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THE POLARIZATION OF NEUTRONS FROM
THE $^{13}$C($^3$He, $n_0)^{15}$O AND $^{24}$Mg($^3$He, $n_0)^{23}$Si
REACTIONS BETWEEN 4 AND 6 MeV

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
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By

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* * * * * *

The Ohio State University
1969

Approved by

T. R. Donoghue
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To my father

whose example provided
the motivation for
this work
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"Polarization of the Neutrons from the $^9$Be($\alpha,n_0)^{12}\text{C}$ and $^9$Be($\alpha,n_1)^{12}\text{C}^*$ ($4.43\text{ MeV}$) Reactions," Bull. Am. Phys. Soc. 12, 500 (1967).

"Polarization of the Neutrons from the $^{12}\text{C}(^3\text{He},n)^{14}\text{O}$ Reaction Between 4.1 and 5.9 MeV," Bull. Am. Phys. Soc. 12, 1198 (1967).


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CHAPTER I

INTRODUCTION

The characteristics of the yield curves and angular distributions of the products of low energy nuclear reactions usually fall into one of two broad categories. The reactions in one of these categories, characterized by resonance behavior in the excitation curves and by differential cross sections which vary considerably with energy, have been most often described in terms of the compound nucleus model. The other class of reactions characterized by yield curves relatively free of structure and by angular distributions which vary slowly with energy and exhibit large forward (and sometimes backward) peaks has led to the development of the direct reaction model in the last two decades. In particular the distorted wave Born approximation (DWBA) theory of direct reactions has been quite successful in describing the cross sections observed in the extensively investigated \((d,p)\) and \((d,n)\) single nucleon stripping reactions\(^{1-12}\) although only limited success has been achieved in describing the polarization produced in these reactions\(^{12-16}\).

The availability of \(^3\)He and triton \((t)\) beams in the last decade has made it possible to study such two nucleon transfer reactions as the \((\text{He},n)\), \((\text{He},p)\) and \((t,p)\) reactions. Previous investigations of such reactions (mainly cross section measurements) have indicated that
a direct reaction process may contribute significantly to the reaction mechanism even at energies as low as 4 or 5 MeV. The primary purpose of the present experiments is to investigate the importance of the two nucleon stripping mechanism in \(^{(3\text{He},n)}\) reactions at bombarding energies of 4 to 6 MeV through measurement of the polarization of the emitted neutrons. Polarization measurements can help to establish the reaction mechanism since the polarization produced in a reaction proceeding through compound nucleus formation should vary considerably with energy near resonances whereas the polarization produced in a direct reaction is expected to be nearly energy independent.

It was further anticipated that if the results of these measurements should indicate the importance of a direct reaction mechanism involving two nucleon transfer, the polarization data together with the cross section data would form the basis for a demanding test of the predictions of the DWBA theory. In such DWBA calculations, these two nucleon transfer reactions have an advantage over the \((d,p)\) and \((d,n)\) reactions in that the ground state \(^3\text{He}\) and \(t\) wave functions are assumed to be pure S-state while the deuteron ground state wave function has a small (4-7\%) D-state component. Although this D-state component is usually ignored in distorted wave calculations, Johnson has shown that it can significantly affect both the cross section at large reaction angles and the polarization angular distribution especially in reactions which involve large \(\bar{t}\) transfers. In the particular case of \(^{(3\text{He},n)}\) and \((t,p)\) reactions, if the two nucleons are assumed to be transferred as a cluster (i.e. a
diproton and a dineutron respectively) then the spin of the cluster is zero and only $\bar{J} = \bar{L}$ is allowed for the transferred cluster. Although the form of the $^3\text{He}$ wave function and the optical potentials used to describe $^3\text{He}$ elastic scattering from various nuclei (which are needed in order to carry out DWBA calculations) are not as well known as they are for the deuteron, distorted wave calculations are feasible and there is a great deal of interest in applying the DWBA theory to two nucleon transfer reactions in view of the success of this theory in describing single nucleon stripping reactions.

Despite the abundant cross section data, previous polarization measurements in mass-3-induced two nucleon transfer reactions have been sparse. Measurements have been made only in the last four years for the ($^3\text{He},p$) reactions with targets of $^6\text{Li}$, $^8\text{B}$ and $^{12}\text{C}$ (Refs. 32-35) and for the($^3\text{He},n$) reactions with targets of $^9\text{Be}$, $^{12}\text{C}$ and $^{13}\text{C}$ (Refs. 36-39). The most thoroughly studied of these reactions have been the $^{12}\text{C}(^3\text{He},p_0)^{14}\text{N}$ and $^{12}\text{C}(^3\text{He},p_1)^{14}\text{N}^*(2.31\text{ MeV})$ reactions investigated at energies up to 5.5 MeV by Marr (35) and the $^{13}\text{C}(^3\text{He},n_0)^{14}\text{O}$ reaction studied between 2.24 and 5.9 MeV by Schaller et al. (37) and Soltesz et al. (38). In these reactions the yield curves exhibit considerable broad structure for bombarding energies up to 10 MeV suggesting that compound nucleus formation may contribute appreciably to the reaction mechanism in the low energy region. In the $^{12}\text{C}(^3\text{He},p_0)^{14}\text{N}$ reaction both the differential cross section and polarization angular distributions show pronounced variations with

---

*See, for example, Refs. 30 and 31.*
energy. Marr has obtained fits to these data below 3.5 MeV by assuming that at any energy no more than three broad overlapping compound nucleus states contribute to the reaction. Marr also attempted to fit the data between 4.5 and 6 MeV from the $^{12}\text{C}(^3\text{He},p)^{14}\text{N}^*$ (2.31 MeV) reaction assuming the reaction mechanism was entirely direct. He was able to fit either the polarization data or the angular distributions (but not both simultaneously) with distorted wave calculations assuming an orbital angular momentum transfer of $L = 0$. In the $^{12}\text{C}(^3\text{He},n)^{14}\text{O}$ reaction the angular distributions show a large forward peak at incident energies above 4.5 MeV and the polarization varies gradually and systematically with energy above 3.5 MeV. However, both DWBA calculations for this reaction performed by Schaller et al. at 3.7 MeV and compound nucleus calculations by Marr at energies up to 3.6 MeV have provided a qualitative description of both the cross section and polarization angular distributions. Thus despite considerable evidence for a two nucleon stripping mechanism in these reactions, in no case has an unambiguous fit to both the polarization and cross section data in a two nucleon transfer reaction been provided by an analysis in terms of a direct reaction mechanism alone.

In the present work the angular and energy dependence of the polarization of the neutrons produced in two ($^3\text{He},n$) reactions has been studied as a means of exploring the reaction mechanism. In addition to experimental advantages, each of these reactions has characteristics which make them attractive from the viewpoint of a
distorted wave analysis. In the $^{13}\text{C}(^3\text{He},n_0)^{15}\text{O}$ reaction the differential cross sections $^{25,26,43}$ measured at energies between 2.5 and 10 MeV show a large peak at 0° characteristic of $\vec{L} = 0$ stripping. The magnitude of this peak appears to vary little with energy above 5 MeV. The yield curves at both forward and backward angles show only a few broad structures superimposed on a cross section which rises slowly with increasing energy. The polarization measurements reported by Stammbach et al. $^{39}$ for energies between 2.95 and 3.85 MeV showed that the polarization varied rapidly in this energy region. The evidence from the cross sections for a large direct reaction amplitude, however, is striking only at energies above 4 MeV. Since this reaction has a large positive Q value (+7.12 MeV), it offers a particular advantage in a distorted wave analysis in that with a beam energy of 4 to 6 MeV the neutrons produced have laboratory energies between 9 and 13 MeV where the optical model parameters used in the exit channel are better determined than they are at lower energies. For this reaction polarization angular distributions were measured for four bombarding energies between 4.2 and 5.7 MeV with targets approximately 450 keV thick.

The $^{24}\text{Mg}(^3\text{He},n_0)^{26}\text{Si}$ reaction is a particularly attractive one to study because (1) it is the first two nucleon transfer reaction for which polarization measurements have been reported using a target with $Z \geq 6$ (i.e. a target with nucleons in orbitals lying above the 1p shell), (2) the cross section measurements show evidence of a sizable direct reaction amplitude and (3) it offers particular advantages
over previously studied two nucleon transfer reactions from the viewpoint of a DWBA analysis. In particular the differential cross section measurements of McMurray et al.\textsuperscript{23} and Ajzenberg-Selove and Dunning\textsuperscript{24} for this reaction at incident energies between 4.9 and 5.6 MeV show a large 0° peak and the shape of the angular distribution changes little over the limited energy range studied, suggesting a large two nucleon stripping contribution to the reaction mechanism.

For a DWBA analysis, this reaction is attractive in that the optical model analysis of elastic scattering data needed to provide the optical potentials for distorted wave calculations becomes more justifiable as the number of target nucleons is increased.\textsuperscript{44} Furthermore, the elastic scattering of \(^3\)He from targets in the mass range 22 \(\leq A \leq 27\) has been extensively investigated at energies between 4 and 12 MeV and thus optical model parameters for DWBA calculations can be directly determined from this data.

From an experimental point of view the \(^{24}\)Mg(\(^3\)He,\(n_0\))\textsuperscript{26}Si reaction is a difficult one to study. Although its Q value (+0.06 MeV) and the separation between the ground state and first excited state of its residual nucleus make it one of the few two nucleon transfer reactions involving a medium A target in which the polarization of the ground state neutrons can be measured with present polarimeters, the low cross section (0.1 to 2.0 mb/sr) necessitates the use of thick targets and large detectors to obtain data in a reasonable amount of time. Because of this difficulty, the measurements were limited to only two angular distributions of polarization at incident energies of 5.0 and 5.8 MeV taken with 400 keV targets.
Both the present polarization data and the previous cross section data from both of these reactions were analyzed in terms of the DWBA theory using the ORNL program JULIE of Bassel, Drisko and Satchler. A short description of the distorted wave theory used in JULIE is given in Chapter II preceded by a brief treatment of the theory of spin 1/2 polarization pertinent to the present measurements. The experimental apparatus and procedures are described in Chapter III and the experimental results are presented in Chapter IV. The results of the distorted wave analyses are presented in Chapter V, followed by a concise summary of the conclusions in Chapter VI.
A. Measurement of Spin Polarization

The spin state of a spin 1/2 particle such as the neutron may be represented by a Pauli spinor, \( \chi = (a_1, a_2) \). This spinor specifies the direction in which the spin is pointing and, relative to a fixed coordinate system, the components \( a_1 \) and \( a_2 \) are given by

\[ a_1 = \cos \frac{\theta}{2} e^{-i\phi_2} \quad \text{and} \quad a_2 = \sin \frac{\theta}{2} e^{i\phi_2} \]  

(II-1)

As has been shown by Wolfenstein,\(^{46,47}\) the polarization of spin 1/2 particles in a given spin state, \( n \), for any choice of a fixed coordinate system is completely specified by the expectation value of the Pauli spin operator;

\[ \bar{P}^{(n)} = \frac{\langle \chi_n^\dagger \sigma_z | \chi_n \rangle}{\langle \chi_n^\dagger \chi_n \rangle} \]  

(II-2)

Since \( \sigma_z^2 \) is the two-by-two identity matrix, \( \chi \) is always an eigenfunction of \( \sigma_z^2 \) and the \( z \) component of the spin for a neutron in a given spin state can take on only two values. Also since only two linearly independent spin states exist for a spin 1/2 system, each spin state
may be written as a linear combination of two particular linearly independent states. If the two eigenfunctions of $\sigma_z$ are chosen, $\chi$ may be written

$$\chi = a_1 (1) + a_2 (0)$$  \hspace{1cm} (II-3)

from which $a_1$ and $a_2$ are identified as the amplitudes (and $|a_1|^2$ and $|a_2|^2$ as the probabilities) for finding the neutron with its $z$ component of spin positive ("up") and negative ("down") respectively. If each neutron in a beam is in the same spin state, the beam is said to be completely polarized since for some particular selection of the coordinate system, $a_1 = 1$ and $a_2 = 0$ for each neutron in the beam.

Neutron beams produced in nuclear reactions are usually only partially polarized and thus to obtain an expression for the beam polarization it is necessary to average over all the different spin states in the beam with their respective statistical weights, $g_n$:

$$\bar{P} = \sum_n g_n \hat{P}^{(n)} \quad \text{with} \quad \sum_n g_n = 1.$$  \hspace{1cm} (II-4)

For each spin state, Eqn. II-3 may be written

$$\chi_n = a_{1n} (1) + a_{2n} (0)$$  \hspace{1cm} (II-5)
Using Eqn. II-5 and following the specification of Eqns. II-2 and II-4, the $z$ component of the beam polarization is

$$P_z = \sum_n g_n P_z^{(n)} = \sum_n g_n \left[ \frac{|a_{1n}|^2 - |a_{2n}|^2}{|a_{1n}|^2 + |a_{2n}|^2} \right] \quad \text{(II-6)}$$

For the case where the beam consists of only two separate oppositely polarized spin states $\chi_1 = \left\{ \begin{array}{l} 1 \end{array} \right\}$ and $\chi_2 = \left\{ \begin{array}{l} 0 \end{array} \right\}$ with $N_+$ particles in state $\chi_1$ and $N_-$ in state $\chi_2$, the statistical weights are

$$g_1 = \frac{N_+}{N_+ + N_-} \quad \text{and} \quad g_2 = \frac{N_-}{N_+ + N_-} \quad \text{(II-7)}$$

and the $z$ component of polarization from Eqn. II-6 is

$$P_z = \frac{N_+ - N_-}{N_+ + N_-} \quad \text{(II-8)}$$

Every partially polarized beam can be considered to be made up of only two such separate states since Eqn. II-6 can be rewritten

$$P_z = \sum_{m=1}^{\infty} \sum_n \frac{g_n |a_{mn}|^2}{\sum_{\xi=1}^{\infty} |a_{\xi n}|^2} (-1)^{m+1} = \sum_{m=1}^{\infty} g_m' P^{(m)} \quad \text{(II-9)}$$

with $P^{(m)} = (-1)^{m+1}$ and $g_m' = \sum_n g_n |a_{mn}|^2$. 
In the case of a reaction involving spin 1/2 particles incident on a spinless target with spin 1/2 particles emerging, the asymptotic form of the final state wave function is

\[ \Psi_f = e^{i \vec{k}_f \cdot \vec{r}} \chi_2^+ + \frac{e^{i \vec{k}_f \cdot \vec{r}}}{r} M(\theta, \phi) \chi_2 \]  

(II-10)

where \( \vec{k}_f = \frac{p_f}{\hbar} \) is the incident wave vector, \( \chi_1 \) is the initial spinor and \( M(\theta, \phi) \) is the spatially dependent operator in spin space which transforms the initial state spinor into the final state spinor. Since \( M \) is a two-by-two matrix in this case, it may be expressed as a linear combination of any four linearly independent two-by-two matrices.

Choosing the identity matrix and the Pauli spin matrices gives for the most general form of \( M \)

\[ M(\theta, \phi) = g(\theta, \phi) + \vec{h}(\theta, \phi) \cdot \vec{\sigma} \]  

(II-11)

where it is understood that \( g(\theta, \phi) = \begin{pmatrix} g(\theta, \phi) & 0 \\ 0 & g(\theta, \phi) \end{pmatrix} \).

If the reaction is produced by a nuclear force, both the total angular momentum and parity are conserved and thus \( M \) must be invariant under rotations and reflections of the coordinate system. Since both the initial and final state spinors are pseudo-vectors, \( M \) must transform like a scalar. Thus \( g(\theta, \phi) \) must be a scalar and \( \vec{h}(\theta, \phi) \) must be a pseudo-vector since \( \vec{\sigma} \) transforms like a pseudo-vector. Neither \( g \) nor \( \vec{h} \) can depend on the choice of the coordinate system; that is \( g \) and \( \vec{h} \) must be expressible in terms of physical vectors. In the reaction of an unpolarized beam with a spinless target there are only two of these,
the incident and final momenta $\vec{p}_1$ and $\vec{p}_f$ respectively. The only scalar these can form is

$$\vec{p}_1 \cdot \vec{p}_f = p_1 p_f \cos \theta$$  \hfill (II-12)

and the only pseudo-vector they can form is

$$\vec{p}_1 \times \vec{p}_f = p_1 p_f \sin \theta \frac{\vec{p}_1 \times \vec{p}_f}{|\vec{p}_1 \times \vec{p}_f|}$$  \hfill (II-13)

where $\theta$ is the polar angle of the reaction in the center of mass system. Thus

$$g = g(E, \theta) \quad \text{and} \quad \hat{h} = h(E, \theta) \frac{\vec{p}_1 \times \vec{p}_f}{|\vec{p}_1 \times \vec{p}_f|} = h(E, \theta) \hat{n}$$  \hfill (II-14)

where $E$ is the bombarding energy and

$$\hat{n} = \frac{\vec{p}_1 \times \vec{p}_f}{|\vec{p}_1 \times \vec{p}_f|}$$  \hfill (II-15)

is the unit normal to the reaction plane chosen in accordance with the Basel convention. If the $z$ axis is now chosen along $\hat{n}$, $M$ is given by

$$M(E, \theta) = g(E, \theta) + h(E, \theta) \hat{n} \cdot \vec{\sigma} \quad \text{or} \quad M = \begin{pmatrix} g + h & 0 \\ 0 & g - h \end{pmatrix}.$$  \hfill (II-16)
Using Eqn. II-16 in Eqn. II-10 and the result of Eqn. II-9 (with $g_1 = g_2$ for an unpolarized incident beam) in Eqns. II-2 and II-4 gives for the final neutron beam polarization

$$\vec{P} = P(E, \Theta)^\hat{\mathbf{n}}$$  \hspace{1cm} (II-17)

where

$$P = \frac{2 Re(g^* h)}{|g|^2 + |h|^2}$$  \hspace{1cm} (II-18)

That is, the polarization produced in the reaction is a function of only the incident energy and the reaction polar angle (and not of the azimuthal angle) and has no component in the reaction plane.\footnote{Using more general arguments Wolfenstein\cite{note9} has shown that these results, expressed in Eqn. II-17, hold whenever an unpolarized beam is incident on an unpolarized target of any spin.}

For elastic scattering at a given energy of a partially polarized beam, characterized by $\vec{P}_1$, from a spinless target such as $^4$He, Wolfenstein\cite{note8} has shown that the cross section has an azimuthal dependence on the incident beam polarization given by

$$\sigma(\Theta, \Phi) = \sigma_u(\Theta) \left[ 1 + P_1 P_2(\Theta) \cos \Phi \right]$$  \hspace{1cm} (II-19)

where $\sigma_u(\Theta)$ is the cross section corresponding to an unpolarized incident beam and where $\Phi$ is the angle between the incident beam
polarization and the normal to the scattering plane as defined in Eqn. II-15. \( P_2 \), called the analyzing power, is the magnitude of the polarization which would be produced in elastic scattering through the same angle \( \theta \) with an unpolarized incident beam. It is this azimuthal dependence of the cross section on the incident beam polarization which makes possible measurement of the polarization by the "double scattering" technique.

The numbers of polarized neutrons scattered to the left \( (\phi = 0) \) and the the right \( (\phi = \pi) \) through the same polar angle \( \theta_2 \) from the analyzer into a detector are, from Eqn. II-19, respectively,

\[
N_L = N f \sigma_0 (\theta_2) [1 + P_1 P_2 (\theta_2)] \\
N_R = N f \sigma_0 (\theta_2) [1 - P_1 P_2 (\theta_2)]
\]

where \( N \) is the total number of polarized neutrons incident on the analyzer and \( f \) is the solid angle fraction subtended by the detector at the analyzer. The left-right ratio is thus

\[
f = \frac{N_L}{N_R} = \frac{1 + P_1 P_2}{1 - P_1 P_2}
\]

The asymmetry, \( \varepsilon \), is therefore

\[
\varepsilon = \frac{N_L - N_R}{N_L + N_R} = P_1 P_2 = \frac{f - 1}{f + 1}
\]
Hence, if the analyzing power, \(P_2(E,\theta_2)\), is known, the beam polarization \(P_1(E,\theta_1)\) produced in the reaction may be determined by measuring the ratio \(r\).

B. Direct Reactions in the Distorted Wave Born Approximation

A brief description of the salient features of the distorted wave theory of direct reactions (and in particular of stripping reactions) following the treatment by Satchler\(^{49}\) is presented below. The equations given are not intended to be complete in any way but are intended to serve only as a guide in indicating how theoretical values for the observed quantities---in this case, cross sections and polarizations---are calculated in this work.

