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CURRICULAR IMPLICATIONS OF MATHEMATICAL
CONCEPTS OF THE PRESCHOOL CHILD

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

Harold Dickerscheid, B.S., M.Ed.

********

The Ohio State University
1969

Approved by

Paul R. Kloden
Adviser
College of Education
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To my wife, Jean, and children Annette, Daniel, and Pamela, a special thanks— for patience, understanding and sacrifice which made possible my fulfillment of a long sought goal.
VITA

January 21, 1927 • • • Born—Hamilton, Ohio

1950 • • • • • • • • B.S., The Ohio State University, Columbus, Ohio

1951-1955 • • • • • • • Teacher, Mathematics, Elm Valley High School, Ashley, Ohio

1955 • • • • • • • M.Ed., Miami University, Oxford, Ohio

1955-1965, 1967-1968 • • • • • • • Teacher, Mathematics, Columbus Public Schools, Columbus, Ohio

1958-1959 • • • • • • National Science Foundation Participant

1965-1967 • • • • • • Teaching Assistant, The Ohio State University, Columbus, Ohio

1968-1969 • • • • • • Assistant Professor, Urbana College, Urbana, Ohio

FIELDS OF STUDY

Major Field: Curriculum and Foundations

Studies in Curriculum and Foundations. Professor Paul R. Klohr

Studies in Mathematics Education. Professor Nathan Lazar

Studies in Higher Education. Professor Karl Openshaw
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CHAPTER I

INTRODUCTION AND STATEMENT OF PROBLEM

Origin of the Problem

The preschool is currently being critically scrutinized by educators, psychologists, sociologists, and many other interested groups in an attempt to reformulate the curriculum. The preschool years are being studied to see what potential for learning there is during these first six years of a child's life. Psychologists are stating that a child's approximate full potential for learning is determined by the time he is five years of age, and hence, the experiences the child has had up to this time are very crucial to the experiences he will have the remainder of his life. Behavioral scientists theorize that if certain intellectual pursuits are not undertaken at this crucial time, some concepts will not be acquired in later years. Other concepts may be only partially acquired. Since research is indicating the early years as the time of greatest physical and intellectual growth, appropriate stimulation must be provided during these years if all children are not to be labeled "culturally deprived."

Although there have been sporadic attempts in the past to place mathematics in the preschool curriculum (Froebel, Dewey, Montessori), the impact on the preschools is negligible.
As late as thirty years ago, there was great disagreement whether arithmetic should be systematic or incidental in the primary grades. This debate has now sifted down to the preschooler and his experiences with arithmetic.¹

Much of the present emphasis on the preschooler may be attributed to a concern for the disadvantaged. Money made available by the Federal government, private endowments, universities, and business has precipitated increased interest in the intellectual capabilities of the disadvantaged preschool child. This, in turn, has caused some persons to become concerned with improving programs for all preschool children.

The time for the pre-school experiment is doubly right. That it is needed for the sake of the excluded and deprived is self-evident; but the movement is gathering momentum at the very moment when educational psychologists are offering persuasive evidence that all children can learn—and often want to learn—much more, much sooner. What has been introduced as a lifesaving device for those at the bottom of our society's ladder may, in time, help to loosen the rigidity of the educational structure as a whole.²

There is a broad spectrum on positions taken by preschools concerning mathematics. Some are concerned solely with child care and do nothing with mathematical learning. Others, such as those based on the "child study" movement, have moved cautiously to incorporate mathematics in the curriculum by


placing their main emphasis on incidental learning. One bold, present day program (Bereiter and Engelmann)\(^3\) places its emphasis on rote memorization of facts.

**Statement of the Problem**

The purpose of this study is to investigate the mathematical concepts of a selected group of preschool children and to design a pilot mathematics curriculum. Three objectives will give direction to this study. They are:

1. To investigate how a child builds his earliest number concepts.
2. To design curricular materials to aid in the learning of mathematical concepts.
3. To identify how mathematical concepts can be effectively incorporated in a nursery school program.

**Importance of the Study**

Bernard Spodek succinctly underlines the importance of critical investigation of the preschooler and curricular experiences that are possibly mandatory if all children are not to be classified as deprived.

Early childhood education is coming to be seen less as a privilege and more as an individual right and possibly even as a responsibility which society owes to all children.\(^4\)

The current trend in preschool education underlines this terse


statement. More and more educators and psychologists are turning toward the preschooler in a search for answers to many educational problems. Research in human development by behavioral scientists in the past decade has indicated that the years up to six may be the most important ones in the child's intellectual growth. Research on early deprivation with animals and subsequently with preschoolers has more and more substantiated the theory that early learning is vital.

Some who advance the critical period theory are Benjamin Bloom, William Fowler, J. McVicker Hunt, and Arthur Jensen. The general consensus of this group is embodied in the following statement by Bloom who points out that the preschool years are the ones in which

... general learning patterns develop most rapidly, and failure to develop appropriate achievement and learning in these years is likely to lead to continual failure throughout the remainder of the individual's school career.

Little has been done to study the way the preschool child progresses through basic mathematical concepts. Piaget and others report what the child at a certain age does or does not do, but what a child can do if sequential patterns different

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9Bloom, op. cit., p. 127.
from those currently being used are utilized has not been studied and reported. For example, rote counting, an abstract concept, is commonly introduced to the child before he learns to interact with concrete materials. An outstanding example of this is the program being conducted at The University of Illinois under the direction of Carl Bereiter and Siegfried Engelmann. This preoccupation with dealing with the abstract in the preschool is a reversal of many proven psychological studies. Yet, it seems that many of these programs are gaining momentum. There need to be many illustrations of sound curricular alternatives. This pilot study will contribute to the alternatives as it undertakes:

(1) to assemble visual as well as manipulative materials to be used in the preschool;

(2) to give direction to nursery school teachers concerning the integration of mathematical concepts in the program;

(3) to serve as a base for additional research of preschool cognitive processes relating to mathematical concepts.

The main emphasis in traditional nursery school training has been on motor and social development. Nursery educators have been largely concerned with the effects of interaction of people and things on the child's personality. The cognitive aspect of the child's development in general, and in particular, mathematics has been largely ignored or left to chance. Many reasons have been advanced in defense of the traditional position. Some of these are:
(1) The child does not have the neurological equipment to enable him to sit for a considerable length of time nor to focus his attention.

(2) The time and energy expended will be much less if attempts at formal learning are delayed until entry into the schools.

(3) Motor skills are a prerequisite to early cognitive skills, therefore, the emphasis at the preschool stage should be on motor skills.

(4) The stress created by programs of learning will stifle the child's personality, creativity, and social-awareness.

Many dangers are inherent in attempts to foster cognition at the preschool level. At the same time, every potential for the early cultivation of cognitive growth of the preschool child must be developed. William Fowler states:

Much if not most of the energy in child psychology and development in late years has been concentrated on the child's personality, perceptual motor, and socioemotional functioning and development. Originating primarily as a reaction to historically inadequate and stringent methods, fears have generalized to encompass early cognitive learning per se as intrinsically hazardous to development . . . . In harking constantly to the dangers of premature cognitive training, the image of the "happy", socially adjusted child has tended to expunge the image of the thoughtful and intellectually educated child.10

Some attempts have been made to incorporate language development and science in nursery school curriculums; however, the planned teaching of mathematical concepts has been practically non-existent. Since Sputnik, many attempts to change the sequence and content of the mathematics curriculum at various

10 Fowler, op. cit., p. 145.
grade levels have been made. These innovations originally were attempted at some intermediate point, such as the ninth grade, and after a brief trial run, the materials were extended toward graduate school in one direction and first grade in the other. Very little has been done at the kindergarten or preschool level. A brief review of the literature reveals few attempts to build a planned program from the preschool stage upward. Starting from the nursery school seems the more logical approach to conceptual learning. Clearly, more research is needed at the formative stages in a child's development. Once more insight into the learning process at this level has been gained, a sounder curricular structure in mathematics can be built at each succeeding developmental level.

Irrelevant and confusing number concepts abound in nursery rhymes, counting procedures, and in children's books that perpetuate difficulties many have with mathematical concepts. For example, the reciting of the numerals "one, two, three . . ." at an early age is at best the memorization of nonsense syllables and at worst a hindrance to the learning of basic mathematics.

Traditional nursery school teachers say they 'prepare' an environment in which the young child can have direct contact with persons and things. In this way the child has sensory experiences with size, shape, proportion and number which provides the basis for later mathematical learning. This approach, although not totally without merit, has one
major deficit. No sequence of learning experience is visible. The child haphazardly encounters learning situations. He may or may not be ready for and profit from his encounters. Little attempt is made to meaningfully match the child and the experience. Arthur Jensen states:

For these children to benefit from nursery education . . . a part of their time in the nursery school must be carefully planned so that it will include many kinds of experiences that will aid the child in developing through the various stages of learning ability. It cannot be a passive experience. It involves mainly the coordination of verbal behavior with other sensorial-motor experience. In essence, the child must see, hear, say, and do all more or less at the same time.11

With the passage of the ESEA of 1965, there has been increasing interest in and emphasis on the importance of the preschool years. Funding for projects has been aimed primarily at a certain economic group—the disadvantaged. Therefore, most of the early work has been with this group. The past few years have seen the introduction of Project Head Start, a resurgence of Montessori schools, increased interest in Piaget's work, and an increased interest in the preschooler by scholars such as Bruner, Suppes, and Gagné, to name a few.

Hopefully, this pilot study will provide a base for continuing research in mathematics education at the preschool level. If the recommended program is accepted and instituted by The Ohio State University nursery school, there will be many areas open to this researcher and others to investigate.

Design of the Study

This investigation was conducted during the summer quarter, 1968, in The Ohio State University Home Economics Child Development Laboratory. Permission to do the pilot study was granted by the director of the laboratory school located in Campbell Hall on The Ohio State University campus. A pre-pilot study in the laboratory school was made by Professor Nathan Lazar.

To investigate how a mathematics curriculum can be incorporated effectively into the framework of a laboratory nursery school, the procedures listed below were followed:

1. Systematic observations were made in the nursery school program to analyze what was being done. Three facets of the program scrutinized were:
   a. Adult-directed activities
   b. Routine activities
   c. Free play situations

2. Direct observations were made of the children to identify some of the characteristic concepts they used in the three facets, above.

3. A search was made of the literature in the field of mathematics education and developmental psychology to identify what mathematical concepts preschool children should know.

4. A conceptual framework was formulated to be used in designing preschool mathematics experiences.

5. A pilot mathematics curriculum for preschoolers was projected with illustrative procedures and materials.

A block of time at the outset of the study was used for the observations cited in (1) and (2). Anecdotal records of what was being done with mathematical concepts were kept. The record included the status of the persons involved (i.e.,
teacher, student teacher, student), the age of the child and what transpired. The record also included instances where mathematical concepts could have been utilized and for some reason were not.

The insights gained from the observation period coupled with the search for mathematical understandings of preschool children gleaned from mathematics education and developmental psychology served as guides in developing a conceptual framework for preschool mathematics. After step four had been accomplished, a pilot mathematics curriculum was projected. The mathematics curriculum is designed to fit into the existing structure of the nursery school. In one sense, it will serve as a curriculum resource guide for further program development and field testing.

Limitations of the Study

Following are the limitations of the study:

1. The sample used was composed of middle class children in a university-operated preschool.
2. The sample population was small.
3. The complete study was performed over one quarter (ten weeks), and comparisons with performances in other quarters were not made.
4. Since the children were not in a single classroom unit but were performing in varied parts of the school, observations, of necessity, were only random.
5. Clues given verbally or through movement by young children are many times difficult to see and to interpret. Direct observation is subject to observer distortion and misinterpretation.

Definition of Terms

The following definitions of terms were used in this study:

1. Disadvantaged. A person deprived of distinctive stimulation.12

2. Incidental learning. Learning that is acquired through chance as the situation presents itself in consonance with other planned learnings.


4. Kindergarten. The unit of the school which enrolls five-year-olds on a regular basis for a year prior to the first grade.13

5. Nursery. A school enrolling children three and four years of age on a regular basis prior to kindergarten which provides a continuous educational program under professionally qualified teachers in cooperation with parents.14

6. **Preschool.** Although it is definitely a misnomer, the term preschool will be used to encompass the nursery and kindergarten.

7. **Rational counting.** Enumerating with meaning.

8. **Rote counting.** To name consecutive integers without relating to a concrete experience.

**Organization of the Dissertation**

Included in Chapter I is a brief introduction to the study, the origin, importance, and limitations of the study, as well as the design of the study and definition of terms. Chapter II presents studies related to: (1) the history of the nursery school, (2) nursery school education, and (3) mathematics education. In Chapter III, an analysis is made of the mathematical concepts in use in the nursery school. Chapter IV contains a conceptual framework for the development of curricular material and delineates illustrative curricular material for use in a pilot preschool mathematics program. Presented in the final chapter are the summary, conclusions, implications, and recommendations for further study.
CHAPTER II

REVIEW OF THE LITERATURE

The review of the literature is divided into five major sections. The first is concerned with the history of the nursery school; the second reviews studies pertaining to child development; the third reviews current preschool mathematics programs; the fourth reviews mathematics content of nursery school texts; and the fifth reviews studies delineating the concepts possessed by the preschooler.

History of the Nursery School

To study the mathematical concepts in current use in the nursery school and to make curricular recommendations it is first necessary to trace the history of the nursery school and the position that mathematics has occupied in the curriculum.

John Amos Comenius (1592-1670) is generally recognized as one of the earliest educators concerned with the education of young children. In School of Infancy, published in 1633, he presented a course of study to be used as a textbook and guide for the child six years of age and younger. The emphasis

\[1\text{John Comenius, The School of Infancy (New York: University of North Carolina Press, 1956).}\]
to be placed on mathematics is illustrated with the following passages.

The child's first instruction in Chronology will be to know what is an hour, a day, a week, a month, a year, what is spring, what is summer.

The foundations of Arithmetic will be to know that something is much or little, to be able to count to twenty, or even all the way to sixty ... and to understand that three are more than two, that three and one make four, and other such simple matters.

Geometry's beginnings will be to know what is small or large, short or long, narrow or broad, thin or thick; likewise what is a span, an ell, and a fathom, and other little measures.²

In *The Great Didactic*,³ written in 1657, he advocated the Mother School for children six years and under. His plan was for every home to be a Mother's School. In 1658 he published the first picture book for children titled *Orbus Pictus*⁴ which illustrated numerous objects and their names. Drawing from his own school experiences Comenius had a great dislike for the headlong rush into the abstract and firmly believed in more use of the concrete for all ages. He ushered in a period in which education was to be warm, friendly, and interesting to the students rather than cold and austere.

Jean Jacques Rousseau (1712-1778), in his book *Emile*,⁵ advocated a preschool experience in which the child is left to his own innate curiosity to wander and to gather up knowledge.

in his quest the child is to be accompanied by a tutor who will merely answer any spontaneous question the child may have. The tutor is not to initiate the learning experience but rather to act as respondent to the child.

Johann Friedrich Oberlin (1740-1826) opened the first infant school at Walbach, France in 1769. Oberlin started this school to enable parents to work without having to spend time on the care of their children. His school was not merely custodial in nature but rather provided instruction in practical needs of the children.

Robert Owen (1771-1858) started the first infant school in Great Britain in 1816. This school was in conjunction with a cotton mill that he owned located at New Lanark. Owen gave the schools his own personal attention in an attempt to raise the level of living in the community. The children had no formal lessons and their main activity was play. In 1826 Owen was instrumental in establishing a new community in New Harmony, Indiana and thereby introducing the first nursery school into the United States. This experiment was short-lived though and by 1828 Owen had lost approximately four-fifths of his wealth and returned to England.

Friedrich Froebel (1782-1852) founded the first kindergarten in 1837. He was convinced that children could learn through play and without formal instruction. Like so many educators before him and since he advanced the idea that the
most needed educational reform was that of the early childhood years.

The young growing human being should, therefore, be trained early for outer work; for creative and productive activity. Play, building, modeling are the first tender blossoms of youth.

Each child in the Froebelian school was given a series of 'gifts'. These gifts were designed with the specific purpose to teach the child the nature of form, number, and measurement. The first 'gift' consisted of six soft colored balls; the second of a cube, a cylinder, and sphere; and the third was a number of rectangular sections which formed a cube.

By manipulating these materials in prescribed fashion, the child learned to count, combine, divide, make fractions out of wholes, arrange in order, measure, and analyze.

Mrs. Carl Schurz, a pupil of Froebel's, established the first kindergarten in the United States at Watertown, Wisconsin, in 1855. This was a German speaking school and five years later Miss Elizabeth Peabody established the first English speaking kindergarten in the United States at Boston, Massachusetts.

Meanwhile in England other infant schools based on Owen's ideas flourished and in 1870 became an integral part of the state system of education. As the years passed more of a

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8 Loc. cit.
scholastic tradition was adopted in which little provision was made for physical activity, sleep, and play. Instead the children were required to sit still in large galleries with their arms folded and without conversing with their neighbors. They recited lessons in which pictures and museum specimens took the place of living plants and animals. Considerable time was spent on the three r's and needlework.\(^9\)

In 1873, Susan Blow and William Harris started the first public school kindergarten in the United States. The program was based on the mother-play sequence initiated by Froebel. The children were given gifts that had a striking similarity to Froebel's. Gift one was a ball on a string. The teachers used this to perform a prescribed sequence of operations with the child. This would include directions to move the ball 'up and down,' 'back and forth,' 'round and round,' and 'side to side' until the teacher thought the child was able to evoke the proper response to the stimulus. The second gift included a cube, cylinder, and a sphere; the third gift was the first in a series of four 'building gifts,' a two-inch wooden cube cut into eight one-inch cubes; the fourth a two-inch cube cut into eight rectangular parallelopipeds; the fifth a three-inch wooden cube cut into 27 one-inch cubes; and the sixth a three-inch wooden cube cut into rectangular parallelopipeds. The children also were instructed with lentils

placed in prescribed patterns to acquaint them with lines, triangles, rectangles, and circles. Designs used in sewing were geometric in form to further acquaint them with geometry.10

Maria Montessori (1870-1952), a physician, became interested in the educational problems of the mentally defective child and the socially deprived child. She established the casa dei bambini in connection with a tenement improvement project in the city of Rome in 1907. She was greatly influenced by the work of Rousseau. In her program, the teacher stayed mainly in the background and the student interacted with materials. Self-education was a keynote, and the material was not designed for the children to interact with each other.

... in one important aspect her method differed from the Froebelians'; instead of placing the responsibility for directing the work upon a teacher, the material itself was supposed to be corrective. Each child was permitted to choose what material he preferred to work with, but he must work with it in such a way that he correctly performed the task set by the material. For instance, one unit of the didactic apparatus is the 'Montessori tower', a set of nine graduated cubes which the child was expected to pile correctly, with the largest at the bottom and the smallest at the top.11

With this system no variance in performance was tolerated. If the child did not use the materials in the prescribed form he was immediately removed from the area and directed to a different task.

Although Montessori started with the atypical child, she soon saw that her system also had potential for the normal

11Ibid., p. 41.
child. The Montessori System was introduced into the United States early in the Twentieth Century but it failed to flourish. During the past decade the system has been revived and renovated and has gained some popularity in the United States.

In 1908, Grace Owen and Margaret McMillan\textsuperscript{12} of England initiated a nursery school program very similar to the one founded by Robert Owen. This time the program started a general rise that has led to the position now held by the nursery school in the educational setup. Margaret McMillan saw the nursery school as an extension of home life and she maintained that the objective of the nursery school was not for formal education for the child but rather for the mental, physical, and social growth of the child. She initiated a three-year training course for nursery teachers and many of the early pioneers in nursery education in the United States were products of this school. In 1918 the nursery school was recognized as a part of the national educational system in Great Britain by the Fisher Education Act.\textsuperscript{13} This was the first official national recognition given to a nursery program. Standards for the administration of a nursery school were defined by law.

