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HYPOTHESIS SAMPLING AND DIMENSION SELECTION

MODELS OF CONCEPT IDENTIFICATION FOR

PROBLEMS WITH TERNARY DIMENSIONS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of the Ohio State University

By

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***

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I. DEFINITION AND LITERATURE REVIEW

Definition of Concept Identification

In the typical concept learning (identification, utilization) experiment, a number of stimuli varying in certain well-defined dimensions (e.g., color, geometric form, number) are presented to the subject (either one-at-a-time, some-at-a-time or all-at-once) and his task is to make one of a series of highly integrated and available categorization responses. Over a series of trials the subject observes the stimuli and the responses to be associated with these stimuli, and on the basis of these observations forms the "concept to be learned". The concept has been learned when the subject can make some number of correct categorization responses in a row. Generally the concept learning experiment requires no stimulus differentiation (however, stimulus predifferentiation is investigated in this situation - e.g., Rommetveit, 1965, i.e., the dimensions and values of the dimensions are highly discriminable and there is no response learning.

In these experiments the stimulus population consists of all possible combinations of N stimulus dimensions with n values of these dimensions, a total of $N^n$ stimuli. The experimenter selects some $I^iN$ values from separate dimensions to be the relevant attributes and then on the basis of the presence and/or absence of these attributes forms mutually exclusive and exhaustive subsets of the stimulus population. These subsets are then mapped to the response
categories. It is the subset-to-response contingency that constitutes the "concept-to-be-learned". The existence of a concept is validated experimentally by the emission of a common categorization response to each member of the subset. A concept may differ from another (second) concept in one of two respects: (1) different subsets of the stimulus population may be formed on the basis of different relevant attributes; (2) these subsets may be paired with different categorization responses. Therefore, Bourne (1966, p. 11) concludes that a concept is a joint function of (1) the relevant stimulus attributes and (2) generating the subsets and assigning them to categories.

Some investigators, such as Osgood (1953), would object to this rather limited definition of a concept. Osgood (1953) argues that a concept should require some degree of abstraction, i.e., that a concept emerges from varying stimuli not necessarily having any features in common. Piaget, whose "concepts" are somewhat in the same spirit, would also probably object. Kendler (1964, p. 219), however, has noted that it is certainly much easier to specify similar stimuli than to specify stimuli having no features in common. Consequently, the definition stated by Bourne seems to be a good working definition (amenable to experimental investigation) and will be the one adopted herein.

Investigators in the area of concept learning have not adopted any precise terminology which might identify the aspects of concept learning with which they are concerned. They have repeatedly used a multiplicity of terms in very unsystematic ways. As a result, one is likely to find members of the set \{concept, attribute, cue, rule, category\} paired with members of the set \{learning, identification, utilization, sorting, labelling, attainment\} in the titles of experiments concerned with
assorted conceptual tasks. Although investigators have not been consistent in the use of the terms they have chosen to describe their experiments, Haygood and Bourne (1965) suggest that, with a careful reading of instructions, some distinction among the aspects of concept learning investigated in various experiments can be made by considering the task required of the subject. Bourne (1966) also suggests that the field of research concerned with conceptual behavior can be subdivided by considering the task required of the subject. With respect to this subdivision, Hunt (1962) has suggested that those experiments requiring the formation of a concept or equivalence class anew be properly termed concept formation experiments, while those requiring the categorization of stimuli whose set of attributes are already known to the learner be termed concept selection experiments. In Bourne's (1966, p. 19) terminology, concept formation is attribute learning and involves those type of tasks in which "the perceptual characteristics of a stimulus array are changed either through enhanced sensitivity to the stimulus dimensions" (e.g., Tighe and Tighe (1966)) or "through learning to detect distinguishing features of stimuli" (e.g., Grant and Cost (1954); Capaldi and Stevenson (1957)). Hunt's concept selection experiments are termed attribute utilization by Bourne and include those tasks which "require the discovery and/or use of already discriminable and labelled attributes, such as concept-identification problems". As a point of information, attribute and concept refer to the same thing; i.e., a subset of the stimulus population formed on the basis of the presence or absence of some values of the relevant dimension. Because a concept is defined both in terms of the focal attributes and the particular combinations of these focal attributes in a given subset, Haygood and Bourne (1965) suggest that
concept learning problems can be considered as two subproblems: (1) identifying the relevant attributes (the task in concept identification experiments) and (2) identifying (or learning) the conceptual rule (i.e., learning which combinations of the focal attributes constitute the positive category). These two structural components of a concept can be manipulated separately by requiring that the S learn the subset-category contingency but (1) identify the relevant attributes and require him to learn the rule (rule learning) or (2) identify the rule and require him to identify the attributes (attribute or concept identification). Tasks which require S to identify the relevant attributes and the conceptual rule are more properly termed complete learning (Haygood and Bourne, 1965).

It is necessary to make these task distinctions in order to relate the present concept identification experiments to the (selected) literature concerned with the same kind of task. In addition, the experiments will serve as tests of several theories. In order for the experiments to provide a proper test of the theories, it is necessary that the experiments satisfy the range of applicability of the theories. Generally, however, the range of applicability of the major theories of concept identification rarely has axiomatic status (see, for example, Bower and Trabasso, 1964; Restle, 1962; Bourne & Restle, 1959), and is usually delineated in discussion of the theory. Consequently one must attend closely to procedures in any test of the theories and in attempting to relate them to experiments performed.

Early Theories of Concept Identification and Discrimination Learning

There are many theories of concept identification and many theories which are potentially generalizable to the concept identification
situation. An exhaustive review and analysis of all theories of concept identification would go beyond the scope of this paper. Only those theories whose extensions are to be tested by the experiments proposed herein will be presented and analyzed in detail.

The first division of theoretical analyses of concept identification to be made separates those theories which construe concept identification as involving a process of conditioning from those theories viewing concept identification as strategy or hypothesis selection. To illustrate the difference between these two approaches, consider the definition of a strategy (or hypothesis) similar to that of Martin (1965): "A strategy is a rule according to which responses are made". The important point is that the selection of a strategy on a given trial implies a particular response on that trial. Thus, there is no conditioning of responses to stimuli, only selection of strategies (Martin, 1965).

Hypothesis theories were first developed within simple discrimination learning experiments with the white rat. The fundamental tenet of these theories asserts that, in this situation, there are many stimulus components competing for the organism's attention. Some of these components are relevant to the solution of the problem (e.g., in form discrimination, the stimulus "triangle" and the stimulus "square") and some are irrelevant (e.g., right-left placement of the stimulus). The organism attends to selected components of the environment. Hypotheses are identifiable in response profiles as systematic ways of responding. Lashley (1929) referred to these persistent ways of responding to position or other irrelevant stimuli as "attempted solutions" to the discrimination problem and

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1Theories of verbal control (e.g., Verplanck, 1962; Dulany and O'Connell, 1963) will not be considered in this review.
Kreschevsky (1932) described the behavior of the rat during the presolution phase of discrimination learning to be that of "testing hypotheses". The psychologists advancing an "hypothesis" account were objecting to accounts of presolution discrimination learning by S-R psychologists as "trial and error" behavior. However, Spence (1936) noted that random responding was not a necessary result of an S-R analysis of presolution responding and that behavior could appear systematic and yet not result from cognitive strategies. Spence's tenet was a typical result of an S-R analysis of learning: that responses are determined by the summation of the excitatory tendencies elicited by the stimulus components impinging on the organism on a trial. Although Kreschevsky stated nothing about how hypotheses are sampled and rejected, Spence (1936) noted one testable implication of the "hypothesis" account of discrimination learning: if the relevant cue is reversed before learning has occurred (e.g., square+ and triangle- to square- and triangle+), there should be no difference in trials to learn between a "reversal" group and a control group. Spence and others have tested this implication (McCulloch and Pratt, 1934; Spence, 1945; Krechevsky, 1938; Lashley and Wade, 1946); the results have been contradictory. Blum and Blum (1949) have suggested that the contradictory results are functions of different experimental designs and have noted those conditions which tend to support each theory. More recently, Walk (1952), Bower and Trabasso (1963), and Erickson et. al. (1966, 1967) have compared presolution shift groups with control group performance with human Ss and have found no differences between these two groups.

It should be noted that these tests for transfer after reversal of reward relations of a stimulus pair at some point before solution of a problem stem from Lashley's (1934) explanation of the learning curve on the
assumption that single associations occur in one trial and are not strengthened with practice. This interpretation of learning has been called the noncontinuity or all-or-none explanation of learning. Many experiments have been performed in an attempt to resolve the continuity (incremental)-noncontinuity (all-or-none) issue. A comprehensive review of this literature would exceed the scope of this paper. The reader is referred to Blum and Blum (1949) for a review of the early literature in this area, including experiments and issues stemming from these two accounts. Restle (1965) reviews the recent literature and comments on the relevance of this issue to the major theories of learning and to theory construction and delineates conditions under which each is observed.

A second theoretical issue arising from the postulation of "hypotheses" to account for behavior in discrimination learning concerns the components of stimulation impinging on the organism's sensorium, or, alternatively, the components of stimulation serving as the bases for the strategy tested. Early hypotheses-testing theories were unidimensional in form (perhaps a result of the simple discrimination learning situation from which they arose) in that either position (right-left) served as the component of stimulation upon which the response was based or the one dimension of the stimulus (e.g. form) served as the element of the hypothesis. With respect to this issue, Spence's contention was that the learning process was a gradual strengthening of the associative tendency of each component of the pattern of stimulation to evoke the reinforced response. The stimulus elements influence the response in proportion to their reinforcement history. Early experiments performed to test these two interpretations involved giving the animal a preliminary set to respond
to one aspect or pattern of the stimulus situation and then testing for
association of other aspects or patterns to the response (Lashley, 1942;
Spence, 1945). Again the results were contradictory; Blum and Blum (1949)
have reviewed these data and on the basis of an experiment they performed
have concluded that animals can acquire associations to aspects of the
stimuli other than the aspects to which they were set to respond. Re­
search along these lines is continuing; the general area of attention
in discrimination learning and concept learning is currently being act­
ively investigated (see, e.g., Lovejoy, 1965; Mackintosh, 1965; Trabasso,
1963; Suchman & Trabasso, 1966; Trabasso and Bower, 1968; Zeaman and

For a review of the early literature in concept learning, including
discussion of the definition of a concept, the methodology and important
theoretical issues, the reader is referred to Humphrey (1951), Johnson
(1955), Leeper (1951), Underwood (1949), and Vinacke (1951, 1952). In
the closely related area of human problem solving, the reader is referred
to Duncan (1959) and Russell (1956).

Definition and Experimental Identification of Hypotheses

The recent work in the area of hypothesis behavior in discrimination
learning which provides the most clearly defined link to the early work is
that of Levine (1959, 1963, 1966, 1969; Levine, Leitenberg, and Richter,
1964). Levine (1959) adopted Krechevesky's definition of an hypothesis,
i.e., it is a "specifiable pattern of responses to a selected stimulus set"
and he was able to identify two types of hypotheses: Prediction hypotheses,
those in which trial-to-trial responding is contingent upon the outcomes of
previous trials and response set hypotheses, those in which responding is
independent of trial outcomes. An example of the former is a "win-shift", "lose-stay" hypothesis (in which the subject responds to the stimulus correctly on the preceding trial) and of the latter, a position or stimulus preference hypothesis (S chooses the right position or the same stimulus on each trial). By making several assumptions about the set of hypotheses he had identified, he was able to account for the traditional learning-to-learn curve in monkeys. In a later article, Levine (1963) analysed hypotheses of humans in discrimination problems and modified his earlier definition of an hypothesis to that in current use, i.e., an hypothesis is the determinant of a response pattern, to take into account the fact that humans mediate their patterns of response.

Levine (1963) introduced two assumptions in order to relate each hypothesis to Ss overt responses. They were: 1. a prediction postulate stating that a subject who predicts rewards attempts to maximize these rewards and 2. a response-set postulate stating that a subject who holds a response-set hypothesis responds according to that hypothesis without regard to trial outcome. On the basis of these and previously stated assumptions, and using the technique he developed in his 1959 model, he was able to derive a set of equations for evaluating the relative strengths of hypotheses on the basis of experimental data. Levine also extended the model to an n-dimensional situation. The extension required the assumption that the set of hypotheses includes only prediction hypotheses contingent upon the stimulus justified for the following two reasons: predictions from his model were invariant over an assumption that response-set hypotheses were zero and instructions and pretraining can minimize the occurrence of response-set hypotheses.

Levine developed a partial feedback technique for monitoring hypotheses and used it to evaluate the strengths of the various hypotheses in a four
binary dimension concept identification problem by presenting four trial nonoutcome problems interspersed with outcome problems. He concluded that the hypothesis data provided "generally good predictions of performance".

Thus, Levine extended the conception of hypothesis testing from the simple discrimination learning situation with animals to an n-dimensional discrimination learning situation with adult humans. In the course of this development, he has made several assumptions which have been incorporated in recent hypothesis testing models relevant to the experiments proposed herein. They include the restriction of the hypothesis set to those hypotheses which serve as possible solutions to the problem (and a residual set which mediates responses to an unrecorded dimension). The occurrence of the residual H set has proved troublesome for later investigators using the hypothesis monitoring technique and has suggested revision of the one-at-a-time sampling axiom (although there are other alternatives which can account for the occurrence of inconsistent H's; see Erickson and Block, 1969 or Erickson, Block, and Rulon, 1969, e.g.)

Descriptive Accounts of Concept Identification

Several accounts of concept identification have appeared which incorporate statements about the nature of "hypothesis change" as S attempts to solve a concept identification problem. These can be at least tentatively divided into theories of behavior in a concept identification problem and analytic descriptions of these behaviors. The former provide predictions, and these predictions can be said to support or not to support the theory. The latter involve comparisons of behavior with an ideal derived from a purely logical analysis of the problem confronting S; there is no independent attempt to converge uniquely upon the factors which account for deviations of Ss performance from the ideal on any trial. Those accounts
of concept identification which compute, e.g., the number of trials to the logical solution of a concept and compare this number with Ss performance can be called hypothesis testing theories because the ideal S is "talked about" as an hypothesis-sampler (or, equivalently, an eliminator) and the set of hypotheses tenable on any trial is a function of reinforcements and determines Ss response. There is no conditioning, only hypothesis selection. The following studies will serve as examples of this approach.¹

The orientation of "descriptive accounts" of concept identification was set by Hovland (1952) in his criticism of Smoke's work. Smoke (1933) concluded that positive instances (i.e., those which illustrate the concept and include all the values essential to the concept) are most important in conveying information about the concept while negative instances (those which do not contain all values relevant to the concept) convey very little, perhaps no information about the concept. That is, Ss learn a concept on the basis of "what it is" rather than "what it is not".

Hovland (1952) criticized Smoke's work by noting that Smoke's Ss had no instruction as to the dimensions of the stimuli nor the nature of the concept to be formed and that the amount of information conveyed by positive and negative instances was not controlled. Hovland derived general formulas for computing the number of tenable conjunctive hypotheses. Using these formulas it was possible to equate the amount of information conveyed by positive and negative instances. Hovland and Weiss (1953) did this and showed that the number of Ss stating the concept was markedly higher for Ss receiving positive instances. More recent investigations of the role of positive and negative instances (Bourne, 1967; Byers, 1965; Conant and

¹Only the major works in this area will be reviewed.
Trabasso, 1965; Friebergs and Tulving, 1961; Haygood and Devine, 1967; Haygood and Stevenson, 1966; Pishkin and Wolfgang, 1965) support these findings.

The second major attempt at an analytic description of behavior was initiated by Bruner, Goodnow, and Austin (1956). Their concern was the extent to which human Ss invoked one of several identified strategies in solving concept problems and how these strategies were modified by variables such as memory, risk, ordered vs. random displays, etc. Bruner et al identify two major classes of these strategies: the wholist strategy and the partist strategy. These strategies differ in two respects: whether all or part of the first exemplar serves as the basis for an initial hypothesis and the manner in which hypotheses are altered when an infirming instance is encountered.

Bruner et al's study of concept learning concentrated on strategies used in conjunctive or disjunctive problems. Haygood and Bourne (1965) have labeled the tasks required of Bruner et al's Ss as attribute identification. Accounts of attribute identification have not remained at the atheoretical level of Bruner et al. The theories of Bourne and Restle (1959), Bower and Trabasso (1964) and others account for attribute identification and will be considered later. A second point to be made with respect to the Bruner et al study is that their preliminary work has led to the empirical study of type of conceptual rule as a variable and to theories which account for the relative difficulty in acquiring different types of concepts, (see, e.g., Hunt, 1962; Hunt, Marin, and Stone, 1966; Neisser and Weene, 1962; Shepard, Hovland, and Jenkins, 1961).
The kinds of theories proposed to account for the difficulty in acquiring various kinds of concepts can be termed "information processing" theories in that they describe concept identification as a sequential decision-making process in which reinforcement modifies the size and the content of the set of potential hypotheses on each trial. Information processing theories are sometimes formalized as computer programs and simulated data are compared to experimental data in tests of these theories (Hunt, 1962; Hunt, Marin, and Stone, 1966). Murdock (1967, p. 151) has noted the major problem with these types of theories: the fact that the "extensive work on computer simulation has not fed back and led to new and insightful experiments in psychological literature". Perhaps this is due to the fact that these models, although explicit, require a number of assumptions about the strategies involved or the number of items in memory, etc. Thus, a fairly complicated learning process is postulated and if the model is not supported by the data, it is difficult to trace the inadequacy of the model to any one assumption.

One other theory of concept learning which has stimulated research is that of Underwood (1952). Underwood postulated the notion of response contiguity as a necessary requirement for concept learning. Close proximity of instances of the same concept insures that the perceptual or mediational representations of the properties of these instances occur contiguously, facilitating the "abstraction" of some common property. Spaced practice should lead to the forgetting of preceding stimulus properties as response contiguity is reduced. Underwood discussed a "mechanical model" of concept attainment whose operation is somewhat akin to hypothesis sampling models: The subject draws from a pool of "representations" of the stimuli (e.g.,
attributes of the stimuli) and when the appropriate (solution) attributes are selected, the problem is solved. Underwood's notions stimulated research on variables important in the acquisition of concepts (e.g., massed vs. spaced practice, instance contiguity, memory requirements, etc.) These data are reviewed by Dominowski (1965).

Hypothesis Sampling Models of Concept Identification

A. No memory models

Among recent theories of concept identification, those of Restle (1962) and Bower and Trabasso (1964) probably have been the most influential. The logical structure of these theories has been carefully analyzed by Martin (1965); only their basic tenents will be presented herein.

It is postulated that solving a concept identification problem can be characterized by a two state Markov process (presolution and solution) with the following properties:

Axiom #1 (stationarity assumption) A subject samples hypotheses, one at a time, randomly, with replacement, from a pool of strategies, some of which always lead to an error (wrong strategies) and some of which lead to a correct response 50% of the time and an incorrect response 50% of the time (irrelevant strategies). The subject in the presolution state is testing irrelevant or wrong strategies and the probability of a correct response in this state is about .50 in a two category problem. The solution state is entered when S selects the correct strategy and continues to use it on the remaining trials.\(^1\)

---

\(^1\)The theory will be discussed in terms of S's holding one strategy at a time. It should be noted that Restle (1962) has shown that some characteristics of the data (e.g., the distribution of errors) are independent of the number of strategies in the sample—if it is additionally assumed that the probability of a response equals the number of strategies in the sample leading to that response and that an error is a recurrent event.
Axiom #2. The opportunity to learn (i.e., to sample the correct strategy) occurs only on error trials. If the sampled strategy leads to an incorrect response, it is rejected and returned to the pool of strategies from which S samples on the next trial. If the sampled strategy leads to a correct response, it is retained and used as the basis for responding on the next trial. Thus, S can leave the presolution state and enter the solution state only on error trials.

Axiom #3. Errors are recurrent events (Feller, 1957). This assumption means that after every error, the probability of learning is a constant.

A Markovian formulation of Restle's model as it appears in his 1962 paper is:

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>I</th>
<th>W</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>1/2 e</td>
<td>1/2i+1/2</td>
<td>1/2(1-(e+i))</td>
<td>1/2</td>
</tr>
<tr>
<td>W</td>
<td>e</td>
<td>i</td>
<td>1-(e+i)</td>
<td>0</td>
</tr>
</tbody>
</table>

in which C = the state in which S has selected the correct strategy.

I = the state in which S is responding on the basis of an irrelevant strategy.

W = the state in which S is responding on the basis of a wrong strategy.

e = p (selecting the correct strategy).

i = p (selecting an irrelevant strategy).

The entries of the matrix are the probabilities that S progresses from a particular row state on trial N to a particular column state on trial N+1. Thus the entries are conditional probabilities, called transition probabi-
ties, since they express the probability of state-to-state changes from one trial to the next according to the model. In the above model, the probability of going from state $W$ to state $C$ is the $p$ (selecting the solution hypothesis) $= e$. The $p(W \text{ to } I)$ is the probability of selecting an irrelevant hypothesis $= i$. The probability of staying in state $W$, i.e., the $P(W \text{ to } W)$ is the probability of selecting a wrong strategy, or, equivalently, the probability of selecting neither the solution strategy nor an irrelevant strategy $= 1-(e+i)$. The $P(I \text{ to } C)$ is the probability that $S$ makes an error and samples the solution $H = (1/2)e$. $P(I \text{ to } I)$ is the probability that $S$ makes an error and samples a wrong hypothesis $= (1/2)i + 1/2$. $P(I \text{ to } W)$ is the probability that $S$ makes an error and samples a wrong hypothesis $= 1/2(1-(e+i))$. Once $S$ has sampled the solution strategy, he continues to use that strategy and does not sample strategies. Thus, $S$ is "absorbed" in the $C$ state.

In order to relate the observable responses to the states of the model, the probability of a correct response in each state of the model must be formed. These probabilities are the entries of the $P(c)$ vector. The $P(c)$ when $S$ is testing a wrong strategy is zero; when $S$ is testing an irrelevant strategy, $P(c) = 1/2$ and when $S$ has sampled the solution $H$, $P(c) = 1.00$.

The probability that $S$ enters each state of the model on the first trial are the entries of $\Pi_0$, the initial trial vector. The probability that $S$ enters $W$, $I$, or $C$ on the first trial is the probability that $S$ samples a $W$, $I$, or $C$ strategy on the first trial.

The Restle strategy selection theory is in effect a no memory theory; when strategies are rejected, they return to the pool and are available for sampling on the next trial.

Bower and Trabasso (1964) view concept identification as a process of selection and conditioning and have constructed a model that formalizes these
two processes. The model is similar to that of Restle in several ways: Performance can be adequately represented as a two state process: presolution and solution. Ss behavior in the presolution state is viewed as a stimulus selection phase in which S comes to attend to the relevant dimension. While S is in the unconditioned state, the probability of a correct response is some chance level, \( p \). Other assumptions are identical to Restle's. Opportunities for learning occur only on error trials and errors are recurrent events. In order to exit the presolution state in this model, Ss must make an error, select the relevant dimension and learn the association between the value of the relevant dimension and the reinforced response. Bower and Trabasso assume that the probability of selecting a dimension is a function of the saliency of that dimension.

The mathematical representation of the binary model with the presolution state divided into chance success and error responses is:

\[
\begin{array}{c|c|c|c|c|c}
\text{C} & \text{E} & \text{S} & \text{P(c)} & \Pi_0 \\
\hline
1 & 0 & 0 & 1 & \text{r}^0 \\
\hline
\text{E} & r & (1-r)q+rq(1-\theta) & (1-r)p+rp(1-\theta) & 0 & q(1-r\theta) \\
\hline
\text{S} & 0 & q & p & 1 & p(1-r\theta) \\
\end{array}
\]

in which \( C \) = solution state
\( E \) = presolution error state
\( S \) = presolution success state
\( r = p(\text{selecting the relevant dimension}) \)
\( \theta = p(\text{conditioning the presented value to the reinforced response}) \)
\( p = p(\text{success in the presolution state}) \)
\( q = p(\text{error in the presolution state}) \).
The assumption of a constant probability of learning on each error trial means the Bower-Trabasso model, like the Restle model, is a no memory model. It is interesting to note that with very slight modifications, the two models are mathematically equivalent. If the probability of a correct response in the presolution state of the Restle model is assumed to be some chance level, \( p \), and the I and W states are aggregated to a general presolution state, \( \overline{C} \), then the following two state model represents Restle's interpretation of concept identification:

\[
\begin{array}{ccc}
C & \overline{C} & P(c) \\
C & 1 & 0 & 1 & \epsilon \\
\overline{C} & q\epsilon & 1-q\epsilon & p & 1-\epsilon \\
\end{array}
\]

in which \( \epsilon = p(\text{selecting the correct strategy}) \)
\( p = p(\text{success in the presolution state}) \)
\( q = p(\text{error in the presolution state}) \).

When \( \theta = 1.00 \), the three state Bower-Trabasso model presented previously (aggregated over states E and S into the general presolution state, \( \overline{C} \)) is equivalent mathematically to the above two state model. Assuming that \( \theta = 1.00 \) means, psychologically, that conditioning is trivial and is tantamount to interpreting Ss presolution behavior as that of selecting and testing strategies.

Approaches to testing these models have invoked direct tests of the assumptions of the models and testing the accuracy of quantitative predictions. The stationarity assumption has repeatedly been supported by the data in binary dimension problems (Bower and Trabasso, 1964; Erickson, Zajkowski and Ehmann, 1966; Erickson and Zajkowski, 1967; Guy, van Fleet, and Bourne, 1966; Grier and Bornstein, 1967; Holstein and Premack, 1965; Trabasso, 1963).
Investigations of the second assumption have shown that it is approximately correct for binary dimension problems. Levine (1966), using his hypothesis monitoring technique, showed that the probability of maintaining the same hypothesis after an error was about .02 and the probability of maintaining that same hypothesis after a correct was about .98. Erickson (1968) obtained the same percentages. Some investigators have cited their data as contradicting this assumption: Suppes and Schlag-Rey (1965) obtained a fairly high probability (.20 in Exp. I, II; .48-.67 in Exp. III) of hypothesis change on a correct trial and Rogers and Haygood (1968) detected hypothesis changes after a series of errorless trials, but these investigators used a "concept identification" task unlike that to which the model was originally designed to apply. Thus, it is difficult to evaluate the validity of their objections to the "win-stay" assumption of Restle's model because of the procedural differences. It is interesting to note that Suppes and Schlag-Rey suggested that in order to account for their data, a model incorporating the sampling of several hypotheses at a time might be appropriate. This notion is explored in some of the recent models of concept identification.

Although it seems fairly safe to attribute these results questioning the "win-stay" assumption to procedural variations in the latter two studies, there are studies to which the Restle and Bower and Trabasso models are applicable in which the results were not compatible with the "win-stay" assumption. Erickson, Block, and Rulon (1969) and Erickson (1969) obtained probabilities of win-shift of approximately .35, much higher than those previously reported. The reasons for the higher "win-shift" probability were somewhat unclear, but it might be due to Ss responding in accord with one hypothesis, yet concurrently testing more than one.
With respect to Axiom #3, Bower and Trabasso (1963) examined reversals prior to solution in concept identification. If errors are recurrent events, an error indicates that S has learned nothing about the situation, so if problem solution is changed on a specified error trial, this should not retard learning. Among three groups, a control, a reversal and a nonreversal shift, they found no significant differences in errors to solution. In other experiments, S-R assignments were switched after every other error. The model predicts that the number of informed errors (opportunities to learn) should be the same for the reversal and control Ss, a prediction which was supported by the data. This result has been replicated several times (Bower and Trabasso, 1964; Trabasso and Bower, 1964a; Erickson, Zajkowski, and Ehmann, 1966; Erickson and Zajkowski, 1967.)

A good deal of recent research has attempted to test the validity of the no memory inference. In one group of studies, it was reasoned that concept identification should not be affected by an initial series of random reinforcememts (RR). However, the typical result of these studies is that solution of the problem is hindered.

