algebraic manipulation may have been, in part, responsible. The group with a semester of calculus scored slightly higher than the group with a full year of calculus. Further details of this comparison will be given in Chapter V.

Analysis of variance was to be employed to compare the means for each of the condition groups on each of the five common examinations and the course grade for Math 151.
<table>
<thead>
<tr>
<th>Group</th>
<th>Number</th>
<th>Mean</th>
<th>S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>83</td>
<td>3.14</td>
<td>1.67</td>
</tr>
<tr>
<td>2</td>
<td>38</td>
<td>3.63</td>
<td>1.86</td>
</tr>
<tr>
<td>3</td>
<td>116</td>
<td>2.97</td>
<td>1.65</td>
</tr>
<tr>
<td>4</td>
<td>75</td>
<td>2.71</td>
<td>1.69</td>
</tr>
<tr>
<td>5</td>
<td>83</td>
<td>4.71</td>
<td>1.89</td>
</tr>
<tr>
<td>6</td>
<td>37</td>
<td>4.84</td>
<td>1.94</td>
</tr>
<tr>
<td>7</td>
<td>64</td>
<td>2.42</td>
<td>1.78</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>2.43</td>
<td>1.99</td>
</tr>
<tr>
<td>9</td>
<td>15</td>
<td>3.53</td>
<td>1.78</td>
</tr>
<tr>
<td>10</td>
<td>13</td>
<td>3.06</td>
<td>2.14</td>
</tr>
<tr>
<td>11</td>
<td>17</td>
<td>2.32</td>
<td>1.69</td>
</tr>
<tr>
<td>12</td>
<td>18</td>
<td>2.58</td>
<td>1.76</td>
</tr>
<tr>
<td>G</td>
<td>68</td>
<td>3.04</td>
<td>1.65</td>
</tr>
<tr>
<td>F₁</td>
<td>89</td>
<td>2.23</td>
<td>1.71</td>
</tr>
<tr>
<td>Total</td>
<td>724</td>
<td>3.15</td>
<td>1.92</td>
</tr>
</tbody>
</table>
Thus it was necessary to have some measure of the abilities of the students involved. It was important the groups have the same estimate of academic success, so that differences, if they occurred, could be attributed to other factors. More specifically, certain background preparations might demonstrate predictive qualities regarding achievement in calculus.

The first-quarter freshmen offered the best prospects of accomplishing the goal of producing a subset of the study population with approximately the same estimated expectancy of success. Background data was most readily available for the newly enrolled freshmen class. They had all taken the Ohio State placement examination and scored at least twelve of a possible twenty-five points. The formula \( y = 2x_4 + x_5 \), mentioned earlier, was used to determine the calculus level students. All of the 567 freshmen scored 55 or higher when the formula was applied. Relying on previous research findings, these particular scores offered the best possible prediction of success and achievement in Math 151. However, it was believed that considerable variation still existed within this group of students.

Several methods were considered in an attempt to insure that the condition groups would have relatively similar estimated expectancy. Each of the five freshmen groups was divided into two subgroups, one with a "high"
expectancy and the other with a "low" expectancy. "High" and "low" were merely relative terms used to indicate that one group should achieve better than the other based on the same criterion factor. The first method by which this was accomplished was to observe that students whose y-value in the formula \( y = 2x_4 + x_5 \) exceeded 64, and those whose total was sixty-four or less, formed nearly equal subsets of each of the five condition groups. It was statistically desirable to have condition groups as large as possible. The sub-divisions of the five freshmen condition groups are shown in Table 9.

**TABLE 9**

**SUBDIVISION OF FRESHMEN CONDITION GROUPS**

<table>
<thead>
<tr>
<th>Group</th>
<th>&quot;High&quot; Group</th>
<th>&quot;Low&quot; Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>44</td>
<td>36</td>
</tr>
<tr>
<td>B</td>
<td>22</td>
<td>12</td>
</tr>
<tr>
<td>C</td>
<td>101</td>
<td>120</td>
</tr>
<tr>
<td>D</td>
<td>57</td>
<td>65</td>
</tr>
<tr>
<td>E</td>
<td>30</td>
<td>39</td>
</tr>
</tbody>
</table>

An identification procedure was adopted that recognized these divisions. For example, \( A_1 \cup A_2 = A \), where \( A_1 \) had the higher expectancy of achievement, \( A_2 \) the lower expectancy, and \( A \) represented the condition group with
a full year of calculus as denoted in Table 6. The condi-
tion groups considered to have nearly the same estimated
expectancy of achievement were \( A_1, B_1, C_1, D_1, \) and \( E_1 \).
Analysis of variance was then employed. Similarly, \( A_2, B_2, C_2, D_2, \) and \( E_2 \) formed another set of condition groups
whose mean variation was analyzed. This set of condition
groups had a slightly lower expectancy. The mean scores
for the groups under this subdivision are reported in
Table 10.

**TABLE 10**

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>High sub-group</td>
<td>71.5</td>
<td>64.1</td>
<td>71.9</td>
<td>70.9</td>
<td>72.1</td>
</tr>
<tr>
<td>Low sub-group</td>
<td>61.3</td>
<td>60.6</td>
<td>59.9</td>
<td>58.9</td>
<td>59.7</td>
</tr>
</tbody>
</table>

Recalling that the students who had studied Math 150
were classified into two groups, one having "A" or "B"
grades and the other with "C" or "D" grades, they were
then considered with the freshmen subgroups. Two sets of
condition groups were formed which combined the non-
freshmen with the freshmen. Further, the final condition
group, \( G \), was combined with \( F_2 \), the "C" - "D" non-
freshmen, and the lower expectancy freshmen groups. A risk
was taken in adding \( F_1, F_2, \) and \( G \) to the two sets of
freshmen groups when the method of determining "high" and "low" expectancy differed as it did. This definitely showed up in the statistical analysis and details will be given in Chapter V.

The results of the analysis of variance will be described in six parts. Variation in the means of the six variables tested were investigated for six sets of condition groups. The six variables were the four mid-term examinations, the final examination and the course grade. The sets of condition groups are shown in Table 11.

TABLE 11
SETS OF CONDITION GROUPS TO BE ANALYZED

<table>
<thead>
<tr>
<th>Set</th>
<th>Groups Included</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A₁, B₁, C₁, D₁, E₁, F₁</td>
</tr>
<tr>
<td>2</td>
<td>A₁, B₁, C₁, D₁, E₁</td>
</tr>
<tr>
<td>3</td>
<td>A₂, B₂, C₂, D₂, E₂, F₂, G</td>
</tr>
<tr>
<td>4</td>
<td>A₂, B₂, D₂, D₂, E₂</td>
</tr>
<tr>
<td>5</td>
<td>A, B, C, D, E</td>
</tr>
<tr>
<td>6</td>
<td>Original freshmen groups numbered 1 through 12</td>
</tr>
</tbody>
</table>

A second approach was taken to the formation of the subdivisions according to estimated expectancy. While the formula \( y = 2x_4 + x_5 \) had proven in previous years to possess predictive qualities, the regression analysis
performed by this writer was expected to supply a new regression equation more pertinent to the current study. This could be used as a predictor of achievement.

After studying the regression analysis, this second approach to estimating expectancy was discontinued. Using the sum of the examination scores as the dependent variable, the sought after equation turned out to be

\[ y = 2.04x_4 + .938x_5 + 12.58, \]

where \( x_4 \) was the placement test score and \( x_5 \) the total quality points for semesters of high school mathematics. This was essentially what had already been used. A \( y \)-value of 75.83 was the population mean.

Summary

After analyzing the information on the two sets of questionnaires and the data collected through personal contact, seventeen initial condition groups were formed. These were later condensed to eight; five involving freshmen and three for non-freshmen. The freshmen groups could be summarized as having background characteristics of a year of calculus, a semester of calculus, a short unit of calculus, no calculus but with a semester of analytic geometry, and no calculus and no clearly defined analytic geometry course. The three upperclass groups were composed of students with prior calculus experience at the college level, pre-calculus experience at the college level with
high achievement, and pre-calculus experience at the college level with low achievement. The five freshmen groups were then subdivided forming two sets of condition groups; one with "high" expectancy and the other with "low" expectancy. This was accomplished through the use of the formula

\[ y = 2x_4 + x_5 \]

and contrasted with a newly developed regression equation utilizing the same variables. The placement test score was represented by \( x_4 \), and \( x_5 \) was the total quality points for semesters of high school mathematics.
CHAPTER V

ANALYSIS OF DATA

The data collected during the course of this study may be subdivided into three parts: preliminary data based on questionnaires, preliminary data supplied by the Office of Evaluation, and data acquired through classroom testing. The questionnaire results were fully described in Chapter IV. The Office of Evaluation provided the data used as independent variables in the analysis. These data were collected prior to student enrollment in classes. The classroom data included five common examination scores and the course grade. The later two phases of data collecting will be discussed in this chapter.

The analysis of data could be subdivided into two major categories: regression analysis and analysis of variance. The regression analysis was employed for the purpose of classifying students according to expectancy and to investigate variation due to instructor differences. The hypotheses of the study were tested by analysis of variance. In this chapter, a summary of the results of the analyses will be presented.
Regression Equations

The task of pursuing relationships pertinent to this study was facilitated by the availability of the IBM 7094 and IBM 360 computers on The Ohio State University campus. This, combined with the advice and assistance of the staff of the mathematics departments' statistics laboratory, made computer processing of the data possible.

Data collected by the Office of Evaluation as an enrollment procedure for all prospective students were requested for utilization in this research. Based on prior studies it was decided that four items would be needed. They were the Mathematics ACT percentile score, the OSU Math D placement test score, quality points for high school mathematics courses, and the high school graduating class rank. The pretest score provided the fifth of the independent variables to be used in the regression analysis.

Data were collected from 526 freshmen and 240 non-freshmen who completed Math 151 in the Autumn Quarter of 1969. While all of these students were considered in the analysis of variance population, it was necessary to eliminate 69 freshmen from the regression population due to incomplete data. Non-freshmen were not considered in this portion of the analysis.

Of the 69 excluded freshmen, 41 could have been eliminated from the study in the beginning, but were
retained so that their course grades could be included in the study. These were students who were enrolled in two special sections, and who did not take any of the common examinations. One group was a special section of high honors consisting of students who had completed supplementary assignments offered by the mathematics department prior to enrollment for the Autumn Quarter. These students did not take the pretest but did respond to the student questionnaire. The other section was especially selected through criteria established by the College of Engineering and labeled "honors" for that department. These students took the pretest and completed the questionnaire but did not take any of the common examinations. The only time the data for these two groups of students were to be utilized was in the analysis of variance among the condition groups with regard to the course grade.

The total of 69 subjects who were not considered in the regression population included 25 students for whom a class rank was not available. There were several students for whom one or more of the common examination scores were not available.

A standard program was employed which was designed to perform a step-wise regression analysis to produce equations of the form,

\[ y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + \cdots + b_ix_i \]
where $x_i$ are the selected independent variables, $y$ is the dependent variable to be predicted, and $b_i$ are the regression coefficients to be estimated. The information provided as output of the program included means, standard deviations, covariance matrix, and correlation matrix for each of the variables utilized. Along with each estimated $b_i$ was a standard error of estimate and the portion of the $F$-value attributed to that variable. For each calculated equation, a multiple coefficient $R$, $R^2$, and the $F$-ratio for the significance of $R$ were included.

The calculated means and standard deviations for each variable are tabulated in Table 12.

One set of statistics provided when prediction equations are calculated is a correlation matrix showing the correlations between all pairs of the variables submitted. Some of these have very little meaning while others provide the vital data necessary in constructing equations. In Table 13 these zero order correlations are shown.

It is of importance that, of the first six variables, it is $x_6$ with which the highest correlations occur with variables $x_7$ through $x_{12}$ and with $x_0$. The variable $x_6$ was derived from the placement test score and the total quality points for high school mathematics. The common examinations were represented by variables $x_7$ through $x_{12}$ and the course grade was $x_0$. 


<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Means</th>
<th>S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>x₁</td>
<td>Pretest</td>
<td>3.293</td>
<td>1.882</td>
</tr>
<tr>
<td>x₂</td>
<td>Math ACT percentile</td>
<td>91.534</td>
<td>7.460</td>
</tr>
<tr>
<td>x₃</td>
<td>H.S. class Rank (%)</td>
<td>90.449</td>
<td>10.210</td>
</tr>
<tr>
<td>x₄</td>
<td>Math &quot;D&quot; Test</td>
<td>16.536</td>
<td>2.838</td>
</tr>
<tr>
<td>x₅</td>
<td>Quality Points</td>
<td>31.302</td>
<td>4.185</td>
</tr>
<tr>
<td>x₆</td>
<td>2x₄ + x₅</td>
<td>64.374</td>
<td>7.023</td>
</tr>
<tr>
<td>x₇</td>
<td>Exam #1</td>
<td>13.954</td>
<td>2.725</td>
</tr>
<tr>
<td>x₈</td>
<td>Exam #2</td>
<td>12.204</td>
<td>3.379</td>
</tr>
<tr>
<td>x₉</td>
<td>Exam #3</td>
<td>11.540</td>
<td>3.358</td>
</tr>
<tr>
<td>x₁₀</td>
<td>Exam #4</td>
<td>12.484</td>
<td>3.140</td>
</tr>
<tr>
<td>x₁₁</td>
<td>Final Exam</td>
<td>25.650</td>
<td>5.659</td>
</tr>
<tr>
<td>x₁₂</td>
<td>Sum x₇ - x₁₁</td>
<td>75.832</td>
<td>13.755</td>
</tr>
<tr>
<td>x₀</td>
<td>Course Grade</td>
<td>2.420</td>
<td>.888</td>
</tr>
</tbody>
</table>
# Table 13

**Zero Order Correlations Among Variables**

<table>
<thead>
<tr>
<th>Variable</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
<th>$x_5$</th>
<th>$x_6$</th>
<th>$x_7$</th>
<th>$x_8$</th>
<th>$x_9$</th>
<th>$x_{10}$</th>
<th>$x_{11}$</th>
<th>$x_{12}$</th>
<th>$x_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>1.000</td>
<td>.146</td>
<td>.015</td>
<td>.184</td>
<td>.147</td>
<td>.237</td>
<td>.204</td>
<td>.271</td>
<td>.260</td>
<td>.208</td>
<td>.180</td>
<td>.292</td>
<td>.260</td>
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<tr>
<td>$x_2$</td>
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<td>.093</td>
<td>.276</td>
<td>.097</td>
<td>.281</td>
<td>.283</td>
<td>.303</td>
<td>.147</td>
<td>.236</td>
<td>.185</td>
<td>.296</td>
<td>.283</td>
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<td>$x_3$</td>
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<td>-.005</td>
<td>.344</td>
<td>.201</td>
<td>.092</td>
<td>.167</td>
<td>.190</td>
<td>.155</td>
<td>.192</td>
<td>.220</td>
<td>.163</td>
<td></td>
<td></td>
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<tr>
<td>$x_4$</td>
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<td>-.009</td>
<td>.803</td>
<td>.380</td>
<td>.355</td>
<td>.247</td>
<td>.288</td>
<td>.321</td>
<td>.420</td>
<td>.369</td>
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<tr>
<td>$x_5$</td>
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<td>.589</td>
<td>.171</td>
<td>.244</td>
<td>.233</td>
<td>.200</td>
<td>.207</td>
<td>.282</td>
<td>.230</td>
<td></td>
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<tr>
<td>$x_6$</td>
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<td>.409</td>
<td>.433</td>
<td>.338</td>
<td>.352</td>
<td>.382</td>
<td>.508</td>
<td>.435</td>
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<td></td>
</tr>
<tr>
<td>$x_7$</td>
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<td>.410</td>
<td>.354</td>
<td>.371</td>
<td>.412</td>
<td>.639</td>
<td>.526</td>
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<tr>
<td>$x_8$</td>
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<td>.491</td>
<td>.462</td>
<td>.512</td>
<td>.763</td>
<td>.676</td>
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<tr>
<td>$x_9$</td>
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<td>.409</td>
<td>.510</td>
<td>.738</td>
<td>.650</td>
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</tr>
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<td>$x_{10}$</td>
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<td>.439</td>
<td>.695</td>
<td>.607</td>
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</tr>
<tr>
<td>$x_{11}$</td>
<td>1.000</td>
<td>.843</td>
<td>.742</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>$x_{12}$</td>
<td>1.000</td>
<td>.873</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_0$</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Correlations are significant at .01 if $|r| > .121$.
Correlations are significant at .05 if $|r| > .093$. 

72
The next phase of the regression analysis was the actual development of the maximum prediction equations using \( x_1, x_2, x_3, x_4, \) and \( x_5 \) as independent variables and considering \( x_7 \) as the dependent variable. Here, and in future references, when a variable \( x_i \) is considered as being dependent, the notation \( y_i \) will be used. Thus, an equation \( y_7 = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + b_5x_5 \) was calculated. From Table 14 the coefficients may be observed from which the equation can be formulated.