Despite Robert Owen's venture at New Harmony, Indiana in the Nineteenth Century, the early 1920's are generally


\textsuperscript{13} Catherine Landreth, \textit{Education of the Young Child} (New York: John wiley & Sons, Inc., 1942), p. 5.
credited with the beginnings of nursery education in the United States. The first schools were started under the auspices of the colleges and universities as the outcome of interest that may be traced to a variety of sources. First there was a scientific interest in early childhood from the applications of the fields of medicine and psychology. Second the fields of child guidance and psychiatry were interested in experimentation. Third there was an interest in improving the educational programs of day nurseries that were already established to care for the children of working mothers. Lastly there was a rapidly expanding background of educational theory. That the varied primary objectives of nursery schools throughout its history include custodial care, parent and teacher education, research, and teacher employment poignantly underlines a basic deficiency in original emphasis.

... throughout the history of nursery education in the United States, the primary objective of the nursery schools has often been the welfare of persons other than the children.14

There were only three nursery schools in the United States in the early 1920's. These were the Iowa Child Welfare Research Station established by a state legislative grant in 1917, the Ruggles Street Nursery of Boston in 1919, and the Merrill-Palmer School of Motherhood and Home Training of Detroit in 1920. It was at Merrill-Palmer in 1922 that a nursery school demonstration center for student study and practice was opened.

The next schools to open were the Child Welfare Institute of Teacher's College, Columbia University, The Fels Institute at Antioch College, the Yale Psycho Clinic, and Cornell University. Private nursery schools were opened throughout the country with an occasional public school system operating one or more laboratories for child study or for parent education purposes. In 1930 the number of nursery schools in the United States had grown to approximately 500.

By the 1930's, the child development point of view had evolved. This has as its basis two tenets—(1) the child has a right to an education and (2) he must be accepted where he is.

In 1933, provision was made for nursery centers as part of the Federal Emergency Relief Administration which later became the WPA. This program was initiated as a means of securing employment for teachers and to provide care for children. This was the first federally connected program for nursery education and by 1942 approximately 2100 of the country's 3000 nursery schools were WPA sponsored. During the following year the number of nursery schools declined rapidly due to the involvement of the United States in World War II. However, due to the rising number of women defense workers and the difficulties of adequate child care, the government enacted the Lanham Act in 1943. This program of child care continued until 1946. When federal money was withdrawn many nurseries closed. Continuation of many of the programs was contingent on receipt of state and local funds.
Starting in the late 50's was an increase in literature pertaining to the preschooler. Jean Piaget,15 Benjamin Bloom,16 Jerome Bruner,17 and J. McVicker Hunt18 are notable among the writers of this period.

In 1960, the Golden Anniversary White House Conference on Children and Youth recommended:

That kindergarten be made an integral part of the tax-supported public school system in all communities and that State departments of education be authorized to extend public education to include nursery schools.19

In 1961, the Council of Chief State School Officers followed suit by voting to:

extend to all children three to six years of age on a voluntary basis equal opportunities for nursery school and kindergarten.20

In 1966 the Educational Policies Commission of the National Education Association stated:

Beginning at what age should the opportunity for education be offered at public expense? . . . the Educational Policies Commission recommends that it begin at the age of four.21

16 Bloom, op. cit.
18 Hunt, op. cit.
20 Council of Chief State School Officers, op. cit., p. 11.
Other important educational bodies that have also endorsed kindergarten and nursery schools are the Association for Childhood Education International, National Society for the Study of Education, and the United States Office of Education. These groups have recommended the inclusion of nursery schools and kindergartens in the regular State program of approval and accreditation.

The growing national concern by parents, teachers, administrators, and other interested groups in a sound educational program for children under six years of age has given impetus to more and more federal support. From the Economic Opportunity Act of 1964, the Elementary and Secondary Education Act of 1965, and other minor legislation there have evolved numerous summer, year round, and short term programs for the nursery-age child. These programs are compensatory in nature and are mainly designed to overcome experiential deprivation. The main emphasis has been on programs for school readiness.

**Child Development Research**

**Critical period**

There are numerous studies concerning critical periods in animals and human beings. These critical periods, it is conjectured, are the prime time for the acquisition of learning and in some instances if this time is missed, learning at a later time is thought impossible.

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Critical periods for animals

Several studies point out that animals handled by humans in their early stages of growth are more capable in learning situations than animals left in their cages. Levine and Denenberg and Karas found that rats handled in the period between birth and weaning learned more rapidly than their siblings who were not touched. Also Hebb and Bennett, Diamond, Krech, and Rosenzweig testing rats and Thompson and Heron testing dogs found that the animals raised as pets learned mazes as adults more readily than did their siblings.

Lorenz found that greylag geese formed a social bond with the first moving object that they saw. In this dramatic experiment the geese would follow a moving box in preference to others of its own kind. In a similar experiment Hess used


decoy ducks with human renditions of the duck call to test the critical period for 'imprinting'. When tested at later periods with decoy ducks using real duck calls it was noted that the experimental ducks would respond to the imitation calls over the real calls and further that the maximum imprinting took place 13 to 16 hours after birth. Each duck was given four trials to make a choice of the two models and although 50% of those imprinted in the 13-16 hour group scored 100% on the test none in the over 20-hour group scored 100%. Next the ducks that were in the older range were given a more concentrated learning period and although their imprinting scores increased slightly there were still none that scored 100%. Two conclusions that may be drawn from this that may also have some relevance for the critical periods of human learning are (1) there are critical periods and (2) remedial learning is possible outside this critical time but at an increase in time and effort and at a decrease in output.

Hunt\(^{30}\) in an early experiment concerning deprivation of animals found that starvation of rats on the 24th day left behavior traits that were readily discernible in adult life whereas starvation at 36 hours did not.

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Scott found that lambs that were taken at birth and raised on the bottle formed relationships with people rather than with sheep.

**Critical periods for children**

The influence of early experience on cognitive development of human beings is currently being researched by many individuals. Bloom hypothesized that a growth variable is most subject to environmental influence during the time that the variable is in the most rapid period of change. Orpet and Meyers are attempting to identify the structure of the intellect of the child under six utilizing Guilford's structure-of-intellect model.

In one of the earlier experiments concerning deprivation Skeels and Fillmore found that older children in seriously underprivileged homes had lower I. Q.'s than younger children in these same homes. Thompson and Melzack, Levine, and a

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32 Bloom, op. cit.


more recent work by Skeels,\textsuperscript{37} have produced more evidence that early deprivation of learning experiences very probably produce irreversible defects in cognitive development. These studies and others point out that instead of remedial programs that are at best only partially workable it is imperative to meet the problem head-on at the point where it is most receptive to outside stimulation, that is, at the critical period. Deprivation literally means that the organism has been denied certain stimuli at particular periods in its existence. There may be physical deprivation such as handling by the mother, exploratory moves such as crawling, feeling, etc., or social deprivation concerning personal contacts, or cognitive deprivation pertaining to materials, language, and other environmental factors. John Bowlby points out that

Maternal deprivation has a differential effect on different processes; most vulnerable seems to be certain intellectual processes, especially language and abstraction.\textsuperscript{38}

**Language**

Language is extremely important for the preschool child. It is through his speech that he is able to communicate his desires and feelings and to assimilate the world about him in some workable conceptual system. It serves as a vehicle in his relations with people and things, and the feedback from his


language ventures determines to a great extent his personality, thought, and capabilities. Language expedites the sorting, categorizing, and storing of concrete experiences into abstract entities. Through motor explorations and sensory experiences the child gains knowledge of similarities and differences of objects, notions of size, shape, and distance, and reactions of persons about him. Orem points out the importance of language.

Human thought cannot grow without language. Not until the child acquires names for things and actions and relationships can he begin to process his world. With language his world can become a part of him, extending beyond the immediate present; he can learn to talk about it, think about it, manipulate it, even when it is not physically there . . . Optimal development of his senses occur within an environment that is rich in sensory and motor experiences as well as in warm, interpersonal communication.39

Marge 40 found that ignoring or rejecting patterns of parents were related to poor language and conceptual ability of the children. In investigating the language patterns of preschoolers Van Alstyne 41 found that slightly over half the time the children were together they tended not to talk with other children while playing with materials. They tended to talk more while working with blocks, crayons, and clay, and

also while engaged in playing with dolls. They talked less while painting or using scissors or books. Hockett con­cluded from his study that older children are the most im­portant environmental force in shaping the younger child's speech habits. Smith found that when children talk to adults their language patterns are more advanced than when they are conversing with other children. These studies sug­gest the possibility of multigrading as well as increasing the adult-child language contacts. The more language the child hears enhances the possibility for developing compe­tency in language which in turn facilitates cognition. More important than quantity of speech is quality of speech, timing (e.g., Hunt's 'match'), and motivational and emotional value.

Martin Deutsch, one of the leaders in the field of the disadvantaged, emphasizes the language development of the preschooler.

Language is probably the most important area for the later development of conceptual systems. If a child is to develop the capabilities for organizing and categorizing concepts, the availability of a wide range of appropriate vocabulary, of appropriate context relationships becomes essential. Sometimes the most productive training can be done in the third and fourth and fifth years of life in the language area.

Teaching strategies

Anderson's work with the dominative and integrative behaviors of adults and the subsequent behavior of the children give direction to certain teaching strategies for the preschooler. In essence Anderson's theory gives rise to the concept of 'circular behavior,' that is, domination breeds domination and integration breeds integration. He also found that teachers had higher mean frequencies of dominative than integrative behaviors and that two out of three teacher initiated contacts were dominative while six out of seven child initiated contacts were integrative.

Rosenblith found that having a model was more effective in helping a child to learn than was simple opportunity for practice. Bandura and Huston found that children who experienced a warm relationship with the adult model displayed more imitative behavior than the child who experienced a cold relationship. In the warm relationship the irrelevant behaviors of the experimenter were imitated. Hartup and Keller reported

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that continuous nurturance was less effective than nurturance followed by nurturance-withdrawal for the performance of girls but with boys there was no clear difference. Gewirtz pointed out in his research that children seek attention more times when the adult is not readily available than when he is.

In a classic experiment concerning systematic guidance versus an incidental approach Thompson planned two different curricula for children. His group A was designed for the teacher to initiate a minimum of contacts with the children whereas in group B the teacher was to be actively involved. The results, favoring group B, demonstrated that B (1) was more constructive, (2) was more ascendant, (3) showed more participation, and (4) showed more leadership. In addition, group B was lower than group A in destructive behavior.

Kol'tsova conducted an investigation using 20-month old children. He used two groups in which the first was presented a doll 1500 times accompanied by one of three statements concerning the doll. A second group was also shown the doll 1500 times but was given 30 statements containing the word doll. Then both groups were tested in selecting a doll from a group of toys. The group that had the more varied experience was


superior in accomplishing this task. This experiment gives impetus to a statement by J. McVicker Hunt in *Intelligence and Experience*:

... the more new things a child has seen and heard, the more things he is interested in seeing and hearing. Moreover, the more variation in reality with which he has coped, the greater is his capacity for coping.\(^{52}\)

In the classic preschool debate concerning incidental versus systematic learning there seems to be a dearth of supportive research on both sides. Suppes and Ginsburg\(^{53}\) did a series of experiments concerning mathematical concept formation in young children and concluded that incidental learning does not seem to be an effective method for learning. Bereiter and Engelmann\(^{54}\) seemingly dismiss the problem with no comment by setting forth a highly systematic program for preschoolers.

Comenius and the Nuffield Project, though separated by many years in the history of nursery education, make similar pleas for a teaching strategy that is cognizant of the child. Comenius says:

Parents must take care as to the method adopted with infants in these several things. Instruction should not be apportioned precisely to certain years and months (as afterwards in other schools) but generally only.\(^{55}\)

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\(^{54}\) Bereiter, *op. cit.*

\(^{55}\) Comenius, *School of Infancy, op. cit.*, p. 74.
In like manner the Nuffield Project in a publication entitled *Pictorial Representation* states:

"It is very important that the teacher should be aware of the various stages of presentation. The correct moment for taking the child into the next stage will be decided by the observant teacher. It is impossible to lay down any defined programme for a child of any given age."

Research on teaching strategies should give direction to the preschool teacher. Moustakas and Berson used a questionnaire in an attempt to assess the teacher and her role in the nursery.

The most prevalent pattern of response, not surprisingly, was child-centered theory and authoritarian practice. Apparently, a considerable gap exists between what nursery school teachers believe and what they report themselves as doing. Presumably, a study of what they actually are doing would show the gap to be wider still.

**Current Preschool Mathematics Programs**

One interesting foreign program is the Nuffield Mathematics Teaching Project, directed by Geoffrey Matthews in London, England. The main emphasis in this program is for the child to learn by discovery. The Nuffield approach uses the 'family plan' wherein one classroom contains children of varying ages. The classroom is arranged comparable to our own science laboratories and the children work in small groups on projects.

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outlined on assignment cards. These projects involve the use of physical materials. The teacher in this setup circulates from group to group giving aid whenever necessary. The main emphasis given by Matthews' project group is the formulation of teachers' guides. The guides concern computation and structure, shape and size, and graphs leading to algebra.

There are teaching centers set up throughout England where the teachers meet to learn more mathematical content, to examine the teaching guides, and to discuss how the guides are to be used. Much of the Nuffield Program is based on the work of Jean Piaget. As a result of the type of program being conducted traditional tests are not feasible and currently a group from the Institut des Sciences de l' Education of Geneva, with Piaget and Inhelder as consultants, are writing a series of 'Check-up Guides.'

Some of the tasks that a preschool child does in the Nuffield Program are:

1. Using a balance beam.

2. Using yarn to connect a picture of the child to a toy or other item that the child owns.

3. Pouring water into jars.

4. Sorting physical objects such as buttons or plastic toys into various groups. (For example, they may be sorted as to size, color, texture, or design.) In utilizing the work of Piaget and others the Nuffield Program attempts to individualize the curriculum of all the children at all age levels.
At Webster College in St. Louis, Donald Bushell and Jo Maiorano are experimenting with computer-assisted learning. The computer presents to the child a large picture which is the problem and the child has three smaller pictures from which he is to choose the correct answer. The child merely has to push on the picture that he believes is the correct response. An outstanding feature of the computer is that it records every student response as well as the time involved in making that response. The student must also press the large screen to advance the next question and this time is also recorded. This gives the researcher or teacher a chance to analyze student difficulties. The Bushell-Maiorano program bases the child's first conception of number on his perceptual discrimination ability rather than on the notion of sets which is being advocated by many preschool theorists.

Two programs that conceptualize the role of mathematics in the preschool on opposing ends of the spectrum of involvement are A. S. Neill's approach and Bereiter and Engelmann's approach. Neill in his book *Summerhill* does not advocate mathematics for the young child. He says:

> My case against mathematics is that the study is too abstract for children. Nearly every child hates

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mathematics. Though every boy understands two apples, few boys can understand \( x \) apples.\(^{62}\)

Bereiter and Engelmann on the other hand advocate a rigorous program in which the child learns mathematics by rote, writes the symbols, and parrots adult language use of mathematical terms before any concrete experiences.

**Nursery School Textbooks**

Thirteen of the most widely used textbooks in the colleges and universities for preschool teachers' programs were studied for mathematical content. It was felt that the emphasis given in the textbooks might give some insight into the emphasis placed on mathematics in the preschool by the eminent authors and leaders in the field. This in turn would give some knowledge concerning the amount of mathematical content introduced in preschools by students using the texts.

Of the thirteen books Allen and Campbell,\(^ {63}\), Christianson,\(^ {64}\) Kellogg,\(^ {65}\) Leavitt,\(^ {66}\) Moore,\(^ {67}\) Moustakas,\(^ {68}\)

\(^{62}\)Neill, op. cit., p. 378.


Murphy, and Rudolph give no indication of a mathematical curriculum. The majority of these authors gave a chapter or a portion of a chapter to science but mathematics was not included in this section.

Landreth and Read devoted two-thirds of a page to arithmetic under the title "Numbers and Simple Arithmetic Processes." One example of two, three, and four items was given. Also the words 'lots', 'few', 'halve', and 'divide' were used. The latter two were used to instruct the children to 'Halve your clay' or 'Divide your clay into three.' From these few examples the authors proceed to say "Such simple experiences with addition, subtraction, division, and fractions are the young child's introduction to arithmetic processes."

Katherine Read, whose works are the most widely used in nursery education, dispenses with preschool mathematics in one short paragraph. She stresses the use of the words 'big', 'little', 'heavy', and 'how many'. She also advocates the use of measuring cups and the ruler.

Ruth Updegraff has one of the few pre-1960 texts containing mathematics but the content seems very difficult for

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71 Landreth, op. cit.
72 Ibid., p. 155.
this age level. For example, from a first experience with recognition of coins she leads the children to monetary equality. From store experiences she expects the child to understand that 20 nickels equal one dollar, 10 dimes equal one dollar, 4 quarters equal one dollar, 2 nickels equal a dime, etc.

Clarice Wills\(^75\) devotes two pages to mathematical concepts. She would have the children count other children, milk containers, windows, crayons, and blocks. She also would introduce the children to large, small, circle, square, triangle, cube, under, over, far, near, high, low, more, less, little, much, all, and some. She also states that the child should know that the clock measures time and be able to recognize money.

Sarah Leeper,\(^76\) in a 1968 book called *Good Schools for Young Children* gives some indication of the future of mathematics in the preschool. She and the co-authors devote one full chapter to mathematics.

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**Mathematical Concepts Possessed by the Preschooler**

A search of the literature produced many articles and books dealing with the mathematical concepts of the kindergarten child but very few pertaining to the nursery school.


Research concerning the cognitive growth of the preschooler is very sketchy and in particular research pertaining to their mathematical learning is almost non-existent. Following the revival of Piaget's research there have been numerous experiments replicating his work but these have merely perpetuated his theories concerning the stages of children's learning. There have been few attempts to tamper with the sequential learning patterns of children to see whether the stages may be advantageously hastened. In general the studies concerned with the mathematical concepts possessed by preschoolers are consistent. At the same time there is a dearth of information concerning how the preschooler obtains these concepts. Many of the researchers seem to pass it off as something that happens with the passage of time and this appears to be a poor answer to learning at this stage.

**Rote counting.**—Rote counting denotes the ability of the child to start with some natural number, usually one, and then repeat in serial order (either by ones, twos, or some other multiple) subsequent natural numbers with no reference to the counting of objects. Researchers have generally noted that rote counting precedes rational counting. That is to say that memorization of beginning counting words comes before the actual counting of things.
Bjonerud\textsuperscript{77} conducted a study of 100 beginning kindergarten students in 1957 and 27 more in a different locale in 1960 and found that the mean for rote counting was 19. Five of the 127 children counted to 100 or more. Twenty-five percent of the group were able to count to 100 by 10's. Brace and Nelson\textsuperscript{78} tested 124 children ranging in age from five years four months to six years five months and found that 28.4\% could count by 10's to 100. Buckingham and MacLatchey\textsuperscript{79} used 2670 children who were entering the first grade for their study. They found that approximately 90\% could count to ten, 60\% to twenty, 20\% to fifty, and 10\% to one hundred. Brownell\textsuperscript{80} likewise tested 631 entering first graders with the results that 52.3\% could count by ones past twenty, 62.8\% from fifteen to nineteen, 90.5\% from ten to fourteen, and 100\% from one to nine. In a similar study by Lehew\textsuperscript{81} in a Head Start program in California using 30 five-year-olds 20\% could count by ones past twenty, 33\% from sixteen to nineteen, 57\% from eleven to


\textsuperscript{80}Brownell, \textit{op. cit.}

fifteen, and 100% from one to nine. Of the 20 four-year-olds tested Leheuw found that 15% could count by ones past twenty, 20% from sixteen to nineteen, 45% from eleven to fifteen, and 100% from one to nine. McDowell gave fourteen three-year-olds, fourteen four-year-olds, and thirty five-year-olds instructions to count as far as they could beginning with one. The results are shown in Table 1.

<table>
<thead>
<tr>
<th>Number Attained</th>
<th>Age of Child</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3</td>
</tr>
<tr>
<td>0-5</td>
<td>14</td>
</tr>
<tr>
<td>6-10</td>
<td></td>
</tr>
<tr>
<td>11-20</td>
<td>3</td>
</tr>
<tr>
<td>21-30</td>
<td>3</td>
</tr>
<tr>
<td>31-40</td>
<td></td>
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<tr>
<td>41-100</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>14</td>
</tr>
</tbody>
</table>

Rational counting.—The last numeral named in rationally counting a set is called the cardinality of the set. In essence this numeral answers the question of 'how many' in the set.