Levine (1962) found that the retarding effect of a series of initial random reinforcement trials was independent of the number of prior random reinforcement trials (from 4 to 60). This finding is in direct opposition to incremental models of concept identification which hold that the more inconsistent reinforcement experienced by Ss, the more adaptation or neutralization of irrelevant cues and the resultant correlation of amount of prior random reinforcement with the degree of negative transfer. Holstein and Premack (1965) replicated these results and their data also lends support to the stationarity assumption of all or none models. These authors suggested that the retarding effects of random reinforcement were due to
(1) Ss temporarily rejecting disconfirmed hypotheses based on stimulus cues experienced during RR or (2) Ss generating irrelevant hypotheses, (e.g., hypotheses based on more complicated conceptual rules than that required to solve the problem, or H's based on outcome sequences). In an attempt to investigate these two conjectured effects of RR, Trabasso and Staudenmeyer (1968) provided conditions in which Ss experienced RR on either the same set or a different set of stimuli from those used in the transfer problem. Their results indicated that if Ss were assured of an elementary solution to the problem, they were less likely to generate complicated irrelevant H's. The authors interpreted their results to lend support (although not exclusively) to Holstein and Premack's first conjecture: that RR effects negative transfer in the solution of a concept identification problem because Ss reject H's based on irrelevant cues and that this rejection is temporary and occurs only if the stimuli or dimensions remain unchanged. It should be noted that Trabasso and Staudenmayer (1968) have demonstrated only that the effects of RR are problem specific, a conclusion which does not rule out the possibility that Ss generate irrelevant H's.

Merryman, Kaufman, Brown, and Dames (1968) have shown that the inhibiting effect of random reinforcement on the learning rate in concept identification is due to incorrect information gathered on "right" trials rather than "wrong" trials. Previous investigators had suggested that noncontingent "wrongs" were responsible for the retarding effects of random reinforcement by (1) relegating a wrong strategy to a lower probability of sampling or (2) increasing the size of the hypothesis pool. It is important to note that the Merryman et al study suggests the processing of information on "right" trials rather than learning exclusively on "wrong" trials as the all-or-none model suggests.
Trabasso and Bower (1964b) questioned their Ss about the solutions of a series of 6 trial binary concept identification problems and found that Ss could recall their current working hypothesis or the solution strategy (recency effect) but also the information contained in the first stimulus presentation (primacy effect).

Levine (1966, 1967, 1969) using his hypothesis monitoring technique found that the size of the hypothesis pool decreased across trials. He suggested that Ss use the information transmitted in a stimulus-response pairing to divide the set of possible solutions into those which are tenable and those which are not. His data suggested that Ss remember the last S-R pairing they have seen and also some information about tenable hypotheses from previous trials. This process has been formalized by Chumbley (1967) and will be considered in a later section. The main implication of his studies is than an error is not a recurrent event; that the learning rate is not constant but increases over trials as stimulus information is gathered.

Memory in concept identification has also been studied under the assumption that if concept identification problems are solved by pure trial and error, then problems solved concurrently should be no more difficult than problems solved successively. Both Restle and Emmerich (1966) and Erickson and Zajkowski (1967) have compared these situations. Both studies found that problems solved concurrently were more difficult than problems solved successively. Both studies postulated the existence of a short term memory state: in the Restle and Emmerich study the suggestion was that Ss remember stimulus-response information; Erickson and Zajkowski suggested memory for hypotheses tried-but-rejected. They noted, in addition, that the concurrent method of solving concept identification problems moves the
behavior closer to that predicted by the no memory models for the learning rate approaches \(1/8\), that probability of randomly selecting the solution strategy in a four binary dimension problem.

In the Erickson, et al (1966) study, choice latency was the major dependent variable and again the no memory assumption was tested. Assuming that latency on a trial is a monotonic function of the size of the hypothesis pool on that trial, then several predictions about latency follow from the random sampling with replacement assumption. Latency data supported the notion that Ss resample on error trials, but did not support the assumption that the size of the hypothesis pool remains constant across presolution trials. Erickson et al suggested continuous revision of the hypothesis pool; that as hypotheses are tried and rejected, they are stored in short term memory until they are "pushed out" by hypotheses more recently tested.

Data gathered by Trabasso and Bower (1966) also question the no memory assumption. Their experiment involved a test of the sampling with replacement axiom in the context of an analysis of the "additivity of cues" (Restle, 1962). The additivity design involves three conditions with a constant set of cues (dimensions) but a varying number of relevant cues. The prediction is that the learning rate in a condition in which both cues 1 and 2 are relevant and redundant is the sum of the learning rates of conditions 1 and 2. This prediction has been supported by animal data in discrimination learning and human data from concept identification experiments (see Trabasso and Bower, 1968, for a review and discussion of additivity results).

In addition to the three standard additivity conditions, Trabasso and Bower (1966) included a fourth: dimension shift condition (DS) in which the relevant dimension (1 or 2) was shifted on every second error according to the S's response on the shift trial. In this way, it was possible for S to
be "switched" in to the correct problem solution if he sampled either cue 1 or 2 on the preceding informed error trial. In theory, therefore, the observed mean informed errors in the DS condition should be equal to the mean number of errors in the condition in which both cues are relevant and redundant. This expected result was not found although the standard additivity results were replicated; DS learning rate was slower than that expected under the assumption of sampling-with-replacement on error trials.

To account for these data, Trabasso and Bower (1966) proposed that some dimensions had been eliminated from the hypothesis pool through a process of consistency checking on an error trial (i.e., recalling the stimulus and its paired reinforcement on trial n-1 and comparing it to the stimulus and reinforcement on trial n and eliminating those dimensions which had been inconsistently reinforced), and that these dimensions remained eliminated from the pool for k trials. It should be noted that this memory process postulated does not contradict the data from the standard additivity experiment since relevant cues never fail a consistency check.

Thus, data accumulated from binary dimension concept identification problems suggests that Ss do not sample with replacement from a pool of hypotheses, but use memory for hypotheses tried-but-rejected and/or stimulus information.

B. Process models: no memory and memory for hypotheses models: the one-at-a-time sampling axiom.

Gregg and Simon (1967) have suggested that if the pool of strategies is assumed to be limited to certain well defined strategies (specifically, those prediction hypotheses defined by Levine, (1963), then a class of models can be constructed from the several memory processes suggested by various investigators and the usual assumptions of the Restle and Bower and Trabasso models. The assumption that S is choosing from a limited pool
of strategies is tenable on the basis of data presented by Levine (1963; 1966) in which Ss received instructions about the nature of the problem solution and pretraining on problems with like solutions. In order that the assumption of random sampling from the "active pool of hypotheses" on each trial be realized, problem solution must be randomized across Ss and the dimensions should be equally salient. In order for presolution stationarity to occur, instances from the whole stimulus population must be presented completely randomly (i.e., randomization with replacement). If these conditions are satisfied, then behavior in a binary concept identification task can be formalized in the following ways: If the state random variable is the particular (one) hypothesis S is using and the response to which it leads on each trial, and Ss sample hypotheses with replacement, then a $4N-2$ state ($N$ is the number of binary dimensions) Markov chain describes behavior. If the state random variable is the kind of strategy S is testing (correct, wrong or irrelevant) and the response it leads to on each trial, then the following Markov chain describes behavior, where

$$\begin{align*}
N &= \text{number of dimensions} \\
1 &= \text{the correct hypothesis} \\
2 &= \text{the wrong hypothesis, or the reversal of the solution hypothesis} \\
W &= \text{an incorrect response} \\
R &= \text{a correct response}.
\end{align*}$$

\[
\begin{array}{cccc}
\text{Trial n} & \text{IR} & \text{AR} & \text{2W} & \text{AW} \\
\hline
\text{IR} & 0 & 0 & 0 & 0 \\
\text{AR} & 0 & 1/2 & 0 & 1/2 \\
\text{2W} & \frac{1}{2N} & \frac{N-1}{2N} & \frac{1}{2N} & \frac{N-1}{2N} \\
\text{AW} & \frac{1}{2N} & \frac{N-1}{2N} & \frac{1}{2N} & \frac{N-1}{2N} \\
\end{array}
\]

\[
P(c) = \begin{bmatrix}
1 & 1 & 0 & 0
\end{bmatrix}
\]
This process model is called \( P_0 \) by Gregg and Simon (1967, p. 256). It is a sampling with replacement model since the probability of solving the problem on any error trial is a constant 1 out of 2N possible solutions to the problem. In this model, Ss have a "little bit of short term memory" since they can remember the hypothesis they are holding until they respond incorrectly. Some other process models which assume a larger storage capacity with revised sampling axioms are:

\( P_1 \): Ss recall the hypothesis they have just tested and do not return it to the pool from which they sample on the trial following their error. Thus, the probability of sampling the solution hypothesis after any error is \( \frac{1}{2N-1} \); this is a local nonreplacement model.

\( P_2 \): S chooses from a pool of hypotheses consistent with the most recent stimulus-reinforcement pairing. The learning rate is thus \( \frac{1}{N} \) after the first error trial; this is a local consistency model.

\( P_3 \): S samples randomly from a subset of hypotheses that are consistent with all prior trials. The learning rate from this assumption increases across trials as information is gathered. The number of possible solution hypotheses as a function of trials, \( n \), is \( H_n = R + 2I(1/2)^n \) where \( R \) is the number of relevant dimensions and \( I \) is the number of irrelevant dimensions. Relevant dimensions will always be consistent with reinforcement; irrelevant dimensions will be consistent with previous reinforcement with probability 1/2 on each trial. This is a global consistency model.
These models make parameter-free predictions of the learning rate in binary concept identification problems. From examination of the data, it is possible to identify which of the memory processes might be operating as Ss solve problems. In their 1967 article, Gregg and Simon compared predicted learning rates from the process models to data obtained by Bower and Trabasso (1964) from several concept identification problems. Models $P_0$ and $P_1$ corresponded fairly well with the data from 5 of the 9 experiments. One experiment could be accounted for by models $P_2$ and $P_3$ but this experiment was very similar in format and task to those for which $P_0$ and $P_1$ offered accurate predictions. Gregg and Simon could offer no explanation for this anomalous result. For three of the experiments, results could not be explained within the framework of any of the models. One experiment did not conform to the models because the task required the assignment of one of two responses to two values of four-valued stimulus dimensions. The models as formulated do not apply to this task. From these data, however, the process models have some support and can fairly accurately predict the average learning rate in binary dimension problems to which the models apply.

Erickson (1968) attempted to determine which of the proposed memory processes could best account for the data in a four binary dimension problem. After being instructed about the nature of solution and solving practice problems with two binary dimensions, Ss solved two four dimension problems, a control and an experimental. Levine's hypothesis monitoring procedure was used. In the second or experimental problem, Ss were given misinformative feedback, i.e., they were told that their first tested hypothesis was incorrect, and it was then made the basis for solution for the problem. It was found that the two problems were separated by approximately two hypotheses:
the experimental problem being more difficult. The data suggested that there is a continuous revision of the hypothesis pool, and that hypotheses reside in short term memory for about two error trials before they are pushed out by new incoming information and return to the hypothesis pool. Predicted distributions of hypotheses to solution derived from various assumptions about the sampling process were compared to the observed data. Local consistency models (like \( P_2 \)) gave the best fit to the data.

C. Hypothesis manipulation models: The some-at-a-time sampling axiom.

In an attempt to increase the generality of a model of Ss behavior as he solves concept identification problems, Trabasso and Bower (1968) have suggested several theoretical alternatives to their 1964 model. These more recent models have been proposed by these authors in order to account for data that the 1964 model is unable to explain. The first theoretical efforts were launched in order to explain the experimental results of relevant and redundant cue (RRC) studies, i.e., that some Ss solve a concept identification problem in which two dimensions are relevant and redundant by learning both relevant cues (rather than solving on the basis of one single cue as the earlier hypothesis sampling model - a "one look" model - suggests). Later efforts were concerned with formulating a model which could, in addition, account for overtraining effects such as those reported by Guy, van Fleet, and Bourne (1966): the later in overtraining a novel cue is introduced, the more it is learned and the better the transfer to it. On the basis of these and other data (reviewed by Trabasso and Bower (1968)), the authors proposed the following model:

On an error trial, S selects a focus sample, of size \( s \) which is a subset of those hypotheses which are consistent with reinforcement on the error trial (i.e., the sample consists of locally consistent H's). In this
respect, the model can be labeled a "multiple look" model since S can attend to more than one dimension on a trial. On the next trial, S responds correctly or incorrectly; the probability of a correct response is the percentage of H's in the focus sample leading to that response. If S responds correctly, he retains in the focus sample those H's that led to that correct response and eliminates those H's which would have led to an error. If however, S responds incorrectly, he rejects all H's in the focus sample on the error trial and draws a new focus sample (of locally consistent H's). Thus, the model evidences a "some-at-a-time" sampling of H's because the focus sample can consist of several H's. It should be noted that this model is consistent with data evidencing information processing on "right" trials (the elimination of H's on correct trials) and with latency declines as success runs increase in length. It is necessary, however, to elaborate further about this model in order to demonstrate that an error is a recurrent event.

In the model, the size of the sample focus remains constant across one acquisition series (this is necessary for mathematical tractibility) and the particular H's in the focus sample are determined probabilistically. The S is viewed as sampling with replacement from locally consistent H's on an error trial; each of the H's has probability $a_i$ of being included in the sample. The $a_i$ are quantities which represent the salience or attention value of each of the dimensions and these $a_i$ also remain fixed across a particular acquisition series. An error is a recurrent event because after each error S draws a "fresh" focus sample of size $s$, the content of which is a function only of the $a_i$.

In this model, as in the 1964 model, S's behavior can be represented as a two state Markov chain, the states of which are defined by the probability
of a correct response. In order to derive the presolution probability of a correct response, it is necessary to form the probability of another error given that S has responded correctly after some reference error. Trabasso and Bower (1968, p. 56) present an expression for the distribution of the length of success runs conditional upon the occurrence of another error. If the random variable $H$ is defined to be the number of successively correct responses between any two errors, $P(H=n) = qp^n$ where $p$ is the probability that the irrelevant cues in the focus sample lead to the same response as the relevant cues, and $q$ is $1-p$. This is the geometric distribution, thus the probability of a correct response following a correct response (in the presolution state) is equal to $p$. The sum of the terms of this distribution from $n=1$ to $n=\infty$, is the probability that $S$ is correct after an error during the presolution state (that is, the length of the success run is at least 1) and $S$ makes at least one more error. This is also equal to $p$, since \[
\sum_{n=1}^{\infty} qp^n = \sum_{n=0}^{\infty} qp^n - q = 1 - q = p.\] Therefore, presolution $P(c)$ is stationary at $p$.

The second state of the 1968 model is the solution state, in which the probability of a correct response is 1.00. The probability that $S$ enters the solution state on any error trial is the probability that $S$ draws a sample focus that eventuates in solution. Trabasso and Bower (p. 56) have shown that the probability of solving on an error trial is the probability of sampling the solution dimension. This probability is independent of the chance proportion correct when irrelevant dimensions are the basis for responding.

So far, the basic tenents of the theory have been reviewed and some similarities to the 1964 model have been noted. Because it is similar to the 1964 model, the 1968 model can account for data consistent with the
former model, e.g., stationary $P(c)$, additivity of cues, independence of presolution responses. However, the probability of solving on each error trial in the recent model remains a constant ($a_k$) while much data show this not to be true (Erickson et al, 1966; Erickson, 1968; Holstein and Premack, 1965; Levine, 1966, 1969; Trabasso and Bower, 1966). In addition, opportunities for learning (exiting the presolution state) occur only on an error trial.

One major difference between the 1964 and 1968 model is that the recent model can account for data from RRC experiments. The earlier model is unable to account for the fact that some Ss learn two cues, since it assumes Ss attend to and test only one dimension at a time. However, the recent model allows one or the other or both of the relevant dimensions to be present in the solution focus. Trabasso and Bower (1968, p. 63) develop equations which express the proportion of Ss solving on one or the other or both dimensions as a function of learning rates in problems with only one of the dimensions relevant. The theory has been tested in concept identification and paired associates learning (Trabasso and Bower, 1968; chs. 3 and 5) with some success. Other predictions of the model such as all-or-none transfer following RRC training and the influence of cue salience and number of irrelevant cues on the frequencies of solution type were tested and generally supported. The model had some shortcomings, however, particularly with respect to observed "blocking" effects (a first learned cue inhibits the learning of a redundant relevant cue added during overtraining) and overtraining effects in RRC problems sometimes resulted in an increased number of Ss learning both cues, but sometimes did not (in their experiments, overtraining results were a function of the type of stimuli in RRC problems). Trabasso and Bower presented several theoretical alternatives to their first
proposed model tailored so that these alternatives could account for the aforementioned data. These will not be examined here, since their predictions have not been worked out in any detail and they have not been tested empirically.

A second general class of hypothesis manipulation models has been proposed by Levine (1966) and Richter (1965) and formalized for the four binary dimension problem and tested in that situation by Chumbley (1967). The essential features of this model are quite different from the process models previously discussed. In these models, S samples more than one hypothesis at a time; an error is not necessarily a recurrent event and Ss can learn on an error or correct trial, but via different processes. These models are in some ways similar to the Trabasso and Bower (1968) model, yet there are differences. In these IM models, the size of the focus sample varies from trial to trial, but, as in the Trabasso and Bower (1968) model, the probability of a correct response is the percentage of H's leading to that response. On the early trials, H's in the focus sample are chosen on the basis of local consistency but as the S experiences more reinforced trials, the H's in the focus sample are chosen on the basis of consistency with reinforcement on more than the most recent trial (trials n-1, n-2, etc.). In addition, the H's in the focus sample are all those H's which are tenable on the basis of what S can remember about the sequence of stimuli and reinforcements: there is no selection of H's on any attention basis. As in the Trabasso and Bower model, H's which are untenable on the basis of the reinforcement are eliminated from the set of tenable H's (or the focus sample); on error trials, the focus sample is constituted anew, but it is not sufficient to describe this new sample as consisting of locally consistent H's, nor, at times, is it correct.
As Chumbley (1967) formalized the model, the process can be explained as follows: Consider a stimulus population formed from four binary dimensions arranged so that in any sequence of three stimuli, the dimensions are internally orthogonal. On the first trial, S considers all hypotheses to be equally likely to be correct (the 8H state). In order to respond to the first stimulus, he must choose between two complimentary subsets of hypotheses (those which label the stimulus A and those which label it B). If he chooses the correct response, e.g., A, then he holds the set of hypotheses which lead to A as a response to the stimulus on that trial (the 4H state). If he makes an error, then he may recover the hypotheses which lead to the opposite response remaining in the 4H state with probability r or return to the 8H state. In the 4H state, S is forced to subdivide this set of hypotheses again; if he responds correctly, then he retains those hypotheses leading to that correct response (the 2H state); if he responds incorrectly, he either recovers the tenable set of hypotheses remaining in the 2H state with probability r or does not recover these hypotheses, in which case he begins the sampling process again (the 8H state). From the 2H state, the set is again subdivided, and if S chooses correctly or recovers, he learns; if not, he begins again. The model can be written as a transition matrix (Chumbley, 1967):

\[
\begin{array}{cccc|c|c}
\text{Trial n} & \text{1H} & \text{2H} & \text{4H} & \text{8H} & \text{P(c)} & \Pi_0 \\
1H & 1 & 0 & 0 & 0 & 1 & 0 \\
2H & p_2 + (1-p_2)r & 0 & 0 & (1-p_2)(1-r) & p_2 & 0 \\
4H & 0 & p_4 + (1-p_4)r & 0 & (1-p_4)(1-r) & p_4 & 0 \\
8H & 0 & 0 & p_8 + (1-p_8)r & (1-p_8)(1-r) & p_8 & 1 \\
\end{array}
\]
where \( r = p(\text{recovering alternative subset following an error}) \)

\[ p_2 = p(\text{choosing correctly in 2H}) \]
\[ p_4 = p(\text{choosing correctly in 4H}) \]
\[ p_8 = p(\text{choosing correctly in 8H}). \]

Chumbley's (1967) experiment provided a test of this "hypothesis manipulation" (HM) model and also the consistency check model proposed by Trabasso and Bower (1966). Certain anomalies in the values of the parameters of the Trabasso-Bower model (e.g., the probability of recalling an S-R pairing one trial back was higher than the probability of recalling an S-R pairing on the current trial) eliminated it from consideration, while the HM model accounted for the data reasonably well. The parameters \( p_2 \), \( p_4 \) and \( p_8 \) were given a priori values and \( r \) was estimated from the data; the HM model could not be rejected on the basis of goodness-of-fit criteria. The values of the recovery parameter made psychological sense across the conditions of his experiment. For example, it decreased as intertrial interval decreased and as number of concurrent problems increased.

Other data from the literature make this model attractive: the suggestion that Ss process information on "right" trials (Suppes and Schlag-Rey, 1965; Merryman et al, 1968; Bourne, Guy, and Wadsworth, 1967; Levine, 1969) and that they test more than one hypothesis per trial (Richter, 1965; Erickson, Block, and Rulon, 1969; Levine, 1969; Trabasso and Bower, 1968).

These various models all of which have received some support, have dealt only with binary concept identification problems. One purpose of this dissertation is to extend these models to more complex problems, specifically to problems with three-level (ternary) dimensions, in order to provide further tests of their assumptions.
II. DEVELOPMENT OF HYPOTHESIS MODELS FOR TERNARY DIMENSIONS

The previous sections have reviewed several models of Ss behavior as he solves binary concept identification problems. Each of these models has some support in the literature. One way to establish the potential usefulness of the processes these models include is to logically extend them to more complex concept identification problems. Empirical tests of these extensions would provide information concerning the validity of the processes in more complex problems and thus serve to define the range of applicability of the models and provide a step toward a general theory of concept identification.

The several hypothesis sampling models reviewed can probably be most easily contrasted by considering two of their basic tenants: the number of hypotheses sampled on a trial; and the function of "right" and "wrong" trials. The class of models in which one hypothesis is held by S on a trial and in which correct responses serve to maintain the hypothesis and incorrect trials lead to its rejection are herein referred to as Process Models (after Gregg and Simon, 1967). Note that in this class of models an error may or may not be a recurrent event. The class of models in which S tests more than one hypothesis on a trial and in which information from the reinforcement can be used to narrow the set of tenable hypotheses on both right and wrong trials are herein referred to as Hypothesis Manipulation (HMI) models (after Chumbley, 1967). In this class of models also, an error is not necessarily a recurrent event since S may "recover" information afforded by the reinforcement.

The following section is the mathematical formalization of these two classes of models to problems with ternary dimensions. The Process Models are developed first for the several memory processes considered earlier.
The HM models are then developed along the lines reviewed in the preceding paragraphs. All models are applied to the following type of simple concept - identification problem. The solution to the problem is based on one dimension. The subject's task is to map the values of the relevant dimension onto the response categories in a one-to-one fashion as defined by the experimenter. The following paragraph explains the representation of the stimuli and defines the various types of hypotheses. These definitions and notations are used throughout the development of the models.

If the values of the dimensions are represented by the numbers 0, 1, 2 and each digit in a number represents a dimension, then stimuli may be represented abstractly as an N digit number, e.g., 0012..., in which dimensions 1 and 2 have values 0 and 0; dimension 3 has value 1, dimension 4 has value 2, etc. For example, in a population of four ternary dimensions there are 81 possible stimuli (four places with three values possible in each = $3^4$). If the response categories are labeled A, B, and C, then a solution to the problem may be of the form 0 = A, 1 = B, 2 = C with dimension 2 relevant. An irrelevant hypothesis could be, e.g., 0 = A, 2 = B, 1 = C on dimension 2 or 1 = A, 0 = B, 2 = C on dimension 1, etc. A "reversal" of the solution hypothesis is what has previously been called a wrong strategy (a wrong strategy always leads to incorrect responses) and for this example problem would be hypotheses based on the relevant dimension but with the response categories completely "switched", e.g., 0 = C, 2 = B, 1 = A or 0 = B, 2 = A, 1 = C on dimension 2.

Process Models The first model developed is a no memory model ($P_0$) for N ternary dimensions with three response categories. For the general problem with N ternary dimensions, there are $6^N$ possible prediction hypotheses (6 response permutations and N dimensions). One of these is the
solution hypothesis; two are "reversals" of the solution, while the rest are irrelevant and lead to a correct response with chance probability of 1/3. If the state random variable is the particular hypothesis S is holding and whether it leads to a correct response or an incorrect response on that trial, then behavior can be described by a multi-state Markov chain (in particular, the chain involves N ternary dimensions and it has thirty-three states). Fortunately, this chain can be aggregated (Kemeny and Snell, 1960; p. 124) to the following chain:

<table>
<thead>
<tr>
<th>Trial n</th>
<th>SC</th>
<th>IC</th>
<th>RE</th>
<th>IE</th>
<th>P(c)</th>
<th>P_0</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>IC</td>
<td>0</td>
<td>1/3</td>
<td>0</td>
<td>2/3</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>RE</td>
<td>1/6N</td>
<td>2N-1/6N</td>
<td>1/3N</td>
<td>2N-1/3N</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>IE</td>
<td>1/6N</td>
<td>2N-1/6N</td>
<td>1/3N</td>
<td>2N-1/3N</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

in which the states are:
- **SC**: S holds the solution hypothesis and makes a correct response
- **IC**: S holds any irrelevant hypothesis and makes a correct response
- **RE**: S holds one of the "reversal" hypotheses and makes a wrong response
- **IE**: S holds any irrelevant hypothesis and makes an incorrect response.

The transition probabilities are formed in the following ways: the probability of going from either RE or IE to SC is simply the probability of sampling the solution hypothesis. For P_0 this probability is constant on each error trial and is the probability that the one sampled hypothesis is the solution hypothesis (out of the entire pool of hypotheses). The
transition probability from RE and IE to IC is the probability that S samples one of the "I" hypotheses (of which there are 6N-3) times the chance probability of a correct response which is 1/3. The probability of remaining in RE or going from IE to RE is the probability that S samples one of the two reversals from the hypothesis pool - which is 2/6N or 1/3N (recall that a reversal leads to an error with probability 1.00). The probability of going from RE or IE to IE is the probability that S samples one of the "I" hypotheses and makes a chance error with probability 2/3. Note that hypotheses are not sampled on a correct choice trial and that SC is an absorbing state.

The starting vector, \( \Pi_0 \), is formed in the following way: The probability of starting in SC is the probability of sampling the solution hypothesis on the first trial (1/6N). The probability of starting in IC is the probability of sampling one of the irrelevant hypotheses and making a chance success (6N-3/6N x 1/3). The probability of starting in IE is formed in the same way, except the "I" hypotheses lead to an error with probability 2/3. The probability of starting in RE is the probability that S samples one of the reversal hypotheses (2/6N).

By aggregating states RE and IE, an analogy to the Bower-Trabasso (1964) three state matrix is:

\[
\begin{array}{ccc|c|c}
SC & IC & E & P(c) & \Pi_0 \\
\hline
SC & 1 & 0 & 0 & 1 & \frac{1}{6N} \\
IC & 0 & 1/3 & 2/3 & 1 & \frac{2N-1}{6N} \\
E & \frac{1}{6N} & \frac{2N-1}{6N} & 2/3 & 0 & \frac{4N}{6N} \\
\end{array}
\]
If it is assumed that Ss remember the hypothesis most recently tried but rejected, then model $P_1$ can be formulated. The transition probabilities are formed in the same manner as the model $P_0$, but the size of the active hypothesis pool is decreased by one hypothesis (the one most recently tried) after the first error trial. In Markovian formation, model $P_1$ takes the following form:

$$
\begin{array}{cccc|c|cccc}
 & SC & IC & RE & IE & P(c) & \pi_0 \\
\hline
SC & 1 & 0 & 0 & 0 & 1 & \frac{1}{6N} \\
IC & 0 & 1/3 & 0 & 2/3 & 1 & \frac{2N-1}{6N} \\
RE & 1 & \frac{2N-1}{6N-1} & \frac{1}{6N-1} & \frac{4N-2}{6N-1} & 0 & \frac{2}{6N} \\
IE & \frac{1}{6N-1} & \frac{2N-1}{6N-1} & \frac{2}{6N-1} & \frac{4N-2}{6N-1} & 0 & \frac{4N-2}{6N} \\
\end{array}
$$

which can be aggregated as in model $P_0$.