In the distorted wave theory of direct interactions the reaction \(A(a,b)B\) is viewed as a single transition between initial and final states which take account of the elastic scattering in the entrance and exit channels. The transition amplitude is thus of the form

\[
T = \mathcal{J} \int d\vec{r}_a \int d\vec{r}_b \phi^{(-)}_{m_a m_b}(\vec{k}_a, \vec{r}_a) \langle b \mid V \mid a A \rangle \phi^{(+)}_{m_2 m_2}(\vec{k}_a, \vec{r}_b) \tag{II-23}
\]

where \(\vec{r}_a\) and \(\vec{r}_b\) are the displacements of \(a\) from \(A\) and \(b\) from \(B\) respectively and \(\mathcal{J}\) is the Jacobian of the transformation to these relative coordinates. The wave functions \(\phi^{(-)}_{m_b m_b}\) and \(\phi^{(+)}_{m_a m_a}\) are plane waves plus either incoming (−) or outgoing (+) spin-dependent spherical scattered waves which describe the elastic scattering of
the particles $b + B$ with relative momentum $\vec{k}_b$ and $a + A$ with relative momentum $\vec{k}_a$ respectively. These distorted wave functions are usually generated from optical model potentials which best describe elastic scattering data for the appropriate channel in a given energy range. The factor $\langle bB|V|aA \rangle$ is the matrix element of the potential responsible for the reaction, $V$, taken between the internal states of the initial and final pairs of particles,

$$\langle bB|V|aA \rangle = \int \psi_b^*\psi_b^* V \psi_a^* \psi_a \, d\tau \quad (II-24)$$

where $\tau$ represents all the internal coordinates. In a stripping reaction the projectile $a$ is considered to be composed of the light reaction product $b$ and a cluster $x$ which is transferred to the target nucleus $A$ to form the residual nucleus $B$. That is $a = b + x$ and $B = A + x$. $V$ is then taken to be $V(r_{bx})$, a central interaction between the light reaction projectile and the stripped cluster.

The matrix element in Eqn. II-24 may be expanded into terms corresponding to transfer of a given total angular momentum $\vec{j}$, composed of specific orbital angular momentum $\vec{l}$ and specific spin angular momentum $\vec{s}$, to the target nucleus $A$. If the spins of the reactants are denoted by $\vec{s}_a$ and those of the products by $\vec{s}_b$, then $\vec{j}$, $\vec{l}$ and $\vec{s}$ are defined by

$$\vec{J}_B - \vec{J}_a = \vec{j} = \vec{l} + \vec{s} \quad (II-25)$$
and

$$\tilde{S} = \tilde{S}_b - \tilde{S}_a$$  \hspace{1cm} (II-26)

and the expansion is

$$\mathcal{G} \left< J_0 M_0 S_b m_b | V | J_a M_a S_a m_a \right> = \sum_{j,l,s} \left< J_a, j ; M_a, M_b-M_a | J_0, M_0 \right> \times \left< l, s ; m, m_a-m_b | j, M_b-M_a \right> \left< S_a, S_b ; m_a, -m_b | S, m_a-m_b \right> \times (-1)^{s-m_b} A_{l s j} f_{l s j m} (\hat{r}_b, \hat{r}_a)$$  \hspace{1cm} (II-27)

where $M_B$, $m_b$, $M_A$ and $m_a$ are the magnetic quantum numbers of the various particles indicated by their subscripts and $m = M_B + m_b - (M_A + m_a)$. $A_{l s j}$ is a spectroscopic factor which depends only on the internal states of the system and $f_{l s j m}$ is a spatial function which behaves like the spherical harmonic $Y^{m_k}_l$ under rotation. Substitution of Eqn. II-27 into Eqn. II-23 gives for the transition amplitude the general expression

$$T = \sum_{l s j} A_{l s j} (-1)^{s-m_b} \left< J_a, j ; M_a, M_b-M_a | J_0, M_0 \right> \times \left< l, s ; m, m_a-m_b | j, M_b-M_a \right> \left< S_a, S_b ; m_a, -m_b | S, m_a-m_b \right> \times \int d\hat{r}_a \int d\hat{r}_b \phi^{(+)}_{m_a m_b} (\hat{r}_a, \hat{r}_b) f_{l s j m} (\hat{r}_b, \hat{r}_a) \phi^{(-)}_{m'_a m'_b} (\hat{r}_b, \hat{r}_a)$$  \hspace{1cm} (II-28)
In the case of a (\(^3\)He,n) reaction, the transferred "particle" is a diproton whose spin is predominantly zero. Eqn. II-25 thus becomes

\[ \vec{J} = \vec{I} = \vec{J}_B - \vec{J}_A. \]  

(II-29)

In the \(^{24}\)Mg(\(^3\)He,n\(^0\))\(^{26}\)Si reaction both the target and residual nuclei have even numbers of protons and neutrons whose spins couple to zero since both nuclei are in their ground states giving \(J_B = J_A = 0\).

Thus \(\vec{J} = \vec{I} = 0\) is the only allowed angular momentum transfer. In the case of \(^{13}\)C(\(^3\)He,n\(^0\))\(^{15}\)O, the ground states \(^{50}\) of both the target and residual nuclei have \(J'' = 1/2^+\). Eqn. II-29 thus gives \(\vec{J} = \vec{I} = 0\) or 1.

As will be shown (see Eqn. II-35) the zero-range approximation insures that the parity change in the reaction is \((-1)^L\). Thus, since both the initial and final states have negative parity, only \(\vec{J} = \vec{I} = 0\) is allowed for this reaction also. For the two reactions studied here, therefore, the triple sums in Eqns. II-27 and II-28 reduce to single terms giving

\[ \mathcal{Y} \left< J_0 M_0 s_b m_b | V | J_0 M_0 s_a m_a \right> = A_{000} f_{0000}(\vec{r}_b, \vec{r}_a) \]  

(II-30)

and

\[ T = A_{000} \int d\vec{r}_b \int d\vec{r}_a \phi^{(-)}_{m_b m_a} (\vec{r}_b, \vec{r}_a) f_{0000} (\vec{r}_b, \vec{r}_a) \phi^{(+)}_{m_b m_a} (\vec{r}_b, \vec{r}_a). \]  

(II-31)
To reduce the six-dimensional integrals appearing in Eqns. II-28 and II-31 to three-dimensional integrals, the zero-range approximation is used. This may be written

\[ \int_{S_{j}m}(\vec{r}_a, \vec{r}_b) = \int_{S_{j}m}(r_a) \gamma_{m}(\theta_a, \phi_a) D\left(r_a - \frac{M_A}{M_B} r_b\right) \]  

where \( M_A \) and \( M_B \) are the masses of the target and residual nucleus respectively. This approximation corresponds to assuming that the particle \( b \) is emitted from the same point at which particle \( a \) is absorbed and that particles \( b \) and \( x \) are in an S-state of relative motion within \( A \). Using the partial wave expansions of the distorted waves \( \phi_{m}^{(-)}(k_b, r_b) \) and \( \phi_{m}^{(+)}(k_a, r_a) \) along with Eqn. II-32 and choosing the \( z \) axis along \( \vec{r}_a \) and the \( y \) axis along \( \vec{k}_a \times \vec{k}_b \) with \( \theta \) the angle between \( \vec{k}_a \) and \( \vec{k}_b \) leads to the following general expression for the transition amplitude:

\[ T = \frac{2 \pi^2}{k_b k_a} \frac{M_B}{M_A} \sum_{k_s, f} (2j+1)^{1/2} \langle J_A, j; M_A, M_B - M_A | J_B, M_B \rangle \]

\[ \times A_{S_{j}m} \sum_{l_1, l_2} \frac{\Gamma_{l_1 l_2 j_1 j_2}}{l_{12} J_{12} J_{22}} \mathcal{P}_{l_1}^{m}(\theta) \]

\[ \times \sum d\tau \phi_{l_1, j_1, l_2, j_2}(k_b, \frac{M_B}{M_A} r) F_{S_{j}m}(r) \phi_{l_2, j_2}(k_a, r) \]

That is, \( T \) is a sum of terms each of which consists of angular momentum coupling coefficients \( (2j+1)^{1/2} \langle J_A, j; M_A, M_B - M_A | J_B, M_B \rangle \).
and \( \Gamma_{L_b J_b L_a J_a}^{j m m_b m_a} \), a spectroscopic coefficient \( A_{j a j} \), an associated Legendre function \( P_L^m(\theta) \) containing the angular dependence and a radial integral containing the radial parts of the various partial distorted waves in the entrance and exit channels and a radial form factor, \( F_{L a J}(r) \), which in the case of a stripping reaction may be identified with the radial eigenfunction \( u_{L a j}(r) \) of the transferred particle bound by some central potential to the target \( A \) (i.e., a shell model eigenfunction).

The transition amplitude can be more conveniently expressed in terms of the "reduced" amplitude, \( \beta_{j a j}^{L m m_b m_a}(\theta) \), which in the zero-range approximation is defined as

\[
\beta_{L a J}^{L m m_b m_a}(\theta) = \frac{2 \pi \hbar}{k_a k_b M_a} \sum_{L_b J_b L_a J_a} \Gamma_{L_b J_b L_a J_a}^{L m m_b m_a} P_L^m(\theta) \\
\times \int dr \phi_{L_b J_b}^{(b)}(k_b, M_a; r) \phi_{L a J_a}^{(a)}(k_a, r)
\]

where the angular momentum coefficient \( \Gamma_{L b J b L a J a}^{j m m_b m_a} \) is given by

\[
\Gamma_{L_b J_b L_a J_a}^{L m m_b m_a} = \frac{i (-1)^{J_b + J_a}}{L_a L_b L J} \frac{(l_b - m)!}{(l_b + m)!} \frac{1}{(2L_b + 1)(2L + 1)} \frac{1}{(2J_b + 1)(2J_a + 1)} \frac{1}{(2S + 1)!} \frac{1}{(2J + 1)!} \frac{1}{(2J_a + 1)!} \langle J_b, j; m_b - m, m, m_b + m_a | J_b, m_a \rangle \\
\times \langle L_b, S_b; O_m, m_2 | J_a, m_2 \rangle \langle L_b, S_b; -m, m_b | J_b, m_b - m \rangle \\
\times \langle L_b, S_b; O_0, 0 | L_b, 0 \rangle \left( \frac{j L S}{J_a L_a S_a} \right) \left( \frac{j L S}{J_b L_b S_b} \right).
\]
The Clebsch-Gordon coefficient \( \langle L_b, L; 0, 0 | L_a, 0 \rangle \) in Eqn. II-35 vanishes unless \( L_b + L_a + \ell \) is even. The parity change in the reaction is therefore \((-1)^\ell\). In terms of the reduced amplitude, the transition amplitude is

\[
T = \sum_{L, s, j} (2j+1)^{1/2} \langle J_a, j; M_a, M_b - M_a | J_b, M_b \rangle A_{L,s,j}^\ell m_b m_a^\ell (\theta) (II-36)
\]

From Eqns. II-34 and II-35 the differential cross section and polarization for an unpolarized beam incident on an unpolarized target may be obtained from the equations

\[
\sigma^-(\theta) = \frac{\mu_a \mu_b}{(2\pi\hbar^2)^2} \frac{k_b}{k_a} \frac{(2J_b + 1)}{(2J_a + 1)(2S_b + 1)} \sum_{J, m, m_a} \left| \sum_{L, s, j} A_{L,s,j}^\ell m_b m_a^\ell (\theta) \right|^2 (II-37)
\]

and

\[
\bar{P}(\theta) = \sum (s_b - m_b)^{1/2} (s_b + m_b + 1)^{1/2} \times \Im \left[ A_{L,s,j}^\ell m_b m_a^\ell (\theta) A_{L',s',j}^{\ell m_b m_a^\ell}(\theta) \right] (II-38)
\]

\[
\times \left\{ s_b \sum A_{L,s,j}^\ell m_b m_a^\ell (\theta) A_{L',s',j}^{\ell m_b m_a^\ell}(\theta) \right\}^{-1} \frac{k_a \times k_b}{|k_a \times k_b|}
\]
where $\mu_a$ and $\mu_b$ are the reduced masses in the entrance and exit channels and both summations indicated in Eqn. II-38 are over the indices $s, s', \ell, \ell', j, m_b, m_a,$ and $m$.

The particular choices of spectroscopic coefficients, form factors, and optical model potentials used in the direct reaction calculations for the $^{24}\text{Mg}(^3\text{He},n_0)^{26}\text{Si}$ and $^{13}\text{C}(^3\text{He},n_0)^{15}\text{O}$ reactions will be discussed in detail in Chapter V.
CHAPTER III

EXPERIMENTAL TECHNIQUE

A. Introduction

The apparatus used to measure the polarizations of the neutrons from the $^{24}\text{Mg}(^{3}\text{He},n)^{26}\text{Si}$ and $^{13}\text{C}(^{3}\text{He},n)^{15}\text{O}$ reactions was the same in both cases and consisted mainly of a spin precession solenoid and a helium gas scintillator as the polarization analyzer. This scintillator was operated in fast coincidence with either of two plastic neutron detectors to reduce backgrounds. A general description of the experimental procedure is given below followed by a detailed description of the principal components of the polarimeter, a discussion of data collection and reduction procedures, an analysis of the various corrections to the data, and checks for instrumental asymmetries.

The beam of singly charged $^{3}\text{He}$ ions from The Ohio State University 6 MeV Van de Graaff Accelerator was analyzed by a $90^\circ$ magnet and defined by a set of slits to an energy resolution, $\Delta E/E$, of less than 0.5%. The quadrupole-focused beam was then defined to an angular spread of less than $0.6^\circ$ by a rectangular slit system and a $3/16''$ diameter circular collimator before striking a target of either $^{24}\text{Mg}$ or $^{13}\text{C}$.

A general schematic of the target, spin precession magnet and
neutron polarimeter is shown in Fig. 1. The polarized neutrons produced at a laboratory polar angle, $\theta_1$, are collimated to an angular spread of 1.5° by a conical brass collimator as they travel down the axis of an air core solenoid. The neutrons are then elastically scattered through an angle $\theta_2 = 121^\circ$ from helium gas contained at high pressure in a gas scintillator and detected by either of two 2" x 3" x 6" rectangular Pilot-B plastic scintillators operated in fast coincidence with the helium cell. As shown in the figure, the neutron detectors are located above and below the reaction plane at a distance of 7 1/2" from the center of the helium scintillator. The plastic detectors are viewed through lucite light pipes by Ampere 56 DVP bi-alkali cathode photomultiplier tubes chosen for their high gain and fast rise time characteristics. The magnet and entire detector mount are supported by a table which is rotated about a vertical axis through the target in order to vary the reaction angle, $\theta_1$ which is determined to within 1/4° with a vernier indicator.

The fast coincidence requirement between the helium cell and each plastic detector is used to gate a pulse height spectrum of the linear signals from the helium recoils. The asymmetry in the scattering is then determined by measuring the ratio of helium recoil pulses in a given pulse-height region which are in coincidence with pulses from a particular plastic neutron detector with one current sense in the spin precession solenoid to those in coincidence when the current sense is reversed. The polarizations are extracted from the measured asymmetries by correcting for the geometrically averaged analyzing
Figure 1. Schematic of neutron polarimeter showing spin precession solenoid, helium scintillation detector and two Pilot B plastic neutron detectors.
power of helium and, in some cases, for incomplete spin precession.

B. Targets

Since both reactions studied here have extremely low cross sections varying between 0.1 and 5.0 mb/sr, it was necessary to use relatively thick targets to increase the yields. As a compromise between yield and energy resolution, targets with a mean thickness of about one-twelfth of the incident beam energy were used.

Self-supporting foils isotopically enriched to 99.96% $^{24}$Mg purchased from Oak Ridge National Laboratory were used as targets for the $^{24}$Mg($^3$He,n)$^{26}$Si reaction. These targets, prepared by reduction of $^{24}$MgO and subsequent evaporation of $^{24}$Mg, had a mean thickness of approximately 400 keV to a 5.0 MeV $^3$He beam as determined by weighing and using the published tables for the energy loss of charged particles in matter. Beam currents were limited to less than 1.5 µA in order to avoid damage to the target foils. After prolonged use (approximately 140 hrs. bombardment) the targets were changed to limit carbon buildup. Although neutrons from the $^{12}$C($^3$He,n)$^{14}$O reaction could not contribute to the recoil spectra in the region of the ground state neutron peak, significant carbon buildup would have decreased the mean neutron energy. Since the Q values for the $^{12}$C($^3$He,n$^0$)$^{14}$O and $^{24}$Mg($^3$He,n$_1$)$^{26}$Si*(1.79 MeV) reactions are less than 600 keV different, the neutrons from the $^{12}$C buildup made it impossible to extract reliable polarizations for the first excited state neutrons from $^{24}$Mg.

Although self-supporting foils are desirable as targets in order to eliminate backgrounds from a backing material, it was not
possible to produce $^{13}$C targets in this form due to the high melting temperature of carbon and the high cost of compounds isotopically enriched in $^{13}$C. Consequently the $^{13}$C targets were prepared by ultrasonically dispersing finely ground amorphous carbon powder enriched to 90% $^{13}$C in a dag suspension. The powder was then allowed to settle out on a 0.005" tungsten backing which was fixed to the end of a 0.025" wall stainless steel tube thus forming part of the Faraday cup. The mean energy loss of the beam in these targets was determined by measuring the effective width of the narrow (75 eV) resonance in the $^{13}$C(p,$\gamma$)$^{14}$N reaction at 1.75 MeV. Targets with mean thicknesses of approximately 425 keV to a 5.0 MeV $^3$He beam were used. 80% of the yield was produced from the thickness region between 250 and 650 keV. The nonuniformity of the target thickness was due to variations in the particle size of the $^{13}$C powder. Water cooling of the backing allowed the use of beam currents of between 3 and 5 $\mu$A with the $^{13}$C targets. Due to the more than 8 MeV difference in the Q values for the ($^3$He,n) reactions with $^{12}$C and $^{13}$C, no neutrons from carbon buildup on these targets could contribute to the pertinent parts of the recoil spectra.

C. Spin Precession Magnet

In a conventional double scattering experiment to determine an asymmetry, two detectors are usually used not only to double the counting rate but also to eliminate any false asymmetry due to inaccurate charge integration when the asymmetry is calculated as

$^4$Purchased from Monsanto Research Corporation, Miamisburg, Ohio.
the geometric mean of the values obtained with the two independent detectors. An elegant method of eliminating all instrumental asymmetries associated with the interchange of the detectors, first pointed out by Hillman, Stafford and Whitehead, is the use of either an electromagnet or a solenoid to precess the neutron magnetic moments and thus their spins before scattering them from the analyzer. This rotates the plane of polarization leaving the detector's fixed in space. This method, discussed in Appendix I, requires only that one maintain a magnetic field stable to within about 1%.

In these experiments a solenoid magnet was chosen because it requires less power and its fields are easily contained allowing for adequate shielding of detectors. This solenoid, capable of precessing the spins of 5.2 MeV neutrons through $\pm 90^\circ$ about their momentum direction, consists of two coils each of which is made up of twelve "pancake" wound sections of $1/4"$ square hollow copper tubing. The tubing is wound with glass tape insulation and the whole coil is bound and vacuum impregnated with epoxy. Both the

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+ The solenoid is modular and may be comprised of one, two or three coils all of the same design. In these experiments the field lengths produced with a two-coil magnet were sufficient to precess the neutrons of interest through substantial angles of 60° or more introducing an accurate correction factor of less than 20% as discussed below. Although 90° precession could have been achieved in all cases with a three-coil magnet, the decrease in the counting rate due to the smaller solid angle subtended by the analyzer at the target would have necessitated data runs approximately 20 to 40% longer in order to achieve the same statistical uncertainty in $P_1$ as with the two-coil magnet.
pancake sections and coils are connected electrically in series and hydraulically in parallel. The magnet requires 22 Kw to produce a maximum axial field of approximately 8 kilogauss. The power is supplied by a 33 Kw motor generator set whose output current was regulated to within 0.3% by a solid-state control circuit described by Sukis. This circuit also allowed the magnetic field to be accurately and remotely reversed in approximately 20 sec. nearly every 30 min. during data collection to average out the effects of any possible long period electronic drift. The magnet is cooled by a closed water system which supplies 4 G.P.M. of distilled water, limiting the temperature rise across the magnet to less than 15° C.

In order to reduce fringing fields which could both deflect the incident charged particle beam and cause gain shifts in the photomultiplier tubes, the solenoid was entirely encased by a 1" thick Armco iron box. In addition all photomultiplier tubes were encased in two layers of conetic + magnetic shielding and mounted in soft iron cylindrical cases. With this shielding gain shifts were reduced to less than 1% which was insignificant in these experiments.

D. Helium Gas Scintillator

The helium gas scintillation detector shown in Fig. 2 is similar to that reported by Shamu and consists of a cylinder of type 304 stainless steel 2" in diameter and 3" in length with 0.100" walls closed at one end with a hemispherical dome. To insure

+Supplied by Perfection Mica Company, Chicago, Illinois.
Figure 2. Cut-away view of the high pressure helium gas scintillator used as the polarization analyzer.
maximum light collection and uniform pulse-height resolution the inner walls of the cell were coated with reflecting layers of first aluminum and then MgO powder deposited by burning a Mg ribbon below the open cell. The cell was then baked at 120°F in a vacuum of \( \approx 10^{-5} \) mm Hg for approximately 50 hours to drive off volatile contaminants.

A layer of p,p'-diphenylstilbene was next evaporated on the inner surface of the cell which served to shift the wavelength of the ultraviolet helium scintillations into the region of maximal sensitivity of the photomultiplier tube. A low-noise high gain RCA 8575 bi-alkali cathode photomultiplier tube was used to view the gas cell through 3/4" crown tempered glass\(^+\) and 1/2" plexiglas windows. The thickness of the diphenylstilbene layer was chosen to give pulse heights as nearly maximal as possible. The distribution of the diphenylstilbene layer, however, was adjusted so as to produce nearly equal pulse heights from different parts of the cell when irradiated by a collimated monoenergetic neutron source and thus insure good pulse height resolution. Thicknesses averaging 0.14 mg/cm\(^2\) at the closed end and decreasing to 0.06 mg/cm\(^2\) at the open end were found to produce the best results. The glass window was also coated with a thin 0.015 mg/cm\(^2\) layer of p,p'-diphenylstilbene.