Mathematically, the process of enumeration is fundamental to arithmetic. Psychologically, it is a sensorimotor chain controlled at every stage by a shifting perceptual organization. Enumeration requires

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a chant (1, 2, 3, . . . ), a shifting indicator response (pointing), and a perceptual grouping of objects into those already counted and those still ahead. 83

Many parents have their children parody the sounds necessary for rote counting and their ability to rationally count is much behind their rote ability. Brace and Nelson state that "Although the large majority of the children could count beyond twenty, they showed an almost complete lack of knowledge of the construction of our system of numeration." 84 In Buckingham and MacLatchey's 85 study 97% could enumerate to five, 90% to ten, 70% to fifteen, and 58% to twenty. Brownell's 86 entering first graders were asked to count up to ten objects. All were able to count five or fewer objects, 94.6% six, 92.5% seven, 91.7% eight, 89.2% nine, and 85.7% ten. McDowell 87 used five tractors and asked the children to count them. None of the three-year-olds could accomplish this task, but ten of the fourteen four-year-olds and twenty-five out of thirty five-year-olds could. Sister Josephine 88 tested thirty children who were four and five years old. She used two different levels of enumeration in her testing. In the first she asked the subjects

84 Brace and Nelson, op. cit., p. 131.
85 Buckingham and MacLatchey, op. cit.
86 Brownell, op. cit.
87 McDowell, op. cit.
to "show me ____ children." In response to the command 100% identified two children, 83% three, 100% five, 90% seven, and 83% eight. In another part of her test she asked the children to "Show me pictures with _______." When the sentence was completed with the words "two pencils" there was a 93% correct response; for three cars it was 90%; four balloons 83%; five blocks, 83%; and seven children, 66%.

McDowell also presented the children with twelve cakes with birthday candles that represented the ages from one through twelve. The subjects were then asked to select the cake that had the same number of candles as their age. Sister Josephina did the same experiment and both researchers found that every child was able to select the proper cake. It is interesting to note in Lehew's testing that the children's ability to rationally count exceeds their rote counting ability. Of the four-year-olds 20% counted twenty, 25% sixteen to nineteen, 55% eleven to fifteen, and 80% to five. Of the five-year-olds 23% counted twenty, 30% sixteen to nineteen, 57% eleven to fifteen, and 90% to five.

Sequences.—A sequence is defined for the purpose of this paper as equivalent to an arithmetic progression of natural numbers. That is, there is a beginning numeral and a specified difference between each of the members of the set.

89McDowell, op. cit.
90Sister Josephina, op. cit.
91Lehew, op. cit.
Bjonerud\textsuperscript{92} found that less than 10\% of the five-year-olds displayed an understanding of a sequence of odd numbers and less than 20\% for even. McDowell\textsuperscript{93} asked what number came after four and found that no three-year-old could tell, but that 50\% of the fours and 50\% of the fives could. In asking what number came before eight again none of the threes could tell whereas 10\% of the fours and 20\% of the fives could. Sister Josephina\textsuperscript{94} shows much higher percentages on similar types of sequences. She found that 97\% could tell what was next in the sequence 1-2-3-4-5, 50\% for 1-3-5-7, 40\% for 5-10-15, and 33\% for 2-4-6. Brace and Nelson\textsuperscript{95} reported that 72.6\% did not respond to counting by 5's or 10's.

**Ordinal.** An ordinal number indicates the position of an object or a number. It fundamentally answers the question "Which one?" An ordinal number is more generally recognized when used as first, second, third, etc. To say that a person is sitting in seat four is an example of ordinal use. The number on the jersey of a football player is considered neither cardinal nor ordinal.

Only two studies were found testing the ordinal concept and neither of these reported the number of objects used. This would have some relevance in locating the middle object since there is a wide variance in locating the middle object of three

\begin{itemize}
  \item \textsuperscript{92}Bjonerud, op. cit.
  \item \textsuperscript{93}McDowell, op. cit.
  \item \textsuperscript{94}Sister Josephina, op. cit.
  \item \textsuperscript{95}Brace and Nelson, op. cit.
\end{itemize}
objects as opposed to fifteen. Bjonerud\textsuperscript{96} found that 95% understood first, 70% middle and last, and 50% second. Sister Josephina's\textsuperscript{97} students scored approximately the same as Bjonerud's. In response to the command "Show me the block that is _____." all were able to do first, 90% last, 86% middle, 66% second from right, and 66% fourth.

**Subitizing.**—Subitizing refers to the visual discrimination of a group of objects without enumerating. For example, a person playing dominoes does not need to count the dots before making a play but may merely recognize the configuration. Two primary sources were used to gain information. Beckwith reported on three studies. Bourdon (1908) "judged that 7 was the limit of the number that could be accurately ascertained at one glance." "Von Szeliski (1924) found that fields up to 6 dots of figures could be perceived without eye movement." Jensen, Reese, and Reese (1950) "concluded that subjects subitize up to 5 or 6 dots."\textsuperscript{98} Lowry, after a search of studies concerning subitizing in children, concludes that:

The number of objects in a group which children can perceive varies widely, and appears to depend on a number of factors which include age, previous experience with numbers, and the type of perceptual materials being used at the time. It would appear that a majority of children from four and one-half to

\begin{flushright}
\textsuperscript{96}Bjonerud, op. cit.
\textsuperscript{97}Sister Josephina, op. cit.
\textsuperscript{98}Beckwith, op. cit., p. 438.
\end{flushright}
six or seven years of age can perceive visually at least 4 objects when they are presented simultaneously in various arrangements.99

Inequalities.--In some situations it seems preferable to select like characteristics to categorize for more effective learning. In the preschool child's introduction to mathematical concepts learning may be enhanced by attention to the dissimilar aspects. For example, the child seems to grasp the concept of inequality much before he can grapple with equality. Equality, in most cases, requires a closer scrutiny and a more discerning eye than does inequality. For a child to say "I am as tall as John" is much more difficult to ascertain than to say that some adult is taller than he is.

McDowell100 had the children compare four cars to six cars and answer which was more. Seventy-one percent of the three-year-olds, 92% of the four-year-olds, and 96% of the five-year-olds were correct. When comparing ten, seven, and five houses for the most, 14% of the threes, 92% of the fours, and 96% of the fives responded accurately. Lehew's101 four- and five-year-olds scored lower than McDowell's. Using a set of yellow and a set of red wooden men Lehew asked "Which has more men in it?" Sixty-five percent of the fours and 77% of the fives answered correctly. Using a set of blue and a set of

100 McDowell, op. cit.
101 Lehew, op. cit.
yellow cars she asked "Which has the most cars?" Once more 65% of the fours and 77% of the fives responded correctly. Lehew did not specify the number of men or cars used for the questioning.

Brownell\textsuperscript{102} used pictures and asked the children to place a mark on the concept he was testing. He presented four dichotomies and obtained the following percentages of correct responses: largest 83.7%, smallest 79.2%, longest 93.9%, shortest 89.5%, most 97.0%, fewest 63.7%, more 83.1%, and less 45.0%. Bjonerud\textsuperscript{103} concluded that 80% of his group understood largest, smallest, tallest, longest, most, inside, and beside, while 50% knew shortest, few, and underneath.

Premeasurement.—The only premeasurement studies found limited themselves to the naming of measuring instruments. Bjonerud\textsuperscript{104} found that one-third of his five-year-olds could name the ruler and one-third the yardstick. In a similar test Davis\textsuperscript{105} found for his four-year-olds that none could name a ruler, 13% named a yardstick, and 9% could show an inch. For the fives 35% named a ruler, 10% a yardstick, and 41% showed an inch.

\textsuperscript{102}Brownell, \textit{op. cit.}
\textsuperscript{103}Bjonerud, \textit{op. cit.}
\textsuperscript{104}Ibid.
Prageometry.—Bjonerud 106 found that 91% recognized a circle and 76% a square.

Fractions.—Bjonerud's 107 students were tested on the recognition of one-half, one-third, and one-fourth of an item. Fifty percent recognized one-half, 89% one-third, and 66% one-fourth.

Time.—Fifty percent of the five-year-olds in Bjonerud's 108 study were able to tell time on the hour while only 30% could do the same on the half hour. O. L. Davis 109 conducted a study using 23 four-year-olds and 29 five-year-olds. When confronted with a clock showing 4, 6, 8, and 12 o'clock no four-year-old recognized the first time and only one recognized the other three. Of the fives only one recognized four o'clock, whereas 21% recognized six, 28% eight, and 52% twelve. Paul Spayde 110 conducted the same test for five-year-olds and came up with surprisingly similar results. He showed 14% for four o'clock, 35% for six, 47% for eight, and 52% for twelve.

Coins.—Bjonerud 111 found that 80% recognized a penny. In a strange finding 38% recognized a nickel and yet 39% knew

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106 Bjonerud, op. cit.
109 Davis, op. cit.
111 Bjonerud, op. cit.
that five pennies equalled a nickel. Fifteen percent knew the number of quarters in a dollar and 10% knew the number of half dollars in a dollar. McDowell\textsuperscript{112} placed six pennies on a table and asked the children to count them. Although no three-year-old could rote count to six in the test previously cited, two of these children were able to count out the six pennies. Of the four-year-olds ten of the fourteen counted correctly and of the fives there were twenty-two of the thirty. Sister Josephina\textsuperscript{113} found that 90% recognized a penny, 73% a dollar bill, 56% a dime, 53% a quarter, 50% a nickel, and 50% a half dollar. Davis\textsuperscript{114} found with the fours that 83% recognized a penny, 52% a dollar bill, 30% a nickel, 26% a dime, 4% a quarter, and none a half dollar. With the five-year-olds 97% recognized a penny, 79% a dollar bill, 79% a dime, 76% a nickel, 35% a quarter, and 31% a half dollar.

Vocabulary.—The vocabulary of the preschooler has been researched and analyzed as to order, magnitude, and type of mathematical words used. For example, Horn\textsuperscript{115} conducted a study of the one thousand three most used words by kindergarten children and arranged the mathematical terms in order. The twenty most used terms in order are one, some, little, all, big, not, no, two, just, more, any, about, three, another, something, something,

\textsuperscript{112}McDowell, op. cit.
\textsuperscript{113}Sister Josephina, op. cit.
\textsuperscript{114}Davis, op. cit.
once, four, first, again, and much. Johnston analyzed the mathematical vocabulary of four- and five-year-old children and found in her sample that they used number words 50% of the time, position words 30%, form words 10%, and measurement words 10%.

Piagetian research.—Piaget's research and replications of his research in the past few years constitute a voluminous amount of work. For this reason only a few of the studies will be discussed here. In Piaget's stages the period from 4 to 7 years of age is called the Intuitive Thought stage. This is a period of gradual coordination of representative relations and thus a growing conceptualization, but the child is still dominated by his perceptions. His conclusions are still at the mercy of 'centrings' which is the overemphasis of one element while others are relatively ignored. For example, a child may attend to length but not volume of a substance. Piaget, in checking for the conservation of number, has determined that up to five years of age there is none, from five to six there is a phenomenistic, unstable notion of conservation, and from six to seven and one-half years there appears a logical, axiomatic certainty of conservation. Invariance of the number of elements in a group means that regardless of the space occupied or how they are distributed the number of elements will remain the same. Piaget states that the nursery age child

"evaluates discontinuous quantities as if they were continuous, i.e., extended quantities. His quantitative judgements are thus based only on the general shape of the set and on global relationships such as 'more or less long', 'more or less wide', etc." This process in which the child focuses on only one criteria such as length, width, density, etc., is known as 'centring'.

Piaget and his associates have established approximate ages at which conservation takes place:

Conservation of numerousness--If a child of five is given two rows of marbles of equal number and spacing he will say that both groups are the same. When one group is given wider spacing he will now say that this row has more marbles. Conservation of numerousness takes place about the age of six.

Conservation of length--If you place two blocks of equal length beside each other and then push the one so that the end projects the child under six will no longer believe that the two blocks are of equal length. Conservation of length takes place about the age of seven.

Conservation of distance--If a wall of a block or a piece of cardboard is placed between two small toy trees a child of five or six will believe that the distance has been reduced. Conservation of distance takes place about the age of seven.
Piaget found that conservation of matter also takes place about the age of seven but that conservation of weight and volume do not take place until the ages of ten and eleven, respectively.\(^{117}\)

If Piaget's stages are fixed and immutable then a pre-school program need attend to only the formation stage of Piaget's theory. But Piaget himself says:

If we accept the fact that there are stages of development, another question arises, which I call 'the American question,' and I'm asked it every time I come here: If there are stages that children reach at given norms of ages, can we accelerate these stages? Do we have to go through each one of these stages, or can't we speed it up a bit? Well, surely, the answer is yes . . . but how far can we speed them up?\(^{118}\)

In order for the child to learn conservation it seems imperative, such as in other learning, that he be presented with numerous situations pertaining to conservation.

The research cited here will pertain to those attempting to hasten the child through the stages. "Piaget theorizes that the transition from nonconservation to conservation occurs through the 'equilibration process', an internal process heavily dependent upon activity and experience."\(^{119}\) In response to Piaget's stance on conservation Sigel, Roeper, and

Hooper taught for conservation of quantity. They reported success and they attributed this to attention to pre-requisite operations. Coxford pretested 60 children and from the results placed them in Piaget's stage 1, 2, or 3. The children ranged in age from 3-6 to 7-5. Twenty-seven tested at stage 1, 28 at stage 2, and 5 at stage 3. Of these 12 pairs were matched in stage 1 and 12 pairs in stage 2. The experimental 12 in each stage were given instruction and then all 48 were given a post-test. The children in stage 1 did not make significant gains whereas those in stage 2 made significant gains at the .05 level. In comparing two groups of children from differing cultural backgrounds, Millie Almy points out the importance of experience in progressing through Piaget's stages.

Piaget's theory leaves no question as to the importance of learning through activity. Demonstrations, pictured illustrations, particularly for the youngest children, clearly involve the child less meaningfully than do his own manipulation and his own experimentation. While the vicarious is certainly not to be ruled out, it is direct experience that is the avenue to knowledge and logical ability.

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Analysis of Concepts from the Literature

Robert B. Davis\(^{123}\) lists six ways that children build their earliest notions concerning number. These are:

a. By abstraction, from experience with two pencils, two dogs, two boys, two apples, and so on, by asking 'what do these have in common?';

b. By experience in performing the act of counting, which is here regarded as a human act that is learned by imitation, much as the child learns to swing a baseball bat, or to hold a spoon; after the act has been performed, it can be discussed, much as a class might subsequently discuss a trip to the zoo, etc.;

c. By getting first the idea of 'more', 'less', and 'equality', and thereafter giving number names to 'as many as I have fingers', etc.;

d. By studying sets, and the various attributes of sets, including the numerousness of the things in the collection (other properties of sets are studied in biological classification schemes, etc.; 'numerousness' is by no means the only property of sets);

e. By studying invariance, as in the fact that rearranging pebbles in a different pattern or order does not change the number of pebbles present;

f. By a gradual deformation of perceptual stimuli.

Lucas and Neufield\(^{124}\) present a good example of learning by abstraction in their book *Developing Pre-Number Ideas*. Shape and color are used throughout the book to encourage the child to categorize and find the commonality of various groups of objects.

\(\ldots\) (1) the child learns to name an object by connecting the selected word with the object; (2) the


child discovers the limitations of the application of the word through experience, and begins to seek the common characteristics of observed objects; (3) the child finds the common characteristics (conceptualizing) and forms a classification of these objects.\textsuperscript{125}

In searching for the common characteristics of two or more objects, there must be certain characteristics that must be denied. For example, if the child is being introduced to 'twoness' he must be able to ignore all the other classes the two objects belong to and attend to this one classification. The nursery teacher may point to the fish in the fishbowl and say, "We have two fish in our bowl." To be able to select the 'twoness' the child must ignore classes such as animals, swimmers, food, smooth skin, etc. Even when dealing with two dishes of the same set on a table we still distinguish between the one in front of Pamela and the one in front of Annette.

Although the child may hear the name for a concept numerous times, the name means nothing until the child is able to sort and classify the objects into a single entity that can be tagged with this name. Once he has acquired the name he still must respond to new situations and determine whether they belong or not. Vygotsky\textsuperscript{126} states that once a child is able to identify an object with a name he does not separate the two. "The word, to the child, is an integral part of the object it denotes."

\textsuperscript{125}Ibid., p. x.
An example of Davis's experience by imitation is the usual elementary school program, and his classification that starts with the concepts of 'more' and 'less' typify the kindergarten approach.

Patrick Suppes and most of the current textbook writers place their emphasis on sets as the beginning concept for children. Jerome Bruner states:

A disparate collection of words or objects, each discriminately different from the others, is presented to a person. It is not automatically the case that he will group or associate them. Whether he will or not group a given set depends on a variety of circumstances. It will depend, if you will, on what he is up to. Indeed, it is a nice question as to what will lead an individual to group things, or to form an association. He may have to pack them in the same suitcase, for example. Or he may want to assemble them in order to build a shelter... There may not, on the other hand, be any reason for him to group the disparate things, and, under the circumstances, there will be a very sharp and quick loss in the ability to report what things were conjointly present just a few minutes before. 127

As his last category pertaining to how children build their earliest notions concerning number Davis cited a gradual deformation of perceptual stimuli. The Madison Project and a project by Donald Bushell and Jo Maiorano are prime examples of this. In the latter program children are introduced to the written numeral 3 through the use of dots. The dots are gradually lightened while the numeral that corresponds to the number of dots is steadily darkened.

Wohlwill attempted to find a hierarchy of learning for the number concept. He theorized a sequential pattern that would be followed and after testing arrived at the following:

(a) abstraction (a selective response to one aspect of a stimulus)
(b) elimination of perceptual clues (requires intervention of symbolic processes, specifically counting)
(c) memory
(d) addition and subtraction
(e) extension
(f) conservation of number
(g) ordinal-cardinal correspondence

From the test he placed children's beginning learning in three stages: initial, intermediate, and final. In the initial stage the child responds to number purely on a perceptual basis. At this stage the child passes none of Wohlwill's tests. In the intermediate stage the child responds on a conceptual basis and is recognized by passing tests (a), (b), and (c). In the final stage the relationship is

conceptualized and the child passes all tests, with the frequent exception of test (g).

Piaget, in a similar fashion, classed the children in stages. The preconceptual stage occurs from approximately two to four years of age. In this stage the child is developing his language and early concept forms. In this preparatory phase the child's representational thought is characterized by animism, concreteness, egocentrism, irreversibility, phenomenism, and transductive reasoning. The next stage Piaget labels the intuitive thought stage and encompasses the period from four to seven. At this stage the child is still dominated by his perceptions. Centring, which is the overemphasis on one property of an object while de-emphasizing or ignoring other properties, is very prevalent at this stage. This is illustrated when the child acknowledges two sets to be equal and then by spreading one out over a longer line changing the response to say the group covering the longer space is more.

It is as though, for the child, quantity depended less on number (a notion which, if our hypothesis is correct, is still only verbal, although the child can count correctly) or on the one-one correspondence between the objects, than on the global appearance of the set, and in particular on the space occupied by it.129

Piaget's work on conservation seems at first glance to say that any child in the pre-concrete stage (generally up to 7) cannot benefit from a program in mathematics. In attempting

to find whether children can profit from early instruction

Celia Stendler says:

1. It has been possible to accelerate the development of logical intelligence by inducing cognitive conflicts in subjects.

2. Training children to recognize that an object can belong to several different classes at once aids in the development of logical classification.

3. There is a tendency for conservation of number to be accelerated in children trained to see that addition and subtraction of elements change numerical value.

4. To help children move from the preoperational stage to the stage of concrete operations, it is helpful to make gradual transformations in the stimulus and to call the child's attention to the effects of a change in one dimension to a change in another.

Angela Pace conducted a study concerning young children's understanding of number conservation and then tested to see what effect instruction had in altering the length of time spent in the pre-concrete stage. She found that her instructional program was effective in accelerating the attainment of the concept of number.