Table 14 also indicates coefficients for the maximum prediction equations for \( y_8, y_9, y_{10}, y_{11}, y_{12}, \) and \( y_0 \).

Many times it is desirable to reduce the number of variables in a prediction equation to make it more practical. The order in which the variables enter into the regression analysis is quite important in this respect. It serves to indicate the predictive efficiency attributed to that particular variable. The best predictor variable enters first. The next variable entered is the one contributing most to the multiple correlation.

Table 15 illustrates the order of the five independent variables for each of the seven dependent variables whose equations have already been determined.
### TABLE 14

**REGRESSION COEFFICIENTS FOR MAXIMUM PREDICTION EQUATIONS**

<table>
<thead>
<tr>
<th></th>
<th>( y_7 )</th>
<th>( y_8 )</th>
<th>( y_9 )</th>
<th>( y_{10} )</th>
<th>( y_{11} )</th>
<th>( y_{12} )</th>
<th>( y_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exam 1</td>
<td>.14647</td>
<td>.29899</td>
<td>.34219</td>
<td>.20830</td>
<td>.28559</td>
<td>1.28164</td>
<td>.07386</td>
</tr>
<tr>
<td>Exam 2</td>
<td>.06006</td>
<td>.08012</td>
<td>.01582</td>
<td>.05540</td>
<td>.04962</td>
<td>.26101</td>
<td>.01815</td>
</tr>
<tr>
<td>Exam 3</td>
<td>.00842</td>
<td>.02945</td>
<td>.04283</td>
<td>.02900</td>
<td>.07605</td>
<td>.18575</td>
<td>.00793</td>
</tr>
<tr>
<td>Exam 4</td>
<td>.30495</td>
<td>.33083</td>
<td>.24087</td>
<td>.25463</td>
<td>.57235</td>
<td>1.70357</td>
<td>.09385</td>
</tr>
<tr>
<td>Final Ex.</td>
<td>.08599</td>
<td>.14086</td>
<td>.12741</td>
<td>.10366</td>
<td>.19166</td>
<td>.64955</td>
<td>.03457</td>
</tr>
<tr>
<td>Course Grade</td>
<td>Mult. R</td>
<td>.4619</td>
<td>.5048</td>
<td>.4086</td>
<td>.4068</td>
<td>.4207</td>
<td>.5718</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>.2134</td>
<td>.2549</td>
<td>.1670</td>
<td>.1655</td>
<td>.1770</td>
<td>.3270</td>
<td>.2468</td>
</tr>
<tr>
<td>S.E.E.</td>
<td>2.4302</td>
<td>2.9333</td>
<td>3.0822</td>
<td>2.8844</td>
<td>5.1623</td>
<td>11.3468</td>
<td>.7747</td>
</tr>
<tr>
<td>F-Ratio</td>
<td>24.466</td>
<td>30.851</td>
<td>18.080</td>
<td>17.884</td>
<td>19.396</td>
<td>43.818</td>
<td>29.550</td>
</tr>
</tbody>
</table>
### TABLE 15
ORDER IN WHICH VARIABLES ENTER THE REGRESSION

<table>
<thead>
<tr>
<th>Variable</th>
<th>Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_7$</td>
<td>$x_4$, $x_2$, $x_5$, $x_1$, $x_3$</td>
</tr>
<tr>
<td>$y_8$</td>
<td>$x_4$, $x_5$, $x_2$, $x_1$, $x_3$</td>
</tr>
<tr>
<td>$y_9$</td>
<td>$x_1$, $x_4$, $x_5$, $x_3$, $x_2$</td>
</tr>
<tr>
<td>$y_{10}$</td>
<td>$x_4$, $x_5$, $x_2$, $x_1$, $x_3$</td>
</tr>
<tr>
<td>$y_{11}$</td>
<td>$x_4$, $x_5$, $x_3$, $x_1$, $x_2$</td>
</tr>
<tr>
<td>$y_{12}$</td>
<td>$x_4$, $x_5$, $x_2$, $x_1$, $x_3$</td>
</tr>
<tr>
<td>$y_0$</td>
<td>$x_4$, $x_5$, $x_2$, $x_1$, $x_3$</td>
</tr>
</tbody>
</table>
The accuracy of a prediction obtained from a regression may be estimated by the coefficient of multiple correlation $R$, or more meaningfully, by $R^2$. The $R$-value may be interpreted as the correlation between actual values of $y_i$ and the values of $y_i$ predicted by the regression equation. The square of the multiple correlation coefficient indicates the percent of variation in the dependent variable which could be attributed to variations in the predictor variables employed.

Another measure of the predictive efficiency of the derived equation is the standard error of estimate. This number signifies that predicted $y_i$ values do not differ from actual $y_i$ values by more than this amount approximately 70 percent of the time.

Prior to the processing of data, it was anticipated that $y_{12}$, the sum of the common examination scores, would be the dependent variable of a regression equation which would serve to partition the condition groups into two sets of condition groups according to expectancy of achievement. An examination of Table 14 reveals that this was the best possible choice. The multiple $R$ value of .5718 indicated that the equation with the greatest efficiency was the one involving $y_{12}$. For this reason the details of the step-by-step regression for this equation will be given. At this stage of the analysis, it was decided that the test of instructor differences would be performed by
considering $x_8$ and $x_0$, the score on the second examination and the course grade. This would be a comparison between actual scores and predicted scores for each section of Math 151. The later variable was chosen because of the effect an instructor might have in the awarding of a letter grade other than by an observation of the total points accumulated on the examinations. However, the high correlation between the course grade and the sum of the test scores, $r = .873$, indicates that this was not a major point of discrepancy. The choice of Examination #2 was made because the correlations between $x_8$ and $x_9$, $x_{10}$, $x_{11}$, $x_{12}$, $x_0$ were quite high. This was especially valuable in that honors sections of calculus were formed partially on the basis of the results of this examination. Further details relating to the honors sections will follow.

A careful examination of the regression equation for $y_{12}$, the sum of the examination scores, reveals some useful information for this study. From Table 15 it can be observed that the order in which the five variables were entered was $x_4$, $x_5$, $x_1$, $x_2$, and $x_3$. The equations, in the order in which they were developed, are as follows:
\( y_{12} = 2.03748x_4 + 42.13959 \)

\( y_{12} = 2.04973x_4 + 0.93794x_5 + 12.57759 \)

\( y_{12} = 1.33280x_1 + 1.88597x_4 + 0.84859x_5 + 13.69327 \)

\( y_{12} = 1.23818x_1 + 0.27876x_2 + 1.69487x_4 + 0.80535x_5 - 6.99779 \)

\( y_{12} = 1.28164x_1 + 0.26101x_2 + 0.18575x_3 + 1.70357x_4 + 0.64955x_5 - 17.58475 \)

The multiple R's are, respectively, .4204, .5081, .5381, .5571, and .5718.

An observation of equation (2) reveals that the formula is very similar to \( y = 2x_4 + x_5 \) which is utilized in the placement procedure at The Ohio State University. Using these new found equations for \( y_{12} \) did not produce significant changes in the subdivisions by ability, from those which were formed by using \( y = 2x_4 + x_5 \).

A further regression using variables \( x_1, x_2, x_3, \) and \( x_6 \) where \( x_6 \) was \( 2x_4 + x_5 \) estimated the coefficients forming the following equations:

\( y_{12} = 0.99439x_6 + 11.81837 \)

\( y_{12} = 1.33095x_1 + 0.91001x_6 + 12.86752 \)

\( y_{12} = 1.23673x_1 + 0.28053x_2 + 0.83226x_6 - 7.49554 \)

The respective multiple R's were .5077, .5376, and .5570 which were slightly lower than when \( x_4 \) and \( x_5 \) were considered separately. The coefficient .99439 of \( x_6 \)
in equation (1) again illustrates the similarity to the equation \( y = 2x_4 + x_5 \).

One additional observation that may be made regarding the equations for \( y_{12} \) is that the variable \( x_1 \) enters as a predictor earlier than in most other instances. This relationship could exist because the pretest was calculus-oriented and Examination #2 was the first which included topics from calculus. Examination #1 included only pre-calculus topics. The other instance where \( x_1 \) held high predictive qualities was for Examination #3. For that variable, the Pretest was the initial variable entered into the regression, both when \( x_1, x_2, x_3, x_4, \) and \( x_5 \) were utilized and when \( x_1, x_2, x_3, \) and \( x_6 \) were the predictor variables. However, the regression equation for Examination #3 also had the lowest multiple R associated with it. As a predictor, the equation was not very efficient.

Honors sections for Math 151 were formed after the first and second examinations were administered. This was done on the basis of high achievement, selecting students who scored 18, 19, or 20 on the 20-item examinations. This procedure proved to have merit. The table of correlations shows that Examination #2 correlated with the final examination, total test points, and course grade better than any other variable. This excludes the high correlation which existed between \( x_{11} \) and \( x_0 \).
Regression equations were calculated for Examination #2 in terms of variables \( x_1, x_2, x_3, x_6, \) and \( x_7 \). Other equations were calculated for Examinations #3 and #4, the Final Examination, and course grade in terms of \( x_1, x_2, x_3, x_6, x_7, \) and \( x_8 \). The Exam 3 score, Exam 4 score, Final Exam score, and the course grade were predicted by equations with relatively high multiple R's. In each case, Exam 2 as a variable was the major contributor in the prediction equation. The two highest contributors in each case were \( x_7 \) and \( x_8 \), the first two examinations. The multiple R for course grade was \( .5467 \), for the final examination was \( .5779 \), for the third examination was \( .5492 \), and for the fourth examination was \( .5467 \).

Also of significance was the fact that Examination #2 was best predicted by \( x_6 \), the combination of the Math D Test score and the quality points for high school mathematics courses. The variable \( x_6 \) was considerably better as a predictor for \( y_8 \) than was \( x_7 \), the first exam. Thus one might conclude that \( x_6 \) is the best indicator of achievement for \( x_8 \), and \( x_8 \) is the best indicator of success for \( x_0 \), the course grade. This is, of course, an oversimplification but it does lend considerable support to the method by which the honor students were chosen during this particular school year.
Analysis of Variance

Recalling from Chapter IV that five freshmen and three non-freshmen condition groups were defined, the principal statistical method applied in this study was an analysis of the variation which occurred among the means of these groups for each of six variables. These six scores, \( x_7, x_8, x_9, x_{10}, x_{11}, \) and \( x_0 \) were the variables considered in a multivariate analysis of variance. Standard output of the program used included means and standard deviations for each condition group from each variable. Also produced were the Mean Square among condition groups, univariate F-values computed for each variable, and the level of significance of each F-value. The Mean Square within the condition groups was indirectly given through application of the formula

\[
F = \frac{MS_a}{MS_e}
\]

where \( MS_3 \) is the Mean Square within the groups and \( MS_a \) is the Mean Square among the groups. However, the important item was the F-value, for this determined whether the group means differed significantly in view of the differences within the separate groups. Duncan's Multiple Comparison Test was used to determine which means were significantly different whenever the F-test indicated that differences existed.
To test the null hypothesis that no significant variation in the means of the condition groups would occur if all groups had the same relative estimated expectancy of achievement, analysis of variance was employed in six phases. Each phase involved the six variables mentioned above, which included the four mid-term examinations, the final examination, and the course grade. The difference phases were necessitated by the establishment of six different sets of condition groups (see Table 11). These different sets of condition groups were created in a desire to provide nearly similar estimates of achievement and success.

The first set of condition groups, A₁, B₁, C₁, D₁, E₁, and F₁ included the freshmen groups with the greatest expectancy, and the non-freshmen with high achievement in Math 150. The second set was formed by deleting F₁ from the first set so that only freshmen would be involved in the analysis. Table 16 lists the means and standard deviations for each group for each of six variables. These means and standard deviations did not change, of course, when F₁ was omitted, but the results of the analysis of variance was considerably altered. The task at this point was to determine if differences among the means were significant.
<table>
<thead>
<tr>
<th>Variable</th>
<th>A1</th>
<th>B1</th>
<th>C1</th>
<th>D1</th>
<th>E1</th>
<th>F1</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>15.000</td>
<td>16.000</td>
<td>15.082</td>
<td>14.514</td>
<td>15.577</td>
<td>11.815</td>
</tr>
<tr>
<td>SD</td>
<td>1.962</td>
<td>1.892</td>
<td>2.408</td>
<td>2.599</td>
<td>2.859</td>
<td>3.096</td>
</tr>
<tr>
<td>SD</td>
<td>3.025</td>
<td>2.936</td>
<td>2.761</td>
<td>3.362</td>
<td>3.899</td>
<td>3.155</td>
</tr>
<tr>
<td>SD</td>
<td>3.414</td>
<td>3.706</td>
<td>3.516</td>
<td>2.655</td>
<td>2.943</td>
<td>3.151</td>
</tr>
<tr>
<td>SD</td>
<td>2.768</td>
<td>3.845</td>
<td>2.998</td>
<td>2.917</td>
<td>2.577</td>
<td>3.249</td>
</tr>
<tr>
<td>SD</td>
<td>3.840</td>
<td>5.509</td>
<td>5.176</td>
<td>4.753</td>
<td>5.382</td>
<td>6.203</td>
</tr>
<tr>
<td>M</td>
<td>2.976</td>
<td>3.050</td>
<td>2.616</td>
<td>2.757</td>
<td>2.846</td>
<td>1.444</td>
</tr>
<tr>
<td>SD</td>
<td>.758</td>
<td>.887</td>
<td>.793</td>
<td>.830</td>
<td>.967</td>
<td>.904</td>
</tr>
</tbody>
</table>
When the first set of condition groups was analyzed, highly significant F-values were found for each of the six variables. These F-values demonstrated that one or more of the differences of means was significant, even in view of the variation within the groups. This test did not reveal which of the differences observed in Table 16 were actually significant, merely that in at least one case this was so. For each examination, as well as the course grade, the F-value was significant beyond the .001 level (see Table 17). Thus, the null hypothesis is easily rejected on this basis.

A close observation of Table 16 reveals that the greatest differences among the means occurs between F₁, the non-freshman group with pre-calculus at the university level, and each of the remaining five condition groups. In view of the method by which F₁ was established, it seemed desirable to delete it as a condition group and again calculate F-values. When this was done, the only significant F-value was for x₈, the second examination (see Table 18). This considerably reduces the statistical evidence necessary to reject the null hypothesis, for it can only be rejected when considering the second examination and not for "achievement" in general. This information is of some value, however, since x₈ had earlier proven highly predictive of final achievement and success in
### TABLE 17
**F-VALUES FOR CONDITION GROUPS (SET ONE)**

<table>
<thead>
<tr>
<th>Variable</th>
<th>F (df: 5,245)</th>
<th>MS (among)</th>
<th>p less than</th>
</tr>
</thead>
<tbody>
<tr>
<td>x7</td>
<td>15.275</td>
<td>99.461</td>
<td>.001</td>
</tr>
<tr>
<td>x8</td>
<td>20.082</td>
<td>196.195</td>
<td>.001</td>
</tr>
<tr>
<td>x9</td>
<td>10.582</td>
<td>112.892</td>
<td>.001</td>
</tr>
<tr>
<td>x10</td>
<td>13.919</td>
<td>128.732</td>
<td>.001</td>
</tr>
<tr>
<td>x11</td>
<td>21.717</td>
<td>591.396</td>
<td>.001</td>
</tr>
<tr>
<td>x0</td>
<td>23.007</td>
<td>16.407</td>
<td>.001</td>
</tr>
</tbody>
</table>

### TABLE 18
**F-VALUES FOR CONDITION GROUPS (SET TWO)**

<table>
<thead>
<tr>
<th>Variable</th>
<th>F (df:4,192)</th>
<th>MS(among)</th>
<th>p less than</th>
</tr>
</thead>
<tbody>
<tr>
<td>x7</td>
<td>1.555</td>
<td>8.805</td>
<td>.188</td>
</tr>
<tr>
<td>x8</td>
<td>4.098</td>
<td>39.825</td>
<td>.003</td>
</tr>
<tr>
<td>x9</td>
<td>1.386</td>
<td>15.070</td>
<td>.240</td>
</tr>
<tr>
<td>x10</td>
<td>.882</td>
<td>7.842</td>
<td>.476</td>
</tr>
<tr>
<td>x11</td>
<td>1.367</td>
<td>32.983</td>
<td>.247</td>
</tr>
<tr>
<td>x0</td>
<td>1.858</td>
<td>1.271</td>
<td>.119</td>
</tr>
</tbody>
</table>
Math 151. Further, it was the first examination to test calculus concepts.