The cell was filled with purified helium gas (2300 p.s.i.) to which a small amount (200 p.s.i.) of purified xenon gas was added to increase the light output of the scintillator. The filling gases

\[ ^+ \text{Supplied by Pittsburgh Plate Glass Company, Pittsburgh, Pa.} \]
were passed through liquid nitrogen, activated charcoal, and Zeolite\textsuperscript{+} traps to purify them.

A $^{210}\text{Po}$ alpha source was mounted in the cell as shown in Fig. 2 to check resolution. Inherent resolutions of 6\% full width at half maximum (FWHM) were achieved with this monoenergetic source. Since only approximately 25-30\% overall resolution was necessary to clearly separate the ground state neutrons from excited state neutrons in both reactions, the high inherent resolution allowed the use of thick targets and large angle detectors to increase the counting rates. The overall energy resolution of the helium scintillator in these experiments was thus between 20 and 25\% FWHM and included contributions of 7-11\% due to the energy spread of neutrons from the target and approximately 8\% due to the finite geometry of the large plastic neutron detectors.

\textbf{E. Electronics}

A block diagram of the electronics is shown in Fig. 3. Fast pulses (approximately 3 ns. rise time) from the anodes of the photomultipliers were clipped with 2 ns. delay lines and fed into E.G.\&G. model TD101 fast differential discriminators operated in a

\textsuperscript{+}Supplied by Linde Division of Union Carbide Corporation.
Figure 3. Block diagram of electronics. The circuit shown by solid lines was used to measure foreground and room-scattered background spectra and that shown by dashed lines was used to measure accidental spectra.
lower level timing mode. A fast coincidence ($2\tau = 12$ ns) was required between the logic pulses from the discriminators associated with the helium cell and either of the two Pilot-B neutron detectors. The slow linear helium recoil pulses were amplified and doubly differentiated by an Ortec model 410 multimode amplifier and then analyzed by a Nuclear Data 512 channel pulse-height analyzer which was gated by the logic signals derived from the two fast coincidence circuits. These logic signals were also used to route the recoil pulses into one of four separate quadrants of the analyzer depending on the particular combination of precession sense and scattering direction. Thus a set of four foreground gated helium recoil spectra were collected in the analyzer for a given data point; one corresponding to neutrons scattered into the top detector with clockwise precession ($T_+$), one corresponding to neutrons scattered into the bottom detector with clockwise precession ($B_+$), and two more corresponding to neutrons scattered into each plastic detector with counterclockwise precession ($T_-$ and $B_-$ respectively).

Coincidence delay curves were taken by varying the variable delays shown in Fig. 3 each time the electronics was set up using the higher yield $^9\text{Be}(\alpha,n)^{12}\text{C}$ reaction at an energy and angle

\[+\text{In this mode an output logic pulse is generated only if the input signal pulse height crosses a high level discriminator (E+$\Delta$E). The timing of the output pulse, however, is generated from the point at which the input pulse crosses a lower level discriminator (E). This reduces the timing difference in output pulses due to input pulses of the same shape but different magnitudes. E was typically set at 50 mv and E+$\Delta$E at 250 mv. The maximum pulse heights from the plastic neutron detectors were nearly 10 V. This discrimination thus primarily eliminated some of the singles rate due to gamma background as well as phototube noise.}\]
selected to make the neutron energy close to those for which the polarizations were to be measured. A typical pair of delay curves is shown in Fig. 4 with the chosen delays indicated by arrows.

F. Recoil Spectra and Backgrounds

Two typical gated helium recoil spectra from the $^{13}\text{C}(^3\text{He},n)^{15}\text{O}$ reaction, one at a forward angle of $\theta_1 = 30^\circ$ where the cross section is reasonably large ($\sim 2\ \text{mb/sr}$) and one at a back angle of $\theta_1 = 130^\circ$ where the cross section is nearly minimal ($< 0.5\ \text{mb/sr}$), are shown in Fig. 5. The peaks labeled $n_0$ and $n_1 + n_2$ are due to recoils from neutrons which left $^{15}\text{O}$ in its ground state and first two excited states respectively. The solid curves represent the foreground spectrum summed channel by channel over all four quadrants while the dashed curves represent the total background spectrum similarly summed.

A typical recoil spectrum from the $^{24}\text{Mg}(^3\text{He},n)^{26}\text{Si}$ reaction is shown in Fig. 6. The peak labeled $n_1$ includes contributions from both the $^{24}\text{Mg}(^3\text{He},n_1)^{26}\text{Si}^*(1.79\ \text{MeV})$ reaction and the $^{12}\text{C}(^3\text{He},n_0)^{14}\text{O}$ reaction which is present due to carbon buildup on the targets and which has a much larger cross section ($\sim 10:1$) than the former reaction and thus produces a comparable yield. Here the solid curve represents the net spectrum summed channel by channel over all four quadrants and the dashed curve represents the similarly summed total background spectrum.

Two types of backgrounds were measured and separately subtracted from the foreground spectra to obtain the net recoil spectra. Accidental coincidences due to uncorrelated events in the helium cell and either plastic were measured by delaying the fast pulse from
Figure 4. Coincidence delay curves for the helium gas scintillator operated in fast coincidence with the top and bottom plastic scintillators respectively. The arrows indicate the method of selection for the variable delays of Fig. 3.
\[ ^{9}\text{Be}(\alpha,n)^{12}\text{C}^*(4.43\text{MeV}) \]
\[ E_\alpha = 3.9\text{MeV} \quad \theta = 0^\circ \]
Figure 5. Two typical gated helium recoil pulse-height spectra from the $^{13}\text{C}(^{3}\text{He},n)^{15}\text{O}$ reaction measured at the bombarding energies and reaction angles indicated in the upper right hand corners. The peaks labeled $n_0$ are due to neutrons which left $^{15}\text{O}$ in its ground state and that labeled $n_1 + n_2$ is due to neutrons which left $^{15}\text{O}$ in its first two excited states.
\(^{13}\text{C} (^{3}\text{He}, n)^{15}\text{O}\)

\[ 5.2 \text{ MeV} \]
\[ \theta_{\text{LAB}} = 30^\circ \]

\[ \text{GROSS SPECTRUM} \]

\[ \text{SUM OF BACKGROUNDS} \]

\[ n_1 + n_2 \]

\[ n_0 \]

\[ 4.7 \text{ MeV} \]
\[ \theta_{\text{LAB}} = 130^\circ \]

\[ 0 \]

\[ 10 \]

\[ 50 \]

\[ 90 \]

\[ 110 \]

\[ 0 \]

\[ 20 \]

\[ 40 \]

\[ 60 \]

\[ 80 \]

\[ 100 \]

\[ 120 \]

\[ 140 \]

\[ 160 \]

\[ \text{CHANNEL} \]

\[ \text{COUNTS} \]
Figure 6. Typical gated helium recoil pulse-height spectrum from the $^{24}\text{Mg}(^{3}\text{He},n)^{26}\text{Si}$ reaction measured at a bombarding energy of 5.0 MeV and a laboratory reaction angle of 70°.
\[ ^{24}\text{Mg}(^{3}\text{He},n)^{26}\text{Si} \]

5.0 MeV
\[ \theta_{\text{LAB}} = 70^\circ \]

- NET SPECTRUM
- --- SUM OF BACKGROUNDS

COUNTS

CHANNEL
the helium cell an extra 100 ns before coincidence as shown in Fig. 3 by the dashed lines. In addition real coincidences due to room-scattered neutrons were measured by inserting a solid brass cylinder in the core of the spin precession solenoid completely blocking all direct air paths from the target to the helium cell. The transmission of this cylinder was less than 0.002 for fast neutrons of up to 15 MeV. Accidental contributions to the room-scattered background spectra in the region of interest amounted to less than 1% of the foreground counts in all cases.

Since the room-scattered background was small and slowly varying with $\theta_1$, this type of background was measured for only approximately one-third of the total integrated charge of a given foreground run and only at every third reaction angle. A charge normalized room-scattered background spectrum at a given angle was then subtracted from the foreground spectrum at the same angle. At angles where no room-scattered background spectrum was measured, a charge normalized average of the background spectra taken at the two nearest angles was subtracted from the foreground spectrum.

The contributions to the recoil spectra in the region of interest from both sources of background were found to be small. The total background in this region varied from 1 to 20% of the foreground counts, average 10% in the $^{24}\text{Mg}(^3\text{He},n_0)^{26}\text{Si}$ reaction and from 1 to 12% of the foreground counts, averaging 5% in the $^{13}\text{C}(^3\text{He},n_0)^{15}\text{O}$ reaction.

G. Data Reduction

Because both the reactions studied had extremely low yields, it
was necessary to use thick targets and large geometry detectors thereby reducing the overall energy resolution as has been discussed above. This resulted in less complete separation of ground state neutrons from excited state neutrons as is evidenced by the finite, though small, non-subtracting portion of the recoil spectra between the peaks in Figs. 5 and 6. In addition the low yields limited the counting statistics so as to make the exact extent of the peak due to a given group of neutrons less certain.

In order to eliminate any dependence of the measured asymmetries on a subjective peak definition, a computer program described in Appendix I was written which calculated the asymmetry as a function of peak width and position. In the present experiments the lower limit, \( L \), and upper limit, \( U \), to the peak area were chosen from the summed net spectrum such as the one shown in Fig. 6 for \(^{24}\text{Mg}(^3\text{He},n)^{26}\text{Si} \) at 5.0 MeV and \( \theta_1 = 70^\circ \). Three different peak widths, \( N \), were then chosen. For each choice the asymmetry was calculated for a peak \( N \) channels wide beginning in channels \( L \), \( L + \Delta \), \( L + 2\Delta \), \ldots, and \( L + \nu\Delta \) where \( \nu \) was chosen such that

\[
L + \nu\Delta \leq U
\]

The left-right ratio, \( r \), was thus calculated as

\[
T_m = \frac{L}{R} = \left[ \frac{\sum_{i=L+1}^{L+m} (T_+)_i \times \sum_{i=L+1}^{L+m} (B_-)_i}{\sum_{i=L+1}^{L+m} (T_-)_i \times \sum_{i=L+1}^{L+m} (B_+)_i} \right]^{\frac{1}{2}}
\]

and the asymmetry, \( \varepsilon \), was given by

\[
\varepsilon = \frac{p_1 p_2 \sin \phi}{(T_m - 1)} = \frac{(T_m - 1)}{(T_m + 1)}
\]

\[
\text{III-1}
\]

\[
\text{III-2}
\]

\[
\text{III-3}
\]
where $\Delta$ is a variable increment which in this case was set to approximately 0.05 (U-L).

The several values for the asymmetry for a given $N$ were then examined for consistency. In no case was the variation of the asymmetry over the peak area greater than the statistical uncertainty. For this $N$ value, the asymmetries were arithmetically averaged. The measured asymmetry was then taken as the mean of the three average values for the three different $N$ values. Any result for which the ratio of the asymmetry measured by one detector to that measured by the other was greater than 1.6 was questioned and the data point checked. In virtually all cases, however, the data point was reproduced. The neutron polarizations were obtained by dividing the measured asymmetries by the average analyzing power of helium and, where necessary, correcting for incomplete spin precession.

H. Helium Analyzing Power

Helium is an exceptional neutron polarization analyzer because of its large analyzing power which is relatively slowly varying with energy over a wide range of energies and the relatively well known phase shifts from which it is calculated which have been derived from the elastic scattering data. By far its most important advantage over other frequently used analyzers such as $^{12}$C and $^{16}$O in the present experiments, however, is its property of scintillation. This property makes it adaptable for use in a gas scintillation detector which through the use of the coincidence technique provides
significant reduction in backgrounds as well as energy information about the incident polarized neutrons.

The analyzing power of helium was calculated from the equation

\[ P_2(\theta) = \frac{AB^* + BA^*}{AA^* + BB^*} \]  \hspace{1cm} \text{III-4} \\

where

\[ A(\theta) = \sum_{\ell=0}^\infty P_\ell(\cos\theta) \left[ (\ell+1) e^{i\delta^+_\ell \sin\delta^+_\ell} + \ell e^{i\delta^-_\ell \sin\delta^-_\ell} \right] \]  \hspace{1cm} \text{III-5} \\

and

\[ B(\theta) = -i \sum_{\ell=0}^\infty P_\ell^{(1)}(\cos\theta) \left[ e^{i\delta^+_\ell \sin\delta^+_\ell} - e^{i\delta^-_\ell \sin\delta^-_\ell} \right] \]  \hspace{1cm} \text{III-6} \\

following Lepore.\textsuperscript{56} Here \( P_\ell(\cos\theta) \) are the ordinary Legendre polynomials and \( P_\ell^{(1)}(\cos\theta) \) the associated Legendre polynomials defined by

\[ P_\ell^{(1)}(\cos\theta) = \sin\theta \frac{d}{d(\cos\theta)} P_\ell(\cos\theta) \]  \hspace{1cm} \text{III-7} \\

The phase shifts, \( \delta^\pm_\ell \), of Hoop and Barschall\textsuperscript{57} determined from \( n - \alpha \) elastic scattering were used in these formulae.

In order to minimize the accelerator time necessary to obtain a value for the polarization with a given relative uncertainty, it is shown in Appendix II that to a first approximation the quantity \( P_2^2\sigma(\theta_2) \) should be maximized. For an ideal point scatterer and point neutron detector, \( P_2^2\sigma(\theta_2) \) at a given neutron energy has two maxima; one at a forward angle near \( \theta_2 = 60^\circ \) and one at a back angle near \( \theta_2 = 120^\circ \). The angular positions of these maxima as functions of energy are shown in Fig. 7.
Figure 7. Angles of the maxima in $P^2 \sigma$ for elastic scattering of neutrons from $^4$He calculated from the phase shifts of Hoop and Barschall.\textsuperscript{57}
ANGLES OF MAXIMUM $P^2 \sigma$ FOR A POINT DETECTOR
Since any real polarimeter has a finite scattering volume and extended rather than point detectors, the average analyzing power \( \bar{P}_2(\theta_2) \) for a given geometry was calculated for each of the two maxima in \( P_2^2\sigma(\theta_2) \) as a function of energy by a program described elsewhere. This program divided the scattering volume defined by the intersection of two perpendicular right circular cylinders into 78 cubical cells and the face of each plastic neutron detector into 64 cells. The analyzing power calculated from the above equations for each such pair of cells was weighted by the n - α scattering cross section and plastic neutron detector efficiency for the appropriate scattering angle and neutron energy. Finer subdivisions of the detectors were found to produce negligible changes in the calculated average analyzing power.

Since \( P_2^2\sigma(\theta_2) \) is a more slowly varying function of \( \theta_2 \) at the back angles than at forward angles, higher values for \( \bar{P}_2 \) were obtained for the maximum in \( P_2^2\sigma \) at approximately 120° than for that at approximately 60°. The position of the back angle maximum in \( P_2^2\sigma \) is a slowly varying function of energy and since the average analyzing power there was large at all energies, it was not deemed necessary to change \( \theta_2 \) during the course of these experiments and a scattering angle of \( \theta_2 = 121° \) was chosen.

The calculated average analyzing power, \( \bar{P}_2 \), for the actual experimental geometry depicted by a solid line and for an ideal point scatterer and detectors depicted by a dashed line at a scattering angle of \( \theta_2 = 121° \) are shown as functions of incident neutron energy.
in Fig. 8 where the zero is suppressed for clarity of comparison.
The regions pertinent to the present experiments are labeled in the
lower portion of the figure. As can be seen, the average analyzing
power and therefore the efficiency, $P_2^2\sigma$, of the polarimeter are
hardly reduced by the use of a large scattering volume and wide angle
detectors ($\Delta\theta \sim 23^\circ$ and $\Delta\phi \sim 43^\circ$) although the counting rates are
significantly increased by a factor of approximately 6 by using
2" x 3" x 6" plastic detectors instead of 1" x 2" x 3" plastic
detectors formerly used. An evaluation of the uncertainty in the
average analyzing power of helium due to the experimental uncer­
tainties in the n-$\alpha$ phase shifts has been made for a geometry similar
to that used in this experiment by Morgan and Walter. On the basis
of this evaluation an uncertainty of 3% in the average analyzing
power was included in the calculation of the experimental uncertainties
in these measurements.

I. Spin Precession Correction

Since not all neutrons traversed the same path in the solenoid,
the actual spin precession angle $\phi$ for a given average neutron energy
and a given magnet current was determined by measuring the asymmetry
for neutrons from the $^{12}\text{C}(d,n_0)^{13}\text{N}$ reaction at a mean deuteron energy
of $E_d = 2.94$ MeV and a lab reaction angle of $\theta_1 = 20^\circ$ for 8 values of
the magnet current I with a self-supporting foil target approximately
280 keV thick (i.e. $\Delta E_d \sim 0.1 E_d$ as in the experiments). For this
purpose the field B was assumed to be directly proportional to the
Figure 8. Calculated $^4$He analyzing power for elastic scattering of polarized neutrons through a laboratory angle of 121°. The dashed line refers to a point scatterer and detector, while the solid line refers to the analyzing power averaged over the detector geometry used in these experiments. The energy regions of interest in the two reactions studied here are indicated by the labeled lines in the lower part of the figure.
\[ \theta_2 = 121^\circ \]

- **POINT DETECTORS**
- **AVERAGED OVER HELIUM CELL AND 2''x3''x6'' PLASTIC**

**Graph:**
- **X-axis:** Neutron Energy (MeV)
- **Y-axis:** Analyzing Power

**Reactions:**
- \( ^{24}\text{Mg}(^3\text{He},n)^{28}\text{Si} \)
- \( ^{13}\text{C}(^3\text{He},n)^{15}\text{O} \)
magnet current $I$ and the 8 measured values of the asymmetry were fit by a function of the form

$$\epsilon(I) = \epsilon_{\text{max}} \sin\left(\frac{I}{I_{\text{max}}} \cdot \frac{\pi}{2}\right)$$

III-8

$\epsilon_{\text{max}}$ and $I_{\text{max}}$ were adjusted to minimize the $\chi^2$ function defined by

$$\chi^2 = \sum_{\Delta I} \left[ \frac{\epsilon_{\text{max}} \sin\left(\frac{I}{I_{\text{max}}} \cdot \frac{\pi}{2}\right) - \epsilon_{\text{exp}}(I_1)}{\Delta \epsilon_{\text{exp}}(I_1)} \right]^2$$

III-9

where $\epsilon_{\text{exp}}(I_1)$ is the measured asymmetry at a given current and $\Delta \epsilon_{\text{exp}}(I_1)$ its associated statistical uncertainty. The relationship between the known mean neutron energy and the optimum precession current $I_{\text{max}}$ was thus determined.

The measured values of $\epsilon$ at each current with their statistical uncertainties are given in Table 1. The data points and the best-fit sine curve are shown in Fig. 9. Within the statistical uncertainties of approximately 1%, the fit is consistent with the assumption of sinusoidal dependence of the asymmetry with the magnet current. The value obtained for the maximum asymmetry was

$$\epsilon_{\text{max}} = -0.3544 \pm 0.0090$$

which, after correcting for the average analyzing power of helium, gave a value for the polarization of

$$P_1 = -0.430 \pm 0.011$$

+A similar curve taken using the $^9$Be(p,n)$^9$B reaction by Sukis with a three-coil magnet shows points corresponding to full 180° precession. In that case too, the integrated field measured with a Hall probe was within 2% of that predicted by the infinite solenoid approximation.
\begin{table}
\centering
\caption{Asymmetry in the $^{12}\text{C}(d,n)^{13}\text{N}$ Reaction as a Function of Magnet Current}
n\begin{tabular}{llll}
$I$ (Amperes) & $\epsilon_{\text{exp}}(I)$ & $\pm$ & $\Delta\epsilon_{\text{exp}}(I)$ \\
0.0 & -0.0035 & & 0.0094 \\
49.2 & -0.1562 & & 0.0093 \\
73.8 & -0.2594 & & 0.0092 \\
98.7 & -0.2964 & & 0.0087 \\
123.6 & -0.3323 & & 0.0085 \\
148.2 & -0.3506 & & 0.0086 \\
172.8 & -0.3397 & & 0.0089 \\
197.4 & -0.3309 & & 0.0091 \\
214.2 & -0.2989 & & 0.0092 \\
\end{tabular}
\end{table}
Figure 9. Asymmetry in the $^{12}$C(d,n)$^{13}$N reaction at a mean bombarding energy of 2.94 MeV and a laboratory reaction angle of 20° measured as a function of current in the spin precession solenoid. The solid curve represents the best fit to the data using a sine function.
$^{12}\text{C}(d,n)^{13}\text{N}$

$E_d = 2.94$ MeV

$\Theta_i = 20^\circ$ LAB

$\epsilon_{\text{max}} = -0.354$

$I_{\text{max}} = 155.6$ amp
in good agreement with the average value of $-0.409 \pm 0.020$ obtained at deuteron energies between 2.90 and 3.00 MeV by Sawers, Purser and Walter.\(^{59}\) The value obtained for $I_{\text{max}}$ was slightly ($\sim 4\%$) higher than that expected from the integrated field measurement along the axis of the magnet (see Appendix III).

