LAW, Charles Herbert, 1943-
THE SUPERSONIC AXISYMMETRIC WAKE ABOUT A TRAILING TOW CABLE.

The Ohio State University, Ph.D., 1970
Engineering, aeronautical

University Microfilms, A XEROX Company, Ann Arbor, Michigan
THE SUPERSONIC AXISYMMETRIC WAKE ABOUT
A TRAILING TOW CABLE

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

Charles Herbert Law, B.A.A.E., M.S.

# * * * * *

The Ohio State University
1970

Approved by

[Signature]
Adviser
Department of Aeronautical
and Astronautical Engineering
ACKNOWLEDGMENTS

The author wishes to acknowledge Dr. Robert M. Nerem of The Ohio State University for conceiving this project and for his much needed helpful advice and direction throughout the past five years. The author also wishes to thank Dr. John D. Lee and Dr. Gerald M. Gregorek for their assistance and many helpful ideas in conducting the experimental program. Sincere thanks also goes to the Staff of the Aerodynamics Laboratory at The Ohio State University for their cooperation, for without their engineering skills and patience this project would never have been completed.
VITA

Nov. 16, 1943. Born - Cumberland, Ohio
June, 1966. B.A.A.E., The Ohio State University, Columbus, Ohio
Dec., 1966. M.S., The Ohio State University, Columbus, Ohio
1966-1970. National Science Foundation Fellow, The Ohio State University, Columbus, Ohio

PUBLICATIONS


FIELDS OF STUDY

Major Field: Aeronautical and Astronautical Engineering

Studies in Reentry Aerodynamics. Dr. Robert M. Nerem, Associate Professor

Studies in Aerodynamics. Professor John D. Lee

Studies in Experimental Methods. Dr. Gerald M. Gregorek, Associate Professor

Studies in Fluid Mechanics. Professor Odus R. Burggraf
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>ii</td>
</tr>
<tr>
<td>VITA</td>
<td>iii</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>vi</td>
</tr>
<tr>
<td>LIST OF ILLUSTRATIONS</td>
<td>vii</td>
</tr>
<tr>
<td>LIST OF SYMBOLS</td>
<td>xiv</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
</tbody>
</table>

Chapter

I. REVIEW OF THEORY                                          | 7    |
   Near Wake Flowfield                                       |
   Recirculating Base Flow                                   |
   Theoretical and Experimental Methods                      |
   Tow Cable-Wake Problem                                    |

II. APPLICATION OF AN INTEGRAL METHOD OF SOLUTION            | 17   |
   Conservation Equations                                    |
   Momentum Integral Equation                                 |
   Velocity Profiles                                          |

III. APPLICATION OF A FINITE-DIFFERENCE METHOD OF SOLUTION    | 24   |
   Stream Function Coordinates                               |
   Finite-Difference Grid Definition                          |
   Momentum Equation, Finite-Difference Form                  |
   iv
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Wind Tunnel Flow Conditions For The Three Test Series</td>
<td>42</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>1.</td>
<td>System of Forebody, Connecting Cable, and Decelerator and Resulting Flowfield at Supersonic Speeds</td>
</tr>
<tr>
<td>2.</td>
<td>Distinct Regions of the High Speed Wake</td>
</tr>
<tr>
<td>3.</td>
<td>Subscript Notation and Grid Point Definition for Finite-Difference Solution</td>
</tr>
<tr>
<td>4.</td>
<td>Model Configuration and Tow Cable Design</td>
</tr>
<tr>
<td>5.</td>
<td>Turbulence Level in the Wake as a Function of Stagnation Pressure</td>
</tr>
<tr>
<td>6.</td>
<td>Conventional Wake Transition Correlation by Pallone</td>
</tr>
<tr>
<td>7.</td>
<td>Wind Tunnel Design Drawing</td>
</tr>
<tr>
<td>8.</td>
<td>Picture of the Wind Tunnel in the Laboratory</td>
</tr>
<tr>
<td>9.</td>
<td>Wind Tunnel Laboratory Configuration.</td>
</tr>
<tr>
<td></td>
<td>Top: View Through Schlieren Window</td>
</tr>
<tr>
<td></td>
<td>Showing Model Configuration and Hot-Wire Anemometer Probe.</td>
</tr>
<tr>
<td></td>
<td>Bottom: Side Mounted Probe Traversing Mechanism</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>10.</td>
<td>Probe Tip Design Drawings. Top: Pitot Probe No. 1. Middle: Pitot Probe No. 2.</td>
</tr>
<tr>
<td></td>
<td>Bottom: Static Pressure Probe.</td>
</tr>
<tr>
<td>11.</td>
<td>&quot;U = 0&quot; Probe Design Drawing.</td>
</tr>
<tr>
<td>12.</td>
<td>Universal Probe Support Design Drawing.</td>
</tr>
<tr>
<td>13.</td>
<td>Complete Probe Systems. Top: Static Pressure Probe. Bottom: &quot;U = 0&quot; Probe.</td>
</tr>
<tr>
<td>16.</td>
<td>Pitot Pressure Profiles at Various Axial Locations for Test Series No. 1 with no Tow Cable</td>
</tr>
<tr>
<td>17.</td>
<td>Total Temperature Profiles at Various Axial Locations with no Tow Cable, for Test Series No. 1</td>
</tr>
<tr>
<td>18.</td>
<td>Mach Number Profiles at Various Axial Locations for Test Series No. 1 with viii</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td></td>
<td>no Tow Cable</td>
</tr>
<tr>
<td>19.</td>
<td>Pitot Pressure Profiles at Various Axial Locations for Test Series No. 1 with a 1/16 inch Tow Cable</td>
</tr>
<tr>
<td>20.</td>
<td>Total Temperature Profiles at Various Axial Locations with a 1/16 inch Tow Cable for Test Series No. 1</td>
</tr>
<tr>
<td>21.</td>
<td>Mach Number Profiles at Various Axial Locations with a 1/16 inch Tow Cable for Test Series No. 1</td>
</tr>
<tr>
<td>22.</td>
<td>Pitot Pressure Profiles at Various Axial Locations with a 1/4 inch Tow Cable for Test Series No. 1</td>
</tr>
<tr>
<td>23.</td>
<td>Total Temperature Profiles at Various Axial Locations with a 1/4 inch Tow Cable for Test Series No. 1</td>
</tr>
<tr>
<td>24.</td>
<td>Mach Number Profiles at Various Axial Locations with a 1/4 inch Tow Cable for Test Series No. 1</td>
</tr>
<tr>
<td>25.</td>
<td>Pitot Pressure Map for Test Series No. 1 with no Tow Cable</td>
</tr>
<tr>
<td>26.</td>
<td>Pitot Pressure Map for Test Series No. 1 with a 1/16 inch Tow Cable</td>
</tr>
<tr>
<td>27.</td>
<td>Pitot Pressure Map for Test Series No. 1 with a 1/4 inch Tow Cable</td>
</tr>
</tbody>
</table>
28. Pitot Pressure Distribution for Test Series No. 1 with no Tow Cable........... 77
29. Pitot Pressure Distribution for Test Series No. 1 with a 1/16 inch Tow Cable............................................. 78
30. Pitot Pressure Distribution for Test Series No. 1 with a 1/4 inch Tow Cable. 79
31. Static Pressure Distribution for Test Series No. 1 with no Tow Cable.......... 80
32. Static Pressure Distribution for Test Series No. 1 with a 1/16 inch Tow Cable. 81
33. Static Pressure Distribution for Test Series No. 1 with a 1/4 inch Tow Cable.. 82
34. Mach Number Distribution for Test Series No. 1 with no Tow Cable............ 83
35. Mach Number Distribution for Test Series No. 1 with a 1/16 inch Tow Cable...... 84
36. Mach Number Distribution for Test Series No. 1 with a 1/4 inch Tow Cable..... 85
37. "U = 0" Line Data and Resulting Rear Stagnation Points for the Three Cases of Test Series No. 1.............................. 88
38. Wake Growth Data for the Three Cases of Test Series No. 1............................ 89
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>39.</td>
<td>Velocity Profiles Calculated from Test Series No. 1 with no Tow Cable</td>
<td>90</td>
</tr>
<tr>
<td>40.</td>
<td>Velocity Profiles Through the Wake for Test Series No. 1 with no Tow Cable</td>
<td>92</td>
</tr>
<tr>
<td>41.</td>
<td>Velocity Profiles Through the Wake for Test Series No. 1 with a 1/16 inch</td>
<td>93</td>
</tr>
<tr>
<td></td>
<td>Tow Cable</td>
<td></td>
</tr>
<tr>
<td>42.</td>
<td>Velocity Profiles Through the Wake for Test Series No. 1 with a 1/4 inch</td>
<td>94</td>
</tr>
<tr>
<td></td>
<td>Tow Cable</td>
<td></td>
</tr>
<tr>
<td>43.</td>
<td>Velocity Profiles at the Rear Stagnation Point for the Three Cases of Test</td>
<td>95</td>
</tr>
<tr>
<td></td>
<td>Series No. 1</td>
<td></td>
</tr>
<tr>
<td>44.</td>
<td>Static Pressure Profiles for Test Series No. 3 with no Tow Cable</td>
<td>99</td>
</tr>
<tr>
<td>45.</td>
<td>Pitot Pressure Profiles for Test Series No. 3 with no Tow Cable</td>
<td>100</td>
</tr>
<tr>
<td>46.</td>
<td>Static Pressure Profiles for Test Series No. 3 with a 1/16 inch Tow Cable</td>
<td>101</td>
</tr>
<tr>
<td>47.</td>
<td>Pitot Pressure Profiles for Test Series No. 3 with a 1/16 inch Tow Cable</td>
<td>102</td>
</tr>
<tr>
<td>48.</td>
<td>Static Pressure Profiles for Test Series No. 3 with a 1/4 inch Tow Cable</td>
<td>103</td>
</tr>
<tr>
<td>49.</td>
<td>Pitot Pressure Profiles for Test Series No. 3 with a 1/4 inch Tow Cable</td>
<td>104</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>50</td>
<td>Mach Number Profiles for Test Series No. 3 with no Tow Cable</td>
<td>105</td>
</tr>
<tr>
<td>51</td>
<td>Mach Number Profiles for Test Series No. 3 with a 1/16 inch Tow Cable</td>
<td>106</td>
</tr>
<tr>
<td>52</td>
<td>Mach Number Profiles for Test Series No. 3 with a 1/4 inch Tow Cable</td>
<td>107</td>
</tr>
<tr>
<td>53</td>
<td>Velocity Profiles Through the Wake for Test Series No. 3 with no Tow Cable</td>
<td>108</td>
</tr>
<tr>
<td>54</td>
<td>Velocity Profiles Through the Wake for Test Series No. 3 with a 1/16 inch Tow Cable</td>
<td>109</td>
</tr>
<tr>
<td>55</td>
<td>Velocity Profiles Through the Wake for Test Series No. 3 with a 1/4 inch Tow Cable</td>
<td>110</td>
</tr>
<tr>
<td>56</td>
<td>Velocity Profiles at the Rear Stagnation Point for the Three Cases of Test Series No. 3</td>
<td>111</td>
</tr>
<tr>
<td>57</td>
<td>Transformed Velocity Profiles at the Rear Stagnation Point for the Three Cases of Test Series No. 3</td>
<td>113</td>
</tr>
<tr>
<td>58</td>
<td>Comparison of Theoretical and Experimental Velocity Profiles for Laminar Flow with a 1/16 inch Tow Cable</td>
<td>115</td>
</tr>
<tr>
<td>59</td>
<td>Comparison of Theoretical and Experimental Velocity Profiles for Laminar Flow with xii</td>
<td></td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>------</td>
</tr>
<tr>
<td>60.</td>
<td>Centerline or Tow Cable Wall Variation of Static Pressure for Test Series No. 3.</td>
<td>117</td>
</tr>
<tr>
<td>61.</td>
<td>Wake Edge Variation of Static Pressure for Test Series No. 3.</td>
<td>118</td>
</tr>
<tr>
<td>62.</td>
<td>Comparison of Experimental and Theoretical Wake Growth for the Three Cases of Test Series No. 3.</td>
<td>120</td>
</tr>
<tr>
<td>63.</td>
<td>Velocity Profile Distribution Theoretically Calculated for the Case of a 1/16 inch Tow Cable.</td>
<td>123</td>
</tr>
<tr>
<td>64.</td>
<td>Comparison of the Wake Growths for Laminar Flow with the Conventional Wake Growth for a 1/16 inch Tow Cable and for a Zero-Diameter Tow Cable.</td>
<td>124</td>
</tr>
</tbody>
</table>
LIST OF SYMBOLS

\[ \begin{align*} 
  a_i & = \text{Velocity profile coefficients} \\
  A & = \text{Function defined in equation 14 of the Appendix} \\
  A^* & = \text{Integral function defined in equation 37 of the Appendix} \\
  b_i & = \text{Total enthalpy profile coefficients} \\
  B & = \text{Function defined in equation 14 of the Appendix} \\
  B^* & = \text{Integral function defined in equation 37 of the Appendix} \\
  C & = \text{Function defined in equation 27 of the Appendix} \\
  C^* & = \text{Integral function defined in equation 37 of the Appendix} \\
  C_p & = \text{Specific heat at constant pressure} \\
  D & = \text{Function defined in equation 27 of the Appendix} \\
  D^* & = \text{Integral function defined in equation 37 of the Appendix} \\
  \text{def} & = \text{Momentum defect defined in equation 33 of the Appendix} \\
  f,F & = \text{Arbitrary functions} \\
  H & = \text{Total enthalpy} \\
  K & = \text{Stability function defined in equation 45 of the text} \\
  M & = \text{Mach number} 
\end{align*} \]
n = Transformed radial coordinate defined in equation 2 of the text

$P_\infty$ = Pressure

$p_B$ = Forebody base pressure

$p_S$ = Static pressure

$P_{t_2}$ = Pitot pressure

$P_{t_\infty}$ = Freestream total pressure

$Pr$ = Prandtl number

$R$ = Gas Constant

$r$ = Radial coordinate measured from the axis of symmetry

$r_S$ = Radial distance measured from the axis of symmetry to the wake or boundary-layer edge

$Re$ = Reynolds number

$r_w$ = Tow cable wall radius

$S$ = Distance measured in the axial direction

$\Delta S$ = Step size in the axial direction

$T$ = Temperature

$T_t$ = Total temperature

$u$ = Axial component of velocity

$v$ = Radial component of velocity

$x$ = Distance measured in the axial direction from the base of the forebody

$y$ = Radial coordinate measured from the tow cable wall
\( \beta = \) Constant defined in equation 10 of the Appendix
\( \beta_1 = \) Constant defined in equation 11 of the Appendix
\( \Delta = \) Transformed boundary-layer or wake thickness
\( \delta = \) Boundary-layer or wake thickness measured from the wall or wake centerline
\( \delta^* = \) Displacement thickness defined in equation 11 of the text
\( \overline{\delta}^* = \) Thickness defined in equation 10 of the text
\( \eta = \) Transformed radial coordinate defined in equation 1 of the text
\( \theta = \) Momentum thickness defined in equation 13 of the text
\( \overline{\theta} = \) Thickness defined in equation 12 of the text
\( \lambda = \) Profile parameter defined in equation 15 of the text
\( \mu = \) Viscosity
\( \psi = \) Transformed coordinate measured in the radial direction; stream function coordinate
\( \Delta \psi = \) Step size in the radial direction
\( \rho = \) Density

Subscripts
\( e = \) Wake or boundary-layer edge condition
\( i = \) Grid point index
\( j = \) Grid point index
\( m = \) Grid point index; point definition
n  =  Grid point index; point definition
o  =  Axial velocity stagnation point
r  =  Rear stagnation point condition
s  =  Static condition
w  =  Wall condition
\infty  =  Freestream condition
C  =  Centerline condition
INTRODUCTION

Since the time of Homer, many scientists have been interested in trying to answer questions about bodies moving through fluids and the associated confusion they leave behind them. It has not been until recently, however, that a combined effort of many authors has led to some understanding of wake phenomena. The motivation for much of this work originally came from military needs for understanding re-entry phenomena and the interaction of hypervelocity vehicles with the atmosphere as a part of the ballistic-missile research program. The advent of landing men on the moon and resulting proposals for travel to other planets required more concentrated efforts by government agencies and private industry which has led to further understanding of wake phenomena, while at the same time uncovering new questions to be answered.