The third process model, the local consistency model, ($P_2$), is formulated by assuming that Ss sample from a pool of hypotheses which are consistent with reinforcement on the most recent error trial. For the ternary problem, $2N$ hypotheses are consistent with each reinforcement ($N$ dimensions with response permutations of the two values of the dimension not experienced on that trial). In this model, the "reversal" hypotheses are never consistent with reinforcement; consequently only states SC, IC, and IE are possible after the first trial. The probability of moving from IE to SC is the probability that S samples the solution hypothesis from the pool of $2N$ locally consistent hypotheses. The p(IE to IC) is formed by considering the probability of choosing an "I" hypothesis ($2N-1/2N$) and making a chance success (1/3). The p(IE to IE) is the probability of choosing an "I" hypothesis and making a chance error (2/3). The starting vector includes the probability that S starts in SC, which is the probability of choosing the solution
hypothesis with no information \( (1/6N) \). S starts in IC by choosing one of the "I" hypotheses \( (6N-3/6N) \) with no information and making a chance success. The probability of starting in IE is the probability of choosing an "I" hypothesis and making a chance error or of starting with one of the reversal hypotheses. Thus, the transition matrix for the local consistency model takes the following form:

<table>
<thead>
<tr>
<th></th>
<th>SC</th>
<th>IC</th>
<th>IE</th>
<th>P(c)</th>
<th>( \Pi_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>( \frac{1}{6N} )</td>
</tr>
<tr>
<td>IC</td>
<td>0</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{2}{3} )</td>
<td>1</td>
<td>( \frac{2N-1}{6N} )</td>
</tr>
<tr>
<td>IE</td>
<td>( \frac{1}{2N} )</td>
<td>( \frac{2N-1}{6N} )</td>
<td>( \frac{2N-1}{3N} )</td>
<td>0</td>
<td>( \frac{4N}{6N} )</td>
</tr>
</tbody>
</table>

The final process model to be considered in this section is \( P_3 \), or the global consistency model. In this model, the hypothesis pool from which S samples on an error trial consists of those hypotheses which are consistent with all previous reinforcements. In general, the hypothesis pool "shrinks" as the number of trials increases. In the binary case, a formula which related the number of hypotheses tenable on a trial to the number of trials experienced by S was presented for random stimulus sequences. Unfortunately, the analogous formula for the ternary case is not a simple extension of the binary case and in order to derive a formula for the ternary case, inductive methods would be required. But one can control the stimulus sequence and the number of hypotheses tenable on a trial are a product of this stimulus control. In the following section on HM models, which are closely related to \( P_3 \), the use of stimulus sequencing accomplishes this result. As the HM model is developed, the number of hypotheses tenable for the early trials in a concept problem are noted.
In the previous section on binary models of concept identification, the Bower and Trabasso (1964) model was presented and it was noted that this model could be reduced to the process models of Gregg and Simon (1967) if several simplifying assumptions were made. But the complete Bower and Trabasso model describes S's behavior as he solves problems with ternary dimensions via a different process. The following paragraphs include a brief explication of the complete Bower-Trabasso model for ternary dimensions and its relation to the process models just considered.

The Bower-Trabasso model contains two phases in learning: stimulus selection and paired associate learning. State 0 is the state during which stimulus selection occurs, before the selection of the relevant attribute. In order to leave state 0, S must make an error, select the relevant dimension, and condition the presented value to the reinforced response. S stays in state 0 if he makes a chance success, or if he makes an error and does not select the relevant dimension or condition the value to the reinforced response. If S leaves state 0, he goes to state 1 where one of the value-response pairs has been learned. To leave state 1, he must experience an unconditioned value of the relevant dimension (if instances from the stimulus population are random and equally frequent, this probability is 1/3) and learn its associated response, thus going to state 2 where two value-response pairings are learned. He may stay in state 1 by not experiencing an unconditioned value or experiencing an unconditioned value but not conditioning it. States 2 and 3 are entered analogously. While S is in state 2, he is assumed to use "restricted guessing", i.e., making that response which is not yet conditioned to the unconditioned value of the relevant dimension. In Markovian formulation:
in which $\theta = p($conditioning the presented value to the reinforced response$)$

$q = p($chance error in state 0$)$,

This model is presented as it appears in Bower and Trabasso (1964).

The parameter, $\varepsilon$, remains to be defined. The authors state (p. 89)
"[In order to leave state 0] .... stimulus selection occurs with probability $\varepsilon$ when the subject makes a chance error with probability $q.$" The next sentence states "Elsewhere we have identified $\varepsilon$ as $r\theta$, where $r$ is the probability of selecting the relevant attribute and $\theta$ is the probability of conditioning the presented value to the reinforced response."

From these two statements, it is difficult to decide just what psychological process takes place with probability $\varepsilon$. If $\varepsilon = r\theta$, then the probability that $S$ leaves state 0 on any error trial is $r\theta$, the probability of selecting the relevant dimension and conditioning the reinforced response to that value presented on the error trial. Such an interpretation suggests that $S$s engage in dimension selection on every error trial before they select the relevant dimension. This does not seem reasonable in problems with more than two values per dimension, since errors occur for two reasons: $S$s are attending to an irrelevant dimension or they are assigning responses to the values of the dimensions improperly. Thus, on an error trial, $S$s may not sample among the dimensions but rather retain the same dimension and "switch the response assignments around".
In order to account for this, it seems reasonable to assume that Ss engage in dimension selection on an error trial with some probability and to include this probability in calculating the learning rate per error trial. Bower and Trabasso's first statement quoted herein suggests that this probability is $\varepsilon$. If $\varepsilon$ is the probability that Ss engage in dimension selection, then there is some probability $r$ that they select the relevant dimension. Reasoning along these lines then implies that the probability of leaving state 0 on an error trial is $\varepsilon r$, rather than $\varepsilon$ as presented in the model. Although it makes no difference quantitatively whether a parameter is expressed as $\varepsilon$ or $\varepsilon r$, any attempt at process interpretations forces these distinctions.

On a later page of the same article (p. 91), Bower and Trabasso (1964) discuss numerical estimates of the parameter $\varepsilon$. They note that the obtained estimate of $\varepsilon$ from an experiment with three dimensions was very close to $(1/3)\theta$ and that the hypothesis $\varepsilon = r \theta$ was confirmed by their data. Apparently Bower and Trabasso do interpret $\varepsilon$ as $r \theta$ rather than the second interpretation of $\varepsilon$ mentioned herein. With this interpretation it is possible to compare their model with the previously presented process model for ternary dimensions.

In order to show the relation of their model to the process models, it is necessary to consider two cases of the model: the case in which $\theta=1$ and $\theta<1$ and to make some process interpretations of the parameters. Assuming that $q=2/3$ (i.e., the probability of an error in state 0 is 2/3) and that $r=1/N$, and additionally that $\theta=1$ leads to some interesting predictions of the learning rate in ternary concept identification problems. On an error trial, the probability that S makes no further errors given the previous assumptions is the joint probability that S selects the
relevant dimension, makes a correct response on the next trial and makes a correct response when he encounters the first of the remaining two values on the relevant dimension. This joint probability is $1/N \cdot 1/3 \cdot 1/2$, or, the probability that, on an error trial, S selects one of the $6N$ possible solutions to the problem. Thus, under the preceding assumptions, the Bower and Trabasso model is equivalent to process model $P_0$, the no memory model. Note that the Bower-Trabasso model is also a no memory model since the learning rate on each trial is a constant and is formed by assuming no memory for recently tested dimensions ($r=1/N$ on each error trial) or for recent stimuli ($1-q=1/3$ on each error trial); the only memory $S$ uses is that for the dimension currently being tested and the reinforced value on that dimension. The no memory model (with $\theta=1$) implies stationary $P(c)$ at $1/3$ while $S$ is in state $0$. If it is assumed that dimension selection does not occur on each error trial in the presolution state, i.e., that on some trials, $S$s are "switching" responses based on an irrelevant dimension, the $P(c)$ after these error trials is also $1/3$ in a random sequence of stimuli. The $P(c)$ remains at $1/3$ even if $S$ has "learned" one value-response pairing on an irrelevant dimension because, in order to make a correct response, both the relevant and irrelevant dimensions must remain the same (with probability $1/3\times 1/3=1/9$) or both must change (with probability $2/3\times 2/3=4/9$); and, if they change, $S$ must choose the correct response (with probability $1/2$). Thus, $1/9+4/9\times 1/2=1/3$. After $S$ leaves state $0$ and before he enters state $2$, the $P(c)$ increases to $2/3$, since, on any trial, $S$ may experience the one value of the relevant dimension that he "knows" (with probability $1/3$) or the other two values (with probability $2/3$). Thus, the $P(c)=1/3\times 1+2/3\times 1/2=2/3$. 
Allowing for memory of the stimulus present on the error trial increases the learning rate. The probability that S makes no more errors after an error trial is the probability that S selects the relevant dimension and makes a correct response when confronted with the two remaining values of that dimension = \(1/N^{1/2}\). This learning rate is the same as that of \(P_2\), the local consistency model.

If \(\theta < 1\), then the \(P(c)\) before the TLE retains the same step function characteristic (from 1/3 to 2/3), but the plateau at \(P(c)=2/3\) should hold for a larger number of trials than for the case in which \(\theta = 1\).

**Hypothesis Manipulation Model**

In formulating an HM model for ternary dimensions it seems desirable to allow S two alternatives on every trial: permitting him to gain new information as each stimulus is experienced or allowing him to "lose" all accumulated information. He can, of course, acquire new information by either responding correctly or responding incorrectly and recovering the complementary set of hypotheses; he can lose accumulated information by not responding correctly and not recovering. Thus, the extended version of the manipulation model encompasses the two main characteristics of Chumbley's (1967) model: there is no partial recovery of hypotheses and S cannot remain in the same informed state from one trial to the next.

To incorporate these characteristics into a testable extension of the model, the sequence of stimuli experienced by S must be constructed in nonrandom fashion. After consideration of several alternatives, the stimulus sequences were constructed with the following characteristics:

1. A stimulus population of 4 ternary dimensions was used.
2. Three dimensions change value on each trial.
(3). A particular dimension could have the same value only twice in succession.

(4). The dimension that stayed the same on trials $n$ and $n+1$ could not, when it changed on $n+2$, take on the same value twice in succession. This means that sequences such as trial

<table>
<thead>
<tr>
<th>Dimension 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
</tr>
<tr>
<td>$n+1$</td>
</tr>
<tr>
<td>$n+2$</td>
</tr>
<tr>
<td>$n+3$</td>
</tr>
</tbody>
</table>

were not permitted.

When the stimuli are arranged in this manner, $S$ can acquire new information (or, equivalently, eliminate alternative hypotheses) on every trial. The $HM$ model formulated for $N=4$ ternary dimensions and takes the following Markovian form:

$$
\begin{array}{cccccc}
\text{trial} & 1H & 2H & 3H & 8H & 24H \\
1H & 1 & 0 & 0 & 0 & 0 \\
2H & 0 & 0 & 0 & 0 & (1-\rho_2)(1-r) \\
3H & \frac{3}{4}(\rho_6+1-\rho_6)r & 0 & 0 & 0 & \frac{3}{4}(1-\rho_6)(1-r) \\
8H & 0 & \frac{3}{4}(\rho_6+1-\rho_6)r & \frac{3}{4}(\rho_6+1-\rho_6)r & 0 & \frac{3}{4}(1-\rho_6)(1-r) \\
24H & 0 & 0 & 0 & 0 & \rho_6+1-\rho_6) r & (1-\rho_6)(1-r) \\
\end{array}
$$

$$
\begin{align*}
\mathbf{p}(c) &= [1] \\
\mathbf{\pi}_o &= [0, 0, 0, 0, 0, 1]
\end{align*}
$$
in which \( r = p(\text{recovering the complementary set of } H'\text{s on an error trial}) \)

\[
\begin{align*}
P_{24} &= p(\text{correct response in } 24H) \\
P_{8U} &= p(\text{correct response in } 8H \text{ when the relevant dimension is the unchanged dimension}) \\
P_{8C} &= p(\text{correct response in } 8H \text{ when the relevant dimension is the changed dimension}) \\
P_{3U} &= p(\text{correct response in } 3H \text{ when the relevant dimension is the unchanged dimension}) \\
P_{3C} &= p(\text{correct response in } 3H \text{ when the relevant dimension is a changed dimension}) \\
P_2 &= p(\text{correct response in } 2H).
\end{align*}
\]

The states of the model are the number of hypotheses which are tenable solutions to the problem on the basis of past reinforcements. \( S \) begins in state 24H; since he has not been reinforced, all possible prediction hypotheses are tenable solutions to the problems. As \( S \) gains information, he can reduce the initial set of hypotheses until only one hypothesis is tenable; that remaining hypothesis is the solution to the problem and \( S \) is in the 1H state. This model evidences certain properties which are quite different from the process models \( (P_0, P_1, P_2) \): \( S \) tests more than one hypothesis at a time on trials before solving the problem and his response determines the current subset of tenable hypotheses. E.g., if \( S \) begins the problem by experiencing 0000 = A and he has made a correct response or an incorrect response but recovered the hypothesis set, and the stimulus on the next trial is 1000, then if \( S \) responds B or C, he is responding according to those tenable \( H'\text{s} \) based on the changed dimension. If he responds A, then he is attending to the unchanged dimensions in accordance with the \( H'\text{s} \) tenable on those dimensions. None of the three process models permit the concurrent testing of more than one hypothesis and \( S \)'s response is
implied by the one hypothesis he is testing. The HM model allows the processing of information on both right and wrong trials, while the only process model which permits this is the global consistency model. With respect to this latter model, it is difficult to draw definite distinctions between it and the HM model without making assumptions about, e.g., the possibility of Ss learning on right trials or the chance probability of a correct response in the presolution state. However, the fact that the learning rate is a function of the number of hypotheses tenable on the basis of all previous reinforcements makes the global consistency model analogous to the HM model.

In the following paragraphs, the HM model is developed in detail. The development requires two things: the number of hypotheses tenable on each trial as a function of the stimulus sequence and the \( \text{a priori} \) probability of a correct response when \( S \) is in each state of the model.

In this model, it is possible for \( S \) to follow one of three paths to the solution or the 1H state. He may proceed from 24H-8H-3H-1H, from 24H-8H-2H-1H, or from 24H-8H-3H-2H-1H. These several paths to the solution state are possible because the number of hypotheses logically tenable on a trial is a function of whether or not the relevent dimension has changed from one trial to the next. In order to explain the possible paths through the various states, it is necessary to examine possible sequences of stimuli and reinforcements and how these sequences determine the number of logically tenable H's on a trial. Figure 1 is an illustration of the allowable stimulus sequences and how these, in combination with sequences of reinforcements, function to determine Ss H-state on a trial. Figure 1 begins with Ss experiencing 0000=A on trial n. If \( S \) has made a correct response or an incorrect response but has recovered those H's consistent with 0000=A,
Figure 1. Flow chart of the various paths S may take to arrive in the 1H state.
then he holds 8H's as possible solutions to the problem, two on each dimension of the form 0=A, 1=B, 2=C or 0=A, 2=B, 1=C. On the next trial, trial n+1, two events may occur: the relevant dimension changes or it remains the same. If it remains the same, then S might experience, e.g., 0111=A. If he has made the correct response or an incorrect response and recovered, then he knows that 2H's on the relevant dimension are possible solutions to the problem: 0=A, 1=B, 2=C or 0=A, 2=B, 1=C. When S is in 2H, he must decide between these two H's, since, by restriction 3 on the stimulus sequences, the relevant dimension must change on trial n+2. If he makes a correct response or an incorrect response but recovers, then he is absorbed into the 1H state. Thus, S has progressed from state 8H on trial n to 2H on trial n+1 to 1H on trial n+2.

The paths resulting from the event that the relevant dimension changes from trial n to n+1 are also outlined in Figure 1. If S has experienced 0000=A and the relevant dimension changes from n to n+1, then S might experience 0111=B on trial n+1. If he responds correctly or incorrectly but recovers, he enters the 3H state in which 3 H's are logically possible solutions to the problem: one H on the relevant dimension and two H's on two changed irrelevant dimensions of the form: 0=A, 1=B, 2=C. On trial n+2, two events may occur: the relevant dimension stays the same or changes. The path to the 1H state is simpler when the relevant dimension stays the same from n+1 to n+2 and so it is considered first. If the relevant dimension stays the same from n+1 to n+2, then S might experience, e.g., 1120=B. Since the relevant dimension stayed the same, then the two H's (from the 3H state) based on the irrelevant dimensions are not consistent with reinforcement; the only H that is consistent with reinforcement is that based on the relevant dimension: 0=A, 1=B, 2=C. If S was correct or incorrect but re-
covered, then he is absorbed into the 1H state. Thus, S has proceeded from 8H-3H-1H.

The remaining case to be considered is that in which S is in the 3H state and the relevant dimension changes from n+1 to n+2. When the relevant dimension changes, it may change to the same value held on trial n; or, to a new (different) value not held on trial n or n+1. In a sense, then, the relevant dimension may be "confounded" with an irrelevant dimension if both take the same values held on n or different values; or "not confounded" when one takes the value held on n and the other does not. If the dimensions are not confounded, then S might experience 1020=A and end in the 1H state on trial n+2 with the solution H 0=A, 1=B, 2=C on dimension 2. If the dimensions are confounded and S experiences, e.g., 2001=A, then he holds two remaining tenable H's: 0=A, 1=B, 2=C on dimensions 2 and 3. The characteristics of the stimulus sequence are such that these two dimensions become "unconfounded" on trial n+3 because either the relevant dimension or the remaining tenable irrelevant dimension changes on the next trial. Thus, S may experience 1022=A and if he responds correctly or incorrectly but recovers, he is absorbed into the 1H state (0=A, 1=B, 2=C on dimension 2). The remaining path from 3H to 1H when the relevant dimension changes to a different value than that held on n and n+1 is directly analogous to the path just explained and is also outlined in Figure 1. Thus, from 3H, it is possible to go to 1H on n+2 or to 2H on n+2 and then to 1H on n+3.

In general, it is possible to go from one state of the model to another state with more information by either making a correct response or making an incorrect response but recovering the complement of the hypothesis set on the error trial. If S does not recover this information, he loses all
previous information and slips back into the 24H state.

Thus far, the number of hypotheses tenable on a trial have been de-
lineated as a function of the stimulus sequence. These hypotheses define
the states of the model. In order to form transition probabilities, the
probability of a correct response in each state of the model must be derived
(those parameters $p_{24}$, $p_{8C}$, $p_{8U}$, etc.) and the probability of recovery on
an error trial must be added.

In general, the transition probabilities are formed by first considering
the probability of a correct response in each state of the model. This $P(c)$
is a function of the H's tenable on a trial and the probability that the rele-
vant dimension changes or remains the same on the next trial. If S makes a
correct response or makes an incorrect response and recovers the complementary
set of H's with probability $r$, then it is possible to enter other states of
the model in which he has more information about the tenable H's. If, however,
S makes an incorrect response and does not recover the complementary set of
H's, then he enters the 24H or no information state. These statements out-
line the general manner in which the state to state transition probabilities
are formed. For a more complete derivation of these probabilities, the in-
terested reader is referred to Appendix A in which a tree diagram of the
transition probabilities is presented and an explanation of the combination
of these probabilities is also included.

Summary of Theoretical Developments and Description of the Experiment

In the previous section, several current models of concept identification
in the binary situation were extended to a problem with ternary dimensions.
In this section, the experiment which provides tests of these models is
described and the particular predictions of the models and data relevant to
these models is discussed.
A. The experimental conditions

1. The stimulus sequences

   It was noted in the theoretical development that hypothesis sampling models of CI are developed with the assumption of a random stimulus sequence. It was also noted that in order to formulate and test a HM model of CI, the sequence of stimuli must be controlled, i.e., presented in nonrandom fashion. In particular, the HM model for ternary dimensions assumed a sequence in which three dimensions changed per trial. Thus, in order to provide tests of these models, it is necessary to provide two conditions of stimulus sequence: random and 3/trial. It is also of some interest to provide a condition in which learning rates for a concept identification problem would differ markedly from the random and 3/trial condition if Ss remember more recent stimulus information than that provided by the most recent stimulus. Thus, a condition in which four dimensions change per trial was added.

2. The stimulus-feedback interval

   Erickson (1969) has suggested that the interval in which the stimulus and the reinforcement appear concurrently is an important interval in the present task because it might affect the memory process in CI. In particular, it seems reasonable to expect that the longer the stimulus-feedback interval, the higher the proportion of Ss using a local consistency strategy (since there is more time to rehearse the dimension value-feedback pairs, e.g.). This particular hypothesis was tested in the present experiment by varying the stimulus-feedback interval at 1 sec., 5 sec., 10 sec. for each stimulus sequence. In addition to providing information with respect to the differential use of several processes in solving CI problems, the variation in this interval should result in variations of the estimates of r, the recovery parameter in the HM model. It seems reasonable to expect that r should be
smaller for the shorter intervals, since Ss have less time to recover the complementary set of H's.

3. The relevant dimension

The previously developed models rest heavily on the assumption of equal salience of the stimulus dimensions. Thus, in any test of these models, the validity of this assumption must be assessed. In order to do so, each of the stimulus dimensions was the relevant dimension an equal amount of time for each sequence by stimulus-feedback interval condition. Thus, the design of the experiment was a 3x3x4 factorial with three levels of stimulus sequence (random, 3/trial, 4/trial), three levels of stimulus-feedback duration (1 sec., 5 sec., 10 sec.) and four stimulus dimensions.

4. The shift conditions

It has recently been shown (Erickson, 1969; Erickson, Block and Rulon, 1969) that Ss in a binary concept ID task tend to reject the entire dimension upon which a recently disconfirmed H was based, and, on the next trial, test H's based on other dimensions (if thoroughly instructed). It was decided that several shift conditions should be added to the present experiment in order to determine whether Ss solving a ternary problem also exhibit this same tendency. This information is very important in any attempt to account for concept ID by the application of H sampling or dimension sampling theory. After solving a ternary dimension problem, Ss were shifted to a reversal of their prior solution, or to an intradimensional or an extradimensional shift condition.

5. The one-A, two-B problem

It is of some interest to examine a condition in which the concept-to-be-learned is similar to the one dimensional ternary concept, but the number of possible solution H's is much smaller. A condition of this sort
55

provides a "link" to the binary dimension problem in which, for the same number of dimensions, the number of solution H's is much smaller than in the ternary problem. A problem which satisfies this requirement is one in which the ternary stimuli are used, but the solution is based on one dimension and involves the matching of one value of that dimension to one response (the "A" response) and the other two values to a second response (the "B" response). If $P_0$ or $P_2$ provide good accounts of the data in the ternary problem, then these data will be analyzed to provide additional checks on the validity of the Process Models.
III. EXPERIMENTAL METHOD

Subjects: One hundred eight male and female students from the introductory psychology course served as Ss. They served in the experiment as part of a course requirement. An additional seven Ss served in the experiment but had to be rejected for one of several reasons: two could not solve the practice problem; one could not solve the first experimental problem within the one hour time limit, and four had to be discarded because of apparatus malfunction.

Procedure: The Ss were given detailed instructions about the nature of solutions of the problems stressing that E would label each stimulus according to the values of one of the dimensions. Ss then solved one practice problem in which stimuli varied on two binary dimensions. Following the practice problem, Ss were given instructions about the first experimental problem in which stimuli varied along four ternary dimensions. Exemplar stimuli were shown to S and the various dimensions and their values were pointed out. Ss were again told that E would label each stimulus "A", "B", or "C" according to the values on one of the dimensions. If S solved the first experimental problem to a criterion of 10 successively correct responses, within one-half hour, he was shifted without warning to a second experimental problem, the solution of which could be (1). a complete reversal (RS) of the first problem solution: the previously relevant dimension remained relevant, but the values of the relevant dimension were assigned new responses or (2). an intradimensional shift (IDS) of first problem solution: the previously relevant dimension remained relevant, but one value of the relevant dimension retained the response assignment of the first problem, while two values were assigned new responses or (3). an extradimensional shift (EDS): the previously relevant dimension became irrelevant and one of the irrelevant dimensions during the first problem contained the solution to the shift problem. The
criterion for solution of the shift problem was ten consecutively correct responses. If these two experimental problems were solved within forty-five minutes, Ss solved a third experimental problem. After solving the shift problem, S was told that he had solved two problems and that the next problem would not contain a shift. The dimensions were reviewed for S and the nature of the solution to the third problem was explained. They were instructed that again one of the dimensions contained the solution to the problem but that E would label one value of that dimension an "A" and the other two values a "B". The criterion for solution of the third problem was 15 consecutively correct responses. Thus, all Ss solved a practice problem and a four ternary dimension experimental problem. Of these Ss, the "fast learners", ones who did not come late, etc. solved one or two additional experimental problems: a shift problem and a "one-A, two-B" problem. Final n's for each group were as follows: 108 solved the first experimental problem; of these, 83 solved a shift problem, 26 solved a RS, 33 solved an IDS, 24 solved an EDS; of these, 69 solved the "one-A, two-B" problem.

Design: Each S experienced one of three types of stimulus sequence: a random sequence, a sequence in which three dimensions changed on every pair of trials, or a sequence in which all four dimensions changed on every pair of trials. These sequences are labeled R, 3/trial, and 4/trial. In addition, each S experienced one of three time intervals during which the stimulus and feedback were paired for 1 sec., 5 sec., or 10 sec.. For each sequence and time interval, each of the four dimensions contained the solution to the first experimental problem for three Ss. Thus, the design for the first experimental problem was a 3x3x4 factorial with three Ss per cell. Within each schedule and dimension condition, each of the six permutations of response and assignment to dimension values occurred once and three randomly
Table 1. 3x3x4 factorial design used in problem 1.

See Table 2 for response assignments for various solution numbers.

<table>
<thead>
<tr>
<th>Stimulus - Feedback Interval</th>
<th>1 sec.</th>
<th>5 sec.</th>
<th>10 sec.</th>
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<tbody>
<tr>
<td>Dimension Sol. no.</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Color 2,5,6&lt;sup&gt;a&lt;/sup&gt;</td>
<td>Color 1,3,4</td>
<td>Color 2,5,6</td>
<td>Color 2,5,6</td>
</tr>
<tr>
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<td>Form 3,4,6</td>
<td>Form 1,2,5</td>
<td>Form 3,4,6</td>
</tr>
<tr>
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</table>

Random Schedule 3/trial

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<th></th>
<th></th>
</tr>
</thead>
<tbody>
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<td>Dot 2,3,4</td>
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</table>

Schedule 4/trial

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<td>Form 1,2,5</td>
<td>Form 3,4,6</td>
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<tr>
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<td>Border 1,5,6</td>
<td>Border 2,3,4</td>
<td>Border 1,5,6</td>
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<td>Dot 2,3,4</td>
<td>Dot 2,3,4</td>
<td>Dot 1,5,6</td>
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</table>
Table 2. Solutions to problem 1.

<table>
<thead>
<tr>
<th>Solution number</th>
<th>Dimension Values</th>
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<tbody>
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<tr>
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<td></td>
</tr>
<tr>
<td>Red</td>
<td>A</td>
</tr>
<tr>
<td>Blue</td>
<td>B</td>
</tr>
<tr>
<td>Green</td>
<td>C</td>
</tr>
<tr>
<td>Form</td>
<td></td>
</tr>
<tr>
<td>Circle</td>
<td>A</td>
</tr>
<tr>
<td>Triangle</td>
<td>B</td>
</tr>
<tr>
<td>Square</td>
<td>C</td>
</tr>
<tr>
<td>Dimension</td>
<td></td>
</tr>
<tr>
<td>Left</td>
<td>A</td>
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<tr>
<td>Right</td>
<td>B</td>
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<tr>
<td>Bottom</td>
<td>C</td>
</tr>
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<tr>
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<td>A</td>
</tr>
<tr>
<td>Solid</td>
<td>B</td>
</tr>
<tr>
<td>Dashed</td>
<td>C</td>
</tr>
</tbody>
</table>
chosen permutations occurred twice. Table 1 is a diagram of this design.

Table 2 includes the response permutations for problem 1.

Each shift problem was solved under the same sequence and time conditions as the original problem. Each of the three Ss in a cell was randomly assigned to a shift condition with the restriction that shift conditions be represented in each cell as equally as possible. In addition, within each shift condition, an attempt was made to include all solutions qualifying as an RS, IDS, or EDS.

The third problem was also solved under the same sequence and time conditions as the original and shift problems. Solutions to the third problem were based on a dimension other than that relevant during the first two problems for RS and IDS Ss. The relevant dimension was counterbalanced across the conditions of the experiment as much as possible. For the EDS subjects, the relevant dimension was approximately equally likely to be a dimension relevant in the previous problems or a previously irrelevant dimension. Of the two dimensions falling into each case, the solution dimension was chosen randomly and an attempt was made to include all solutions on each dimension across the conditions of the experiment.