As a result of the energy spread of the neutrons due to the finite target thickness, not all neutrons could be precessed through the same angle. The magnet current, $I$, was thus selected to precess neutrons of the median energy through a certain angle $\phi$ and the correction factor $[\sin \phi]^{-1}$ was applied to the measured asymmetry. For precession angles $\phi < \pi/2$ as in the case of the $^{13}\text{C}(^{3}\text{He},n)^{15}\text{O}$ measurements no correction for differing precession angles is necessary since to first order

$$\frac{1}{2\Delta \phi} \int_{\phi-\Delta \phi}^{\phi+\Delta \phi} \sin \phi \times dx = \sin \phi$$  \hspace{1cm} \text{III-10}$$

For $\phi = \pi/2$, however, as in the case of the $^{24}\text{Mg}(^{3}\text{He},n)^{26}\text{Si}$ measurements, Eqn. III-10 no longer holds. Since in the classical approximation $\phi \sim E_{n}^{-1/2}$, for $\Delta E/E \approx 0.1$ the maximum difference in the correction factor, $\sin \phi$, at $\phi = \pi/2$ is less than 0.1%.

With 0.3% current regulation, the maximum difference in the spin precession correction factor due to varying field strength was calculated to be less than 0.2%.

Depolarization effects arising from field components not parallel to the neutron momentum direction were estimated from consideration...
of the solenoid geometry to be less than 1% using the results of Atkinson and Sherwood. Since all these possible sources of error combined were limited to less than 1.3% of the measured asymmetries, the accuracy of these measurements was principally limited by uncertainties due to counting statistics which averaged 12-13% in the case of the $^{24}\text{Mg}(^{3}\text{He},n)^{26}\text{Si}$ measurements and 9 to 10% in the case of the $^{13}\text{C}(^{3}\text{He},n)^{15}\text{O}$ measurements.

J. Polarimeter Checks

In order to check the apparatus for instrumental asymmetries, the polarizations of neutrons from the high yield $^{9}\text{Be}(\alpha,n)^{12}\text{C}$ and $^{9}\text{Be}(\alpha,n_{1})^{12}\text{C}^{*}(4.43 \text{ MeV})$ reactions were measured at $E_{\alpha} = 4.70 \text{ MeV}$ and $\theta_{1} = 0^\circ$. Assuming the target is unpolarized, the polarization at $0^\circ$ must be zero. The measurements are consistent with this value within an uncertainty of approximately 1%. The neutron energies, polarizations and their statistical uncertainties are presented in Table 2.

In addition the polarizations at various energies and angles were measured at equal reaction angles to the left ($+\theta_{1}$) and to the right ($-\theta_{1}$) of the incident momentum direction. The values obtained were consistent to within an average uncertainty of approximately 2%. The reaction energies, angles, polarizations and statistical uncertainties are given in Table 3.

A further check was provided by measuring a component of the polarization in the reaction plane, which must be zero if the charged particle beam is unpolarized, for the neutrons from the $^{12}\text{C}(d,n_{0})^{13}\text{N}$
TABLE 2

$^0\degree$ POLARIZATION CHECKS WITH THE $^9$Be($\alpha,n_0$)$^{12}$C AND $^9$Be($\alpha,n_1$)$^{12}$C*(4.43 MeV) REACTIONS AT 4.70 MeV

<table>
<thead>
<tr>
<th>Measurement</th>
<th>$^9$Be($\alpha,n_0$)$^{12}$C</th>
<th>$^9$Be($\alpha,n_1$)$^{12}$C*(4.43 MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_n$(MeV) $P_1 \pm \Delta P_1$</td>
<td>$E_n$(MeV) $P_1 \pm \Delta P_1$</td>
</tr>
<tr>
<td>1</td>
<td>10.30 $+0.048 \pm 0.054$</td>
<td>5.66 $-0.003 \pm 0.010$</td>
</tr>
<tr>
<td>2</td>
<td>10.30 $-0.001 \pm 0.025$</td>
<td>5.66 $+0.007 \pm 0.005$</td>
</tr>
<tr>
<td>Measurement</td>
<td>$E_\alpha$(MeV)</td>
<td>$\theta_{\text{lab}}$(deg)</td>
</tr>
<tr>
<td>-------------</td>
<td>----------------</td>
<td>-----------------------------</td>
</tr>
<tr>
<td>1</td>
<td>4.70</td>
<td>+30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-30</td>
</tr>
<tr>
<td>2</td>
<td>4.70</td>
<td>+45</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-45</td>
</tr>
</tbody>
</table>

$^9\text{Be}(\alpha,n_0)^{12}\text{C}$

| 3           | 4.50           | +15                         | 5.42       | +0.057 ± 0.012       |
|             |                | -15                         |            | +0.058 ± 0.016       |
| 4           | 4.50           | +30                         | 5.25       | +0.072 ± 0.010       |
|             |                | -30                         |            | +0.067 ± 0.015       |
| 5           | 4.70           | +30                         | 5.42       | +0.044 ± 0.008       |
|             |                | -30                         |            | +0.036 ± 0.010       |
| 6           | 4.70           | +45                         | 5.14       | -0.032 ± 0.013       |
|             |                | -45                         |            | -0.048 ± 0.012       |
| 7           | 5.00           | +30                         | 5.68       | -0.086 ± 0.018       |
|             |                | -30                         |            | -0.087 ± 0.009       |
| 8           | 5.00           | +45                         | 5.39       | -0.077 ± 0.018       |
|             |                | -45                         |            | -0.088 ± 0.013       |
| 9           | 5.00           | +105                        | 3.90       | -0.268 ± 0.028       |
|             |                | -105                        |            | -0.243 ± 0.010       |
| 10          | 5.00           | +120                        | 3.60       | -0.158 ± 0.027       |
|             |                | -120                        |            | -0.112 ± 0.015       |
| 11          | 5.00           | +130                        | 3.43       | +0.048 ± 0.017       |
|             |                | -130                        |            | +0.062 ± 0.023       |

$^9\text{Be}(\alpha,n_1)^{12}\text{C}^*$(4.43 MeV)
reaction at $E_d = 2.94$ MeV and $\theta_1 = 20^\circ$. The value obtained was

$$P_1 = -0.0035 \pm 0.0094.$$ 

K. Data Collection Procedure

In summary, after setting up the electronics and taking a delay curve, four gated helium recoil foreground spectra were taken simultaneously with four accidental spectra. At every third angle at each energy four room-scattered background spectra were taken for about one-third the average total charge collected in the foreground runs. The accidental and charge-normalized room-scattered background spectra were subtracted channel by channel from the foreground spectra to produce four net recoil spectra. The measured asymmetry was extracted from these net spectra using the procedure described above in section G. The polarizations were finally calculated by correcting the measured asymmetries for the analyzing power of helium and for less than 90° spin precession in some cases.

On the average each data point for the $^{13}C(^3He,n)^{15}O$ reaction required about 9 hours of accelerator time while each point for the $^{24}Mg(^3He,n)^{26}Si$ reaction required nearly 16 hours of accelerator time. During each run the current sense in the spin precession solenoid was changed approximately every 30 minutes to minimize any false asymmetry due to possible long term electronic drift.

The $^{13}C(^3He,n)^{15}O$ measurements were made for eight angles back to 130° lab at four bombarding energies separated from each other.
by 500 keV. This energy interval was chosen to cover the range of energy studied with the relatively thick targets used.

The $^{24}\text{Mg}(^{3}\text{He},n)^{26}\text{Si}$ measurements were made for nine angles back to 135° lab at two bombarding energies separated by 800 keV. The range of investigation here was limited to energies above 4.5 MeV by the Coulomb barrier and to energies below 6 MeV by the accelerator. Due to the amount of accelerator time necessary to obtain one data point, the energy interval between polarization angular distributions was chosen somewhat larger than the target thickness so as to observe any large variations of the polarization with energy. Measurements at selected angles were made at other energies to investigate the energy variation of the polarization more fully.
CHAPTER IV
EXPERIMENTAL RESULTS

A. $^{13}\text{C}^{(3}\text{He},n)^{15}\text{O}$

Angular distributions of the polarization of the neutrons from the $^{13}\text{C}^{(3}\text{He},n)^{15}\text{O}$ reaction were measured at a minimum of eight angles back to 130 degrees lab for four bombarding energies between 4.2 and 5.7 MeV where data at 4.70 and 4.73 MeV were combined. All the data are presented in Table 4 where the bombarding energy $E_{^3\text{He}}$, the reaction angle $\theta$ in the laboratory and center of mass systems, the neutron energy $E_n$, the mean target thickness $\Delta E$, and the measured asymmetry $\epsilon$ are listed along with the polarization $P_1$ and its associated uncertainty $\Delta P_1$. The uncertainties given are due primarily to counting statistics but include as well such other minor uncertainties discussed in sections H and I of Chapter III as an uncertainty of 3% in the analyzing power of $^4\text{He}$ and uncertainties of 1.3% in the asymmetry due to different spin precession angles and the effects of depolarizing fields. The geometrically averaged analyzing power varied between 0.81 and 0.86 for these measurements. The angular distributions of polarization are shown in Fig. 10 where the error bars shown refer to the uncertainties given in Table 4. The scales in Fig. 10 alternate sides to indicate to which angular distribution each particular scale applies.

The polarization varies considerably with angle in this energy region, there being at least three extrema in each of the four angular
Figure 10. Polarization angular distributions of the neutrons from the $^{13}\text{C}(^{3}\text{He},n^{0})^{15}\text{O}$ reaction measured between 4.2 and 5.7 MeV.
Polarization

$^{13}$C$(^{3}$He, n$) ^{15}$O

4.2 MeV

4.7 MeV

5.2 MeV

5.7 MeV

$\theta_{Lab}$ (deg)
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distributions of polarization. Large polarizations of up to $+0.68 \pm 0.10$ at 5.2 MeV and 70° lab were observed. Large polarizations with rapid angular variation were also observed in the previous measurements by Stammbach et al. between 2.95 and 3.85 MeV and the 4.2 MeV polarization angular distribution is similar in shape to that measured at 3.85 MeV by those authors. The polarization angular distributions above 4 MeV vary in a systematic fashion with energy in such a way that the entire pattern of polarization appears to move toward more forward angles with increasing energy. This systematic energy variation is also evident in Fig. 11 which shows the excitation functions of polarization between 4.2 and 5.7 MeV at 20, 70 and 130 degrees lab. As the energy increases, the polarization at each angle passes through a broad extremum as each peak in the polarization angular distribution moves toward more forward angles. No violent fluctuations of the polarization with energy characteristic of the influence of pronounced resonances are evident although this may in part be due to the thickness of the targets or to their non-uniformity. In Fig. 11 the horizontal error bars indicate the mean target thickness of the measurements.

A more complete representation of all the polarization data from 2.95 to 5.7 MeV is given in the contour plot shown in Fig. 12 which was prepared from 70 data points from both the present and previous measurements taken with targets ranging in thickness from 200 to 700 keV. The contours represent constant values of the polarization plotted as a function of the reaction angle and bombarding energy. The contours were constructed by linear angular and energy interpolation between data points. The interval between contours is 0.10 and
Figure 11. Excitation functions of polarization for the neutrons from the $^{13}\text{C}(^{3}\text{He},n)^{15}\text{O}$ reaction measured at laboratory reaction angles of 20°, 70°, and 130°. The solid curves are presented only to guide the eye. The horizontal error bars represent the mean energy spread.
Polarization diagram for $^{13}\text{C}(^{3}\text{He},n)^{15}\text{O}$ reaction.
Figure 12. A contour plot of the neutron polarization in the $^{13}\text{C}({}^3\text{He},n)^{\text{15}O}$ reaction constructed from measurements made between 2.95 and 5.7 MeV. Dashed contours alternate with solid ones for clarity in presentation. The 0° excitation function measured by Din and Weil$^{25}$ for this reaction is sketched above the contour plot.
dashed lines alternate with solid lines for clarity. Above the con-
tour plot is shown the 0° yield curve from the work of Din and Weil.\textsuperscript{25} Although they are subject to both experimental uncertainties and un-
certainties in interpolation, contour plots illustrate the salient
features of the polarization over a wide range of energies and angles
and can thus be useful in making general observations on the importance
of various reaction mechanisms in producing the polarization. For
instance, sharp fluctuations in the polarization characterized either
by nearly vertical contours or by narrow closed regions at a given
energy such as the three closed regions near 3.5 MeV in Fig. 12 are
generally interpreted as indicating the influence of a particular
resonance at that energy. In this case the 0°, 90° and 150° yield
curves for the \( \text{^{13}C(}^{3}\text{He,nO)} \) \text{^{15}O} reaction all exhibit peaks at an energy
near 3.5 MeV and peaks corresponding to the same resonance in \( \text{^{16}O} \) are
also observed in the yield curves\textsuperscript{40} from the \( \text{^{13}C(}^{3}\text{He,pO)} \text{^{15}N} \) and
\( \text{^{13}C(}^{3}\text{He, } \alpha \nu_{15.1}) \) reactions.\textsuperscript{62} It thus appears that the polarization
in this region is influenced by a resonance corresponding to a bom-
barding energy near 3.5 MeV. On the other hand, the more gradual
variation of the polarization with energy characterized by nearly
horizontal contours such as those in the region above 4.3 MeV in
Fig. 12 are usually interpreted as indicative of a significant con-
tribution from a direct reaction mechanism.

B. \( \text{^{24}Mg(}^{3}\text{He,nO)} \text{^{26}Si} \)

Angular distributions of the polarization of the \( \text{^{24}Mg(}^{3}\text{He,nO)} \text{^{26}Si} \)
neutrons were measured back to 135 degrees lab for bombarding energies
of 5.0 (at nine angles) and 5.8 MeV (at ten angles). In addition the
polarization at some forward angles was measured at other energies between 4.6 and 5.4 MeV to obtain more detailed information on the energy dependence of the polarization. These data are all listed in Table 5 where the column headings are the same as those in Table 4. The two angular distributions of polarization are shown in Fig. 13. Solid curves are drawn through the data points to suggest the angular dependence of the polarization and the vertical error bars refer to the uncertainties given in Table 5. A break is shown in the curve through the 5.0 MeV data because the ratio of the cross section for the first excited state neutrons to that for the ground state neutrons in the angular region between 20 and 50 degrees lab at this energy was too large to permit extraction of a meaningful value for the polarization of the ground state neutrons from the helium recoil spectra. The data in this same angular region at 5.8 MeV and the similarity in the shapes of the two polarization angular distributions at other angles indicate that a smooth connection of the two data points at 20 and 50 degrees lab in the 5.0 MeV angular distribution would be misleading. In both of these angular distributions the polarization shows a similar angular dependence, being positive at very forward angles, negative at angles near 90 degrees lab and positive again at back angles. In this reaction as in the $^{13}$C($^3$He,n$^0$)$^{15}$O reaction large polarizations of up to $-0.39 \pm .12$ at forward angles and up to $+0.71 \pm .16$ at back angles were observed.

Two excitation functions of polarization at 20 and 70 degrees lab plotted in Fig. 14 show that the polarization varies systematically with energy over the limited range of energy investigated. The
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Figure 13. Polarization angular distributions for the neutrons from the $^{24}\text{Mg}(^3\text{He},n)^{26}\text{Si}$ reaction measured at 5.0 and 5.8 MeV. The curves are intended only to guide the eye. The gap in the curve through the 5.0 MeV data is explained in the text.
Polarization

\[ \Theta_{\text{LAB}} (\text{deg}) \]

\[ \begin{array}{c}
\text{5.8 MeV} \\
\text{5.0 MeV}
\end{array} \]

\[ ^{24}\text{Mg}(^{3}\text{He},n)^{26}\text{Si} \]
Figure 14. Polarization excitation functions for the neutrons from the $^{24}$Mg($^3$He, $n_0$)$^{26}$Si reaction measured at laboratory reaction angles of 20° and 70°. Solid curves are drawn through the data points. The dashed point shown at 5.8 MeV and 70° is angularly interpolated from data points at 65° and 80°.
$^{24}\text{Mg}(^{3}\text{He},n_{o})^{26}\text{Si}$

![Graph showing polarization vs. $E_{^{3}\text{He}}$ (MeV) with $\theta_{LAB}=20^\circ$ and $\theta_{LAB}=70^\circ$.](image)
horizontal error bars in Fig. 14 represent the target thicknesses and the curves here serve only to guide the eye. The point indicated at 5.8 MeV and 70 degrees lab is linearly interpolated from the two points at 65 and 80 degrees lab in the 5.8 MeV angular distribution where the polarization appears to vary little with angle. The gradual variation of the polarization with energy along with the strong peaking at 0 degrees and energy independence of the differential cross sections suggest a large direct reaction contribution to this reaction.

In summary, large polarizations which appear to vary systematically and gradually with energy are observed in both the $^{13}$C($^3$He, n$_o$)$^{15}$O and $^{24}$Mg($^3$He, n$_o$)$^{26}$Si reactions. The large forward peaks in the differential cross sections and the slow variation of their shapes with energy above 4.5 MeV indicate a substantial amplitude for two-nucleon stripping in each case. The systematic energy dependence of the polarization suggests that a direct reaction mechanism plays an important role as well in producing the polarization in these reactions.
CHAPTER V
ANALYSIS AND DISCUSSION

A. General Discussion of Analysis

Both reactions studied here exhibit characteristics of a direct reaction process, namely, large forward angle peaks in the differential cross sections with magnitudes which vary little with energy and polarizations (which are large) that also vary slowly with energy. DWBA calculations were therefore performed in an effort to determine how well the cross section and polarization data might be described by the assumption of a pure direct reaction mechanism for each reaction. These calculations were carried out in the zero-range approximation using ORNL computer program JULIE\textsuperscript{45} which computes the reaction differential cross section and polarization using the formalism presented in section B of Chapter II.

For specific calculations, the spectroscopic coefficient $A_{lsj}$, the radial form factor $F_{lsj}(r)$ and the distorted wave functions $\Phi_{Lj}(k_1, r)$ in both the entrance and exit channels must all be specified. In the particular cases considered here (i.e. two nucleon transfer with $\sqrt{s_x} = 0$ and $\sqrt{j_x} = 2x = 0$), the spectroscopic coefficient of the one contributing term enters only as a normalization constant in both the differential cross section (Eqn. II-37) and the polarization (Eqn. II-38). Therefore, although it affects the magnitude of
the total reaction cross section, $A_{\lambda S}|^2$ affects neither the shape of the angular distribution nor that of the polarization angular distribution in any way. Previous results for two nucleon stripping reactions have indicated that $|A_{\lambda S}|^2$ can have values ranging between 20 and 70 for reactions of the type studied here when the calculated cross section is expressed in mb/sr. A value near $|A_{\lambda S}|^2 = 30$ is suggested by Drisko\textsuperscript{63} as being most appropriate for the present calculations.

The form factor $F_{\lambda S}(r)$ is calculated by JULIE as the eigenfunction of the Schrödinger equation for the transferred cluster $x$ (with each nucleon of $x$ in a specified configuration) bound to the core $c$ by the potential

$$U(r) = U_{\text{coul}} - V_o f'(r) - \alpha \langle \ell \rangle \lambda_{\sigma_0} \left( \frac{\lambda_{\sigma_0}}{\lambda_{\sigma_0} m_c} \right)^2 \frac{d}{dr} f'(r) \quad \text{(V-1)}$$

where

$$f'(r) = \left[ 1 + e^{(r - R_{oc} A_{c}^{1/3}) / \alpha} \right]^{-1} \quad \text{(V-2)}$$

and where, for the single particle configuration,

$$\alpha \langle \ell \rangle = \ell \quad \text{if} \quad j = \ell + \frac{1}{2}$$

$$= -(\ell + 1) \quad \text{if} \quad j = \ell - \frac{1}{2} \quad \text{(V-3)}$$

$$U_{\text{coul}} = \frac{Z_x Z_s (3 - \frac{r}{R_c})}{2 R_c} \quad \text{if} \quad r \leq R_c$$

$$= \frac{Z_x Z_s}{r} \quad \text{if} \quad r \geq R_c \quad \text{(V-4)}$$

is the Coulomb potential for a point charge interacting with a uniform charge distribution within a sphere of radius $R_c = r_{oc} A_{c}^{1/3}$. The depth
of the real potential \( V_0 \) is adjusted to give an eigenvalue equal to the binding energy \( B_{E_x} \) for each nucleon in \( x \) bound to the target which is determined as half the difference between the binding energy of the residual nucleus and that of the target nucleus. The expected single particle shell model configurations on the basis of the Nilsson model\(^6\) for the captured protons are \((1p_{\frac{3}{2}})^2\) in the \(^{13}\text{C}(^{3}\text{He},n_0)^{15}\text{O}\) reaction and either \((1d_{\frac{5}{2}})^2\) or possibly \((2s_{\frac{1}{2}})^2\), for a large deformation, in the \(^{24}\text{Mg}(^{3}\text{He},n_0)^{26}\text{Si}\) reaction. The binding well parameters for two protons bound in \(1p_{\frac{3}{2}}\) configurations to a \(^{13}\text{C}\) core and for two protons bound in \(1d_{\frac{5}{2}}\) and \(2s_{\frac{1}{2}}\) configurations respectively to a \(^{24}\text{Mg}\) core are given in Table 6 where energies are given in MeV and lengths in fermi's (\(1f = 10^{-13}\) cm). These three form factors are plotted in Fig. 15 where each has been normalized to unity at \( r = 0 \) for purposes of comparison.