One potentially attractive technique for efficiently accomplishing spacecraft module deceleration to effect orbital capture on return from long missions is to fly through the atmosphere in a manner sufficient to produce the required deceleration (braking by aerodynamic drag) from the high re-entry velocities. Other developments arising from the accelerated wake programs have been
proposals for use of the decelerator system for recovery of spent rocket stages and safely returning pilots to earth after high speed and high altitude ejection from aircraft or space capsules. These latter developments have been successfully accomplished by low Mach number decelerator deployment.

Many types of decelerators have been considered, varying from supersonic parachutes to the Ballute-type configuration proposed by the Goodyear Aerospace Corporation. All of these drag devices have certain advantages and disadvantages depending upon their particular applications. Choice of the final design would depend upon such factors as system weight, aerodynamic drag efficiency, stability characteristics, and the complexities of the design. These decelerators are in general designed such that they are deployed in the wake of a high-speed vehicle; and in determining the performance of such a decelerator, it is necessary to have an understanding of the wake phenomena that exist behind a high-speed vehicle and the interaction of this wake with the decelerator. The efficiency of any decelerator placed behind a body traveling at high supersonic speeds will thus be highly dependent upon the wake characteristics of the resulting system of forebody, connecting cable, and afterbody. The problems associated with the analysis
include obtaining a configuration which is stable under specific design conditions and also obtaining one which has a high efficiency to produce drag and yet be within the limits of maximum design weight.

Much theoretical and experimental research has been done in the past; and from a fluid mechanical viewpoint, the general features of the conventional high-speed wake in the absence of a trailing decelerator have been well defined. However, if an aerodynamic decelerator is deployed into such a high-speed wake behind a typical vehicle, the questions that arise are: "What will the effect of the decelerator be?" "What will be the alteration in the wake characteristics due to the presence of a decelerator and a tow cable?" Obviously, the wake region downstream of the decelerator will be greatly affected by decelerator deployment, but the extent to which the wake region in front of it is disturbed is not so clearly defined.

In order to be able to predict the performance of any decelerator, the forebody wake properties must be clearly defined because it is the trailing viscous and inviscid wake of the forebody which represents the effective freestream in which the decelerator will reside. At the same time, the presence of the decelerator and the tow cable connecting it to the forebody have undefined effects on the forebody wake properties. In addition,
the wake properties behind the forebody depend directly on the characteristics of the forebody flow field. Thus, whether a study is undertaken from a theoretical or an experimental point of view, there are a number of highly coupled individual problems which must be considered.

One system variable which was considered to be of primary importance was the size of the tow cable and its effect on the overall performance of the decelerator. Early experimental work by this author and even recent work by other people experimenting in this area could not be considered necessarily valid if this effect was substantial because while attempting to minimize support interference the tow cable was made many times oversized in order to support the forebody. It had to be determined, either experimentally or theoretically, if it was necessary to accurately match the scaled tow cable diameter. A schlieren photograph of the system of forebody, connecting cable, and decelerator and the resulting flowfield at supersonic speeds is shown in Figure 1.

The goals of this study were to investigate the effect of the presence of a tow cable and of tow cable diameter on the characteristics of a high speed axisymmetric wake by developing a finite-difference solution to the appropriate equations of motion and undertaking an experimental program to provide initial conditions for
Figure 1. System of Forebody, Connecting Cable, and Decelerator and Resulting Flowfield at Supersonic Speeds.
the theoretical solution, to provide data for verification of the calculations, and to develop in general a better understanding of tow-cable effects. The experimental investigations included both laminar and turbulent flow, although only a laminar theoretical solution was developed.
CHAPTER I

REVIEW OF THEORY

To predict accurately the behavior of a trailing decelerator and the effect of its connecting or tow cable, it is necessary to have a clear understanding of the properties of the wake into which the decelerator and connecting cable will be deployed. The conventional wake behind a high speed vehicle has five distinct regions:

1) The recirculation base flow
2) The free shear layer
3) The neck region
4) The inner viscous wake downstream of the neck
5) The outer inviscid wake

These regions are shown in Figure 2. The recirculating base flow and the free shear layer both contain fluid which has come from the boundary layer of the forebody. The free shear layer initiates the development of the neck region and the viscous inner wake which can be described by the conventional boundary layer equations. An additional characteristic of the wake is that it is either a laminar wake or a turbulent wake, either of these retaining the same general features as above.

In spite of the emergence of a good qualitative
Figure 2. Distinct Regions of the High Speed Wake.
picture of the near-wake some years ago, there is no rational theory which completely explains all of the major observed phenomena of the recirculating base flow. Various experimental studies have revealed the following general description of the base flow problem. During the initial stage of development of the recirculating base flow, the momentum of the shear layer is enriched as high-velocity fluid enters the shear layer and, by diffusion, imparts momentum to the low-velocity viscous flow. The velocity along the streamline dividing that fluid which finally enters the viscous region downstream of the neck from the fluid which is trapped and remains in the base region is increased, preparing the flow for the compression region downstream of the body. The shear layers generated by the upper and lower sides of the body begin to interact upstream of the rear stagnation point, and the flow gradually turns toward a direction parallel to the wake centerline. As the flow is compressed during this turning, the velocity along the dividing streamline is decelerated and finally brought to rest at the rear stagnation point. Fluid below the dividing streamline is turned back by the compression and fluid above the dividing streamline continues to flow downstream to form the inner viscous wake flow.

The most significant contribution to the solution of the base flow problem was Chapman's recognition of the
importance of the shear layers shed from the body surface. He introduced the concept of the "dividing streamline" separating the recirculation fluid from the fluid entrained by mixing with the "external" inviscid flow. Chapman assumed that the flow along this streamline is brought to rest at the rear stagnation point in a distance so short that the compression can be regarded as isentropic. At the same time, the flow separation from the body surface is assumed to occur over a very short distance and the initial boundary-layer thickness is supposed to be vanishingly small. The most successful application of Chapman's theory is to the pressure rise that occurs when a "free" shear layer reattaches to a solid surface, provided the initial boundary layer at separation is extremely thin.

The "mixing" theory devised by Crocco begins with the concept that the compression in the near wake at supersonic speeds is a smooth, viscous-inviscid interaction rather than a sudden process. Transport of momentum from the "external" inviscid flow to the internal dissipative flow is regarded as a key element in this interaction. An important property of this wake flow theory is the existence of a mathematical "critical point" or "throat" somewhere downstream of the rear stagnation point. The base pressure is determined by the requirement
that the correct solution must pass through this throat.
This theory gave a qualitatively correct description of
the variation of base pressure with Reynolds number on
blunt trailing-edge airfoils and ogival bodies, including
the effect of transition to turbulent flow in the wake.

Reeves and Lees\(^3\) formulated a theory capable of in­
cluding supersonic laminar, preseparated, separated, and
reattaching flows within a single framework, without
introducing semiempirical features. The essential element
of viscous-inviscid interaction is retained, but the semi­
empirical "mixing rate" of the Crocco-Lees theory is re­
placed by an additional moment of the momentum equation
across the viscous flow. This approach was later applied
to laminar near wakes behind blunt bodies at hypersonic
speeds. It was deduced that because of the transfer of
momentum from the "external" inviscid stream to the vis­
cous flow above the dividing streamline, the velocity
along this streamline increases continuously in the down­
stream direction some distance downstream of separation.
More and more fluid below the dividing streamline is
turned back as the flow proceeds downstream, until the
velocity along this streamline itself is brought to rest
at the reattachment point. Downstream of this point, the
fluid above the dividing streamline forms a new boundary
layer. This layer reaches a minimum section, or "neck"
before relaxing to the "normal" state corresponding to
a weak, supersonic viscous interaction at the new Mach number.

Although there is little doubt that the boundary-layer equations are reasonable approximations of the Navier-Stokes equations some distance past the rear stagnation point, Weiss has shown that the boundary-layer equations are inadequate in the base region at moderate Reynolds numbers. Weiss was able to predict the variation of base pressure with Reynolds number by assuming a boundary-layer region to exist only outside the dividing streamline and to use a Stokes solution of the recirculation region inside the dividing streamline.

Weiss was able to complete a detailed solution of the near wake by dividing the flowfield into three sub-regions, solving each region separately, and then matching certain flow conditions on the dividing streamline. The outer flow resulting from the expansion of the boundary layer is solved by the method of characteristics. The viscous layer above the dividing streamline is solved by a modified Oseen solution of the boundary-layer equations. A finite difference solution of the full Navier-Stokes equations is used to solve the recirculation region inside the dividing streamline.

There is a vast amount of experimental data available concerning the recirculation and near wake regions.
of high speed bodies, although very little general data presently exists. Making base flowfield measurements for axisymmetric and nonaxisymmetric bodies is difficult primarily because of the disturbing effects of sting supports and the interference of various wire support systems.

6 Zakkay and Cresci used a wire support system to experimentally analyze the near wake of a slender cone in a hypersonic laminar flow. Martellucci, Trucco, and Agnone performed similar experiments for turbulent flow with an additional interest in comparisons of wire support systems on interference effects.

One of the more interesting investigations was made by Bauer, who experimented with slender cones in supersonic laminar flow. The cone models were supported by a streamwise tube cantilevered into the test section through the nozzle throat to eliminate support interference effects. Although Bauer's immediate concern was the effect of mass exchange between the vehicle and the fluid around it on the near wake flow, some contributions were made toward a clearer understanding of the conventional high speed wake. The experimental techniques used offered an undisturbed wake which could be probed to determine the variation of any parameter.

Returning to a review of the theoretical work, downstream of the rear stagnation point any detailed
solution of the wake requires solving the appropriate boundary-layer equations by either an integral method or a finite-difference method. In the former, the boundary-layer equations are integrated across the boundary layer, or wake, to obtain a momentum integral equation which can then be solved by a stepwise and iterated solution in the downstream direction. In some wake integral methods velocity and total enthalpy profiles are chosen with the coefficients of the polynomials being determined from local boundary conditions at the wake centerline and at the wake edge. It is a basic assumption associated with this integral technique that a finite series will suffice to express the properties within the boundary layer or wake.

The finite-difference method represents an exact method of solution of the governing equations for arbitrarily prescribed initial and boundary conditions. Using a finite step method of small size affords an opportunity to describe flow details with precision throughout the flow field while matching specific initial conditions and allowing for nonsimilar flow behavior. A large number of steps of increasingly smaller size are required for marked improvement, and in the limit the problem approaches a finite-difference calculation.

Availability of the high speed computer has led many researchers away from integral methods which have been
unable to predict many observed features of boundary layers and wakes in favor of the more accurate finite-difference methods. Approximate velocity and total enthalpy profiles need not be assumed, and the boundary-layer equations are solved by an exact method rather than on an average across the boundary layer or wake.

No matter which method is used to calculate the properties of the wake downstream of the rear stagnation point, all initial conditions must be specified at the point where the calculations will begin. A purely theoretical solution could be obtained if one of the previously discussed analytical methods were used to calculate the flow properties in the base region up to the rear stagnation point. This would offer a very general solution since all freestream conditions and forebody configurations would be covered. Much less general solutions could be obtained for specific freestream conditions and forebody configurations if the experimental data previously compiled were used to specify the required initial conditions. This semiempirical solution would still offer a means of determining effects of specific parameters on the wake downstream of the rear stagnation point independent of changes in the base flow region.

A majority of the related literature deals with the conventional high speed wake problem and only a very
limited amount of research has been carried out in the area of the tow cable problem. The integral method has been used by many to solve the conventional wake problem and by Nerem\textsuperscript{9,10,11} to solve the limiting case of a wake with a zero-diameter tow cable. For the latter case the presence of the cable at the centerline of the wake alters the wake only by forcing a zero centerline velocity along the entire cable length. Few other contributions have been made toward solving the problem of a small finite diameter tow cable immersed in the inner viscous wake behind a high speed body.
CHAPTER II
APPLICATION OF AN INTEGRAL METHOD OF SOLUTION

The one method that has been used successfully to solve a variety of boundary layer and wake problems is the integral method. This method was considered first in the attempt to solve the wake-tow cable problem because of its relative simplicity and because of its similarity to other related theoretical investigations. The basic assumption associated with any integral technique is that a finite series will suffice to express the properties within the boundary layer (which the inner viscous wake becomes when it reattaches to the tow cable). In the case of compressible boundary layers, the Dorodnitzen transformation for axisymmetric flow has been used in the past to simplify and transform the governing equations into incompressible form. The transformation is

\begin{equation}
\rho e n d\eta = \rho r dr
\end{equation}

To obtain a non-dimensional coordinate, \( \eta \), is defined as

\begin{equation}
\eta = \frac{n-n_w}{\Delta}
\end{equation}

where \( \Delta \) is the transformed boundary layer thickness; and \( \eta = 0 \) at the wall, \( \eta = 1 \) at the boundary layer edge.
Then

\[ \eta d\eta = \frac{n-n_w}{\Delta} \frac{dn}{\Delta} \] ; \quad d\eta = \frac{dn}{\Delta} \]

and,

\[ \eta d\eta + \frac{n_w}{\Delta} d\eta = \frac{n dn}{\Delta} \]

Or,

\[ \left[ \eta + \frac{r_w}{\Delta} \right] d\eta = \frac{1}{\Delta^2} \frac{\rho_c}{\rho} r dr \]

Now the \( \eta \)-coordinate can be obtained by integrating the last equation to get

\[ \frac{x^2}{2} + \frac{r_w}{\Delta} = \frac{1}{\Delta^2} \int_{r_w}^{r} \frac{\rho_c}{\rho} r dr \]

Once the boundary-layer problem has been solved in the \( \eta \)-plane, the inverse transformation is used to transform into the physical or \( r \)-plane.

The momentum integral technique reduces the boundary-layer equations into a form which can be programmed on a digital computer. Probstein and Elliot have written the basic equations for the case of thick, axisymmetric laminar boundary layers. The continuity equation is

\[ \frac{\partial}{\partial s}[\rho u r] + \frac{\partial}{\partial y}[\rho v r] = 0 \]

and the momentum equation is
(8) \( \rho_u \frac{\partial u}{\partial S} + \rho_v \frac{\partial u}{\partial Y} = -\frac{\partial p}{\partial S} + \frac{\partial}{\partial Y} \left( \mu \frac{\partial u}{\partial Y} \right) + \frac{\mu}{r} \frac{\partial r}{\partial Y} \frac{\partial u}{\partial Y} \)

Using the Dorodnitzen transformation and integrating across the boundary layer, the momentum integral equation which results is

(9) \( \frac{d\bar{\theta}}{dS} + \frac{\bar{\theta}}{\rho u_e u_w} \frac{d}{dS} \left[ \rho e u_w u_e \right] + \frac{\bar{\delta}^*}{u_e} \frac{d u_e}{dS} = \frac{c_e}{2} \)

where \( \bar{\delta}^* \) is defined as

(10) \( \bar{\delta}^* = \int_0^\delta \frac{r}{r_w} \left[ 1 - \frac{\rho u}{\rho e u_e} \right] dY = \delta^* + \frac{\delta^*}{2r_w} \)

and \( \delta^* \) is the displacement thickness defined by

(11) \( \int_0^{\delta^*} 2\pi r e u_e dY = \int_0^\delta 2\pi r \left( \rho e u_e - \rho u \right) dY \)

Also, \( \bar{\theta} \) is defined as

(12) \( \bar{\theta} = \int_0^\delta \frac{r}{r_w} \left[ 1 - \frac{u}{u_e} \right] \frac{\rho u}{\rho e u_e} dY = \theta + \frac{\theta^2}{2r_w} \)

and \( \theta \) is the momentum thickness defined by

(13) \( \int_0^\theta 2\pi r e u_e^2 dY = \int_0^\delta 2\pi r [u e u_e - u^2] dY \)

Equation (9) is in a form which can be used in a numerical integration procedure to solve for the boundary-layer characteristics using an assumed velocity profile.

The form of the assumed velocity profile is
where \((N+1)\) boundary conditions are needed to determine the coefficients \(a_i\). For a fourth order polynomial, there are five boundary conditions to be imposed, three at the edge of the layer and two at the tow cable wall.

At the edge, \(\eta = 1\),

1. \(\frac{u}{u_e} = 1\)
2. \(\frac{d}{d\eta}\left(\frac{u}{u_e}\right) = 0\)
3. \(\frac{d^2}{d\eta^2}\left(\frac{u}{u_e}\right) = 0\)

At the tow cable wall, \(\eta = 0\),

4. \(\frac{u}{u_e} = 0\)
5. \(\frac{d}{d\eta}\left(\frac{u}{u_e}\right) = -\frac{\kappa_w}{\Delta} \frac{1}{2 \frac{P_e}{P_w} - 1} \frac{d^2}{d\eta^2}\left(\frac{u}{u_e}\right)\)

Boundary condition 5 is a requirement that the momentum equation (8) be satisfied at the tow cable surface and it is further assumed that there is zero pressure gradient in the axial direction.