Apparatus and Materials: The 81 stimuli varied on four ternary dimensions: color (red, green, or blue); form (circle, triangle, or square); border (none, solid, dashed); and position of a dot outside the figure (right, left or bottom.) The stimuli were placed on 35 mm. slides and were rear projected onto a screen at the front of Ss booth. The S sat at a table with a response panel containing three buttons and three feedback lights. The lights directly above the buttons; in the space between the buttons and the lights were the labels "A", "B", and "C". In the practice problem, only the "A" and "B" buttons were used. Stimulus sequences were preprogrammed and punched on paper tape which was read by a Friden tape reader. A system of relays, timers,
and associated electronic equipment presented stimuli and feedback at the proper intervals and allowed the recording of responses and choice latencies. The time relations were as follows: feedback was given 2 sec. after the response and remained on for 1 sec., 5 sec., or 10 sec. after which both the feedback light and slide went off. Ten seconds later the next slide came on. Slide presentation started a timer which stopped with S's response. Choice latencies and responses were recorded manually. The Ss wore earphones into which low level white noise was projected to eliminate distracting noise from the projector, tape feed, relays, etc.

For each stimulus sequence condition, three different 162 trial stimulus sequences were prepared. In the R condition, stimuli were randomly ordered with the following restrictions:

(1). During the 162 trials, each of the 81 stimuli appeared twice.
(2). A stimulus could appear only once in each set of four consecutive stimuli.

The 4/trial sequences were randomly ordered with the above two restrictions and one additional restriction:

(3). On every pair of trials, all four dimensions had different values.

The 3/trial sequences were randomly ordered maintaining the two restrictions on the R sequences and three additional restrictions:

(3). Three dimensions had different values on every pair of trials.
(4). A particular dimension could have the same value only twice in succession.
(5). The dimension that stayed the same on trials n, n+1, could not, when it changed on n+2, have the same value twice in succession.

In each cell of the experiment, one of the three stimulus schedules was paired with one solution. When a solution was repeated within one sequence condition, it was paired with a different schedule for the same condition. Each S experienced one schedule during all three problems. For the third problem, the schedule was advanced one trial from the end of the shift problem.
IV. RESULTS AND DISCUSSION

Data relevant to tests of the various models formulated for the ternary situation are presented in this section. These models make varying predictions about presolution responding, learning rates and response latency in this situation; each model prediction is presented and the models are discussed in light of the data.

The aspects of presolution responding for which the models under consideration herein make differing predictions are the stationarity of presolution $P(c)$, the probability of a correct response conditional upon preceding consecutively correct responses, and the probability of a correct response conditional upon a preceding correct or error response.

Stationarity Tests

The Process Models $P_0$, $P_1$, $P_2$ which assume that Ss solve concept identification problems by sampling hypotheses on error trials predict that the probability of a correct response before the TLE should be stationary, or constant, before the TLE. The a priori $P(C)$ before the TLE predicted by these models is about 1/3 for the random condition. This prediction was tested by Vincentizing the $P(C)$ before the TLE into four quartiles in the random condition. Figure 2 is a graph of Vincentized $P(C)$ for each stimulus sequence condition. The figure reveals that Vincentized $P(C)$ in the random condition exhibited a rather distinct upward trend across presolution quartiles; $P(C)$ began a little below chance and rose to a presolution maximum of about 2/3. The mean $P(C)$ before the TLE in the random condition was .419, significantly above the a priori value ($z = 2.965$, $P<.01$). A $\chi^2$ test of these Vincentized data of 31 Ss in the random condition verified the nonstationary ($\chi^2 = 21.75$, $P<.005$).
Figure 2. Backward learning curve of the probability of a correct response before the TLE Vincentized into four parts for each stimulus sequence.
According to the HM model, while Ss are still making errors, they could be in any state of the model except the solution (1H) state. The probability of a correct response in the presolution states ranges from 1/3 (the 21H state) to 1/2 (the 2H state). Thus, presolution P(C) should range from 1/3 to 1/2 before the TLE. The Vincentized data from the 3/trial condition indicated that this was not the case; presolution P(C) of 34 Ss was rather low (below chance) in the first quartile and exhibited the same upward trend as that evidenced in the random condition; terminal P(C) in the 3/trial condition was about .60 which was above the a priori maximum predicted by the model. Vincentized $\chi^2$ verified the nonstationarity in these data ($\chi^2_3 = 20.40, p<.005$). It is interesting to note the form of the P(C) trend before the TLE in the random and 3/trial conditions. In the random condition, the P(C) curve is relatively stationary until the fourth quartile, not unlike previous data (Suppes and Ginsberg, 1963). In the 3/trial condition, the P(C) curve increased more gradually to a maximum of .60 in the third and fourth quartiles. Table 12 in the appendix shows ANOVAs relevant to these trends.  

Analyses of variance on presolution P(C) Vincentized in four parts for the random and 3/trial conditions verified the differing proportion of correct responses in the 4 quartiles in these data ($F_{3,90} = 7.228, p<.005$ for the random condition; $F_{3,99} = 5.371, p<.005$ for the 3/trial condition) and Scheffé contrasts of the means verified the aforementioned trends; the mean of the quartile totals in quartiles 1,2,3 was significantly different from the total correct responses in quartile 4 in the random condition ($p<.01$) and the mean of total correct responses in quartiles 1 and 2 was significantly different from the mean total correct in quartiles 3 and 4 ($p<.01$).

Since every dimension changed from trial n to n+1 in the 4/trial condition, process models $P_0$ and $P_2$ predict differing probabilities of a correct response

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1 All analyses of variance tables can be found in Appendix B.
following correct or error responses for trials before the TLE. $P_0$, the no
memory model, predicts that $P(C_{n+1}/E_n)$ should be about 1/3 whereas $P(C_{n+1}/C_n)$
should be about 1/2. $P_2$, the local consistency model, predicts these two condi-
tional probabilities to be equal to 1/2. Thus, under $P_0$, $P(C)$ should range
between 1/3 and 1/2; while under $P_2$, $P(C)$ should be stationary at about 1/2.
Neither model can be rejected from the 4/trial data in Figure 2. The Vincent
curve appeared stationary (within the range of $P(C)$ of .40 to .50) at an
average $P(C)$ of .439; however, this $P(C)$ was significantly different from the
*a priori* .50, $z = 2.541$, $p < .01$. A $\chi^2$ test of the Vincentized data was not
significant ($\chi^2_3 = 4.54$, $0.10 < p < .25$).

It is possible that the nonstationarity evidenced in the data from the
random and 3/trial conditions arose because either $P(C_n/E_{n-1})$ or $P(C_n/C_{n-1})$
increased. Additional analyses were performed to determine the locus of the
nonstationarity and to provide additional tests of the process models $P_0$ and $P_2$, each of which predicts $P(C_n/E_{n-1})$ should be constant across trials before
the TLE for both the random and 3/trial conditions. With respect to
$P(C_n/C_{n-1})$ across presolution trials, both $P_0$ and $P_2$ predict this conditional
should be constant in the random condition, however, due to the arrangement
of the stimuli in 3/trial condition, $P(C_n/C_{n-1})$ is not independent of the
number of the preceding consecutive correct responses (according to $P_0$ and $P_2$),
consequently, *a priori* stationarity predictions are difficult to derive.\(^2\)

Both $P_0$ and $P_2$ predict that both conditional probabilities of a correct
response should be stationary across presolution trials in the 4/trial con-
dition.

---

\(^1\)The fact that $P(C_n)$ varies as a function of the number of preceding con-
secutive correct responses in the 3/trial condition is discussed more fully
in a later section and in Appendix G.
The first analysis of $P(C_n/E_{n-1})$ was conducted by Vincentizing each Ss presolution trials into two parts and including in the analysis only those Ss who had at least one error (followed by an error or a correct response) in each part. (Neither the median trial nor the TLE were included in this analysis.) Table 3 shows $P(C_n/E_{n-1})$ for each sequence condition and also the number of Ss in this analysis. For the random and 3/trial conditions, $P(C_n/E_{n-1})$ increased from the first to the last half of learning; in the 4/trial condition, this probability decreased. Chi square tests showed the change was significant in the 3/trial condition ($\chi^2 = 7.985, p<.005$); and approached significance in the random condition ($\chi^2 = 2.787, .054 < p < .10$); but was not significant in the 4/trial condition ($\chi^2 = .140, p>.50$).

Since so few Ss could be included in the previous analysis, two additional analyses of responses before the TLE were performed. In these analyses, pairs of consecutive responses were Vincentized across presolution trials. The analyses were performed on $C_n$, $C_{n+1}$ and on $E_n$, $E_{n+1}$ pairs by recoding Ss' protocols into $C_n$, $C_{n+1}$ and $E_n$, $E_{n+1}$ pairs and Vincentizing the recoded sequences in the usual manner. In this analysis there were 22 Ss in the random condition, 17 Ss in the 3/trial, and 22 Ss in the 4/trial condition.

The analysis of $E_n$, $C_{n+1}$ and $E_n$, $E_{n+1}$ pairs proceeded in the same fashion. In the random, 3/trial and 4/trial conditions, 30, 29 and 29 Ss respectively qualified for this analysis. This technique of Vincentizing sequences of responses essentially divides pre-TLE trials into parts which have the same number of correct or error responses rather than the same number of trials.

Figures 3 and 4 present the Vincentized sequences for each stimulus condition. Figure 3 presents the ratio of $C_n$, $C_{n+1}$ to $E_n$, $E_{n+1}$ pairs across quartiles before the TLE. In both the random and 3/trial conditions
Table 3. The probability of a correct response after an error response for early and late presolution trials for each stimulus sequence.

<table>
<thead>
<tr>
<th>Schedule</th>
<th>Early</th>
<th>Late</th>
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</tr>
</thead>
<tbody>
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<td>.511</td>
<td>.428</td>
</tr>
<tr>
<td>N</td>
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<td>16</td>
<td></td>
</tr>
<tr>
<td>3 /trial</td>
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<td>.583</td>
<td>.394</td>
</tr>
<tr>
<td>N</td>
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<td>16</td>
<td></td>
</tr>
<tr>
<td>4 /trial</td>
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<tr>
<td>X</td>
<td>.326</td>
<td>.375</td>
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</tbody>
</table>
Figure 3. Backward learning curve for sequences of two correct responses Vincentized into 4 parts before the TLE for each sequence condition.
Figure 4. Backward learning curve for the probability of a correct response on trial n after an error on trial n-1 Vincentized before the TLE for each stimulus sequence.
P(C_n, C_{n+1}) increased throughout the precriterion period. ($\chi^2 = 8.667$, and 8.419 for the random and 3/trial conditions respectively, both $p < .05$.)

A test of nonstationarity in the 4/trial condition resulted in a $\chi^2$ of 5.054 which was not significant ($10 < p < .25$).

Figure 4 shows the Vincentized $P(E_n, C_{n+1})$ sequences; again, both the 3/trial and the random conditions exhibit increasing trends across presolution trials; ($\chi^2 = 10.152$, and 13.235 respectively for the random and 3/trial conditions, both $p < .025$). A $\chi^2$ test of nonstationarity in the 4/trial condition was not significant ($\chi^2 = 2.093, p > .50$).

In summary, the tests of stationarity in the three stimulus sequence conditions showed that presolution $P(C)$ was not stationary in the random nor the 3/trial condition, but was stationary in the 4/trial condition. It was also found that $P(C_n/E_{n-1})$ was stationary in only the 4/trial condition, and that both $C_n, C_{n+1}$ and $E_n, C_{n+1}$ response pairs increased across presolution trials in the random and 3/trial conditions, but not in the 4/trial condition. Thus, the stationary predictions of $P_0$ and $P_2$ were not supported in the random condition, but were supported in the 4/trial condition. The nonstationarity observed in the random and 3/trial conditions was probably due to both the increasing probability of a correct response after corrects and the increasing probability of a correct response after errors. In addition, the HM model predicted a range for presolution $P(C)$; since this range was exceeded in the 3/trial condition, the prediction was not supported.

Polson and Greeno (1969) have discussed the relevance of nonstationary tests of performance to inferences about all-or-none learning. These authors have demonstrated that assumptions of Bernoulli presolution trials is in fact not necessary to the occurrence of all-or-none learning and that data may exhibit nonstationarity characteristics through the influence of response
biases or other performance variables even though the learning process is all-or-none. In order to test for the existence of these factors in the present data, some indices were computed. The percentage of first responses that were As, Bs, or Cs were 42% As, 34% Bs, and 24% Cs. Thus, at the start of the experiment, Ss preferred the A response. However, when the per cent of each response was computed across all presolution trials in problem 1, these percentages were almost equal: 34% As, 36% Bs, and 30% Cs. On the basis of these data, there seems to be at least no strong marginal response bias. While these data do not exclude the possibility of more complex kinds of response dependencies, it is probably safe to accept the stationarity data as it stands.

Sequences of Presolution Responses

According to the Process Models $P_0$, $P_1$, and $P_2$, Ss begin a concept identification problem by sampling one of the 24 possible Hs and respond on the basis of this H until they reach criterion or make a later error at which time they again sample from a pool of Hs, the Hs in the pool being defined by the particular memory process assumed. This process continues until Ss sample the solution H. Thus, on the trials before the last error, Ss are sampling and responding on the basis of irrelevant or wrong Hs. Using these assumptions, it is possible to derive the probability of successive confirmations of presolution Hs$^1$ and the probability of various numbers of successive correct responses before the TLE for each stimulus sequence. These predictions can be seen in Table 4 along with the observed probabilities. In the random condition, regardless of the amount of memory assumed (for previous stimuli

$^1$Levine et al. (1968) have used this technique in deriving the probability of successively correct responses given a pool of presolution Hs for internally orthogonal stimuli.
Table 4. The probability of a correct response contingent upon 0, 1, or 2 previous consecutive correct responses estimated from all responses before the TLE or from all responses after the first error and before the TLE from the 3 sequence conditions.

<table>
<thead>
<tr>
<th></th>
<th>$P_0$</th>
<th>$P_2$</th>
<th>HM</th>
<th>All responses</th>
<th>$N$</th>
<th>N first error</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P(C_n)$</td>
<td>.304</td>
<td>.333</td>
<td>-</td>
<td>.419</td>
<td>26</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$P(C_n/C_{n-1})$</td>
<td></td>
<td></td>
<td></td>
<td>.425</td>
<td>17</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$P(C_n/ C_{n-1}, C_{n-2})$</td>
<td>.333</td>
<td>.333</td>
<td>-</td>
<td>.444</td>
<td>11</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3/trial</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P(C_n)$</td>
<td>.304</td>
<td>.321</td>
<td>&gt;.333</td>
<td>.441</td>
<td>28</td>
<td>.454</td>
<td>19</td>
</tr>
<tr>
<td>$P(C_n/C_{n-1})$</td>
<td>.250</td>
<td>.148</td>
<td>&gt;.344</td>
<td>.462</td>
<td>15</td>
<td>.507</td>
<td>12</td>
</tr>
<tr>
<td>$P(C_n/ C_{n-1}, C_{n-2})$</td>
<td>.143</td>
<td>.000</td>
<td>&gt;.444</td>
<td>.500</td>
<td>8</td>
<td>.500</td>
<td>7</td>
</tr>
<tr>
<td>4/trial</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P(C_n)$</td>
<td>.304</td>
<td>.429</td>
<td>-</td>
<td>.439</td>
<td>28</td>
<td>.445</td>
<td>23</td>
</tr>
<tr>
<td>$P(C_n/C_{n-1})$</td>
<td>.429</td>
<td>.500</td>
<td>-</td>
<td>.503</td>
<td>22</td>
<td>.520</td>
<td>20</td>
</tr>
<tr>
<td>$P(C_n/ C_{n-1}, C_{n-2})$</td>
<td>.500</td>
<td>.500</td>
<td>-</td>
<td>.506</td>
<td>15</td>
<td>.545</td>
<td>14</td>
</tr>
</tbody>
</table>
and/or previous Hs tested), the probability of a correct response contingent upon 1 preceding consecutive correct responses remains constant at .333.

In the case of $P_0$, the unconditional $P(C_n)$ is lower than the conditional $P(C_n)$ because reversal or wrong Hs cannot lead to a correct response and are excluded from the computation of the conditional $P(C_n)$.

Because $P_0$ and $P_2$ predict the same conditional probability of a correct response (and nearly the same unconditional probability), all presolution responses were used to estimate these probabilities. The observed probabilities are in Table 4; $N$ is the number of Ss making 1, 2, or 3 consecutively correct responses. These observed probabilities in the random condition were significantly higher than the \textit{a priori} probabilities ($\chi^2 = 18.164, p < .005$) and remained relatively constant as the number of preceding corrects increased. A $\chi^2$ test of the independence of the probability of a correct response of the number of preceding correct responses supported the hypothesis of independence ($\chi^2 = .131, p > .50$).

In the 3/trial conditions, models $P_0$ and $P_2$ predict that the conditional probability of a correct response should decrease as the number of previous corrects increase. Also, the probability that a success run continues decreases more sharply as more memory is assumed (i.e., $P_2$ vs. $P_0$). From the data in Table 4 it can be seen that for all responses and responses after the first error\footnote{Model $P_2$ holds only after the first error.} the conditional probability of a correct response increased slightly rather than decreased as predicted by the Process Models, but the increase was not significant ($\chi^2 = 1.013$ for all responses and 1.187 for responses after the first error, both $p > .25$). Chi square tests of predicted and observed probabilities resulted in significant differences from $P_0$ ($\chi^2 = 89.135$) and $P_2$ ($\chi^2 = 90.008$), both $p < .005$. In addition, if Ss were learning as prescribed
by $P_2$, then there should be no success runs longer than 3 successes between
the first error and the TLE. However, in the 3/trial condition, there were
39 occurrences of 3 or more corrects during this period. Thus, the predic-
tions of the Process Models were not confirmed; the probability of a correct
response did not decrease as the number of preceding errors increased.

In the 4/trial condition, $P_0$ predicts an increasing probability of a
correct response as the number of consecutive corrects increases from 1 to 2;
the data from all responses indicates that these probabilities did increase
($\chi^2_1 = 3.794, p<.05$) although observed probabilities were significantly higher
than predicted probabilities ($\chi^2_4 = 36.127, p<.005$). $P_2$ also predicts that
these probabilities should increase from 1 to 2 previous corrects; and the
data from responses after the first error showed a significant increase
($\chi^2_1 = 6.536, p<.025$). In addition, a test of observed and predicted proba-
bilities from $P_2$ indicated agreement between these two in this condition
($\chi^2_4 = 1.262, p>.50$).

The conditional data in the 3/trial condition are relevant to tests of
the HM model. The HM model predicts that the probability of a correct response
should increase as the number of preceding consecutively correct response
increases since when Ss make a correct response they proceed to a state of the
model in which they have more information about the solution $H$, and have a
higher probability of making a correct response. The predictions for the HM
model in this condition are that shown in Table 4. Recall that the increasing
trend appeared, but was not significant. It should be noted that since these
probabilities were computed before the TLE, they do not include Ss who solved
with no errors, or those Ss who "solved" after the TLE; thus, the probabili-
ties are underestimated.
The HM model also predicts that after S makes four correct responses, he should make no more errors, since four successive correct trials are sufficient for eliminating all Hs except the solution H. In the 3/trial condition, there were 7 cases where there were at least four correct responses followed by an error.

With respect to these data, it is clear that, in general, predictions of the Process Models were not supported. Tests of the HM model led to ambiguous results; although presolution $P(C)$ increased slightly with increasing corrects, a second prediction of the model did not hold, Ss made more consecutively correct responses than expected before the TLE.

**Independence of Response Tests**

On trials before the TLE, the Process Models make differing predictions about the probability of a correct response on trial $n$ contingent upon a correct or an error response on trial $n-1$. For these conditions, according to the Process Models, one would expect the following trends to appear: in the random condition, $P(C_n/E_{n-1}) = P(C_n/C_{n-1})$; in the 3/trial condition $P(C_n/E_{n-1}) > P(C_n/C_{n-1})$; in the 4/trial condition $P(C_n/E_{n-1}) < P(C_n/C_{n-1})$. The data are shown in Table 5 for each sequence condition. In general, the same differences in the conditional probabilities appeared for each condition: $P(C_n/C_{n-1}) > P(C_n/E_{n-1})$. In the random condition, the two probabilities were not significantly different ($\chi^2 = .654, p<.25$). In both the 3/trial and the 4/trial conditions, the difference was significant; $\chi^2 = 4.330, p<.05$ and $\chi^2 = 11.156, p<.005$ respectively.

These conditional data from the 3/trial condition are relevant to predictions of the HM model. The HM model predicts that $P(C_n/C_{n-1}) > P(C_n/E_{n-1})$.

---

1These trends are expected on the basis of the derivations of the probability of a correct response given 0 or 1 preceding corrects shown in Table 4 and derived in the appendix.
Table 5. Probability of a correct response or error response on trial n contingent upon a correct or error on trial n-1. Data from all presolution responses.

<table>
<thead>
<tr>
<th>Response on trial n-1</th>
<th>Random</th>
<th>3/trial</th>
<th>4/trial</th>
</tr>
</thead>
<tbody>
<tr>
<td>trial n</td>
<td>trial n</td>
<td>trial n</td>
<td>trial n</td>
</tr>
<tr>
<td>Correct</td>
<td>.425</td>
<td>.462</td>
<td>.503</td>
</tr>
<tr>
<td>Error</td>
<td>.374</td>
<td>.322</td>
<td>.333</td>
</tr>
<tr>
<td>Correct</td>
<td>.575</td>
<td>.538</td>
<td>.497</td>
</tr>
<tr>
<td>Error</td>
<td>.626</td>
<td>.678</td>
<td>.667</td>
</tr>
</tbody>
</table>
because, of those Ss making an error on trial \( n \), some of them will slip back into the 24H state where the probability of a correct response is lower than in the other states of the model, while all those Ss making a correct response will proceed into states 8H, 3H, or 2H where \( P(C) > 1/3 \). The data from the 3/trial condition indicate that \( P(C_n/C_{n-1}) \) is significantly greater than \( P(C_n/E_{n-1}) \) supporting this prediction. Thus, the predictions of \( P_0 \) and \( P_2 \) were confirmed in the random condition and the 4/trial condition, but not confirmed in the 3/trial condition. Data from the 3/trial condition supported predictions of the HM model.

Parameter Estimation

In order to compare the models to additional data, it is necessary to estimate the parameter of the HM model, \( r \). In order to estimate \( r \), a minimum \( \chi^2 \) procedure (Atkinson, Bower, and Crothers, 1965; Sternberg, 1963) was carried out on the frequencies of the eight possible sequences of correct and error responses for the first three trials in the 3/trial condition. Four estimates of \( r \) were obtained: one estimate for each of the S–R interval conditions and one estimate based on the sequence totals across the time conditions. Estimates of \( r \) from the 1, 5, and 10 sec. stimulus–feedback intervals and for all these combined were .149, .429, 1.00 and .650 respectively. Unfortunately, these \( \chi^2 \) were based on a rather small number of observations (12 in each interval group) and although these parameters are ordered in the way one might expect on the basis of the HM model (i.e., the probability of recovery increases as \( S \) has more time to compare the stimulus with the feedback) an additional check on these estimates seemed desirable. Therefore, values of \( r \) were also estimated by maximizing the fit of the HM model to the distribution of total errors. The smallest \( \chi^2 \)'s occurred at \( r \) values of about .50 in the
1 sec. and 5 sec. interval conditions and at $r = .75$ in the 10 sec. condition. These are quite discrepant from the estimates obtained previously, however, they were ordered in the same way. The total error $\hat{r}$ based on all Ss in the 3/trial condition was $\hat{r} = .55$ rather than the previous $\hat{r} = .65$. These two estimates seem reasonably close.

Additional aspects of the data for which these models make varying predictions include portions of the data which are predicted to be functions of the learning rate of the Process Models or functions of the parameter of the HM model. These data include the distribution of errors before the first success, error runs, and the distributions of trials to criterion and total errors.

**Distribution of Errors before the First Success**

The distribution of errors before the first success is important for testing $P_0$ and $P_2$ because it is a function of the probability of sampling the solution $H$ on each error trial. Figure 5 compares distribution predicted from $P_0$ and $P_2$ to the data from the random condition. There are large discrepancies between the predicted and observed values at both $i=0$ and $i=2$, especially at $i=2$ where both $P_0$ and $P_2$ underestimate the actual proportion of Ss making two errors before the first success by a factor of 2:1. The predicted distribution of the number of errors before the first success has the form

$$P(I=i) = q(1-\epsilon)\epsilon^{i-1} \text{ for } i > 0$$

where $I$ is the random variable and $\epsilon$ is the learning rate per error trial and $q$ is the probability of an error on a trial. Varying $q$ (e.g., by using $q$ estimated from the data) changes the level of the curve. Varying $\epsilon$ changes the slope of the curve. Neither of these changes would produce a distribution close to that obtained. Thus, in the random condition, neither $P_0$ nor $P_2$ nor any model of the same family could account for the distribution of errors before the first success.
$i =$ no. of errors before the first success

Figure 5. Observed distribution of the number of errors before the first success in the Random Condition compared to the prediction of $P_0$, $P_1$ and $P_2$. 
A distribution of the number of errors before the first success is also predicted by the HM model. This distribution is shown in Figure 6. Since the predictions for \( \hat{r} = .65 \) and \( \hat{r} = .55 \) were so similar, only one predicted distribution is shown. Also shown is the observed distribution from the 3/trial condition. The observed distribution follows somewhat the predicted distribution except for the discrepancy at \( i=1 \); fewer Ss than expected made one error before the first success. The observed mean number of errors before the first success was 2.167; the expected mean number was 1.774 (for \( \hat{r} = .65 \) and \( \hat{r} = .55 \) averaged). These statistics indicate that Ss made more errors before the first success than expected, however, the discrepancy between the mean number of errors expected and observed is not large and is in line with other published data.

Error Run Statistics

Error run statistics are a function of learning rate per error trial and as such provide an additional basis for testing \( P_0 \) and \( P_2 \); they also provide additional tests of the HM model. Table 6 includes the observed mean number of error runs of varying lengths observed in the random and 3/trial conditions. Included also are the predictions of \( P_0 \) and \( P_2 \) and the HM model. With respect to \( P_0 \) and \( P_2 \), it can be noted that with more memory, predicted error runs of any length decrease. When the predictions are compared to the data from the random condition, it is clear that both \( P_0 \) and \( P_2 \) seriously overestimate the mean number of error runs of any length and also of each particular length \( j \). Again, \( P_0 \) and \( P_2 \) seemed to underestimate the amount of memory used by Ss in this situation.

When predictions of the HM model for \( \hat{r} = .65 \) and \( \hat{r} = .55 \) are compared to the data from the 3/trial condition, the expected mean number of error runs of any length was somewhat lower than that observed. In particular, more runs of almost every length were observed than were predicted and the discrepancy
Figure 6. Observed distribution of number of errors before the first success in the 3/trial condition compared to the predicted distribution of the HM model.
<table>
<thead>
<tr>
<th>Mean no error runs of any length.</th>
<th>Predicted</th>
<th>Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_0$</td>
<td>$P_2$</td>
</tr>
<tr>
<td>Mean no error runs of length $j$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$j=1$</td>
<td>2.555</td>
<td>1.331</td>
</tr>
<tr>
<td>2</td>
<td>1.704</td>
<td>0.776</td>
</tr>
<tr>
<td>3</td>
<td>1.135</td>
<td>0.453</td>
</tr>
<tr>
<td>4</td>
<td>0.757</td>
<td>0.264</td>
</tr>
<tr>
<td>5</td>
<td>0.504</td>
<td>0.154</td>
</tr>
<tr>
<td>6</td>
<td>0.337</td>
<td>0.089</td>
</tr>
<tr>
<td>7</td>
<td>0.224</td>
<td>0.052</td>
</tr>
<tr>
<td>8</td>
<td>0.149</td>
<td>0.031</td>
</tr>
<tr>
<td>9</td>
<td>0.099</td>
<td>0.018</td>
</tr>
<tr>
<td>10</td>
<td>0.066</td>
<td>0.010</td>
</tr>
</tbody>
</table>

Table 6. Observed mean number error runs in the random and 3/trial conditions compared to predicted mean no error runs from $P_0$, $P_2$ and the HM model.
between observed and predicted was larger for $r = .65$. Generally, however, these discrepancies are not large enough to represent any serious deviation from the prediction of the model.

**Analyses of Trials to Criterion and Total Errors**

Thus far the HM model has been compared to distributions which were mainly a function of errors. It is of some interest to also compare the model to the distribution of the TLE, which is a function of both corrects and errors. Figure 7 shows the observed cumulative distribution of the TLE in the 3/trial condition compared to the predicted distributions from the HM model with $r = .55$ and $r = .65$. Both predicted distributions are above the observed distribution indicating that the cumulative proportion of Ss expected to reach a particular TLE was somewhat greater than that observed i.e., Ss learned faster than predicted. The fit of the $r = .65$ distribution was poor; however, the fit of $r = .55$ distribution was much better and accounts fairly well for the distribution of the TLE.