The distorted waves \( \phi^{(1)}_{L,J,J_1}(k_1,r) \) in both the entrance and exit channels were generated from optical potentials of the form

\[
U(r) = V_c(r) - V f(r) - i \left[ W - W' a' \frac{d}{dr} \right] g(r) \\
+ V_s \left( \frac{\phi}{m_r c} \right)^2 \hat{r} \cdot \hat{\sigma} \frac{1}{r} \frac{d}{dr} f(r)
\]

\[(V-5)\]

where either the volume imaginary well depth \( W \) or the surface imaginary well depth \( W' \) was set equal to zero (i.e. either volume or surface absorption was used). The variables \( f(r) \) and \( g(r) \) are independent Saxon-Woods form factors given by

\[
f(r) = \left[ 1 + e^{(r-R_s)/a} \right]^{-1} \quad \text{and} \quad g(r) = \left[ 1 + e^{(r-R'_s)/a'} \right]^{-1}
\]

\[(V-6)\]
### TABLE 6
PARAMETERS OF SAXON-WOODS POTENTIAL WELLS
USED TO GENERATE FORM FACTORS

<table>
<thead>
<tr>
<th></th>
<th>$^{13}\text{C}(^{3}\text{He},n_0)^{15}\text{O}$</th>
<th></th>
<th>$^{24}\text{Mg}(^{3}\text{He},n_0)^{26}\text{Si}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$^{1p}_{1/2}$</td>
<td>$^{1d}_{5/2}$</td>
<td>$^{2s}_{1/2}$</td>
</tr>
<tr>
<td>$B_{\text{ex}}$</td>
<td>7.42</td>
<td>3.89</td>
<td>3.89</td>
</tr>
<tr>
<td>$V_0$</td>
<td>52.45</td>
<td>49.25</td>
<td>54.17</td>
</tr>
<tr>
<td>$r_{\text{on}}$</td>
<td>1.25</td>
<td>1.25</td>
<td>1.25</td>
</tr>
<tr>
<td>$a$</td>
<td>0.65</td>
<td>0.65</td>
<td>0.65</td>
</tr>
<tr>
<td>$r_{\text{oc}}$</td>
<td>1.25</td>
<td>1.25</td>
<td>1.25</td>
</tr>
<tr>
<td>$\lambda_{\text{so}}$</td>
<td>25.</td>
<td>25.</td>
<td>25.</td>
</tr>
</tbody>
</table>
Figure 15. Calculated radial form factors used in the DWBA calculations for the $^{13}\text{C}(^{3}\text{He},n)^{15}\text{O}$ and $^{24}\text{Mg}(^{3}\text{He},n)^{26}\text{Si}$ reactions.
$F_{lsj}(r)$ (normalized to 1.0 at 0 fermis)
with \( R_0 = r_0 A^{1/3} \) and \( R'_0 = r'_0 A^{1/3} \). The other parameters are \( V \), the real well depth, \( r_0 \) and \( r_0' \), the radius parameters of the real and imaginary wells respectively, \( a \) and \( a' \) the diffuseness parameters of the real and imaginary wells respectively, \( A \), the mass number of the target nucleus (entrance channel) or residual nucleus (exit channel), \( \text{i} \) the relative orbital angular momentum operator \( \overrightarrow{r} \times \overrightarrow{\mathbf{L}} \) and \( \sigma \) the Pauli spin operator. \( V_c(r) \) is the Coulomb potential given in Eqn. (V-4) above. Some 20 partial waves were included in all the calculations. The six variable parameters \( V \), \( W \), or \( W' \), \( r_0 \), \( r_0' \), \( a \) and \( a' \) were determined by requiring that as a first approximation the optical model potentials give optimum fits to the elastic scattering data for the appropriate channel.

Drisko\(^64\) has shown that the optical model analysis of elastic scattering data for heavily absorbed composite particles such as \(^3\)He is sensitive only to the asymptotic form of the wave function which unfortunately leads to discrete ambiguities in the optical potential. That is, different sets of optical model parameters fit the elastic scattering data equally well, the asymptotic form of their wave functions for each partial wave being essentially the same. The wave functions from these different families of potentials, however, are different in the nuclear interior region where the \( F_{lsj}(r) \) are large (see Fig. 15). This difference can give rise to large variations in the radial integral of Eqn. II-33 and can thus be important in distorted wave calculations. Previous calculations\(^66,67\) indicate that the physically correct potential for elastic scattering of a composite
particle of \( n \) nucleons has a real well depth \( n \) times the depth of the potential \( (V \sim 50 \text{ MeV}) \) which best describes single nucleon scattering from the same target. The other sets of potentials, however, cannot be ruled out on the basis of elastic scattering data alone. A further difficulty involved in the optical model analysis of elastic scattering data is that equivalent fits may be obtained in the neighborhood of each particular discrete ambiguity by varying only a few of the parameters in such a way that their effects compensate each other. These are the continuous ambiguities the most well-known example of which is the \( V_{rc}^N \) ambiguity where \( n \) is a number near two. \(^{44}\) The spin-orbit term only mildly affects the fit to the elastic scattering angular distributions at large angles whereas it can give rise to large polarizations in the reaction calculations. Hence no attempt was made to determine \( V_{so} \) from the elastic scattering cross section data. In the analysis of each reaction, several sets of entrance channel optical model potentials were used in conjunction with two sets of exit channel parameters and the spin-orbit potential depth \( V_{so} \) was treated as a free variable in both the entrance and exit channels.

B. The \( ^{13}\text{C}(^{3}\text{He},n\alpha)^{15}\text{O} \) Reaction Calculations

1. Optical Model Parameter Selection

The optical model parameters are generally derived from elastic scattering data. However, only three angular distributions have been reported\(^{68}\) at low energies \((6, 7 \text{ and } 8 \text{ MeV})\) for \(^3\text{He} \) elastic scattering from \( ^{13}\text{C} \). Furthermore these angular distributions show considerable
variations with energy suggesting the presence of significant compound elastic effects. To avoid the influence these effects might have on the DWBA calculations, the sets of optical model parameters chosen for the entrance channel were obtained by averaging potentials derived by various authors\textsuperscript{37,68-78} to fit \(^3\text{He}\) elastic scattering cross section data involving a variety of low and medium \(A\) targets at bombarding energies from 3.7 to 37.7 MeV. This method of selection should tend to average out the distortions which might occur in any specific parameter set due to significant compound elastic contributions to the elastic scattering data fitted with that particular set. It has been shown\textsuperscript{79,80} that equivalent optical model fits to \(^3\text{He}\) and \(t\) elastic scattering data can be obtained with either a volume imaginary or surface imaginary term to account for inelastic events.

Since in the majority of recent optical model analyses of such data a volume imaginary term has been employed, only potentials using a volume imaginary term were considered. These published "best-fit" potentials, listed in Table 7, were divided into three discrete families by their values of the quantity\textsuperscript{\footnote{The value of 1.96 for the exponent here is the value suggested by Bassel.\textsuperscript{69}}} \(V_{r_0}^{1.96}\) and values near the average value for each parameter from each family were then selected. Because the selected values for \(r_0\) were nearly the same in all three cases, the three chosen potential sets differed mainly in the depth of their real potential well \(V\). The three selected parameter sets, labeled H-1, H-2 and H-3 in descending order of their
**TABLE 7**

OPTICAL MODEL POTENTIALS USED TO OBTAIN ENTRANCE CHANNEL PARAMETER SETS FOR $^{13}$C($^3$He, n)$^{15}$O DWBA CALCULATIONS

<table>
<thead>
<tr>
<th>Target Nuclei</th>
<th>Reference</th>
<th>$E_{3He} (\text{MeV})$</th>
<th>$V (\text{MeV})$</th>
<th>$V (\text{MeV})$</th>
<th>$r_o$ ($\AA$)</th>
<th>$a$ ($\AA$)</th>
<th>$r'_o$ ($\AA$)</th>
<th>$a'$ ($\AA$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{40}$Ca, $^{58}$Ni</td>
<td>68</td>
<td>37.7</td>
<td>172</td>
<td>18</td>
<td>1.15</td>
<td>0.70</td>
<td>1.57</td>
<td>0.92</td>
</tr>
<tr>
<td>$^{39}$K</td>
<td>69</td>
<td>9</td>
<td>181</td>
<td>16.1</td>
<td>1.07</td>
<td>0.85</td>
<td>1.81</td>
<td>0.59</td>
</tr>
<tr>
<td>$^{24}$Mg</td>
<td>74</td>
<td>29</td>
<td>182.9</td>
<td>10.4</td>
<td>1.15</td>
<td>0.78</td>
<td>1.84</td>
<td>0.66</td>
</tr>
<tr>
<td>$^{12}$C</td>
<td>72</td>
<td>15</td>
<td>172</td>
<td>25</td>
<td>1.20*</td>
<td>0.71</td>
<td>1.20*</td>
<td>0.71</td>
</tr>
<tr>
<td>$^{12}$C</td>
<td>37</td>
<td>3.7</td>
<td>189.6</td>
<td>47.4</td>
<td>1.11</td>
<td>0.57</td>
<td>1.11</td>
<td>0.57</td>
</tr>
<tr>
<td>$^{10}$B</td>
<td>70</td>
<td>32.5</td>
<td>171</td>
<td>12.8</td>
<td>1.25</td>
<td>0.58</td>
<td>1.78</td>
<td>0.94</td>
</tr>
<tr>
<td><strong>Values Selected</strong></td>
<td></td>
<td></td>
<td>176.0</td>
<td>18.5</td>
<td>1.14</td>
<td>0.70</td>
<td>1.75</td>
<td>0.80</td>
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<tr>
<td>$^{22}$Ne</td>
<td>77</td>
<td>9.74</td>
<td>140.1</td>
<td>21.3</td>
<td>1.07*</td>
<td>0.85*</td>
<td>1.81*</td>
<td>0.65*</td>
</tr>
<tr>
<td>$^{12}$C</td>
<td>72</td>
<td>5-12</td>
<td>132.1</td>
<td>17.3</td>
<td>1.20*</td>
<td>0.64</td>
<td>1.20*</td>
<td>0.64</td>
</tr>
<tr>
<td>$^{12}$C</td>
<td>72</td>
<td>3.7</td>
<td>110</td>
<td>24.0</td>
<td>1.20*</td>
<td>0.60</td>
<td>1.20*</td>
<td>0.60</td>
</tr>
<tr>
<td>$^{9}$Be</td>
<td>71</td>
<td>6-8</td>
<td>140.9</td>
<td>28.1</td>
<td>1.15</td>
<td>0.78</td>
<td>1.88</td>
<td>0.61</td>
</tr>
<tr>
<td><strong>Values Selected</strong></td>
<td></td>
<td></td>
<td>137.7</td>
<td>22.2</td>
<td>1.14</td>
<td>0.75</td>
<td>1.85</td>
<td>0.65</td>
</tr>
<tr>
<td>Si</td>
<td>75</td>
<td>28.7</td>
<td>101.8</td>
<td>14.2</td>
<td>1.07</td>
<td>0.85</td>
<td>1.81</td>
<td>0.70</td>
</tr>
<tr>
<td>$^{27}$Al</td>
<td>75</td>
<td>28.9</td>
<td>99.6</td>
<td>14.5</td>
<td>1.07</td>
<td>0.85</td>
<td>1.81</td>
<td>0.69</td>
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<tr>
<td>$^{27}$Al</td>
<td>76</td>
<td>5.5-10</td>
<td>105</td>
<td>13</td>
<td>1.07</td>
<td>0.85</td>
<td>1.80</td>
<td>0.65</td>
</tr>
<tr>
<td>Mg</td>
<td>75</td>
<td>28.7</td>
<td>108.6</td>
<td>15.4</td>
<td>1.07</td>
<td>0.85</td>
<td>1.80</td>
<td>0.76</td>
</tr>
<tr>
<td>$^{24}$Mg</td>
<td>74</td>
<td>29</td>
<td>107.1</td>
<td>14.3</td>
<td>1.15</td>
<td>0.78</td>
<td>1.81</td>
<td>0.74</td>
</tr>
<tr>
<td>$^{24}$Mg</td>
<td>73</td>
<td>12</td>
<td>101.2</td>
<td>14.8</td>
<td>1.07*</td>
<td>0.85*</td>
<td>1.81*</td>
<td>0.59*</td>
</tr>
<tr>
<td>$^{22}$Ne</td>
<td>77</td>
<td>9.74</td>
<td>115.7</td>
<td>17.2</td>
<td>1.07*</td>
<td>0.85*</td>
<td>1.81*</td>
<td>0.65*</td>
</tr>
<tr>
<td>$^{12}$C</td>
<td>67</td>
<td>6.0</td>
<td>139.7</td>
<td>5.5</td>
<td>0.93*</td>
<td>0.81</td>
<td>2.25</td>
<td>0.65*</td>
</tr>
<tr>
<td>$^{10}$B</td>
<td>70</td>
<td>32.5</td>
<td>104.4</td>
<td>26.6</td>
<td>1.15</td>
<td>0.78</td>
<td>1.14</td>
<td>0.91</td>
</tr>
<tr>
<td><strong>Values Selected</strong></td>
<td></td>
<td></td>
<td>105.0</td>
<td>16.8</td>
<td>1.10</td>
<td>0.80</td>
<td>1.80</td>
<td>0.70</td>
</tr>
</tbody>
</table>

*These parameters were not allowed to vary in searching for the best fit to the data.*
real potential well depth, are listed in Table 8. Another set, labeled H-4, is the one which Weller et al. found gave the best fit to the $^3$He elastic scattering angular distribution from $^{13}$C at 6.0 MeV.

In the exit channel the primary set of parameters used was the energy dependent set of Perey labeled N-1 in Table 8. This set of parameters was derived from a simultaneous analysis of 35 proton elastic scattering angular distributions at energies between 9.4 and 22 MeV from various target nuclei with $Z \geq 13$. A parameter set derived from an analysis of both proton and neutron elastic scattering cross section and polarization data from targets with $A > 50$ over a wider range of energies has recently been reported by Becchetti and Greenlees. This set, labeled N-2 in Table 8, was made available as the calculations were nearing completion and was thus used only in the latter calculations. The spin-orbit potential well depths were varied from 0 to 5 MeV in the entrance channel and from 0 to 10 MeV in the exit channel, which covers the range of commonly accepted values.

2. DWBA Reaction Calculations

The differential cross section angular distributions ($\sigma(\theta)$) and polarization angular distributions ($P(\theta)$) calculated at the indicated energies using JULIE are compared in Fig. 16 with the present polarization data at 5.2 MeV and the previously measured cross section data at 5.0 MeV. In these calculations, each of the four entrance channel parameter sets was used together with the exit channel set N-1.
### Table 8

**Optical Model Parameters Used in \(^{13}\text{C}(^{3}\text{He},n)^{15}\text{O} Calculations**

<table>
<thead>
<tr>
<th>Set Label</th>
<th>(V) (MeV)</th>
<th>(r_0) (f)</th>
<th>(a) (f)</th>
<th>(r_s) (f)</th>
<th>(W) (MeV)</th>
<th>(r'_0) (f)</th>
<th>(a') (f)</th>
<th>(W') (MeV)</th>
<th>(V_{so}) (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H-1</td>
<td>176.0</td>
<td>1.14</td>
<td>0.70</td>
<td>1.40</td>
<td>18.5</td>
<td>1.75</td>
<td>0.80</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>H-2</td>
<td>137.7</td>
<td>1.14</td>
<td>0.75</td>
<td>1.40</td>
<td>22.2</td>
<td>1.85</td>
<td>0.65</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>H-3</td>
<td>105.0</td>
<td>1.10</td>
<td>0.80</td>
<td>1.40</td>
<td>16.8</td>
<td>1.80</td>
<td>0.70</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>H-4</td>
<td>173.9</td>
<td>0.93</td>
<td>0.81</td>
<td>1.40</td>
<td>4.55</td>
<td>2.26</td>
<td>0.65</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>N-1</td>
<td>46.0-46.9</td>
<td>1.25</td>
<td>0.65</td>
<td>1.25</td>
<td>0</td>
<td>1.25</td>
<td>0.47</td>
<td>54.0</td>
<td>0-10</td>
</tr>
<tr>
<td>N-2</td>
<td>53.8-54.3</td>
<td>1.17</td>
<td>0.75</td>
<td>1.25</td>
<td>0</td>
<td>1.26</td>
<td>0.58</td>
<td>42.5-44.0</td>
<td>6</td>
</tr>
</tbody>
</table>
Figure 16. Comparison of differential cross section and polarization data with DWBA calculations for the $^{13}$C($^3$He,$n_0$)$^{15}$O reaction using the four sets of entrance channel optical model parameters listed in Table 7 together with exit channel parameter set N-1. The spin-orbit well depths used were 2.5 MeV in the entrance channel and 5.5 MeV in the exit channel.
Spin-orbit potential depths of 2.5 MeV in the entrance channel 
($V_{so}(\text{He}) = 2.5 \text{ MeV}$) and 5.5 MeV in the exit channel ($V_{so}(\text{n}) = 5.5 \text{ MeV}$) were used. Because of the uncertainty in $|A_{\text{lab}}|^2$, the 
differential cross sections are all normalized to unity at 0° to 
allow convenient comparison of the shapes of the angular distributions.

The three entrance channel optical model potentials H-1, H-2 and H-3 
give reasonably good fits to the shape of the angular distributions 
measured between 3.83 and 5.35 MeV. The fit with set H-4 is somewhat 
poorer than those obtained with the three average potentials in the 
region of the stripping peak, especially at energies above 4.5 MeV.

The calculated $P(\theta)$ curves all have similar shapes which resemble 
the shape of the measured $P(\theta)$. It must be noted that while the 
calculations are done at a precise energy and vary slowly with energy, 
the experimental values represent average values over an energy 
interval of a few hundred keV. Thus, the magnitudes of polarization 
obtained with sets H-3 and H-4 are too small to account for the large 
observed values. The best fits to both the cross section and polarization data were obtained with entrance channel sets H-1 and H-2 both 
of which have real potential well depths approximately three times 
the real well depth used to describe single nucleon elastic scattering.

The effects of varying $V_{so}$ in the entrance and exit channels 
are shown in Figs. 17 and 18 respectively. In both figures calculations carried out at the indicated energies with sets H-2 and N-1 
are compared to the data at the same energies.† In Fig. 17, $V_{so}(\text{n})$

†Unless otherwise noted, all calculations are done at the energies indicated in the figures at which the data were taken.
Figure 17. Effect on the DWBA calculations of the cross section and polarization angular distributions for the $^{13}\text{C}(^{3}\text{He},n^{0})^{15}\text{O}$ reaction produced by varying the entrance channel spin-orbit well depth. Parameter sets H-2 and N-1 were used in the entrance and exit channels respectively and the spin-orbit well depth in the exit channel was 3.0 MeV. The relative cross sections for the three cases are indistinguishable in the figure.
$^{13}\text{C} \left(^3\text{He}, n_0\right)^{15}\text{O}$

△ DATA OF DIN AND WEIL AT 5.00 MeV

$V_{s0}(\text{He}) = 0.0$ MeV
$V_{s0}(\text{He}) = 2.5$ MeV
$V_{s0}(\text{He}) = 5.0$ MeV

Polarization vs. $\theta_{\text{CM}}$ (deg) at 5.20 MeV.
Figure 18. Effect on the DWBA calculations of the cross section and polarization angular distributions for the $^{13}\text{C}(^{3}\text{He},n^{0})^{15}\text{O}$ reaction produced by varying the exit channel spin-orbit well depth. The calculations were made using optical potentials H-2 and N-1 with an entrance channel spin-orbit well depth of 2.5 MeV.
$^{13}\text{C}\left(^3\text{He}, n_0\right)^{15}\text{O}$

Data of Din and Weil at 5.00 MeV

Polarization

$\sigma_{\text{CM}}(\theta) / \sigma_{\text{CM}}(0^\circ)$

$\theta_{\text{CM}}$ (deg)

$V_{s0}(n) = 0$ MeV

$V_{s0}(n) = 3$ MeV

$V_{s0}(n) = 6$ MeV

$V_{s0}(n) = 9$ MeV

5.20 MeV
was held constant at 3.0 MeV and in Fig. 18 $V_{so}(\text{He})$ was held constant at 2.5 MeV. Calculations with the spin-orbit term in both the entrance and exit channels set equal to zero gave maximum polarizations much less than 0.01 indicating the necessity of spin-dependent distortions to account for the observed magnitudes of polarization. As has been observed by Schaller et al.\textsuperscript{37} for the $^{12}\text{C}(^{3}\text{He},n_{0})^{14}$ reaction and by Marr\textsuperscript{35} for the $^{12}\text{C}(^{3}\text{He},p_{l})^{14}\text{N}^{*}(2.31 \text{ MeV})$ reaction, the polarization appears to be nearly entirely produced by the spin-orbit potential in the exit channel. In particular, variations of $V_{so}(\text{He})$ from 0 to 5 MeV (which is near the maximum value expected from former work\textsuperscript{85}) hardly affect either the cross section or the polarization angular distributions. Therefore, no conclusion could be drawn about the magnitude of $V_{so}(\text{He})$ and a value of $V_{so}(\text{He}) = 2.5 \text{ MeV}$ was chosen for all subsequent calculations.