Letting \(\lambda\) be defined by

\[
\lambda = 2 \frac{\kappa_w}{\Delta} \frac{1}{2 \frac{P_e}{P_w} - 1}
\]

it is found that the coefficients are given by
In the limit that \( \lambda \to 0 \), or \( r_w \to 0 \) (riser line) the coefficients are

\[
\begin{align*}
a_0 &= 0 \\
a_1 &= 0 \\
a_2 &= \frac{-6 \lambda}{1-3\lambda} \\
a_3 &= \frac{6\lambda - 8}{1-3\lambda} \\
a_4 &= \frac{3 - 3\lambda}{1-3\lambda}
\end{align*}
\]

(16)

which is the result Nerem obtained for the zero-diameter case. In the limit that \( \lambda \to \infty \), or \( r_w \to \infty \) (two-dimensional flat plate) the coefficients are

\[
\begin{align*}
a_0 &= 0 \\
a_1 &= 2 \\
a_2 &= 6 \implies \frac{u}{u_e} = 6\eta^2 - 8\eta^3 + 3\eta^4 \\
a_3 &= -8 \\
a_4 &= 3
\end{align*}
\]

(17)

which agrees with the Pohlhausen solution.

This scheme works well for the limits of \( r_w = 0 \)
(riser line) and $r_w = \infty$ (two-dimensional flat plate) where the assumed velocity profiles are meaningful, but such is not the case when $r_w$ is finite. For small finite values of wall radius, the slope of the velocity profile is negative at the wall indicating reversed flow in a thin layer near the wall. Therefore, this is not a realistic representation of the velocity profile in the wake downstream of the rear stagnation point. The fallacy of the approach appears to be in the use of a fourth order polynomial for a velocity profile. Although all of the boundary conditions specified to calculate the coefficients are valid, apparently not enough of them were used. A sixth order polynomial was also tried but did not give any improvement in providing a realistic representation for the velocity profiles for small but finite values of tow cable radius.

Since the integral method did not provide an acceptable solution or provide an analytical method for investigating the effect of tow cable diameter, it was necessary to use some other means for solving the problem. A finite-difference solution was thus developed which will be discussed in the next section. This method of solution did not require assuming an invariant or similar velocity profile. It did, however, require the specification of all initial conditions, which must either be assumed or obtained experimentally. This disadvantage is offset by the
increased accuracy of the calculations. No averaging across the boundary layer is necessary, and the only error that results follows directly from the initial data input and limitations of the finite-difference network.
CHAPTER III
APPLICATION OF A FINITE DIFFERENCE METHOD OF SOLUTION

The flow field under analysis is the region down-stream of the wake rear stagnation point corresponding to the reattachment point on the tow cable surface, but up-stream of any disturbances caused by the interaction of the decelerator with the wake. The governing compressible axisymmetric conservation equations for mass and momentum were written, after the usual boundary layer assumptions had been made, by Probstein and Elliot\(^{12}\) as equations (7) and (8), and rewritten here in slightly different form:

\[
\frac{\partial}{\partial x}[\rho u] + \frac{\partial}{\partial y}[\rho v] = 0 \tag{19}
\]

\[
\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{r} \frac{\partial}{\partial y} \left[ r \mu \frac{\partial u}{\partial y} \right] \tag{20}
\]

with \(x\) and \(y\) being the physical streamline and radial axisymmetric coordinates; \(y\) being measured from the tow cable surface and \(r\) being measured from the axis of symmetry.

Equations (19) and (20) are then transformed to the Von Mises coordinates \(S\) and \(\psi\) defined by

\[
\psi \frac{\partial \psi}{\partial y} = \rho u r ; \quad \psi \frac{\partial \psi}{\partial x} = \rho v r \tag{21}
\]

and

\[S = x\]
The use of the stream function $\psi$ so defined automatically satisfies the overall continuity equation (19). With these transformations,

$$
(22) \quad \frac{\partial u}{\partial x} = \frac{\partial u}{\partial S} \frac{\partial S}{\partial x} + \frac{\partial u}{\partial \psi} \frac{\partial \psi}{\partial x} = \frac{\partial u}{\partial S} - \frac{\rho u r}{\psi} \frac{\partial u}{\partial \psi}
$$

$$
(23) \quad \frac{\partial u}{\partial y} = \frac{\partial u}{\partial S} \frac{\partial S}{\partial y} + \frac{\partial u}{\partial \psi} \frac{\partial \psi}{\partial y} = 0 + \frac{\rho u r}{\psi} \frac{\partial u}{\partial \psi}
$$

the momentum equation (20) then becomes

$$
(24) \quad \rho u \frac{\partial u}{\partial S} - \rho \frac{u r}{\psi} \frac{\partial u}{\partial \psi} + \frac{\rho u r}{\psi} \frac{\partial u}{\partial \psi} = -\frac{\partial p}{\partial x} + \frac{1}{r} \frac{\rho u r}{\psi} \frac{\partial}{\partial \psi} \left( \mu \frac{\rho u r^2}{\psi} \frac{\partial u}{\partial \psi} \right)
$$

or, written,

$$
(25) \quad \frac{\partial u}{\partial S} = -\frac{1}{\rho u} \frac{\partial p}{\partial S} + \frac{1}{\psi} \frac{\partial}{\partial \psi} \left[ \mu \frac{\rho u r^2}{\psi} \frac{\partial u}{\partial \psi} \right]
$$

A streamwise pressure gradient, which is assumed to be specified, is included to allow the possibility of a radially uniform but streamwise varying external flow. The boundary conditions are those of zero velocity at $\psi = 0$, and matching to the freestream as $\psi \to \infty$. The initial conditions at $S = 0$ are specified as a function of $\psi$.

The finite-difference formulation of the governing conservation equations is accomplished by employing the following explicit difference relations for a typical variable $F$:

$$
(26) \quad \frac{\partial F}{\partial S} = \frac{F[S+\Delta S, \psi] - F[S, \psi]}{\Delta S}
$$
\[ \frac{\partial F}{\partial \psi} = \frac{F(S, \psi + \Delta \psi) - F(S, \psi - \Delta \psi)}{2\Delta \psi} \]

and
\[ \frac{\partial}{\partial \psi} \left[ a \frac{\partial F}{\partial \psi} \right] = \frac{a(S, \psi + \frac{1}{2} \Delta \psi) [F(S, \psi + \Delta \psi) - F(S, \psi)]}{(\Delta \psi)^2} \]

\[ - \frac{a(S, \psi - \frac{1}{2} \Delta \psi) [F(S, \psi) - F(S, \psi - \Delta \psi)]}{(\Delta \psi)^2} \]

where
\[ a(S, \psi \pm \frac{1}{2} \Delta \psi) = \frac{1}{2} \left[ a(S, \psi) + a(S, \psi \pm \Delta \psi) \right] \]

It should be pointed out that equation (24) is "numerically singular" on the tow cable surface \( \psi = 0 \). However, alternate forms of these equations suitable for computation of \( \psi = 0 \) are readily derived with the aid of L'Hôpital's rule and the cylinder surface conditions, i.e., at \( \psi = 0, u = 0 \).

Following the subscript notation designated in Figure 3, the finite-difference formulation of the momentum equation is accomplished by employing the following explicit difference relations for the variable \( F \):

\[ \left( \frac{\partial F}{\partial S} \right)_{m,n} = (F_{m+1,n} - F_{m,n}) \frac{1}{\Delta S} \]

\[ \left( \frac{\partial F}{\partial \psi} \right)_{m,n} = \frac{1}{2} (F_{m,n+1} - F_{m,n-1}) \frac{1}{\Delta \psi} \]

\[ \left( \frac{\partial}{\partial \psi} \left[ a \frac{\partial F}{\partial \psi} \right] \right)_{m,n} = \frac{1}{2} \frac{1}{\Delta \psi^2} \left\{ (a_{m,n} + a_{m+1,n})(F_{m,n+1} - F_{m,n}) - (a_{m,n} + a_{m,n-1})(F_{m,n} - F_{m,n-1}) \right\} \]

or, simplifying,
Figure 3. Subscript Notation and Grid Point Definition for Finite-Difference Solution.
The momentum equation (24) then becomes, in finite-difference form,

\[
\begin{align*}
\left( \frac{\partial}{\partial \psi} \left( \frac{\partial F}{\partial \psi} \right) \right)_{m,n} &= \frac{1}{2} \frac{1}{(\Delta \psi)^2} \left\{ F_{m,n-1} (a_{m,n} - a_{m,n-1}) \\
&\quad - F_{m,n} (a_{m,n+1} + 2a_{m,n} + a_{m,n-1}) \\
&\quad + F_{m,n+1} (a_{m,n} + a_{m,n+1}) \right\} \\
\end{align*}
\]

(32)

Adopting the assumption that \( \rho \mu \) remains constant through the layer allows the equation to be factored and reduced to

\[
\begin{align*}
\nu_{m+1,n} &= \nu_{m,n} - \frac{\Delta S}{\rho_{m,n} \nu_{m,n}} \left( \frac{dp}{dS} \right)_{m} \\
&\quad + \frac{\Delta S}{2(\nu_{m,n}(\Delta \psi))} \left\{ u_{m,n-1} \left[ \left( \frac{\mu u r^2}{\psi} \right)_{m,n} + \left( \frac{\mu u r^2}{\psi} \right)_{m,n-1} \right] \\
&\quad - u_{m,n} \left[ \left( \frac{\mu u r^2}{\psi} \right)_{m,n+1} + \left( \frac{2\mu u r^2}{\psi} \right)_{m,n} + \left( \frac{\mu u r^2}{\psi} \right)_{m,n-1} \right] \\
&\quad + u_{m,n+1} \left[ \left( \frac{\mu u r^2}{\psi} \right)_{m,n} + \left( \frac{\mu u r^2}{\psi} \right)_{m,n+1} \right] \right\} \\
\end{align*}
\]

(33)

(34)
Rewriting this equation in matrix notation, and being completely general by only considering $m = 1$ (a computer program only performs one step at a time), the momentum equation becomes

$$u(2, n) = u(1, n) - \frac{\Delta S}{\rho(1, n) u(1, n)} \left( \frac{\partial P}{\partial S} \right)$$

$$+ \frac{\Delta S \mu(1,1)}{2\psi(1, n)(\Delta \psi)^2} \left[ u(1, n-1) \left( \frac{u(1, n) \psi(1, n-1)}{\psi(1, n)} + \frac{u(1, n-1) \psi(1, n)}{\psi(1, n-1)} \right) \right]$$

$$- u(1, n) \left[ \frac{u(1, n+1) \psi(1, n+1)}{\psi(1, n+1)} + 2 \frac{u(1, n) \psi(1, n)}{\psi(1, n)} + \frac{u(1, n-1) \psi(1, n-1)}{\psi(1, n-1)} \right]$$

$$+ u(1, n+1) \left[ \frac{u(1, n) \psi(1, n)}{\psi(1, n)} + \frac{u(1, n+1) \psi(1, n+1)}{\psi(1, n+1)} \right]$$

$$+ u(1, n+1) \left[ \frac{u(1, n+1) \psi(1, n+1)}{\psi(1, n+1)} + \frac{u(1, n) \psi(1, n)}{\psi(1, n+1)} \right].$$

(35)

Use of this equation to determine $u(2, n)$ is straightforward for all points except $n = 1, 2$ where singularities in the equation exist. For instance, the first term on the right side of the equation has a singularity for $n = 1$ since $\psi(1, 1) = 0$. However, this problem is eliminated since all flow properties are prescribed not only at the initial points but also at all points on the tow cable surface, and hence equation (35) is never used for $n = 1$.

Singularities also exist for the point $n = 2$. In several terms on the right side of equation (35) the term $\psi(1, 1)$ appears in the denominator. However, the quantity $u(1, 1)$ appears in the numerator of the same terms.
It is then necessary to determine the limiting value of the term \( u(1,1)/\psi(1,1) \).

Calculating the velocity at the second grid points depends on obtaining a value of \( u/\psi \) in the limit as \( \psi \) goes to zero. The nature of the assumed profile at the starting line requires this ratio to be zero at \( S = 0 \) since the velocity profile has initially zero slope at the wall (only at the rear stagnation point). However, at the first and succeeding steps downstream, the velocity profiles will have finite slope at the wall which results in some finite value of \( u/\psi \) as \( \psi \) goes to zero. The approximation used in the present analysis is given by,

\[
(36) \quad \frac{u(2,1)}{\psi(2,1)} = 3 \frac{u(2,2)}{\psi(2,2)} - 3 \frac{u(2,3)}{\psi(2,3)} + \frac{u(2,4)}{\psi(2,4)}
\]

Although this is an approximation, it can be made as accurate as is desired simply by reducing the radial distance between grid points.

Given all the necessary initial conditions, equation (35) will give the velocities at all grid points one step downstream. It is necessary to also calculate the corresponding values of the radius at the grid points at the new location. From equation (21),

\[
(37) \quad \psi d\psi = \rho udr
\]

which becomes after integration,
Knowing the values of the stream function, velocity, and temperature at each grid point at a specific value of $S$, a numerical integration method is then used to evaluate the integral and give the values of the radius at each grid point. Once these calculations have been made, all the necessary information at this specific axial location has been obtained; and another step downstream to a new value of $S$ can be made. The whole process is then repeated at this new location.

The numerical integration approximation used in this analysis to calculate the radius is given by

$$\int_{x_{j-2}}^{x_j} f(x) \, dx \approx \frac{1}{3} \left( f_{j-2} + 4f_{j-1} + f_j \right) \Delta x \tag{39}$$

The first three points near the tow cable surface are calculated separately by another method, as are the values of the velocity. The first value of the radius is given as the radius of the tow cable. The value of the radius at the second grid point from the wall is calculated by approximating the integral in equation (38) by the trapezoid rule given by

$$\int_{x_{j-1}}^{x_j} f(x) \, dx \approx \frac{1}{2} \left( f_{j-1} + f_j \right) \Delta x \tag{40}$$
The third value of the radius is calculated by using the inverse of equation (37) in the numerical integration of equation (38) by equation (39).

No further information is necessary if it is assumed that Prandtl number equals one because the temperature is then given as a direct function of the velocity. The density at any point can be found knowing the temperature since it has been assumed that static pressure does not vary across the layer in the radial direction.