Analysis of Individual Concepts

Rote counting.—When should a child be presented with rote counting? Should it precede any rational counting? Should it come after he has a command of two, three, four, and five? Or maybe it should come after he has some command of two, three, etc., up to twenty. Although serial order is


eventually a necessity it is doubtful that rote counting should be one of the earliest arithmetical processes encountered. Rote counting does not give the child a tag to place on something in his real world such as the other words he has been accumulating do. If he is taught to rote count he is running around with a name for something which he is eventually asked to assign to certain sets of objects. In essence, at this point they are names with no referent. Would it not be better to wait until a need arises? For example, if the child were to acquire a knowledge of 'twoness', 'threeness', 'fourness', 'fiveness', and 'sixness', he could possibly surmise the serial order by their differences. Lowry points out that if we are to follow this approach then there needs to be a revision at the elementary school level.

Pre-school children and beginning first-grade children use their abilities to perceive the number of objects in a group in their solutions of quantitative situations. After they have been exposed to the usual emphasis on counting in early number work, however, they tend to use counting procedures even in situations where grouping is easier, just as reliable, and much quicker.\textsuperscript{132}

If the child is taught to learn the groups up to a point like five and as a byproduct is able to rationally count these groups, the time may be ripe to learn the number names beyond in rote fashion. At this stage the child's ability to rationally count to ten, with the addition of the new words, may enable him to rationally count far beyond ten. Although

\textsuperscript{132}Lowry, \textit{loc. cit.} p. 347.
there are ten words that are used to count to ten, only seven new words and/or syllables are needed to count to ninety-nine. These are "eleven", "twelve", "thir", "teen", "fif", "twen", and "ty". Also to go from one hundred to nine hundred ninety-nine only one new word is needed.

In summary, the words of Ned Russell seem apropos:

It would appear, also, that the attempt to have the child count is a mistaken procedure until the child gives an adequate conception of the magnitude or quantity he is differentiating. True counting is not the cause of number development, but is the result of the development.133

**Rational counting.**—One-to-one correspondence is a prerequisite for rational counting since the child must associate a word for each object in the set being counted. One-to-one also requires the child to associate the cardinal number to the set while actually using the ordinal concept to reach the last object. For example, to count eight objects the visual pattern is to arrange the objects in some serial manner and then proceed to assign to each group that has been counted an ordinal number. This may be accomplished in several ways. One way would be to group the objects into those that have been counted and those to be counted.

\[ \begin{array}{ccc}
0 & \text{one} & 0000000 \\
00 & \text{two} & 000000 \\
000 & \text{three} & 00000 \\
\text{etc.} & & \\
\end{array} \]

They may also be subitized and then paired with the appropriate

---

word until eight is attained. Both of these methods seem superior to the usual method in which the child touches each object without moving it and gives the ordinal number associated with the object.

0 0 0 0 0 0 0 0
one two three four five six seven eight

The danger in this approach is that the child may associate the word four to the fourth object instead of the group of four.

Another level of knowing a certain group, say four, is by an internal grouping. For example, three and one or two and two. From this stage children may progress to partial counting. That is, if there are four items and two more to be put together they may say 'four, five, six,' or 'two, three, four, five, six.'

Children need to handle and arrange manipulative materials if they are to understand the ideas of number and counting. "K and 1st grade teachers have long realized that 'number' and 'counting' are abstract ideas to be developed patiently and carefully. They know that most see many collections of three things before they sense in all these situations a common property we call 'threeness.'" 134 Most young children seem to acquire a 'oneness', 'twoness', 'threeness', etc. without benefit of a counting ability.

No doubt the child first sees two as a pair and three as a group of three and would develop his conception to include a group of four; but before he gets a chance to understand them they are lost in the serial relationships forced upon him by benevolent elders.\textsuperscript{135}

Children may rationally count in response to commands pertaining to identification and reproduction. Identification is typified by response to the question "How many apples have I?" and reproduction in response to the command, "Give me five apples." The answers may also be obtained from a one-to-one correspondence with a known set or from the configuration.

One-to-one correspondence.---One-to-one correspondence is a prerequisite for rational counting. To be able to assign the proper numeral to a group of objects, especially over six of them, there must be a one-to-one assignment of words to the objects. Whereas rational counting required attention to those already counted versus those to be counted in one set, the one-to-one correspondence is mainly concerned with the comparison of two sets and whether both sets are exhausted at the same time or one has some members still remaining. One-to-one correspondence then concerns itself with the equality-inequality aspect of two sets.

Piaget has worked extensively with the concept of one-to-one correspondence. He maintains that a child of five or six may be readily taught to name the numerals from 1 to 10 and

if ten stones are laid in a row he can count them correctly. But given eight blue chips and eight red chips he will say they are the same if placed in a one-to-one correspondence but if one is lengthened as

```
0 0 0 0 0 0 0
X x x x x x x x
```

he will go for the longer grouping. In like fashion if for each blue bead placed in one jar a red bead is placed in another jar the child will say both jars have the same number. But if the blue beads are poured in a third receptacle of a different size he thinks the number has changed. By the age of seven this is comprehended.

Dichotomies—The egocentric nature of the three to five-year-old may give impetus to his seeming better grasp of bigger, longer, higher, etc., versus smaller, shorter, lower, etc. Ned Russell found in checking the vocabulary of seven-year-olds that they had a general knowledge of many, more, and most and some understanding of less and least. In all the studies concerning the dichotomies concepts such as largest, longest, most, and more were answered correctly more times than smallest, shortest, fewest, and least. From the evidence it seems that it is not necessarily true that to know one of the parts of a dichotomy implies knowledge of the other. For example, to know more does not denote a knowledge of less. At this stage it seems that the child learns each concept distinct from the others. They are not coordinated one with the other.
In order to work with the dichotomies the child must be introduced to numbers of objects, or lengths, or weights, etc., that differ greatly so as to emphasize the differences. Only after the child has acquired a facility in comparing objects that differ greatly should he be required to make finer and finer distinctions down to the point of equality.

Ordinal.—From the research cited the three best known ordinal concepts in order are middle, first, and last. Whereas cardinal numbers denote quantity, ordinals state position. An important part of the process of naming ordinals is knowing where the starting place is and the direction to proceed. Then it becomes a process of a one-to-one correspondence between the ordinal words and each individual object in the prescribed order.

Adults so rarely use the ordinals beyond third that it is understandable that the child is lacking in the use of ordinals.

Addition.—It is difficult to ascertain at what stage the child is said to be using addition. For example, to go from two to three in rational counting requires an addition of two and one. Suppes and Groen investigated the ways children solved problems requiring addition. They determined that there were five approaches a child could take to a situation requiring the addition of 3 and 2.

1. Start at zero, add 3, and then add 2.
2. Start with first number (3) and add second (2).
3. Start with second number (2) and add first (3).
4. Start at minimum (2) and add maximum (3).
5. Start at maximum (3) and add minimum (2).

Premeasurement.—Premeasurement at the preschool level is generally related to the body as a measuring device or some handy object. Instead of using a ruler or yardstick to make a measurement the child will compare a length to his own height or the length of an arm or to the length of a block. The first delvings into the realm of the weight of an object concerns heavy and light. This is generally accomplished by either lifting the object or noting its size. That is, to a preschool child a larger object is necessarily the heavier object.

Pregeometry. Although rectangles are possibly more abundant in the physical world than any other plane geometric figures the child is able to recognize and name squares and circles with more certainty. This is very probably due to the use of these concepts more often by the adults in the child's presence.

Fractions.—To visualize a fraction the child must attend to certain properties and overlook others. For example, if a candy bar is broken into two parts the child receives 'one' piece of candy but it is now designated 'one-half.'

Coin recognition.—As cited in the previous chapter the order of recognition of the coins and bills is a penny, dollar
bill, dime, nickel, quarter, and half-dollar. The order follows closely to what the child experiences. Very probably his own spending habits have already included pennies for chewing gum and candy and his parents will generally bring out bills which they label as 'dollars' in payment for most other transactions.

Reading and writing numerals.—Buchanan\textsuperscript{137} found that the kindergarten pupils were able to write the numerals but that there was no evidence that this facilitated arithmetic conceptualization. Since writing requires fine muscle control this is best delayed for the majority. Like anything else, though, a test for readiness could be devised to see whether the individual is ready for this experience. In regards to reading numerals it is very possible that many of the children already know some of them from experience with television, house number, telephone number, etc. This is one item that was not checked in the nursery but since there are only ten different symbols to learn this may not be a difficult task. Bushell's perceptual method (see page 57) could be utilized to expedite learning.

CHAPTER III

ANALYSIS OF MATHEMATICAL CONCEPTS IN USE IN THE
CHILD DEVELOPMENT LABORATORY

During the spring and summer quarters in 1968 the children in the Child Development Laboratory of The Ohio State University were observed and a mathematics test given. During this same period, two Head Start centers and a day-care center were also observed and the test administered at the Head Start centers. The results of the observations and tests are recorded in this chapter. The results of the observations and tests are then analyzed.

**Exploratory Study**

A pre-pilot study was conducted by Dr. Nathan Lazar during the spring quarter of 1968 in the laboratory. The purpose of this exploration was to give direction to the pilot study. The pilot study, in turn, will recommend curricular alternatives to an improved mathematics program in the laboratory.

In the pre-pilot exploration, a graduate student in Family and Child Development administered the mathematics section of Bettye M. Caldwell's *Preschool Inventory* to fourteen three-year-olds and fourteen four-year-olds. This same test was administered in the summer quarter to eleven threes and
fifteen fours in the laboratory as well as thirteen Head Start children in the Columbus area. The section of the inventory that was administered follows.

Preschool Inventory

The questions used in this study were taken from Bettye Caldwell's *Preschool Inventory* written for Head Start evaluations. The total test had 85 items. Nineteen items that pertained to mathematics were selected and used in this investigation. The questions and instructions for administering are given below.

1-9. In answering the questions requiring a number as the answer, a child may often hold up the correct number of fingers. If this is done the examiner may say "HOW MANY IS THAT?" A child may also give a correct answer such as "2 in front and 2 in back;" if this is done, the examiner may say, "HOW MANY ALL TOGETHER?" In both cases, if the correct answer is given it is credited.

1. HOW MANY EYES DO YOU HAVE? Credit 2 only.
2. HOW MANY NOSES DO YOU HAVE? Credit 1 only.
3. HOW MANY HANDS DO YOU HAVE? Credit 2 only.
4. HOW MANY TOES DO YOU HAVE? Credit 10 only.
5. HOW MANY WHEELS DOES A CAR HAVE? Credit 4 only.
6. HOW MANY WHEELS DOES A BICYCLE HAVE? Credit 2 only.
7. HOW MANY WHEELS DOES A TRICYCLE HAVE? Credit 3 only.
8. HOW MANY WHEELS DOES A WHEELBARROW HAVE? Credit 1 only. (If child says "2" get him to describe and make certain he is referring to the new style.)

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9. HOW MANY WHEELS DOES A ROWBOAT HAVE? Credit 0 only.

10. LET'S HEAR YOU COUNT OUT LOUD. If no response, start child by saying "ONE--." Give credit if child counts to five. If child stops before 5, say, "CAN YOU COUNT ANY MORE?"

11. Hold up a blank piece of paper. Say, "HOW MANY CORNERS DOES THIS SHEET OF PAPER HAVE?" Credit 4. (Let child count if he can and needs to.)

12-14. Take out the box of 12 checkers, all the same color. Give the child the opportunity to manipulate them briefly. In establishing the groups to be judged, make certain that all the checkers are bunched together, all touching but not lined up, and all flat on the table. Put the checkers in two groups in front of the child and ask, first pointing to the group represented by the first number and then to the other:

12. 2 & 8 WHICH HAS MORE CHECKERS IN IT? Credit correct response.

13. 6 & 6 WHICH HAS MORE CHECKERS IN IT? Credit "Both" or "Neither" etc.

14. 2 & 8 WHICH HAS FEWER CHECKERS IN IT? Credit correct response.

15-19. Take away all but 5 of the checkers. Instruct the child as follows: "PUT THESE CHECKERS NEXT TO EACH OTHER IN A ROW." Following the pattern set by the previous item, the child may have all checkers touching. If so, see to it that a half-inch space is left between each two checkers. Give whatever guidance is needed to yield a fairly straight row. Credit first-last in terms of a child's choice--i.e., either end of the row of checkers with all subsequent choices consistent with that choice. Return the checker to the appropriate place after each response. Credit the correct response. Say:

15. POINT TO THE MIDDLE ONE.

16. POINT TO THE FIRST ONE.

17. POINT TO THE LAST ONE.

18. POINT TO THE SECOND ONE.

19. POINT TO THE NEXT TO THE LAST ONE.
Results of Preschool Inventory in the Child Development Laboratory

Individual results of the Preschool Inventory will be found in Tables 2, 3, 4, and 5. In analyzing the results of the test, it was found that the fours scored higher as a group than the threes. See Table 6. The order of difficulty of the concepts as judged by correct responses was remarkably alike. See Table 7. When placed in serial order from the one least missed to the one most missed, twelve of the nineteen questions occupied the same position and four others were only one removed. The one question in which the responses did not fit this prevailing pattern was the one requiring knowledge of the number of wheels on a car. This was in 16th place for the threes and in 11th place for the fours.

Item analysis.—Two items dealt with the ability to recognize oneness. These were item two concerning the number of noses and item eight for wheels on a wheelbarrow. Seventy-eight percent of the threes and 91% of the fours gave correct responses. There were three items pertaining to twoness. These were items one, three, and six concerning eyes, hands, and wheels on a bicycle. Seventy-two percent of the threes and 87% of the fours accomplished these. Number seven was the only question testing threeness through knowledge of a tricycle. Forty percent of the threes and 72% of the fours responded correctly. Two items, five and eleven, were used to identify fourness. Thirty percent of the threes and 67% of the fours were able to do this. The only other cardinal tested was ten.
TABLE 2. Results of Preschool Inventory with the three-year-olds at laboratory school during summer quarter, 1968, with 1 denoting a correct response

<table>
<thead>
<tr>
<th>Name</th>
<th>Previous Quarters in Nursery</th>
<th>Age</th>
<th>Question Number</th>
<th>Total Score</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1 2 3 4 5 6 7 8</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>9 10 11 12 13 14 15 16 17 18 19</td>
<td></td>
</tr>
<tr>
<td>Lisette</td>
<td>0</td>
<td>3-1-2</td>
<td>1 1 1 0 0 0 0 0</td>
<td>5</td>
</tr>
<tr>
<td>Joshua</td>
<td>0</td>
<td>3-1-23</td>
<td>1 1 0 0 0 0 1 0</td>
<td>3</td>
</tr>
<tr>
<td>Julane</td>
<td>0</td>
<td>3-1-24</td>
<td>1 1 1 0 0 0 0 0</td>
<td>3</td>
</tr>
<tr>
<td>Mark V.</td>
<td>0</td>
<td>3-2-7</td>
<td>1 1 1 0 0 0 0 1</td>
<td>13</td>
</tr>
<tr>
<td>Debi</td>
<td>0</td>
<td>3-4-26</td>
<td>0 0 0 0 0 0 0 1</td>
<td>1</td>
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<td>Dana</td>
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<td>1 1 1 0 0 0 0 1</td>
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</tr>
<tr>
<td>Mark F.</td>
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<td>3-6-14</td>
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</tr>
<tr>
<td>Gregg</td>
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<td>1 1 0 0 0 1 1 0</td>
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</tr>
<tr>
<td>John B.</td>
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</tr>
<tr>
<td>JoDee</td>
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</table>

TOTAL CORRECT RESPONSES 9 10 8 0 2 4 2 4 2 6 2 4 0 3 3 1 2 2 1
TABLE 3. Results of Preschool Inventory with the three-year-olds at laboratory school during spring quarter, 1968, with 1 denoting a correct response

<table>
<thead>
<tr>
<th>Name</th>
<th>Previous Quarters in Nursery</th>
<th>Age</th>
<th>1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19</th>
<th>Question Number</th>
<th>Total Score</th>
</tr>
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<td>Stephanie</td>
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<td>Isaac</td>
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<tr>
<td>Steven</td>
<td>3 3-11-24</td>
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</tr>
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<tr>
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<td></td>
</tr>
<tr>
<td>Patrick</td>
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<td></td>
</tr>
<tr>
<td>TOTAL CORRECT RESPONSES</td>
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</tbody>
</table>
TABLE 4. Results of Preschool Inventory with the four-year olds at laboratory school during summer quarter, 1960, with 1 denoting a correct response

<table>
<thead>
<tr>
<th>Name</th>
<th>Previous Quarters in Nursery</th>
<th>Age</th>
<th>Question Number</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sara</td>
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<td>3-11-24</td>
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<td></td>
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<tr>
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<td>4-1-5</td>
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<tr>
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<td>4-1-10</td>
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</tr>
<tr>
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<tr>
<td>Patrick</td>
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<td>4-3-24</td>
<td>1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 0 17</td>
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<tr>
<td>David</td>
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</tr>
<tr>
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<tr>
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<tr>
<td>Bruce</td>
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</tr>
<tr>
<td>Marc</td>
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<td>4-11-24</td>
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<td>14 15 13 3 8 9 8 13 4 8 9 11 2 6 9 8 7 6 3</td>
<td></td>
<td></td>
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</tbody>
</table>
### TABLE 5. Results of Preschool Inventory with the four-year-olds at laboratory school during spring quarter, 1968, with 1 denoting a correct response

<table>
<thead>
<tr>
<th>Name</th>
<th>Previous Quarters in Nursery</th>
<th>Age</th>
<th>Question Number</th>
<th>Total Score</th>
</tr>
</thead>
<tbody>
<tr>
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<td>4-3-12</td>
<td>1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19</td>
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</tr>
<tr>
<td>Wanda</td>
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<td>4-5-13</td>
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<td>14 14 14 4 12 12 13 11 9 13 10 14 7 5 12 13 11 9 8</td>
</tr>
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<td>14 14 14 4 12 12 13 11 9 13 10 14 7 5 12 13 11 9 8</td>
</tr>
<tr>
<td>Betsy</td>
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<td>4-5-29</td>
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<td>14 14 14 4 12 12 13 11 9 13 10 14 7 5 12 13 11 9 8</td>
</tr>
<tr>
<td>Diane</td>
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<td>4-7-20</td>
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</tr>
<tr>
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<td>14 14 14 4 12 12 13 11 9 13 10 14 7 5 12 13 11 9 8</td>
</tr>
<tr>
<td>Marc</td>
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<td>4-8-27</td>
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<td>14 14 14 4 12 12 13 11 9 13 10 14 7 5 12 13 11 9 8</td>
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<tr>
<td>Robin</td>
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<td>4-9-24</td>
<td>1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19</td>
<td>14 14 14 4 12 12 13 11 9 13 10 14 7 5 12 13 11 9 8</td>
</tr>
<tr>
<td>Susan C.</td>
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<tr>
<td>Jeremy</td>
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<td>4-10-20</td>
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<td>14 14 14 4 12 12 13 11 9 13 10 14 7 5 12 13 11 9 8</td>
</tr>
<tr>
<td>Chris</td>
<td>7</td>
<td>4-10-23</td>
<td>1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19</td>
<td>14 14 14 4 12 12 13 11 9 13 10 14 7 5 12 13 11 9 8</td>
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<td>14 14 14 4 12 12 13 11 9 13 10 14 7 5 12 13 11 9 8</td>
</tr>
<tr>
<td>Robert</td>
<td>7</td>
<td>5-2-24</td>
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<td>14 14 14 4 12 12 13 11 9 13 10 14 7 5 12 13 11 9 8</td>
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</table>
TABLE 6. Total number of correct responses to the Preschool Inventory by 25 three-year-olds and 29 four-year-olds of the laboratory school

<table>
<thead>
<tr>
<th>Age</th>
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<th>4</th>
<th>5</th>
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<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
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</thead>
<tbody>
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<td>22</td>
<td>24</td>
<td>20</td>
<td>2</td>
<td>5</td>
<td>12</td>
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<td>11</td>
<td>10</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>Fours</td>
<td>28</td>
<td>29</td>
<td>27</td>
<td>7</td>
<td>20</td>
<td>21</td>
<td>21</td>
<td>24</td>
<td>13</td>
<td>21</td>
<td>19</td>
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<td>15</td>
<td>11</td>
</tr>
<tr>
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<td>47</td>
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<td>25</td>
<td>33</td>
<td>31</td>
<td>39</td>
<td>19</td>
<td>34</td>
<td>28</td>
<td>39</td>
<td>12</td>
<td>20</td>
<td>33</td>
<td>32</td>
<td>28</td>
<td>25</td>
<td>15</td>
</tr>
</tbody>
</table>
When asked to state how many toes they had, 8% of the threes and 24% of the fours were correct.