Changes in the exit channel $V_{so}$ produce large differences in $P(\theta)$ as is shown in Fig. 18 although they hardly affect $\sigma(\theta)$. Thus for each of the four entrance channel parameter sets, $V_{so}(n)$ was varied from 1 to 10 MeV in 1 MeV steps and DWBA calculations were performed at each of the four energies where a polarization angular distribution was measured. The quantity $\chi^2$ defined by

$$
\chi^2 = \sum_{l=1}^{4} \sum_{j=1}^{5} \left[ \frac{P_{\text{calc}}(E_{l},\theta_{j}) - P_{\text{exp}}(E_{l},\theta_{j})}{\Delta P_{\text{exp}}(E_{l},\theta_{j})} \right]^2 \tag{V-7}
$$

was calculated for each value of $V_{so}(n)$. For the two entrance channel parameter sets which gave the best fit to the polarization data, the $\chi^2$ functions had shallow minima at $V_{so}(n) = 3 \text{ MeV}$ for H-2.
and at $V_{80}(n) = 5.5\text{ MeV}$ for H-1. The best fit to the polarization data at all four energies was obtained with set H-2 for which the minimum value was $\chi^2 = 5.2$. Neither using the independent spin-orbit potential well geometry suggested by Perey\textsuperscript{83} in the exit channel nor adding an absorptive spin-orbit term to the exit channel optical potential resulted in improved fits to the experimental values for the polarization.

In general the optical model fits to elastic scattering data are relatively insensitive to the magnitude of the absorptive potential well (either $W$ or $W'$). Therefore the sensitivity of the reaction calculations to variations in $W(^3\text{He})$ and $W'(n)$ was studied. The effects on both the cross section and polarization were similar in both cases although the calculations were more sensitive to variations of the imaginary potential depth in the exit channel. Fig. 19 shows a comparison between the data and the curves calculated with three different values of $W'(n)$ using sets H-2 and N-1. Although extreme variations in either $W(\text{He})$ or $W'(n)$ lead to the appearance of a "shoulder" on the stripping peak in the calculated differential cross section, these calculations indicate that variations of as much as 50% can be made in $W(\text{He})$ or $W'(n)$ without significantly affecting the fits to the shapes of $\sigma(\theta)$ and $P(\theta)$. Variations in these parameters, however, do produce large variations in the reaction total cross section as expected. In fact, the total cross section with $W'(n) = 20\text{ MeV}$ is approximately 2.5 times that obtained with $W'(n) = 54\text{ MeV}$. On the other hand, with $W(\text{He}) = 5\text{ MeV}$, the total cross
Figure 19. Effect of varying the surface imaginary well depth in the exit channel on the DWBA calculations of the cross section and polarization angular distributions for the $^{13}\text{C}(^3\text{He},n)^{15}\text{O}$ reaction. The calculations were made using sets H-2 and N-1 with spin-orbit strengths of 2.5 MeV and 3.0 MeV in the entrance and exit channels respectively.
$^{13}\text{C} (^3\text{He}, n_0)^{15}\text{O}$

Data of Din and Weil at 5.00 MeV

\[ \frac{\sigma_{\text{CM}}(\theta)}{\sigma_{\text{CM}}(0^\circ)} \]

- \( W'(n) = 20 \)
- \( W'(n) = 37 \)
- \( W'(n) = 54 \)

\[ 5.20 \text{ MeV} \]

\[ \theta_{\text{CM}} (\text{deg}) \]
section is approximately 5.5 times the value obtained with \( W(\text{He}) = 22 \text{ MeV} \) when all other parameters are left unchanged.

Since the polarization is most sensitive to the parameters used in the exit channel, the sensitivity of the calculations to the exit channel parameters was checked by comparing the results obtained with the Perey parameters (N-1) with those obtained with the more recent Becchetti and Greenlees parameters (N-2). The latter set has a slightly smaller \( r_0 \) and a correspondingly larger \( V \) than the Perey parameter set. These calculations were carried out using set H-2 in the entrance channel at 5.2 MeV in both cases with \( V_{so}(\text{He}) = 2.5 \text{ MeV} \) and \( V_{so}(\text{n}) = 3 \text{ MeV} \). The results are shown in Fig. 20.

Both exit channel sets give similar results but with the set N-2, the angular positions of the main features of both \( \sigma(\theta) \) and \( P(\theta) \) are displaced approximately 5° toward more forward angles from their positions using set N-1 which slightly improves the fits to the data.

The fits to all four measured angular distributions of polarization as well as to the angular distributions measured by Din and Weil\(^{25}\) at 4.40, 4.67, 5.00 and 5.35 MeV obtained using sets H-1 and H-2 in the entrance channel with set N-1 in the exit channel are presented in Fig. 21. In both cases a value of 2.5 MeV was used for \( V_{so}(\text{He}) \) while the optimum values of 5.5 and 3 MeV were used for \( V_{so}(\text{n}) \) with sets H-1 and H-2 respectively. In view of the naive assumption that the reaction mechanism is entirely direct, the generally good agreement between the calculations and the shapes of both \( \sigma(\theta) \) and \( P(\theta) \) throughout the energy range clearly indicates
Figure 20. Comparison of the calculated cross section and polarization angular distributions for the $^{13}\text{C}(^{3}\text{He},n_o)^{15}\text{O}$ reaction at 5.2 MeV using set H-2 in the entrance channel with a spin-orbit strength of 2.5 MeV and sets N-1 and N-2 respectively in the exit channel with a spin-orbit strength of 3.0 MeV in each case.
$^{13}\text{C}(^{3}\text{He}, n_0)^{15}\text{O}$

5.20 MeV

$\sigma_{\text{c.m.}}(\theta)/\sigma_{\text{c.m.}}(0^\circ)$

- SET N-1
- SET N-2

Polarization

$\theta_{\text{c.m.}}$ (deg)
Figure 21. DWBA fits to cross section and polarization angular distributions from the $^{13}$C($^3$He,n$^0$)$^{15}$O reaction carried out at the indicated energies between 4.2 and 5.7 MeV using each of the two best-fit sets of entrance channel optical model parameters with set N-1 in the exit channel. All the cross section data is taken from the work of Din and Weil. 25
the importance of the two nucleon stripping mechanism in this reaction, especially at energies above 4.4 MeV. In reality, other mechanisms such as compound nucleus formation can be expected to contribute to the reaction in this low energy region. The improvement in the fits to $\sigma(\theta)$ with increasing energy suggests that the stripping mechanism becomes relatively more important with increasing energy. The lack of detailed agreement between the calculations and the data particularly at the extreme energies of 4.2 and 5.7 MeV may be due to the effects of competing compound nucleus formation since the previous polarization data\(^{39}\) gives evidence of a significant compound nucleus contribution\(^{4}\) at energies near 3.5 MeV and resonance behavior has been observed in the yield curves from the \(^{13}\text{C}(^{3}\text{He},\alpha_{o})^{12}\text{C},\) \(^{13}\text{C}(^{3}\text{He},\alpha_{1})^{12}\text{C}^{*}(4.43\text{ MeV}),\) \(^{13}\text{C}(^{3}\text{He},p_{o})^{15}\text{N}\) and \(^{13}\text{C}(^{3}\text{He},^{3}\text{He})^{13}\text{C}\) reactions by Weller et al.\(^{68}\) at a bombarding energy near 6 MeV.

It is particularly difficult to compare the calculated total cross section to experimental values since such a comparison requires a knowledge of $|A_{\lambda\delta j}|^2$ which, as has been mentioned above, is not well-known. As has been noted, comparisons between DWBA calculations\(^{63}\) and cross section data from previous two nucleon transfer reactions have indicated that a value between 20 and 70 is appropriate for the \(^{13}\text{C}(^{3}\text{He},n_{o})^{15}\text{O}\) reaction with the form factor used here. A comparison of the calculated reaction total cross sections with the

\(^{4}\)See Chap. IV, Section A and Fig. 12.
data is shown in Fig. 22. In these calculations each of the four entrance channel parameter sets were used with set N-1 in the exit channel and a value of $|A_{lsj}|^2 = 30$ was assumed. The experimental values were obtained by integrating the angular distributions generated from the set of Legendre Polynomial coefficients which were extracted by Din and Weil from their measured angular distributions. Their 0° yield curve was used for normalization. There is a considerable difference once again between the results obtained with entrance channel parameter sets H-1 and H-2 and those obtained with sets H-3 and H-4. As can be seen in Fig. 22, the entrance channel parameter sets (H-1 and H-2) which give the best description of the shapes of $\sigma(\theta)$ and $P(\theta)$ also give the best description of the energy dependence of the total reaction cross section. However, these sets give a value for the total cross section some 35 to 40 times too small whereas sets H-3 and H-4 give results consistent with the measurements. As has been noted, the calculated total cross section is particularly sensitive to variations in the imaginary well depths of the optical potentials used in both the entrance and exit channels whereas the shapes of the calculated cross section and polarization angular distributions are little affected by variations of these parameters. Furthermore, these particular parameters are not well determined from fits to elastic scattering cross section data alone. In addition, the total cross section would also clearly be sensitive to small variations in the radial form factor such as those which would result from the
Figure 22. Comparison of calculated total reaction cross sections with the data of Din and Weil\textsuperscript{25} for the $^{13}\text{C}(^{3}\text{He},n)^{15}\text{O}$ reaction. Each of the four entrance channel parameter sets was used with the set N-1 in the exit channel. The scale factors shown assume a value of 30 for $|A_{ls}|^2$. 
$^{13}\text{C}(^{3}\text{He}, n_0)^{15}\text{O}$

DATA OF DIN AND WEIL

- SET H-1 (X50)
- SET H-3 (X2)
- SET H-2 (X50)
- SET H-4 (X2)

$|A_{ls}|^2 = 30$

\[ \sigma_{\text{TOT}} \text{ (mb)} \]

\[ E_{^{3}\text{He}} \text{ (MeV)} \]
inclusion of finite range effects. Such variations would in general, however, also alter the shapes of the calculated cross section and polarization angular distributions. Thus the values for the total cross section calculated using sets H-1 and H-2 and the observed value are not irreconcilable though they are hardly in agreement.

3. Other Calculations

Since other reaction mechanisms are expected to contribute to this reaction in this energy region, calculations were made to see if the simple assumption of some other pure reaction mechanism would lead to a satisfactory description of the data. In particular, contributions from compound nucleus formation were considered in two extreme cases. For beam energies between 4 and 6 MeV, the corresponding excitation energies in the compound nucleus $^{16}$O would be 26 to 28 MeV. Since the density of states at these high excitation energies is expected to be large, the possibility of contributions from a statistical process was considered. Under this assumption the previously noted relatively broad structure in the cross section which occurs at energies near 3.5 and 6 MeV might be interpreted as due to intermediate structure. A Hauser-Feshbach calculation of the differential cross section angular distribution was performed by R. G. Seyler using the computer program BARBARA of Sheldon. This calculation, considering 35 final states for which the spins and parities of the residual nucleus are known, gave a total cross section of approximately 30 mb. This calculation, however, overestimates the total cross section since some of the open exit channels were
ignored (for example, the deuteron and alpha channels) due to limitations of the program. The calculated angular distribution decreases monotonically from 0° to 90° with a 0° to 90° ratio of \( \sigma(0°)/\sigma(90°) \sim 9 \). Since the statistical model predicts cross sections symmetric about 90°, the calculation not only fails to reproduce the secondary maximum observed near 100° but also gives a large back angle peak which is not observed in the measurements of Din and Weil extending back to 160°. Although the program does not calculate the polarization, the statistical assumption of simultaneous excitation of a large number of strongly interfering states of random phase guarantees that no vector polarization can result from a statistical model mechanism. The experimental evidence thus indicates that despite the high excitation energies involved, the statistical model is not applicable in this energy range.

The broad structure in the cross section might alternatively be interpreted as evidence that the reaction proceeds primarily through a few broad overlapping resonances. The consequences of this possibility were investigated using the search program DEBBIE of Marr. In this program the magnitudes and phases of the scattering matrix elements are arbitrarily varied to optimize the fit to either cross section or polarization data or both. Calculations assuming three broad overlapping resonances in \(^{160}\text{O}\) with spins and parities of \( 1^- \), \( 1^+ \) and \( 2^+ \) gave cross section and polarization angular distributions qualitatively similar to those observed at 4.40 and 4.20 MeV respectively. The calculations at higher energies, however, all
gave large back angle peaks in \( \sigma(\theta) \) which have not been observed. From the overall fit to the data in this energy region, it appears that a compound nucleus mechanism proceeding through only a few overlapping states can account for at most only a small fraction of the cross section which leaves the magnitude of the total cross section still largely unexplained. Among the reaction mechanisms considered here, the zero-range DWBA theory provides the best description of the observed gradual rise in the total cross section and the observed general angular dependence of the differential cross section and polarization in the energy region between 4.2 and 5.7 MeV.

C. The \(^{24}\text{Mg}(^{3}\text{He},n)^{26}\text{Si} \) Reaction Calculations

1. Optical Model Parameter Selection

Several authors \(^{77,78,81,82}\) have reported on the elastic scattering of \(^3\text{He}\) from \(^{24}\text{Mg}\) and other nuclei in the mass region \(22 \leq A \leq 27\) at bombarding energies below 10 MeV. Thus in order to find appropriate entrance channel optical potentials for the DWBA analysis of this reaction, an optical model analysis of two selected sets of data was made using the ABACUS-2 search program written by E. H. Auerbach.\(^{89}\) This program calculates among other things the elastic differential cross section due to an optical potential of the form given in Eqns. V-5 and V-6 and compares this calculated cross section to experimental values, varying up to five of the parameters of the optical potential simultaneously so as to minimize the quantity \( \chi^2 \) defined by
\[ \chi^2 = \sum_{i=1}^{M} \frac{1}{N_i} \sum_{j=1}^{N_i} \left[ \frac{\sigma_{\text{calc}}(\theta_j) - \sigma_{\text{exp}}(\theta_j)}{\Delta \sigma_{\text{exp}}(\theta_j)} \right]^2 \]  

where \( M \) is the number of elastic angular distributions at different energies which are to be fit.

The \(^3\text{He}\) elastic scattering data chosen for this analysis were the two angular distributions measured at 5.50 and 7.00 MeV by Bray et al.\(^{77}\) with a target of \(^{27}\text{Al}\) and the six angular distributions measured at energies between 4.0 and 5.5 MeV with targets of Mg and Al by Stickler.\(^{81}\) The simultaneous fitting of several angular distributions taken at various energies with more than one target should reduce distortions in the derived optical potentials due to compound elastic effects at a given energy in a particular reaction. These data were separated into three groups as follows:

1. the two angular distributions at 5.50 and 7.00 MeV measured by Bray et al. with \(^{27}\text{Al}\),
2. the three angular distributions at 4.0, 4.5 and 5.0 MeV measured by Stickler with Mg and
3. the three angular distributions at 4.5, 5.0 and 5.5 MeV measured by Stickler with Al.

Each of the data groups was separately analyzed. An idea of the best-fit potentials was provided by the optical model analysis of their data by Bray et al.\(^{77}\) at each individual energy. In addition a potential was sought with a somewhat larger real well depth which
might satisfy the sum rule mentioned above. Since only five parameters can be simultaneously varied by ABACUS-2 and since the fits to elastic scattering cross sections are relatively insensitive to \( V_{80} \), \( V_{80} \) was set equal to zero for this analysis. Then for a given potential set, best-fits were obtained for each of the three data groups by fixing either \( r_0 \) or \( W \) and letting the other vary along with \( V, A, r_0' \) and \( a' \). Partial waves up to \( \ell = 20 \) were included in each case.

Three optical potentials were found which gave equivalent \( \chi^2 \) values for the three data groups. For each of these three potentials the best-fit parameters obtained for each data group were averaged to give three final average parameter sets. The best-fit potentials for each of the three data groups and the three final average potentials are tabulated in Table 9.

Fig. 23 shows the fits of each of the final average potentials to all the data points. The 10\% uncertainties in the data of Stickler are not shown. The labels He-1, He-2 and He-3 refer to the average potentials with \( V = 179.6, 156.9 \) and \( 112.2 \) MeV respectively. As can be seen from Table 9, the parameter sets He-2 and He-3 are from the same discrete ambiguity, the larger \( r_0 \) in set He-3 being compensated by a smaller \( V \).

\[ + \text{See Chap. V, section A.} \]
### TABLE 9

$^3$He ELASTIC SCATTERING BEST-FIT AND AVERAGE OPTICAL MODEL PARAMETERS

<table>
<thead>
<tr>
<th>Target Nucleus</th>
<th>Reference</th>
<th>Energies (MeV)</th>
<th>$V$ (MeV)</th>
<th>$r_0$ (f)</th>
<th>$a$ (f)</th>
<th>$W$ (MeV)</th>
<th>$r_0'$ (f)</th>
<th>$a'$ (f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{27}$Al</td>
<td>77</td>
<td>5.50,7.00</td>
<td>174.7</td>
<td>1.126</td>
<td>0.702</td>
<td>20.15+</td>
<td>1.462</td>
<td>0.716</td>
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<tr>
<td>Al</td>
<td>81</td>
<td>4.50,5.00,5.50</td>
<td>181.5</td>
<td>1.150+</td>
<td>0.641</td>
<td>20.21</td>
<td>1.595</td>
<td>0.737</td>
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<tr>
<td>Mg</td>
<td>81</td>
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<td>182.5</td>
<td>1.150+</td>
<td>0.758</td>
<td>20.21</td>
<td>1.589</td>
<td>0.749</td>
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<td>Average</td>
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<td></td>
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<td>0.700</td>
<td>20.19</td>
<td>1.549</td>
<td>0.734</td>
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<td>$^{27}$Al</td>
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<td>156.0</td>
<td>1.084</td>
<td>0.758</td>
<td>20.00+</td>
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<td>0.612</td>
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<td>156.9</td>
<td>1.068</td>
<td>0.776</td>
<td>20.02</td>
<td>1.614</td>
<td>0.638</td>
</tr>
<tr>
<td>$^{27}$Al</td>
<td>77</td>
<td>5.50,7.00</td>
<td>113.1</td>
<td>1.226</td>
<td>0.660</td>
<td>18.00+</td>
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<td>0.937</td>
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<tr>
<td>Al</td>
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<td>110.2</td>
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<tr>
<td>Mg</td>
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<td></td>
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<td>1.242</td>
<td>0.675</td>
<td>18.29</td>
<td>1.664</td>
<td>0.865</td>
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+ This parameter held fixed at the indicated value in search.
Figure 23. Optical model fits to the $^{27}\text{Al}(^{3}\text{He},^{3}\text{He})^{27}\text{Al}$ and $^{37}\text{Mg}(^{3}\text{He},^{3}\text{He})^{37}\text{Mg}$ data of Bray et al.\textsuperscript{77} and Stickler\textsuperscript{81} obtained with each of the three final average entrance channel parameter sets listed in Table 9. The statistical uncertainties in the data of Bray et al. are shown in the figure while the 10% uncertainties in the data of Stickler are not shown.
$^3$He ELASTIC SCATTERING

**DATA OF BRAY et al.**

**DATA OF STICKLER**
The parameters chosen for the exit channel were again those of Perey\(^3\) (set N-1) and Becchetti and Greenlees\(^4\) (set N-2). Table 10 lists all the optical potential parameter sets used in both the entrance and exit channels in the DWBA calculations for the \(^{24}\text{Mg}(^{3}\text{He},n_0)^{26}\text{Si}\) reaction. As in the \(^{13}\text{C}(^{3}\text{He},n_0)^{15}\text{O}\) calculations, the spin-orbit potentials in both channels were allowed to vary as indicated in Table 10.

2. DWBA Reaction Calculations

The calculated \(\sigma(\theta)\) at 5.6 MeV and \(P(\theta)\) at 5.8 MeV using each of the three entrance channel parameter sets with the exit channel set N-1 and using \(V_{so}(\text{He}) = 2.5\) MeV and \(V_{so}(n) = 5\) MeV are compared in Fig. 24 with the polarization data at 5.8 MeV and the experimental \(\sigma(\theta)\) measured at 5.6 MeV by McMurray et al.\(^{23}\) The form factor calculated assuming a \((1d_{5/2})^2\) configuration for the two stripped protons was used. Here again all the cross sections are normalized to unity at 0° for ease in comparing the shapes of the calculated angular distributions to that of the experimental angular distribution. The entrance channel sets all produce similar shapes for both \(\sigma(\theta)\) and \(P(\theta)\), the major differences being that using set He-3 gives a secondary maximum in \(\sigma(\theta)\) more than twice as large as is given by using either set He-1 or set He-2 and that the positions of the main features of \(P(\theta)\) calculated using set He-3 are displaced about 15° toward more forward angles from their positions when calculated using the other two entrance channel optical potentials. More satisfactory
### Table 10

**Optical Model Parameters Used in $^{24}$Mg($^3$He, 3n) $^{26}$Si Calculations**

<table>
<thead>
<tr>
<th>Set Label</th>
<th>$V$ (MeV)</th>
<th>$r_0$ (fm)</th>
<th>$a$ (fm)</th>
<th>$r_c$ (fm)</th>
<th>$W$ (MeV)</th>
<th>$r_0'$ (fm)</th>
<th>$a'$ (fm)</th>
<th>$W'$ (MeV)</th>
<th>$V_{SO}$ (MeV)</th>
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<tr>
<td>He-1</td>
<td>179.6</td>
<td>1.142</td>
<td>0.700</td>
<td>1.40</td>
<td>20.19</td>
<td>1.549</td>
<td>0.734</td>
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<tr>
<td>He-2</td>
<td>156.9</td>
<td>1.068</td>
<td>0.776</td>
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<td>1.614</td>
<td>0.638</td>
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<tr>
<td>He-3</td>
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<td>1.664</td>
<td>0.865</td>
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<td>0.47</td>
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<tr>
<td>N-2</td>
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<td>1.25</td>
<td>0</td>
<td>1.26</td>
<td>0.58</td>
<td>49.9-50.8</td>
<td>2-11</td>
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</tbody>
</table>
Figure 24. Comparison of differential cross section and polarization data with DWBA calculations for the $^{24}\text{Mg}(^{3}\text{He},n)^{26}\text{Si}$ reaction made using each of the three final average sets of entrance channel optical model parameters with the set N-1 (see Table 10) of exit channels parameters. The $(1d_{5/2})^2$ form factor was used and the spin-orbit well depths used were 2.5 MeV and 5.0 MeV in the entrance and exit channels respectively.
$^{24}\text{Mg}(^{3}\text{He},n)^{26}\text{Si}$

DATA OF McMURRAY et al.