The most general case would be that where Prandtl number is not equal to one. Although it has been assumed that \( \rho \mu \) is constant, it is necessary to use the conservation of energy equation to solve for the temperature and hence the density. The governing energy equation is written as

\[
(41) \quad C_p \frac{dT}{dS} = \frac{1}{\rho} \frac{d\rho}{dx} + \frac{1}{\Psi} \frac{d}{d\Psi} \left[ C_p \frac{\mu \rho u^2}{\Psi} \frac{dT}{d\Psi} + \frac{\mu \rho u^2}{\Psi^2} \left( \frac{du}{d\Psi} \right)^2 \right]
\]

Rewriting, using \( \rho \mu = \text{constant} \),

\[
(42) \quad \frac{dT}{dS} = \frac{1}{\rho C_p} \frac{d\rho}{dS} + \frac{\mu \rho}{\Psi Pr} \frac{d}{d\Psi} \left[ \frac{u r^2}{\Psi} \frac{dT}{d\Psi} \right] + \frac{\mu \rho u^2}{C_p \Psi^2} \left( \frac{du}{d\Psi} \right)^2
\]

In finite-difference form, the energy equation yields the following for calculating the temperature \( T \):
\[ T(m+1, n) = T(m, n) + \frac{\Delta S}{\rho(m,n) C_p} \left[ \frac{dS}{dT} \right]_m \]

\[ + \frac{\Delta S \mu(l,l) \rho(l,l)}{2 \nu(m,n) \Pr(\Delta \psi)^2} \left\{ T(m, n-1) \left[ \frac{u(m,n) r^2(m,n)}{\psi(m,n)} + \frac{u(m,n-l) r^2(m,n-l)}{\psi(m,n-l)} \right] \right. \]

\[ - T(m, n) \left[ \frac{u(m,n+1) r^2(m,n+1)}{\psi(m,n+1)} + \frac{2 u(m,n) r^2(m,n)}{\psi(m,n)} + \frac{u(m,n-l) r^2(m,n-l)}{\psi(m,n-l)} \right] \]

\[ + T(m, n+1) \left[ \frac{u(m,n) r^2(m,n)}{\psi(m,n)} + \frac{u(m,n+1) r^2(m,n+1)}{\psi(m,n+1)} \right] \}

\[ + \frac{\Delta S \mu(l,l) \rho(l,l) u(m,n) r^2(m,n)}{4 C_p \nu^2(m,n) (\Delta \psi)^2} \left\{ u^2(m,n+1) - 2 u(m,n+1) u(m,n-l) + u^2(m,n-l) \right\} \]

Equation (43) is used to calculate the temperature \( T \) at all grid points in the same way as equation (35) is used to calculate the velocity. The same procedure for handling singularities applies. In a form comparable to equation (35), equation (43) is rewritten as

\[ T(2, n) = T(1, n) + \frac{\Delta S}{\rho(1,n) C_p} \left[ \frac{dS}{dT} \right]_1 \]

\[ + \frac{\Delta S \mu(l,l) \rho(l,l)}{2 \nu(1,n) \Pr(\Delta \psi)^2} \left\{ T(1, n-1) \left[ \frac{u(l,n) r^2(l,n)}{\psi(l,n)} + \frac{u(l,n-l) r^2(l,n-l)}{\psi(l,n-l)} \right] \right. \]

\[ - T(1, n) \left[ \frac{u(l,n+1) r^2(l,n+1)}{\psi(l,n+1)} + \frac{2 u(l,n) r^2(l,n)}{\psi(l,n)} + \frac{u(l,n-l) r^2(l,n-l)}{\psi(l,n-l)} \right] \]

\[ - T(1, n+1) \left[ \frac{u(l,n) r^2(l,n)}{\psi(l,n)} + \frac{u(l,n+1) r^2(l,n+1)}{\psi(l,n+1)} \right] \}

\[ + \frac{\Delta S \mu(l,l) \rho(l,l) u(l,n) r^2(l,n)}{4 C_p \psi^2(l,n) (\Delta \psi)^2} \left\{ u^2(l,n+1) - 2 u(l,n+1) u(l,n-l) + u^2(l,n-l) \right\} \]

In the previous discussion of the integral solution
it was stated that the basic assumption associated with the integral technique was that a finite series will suffice to express the properties within the boundary layer. Similarly for the finite-difference solution, the assumed starting profile series will be of the form:

\[
\frac{u}{u_e} = \sum_{i=0}^{N} a_i \eta^i
\]

where \(N\) is a finite number fixed by the method used to obtain the series, whether it be an experimental or a theoretical method. \(\eta\) is the non-dimensional coordinate defined earlier and is employed to allow the use of the previously discussed integral solution as a means of supplying initial conditions when no other means is available. Using starting profiles of this form, which are known exactly for the limiting cases of zero and infinite tow cable diameters, furnishes a means by which both theoretical and experimental initial conditions can be introduced and still retain the non-dimensional similar profile form used in the general solution. A complete description of the calculation procedure for specifying the initial conditions is presented in the Appendix.

The general problem to be solved and a finite-difference method of solution for the problem have been discussed in this section. Enough information has been given and the equations have been written in such a form
that writing a computer program for calculating the solution is a straightforward procedure. The Fortran IV program written by this author and a brief description of its use are presented in the Appendix. This program is designed to handle only those cases where a tow cable is present. Some modification is needed to solve the conventional wake problem with this program. The initial conditions, velocity profile and enthalpy profile, along with the freestream conditions and wake edge conditions are the required inputs to the program.

The finite-difference steps in both the radial and the axial directions must be specified. The accuracy and stability of the calculations can only be determined by varying the step sizes. Of course, computer time available may limit the step size which can be used; smaller step sizes mean longer calculation time.

The governing conservation equations are parabolic, and thus, the use of the explicit finite-difference method subjects the system of equations to stability conditions that govern the permissible relative grid dimensions. Furthermore, in the present nonlinear problem, it is not possible to determine by analytical means the precise stability requirements. It is possible, however, to provide estimates based on the application of linear theory. These estimates, plus numerous trial calculations, provide the basis for the determination of
the stability conditions pertinent to the present problem. The analytical estimate, as determined by procedures established by Richtmyer, is given by

\[ \Delta S \leq K(r_w)(\Delta \psi)^2 \]

where \( K \) is constant for any given tow cable radius and freestream flow conditions. Changing the tow cable radius or the freestream conditions does change \( K \), but in an undefined manner. Equation (46) is of use only when more accurate or less time consuming calculations are desired. When \( \Delta S \) is changed, then \( \Delta \psi \) must also be changed accordingly.

Use of the computer program to obtain analytical results can only provide limited accuracy which can be determined only by trial and error. At some point, decreasing the step sizes requires so many individual calculations that computer accumulated error becomes more important than analytical procedural error, and reducing the step sizes serves no useful purpose.
CHAPTER IV

EXPERIMENTAL PROCEDURE

Before a finite difference solution to the conventional wake problem or the tow cable-wake problem could be obtained, the initial conditions and edge conditions of the specific problem had to be exactly defined. The means by which these starting conditions were to be obtained, whether by experiment or theory, was not important, although it should be stressed that the degree of exactness of the finite-difference solution depends to a large extent on the degree of exactness of the starting conditions. The exact location in the wake where the initial conditions are specified is not important, so long as this location is known precisely. The most convenient and common point within the wake to begin the calculations is the rear stagnation point. This is the nearest point to the forebody where calculations can begin because the finite-difference solution is not capable of handling a reversed flow problem, or more specifically, the recirculating base flow problem.

A very thorough search through the literature did not reveal either a theoretical method or experimental data which could be useful toward specifying the initial
conditions and edge conditions for a variety of tow cable diameters, as well as for the conventional wake case. The only extensive work done in this area pertinent to the present analysis was that of A.B. Bauer, who did furnish some useful experimental conventional wake data. It was therefore necessary to undertake an experimental program with the purpose being "to obtain the initial conditions at the rear stagnation point in the wake" and supply this data to obtain a finite-difference solution under the same conditions as the experimental flow problem. At the same time, data was obtained at other points within the wake to determine the accuracy of the finite-difference solution and make whatever other contributions possible to more clearly define the near wake flow problem when a tow cable is present.

To obtain meaningful experimental data, it was necessary to construct a facility which provided the desired freestream conditions and at the same time minimized model support interference to the freestream and to the wake behind the model. The model was nominally chosen as a ten degree half-angle cone faired at the apex into a circular cylinder support rod, for which there was considerable wake data available over a wide range of freestream Mach numbers and Reynolds numbers. It was necessary to support the model by some means other than from the rear, since the experimental program called for a
wide range of tow cable radii, including the conventional case of no tow cable present. Extensive examination of the literature disclosed several methods which had been used in the past, which included wire support systems and strut support systems. However, the most attractive support system was that used by Bauer, where the cone models were supported by a streamwise tube cantilevered into the test section through the nozzle throat. This tube was faired smoothly into the nose of each model. Since Bauer was trying to determine the effect of vehicle and fluid injection geometries on the near-wake flow field structure, the support tube was necessary to carry the injectant gas to the model base. For the present experimental tests, however, the support tube serves only as a means to measure the cone base pressure from a point outside the wind tunnel in addition to supporting the cone model. This support system offers the most interference free configuration available without disturbing the region downstream of the cone base.

Three model configurations were designed to be tested. As was stated previously, the forebody remained fixed as a faired ten degree half-angle cone. The three cases to be explored were that of no tow cable, a 1/16 inch diameter tow cable, and a 1/4 inch diameter tow cable, compared to the one inch diameter forebody base. The model configuration is shown in Figure 4.
Figure 4. Model Configuration and Tow Cable Design.
The wind tunnel facility was designed to run under the conditions specified in Table 1. Air was supplied to the stagnation chamber at pressures varying from 1.50 to 150+ psia at a stagnation temperature near room temperature. Although there was some variation in stagnation temperature between runs, this variation was small and the value given in Table 1 was a representative value. It was possible to run the wind tunnel over two orders of magnitude in freestream Reynolds number because of the wind tunnel exit design. For the higher Reynolds number conditions the tunnel exited to the atmosphere, whereas for the lower Reynolds number conditions the tunnel exited to continuous flow vacuum pumps. For all conditions, the run time was continuous up to several hours.

Test series number one was initiated before any attempts were made to determine the freestream conditions necessary to insure laminar flow throughout the near wake region. The results obtained from test series number one indicated that the wake was turbulent and that further tests should be conducted to confirm or deny this conclusion. Test series number two was then initiated with results similar to those of test series number one, again indicating a turbulent wake behavior. The results of test series number three were, however, much different from those of either test series number
TABLE 1

Wind Tunnel Flow Conditions for the Three Test Series

<table>
<thead>
<tr>
<th>Flow Condition</th>
<th>Test Series 1</th>
<th>Test Series 2</th>
<th>Test Series 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_\infty$</td>
<td>3.65</td>
<td>3.65</td>
<td>3.65</td>
</tr>
<tr>
<td>$T_{t_\infty}$</td>
<td>475.2°R</td>
<td>475.2°R</td>
<td>475.2°R</td>
</tr>
<tr>
<td>$P_{t_\infty}$</td>
<td>109.2psia</td>
<td>11.85psia</td>
<td>1.84psia</td>
</tr>
<tr>
<td>$T_{exit}$</td>
<td>129.7°R</td>
<td>129.7°R</td>
<td>129.7°R</td>
</tr>
<tr>
<td>$P_{exit}$</td>
<td>1.16psia</td>
<td>0.1258psia</td>
<td>0.0195psia</td>
</tr>
<tr>
<td>$U_{exit}$</td>
<td>2037.2ft/sec</td>
<td>2037.2ft/sec</td>
<td>2037.2ft/sec</td>
</tr>
<tr>
<td>Re</td>
<td>455,000</td>
<td>49,400</td>
<td>7,670</td>
</tr>
</tbody>
</table>

Nature of Wake

| Turbulent       | Transitional   | Laminar       |

Approximate Transition Reynolds Number = 8,000
one or two, although these results still did not confirm the laminar or turbulent nature of the wake with respect to the freestream conditions. Hot wire anemometer tests were then conducted and the results of these tests are shown in Figure 5. The turbulence level is plotted as a function of stagnation pressure rather than Reynolds number because the reference condition used to calculate Reynolds number has not been clearly defined. Pallone\textsuperscript{13} correlated conventional wake transition data and arrived at the results presented in Figure 6. The flow conditions for test series one, two, and three are plotted in this figure and show good agreement with Pallone's prediction. It would be expected beforehand then that the data and velocity profiles obtained from test series number three would be applicable to the laminar wake study. The results of test series number one and two will also be presented and discussed for the sake of completeness.

The vertical positioning of the tunnel was necessary to facilitate access to the vacuum pumps and exit to the atmosphere. This positioning was also required to reduce possible drooping of the model if it were mounted in a horizontal position. A sketch of the wind tunnel configuration is shown in Figure 7.

The test cabin was equipped with one probe hatch and two 12 inch schlieren windows. The schlieren system was that of the single path type using a strobe light source.
Figure 5. Turbulence Level in the Wake as a Function of Stagnation Pressure.
Figure 6. Conventional Wake Transition Correlation by Pallone.
Figure 7. Wind Tunnel Design Drawing.
A permanent probe support system was used which manually traversed in the axial direction as well as in the radial direction. It was therefore possible to make continuous data measurements at any axial station across the entire air stream. Use of a universal probe support system to measure total temperature, total pressure, static pressure, points of zero axial velocity, and turbulence level gave data with a minimum of change in probe interference, since only the probe tip design varied between runs. Figures 8 and 9 show the wind tunnel, access windows, model position, and probe support system.

Two probes were used to measure the pitot pressure. The design drawings for these probes are shown in Figure 10. Other researchers have shown experimentally that pitot probe number one is more accurate than probe number two when measuring pitot pressures in uniform flow at small angles of attack, primarily because of the probe inlet configuration. However, after a few trial runs it became apparent that at no point in the wake of the cone was the angle of flow large enough to be able to record any difference in the measurements of the two probes. Pitot probe number two was used throughout the remainder of the tests because its smaller exposed tip reduces the averaging effect of the probe on the velocity profiles.

The probe used to measure the static pressure is shown in Figure 10. Two pressure sensing holes are
Figure 8. Picture of the Wind Tunnel in the Laboratory.
Figure 9. Wind Tunnel Laboratory Configuration. Top: View Through Schlieren Window Showing Model Configuration and Hot-Wire Anemometer Probe. Bottom: Side Mounted Probe Traversing Mechanism.
Figure 10. Probe Tip Design Drawings. Top: Pitot Probe No. 1. Middle: Pitot Probe No. 2. Bottom: Static Pressure Probe.
located eight probe diameters downstream of the nose to reduce the nose interference effects. Whereas the point of measurement for pitot pressure is the nose of the pitot probe, the point of measurement for static pressure is the position of the sensing holes on the static probe. The experimental data obtained was labeled according to the distance of measurement from the forebody base; this distance was measured from the forebody base to the pitot probe tip for the pitot pressure profiles, and from the forebody base to the sensing holes on the static pressure probe for the static pressure profiles. The static pressure probe actually averages the static pressure over a finite region and assigns this average value to the midpoint of the probe. There is very little error involved here except at the point of intersection of a shock wave, since there is little change in static pressure in the radial direction. Intersection of the static probe and a shock wave causes a shock wave-boundary layer interaction which has more influence on static pressure reading accuracy than does probe interference with a viscous layer or an inviscid layer alone. The specific design of a conical nose was used because it gives more accurate readings over both subsonic and supersonic flow fields, regardless of any viscosity effects.

The "U = 0" probe is shown in Figure 11. This probe was used to find those points in the wake where the axial velocity was zero. Only at these points of stag-
Figure 11. "U = 0" Probe Design Drawing.
nant flow in the axial direction will the pressures at both sensing holes shown in the blow-up insert of Figure 9, be equal. The accuracy of this probe was found to be very limited due to the rather bulky construction of the probe in the axial direction. Although no large interference effects were observed through schlieren photographs or base pressure monitoring, accuracy could not be expected to be better than 10-20%, at least near the rear stagnation point where variations in velocity are largest.

The total temperature probe was a simple exposed thermocouple junction placed at the tip of the pitot pressure probe with the lead-out wires running through the probe to a temperature recorder outside the tunnel. The total temperature was not high enough that radiation was considered to be important, and by providing enough time for exposed surfaces to reach their equilibrium temperatures any conduction effects within the probe itself were minimized.

The standard probe support is shown in Figure 12. The probe tips shown in Figure 10 were attached to this support which was in turn attached to the traversing mechanism shown in Figure 9. Pictures of the standard probe support with static pressure tip attached and the "U = 0" probe are shown in Figure 13. As a reference dimension for these photographs, the support bases are
Figure 12. Universal Probe Support Design Drawing.
Figure 13. Complete Probe Systems. Top: Static Pressure Probe. Bottom: "\( U = 0 \)" Probe.
It was stated earlier that the primary purpose of this experimental investigation was to first locate the rear stagnation point and then determine the velocity and total enthalpy profiles at that point for the three cases of no tow cable, a 1/16 inch diameter tow cable, and a 1/4 inch diameter tow cable. Two methods were available for providing information on the location of the rear stagnation point: location of the zero axial velocity points in the wake with the aid of the "U = 0" probe, and locating the zero axial velocity points as those points in the wake where the pitot pressure equals the stagnation pressure.

The schlieren photographs gave the quickest overall picture of the entire wake region, but they did not give any insight into the behavior of the recirculating base flow or the location of the rear stagnation point. Measurements made from these photographs accurately located the position of the wake neck, which was then specified as the farthest point downstream of the base where the rear stagnation point could be located.

A much more time consuming process was to use the "U = 0" probe to locate the locus of points where the axial component of velocity was zero. For the case of no tow cable, or the conventional wake case, the rear stagnation point was determined as that point on the center-
line of the flow which intersects the line of zero axial velocity points. For the cases of a tow cable present, the point of intersection of the line of zero axial velocity points and the tow cable wall in the rear stagnation point of the wake, and is also the point of reattachment of the base separated flow.