Duncan's Multiple Comparison Test was employed to detect those mean differences which were significant. The procedure followed in utilizing this test was to calculate a "q" value for each difference. The critical value for which "q" is significant at some given level is a function of the Rank Difference and the Degrees of Freedom (see Appendix E). The formula used to find "q" is

\[ q = \frac{M_i - M_j}{\sqrt{\frac{M_{Se}}{n}}} \]

where \( M_i \) and \( M_j \) are the group means being contrasted, \( M_{Se} \) is the mean square within the condition groups, and \( n \) is an "average" of the number of subjects in the two groups being compared. The value of \( n \) was determined by

\[ n = \frac{2}{\frac{1}{N_i} + \frac{1}{N_j}} \]

where \( N_i \) and \( N_j \) are the numbers of subjects in the two condition groups.

The significance of "q" depends on Rank Difference. The means for the condition groups are ranked in numerical order. Whenever the rank difference increases, the q-value must increase to become significant at some given level.
For example, if \( q > 3.98 \) for the analysis described in Table 18, all differences are significant at the .01 level, but "q" need only exceed 3.64 if the rank-difference is 2 instead of 5.

Application of the Multiple Comparison Test to the means calculated for the first set of condition groups revealed that the mean for \( F_1 \), the non-freshman group, differed significantly (beyond the .01 level) from each of the other five groups for all six variables.

When \( F_1 \) was deleted the only variable for which significant differences of means could be found was \( x_8 \), the second examination score. Among the five condition groups, \( B_1 \) had the highest mean while \( D_1 \) had the lowest for variable \( x_8 \). The calculated "q" for these two groups was 4.74 which was significant at .01 when the degrees of freedom were 245 and the rank-difference was 5. Of the ten differences possible among the five means, seven were significant. The significant differences were between:

- \( B_1 \) and \( D_1 \) at .01
- \( A_1 \) and \( D_1 \) at .01
- \( C_1 \) and \( D_1 \) at .01
- \( B_1 \) and \( E_1 \) at .05
- \( B_1 \) and \( C_1 \) at .05
- \( A_1 \) and \( E_1 \) at .05
- \( A_1 \) and \( C_1 \) at .05
Differences significant at the .01 level occurred between 
D₁, the no-calculus and no-analytic geometry group, and 
each of the three groups A₁, B₁, and C₁, all of which had 
at least a unit of calculus in the background preparation. 
D₁ was significantly lower than the others.

Additional significant differences occurred when 
A₁ and B₁, the groups with a year of calculus and a semester of calculus, were contrasted with C₁ and E₁, the 
groups with a short unit of calculus and a semester of analytic geometry. No significant differences were 
detected between A₁ and B₁, C₁ and E₁, and D₁ and E₁.

The next phase of the analysis of variance was a 
consideration of the third and fourth sets of condition groups. These groups consisted of subjects with a lower 
epectancy of achievement than was the case for the first two sets. The statistical results were almost identical. 
When A₂, B₂, C₂, D₂, E₂, F₂, and G were analyzed, highly 
significant F-values (beyond .001) were calculated for 
each of the six variables. Observation of the means in 
Table 19 reveals that once again, the greatest differences 
occur between the freshmen groups A₂, B₂, C₂, D₂ and E₂, 
and the non-freshmen groups F₂ and G.

When F₂ and G were deleted, the F-values were 
significant only for variables x₈ and x₉, the second and 
third examination scores. These F-values are found in 
Table 20 (Set 3) and Table 21 (Set 4).
<table>
<thead>
<tr>
<th>Variable</th>
<th>$A_2$</th>
<th>$B_2$</th>
<th>$C_2$</th>
<th>$D_2$</th>
<th>$E_2$</th>
<th>$F_2$</th>
<th>$G$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.438</td>
<td>2.716</td>
<td>2.801</td>
<td>2.570</td>
<td>3.017</td>
<td>2.745</td>
<td>2.406</td>
</tr>
<tr>
<td></td>
<td>3.479</td>
<td>1.491</td>
<td>2.923</td>
<td>3.383</td>
<td>3.240</td>
<td>3.095</td>
<td>2.995</td>
</tr>
<tr>
<td>$x_9$</td>
<td>12.000</td>
<td>11.700</td>
<td>10.802</td>
<td>10.158</td>
<td>10.250</td>
<td>7.961</td>
<td>6.722</td>
</tr>
<tr>
<td></td>
<td>2.859</td>
<td>3.057</td>
<td>3.111</td>
<td>3.086</td>
<td>3.288</td>
<td>3.156</td>
<td>4.084</td>
</tr>
<tr>
<td>$x_{10}$</td>
<td>12.714</td>
<td>12.100</td>
<td>11.339</td>
<td>11.697</td>
<td>11.300</td>
<td>8.490</td>
<td>9.056</td>
</tr>
<tr>
<td></td>
<td>3.223</td>
<td>2.470</td>
<td>2.931</td>
<td>3.418</td>
<td>2.255</td>
<td>3.776</td>
<td>3.316</td>
</tr>
<tr>
<td>$x_{11}$</td>
<td>24.886</td>
<td>22.900</td>
<td>24.058</td>
<td>24.263</td>
<td>22.950</td>
<td>17.137</td>
<td>16.500</td>
</tr>
<tr>
<td></td>
<td>5.268</td>
<td>8.439</td>
<td>5.738</td>
<td>4.997</td>
<td>5.996</td>
<td>6.283</td>
<td>7.786</td>
</tr>
<tr>
<td>$x_0$</td>
<td>2.400</td>
<td>2.400</td>
<td>2.140</td>
<td>2.145</td>
<td>2.050</td>
<td>1.059</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>.847</td>
<td>.516</td>
<td>.799</td>
<td>.795</td>
<td>.783</td>
<td>.925</td>
<td>1.085</td>
</tr>
</tbody>
</table>
### TABLE 20

**F-VALUES FOR CONDITION GROUPS (SET THREE)**

<table>
<thead>
<tr>
<th>Variable</th>
<th>F (df: 6,344)</th>
<th>MS</th>
<th>P less than</th>
</tr>
</thead>
<tbody>
<tr>
<td>x7</td>
<td>8.363</td>
<td>61.645</td>
<td>.001</td>
</tr>
<tr>
<td>x8</td>
<td>8.636</td>
<td>84.250</td>
<td>.001</td>
</tr>
<tr>
<td>x9</td>
<td>10.853</td>
<td>108.542</td>
<td>.001</td>
</tr>
<tr>
<td>x10</td>
<td>9.376</td>
<td>93.027</td>
<td>.001</td>
</tr>
<tr>
<td>x11</td>
<td>13.818</td>
<td>474.562</td>
<td>.001</td>
</tr>
<tr>
<td>x0</td>
<td>17.748</td>
<td>12.256</td>
<td>.001</td>
</tr>
</tbody>
</table>

### TABLE 21

**F-VALUES FOR CONDITION GROUPS (SET FOUR)**

<table>
<thead>
<tr>
<th>Variable</th>
<th>F (df: 4,277)</th>
<th>MS</th>
<th>P less than</th>
</tr>
</thead>
<tbody>
<tr>
<td>x7</td>
<td>.316</td>
<td>2.348</td>
<td>.867</td>
</tr>
<tr>
<td>x8</td>
<td>2.786</td>
<td>27.399</td>
<td>.027</td>
</tr>
<tr>
<td>x9</td>
<td>2.619</td>
<td>25.143</td>
<td>.035</td>
</tr>
<tr>
<td>x10</td>
<td>1.607</td>
<td>14.579</td>
<td>.173</td>
</tr>
<tr>
<td>x11</td>
<td>.701</td>
<td>22.309</td>
<td>.592</td>
</tr>
<tr>
<td>x0</td>
<td>1.233</td>
<td>.788</td>
<td>.297</td>
</tr>
</tbody>
</table>
The Multiple Comparison Test indicated that differences among means of the seven groups in Set 3 were principally between freshmen groups and non-freshmen groups. When only the freshmen condition groups were analyzed, significant differences of means for variables $x_8$ and $x_9$ were found to occur between calculus groups and non-calculus groups. No significant difference of means was detected between any two of the three groups $C_2$, $D_2$, and $E_2$.

The method by which the condition groups were divided according to estimated expectancy of achievement, proved reliable in that the mean for $A_1$ was significantly higher than the mean for $A_2$ for all six variables tested. This was true for each of the five freshmen groups as well as for $F_1$ and $F_2$. It supports the previous research evidence that the method utilized can identify superior students in beginning calculus.

The next phase of the analysis entailed the combining of $A_1$ and $A_2$ to obtain condition group $A$. Similarly, $B$, $C$, $D$, and $E$ were formed to produce Set 5 of condition groups. These groups did not have the "expectancy of success" as clearly defined as for the earlier groups, but in a practical sense, having only this set of condition groups would be simpler to work with if the results proved meaningful. Statistical results were slightly different than when the groups were subdivided.
The F-values for the analysis of Set 5 were highly significant for all variables except \(x_{11}\), the final examination. The means and standard deviations for each condition group for each variable score are tabulated in Table 22. Immediately following, Table 23 lists the F-values for Set 5.

**TABLE 22**

**MEANS AND STANDARD DEVIATIONS FOR SET 5**

<table>
<thead>
<tr>
<th>Variable</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_7)</td>
<td>M</td>
<td>14.253</td>
<td>15.133</td>
<td>13.742</td>
<td>13.398</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>2.343</td>
<td>2.488</td>
<td>2.851</td>
<td>2.684</td>
</tr>
<tr>
<td>(x_8)</td>
<td>M</td>
<td>13.613</td>
<td>14.167</td>
<td>12.026</td>
<td>11.522</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>3.357</td>
<td>2.960</td>
<td>3.007</td>
<td>3.418</td>
</tr>
<tr>
<td>(x_9)</td>
<td>M</td>
<td>12.853</td>
<td>12.600</td>
<td>11.289</td>
<td>10.858</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>3.245</td>
<td>3.510</td>
<td>3.320</td>
<td>3.108</td>
</tr>
<tr>
<td>(x_{10})</td>
<td>M</td>
<td>13.467</td>
<td>13.400</td>
<td>12.072</td>
<td>12.168</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>3.068</td>
<td>3.529</td>
<td>3.097</td>
<td>3.319</td>
</tr>
<tr>
<td>(x_{11})</td>
<td>M</td>
<td>26.867</td>
<td>26.867</td>
<td>25.098</td>
<td>25.575</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>4.842</td>
<td>7.080</td>
<td>5.680</td>
<td>5.249</td>
</tr>
<tr>
<td>(x_0)</td>
<td>M</td>
<td>2.720</td>
<td>2.833</td>
<td>2.320</td>
<td>2.345</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>0.847</td>
<td>0.834</td>
<td>0.828</td>
<td>0.853</td>
</tr>
</tbody>
</table>
TABLE 23
F-VALUES FOR CONDITION GROUPS (SET FIVE)

<table>
<thead>
<tr>
<th>Variable</th>
<th>F (df:4,473)</th>
<th>MS</th>
<th>P less than</th>
</tr>
</thead>
<tbody>
<tr>
<td>x7</td>
<td>2.973</td>
<td>22.715</td>
<td>.019</td>
</tr>
<tr>
<td>x8</td>
<td>8.785</td>
<td>93.305</td>
<td>.001</td>
</tr>
<tr>
<td>x9</td>
<td>5.547</td>
<td>59.388</td>
<td>.001</td>
</tr>
<tr>
<td>x10</td>
<td>3.665</td>
<td>35.686</td>
<td>.006</td>
</tr>
<tr>
<td>x11</td>
<td>1.789</td>
<td>57.230</td>
<td>.130</td>
</tr>
<tr>
<td>x0</td>
<td>5.033</td>
<td>3.663</td>
<td>.001</td>
</tr>
</tbody>
</table>

With the knowledge obtained from Table 23 that significant differences occurred, once again Duncan's Multiple Comparison Test was employed to determine which of the ten differences actually were significant. Table 24 lists the differences of means and the level of significance. Very significant differences occurred between the calculus groups and the non-calculus groups. Groups A and B differed from groups C, D, and E for all five variables. However, in no cases did any two of the means of C, D, and E demonstrate significant difference.
TABLE 24
DIFFERENCES OF MEANS AMONG GROUPS IN SET 5

<table>
<thead>
<tr>
<th>Compared Variables</th>
<th>x_7</th>
<th>x_8</th>
<th>x_9</th>
<th>x_10</th>
<th>x_0</th>
</tr>
</thead>
<tbody>
<tr>
<td>B &amp; D</td>
<td>1.735**</td>
<td>2.645**</td>
<td>1.742**</td>
<td>1.232</td>
<td>.488**</td>
</tr>
<tr>
<td>B &amp; C</td>
<td>1.391*</td>
<td>2.141**</td>
<td>1.311*</td>
<td>1.328*</td>
<td>.513**</td>
</tr>
<tr>
<td>B &amp; E</td>
<td>1.027</td>
<td>2.834**</td>
<td>1.464*</td>
<td>1.112</td>
<td>.469*</td>
</tr>
<tr>
<td>A &amp; D</td>
<td>.855*</td>
<td>2.091**</td>
<td>1.995**</td>
<td>1.299**</td>
<td>.375**</td>
</tr>
<tr>
<td>A &amp; C</td>
<td>.511</td>
<td>1.587**</td>
<td>1.564**</td>
<td>1.395**</td>
<td>.400**</td>
</tr>
<tr>
<td>E &amp; D</td>
<td>.708</td>
<td>.189</td>
<td>.278</td>
<td>.120</td>
<td>.009</td>
</tr>
<tr>
<td>B &amp; A</td>
<td>.880</td>
<td>.554</td>
<td>.253</td>
<td>.067</td>
<td>.113</td>
</tr>
<tr>
<td>A &amp; E</td>
<td>.147</td>
<td>2.280**</td>
<td>1.717**</td>
<td>1.179*</td>
<td>.356*</td>
</tr>
<tr>
<td>E &amp; C</td>
<td>.364</td>
<td>.693</td>
<td>.153</td>
<td>.216</td>
<td>.044</td>
</tr>
<tr>
<td>C &amp; D</td>
<td>.344</td>
<td>.504</td>
<td>.431</td>
<td>.096</td>
<td>.025</td>
</tr>
</tbody>
</table>

*Significant at .05 level.
**Significant at .01 level.
The final phase of the analysis of variance testing was a comparison of the means of the original twelve freshmen condition groups (Set 6). The means and standard deviations are found in Table 25 and F-values associated with the six variables are listed in Table 26. Once again, \(x_8\) demonstrated the most significant F-value but \(x_9\) and \(x_0\) were also significant beyond the .01 level.

Observation of Table 25 reveals that major differences again occurred between the calculus groups and the remaining ten groups. Duncan's test verified this observation, with groups 5 and 6 differing significantly from each of the other groups except 9 and 10. The difficulty in obtaining significant tests for groups 9 and 10 resulted from the relatively small number of subjects associated with each. The ten non-calculus groups had no significant mean differences among them for any of the variables.