$^{5.6\text{ MeV}}$

$\sigma_{c.m.}(\theta)/\sigma(0)$

$^{5.8\text{ MeV}}$

POLARIZATION

$\theta_{c.m.}$ (degrees)
fits to the shape of $\sigma(\theta)$ are achieved with sets He-1 and He-2 than with the set He-3 which again may indicate a preference for entrance channel optical model potentials with $V$ approximately three times the value appropriate for single nucleon scattering. None of the calculated $P(\theta)$ curves, however, resembles the experimental $P(\theta)$.

Figs. 25 and 26 show the changes in the calculated $\sigma(\theta)$ and $P(\theta)$ produced by varying $V_{SO}(\text{He})$ and $V_{SO}(n)$ respectively. In both cases the $(1d_{3/2})^2$ form factor was used and the sets He-2 and N-1 were chosen for the entrance and exit channels respectively. The effects produced in $P(\theta)$, however, are independent of the choice of the entrance channel optical potential and similar results are obtained with both the $(1d_{3/2})^2$ and $(2s_{1/2})^2$ form factors. Values of $V_{SO}(n) = 5 \text{ MeV}$ and $V_{SO}(\text{He}) = 2.5 \text{ MeV}$ were used for the calculations shown in Figs. 25 and 26 respectively. As might have been anticipated from the results of the $^{13}\text{C}(^{3}\text{He},n)^{15}\text{O}$ calculations, variations of $V_{SO}(\text{He})$ have a negligible effect on $\sigma(\theta)$ and $P(\theta)$ and no conclusions concerning the correct magnitude of $V_{SO}(\text{He})$ or even the necessity of a non-zero $V_{SO}(\text{He})$ could be reached. The value of $V_{SO}(\text{He}) = 2.5 \text{ MeV}$ was thus chosen for all succeeding calculations.

Although variations in $V_{SO}(n)$ produced differences in the magnitude of the calculated polarizations, they did not change the angular pattern of $P(\theta)$ and so were ineffective in producing a fit to the observed angular dependence of the polarization.

The differences in $P(\theta)$ and $\sigma(\theta)$ produced by using the $(2s_{1/2})^2$ form factor instead of the $(1d_{3/2})^2$ form factor using set He-2 in
Figure 25. Effect on the DWBA calculations of the $^{24}\text{Mg}(^{3}\text{He},n^{0})^{26}\text{Si}$ cross section and polarization produced by varying the entrance channel spin-orbit well depth. The $(1d_{5/2})^{2}$ form factor was used and parameter sets He-2 and N-1 were used in the entrance and exit channels respectively with an exit channel spin-orbit well depth of 5.0 MeV. The relative cross sections for the three cases are indistinguishable in the figure.
$^{24}\text{Mg}(^{3}\text{He},n_0)^{26}\text{Si}$

$5.6\text{ MeV}$

$5.8\text{ MeV}$

DATA OF McMURRAY et al.

$\sigma^{C.M.}(\theta)/\sigma^{(0)}$

$\theta_{C.M.}$ (degrees)
Figure 26. Effect on the DWBA calculations of the $^{24}\text{Mg}(^{3}\text{He},n)^{26}\text{Si}$ cross section and polarization produced by varying the exit channel spin-orbit well depth with the entrance channel spin-orbit well depth held fixed at 2.5 MeV. The calculations were performed with the $(1d_{5/2})^{2}$ form factor using parameter sets He-2 and N-1 in the entrance and exit channels respectively.
$\textbf{POLARIZATION (Tu(0)/CT(o*)}$

$^{24}\text{Mg}(^{3}\text{He},n_o)^{26}\text{Si}$

$5.6 \text{ MeV}$

DATA OF McMURRAY et al.

$V_{SO}(n) = 2 \text{ MeV}$

$V_{SO}(n) = 5 \text{ MeV}$

$V_{SO}(n) = 8 \text{ MeV}$

$V_{SO}(n) = 11 \text{ MeV}$
the entrance channel and using set N-1 with $V_{so}(n) = 5 \text{ MeV}$ in the exit channel are shown in Fig. 27. Once again, very little effect is noted on the angular dependence of the polarization as well as the cross section. The similarity between both $\sigma(\theta)$ and $P(\theta)$ calculated with the $(1d_{\frac{3}{2}})^2$ form factor and $\sigma(\theta)$ and $P(\theta)$ calculated with the $(2s_{\frac{1}{2}})^2$ form factor was found to be independent of the choice of entrance and exit channel optical potentials and thus the $(1d_{\frac{3}{2}})^2$ form factor was used for all the following calculations.

A comparison between the $\sigma(\theta)$ and $P(\theta)$ obtained with the two exit channel parameter sets again using set He-2 in the entrance channel with $V_{so}(n) = 5 \text{ MeV}$ is shown in Fig. 28. As in the case of the $^{13}\text{C}(^{3}\text{He},n)^{15}\text{O}$ reaction calculations, the use of set N-2 instead of set N-1 in the exit channel results in a shift of some $5^\circ$ toward more forward angles in the main features of both $\sigma(\theta)$ and $P(\theta)$.

In an attempt to obtain satisfactory fits to the polarization observed in this reaction, the effects of radial lower cutoff were investigated as a crude way of approximating finite range effects which, in general, reduce contributions from the nuclear interior region. In these calculations the radial integral of Eqn. II-33 is done from a specified lower limit $r_L$ out to the asymptotic region where $|F_{j\ell j}(r)| \leq 10^{-20}$. The $\sigma(\theta)$ and $P(\theta)$ predicted with $r_L = 3.2$, 5.2 and 7.2 $\text{fm}$ as well as those obtained with no cutoff (i.e. $r_L = 0$) are shown in Fig. 29. Here again set He-2 was used in the entrance channel ($V_{so}(\text{He}) = 2.5 \text{ MeV}$) and set N-1 in the exit channel ($V_{so}(n) = 5 \text{ MeV}$). The effect of radial lower cutoff on the shapes
Figure 27. Comparison of the data with the DWBA calculations of the $^{24}\text{Mg}(^3\text{He},n^0)^{26}\text{Si}$ cross section and polarization obtained with each of the two form factors shown in Fig. 15 for this reaction. The calculations were made with parameter sets He-2 ($V_{\text{80}}(\text{He}) = 2.5 \text{ MeV}$) and N-1 ($V_{\text{80}}(n) = 5.0 \text{ MeV}$) in the entrance and exit channels respectively.
\[ ^{24}\text{Mg}(^{3}\text{He},n_{0})^{26}\text{Si} \]

5.6 MeV

\[ \sigma_{\text{C.M.}}(\theta) / \sigma(\theta) \]

- Data of McMurray et al.
- \((1d_{5/2})_{2}^{2}\)
- \((2s_{1/2})_{2} \)

5.8 MeV

Polarization

\[ \theta_{\text{C.M.}} \text{ (degrees)} \]
Figure 28. Comparison of the data with the DWBA calculations of the \(^{24}\text{Mg}(^3\text{He},n)^{26}\text{Si}\) cross section and polarization angular distributions using parameter set He-2 in the entrance channel (with \(V_{so}(\text{He}) = 2.5\ \text{MeV}\)) and sets N-1 and N-2 respectively in the exit channel (with \(V_{so}(\text{n}) = 5.0\ \text{MeV}\) in each case). The \((1d_{5/2})^2\) form factor was used.
$^{24}\text{Mg}(^{3}\text{He},n_{e})^{26}\text{Si}$

5.6 MeV

DATA OF McMURRAY et al.

$\sigma_{\text{C.M.}}(\theta)/\sigma(0)$

$\theta_{\text{C.M.}}$ (degrees)

5.8 MeV

POLARIZATION

---

\[ \text{POLARIZATION} \]
Figure 29. Effect of introducing lower cutoff in the DWBA calculations of the cross section and polarization for the $^{24}\text{Mg} \left( ^3\text{He}, n_0 \right) ^{26}\text{Si}$ reaction. The $(1d_{5/2})^2$ form factor was used with parameter sets He-2 ($V_{80}(\text{He}) = 2.5$ MeV) and N-1 ($V_{80}(n) = 5.0$ MeV) in the entrance and exit channels respectively.
$^{24}\text{Mg}(^3\text{He},n_0)^{26}\text{Si}$

$5.6 \text{ MeV}$

\[ \sigma_{\text{C.M.}}(\theta) / \sigma(0) \]

- DATA OF McMURRAY et al.
- NO CUTOFF
- $3.2 \text{ f}$
- $5.2 \text{ f}$
- $7.2 \text{ f}$

Polarization

$\theta_{\text{C.M.}}$ (degrees)
of $\sigma(\theta)$ and $P(\theta)$ is surprisingly small. In particular it does not provide a better description of the polarization data than that obtained with no lower cutoff.

A further attempt was made to achieve a fit to the experimental $P(\theta)$ at 5.8 MeV by using two entrance channel optical model potentials derived from higher energy (12-29 MeV) $^{24}$Mg + $^3$He elastic scattering data\cite{74,75,76} with set N-1 in the exit channel. In both cases the shapes of the calculated $P(\theta)$ were similar to those obtained with entrance channel sets He-1, He-2 and He-3 and in neither case was the fit to the shape of $\sigma(\theta)$ as good as those achieved with the entrance channel optical potentials obtained from the lower energy elastic scattering data.

The calculated cross sections and polarizations were found to vary gradually with energy as was the case in the $^{13}$C($^3$He,$n\alpha$)$^{15}$O calculations. In view of the similarity between the 5.0 and 5.8 MeV polarization data and the slow energy dependence of the calculated quantities, no attempt was made to specifically fit the 5.0 MeV polarization data.

Although the DWBA calculations give a good fit to $\sigma(\theta)$, they fail to provide a description of the angular variation of the observed polarization. In fact, as has been shown, the angular dependence of the calculated polarization is surprisingly insensitive to variations of several of the calculation parameters. The results of these DWBA calculations are quite disappointing in view of the availability of elastic scattering data for the entrance
channel for this reaction and the higher A region studied here as compared to the case of $^{13}\text{C}(^{3}\text{He},n^{0})^{15}\text{O}$. Considering the reasonable agreement with the polarization data achieved with the DWBA calculations for the $^{13}\text{C}(^{3}\text{He},n^{0})^{15}\text{O}$ reaction, the low incident energy in itself can hardly be held responsible for the lack of agreement in this case. The most striking difference between these two reactions besides the difference in target masses is the difference in their Q values which leads to markably different neutron energies in the exit channels of the two reactions. Since the calculated polarization is most strongly influenced by the exit channel optical model parameters, perhaps the energy dependence of the parameters of sets N-1 and N-2 which are derived principally from data at higher energies is incorrect when extrapolated to the low energy region (i.e., $E_{N} = 3.5 \text{ to } 6 \text{ MeV}$) studied in the $^{24}\text{Mg}(^{3}\text{He},n^{0})^{26}\text{Si}$ reaction. On the other hand, contributions from other reaction mechanisms might help to provide a better description of the polarization data.

3. Other Calculations

Calculations were undertaken with program DEBBIE$^{35}$ to see if the compound nucleus model could describe the measured angular distributions under the assumption that only two broad overlapping resonances contribute to the reaction. Since the measured angular distributions are not symmetric about $90^\circ$, all pairs of levels with opposite parity accessible with $\lambda \leq 4$ were tried. Table I lists the values for the spins and parities used, the $\chi^{2}$ of the fit to
| $J_1^\pi$ | $J_2^\pi$ | $\chi^2_{\text{min}}$ | $|P_{\text{max}}|$ |
|----------|----------|----------------|----------------|
| 1/2^-    | 1/2^+   | 14.2           | 0.132          |
| 1/2^-    | 3/2^+   | 14.1           | 0.135          |
| 1/2^-    | 5/2^+   | 13.3           | 0.242          |
| 1/2^-    | 7/2^+   | 13.3           | 0.247          |
| 1/2^-    | 9/2^+   | 11.6           | 0.002          |
| 3/2^-    | 1/2^+   | 14.8           | 0.222          |
| 3/2^-    | 3/2^+   | 14.8           | 0.196          |
| 3/2^-    | 5/2^+   | 15.1           | 0.193          |
| 3/2^-    | 7/2^+   | 13.2           | 0.015          |
| 3/2^-    | 9/2^+   | 13.3           | 0.274          |
| 5/2^-    | 1/2^+   | 13.3           | 0.241          |
| 5/2^-    | 3/2^+   | 15.1           | 0.193          |
| 5/2^-    | 5/2^+   | 9.3            | 0.023          |
| 5/2^-    | 7/2^+   | 11.5           | 0.153          |
| 5/2^-    | 9/2^+   | 11.5           | 0.084          |
| 7/2^-    | 1/2^+   | 13.3           | 0.247          |
| 7/2^-    | 3/2^+   | 10.1           | 0.018          |
| 7/2^-    | 5/2^+   | 10.2           | 0.160          |
| 7/2^-    | 7/2^+   | 11.1           | 0.054          |
| 7/2^-    | 9/2^+   | 10.0           | 0.151          |
\( \sigma(\theta) \) at 5.2 MeV as defined in Eqn. V-8, and the absolute value of the maximum calculated polarization for each pair of states considered at the minimum in \( \chi^2 \). The best fits obtained to \( \sigma(\theta) \) at 5.2 MeV were almost as good as those achieved with the DWBA calculations. In each case, however, for the best fit to \( \sigma(\theta) \), the magnitudes of polarization predicted were much smaller than those observed (maximum \( P = -0.71 \pm .16 \) at 5.0 MeV and 135° lab) and the calculated \( P(\theta) \) did not resemble the observed angular distributions of polarization at 5.0 and 5.8 MeV. Furthermore, the \( \chi^2 \) values for the fits to \( \sigma(\theta) \) showed no apparent preference for a state with any particular spin and parity suggesting that the reasonably good fit to \( \sigma(\theta) \) achieved may be merely due to the wide limits within which the program could search.

Because of these results and the slow energy dependence of the measured cross section and polarization with energy, no further calculations assuming only a few compound states contribute to the reaction were attempted.

Here, as in the \(^{13}\text{C}(\text{^3He,n}_0)^{15}\text{O} \) reaction, the lack of symmetry about 90° in the measured cross section angular distributions and the reasonably large magnitudes of polarization observed rule out significant contributions from a statistical model mechanism. Since no incoherent sum of the polarizations calculated on the basis of the pure reaction mechanisms considered here would account for the angular variation of the polarization in this reaction, it remains to be satisfactorily explained.
CHAPTER VI

SUMMARY AND CONCLUSIONS

In order to investigate the reaction mechanism in two nucleon transfer reactions, the angular and energy dependence of the polarization of the neutrons produced in the $^{13}\text{C}(^3\text{He},n)^{15}\text{O}$ and $^{24}\text{Mg}(^3\text{He},n)^{26}\text{Si}$ reactions was studied. The neutrons produced in the former reaction were found to have large magnitudes of polarization and their angular distributions of polarization varied gradually and systematically with energy from 4.2 to 5.7 MeV. Large neutron polarizations were also observed in the latter reaction and angular distributions of polarization measured at 5.0 and 5.8 MeV indicate a systematic variation of the polarization with energy in this reaction as well.

Because of the gradual energy dependence of both the polarization and cross section data as well as the large $J = 0$ stripping peaks observed in the angular distributions of both reactions, zero-range DWBA calculations were carried out to determine if the assumption of a simple two nucleon stripping mechanism in each case could account for all the experimental data in the energy region studied.

The distorted wave calculations for the $^{13}\text{C}(^3\text{He},n)^{15}\text{O}$ reaction reproduced the angular dependence of both the differential cross section and polarization reasonably well throughout the energy range.
investigated. These calculations also fit the energy dependence of the reaction total cross section but they gave a magnitude for the total cross section which appears to be far too small. Despite the considerable freedom introduced by the lack of information about specific states which might contribute, calculations assuming a compound nucleus mechanism with three broad overlapping states contributing to the reaction did not give satisfactory fits to the angular dependence of the differential cross section and polarizations, particularly at the higher energies. The large polarizations observed with thick targets in both reactions rule out any significant contribution to the total cross section from a compound nuclear process which involves many narrow resonances. On the whole, the evidence for the two nucleon stripping mechanism in this reaction is quite good.

The DWBA calculations for the $^{24}_{\text{Mg}}(^{3}_{\text{He}},n_{o})^{26}_{\text{Si}}$ reaction gave a good account of the angular dependence of the differential cross section but did not reproduce the angular dependence of the polarization at 5.8 MeV. Calculations based entirely on a compound nucleus mechanism could also be made to fit the cross section but not the angular distribution of polarization at 5.8 MeV. Despite the evidence for a direct reaction mechanism from the gradual energy variation of both the cross section and polarization, apparently none of the simple reaction mechanisms considered here can describe the polarization data from this reaction. Furthermore, no incoherent combination of the results obtained with any two simple mechanisms
considered here would account for the angular variation of the observed polarization.

The DWBA calculations themselves indicated that in a \((^3\text{He},n)\) reaction which proceeds by two nucleon stripping, the polarization is essentially entirely produced by the spin-orbit coupling in the exit channel as has been noted previously.\(^{35,37}\) These calculations also suggest that DWBA reaction calculations may provide a means for differentiating between optical model potentials which give equivalent fits to the elastic scattering data for composite particles. Finally, the insensitivity of the angular dependence of these calculated polarizations to changes in various parameters (e.g. entrance or exit channel optical potential parameters and form factors) suggests that polarization measurements provide a stern test for the DWBA theory. The lack of agreement between the polarization data and the DWBA calculations in the case of the \(^{24}\text{Mg}(^3\text{He},n_n)^{26}\text{Si}\) reaction emphasizes the danger involved in assessing reaction mechanisms on the basis of angular distribution measurements alone. Further polarization measurements in two nucleon transfer reactions can be of great help in determining the mechanisms for these reactions.
APPENDIX I

DESCRIPTION OF DATA REDUCTION PROGRAM

A. General Description

This program, written in Fortran IV for use on the IBM 7094 computer, provides four basic options for neutron polarization data reduction. For each data point the input consists of several spectra preceded by three control cards specifying which of these options are to be used and containing all input information. The four basic options are:

1. Spectrum Print Out

Provides channel by channel print out of all the following spectra:

(a) as many as four separate foregrounds as read into the computer,
(b) as many as two separate accidentals as read into the computer,
(c) as many as two separate room-scattered backgrounds as read into the computer,
(d) total foreground (equal to the sum of all four foregrounds),
(e) net normalized background (equal to the sum of both accidentals times the accidental multiplier plus
the sum of both room-scattered backgrounds times
the room-scattered background multiplier),

(f) net spectrum (equal to total foreground minus net
normalized background).

2. Running Average Asymmetry Calculation

Provides calculation and print out of the following
information:

(a) $\varepsilon$ and $\Delta \varepsilon$, the measured asymmetry and absolute
statistical uncertainty associated with it, un-
corrected for spin precession,

(b) net counts in the peak of interest in each quadrant,

(c) net normalized background counts in the peak of
interest in each quadrant,

(d) $R$, the ratio of the observed asymmetry measured by
one detector (top) to that measured by the other
(bottom),

(e) net normalized accidental counts, net normalized
room-scattered background counts, and net normalized
total background counts each summed over the peak
and over all four quadrants and expressed as a
percentage of the total foreground counts similarly
summed.

This information is printed out $n + 1$ times for peaks $N$
channels wide starting in the $n + 1$ channels $IPC, IPC + \Delta,$
$IPC + 2\Delta, \ldots, IPC + n\Delta$ such that $IPC - 1 + N + (n + 1)\Delta >$
LPC where IPC, LPC, N and Δ are all input data defined below and contained on the control cards.

3. Central Peak Asymmetry Calculation

Provides calculation and print out of the same information listed under option (2) m + 1 times for peaks LPC - IPC + 1, LPC - IPC + 1 - 2Γ, LPC - IPC + 1 - 4Γ, ..., LPC - IPC + 1 - 2mΓ channels wide starting in channels IPC, IPC + Γ, IPC + 2Γ, ..., IPC + mΓ respectively such that IPC + 1 + 2(m + 1)Γ > LPC where Γ is also specified on the control cards.