The most accurate method to determine the rear stagnation point was to compare the data obtained from the pitot probe with that obtained from the static pressure probe. Those points in the flow where pitot pressure equals static pressure are points of zero axial velocity. The same procedure as that used with the "U = 0" probe method is then used to find the rear stagnation point. Some error is inherent in this method due to the process in which data is obtained. Instead of obtaining the static and pitot pressure data concurrently, this separate data is collected during two different runs; and it is therefore possible that inconsistencies in stagnation conditions and probe positions will result in a slight error when comparing small pressures which are very nearly equal.
CHAPTER V

PRESENTATION OF EXPERIMENTAL RESULTS

The first experiments performed were those designat- ed as Test Series No. 1 and the flow conditions are those specified in Table 1. These were the only tests per­formed where the density was high enough to obtain good quality schlieren photographs. These photographs are presented in Figure 14. Although the wake was later determined to be turbulent from velocity profile analysis and hot-wire anemometer tests, these schlieren photo­graphs do give a good picture of the flowfield under consideration even though the wake is turbulent. Qual­itatively, no useful measurements such as turbulence level, position of rear stagnation point, or tow cable diameter effects could be obtained from these photographs. The wake shock is clearly defined, but the inner viscous wake can not be located precisely. However, these photographs did serve the original purpose of defining an undisturbed, fully developed near wake flowfield.

For the lower Reynolds number tests, Test Series Nos. 2 and 3, the density was too low to obtain good quality schlieren photographs. Even for the laminar wake, the general features of the near wake flowfield
Figure 14. Schlieren Photographs of Near Wake Flowfield. Top: No Tow Cable. Middle: 1/16 inch Tow Cable. Bottom: 1/4 inch Tow Cable.
were not expected to change from those of Figure 14, although there would be some variation in wake thickness and position of the rear stagnation point and neck.

Some concern was initially expressed as to the interference effects of inserting the standard probe into the near wake region. Figure 15 shows a comparison of the flowfield both with and without the standard probe inserted into the flow. This comparison and the results of monitoring the base pressure before and during probe insertion indicated that the probe interference was minimal. The maximum variation in base pressure directly caused by probe insertion was less than one percent, and in most cases was not even noticeable. Although schlieren comparison was impossible at the lower Reynolds numbers, monitoring of the base pressure was considered sufficient to show negligible probe interference.

For Test Series No. 1 the entire near wake region was probed from the forebody base downstream to an $x/D = 2.25$ to obtain profiles of total pressure $p_{t_2}$, static pressure $p_s$, and total temperature $T_t$. Although continuous data was obtained in the radial direction at each axial station, only a finite number of axial stations were explored. The data was then faired to obtain information at points between these axial stations. Figures 16, 17, and 18 show the experimental procedure and the data obtained for the case of no tow cable
Figure 15. Schlieren Photographs of Probe Interference. Top: Probe out of Flow. Bottom: Probe in the Flow.
Figure 16. Pitot Pressure Profiles at Various Axial Locations for Test Series No. 1 with no Tow Cable.
Figure 17. Total Temperature Profiles at Various Axial Locations with no Tow Cable, for Test Series No. 1.
Figure 18. Mach Number Profiles at Various Axial Locations for Test Series No. 1 with no Tow Cable.
present, or the conventional wake case. The scale used to present the data has been reduced considerably and should not be used to make quantitative measurements; only a general picture of the procedure is intended. Figures 19, 20, and 21 show similar results for the case of \( r_w = 0.03125 \) and Figures 22, 23, and 24 for the case of \( r_w = 0.125 \).

The procedure used to obtain data for Test Series Nos. 2 and 3 were basically the same as that given above, except that the axial probing region was reduced to a smaller region near the rear stagnation point and the distance between axial stations was reduced to 1/8 inch instead of the 1/4 inch used above.

Figures 25, 26, and 27 present the pitot pressure "maps" of the flowfield for the three cases of Test Series No. 1. These are presented here to give another picture of the flowfield under consideration. The flow expansion around the cone base corner is clearly evident, in as much as the pitot pressure is known to decrease as the probe is moved to a region of larger Mach number in hom-entropic or near-homentropic flows. The recirculation region is largely within the line labeled "1.0", whereas the shear layer is just outside the recirculation zone. The neck region cannot be located precisely, but it is in the neighborhood of \( x/D = 1.0 \), where \( x/D \) is the number of base diameters behind the cone base. The wake
Figure 19. Pitot Pressure Profiles at Various Axial Locations for Test Series No. 1 with a 1/16 inch Tow Cable.
Figure 20. Total Temperature Profiles at Various Axial Locations with a 1/16 inch Tow Cable for Test Series No. 1.
Figure 21. Mach Number Profiles at Various Axial Locations with a 1/16 inch Tow Cable for Test Series No. 1.
Figure 22. Pitot Pressure Profiles at Various Axial Locations with a 1/4 inch Tow Cable for Test Series No. 1.
Figure 23. Total Temperature Profiles at Various Axial Locations with a 1/4 inch Tow Cable for Test Series No. 1.
Figure 24. Mach Number Profiles at Various Axial Locations with a 1/4 inch Tow Cable for Test Series No. 1.
Figure 25. Pitot Pressure Map for Test Series No. 1 with no Tow Cable.
Figure 26. Pitot Pressure Map for Test Series No. 1 with a 1/16 inch Tow Cable.
Figure 27. Pitot Pressure Map for Test Series No. 1 with a 1/4 inch Tow Cable.
shock emerges from the neck region as a strip of high pitot pressure gradient. For all cases, the pitot pressure "map" features remain basically unchanged. The presence of a center rod appears to have no effect other than to displace the flowfield outward in the radial direction. For this series of tests, the recirculation zone becomes shorter with increasing wake rod diameter, as does the inner wake thickness near the neck region.

Figures 28, 29, and 30 show the pitot pressure data obtained for the three cases of Test Series No. 1. Figures 31, 32, and 33 show the static pressure data obtained. The only data presented is that through the inner viscous wake even though data was obtained further out in the stream. This pitot and static pressure data was then used to calculate the Mach number distribution from the computed ratio $p_s/p_{t2}^2$. The results of these calculations are presented in Figures 34, 35, and 36. The stations nearest the base were omitted because the flow nearest the wake centerline, or tow cable wall, was reversed and the exact Mach number could not be determined within the capabilities of the experimental test arrangement.

The pitot pressure data shows an abrupt change in the pitot pressure $p_{t2}$ near the midpoints in almost all of the profiles. These abrupt changes may be interpreted as the positions of the wake shock, which at first appears
Figure 28. Pitot Pressure Distribution for Test Series No.1 with no Tow Cable.
Figure 29. Pitot Pressure Distribution for Test Series No. 1 with a 1/16 inch Tow Cable.
Figure 30. Pitot Pressure Distribution for Test Series No. 1 with a 1/4 inch Tow Cable.
Figure 31. Static Pressure Distribution for Test Series No. 1 with no Tow Cable.
Figure 32. Static Pressure Distribution for Test Series No. 1 with a 1/16 inch Tow Cable.
Figure 33. Static Pressure Distribution for Test Series No. 1 with a 1/4 inch Tow Cable.
Figure 34. Mach Number Distribution for Test Series No. 1 with no Tow Cable.
Figure 35. Mach Number Distribution for Test Series No. 1 with a 1/16 inch Tow Cable.
Figure 36. Mach Number Distribution for Test Series No. 1 with a 1/4 inch Tow Cable.
thicker than an ideal shock wave for two reasons:

1. The thickness of the pitot pressure probe adds to the apparent thickness because it "averages" the pressure over a finite region.

2. Small amplitude motions of the shock and small motion of the model and the boundary layer add to the apparent thickness of the shock wave. The static pressure jump near the shock is even more diffuse than that of the pitot pressure because of the greater tendency for the shock wave-boundary layer interference between the probe and the wave.

The total temperature distribution was obtained, as presented in Figures 17, 20, and 23, after allowing sufficient time for the model and probe system to reach their steady-state temperatures. The maximum variation in the total temperature in the inner wake was less than five percent. Since the velocity distribution is proportional to the square root of the total temperature distribution, it was well within experimental accuracy to make the assumption of a constant total temperature equal to the wake edge value throughout the inner wake. Hence, the total temperature data, which was obtained and presented here for completeness, was not used directly in making experimental velocity profile calculations. This
same assumption was applied to the later tests of Test Series Nos. 2 and 3.

Figure 37 presents the "U = 0" line data (points of zero axial velocity) and the resultant relative positions of the rear stagnation point for all three cases. Figure 38 presents the calculated wake or boundary layer growth obtained from the results of the velocity profile calculations. The boundary-layer edge is roughly defined here as the point in the velocity profile where there is zero slope (except at the tow cable wall). The edge of the boundary layer, or wake, could not be determined from any of the raw data obtained. Only after the Mach number and velocity profiles had been calculated could the viscous edge be found. The variation in velocity outside the viscous layer is caused not by viscous effects but by the non-homentropic nature of the flow field. To serve as a means for showing how the boundary layer edge was determined, Figure 39 is included which shows the velocity distribution for the conventional wake case from the centerline out to an arbitrary distance of r/D = 0.5. Each velocity profile has a point off of the centerline where the slope is either zero or near zero. The points of zero slope are presumed to be the corresponding wake edges for the specific axial stations. For the very few profiles where the slope did not become zero, the profiles were extrapolated to find the edge locations.
Figure 37. "\(U = 0\)" Line Data and Resulting Rear Stagnation Points for the Three Cases of Test Series No. 1.
Figure 38. Wake Growth Data for the Three Cases of Test Series No. 1.
Figure 39. Velocity Profiles Calculated from Test Series No. 1 with no Tow Cable.
Figures 40, 41, and 42 present the results of the calculations for the velocity profiles at various stations downstream of the base, as obtained from the Mach number calculations and total temperature data. The viscous wake or boundary-layer thickness has been designated as $\delta$ and the edge velocity as $u_e$. Each profile has been non-dimensionalized by a different edge velocity because it is the relative variation in velocity that is important. The variation in edge velocity is caused by non-homentropic flow and slight variation in stagnation temperature for each wind tunnel run.

The velocity profiles have been plotted as a function of the non-dimensional radial coordinate $(r-r_w)/\delta$ which has minimum and maximum values of 0 and 1 no matter what the tow cable radius or what the boundary-layer thickness is. For the conventional wake case, the centerline velocity is zero only at two points, the cone base and the rear stagnation point at $x/D = 0.73$. For those cases where a tow cable is present, the no slip condition requires that the velocity be zero along the entire surface of the tow cable. Comparing the velocity profiles for the three cases at any particular axial station, the change in velocity caused by the presence of a tow cable is confined to a relatively narrow region near the tow cable wall. Outside this region there is very little difference. Figure 43 shows a comparison of the velocity
Figure 40. Velocity Profiles Through the Wake for Test Series No. 1 with no Tow Cable.
Figure 41. Velocity Profiles Through the Wake for Test Series No. 1 with a 1/16 inch Tow Cable.
Figure 42. Velocity Profiles Through the Wake for Test Series No. 1 with a 1/4 inch Tow Cable.
Figure 4-3. Velocity Profiles at the Rear Stagnation Point for the Three Cases of Test Series No. 1.
profiles for the three cases at the rear stagnation point. At this axial station the presence of a tow cable has very little effect on the velocity distribution through the viscous layer. In the figure only a finite number of discrete points are plotted, but the scatter is within experimental error of the faired curve for all three cases.

The results of Test Series No. 1 indicate that for small tow cables there is very little difference indicated between the measured profiles of the conventional turbulent wake and a wake under identical conditions with a tow cable present. The turbulent wake in the presence of a tow cable includes both a sublayer region and an outer turbulent mixing layer, and the effects of the tow cable are limited to a sublayer type region.

Only the results of Test Series No. 1 have thus far been discussed in detail. The experimental procedures and data reduction methods have been established and were the same for the remaining Test Series Nos. 2 and 3. However, the results of Test Series No. 1 were not applicable to the previous laminar finite-difference solution to the wake problem. The results obtained were enlightening and provided an increased understanding of this particular wake problem, but further experiments at lower Reynolds numbers were necessary to obtain laminar wake data.
Test Series No. 2 was initiated under the flow conditions as specified in Table 1. After obtaining experimental data and calculating velocity profiles by the same procedure as in Test Series No. 1, the data indicated that a turbulent, or at least transitional, wake existed even at this lower Reynolds number flow. An extensive presentation of the results of this test series is not presented because the results are very similar to those of Test Series No. 1.

The lowest Reynolds number case explored was that of Test Series No. 3. The flow conditions of this test series are specified in Table No. 1. The results of hot wire anemometer tests indicated that the near wake flow was laminar up to a freestream Reynolds number of about 8,000 and transitional up to a Reynolds number of at least 80,000. It was not possible to determine if the highest Reynolds number case of 455,000 was fully developed turbulent flow, or just transitional. The experimental equipment available was not capable of determining the effect of the presence of a tow cable on turbulence, although it was felt that the presence of a surface at the wake centerline should tend to stabilize the viscous layer.

The experimental data was obtained in Test Series No. 3 in much the same manner as for the previous test series. The region near the rear stagnation point was
probed at axial stations spaced 1/8 inches apart, and at the calculated rear stagnation point after the initial data was obtained and this point could be located exactly. The basic form of the experimental data obtained is similar to that obtained in the other test series and the pitot pressure and static pressure data is presented in Figures 44 through 49 for the three cases of no tow cable, and 1/4 inch diameter tow cable, and a 1/16 inch diameter tow cable. The calculated Mach number distributions are shown in Figures 50, 51, and 52 for the three cases considered. The calculated velocity profiles are presented in Figures 53, 54, and 55 for all three cases. The velocity profiles at the rear stagnation point for all three cases are presented in Figure 56. Very little difference between these profiles is noticed especially for the cases with a tow cable present.

A comparison of the velocity profiles shown in Figure 56 shows very little effect of the presence of the tow cable or its variable diameter. There is also very little difference between these laminar profiles and the turbulent profiles shown in Figure 43. These limited effects of the presence of a tow cable are confined only to the rear stagnation point, however.

For laminar flow, the rear stagnation point and neck are further downstream from the base in terms of number of forebody base diameters for all cases of tow cable
Figure 44. Static Pressure Profiles for Test Series No. 3 with no Tow Cable.
Figure 45. Pitot Pressure Profiles for Test Series No. 3 with no Tow Cable.
Figure 46. Static Pressure Profiles for Test Series No. 3 with a 1/16 inch Tow Cable.
Figure 47. Pitot Pressure Profiles for Test Series No. 3 with a 1/16 inch Tow Cable.
Figure 48. Static Pressure Profiles for Test Series No. 3 with a 1/4 inch Tow Cable.
Figure 49. Pitot Pressure Profiles for Test Series No. 3 with a 1/4 inch Tow Cable.
Figure 50. Mach Number Profiles for Test Series No. 3 with no Tow Cable.
Figure 51. Mach Number Profiles For Test Series No. 3 with a 1/16 inch Tow Cable.
Figure 52. Mach Number Profiles for Test Series No. 3 with a 1/4 inch Tow Cable.
Figure 53. Velocity Profiles Through the Wake for Test Series No. 3 with no Tow Cable.
Figure 54. Velocity Profiles Through the Wake for Test Series No. 3 with a 1/16 inch Tow Cable.
Figure 55. Velocity Profiles Through the Wake for Test Series No. 3 with a 1/4 inch Tow Cable.
Figure 56. Velocity Profiles at the Rear Stagnation Point for the Three Cases of Test Series No. 3.
diameter when compared to the results of the turbulent and transitional flow of Test Series Nos. 1 and 2. At this lower Reynolds number the thickness of the wake in the neck region is considerably greater, as later presentation of results will show.

The effect of tow cable presence on the velocity profiles at the rear stagnation point becomes much more evident when the profiles are expressed in terms of the non-dimensional transformed coordinate \( \eta \), as presented in Figure 57. The definite trend established by this representation is primarily caused by the physical transformation with non-zero tow cable radius to the coordinate \( \eta \) and not by any difference in physical velocity profiles.

Figure 57 also shows the velocity profile established by the fourth-order integral representation

\[
\frac{u}{u_e} = 6\eta^2 - 8\eta^3 + 3\eta^4
\]

for the limiting case of \( r_w = 0.0 \). Although this profile is nearly within experimental accuracy of all three experimental profiles, it is not evident yet that this simple representation for the velocity profiles at the rear stagnation point is universal for all tow cable radii, even if \( r_w \) remains small.
Figure 57. Transformed Velocity Profiles at the Rear Stagnation Point for the Three Cases of Test Series No. 3.
CHAPTER VI

PRESENTATION OF THEORETICAL RESULTS

Once the experimental program had been completed and the initial data and boundary conditions obtained, it was then possible to complete the theoretical calculations using the finite-difference method of solution discussed in Chapter III.