Analyses of variance were performed on trials to criterion and on total errors. The results of the two analyses were the same and only the results of the analyses on errors will be repeated herein.¹ The main effect of dimension was significant ($F_{3,72} = 3.158, p < .05$) while the main effects of stimulus sequence and S-R duration interval approached significance ($F_{2,72} = 2.518, .05 < p < .10$ and $F_{2,72} = 2.259, .10 < p < .25$ respectively). Two interactions approached significance, the duration by dimension interaction and the three way sequence by duration by dimension. Table 7 shows the mean total errors for each pair of the three main variables and also the marginal error means. From these means, it can be seen that dot was generally the most salient dimension.

¹Table 13 in Appendix B is the ANOVA table for these variables.
Figure 7. Observed cumulative distribution of TLE in the 3/trial condition compared to predictions from the HM model for $r = .55$ and $r = .65$. 

- • observed
- ◻ ◻ HM, $r = .55$
- △ △ HM, $r = .65$

Figure 7. Observed cumulative distribution of TLE in the 3/trial condition compared to predictions from the HM model for $r = .55$ and $r = .65$. 

- • observed
- ◻ ◻ HM, $r = .55$
- △ △ HM, $r = .65$
Table 7. Mean total errors in the first problem for the schedule, interval and dimension variables.

<table>
<thead>
<tr>
<th>S-R interval</th>
<th>1 sec</th>
<th>5 sec</th>
<th>10 sec</th>
<th>$\bar{X}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random</td>
<td>6.500</td>
<td>5.416</td>
<td>3.083</td>
<td>5.000</td>
</tr>
<tr>
<td>Sequence</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3/trial</td>
<td>4.583</td>
<td>5.250</td>
<td>3.000</td>
<td>4.278</td>
</tr>
<tr>
<td>4/trial</td>
<td>8.500</td>
<td>6.083</td>
<td>6.000</td>
<td>6.861</td>
</tr>
<tr>
<td>$\bar{X}$</td>
<td>6.528</td>
<td>5.583</td>
<td>4.028</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Color</th>
<th>Form</th>
<th>Border</th>
<th>Dot</th>
<th>$\bar{X}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random</td>
<td>4.222</td>
<td>7.222</td>
<td>6.778</td>
<td>1.778</td>
<td>5.000</td>
</tr>
<tr>
<td>Sequence</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3/trial</td>
<td>4.778</td>
<td>4.222</td>
<td>5.333</td>
<td>2.778</td>
<td>4.278</td>
</tr>
<tr>
<td>$\bar{X}$</td>
<td>5.741</td>
<td>6.963</td>
<td>5.889</td>
<td>2.926</td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Color</th>
<th>Form</th>
<th>Border</th>
<th>Dot</th>
<th>$\bar{X}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 sec</td>
<td>8.778</td>
<td>9.778</td>
<td>5.889</td>
<td>1.667</td>
<td>6.528</td>
</tr>
<tr>
<td>S-R interval</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
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<td>6.000</td>
<td>8.778</td>
<td>2.667</td>
<td>5.583</td>
</tr>
<tr>
<td>10 sec</td>
<td>3.556</td>
<td>5.111</td>
<td>3.000</td>
<td>4.444</td>
<td>4.028</td>
</tr>
<tr>
<td>$\bar{X}$</td>
<td>5.741</td>
<td>6.963</td>
<td>5.889</td>
<td>2.926</td>
<td></td>
</tr>
</tbody>
</table>
in that Ss whose relevant dimension was the dot dimension solved the problem faster than Ss having another dimension relevant. Post hoc comparisons among the mean total errors for each pair of relevant dimensions were made using a Newman-Keuls procedure. The only significant comparison occurred between the form and dot dimensions (p < .05).

Although the sequence and duration variables did not reach significance, the ranking of the marginal means of these variables is interesting. Mean total errors was smallest in the 3/trial condition and largest in the 4/trial condition. Also, as the S-R interval increased, mean total errors tended to decrease. Thus, from these trends, there is some indication that stimulus sequence and S-R duration interval affect total errors in concept identification, although these trends were not significant.

An analysis of variance was also performed on total errors in the shift problem. None of the previous main effect variables significantly affected total errors; the only variable to approach significance was the main effect of sequence. The main effect of shift was not significant. None of the three way interactions were tested because of the unequal n across the cells of the ANOVA. Table 8 presents mean total errors for the shift condition paired with the S-R interval. The trends evidenced by the means are again of some interest. Although the marginal means in the shift condition indicated that the reversal problem was more difficult to solve than the intradimensional or extradimensional shifts, the ordinal relationships among these means were somewhat dependent upon the sequence and S-R interval conditions. Also, the marginal means of the sequence and interval conditions were ordered differently than in the first problem: Ss solved the shift problem faster in the random condition, although the 4/trial condition remained the most difficult for

---

1 This analysis is Table 14 in Appendix B.
Table 8. Mean total errors in the second experimental problem for the schedule, interval, and shift variables.

<table>
<thead>
<tr>
<th>S-R interval</th>
<th>1 sec</th>
<th>5 sec</th>
<th>10 sec</th>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random</td>
<td>5.875</td>
<td>5.000</td>
<td>4.444</td>
<td>5.011</td>
</tr>
<tr>
<td>3/trial</td>
<td>6.778</td>
<td>7.600</td>
<td>7.100</td>
<td>7.175</td>
</tr>
<tr>
<td>4/trial</td>
<td>7.875</td>
<td>12.250</td>
<td>8.091</td>
<td>9.645</td>
</tr>
<tr>
<td>X</td>
<td>6.840</td>
<td>9.185</td>
<td>6.667</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Shift</th>
<th>R</th>
<th>IAD</th>
<th>ED</th>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random</td>
<td>7.571</td>
<td>3.714</td>
<td>4.125</td>
<td>5.091</td>
</tr>
<tr>
<td>3/trial</td>
<td>7.800</td>
<td>7.833</td>
<td>5.143</td>
<td>7.172</td>
</tr>
<tr>
<td>X</td>
<td>9.269</td>
<td>7.871</td>
<td>5.360</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Shift</th>
<th>R</th>
<th>IAD</th>
<th>ED</th>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 sec</td>
<td>11.857</td>
<td>5.556</td>
<td>4.222</td>
<td>6.840</td>
</tr>
<tr>
<td>5 sec</td>
<td>8.125</td>
<td>11.364</td>
<td>7.250</td>
<td>9.185</td>
</tr>
<tr>
<td>10 sec</td>
<td>8.455</td>
<td>6.273</td>
<td>4.750</td>
<td>6.667</td>
</tr>
<tr>
<td>X</td>
<td>9.269</td>
<td>7.871</td>
<td>5.360</td>
<td></td>
</tr>
</tbody>
</table>
shift solution; also, the 5 sec. interval proved to be the most difficult condition for shift problem solution. None of these trends were significant, however.

Distributions of Total Errors

Although the process Models and the HM model were derived assuming equal salience of stimulus dimension, the formulations of these models presented herein also hold for the more general case of unequally salient dimensions.¹

The distributions of total errors were computed for the sequence and S-R interval conditions. Figure 8 is a graph of these distributions for each sequence condition. Generally, the distributions of total errors reflect

¹ In particular if p_i is the probability of sampling dimension i and Σ_i=1^4 p_i = 1.00 on each error trial, then the probability of sampling the relevant dimension (D*) on each error trial is p_1 Pr(D_1=D*) + p_2 Pr(D_2=D*) + p_3 Pr(D_3=D*) + p_4 Pr(D_4=D*). Since the probability that each dimension is the relevant dimension is 1/4, this probability is 1/4p_1+1/4p_2+1/4p_3+1/4p_4 = 1/4(Σ_i=1^4 p_i) = 1/4. Thus the probability of sampling the solution H for P_0 is

P(sampling the relevant dimension)XP(selecting 1 of 3 responses correctly)XP(selecting 1 of 2 responses correctly) = (1/4)(1/3)(1/2) = (1/24).

For P_2, the probability of sampling the solution H is

P(sampling the relevant dimension)XP(selecting 1 of 2 responses correctly) = (1/4)(1/2) = 1/8.

The analogous development for the HM model is shown in Appendix C. The general result is the same, i.e., the probability of a correct response in each state of the model, and consequently the transition probabilities of the model, remain the same under the more general condition of unequal salience of dimensions. It should be noted that this assumption allows the p_i to change across trials.
Figure 8. Observed cumulative distribution of errors for the random, 3/trial and 4/trial conditions.
the difference among these conditions found in the ANOVA; learning was somewhat faster in the random and 3/trial conditions than in the 4/trial condition, although Kolmogorov-Smirnov two sample tests showed no significant differences between these distributions ($K_D = 5$ for random vs. 3/trial, $p > .05$; $K_D = 6$ for the random vs. 4/trial, $p > .05$; $K_D = 7$ for the 3/trial vs. 4/trial, $p > .05$).

Figure 9 is the distribution of total errors for the S-R interval conditions; the distribution of errors in the 5 sec. and 1 sec. conditions were most similar; however, the distribution in the 10 sec. condition indicated somewhat faster learning in this condition. However, Kolmogorov-Smirnov tests showed pairs of these distributions were not significantly different ($K_D = 4$ for the 1 sec. vs. 5 sec.; $K_D = 8$ for the 5 sec. vs. 10 sec.; $K_D = 8$ for the 1 sec. vs. 5 sec., all $p > .05$).

The predicted distribution of total errors from the HM model is compared to the observed distribution form the 3/trial condition in Figure 10. Also included is the predicted distribution of total errors from the HM model for $r = 1.00$ (which is equivalent to the global consistency model, $P_3$). It is clear that the global consistency model does not fit the data well; Ss learned more slowly than the model predicts.

The distribution of total errors for $\hat{r} = .65$ does not fit the observed distribution well; Ss learned more slowly than predicted. Thus, $r$ estimated from the response sequences early in learning does not predict the course of learning well. When $\hat{r} = .55$, however, there is a good fit of the predicted distribution of total errors to the observed distribution (but it should be noted that the $\hat{r} = .55$ estimate was based on this distribution).

One important difference between $P_0$ and $P_2$ is the learning rate per error trial. The predicted distribution of total errors is a function of only the learning rate. Thus it provides a clear basis for distinction between
Figure 9. Distribution of total errors in each S-R duration interval condition.
Figure 10. Observed mean total errors in the 3/trial condition compared to predicted mean total errors. Predictions from the HM model for $\hat{r} = .55$ and $\hat{p} = .65$ and $r = 1.00$ (global consistency).
In Figure 11, the distributions of total errors predicted from $P_0$ and $P_2$ are compared to the data from the random condition. The distribution from $P_0$ is clearly quite discrepant from the data. $P_2$ comes closer to the data, but still does not adequately account for the data; Ss apparently used more memory in this problem than provided for by $P_0$ and $P_2$. Other data presented herein were also consistent with this suggestion; thus, in order to account for these data, additional memory processes must be assumed.

There have been many suggestions in the literature about memory processes in concept identification. One recent suggestion was that on an error trial, Ss eliminate the dimension most recently tested and sample among the remaining dimensions (Erickson, 1969). Combining this with local consistency, the probability that Ss sample the solution $H(P(S_H))$ on any error trial is

$$P(S_H) = P(S \text{ eliminates the relevant dimension}) \times P(\text{selects solution } H) + P(S \text{ eliminates an irrelevant dimension}) \times P(\text{selects solution } H)$$

$$= \left(\frac{1}{4}\right) \cdot 0 + \left(\frac{3}{4}\right) \cdot \left(\frac{1}{6}\right) = \frac{1}{8}.$$ 

Thus, the probability of sampling the solution $H$ given that $S$ eliminates the most recently tested dimension is equivalent to the probability of sampling the solution $H$ from $P_2$, and the predicted distribution of errors from $P_2$ did not adequately account for the data. In addition, data from the shift condition indicated no significant differences between the reversal and extradimensional shift conditions, a result which would be expected if this memory process were operating.

Other memory processes suggested in the literature include the constant revision process first suggested by Erickson et al. (1966). This process assumes Ss remember Hs tried and rejected and hold these in short term memory until they are pushed out by other incoming information. This formulation can be extended to the ternary situation. It was clear from the data that $P_2$, the local consistency model, underestimated the amount of memory used by Ss in
Figure II. Observed cumulative total errors, for the random condition compared to predicted distribution from $P_0$, $P_2$. 
this situation; thus S might remember Hs tried but rejected in addition to local consistency. This formulation has been tested in the binary situation (Erickson, 1968) and has shown a good fit to those data.

Extending the notion of local consistency plus H memory to the ternary situation requires "keeping track" of which Hs tested are redundant with information given by the S-R pair. The H that leads to an error is by definition inconsistent with the information on the S-R pair (otherwise it would not have led to an error); thus, the H most recently tested does not reduce the pool of locally consistent Hs. However, more remotely tested Hs do allow reduction of the 8H locally consistent pool. In particular, if it is assumed Ss remember the two most recently tested Hs (that H leading to the error trial and one additional H), one can estimate the probability of sampling the solution H on trial i (i ≥ 2), by noting whether or not the value of the dimension tested by the more remote H appears on the error trial and whether or not the reinforcement on the error trial is the same as that in the remote H.¹

Figure 12 compares the observed distribution in the random condition to that predicted by local consistency plus 2H memory. The predicted distribution comes closer to fitting the data than \( P_2 \), however, the fit is still not adequate. Adding another H in memory² (local consistency plus 3H memory) comes even closer to fitting the data as also shown in Figure 12, but the proportion of Ss learning on each error trial is still somewhat higher than that predicted. Although it is possible to add more Hs to memory and to more closely approximate the learning rate, this does not seem particularly fruitful,

¹The derivation of the learning rate assuming local consistency plus 2H memory is in Appendix D.

²The derivation of the learning rate for this process follows in the same manner as for local consistency plus 2H memory and is in Appendix E.
Figure 12. Observed distribution of total errors in the random condition compared to predicted distribution assuming local consistency + 2H memory and local consistency + 3H memory.
especially since data from the binary situation suggest that local consistency plus 2H memory is sufficient. It seems Ss are not using this particular memory process in the ternary situation.

A second memory process suggested by investigators of the binary situation is the consistency check process. In the ternary situation it might be easier for Ss to organize and remember past information on the basis of dimensions only rather than on the basis of both the particular dimension and the values that did not occur on a particular trial, i.e., in the ternary situation, remembering $0 \neq B$ on error trial $i$ implies: $0 = C$ or $0 = A$ and this must be recalled on error trial $i+2$ in order to be used. However, recalling which dimensions were consistently reinforced allows one to eliminate entire dimensions on most error trials, eliminating the necessity for recalling the $H$ tested and its implications. In order to test the consistency check process in the ternary problem, it is necessary to compute distributions contingent upon Ss making a correct response or an incorrect response on the first trial. If the probability of sampling the solution $H$ on trial 1 is $\varepsilon_1$, and if $\varepsilon$ is the probability that Ss make no more errors after carrying out the consistency check, $k$ is the probability that Ss make no more errors after an error on trial 1, $E_1$ is the event of an error on trial 1, and $C_1$ is the event of a correct on trial 1 and a later error, then the distribution of total errors is given by:

$$P(T=0) = \varepsilon_1$$

$$P(T=1/C_1) = (1-\varepsilon_1)\varepsilon$$
$$P(T=1/E_1) = (1-\varepsilon_1)k$$

$$P(T=2/C_1) = (1-\varepsilon_1)(1-\varepsilon)\varepsilon$$
$$P(T=2/E_1) = (1-\varepsilon_1)(1-k)\varepsilon$$

$$P(T=n/C_1) = (1-\varepsilon_1)(1-\varepsilon)^{n-1}$$
$$P(T=n/E_1) = (1-\varepsilon_1)(1-k)(1-\varepsilon)^{n-2}$$
The distribution of total errors for all Ss is:

\[ P(T=n) = P(T=n/C_1)P(C_1) + P(T=n/E_1)P(E_1) \]

In the 3/trial condition, ξ is easily found. In this sequence, on an error trial, S can eliminate 1 dimension or 3 dimensions depending upon whether the relevant dimension has changed or remained the same from trial n-1 to n. If the relevant dimension remains the same (with probability 1/4), then the probability that S selects the solution H is 1/2, the probability of choosing correctly among the remaining responses. If the relevant dimension changes, the probability that S samples the relevant dimension among the three remaining dimensions is 1/3; the probability that S selects the solution H is 1/6. Thus, the estimate of ξ for the 3/trial condition is 1/4 * 1/2 + 3/4 * 1/6 = 1/4. The remaining terms of the distribution of total errors are: \( \epsilon_1 = 1/24 \) (the probability of choosing the solution H at the start of the problem); \( k = 1/8 \) (the probability of choosing the solution H from the 8 locally consistent Hs on the first error trial); \( P(C_1) \) = the proportion of Ss making a correct response on trial 1 and a later error (this was 1/3 in the 3/trial condition); and \( P(E_1) = 1 - P(C_1) \) since all Ss made at least 1 error. Figure B compares the observed distribution of total errors in the 3/trial condition to the consistency check predicted distribution. The fit is extremely good, much better than the fit of the Process Models assuming memory for the Hs to the data in the random condition.

It is also possible to form the distribution of total errors in the random condition assuming a consistency check process. The same equation used in the 3/trial condition is used to form this distribution; the terms of the equation which change are \( \epsilon \) and the proportion of Ss making an error on the first trial or a correct on the first trial and a later error. The latter \( P(C_1) \) and \( P(E_1) \) can be estimated from the data. In the random condition, \( \epsilon \) is estimated by
Figure 13. Observed cumulative distribution of total errors in the 3/trial condition compared to the predicted distribution from the consistency check process.
computing the probability that 0, 1, 2, or 3 dimensions are eliminated and multiplying these probabilities by the probability of sampling the solution $H$ given that 0, 1, 2, or 3 dimensions are eliminated. These derivations are in Appendix F. In the random condition $c$ is .226, somewhat lower than in the 3/trial condition. Figure 14 compares the observed and predicted distributions in the random condition. Again, the fit of the consistency check process to the data is quite good, and accounts well for the data from this condition.

The 4/trial condition provides some additional information about the consistency check process hypothesized for these data. In the 4/trial condition, since every dimension changes on every pair of trials, every dimension must pass a consistency check. Thus, this condition gives some information about the extent to which Ss recall stimulus information from the trial preceding an error trial. The data from Figure 15 shows that Ss in the 4/trial condition learn faster than predicted by $P_2$, the local consistency model; thus, they recall more information than that on the error trial. Also, in Figure 15, a predicted distribution was found assuming memory for the two most recent stimuli; clearly the curve predicts faster learning than that observed. Thus, on these problems, Ss do not recall all the stimulus information from the two most recent trials on an error trial; they recall some portion of that information.

Thus, on the basis of these data, it is possible to suggest some processes used by S in carrying out a consistency check. One way Ss might carry out this

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1 This curve was computed by combining the conditional distribution of the total number of errors as in the consistency check models; however, $c$ for two stimulus memory is .250, $P(C_1)$ and $P(E_1)$ were estimated from the data, the other two parameters remain the same.
Figure 14. Observed distribution of total errors in the random condition compared to the distribution predicted by the consistency check process.
Figure 15. Observed cumulative distribution of errors in the 4/trial condition compared to prediction from $P_2$ and 2 stimulus memory model.
check (consistent with these data) is the following: on trial n, S remembers the specific stimulus and reinforcement; on trial n+1, S compares this stimulus to the stimulus on trial n and notes which dimensions are similar and which are different, then S responds. If his response is the same as the feedback from trial n and leads to an error, he can eliminate all unchanged dimensions. However, if his response is different from the feedback on trial n and leads to an error he cannot eliminate all changed dimensions but must sample via local consistency among the dimensions. It should be noted that this suggested process does not require memory of the stimulus on trial n-1, per se; it assumes that at the time of the response, S has coded the dimensions of the stimulus on a trial as "same" or "different" from the prior trial. Although this hypothesized process could account for the distribution of total errors observed herein, the process is not sufficiently well developed to be tested by other aspects of the data, and remains speculative and suggestive of future theoretical development.

Latency Analyses.

If it is assumed that the latency of a response is a monotonic function of the number of Hs among which S chooses when he makes a response, then some interesting predictions follow from the Process Models and the HM model. In particular, according to the Process Models, latency after a correct should be shorter than latency after an error, since, after a correct response S retains the H which led to the correct, while after an error S samples from a larger pool of hypotheses. The HM model also predicts that latency after a correct should be less than latency after an error, since Ss need time to recover the complementary H set. Furthermore, if the memory process used by Ss is one in which Hs or dimensions are gradually eliminated from the pool of available Hs for sampling on error trials, then latencies after errors
should decline across presolution error trials. If, however, the memory process is one in which Ss recalled a constant amount of information on each error trial (as suggested by any model postulating memory for a constant amount of stimulus information), then latency should not decline across presolution trials.

According to the HM model, latency after a correct response should decline as the number of preceding consecutive correct responses increases. Since as corrects increase, the number of tenable hypotheses decreases, and S has fewer Hs from which to choose when he makes his next response. According to the Process Models, however, latencies after corrects should remain constant as consecutive corrects increase, since S retains the same H and responds according to this H.

Pre-TLE Choice Latencies

The first latency analysis performed was an ANOVA on mean choice latencies before the TLE excluding trial 1. Table 15 of Appendix B shows the ANOVA table; there were no significant main effects or two way interactions. Three way interactions were not tested because of the unequal n across the cells of the design (some Ss solved with none or one pre-TLE trial). Table 9 presents the mean pre-TLE choice latencies for the S-R interval and the sequence conditions.

Latencies on Correct and on Error Trials

In order to make inferences about latencies after errors and after corrects, latencies on correct trials and on error trials must be analyzed. This analysis must be performed, since the choice data from Table 5 indicated that errors were more likely to follow errors, and thus longer after error latencies might be due to longer latencies on errors.
Table 9. Mean Choice Latencies before the TLE for the schedule and duration variables.

<table>
<thead>
<tr>
<th>S-R interval</th>
<th>Random</th>
<th>3/trial</th>
<th>4/trial</th>
<th>$\bar{x}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 sec</td>
<td>4.588</td>
<td>5.237</td>
<td>3.554</td>
<td>4.491</td>
</tr>
<tr>
<td>5 sec</td>
<td>3.753</td>
<td>4.425</td>
<td>4.291</td>
<td>4.179</td>
</tr>
<tr>
<td>10 sec</td>
<td>3.851</td>
<td>6.901</td>
<td>4.015</td>
<td>4.781</td>
</tr>
<tr>
<td>$\bar{x}$</td>
<td>4.075</td>
<td>5.388</td>
<td>3.967</td>
<td></td>
</tr>
</tbody>
</table>
Backward learning curves were computed for latencies on correct and on error trials for all Ss in the first problem. This curve can be seen in Figure 16.

Several interesting points can be noted about Figure 16. Latencies after the TLE declined; these latencies were analysed by a repeated measures ANOVA (shown in Table 16 of Appendix B) in which the main effect of trials was highly significant ($F_{3,306} = 13.605, p<.005$). These data are in line with other studies in the binary situation which demonstrated this decline (Erickson, et al, 1966; Erickson and Zajkowski, 1967; Levine, 1969).

The backward learning curve also revealed that latencies on error trials were longer than on correct trials particularly as the TLE was approached. To investigate possible differences between latencies on corrects and latencies on errors, several analyses were conducted. First, for all Ss with at least one correct and one error trial before the TLE, median latencies on corrects and on errors were computed for each S. There were 73 Ss in this analysis; a t test of the difference scores indicated that these scores were not significantly different from zero ($t_{72} = 1.177, p>.25$). It was noted from the backward learning curve that pre-TLE latencies on corrects and errors seemed to become increasingly different as the TLE was approached; thus, in order to test for the possibility that latencies on corrects and on errors did not differ in the early stage of learning but were different in a later stage, a second analysis was run.

In order to define an early and a late stage of learning, Ss pre-TLE trials were Vincentized into halves. Those Ss having at least one correct and one error in each half of these pre-TLE trials were included in the analysis (N was 33 for this analysis). Median latencies on corrects and on errors were computed for the early and late stages of learning and were
Figure 16. Backward learning curve for latencies: mean latency on correct and error trials before the TLE; mean latency on trials after the TLE. N is the number of Ss at each point on the graph.
analyzed by ANOVA (shown in Table 17 in the Appendix B). Table 10 shows the mean of the median latencies for the type of response and the stage of learning. Again the latency on a correct was not different from the latency on an error response ($F = 1.00$) and median latencies on early trials were not significantly different from median latencies on late trials ($F_{1,32} = 1.799$, $0.10 < \rho < 0.25$). The interactions between these variables approached significance ($F_{1,32} = 3.072$, $0.05 < \rho < 0.10$) and Table 10 clarifies the nature of this interaction: median latencies on corrects were somewhat longer than on errors for the early stage of learning, but for the later stage, the relationship was reversed.

**Latencies after Corrects and after Errors**

Backward learning curves for latencies after errors and after corrects were also obtained and are shown in Figure 17. There are some interesting points revealed by the backward curve in Figure 17. Latencies after errors were longer than latencies after corrects for most of the pre-TLE period. Also, when linear regression lines were fitted to these data, two interesting trends were revealed: latencies after corrects decreased while latencies after errors increased; however, t tests on the regression coefficients revealed they were not significantly different from zero ($t_8$ on the after error latencies was $-0.406$, $t_8$ on the after correct latencies was $0.728$, both $p > 0.25$).

Analyses similar to those conducted for latencies on correct and on error trials were conducted for latencies after correct and after error trials in order to verify the relationships suggested by the backward curve. For those Ss making at least one correct and at least one error before the TLE, median latencies after corrects and after errors were computed. The mean of the
Figure 17. Backward learning curve for latencies. Mean latency after a correct response or after an error on trials before the TLE. Mean latency on trials after the TLE.
Table 10. Means of median latencies on a correct or an error response before the TLE for early and late stages of learning.

<table>
<thead>
<tr>
<th>Type of Response</th>
<th>Early</th>
<th>Late</th>
<th>( \bar{X} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>on correct</td>
<td>4.682</td>
<td>3.702</td>
<td>( \bar{X} )</td>
</tr>
<tr>
<td>on error</td>
<td>4.139</td>
<td>4.273</td>
<td>4.192</td>
</tr>
<tr>
<td>( \bar{X} )</td>
<td>4.411</td>
<td>3.988</td>
<td>4.206</td>
</tr>
</tbody>
</table>
median latencies after errors was significantly greater than those after corrects ($t_{72} = 2.95, p < .005$).

An ANOVA on median latencies after corrects and after errors for the data Vincentized into early and late stages of learning for those Ss having at least one correct and one error in each stage was also conducted; the number of Ss in this analysis was 33. The results of this ANOVA are in Table 18 of Appendix B. Neither the main effect of stage or type of response nor their interaction were significant. Table 11 shows the means of these median latencies. While these means were ordered in the way suggested by the backward curve: early in learning, the after error and after correct latencies were approximately the same; late in learning, the difference between these latencies increased, the latency after a correct response being shorter than after an error, these trends were not significant.

Thus, while significant differences in median latencies after errors and after corrects could be demonstrated for the large group of 73 Ss, these differences could not be demonstrated for the more restricted group of 33 Ss having more trials in the presolution period. Since other studies (Erickson et al, 1966; and Erickson and Zajkowski, 1967) have found significant differences between after error and after correct latencies it seems safe to conclude that the differences found in the present study are real.

Pre-TLE Latencies across Consecutive Corrects

Latencies as the number of consecutive correct responses increased were analyzed by selecting all Ss with three correct responses in a row before the TLE. For the 39 Ss in this analysis, latencies after their first $i$ correct responses ($i = 1, 2, 3$) were averaged across Ss. These mean latencies were $3.567, 3.073, 3.208$ for $i = 1, 2, 3$ respectively. The means indicated that latencies decreased from $i=1$ to $i=2$ and then increased slightly at $i=3$. A
Table 11. Means of median latencies for after errors and after corrects for the early and late stages of learning during the presolution period.