In a test of the concept of zero, the children were asked to identify the number of wheels on a rowboat. Twenty-four percent of the threes and 46% of the fours were able to
do so. The ability to rote count to five was accomplished by 52% of the threes and 72% of the fours.

Three questions tested equality-inequality. When two and eight checkers were presented, the child was asked which was more. This was followed by a presentation of six and six with the same question. Then two and eight were presented once more and the child was to select the group that was fewer. With the two and eight checkers, the threes were 56% correct for more and 36% for fewer while the fours were 86% and 38%, respectively. On the equality 12% of the threes and 31% of the fours were successful.

The last five questions of the inventory were concerned with ordinality. First and middle evoked comparable responses from the children. Forty-eight percent of the threes understood middle, and 44%, first. With the fours, 72% understood middle, and 72% understood first. Similarly, second and last had approximately the same percentages. Forty percent of the threes selected the second, and 40%, last. With the fours, 62% identified last, one and 52% second. The most difficult ordinal was to identify the next-to-the-last. Only the threes, 16% were able to make the identification and 38% of the fours were also.

A composite of the threes and fours displayed the following order from the highest correctly answered to the one least correctly answered: one nose, two eyes, two hands, one wheel, more (of 2 and 8), count to five, two wheels, middle (of 5 counters), first (of five counters), three wheels, last (of
5 counters), four corners, four wheels, second (of 5 counters), fewer (of 2 and 8), zero wheels, next-to-last (of 5 counters), more (of 6 and 6), and ten toes. The ability to handle cardinal concepts seemed to follow the serial order. The ordinals appeared to also follow a similar pattern but with middle and last being among the first to be learned. The ability to handle the ordinals seemed to be slightly out of phase with the cardinals, but not to a degree that warrants their being learned separately. From the way the children performed, the two concepts should not be learned consecutively, but rather, concurrently. As with the research cited in the previous chapter, the concept "more" was substantially better known than was "fewer."

Results of Preschool Inventory in Head Start Centers

Thirteen Head Start children were administered the Preschool Inventory with six of these being threes. Individual results will be found in Table 8. As with the children in the laboratory, the fours scored better than the threes and once more the order of difficulty of the concepts was remarkably alike. See Table 9. Only on rote counting ability and in identifying the middle checker were the fours noticeably different from the threes.

Item analysis.--In the two items concerning oneness 17% of the threes and 50% of the fours gave correct responses. With the three items concerning twoness 39% of the threes and
TABLE 8. Results of Preschool Inventory with the three- and four-year-olds in Operation Head Start, summer, 1965, with 1 denoting a correct response

<table>
<thead>
<tr>
<th>Name</th>
<th>Previous Quarters in Nursery</th>
<th>Age</th>
<th>Question Number</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mimi</td>
<td>0</td>
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<td>Aisha</td>
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<td>17</td>
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<td>Tyrone</td>
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</table>
TABLE 9. Total number of correct responses to the Preschool Inventory by six three-year-olds and seven four-year-olds of Head Start

<table>
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<th>7</th>
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<th>10</th>
<th>11</th>
<th>12</th>
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</tbody>
</table>
52% of the fours accomplished these. On the concept "three," no three responded correctly, and only one four was able to do so. With the two items pertaining to "fourness," 17% of the threes and none of the fours responded correctly. No Head Start child was able to give the correct answer regarding the number of toes he had. The same was true when testing the zero concept, and more when two items were, in fact, equal. When asked to rote count, 17% of the threes and 57% of the fours were successful. When tested for "more" and "fewer," 50% of the threes were successful with the former and 17% with the latter. Fifty-seven percent of the fours were successful with "more," while 29% were successful with "fewer." In dealing with the ordinals: first, second, and next-to-the-last, no threes or fours were able to make these distinctions, and only one pointed to the last object.

A composite of the threes and fours displayed the following order from the one most answered correctly to the one least answered: one nose, two eyes, more (of 2 and 8), two hands, two wheels, middle (of 5 counters), count to five, fewer (of 2 and 8), three wheels, four wheels, one wheel, and last (of 5 counters). The remaining six items—ten toes, zero wheels, more (of 6 and 6), first (of 5 counters), second (of 5 counters), and next-to-the-last (of 5 counters)—were missed by all the threes and fours. Although most of this order followed closely the order of the children in the laboratory school, the
use of the ordinal concept was almost totally lacking in the Head Start children.

**Comparison of Child Development Laboratory and Head Start.**—Although the sampling of Head Start children was small compared to the Child Development Laboratory children, some observations are in order. On every item, the laboratory threes scored better than the Head Start threes. The fours did likewise. The laboratory threes actually had better percentages than the Head Start fours on every item except two. When comparing two and eight checkers for "moreness," both groups had the same percentage. In rote counting, the advantage was in the Head Start favor by a 57% to 52% count. In the Head Start group, six of the nineteen categories were missed by all thirteen children; whereas, in the laboratory no category was missed by all the children in any one of the four groups tested.

**Analysis of tests of those who missed either one, two, three, or four questions.** Only one child (4-10-20) missed just one question and that pertained to the number of wheels on a wheelbarrow. This same question was not missed by any of the fourteen persons missing either two, three, or four questions so this result is not significant.

Of the six children who missed two questions, four responded incorrectly to "fewer," two to "more" when the sets were equal, two to "next-to-the-last-one," and one each on four other questions. Only one person missed three questions and these included "more" and "fewer" referred to in the previous
sentence. Of the seven children who missed four questions six were unable to state the number of toes they had, five the wheels on a rowboat, five for "more," three for wheels on a bicycle, and three each for second and next-to-the-last-one. It is interesting to note that none missed the question pertaining to tricycle, but that three did not know the number of wheels on a bicycle. Since the tricycle is the vehicle of this age child, it is probable that familiarity played an important part in this phenomenon. At the same time, it is puzzling to note that in the whole sampling, the two concepts are almost even.

Table 10 shows the questions missed by those with scores ranging from 15 to 19. Fifteen of the 67 children were in this category.

TABLE 10. Total number of questions missed on the Preschool Inventory by children having a score of fifteen to nineteen

<table>
<thead>
<tr>
<th>Question Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total number of misses by 15 children</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>8</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>6</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>8</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

This shows that all the children responded correctly to "two eyes," "one nose," "two hands," "four corners," "more" when there were more, "middle," and "first." As with the whole sampling the most missed items were "How many toes do you have?" and "Which has more checkers in it?" when the number of checkers was even.
This further solidifies the possibility that these are the most difficult concepts of the ones tested for this age child.

Analysis of tests of those who responded in terms of either one, two, three, or four correct.--No one responded with only one right, and one boy responded with only two. These two concerned four wheels on a car and the pile of checkers that was more. This child was in one of the Head Start programs and had spent a considerable amount of time telling about his father's automobile. He made references to the carburetor and muffler as well as describing how he helped in fixing the car. The one vehicle that he monopolized on the playground was an automobile that he could pedal. His high interest may account for his knowing that a car had four wheels and at the same time could not answer supposedly easier questions.

Of the eight children who were correct on only three of the questions, six were correct on the number of noses, six on the number of eyes, and four on the number of hands. This means that sixteen of their twenty-four correct responses pertained to knowledge of one and two. Of those who were correct on four items, there was agreement with those who had three correct with the addition of the ability to rote count.

Table 11 shows the questions that were correct by those who had scores ranging from one to four. Fourteen of the 67 children were in this category.
TABLE 11. Total number of questions correct on the Preschool Inventory by children having a score of one to four

<table>
<thead>
<tr>
<th>Question Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total number correct by 14 children</td>
<td>9</td>
<td>1</td>
<td>1</td>
<td>7</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

As noted, none knew the number of toes they had, the number of wheels on a rowboat, the number of corners on a sheet of paper, more when the same number of checkers were used, fewer, and any ordinal except middle. The number of wheels on a rowboat could be an economic one or these children could have a very vague concept of zero, or nothing.

Rote counting to five and counting four corners.—Twenty children could do neither task. Twenty-three could do both. Seventeen could rote count, but not count the corners, and seven could not rote count but counted the four corners. Some of the children who could not rote count but counted the corners did so by pointing to each corner and repeating "one, two, three, four." It is not known whether some of the others did the same thing or had memorized this fact. These phenomena could substantiate the fact that rote counting is not a necessary prerequisite for counting ability. Although the mean score for all 67 children was 9.4, the mean for those who could not rote count but yet counted four corners was 10.1, and those who could rote count but could not count the corners was 8.0.
Oneness.—Two questions tested for "oneness." The first questioned the child as to the number of noses he had, and the second related to the number of wheels on a wheelbarrow. Six were unable to respond to either one, and 40 responded correctly to both situations. The remaining 21 answered correctly to noses and incorrectly to wheelbarrow. No child answered correctly for wheelbarrow and missed noses. This possibly points out the varying degrees of oneness that children may have.

Twoness.—Three questions tested for "twoness." The first questioned the number of eyes, the second the number of hands, and the third the number of wheels on a bicycle. Six children missed all three and 31 missed none. Of the remaining 30, 19 gave correct responses to the first two and missed the last one. The number of correct responses followed the same order as presentation of questions, that is, number of eyes, number of hands, and lastly the number of wheels on a bicycle. This once more points out that mathematical concepts are first learned when objects are in close proximity pointing out the use of the child's own body in presenting mathematics. Finger counting, which has been frowned upon by so many teachers of young children, seems to be a good vehicle for beginning number concepts.

Threeness. Of the 31 who had perfect scores on "twoness," nine faltered on the three wheels of a tricycle. At the same time, 11 other children gave correct responses, of which seven had missed only the number of wheels on a bicycle.
concerning twoness. This could possibly be explained by the fact that the tricycle is the vehicle of the threes and fours and hence may have more meaning to some of them.

Fourness. — Two test items dealt with fourness—wheels on a car and corners on a piece of paper. Although 21 were able to answer both correctly, 31 were unable to do so. Nine of the remaining knew the number of corners on the piece of paper while six knew the number of wheels on an automobile. It would be interesting to see how the results may have differed if a real automobile were present at the testing. Another aspect of this type of problem that bears researching is how many of the answers are pure memorization of facts with no mathematical competency as such involved.

More, equal, fewer. — Sixteen were unable to get any one of the questions pertaining to "more," "equal," or "fewer" correct, while only ten were able to do so. Seventeen had "more" and "fewer" only; six "more" and "equal" only, and none had "equal" and "fewer" only. Fourteen had "more" only, none "equal" only, and four "fewer" only. Of the 48 who knew "more," only 19 knew "fewer." Similarly, of the 22 who knew "fewer," 19 also knew "more." It is not true that to know "more" is to know "less," but it is more possibly true to know "less" implies that "more" has been learned. Of the 48 who knew "more," only 13 knew "equality." Of 13 who knew "equality" all knew "more." An interpretation of these data tends to substantiate that the child learns "inequality" before "equality."
Also, "moreness" is learned before "fewness" and not together as a dichotomy as is so often the teaching strategy. In needing of further research is the possibility that most, if not all, of the dichotomies are first learned as separate entities and not as concepts dependent on each other. For instance, to know that some object is bigger than a second object does not necessarily imply in the young child's mind that the second object is, as a consequence, smaller.

Striking support is given to this proposition in the fact that all ten children who scored perfectly were in the one class that had the longest experience in the preschool. Although their scores in most other categories were in line with the total, this finding was conspicuous. It is quite possible that the teachers, without realizing it, were using and promoting the concepts of more, fewer, and equal.

First and last.—Of the 33 who knew "first," 22 also knew "last," while of the 28 who knew "last," 22 also knew "first." It is quite possible that the concepts of "first" and "last" are learned separately and at about the same time.

Optimal sequence for introducing concepts.—Although there are numerous sequences that beginning mathematical concepts could follow, it is conjectured that there are several that could be followed for optimal learning. Further research needs to be done to ascertain these sequences. Some possible leads are presented in the following paragraphs. In this study, 48 children understood "more." In comparing this to the
cardinals from one to four, there were 61 who responded correctly to "one" down to 27 who responded to "four," with the cardinal concepts being in order from one to four. The total correct responses for "two and more" were approximately the same. This suggests the possibility that "more" is not one of the very earliest concepts learned but is learned when the need arises to go from "one" to "one more,"—namely, "two." In a similar analogy using "first" instead of "more," the point at which the number of correct responses practically coincide is "three." "Second" and "four" also approximate one another. In sequencing concepts, this would possibly place one, two, more, three, first, four, second, and five in that order.

Most current elementary school textbooks place the cardinals in order from one to ten before the children are even introduced to ordinals. If intertwining the cardinals and ordinals bears out in further research, then the programs would have to be altered to run the two concurrently rather than consecutively.

Age, score, and quarters in preschool program.--The mean age, score on the inventory, and quarters in the preschool for each of the five groups of children are presented in Table 12. As noted as age and quarters increased, the general trend was an increase in score. The main exception to this was the group from the Head Start program. Also, there was a slight disparity between the spring threes and the summer fours.
TABLE 12. Mean age and mean score on Preschool Inventory, and mean quarters in preschool of five classes

<table>
<thead>
<tr>
<th>Class</th>
<th>Mean Age</th>
<th>Mean Score</th>
<th>Mean Quarters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head Start</td>
<td>4-2-18</td>
<td>4.0</td>
<td>0</td>
</tr>
<tr>
<td>Summer threes</td>
<td>3-5-9</td>
<td>5.9</td>
<td>0</td>
</tr>
<tr>
<td>Spring threes</td>
<td>3-11-0</td>
<td>11.0</td>
<td>2.75</td>
</tr>
<tr>
<td>Summer fours</td>
<td>4-4-15</td>
<td>10.4</td>
<td>2.9</td>
</tr>
<tr>
<td>Spring fours</td>
<td>4-9-16</td>
<td>14.6</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Although the summer fours were approximately five months older, and had about .15 more quarters per person, the spring threes scored .6 of a point better on the test. In examining the number of quarters in the preschool, the spring threes each had one, two, or three quarters experience whereas in the summer fours there were four who had no experience while eight had four or more quarters experience. This possibly points out the mathematical benefits that may be derived from an incidental program. A planned program in the next year could be correlated with the results from this year's testing.

Those with zero or one quarter experience among the four-year-olds and those with six or seven quarters experience were compared. There were six with zero or one quarter experience, and their mean quarter experience was .33. The mean age was 4-8-6, mean score 9.66, and the range was from three to sixteen. There were eight with six or seven quarters experience with the mean being 6.5. The mean age was 4-11-12, mean score 14.75, and the range was from 11 to 17. Although the mean age
differed by only three months, there was slightly more than a five point difference in the test score. Also, the narrower range indicates a possible better consistency in favor of those with more preschool experience.

Boys versus girls on inventory.—Table 13 compares the scores of the boys versus the girls in the individual classes.

<table>
<thead>
<tr>
<th>Class</th>
<th>Boys</th>
<th>Girls</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head Start</td>
<td>3.4</td>
<td>4.5</td>
</tr>
<tr>
<td>Summer threes</td>
<td>7.0</td>
<td>4.6</td>
</tr>
<tr>
<td>Spring threes</td>
<td>12.1</td>
<td>9.9</td>
</tr>
<tr>
<td>Summer fours</td>
<td>10.5</td>
<td>10.4</td>
</tr>
<tr>
<td>Spring fours</td>
<td>14.7</td>
<td>14.6</td>
</tr>
</tbody>
</table>

As Table 13 shows, the mean scores of the girls and boys do not differ radically within each set but there are differences between the same age groups and classes. This may be attributed to either the child's age, length of time in program, or the influence of certain teachers. Further research needs to be done to ascertain the influence of individual teachers on the cognitive learnings of the preschooler. Overall, the mean score of the boys is 9.7 while that of the girls is 9.2.

In examining their knowledge of wheel toys, there was an 18 to 15 differential by the boys over the girls on tricycles. This was approximately the same ratio as boys to girls in the
Concerning knowledge of the bicycle, there were 14 girls. However, the boys showed a sharp rise up to 24. It may be that the boys at this age are motivated toward having a bicycle of their own or in some cases may already have one.

Observations.—The students and teachers in the nursery school were introduced to what was being attempted in the pilot study during a one-hour presentation the third week. They were then instructed to look for and record mathematical occurrences in the classroom. This group of teachers worked with 14 threes in the morning session. In the five-day period there were 45 recorded observations, of which 29 were child-initiated. Of these 29, 21 were initiated by the two students with the top scores on the Preschool Inventory. Seven of the 16 teacher-initiated contacts were also directed to these two children. Overall the child-initiated contacts averaged six per one-half day and the teacher-initiated contacts three a day. Also, the mean teacher-initiated contacts were one per week and the mean child-initiated contacts were two per week. These observations paralleled the observations taken at random during the first week of the summer term by the writer. During this period, were recorded 45 mathematical explorations of which 33 were child-initiated and 12 teacher-initiated. Of the child-initiated contacts, 22 were by the two high scorers on the Preschool Inventory. Of the teacher-initiated, 7 of the 12 were directed to these same two children.
In summary, it seems that the children who have the beginning concepts use them liberally in their play and, consequently, elicit questions from the teachers which further solidify and build on the concepts.

Illustrative anecdotal accounts of students' responses to Caldwell's Inventory.—In questioning Glen, the boy who was interested in automobiles and got only four wheels and more correct, reversal of one and two were exhibited. When asked how many eyes he had, he replied "One," how many hands—"One," and how many noses—"Two." Marchane, who had the highest score among the Head Start children, missed every item but one pertaining to objects with wheels.

Examiner: How many toes do you have?
Mark V.: (No answer. Looks around the room as if searching.)

Examiner: How many wheels does a car have?
Mark V.: Two on both sides.

Examiner: How many all together?
Mark V.: (No answer.)

Examiner: How many wheels does a tricycle have?
Mark V.: Two on both sides; one in front.

Examiner: How many all together?
Mark V.: (No answer.)

Examiner: Let's hear you count out loud.
Mark V.: I can't count.

Examiner: One, . . .

Mark V.: I told you that I can't count.
Debi responded correctly to only one question in the entire test and that was to rote count. In response to all ten questions asking how many Debi responded with the same answer every time. This answer was "Two, four, five." In response to all the remaining questions that required pointing to the ordinals and pointing to unequal groups she did not point but answered "two" for every one.

Mayank, who had a score of four, got the three questions pertaining to twoness correct and was able to rote count. He also responded to the question concerning a car by saying "two in back" and "two in front," but could not come up with a total of four. Mayank was the only child to get "twoness" without getting oneness.

Examiner: How many eyes do you have?
Autumn: Three.

Examiner: How many hands do you have?
Autumn: Three.

Examiner: How many wheels does a bicycle have?
Autumn: Two.

Examiner: How many wheels does a tricycle have?
Autumn: Three.
Examiner: How many wheels does a car have?
Isaac: Six.
Examiner: How many wheels does a bicycle have?
Isaac: Six.
Examiner: How many wheels does a tricycle have?
Isaac: Fourteen. Two small ones and one big one.

Examiner: How many toes do you have?
Steven: (Started to count out loud but then said "I don't know.")
Examiner: How many wheels does a car have?
Steven: Two wheels and two more.
Examiner: How many all together?
Steven: (No answer.)
Examiner: How many wheels does a tricycle have?
Steven: Two and a big one up front.
Examiner: How many all together?
Steven: I don't know.

Illustrative anecdotal accounts of students while at play.—

Teacher: How many cars do you have?
Debi: Two, three, four, five. (Debi had answered all questions on the test that asked how many by answering two, four, five. In this instance Debi was playing with three play cars she had brought from home.)