Figures 58 and 59 show the comparisons of the experimental results and the theoretical results obtained from the finite-difference solution using the computer program. These comparisons of results are good and are within the expected experimental accuracy of the system. The differences in experimental and theoretical results are presumably caused by slight inaccuracies in the assumption of constant static pressure across the layer. Small errors in the calculations also result from slight variation in edge velocity caused by non-homentropic flow and small variations in the axial variation of edge static pressure which could not be entirely taken into account in the theoretical calculations. Figures 60 and 61 show the edge variations and centerline or tow cable wall variations in static pressure. The static pressure at any given axial station was approximated as 114
Figure 58. Comparison of Theoretical and Experimental Velocity Profiles for Laminar Flow with a 1/16 inch Tow Cable.
Figure 59. Comparison of Theoretical and Experimental Velocity Profiles for Laminar Flow with a 1/4 inch Tow Cable.
Figure 60. Centerline or Tow Cable Wall Variation of Static Pressure for Test Series No. 3.
Figure 61. Wake Edge Variation of Static Pressure for Test Series No. 3.
a representative constant value and axial variation of this static pressure was approximated by a straight line variation. These approximations become more accurate further downstream where both the axial and radial variations in static pressure become much smaller. Even near the rear stagnation point, where the calculations begin, the variations in static pressure have damped considerably from those in the base region.

Figure 62 shows the experimental results of determining the "U = 0" line (points of zero axial velocity) and the boundary-layer or wake edge. The circles are those points computed by the theoretical program and show good agreement. The theoretical program not only accurately predicts the boundary-layer edge but also predicts the formation of the neck. The accuracy is somewhat surprising since only an approximate variation in edge static pressure is specified. Apparently the initial starting velocity profile at the rear stagnation point has a great deal to do with initiating the formation of the neck because even when a constant edge static pressure distribution is assumed the theoretical calculations predict the neck formation with surprising accuracy. The distances from the forebody base considered here were relatively short and variations in wake thickness were quite small so that theoretical calculations could not be too far from experimental observations even
Figure 62. Comparison of Experimental and Theoretical Wake Growth for the Three Cases of Test Series No. 3.
if neck formation in the theoretical calculations was not obtained.

One should not be too hasty in complimenting the results of the accurate prediction of the wake edge, however. The means of specifying the boundary-layer edge are somewhat arbitrary both in the experimental sense and the theoretical sense. Care must be taken that the procedures of each be consistent. The comparison of experiment and theory look good here; but this may be because the region of investigation was comparatively small and the variation of wake or boundary-layer edge in the axial direction, or the wake growth, was very small.

Theoretical calculations were also performed using the conventional wake case as a reference point to calculate the non-dimensional momentum defect as in the "first" case described in the Appendix. The solutions were then obtained for tow cable radii of $r_w = 0.03125$ and $r_w = 0.125$ as "second" cases. The theoretical results were within four place accuracy of those already presented. Care must be taken in non-dimensionalizing the momentum defect because of small variations in edge velocities for the three cases, especially when computing the wake growth.

Theoretical calculations were also made for the two cases of a zero diameter tow cable and a 1/16 inch dia-
meter tow cable downstream to $x/D = 10.0$. The results of these calculations are shown in Figures 63 and 64. To make the calculations for the zero diameter tow cable case, it was assumed that the starting profile obtained for the conventional wake at the rear stagnation point could be used without appreciable loss in accuracy, as concluded in previous discussion.

Figure 63 presents the variation in velocity profiles in the axial direction for the one case of a 1/16 inch diameter tow cable. As for all cases, the largest variation in the velocity profiles occurs in the neck region and downstream of $x/D = 5$ there is only a slight change.

Figure 64 presents the wake growths for the two cases considered. Also shown in this figure are curves nearly representing the conventional wake growth downstream of the neck for laminar flow ($\approx x^{1/4}$) and turbulent flow ($\approx x^{1/2}$) as obtained from a combination of theory and experiment. Addition of a small tow cable reduces the wake width at any given station and initially reduces the wake growth rate, at least for the case of laminar flow. Far downstream, however, the wake growth rate approaches the conventional wake growth rate proportional to the $1/2$ power in axial distance. Figure 64 also indicates that increasing the size of the tow cable increases the
Figure 63. Velocity Profile Distribution Theoretically Calculated for the Case of a 1/16 inch Tow Cable.
Figure 64. Comparison of the Wake Growths for Laminar Flow with the Conventional Wake Growth for a 1/16 inch Tow Cable and for a Zero-diameter Tow Cable.
wake width at any given station, at least for small tow cables, and assuming initially constant momentum defect at the rear stagnation point independent of the size of the tow cable. The conventional wake growth rate is also more rapidly reached for larger tow cables in a laminar wake.
CHAPTER VII
CONCLUSIONS AND SUMMARY

The general conclusions about the wake behind a 10° half-angle cone in supersonic flow and the effects of a tow cable on this wake derived from this study are listed below:

1. Significant contributions have been made by the experimental program of this study to generally describe the wake characteristics in supersonic flow and the effects of the presence of a tow cable on this wake. The data which was obtained adds considerable to an area of interest which has not been given much attention in the past.

2. A theoretical method for the solution of the tow cable-wake problem has been developed which will add a useful tool for future investigations in this area.

3. The turbulent experimental data indicates that the effects of the tow cable are confined to a very thin sublayer near the cable surface.
4. The laminar experimental data indicates that the effect of the presence of a tow cable for this case is not confined to a thin layer as for the turbulent case, but extends over the entire wake thickness.

5. The physical velocity profiles at the rear stagnation points for both laminar and turbulent flows are characterized by zero slope at the centerline or tow cable wall and these profiles do not vary greatly from those of the conventional wake case so long as the tow cable is small compared to the wake thickness.

More specific conclusions drawn from the overall aspects of this study are listed below:

1. The finite-difference method of solution accurately predicts the axial variation in the velocity profiles and the boundary-layer or wake growth for laminar flow behind a 10° half-angle cone in supersonic flow at a Mach number of 3.65, at least up to an $x/D = 0.5$ downstream of the rear stagnation point.

2. The accuracy of the finite-difference solution to the tow cable-wake problem is good only if extreme care is taken to specify
the initial starting conditions and wake edge conditions.

3. For all tow cable sizes considered and for the conventional wake case, the largest variations in the velocity profiles occur near the rear stagnation point, and downstream of $x/D = 5.0$ there is very little change in the profile shape.

4. The variation in the velocity profile shape at the tow cable wall downstream of the rear stagnation point is greater for increasing tow cable diameter. The velocity profile slope at the wall increases more rapidly for larger tow cables, which perform correspondingly more work on the viscous layer.

5. The finite-difference method of solution accurately predicts the formation of the neck downstream of the rear stagnation point even with the assumption of zero static pressure gradient for all tow cable sizes considered.

6. Contrary to the predictions made by Nerem that a cylindrical wake with no appreciable wake growth would result for the case of a zero-diameter tow cable, the finite-dif-
ference method of solution does predict some wake growth. It is, however, much less than for a conventional wake. Whereas the integral method approach of Nerem assumes that the wake velocity profiles can be assumed invariant in the axial direction, the finite-difference method of solution allows axial variation of the velocity profiles, and shows that only far downstream do the velocity profiles become invariant.

7. The results of the theoretical analysis indicates that even for the case of the zero-diameter tow cable, there is some wake growth, and that this wake growth increases with increasing tow cable diameter under the conditions of a constant diameter forebody base with constant free-stream conditions. For small tow cables, the conventional wake case with no tow cable present represents the greatest wake growth rate downstream of the neck, which has previously been shown to be proportional to the 1/2 power of the axial distance from the forebody base. Increasing the tow cable diameter increases the wake
thickness at any given axial station, and also increases the wake growth rate at that point, at least when comparing calculations made for a 1/16 inch tow cable and a zero-diameter tow cable. The zero-diameter tow cable caused the lowest wake growth rate of all the cases considered.

The disadvantage of using the finite-difference method of solution is that its accuracy is highly dependent upon the accuracy with which the initial starting conditions are specified. Unfortunately it was not possible to obtain a clearcut dependency of these starting conditions at the rear stagnation point on variable tow cable diameter and it was therefore necessary to obtain experimental starting conditions for each tow cable radius considered. The finite-difference method of solution could be a much more useful method if in the future an analytical means for specifying the starting conditions at the rear stagnation point could be developed covering a larger spectrum of forebody shapes, freestream conditions, and tow cable diameters.
APPENDIX

A computer program was written to solve the tow cable‐wake problem by the finite-difference method discussed in Chapter III of the text. The equations which were derived in that section to solve for the velocity and temperature distributions came from the momentum and energy conservation equations, reduced to finite-difference form and finally written in matrix notation for direct use in a computer program as equations 35 and 44 of the text. Although the experimentally supplied version of the computer program, written in Fortran IV language, is presented and discussed in this Appendix, there was also a purely theoretical version written; but there are only slight modifications between these two versions.

The body configuration, wind tunnel stagnation conditions, freestream conditions, and initial wake edge conditions are first supplied as experimental data. Calculations are then made to specify freestream and initial wake edge pressures, temperatures, and velocities.

To completely define the initial starting conditions, the velocity and enthalpy profiles are specified by supplying the coefficients of the assumed transformed
velocity and enthalpy profiles obtained by another separate program which uses the experimental data. The procedures used in this program are described in the discussion that follows.

When the boundary layer initial conditions are specified in the $\eta$-plane, the inverse transformation is needed to transform into the physical or $r$-plane. Since the static pressure was assumed constant through the layer, the simple relation between static temperature $T$ and density is

\[ \frac{T}{T_e} = \frac{\rho_e}{\rho} \]

When equation 1 is substituted into equation 5 of the text and integrated,

\[ \frac{r^2}{2} - \frac{r_w^2}{2} = \Delta^2 \int_0^{\eta} \frac{T}{T_e} \left[ \eta + \frac{r_w}{\Delta} \right] d\eta \]

Or, since $n$ is defined such that $n_w = r_w$,

\[ \frac{r^2}{2} - \frac{r_w^2}{2} = \Delta^2 \int_0^{\eta} \frac{T}{T_e} \left[ \eta + \frac{r_w}{\Delta} \right] d\eta \]

A relation for obtaining the transformed thickness $\Delta$ in terms of the boundary-layer thickness is then obtained from equation 3 at the edge:

\[ \frac{r_e^2}{2} - \frac{r_w^2}{2} = \Delta^2 \int_0^1 \frac{T}{T_e} \left[ \eta + \frac{r_w}{\Delta} \right] d\eta \]
or,

\[
\frac{r_8 - r_w}{2\Delta^2} = \int_0^1 \frac{T}{T_e} \left[ \eta + \frac{r_w}{\Delta} \right] d\eta
\]

However, solution of this equation at this point would require some iterative procedure to obtain a value for the transformed thickness.

The form of the assumed velocity profile is

\[
\frac{u}{u_e} = \sum_{i=0}^{6} a_i \eta^i
\]

where either seven boundary conditions are needed to determine the coefficients \(a_i\), or at least seven experimental data points are needed. A total enthalpy profile, for the general case of Prandtl number not equal to one, is assumed of the form

\[
\frac{H}{H_e} = \sum_{i=0}^{6} b_i \eta^i
\]

where the \(b_i\) coefficients are determined to be consistent with the velocity coefficients \(a_i\).

The temperature distribution can then in general be calculated by

\[
\frac{T}{T_e} = \left[ 1 + \frac{u_e^2}{2C_p T_e} \right] \sum_{i=0}^{6} b_i \eta^i - \frac{u_a^2}{2C_p T_e} \left[ \sum_{i=0}^{6} a_i \eta^i \right]^2
\]
or, more precisely,

\[
\frac{T}{T_e} = \beta \left( b_0 + b_1 \eta + b_2 \eta^2 + b_3 \eta^3 + b_4 \eta^4 + b_5 \eta^5 + b_6 \eta^6 \right) - \beta \left[ a_0 \left( a_0 + 2 a_1 \eta + 2 a_2 \eta^2 + 2 a_3 \eta^3 + 2 a_4 \eta^4 + 2 a_5 \eta^5 + 2 a_6 \eta^6 \right) + a_1 \eta \left( a_1 \eta + 2 a_2 \eta^2 + 2 a_3 \eta^3 + 2 a_4 \eta^4 + 2 a_5 \eta^5 + 2 a_6 \eta^6 \right) + a_2 \eta^2 \left( a_2 \eta^2 + 2 a_3 \eta^3 + 2 a_4 \eta^4 + 2 a_5 \eta^5 + 2 a_6 \eta^6 \right) + a_3 \eta^3 \left( a_3 \eta^3 + 2 a_4 \eta^4 + 2 a_5 \eta^5 + 2 a_6 \eta^6 \right) + a_4 \eta^4 \left( a_4 \eta^4 + 2 a_5 \eta^5 + 2 a_6 \eta^6 \right) + a_5 \eta^5 \left( a_5 \eta^5 + 2 a_6 \eta^6 \right) + (a_6 \eta^6)^2 \right]
\]

(9)

where,

\[
\beta = 1 + \frac{u_e^2}{2C_p T_e}
\]

(10)

and,

\[
\beta_1 = \beta - 1
\]

(11)

or finally,

\[
\frac{T}{T_e} = \beta \sum_{i=0}^{6} b_i \eta^i - \beta_1 \left[ \sum_{i=0}^{6} (a_i \eta^i)^2 + 2 \sum_{i=0}^{5} \sum_{j=i+1}^{6} (a_i \eta^i)(a_j \eta^j) \right]
\]

(12)

Using equation 12 in equation 3 the following expression is obtained for the physical dimension \( r \):
\[ \frac{r^2 - r_w^2}{2\Delta^2} = \eta^2 \left\{ \beta \sum_{i=0}^{6} \frac{b_i \gamma^i}{i+2} - \beta \left[ \sum_{i=0}^{6} \frac{(a_i \gamma^i)^2}{2i+2} + 2 \sum_{i=0}^{5} \sum_{j=i+1}^{6} \frac{(a_i \gamma^i)(a_j \gamma^j)}{i+j+2} \right] \right\} + \frac{r_w \eta}{\Delta} \left\{ \beta \sum_{i=0}^{6} \frac{b_i \gamma^i}{i+1} - \beta \left[ \sum_{i=0}^{6} \frac{(a_i \gamma^i)^2}{2i+1} + 2 \sum_{i=0}^{5} \sum_{j=i+1}^{6} \frac{(a_i \gamma^i)(a_j \gamma^j)}{i+j+1} \right] \right\} \]

(13)

Setting \( \eta = 1 \) in equation 13, an expression for \( \Delta \) can be obtained in the form,

\[ \frac{r^2 - r_w^2}{2\Delta^2} = A + \frac{r_w}{\Delta} B \Rightarrow A \Delta^2 + r_w B \Delta - \frac{r^2 - r_w^2}{2} = 0 \]

and hence, solving for \( \Delta \),

\[ \Delta = \frac{-r_w B \pm \sqrt{r_w^2 B^2 + 2(r_w^2 - r_w^2)A}}{2A} \]

(15)

which yields two values for \( \Delta \), one positive and one negative, the positive one being physically meaningful.