Summarizing the six phases of variance analysis, it appears that the null hypothesis can be rejected ... sometimes. This is especially true when A, B, C, D, and E are the condition groups in consideration or for the variables involving the earliest work with calculus concepts. Differences were not predominant for variable \(x_7\), the first examination, but this is perhaps due to the fact that the test included only pre-calculus concepts. Examination Two was based largely on the initial concepts of calculus, and this is where the highly significant
<table>
<thead>
<tr>
<th>Group</th>
<th>( x_7 )</th>
<th>( x_8 )</th>
<th>( x_9 )</th>
<th>( x_{10} )</th>
<th>( x_{11} )</th>
<th>( x_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3.113</td>
<td>2.669</td>
<td>3.256</td>
<td>3.095</td>
<td>5.683</td>
<td>.819</td>
</tr>
<tr>
<td></td>
<td>3.146</td>
<td>3.127</td>
<td>3.245</td>
<td>2.604</td>
<td>5.723</td>
<td>.846</td>
</tr>
<tr>
<td></td>
<td>2.584</td>
<td>3.143</td>
<td>3.358</td>
<td>3.237</td>
<td>5.679</td>
<td>.825</td>
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<td>2.343</td>
<td>3.357</td>
<td>3.245</td>
<td>3.068</td>
<td>4.842</td>
<td>.847</td>
</tr>
<tr>
<td>6</td>
<td>15.133</td>
<td>14.167</td>
<td>12.600</td>
<td>13.400</td>
<td>26.867</td>
<td>2.833</td>
</tr>
<tr>
<td></td>
<td>2.488</td>
<td>2.960</td>
<td>3.510</td>
<td>3.529</td>
<td>7.080</td>
<td>.834</td>
</tr>
<tr>
<td></td>
<td>3.007</td>
<td>3.031</td>
<td>3.070</td>
<td>3.553</td>
<td>4.592</td>
<td>.784</td>
</tr>
<tr>
<td>8</td>
<td>14.143</td>
<td>10.857</td>
<td>10.286</td>
<td>12.000</td>
<td>25.429</td>
<td>2.429</td>
</tr>
<tr>
<td></td>
<td>1.574</td>
<td>3.671</td>
<td>4.608</td>
<td>2.517</td>
<td>4.117</td>
<td>1.134</td>
</tr>
<tr>
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<td>1.978</td>
<td>3.634</td>
<td>2.987</td>
<td>2.921</td>
<td>5.300</td>
<td>.745</td>
</tr>
<tr>
<td>10</td>
<td>12.300</td>
<td>10.800</td>
<td>10.700</td>
<td>12.000</td>
<td>24.800</td>
<td>2.200</td>
</tr>
<tr>
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<td>2.003</td>
<td>3.615</td>
<td>3.199</td>
<td>3.197</td>
<td>8.135</td>
<td>1.033</td>
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<tr>
<td></td>
<td>1.567</td>
<td>4.503</td>
<td>3.118</td>
<td>3.525</td>
<td>5.900</td>
<td>1.087</td>
</tr>
<tr>
<td></td>
<td>3.250</td>
<td>3.833</td>
<td>2.207</td>
<td>3.349</td>
<td>5.087</td>
<td>.786</td>
</tr>
</tbody>
</table>
### TABLE 26
F-VALUES FOR CONDITION GROUPS (SET SIX)

<table>
<thead>
<tr>
<th>Variable</th>
<th>F (df:11,466)</th>
<th>MS</th>
<th>P less than</th>
</tr>
</thead>
<tbody>
<tr>
<td>x_7</td>
<td>1.702</td>
<td>13.010</td>
<td>.070</td>
</tr>
<tr>
<td>x_8</td>
<td>3.764</td>
<td>40.036</td>
<td>.001</td>
</tr>
<tr>
<td>x_9</td>
<td>2.730</td>
<td>29.179</td>
<td>.002</td>
</tr>
<tr>
<td>x_10</td>
<td>1.653</td>
<td>16.213</td>
<td>.081</td>
</tr>
<tr>
<td>x_11</td>
<td>1.237</td>
<td>39.635</td>
<td>.259</td>
</tr>
<tr>
<td>x_0</td>
<td>2.308</td>
<td>1.686</td>
<td>.009</td>
</tr>
</tbody>
</table>
differences occurred for every variance test. Examination Three was next in order of revealing meaningful differences. Examination Four showed up only once with differing means, and the final examination never entered as significant whenever the freshmen groups were analyzed alone. The course grade did reflect some of the differences experienced on earlier scores. In nearly all cases the measurable differences were between calculus groups and non-calculus groups, and these differences appeared to diminish as the work in the course progressed. In other words, the students with a high school calculus background achieved significantly better than those without that background. However, this difference in achievement became less noticeable as work in the course progressed.

Instructor Differences

Regardless of the efforts made to have strong instructor personnel in each of the Math 151 classrooms, differences are inevitable. Some instructors are "better" than others, which in many cases might mean that they are "better preparers for examinations." It need not reflect superior pedagogical capabilities and techniques. To obtain some measure of confidence that instructor differences were not a major source of error in this study, these differences were compared for significance.

Using the maximum prediction equations from the earlier regression analysis for variables $y_8$ and $y_0$
(and $y_9$, $y_{10}$, $y_{11}$ for honors sections), the Second Examination score and the course grade were predicted for each subject. The "expected" mean score for each instructor-section was compared with the "actual" mean score by employing a t-test. Thirty-seven instructors were considered in this analysis and in six cases the actual mean was significantly lower than the predicted mean. Two of these differences were for $x_8$, the examination score, and the other four differences were for $x_0$, the course grade. In no case was there a difference for the examination score and the course grade of the same instructor. However, there was one example of an instructor having two sections and both classes scored significantly lower than was predicted.

The only situation where an instructor's "actual" mean exceeded his "expected" mean was in three of the four honors sections. The course grades predicted for students in these classes were much higher than for any other sections and yet the differences by which they actually exceeded prediction were significant beyond the .01 level. When predicted scores for $x_8$ were calculated for two of the honors sections, it was found that one of them exceeded prediction significantly. The predicted score for Examination Three was exceeded significantly at the .01 level by all four sections. Yet, when these students were
interspersed in the regular classes for the first two examinations, no classes achieved higher then predicted.

These variations were not excessively higher than could be expected by chance. Thus, the claim was made that instructor differences were not a significant factor in this study.

**Analysis of Grade Distribution**

Grade distribution for the Autumn Quarter freshmen is shown in Table 27, according to condition groups without regard to estimated expectancy.

**TABLE 27**

<table>
<thead>
<tr>
<th>Course Grade</th>
<th>Group A (%)</th>
<th>Group B (%)</th>
<th>Group C (%)</th>
<th>Group D (%)</th>
<th>Group E (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>17 (21)</td>
<td>9 (27)</td>
<td>29 (13)</td>
<td>14 (11)</td>
<td>11 (16)</td>
</tr>
<tr>
<td>B</td>
<td>34 (43)</td>
<td>15 (44)</td>
<td>58 (26)</td>
<td>35 (29)</td>
<td>20 (29)</td>
</tr>
<tr>
<td>C</td>
<td>26 (33)</td>
<td>9 (27)</td>
<td>108 (49)</td>
<td>59 (49)</td>
<td>28 (41)</td>
</tr>
<tr>
<td>D</td>
<td>3 (3)</td>
<td>1 (2)</td>
<td>23 (10)</td>
<td>11 (9)</td>
<td>8 (11)</td>
</tr>
<tr>
<td>E</td>
<td>0 (0)</td>
<td>0 (0)</td>
<td>3 (2)</td>
<td>3 (2)</td>
<td>2 (3)</td>
</tr>
<tr>
<td>Total</td>
<td>80</td>
<td>34</td>
<td>221</td>
<td>122</td>
<td>69</td>
</tr>
</tbody>
</table>
Sixty-four percent of the students with a year of high school calculus and 71 percent of those with a semester of high school calculus received a grade of "A" or "B" for the course. This was considerably higher than for any of the other groups where the percents were 39, 40, and 45. The differences among the latter three percentages are negligible. From another point of view, of the eighty "A" grades for Autumn Quarter, 30 percent were by students who had studied high school calculus while only 22 percent of the study population had studied a semester or more of calculus.

Only eight members of the freshman data population received a grade of "E" and only forty-six received a grade of "D." This represents approximately 10 percent of the total.

A close observation of those subjects chosen for honors sections revealed that all five condition groups were represented proportionally except for Group D. The four honors sections included 21 students with a semester or year of calculus in high school, 19 with the "foundations" background, 10 with a semester of analytic geometry, but only 8 from Group D which had neither calculus nor analytic geometry in the background. Group D was the second largest, but the least represented.

In the honors sections, all students but one received a grade of "A" or "B." The one section of honors,
which did not succeed higher than predicted, contained only two students with prior calculus training and two others with the "foundations" background. The other three honors sections had a significantly higher percentage of students from these two groups.

One fact that could be of importance to placement personnel is that of the 58 students chosen for honors, only five had scored lower than 64 when the formula $y = 2x_4 + x_5$ was applied. The only student with a score lower than 62 eventually stopped coming to class and did not take the final examination. His placement into the honors section was based upon a very high score on the first examination. It appears that exceeding 63 is a necessary but not a sufficient condition for high achievement in Math 151 honors at The Ohio State University.

**Item Analyses**

An item analysis was run for each of the common examinations and for the pretest. Standard data provided through this analysis included frequency distribution of raw scores, mean, median, mode, skewness, kurtoisis, and Kuder-Richardson 20 and 21 reliability estimates yielding indices of internal consistency. For each test item a tally of responses is given. These responses are divided into three categories; responses for the upper 27.5 percent of the cases, the lower 27.5 percent of the cases, and the total. Relative difficulty is the percentage of
students missing an item. The obtained D, or the discrimination index, reflects the degree to which the item discriminates between the upper and lower groups. The efficiency of an item is the ratio of the obtained D to the maximum D which could hypothetically be expected between the groups.

According to Garrett (20,368) items with a discrimination index of .20 or higher are regarded as satisfactory. All examinations used in this study had mean item discrimination indices exceeding .40. Abbreviated summaries of each item analysis are found in Appendix D.

Summary

Regression analysis was utilized to determine the most appropriate method by which groups with the same relative estimated expectancy of achievement could be established. Two of the calculated maximum prediction equations were used to investigate instructor differences by comparing "expected" means with "actual" means for each instructor.

The employment of analysis of variance techniques demonstrated that some significant differences of means were experienced, and Duncan's Multiple Comparison Test showed that differences, when they occurred, were between calculus and non-calculus groups. No single pre-calculus course demonstrated having a quality of guaranteeing greater success in beginning calculus.
Significant differences in achievement were discovered for Examination Two on all of the variance analyses. Examination Three yielded significant differences several times, Examination Four only once, and the final examination never produced significant differences.

In brief, students with a year or semester of high school calculus as background preparation achieved significantly higher than those without that preparation. The differences in achievement, apparent early in the quarter, appeared to diminish as the quarter progressed.
CHAPTER VI

ANALYSIS OF COURSE GRADES--SECOND QUARTER CALCULUS

During the Winter Quarter of 1970, the progress of the freshmen study population was observed. Of the 526 subjects completing Math 151, only 317 also completed Math 152, the second course in calculus. This represented approximately 60 percent of the Autumn total. The numbers of students in each of the condition groups A, B, C, D, and E were substantially reduced but were sufficient to again be considered in an analysis of variance. The objective in doing this was to determine if differences, which were prevalent during the first quarter, tended to diminish during the second quarter. No expectancy subdivisions were considered.

Table 28 shows a comparison of the numbers in each group between Autumn Quarter and Winter Quarter.

The grade distribution for the second quarter of calculus proved to be of interest. During the Autumn Quarter, approximately 18 percent of the freshmen received grades of "D" or "E"; for the Winter Quarter this increased to 23 percent. During Autumn Quarter approximately 15 percent of the freshmen received a grade of "A"; for
Winter Quarter it increased to 22 percent. Table 29 shows the Math 152 grade distribution.

**TABLE 28**

CONDITION GROUP DISTRIBUTION FOR MATH 152

<table>
<thead>
<tr>
<th>Groups</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math 151</td>
<td>80</td>
<td>34</td>
<td>221</td>
<td>122</td>
<td>69</td>
<td>526</td>
</tr>
<tr>
<td>Math 152</td>
<td>55</td>
<td>20</td>
<td>130</td>
<td>74</td>
<td>38</td>
<td>317</td>
</tr>
<tr>
<td>Percent Continuing</td>
<td>69</td>
<td>58</td>
<td>58</td>
<td>60</td>
<td>55</td>
<td>60</td>
</tr>
</tbody>
</table>

**TABLE 29**

GRADE DISTRIBUTION FOR MATH 152

<table>
<thead>
<tr>
<th>Letter Grade</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10</td>
<td>10</td>
<td>26</td>
<td>17</td>
<td>7</td>
<td>70</td>
</tr>
<tr>
<td>B</td>
<td>19</td>
<td>4</td>
<td>32</td>
<td>11</td>
<td>8</td>
<td>74</td>
</tr>
<tr>
<td>C</td>
<td>20</td>
<td>3</td>
<td>33</td>
<td>28</td>
<td>15</td>
<td>99</td>
</tr>
<tr>
<td>D</td>
<td>4</td>
<td>3</td>
<td>24</td>
<td>14</td>
<td>7</td>
<td>52</td>
</tr>
<tr>
<td>E</td>
<td>2</td>
<td>0</td>
<td>15</td>
<td>4</td>
<td>1</td>
<td>22</td>
</tr>
<tr>
<td>Total</td>
<td>55</td>
<td>20</td>
<td>130</td>
<td>74</td>
<td>38</td>
<td>317</td>
</tr>
</tbody>
</table>
Table 30 indicates the grade distribution for Math 152 by percent rather than by numbers.

### TABLE 30
GRADE DISTRIBUTION FOR MATH 152 BY PERCENT

<table>
<thead>
<tr>
<th>Letter Grade</th>
<th>Condition Groups (percent)</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td>18</td>
<td>50</td>
<td>20</td>
<td>23</td>
<td>18</td>
<td>22</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>34</td>
<td>20</td>
<td>25</td>
<td>15</td>
<td>21</td>
<td>23</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>37</td>
<td>15</td>
<td>26</td>
<td>38</td>
<td>40</td>
<td>31</td>
</tr>
<tr>
<td>D</td>
<td></td>
<td>7</td>
<td>15</td>
<td>18</td>
<td>19</td>
<td>18</td>
<td>17</td>
</tr>
<tr>
<td>E</td>
<td></td>
<td>4</td>
<td>0</td>
<td>11</td>
<td>5</td>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

Fifty-three percent of the students with a year of high school calculus received "A" or "B" in Math 152 while 70 percent of those with a semester of high school calculus achieved similarly. For groups C, D, and E the percents were, respectively, 45, 38, and 39. The highest percent of failure occurred within group C where nearly 12 percent received a grade of "E." This group also had the highest total in the combined "D" and "E" range; 30 percent.

Twenty-two failures were recorded among the 317 freshmen in Math 152. Of this total, eight had received a grade of "D" in Math 151, 11 had received "C" two had
received "B" and one student actually dropped from an "A" in Math 151 to an "E" in Math 152. The second course in calculus appears to serve as a critical discriminating agent, more clearly identifying the highly capable and the less capable as pertaining to mathematics.

Of the 46 students who received a grade of "D" in Math 151, only 17 continued in Math 152 and eight of them dropped to a grade of "E" in the second course. The remaining nine students again received a "D" or, in several instances, raised their grade to a "C." Only six of the 80 freshmen with "A" in Math 151 did not continue in Math 152. Also, 43 of 162 in the "B" range and 109 of 230 in the "C" range did not continue in the second quarter of calculus.

The analysis of variance technique was applied, using the five condition groups mentioned at the beginning of this chapter and the Math 152 course grade as a single variable. Means and standard deviations are shown in Table 31.
The earlier analyses had shown that group A and group B were generally quite close, with neither consistently outscoring the other. However, in this final analysis, group B was considerably higher than group A. Statistical evidence provided by the Multiple Comparison Test indicated that this difference was not actually significant at the .05 level. The F-value obtained for degrees of freedom of 4 and 310 was 2.5225. The mean square between groups was 3.5654 and within groups was 1.4115. This F-value is significant at the .05 level but not at the .01 level.

Since the F-value was significant, the pair-wise differences were all investigated to determine significance. The group with a semester of calculus as a background proved to have a significantly higher mean than group C, group D, and group E. The group of students with a year of calculus as background did not score significantly higher than any of the other groups. Group
means for C, D, and E were very similar. In fact, group D had a mean identical to that of group E.

Investigation of the twenty individuals constituting group B revealed that seventeen of the twenty students were included in the sub-group B1 which was the "high" achievement section. Furthermore, Table 10 revealed these individuals had initially been quite high within that subgroup. The other condition groups all had higher percents of students from the lower expectation subdivisions. All of the mean differences are shown in Table 32.

While the variation in mean scores for the Math 152 calculus course grades was less than for the first course, this cannot necessarily be attributed to one single cause. It does appear that the high school background tends to fade as a contributor to relationships associated with achievement. The first calculus grade provides a better indication of success in the subsequent course.

The make-up of group A was somewhat different from that of group B which most likely accounts for the results observed in Table 32. It had been pointed out that 85 percent of group B had been placed in the B1 subgroup. For group A, it was approximately 57 percent. Twenty-four of the fifty-five subjects in group A had been classified in A2, the "low" expectancy group for
TABLE 32
MEAN DIFFERENCES--COURSE GRADES IN MATH 152

<table>
<thead>
<tr>
<th>Groups</th>
<th>Mean Difference</th>
<th>Sign. Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>B &amp; C</td>
<td>.8192</td>
<td>.01</td>
</tr>
<tr>
<td>B &amp; D</td>
<td>.7257</td>
<td>.05</td>
</tr>
<tr>
<td>B &amp; E</td>
<td>.7257</td>
<td>.05</td>
</tr>
<tr>
<td>B &amp; A</td>
<td>.4759</td>
<td>n.s.</td>
</tr>
<tr>
<td>A &amp; C</td>
<td>.3433</td>
<td>n.s.</td>
</tr>
<tr>
<td>A &amp; D</td>
<td>.2498</td>
<td>n.s.</td>
</tr>
<tr>
<td>A &amp; E</td>
<td>.2498</td>
<td>n.s.</td>
</tr>
<tr>
<td>C &amp; D</td>
<td>.0935</td>
<td>n.s.</td>
</tr>
<tr>
<td>C &amp; E</td>
<td>.0935</td>
<td>n.s.</td>
</tr>
<tr>
<td>D &amp; E</td>
<td>.0000</td>
<td>n.s.</td>
</tr>
</tbody>
</table>

the earlier analyses. These are the students who achieved at the lowest level in Math 152. In fact, two students in group A received grades of "E." This tends to give greater credibility to the formula used in placing new students into appropriate mathematics courses. While this study tends to reveal that students with a high school calculus background achieve above the average in beginning calculus, this is not as apparent in the second course. In fact, there is every indication that significant differences no longer exist except when it is obvious that one group is substantially higher when ranked by the place-
ment criterion. That is, whenever a group of students scores in excess of 64 using the formula $y = 2x_4 + x_5$, they will achieve higher than a group of students using a cut-off score of 55.
CHAPTER VII

SUMMARY, CONCLUSIONS, AND INTERPRETATIONS

Recent research investigations have substantiated the claim that students with extended high school calculus experience will achieve significantly higher in college calculus than students without a high school calculus background. While a short unit of calculus appears to have little relationship to achievement at the college level, a semester or more has proven to have a definite relationship.