<table>
<thead>
<tr>
<th>Stage of Learning</th>
<th>Type of Response</th>
<th>After Correct</th>
<th>After Error</th>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td>Early</td>
<td></td>
<td>4.273</td>
<td>4.313</td>
<td>4.213</td>
</tr>
<tr>
<td>Late</td>
<td></td>
<td>3.543</td>
<td>4.217</td>
<td>3.880</td>
</tr>
<tr>
<td>X</td>
<td></td>
<td>3.908</td>
<td>4.265</td>
<td></td>
</tr>
</tbody>
</table>
"t" test was performed on mean latencies for i=1 and i=2; the decrement was not significantly different from 0 ($t_{38}^g = 1.182, .10 < p < .20$). An additional between Ss analysis for all Ss and all conditions for mean latencies after 1, 2, 3 or 4 correct responses before the TLE was also computed. These mean latencies were 3.764, 3.559, 3.516, and 3.555 with Ns of 364, 171, 84 and 44 respectively. These means show a slight decrement from i=1 to i=2 and latencies remained constant for i>2. Thus, on the basis of this and the preceding result, it can be concluded that latencies remained fairly constant as the number of correct responses increased before the TLE. However, since latencies after the TLE are based on more Ss, these latencies are probably more stable, and the decline after the TLE would support the basic process assumed by the HM model rather than that suggested by the Process Models.
V. CONCLUSIONS

The stimulus sequence variable was manipulated in this study in order to test several models of concept identification. With respect to the Hn model, the data form the 3/trial condition were generally in line with the basic assumptions of the model: an error was not necessarily a recurrent event in the presolution phase and some learning occurred on presolution correct trials. The latency data were also nicely consistent with the processes suggested by the Hn model. However, portions of the data are not in agreement with the model as presently formulated.

In particular, it was expected on the basis of the model that if S makes four correct responses in a row he should make no more errors; this prediction was not supported by the data. Also, there was some parameter variance; parameters estimated on the basis of response sequences did not agree with parameters estimated using distributions of total errors within each S-R interval condition; however, overall estimates were fairly close. The estimate based on response sequences predicted less errors than observed.

While it is difficult to state the exact nature of the revision of the Hn model suggested by these data, it is possible to provide some guidelines for future theoretical development of this class of models. First, on the basis of these data, Ss seemed to remain in the error states of the model longer than expected. This might be due to the fact that Ss are less likely to recover larger complementary H sets which allows them to reside longer in the H states with which lower P(C)s are associated. Also, it seems that Ss might not reduce the tenable H set on every correct trial, either by not attending to all dimensions of the stimulus across trials or by failing to keep track of previously eliminated Hs.
Although the HM model accounted well for several portions of the data, it should be noted that this general class of models is difficult to work with in the sense of easily obtaining predictions for aspects of the choice and latency data. For example, in order to obtain choice statistics or latency predictions conditional upon preceding errors, it is necessary either to assume a distribution of states across each error trial, or to attempt to work with complicated analytic expressions. Also, as has been noted, because of the somewhat complicated structure of this model, deviations from the model are not uniquely identifiable in this experimental situation. The model, however, accounts for some data from this experiment and probably merits future investigation in experiments which permit process interpretations.

With respect to the Process Models tested herein, neither $P_0$ nor $P_2$ could account for the data.

The presolution choice data present particular difficulties for the Process Models. They cannot account for the nonstationarity observed in the random condition or for the probability of a correct response conditional upon preceding consecutive corrects observed in the 3/trial condition. In addition, the Process Models were not entirely consistent with the latency data; they cannot account for the latency decline after the TLE. Thus, although some Process Models can account for the learning rate data, there are other aspects of the data which are not consistent with any Process Models.

It should be noted that since the choice predictions of the Process Models are equivalent to predictions of the multiple-look and one-look hypothesis sampling models of Bower and Trabasso (1964) and Trabasso and Bower (1968), these too are inadequate to account for the ternary situation.
These data provide some information as to models which might account for learning in the ternary situation. The nonstationarity observed in the random and 3/trial conditions can be accounted for by a model such as the model for three responses provided by Bower and Trabasso (1964). The nonstationarity in the random and 3/trial condition is consistent with their notion of a P-A stage; the stationarity in the 4/trial condition does not contradict the assumption of a P-A stage, since the probability that the conditioned value occurs is .50, a value which was close to presolution P(C). The Bower and Trabasso (1964) model should be revised, however, to more clearly specify the way in which this stage is entered; presumably S can try to learn value response pairings on both relevant and irrelevant dimensions. Thus, there should be some probability s that S enters the P-A stage; with probability 1-s, the S samples among the dimensions. The nonstationarity data also indicated some interesting differences for the random and 3/trial conditions which are interpretable within the Bower-Trabasso theory: apparently Ss took longer to discover the relevant dimension in the random condition and less time to learn the value response pairings, while in the 3/trial condition, Ss discovered the relevant dimension sooner, but took more trials to learn it. However, since the probability that any value is repeated in the next trial in the 3/trial condition is 1/4, less than the probability of a repetition in the random condition, then these data imply that θ is less in the random condition, a result which is contradictory to Greeno's (1964, 1967) data.

It is also possible that the nonstationarity observed in the data could be accounted for by a forgetting process. Errors after long sequences of correct responses might occur because Ss forget a value response pairing. Thus a model which allows forgetting of previously learned value response pairings could also account for the nonstationarity observed.
One model was formulated which could account well for the learning rate in the ternary situation; this was the consistency check model. It is possible for this model to account, additionally, for the nonstationarity observed in these data, if it is assumed, as was suggested in the initial formulation, that Ss' responses are based on similarities and differences between the stimuli on two consecutive trials.

In order to suggest one reason why the S-R duration interval did not significantly affect learning rates, consideration must be given to recent studies of this interval. First, Bourne and Bunderson (1963) demonstrated that the length of "blank" postfeedback intervals (0-8 seconds) significantly affected learning rates in concept identification. Bourne et al (1965) demonstrated that for complex problems with five relevant dimensions, the optimal "blank" postfeedback interval was about 17 seconds, and, for longer or shorter intervals, including in these intervals either the stimulus or the reinforcement and the stimulus, mean total errors decreased. Thus, it seems that if the stimulus is available during the postfeedback interval, Ss use it in task related activities, perhaps as a memory aid, and this stimulus availability decreases total errors. Thus, one might expect that by increasing stimulus availability during the postfeedback interval, as was performed in this study, mean total errors would decrease.

The results of this study, at first glance, seem to be contradictory to those of the previous two studies. Postfeedback interval did not affect learning rate nor did increased stimulus availability. However, in this study, two events occurred in the postfeedback interval: one was the simultaneous occurrence of the stimulus and feedback, the second was a blank period between the offset of the previous event and the onset of the next stimulus. Thus, in effect, "blank" postfeedback intervals were not manipulated in this study; and they did not affect mean total errors, a result which is consistent with
the Bourne and Bunderson (1963) study. In the Bourne et al (1965) study, the stimulus was available for the entire interval after the feedback, whereas, in this study that were not; thus it seems possible that Ss only use the stimulus in task related activities if they are "forced" to, i.e., if the entire interval after the feedback includes the stimulus and there is no blank period.

The term "task related activities" is used as a general term which refers to covert processes used by Ss in solving concept identification problems. Recent theories of concept identification have not yet been sufficiently well formulated in the direction of stating the actual processes which take place at various places during the intertrial interval, although a recent exception to this is the work of Levine (1969). The data from this and other studies support Levine's notion that latency after a correct should be shorter than after an error (since Ss need time to recover the complementary H set). Levine has also stated that the Ss probably do not code tenable H sets until after the feedback and that the presence of the stimulus after the outcome should decrease if the stimulus appears after the outcome. Taking this notion a little further implies that increasing the time the stimulus appears after the outcome should facilitate the coding process; thus, if responses to the next stimulus are based on the tenable H set, one would expect mean choice latency on pre-TLE trials to be shorter for those "well-coded" H sets. This prediction, however, was not supported by the data in this study since mean pre-TLE choice latencies were not different for the varying durations of the stimulus and reinforcement.

Thus, it seems plausible that whatever task related activities occur in this CI problem, these processes begin after the termination of the stimulus and reinforcement (in the "blank" period) and that there is a constant amount
of carry-over into the next trial regardless of the duration of the stimulus after the outcome, although the carry-over processing time is different if the previous response was a correct response or an error response.

A second interpretation that is possible is that whatever processes Ss engage in while the stimulus appears with the outcome, the intervals manipulated in this study allowed sufficient time for these. For example, if Ss only engage in coding the stimulus during the time it appears with the outcome and engage in processing the information from the coded stimulus during the blank period into the next trial when a response decision must be made, then perhaps for the easily coded stimuli used in this study, one second was sufficient time for coding the stimulus and 5 sec. and 10 sec. were simply redundant for the coding process.
APPENDIX A

Transition probabilities of the HM model are derived and illustrated in Figure 18.

If S is in 24H, the probability of a correct response is 1/3 because S is equally likely to make any of three responses with no information.

Suppose S experiences 0000=A on the first trial. If he makes a correct response or an incorrect response and recovers, he goes to 8H and holds 8 hypotheses (H's) of the form 0=A, 1=B, 2=C or 0=A, 2=B, 1=C with respect to each dimension. Thus,

\[
P(24H \text{ to } 8H) = \frac{1}{3} + \frac{2}{3}r.
\]

If S is in 8H, suppose the stimulus on the next trial is 0111. S makes a response on the basis of the 8 H's which are currently tenable: two of these H's are based on the unchanged dimension and six are based on the changed dimensions. In order to calculate the p(c) when S is in 8H, two possibilities must be considered: the relevant dimension remains unchanged on the next trial or the relevant dimension changes. The relevant dimension changes on the next trial with probability 1/4 since one of the four dimensions remains the same. The probability of a correct response if the relevant dimension remains unchanged on the next trial is formed by considering the p(S attends to each dimension) and the p(c) when he attends to a particular dimension. If the relevant dimension remains unchanged, then, the probability of a correct response given that S is in 8H and the relevant dimension remains unchanged (p_{8U}) is

\[
p_{8U} = \frac{1}{4} \cdot 1 + \frac{1}{4} \cdot 0 + \frac{1}{4} \cdot 0 + \frac{1}{4} \cdot 0 = \frac{1}{4}.
\]

If the relevant dimension changes on the next trial (with probability 3/4), then the probability of a correct response (p_{8C}) is equal to

\[
p_{8C} = \frac{1}{4} \cdot 0 + \frac{1}{4} \cdot 1 + \frac{1}{4} \cdot 1 + \frac{1}{4} \cdot 1 = \frac{3}{8},
\]

since S is correct with probability 1/2 if he attends to any of the changed dimensions.
Figure 18. Tree diagram of the transition probabilities of the HM model.
The number of hypotheses tenable after the second trial depends upon whether the relevant dimension changed from the first to the second trial. The following example illustrates this dependency. If $S$ has experienced 0000=$A$ and 0111=$A$ and has responded correctly or responded incorrectly and recovered, then there are two $H$'s tenable: $0=A, 1=B, 2=C$ or $0=A, 2=B, 1=C$ on the first dimension (the 2H state). If, however, $S$ has experienced 0000=$A$, and 1011=$B$, then there are three tenable $H$'s of the form $0=A, 1=B, 2=C$ on each of the changed dimensions (the 3H state). Thus, $S$ can go from 8H to 2H if the relevant dimension did not change, or from 8H to 3H if the relevant dimension changed. To form the transition probabilities out from 8H, both the probability that the relevant dimension remains the same (or changed) and the probability of a correct response (or an incorrect response and recovery) contingent upon the changing of the relevant dimension must be considered. Therefore, the

$$P(8H \text{ to } 2H) = \frac{1}{4} \cdot (\frac{1}{4}+\frac{3}{4}r),$$

and

$$P(8H \text{ to } 3H) = \frac{3}{4} \cdot (\frac{3}{8}+\frac{5}{8}r).$$

To form the $P(2H \text{ to } 1H)$, again the probability of a correct response in 2H must be formed. Suppose $S$ has experienced 0000=$A$, 0111=$A$ and is in 2H. By restriction (3) on the stimulus sequence, the relevant dimension must change on the next trial. Thus, the stimulus on the next trial might be 2102, and $S$ is forced to choose between two tenable $H$'s. The probability of a correct response when $S$ is in 2H ($p_2$) is 1/2 because $S$ chooses between two possible responses. If he responds correctly or incorrectly and recovers, he is absorbed into the 1H state. Therefore,

$$P(2H \text{ to } 1H) = 1/2+1/2r.$$
If $S$ is in $3H$ and has experienced, e.g., $0000=A$ and $1011=B$, then the probability of a correct response when $S$ is in $3H$ is again formed by considering whether the relevant dimension changes on the next trial. Since three dimensions change on every trial, and, in the aforementioned sequence, the second dimension must change on the next trial (restriction (3) on these sequences), then the probability that the relevant dimension remains unchanged on the next trial is $1/3$ (one out of three dimensions remains unchanged). The probability of a correct response if the relevant dimension remains unchanged is formed by considering the probability that $S$ attends to each of these three dimensions and the probability that $S$ is correct if he attends to a particular dimension. The stimulus on the third trial might be $1220$. The $p_{3U}$ is thus equal to

$$p_{3U} = \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot 0 = \frac{1}{3}. $$

(if he attends to dimensions 2 or 3 which must change).

If, however, $S$ is in $3H$ and the relevant dimension changes on the next trial (with probability $2/3$), then it is possible for the relevant dimension to change to the same value it held on the first trial (trial $n$) or to a value not yet held on $n$ or $n+1$ (each with probability $1/2$). For example, if dimension 1 is the relevant dimension, then in the following sequence $0000=A$, $1011=B$, on the third trial, dimension 1 may hold either 0 or 2. If dimension 1 changes, dimension 2 must change (by restriction (3)), then of dimensions 3 and 4, one must change and the other must stay the same. Of the irrelevant dimensions upon which two of the three tenable $H$'s are based (3 and 4), the one that changes can change to the value held on the first trial or to a value not yet held - these possibilities must be taken into account when the probability of a correct response when $S$ is in $3H$ and the relevant dimension changes ($p_{3C}$) is calculated.
\[ P_{3C} = \frac{1}{2} \left[ \frac{1}{3} \cdot 1 + \frac{1}{3} \cdot 0 + \frac{1}{3} \cdot \frac{1}{2} \right] + \frac{1}{2} \left[ \frac{1}{3} \cdot 1 + \frac{1}{3} \cdot 0 + \frac{1}{3} \cdot \frac{1}{2} \right] \]

(Rel Dim changes to value on n) (Rel Dim changes to value not held yet)

If S is in 3H and the relevant dimension does not change on the next trial and S responds incorrectly and recovers, the S goes to 1H. For example, if S experiences 0000=A, 1011=B, 1220=B, then one tenable H of the form 0=A, 1=B, 2=C remains on dimension 1.

If S is in 3H and the relevant dimension changes on the next trial and S makes a correct response or an incorrect response and recovers, then he may go to either 1H or 2H. For example, if S has experienced 0000=A, 1021=B, 0101=A, then he holds two H's: 0=A, 1=B, 2=C on dimension 1 and 0=A, 2=B, 1=C on dimension 3. If, however, the third stimulus is 0111, then S holds one H on dimension 1. Whether S progresses to 2H or 1H if he makes a correct response or an incorrect response and recovers while in 3H depends upon whether the irrelevant changed dimension (upon which one of the three H's is based) takes a value which "confounds" it with the relevant dimension. It is possible for the irrelevant dimension to be confounded with the relevant dimension in two ways: they both take the same values as on trial n or they both take different values from trial n or n+1. In the previous sequence, 0000=A, 1021=B, dimension 1 is the relevant dimension and it changes on the next trial; dimension 2 must change, and dimension 3 or 4 must change. Thus, 8 stimuli are possible on the next trial (2x1x2x2) for each dimension value, and, in four of these, the relevant dimension and an irrelevant dimension are confounded (either dimension 3 or 4 is confounded with the relevant dimension). Thus, the probability that an irrelevant dimension is confounded with the relevant dimension is 1/2. The \( P_{3C} \) which was derived earlier can be verified by considering these two cases. If one irrelevant dimension is confounded
with the relevant dimension (with probability 1/2), then the probability of a correct response is 2/3, since two out of three tenable H's lead to the same (correct) response. If the dimensions are not confounded, then the probability of a correct response is 1/3, since each of the three H's leads to a different response. Thus, the

\[ P_{3C} = \frac{1}{2} \cdot \frac{2}{3} + \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{2}. \]

In order for S to proceed from 3H to 2H, the relevant dimension must change on the next trial; the dimensions must be confounded, and S must make a correct response or an incorrect response and recover; thus,

\[ P(3H \text{ to } 2H) = \frac{2}{3} \cdot \frac{1}{2} \cdot (\frac{2}{3} + \frac{1}{3}). \]

S can proceed from 3H to 1H if the relevant dimension remains the same on the next trial or if it changes and is not confounded with an irrelevant dimension (and, for each of these cases, S makes a correct response or an incorrect response and recovers). Thus,

\[ P(3H \text{ to } 1H) = \frac{1}{3}(\frac{1}{3} + \frac{2}{3}) + \frac{2}{3} \cdot \frac{1}{2} \cdot (\frac{1}{3} + \frac{2}{3}) = \frac{2}{3}(\frac{1}{3} + \frac{2}{3}). \]

If S is in 2H by trial n+2, then he can, if he makes a correct response or an incorrect response and recovers, proceed to the 1H state on the next trial. Suppose S has experienced 0000=A, 1021=B, and 0101=A and is in the 2H state (two H's, one each on dimension 1 - the relevant dimension - and dimension 3). By restrictions on the stimulus sequence, dimension 4 must change (restriction (3)) and dimension 2 must change (restriction (2)), so either dimension 1 or dimension 3 must change. Thus, the probability of a correct response is 1/2, since S makes one out of two responses. The information on the n+3rd trial enables S to enter the 1H state. Thus, the

\[ P(2H \text{ to } 1H) \text{ is the same whether } S \text{ arrived in } 2H \text{ from } 3H \text{ or } 8H. \]

S slips back into the 2H state if he makes an incorrect response and does not recover information from that trial and all previous trials. The
probability that S slips back into the 24H state from each state of the model (except 1H, which is absorbing) is formed by considering the probability that the relevant dimension changes (or remains the same) and the probability of an incorrect response contingent upon what happened to the relevant dimension and the probability that S does not recover. For example, the

\[ P(\text{8H to 24H}) = \frac{1}{4}(\frac{3}{4})(1-r) + \frac{3}{4}(\frac{5}{8})(1-r) \]

(\text{Rel Dim unchanged}) (Rel Dim changes)

Thus, the transition probabilities of the HM model have been formed and the model takes the following form:

\[
\begin{array}{cccccccc}
1H & 2H & 3H & 4H & 8H & 24H \\
1H & 1 & 0 & 0 & 0 & 0 & 0 \\
2H & \frac{1}{2} + \frac{3}{8} r & 0 & 0 & 0 & \frac{1}{2}(1-r) \\
3H & \frac{3}{2}(\frac{1}{2} + \frac{3}{8} r) & \frac{3}{2}(\frac{1}{2} + \frac{3}{8} r) & 0 & 0 & \frac{3}{2} \cdot \frac{3}{8}(1-r) + \frac{1}{2} \cdot \frac{3}{8}(1-r) + \frac{3}{2} \cdot \frac{3}{8}(1-r) \\
8H & 0 & \frac{3}{4}(\frac{1}{2} + \frac{3}{4} r) & \frac{3}{4}(\frac{1}{2} + \frac{3}{4} r) & 0 & \frac{3}{4}(\frac{3}{8})(1-r) + \frac{3}{4}(\frac{5}{8})(1-r) \\
24H & 0 & 0 & 0 & \frac{1}{2} + \frac{3}{8} r & \frac{3}{2}(1-r) \\
\end{array}
\]

To form the correct probability vector, the probability of a correct response when S is in each state of the model, these events must be considered: the probability that the relevant dimension can change or not change on the next trial, in addition, the probability of a correct response contingent upon this event.

\[
p(C/24H) = \frac{1}{3} \quad p(c) = \begin{bmatrix} 1 \\ 1/2 \\ 4/9 \\ 11/32 \\ 1/3 \end{bmatrix}
\]

\[
p(C/\text{8H}) = \frac{1}{4} \cdot p_{8U} + \frac{3}{4} \cdot p_{8C} = \frac{1}{4} \cdot \frac{1}{4} + \frac{3}{4} \cdot \frac{3}{8} = \frac{11}{32}
\]

\[
p(C/3H) = \frac{1}{3} \cdot p_{3U} + \frac{2}{3} \cdot p_{3C} = \frac{1}{3} \cdot \frac{1}{3} + \frac{2}{3} \cdot \frac{1}{2} = \frac{4}{9}
\]

\[
p(C/2H) = 1/2
\]

\[
p(C/\text{1H}) = 1
\]
$\Pi_0$, the initial vector, is formed by assuming that $S$ starts in the 24H state. Thus,

$$\Pi_0 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$
APPENDIX B. Analyses of Variance Tables
Table 12. Analysis of Variance on Vincentized correct responses before the TLE in the random condition (top table) and the 3/trial condition (bottom table).

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Ss</td>
<td>79.887</td>
<td>30</td>
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<tr>
<td>Within Ss</td>
<td>55.626</td>
<td>93</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Quartiles</td>
<td>10.802</td>
<td>3</td>
<td>3.600</td>
<td>7.228</td>
<td>&lt;.05</td>
</tr>
<tr>
<td>Residual</td>
<td>44.824</td>
<td>90</td>
<td>.498</td>
<td></td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Ss</td>
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<td>33</td>
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<td></td>
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<tr>
<td>Within Ss</td>
<td>55.76</td>
<td>102</td>
<td></td>
<td></td>
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<tr>
<td>Quartiles</td>
<td>7.80</td>
<td>3</td>
<td>2.600</td>
<td>5.371</td>
<td>&lt;.05</td>
</tr>
<tr>
<td>Residual</td>
<td>47.96</td>
<td>99</td>
<td>.484</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 13. Analyses of Variance on errors in problem 1 (top) and trials to criterion in problem 1 (bottom).

<table>
<thead>
<tr>
<th>Effects</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
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<td>Sequence</td>
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<td>127.907</td>
<td>63.954</td>
<td>2.518</td>
<td>.05&lt;p&lt;.10</td>
</tr>
<tr>
<td>Duration of S and R</td>
<td>2</td>
<td>114.741</td>
<td>57.370</td>
<td>2.259</td>
<td>.10&lt;p&lt;.25</td>
</tr>
<tr>
<td>Dimension</td>
<td>3</td>
<td>240.769</td>
<td>80.256</td>
<td>3.158</td>
<td>p&lt;.05</td>
</tr>
<tr>
<td>Sequence x Duration</td>
<td>4</td>
<td>38.870</td>
<td>9.718</td>
<td>&lt;1</td>
<td>-</td>
</tr>
<tr>
<td>Sequence x Dimension</td>
<td>6</td>
<td>118.315</td>
<td>19.719</td>
<td>&lt;1</td>
<td>-</td>
</tr>
<tr>
<td>Duration x Dimension</td>
<td>6</td>
<td>314.148</td>
<td>52.358</td>
<td>2.061</td>
<td>.05&lt;p&lt;.10</td>
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<tr>
<td>Sequence x Duration x Dimension</td>
<td>12</td>
<td>400.018</td>
<td>33.335</td>
<td>1.312</td>
<td>.10&lt;p&lt;.25</td>
</tr>
<tr>
<td>Error within</td>
<td>72</td>
<td>1828.666</td>
<td>25.398</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Total 107 3183.435

ANOVA on trials to criterion in problem 1

<table>
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<tr>
<th>Effects</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>p</th>
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</thead>
<tbody>
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<td>154.111</td>
<td>2.203</td>
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<tr>
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<td>588.296</td>
<td>196.099</td>
<td>2.803</td>
<td>p&lt;.05</td>
</tr>
<tr>
<td>Sequence x Duration</td>
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<td>224.722</td>
<td>56.180</td>
<td>41</td>
<td>-</td>
</tr>
<tr>
<td>Sequence x Dimension</td>
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<td>434.426</td>
<td>72.404</td>
<td>1.035</td>
<td>p&gt;.25</td>
</tr>
<tr>
<td>Duration x Dimension</td>
<td>6</td>
<td>571.481</td>
<td>95.246</td>
<td>1.362</td>
<td>.10&lt;p&lt;.25</td>
</tr>
<tr>
<td>Sequence x Duration x Dimension</td>
<td>12</td>
<td>1188.462</td>
<td>99.0385</td>
<td>1.416</td>
<td>.10&lt;p&lt;.25</td>
</tr>
<tr>
<td>Error within</td>
<td>72</td>
<td>5036.666</td>
<td>69.9537</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Total 107 8774.6642
Table 14. Analysis of Variance on shift problem total errors.

<table>
<thead>
<tr>
<th>Effects</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
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<tbody>
<tr>
<td>Sequence</td>
<td>2</td>
<td>271.349</td>
<td>135.675</td>
<td>1.526</td>
<td>.10 &lt; p &lt; .25</td>
</tr>
<tr>
<td>Duration of S-R</td>
<td>2</td>
<td>77.487</td>
<td>38.744</td>
<td>&lt;1</td>
<td>-</td>
</tr>
<tr>
<td>Shift</td>
<td>2</td>
<td>236.024</td>
<td>118.012</td>
<td>1.327</td>
<td>p &gt; .25</td>
</tr>
<tr>
<td>Dimension</td>
<td>3</td>
<td>249.926</td>
<td>83.309</td>
<td>&lt;1</td>
<td>-</td>
</tr>
<tr>
<td>Sequence x Duration</td>
<td>4</td>
<td>66.1496</td>
<td>16.5374</td>
<td>&lt;1</td>
<td>-</td>
</tr>
<tr>
<td>Sequence x Shift</td>
<td>4</td>
<td>85.36471</td>
<td>21.5912</td>
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<td>-</td>
</tr>
<tr>
<td>Sequence x Dimension</td>
<td>6</td>
<td>307.0122</td>
<td>51.1687</td>
<td>&lt;1</td>
<td>-</td>
</tr>
<tr>
<td>Duration x Shift</td>
<td>4</td>
<td>112.8953</td>
<td>28.2238</td>
<td>&lt;1</td>
<td>-</td>
</tr>
<tr>
<td>Duration x Dimension</td>
<td>6</td>
<td>294.9625</td>
<td>49.1604</td>
<td>&lt;1</td>
<td>-</td>
</tr>
<tr>
<td>Shift x Dimension</td>
<td>6</td>
<td>156.3946</td>
<td>26.0657</td>
<td>&lt;1</td>
<td>-</td>
</tr>
<tr>
<td>Error</td>
<td>42</td>
<td>3734.232</td>
<td>88.910</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>81</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
Table 15. Analysis of Variance on Mean latencies before the TLE

<table>
<thead>
<tr>
<th>Effect</th>
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<th>MS</th>
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<th>p</th>
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<tbody>
<tr>
<td>Schedule</td>
<td>35.342</td>
<td>2</td>
<td>17.671</td>
<td>2.068</td>
<td>.10&lt;p&lt;.15</td>
</tr>
<tr>
<td>Dimension</td>
<td>13.953</td>
<td>3</td>
<td>4.651</td>
<td>.544</td>
<td>p&lt;.50</td>
</tr>
<tr>
<td>Time</td>
<td>8.168</td>
<td>2</td>
<td>4.084</td>
<td>.478</td>
<td>p&lt;.50</td>
</tr>
<tr>
<td>Schedule x Dimension</td>
<td>49.669</td>
<td>6</td>
<td>8.278</td>
<td>.969</td>
<td>.50&lt;p&lt;.40</td>
</tr>
<tr>
<td>Schedule x Time</td>
<td>28.443</td>
<td>4</td>
<td>7.111</td>
<td>.832</td>
<td>.50&lt;p&lt;.40</td>
</tr>
<tr>
<td>Dimension x Time</td>
<td>47.489</td>
<td>6</td>
<td>7.915</td>
<td>.926</td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>435.788</td>
<td>51</td>
<td>8.545</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 16. ANOVA on latencies on TLE+1, TLE+2, TLE+3 and TLE+4.