An expression for the coordinate \( \psi \) can also be obtained in terms of \( \eta \). From equation 21 of the text

\[ \psi d\psi = \rho u r dr \]

(16)

and equation 5 of the text

\[ \left[ \eta + \frac{r_w}{\Delta} \right] d\eta = \frac{1}{\Delta^2} \frac{\rho}{\rho_e} r dr \]

(17)

the result of combining these two equations is

\[ \psi d\psi = \Delta^2 \rho_e u \left[ \eta + \frac{r_w}{\Delta} \right] d\eta \]

(18)
or,

\[
\frac{\psi^2}{2\Delta^2 \rho u_e} = \int_0^1 \frac{u}{u_e} \left[ \eta + \frac{r_w}{\Delta} \right] d\eta
\]

Integrating this equation yields the expression

\[
\frac{\psi^2}{2\Delta^2 \rho u_e} = \eta^2 \left[ \frac{a_0}{2} + \frac{a_1 \eta}{3} + \frac{a_2 \eta}{4} + \frac{a_3 \eta}{5} + \frac{a_4 \eta}{6} + \frac{a_5 \eta}{7} + \frac{a_6 \eta}{8} \right] + \frac{r_w}{\Delta} \eta \left[ \frac{a_0}{2} + \frac{a_1 \eta}{3} + \frac{a_2 \eta}{4} + \frac{a_3 \eta}{5} + \frac{a_4 \eta}{6} + \frac{a_5 \eta}{7} + \frac{a_6 \eta}{8} \right]
\]

or finally,

\[
\frac{\psi^2}{2\Delta^2 \rho u_e} = \eta^2 \sum_{i=0}^{6} \frac{a_i \eta^i}{i+2} + \frac{r_w}{\Delta} \eta \sum_{i=0}^{6} \frac{a_i \eta^i}{i}
\]

The momentum thickness is defined by

\[
\rho_e u_e^2 [\pi (r_w + \theta)^2 - \pi r_w^2] = 2\pi \int_{r_w}^r \rho u (u_e - u) r dr
\]

or,

\[
\theta^2 + 2r_w \theta + r_w^2 - r_w^2 = 2 \int_{r_w}^r \frac{u}{u_e} \left[ 1 - \frac{u}{u_e} \right] \frac{\rho}{\rho_e} r dr
\]

Making use of the transformation from \(r\) to \(\eta\), the following expression results:

\[
\frac{\theta^2 + 2r_w \theta}{\Delta^2} = 2 \int_0^1 \frac{u}{u_e} \left[ \eta + \frac{r_w}{\Delta} \right] d\eta
\]

Completing the integration,
\[
\frac{\theta^2 + 2\theta r_w}{2\Delta^2} = \left( \frac{a_0 + a_1}{2} + \frac{a_2}{3} + \frac{a_3}{4} + \frac{a_4}{5} + \frac{a_5}{6} + \frac{a_6}{8} \right) - \left[ a_0 \left( \frac{a_0 + 2a_1}{2} + \frac{2a_2}{3} + \frac{2a_3}{4} + \frac{2a_4}{5} + \frac{2a_5}{6} + \frac{2a_6}{8} \right) \right]
\]
\[
+ a_1 \left( \frac{a_1}{4} + \frac{2a_2}{5} + \frac{2a_3}{6} + \frac{2a_4}{7} + \frac{2a_5}{8} + \frac{2a_6}{9} \right)
\]
\[
+ a_2 \left( \frac{a_2}{6} + \frac{2a_3}{7} + \frac{2a_4}{8} + \frac{2a_5}{9} + \frac{2a_6}{10} \right)
\]
\[
+ a_3 \left( \frac{a_3}{8} + \frac{2a_4}{9} + \frac{2a_5}{10} + \frac{2a_6}{11} \right)
\]
\[
+ a_4 \left( \frac{a_4}{10} + \frac{2a_5}{11} + \frac{2a_6}{12} \right) + a_5 \left( \frac{a_5}{12} + \frac{2a_6}{13} \right) + \frac{(a_6)^2}{14}
\]

\[
(25) + \frac{r_w}{\Delta} \left\{ a_0 \left( \frac{a_0 + a_1}{2} + \frac{a_2}{3} + \frac{a_3}{4} + \frac{a_4}{5} + \frac{a_5}{6} + \frac{a_6}{7} \right) \right\}
\]
\[
- \left[ a_0 \left( \frac{a_0 + 2a_1}{2} + \frac{2a_2}{3} + \frac{2a_3}{4} + \frac{2a_4}{5} + \frac{2a_5}{6} + \frac{2a_6}{8} \right) \right]
\]
\[
+ a_1 \left( \frac{a_1}{3} + \frac{2a_2}{4} + \frac{2a_3}{5} + \frac{2a_4}{6} + \frac{2a_5}{7} + \frac{2a_6}{8} \right)
\]
\[
+ a_2 \left( \frac{a_2}{5} + \frac{2a_3}{6} + \frac{2a_4}{7} + \frac{2a_5}{8} + \frac{2a_6}{9} \right)
\]
\[
+ a_3 \left( \frac{a_3}{7} + \frac{2a_4}{8} + \frac{2a_5}{9} + \frac{2a_6}{10} \right)
\]
\[
+ a_4 \left( \frac{a_4}{9} + \frac{2a_5}{10} + \frac{2a_6}{11} \right) + a_5 \left( \frac{a_5}{11} + \frac{2a_6}{12} \right) + \frac{(a_6)^2}{13}\}
\]

which can be more conveniently written as
\[ \frac{\theta^2 + 2\theta r_w}{2\Delta^2} = \sum_{i=0}^{6} \frac{a_i}{i+2} - 2 \sum_{i=0}^{5} \sum_{j=i+1}^{6} \frac{a_ia_j}{i+j+2} - \sum_{i=0}^{6} \frac{(a_i)^2}{2i+2} \]

(26)

\[ + \frac{r_w}{\Delta} \left[ \sum_{i=0}^{6} \frac{a_i}{i+1} - 2 \sum_{i=0}^{5} \sum_{j=i+1}^{6} \frac{a_ia_j}{i+j+1} - \sum_{i=0}^{6} \frac{(a_i)^2}{2i+1} \right] \]

Equation 26 can also be written in the form

(27) \[ \frac{\theta^2 + 2\theta r_w}{2\Delta^2} = C(a_i) + D(a_i) \frac{r_w}{\Delta} \]

or,

(28) \[ 2C\Delta^2 + 2r_wD\Delta - (\theta^2 + 2\theta r_w) = 0 \]

which yields an expression for \( \Delta \) in terms of \( \theta, r_w, \) and \( a_i \) as

(29) \[ \Delta = \frac{-2r_wD \pm \sqrt{4r_w^2D^2 + 4(\theta^2 + 2\theta r_w)(2C)}}{4C} \]

or, choosing the physically meaningful value,

(30) \[ \Delta = \frac{-r_wD + \sqrt{r_w^2D^2 + 2C(\theta^2 + 2\theta r_w)}}{2C} \]

For the "first" case where \( r_w, r_b, a_1, \) and \( b_1 \) are specified, equation 15 is used to calculate \( \Delta \).

Rewriting equation 28,

(31) \[ \theta^2 + 2\theta r_w - 2\Delta^2(C + D \frac{r_w}{\Delta}) = 0 \]

\( \theta \) may be calculated as
The momentum defect is given by

\[ \theta = -r_w \pm \sqrt{r_w^2 + 2(C + D\frac{r_w}{\Delta})\Delta^2} \]

or, using the transformation to \( \eta \),

\[ \text{def} = 2\pi \int_{r_w}^{r_0} \rho u (u_e - u) r \, dr \]

This expression is defined as the non-dimensional momentum defect. Hence, expanding equation 34,

\[ \frac{\text{def}}{2\pi \rho u_e^2} = \Delta^2 \int_0^1 \frac{u}{u_e} \left[ 1 - \frac{u}{u_e} \left( \eta + \frac{r_w}{\Delta} \right) \right] \, d\eta \]

For a given velocity profile, all of the integrals can be calculated exactly, in terms of the \( a_i \) coefficients. Then,

\[ \frac{\text{def}}{2\pi \rho u_e^2} = \Delta^2 \left\{ \int_0^1 \frac{u}{u_e} \eta \, d\eta + \frac{r_w}{\Delta} \int_0^1 \frac{u}{u_e} \, d\eta - \int_0^1 \left[ \frac{u}{u_e} \right]^2 \eta \, d\eta - \frac{r_w}{\Delta} \int_0^1 \left[ \frac{u}{u_e} \right]^2 \, d\eta \right\} \]

where,

\[ A(a_i) = \int_0^1 \frac{u}{u_e} \eta \, d\eta \quad ; \quad C(a_i) = \int_0^1 \left[ \frac{u}{u_e} \right]^2 \eta \, d\eta \]

and

\[ B(a_i) = \int_0^1 \frac{u}{u_e} \, d\eta \quad ; \quad D(a_i) = \int_0^1 \left[ \frac{u}{u_e} \right]^2 \, d\eta \]
The equation used to calculate the non-dimensional momentum defect is

\[ \text{def} \frac{1}{2 \pi \rho u_e^2} = (A^* - C^*) \Delta^2 + r_w (B^* - D^*) \Delta \]  

For the "second" case, the value of the non-dimensional momentum defect found in the "first" case is specified along with \( r_w, a_i, \) and \( b_i \). Equation 38 is then rewritten to solve for \( \Delta \) as

\[ \Delta = \frac{(D^* - B^*) r_w + \sqrt{r_w^2 (B^* - D^*)^2 + 4 (A^* - C^*) \text{def} / 2 \pi \rho u_e^2}}{2 (A^* - C^*)} \]  

Equation 15 can be rewritten to solve for \( r_\delta \) by

\[ r_\delta = \sqrt{r_w^2 + 2 \Delta^2 (A^* + B^* r_w / \Delta)} \]  

and equation 32 is used to calculate \( \theta \).

The procedure for making an analytical calculation is to choose one value of tow cable radius \( r_w \) as a point of reference, or the "first" case. Calculations for other values of tow cable radius are referred to the "first" case by holding the non-dimensional momentum defect constant for all succeeding "second" cases.

The non-dimensional momentum defect is the constant parameter used to define the characteristics of the wake at the rear stagnation point because this is the only parameter which does not vary with tow cable radius, so long as the freestream flow conditions remain fixed.
The other parameters, $\theta$, $\zeta$, $\Delta$, $a_i$, and $b_i$, are allowed to change with tow cable radius, even at the rear stagnation point.

The remaining variables to be specified in the finite-difference solution program are the step sizes to be taken in the axial and radial directions. Specifying the axial step size presents no problem because of its physical nature. However, the radial step size cannot be readily specified because it is not known beforehand what the absolute magnitude of the coordinate $\psi$ will be. Hence, rather than specifying $\Delta \psi$, the number of steps across the wake in the radial direction is specified as NSTP in the program. Equation 21 is then used to calculate the wake edge value of the coordinate $\psi$, and the quantity $\psi_e/NSTP$ defines the step size in the radial direction.

Once the initial starting and edge conditions have been supplied and the radial step size calculated, all the variables at the starting line, or first axial grid station, can be calculated at each radial grid point. The procedure is to specify several values of the transformed coordinate $\eta$, calculate the corresponding values of velocity and temperature from equations 6 and 12, calculate the radius from equation 13, and finally calculate the corresponding values of $\psi$ from equation 21. A numerical interpolation scheme is then used to calculate the velocity, temperature, and radius as a
function of the coordinate $\psi$ at each radial grid location.

Equations 35 and 44 of the text are then used to calculate the values of velocity and temperature at the next axial grid location, a distance $\Delta S$ downstream of the first station. Because of the grid definition, the values of $\psi(2,i)$ remain the same as $\psi(1,i)$ at each radial grid point. The numerical integration methods discussed in Chapter III of the text are then used to calculate the radii for the new grid locations.

The same procedure is then used at the next axial grid station downstream, and so on until the farthest axial station downstream which was initially specified is reached.

The complete calculation procedure has been described above and should be sufficient to obtain a finite-difference solution to the tow cable-wake problem using the computer program written by this author. The Fortran IV program follows with a definition of the notation used so that the specific methods used may be more clearly presented to those interested.

Symbols used in the program are defined as follows:

A(I) = Initial velocity profile coefficients
AINTR = Interpolation scheme subroutine
AMACHE = Initial wake edge Mach number
AMEX = Freestream Mach number
B(I) = Initial total enthalpy profile coefficients
CBETA = Constant $\beta$ defined in equation 10
CBETAI = Constant $\beta_1$ defined in equation 11
DEFECT = Non-dimensional momentum defect defined in equation 35
DEL = Wake edge thickness measured from tow cable surface
DELS = Axial step size $\Delta S$
DELPsi = Radial step size $\Delta \psi$
DELSTR = Wake edge thickness measured from tow cable surface
DELTA = Transformed wake thickness defined in equation 30
ETA = Non-dimensional transformed radial coordinate defined in equation 17
FNCT1 = Numerical integration subroutine
NSTP = Number of radial grid divisions
PDIF = Wake edge pressure distribution
PE = Wake edge static pressure
PEX = Freestream static pressure
PTOTE = Wake edge total pressure
PSIE = Edge value of the coordinate $\psi$
PSRAT(I) = Initial ratio $\psi / \psi_e$
PSTAG = Stagnation pressure
R(I,J) = Radii at grid location i,j; or $r_{i,j}$
RDEL = Value of radius at initial wake edge
RHOE = Initial wake edge density
RHOEX = Freestream density
RRATI(I) = Initial non-dimensional radial coordinate, \((r-r_w)/\delta\)
RWALL = Tow cable radius
S = Axial coordinate measured from rear stagnation point, or initial starting line
SCHK = Axial coordinate used to define the print-out interval
T(I,J) = Temperature at grid location i,j; or \(T_{i,j}\)
TE = Wake edge static temperature
TEX = Freestream static temperature
THETA = Initial momentum thickness \(\theta\)
TRATI(I) = Initial non-dimensional temperature, \(T/T_e\)
TTOTE = Initial wake edge total temperature
TSTAG = Stagnation temperature
TW = Tow cable wall temperature
U(I,J) = Velocity at grid location i,j; or \(u_{i,j}\)
UEX = Freestream velocity
UE = Initial wake edge velocity
URATI(I) = Initial non-dimensional velocity \(u/u_e\)
XM(I) = Initial Mach number data
XR(I) = Initial radius data
XT(I) = Initial temperature data
XSTRT = Initial starting location
CALCULATION OF STING WAKE BY FINITE DIFFERENCE

TECHNIQUE USING EXPERIMENTALLY DETERMINED STARTING PROFILE

DIMENSION XR(10),XM(10),XT(10),X(10),Y(10),Z(10),
A(7),B(7),URATI(111),TRATI(111),RRATI(111),
PSRATI(111),U(2,400),T(2,400),R(2,400),XXB(11),
YYB(11)

WRITE(6,100)

C BODY CONFIGURATION
READ(5,201)RWALL
READ(5,201)XSTRT
PDIF=0.0
WRITE(6,101)RWALL,XSTRT

C STAGNATION CONDITIONS
READ(5,201)TSTAG
READ(5,201)PSTAG
WRITE(6,102)TSTAG,PSTAG

C EXIT CONDITIONS
READ(5,201)AMEX
PEX=PSTAG/((1.0+0.2*AMEX**2)**3.5)
TEX=TSTAG/(1.0+0.2*AMEX**2)
UEX=AMEX*SQR(2402.4*TEX)
RHOEX=PEX/(1716.0*TEX)
WRITE(6,103)AMEX,PEX,TEX,UEX,RHOEX

C EDGE CONDITIONS AT DATA LOCATION
READ(5,201)AMACHE
READ(5,201)PE
READ(5,201)TTOTE
READ(5,201)DELSTR
PTOTE=PE*((1.0+0.2*AMACHE**2)**3.5)
TE=TTOTE/(1.0+0.2*AMACHE**2)
UE=AMACHE*SQR(2402.4*TE)
RHOE=PE/(1716.0*TE)
CBETAI=UE*UE/12012.0/TE
CBETA=1.0+CBETAI
WRITE(6,104)AMACHE,PE,TTOTE,PTOTE,TE,UE,RHOE,
DELSTR,CBETAI,CBETA
WRITE(6,105)

C CALCULATION FOR FOLLOWING DATA
READ(5,202)N
N IS THE NUMBER OF DATA POINTS(NOT TO EXCEED 10)
READ(5,203)(XR(I),I=1,N)
XR(I) ARE THE DATA RADIUS POINTS
READ(5,203)(XM(I),I=1,N)
XM(I) ARE THE DATA MACH NO. POINTS
READ(5,203)(XT(I),I=1,N)
XT(I) ARE THE DATA TOTAL TEMPERATURE POINTS
DO 1 I=1,N
1 WRITE(6,106)I,XR(I),XM(I),XT(I)
DO 2 I=1,N
2 \[ X(I) = \frac{(XR(I) - RWALL)}{DELSTR} \]
DO 3 I = 1, N

3 \[ Y(I) = \frac{XM(I) \times SQRT(2402.4 \times XT(I)/(1.0 + 0.2 \times XM(I)^2))}{UE \times 2.402.1} \]
DO 4 I = 1, N