This research study was designed to analyze differences in achievement in a beginning calculus course of students with varying mathematics preparatory programs. If some differences can be expected, as in the case of a high school calculus background, perhaps other differences could be discovered which might shed some light on the often perplexing problem of selecting an appropriate high school mathematics program for seniors.

The study utilized programs developed for the IBM 7094 and IBM 350 computers available on The Ohio State University campus to derive prediction equations. The objective was to group students as accurately as possible.
according to expectancy of achievement. The major statistical analysis involved an investigation of the variance in mean scores on summary examinations which occurred among the selected condition groups. The differences in test forms and instructor differences were analyzed to determine the effect which they might have had on the statistical results.

Summary

The current study was confined to an appropriate subset of the students enrolled in Math 151 at The Ohio State University for the Autumn Quarter of 1969. This included 567 first-quarter freshmen among the approximately 1100 students enrolled in the course. Also considered, in one phase of the analysis, were 351 non-freshmen. Of these numbers, 41 freshmen and 111 non-freshmen did not complete the course. Achievement was measured by four common examinations which were administered at two week intervals, and a final examination. Success was measured by the course grade received at the end of the quarter.

Condition Groups

The initial phase in the development of the study was to determine accurately, the condition groups, based on high school mathematics preparatory programs. A questionnaire was completed by all students enrolled in the Math 151 beginning calculus course on the first day of
class. This provided information about the high school mathematics courses which the students had taken, and the name and location of the high school they had attended. A second questionnaire was mailed to a sample of the high schools represented by freshmen subjects in the study. This served to classify more accurately some students who were uncertain about the exact nature of their mathematics background, and also served to verify the responses given by other students on the first questionnaire.

The 567 freshmen subjects in the research population were classified into twelve initial categories. They included 83 students with a year of high school calculus, 37 with a semester of high school calculus, and 237 who claimed to have studied "advanced mathematics" with perhaps a short introductory unit of calculus. The majority of these 237 freshmen had studied from five textbooks identified as "modern." The remaining seven groups included 75 students with a semester of high school analytic geometry, 64 with an algebra-trigonometry background, 18 with a semester of solid geometry (with trigonometry), 13 with a semester of probability, 17 with only trigonometry, 15 with a year of advanced algebra, and 8 who had studied linear algebra and algebraic systems.

These twelve groups were then condensed to five condition groups under the headings of: a year of calculus, a semester of calculus, a semester of analytic
geometry, "foundations" background, and non-calculus, non-analytic geometry with algebra and trigonometry the primary contributor to the background. The numbers of students in each category were, respectively, 83, 37, 75, 221, and 122. Combined with these groups, were non-freshmen groups who had studied pre-calculus mathematics at the university level.

A ten-item test of fundamental calculus concepts was given to all students enrolled in Math 151 on the first day of class in October. The results were used as a reliability check on the claim that prior calculus had been studied. Support was given to these claims. The pretest was also to be utilized as an independent variable in a regression analysis to determine if it might have a quality predictive of success in college calculus.

The final phase of work with the condition groups was to partition the groups into "high" and "low" expectancy groups to gain support for the claim of similar estimates of expectancy.

**Regression Analysis**

The hypothesis being tested was that students with one given preparatory program as background would not achieve at a level significantly different from students who studied in an alternate program during the senior year.
in high school. Before this could be investigated, it was necessary to group the subjects of the study according to expectancy of achievement.

Five independent variables were chosen, based on prior research investigations, to be used in a regression analysis designed to calculate formulae for predicting scores on five common examinations, the sum of the examination scores, and the course grade. These variables were the pretest ($x_1$), the Mathematics ACT percentile ($x_2$), high school class rank ($x_3$), the mathematics "D" placement test score ($x_4$), and the total quality points for semesters of high school mathematics ($x_5$). The maximum regression equation predicting the sum of the scores of the five common examinations was considered the most reasonable as a partitioning instrument for the condition groups. The similarity of the derived equation to the formula $y = 2x_4 + x_5$ resulted in the decision to utilize the simpler equation. Investigation revealed that the choice of a $y$-value of 64 most satisfactorily served as a basis for which the "high" achievers were separated from the "low" achievers.

**Analysis of Variance**

Each of the five freshmen condition groups were sub-divided, by expectancy, forming two sets of condition groups. When the "high" and "low" expectancy groups of
non-freshmen were added, two additional sets of condition groups became established. A fifth set consisted of the five classifications without the finer ability subdivisions. Finally, all twelve of the original freshmen groups were considered as a sixth set of groups.

Analysis of variance techniques were employed for each of the six sets mentioned above. The mean scores for six different variables were analyzed. Whenever the F-value was significant, indicating variation, Duncan's Multiple Comparison Test was employed to determine which differences among the condition groups were significant.

Findings and Conclusions
The most efficient prediction equation, in terms of multiple correlation, was the one intended to be utilized in predicting course achievement. The multiple correlation obtained for the maximum regression equation predicting the sum of the examination scores, was .5718. The order in which the five independent variables entered the regression was $x_4$, $x_5$, $x_2$, $x_1$, and $x_3$. When the y-value of 75.83 was used to partition the condition groups, essentially the same divisions occurred as when $y = 2x_4 + x_5$ was used. The multiple R's of the equations for $y_7$, $y_8$, $y_9$, $y_{10}$, $y_{11}$, and $y_0$, the five common examination scores and the course grade, were, respectively, .4619, .5048, .4086, .4068, .4207, and .4968.
The conclusions resulting from the employment of analysis of variance are complex. When the two sets of condition groups (one with "high" and one with "low" expectancy) which included non-freshmen classifications were analyzed, significant F-values were calculated for all six of the variables. Observation of group means clearly showed that the greatest differences occurred between freshmen and non-freshmen groups. Thus, a concentration was focused on the investigation of mean-differences among the freshmen groups. For the "high" expectancy set of condition groups, only the variable \( X_g \), representing the second examination, had an associated F-value that was significant. For the "low" expectancy groups, \( X_g \) and \( x_g \) had significant F-values. When the "high" and "low" groups were considered as one set of condition groups, \( X_7 \), \( X_g \), \( X_9 \), \( x_{10} \), and \( X_0 \) all produced significant F-values. This included all examination scores except the final examination. The initial set of condition groups, consisting of the original twelve classifications, produced significant F-values for variables \( X_g \), \( x_g \), and \( X_0 \).

Applying Duncan's Multiple Comparison Test in each case for which there was a significant F-value, demonstrated that differences which were significant were those between calculus and non-calculus groups. Students with a year or a semester of calculus achieved
significantly higher than students whose background included analytic geometry, algebra-trigonometry, or "foundations" course work. In one instance, for the highest expectancy groups, the C group ("foundations") achieved significantly higher than the D group (algebra-trigonometry). Differences of means for examinations ranged from 2.645 to .060 and from .513 to .009 for the course grade.

An analysis of the rank order for the twelve freshmen groupings revealed several inconsistent or unusual circumstances. For example, those students with a semester of calculus scored higher than those students with a year of calculus, on three of the six variable scores. The small group with linear algebra preparation ranged from fifth to twelfth in the rankings with no noticeable trend in the variation. One trend that was quite noticeable was observed for group 2, associated with a particular text which was the adoption of the Columbus City Schools. The mean for the group was relatively high early in the quarter and on the pretest, but decreased steadily in rank on subsequent examinations. The rank for this group of the final examination and the course grade was twelfth. One consistency in the rank orderings was the continual high ranking of the two calculus groups. Also consistently ranking next in order were groups 1 and 9, associated with a second textbook and with a college
algebra background. They generally ranged from first to fifth. Group 3, formed from the collection of "advanced mathematics" texts programs, consistently scored in the middle ranks, and groups with probability and solid geometry always ranked ninth or lower.

Analysis of the differences of mean scores for the 3:00 P.M. and the 9:00 A.M. sections revealed no significant differences, supporting the claim of nearly similar examinations. When the final examination was analyzed, again no significant difference was calculated. For that examination, all students took the same examination.

Utilizing the maximum prediction equation for the second examination and the course grade, predicted mean scores for each instructor-class were contrasted with the actual mean scores. In two cases, an instructor's actual mean for the second examination was significantly lower than his expected mean. In four cases an instructor's course grade was significantly lower than his predicted mean. Only in the case of three honors sections did a class achieve significantly higher than predicted. Since 37 instructor-sections were involved it was concluded that this was not excessively higher than would be expected by chance and instructor difference was not a major source of error for the study.
Item analyses of each examination revealed sufficiently high discrimination indices to claim test-item validity. The mean item discrimination indices ranged from .430 to .496. For the pretest, the mean discrimination index was .528.

An important factor, the influence of which was felt during this study, pertained to examination content. The first common examination contained only pre-calculus concepts. It was on the second examination that test items involving calculus concepts first were used. Several statistical conclusions were reached regarding this variable. The output data received in the employment of regression analysis included correlations between all pairs of variables entered. Examination Two proved to have relatively high correlations with the sum of the examination scores, the final examination, and the course grade. Since this examination was utilized in the placement of students into honors classes, it was not surprising that the majority of the students so chosen were successful in the calculus course.

The second examination, as a variable, provided significant differences of means among the condition groups for each analysis of variance test employed. The null hypothesis can be rejected when this criterion of achievement is considered. That is, groups with similar estimated expectancy, but with differing high school
background preparations will register significant differences in achievement in a beginning calculus course at The Ohio State University. When the first examination was the criterion, significant differences seldom occurred. In testing pre-calculus concepts, it does not appear to make a difference how or where these concepts were established. Examination Three revealed fewer differences than Test Two and the Fourth Examination revealed significant differences only once. The final examination never did.

Interpretations

The findings of this study substantiated the conclusions expressed by researchers investigating the relationship of high school calculus to achievement in beginning calculus in college. There is higher achievement associated with students who have studied a year or a semester of calculus than with students who have not. There seems to be an indication toward slightly better performances by students who have studied from such texts as Modern Introductory Analysis (14), and Advanced High School Mathematics (45). Students whose high school program was concluded with trigonometry and solid geometry or with trigonometry and probability tended to score slightly lower than students with other preparations, but significant variation was lacking to support such a claim.
Findings from the regression analysis support the conclusion that the two variables with the greatest predictive values for achievement in calculus at The Ohio State University are the mathematics "D" placement test score and the total quality points for semesters of high school mathematics. Regarding the placement formula, the question of cut-off scores can always be reinvestigated and in this study a score of 64 or higher proved quite essential to high achievement in the honors program. This compares with the score of 55 required to be placed into Math 151 as a first quarter freshman.

In the findings it was revealed that the second examination correlated quite well with the final examination and the course grade. Variables $x_4$ and $x_5$ contributed most to the efficiency of the regression equation predicting results on examination two. Thus it might be argued that quality points and the placement test score relate best to achievement on examination Two and examination Two relates best to final success and achievement.

Since the findings also revealed that differences in achievement were noticeable for examination Two, but decreased for examination Three and examination Four and never appeared for the final examination, there is a basis for further interpretation. It is known that students with extended calculus as a high school background
achieve significantly better than those without, early in the quarter, but it appears that these differences tended to disappear as work in the first quarter of calculus progressed. This trend continued in the second quarter of calculus.

Recommendations

The results of the statistical analysis for this study do not clearly indicate that any one or two senior high school mathematics courses will guarantee success in college mathematics. Indeed, there are many variables which play a role in determining whether or not a student succeeds in college. Even when statistics clearly show existing relationships between students with high school calculus and high achievement in college calculus, one cannot say that all high schools should now offer calculus to seniors.

Probably the most significant recommendations which can be made as a result of this study relate to scheduling and counseling procedures. This extends to the selection of honors sections in mathematics. The practice of selecting students for such classes based on achievement on early examinations appears sound. It is recommended that greater emphasis be placed on an observation of the results of the first examination which involves calculus concepts. During the Autumn Quarter of
1969 this was the second of four midterm examinations. Further, the scores derived from the formula $y = 2x_4 + x_5$ should be observed at the time of selection. It is recommended that, unless other favorable factors are prevalent, this score should exceed 63.

Because this study was conducted at a single state supported university which draws a majority of its students from the state of Ohio, further studies performed in other locations and in other types of institutions would provide a comparison which could prove useful to a wider range of students and school personnel. There is a need to observe more closely the concepts which are imbedded in a high school mathematics program. Rather than considering analytic geometry as a unit, a more detailed analysis of the concepts within the course might reveal relationships to success in college calculus.

Locally, there is a continual need to investigate more fully the methods by which honors classes are chosen so as not to do a disservice to students not actually qualified for such intensification. This could be investigated through a procedure similar to the manner in which instructor differences were examined. The phenomena of over-achieving and under-achieving as individuals and as a class reveals more than instructor differences. When high achievement is predicted and significantly higher results actually occur, the factors causing this can be investigated.
Further studies of high school mathematics programs should be performed. Many schools still are groping in the dark for a satisfactory senior course and often look to college personnel for solutions. Several such requests resulted from the high school questionnaire which was utilized in this study. A method which could supply additional information would be to contrast different pre-calculus content in a given high school so that achievement in a subsequent calculus course offered by that high school could be analyzed. This would permit a more accurate enumeration and description of the concepts actually presented prior to the calculus course.
APPENDIXES

A. Student Questionnaire to Obtain High School Mathematics Background
   Letter to Instructors

B. High School Questionnaire to Obtain Additional Information and to Verify Student Responses
   Letter Accompanying High School Questionnaire
   List of High Schools Contacted

C. Pretest
   Five Common Examinations

D. Summaries of Item Analyses

E. Values for Duncan's Multiple Comparison Test
### STUDENT QUESTIONNAIRE

**Math 151**  
**Autumn 1969**

Name: ...........................................  
Expected Major: .................................  
Class:  Fr... So... Jr... Sr...  
Hometown: ........................................

Name of high school:.............................

How many years of mathematics did you have, prior to attending college, beginning with first year algebra?  
List the mathematics courses (and grades received) taken at OSU, or the course credit received if taken at another university:

<table>
<thead>
<tr>
<th>COURSE</th>
<th>GRADE LEVEL TAKEN</th>
<th>Grade Received</th>
</tr>
</thead>
<tbody>
<tr>
<td>First year algebra</td>
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<td></td>
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<tr>
<td>Plane geometry</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Second year algebra</td>
<td></td>
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<td>Advanced algebra</td>
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<td>Integrated algebra-trig.</td>
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<td>Solid geometry</td>
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<tr>
<td>Probability</td>
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<td></td>
</tr>
<tr>
<td>Analytic geometry</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculus (full year)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Calculus with analytic geom.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculus (semester)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculus (less than semester)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>weeks</td>
<td></td>
</tr>
</tbody>
</table>

List your highest level math courses if not mentioned above.  
(e.g. Matrix Algebra, Logic)

|                        |                   |                |

Describe the content of your highest level high school math course if it cannot be done above:

If you studied calculus, did you take the Advanced Placement Examination last Spring?  
if "yes", was it AB.... or BC....

129
September 30, 1969

To all Math 151 Instructors:

Please have these questionnaires completed by all students enrolled on the first day of classes (October 1). Return them to the Mathematics Office (Room 150) immediately after class.

On October 2, have the late entries complete the form. Bring the completed forms to the Office as soon as possible and return all unused copies of the questionnaire.

The second page (calculus questions) is merely for general information and has nothing to do with a grade in the course. Emphasize this. They should answer the questions if they can but many will be unable to. Do not answer questions for them but merely indicate they should interpret as best they can on their own.
Dear Mathematics Chairman:

are former students from your school who are currently enrolled in the first quarter calculus course at The Ohio State University. They are subjects in a research project which requires a careful investigation of their mathematical background. The purpose of the study is to investigate the relation of previous mathematical training to achievement in college mathematics.

Of critical importance is information concerning the name, text, and content of their 12th grade mathematics course. Content is the major item but knowing the text will, in most cases, be sufficient. Your cooperation in supplying this information will be appreciated.

Please complete the enclosed form and return it as soon as possible.

Sincerely,

William Paul

William Paul
The Ohio State University
231 West 18th Ave.
Columbus, Ohio 43210
Please check the following courses which your school offers and list the text books.