At manipulative table John has five pegs in the pegboard when the teacher walks up.
Teacher: How many pegs do you have in the board, John?

John Z.: One, two, three, four, five. (Smiles at teacher and puts in another.)

Teacher: How many do you have now?

John Z.: One, two, three, four, five, six. (Adds another.) One, two, three, four, five, six, seven. (Adds another.) Eight. (Adds another.) Nine. (Adds another.) Ten. (John takes all the pegs out of the board and looks very pleased with himself.)

Approximately one month after taking the test Debi was in the science corner. At this time she displayed an improvement in her mathematical knowledge.

Teacher: How many gerbils are there, Debi?

Debi: Two inside, one outside.

Teacher: How many in all?

Debi: (No answer.)

Teacher: There's three.

Debi: Three.

There were only two recorded use of ordinals by either a teacher or a child.

John B.: You're the second person I saw first. I saw Julane first and then you first.

Mark V.: Why does this one always come out first?

Teacher: (No answer.)

Mark V.: One, two, three. (Counting free play posts. Picks up one more.) How many is this?

Teacher: Four.

Mark V.: (Picks up one more.) And this?
Five.
That's a lot.
(Outdoors and has three shovels all the same size. He picks up one.) Is this the biggest one?
No.
(Picks up another one.) Is this one?
No.
(Picks up last one.) This one?
No, they are all the same size.
(Places them side by side.) Uh, huh. They are.
(Looking at a wooden airplane that is a push toy.) It has wheels like a real airplane.
(No comment.)
(Looking at a wooden boat that has wheels for pushing.) Why are there wheels on this boat?
(No reply.)
(Playing with plastic bottles in a tub filled with water.) Who put so many bottles in here?
(No reply.)
Why aren't there very many children? (Only four were on hand the first day and John had been in the preschool previously. New ones were being inducted at the rate of two a day.)
More will be coming tomorrow.
Only two references to measurement were made and both of these involved Mark V.
Mark V.: (Using a small ladder outside.) I will go up one, two, three, four steps and be taller than a teacher.

Mark V.: (Mark and Joshua were digging tunnels outside.) How deep is it? Five inches? (Joshua took out another scoopful.) Now how deep? Ten inches?

John Z.: (Playing with wooden pieces—screws and nuts. He had three screws and asked the teacher for nuts. She intentionally put out two to see his reaction.) I'll need three. There are three screws.

Teacher: How many wheels on this wagon? (Wagon is on the playground.)

Jimmy: Five.

Teacher: Are you sure?

Jimmy: Here's one, here's one, here's one, here's one. (At this point he looks puzzled and then pointing to the side of the wagon he looks triumphant.) And here's one.

Teacher: How many wheels on this wagon, Bryant?

Bryant: Two. One, two, three, four. Two.

Teacher: He's kidding.

Teacher: How many children in your family?

Carolyn: Eleven.

Teacher: How many girls?

Carolyn: Four.

Teacher: How many boys?

Carolyn: (Starts naming them and successfully names seven.)

Teacher: (In one Head Start classroom the teacher held up a picture of three kittens.) How many kittens? Hold up the number of fingers that tells the number of kittens.
Of nine children one held up three fingers, six held up four, one held up five, and one held up eight.

Avoidance of teachers and/or questions.—Several methods were employed by the children when an answer was not known. One was to guess an answer, another was to not answer, and another was to redirect the questioner by pointing out something in the room or to ask a non-pertinent question in return. Questions of this sort included "Why is that sink in here?" and "Why are there chairs in this room?"

In situations outside the test, redirection was not employed but not responding was a favorite strategy to avoid questions with which the child felt uncomfortable. Noteworthy also was the number of times that the adults were queried by the children, and they made no reply.

Summary.—The children in The Ohio State University Laboratory made higher scores on all items in the Preschool Inventory than children of comparable ages in the Head Start centers. The order of difficulty of the various concepts did not vary significantly from the three-year-olds to the four-year-olds in either group.

In the observations, it was noted that the number of child-initiated contacts concerning mathematical concepts was in a two-to-one ratio with teacher-initiated contacts.
Mathematical concepts held by the preschooler represent a wide range of abilities and capabilities. The child has been developing the rudiments of mathematics in his interactions with his parents and friends. Some come to nursery school with the ability to add and subtract, while others do not have the idea of "oneness." The children have varied experiences related to reasoning, proof, and many other arithmetical explorations. They have seen their parents use money; and in many cases, they have already purchased candy, gum, and other items themselves. They have estimated distances by riding tricycles and have played with geometrical shapes. There have been experiences with food and toys related to the concepts "more" and "less."

A formal test when the child enters the nursery school as well as sensitive observations of the child in the nursery setting should supply the needed information for an individualized program. The curriculum guides formulated heuristically in this chapter are to be utilized for prearranged learning experiences. At the same time, they should provide an adequate framework for the unplanned experiences that arise. The
teacher needs to use and speak arithmetic if the child is expected to do so. From this perspective, the teacher must stimulate the children's thinking. The children need a sequence of carefully-organized learning experiences. Ideas develop gradually at this stage, and it is only through many varied experiences that the concepts become more precise.

Appropriate guides must be conceptualized if the environment in the nursery school is to become "responsive."

A "responsive environment" curriculum should give the child an opportunity to manipulate concrete and semiconcrete materials, sharpen and extend his vocabulary, and make him more competent in expressing mathematical ideas. An important aspect of such an environment is that the teachers encourage the children to make generalizations that are in sentences, not just one word answers to direct questions. It is not simply a task of assigning a word to a concept, but rather, a complete generalization concerning the concept.

In summary, the teacher needs to (1) pretest the child to ascertain roughly his mathematical abilities, (2) structure an individualized program, (3) consciously plan situations for learning experiences and carry them out, (4) take advantage of unplanned experiences, and (5) continually evaluate the child's progress for further individualization of the program.

The Home Economics Child Development Laboratory.—The curriculum guides that have been developed and which are included in this chapter were designed for use in the Home
Economics Child Development Laboratory of The Ohio State University. The School of Home Economics, assisted by members of the Merrill-Palmer School and Dr. Earl Baxter of the College of Medicine, introduced a course in child development in 1923. Two years later, the child development laboratory was added. The main purposes of the laboratory are to give students experience in observing and evaluating the growth patterns of young children and to gain experience in directing preschool activities. The Child Development Laboratory is located on the ground floor of Campbell Hall. There is also a playground located on the south end of the building. Approximately 14 children between the ages of three and four are enrolled in the morning session and 14 between the ages of four and five for the afternoon session. The selections are equally divided between boys and girls in each three-month age level.

The nursery school schedule gives a basic framework for the curriculum. It has some flexibility so as to make adaptations to an individual or a group or for variations in the weather. Following is the nursery school schedule.

**Nursery School Schedule**

**Morning**

8:30-9:00 Arrival  
8:30-9:40 Free play period  
9:40-9:50 Toilet time--juice  
9:50-10:00 Music  
10:00-10:40 Play period (outdoors, weather permitting)  
10:40-10:45 Toilet time  
10:45-11:00 Stories  
11:15-12:00 Rest
Afternoon

1:00-2:15 Arrival—free play (outdoors, weather permitting)
2:15-2:30 Toilet time—snack
2:30-2:50 Stories
2:50-3:00 Rest
3:00-3:10 Music
3:10-3:45 Free play
3:45-4:00 Departure and free play (outdoors, weather permitting)

In addition to the program outlined, science activities are sometimes included. These generally are conducted during the free play periods. The preschool program is divided into three types of activities. These are (1) adult directed, (2) free play, and (3) routines. Adult directed activities include science, music, and reading. Free play indicates that the child has a choice of areas that he may go to to play. These areas are (1) art, (2) block, (3) book, (4) creative, (5) housekeeping, (6) music, (7) tables for small muscle work, and (8) woodworking. Routines include toilet time, snack and lunch time, and rest.

Curricular packet.—As a consequence of the observations, testing, and readings which constitute the main body of this investigation, a curricular packet for use by the Family and Child Development Laboratory School of The Ohio State University is outlined in this chapter. The curricular packet includes (1) a teacher's guide, (2) a pretest, (3) the instructional guides, and (4) a cumulative card. The teacher's
guide supplies information as to the use of the pretest, guides, and cumulative card. The pretest gives information to enable the teacher to outline a program for the individual child, and the cumulative card supplies a continuous record of how the child is responding to this program.

Teacher's guide to curricular packet.—The curricular packet contains a pretest, curricular guides, and a cumulative record card. The pretest contains 52 items for testing many, but not all, the concepts used in the guides. The pretest is based primarily on visual recognition of the concepts using concrete objects rather than pictures. Although words are used they are in conjunction with the physical objects rather than basing the referent on past experience.

The cumulative record card is to be used to record the items from the pretest as well as observations. The card is also to be used to formulate next steps for the individual child in respect to increasing mathematical competency. The guides supply teaching situations and indications of learning that the child may have. When the concept has been sufficiently mastered the information is recorded on the cumulative record card. These cards are to be scrutinized every two weeks of the quarter to ascertain whether any progress has been made and also to gain further insight into a more viable program.

To use the packet and to teach mathematical concepts more effectively, the teacher needs to be familiar with the
material covered in the guides. For this reason a brief dis-
cussion of the various topics will be incorporated in this
section.

Sets.—A set is a group or a collection of objects
considered as an entity. The group must be clearly defined
so that one is able to tell whether a given object is or is
not a member of the set. The members of a set do not have to
be a part of some special class, and need only share one common
attribute, namely that they were selected to share the same
set. Young children have been using the word set and the
teacher must extend the concept.

More.—By comparing two sets the child can acquire the
idea of moreness. It is advisable to match two sets that
differ radically at first to emphasize the concept more. As
the child gains more competence the difference between the two
sets is reduced thereby causing the child to either pair in a
one-to-one correspondence or to count the two sets rationally.

One-to-one correspondence.—Two sets can be placed in
a one-to-one correspondence to test whether one of the sets is
more than, less than, or equal in quantity to the other. The
correspondence may be done without counting and hence the
beginnings of the correspondence may be accomplished before
any rational counting. The order in which inequalities and
equalities seem to be learned is the inequalities, more followed
by less, and then equality. When working with equivalent sets
stress that one-to-one correspondence is independent of
arrangement. The child needs to see that making one set longer does not increase the number of items in that set.

Cardinal numbers.--Most children can recognize sets up to five or six items by sight and thereafter have to count. The acquisition of the cardinals seems to follow the same sequence as their order. Rote counting should not be encouraged but rather the child should be encouraged to either see groupings or place a given set in a one-to-one correspondence with a set for which the child already has a cardinal number.

Measurement.--When comparing two objects to see which is longer be sure to first place the two objects so one end of each is together. Then proceed to test the child's ability to conserve length by moving one of the two. After the child has a grasp of the concept longer he should be exposed to measurement of distances with his body, followed by any handy object, and eventually with rulers.

Ordinals. The ordinals such as first, second, last, etc., necessitate an ordering and a direction for a given line. Ordinals should not be used in a competitive sense but rather as defining a certain position of a child or of an object.

Recognition of digits.--Most children will recognize several digits by the time they leave the nursery school. The digits that have relevance to them are most easily learned. This would include their own age, a school room number, their locker, etc.
Pretest.—The pretest that follows is to be used in place of Bettye Caldwell's Preschool Inventory for the coming year. Although this test will not reveal all of the child's capabilities in mathematics it will act as a base for constructing an individualized curriculum. An ideal place for conducting the test is in the observation room with the one-way glass. By using this room the observation room can be used by the person recording the test. This would leave the second person free to conduct the test questions without stopping to record the results. Materials for conducting the test must be placed in the room ahead of the testing period.

Materials that will be needed are:

- Fifteen blocks—same size and shape
- Toy boat
- Toy wheelbarrow
- Bicycle
- Tricycle
- Wagon
- Sixteen pennies
- Soft drink carton
- Eight-cup muffin tin
- Nine nails in a board
- Ten small chairs
- Five toy animals—wooden
- Two blocks—same size and shape with one much heavier
- Ten cards with individual digits
- Nickel, dime, quarter, half-dollar, and dollar
Pretest

1. How many hats do you have on your head?
2. How many noses do you have?
3. How many eyes do you have?
4. How many fingers am I holding? (Place your hand around three of his fingers making sure that he can still see the ends. Work questions 4 through 11 only until a child misses two in a row.)
5. How many fingers am I holding? (Hold four of them.)
6. How many fingers am I holding? (Hold five of them.)
7. How many fingers am I holding? (Hold six of them.)
8. How many fingers am I holding? (Hold seven of them.)
9. How many fingers am I holding? (Hold eight of them.)
10. How many fingers am I holding? (Hold nine of them.)
11. How many fingers am I holding? (Hold ten of them.)

Place fifteen blocks of the same size and shape on a table in front of the child. These will be used for questions 12 through 21. Replace blocks on the table after each response. Extend child until he makes two mistakes in a row.

12. Give me one block.
13. Give me two blocks.
14. Give me three blocks.
15. Give me four blocks.
16. Give me five blocks.
17. Give me six blocks.
18. Give me seven blocks.
19. Give me eight blocks.
20. Give me nine blocks.
21. Give me ten blocks.
Collect the following items and place them in the observation room before testing. For this part of the test there will have to be an aide scoring while one person is doing the testing (22 through 32).

22. How many wheels on this boat? (Make sure it is a simple boat with no steering wheel or life preservers.)

23. How many wheels on this wheelbarrow?

24. How many wheels on this bicycle?

25. How many wheels on this tricycle?

26. How many wheels on this wagon?

27. How many pennies do I have in my hand? (Hold five of them.)

28. How many bottles are there in this soft drink carton?

29. How many blocks are in this box? (Have seven in the box.)

30. How many muffins will this hold? (Have a muffin tin for eight.)

31. How many nails are on this board? (Have a board with a square array with three rows and three columns.)

32. How many chairs are there in this room? (Be sure there are exactly ten chairs in the room.)

Place three toy animals in a row facing in the same direction.

33. Point to the animal that is first in line.

34. Point to the middle animal.

35. Point to the last animal in line.

36. Point to the second animal in line.

37. Point to the third animal in line.

Place five toy animals in a row facing in the same direction. This part of the test is to be used only with those who have successfully completed 33 through 37.

38. Point to the animal that is first in line.

39. Point to the middle animal.
40. Point to the last animal in line.

41. Point to the fourth animal in line.

42. Point to the second animal in line.

43. Point to the third animal in line.

In questions 44 through 49 pennies will be used to differentiate inequality and equality.

44. What can you say about this set when comparing it to this other set? (This set refers to five pennies while the other set has two pennies.)

45. What can you say about this set when comparing it to this other set? (This set has six pennies while the other has five.)

46. What can you say about this set when comparing it to this other set? (This set refers to one penny while the other set is four pennies.)

47. What can you say about this set when comparing it to this other set? (This set has four pennies while the other has five.)

48. What can you say about this set when comparing it to this other set? (Both sets have two pennies.)

49. What can you say about this set when comparing it to this other set? (Both sets have eight pennies.)

Use two blocks that appear to be the same but where one has a heavy metal center.

50. Which of these two blocks is heavier?

51. Have the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 each on separate four by four cards and have the child identify as many as he can.

52. Place a penny, nickel, dime, quarter, half-dollar, and dollar bill on the table and ask the child to identify as many as he can.
Teaching guides. -- The teaching guides are intended as their name states, as guides not as a standard for every child. The wide differences in ability must be recognized. The main purpose of the guides is to take the child from where he is to a higher level of achievement through educational experiences that are suited to his specific abilities and needs. It is the nursery teacher's judgment as to what materials and experiences to make available for the optimal growth of the child's intellectual capacities.

The curriculum guides provide a semi-sequential pattern for the nursery teacher. Since there is no conclusive evidence as yet concerning a precise sequential pattern for learning beginning mathematical concepts the teacher needs to investigate the feasibility of various pathways. By following the sequence recommended in this pilot study the teacher should note the difficulties the children may have and then study these for possible revision of the sequence. Although many mathematical concepts form sequential patterns, others may not exhibit any relationship to each other and hence may be learned before, after, or concurrently with each other. By structuring the learning situation for the child and listening attentively to the feedback the teacher will be better able to make further plans for activities for the child. With no preconceived plans the teacher is not able to effectively cope with incidental mathematical activities in which the children may participate.
In order to use the guides more effectively a tentative sequential pattern has been formulated. This was derived from the observations and the results of the Preschool Inventory, as well as personal insight. After the child has been tested and the results placed on the cumulative record card the teacher should select those activities that correspond to the concepts that are shown to be deficient.

The recommended order of the concepts in the guides is as follows: sets, more, one-to-one correspondence, one, two, longer, three, first, middle, four, last, five, second, fewer, six, shorter, seven. Five items that do not seem to follow a sequential pattern within the framework exhibited and hence can be used whenever appropriate are: recognition of the digits, recognition of coins, recognition of geometric shapes, weight, and distance. Within the framework of several of these is a possible sequential pattern. For example, in recognition of coins the order to be followed is (1) penny, (2) dollar, (3) dime, (4) nickel, (5) quarter, and (6) half-dollar. Recognition of the digits should proceed from ones that seem to have pertinence for the child such as his age or house number and then possibly in numerical sequence. In using weight and distance the first activities should utilize the child's own body first. He should experience heavy objects by lifting them before attempting any sort of weighing process with a scales. Similarly, distance can be measured with the hands, a step, etc., before proceeding through measuring devices such
as a board or a pencil, and finally to standardized devices such as a ruler or a yardstick.

General objectives of the guides.—The guides are not intended as something to be learned by all but rather as "next steps" in the child's explorations with his environment. The acquisition of mathematical concepts should be in consonance with the child's search to better enable him to interpret the world. The general objectives, which are described more specifically in the guides, are to have the child:

1. Recognize and use sets as a base for further mathematical concepts.

2. Increase his mathematical vocabulary. The beginning words on which many other concepts are built include above, all, below, big, empty, full, heavy, large, light, little, long, many, more, most, once, short, small, tall, thin, and under.

3. Place sets in a one-to-one correspondence with each other in determining more and less and also equality.

4. Recognize small groups of two, three, four, and possibly five and six without counting.

5. Acquire a facility in use of the cardinals by noting the oneness, twoness, threeness, etc., of a set.

6. Develop the ordinals first, second, etc., to include first, last, and middle.

7. Recognize the ten digits in the Hindu-Arabic system.

8. Recognize the circle, square, rectangle, and triangle and see their uses in the environment.

9. Recognize the penny, nickel, dime, quarter, half-dollar, and dollar.

10. Extend the concept of weight from a kinesthetic approach to using scales.
11. Extend the concept of length by first measuring with own body, then objects, and finally with measuring devices.

12. Develop beginning concepts of the basic operations—addition and subtraction.
SETS

Objective: To include the word set in the child's vocabulary and to be able to differentiate sets.

Note to teacher. Use the word set liberally in situations that were denoted by other words such as group, some, the, etc. Following are several examples of situations where the word set may be used. Instead of saying "Here are some pencils," say "Here is a set of pencils." Instead of saying "Are you playing with the dishes on the table?", say "Are you playing with the set of dishes on the table?" At the science table say "Robert brought in a set of rocks for our science table," instead of "Robert brought rocks for our science table."

Teaching situations:

(1) Have the child place a string around a given set. For example, with six toy autos ask him to place the string around the red ones or use some other distinguishing characteristic. Use numbers only if the child has the given number in his repertoire or if you are attempting to move him to the next concept in numerosness.

(2) Have a box that is partitioned so that it has two parts. This is a set box. Place colored balls or any of numerous other articles in each side. Ask questions using the concept of set to elicit responses from the child. The set box can be used in the number concepts, more and less, pre-addition concepts, and in numerous other ways.

(3) Two mats or trays can be used similarly to the set box. Give the child directions concerning the placing or removal of items on the mats or trays. The mats and trays serve to focus the child's attention to the concept that the items placed on a particular tray have some common property which places them in the same set.

Evaluation: The child should respond correctly to two out of three situations.
Objective: To compare two sets to determine moreness in terms of numerosity. The children should observe without counting which set is more.

Note to teacher. It is advisable to compare groups that differ radically at first and to then make differences less.