4 \[ Z(I) = \frac{XT(I)}{TTOTE} \]
WRITE (6, 107)
DO 5 I = 1, N

5 WRITE (6, 106) I, X(I), Y(I), Z(I)

C

STARTING PROFILE
READ (5, 203) (A(I), I = 1, 7)
READ (5, 203) (B(I), I = 1, 7)
WRITE (6, 108)
DO 6 I = 1, 7

6 WRITE (6, 109) I, A(I), B(I)

DEL = DELSTR
RDEL = DEL + RWALL
ETA = 1.0
CALL FNCT1 (A, B, ETA, VAL1, VAL2, VAL3, VAL4, VAL5, VAL6, VAL7, VAL8)
CONSTA = CBETA * VAL3 - CBETAI * VAL4
CONSTB = CBETA * VAL5 - CBETAI * VAL6
CONSTC = VAL7 - VAL4
CONSTD = VAL8 - VAL6
RADCAL = SQRT ((RWALL * CONSTB)^2 + 2.0 * (RDEL^2 - RWALL^2) + CONSTA)
DELTA = (-RWALL * CONSTB + RADCAL) / (2.0 * CONSTA)
THETA = -RWALL + SQRT (RWALL^2 + 2.0 * (CONSTC + CONSTD * RWALL / DELTA) * DELTA^2)
DEFECT = CONSTC * DELTA^2 + RWALL * CONSTD * DELTA
PSIE = SQRT (2.0 * RHOE * UE * (DELTA^2) * (VAL7 + RWALL * VAL8 / DELTA)) / PSIE
WRITE (6, 110) DEFECT, DELTA, DEL, THETA, PSIE
WRITE (6, 111)
DO 12 N = 1, 111
ETA = FLOAT (N - 1) * 0.01
IF (N .EQ. 1) GO TO 8
URATI (N) = 0.0
DO 7 I = 1, 7

7 URATI (N) = URATI (N) + A(I) * ETA^2
GO TO 9

8 URATI (N) = A(I)

9 CALL FNCT1 (A, B, ETA, VAL1, VAL2, VAL3, VAL4, VAL5, VAL6, VAL7, VAL8)
TRATI (N) = CBETA * VAL1 - CBETAI * VAL2
RRATI (N) = (SQRAT (2.0 * (DELTA^2) * ETA * (ETA * CBETA * VAL3 - ETA * VAL4 + RWALL / DELTA) * (CBETA * VAL5 - CBETAI * VAL6))) + (RWALL^2 - RWALL) / DELTA
PSRATI (N) = SQRT (2.0 * RHOE * UE * (DELTA^2) * ETA * (ETA * VAL7 + RWALL * VAL8)) / PSIE
IF (URATI (N) .EQ. 0.0) GO TO 10
VALUE = PSRATI (N) * TRATI (N) / URATI (N)
GO TO 11
10 VALUE=0.0
11 WRITE(6,112)ETA,PSRATI(N),RRATI(N),URATI(N),TRATI(N),
1VALUE
12 CONTINUE
C FINITE DIFFERENCE STEPS
NSTP=101
DELS=0.005
C BEGIN CALCULATIONS DOWNSTREAM
S=0.0
SCHK=0.0
SW=B(1)#TE*CBETA
U(1,1)=0.0
T(1,1)=SW
R(1,1)=RWALL
NSTP1=NSTP-1
DO 21 K=2,NSTP1
PBAR=FLOAT(K-1)/FLOAT(NSTP1)
DO 13 J=1,101
IF(PSRATI(J) .GT. PBAR)GO TO 14
13 CONTINUE
GO TO 22
14 IF(J .GE. 6)GO TO 15
NB=1
GO TO 16
15 NB=J-5
16 NO=10
17 DO 18 M=1,NO
MMM=NB+M-1
XXB(M)=PSRATI(MMM)
18 YYB(M)=URATI(MMM)
CALL AINTR(XXB,YYB,NO,PBAR,UBAR)
U(1,K)=UBAR*UE
DO 19 M=1,NO
MMM=NB+M-1
XXB(M)=PSRATI(MMM)
19 YYB(M)=RRATI(MMM)
CALL AINTR(XXB,YYB,NO,PBAR,RBAR)
R(1,K)=RBAR*DEL+RWALL
DO 20 M=1,NO
MMM=NB+M-1
XXB(M)=PSRATI(MMM)
20 YYB(M)=TRATI(MMM)
CALL AINTR(XXB,YYB,NO,PBAR,TBAR)
T(1,K)=TBAR*TE
21 CONTINUE
22 U(1,NSTP)=UE
T(1,NSTP)=TE
R(1,NSTP)=DEL+RWALL
WRITE(6,113)
MCHE=NSTP+NSTP
DO 27 M=1,NSTP
GURAT= U(1,M)/UE
GRRAT=(R(1,M)-RWALL)/DEL
GTRAT = T(1,M)/TE
IF(GURAT .EQ. 0.0) GO TO 23
VALUE = FLOAT(M-1)/FLOAT(NSTP1)*GTRAT/GURAT
GO TO 24
23 VALUE = 0.0
24 IF(M .LT. MCHECK) GO TO 25
GO TO 27
25 IF(GURAT .GT. 0.990) GO TO 26
GO TO 27
26 MCHECK = M
BLEEDGE = GRRAT - ((GURAT-0.990)/(GURAT-U(1,M-1)/UE))*GURAT
WRITE(6,114) BLEDGE
27 WRITE(6,115) M, GRRAT, GURAT, GTRAT, VALUE
28 DO 29 I = 1, 100
IO = NSTP + I
U(1,IO) = UE
T(1,IO) = TE
R(1,IO) = 3.0*R(1,IO-1) - 3.0*R(1,IO-2) + R(1,IO-3)
YRATIO = (R(1,IO) - RWALL)/DEL
IF(YRATIO .GE. 1.250) GO TO 30
29 CONTINUE
30 NSTEPS = IO - 1
DELPSI = PSIE/FLOAT(NSTP1)
31 S = S + DELS
SCHK = SCHK + DELS
UE = UE - ((DELS*PE*UE*PDIF)/(1716.0*TE))
PE = PE + PDIF
U(2,1) = 0.0
T(2,1) = T(1,1)
R(2,1) = RWALL
DO 36 N = 2, NSTEPS
UPO = (U(1,N)*R(1,N)**2)/(DELPSI*FLOAT(N-1))
UP1 = (U(1,N+1)*R(1,N+1)**2)/(DELPSI*FLOAT(N))
IF(N .EQ. 2) GO TO 32
UP2 = (U(1,N-1)*R(1,N-1)**2)/(DELPSI*FLOAT(N-2))
GO TO 33
32 UP2 = (R(1,1)**2)/DELPSI*(3.0*U(1,2)-1.5*U(1,3)+U(1,4))/3.0
33 UP = U(1,N-1)*(UPO+UP2)-U(1,N)*(UP1+2.0*UPO+UP2)+U(1,N+1)*(UPO+UP1)
TP = T(1,N-1)*(UPO+UP2)-T(1,N)*(UP1+2.0*UPO+UP2)+T(1,N+1)*(UPO+UP1)
CONST1 = (2.270*(1.E-8)*SQRT(TE)*(PE-PDIF)*DELS)/
1(3432.0*(TE+198.6)*(DELPSI**3)*FLOAT(N-1))
CONST2 = DELS*PDIF**1716.0*T(1,N)/(PE-PDIF)
U(2,N) = U(1,N) - (CONST2/U(1,N)) + (CONST1*UP)
T(2,N) = T(1,N) + (CONST2/5006.0) + (CONST1*TP/0.72) +
1(CONST1*UPO/12012.0*((U(1,N+1)**2)-(2.0*U(1,N+1)*
2U(1,N+1)*U(1,N-1)+(U(1,N-1)**2))
IF(U(2,N) .GE. UE) GO TO 34
60 IF(T(2,N) .LE. TE) GO TO 35
GO TO 36
34  \(U(2,N) = UE\)
GO TO 60
35  \(T(2,N) = TE\)
36  CONTINUE
R(2,1) = RWALL
CKNST2 = (PSIE**2)/((1.5*TE*RHOE*FLOAT(NSTP1)**2)
R(2,2) = SQRT(((R(2,1)**2 + CKNST2*1.5*(T(2,2)/U(2,2) +
1*T(2,1)/(3.0*U(2,2) - 1.5*U(2,3) + U(2,4)/3.0)))
R(2,3) = SQRT(((R(2,1)**2 + CKNST2*(T(2,1)/(3.0*U(2,2) -
1.5*U(2,3) + U(2,4)/3.0) + 4.0*T(2,2)/U(2,2) + 2.0*T(2,3)/
2*U(2,3)))
DO 37 K = 4, NSTEPS
IF(U(2,K-2) .EQ. 0.0) GO TO 37
IF(U(2,K-1) .EQ. 0.0) GO TO 37
IF(U(2,K) .EQ. 0.0) GO TO 37
R(2,K) = SQRT(((R(2,K-2)**2 + CKNST2*(T(2,K-2)*FLOAT(K-3)
1/U(2,K-2) + 4.0*T(2,K-1)*FLOAT(K-2)/U(2,K-1)*FLOAT(K-1)
2/U(2,K)))
37 CONTINUE
NNN = NSTEPS + 1
U(2,NNN) = UE
T(2,NNN) = TE
R(2,NNN) = 3.0*R(2,NNN-1) - 3.0*R(2,NNN-2) + R(2,NNN-3)
IF(SCHK .GE. 0.9999) GO TO 38
GO TO 42
38 SCHK = 0.0
WRITE(6,116) S, UE, TE
KCHECK = NSTEPS + 5
DO 41 K = 1, NSTEPS
YRATIO = (R(2,K) - RWALL)/DEL
URATIO = U(2,K)/UE
TRATIO = T(2,K)/TE
IF(K .LT. KCHECK) GO TO 39
GO TO 41
39 IF(URATIO .GT. 0.990) GO TO 40
GO TO 41
40 KCHECK = K
BLEDGE = YRATIO - ((URATIO - 0.990)/(URATIO - U(2,K-1)/UE))*
1(YRATIO - (R(2,K-1) - RWALL)/DEL)
WRITE(6,114) BLEDGE
41 WRITE(6,117) K, YRATIO, URATIO, TRATIO
42 DO 43 I = 1, NNN
R(I,1) = R(2,I)
U(I,1) = U(2,I)
T(I,1) = T(2,I)
IF(S .GE. 10.00) GO TO 44
GO TO 31
44 STOP
100 FORMAT(1H1, 10X, 1IH RUN NUMBER, 10X, 12H DECK NUMBER)
101 FORMAT(1H0, 5X, 37H EMPIRICAL STING WAKE CALCULATION
1/1H0, 10X, 37H THE BODY CONFIGURATION IS AS FOLLOWS/
21H, 15X, 21H 1/2 INCH SUPPORT ROD/1H, 15X, 35H 10
3DEGREE HALF ANGLE CONE FOREBODY/1H ,15X,22H TRAILING 
4ROD RADIUS = 1PE11.4/1H ,15X,29H PROBE SWEEP AXIAL 
5LOCATION = 1PE11.4) 
102 FORMAT(1H0,10X,43H THE WIND TUNNEL STAGNATION CONDIT 
1IONS WERE/1H ,15X,10H STAG. T = 1PE11.4/1H ,15X,10H 
2 STAG. P = 1PE11.4) 
103 FORMAT(1H0,10X,37H THE WIND TUNNEL EXIT CONDITIONS 
1WERE/1H ,15X,16H EXIT MACH NO. = 1PE14.7/1H ,15X,9H 
2EXIT P =1PE14.7/1H ,15X,9H EXIT RHO = 1PE14.7) 
104 FORMAT(1H0,10X,49H THE EDGE CONDITIONS AT PROBE SWEEP 
1LOCATION WERE/1H ,15X,16H EDGE MACH NO. = 1PE14.7/ 
21H ,15X,16H EDGE P = 1PE14.7/1H ,15X,12H EDGE TTOT = 
31PE14.7/1H ,15X,12H EDGE PTOT =1PE14.7/1H ,15X,9H 
4EDGE T =1PE14.7/1H ,15X,9H EDGE U =1PE14.7/1H ,15X, 
511H EDGE RHO = 1PE14.7/1H ,15X,27H BOUNDARY LAYER 
6THICKNESS = 1PE14.7/1H ,15X,9H CBETAI = 1PE14.7/1H , 
715X,8H CBETA = 1PE14.7) 
105 FORMAT(1H0,10X,32H THE FOLLOWING DATA WAS OBTAINED/ 
11H ,15X,10H POINT NO.,6X,&h RADIUS,10X,9H MACH NO., 
27H,11H TOT. TEMP.) 
106 FORMAT(1H ,18X,I2,5X,3(1PE17.7)) 
107 FORMAT(1H0,10X,52H THE VELOCITY AND ENTHALPY PROFILES 
1WERE OBTAINED /1H ,10X,35H FROM THE FOLLOWING CAL 
2CULATED DATA/1H ,15X,10H POINT NO.,3X,14H (R-RWALL)/ 
3DEL,8X,5H U/UE ,12X,5H H/HE) 
108 FORMAT(1H0,10X,55H THE PROFILE COEFFICIENTS WERE CAL 
1CULATED TO BE /1H ,14X,12H COEF. NO. I,8X,5H A(I), 
211X,5H B(I)) 
109 FORMAT(1H ,18X,I2,5X,2(1PE17.7)) 
110 FORMAT(1H0,15X,9H DEFECT = 1PE14.7/1H ,15X,8H DELTA 
1= 1PE14.7/1H ,15X,6H DEL = 1PE14.7/1H ,15X,8H THETA 
2= 1PE14.7/1H ,15X,7H PSIE = 1PE14.7) 
111 FORMAT(1H1,5X,40H CALCULATED POINTS TO BE INTERPOLA 
1TED ON/1H0,6X,4H ETA,10X,9H PSI/PSIE,6X,14H (R-RWALL) 
2/DDEL,7X,5H U/UE,12X,5H T/TE,12X,6H VALUE) 
112 FORMAT(1H ,6(1PE17.7)) 
113 FORMAT(50HPROFILE OBTAINED BY ITERATED LINEAR INTER 
1POLATION/1H0,5X,8H S = 0.0/1H ,1X,2H M ,6X,6H Y/DDEL, 
211X,5H U/UE,12X,5H T/TE,11X,6H VALUE) 
114 FORMAT(1H ,20X,22H BOUNDARY LAYER EDGE = 1PE17.7) 
115 FORMAT(1H ,14,4(1PE17.7)) 
116 FORMAT(4H1S = 1PE11.4,3X,5H UE =1PE14.7,3X,5H TE = 
11PE14.7/4HO M,7X,6H Y/DDEL,12X,5H U/UE,12X,5H T/TE) 
117 FORMAT(1H ,I5,3(1PE17.7)) 
201 FORMAT(1PE11.4) 
202 FORMAT(I3) 
203 FORMAT(5(1PE11.4)/5(1PE11.4)) 
END
SUBROUTINE AINTR(XX, YY, NN, XB, YB)
DIMENSION XX(10), YY(10), X(10), F(10, 10)
XBAR=XB
DO 1 I=1, NN
  X(I)=XX(I)
  F(I, 1)=YY(I)
1 DO 3 J=2, NN
  IF(J .LT. I) GO TO 2
  F(J, I)=((X(J)-XBAR)*F(I-1, I-1)-(X(I-1)-XBAR)*F(J, I-1))/(X(J)-X(I-1))
2 F(J, I)=0.0
3 CONTINUE
YB=F(NN, NN)
RETURN
END

SUBROUTINE FNCT1(FA, FB, FETA, VALUE1, VALUE2, VALUE3,
1 VALUE4, VALUE5, VALUE6, VALUE7, VALUE8)
DIMENSION FA(7), FB(7), AA(7), BB(7)
AA(1)=FA(1)
BB(1)=FB(1)
DO 1 I=2, 7
  AA(I)=FA(I)*FETA**(I-1)
1  BB(I)=FB(I)*FETA**(I-1)
VALUE=0.0
VALUE2=0.0
VALUE3=0.0
VALUE4=0.0
VALUE5=0.0
VALUE6=0.0
VALUE7=0.0
VALUE8=0.0
DO 2 I=1, 7
  VALUE1=VALUE1+BB(I)
  VALUE2=VALUE2+AA(I)**2
  VALUE3=VALUE3+BB(I)/FLOAT(I+1)
  VALUE4=VALUE4+(AA(I)**2)/FLOAT(I+1)
  VALUE5=VALUE5+BB(I)/FLOAT(I)
  VALUE6=VALUE6+(AA(I)**2)/FLOAT(I+I-1)
  VALUE7=VALUE7+AA(I)/FLOAT(I+1)
2 VALUE8=VALUE8+AA(I)/FLOAT(I)
DO 3 I=1.6
  II=I+1
3 DO 4 J=II, 7
  VALUE2=VALUE2+2.0*AA(I)*AA(J)
  VALUE4=VALUE4+2.0*AA(I)*AA(J)/FLOAT(I+J)
3  VALUE6=VALUE6+2.0*AA(I)*AA(J)/FLOAT(I+J-1)
RETURN
END
LIST OF REFERENCES


29. Chapman, D.R., "An Analysis of Base Pressure at Supersonic Velocities and Comparison with