4. Energy Dependent Peak Asymmetry Calculation

Provides calculation and print out of the same information listed under option (2) k + 1 times for peaks LPC - IPC + 1, LPC - IPC + 1 - β, LPC - IPC + 1 - 2β, ..., LPC - IPC + 1 - kβ channels wide starting in channels IPC, IPC + β, IPC + 2β, ..., IPC + kβ respectively and all ending in channel LPC such that IPC - 1 + (k + 1)β > LPC where β is also specified on the control cards.

B. Input Information

For a single data point the card sequence is as follows:

Card 1. (Format 414) Option Selection Card

Each of the first four fields of four columns each on this card is associated with one specific option; the first field with option (1) above, the second with option (2), etc. To select a given option, a non-zero integer must be placed in its associated field on this
card. To bypass a given option, an integer zero must be placed in
the field associated with that option on this card.

Card 2. (Format 3A6, 7F7.2) Decimal Information Card

The first 18 columns of this card are reserved for alphanumeric information identifying the data such as date, run number, reaction title, etc. The next seven fields of seven columns each contain the following information respectively:

1. EB, bombarding energy in MeV,
2. AN, lab reaction angle, $\theta_1$, in degrees,
3. ETH, target thickness in keV,
4. EN, lab neutron energy in MeV,
5. AA, lab scattering angle, $\theta_2$, in degrees,
6. AM, accidental multiplier (sum of integrated charge for all foregrounds divided by sum of integrated charge for all accidentals),
7. SBM, room-scattered background multiplier (sum of integrated charge for all foregrounds divided by sum of integrated charge for all room-scattered backgrounds).

Card 3. (Format 15I4) Integer Information Card

The first 15 fields of four columns each contain the following information respectively:

1. NOFGD, the number of foreground spectra to be read in (restriction: $1 \leq \text{NOFGD} \leq 4$),
2. NOACC, the number of accidental spectra to be read in (restriction: $0 \leq \text{NOACC} \leq 2$),
3. NOSB, the number of room-scattered background spectra to be read in (restriction: $0 \leq NOSB \leq 2$),

4. IC, initial channel number of all the spectra to be read in,

5. LC, final channel number of all the spectra to be read in,

6. IPC, lower peak limit channel number,

7. LPC, upper peak limit channel number,

8. through 12. NWID (J), $J = 1, 5$, as many as five different peak widths for option (2) above (these are the values of $N$ above),

13. through 15. NSTEP (J), $J = 1, 3$, running average, central peak, and energy dependent peak increments respectively (these are $\Delta$, $\Gamma$ and $\beta$ above respectively).

Cards 4 to 3 + P (Format 2X, 10F7.2) First Foreground Spectrum

These cards contain the channel by channel data of the first foreground spectrum, 10 channels to a card, in the following order:

1. top detector and positive current sense, channels IC through LC,

2. bottom detector and positive current sense, channels IC through LC,

3. top detector and negative current sense, channels IC through LC,

4. bottom detector and negative current sense, channels IC through LC.
The number \( P \) will be one plus the greatest integer in \( 0.4 \ (LC - IC + 1) - 0.1 \).

Cards 4 + P to 3 + Px(NOFGD)(Format 2X, 10F7.2) Other Foreground Spectra

These cards contain the channel by channel data for the rest of the foreground spectra each of which is assembled in the same order as described above for the first foreground spectrum.

Cards 4 + Px(NOFGD) to 3 + Px(NOFGD + NOACC) (Format 2X, 10F7.2) Accidental Spectra

These cards contain the channel by channel data of the accidental spectra, each of which is assembled in the same order as described above for the first foreground spectrum.

Cards 4 + Px(NOFGD + NOACC) to 3 + Px(NOFGD + NOACC + NOSB) (Format 2X, 10F7.2) Room-Scattered Background Spectra

These cards contain the channel by channel data of the room-scattered background spectra, each of which is assembled in the same order as described above for the first foreground spectrum.

This completes the input data deck for a single data point. Any number of such data decks may follow each other. Each must be assembled, however, as described above.

C. Listing

The Fortran IV statement listing and a sample input data deck listing follows.
NEUTRON POLARIZATION DATA REDUCTION PROGRAM BY D. C. DEMARTINI

1001 FORMAT(4I4)
1002 FORMAT(JA6*7F7.2)
1003 FORMAT(15I4)
1004 FORMAT(2X*10F7.0)
1005 FORMAT(I1H+3A6*20X+F4.2+9H MEV +F5.1+8H DEGREES)
1006 FORMAT(I1H+F5.2+9H MEV +F4.0+9H KEV +F5.1+8H DEGREES)
1007 FORMAT(I1H +3A4+6X+13+2X+13+5X+13+2X+13+8X+513+5X+313)
1008 FORMAT(I1H+I3.1X+F5.0+2X+F5.0+2X+F5.0+5X+F5.0+2X+F5.0+5X+F5.0+1)
1009 FORMAT(I1H+75H N F1 F2 F3 F4 A1 A2 SB1 SB2
FT NB NS/)
1010 FORMAT(25X+BHTOP PLUS/) 1011 FORMAT(I1H+24X+11HBOTTOM PLUS/) 1012 FORMAT(I1H+25X+9HBOTTOM MINUS/) 1013 FORMAT(I1H+23X+12HBOTTOM MINUS/) 1014 FORMAT(I1H+50F8.5+FROM TO E DELTA E TP BP TM BM)
1015 FORMAT(I1H+14+1X+13+1X+F7.4+3X+F6.4+8X+F5.0+2X+F5.0+2X+F5.0+2X+F5.0+10)
1016 FORMAT(I1H+B 1+HAT LEAST ONE QUADRANT IS NEGATIVE/) 1017 FORMAT(20X+F4.1+5X+F4.1+5X+F4.1)
1018 FORMAT(I1H+10X+3HAT LEAST ONE QUADRANT IS NEGATIVE/) 1019 FORMAT(I4HNUMBER OF FOREGROUNDS IS GREATER THAN 4)
1020 FORMAT(I4HNUMBER OF ACCIDENTALS IS GREATER THAN 2)
1021 FORMAT(I4HNUMBER OF SHADOW BARS IS GREATER THAN 2)
1022 FORMAT(I4HNUMBER OF ForeGROUNDS IS LESS THAN OR EQUAL TO 0)
1023 FORMAT(I5HDO MANY NWIDS)
1024 FORMAT(I5HDO OTHER NUMBER OF ACCIDENTALS OR NUMBER OF SHADOW BARS IS NEGATIVE)
1025 FORMAT(I7HONDID IS NEGATIVE)
1026 FORMAT(I3HONSTEP(1+2+OR3) IS 0 OR NEGATIVE)
1027 FORMAT(I4HINITIAL OR FINAL CHANNEL IS 0 OR NEGATIVE)
1028 FORMAT(I6HPEAK EXTENDS BEYOND CHANNELS FEED IN)

DIMENSION NOPT(4), NSTEP(3), S(7:512), A(3:512), SB(3:512), ON(4), OA(4),
OSB(4), OR(4), REAC(J), NWID(5)
1 READ(5,1001)(NOPT(J), J=1,4)
2 READ(5,1002)(REAC(J), J=1,3), EB, AN, ETH, EN, AA, AM, SBM
3 READ(5,1003)NOFGD, NOACC, NSS, LC, IPC, LPC, NWID(M), M=1,5, (NSTEP(1)
J=1,3)
800 IF(NOFGD=EO=0)GOTO314 901 IF(NOSB LE-1 OR NOACC LE-1)GOTO316
906 IF(I1C LE-0 OR ILC LE-0)GOTO319
910 IF(IIPC LE-1 OR LPC GE-LC+1)GOTO320
4 NA=LC-IC+1
5 NB=4*NA
6 DDA00M=1+NB
7 S(1,M)=0
8 S(2,M)=0
9 S(3,M)=0
10 S(4,M)=0
11 A(1,M)=0
12 A(2,M)=0
13 SB(1,M)=0
14 SB(2,M)=0
400 CONTINUE
15 READ(5,1004)(S(1,J), J=1+NB)
16 IF(NOFGD=EO=1)GOTO300
17 READ(5,1004)(S(2,J), J=1+NB)
18 IF(NOFGD=EO=2)GOTO300
19 READ(5,1004)(S(3,J), J=1+NB)
20 IF(NOFGD=EO=3)GOTO300
21 READ(5,1004)(S(4,J), J=1+NB)
22 IF(NOFGD=EO)=4)GOTO311
154
300 CONTINUE
23 IF(NOACC.EQ.0)GOTO301
24 READ(5,1004)(A(1,J),J=1,NB)
25 IF(NOACC.EQ.1)GOTO301
26 READ(5,1004)(A(2,J),J=1,NB)
27 IF(NOACC.EQ.2)GOTO312
301 CONTINUE
28 IF(NOSB.EQ.0)GOTO302
29 READ(5,1004)(SB(1,J),J=1,NB)
30 IF(NOSB.EQ.1)GOTO302
31 READ(5,1004)(SB(2,J),J=1,NB)
32 IF(NOSB.EQ.2)GOTO313
302 CONTINUE
33 DO401 J=1,NB
35 A(3,J)=A(1,J)+A(2,J)
36 SB(3,J)=SR(1,J)+SB(2,J)
37 S(6,J)=AM*A(3,J)+SBM*SB(3,J)
38 S(7,J)=S(5,J)-S(6,J)
401 CONTINUE
39 WRITE(6,1005)(REA(J),J=1,3)EAN
40 WRITE(6,1006)ENETH,AA,AM,SBM
41 WRITE(6,1007)NOFGD,NOACC,NOSB,IC,LC,IPC,LPC,(NWID(M),M=1,5),(NSTEP
1(J),J=1,3)
42 IF(NOPT(EQ.0)GOTO307
43 WRITE(6,1009)
44 WRITE(6,1010)
45 D0402 M=1,NA
46 NO=IC+M-1
47 WRITE(6,1008)NO,S(1,M),S(2,M),S(3,M),S(4,M),A(1,M),A(2,M),SB(1,M),
1SB(2,M),S(5,M),S(6,M),S(7,M)
402 CONTINUE
48 WRITE(6,1011)
49 NINA=NA+1
50 NFIA=2*NA
51 DO403 M=NINA,NFIA
52 NO=IC+M-NA-1
53 WRITE(6,1008)NO,S(1,M),S(2,M),S(3,M),S(4,M),A(1,M),A(2,M),SB(1,M),
1SB(2,M),S(5,M),S(6,M),S(7,M)
403 CONTINUE
54 WRITE(6,1012)
55 NINA=NFIA+1
56 NFIA=NFIA+NA
57 DO404 M=NINA,NFIA
58 NO=IC+M-NA-1
59 WRITE(6,1008)NO,S(1,M),S(2,M),S(3,M),S(4,M),A(1,M),A(2,M),SB(1,M),
1SB(2,M),S(5,M),S(6,M),S(7,M)
404 CONTINUE
60 WRITE(6,1013)
61 NINA=NFIA+1
62 NFIA=NFIA+NA
63 DO405 M=NINA,NFIA
64 NO=IC+M-3*NA-1
65 WRITE(6,1008)NO,S(1,M),S(2,M),S(3,M),S(4,M),A(1,M),A(2,M),SB(1,M),
1SB(2,M),S(5,M),S(6,M),S(7,M)
405 CONTINUE
307 CONTINUE
67 IF(NOPT(2).EQ.0)GOTO304
904 IF(NSTEP(1).LE.0)GOTO318
801 JNO=1
68 L=0
908 IF(NWID(JNO).LE.-1)GOTO317
69 WRITE(6,1014)
70 IB=IPC+L*NSTEP(1)
155

71 DO 07 M = 1, 4
72 ON ( M ) = 0
73 QA ( M ) = 0
74 QB ( M ) = 0
840 NWIDTH = NWID ( JNO )
75 DO 06 J = 1, NWIDTH
260 NO = 18 - IF + ( M - 1 ) * NA + J
76 ON ( M ) = GN ( M ) + S ( 7 , NO )
77 QA ( M ) = QA ( M ) + A ( 3 , NO )
78 QB ( M ) = QB ( M ) + SB ( 3 , NO )
406 CONTINUE
79 QA ( M ) = AM * QA ( M ) + SBM * QS3 ( M )
407 CONTINUE
92 ID = ID + NWID ( JNO ) - 1
760 IF ( ON ( 1 ) LE .0 OR ON ( 2 ) LE 0 ) GOTO 0500
761 IF ( ON ( 3 ) LE .0 OR ON ( 4 ) LE 0 ) GOTO 0500
80 SACC = OA ( 1 ) + QA ( 2 ) + QA ( 3 ) + QA ( 4 )
81 SB = OSB ( 1 ) + OSB ( 2 ) + OSB ( 3 ) + OSB ( 4 )
82 SNET = ON ( 1 ) + ON ( 2 ) + ON ( 3 ) + ON ( 4 )
83 SACK = AM * SACC + SBM * SB
84 PERR = 1 .0 * ( SACC * AM ) / ( SNET + SACK )
85 PERA = 1 .0 * ( SS ? * SBM ) / ( SNET + SACK )
86 PERR = PERA + PERR
87 RATIO = ( ON ( 1 ) * ON ( 2 ) ) / ( ON ( 3 ) * ON ( 4 ) )
88 R = SORT (( ON ( 1 ) * ON ( 2 ) ) / ( ON ( 3 ) * ON ( 4 ) ))
89 EPP = ( R - 1 ) / ( R + 1 )
90 DEPP = ( 1 * ( QN ( 1 ) * QN ( 4 )) + ON ( 2 ) ) * R
91 DEPP = 2 * ( ON ( 1 ) / ( ON ( 3 ) * ON ( 4 ) )) * DEPP / ( ( SORT ( ON ( 1 ) * ON ( 2 ) ) + SORT ( ON ( 3 ) * ON ( 4 ) ))
920 GOTO 0503
590 CONTINUE
501 WRITE ( 6 , 1018 )
502 GOTO 096
503 CONTINUE
93 WRITE ( 6 , 1015 ) IR , ID , EPP , DEPP , ON ( 1 ) , ON ( 2 ) , ON ( 3 ) , ON ( 4 )
94 WRITE ( 6 , 1016 ) RATIO , OS ( 1 ) , CB ( 2 ) , CB ( 3 ) , CB ( 4 )
95 WRITE ( 6 , 1017 ) PERA , PERR , PER
96 L = L + 1
97 IF ( ID + NSTEP ( 1 ) LE LPC ) GOTO 069
802 JNO = JNO + 1
804 IF ( JNO GE 5 ) GOTO 0315
803 IF ( NWID ( JNO ) .EQ .0 ) GOTO 068
304 CONTINUE
99 IF ( NOPT ( 3 ) .EQ .0 ) GOTO 0395
995 IF ( NSTEP ( 2 ) LE 0 ) GOTO 0318
99 L = 0
100 WRITE ( 6 , 1014 )
101 LB = IPC + L * NSTEP ( 2 )
102 ID = LPC - 1 * NSTEP ( 2 )
103 NEND = LPC + 1 - IPC - 2 * L * NSTEP ( 2 )
104 NO = ON ( M ) + 4
105 ON ( M ) = 0
106 QA ( M ) = 0
107 QB ( M ) = 0
108 DO ( 3 ) = 1 + NEND
201 NO = 18 - IF + ( M - 1 ) * NA + J
209 ON ( M ) = GN ( M ) + S ( 7 , NO )
110 QA ( M ) = QA ( M ) + A ( 3 , NO )
111 QB ( M ) = QB ( M ) + SB ( 3 , NO )
154 DEP = (1.0 / (2.0 * SORT (ON (1) * ON (4) * ON (2) * ON (3))) * SORT (AM * (AM + 1) * (ON (4) * ON (1) * ON (4) * ON (2)) * (ON (4) / ON (2)) * OA (2))) + SBM * (SBM + 1) * (ON (4) * ON (4) * QA (1) / ON (1) / ON (3)) * OA (3)) + ON (1) / ON (1) * OA (4).

155 DEPP = 2.0 * (ON (3) / ON (4)) + DEP / ((SORT (ON (1) / ON (2)) + SORT (ON (3) / ON (4))) * (SORT (ON (1) / ON (2)) + SORT (ON (3) / ON (4))))

160 IF (ID <= LPC - NSTEP (3)) GOT0 132

STOP
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APPENDIX II

DERIVATION OF MAXIMUM $P^2\sigma$ CRITERION

FOR POLARIZATION ANALYZER

For a left-right scattering of a beam of perpendicularly polarized neutrons characterized by polarization $P_1$ from a scatterer with analyzing power $P_2$, one has for the measured asymmetry;

$$\xi = \frac{P_1 P_2}{L+R} = \frac{L-R}{L+R} = \frac{T-1}{T+1} \quad \text{AII-1}$$

where $L$ is the number of neutrons scattered into the left detector and $R$ is the number of neutrons scattered into the right detector and $T = \frac{L}{R}$. The statistical uncertainty is therefore

$$\Delta \xi = \left[ \left( \frac{\partial \xi}{\partial L} \right)^2 (\Delta L)^2 + \left( \frac{\partial \xi}{\partial R} \right)^2 (\Delta R)^2 \right]^{1/2} \quad \text{AII-2}$$

since

$$\frac{\partial \xi}{\partial L} = \frac{1}{L+R} - \frac{L-R}{(L+R)^2} = \frac{2R}{(L+R)^2} \quad \text{AII-3}$$

while

$$\frac{\partial \xi}{\partial R} = -\frac{1}{L+R} - \frac{L-R}{(L+R)^2} = -\frac{2L}{(L+R)^2} \quad \text{AII-4}$$

using $\Delta L = \sqrt{L}$ and $\Delta R = \sqrt{R}$

to

$$\Delta \xi = \frac{2\sqrt{LR}}{(L+R)^{3/2}} \quad \text{AII-5}$$
If one considers \( N \) polarized neutrons incident per unit time on the analyzer, one has:

\[
L = \frac{N}{2} \sigma(\Theta_1) [1 + P_1 P_2] \quad \text{and} \quad R = \frac{N}{2} \sigma(\Theta_2) [1 - P_1 P_2] \quad \text{AII-6}
\]

where \( \sigma(\Theta) \) is the cross section for an unpolarized beam incident on the analyzer. Substituting AII-6 into AII-5 gives:

\[
\Delta \varepsilon = \left[ \frac{1 - P_1^2 P_2^2}{N \sigma} \right]^{1/2} \quad \text{AII-7}
\]

Using AII-1 and AII-7 one has for the relative statistical uncertainty:

\[
\frac{\Delta \varepsilon}{\varepsilon} = \left[ \frac{1 - P_1^2 P_2^2}{N \sigma P_1^2 P_2^2} \right]^{1/2} \quad \text{AII-8}
\]

To a first approximation \( P_1^2 P_2^2 \ll 1 \) and \( \Delta \varepsilon / \varepsilon \) is minimized for a given incident polarization \( P_1 \) by maximizing the quantity \( P_2^2 \sigma(\Theta_2) \).
APPENDIX III

PRINCIPLE OF SPIN PRECESSION

The neutron magnetic moment is

\[ \mu = -1.91316 \frac{e}{2M_p c} \]  

where \( e \) is the electronic charge, \( c \) the speed of light and \( M_p \) the proton rest mass. In a uniform magnetic field, \( B \), the Larmor precession frequency of the neutron magnetic moment about the field direction is

\[ \vec{\omega} = \frac{\mu}{J} \vec{B} \]

where \( J \) is the magnitude of the total angular momentum of the neutron. For a free neutron \( J \) is the magnitude of the intrinsic angular momentum, \( J = s \hbar \), and since the neutron has spin \( s = 1/2 \), we have

\[ \vec{\omega} = \frac{2 \mu}{\hbar} \vec{B} \]

where \( \hbar \) is Planck's constant divided by \( 2\pi \).

If \( \tau \) is the transit time of the neutron in the magnetic field, the total angle of precession, \( \phi \), is

\[ \phi = \int_0^\tau \vec{\omega} \, dt = \frac{1}{\nu} \int_0^L \vec{\omega} \cdot d\vec{l} \]

where \( L \) is the distance from the target to the center of scattering.
from helium. Since for a solenoid the only significant field component is along the magnet axis and since the neutron travels along this axis, substituting Eqn. AIII-3 into Eqn. AIII-4 gives

$$\phi = \frac{2\mu_0}{h} \int_0^L B_x \, dl$$

where $B_x$ is the axial component of the field. The precession angle is thus proportional to the integrated field, $\int_0^L B_x \, dl$.

$B_L$ was measured with a Hall effect probe for various currents in 0.5" steps along the axis of the solenoid. The quantity $\int_0^L B_x \, dl$ was then evaluated by a sum approximation and compared to the value obtained from the infinite solenoid approximation, $B_L = \frac{4\pi}{10} NI$ ampere-turns. Values of up to $3 \times 10^5$ gauss-cm were obtained with the two-coil magnet. It was found that when the actual integrated field was expressed by $\int_0^L B_x \, dl = K(I) \frac{4\pi}{10} NI$, $K(I)$ varied from 0.99 to 1.03 for magnet currents from 80 to 210 amperes. Thus the precession angle, $\phi$, was essentially a linear function of the magnet current over a wide range of currents.
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REFERENCES (continued)


REFERENCES (continued)


REFERENCES (continued)


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