Teaching situations:

(1) Compare a group of children at one table to a group at another table. Make sure one table has more than the other before comparing. Ask the question, "Which table has more children?" If the answer is correct you may introduce cardinality by asking "How many more?"

(2) Place two sets of crayons on the art table and while the child is engaged in crayoning ask him to select the set that has more crayons in it.

(3) Compare crackers, cookies, etc., at snack or lunch time if there are more of these items than there are children.

(4) Intentionally bring in not enough plates, food, or utensils at snack or lunch time and ask the children which is more, the children or the item.

(5) Place small toy objects on wood bases, e.g., pigs, cars, etc., and ask child which set has more. An alternate would be to create a farmyard scene with small animals inside fenced enclosures and compare sets.

(6) Use Set Box (described on previous page) or two egg cartons or two soft drink cartons or two muffin tins with objects in each.

(7) On two separate place mats place small objects such as toy parts, coins, or geometric cutouts and compare for moreness.

(8) Large dominoes can be utilized to determine which side has the greater number of dots.

(9) In carpenter corner nails may be compared to the number of hammers or the number of screws to screwdrivers, etc.
(10) At the puzzle and construction table compare construction blocks, plastic parts of interlocking sets, checkers, buttons, etc.

(11) At the science table compare legs of four-legged animals to the child.

(12) Compare a set that has two objects to one that has three.

(13) Compare a set that has nine objects and one that has ten.

(14) Use pictures such as those generally found in kindergarten and first grade books to check moreness.

Game: This game is to be used after the child has been exposed to many of the previous situations or the teacher believes the child to have a fair grasp of moreness. The child receives one deck of cards and the teacher uses another. Each deck has eighteen cards of which there are three blank cards, three with a picture of one item, three with two items, up to five. Teacher should pre-shuffle both decks and place them face down. Each person turns over top card. If child responds accurately to who has more he receives both cards. If not the cards are turned under the respective stacks.

Books: Brustlein, D. One Two Three—Going to Sea.

Evaluation: A child should be checked on five of the above fourteen situations before he is given credit for knowing more.
ONE-TO-ONE CORRESPONDENCE

Objective: To place two sets A and B in a correspondence so that to each member of A there is one and only one member of B assigned to it with no members of B left over.

Note to teacher. There are numerous situations in which the child is involved daily in which he is in a one-to-one correspondence with objects (e.g., hat, spoon, etc.).

Teaching situations:

(1) Make comparisons using the child's body. For example, see if he knows that he has a nose and that there is exactly one nose for every person. See if he can extend this same concept to some other part of his body such as one person—one mouth, one person—one head, etc.

(2) Set the table for snack or lunch time with a plate. Ask one of the children to place a fork or spoon at each plate. Ask another child to place a napkin at each plate. These items should be on a tray next to the table. Later on, after the child has successfully performed this task, have the child bring items from the kitchen that require a one-to-one matching. For example, they could bring straws or even the plates. The children could also be involved in serving food items that are distinct such as carrots, cookies, crackers, etc. (If the child has mastered one-to-one and also knows two and three he could be involved in many-to-one situations at the table.)

(3) Use the musical instruments and have the children reproduce the beat. The teacher should begin with a very simple beat and gradually make it more difficult.

(4) Use the drum and give instructions such as "Take as many giant (baby, duck, etc.) steps as I hit the drum."

(5) In coat weather compare the number of buttons with buttonholes. At all other times do the same with shirts and dresses.
(6) Place a small set of blocks on a table. Select some and place on the end of the table. Ask the child to select the same amount.

(7) In the dress-up area point out that there is one play hat for each child.

(8) For six children seated at table have one of the children go for enough paper for each to have one sheet.

(9) Arrange a table setting with plate, silverware and napkin in housekeeping area before children arrive. Observe the children playing in the area during the day to see whether they have copied the model.

(10) Have the child hold a string that is attached to a pull toy. Do the same for three or four more. Ask if there is one toy for each child. Then have the toys moved so they cover more length than the children. Ask the same thing. If it is missed bring the objects back closer to the children. Repeat. From this move to a table with objects that are to be matched one-to-one. Make sure they have the same length and then move one row so that it is longer. Ask if there is a matching. If the child centers on the length bring them back together saying, "See, they are the same."

Books: Martin, B. Sounds of Numbers, p. 32. McLeod, E. One Snail and Me.

Evaluation: The child must respond correctly to four out of the first nine and also to the tenth one.
Objective: The child will recognize oneness in a situation by being able to respond correctly to the following types of situations:

(a) Show me one ______.
(b) Give me one ______.
(c) How many ______ are there?

Note to teacher. Most nursery children will understand the concept "one" and this can be used to build many of the other mathematical concepts. For example, "one more than" can supply the cardinals and lead toward addition.

Teaching situations:

(1) The child should be cognizant of the fact that he has one head, one nose, and one mouth.

(2) Use the word one in sentences for "the" and the indefinite articles "a" and "an" whenever the change can be easily made. For example, "Give me one crayon," not "Give me a crayon." Be sure to use situations where there is more than one object to select.

(3) Use crayons, checkers, pieces of paper, books, toys, etc., to test the child in each of the three situations described in the objectives.

Books:

- Kay, H. *One Mitten*
- Lewis, L. *Davy Goes Places*
- Lenski, L. *Davy Goes Places*
- Martin, B. *Surprise for Davy*
- Martin, B. *Sounds of Numbers*, p. 30
- Martin, B. *Sounds of Numbers*, p. 139
- Tudor, T. *1 is One*

Evaluation: The child must respond correctly to all situations.
TWO

Objective: To be able to recognize twoness in various situations and to comprehend the "one more than" one of two.

Note to teacher: Since so many animals have two or four of various parts of their body it seems imperative to orient the child to two by first using his own body and then to look for twoness in the animals. This could then be followed by the twoness in other physical objects and finally by referring to pictures of twoness.

Teaching situations:

(1) Have the child raise the number of fingers that you have raised. (Raise one.) If he says the correct number say "Raise one more." Say "How many is that?" If correct, go on to three. If incorrect, say "There are two fingers." Place one hand around two fingers to solidify the set idea and to keep the child from thinking that the new finger is "two." Use the concept of more at this point by saying "Which is more, one finger or two fingers?"

(2) Prepare before class time groupings of two throughout the room. For example, you may place two chairs at one table, two pictures on the bulletin board, two fish in the fishbowl, two books on a table, etc. Point out one or two of the most difficult examples of two to the child, such as the fish in the fishbowl, and then ask the child to find some other examples of two objects.

(3) Ask the child to identify something concerning himself that comes in twos. If he is unable to do so, point out several and then give him a chance to continue. Some examples are arms, legs, hands, feet, eyes, and ears. He may also say socks, shoes, or gloves.

(4) At snack or lunch time pass a tray of cookies (crackers, carrot strips, etc.) and say "Take only two, please."

(5) Have the child close his eyes, reach in a sack, and tell how many objects there are. This can also be used for working with the other cardinal numbers.
(6) In sand and water areas use containers that are in units. For example, use transparent 1-cup, 2-cup, and 3-cup containers. Use the smaller one to fill the other two. Question the number of cups needed to fill these two containers.

(7) Teacher either sets up in the doll corner two items of a kind before they arrive or waits for the children to play together to point out two items of a kind. For example, "I see that you have two chairs at your table," or "How many dishes are at your table?", or "I see there is one spoon on this side of the table and one on that side. How many spoons are there on the table?"

(8) Arrange three, four, five, or six objects on a table or on a tray. For example, have in your set two of the same color, two of the same shape, two of the same texture, etc. Identify the pair you wish selected by signifying their common characteristic.

(9) Engage the children in conversation concerning the live animals and insects brought into the class. Find out similarities and dissimilarities between the children and the animals and insects. For example, point out two eyes, two legs, four legs, two wings, etc.

(10) In the music area say "Clap your hand two times," and beat the drum twice.

(11) Have toy farm animals or toy zoo animals in pairs and have them placed in separate pens. Emphasize the "twoness."

Books:

Kay, H. One Mitten Lewis.
Lenski, L. Davy Goes Places.
Lenski, L. Surprise for Davy.
Rey, H. Curious George Rides a Bike.
Ylla. Two Little Bears.

Evaluation: The child must respond correctly to five out of the eleven situations.
LONGER

Objective: To compare two objects to determine which is the longer.

Note to teacher. Piaget's studies cast some doubts on the preschooler's ability to comprehend fully this premeasurement concept.

Teaching situations:

(1) Use two objects that are definitely different in size, that is, the difference is not minimal. For example, use a Tinkertoy part that is eight inches long and compare it to one that is three inches long. The following sequential steps should be adhered to in presenting the concept longer. The material need not be given all at one time.

(a) Use objects that are in the same class. For example, use the two Tinkertoy parts and place them so they are like the following drawing.

```
A------B
C-------------D
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Ask which is longer. If the child answers correctly go on to (b). If not, point out the longer one and give several other examples using objects that are placed parallel with one end of each opposite. Other items that may be used are building blocks, pencils, pieces of paper, strings, etc. Have the child respond to several examples after he has seen the teacher's examples.

(b) Use objects that are in differing classes. For example, compare a pencil's length with a crayon. Place the objects as in (a) and repeat the process from (a).

(c) Use objects that are in the same class. Place them as in (a) and then move A to the right in the child's presence so that it now looks like the following.

```
A------B
C-------------D
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Ask the child which is longer. If he says AB, shift it back to its previous position, pointing out that CD is longer. Then very slowly move AB to the right stopping along the way to ask which
is longer. Repeat several times if the child does not respond accurately.

(d) Compare two objects that are separated. For example, hold a paper clip in one hand and a hat pin in the other. Ask which is longer. If the child makes a mistake place them as in (a) and ask the same question.

(e) Compare objects that the child cannot see. Ask him which is longer, a crayon or a car, a loaf of bread or a needle, etc.

(f) Extend the concept longer to include longest by introducing three or more objects.

(2) Use two objects that are approximately the same length. For example, a 3d and a 4d nail. Do the same sequence as in (1). These two sequences should not be performed on the same day.


Evaluation: Credit longer if successfully passing the whole sequence from a through e in both (1) and (2). Credit longest if successful in (1)f and (2)f.
Objective: To recognize the various sets that share the equivalence class of three members.

Note to teacher. The child should not be forced to count one, two, three, but rather should be given encouragement to recognize three without counting.

Teaching situations:

(1) Ask the child to raise two fingers. If he does, ask him to raise one more. If he does, ask him how many he has raised. If he is unable to answer correctly tell him the number. Then point out other examples of three in the room. Pre-arrange sets of three throughout the room.

(2) Use shells, seeds, pine cones, stones, etc., on a tray or mat and select three of one of the sets and ask the child to do likewise.

(3) Compare three in a set to one in a set and also three to two using the idea of more.

(4) Use six, nine, or twelve objects such as checkers, seeds, dominoes, etc., and ask the child to separate them into sets of three. If successful at this task use sets that would have some left over, such as eleven or thirteen.

(5) Space items on a sheet of paper so there is one separated from a set of two. See whether the child is able to tell you how many there are on the sheet of paper. This should aid in making the child more aware of the one moreness of three as compared to two and to also serve as a step toward addition.

(6) On the playground ask the child the number of small wheels on a tricycle and then ask the number of large wheels. If these are both correct then ask him how many there are all together.

Books:

- Austin, M. The Three Silly Kittens.
- Becker, C. Three Little Steps and the Party.
- Martin, B. Sounds of Numbers, p. 12 and p. 32.

Evaluation: The child must perform correctly on four out of six.
ORDINALS

Objective: To learn the ordinals from one to six plus middle and last.

Note to teacher. Most children learn the ordinals in the following sequence: middle, first, last, second, third, fourth, etc.

Teaching situations:

(1) Line up children to go outside. Ask who is first, second, third, middle, last, etc. If no answer point out the ordinal positions. Line them up once more when returning to the building and check them again.

(2) If a child is stacking blocks say the ordinals after each one is placed.

(3) At snack or lunch time say "We are serving Dan first, Jean second, Betty third, and John last.

(4) When children ask to be pushed on the swings, say "I will swing Barbara first and Joe will be second.

(5) Place the play hats in a row on top of the counter. As the children take them off to play say the name of the child and tell which ordinal corresponds to that child.

Books:
- Duvoisin, R. Two Lonely Ducks.
- Friskey, M. Chicken Little Count-to-Ten.
- Friskey, M. Seven Diving Ducks.
- Ipcar, D. Ten Big Farms.
- Martin, B. Sounds of Numbers, p. 12, p. 32, and p. 108.

Evaluation: Credit a child with any ordinal only after hearing the child use it correctly at play. The child may be guided toward responding.
Objective: To recognize at a glance groups of objects having cardinality of four.

Note to teacher. Some children will start rational counting at this stage while others will continue recognizing groups on up to six. Encourage group recognition but, at the same time, do not discourage counting.

Teaching situations:

(1) Place four glasses with four spoons in them on the table. If a child can tell that there are four glasses ask him how many spoons there are. If he tells you immediately he has very probably used a one-to-one correspondence. If he waits for a while before answering he may have counted them one by one. If the child was unable to tell you there were four glasses tell him there are four and then continue questioning.

(2) Check to see how many buttons the child has on his shirt, coat, or dress. If there are four have him count them.

(3) Point out things such as tables, chairs, animals, wagons, trucks, automobiles, etc., that have some apparent item representing the class of four.

(4) To check the concept of two and two ask how many eyes there are all together on the child and the teacher. The same thing can be done with hands, arms, feet, legs, ears, etc.

(5) Place three items in one side of the set box and one in the other. Check the child to see whether he can verbalize the one moreness of four as compared to three.

(6) Place four empty bottles in a soft drink carton. Observe the child counting the bottles. He may get the answer by observing the pattern, or by counting or by adding two and two. This type of problem serves as a beginning toward addition and multiplication.


Evaluation: The child must respond correctly to four out of six situations.
Objective: To recognize and/or count groups that contain five members.

Note to teacher. At this stage the child is generally ready for the idea of "one more than" and subgroupings of sets that make up five, which is a step toward addition.

Teaching situations:

(1) First point out that there are five fingers on one hand. Ask how many there are on the other hand. After he is able to discern that he has five fingers, which he can see, check to see whether he is able to tell how many toes on each foot, which he cannot see.

(2) Group pennies into sets of four and one. Check fiveness. Proceed to three and two. Then two, two, and one.

(3) Use a large domino. Proceed from recognition of this to the perception of the dots changing to the numeral.

(4) Roll six blocks with each of the six faces painted a different color. To check the knowledge of the cardinals from one to six ask how many of the blocks have a top face of a given color.

Books: Fehr, H. *Five is 5.*

Evaluation: The child must respond correctly to three out of four situations.
Objective: To compare two sets to determine which is less in terms of cardinality. The children should observe without counting which set is less.

Note to teacher. If the child has not already mastered more, it is doubtful that he will comprehend less.

Teaching situations:

(1) Compare two groups of children. If the child answers correctly to which group is more, then ask which has fewer.

(2) Place two sets of crayons on the art table and while the child is engaged in crayoning ask him to give you the set that has less crayons than the other set.

(3) While in the play yard ask the child whether there are fewer children or fewer tricycles. (Use swings, wagons, etc., as alternates.)

(4) In housekeeping area compare the number of ironing boards, or irons, or refrigerators to the number of children in the area.

(5) Compare two sets of zoo animals that are enclosed.

(6) In woodworking area compare the number of saws to the number of pieces of wood in the bin.

Evaluation: The child must respond to three out of six situations.
SHORTER

Objective: To compare two objects to determine which is the shorter.

Note to teacher. Knowing longer seems to be a prerequisite for knowing shorter so check to see that the child understands longer before teaching this concept.

Teaching situations:

(1) Use the procedure for longer only substituting the word shorter for longer.

(2) Use two pipe cleaners of differing lengths and check the longer-shorter dichotomy. Use statements like "If this one is longer, then the other one is what?" and vice versa.


Evaluation: Credit is to be given for passing la through e and 2a through e from the guide for 'longer.'
SIX, SEVEN, EIGHT, NINE

Objective: To extend the cardinals beyond five to include nine.

Note to teacher. Recognition of the group stops by this point in most cases. Child must now count. Encourage child to continue recognizing subgroups of two to five objects as much as possible.

Teaching situations:

(1) Place a group of objects on a table with subsets arranged like the dots on a domino. For example, nine may be placed as six and three or five and four. This would encourage the child to spot one of the two sets and then to count from that point on. For example, with five and four he may say five, six, seven, eight, nine.

(2) Have the child count the number of holes in a muffin tin that has six or eight holes.

(3) While in the play yard have the child count the number of wheels on two wagons.

(4) Count objects such as the following: tongue depressors, pegs on a pegboard, beads, juice cups, collections of stones, blocks in block area, crayons, books, children, etc.

(5) Have the children be responsible for bringing the correct number of scissors or crayons or paper, etc., for a table. First make them available so the child and the objects are in close proximity. If this is accomplished, at a later time place them out of sight so the child has to use some means of placing the objects in a one-to-one relationship with the children at the table. This may mean bringing the incorrect number and having to take some back or to pick up more. To accomplish this task the child may rationally count, make a one-to-one correspondence, or guess.

Books:

(The first group of books contain concepts from one to seven, the second from one to ten, and the last group from one to twelve.)

I. Barr, K. Seven Chicks Missing.
   Seignobosc, F. Jeanne-Marie Counts Her Sheep.
II. Brustlein, D. One, Two, Three—Going to Sea.
Duvoisin, R. Two Lonely Ducks.
Friskey, M. Chicken Little Count-to-Ten.
LeSieg, T. Ten Apples Up on Top.
Martin, B. Sounds of Numbers, p. 80.
McLeod, E. One Snail and Me.
Moore, L. My Big Golden Counting Book.
Watson, N. What is One?
Wolff, J. Let's Imagine Numbers.

III. Martin, B. Sounds of Numbers, p. 108.
Ziner, F. Counting Carnival.
Zolotow, C. One Step, Two . . .

Evaluation: Credit for any one of these cardinals if the child is able to successfully count a certain number of objects in the same equivalence class.
RECOGNITION OF DIGITS

Objective: The child will be able to recognize the ten digits which will be indicated by his ability to (1) verbally associate the correct word with the digit and (2) to point to the correct digit when asked "Which one is _____?"

Note to teacher. The learning of the digits does not seem to follow a sequential pattern. The child may learn the digit associated with his own age first or the digit associated with a favorite television channel. A recognition of the complete set of digits should not be expected of anyone but the highest achievers in the four-year-old group.

Teaching situations:

(1) Place the numerals from 1 to 14 on the children's lockers. Do not stress that they have to know these but use them to give them experience in seeing numerals used.

(2) As you go by rooms with numerals on the doors on the way to or from the play yard read the digits on some of the doors. Be alert to children who begin to do the same.

(3) Point out numerals on the clock and on the calendar.

(4) Have the child look at the digits in his own house number.

(5) Read stories containing digits and let the children see them as you read. Also point out how they are used in the page numbers.

(6) Use interlocking puzzle shown below to associate objects with numerals.

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* * 3
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(7) Use cards like the following that start with a number of dots which gradually fade into the digit.

![Image of cards with dots fading into a digit]

(8) Glue felt or sandpaper numerals on cardboard. Have child feel them as you sound the names.

(9) Place a temporary number line on the floor using masking tape. Number squares such as the following could also be used.

![Image of number squares]

Books:
- Barr, K. Seven Chicks Missing.
- Brustlein, D. One Two Three—Going to Sea.
- Duvoisin, R. Two Lonely Ducks.
- Friskey, M. Chicken Little Count-to-Ten.
- Martin, B. Sounds of Numbers, p. 108.
- Moore, L. My Big Golden Counting Book.
- Seignobosc, F. Jeanne-Marie Counts Her Sheep.
- Simon, N. A Tree for Me.
- Wolff, J. Let’s Imagine Numbers.
- Ziner, F. Counting Carnival.

Evaluation: Credit is to be given for responding correctly to a given digit in any situation.
RECOGNITION OF COINS

Objective: To acquaint the child with a penny, nickel, dime, quarter, half-dollar, and a dollar bill. The task is to be able to recognize the coin, not its value.