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Ss</td>
<td>515.546</td>
<td>102</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Within Ss</td>
<td>845.627</td>
<td>309</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trials</td>
<td>99.510</td>
<td>3</td>
<td>33.170</td>
<td>13.605</td>
<td>&lt;.005</td>
</tr>
<tr>
<td>Residual</td>
<td>746.117</td>
<td>306</td>
<td>2.438</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 17. ANOVA on median latencies on a correct or on an error response in the early and late stages of learning before the TLE

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Ss</td>
<td>32</td>
<td>759.377</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Within Ss</td>
<td>99</td>
<td>284.315</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stage of learning (early or late)</td>
<td>1</td>
<td>6.367</td>
<td>6.367</td>
<td>1.779</td>
<td>.10&lt;p&lt;.25</td>
</tr>
<tr>
<td>Stage x Ss</td>
<td>32</td>
<td>114.547</td>
<td>3.579</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type of Response (correct or error)</td>
<td>1</td>
<td>.460</td>
<td>.460</td>
<td>&lt;1</td>
<td></td>
</tr>
<tr>
<td>Type x Ss</td>
<td>32</td>
<td>51.455</td>
<td>1.608</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type x Stage</td>
<td>1</td>
<td>9.763</td>
<td>9.763</td>
<td>3.072</td>
<td>.05&lt;p&lt;.10</td>
</tr>
<tr>
<td>Residual</td>
<td>32</td>
<td>101.723</td>
<td>3.178</td>
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</table>
Table 1 & ANOVA on means of median latencies after a correct response or after an error in the early and late stages of learning of problem 1.

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Ss</td>
<td>32</td>
<td>584.432</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Within Ss</td>
<td>99</td>
<td>366.522</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stage (early or late)</td>
<td>1</td>
<td>5.636</td>
<td>5.636</td>
<td>1.863</td>
<td>.10&lt;p&lt;.25</td>
</tr>
<tr>
<td>Stage x Ss</td>
<td>32</td>
<td>96.787</td>
<td>3.025</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type of Response (correct or error)</td>
<td>1</td>
<td>4.219</td>
<td>4.219</td>
<td>1.060</td>
<td>p&gt;.25</td>
</tr>
<tr>
<td>Type x Ss</td>
<td>32</td>
<td>127.330</td>
<td>3.979</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type x Stage</td>
<td>1</td>
<td>3.294</td>
<td>3.294</td>
<td>&lt;1</td>
<td></td>
</tr>
<tr>
<td>Residual</td>
<td>32</td>
<td>129.256</td>
<td>4.039</td>
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<td></td>
</tr>
</tbody>
</table>
Appendix C

Proof that the transition probabilities of the HM model are invariant across the condition that the $p_i$ are unequal but sum to 1.00 when $p_i$ is probability that $S$ selects dimension $i$.

Let

- $C = \{\text{correct response}\}$
- $U = \{\text{relevant dimension is unchanged}\}$
- $C_1 = \{\text{relevant dimension is changed}\}$
- $D_i^* = \{i \text{ is the unchanged dimension}\}$
- $8C_1 = \{C_1 \land 8H\}$

$$P(C/8H) = P(C \land U/8H) + P(C \land C_1/8H) = P(U/8H)P(C/U \land 8H) + P(C_1/8H)P(C/C_1 \land 8H),$$

(1) $P(C/8H) = 1/4 \cdot P_{8U} + 3/4 \cdot P_{8C}.$

Equation (1) is that presented for the HM model herein.

$$P_{8U} = 1/4 \cdot p_1 + 1/4 \cdot p_2 + 1/4 \cdot p_3 + 1/4 \cdot p_4 = 1/4 \left( \sum_{i=1}^{4} p_i \right) = 1/4.$$

$$P_{8C} = P(C/8C_1) = P(C \land D_i^*/8C_1) + P(C \land D_i^*/8C_1) + P(C \land D_i^*/8C_1) + P(C \land D_i^*/8C_1) + P(C \land D_i^*/8C_1) + P(C \land D_i^*/8C_1) + P(C \land D_i^*/8C_1) + P(C \land D_i^*/8C_1) + P(C \land D_i^*/8C_1) + P(C \land D_i^*/8C_1)$$

and, e.g.,

$$P(C \land D_i^*/8C_1) = P(D_i^*/8C_1)P(C/D_i^* \land 8C_1)$$

$$P(D_i^*/8C_1) = 1/4,$$

and

$$P(C/D_i^* \land 8C_1) = p_2 \cdot 1/2 + p_3 \cdot 1/2 + p_4 \cdot 1/2.$$  

Thus,

$$P_{8C} = 1/4(1/2(p_2+p_3+p_4) + 1/2(p_1+p_3+p_4) + 1/2(p_1+p_2+p_4) + 1/2(p_1+p_2+p_3) + 1/2(p_1+p_2+p_3) + 1/2(p_1+p_2+p_3) + 1/2(p_1+p_2+p_3) + 1/2(p_1+p_2+p_3) + 1/2(p_1+p_2+p_3)) = 3/8.$$  

Thus, $P(C/8H) = 11/32.$
For $P(C/3H)$ the following notation is needed:

- $D_i' = \{ i \text{ is the unchanged dimension from } n+1 \text{ to } n+2 \}$
- $D_c = \{ \text{relevant dimension changed from } n \text{ to } n+1 \}$
- $D_s = \{ \text{relevant dimension remained the same from } n \text{ to } n+1 \}$
- $3U = \{ U \cap 3H \}$
- $3C_1 = \{ C_1 \cap 3H \}$

$P(C/3H) = 1/3 \cdot P_{3U} + 2/3 \cdot P_{3C}$ for the model developed herein.

$P_{3U} = P(C/3U) = P(C \cap D_i'/3U) + P(C \cap D_2'/3U) + \ldots + P(C \cap D_4'/3U)$

and, e.g.,

$$P(C \cap D_i'/3U) = P(C/3U \cap D_i')P(D_i'/3U)$$

$$= \left[ P(C \cap D_2'/3U \cap D_i') + \ldots + P(C \cap D_4'/3U \cap D_i') \right] \cdot 1/4$$

$$= \left[ P(C/3U \cap D_2' \cap D_i')P(D_2'/3U \cap D_i') + \ldots + P(C/3U \cap D_4' \cap D_i')P(D_4'/3U \cap D_i') \right] \cdot 1/4$$

$$= \left[ \frac{P_2}{P_2 + P_3 + P_4} \cdot 1/3 + \frac{P_3}{P_2 + P_3 + P_4} \cdot 1/3 + \frac{P_4}{P_2 + P_3 + P_4} \cdot 1/3 \right] \cdot 1/4 = 1/12.$$

Thus,

$$P_{3U} = 1/12 + 1/12 + 1/12 + 1/12 = 1/3.$$

$P_{3C} = 1/2 \left[ P(C/3C_1 \cap D_c) \right] + 1/2 \left[ P(C/3C_1 \cap D_s) \right].$

$P(C/3C_1 \cap D_c) = P(C \cap D_i'/3C_1 \cap D_c) + \ldots + P(C \cap D_4/3C_1 \cap D_c)$

$$= P(C/3C_1 \cap D_c \cap D_i')P(D_i'/3C_1 \cap D_c) + \ldots$$

$$+ P(C/3C_1 \cap D_c \cap D_4')P(D_4'/3C_1 \cap D_c).$$

Now,

$$P(D_i'/3C_1 \cap D_c) = 1/4$$

and, e.g.,

$$P(C/3C_1 \cap D_c \cap D_i') = P(C \cap D_2'/3C_1 \cap D_c \cap D_i')$$

$$+ P(C \cap D_3'/3C_1 \cap D_c \cap D_i')$$

$$+ P(C \cap D_4'/3C_1 \cap D_c \cap D_i').$$

Thus,

$$P(C/3C_1 \cap D_c) = 1/2 \cdot 1/4 + 1/2 \cdot 1/4 + 1/2 \cdot 1/4 + 1/2 \cdot 1/4 = 1/2$$
and similarly,
\[ P(C/3C_1 \land D_3) = 1/2. \]

Thus, \( P_{3C} = 1/2 \) and \( P(C/3H) = 4/9. \)

\( P(C/2H) \) is calculated as in the previous computation.

If \( D_i = \{ i \text{ is the relevant dimension} \} \)

Then, for example,
\[
P(C/2H \cap D_1^* \cap D_2^*) = P(C \cap D_3/2H \cap D_1^* \cap D_2^*) + P(C \cap D_4/2H \cap D_1^* \cap D_2^*)
\]
\[
= P(C/2H \cap D_1^* \cap D_2^* \cap D_3)P(D_3/2H \cap D_1^* \cap D_2^*)
\]
\[
+ P(C/2H \cap D_1^* \cap D_2^* \cap D_4)P(D_4/2H \cap D_1^* \cap D_2^*),
\]
\[
= \frac{P_3}{P_3 + P_4} \cdot \frac{1}{2} + \frac{P_4}{P_3 + P_4} \cdot 1/2 = 1/2.
\]

Thus, \( P(C/2H) = 1/2. \)
Appendix D

The distribution of total errors in the random condition assuming local consistency plus 2H memory.

The predicted distribution of total errors assuming local consistency plus 2H memory assumes the following process:

S starts a concept identification problem by selecting 1 of 24 possible H's; thus,

\[ P(T=0) = \frac{1}{24}. \]

On the first error trial, the probability of selecting the solution H is the probability S selects the solution H from one of the 8 locally consistent H's. Thus,

\[ P(T=1) = \left(1 - \frac{1}{24}\right) \cdot \left(\frac{1}{8}\right). \]

The probability of sampling the solution H on error trial \( i \geq 2 \) is the probability S selects the solution H from the pool of H's locally consistent with information on error trial \( i \) minus those H's which are eliminated on the basis of the recently tested H. If this probability is \( e \), then the distribution of total errors for \( i \geq 2 \) is

\[ P(T=n) = \left(1 - \frac{1}{24}\right) \cdot \left(\frac{1}{8}\right) \cdot (1-e)^{n-2}. \]

In order to compute \( e \), it is necessary to consider the following cases:

Suppose S has tested 0=C on irrelevant dimension 3 and found that it led to an error on trial n. Thus, on the \( (n+1)^{\text{st}} \) error trial, the following events can occur:

- \( E_1 \): 0 appears in \( D_3 \) and C is the reinforcement, e.g., 1101=C
- \( E_2 \): 1 or 2 appears in \( D_3 \) and C is the reinforcement, e.g., 1111=C
- \( E_3 \): 1 or 2 appears in \( D_3 \) and Bor A is the reinforcement, e.g., 1111=B
- \( E_4 \): 0 appears in \( D_3 \) and Bor A is the reinforcement, e.g., 1101=A.
If Event 1 occurs, the probability of sampling the solution $H(p_1)$ is $1/6$ since recalling $O^C$ on dimension 3 allows $S$ to eliminate dimension 3. If $E_2$ occurs, $p_2 = 1/8$, since $O^C$ on dimension 3 does not allow $S$ to eliminate any of the locally consistent $H$'s on dimension 3. If $E_3$ occurs, $p_3 = 1/7$, since $O^C$ eliminates one $H$ on dimension 3, i.e., 1=B, 0=C, 2=A. If $E_4$ occurs, $p_4 = 1/8$, since $O^C$ is redundant with 0=A or B on dimension 3. Thus, if $S$ has tested an $H$ on an irrelevant dimension, the probability of sampling the solution $H$ is

$$4 \sum_{i=1}^{4} P(E_i) p_i = 1/3 \cdot 1/6 + 2/3 \cdot 1/8 + 2/3 \cdot 2/3 \cdot 1/7 + 1/3 \cdot 2/3 \cdot 1/8$$

$$= .137.$$  

The probability, .137, is the probability $S$ selects the solution $H$ given that the $H$ tested and recalled is based on an irrelevant dimension.

If the $H$ tested was based on the relevant dimension, i.e., if dimension 3 were the relevant dimension and $O^C$, then the probability of sampling the solution $H$ depends upon whether a 0 or a 1 or 2 appears in dimension 3. If a 0 appears in dimension 3 (with probability $1/3$) then the probability of sampling the solution $H$ is $1/8$ (since the $H$ recalled is redundant with the stimulus information on dimension 3). If a 1 or 2 appears (with probability $2/3$), the probability of sampling the solution $H$ is $1/7$, since the recalled $H$ eliminates one locally consistent $H$. Thus, the probability of sampling the solution $H$ given $S$ has recalled an $H$ based on the relevant dimension is:

$$1/3 \cdot 1/8 + 2/3 \cdot 1/7 = .137.$$


Thus, $\epsilon$, the probability of sampling the solution $H$ given $S$ recalls a recently tested $H$ is found by weighting the probabilities of sampling the solution $H$ conditional upon whether $S$ has tested an $H$ based on a relevant or irrelevant dimension by the probability that $S$ tests the relevant or an irrelevant dimension on an error trial; this probability is $1/4$. Thus,

$$\epsilon = \frac{1}{4}(.137) + \frac{3}{4}(.137) = .137.$$ 

This $\epsilon$ was substituted into the formula for the distribution of total errors to form the predicted distribution of total errors for local consistency plus $2H$ memory found in the text.
Appendix E

The predicted distribution of total errors assuming local consistency plus memory for 3H's in the random condition.

The predicted distribution of total errors assuming local consistency plus memory for 3H's is equivalent to the predicted distribution of total errors for local consistency plus memory for 2H's for P(T=n), n=0,1,2. For n>=3, the distribution is:

\[ P(T=n) = (1-1/24)(1-1/8)(1-.137)(1-\varepsilon)^{n-3}. \]

The distribution for n>3 is found by multiplying the probability that S does not select the solution H at the start of the problem by the probability S does not select the solution H on the first or second error trial or for n-3 error trials on which he has memory for 3 previously tested H's multiplied by the probability S selects the solution H on the n\textsuperscript{th} error trial (which is \( \varepsilon \) in the above expression).

In order to calculate \( \varepsilon \), several cases of H's previously tested must be considered. There are 16 possible pairs of dimensions on which two previously tested H's may be based. Of these, 12 pairs of H's are based on different dimensions and 4 pairs of H's are based on the same dimension. Of the 12 pairs based on two different dimensions, 6 of these are based on irrelevant dimensions (Case 21) and 6 are based on one irrelevant and one relevant dimension (Case I and R).

Case 21

Suppose the hypothesis that 0=C on dimension 3 has led to error trial \( n \) and the hypothesis 1=B on dimension 2 has led to error trial \( n+1 \). Thus, on error trial \( n+2 \), when S is recalling 0\#C on \( D_3 \) and 1\#B on \( D_2 \), the following events can occur where \( p_i \) is the probability of selecting the solution H given event \( i \):
$E_1$: 2101=B \quad p_1=1/6, \text{ since } D_2 \text{ is eliminated by } 1\neq B.

$E_2$: 2101=C \quad p_2=1/6, \text{ since } D_3 \text{ is eliminated by } 0\neq C.

$E_3$: 2101=A \quad p_3=1/8, \text{ since no } H's \text{ are eliminated.}

$P(E_i)$ for $i=1,2,3 = 1/3 \cdot 1/3 \cdot 1/3$, since the probability $D_2$ and $D_3 = 1$ and $0$ respectively is $1/3 \cdot 1/3$ and $P(A), P(B), P(C) = 1/3$.

Additional events which may occur include the cases in which $D_2$ takes on one of two values which are different from the value in the $H$ recalled on $D_2$ while $D_3$ takes on the value of the $H$ recalled. These are:

$E_4$: 2(2 or 0) 01 = C \quad p_4 = 1/5, \text{ since } D_3 \text{ is eliminated and } 1H \text{ is eliminated on } D_2

$E_5$: 2(2 or 0) 01 = B \quad p_5 = 1/8, \text{ since no } H's \text{ are eliminated}

$E_6$: 2(2 or 0) 01 = A \quad p_6 = 1/7, \text{ since } 1H \text{ on } D_2 \text{ is eliminated.}

$P(E_i)$ for $i=4,5,6$ is $2/3 \cdot 1/3 \cdot 1/3$, since the probability of a 0 or a 2 on $D_2$ is $2/3$; the probability of a 0 in $D_3 = 1/3$ and the probability of an $A$, $B$, or $C$ occurring is $1/3$.

If $D_2$ takes on the same value as in the $H$ recalled on $D_2$, and $D_3$ changes to values other than that in the $H$ recalled on $D_3$, then these events occur:

$E_7$: 21(2 or 1)1 = B \quad p_7 = 1/5, \text{ since } D_2 \text{ is eliminated and } 1H \text{ on } D_3 \text{ is eliminated.}

$E_8$: 21(2 or 1)1 = C \quad p_8 = 1/8, \text{ since no } H's \text{ are eliminated.}

$E_9$: 21(2 or 1)1 = A \quad p_9 = 1/7, \text{ since } 1H \text{ on } D_3 \text{ is eliminated.}

$P(E_i)$ for $i=7,8,9 = 1/3 \cdot 2/3 \cdot 1/3$ for reasons similar to $P(E_i)$ for $i=4,5,6$.

If $D_2$ and $D_3$ take on values different from those in the $H$'s recalled, then these events occur:

$E_{10}$: 2(0 or 2)(2 or 1)1=A \quad p_{10}=1/6, \text{ since } 1H \text{ is eliminated on both } D_2, D_3.

$E_{11}$: 2(0 or 2)(2 or 1)1=B \quad p_{11}=1/7, \text{ since } 1H \text{ is eliminated on } D_3.

$E_{12}$: 2(0 or 2)(2 or 1)1=C \quad p_{12}=1/7, \text{ since } 1H \text{ is eliminated on } D_2.
P(E_i) for i=10,11,12 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{3}, since the probability that both D_2 and D_3 take on values different from those in the H's recalled is \frac{2}{3} \cdot \frac{2}{3} which is then multiplied by the probability of an A, B, or C.

To form an average probability of sampling the solution H after S has tested 2H's on different irrelevant dimensions, P(E_i) is multiplied by the p_i for each E_i and summed across i. That is,

\[
\sum_{i=1}^{12} P(E_i) p_i = \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} (\frac{1}{6} + \frac{1}{6} + \frac{1}{8}) + \frac{2}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} (\frac{1}{5} + \frac{1}{8} + \frac{1}{7})
\]

\[
+ \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{1}{3} (\frac{1}{5} + \frac{1}{8} + \frac{1}{7}) + \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{3} (\frac{1}{6} + \frac{1}{7} + \frac{1}{7}) = .154.
\]

Thus, for Case 2I, the probability of sampling the solution H is .154.

For Case I and R, it is necessary to consider the value of the relevant dimension on the n+2^{nd} trial. With probability 1/3, the value of the relevant dimension is the same as that in the H based on the relevant dimension. So, e.g., if S has recalled 0\#C on dimension 3 (and it is the relevant dimension) and recalled 1\#B on dimension 2, then the following events must be considered in computing the probability of sampling the solution H:

E_1: 1101=A p_1=\frac{1}{8}, since no H's are eliminated.

E_2: 1(0 or 2)01=A p_2 = \frac{1}{7}, since 1H is eliminated on D_2.

P(E_1) = \frac{1}{3} and P(E_2) = \frac{2}{3}, so the average probability of sampling the solution H given Case I and R and the relevant dimension is the same on the n+2^{nd} error trial as on the n^{th} is:

\[
\frac{1}{3} \cdot \frac{1}{8} + \frac{2}{3} \cdot \frac{1}{7} = .137.
\]

If Case I and R holds and the relevant dimension has a different value on n+2^{nd} error trial from that on the n^{th}, then these events must be considered:
E_1: 1111=B  \quad p_1 = 1/5, \text{ since } D_2 \text{ is eliminated and also } 1H \text{ on } D_3

E_2: 1(0 \text{ or } 2)11=B  \quad p_2 = 1/7, \text{ since } 1H \text{ is eliminated on } D_3

E_3: 1121=C  \quad p_3 = 1/8, \text{ since no } H's \text{ are eliminated}

E_4: 1(0 \text{ or } 2)21=C  \quad p_4 = 1/7, \text{ since } 1H \text{ is eliminated on } D_3.

P(E_1) \text{ and } P(E_4) = \text{ probability } D_2 \text{ takes same value as in } H \text{ tested based on } D_2 \text{ times the probability the relevant dimension takes 1 of 2 changed values = } 1/3 \cdot 1/2 = 1/6. \quad P(E_2) \text{ and } P(E_4) = 2/3 \cdot 1/2 = 2/6, \text{ since } D_2 \text{ takes on either value on which the } H \text{ tested is not based.}

Thus, the probability of sampling the solution } H \text{ given Case I and } R \text{ and the relevant dimension takes a value different from the } n^{th} \text{ error trial =}

\frac{1}{6} \cdot \frac{1}{5} + \frac{2}{6} \cdot \frac{1}{7} + \frac{1}{6} \cdot \frac{1}{8} + \frac{2}{6} \cdot \frac{1}{7} = .148.

If both dimensions tested by } S \text{ are the same dimension, i.e., the } H's \text{ recalled are based on the same dimensions, then with probability } 1/4, \text{ both } H's \text{ are based on the one relevant dimension (Case 2R). To compute the probability of sampling the solution } H \text{ given Case 2R, suppose } S \text{ has tested } 0=C \text{ and } 1=C \text{ on the relevant dimension 3. Thus, these events can occur:}

E_1: 1101=A  \quad p_1 = 1/7, \text{ since } 1H \text{ is eliminated on } D_3

E_2: 1111=B  \quad p_2 = 1/7, \text{ since } 1H \text{ is eliminated on } D_3

E_3: 1121=C  \quad p_3 = 1/8, \text{ since the } 2H's \text{ recalled are redundant with } 2=C \text{ on } D_3.

The } P(E_1) = 1/3, \text{ since } A,B,C \text{ are equally likely on the relevant dimension.}

Thus, the probability of sampling the solution } H \text{ given Case 2R is:}

\frac{1}{3} \cdot \frac{1}{7} + \frac{1}{3} \cdot \frac{1}{7} + \frac{1}{3} \cdot \frac{1}{8} = .138.

If both } H's \text{ tested are based on the same irrelevant dimension with probability } 3/4 \text{ (Case II), then the values of the irrelevant dimensions on the } n^{th} \text{ and } n+1^{th} \text{ error trial must be considered in computing the probability of sampling the solution } H. \text{ If the irrelevant dimension has the same value with}
probability 1/3 and suppose S has tested 0=C and 0=A on irrelevant dimension.

3. Then these events may occur:

E₁: 1101=A or C  \( p₁ = \frac{1}{6} \), since \( D₃ \) is eliminated
E₂: 1101=B  \( p₂ = \frac{1}{8} \), since no H's can be eliminated
E₃: 11(1 or 2)1=A or C  \( p₃ = \frac{1}{7} \), since 1H on \( D₃ \) is eliminated
E₄: 11(1 or 2)1=B  \( p₄ = \frac{1}{6} \), since \( D₃ \) is eliminated.

\( P(E₁) \) and \( P(E₄) \) = 2/9 while \( P(E₂) \) = 1/9 and \( P(E₃) \) = 4/9. Thus, the probability of sampling the solution dimension given Case II and the irrelevant dimension holds the same value on the \( n^{th} \) and \( n+1^{st} \) error trial is

\[
\frac{1}{6} \cdot \frac{2}{9} + \frac{1}{8} \cdot \frac{1}{9} + \frac{1}{7} \cdot \frac{4}{9} + \frac{1}{6} \cdot \frac{2}{9} = .151.
\]

If both H's tested are on the same irrelevant dimension, but that value is different from the \( n^{th} \) to the \( n+1^{st} \) error trial with probability 2/3, then 9 events must be considered. If S has tested 0=C and 1=B on \( D₃ \), then these events may occur:

E₁: 1101=A  \( p₁ = \frac{1}{7} \), since 1H on \( D₃ \) is eliminated
E₂: 1111=A  \( p₂ = \frac{1}{7} \), since 1H on \( D₃ \) is eliminated
E₃: 1111=C  \( p₃ = \frac{1}{8} \), since no H's are eliminated
E₄: 1121=A, B or C  \( p₄ = \frac{1}{7} \), since 1H on \( D₃ \) is eliminated
E₅: 1101=C  \( p₅ = \frac{1}{6} \), since \( D₃ \) is eliminated
E₆: 1111=B  \( p₆ = \frac{1}{6} \), since \( D₃ \) is eliminated
E₇: 1101=B  \( p₇ = \frac{1}{8} \), since no H's are eliminated.

If S had tested 0=C and 1=C with probability 1/3 (i.e., same response in the H), then these events would occur:

E₁: 1101=A or B  \( p₁ = \frac{1}{7} \), since 1H on \( D₃ \) is eliminated
E₂: 1121=C  \( p₂ = \frac{1}{8} \), since no H's are eliminated
E₃: 1111=A or B  \( p₃ = \frac{1}{7} \), since 1H is eliminated
E₄: 1121=A \[ p₄=1/6, \text{ since } D₃ \text{ is eliminated} \]

E₅: 1121=B \[ p₅=1/6, \text{ since } D₃ \text{ is eliminated} \]

E₆: 11(0 \text{ or } 1)1=C \[ p₆=1/6, \text{ since } D₃ \text{ is eliminated} \]

Thus, the probability of sampling the solution H given Case II is:

\[
\frac{1}{3}(0.151) + \frac{2}{3}\left[ \frac{2}{3}(5/9 \cdot 1/7 + 2/9 \cdot 1/8 + 2/9 \cdot 1/6) + \frac{1}{3}(4/9 \cdot 1/7 + 1/9 \cdot 1/8 + 4/9 \cdot 1/6) \right]
\]

Thus, the probability of sampling the solution H is formed in the following manner:

\[
P = P(\text{H's based on 2 different dimensions}) \times \left\{ P(\text{sampling the solution H given Case 2I}) \times P(\text{Case 2I}) \right\}
\]

\[
+ P(\text{Case I and R}) \left\{ P(\text{relevant dimension same value on } n, \text{ n+2nd errors}) \times P(\text{sampling the solution H given relevant dimension same on } n, \text{ n+2nd}) + P(\text{relevant dimension different value on } n, \text{ n+2nd error trial}) \times P(\text{sampling the solution H given relevant dimension different on } n, \text{ n+2nd errors}) \right\}
\]

\[
+ P(\text{H's based on same dimension}) \left\{ P(\text{sampling solution H given Case 2R}) \times P(\text{Case 2R}) \right\}
\]

\[
+ P(\text{Case II}) \left\{ P(\text{Irrelevant dimension has same value on } n, \text{ n+1st error trial}) \times P(\text{sampling the solution H given irrelevant dimension has same value on } n, \text{ n+1st error trials}) + P(\text{Irrelevant dimension has different value on } n, \text{ n+1st error trials}) \times P(\text{sampling the solution H given irrelevant dimension has different value on } n \text{ and n+1st error trial}) \right\}
\]

\[
= \frac{12}{16} \left\{ \frac{6}{12}(0.154) + \frac{6}{12}\left[ \frac{1}{3}(0.137) + \frac{2}{3}(0.148) \right] \right\} + \frac{4}{16} \left\{ \frac{1}{4}(0.138) + \frac{3}{4}(0.148) \right\} = 0.148.
\]

Thus, the predicted distribution of total errors is formed using \( \xi = 0.148 \).
The learning rate, \( \xi \), for the consistency check process in the random condition can be formed in the following way:

From trial \( n \) to trial \( n+1 \), either the relevant dimension remains the same (with probability = \( 1/3 \)) or it changes with probability \( 2/3 \).

If the relevant dimension remains the same on \( n+1 \) then there are 26 possible stimuli in which the relevant dimension remains the same. In 6 of the 26, one irrelevant dimension changes; thus the probability of sampling the solution \( H \) is \( 1/6 \) (eliminating the one changed dimension). In 12 of the stimuli, two irrelevant dimensions change, and the probability of sampling the solution \( H \) is \( 1/4 \) (eliminating two changed dimensions). In 8 of the stimuli, three dimensions change and the probability of sampling the solution \( H \) is \( 1/2 \).

If the relevant dimension changes from \( n \) to \( n+1 \), then there are 54 possible stimuli which can follow the stimulus on trial \( n \). In 24 of these, 1 irrelevant dimension remains the same and two change, thus the probability of sampling the solution \( H \) is \( 1/6 \) (eliminating the one dimension that remained the same). In 12 of the stimuli, two dimensions remain the same, thus the probability of sampling the solution \( H \) is \( 1/4 \). In 2 of the stimuli, 3 dimensions remain the same, thus the probability of sampling the solution \( H \) is \( 1/2 \) (one of the 2H's on the relevant dimension). In 16 of the stimuli, no dimensions remain the same, thus the probability of sampling the solution \( H \) is \( 1/8 \) (since no dimensions are eliminated by the consistency check).
The average probability of sampling the solution H, \( \xi \), is the probability 0,1,2,3 dimensions are eliminated given that the relevant dimension changes or remains the same multiplied by the probability of sampling the solution H for each of these joint events. Thus,

\[
\xi = \frac{1}{3}(6/26 \times 1/6 + 12/26 \times 1/4 + 8/26 \times 1/2) + \\
\frac{2}{3}(24/54 \times 1/6 + 12/54 \times 1/4 + 2/54 \times 1/2 + 16/54 \times 1/8) = .226.
\]
Appendix G

Derivations of the probability of i consecutive correct responses from $P_0$, $P_2$ for $i=0,1,2$.