<table>
<thead>
<tr>
<th>COURSE</th>
<th>TITLE OF TEXT</th>
<th>AUTHOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advanced Algebra</td>
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</tr>
<tr>
<td>Calculus</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

List your highest level mathematics courses if not mentioned above.

- 
- 
- 

Did the previously named students participate in your most advanced twelfth grade program? Yes... No... *If not, indicate the course which each did have.*

If calculus is offered is it taught for (check one):

- Full year
- Semester
- Less than a semester

Was it taught for Advanced placement? Yes... No...

If taught for less than a semester, what topics were presented?

Name ..................................................

School ..............................................

Location .........................................
High School Questionnaire

Schools Contacted By Mail

* Indicates No Response

<table>
<thead>
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<th>School</th>
<th>Location</th>
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<tbody>
<tr>
<td>Alliance High School</td>
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</table>
Appendix C

Pretest

1. The definition of DERIVATIVE is not usually associated with:
   (a) limit (b) slope (c) rate of change (d) area

2. A function which has a derivative of 1 when x = 2 most resembles:
   (a) \[ \] \[ \] (b) \[ \] \[ \] (c) \[ \] \[ \] (d) \[ \] \[ \]

3. Find the value of \[ \lim_{x \to -1} \frac{x^3 + 1}{x + 1} \]. (a) \( \frac{1}{2} \) (b) 3 (c) 0 (d) 1

4. Given an expression for acceleration in terms of time, finding an expression for velocity in terms of time involves:
   (a) differentiation (b) Newton's Method (c) Mean Value Theorem (d) integration

5. For which of the following will a maximum or minimum point of a graph not occur? Where
   (a) the derivative is 0 (b) the derivative does not exist (c) \( x, y \) is an end point of the graph (d) \( f(x) \) doesn't exist

6. How many functions exist which have a derivative of \( 2x + 3 \)?
   (a) infinitely many (b) 0 (c) 1 (d) 2

7. If \( f \) is a function with second derivative greater than zero when \( x = 2 \), then
   (a) \( f \) is increasing at \( x = 2 \) (b) \( f \) is decreasing at \( x = 2 \) (c) graph of \( f \) is concave up at \( x = 2 \) (d) graph of \( f \) is concave down at \( x = 2 \).

8. If \( f \) is differentiable at \( x \), then
   (a) \( f \) is continuous at \( x \) (b) \( f \) is discontinuous at \( x \) (c) the slope at \( x \) is 0 (d) \( f(x) = 0 \).

9. If \( \lim_{x \to a} f(x) = A \) and \( \lim_{x \to a} g(x) = B \), then \( \lim_{x \to a} f(x) \cdot g(x) \) is
   (a) \( AB \) (b) \( A + B \) (c) \( \frac{A + B}{AB} \) (d) None of these

10. To find the volume of a solid of revolution you most likely would apply:
    (a) integration (b) differentiation (c) both integration and differentiation (d) neither
Fill in your name and instructor and test form (A or B) on your answer sheet. Indicate the one best response to each question on the answer sheet. Response "(e) None" means "None of the above."

1. The graph of \((t, \sin 2t) \mid t \in [0, \frac{\pi}{2}]\) is
   - (a) 
   - (b) 
   - (c) 
   - (d) 
   - (e) None.

2. The set \([t \mid 2 > |3t - 4|]\) can be described using interval notation as:
   - (a) \((2/3, 2)\)
   - (b) \((-\infty, -2/3) \cup (2, \infty)\)
   - (c) \((-2, -2/3)\)
   - (d) \((-\infty, -2) \cup (-2/3, \infty)\)
   - (e) None.

3. The inequality \(|2 - 3x| \geq 7\) is equivalent to:
   - (a) \(3 \leq x \leq -5/3\)
   - (b) \(2/3 \geq x \geq -5/3\)
   - (c) \(-5/3 \leq x \leq 3\)
   - (d) \(x \geq 3\) or \(x \leq -5/3\)
   - (e) None.

4. The set \([x \mid \|x\| = 1]\) can be described using interval notation as:
   - (a) \((-1,1)\)
   - (b) \([-1,0) \cup [1,2)\)
   - (c) \((-2,-1) \cup [1,2)\)
   - (d) \([1,2)\)
   - (e) None.
5. The sketch represents the graph of:

(a) $2 > 3 - y$  (b) $|y-1| > 0$  (c) $y-x+1 > 0$  (d) $2-x > 3+2x$  (e) None.

6. The number $\sin \frac{4\pi}{3}$ is equal to the number

(a) $\cos \frac{2\pi}{3}$  (b) $\sin \frac{\pi}{3}$  (c) $\cos \left(-\frac{7\pi}{6}\right)$  (d) $-\tan^2 \frac{\pi}{4}$  (e) None.

7. The sketch represents the graph of

(a) $xy = 1$  (b) $(x-|x|)(y-|y|) = 0$  (c) $(|x|-1)(|y|-1) = 1$
(d) $(x^2-1)(y^2-1) = 0$  (e) None.

8. The point $1/4$ of the way along the line segment from $(2,-3)$ to $(10,1)$ is:

(a) $(8,0)$  (b) $(4,-2)$  (c) $(6,-1)$  (d) $(-6,1)$  (e) None.

9. The distance between the points $(a+b, b)$ and $(b, a-b)$ is

(a) $\sqrt{a^2 + b^2}$  (b) $|a-b|$  (c) $\sqrt{2a^2 - 4ab + 4b^2}$
(d) $\sqrt{2a^2 - 4b^2}$  (e) None.

10. Which of the following is true?

(a) $\cos 2 > \sin 3$  (b) $\sin (-3) > \cos 3$  (c) $\tan 3 > \cos 6$
(d) $\tan \pi = 1$  (e) None.

11. The set $\{x \mid \frac{1}{x} - 3 < 4\}$ can be described using interval notation as:

(a) $(-1, \frac{1}{7})$  (b) $(-\infty,-1) \cup \left(\frac{1}{7},\infty\right)$  (c) $(-3,4)$
(d) $(-\infty,-1) \cup (0, \frac{1}{7})$  (e) None.
12. The equation that represents the set of all points more than two units from the origin is:
   (a) $\frac{x^2}{4} + \frac{y^2}{4} > 1$  (b) $|x| + |y| > 2$  (c) $(x - y)^2 > 4$
   (d) $x^2 + y^2 = 2$  (e) None.

13. A function $f$ is called an even function if, for each $x$ in its domain, $f(-x) = f(x)$. In which of the following cases is $f$ an even function:
   (a) $f(x) = \sin x$  (b) $f(x) = -\sin x$  (c) $f(x) = \cos x - x$
   (d) $f(x) = x \cos x$  (e) None.

14. $\tan x + \cot x$ is equivalent to
   (a) $\sec x \csc x$  (b) $\sec^2 x + 1$  (c) $\sec^2 x - 1$
   (d) $\sec x$  (e) None.

15. The understood domain of the function defined by $f(x) = \sqrt{\frac{x-1}{x}}$ is
   (a) $(-\infty, 0)$  (b) $(1,\infty)$  (c) $\{x \mid x \neq 0\}$  (d) $[1,\infty)$  (e) None.

16. Which of the following describes a function?
   (a) $\{ (x,y) \mid \frac{2-3x^2}{y^2 - 4} = 1\}$  (b) $\{ (x,y) \mid x + x^2 y^2 = 1\}$
   (c) $\{ (x,y) \mid \sin y = \cos x\}$  (d) $\{ (x,y) \mid |x| = |y| - 1\}$  (e) None.

17. If $f(x) = x^2 - 3$ and $g(x) = 5 - 2x$ then $f(g(3))$ equals
   (a) -7  (b) -6  (c) -4  (d) -2  (e) None.

18. If $f(x) = (x+1)^2 + \sqrt{x}$ then $f(x^2 - 1)$ equals
   (a) $x^2 + \sqrt{x}$  (b) $x^4 + \sqrt{x^2 - 1}$  (c) $(x^2 - 1)^2 + |x^2 - 1|
   (d) $x^2 + |x^2 - 1|$  (e) None.

19. For $f(x) = x^2$, $A = [0,4]$ and $B = [-2,1]$, which of the following is true
   (a) $f(A \cup B) \subseteq A \cup B$  (b) $f(B) \subseteq A$  (c) $f(A \cap B) = A \cup B$
   (d) $f(A) \subseteq B$  (e) None.
Fill in your name and instructor on your answer sheet; do not detach. Indicate the one best response to each question on the answer sheet. The response "None" means "None of the above".

1. The sketch of \( 2 + y > 3 \) is represented by

   (a) \[
   \begin{array}{c}
   \hline
   \end{array}
   \]

   (b) \[
   \begin{array}{c}
   \hline
   \end{array}
   \]

   (c) \[
   \begin{array}{c}
   \hline
   \end{array}
   \]

   (d) \[
   \begin{array}{c}
   \hline
   \end{array}
   \]

   (e) None.

2. The inequality \( |2x - 3| > 7 \) is equivalent to:

   (a) \(-2 \leq x \leq 5\)  
   (b) \(5 \leq x \leq -2\)  
   (c) \(x \geq 5\) or \(x \leq -2\)  
   (d) \(3/2 \leq x \leq 5\)  
   (e) None.

3. The set \( \{ x | |x| = 0 \} \) can be described using interval notation as:

   (a) \((-\infty,-1) \cup (1,\infty)\)  
   (b) \((-1,1)\)  
   (c) \([0,1]\)  
   (d) \((-1,0]\)  
   (e) None.

4. The set \( \{ t | -2 > |3t - 4| \} \) can be described using interval notation as:

   (a) \((2/3, 2)\)  
   (b) \((-\infty, -2/3) \cup (2,\infty)\)  
   (c) \((-2, -2/3)\)  
   (d) \((-2,3)\)  
   (e) None.
5. The graph represents

- \( \frac{\pi}{2} \)

(a) \( \{(x, y) \mid x = \sin y\} \)
(b) \( \{(t, \sin t) \mid t \in [0, \pi]\} \)
(c) \( \{(t, \sin t) \mid t \in [0, \pi/2]\} \)
(d) \( \{(t, |\sin 2t|) \mid t \in [0, \pi/2]\} \)
(e) None.

6. The number \( \cos \frac{2\pi}{3} \) is equal to the number

(a) \( \cos \frac{\pi}{3} \)
(b) \( \sin \frac{4\pi}{3} \)
(c) \( -\tan^2 \left(\frac{\pi}{4}\right) \)
(d) \( -\sin^2 \left(\frac{\pi}{4}\right) \)
(e) None.

7. The sketch represents the graph of:

(a) \( xy = 1 \)
(b) \( (x - |x|)(y - |y|) = 0 \)
(c) \( (|x| - 1)(|y| - 1) = 0 \)
(d) \( (x^2 - 1)(y^2 - 1) = 1 \)
(e) None.

8. The point \( \frac{1}{4} \) of the way along the line segment from \( (10, 1) \) to \( (2, -3) \) is

(a) \( (8, 0) \)
(b) \( (4, -2) \)
(c) \( (6, -1) \)
(d) \( (-6, 1) \)
(e) None.

9. The distance between the points \( (u, u + v) \) and \( (v, u - v) \) is

(a) \( u + v \)
(b) \( \sqrt{3u^2 + 3v^2} \)
(c) \( \sqrt{u^2 - 2uv + 5v^2} \)
(d) \( |u| \)
(e) None.

10. The set \( \{x \mid |2 - \frac{1}{x}| < 3\} \) can be described using interval notation as:

(a) \( (-2, 3) \)
(b) \( (-\infty, -1) \cup (1/5, \infty) \)
(c) \( (-1, 1/5) \)
(d) \( (-\infty, -2) \cup (3, \infty) \)
(e) None.

11. Which of the following is true:

(a) \( ||\sin 3|| > 1 \)
(b) \( \cos 0 < \sin 3 \)
(c) \( \sin 3 > \cos 4 \)
(d) \( \tan 3 > \sin 1 \)
(e) None.
12. Which of the following describes a function
(a) \[(x,y) \mid y^2 + x^2 = 4\]  
(b) \[(x,y) \mid y^4 = 2 + 3x^2\]  
(c) \[(x,y) \mid \cos y = \sin x\]  
(d) \[(x,y) \mid |x| = |y|\]  
(e) None.

13. A function \( f \) is called an odd function if, for each \( x \) in its domain, \( f(-x) = -f(x) \). In which of the following cases is \( f \) an odd function
(a) \( f(x) = \cos x \)  
(b) \( f(x) = -\sin x \)  
(c) \( f(x) = x\sin x \)  
(d) \( f(x) = x - \sin x^2 \)  
(e) None.

14. \( \sec^2 x + \csc^2 x \) is equivalent to
(a) \( \sec^2 x \csc^2 x \)  
(b) \( \cos^2 x + \cot^2 x \)  
(c) \( \sin x \tan x \)  
(d) \( \sin^2 x + \tan^2 x \)  
(e) None.

15. If \( f(x) = x^2 + 3 \) and \( g(x) = 2x - 5 \) then \( f(g(3)) \) equals
(a) 0  
(b) 4  
(c) 12  
(d) 19  
(e) None.

16. The "understood" domain of the function defined by \( f(x) = \sqrt{3x - 9} \) is the interval
(a) \((-\infty, -3]\)  
(b) \([3, \infty)\)  
(c) \([\sqrt{3}, 3]\)  
(d) \((3, \infty)\)  
(e) None.

17. If \( f(x) = (x-1)^2 + \sqrt{x} \), then \( f((x+1)^2) \) equals
(a) \( x^4 + 4x^2 + x + 1 \)  
(b) \( x^2(x+2)^2 + |x+1| \)  
(c) \( x^2 + |x + 1| \)  
(d) \( x^2 + 2x + 3 \)  
(e) None.

18. The equation that represents the set of points 2 units from the point \((-1, 3)\) is
(a) \( x^2 + 2x + y^2 - 6y + 6 = 0 \)  
(b) \( (x-1)^2 + (y+3)^2 = 2 \)  
(c) \( x^2 - 2x + y^2 + 6y + 6 = 0 \)  
(d) \( (x-1)^2 + (y+3)^2 = 4 \)  
(e) None.

19. For \( f(x) = x^2 \), \( A = [-2, 1] \), and \( B = [0, 4] \), which of the following is false:
(a) \( f(A \cap B) = A \cap B \)  
(b) \( f(A \cup B) = B \)  
(c) \( B \subseteq f(B) \)  
(d) \( f(A) \subseteq B \)  
(e) None.

20. If \( f(x) = \frac{-2}{x} \), and \( A = [-4, 0) \cup (2, 3) \), then \( f(A) \) equals
(a) \((0, 8) \cup (-4, -6)\)  
(b) \([-2, -2/3) \cup (0, 2]\)  
(c) \([2, 0) \cup (-1, -3/2)\)  
(d) \((2/3, 2) \cup [-2, 0]\)  
(e) None.
Write your name above; do not detach. Block in your name, section number, and form (A or B) on the answer sheet. Indicate the one best response to each question on the answer sheet. Your score is the number correct. "None" means "None of the above."

1. The product of the slope and \( Y \)-intercept of the line whose equation is \( 4x - 2y + 5 = 0 \) is
   (a) \( \frac{2}{5} \)  (b) \( \frac{5}{4} \)  (c) \( \frac{5}{2} \)  (d) \( 5 \)  (e) None.

2. The equation of the line through \((a,b)\) which is perpendicular to the line \( Ax + By + C = 0 \) is
   (a) \( Ay - Bx + Bb - Aa = 0 \)  (b) \( Ay - Bx + Ba - Ab = 0 \)
   (c) \( By - Ax - Bb + Aa = 0 \)  (d) \( By - Ax - Ba + Ab = 0 \)  (e) None.

3. The line graph has for its equation
   (a) \( 2y - x = 1 \)  (b) \( 2y + x = 1 \)  (c) \( 2y - x = 2 \)  (d) \( 2y + x = 2 \)  (e) None.

4. Suppose \( f \) and \( g \) are linear functions. Which of the following formulas for \( h(x) \) does not define a linear function \( h \)?
   (a) \( f(x) + g(x) \)  (b) \( 3f(x) + 2g(x) \)  (c) \( 3f(x) + 2 \)  (d) \( f(g(x)) \)
   (e) None.

5. \( \lim_{x \to 1} \frac{1 - \sqrt{x}}{1 - x} \) is
   (a) 0  (b) \( \frac{1}{2} \)  (c) 1  (d) does not exist  (e) None.

6. \( \lim_{x \to 0} \frac{(3+x)^{-1} - 3^{-1}}{x} \) is
   (a) -1  (b) 0  (c) 1  (d) does not exist  (e) None.
Questions 7 through 10 involve the function \( F \) defined as follows:

\[
F(x) = \begin{cases} 
1 & \text{if } x \leq -2 \\
x + 2 & \text{if } -2 < x \leq -1 \\
|x| & \text{if } -1 < x 
\end{cases}
\]

7. The function \( F \) is discontinuous for the following values of \( x \):
(a) -2 (b) -2 and -1 (c) -1 and 0 (d) -2, -1, and 0 (e) None.