Note to teacher. The approximate order in which most children learn to recognize coins is the following: penny, dollar, dime, nickel, quarter, and half-dollar. Comparative value of the coins has little meaning to the three- and four-year-old.

Teaching situations:

(1) Use a set of one of the coins in recognition of the cardinals from one to nine.

(2) Use a set of one of the coins in recognition of the ordinals from one to six.

(3) Use the coins to reinforce the idea of more.

(4) Use the coins to reinforce the idea of less.

(5) Place a set of the coins that has been placed in plastic on the science table to generate questions.


Evaluation: Recognition of a coin only once is sufficient for credit.
Objective: To compare the weight of two objects by a kinesthetic approach and to compare later by actually using the balance scale.

Note to teacher. The child needs much experience in comparing objects by their respective weight kinesthetically. It is advantageous to use two objects that are definitely different in weight rather than two that are approximately the same. Do not hasten the use of the scales.

Teaching situations:

(1) Use one, two, three, four, and five pound wooden blocks whose respective lengths are in the same ratio as their weight. Place different weights in the child's hands and have him tell you which is heavier.

(2) Do the same as in (1) only use two items where the larger is the lighter such as a block of balsa and a piece of iron.

(3) Have the child guess which of two sacks is the heavier. Fill a larger one with crumpled paper and a smaller one with a lead weight. After he has guessed have him pick them up and ask him again.

(4) Use the blocks cited in (1) on a balance scale to compare objects. This is useful for solidifying the cardinal values as well as paving the way for basic addition facts.

Books: Schlein, M. Heavy is a Hippopotamus.

Evaluation: The child must successfully perform (1), (2), and (4).
MEASUREMENT OF DISTANCE

Objective: To introduce measurement of distance by units related to the body (e.g., steps, arm's length, etc.) followed by applying physical objects.

Note to teacher. Measurement requires a facility with the cardinal numbers since a reiteration of the measuring device as well as a means of noting these applications is required.

Teaching situations:

(1) Use the one, two, and four unit length blocks. Point out that it takes two lengths of one to be the same as the next block.

(2) Step off distances on the playground or in the room to find out how far it is between two objects.

(3) Using the hand as a measuring instrument find out how many hands wide a desk is. A pencil, book, or other such instrument may be used to measure objects.

(4) Use a ruler to measure lengths but do not attempt to go into inches or fractions of inches.

Books: Myllar, R. How Big Is a Foot?

Evaluation: Credit is to be given if the first three situations are successfully accomplished.
GEOMETRIC SHAPES

Objective: To recognize the square, circle, triangle, and rectangle.

Note to teacher. Recognition of these basic shapes can give the child a means of describing many things in his environment and hence better able to grasp other concepts.

Teaching situations:

(1) Point out things in classroom having the given shape.
   a. Square—cabinet doors
   b. Circle—cups, glasses, sand toys, water toys, records, plates, clock, coins
   c. Rectangle—windows, blackboard, books, piano, cots, blocks
   d. Triangle—climbing apparatus

(2) Point out things in play yard having the given shape.
   a. Square—side of tool building
   b. Circle—wheels on vehicles, center of playground, sandbox
   c. Rectangle—jungle gym, sides of two buildings
   d. Triangle—braces on swings

(3) Make cutouts of masonite, or cardboard, or of cloth. Have children feel the texture and identify.

(4) Use the game box that has a square, triangle, and circle and have the child identify each as he drops them through the slot.

(5) Place geometric designs in a sack and have the child feel it and identify.

(6) Use geometric cloth or paper cutouts of circles, squares, triangles, and rectangles for a collage.

(7) Have patterns of the square, circle, triangle, and rectangle stamped onto paper using sponges shaped like these figures.
(8) Give the child four cards with the basic figures. Spin the pointer on the following and instruct the child to show the card that has the same figure.

(9) If successful with (8) use a different spinner with color and shape both involved. Give the child eight cards to match this time.

Books:
Borten, H. Do You See What I See?
Budney, B. A Kiss is Round.
Jean, P. Patty Round and Wally Square.
Kohn, B. Everything Has a Shape.
Martin, J. Round and Square.
Schlein, M. Shapes.
Shapur, F. Round and Round and Square.
Ungerer, T. Snail, Where Are You?

Evaluation: Present a square, circle, triangle, or rectangle to the child whenever he seems ready. He does not have to pass all at the same time.
Books. Books are useful in presenting concepts to the child since they are entertaining. They also reinforce concepts he already has as well as providing a source for new information built on the old. The books cited in the teaching guides may be used for a group of children during the story time but a more advantageous use would be to the individual child who seems ready for advancing to a new concept. For example, if a child has a grasp of the cardinals up to two then a story concerning one, two, and three would be appropriate. Many counting books go from one to ten and these would seem more appropriate to a group whereas books pertaining to only one concept, such as twoness, seems more apropos to the individual child. A list follows citing the books containing mathematical concepts that are currently in the laboratory library as well as recommended additions.

Books Containing Mathematical Concepts
Currently in Laboratory Library


Brown, Margaret. *The Little Fisherman* (N.Y.: Wm. Scott, 1945), Big and little.


Friskey, Margaret. *Chicken Little Count-to-Ten* (Chicago: Children's Press, 1946), Cardinal 1-10 and digits 1-10.


Recommended Additions to the Laboratory Library


Berkley, Ethel. *Big and Little, Up and Down* (N.Y.: Wm. R. Scott, 1960) Little, big, long, tall, wide, narrow, short, up, down, high, low, bottom, over, under.


Brustlein, Daniel. *One Two Three--Going to Sea* (N.Y.: Wm. R. Scott, 1965) Cardinals one to ten, "one more than," digits 1 to 10.

Duvoisin, Roger. Two Lonely Ducks (N.Y.: Alfred Knopf, 1955)
Cardinals one to ten, ordinals first to tenth, and
digits 1 to 10.

Federico, Helen. The Golden Happy Book of Numbers (N.Y.: Golden
Press, 1963) Cardinals from one to ten.

Fehr, Howard. Five is 5 (N.Y.: Holt, Rinehart and Winston,

Fisher, Margery M. One and One (N.Y.: Dial Press, 1963)
Cardinal numbers.

Friskey, Margaret and Katherine Evans. Seven Diving Ducks
(Chicago: Children's Press, 1965) Six and seven;
sixth and seventh.

Gregor, Arthur. 1, 2, 3, 4, 5, Verses (Philadelphia: Lippincott,
1956) Cardinal numbers; counting and numerical
order.

Ipcar, Dahlov. Ten Big Farms (N.Y.: Alfred Knopf, 1958)
Ordinal numbers one to ten.

Jean, Priscilla. Patty Round and Wally Square (N.Y.: Obolensky,
1965) Round and square.

Kaufman, Joe. Big and Little (N.Y.: Golden Press, 1966) Big,
little, tall, small.

Kohn, Bernice. Everything Has a Shape (Englewood Cliffs, N.J.:
Prentice-Hall, 1964) Patterns and shapes.

Cardinals one to ten.

MacDonald, Golden. Big Dog, Little Dog (Garden City, N.Y.:
Doubleday and Co., 1943) Big and little.

Martin, Bill. Sounds of Numbers (N.Y.: Holt, Rinehart and
Winston).

Martin, Janet. Round and Square (N.Y.: Platt and Munk, 1965)
Circle, square, and rectangle.

McLeod, Emilie. One Snail and Me (Boston: Little Brown, 1961)
Cardinal one to ten and one-to-one correspondence.

Moore, Lilian. My Big Golden Counting Book (N.Y.: Simon and
Schuster, 1957) Cardinal one to ten and digits 1-10.
Premeasurement using a person's foot.


Tudor, Tasha. 1 is One (N.Y.: Henry Walck, 1956).


Wildsmith, Brian. Brian Wildsmith's 1, 2, 3's (N.Y.: Franklin Watts, 1965) Counting and numerical order.


Materials for an effective mathematics program in the nursery school.—Materials aid the child in assimilating messages from the environment and relating himself to his world. Through acquisition of concepts he clarifies hazy and incomplete ideas and transforms them into a clearer picture. The nursery school, through materials, can simulate the world for
the child. Through intelligent selection of materials the nursery teacher can maximize the amount of learning that takes place. The teacher needs to be aware of the possible materials to be used and then must be willing to try them with the children. To check whether a certain object is doing what it is purported to do the child and the object must come into contact. It is then the teacher's responsibility to see whether it is doing what it is supposed to and if it is not to eliminate it from the program.

There are fundamentally three ways that materials may be used. These are (1) manipulative, (2) representational, and (3) rule-bound. As the names intimate manipulative embodies using the hands and eyes in rearranging materials, representational places emphasis on acting out parts, and rule-bound supplies rules for a game to be followed. Although nursery age children mainly do manipulative and representational play the materials cited in this chapter refer primarily to manipulative. The two lists that follow cite materials that are currently in the Family and Child Development Laboratory and the recommended materials to be added to the laboratory.

### Materials currently in the Family and Child Development Laboratory

<table>
<thead>
<tr>
<th>Arithmetic tangibles</th>
<th>Balance scales</th>
<th>Blocks—plastic</th>
<th>Blocks—wooden</th>
<th>Blocks—hollow wood building</th>
<th>Domino blocks</th>
<th>Farm animals</th>
</tr>
</thead>
</table>

Flannel board
Freeplay posts
Hats
Lock-a-blocks
Number sorter
Parquetry blocks
Pegboard
Pegs--wooden
Shape sorter
Snap blocks
Tinker toys
Unit blocks 1, 2, 4, and 8 unit lengths
Zoo--wooden
Zoo animals

Recommended materials to be added to the laboratory

The recommended materials will be listed under the names of the companies that supply them. A brief description will follow materials whose titles do not fully suggest what they are.

Community Playthings, Rifton, New York, 12471

Large block-size dominoes
Pots and pans (contains graduated measuring cups and measuring spoons)

Creative Playthings, Princeton, New Jersey, 08540

Aluminum sand cans and sifter
Arithmetic tangibles (four sizes of squares, circles, and triangles to place in slots)
Blockmobiles (block automobiles with pegs to fit in holes. Will hold one to seven pegs)
Matchmates (number on top corresponds to number of objects in bottom half)

Fisher Price, East Aurora, New York

Creative blocks (circles, squares, and triangles)
Milton Bradley, Springfield, Massachusetts

Flannel board cut outs
Flannel board numbers
Introduction to sets and numbers
Number concept cards

Novo, 585 Sixth Ave., New York, 10011

Geometric insert sets
Self-correcting learning numbers
Self-correcting number puzzle

Teaching Aids, 159 W. Kinzie St., Chicago, Illinois, 60610

Basic weight tablets (wood of same dimensions but differing weights)
Counting box and spindles (counting tray for one to ten)
Sandpaper numbers
The long stair (rods in 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 unit lengths)

The materials cited in the list for Teaching Aids are Montessori type aids. In addition to the listed materials some were not found on the market and hence could possibly be made for use in the laboratory school. These materials will now be designated and described.

Abacounter
Matching board (nine 3-inch squares and one 9" x 9" board. Each square contains one, two, or three objects to be matched with ones on the board)
Measuring devices--cup, quart, half gallon
Number-numeral chart (to be used on the bulletin board. Number objects for one through nine to be changed weekly)
Set box (a box partitioned into two sections. Used for differentiating two sets)
Tray (to be used for placing one, two, three, and four objects in the four compartments. Each section will contain the numeral written out (e.g., three), the digit (e.g., 3), and the number of dots corresponding to the numeral (e.g., ).
Weight blocks (one, two, and four pound wooden blocks for use on scales)
Cumulative card.—The cumulative card is to be used to record the results of the pretest and then to record the mastery of the various concepts as determined by the criteria set forth on the curriculum guides. A child is to be given credit for having knowledge of a concept if he passes all parts of the test pertaining to the concept. Subsequent information the teacher may gather may show that the child acquired a correct answer on the test without having the required concept, and consequently the child should be exposed to the material covered in the guides. Following are the concepts and the question numbers that must be correct in order to give credit.

- More 44,45
- Zero 1,22
- One 2,12,23
- Two 3,13,24
- Three 4,14,25
- First 33,38
- Middle 34,39
- Four 5,15,26
- Last 35,40
- Five 6,16,27
- Second 36,42
- Fewer 46,47
- Six 7,17,28
- Seven 8,18,29
- Equal 48,49
- Eight 9,19,30
- Nine 10,20,31
- Ten 11,21,32
- Digits 51
- Coins 52

The cumulative cards for all children should be reviewed at the end of every second week during the first year of the use of the guides to evaluate pupil progress as well as to evaluate the program.
CUMULATIVE CARD

Name_________________________________ Birthdate____________________________

Date Mathematics Test Given________________________

Place an X in the space following each number that was correct on the mathematics test.

27. ___ 28. ___ 29. ___ 30. ___ 31. ___ 32. ___ 33. ___ 34. ___
35. ___ 36. ___ 37. ___ 38. ___ 39. ___ 40. ___ 41. ___ 42. ___
43. ___ 44. ___ 45. ___ 46. ___ 47. ___ 48. ___ 49. ___ 50. ___
51. ___ 52. ___

Give the date when the following concepts were mastered. Use instructions given with the curriculum guides.

Sets
1. ___ 2. ___ 3. ___

More
1. ___ 2. ___ 3. ___ 4. ___ 5. ___ 6. ___ 7. ___

One-to-one
1. ___ 2. ___ 3. ___ 4. ___ 5. ___ 6. ___ 7. ___
8. ___ 9. ___ 10. ___

One
1. ___ 2. ___ 3. ___

Two
1. ___ 2. ___ 3. ___ 4. ___ 5. ___ 6. ___ 7. ___
8. ___ 9. ___ 10. ___ 11. ___

Longer
1. ___ 2. ___

Three
1. ___ 2. ___ 3. ___ 4. ___ 5. ___ 6. ___

First
1. ___ 2. ___ 3. ___ 4. ___ 5. ___

Middle
1. ___ 2. ___ 3. ___ 4. ___ 5. ___
<table>
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Observations of use of other mathematical concepts. This would include time, units of measurement, fractions, etc.
CHAPTER V

SUMMARY, CONCLUSIONS, AND IMPLICATIONS

Summary

This pilot study was undertaken to design a set of mathematics curriculum guides to be used by the Child Development Laboratory School of The Ohio State University. To have a sound theoretical and empirical base for these guides, it was first necessary to investigate the nature of the mathematical abilities of the particular children involved and also from the literature to find what researchers have found the three- to five-year-old is capable of learning.

The mathematics section of the Preschool Inventory was administered to 54 children from the laboratory and 13 from two Head Start centers. Observations were made at the laboratory, two Head Start centers, and one day-care center during the summer quarter, 1968. As evidenced in the testing and observations, the children exhibited a wide range of abilities in handling mathematical concepts. Clearly, this wide range implies the need for an individualized program. Meeting this need does not eliminate some group activity, but the main emphasis should be on a planned program in which the teacher causes the child to interact with materials and ideas. In
effect, a "responsive environment" requires pre-planning for mathematical understanding at the preschool level.

Analyses of the test results and observations were instrumental in the material selected for the curriculum guides as well as the sequential pattern. The guides are to be presented to the staff of the Family and Child Development division for possible adoption during the coming year. The subsequent program will be evaluated continuously for possible revisions, and new insights into the nature of beginning concepts of mathematics.

General findings

1. The order that the concepts were learned, using the Preschool Inventory results, was similar for the threes and fours in both the laboratory and Head Start programs. This suggests an optimal learning sequence for the basic mathematical concepts.

2. In the laboratory, the fours were more advanced than the threes and the same result was evidenced at the Head Start centers. In addition, the laboratory threes were more advanced than the Head Start fours in comparing mathematical capabilities.

3. The number of quarters of attendance in a nursery school seemed to have some influence on mathematical competency. This could be attributed to the teachers stressing certain mathematical ideas either consciously or unconsciously, while ignoring others.
4. The children who scored higher in mathematics competency on the test exhibited more use of mathematics in encounters in the nursery school. These children initiated more mathematical contacts with their teachers, other children, and objects than did the children who scored lower on the test. The teachers also initiated more mathematical contacts with these children, hence perpetuating a cycle.

5. Boys and girls in a class did not differ significantly but the difference between classes of the same age was evident. This could possibly point out the relevance of the role of the teacher.

6. Approximately one-half of the threes were able to comprehend groups up to three, while about one-half of the fours reached "five." There were wide variations within the group concerning cardinality. Some did not understand "one-ness" while others tested correctly as high as "twenty." Individuals in this latter category were not studied further to see the upper limits of their concepts.

7. Approximately one-half of the threes were able to rote count to five or more, while three-fourths of the fours could. The rote counting ability of most exceeded the ability to rationally count. In several cases, children who could not rote count were very adept with rational counting. Head Start children compared favorably with the laboratory children in the ability to rote count but were noticeably deficient in all other categories.
8. The dichotomies, which are generally thought of as premathematical concepts, were found to possibly not exist dependently on each other but rather as separate entities. For example, "more" seems to be learned first to some degree before "less" is begun. Some facility with "more" and then "less" seems a prerequisite for learning equality.

Within the dichotomies, an emerging pattern seemed to point out the initial learning of the concepts such as "more," "longer," "bigger," "heavier," "higher," and the like over their counterparts.

9. "Middle" and "first" seemed to be the first ordinals that the nursery age child understood. The ordinals seemed to start about the time the child was comprehending the cardinal number "three" and then to remain about this far behind as each cardinal and ordinal was grasped. All the current programs that were reviewed had the child engage in a sequence of exercises with the cardinals from "one" to "ten" before doing the same thing with the ordinals. If the finding in this investigation is valid, it would suggest the learning of the two in numerical order but slightly out of phase with each other.

**Implications**

1. The child's first encounters with mathematics should be with concrete materials rather than with printed materials. Experience with the written symbols is not
appropriate except for some one exceptional child who may be a member of the group.

2. Teachers need to initiate more mathematical contacts. This deliberate arranging of the environment, in turn, causes the child to acquire more information which, in turn, causes him to initiate contacts, thereby fostering a learning cycle.

3. Each concept must be presented in a variety of ways until the child is able to grasp it firmly. Then, the concept needs to be nurtured and enriched so that other concepts can be built on it. In modern, technological terminology the "spin off" idea helps to describe this relationship.

4. Individualized programs and accurate records are needed to maximize learning. Instead of presenting material to a group of children without regard to the individual variations, knowledge of the individual child must be used in involving him in a program.

5. Knowledge of the sequential learning patterns within the mathematical framework should aid the teacher in designing an appropriate program for the child. Accurate records should give the teacher more insight into refining the sequential patterns.

Suggestions for Further Study

From the general findings and implications of this study, some suggestions for further investigation are in order. They are:
1. Make a longitudinal study of one child in the three-to-four and one in the four-to-five year age bracket over a one year span. Find out from language and interactions with others how mathematical concepts grow. Keep anecdotal records at school and at home.

2. Investigate the first three years concerning acquisition of mathematical concepts.

3. Search for more precise sequential learning patterns.

4. Attempt to find out what is involved in a child progressing from one concept to a subsequent one, such as from one to two, two to three, and so on.

5. Find out what effect introduction of concepts at the preschool level has on subsequent learning.

6. Explore to see whether there are critical periods for learning certain mathematical concepts.

7. Inquire into the learning of the dichotomies to see whether they actually are learned separately or together.

8. Investigate to see whether certain concepts can be effectively learned in a group setting.

9. Explore the results of a planned mathematical period which is conducted either once a day for a short time (e.g., ten minutes) or two or three times a week.

10. Develop a program for nursery school teachers and test their subsequent influence on the children. This may
prove to be the most effective way to improve the children's
cognitive mathematical growth.

11. Explore the use of pictures, film loops, and the
computer, using them much like O. K. Moore did with his
typewriter.

12. Investigate how the parents may be involved in
helping the child learn.

13. Inquire into the conditions in the home in an
attempt to see what differences there are between high and low
ability children in mathematics.

The findings of this study which were intended to
provide an initial, heuristic base and some beginning cur-
riculum guides for individualizing programs on this base
should serve as one set of bench work ideas for launching
any one or a number of the above investigations.
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