The $P(C)$ before the TLE for a four ternary dimension problems:

According to H sampling models, before the TLE Ss are testing irrelevant H's on the relevant dimension ($I_R$) or irrelevant H's based on irrelevant dimensions ($I_J$) or reversal H's ($R$). It is possible to compute the $P(C)$ given that S is testing one of these three types of H's and, since the probability of sampling each of these three types of H's is known it is therefore possible to compute the $P(C)$ before the TLE.

Thus,

$$P(C) \text{ before the TLE} = P(C/I_R U I_J U R) = \frac{P(C/I_R)P(I_R) + P(C/I_J)P(I_J) + P(C/R)P(R)}{P(I_R) + P(I_J) + P(R)}.$$

Depending upon the particular memory process assumed in an H sampling model and the sequential changes in the stimulus dimensions, the $P(C)$ before the TLE will differ. It is the purpose of this section to derive this statistic for the no memory and the local consistency H sampling models for each of the sequence conditions in the experiment. It is also of interest to derive the $P(C)$ as the number of successively correct responses increases.

The succeeding derivations are the $P(C)$ before the TLE and the $P(C/$ several previous corrects) for the no memory H sampling model for the random, 3/trial and 4/trial sequences:

**Random stimulus sequence:** In a Random stimulus the probability that a particular dimension holds a 0, 1, or 2 $= 1/3$ on each trial.

On an error trial, S is assumed to sample an H randomly from the set of 24 H's (6H's on each of 4 dimensions). In this set there are the
the solution $H$, $18I_I$'s, $3I_R$'s, and 2 reversals. These $H$'s are noted in
the following diagram:

<table>
<thead>
<tr>
<th>Relevant Dimension $H$'s</th>
<th>Irrelevant Dimension $H$'s</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_R$ $I_R$ $R$ $R$ $I_R$</td>
<td>$0$ $A$ $A$ $B$ $C$ $C$</td>
</tr>
<tr>
<td>Values</td>
<td>$1$ $B$ $C$ $A$ $C$ $A$ $B$</td>
</tr>
<tr>
<td></td>
<td>$2$ $C$ $B$ $C$ $A$ $B$ $A$</td>
</tr>
</tbody>
</table>
Thus, $P(I_I) = 18/24$, $P(I_R) = 3/24$, $P(R) = 2/24$. The $P(C/I_I) = P(irrelevant$ $H$ sampled leads to the same response as the solution $H) = 6/18 = 1/3$. $P(C/I_R) = P(irrelevant$ $H$ on the relevant dimension leads to the same response as the solution $H) = 1/3$ since one out of 3 $H$'s leads to an $A$, $B$, or $C$. The $P(C/R)=0$ since reversal $H$'s never lead to a correct response. Thus, with these values substituted, equation (1) becomes

$$P(C) \text{ before the TLE} = \frac{1/3 \cdot 3/24 + 1/3 \cdot 18/24 + 0 \cdot 2/24}{3/24 + 18/24 + 2/24} = \frac{1/3 \cdot (21/24)}{23/24} = 7/23,$$
below $1/3$.

3/trial and 4/trial sequences: The $P(C \text{ before the TLE}) = 7/23$ for both the
3/trial and 4/trial sequences. This probability holds because regardless of
whether the relevant dimension changes or does not change on the next trial,
there is still one of the $3I_R$ which leads to a correct response and 6 of the
$18I_I$ which lead to a correct response. Thus the $P(C \text{ before the TLE})$ is
formed using the same value substitutions as for the random sequence in
equation (2).

Random sequence. The $P(C/C \text{ before the TLE})$ is computed by considering the
probability of two successively correct responses if $S$ is testing either an $I_R$
or an $I_I$ (since $S$ must make one correct on the $H$ he is testing, $R$'s are ex-
cluded). Thus, in set notation, the problem is to find:
\[ P(C_{n+1}/C_n \land (I_R \cup I_I)) \] which, in turn, =

\[ P(C_{n+1}/C_n \land I_R) \lor (C_n \land I_I) = \frac{P(C_{n+1} \land C_n \land I_R) + P(C_{n+1} \land C_n \land I_I)}{P(C_n \land I_R) + P(C_n \land I_I)} \]

\[ = \frac{P(C_{n+1}/C_n \land I_R)P(I_R) + P(C_{n+1}/C_n \land I_I)P(I_I)}{P(C_n/I_R)P(I_R) + P(C_n/I_I)P(I_I)}. \]

To fill in the preceding equation \( P(C_n/I_R) = 1/3 \) from preceding arguments

\[ P(I_R) = 3/24 \]
\[ P(C_n/I_I) = 1/3 \]
\[ P(I_I) = 18/24. \]

In order to form the \( P(C_{n+1}/C_n \land I_R \text{ or } I_I) \), the probability that the relevant dimension changes must be considered. Let \( S \) be the event that the relevant dimension remains the same from \( n \) to \( n+1 \) and \( D \) be the event that the relevant dimension is different.

Thus, \( P(C_{n+1}/C_n \land I_R) = P(C_{n+1} \land S/C_n \land I_R) + P(C_{n+1} \land D/C_n \land I_R) \)

\[ = P(C_{n+1}/C_n \land I_R \land S) \cdot P(S/C_n \land I_R) + P(C_{n+1}/C_n \land I_R \land D) \cdot P(D/C_n \land I_R). \]

In the Random sequence, the probability that a dimension remains the same on trial \( n+1 = 1/3 \) and the probability that it changes = \( 2/3 \), and is independent of Ss response.

Thus, \( P(S/C_n \land I_R) = 1/3 \)
\[ P(D/C_n \land I_R) = 2/3. \]

The probability of a second successively correct response conditional upon the dimension remaining the same and Ss responding on the basis of an \( I_R \) \( H \) is 1.00, since, to get a correct on trial \( n \), S responded on the basis of the one \( I_R \) \( H \) that led to a correct response. Since S was correct on trial \( n \), he retains the same \( H \) and is correct on trial \( n+1 \) since the relevant dimension remained the same. If the relevant dimension changes, then the S responding according to a previously correct \( I_R \) \( H \) will not be correct on the next trial
since the $I_R$'s share only one response in common with the solution $H$.

Thus, equation (5) becomes

$$P(C_{n+1}/C_n \cap I_R) = (1.00)(1/3) + (0) 2/3 = 1/3.$$ 

The $P(C_{n+1}/C_n \cap I_I)$ must now be formed in the same manner as that for Ss responding on the basis of $I_R$ H's. The analogy to equation (5) is

$$(6) \quad P(C_{n+1}/C_n \cap I_I) = \frac{P(C_{n+1}/C_n \cap I_I \cap S) \cdot P(S/C_n \cap I_I) + P(C_{n+1}/C_n \cap I_I \cap D) \cdot P(D/C_n \cap I_I)}{P(S/C_n \cap I_I) = 1/3 \text{ and } P(D/C_n \cap I_I) = 2/3 \text{ because of the random stimulus sequences.}}$$

The probability of a second successively correct response if S is testing on $I_I$ H and the relevant dimension changes or does not change is now computed:

$$P(C_{n+1} \cap S/C_n \cap I_I) = P(\text{the dimension upon which the } I_R \text{ is based remains the same}) = 1/3.$$ 

$$P(C_{n+1} \cap D/C_n \cap I_I) = P(\text{the dimension upon which the } I_I \text{ is based changes and the } I_I \text{ leads to the same response as the solution } H) = 2/3 \cdot 3/6 \text{ (3 of the 6 } I_I \text{ H's lead to a correct response on n+1) = 1/3}$$

and equation (6) becomes

$$P(C_{n+1}/C_n \cap I_I) = 1/3 \cdot 1/3 + 2/3 \cdot 1/3 = 3/9. \text{ Thus, the equation becomes:}$$

$$P(C_{n+1}/C_n \cap (I_R \cup I_I)) = \frac{1/3 \cdot 1/3 \cdot 3/24 + 1/3 \cdot 1/3 \cdot 18/24}{1/3 \cdot 3/24 + 1/3 \cdot 3/24} = \frac{1/9(21/24)}{1/3(21/24)} = 1/3.$$ 

3/trial sequence:

The terms of equation (3) must be filled in.

$$P(C_{n+1}/C_n \cap (I_R \cup I_I)) = \frac{P(C_{n+1}/C_n \cap I_R)P(C_n/I_R)P(I_R) + P(C_{n+1}/C_n \cap I_I)P(C_n/I_I)P(I_I)}{P(C_n/I_R)P(I_R) + P(C_n/I_I)P(I_I)},$$

Some terms remain the same as for the random sequence:

$$P(C_n/I_R) = 1/3$$

$$P(I_R) = 3/24$$

$$P(C_n/I_I) = 1/3$$

$$P(I_I) = 18/24$$

but $P(C_{n+1}/C_n \cap I_R)$ and $P(C_{n+1}/C_n \cap I_I)$ are different.
When 3 dimensions change per trial,
\[ P(S/C_n \land I_R) = 1/4 \quad \text{and} \quad P(D/C_n \land I_R) = 3/4. \]
The \( P(C_{n+1}/C_n \land I_R \land S) = 1.00. \) Since the relevant dimension does not change, the one previous correct \( I_R \) will lead to the same (correct) response as on trial \( n \).

If the relevant dimension changes and \( S \) holds an \( I_R \), then
\[ P(C_{n+1}/C_n \land I_R \land D) = 0 \] because the \( I_R \) and the solution \( H \) lead to the same response for only one of the three values of the relevant dimension (the value on trial \( n \)).

Thus, equation \((4)\) becomes
\[ P(C_{n+1}/C_n \land I_R) = 1 \cdot 1/4 + 0 \cdot 3/4 = 1/4. \]

If \( S \) holds an \( I_T \), then
\[ P(C_{n+1}/C_n \land I_T) = P(C_{n+1} \land S/C_n \land I_T) + P(C_{n+1} \land D/C_n \land I_T) \]
\[ = P(C_{n+1}/C_n \land I_T \land S)P(S/C_n \land I_T) + P(C_{n+1}/C_n \land I_T \land D)P(D/C_n \land I_T). \]
\[ P(C_{n+1}/C_n \land I_T \land S) \] is the probability that \( S \) makes a second correct response when the relevant dimension remains the same and he holds an \( I_T \) is 0 because since 3 dimensions change per trial, the irrelevant dimensions change and \( S \)'s response is different from that on the previous trial and his response is not correct. \( P(C_{n+1}/C_n \land I_T \land D) \) is more difficult to calculate. If the relevant dimension changes, two of the irrelevant dimensions change and one remains the same. The two \( I_T \)'s based on the irrelevant dimension that stays the same lead to an incorrect response. Of the four \( I_T \)'s based on the irrelevant dimensions (two per dimension were correct on \( n \)), two lead to the same response as the solution \( H \) and two lead to a different response. Thus
\[ P(C_n/C_n \land I_T \land D) = 2/6 = 1/3 \] since the probability of a correct response is equal to the probability that \( S \) holds one of the \( H \)'s that lead to a correct
response on trial \( n+1 \) (=2/6). \( P(C_{n+1}/C_n \land I) = 0 \cdot 1/4 + 1/3 \cdot 3/4 = 1/4 \).

Now the basic equation (3) can be filled in:

\[
P(C_{n+1}/C_n \land (I_R \cup I_I)) = \frac{1/4 \cdot 1/3 \cdot 3/24 + 1/4 \cdot 1/3 \cdot 18/24}{1/3 \cdot 3/24 + 1/3 \cdot 18/24} = \frac{1/4(21/24)}{(21/24)} = 1/4.
\]

4/trial sequences:

In these sequences, the relevant dimension changes on each trial. The basic equation (3) can be filled in if two terms are formed.

\[
P(C_{n+1}/C_n \land I_R) \quad \text{and} \quad P(C_{n+1}/C_n \land I_I).
\]

\[
P(C_{n+1}/C_n \land I_R) = 0 \quad \text{since the } I_R \text{ which led to a correct response on } n \text{ leads to a different response on } n+1 \text{ from the solution } H.
\]

\[
P(C_{n+1}/C_n \land I_I) = 3/6 = 1/2 \quad \text{since of the six previously correct } I_I \text{'s, three of these } H's \text{ lead to the same response as the solution } H \text{ on trial } n+1.
\]

Thus equation (3) becomes

\[
P(C_{n+1}/C_n \land (I_R \cup I_I)) = \frac{0 \cdot 1/3 \cdot 3/24 + 1/2 \cdot 1/3 \cdot 18/24}{1/3 \cdot 3/24 + 1/3 \cdot 18/24} = \frac{9/24}{21/24} = 9/21 = 3/7.
\]

To form the probability of a third successively correct response for each of the sequences, the basic equation:

\[
P(C_{n+2}/C_n \land C_n \land (I_R \cup I_I)) = \frac{P(C_{n+2}/C_n \land C_n \land I_R) + P(C_{n+2}/C_n \land C_n \land I_I)}{P(C_{n+1}/C_n \land I_R) + P(C_{n+1}/C_n \land I_I)}
\]

is used. For each of the 3 types of sequences, all terms have been previously derived except the probability of a third consecutively correct response contingent upon an \( I_R \) or an \( I_I \). This expression is derived for the following:

Random sequence:

\[
P(C_{n+2}/C_n \land I_R) = P(I_R \text{ leads to the same response as the solution } H) = 1/3.
\]

\[
P(C_{n+2}/C_n \land I_I) = P(I_I \text{ leads to the same response as the solution } H) = 6/18 = 1/3.
\]

Thus, equation (6) for the random sequence is:

\[
P(C_{n+2}/C_n \land (I_R \cup I_I)) = \frac{1/3 \cdot 1/3 \cdot 1/24 + 1/3 \cdot 1/3 \cdot 1/4}{1/3 \cdot 1/24 + 1/3 \cdot 1/4} = \frac{1/3(7/24)}{7/24} = 1/3.
\]
3/trial sequences:

S can only get two correct responses in a row on an \( I_R \) only if the relevant dimension has stayed the same from \( n \) to \( n+1 \). Because of the restrictions on the sequences, the relevant dimension must change on \( n+3 \) and if \( S \) holds on \( I_R \), he is incorrect, i.e., \( P(C_{n+2}/C_{n+1} \land C_n \land I_R) = 0 \).

\( S \) can get two correct responses in a row on an \( I_I \) only if the relevant dimension has changed from trial \( n \) to \( n+1 \). Correct \( I_I H \)'s are based on the two irrelevant dimensions that changed from trial \( n \) to \( n+1 \) and these \( I_I H \)'s led to the same response as the solution \( H \) - thus, there remain only 2\( I_I H \)'s which led to two previously correct responses (one on each of the irrelevant changed dimensions). The probability that these two \( I_I H \)'s lead to a third correct response is found in the following way: The previously unchanged irrelevant dimension must change from \( n+1 \) to \( n+2 \), thus either the relevant dimension or one of the irrelevant dimensions must remain the same. If the relevant dimension remains the same (\( pr = 1/3 \)), then the \( I_R H \)'s lead to incorrect responses (since the dimension upon which they are based change from \( n+1 \) to \( n+2 \)). If the relevant dimension changes (\( pr = 2/3 \)), then one of the two irrelevant dimensions remains the same (and its \( I_R \) leads to an incorrect response). The other irrelevant dimension changes from \( n+1 \) to \( n+2 \) and the \( I_R \) based on this dimension can lead to the same response as the solution \( H \) or to a different response (each with \( pr = 1/2 \)). Thus,

\[
P(C_{n+2}/C_{n+1} \land C_n \land I_I) = 2/3 \cdot 1/2 \cdot 1/2 = 1/6.
\]

Thus, equation (6) for the 3/trial sequence is:

\[
P(C_{n+2}/C_{n+1} \land C_n \land (I_R \cup I_I)) = \frac{0 \cdot 1/4 \cdot 1/24 + 1/6 \cdot 1/4 \cdot 1/4}{1/4 \cdot 1/24 + 1/4 \cdot 1/4} = \frac{1/24}{7/24} = 1/7.
\]
Since $P(C_{n+1}/C_n \cap I_R) = 0$ in the 4/trial sequences and because the
$\bar{I}_R^H$ leads to the same response as the solution $H$ for only 1 value of the
relevant dimension, only the $P(C_{n+3}/C_n \cap C_n \cap I_I)$ must be found.
Since four dimensions change per trial, the relevant dimension may take the
value held on trial $n$, or a value different from trial $n$, on trial $n+3$, each
with $p=1/2$. The same reasoning applies to each of the irrelevant dimensions.
Thus the $P(C_{n+2}/C_n \cap C_n \cap I_I) = 1/2 \cdot 1/2 + 1/2 \cdot 1/2 = 1/2$, and equation (6)
becomes:

$$P(C_{n+2}/C_n \cap C_n \cap I_I) = \frac{0 + 1/2 \cdot 1/2 \cdot 1/4}{0 \cdot 1/4 + 1/2 \cdot 1/4} = 1/2.$$ 

Thus, the $P_0$ predicts

<table>
<thead>
<tr>
<th></th>
<th>$P(C_n)$</th>
<th>$P(C_n/C_{n-1})$</th>
<th>$P(C_n/C_{n-1}, C_{n-2})$</th>
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</thead>
<tbody>
<tr>
<td>Rand</td>
<td>7/23=.304</td>
<td>1/3=.333</td>
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</tr>
<tr>
<td>3/tr</td>
<td>7/23=.304</td>
<td>1/4=.250</td>
<td>1/7=.143</td>
</tr>
<tr>
<td>4/tr</td>
<td>7/23=.304</td>
<td>3/7=.429</td>
<td>1/2=.500</td>
</tr>
</tbody>
</table>

It should be noted that the no memory model in a random stimulus sequence
predicts that the probability of a correct response (after the first) is
independent of the number of correct responses preceding it. This illustrates
the general "independence of responses" predicted by $H$ sampling models for
these stimulus sequences.

According to the local consistency $H$ sampling model, $S$s sample from a
pool of $H$'s consistent with reinforcement on the most recent error trial.
This section develops the same sequential statistics as the last section,
but on the assumption that $S$s choose from a pool of locally consistent $H$'s.
On every error trial, $S$ samples from a pool of 8 locally consistent $H$'s, e.g.,
if $S$ makes an error on a trial in which he has experienced 0000=A, then he
chooses one of 8 possible H's: the solution H or an IR based on dimension 1 or one of 6 I_H's (2 on each irrelevant dimension). All irrelevant H's are of the form 0=A, 1=B, 2=C or 0=A, 2=B, 1=C in this example.

**Random sequence:**

\[
P(C_n \text{ after the first error and before TLE}) = P(C/I_R \cup I_I) = \frac{P(C/I_R)P(I_R) + P(C/I_I)P(I_I)}{P(I_R) + P(I_I)}
\]

\[P(I_R) = 1/8\]

\[P(I_I) = 6/8 = 3/4\]

\[P(C/I_R) = 1/3 = P(\text{solution H and I_R lead to same response on n}) = P(\text{"0" appears in relevant dimension})\]

\[P(C/I_I) = P(C_n | S \land I_I) \cdot P(S/I_I) + P(C_n/D \land I_I)P(D/I_I)\]

\[P(S/I_I) = P(S) = 1/3 \text{ [independent]}\]

\[P(D/I_I) = P(D) = 2/3\]

\[P(C_n/S \land I_I) = P(\text{value held by irrelevant dimension on trial n is repeated on n+1}) = 1/3\]

\[P(C_n/D \land I_I) = P(\text{irrelevant dimension changes and I_I leads to same response as solution H}) = 2/3 \cdot 1/2 = 1/3.\]

Thus \[P(C \text{ after the first error and before TLE}) = \frac{1/3 \cdot 1/8 + 1/3 \cdot 3/4}{7/8} = \frac{1/3(7/8)}{7/8} = 1/3.\]

**3/trial sequence:**

Again, \[P(I_R) = 1/8\]

\[P(I_I) = 3/4\]

\[P(C_n/I_R) = P(C_n \land S/I_R) + P(C_n \land D/I_R) = P(C_n/S \land I_R)P(S) + P(C_n/D \land I_R)P(D).\]

In the 3/trial sequences, \[P(S) = 1/4; P(D) = 3/4\]

\[= (1.00)(1/4) + 3/4 \cdot 0\]

\[= 1/4.\]

\[P(C_n/S \land I_I) = P(\text{irrelevant dimension remains the same on n+1}) = 1/3.\]
\[ P(C_n / I_j) = P(C_n / S \land I_j)P(S) + P(C_n / D \land I_j)P(D). \]

\[ = (1/3)(1/4) + 1/3 \cdot (3/4) = 1/3. \]

\[ P(C_n / D \land I_j) = P(\text{irrelevant dimension changes and } I_j \text{H leads to same response solution } H) = 2/3 \cdot 1/2 = 1/3. \]

Thus, \[ P(C_n \text{ after the first error and before TLE}) = \frac{1/4 \cdot 1/8 + 1/3 \cdot 3/4}{7/8} = \frac{1/4(9/8)}{7/8} = \frac{9}{28}. \]

4 trial sequences:

\[ P(C_n / I_R) = 0 \text{ because } I_R \text{ and the solution } H \text{ have only one response in common (that response correct on } n). \]

\[ P(C_n / I_j) = P(I_j \text{H leads to same response as the solution } H) = 1/2 \]

Thus

\[ P(C_n \text{ after the first error and before TLE}) = \frac{0 \cdot 1/8 + 1/2 \cdot 3/4}{7/8} = \frac{3/8}{7/8} = 3/7. \]

For the probability of 2 consecutively correct responses in each sequence,

the basic formula

\[ P(C_{n+1} / C_n \land (I_R \cup I_j)) = \frac{P(C_{n+1} / C_n \land I_R)P(I_R) + P(C_{n+1} / C_n \land I_j)P(I_j)}{P(C_n / I_R)P(I_R) + P(C_n / I_j)P(I_j)} \]

is used.

Random sequences: All terms in equation (3) are known except \( P(C_{n+1} / C_n \land I_R) \) and \( P(C_{n+1} / C_n \land I_j) \).

\[ P(C_{n+1} / C_n \land I_R) = P(\text{relevant dimension takes same value as on } n) = 1/3 \]

\[ P(C_{n+1} / C_n \land I_j) = P(C_{n+1} / C_n \land I_j \land S) \cdot P(S / C_n \land I_j) + P(C_{n+1} / C_n \land I_j \land D) \cdot P(D / C_n \land I_j) + P(C_{n+1} / C_n \land I_j \land D) = P(\text{irrelevant dimension remains the same on } n+1 \text{ as on } n) = 1/3 \]

\[ P(C_{n+1} / C_n \land I_j \land D) = P(\text{irrelevant dimension changes and } I_j \text{H leads to same response as solution } H) = 2/3 \cdot 1/2 = 1/3 \]

Thus,

\[ P(C_{n+1} / C_n \land (I_R \cup I_j)) = \frac{1/3 \cdot 1/3 \cdot 1/8 + 1/3 \cdot 1/3 \cdot 3/4}{1/3 \cdot 1/8 + 1/3 \cdot 3/4} = \frac{1/3(7/8)}{7/8} = 1/3. \]
3/trial sequences:

\[ P(C_{n+1}/C_n \land I_R) = P(C_{n+1}/C_n \land I_R \land S) \cdot P(S/C_n \land I_R) \]
\[ + P(C_{n+1}/C_n \land I_R \land D) \cdot P(D/C_n \land I_R). \]

S was correct on an \( I_R \) on trial \( n \) only if the relevant dimension has remained the same from the error trial to trial \( n \). Thus, the relevant dimension must change from \( n \) to \( n+1 \) and Ss holding the \( I_R \) are incorrect. Thus, \( P(C_{n+1}/C_n \land I_R) = 0 \). To form \( P(C_{n+1}/C_n \land I_T) \), consider that S was correct on an \( I_T \) only if the relevant dimension has changed from the error trial to trial \( n \). Thus the relevant dimension can stay the same (pr=1/3) or change (pr=2/3) from \( n \) to \( n+1 \). S can only be correct if the relevant dimension changes from \( n \) to \( n+1 \) and only if his \( I_T \) is based on the one irrelevant dimension that did not remain the same from the error trial to \( n \) (pr=1/2) and the \( I_T \) leads to the same response as the solution \( H \) (pr=1/2). Thus,

\[ P(C_{n+1}/C_n \land I_T) = 1/2 \cdot 1/2 \cdot 2/3 = 1/6. \]

Thus equation (3) becomes

\[ P(C_{n+1}/C_n \lor (I_R \land I_T)) = 0 = 1/4(9/8) = 8/54 = 4/27. \]

4/trial sequence:

Recall that \( P(C_n/I_R) = 0 \).

\[ P(C_{n+1}/C_n \land I_T) = P(I_T \text{ leads to same response as solution } H) = 1/2. \]

Thus, \( P(C_{n+1}/C_n \land (I_R \lor I_T)) = 0 + 1/2 \cdot 1/2 \cdot 3/4 = 1/2. \)

The basic equation for the probability of a third consecutively correct response is:

\[ P(C_{n+2}/C_{n+1} \land C_n \land I_R) = \frac{P(C_{n+1}/C_n \land I_R) P(C_n/I_R) P(C_{n-1}/I_R)}{P(C_{n+1}/C_n \land I_R) P(C_n/I_R) + P(C_{n+1}/C_n \land I_R) P(C_n/I_T)} \]

Random sequence:

\[ P(C_{n+2}/C_{n+1} \land C_n \land I_R) = 1/3 = P(\text{relevant dimension is the same on trials } n, n+1), \]
\[ P(C_{n+2}/C_{n+1} \land C_n \land I_T) = 1/3 \cdot 1/3 + 2/3 \cdot 2/3 \cdot 1/2 = 1/3. \]
thus $P(C_{n+2}/C_{n+1} \land C_n \land (I_R \cup I_I)) = \frac{1/3 \cdot 1/3 \cdot 1/3 \cdot 1/8 + 1/3 \cdot 1/3 \cdot 3/4}{1/3 \cdot 1/3 \cdot 1/8 + 1/3 \cdot 1/3 \cdot 3/4} = \frac{1/3(1/8+3/4)}{(1/8+3/4)} = 1/3.$

3/trial sequence:

Recall $P(C_{n+1}/C_n \land I_R) = 0$.

$P(C_{n+2}/C_{n+1} \land C_n \land I_I) = 0$ also because the restrictions on the stimulus sequence require that either the relevant dimension or the irrelevant dimension upon which the $I_I$ is based must change and the other must remain the same; thus, it is impossible that the $I_I$ lead to the same response as the situation $H$.

Thus, $P(C_{n+2}/C_{n+1} \land C_n \land (I_R \cup I_I)) = 0$.

4/trial sequence:

Recall $P(C_n/I_R) = 0$

$P(C_{n+2}/C_{n+1} \land C_n \land I_I) = P($solution $H$ and $I_I$ lead to the same response$) = 1/2$.

Thus

$P(C_{n+2}/C_{n+1} \land C_n \land (I_I \cup I_R)) = \frac{0+1/2 \cdot 1/2 \cdot 1/2 \cdot 3/4}{0+1/2 \cdot 1/2 \cdot 3/4} = 1/2.$

Thus, for $P_2$:

<table>
<thead>
<tr>
<th></th>
<th>$P(C_n)$</th>
<th>$P(C_{n+1}/C_n)$</th>
<th>$P(C_{n+2}/C_{n+1}; C_n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rand</td>
<td>1/3=.333</td>
<td>1/3=.333</td>
<td>1/3=.333</td>
</tr>
<tr>
<td>3/tr</td>
<td>9/28=.321</td>
<td>4/27=.148</td>
<td>0</td>
</tr>
<tr>
<td>4/tr</td>
<td>3/7=.439</td>
<td>1/2=.500</td>
<td>1/2=.500</td>
</tr>
</tbody>
</table>
REFERENCES


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Neisser, V. and Weene, P. Hierarchies in concept attainment. J. exp. Psychol., 1962, 64, 640-645.


REFERENCES


Suppes, P. and Ginsberg, R. A fundamental property of all-or-none models, binomial distribution of responses prior to conditioning, with application to concept formation in children. Psychol. Rev., 1963, 70, 139-161.


REFERENCES