8. \( \lim_{x \to -1} F(x) \) is (a) -1 (b) 0 (c) 1 (d) does not exist (e) None.

9. The values of \( x \) for which \( F'(x) \) does not exist are
(a) -2 (b) -2 and -1 (c) -1 and 0 (d) -2, -1, and 0 (e) None.

10. \( \{x \mid F'(x) = 1\} \) is
(a) \((-\infty, -2)\) (b) \((-2, -1)\) (c) \((0, \infty)\) (d) \((-2, -1) \cup (0, \infty)\) (e) None.

11. The derivative of \( \sqrt[3]{x} \) equals (a) \( \sqrt[3]{x^2} \) (b) \( \frac{\sqrt{x}}{4x} \) (c) 1 (d) \( \frac{1}{\sqrt{x}} \) (e) None.

12. The equation of the tangent line to \( y = \frac{3}{\sqrt{x}} \) at the point where \( x = 8 \) is
(a) \( 12y - x = 16 \) (b) \( 12y + x = 32 \) (c) \( 3x + 4y = 32 \) (d) \( 3y + 4x = 16 \) (e) None.

13. The equation of the normal line (perpendicular to the tangent line) to \( y = x^3 \) at the point where \( x = 2 \) is
(a) \( 12x - y = 16 \) (b) \( 12x + y = 32 \) (c) \( 12y - x = 94 \) (d) \( 12y + x = 98 \) (e) None.

14. Let \( f(x) = \sqrt{2x + 1} \). Then using some algebra we find that
\[
\frac{f(x+h) - f(x)}{h} = 2(\sqrt{2x + 2h + 1} + \sqrt{2x + 1})^{-1}.
\] Thus \( f'(x) \) equals
(a) 1/6 (b) 1/3 (c) 6 (d) 12 (e) None.
15. A particle moves along an s-scale according to the equation \( s = t^2 - \pi t \). The velocity of the particle at \( t=1 \) is (a) \( 1-\pi \) (b) \( 1 \) (c) \( 2-\pi \) (d) 0 (e) None.

16. Let \( f(x) = |x| + \frac{|x|}{x} \). Which of the following statements (a, b, c, or d) is false? (a) \( \lim_{x \to 0} f(x) = 1 \) (b) \( \lim_{x \to 0} f(x) = -1 \) (c) \( \lim_{x \to 1} f(x) = 2 \) (d) \( \lim_{x \to 0} f(x) \) does not exist (e) None of the above are false.

17. Let \( g(x) = \sqrt{x} \). The definition of the derivative can be used to find that \( g'(-1) \) equals (a) -1 (b) 0 (c) 1/2 (d) 1 (e) None.

18. Which of the following graphs best represents the function \( G \) such that \( G(2) = G'(1) = 0 \), \( G(0) = G'(0) \), and \( G'(2) < G'(3) \). (a) (b) (c) (d) (e).

19. If \( f(x) = x^3 \) and \( g(y) = \sqrt{y} \), then \( f'(2) + \frac{1}{g'(4)} \) equals (a) 6 (b) 14 (c) 16 (d) 52 (e) None.

20. A cube has volume \( V \). Let \( x \) denote the length of an edge and \( A \) the area of one face - then the rate of change of volume with respect to edge length is (a) \( 3x \) (b) \( 3A \) (c) \( 3xA \) (d) \( 3A^2 \) (e) None.
Write your name above; do not detach. Block in your name, section number, and form (A or B) on the answer sheet. Indicate the one best response to each question on the answer sheet. Your score is the number correct. "None" means "None of the above."

1. The equation of the line containing the point \((a, b)\) and parallel to the line \(Ax + By + C = 0\) is

   (a) \(Ay - Bx + Aa - Bb = 0\)  
   (b) \(Ay + Bx - Ab - Ba = 0\)  
   (c) \(Ax - By - Aa + Bb = 0\)  
   (d) \(Ax + By - Bb - Aa = 0\)  
   (e) None.

2. Let \(f\) and \(g\) be linear functions. Which of the following formulas for \(h(x)\) does not define a linear function \(h\) ?

   (a) \(2f(x)\)  
   (b) \(3f(x) + 2g(x)\)  
   (c) \(\frac{f(x)}{g(x)}\)  
   (d) \(f(g(x))\)  
   (e) None.

3. The product of the slope and \(Y\)-intercept of the line whose equation is \(4x + 2y - 3 = 0\) is

   (a) \(-\frac{3}{2}\)  
   (b) \(-3\)  
   (c) \(-5\)  
   (d) \(-12\)  
   (e) None

4. The line graph has for its equation

   (a) \(2x + y = 2\)  
   (b) \(2y + x = 2\)  
   (c) \(2x + y = 1\)  
   (d) \(2x - y = 1\)  
   (e) None.

5. The equation of the tangent line to \(y = x^3\) at the point at which \(x = 2\) is

   (a) \(x + 12y = 16\)  
   (b) \(12x - y = 16\)  
   (c) \(x + 12y = 32\)  
   (d) \(12x - y = 32\)  
   (e) None.

6. The equation of the normal line (perpendicular to tangent line) to \(y = \frac{4}{\sqrt{x}}\) at the point \((16, 2)\) is

   (a) \(x - 32y + 51\frac{1}{2} = 0\)  
   (b) \(32x - y + 51\frac{1}{2} = 0\)  
   (c) \(y + 32x = 51\frac{1}{2}\)  
   (d) \(y - 32x = 510\)  
   (e) None.
Questions 7, 8, 9, 10 involve the function $F$ described below:

$$F(x) = \begin{cases} 
1 & x < -1 \\
|x| & -1 \leq x \leq 1 \\
x - 1 & 1 < x 
\end{cases}$$

7. The function $F$ is discontinuous for the following values of $x$:
   (a) -1, 0, and 1  (b) -1 and 1  (c) 0  (d) -1  (e) None

8. $\lim_{x \to 1} F(x)$ is (a) 1  (b) 0  (c) -1  (d) does not exist  (e) None.

9. The values of $x$ for which $F'(x)$ does not exist are
   (a) -1, 0, and 1  (b) -1 and 1  (c) 0  (d) -1  (e) None.

10. Using interval notation, the set of numbers such that $F'(x) = 1$ is
    (a) $(-\infty, -1) \cup (1, \infty)$  (b) (0, 1)  (c) (0, 1) $\cup (1, \infty)$  (d) $(-\infty, -1) \cup (0, \infty)$
        (e) None.

11. $\lim_{x \to 0} \frac{(3x)^{-1} - 3^{-1}}{x}$ is (a) -1  (b) 0  (c) 1  (d) does not exist  (e) None.

12. Let $f(x) = \sqrt{2x + 1}$. Then using some algebra we find that
    $$\frac{f(x+h) - f(x)}{h} = 2(\sqrt{2x + 2h + 1} + \sqrt{2x + 1})^{-1}.$$ Thus it follows
    that $f'(4)$ equals.
    (a) 1/6  (b) 1/3  (c) 6  (d) 12  (e) None.

13. $\lim_{x \to 0} \frac{1 - \sqrt{x+1}}{x}$ is (a) 0  (b) 1/2  (c) 1  (d) does not exist  (e) None.

14. The derivative of $\frac{3}{\sqrt[3]{x}}$ is equal to (a) $\frac{3\sqrt[3]{x}}{6x}$  (b) $\frac{1}{5} x^{-4/5}$
    (c) 1  (d) $\frac{1}{3\sqrt[3]{x}}$  (e) None.
15. If \( g(x) = |x|^{-1} \), the definition of the derivative can be used to find that 
\( g'(-1) \) is 
(a) -1  (b) 0  (c) 1/2  (d) 1  (e) None.

16. Let \( f(x) = |x| + \frac{|x|}{x} \). Which of the following statements (a) - (d) is false?
(a) \( \lim_{x \to 0} f(x) = 1 \)  (b) \( \lim_{x \to 0} f(x) = -1 \)  (c) \( \lim_{x \to 0} f(x) \) does not exist  
(d) \( \lim_{x \to 1} f(x) = 2 \)  (e) None of the above is false.

17. Which of the following graphs best represents the function \( G \) if:
\( G(2) = G'(1) = 0, \ G(0) = G'(0), \) and \( G'(2) < G'(3) \)
(a)  
(b)  
(c)  
(d)  
(e)  

18. A particle moves along the s-axis according to the equation 
\( s = t^3 - t^2 \). The velocity of the particle at \( t = 1 \) is 
(a) \( 3\pi^2 - 2 \)  (b) \( \pi^3 - 1 \)  (c) -1  (d) \( \frac{1}{2}(\pi^3 - 1) \)  (e) None.

19. A sphere of radius \( r \) has volume \( V = \frac{4}{3}\pi r^3 \) and surface area \( A = 4\pi r^2 \). 
The rate of change of volume with respect to radius is 
(a) \( 3A \)  (b) \( \frac{V}{r} \)  (c) \( 8\pi r \)  (d) \( \frac{3V}{r} \)  (e) None.

20. If \( f(x) = \sqrt[3]{x} \) and \( g(y) = y^2 \), then \( \frac{1}{f'(8)} + g'(3) \) equals 
(a) \( 3 \frac{1}{12} \)  (b) \( 3\frac{1}{2} \)  (c) \( 9\frac{1}{2} \)  (d) \( 18 \)  (e) None.
Write your name above; black in your name, section number, and test form (A or B) on your answer sheet. Indicate the one best response to each question on your answer sheet.

Your score is \( A - \frac{1}{4} B \) where \( A \) = number correct responses, \( B \) = number of incorrect responses and \( C \) = number unanswered.

1. Let \( g(t) = \frac{t+1}{t-1} \). Then \( g'(3) \) equals
   (a) -1/16  (b) -1/8  (c) -1/4  (d) -1/2  (e) none

2. If \( y = \sqrt{1 - 2x} \), then \( y' \) equals
   (a) -1/2\sqrt{1-2x}  (b) 1/2\sqrt{1-2x}  (c) -1/\sqrt{1-2x}  (d) 1/\sqrt{1-2x}  (e) none

3. Let \( f(u) = (1 - \sqrt{u})^2 \). Then \( f'(\frac{1}{4}) \) equals
   (a) -2  (b) -1  (c) 1  (d) 2  (e) none

4. If \( f(x) = \cot \pi x \), then \( f'(1/2) \) equals
   (a) -\pi  (b) -1  (c) 1  (d) \pi  (e) none

5. The best approximation to the slope of the tangent line to the cosine curve at the point \((1, \cos 1)\) is
   (a) -1  (b) -.8  (c) -.5  (d) -.1  (e) .1

6. If \( u = g(x) \), then \( D_x \log g(x) \) equals
   (a) \( D_u \log g(u) D_x u \)  (b) \( D_u \log u D_x u \)  (c) \( D_x \log x D_u g(u) \)
   (d) \( D_x \log u D_u x \)  (e) none
7. If \( y = \sqrt{2x - x^2} \), then \( y' \) equals
   
   (a) 1-x  \hspace{1cm} (b) 1/2  \hspace{1cm} (c) 1  \hspace{1cm} (d) -x  \hspace{1cm} (e) none

8. If \( y = \sin^2 x \) then \( y' \) equals
   
   (a) \cos 2x \hspace{1cm} (b) \sin 2x \hspace{1cm} (c) 2 \sin x \hspace{1cm} (d) 2x \sin x \hspace{1cm} (e) none

9. If \( G(t) = \cos (\pi/t) \), then \( G'(2) \) equals
   
   (a) 1/4 \hspace{1cm} (b) -1/4 \hspace{1cm} (c) \pi/4 \hspace{1cm} (d) -\pi/4 \hspace{1cm} (e) none

10. If \( y = x^2/(1+x^3) \), then \( y' \) equals
    
    (a) \( x(2+5x^3)/(1+x^3)^2 \) \hspace{1cm} (b) \( x(2+5x^3)/(1+x^6) \) \hspace{1cm} (c) \( x(2-x^3)/(1+x^6) \) \hspace{1cm} (d) \( x(2-3x^3)/(1+x^3)^2 \) \hspace{1cm} (e) none

11. If \( g(t) = (1+t^2)^5 \), then \( g''(0) \) equals
    
    (a) 0 \hspace{1cm} (b) 5 \hspace{1cm} (c) 10 \hspace{1cm} (d) 20 \hspace{1cm} (e) 40

12. If \( y = x^3(1+x)^4 \), then \( y' \) equals
    
    (a) \( x^2(1+x)^3(7x+3) \) \hspace{1cm} (b) \( x^2(3+7x)^4 \) \hspace{1cm} (c) \( 12x^2(1+x)^3 \) \hspace{1cm} (d) \( x^2(1+x)^3(3-x) \) \hspace{1cm} (e) none

13. Let \( y = \cos 3x \). Then \( y'' + Ay' + By'' = 0 \) if
    
    (a) \( A=3 \) and \( B=9 \) \hspace{1cm} (b) \( A=9 \) and \( B=3 \) \hspace{1cm} (c) \( A=0 \) and \( B=9 \) \hspace{1cm} (d) \( A=0 \) and \( B=3 \) \hspace{1cm} (e) none

14. Let \( f(x) = \tan x \). There is a number \( m \) in the open interval \((0, \pi/4)\) such that \( f'(m) \) is equal to
    
    (a) \( \pi/4 \) \hspace{1cm} (b) \( 4/\pi \) \hspace{1cm} (c) 1 \hspace{1cm} (d) 2 \hspace{1cm} (e) none
15. \[ \lim_{{h \to 0}} \frac{\sin h^2}{h} \] is
(a) 0  (b) 1  (c) 2  (d) 1/2  (e) none

16. Given: \( g \) is continuous for all \( x \) and \( g(x) = \frac{\sin x}{x} \) if \( x \neq 0 \).
Then \( g([-\pi, \pi]) \) is the interval
(a) \([0,0]\)  (b) \([0,1]\)  (c) \([0,1]\)  (d) \((0,1]\)  (e) \((0,1]\)

17. The graph of \( f \) consists of straight line segments joining (consecutively)
the points \((\pm 2,1)\) to \((-1,0)\) to \((1,0)\) to \((2,1)\). Which of the following statements concerning the hypothesis and the conclusion of Rolle's Theorem for the interval \([-2,2]\) is true?
(a) Hypothesis true, and conclusion true.
(b) Hypothesis false, and conclusion false.
(c) Hypothesis false, but conclusion true.
(d) Hypothesis true, but conclusion false.
(e) None of the preceding is true.

18. Let \( f(x) = 0 \) if \( x < 0 \), and \( f(x) = x^2 \) if \( x \geq 0 \). Which of the following statements concerning the hypothesis and conclusion of the Mean Value Theorem (Theorem of the Mean) for the interval \([-1,1]\) is true?
(a) Hypothesis true, but conclusion false.
(b) Hypothesis false, but conclusion true.
(c) Hypothesis false, and conclusion false.
(d) Hypothesis true, and conclusion true.
(e) None of the preceding is true.

19. Let \( F(x) = \lfloor x \rfloor x^2 \). Which one of the following statements is true?
(a) \( F''(0) = 0 \)  (b) \( \lim_{{x \to 0}} F''(x) = 0 \)  (c) \( F'''(0) = 0 \)
(d) \( \lim_{{x \to 0}} F'''(x) = 0 \)  (e) none of the preceding statements is true

20. Let \( F(x) = \lfloor x \rfloor x^2 \). Which one of the following statements is false?
(a) \( F([-1,1]) \subseteq [-1,1] \)  (b) \( F \) is continuous in \((-1,1)\)
(c) \( F \) is differentiable in \((-1,1)\)  (d) \( F' \) is continuous in \((-1,1)\)
(e) \( F' \) is differentiable in \((-1,1)\)
In the first four questions $F(x) = 2x^3 - 3x^2 + 2$.

1. A (relative) minimum point on the graph of $F$ is
   (a) $(1,1)$  (b) $(1,2)$  (c) $(0,2)$  (d) $(0,0)$  (e) none

2. A (relative) maximum point on the graph of $F$ is
   (a) $(1,1)$  (b) $(1,2)$  (c) $(0,2)$  (d) $(0,0)$  (e) none

3. $F$ is increasing for numbers in the set
   (a) $(0,1)$  (b) $(0,\infty)$  (c) $(-\infty,0) \cup (1,\infty)$  (d) $(1/2,\infty)$  (e) $(\infty,\infty)$

4. The graph of $F$ is concave up for $x$ in the interval
   (a) $(-\infty,1/2)$  (b) $(0,1/2)$  (c) $(1/2,\infty)$  (d) $(0,1)$  (e) $(2,\infty)$
5. A part of the graph of $y = x - \frac{1}{x}$ is

(a) | (b) | (c) | (d) | (e) |
---|---|---|---|---|

Questions 6 and 7 are about a body moving along a number scale. Its displacement $s$ from the origin at time $t$ is given by the equation $s = \cos \left(\frac{\pi t}{2}\right)$.

6. In which of the following time intervals is the displacement negative and the velocity positive?
   (a) (0,1) (b) (1,2) (c) (2,3) (d) (3,4) (e) none

7. In which of the following time intervals is the velocity positive and the acceleration negative?
   (a) (0,1) (b) (1,2) (c) (2,3) (d) (3,4) (e) none
8. The graph of $G$ is shown at the right. In which interval is $f'(x) < 0$ and $f''(x) < 0$?

(a) (0,1) (b) (0,2) (c) (1,2) (d) (1,3) (e) (2,3)

9. A part of the graph of the equation $y = x \sqrt{4 - x^2}$ is:

(a) (b) (c) (d) (e)

10. A triangle with two equal sides of length 2 units is drawn so as to make the area a maximum. That maximum area is

(a) 1  (b) $\sqrt{2}$  (c) 3  (d) 2  (e) $2\sqrt{2}$

11. Repeat your answer to question 10.
In questions 12 and 13 use the approximation for \( f(x + h) \) in terms of \( f(x) \) and \( f'(x) \).

12. Which of the following numbers is the best approximation to \( \sqrt[3]{9} \)?
   (a) \( 9/4 \)  (b) \( 13/6 \)  (c) \( 25/12 \)  (d) \( 33/16 \)  (e) \( 49/24 \)

13. Which of the following numbers is the best approximation to \( \cos 6° \)?
   (a) \( .7 \)  (b) \( .8 \)  (c) \( .9 \)  (d) \( 1 \)  (e) \( 1.1 \)

14. Using a single application of Newton's Method and an initial guess of zero, an approximation to a solution of the equation \( 2x^3 + 2x - 1 = 0 \) is
   (a) \( .4 \)  (b) \( -.4 \)  (c) \( .5 \)  (d) \( -.5 \)  (e) \( 2 \)

15. A 13 foot ladder leans against the wall of a house. Someone pulls the base of the ladder away from the house at the rate of 2 feet/sec. The speed of the top of the ladder sliding down the wall when the top is 12 feet from the ground is how many ft/sec?
   (a) \( 24/5 \)  (b) \( 5/6 \)  (c) \( 5/12 \)  (d) \( 1 \)  (e) \( 12/5 \)

16. Which of the following numbers is the \( x \)-coordinate of a maximum point on the graph of \( y = x - \cos x \)?
   (a) \( \pi/2 \)  (b) \( \pi \)  (c) \( 3\pi/2 \)  (d) \( 2\pi \)  (e) none

17. An open trough is to be made in the shape of a half-cylinder whose vertical ends are halves of circular disks. The volume of the trough is to be \( V \) units. Let \( R \) be the radius of the ends, and \( L \) the length of the trough. The expression to be minimized in order to find the radius of the trough of least surface material is
   (a) \( RL + \pi R^2 \)  (b) \( 2\pi R + LR^2 \)  (c) \( \pi R^2 + \frac{2V}{R} \)  (d) \( \pi R^2 + \frac{V}{R} \)
   (e) \( 2\pi R^2 + \frac{V}{R} \)
18. A box with a square base and no top is to be constructed to hold a fixed volume $V$. The material to be used to construct the bottom of the box costs twice as much as the material to be used to make the sides. The cost of the box is least if the length of a side of the base equals
(a) one fourth the height  (b) one half the height  (c) the height  
(d) twice the height  (e) none

19. Repeat your answer to question 18.

20. Suppose $y = x^2(1 - 2x)$, and that $x$ increases at a constant rate of 2 units per second. Which of the following statements is true?
(a) $y$ never decreases  (b) $y$ always decreases  (c) if $x > 0$, 
then $y$ decreases  (d) if $x < 1/2$, then $y$ increases 
(e) if $x > 1/3$, then $y$ decreases
Write your name and instructor's name above. Black in your name, section number, and form B on your answer sheet. Indicate the one best response to each question. Your score is the number correct.

1. The "understood" domain of the function defined by \( f(x) = \sqrt{\frac{x+1}{x^3}} \) is
   (a) \((0,\infty)\)
   (b) \((-\infty,-1] \cup (0,\infty)\)
   (c) \((-\infty,0) \cup [1,\infty)\)
   (d) \([1,\infty)\)
   (e) \((-\infty,0)\)

2. The figure at right shows part of
   the graph of
   (a) \(y = \cos x\)
   (b) \(y = \sin x\)
   (c) \(y = \sin^2 x\)
   (d) \(y = 1 - \cos x\)
   (e) \(y = \cos x\)

3. Which of the following describes a function?
   (a) \(\{(x,y)|x^2 = y^2\}\)
   (b) \(\{(x,y)|xy = 1\}\)
   (c) \(\{(x,y) \mid \sin y = \cos x\}\)
   (d) \(\{(x,y)|x^2 = |y|\}\)
   (e) none

4. Suppose the graph of \(f\) is the parabola \(y = 1 - x^2\). If \(A = [-1,1]\) and \(B = [0,1]\), which of the following is true?
   (a) \(A \subseteq B\)
   (b) \(f(B) \subseteq A\)
   (c) \(A \subseteq f(B)\)
   (d) \(A \cup B \subseteq f(B)\)
   (e) none

5. The equation of the circle with radius 2 and center \((-1,3)\) is
   (a) \(x^2 + y^2 = 2\)
   (b) \((x-1)^2 + (y+3)^2 = 2\)
   (c) \((x-1)^2 + (y+3)^2 = 4\)
   (d) \(x^2 + 2x + y^2 - 6y + 6 = 0\)
   (e) \(x^2 - 2x + y^2 + 6y + 10 = 0\)
6. \( \lim_{x \to -1} \frac{|x| - 1}{1 + x} \) is (a) 1 (b) 0 (c) -1 (d) does not exist (e) none

7. Suppose that \( \frac{f(x+h) - f(x)}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h} \). Then \( f'(x) \) equals:
   (a) 0 (b) 2 (c) 1/2 (d) 1/4 (e) none

8. A particle moves along an s-scale according to the equation \( s = \sin 2\pi t^2 \).
   The velocity of the particle at \( t=2 \) is
   (a) 2\pi (b) 4 (c) 8\pi (d) 0 (e) none

9. The equation of the tangent line to the parabola \( y = x^2 - x \) at the point \( (1,0) \) is
   (a) \( y-x+1=0 \) (b) \( x+y+1=0 \) (c) \( x-y+1=0 \)
   (d) \( x+y-1=0 \) (e) none

10. The slope of the normal line to the curve \( y = \sin 2x - \cos x \) at the point \( (\pi,1) \) is
    (a) -1 (b) 1 (c) -1/2 (d) 1/2 (e) none

Questions 11 through 15 involve the function \( F \) defined as follows:

\[
F(x) = \begin{cases} 
|x| & \text{if } x \geq 0 \\
|x+1| & \text{if } x < 0
\end{cases}
\]

11. \( \lim_{x \to 0} F(x) \) is (a) 1 (b) 0 (c) -1 (d) does not exist (e) none

12. The function \( F \) is discontinuous for the following values of \( x \):
    (a) 1 (b) 0 (c) -1 (d) 0 and -1 (e) none of the above

13. \( \{x | F'(x) \text{ does not exist}\} \) is
    (a) \{1\} (b) \{0\} (c) \{-1\} (d) \{-1,0\} (e) none

14. \( \{x | F'(x) = 1\} \) is
    (a) \((0,\infty)\) (b) \((-1,\infty)\) (c) \((-1,0) U (0,\infty)\)
    (d) \((-\infty,-1) U (0,\infty)\) (e) none

15. If \( f(x) = \sqrt{2x^2 - 1} \) then \( f'(1) \) equals
    (a) 0 (b) 1 (c) 2 (d) 4 (e) none
16. If \( y = (\sqrt{2x - 1})^{1/2} \) then
(a) \( 2y (y^2 + 1) = y' \)
(b) \( 2y (y^2 + 1) y' = 1 \)
(c) \( 4y (y^2 + 1) y' = 1 \)
(d) \( (y^2 + 1) y' = y^{-1} \)
(e) \( 4(y^2 + 1) = yy' \)

17. If \( f(x) = \sec (\pi x) \), then \( f'(1/4) \) equals
(a) \( \pi \)
(b) \( \pi \sqrt{2} \)
(c) \( \pi / \sqrt{2} \)
(d) \( \sqrt{2} \)
(e) none

18. \( x \sqrt{g(x)} \) equals (a) \( \sqrt{g'(x)} \)
(b) \( 1/2 \sqrt{g(x)} \)
(c) \( g'(x)/2\sqrt{g(x)} \)
(d) \( g'(x) \sqrt{g(x)} \)
(e) none

19. If \( g(t) = \frac{t}{\sqrt{t^2 - 1}} \), then \( g''(2) \) equals
(a) \( 5\sqrt{3}/9 \)
(b) \( 2\sqrt{3}/9 \)
(c) \( -5 \sqrt{3}/9 \)
(d) \( -10 \sqrt{3}/9 \)
(e) none

20. If \( f(x) = 2x^3 - 3x^2 \) and \( A = [0,2] \) then \( f(A) \) equals
(a) \([0,4]\)
(b) \([0,2]\)
(c) \([-1,4]\)
(d) \([-2,4]\)
(e) none

21. If \( x^2 + y^3 = 1 \) then \( y' \) equals
(a) \( -3y^2/2x \)
(b) \( 3\sqrt{1-x^2}/3y^2 \)
(c) \( -2x/3y^2 \)
(d) \( (1-3y^2)/2x \)
(e) none

22. A relative minimum point on the graph of \( y = x^2(x+2)^2 \) is
(a) \( (0,0) \)
(b) \( (-1,1) \)
(c) \( (1,9) \)
(d) \( (2,64) \)
(e) none

23. \( \lim_{h \to 0} \frac{\sin \frac{h^2}{h} - \sin \frac{2h}{h}}{h} \) is
(a) \(-1\)
(b) \(-2\)
(c) \(0\)
(d) does not exist
(e) none

24. Let \( f(x) = x - \frac{|x|}{x} \). Which of the following is \textit{false}?
(a) \( \lim_{x \to 0} f(x) = -1 \)
(b) \( \lim_{x \to 0} f(x) = 1 \)
(c) \( f'(-1) = -1 \)
(d) \( f'(0) = 0 \)
(e) \( f'(1) = 1 \)

25. The graph of \( y = \left( x^4 + 2x^3 - 12x^2 \right) \) is concave up in the interval
(a) \((-2,-1)\)
(b) \((-1,0)\)
(c) \((0,1)\)
(d) \((1,\infty)\)
(e) none
33. The graph of \( f \) is shown at the right. Which of the following is the longest interval in which the product \( f(x) \cdot f'(x) \cdot f''(x) \) is positive?

(a) \((0, 1)\)  
(b) \((0, 2)\)  
(c) \((1, 3)\)  
(d) \((1, 4)\)  
(e) \((2, 4)\)

34. The area of a segment of a circle (a pie-shaped piece) of radius \( r \) and angle \( \theta \) radians is \( r^2 \theta / 2 \). The length of a circular arc is \( r \theta \). If the perimeter of the segment is fixed at \( P \) units, then the radian measure of the angle which maximizes the area of the segment is

(a) \( \pi / 2 \)  
(b) \( \pi / 4 \)  
(c) \( 2 \)  
(d) \( 1 \)  
(e) none

35. Repeat your answer to number 34.

36. A 13-foot ladder leans against a wall. The base of the ladder is pulled away from the wall at the rate of 4 ft./sec. The speed (in ft./sec) of the top of the ladder sliding down the wall when the top is 12 ft. from the ground is

(a) \( 48/5 \)  
(b) \( 5/4 \)  
(c) \( 5/3 \)  
(d) \( 3/2 \)  
(e) \( 12/5 \)

37. A box with a square base and top is to hold a fixed volume \( V \). The material used for the bottom costs twice as much as that used for the sides, and the material for the top costs half as much as that for the sides. The cost of the box is least if the length of a side of the base equals

(a) the height  
(b) \( 4/5 \) the height  
(c) \( 5/4 \) the height  
(d) \( 3/4 \) the height  
(e) none

38. Repeat your answer to question 37.

The next two questions deal with a function \( F \) with domain \([-c, c]\) such that \( F(0) = 0 \), and with the following statements:

I. \( F'(0) = \frac{F(c) - F(-c)}{2c} \)  
II. \( \lim_{x \to 0} F(x) = 0 \)  
III. \( \lim_{x \to 0} \frac{F(x)}{x} = 0 \)  
IV. \( F \) is differentiable at 0; \( V \) \( F \) is continuous at 0.

39. Which of the following statements is true?

(a) II implies III.  
(b) III implies I.  
(c) II implies V.  
(d) V implies I.  
(e) none.

40. Which of the following statements is true?

(a) IV implies I.  
(b) IV implies III.  
(c) III implies II.  
(d) V implies III.  
(e) none
26. The vertex of the parabola \( y^2 - 4y - x + 5 = 0 \) is
   (a) (5,0)  (b) (2,1)  (c) (1,2)  (d) (0,5)  (e) none

27. The hyperbola \((x-1)^2 - 4y^2 = 4\) has one asymptote with slope
   (a) 1/4  (b) 1/2  (c) 4  (d) 2  (e) none

28. The X-coordinate of one focus of the ellipse \(4x^2 + 9y^2 = 36\) is
   (a) 2  (b) 3  (c) 4  (d) 9  (e) none

29. The eccentricity of the conic \(7x^2 - 6x + 19 = 5y^2 + 8y + 13\) is
   (a) 0  (b) \(\sqrt{7}/12\)  (c) 1  (d) \(\sqrt{12}/7\)  (e) -1

30. Suppose \(G'(0) = G'(2), G'(1) \geq G'(3)\) and suppose \(G''(0)\) and \(G''(2)\)
   have opposite signs. Which of the following graphs best represents \(G\)?
   (a)  (b)  (c)  (d)  (e) 

   ![Graphs](image)

31. An ellipse with center \((0,1)\) and semidiameters of 2 and 1 has for its
   equation \(x + By^2 + Cy = 0\). The quotient \(C/B\) equals
   (a) 1/2  (b) -1/2  (c) 2  (d) -2  (e) none

32. The graph of \(2x^2 - 3x - 3y^2 - 2y - 7 = 0\) is a(an)
   (a) circle  (b) ellipse  (c) parabola  (d) hyperbola  (e) none
APPENDIX D

ITEM ANALYSES

1. Pre-test
   Kuder-Richardson 20: .538
   Kuder-Richardson 21: .462
   Mean Item Difficulty: .685
   Item Discrimination Distribution:
   81-1.00 1 10%
   61-.80 1 10%
   41-.60 6 60%
   21-.40 2 20%
   Mean Item Discrimination: .528

2. Final Examination
   Kuder-Richardson 20: .835
   Kuder-Richardson 21: .804
   Mean Item Difficulty: .433
   Item Discrimination Distribution:
   81-1.00 0 0%
   61-.80 1 2%
   41-.60 23 57%
   21-.40 14 35%
   00-.20 2 5%
   Mean Item Discrimination: .430

163
3. Examination One (9:00)

Kuder-Richardson 20: .672
Kuder-Richardson 21: .618
Mean Item Difficulty: .386

Item Discrimination Distribution:
- .81-1.00 0 0%
- .61-.80 2 10%
- .41-.60 11 55%
- .21-.40 5 25%
- .00-.20 2 10%
Mean Item Discrimination: .430

4. Examination One (3:00)

Kuder-Richardson 20: .697
Kuder-Richardson 21: .659
Mean Item Difficulty: .410

Item Discrimination Distribution:
- .81-1.00 0 0%
- .61-.80 2 10%
- .41-.60 11 55%
- .21-.40 6 30%
- .00-.20 1 5%
Mean Item Discrimination: .462
5. Examination Two (9:00)

Kuder-Richardson 20: .667
Kuder-Richardson 21: .614

Mean Item Difficulty: .501

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Mean Item Discrimination: .448

6. Examination Two (3:00)

Kuder-Richardson 20: .767
Kuder-Richardson 21: .741

Mean Item Difficulty: .465

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Mean Item Discrimination: .496
7. Examination Three (9:00)

Kuder-Richardson 20: .775
Kuder-Richardson 21: .692

Mean Item Difficulty: .498

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Mean Item Discrimination: .487

8. Examination Three (3:00)

Kuder-Richardson 20: .739
Kuder-Richardson 21: .673

Mean Item Difficulty: .505

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Mean Item Discrimination: .472
9. Examination Four

Kuder-Richardson 20: \(0.707\)
Kuder-Richardson 21: \(0.644\)

Mean Item Difficulty: \(0.444\)

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